CS8803 Logic in Computer Science
Interim Report

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1 Propositions

The puzzle can be considered as finding which color, national, drink, cigar and pet is assigned to each of the houses. There is a bijection between every artifact mentioned above and the house. Thus, there are 5*5*5 number of propositions.

Let the propositions be $H_{i,Cj}$ where $i \in$ set of houses i.e. H={1,2,3,4,5} and $Cj \in$ set of colors i.e. {C1,C2,C3,C4,C5}. Similarly, define $H_{i,Nj}$, $H_{i,Pj}$, $H_{i,Dj}$, $H_{i,Gj}$ where Nj, Pj, Dj, Gj are jth national, pet, drink and cigar respectively.
Let the functions c(i), n(i), p(i), d(i), g(i) be functions from $H- > C1, C2, C3, C4, C5$, $H- > N1, N2, N3, N4, N5$, $H- > P1, P2, P3, P4, P5$, $H- > D1, D2, D3, D4, D5$, $H- > G1, G2, G3, G4, G5$ respectively. We follow the order:

j	1	2	3	4	5
color (C)	red	green	white	yellow	blue
national (N)	Brit	Swede	Dane	Norwegian	German
pet (P)	dog	birds	cats	horse	fish
drinks (D)	tea	coffee	milk	beer	water
cigar (G)	Pall Mall	Dunhill	Blends	Bluemaster	Prince

Table 1: Mapping for propositions

Assignment is

$$H_{i,Cj} = \begin{cases} T, & \text{if } c(i) = Cj \\ 0, & \text{otherwise} \end{cases}$$

$$H_{i,Nj} = \begin{cases} T, & \text{if } n(i) = Nj \\ 0, & \text{otherwise} \end{cases}$$

$$H_{i,Pj} = \begin{cases} T, & \text{if } p(i) = Pj \\ 0, & \text{otherwise} \end{cases}$$

$$H_{i,Dj} = \begin{cases} T, & \text{if } d(i) = Dj \\ 0, & \text{otherwise} \end{cases}$$

$$H_{i,Gj} = \begin{cases} T, & \text{if } g(i) = Gj \\ 0, & \text{otherwise} \end{cases}$$

The encoding of the problem should satisfy:

- No two houses have the same color, national pet, drink, cigar.
- Each house two house has at least one color, one national, one pet, one drink, one cigar.
- Each had at most one color, one national, one pet, one drink, one cigar.
- All the hints.

Constraint 1: For each pair of houses $(m, n) \in H$, we want, $(\neg H_{m,C1} \vee H_{n,C1}) \wedge (\neg H_{m,C2} \vee H_{n,C2}) \wedge (\neg H_{m,C3} \vee H_{n,C3}) \wedge (\neg H_{m,C4} \vee H_{n,C4}) \wedge (\neg H_{m,C5} \vee H_{n,C5})$ i.e. no two pair of houses can have color 1, color 2, color 3, color 4 and color 5. So we get

$$\psi_1 = \bigwedge_{(m,n)\in H, m\neq n} \left(\bigwedge_{Cj} (\neg H_{m,Cj} \vee \neg H_{n,Cj}) \right) \tag{1}$$

Similarly, for national, pet, drink and cigar, we get:

$$\psi_2 = \bigwedge_{(m,n)\in H, m\neq n} \left(\bigwedge_{Nj} (\neg H_{m,Nj} \vee \neg H_{n,Nj}) \right) \tag{2}$$

$$\psi_3 = \bigwedge_{(m,n)\in H, m\neq n} \left(\bigwedge_{Pj} (\neg H_{m,Pj} \vee \neg H_{n,Pj}) \right) \tag{3}$$

$$\psi_4 = \bigwedge_{(m,n)\in H, m\neq n} \left(\bigwedge_{Dj} (\neg H_{m,Dj} \vee \neg H_{n,Dj}) \right) \tag{4}$$

$$\psi_5 = \bigwedge_{(m,n)\in H, m\neq n} \left(\bigwedge_{Gj} (\neg H_{m,Gj} \vee \neg H_{n,Gj}) \right) \tag{5}$$

Constraint 2: For every house i, at least one of $H_{i,C1}, H_{i,C2}, H_{i,C3}, H_{i,C4}, H_{i,C5}$ should be true. Same for other artifacts.
So we get,

$$\psi_6 = \bigwedge_{i\in H} (H_{i,C1} \vee H_{i,C2} \vee H_{i,C3} \vee H_{i,C4} \vee H_{i,C5})) \tag{6}$$

$$\psi_7 = \bigwedge_{i\in H} (H_{i,N1} \vee H_{i,N2} \vee H_{i,N3} \vee H_{i,N4} \vee H_{i,N5})) \tag{7}$$

$$\psi_8 = \bigwedge_{i\in H} (H_{i,P1} \vee H_{i,P2} \vee H_{i,P3} \vee H_{i,P4} \vee H_{i,P5})) \tag{8}$$

$$\psi_9 = \bigwedge_{i \in H} (H_{i,D1} \vee H_{i,D2} \vee H_{i,D3} \vee H_{i,D4} \vee H_{i,D5})) \tag{9}$$

$$\psi_{10} = \bigwedge_{i \in H} (H_{i,G1} \vee H_{i,G2} \vee H_{i,G3} \vee H_{i,G4} \vee H_{i,G5})) \tag{10}$$

Constraint 3: Every house i must be assigned at most one color i.e. no two pair of color for house i can be simultaneously true.
So we get,

$$\psi_{11} = \bigwedge_{i \in H} \left(\bigwedge_{(Cm,Cn) \in C} (\neg H_{i,Cm} \vee \neg H_{i,Cn}) \right) \tag{11}$$

Similarly for other artifacts,

$$\psi_{12} = \bigwedge_{i \in H} \left(\bigwedge_{(Nm,Nn) \in N} (\neg H_{i,Nm} \vee \neg H_{i,Nn}) \right) \tag{12}$$

$$\psi_{13} = \bigwedge_{i \in H} \left(\bigwedge_{(Pm,Pn) \in P} (\neg H_{i,Pm} \vee \neg H_{i,Pn}) \right) \tag{13}$$

$$\psi_{14} = \bigwedge_{i \in H} \left(\bigwedge_{(Dm,Dn) \in D} (\neg H_{i,Dm} \vee \neg H_{i,Dn}) \right) \tag{14}$$

$$\psi_{15} = \bigwedge_{i \in H} \left(\bigwedge_{(Gm,Gn) \in G} (\neg H_{i,Gm} \vee \neg H_{i,Gn}) \right) \tag{15}$$

Now we encode hints given for solving the puzzle.

Hint 1: Brit lives in the red house. This means brit lives in house i iff house i is red, $i \in H$. Iff is xnor operation. $a \odot b = (\neg a \vee b) \wedge (a \vee \neg b)$. We know that at least one house will be assigned red because of bi-jection and hence the final formula is just logical AND of all xnor.
So we get,

$$\psi_{16} = \bigwedge_{i \in H} ((H_{i,C1} \vee \neg H_{i,N1}) \wedge (\neg H_{i,C1} \vee H_{i,N1})) \tag{16}$$

Hint 2: Swede keeps dogs. This means swede lives in house i iff house i has dogs, $i \in H$. Same justification as hint 1. So we get,

$$\psi_{17} = \bigwedge_{i \in H} ((H_{i,N2} \vee \neg H_{i,P1}) \wedge (\neg H_{i,N2} \vee H_{i,P1})) \tag{17}$$

Hint 3: Dane drinks. This means Dane lives in house i iff house i is assigned tea, $i \in H$. Same justification as hint 1. So we get,

$$\psi_{18} = \bigwedge_{i \in H} ((H_{i,N3} \vee \neg H_{i,D1}) \wedge (\neg H_{i,N3} \vee H_{i,D1})) \tag{18}$$

Hint 4: Green is to the left of white. So house 1 is assigned green iff house 2 is white, house 2 is assigned green iff house 3 is white, house 2 is assigned green iff house 4 is white and house 4 is green iff house 5 is white. Note house 5 can not be green. Since house 5 is not green, exactly one of house1,2,3,4 will be assigned green and then with iff condition the house to its right will be white. So we get,

$$\psi_{19} = \bigwedge_{i \in \{1,2,3,4\}} \left((H_{i,C2} \vee \neg H_{i+1,C3}) \wedge (\neg H_{i,N3} \vee H_{i,D1}) \right) \wedge \neg H_{5,C2} \quad (19)$$

Hint 5: Green house is assigned coffee. This means house i is green iff house i is assigned coffee, $i \in H$. Same justification as hint 1.
So we get,

$$\psi_{20} = \bigwedge_{i \in H} \left((H_{i,C2} \vee \neg H_{i,D2}) \wedge (\neg H_{i,C2} \vee H_{i,D2}) \right) \quad (20)$$

Hint 6: House with Pall Mall is assigned birds. This means house i owner smoked Pall Mall iff house i has has birds $i \in H$. Same justification as hint 1. So we get,

$$\psi_{21} = \bigwedge_{i \in H} \left((H_{i,G1} \vee \neg H_{i,P2}) \wedge (\neg H_{i,G1} \vee H_{i,P2}) \right) \quad (21)$$

Hint 7: House i is yellow iff it is assigned Dunhill cigar.
So we get,

$$\psi_{22} = \bigwedge_{i \in H} \left((H_{i,C4} \vee \neg H_{i,G2}) \wedge (\neg H_{i,C4} \vee H_{i,G2}) \right) \quad (22)$$

Hint 8: Milk is assigned to house 3, since middle. So we get,

$$\psi_{23} = H_{3,D3} \quad (23)$$

Hint 9: Norwegian lives in H1, since first. So we get,

$$\psi_{24} = H_{1,N4} \quad (24)$$

Hint 10: House which has cats has neighbor who smokes Blends. We know one of house 1,2,3,4 or 5 is assigned cats and blends. So we want negation of: if house 1 is assigned blends and house 2 does not have cats, or house 2 assigned blends and house 1,3 do not have cats, or if house 3 is assigned blends and house 2, 4 do not have cats, or house 4 assigned blends and house 3,5 do not have cats, or house 5 is assigned blends and house 4 does not have cats.

So we get,

$$\psi_{25} = (\neg H_{1,G3}) \vee H_{2,P3}) \wedge (\neg H_{2,G3} \vee H_{1,P3} \vee H3, P3) \wedge (\neg H_{3,G3} \vee H_{2,P3} \vee H_{4,P3}) \\ \wedge (\neg H_{4,G3} \vee H_{3,P3} \vee H_{5,P3}) \wedge (\neg H_{5,G3} \vee H_{4,P3}) \quad (25)$$

Hint 11: Same as above with horse and Dunhill instead of cats and Blends.

$$\psi_{26} = (\neg H_{1,P4}) \vee H_{2,G2}) \wedge (\neg H_{2,P4} \vee H_{1,G2} \vee H3, G2) \wedge (\neg H_{3,P4} \vee H_{2,G2} \vee H_{4,G2}) \\ \wedge (\neg H_{4,P4} \vee H_{3,G2} \vee H_{5,G2}) \wedge (\neg H_{5,P4} \vee H_{4,G2}) \quad (26)$$

Hint 12: House i is assigned beer iff it is assigned Bluemaster. Similar to hint 1, we get,

$$\psi_{27} = \bigwedge_{i \in H} \left((H_{i,G4} \vee \neg H_{i,D4}) \wedge (\neg H_{i,G4} \vee H_{i,D4}) \right) \tag{27}$$

Hint 13: House i has German iff it is assigned Prince. Similar to hint 1, we get,

$$\psi_{28} = \bigwedge_{i \in H} \left((H_{i,N5} \vee \neg H_{i,G5}) \wedge (\neg H_{i,N5} \vee H_{i,G5}) \right) \tag{28}$$

Hint 14: Similar to hint 10, but with Norwegian and blue house we get

$$\begin{aligned} \psi_{29} = (\neg H_{1,N4}) \vee H_{2,C5}) \wedge (\neg H_{2,N4} \vee H_{1,C5} \vee H3, C5) \wedge (\neg H_{3,N4} \vee H_{2,C5} \vee H_{4,C5}) \\ \wedge (\neg H_{4,N4} \vee H_{3,C5} \vee H_{5,C5}) \wedge (\neg H_{5,N4} \vee H_{4,C5}) \end{aligned} \tag{29}$$

Hint 15: Similar to hint 10, but with Blends and water as drink, we get

$$\begin{aligned} \psi_{30} = (\neg H_{1,G3}) \vee H_{2,D5}) \wedge (\neg H_{2,G3} \vee H_{1,D5} \vee H3, D5) \wedge (\neg H_{3,G3} \vee H_{2,D5} \vee H_{4,D5}) \\ \wedge (\neg H_{4,G3} \vee H_{3,D5} \vee H_{5,D5}) \wedge (\neg H_{5,G3} \vee H_{4,D5}) \end{aligned} \tag{30}$$

Fig 1 is my solution obtained using the DPLL implementation. Here positive value means, the variable is assigned T, negative means the variable is assigned F. Propositions are numbered in increasing order as in the definition. For ex-

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[  -1.   -2.   -3.    4.   -5.   -6.   -7.   -8.   -9.   10.   11.  -12.
  -13.  -14.  -15.  -16.   17.  -18.  -19.  -20.  -21.  -22.   23.  -24.
  -25.  -26.  -27.  -28.   29.  -30.  -31.  -32.   33.  -34.  -35.   36.
  -37.  -38.  -39.  -40.  -41.  -42.  -43.  -44.   45.  -46.   47.  -48.
  -49.  -50.  -51.  -52.   53.  -54.  -55.  -56.  -57.  -58.   59.  -60.
  -61.   62.  -63.  -64.  -65.  -66.  -67.  -68.  -69.   70.   71.  -72.
  -73.  -74.  -75.  -76.  -77.  -78.  -79.   80.   81.  -82.  -83.  -84.
  -85.  -86.  -87.   88.  -89.  -90.  -91.   92.  -93.  -94.  -95.  -96.
  -97.  -98.   99. -100. -101.  102. -103. -104. -105. -106. -107.  108.
 -109. -110.  111. -112. -113. -114. -115. -116. -117. -118. -119.  120.
 -121. -122. -123.  124. -125.]
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Figure 1: Solution to puzzle

ample -1,-2,-3,4,-5 implies house 1 is yellow. In CNF encoding, color variables for house 1: 1-5, house 2: 6-10, house 3: 11-15, house 4: 16-20, house 5: 21-25. Nationality variables for house 1: 26-30, house 2: 31-35, house 3: 36-40, house 4: 41-45, house 5: 46:50. Pet variables for house 1: 51-55, house 2: 56-60, house 3: 61-65, house 4: 66-70, house 5: 71-75. Drink variables for house 1: 76-80, house 2: 81-85, house 3: 86-90, house 4: 91-95, house 5: 96-100. Cigar variable

for house 1: 101-105, house 2: 106-110, house 3: 111-115, house 4: 116-120, house 5: 121-125.
So mapping solution to values, we get,

House / Artifact	1	2	3	4	5
color	yellow	blue	red	green	white
national	Norwegian	Dane	Brit	German	Swede
pet	cats	horse	birds	fish	dog
drinks	water	tea	milk	coffee	beer
cigar	Dunhill	Blend	Pall Mill	Prince	Bluemaster

Table 2: Solution mapping

Hence, German owns the fish.