Signals and Systems

Programming Assignment

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Motivation of the problem

x[n] denotes the samples of the true value of temperature recorded by a sensor. But due to limitations of hardware, the sensor memory needs to be cleared periodically and this is done by transmitting the stored values to a base unit.

Hence received signal y[n] at the base unit suffers from blur distortions and noise (additive).

The main motivation of this problem is to **recover back x[n] from the distorted signal y[n]** using the concepts of Deblurring and Denoising learnt throughout the signals and systems course.

Theoretical Aspects which are covered

1.) Signal Denoising:

Main strategy for removing noise can be done by using two approaches :

(a.) Convolution of Kernel with the signal from which we have to remove the noise.

(b.) Averaging out the Neighbouring signal values :

In=Io+E

where Io is the clean signal

In is the noisy signal and E is the noise factor / Gaussian Term which adds and hence it is said to be additive noise.

In both approaches the denoising method works and basically these mathematical calculations make the **E tending to zero and we are able to get Denoised Signal.**

2.) Signal Deblurring:

Blur in a signal corresponds to the **low pass filtering** and it signifies that the received signal is modified from the original signal due to the convolutions of the original signal with the some blurring kernel. Hence Deblurring could be done by using the concepts of Fourier Transform and then taking out the inverse fourier of the Deblurred signal to get the desired output. The mathematical relations corresponding to this concept are shown next.

$$\mathbf{I}_b = \mathbf{I}_c * \mathbf{h}.$$

Given blurred signal (Ib) and h we can recover out the clean signal (Ic).

Taking Fourier Transform on both sides , using convolution property, it changes over to multiplication form and then we can take its inverse.

$$\mathcal{F}(\mathbf{I}_0) = \frac{\mathcal{F}(\mathbf{I}_d)}{\mathcal{F}(\mathbf{h})}$$

Now taking the inverse of the above form would give us the unblurred signal which we needed.

Formula of Fourier and Inverse Fourier Transform which have used are as follow :

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n}$$

(Fourier Transform)

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega})e^{j\omega n} d\omega$$

(Inverse Fourier Transform)

h[n] in this case is just as similar as to the **Weighted Averaging.**

3.) Spectral Leakage in DTFT case:

As in the case of the DTFT, this spectral leakage is caused by **finite-length sampling (Windowing)** that occurs for any practical application.

Increasing the sampling frequency, thereby generating longer discrete-time sequences for equivalent sampling times, reduces spectral leakage, but does not eliminate the problem .

Ideation and proposed solution

In the question, signal y[n] is given which is the blurred and noisy version of signal x[n] for which the value of n varies from index 0 to 192 (193 corresponding values are present).

For the part of denoising, the **concept of mean averaging** is used and we have also checked out our answers through convolution kernel method as well, any one of them can be used aptly as per need. Efficiency is almost similar in both cases.

Now, we will keep averaging the 193 given values of the signal to be taken with their neighbourhood values (two above and two below) so it will be divided by 5, but in the first and last term, five values average is not possible, hence three value average for them is taken and in case of second and second last term four value average is taken so as to keep it distinct from the previous case and for all the rest cases, simple for loop is being runned for averaging of fives.

Now in the case of Deblurring we will use the formula and conceptual aspects as shown in the above theoretical aspects. This involves summation and the ratio of F(Id)/F(h), but this ratio approaches infinity **for the cases of $\Omega=n\pi$,** so in that case the ratio is not solvable and hence sampling is used to get out the approximative values. As Ω nears to $n\pi$, the ratio becomes very large. So to solve this case, we have to **ignore some values** near the threshold part.

Seeing the data from the graphs and doing some trials, a threshold value of **0.59** for the case of this ratio is chosen, which will provide a good approximation for the deblurred signal .

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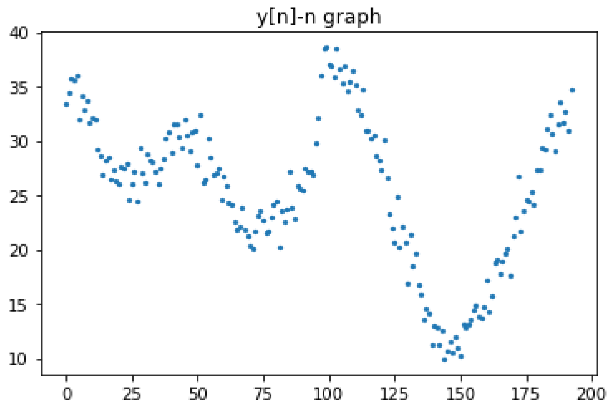
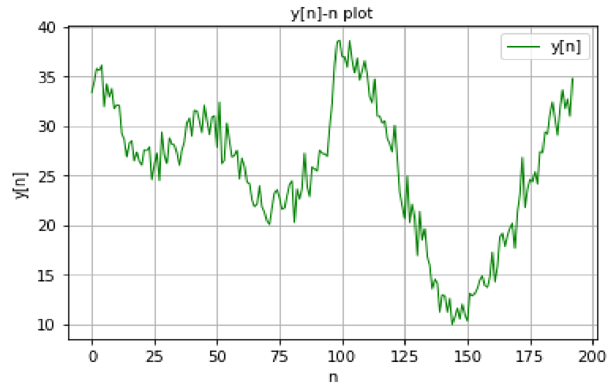
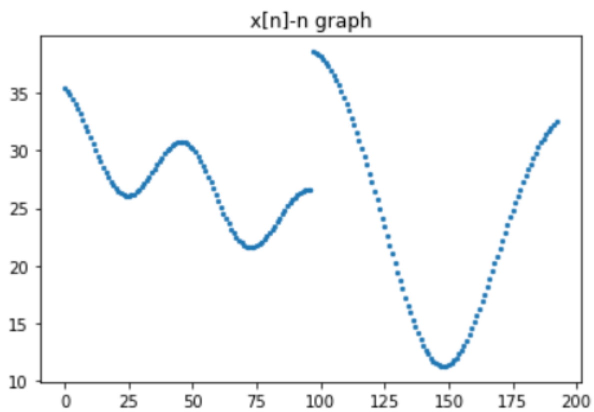
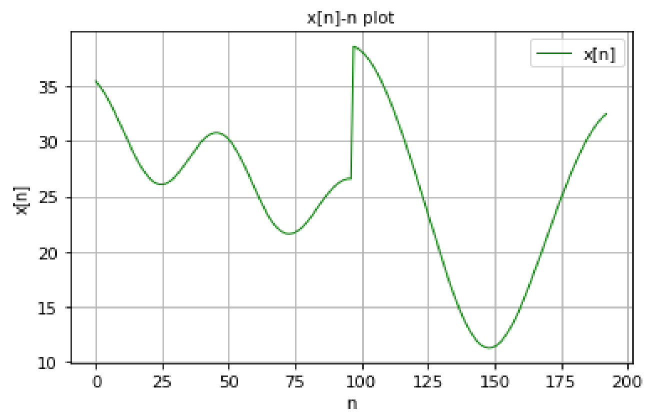
Evaluation results based on the graphs and elements for comparison

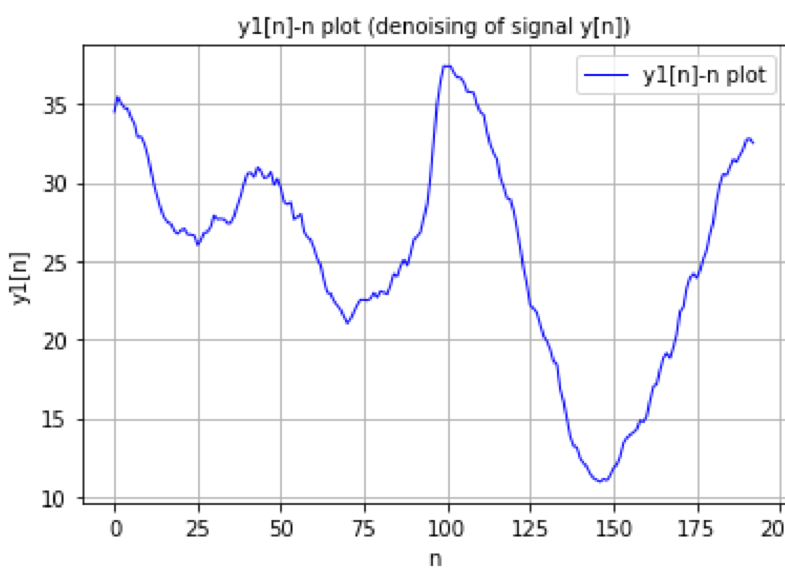




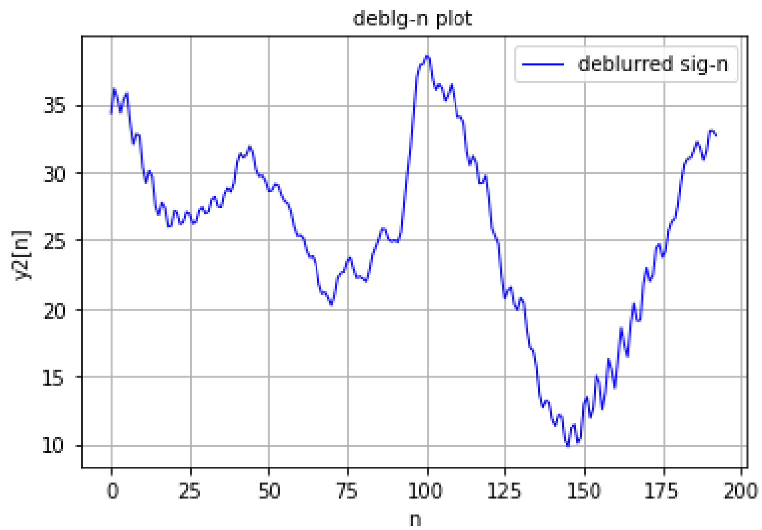




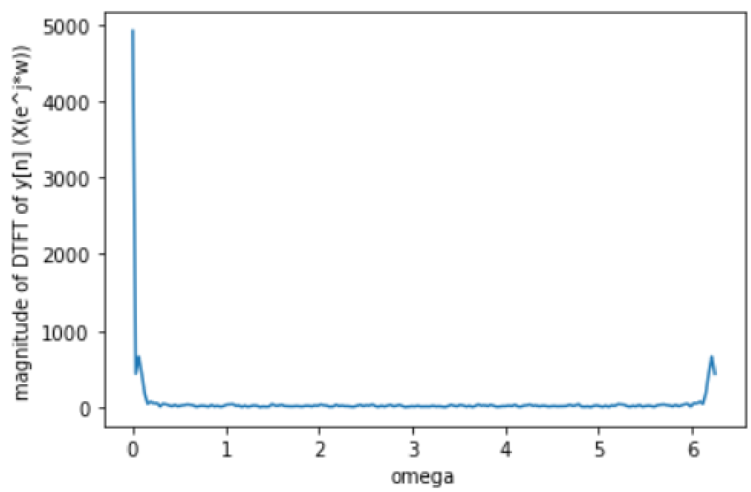
Following Graphs have been made from the data given



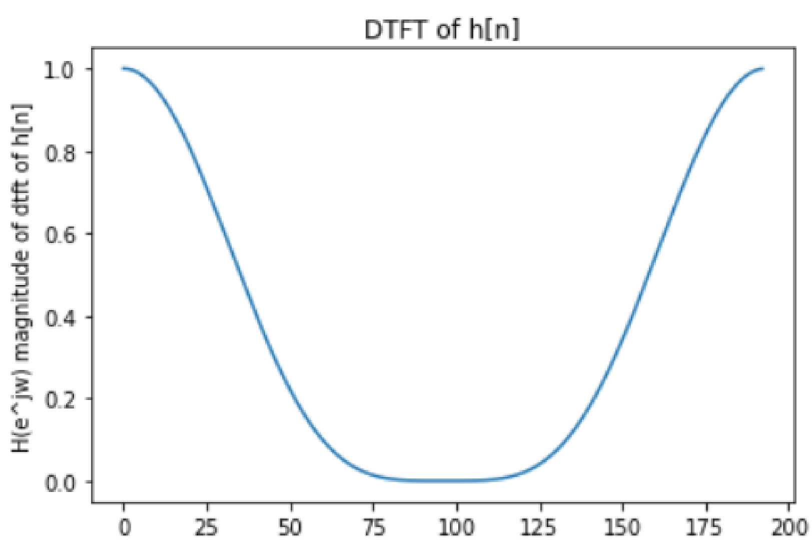
y1[n] in this case corresponds to denoised signal in position of step to obtain x1[n]



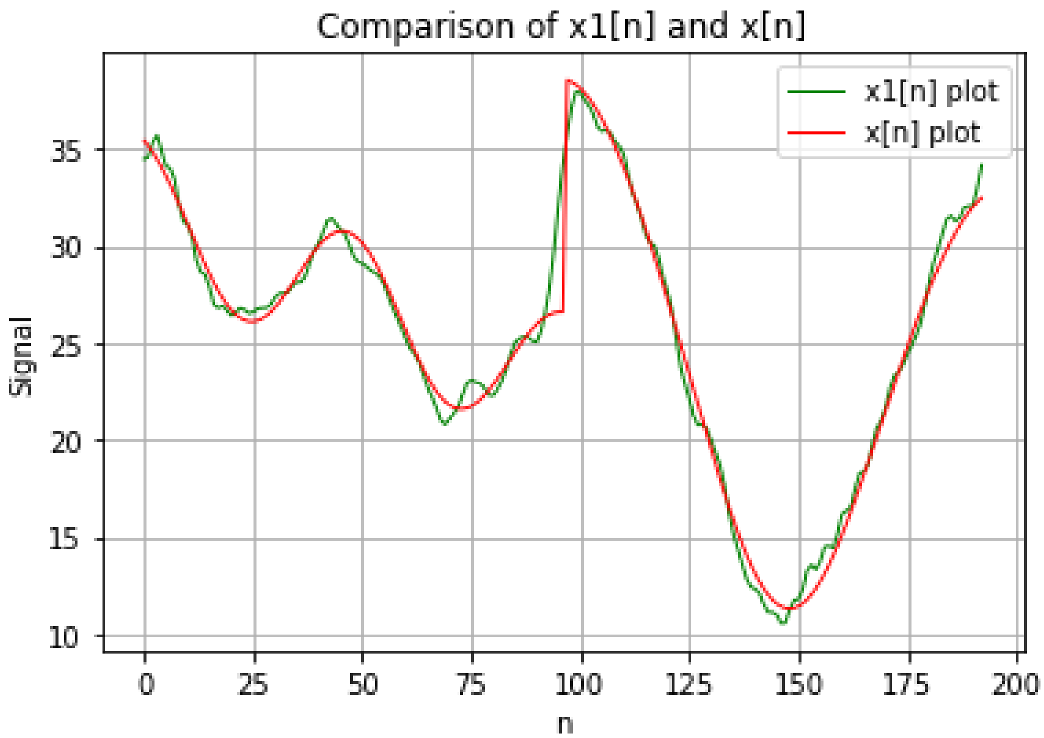
y2[n] in this case corresponds to deblurred signal in position of step to obtain x2[n]

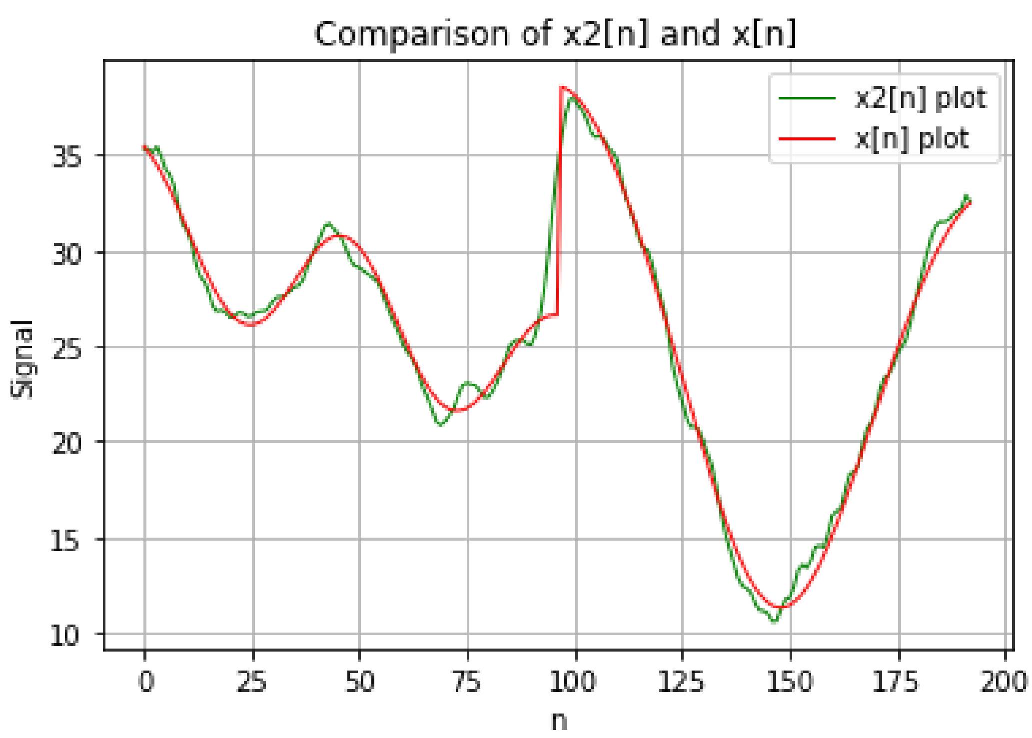


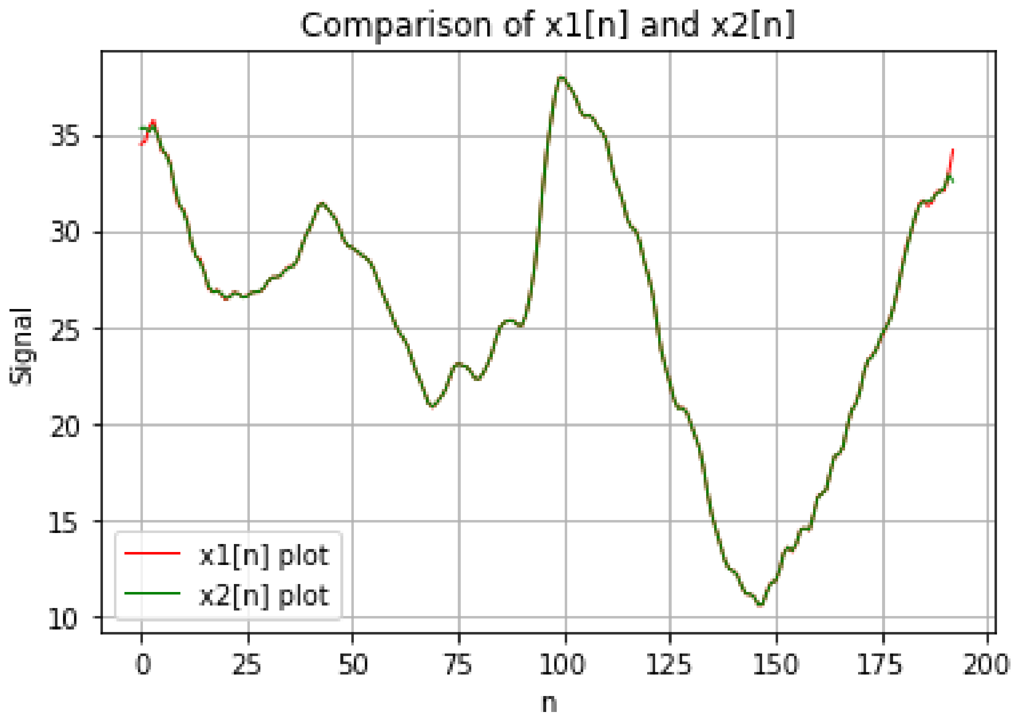
DTFT of y[n]

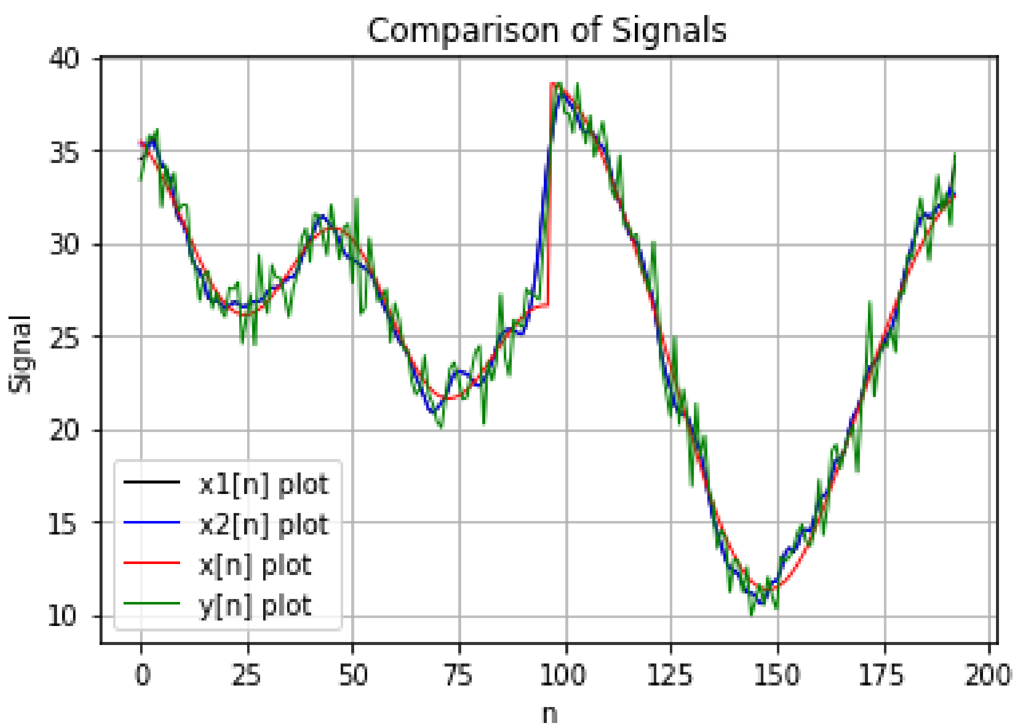


DTFT of h[n]


Comparison of x1[n] and x[n]
x1[n] plot
x[n] plot
Signal
n
35
30
25
20
15
10
0
25
50
75
100
125
150
175
200


Comparison of x2[n] and x[n]
x2[n] plot
x[n] plot
Signal
n
35
30
25
20
15
10
0
25
50
75
100
125
150
175
200





It can be observed from the above graphs that x1[n] and x2[n] are quite similar to each other towards the middle / mean values and are approximately similar to the original signal x[n].

Conclusions

After going through various graphs, scatter plots and theoretical concepts, the following conclusions can be obtained:

1.) Choosing **an appropriate Average function /Kernel value** is must to get out the optimum output for your signal while denoising so to remove out the Noise factor as explained above .

2.) For the fact of spectral leakage, **things of taking N** in sampling case using proper conditions must be considered such that the amount of error in the answer comes out to be a bare minimum.

3.) After analyzing the obtained graphs, it is clearly evident that values of x1[n] and x2[n] are similar in nature but as expected by the mean average value theorem of mathematics, these values are much closer and indistinguishable in the middle and somewhat more distorted and separated towards the border end .

4.) Hence the following observations can be made :
x2[n] is found smoother, closer to original x[n] and better as compared to x1[n] and this can also be said theoretically correct by the following facts :
a.) In case of x1[n], denoising is done before deblurring and hence the slightly blurred noises that were present in y[n] are only partially removed .
b.) Whereas in this case of x2[n], deblurring filters are being applied earlier which helps in removing blurred noises earlier and hence it leads to more removal of noises in a slightly better way .

5.) As we obtain back x[n] from y[n], it is not exactly similar to the original x[n] and this is true and expected to happen because the retrieved signal always has some missed data or signals and cases where the original data can't be retrieved, it is somewhat related to the topic '**the loss of data during the inverse systems**' studied in the course.

P.T.O

Contribution made by each group member

This project brought out the gates of horizon to our brain and it helped both of us in understanding the details of this course in greater depths . It was truly great that we got this amazing chance through this course to work together as a fantastic team .

Work was distributed equally among the both of us and in total we had held 10-12 meeting hours to discuss the logic, distribute the work, writing of codes and comments, making factual checks and all the necessities that were required through this project .

Though both of us equally participated in this project with full zeal and enthusiasm but for the sake of the report, here we sub categorise our work, though again some intersection exists in all work as after all it was a teamwork .

a.) **Surabhi Lambe(B20MT026)** - Major contribution in coding, commenting the codes and looking over all the data obtained by programming in Python with minor contributions in Report Writing , Logic and System Designing and after all making out the system a feasible and logical one.

b.) **Mitarth Arora(B20MT027) -** Major contributions in Report Writing, Logic and System Designing and after all making out the system a feasible and logical one with minor contributions in Coding, commenting the codes and looking over all the data and stuff over the programmed level in Python.

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END OF THE REPORT