A function = Vector

$f(x)$ — $\vec{V}$

infinite values (components). — finite values (No. of components).

dot product

$$(f(x), g(x)) = \int_a^b f(x)\, g(x)\, dx$$

integration

$$\vec{V} \cdot \vec{W} = \sum V_i W_i$$

sum

Gram-Schmidt:

given a set of vectors, create an orthonormal set of vectors..

~~Aryan~~ Aarya & Aatrey:

Claim: Just like any vector can be decomposed in terms of its basis vector ~~compomon~~ components, a function can be decomposed in terms of its Fourier components.

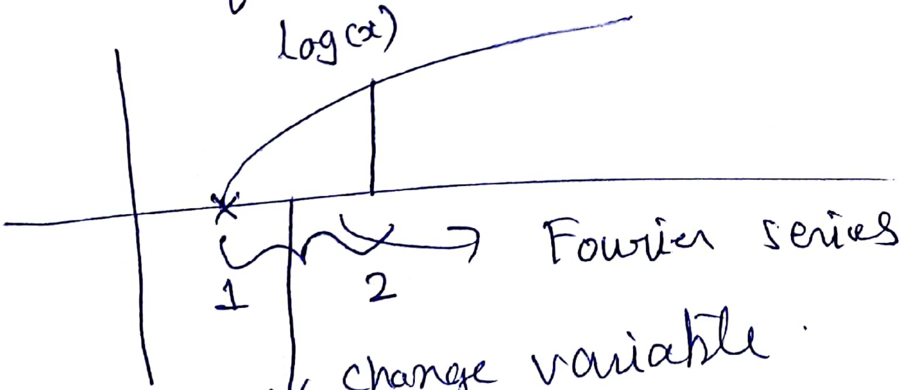
What can go wrong?

→ Non periodic functions can not be decomposed in terms of sin and cos functions.

→ A function like log(x) has no fourier ~~ta~~ series.

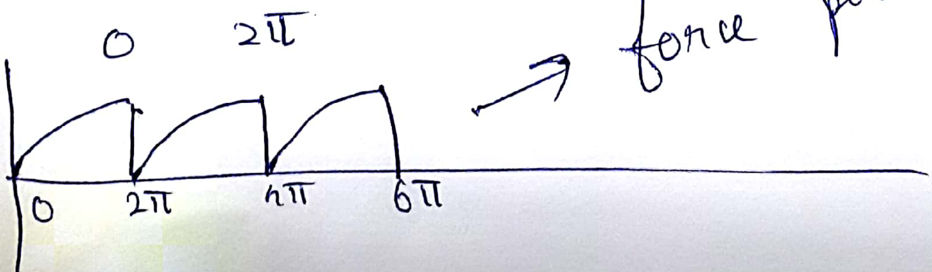
→ In some situations periodicity is not required.

Jainam K: If a function is restricted to a finite interval & repeated



→ Fourier series needed.

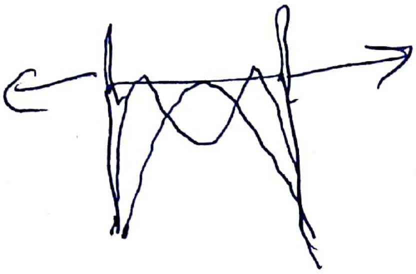
change variable.



→ force periodicity

What can go wrong with the above approach?

~~Aryaa~~: The series goes wrong at the
Aarya point of discontinuity



disagree.

Just like $V_x = \vec{V} \cdot \hat{x}$

the Fourier components of a function can be defined in terms of its dot product with sin, cos & 1.

→ what wrong in the above statement

The function set 1, $\sin x$, $\sin 2x$, ...
$\cos x$, $\cos 2x$, ...
is orthogonal but not orthonormal.

L-26.

$$(\sin x, \sin x) = \int_0^{2\pi} \sin^2 x \, dx$$

$$= \pi$$

$$\| \sin x \|^2 = \pi$$

$$\| \sin x \| = \sqrt{\pi}$$

the basis function $\sin x$ needs to be divided by $\sqrt{\pi}$ to make is ortho normal.

$1, \sin x, \sin 2x, \ldots, \cos x, \cos 2x, \ldots$ are not the only orthonormal complete basis set.

→ Given any set of functions one can use the Gram-Schmidt orthogonalization to create an orthonormal basis set.

Given the three functions $1, x, x^2$ create an orthonormal set of functions on the interval $[-1, 1]$

Find magnitude of 1 on $[-1, 1]$.

$\| 1 \| = \sqrt{2}$, The orthonormal function is $\frac{1}{\sqrt{2}}$

$$\int_{-1}^{1} 1 \cdot x = 0$$

The function 1 & the function x are orthogonal to each-other on $[-1, 1]$.

Find

$$\|x\| = \sqrt{2/3}$$