Linear Algebra, in context

Mitaxi Mehta: Lecture 2

Linear equations and solutions

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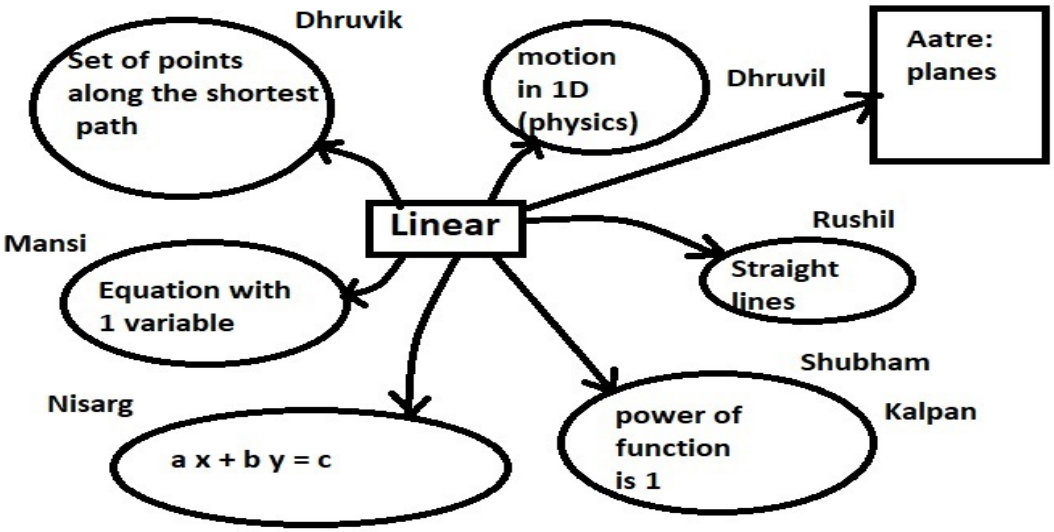

Figure: Mind map: linear

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- In answer to, can there be two variable equations connected to the word linear?, Nisarg came up with the general equation $ax + by = c$. Note that when b is not zero, one can always divide the equation by b to get the standard equation of line in the form $y = mx + c'$, where $m = -a/b$ and $c' = c/b$.

discussion

- Shubham came up with the unusual idea that any function with power 1 can be associated with the word linear. Later in the course we shall treat general functions like sin, cosine and Hermite and other polinomials within linear algebra.

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- Aatre started out with arguing that definition of a line in terms of points along the shortest path between two points, will not work in 3D. I convinced the class that the definition works in 3D too. Aatre was thinking about planes in connection with linear algebra, which is quite a well placed thought, We shall talk a lot more about planes and hyperplanes later in the course.

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- The same word may mean different things in different contexts. For example a vector means an n-tuple in mathematics, an n-tuple with specific transformation properties in physics and a virus in Biology.
- That is the reason that you need to know your audience before you talk and companies need to understand their customers before designing their products.

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- In the race between Achilles and tortoise where the Tortoise has been given a head start, why does Achiles still win ?
- The enmity between Newton and Liebnitz, non-standard analysis and its relevance.
- A slide entitled “tangent”, like this one, is just an aside and does not form a part of the course

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- The furure workplaces will value creativity, orginality, problem solving skills and team work much more than memory.

Mind-map Algebra

- Mind map of algebra

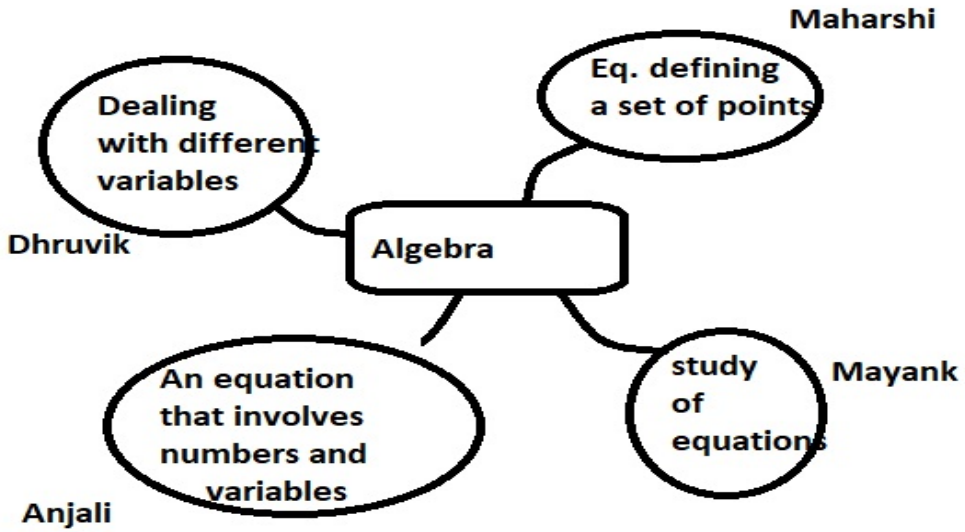

Figure: Mind map: Algebra

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- What can you say about the intersection points for such random choice of 3 lines?
- Discuss possible number of intersection points for two randomly selected lines.

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- Some of the methods used for finding intersection points are, (1) geometric (2) computational (3) substitution (4) elimination (5) Inversion of a matrix.
- Some thoughts on the computational methods of solution: While computational methods of arriving at solutions are getting more accessible and faster in time, mindless use of such methods is likely to weaken your mental abilities. Do apply your mind before using the computer.

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- Elimination. Multiply the second equation by 3 and add to the first equation.

- Inversion of a matrix. Given the equation of the form $A\vec{x} = \vec{b}$,

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The solution is $\vec{x} = A^{-1}\vec{b}$.

$$\begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{5} \begin{pmatrix} -1 & -3 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 5 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

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$$\begin{aligned} x + y &= 1 \\ 3x + 2y &= 5 \end{aligned}$$