

Column space and Null space

Mitaxi Mehta: Lecture 7

- Consider a matrix A .

$$\begin{pmatrix} a_{11} & \dots & a_{1n} \\ \dots & \dots & \dots \\ a_{n1} & \dots & a_{nn} \end{pmatrix}$$

- Consider a matrix A .

$$\begin{pmatrix} a_{11} & \dots & a_{1n} \\ \dots & \dots & \dots \\ a_{n1} & \dots & a_{nn} \end{pmatrix}$$

- Think of each column as a vector.

- Consider a matrix A .

$$\begin{pmatrix} a_{11} & \dots & a_{1n} \\ \dots & \dots & \dots \\ a_{n1} & \dots & a_{nn} \end{pmatrix}$$

- Think of each column as a vector.
- Consider linear combination of the column vectors,

$$c_1 \mathbf{v}_1 + \dots + c_n \mathbf{v}_n$$

for $c_1, \dots, c_n \in R$.

- Consider a matrix A .

$$\begin{pmatrix} a_{11} & \dots & a_{1n} \\ \dots & \dots & \dots \\ a_{n1} & \dots & a_{nn} \end{pmatrix}$$

- Think of each column as a vector.
- Consider linear combination of the column vectors,

$$c_1 \mathbf{v}_1 + \dots + c_n \mathbf{v}_n$$

for $c_1, \dots, c_n \in R$.

- All such vectors are members of the column space.

- Exercise: Give the row and column picture interpretation of the following equation.

$$\begin{pmatrix} 2 & 6 \\ 3 & 9 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

- Exercise: Give the row and column picture interpretation of the following equation.

$$\begin{pmatrix} 2 & 6 \\ 3 & 9 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

- What is the column space Geometrically ? Draw it.

- Exercise: Give the row and column picture interpretation of the following equation.

$$\begin{pmatrix} 2 & 6 \\ 3 & 9 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

- What is the column space Geometrically ? Draw it.
- Draw the RHS vector, Does the RHS belong to the column space ?

- Exercise: Give the row and column picture interpretation of the following equation.

$$\begin{pmatrix} 2 & 6 \\ 3 & 9 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

- What is the column space Geometrically ? Draw it.
- Draw the RHS vector, Does the RHS belong to the column space ?
- Is the equation solvable?

- Consider the following equation.

$$\begin{pmatrix} 2 & 4 \\ 3 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$$

- Consider the following equation.

$$\begin{pmatrix} 2 & 4 \\ 3 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$$

- Answer the same questions as in the previous slide.

- When the RHS is an element of the $C(A)$ the equation is solvable and not solvable otherwise.

- When the RHS is an element of the $C(A)$ the equation is solvable and not solvable otherwise.
- Row space of A is column space of A^T , Thus while solving $A^T x \uparrow = b \uparrow$, the equation has a solution if $b \uparrow$ is an element of the $R(A)$.

- The null space of a matrix A consists of all such vectors x such that $Ax = 0$

- The null space of a matrix A consists of all such vectors x such that $Ax = 0$
- Find the null space of the matrix in the first problem and draw it.