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Gram-Schmidt orthonormalization.

Given vectors $a\uparrow$, $b\uparrow$, $c\uparrow$, ...
form an orthonormal set of vectors
$e_1\uparrow$, $e_2\uparrow$, ...

(1) $$e_1\uparrow = \frac{a\uparrow}{\|\vec{a}\|}$$

(2) $$B\uparrow = b\uparrow - (b\uparrow \cdot e_1\uparrow)\, e_1\uparrow$$

$$e_2\uparrow = \frac{B\uparrow}{\|\vec{B}\|}$$

(3) $$C\uparrow = c\uparrow - (c\uparrow \cdot e_1\uparrow)\, e_1\uparrow - (c\uparrow \cdot e_2\uparrow)\, e_2\uparrow$$

$$e_3\uparrow = \frac{C\uparrow}{\|\vec{C}\|}$$

$e_1\uparrow \perp e_2\uparrow$, $e_2\uparrow \perp e_3\uparrow$, ...
$\|\vec{e_1}\| = \|\vec{e_2}\| = \|\vec{e_3}\|$.

generalize.

Orthonormalize the vectors. ② 10 March

$a\uparrow = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $b\uparrow = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$, $c\uparrow = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

find $e_1\uparrow$, $e_2\uparrow$, $e_3\uparrow$

$e_1\uparrow = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $e_2\uparrow = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $e_3\uparrow = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$a\uparrow = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $b\uparrow = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ $c\uparrow = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Orthonormalize.

$e_1\uparrow = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}$ $e_2\uparrow = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}$ $e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Check $e_i\uparrow \cdot e_j\uparrow = 0$ if $i \neq j$

equivalently. $\vec{e_i}\, e_j\uparrow = 0$.

$e_i\uparrow \cdot e_j\uparrow = 1$ ~~if i=j~~

if $i = j$

In short $e_i\uparrow \cdot e_j\uparrow = \delta_{ij}$

~~Ch~~ Croneker delta symbol.

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10 March.

Using linear algebra to do differentiation of a polynomial.

Representing a polynomial in linear algebra:

$$P_n(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$$

$\left.\begin{array}{l} 1 \\ x \\ x^2 \\ \vdots \\ x^n \end{array}\right\}$ are treated like basis vectors.

$$P_2(x) = a_0 + a_1x + a_2x^2$$

$$= a_0\begin{bmatrix}1\\0\\0\end{bmatrix} + a_1\begin{bmatrix}0\\1\\0\end{bmatrix} + a_2\begin{bmatrix}0\\0\\1\end{bmatrix}$$

Do addition

Represent $Q_2(x) = b_0 + b_1x + b_2x^2$

add: $P_2(x) + Q_2(x)$