L-28 23 March. ①

$\sqrt{i}$: Two solutions $e^{\pm \frac{i\pi}{4}}$.

why are there exactly two solutions & not more or less?

$w^2 = i$. Second degree eqn. Two roots. $e^{\pm\frac{i\pi}{4}}$?

which are solutions other than

$e^{\pm \frac{i\pi}{4} \pm 2n\pi i}$

The solutions to the equation

$w^n = 1.$

are called roots of unity.

$e^{\frac{2\pi i}{N} j}$ where $j = 0, \ldots, N-1$

the same as $j = 1, \ldots, N$

$j = 0$ case the same as $j = N$ case

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Result: Roots of unity always add up to give zero.

Assume you have solved

$$w^n = 1.$$

roots. $e^{\frac{2\pi i}{n} j}$ $\quad j = 0, \dots, n-1.$

$w_n = e^{\frac{2\pi i}{N}}$ is the root where $j = 1$.

roots are $1, w_n, w_n^2, \dots, w_n^{n-1}$

The sum is

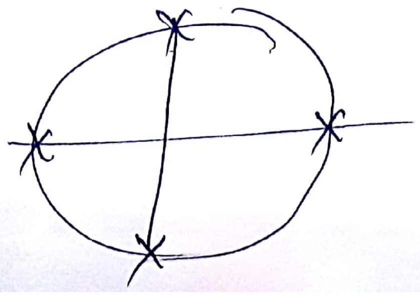
$$S = 1 + w_n + w_n^2 + \dots + w_n^{n-1}$$

$$S w_n = w_n + w_n^2 + \dots + w_n^n = 1.$$

$$= S$$

$\Rightarrow S = 0$ any other number does not have this property

$n =$ even



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A periodic function can be expanded as

$$f(x) = a_0 + a_1 \sin x + a_2 \sin 2x + \cdots$$

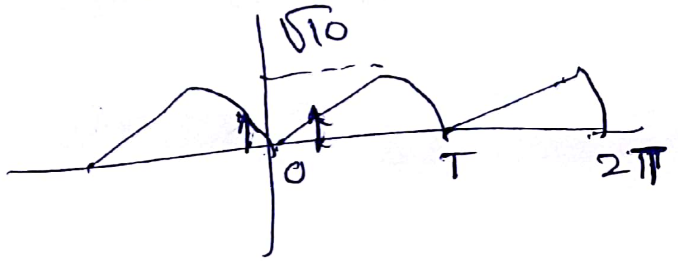
or

$$\cancel{= b_0 + b_1 \cos x + b}$$

$$= b_0 + b_1 \cos x + b_2 \cos 2x + \cdots$$

$$= a_0 + \sum_{n=1}^{N} a_n \sin nx + \sum_{n=1}^{N} b_n \sin nx$$

Experimental data



Shubham: Sine series. because sin is zero at zero.

Criteria: Look at the symmetry of the signal

odd: sine, even: cosine

otherwise: both.

* How will you handle T?

Abu

take $x \to \frac{2\pi x}{T}$

How does that help?

Aatre take $x \to \frac{Tx}{2\pi}$.

does $\sin \frac{2\pi x}{T}$ has the right period

or $\sin \frac{Tx}{2\pi}$ has the right period

Aim is at ~~$x=2\pi$~~ $x=T$ the function should have the same value.

A different periodic series is

$$f(x) = \sum_{n=0}^{N} c_n e^{inx}$$ why use this?

basis functions are e^{inx} $n=0,\dots,N$

are they $\perp$ over $[0, 2\pi]$

(e^{2ix}, e^{3ix}).

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$$\int_0^{2\pi} e^{2ix} e^{3ix} \, dx$$

$$= \int_0^{2\pi} e^{5ix} \, dx = \left[\frac{e^{5ix}}{5i} \right]_0^{2\pi} = \frac{1-1}{5i} = 0$$

$$= \int_0^{2\pi} \underbrace{\cos 5x + i \sin 5x}_{\text{periodic}} \, dx$$

$$= 0$$

For complex functions

$$(f(x), g(x)) = \int_a^b f^*(x) \, g(x) \, dx$$

Using this calculate

$$(e^{2ix}, e^{3ix}) = \int_0^{2\pi} e^{-2ix} e^{3ix} \, dx = \int_0^{2\pi} e^{ix} \, dx$$

$$= 0$$

$$\|e^{4ix}\|_{L^2} \, (e^{4ix}, e^{4ix}).$$

$$= \int_0^{2\pi} e^{-4ix} e^{4ix} \, dx = \int_0^{2\pi} e^0 \, dx = \int_0^{2\pi} 1 \, dx$$

$$= 2\pi$$

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A complex series is easier to work with because

* Easy to integrate
* Easy to differentiate.
* Orthogonality is easy to see.

Where would one want to use $f(x) = \sum_{n=0}^{N-1} c_n e^{inx}$?

Write the series for

(1) $f(x) = \sin x$. $= \frac{1}{2i} e^{ix} + \left(\frac{-1}{2i}\right) e^{-ix}$

what are the amplitudes. [c_n values]

(2) $f(x) = \cos x$

"

(1) ~~$c_1 = ?$~~ $\sin x$ requires both e^{ix} & e^{-ix}, whereas the series has only +ve exponents.

How do we handle this?

It is sufficient to work with only +ve exponents.

For a real function the amplitude for complex conjugate ~~b~~ part is c_n^*

For example:

sinx has $c_1 = \frac{1}{2i}$

$c_{-1} = \frac{1}{-2i}$

It is ~~underst~~ understood that for a real function for every c_n there exists $c_{-n} = c_n^*$

What are the amplitudes for cos x ?

$\cos x = \frac{e^{ix} + e^{-ix}}{2}$ $\quad c_1 = \frac{1}{2}$ $\quad c_{-1} = \frac{1}{2}$

What are the Fourier amplitudes of:

a cosx + b sinx ? $\quad$ Assume ~~A & B~~ a & b ~~are~~ are real.

$c_1 = \frac{a}{2} + \frac{b}{2i}$ $\quad c_{-1} = \frac{a}{2} + \frac{b}{-2i}$