L - 34 ① 9-4-18

Find the eigenvalues & eigenvectors of

$$A = \begin{bmatrix} -2 & -6 \\ 2 & 5 \end{bmatrix}$$

$\lambda = 2, 1$

eigenvectors ?

$\lambda = 1 \quad e_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ or $\begin{bmatrix} -4 \\ 2 \end{bmatrix}$ or $\begin{bmatrix} 6 \\ -3 \end{bmatrix}$

$\lambda = 2 \quad e_2 = \begin{bmatrix} -3/2 \\ 1 \end{bmatrix}$ or. $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$ or $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$ or ....

$$A \begin{bmatrix} -3 \\ 2 \end{bmatrix} = 2 \begin{bmatrix} -3 \\ 2 \end{bmatrix} \quad \text{--- (1)}$$

$$A \begin{bmatrix} 2 \\ -1 \end{bmatrix} = 1 \begin{bmatrix} 2 \\ -1 \end{bmatrix} \quad \text{--- (2)}$$

(1) & (2) can be combined as

$$A \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow AS = SD$$

$\Rightarrow S^{-1} A S = D$.

If S is selected as a matrix whose columns are eigenvectors of A then $S^{-1} A S$ is a diagonal matrix D whose entries are eigenvalues.

* Calculate A^3

$$y = A^3$$
$$= A \cdot A \cdot A$$

$$\Rightarrow S^{-1} y S = S^{-1} A \underbrace{S S^{-1}}_{I} A \underline{S S^{-1}} A \cdot S$$
$$= (S^{-1} A S)(S^{-1} A S)(S^{-1} A S).$$

$$S^{-1} y S = D^3$$ — very easy to calculate.

$$\Rightarrow y = S D^3 S^{-1}$$

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A kind of beetle lives 3 years.

1. After the first year. the survival probability is $\frac{1}{2}$
2. After second year the survival probability is $\frac{1}{3}$
3. At the end of third year each beetle gives birth to 6 babys.

To start with (at year 0)

$y_1(0)$: Beetles, new borns.

$y_2(0)$: " one year old.

$y_3(0)$: " two year olds.

At year 1

$y_1(1)$: Beetles are new borns

$y_2(1)$: " one year old.

$y_3(1)$: " two " ".

$$\begin{bmatrix} y_1(1) \\ y_2(1) \\ y_3(1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 6 \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \end{bmatrix} \begin{bmatrix} y_1(0) \\ y_2(0) \\ y_3(0) \end{bmatrix}$$

Find the matrix.

$$A^3 = I$$

$$\Rightarrow \quad A^{300} = I$$

Following Beetle population through years is easy.

Diagonalisation of A is achieved by $D = S^{-1} A S$.

Can any matrix A be diagonalized?

NO, why?

→ If two eigenvalues are equal then eigen vectors are equal., S will be singular S^{-1} can't be calculated. (Akash).

→ It is not necessarily true that same eigenvalues imply same eigenvectors.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 1 \begin{bmatrix} x \\ y \end{bmatrix}$$

Any vector is an eigen-vector.

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when eigenvalues are same they are called degenerate eigenvalues.

→ The diagonalisation may fail in this case.

→ If eigenvalues are distinct then diagonalization is gaurenteed to work.

→ If $\lambda_1 \neq \lambda_2$ then $\hat{e_1}$ & $\hat{e_2}$ are linearly independent [True also for n dimensions]

Assume that $\hat{e_1}$ & $\hat{e_2}$ are ~~not~~ linearly ~~in~~dependent

$\Rightarrow$ $c_1 \neq 0$, $c_2 \neq 0$ exist so that

$$c_1 \hat{e_1} + c_2 \hat{e_2} = \hat{0} \quad (1)$$

Apply A on both sides.

$$c_1 \lambda_1 \hat{e_1} + c_2 \lambda_2 \hat{e_2} = 0 \quad (2)$$

take λ_2 time (1)

$$c_1 \lambda_2 \hat{e_1} + c_2 \lambda_2 \hat{e_2} = 0 \quad (3)$$

(2) − (3) $\Rightarrow$ $c_1(\lambda_1 - \lambda_2)\hat{e_1} = 0 \Rightarrow \lambda_1 = \lambda_2$

contradiction

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Starting with $\lambda_1 \neq \lambda_2$
we arrived at $\lambda_1 = \lambda_2$
because the assumption that
$e_1\uparrow$ & $e_2\uparrow$ are linearly dependent
is wrong.
$\Rightarrow$ $e_1\uparrow$ & $e_2\uparrow$ have to be linearly
independent.