Linear Algebra: 3D geometry

Mitaxi Mehta: Lectures 3 and 4

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- To solve multiply by A^{-1} to get $\vec{x} = A^{-1}\vec{b}$.

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- The row and column vector are called dual to each other.

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- Aatre: Determinant of A should be nonzero.
- Jainam: The reason for that is that A^{-1} calculation requires division by $Det(A)$.
- Preet: Two columns or two rows being the same would make the determinant zero.

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- The 3D analogue of the above will be parallel planes.
- When $Det(A) = 0$, A is called a singular matrix.

Example

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- Write the matrix equation and check if $Det(A) = 0$. How many solutions are there ?

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- Answer all the questions on the previous slide.

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- Learn whtever you are interested in,sometimes very diverse fields come together to create very effective technniques.
- Reading suggestions: “The blind watchmaker” and “The selfish gene” by Richard Dawkins.

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- Milind: A plane parallel to the y-z plane passing through $x = 3$

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- On 2D plane, it is a line parallel to the y axis.
- In 3D space, it is a plane parallel to the yz plane.

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- The solution gives the intersection points of the three planes.

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- Tangent: When needed, always check your answer for correctness.

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- Arya: Trial and error gives one point as $(3, 1, 4)$, check it out.
- Is there a better way for finding points on a plane?