Images, Dimensionality of Spaces

Mitaxi Mehta: Lecture 12

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- Drashti Shah: Create a matrix whose first column and last row have ones and rest of the entries are zeros.
- What should be the size of the matrix (Number or rows and columns)?
- Jainam K: Depends on the aspect ratio of the device, number of pixels in rows and columns.

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- The entries are between 0 and 1 with the number corresponding to the intensity of the light (gray levels for black and white image).

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- Solve,

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- Manthan Shah: There are infinite number of solutions.

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- For $y = 0$, $x = 1$ hence $(1, 0)$ is a solution.

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- A plane requires two free variables to define it, the dimensions of a plane are 2.

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- There are extra conditions that apply to the above criteria but we shall not go into them explicitly.

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- If you allow x and y to take complex values then x can be treated as a unbounded free variable. On real line x has to be bounded on a continuous interval $[-1, 1]$.
- Since there is one free variable, the circle is a one dimensional object.

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- To be one dimensional an equation should have one free variable, a variable that can take values on a continuous interval.
- $\sin x = 1$ is satisfied by discrete value $x = 2n\pi + \pi/2$ where n is an integer. Hence x is not a free variable.

- Find $R(A)$, $C(A)$ and $N(A)$ for the equation $A\vec{x} = \vec{b}$.

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- Arya Gohil and Shubham: The row space is a line containing vectors of the form,

$$\vec{v} = c_1 \begin{pmatrix} 3 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 0 \end{pmatrix} = c_1 \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

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- The equation of the line is $3y = x$.

- The column space contains vectors of the form,

$$v \uparrow = c_1 \begin{pmatrix} 3 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} = (3c_1 + c_2) \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

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- Thus the column space is the $x-$axis.

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- Akash Prasad: Null space is a point, the zero vector.
- Dhuval Mehta: the vector $(1, -3)$ is also in $N(A)$.
- Arya Gohil: $N(A)$ is a line, $y = -3x$, which is the correct answer.

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- If the null space is not a point then $Ax \uparrow = b \uparrow$ has infinite number of solutions.

Tangent

- Remember the course ”Calculus and Differential equations“ where the general solution of an inhomogeneous ODE is given as $y = y_c + y_p$ where y_p is the particular solution (like our $x^* \uparrow$) and y_c is the general solution of the homogeneous equation (RHS =0), (like our x_n).

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- In the course Discrete Mathematics too while finding closed form solutions to linear recurrence equations (compare with ”Linear algebra“ and ”linear ODEs“) a similar construct is used. See for example the lecture notes ”Mathematics for computer science“ by Lehman, Leighton and Meyer” on MIT, OCW