2 - 31 ① 2nd April,

Eigen values & ~~eigenf~~ eigenvectors.

- Matrix multiplied to a vector

3 geometric pictures.

$$A \vec{x} = \vec{b}$$

① The matrix A represents a transformation

It changes $\vec{x}$ to $\vec{b}$

Mayank

examples: Scaling, reflection, projection, rotation

② Akash: Intersection of lines, planes, hyperplanes.

③ Scaled sum of vectors.

$$x_1 \begin{bmatrix} c_1 \end{bmatrix} + x_2 \begin{bmatrix} c_2 \end{bmatrix} + \dots = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}.$$

Eigenvector $\vec{e}$ is defined by the equation

$$A \vec{e_1} = \lambda_1 \vec{e_1}$$

λ_1 is a constant.

$\vec{e_i}$ are eigen-vectors

λ_i are eigenvalues.

Markov processes:

Consider population of Ahmedabad on 1^{st} April of every year.

$\frac{1}{10}$ of people outside move in

$\frac{2}{10}$ " " inside move out.

y_i : The population of Ahmedabad in the ith year

z_i : The population of outside Ahd in the ith year.

You are given the initial condition

z_0, y_0.

$y_1 = \underline{.1 z_0} + \underline{.8 y_0}$. → Came from Ahd. → Left outside.

~~$z_1 = .2 y_0 +$~~

$z_1 = \underline{.9 z_0} + .2 \underline{y_0}$ → Come to Ahd.

↙ remains in Ahd.

Describes populations in year 1, given the population in year 0.

Consider 3 countries A, B and C.

3

→ every year the country A, loses
10% of its cricketers to B and
5% " " to C.

→ Every year " B loses.
20% of its cricketers to A
and 10% " C

→ " C loses
5% to A
& 5% to B.

In year 0 the No. of cricketer in
A is a_0, B is b_0, C is c_0.
Write the # of cricketers in year 1.

$a_1 =$ ___ $a_0 +$ ___ $b_0 +$ ___ c_0

$b_1 =$ ___ $a_0 +$ ___ $b_0 +$ ___ c_0

$c_1 =$ ___ $a_0 +$ ___ $b_0 +$ ___ c_0

(4)

$$a_1 = .85\, a_0 + .2\, b_0 + .05\, c_0$$

$$b_1 = .1\, a_0 + .7\, b_0 + .05\, c_0$$

$$c_1 = .05\, a_0 + .1\, b_0 + .9\, c_0$$

$$\begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} = \underbrace{\begin{bmatrix} .85 & .2 & .05 \\ .1 & .7 & .05 \\ .05 & .1 & .9 \end{bmatrix}}_{A.} \begin{bmatrix} a_0 \\ b_0 \\ c_0 \end{bmatrix}.$$

- Every column adds up to 1.
- Every entry is positive.

Such matrices are called ~~to~~ Markov matrices. In nth year

$$\begin{bmatrix} a_n \\ b_n \\ c_n \end{bmatrix} = A^n \begin{bmatrix} a_0 \\ b_0 \\ c_0 \end{bmatrix}.$$

$$\begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} = A \begin{bmatrix} a_0 \\ b_0 \\ c_0 \end{bmatrix}.$$

Letus make a change of variables.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = S \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

x, y, z are the new variables.

$$\Rightarrow S \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} = S A \begin{bmatrix} a_0 \\ b_0 \\ c_0 \end{bmatrix}.$$

$$\Rightarrow \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = SAS^{-1} \underbrace{S}_{I} \begin{bmatrix} a_0 \\ b_0 \\ c_0 \end{bmatrix}$$

rewrite

$$= B \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$$

If S is chosen in such a way that SAS^{-1} is a diagonal matrix then iterations of $\boxed{X_n = B X_{n-1}}$ become very simple

$$\text{If } B = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

$$B^n = \begin{bmatrix} \lambda_1^n & 0 & 0 \\ 0 & \lambda_2^n & 0 \\ 0 & 0 & \lambda_3^n \end{bmatrix}$$

$$x_n = \lambda_1^n x_0$$

$$y_n = \lambda_2^n y_0$$

$$z_n = \lambda_3^n z_0 .$$

$$\begin{bmatrix} a_n \\ b_n \\ c_n \end{bmatrix} = S^{-1} \begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix}.$$