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Solve

$$\frac{dx}{dt} = -2x + y$$

$$\frac{dy}{dt} = x - 2y.$$

$$\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\Rightarrow \frac{d\vec{X}}{dt} = A\vec{X}$$

$$\Rightarrow \frac{d}{dt}(B\vec{X}) = BAB^{-1}B\vec{X}$$

$B\vec{X} = \vec{Y}$

matrix

$$\Rightarrow \frac{d\vec{Y}}{dt} = BAB^{-1}\vec{Y}$$

$$= D\vec{Y}$$

y equations are decoupled

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For $\frac{d\hat{x}}{dt} = A\hat{x}$

given that $\hat{X(0)} = \hat{X_0}$ initial conditions.

The general solution is:

$$\boxed{\hat{X(t)} = c_1 \hat{e_1} e^{\lambda_1 t} + c_2 \hat{e_2} e^{\lambda_2 t}.}$$

$\hat{e_1}$ & $\hat{e_2}$ are eigenvectors of A, λ_1 & λ_2 are eigenvalues of A and c_1 and c_2 can be found from initial conditions.

Solve

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \vec{x}$$

for $\vec{X(0)} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$.

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for $A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$ $\lambda_1 = -1$

$\lambda_2 = -3$.

$\vec{e_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\vec{e_2} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

done in the last class.

$$X(t) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-1t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-3t}$$

Use $X(t=0) = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 1 \\ 5 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$c_1 + c_2 = 1$$

$$c_1 - c_2 = 5.$$

$$c_1 = 3 \qquad c_2 = -2.$$

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = 3\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-t} + (-2)\begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-3t}$$

$$x(t) = 3e^{-t} - 2e^{-3t}$$

$$y(t) = 3e^{-t} + 2e^{-3t}$$

The eigenvector matrix is

$$S = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad S^{-1} = -\frac{1}{2}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$S^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Define a change of variables.

$$\begin{bmatrix} X \\ Y \end{bmatrix} = S^{-1}\begin{bmatrix} x \\ y \end{bmatrix}$$

The new variables are

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}.$$

$$= \begin{bmatrix} \frac{x+y}{2} \\ \frac{x-y}{2} \end{bmatrix}$$

$$\frac{dX}{dt} = \frac{1}{2}\left(\frac{dx}{dt} + \frac{dy}{dt}\right).$$

$$= \frac{1}{2}\left((-2x+y) + (x-2y)\right)$$

$$= -\frac{(x+y)}{2} = -X..$$

The new equation is

$$\frac{dX}{dt} = -X = (-1)X = \lambda_1 X$$

Similarly

$$\frac{dY}{dt} = -3Y. = (-3)Y = \lambda_2 Y.$$

X, Y are normal coordinates.

Find x & y from X and Y