L-24. 1 12 March 18

Differentiating a polynomial

$$P_3(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

$$= a_0 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + a_1 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + a_2 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + a_3 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$\frac{d}{dx} P_3(x) = 0 + a_1 + 2a_2 x + 3a_3 x^2$$

$$= a_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + 2a_2 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + 3a_3 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

Find a matrix which does the differenti-
-ation.

$$P_3(x) = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}.$$

Find a matrix A such that → Shubham

↓

$$\underline{\underline{A}}\, P_3(x) = \frac{d}{dx} P_3(x) = \begin{bmatrix} a_1 \\ 2a_2 \\ 3a_3 \\ 0 \end{bmatrix}$$ ⇄ Jainam

Akash: Find the effect on unit vectors.

12 March 18

On differentiation

1. $\frac{d}{dx} 1 = 0$

$$A \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

2. $\frac{d}{dx} x = 1$

$$A \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

3. $\frac{d}{dx} x^2 = 2x$

$$A \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

4. $\frac{d}{dx} x^3 = 3x^2$

$$A \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \end{bmatrix}$$

 12 March 18.

- $1, x, x^2, x^3$ are the building blocks of any 3rd degree polynomial

- $\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$

are building blocks of any vector.

A third degree polynomial can be represented by a 4 dimensional vector.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{1n} \\ a_{21} & a_{22} & & \\ a_{31} & & & \\ a_{41} & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix}1\\0\\0\\0\end{bmatrix} = \begin{bmatrix}a_{11}\\a_{21}\\a_{31}\\a_{41}\end{bmatrix}$$

$$A\begin{bmatrix}1\\0\\0\\0\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix} \Rightarrow \begin{bmatrix}a_{11}\\a_{21}\\a_{31}\\a_{41}\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

4

12 March 18

A

$$\frac{d}{dx} x = 1$$

$$A \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{14} \\ a_{21} & a_{22} & \cdots & \cdot \\ a_{31} & a_{32} & \cdots & \cdot \\ a_{41} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \\ a_{n2} \end{bmatrix}.$$

$$\Rightarrow \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \\ a_{n2} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

What is the third column of A?

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

 12 March 18

Do the derivative of

$P_3(x) = 5 + 2x + 3x^2 + x^3$

In two ways

1. take usual differentiation

2. Write $\vec{a} = P_3(x)$ in vector form

multiply $A\vec{a} = \vec{b}$

write $\vec{b}$ in polynomial form.

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 3 \\ 0 \end{bmatrix}.$$

$$P_3(x) = 5 \cdot \underline{\underline{1}} + 2 \cdot \underline{\underline{x}} + 3\,\underline{\underline{x^2}} + 1\,\underline{\underline{x^3}}$$

$$= 5\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + 3\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + 1\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 2 \\ 3 \\ 1 \end{bmatrix} = P_3(x).$$

12 March 18

The differentiation

$$\frac{d}{dx} P_3(x) = P_2(x)$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$A \begin{bmatrix} 5 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \text{to be found.}$$

$$\Rightarrow \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 3 \\ 0 \end{bmatrix}.$$

Rewrite the vector in the polynomial form

$$\begin{bmatrix} 2 \\ 6 \\ 3 \\ 0 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + 6 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + 3 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= 2 \cdot 1 + 6 \cdot x + 3 \cdot x^2$$

7 12 March 18

Integration of a polynomial.

$$\int P_2(x)\,dx = P_3(x)$$

define $\int 1\,dx = x$

$$\int x\,dx = \frac{x^2}{2}$$

matrix → B

unit vectors

Results.

Construct B such that it gives required results.

$$\int 1\,dx = x$$

$$B \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Find B.

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

 12 March.

B should be 4×3 matrix because. B acts on a 2^{nd} degree polynomial to give a third degree polynomial.

(1) $\int 1 \, dx = x$

$$B\begin{bmatrix}1\\0\\0\\0\end{bmatrix} = \begin{bmatrix}0\\1\\0\\0\end{bmatrix} = \begin{bmatrix}B_{11}\\B_{21}\\B_{31}\\B_{41}\end{bmatrix}$$

(2) $\int x \, dx = \frac{x^2}{2}$

$$B\begin{bmatrix}0\\1\\0\\0\end{bmatrix} = \frac{1}{2}\begin{bmatrix}0\\0\\1\\0\end{bmatrix} = \begin{bmatrix}B_{12}\\B_{22}\\B_{32}\\B_{42}\end{bmatrix}$$

(3) $\int x^2 \, dx = \frac{x^3}{3}$

$$B\begin{bmatrix}0\\0\\1\\0\end{bmatrix} = \frac{1}{3}\begin{bmatrix}0\\0\\0\\1\end{bmatrix} = \begin{bmatrix}B_{13}\\B_{23}\\B_{33}\\B_{43}\end{bmatrix}$$

The matrix is (9) 12^{th} March.

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

Write the polynomial

→ $P_2(x) = 3 + 4x + 5x^2$ → Three component vector.

in vector form

→ multiply B to the vector

→ Write the resulting vector as a polynomial.

→ Check that the answer is sensible.

$$3 \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 4 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 5 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$