\ L - 27 (1) 21^{st} March 18

Gram - Schmidt orthogonalization

Orthogonalise the functions, $1, x, x^2$ on the interval $(-1, 1)$.

$f_1(x) = 1$

$$(1, x) = \int_{-1}^{1} x \, dx = 0$$

$= 0$ (odd function, symmetric interval).

1 & x are orthogonal to each other.

$f_2(x) = x$

$$f_3(x) = x^2 - \frac{(x^2, 1)}{(1,1)} 1 - \frac{(x^2, x)}{(x, x)} x$$

$$= x^2 - \frac{1}{3}$$

f_1, f_2, f_3 are $\perp$ to each other on $[-1, 1]$. Legendre polynomials.

Fourier transforms.

$1, \sin x, \sin 2x, \ldots, \sin nx$

$\cos x, \cos 2x, \ldots, \cos nx$

are orthogonal functions.

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \sin nx + \sum_{n=1}^{\infty} b_n \cos nx$$

$$a_n = \frac{(f(x), \sin nx)}{(\sin nx, \sin nx)}$$ → Depends on convention

dot product is over $[0, 2\pi]$.

$$b_n = \frac{(f(x), \cos nx)}{(\cos nx, \cos nx)}$$

$$a_0 = \frac{(f(x), 1)}{(1, 1)}$$

$\sqrt{-1} = i$

$\sqrt{i} = ?$

Jainam K. : Treat i as a vector.
claim is $\sqrt{i}$ will be in the same direction as i.

? How do you take square-root of a vector ?

Milind :

$\sqrt{i} = z \Rightarrow i = z^2 = (a+ib)^2$

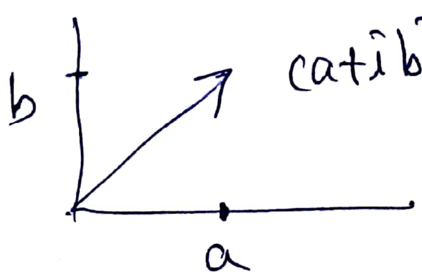
$\sqrt{i} = -1. \qquad = a^2 - b^2 + 2iab.$

$a^2 - b^2 = 0$
$2ab = 1$

$\Rightarrow i = 1$

$a = b = \frac{1}{\sqrt{2}}$ or $a = b = -\frac{1}{\sqrt{2}}.$

A complex number can be expressed as $(a+ib)$



$(a+ib). = Re^{i\theta}$

$R = \sqrt{a^2+b^2}$

$\theta = \tan^{-1}\frac{b}{a}.$

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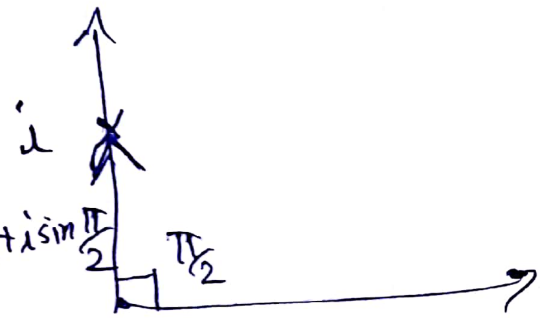
$z_1 = R_1 e^{i\theta_1}$ $\qquad$ $z_2 = R_2 e^{i\theta_2}$

$= a_1 + i b_1$ $\qquad$ $= a_2 + i b_2.$

$$z_1 z_2 = (R_1 R_2)\, e^{i(\theta_1 + \theta_2)}$$

$z_1^2 = R_1^2 e^{2i\theta_1}$ $\qquad$ $i = R e^{i\theta}$

$R = 1, \quad \theta = \pi/2$



$= \cos\frac{\pi}{2} + i \sin\frac{\pi}{2}$

$= e^{i\pi/2}$

$$i = z_1^2 = R_1^2 e^{2i\theta_1}$$

$R_1^2 = 1 \Rightarrow R_1 = 1.$

$2\theta_1 = \pi/2 \Rightarrow \theta_1 = \frac{\pi}{4}.$

$$\sqrt{i} = z_1 = 1\, e^{i\frac{\pi}{4}} = \cos\frac{\pi}{4} + i \sin\frac{\pi}{4}$$

$= a + ib$

$a = \frac{1}{\sqrt{2}} \qquad b = \frac{1}{\sqrt{2}}$

$i^{\frac{1}{3}} = e^{\frac{i\pi}{6}}$: Maharshi.

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$\sqrt{1} = \pm 1$

There are two solutions to $z^2 = 1$.

$z^4 = 1$ How many solutions are expected?

4^{th} degree equation has 4 complex solution.

→ Tangent: An n^{th} degree equation has n roots, why? Think

$z^n = 1$, Give one solution.

a basic complex root is $e^{\frac{2\pi i}{n}}$

even $e^{2\frac{2\pi i}{n}}$ is a solution.

$e^{3\frac{2\pi i}{n}}$ " "

$e^{(n-1)\frac{2\pi i}{n}}$ " ", 1.

roots of unity

$$1 = e^{n \frac{2\pi i}{n}} = e^{0 \frac{2\pi i}{n}}$$

Depending on the convention the n roots of unity are

$$e^{j \frac{2\pi i}{n}} \quad \text{with } j = 0, \dots, n-1$$

$$\text{or } j = 1, \dots, n.$$