L-18 26th Feb.

①

| Shubham: Clean things. Spying.

Akash: To fit small parts.

Prashti: Geological survey.

Spanning:

Given vectors $\vec{a}$ and $\vec{b}$ are said to be spanning a vector space V such that V contains all linear combi-nations of $\vec{a}$ and $\vec{b}$

like $c_1 \vec{a} + c_2 \vec{b}$.

Where $c_1, c_2 \in \mathbb{R}$.

Consider the space spanned by $(1,0,0,0) = \vec{a}$ and $(1,1,0,0) = \vec{b}$, which space do they span?

If the space is (x_1, x_2, x_3, x_4) then the given vectors span the $x_1 x_2$ plane.

Create $(5, 8, 0, 0)$ using $\vec{a}$ & $\vec{b}$

$$c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

Orthogonal complement :

Consider $U \subset R^n$ then all the vectors that are orthogonal to vectors in U make up an orthogonal complement $U^\perp$. (U-perp).

Example :

U : Vectors on the x- axis.

R^3 : The 3D Eucledian plane

Anjali : $U^\perp$ is all vectors on the y-axis.

Prashti : All vectors on y as well as all vectors on z axis.

Preet : All vectors on the yz plane. ✓

Fundamental theorem of linear algebra.

Given an $n \times m$ matrix [n: No. of rows, m: No. of columns]

On which space does A act? row or column?

Druv : Row space?

To write $A\vec{x}$ x has to be m- dimensional

Any $\vec{x} \in \mathbb{R}^m$

L-18 ③

A acts on an m-dimensional vecter $\vec{x} \in \mathbb{R}^m$

and gives an n-dimensional vecter

A is $n \times m$.

Row space dimension is:

A row space vector has m components.

So the max. poss. dimension is m.

A general vecter in $\mathbb{R}^m$ can be written

as $\vec{x} = \vec{x_1} + \vec{x_N}$

$\vec{x_1}$ ↳ in the row space

$\vec{x_N}$ ↳ in the null space.

Row space & Null space are orthogonal complement spaces thus any vecter ~~space~~ in $\mathbb{R}^m$ can be decompored as above.

$A \vec{x_1} \longrightarrow$ A vector in the column space

$A \vec{x_N} \longrightarrow \vec{0}$

Scanned by CamScanner

$\dim(C(A)) + \dim(N(A^T)) = m$

$\dim(R(A)) + \dim(N(A)) = n.$

Show that the following statement is false:

if V is $\perp$ to W then

$V^\perp$ is $\perp$ to $W^\perp$.

Arya: take V: x-axis in $\mathbb{R}^3$.

W: y-axis

$V^\perp$: is the yz plane

$W^\perp$: is the xz plane

v_1 in $V^\perp$: $(0,1,2)$.

w_1 in $W^\perp$: $(1,0,1)$.

$\vec{v_1}\, \vec{w_1} \neq 0 \qquad (v_1^T w \neq 0)$.

Show that $\vec{y} - \vec{x}$ is $\perp$ to $\vec{y} + \vec{x}$ if

$\|\vec{x}\| = \|\vec{y}\|$ where $\vec{x}, \vec{y} \in \mathbb{R}^n$

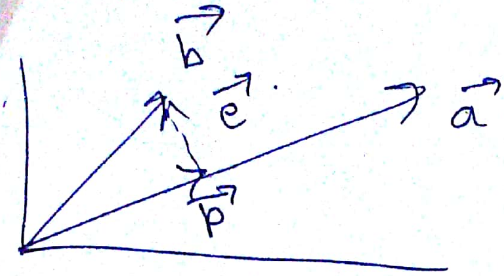
$$(\vec{y} - \vec{x})^T (\vec{y} + \vec{x}) = 0$$

$$\text{L.H.S.} = \vec{y}\,\vec{y} + \underline{\vec{y}\,\vec{x}} - \underline{\vec{x}\,\vec{y}} - \vec{x}\,\vec{x}$$

$$= \|\vec{y}\|^2 - \|\vec{x}\|^2 \quad \text{if zero}$$

then $\|\vec{y}\| = \|\vec{x}\|$

Projection matrix



$$\vec{e} = \vec{b} - \vec{p}$$

and $\vec{e} \perp \vec{a}$.

$$\boxed{p\uparrow = \frac{\vec{a}\, b\uparrow}{\vec{a}\, a\uparrow}\, a\uparrow.} \qquad (1)$$

$$= a\uparrow \frac{\vec{a}\, b\uparrow}{\vec{a}\, a\uparrow} \qquad (2)$$

For two matrices $AB \neq BA$

" 3 " $A(BC) = (AB)C$

matrix ← $= \boxed{\dfrac{a\uparrow \vec{a}}{\vec{a}\, a\uparrow}}\, b\uparrow$ → Vector (3)

$$a\uparrow = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$a\uparrow\, \vec{a}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$$

$$\vec{p} = P\, b\uparrow \qquad P = \frac{a\uparrow\, \vec{a}}{\|\vec{a}\|^2}.$$

Scanned by CamScanner