1 Fitting data to linear equations.

Suppose the data is

i/p variables			output
x_1	x_2	x_3	y
=	=	=	=

There are m data points

x_1^m, x_2^m, x_3^m, y^m is the last point.

The ~~model~~ model is

$$y = a_1x_1 + a_2x_2 + a_3x_3 + c$$

In $A\vec{a} = \vec{y}$

identify A, $\vec{a}$ and $\vec{y}$

Winner: Abu Tibyan.

$$\underbrace{\begin{bmatrix} x_1^1 & x_2^1 & x_3^1 & 1 \\ x_1^2 & x_2^2 & x_3^2 & 1 \\ \vdots & & & \\ x_1^m & x_2^m & x_3^m & 1 \end{bmatrix}}_{\text{i/p data}} \underbrace{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ c \end{bmatrix}}_{\text{Fit parameters}} = \underbrace{\begin{bmatrix} y^1 \\ y^2 \\ y^3 \\ \vdots \\ y^m \end{bmatrix}}_{\text{O/P data}}$$

Construct the rotation matrix. $R(\theta)$, when $R(\theta)$ multiplies a vector it rotates it counter-clockwise by angle θ.

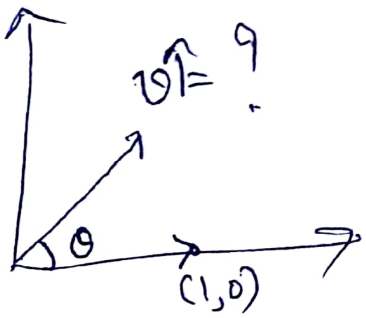
A 2D vector

$$\begin{pmatrix} a \\ b \end{pmatrix} = a\begin{pmatrix} 1 \\ 0 \end{pmatrix} + b\begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

$$A\begin{pmatrix} a \\ b \end{pmatrix} = a\,\underline{\underline{A\begin{pmatrix} 1 \\ 0 \end{pmatrix}}} + b\,\underline{\underline{A\begin{pmatrix} 0 \\ 1 \end{pmatrix}}}.$$

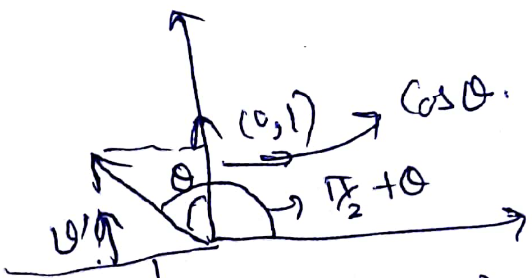
$\downarrow$ $\Leftarrow$ if known

known



$$\vec{v} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

$$A\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$



$\cos(\frac{\pi}{2}+\theta) = -\sin\theta)$

$$\vec{v'} = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix}$$

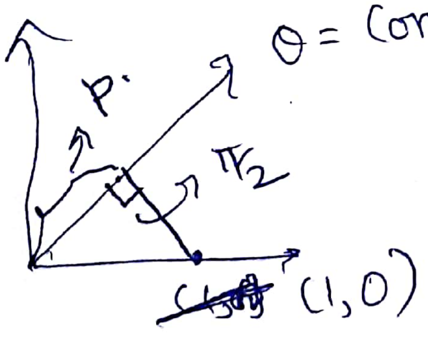
$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} a_{12} \\ a_{22} \end{bmatrix}$$

Hence the rotation matrix

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

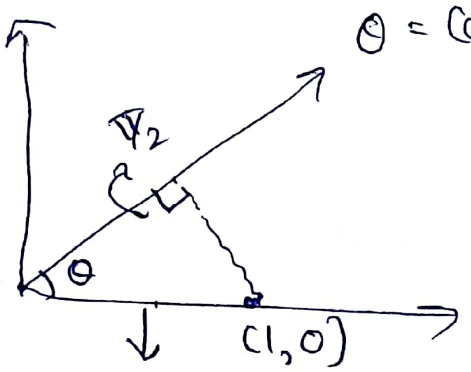
Projection along $\theta =$ constant line.

Find the matrix by considering the effect on unit vectors.



$P = \begin{bmatrix} \ \end{bmatrix}$

Atre



Hypoteneous.

In a right angle triangle.

$$\cos\theta = \frac{\text{Near side}}{\text{Hypo.}} \qquad \sin\theta = \frac{\text{far side}}{\text{Hypo}}$$

The length of the near side

$$\|p\uparrow\| = \cos\theta \cdot (\text{Hypo.}).$$
$$= \cos\theta.$$

Components of ANY vector of length $\|v\uparrow\|$ and angle θ with the x- axis are $\|v\uparrow\|\cos\theta$ and $\|v\uparrow\|\sin\theta$.

$$p\uparrow = \begin{bmatrix} \cos^2\theta \\ \cos\theta\sin\theta \end{bmatrix}$$

Projection of $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is $\begin{bmatrix} \cos\theta\sin\theta \\ \sin^2\theta \end{bmatrix}$

The projection matrix is.

$$P = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

Do polynomial fit to the given data.

given m data points.

$(x_1, y_1), (x_2, y_2), \ldots, (x_m, y_m)$.

Fit the polynomial

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$ to

the data.

Let us convert this non linear problem to a linear problem.

$x = x_1$, $x^2 = x_2$, $x^3 = x_3$. $x_0 = 1$.

Create the five dimensional data points.

x_0	x_1	x_2	x_3	y.
1	x	x^2	x^3	y.

} m such 5D data points

Write the corresponding linear equation

$$A\, a\uparrow = y\uparrow$$

Identify A, $a\uparrow$ and $y\uparrow$

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3.$$

$$= a_0 x_0 + a_1 x_1 + a_2 x_2 + a_3 x_3.$$

m data points, give m equations which can be written (with $x_0 = 1$) as.

$$\underbrace{\begin{bmatrix} 1 & x_1^1 & x_2^1 & x_3^1 \\ \vdots & & & \\ 1 & x_1^m & x_2^m & x_3^m \end{bmatrix}}_{\text{Input data}} \underbrace{\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}}_{\text{Unknown parameters.}} = \underbrace{\begin{bmatrix} y^1 \\ y^2 \\ \vdots \\ y^m \end{bmatrix}}_{\text{O/P data}}$$

The best fit solution is.

$$p\uparrow = A(A^T A)^{-1} A^T y.$$

The fit parameters $\hat{a}$ are found from $\boxed{(A^T A)\hat{a}\uparrow = A^T y\uparrow}$

Nonsingular
invertible

Remember the matrix

A is $m \times n$ where m is the number of data points and n number of parameters.

A : Not square, not invertible.

$A^T A$: Is square