

# Column and Row pictures

Mitaxi Mehta: Lecture 5

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- One could as well choose various  $y$  values and calculate the corresponding  $x$  coordinates.
- The variable that can take any arbitrary value on a chosen interval is called a free variable. The variable whose value depends on the free variables is called the dependent variable.

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- Anjali: Plug-in  $(x, y)$  values to get  $z$ .
- There are two free variables, plane is a two dimensional object.
- The method works for nonlinear equations too, one can draw the curve for  $x \sin(y) + e^y \cos(y) = 10$  by inputting  $y$  values and calculating  $x$ , to generate the corresponding one dimensional curve.

# Row space

- Row space of a matrix  $A$ ,  $R(A)$  constitutes of all possible vectors of the form,  $c_1 R_1 \uparrow + c_2 R_2 \uparrow + \dots$ , where,  $R_1 \uparrow, R_2 \uparrow, \dots$ , are row vectors of the matrix and  $c_1, c_2, \dots \in R$ .

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- A set of equations representing two lines can also be written as, for example,

$$x \begin{pmatrix} 2 \\ 0 \end{pmatrix} + y \begin{pmatrix} 1 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

- The above type of representation is known as the column picture.

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- The row picture and the column picture are useful for higher dimensions too.

- Consider the matrix equation,

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- In column picture it translates to sum of three vectors.
- When  $\text{Det}(A) = 0$ ,  $A$  is called a singular matrix.

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- Solve section 1.3 problem 3 from Strang.