Exercise:

Write the polynomial $3 + x^2$ in Vector form &

* find its differentiation
* " " Integration.

$$3.1 + x^2$$

$$= 3\begin{bmatrix}1\\0\\0\end{bmatrix} + 0\begin{bmatrix}0\\1\\0\end{bmatrix} + 1\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$= \begin{bmatrix}3\\0\\1\end{bmatrix}$$

The differentiation matrix

$$A = \begin{bmatrix}0 & 1 & 0\\0 & 0 & 2\\0 & 0 & 0\end{bmatrix}$$

$$A\begin{bmatrix}3\\0\\1\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\0 & 0 & 2\\0 & 0 & 0\end{bmatrix}\begin{bmatrix}3\\0\\1\end{bmatrix} = \begin{bmatrix}0\\2\\0\end{bmatrix}$$

$$= 0\begin{bmatrix}1\\0\\0\end{bmatrix} + 2\begin{bmatrix}0\\1\\0\end{bmatrix} + 0\begin{bmatrix}0\\0\\1\end{bmatrix} = 2x$$

Functions as vectors.

$f(x) = e^x$

→ For a vecten V↑ how many numbers are needed to define it?

JainamK : As many dimensions.
3D vectors have 3 components . . . -

→ To define $f(x) = e^x$
one needs infinite numbers.

→ Vectors are usually defined over a domain
examples : real numbers, complex numbers etc.

→ Similarly functions are defined over a domain.

• Which is a good domain of definition for $\sin x$?

Shubham : $[-\pi/2, \pi/2]$.
also $[0, 2\pi]$

Length of a vector is

$\| \vec{v} \|^2 = \vec{v}\ v\uparrow$ (our notation. <u>not</u> $v\uparrow \vec{v}$

$= v^T v$ (Strang notation).

$= \vec{v} \cdot \vec{v}$ (usual notation)

$= v\uparrow \cdot v\uparrow$ (our notation).

How would you find $\| \sin x \|^2$?

Take a dot product of $\sin x$ with $\sin x$

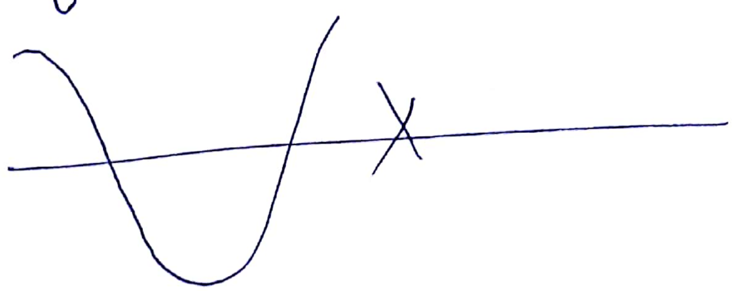
Notation

$\| \sin x \|^2 = (\sin x, \sin x)$.

$[-1, 1]$ $h = 0.1$

$\sin^2(-1) + \sin^2(-0.9) + \sin^2(-0.8) + \sin^2(-0.7)$
$+ \sin^2(-0.6) + \sin^2(-0.5) + \dots$

If the sum is from 0 to 2π



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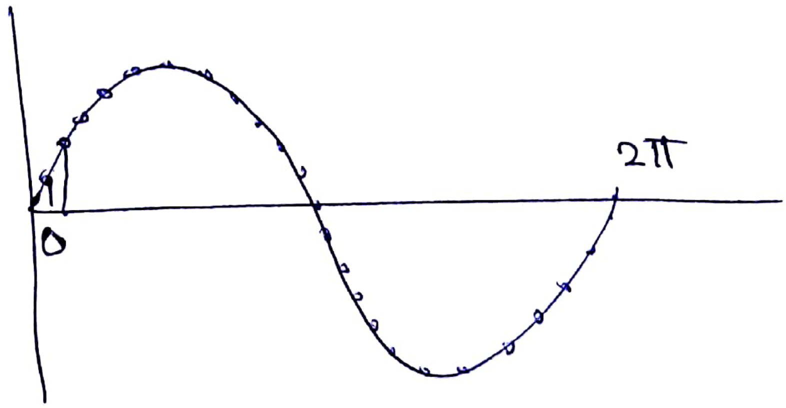


approximate an infinite dimensional vector with a finite dimensional vector.

$$\sum_{j=1}^{N} \sin^2\left(\frac{2\pi j}{N}\right).$$ an approximation to

$= (\sin x, \sin x)$

$= \int_0^{2\pi} \sin^2 x \, dx$

always true

[definition of dot product of sinx with sinx]

Jainam : The sum does not represent an area

Rutvik : If dx in the sum then it will become the area.

In general the dot product of any two functions $f(x)$ and $g(x)$ is defined as

$$(f(x), g(x)) = \int_a^b f(x)\, g(x)\, dx$$

↓

Compare with dot product of tangent vectors:

$\vec{v}$ v^T → The vectors are dual to each other

For complex functions

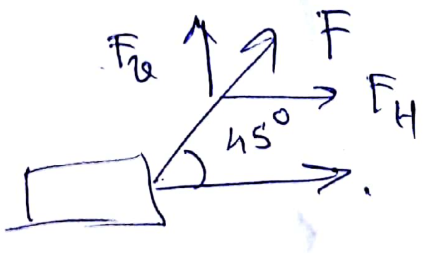
$$(f(x), g(x)) = \int_a^b f^*(x)\, g(x)\, dx.$$

↓

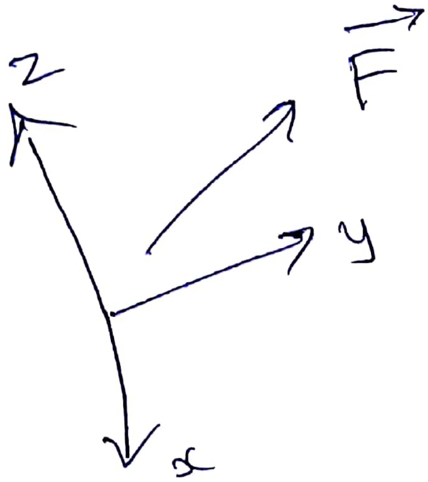
For real functions $f^*(x) = f(x)$.

The use of dot product:

Given $(2, 3)$ what is it's component in $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ direction?



Given an arbitrary vector $\vec{F}$.



→ Lab frame.

$$\vec{F} = (\vec{F} \cdot \hat{x})\,\hat{x} + (\vec{F} \cdot \hat{y})\,\hat{y} + (\vec{F} \cdot \hat{z})\,\hat{z}$$

Force written in the lab frame.

Similarly for functions.

If a given basis set is complete.

$(f_1, f_2, \ldots,$ [illegible]$)$.

Any function can be decomposed in the basis set functions.

$$g(x) = (g(x), f_1(x))\, f_1(x) + (g(x), f_2(x))\, f_2(x) + \ldots$$

The basis has to be orthonormal.

$$(f_i, f_j) = \int_a^b f_i(x)\, f_j(x)\, dx$$

$$= \delta_{ij}$$

It turns out that the functions

$1, \quad \sin x, \quad \sin 2x, \quad \ldots\ldots$

$\cos x, \quad \cos 2x, \quad \ldots\ldots$

are orthogonal to each other.
over the domain $[0, 2\pi]$.

$$(1, \sin nx) = \int_0^{2\pi} \sin nx \, dx = 0$$

$= 0$ [Area under the curve for periodic function with Sym $\pm$ values].

Find over $[0, 2\pi]$

$(\sin x, \cos x) = 0$

$(\sin x, \sin x) = \pi$

$(\sin x, \sin 2x) = 0$

Orthogonal but not orthonormal.