Exercises

Mitaxi Mehta: Lectures 15

- Let us take an example of a linear equation.

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$$\begin{pmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 7 \\ -1 & -3 & 3 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

- After doing the row operations to bring the matrix into the Echelon form on both sides of the equation,

$$\begin{pmatrix} 1 & 3 & 3 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 - 2b_1 \\ b_3 - 2b_2 + 5b_1 \end{pmatrix}$$

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- Note that the vector $x \uparrow$ remains unchanged. Veify this by doing the same operations on equations of lines (eliminations) and then writing the corresponding matrix equation.

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- If (b_1, b_2, b_3) are taken to be $(1, 5, 5)$ then,

$$\begin{pmatrix} 1 & 3 & 3 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 - 2b_1 \\ b_3 - 2b_2 + 5b_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}$$

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- This is called the consistency conditiion for the solution.
- For a different choice of b_1, b_2, b_3, the consistency condition will not be satisfied and the equation would not be solvable (In case there are two or more parallel hyperplanes).

- Solving the first two equations give, $x_3 = 1 - x_4$ and $x_1 = -2 - 3x_2 + x_4$

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$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -2 - 3x_2 + x_4 \\ x_2 \\ 1 - x_4 \\ x_4 \end{pmatrix}$$

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$$\begin{pmatrix} -2-3x_2+x_4 \\ x_2 \\ 1-x_4 \\ x_4 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 1 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix}$$

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- The first vector on RHS is called the particular solution and the next two vectors can be taken as basis vectors of the null space.

- Find the general solution of the following equation.

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$$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 4 & 5 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

- Answer:

$$\begin{pmatrix} \text{u} \\ \text{v} \\ \text{w} \end{pmatrix} = \begin{pmatrix} \text{-3} \\ 0 \\ 2 \end{pmatrix} + u \begin{pmatrix} \text{-2} \\ 1 \\ 0 \end{pmatrix}$$