L-33. 6-4-18

Economy models

Read 2-5 equations . once

→ $\lambda p = Ap$

Stab.
$\lambda_1 > 1$ divergent
$\lambda_1 = 1$ neutral
$\lambda_1 < 1$ convergent

power , coal , transportation

p_1 ~~input~~ output. Ap is the input
p_2
p_3

$p - Ap$ gross O/P.

→. For a big industry this may have 80 × 80 matrix

Ap I/P
p O/P

$p - Ap$ gross o/p.

$y = p - Ap$

$p = (I-A)^{-1} y$.

of A

$\lambda_1 > 1$ $(I-A)^{-1}$ is not ~~non~~ positive (non negative).

$\lambda_1 = 1$ $(I-A)^{-1}$ does not exist.

$\lambda_1 < 1$ $(I-A)^{-1}$ is a non neg. matrix

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price fixing in closed economy

O/P = I/P becomes markov matrix

p_0 vector of prices

Ap_0 prices times amounts . to produce amounts.

values

total cost is O/P.

Iterate to get

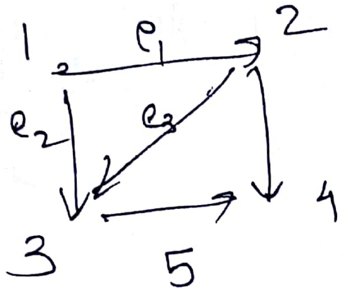
$A^2 p_0$.

$p = Ap$ possible?

produced.

$$\begin{array}{c|c}
 & \text{cc} \quad \text{ce} \quad \text{ct} \\
\text{coal.} & \\
\text{ele.} &
\end{array}
\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}
\begin{bmatrix} \\ \\ \\ \end{bmatrix}
\begin{array}{l} \text{coal} \\ \text{elec} \\ \text{trans.} \end{array}$$

Edge node matrix



$$A = \begin{array}{cc} & \begin{array}{cccc} & & & \end{array} \\ e \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \\ & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \end{array}$$

node

$$N(A) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

any $\quad Ax = b$ $\quad$ b pot. difference across edges

x ↓ voltage

column space :

$b_1 - b_2 + b_3 = 0. \qquad b_3 + b_5 - b_4 = 0$

Kirchoff's voltage law.

left Null space $N(A^T)$.

current laws

Node 1 $\quad -y_1 - y_2 = 0.$

Node 2. $\quad y_1 - y_2 - y_3 = 0.$

Row space : 1, 2 & 4 are ind. basis.

$f_1 + f_2 + f_3 + f_4 = 0$ every row.

$R(A) \perp N(A)$.

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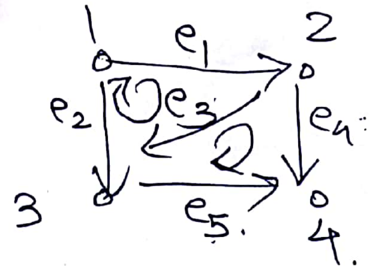
L - 33

Electrical circuit incomplete example.



Edge - Node matrix

$$A = \text{Edges} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$\begin{array}{cccc} 1 & 2 & 3 & 4 \end{array}$$

Nodes.

Node 3 has edges 2 & 3 incoming
~~edge~~ edge 5 going out

$A x\uparrow = b.\uparrow$

b_1 to b_5 were potential differences
~~betwee~~ across the edges.

x_1 to x_n, potential values at the nodes.

$$N(A). = c \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

The general solution is

$y\uparrow = x^* \uparrow + x_N \uparrow$

→ Null vector

if $x^*\uparrow$ are potential values at the nodes for which $b\uparrow$ are the potential differences across edges.

If constant values $c \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

are added to each potential at the node. one would get the same potential difference across the edges.

$$5 \xrightarrow{e_1} 8$$

$x_1 = 5$

$x_2 = 8$

e_1 gives $b_1 = 8 - 5 = 3.$

take $c = 10.$

the New potentials are

$y_1 = 15$ $\quad$ $y_2 = 18$ $\quad$ $b = y_2 - y_1 = 3.$

What is the column space of A?

Defined by conditions on possible entries on b_1 to b_5, can be found by reduction to echelon form & getting a solution.

$$\left.\begin{aligned} b_1 - b_2 + b_3 &= 0. \\ b_3 + b_5 - b_4 &= 0. \end{aligned}\right\} \text{Kirchoff's law for voltages.}$$

Advantage: Given any matrix, make a circuit, write Kirchoff's law conditions

→ get column space

When can you do this?

Conditions on A. tangent.

1. # of edges > # of Nodes.

No self loops? Not necessarily a condition.

Outgoing & incoming edges should balance.

~~Row space of A, R(A).~~

The left Null space of A, $N(A^T)$.

→ Kirchoff's law for currents.

Node 1 $\quad -i_1 - i_2 = 0.$

Node 2 $\quad i_1 - i_3 - i_4 = 0.$

Read up section 2.5.

$N(A)$, $C(A)$, $N(A^T)$.

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Announcement:

→ ~~To~~ Two feedbacks, uni & mine in the next week and one quiz.

→ Both feedbaks are anomymous, do give your opinions, honestly & constructively

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Matrices in economics.

Consider an industry that produes power ~~Input are~~ output

$$\begin{bmatrix} \text{coal.} \\ \text{power} \\ \text{transport.} \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

~~output $p' = Ap$.~~

~~A is the processing matrix that processes the input to give the output~~

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output of an industry.

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} \begin{array}{l} \text{coal} \\ \text{electricity.} \\ \text{petroleum products} \end{array}$$

input used are

$$A \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}.$$

The gross output is

$$\underbrace{p\uparrow - A p\uparrow}\ = y\uparrow = \underbrace{(I-A)\,p\uparrow}.$$

You want $y\uparrow$ to be a positive vector, defined by you.

$$p\uparrow = (I-A)^{-1}\, y\uparrow$$

If the largest eigenvalue λ_1 of A.

$\lambda_1 > 1 \quad \Rightarrow p\uparrow$ is not +ve

$\lambda_1 = 1. \quad \Rightarrow (I-A)^{-1}$ does not exist

$\lambda_1 < 1 \quad \Rightarrow (I-A)^{-1}$ exists, is +ve

$p\uparrow$ can be calculated.

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