1 18-4-18

L- 38

P. 203

Properties of determinants.

1. Det I = 1

2. Row exchange ~~makes~~ causes a factor of -1 in the det A.

$$\det A \rightarrow -\det A$$

Jainam: Try it out on 2x2 unit matrix.

[An example is not a rule.
However a quick check may increase your confidence in the rule]

$\vec{a} \times \vec{b} \rightarrow$ $\vec{a} \times \vec{b} \rightarrow -\vec{b} \times \vec{a}$.

Flux through the boundaries.

(3) The determinant is linear in it's first row.

In 2x2 case this means:

$$\begin{vmatrix} a+a' & b+b' \\ c & d \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} a' & b' \\ c & d \end{vmatrix}.$$

and
$$\begin{vmatrix} ta & tb \\ c & d \end{vmatrix} = t \begin{vmatrix} a & b \\ c & d \end{vmatrix}.$$

$$(\vec{a}+\vec{b}) \times \vec{c} = \vec{a}\times\vec{c} + \vec{b}\times\vec{c}.$$

(4) equal rows $\Rightarrow$ det $A = 0$

Akash : 2x2 case $\vec{a}\times\vec{a} = 0$.

Use the fact that under row exchange det $A \rightarrow$ $\boxed{-\det A.}$

if same rows det $A \rightarrow$ det A

Jainam : ↑ The proof

Row operations leave the determinant unchanged.

$R_i \rightarrow R_i - c R_j$

6) A zero row $\rightarrow$ zero determinant.

7) A triangular then det A is multiplication of diagonal entries.

Jainam: Proof Use linearity for the first, row exchange and a -ve sign for any other row.

8) For invertible matrices.

$\det A \neq 0$

9) $\text{Det } |AB| = \det A \det B$.

10) $\text{Det } A^T = \text{Det } A$.

If A is a 4×4 matrix with determinant $= \frac{1}{2}$.

Calculate $\det(2A)$, $\det(-A)$, $\det(A^2)$, $\det(A^{-1})$.

1. $\det 2A = 2^4 \det A = 16\left(\frac{1}{2}\right) = 8$

[Cross check with simple examples if in doubt, use identity matrix or 2×2 matrix]

2. $\det(-A) = (-1)^4 \det A = \det A$.

3. $\det(A^2) = \det A \det A = \frac{1}{4}$

4. $\det(A^{-1}) =$

$\det(AB) = \det A \det B$

$\Rightarrow \det(AA^{-1}) = \det(I) = \det A \det A^{-1}$

$\Rightarrow \det A^{-1} = \frac{1}{\det A} = 2.$

5

If a 3×3 matrix A has

$\det A = -1$,

$\det(\frac{1}{3}A)$, $\det(-A)$, $\det(A^2)$

$\det(A^{-1})$.

4 Find the determinant of

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \end{bmatrix}$$

5 Find the determinant of a reverse triangular matrix

$$\begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 4 \\ 0 & 2 & 3 & 1 \\ 4 & 1 & 1 & 2 \end{bmatrix} \rightarrow$$

Two row exchanges makes it U:
and $(-1)(-1)\ 4\cdot2\cdot1\cdot2$
$= 16$

Use the fact that for upper & lower triangular matrices $\det A$ is just multiplication of diagonal entries.

Mayank :

$$A = \begin{bmatrix} a & g & h & i \\ 0 & b & j & k \\ 0 & 0 & c & \lambda \\ 0 & 0 & 0 & d \end{bmatrix}$$

$\Rightarrow$ $\boxed{\lambda \text{ values are } a, b, c, d.}$

$\boxed{\det A = abcd.}$ 2 ✓

$\text{Det}(A - \lambda I) = 0$ condition for λ being the eigenvalue.

$\Rightarrow \text{Det}(A) - \text{Det}\,\lambda I = 0$

not true.

only one λ not all λ.

$\text{Det}(A+B) = \text{Det}\,A + \text{Det}\,B$ ✗