LU decomposition: Exercise

Mitaxi Mehta: Lecture 9

- Exercise: Do LU decomposition of the following matrix,

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- Treat the element other than the identity matrix entries as an unknown x, to fix the matrices for raw operations.

- Writing $EA = B$ as follows,

$$\begin{pmatrix} 1 & 0 & 0 \\ x & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & 5 \\ 1 & 1 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & a & b \\ 1 & 1 & 5 \end{pmatrix}$$

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- The product $(x, 1, 0) \cdot (1, 2, 1) = 0$ gives $x = -2$, the values of a and b are defined accordingly.

- Solving for E as in previous slide and calculating $E\,A = B$ gives,

$$B = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 1 & 1 & 5 \end{pmatrix}$$

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- The next steps are to find F such that $F\,B = C$ where the entry $(3,1)$ of C is zero, and G such that $G\,C = D$ with the entry $(3,2)$ of D is zero. Making D an upper triangular matrix.

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- Check if $UL = LU$.

- Solve problem 29 on page 43 of Strang, Solve $Lc \uparrow = b \uparrow$ to find $c \uparrow$ and $Ux \uparrow = c \uparrow$ to find $x \uparrow$.

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- Verify if your answer is correct by substitution in the original equation.

- Solve problem 29 on page 43 of Strang, Solve $L\vec{c} = \vec{b}$ to find $\vec{c}$ and $U\vec{x} = \vec{c}$ to find $\vec{x}$.

- Verify if your answer is correct by substitution in the original equation.

- Note that $A^{-1} = U^{-1}L^{-1}$.