Use of linear algebra.

① Representation of networks.

Akash: Networks have some thing to do with nodes & edges.

Anjali: Connections between things.

A collection of nodes & edges.

Nodes: Entities being connected

Edges: Connections.

Examples:

* A Road network
  Nodes: Cities (or places).
  Edges: Roads.

* Railway.

* Internet
  Nodes: Webpages.
  Edges: links.

* Face book
  Nodes: people
  Edges: Friendships.

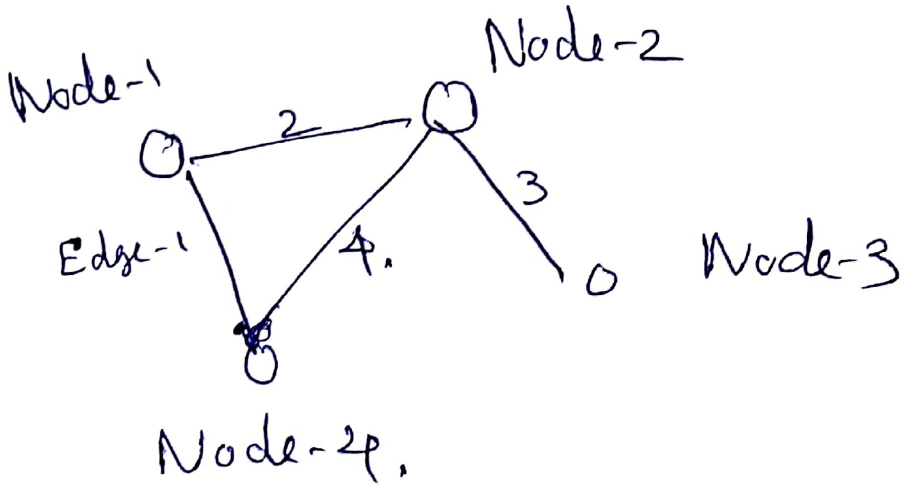
* Food chain
  Nodes: Animals
  Edges: A connection if one eats the other.

* Power distribution:
  Nodes: Transformers.
  Edges: Electrical connection

* Water distribution network
  Nodes: Sources. (tanks) sinks.
  Edges: Pipe lines.

* Chemical Eng.
  Molecular interactions.
  Nodes: Atomic elements.
  Edges: bonds.

The adjacency matrix of a network.



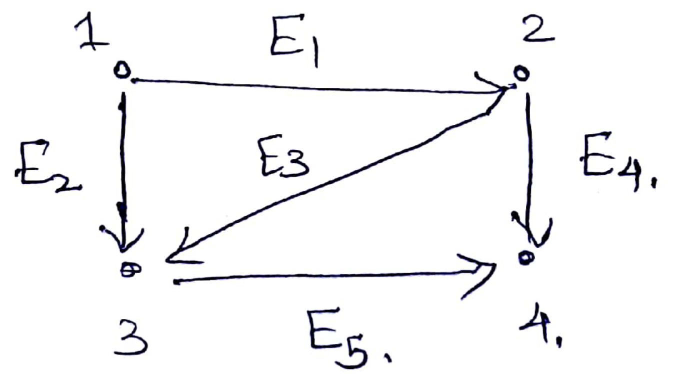
Inside a computer, one of the ways in which the network gets represented is in terms of the adjacency matrix.

$a_{ij} = 1$ if i^{th} & j^{th} nodes are connected

$a_{ij} = 0$ otherwise.

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

Electrical circuit



Edge node matrix is used to represent this network.

$$
\begin{array}{c c}
\text{Edges} & \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} &
\begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \\
\text{Nodes :} & \begin{array}{cccc} 1 & 2 & 3 & 4. \end{array}
\end{array}
$$

If a node has an outgoing edge the edge entry is -1

An incoming edge entry is $+1$

No edge of the given index, the entry is 0.

The meaning of the matrix equation

$A x\uparrow = b\uparrow$

The entries in $b\uparrow$ are potential differences across the edges.

The entries in $x\uparrow$ are potentials at the node

Given the potential differences, find the potentials on the nodes.

$$\begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{bmatrix}$$

$x_2 - x_1 = b_1$ The potential difference between node 2 & node 1 is b_1

What would $N(A)$ mean?

Find the $N(A)$.

$x_1 = x_2 = x_3 = x_4$.

A general null vector $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

The Null space is 1 dimensional. (one free variable), it is the line

$$x_1 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

How does the N(A) affect the solution of $A\vec{x} = \vec{b}$?

Given a solution x^*, another solution y^* can be found by adding a null vector to x^*.

If $A\vec{x^*} = \vec{b}$

then $A(\vec{x^*} + \vec{x_n}) = A\vec{y^*} = \vec{b}$

because $A\vec{x_n} = 0$

You can find all the infinite solutions by knowing only one solution.

Tangent: An example of ~~of~~ gauge transformation. Potentials can be defined only upto an arbitrary constant.

What is $C(A)$?

All such vectors $\vec{b}$ for which $A\vec{x} = \vec{b}$ is solvable.

* Exercise find $C(A)$ in terms of conditions on b_1 to b_5