The Pendulum Equation

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Lecture 3

Working with a pendulum

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- What about human dynamics?
- Why should chemical engineers study a pendulum?
- Do an experiment at your leisure, Take a pendulum and make it oscillate. Does the oscillation time period depend on the amplitude?

- Please note that for a usual pendulum (a bob of mass m, hung by a rod of length l, having negligible mass and pivoted at a fixed point) the time period of the pendulum depends on the amplitude.

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- Only in the limit of small oscillations, the time period is independent of the amplitude.

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- Let us re-frame the question, what is the force under which the bob accelerates?

- If you were a bob of mass m what are the forces that you will feel?

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- Which is the force that actually accelerates the bob at any given point of time?

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- Can you explain why did I write the acceleration as above? Hint: Arc length = $l\theta$

- In other words,

$$\ddot{\theta} = -\frac{g}{l}\sin\theta$$

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- This differential equation is not easy to solve (for a person working with second order ODE for the first time), the solution can be written in terms of Jacobi Elliptic function.

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- What is the dependent variable and what is the independent variable in the pendulum equation?
- How do we decide? Give some examples. What can you say about position, velocity and acceleration of a point mass are they dependent or independent?
- Example: Mass attached to a spring. Draw a coordinate system define variables and write the differential equation.

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- Exercise: Consider 2 simple dynamical systems in your surrounding, describe them, draw appropriate coordinate system and write down the corresponding differential equations. (It is alright to search the web if you can't start, but please give the webpage URL you have used and highlight if you have made any changes in that information).

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- You already know how to solve this.

- However there is a new way of thinking about this equation.

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- Consider $\frac{d}{dx}$, Is this a number ? Is this a function ?
- $\frac{d}{dx}$ is a mathematical operation, which when it acts on a function, gives another function. Such an expression is called an operator. We shall denote $D = \frac{d}{dx}$

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$$D3 = 0, \qquad 3D = 3\frac{d}{dx}$$

$$Df(x) = \frac{df(x)}{dx}$$

- However, note that the following kind of commutation works.

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- What is the value of $(D-3)$ operating on zero?