Approximation of functions

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Lecture 6

- In the pendulum equation, why is $\sin\theta$ replaced by θ (and not θ^2 or 2θ) ? Give as many explanations as you can.

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- Suppose you were asked to approximate $\sin\theta$ in the neighborhood of $\pi/4$ how would you go about doing it?

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- Did you notice that a large number of statements so far have been questions?
- Probing knowledge through interrogation is known as Socratic (sometimes also Platonic) dialogue, I picked this up from my late teacher Prof. C S R Murthy of PRL, who taught me statistics.

- How much information do you have of $\sin\theta$ about $\theta = pi/4$?

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- Can that information be used to gain better approximation of $\sin\theta$?

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- Select Δx to be as small as possible.

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- The actual answer is close to 1.9831.

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- Substitute in the formula.

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What is wrong ?

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\Rightarrow \quad & \sin(1^\circ) \approx 0 + .0174 \times 1 = .0174.
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- What are the dimensions of an angle ?

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- How many different interpretations (Geometric or otherwise) can you find for the $\sin$ and $\cos$ functions ?

Some interpretations of sin and cos.

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- Projection on the x and y axis, of a point on the unit circle.
- Summation of infinite power series.
- Solutions of the differential equation, $\frac{d^2u}{dx^2} = -u$.
- Often the best way of interpreting a new transcendental function is as a solution of a particular differential equation.

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$$V = \pi r^2 h \quad \Rightarrow \quad \Delta v \approx 2\pi r h \Delta r = 10\pi cm^3$$

- In the previous example find the relative error in the volume.

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- Find the percentage error in the volume.

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$$\text{If } Flow \propto r^4,$$
$$\text{10\% change in the radius} \quad \Rightarrow \quad \text{40\%.change in the flow.}$$