Oral Exam Syllabus

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Algebraic Topology

[**Hatcher**]

- Fundamental group
  - Covering spaces, their classification, deck transformations
  - Seifert-van Kampen theorem
- Homology
  - Simplicial and singular homology
  - Exact sequences and excision, the Mayer-Vietoris sequence
  - Cellular homology
  - Homology with coefficients, universal coefficient theorem
- Cohomology
  - Cup product and the cohomology ring
  - Künneth formula
  - Poincaré and Lefschetz duality
  - de Rham cohomology, de Rham's theorem

Riemann Surfaces

[**Donaldson**, **Farkas - Kra**]

- Examples of Riemann surfaces, properties of holomorphic maps
  - Quotients of group actions, algebraic curves, analytic continuation
  - Definition of holomorphicity, significance of meromorphic functions
  - Local behavior of holomorphic maps, degree, the Riemann-Hurwitz formula
  - Monodromy of covering spaces
- Calculus on Riemann surfaces
  - Differential forms, Stokes' theorem
  - Complexified tangent space, meromorphic 1-forms, the residue theorem
  - Poisson equation and its significance, Weyl's lemma
- Major results: Riemann-Roch formula, Abel-Jacobi map, Poincaré-Koebe uniformization, Riemann's existence theorem

Hyperbolic Geometry

[**Hubbard**]

- Plane hyperbolic geometry
  - Hyperbolic metric, geodesics
  - Normal form for automorphisms
  - Curvature of a metric
  - Hyperbolic trigonometry
- Hyperbolic geometry of Riemann surfaces
  - Geometric function theory: Schwarz lemma, length-area method, extremal length, the moduli of quadrilaterals and annuli
  - The hyperbolic metric of a Riemann surface, its properties, properties of geodesics
  - Right-angled hexagons and the trouser decomposition
  - The ideal boundary of a hyperbolic surface
  - Fundamental domain of a group action, Picard's little theorem

References

[1] Simon Donaldson, *Riemann Surfaces*, Oxford University Press, 2011.

[2] Hershel M. Farkas, Irwin Kra, *Riemann Surfaces*, Springer, 1991.

[3] Allen Hatcher, *Algebraic Topology*, Cambridge University Press, 2001.

[4] John H. Hubbard, *Teichmüller theory and Applications to Geometry, Topology, and Dynamics, Volume 1: Teichmüller theory*, Matrix Editions, 2006.