Assume well-separated "modes" in $P(i)$.

instead of

Model $P(i)$ as sum of 2 truncated Gaussians:

$$f(i) = \begin{cases} \dfrac{g_1}{\sigma_1\sqrt{2\pi}}\, e^{-(i-\mu_1)^2/2\sigma_1^2} & \text{if } i \le t \\[2ex] \dfrac{g_2}{\sigma_2\sqrt{2\pi}}\, e^{-(i-\mu_2)^2/2\sigma_2^2} & \text{if } i > t \end{cases}$$

$$H(t) = -\sum_{i=0}^{t} P(i) \log\left[\frac{g_1}{\sigma_1\sqrt{2\pi}}\, e^{-(i-\mu_1)^2/2\sigma_1^2}\right]$$

$$-\sum_{i=t+1}^{255} P(i) \log\left[\frac{g_2}{\sigma_2\sqrt{2\pi}}\, e^{-(i-\mu_2)^2/2\sigma_2^2}\right]$$

$$= -\sum_{i=0}^{t} P(i)\left[\log q_1 - \log \sigma_1 - \log\sqrt{2\pi} - \frac{(i-\mu_1)^2}{2\sigma_1^2}\right]$$

$$-\sum_{i=t+1}^{255} P(i)\left[\log q_2 - \log \sigma_2 - \log\sqrt{2\pi} - \frac{(i-\mu_2)^2}{2\sigma_2^2}\right]$$

$$= -q_1 \log q_1 + q_1 \log \sigma_1 + q_1 \log\sqrt{2\pi} + \frac{1}{2} q_1$$

$$\quad -q_2 \log q_2 + q_2 \log \sigma_2 + q_2 \log\sqrt{2\pi} + \frac{1}{2} q_2$$

$$= \frac{1}{2} + \frac{1}{2}\log 2\pi - q_1 \log q_1 - q_2 \log q_2$$

$$\quad + q_1 \log \sigma_1 + q_2 \log \sigma_2 \quad = H(t)$$

Choose the t that minimizes $H(t)$.

Steps:

1. Compute histogram, $h(i)$
2. Normalize: $P(i) = \dfrac{h(i)}{\#\text{pixels}}$
3. For each t, compute $H(t)$.
4. Pick the t having the smallest $H(t)$.

Recursive Calculation

$$q_1(0) = P(0), \quad q_1(t) = q_1(t-1) + P(t)$$

$$q_2(t) = 1 - q_1(t)$$

$$\mu_1(0) = 0$$

$$\mu_1(t) = \frac{\sum_{i=0}^{t} i P(i)}{q_1(t)}$$

$$= \frac{\sum_{i=0}^{t-1} i P(i) + t P(t)}{q_1(t)}$$

$$= \frac{q_1(t-1)\mu_1(t-1) + t P(t)}{q_1(t)}$$

$$\mu_2(t) = \frac{\mu - q_1(t)\mu_1(t)}{q_2(t)}$$

where $\mu = \sum_{i=0}^{255} i P(i)$

$$\mu_2(0) = \frac{\mu}{q_2(0)}$$

$$\sigma_1^2(0) = 0$$

$$\sigma_1^2(t) = \frac{1}{q_1(t)} \Big(q_1(t-1) \left\{ \sigma_1^2(t-1) + [\mu_1(t-1) - \mu_1(t)]^2 \right\} + P(t)\,[t - \mu_1(t)]^2 \Big)$$

$$\sigma_2^2(0) = \sum_{i=1}^{255} [i - \mu_2(0)]^2 \frac{P(i)}{q_2(0)}$$

$$\sigma_2^2(t) = \frac{1}{q_2(t)} \Big(q_2(t-1) \left\{ \sigma_2^2(t-1) + [\mu_2(t-1) - \mu_2(t)]^2 \right\} - P(t)\,[t - \mu_2(t)]^2 \Big)$$

Note: • Use double-precision for μ, q, σ's

• Problem at ends of histogram:
$\log \sigma_1 = \log 0 \Rightarrow$ bad
Similar prob. at rt end. with σ_2.

Let "first" = index of first-occupied hist. bin.

"last" = index of last-occupied hist bin.

Assume hist[first+1] $\neq 0$

hist[last-1] $\neq 0$

In the iteration, only try candidate threshold values in

$$\text{first}+1 \leq t \leq \text{last}-2$$

If we begin with $t = \text{first} + 1$,

then $\mu_1(\text{first}+1) = \sum_{i=\text{first}}^{\text{first}+1} i\,P(i) / q_1(\text{first}+1)$

where $q_1(\text{First}+1) = P(\text{First}) + P(\text{First}+1)$

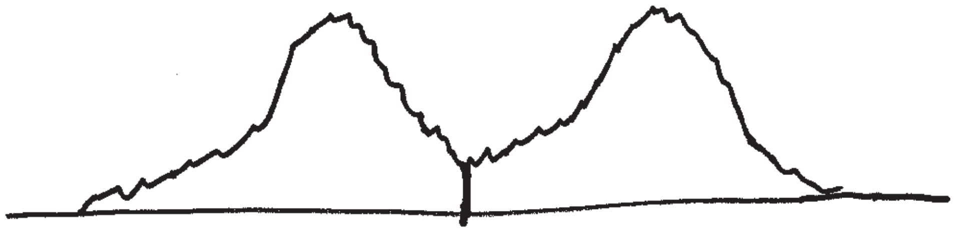
Can determine similar init. for other parms.

Cho, Haralick, & Yi

Pattern Recog., 1989

Added a correction term.

Kittler's model: sum of truncated Gaussians:



μ_1 in Kittler's alg is computed using a distribution whose right tail was truncated. So, μ_1 is biased too small.

Similarly, μ_2 is biased too large.

Cho's correction terms:

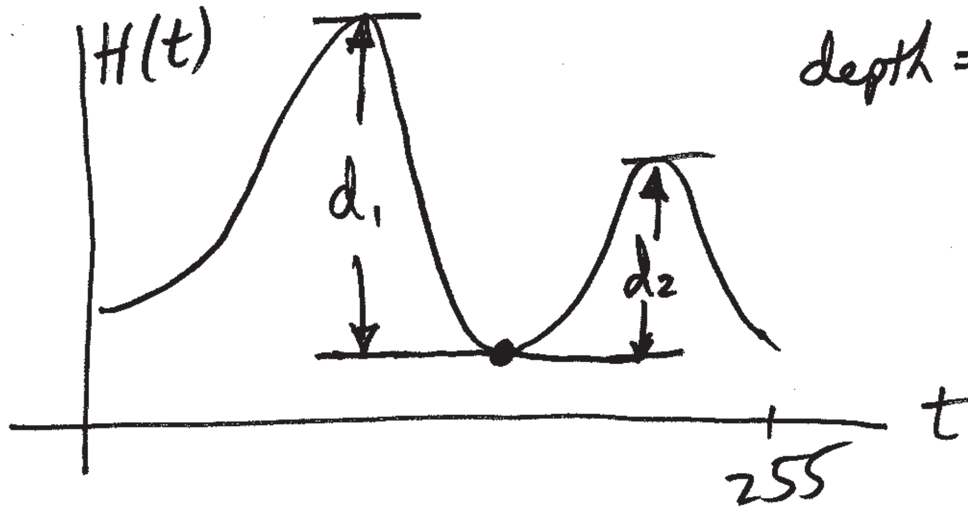
$$\hat{\mu}_1 = \mu_1 + \frac{\sigma_1}{\phi\left(\frac{t-\mu_1}{\sigma_1}\right)} \cdot \frac{1}{\sqrt{2\pi}} e^{-(t-\mu_1)^2/2\sigma_1^2}$$

where
$$\phi(x) = 1 - \mathrm{erfc}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-u^2/2}\, du$$

$$\hat{\mu}_2 = \mu_2 - \frac{\sigma_2}{1-\phi\left(\frac{t-\mu_2}{\sigma_2}\right)} \frac{1}{\sqrt{2\pi}} e^{-(t-\mu_2)^2/2\sigma_2^2}$$

Cho: Minimum Finding

For more robustness, they found the deepest valley instead of the global min. of $H(t)$.



$$\text{depth} = \frac{d_1 + d_2}{2}$$

Multi-Level Thresholding

See Kittler.

Other Histogram-Based Segm. Methods

Li, Gong, & Chen

Pattern Recog, 1997

They found that a hist. of local ave. gray level gave better thresholding performance.

Chow & Kaneko

1972 — See Haralick & Shapiro, Computer & Robot Vision, v. 1.

Local histogramming (adaptive thresh.)

- divide img into disjoint blocks (e.g., 33x33 or 65x65)
- compute local hist ∀ block
- determine a thresh. for each block
- Associate the thresholds with the block centers.
- Interpolate the thresholds. to get thresholds for other pixels.

Weszka J.S, R.N. Nagel, & A. Rosenfeld

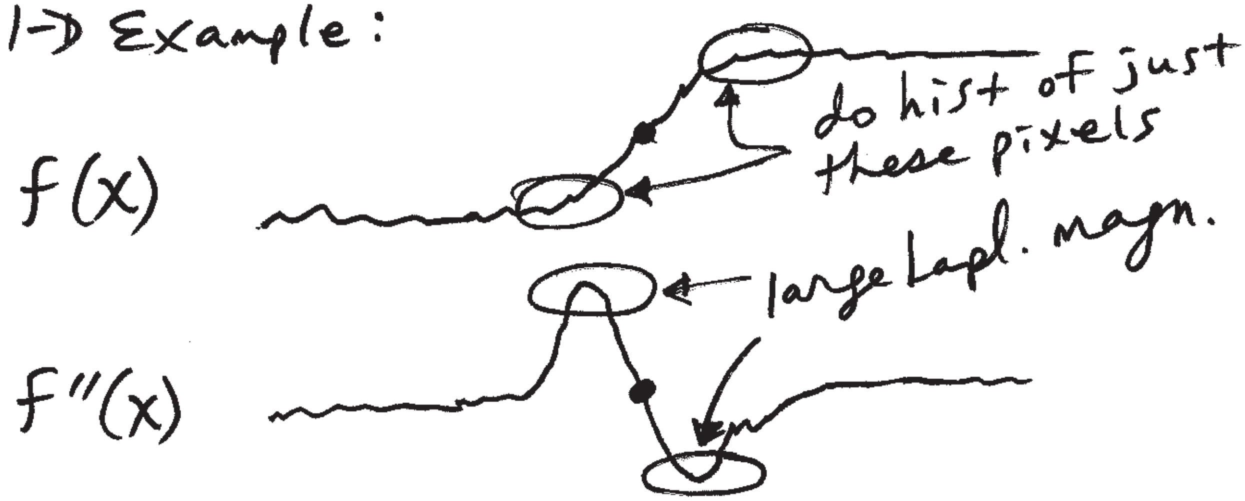
"A Threshold Selection Techn."

IEEE T-Computers v. C-23, 1974, pp 1322-6.

Laplacian-based method.

Do hist. thresholding for only the pixels with large Laplacian magnitude.

1-D Example:



- This includes the pixels adj. to the edges, but not the edges themselves.
- The resulting hist should have a sharper valley.

• This will involve approx equal numbers of fg & bg pixels, making the modes about the same size.

Watanabe, S.

CGIP, v. 3, 1974, pp 350-8.

Gradient-based scheme.

Choose the threshold that maximizes the sum of gradients at all pixels whose gray level equal the thresh.

→ i.e., grad. magn.

Weska & Rosenfeld

T-SMC, v. 8, 1978, p 622-9.

Busyness-based scheme.

• ∀ candidate thresh, compute busyness = % of pixels that are border pixels (either fg or bg border pixels).

f(x)

• Choose the thresh that minimized busyness.

Kohler's Method

"A Segm. Sys. Based on Thresholding," CGIP, v. 15, 1981, pp 319–38.

Contrast-based scheme.

- Let T = thresh.
- Define edges to be

$$E(T) = \{[(i,j),(k,l)]:$$

pixels (i,j) & (k,l) are neighbors, and $\min\{I(i,j), I(k,l)\} \le T \le \max\{I(i,j), I(k,l)\}\}$

- Define total contrast to be

$$C(T) = \sum_{\substack{[(i,j),(k,l)] \\ \in E(T)}} \min\{|I(i,j)-T|, |I(k,l)-T|\}$$

- Ave contrast of edges detected by threshold T is $\dfrac{C(T)}{\#E(T)}$

- Choose the thresh that maximizes ave. contrast.