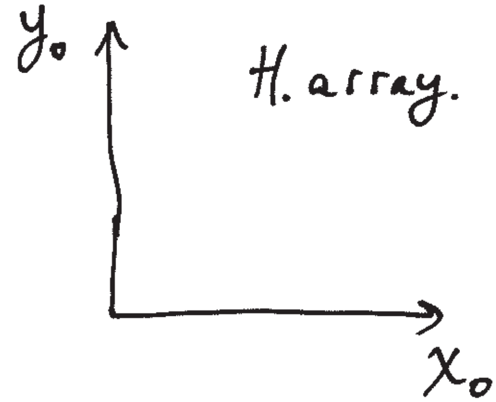
For each edge pt $(x_i, y_i)$ in the edge map, increment the bins at $(x_i + R_{x_j}, y_i + R_{y_j})$ in the Hough array, for each $(R_{x_j}, R_{y_j})$ in the R-table.

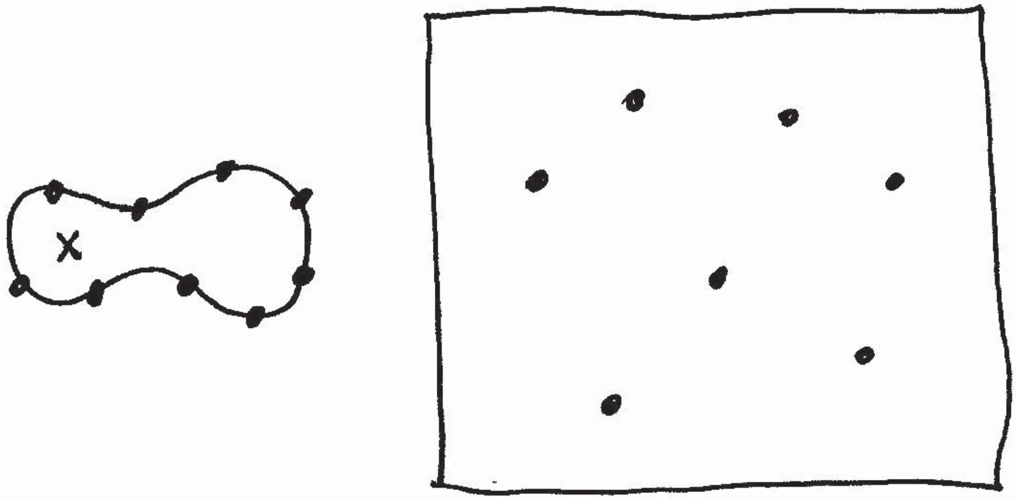


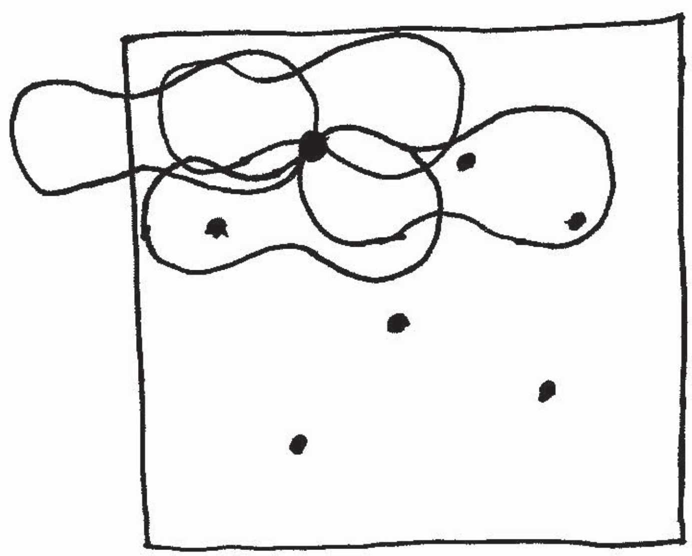

I.e.,

For each $(x_i, y_i)$:

$$x_0 = x_i + R_{x_j}$$
$$y_0 = y_i + R_{y_j}$$
$$H(x_0, y_0)\,{+}{+}$$

Afterward, we threshold the H. array and do NMS to find the ref. pts. for the strongest detected shapes.

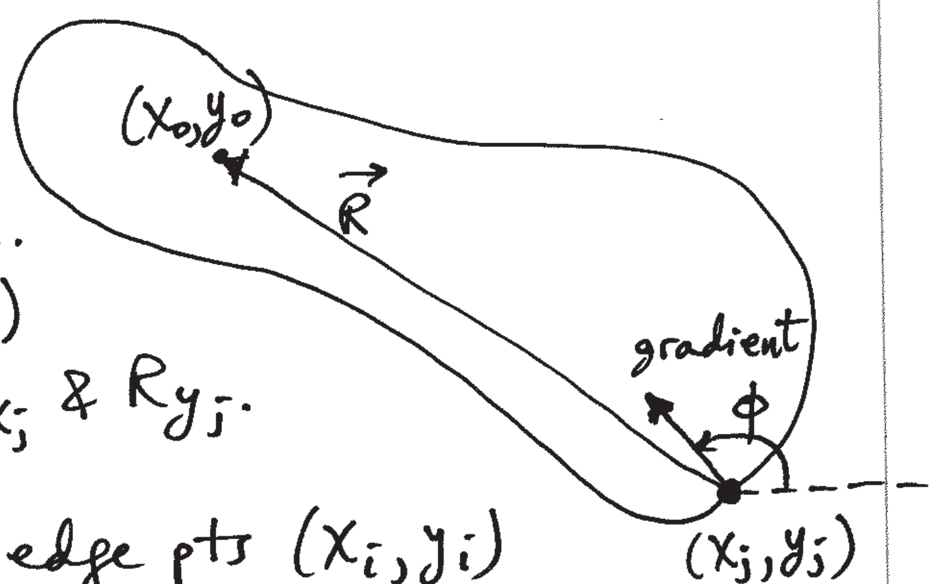

X



## Ballard's Alg (1981):

Assume we know edge orient. or grad. dir. from the edge detector.

When we build the R-table, list the grad. dir. $\phi(x_j, y_j)$ along with $R_{x_j}$ & $R_{y_j}$.




Then scan all edge pts $(x_i, y_i)$ as before, calculating the corresponding $(x_0, y_0)$ each time. But we will only incr. those entries in the R-table that correspond to cases where $\phi(x_j, y_j)$ matches the estimated grad. dir $\theta_i$ at edge pt $(x_i, y_i)$.

Note: Can extend to arbitrary scale & orient. See Sonka.

# Image Segmentation Example 1

(from https://www.mathworks.com/help/images/ref/graythresh.html)

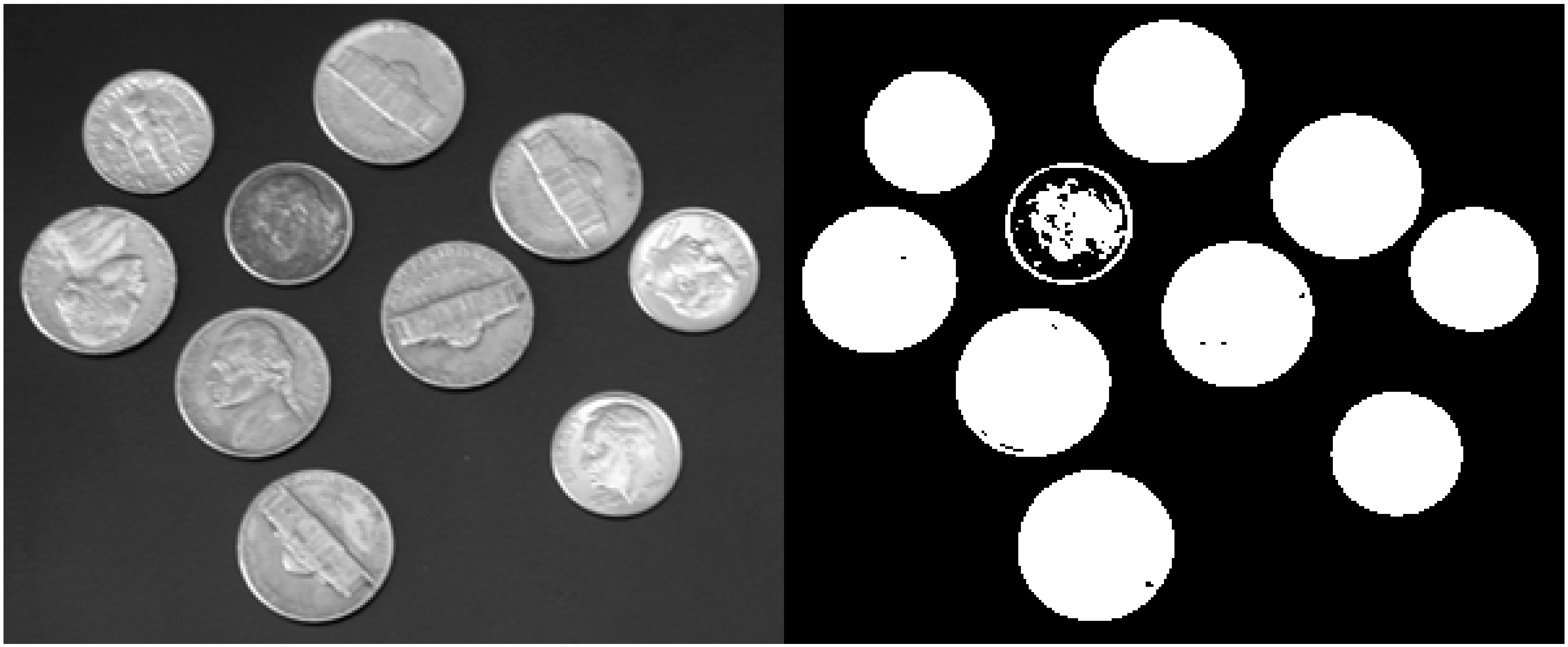
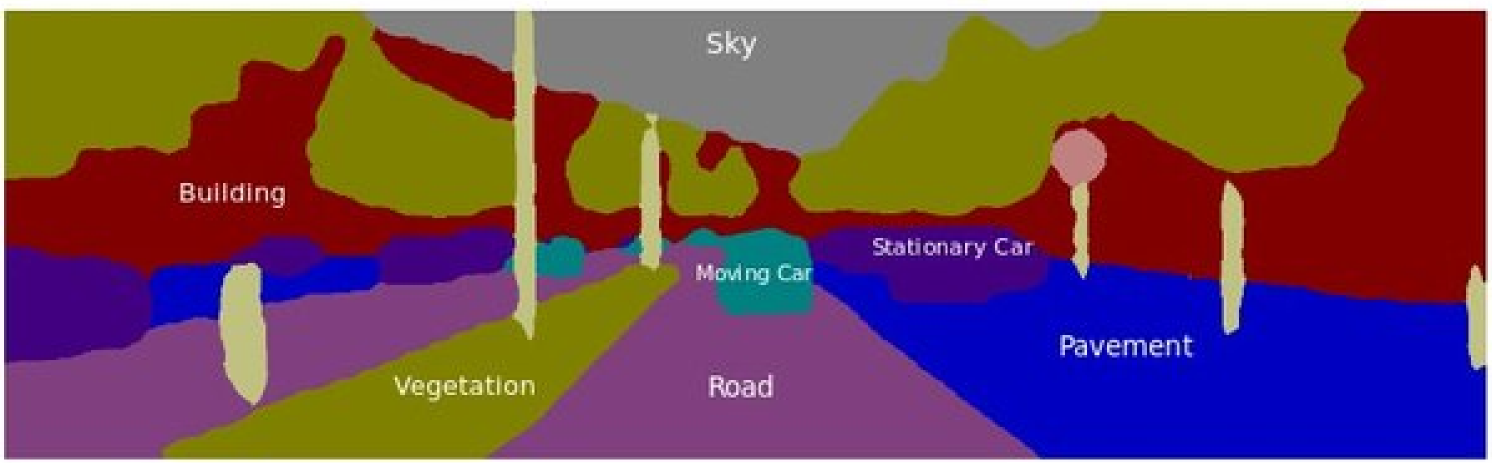


Image Segmentation Example 2

(from https://arxiv.org/pdf/1704.08331.pdf;
https://www.researchgate.net/publication/316538613_Joint_Semantic_and_Motion_Segmentation_for_dynamic_scenes_using_Deep_Convolutional_Networks/figures?lo=1)



# Image Segmentation

Img. segm. is the dual of edge det.

Goal: To partition the (grayscale) image into homogeneous regions.

Def: $\{A_K ; K=1,\dots,N\}$ is a partition of $A$ if

$$A_i \cap A_j = \phi \quad \text{for } i \neq j$$

and

$$\bigcup_{K=1}^{N} A_K = A$$

$A$: all pixels in img

$A_K$: all pixels in region $K$

Segmentation into Homog. Regions:

Let $\mathcal{H}$ be some homogeneity measure (e.g., intensity similarity, color similarity, texture similarity).

Then a segm. of img $A$ is a partition $\{A_K\}$ satisfying:

(1) $\mathcal{H}[A_k] = \text{True} \quad \forall k$

(2) $A_i$ adj. $A_j \Rightarrow \mathcal{H}[A_i \cup A_j] = \text{False}$ for $i \neq j$

(3) $A_k$ is connected (optional)

Also, it would be nice if

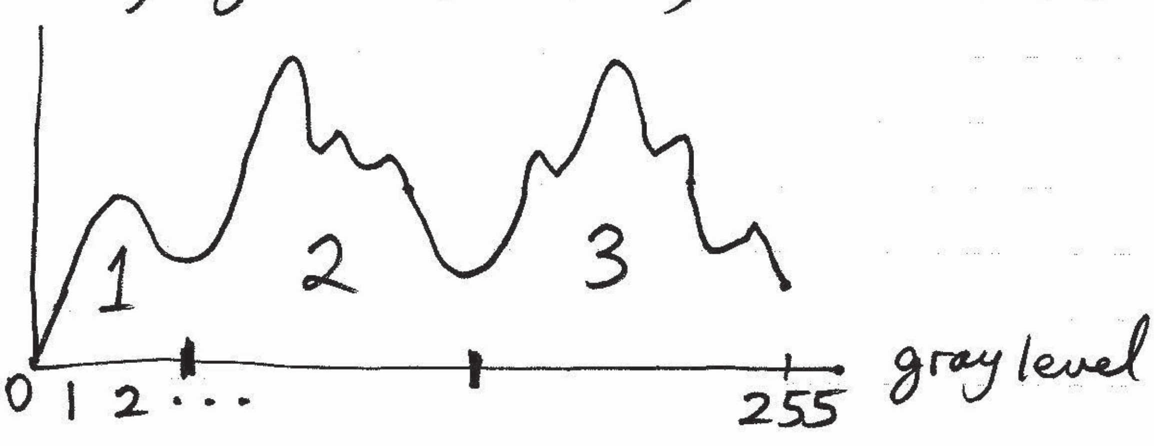
(1) regions don't have too many small holes

(2) region boundaries are not too ragged

(3) the region boundaries are consistent with actual (physical) boundaries between material surfaces.

## Thresholding (See Sonka 6.1)

Assume a grayscale img.
Assume gray level similarity is the $\mathcal{H}$. test.



Problems:

(1) Number of clusters in histogram is not clear (2 or 3 or 8?).

(2) Optimal thresh locations not clear.

(3) Pixels with similar gray level may not be connected in spatial domain. (disconnected regions)

## Sonka's Alg. 6.2

"Iterative threshold selection"

Assume 2 regions: fg & bg.

Assume fg is near the center of the img & is surrounded by bg.

Step 1: Initialize the bg region to be the 4 corners of the img (or something similar). Let the fg region be the remainder.

Step 2: Compute

$\mu_{bg}^{k}$ = mean bg gray level

$\mu_{Fg}^{k}$ = mean Fg gray level

k = iteration number

$$\mu_{bg}^{k} = \frac{\sum_{(m,n)\in bg} f(m,n)}{\# \text{ bg pixels}}$$

$$\mu_{Fg}^{k} = \frac{\sum_{(m,n)\in Fg} f(m,n)}{\# \text{ Fg pixels}}$$

Step 3: Compute the next threshold.

$$T^{k+1} = \frac{\mu_{bg}^{k} + \mu_{Fg}^{k}}{2}$$

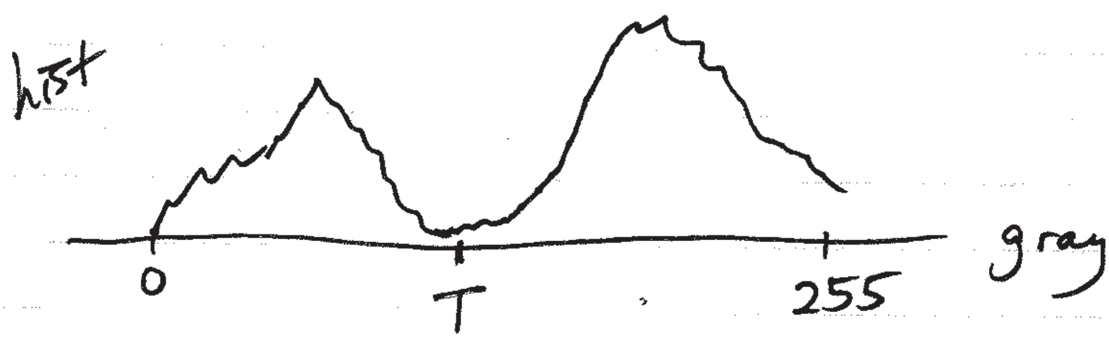
Step 4: If $T^{k+1} = T^{k}$ then stop, else return to Step 2 with the newly thresholded bg & Fg regions.

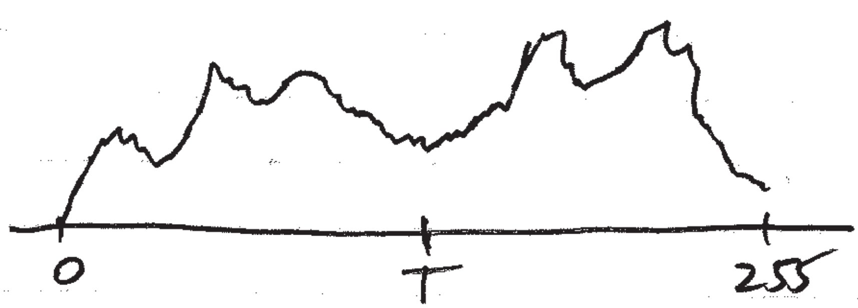
# Kittler & Illingworth's Optimal Thresholding Alg.

J. Kittler & J. Illingworth, "Minimum Error Thresholding," Pattern Recognition, v. 19, no. 1, 1986, pp. 41-47.

C.A. Glasbey, "An Analysis of Histogram-Based Thresholding Algs," CVGIP: Graphical Models & Img. Proc., v. 55, no. 6, Nov 93, pp 532-37. [compared 11 algs & found Kittler was best.]

Assume a bimodal histogram.
Find optimal threshold to partition the hist. into 2 clusters.



Model the hist. as the sum of 2 Gaussians:

$$f(i) = \frac{g_1}{\sigma_1\sqrt{2\pi}} e^{-(i-\mu_1)^2/2\sigma_1^2} + \frac{g_2}{\sigma_2\sqrt{2\pi}} e^{-(i-\mu_2)/2\sigma_2^2}$$

Here, $g_1$ & $g_2$ give us the relative mixture of fg & bg.

Let the observed "probability distr." be

$$P(i) = \frac{h(i)}{\text{total \# pixels}} \Rightarrow \sum P(i) = 1$$

where $h(i)$ = histogram.

Let $t$ = candidate threshold.
This defines a left side & right side of the observed hist. Do curve fitting.

$$q_1(t) = \sum_{i=0}^{t} P(i)$$

$$q_2(t) = \sum_{i=t+1}^{255} P(i) = 1 - q_1(t)$$

$$\mu_1(t) = \sum_{i=0}^{t} i\, P(i) / q_1(t)$$

$$\mu_2(t) = \sum_{i=t+1}^{255} i\, P(i) / q_2(t)$$

$$\sigma_1^2(t) = \sum_{i=0}^{t} [i - \mu_1(t)]^2 P(i) / q_1(t)$$

$$\sigma_2^2(t) = \sum_{i=t+1}^{255} [i - \mu_2(t)]^2 P(i) / q_2(t)$$

Minimize the Kullback information distance (min. Bayesian classif. error):

$$J = \sum_{i=0}^{255} P(i) \log \frac{P(i)}{f(i)} \quad \begin{matrix} \leftarrow \text{actual} \\ \leftarrow \text{model} \end{matrix}$$

Note: $J \geq 0$ for all prob. distr. ~~fcts.~~
and $J = 0$ iff $P = f$.

Find $t$ such that $J$ is min.

$$J = \sum_{i=0}^{255} P(i) \log P(i)$$ does not vary with $t$

$$- \sum_{i=0}^{255} P(i) \log f(i)$$ varies as $t$ varies

So, find $t$ that minimizes

$$H(t) = - \sum_{i=0}^{255} P(i) \log f(i)$$