

The R names for the probability distributions are given in **RED**. That is, these names can be used after the letters **d**, **p**, **q** and **r** to find the density (PDF), distribution function (CDF), quantile function (inverse CDF), and random generation for a specified probability distribution.

### Discrete Distributions

#### 1. Bernoulli Distribution

$$\begin{aligned} X &\sim \text{Bernoulli}(\pi) \\ P(X = x|\pi) &= \pi^x(1 - \pi)^{1-x}, \quad x = 0, 1 \\ E[X] &= \pi \\ \text{Var}[X] &= \pi(1 - \pi) \\ M_X(t) &= \pi e^t + \varrho \end{aligned}$$

#### 2. Binomial Distribution (binom)

$$\begin{aligned} X &\sim \text{Bin}(n, \pi) \\ P(X = x|n, \pi) &= \binom{n}{x} \pi^x (1 - \pi)^{n-x}, \quad x = 0, 1, 2, \dots, n. \\ E[X] &= n\pi \\ \text{Var}[X] &= n\pi(1 - \pi) \\ M_X(t) &= (\pi e^t + \varrho)^n \end{aligned}$$

#### 3. Poisson Distribution (pois)

$$\begin{aligned} X &\sim \text{Pois}(\lambda) \\ P(X = x|\lambda) &= \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, \dots \\ E[X] &= \lambda \\ \text{Var}[X] &= \lambda \\ M_X(t) &= e^{\lambda(e^t - 1)} \end{aligned}$$

#### 4. Geometric Distribution (geom)

$$\begin{aligned} X &\sim \text{Geo}(\pi) \\ P(X = x|\pi) &= \pi \varrho^x, \quad x = 0, 1, \dots \\ E[X] &= \frac{\varrho}{\pi} \\ \text{Var}[X] &= \frac{\varrho}{\pi^2} \\ M_X(t) &= \frac{\pi}{1 - \varrho e^t} \end{aligned}$$

#### 5. Negative Binomial Distribution (nbinom)

$$\begin{aligned} X &\sim \text{NB}(r, \pi) \\ P(X = x|r, \pi) &= \binom{x+r-1}{r-1} \pi^r \varrho^x, \quad x = 0, 1, 2, \dots \\ E[X] &= r \frac{\varrho}{\pi} \\ \text{Var}[X] &= r \frac{\varrho}{\pi^2} \\ M_X(t) &= \pi^r (1 - \varrho e^t)^{-r} \end{aligned}$$

#### 6. Hypergeometric Distribution (hyper)

$$\begin{aligned} X &\sim \text{Hyper}(m, n, k) \\ P(X = x|m, n, k) &= \frac{\binom{m}{x} \binom{n}{k-x}}{\binom{N}{k}}, \end{aligned}$$

for  $x = \max\{0, k - n\}, \dots, \min\{m, k\}$ , where  $N = m + n$

$$\begin{aligned} E[X] &= \frac{m \times k}{N} \\ \text{Var}[X] &= \frac{m \times n \times k \times (N - k)}{N^2 \times (N - 1)} \end{aligned}$$

### Continuous Distributions

#### 7. Uniform Distribution (unif)

$$\begin{aligned} X &\sim \text{Unif}(a, b) \\ f(x|a, b) &= \frac{1}{b - a}, \quad a \leq x \leq b \\ E[X] &= \frac{b + a}{2} \\ \text{Var}[X] &= \frac{(b - a)^2}{12} \\ M_X(t) &= \begin{cases} \frac{e^{tb} - e^{ta}}{t(b - a)} & \text{if } t \neq 0 \\ 1 & \text{if } t = 0 \end{cases} \end{aligned}$$

#### 8. Exponential Distribution (exp)

$$\begin{aligned} X &\sim \text{Exp}(\lambda) \\ f(x|\lambda) &= \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} \\ E[X] &= \frac{1}{\lambda} \\ \text{Var}[X] &= \frac{1}{\lambda^2} \\ M_X(t) &= (1 - \lambda^{-1}t)^{-1} \text{ for } t < \lambda \end{aligned}$$

#### 9. Gamma Distribution (gamma)

$$\begin{aligned} X &\sim \Gamma(\alpha, \lambda) \\ f(x|\alpha, \lambda) &= \begin{cases} \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} \\ E[X] &= \frac{\alpha}{\lambda} \\ \text{Var}[X] &= \frac{\alpha}{\lambda^2} \\ M_X(t) &= (1 - \lambda^{-1}t)^{-\alpha} \text{ for } t < \lambda \end{aligned}$$

#### 10. Normal Distribution (norm)

$$\begin{aligned} X &\sim N(\mu, \sigma) \\ f(x|\mu, \sigma) &= \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty, \\ &\text{where } -\infty < \mu < \infty \text{ and } 0 < \sigma < \infty. \\ E[X] &= \mu \\ \text{Var}[X] &= \sigma^2 \\ M_X(t) &= e^{\mu t + \frac{\sigma^2 t^2}{2}} \end{aligned}$$

Distributions Associated with the Normal Distribuiton
---

11. **Chi-Square Distribution** (chisq)

$$X \sim \chi_n^2$$

$$f(x) = \begin{cases} \frac{1}{\Gamma\left(\frac{n}{2}\right) 2^{\frac{n}{2}}} \cdot x^{\frac{n}{2}-1} e^{-\frac{x}{2}} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

$$E[X] = n$$

$$Var[X] = 2n$$

$$M_X(t) = (1 - 2t)^{-\frac{n}{2}} \text{ for } t < \frac{1}{2}$$

12. **t-Distribution** (t)

$$X \sim t_\nu$$

$$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi\nu}\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}} \text{ for } -\infty < x < \infty$$

$$E[X] = 0$$

$$Var[X] = \frac{\nu}{\nu-2} \text{ for } \nu > 2$$

13. **F-Distribution** (f)

$$X \sim F_{\nu_1, \nu_2}$$

$$f(x) = \frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right)}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)} \left(\frac{\nu_1}{\nu_2}\right)^{\frac{\nu_1}{2}} x^{\frac{\nu_1}{2}-1} \left(1 + \frac{\nu_1}{\nu_2}x\right)^{-\frac{1}{2}(\nu_1+\nu_2)}, \quad x > 0$$

$$E[X] = \frac{\nu_2}{\nu_2 - 2}$$

$$Var[X] = \frac{2\nu_2^2(\nu_1 + \nu_2 - 2)}{\nu_1(\nu_2 - 2)^2(\nu_2 - 4)} \text{ provided } \nu_2 > 4$$