MATH S1202: Calculus IV
Quiz 4
June 16, 2016

Define curves in the following way.

- Let D be the region *in the upper half xy-plane* between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

- Let C denote the boundary of D with the positive orientation. (Note that C consists of 4 arcs.)

- Let C_1 denote the piece of C lying along the circle $x^2 + y^2 = 4$.

1. Determine whether the vector field is conservative *and* if it is conservative, find a potential function.

(a) $\vec{F}(x, y) = (x^3, y^3)$

(b) $\vec{G}(x, y) = (-y^3, x^3)$

2. Compute the line integral of the vector field over the curve C_1.

(a) $\vec{F}(x, y) = (x^3, y^3)$

(b) $\vec{G}(x, y) = (-y^3, x^3)$

Hint: For (b), you may use the trig identity

$$\sin^4 t + \cos^4 t = \frac{1}{4} \left(\cos(4t) + 3 \right)$$

3. Compute the line integral of the vector field over the closed curve C.

(a) $\vec{F}(x, y) = (x^3, y^3)$

(b) $\vec{G}(x, y) = (-y^3, x^3)$