MATH 2610-01

Take-Home #1

Due February 11, 2022

1. Snow started falling at 5am and continued to fall at a constant rate. A snowplow departed its garage at 6am, and another departed on the same route at 7am. At what time did the snowplows crash? Assume the rate at which a snowplow can clear the snow is inversely proportional to the height of the snow.

2. Consider the IVP

$$\begin{cases} y' = 5y^{4/5} \\ y(3) = 1 \end{cases}.$$

Mary makes the following claim: For each real number $a \leqslant 1$, there is a solution φ_a to the IVP satisfying

$$\varphi_a(a) = -1.$$

Is she correct?

3. The position x of a particle satisfies the differential equation

$$\frac{dx}{dt} = t^2 - x^2 + 6x - 9.$$

If the particle is located at $x = 5$ when $t = 3.5$, can it reach the location $x = 4$ at a later time? (Hint: I am seeking a thorough and detailed proof, and the most thorough way to show that something is impossible is to show that its occurrence would lead to a contradiction.)