MATH 2610-01 **Name:** ____________________

Spring 2022

Midterm 2

March 18, 2022

Time Limit: 50 minutes

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
Total	70	

1. [10 points] Solve the initial value problem

$$\begin{cases} y'' - 5y' + 6y = 0 \\ y(0) = 3 \\ y'(0) = 5 \end{cases}$$

2. [10 points] For the equation

$$(D+I)(D-3I)^2(D^2+2D+5I)[y] = f(t),$$

complete the table with an appropriate guess $\varphi(t)$ for a particular solution corresponding to the given right-hand side $f(t)$.

$f(t)$	$\varphi(t)$
$7t^2\cos(2t)$	$(A_2t^2 + A_1t + A_0)\cos(2t) + (B_2t^2 + B_1t + B_0)\sin(2t)$
$-te^{-t}\sin(2t)$	
$2t^3e^{-t}$	
$\pi te^{3t} + 1$	
t^2	
$(t-3)^2e^{-3t}\sin(2t)$	

3. [**10 points**] Find a particular solution to

$$t^3 y''' = t^2, \qquad\qquad t > 0$$

given that $\{1, t, t^2\}$ solve the corresponding homogeneous equation.

4. [**10 points**] Find a solution $\varphi(t)$ to the non-linear autonomous equation

$$y'' = 12y^{5/3}$$

satisfying $\varphi(1) = 1$.

5. [10 points] In my recent research, I have encountered the linear differential operator L defined by

$$L[y](t) = \frac{d^2}{dt^2}\big((1+t)y(t)\big).$$

Determine, with complete reasoning, the largest interval on which you can guarantee there exists a unique solution φ to the equation

$$L[y](t) = \frac{1}{9-t^2}$$

satisfying

$$\varphi(0.5) = 1 \qquad \text{and} \qquad \varphi'(0.5) = -7.$$

6. [**10 points**] With Chief Engineer La Forge currently abducted by the Borg, the Starship *Enterprise* is immobile. Fortunately, you have found La Forge's notebook, where it is indicated that the propulsion mechanism of the starship involves a third-order constant-coefficient homogeneous equation. In the notebook, it is written that

$$\begin{aligned} y_1(t) &= 9\pi e^{4t} - \blacksquare e^{\blacksquare t}\sin(3t) \\ y_2(t) &= \blacksquare e^{2t}\cos(\blacksquare t) - 5\pi e^{\blacksquare t} \\ y_3(t) &= \blacksquare e^{4t} \end{aligned}$$

all solve a third-order constant-coefficient equation

$$\blacksquare\, y''' - 16y'' + \blacksquare\, y' - 104y = 0, \tag{1}$$

but many of the numbers are smudged out. Determine the equation (1) anyway.

7. [10 points] All that is known about a mysterious second-order linear equation

$$y'' + p(t)y' + q(t)y = g(t)$$

is that three solutions are

$$\begin{aligned}\varphi_1(t) &= \cos(2t) + e^{3t}\\ \varphi_2(t) &= t + e^{3t}\\ \varphi_3(t) &= \cos(2t) + t + e^{3t}.\end{aligned}$$

Using this, solve the IVP

$$\begin{cases} y'' + py' + qy = g(t) \\ y(0) = 0 \\ y'(0) = 5 \end{cases}.$$