MATH S1202: Calculus IV
Homework 10
Due: June 27

1. Show that

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}.$$

(Hint: First compute $\iint_{\mathbb{R}^2} e^{-x^2-y^2}\,dA$ by switching to polar coordinates.)

2. Let C be the portion of the parabola $y = x^2$ starting at 0 and ending at $1+\sqrt{-1}$. Compute the contour integral $\int_C z^2\,dz$.

3. Compute the contour integral

$$\int_C \frac{1}{z-a}\,dz$$

where C is the circle of radius r centered at $a \in \mathbb{C}$, parameterized in a counter-clockwise fashion.

4. More generally, compute the contour integral

$$\int_C (z-a)^n\,dz$$

for each integer n, where C is as in Problem 3.

5. Let C be the circle of radius $1/2$ centered at the origin, parametrized counterclockwise. Compute

$$\int_C \frac{1}{z^2+1}\,dz.$$

(Hint: Cauchy's theorem.)

6. Let C be the circle of radius $1/2$ centered about $\sqrt{-1}$, parametrized counterclockwise. Compute

$$\int_C \frac{1}{z^2+1}\,dz.$$

(Hint: It might be useful to use a partial fraction decomposition of the integrand. Cauchy's theorem might also be useful.)

7. Let C be the circle of radius 2 centered about the origin, parametrized counterclockwise. Compute

$$\int_C \frac{1}{z^2+1}\,dz.$$

(Hint: See the first part of the Hint of Problem 6.)