MATH S1202: Calculus IV
Quiz 5
June 21, 2018

1. Let E be the solid region above the xy-plane, inside the cylinder $x^2 + z^2 = 9$, and bounded by the planes $y = -4$ and $y = 4$. Let S_1 be the portion of the boundary of E lying along the cylinder $x^2 + z^2 = 9$.

(a) Use cylindrical coordinates about the y-axis with fixed radius $r = 3$ to give a parametrization $\mathbf{r}(\theta, y)$ of S_1. In particular, specify the domain D of your parametrization $\mathbf{r}(\theta, y)$.

(b) Find the area of S_1 using (a).

(c) If a thin sheet occupies S_1 with density $\rho(x, y, z) = 1$, find the z-coordinate $\bar{z}$ of the center of mass $(\bar{x}, \bar{y}, \bar{z})$.

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2. Let S_1 be as above and equip S_1 with the induced orientation as a portion of the boundary of the solid region E. Let $\mathbf{F}$ be the vector field $\mathbf{F}(x, y, z) = (x, x + y, z)$.

(a) Compute the flux of $\mathbf{F}$ across S_1. (Hint: Pay attention to orientation. In particular, is the parametrization $\mathbf{r}(\theta, y)$ of S_1 you gave in Problem 1 compatible with the orientation specified on S_1? If not, you may need a minus sign somewhere.)

(b) Let C_1 be the boundary curve of S_1 with induced orientation. Compute the work done by $\mathbf{F}$ on a particle traveling along C_1. (Hint: Again, pay attention to the orientation of C_1.)