MATH S1202: Calculus IV **Name:** ____________________

Summer 2018

Final Exam

June 28, 2018

Time Limit: 95 minutes

Question	Points	Score
1	10	
2	20	
3	30	
4	40	
Total	100	

1. [10 points] Let C be a circle of radius R centered about the point (a, b) in the xy-plane. If f is the function $f(x, y) = x + y + 1$, compute the line integral $\int_C f\, ds$.

2. **[20 points]** Let $\mathbf{G}$ denote the vector field $\mathbf{G}(x, y, z) = (2(x + y), 2(x + y) + e^z, ye^z)$ defined on $\mathbb{R}^3$.

(a) **[10 points]** If $\mathbf{G}$ is conservative, find a function g such that $\nabla g = \mathbf{G}$. If $\mathbf{G}$ is not conservative, explain why not.

(b) [10 points] Compute the work done by $\mathbf{G}$ on a particle moving along $\mathbf{r}(t) = (\sin(t), \cos(t), t)$ for $0 \leqslant t \leqslant 2\pi$.

3. **[30 points]** Let E be the solid region under the paraboloid $z = 1 - x^2 - y^2$ lying in the first octant. Let S denote the boundary of E with the induced orientation.

(a) **[10 points]** Compute the flux of the vector field $\mathbf{H}(x, y, z) = (x, y, z - x^2 - y^2)$ over S.

(b) [10 points] Let S_1 denote the portion of the boundary of E lying along the paraboloid $z = 1 - x^2 - y^2$ with induced orientation. Compute the flux of $\mathbf{H}(x, y, z) = (x, y, z - x^2 - y^2)$ over S_1.

(c) [10 points] Let C_1 denote the boundary of S_1 with the corresponding induced orientation. (Note that C_1 consists of 3 arcs.) If $\mathbf{H}$ is the vector field $\mathbf{H}(x, y, z) = (x, y, z - x^2 - y^2)$, compute the work done by $\mathbf{H}$ on a particle moving along C_1.

4. [40 points] Let S_2 denote the portion of the cylinder $x^2 + y^2 - 4y = 0$ lying between the planes $z = 0$ and $z = y$.

(a) [10 points] Compute the area of S_2.

(b) [10 points] If a thin sheet occupies the surface S_2 with constant density $\rho(x, y, z) = c$, then compute the x-coordinate $\bar{x}$ of the center of mass.

(c) **[10 points]** Find an equation of the tangent plane to S_2 at the point $(0, 4, 1)$.

(d) [10 points] If $\mathbf{F}$ is the vector field $\mathbf{F}(x, y, z) = (2yz - 2z, 0, xy)$, then compute the flux $\iint_{S_2} \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$, where S_2 is oriented so that the unit normal vector points *away* from the inside of the cylinder $x^2+y^2-4y = 0$.