Probabilistic Program Analysis

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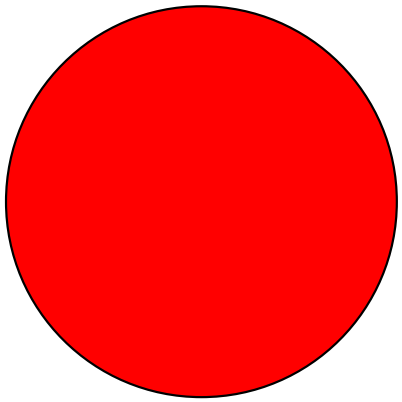
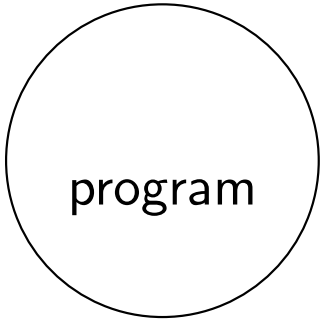
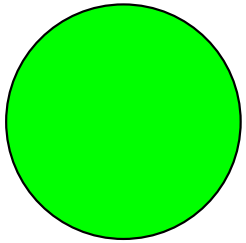
Program Analysis in a nutshell



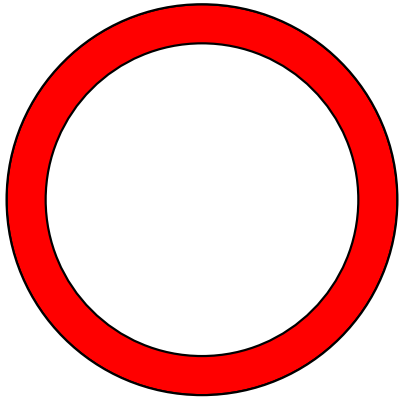
over-approximation



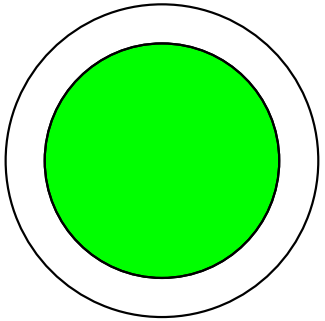


under-approximation

Program Analysis in a nutshell

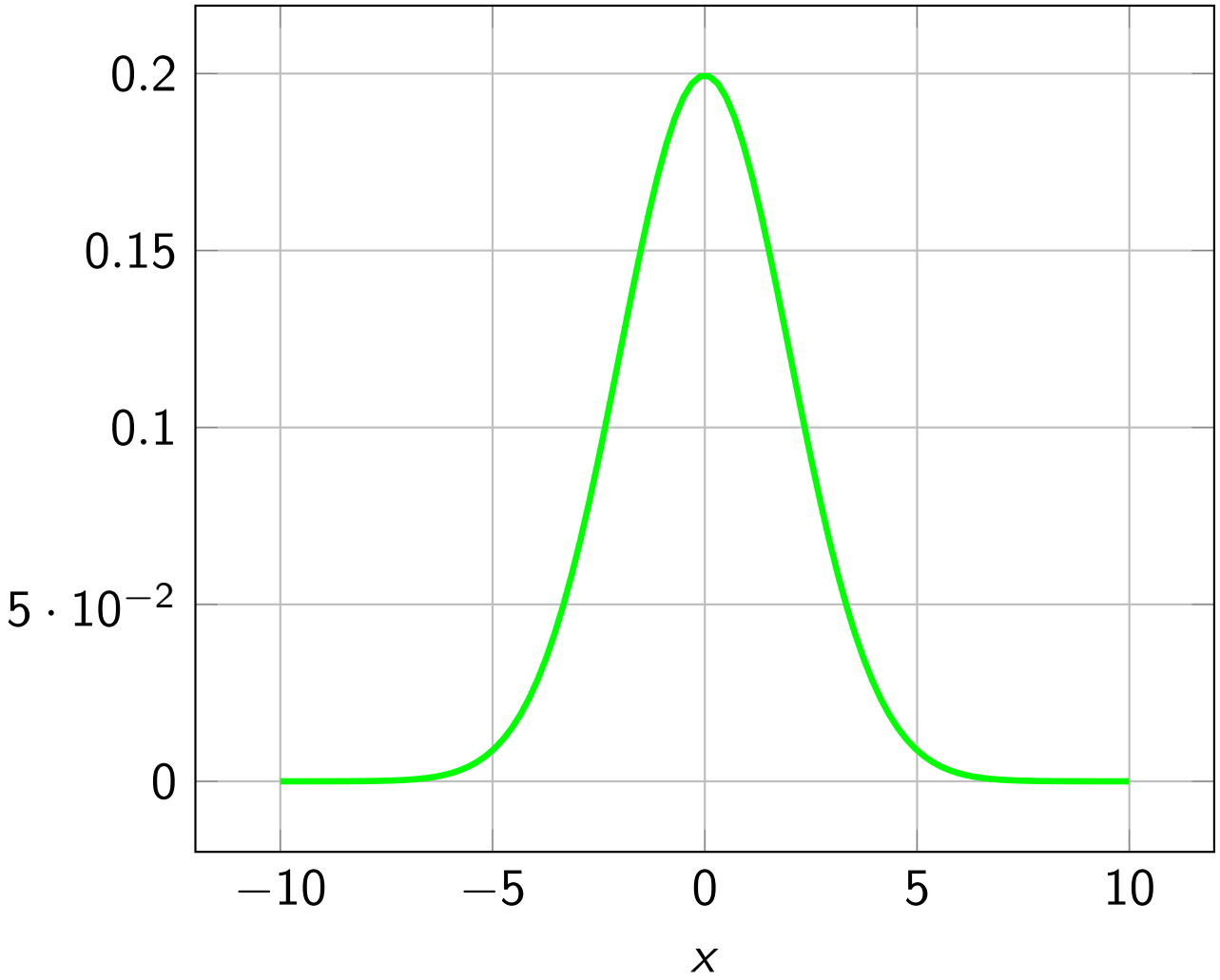


extra behavior



missing behavior

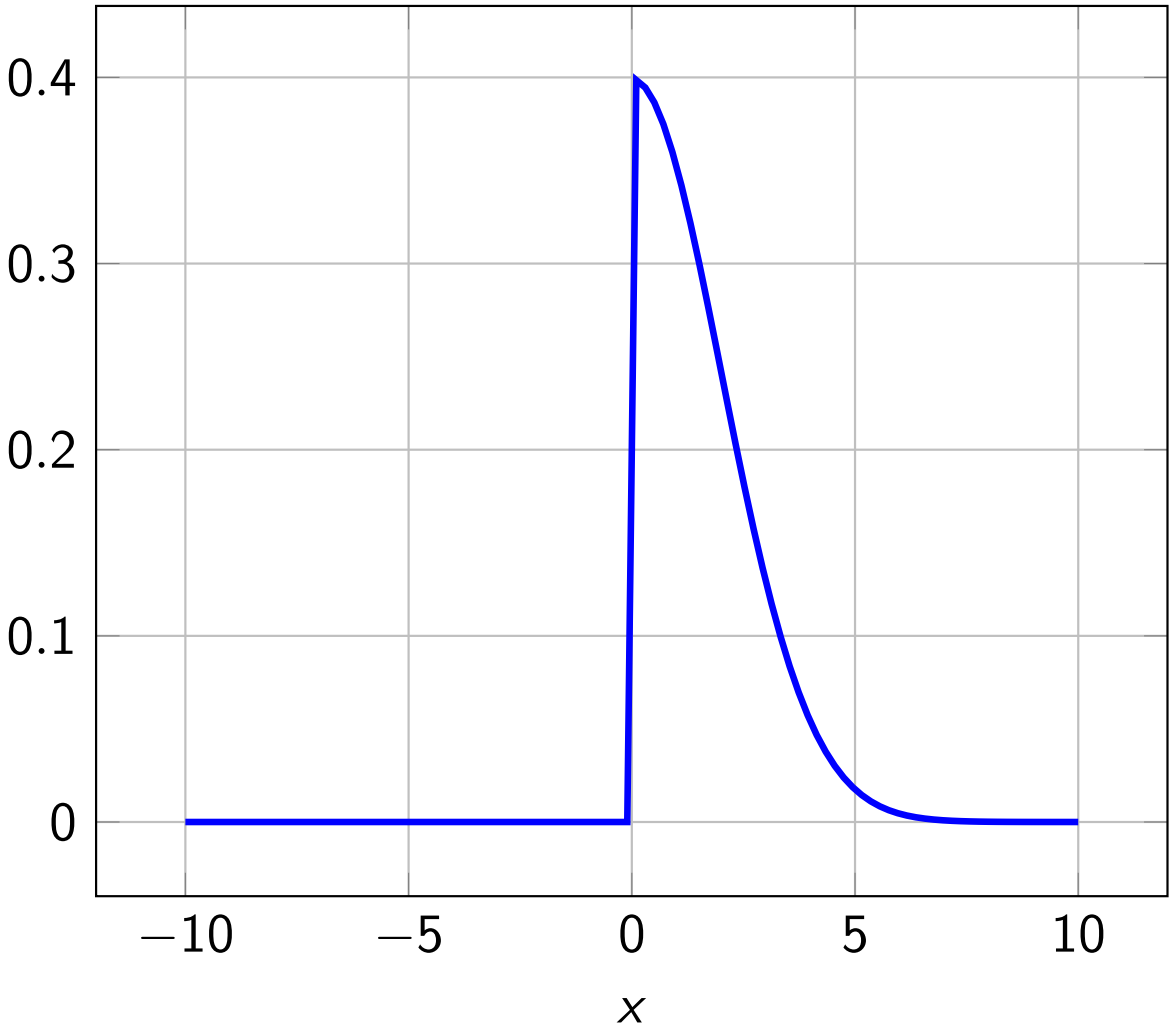
Imagine a normally distributed integer



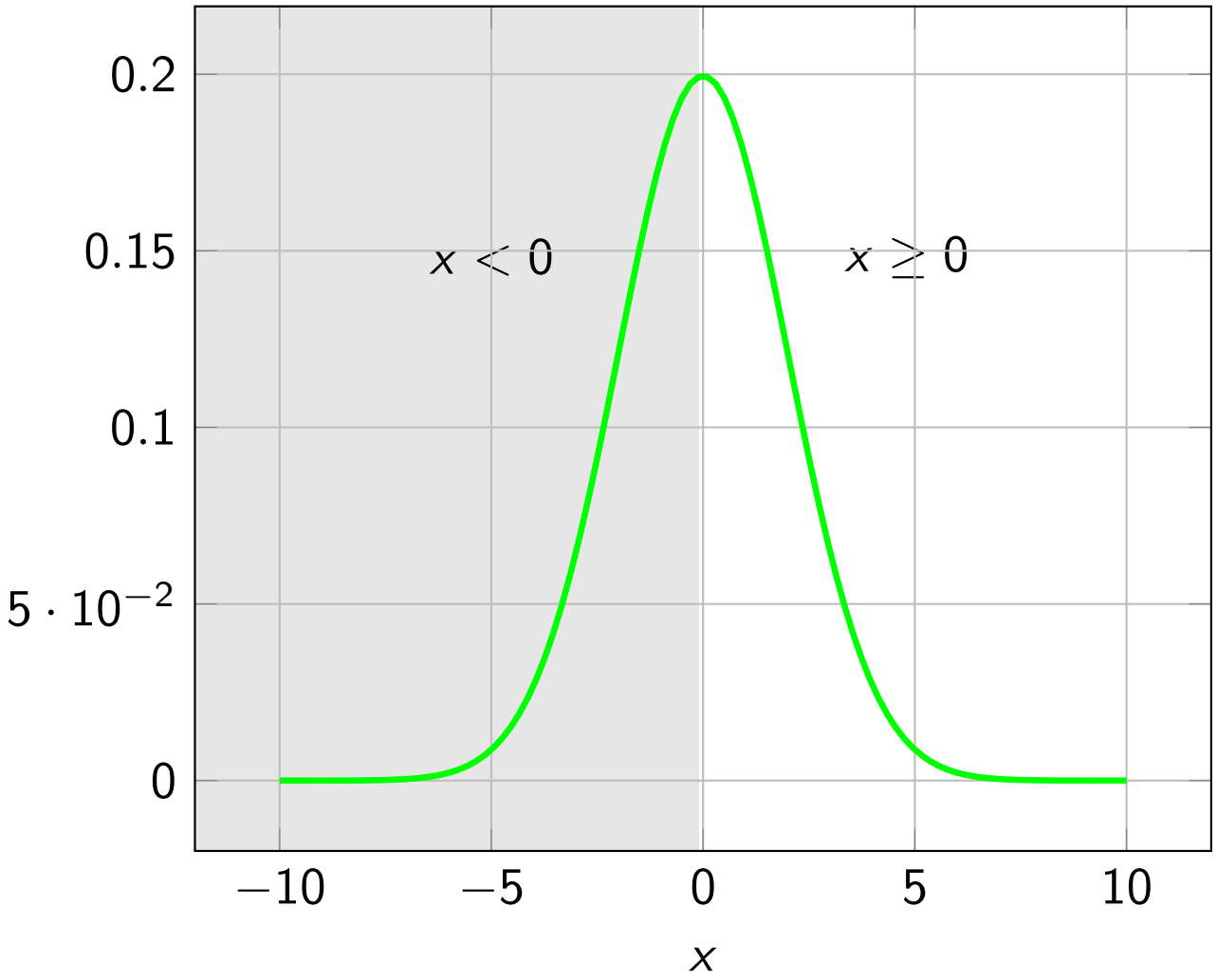
A trivial program

```
int abs(int x) {
  if (x<0)
    return -x;
  else
    return x;
}
```

Here is the output distribution

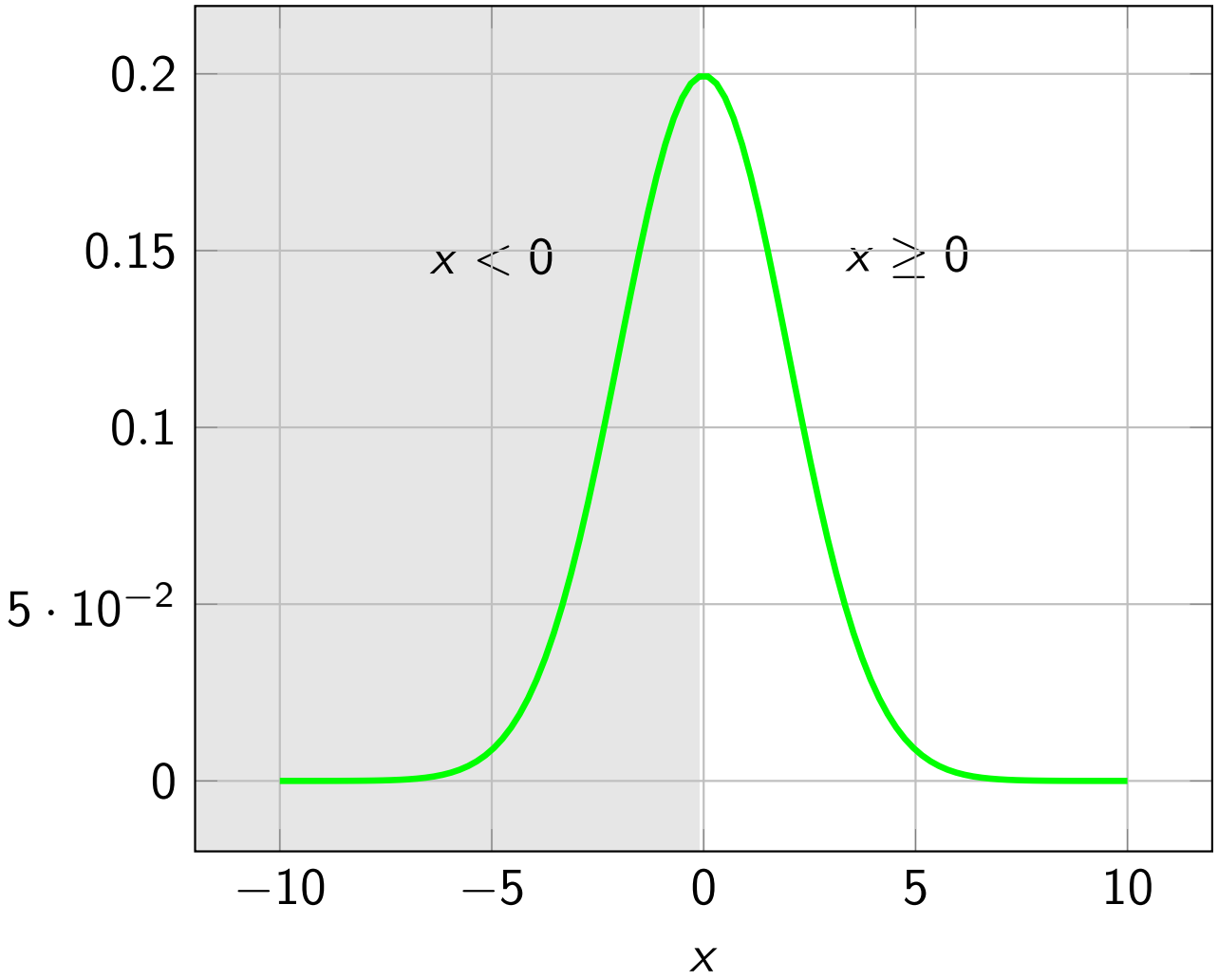


What's going on here?



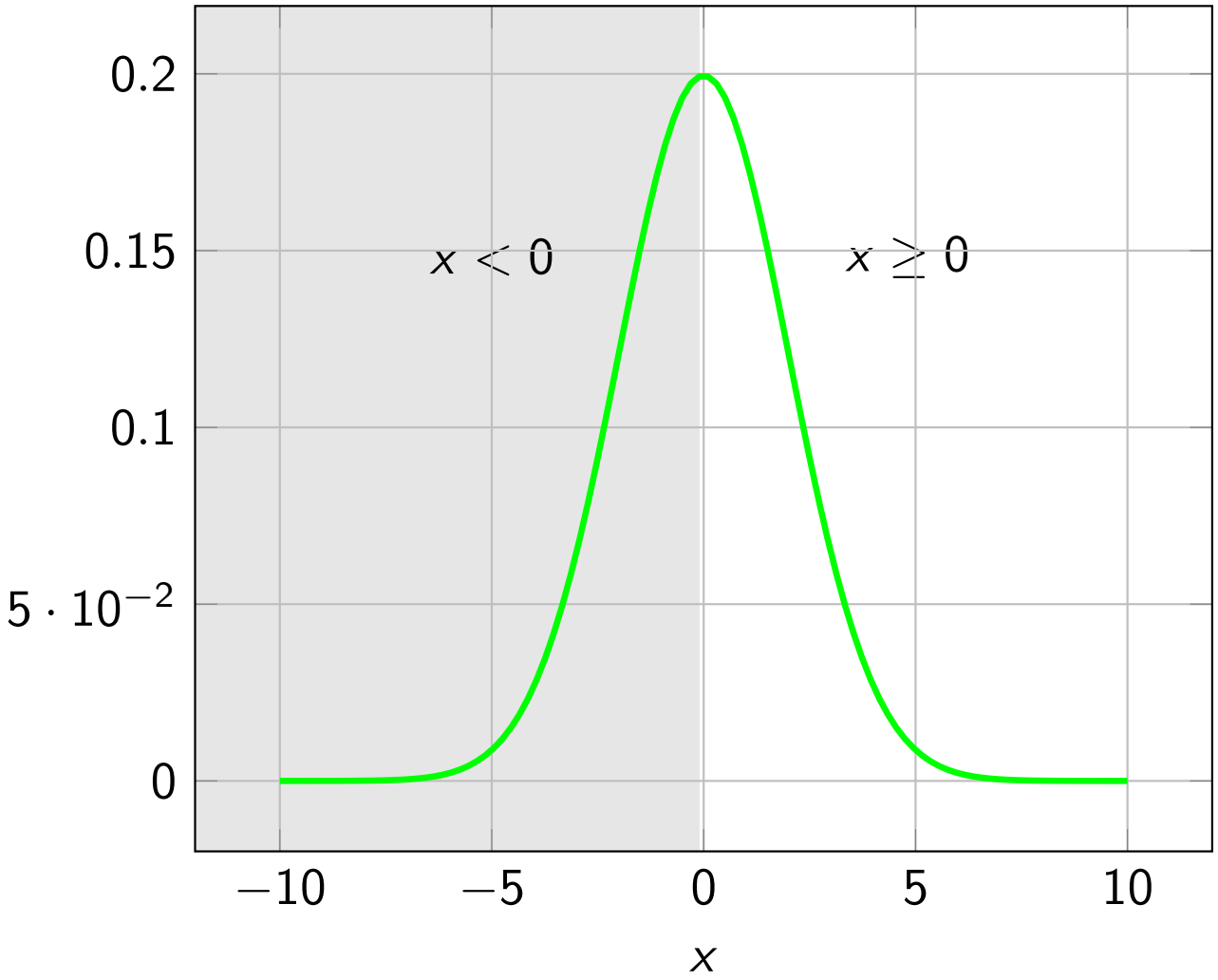
Let's think in terms of a very coarse division of the input

What's going on here?



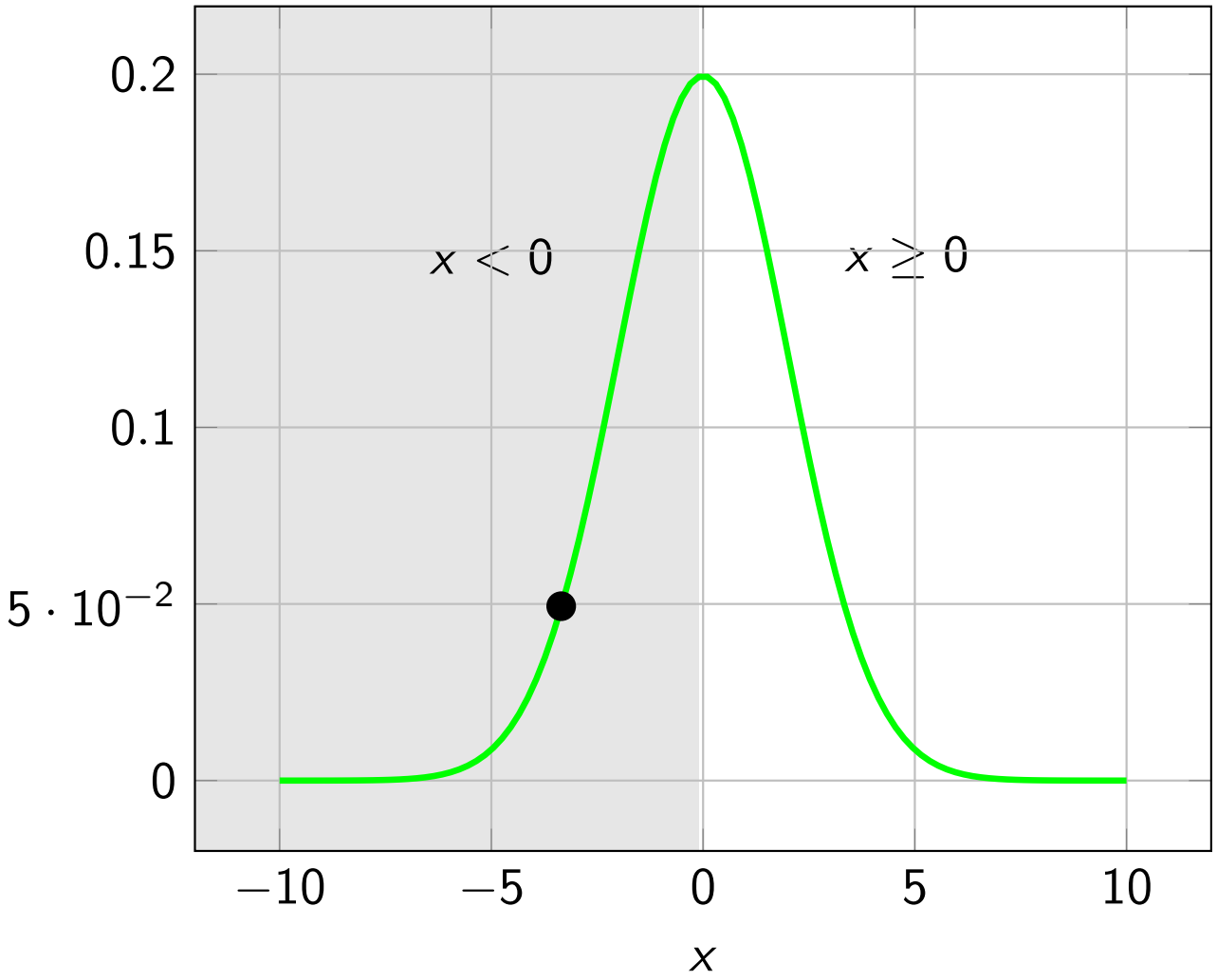
Input values $x \geq 0$ appear on the output unchanged.

What's going on here?



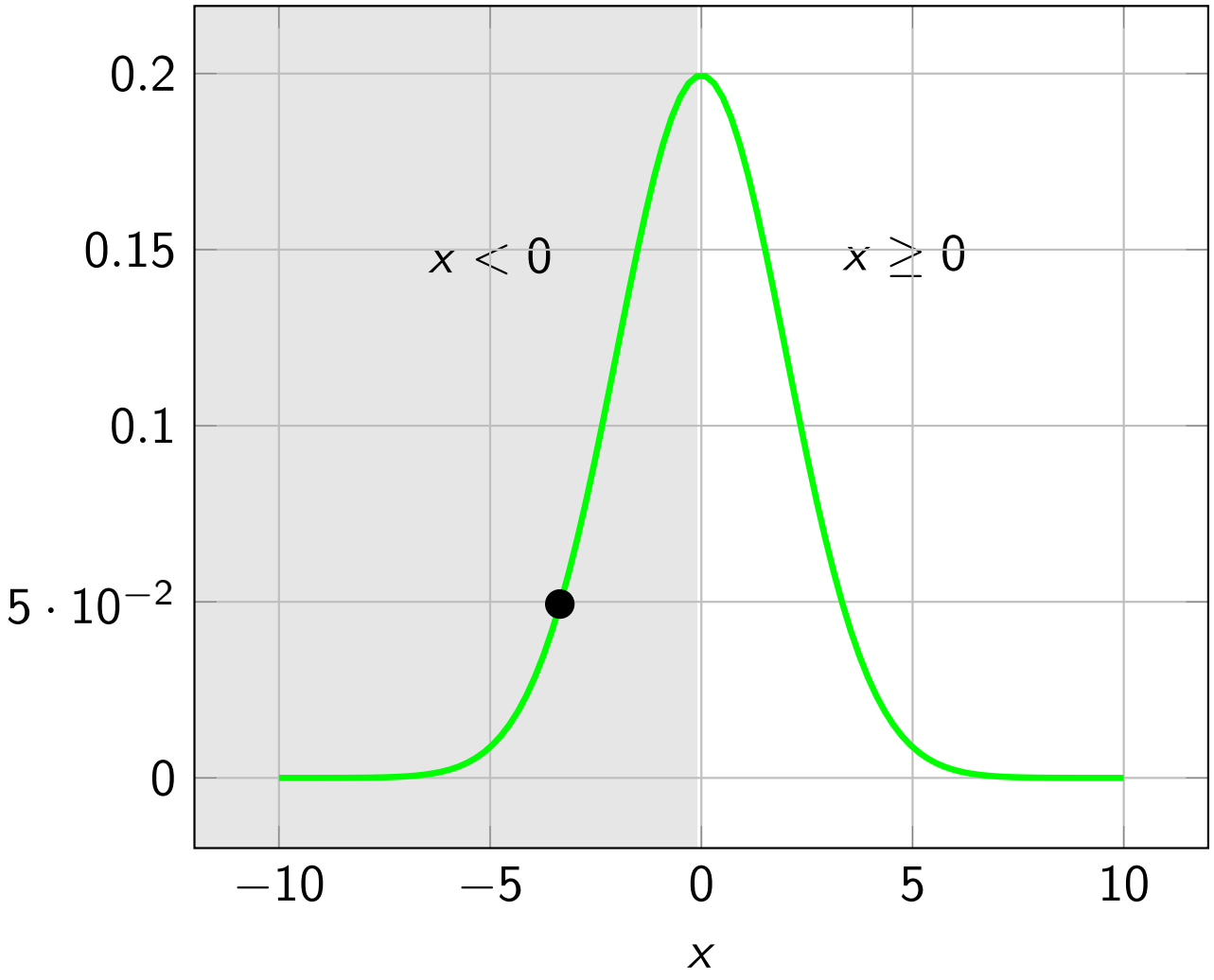
Input values $x \geq 0$ appear on the output unchanged.
Their mass in the input distribution propagates to the output.

What's going on here?



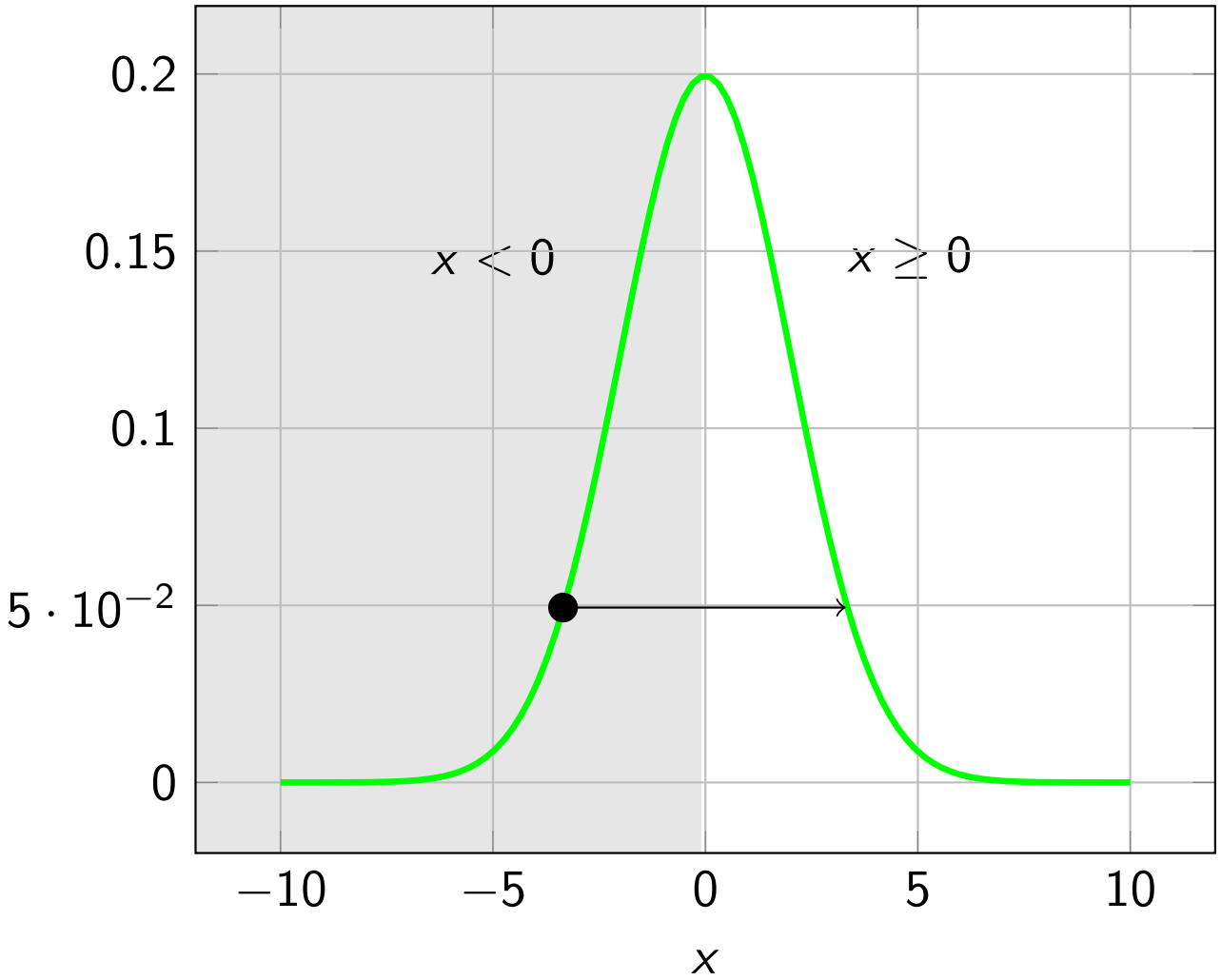
Input values $x < 0$ are transformed.

What's going on here?



Input values $x < 0$ are transformed.
Their mass in the input distribution is shifted to $-x$

What's going on here?



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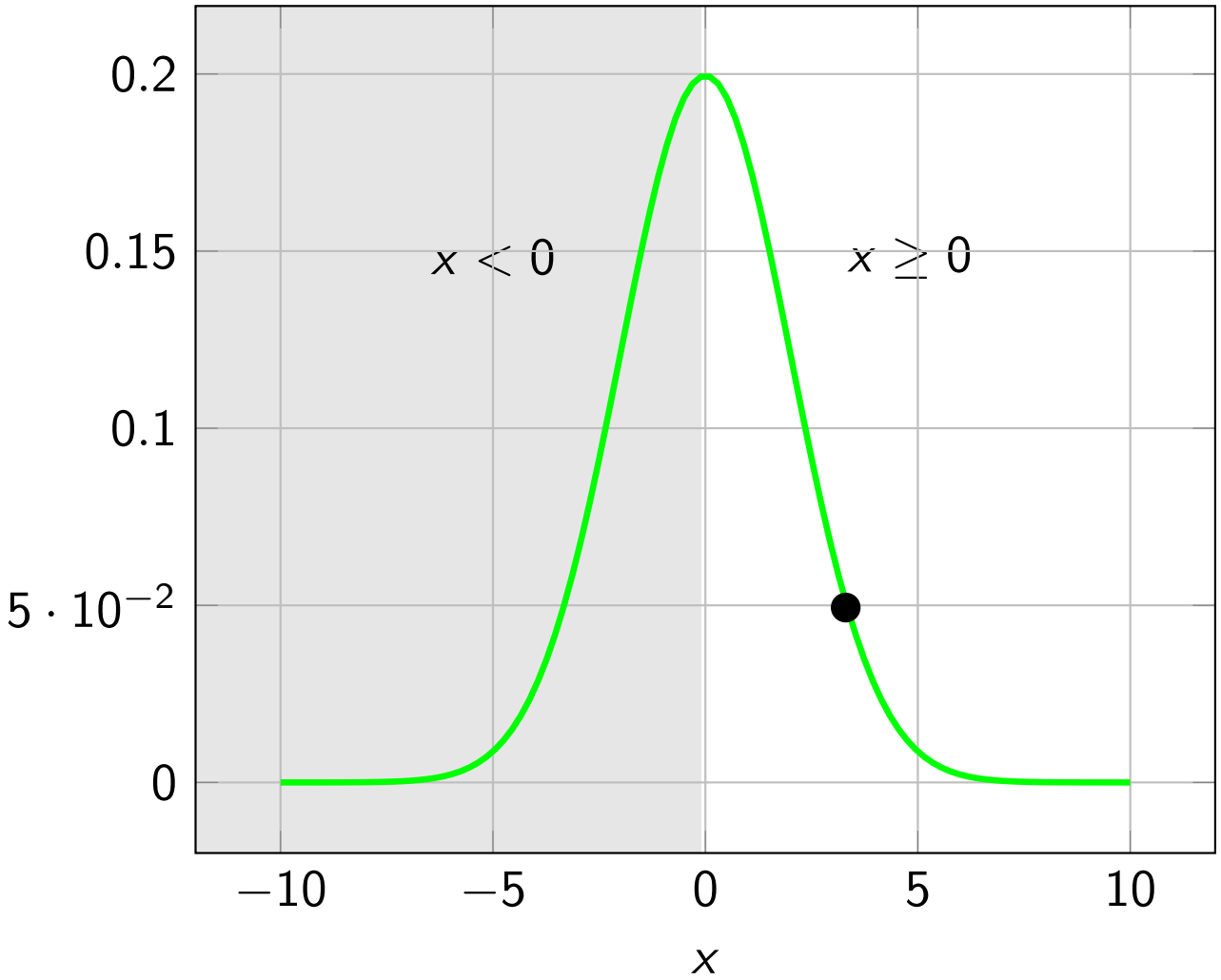
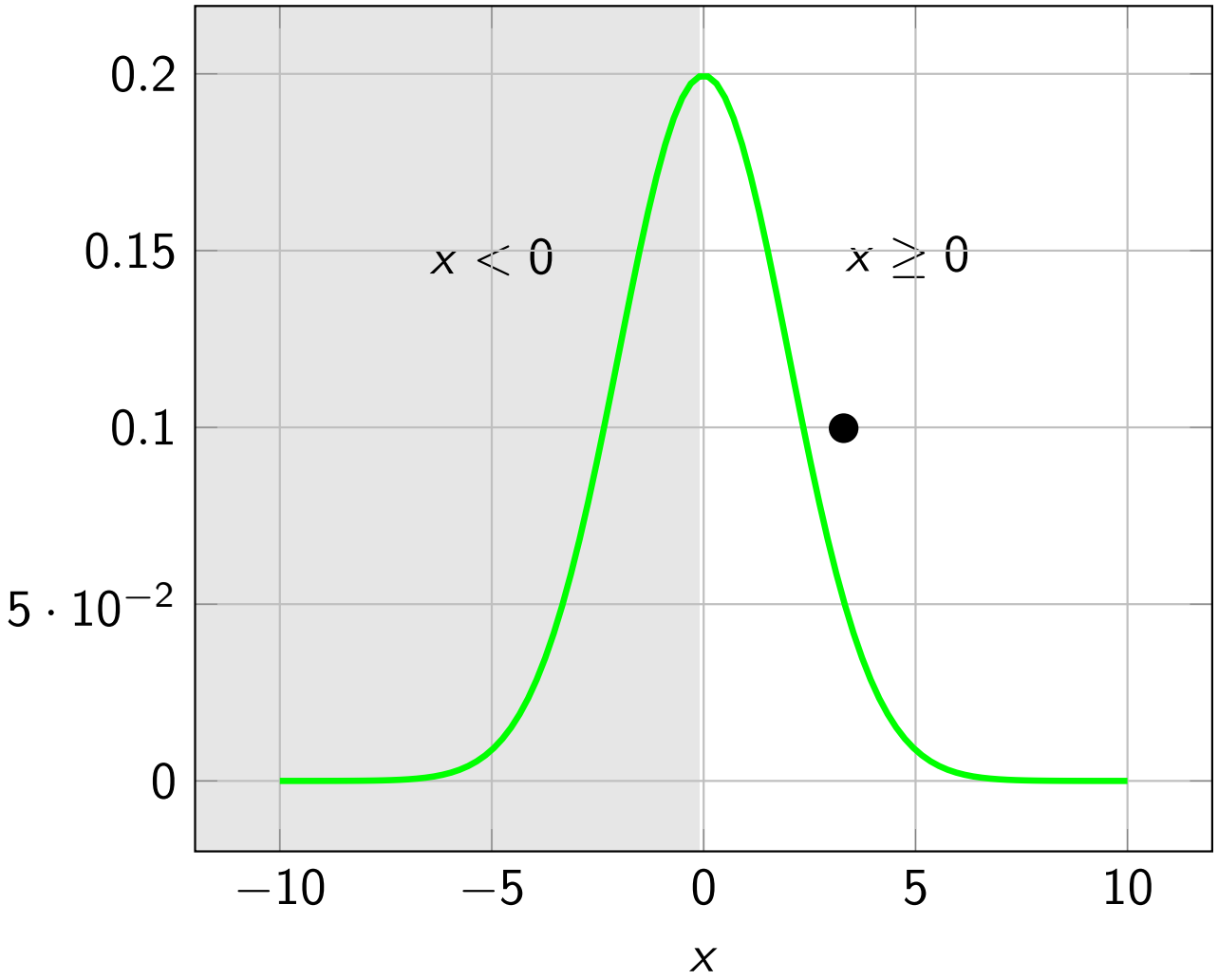


Input values $x < 0$ are transformed.
Their mass in the input distribution is shifted to $-x$ and accumulates in the output distribution

What's going on here?



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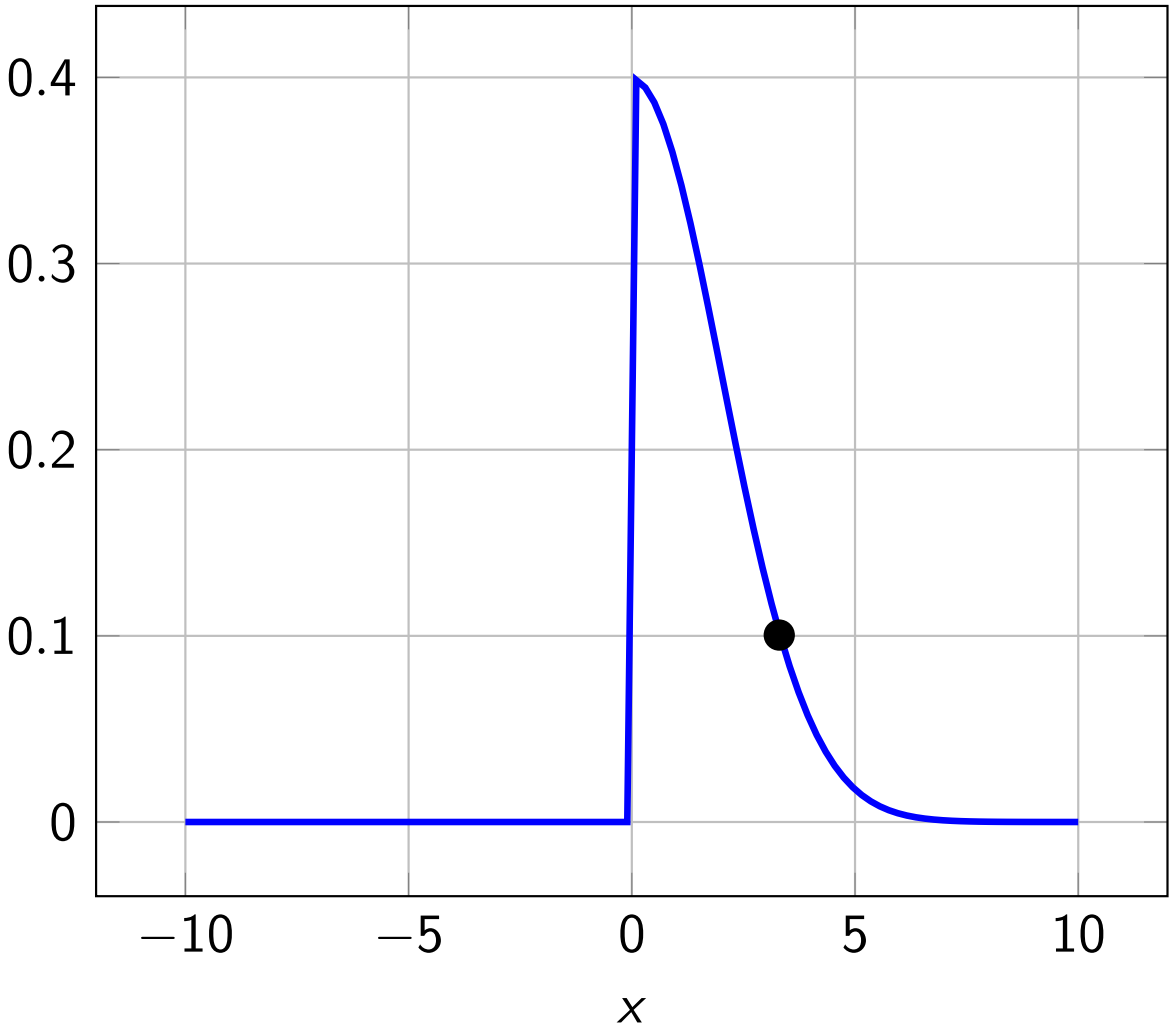
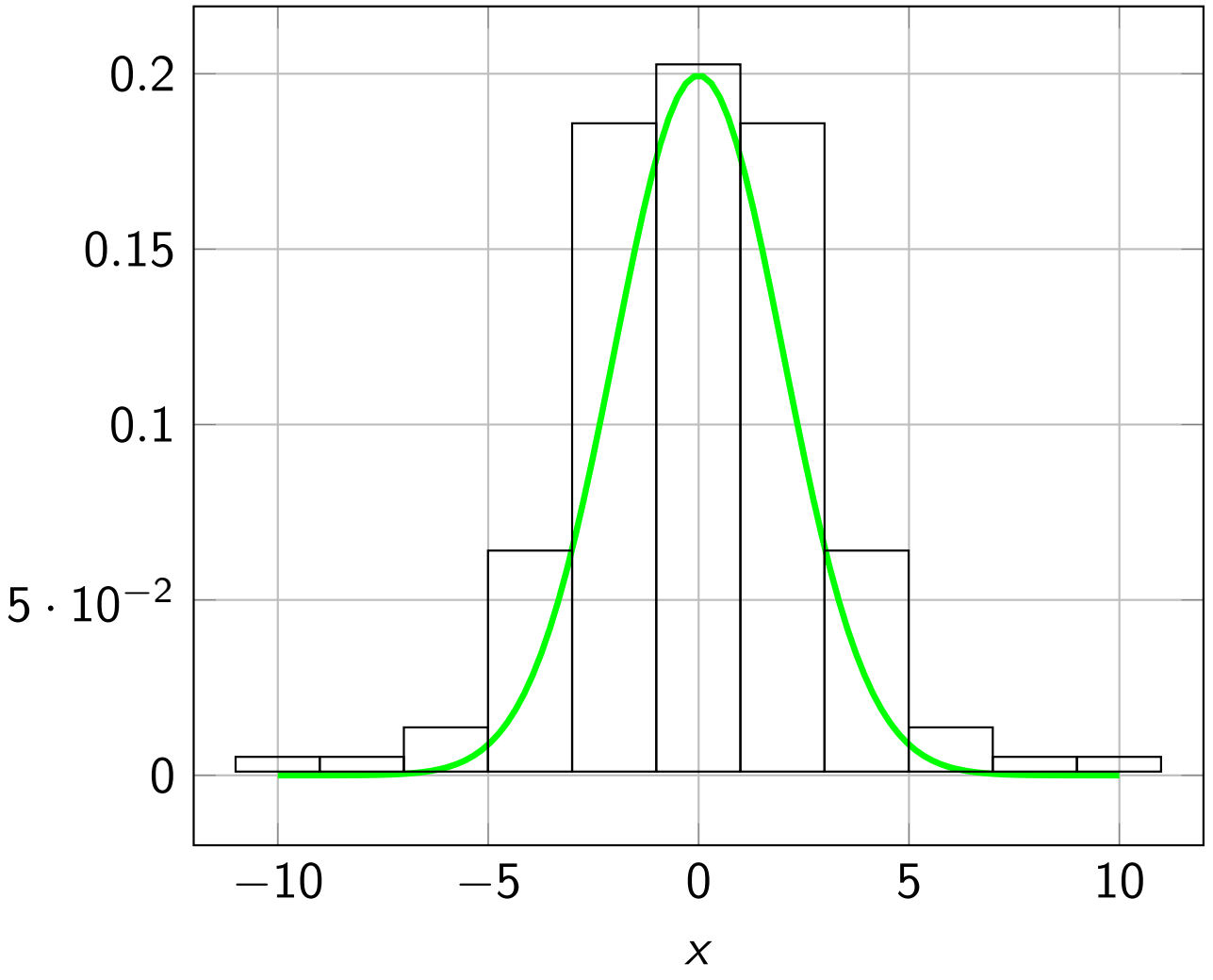
Here is the output distribution



Bounding distribution



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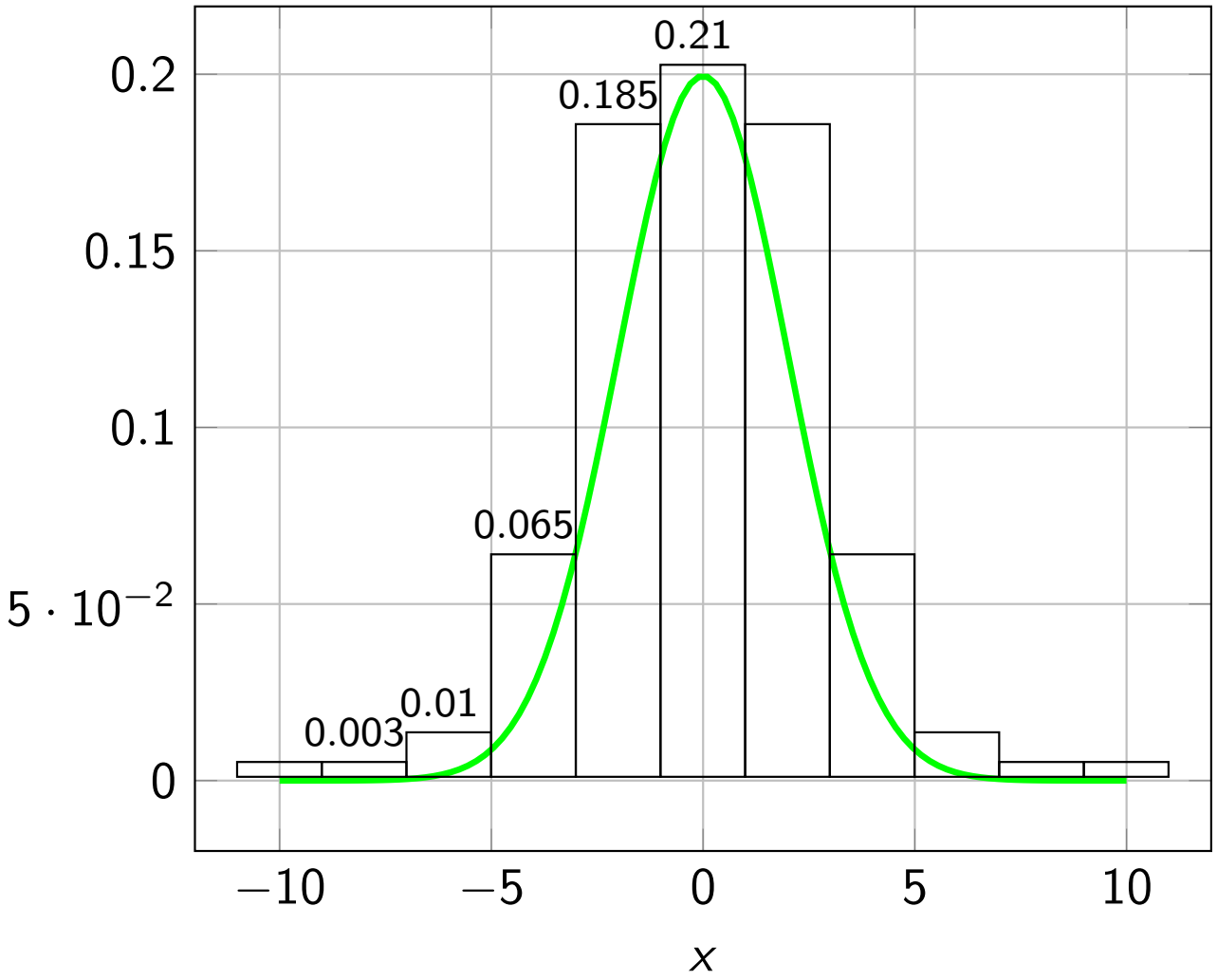
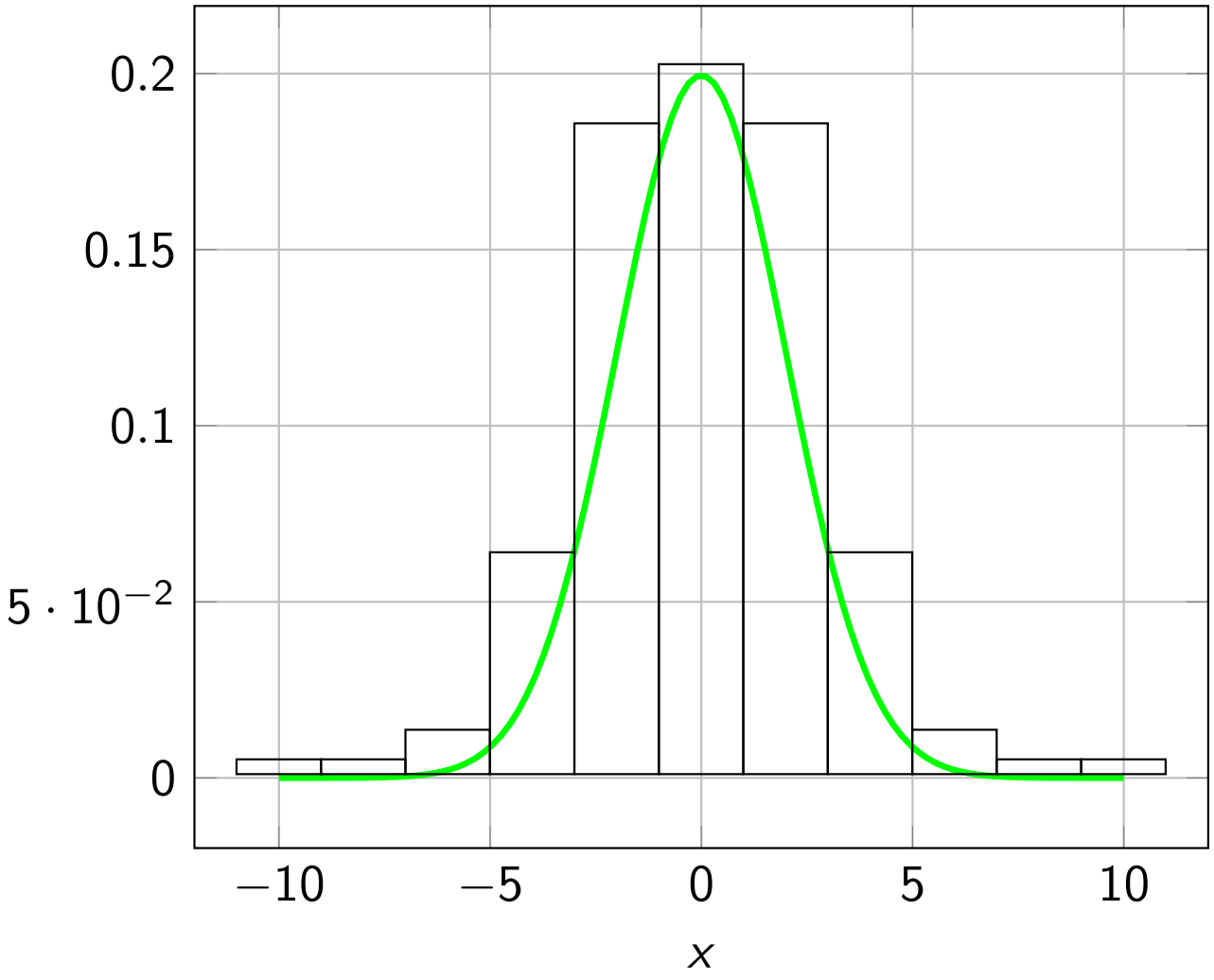
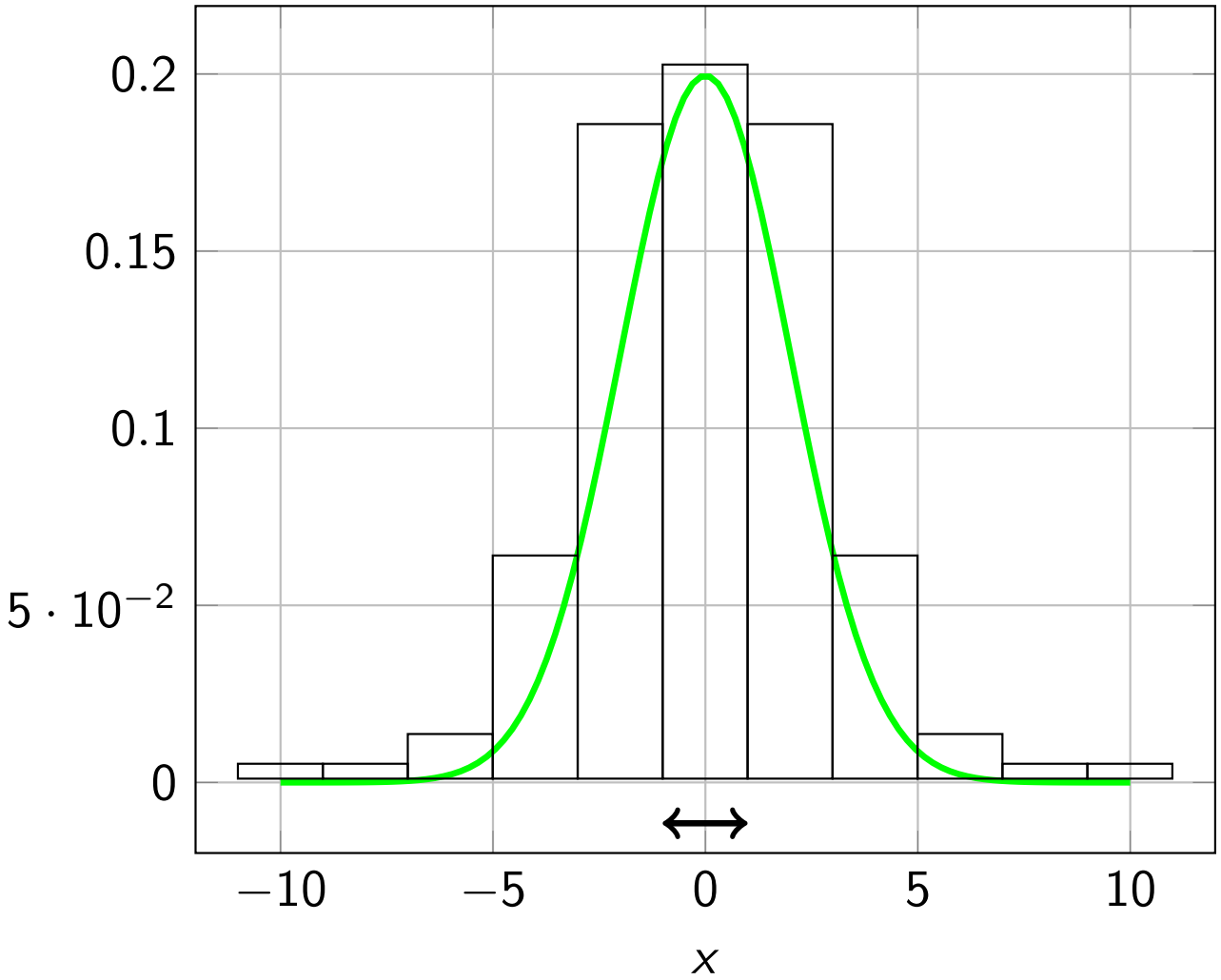


Bounding $Pr([-1, 1])$



How many elements are in the domain? 3
What is the mass of each element? ≤ 0.21
$Pr([-1, 1]) \leq 0.63 = 3 * 0.21$

Bounding $Pr([-1, 1])$



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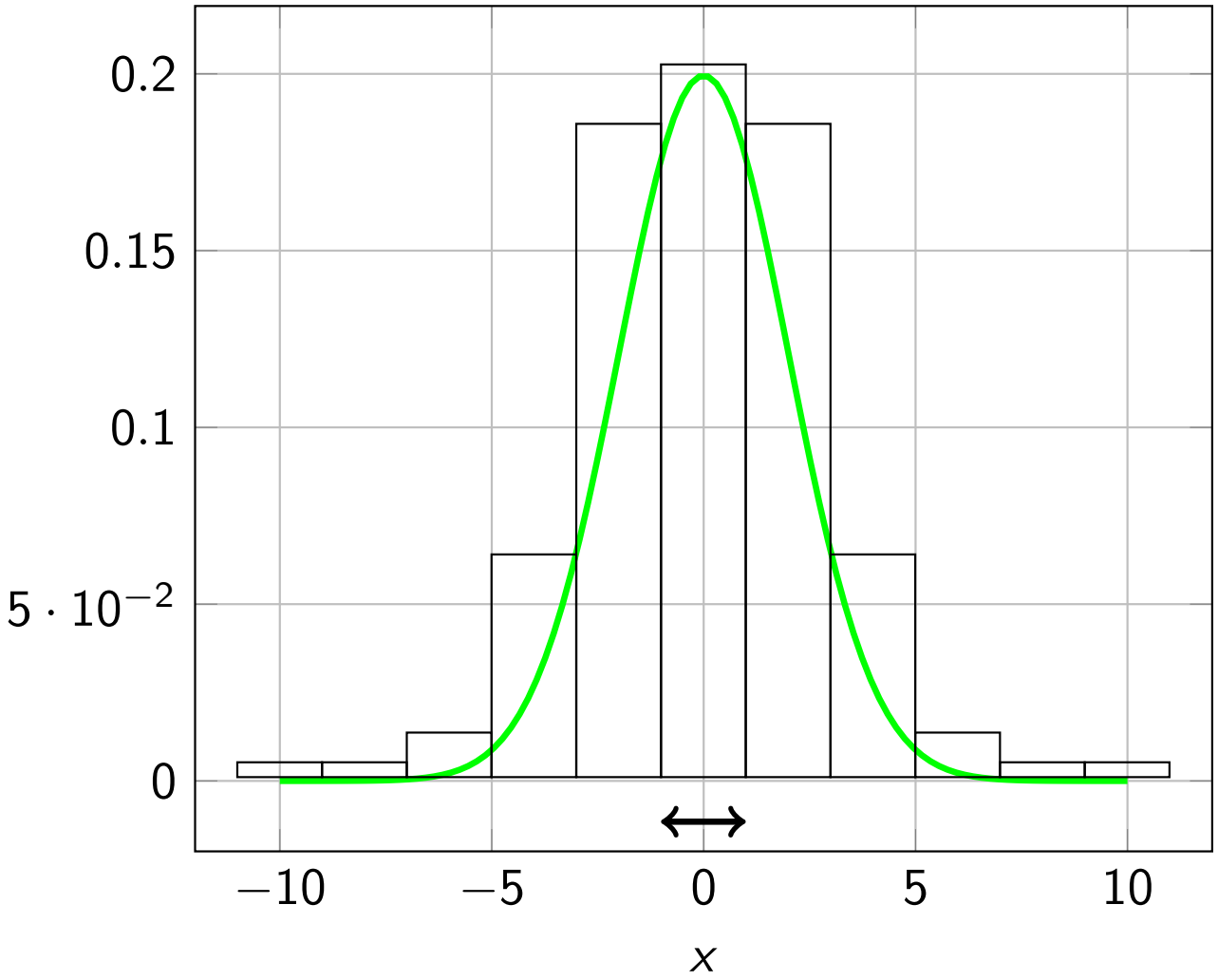

How many elements are in the domain?

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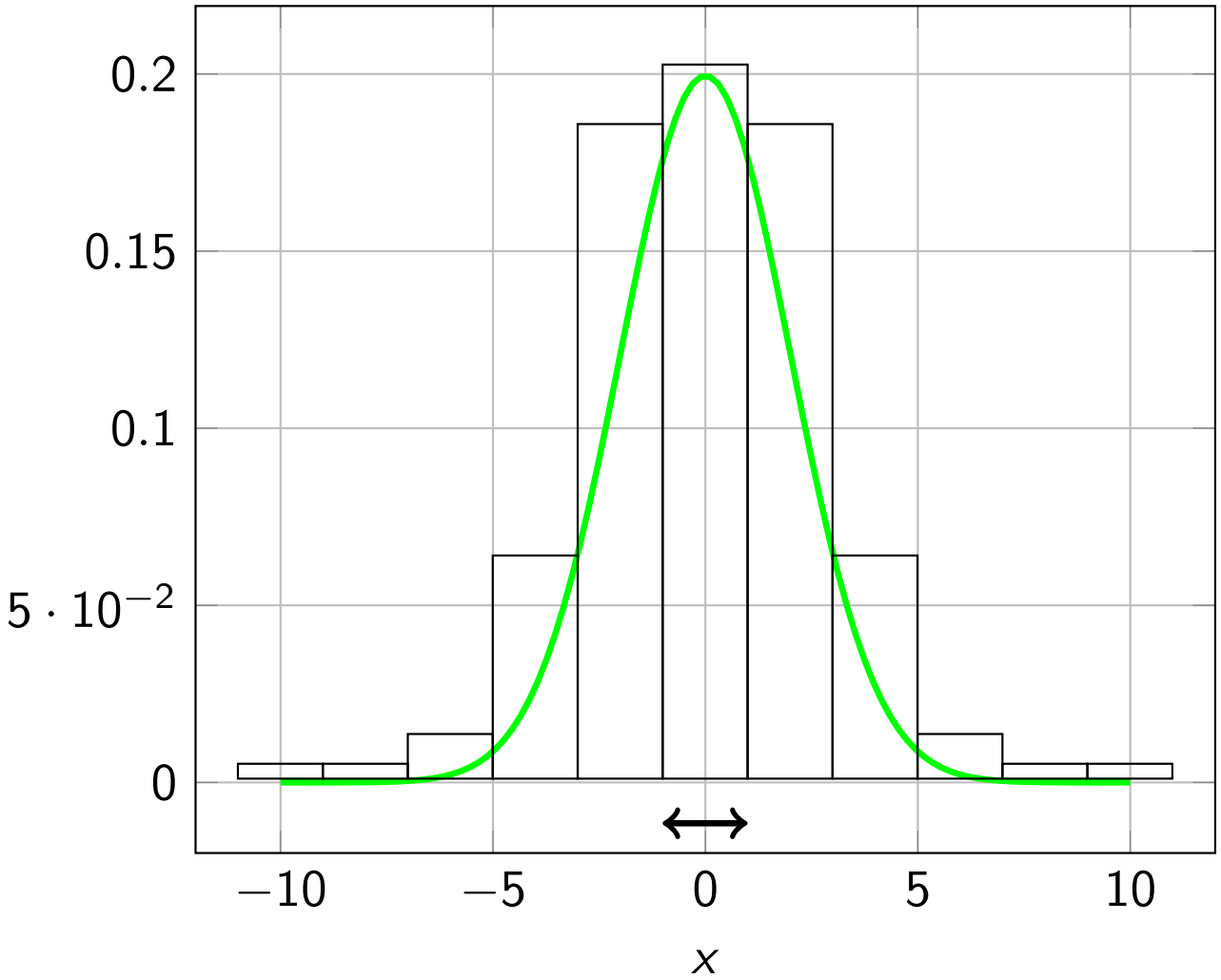

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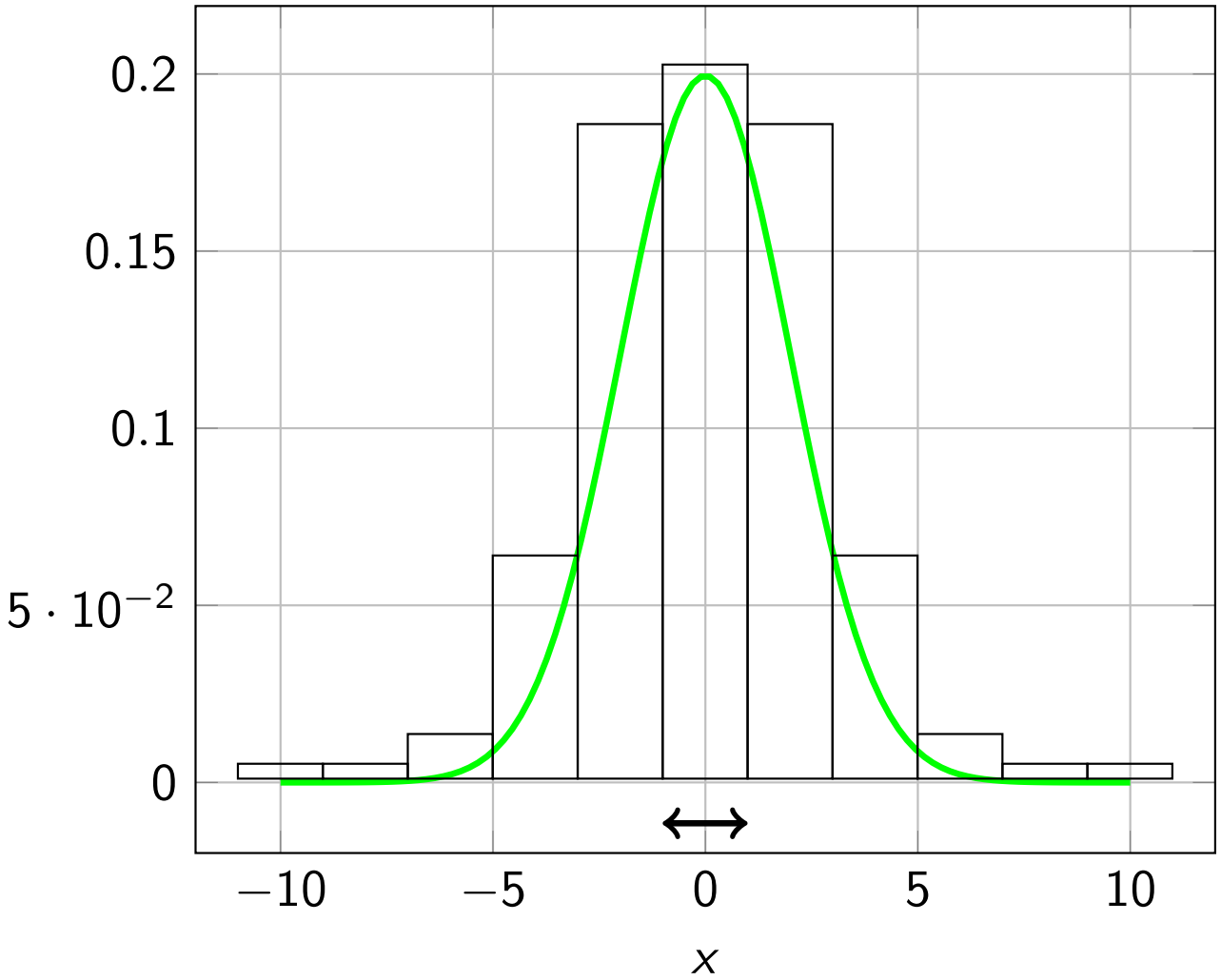
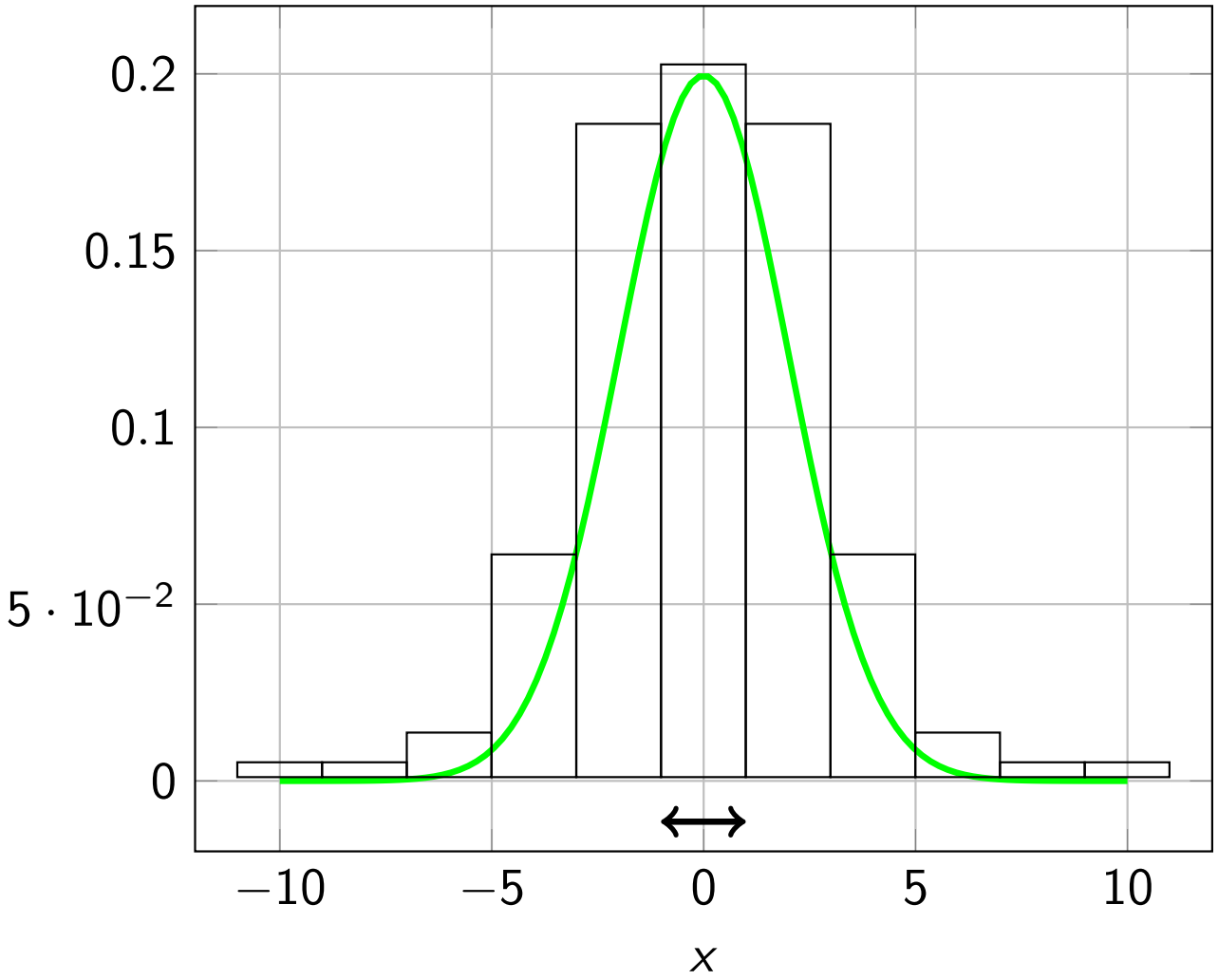


How many elements are in the domain? 3
What is the mass of each element?

Bounding $Pr([-1, 1])$



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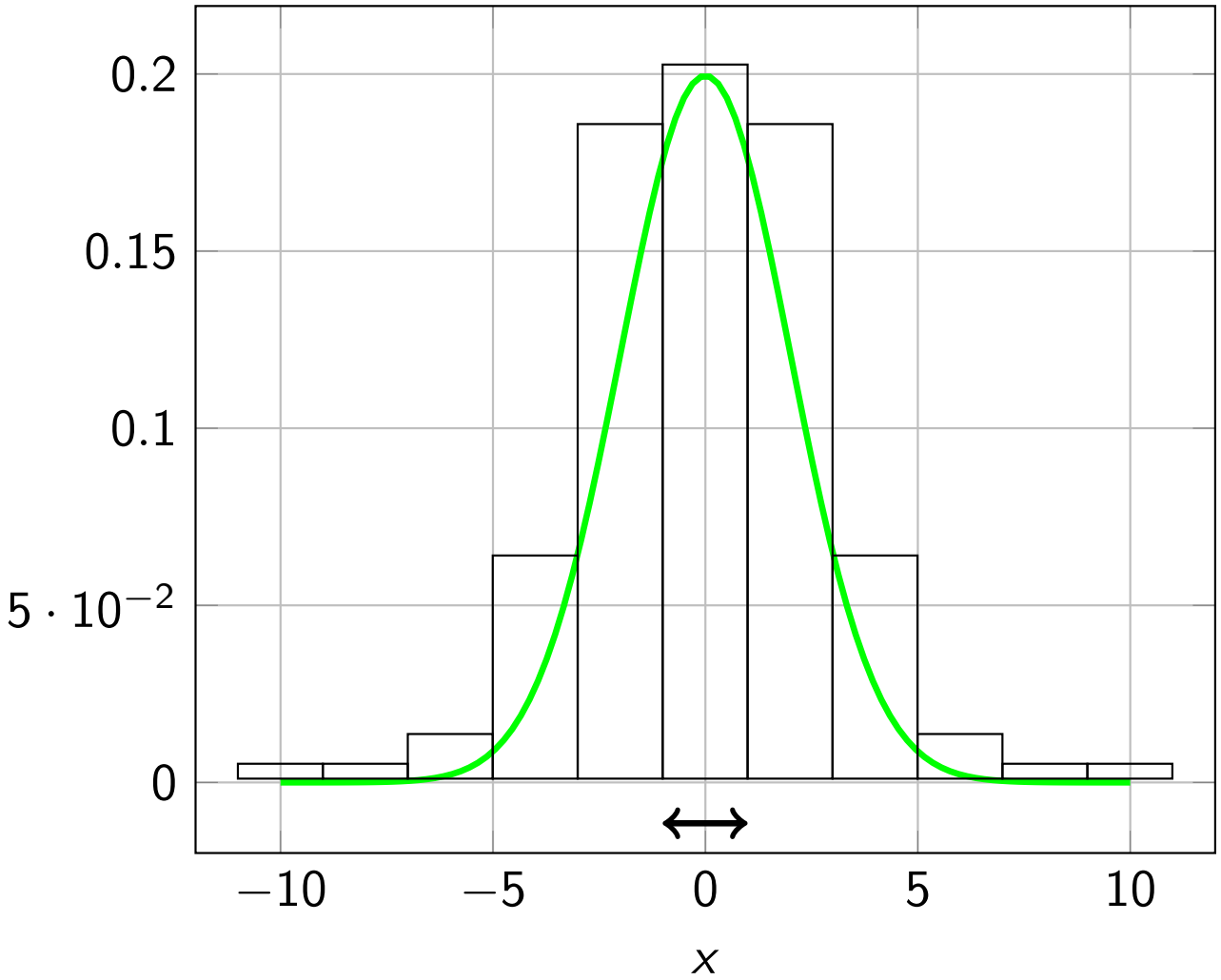
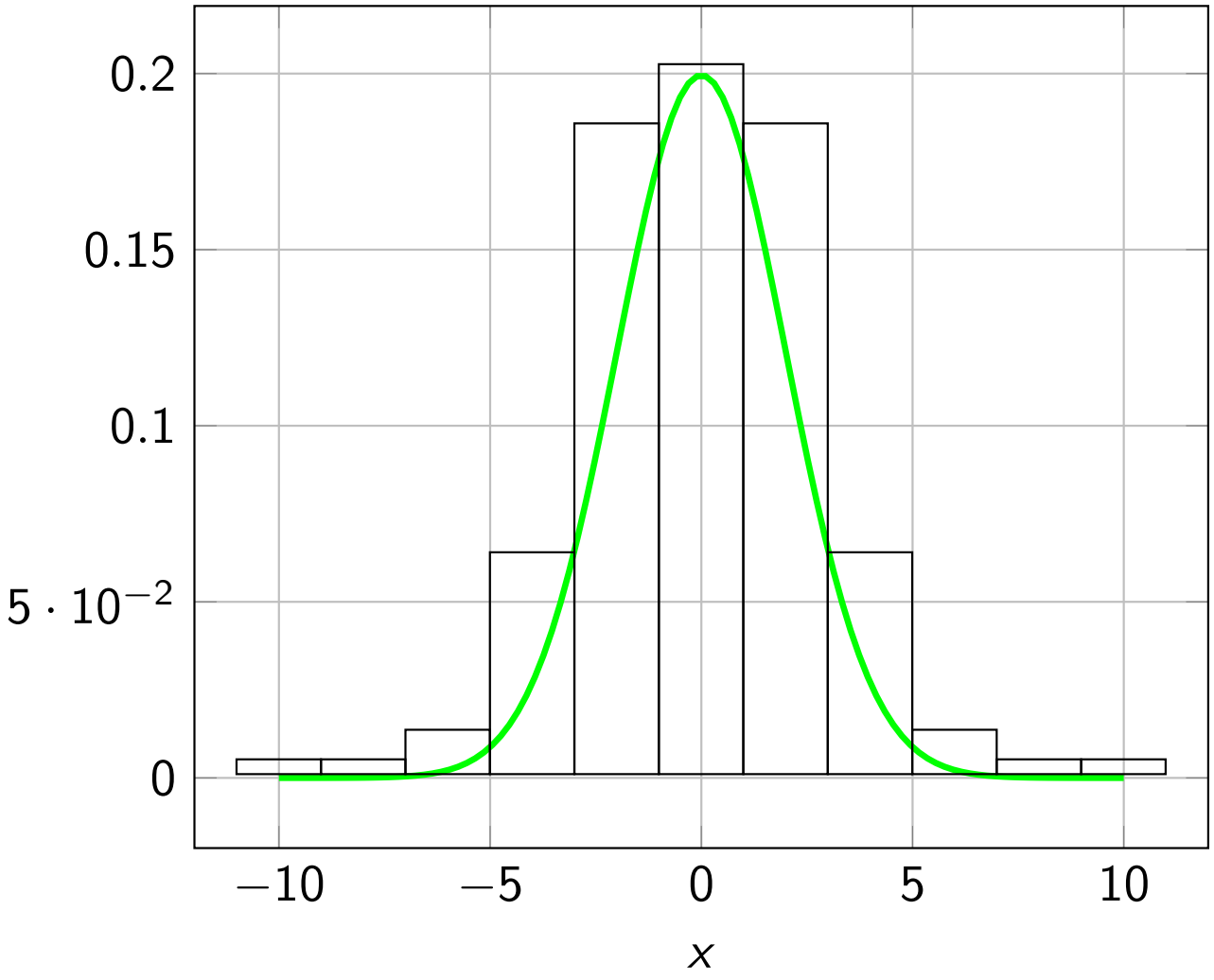


How many elements are in the domain? 3
What is the mass of each element? ≤ 0.21
$Pr([-1, 1]) \leq 0.63 = 3 * 0.21$

Bounding $Pr([0,5])$

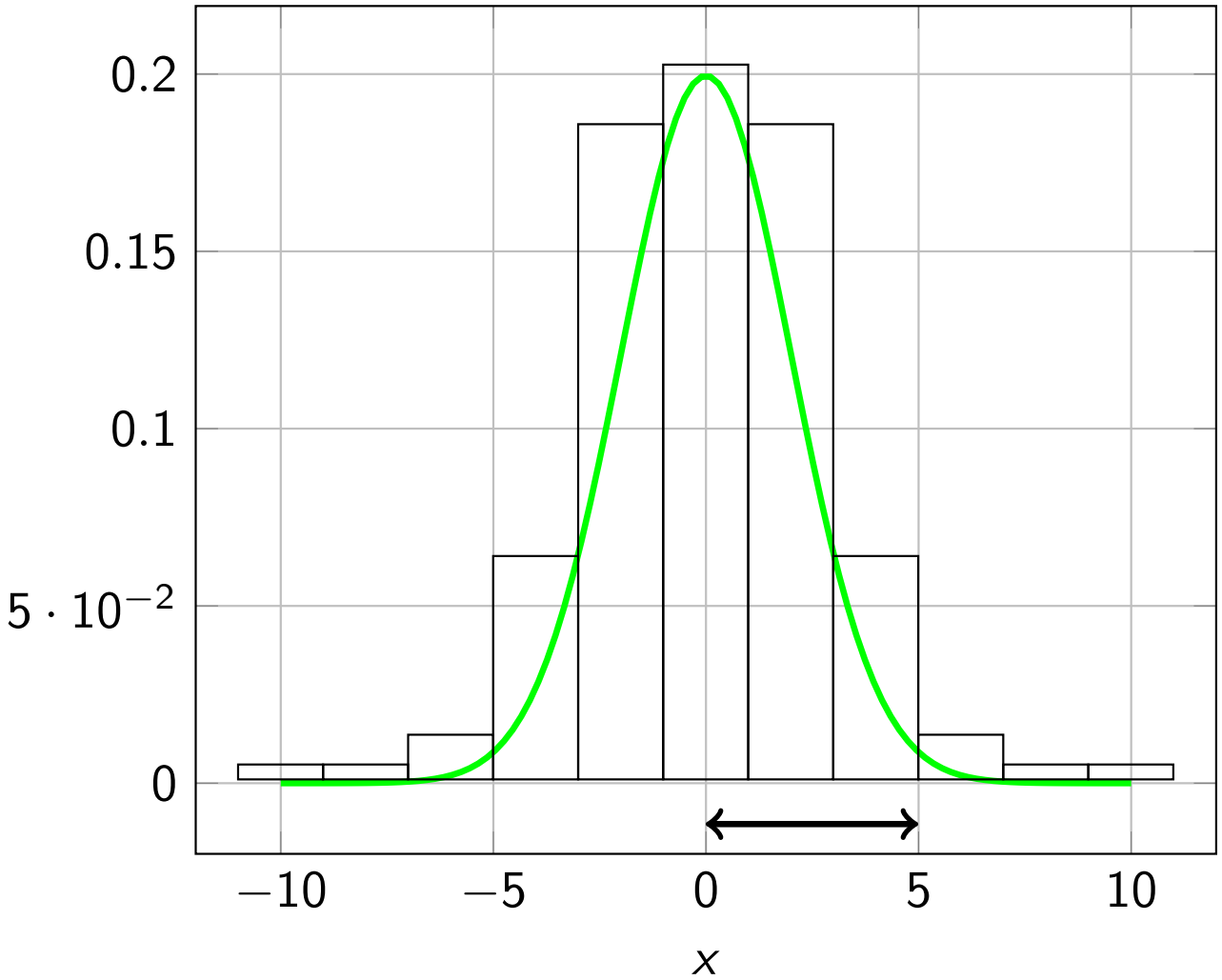


How many elements are in the domain? 5

What is the mass of each element? $\leq 0.21, \leq 0.185, \leq 0.065$

$Pr([0,5]) \leq 0.92 = 2 * 0.21 + 2 * 0.185 + 2 * 0.065$

Bounding $Pr([0, 5])$



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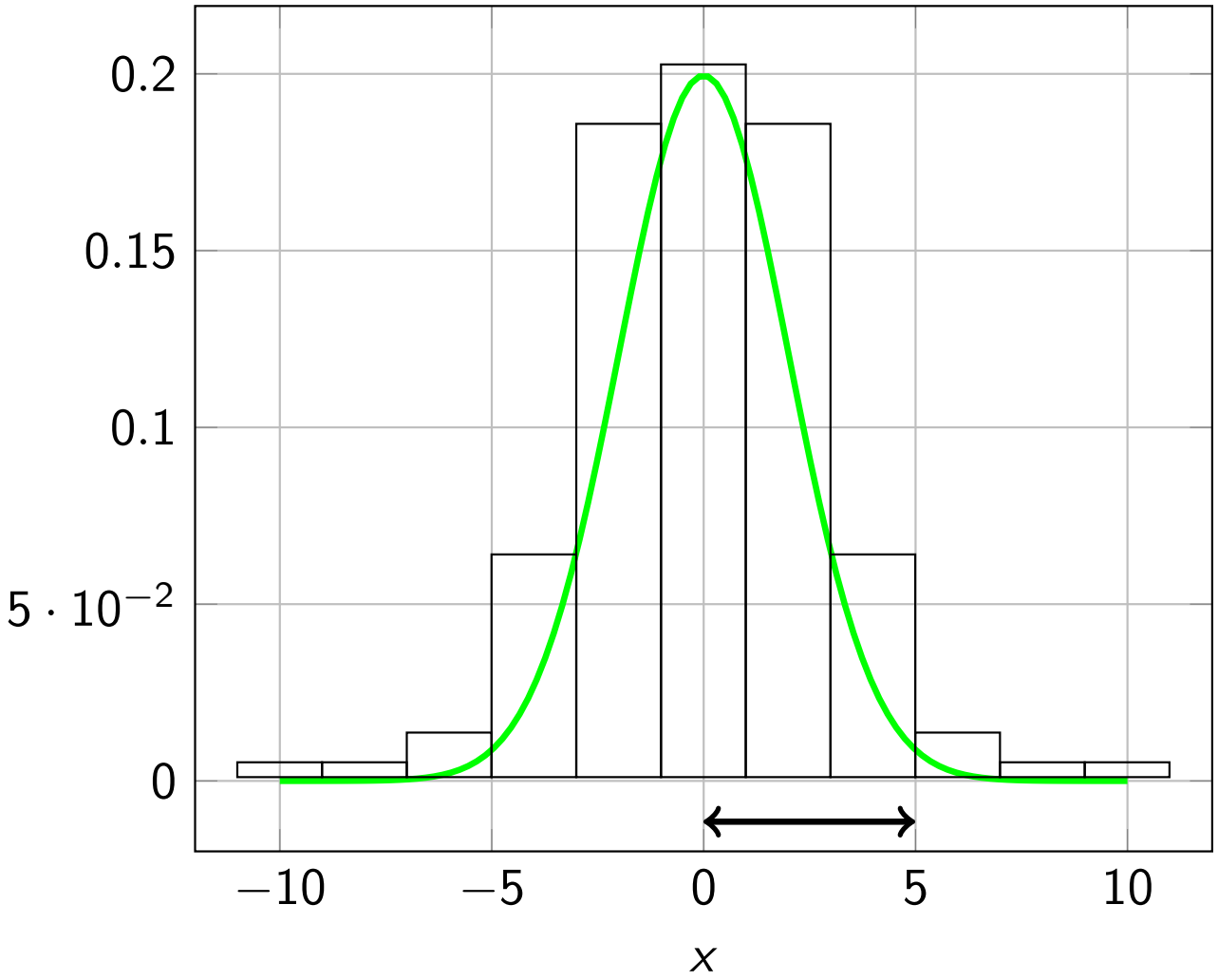

How many elements are in the domain?

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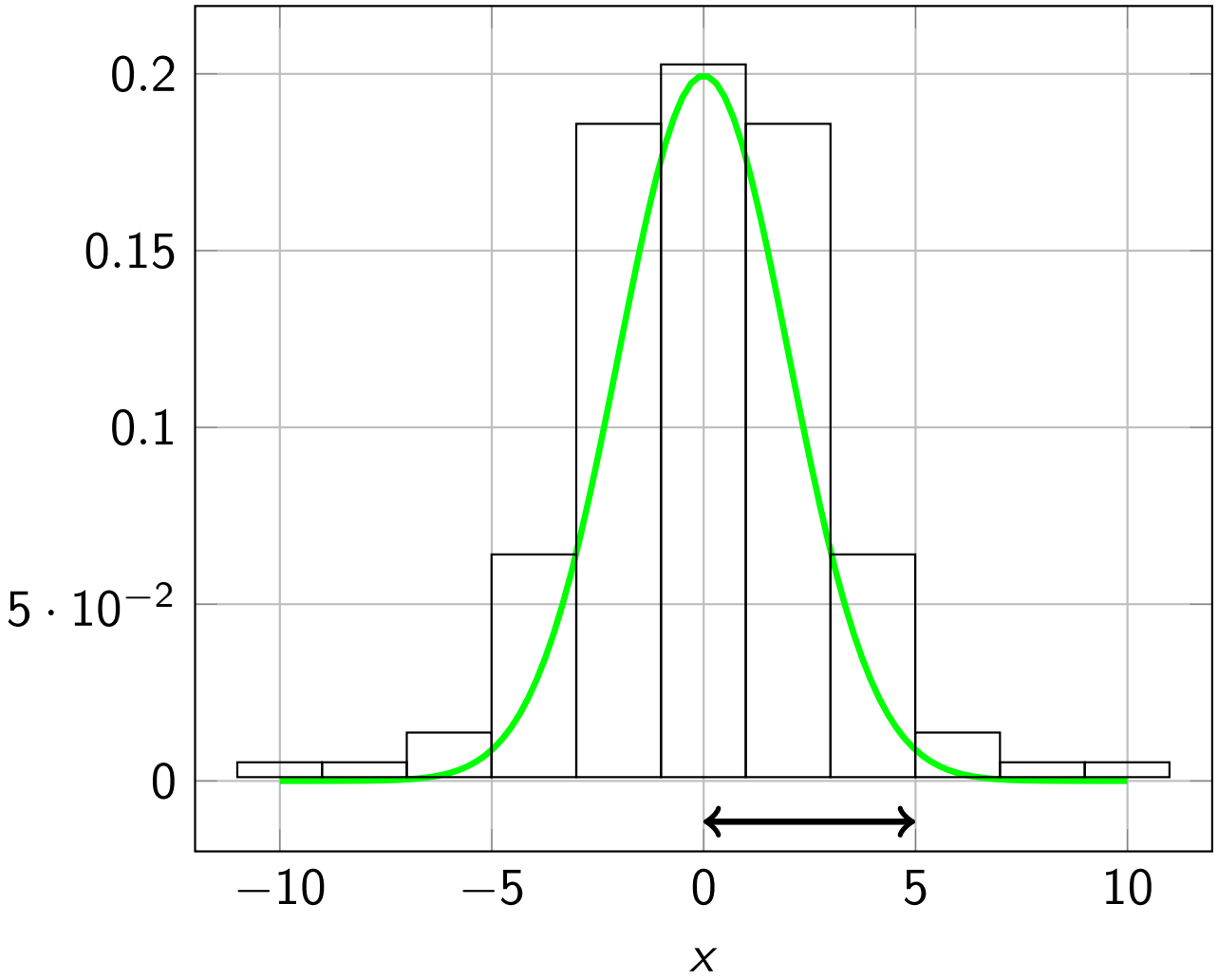

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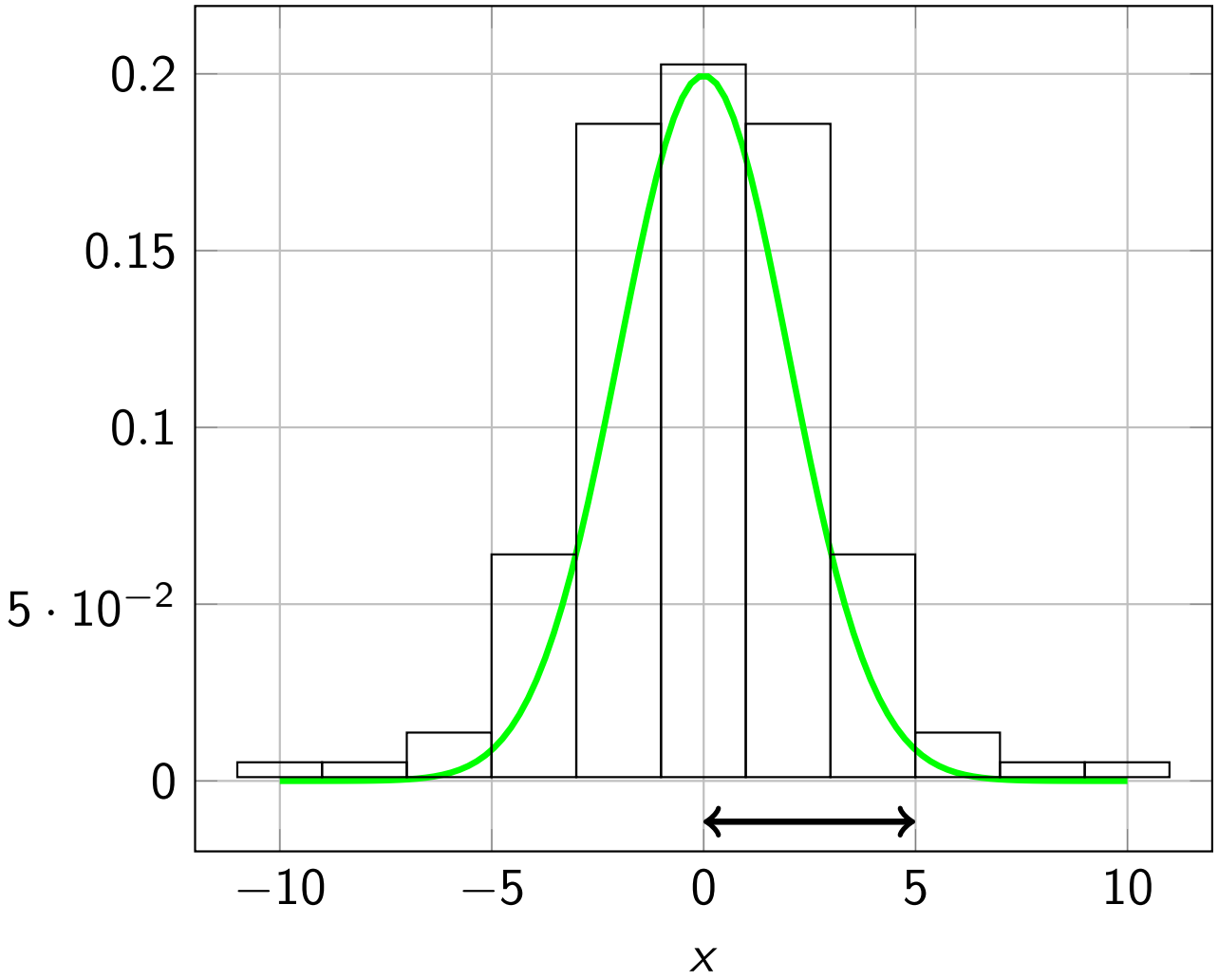

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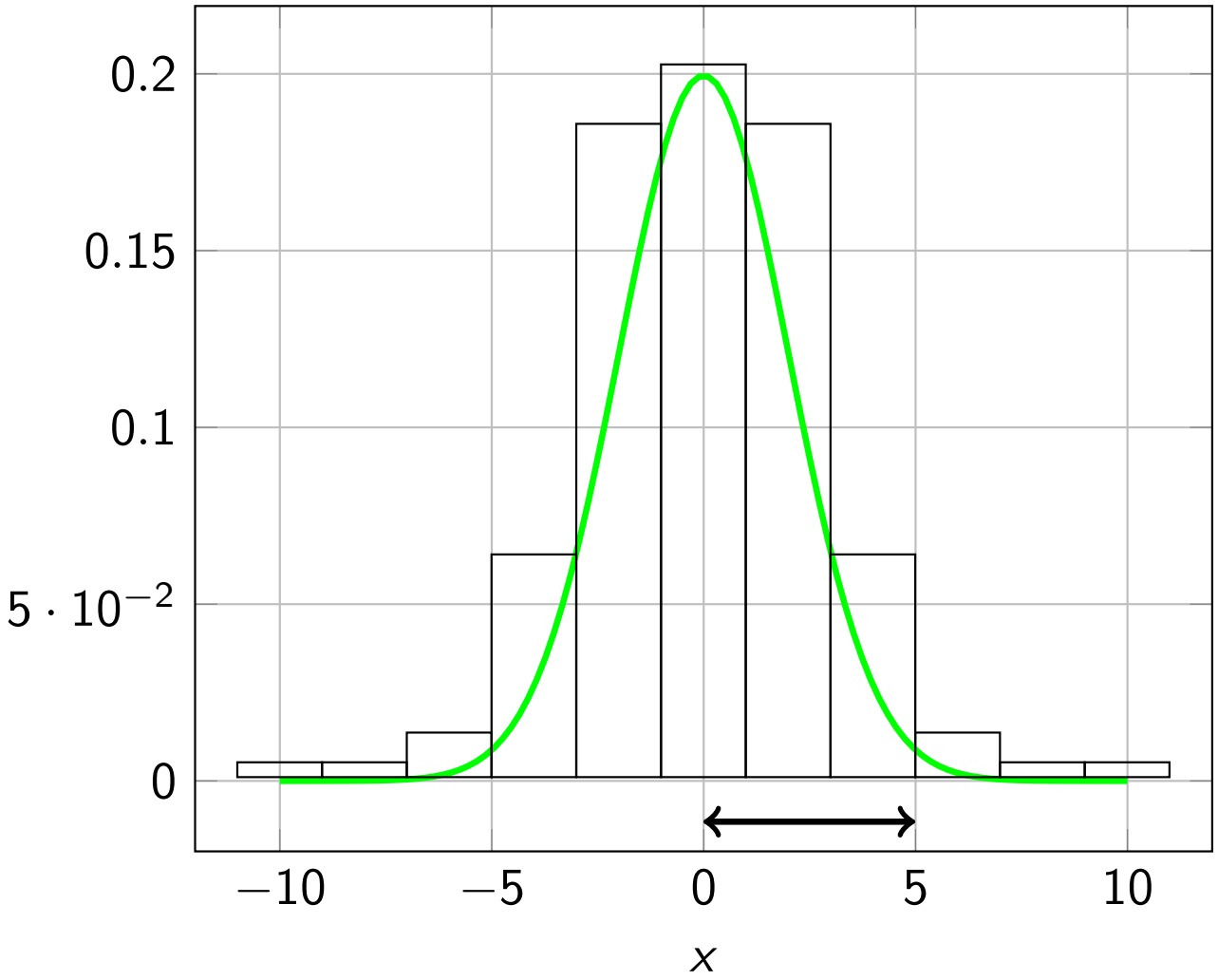

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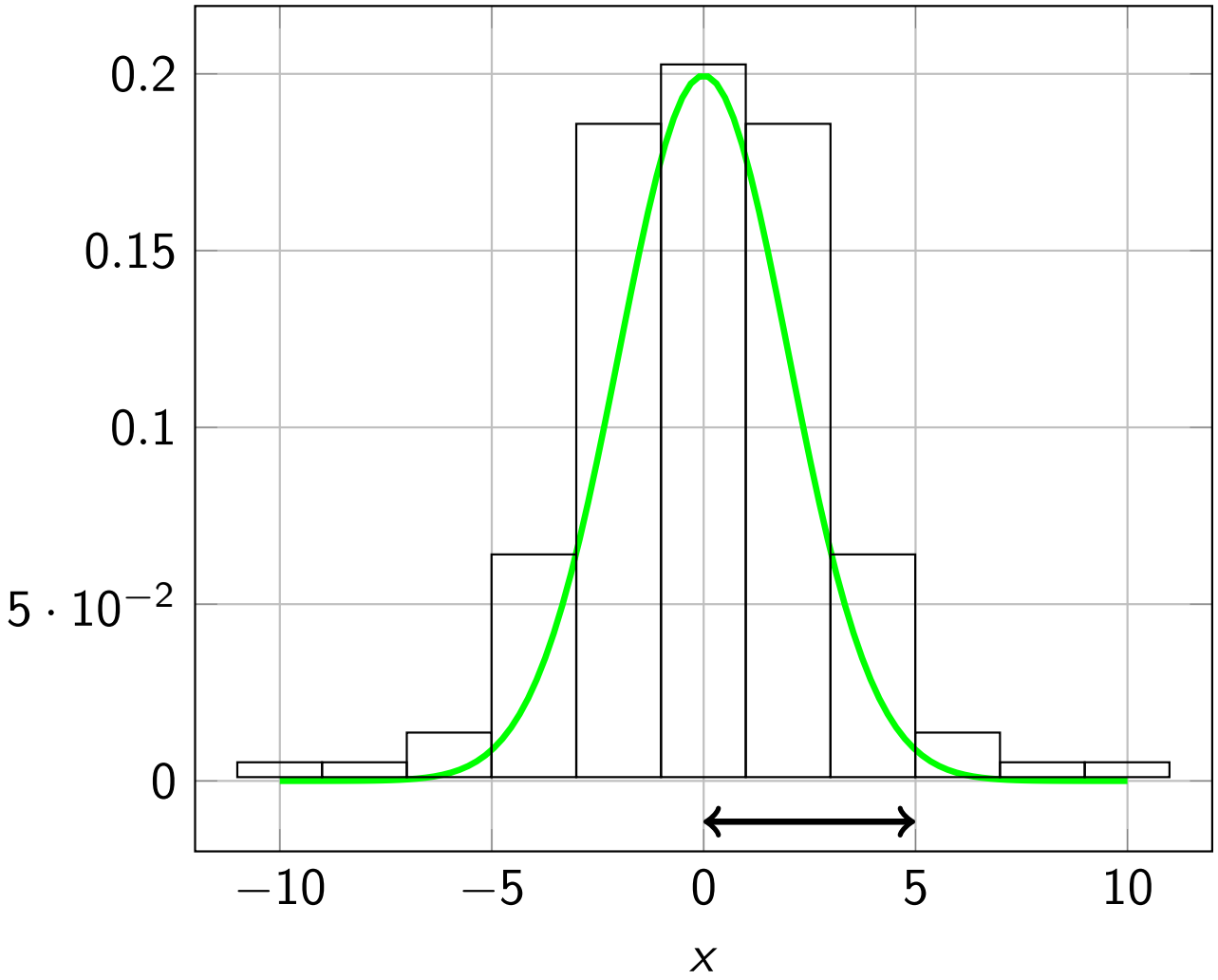

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$Pr([0,5]) \leq 0.92 = 2 * 0.21 + 2 * 0.185 + 2 * 0.065$

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