Probabilistic Program Analysis

Symbolic Execution + Model Counting + Reinforcement Learning

Matthew B. Dwyer

Department of Computer Science and Engineering
University of Nebraska - Lincoln
Lincoln, Nebraska USA

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Program Correctness

How do you tell if your program works correctly?

(hint: this is a two part answer)

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- Provide evidence that the program's behavior matches the specification

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- testing: $completeTriple(3, 4) = c \iff \sqrt{3^2 + 4^2} = \sqrt{25} = 5 = c \lor \ldots$

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Wouldn't it be nice if you could systematically check all/most of the program's behavior against the specification?

Program analysis in a nutshell

Consider the following program fragment:

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int classify(int a, int b, int c) {
  if (a==b)
    type+=1;
  if (a==c)
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  assert type != 6; ...
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A **set** of program traces

$$\{[1,2,4,6,8],\ [1,2,3,4,6,8],\ [1,2,4,5,6,8],\ [1,2,4,6,7,8],\ \ldots\}$$

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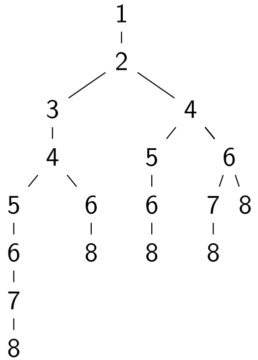
A **tree** of program traces



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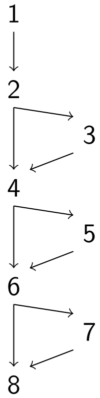

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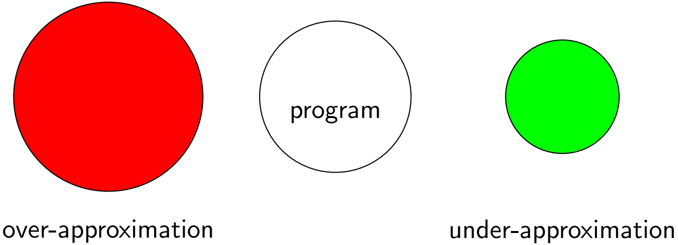

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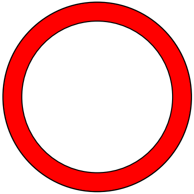
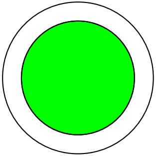


extra behavior



missing behavior

Where is the overapproximation?

```
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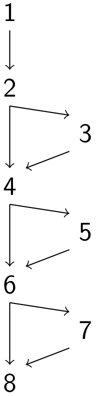

Underapproximating Symbolic Execution

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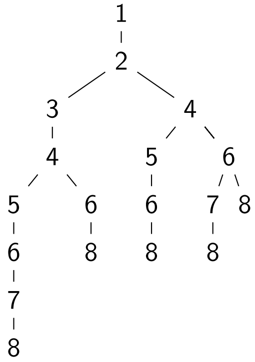

Basic symbolic execution algorithm

Algorithm 1 `symbolicExecute`(l, ϕ, m)

while $\neg branch(l)$ **do**
    $m \leftarrow m\langle v, e\rangle$
    $l \leftarrow next(l)$
end while
$c \leftarrow m[cond(l)]$
if $\mathrm{SAT}(\phi \wedge c)$ **then**
    `symbolicExecute`$(target(l), \phi \wedge c, m)$
end if
if $\mathrm{SAT}(\phi \wedge \neg c)$ **then**
    `symbolicExecute`$(next(l), \phi \wedge \neg c, m)$
end if

A simple example ...

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int classify(int a, int b, int c) {
  if (a<=0 || b<=0 || c<=0) return 4;
  int type=0;
  if (a==b) type+=1;
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  if (type==0) {
    if (a+b<=c || b+c<=a || a+c>=b) type=4;
    else type=1;
    return type;
  }
  if (type>3) type=3;
  else if (type==1 && a+b>c) type=2;
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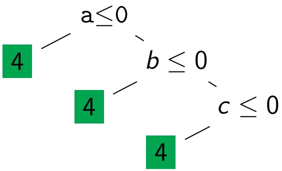
Symbolic execution tree



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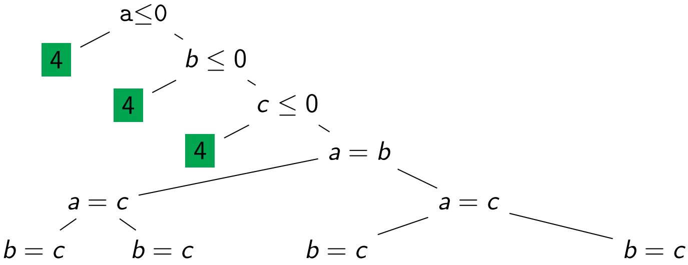

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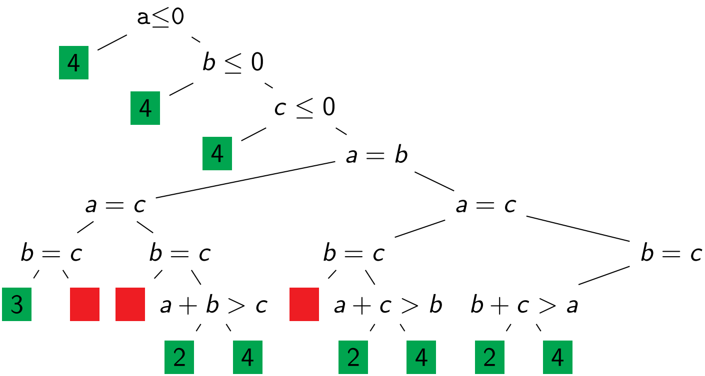

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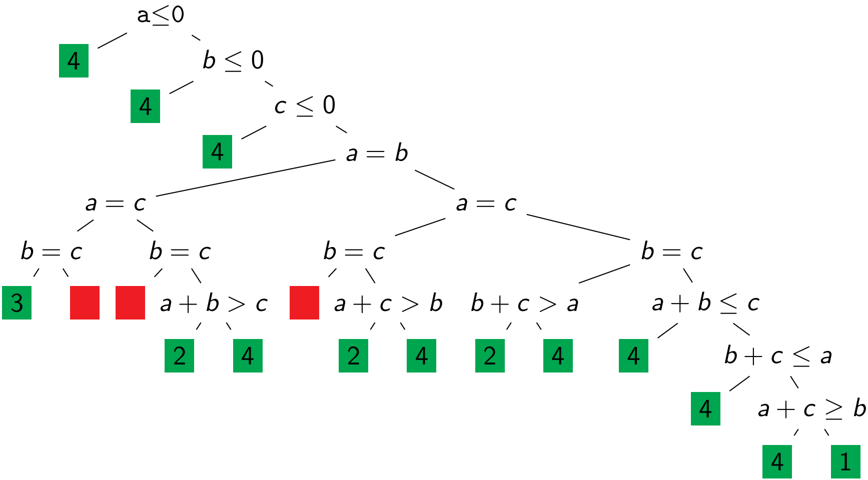

Some observations

- There are 14 distinct paths (Green): from 1 to 9 branches
- 1 path returns "scalene" (1); 3 return "isoscelese" (2); and 1 returns "equilateral" (3)
- 3 paths are pruned because constraints are unsat (Red)
- Interesting symmetries are involved in the "isoscelese" and unsat cases

Analyzing program behavior

The symbolic execution could be used to ...

- check contracts, e.g., if $a = b = c$ then $return = 3$
- generate a suite of 14 thorough tests
- demonstrate that it is possible to return values $[1, 4]$

... and support answering other yes/no questions about program behavior.

Moving beyond yes/no questions

We are interested in exploring how to ...

- determine the chance that a contract holds (1 is a special case)
- focus testing on *rare* paths (likely ones are easy to hit)
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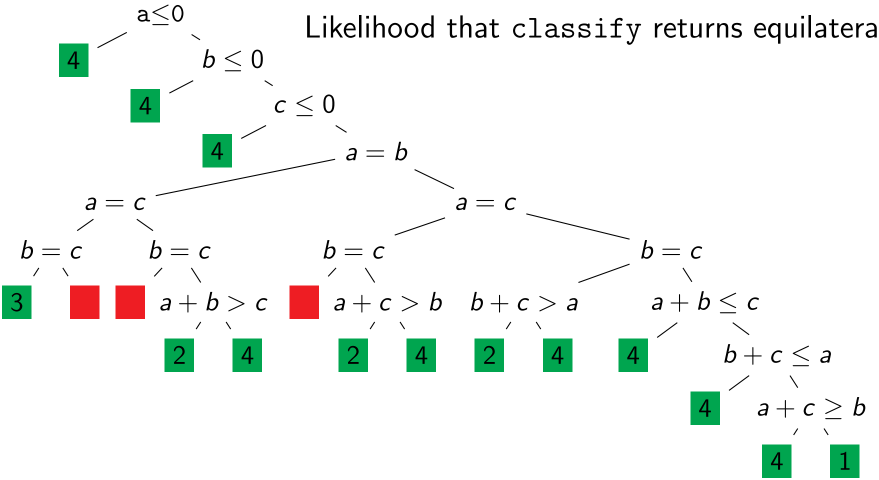
When yes/no questions cannot be answered can we quantify what was discovered during program analysis?

Assume `ints` are drawn uniformly from $[-1000, 1000]$

Likelihood that `classify` returns equilateral?



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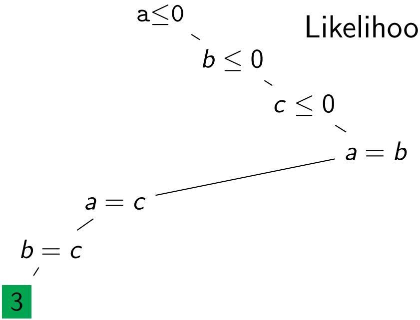

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$b \leq 0$

$c \leq 0$

$a = b$

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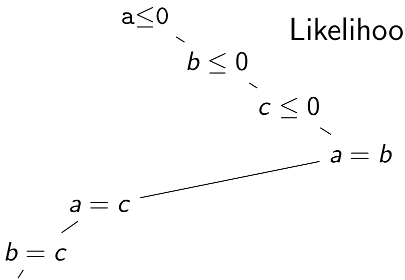

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$1.25e^{-7}$

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  - returns a value, reaches a statement, ...
- Adapting symbolic execution to perform these computations involves ...
  - calculating the paths of interest
  - calculating the probability of taking those paths
  - combining those probabilities appropriately

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Probabilistic symbolic execution algorithm

Algorithm 2 `probSymbolicExecute`$(l, \phi, m, \boxed{\texttt{p}})$

while $\neg branch(l)$ **do**
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  $l \leftarrow next(l)$
end while
$c \leftarrow m[cond(l)]$
$\boxed{\phi' \leftarrow slice(\phi, c)}$
$\boxed{p_c \leftarrow prob(\phi' \wedge c)/prob(\phi')}$
if $\boxed{p_c > 0}$ **then**
  `probSymbolicExecute`$(target(l), \phi \wedge c, m, \boxed{p * p_c})$
end if
if $\boxed{p_c < 1}$ **then**
  `probSymbolicExecute`$(next(l), \phi \wedge \neg c, m, \boxed{p * (1 - p_c)})$
end if

Key algorithmic features

Slicing the path condition, i.e., $\mathit{slice}(\phi, c)$

- reduces formula size which reduces cost of $\mathit{prob}(\cdot)$
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- requires model counting of path condition
- determines satisfiability of branches, i.e., $p_c > 0 \implies SAT(\phi \wedge c)$
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This allows the algorithm to compute path probabilities cost-effectively.

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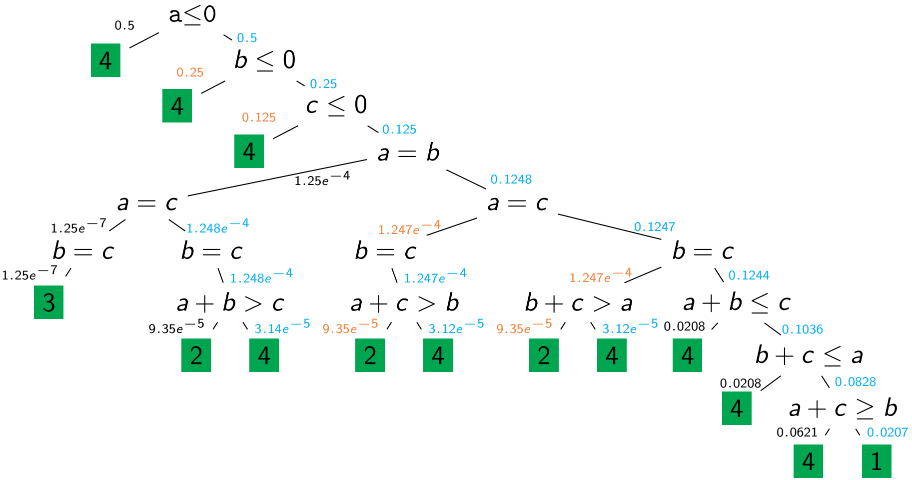
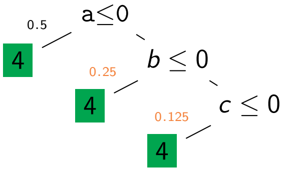


29 branches: 8 counting queries, 6 reused, 15 inferred

Probabilistic symbolic execution tree



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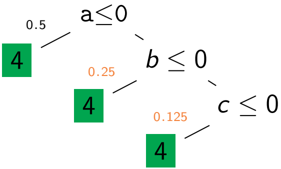

$$slice(true, a \leq 0) = true$$

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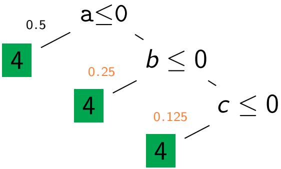

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$$p_c = prob(a \leq 0)/prob(true)$$

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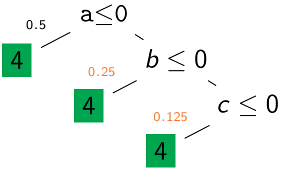

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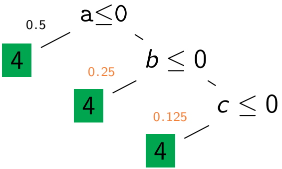

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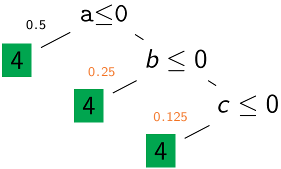

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Normalization of constraints, e.g., $a \mapsto v_1$, $c \mapsto v_1$, enables reuse in calculating p_c

Calculating $prob(\cdot)$

We report on support for linear integer arithmetic (LIA) constraints using LattE

- computes the number of *lattice* points in a convex polytope;
- constraints encoded as system of inequalities, $Ax \leq B$;
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Our calculation relies on "counting" the number of solutions of a set of related constraints using LattE and combining the results.

- $count = count_{\wedge}(\bigwedge_{ineqSet}) - count_{\vee}(\bigvee_{exSet})$
- `return` $count / \prod_{v \in vars} dom(v)$

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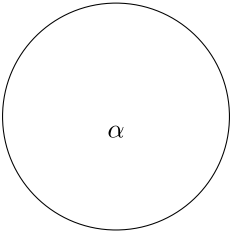

Count the solutions to α

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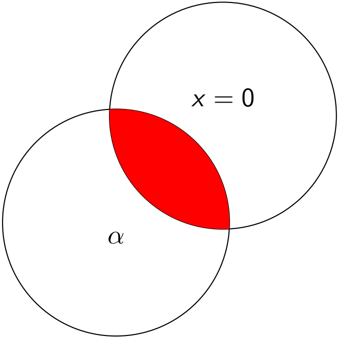

Remove the count of solutions to $\alpha \wedge (x = 0)$

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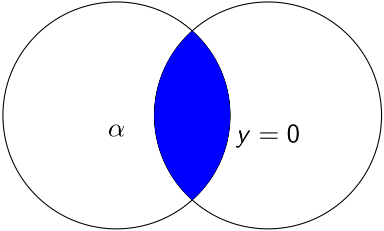

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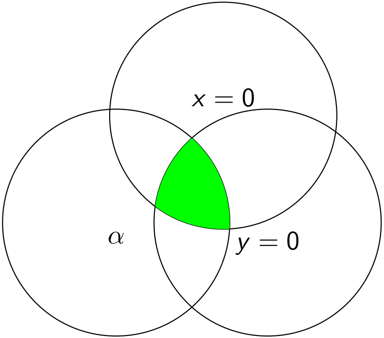

Add back the count of solutions to $\alpha \wedge (x = 0) \wedge (y = 0)$

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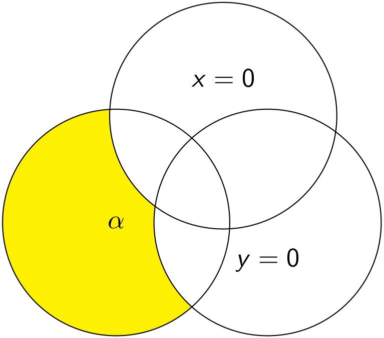

This results in the count of $\alpha \wedge (x \neq 0) \wedge (y \neq 0)$

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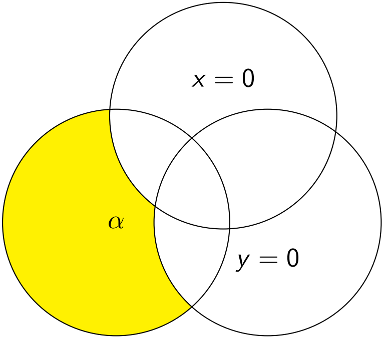

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Complexity is exponential in number of disequality constraints

Optimizing $count_{\wedge}(\cdot)$

Our experience with LattE revealed that its execution time

- is not dependent on the size of variable domains;
- is highly dependent on the number of variables (dimension of the polytope);
- is highly dependent on the number of constraints (faces of the polytope);

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Our implementation of $count_{\wedge}(\cdot)$

- normalizes the inequality system
- caches the counts computed for each system; and
- checks the cache before invoking LattE.

How well does this work?

Our ISSTA 2012 paper describes several usage scenarios ...

- finding a bug by looking at anomalies in path probabilities;
- assessing the probability of covering lines of code; and
- characterizing the likelihood of detected bugs

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I'll focus on the necessity of the optimizations we've developed.

Memoization and Slicing are key

- Ran on Binomial Heap and TreeMap collections with all possible input sequences (adds, removes, etc.) of length 4;
- times reported in seconds

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Subject	Memoize	Slice	PC Red.	Var Red.	*prob*(·)	LattE	Reused	LattE time	Total time
Binomial	✔	✔	55%	67%	634	518	370	35	57
	✗	✔	55%	67%	634	888	0	61	84
	✔	✗	0%	0%	634	3160	698	388	414
TreeMap	✔	✔	44%	55%	766	2264	562	118	145
	✗	✔	44%	55%	766	2826	0	150	178
	✔	✗	0%	0%	766	12108	4965	1028	1056

Probabilistic Symbolic Execution for Concurrency

Non-deterministic choice is used to model a lack of information

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Two additional challenges

- What conditional probability should we use for a non-deterministic choice?
- State space explosion

We use a sampling-based analysis with reinforcement learning

Probabilistic Symbolic Execution for Concurrency

View program's behavior as a tree-structured Markov Decision Process

- symbolic execution tree with non-deterministic choice nodes

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Fastest and most precise method to date (submitted to ASE 2014)

Availability and Ongoing work

An SPF extension that implements our approach is available

- http://probsym.googlecode.com
- improved over results in paper; uses the Green solver interface
- http://green-solver.googlecode.com
- multiple MDP-based algorithms (exact and sampling based)

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We have added ...

- confidence-based model counting when exact counters are inefficient (PLDI'14);
- support for user defined input probability distributions; and
- parallelism to exploit sample independence

Probabilistic Program Analysis

Symbolic Execution + Model Counting + Reinforcement Learning

Matthew B. Dwyer

Department of Computer Science and Engineering
University of Nebraska - Lincoln
Lincoln, Nebraska USA

Summer 2014