

Name _____

This in-class exam is worth 90 points. SHOW ALL WORK ON THE EXAM IN ORDER TO RECEIVE CREDIT. For problems that require solving systems of linear equations, you must use augmented matrices and put them in reduced row echelon form. You must use the techniques that were discussed in class in order to receive full credit. All determinants must be evaluated using cofactor expansion. No credit for guessing. NO CALCULATORS OR CELL PHONES. NO NOTES. You are on the DVC honor code. You may not work with classmates. Cameras must be on for the duration of the exam in order to receive credit

1. Given the function below

$$\langle \mathbf{u}, \mathbf{v} \rangle = u_1 v_1 - 2u_2 v_2 + u_3 v_3, \text{ where } \mathbf{u} = (u_1, u_2, u_3) \text{ \& } \mathbf{v} = (v_1, v_2, v_3)$$

This operation does not define an *inner product* on \mathbb{R}^3

a) Prove Axiom 2 holds in the definition of Inner Product Space for all vectors in \mathbb{R}^3

note: Axiom 2: $\langle \mathbf{u}, \mathbf{v} + \mathbf{w} \rangle = \langle \mathbf{u}, \mathbf{v} \rangle + \langle \mathbf{u}, \mathbf{w} \rangle$

1A) let $\tilde{\mathbf{w}} = (w_1, w_2, w_3)$

$$\begin{aligned} \langle \tilde{\mathbf{u}}, \tilde{\mathbf{v}} + \tilde{\mathbf{w}} \rangle &= \langle (u_1, u_2, u_3), (v_1, v_2, v_3) + (w_1, w_2, w_3) \rangle \\ &= \langle (u_1, u_2, u_3), (v_1 + w_1, v_2 + w_2, v_3 + w_3) \rangle \\ &= u_1(v_1 + w_1) - 2u_2(v_2 + w_2) + u_3(v_3 + w_3) \\ &= u_1 v_1 + u_1 w_1 - 2u_2 v_2 - 2u_2 w_2 + u_3 v_3 + u_3 w_3 \\ &= u_1 v_1 - 2u_2 v_2 + u_3 v_3 + u_1 w_1 - 2u_2 w_2 + u_3 w_3 \\ &= \langle \tilde{\mathbf{u}}, \tilde{\mathbf{v}} \rangle + \langle \tilde{\mathbf{u}}, \tilde{\mathbf{w}} \rangle \end{aligned}$$

b) Show by counterexample that Axiom 4 does not hold

Note: Axiom 4: $\langle \mathbf{u}, \mathbf{u} \rangle \geq 0$ and $\langle \mathbf{u}, \mathbf{u} \rangle = 0$ iff $\mathbf{u} = \mathbf{0}$

1B) let $\tilde{\mathbf{v}} = (1, 4, 1)$

$$\langle \tilde{\mathbf{v}}, \tilde{\mathbf{v}} \rangle = 1^2 - 2 \cdot 4^2 + 1^2 < 0 \quad \therefore \langle \tilde{\mathbf{v}}, \tilde{\mathbf{v}} \rangle \geq 0 \text{ fails}$$