workshop.nectric.com.au/tidyfc2024

Time Series Analysis & Forecasting Using R

8. ARIMA models

tsibble feasts fable

Outline

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ARIMA models

AR: autoregressive (lagged observations as inputs)
I: integrated (differencing to make series stationary)
MA: moving average (lagged errors as inputs)

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An ARIMA model is rarely interpretable in terms of visible data structures like trend and seasonality. But it can capture a huge range of time series patterns.

Stationarity

Definition

If $\{y_t\}$ is a stationary time series, then for all s, the distribution of $(y_t, \dots, y_{t+s})$ does not depend on t.

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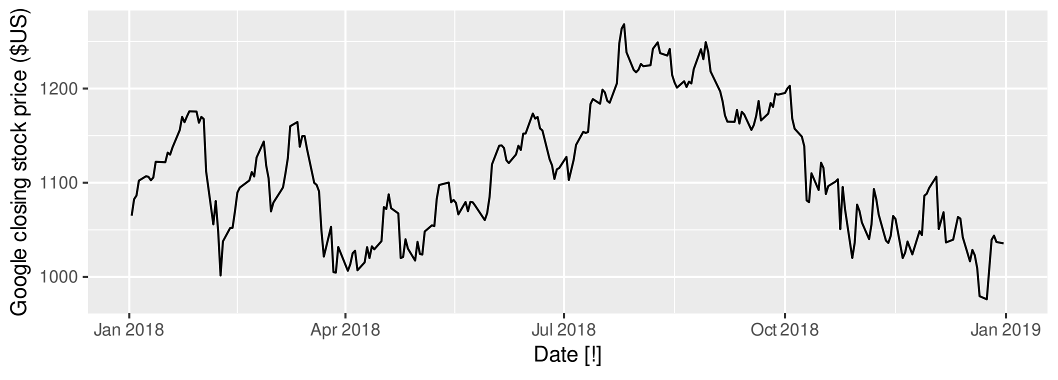
A **stationary series** is:

- roughly horizontal
- constant variance
- no patterns predictable in the long-term

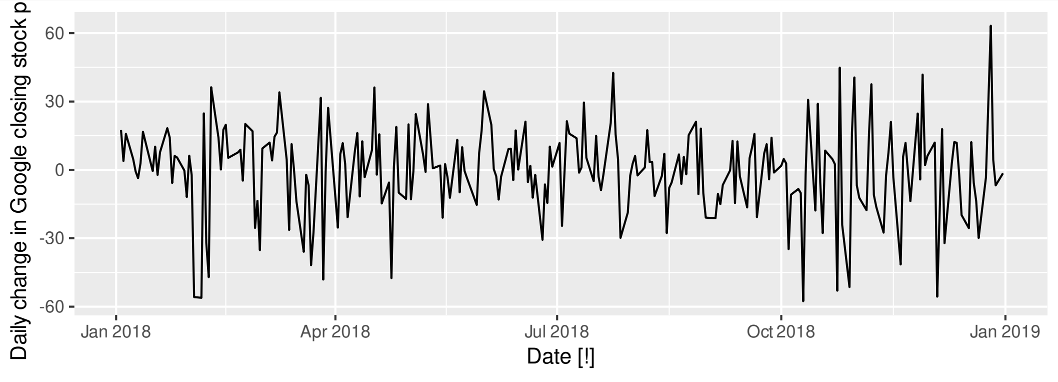
Stationary?

```
gafa_stock |>
  filter(Symbol == "GOOG", year(Date) == 2018) |>
  autoplot(Close) +
  labs(y = "Google closing stock price ($US)")
```



Stationary?

```
gafa_stock |>
  filter(Symbol == "GOOG", year(Date) == 2018) |>
  autoplot(difference(Close)) +
  labs(y = "Daily change in Google closing stock price")
```



Differencing

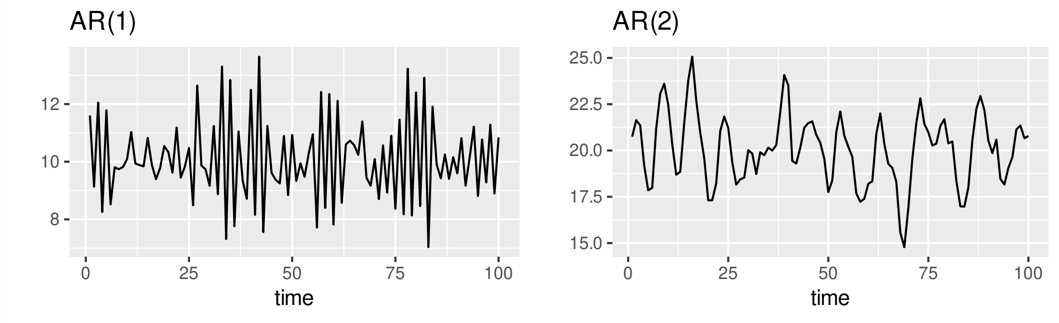
- Differencing helps to **stabilize the mean**.
- The differenced series is the *change* between each observation in the original series.
- Occasionally the differenced data will not appear stationary and it may be necessary to difference the data a second time.
- In practice, it is almost never necessary to go beyond second-order differences.

Autoregressive models

Autoregressive (AR) models:

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t,$$

where ε_t is white noise. A multiple regression with **lagged values** of y_t as predictors.

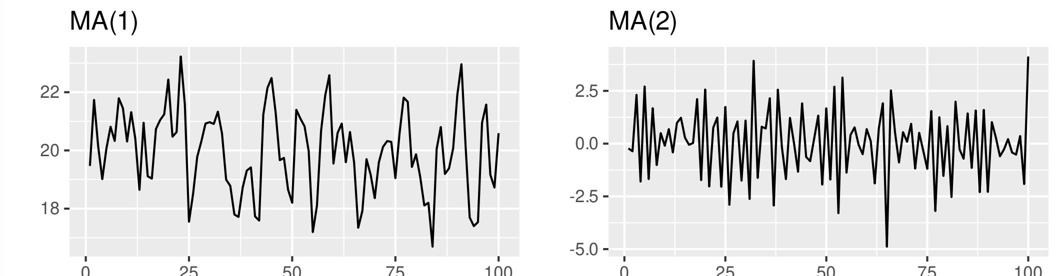


Moving Average (MA) models

Moving Average (MA) models:

$$y_t = c + \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \cdots + \theta_q\varepsilon_{t-q},$$

where ε_t is white noise. A multiple regression with **lagged *errors*** as predictors. *Don't confuse with moving average smoothing!*



ARIMA models

Autoregressive Moving Average models:

$$
\begin{aligned}
y_t = c + \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} \\
+ \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} + \varepsilon_t.
\end{aligned}
$$

ARIMA models

Autoregressive Moving Average models:

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- Predictors include both **lagged values of y_t and lagged errors.**

ARIMA models

Autoregressive Moving Average models:

$$\begin{aligned} y_t = c + \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} \\ + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} + \varepsilon_t. \end{aligned}$$

- Predictors include both **lagged values of y_t and lagged errors.**

Autoregressive Integrated Moving Average models

- Combine ARMA model with **differencing**.
- d-differenced series follows an ARMA model.
- Need to choose p, d, q and whether or not to include c.

ARIMA models

ARIMA(p, d, q) model

AR:	p = order of the autoregressive part
I:	d = degree of first differencing involved
MA:	q = order of the moving average part.

- White noise model: ARIMA(0,0,0)
- Random walk: ARIMA(0,1,0) with no constant
- Random walk with drift: ARIMA(0,1,0) with const.
- AR(p): ARIMA(p,0,0)
- MA(q): ARIMA(0,0,q)

Example: National populations

```
fit <- global_economy |>
  model(arima = ARIMA(Population))
fit
```

```
# A mable: 263 x 2
# Key:     Country [263]
   Country                                    arima
   <fct>                                    <model>
 1 Afghanistan                        <ARIMA(4,2,1)>
 2 Albania                            <ARIMA(0,2,2)>
 3 Algeria                            <ARIMA(2,2,2)>
 4 American Samoa                     <ARIMA(2,2,2)>
 5 Andorra               <ARIMA(2,1,2) w/ drift>
 6 Angola                             <ARIMA(4,2,1)>
 7 Antigua and Barbuda <ARIMA(2,1,2) w/ drift>
 8 Arab World                         <ARIMA(0,2,1)>
 9 Argentina                          <ARIMA(2,2,2)>
```

Example: National populations

```
fit |>
  filter(Country == "Australia") |>
  report()
```

```
Series: Population
Model: ARIMA(0,2,1)

Coefficients:
         ma1
      -0.661
s.e.   0.107

sigma^2 estimated as 4.063e+09:  log likelihood=-699
AIC=1401   AICc=1402   BIC=1405
```

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```

$$y_t = 2y_{t-1} - y_{t-2} - 0.7\varepsilon_{t-1} + \varepsilon_t$$
$$\varepsilon_t \sim \text{NID}(0, 4 \times 10^9)$$

Understanding ARIMA models

- If $c = 0$ and $d = 0$, the long-term forecasts will go to zero.
- If $c = 0$ and $d = 1$, the long-term forecasts will go to a non-zero constant.
- If $c = 0$ and $d = 2$, the long-term forecasts will follow a straight line.
- If $c \neq 0$ and $d = 0$, the long-term forecasts will go to the mean of the data.
- If $c \neq 0$ and $d = 1$, the long-term forecasts will follow a straight line.
- If $c \neq 0$ and $d = 2$, the long-term forecasts will follow a quadratic trend.

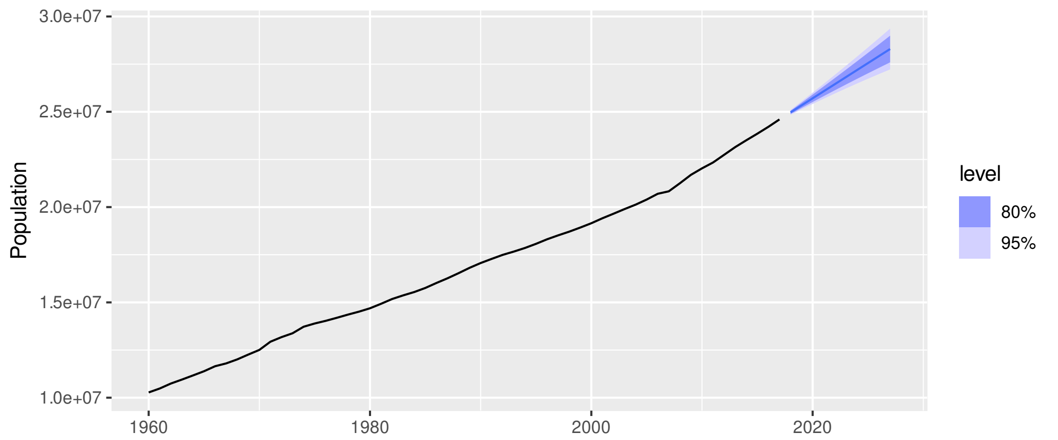
Understanding ARIMA models

Forecast variance and d

- The higher the value of d, the more rapidly the prediction intervals increase in size.
- For $d = 0$, the long-term forecast standard deviation will go to the standard deviation of the historical data.

Example: National populations

```
fit |>
  forecast(h = 10) |>
  filter(Country == "Australia") |>
  autoplot(global_economy)
```



How does ARIMA() work?

Hyndman and Khandakar (JSS, 2008) algorithm:

- Select no. differences d via KPSS test.
- Select p, q and inclusion of c by minimising AICc.
- Use stepwise search to traverse model space.

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$$\text{AICc} = -2\log(L) + 2(p+q+k+1)\left[1 + \frac{(p+q+k+2)}{T-p-q-k-2}\right]$$

where L is the maximised likelihood fitted to the *differenced* data, $k = 1$ if $c \neq 0$ and $k = 0$ otherwise.

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where L is the maximised likelihood fitted to the *differenced* data, $k = 1$ if $c \neq 0$ and $k = 0$ otherwise.

Note: Can't compare AICc for different values of d.

How does ARIMA() work?

Step1: Select current model (with smallest AICc) from:

ARIMA$(2, d, 2)$
ARIMA$(0, d, 0)$
ARIMA$(1, d, 0)$
ARIMA$(0, d, 1)$

How does ARIMA() work?

Step1: Select current model (with smallest AICc) from:
ARIMA$(2, d, 2)$
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Step 2: Consider variations of current model:

- vary one of $p, q,$ from current model by ± 1;
- p, q both vary from current model by ± 1;
- Include/exclude c from current model.

Model with lowest AICc becomes current model.

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Step 2: Consider variations of current model:

- vary one of $p, q,$ from current model by ± 1;
- p, q both vary from current model by ± 1;
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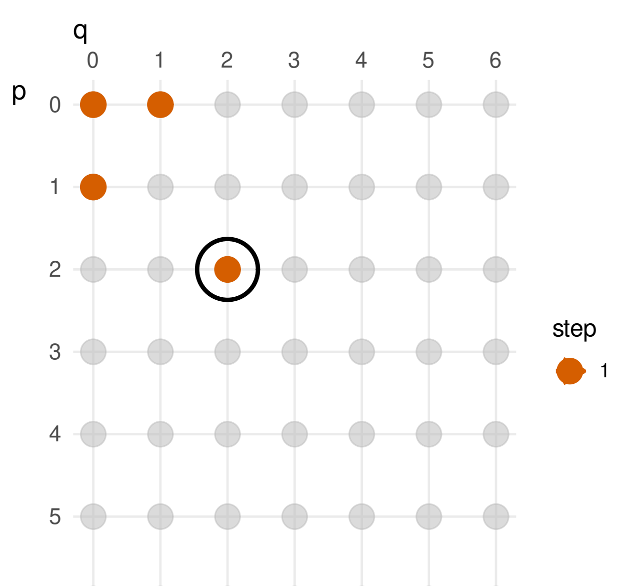
Model with lowest AICc becomes current model.

Repeat Step 2 until no lower AICc can be found.

How does ARIMA() work?



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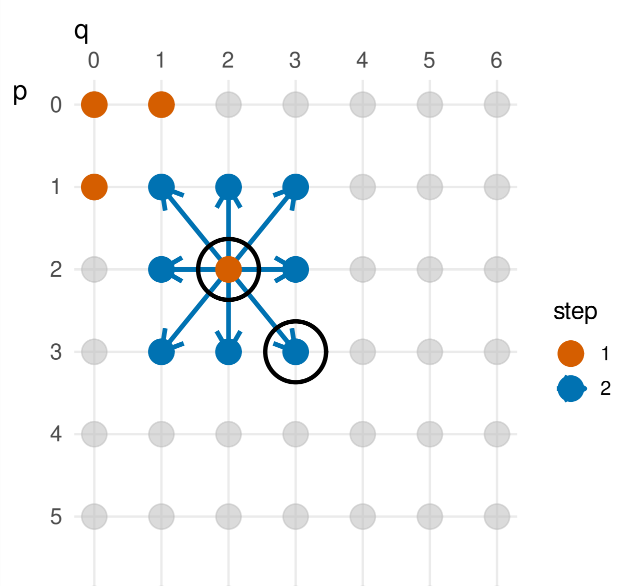

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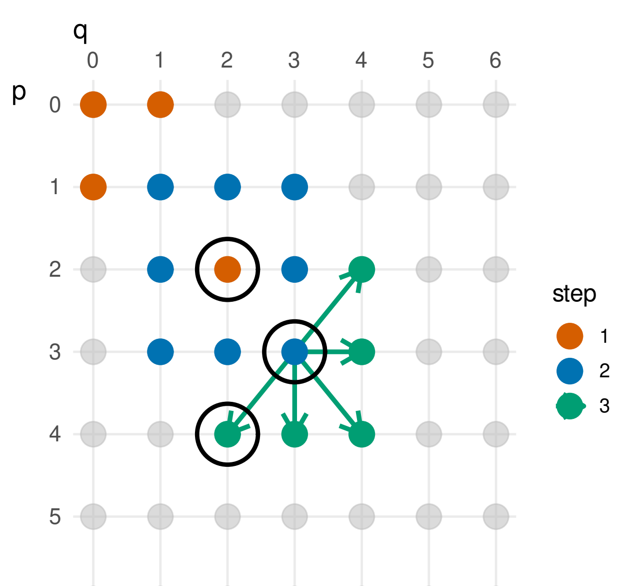
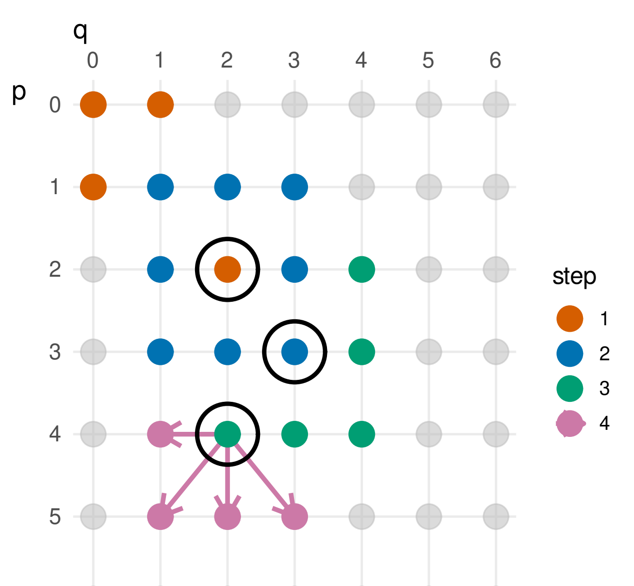


How does ARIMA() work?



Outline

Lab Session 14

For the total number of students in Government, Catholic, and Independent schools:

```
enrolment_affiliation <- students |>
  group_by(`Affiliation (Gov/Cath/Ind)`) |>
  summarise(enrolments = sum(`All Full-time and Part-time Student count`))
```

- Fit a suitable ARIMA model (including possible transformation) for the data.
- Produce forecasts of your fitted model. Do the forecasts look reasonable?

Outline

1. ARIMA models
2. Lab Session 14
3. Seasonal ARIMA models
4. Lab Session 15
5. Forecast ensembles

Seasonal ARIMA models

ARIMA	$\underbrace{(p, d, q)}$	$\underbrace{(P, D, Q)_m}$
	$\uparrow$ Non-seasonal part of the model	$\uparrow$ Seasonal part of of the model

- m = number of observations per year.
- d first differences, D seasonal differences
- p AR lags, q MA lags
- P seasonal AR lags, Q seasonal MA lags

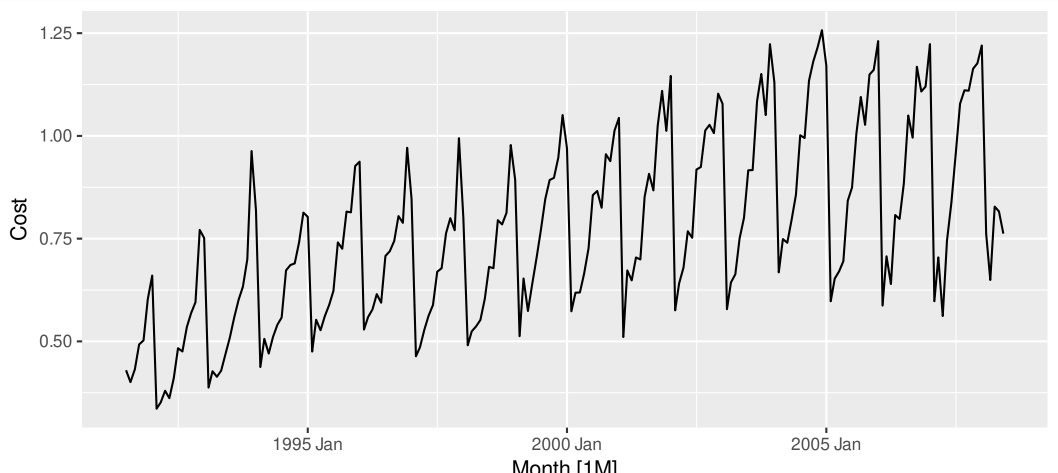
Seasonal and non-seasonal terms combine multiplicatively

Cortecosteroid medication subsidies

```r
cortecosteroid_subsidies <- PBS |>
  filter(ATC2 == "H02") |>
  summarise(Cost = sum(Cost) / 1e6)
```

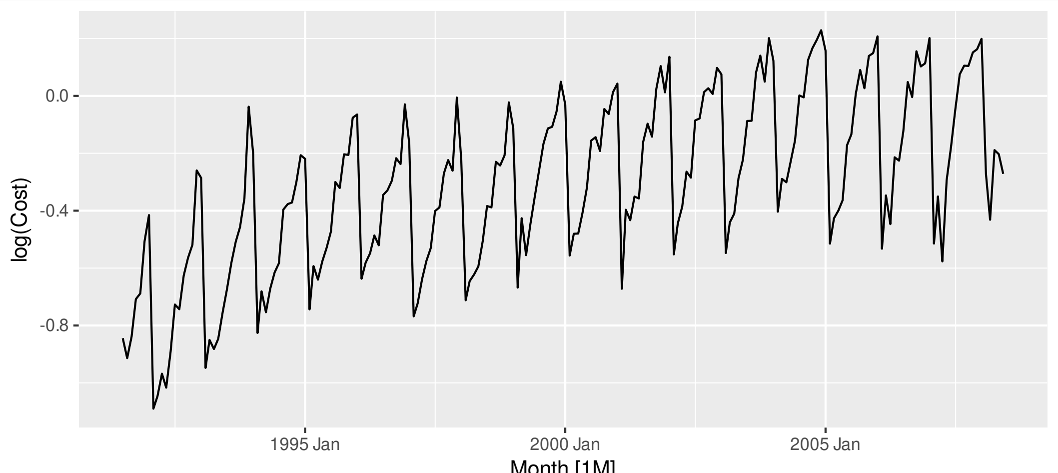
Cortecosteroid medication subsidies

```
cortecosteroid_subsidies |> autoplot(
  Cost
)
```



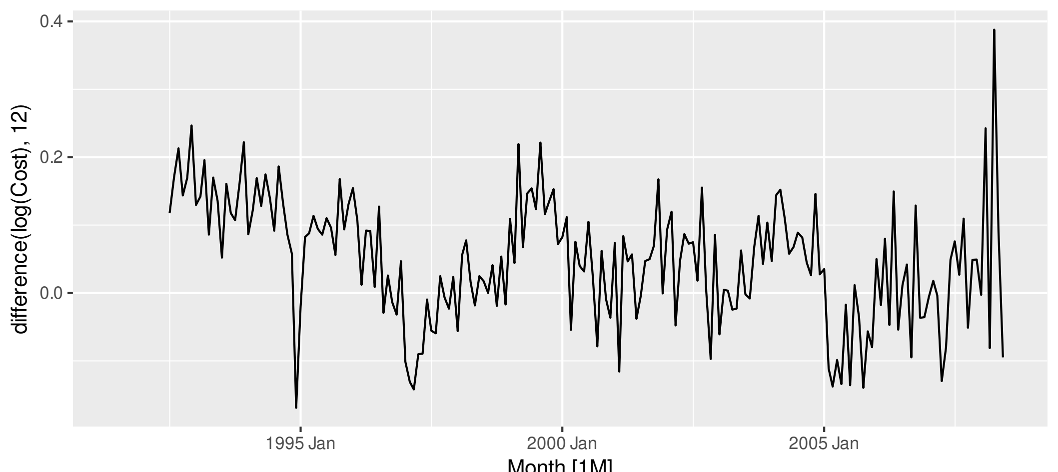
Cortecosteroid medication subsidies

```
cortecosteroid_subsidies |> autoplot(
  log(Cost)
)
```



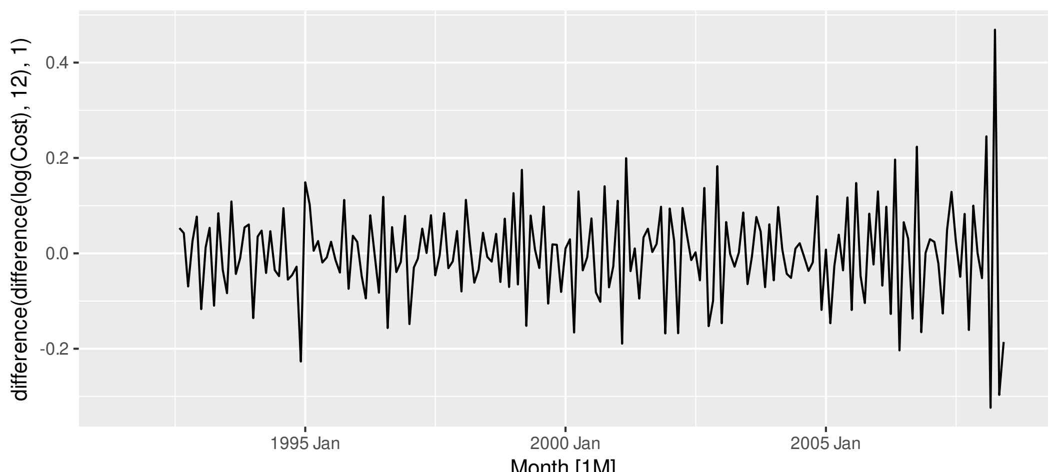
Cortecosteroid medication subsidies

```
cortecosteroid_subsidies |> autoplot(
  log(Cost) |> difference(12)
)
```



Cortecosteroid medication subsidies

```r
cortecosteroid_subsidies |> autoplot(
  log(Cost) |> difference(12) |> difference(1)
)
```



Cortecosteroid medication subsidies

```r
fit <- cortecosteroid_subsidies |>
  model(best = ARIMA(log(Cost),
    stepwise = FALSE,
    approximation = FALSE,
    order_constraint = p + q + P + Q <= 8
  ))
report(fit)
```

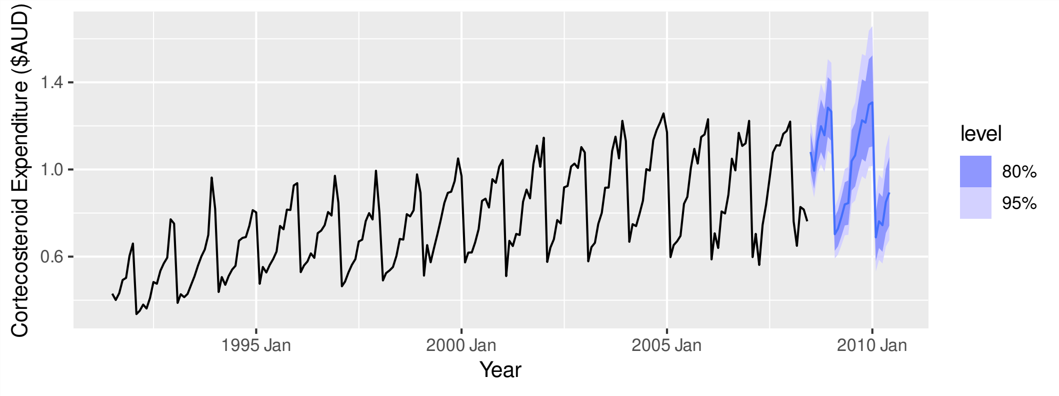
```
Series: Cost
Model: ARIMA(4,1,1)(2,1,1)[12]
Transformation: log(Cost)

Coefficients:
          ar1    ar2    ar3      ar4     ma1   sar1     sar2    sma1
      -0.0099  0.248  0.225  -0.2429  -0.776  0.110  -0.2022  -0.671
s.e.   0.1760  0.150  0.103   0.0805   0.165  0.125   0.0995   0.110

sigma^2 estimated as 0.004098:  log likelihood=253
AIC=-488   AICc=-487   BIC=-459
```

Cortecosteroid medication subsidies

```
fit |>
  forecast() |>
  autoplot(cortecosteroid_subsidies) +
  labs(y = "Cortecosteroid Expenditure ($AUD)", x = "Year")
```



Outline

Lab Session 15

Find an ARIMA model for the each school attendance type for people aged 15-24 in Australia (`student_labour`).

```
youth_learning <- student_labour |>
  group_by(attendance) |>
  summarise(persons = sum(persons))
```

- Fit suitable ARIMA models for each time series.
- Produce forecasts of your fitted models.
- Check the forecasts for each of the school `attendance` category, do they look reasonable?

Outline

Is ARIMA or ETS best?

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It depends!

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Capturing patterns

- Both handle time varying trends and seasonality.
- ETS directly captures multiplicative patterns.
- ARIMA can forecast cyclical patterns.

Is ARIMA or ETS best?

It depends!

Evaluate accuracy

We can determine which model works best on a specific dataset using accuracy evaluation.

Is ARIMA or ETS best?

It depends!

Evaluate accuracy

We can determine which model works best on a specific dataset using accuracy evaluation.

Or, combine them with ensembles!

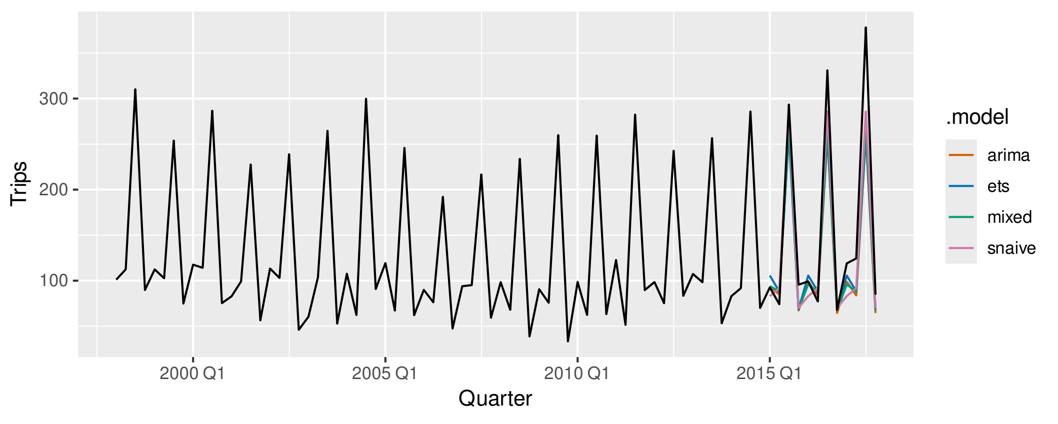
Forecast ensembles

```
train <- tourism |>
  filter(year(Quarter) <= 2014)
fit <- train |>
  model(
    ets = ETS(Trips),
    arima = ARIMA(Trips),
    snaive = SNAIVE(Trips)
  ) |>
  mutate(mixed = (ets + arima + snaive) / 3)
```

- Ensemble forecast `mixed` is a simple average of the three fitted models.
- `forecast()` will produce distributional forecasts taking into account the correlations between the forecast errors of the component models.

Forecast ensembles

```
fc <- fit |> forecast(h = "3 years")
fc |>
  filter(Region == "Snowy Mountains", Purpose == "Holiday") |>
  autoplot(tourism, level = NULL)
```



Forecast ensembles

```r
accuracy(fc, tourism) |>
  group_by(.model) |>
  summarise(
    RMSE = mean(RMSE),
    MAE = mean(MAE),
    MASE = mean(MASE)
  ) |>
  arrange(RMSE)
```

```
# A tibble: 4 x 4
  .model  RMSE   MAE  MASE
  <chr>  <dbl> <dbl> <dbl>
1 mixed   19.8  16.0 0.997
2 ets     20.2  16.4 1.00
3 snaive  21.5  17.3 1.17
4 arima   21.9  17.8 1.06
```

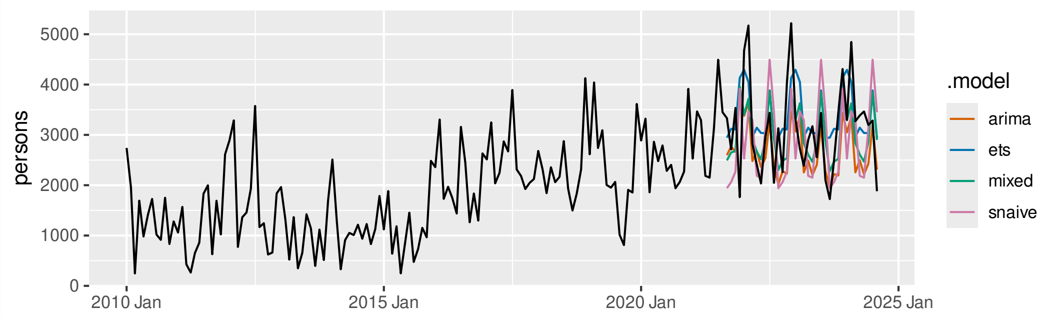
Forecast ensembles

```
train <- student_labour |>
  filter(month <= yearmonth("2021 Aug"))
fit <- train |>
  model(
    ets = ETS(persons),
    arima = ARIMA(persons),
    snaive = SNAIVE(persons)
  ) |>
  mutate(mixed = (ets + arima + snaive) / 3)
```

- Ensemble forecast `mixed` is a simple average of the three fitted models.
- `forecast()` will produce distributional forecasts taking into account the correlations between the forecast errors of the component models.

Forecast ensembles

```
fc <- fit |> forecast(h = "3 years")
fc |>
  filter(
    state == "Australian Capital Territory",
    attendance == "Attending full-time education",
    status == "Employed full-time"
  ) |>
  autoplot(student_labour |> filter(month >= yearmonth("2010 Jan")), level = NULL)
```



Forecast ensembles

```
accuracy(fc, student_labour) |>
  group_by(.model) |>
  summarise(
    RMSE = mean(RMSE),
    MAE = mean(MAE),
    MASE = mean(MASE)
  ) |>
  arrange(RMSE)
```

```
# A tibble: 4 x 4
  .model  RMSE   MAE  MASE
  <chr>  <dbl> <dbl> <dbl>
1 mixed  6783. 5835.  1.54
2 ets    6979. 5975.  1.57
3 arima  7135. 6222.  1.61
4 snaive 7729. 6485.  1.72
```