



# Time Series Analysis & Forecasting Using R

## 3. Transformations



# Outline

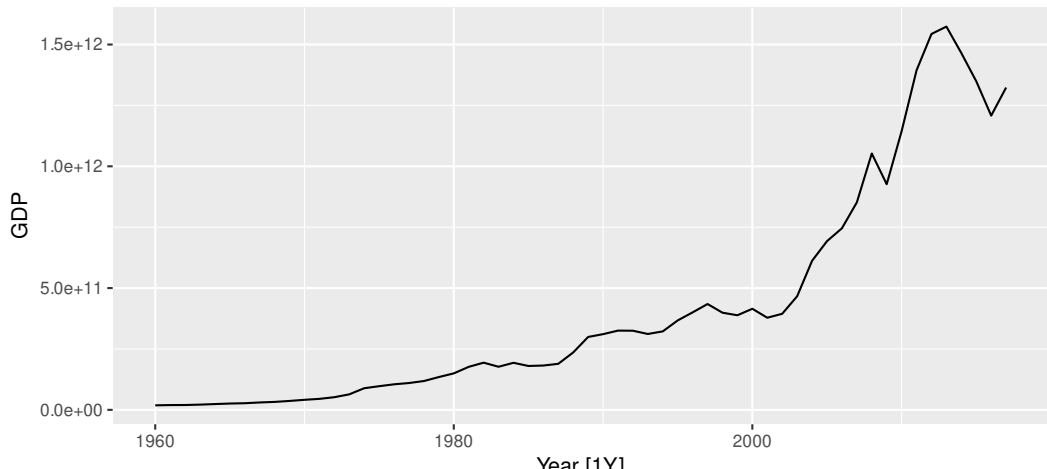
- 1 Per capita adjustments
- 2 Lab Session 6
- 3 Inflation adjustments
- 4 Mathematical transformations
- 5 Lab Session 7

# Outline

- 1 Per capita adjustments
- 2 Lab Session 6
- 3 Inflation adjustments
- 4 Mathematical transformations
- 5 Lab Session 7

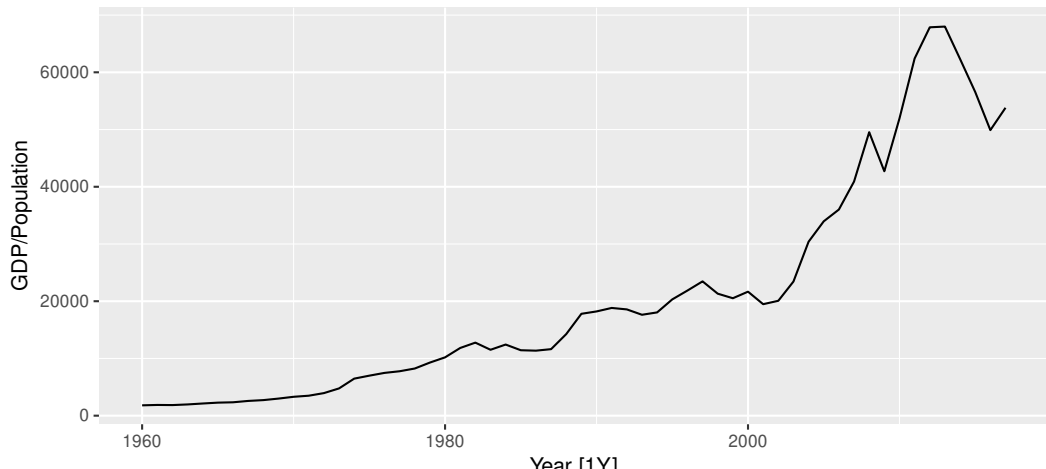
# Per capita adjustments

```
global_economy |>  
  filter(Country == "Australia") |>  
  autoplot(GDP)
```



# Per capita adjustments

```
global_economy |>  
  filter(Country == "Australia") |>  
  autoplot(GDP / Population)
```



# Outline

- 1 Per capita adjustments
- 2 Lab Session 6
- 3 Inflation adjustments
- 4 Mathematical transformations
- 5 Lab Session 7

## Lab Session 6

Consider the GDP information in `global_economy`. Plot the GDP per capita for each country over time. Which country has the highest GDP per capita? How has this changed over time?

# Outline

- 1 Per capita adjustments
- 2 Lab Session 6
- 3 Inflation adjustments**
- 4 Mathematical transformations
- 5 Lab Session 7

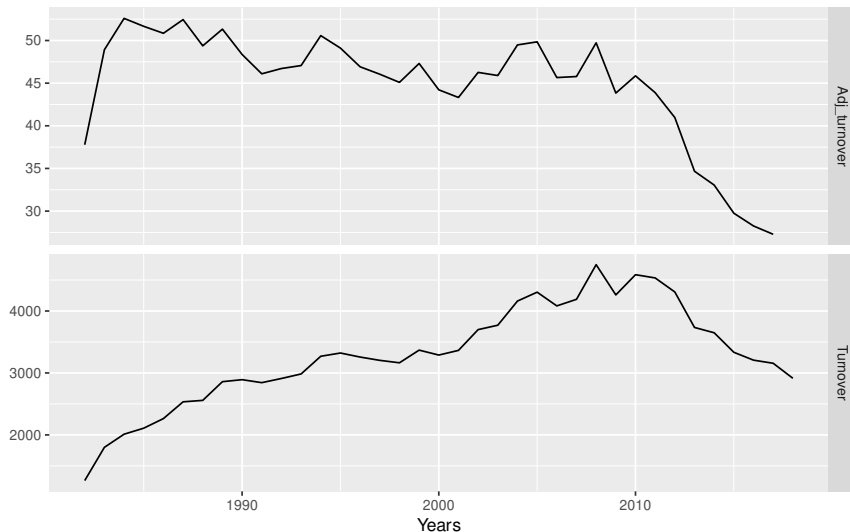


# Inflation adjustments

```
print_retail <- aus_retail |>
  filter(Industry == "Newspaper and book retailing") |>
  group_by(Industry) |>
  index_by(Year = year(Month)) |>
  summarise(Turnover = sum(Turnover))
aus_economy <- filter(global_economy, Country == "Australia")
print_retail |>
  left_join(aus_economy, by = "Year") |>
  mutate(Adj_turnover = Turnover / CPI) |>
  pivot_longer(c(Turnover, Adj_turnover),
    names_to = "Type", values_to = "Turnover"
  ) |>
  ggplot(aes(x = Year, y = Turnover)) +
  geom_line() +
  facet_grid(vars(Type), scales = "free_y") +
  labs(x = "Years", y = NULL,
    title = "Turnover: Australian print media industry")
```

# Inflation adjustments

Turnover: Australian print media industry



# Outline

- 1 Per capita adjustments
- 2 Lab Session 6
- 3 Inflation adjustments
- 4 Mathematical transformations**
- 5 Lab Session 7

# Variance stabilization

If the data show different variation at different levels of the series, then a transformation can be useful.

# Variance stabilization

If the data show different variation at different levels of the series, then a transformation can be useful.

Denote original observations as  $y_1, \dots, y_n$  and transformed observations as  $w_1, \dots, w_n$ .

# Variance stabilization

If the data show different variation at different levels of the series, then a transformation can be useful.

Denote original observations as  $y_1, \dots, y_n$  and transformed observations as  $w_1, \dots, w_n$ .

## Mathematical transformations for stabilizing variation

Square root	$w_t = \sqrt{y_t}$	↓
Cube root	$w_t = \sqrt[3]{y_t}$	Increasing
Logarithm	$w_t = \log(y_t)$	strength

# Variance stabilization

If the data show different variation at different levels of the series, then a transformation can be useful.

Denote original observations as  $y_1, \dots, y_n$  and transformed observations as  $w_1, \dots, w_n$ .

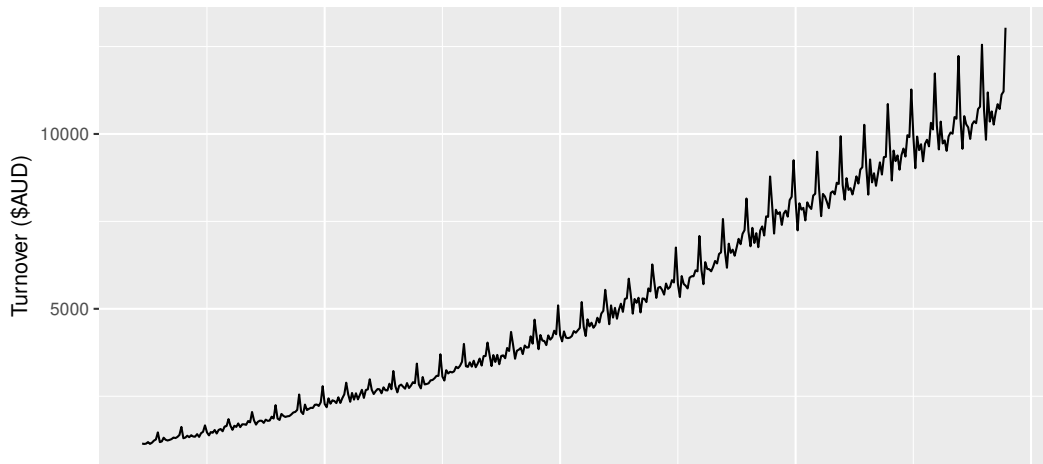
## Mathematical transformations for stabilizing variation

Square root	$w_t = \sqrt{y_t}$	↓
Cube root	$w_t = \sqrt[3]{y_t}$	Increasing
Logarithm	$w_t = \log(y_t)$	strength

Logarithms, in particular, are useful because they are more interpretable: changes in a log value are **relative (percent) changes**

# Variance stabilization

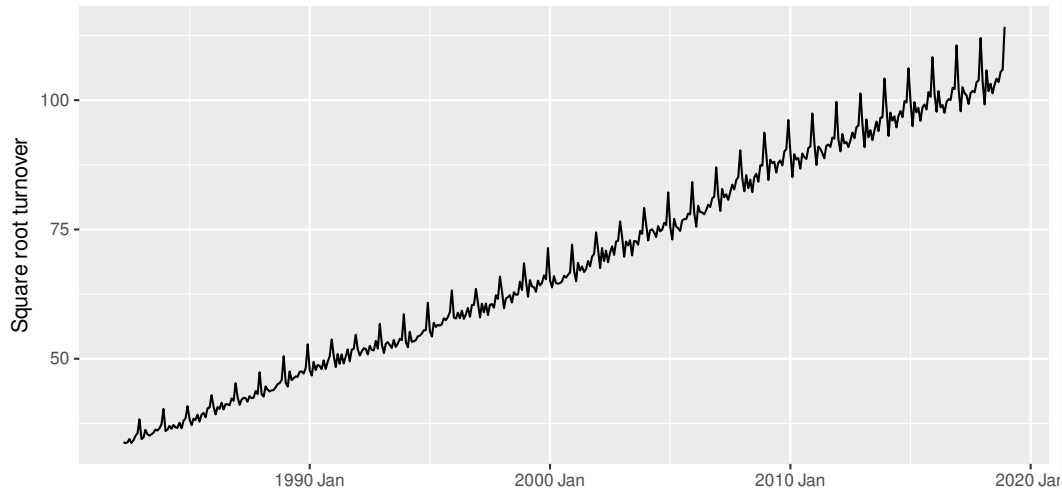
```
food <- aus_retail |>  
  filter(Industry == "Food retailing") |>  
  summarise(Turnover = sum(Turnover))
```





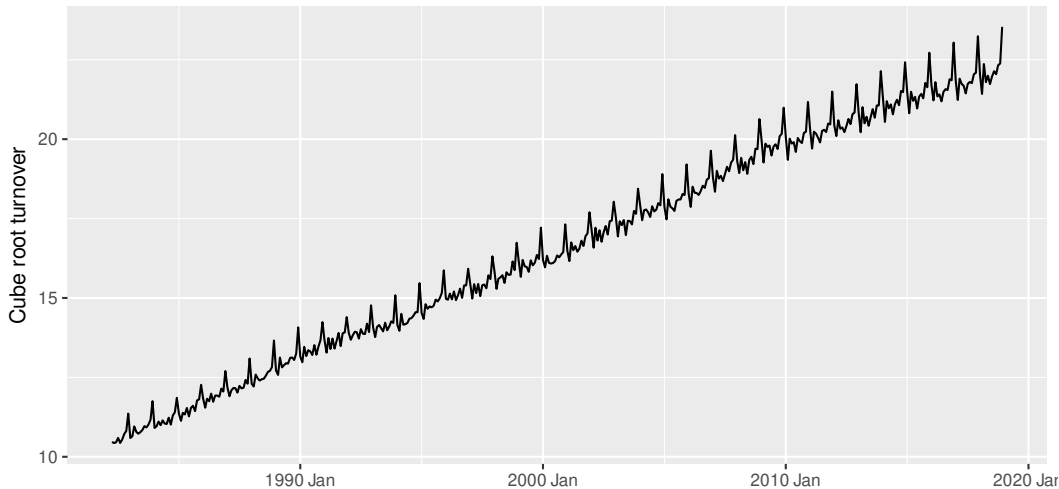
# Variance stabilization

```
food |> autoplot(sqrt(Turnover)) +  
  labs(y = "Square root turnover")
```



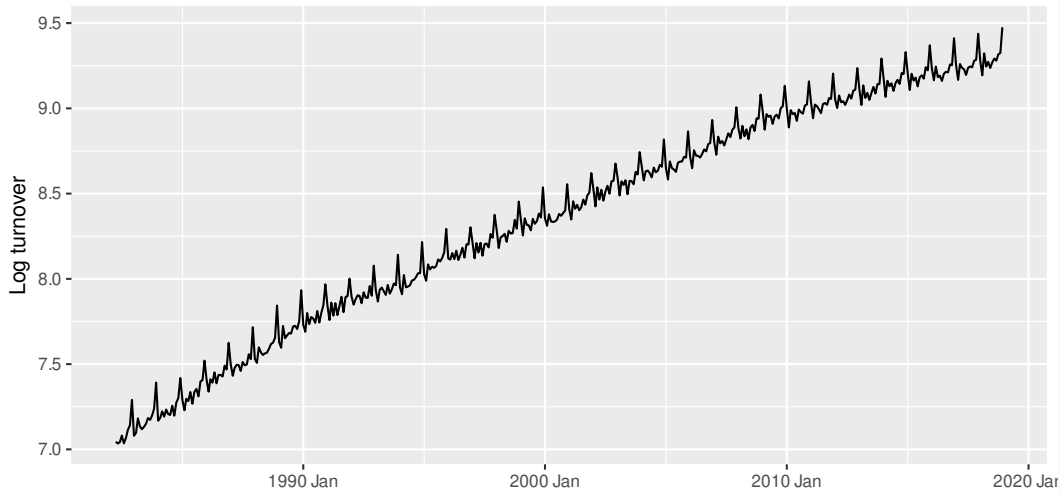
# Variance stabilization

```
food |> autoplot(Turnover^(1 / 3)) +  
  labs(y = "Cube root turnover")
```



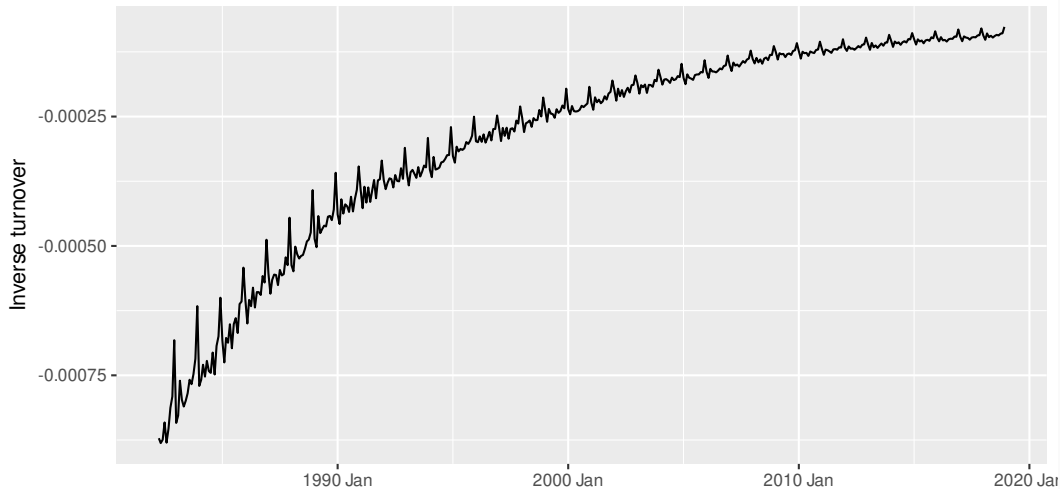
# Variance stabilization

```
food |> autoplot(log(Turnover)) +  
  labs(y = "Log turnover")
```



# Variance stabilization

```
food |> autoplot(-1 / Turnover) +  
  labs(y = "Inverse turnover")
```



# Box-Cox transformations

Each of these transformations is close to a member of the family of **Box-Cox transformations**:

$$w_t = \begin{cases} \log(y_t), & \lambda = 0; \\ (\text{sign}(y_t)|y_t|^\lambda - 1)/\lambda, & \lambda \neq 0. \end{cases}$$

# Box-Cox transformations

Each of these transformations is close to a member of the family of **Box-Cox transformations**:

$$w_t = \begin{cases} \log(y_t), & \lambda = 0; \\ (\text{sign}(y_t)|y_t|^\lambda - 1)/\lambda, & \lambda \neq 0. \end{cases}$$

- Actually the Bickel-Doksum transformation (allowing for  $y_t < 0$ )
- $\lambda = 1$ : (No substantive transformation)
- $\lambda = \frac{1}{2}$ : (Square root plus linear transformation)
- $\lambda = 0$ : (Natural logarithm)
- $\lambda = -1$ : (Inverse plus 1)

# Box-Cox transformations

# Box-Cox transformations

```
food |>  
  features(Turnover, features = guerrero)
```

```
# A tibble: 1 x 1  
  lambda_guerrero  
          <dbl>  
1             0.0895
```



# Box-Cox transformations

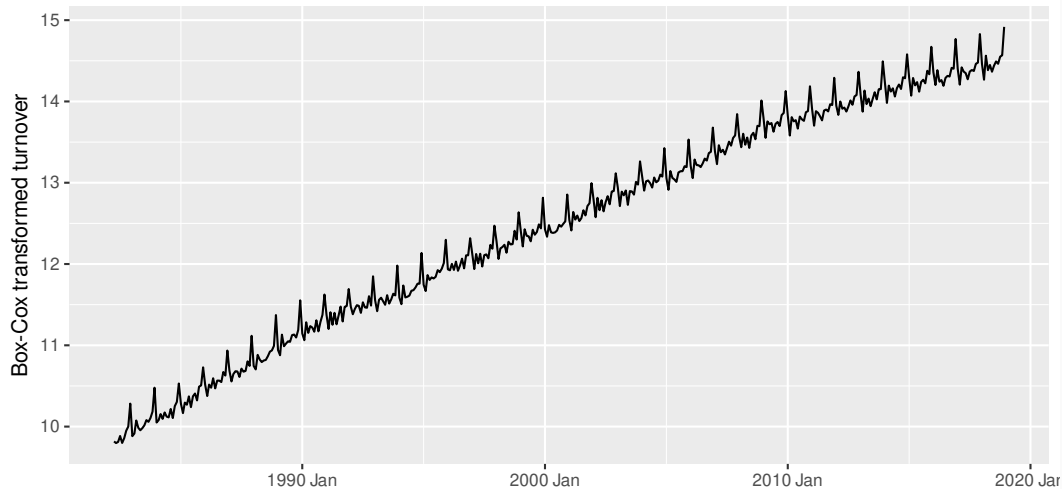
```
food |>  
  features(Turnover, features = guerrero)
```

```
# A tibble: 1 x 1  
  lambda_guerrero  
          <dbl>  
1          0.0895
```

- This attempts to balance the seasonal fluctuations and random variation across the series.
- Always check the results.
- A low value of  $\lambda$  can give extremely large prediction intervals.

# Box-Cox transformations

```
food |> autoplot(box_cox(Turnover, 0.0895)) +  
  labs(y = "Box-Cox transformed turnover")
```



# Transformations

- Often no transformation needed.
- Simple transformations are easier to explain and work well enough.
- Transformations can have very large effect on PI.
- If some data are zero or negative, then use  $\lambda > 0$ .
- `log1p()` can also be useful for data with zeros.
- Choosing logs is a simple way to force forecasts to be positive
- Transformations must be reversed to obtain forecasts on the original scale. (Handled automatically by `fable`.)

# Outline

- 1 Per capita adjustments
- 2 Lab Session 6
- 3 Inflation adjustments
- 4 Mathematical transformations
- 5 Lab Session 7

# Lab Session 7

- 1 For the following series, find an appropriate transformation in order to stabilise the variance.
  - ▶ United States GDP from `global_economy`
  - ▶ Slaughter of Victorian “Bulls, bullocks and steers” in `aus_livestock`
  - ▶ Victorian Electricity Demand from `vic_elec`.
  - ▶ Gas production from `aus_production`
- 2 Why is a Box-Cox transformation unhelpful for the `canadian_gas` data?