



Time Series Analysis & Forecasting Using R

6. Introduction to forecasting



Outline

1 Statistical forecasting

2 Benchmark methods

3 Lab Session 11

4 Residual diagnostics

5 Lab Session 12

6 Forecast accuracy measures

7 Lab Session 13

Outline

1 Statistical forecasting

2 Benchmark methods

3 Lab Session 11

4 Residual diagnostics

5 Lab Session 12

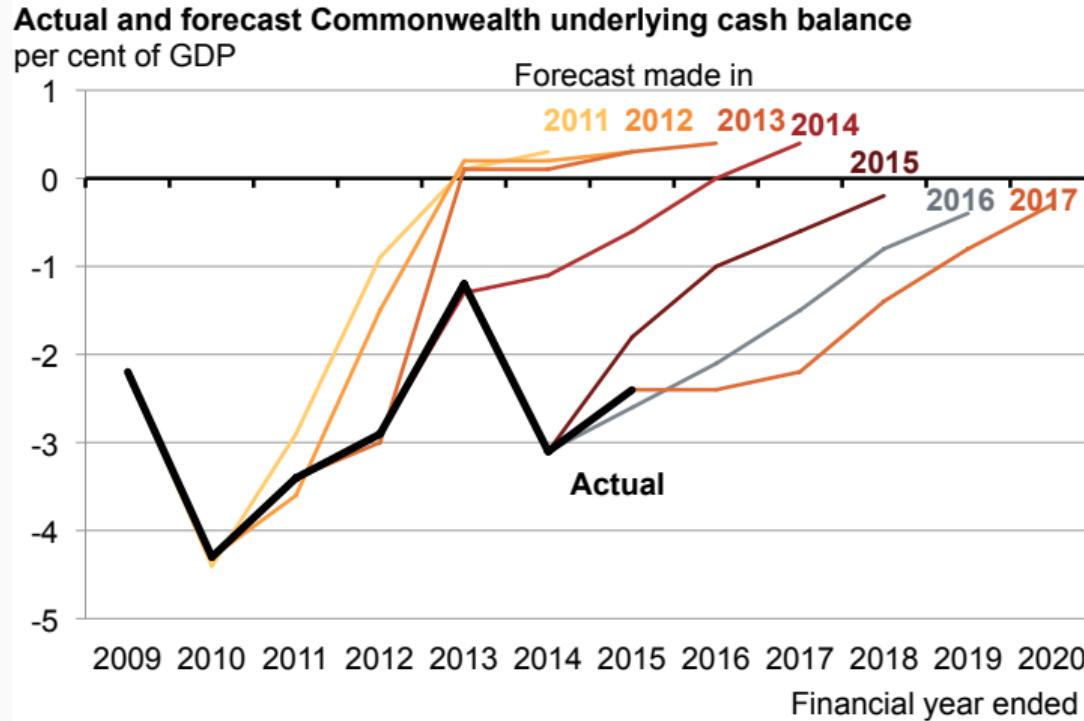
6 Forecast accuracy measures

7 Lab Session 13

Forecasting is difficult

Commonwealth plans to drift back to surplus
show the triumph of experience over hope

GRATTAN
Institute



What can we forecast?



What can we forecast?



What can we forecast?

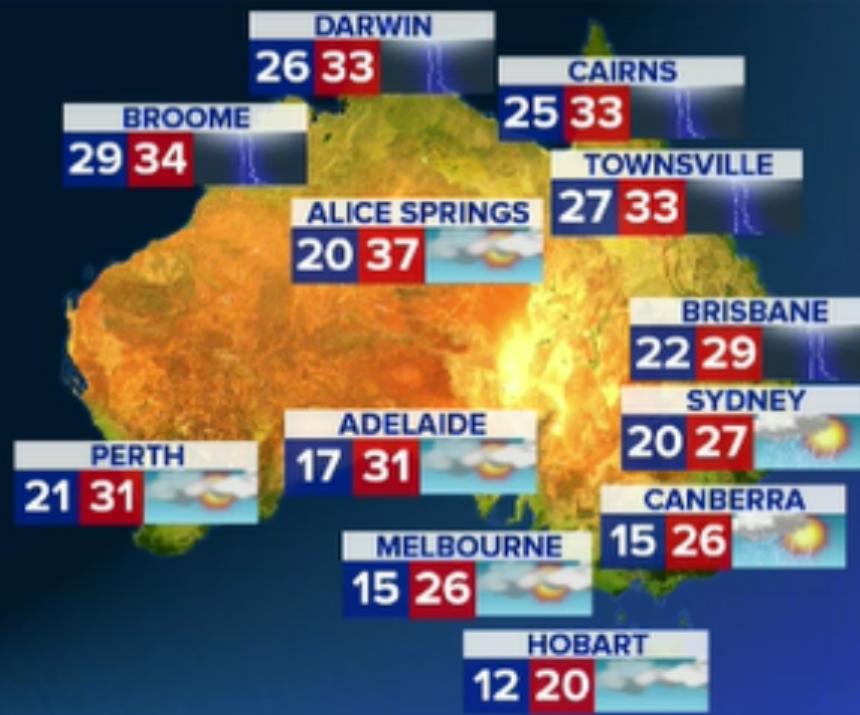


What can we forecast?



What can we forecast?

TOMORROW



What can we forecast?



What can we forecast?



Which is easiest to forecast?

- 1 daily electricity demand in 3 days time
- 2 timing of next Halley's comet appearance
- 3 time of sunrise this day next year
- 4 Google stock price tomorrow
- 5 Google stock price in 6 months time
- 6 maximum temperature tomorrow
- 7 exchange rate of \$US/AUS next week
- 8 total sales of drugs in Australian pharmacies next month

Which is easiest to forecast?

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-
- how do we measure “easiest”?
 - what makes something easy/difficult to forecast?

Factors affecting forecastability

Something is easier to forecast if:

- we have a good understanding of the factors that contribute to it
- there is lots of data available;
- the forecasts cannot affect the thing we are trying to forecast.
- there is relatively low natural/unexplainable random variation.
- the future is somewhat similar to the past

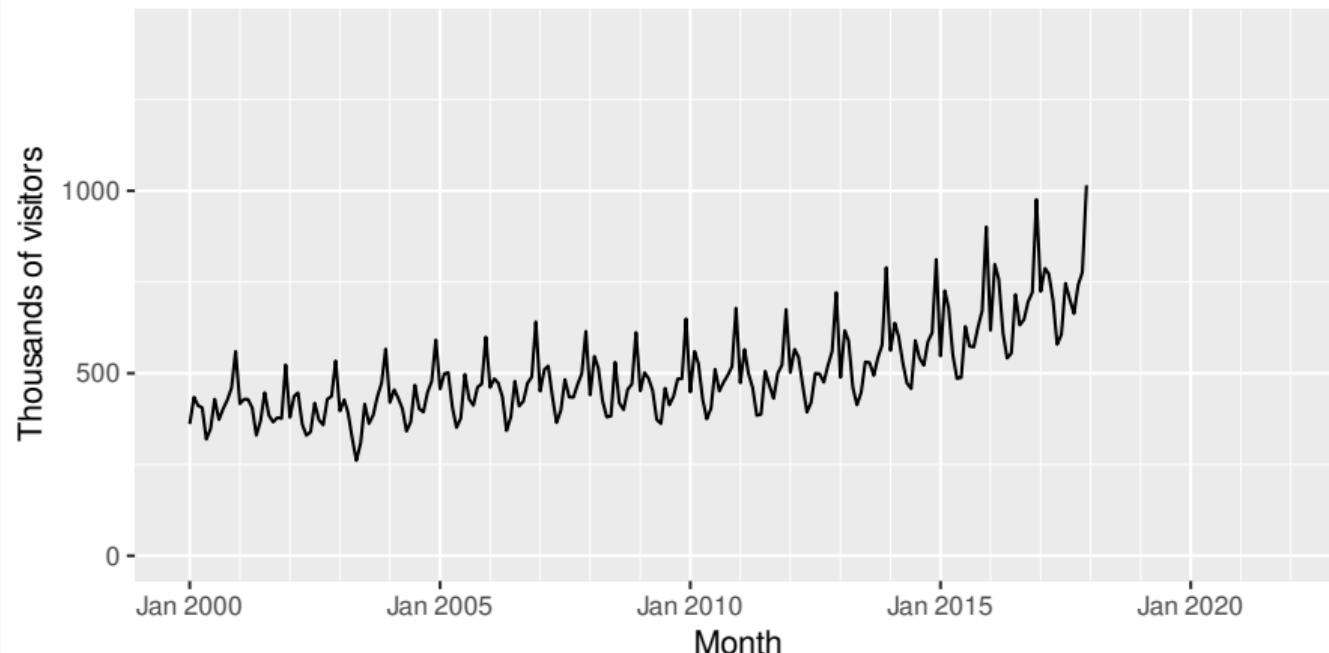
Random futures

A forecast is an estimate of the probabilities of possible futures.

Random futures

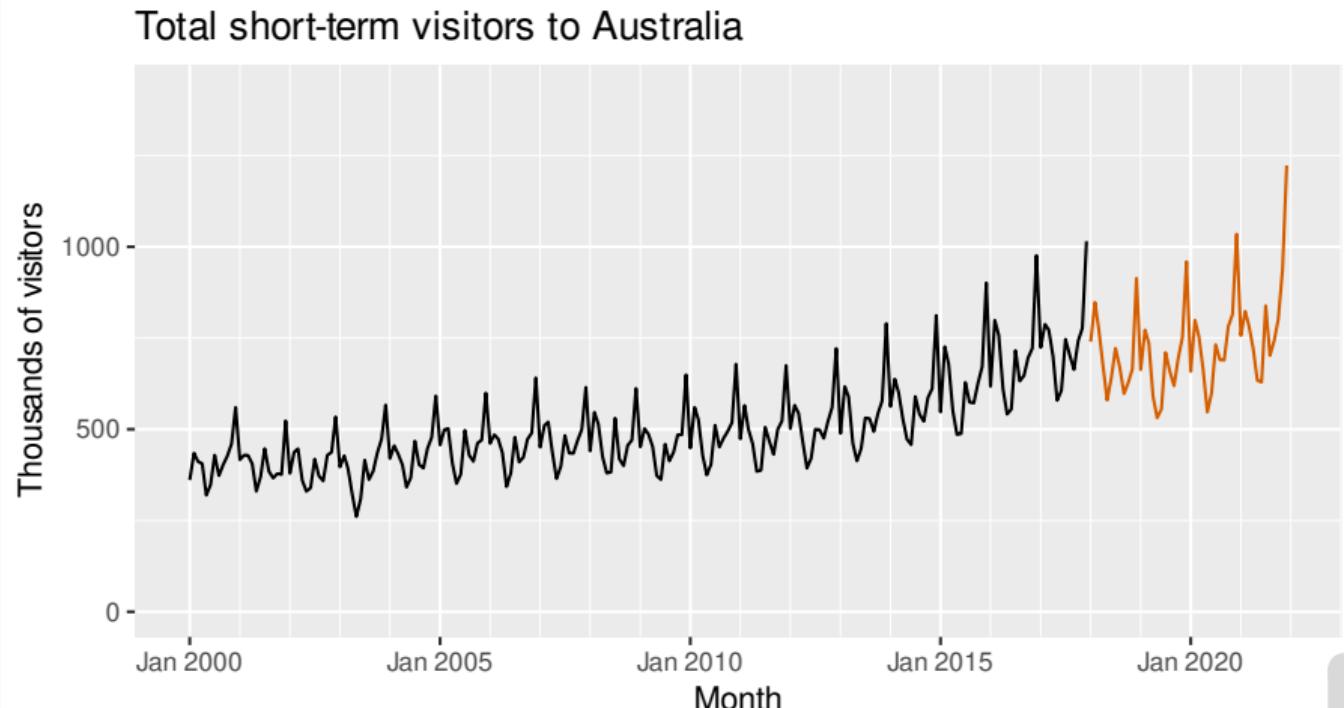
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Total short-term visitors to Australia



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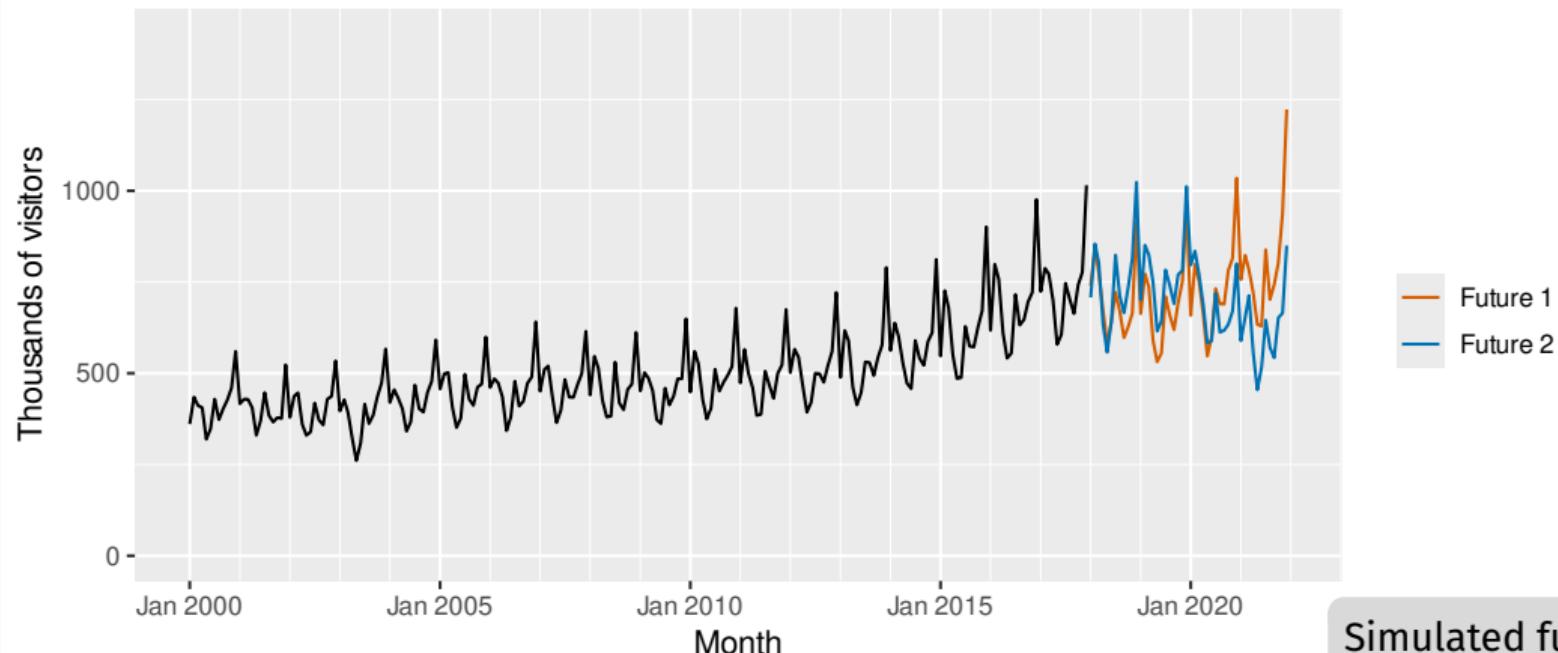


Simulated futures
from an ETS model

Random futures

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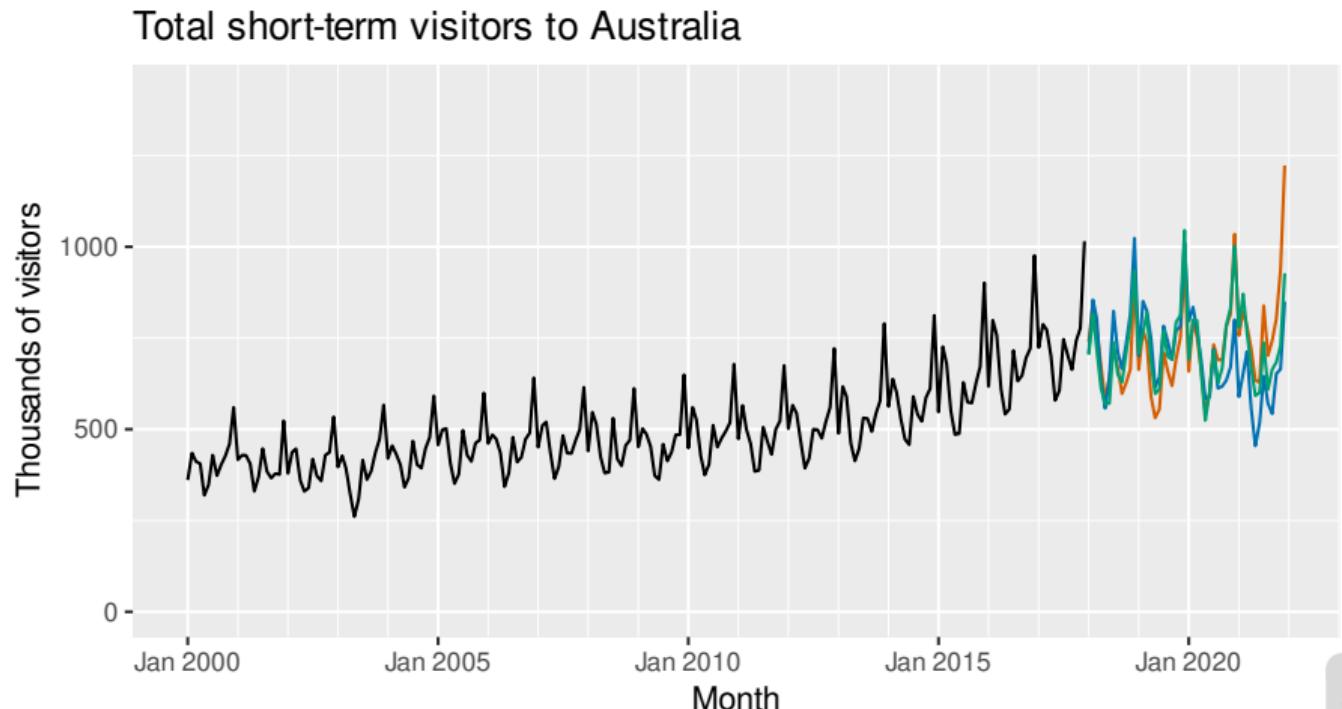
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Random futures

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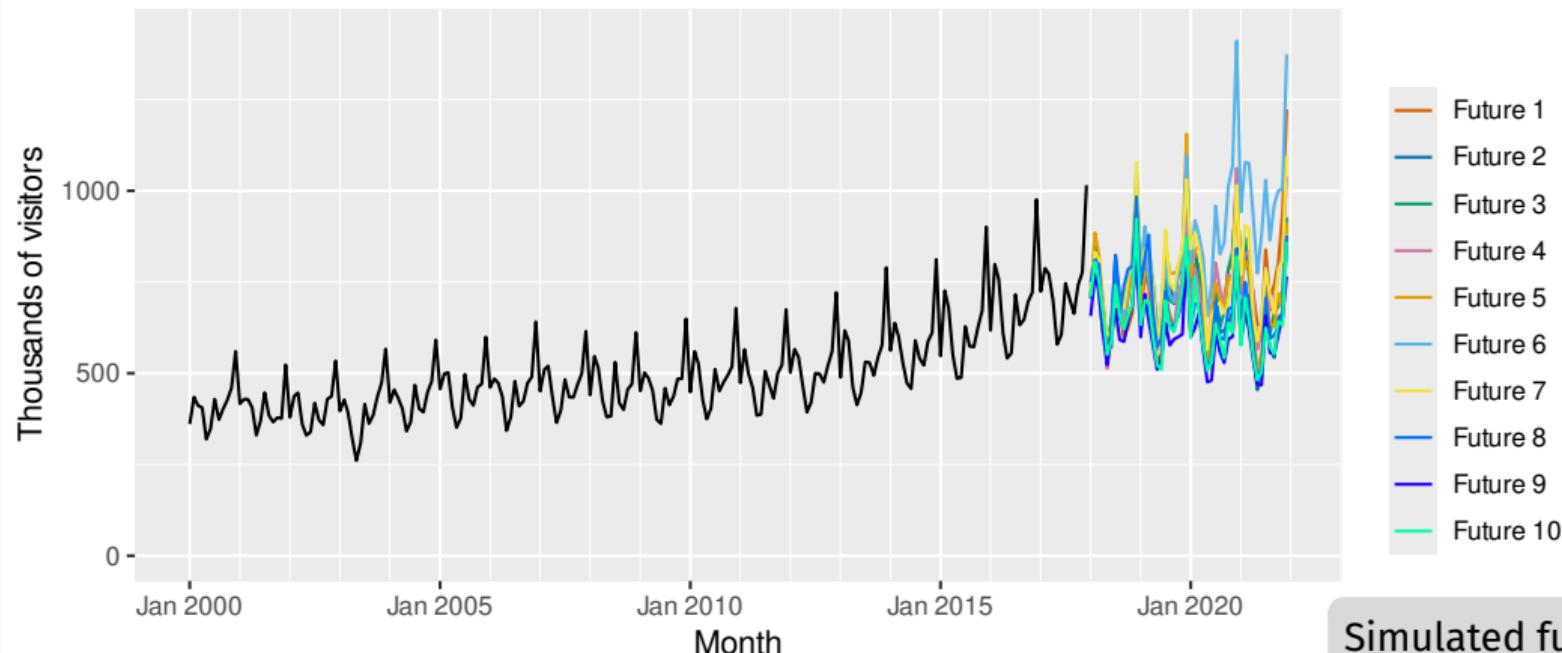


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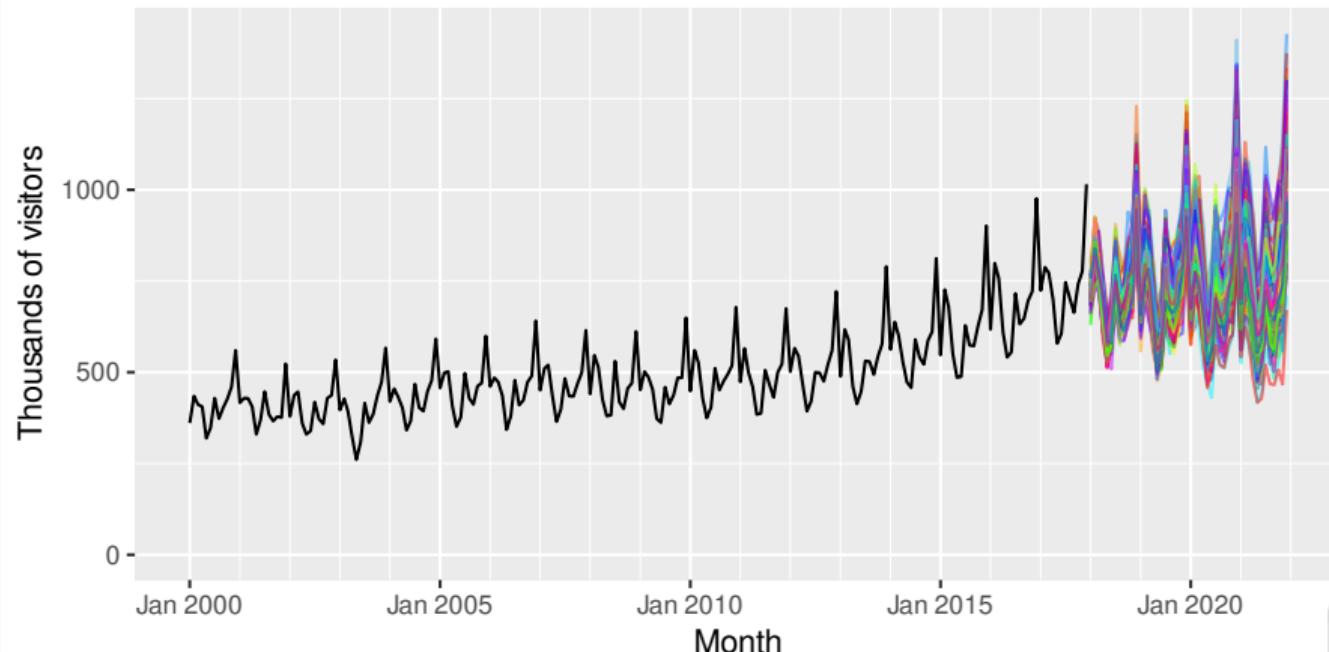


Simulated futures
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Random futures

A forecast is an estimate of the probabilities of possible futures.

Total short-term visitors to Australia

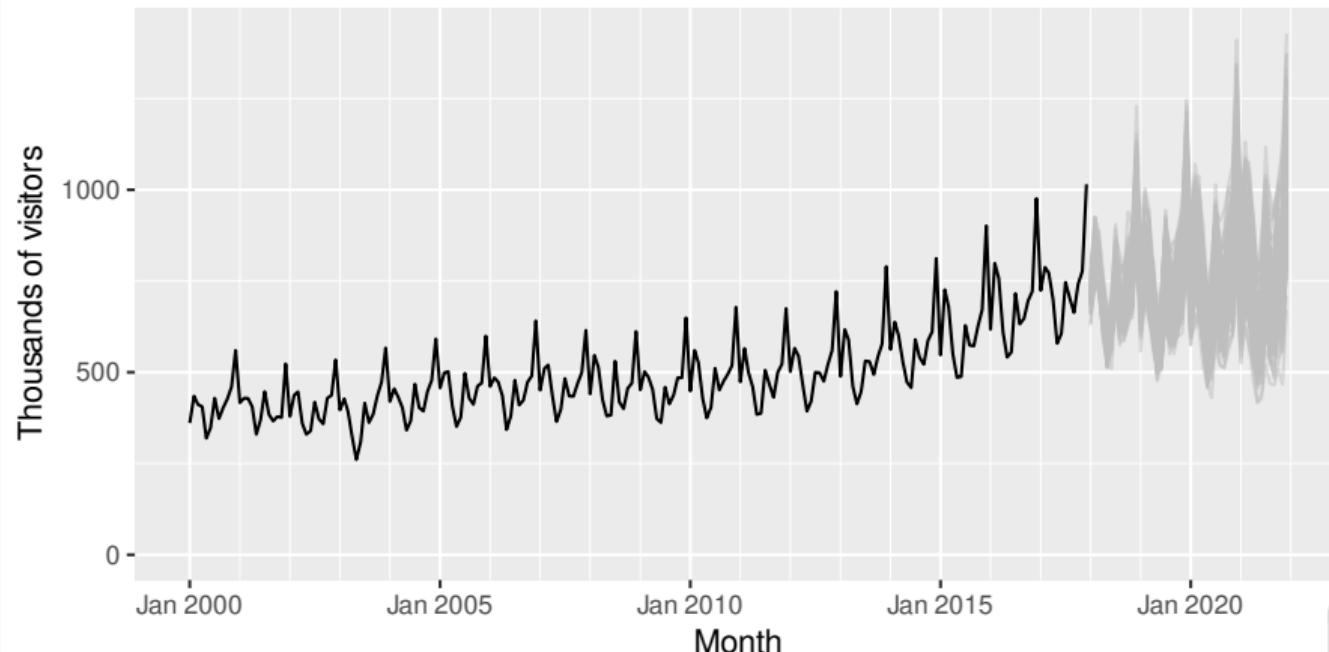


Simulated futures
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Random futures

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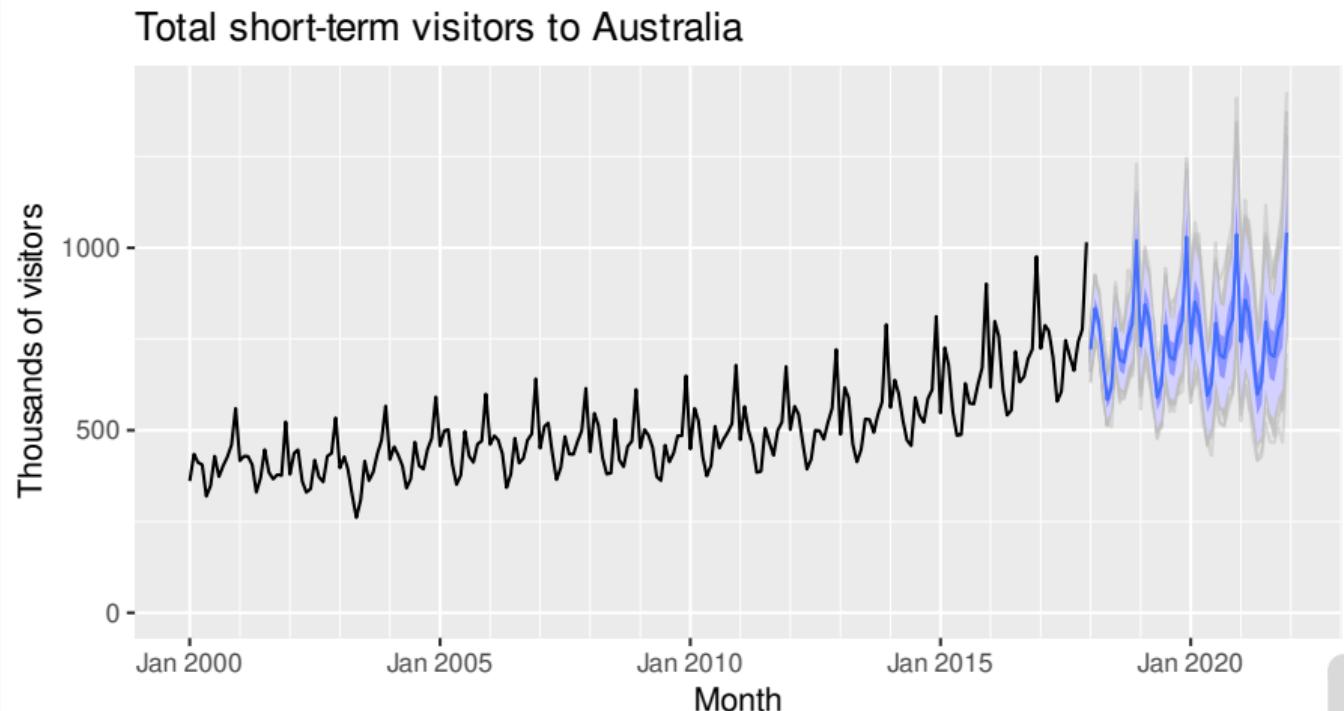
Total short-term visitors to Australia



Simulated futures
from an ETS model

Random futures

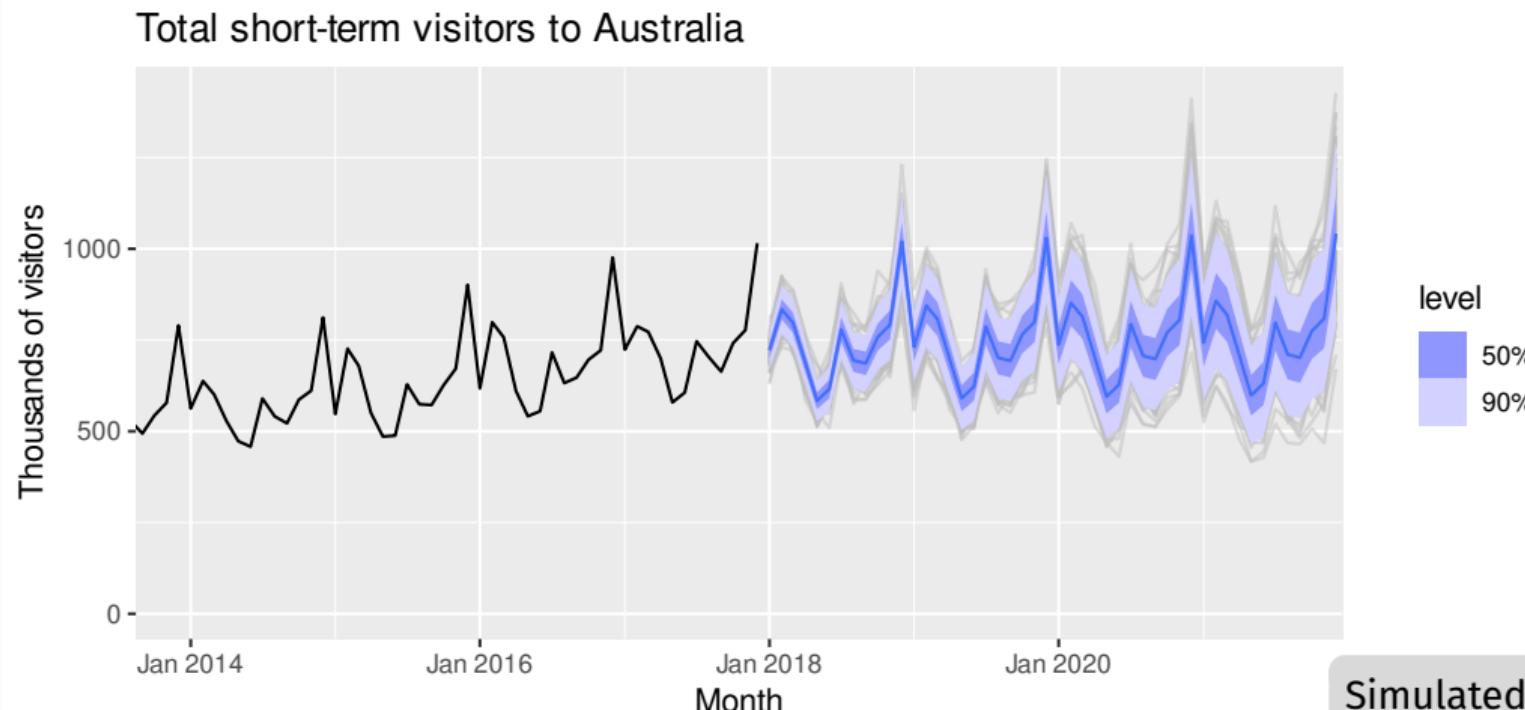
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Simulated futures
from an ETS model

Random futures

A forecast is an estimate of the probabilities of possible futures.

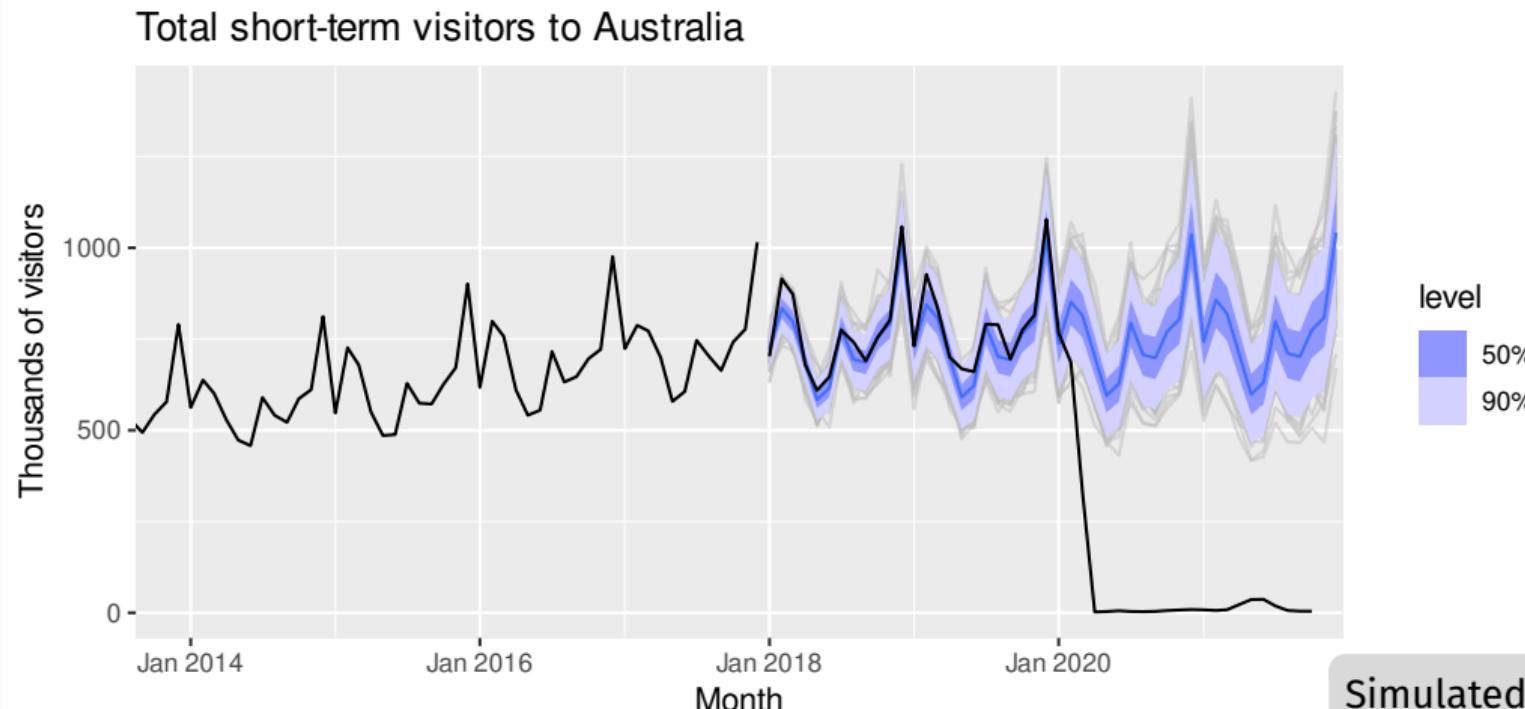


Month

Simulated futures
from an ETS model

Random futures

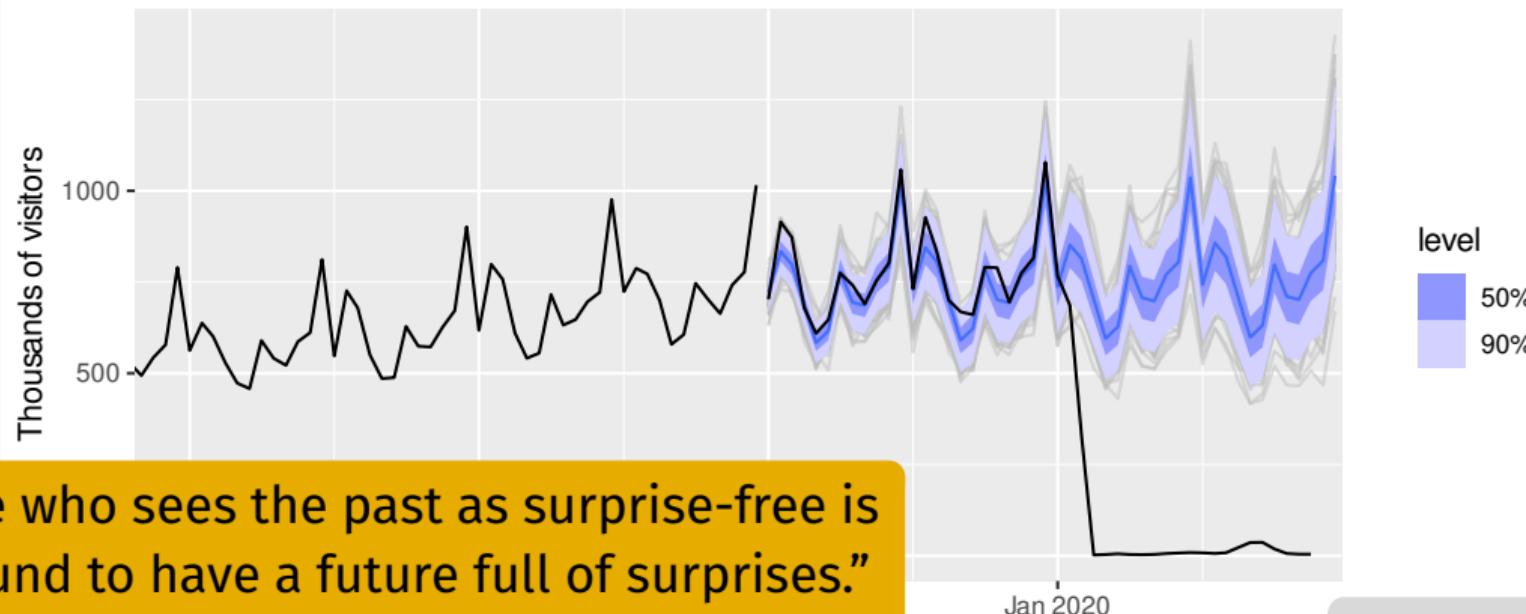
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Random futures

A forecast is an estimate of the probabilities of possible futures.

Total short-term visitors to Australia



“He who sees the past as surprise-free is bound to have a future full of surprises.”

(Amos Tversky)

Simulated futures
from an ETS model

Statistical forecasting

- Thing to be forecast: y_{T+h} .
- What we know: y_1, \dots, y_T .
- Forecast distribution: $y_{T+h|t} = y_{T+h} \mid \{y_1, y_2, \dots, y_T\}$.
- Point forecast: $\hat{y}_{T+h|T} = E[y_{T+h} \mid y_1, \dots, y_T]$.
- Forecast variance: $\text{Var}[y_t \mid y_1, \dots, y_T]$
- Prediction interval is a range of values of y_{T+h} with high probability.

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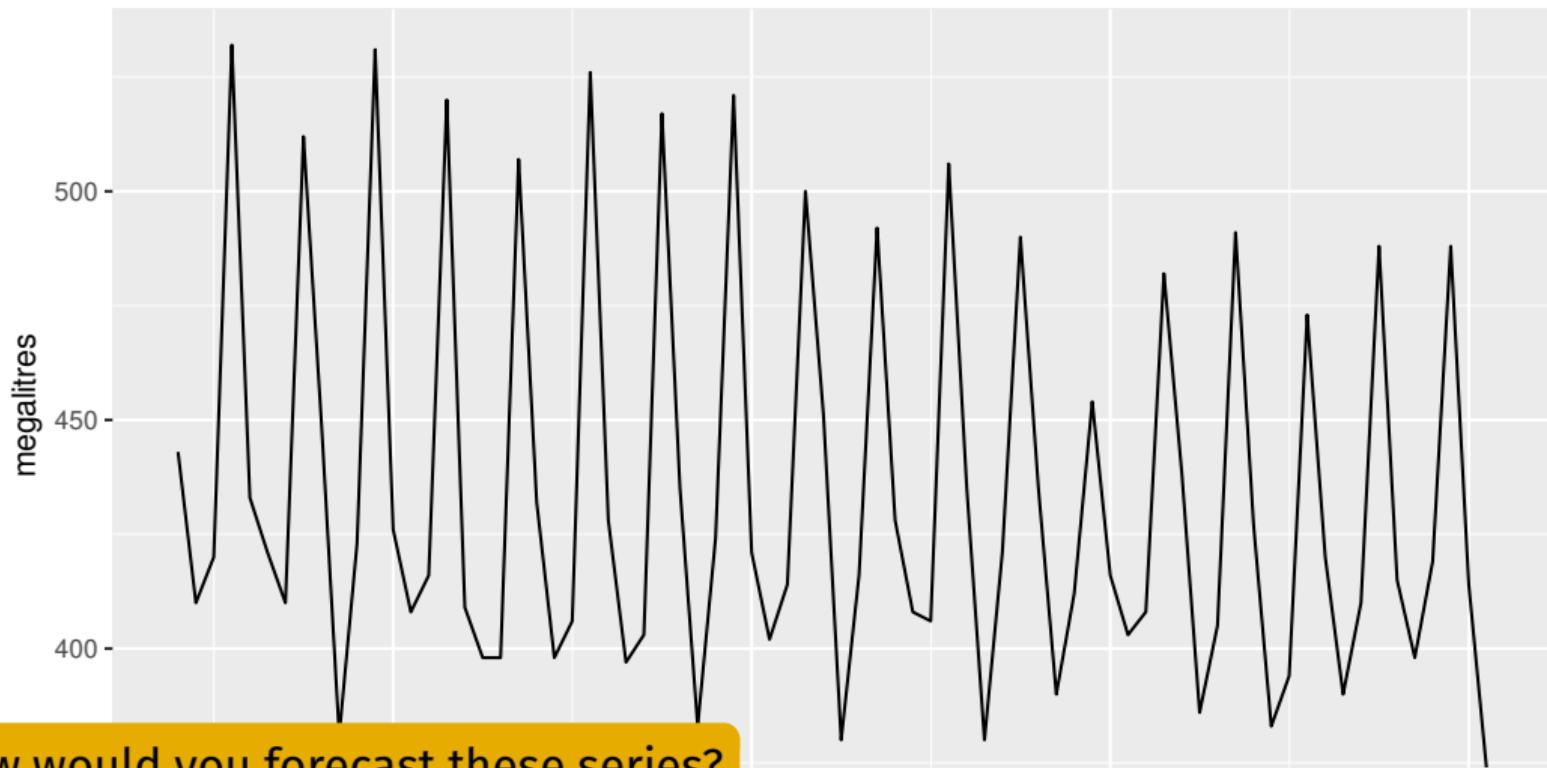
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7 Lab Session 13

Some simple forecasting methods

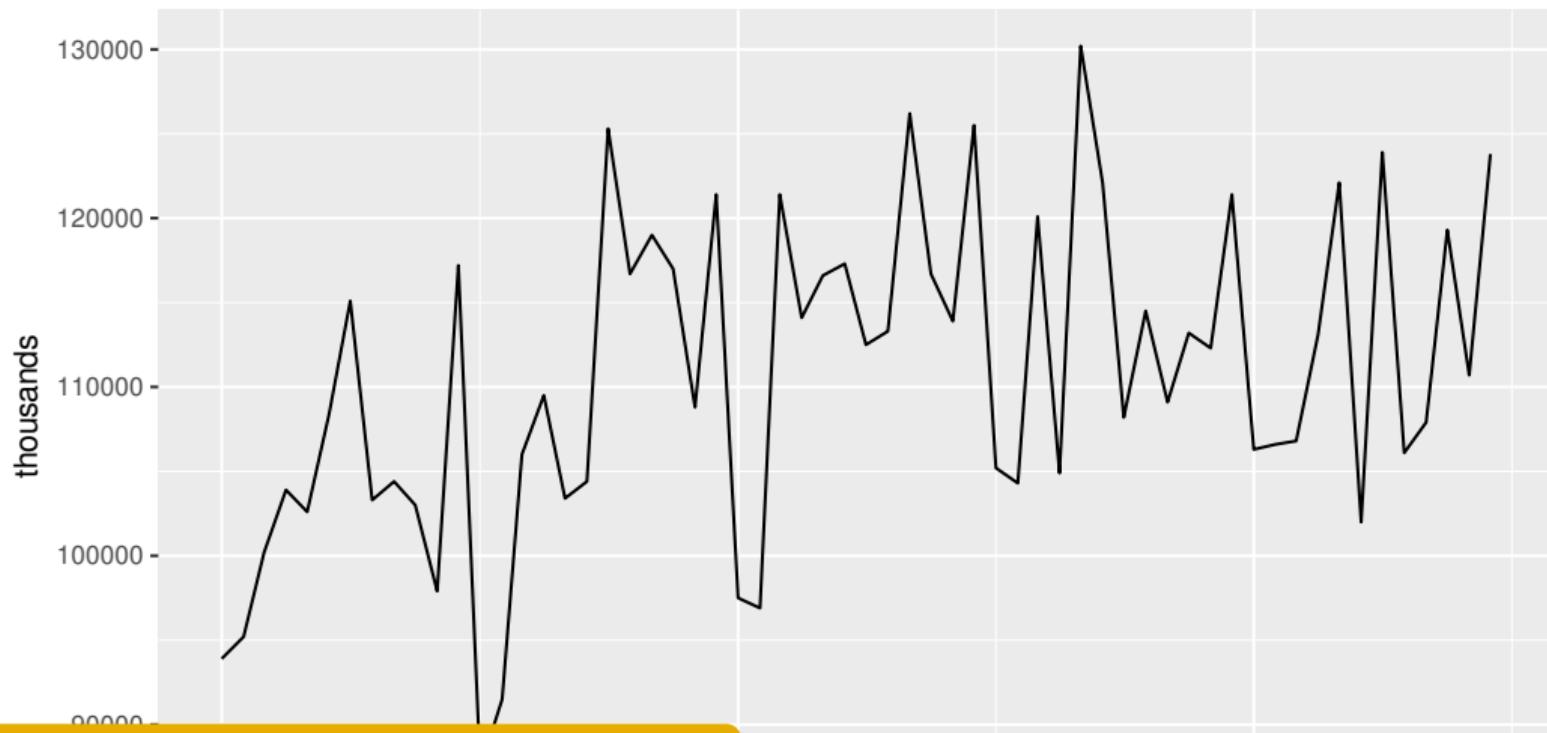
Australian quarterly beer production



How would you forecast these series?

Some simple forecasting methods

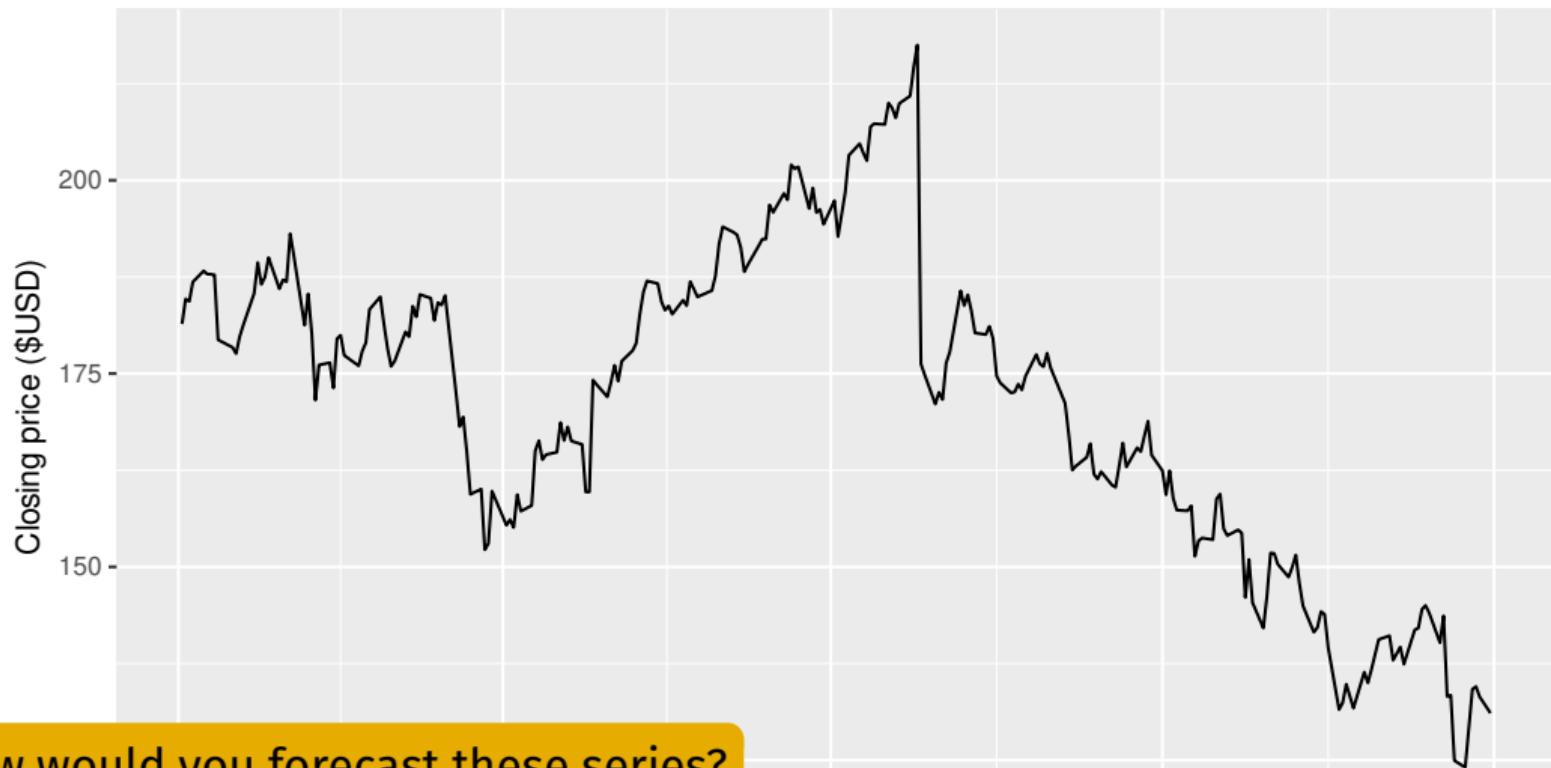
Number of pigs slaughtered in Victoria, 1990-1995



How would you forecast these series?

Some simple forecasting methods

Facebook closing stock price in 2018



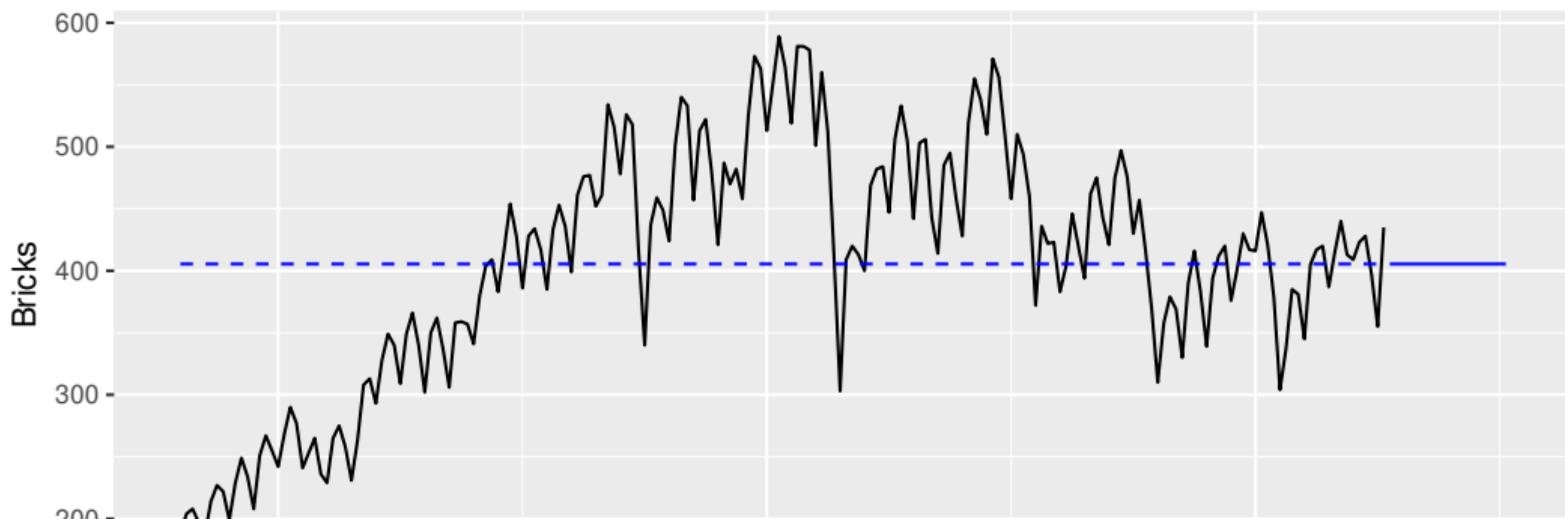
How would you forecast these series?

Some simple forecasting methods

MEAN(y): Average method

- Forecast of all future values is equal to mean of historical data $\{y_1, \dots, y_T\}$.
- Forecasts: $\hat{y}_{T+h|T} = \bar{y} = (y_1 + \dots + y_T)/T$

Clay brick production in Australia

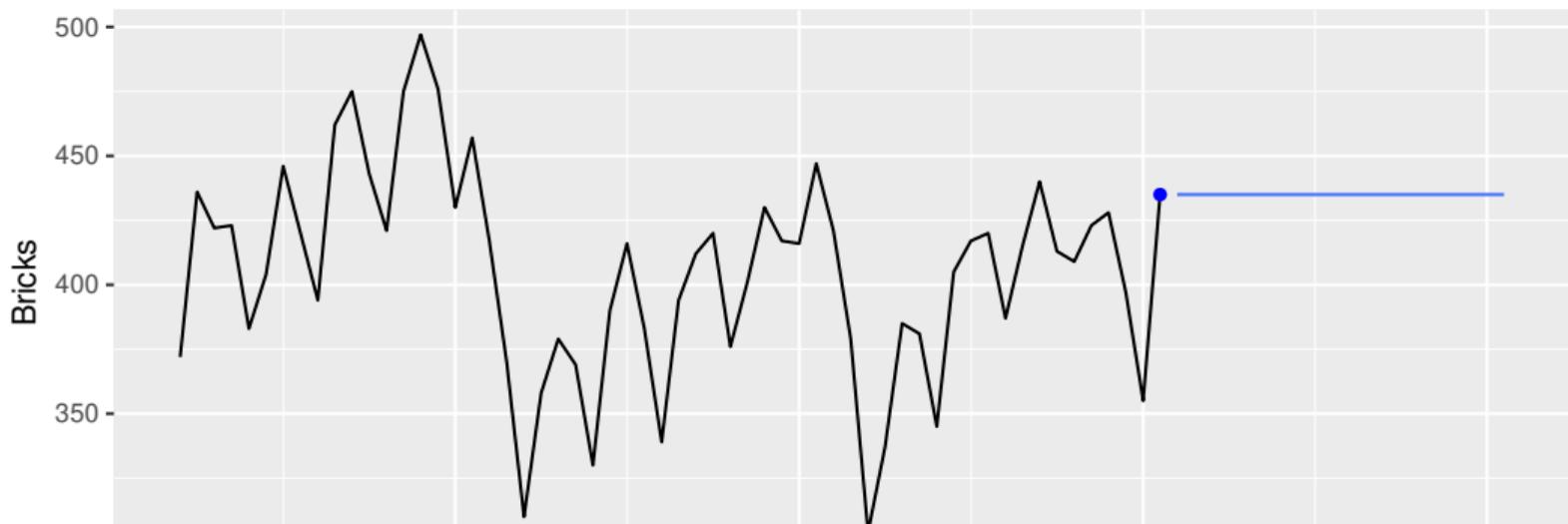


Some simple forecasting methods

NAIVE(y): Naïve method

- Forecasts equal to last observed value.
- Forecasts: $\hat{y}_{T+h|T} = y_T$.
- Consequence of efficient market hypothesis.

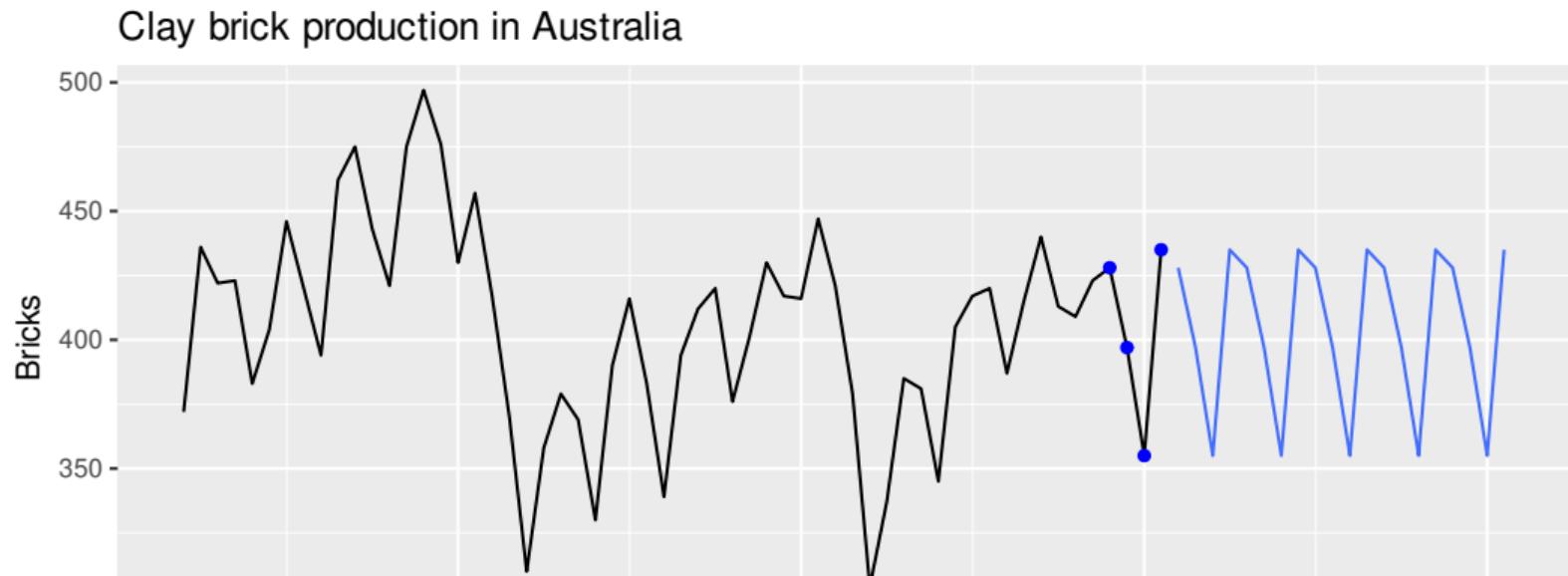
Clay brick production in Australia



Some simple forecasting methods

SNAIVE($y \sim \text{lag}(m)$): Seasonal naïve method

- Forecasts equal to last value from same season.
- Forecasts: $\hat{y}_{T+h|T} = y_{T+h-m(k+1)}$, where m = seasonal period and k is the integer part of $(h - 1)/m$.



Some simple forecasting methods

RW(y ~ drift()): Drift method

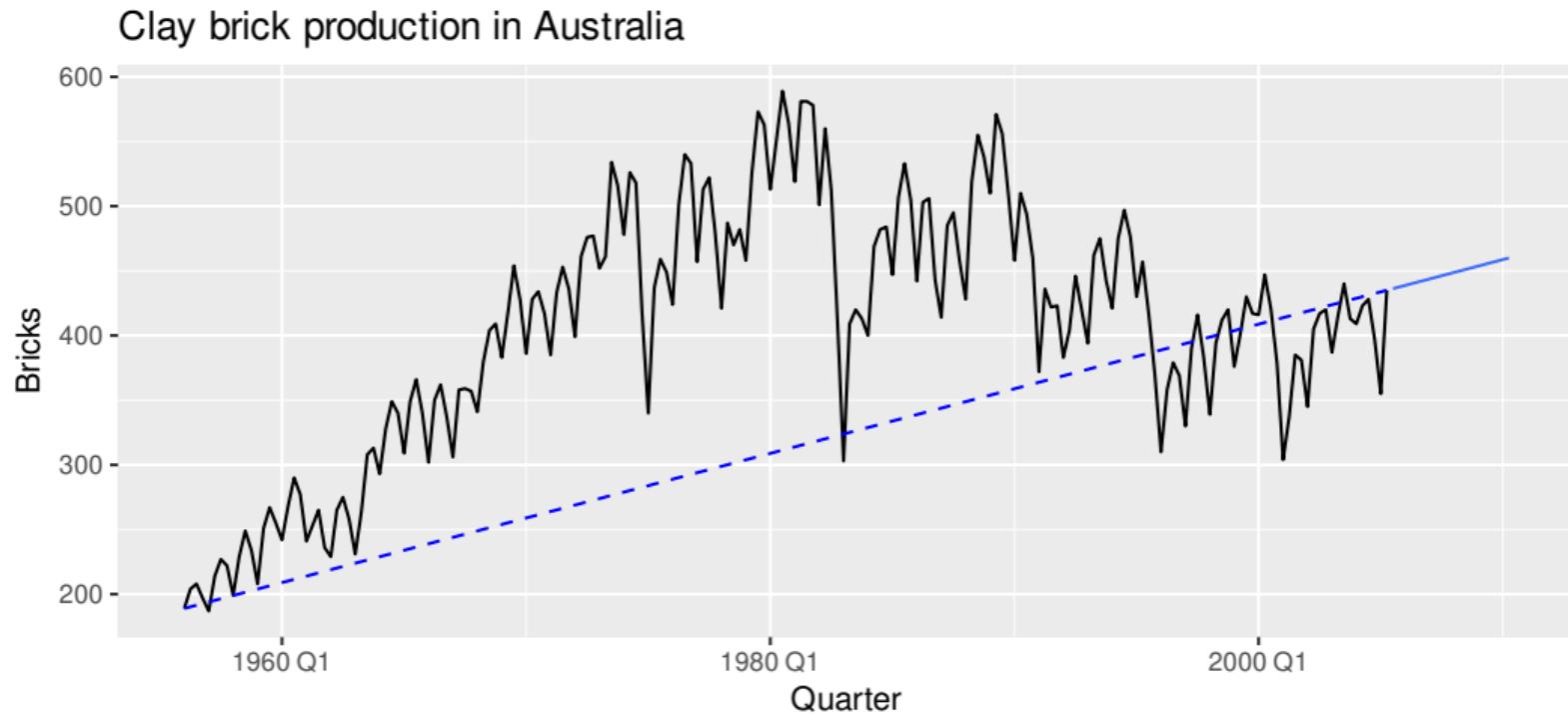
- Forecasts equal to last value plus average change.
- Forecasts:

$$\begin{aligned}\hat{y}_{T+h|T} &= y_T + \frac{h}{T-1} \sum_{t=2}^T (y_t - y_{t-1}) \\ &= y_T + \frac{h}{T-1} (y_T - y_1).\end{aligned}$$

- Equivalent to extrapolating a line drawn between first and last observations.

Some simple forecasting methods

Drift method



Model fitting

The `model()` function trains models to data.

```
brick_fit <- aus_production |>
  filter(!is.na(Bricks)) |>
  model(
    `Seasonal_naïve` = SNAIVE(Bricks),
    `Naïve` = NAIVE(Bricks),
    Drift = RW(Bricks ~ drift()),
    Mean = MEAN(Bricks)
  )
```

```
# A mable: 1 x 4
`Seasonal_na\u00eefve` `Na\u00eefve`          Drift      Mean
                <model>      <model>      <model> <model>
1              <SNAIVE>     <NAIVE>   <RW w/ drift> <MEAN>
```

A `mable` is a model table, each cell corresponds to a fitted model.

Producing forecasts

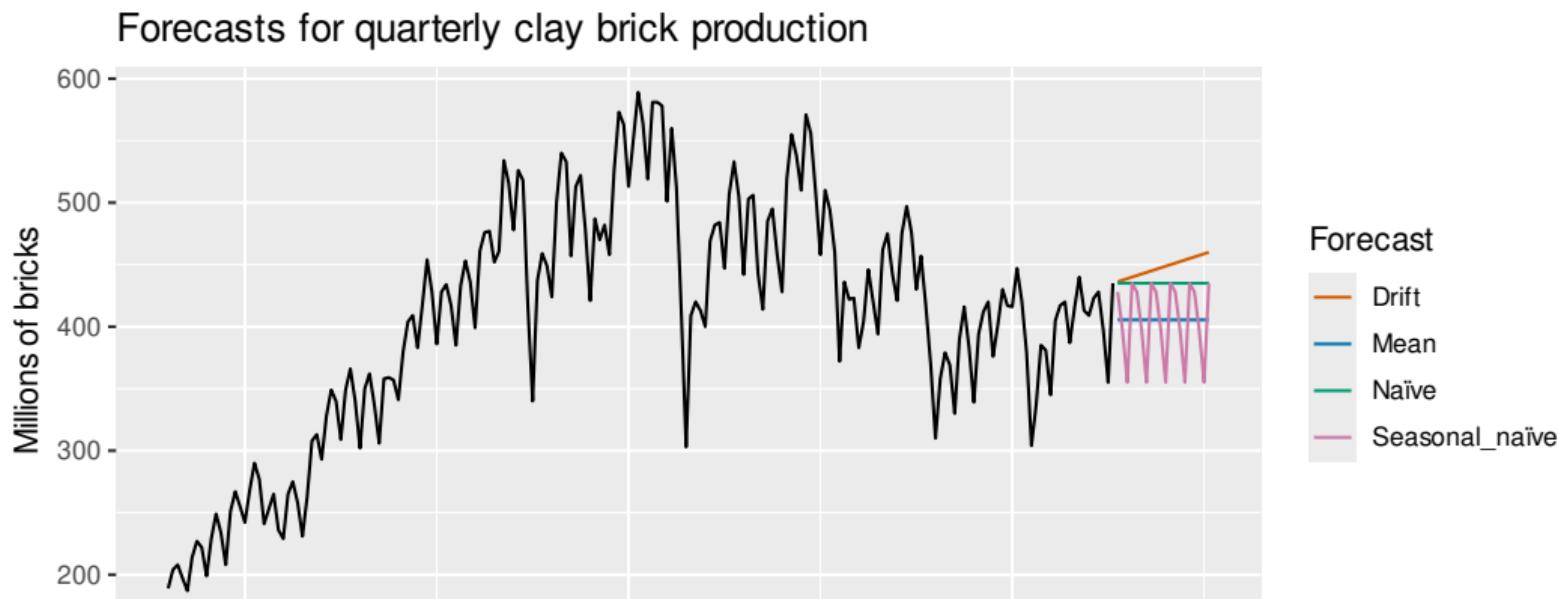
```
brick_fc <- brick_fit |>  
  forecast(h = "5 years")
```

```
# A fable: 80 x 4 [1Q]  
# Key:     .model [4]  
  
.model          Quarter      Bricks .mean  
<chr>           <qtr>       <dist> <dbl>  
1 "Seasonal_na\u00efve" 2005 Q3 N(428, 2336) 428  
2 "Seasonal_na\u00efve" 2005 Q4 N(397, 2336) 397  
3 "Seasonal_na\u00efve" 2006 Q1 N(355, 2336) 355  
4 "Seasonal_na\u00efve" 2006 Q2 N(435, 2336) 435  
# i 76 more rows
```

A fable is a forecast table with point forecasts and distributions.

Visualising forecasts

```
brick_fc |>  
  autoplot(aus_production, level = NULL) +  
  labs(title = "Forecasts for quarterly clay brick production",  
       x = "Year", y = "Millions of bricks") +  
  guides(colour = guide_legend(title = "Forecast"))
```



Prediction intervals

```
brick_fc |>  
  hilo(level = c(50, 75))
```

#	.model	Quarter	Bricks	.mean	`50%`	`75%`
	<chr>	<qtr>	<dist>	<dbl>	<hilo>	<hilo>
1	"Seasonal_na\ufe0f~	2005 Q3	N(428, 2336)	428	[395, 461]	[372, 484]
2	"Seasonal_na\ufe0f~	2005 Q4	N(397, 2336)	397	[364, 430]	[341, 453]
3	"Seasonal_na\ufe0f~	2006 Q1	N(355, 2336)	355	[322, 388]	[299, 411]
4	"Seasonal_na\ufe0f~	2006 Q2	N(435, 2336)	435	[402, 468]	[379, 491]
5	"Seasonal_na\ufe0f~	2006 Q3	N(428, 4672)	428	[382, 474]	[349, 507]
6	"Seasonal_na\ufe0f~	2006 Q4	N(397, 4672)	397	[351, 443]	[318, 476]
7	"Seasonal_na\ufe0f~	2007 Q1	N(355, 4672)	355	[309, 401]	[276, 434]
8	"Seasonal_na\ufe0f~	2007 Q2	N(435, 4672)	435	[389, 481]	[356, 514]
9	"Seasonal_na\ufe0f~	2007 Q3	N(428, 7008)	428	[372, 484]	[332, 524]
10	"Seasonal_na\ufe0f~	2007 Q4	N(397, 7008)	397	[341, 453]	[301, 492]

Prediction intervals

```
brick_fc |>
  hilo(level = c(50, 75)) |>
  mutate(lower = `50%`$lower, upper = `50%`$upper)
```

#	A tsibble: 80 x 8 [1Q]	# Key:	.model	[4]	.model	Quarter	Bricks	.mean	`50%`	`75%`	lower	upper
					<chr>	<qtr>	<dist>	<dbl>	<hilo>	<hilo>	<dbl>	<dbl>
1	"Seasonal"	2005	Q3	$N(428, 2336)$	428	[395, 461]	50	[372, 484]	75	395.	461.	
2	"Seasonal"	2005	Q4	$N(397, 2336)$	397	[364, 430]	50	[341, 453]	75	364.	430.	
3	"Seasonal"	2006	Q1	$N(355, 2336)$	355	[322, 388]	50	[299, 411]	75	322.	388.	
4	"Seasonal"	2006	Q2	$N(435, 2336)$	435	[402, 468]	50	[379, 491]	75	402.	468.	
5	"Seasonal"	2006	Q3	$N(428, 4672)$	428	[382, 474]	50	[349, 507]	75	382.	474.	
6	"Seasonal"	2006	Q4	$N(397, 4672)$	397	[351, 443]	50	[318, 476]	75	351.	443.	
7	"Seasonal"	2007	Q1	$N(355, 4672)$	355	[309, 401]	50	[276, 434]	75	309.	401.	
8	"Seasonal"	2007	Q2	$N(435, 4672)$	435	[389, 481]	50	[356, 514]	75	389.	481.	
9	"Seasonal"	2007	Q3	$N(428, 7008)$	428	[372, 484]	50	[332, 524]	75	372.	484.	

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Lab Session 11

- Produce forecasts using an appropriate benchmark method for student enrolments in Australia (as shown yesterday). Plot the results using `autoplot()`.
- Produce forecasts using an appropriate benchmark method for total Australian retail turnover (aggregate `aus_retail`). Plot the results using `autoplot()`.

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Fitted values

- $\hat{y}_{t|t-1}$ is the forecast of y_t based on observations y_1, \dots, y_{t-1} .
- We call these “fitted values”.
- Sometimes drop the subscript: $\hat{y}_t \equiv \hat{y}_{t|t-1}$.
- Often not true forecasts since parameters are estimated on all data.

For example:

- $\hat{y}_t = \bar{y}$ for average method.
- $\hat{y}_t = y_{t-1} + (y_T - y_1)/(T - 1)$ for drift method.

Forecasting residuals

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

Forecasting residuals

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Assumptions

- 1 $\{e_t\}$ uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.
- 2 $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

Forecasting residuals

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

Assumptions

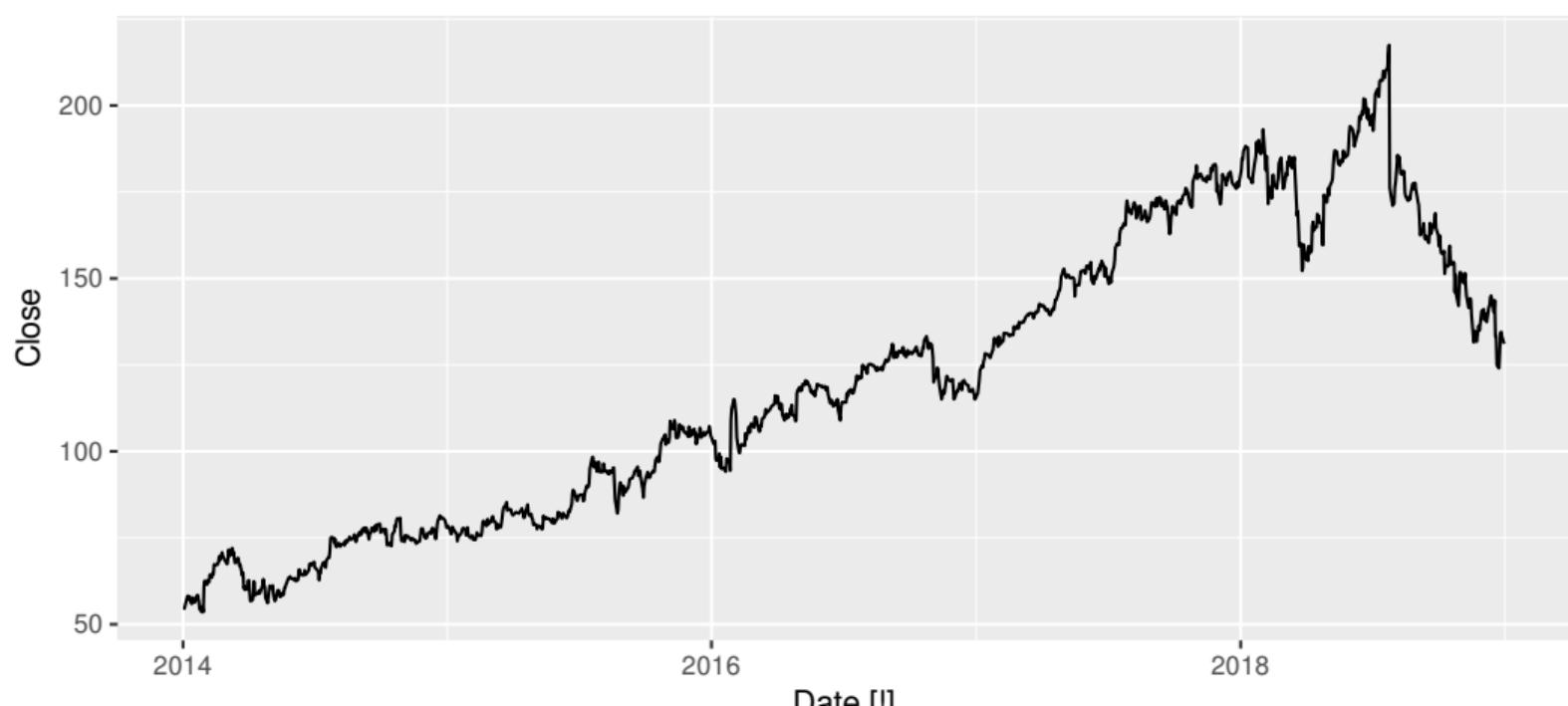
- 1 $\{e_t\}$ uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.
- 2 $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

Useful properties (for prediction intervals)

- 3 $\{e_t\}$ have constant variance.
- 4 $\{e_t\}$ are normally distributed.

Facebook closing stock price

```
fb_stock <- gafa_stock |>  
  filter(Symbol == "FB")  
fb_stock |> autoplot(Close)
```



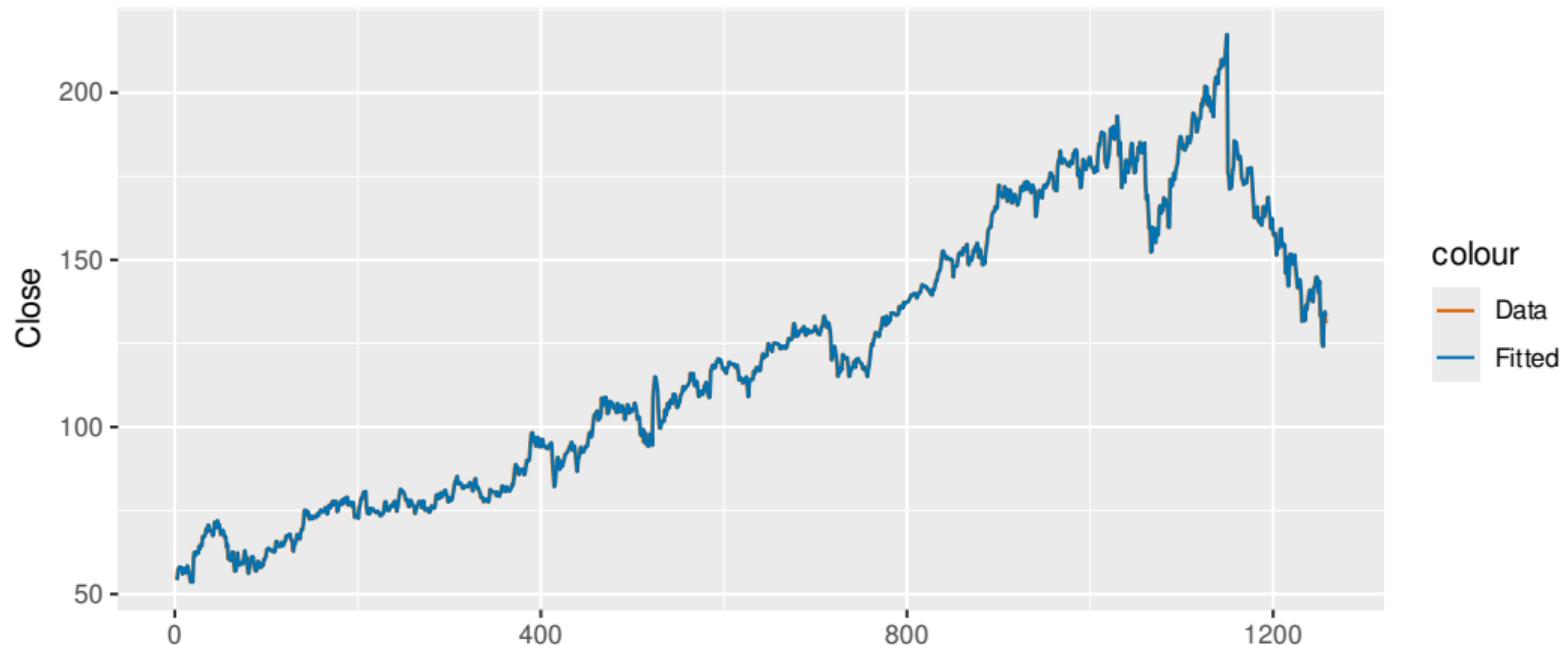
Facebook closing stock price

```
fb_stock <- fb_stock |>  
  mutate(trading_day = row_number()) |>  
  update_tsibble(index = trading_day, regular = TRUE)  
fit <- fb_stock |> model(NAIVE(Close))  
augment(fit)
```

```
# A tsibble: 1,258 x 7 [1]  
# Key:     Symbol, .model [1]  
  Symbol .model      trading_day Close .fitted .resid .innov  
  <chr>  <chr>        <int>  <dbl>   <dbl>   <dbl>   <dbl>  
1 FB     NAIVE(Close)      1  54.7    NA    NA    NA  
2 FB     NAIVE(Close)      2  54.6  54.7 -0.150 -0.150  
3 FB     NAIVE(Close)      3  57.2  54.6   2.64   2.64  
4 FB     NAIVE(Close)      4  57.9  57.2   0.720  0.720  
5 FB     NAIVE(Close)      5  58.2  57.9   0.310  0.310  
6 FB     NAIVE(Close)      6  57.2  58.2  -1.01  -1.01  
7 FB     NAIVE(Close)      7  57.9  57.2   0.720  0.720  
8 FB     NAIVE(Close)      8  55.9  57.9  -2.03  -2.03  
9 FB     NAIVE(Close)      9  57.7  55.9   1.83   1.83
```

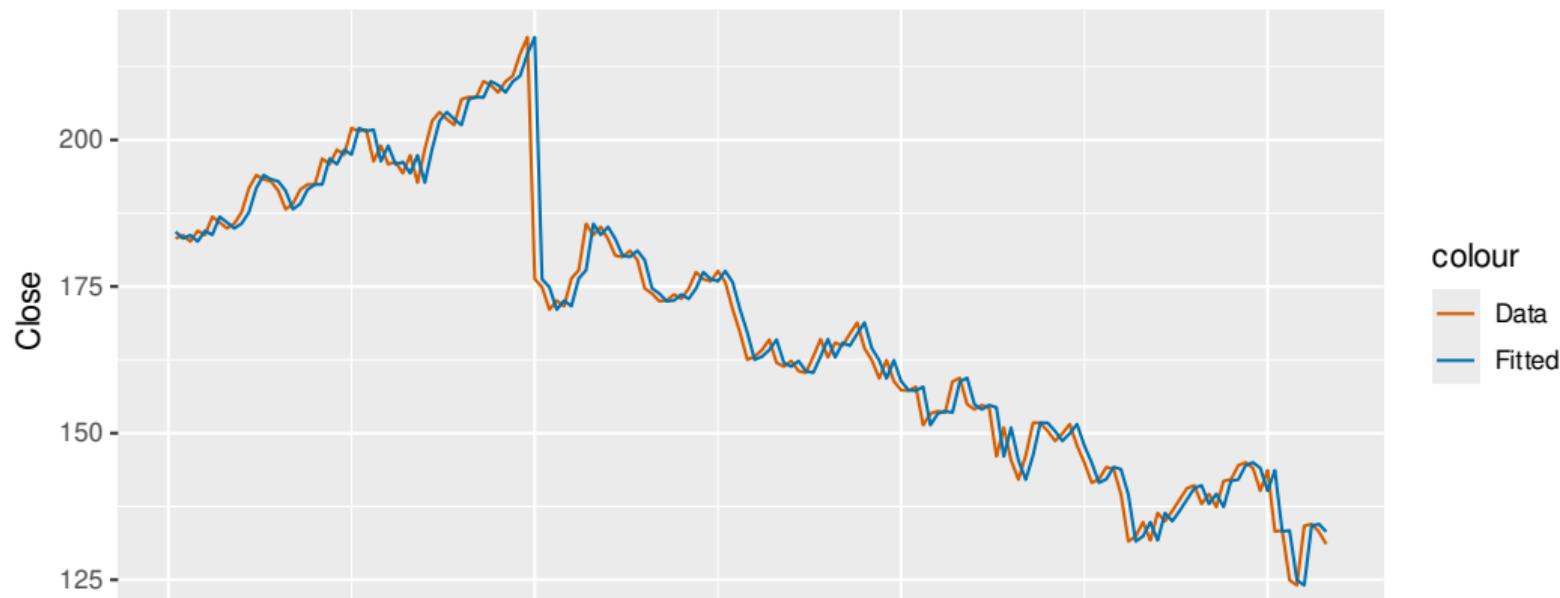
Facebook closing stock price

```
augment(fit) |>  
  ggplot(aes(x = trading_day)) +  
  geom_line(aes(y = Close, colour = "Data")) +  
  geom_line(aes(y = .fitted, colour = "Fitted"))
```



Facebook closing stock price

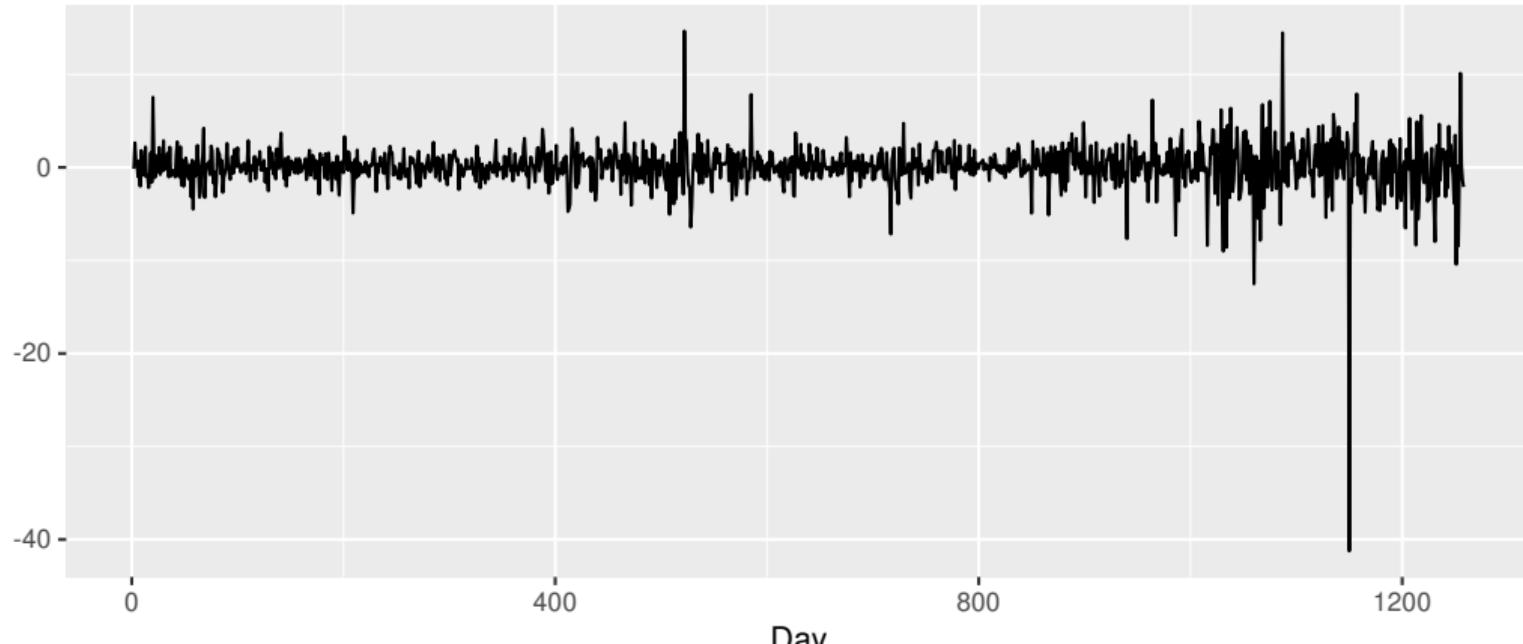
```
augment(fit) |>  
  filter(trading_day > 1100) |>  
  ggplot(aes(x = trading_day)) +  
  geom_line(aes(y = Close, colour = "Data")) +  
  geom_line(aes(y = .fitted, colour = "Fitted"))
```



Facebook closing stock price

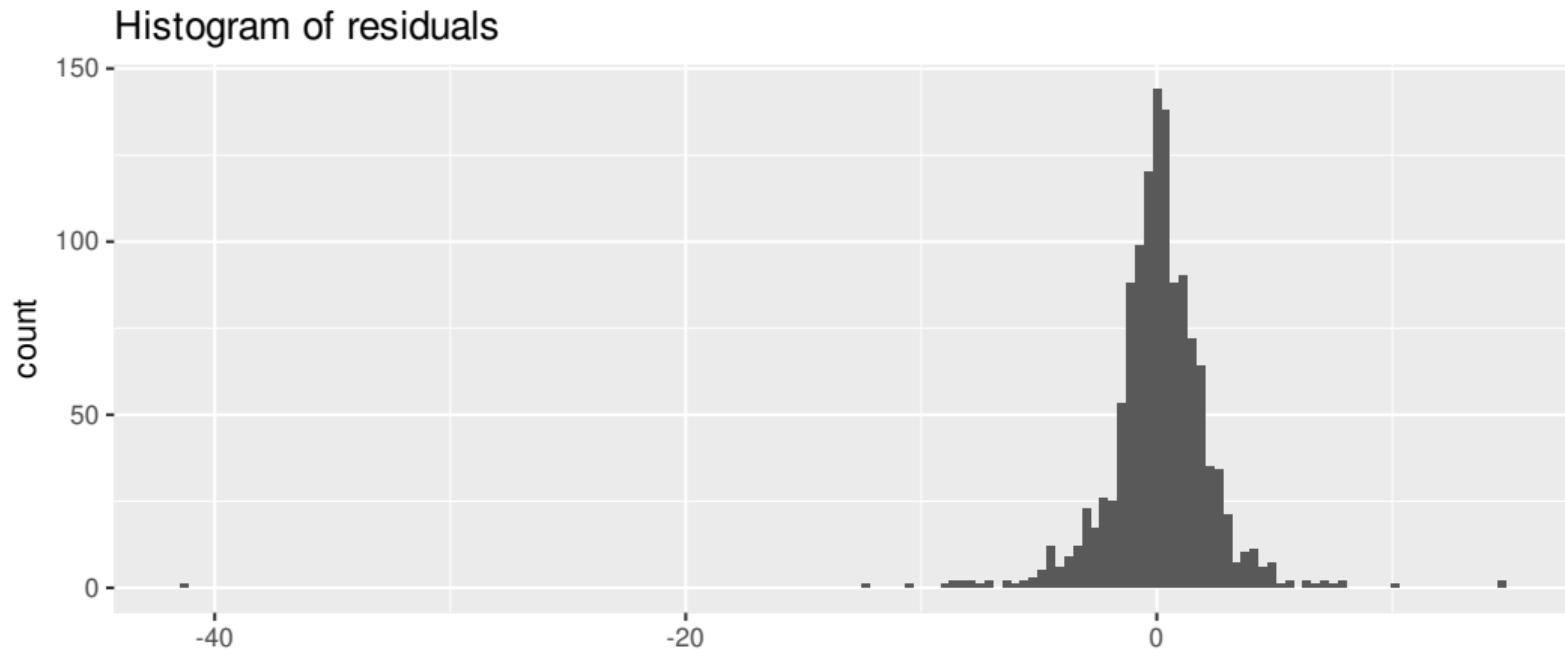
```
augment(fit) |>  
  autoplot(.resid) +  
  labs(x = "Day", y = "", title = "Residuals from naïve method")
```

Residuals from naïve method



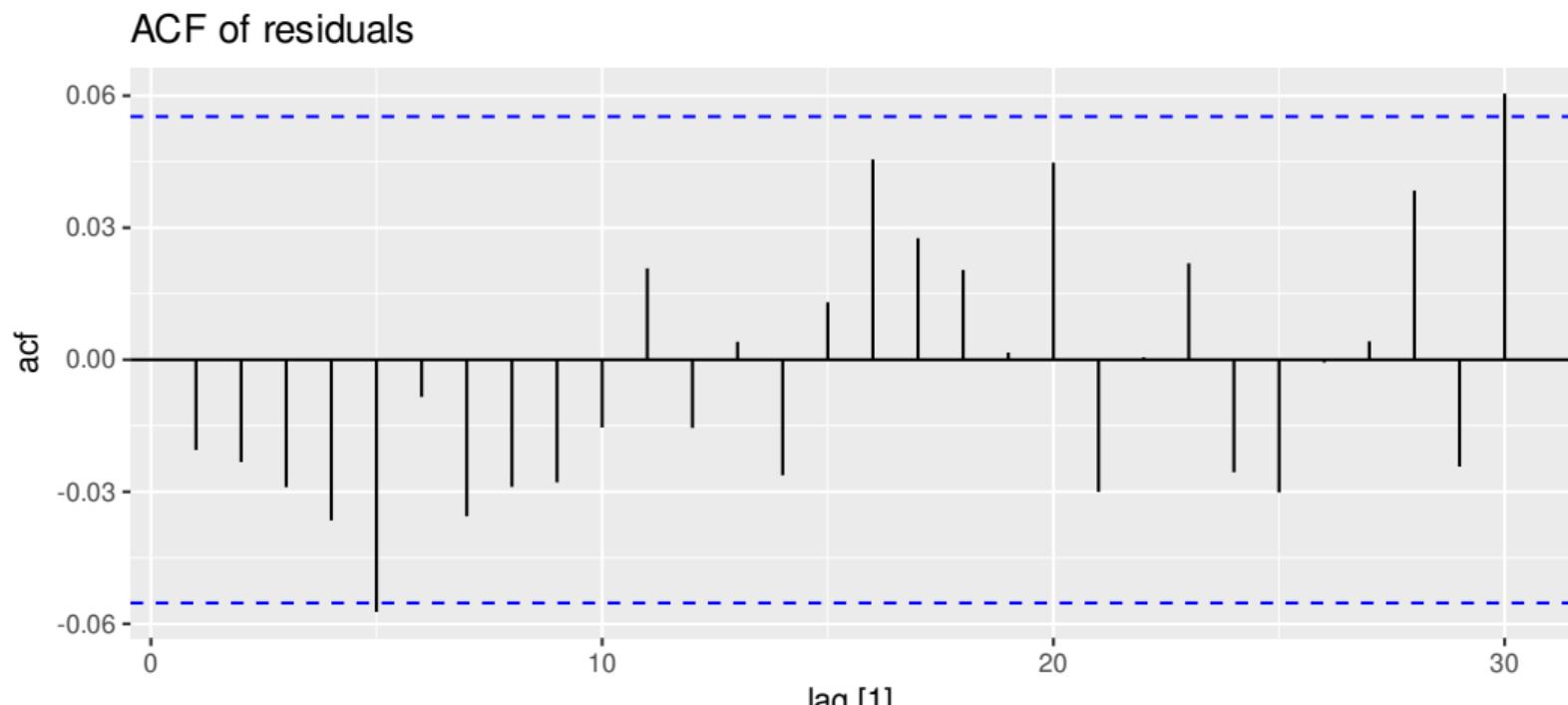
Facebook closing stock price

```
augment(fit) |>  
  ggplot(aes(x = .resid)) +  
  geom_histogram(bins = 150) +  
  labs(title = "Histogram of residuals")
```



Facebook closing stock price

```
augment(fit) |>  
  ACF(.resid) |>  
  autoplot() + labs(title = "ACF of residuals")
```

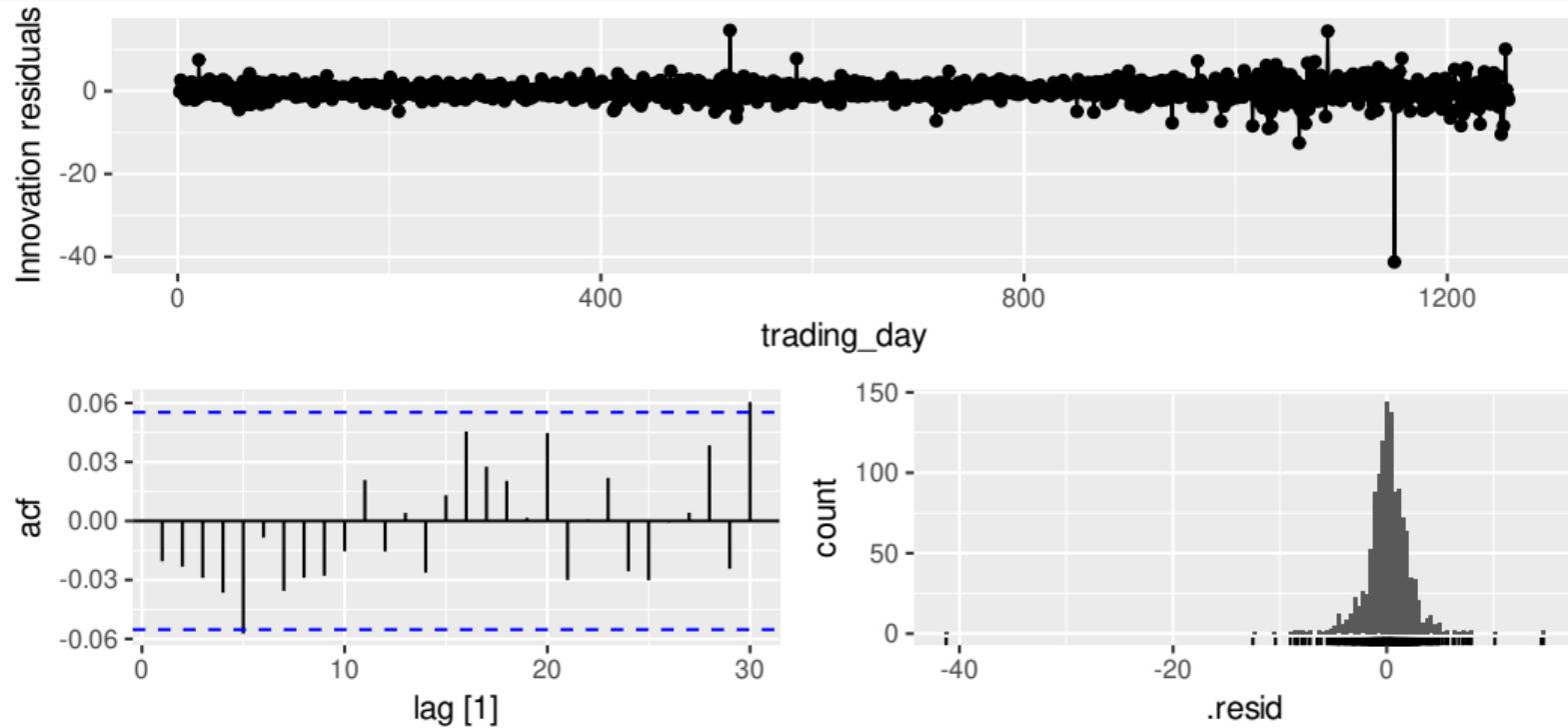


ACF of residuals

- We assume that the residuals are white noise (uncorrelated, mean zero, constant variance). If they aren't, then there is information left in the residuals that should be used in computing forecasts.
- So a standard residual diagnostic is to check the ACF of the residuals of a forecasting method.
- We *expect* these to look like white noise.

Combined diagnostic graph

```
fit |> gg_tsresiduals()
```



Ljung-Box test

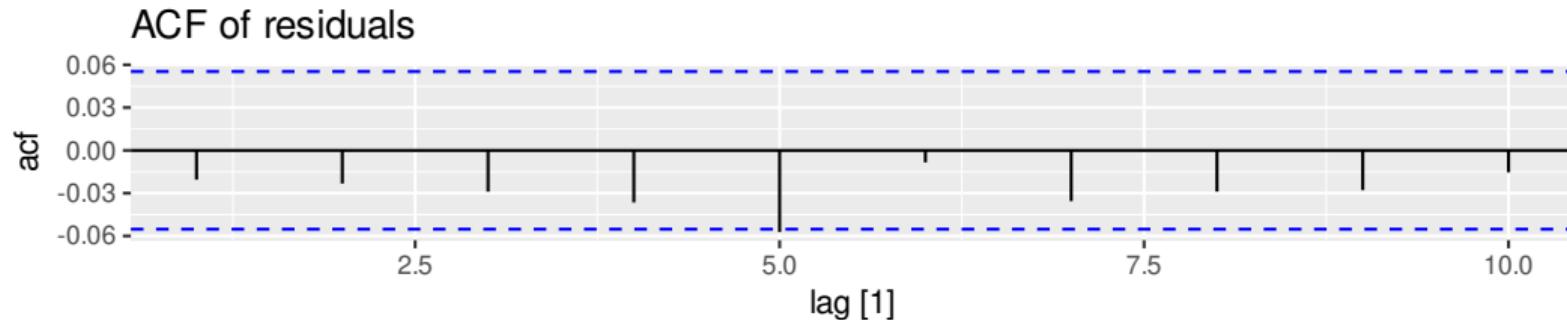
Test whether *whole set* of r_k values is significantly different from zero set.

$$Q = T(T + 2) \sum_{k=1}^{\ell} (T - k)^{-1} r_k^2 \quad \text{where } \ell = \text{max lag and } T = \# \text{ observations}$$

- If each r_k close to zero, Q will be **small**.
- If some r_k values large (+ or -), Q will be **large**.
- My preferences: $h = 10$ for non-seasonal data, $h = 2m$ for seasonal data.
- If data are WN and T large, $Q \sim \chi^2$ with ℓ degrees of freedom.

Ljung-Box test

$$Q = T(T + 2) \sum_{k=1}^{\ell} (T - k)^{-1} r_k^2 \quad \text{where } \ell = \max \text{ lag and } T = \# \text{ observations.}$$



```
# lag = h
augment(fit) |> features(.resid, ljung_box, lag = 10)
```

```
# A tibble: 1 x 4
  Symbol .model      lb_stat lb_pvalue
  <chr>  <chr>       <dbl>     <dbl>
1 FB     NAIVE(Close) 12.1      0.276
```

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Lab Session 12

- Compute RW w/ drift forecasts for total student enrolments in Australia.
- Test if the residuals are white noise. What do you conclude?

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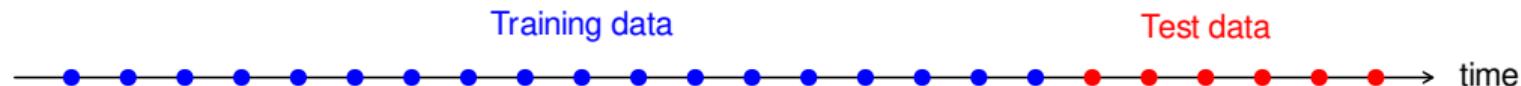
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Training and test sets



- A model which fits the training data well will not necessarily forecast well.
- Forecast accuracy is based only on the test set.

Forecast errors

Forecast “error”: the difference between an observed value and its forecast.

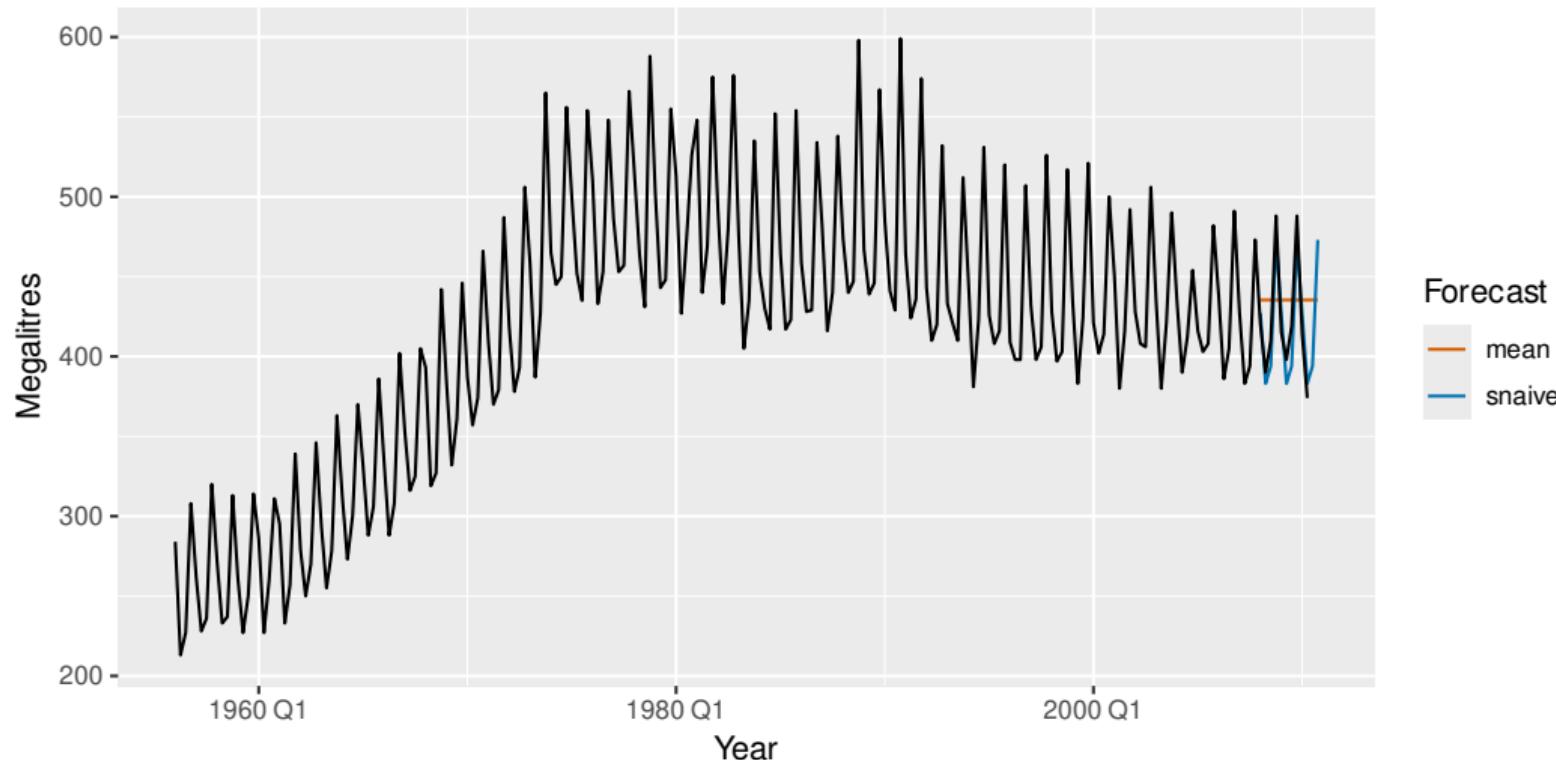
$$e_{T+h} = y_{T+h} - \hat{y}_{T+h|T},$$

Measures of forecast accuracy

```
beer_fit <- aus_production |>
  filter(between(year(Quarter), 1992, 2007)) |>
  model(
    snaive = SNAIVE(Beer),
    mean = MEAN(Beer)
  )
beer_fit |>
  forecast(h = "3 years") |>
  autoplot(aus_production, level = NULL) +
  labs(title ="Forecasts for quarterly beer production",
       x ="Year", y ="Megalitres") +
  guides(colour = guide_legend(title = "Forecast"))
```

Measures of forecast accuracy

Forecasts for quarterly beer production



Measures of forecast accuracy

y_{T+h} = $(T + h)$ th observation, $h = 1, \dots, H$

$\hat{y}_{T+h|T}$ = its forecast based on data up to time T .

$e_{T+h} = y_{T+h} - \hat{y}_{T+h|T}$

MAE = $\text{mean}(|e_{T+h}|)$

MSE = $\text{mean}(e_{T+h}^2)$

RMSE = $\sqrt{\text{mean}(e_{T+h}^2)}$

MAPE = $100\text{mean}(|e_{T+h}|/|y_{T+h}|)$

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- MAE, MSE, RMSE are all scale dependent.
- MAPE is scale independent but is only sensible if $y_t \gg 0$ for all t , and y has a natural zero.

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Mean Absolute Scaled Error

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- For seasonal series, scale uses seasonal naïve forecasts:

$$Q = \frac{1}{T-m} \sum_{t=m+1}^T |y_t - y_{t-m}|$$

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Proposed by Hyndman and Koehler (IJF, 2006).

Measures of forecast accuracy

Root Mean Squared Scaled Error

$$\text{RMSSE} = \sqrt{\text{mean}(e_{T+h}^2/Q)}$$

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where m is the seasonal frequency

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Measures of forecast accuracy

```
beer_fc <- forecast(beer_fit, h = "3 years")
accuracy(beer_fc, aus_production)
```

```
# A tibble: 2 x 10
  .model .type     ME   RMSE    MAE    MPE   MAPE   MASE RMSSE      ACF1
  <chr>  <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
1 mean    Test -13.8  38.4  34.8 -3.97  8.28  2.20  1.96 -0.0691
2 snaive  Test   5.2  14.3  13.4  1.15  3.17  0.847 0.729  0.132
```

Outline

1 Statistical forecasting

2 Benchmark methods

3 Lab Session 11

4 Residual diagnostics

5 Lab Session 12

6 Forecast accuracy measures

7 Lab Session 13

Lab Session 13

- Create a training set for Australian student enrolments by withholding the last four years as a test set.
- Fit all the appropriate benchmark methods to the training set and forecast the periods covered by the test set.
- Compute the accuracy of your forecasts. Which method does best?
- Repeat the exercise using the Australian takeaway food turnover data (`aus_retail`) with a test set of four years.