Time Series Analysis & Forecasting Using R

12. Accuracy evaluation

Outline

1. Residual diagnostics
2. Lab Session 10
3. Forecast accuracy measures
4. Lab Session 11

Outline

Fitted values

- $\hat{y}_{t|t-1}$ is the forecast of y_t based on observations $y_1, \ldots, y_t$.
- We call these "fitted values".
- Sometimes drop the subscript: $\hat{y}_t \equiv \hat{y}_{t|t-1}$.
- Often not true forecasts since parameters are estimated on all data.

For example:

- $\hat{y}_t = \bar{y}$ for average method.
- $\hat{y}_t = y_{t-1} + (y_T - y_1)/(T-1)$ for drift method.

Forecasting residuals

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

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Assumptions

1. $\{e_t\}$ uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.
2. $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

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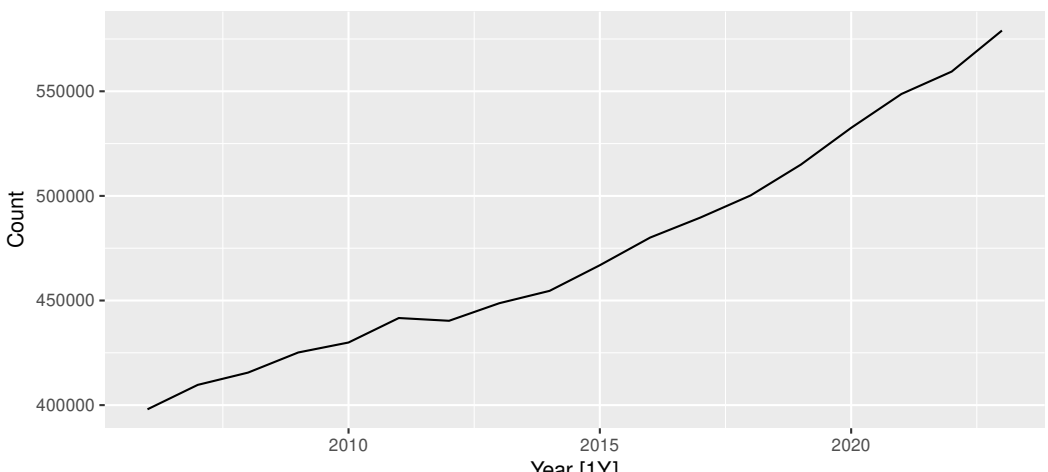
Useful properties (for prediction intervals)

3. $\{e_t\}$ have constant variance.
4. $\{e_t\}$ are normally distributed.

Total in-school staff

```
total_staff <- staff |>
  summarise(Count = sum(`In-School Staff Count`))
total_staff |> autoplot(Count)
```



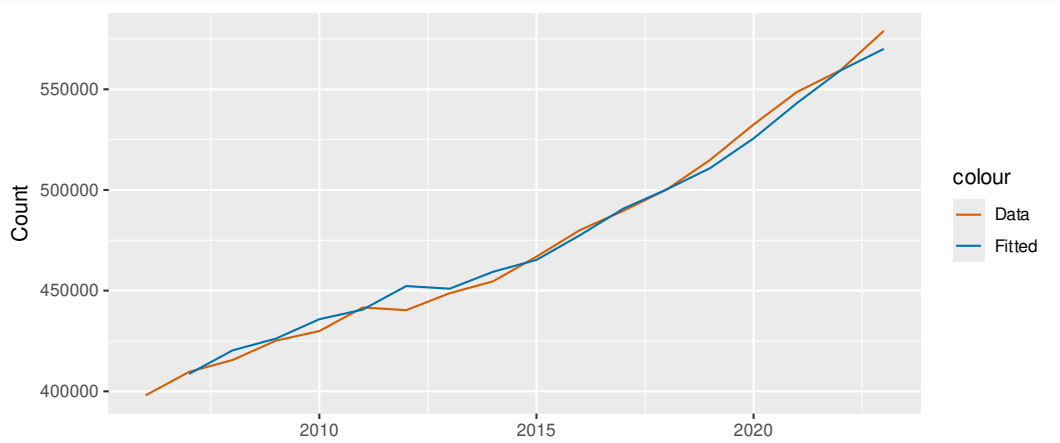
Total in-school staff

```
fit <- total_staff |> model(RW(Count ~ drift()))
augment(fit)
```

```
# A tsibble: 18 x 6 [1Y]
# Key:       .model [1]
   .model                Year  Count .fitted   .resid   .innov
   <chr>                <dbl>  <dbl>   <dbl>    <dbl>    <dbl>
 1 RW(Count ~ drift())  2006 398003      NA       NA       NA
 2 RW(Count ~ drift())  2007 409678 408652.   1026.    1026.
 3 RW(Count ~ drift())  2008 415541 420327.  -4786.   -4786.
 4 RW(Count ~ drift())  2009 425166 426190.  -1024.   -1024.
 5 RW(Count ~ drift())  2010 429933 435815.  -5882.   -5882.
 6 RW(Count ~ drift())  2011 441631 440582.   1049.    1049.
 7 RW(Count ~ drift())  2012 440313 452280. -11967.  -11967.
 8 RW(Count ~ drift())  2013 448711 450962.  -2251.   -2251.
 9 RW(Count ~ drift())  2014 454615 459360.  -4745.   -4745.
10 RW(Count ~ drift())  2015 466867 465264.   1603.    1603.
11 RW(Count ~ drift())  2016 480077 477516.   2561.    2561.
12 RW(Count ~ drift())  2017 489645 490726.  -1081.   -1081.
```

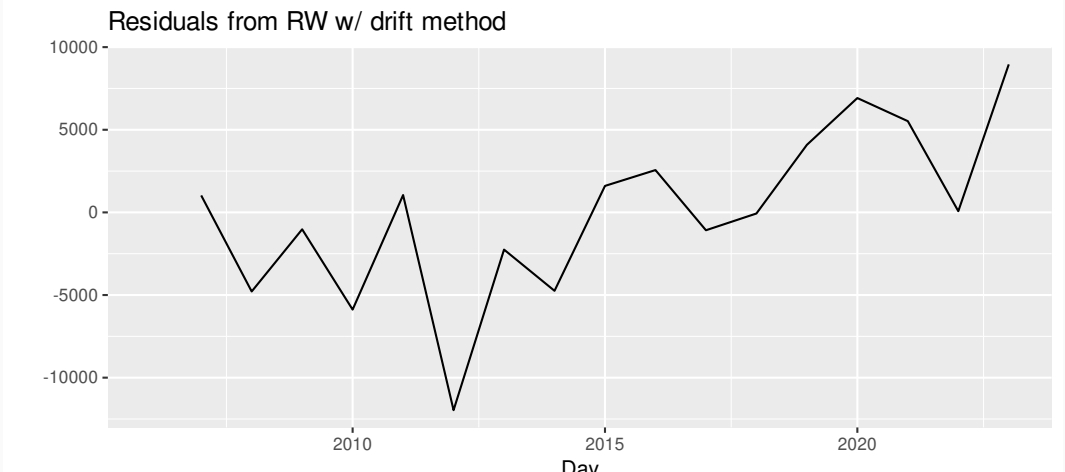
Total in-school staff

```
augment(fit) |>
  ggplot(aes(x = Year)) +
  geom_line(aes(y = Count, colour = "Data")) +
  geom_line(aes(y = .fitted, colour = "Fitted"))
```



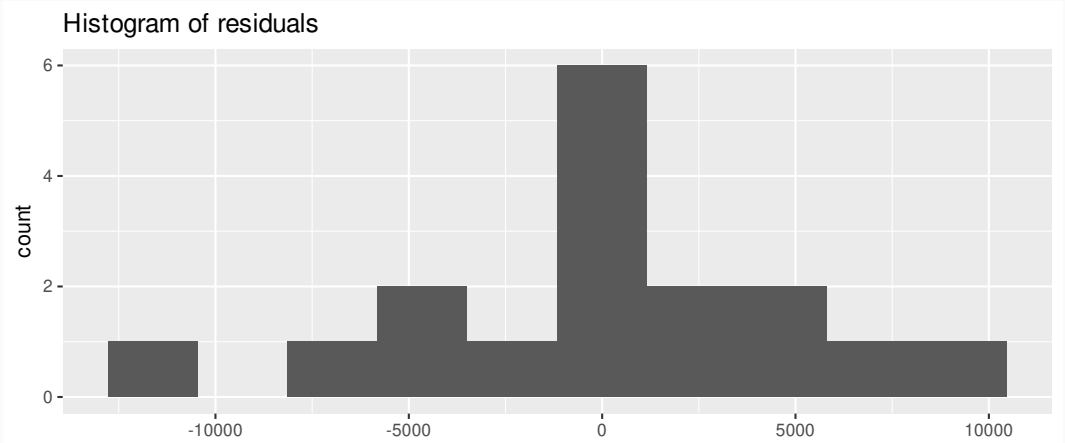
Total in-school staff

```
augment(fit) |>
  autoplot(.resid) +
  labs(x = "Day", y = "", title = "Residuals from RW w/ drift method")
```



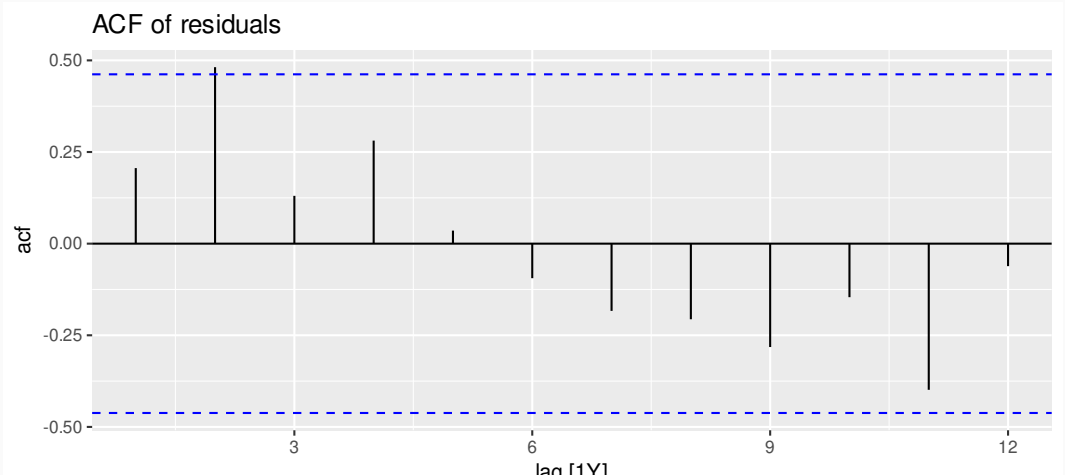
Total in-school staff

```
augment(fit) |>
  ggplot(aes(x = .resid)) +
  geom_histogram(bins = 10) +
  labs(title = "Histogram of residuals")
```



Total in-school staff

```
augment(fit) |>
  ACF(.resid) |>
  autoplot() + labs(title = "ACF of residuals")
```

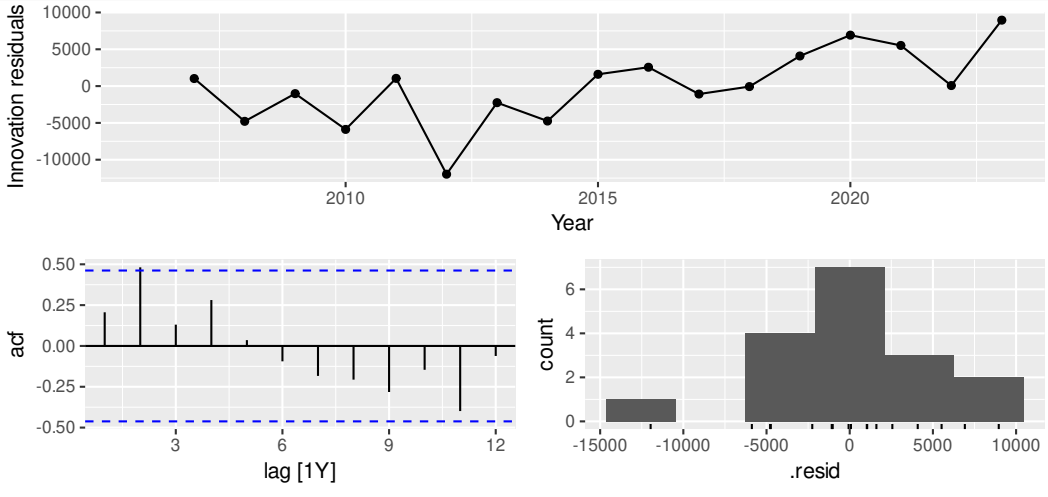


ACF of residuals

- We assume that the residuals are white noise (uncorrelated, mean zero, constant variance). If they aren't, then there is information left in the residuals that should be used in computing forecasts.
- So a standard residual diagnostic is to check the ACF of the residuals of a forecasting method.
- We *expect* these to look like white noise.

Combined diagnostic graph

```
fit |> gg_tsresiduals()
```



Ljung-Box test

Test whether *whole set* of r_k values is significantly different from zero set.

$$Q = T(T+2)\sum_{k=1}^{\ell}(T-k)^{-1}r_k^2 \qquad \text{where } \ell = \text{max lag and } T = \#\text{ observations}$$

- If each r_k close to zero, Q will be **small**.
- If some r_k values large (+ or $-$), Q will be **large**.
- My preferences: $h = 10$ for non-seasonal data, $h = 2m$ for seasonal data.
- If data are WN and T large, $Q \sim \chi^2$ with ℓ degrees of freedom.

Ljung-Box test

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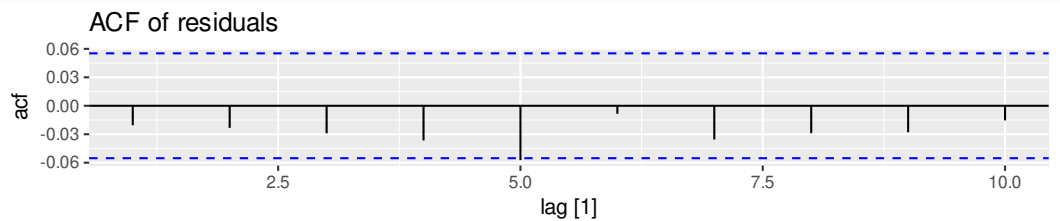



```
# lag = h
augment(fit) |> features(.resid, ljung_box, lag = 10)
```

```
# A tibble: 1 x 3
  .model               lb_stat lb_pvalue
  <chr>                  <dbl>     <dbl>
1 RW(Count ~ drift())     15.3     0.121
```

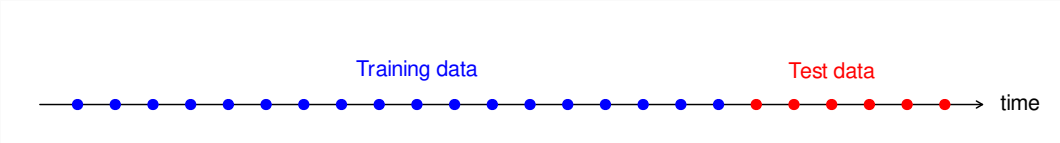
Outline

Lab Session 10

- Compute RW w/ drift forecasts for total student enrolments in Australia (`students`).
- Test if the residuals are white noise. What do you conclude?

Outline

Training and test sets



- A model which fits the training data well will not necessarily forecast well.
- Forecast accuracy is based only on the test set.

Forecast errors

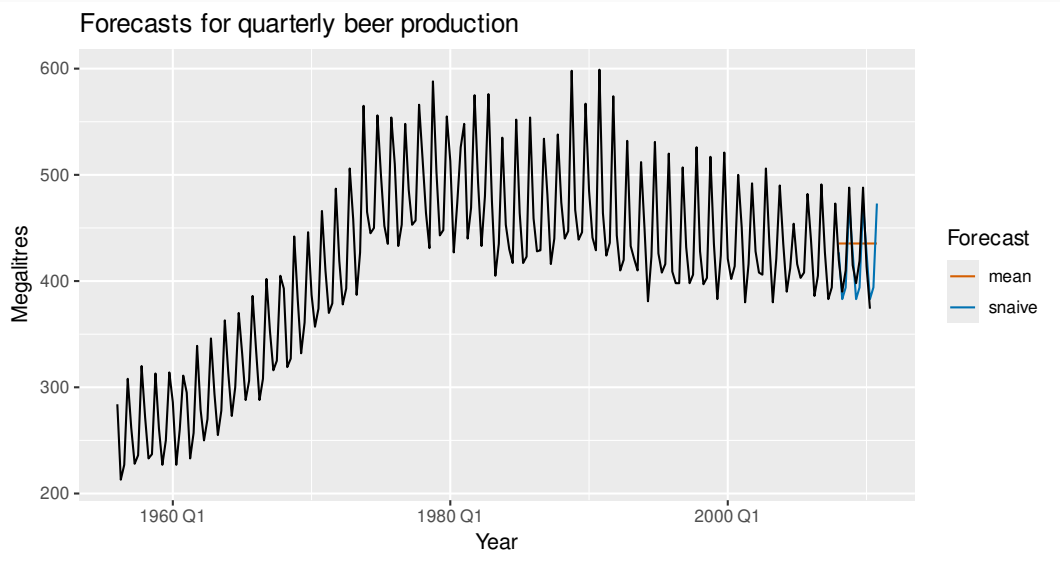
Forecast “error”: the difference between an observed value and its forecast.

$$e_{T+h} = y_{T+h} - \hat{y}_{T+h|T},$$

Measures of forecast accuracy

```
beer_fit <- aus_production |>
  filter(between(year(Quarter), 1992, 2007)) |>
  model(
    snaive = SNAIVE(Beer),
    mean = MEAN(Beer)
  )
beer_fit |>
  forecast(h = "3 years") |>
  autoplot(aus_production, level = NULL) +
  labs(title ="Forecasts for quarterly beer production",
       x ="Year", y ="Megalitres") +
  guides(colour = guide_legend(title = "Forecast"))
```

Measures of forecast accuracy



Measures of forecast accuracy

y_{T+h} = $(T+h)$th observation, $h = 1, \ldots, H$

$\hat{y}_{T+h|T}$ = its forecast based on data up to time T.

e_{T+h} = $y_{T+h} - \hat{y}_{T+h|T}$

$$\text{MAE} = \text{mean}(|e_{T+h}|)$$

$$\text{MSE} = \text{mean}(e_{T+h}^2) \qquad \text{RMSE} = \sqrt{\text{mean}(e_{T+h}^2)}$$

$$\text{MAPE} = 100\text{mean}(|e_{T+h}|/|y_{T+h}|)$$

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- MAE, MSE, RMSE are all scale dependent.
- MAPE is scale independent but is only sensible if $y_t \gg 0$ for all t, and y has a natural zero.

Measures of forecast accuracy

Mean Absolute Scaled Error

$$\text{MASE} = \text{mean}(|e_{T+h}|/Q)$$

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- For non-seasonal series, scale uses naïve forecasts:

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- For seasonal series, scale uses seasonal naïve forecasts:

$$Q = \frac{1}{T-m}\sum_{t=m+1}^{T}|y_t - y_{t-m}|$$

where m is the seasonal frequency

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where m is the seasonal frequency

Proposed by Hyndman and Koehler (IJF, 2006).

Measures of forecast accuracy

Root Mean Squared Scaled Error

$$\text{RMSSE} = \sqrt{\text{mean}(e_{T+h}^2/Q)}$$

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$$Q = \frac{1}{T-1}\sum_{t=2}^{T}(y_t - y_{t-1})^2$$

- For seasonal series, scale uses seasonal naïve forecasts:

$$Q = \frac{1}{T-m}\sum_{t=m+1}^{T}(y_t - y_{t-m})^2$$

where m is the seasonal frequencyq

Proposed by Hyndman and Koehler (IJF, 2006).

Measures of forecast accuracy

```
beer_fc <- forecast(beer_fit, h = "3 years")
accuracy(beer_fc, aus_production)
```

```
# A tibble: 2 x 10
  .model .type    ME  RMSE   MAE   MPE  MAPE  MASE RMSSE    ACF1
  <chr>  <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>   <dbl>
1 mean   Test  -13.8  38.4  34.8 -3.97  8.28 2.20  1.96  -0.0691
2 snaive Test    5.2  14.3  13.4  1.15  3.17 0.847 0.729  0.132
```

Outline

Lab Session 11

- Create a training set for employed Australian students (`student_labour`) by withholding the last four years as a test set.
- Fit all the appropriate benchmark methods to the training set and forecast the periods covered by the test set.
- Compute the accuracy of your forecasts. Which method does best?

Finished early?

Repeat the exercise using the Australian takeaway food turnover data (`aus_retail`) with a test set of four years.