



# Time Series Analysis & Forecasting Using R

## 9. Dynamic regression



# Outline

- 1 Regression with ARIMA errors
- 2 Some useful predictors
- 3 Dynamic harmonic regression
- 4 Lab Session 16
- 5 Forecasting with regressors
- 6 Lab Session 17

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# Time series regression

## **i** Regression models

$$y_t = \beta_0 + \beta_1 X_{1,t} + \cdots + \beta_k X_{k,t} + \varepsilon_t,$$

- $y_t$  modeled as function of  $k$  explanatory variables
- In regression, we assume that  $\varepsilon_t$  is white noise.

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- In regression, we assume that  $\varepsilon_t$  is white noise.

Specify this model with `TSLM()`.

Much like `lm()`, regressors are specified on the formula's right.

# Regression with ARIMA errors

## **i** RegARIMA models

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t,$$
$$\eta_t \sim \text{ARIMA}$$

- Residuals are from ARIMA model.
- Estimate model in one step using MLE
- Select model with lowest AICc value.

Simply add regression terms to the `ARIMA()` formula's right.

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# Some useful predictors

## **i** Linear trend

The time index is an effective predictor for trends.  
It can be added to models as a regressor with `trend()`.



# Some useful predictors

## Linear trend

The time index is an effective predictor for trends.  
It can be added to models as a regressor with `trend()`.

## Piecewise linear trend

Trends often change over time.  
We can add changepoints in the trend with  
`trend(knots = <times>)`.

# Some useful predictors

## **i** Dummy variables

Identify categories of observations with  $\{0, 1\}$  indicators.  
Useful for public holidays, special events & policy changes.

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## **i** Dummy variables

Identify categories of observations with  $\{0, 1\}$  indicators.  
Useful for public holidays, special events & policy changes.

## **i** Dummy seasonality

Use dummy variables for each season.  
It can be added to models as a regressor with `season()`.

# Some useful predictors

## Fourier seasonality

Fourier terms use sine and cosine harmonics to model seasonality.

It offers key advantages over dummy seasonality:

- Reduce model complexity
- Non-integer seasonality

Use `fourier(K = ???)` to add fourier seasonality with  $\kappa$  harmonics to the model.

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# Dynamic harmonic regression

Capture seasonality with fourier terms instead of ARIMA PDQ().

## Advantages

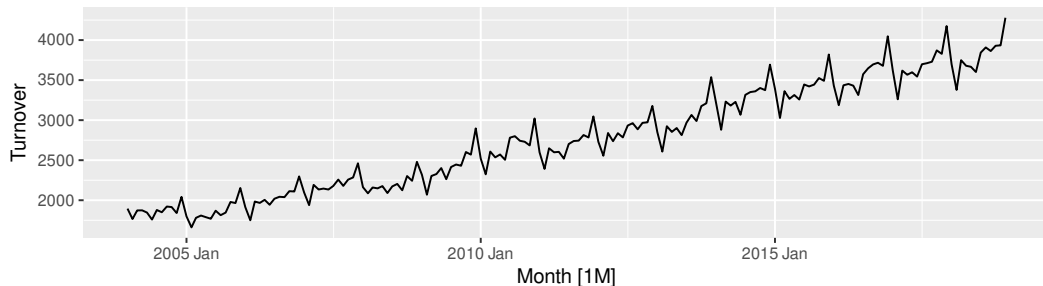
- all the benefits of fourier terms;
- supports multiple seasonality via multiple fourier terms;
- capture remaining dynamics with a simple ARMA model.

## Disadvantages

- seasonality cannot change over time.

# Eating-out expenditure

```
aus_cafe <- aus_retail |>
  filter(
    Industry == "Cafes, restaurants and takeaway food services",
    year(Month) %in% 2004:2018
  ) |>
  summarise(Turnover = sum(Turnover))
aus_cafe |> autoplot(Turnover)
```



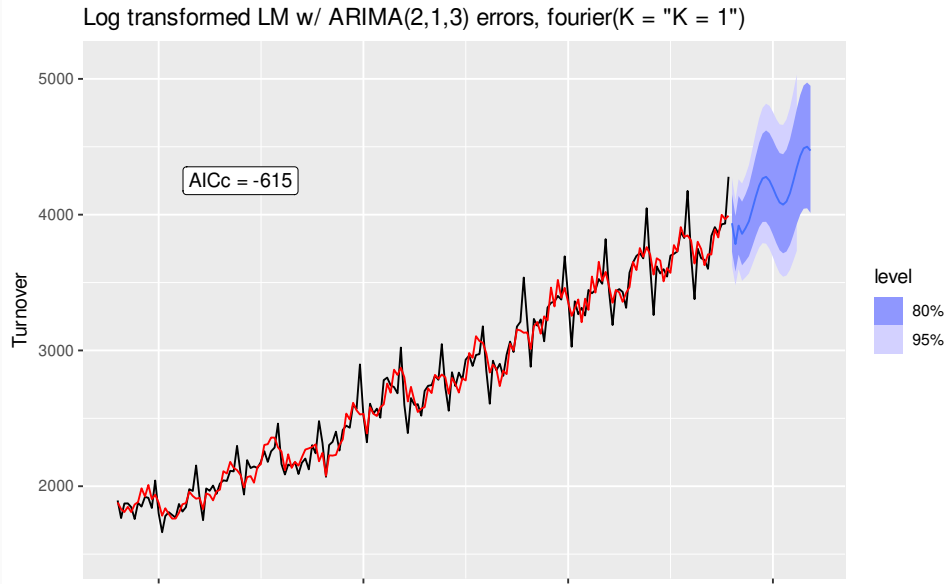
# Eating-out expenditure

```
fit <- aus_cafe |> model(  
  `K = 1` = ARIMA(log(Turnover) ~ fourier(K = 1) + PDQ(0, 0, 0)),  
  `K = 2` = ARIMA(log(Turnover) ~ fourier(K = 2) + PDQ(0, 0, 0)),  
  `K = 3` = ARIMA(log(Turnover) ~ fourier(K = 3) + PDQ(0, 0, 0)),  
  `K = 4` = ARIMA(log(Turnover) ~ fourier(K = 4) + PDQ(0, 0, 0)),  
  `K = 5` = ARIMA(log(Turnover) ~ fourier(K = 5) + PDQ(0, 0, 0)),  
  `K = 6` = ARIMA(log(Turnover) ~ fourier(K = 6) + PDQ(0, 0, 0))  
)  
glance(fit)
```

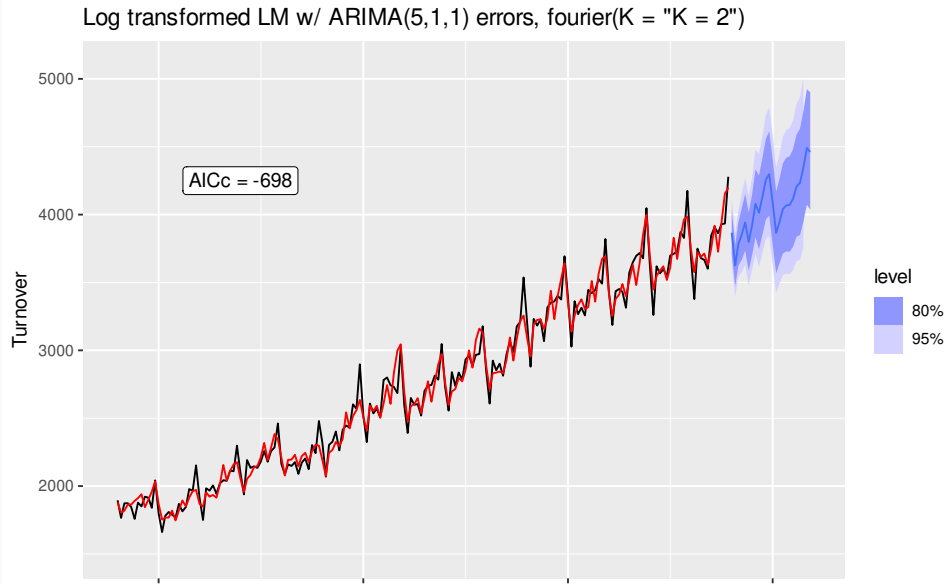
.model	sigma2	log_lik	AIC	AICc	BIC
K = 1	0.002	317	-616	-615	-588
K = 2	0.001	362	-700	-698	-661
K = 3	0.001	394	-763	-761	-725
K = 4	0.001	427	-822	-818	-771
K = 5	0.000	474	-919	-917	-875
K = 6	0.000	474	-920	-918	-875



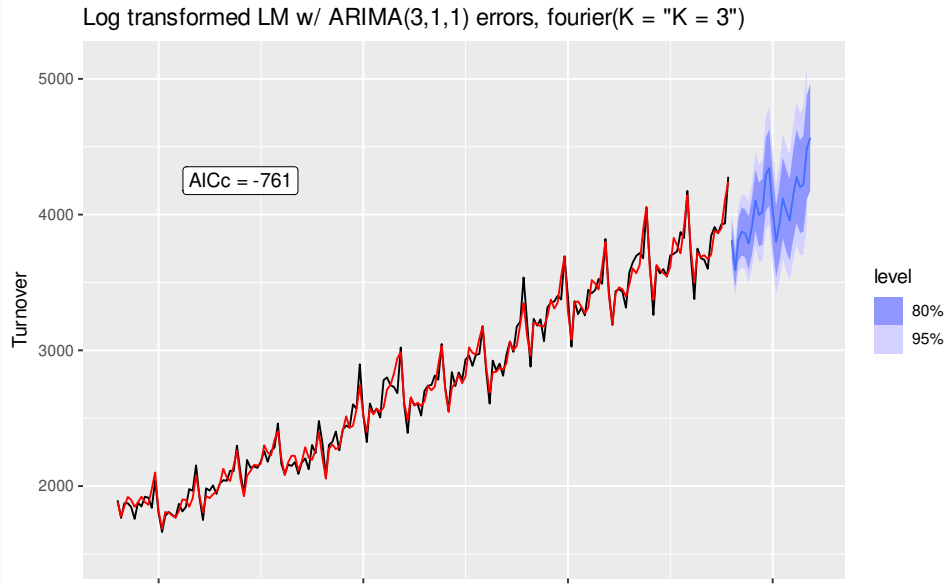
# Eating-out expenditure



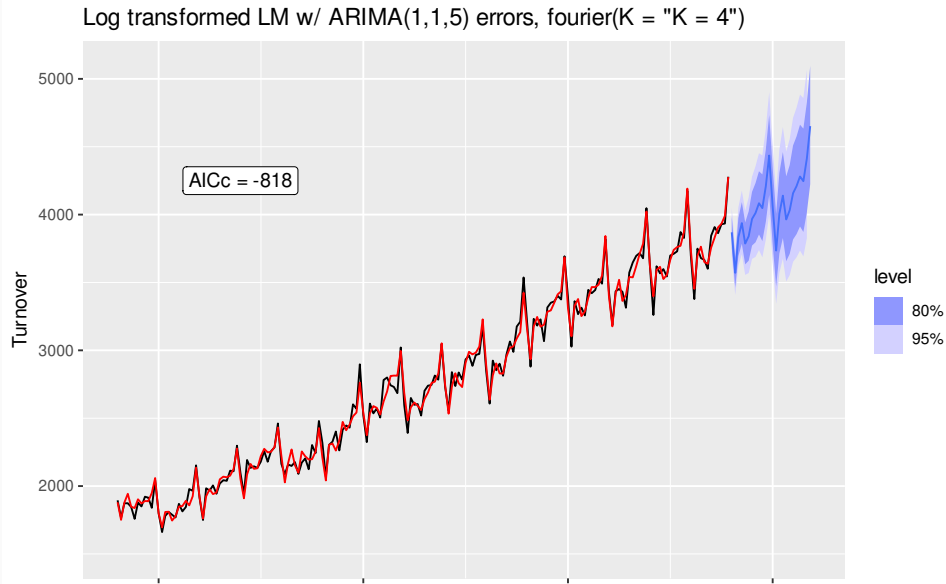
# Eating-out expenditure



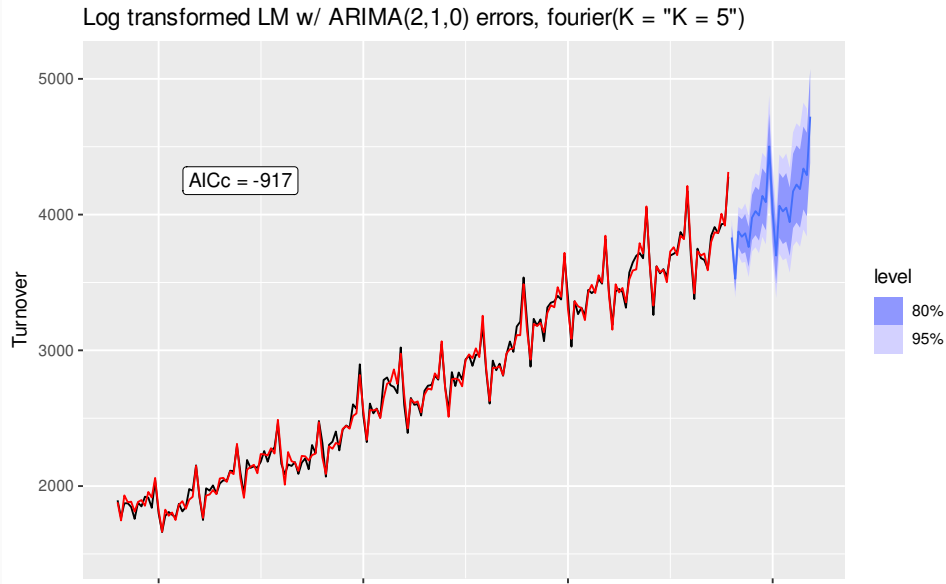
# Eating-out expenditure



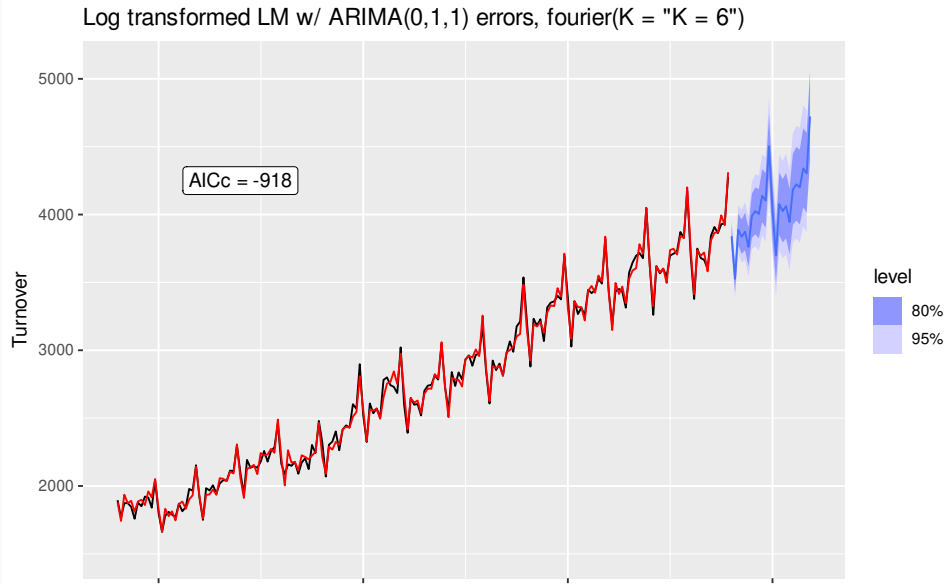
# Eating-out expenditure



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# Lab Session 16

Produce forecasts of preschool and school education jobs from the ABS payroll data (payroll\_education).

```
payroll_education |>  
  filter(Industry == "Preschool and School Education")
```

- 1 Estimate a TSLM model with appropriate regression terms (trend, fourier harmonics, ...)
- 2 Produce and visualise forecasts from this model
- 3 Perform residual diagnostic checks on the model
- 4 Instead use dynamic harmonic regression
- 5 Do the residuals and forecasts look better?



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# Forecasting with regressors

## **i** Additional regressors

Using additional information from other variables is a great way to enhance your time series model.

Add them the the formula just like  $\tau_m()$ .

Additional regressors in forecasting models can make it harder to produce forecasts. Why?

# Forecasting with regressors

## ! Future values

Future regressor values need to be given for forecasting.

# Forecasting with regressors

## ! Future values

Future regressor values need to be given for forecasting.

## Advantages

- Future values could be known in advance.
- Forecasts under different scenarios can be compared.

## Disadvantages

- Unknown future values also need forecasting.
- Forecasts ignore the uncertainty in predictors.

# Lagged predictors

Sometimes a change in  $x_t$  does not affect  $y_t$  instantaneously

- $y_t$  = sales,  $x_t$  = advertising.
- $y_t$  = stream flow,  $x_t$  = rainfall.
- $y_t$  = size of herd,  $x_t$  = breeding stock.

# Lagged predictors

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  - $y_t$  = size of herd,  $x_t$  = breeding stock.
- 
- These are dynamic systems with input ( $x_t$ ) and output ( $y_t$ ).
  - $x_t$  is often a leading indicator.
  - There can be multiple predictors.

# Lagged predictors

The model include present and past values of predictor:

$X_t, X_{t-1}, X_{t-2}, \dots$

$$y_t = a + \nu_0 X_t + \nu_1 X_{t-1} + \dots + \nu_k X_{t-k} + \eta_t$$

where  $\eta_t$  is an ARIMA process.

# Lagged predictors

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$X_t, X_{t-1}, X_{t-2}, \dots$

$$y_t = a + \nu_0 X_t + \nu_1 X_{t-1} + \dots + \nu_k X_{t-k} + \eta_t$$

where  $\eta_t$  is an ARIMA process.

- $x$  can influence  $y$ , but  $y$  is not allowed to influence  $x$ .

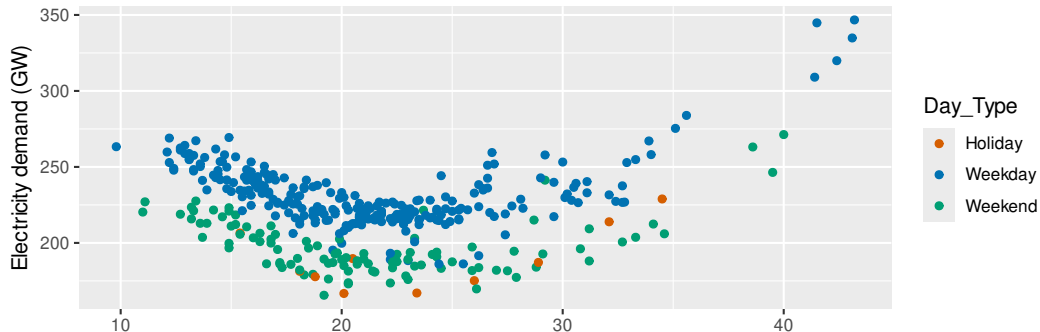
Use `lag()` on model regressors to lag them.



# Daily electricity demand

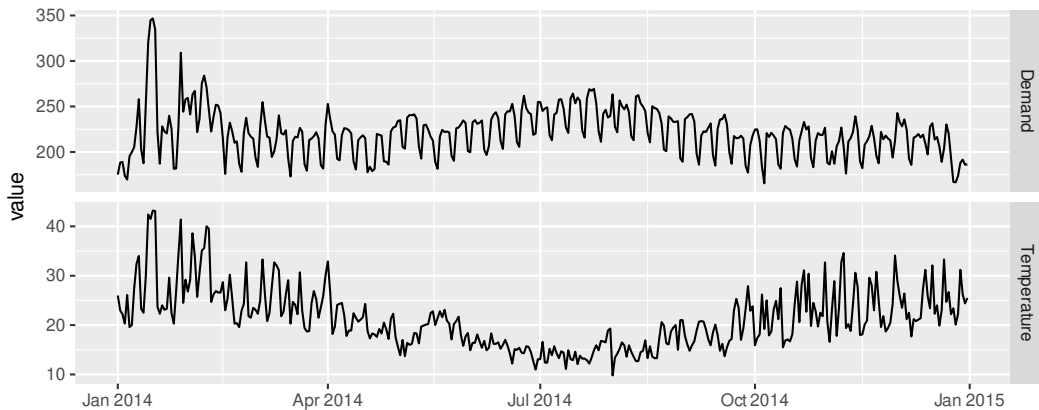
Model daily electricity demand as a function of temperature using quadratic regression with ARMA errors.

```
vic_elec_daily |>  
  ggplot(aes(x = Temperature, y = Demand, colour = Day_Type)) +  
  geom_point() +  
  labs(x = "Maximum temperature", y = "Electricity demand (GW)")
```



# Daily electricity demand

```
vic_elec_daily |>  
  pivot_longer(c(Demand, Temperature)) |>  
  ggplot(aes(x = Date, y = value)) +  
  geom_line() +  
  facet_grid(vars(name), scales = "free_y")
```



# Daily electricity demand

```
fit <- vic_elec_daily |>
  model(fit = ARIMA(Demand ~ Temperature + I(Temperature^2) +
    (Day_Type == "Weekday")))
report(fit)
```

Series: Demand

Model: LM w/ ARIMA(2,1,2)(2,0,0)[7] errors

Coefficients:

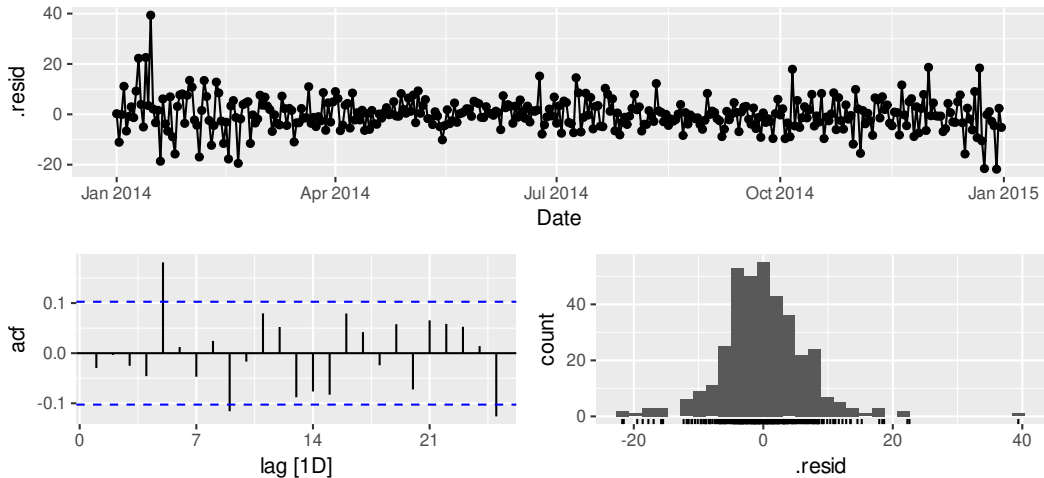
	ar1	ar2	ma1	ma2	sar1	sar2	Temperature
	-0.1093	0.7226	-0.0182	-0.9381	0.1958	0.417	-7.614
s.e.	0.0779	0.0739	0.0494	0.0493	0.0525	0.057	0.448
	I(Temperature^2)						Day_Type == "Weekday"TRUE
		0.1810				30.40	
s.e.		0.0085				1.33	

sigma^2 estimated as 44.91: log likelihood=-1206

AIC=2432 AICc=2433 BIC=2471

# Daily electricity demand

```
augment(fit) |>  
  gg_tsdisplay(.resid, plot_type = "histogram")
```



# Daily electricity demand

```
fit |>  
  forecast(h = 14)
```

```
Error in `mutate()`:  
i In argument: `fit = (function (object, ...) ...`.  
Caused by error in `value[[3L]]()`:  
! object 'Temperature' not found  
  Unable to compute required variables from provided `new_data`.  
  Does your model require extra variables to produce forecasts?
```

## ! More information needed

Our model depends on Temperature and Day\_Type.  
To produce forecasts, we need to also provide future values  
for these variables.

# Daily electricity demand

```
# Forecast one day ahead.
vic_next_day <- new_data(vic_elec_daily, 1) |>
  mutate(Temperature = 26, Day_Type = "Holiday")
forecast(fit, vic_next_day)

# A tibble: 1 x 6 [1D]
# Key:   .model [1]
#   .model Date          Demand .mean
#   <chr>   <date>         <dbl> <dbl>
1 fit     2015-01-01 N(161, 45)  161.

# i 2 more variables: Temperature <dbl>, Day_Type <chr>
```

# Daily electricity demand

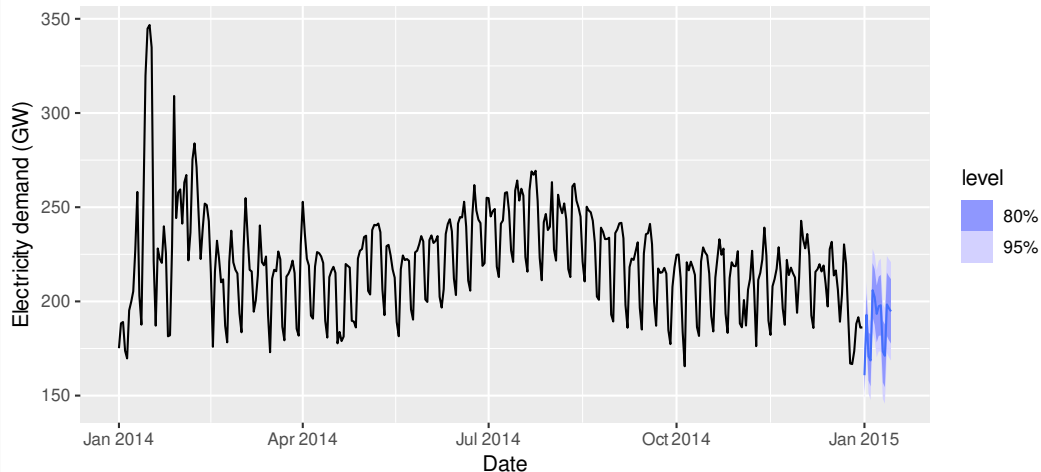
```
# Forecast two weeks ahead.  
vic_elec_future <- new_data(vic_elec_daily, 14) |>  
  mutate(  
    Temperature = 26,  
    Holiday = c(TRUE, rep(FALSE, 13)),  
    Day_Type = case_when(  
      Holiday ~ "Holiday",  
      wday(Date) %in% 2:6 ~ "Weekday",  
      TRUE ~ "Weekend"  
    )  
  )
```

## Scenario forecasting

Instead of forecasting most-likely values for regressors, it can be worthwhile forecasting worst-case scenarios to adequately prepare.

# Daily electricity demand

```
forecast(fit, vic_elec_future) |>  
  autoplot(vic_elec_daily) + labs(y = "Electricity demand (GW)")
```

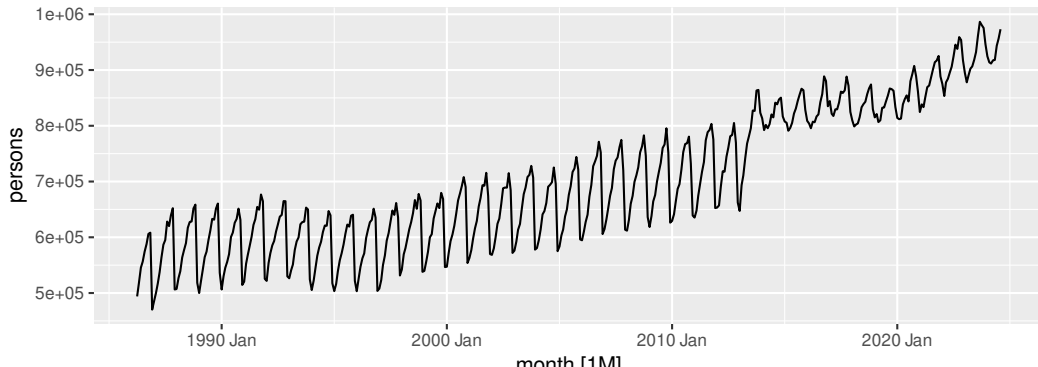




# Scenarios with policy decisions

Consider the total school students aged 15-19 in Australia.

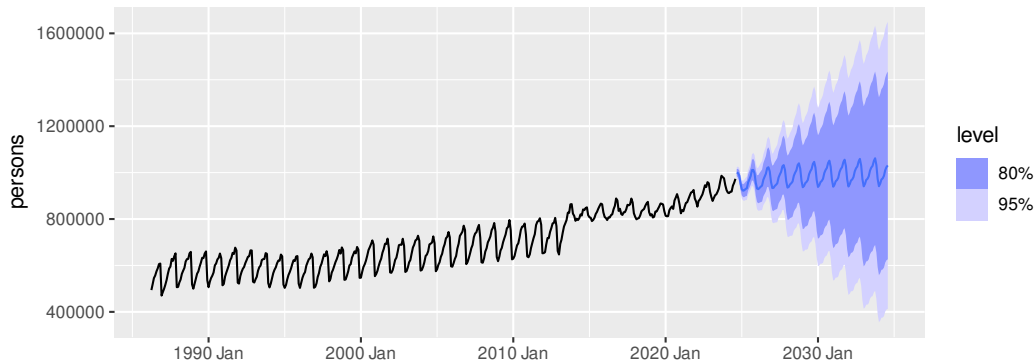
```
working_age_school_students <- student_labour |>  
  filter(attendance == "Attending school (aged 15-19 years)") |>  
  summarise(persons = sum(persons))  
working_age_school_students |> autoplot(persons)
```



# Scenarios with policy decisions

Without capturing the policy change, the forecasts are biased.

```
working_age_school_students |>  
  model(ARIMA(persons ~ fourier(K = 3))) |>  
  forecast(h = "10 years") |>  
  autoplot(working_age_school_students)
```



# Scenarios with policy decisions

Add dummy variable for the change in 2013 that interacts with the seasonality.

```
fit_policy <- working_age_school_students |>
  mutate(new_policy = month >= yearmonth("2013 Jan")) |>
  model(ARIMA(persons ~ new_policy*fourier(K = 3)))
report(fit_policy)
```

Series: persons

Model: LM w/ ARIMA(0,1,1)(2,0,0)[12] errors

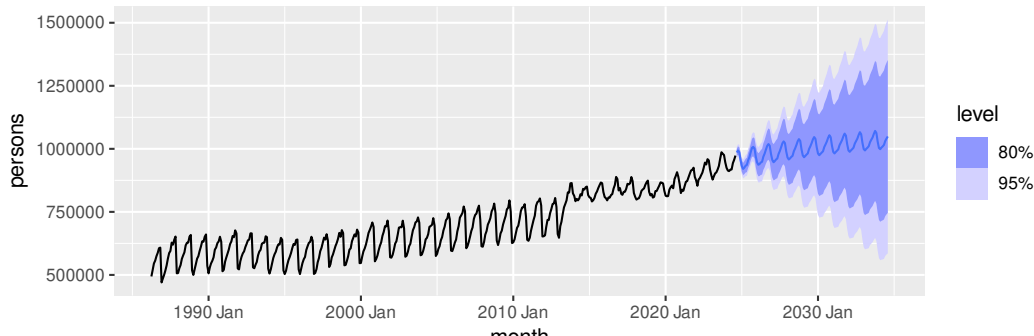
Coefficients:

	ma1	sar1	sar2	new_policyTRUE	fourier(K = 3)C1_12
	-0.3537	0.4544	0.3826	-38178	-47610
s.e.	0.0463	0.0436	0.0445	10015	5302
	fourier(K = 3)S1_12	fourier(K = 3)C2_12	fourier(K = 3)S2_12		
	-41123	-24555	2573		
s.e.	5346	3368	3371		

# Scenarios with policy decisions

The policy is expected to continue into the future.

```
future_policy <- new_data(working_age_school_students, 120) |>  
  mutate(new_policy = TRUE)  
fit_policy |>  
  forecast(new_data(working_age_school_students, 120) |> mutate(new_policy = TRUE))  
autoplot(working_age_school_students)
```



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## Lab Session 17

What if the 2013 policy was later reverted, what would you expect the forecasts to be?

- 1 Produce forecasts from a scenario in which this policy was reverted in 2030.

Hint: Use `new_data()` to create the future time points, and `mutate()` a date comparison to create the future dummy variable values.

- 2 Visualise the forecasts, are they realistic?

# Forecasting unknown scenarios

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## International student commencements to be capped at 270,000 next year

By political reporter [Maani Truu](#)

Posted Tue 27 Aug 2024 at 12:05pm, updated Tue 27 Aug 2024 at 4:18pm

[abc.net.au/news/international-student-ca...](https://abc.net.au/news/international-student-ca...)

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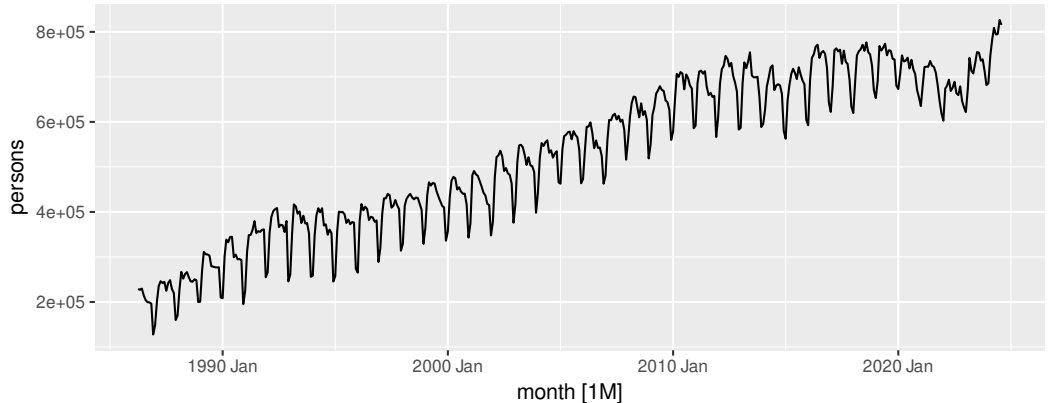
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- **In short:** New international student commencements will be capped at 270,000 across Australian higher education and vocational providers for the 2025 calendar year.
- Education Minister Jason Clare said the changes — if passed by parliament — would make the international student sector "better and fairer".
- **What's next?** The federal government will consult with universities about individual caps.

# Forecasting unknown scenarios

```
student_labour |>  
  filter(attendance == "Attending tertiary educational institution full-time") |>  
  summarise(persons = sum(persons)) |>  
  autoplot(persons)
```





# Forecasting unknown scenarios

💡 How would we forecast something without history?

- Judgemental forecasting with expert opinions
- Incorporate limits into the model  
(will these limits be met by demand?)
- Forecast more disaggregated data separately