

Image Processing

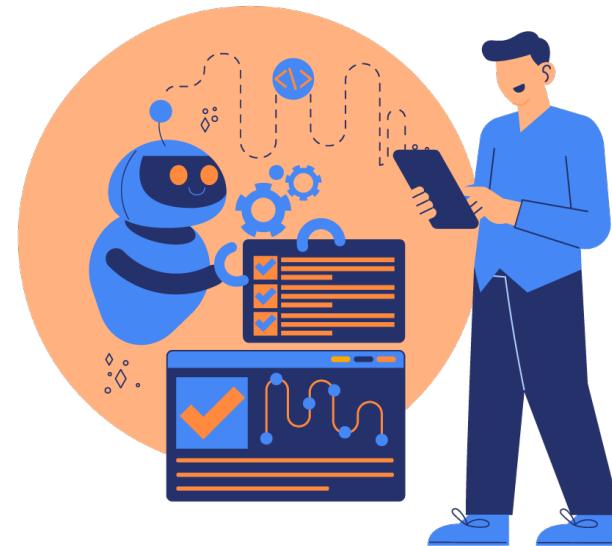
Dr. Mitchell Olsthoorn

Dr. Annibale Panichella



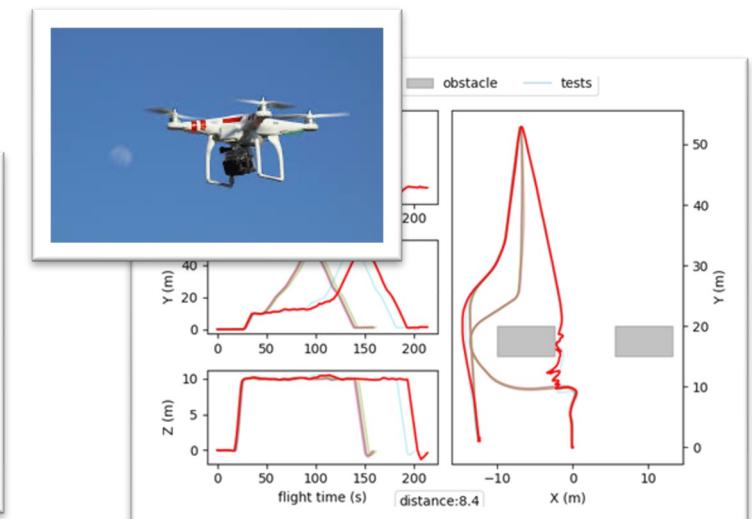
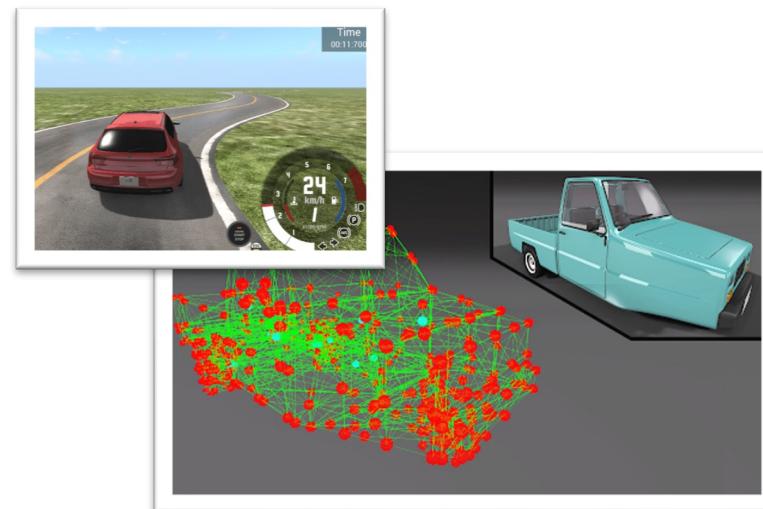


Dr. Mitchell Olschoorn





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Overview

01

Digital Images



02

Image
Filtering



03

Advanced
Use Cases



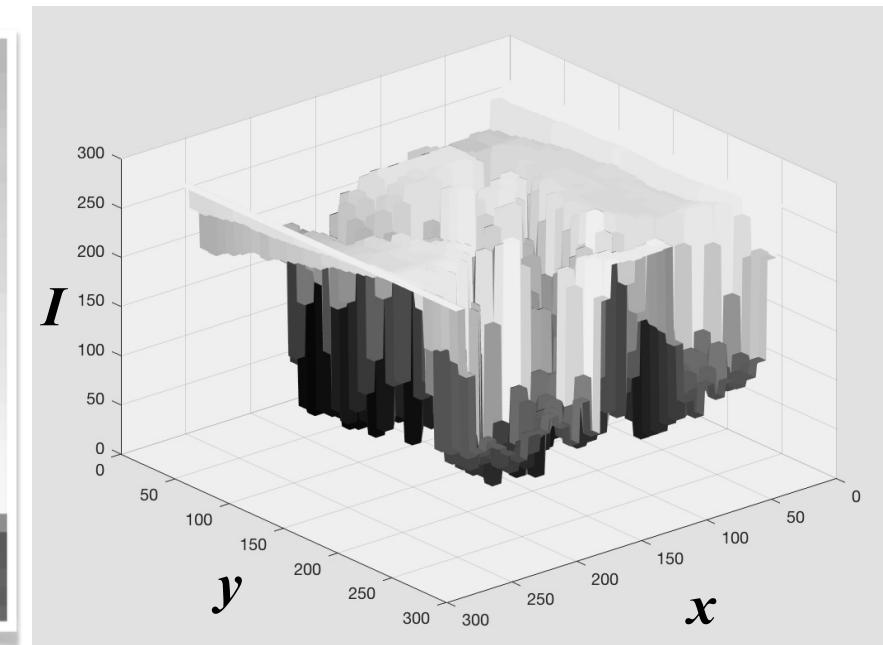


01

What is an Image?

Digital Images

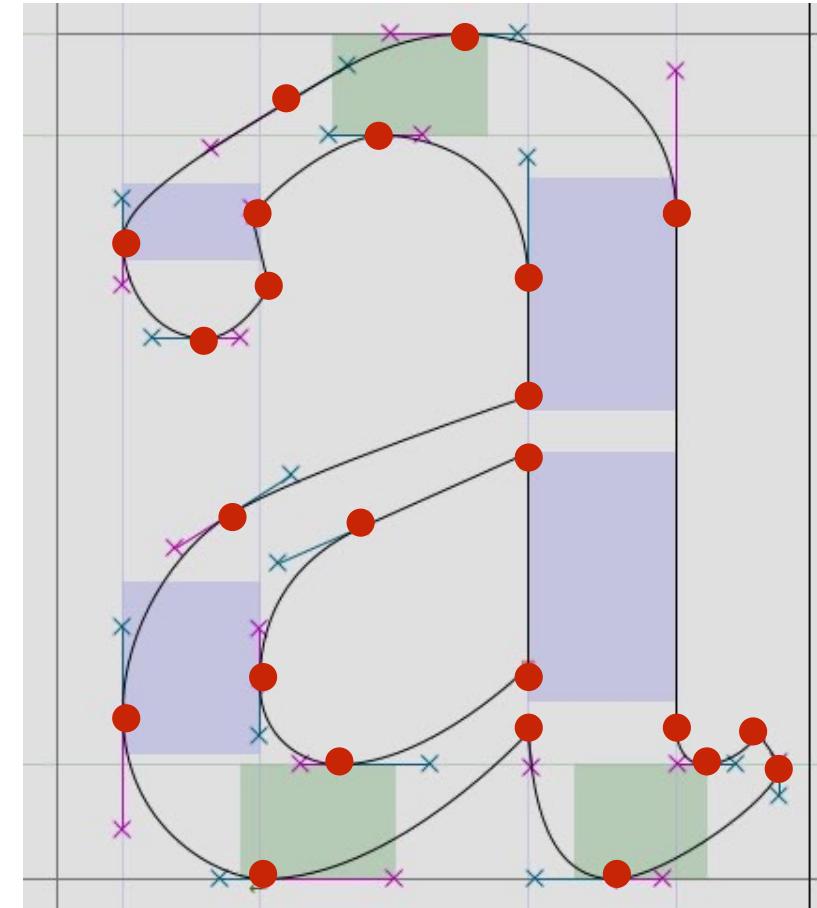
- A digital images is a two-dimensional arrays of numbers
- Each number (pixel) represents the intensity (I) of the colors



Vector Images

- Images are modeled using mathematical formulas: like polygons, curves, lines.
- The quality of the vector images is preserved when they are zoomed in
- Used by word processors (Microsoft Word), video-games, etc.

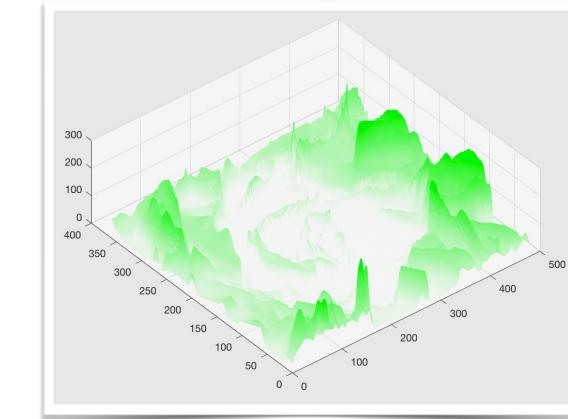
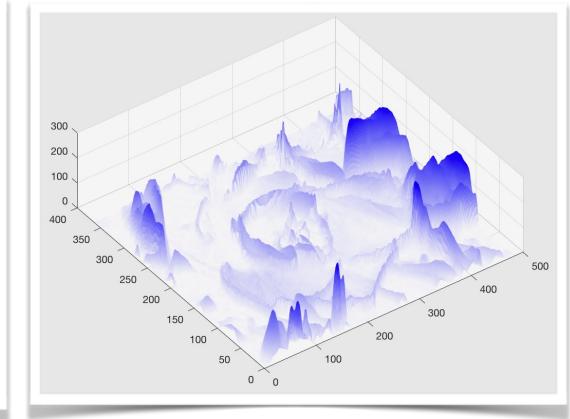
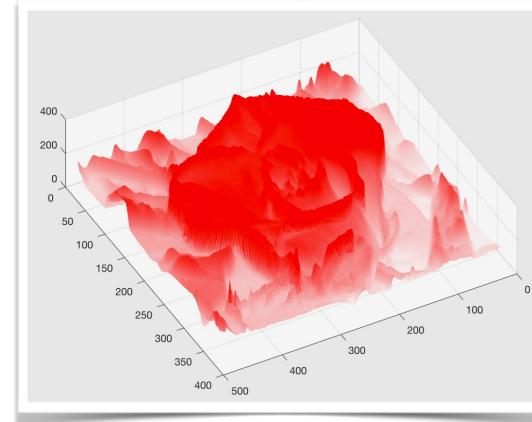
Control Points



Color Digital Images

- A color images is a three-dimensional grid: each pixel is described with three colors =>

$$I(x,y) = \mathbf{R}(x,y) + \mathbf{G}(x,y) + \mathbf{B}(x,y)$$



Demonstration – What is an Image?





02

What is Image Noise?

Image Noise

- Noise: any random degradation (variation of brightness or color information) on an image caused by external disturbance

$$I(x,y) = R(x,y) + G(x,y) + B(x,y) + D(x,y)$$



Source of Noise

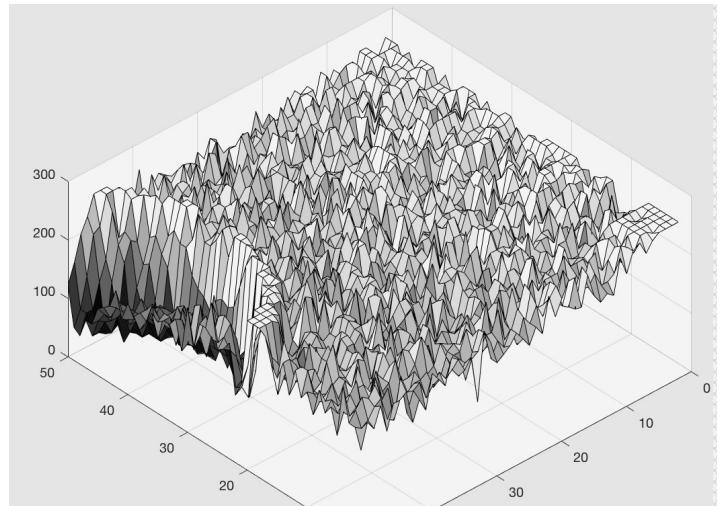
- **Limits of the device** used to take the picture: (ISO factor) how quickly the camera's sensor absorbs lights
- Errors in the **transmission** (e.g., if an image is sent from a satellite to a phone)
- Memory cell **failures**
- **Sensor heat** while clicking an image



Type of Noise



Salt and Pepper



Uniform Noise



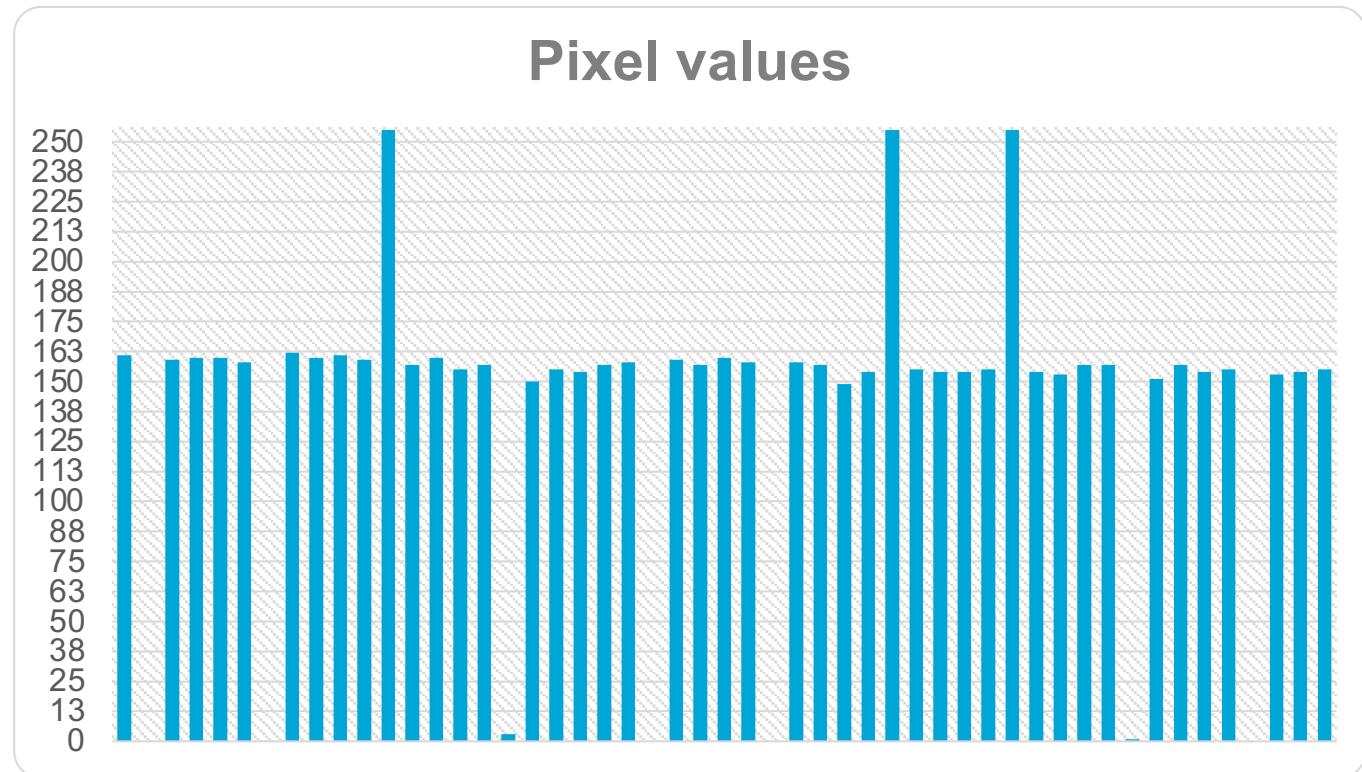
Blurry Images



03

What is the Intuition Behind Image Filtering?

Salt and Pepper Noise



Median Filter



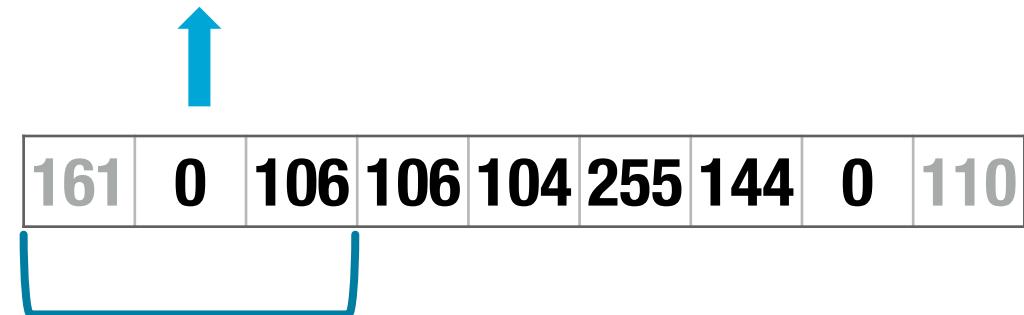
161	0	106	106	104	255	144	0	110
-----	---	-----	-----	-----	-----	-----	---	-----

Kernel

Median Filter



$$\text{Median}(0, 106, 161) = 106$$



Kernel

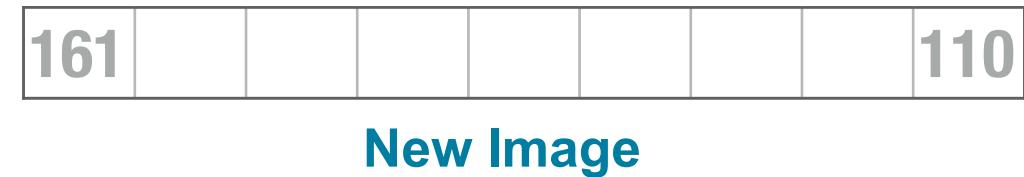
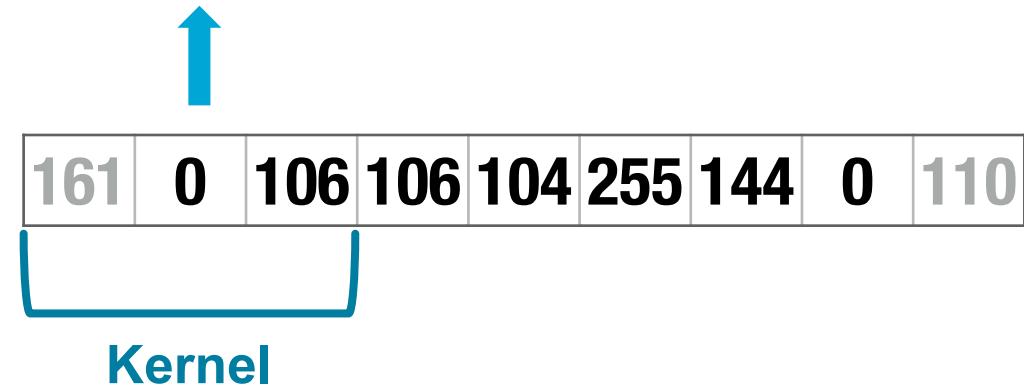
161								110
-----	--	--	--	--	--	--	--	-----

New Image

Median Filter



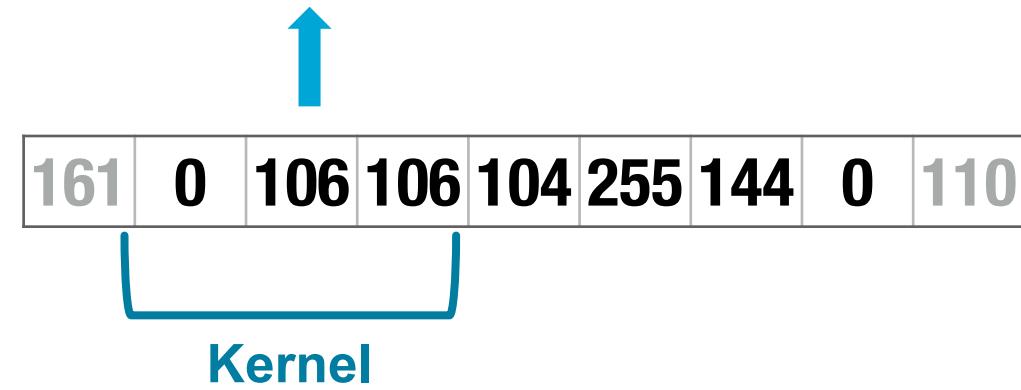
$$\text{Median}(0, 106, 161) = 106$$



Median Filter



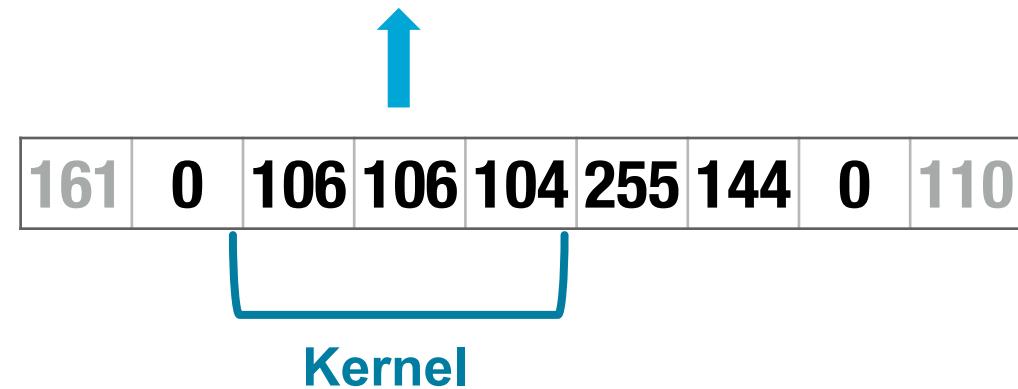
$$\text{Median}(0,106,106) = 106$$



Median Filter



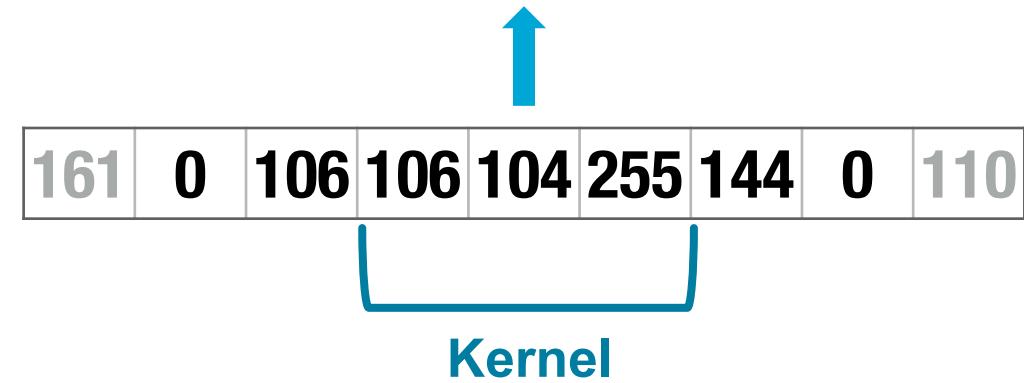
$$\text{Median}(104, 106, 106) = 106$$



Median Filter



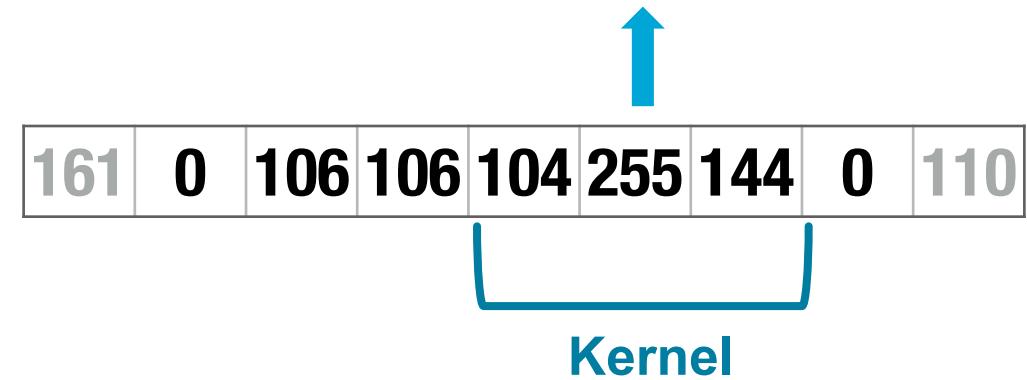
$$\text{Median}(104, 106, 255) = 106$$



Median Filter



$$\text{Median}(104, 144, 255) = 144$$



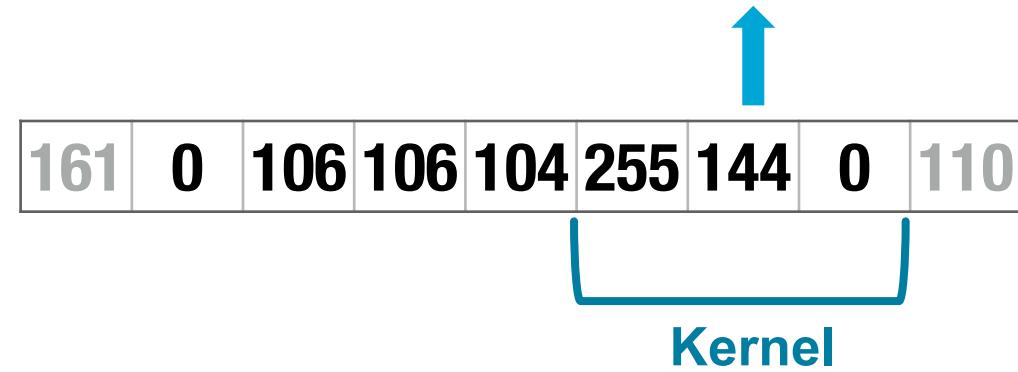
New Image

161	106	106	106	106	144			110
-----	-----	-----	-----	-----	-----	--	--	-----

Median Filter



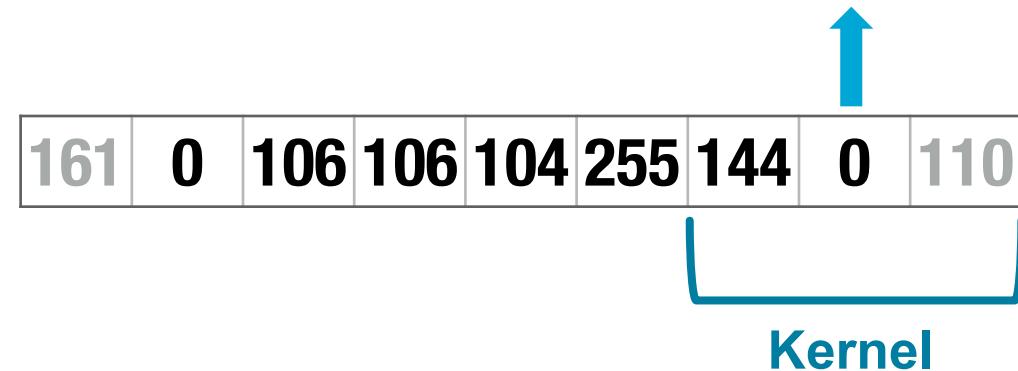
$$\text{Median}(0, 144, 255) = 144$$



Median Filter



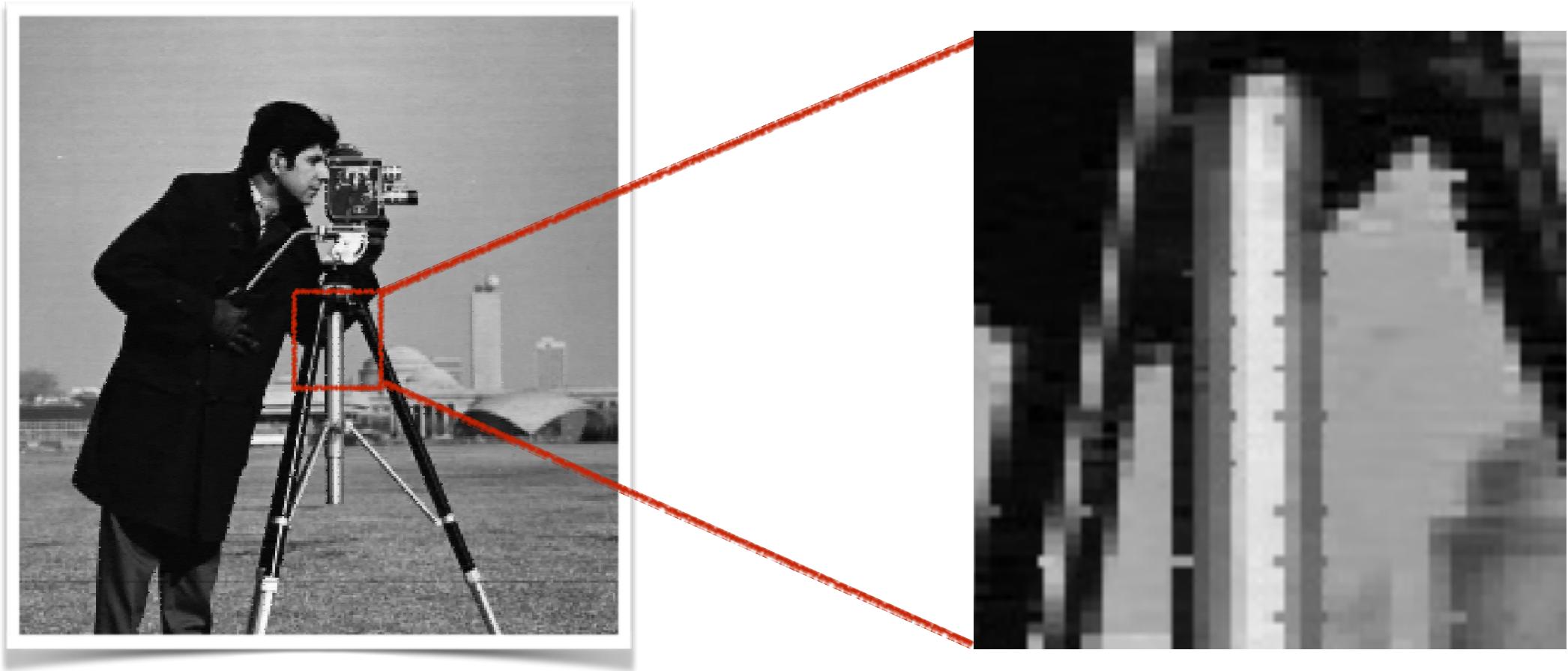
$$\text{Median}(0,110,144) = 110$$



New Image

161	106	106	106	106	144	144	110	110
-----	-----	-----	-----	-----	-----	-----	-----	-----

Median Filter



2D Median Filter

Kernel

108	107	100	103	108	108	110	111	109
161	0	106	106	104	255	144	0	113
110	106	106	106	104	103	0	113	110
115	108	109	108	107	112	110	106	111
115	115	111	111	110	108	122	119	122
112	116	113	113	114	112	120	126	126
118	118	111	109	113	111	117	118	123
117	0	117	113	114	113	0	120	116
123	115	117	116	116	117	121	121	124

Median(0,100,106,106,...
...106,107,108,110,161)

2D Median Filter

Kernel

108	107	100	103	108	108	110	111	109
161	106	106	106	104	255	144	0	113
110	106	106	106	104	103	0	113	110
115	108	109	108	107	112	110	106	111
115	115	111	111	110	108	122	119	122
112	116	113	113	114	112	120	126	126
118	118	111	109	113	111	117	118	123
117	0	117	113	114	113	0	120	116
123	115	117	116	116	117	121	121	124

Median(0,100,106,106,...
... 106,107,108,110,161)

Kernel =

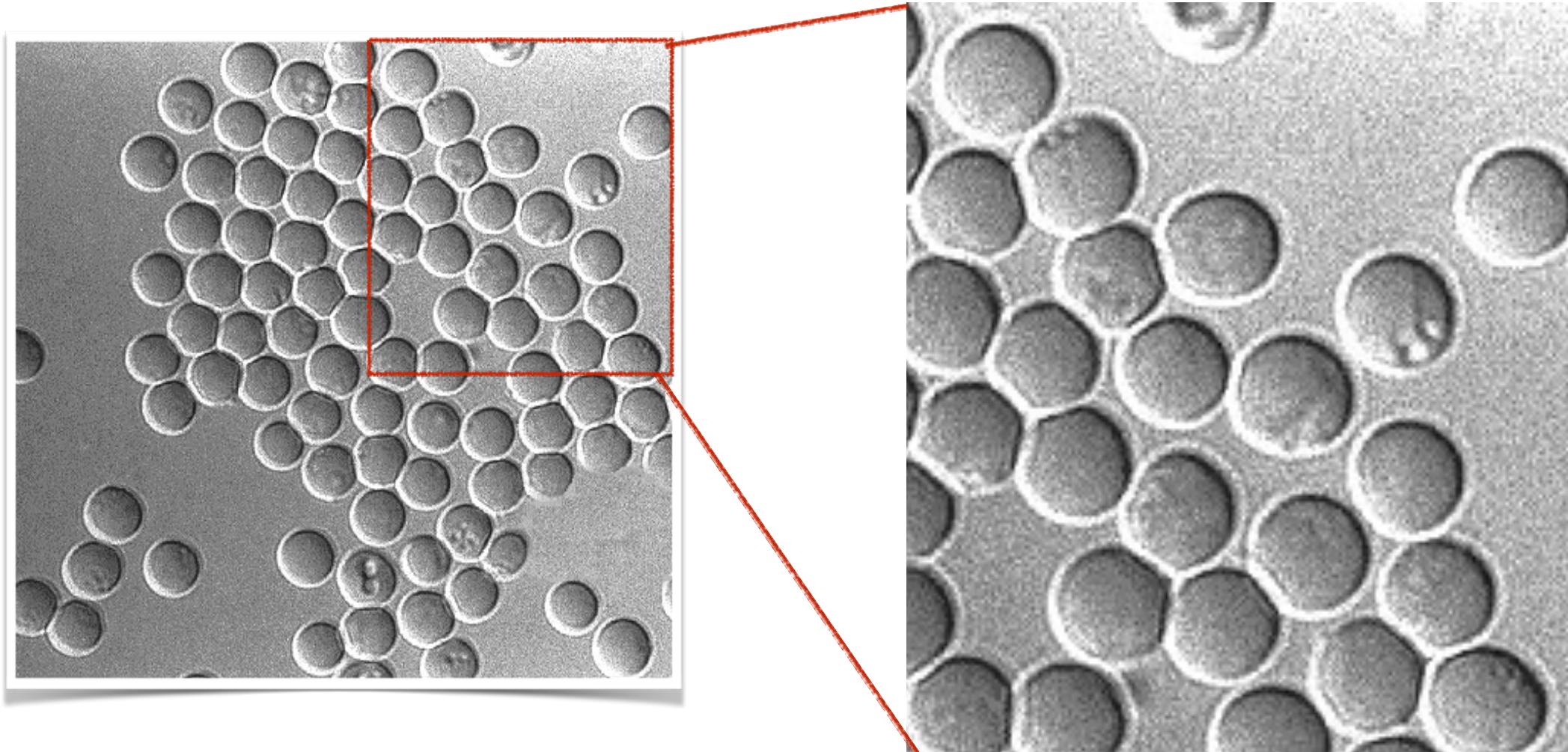
$P(i-1,j-1)$	$P(i-1,j)$	$P(i-1,j+1)$
$P(i,j-1)$	$P(i,j)$	$P(i,j+1)$
$P(i+1,j-1)$	$P(i+1,j)$	$P(i+1,j+1)$

$$P'_{(i,j)} = \text{med}\{P_{(i-1,j-1)}, \dots, P_{(i+1,j+1)}\}$$

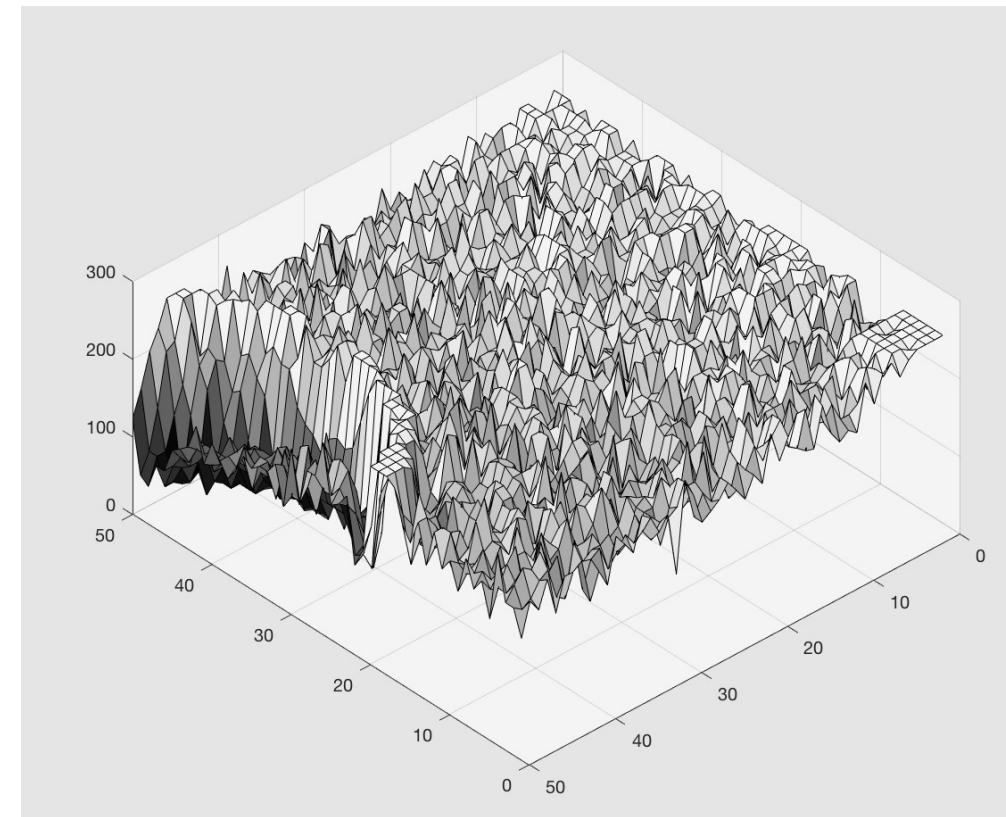
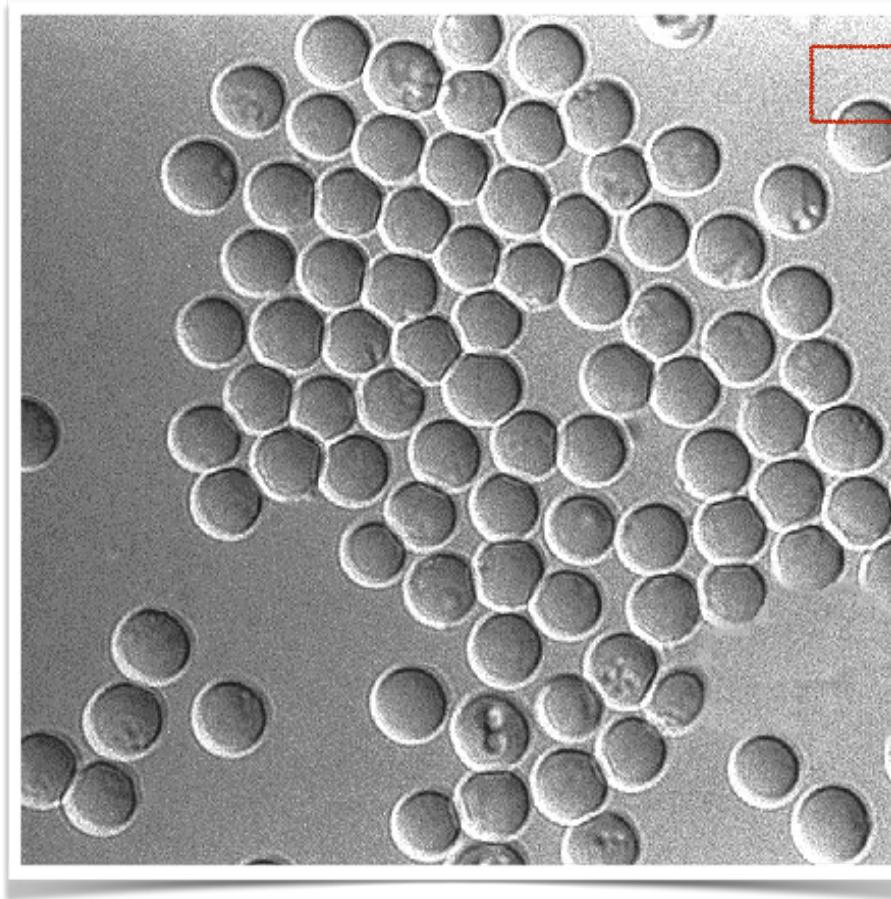
Demonstration – Median Filter



Uniform Noise



Uniform Noise



Arithmetic Mean Filter

Kernel

108	107	100	103	108	108	110	111	109
161	106	106	106	104	255	144	0	113
110	106	106	106	104	103	0	113	110
115	108	109	108	107	112	110	106	111
115	115	111	111	110	108	122	119	122
112	116	113	113	114	112	120	126	126
118	118	111	109	113	111	117	118	123
117	0	117	113	114	113	0	120	116
123	115	117	116	116	117	121	121	124

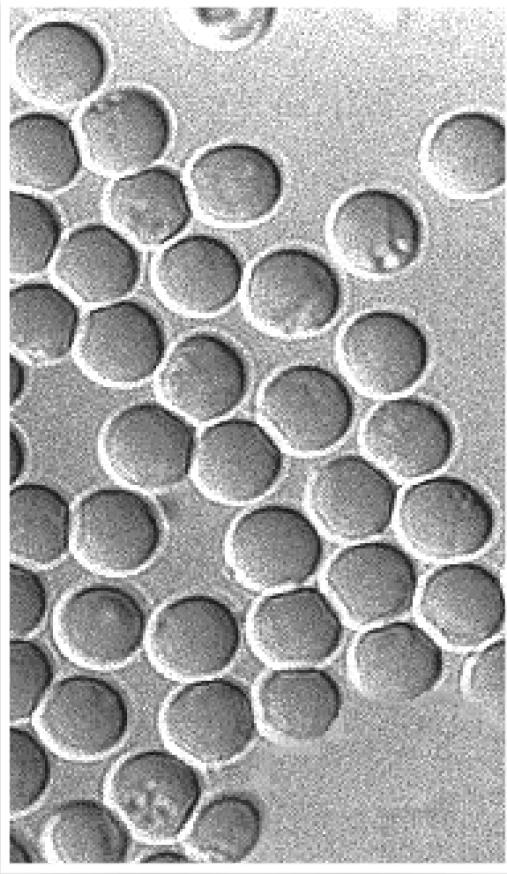
Kernel =

$P(i-1,j-1)$	$P(i-1,j)$	$P(i-1,j+1)$
$P(i,j-1)$	$P(i,j)$	$P(i,j+1)$
$P(i+1,j-1)$	$P(i+1,j)$	$P(i+1,j+1)$

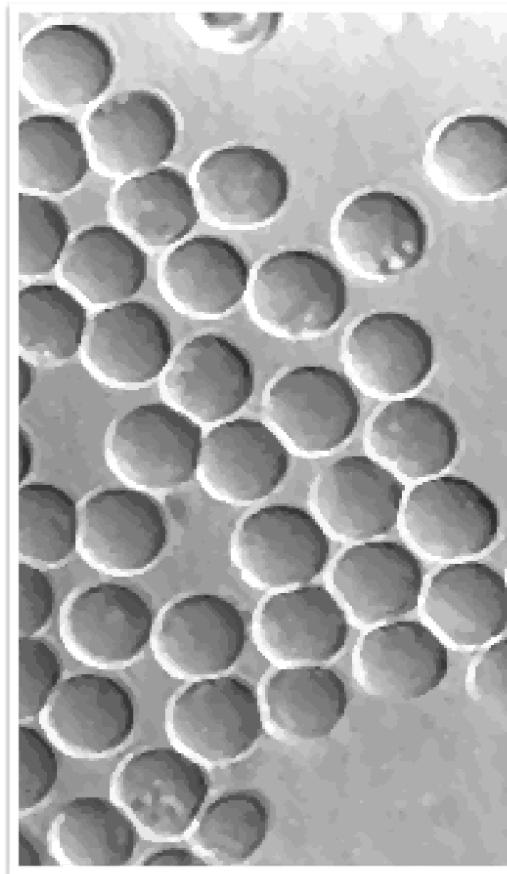
$$P'_{(i,j)} = \frac{1}{9} * \sum_{R=i-1}^{i+1} \left(\sum_{C=j-1}^{j+1} P_{(R,C)} \right)$$

Arithmetic Mean Filter

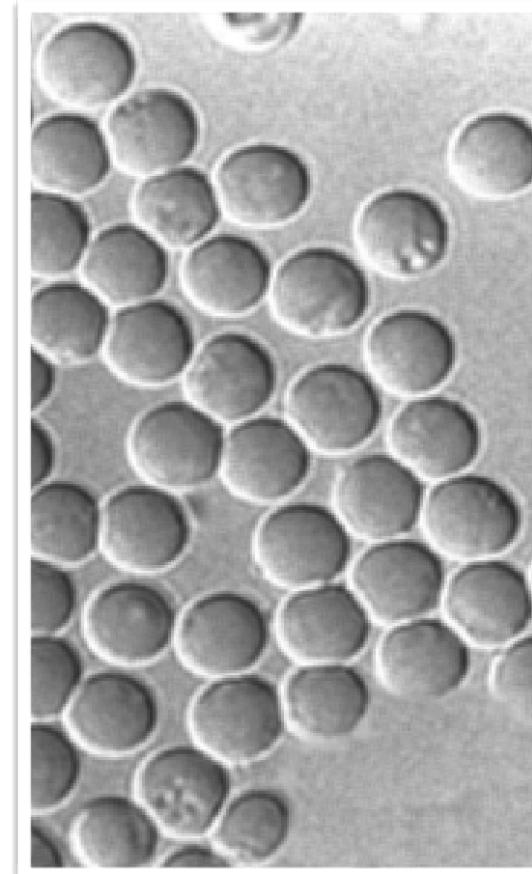
Uniform Noise



3x3 Median

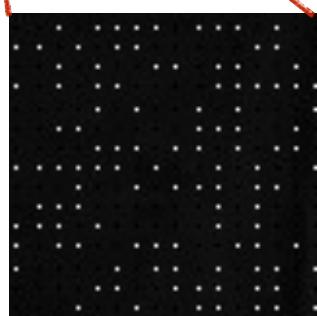
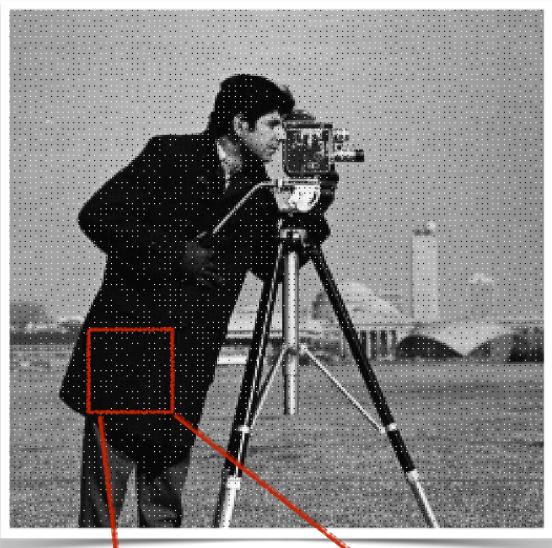


3x3 Mean



Arithmetic Mean Filter

Salt and Pepper Noise



3x3 Median



3x3 Mean



Weighted Mean (Linear) Filter

5x5 Kernel

108	107	100	103	166	108	110	111	109
161	0	106	106	164	255	144	0	113
110	106	0	106	164	103	0	113	110
115	108	109	108	158	112	110	106	111
158	115	143	136	165	108	122	119	122
112	116	113	113	114	112	120	126	126
118	118	111	109	113	111	117	118	123
117	0	117	113	114	113	0	120	116
123	115	117	116	116	117	121	121	124

X

Mask

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

=

Output

108	107	100	103	166
161	0	106	106	164
110	106	0	106	164
115	108	109	108	158
158	115	143	136	165

25

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Weighted Mean (Linear) Filter

5x5 Kernel

108	107	100	103	166	108	110	111	109
161	0	106	106	164	255	144	0	113
110	106	0	106	164	103	0	113	110
115	108	109	108	158	112	110	106	111
158	115	143	136	165	108	122	119	122
112	116	113	113	114	112	120	126	126
118	118	111	109	113	111	117	118	123
117	0	117	113	114	113	0	120	116
123	115	117	116	116	117	121	121	124

Normalization

$$P'_{(i,j)} = \frac{1}{\sum_{R=i-2}^{i+2} \sum_{j=2}^{j+2} W_{(i,j)}} * \left(\sum_{R=i-2}^{i+2} \sum_{j=2}^{j+2} W_{(i,j)} * P_{(i,j)} \right)$$

Weights



$$P'_{(i,j)} = \frac{1}{25} * \left(\sum_{R=i-2}^{i+2} \sum_{j=2}^{j+2} 1 * P_{(i,j)} \right)$$

5x5 Mean Filter when all $W(i,j) = 1$

Weighted Mean (Linear) Filter

5x5 Kernel

i-2	108	107	100	103	166
i-1	161	0	106	106	164
i	110	106	0	106	164
i+1	115	108	109	108	158
i+2	158	115	143	136	165

j-2 j-1 j j+1 j+2

Mask

x

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

Weighted Mean (Linear) Filter

5x5 Kernel

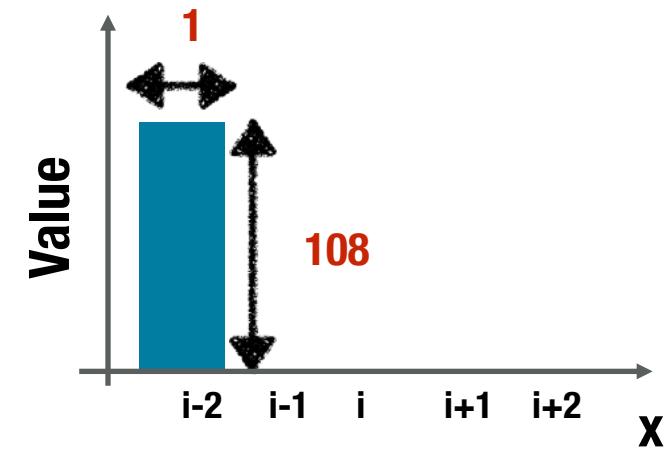
i-2	108	107	100	103	166
i-1	161	0	106	106	164
i	110	106	0	106	164
i+1	115	108	109	108	158
i+2	158	115	143	136	165

j-2 j-1 j j+1 j+2

x

Mask

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1



Weighted Mean (Linear) Filter

5x5 Kernel

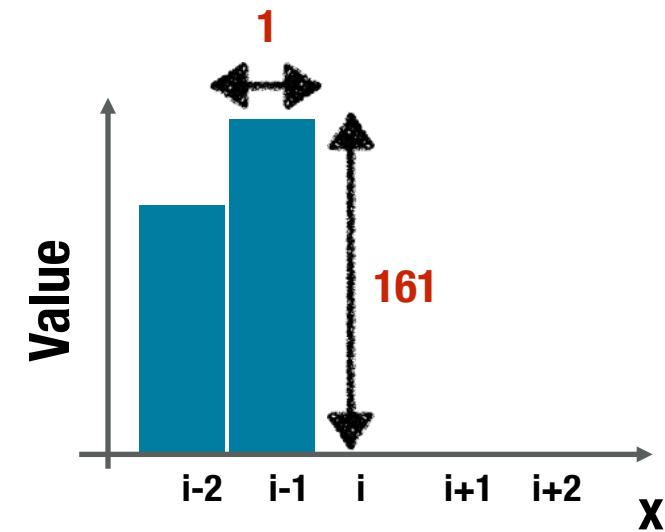
i-2	108	107	100	103	166
i-1	161	0	106	106	164
i	110	106	0	106	164
i+1	115	108	109	108	158
i+2	158	115	143	136	165

j-2 j-1 j j+1 j+2

x

Mask

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1



Weighted Mean (Linear) Filter

5x5 Kernel

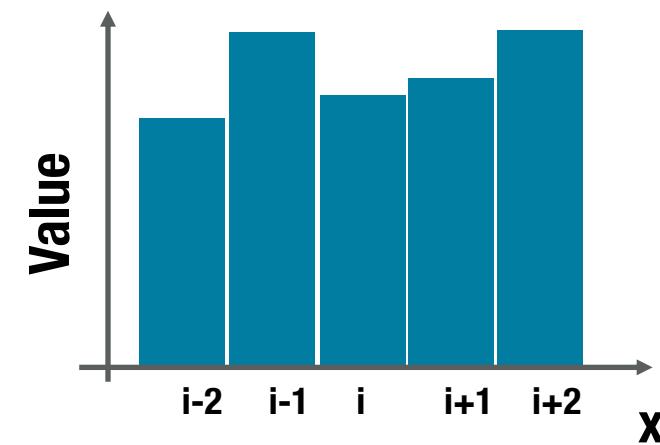
i-2	108	107	100	103	166
i-1	161	0	106	106	164
i	110	106	0	106	164
i+1	115	108	109	108	158
i+2	158	115	143	136	165

x

Mask

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

Area Under Curve



Weighted Mean (Linear) Filter

5x5 Kernel

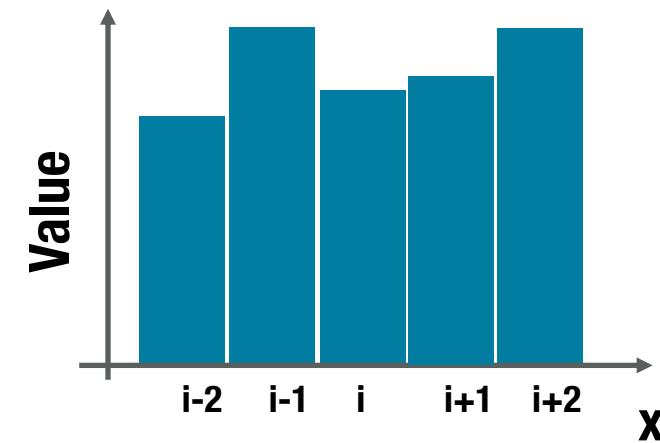
i-2	108	107	100	103	166
i-1	161	0	106	106	164
i	110	106	0	106	164
i+1	115	108	109	108	158
i+2	158	115	143	136	165

x

Mask

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

Area Under Curve



$$\int_{i-2}^{i+2} Kernel(x, y) \ dx$$

Weighted Mean (Linear) Filter

5x5 Kernel

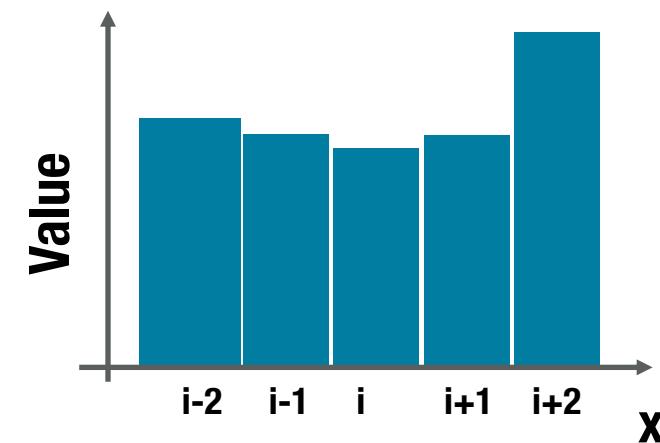
i-2	108	107	100	103	166
i-1	161	0	106	106	164
i	110	106	0	106	164
i+1	115	108	109	108	158
i+2	158	115	143	136	165

x

Mask

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

Area Under Curve



$$\int_{j-2}^{j+2} Kernel(x, y) \ dy$$

Weighted Mean (Linear) Filter

5x5 Kernel

108	107	100	103	166	108	110	111	109
161	0	106	106	164	255	144	0	113
110	106	0	106	164	103	0	113	110
115	108	109	108	158	112	110	106	111
158	115	143	136	165	108	122	119	122
112	116	113	113	114	112	120	126	126
118	118	111	109	113	111	117	118	123
117	0	117	113	114	113	0	120	116
123	115	117	116	116	117	121	121	124

Mask

X

1	2	3	2	1
2	5	6	5	2
3	6	8	6	3
2	5	6	5	2
1	2	3	2	1

84

Output

108	107	100	103	166
161	0	106	106	164
110	106	0	106	164
115	108	109	108	158
158	115	143	136	165

=

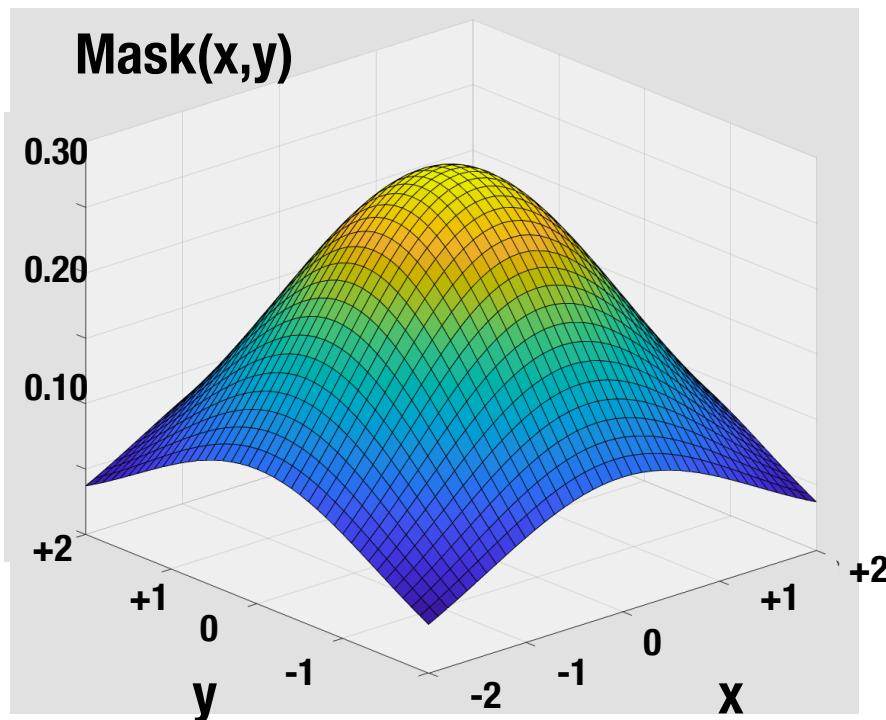
8438

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Gaussian Filter

Mask

$$\frac{1}{84} * \begin{matrix} 1 & 2 & 3 & 2 & 1 \\ 2 & 5 & 6 & 5 & 2 \\ 3 & 6 & 8 & 6 & 3 \\ 2 & 5 & 6 & 5 & 2 \\ 1 & 2 & 3 & 2 & 1 \end{matrix}$$



2-D Gaussian distribution with mean (0,0) and $\sigma = 1.4$

$$G(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Demonstration – Linear Filters





04

How Do We Sharpen Images?

From smoothing to sharpening

- We can use positive kernel to smooth images
 - Smoothing => averaging the pixel => Integration
-
- How can we sharpen images using kernels?
 - Ideas?



Kernel with Negative Values

Image to filter

108	107	100	103	108	108	110	111	109
161	0	106	106	104	255	144	0	113
110	106	106	106	104	103	0	113	110
115	108	109	108	107	112	110	106	111
115	115	111	111	110	108	122	119	122
112	116	113	113	114	112	120	126	126
118	118	111	109	113	111	117	118	123
117	0	117	113	114	113	0	120	116
123	115	117	116	116	117	121	121	124

Kernel

0	-1	0
-1	4	-1
0	-1	0

Output

-480	112	5	-157	561	211	-481		
100	-3	0	-4	-59	-470	236		
-13	3	-1	-6	20	100	-29		
10	-4	2	0	-24	31	0		
6	1	5	8	-5	3	21		
127	-13	-14	4	-11	119	-14		
-467	127	-4	1	110	-471	125		

Values after the filter may be negative or larger than 255.

Mask with Negative Values

Image to filter

108	107	100	103	108	108	110	111	109
161	0	106	106	104	255	144	0	113
110	106	106	106	104	103	0	113	110
115	108	109	108	107	112	110	106	111
115	115	111	111	110	108	122	119	122
112	116	113	113	114	112	120	126	126
118	118	111	109	113	111	117	118	123
117	0	117	113	114	113	0	120	116
123	115	117	116	116	117	121	121	124

Kernel

0	-1	0
-1	4	-1
0	-1	0

Output

0	112	5	0	255	211	0		
100	0	0	0	0	0	0	236	
0	3	0	0	20	100	0		
10	0	2	0	0	31	0		
6	1	5	8	0	3	21		
127	0	0	4	0	119	0		
0	127	0	1	110	0	125		

Values after the filter may be negative or larger than 255.

Values must be remapped to [0; 255]

Mask with Negative Values

Image to filter

108	107	100	103	108	108	110	111	109
161	0	106	106	104	255	144	0	113
110	106	106	106	104	103	0	113	110
115	108	109	108	107	112	110	106	111
115	115	111	111	110	108	122	119	122
112	116	113	113	114	112	120	126	126
118	118	111	109	113	111	117	118	123
117	0	117	113	114	113	0	120	116
123	115	117	116	116	117	121	121	124

Local peaks

Kernel

0	-1	0
-1	4	-1
0	-1	0

Output

0	112	5	0	255	211	0		
100	0	0	0	0	0	0	236	
0	3	0	0	20	100	0		
10	0	2	0	0	31	0		
6	1	5	8	0	3	21		
127	0	0	4	0	119	0		
0	127	0	1	110	0	125		

Remove negative values

Replacing values > 255

Mask with Negative Values

Image to filter

108	107	100	103	108	108	110	111	109
161	0	106	106	104	255	144	0	113
110	106	106	106	104	103	0	113	110
115	108	109	108	107	112	110	106	111
115	115	111	111	110	108	122	119	122
112	116	113	113	114	112	120	126	126
118	118	111	109	113	111	117	118	123
117	0	117	113	114	113	0	120	116
123	115	117	116	116	117	121	121	124

Kernel

0	-1	0
-1	4	-1
0	-1	0

Output

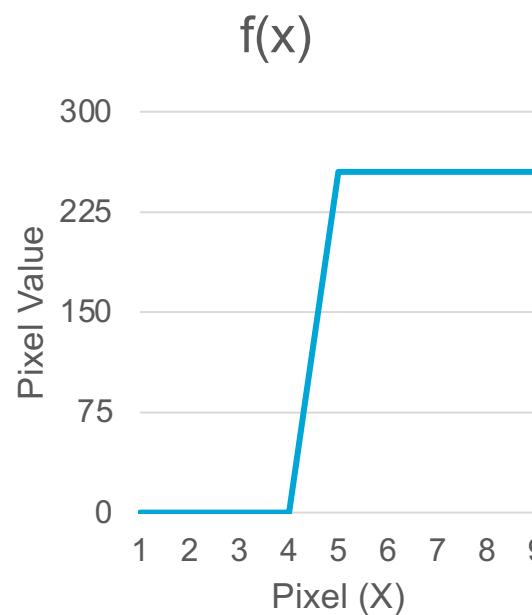
	0	112	5	0	255	211	0	
	100	0	0	0	0	0	0	236
	0	3	0	0	20	100	0	
	10	0	2	0	0	31	0	
	6	1	5	8	0	3	21	
	127	0	0	4	0	119	0	
	0	127	0	1	110	0	125	

Effect = Highlighting pixels that are “local” **peaks** in the image

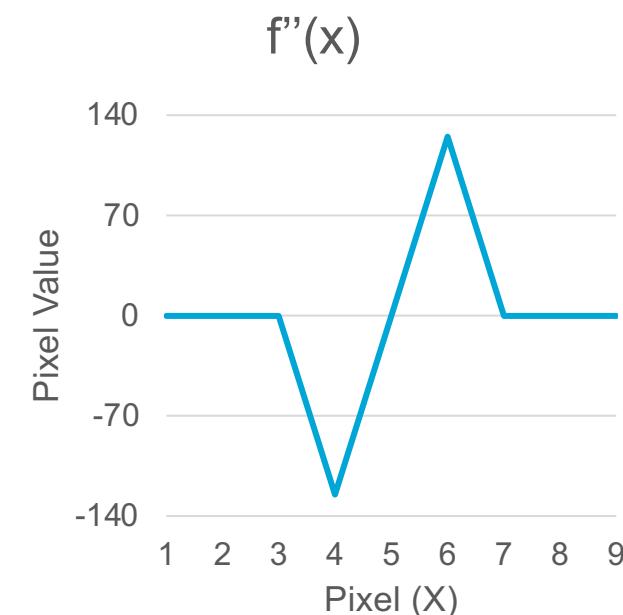
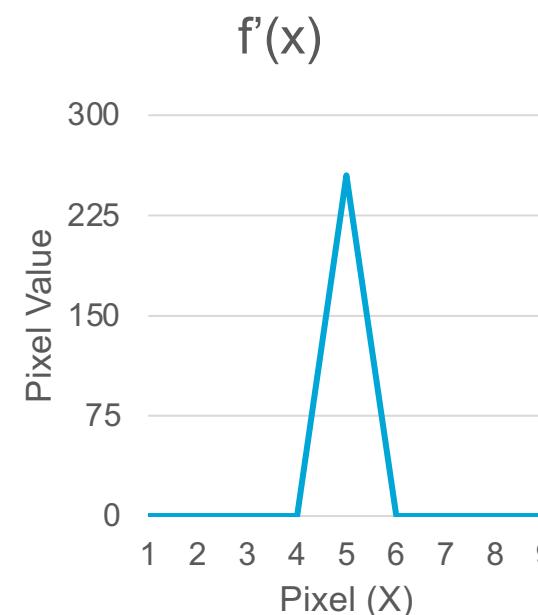
Sharpening Filter

Laplacian Mask

$$\begin{matrix} & y \\ \begin{matrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{matrix} & x \end{matrix}$$



Coefficients of the Laplacian (2nd derivative)



Sharpening Filter

Laplacian Mask

$$\mathbf{y} \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} \mathbf{x}$$

Coefficients of the Laplacian (2nd derivative)

$$\begin{bmatrix} -1 & 2 & -1 \end{bmatrix}$$

$$\frac{\partial^2 f}{\partial^2 y} = -f_{(i+1,j)} + 2f_{(i,j)} - f_{(i-1,j)}$$

$$\begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$$

$$\frac{\partial^2 f}{\partial^2 x} = -f_{(i,j+1)} + 2f_{(i,j)} - f_{(i,j-1)}$$

Demonstration – Sharpening Filters





05

What Can We Do With These Techniques?

Demonstration – Advanced Usages





Thank you for your attention!