A Report on Uwe Fest 2024

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Summary of Events

Uwe Fest 2024 was a conference on commutative algebra and its interaction with algebraic geometry and algebraic combinatorics, organized in celebration of Uwe Nagel's 60th birthday. The conference was held at The University of Notre Dame from August 12th through 16th, and featured 50 minute talks from 19 different speakers; the underlying theme of these talks was Uwe's contributions to commutative algebra, algebraic geometry, and algebraic combinatorics, with many speakers presenting research that was inspired by Uwe's work, or done jointly with him. In addition to these talks, there were two poster sessions and 19 poster presenters. A full list of speakers, poster presenters, and other participants can be found on the conference's website https://sites.nd.edu/uwefest2024/ along with a detailed schedule.

In addition to the many talks and poster sessions, there was a reception after the main events of the first day, and on Wednesday the 14th, a banquet was held in celebration of Uwe's birthday. At the banquet, speakers including Heide Gluesing-Luerssen (Uwe's wife, who is also a mathematician), Juan Migliore (Uwe's collaborator on more than 50 papers), and former students of Uwe, including Sonja Petrović, gave speeches honoring Uwe and his contributions to mathematics.

While attending Uwe Fest, I had the opportunity to meet many faculty, postdocs, and graduate students including Bek Chase (student of Giulio Caviglia at Purdue), Caitlin Davis (student of Daniel Erman at UW Madison), Juliann Geraci (student of Alexandra Seceleanu at UNL), Sean Grate (student of Hal Schenck at Auburn), Tài Huy Hà (Tulane), Jake Kettinger (postdoc at CSU), Chris Peterson (CSU), Claudiu Raicu (Notre Dame), Henry Potts-Rubin (student of Claudia Miller at Syracuse), Hal Schenck (Auburn), Alexandra Seceleanu (UNL), Adam Van Tuyl (McMaster), and Shahriyar Roshan-Zamir (student of Alexanra Seceleanu at UNL).

Notable Talks

Many of the talks were rather advanced, aimed towards the experts in the audience, but there were several more accessible talks. I will summarize a few of those which I most enjoyed.

Chris Peterson (CSU) - *Two problems involving groups, graphs, and combinatorics.*

Chris Peterson gave an accessible and example driven talk about Cayley graphs of finite groups and possible connections to the Hilbert functions of Artinian algebras. Given a group G generated by a finite subset $S \subseteq G$, the Cayley graph $\Gamma(G, S)$ has vertices the elements of G, and edges $E = \{(g, gs) : s \in S\}$. For this talk, S was assumed to be closed under inverses and to not contain the identity, so that $\Gamma(G, S)$ is simple and undirected. Now, given a starting node $g \in G$, one can define a sequence $a_1, a_2, \cdots$ where $a_n = |\{h \in G : d(h, g) = n\}|$ is the number of nodes at distance n from g, and distance $d(h, g)$ is the length of the shortest path from h to g (i.e. d is the path metric). The sequence (a_n) does not depend on the chosen starting node, suggesting a connection between the pair (G, S) and the Artinian algebra having Hilbert function (a_n). In many cases, the sequence (a_n) is symmetric, corresponding to the Hilbert function of an Artinian Gorenstein algebra. The talk was driven by computations in GAP and Maple, which visualized the Cayley graphs and their corresponding sequences. A pattern was observed that the Cayley graph is Polyhedral exactly when its sequence is symmetric.

Victor Reiner (UMN) - *Macaulay inverse systems and graded Ehrhart theory.*

Given a lattice polytope $P \subseteq \mathbb{R}^n$ (the convex hull of a finite set of points in $\mathbb{Z}^n$), we define $mP = \{mp : p \in P\}$ for $m \geq 0$, and $i_P(m) = |mP \cap \mathbb{Z}^n|$, the number of integer lattice points contained in mP. We can then construct the *Ehrhart series* $E_P(t) = \sum_{m=0}^{\infty} i_P(m) \cdot t^m$. Victor Reiner and collaborators are interested in *q-counting* the number of integer lattice points contained in mP – that is, introducing a new parameter q into the Ehrhart series such that, when $q = 1$, we obtain the original Ehrhart series. To do so, they introduce a ring $R(mP \cap \mathbb{Z}^n)$, which is the quotient of $k[x_1, \cdots, x_n]$ by a particular homogeneous ideal such that $\dim_k R(mP \cap \mathbb{Z}^n) = i_P(m)$. Then, they introduce the q-Ehrhart series

$$E_P(t, q) = \sum_{m=0}^{\infty} H_P(m, q) \cdot t^m, \quad \text{where } H_P(m, q) = \sum_{d=0}^{\infty} \dim_k R(mP \cap \mathbb{Z}^n)_d \cdot q^d.$$

Evaluating at $q = 1$ gives $E_P(t, 1) = \sum_{m=0}^{\infty} i_P(m) \cdot t^m = E_P(t)$ as desired. Having introduced the

q-analogue of the Ehrhart series, Reiner posed some conjectures about the q-Ehrhart series which mirror the classical Ehrhart theory, and formulated these conjectures in terms of questions about the properties of an algebra associated to P.

Hal Schenck (Auburn) - *Syzygies of permanental ideals.*

Given an $n \times m$ matrix of indeterminates, say x_{ij} for $1 \leq i \leq n$, $1 \leq j \leq m$, and some d smaller than the lesser of n and m, one can construct an ideal of the polynomial ring $k[x_{ij} : 1 \leq i \leq n, 1 \leq j \leq m]$ generated by determinants of the $d \times d$ minors; these so called *determinantal ideals* are well behaved. In this talk, Hal Schenck introduced the *permanental ideal* of a matrix, generated by the *permanents* of square minors – the permanent of a square matrix is a polynomial in its entries, obtained by taking the determinant and "forgetting" the minus signs. These ideals turn out to be quite complicated, and the rest of the talk was dedicated to studying the ideal generated by the permanents of 2×2 minors of a $2 \times n$ matrix. I struggled to follow the latter half of the talk, which described the minimal free resolution of this ideal, but I found it surprising and intriguing to see just how badly behaved permanental ideals can be, considering how nice determinantal ideals are.

General Impressions of the Talks.

Many of the talks I enjoyed most had a combinatorial flavor (in addition to those mentioned above, I liked Adam Van Tuyl's talk on vertex decomposable simplicial complexes); perhaps it was just that these talks were most accessible, but this first exposure to algebraic combinatorics piqued my interests. I was attracted to the talks with a geometric focus as well – for example, Brian Harbourne's talk on Geproci sets – but these tended to expect more from the audience in terms of background. This challenge has motivated me to learn some algebraic geometry.

Poster Sessions

Poster sessions were held on Tuesday and Thursday afternoon from 2:00 to 3:30 PM. Presenters were organized into two groups alphabetically, with the first 10 presenting on Tuesday, and the remaining 9 presenting on Thursday. Poster presenters on Tuesday included

- Bek Chase (student of Giulio Caviglia at Purdue), who presented their work on the strong Lefschetz property for modules over Clements-Lindström rings,
- Thiago de Holleben (student of Sara Faridi at Dalhousie University), who presented his work on the behavior of h-vectors of Artinian Gorenstein algebras,
- Juliann Geraci (student of Alexandra Seceleanu at UNL), who presented her work on boolean matrix factorization and applications to neural codes,
- and Jake Kettinger (postdoc at CSU, formerly a student of Brian Harbourne at UNL) who presented his work on Geproci sets in positive characteristic.

On Thursday afternoon, I presented *A bi-Lipschitz Coorbit Embedding of Point Clouds* (PDF), a poster resulting from an REU-RTG project with Ang Barrett and Dan Kiem, conducted under the supervision of Shengnan Huang (Ph.D. student at Northeastern) and Prof. Harm Derksen. While presenting, I had nice conversations with Chris Peterson, Hal Schenck, Claudiu Raicu, Tài Huy Hà, and Uwe Nagel, to name a few of the participants I spoke with. Chris Peterson mentioned a possible connection to multi-dimensional scaling, Hal Schenck asked about the computational complexity

of our construction, Claudiu Raicu asked about plans to connect the coorbit embedding back to invariant theory, Tài Huy Hà asked about the configuration of templates as d (the dimension of the space points are drawn from) becomes larger, and Uwe Nagel was particularly interested in the generalization of the co-orbit embedding to infinite subgroups of the orthogonal group given in [1]. In total, I think the poster presentation went well, although in hind sight, I would have put less text on the poster, as most participants did not read it carefully, preferring to listen to me talk.

Advice to Students

My advice to students attending a conference for the first time would be to attend all the talks if possible (that is, if you feel up for it, doing so can be a bit overwhelming), and take notes so you can clarify things you did not understand, and follow up with speakers whose work really interested you. The first day of the conference can be awkward if you do not know anyone, but don't hesitate to introduce yourself to new people, and try to participate in extracurricular activities like going to lunch with a group. It is helpful to befriend other students, perhaps including some who can help resolve your questions about the talks of the day, but do not limit yourself to interacting only with those you meet on the first day.

Advice on Presenting a Poster

For those who have not presented a poster before, let me outline the basic form of the presentation: you will stand with your poster as participants walk the room, and when someone approaches you, introduce yourself and give a short presentation guiding them through your poster. Afterwards, you may offer to elaborate on any sections of particular interest, and you may also let the audience read on their own. Most participants will have questions, so be prepared for common ones. It is helpful to practice talking about your poster at varying levels of detail, and change your presentation based on each person or group's interests and background.

When it comes to preparing a poster, I would recommend using LaTeX and the `beamerposter` package (there is also `tikzposter`, but I have no experience with it). There are a number of nice templates available online including gemini, most of which are easily adapted to fit Northeastern's design guidelines. I have modified the gemini template to use Northeastern colors and fonts, and include Northeastern brand elements; the resulting template is available on github here along with an example. If you do plan to use this template (or gemini), you will need to compile with LuaTeX, which is included with most TeX distributions and available on Overleaf.

Common poster sizes range from 24in×36in to 36in×48in, but you may want to ask the conference organizers for more specific instructions regarding poster sizes. Poster printing is available at a number of locations near Northeastern but most cannot do same day printing, so plan ahead; should you need same day printing, try the Staples near government center. If you plan to fly with your poster, I recommend getting a poster tube (there are a number of good options at Blick in Fenway). This will allow you to carry your poster on the plane, and keep it safe while traveling. It is also possible (perhaps advisable) to print your poster at the conference location, although I did not personally pursue this option because I was a bit nervous about how it would come out, and wanted time to make revisions if necessary.

Advice Specific to Notre Dame

If you are attending a conference at Notre Dame, you are sure to have a great time. Notre Dame has a large and beautiful campus, and is within walking distance of many hotels and restaurants. Popular hotel choices for Uwe Fest 2024 included the Ivy Court Inn and Suites, the Fairfield Inn, and the Embassy Suites by Hilton. When deciding on a place to stay, contact Ms. Nikki Stapleton (nstaplet@nd.edu) of the math department to see if they have special department rates. Just south of Notre Dame's campus is Eddy Street, which hosts a number of local restaurants, and at the end of the street (less than 1/2 a mile) there is a Trader Joe's. There are also two student centers on campus which have a few restaurants each. Travel to and from South Bend International Airport is about 15 minutes by car, there is also train and bus service from Chicago. More travel information is available here https://transportation.nd.edu/travel-to-and-from-campus/.

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References

[1] Dustin G. Mixon and Yousef Qaddura. *Stable Coorbit Embeddings of Orbifold Quotients*. 2024. arXiv: 2403.14042 [math.FA].