Homework 2 and 3

Mitchell Meier

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1. (a)

$$H_0 : P = 0.05$$

$$H_1 : P < 0.05$$

(b)

$$Z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

$$Z_0 = \frac{0.02 - 0.05}{\sqrt{\frac{0.05(1-0.05)}{200}}}$$

$$Z_0 = -1.946$$

(c)

$$P(Z < -1.95) = 0.0256$$

$$p = 0.0256$$

$$\alpha = 0.05$$

$$p < \alpha \text{ ,therefore reject } H_0$$

(d)

$$Z_0 = -1.9467$$

$$\text{p-value} = 0.0258$$

$$p < \alpha \text{ ,therefore reject } H_0$$

(e)

$$\hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.02 \pm Z_{0.025} * 0.0099$$

95 % two sided Confidence Interval = (0.0006 - 0.039)

(f) 95 % two sided Confidence Interval = (0.0006 , 0.0394)

2. (a)

$$H_0 : P = 0.1$$

$$H_1 : P > 0.1$$

(b)

$$Z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

$$Z_0 = \frac{0.08 - 0.1}{\sqrt{\frac{0.1(1-0.1)}{200}}}$$

$$Z_0 = -0.943$$

(c)

$$P(Z < -0.94) = 0.174$$

$$p = 0.827$$

$$\alpha = 0.05$$

$p > \alpha$,therefore fail to reject H_0

(d)

$$Z_0 = -0.9428$$

$$\text{p-value} = 0.8271$$

$$p > \alpha \text{ ,therefore fail to reject } H_0$$

(e)

$$\hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.08 \pm Z_{0.025} * 0.019$$

$$0.08 \pm 0.0376$$

95 % two sided Confidence Interval = (0.0424 , 0.1176)

(f) 95 % two sided Confidence Interval = (0.0424 , 0.1176)

3. (a)

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2$$

(b)

$$T_0 = \frac{\bar{x_1} - \bar{x_2}}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$T_0 = \frac{92.255 - 92.733}{2.70\sqrt{\frac{1}{8} + \frac{1}{8}}}$$

$$T_0 = 0.35$$

(c)

$$\text{Degrees of freedom} = 7$$

$$0.25 < p < 0.4$$

$$p > \alpha, \text{ therefore fail to reject } H_0$$

(d) N/A

(e)

$$(x_1 - x_2) \pm t_{\alpha/2, n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$0.478 \pm 2.145 * 1.35$$

95% two sided Confidence Interval = (-2.42 , 3.37)

4. (a)

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2$$

(b)

$$T_0 = \frac{\bar{x_1} - \bar{x_2}}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

$$T_0 = \frac{12.5 - 27.5}{\sqrt{\frac{7.63^2}{10} + \frac{15.3^2}{10}}}$$

$$T_0 = -2.77$$

(c)

$$\text{Degrees of freedom} = 13$$

$$0.01 < p < 0.02$$

$$p < \alpha, \text{ therefore reject } H_0$$

(d) N/A

(e)

$$(x_1 - x_2) \pm t_{\alpha/2, n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$(12.5 - 27.5) \pm t_{0.025,13} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$15 \pm 2.4178$$

95% two sided Confidence Interval = (12.58 , 17.42)
H_0 is outside the confidence interval, therefore we can reject H_0

5. (a)

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2$$

(b)

$$T_0 = \frac{\bar{D}}{\frac{S_D}{\sqrt{n}}}$$

$$\bar{D} = \frac{\sum_{i=1}^{n} D_i}{n}$$

$$\bar{D} = \frac{2.462}{9} = 0.274$$

$$S_D = \sqrt{\frac{0.82 - \frac{2.462^2}{9}}{8}}$$

$$S_D = 0.135$$

$$T_0 = \frac{0.274}{\frac{0.135}{\sqrt{9}}}$$

$$T_0 = 6.089$$

(c)

$$\text{Degrees of freedom} = 8$$

$$p < 0.0005$$

$$p < \alpha, \text{ therefore reject } H_0$$

(d) N/A

(e)

$$\bar{D} \pm t_{\alpha/2,n-1} * \frac{S_D}{\sqrt{n}}$$

$$0.274 \pm 0.1034$$

95% two sided Confidence Interval = (0.17 , 0.38)
H_0 is outside the confidence interval, therefore we can reject H_0