Exam 2

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Problem 1

a.

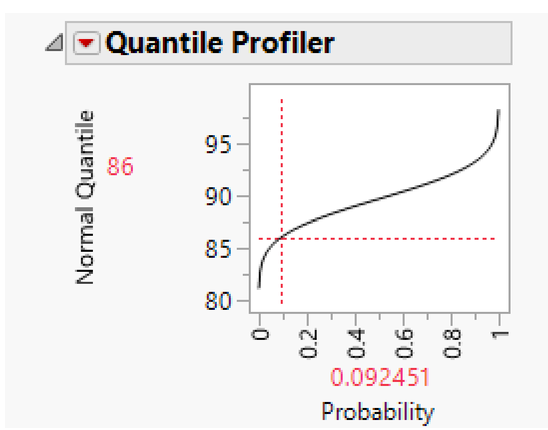
(i) The most appropriate distribution that can be used to model STAT 3113 scores from Section A is a normal distribution, according to its AIC score

(ii) The mean parameter, μ is estimated to be 89.666304
The standard deviation parameter, σ, is estimated to be 2.7653256
The variance parameter, σ^2 is estimated to be 7.647025674

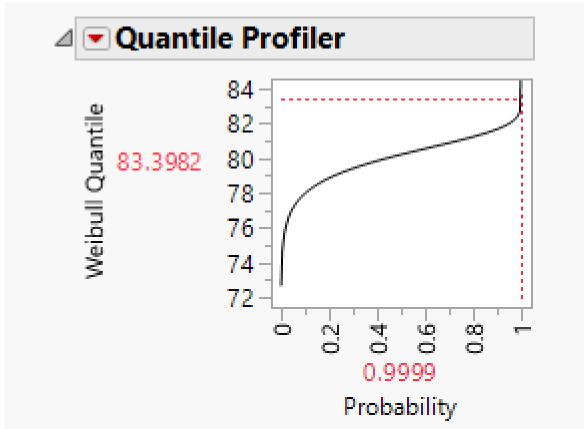
(iii) The most appropriate distribution that can be used to model STAT 3113 scores from Section B is a weibull distribution, according to its AIC score

(iv) The mean parameter, μ is estimated to be 79.9657
The standard deviation parameter, σ, is estimated to be 1.53
The variance parameter, σ^2 is estimated to be 2.34129
The scale parameter, α, is estimated to be 80.650259
The shape parameter, β, is estimated to be 66.269092

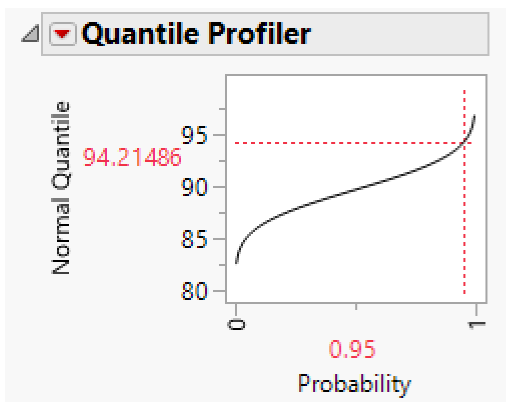
(v) The probability that a randomly picked student from Section A scores more than 86 marks is $1 - 0.092451 = 0.907549$, or 90.75 percent



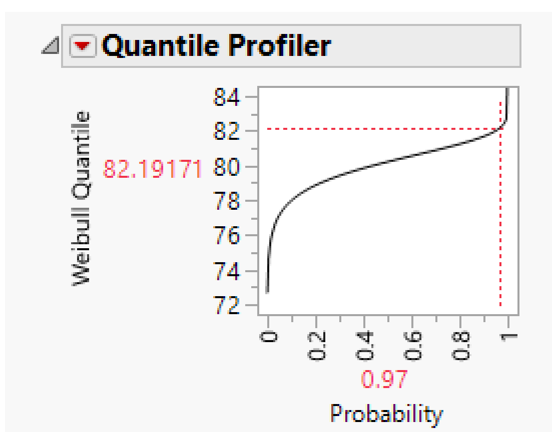
(vi) The probability that a randomly picked student from Section B scores less than 86 marks is 1, or 100 percent



(vii) The lowest score you can achieve and still be in the top 5 percent of Section A would be a score of 94.22 percent



(viii) The lowest score you can achieve and still be in the top 5 percent of Section B would be a score of 82.20 percent

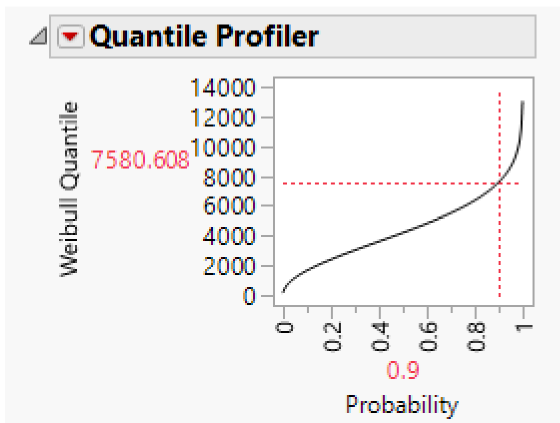


b.

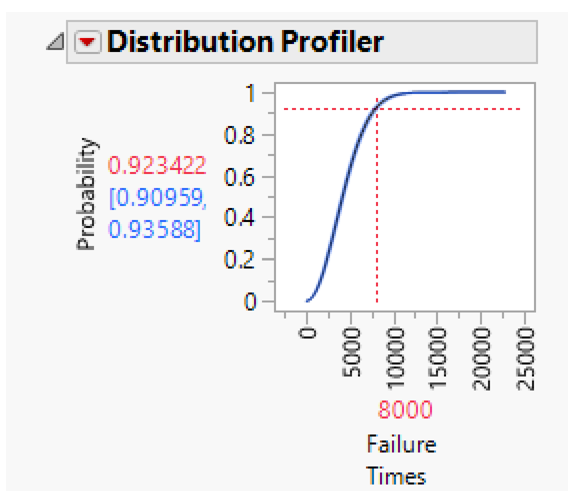
(i) The most appropriate distribution that can be used to model failure times of the bearings is a weibull distribution, according to its AIC score

(ii) The mean parameter, μ is estimated to be 4459.2
The standard deviation parameter, σ, is estimated to be 2293.35
The variance parameter, σ^2 is estimated to be 5259442
The scale parameter, α, is estimated to be 5033.1405
The shape parameter, β, is estimated to be 2.0364645

(iii) 90 percent of the bearings would have failed at the time value equal to 7580.608 hours



(iv) The probability that a randomly selected bearing last more than 8000 hours is is $1 - 0.923422 = 0.076578$, or 7.7 percent

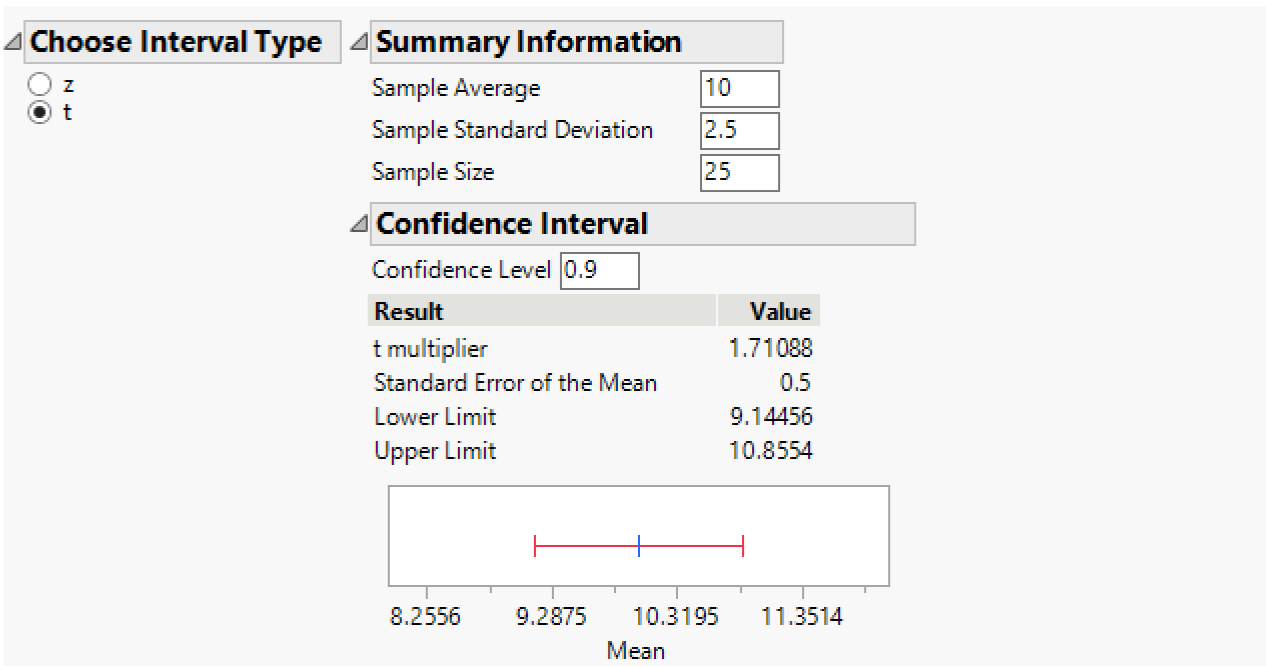


Problem 2

(a) 1. The population parameter given is the mean, μ, of time spent watching TV each week by students at Missouri S&T (individually), and the $\mu = 8$ hours

2. $H_0 : \mu = 8$
   $H_1 : \mu \neq 8$

3. With a sample size of $n = 25$
   sample mean of $\bar{x} = 8$
   sample standard deviation of $s = 2.5$
   and 90% confidence interval, our JMP output is as follows:

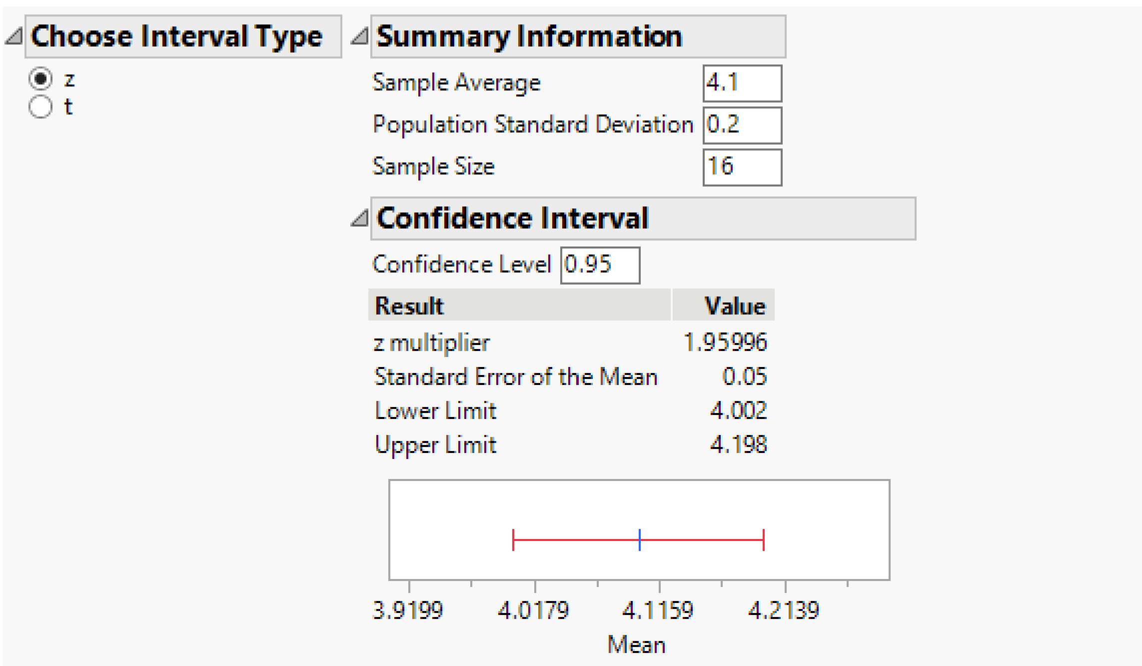


4. We have sufficent evidence to conclude that the average time spent watching TV each week by Missouri S&T students is not equal to 8 hours, since 8 hours does not fall in the range of our confidence interval. Therefore, we have sufficent evidence to reject H_0

1. The population parameter given is the mean, μ, of time an artifical heart's battery pack needs to be recharged and the $\mu = 4$ hours
   The population standard deviation, σ, is given as 0.2 hours

2. $H_0 : \mu = 4$
   $H_1 : \mu \neq 4$

3. With a sample size of $n = 16$
   sample mean of $\bar{x} = 4.1$
   population standard deviation of $\sigma = 0.2$
   and 90% confidence interval, our JMP output is as follows:



4. We have sufficent evidence to conclude that the battery life of an artifical heart is not equal to 4 hours, since 4 hours does not fall in the range of our confidence interval. Therefore, we have sufficent evidence to reject H_0

Problem 3

(a)

$$f(x,\theta) = \frac{x}{\theta^2} e^{\frac{-x^2}{2\theta^2}}, x > 0$$

$$\text{pdf of x } = \int_0^\infty \frac{x}{\theta^2} e^{\frac{-x^2}{2\theta^2}} dx$$

$$u = \frac{-x^2}{2\theta^2}, du = \frac{-x}{\theta^2} dx$$

$$-\int_0^\infty e^u du = -e^u + C = -e^{\frac{-x^2}{2\theta^2}}$$

$$-e^{\frac{-\infty^2}{2\theta^2}} - -e^{\frac{0^2}{2\theta^2}} = 0 - (-1) = 1$$

Since the integral of our pdf is equal to 1, this is a valid pdf

(b)

$$F(x) = 1 - e^{(\frac{-x}{\alpha})^\beta}$$

$$F(x) = 1 - e^{-x^\beta}, \text{since } \alpha = 1$$

$$F(x) = 1 - e^{-x^2}, \text{since } \beta = 2$$

(c)

$$\mu = 1\gamma(1 + \frac{1}{2}) = \gamma\frac{3}{2} = 0.886$$

$$\sigma^2 = 1^2\gamma(1 + \frac{2}{2} - 1^2(\gamma 1 + \frac{1}{2})^2$$

$$\sigma^2 = \gamma(2) - (\gamma(\frac{3}{2}))^2 = 1 - 0.886 = 0.214$$