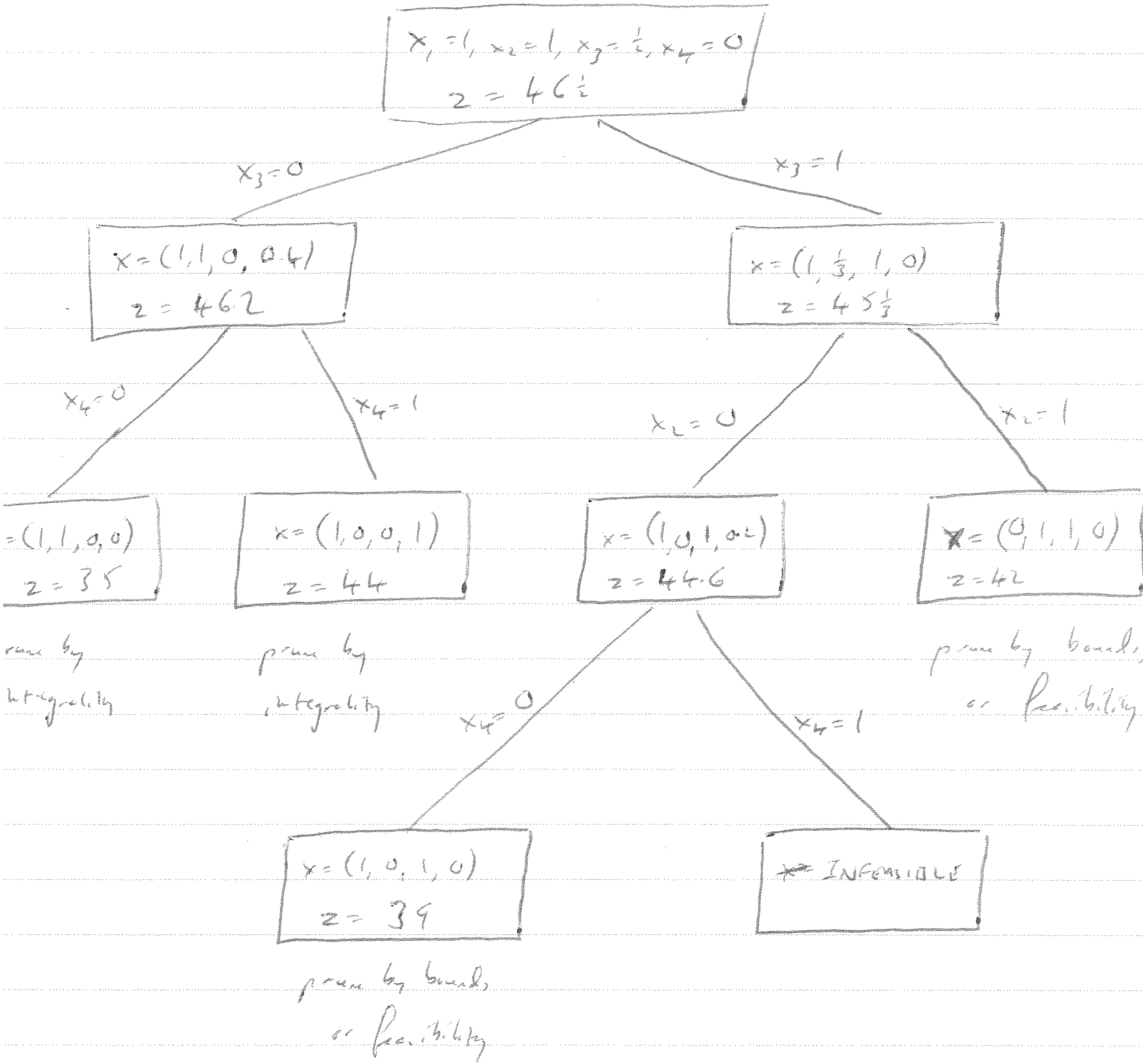
1. NGW, 381, Q3



So $x = (1, 0, 0, 1)$ is optimal, with value $z = 44$.

2. Soln to LP relaxation: $x = (1, 1, \frac{1}{2}, 0)$

Violated cover inequality: $x_1 + x_2 + x_3 \leq 2$

Lift: $\max \; x_1 + x_2 + x_3$

$$\text{s.t. } 2x_1 + 3x_2 + 4x_3 + 5x_4 \leq 7$$
$$x_4 = 1$$
$$x_i \text{ binary.}$$

Soln: $x = (1, 0, 0, 1)$, value $= 1$.

So lifting coefficient is $2 - 1 = 1$.

Lifted constraint:

$$x_1 + x_2 + x_3 + x_4 \leq 2.$$

Add lifted constraint:

$$\max \; 16x_1 + 19x_2 + 23x_3 + 28x_4$$
$$\text{s.t. } 2x_1 + 3x_2 + 4x_3 + 5x_4 \leq 7$$
$$x_1 + x_2 + x_3 + x_4 \leq 2$$
$$x_i \text{ binary.}$$

Soln: $x = (1, 0, 0, 1)$, $z = 44$.

3. NoW, p.381, Q8:

Let x_0 be the integral variable

Solve LP relaxation; if $\bar{x}_0$ is integral, then solved with only one node.

Else branch on $x_0 \le \lfloor \bar{x}_0 \rfloor$, $x_0 \ge \lceil \bar{x}_0 \rceil$.

The added constraints are active at an optimal soln to the subproblems.

(Note: to ensure we get x_0 integral in the child problems, could impose the equality constraints $x_0 = \lfloor \bar{x}_0 \rfloor$, $x_0 = \lceil \bar{x}_0 \rceil$.)

4. N&W, p. 346, Q13.

(i) $Z_{LR}(\lambda) = \min\ 7x_1 + 6x_2 + 2x_3 \cancel{+} \lambda_1(3x_1 + 3x_2 + x_3 - 5)$
$\quad$ s.t. $3x_1 + x_2 + 2x_3 \geq 4$
$\quad\quad x \in \mathbb{Z}^3_+$.

$Z_{LD} = \max_{\lambda \geq 0} Z_{LR}(\lambda)$

$Z_{LR}(\lambda_1) = \min\ (7-3\lambda_1)x_1 + (6-3\lambda_1)x_2 + (2-\lambda_1)x_3 + 5\lambda_1$
$\quad$ s.t. $3x_1 + x_2 + 2x_3 \geq 4$
$\quad\quad x \in \mathbb{Z}^3_+$.

$\lambda_1 > 2 \Rightarrow Z_{LR}(\lambda_1) = -\infty$

Extreme points in x: $(2,0,0)$, $(0,4,0)$, $(0,0,2)$, $(1,1,0)$
Values: $14-6\lambda_1+5\lambda_1$, $24-12\lambda_1+5\lambda_1$, $4-2\lambda_1+5\lambda_1$, $13-6\lambda_1+5\lambda_1$
$= 14-\lambda$, $24-7\lambda$, $4+3\lambda$, $13-\lambda$

Optimal is $\lambda_1 = 2$, giving $Z_{LD} = 10$, achieved by both $x = (0,4,0)$ and $x = (0,0,2)$.

q4 p2

ii) $z_{LR}(\lambda_2) = \min\ (7-3\lambda_2)x_1 + (6-\lambda_2)x_2 + (2-2\lambda_2)x_3 + 4\lambda$

s.t. $\quad 3x_1 + 3x_2 + x_3 \geq 5.$

$z_{LR}(\lambda_2) = -\infty$ if $\lambda > 1.$

Extreme points: $(2,0,0)$ $\quad (0,2,0)$ $\quad (0,0,5)$ $\quad (1,0,2)$ $\quad (0,1,2)$

· Value $\quad 14-2\lambda$ $\quad 12+2\lambda$ $\quad 10-6\lambda$ $\quad 11-3\lambda$ $\quad 10-\lambda$

So $z_{LR}(\lambda_2) = 10-6\lambda_2$ for $0 \leq \lambda_2 \leq 1.$

So $z_{LD} = 10$, achieved by $\lambda = 0$ and $x = (0,0,5)$ and $x(0,1,2)$.

iii) Sensitivity analysis:

From part (i): $\lambda_1^* > 0$, so need $b_1 = 5$.

From part (ii): $\lambda_2^* = 0$, with $x = (0,0,5)$ or $(0,1,2)$.

So can take ~~$b_2 \leq$ [illegible] and still get the same~~ x.

$b_2 \leq (3,1,2)\begin{pmatrix}0\\0\\5\end{pmatrix} = 10$ and still get $x = (0,0,5)$

or $b_2 \leq (3,1,2)\begin{pmatrix}0\\1\\2\end{pmatrix} = 5$ and still get $x = (0,1,2)$.

So easily solved for $b_2 \leq 10$.

J. NoW p347 Q16

$$\min \sum_{i \in M} \sum_{j \in N} h_{ij} y_{ij} + \sum_{j \in N} c_j x_j$$

$$\text{s.t.} \quad \sum_{j \in N} y_{ij} = a_i \quad \text{for } i \in M \qquad (1)$$

$$\sum_{i \in M} y_{ij} \le b_j x_j \quad \text{for } j \in N \qquad (2)$$

$$y_{ij} \le d_{ij} x_j \quad \text{for } i \in M, j \in N \qquad (3)$$

$$y \in \mathbb{R}^{mn}_+, \; x \in \mathbb{B}^n \qquad (d_{ij} = \min\{a_i, b_j\}).$$

Relax (1):

Easy to solve subproblem: it separates into different subproblem for each $j \in N$, with only one integer variable in the subproblem. Each subproblem can be solved by solving the LP relaxation.

Value is no better than LP relaxation.

Relax (2)

Similar to an uncapacitated facility location problem.
Hard to solve, gives good bound.

Relax (3)

(1) & (2) imply (3), so this is equivalent to the original problem.
So hardest to solve, but gives the best bound (i.e., the original optimal value)