MATP6640/DSES6770 Linear Programming, Homework 1.

Due: Monday, January 28, 2008.

1. Let (P) be the standard form LP, $\min\{c^T x : Ax = b, x \geq 0\}$. In class, we saw that if a problem of the form (P) has a feasible solution then it has a basic feasible solution. Construct a similar proof to show that if (P) has an optimal solution, it has a basic feasible solution that is optimal.

2. Consider the standard form linear programming problem

$$\begin{array}{lrcl} \min & c^T x & & \\ \text{s.t.} & Ax & = & b \\ & x & \geq & 0. \end{array} \qquad (P)$$

   Here, $A \in \mathbb{R}^{m \times n}$, the dimensions of x, c, and b are defined appropriately, and $1 \leq m \leq n \leq 3$. Let K be the feasible region of (P).

   (a) Construct a linear programming problem of the form (P) with $\dim(K) > n - m$.

   (b) Construct a feasible linear programming problem of the form (P) with $\dim(K) < n - m$, $b \neq 0$, and $\text{rank}(A) = m$.

   (c) In part (b), the linear program you defined has a degenerate basic feasible solution. What are the bases associated with that bfs?

3. What is the dual of the following linear programming problem?

$$\begin{array}{lrcrcrcr} \min & 3x_1 & - & 5x_2 & + & 8x_3 & & \\ \text{s.t.} & x_1 & & & - & x_3 & = & 6 \\ & & & 2x_2 & - & x_3 & \geq & -5 \\ & x_1 & + & x_2 & & & \leq & 8 \\ & & & x_2 & \leq 0, & x_3 & \geq & 0. \end{array}$$

   Use complementary slackness to show that $x = (6, 0, 0)$ is optimal for this problem. Find a linear programming problem in standard form which is equivalent to this problem.

4. Let $A \in \mathbb{R}^{m \times n}$ and $c \in \mathbb{R}^n$. Show that exactly one of the following holds:

   (a) $\exists y \in \mathbb{R}^m$ s.t. $A^T y = c$

   (b) $\exists x \in \mathbb{R}^n$ s.t. $Ax = 0, c^T x \neq 0$.

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