MATP6640/DSES6770 Linear Programming, Homework 2.

Due: Monday, February 11, 2008.

1. Let the polyhedron $Q = \{x \in I\!R^2 : 3x_1 + 5x_2 \geq 15, 2x_1 - x_2 \leq 2, x_2 \geq 2, -3x_1 + x_2 \leq 6\}$. Graph this polyhedron, and hence find matrices B and C such that $Q = \{x \in I\!R^2 : x = By + Cz, y \geq 0, z \geq 0, \sum_i z_i = 1\}$.

2. Consider the linear programming problem

$$
\begin{array}{lccccccccccccc}
\min & x_1 & - & 3x_2 & + & x_3 & + & x_4 & + & x_5 & & \\
\text{s.t.} & x_1 & & & + & x_3 & & & - & 2x_5 & = & 10 \\
 & & - & 3x_2 & + & x_3 & + & x_4 & + & 2x_5 & = & 7 \\
 & x_1 & + & x_2 & & & & & - & 3x_5 & = & 6 \\
 & & & & & x_i & \geq & 0, & & i & = & 1, ..., 5.
\end{array}
$$

   Show that this problem has unbounded objective function value by using the revised simplex algorithm starting from the basic feasible solution $x = [6, 0, 4, 3, 0]^T$. Use the eta factorization of the inverse, so you should first factorize the initial basis B as $LB = U$, where L is lower triangular and U is upper triangular. On subsequent iterations, update the basis matrix by using eta matrices. What is the ray that you find? (Hint: you should find the ray on the second iteration.)

3. Use **AMPL** or another linear programming package to solve the linear programming problem

$$
\begin{array}{lccccccccccccc}
\min & x_1 & + & 3x_2 & + & x_3 & & & - & 4x_5 & & \\
\text{s.t.} & x_1 & & & + & x_3 & & & - & 2x_5 & = & 10 \\
 & & - & 3x_2 & + & x_3 & + & x_4 & + & 2x_5 & = & 7 \\
 & x_1 & + & x_2 & & & & & - & 3x_5 & = & 6 \\
 & & & & & x_i & \geq & 0, & & i & = & 1, ..., 5.
\end{array}
$$

   (See the course webpage for more information on AMPL.)

4. Construct a primal-dual pair of linear programs of the form

$$
\begin{array}{llcll}
\min & c^T x & & & \\
\text{s.t.} & Ax & = & b & (P) \\
 & x & \geq & 0 &
\end{array}
\qquad\qquad
\begin{array}{llcccll}
\max & b^T y & & & & & \\
\text{s.t.} & A^T y & + & s & = & c & (D) \\
 & & & s & \geq & 0 &
\end{array}
$$

   satisfying the following three requirements:

   - A is an $m \times n$ matrix with rank equal to m.
   - The optimal solution to (P) is unique.
   - One optimal solution to (D) has more than m components of s equal to zero.

5. The dual to the linear program in question 2 is infeasible. Taking nonnegative linear combinations of the dual constraints gives further valid dual constraints. Show how the ray you found for the linear programming problem in question 2 can be used to construct a linear combination of the dual constraints that is clearly infeasible.

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