Restoration of Services in Interdependent Infrastructure Systems: A Network Flows Approach

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***Abstract* —** Modern society depends on the operations of civil infrastructure systems, such as transportation, energy, telecommunications and water. Clearly, disruption of any of these systems would present a significant detriment to daily living. However, these systems have become so interconnected, one relying on another, that disruption of one may lead to disruptions in all. The focus of this research is on developing techniques which can be used to respond to events that have the capability to impact interdependent infrastructure systems. As discussed in the paper, infrastructure interdependencies become critical when an impact on one infrastructure system affects one or more other infrastructure systems. The approach is to model the salient elements of these systems and provide decision makers with a means to manipulate the model.

Definitions of five types of interdependency identified during the research are presented and incorporated into a network flows mathematical representation. The mathematical representation, i.e. an interdependent layer network model, is described, including the formulation of the infrastructure interdependencies. Using the lower Manhattan region of New York, USA for illustrative purposes, implementation of the model is shown. First the data requirements are presented with realistic data on the interdependent infrastructure systems of power, telecommunications and subways. Next, a scenario is given that causes major disruption in the services provided by these infrastructures. Finally, three cases of possible restoration of services

are presented depicting progressive stages of service recovery following a disruption. The paper concludes with a discussion of accomplishments and opportunities for future work.

Index Terms — Civil Infrastructure Systems, Emergency Management, Mathematical Programming, Networks.

I. BACKGROUND

Modern society relies on the operations of a set of human-built systems and their processes. The set of systems which is investigated by this research is referred to as civil infrastructure systems. These systems are typically considered to be transportation (including roads, bridges, water and rail); energy (including electric power, gas and liquid fuels); telecommunications (including telegraph, telephone, wireless and internet/digital); and finally, water (including wastewater facilities and water supplies). All civil infrastructures systems rely on a constructed system in order to provide services, such as power delivery, voice and data transmission. Each system's components can only be used to support services of their respective group (communications lines cannot be used for energy transmission and vice versa; water system pipelines are not readily available for energy products such as gas or fuel).

This set of systems is so essential that they have been called our "lifelines" [1] and is also included in the broader set of critical infrastructures defined by the President's Council on Critical Infrastructure Protection (PCCIP). As critical infrastructure systems, they are considered "so vital that their incapacity or destruction would have a debilitating effect on our defense and economic security"[2]. This research focuses on the interconnectedness of these lifeline systems. While all the systems characterized by the PCCIP report are considered critical, some of the systems, such as banking or emergency services, rely upon civil infrastructure systems in order to deliver their services. Therefore, disruption in civil infrastructure systems can cause disruption in these critical infrastructure systems, e.g., disruptions in power and communications after the 2001 World Trade Center (WTC) attack forced the closing of the New York Stock Exchange, part of the banking and finance critical infrastructure [3]. However, this paper will focus only on interdependencies among the civil infrastructure systems. This paper first presents a discussion of infrastructures and interdependencies. The mathematical representation, i.e. an interdependent layer network model, is described, including the formulation of the infrastructure interdependencies. Using the lower Manhattan region of New York, USA for illustrative purposes, implementation of the model is shown. First the data requirements are presented with realistic data on the interdependent infrastructure systems of power, telecommunications and subways. Next, a scenario is given that causes major disruption in the services provided by these infrastructures. The paper concludes a summary and a discussion of opportunities for future work.

A. Historical Perspective

The development of any one of these civil infrastructure systems has historically been made possible in most cases by it relying on another system [4, 5].

- Transportation is America's oldest infrastructure [6]. Horse paths and wagon trails led to rail systems stretching across the country, connecting, and in some cases creating, our cities. Within those cities, the installation of rails led to the replacement of horse-drawn omnibuses with horse-drawn trolleys [7] which were later electrified as the power systems grew. Subways appeared at the end of the nineteenth century and the twentieth century brought cars, trucks, and buses which all necessitated growth of the road infrastructure - all requiring energy systems in order to provide their service.

- Gas was the first energy infrastructure. Small, local coal gas plants produced the gas which was distributed via a dedicated piping system to homes and businesses [4, 5]. Electricity followed with its system of generators, transmission and delivery networks [4, 5]. Natural gas and petroleum pipelines and refineries complete the set of energy systems. The pipeline systems rely on power for compressors and on communications for data acquisition and control systems.

- The growth of cities led to the need for increased water supplies. Gravity-fed or pumped water delivery from lakes, ponds and springs were followed by dams, reservoirs and the piping systems necessary to deliver the water where needed[4, 5, 8]. Distribution networks then delivered the water to where it was needed, relying on power for pumps when gravity feed was not sufficient.

- Telecommunications was the last of the lifeline systems to appear. These systems began with the telegraph; telephones followed, evolving from operators and local switchboards to worldwide networks with high speed digital switches [5]. The internet and wireless technologies have become the newest telecommunications infrastructures.

Early power, water, sewer and gas systems were designed to serve a local populace. All such systems, with the exception of roads, were initially privately owned with customers paying for the service they received. Government at the state and federal levels took responsibility for the road systems from the beginning, using taxes and tolls to build and maintain them for the common good [5].

Each agency or company that owned or managed these systems developed its own control and monitoring systems. As the infrastructure systems grew to cover larger regions and to serve growing populations, more advanced monitoring was required. Greater efficiency was gained in systems such as communications when computers began to aid operators in decision making and control. The use of leased communication lines allowed companies to use an existing infrastructure system instead of using proprietary systems. However, reliance on another companies' systems caused interdependencies.

B. Managing Disruptions to Critical Infrastructure Systems

When an event occurs that may cause disruptions to more than one infrastructure system or is considered to be beyond the management capability of normal staff, emergency response organizations are activated. Emergency Response Organizations (EROs) exist not only at the federal, state, county or city level, but within organizations responsible for operation of the infrastructure systems [9, 10]. Immediately after the September 11, 2001 attacks in New York City, many emergency response organizations were activated. For New York City, the ERO is the Office of Emergency Management (NYCOEM); at the state level, it is the Emergency Management Office (NYSEMO); within Consolidated Edison (the principal supplier of power), it is the Corporate Emergency Response Center; for Verizon, a telecommunications provider, it is the Emergency Command Center. No matter the name, each of these emergency response organizations is established for the same basic reasons: to set priorities, coordinate response efforts, collect information and keep informed all relevant parties, both within and external to the organization [11]. For example, following the 9/11 attacks, ConEd established initial responses priorities for crews and kept NYCOEM informed. As NYCOEM became aware of needs, requests were made to responsible agencies or companies. Additionally, coordination of resources was made at NYCOEM as they were made aware of the resources each agency or company had available for response and restoration of services. When a priority was established by federal, state or city government officials, it was the responsibility of NYCOEM to make this priority clear to all member agencies.

The present research focuses on supporting the EROs who exist at the city, county or state level in setting priorities and coordinating activities [12]. Additionally, support is provided for EROs in the organizations responsible for managing civil infrastructure systems in responding to events that disrupt services provided by the systems they manage. The decision makers in both types of ERO are responsible for developing strategies for response and restoration and proposing them for review by stakeholders or regulators both within and external to their organization [13]. Once a strategy has been determined, it is implemented by field personnel. The methodology proposed in this research enables both the independent system perspective for managers of each system, as well as providing the interdependent view for persons charged with setting priorities and directing restoration activities when an event impacts two or more of these systems simultaneously, e.g. the New York City Office of Emergency Management.

II. MOTIVATION

While much has been written about our infrastructures systems and their essentiality in meeting the needs of daily life, the phrase “critical infrastructures” as used in this research comes from the 1997 report *Critical Foundations - Protecting America's Infrastructure*” [2]by the President’s Commission on Critical Infrastructure Protection. This Commission was established by President Clinton following the 1993 bombing of the World Trade Center and the 1995 bombing of the Murrah Federal Building in Oklahoma City. *Critical Foundations* and the subsequent *Presidential Decision Directive 63, The Clinton Administration Policy on Critical Infrastructure Protection* [14], identified the set of critical infrastructure systems and their vulnerabilities, established the need for and outlined a national strategy for action. These documents also led to the establishment of the Critical Infrastructure Advisory Council which is now a part of the Department of Homeland Security.

In *Critical Foundations*, the following definition is given:

> *Infrastructure: a network of independent, mostly privately-owned, manmade systems and processes that function collaboratively and synergistically to produce and distribute a continuous flow of essential goods and services* [2, p. 3].

Following the September 11, 2001 attacks, the USA Patriot Act revised the definition of critical infrastructure to

> *…systems and assets, whether physical or virtual, so vital to the United States that the incapacity or destruction of such systems and assets would have a debilitating impact on security, national economic security, national public health or safety, or any combination of these matters.* [15]

The 2002 National Strategy for Homeland Security [16] established the following critical infrastructure sectors, which includes their assets, functions and systems: Agriculture; Food; Water; Public Health; Emergency Services; Government; Defense Industrial Base; Information and Telecommunications; Energy; Transportation; Banking and Finance; Chemical Industry; and Postal and Shipping. This report also identified the key asset categories of: National Monuments and Icons; Nuclear Power Plants; Dams; Government Facilities; and Commercial Key Assets.

There are a myriad of publications from Federal agencies and other public organizations in addition to those already cited: see, for example, *Identifying, Understanding, and Analyzing Critical Infrastructure Interdependencies* [17]; *Controlling Cascading Failure: Understanding the Vulnerabilities of Interconnected Infrastructures* [18]; *Critical Infrastructure: Interlinked and Vulnerable* [19] and *Making the Nation Safer: The Role of Science and Technology in Countering Terrorism* [20]). These documents framed the discussion following the attacks of September 11, 2001 and guided the direction of the new Department of Homeland Security regarding matters of critical infrastructures and their protection.

A. Management Of Systems

As previously noted, each of these systems has essentially evolved independently. However as technology has advanced, each infrastructure system has become interconnected to others. The reliance of any of these systems on power is obvious. Remote monitoring and control systems, essential for safe operation of each, rely on a variety of communication paths. Failures within the communications network in one locale, by whatever cause, may have far reaching effects across many systems. This is specifically noted in *The National Strategy for the Physical Protection of Critical Infrastructures and Key Assets* [21] (referred to later as the *National Strategy*).

> *Most industry officials have a fairly complete understanding of their own operations and associated vulnerabilities. However, many of these enterprises require assistance to identify their dependencies on other sectors and the degree of risk to which they are exposed as a function of those interdependencies. The potential impact of such interdependencies hit home for the banking and financial services sector on September 11, when the collapse of the World Trade Center*

> *towers interrupted telecommunications services in lower Manhattan. The disruption brought electronic financial transactions to a halt, with long-term economic impacts still being felt more than a year later.[21, p.34]*

As noted in the *National Strategy*, many emergency managers fail to recognize this “interconnectedness” or interdependence of infrastructures in responding to an incident. Infrastructure management systems did not allow a manager of one system to “see” the operations and conditions of another system. Existing modeling systems also failed to include this “interconnectedness”. This research provides a “system of systems” view to better understanding the interdependent nature of these systems with respect to mitigation and post-disruption response and recovery.

Critical Foundations and the subsequent call for action led to the Complex Interactive Network Systems Initiative (CIN/SI), a joint endeavor between the Department of Defense, academia and the Electric Power Research Institute (EPRI). Discussion of the CIN/SI program is found in Hasse [22] and in the annual reports of the initiative consortia. Amin has written extensively on issues relating to modeling, control, reliability and vulnerability of our electric power systems [23-27]. Liu [28]and Salmeron [29] discuss analytic techniques for identifying key system components in electric power grids. Work on vulnerability of water systems was conducted by Haimes [30] and identifying and reducing vulnerability in petroleum systems by the National Petroleum Council [31]. Kuhn [32] looked at effects of failures of components in the telephone system. Chamberland [33] discussed design of multi-technology data networks. Cremer [34] looked at issues relating to the physical construction of the Internet.

B. System Of Systems

Past research has studied interconnected systems. Haimes and Jiang [35] present a Leontief-based input-output model called the inoperability input-output model (IIM) which enabled the accounting for interconnectedness among infrastructure systems. However, this approach worked at a macroscopic level and while useful for vulnerability assessment, it would be difficult to extend this approach to restoration activities. In a more recent work, [36], they continue the development of the IIM and its ability to measure economic impact among various sectors in the economy by analyzing both the initial disruption and the ripple effects. Carullo [37] presents experimental studies in electrical power systems with an embedded communication system for transmission of network conditions. However, this work only looked at control issues due to communication system delay issues. Holmgren [38] also presents issues in power control systems and the associated communication systems. Jha and Wing [39] develop a constrained Markov decision process method to investigate survivability within infrastructures systems which rely on computers and computer networks. While the work refers to critical infrastructures and measuring impacts of disruptions, the work consists of computer network survivability analysis as those networks relate to a specific system, in this case, banking and finance.

Additionally, there is a body of work in what could be labeled “hybrid systems”. Nagurney’s work in supernetworks would be an example [40-42]. These hybrid systems are new networks formed from portions of the underlying civil systems. A supernetwork might consist of one set of arcs between an origin and destination representing traditional purchasing and goods delivery systems and a parallel set of arcs representing e-business transactions. Similarly, Cetin [43] integrates physical flow layers of transportation system models with the data layers they rely on for control systems by extracting portions of information and transportation systems.

Peeta, Zhang and Friesz [44] present a preliminary model of dynamic multilayer infrastructure networks in the context of telecommuting where three coupled network layers – automobiles, urban freight and data – were modeled as dynamic agents. The work looked at "switchable flows". Commuters could choose to either commute to work based on information and experience on traffic conditions or telecommute from home based on existing efficiency within the telecommunications subsystem. This work did assume the presence of a super authority responsible for providing information to commuters and freight operators. This super authority would also be a controlling entity for improvements in both the transportation and information networks. In Zhang, Peeta and Friesz [45], a game theoretic formulation is presented where investments in the three infrastructure systems were controlled by the super authority.

Significant effort is being expended in the development of simulations of infrastructure interdependencies. On going work at Argonne, Sandia and Los Alamos National Laboratories including the Simulation Object Framework for Infrastructure Analysis (SOFIA), the Energy Interdependence Simulator (EISIM), and the Interdependence Energy Infrastructure Simulation System (IEISS) projects (see http://public.lanl.gov/bwb/#sofia). As simulations they can improve understanding of system response to an event or scenario and can be useful in vulnerability studies.

Gursesli and Desrochers [46] propose Petri nets for modeling infrastructure interdependencies. However, their work models an entire infrastructure system (such as electric power or transportation) as a single node. While useful in showing relationships, it lacks sufficient detail to be useful for either planning and mitigation or response and restoration activities.

C. Infrastructure Interdependencies

The PCCIP report discussed the reliance or dependence of the critical systems but Rinaldi, Peerenboom and Kelly [17] formalized the definitions within this on-going discussion of critical infrastructure interdependencies:

> *Dependency: A linkage or connection between two infrastructures, through which the state of one infrastructure influences or is correlated to the state of the other.*
>
> *Interdependency: A bi-directional relationship between two infrastructures through which the state of each influences or is correlated to the state of the other. More generally, two infrastructures are interdependent when each is dependent on the other* [17, p. 14].

Also, they defined four classes of interdependency:

> **Physical** – *…a physical interdependency arises from a physical linkage between the inputs and outputs of two agents: a commodity produced or modified by one infrastructure (an output) is required by another infrastructure for it to operate (an input).*
>
> **Cyber** – *An infrastructure has a cyber interdependency if its state depends on information transmitted through the information infrastructure*

Geographic – *Infrastructures are geographically interdependent if a local …event can create state changes in all of them.*

Logical – *Two infrastructures are logically interdependent if the state of each depends on the state of the other via a mechanism that is neither physical, cyber nor geographic connection* [17, p.14-16].

Due to the number of different types of dependencies and interdependencies, Rinaldi, Peerenboom and Kelly [17] classifies the entire family of interrelationships between systems as interdependencies, an approach retained in this paper. The objective of the definitions is to aid in the discussion of policies for addressing the vulnerability of infrastructures to natural, technological and intentional human-induced hazards [2, 17].

In [47], the following definitions were established for the study of critical interdependent infrastructures. An *infrastructure* is defined as a linked set of physical components with associated activities. *Physical component*s are the built part of an infrastructure; *activities* are tasks necessary to operate physical components of the infrastructure. An *intersection* is the area where two or more physical components meet or are joined. An intersection circumscribes the activities and physical components necessary to manage the connection between the joined physical components. As an example, the intersection of two roadways may have one or more physical components (e.g., a traffic signal) and activities (e.g., manipulation of the signal via sensors embedded in the roadway). All intersections in a given infrastructure must have a physical component.

A *service* is something made available by the infrastructure for use or consumption. A service may be used by people or by other infrastructures: it is provided in order to meet a real or perceived need. An infrastructure can provide one or more services. *Material* is any physical entity or “substance or substances out of which a thing is or can be made” [48, p.837]. Examples include electrons, people, product, and electromagnetic signals. Provision of a service requires activities such as movement, collection, transformation or storage of material. Activities may be initiated at one or many locations and may be terminated at one or many locations. Assuming that traversal of a connection between two intersections requires a set of activities from beginning to end, management activities are necessary when provision of the service requires traversal of more than one intersection.

A *disruption* in an infrastructure is said to occur when one or more of the physical components or one or more of the activities needed to operate a physical component cannot function at prescribed levels. Disruption may or may not result in service degradation. *Service degradation* is said to occur when the service itself cannot be provided at its prescribed level.

The current research identifies five types of interrelationship between infrastructure systems:

- *Input*: the infrastructure requires as input one or more services from another infrastructure in order to provide some other service.
- *Mutually dependent*: at least one of the activities of each infrastructure in a collection of infrastructures is dependent upon each of the other infrastructures. (An example of mutual dependence involving two infrastructures occurs when an output of infrastructure A is an input to infrastructure B, and an output of infrastructure B is an input to infrastructure A.)
- *Co-located*: any of their physical components are situated within a prescribed geographical region.
- *Shared (AND)*: some physical components or activities of the infrastructure used in

providing the services are shared.

- *Exclusive-or* (*XOR*): only one of two or more services can be provided by an infrastructure. (Note that a disturbance in an infrastructure that is dependent on another by virtue of its inability to operate if the other infrastructure is operating will effect just its own provision of service.)

Collectively, these five conditions—input, mutual dependence, co-location, shared and exclusive-or—will be denoted types of *interdependence*, since all imply that an impact on one infrastructure system is also an impact on one or more other infrastructure systems[49]. These definitions of civil infrastructure systems and their interdependencies form the basis of the mathematical formulations developed in the next section.

III. THE MODELING PARADIGM: NETWORK FLOWS

Interdependent infrastructures are viewed as networks, with movement of commodities (i.e. material) corresponding to flows and with services corresponding to a desired level of these flows. For ease of representation, each network, or infrastructure system, is defined as a collection of nodes and arcs with commodities flowing from node to node along paths in the network. Activities, physical components and intersections are considered to be contained within a node. Similarly, management activities are not considered in traversal of an arc; they are contained within the arc itself. Fundamentals of network flow problems are fairly uniform within the literature and texts on the subject [50]. For each commodity, each node is either a supply node which is a source for the commodity; a demand node which is a point that requires some amount of the commodity; or a transshipment node which is a point that neither produces nor requires the commodity but serve as a point though which the commodity passes. Arcs may, of course, have limited capacities. Infrastructure systems operate in an environment subject to disruptions, natural, human-caused or willful acts. Based upon performance criteria, an infrastructure system can be designed to minimize possible service degradation following a disruption. In addition, once a disruption occurs, alternative ways of restoring service can be determined.

Mathematically, a collection of infrastructure systems is represented as follows. Let I denote the set of infrastructures. Infrastructure $i \in I$ has nodes V^i and directed arcs E^i. Associated with each node $j \in V^i$ is a scalar b^i_j representing its supply or demand. If node $j \in V^i$ is a demand point then $b^i_j < 0$; if it is a supply point then $b^i_j > 0$; and if it is a transshipment node then $b^i_j = 0$. If $j \in V^i$ is a supply node then b^i_j equals the maximum possible amount that could be produced at that node. A nonnegative vector of variables, x^i_e, represents the flow on each arc e of the infrastructure. Associated with each arc e in E^i are non-negative scalars of costs c^i_e and capacities u^i_e, where $0 \leq x^i_e \leq u^i_e$.

Arcs are represented using either the endpoints of the arc or the index of the arc. For a node $l \in V^i$ for some infrastructure $i \in I$, let $\delta^+(l)$ denote the set of arcs in E^i that enter node l and let $\delta^-(l)$ denote the set of arcs in E^i that leave node l. Define $\delta(l) := \delta^+(l) \cup \delta^-(l)$, the set of all arcs incident to node l. Without loss of generality, assume that every supply node has no incoming arcs (i.e., $\delta^+(l) = 0$ if $b^i_l > 0$) and that demand nodes have no outgoing arcs, (i.e., $\delta^-(l) = 0$ if $b^i_l < 0$). Define the set $V^{i,+} \subseteq V^i$ to be the nodes $j \in V^i$ with $b^i_j > 0$ (supply nodes). Sets $V^{i,=} \subseteq V^i$ (transshipment nodes) and $V^{i,-} \subseteq V^i$ (demand nodes) are defined similarly.

A transshipment node j may have a limited capacity, w_j^i, modeled by placing an upper bound on the total flow across the arcs $\delta^+(l)$. Included in the model are *flow conservation constraints* that (i) for supply nodes ensure that total flow out of the node is no greater than the available supply, (ii) for demand nodes ensure that demand is met, and (iii) for transshipment nodes ensure that flow into the node equals flow out of the node. The structural requirements are modeled by constraints on the capacities of arcs and transshipment nodes.

Network flow models can also be characterized as single or multicommodity. Infrastructures such as water, power, gas and sewer would be single commodity systems, where material moves from one or more supply points, through the set of arcs and nodes, subject to constraints on capacity, and reaches one of more demand points, in an optimal fashion. However, systems like transportation and telecommunications have additional requirements. In these cases, commodities moving across the system have specific origin and destination requirements. For example, passengers arriving at a subway station may each have unique destinations and the needs of each passenger must be met. However, these multiple commodities are not moving independently of each other. These additional requirements of multicommodity systems must be accounted for [50].

As before, let I denote the set of infrastructures. Infrastructure $i \in I$ has nodes V^i and directed arcs E^i. Set O^i represents the set of all origin – destination pairs in infrastructure i. Associated with each origin-destination (O-D) pair, $o \in O^i$ is a market, m_o^i, the amount of a commodity which must flow between that O-D pair. Between each O-D pair is a set of possible paths, P_o. Each path p in P_o is comprised of a subset of the arcs in E. The flow on any path is f and the flows across all the paths in P_o must equal m. The flow x on an arc e is determined by summing the flow on all paths which contain e and is constrained by its capacity, u.

A. Mathematical Representation Of Interdependencies

This research has developed a formal, mathematical representation of the set of civil infrastructure systems that explicitly incorporates the interdependencies among them and will be referred to as an interdependent layered network (ILN) model. This ILN is a mixed-integer, network-flow based model which has been implemented in software that enables the resulting model to be exercised.

While much has been written about the need for models of the interconnected networks as was discussed in Section II.A, the vast majority of the work remains focused on one or possibly two systems. This research has developed a network flow formulation which clearly identifies effects of a disruptive event across the set of infrastructure systems. This model which includes the interdependencies defined in Section II.C will be presented in the material to follow.

1) Input: An infrastructure is input dependent when it requires as input one or more services from another infrastructure in order to provide some other service. As an example, in the case of a telephone switching station, the switching station itself is a transshipment node within the telecommunications network. However, this same switching station from the perspective of the electrical network is seen as a demand node since it needs an adequate source of electricity to operate. This situation may be represented more formally as follows. Denote the demand node for the switching station in the electrical network to be node j. If there is an adequate flow of electric power into node j, the switching station can function; otherwise, the switching station fails. A connector variable, y, is used in this case to represent the two states of the switching station. If adequate power is available at j, then $y = 1$; if not, then $y = 0$. The phone switching station also has some maximum capacity within the telecommunications network. The station's capacity can be represented as the product of the connector variable y and the rated capacity. When adequate power is available the station can operate to its rated capacity (since $y = 1$). On the other hand, if adequate power is not available then the capacity of the station is 0. The value of the connector is set by the conditions existing in one system, and affects the operating characteristics of a second system. Events affecting the power network that have an effect on node j in turn impact a node in the model of the telecommunications network. The effect on any set of systems can be analyzed in a similar manner. Note that some interdependent infrastructure system failures may result in reducing capacity to some value other than zero. For example, loss of supervisory control systems in a subway system may result in operators exercising greater care and slowing trains. Therefore, the post-disruption capacity may be lower than normal. In this case, the connector variable y would shift from 1 to a lower value. Mathematically, this is as follows. If the capacity, u, of a system is reduced to x% when the system it relies on is lost, then the value of capacity becomes $u_{NEW} = u_{ORIG}(\frac{x}{100} + y(1 - \frac{x}{100}))$. The exact effect of each disruption must be evaluated during impact assessment.

In general, input dependency is represented as follows: Let $D(i,h) \subseteq V^{i,-}$ be the set of nodes in i that another infrastructure h depends upon (parent nodes). Let $D^i := \cup_{h \in I, h \neq i} D(i,h)$ be the interdependent nodes in infrastructure i. The remaining nodes in $V^{i,-}$ will be referred to as the independent nodes. For $j \in D(i,h)$, the binary variable $y_{h,l}^{i,j}$ is the connection between node j in infrastructure *i* (where it is a demand node) and node *l* in infrastructure *h*, where it may be either a supply, demand or transshipment node.

Let $C(h,i) \subseteq V^h$ be the set of nodes in h that depend on some other infrastructure *i*, (child nodes) and let $C^h := \cup_{i \in I, i \neq h} C(h,i)$ .Without loss of generality, all nodes have been disaggregated to the point where, given infrastructures *i* and *h*, and node *j* in $D(i,h)$, there is a unique node *l* in $C(h,i)$ such that $y_{h,l}^{i,j}$ is defined, and given infrastructures *i* and *h* and node *l* in $C(h,i)$, there is a unique node *j* in $D(i,h)$, such that $y_{h,l}^{i,j}$ is defined. Let $F(i,h)$ be the set of ordered pairs (*j,l*) associated with node *j* in $D(i,h)$ and node *l* in $C(h,i)$ for each $y_{h,l}^{i,j}$.

Input dependency changes the basic network flow model as follows. Following a disruption, the flow into demand nodes may be insufficient. The slack variable, *s*, represents the shortfall. The constraint for all demand nodes becomes

$$s_j^i + \sum_{e \in \delta^+(j)} x_e^i = -b_j^i \qquad \forall j \in V^{i,-}, \forall i \in I \tag{1}$$

and in the objective function, the weighted slack is:

$$\sum_{i \in I} \sum_{j \in V^{i,-} \setminus D^i} k_j^i s_j^i \quad \text{for demand nodes} \tag{2}$$

and

$$\sum_{i \in I} \sum_{j \in V^{i,+}} k_j^i s_j^i \tag{3}$$

The existence of slack at an interdependent node (parent system) acts as a control switch for the connector variable, *y*:

$$s_j^i \leq (1 - y_{h,l}^{i,j})(-b_j^i) \qquad \forall (j,l) \in F(i,h), \forall i,h \in I, i \neq h \tag{4}$$

When s is greater than 0, y must be 0 in order to satisfy this constraint.

The following component of the objective function tends to push y back to 1:

$$\sum_{i\in I}\sum_{j\in D^i}\sum_{\substack{h\in I\\ h\neq i}}\sum_{\substack{(j,l)\in\\ F(i,h)}} k_j^i b_j^i (1-y_{h,l}^{i,j}) \tag{5}$$

When a parent node has unmet demand, the corresponding capacity of its child node in the dependent system is reduced. The following constraints are required:

For supply nodes -

$$\sum_{e\in\delta^-(l)} x_e^h \le b_l^h y_{h,l}^{i,j} \qquad \forall (j,l)\in F(i,h) \text{ with } b_l^h > 0, \forall i,h\in I, i\neq h \tag{6}$$

For transshipment nodes -

$$\sum_{e\in\delta^+(l)} x_e^h \le w_l^h y_{h,l}^{i,j} \qquad \forall (j,l)\in F(i,h) \text{ with } b_l^h = 0, \forall i,h\in I, i\neq h \tag{7}$$

For demand nodes –

$$s_l^h + \sum_{e\in\delta^+(l)} x_e^h \le -b_l^h y_{h,l}^{i,j} \qquad \forall (j,l)\in F(i,h) \text{ with } b_l^h = 0, \forall i,h\in I, i\neq h \tag{8}$$

2) Mutual Dependence: A collection of infrastructures is said to be mutually dependent if at least one of the activities of one infrastructure system is dependent upon any other infrastructure system and at least one of the activities of this other infrastructure system is dependent upon the first infrastructure system. So in the case of two systems i and h , mutual dependence would occur if there is at least one $y_{h,l}^{i,j}$ (connection between node j in infrastructure i (where it is a demand node) and node l in infrastructure h) and at least one $y_{i,n}^{h,m}$ (connection between node m in infrastructure h (where it is a demand node) and node n in infrastructure i). Consider a natural gas system compressor and a gas-fired electric power generator. From the perspective of the natural gas system, the compressor is a transshipment node and the generator is a demand node. From the perspective of the electrical network, the generator is a supply node and the compressor is a demand node. The generator needs gas to produce electricity; the compressor needs electric power to deliver gas through the system to the generator. In this case, $y_{h,l}^{i,j}$ would be the connection from the power system (node j in infrastructure i) to the compressor in the gas system (node l in infrastructure h), and $y_{i,n}^{h,m}$, the connection from the gas system (node m in infrastructure h) to the generator in the power system (node n in infrastructure i). Failure of one component causes its corresponding binary variable to be set to zero, thus reducing the effective capacity of the other component to zero. Mathematically, the mutual dependence is represented as two input interdependencies.

3) Shared (AND): Shared interdependence occurs when some physical components and/or activities of the infrastructure used in providing the services are shared. Phone lines could be considered in the AND interdependency. Each phone line carries two types of calls, incoming and outgoing. Therefore, each cable section whether it be the connection from a single home to a distribution line or the feeder cable connecting a CEV to a central office, would have some maximum capacity. For example, if the capacity of some section is 50, this could be 50 incoming calls or 50 outgoing calls or some combination totaling 50.

Mathematically, let $x_{e,c1}^{i}$ and $x_{e,c2}^{i}$ be the amount of commodity *c1* and *c2* carried across arc *e* of infrastructure *i.* Additionally, arc *e* has total capacity *u*. Therefore, the shared interdependency would add a constraint of the form:

$$x^i_{e,c1} + x^i_{e,c2} \le u^i_e \tag{9}$$

Similarly for nodes, their capacity is limited to the sum of flow of all commodities into the node. Constraints of this form will have to exist for all arcs and nodes that carry multiple commodities.

4) Exclusive-or (XOR): When multiple services share infrastructure component(s), but the component can only be used by one service at a time, exclusive-or interdependence occurs. In the first few days following the WTC attacks, streets (i.e., shared components) could not be used by both the emergency response personnel and financial district workers. This conflict had to be resolved prior to reopening the New York Stock Exchange [51]. Exclusive-or interdependencies are modeled by selecting additional constraints to restrict flow to one commodity or the other.

A binary variable r^i_{e,c_n} will indicate whether or not commodity n is being carried on arc e in infrastructure i (r=1 means the arc is carrying commodity c_n, 0 its not). Mathematically, this is

$$x^i_{e,c_n} \le u^i_e r^i_{e,c_n} \tag{10}$$

$$\sum_n r^i_{e,c_n} \le 1 \tag{11}$$

5) Co-located: The co-located interdependency occurs when any of the physical components or activities of the civil infrastructure systems are situated within a prescribed geographical region. It was previously noted that managers of individual infrastructure systems would identify the components of their respective system at or near the site of the incident which may have been affected by the event. Based on further investigation, the status of these components will be adjusted. However, since only those EROs who are responsible for coordinating activities across multiple agencies maintain the complete view of all civil infrastructure systems, it is ultimately their responsibility to ensure that all co-located interdependencies have been considered and the models of the affected infrastructures revised as appropriate.

B. The Objective Function and Constraints of an Interdependent Layered Network Model

The objective function of this interdependent layered network model combines the minimum cost terms from traditional network flow models, both single and multicommodity, and the weighted slacks described in equations (2) for independent demand nodes and (3) for supply nodes. Each situation will require whether one or both are used. Additionally, equation (5) will be included with its own weighting to shift available commodities to or away from parent demand nodes in interdependent systems.

The constraint set also includes the typical flow conservation and capacity constraints from single and multicommodity network flow models, including the slack included in equation (1). Additionally, equation (4) is included as the control switch of the connector variable, y, so when there is unmet demand at a parent node, the effect is reflected in the operation of the child node among interdependent systems, as earlier described with equations (6),(7) and (8). Because these interdependent nodes control the operation of nodes in other infrastructure systems, if they are not fully operational then they are in a failed condition: there is no benefit to partially meeting the requirement. When the parent node has unmet demand, the flow to that node is reduced to zero, so the remaining flow may be diverted to other demand nodes. Mathematically, this is

$$\sum_{e \in \delta^{+}(l)} x_e^i = b_j^i y_{h,l}^{i,j} \tag{12}$$

and is an additional constraint. Shared interdependency constraints (equation (9)) and exclusive-or constraints (equations (10) and (11)) are added as appropriate as well as constraints imposed by the particular situation.

IV. IMPLEMENTATION

The foregoing presents a formulation of an interdependent layered model in the context of interdependent critical infrastructure systems. The section to follow will discuss its implementation in terms of data requirements, software to run the model, and a human-computer interface for interacting with the model.

A. Data Requirements

In constructing the data set for these systems, the modeler must be provided with both the physical layout of the system and the supplies, demands and capacities for the components of the system. However, systems like transportation and telecommunications have additional requirements. As stated earlier, commodities moving across the system have specific origin and destination requirements. For example, passengers arriving at a subway station may each have unique destinations and the needs of each passenger must be met. However, these multiple commodities are not moving independent of each other. In this case, the modeler not only needs the physical layout and capacities of system components, but the origin and destination requirements (commonly referred to as an O-D matrix) for the system.

1) Power System: In general, power is produced at generating stations at voltages in the range of 13.8 – 22kV. It is transformed up to 345 – 500kV for transmissions, although some lower voltage transmission systems exist. It is reduced in steps for distribution and is delivered to most homes and businesses as 120V and 220V.

Typically, the U.S. power distribution is of a radial design from the substation. Each feeder distributes power to a portion of the service area. Transformers along the feeder provide 120/220V service to homes and businesses. Transformer failures will cause loss to a few homes or businesses, feeder disruptions to a greater number, but only in the sub-area serviced by the feeder. In developing the 120/220V service grid for Consolidated Edison, a different design was used.

In Manhattan, the high voltages of transmission are first transformed down to an intermediate voltage of 69 – 123 kV and further reduced at area substations to 13.5 kV. The power is distributed for the substation on a network of 8 to 28 feeders. Each substation serves two service areas, each area having its own set of feeders. So there might be as many as 56 feeders exiting a substation.

The power system below 60th Street in Manhattan contains 15 substations covering 30 service areas. Each substation has an average capacity of 400 MVA. Each feeder has a capacity load of 15 MVA. Along each feeder 20 to 40 transformers reduce feeder voltage to 120-220V. The output of several hundred transformers are connected in an elaborate 120/220 sub-network. The system is designed to maintain 120/220V service throughout the area even if as many as 2 feeders fail. This design provides for extremely high reliability. The ConEd website provided the following information of the effect on reliability

> *"Our electric system is seven times more reliable than the average of all other New York state utilities, and on average, approximately nine*

> *times more reliable than any other utility in the country. Industry experts have recognized Con Edison as the most reliable electric utility in North America."* (http://www.coned.com)

At some key facilities, such as hospitals, phone switching centers, etc. these feeders will each provide power to sets of transformers at that location again assuring highly reliable service. Non-key facilities are connected directly to the 120/220V grid.

The tracks of New York's subway system are powered from DC rectifiers, transforming feeder voltage of 13.5kV ac to 600V dc Each rectifier has a capacity of about 3MW and at least two rectifiers power at each track section (typically 100-1500 ft.). Each rectifier has sufficient capacity to power its respective track section, so the additional rectifiers(s) are providing redundancy and reliability. Each rectifier is connected to 2 feeders and each feeder is capable of carrying the full demand of the rectifier.

The model of the power system for Lower Manhattan contains 15 substations and 30 service areas in Manhattan south of 60th Street. The geographic boundaries of the service areas differ from the actual as the model is intended to provide a realistic view because actual data is not available due to security concerns. Each substation distributes power along 8-24 feeders to 17 phone switching centers, 178 ac/dc rectifiers for the subways and service to all residences and businesses in the area. The 120/220V grid was modeled in an aggregate fashion since none of the other major facilities (phone switching centers or subway tracks) relied on it to provide service. Instead of 20-40 transformers along the feeders, a transformer (node) was placed at the ends of the feeder. The 110/220V demands of a service area were aggregated into a single node fed from all the feeder ends. Blocks in the study were then classified residential or commercial. Census data for the 1892 blocks were plotted in bins of 120. It was noted that 633 of 1892 blocks (33%) had populations less than 120. These blocks were classified commercial; the remaining blocks residential. Power demand for a residential block is 2.33 MW and commercial is 10 MW.

2) Model of Telecommunications: The phone system is of a spoke and hub construction. Customers may be connected via a Controlled Environment Vault (CEV) by a distribution cable serving dozens of customers. Distribution cables are combined at the CEV into a feeder cable. Each feeder typically carries thousands of lines. The feeder enters a central office (CO) at the cable vault. In many areas of Manhattan, customers are connected directly to the central office. In the case of 140 West Street, there were 300,000 wire pairs in the cable vault. From a CO, calls are digitally switched to the intended destination, either directly or via a CEV if the call originates and terminates with the service area of the CO. In high density urban areas, like Manhattan, one CO may be connected to other nearby CO's via a trunk line. If the call is for a destination outside the CO and nearby CO's, it is directed to the tandem. Tandems connect to other tandems and to the national and international networks.

Calls are modeled as multicommodity flow problems [50] with a certain market (number of calls) between each origin and destination (O-D) pair. Within the model, since there were no instances of systems that would fail when phones fail, the model only looked at O-D modeling between CO's and any outages would reflect as the entire service area of the CO. For each O-D pair, a set of paths must be generated and the set of arcs comprising each path is additional data for the mode.. In the case of the phone system, two paths exist between most central offices.

If cellular service was to be incorporated in the analysis, a tower would be used in the place of a CEV. Cellular calls are connected to the tower and are then transferred via landline to the cellular company CO. From there it is connected to other CO's or tandems. Seventeen CO's and service areas were included in the model for Lower Manhattan.

3) Model of the Subway System: The New York City subway system serves 7,000,000 users per day. Information regarding station and track locations was taken from existing system maps from the Metropolitan Transportation Authority. Stations are individual nodes and local and express tracks are arcs.

Similar to phone, passengers are modeled as a multicommodity flow problem. Modeling the flow through the system starts with determining an O-D matrix for a time period of concern. Data was received from the MTA including turnstile counts from each station in the system and data from an annual cordon count. The cordon count is an hour by hour measure of the passenger flow in and out of Manhattan from above (north of 60th St.) from Queens and from Brooklyn. The morning commute is considered by the MTA to be the period from 6am- 9 am and the evening from 4pm – 9pm.

Travel times were determined between each station pair based on track length, an average train speed of 45 mph and 20 seconds for the time stopped at a station. For trips which required transfers at a station from one line to another, the arcs were included in the path with a travel time of one half the advance for the train line the person was transferring to. (The advance is the time between train arrivals; if a line has 20 trains per hour, the advance is 3 minutes.)

Using methods outlined in Fricker [52], an O - D matrix was developed for the 115 stations included in the model. Within the dataset provided by the MTA, no trips originated or terminated from Brooklyn on the B or D trains. This was due to bridge maintenance during the time frame. Paths for each O-D pair were also developed. The path set included the shortest path and alternatives that did not exceed 150% of the shortest path time to a maximum of three paths. For example, if the shortest path for an O-D pair was 20 minutes, up to two alternative paths would be generated as long as the travel time on the alternatives did not exceed 30 minutes. The set of arcs comprising each path are included within the data for the model. Line capacities were based on MTA data.

B. Software Requirements

The model has been implemented in AMPL [53] using the CPLEX [54] solver. All of the model's data is stored in a Microsoft Access database. The database contains the component attributes such as a name, their capacity and their priority, as well as spatial attributes, such as location and length. These spatial characteristics are generated automatically by the GIS software, ESRI's ArcGIS [55] in this case. The remaining attributes are added by the modeler. Changes to attributes due to disruption can easily be made. Some of the tables and attributes are shown in Figure 1.

Insert Figure 1 here.

C. The User Interface

A geographic information system (GIS) was selected as the user interface as this seemed to be the most natural method of displaying systems and determining affected areas to ERO personnel. The interface allows the operator to update the conditions of the components of the set of systems modeled, to add temporary systems during restoration, and to display areas affected by an inability to meet demands.

D. Scenario

In order to demonstrate the model's usefulness and since our work was related to the September 11 attacks on the World Trade Center (WTC), a scenario with damage of a magnitude similar to the effects of those attacks was developed. After the collapse of the North and South towers on the morning of September 11 and the collapse that afternoon of WTC Building 7 on Vessey St, Consolidated Edison (Con Ed), Verizon and the Metropolitan Transit Authority were faced with the following conditions. For Con Ed, the collapse of WTC 7 resulted in the loss of a power substation and a resulting power outage south of Canal St., an area which included the Verizon switching center on West St, the American Stock Exchange and the New York Stock Exchange. Con Ed restored power throughout most of the area in 8 days using 80 temporary generators and miles of enclosed shunts (shunts are power lines run along streets which connected working portions of the power grid in southern Manhattan to the feeders that had been distributing the power from the destroyed substation in order to restore power).

For Verizon, the power outage would not have normally affected operations due to the presence of emergency generators for the building. However, this was not normal circumstances. Broken water mains and firefighting operations flooded the basement of the building preventing operations of emergency generators there while the extensive dust and debris outside the building made the remaining generator inoperable. Falling debris from all three buildings severed cables leading to and from the switching center and the cable vault where all the feeder cables entered the building was flooded. These combined to result in loss of 300,000 voice lines and 4.4 million data lines. Verizon connected temporary phone lines to the digital switches in the upper floors on the 140 West St office, ran the lines down along the outside of the building and then connected them to undamaged lines approximately one block from the building along Manhattan streets (any temporary line for power or phone will be referred to as a shunt).

The MTA temporarily lost power to all the subway stations south of Canal St due to the power outages with the long term impact being the destruction of the Cortlandt St station on the 1,9 lines located under the World Trade Center. Service south of the WTC complex on the 1,9 line was not available until the station was reconstructed.

To exercise the model, the following scenario has been developed. Some event (no matter the cause) has resulted in the collapse of the Brooklyn – Battery Tunnel. Power and phone lines as well as subway tracks passing over the area are either damaged or severed. The electrical substation at the southern end of Manhattan is also damaged with repairs estimated at 4 to 6 weeks. The resulting outage affects the area west of Broadway north to nearly Vessey St and the area east of Broadway north to Fulton. Within the subway system, this power outage impacts the Rector St station on the R and W lines; the Wall St, Bowling Green and Fulton St. stations on the 4 and 5 lines; the Wall St and Fulton St. stations on the 2 and 3 lines, the Rector St. and South Ferry St stations on the 1 and 9 lines; and the Broad St and Fulton St stations on the J and Z lines, as well as all connecting local and express tracks. Also affected by the power outage is the phone central office between Bowling Green and Battery Place. Other directly affected points of concern are the New York Stock Exchange (NYSE) and the Goldman Sachs offices at Broad St and William St. as well as the residential and commercial customers. An overview map of Manhattan showing the affected area and a detailed map are shown in figures 2 and 3.

(Insert figure 2 here)

(insert Figure 3 here)

If they did not have emergency backup power supplies, the power outage to the two phone central offices would have resulted in a loss of phone service over a similar area to the power outage. Simulating the WTC event, backup power at the CO at Bowling Green / Battery Place is lost, resulting in a loss of phone service to the Merrill Lynch and Lehman Brothers offices along Vessey St. Additionally, the trunk line between the Bowling Green central office and the central office along Fulton St is severed by the event. This is significant because this trunk line is in the primary or alternate communication path to the NYSE for the trading offices of Merrill Lynch and Lehman Brothers in southern Manhattan and Morgan Stanley along Broadway between 48th and 49th Streets. The reason for having both the primary and alternate paths allows for rerouting during periods of traffic congestion.

Based on discussions with the Metropolitan Transit Authority (MTA), a phone system outage would have little effect on subway operations. Commercial phone lines only support communications between supervisory personnel and the information booth attendants. The subway system operations rely on a control and monitoring system which is independent. The tunnel collapse does result in disruption of the tracks connecting the Rector St and South Ferry St stations on the 1 and 9 lines.

The impacts are entered into the model via the GIS interface to the database. In this case, the available supply at substation 15 is reduced to zero as well as portions of eleven feeders as reports of their condition are received. The 1 and 9 line tracks connecting the South Ferry St and

Rector St stations also have their capacity reduced to zero as well as the trunk line between the two central offices (note: since the only individual phone lines lost did not impact either the NYSE or the trading offices of concern or have an impact on subway or power system operation, we will not be including them in this analysis). The outputs of the model are lists of unmet demands for service - the power and phone outage areas and the number of subway passengers affected (those who can no longer board or depart at the South Ferry St. station). Sufficient redundancy exists in the phone system so that there is no loss of service outside of the area served by the CO at Bowling Green. Demands for power, phone and subway service across the rest of Manhattan are being met.

Affected agencies and businesses start making restoration plans. Power and phone systems would have similar plans consisting of laying temporary shunts along streets, connecting them to intact portions of the original network. Con Ed intends to utilize the reserve capacity from the four substations nearest to the affected area, running shunts from them to the outage area. The model will then aid in determining where individual shunts should be run within the outage area. Similarly, Verizon will run a new trunk line from the Bowling Green / Battery Place central office and tie into the existing line somewhere along Broadway. At the Emergency Operation Center, Con Ed and Verizon are asked to work collaboratively, running lines along the same street sections to the maximum extent possible while each company meets their goal of running the least amount of temporary shunt.

This first restoration plan is formulated similar to the single source fixed charge network flow problem [56]. The objective is to select the smallest set of arcs which meet the flow and capacity constraints. With single commodity systems like power, the set of possible temporary arcs is added to the system's existing arc set and the model will find the new paths through the system. However, in multicommodity systems like phone, the system must first determine the set of temporary arcs that will comprise the shunt. Then the set of paths between each origin and destination must be revised to reflect the installation of the shunt.

The model is used in two steps. First, it must determine where the temp arcs go. In this scenario, each shunt has a fixed cost, q, which is a function of its length. Since the stated goal was to minimize the amount of shunt used, while also minimizing the number of street sections distributed, a new binary variable, z, is also introduced. Let $z_e^i = 1$when arc e in infrastructure I has flow on it $x_e^i > 0$. So, when $z_e^i = 1$, a shunt must be installed at this location and its fixed charge will be incurred. The sum of all the products of z and q represent the total fixed charge and the sum of z represents the number of street sections used. Both of these sums are added to the model's objective function, while a new constraints is added

$$x_e^i \le u_e^i z_e^i \tag{13}$$

After the set of temp arcs is determined, the new set of paths for the multicommodity system is generated. The model is run again to verify that service is in fact restored.

The proposed solution (called case 1) is shown below and restores power to the affected area in sufficient quantity by using 29 street sections, totaling about two miles of shunts, and restores the phone trunk line using only 8 arcs, i.e. street sections.

Insert Figure 4 here

Upon further review of the Case 1 solution, Con Ed notes that one of the substations selected for this proposed plan is operating at full capacity. This condition is not acceptable and they specify that there must be at least 90 MW reserve capacity. This is added to the model by constraining the slack at the node to be greater than or equal to 90. OEM adds a new requirement that no temporary lines may cross West St. This constraint is added to the model as follows. First, all nodes along West St are transshipment nodes. Therefore, if you sum the installation index, z ,for the temporary arcs which either originate or terminate at nodes in the subset, this sum must be less than or equal to one if no arc crosses West St.

$$\sum z^i_{j,k} + \sum z^i_{k,l} \leq 1 \qquad \text{for all } k \text{ in the subset of concern for each infrastructure system} \quad (14)$$

Adding these constraints (case 2), results in the next solution shown in figure 5.

Insert figure 5 here

This restoration plan uses 34 arcs, i.e. street sections, covering about two and a half miles.

Based on new information from the scene and the advice of other agencies, OEM directs that no lines will be run from West Side substations. Also, because of the heavy equipment that will be involved in restoration and recovery at the tunnel site, the number of power shunts crossing and running along streets will be limited to a capacity of 150 MW. When these constraints are added, the solution results in unmet demands within the outage area. Reducing power shunt capacity to 150MW and not allowing flow from the two West Side substations gives the following solution (case 3) is shown in figure 6. With the slack weighting factor, *k*, for all power nodes constant, outages exist at the subway rectifier for the subway tracks on the R and W lines between the Rector St and Cortlandt St stations. Power would also not be available to the Bowling Green central office and outages would exist in some areas west of Broadway.

Insert figure 6 here

This restoration plan involves 50 street sections totaling about three and a third miles. Now, priorities must be set and tradeoffs must be made. Since this scenario has been focused on the financial sector, restoring phone service and power to them is of primary importance. Workers are also needed to support restoration of other services in sector; a fully operating subway system is needed (except for the station at South Ferry whose tracks require repair). In order to satisfy the priority of restoring service to the financial sector and making workers available, the slack weighting factors are adjusted to preferentially shift power to the phone and subway systems at the expense of residential and commercial customers. (The remaining area of outage would be of about 39 MW. This amount of demand corresponds to the area highlighted). This last restoration plan (case 4 and shown in figure 7) utilizes 53 street sections and covers over three and a half miles. Con Ed would make the final decisions on which areas remain without service with the advice and consent of OEM. Alternatives could include the use of temporary generators or running shunts from a third East Side substation.

V. Summary and Significance

A. Summary

Models can provide powerful means of understanding [57], monitoring and controlling large-scale infrastructure systems [58]. The need for powerful but parsimonious models is particularly acute as infrastructures increase in complexity, such as when infrastructures are interdependent. The particular focus of this work is on developing techniques that can be used to respond to and restore from events that have the capability of impacting interdependent infrastructure systems. The approach taken is to model salient elements of interdependent critical infrastructure systems and to provide decision makers with means of manipulating this model for purposes of response and restoration of service, i.e. a decision support system.

Definitions of various types of infrastructure interdependencies were developed and incorporated into a mathematical representation of interdependent infrastructure systems. This representation permits the development and use of algorithms for identifying solutions to problems associated with disruptions to interdependent critical infrastructures. The models allow representation of infrastructures under conditions of normal operations, post-disruption impact assessment and finally, restoration.

B. Opportunities for Future Work

1) Decision Support System: These models are designed to be imbedded in a decision support system that will employ a database management system for storing data and information on response and restoration resources and have as the human-machine interface, a geographical information system. In its current form, the modeler must change the AMPL code in order to add a constraint. It is envisioned that these operations could be accomplished by additional features, such as drop down menus, which would make the model easier to use by decision makers. Additionally, it is envisioned that this decision support system will have the capability of aiding system designers in increasing the resilience of their systems and increasing their awareness of the effect interdependency plays in the design and operation of these complex systems.

2) Time-Expanded Networks: An approach for time-expanded networks has been conceptualized but must be fully developed. The time-expanded networks will be advanced applications of scheduling problems. Each repair activity, whether it be installation of a temporary component or repair of existing components, is a task which will require resources. Three scenarios for time-expanded networks are foreseen. The existing model is sufficient for the first scenario where all damage and resources are known. It also assumes that all the work proposed can be completed within some acceptable time frame and sufficient resources exist to meet the restoration requirements. The next scenario of the time expanded networks will be based on a fixed set of damage occurring at time t=0, and a set of available resources which vary over time. The time interval t will represent the time needed to complete the shortest task. The objective function will be computed at the end of each time period and the sum of the objective functions will be minimized over all the time periods. It is expected that tasks will be scheduled in the order that has the greatest impact on reducing unmet demand within the resource constraints.

The third case of the time-expanded network will deal with damage which occurs at multiple points in time across the period of analysis. An example of this is a primary shock and aftershocks from an earthquake. When another damage-causing event (such as an aftershock of an earthquake) occurs say at time *t*, it is envisioned that the model would report the set of tasks that were in progress at *t*. The operator will input the new set of damage based on the aftershock and a new set of restoration activities would be proposed. The model, using initial damage reports, would then schedule this new set of tasks along with those tasks partially completed or not yet started.

3) Algorithmic Choices: Solution procedures will be able to take advantage of the structure of the network formulation. Our models have a number of network constraints, which can be exploited to speed up solution both of the linear programming relaxations (through the use of specialized linear programming algorithms, based on either the simplex method [50] or on interior point methods [59]) and of the integer programming problem (through exploitation of the total unimodularity property of network constraint matrices). Algorithms available for solving the integer programming problem include branch-and-cut (see [60, 61] for recent surveys.) and branch-and-price [62]These approaches use constraint and/or column generation; for larger problems, interior point methods for solving the linear programming relaxations should be considered [63, 64].

C. Conclusion

The anticipated results of the research will improve society's ability to withstand the impact of and respond to events that can disrupt the provision of services that are required for the health, safety and economic well being of the citizenry. Managers of critical infrastructures and emergency response officials will be able to model different event scenarios and assess their impact on the services provided by critical infrastructure systems. With this knowledge, mitigation and preparedness strategies can be formulated and evaluated for their ability to prevent an emergency from escalating into a disaster and, if a disaster does occur, ensure a rapid restoration of critical services.

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APPENDIX
GLOSSARY OF SYMBOLS

i, h	Infrastructure systems in the set *I*
I	The collection of all infrastructure systems
b_j^i	The supply or demand at node *j* in infrastructure *i*
V^i	The complete set of nodes in infrastructure *i*
$V^{i,+}$	The set of supply nodes in *i*.
$V^{i,=}$	The set of transshipment nodes in *i*.
$V^{i,-}$	The set of demand nodes in *i*.
$V^{i,t}$	The complete set of nodes in infrastructure *i* at time *t*
e	An arc in infrastructure *i*
E^i	The complete set of arcs in infrastructure *i*
x_e^i	The flow on arc *e* in infrastructure *i*
c_e^i	The cost associated with flow along arc *e* in infrastructure *i*
u_e^i	The capacity of arc *e* in infrastructure *i*
$\delta^+(l)$	The set of arcs that enter node *l*
$\delta^-(l)$	The set of arcs that leave node *l*
$\delta(l)$	The set of arcs incident to node *l*
w_j^i	The capacity of node *j* in infrastructure *i*
s_j^i	The slack associated with node *j* in infrastructure *i*
k_j^i	Weighting factor for node *j* in infrastructure *i*
D^i	The set of all nodes in *i* upon which any other infrastructure nodes depend
$D(i,h)$	The set of nodes in *i* that some other infrastructure h depend upon
C^{i_1}	The set of all nodes in i_1 which depend on any other infrastructure nodes
$C(h,i)$	The set of nodes in h that depend on some other infrastructure *i,*
$F(i,h)$	The set of ordered pairs (*j,l*) associated with node *j* in *D*(*i,h*) and node *l* in *C*(*h,i*)
$y_{h,l}^{i,j}$	The connector between node *j* in infrastructure *i* (where it is a demand node) and node *l* in infrastructure *h*

r^i_{e,c_n}	Variable used in exclusive-or interdependency to indicate if arc *e* is carrying commodity *n*.
q^i_e	Fixed charge associated with temporary arc *e*
z^i_e	Variable indicating whether or not a shunt *e* is to be in infrastructure *i*

Figure 1 Sample of database tables and attributes

Figure 2. Overview of Manhattan showing area affected by outages.

Figure 3. Close up of affected area showing key locations described in the scenario

Figure 4 Restoration plan for case 1 showing locations of power and phone shunts

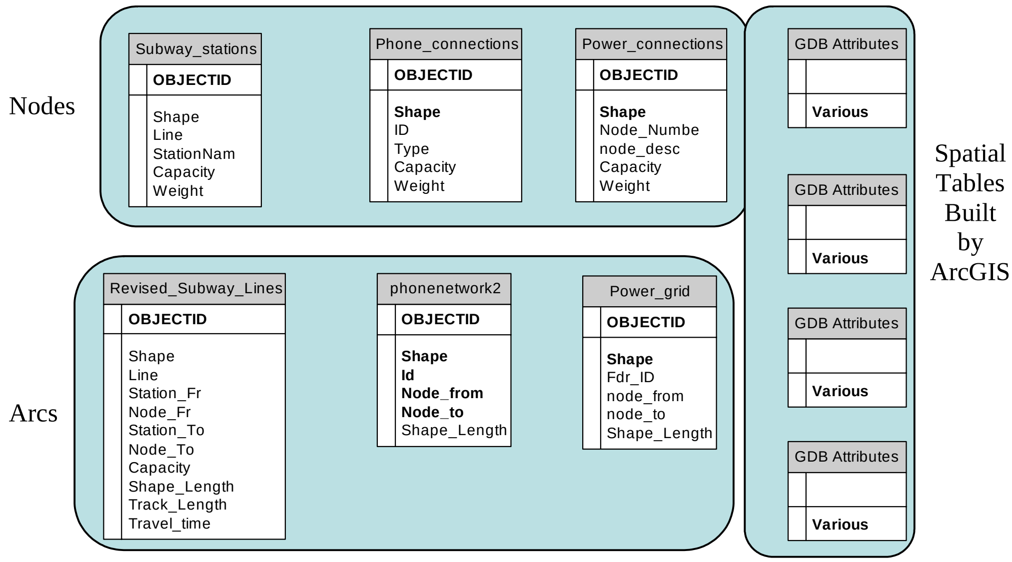
Figure 5 Location of power and phone shunts for case 2

Figure 6 Restoration plan for case 3

Figure 7 Final restoration plan showing area where power outage still exists

Footnotes

"Manuscript received……." The authors are Research Associate, Rensselaer Polytechnic Institute, Troy, NY; Professor of Mathematics, Rensselaer Polytechnic Institute; and Professor, Department of Decision Sciences and Engineering Systems, Rensselaer Polytechnic Institute, Troy, NY. This research has been supported by NSF grants CMS 0139306, Impact of the World Trade Center Attack on Critical Infrastructure Interdependencies; DMII 0228402, Disruptions in Interdependent Infrastructures, A Network Flows Approach and CMS 0301661, Decision Technologies for Managing Critical Infrastructure Interdependencies.


Nodes
Subway_stations
OBJECTID
Shape
Line
StationNam
Capacity
Weight
Phone_connections
OBJECTID
Shape
ID
Type
Capacity
Weight
Power_connections
OBJECTID
Shape
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Capacity
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Arcs
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OBJECTID
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Node_To
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Id
Node_from
Node_to
Shape_Length
Power_grid
OBJECTID
Shape
Fdr_ID
node_from
node_to
Shape_Length
GDB Attributes
Various
GDB Attributes
Various
GDB Attributes
Various
GDB Attributes
Various
Spatial Tables Built by ArcGIS

Figure 1

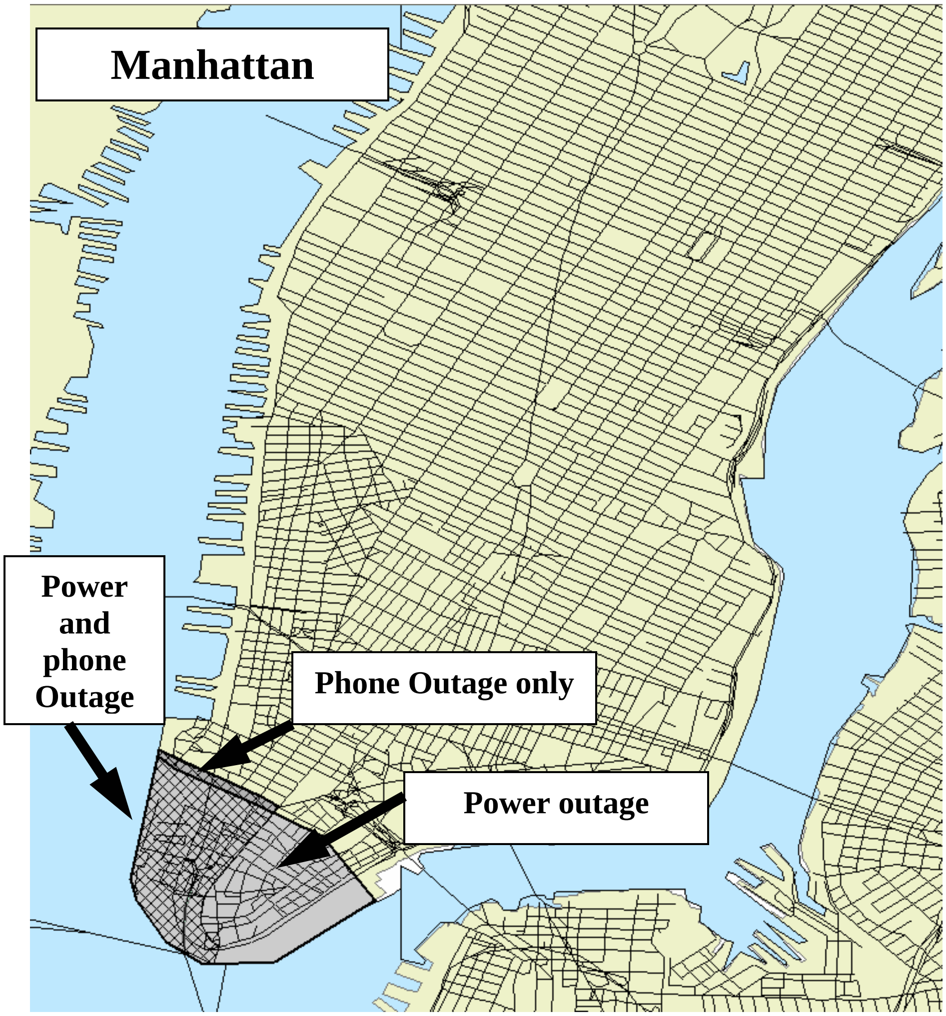
Manhattan
Power and phone Outage
Phone Outage only
Power outage

Figure 2.

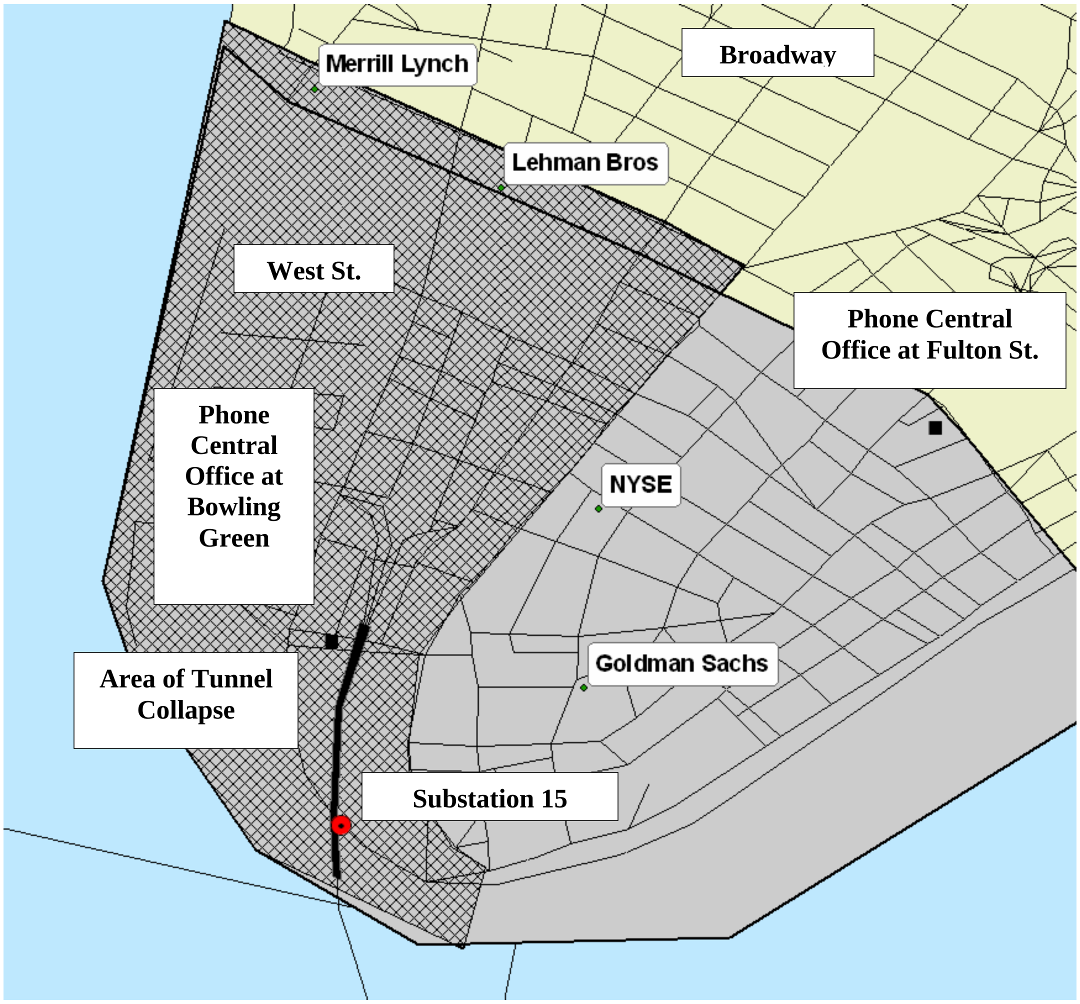
Merrill Lynch
Broadway
Lehman Bros
West St.
Phone Central Office at Fulton St.
Phone Central Office at Bowling Green
NYSE
Goldman Sachs
Area of Tunnel Collapse
Substation 15

Figure 3

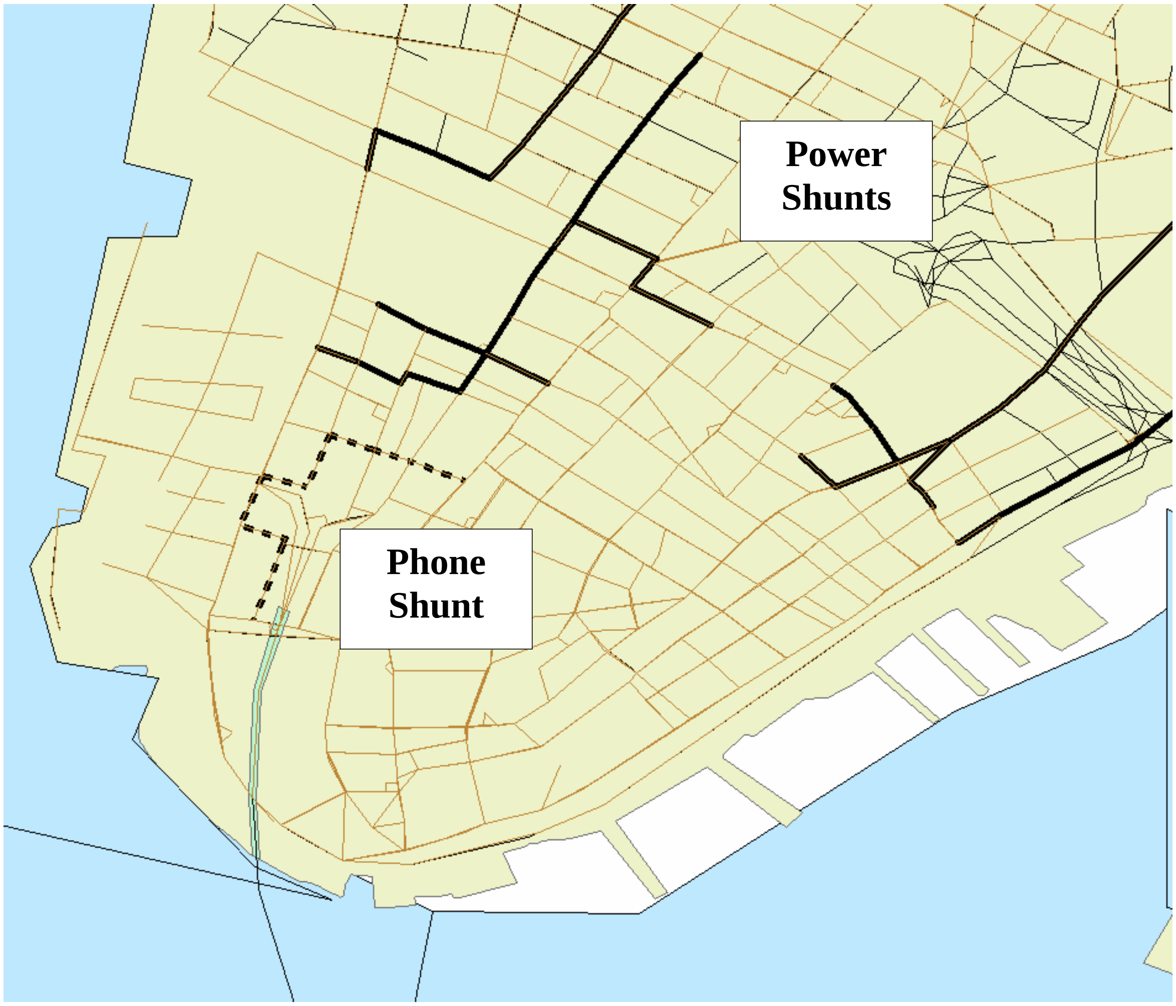

Figure 4

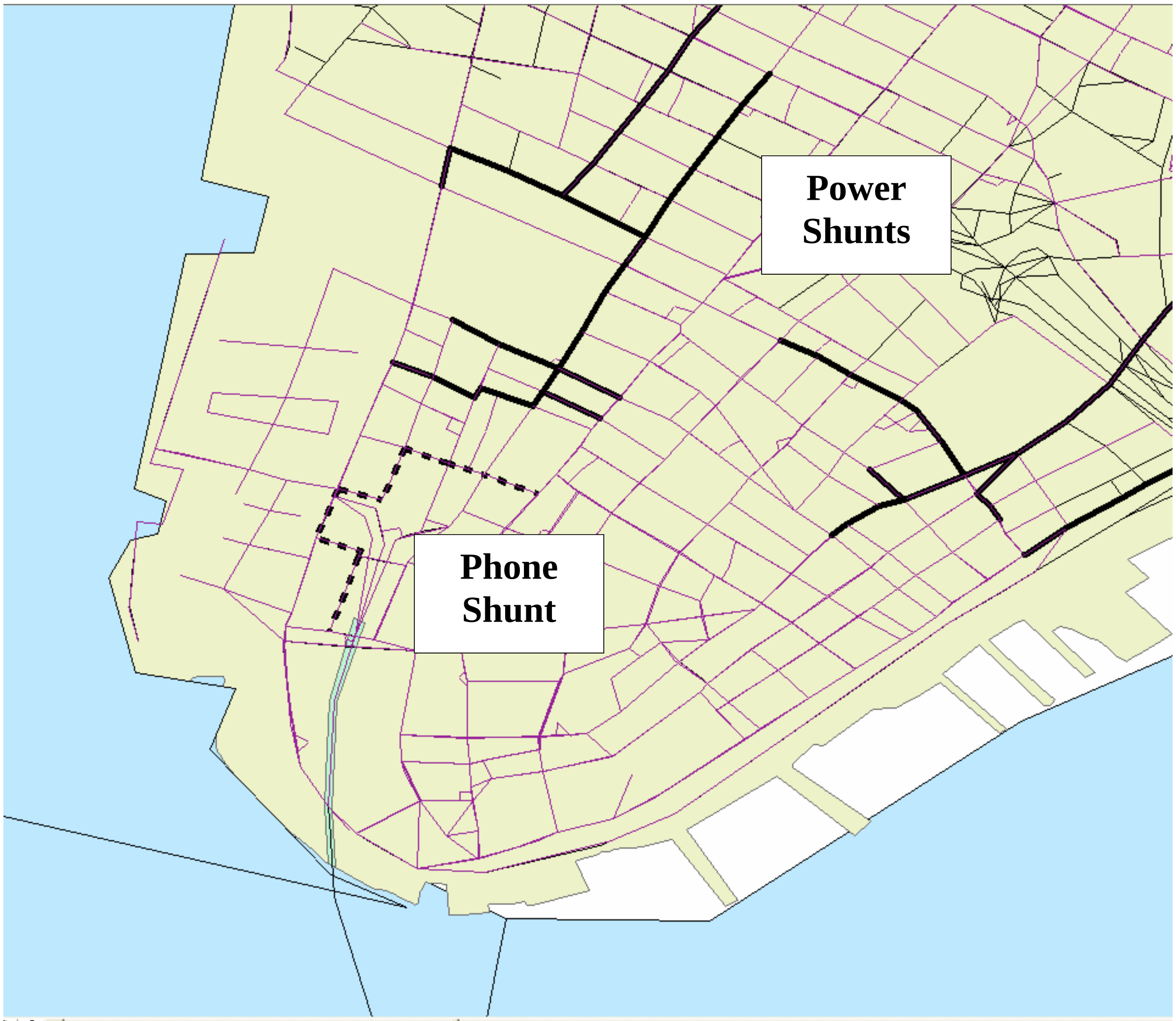

Figure 5

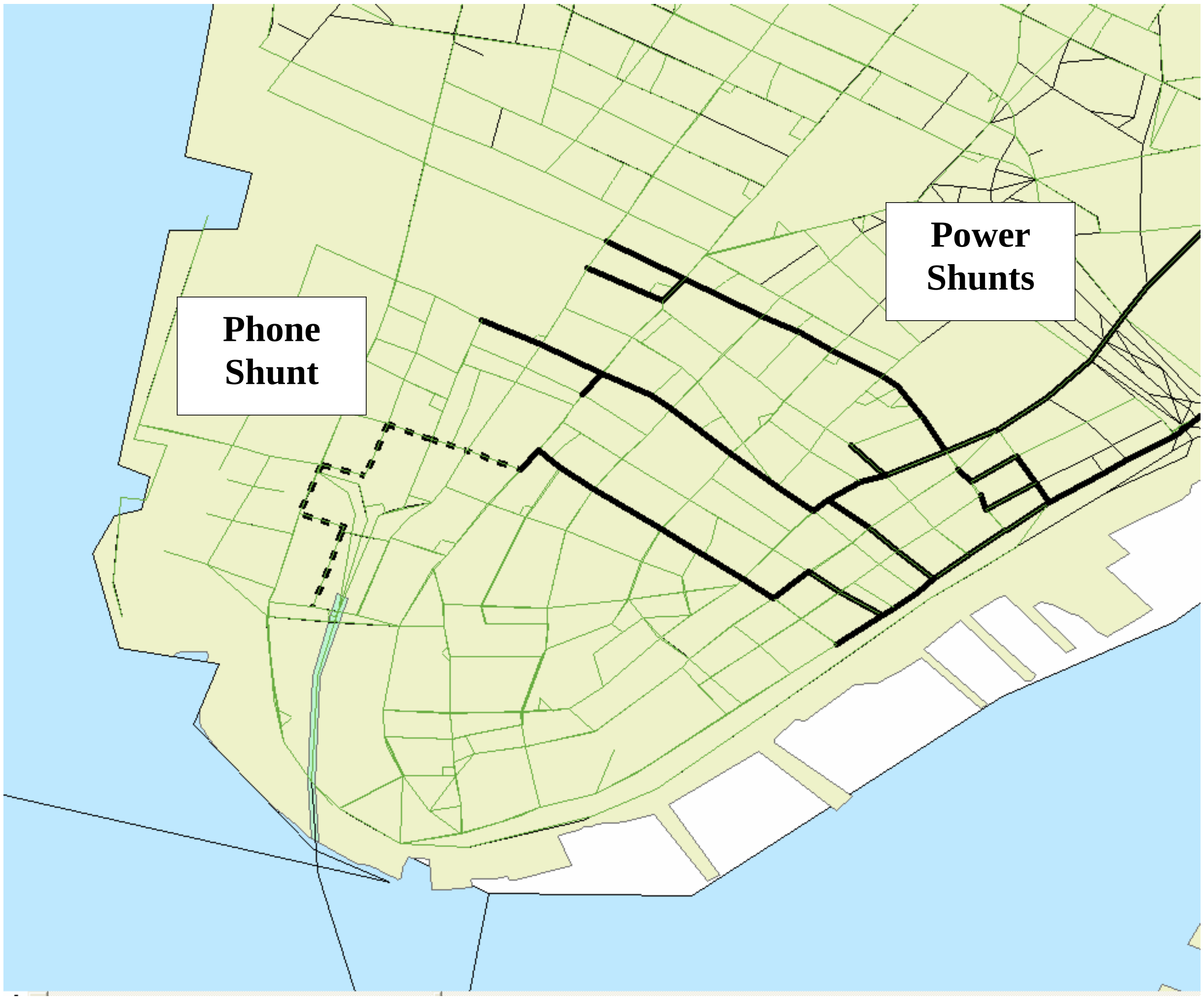

Figure 6

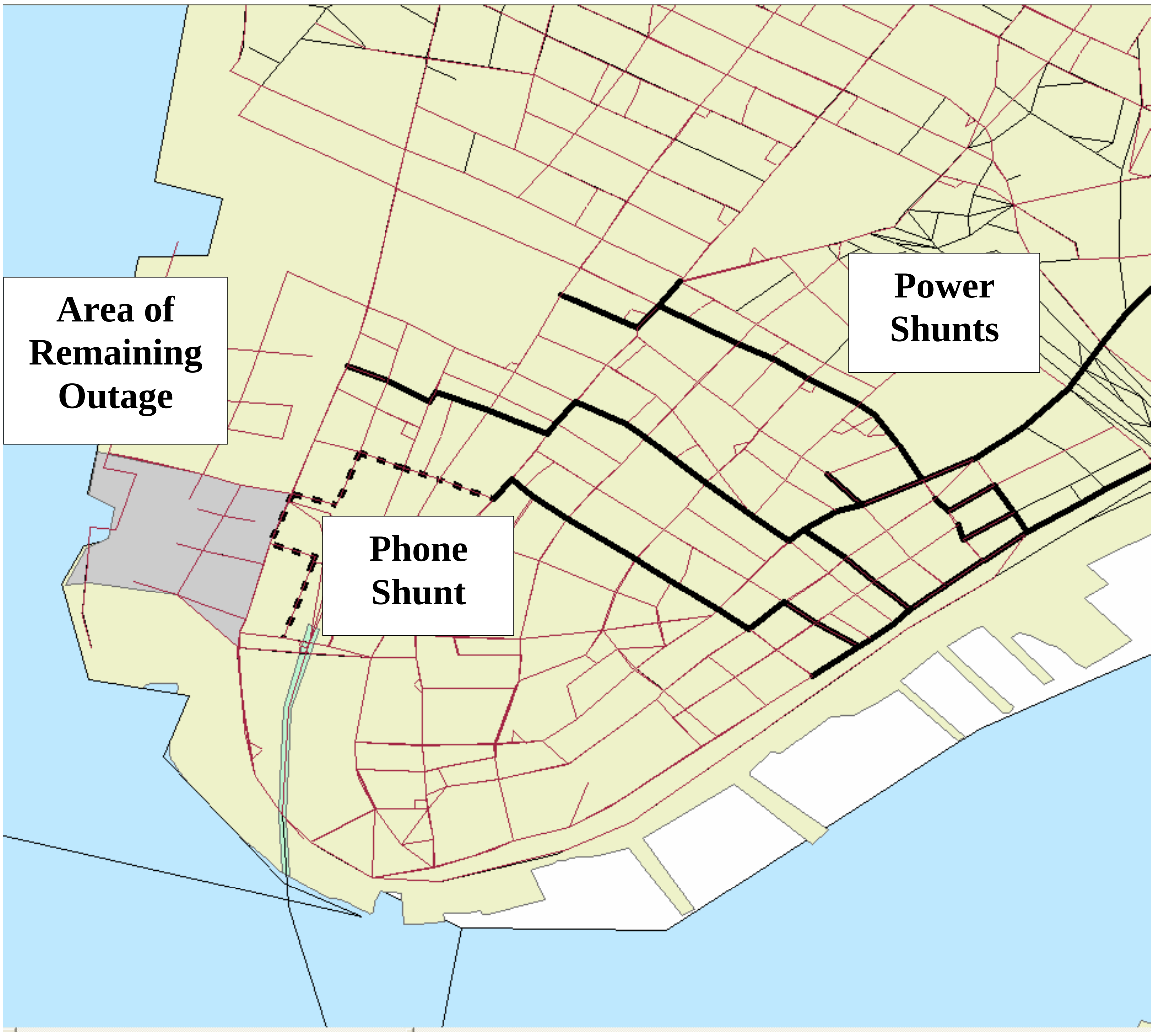

Figure 7