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Re: Manuscript AOR02-10 (2nd Round)
Rebalancing an investment portfolio in the presence of transaction costs

Dear John,

I enclose four referee reports on the revised paper mentioned above that you had submitted to the *Annals of Operations Research* (volume on “Financial Modeling”). Until very recently only one of the initial referees had sent me comments on the revised paper. I received a second response just yesterday; the other referees have yet to indicate whether they would review the revised paper despite a couple of inquiries. Hence, I was forced to seek feedback from two additional referees.

Based on the enclosed evaluations, the paper cannot be accepted for publication. The main point of contention by three of the four referees concerns the economic rationality for rescaling the variance of the portfolio value. The treatment of the fractional quadratic program that results from this rescaling is the major point of discussion in the paper; as assessed by the referees this is not a significant technical development. Obviously, the paper could be reconsidered only if the use of the rescaled objective function can be backed by proper economic arguments. The referees also comment that the empirical tests are scant and do not generate important insights for modeling or practical purposes.

Obviously I must comply with the recommendations of the referees. If you think that the proposed model can be convincingly supported, and that the study can be further strengthened with additional empirical tests (perhaps on a realistic case study) then please let me know. I will then see if I can arrange re-evaluation of another revision within the timeframe left for the completion of the volume.

Kind regards,

Hercules Vladimirov
Editor, Annals of OR (volume on “Financial Modeling”)

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Referee's Report
Rebalancing an Investment Portfolio in the Presence of
Transaction Costs (Revision)
by J. E. Mitchell and S. Braun

1 General Comments

I am generally satisfied with the revised and much shortened version of this paper. Most of the “objectionable” material is gone and what is left is accurate and moderately interesting. I have noted just a couple of corrections below. The only other suggestion I have is about the computational results section. The commentary is weak in this section, going little beyond saying that optimal solutions change when one considers transaction costs. In my original review I had suggested that some comments/comparisons about the magnitude of transaction costs one may incur by ignoring vs. incorporating transaction costs into the optimization model would be useful. I will reiterate this suggestion.

2 Itemized Comments

1. p.4: “Analytically, notice that ...”—bad sentence.
2. p.8: Equation (17) appears to be missing a $\rho\hat{y}$ term inside the parenthesis.

Referee Report on Revised Paper AOR02-10

Rebalancing an investment portfolio in the presence of transaction costs

The revised paper has been shortened, the exposition has been improved and the list of citations has been streamlined. These are indeed improvements, but of secondary importance.

After careful consideration I am led to a more critical evaluation of this paper, compared to the first round. The reason is that I do not see what material scientific contribution this paper has to offer.

In my view, the paper does not develop any new theory on portfolio management, it does not develop a novel solution (algorithmic) procedure, and it does not offer any significant new modeling insights. The empirical results of the paper - confined to a single figure - simply demonstrate the obvious: that increasing values of transaction costs affect the mean-variance efficient frontier, and the impact is larger at the end of the spectrum that corresponds to riskier portfolios (i.e., for higher target returns). The authors employ unrealistically large values of transaction costs in their computational tests in order to illustrate this intuitive result. The observation, on page 9, that the inclusion of transaction costs leads to efficient portfolios that have substantially different compositions compared to the efficient portfolios without the transaction costs is also intuitively obvious; this observation has been pointed out in previous studies.

The portfolio management problem examined in this paper lies in the mean-variance framework. Linear (proportional) transaction costs in portfolio rebalancing problems have been applied in numerous previous studies. Linear costs do not alter the structure of the optimization model and do not cause any major complications. The use of concave transaction cost functions, or fixed + linear transaction costs, or the determination of transaction lot sizes that lead to integer programming models are more interesting and challenging.

Much of the paper is devoted to the formulation of a fractional QP and a procedure for solving this model, which - as the authors state on page 5 - is a straightforward extension of a technique by Charnes and Cooper from the early 1960s.

The new proposition of this paper - which is the source of the fractional QP - is to rescale the variance of the portfolio's end-of-period return (i.e., to divide the variance by the initial value of the portfolio after the transaction costs are deducted). Yet, the investor is concerned with the total initial cost of her portfolio (including transaction costs) and not the net value of the portfolio holdings after the transactions. If a convincing economic justification could be provided for the use of the suggested objective function (rescaled variance) then I would be inclined to accept the paper. But in the absence of any such justification the whole exercise in this paper is pointless.

In my view, rescaling the variance of the portfolio's final return as suggested by the authors contravenes conventional portfolio theory and practice. What is important in portfolio management models is the distribution of portfolio return (or value) at the end of the holding period, and the associated risk-return tradeoff. That is, the tradeoff between expected portfolio return and some risk measure (e.g., variance in the case of a mean-variance setting, or shortfall below a threshold, or a critical fractile, etc.), or gauging the certainty equivalent value when an alternative utility function is employed. Rescaling the variance by the net initial portfolio value as suggested here distorts the portfolio's risk measure. Of course I am willing to be convinced, if the authors can defend their suggested approach on the basis of proper economic arguments.

Referee Report for “Rebalancing an Investment Portfolio in the Presence of Transaction Costs”

This paper considers a mean-variance one-period model with proportional transaction costs. Basing on the comments listed below, I have to conclude that this paper lacks economic content and has very marginal technical contribution, if any. Thus, it does not meet the standard of *Annals of Operations Research*.

My main comments are as follows:

1. The rescaling of the variance by the portfolio value does not make sense to me. I understand that the authors intend to adjust for the fact that the portfolio value after transaction costs is less than \$1. However, an investor with \$1 to start with cares about the expected return after the transaction costs and the variance of the return after the transaction costs. The rescaling thus seems incorrect and distorts the efficient frontier. All the main results in the paper are for the rescaled variance minimization and thus distorted.

2. Even granting the rescaled objective, I do not see any significant contribution over the existing literature such as Yoshimoto (1996), Konno and Wijayanayake (2001). Neither the modelling nor the mathematical programming technique is new.

3. This paper finds that “Theoretically, one way to reduce the measure of risk is to buy and sell the same security,” This seems to suggest that the model in the paper is not well specified because no investor would buy and sell the same stock simultaneously which incurs higher transaction costs and reduces return (like burning money).

4. In addition to changing the objective function, I would also suggest the authors to conduct an extensive comparative statics on the effect of the transaction costs, portfolio constraints and initial positions on the efficient frontier.

Rebalancing an Investment Portfolio in the Presence of Transaction Costs
John E. Mitchell and Stephen Braun

The main thing I got out of reading this paper was to observe that the problem of optimizing (the square of the inverse of) the ‘ex-ante’ Sharpe ratio can be written as a matrix-fractional program. In the authors’ notation, this is

$$f(x) := \frac{\frac{1}{2}x^T Q x}{(\rho y + \mu^T x)^2}.$$

(See also Golfarb and Iyengar 2003, and the recently submitted manuscript Tutuncu and Pinar 2004.) There is no need for the Sharpe ratio in order to trace the efficient frontier (the frontier does not change for any of these objectives), but given the widespread use of the Sharpe ratio as a benchmark in the financial industry this objective might be useful in the automation of finding a desirable point on the frontier.

I was not aware of the method the authors describe for handling matrix-fractional programs (from Charles and Cooper 1962), and read it with interest. I’d be curious to see it compared to other solution strategies (such as embedding in other convex cones, see for example Lobo et al. 1998), especially regarding the ability to handle nonlinear constraints.

It is well known that linear costs preserve linear structure in the program, and that handling such costs is computationally ‘easy’.

The suggestion of rescaling the portfolio variance based on value after transaction costs is nonsensical, and runs counter to the Markowitz framework. The risk relative to value after transaction costs is irrelevant. What we care about is the overall risk profile (that is, the probability distribution of the end-of-period portfolio value) associated with a particular set of transaction decisions. It is true that the cash lost to transaction costs will lead to a slightly lower variance (since we then have less to invest). But this is part of how the distribution of end-of-period value changes as a consequence of our decisions, and the effect should not be corrected out. While this reduction of the variance is in some sense a ‘positive’ for transaction costs, it will be far outweighed by the reduction in expected value.