1. Define a matrix Y which is $n \times k$:

Given a partition, let

$$Y_{ib} = \begin{cases} 1 & \text{if } i \text{ in set } b \\ 0 & \text{otherwise.} \end{cases}$$

Let $X = YY^T$.

Then X is feasible in the SDP, and $\frac{1}{2}\sum_i \sum_j w_{ij} X_{ij}$ is equal to the value of the partition.

2. Greedy

Two possible choices:

(A) Choose vertex that has no neighbour already selected and that has maximum weight

(B) Choose vertex that has no neighbour already selected and that maximizes the ratio

$$\frac{\text{weight of vertex}}{\text{weight of all neighbours that could still be chosen}}$$

3. GRASP: Many choices possible.

Eg: randomly pick one of the best 5 choices

Eg: randomly pick a choice within 20% of best greedy choice

4. Eg: valid solutions are packings, so never have any violated constraints.

Neighbours: any packing that can be obtained by deleting at most one vertex from the current packing

Tabu criteria: an added vertex cannot be dropped for 2 moves.
a dropped vertex cannot be added for 2 moves.

5. (a) Let $\bar{x}$ solve the LP relaxation.

Let $\tilde{x} = \lceil \bar{x} \rceil$.

Then $\tilde{x}$ is feasible in (IP)

(b) Replace a_{ij} by $\bar{a}_{ij} = \begin{cases} 1 & \text{if } a_{ij} > 0 \\ 0 & \text{otherwise.} \end{cases}$

Replace b_j by 1.

Then we have a set covering problem equivalent to (IP):

$$\begin{aligned} \min\ & c^T x \\ \text{s.t. } & \bar{A} x \geq e \\ & x \text{ binary.} \end{aligned}$$

Solving the LP relaxation of this and rounding up gives a soln within a factor of p of optimal.