COMPSCI 289A Homework 2 **Mitchell Negus**
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Problem 1

a.)

Say $P(T)$ is the probability of hitting the target, and $P(W)$ is the probability of it being windy. We are given that $P(T|W) = 0.4$ and $P(T|W^C) = 0.7$, as well as that $P(W) = 0.3$. We note that conditional probability theory states that in general, $P(A|B) = \frac{P(A\cap B)}{P(B)}$.

(i) on a given shot there is a gust of wind and she hits her target; $P(T \cap W)$

$$P(T|W) = \frac{P(T\cap W)}{P(W)}$$
$$P(T\cap W) = P(T|W)\cdot P(W)$$
$$P(T\cap W) = (0.4)(0.3)$$
$$\boxed{P(T\cap W) = 0.12}$$

(ii) she hits the target with her first shot; $P(T)$

$$P(T) = P(T\cap W) + P(T\cap W^C)$$
$$P(T) = P(T|W)\cdot P(W) + P(T|W^C)\cdot P(W^C),\ \ P(W^C) = 1 - P(W) = 0.7$$
$$P(T) = (0.4)(0.3) + (0.7)(0.7)$$
$$\boxed{P(T) = 0.61}$$

(iii) she hits the target exactly once in two shots; $P(T)\cdot P(T^C) + P(T^C)\cdot P(T)$

$$P(T)\cdot P(T^C) + P(T^C)\cdot P(T) = 2\cdot P(T)\cdot P(T^C),\ \ P(T^C) = 1 - P(T) = 0.39$$
$$P(T)\cdot P(T^C) + P(T^C)\cdot P(T) = 2(0.61)(0.39)$$
$$\boxed{P(T)\cdot P(T^C) + P(T^C)\cdot P(T) = 0.4758}$$

(iv) there was no gust of wind on an occasion when she missed; $\frac{P(T^C\cap W^C)}{P(T^C)}$

$$\frac{P(T^C\cap W^C}{P(T^C} = \frac{P(T^C|W^C)P(W^C)}{P(T^C)},\ \ P(T^C|W^C) = 1 - P(T|W^C) = 0.3$$
$$\frac{P(T^C\cap W^C)}{P(T^C)} = \frac{(0.3)(0.7)}{(0.39)}$$
$$\boxed{\frac{P(T^C\cap W^C)}{P(T^C)} = 0.538}$$

b.)

We are given

$$P(A|B,C) = P(A|B \cap C) > P(A|B)$$

and from the properties of conditional probability we find

$$P(A|B \cap C) = \frac{P(A \cap B \cap C)}{P(B \cap C)} > \frac{P(A \cap B)}{P(B)}. \tag{1}$$

We are looking to show

$$P(A|B,C^C) = P(A|B \cap C^C) < P(A|B).$$

or equivalently

$$P(A|B \cap C^C) = \frac{P(A \cap B \cap C^C)}{P(B \cap C^C)} < \frac{P(A \cap B)}{P(B)}. \tag{2}$$

Returning to equation 1, we find through algebraic manipulation that

$$\begin{aligned}
P(A \cap B \cap C) &> \frac{P(A \cap B)P(B \cap C)}{P(B)} \\
-P(A \cap B \cap C) &< -\frac{P(A \cap B)P(B \cap C)}{P(B)} \\
P(A \cap B) - P(A \cap B \cap C) &< P(A \cap B) - \frac{P(A \cap B)P(B \cap C)}{P(B)} \\
P(A \cap B) - P(A \cap B \cap C) &< \frac{P(A \cap B)}{P(B)}\left(P(B) - P(B \cap C)\right) \\
\frac{P(A \cap B) - P(A \cap B \cap C)}{P(B) - P(B \cap C)} &< \frac{P(A \cap B)}{P(B)}. \qquad (3)
\end{aligned}$$

Since in general,

$$P(X \cap Y) + P(X \cap Y^C) = P(X),$$

(also expressed as $P(X \cap Y^C) = P(X) - P(X \cap Y)$)

we can express equation 3 as

$$\frac{P(A \cap B \cap C^C)}{P(B \cap C^C)} < \frac{P(A \cap B)}{P(B)},$$

equivalent to equation 2. ■

Problem 2

a.)

Assume we are considering a positive semidefinite matrix $A \in \mathbb{R}^{n \times n}$ so that $x^\top A x \geq 0$. By definition, $A \succeq 0$ (i).