CS289A_HW03_Prob6c

February 28, 2017

Load modules to be used in the execution of the problem.

In [1]:
```
%load_ext autoreload
```

In [2]:
```
%autoreload 2
```

In [3]:
```
import math
import HW03_utils as ut
import numpy as np
from matplotlib import pyplot as plt
```

In [4]:
```
def normalize_images(image_vectors):
# Function to normalize pixel contrast of images

        magnitudes = np.linalg.norm(image_vectors,axis=1)
        normalized_ims = image_vectors/magnitudes[:,None]
        return normalized_ims
```

In [5]:
```
def get_class_bounds(classid,labels):
# Function to extract index bounds of the specified class from the dataset

    for i in range(len(labels)):
        if labels[i] == classid:
            startindex = i
            break
    stopindex = len(labels)
    for i in range(i,len(labels)):
        if labels[i] != classid:
            stopindex = i
            break

    return startindex,stopindex
```

In [6]:
```
def get_class_from_data(classid,data,labels):
# Find the start (inclusive) and end (exclusive) of a class within the data

    startindex,stopindex = get_class_bounds(classid,labels)
```

```
        # Separate the specified class
        class_data = data[startindex:stopindex]

        return class_data
```

```
In [7]: def mean_of_class(classid,data,labels):
    # Calculate the mean value when the class is fit to a normal distribution

        class_data = get_class_from_data(classid,data,labels)
        # Calculate the mean of the class data
        class_mu = np.mean(class_data,axis=0)

        return class_mu
```

```
In [8]: def cov_of_class(classid,data,labels):
    # Calcualte the covariance matrix when the class is fit to a normal distrib

        class_data = get_class_from_data(classid,data,labels)
        # Calculate the covariance matrix from the class data
        class_Sigma = np.cov(class_data,rowvar=False)

        return class_Sigma
```

```
In [9]: def calc_SigmaHat(data,labels,muCs):
    # Function to calculate the average covariance matrix for the distribution

        SigmaHat = np.zeros((len(data[0]),len(data[0])))
        for i in range(len(data)):
            Xi_minus_muC = data[i] - muCs[labels[i]]
            SigmaHat += np.outer(Xi_minus_muC,Xi_minus_muC)

        SigmaHat = SigmaHat/len(labels)

        return SigmaHat
```

```
In [10]: def zero_rows(sym_matrix):
     # Take a symmetric matrix and find rows/columns that are empty

         zero_rows = []
         for i in range(len(sym_matrix)):
             if not np.any(sym_matrix[i]):
                 zero_rows.append(i)

         return zero_rows
```

```
In [11]: def makeInvertible(sym_matrix):
     # Take a symmetric non-invertible matrix, and eliminate rows/columns to ma

         ZR = zero_rows(sym_matrix)
```

```
        newlen = len(sym_matrix)-len(ZR)
        invmatrix = np.empty((newlen,newlen))
        I = 0
        for i in range(len(sym_matrix)):
            if i in ZR:
                continue
            J = 0
            for j in range(len(sym_matrix)):
                if j in ZR:
                    continue
                invmatrix[I,J] = sym_matrix[i,j]
                J += 1
            I += 1

        return invmatrix
```

```
In [12]: def removeZeroVariance(cov_matrix,valdata):
        # Remove variables with zero variance in the covariance matrix from the va
        #      -valdata is a nxd array with n rows of samples and d-variables per
        #      -Cov_matrix is a dxd matrix giving the covariances of the d-variabl

            ZR = zero_rows(cov_matrix)
            # Create a new array with validation data corresponding to variables i
            NZV_data = np.empty((len(valdata),len(valdata[0])-len(ZR)))
            columnI = 0
            for columni in range(len(valdata[0])):
                if columni in ZR:
                    continue
                NZV_data[:,columnI] = valdata[:,columni]
                columnI += 1

            return NZV_data
```

```
In [13]: def MuAndPi(train_labels,muCs,cov_matrix,ZR):
            mu_i = []
            pi_i = []
            for i in range(10):
                # Calculate the mean (without zero variance variables)
                mu_i.append(np.zeros(np.shape(cov_matrix)[0]))
                J = 0
                for j in range(np.shape(muCs[i])[0]):
                    if j in ZR:
                        continue
                    mu_i[i][J] = muCs[i][j]
                    J += 1

                # Calculate the prior probability
                startindex,stopindex = get_class_bounds(i,train_labels)
```

```
            nPoints = stopindex-startindex
            pi_i.append(nPoints/np.shape(train_labels)[0])

        return mu_i,pi_i
```

In [14]:
```
def LDF_solve(X,mu_C,Sigma,pi_C=0.1):
    # Function to solve the linear discriminant function for class C (will con

        LDFs_C = np.zeros(len(X))
        muCinvSigma = np.dot(mu_C,np.linalg.pinv(Sigma))
        muCinvSigmamuC = np.dot(muCinvSigma,mu_C)
        logpiC = math.log(pi_C)
        for i in range(len(X)):
            x = X[i]
            LDFs_C[i] = np.dot(muCinvSigma,x) - 0.5*muCinvSigmamuC + logpiC

        return LDFs_C
```

In [15]:
```
def maximize_LDFs(valdata,mu_i,cov_matrix,pi_i):
        lin_disc_fns = np.empty((len(valdata),10))
        for i in range(10):
            mu_C = mu_i[i]
            pi_C = pi_i[i]
            lin_disc_fns[:,i] = LDF_solve(valdata,mu_C,cov_matrix,pi_C)
        max_LDF_indices = np.empty(len(valdata))
        for i in range(len(valdata)):
            max_LDF_indices[i] = np.argmax(lin_disc_fns[i])

        return max_LDF_indices
```

In [16]:
```
CS_DIR = r"/Users/mitch/Documents/Cal/2 - 2017 Spring/COMPSCI 289A - Intro
```

In [17]:
```
# Load MNIST data
data_array = ut.loaddata("hw3_mnist_dist/hw3_mnist_dist/train.mat",CS_DIR+
```

In [18]:
```
# Shuffle data and set aside validation set
np.random.shuffle(data_array)

trainarray = data_array[:-10000]
valarray = data_array[-10000:]
```

In [19]:
```
def main(traindata,trainlabels,valdata,vallabels):
    # Main block of code

        # Create a list of the means for each class
        muCs = np.empty((10,len(traindata[0])))
        for i in range(10):
            muCs[i] = mean_of_class(i,traindata,trainlabels)
```

```
        SigmaHat = calc_SigmaHat(traindata,trainlabels,muCs)

        newcov = makeInvertible(SigmaHat)
        newvaldata = removeZeroVariance(SigmaHat,valdata)

        ZR = zero_rows(SigmaHat)

        mu_i,pi_i = MuAndPi(trainlabels,muCs,newcov,ZR)

        digitPicks = maximize_LDFs(newvaldata,mu_i,newcov,pi_i)
        count, total = 0,0
        for i in range(len(digitPicks)):
            if digitPicks[i] == vallabels[i]:
                count += 1
            total += 1

        # VERBOSE COMMANDS FOR WATCHING PROGRESS [OPTIONAL]
        #    if total%200 == 0:
        #        print(total,'points evaluated; current score =',count/total)

        score = count/total

        return score
```

```
In [20]: # Organize array by digit
         trainarray_byclass = trainarray[trainarray[:,-1].argsort()]
         valarray_byclass = valarray[valarray[:,-1].argsort()]

         train_data = trainarray_byclass[:,:-1]
         train_labels = trainarray_byclass[:,-1]

         val_data = valarray_byclass[:,:-1]
         val_labels = valarray_byclass[:,-1]

         normalized_traindata = normalize_images(train_data)
         normalized_valdata = normalize_images(val_data)
```

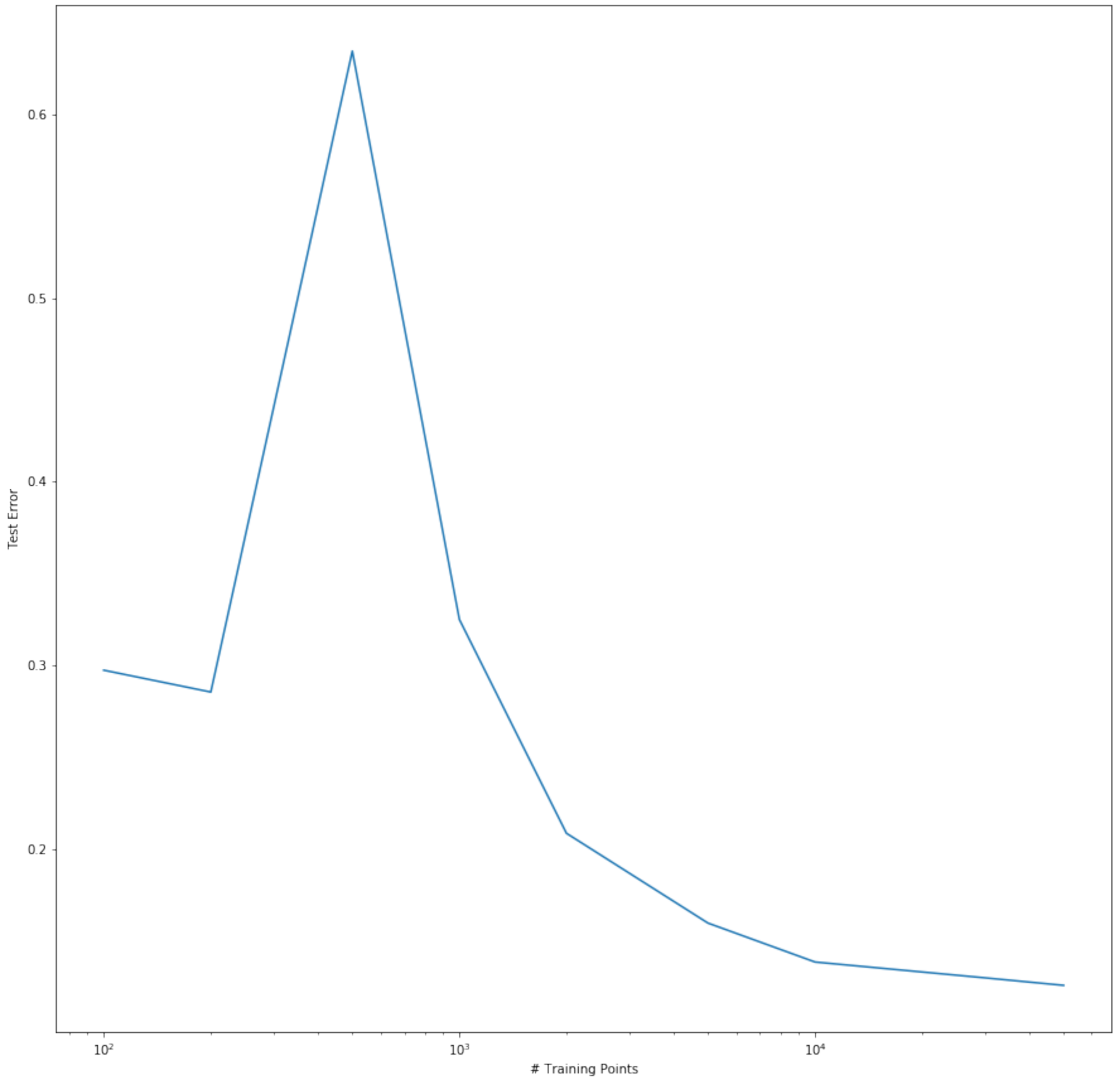
```
In [25]: samples = [100,200,500,1000,2000,5000,10000,30000,50000]
```

```
In [21]: # Train on subsets of full training data set
         scores = []
         for number in samples:
             trainarraysubset = trainarray[:number]

             # Organize array by digit
             trainarray_byclass = trainarraysubset[trainarraysubset[:,-1].argsort()
```

```python
            valarray_byclass = valarray[valarray[:,-1].argsort()]

            # Separate data and labels
            train_data = trainarray_byclass[:,:-1]
            train_labels = trainarray_byclass[:,-1]
            val_data = valarray_byclass[:,:-1]
            val_labels = valarray_byclass[:,-1]

            # Normalize training and validation data
            normalized_train_data = normalize_images(train_data)
            normalized_val_data = normalize_images(val_data)

            print(number,"training samples: ")
            score = main(normalized_train_data,train_labels,normalized_val_data,va
            scores.append(score)
            print(score)
```

```
100 training samples:
0.7026
200 training samples:
0.7145
500 training samples:
0.3655
1000 training samples:
0.675
2000 training samples:
0.7914
5000 training samples:
0.8403
10000 training samples:
0.8615
30000 training samples:
0.8701
50000 training samples:
0.8742
```

In [30]:
```python
errors = np.ones(len(scores))-np.array(scores)

fig = plt.figure(figsize=(15,15))
plt.semilogx(samples,error)
plt.xlabel("# Training Points")
plt.ylabel("Test Error")
plt.savefig("LDA_errors.jpg")
plt.show()
```




```
In [31]: print(errors)

[ 0.2974  0.2855  0.6345  0.325   0.2086  0.1597  0.1385  0.1299  0.1258]


In [ ]:
```