COMPSCI 289A Homework 4 **Mitch Negus**
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Problem 1

1.)

The gradient of the cost function $J(w)$ is given by

$$\nabla_w J(w) = \nabla_w \left(\lambda |w|_2^2 - \sum_{i=1}^{n} y_i \ln s_i + (1 - y_i) \ln(1 - s_i) \right)$$

$$\nabla_w J(w) = \lambda \nabla_w (|w|_2^2) - \nabla_w \left(\sum_{i=1}^{n} y_i \ln s_i + (1 - y_i) \ln(1 - s_i) \right)$$

Using our result from homework 2, that $\nabla_x (x^T A x) = 2Ax$, we can state that $|w|_2^2 = w^T \mathbb{1} w$ and so $\nabla_x (|x|_2^2) = 2\mathbb{1}w = 2w$. Then

$$\nabla_w J(w) = 2\lambda w - \sum_{i=1}^{n} \left(y_i \nabla_w (\ln s_i) + (1 - y_i) \nabla_w \ln(1 - s_i) \right)$$

$$\nabla_w J(w) = 2\lambda w - \sum_{i=1}^{n} \left(\frac{y_i}{s_i} \nabla_w s_i - \frac{1 - y_i}{1 - s_i} \nabla_w s_i \right)$$

We know $s_i = s(X_i \cdot w) = 1/(1 + e^{-X_i \cdot w})$, so

$$\nabla_w s_i = \nabla_w \left(\frac{1}{1 + e^{-X_i \cdot w}} \right)$$

$$\nabla_w s_i = \frac{X_i e^{-X_i \cdot w}}{(1 + e^{-X_i \cdot w})^2}$$

$$\nabla_w s_i = X_i s_i \frac{e^{-X_i \cdot w}}{1 + e^{-X_i \cdot w}}$$

$$\nabla_w s_i = X_i s_i (1 - s_i)$$

and thus

$$\nabla_w J(w) = 2\lambda w - \sum_{i=1}^{n} \left(\frac{y_i}{s_i} (X_i s_i (1 - s_i)) - \frac{1 - y_i}{1 - s_i} (X_i s_i (1 - s_i)) \right)$$

$$\nabla_w J(w) = 2\lambda w - \sum_{i=1}^{n} \left(y_i X_i (1 - s_i) - (1 - y_i) X_i s_i \right)$$

$$\nabla_w J(w) = 2\lambda w - \sum_{i=1}^{n} \left(y_i X_i - y_i X_i s_i - X_i s_i + y_i X_i s_i \right)$$

$$\nabla_w J(w) = 2\lambda w - \sum_{i=1}^{n} \left(y_i X_i - X_i s_i \right)$$

$$\nabla_w J(w) = 2\lambda w - \sum_{i=1}^{n} \left(y_i - s_i \right) X_i$$

$$\boxed{\nabla_w J(w) = 2\lambda w - X^T (y - s)}$$

2.)

The Hessian of $J(w)$ is

$$\nabla_w^2 J(w) = \nabla_w(\nabla_w J(w)) = \nabla_w(2\lambda w - X^T(y-s))$$

$$\nabla_w^2 J(w) = 2\lambda\mathbb{1} - \nabla_w\left(\sum_{i=1}^{n}(y_i - s_i)\,X_i\right)$$

$$\nabla_w^2 J(w) = 2\lambda\mathbb{1} + \sum_{i=1}^{n}\nabla_w(s_i X_i)$$

$$\nabla_w^2 J(w) = 2\lambda\mathbb{1} + \sum_{i=1}^{n}\nabla_w(s_i) X_i$$

$$\nabla_w^2 J(w) = 2\lambda\mathbb{1} + \sum_{i=1}^{n} s_i(1-s_i) X_i X_i^T$$

$$\boxed{\nabla_w^2 J(w) = 2\lambda\mathbb{1} + X^T\Omega X}, \text{ where } \Omega = \begin{bmatrix} s_1(1-s_1) & 0 & \dots & 0 \\ 0 & s_2(1-s_2) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & s_n(1-s_n) \end{bmatrix}$$

3.)

Expanding the Taylor series as discussed in lecture and considering only the first 2 terms as a quadratic approximation to the cost function at our point w,

$$J(w') = J(w) + (\nabla J(w))(w' - w) + \dots .$$

We can find a minimum for the new point, w', by taking the gradient of $J(w')$, and setting it equal to zero:

$$\nabla J(w') = \nabla J(w) + (\nabla^2 J(w))(w' - w) + O(|w' - w|^2)$$

Disregarding small terms,

$$0 = \nabla J(w) + (\nabla^2 J(w))(w' - w)$$

$$0 = \nabla J(w) + (\nabla^2 J(w))(w' - w)$$

$$-\nabla^2 J(w) w' = \nabla J(w) - \nabla^2 J(w) w$$

$$w' = w - (\nabla^2 J(w))^{-1}(\nabla J(w)).$$

Since we would like to avoid calculating the inverse of $\nabla^2 J(w))$, we can substitute e in for the second term so that w' would be given by:

$$w' = w + e, \text{ where } e \text{ is the solution to the equation } (\nabla^2 J(w))e = -\nabla J(w).$$

Then, modifying the solution for w' to be an update rule, we can incorporate our calculated gradient and Hessian to state

$$\boxed{w \leftarrow w + e, \text{ where } e \text{ is the solution to the equation } (X^T\Omega X + 2\lambda\mathbb{1})e = X^T(y-s) - 2\lambda w}.$$

.

4.)

(a)

We have defined $s = \begin{bmatrix} s_1 & s_2 & \dots & s_n \end{bmatrix}^T$ where $s_i = s(X_i \cdot w)$. In other words,

$$s = \begin{bmatrix} s(X_1 \cdot w) & s(X_2 \cdot w) & \dots & s(X_n \cdot w) \end{bmatrix}^T = s\left(\begin{bmatrix} X_1 \cdot w & X_2 \cdot w & \dots & X_n \cdot w \end{bmatrix}^T \right).$$

Furthermore, we can consolidate this a the single matrix-vector product

$$s = s(Xw).$$

We are given that

$$x = \begin{bmatrix} 0 & 3 & 1 \\ 1 & 3 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \text{ and } w = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

where we have included the fictitious dimension as the last column of X. Then,

$$s^{(0)} = s(Xw^{(0)}) = s\left(\begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix} \right) = \begin{bmatrix} 1/(1+e^{-3}) \\ 1/(1+e^{-1}) \\ 1/(1+e^{-1}) \\ 1/(1+e) \end{bmatrix}$$

(b)

$$w^{(1)} = w^{(0)} + e^{(0)}$$

$e^{(0)}$ is the solution to the equation $(X^T \Omega^{(0)} X + 2\lambda \mathbb{1}) e^{(0)} = X^T (y - s^{(0)}) - 2\lambda w^{(0)}$ which can be numerically calculated to be

$$e = \begin{bmatrix} 1.613 \\ 0.404 \\ -2.284 \end{bmatrix}$$

and so

$$w^{(1)} = \begin{bmatrix} -0.387 \\ 1.404 \\ -2.284 \end{bmatrix}$$

(c)

We can then calculate $s^{(1)}$ using our calculated value of $w^{(1)}$, using the formula

$$s^{(1)} = s(Xw^{(1)}).$$

The result is

$$s^{(1)} = \begin{bmatrix} 0.873 \\ 0.824 \\ 0.293 \\ 0.220 \end{bmatrix}.$$

(d)

Following our procedure further, now using $w^{(2)} = w^{(1)} + e^{(1)}$ and recalculating $e^{(1)}$ using $Omega^{(1)}, s^{(1)},$ and $w^{(1)}$,

$$w^{(2)} = \begin{bmatrix} -0.512 \\ 1.453 \\ -2.163 \end{bmatrix}$$

Problem 2

1.)

$$J(w) = |Xw - y|^2 + \lambda ||w||_1$$

$$J(w) = (Xw - y)^T (Xw - y) + \lambda \sum_{i=1}^{d} |w_i|$$

$$J(w) = (Xw)^T Xw - y^T Xw - (Xw)^T y + y^T y + \lambda \sum_{i=1}^{d} |w_i|$$

$$J(w) = |y|^2 - 2(Xw)^T y + w^T X^T Xw + \lambda \sum_{i=1}^{d} |w_i|$$

Because the sample data (X) has been centered and whitened, we know $X^T X = n\mathbb{1}$.

$$J(w) = |y|^2 - 2(Xw)^T y + nw^T w + \lambda \sum_{i=1}^{d} |w_i|$$

We note that $(Xw)^T = \begin{bmatrix} \sum_{i=1}^{d} X_{1i} w_i & \sum_{i=1}^{d} X_{2i} w_i & \dots & \sum_{i=1}^{d} X_{ni} w_i \end{bmatrix} = \sum_{i=1}^{d} w_i \begin{bmatrix} X_{1i} & X_{2i} & \dots & X_{ni} \end{bmatrix}$. Furthermore, $\begin{bmatrix} X_{1i} & X_{2i} & \dots & X_{ni} \end{bmatrix} = X_{*i}^T$, so

$$J(w) = |y|^2 - 2 \left(\sum_{i=1}^{d} w_i X_{*i}^T \right) y + n \sum_{i=1}^{d} w_i^2 + \lambda \sum_{i=1}^{d} |w_i|$$

$$J(w) = |y|^2 - \sum_{i=1}^{d} \left(2 w_i X_{*i}^T y + n w_i^2 + \lambda |w_i| \right)$$

2.)

We can find w_i^* by noting that

$$0 = \nabla_{w_i} J(w) \text{ for } w_i = w_i^*$$

$$0 = \nabla_{w_i} \left(|y|^2 - \sum_{i=1}^{d} \left(2 w_i X_{*i}^T y + n w_i^2 + \lambda |w_i| \right) \right)$$

$$0 = -\nabla_{w_i} \sum_{i=1}^{d} \left(2 w_i X_{*i}^T y + n w_i^2 + \lambda |w_i| \right)$$

$$0 = 2 X_{*i}^T y + 2 n w_i^* + \lambda \frac{w_i^*}{|w_i^*|}$$

In the case that $w_i^* > 0$,

$$0 = 2 X_{*i}^T y + 2 n w_i^* + \lambda$$

$$-2 n w_i^* = 2 X_{*i}^T y + \lambda$$

$$w_i^* = -\frac{\lambda}{2n} - \frac{1}{n} X_{*i}^T y$$

3.)

In the case that $w_i^* < 0$,

$$\begin{aligned} 0 &= 2X_{*i}^T y + 2nw_i^* - \lambda \\ -2nw_i^* &= 2X_{*i}^T y - \lambda \\ -2nw_i^* &= 2X_{*i}^T y - \lambda \\ w_i^* &= \frac{\lambda}{2n} - \frac{1}{n} X_{*i}^T y \end{aligned}$$

4.)

We can note that in the event that $w_i^* > 0$,

$$\begin{aligned} -\frac{\lambda}{2n} - \frac{1}{n} X_{*i}^T y &> 0 \\ -\frac{1}{n} X_{*i}^T y &> \frac{\lambda}{2n} \\ X_{*i}^T y &< \frac{-\lambda}{2} \end{aligned}$$

Similarly, we can note that in the event that $w_i^* < 0$,

$$\begin{aligned} \frac{\lambda}{2n} - \frac{1}{n} X_{*i}^T y &< 0 \\ -\frac{1}{n} X_{*i}^T y &< -\frac{\lambda}{2n} \\ X_{*i}^T y &> \frac{\lambda}{2} \end{aligned}$$

We may then conclude that if neither $X_{*i}^T y < \frac{-\lambda}{2}$ nor $X_{*i}^T y < \frac{-\lambda}{2}$, or equivalently $\frac{-\lambda}{2} \leq X_{*i}^T y \leq \frac{\lambda}{2}$, then $w_i^* \not> 0$ and $w_i^* \not< 0$, so $w_i^* = 0$.

5.)

Now, considering ridge regression with regularization term $\lambda |w|^2$, we find

$$J(w) = |Xw - y|^2 + \lambda |w|^2.$$

Just as we did in part 1, we can reexpress this statement by following the same general procedure, but now substituing $\sum_{i=1}^d w_i^2$ for $\sum_{i=1}^d |w_i|$. We find

$$J(w) = |y|^2 - \sum_{i=1}^{d} \left(2w_i X_{*i}^T y + nw_i^2 + \lambda w_i^2 \right).$$

Then, by taking the gradient and setting it equal to zero and solving for w_i^*

$$\begin{aligned} 0 = \nabla_{w_i} J(w) &= \nabla_{w_i} \left(|y|^2 - \sum_{i=1}^{d} \left(2w_i X_{*i}^T y + nw_i^2 + \lambda w_i^2 \right) \right) \\ 0 &= 2X_{*i}^T y + 2nw_i^* + 2\lambda w_i^* \\ 0 &= X_{*i}^T y + (n + \lambda) w_i^* \\ w_i^* &= \frac{X_{*i}^T y}{n + \lambda} \end{aligned}$$

Now, we see that for $w_i^* = 0$, $X_{*i}^T y = 0$. This is a more stringent condition than what we found in (4) since $X_{*i}^T y = 0$ was only a subset of that interval ($\frac{-\lambda}{2} \leq X_{*i}^T y \leq \frac{\lambda}{2}$).

Problem 3

a.)

$$\nabla|w|^4 = \nabla((w^T w)(w^T w))$$
$$\nabla|w|^4 = (w^T w)\nabla(w^T w) + \nabla(w^T w)(w^T w)$$
$$\nabla|w|^4 = (w^T w)2w + 2w(w^T w)$$
$$\nabla|w|^4 = 2(w^T ww + ww^T w)$$
$$\nabla|w|^4 = 2|w|(w + w)$$
$$\boxed{\nabla|w|^4 = 4|w|w}$$

$$\nabla_w|Xw - y|^4 = \nabla_w(((Xw - y)^T(Xw - y))((Xw - y)^T(Xw - y)))$$
$$\nabla_w|Xw - y|^4 = (Xw - y)^T(Xw - y)\nabla_w((Xw - y)^T(Xw - y)) + \nabla_w((Xw - y)^T(Xw - y))(Xw - y)^T(Xw - y)$$
$$\nabla_w|Xw - y|^4 = |Xw - y|^2\nabla_w((Xw - y)^T(Xw - y)) + \nabla_w((Xw - y)^T(Xw - y))|Xw - y|^2$$
$$\nabla_w|Xw - y|^4 = 2|Xw - y|^2\nabla_w((Xw - y)^T(Xw - y))$$
$$\nabla_w|Xw - y|^4 = 2|Xw - y|^2\nabla_w(wX^T Xw - 2w^T X^T y + y^T y)$$
$$\nabla_w|Xw - y|^4 = 2|Xw - y|^2(\nabla_w(wX^T Xw) - 2\nabla_w(w^T X^T y) + \nabla_w(y^T y))$$
$$\nabla_w|Xw - y|^4 = 2|Xw - y|^2(2X^T Xw - 2X^T y)$$
$$\boxed{\nabla_w|Xw - y|^4 = 4|Xw - y|^2 X^T(Xw - y)}$$

b.)

We can take the value of w which minimizes the objective function to be w^*, the solution to $0 = \nabla_w(|Xw - y|^4 + \lambda|w|^2)$.

$$0 = \nabla_w(|Xw - y|^4) + \nabla_w(\lambda|w|^2)$$
$$0 = 4|Xw^* - y|^2 X^T(Xw^* - y) + \lambda\nabla_w(w^T w)$$
$$0 = 4|Xw^* - y|^2 X^T(Xw^* - y) + 2\lambda w^*$$
$$-2\lambda w^* = 4|Xw^* - y|^2 X^T(Xw^* - y)$$
$$w^* = -2|Xw^* - y|^2 X^T(Xw^* - y)$$
$$w^* = -2X^T\left(\sum_{i=1}^{n}(X_i w^* - y_i)^2\right)(Xw^* - y)$$

(...)

Problem 4

1.)

The cost function for logistic regression with ℓ_2 regularization is

$$J(w) = \lambda|w|_2^2 - \sum_{i=1}^{n} y_i \ln s_i + (1 - y_i)\ln(1 - s_i)$$

$$J(w) = \lambda|w|_2^2 - \sum_{i=1}^{n} y_i \ln s_i + \ln(1 - s_i) - y_i \ln(1 - s_i)$$

$$J(w) = \lambda|w|_2^2 - \sum_{i=1}^{n} y_i(\ln s_i - \ln(1 - s_i)) + \ln(1 - s_i)$$

The gradient of this cost function was calculated in problem 1.1 to be

$$\nabla_w J(w) = 2\lambda w - X^T(y - s)$$

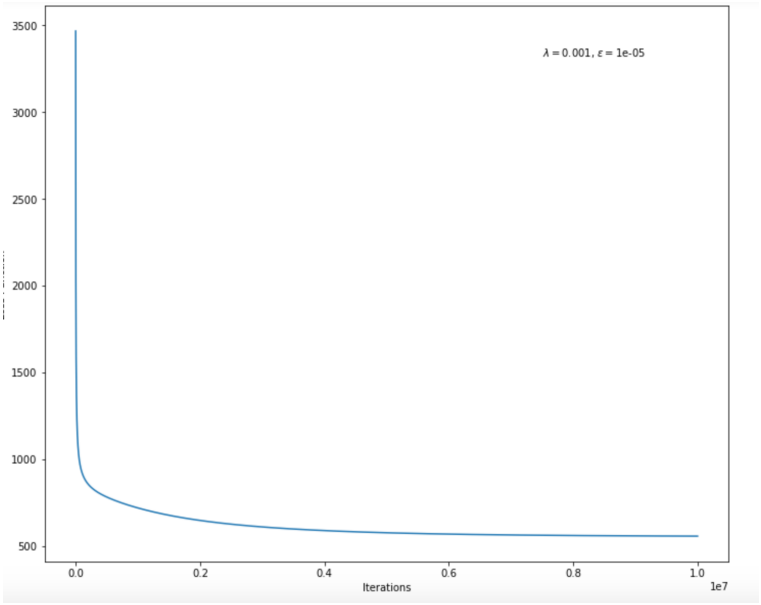
As given in lecture, the general procedure for batch gradient descent is as follows:

$w \leftarrow$ arbitrary starting point
while $J(w) > 0$
s $\quad w \leftarrow w - \epsilon \nabla_w J(w)$
return w

Using our equation for the gradient of the cost function, we can express the update rule more specifically as

$$w \leftarrow w - \epsilon(2\lambda w - X^T(y - s))$$

Executing the gradient descent procedure with this update rule (for regularization parameter $\lambda = 0.001$ and learning rate $\epsilon = 1 \times 10^{-5}$; chose using validation) we find the following relationship between the loss function and the number of iterations through the procedure.



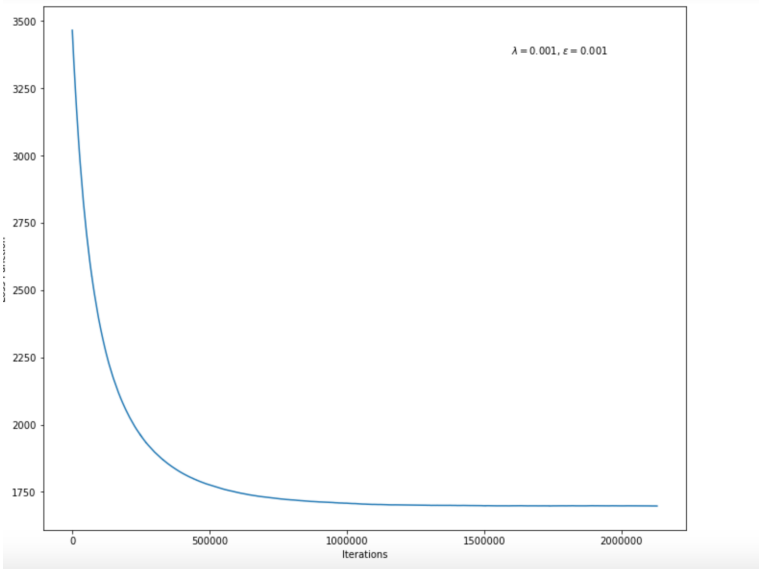
2.)

For stochastic gradient descent we use a similar procedure, but now use the update rule

$$w \leftarrow w - \epsilon(2\lambda w - X_i^T(y_i - s_i)).$$

Here, X_i is now a single randomly chosen data point, y_i is the classification of that point.

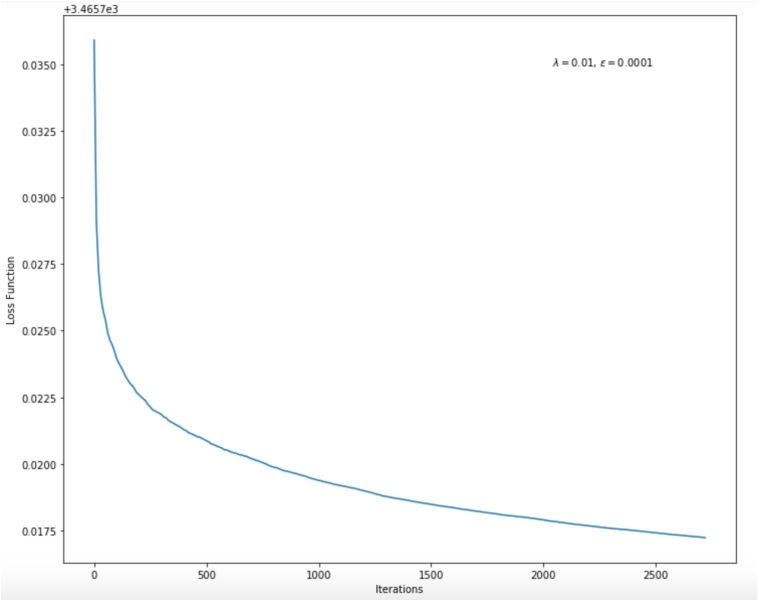
The relationship of the loss function per iteration is now given in the following plot (using hyperparameters λ, ϵ again chosen through validation).



We see here that the convergence of the loss function occurs much faster for stochastic gradient descent.

3.)

Now, when the learning rate decreases proportionally to $1/t$, we find



4.)

Hyperparameters were chosen using validation (sampling from values of λ and ϵ spaced evenly on a logarithmic scale from $1 \times 10^{-3} \leq \lambda \leq 10$ and $1 \times 10^{-5} \leq \epsilon \leq 1 \times 10^{-3}$). The regularization parameter was chosen to be $\lambda = 0.001$ and $\epsilon = 0.001$. Notably, on the first Kaggle submission my score was significantly lower than my predicted scores (12% error as opposed to 3% training error). After hypothesizing that this difference was likely due to a low regularization parameter causing overfitting to the training data, I chose a larger regularization parameter ($\lambda = ...$

Kaggle Submission:

Username: **mnegus**
Score: **95.565%**

Problem 5

If the spike in spam takes place both immediately before and after midnight, then by adding a feature of number of milliseconds since previous midnight, the values of this feature indicating spam will be both small (just a few milleseconds right after midnight) and very large (approximately 86.4 million right before the next midnight). A linear SVM will not be able to construct a margin that separates these two "spam" groups of points from those classified as "ham."

To remedy this issue, Daniel could change his feature so that it is simply the number of milliseconds from the closest midnight (in either direction, before or after). Then, to improve separation from these points (as a linear classifier will still struggle to separate this grouping from the remainder of data points), Daniel could add a feature being the square of the number of milliseconds from the closest midnight. Then the data would form a paraboloid with number of seconds from midnight being near the minimum, which would allow a linear SVM to easily separate points near midnight.

CS289A_HW04_Prob4

March 13, 2017

1 CS 289A Homework 4

Start with program overhead: load modules (and reload them as they are modified)

```
In [1]: %load_ext autoreload
```

```
In [2]: %autoreload 2
```

```
In [3]: import HW04_utils as ut
        import numpy as np
        from scipy import special as spsp
        from matplotlib import pyplot as plt
```

Next, we give a couple paths specifying where to find the data set on the local machine. **A user must change this to reflect the path to their data.**

```
In [4]: BASE_DIR = "/Users/mitch/Documents/Cal/2_2017_Spring/COMPSCI 289A - Intro t
        DATA_PATH = "Data/data.mat"
```

Then, load the data using the custom utilities module:

```
In [5]: # Load training data
        descriptions = ut.loaddata(DATA_PATH,BASE_DIR,'description')
        X = ut.loaddata(DATA_PATH,BASE_DIR,'X')
        y = ut.loaddata(DATA_PATH,BASE_DIR,'y')

        # Shuffle training data
        data = np.concatenate((X,y),axis=1)
        np.random.shuffle(data)
        X = data[:,:-1]
        y = data[:,-1]

        # Normalize training data
        meanX = np.tile(np.mean(X,axis=0),(len(X),1))
        minX = np.tile(np.amin(X,axis=0),(len(X),1))
        maxX = np.tile(np.amax(X,axis=0),(len(X),1))
        X = (X-meanX)/(maxX-minX)
```

```
# Separate a validation set that is a given fraction of the training data
frac = 1/6
n = int(len(X)-frac*len(X))
X_train = X[:n]
X_val = X[n:]
y_train = y[:n]
y_val = y[n:]

X = X_train
y = y_train

# Load test data
X_test = ut.loaddata(DATA_PATH,BASE_DIR,'X_test')

# Normalize test data
meanXt = np.tile(np.mean(X_test,axis=0),(len(X_test),1))
minXt = np.tile(np.amin(X_test,axis=0),(len(X_test),1))
maxXt = np.tile(np.amax(X_test,axis=0),(len(X_test),1))
X_test = (X_test-meanXt)/(maxXt-minXt)
```

1.1 Part 1

We use the following procedure to find the optimal w using batch gradient descent:

(1) $w \leftarrow$ arbitrary starting point

(2) while $J(w) > 0$

$w \leftarrow w - \epsilon(2\lambda w - X^T(y - s))$

(3) return w

Step (1)

```
In [6]: w = np.zeros(len(descriptions))
```

Step (2)

```
In [7]: def update_w_batch(w,X,y,lam,eps):
            s = spsp.expit(np.dot(X,w))
            w_prime = w - eps*(2*lam*w-np.dot(X.T,(y-s)))

            return w_prime

In [8]: def costfnJ(w,X,y,lam):
            s = spsp.expit(np.dot(X,w))
            J = lam*np.linalg.norm(w)**2 - np.sum(y*np.log(s) + (np.ones_like(y)-y)

            return J

In [9]: def whileloop(w,X,y,lam,eps,tol,update_fn):
            i=0
            iters,Js = [],[]
            J = costfnJ(w,X,y,lam)
```

```
    lastJ = J+1  #dummy condition to pass while condition on first run
    while J>0 and i<=1e7 and np.absolute(lastJ-J)>tol:
        w_prime = update_fn(w,X,y,lam,eps)
        w = w_prime
        if i%10==0:
            if i%500000==0:
                print(str(i)+":\tJ =",str(J))
            iters.append(i)
            Js.append(J)
        lastJ = J
        J = costfnJ(w,X,y,lam)
        i+=1

    return w,iters,Js
```

Step (3)

Here we try several values of hyperparameters λ and ϵ to find the optimal values. We also introduce a convergence tolerance that is used in the case that data is not linearly separable.

```
In [10]: lambdas = np.logspace(-3,1,5)
         epsilons = np.logspace(-5,-3,3)
         tol = 1e-6

In [11]: # Collect loss function as f'n of iteration number for each combo
         optima_batch = {}
         LvIs_batch = {}
         for lam in lambdas:
             optima_batch[lam]={}
             LvIs_batch[lam]={}
             for eps in epsilons:
                 print("Lambda:",lam,"\tEpsilon:",eps)
                 w_star,iters,Js = whileloop(w,X,y,lam,eps,tol,update_w_batch)
                 optima_batch[lam][eps]= w_star
                 LvIs_batch[lam][eps] = [iters,Js]
```

```
Lambda: 0.001         Epsilon: 1e-05
0:        J = 3465.7359028
500000:        J = 767.198352822
1000000:        J = 704.407443175
1500000:        J = 662.748898799
2000000:        J = 633.433517029
2500000:        J = 612.32966552
3000000:        J = 596.759285111
3500000:        J = 585.04403722
4000000:        J = 576.096241789
4500000:        J = 569.178297095
5000000:        J = 563.773879883
5500000:        J = 559.513515784
6000000:        J = 556.128237005
```

```
6500000:        J = 553.419368524
7000000:        J = 551.238253803
7500000:        J = 549.472364806
8000000:        J = 548.035622922
8500000:        J = 546.861552511
9000000:        J = 545.898371535
9500000:        J = 545.105426505
10000000:        J = 544.450573101
Lambda: 0.001         Epsilon: 0.0001
0:        J = 3465.7359028
500000:        J = 563.773821406
1000000:        J = 544.450560248
1500000:        J = 541.715934248
Lambda: 0.001         Epsilon: 0.001
0:        J = 3465.7359028
Lambda: 0.01         Epsilon: 1e-05
0:        J = 3465.7359028
500000:        J = 792.373230383
1000000:        J = 744.938598164
1500000:        J = 718.85368012
2000000:        J = 703.760004371
2500000:        J = 694.932644597
3000000:        J = 689.700802893
3500000:        J = 686.560299793
4000000:        J = 684.656004021
4500000:        J = 683.49248145
5000000:        J = 682.777510852
Lambda: 0.01         Epsilon: 0.0001
0:        J = 3465.7359028
500000:        J = 682.777498732
Lambda: 0.01         Epsilon: 0.001
0:        J = 3465.7359028
Lambda: 0.1         Epsilon: 1e-05
0:        J = 3465.7359028
500000:        J = 967.159795304
1000000:        J = 963.570102976
Lambda: 0.1         Epsilon: 0.0001
0:        J = 3465.7359028
Lambda: 0.1         Epsilon: 0.001
0:        J = 3465.7359028
Lambda: 1.0         Epsilon: 1e-05
0:        J = 3465.7359028
Lambda: 1.0         Epsilon: 0.0001
0:        J = 3465.7359028
Lambda: 1.0         Epsilon: 0.001
0:        J = 3465.7359028
Lambda: 10.0         Epsilon: 1e-05
0:        J = 3465.7359028
```

```
Lambda: 10.0          Epsilon: 0.0001
0:        J = 3465.7359028
Lambda: 10.0          Epsilon: 0.001
0:        J = 3465.7359028
```

Print out and save a list of the accuracies corresponding to the optimum w* for each combination of λ and ϵ.

```
In [12]: def HyperparameterAccs(lambdas,epsilons,optima,valdata,vallabels):
             Accs = np.zeros((len(lambdas)*len(epsilons),3))
             i=0
             for lam in optima:
                 for eps in optima[lam]:
                     w_star = optima[lam][eps]
                     probs = spsp.expit(np.dot(X_val,w_star))
                     tally = 0
                     total = 0
                     for j in range(len(probs)):
                         if probs[j] >= 0.5:
                             prob = 1
                         if probs[j] < 0.5:
                             prob = 0
                         if prob == y_val[j]:
                             tally += 1
                         total += 1
                     acc = tally/total
                     Accs[i] = [acc,lam,eps]
                     i+=1
                     print('lam = '+str(lam)+'\teps =',eps,'\t\tAccuracy: ',acc)

             return Accs

In [13]: Accs_batch = HyperparameterAccs(lambdas,epsilons,optima_batch,X_val,y_val)
```

```
lam = 0.001        eps = 1e-05                Accuracy:  0.955
lam = 0.001        eps = 0.0001               Accuracy:  0.958
lam = 0.001        eps = 0.001                Accuracy:  0.959
lam = 0.01         eps = 1e-05                Accuracy:  0.952
lam = 0.01         eps = 0.0001               Accuracy:  0.951
lam = 0.01         eps = 0.001                Accuracy:  0.952
lam = 0.1          eps = 1e-05                Accuracy:  0.935
lam = 0.1          eps = 0.0001               Accuracy:  0.935
lam = 0.1          eps = 0.001                Accuracy:  0.935
lam = 1.0          eps = 1e-05                Accuracy:  0.922
lam = 1.0          eps = 0.0001               Accuracy:  0.921
lam = 1.0          eps = 0.001                Accuracy:  0.921
lam = 10.0         eps = 1e-05                Accuracy:  0.913
lam = 10.0         eps = 0.0001               Accuracy:  0.913
```

```
lam = 10.0        eps = 0.001                  Accuracy:  0.913
```

```
In [14]: print(optima_batch[0.001][0.001])
         print(Accs_batch)
```

```
[  -9.76375855   13.46225692   -5.2732488   -66.13364483   10.78171502
   19.51816531  -26.64844479  146.9055984    -5.99243176    2.76490805
   19.14298793    3.53846425]
[[  9.55000000e-01   1.00000000e-03   1.00000000e-05]
 [  9.58000000e-01   1.00000000e-03   1.00000000e-04]
 [  9.59000000e-01   1.00000000e-03   1.00000000e-03]
 [  9.52000000e-01   1.00000000e-02   1.00000000e-05]
 [  9.51000000e-01   1.00000000e-02   1.00000000e-04]
 [  9.52000000e-01   1.00000000e-02   1.00000000e-03]
 [  9.35000000e-01   1.00000000e-01   1.00000000e-05]
 [  9.35000000e-01   1.00000000e-01   1.00000000e-04]
 [  9.35000000e-01   1.00000000e-01   1.00000000e-03]
 [  9.22000000e-01   1.00000000e+00   1.00000000e-05]
 [  9.21000000e-01   1.00000000e+00   1.00000000e-04]
 [  9.21000000e-01   1.00000000e+00   1.00000000e-03]
 [  9.13000000e-01   1.00000000e+01   1.00000000e-05]
 [  9.13000000e-01   1.00000000e+01   1.00000000e-04]
 [  9.13000000e-01   1.00000000e+01   1.00000000e-03]]
```

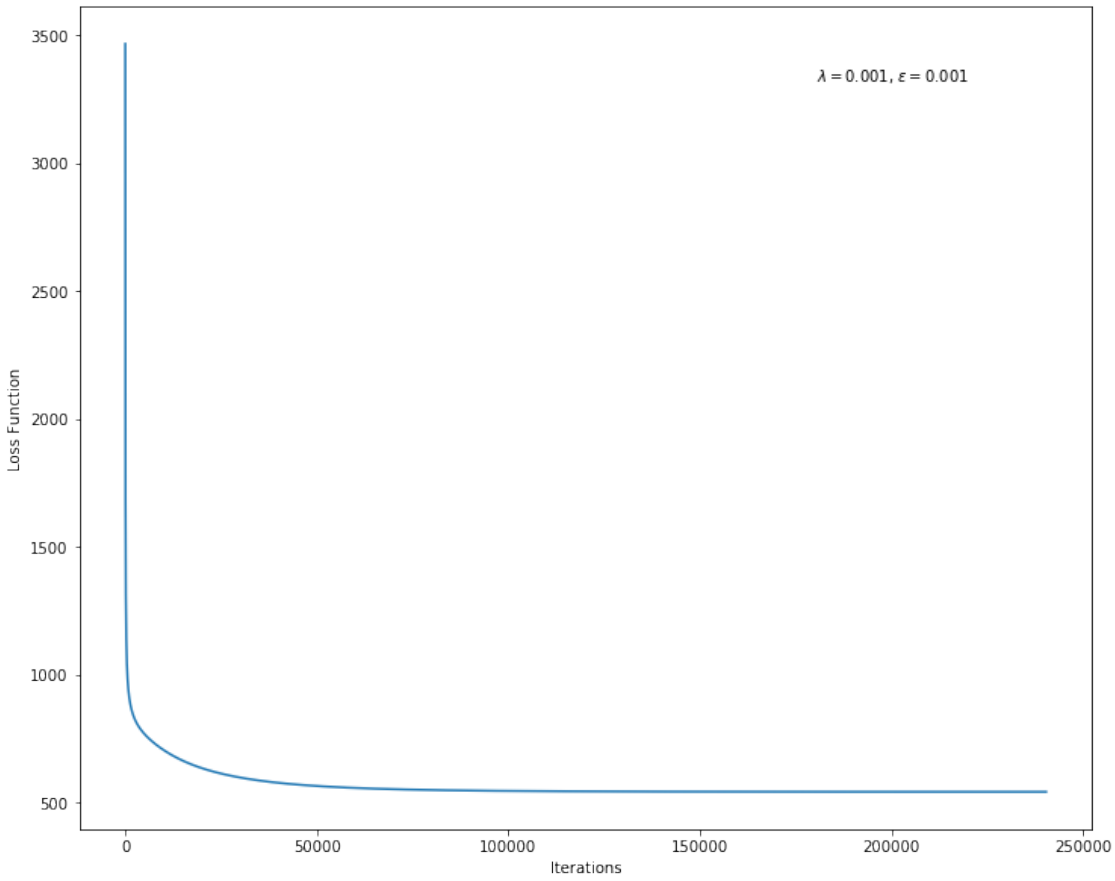
Plot the loss function vs. iteration number for the best combination of λ, ϵ.

```
In [15]: def plot_LFvIt(Accs,LvIs):
             fig = plt.figure(figsize=(12,10))
             plt.clf()

             # Find the most accurate lam,eps combo
             imax = np.argmax(Accs[:,0])
             lam = Accs[imax,1]
             eps = Accs[imax,2]

             Iters = LvIs[lam][eps][0]
             LossFn = LvIs[lam][eps][1]
             plt.plot(Iters,LossFn)
             plt.xlabel('Iterations')
             plt.ylabel('Loss Function')
             plt.text(0.75*(np.amax(Iters)-np.amin(Iters))+np.amin(Iters),0.95*(np.
             plt.show()
```

```
In [16]: plot_LFvIt(Accs_batch,LvIs_batch)
```



1.2 Part 2

Instead of batch descent, we can use the following procedure to find the optimal w using stochastic gradient descent:

(1) $w \leftarrow$ arbitrary starting point

(2) while $J(w) > 0$

$w \leftarrow w - \epsilon(2\lambda w - X_i^T(y_i - s_i))$

(3) return w

Step (1) - Same as in batch gradient descent

```
In [17]: w = np.zeros(len(descriptions))
```

Step (2) - we can reuse the functions for calculating the cost function and the looping process defined in part 1, step 2; we define a new function for the stochastic update rule

```
In [18]: def update_w_stoch(w,X,y,lam,eps):
             i = np.random.choice(len(X))
             y_i = y[i]
             X_i = X[i]
             s_i = spsp.expit(np.dot(X_i,w))
```

```
            w_prime = w - eps*(2*lam*w-X_i*(y_i-s_i))

            return w_prime
```

Step (3)

Again, we try several values of hyperparameters λ and ϵ to find the optimal values and we introduce a convergence tolerance that is used in the case that data is not linearly separable.

```
In [19]: tol = 1e-9
```

```
In [20]: # Collect loss function as f'n of iteration number for each combo
         optima_stoch = {}
         LvIs_stoch = {}
         for lam in lambdas:
             optima_stoch[lam]={}
             LvIs_stoch[lam]={}
             for eps in epsilons:
                 print("Lambda:",lam,"\tEpsilon:",eps)
                 w_star,iters,Js = whileloop(w,X,y,lam,eps,tol,update_w_stoch)
                 optima_stoch[lam][eps]= w_star
                 LvIs_stoch[lam][eps] = [iters,Js]
                 print(iters[len(iters)-1])
```

```
Lambda: 0.001         Epsilon: 1e-05
0:        J = 3465.7359028
500000:        J = 3364.07844802
751460
Lambda: 0.001         Epsilon: 0.0001
0:        J = 3465.7359028
407340
Lambda: 0.001         Epsilon: 0.001
0:        J = 3465.7359028
500000:        J = 1780.02392783
957690
Lambda: 0.01         Epsilon: 1e-05
0:        J = 3465.7359028
500000:        J = 3368.30796372
672060
Lambda: 0.01         Epsilon: 0.0001
0:        J = 3465.7359028
359080
Lambda: 0.01         Epsilon: 0.001
0:        J = 3465.7359028
500000:        J = 2785.21595913
1000000:        J = 2787.32786271
1500000:        J = 2785.51122138
2000000:        J = 2785.11129489
2500000:        J = 2787.0705809
3000000:        J = 2785.28963853
```

```
3500000:        J = 2785.803959
4000000:        J = 2787.22372734
4500000:        J = 2788.00847534
5000000:        J = 2786.29573236
5500000:        J = 2785.48439103
6000000:        J = 2788.66480945
6005030
Lambda: 0.1         Epsilon: 1e-05
0:        J = 3465.7359028
214990
Lambda: 0.1         Epsilon: 0.0001
0:        J = 3465.7359028
500000:         J = 3365.64013098
1000000:        J = 3365.10773544
1500000:        J = 3365.41298886
2000000:        J = 3365.44933867
2500000:        J = 3365.16683129
3000000:        J = 3365.2569922
3238330
Lambda: 0.1         Epsilon: 0.001
0:        J = 3465.7359028
500000:         J = 3365.02642054
1000000:        J = 3364.96104161
1500000:        J = 3365.04799404
2000000:        J = 3365.24070245
2500000:        J = 3364.91826271
3000000:        J = 3365.17746645
3500000:        J = 3365.1079893
4000000:        J = 3364.48461212
4500000:        J = 3366.13618097
5000000:        J = 3364.66628881
5500000:        J = 3365.56950205
6000000:        J = 3365.13812904
6500000:        J = 3366.38725617
7000000:        J = 3366.13648221
7500000:        J = 3365.20638599
8000000:        J = 3365.1508241
8500000:        J = 3364.84247723
9000000:        J = 3365.38729735
9500000:        J = 3365.47776594
10000000:        J = 3364.7427037
10000000
Lambda: 1.0         Epsilon: 1e-05
0:        J = 3465.7359028
500000:         J = 3455.19947615
892190
Lambda: 1.0         Epsilon: 0.0001
0:        J = 3465.7359028
```

```
500000:        J = 3455.22516758
806130
Lambda: 1.0         Epsilon: 0.001
0:        J = 3465.7359028
500000:        J = 3455.45095814
1000000:        J = 3455.32709589
1500000:        J = 3454.99078137
2000000:        J = 3455.46400094
2500000:        J = 3454.41694654
3000000:        J = 3454.82719222
3500000:        J = 3455.04570563
4000000:        J = 3455.11178217
4500000:        J = 3455.63697838
5000000:        J = 3455.37833326
5500000:        J = 3455.22887265
6000000:        J = 3454.9626366
6500000:        J = 3455.19824855
7000000:        J = 3455.31077341
7500000:        J = 3455.36412691
8000000:        J = 3455.8766627
8500000:        J = 3455.37583182
9000000:        J = 3455.11135474
9500000:        J = 3455.25644384
10000000:        J = 3455.08109678
10000000
Lambda: 10.0         Epsilon: 1e-05
0:        J = 3465.7359028
355830
Lambda: 10.0         Epsilon: 0.0001
0:        J = 3465.7359028
500000:        J = 3464.65317259
974090
Lambda: 10.0         Epsilon: 0.001
0:        J = 3465.7359028
500000:        J = 3464.63245865
1000000:        J = 3464.71713815
1500000:        J = 3464.75740153
2000000:        J = 3464.74406301
2500000:        J = 3464.59253951
3000000:        J = 3464.63204545
3500000:        J = 3464.68100638
4000000:        J = 3464.7078865
4500000:        J = 3464.61943668
5000000:        J = 3464.63747874
5500000:        J = 3464.67095198
6000000:        J = 3464.73626162
6500000:        J = 3464.63546903
7000000:        J = 3464.63242981
```

```
7500000:        J = 3464.47215421
8000000:        J = 3464.76204115
8500000:        J = 3464.5792482
9000000:        J = 3464.60974749
9500000:        J = 3464.77891622
10000000:        J = 3464.72769749
10000000
```

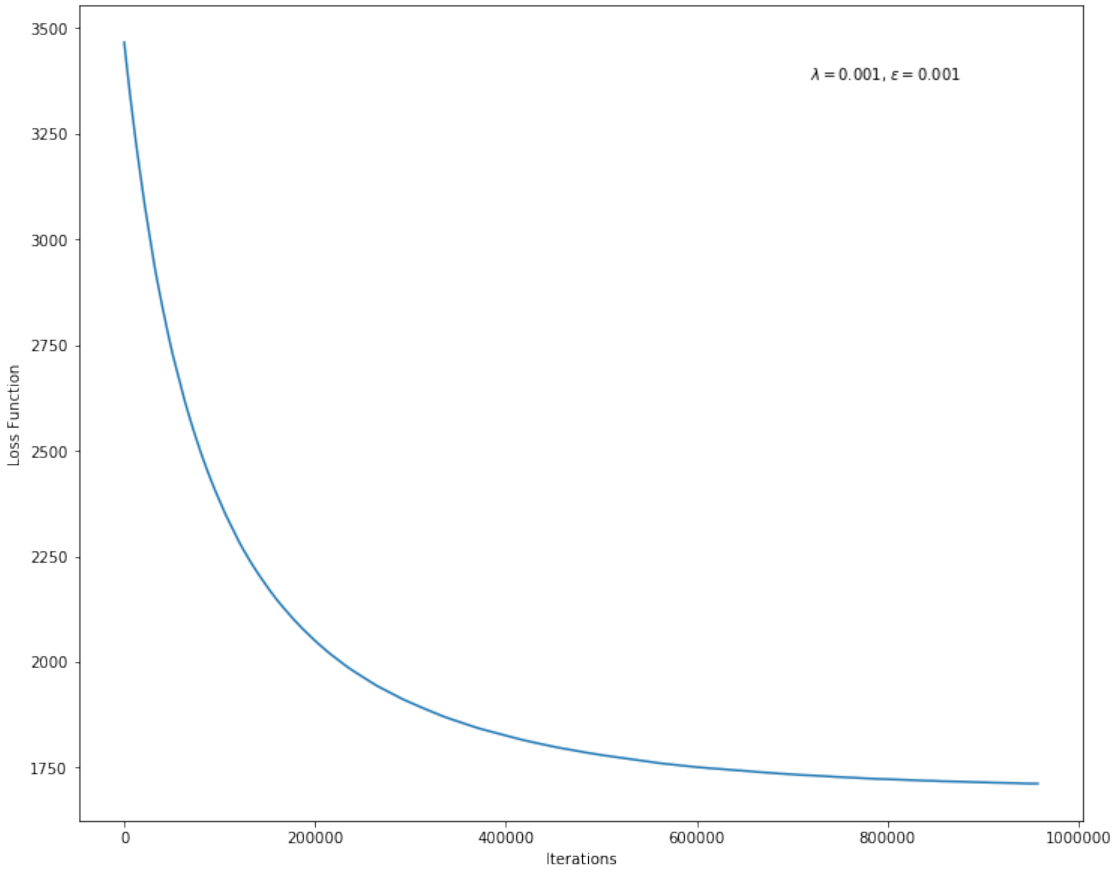
Print out a list of the optimum w* for each combination of λ and ϵ. Save the accuracies in a list.

```
In [21]: Accs_stoch = HyperparameterAccs(lambdas,epsilons,optima_stoch,X_val,y_val)
```

```
lam = 0.001        eps = 1e-05               Accuracy:  0.902
lam = 0.001        eps = 0.0001               Accuracy:  0.903
lam = 0.001        eps = 0.001               Accuracy:  0.913
lam = 0.01        eps = 1e-05               Accuracy:  0.902
lam = 0.01        eps = 0.0001               Accuracy:  0.903
lam = 0.01        eps = 0.001               Accuracy:  0.904
lam = 0.1        eps = 1e-05               Accuracy:  0.901
lam = 0.1        eps = 0.0001               Accuracy:  0.901
lam = 0.1        eps = 0.001               Accuracy:  0.9
lam = 1.0        eps = 1e-05               Accuracy:  0.903
lam = 1.0        eps = 0.0001               Accuracy:  0.902
lam = 1.0        eps = 0.001               Accuracy:  0.899
lam = 10.0        eps = 1e-05               Accuracy:  0.904
lam = 10.0        eps = 0.0001               Accuracy:  0.889
lam = 10.0        eps = 0.001               Accuracy:  0.893
```

Plot the loss function vs. iteration number for the best combination of λ, ϵ.

```
In [22]: plot_LFvIt(Accs_stoch,LvIs_stoch)
```



1.3 Part 3

Now we wish to repeat part 2 (stochastic gradient descent) but using a variable ϵ.
We can accomplish this by redefining our while loop to decrease ϵ such that $\epsilon \propto 1/t$.

```
In [23]: def whileloop_deceps(w,X,y,lam,eps,tol,update_fn):
             i=0
             iters,Js = [],[]
             J = costfnJ(w,X,y,lam)
             lastJ = J+1  #dummy condition to pass while condition on first run
             while J>0 and i<=1e7 and np.absolute(lastJ-J)>tol:
                 w_prime = update_fn(w,X,y,lam,eps/(i+1))
                 w = w_prime
                 if i%10==0:
                     if i%500000==0:
                         print(str(i)+":\tJ =",str(J))
                     iters.append(i)
                     Js.append(J)
                 lastJ = J
                 J = costfnJ(w,X,y,lam)
```

```
            i+=1

        return w,iters,Js
```

Then, we call that function using the same procedure used before.

```
In [24]: # Collect loss function as f'n of iteration number for each combo
         optima_stoch_deceps = {}
         LvIs_stoch_deceps = {}
         for lam in lambdas:
             optima_stoch_deceps[lam]={}
             LvIs_stoch_deceps[lam]={}
             for eps in epsilons:
                 print("Lambda:",lam,"\tEpsilon:",eps)
                 w_star,iters,Js = whileloop_deceps(w,X,y,lam,eps,tol,update_w_stoc
                 optima_stoch_deceps[lam][eps]= w_star
                 LvIs_stoch_deceps[lam][eps] = [iters,Js]
                 print(iters[len(iters)-1])
```

```
Lambda: 0.001         Epsilon: 1e-05
0:        J = 3465.7359028
990
Lambda: 0.001         Epsilon: 0.0001
0:        J = 3465.7359028
1400
Lambda: 0.001         Epsilon: 0.001
0:        J = 3465.7359028
3500
Lambda: 0.01         Epsilon: 1e-05
0:        J = 3465.7359028
1040
Lambda: 0.01         Epsilon: 0.0001
0:        J = 3465.7359028
1430
Lambda: 0.01         Epsilon: 0.001
0:        J = 3465.7359028
1860
Lambda: 0.1         Epsilon: 1e-05
0:        J = 3465.7359028
840
Lambda: 0.1         Epsilon: 0.0001
0:        J = 3465.7359028
990
Lambda: 0.1         Epsilon: 0.001
0:        J = 3465.7359028
23150
Lambda: 1.0         Epsilon: 1e-05
0:        J = 3465.7359028
```

```
290
Lambda: 1.0         Epsilon: 0.0001
0:        J = 3465.7359028
2940
Lambda: 1.0         Epsilon: 0.001
0:        J = 3465.7359028
3020
Lambda: 10.0         Epsilon: 1e-05
0:        J = 3465.7359028
1180
Lambda: 10.0         Epsilon: 0.0001
0:        J = 3465.7359028
3930
Lambda: 10.0         Epsilon: 0.001
0:        J = 3465.7359028
3880
```

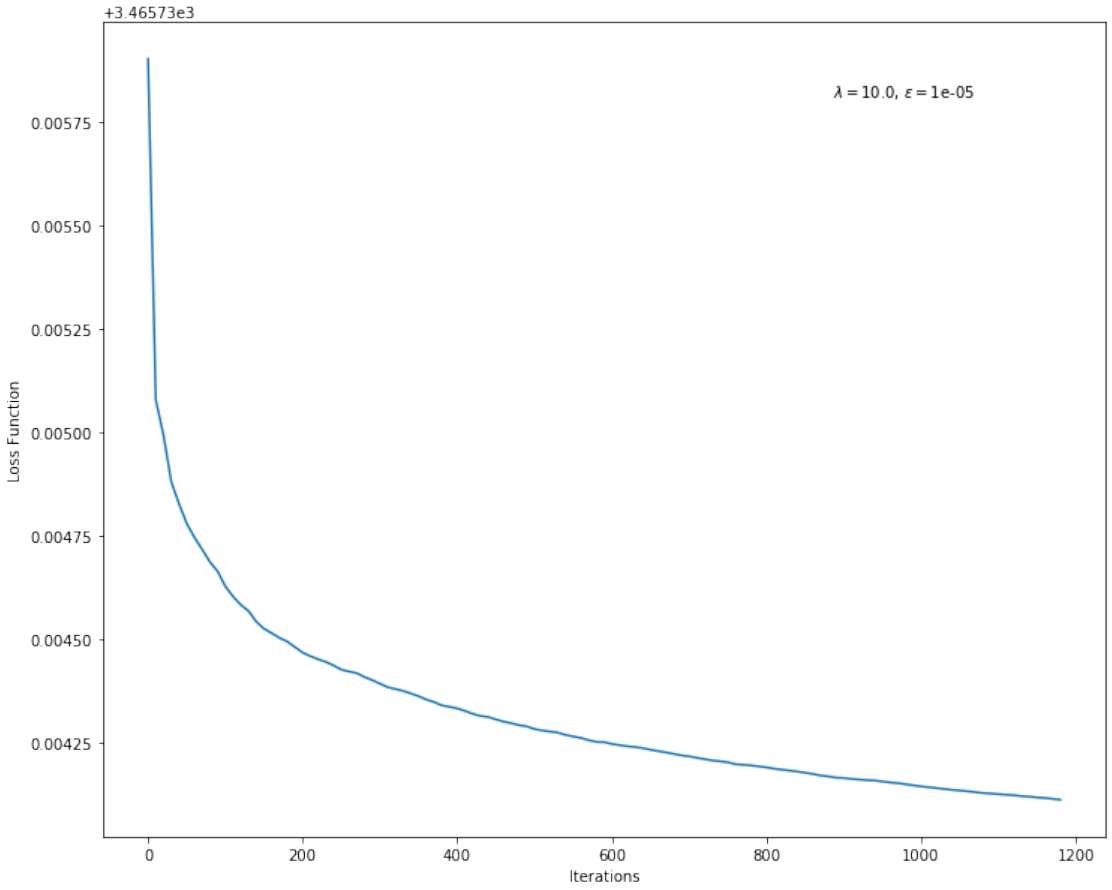
Print out a list of the optimum w* for each combination of λ and ϵ. Save the accuracies in a list.

```
In [25]: Accs_stoch_deceps = HyperparameterAccs(lambdas,epsilons,optima_stoch_decep
```

```
lam = 0.001        eps = 1e-05                Accuracy:  0.88
lam = 0.001        eps = 0.0001                Accuracy:  0.884
lam = 0.001        eps = 0.001                Accuracy:  0.875
lam = 0.01        eps = 1e-05                Accuracy:  0.884
lam = 0.01        eps = 0.0001                Accuracy:  0.769
lam = 0.01        eps = 0.001                Accuracy:  0.854
lam = 0.1        eps = 1e-05                Accuracy:  0.877
lam = 0.1        eps = 0.0001                Accuracy:  0.871
lam = 0.1        eps = 0.001                Accuracy:  0.883
lam = 1.0        eps = 1e-05                Accuracy:  0.761
lam = 1.0        eps = 0.0001                Accuracy:  0.874
lam = 1.0        eps = 0.001                Accuracy:  0.869
lam = 10.0        eps = 1e-05                Accuracy:  0.892
lam = 10.0        eps = 0.0001                Accuracy:  0.864
lam = 10.0        eps = 0.001                Accuracy:  0.887
```

Plot the loss function vs. iteration number for the best combination of λ, ϵ.

```
In [26]: plot_LFvIt(Accs_stoch_deceps,LvIs_stoch_deceps)
```



Finally, we use our most successful training algorithm (in this case, batch gradient descent for $\lambda = 0.001$ and $\epsilon = 0.001$) to predict on the test data.

```
In [51]: lam,eps = 0.001,0.001
         w_star = optima_batch[lam][eps]
         preds = spsp.expit(np.dot(X_test,w_star))
         predictions = np.rint(preds)
```

We save these predictions to the csv file for Kaggle submission.

```
In [38]: IDs = np.arange(len(predictions))
         numpycsv = np.c_[IDs,predictions]
         np.savetxt(BASE_DIR+'/'+'Prob4_testpredictions.csv',numpycsv,fmt='%i',deli
```

Noting that this submission on Kaggle produced a test error of almost 12% (significantly more than the training error of 3% for the same hyperparameters), I chose a new set of hyperparameters with λ greater than the first submission. The intention of this was to reduce overfitting, as I had presumably been overfitting, giving an excellent training error but mediocre test error.

```
In [50]: lam,eps = 0.1,0.001
         w_star2 = optima_batch[lam][eps]
```

```
preds2 = spsp.expit(np.dot(X_test,w_star2))
predictions2 = np.rint(preds2)
```

Again, we save these predictions to the csv file for Kaggle submission.

```
In [45]: IDs = np.arange(len(predictions))
         numpycsv = np.c_[IDs,predictions]
         np.savetxt(BASE_DIR+'/'+'Prob4_testpredictions2.csv',numpycsv,fmt='%i',del
```

The hypothesis seems accurate. This submission (username **mnegus**) gave a score of **95.565%**.