CS289A_HW04_Prob4

March 13, 2017

1 CS 289A Homework 4

Start with program overhead: load modules (and reload them as they are modified)

```
In [1]: %load_ext autoreload
```

```
In [2]: %autoreload 2
```

```
In [3]: import HW04_utils as ut
        import numpy as np
        from scipy import special as spsp
        from matplotlib import pyplot as plt
```

Next, we give a couple paths specifying where to find the data set on the local machine. **A user must change this to reflect the path to their data.**

```
In [4]: BASE_DIR = "/Users/mitch/Documents/Cal/2_2017_Spring/COMPSCI 289A - Intro t
        DATA_PATH = "Data/data.mat"
```

Then, load the data using the custom utilities module:

```
In [5]: # Load training data
        descriptions = ut.loaddata(DATA_PATH,BASE_DIR,'description')
        X = ut.loaddata(DATA_PATH,BASE_DIR,'X')
        y = ut.loaddata(DATA_PATH,BASE_DIR,'y')

        # Shuffle training data
        data = np.concatenate((X,y),axis=1)
        np.random.shuffle(data)
        X = data[:,:-1]
        y = data[:,-1]

        # Normalize training data
        meanX = np.tile(np.mean(X,axis=0),(len(X),1))
        minX = np.tile(np.amin(X,axis=0),(len(X),1))
        maxX = np.tile(np.amax(X,axis=0),(len(X),1))
        X = (X-meanX)/(maxX-minX)
```

```
# Separate a validation set that is a given fraction of the training data
frac = 1/6
n = int(len(X)-frac*len(X))
X_train = X[:n]
X_val = X[n:]
y_train = y[:n]
y_val = y[n:]

X = X_train
y = y_train

# Load test data
X_test = ut.loaddata(DATA_PATH,BASE_DIR,'X_test')

# Normalize test data
meanXt = np.tile(np.mean(X_test,axis=0),(len(X_test),1))
minXt = np.tile(np.amin(X_test,axis=0),(len(X_test),1))
maxXt = np.tile(np.amax(X_test,axis=0),(len(X_test),1))
X_test = (X_test-meanXt)/(maxXt-minXt)
```

1.1 Part 1

We use the following procedure to find the optimal w using batch gradient descent:

(1) $w \leftarrow$ arbitrary starting point

(2) while $J(w) > 0$

$w \leftarrow w - \epsilon(2\lambda w - X^T(y - s))$

(3) return w

Step (1)

```
In [6]: w = np.zeros(len(descriptions))
```

Step (2)

```
In [7]: def update_w_batch(w,X,y,lam,eps):
            s = spsp.expit(np.dot(X,w))
            w_prime = w - eps*(2*lam*w-np.dot(X.T,(y-s)))

            return w_prime
```

```
In [8]: def costfnJ(w,X,y,lam):
            s = spsp.expit(np.dot(X,w))
            J = lam*np.linalg.norm(w)**2 - np.sum(y*np.log(s) + (np.ones_like(y)-y)

            return J
```

```
In [9]: def whileloop(w,X,y,lam,eps,tol,update_fn):
            i=0
            iters,Js = [],[]
            J = costfnJ(w,X,y,lam)
```

```
        lastJ = J+1  #dummy condition to pass while condition on first run
        while J>0 and i<=1e7 and np.absolute(lastJ-J)>tol:
            w_prime = update_fn(w,X,y,lam,eps)
            w = w_prime
            if i%10==0:
                if i%500000==0:
                    print(str(i)+":\tJ =",str(J))
                iters.append(i)
                Js.append(J)
            lastJ = J
            J = costfnJ(w,X,y,lam)
            i+=1

        return w,iters,Js
```

Step (3)

Here we try several values of hyperparameters λ and ϵ to find the optimal values. We also introduce a convergence tolerance that is used in the case that data is not linearly separable.

```
In [10]: lambdas = np.logspace(-3,1,5)
         epsilons = np.logspace(-5,-3,3)
         tol = 1e-6
```

```
In [11]: # Collect loss function as f'n of iteration number for each combo
         optima_batch = {}
         LvIs_batch = {}
         for lam in lambdas:
             optima_batch[lam]={}
             LvIs_batch[lam]={}
             for eps in epsilons:
                 print("Lambda:",lam,"\tEpsilon:",eps)
                 w_star,iters,Js = whileloop(w,X,y,lam,eps,tol,update_w_batch)
                 optima_batch[lam][eps]= w_star
                 LvIs_batch[lam][eps] = [iters,Js]
```

```
Lambda: 0.001          Epsilon: 1e-05
0:         J = 3465.7359028
500000:         J = 767.198352822
1000000:         J = 704.407443175
1500000:         J = 662.748898799
2000000:         J = 633.433517029
2500000:         J = 612.32966552
3000000:         J = 596.759285111
3500000:         J = 585.04403722
4000000:         J = 576.096241789
4500000:         J = 569.178297095
5000000:         J = 563.773879883
5500000:         J = 559.513515784
6000000:         J = 556.128237005
```

```
6500000:        J = 553.419368524
7000000:        J = 551.238253803
7500000:        J = 549.472364806
8000000:        J = 548.035622922
8500000:        J = 546.861552511
9000000:        J = 545.898371535
9500000:        J = 545.105426505
10000000:        J = 544.450573101
Lambda: 0.001         Epsilon: 0.0001
0:        J = 3465.7359028
500000:        J = 563.773821406
1000000:        J = 544.450560248
1500000:        J = 541.715934248
Lambda: 0.001         Epsilon: 0.001
0:        J = 3465.7359028
Lambda: 0.01         Epsilon: 1e-05
0:        J = 3465.7359028
500000:        J = 792.373230383
1000000:        J = 744.938598164
1500000:        J = 718.85368012
2000000:        J = 703.760004371
2500000:        J = 694.932644597
3000000:        J = 689.700802893
3500000:        J = 686.560299793
4000000:        J = 684.656004021
4500000:        J = 683.49248145
5000000:        J = 682.777510852
Lambda: 0.01         Epsilon: 0.0001
0:        J = 3465.7359028
500000:        J = 682.777498732
Lambda: 0.01         Epsilon: 0.001
0:        J = 3465.7359028
Lambda: 0.1         Epsilon: 1e-05
0:        J = 3465.7359028
500000:        J = 967.159795304
1000000:        J = 963.570102976
Lambda: 0.1         Epsilon: 0.0001
0:        J = 3465.7359028
Lambda: 0.1         Epsilon: 0.001
0:        J = 3465.7359028
Lambda: 1.0         Epsilon: 1e-05
0:        J = 3465.7359028
Lambda: 1.0         Epsilon: 0.0001
0:        J = 3465.7359028
Lambda: 1.0         Epsilon: 0.001
0:        J = 3465.7359028
Lambda: 10.0         Epsilon: 1e-05
0:        J = 3465.7359028
```

```
Lambda: 10.0         Epsilon: 0.0001
0:        J = 3465.7359028
Lambda: 10.0         Epsilon: 0.001
0:        J = 3465.7359028
```

Print out and save a list of the accuracies corresponding to the optimum w* for each combination of λ and ϵ.

```
In [12]: def HyperparameterAccs(lambdas,epsilons,optima,valdata,vallabels):
             Accs = np.zeros((len(lambdas)*len(epsilons),3))
             i=0
             for lam in optima:
                 for eps in optima[lam]:
                     w_star = optima[lam][eps]
                     probs = spsp.expit(np.dot(X_val,w_star))
                     tally = 0
                     total = 0
                     for j in range(len(probs)):
                         if probs[j] >= 0.5:
                             prob = 1
                         if probs[j] < 0.5:
                             prob = 0
                         if prob == y_val[j]:
                             tally += 1
                         total += 1
                     acc = tally/total
                     Accs[i] = [acc,lam,eps]
                     i+=1
                     print('lam = '+str(lam)+'\teps =',eps,'\t\tAccuracy: ',acc)

             return Accs

In [13]: Accs_batch = HyperparameterAccs(lambdas,epsilons,optima_batch,X_val,y_val)
```

```
lam = 0.001       eps = 1e-05               Accuracy:  0.955
lam = 0.001       eps = 0.0001               Accuracy:  0.958
lam = 0.001       eps = 0.001               Accuracy:  0.959
lam = 0.01       eps = 1e-05               Accuracy:  0.952
lam = 0.01       eps = 0.0001               Accuracy:  0.951
lam = 0.01       eps = 0.001               Accuracy:  0.952
lam = 0.1       eps = 1e-05               Accuracy:  0.935
lam = 0.1       eps = 0.0001               Accuracy:  0.935
lam = 0.1       eps = 0.001               Accuracy:  0.935
lam = 1.0       eps = 1e-05               Accuracy:  0.922
lam = 1.0       eps = 0.0001               Accuracy:  0.921
lam = 1.0       eps = 0.001               Accuracy:  0.921
lam = 10.0       eps = 1e-05               Accuracy:  0.913
lam = 10.0       eps = 0.0001               Accuracy:  0.913
```

```
lam = 10.0        eps = 0.001                Accuracy:  0.913
```

```
In [14]: print(optima_batch[0.001][0.001])
         print(Accs_batch)
```

```
[  -9.76375855   13.46225692   -5.2732488   -66.13364483   10.78171502
   19.51816531  -26.64844479  146.9055984    -5.99243176    2.76490805
   19.14298793    3.53846425]
[[  9.55000000e-01   1.00000000e-03   1.00000000e-05]
 [  9.58000000e-01   1.00000000e-03   1.00000000e-04]
 [  9.59000000e-01   1.00000000e-03   1.00000000e-03]
 [  9.52000000e-01   1.00000000e-02   1.00000000e-05]
 [  9.51000000e-01   1.00000000e-02   1.00000000e-04]
 [  9.52000000e-01   1.00000000e-02   1.00000000e-03]
 [  9.35000000e-01   1.00000000e-01   1.00000000e-05]
 [  9.35000000e-01   1.00000000e-01   1.00000000e-04]
 [  9.35000000e-01   1.00000000e-01   1.00000000e-03]
 [  9.22000000e-01   1.00000000e+00   1.00000000e-05]
 [  9.21000000e-01   1.00000000e+00   1.00000000e-04]
 [  9.21000000e-01   1.00000000e+00   1.00000000e-03]
 [  9.13000000e-01   1.00000000e+01   1.00000000e-05]
 [  9.13000000e-01   1.00000000e+01   1.00000000e-04]
 [  9.13000000e-01   1.00000000e+01   1.00000000e-03]]
```

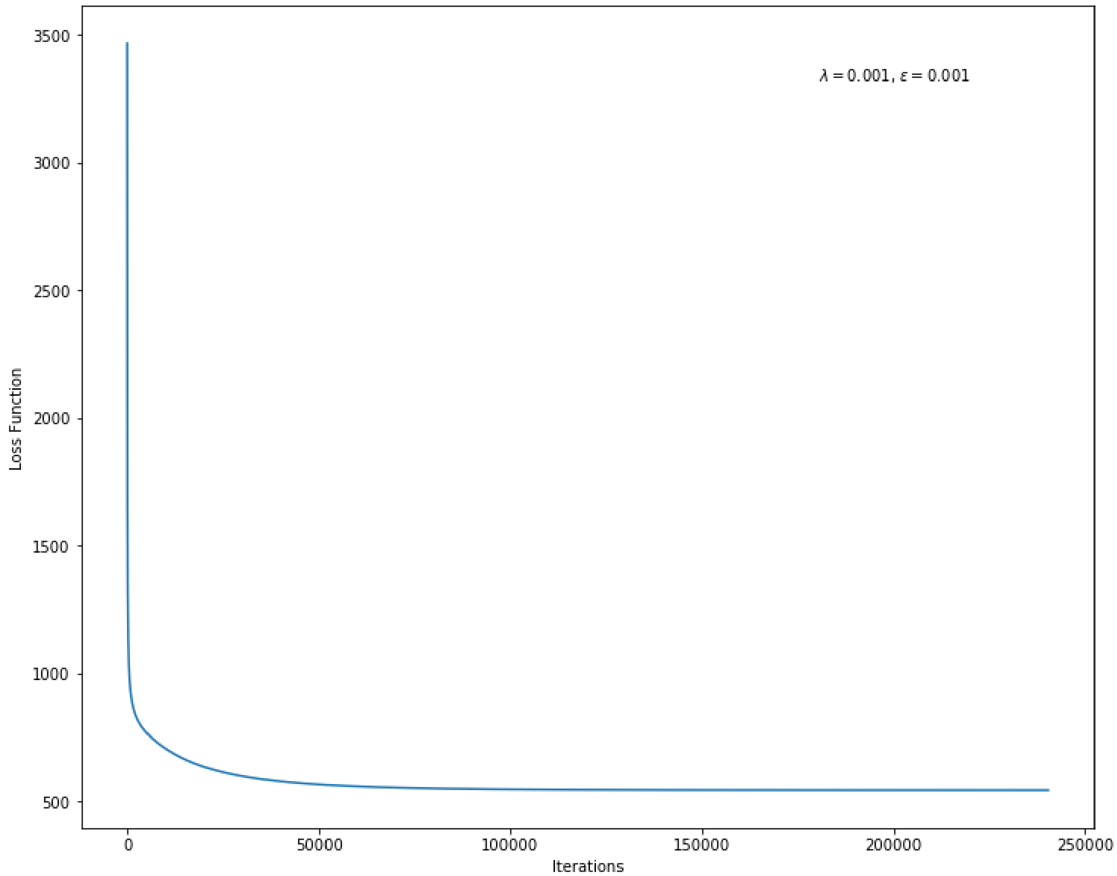
Plot the loss function vs. iteration number for the best combination of λ, ϵ.

```
In [15]: def plot_LFvIt(Accs,LvIs):
             fig = plt.figure(figsize=(12,10))
             plt.clf()

             # Find the most accurate lam,eps combo
             imax = np.argmax(Accs[:,0])
             lam = Accs[imax,1]
             eps = Accs[imax,2]

             Iters = LvIs[lam][eps][0]
             LossFn = LvIs[lam][eps][1]
             plt.plot(Iters,LossFn)
             plt.xlabel('Iterations')
             plt.ylabel('Loss Function')
             plt.text(0.75*(np.amax(Iters)-np.amin(Iters))+np.amin(Iters),0.95*(np.
             plt.show()
```

```
In [16]: plot_LFvIt(Accs_batch,LvIs_batch)
```



1.2 Part 2

Instead of batch descent, we can use the following procedure to find the optimal w using stochastic gradient descent:

(1) $w \leftarrow$ arbitrary starting point

(2) while $J(w) > 0$

$$w \leftarrow w - \epsilon(2\lambda w - X_i^T(y_i - s_i))$$

(3) return w

Step (1) - Same as in batch gradient descent

```
In [17]: w = np.zeros(len(descriptions))
```

Step (2) - we can reuse the functions for calculating the cost function and the looping process defined in part 1, step 2; we define a new function for the stochastic update rule

```
In [18]: def update_w_stoch(w,X,y,lam,eps):
             i = np.random.choice(len(X))
             y_i = y[i]
             X_i = X[i]
             s_i = spsp.expit(np.dot(X_i,w))
```

```
        w_prime = w - eps*(2*lam*w-X_i*(y_i-s_i))

        return w_prime
```

Step (3)

Again, we try several values of hyperparameters λ and ϵ to find the optimal values and we introduce a convergence tolerance that is used in the case that data is not linearly separable.

```
In [19]: tol = 1e-9
```

```
In [20]: # Collect loss function as f'n of iteration number for each combo
         optima_stoch = {}
         LvIs_stoch = {}
         for lam in lambdas:
             optima_stoch[lam]={}
             LvIs_stoch[lam]={}
             for eps in epsilons:
                 print("Lambda:",lam,"\tEpsilon:",eps)
                 w_star,iters,Js = whileloop(w,X,y,lam,eps,tol,update_w_stoch)
                 optima_stoch[lam][eps]= w_star
                 LvIs_stoch[lam][eps] = [iters,Js]
                 print(iters[len(iters)-1])
```

```
Lambda: 0.001          Epsilon: 1e-05
0:         J = 3465.7359028
500000:         J = 3364.07844802
751460
Lambda: 0.001          Epsilon: 0.0001
0:         J = 3465.7359028
407340
Lambda: 0.001          Epsilon: 0.001
0:         J = 3465.7359028
500000:         J = 1780.02392783
957690
Lambda: 0.01          Epsilon: 1e-05
0:         J = 3465.7359028
500000:         J = 3368.30796372
672060
Lambda: 0.01          Epsilon: 0.0001
0:         J = 3465.7359028
359080
Lambda: 0.01          Epsilon: 0.001
0:         J = 3465.7359028
500000:         J = 2785.21595913
1000000:         J = 2787.32786271
1500000:         J = 2785.51122138
2000000:         J = 2785.11129489
2500000:         J = 2787.0705809
3000000:         J = 2785.28963853
```

```
3500000:        J = 2785.803959
4000000:        J = 2787.22372734
4500000:        J = 2788.00847534
5000000:        J = 2786.29573236
5500000:        J = 2785.48439103
6000000:        J = 2788.66480945
6005030
Lambda: 0.1         Epsilon: 1e-05
0:        J = 3465.7359028
214990
Lambda: 0.1         Epsilon: 0.0001
0:        J = 3465.7359028
500000:        J = 3365.64013098
1000000:        J = 3365.10773544
1500000:        J = 3365.41298886
2000000:        J = 3365.44933867
2500000:        J = 3365.16683129
3000000:        J = 3365.2569922
3238330
Lambda: 0.1         Epsilon: 0.001
0:        J = 3465.7359028
500000:        J = 3365.02642054
1000000:        J = 3364.96104161
1500000:        J = 3365.04799404
2000000:        J = 3365.24070245
2500000:        J = 3364.91826271
3000000:        J = 3365.17746645
3500000:        J = 3365.1079893
4000000:        J = 3364.48461212
4500000:        J = 3366.13618097
5000000:        J = 3364.66628881
5500000:        J = 3365.56950205
6000000:        J = 3365.13812904
6500000:        J = 3366.38725617
7000000:        J = 3366.13648221
7500000:        J = 3365.20638599
8000000:        J = 3365.1508241
8500000:        J = 3364.84247723
9000000:        J = 3365.38729735
9500000:        J = 3365.47776594
10000000:        J = 3364.7427037
10000000
Lambda: 1.0         Epsilon: 1e-05
0:        J = 3465.7359028
500000:        J = 3455.19947615
892190
Lambda: 1.0         Epsilon: 0.0001
0:        J = 3465.7359028
```

```
500000:        J = 3455.22516758
806130
Lambda: 1.0         Epsilon: 0.001
0:        J = 3465.7359028
500000:        J = 3455.45095814
1000000:        J = 3455.32709589
1500000:        J = 3454.99078137
2000000:        J = 3455.46400094
2500000:        J = 3454.41694654
3000000:        J = 3454.82719222
3500000:        J = 3455.04570563
4000000:        J = 3455.11178217
4500000:        J = 3455.63697838
5000000:        J = 3455.37833326
5500000:        J = 3455.22887265
6000000:        J = 3454.9626366
6500000:        J = 3455.19824855
7000000:        J = 3455.31077341
7500000:        J = 3455.36412691
8000000:        J = 3455.8766627
8500000:        J = 3455.37583182
9000000:        J = 3455.11135474
9500000:        J = 3455.25644384
10000000:        J = 3455.08109678
10000000
Lambda: 10.0         Epsilon: 1e-05
0:        J = 3465.7359028
355830
Lambda: 10.0         Epsilon: 0.0001
0:        J = 3465.7359028
500000:        J = 3464.65317259
974090
Lambda: 10.0         Epsilon: 0.001
0:        J = 3465.7359028
500000:        J = 3464.63245865
1000000:        J = 3464.71713815
1500000:        J = 3464.75740153
2000000:        J = 3464.74406301
2500000:        J = 3464.59253951
3000000:        J = 3464.63204545
3500000:        J = 3464.68100638
4000000:        J = 3464.7078865
4500000:        J = 3464.61943668
5000000:        J = 3464.63747874
5500000:        J = 3464.67095198
6000000:        J = 3464.73626162
6500000:        J = 3464.63546903
7000000:        J = 3464.63242981
```

```
7500000:         J = 3464.47215421
8000000:         J = 3464.76204115
8500000:         J = 3464.5792482
9000000:         J = 3464.60974749
9500000:         J = 3464.77891622
10000000:         J = 3464.72769749
10000000
```

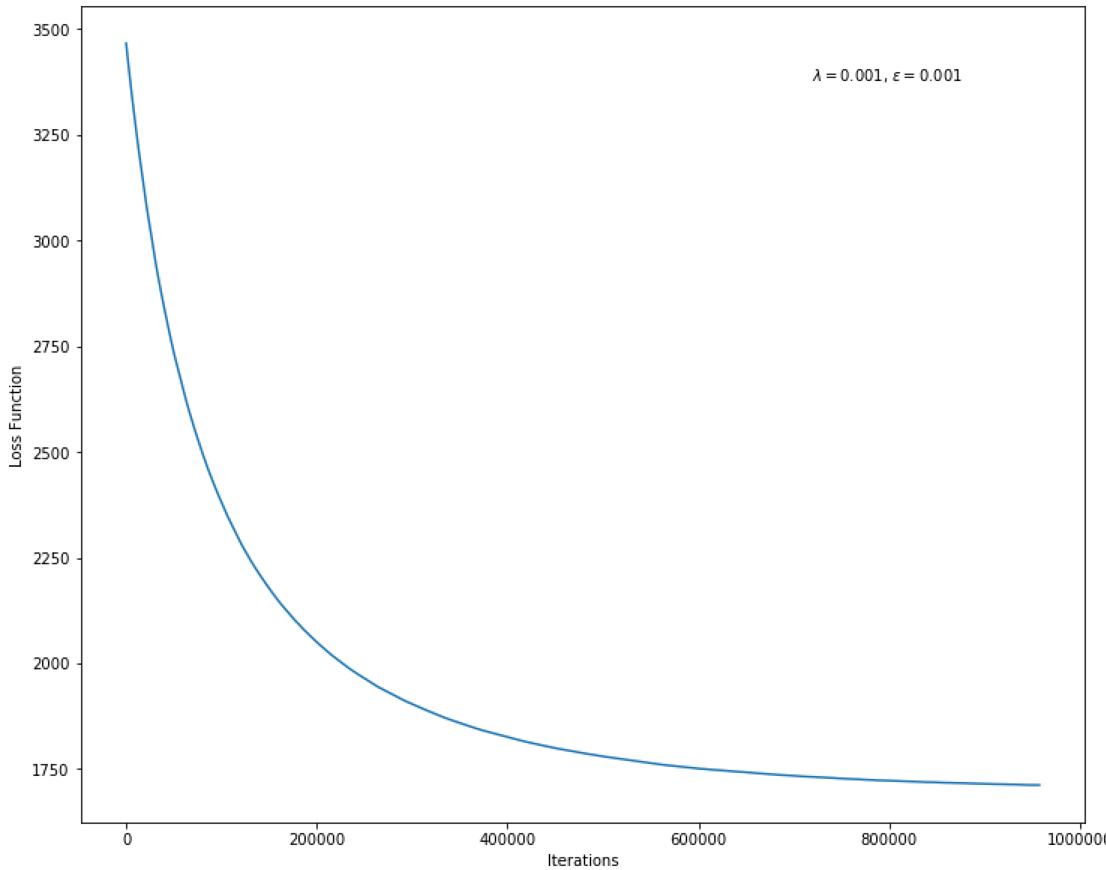
Print out a list of the optimum w* for each combination of λ and ϵ. Save the accuracies in a list.

```
In [21]: Accs_stoch = HyperparameterAccs(lambdas,epsilons,optima_stoch,X_val,y_val)
```

```
lam = 0.001         eps = 1e-05                    Accuracy:  0.902
lam = 0.001         eps = 0.0001                    Accuracy:  0.903
lam = 0.001         eps = 0.001                    Accuracy:  0.913
lam = 0.01         eps = 1e-05                    Accuracy:  0.902
lam = 0.01         eps = 0.0001                    Accuracy:  0.903
lam = 0.01         eps = 0.001                    Accuracy:  0.904
lam = 0.1         eps = 1e-05                    Accuracy:  0.901
lam = 0.1         eps = 0.0001                    Accuracy:  0.901
lam = 0.1         eps = 0.001                    Accuracy:  0.9
lam = 1.0         eps = 1e-05                    Accuracy:  0.903
lam = 1.0         eps = 0.0001                    Accuracy:  0.902
lam = 1.0         eps = 0.001                    Accuracy:  0.899
lam = 10.0         eps = 1e-05                    Accuracy:  0.904
lam = 10.0         eps = 0.0001                    Accuracy:  0.889
lam = 10.0         eps = 0.001                    Accuracy:  0.893
```

Plot the loss function vs. iteration number for the best combination of λ, ϵ.

```
In [22]: plot_LFvIt(Accs_stoch,LvIs_stoch)
```



1.3 Part 3

Now we wish to repeat part 2 (stochastic gradient descent) but using a variable ϵ.
We can accomplish this by redefining our while loop to decrease ϵ such that $\epsilon \propto 1/t$.

```
In [23]: def whileloop_deceps(w,X,y,lam,eps,tol,update_fn):
             i=0
             iters,Js = [],[]
             J = costfnJ(w,X,y,lam)
             lastJ = J+1  #dummy condition to pass while condition on first run
             while J>0 and i<=1e7 and np.absolute(lastJ-J)>tol:
                 w_prime = update_fn(w,X,y,lam,eps/(i+1))
                 w = w_prime
                 if i%10==0:
                     if i%500000==0:
                         print(str(i)+":\tJ =",str(J))
                     iters.append(i)
                     Js.append(J)
                 lastJ = J
                 J = costfnJ(w,X,y,lam)
```

```
            i+=1

        return w,iters,Js
```

Then, we call that function using the same procedure used before.

```
In [24]: # Collect loss function as f'n of iteration number for each combo
         optima_stoch_deceps = {}
         LvIs_stoch_deceps = {}
         for lam in lambdas:
             optima_stoch_deceps[lam]={}
             LvIs_stoch_deceps[lam]={}
             for eps in epsilons:
                 print("Lambda:",lam,"\tEpsilon:",eps)
                 w_star,iters,Js = whileloop_deceps(w,X,y,lam,eps,tol,update_w_stoc
                 optima_stoch_deceps[lam][eps]= w_star
                 LvIs_stoch_deceps[lam][eps] = [iters,Js]
                 print(iters[len(iters)-1])
```

```
Lambda: 0.001         Epsilon: 1e-05
0:         J = 3465.7359028
990
Lambda: 0.001         Epsilon: 0.0001
0:         J = 3465.7359028
1400
Lambda: 0.001         Epsilon: 0.001
0:         J = 3465.7359028
3500
Lambda: 0.01          Epsilon: 1e-05
0:         J = 3465.7359028
1040
Lambda: 0.01          Epsilon: 0.0001
0:         J = 3465.7359028
1430
Lambda: 0.01          Epsilon: 0.001
0:         J = 3465.7359028
1860
Lambda: 0.1           Epsilon: 1e-05
0:         J = 3465.7359028
840
Lambda: 0.1           Epsilon: 0.0001
0:         J = 3465.7359028
990
Lambda: 0.1           Epsilon: 0.001
0:         J = 3465.7359028
23150
Lambda: 1.0           Epsilon: 1e-05
0:         J = 3465.7359028
```

```
290
Lambda: 1.0         Epsilon: 0.0001
0:         J = 3465.7359028
2940
Lambda: 1.0         Epsilon: 0.001
0:         J = 3465.7359028
3020
Lambda: 10.0         Epsilon: 1e-05
0:         J = 3465.7359028
1180
Lambda: 10.0         Epsilon: 0.0001
0:         J = 3465.7359028
3930
Lambda: 10.0         Epsilon: 0.001
0:         J = 3465.7359028
3880
```

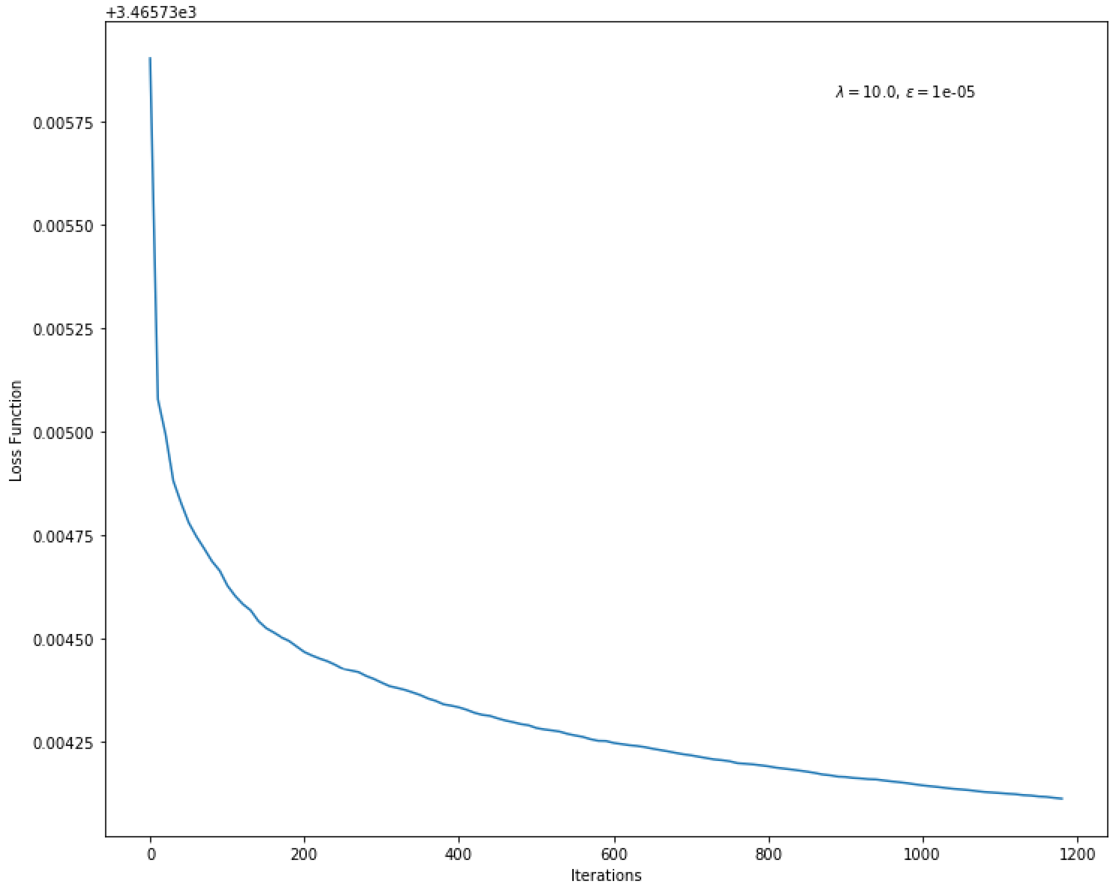
Print out a list of the optimum w* for each combination of λ and ϵ. Save the accuracies in a list.

```
In [25]: Accs_stoch_deceps = HyperparameterAccs(lambdas,epsilons,optima_stoch_decep
```

```
lam = 0.001         eps = 1e-05                 Accuracy:  0.88
lam = 0.001         eps = 0.0001                 Accuracy:  0.884
lam = 0.001         eps = 0.001                 Accuracy:  0.875
lam = 0.01         eps = 1e-05                 Accuracy:  0.884
lam = 0.01         eps = 0.0001                 Accuracy:  0.769
lam = 0.01         eps = 0.001                 Accuracy:  0.854
lam = 0.1         eps = 1e-05                 Accuracy:  0.877
lam = 0.1         eps = 0.0001                 Accuracy:  0.871
lam = 0.1         eps = 0.001                 Accuracy:  0.883
lam = 1.0         eps = 1e-05                 Accuracy:  0.761
lam = 1.0         eps = 0.0001                 Accuracy:  0.874
lam = 1.0         eps = 0.001                 Accuracy:  0.869
lam = 10.0         eps = 1e-05                 Accuracy:  0.892
lam = 10.0         eps = 0.0001                 Accuracy:  0.864
lam = 10.0         eps = 0.001                 Accuracy:  0.887
```

Plot the loss function vs. iteration number for the best combination of λ, ϵ.

```
In [26]: plot_LFvIt(Accs_stoch_deceps,LvIs_stoch_deceps)
```



Finally, we use our most successful training algorithm (in this case, batch gradient descent for $\lambda = 0.001$ and $\epsilon = 0.001$) to predict on the test data.

```
In [51]: lam,eps = 0.001,0.001
         w_star = optima_batch[lam][eps]
         preds = spsp.expit(np.dot(X_test,w_star))
         predictions = np.rint(preds)
```

We save these predictions to the csv file for Kaggle submission.

```
In [38]: IDs = np.arange(len(predictions))
         numpycsv = np.c_[IDs,predictions]
         np.savetxt(BASE_DIR+'/'+'Prob4_testpredictions.csv',numpycsv,fmt='%i',deli
```

Noting that this submission on Kaggle produced a test error of almost 12% (significantly more than the training error of 3% for the same hyperparameters), I chose a new set of hyperparameters with λ greater than the first submission. The intention of this was to reduce overfitting, as I had presumably been overfitting, giving an excellent training error but mediocre test error.

```
In [50]: lam,eps = 0.1,0.001
         w_star2 = optima_batch[lam][eps]
```

```
preds2 = spsp.expit(np.dot(X_test,w_star2))
predictions2 = np.rint(preds2)
```

Again, we save these predictions to the csv file for Kaggle submission.

```
In [45]: IDs = np.arange(len(predictions))
         numpycsv = np.c_[IDs,predictions]
         np.savetxt(BASE_DIR+'/'+'Prob4_testpredictions2.csv',numpycsv,fmt='%i',del
```

The hypothesis seems accurate. This submission (username **mnegus**) gave a score of **95.565%**.