

Problem 1

Python module: **decisiontree.py**

```
"""
decisiontree.py
=====
Train a decision tree and predict on test data
=====

This python module contains the decision tree class, which can be trained on
labeled data (input as numpy arrays: for data, an Nxd matrix with N rows
corresponding to N sample points and d columns corresponding to d features; for
labels, an N vector with labels corresponding to each of the N sample points)
"""

import numpy as np

class DecisionTree:
    """
    Build and store a decision tree, based on supplied training data.
    Use this tree to predict classifications.
    - treedepth: an integer for the max depth of the tree
    - verbose: a boolean for descriptive output
    """

    def __init__(self, treedepth=10, verbose=False, params=None):
        self.depth = treedepth
        self.verbose = verbose
        self.tree = self.Node()
        if type(treedepth) is not int:
            print('ERROR: Tree depth must be an integer.')

    def entropy(self, C, D, c, d):
        """
        Calculate entropy based on classifications above and below the
        splitrule.
        - C: sample points in-class below splitrule (left)
        - D: sample points not-in-class below splitrule (left)
        - c: sample points in-class above splitrule (right)
        - d: sample points not-in-class above splitrule (right)
        Returns the entropy.
        """

        if C != 0:
            Cfactor = -(C/(C+D))*np.log2(C/(C+D))
        else:
            Cfactor = 0
        if D != 0:
            Dfactor = -(D/(C+D))*np.log2(D/(C+D))
        else:
            Dfactor = 0
        if c != 0:
            cfactor = -(c/(c+d))*np.log2(c/(c+d))
        else:
            cfactor = 0
        if d != 0:
            dfactor = -(d/(c+d))*np.log2(d/(c+d))
        else:
            dfactor = 0
        H_left = Cfactor + Dfactor
        H_right = cfactor + dfactor
        H = ((C+D)*H_left + (c+d)*H_right)/(C+D+c+d)

        return H

    def segment(self, data, labels):
        """
        March through data and determine split which maximizes info gain.
```

```

Returns the ideal splitrule as a length-2 list where the first element
is the index of the splitting feature and the second element is the
value of that feature to split on.
"""

totals = np.bincount(labels)
if len(totals)==1:
    totals = np.append(totals,[0])
# Quick safety check
if len(labels) != len(data):
    print('ERROR_(DecisionTree.segment):_There_must_be_the_same_number_of_labels_as_datapoints.')

# Calculate the initial entropy, used to find info gain
C,D = 0,0 # C = in class left of split; D = not in class left of split
c,d = totals[1],totals[0] # c = in class right of split; d = not in class right of split
H_i = self.entropy(C,D,c,d) # the initial entropy, before any splitting

# Initialize objects to store optimal split rules for iterative comparison
maxinfo gain = 0
splitrule = []

for feature_i in range(len(data[0])):
    # Order the data for determining ideal splits
    l1dat = np.concatenate([data[:,feature_i],[labels]],axis=0)
    fv = np.sort(l1dat.T,axis=0)
    lastfeature = np.array([' ',' '])

    C,D = 0,0 # Reset the counters
    c,d = totals[1],totals[0]

    for point_i in range(len(fv)-1):

        # Update C,D,c,d to minimize runtime of entropy calc (keep at O(1) time)
        if fv[point_i,1] == 1:
            C += 1
            c -= 1
        elif fv[point_i,1] == 0:
            D += 1
            d -= 1
        else:
            print("ERROR_(DecisionTree.segment):_Classifications_can_only_be_0_or_1.")

        # Skip splitting values that are not separable
        if fv[point_i,0] == fv[point_i+1,0]:
            continue
        else:
            H_f = self.entropy(C,D,c,d)
            info gain = H_i-H_f
            if info gain > maxinfo gain:
                maxinfo gain = info gain
                splitrule = [feature_i,fv[point_i,0]]

    return splitrule

def train(self,data,labels,node=1,deep=0):
    """
    Train the decision tree on input data
    - data: Nxd numpy array with N sample points and d features
    - labels: 1D, length-N numpy array with labels for the N sample points
    - node: node class passed to function; default is 1, a flag for the head node (INTERNAL USE ONLY)
    - deep: a counter to determine current depth in tree (INTERNAL USE ONLY)
    """

    # Ensure labels are integers
    labels = labels.astype(int)

    # On the first training cycle, set the current node to the head node
    if node==1:
        node=self.tree

    # Grow decision tree
    depthlim = self.depth
    if deep < depthlim:
        splitrule = self.segment(data,labels)
    else:
        splitrule = []
    if self.verbose is True:
        print(data,labels)
    node.isleaf(data,labels,splitrule)

    # Train deeper if the node splits
    if node.nodetype == 'SplitNode':
        if self.verbose is True:

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        print('rule:',node.rule)
        print('Splitting node left and right')
        deep += 1
        node.left=self.Node()
        node.right=self.Node()
        self.train(node.leftdata,node.leftlabels,node.left,deep)
        self.train(node.rightdata,node.rightlabels,node.right,deep)
    elif node.nodetype == 'LeafNode':
        if self.verbose is True:
            print('You made a leaf node! It has value',node.leaflabel,'and',node.leafcount,'items.')
    else:
        print('ERROR (DecisionTree.train): The node type could not be identified!')

def predict(self,testdata):
    """
    Predict classifications for unlabeled data points using the previously
    trained decision tree.
    - testdata: Nxd numpy array with N sample points and d features
      *Note, dimensions N and d must match those used
      for data array in DecisionTree.train*
    Returns a 1D, length-N numpy array of predictions (one prediction per point)
    """
    npoints = len(testdata)
    predictions = np.empty(npoints)
    for point_i in range(npoints):

        # Print out decisions for a point
        if point_i<10:
            if self.verbose == 'path10':
                print('Display of choices for point %i' %point_i)

        ParentNode = self.tree
        Rule = ParentNode.rule
        while Rule is not None:
            splitfeat_i = Rule[0]
            splitval = Rule[1]
            if testdata[point_i,splitfeat_i] <= splitval:
                ChildNode = ParentNode.left
                if point_i<10:
                    if self.verbose == 'path10':
                        print('Feature #'+str(splitfeat_i+1)+':<=',testdata[point_i,splitfeat_i], '<=',splitval)
            else:
                if point_i<10:
                    if self.verbose == 'path10':
                        print('Feature #'+str(splitfeat_i+1)+':>=',testdata[point_i,splitfeat_i], '>=',splitval)
                ChildNode = ParentNode.right
            ParentNode = ChildNode
            Rule = ParentNode.rule
        predictions[point_i]=ParentNode.leaflabel
        if point_i<10:
            if self.verbose == 'path10':
                print('Point labeled as',ParentNode.leaflabel)
    return predictions.astype(int)

class Node:
    """
    Store a decision tree node, coupled in series to construct tree;
    includes a left branch, right branch, and splitrule
    """

    def __init__(self):
        self.rule = None
        self.left = None
        self.leftdata = None
        self.leftlabels = None
        self.right = None
        self.rightdata = None
        self.rightlabels = None
        self.leaflabel = None
        self.leafcount = None
        self.nodetype = None

    def isleaf(self,data,labels,splitrule):
        """Determine if this is a leaf node"""
        if splitrule:
            indsabove = self.datainds_above_split(data,splitrule)
            self.rule = splitrule
            self.leftdata,self.leftlabels = self.leftDL(data,labels,indsabove)

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        self.rightrightdata, self.rightrightlabels = self.rightrightDL(data, labels, indsabove)
        self.nodetype = 'SplitNode'

    elif not splitrule:
        self.leaflabel = np.bincount(labels).argmax()
        self.leafcount = len(labels)
        self.nodetype = 'LeafNode'

def datainds_above_split(self, data, splitrule):
    """
    Collect indices of points with values of the splitting feature
    greater than the split rule
    """

    indsabove = []
    fv = data[:, splitrule[0]]
    for point_i in range(len(fv)):
        if fv[point_i] > splitrule[1]:
            indsabove.append(point_i)

    return indsabove

def leftDL(self, data, labels, indsabove):
    """Return arrays of only left data and labels"""

    leftdata = np.delete(data, indsabove, axis=0)
    leftlabels = np.delete(labels, indsabove, axis=0)

    return leftdata, leftlabels

def rightDL(self, data, labels, indsabove):
    """Return arrays of only right data and labels"""

    rightdata = data[indsabove]
    rightlabels = labels[indsabove]

    return rightdata, rightlabels
```

Problem 2

Python module: randomforest.py

```

"""
randomforest.py
=====
Train a random forest and predict on test data
=====

This python module contains the random forest class, which can be trained on
labeled data (input as numpy arrays: for data, an Nxd matrix with N rows
corresponding to N sample points and d columns corresponding to d features; for
labels, an N vector with labels corresponding to each of the N sample points)
"""

import numpy as np

class RandomDecisionTree:
    """
    Build and store a random decision tree, based on supplied training data.
    Use this tree to predict classifications.
    - treedepth: an integer for the max depth of the tree
    - mfeatures: an integer number of random features tested for splits per node
    - verbose: a boolean for descriptive output
    """

    def __init__(self, treedepth=10, mfeatures=None, verbose=False):
        self.depth = treedepth
        self.mfeatures = mfeatures
        self.nfeatures = None
        self.verbose = verbose
        self.tree = self.Node()
        if type(treedepth) is not int:
            print('ERROR(RandomDecisionTree): Tree depth must be an integer.')
        if mfeatures and type(mfeatures) is not int:
            print('ERROR(RandomDecisionTree): The number of random features must be an integer.')

    def entropy(self, C, D, c, d):
        """
        Calculate entropy based on classifications above and below the
        splitrule.
        - C: sample points in-class below splitrule (left)
        - D: sample points not-in-class below splitrule (left)
        - c: sample points in-class above splitrule (right)
        - d: sample points not-in-class above splitrule (right)
        Returns the entropy.
        """

        if C != 0:
            Cfactor = -(C/(C+D))*np.log2(C/(C+D))
        else:
            Cfactor = 0
        if D != 0:
            Dfactor = -(D/(C+D))*np.log2(D/(C+D))
        else:
            Dfactor = 0
        if c != 0:
            cfactor = -(c/(c+d))*np.log2(c/(c+d))
        else:
            cfactor = 0
        if d != 0:
            dfactor = -(d/(c+d))*np.log2(d/(c+d))
        else:
            dfactor = 0
        H_left = Cfactor + Dfactor
        H_right = cfactor + dfactor
        H = ((C+D)*H_left + (c+d)*H_right)/(C+D+c+d)

        return H

    def pick_random_features(self):
        """Randomly choose a set of m features out of n total features"""

        mrandomfeatures = -1*np.ones(self.mfeatures)
        for i in range(self.mfeatures):
            while mrandomfeatures[i] == -1:
                feature_i = np.random.randint(self.nfeatures)
                if feature_i not in mrandomfeatures:

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        mrandomfeatures[i] = feature_i
    mrandomfeatures = np.sort(mrandomfeatures).astype(int)

    return mrandomfeatures

def segment(self, data, labels):
    """
    March through data and determine split which maximizes info gain.
    Returns the ideal splitrule as a length-2 list where the first element
    is the index of the splitting feature and the second element is the
    value of that feature to split on.
    """

    totals = np.bincount(labels)
    if len(totals)==1:
        totals = np.append(totals, [0])
    # Quick safety check
    if len(labels) != len(data):
        print('ERROR(RandomForest.segment): There must be the same number of labels as datapoints.')

    # Calculate the initial entropy, used to find info gain
    C,D = 0,0 # C = in class left of split; D = not in class left of split
    c,d = totals[1], totals[0] # c = in class right of split; d = not in class right of split
    H_i = self.entropy(C,D,c,d) # the initial entropy, before any splitting

    # Initialize objects to store optimal split rules for iterative comparison
    maxinfo gain = 0
    splitrule = []

    mrandomfeatures = self.pick_random_features()
    for feature_i in mrandomfeatures:
        # Order the data for determining ideal splits
        lbldat = np.concatenate([[data[:,feature_i]], [labels]], axis=0)

        fv = np.sort(lbldat.T, axis=0)
        lastfeature = np.array(['', ''])

        C,D = 0,0 # Reset the counters
        c,d = totals[1], totals[0]

        for point_i in range(len(fv)-1):

            # Update C,D,c,d to minimize runtime of entrop calc (keep at O(1) time)
            if fv[point_i,1] == 1:
                C += 1
                c -= 1
            elif fv[point_i,1] == 0:
                D += 1
                d -= 1
            else:
                print("ERROR(RandomForest.segment): Classifications can only be 0 or 1.")

            # Skip splitting values that are not separable
            if fv[point_i,0] == fv[point_i+1,0]:
                continue
            else:
                H_f = self.entropy(C,D,c,d)
                info gain = H_i - H_f
                if info gain > maxinfo gain:
                    maxinfo gain = info gain
                    splitrule = [feature_i, fv[point_i,0]]

    return splitrule

def train(self, data, labels, node=1, deep=0):
    """
    Train the random decision tree on input data
    - data: Nxd numpy array with N sample points and d features
    - labels: 1D, length-N numpy array with labels for the N sample points
    - node: node class passed to function; default is 1, a flag for the head node (INTERNAL USE ONLY)
    - deep: a counter to determine current depth in tree (INTERNAL USE ONLY)
    """

    # Ensure labels are integers
    labels = labels.astype(int)

    # On the first training cycle, set the current node to the head node
    # If the number of random features has not yet been set, set that too.
    if node==1:
        node=self.tree
        if self.mfeatures is None:
            self.mfeatures = np.int(np.round((np.sqrt(len(data[0]))))) # m random features

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        self.nfeatures = len(data[0])                # n total features
        if self.mfeatures > self.nfeatures:
            print('WARNING: The number of random features to choose is greater than the total number of features. Using a
                  self.mfeatures = self.nfeatures
# Grow decision tree
depthlim = self.depth
if deep < depthlim:
    splitrule = self.segment(data, labels)
else:
    splitrule = []
if self.verbose is True:
    print(data, labels)
node.isleaf(data, labels, splitrule)

# Train deeper if the node splits
if node.nodetype == 'SplitNode':
    if self.verbose is True:
        print('rule:', node.rule)
        print('Splitting node left and right')
    deep += 1
    node.left = self.Node()
    node.right = self.Node()
    self.train(node.leftdata, node.leftlabels, node.left, deep)
    self.train(node.rightdata, node.rightlabels, node.right, deep)
elif node.nodetype == 'LeafNode':
    if self.verbose is True:
        print('You made a leaf node! It has value', node.leaflabel, 'and', node.leafcount, 'items.')
    else:
        print('ERROR (RandomForest.train): The node type could not be identified!')

def predict(self, testdata)::
    """
    Predict classifications for unlabeled data points using the previously
    trained random decision tree.
    - testdata: Nxd numpy array with N sample points and d features
      *Note, dimensions N and d must match those used
      for data array in DecisionTree.train*
    Returns a 1D, length-N numpy array of predictions (one prediction per point)
    """

    npoints = len(testdata)
    predictions = np.empty(npoints)
    for point_i in range(npoints):
        ParentNode = self.tree
        Rule = ParentNode.rule
        while Rule is not None:
            splitfeat_i = Rule[0]
            splitval = Rule[1]
            if testdata[point_i, splitfeat_i] <= splitval:
                ChildNode = ParentNode.left
            else:
                ChildNode = ParentNode.right
            ParentNode = ChildNode
            Rule = ParentNode.rule
        predictions[point_i] = ParentNode.leaflabel

    return predictions.astype(int)

class Node:
    """
    Store a decision tree node, coupled in series to construct tree;
    includes a left branch, right branch, and splitrule
    """

    def __init__(self):
        self.rule = None
        self.left = None
        self.leftdata = None
        self.leftlabels = None
        self.right = None
        self.rightdata = None
        self.rightlabels = None
        self.leaflabel = None
        self.leafcount = None
        self.nodetype = None

    def isleaf(self, data, labels, splitrule)::
        """Determine if this is a leaf node"""

        if splitrule:

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```

        indsabove = self.datainds_above_split(data, splitrule)
        self.rule = splitrule
        self.leftdata, self.leftlabels = self.leftDL(data, labels, indsabove)
        self.rightdata, self.rightlabels = self.rightDL(data, labels, indsabove)
        self.nodetype = 'SplitNode'

    elif not splitrule:
        self.leaflabel = np.bincount(labels).argmax()
        self.leafcount = len(labels)
        self.nodetype = 'LeafNode'

def datainds_above_split(self, data, splitrule):
    """
    Collect indices of points with values of the splitting feature
    greater than the split rule
    """

    indsabove = []
    fv = data[:, splitrule[0]]
    for point_i in range(len(fv)):
        if fv[point_i] > splitrule[1]:
            indsabove.append(point_i)

    return indsabove

def leftDL(self, data, labels, indsabove):
    """Return arrays of only left data and labels"""

    leftdata = np.delete(data, indsabove, axis=0)
    leftlabels = np.delete(labels, indsabove, axis=0)

    return leftdata, leftlabels

def rightDL(self, data, labels, indsabove):
    """Return arrays of only right data and labels"""

    rightdata = data[indsabove]
    rightlabels = labels[indsabove]

    return rightdata, rightlabels

class RandomForest:
    """
    Build and store a random forest, based on supplied training data.
    Use this tree to predict classifications.
    - treedepth: an integer for the max depth of any tree in the forest
    - mfeatures: an integer number of random features tested for splits per node
    - verbose: a boolean for descriptive output
    """

    def __init__(self, treedepth=10, ntrees=None, mfeatures=None, subsize=None, verbose=False):
        self.treedepth = treedepth
        self.mfeatures = mfeatures
        self.subsize = subsize
        self.verbose = verbose
        self.treecount = ntrees
        self.forest = []
        if type(treedepth) is not int:
            print('ERROR(RandomForest): Tree depth must be an integer.')
        if mfeatures and type(mfeatures) is not int:
            print('ERROR(RandomForest): The number of random features must be an integer.')

    def train(self, data, labels):
        """
        Train (grow) the random forest on input data
        - data: Nxd numpy array with N sample points and d features
        - labels: 1D, length-N numpy array with labels for the N sample points
        """
        if self.subsize is None:
            self.subsize = len(data)
        if self.treecount is None:
            self.treecount = int(np.sqrt(len(data)))
        elif type(self.treecount) is not int:
            print('ERROR(RandomForest): The number of trees must be an integer.')

        for tree_i in range(self.treecount):
            # choose a random subset of the data, size "subsize", for BAGGING
            subsetindices = np.random.randint(0, self.subsize, self.subsize)
            baggeddata = data[subsetindices]

```



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        baggedlabels = labels[subindices]
        tree = RandomDecisionTree(self.treedepth, self.mfeatures, self.verbose)
        tree.train(baggeddata, baggedlabels)
        self.forest.append(tree)
        if tree_i % 5 == 0:
            print('Finished training %i out of %i' % (tree_i, self.treecount))

def predict(self, testdata):
    """
    Predict classifications for unlabeled data points using the previously
    trained random forest.
    - testdata: Nxd numpy array with N sample points and d features
      *Note, dimensions N and d must match those used
      for data array in DecisionTree.train*
    Returns a 1D, length-N numpy array of predictions (one prediction per point)
    """

    aggregatedpredictions = np.empty((self.treecount, len(testdata)))
    for tree_i in range(self.treecount):
        treepredictions = self.forest[tree_i].predict(testdata)
        aggregatedpredictions[tree_i] = treepredictions
    forestpredictions = np.round(np.average(aggregatedpredictions, axis=0)).astype(int)

    return forestpredictions

```

Problem 3

a.)

For the Titanic data set, both cabin and ticket number were removed because they were either sparse or not in a common format, so would be difficult (and essentially meaningless) to vectorize.

For the census data set, we removed the final-weight (fnlwgt) category. According to the README, this parameter *only* indicates similarity of demographics for a given state. Without knowing the location of the people in the census data, we cannot be sure that this parameter is valuable.

Other missing values were imputed. Since the vast majority of missing data points in both census and Titanic datasets seemed to be categorical, they were replaced by the mode of their respective feature. Taking the mean of binary vectorized features would not make sense as it would always tend to give a value of zero unless there were either only two classes (so the mean, rounded to the nearest integer 0 or 1, would be the mode). Similarly, it is impossible to take the mean of discrete categories before vectorization.

For the full preprocessing method, see the Jupyter notebook for preprocessing below.

b.)

The stopping criteria (and formation of a leaf node) occurred when either (1) no entropy gain was found after trying every feature over every possible split, or (2) the branch of the tree reached a maximum user-specified depth.

c.)

I did not include any special features to speed up training, other than the common sense approach to entropy evaluation over incremental splits, keeping the runtime at $O(1)$. The code does provide the user with the capacity to adjust all hyperparameters—tree depth for decision trees; tree depth, quantity of random sample points for bagging, random forest feature count, and number of random forest trees—and reducing any of these quantities will achieve a faster run time, though perhaps at a cost of accuracy.

d.)

I implemented random forests by modifying my decision tree class. I created a random forest class which generated a list of "random tree" classes. "Random trees" were decision trees that allowed for a random subsample of features to be used in generating the tree. Furthermore, "random trees" were trained on a bagged (random set, with replacement) set of data by the random forest class.

e.)

Nothing else was implemented.

Problem 4

PERFORMANCE EVALUATION

Spam	Census	Titanic
Decision Tree Training Accuracy: 84.9709 Validation Accuracy: 85.2743	Decision Tree Training Accuracy: 81.5496 Validation Accuracy: 81.9377	Decision Tree Training Accuracy: 72.7778 Validation Accuracy: 71.0000
Random Forest Training Accuracy: 82.3833 Validation Accuracy: 81.9409	Random Forest Training Accuracy: 85.9942 Validation Accuracy: 84.7188	Random Forest Training Accuracy: 88.0000 Validation Accuracy: 87.0000
Kaggle: mnegus 0.79760	Kaggle: mnegus 0.76498	Kaggle: mnegus 0.83226

Problem 5

a.)

No additional packages/features/feature transformations were used.

b.)

Below is the path through the decision tree taken by one of the data points classified as ham:

("[" = 11) > 0
("[" = 1) > 0
("[" = 11) > 1
("[" = 1) ≤ 1
("[" = 11) > 4
("#" = 1) ≤ 0
("[" = 11) > 6
("[" = 11) ≤ 11
("energy" = 0) ≤ 0
("\$" = 10) > 0
("bank" = 0) ≤ 0
("featured" = 0) ≤ 0
Point correctly labeled as 0 (ham)

Below is the path through the decision tree taken by one of the data points classified as spam:

("[" = 3) > 0
("[" = 0) ≤ 0
("message" = 2) > 0
("&" = 0) ≤ 0
("#" = 1) > 0
("[" = 3) > 2
("[" = 3) ≤ 3
("#" = 1) ≤ 1
("drug" = 5) > 0
Point correctly labeled as 1 (spam)

Problem 6

a.)

No additional packages/features/feature transformations were used.

b.)

Below is the path through the decision tree taken by one of the data points classified as making over \$50,000:

Display of choices for point 0 (Education Number = 9.0) ≤ 12

(Hours/Wk = 40.0) ≤ 49

(Age = 33) ≤ 55

(Occupation = Exec/Manag = 0) ≤ 0

(Race = Black = 0) ≤ 0

(Age = 33) ≤ 47

(Relationship = Unmarried = 1) > 0

(Capital Gains = 0) ≤ 3325

Point correctly labeled as 0 ($< \$50,000$)

Below is the path through the decision tree taken by one of the data points classified as making over \$50,000:

(Education Number = 13) > 12

(Age = 58) > 40

(Age = 58) > 46

(Age = 58) > 51

(Age = 58) $58 > 56$

(Age = 58) $58 \leq 61$

(Relationship = Husband = 1) > 0

(Age = 58) > 57

Point correctly labeled as 1 ($> \$50,000$)

Other Code

Python module: **HW05_utils.py**

```
#HW05_utils.py
#-----
# Python module for CS289A HW05
#-----
#-----

import numpy as np
from scipy import io as spio

def load_data(datapath, BASE_DIR, dictkey):
    #Load data
    data_dict = spio.loadmat(BASE_DIR+datapath)
    data = data_dict[dictkey]

    return data

def shuffle_data(data, labels):
    datlbl = np.concatenate((data, labels), axis=1)
    np.random.shuffle(datlbl)
    shuffleddata = datlbl[:, :-1]
    shuffledlabels = datlbl[:, -1]

    return shuffleddata, shuffledlabels

def val_partition(data, valfrac):
```

```
# Separate <valsetsize> items for validation
valsetsize = int(valfrac*len(data))
valset = data[:valsetsize]
trainset = data[valsetsize:]

return trainset, valset

def val_accuracy(predictions, truelabels):
    count, total = 0, 0
    for i in range(len(predictions)):
        if predictions[i] == truelabels[i]:
            count += 1
        total += 1
    valAcc = count/total

    return valAcc
```