

## Problem 1

The radioactive isotope  $^{233}\text{Pa}$  can be produced following neutron capture by  $^{232}\text{Th}$  when the resulting  $^{233}\text{Th}$  decays to  $^{233}\text{Pa}$ . In the neutron flux of a typical reactor, neutron capture in 1 g of  $^{232}\text{Th}$  produces  $^{233}\text{Th}$  at a rate of  $2.0 \times 10^{11} \text{ s}^{-1}$ .

- What are the activities (in Ci) of  $^{233}\text{Th}$  and  $^{233}\text{Pa}$  after this sample is irradiated for 1.5 hours?
- The sample is then placed in storage with no further irradiation so that the  $^{233}\text{Th}$  can decay away. What are the activities (in Ci) of  $^{233}\text{Th}$  and  $^{233}\text{Pa}$  after 48 hours of storage?
- The decay of  $^{233}\text{Pa}$  results in  $^{233}\text{U}$ , which is also radioactive. After the above sample has been stored for 1 year what is the  $^{233}\text{U}$  activity in Ci? (Hint: it should not be necessary to set up an additional differential equation to find the  $^{233}\text{U}$  activity.)

Recall:  $1 \text{ Ci} = 3.7 \times 10^{10} \text{ s}^{-1}$

## Problem 2

Use the following masses for parts (a) and (b):

n: 1.008665 u

$^1\text{H}$ : 1.007825 u

$^2\text{H}$ : 2.014102 u

$^{56}\text{Fe}$ : 55.934939 u

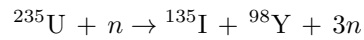
$^{98}\text{Y}$ : 97.922203 u

$^{135}\text{I}$ : 134.910048 u

$^{235}\text{U}$ : 235.043924 u

Also, recall:  $1\text{u} \cdot c^2 = 931.502\text{ MeV}$

a) Calculate the  $Q$ -value of the reaction:



b) Calculate the average binding energy per nucleon in MeV of  $^2\text{H}$ ,  $^{56}\text{Fe}$ , and  $^{235}\text{U}$ .

### Problem 3

- a) Solve the first order differential equation

$$\frac{dy}{dx} + 3y = 0$$

- b) Solve the second order differential equation ( $A$  and  $B$  are constants)

$$\frac{d^2y}{dx^2} - A^2y = B$$

The boundary condition is  $y(\pm\frac{1}{A}) = 0$ .

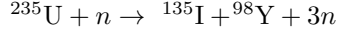
## Problem 4

# Answers

## Problem 2

a) The  $Q$ -value of a reaction is given by:

$$Q = [m(x) + m(X) - m(y) - m(Y)]c^2.$$



$$Q = [m(^{235}\text{U}) + m(n) - m(^{135}\text{I}) - m(^{98}\text{Y}) - 3m(n)]c^2$$

$$Q = [235.043924\text{u} + 1.008665\text{u} - 134.910048\text{u} - 97.922203\text{u} - 3(1.008665\text{u})]c^2$$

$$Q = 0.194343\text{u} \cdot c^2$$

$$Q = 181.031 \text{ MeV}$$

b) The binding energy  $B(Z, A)$  for any nuclei can be found approximately from the equation:

$$M(Z, A) = Zm(^1\text{H}) + (A - Z)m_n - B(Z, A)/c^2,$$

where  $M(Z, A)$ ,  $Zm(^1\text{H})$  and  $m_n$  are experimentally calculated values. Per nucleon, the binding energy can be expressed as:

$$B_A(Z, A) = [Zm(^1\text{H}) + (A - Z)m_n - M(Z, A)]c^2/A.$$

**$^2\text{H}$**

$$B_A(1, 2) = [m(^1\text{H}) + (2 - 1)m_n - M(1, 2)]c^2/2$$

$$B_A(1, 2) = [(1.007825\text{u}) + (1.008665\text{u}) - (2.014102\text{u})]c^2/2$$

$$B_A(1, 2) = 0.001194\text{u} \cdot c^2$$

$$B_A(1, 2) = 1.112 \text{ MeV}$$

**$^{56}\text{Fe}$**

$$B_A(26, 56) = [26m(^1\text{H}) + (56 - 26)m_n - M(26, 56)]c^2/56$$

$$B_A(26, 56) = [26(1.007825\text{u}) + 30(1.008665\text{u}) - (55.934939\text{u})]c^2/56$$

$$B_A(26, 56) = 0.009437\text{u} \cdot c^2$$

$$B_Z(26, 56) = 8.791 \text{ MeV}$$

**$^{235}\text{U}$**

$$B_A(92, 235) = [92m(^1\text{H}) + (235 - 92)m_n - M(92, 235)]c^2/235$$

$$B_A(92, 235) = [92(1.007825\text{u}) + 143(1.008665\text{u}) - (235.043924\text{u})]c^2/235$$

$$B_A(92, 235) = 0.00814924\text{u} \cdot c^2$$

$$B_Z(92, 235) = 7.591 \text{ MeV}$$

### Problem 3

a)

$$\frac{dy}{dx} = -3y \quad (1)$$

$$\frac{dy}{y} = -3 dx \quad (2)$$

$$\int \frac{dy}{y} = -3 \int dx \quad (3)$$

$$\ln y = -3x + C \quad (4)$$

$$y = e^{-3x+C} \quad (5)$$

$$\boxed{y = Ce^{-3x}} \quad (6)$$

Solve the second order differential equation

$$\frac{d^2y}{dx^2} - A^2y = B$$

For the homogeneous equation,  $\frac{d^2y}{dx^2} - A^2y = 0$ , try  $y_c = X_1e^{Ax} + X_2e^{-Ax}$  as the complementary solution ( $X_1$  and  $X_2$  are constants). Then

$$\frac{d^2y}{dx^2} = X_1A^2e^{Ax} + X_2A^2e^{-Ax}$$

and

$$\begin{aligned} (X_1A^2e^{Ax} + X_2A^2e^{-Ax}) - A^2(X_1e^{Ax} + X_2e^{-Ax}) &= 0 \\ A^2(X_1e^{Ax} + X_2e^{-Ax}) - A^2(X_1e^{Ax} + X_2e^{-Ax}) &= 0 \end{aligned}$$

This is true.

For the inhomogeneous equation,  $\frac{d^2y}{dx^2} - A^2y = B$ ,  $y_P = -B/A^2$  is the only particular solution satisfying the equation. The general solution is the sum of the complementary and particular solutions,  $y = y_c + y_P$ .

$$y = X_1e^{Ax} + X_2e^{-Ax} - \frac{B}{A^2}$$

Now we can solve for  $X_1$  and  $X_2$ .

$$y\left(\frac{1}{A}\right) = X_1e^{A(\frac{1}{A})} + X_2e^{-A(\frac{1}{A})} - \frac{B}{A^2} = 0$$

and

$$y\left(-\frac{1}{A}\right) = X_1e^{A(-\frac{1}{A})} + X_2e^{A(\frac{1}{A})} - \frac{B}{A^2} = 0$$

We can note that in these two equations,  $X_1$  and  $X_2$  can be interchanged freely, and so must be equal. We will say,  $X_1 = X_2 = X$ . Then, we have

$$0 = Xe^{A(-\frac{1}{A})} + Xe^{A(\frac{1}{A})} - \frac{B}{A^2}$$

$$0 = X(e^{-1} + e^1) - \frac{B}{A^2}$$

$$X = \frac{B}{A^2(\frac{1}{e} + e)}$$

The we plug this  $X$  into the final solution, which gives

$$y = \frac{B}{A^2(\frac{1}{e} + e)}(e^{Ax} + e^{-Ax}) - \frac{B}{A^2}$$

Note that  $e^{Ax} + e^{-Ax}$  is similar in form to  $\cosh(Ax)$  which we could have also used to solve this problem.