

Nuclear Engineering 150 – Discussion Section
Team Exercises #10

Problem 1

Consider an infinite slab of material described by diffusion coefficient D and macroscopic cross section Σ_a . The material extends infinitely in two dimensions, but has vacuum boundaries at $x = a$ and $x = 0$. At $x = 0$ there is also a uniformly distributed source plane with strength s'' [neutrons per area]. Find the flux in this geometry. (You may use the substitution $L = \sqrt{\frac{D}{\Sigma_a}}$.)

PROBLEM 1 - PLANAR SOURCE, 2D SLAB

Start with diffusion equation $-D \nabla^2 \Phi(\vec{r}) + \sum_a \Phi(\vec{r}) = S(\vec{r})$

With a 1D problem, simplify this to one dimension, x

$$-D \frac{d^2}{dx^2} \Phi(x) + \sum_a \Phi(x) = S(x)$$

Given a plane source, $S(x) = S_0 \delta(x)$, and at all points $x > 0, x < 0$ then $S(x) = 0$

In those regions our equation becomes

$$-D \frac{d^2}{dx^2} \Phi(x) + \sum_a \Phi(x) = 0$$

which we can simplify (using $L = \sqrt{D/\Sigma_a}$) to

$$\frac{d^2}{dx^2} \Phi(x) - \frac{1}{L^2} \Phi(x) = 0$$

An equation of this form has solutions $\Phi(x) = A e^{x/L} + B e^{-x/L}$, and now we must use boundary conditions to find A and B . The two boundary conditions correspond to the source plane at $x=0$ and the vacuum boundary at $x=a$

$$(1) J^-(a) = 0 \quad \leftarrow \text{vacuum}$$

$$(2) J^+(0) = S''/2 \quad \leftarrow \text{plane source (distributed } \frac{1}{2} \text{ in each direction)}$$

Since we're given partial currents, we must be able to convert these into fluxes. Using the definition of partial current provided by the diffusion approximation,

$$J^\pm(x) = \frac{1}{4} \Phi(x) \pm \frac{D}{2} \frac{d}{dx} \Phi(x)$$

We have $\Phi(x)$, we can solve for $\frac{d\Phi}{dx}$ in terms of constants A and B , and then can plug in our two boundary conditions (choosing either $x > 0$ or $x < 0$) to solve.

Problem 2

Consider an infinite slab of material described by diffusion coefficient D and macroscopic cross section Σ_a . The material extends infinitely in two dimensions, but has vacuum boundaries at $x = \pm a$. Inside the slab is a uniformly distributed source with strength s''' [neutrons per volume]. Find the flux in this geometry. (You may use the substitution $L = \sqrt{\frac{D}{\Sigma_a}}$.)

PROBLEM 2 - UNIFORM SOURCE, 1D SLAB

Start with diffusion equation $-D \nabla^2 \phi(r) + \Sigma_a \phi(r) = S(r)$

With a 1D problem, simplify this to one dimension, x

$$-D \frac{d^2}{dx^2} \phi(x) + \Sigma_a \phi(x) = S(x)$$

Given a uniform source, $S(x) = S''$ our equation becomes

$$-D \frac{d^2}{dx^2} \phi(x) + \Sigma_a \phi(x) = S''$$

which we can simplify (using $L = \sqrt{D/\Sigma_a}$) to

$$\frac{d^2}{dx^2} \phi(x) - \frac{1}{L^2} \phi(x) = \frac{S''}{D}$$

This is an inhomogeneous differential equation, so we must find both a particular and complementary solution to construct the general solution.

COMPLEMENTARY $\frac{d^2}{dx^2} \phi_c(x) - \frac{1}{L^2} \phi_c(x) = 0$

with solutions $\phi_c(x) = A e^{x/L} + B e^{-x/L}$

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$$\frac{d^2}{dx^2} \phi_p(x) - \frac{1}{L^2} \phi_p(x) = \frac{S''}{D}$$

with solution $\phi_p(x) = \frac{S'' L^2}{D}$

← only a constant, when subtracted from its second derivative, will be equal to a constant

The general solution is the sum of the complementary and particular solutions

$$\phi(x) = \phi_c(x) + \phi_p(x)$$

$$\phi(x) = A e^{x/L} + B e^{-x/L} + \frac{S'' L^2}{D}$$

Now we must use boundary conditions to solve for A and B. The two boundary conditions correspond to the vacuum boundaries at $x = \pm a$, as well as a symmetry term.

$$(1) J_{\vec{r}}(a) = 0$$

$$(2) J_{\vec{r}}(0) = 0$$