NE 150 Discussion 10

PROBLEM 1 - PLANAR SOURCE, 1D SLAB

Start with diffusion equation $-D\nabla^2 \Phi(\vec{r}) + \Sigma_a \Phi(\vec{r}) = S(\vec{r})$

With a 1D problem, simplify this to one dimension, x

$$-D\frac{d^2}{dx^2}\Phi(x) + \Sigma_a \Phi(x) = S(x)$$

Given a plane source, $S(x) = S_0\,\delta(x)$, and at all points $x>0, x<0$ then $S(x) = 0$

In those regions our equation becomes

$$-D\frac{d^2}{dx^2}\Phi(x) + \Sigma_a \Phi(x) = 0$$

which we can simplify (using $L = \sqrt{D/\Sigma_a}$) to

$$\frac{d^2}{dx^2}\Phi(x) - \frac{1}{L^2}\Phi(x) = 0$$

An equation of this form has solutions $\Phi(x) = Ae^{x/L} + Be^{-x/L}$, and now we must use boundary conditions to find A and B. The two boundary conditions correspond to the source plane at $x=0$ and the vacuum boundary at $x=a$

(1) $J^-(a) = 0$ ← vacuum

(2) $J^+(0) = S''/2$ ← plane source (distributed $\frac{1}{2}$ in each direction)

Since we're given partial currents, we must be able to convert these into fluxes. Using the definition of partial current provided by the diffusion approximation,

$$J^{\mp}(x) = \frac{1}{4}\Phi(x) \pm \frac{D}{2}\frac{d}{dx}\Phi(x)$$

We have $\Phi(x)$, we can solve for $\frac{d\Phi(x)}{dx}$ in terms of constants A and B, and then can plug in our two boundary conditions (choosing either $x>0$ or $x<0$) to solve.

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PROBLEM 2 - UNIFORM SOURCE, 1D SLAB

Start with diffusion equation $-D\nabla^2\phi(\vec{r}) + \Sigma_a\phi(\vec{r}) = S(\vec{r})$

With a 1D problem, simplify this to one dimension, x

$$-D\frac{d^2}{dx^2}\phi(x) + \Sigma_a\phi(x) = S(x)$$

Given a uniform source, $S(x) = S'''$ our equation becomes

$$-D\frac{d^2}{dx^2}\phi(x) + \Sigma_a\phi(x) = S'''$$

which we can simplify (using $L = \sqrt{D/\Sigma_a}$) to

$$\frac{d^2}{dx^2}\phi(x) + \frac{1}{L^2}\phi(x) = \frac{S'''}{D}$$

This is an inhomogeneous differential equation, so we must find both a particular and complementary solution to construct the general solution.

COMPLEMENTARY

$$\frac{d^2}{dx^2}\phi_c(x) - \frac{1}{L^2}\phi_c(x) = 0$$

with solutions $\phi_c(x) = Ae^{x/L} + Be^{-x/L}$

PARTICULAR

$$\frac{d^2}{dx^2}\phi_p(x) - \frac{1}{L^2}\phi_p(x) = \frac{-S'''}{D}$$

with solution $\phi_p(x) = \frac{S'''L^2}{D}$ ← only a constant, when subtracted from its second derivative, will be equal to a constant

The general solution is the sum of the complementary and particular solutions

$$\phi(x) = \phi_c(x) + \phi_p(x)$$

$$\phi(x) = Ae^{x/L} + Be^{-x/L} + \frac{S'''L^2}{D}$$

Now we must use boundary conditions to solve for A and B. The two boundary conditions correspond to the vacuum boundaries at $x = \pm a$, as well as a symmetry term.

(1) $J^{\mp}(a) = 0$

(2) $\vec{J}(0) = 0$