Nuclear Engineering 150 – Discussion Section
Team Exercise Solutions #4

Problem 1

A reactor contains 5% (by atom) enriched uranium dioxide, which is 15% of the entire core by volume.

a) Calculate the macroscopic cross section for this core if we were to treat it as a homogeneous volume.

b) If the reactor were a cube with a side length of 4 m and a beam of 10^{15} thermal neutrons were incident on one face of the cube, how many neutrons would we expect to make it through to the other side uncollided?

Nucleus	Thermal σ_t	Mass
^{1}H	20.8 b	x
^{16}O	3.5 b	x
^{235}U	607.5 b	x
^{238}U	11.8 b	x
Compound	ρ	
H_2O	1 g/cm^3	
UO_2	10.4 g/cm^3	

Problem 1 Solution

a) A macroscopic cross section for a homogeneous mixture of materials can be found by summing the cross sections of those components, weighted by their volume fractions:

$$\Sigma = \sum_i \frac{V_i}{V_t}\Sigma_i \tag{1}$$

where Σ_i is the macroscopic cross section of the i^{th} material, V_i is the volume of the i^{th} material, and V_t is the total volume. For this problem, we are told that the fraction $\frac{V_{\text{UO2}}}{V_t} = 0.15$ and $\frac{V_{\text{H2O}}}{V_t} = 0.85$. Using these in equation (1) the total macroscopic cross section of our reactor can be expressed just in terms of the macroscopic cross sections of UO_2 and H_2O.

$$\Sigma = 0.15\Sigma_{\text{UO2}} + 0.85\Sigma_{\text{H2O}} \tag{2}$$

Calculating these two macroscopic cross sections requires knowledge of each of the individual microscopic cross sections of the compounds, since the macroscopic cross section is defined as

$$\Sigma_i = n_i\sigma_i, \tag{3}$$

where n_i and σ_i are respectively the number density and total microscopic cross sections of the compounds. While neither of these quantities are given explicitly, we can calculate each from the information provided.

We can find a compound's number density by dividing the weight density by the mass of one molecule of the compound.

$$n_i = \frac{\rho_i}{m_i}$$

The densities for UO_2 and H_2O are provided, and we can find the mass of a molecule by dividing the molar mass of the material by Avogadro's number, $N_A = 6.022 \times 10^{23}$ molecules/mol.

$$m_i = \frac{M_i}{N_A},$$

and so

$$n_i = \frac{\rho_i N_A}{M_i}.$$

For our two compounds, UO_2 and H_2O these number densities are

$$n_{\mathrm{UO_2}} = \frac{\rho_{\mathrm{UO_2}} N_A}{M_{\mathrm{UO_2}}} \qquad \text{and} \qquad n_{\mathrm{H_2O}} = \frac{\rho_{\mathrm{H_2O}} N_A}{M_{\mathrm{H_2O}}}.$$

Before moving on to finding the microscopic cross sections, we can note that the molar mass of the compounds can be broken into the sum of the molar masses of their constituents according to their fraction in the compound.

$$n_{\mathrm{UO_2}} = \frac{\rho_{\mathrm{UO_2}} N_A}{f_{\mathrm{U5}} M_{\mathrm{U235}} + f_{\mathrm{U8}} M_{\mathrm{U238}} + 2M_{\mathrm{O}}} \qquad \text{and} \qquad n_{\mathrm{H_2O}} = \frac{\rho_{\mathrm{H_2O}} N_A}{2M_{\mathrm{H}} + M_{\mathrm{O}}}.$$

Next, we look at microscopic cross sections. The microscopic cross section of a compound is given by the sum of the microscopic cross sections of it's elemental components. The complete microscopic cross section for water can be found using this fact.

$$\sigma_{\mathrm{H2O}} = 2\sigma_{\mathrm{H}} + \sigma_{\mathrm{O}}$$

The microscopic cross section for UO_2 can be found similarly, however the calculation is slightly more complicated due to its enrichment in ^{235}U. First, we find the average microscopic cross section for uranium by weighting the cross sections of ^{235}U and ^{238}U by their abundance.

$$\sigma_{\mathrm{U}} = f_{\mathrm{U5}}\sigma_{\mathrm{U5}} + f_{\mathrm{U8}}\sigma_{\mathrm{U8}}$$

where f_i is the atomic fraction of material i in the compound (for uranium in this case, this fraction is just the enrichment in atom %). Then, we can complete the process exactly as we did for H_2O,

$$\begin{aligned} \sigma_{\mathrm{UO_2}} &= \sigma_{\mathrm{U}} + 2\sigma_{\mathrm{O}} \\ &= f_{\mathrm{U5}}\sigma_{\mathrm{U5}} + f_{\mathrm{U8}}\sigma_{\mathrm{U8}} + 2\sigma_{\mathrm{O}}. \end{aligned}$$

Combining these number densities and macroscopic cross sections, we can expand and rewrite the macroscopic cross sections described by equation (3) as

$$\Sigma_{\mathrm{UO_2}} = \frac{\rho_{\mathrm{UO_2}} \left(f_{\mathrm{U5}}\sigma_{\mathrm{U5}} + f_{\mathrm{U8}}\sigma_{\mathrm{U8}} + 2\sigma_{\mathrm{O}}\right) N_A}{f_{\mathrm{U5}} M_{\mathrm{U235}} + f_{\mathrm{U8}} M_{\mathrm{U238}} + 2M_{\mathrm{O}}} \qquad \text{and} \qquad \Sigma_{\mathrm{H_2O}} = \frac{\rho_{\mathrm{H_2O}} \left(2\sigma_{\mathrm{H}} + \sigma_{\mathrm{O}}\right) N_A}{2M_{\mathrm{H}} + M_{\mathrm{O}}}$$

Now we can substitute the given values.

$$\Sigma_{\mathrm{UO_2}} = \frac{\left(10.4 \text{ g/cm}^3\right) \left(0.05(607.5 \text{ b}) + 0.95(11.8 \text{ b}) + 2(3.5 \text{ b})\right) \left(6.022 \times 10^{23} \text{ mol}^{-1}\right)}{0.05(235.044 \text{ g/mol}) + 0.95(238.050 \text{ g/mol}) + 2(15.995 \text{ g/mol})}$$

$$\Sigma_{\mathrm{UO_2}} = 112.7 \text{ m}^{-1}$$

$$\Sigma_{\mathrm{H_2O}} = \frac{\left(1 \text{ g/cm}^3\right) \left(2(20.8 \text{ b}) + 3.5 \text{ b}\right) \left(6.022 \times 10^{23} \text{ mol}^{-1}\right)}{2(1.008 \text{ g/mol}) + 15.995 \text{ g/mol}}$$

$$\Sigma_{\mathrm{H_2O}} = 150.8 \text{ m}^{-1}$$

and plug these into equation (2) for the total cross section of the core.

$$\Sigma = 0.15(112.7 \text{ m}^{-1}) + 0.85(150.8 \text{ m}^{-1})$$

$$\boxed{\Sigma = 145.085 \text{ m}^{-1}}$$

b) Neutron attentuation follows a decaying exponential according to the equation

$$N = N(0)\, e^{-\Sigma x}$$

where N is the number of neutrons (you may be more familiar with this equation in terms of intensity; we have just eliminated the area and rate components from both sides).

We let $N(0)$ be our initial number of incident particles, 10^{15}, Σ be the macroscopic cross section calculated in part (a), and x be the distance traveled by the neutrons, 4 m.

$$N = 10^{15} e^{(-145.085 \text{ m}^{-1})(4 \text{ m})}$$

This works out to be about 9.153×10^{-238}, or zero particles.

$$N = 0$$

Note that this number only indicates the quantity of particles which make it through the core uncollided. Since many of these interactions are scattering collisions, and not absorptive events, it is quite possible that some neutrons will eventually make it through the reactor. A more complicated derivation would be required in that case.

Problem 2

A monoenergetic neutron beam with an intensity of 2×10^{12} neutrons/(cm^2·s) is incident on an unknown shielding material and has a beam spot of 5 cm^2. The shielding material has a thickness of 10 cm.

a) On average, 3.0×10^9 neutrons/s make it through the shield uncollided. What is the macroscopic cross section of the shield material?

b) What is the mean free path of a neutron in the shielding material?

c) If a single beam pulse is 10 μs, how many collisions are expected to take place in the shielding material?

Problem 2 Solution

a) The transmitted uncollided intensity of a neutron beam through a material with macroscopic cross section Σ is given by

$$I = I(0)\, e^{-\Sigma x}.$$

Solving for Σ, we find

$$\Sigma = \frac{1}{x} \ln\left(\frac{I(0)}{I}\right)$$

Using the values provided (and considering the intensity as per the target, rather than per cm^2), we have

$$\Sigma = \frac{1}{10 \text{ cm}} \ln\left(\frac{2 \times 10^{12} \text{ neutrons/(cm}^2\text{·s)} \left(5 \text{ cm}^2\right)}{3.0 \times 10^9 \text{ neutrons/s}}\right)$$

$$\Sigma = \frac{1}{10 \text{ cm}} \ln\left(3.33 \times 10^3\right)$$

$$\boxed{\Sigma = 0.811 \text{ cm}^{-1}}$$

b) The mean free path of a particle is defined as $\lambda \equiv \frac{1}{\Sigma}$.

$$\boxed{\lambda = 1.233 \text{ cm}}$$

c) We are told that the beam pulse is 10 μs, so when considered in conjunction with the known intensity of the neutron beam, we can find the total number of particles produced by the beam per area. We define the neutron fluence, Φ, as the number of neutrons per cm^2, calculated as

$$\Phi = It = (2 \times 10^{12} \text{ neutrons/(cm}^2 \cdot \text{s)})(10^{-5} \text{ s}) = 2 \times 10^7 \text{ neutrons/cm}^2$$

The number of interactions, R, is

$$R = \Phi \Sigma V$$

where Φ is the incident neutron fluence in $\left[\frac{\text{neutrons}}{\text{cm}^2}\right]$, Σ is the macroscopic cross section of the shielding material in $\left[\frac{1}{\text{cm}}\right]$, and V is the volume of the shield in $\left[\text{cm}^3\right]$.

We have already found Φ, we calculated Σ in part (a), and we can determine V by multiplying the area of the beam spot by the thickness of the target.

$$V = 5 \text{ cm}^2 \times 10 \text{ cm} = 50 \text{ cm}^3$$

The total number of collisions is then

$$R = (2 \times 10^7 \text{ cm}^{-2})(0.811 \text{ cm}^{-1})(50 \text{ cm}^3)$$

$$\boxed{R = 8.11 \times 10^8 \text{ collisions}}$$