

Nuclear Engineering 150 – Discussion Section

Team Exercise Solutions #1

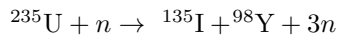
*Problems 1 & 2 borrowed from Nuclear Engineering 101 homework problem sets, Fall 2016

Problem 1 Solution

Problem 2 Solution

a) The Q -value of a reaction is given by:

$$Q = [m(x) + m(X) - m(y) - m(Y)]c^2.$$



$$Q = [m(^{235}\text{U}) + m(n) - m(^{135}\text{I}) - m(^{98}\text{Y}) - 3m(n)]c^2$$

$$Q = [235.043924\text{u} + 1.008665\text{u} - 134.910048\text{u} - 97.922203\text{u} - 3(1.008665\text{u})]c^2$$

$$Q = 0.194343\text{u} \cdot c^2$$

$Q = 181.031 \text{ MeV}$

b) The binding energy $B(Z, A)$ for any nuclei can be found approximately from the equation:

$$M(Z, A) = Zm(^1\text{H}) + (A - Z)m_n - B(Z, A)/c^2,$$

where $M(Z, A)$, $Zm(^1\text{H})$ and m_n are experimentally calculated values. Per nucleon, the binding energy can be expressed as:

$$B_A(Z, A) = [Zm(^1\text{H}) + (A - Z)m_n - M(Z, A)]c^2/A.$$

^2H

$$B_A(1, 2) = [m(^1\text{H}) + (2 - 1)m_n - M(1, 2)]c^2/2$$

$$B_A(1, 2) = [(1.007825\text{u}) + (1.008665\text{u}) - (2.014102\text{u})]c^2/2$$

$$B_A(1, 2) = 0.001194\text{u} \cdot c^2$$

$B_A(1, 2) = 1.112 \text{ MeV}$

^{56}Fe

$$B_A(26, 56) = [26m(^1\text{H}) + (56 - 26)m_n - M(26, 56)]c^2/56$$

$$B_A(26, 56) = [26(1.007825\text{u}) + 30(1.008665\text{u}) - (55.934939\text{u})]c^2/56$$

$$B_A(26, 56) = 0.009437\text{u} \cdot c^2$$

$B_Z(26, 56) = 8.791 \text{ MeV}$

^{235}U

$$B_A(92, 235) = [92m(^1\text{H}) + (235 - 92)m_n - M(92, 235)]c^2/235$$

$$B_A(92, 235) = [92(1.007825\text{u}) + 143(1.008665\text{u}) - (235.043924\text{u})]c^2/235$$

$$B_A(92, 235) = 0.00814924 \text{ u} \cdot c^2$$

$$B_Z(92, 235) = 7.591 \text{ MeV}$$

Problem 3 Solution

a)

$$\frac{dy}{dx} = -3y \quad (1)$$

$$\frac{dy}{y} = -3 dx \quad (2)$$

$$\int \frac{dy}{y} = -3 \int dx \quad (3)$$

$$\ln y = -3x + C \quad (4)$$

$$y = e^{-3x+C} \quad (5)$$

$$(6)$$

$$y = Ce^{-3x}$$

Solve the second order differential equation

$$\frac{d^2y}{dx^2} - A^2y = B$$

For the homogeneous equation, $\frac{d^2y}{dx^2} - A^2y = 0$, try $y_c = X_1e^{Ax} + X_2e^{-Ax}$ as the complementary solution (X_1 and X_2 are constants). Then

$$\frac{d^2y}{dx^2} = X_1A^2e^{Ax} + X_2A^2e^{-Ax}$$

and

$$(X_1A^2e^{Ax} + X_2A^2e^{-Ax}) - A^2(X_1e^{Ax} + X_2e^{-Ax}) = 0$$

$$A^2(X_1e^{Ax} + X_2e^{-Ax}) - A^2(X_1e^{Ax} + X_2e^{-Ax}) = 0$$

This is true.

For the inhomogeneous equation, $\frac{d^2y}{dx^2} - A^2y = B$, $y_P = -B/A^2$ is the only particular solution satisfying the equation. The general solution is the sum of the complementary and particular solutions, $y = y_c + y_P$.

$$y = X_1e^{Ax} + X_2e^{-Ax} - \frac{B}{A^2}$$

Now we can solve for X_1 and X_2 .

$$y\left(\frac{1}{A}\right) = X_1e^{A(\frac{1}{A})} + X_2e^{-A(\frac{1}{A})} - \frac{B}{A^2} = 0$$

and

$$y\left(-\frac{1}{A}\right) = X_1e^{A(-\frac{1}{A})} + X_2e^{A(\frac{1}{A})} - \frac{B}{A^2} = 0$$

We can note that in these two equations, X_1 and X_2 can be interchanged freely, and so must be equal. We will say, $X_1 = X_2 = X$. Then, we have

$$0 = Xe^{A(-\frac{1}{A})} + Xe^{A(\frac{1}{A})} - \frac{B}{A^2}$$

$$0 = X(e^{-1} + e^1) - \frac{B}{A^2}$$

$$X = \frac{B}{A^2(\frac{1}{e} + e)}$$

The we plug this X into the final solution, which gives

$$y = \frac{B}{A^2(\frac{1}{e} + e)}(e^{Ax} + e^{-Ax}) - \frac{B}{A^2}$$

Note that $e^{Ax} + e^{-Ax}$ is similar in form to $\cosh(Ax)$ which we could have also used to solve this problem.

Problem 4 Solution

- (4) ^{238}U fission (high fast region)
- (5) ^{239}Pu fission (resonance peak)
- (1) ^{155}Gd absorption (consistently high absorption)
- (2) ^{235}U fission (high thermal fission cross section)
- (3) ^{238}U absorption