NE 150 Discussion 10

SOLUTIONS TO THE PRKE - THE INHOUR EQUATION

$$\frac{dP}{dt} = \frac{\rho_0 - \beta}{\Lambda} P(t) + \sum_{j=1}^{6} \lambda_j C_j(t)$$

$$\frac{dC_j}{dt} = \frac{\beta_j}{\Lambda} P(t) - \lambda_j C_j(t), \quad j = 1 \ldots 6$$

We had solutions to this of the form $P(t) = Pe^{st}$ and $C_j(t) = C_j e^{st}$,

↳ this gives a system of equations which you can solve as you choose

$$sP = \frac{\rho_0 - \beta}{\Lambda} P + \sum_{j}^{6} \lambda_j C_j$$

$$sC_j = \frac{\beta_j}{\Lambda} P - \lambda_j C_j$$

we have divided out e^{st} factors from both sides

Now, take the second equation and solve for C_j:

$$C_j = \frac{\beta_j P}{\Lambda(s + \lambda_j)}$$

then plug back into the first equation

$$sP = \frac{\rho_0 - \beta}{\Lambda} P + \sum_{j}^{6} \lambda_j \frac{\beta_j P}{\Lambda(s + \lambda_j)}$$

$$s = \frac{\rho_0}{\Lambda} - \frac{\beta}{\Lambda} + \sum_{j}^{6} \lambda_j \frac{\beta_j}{\Lambda(s + \lambda_j)}$$

and solve for ρ_0 (noting that $\beta = \sum_{j}^{6} \beta_j$)

$$s = \frac{\rho_0}{\Lambda} - \frac{1}{\Lambda}\left(\sum_{j}^{6} \frac{\beta_j(s + \lambda_j)}{s + \lambda_j} - \frac{\lambda\beta_j}{s + \lambda_j}\right)$$

$$s = \frac{\rho_0}{\Lambda} - \frac{1}{\Lambda}\left(\sum_{j}^{6} \frac{\beta_j s}{s + \lambda_j}\right)$$

$$\rho_0 = \Lambda s + \sum_{j}^{6} \frac{\beta_j s}{s + \lambda_j}$$

⟵ this is the inhour equation