Nuclear Engineering 150 – Discussion Section
Team Exercise Solutions #1

*Problems 1 & 2 borrowed from Nuclear Engineering 101 homework problem sets, Fall 2016

Problem 1 Solution

Problem 2 Solution

a) The Q-value of a reaction is given by:

$$Q = [m(\mathrm{x}) + m(\mathrm{X}) - m(\mathrm{y}) - m(\mathrm{Y})]c^2.$$

$^{235}\mathrm{U} + n \rightarrow\ ^{135}\mathrm{I} + ^{98}\mathrm{Y} + 3n$

$$Q = [m(^{235}\mathrm{U}) + m(n) - m(^{135}\mathrm{I}) - m(^{98}\mathrm{Y}) - 3m(n)]c^2$$

$$Q = [235.043924\mathrm{u} + 1.008665\mathrm{u} - 134.910048\mathrm{u} - 97.922203\mathrm{u} - 3(1.008665\mathrm{u})]c^2$$

$$Q = 0.194343\mathrm{u} \cdot c^2$$

$$\boxed{Q = 181.031\ \mathrm{MeV}}$$

b) The binding energy $B(Z, A)$ for any nuclei can be found approximately from the equation:

$$M(Z, A) = Zm(^1\mathrm{H}) + (A - Z)m_n - B(Z, A)/c^2,$$

where $M(Z, A)$, $Zm(^1\mathrm{H})$ and m_n are experimentally calculated values. Per nucleon, the binding energy can be expressed as:

$$B_A(Z, A) = [Zm(^1\mathrm{H}) + (A - Z)m_n - M(Z, A)]c^2/A.$$

^{2}**H**

$$B_A(1, 2) = [m(^1\mathrm{H}) + (2 - 1)m_n - M(1, 2)]c^2/2$$

$$B_A(1, 2) = [(1.007825\mathrm{u}) + (1.008665\mathrm{u}) - (2.014102\mathrm{u})]c^2/2$$

$$B_A(1, 2) = 0.001194\mathrm{u} \cdot c^2$$

$$\boxed{B_A(1, 2) = 1.112\ \mathrm{MeV}}$$

^{56}**Fe**

$$B_A(26, 56) = [26m(^1\mathrm{H}) + (56 - 26)m_n - M(26, 56)]c^2/56$$

$$B_A(26, 56) = [26(1.007825\mathrm{u}) + 30(1.008665\mathrm{u}) - (55.934939\mathrm{u})]c^2/56$$

$$B_A(26, 56) = 0.009437\mathrm{u} \cdot c^2$$

$$\boxed{B_Z(26, 56) = 8.791\ \mathrm{MeV}}$$

^{235}**U**

$$B_A(92, 235) = [92m(^1\mathrm{H}) + (235 - 92)m_n - M(92, 235)]c^2/235$$

$$B_A(92, 235) = [92(1.007825\mathrm{u}) + 143(1.008665\mathrm{u}) - (235.043924\mathrm{u})]c^2/235$$

$B_A(92,235) = 0.00814924u\text{u} \cdot c^2$

$\boxed{B_Z(92,235) = 7.591 \text{ MeV}}$

Problem 3 Solution

a)

$$\frac{dy}{dx} = -3y \tag{1}$$
$$\frac{dy}{y} = -3\,dx \tag{2}$$
$$\int \frac{dy}{y} = -3\int dx \tag{3}$$
$$\ln y = -3x + C \tag{4}$$
$$y = e^{-3x+C} \tag{5}$$
$$\tag{6}$$

$$\boxed{y = Ce^{-3x}}$$

Solve the second order differential equation

$$\frac{d^2y}{dx^2} - A^2 y = B$$

For the homogeneous equation, $\frac{d^2y}{dx^2} - A^2y = 0$, try $y_c = X_1 e^{Ax} + X_2 e^{-Ax}$ as the complementary solution (X_1 and X_2 are constants). Then

$$\frac{d^2y}{dx^2} = X_1 A^2 e^{Ax} + X_2 A^2 e^{-Ax}$$

and

$$(X_1 A^2 e^{Ax} + X_2 A^2 e^{-Ax}) - A^2(X_1 e^{Ax} + X_2 e^{-Ax}) = 0$$
$$A^2(X_1 e^{Ax} + X_2 e^{-Ax}) - A^2(X_1 e^{Ax} + X_2 e^{-Ax}) = 0$$

This is <u>true</u>.

For the inhomogeneous equation, $\frac{d^2y}{dx^2} - A^2y = B$, $y_P = -B/A^2$ is the only particular solution satisfying the equation. The general solution is the sum of the complementary and particular solutions, $y = y_c + y_p$.

$$y = X_1 e^{Ax} + X_2 e^{-Ax} - \frac{B}{A^2}$$

Now we can solve for X_1 and X_2.

$$y(\frac{1}{A}) = X_1 e^{A(\frac{1}{A})} + X_2 e^{-A(\frac{1}{A})} - \frac{B}{A^2} = 0$$

and

$$y(-\frac{1}{A}) = X_1 e^{A(-\frac{1}{A})} + X_2 e^{A(\frac{1}{A})} - \frac{B}{A^2} = 0$$

We can note that in these two equations, X_1 and X_2 can be interchanged freely, and so must be equal. We will say, $X_1 = X_2 = X$. Then, we have

$$0 = Xe^{A(-\frac{1}{A})} + Xe^{A(\frac{1}{A})} - \frac{B}{A^2}$$
$$0 = X(e^{-1} + e^{1}) - \frac{B}{A^2}$$

$$X = \frac{B}{A^2(\frac{1}{e} + e)}$$

The we plug this X into the final solution, which gives

$$y = \frac{B}{A^2(\frac{1}{e} + e)}(e^{Ax} + e^{-Ax}) - \frac{B}{A^2}$$

Note that $e^{Ax} + e^{-Ax}$ is similar in form to $\cosh(Ax)$ which we could have also used to solve this problem.

Problem 4 Solution

(4) ^{238}U fission (high fast region)
(5) ^{239}Pu fission (resonance peak)
(1) ^{155}Gd absorption (consistently high absorption)
(2) ^{235}U fission (high thermal fission cross section)
(3) ^{238}U absorption