NUCENG 150 **Mitch Negus**

Group Problems Week 1

Problem 1

The radioactive isotope ^{233}Pa can be produced following neutron capture by ^{232}Th when the resulting ^{233}Th decays to ^{233}Pa. In the neutron flux of a typical reactor, neutron capture in 1 g of ^{232}Th produces ^{233}Th at of a rate of 2.0×10^{11} s^{-1}.

a) What are the activities (in Ci) of ^{233}Th and ^{233}Pa after this sample is irradiated for 1.5 hours?

b) The sample is then placed in storage with no further irradiation so that the ^{233}Th can decay away. What are the activities (in Ci) of ^{233}Th and ^{233}Pa after 48 hours of storage?

c) The decay of ^{233}Pa results in ^{233}U, which is also radioactive. After the above sample has been stored for 1 year what is the ^{233}U activity in Ci? (Hint: it should not be necessary to set up an additional differential equation to find the ^{233}U activity.)

Recall: 1 Ci $= 3.7 \times 10^{10}$ s^{-1}

Problem 2

Use the following masses for parts (a) and (b):

n: 1.008665 u
^{1}H: 1.007825 u
^{2}H: 2.014102 u
^{56}Fe: 55.934939 u
^{98}Y: 97.922203 u
^{135}I: 134.910048 u
^{235}U: 235.043924 u

Also, recall: $1\text{u} \cdot c^2 = 931.502$ MeV

a) Calculate the Q-value of the reaction:

$$^{235}\text{U} + n \rightarrow {}^{135}\text{I} + {}^{98}\text{Y} + 3n$$

b) Calculate the average binding energy per nucleon in MeV of ^{2}H, ^{56}Fe, and ^{235}U.

Problem 3

a) Solve the first order differential equation

$$\frac{dy}{dx} + 3y = 0$$

b) Solve the second order differential equation (A and B are constants)

$$\frac{d^2y}{dx^2} - A^2y = B$$

The boundary condition is $y(\pm\frac{1}{A}) = 0$.

Problem 4

Answers

Problem 2

a) The Q-value of a reaction is given by:

$$Q = [m(\mathrm{x}) + m(\mathrm{X}) - m(\mathrm{y}) - m(\mathrm{Y})]c^2.$$

$^{235}\mathrm{U} + n \rightarrow \ ^{135}\mathrm{I} + ^{98}\mathrm{Y} + 3n$

$$Q = [m(^{235}\mathrm{U}) + m(n) - m(^{135}\mathrm{I}) - m(^{98}\mathrm{Y}) - 3m(n)]c^2$$

$$Q = [235.043924\mathrm{u} + 1.008665\mathrm{u} - 134.910048\mathrm{u} - 97.922203\mathrm{u} - 3(1.008665\mathrm{u})]c^2$$

$$Q = 0.194343\mathrm{u} \cdot c^2$$

$$\boxed{Q = 181.031 \text{ MeV}}$$

b) The binding energy $B(Z, A)$ for any nuclei can be found approximately from the equation:

$$M(Z, A) = Zm(^1\mathrm{H}) + (A - Z)m_n - B(Z, A)/c^2,$$

where $M(Z, A)$, $Zm(^1\mathrm{H})$ and m_n are experimentally calculated values. Per nucleon, the binding energy can be expressed as:

$$B_A(Z, A) = [Zm(^1\mathrm{H}) + (A - Z)m_n - M(Z, A)]c^2/A.$$

^{2}**H**

$$B_A(1, 2) = [m(^1\mathrm{H}) + (2 - 1)m_n - M(1, 2)]c^2/2$$

$$B_A(1, 2) = [(1.007825\mathrm{u}) + (1.008665\mathrm{u}) - (2.014102\mathrm{u})]c^2/2$$

$$B_A(1, 2) = 0.001194\mathrm{u} \cdot c^2$$

$$\boxed{B_A(1, 2) = 1.112 \text{ MeV}}$$

^{56}**Fe**

$$B_A(26, 56) = [26m(^1\mathrm{H}) + (56 - 26)m_n - M(26, 56)]c^2/56$$

$$B_A(26, 56) = [26(1.007825\mathrm{u}) + 30(1.008665\mathrm{u}) - (55.934939\mathrm{u})]c^2/56$$

$$B_A(26, 56) = 0.009437\mathrm{u} \cdot c^2$$

$$\boxed{B_Z(26, 56) = 8.791 \text{ MeV}}$$

^{235}**U**

$$B_A(92, 235) = [92m(^1\mathrm{H}) + (235 - 92)m_n - M(92, 235)]c^2/235$$

$$B_A(92, 235) = [92(1.007825\mathrm{u}) + 143(1.008665\mathrm{u}) - (235.043924\mathrm{u})]c^2/235$$

$$B_A(92, 235) = 0.00814924u\mathrm{u} \cdot c^2$$

$$\boxed{B_Z(92, 235) = 7.591 \text{ MeV}}$$

Problem 3

a)

$$\frac{dy}{dx} = -3y \tag{1}$$
$$\frac{dy}{y} = -3\,dx \tag{2}$$
$$\int \frac{dy}{y} = -3\int dx \tag{3}$$
$$\ln y = -3x + C \tag{4}$$
$$y = e^{-3x+C} \tag{5}$$
$$\tag{6}$$

$$\boxed{y = Ce^{-3x}}$$

Solve the second order differential equation

$$\frac{d^2y}{dx^2} - A^2y = B$$

For the homogeneous equation, $\frac{d^2y}{dx^2} - A^2y = 0$, try $y_c = X_1e^{Ax} + X_2e^{-Ax}$ as the complementary solution (X_1 and X_2 are constants). Then

$$\frac{d^2y}{dx^2} = X_1A^2e^{Ax} + X_2A^2e^{-Ax}$$

and

$$(X_1A^2e^{Ax} + X_2A^2e^{-Ax}) - A^2(X_1e^{Ax} + X_2e^{-Ax}) = 0$$
$$A^2(X_1e^{Ax} + X_2e^{-Ax}) - A^2(X_1e^{Ax} + X_2e^{-Ax}) = 0$$

This is <u>true</u>.

For the inhomogeneous equation, $\frac{d^2y}{dx^2} - A^2y = B$, $y_P = -B/A^2$ is the only particular solution satisfying the equation. The general solution is the sum of the complementary and particular solutions, $y = y_c + y_p$.

$$y = X_1e^{Ax} + X_2e^{-Ax} - \frac{B}{A^2}$$

Now we can solve for X_1 and X_2.

$$y(\frac{1}{A}) = X_1e^{A(\frac{1}{A})} + X_2e^{-A(\frac{1}{A})} - \frac{B}{A^2} = 0$$

and

$$y(-\frac{1}{A}) = X_1e^{A(-\frac{1}{A})} + X_2e^{A(\frac{1}{A})} - \frac{B}{A^2} = 0$$

We can note that in these two equations, X_1 and X_2 can be interchanged freely, and so must be equal. We will say, $X_1 = X_2 = X$. Then, we have

$$0 = Xe^{A(-\frac{1}{A})} + Xe^{A(\frac{1}{A})} - \frac{B}{A^2}$$
$$0 = X(e^{-1} + e^{1}) - \frac{B}{A^2}$$
$$X = \frac{B}{A^2(\frac{1}{e} + e)}$$

The we plug this X into the final solution, which gives

$$y = \frac{B}{A^2(\frac{1}{e} + e)}(e^{Ax} + e^{-Ax}) - \frac{B}{A^2}$$

Note that $e^{Ax} + e^{-Ax}$ is similar in form to $\cosh(Ax)$ which we could have also used to solve this problem.