DISCUSSION 12 - PROBLEM 1

Spherical Core: $D_c, \Sigma_{a,c}, \Sigma_f, \nu$ $\left(k_c^2 = \frac{\nu\Sigma_f - \Sigma_{a,c}}{D_c}\right)$ with radius R

Reflector: $D_R, \Sigma_{a,R}$ $\left(L_R = \sqrt{\frac{D_R}{\Sigma_{a,R}}}\right)$ with thickness $\frac{R}{2}$

a.) The Spherical Laplacian: $\nabla^2 f(r,\theta,\phi) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$

$\hookrightarrow$ simplifies in 1D $\nabla^2 f(r) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right)$

The general diffusion equation is

$$-D_c\nabla^2\Phi_c(r) + \Sigma_{a,c}\Phi_c(r) = \nu\Sigma_f\,\Phi_c(r) \quad \text{in the core } (0 < r < R)$$

and

$$-D_R\nabla^2\Phi_R(r) + \Sigma_{a,R}\Phi_R(r) = 0 \quad \text{in the reflector } \left(R < r < \frac{3R}{2}\right)$$

Consolidating terms and dividing by the diffusion coefficient (and expanding ∇^2)

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\Phi_c(r)\right) + k_c^2\Phi_c(r) = 0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\Phi_R(r)\right) - \frac{1}{L_R^2}\Phi_R(r) = 0$$

We will need four boundary conditions. They are

(1) $\Phi_R\left(\frac{3R}{2}+d\right) = 0$ vacuum boundaries

*d is the extrapolated distance

(2) $\Phi_R(R) = \Phi_c(R)$ continuous flux

(3) $J_R(R) = J_c(R)$ continuous current

(4) $\lim_{r\to 0}\Phi_c(r) < \infty$ flux finiteness

b.) CORE $(0 < r < R)$

The flux shape is given by a solution of the form

$$\phi_c(r) = A_c \frac{\sin(kr)}{r} + B_c \frac{\cos(kr)}{r}$$

Noticing that $\cos(kr)/r$ goes to ∞ as $r \to 0$, we conclude from BC (4) that $B_c = 0$. Then we have

$$\phi_c(r) = A_c \sin(kr)/r$$

REFLECTOR $\left(R < r < \frac{3R}{2}\right)$

The flux shape is given by a solution of the form

$$\phi_R(r) = \frac{A_R}{r} e^{-r/L_R} + \frac{B_R}{r} e^{r/L_R}$$

Using the vacuum boundary condition,

$$\phi_R\left(\frac{3R}{2}+d\right) = \frac{A_R}{\left(\frac{3R}{2}+d\right)} e^{\frac{-\left(\frac{3R}{2}+d\right)}{L_R}} + \frac{B_R}{\left(\frac{3R}{2}+d\right)} e^{\frac{\frac{3R}{2}+d}{L_R}} = 0$$

and so multiplying by $\left(\frac{3R}{2}+d\right)$ and moving the second term to the right side,

$$A_R e^{-(3R/2+d)/L_R} = -B_R e^{(3R/2+d)/L_R}$$

$$A_R = -B_R e^{(3R+2d)/L_R}$$

and so

$$\phi_R(r) = \frac{B_R}{r}\left[e^{r/L_R} - e^{-(3R+2d)-r/L_R}\right]$$

- Then, use BCs (2) and (3) to find A_c and B_R ...

DISCUSSION 12 – PROBLEM 2

a.) The steady-state, continuous energy neutron diffusion equation is (for homogeneous core)

$$-\nabla D(E)\nabla\phi(E)+\Sigma_t(E)\phi(E)=\int_0^\infty \Sigma_s(E'\to E)\phi(E')dE'+\chi(E)\int_0^\infty \nu(E')\Sigma_f(E')\phi(E')dE'$$

If D and ν are constant, this is

$$-D\nabla^2\phi(E)+\Sigma_t(E)\phi(E)=\int_0^\infty \Sigma_s(E'\to E)\phi(E')dE'+\chi(E)\nu\int_0^\infty \Sigma_f(E')\phi(E')dE'$$

b.) Now, we separate the scattering integrals into 3 groups of (flux weighted) averages:

$$\int_0^\infty \Sigma_s(E'\to E)\phi(E')dE' = \int_1 \Sigma_s(E'\to E)\phi(E')dE'+\int_2 \Sigma_s(E'\to E)\phi(E')dE'+\int_3 \Sigma_s(E'\to E)\phi(E')dE'$$

$$=\sum_{g=1}^{3}\Sigma_{s,1\to g}\phi_1+\sum_{g=1}^{3}\Sigma_{s,2\to g}\phi_2+\sum_{g=1}^{3}\Sigma_{s,3\to g}\phi_3$$

which due to direct coupling simplifies to (recall, we also neglected inscattering)

$$=\Sigma_{s,1\to 2}\phi_1+\Sigma_{s,2\to 3}\phi_2$$

We also separate the fission integral similarly:

$$\chi(E)\nu\int_0^\infty \Sigma_f(E')\phi(E')dE' = \chi_1\nu\int_1 \Sigma_f(E')\phi(E')dE'+\chi_2\nu\int_2 \Sigma_f(E')\phi(E')dE'+\chi_3\nu\int \Sigma_f(E')\phi(E')dE'$$

$$=\chi_1\nu\Sigma_{f,1}\phi_1+\chi_2\nu\Sigma_{f,2}\phi_2+\chi_3\nu\Sigma_{f,3}\phi_3$$

Where fission only occurs in the slow group, and only feeds the fast group

$$=\chi_1\nu\Sigma_{f,3}\phi_3$$

If we separate the loss terms on the left of our continuous energy diffusion equation accordingly

GROUP 1: $\quad -D\nabla^2\phi_1+\Sigma_{t,1}\phi_1=\chi_1\nu\Sigma_{f,3}\phi_3$

GROUP 2: $\quad -D\nabla^2\phi_2+\Sigma_{t,2}\phi_2=\Sigma_{s,1\to 2}\phi_1$

GROUP 3: $\quad -D\nabla^2\phi_3+\Sigma_{t,3}\phi_3=\Sigma_{s,2\to 3}\phi_2$