

Problem 1**a.)**

We are attempting to prove

$$\begin{aligned}
\mu \frac{\partial \psi}{\partial r} + \frac{(1-\mu^2)}{r} \frac{\partial \psi}{\partial \mu} &= \frac{\mu}{r^2} \frac{\partial(r^2 \psi)}{\partial r} + \frac{\partial}{\partial \mu} \left[\frac{(1-\mu^2)\psi}{r} \right] \\
&= \frac{\mu}{r^2} \left[r^2 \frac{\partial \psi}{\partial r} + \frac{\partial(r^2)}{\partial r} \psi \right] + \frac{1}{r} \left[(1-\mu^2) \frac{\partial \psi}{\partial \mu} + \frac{\partial(1-\mu^2)}{\partial \mu} \psi \right] \\
&= \frac{\mu}{r^2} \left[r^2 \frac{\partial \psi}{\partial r} + 2r\psi \right] + \frac{1}{r} \left[(1-\mu^2) \frac{\partial \psi}{\partial \mu} - 2\mu\psi \right] \\
&= \mu \frac{\partial \psi}{\partial r} + \frac{2\mu}{r} \psi + \frac{(1-\mu^2)}{r} \frac{\partial \psi}{\partial \mu} - \frac{2\mu}{r} \psi \\
\mu \frac{\partial \psi}{\partial r} + \frac{(1-\mu^2)}{r} \frac{\partial \psi}{\partial \mu} &= \mu \frac{\partial \psi}{\partial r} + \frac{(1-\mu^2)}{r} \frac{\partial \psi}{\partial \mu}
\end{aligned}$$

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b.)**Problem 2****Problem 3****Problem 4****Problem 5****Problem 6****Problem 7**

For cylindrical geometries, the diffusion equation takes the form:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \phi(\vec{r})}{\partial r} \right) + \frac{\partial^2 \phi(\vec{r})}{\partial z^2} + B^2 \phi(\vec{r}) = 0$$

where $B^2 = \frac{\nu \Sigma_f - \Sigma_a}{D}$ near criticality. Since our cylinder is infinite we lose dependence on z in our equations.

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \phi(r)}{\partial r} \right) + B^2 \phi(r) = 0$$

Solutions to this equation are zeroth order Bessel functions of first and second kind, $J_0(r)$ and $Y_0(r)$. Then,

$$\phi_b(r) = A_1 J_0(Br) + A_2 Y_0(Br).$$

Our boundary conditions demand that the flux is finite at $r = 0$, and so $A_2 = 0$. Letting $A_1 = A$, we have

$$\phi_b(r) = A J_0(Br).$$

For a bare reactor, boundary conditions force the flux to go to zero at \tilde{R}_b ,

$$\phi(\tilde{R}_b) = 0,$$

and since the flux must be nonnegative, only the first zero of J_0 can satisfy this condition. This occurs at $B\tilde{R}_b = 2.4048$. $B = B_g = \frac{2.4048}{\tilde{R}_b}$. The criticality condition for the bare core is then

$$\left(\frac{\nu \Sigma_f - \Sigma_a}{D} \right)^2 = \left(\frac{2.4048}{\tilde{R}_b} \right)^2$$

and the critical radius is

$$\tilde{R}_b = \frac{2.4048D}{\nu \Sigma_f - \Sigma_a}.$$

For a reflected core, our initial equation and the form of the solutions stay the same within the core region. With a finite flux at $r = 0$, $A_2 = 0$ and if $A_1 = A$, then in the core

$$\phi_r(r) = A J_0(Br), \quad r < R.$$

In the reflected region, we still have no z dependence, but our diffusion equation is just

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \phi(r)}{\partial r} \right) - \frac{1}{L^2} \phi(r) = 0$$

without the dependence on fission. Solutions to this equation are zeroth order modified bessel functions of the first and second kind, $I_0(r)$ and $K_0(r)$. In the reflector

$$\phi_r(r) = C_1 I_0\left(\frac{r}{L}\right) + C_2 K_0\left(\frac{r}{L}\right), \quad R < r < a$$

We can now impose the boundary condition that the flux must go to zero at the extrapolated distance in the reflector,

$$\phi_r(R + \tilde{a}) = 0.$$

We find

$$0 = C_1 I_0\left(\frac{R + \tilde{a}}{L}\right) + C_2 K_0\left(\frac{R + \tilde{a}}{L}\right).$$

$$C_2 = -C_1 \frac{I_0\left(\frac{R + \tilde{a}}{L}\right)}{K_0\left(\frac{R + \tilde{a}}{L}\right)}$$

If we let $C_1 = C$, then in the reflector

$$\phi_r(r) = C I_0\left(\frac{r}{L}\right) - C \frac{I_0\left(\frac{R + \tilde{a}}{L}\right)}{K_0\left(\frac{R + \tilde{a}}{L}\right)} K_0\left(\frac{r}{L}\right), \quad R < r < a$$

Finally, we use our boundary condition that the flux must be equal at the boundaries.

$$\phi_r(R) = A J_0(BR) = C I_0\left(\frac{R}{L}\right) - C \frac{I_0\left(\frac{R + \tilde{a}}{L}\right)}{K_0\left(\frac{R + \tilde{a}}{L}\right)} K_0\left(\frac{R}{L}\right)$$

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Problem 8

The one-dimensional, single energy diffusion equation in each of the material regions are as follows:

Multiplying Core:

$$\frac{d\phi(x)}{dx} - B^2\phi(x) = 0$$

where $B^2 = \frac{\Sigma_f - \Sigma_a}{D}$.

Uniform Source Reflector:

Problem 9