NE250 Homework 6 **Mitch Negus**
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Problem 1

a.)

Expected Value/Mean:

$$
\begin{aligned}
\mu &= \int_{-\infty}^{\infty} x f(x)\, dx \\
&= \int_0^a \frac{x}{a}\, dx \\
&= \frac{1}{2a}\left[x^2\right]_0^a
\end{aligned}
$$

$$\boxed{\mu = \frac{a}{2}}$$

Variance:

$$
\begin{aligned}
\sigma^2 &= E[(X-\mu)^2] \\
&= \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx \\
&= \int_0^a \left(x - \frac{a}{2}\right)^2 \left(\frac{1}{a}\right) dx \\
&= \int_0^a \left(x^2 - ax + \frac{a^2}{4}\right)\left(\frac{1}{a}\right) dx \\
&= \int_0^a \frac{x^2}{a}\, dx - \int_0^a x\, dx + \int_0^a \frac{a}{4}\, dx \\
&= \left[\frac{x^3}{3a}\right]_0^a - \left[\frac{x^2}{2}\right]_0^a + \left[\frac{ax}{4}\right]_0^a \\
&= \frac{a^2}{3} - \frac{a^2}{2} + \frac{a^2}{4}
\end{aligned}
$$

$$\boxed{\sigma^2 = \frac{a^2}{12}}$$

Cumulative Distribution Function:

$$
\begin{aligned}
F(x) &= \int_{-\infty}^{x} f(x')dx' \\
&= \int_0^x \frac{1}{a}\, dx' \\
&= \frac{1}{a}\left[x'\right]_0^x \\
&= \frac{x}{a}
\end{aligned}
$$

$$\boxed{F(x) = \begin{cases} 0, & x < 0, x > a \\ \frac{x}{a}, & 0 < x < a \end{cases}}$$

b.)

Expected Value/Mean:

$$
\begin{aligned}
\mu &= \int_{-\infty}^{\infty} x f(x)\, dx \\
&= \int_{0}^{\infty} \lambda x e^{-\lambda x}\, dx \\
(\text{let } u = \lambda x, du &= \lambda\, dx, dv = e^{-\lambda x}, v = -e^{-\lambda x}/\lambda) \\
&= uv|_0^{\infty} - \int_0^{\infty} v\, du \\
&= -xe^{-\lambda x}\Big|_0^{\infty} - \int_0^{\infty} \frac{-\lambda e^{-\lambda x}}{\lambda}\, dx \\
&= 0 + \left[\frac{-e^{-\lambda x}}{\lambda}\right]_0^{\infty}
\end{aligned}
$$

$$\boxed{\mu = \frac{1}{\lambda}}$$

Variance:

$$
\begin{aligned}
\sigma^2 &= E[(X-\mu)^2] \\
&= E[X^2] - \mu^2 \\
&= \int_0^{\infty} x^2 f(x)\, dx - \frac{1}{\lambda^2} \\
&= \int_0^{\infty} \lambda x^2 e^{-\lambda x}\, dx - \frac{1}{\lambda^2} \\
(\text{let } u = \lambda x^2, du &= 2\lambda x\, dx, dv = e^{-\lambda x}, v = -e^{-\lambda x}/\lambda) \\
&= uv|_0^{\infty} - \int_0^{\infty} v\, du - \frac{1}{\lambda^2} \\
&= \left[-x^2 e^{-\lambda x})\right]_0^{\infty} + \int_0^{\infty} 2xe^{-\lambda x}\, dx - \frac{1}{\lambda^2} \\
&= 0 + 2\int_0^{\infty} xe^{-\lambda x}\, dx - \frac{1}{\lambda^2} \\
&= \frac{2}{\lambda^2}\left[e^{-\lambda x}(-\lambda x - 1)\right]_0^{\infty} - \frac{1}{\lambda^2} \\
&= \frac{2}{\lambda^2} - \frac{1}{\lambda^2}
\end{aligned}
$$

$$\boxed{\sigma^2 = \frac{1}{\lambda^2}}$$

Cumulative Distribution Function:

$$
\begin{aligned}
F(x) &= \int_{-\infty}^{x} f(x')dx' \\
&= \int_0^{x} \lambda e^{-\lambda x'}\, dx' \\
&= \left[-e^{-\lambda x'}\right]_0^{x} \\
&= \left[1 - e^{-\lambda x}\right]
\end{aligned}
$$

$$\boxed{F(x) = 1 - e^{-\lambda x}}$$

Problem 2

Problem 3

Problem 4

Problem 5

Problem 6

Problem 7

***a.*)**

***b.*)**

***c.*)**

***d.*)**

***e.*)**