# NE250_HW02_mnegus-prob6

October 5, 2017

## 1 NE 250 – Homework 2

### 1.1 Problem 6

10/6/2017

***a)*** Like in Problem 5,

$$k_\infty = \frac{\int_0^\infty \nu\Sigma_f(E)\phi(E)\,dE}{\int_0^\infty \Sigma_a(E)\phi(E)\,dE}.$$

Again, since we are only considering cross sections averaged over the fast spectrum, we can state that $\Sigma_X(E) = \Sigma_{X,F}$, and then

$$k_\infty = \frac{\nu\Sigma_{f,F}\int_0^\infty \phi(E)\,dE}{\Sigma_{a,F}\int_0^\infty \phi(E)\,dE} = \frac{\nu\Sigma_{f,F}}{\Sigma_{a,F}}.$$

Next, we repeat our decomposition of the numerator and denominator. For neutron production, we find

$$\nu\Sigma_f = \sum_i \nu_i n_i \sigma_{f,i}$$

where $i$ represents the $i^{\text{th}}$ fuel.

```
In [1]: def production(fuels,nu,n,xsf):
            p = [nu[i]*n[i]*xsf[i] for i in fuels]
            return sum(p)
```

For neutron absorption,

$$\Sigma_a = \sum_j n_j \sigma_{a,j}$$

and $j$ represents the $j^{\text{th}}$ material in the reactor.

```
In [2]: def absorption(materials,n,xsa):
            a = [n[j]*xsa[j] for j in materials]
            return sum(a)
```

We substitute these into our equation for $k_\infty$ and solve.

```
In [3]: def k_infinity(fuels,materials,nu,n,xsf,xsa):
            k_inf = production(fuels,nu,n,xsf)/absorption(materials,n,xsa)
            return k_inf
```

We are given the material densities in $[\mathrm{g/cm^3}]$, the volume fractions of the materials, and the microscopic cross sections $[1/\mathrm{cm}^2]$.

```
In [4]: densities = {'PuO2': 11.0,
                     'UO2':  11.0,
                     'Na':   0.97,
                     'Fe':   7.87
                    }
        vol_fracs = {'PuO2': 0.15*0.30,
                     'UO2':  0.85*0.30,
                     'Na':   0.500,
                     'Fe':   0.200
                    }
        xsf = {'PuO2': 1.95e-24,
               'UO2':  0.05e-24,
              }
        xsa = {'PuO2': 2.4000e-24,
               'UO2':  0.4040e-24,
               'Na':   0.0018e-24,
               'Fe':   0.0087e-24
              }
```

The number densities can be found by using the molar masses of the materials [g/mol] and the formula, $n_j = f_j \frac{\rho_j N_A}{m_j}$.

```
In [5]: m = {'PuO2': 244+15.999*2,
             'UO2':  238.03+15.999*2,
             'Na':   22.990,
             'Fe':   55.845
            }
        n = {material: vol_fracs[material]*densities[material]*6.022e23/m[material]
```

We are also given $\nu_{\mathrm{PuO_2}}$ and $\nu_{\mathrm{UO_2}}$.

```
In [6]: nu = {'PuO2': 2.98,
              'UO2':  2.47
             }
```

Now we can solve for $k_\infty$

```
In [7]: fuels = ['PuO2','UO2']
        materials = ['PuO2','UO2','Na','Fe']
        print('k_infinity =',k_infinity(fuels,materials,nu,n,xsf,xsa))

k_infinity = 1.3324856823514228
```

***b)*** The masses of the total core and fuel are given by $\sum_k f_k \rho_k$, where $k$ represents the $k^{\text{th}}$ material or fuel.

```
In [8]: def mass(components,vol_fracs,densities):
            return sum([vol_fracs[k]*densities[k] for k in components])
```

```
In [9]: total_mass = mass(materials,vol_fracs,densities)
        fuel_mass = mass(fuels,vol_fracs,densities)
        frac = fuel_mass/total_mass
        print('The fraction of the core which is fuel is {}%'.format(round(100*frac
```

```
The fraction of the core which is fuel is 61.579%
```

***c)*** If $k = 1$ and the non-leakage probability is 0.90, then we can reexpress our earlier equation as

$$k = 1 = P_{NL} \frac{\nu \Sigma_{f,F} \int_0^\infty \phi(E)\, dE}{\Sigma_{a,F} \int_0^\infty \phi(E)\, dE} = P_{NL} \frac{\nu \Sigma_{f,F}}{\Sigma_{a,F}},$$

or

$$P_{NL} \nu \Sigma_{f,F} = \Sigma_{a,F}.$$

When separated into individual materials, this is

$$P_{NL} \sum_i \nu_i n_i \sigma_{f,i} = \sum_j n_j \sigma_{a,j}.$$

or explicitly

$$P_{NL} \left( \nu_{\text{PuO}_2} n_{\text{PuO}_2} \sigma_{F,\text{PuO}_2} + \nu_{\text{UO}_2} n_{\text{UO}_2} \sigma_{F,\text{UO}_2} \right) = n_{\text{PuO}_2} \sigma_{a,F,\text{PuO}_2} + n_{\text{UO}_2} \sigma_{a,F,\text{UO}_2} + n_{\text{Fe}} \sigma_{a,F,\text{Fe}} + n_{\text{Na}} \sigma_{a,F,\text{Na}}$$

We recall that $n_{\text{PuO}_2} = f_{\text{f,PuO}_2}(0.3)\frac{\rho_{\text{PuO}_2} N_A}{m_{\text{PuO}_2}}$ and $n_{\text{UO}_2} = (1 - f_{\text{f,PuO}_2})(0.3)\frac{\rho_{\text{UO2}} N_A}{m_{\text{UO}_2}}$, while $n_{\text{Na}} = (0.5)\frac{\rho_{\text{Na}} N_A}{m_{\text{Na}}}$ and $n_{\text{Fe}} = (0.2)\frac{\rho_{\text{Fe}} N_A}{m_{\text{Fe}}}$.

Then,

$$P_{NL} \left( \nu_{\text{PuO}_2} f_{\text{f,PuO}_2} (0.3) \frac{\rho_{\text{PuO}_2} N_A}{m_{\text{PuO}_2}} \sigma_{F,\text{PuO}_2} + \nu_{\text{UO}_2} (1 - f_{\text{f,PuO}_2})(0.3) \frac{\rho_{\text{UO2}} N_A}{m_{\text{UO}_2}} \sigma_{F,\text{UO}_2} \right) = f_{\text{f,PuO}_2} (0.3) \frac{\rho_{\text{PuO}_2} N_A}{m_{\text{PuO}_2}} \sigma_{a,F,\text{PuO}_2}$$

which can be expanded to

$$f_{\text{f,PuO}_2} P_{NL} \nu_{\text{PuO}_2} (0.3) \frac{\rho_{\text{PuO}_2}}{m_{\text{PuO}_2}} \sigma_{F,\text{PuO}_2} + P_{NL} \nu_{\text{UO}_2} (0.3) \frac{\rho_{\text{UO2}}}{m_{\text{UO}_2}} \sigma_{F,\text{UO}_2} - f_{\text{f,PuO}_2} P_{NL} \nu_{\text{UO}_2} (0.3) \frac{\rho_{\text{UO2}}}{m_{\text{UO}_2}} \sigma_{F,\text{UO}_2} = f_{\text{f,PuO}_2} (0.3)$$

and simplified to

$$f_{\text{f,PuO}_2} \left( P_{NL} \nu_{\text{PuO}_2} (0.3) \frac{\rho_{\text{PuO}_2}}{m_{\text{PuO}_2}} \sigma_{F,\text{PuO}_2} - P_{NL} \nu_{\text{UO}_2} (0.3) \frac{\rho_{\text{UO2}}}{m_{\text{UO}_2}} \sigma_{F,\text{UO}_2} - (0.3) \frac{\rho_{\text{PuO}_2}}{m_{\text{PuO}_2}} \sigma_{a,F,\text{PuO}_2} + (0.3) \frac{\rho_{\text{UO2}}}{m_{\text{UO}_2}} \sigma_{a,F,\text{UO}_2} \right)$$

$$f_{\text{f,PuO}_2} = \frac{(0.3)\frac{\rho_{\text{UO2}}}{m_{\text{UO}_2}} \sigma_{a,F,\text{UO}_2} + (0.2)\frac{\rho_{\text{Fe}}}{m_{\text{Fe}}} \sigma_{a,F,\text{Fe}} + (0.5)\frac{\rho_{\text{Na}}}{m_{\text{Na}}} \sigma_{a,F,\text{Na}} - P_{NL} \nu_{\text{UO}_2} (0.3) \frac{\rho_{\text{UO2}}}{m_{\text{UO}_2}} \sigma_{F,\text{UO}_2}}{0.3 \left( P_{NL} \nu_{\text{PuO}_2} \frac{\rho_{\text{PuO}_2}}{m_{\text{PuO}_2}} \sigma_{F,\text{PuO}_2} - P_{NL} \nu_{\text{UO}_2} \frac{\rho_{\text{UO2}}}{m_{\text{UO}_2}} \sigma_{F,\text{UO}_2} - \frac{\rho_{\text{PuO}_2}}{m_{\text{PuO}_2}} \sigma_{a,F,\text{PuO}_2} + \frac{\rho_{\text{UO2}}}{m_{\text{UO}_2}} \sigma_{a,F,\text{UO}_2} \right)}$$

In [10]:
```
P_NL = 0.90
numerator = 0.3*xsa['UO2']*densities['UO2']/m['UO2'] + 0.2*xsa['Fe']*densi
denominator = 0.3*(P_NL*nu['PuO2']*xsf['PuO2']*densities['PuO2']/m['PuO2']
f = numerator/denominator
print('The reactor will be critical if {}% of the fuel volume is PuO2 ({}%
```

```
The reactor will be critical if 10.322% of the fuel volume is PuO2 (3.097% of the c
```