NE 250 Fall 2017
Homework #2 Due October 6th, 2017

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1. (10 points) The energy distribution for fission neutrons from ^{235}U is calculated from the following analytical approximation:

$$\chi(E) = 0.453e^{-1.036E}\sinh(\sqrt{2.29E})$$

   Find the most probable and the average fission neutron energy.

2. (10 points) An underlying assumption in neutron transport (and diffusion) theory is that the number of neutron-neutron collisions is negligible compared to neutron-nucleus collisions. Prove that this is true even at high flux conditions (10^16 n/cm^{2}s) comparing the neutron-neutron reaction rate ($\sigma_{nn} = 10$ b) to the total reaction rate with UO_2. Assume all neutrons are at thermal energies.

3. (10 points) Consider a thermal neutron incident on a slab-shaped shield of concrete 1 m in thickness and determine the probability that the neutron:

   a. will pass through the shield without a collision;
   b. will ultimately diffuse through the shield;
   c. will be reflected back from the shield.

   Assume concrete is composed of 10 wt% H_2O, 50 wt% calcium, and 40 wt% silicon.

4. (10 points) Explicitly demonstrate by integration that $\bar{\mu}_s = 2/(3\text{A})$ for elastic scattering from stationary nuclei when such scattering is assumed to be isotropic in the center of mass system.

5. (10 points) Consider an infinitely large homogeneous mixture of ^{235}U and a moderating material. Determine the ratio of fuel-to-moderator density that will render this system critical for the following moderators:

   1. graphite;
   2. beryllium;
   3. water;
   4. heavy water (D_20).

   Use thermal cross section data.

Material	σ_f, b	σ_a, b	σ_t, b	Density, g/cm^3
PuO_2	1.95	2.4000	8.6	11.0
UO_2	0.05	0.4040	8.2	11.0
Na	—	0.0018	3.7	0.97
Fe	—	0.0087	3.6	7.87

Table 1: Cross section data

6. (10 points) A sodium-cooled fast reactor is fueled with PuO_2, mixed with depleted UO_2. The structural material is iron. Averaged over the spectrum of fast neutrons, the microscopic cross sections and densities are in Table 1.

   The fuel is 15% PuO_2 and 85% UO_2 by volume. The volumetric composition of the core is 30% fuel, 50% coolant, and 20% structural material.

   1. Calculate k_∞ assuming that the values of ν for plutonium and uranium in the fast spectrum are 2.98 and 2.47, respectively, and that the cross sections of oxygen can be neglected.
   2. What fraction of the mass of the core does the fuel account for?
   3. Suppose the non-leakage probability for this sodium-cooled fast reactor is 0.90. Determine the volume fractions of PuO_2 and UO_2 in the fuel so that $k = 1.0$.

7. (10 points) Albedo, or the reflection coefficient, is defined as the ratio between the current out of a reflecting region to the current into the reflecting region: $\alpha = \frac{J_{out}}{J_{in}}$. Derive an expression of albedo as a function of reflector thickness (a), diffusion coefficient (D), and diffusion length (L), when such a reflector is placed on both sides of a slab containing a uniformly distributed source of intensity S_0 n/(cm^3 s).

8. (10 points) One defines the blackness coefficient characterizing a region as:

$$\beta = \frac{J_+(a) - J_-(a)}{J_+(a)}$$

   where a denotes the surface of the region. Yet another useful parameter characterizing interfaces is the ratio of the current density J to the flux at the interface

$$\gamma = \frac{J(a)}{\phi(a)}$$

   Determine the relation between these parameters and the albedo (get β and γ each in terms of α, and α in terms of β and γ), assuming that diffusion theory can be used to describe the material adjacent to the region of interest.

When converting J_{out} and J_{in} to + and − please do this for a right hand side boundary.

It should be remarked that one frequently uses these concepts to characterize very highly absorbing regions such as fuel elements or control rods in which diffusion theory will usually not be valid in the highly absorbing region.

9. (10 points) Use the method of variation of constants to determine the flux in a finite slab that contains a uniformly distributed neutron source.

10. (10 points) Show that

$$J_{-,1} = \int_{2\pi-} d\hat{\Omega}\, \hat{\Omega} \cdot \hat{e}_s \psi_1(\vec{r}, E, \hat{\Omega}, t) \approx \frac{1}{4}\phi_1(\vec{r}, t) + \frac{D_1(\vec{r})}{2}\hat{e}_s \cdot \nabla\phi_1(\vec{r}, t) = 0\,.$$

11. (10 points) A purely absorbing, homogeneous medium extends over the left half of the plane $-\infty < x \leq 0$ and contains a uniformly-distributed, isotropic, fixed source, $s(E)$, with vacuum in the right half plane ($0 \leq x < \infty$). Solve the steady state neutron transport equation for $\psi(x, \mu, E)$ over the left-half plane, then determine $\psi(0, \mu, E)$ and $\vec{J}(0, E)$.

12. (10 points) In circulating fuel systems, the multi-group diffusion eigenvalue problem is modified to include the effect of the velocity flow:

$$\begin{aligned} \nabla D_g \nabla \phi_g - \Sigma_{a,g}\phi_g - \sum_{g' \neq g} \Sigma_{s,gg'}\phi_g + \sum_{g' \neq g} \Sigma_{s,g'g}\phi_{g'} + (1-\beta_0)\,\chi_{p,g} \sum_{g'=1}^{n} \frac{1}{k_{eff}} (\nu\Sigma_f)_{g'}\,\phi_{g'} & \\ + \sum_{i=1}^{m} \chi_{d,g}\lambda_i c_i = 0 & \\ - \nabla(\mathbf{u}c_i) - \lambda_i c_i + \beta_{0,i} \sum_{g=1}^{n} \frac{1}{k_{eff}} (\nu\Sigma_f)_g\,\phi_g = 0 & \end{aligned}$$

where $\mathbf{u}$ is the flow velocity vector.

1) Describe the term $\nabla(\mathbf{u}c_i)$

2) What is the impact of fuel motion on the effective delayed neutron fraction? Is the effective delayed neutron fraction increased or decreased by fuel motion?

3) What are typical values of fuel recirculation times for which the effect of fuel motion is more important? Why don't solid fuel molten salt reactors (i.e. pebble bed FHR) see impact from delayed neutron precursors' motion? Hint: Compare fuel recirculation time and delayed neutron emission time constants.