

**Problem 1**

If the energy distribution for fission neutrons from  $^{235}\text{U}$  follows the functional approximation (for energy in MeV)

$$\chi(E) = 0.453e^{-1.036E} \sinh(\sqrt{2.29E}),$$

then the most probable energy of a neutron corresponds to the maximum of the function. A maximum value will be found at a critical point of the function, which can be found via differentiation (specifically when  $\frac{d\chi}{dE} = 0$ ):

$$\begin{aligned} \frac{d\chi}{dE} &= 0 = \frac{d}{dE} \left( 0.453e^{-1.036E_{\max}} \sinh(\sqrt{2.29E_{\max}}) \right) \\ 0 &= 0.453 \frac{d}{dE} \left( e^{-1.036E_{\max}} \sinh(\sqrt{2.29E_{\max}}) \right) \\ 0 &= 0.453 \left[ e^{-1.036E} \frac{d}{dE} \sinh(\sqrt{2.29E_{\max}}) + \frac{d}{dE} (e^{-1.036E_{\max}}) \sinh(\sqrt{2.29E_{\max}}) \right] \\ 0 &= 0.453 \left[ e^{-1.036E} \cosh(\sqrt{2.29E}) \frac{d}{dE} (\sqrt{2.29E_{\max}}) - 1.036e^{-1.036E} \sinh(\sqrt{2.29E}) \right] \\ 0 &= 0.453 \left[ e^{-1.036E} \cosh(\sqrt{2.29E}) \frac{\sqrt{2.29}}{2\sqrt{E_{\max}}} - 1.036e^{-1.036E} \sinh(\sqrt{2.29E}) \right] \\ 0 &= \frac{\sqrt{2.29} \cosh(\sqrt{2.29E})}{2\sqrt{E_{\max}}} - 1.036 \sinh(\sqrt{2.29E}) \\ 0 &= 1 - 1.369\sqrt{E_{\max}} \tanh(\sqrt{2.29E_{\max}}) \\ 1 &= 1.369\sqrt{E_{\max}} \tanh(\sqrt{2.29E_{\max}}) \end{aligned}$$

$$\boxed{E_{\max} = 0.724 \text{ MeV}}$$

The average energy can be found by finding the expected value of the function on the domain  $[0, \infty)$ .

$$\begin{aligned} E_{\text{ave}} &= \int_0^{\infty} E \chi(E) dE \\ &= \int_0^{\infty} E \left( 0.453e^{-1.036E} \sinh(\sqrt{2.29E}) \right) dE \\ &= 0.453 \int_0^{\infty} E e^{-1.036E} \sinh(\sqrt{2.29E}) dE \end{aligned}$$

This integral cannot be solved analytically. Solving numerically (with Wolram Alpha),

$$\boxed{E_{\text{ave}} = 1.98 \text{ MeV}}$$

## Problem 2

A general reaction rate for process  $x$  as a function of energy can be defined as

$$R_x(E) = \Sigma_x(E)\phi(E)$$

where  $\Sigma_x(E)$  is the macroscopic cross section for reaction  $x$  and  $\phi$  is the neutron flux, both at energy  $E$ . The macroscopic cross section can be further decomposed, so that

$$R_x(E) = n_x \sigma_x(E) \phi(E).$$

Comparing neutron-neutron reactions with all neutron-nuclei reactions, the ratio of reaction rates is

$$\frac{R_{nn}(E)}{R_{tot}(E)} = \frac{n_n \sigma_{nn}(E) \phi(E)}{n_{\text{UO}_2} \sigma_{tot}(E) \phi(E)}.$$

We can know that the neutron flux is  $\phi(0.025 \text{ eV}) = 10^{16} \text{ neutrons}/(\text{cm}^2 \cdot \text{s})$  and they are at thermal energies ( $E = 0.025 \text{ eV}$  and traveling at  $v = \sqrt{\frac{2(0.025 \text{ eV})}{m_n}} = 2.190 \times 10^5 \text{ cm/s}$ ). The neutrons that are then in a  $1 \text{ cm}^3$  volume at any given second is

$$n_n = \frac{\phi(0.025 \text{ eV})}{v} = \frac{10^{16} \text{ neutrons}/(\text{cm}^2 \cdot \text{s})}{2.190 \times 10^5 \text{ cm/s}} = 4.566 \times 10^{10} \text{ neutrons}/\text{cm}^3$$

For  $\text{UO}_2$ ,  $\rho = 10.97 \text{ g/cm}^3$ ,  $m_{\text{O}} = 16.0 \text{ g/mol}$ ,  $m_{\text{U8}} = 238.05 \text{ g/mol}$ , and  $m_{\text{U5}} = 235.04 \text{ g/mol}$ . If we use an enrichment of 5% (atom percent), then  $m_{\text{UO}_2} = 0.95(238.05) + 0.05(235.04) + 2(16.0) = 269.90 \text{ g/mol}$ . For number density, we find

$$n_{\text{UO}_2} = \frac{\rho N_A}{m_{\text{UO}_2}} = \frac{(10.97 \text{ g/cm}^3)(6.022 \times 10^{23})}{269.90 \text{ g/mol}} = 2.448 \times 10^{22} \text{ molecules/cm}^3$$

Additionally, we're given that  $\sigma_{nn} = 10 \text{ b}$ , and we can determine the microscopic cross section for  $\text{UO}_2$  from tabulated data. (From ENDF/B-VII.1 at  $0.025 \text{ eV}$ :  $\sigma_{tot, \text{U8}} = 11.962 \text{ b}$ ,  $\sigma_{tot, \text{U5}} = 698.856 \text{ b}$ , and  $\sigma_{tot, \text{O}} = 3.852 \text{ b}$ )

$$\sigma_{tot} = 0.95\sigma_{tot, \text{U8}} + 0.05\sigma_{tot, \text{U5}} + 2\sigma_{tot, \text{O}}$$

$$\sigma_{tot} = 0.95(11.962 \text{ b}) + 0.05(698.856 \text{ b}) + 2(3.852 \text{ b})$$

$$\sigma_{tot} = 54.011 \text{ b}$$

We can now solve for the ratio of reaction rates (noting that  $\phi(E)$  cancels in the numerator and denominator)

$$\begin{aligned} \frac{R_{nn}(E)}{R_{tot}(E)} &= \frac{n_n \sigma_{nn}(E)}{n_{\text{UO}_2} \sigma_{tot}(E)} = \frac{(4.566 \times 10^{10} \text{ neutrons/cm}^3)(10 \text{ b})}{(2.448 \times 10^{22} \text{ molecules/cm}^3)(54.011 \text{ b})} \\ \frac{R_{nn}(E)}{R_{tot}(E)} &= 3.40 \times 10^{-13} \end{aligned}$$

The rate of neutron-neutron collisions is 13 orders of magnitudes less than the rate of neutron- $\text{UO}_2$  collisions.

## Problem 3

## Problem 4

First, we define the average scattering cosine  $\bar{\mu}_0$  as the average dot product,  $\langle \hat{\Omega} \cdot \hat{\Omega}' \rangle$ . When normalized by  $4\pi\Sigma_s$ , the total of cross sections for scattering from any angle  $\hat{\Omega}$  to any other angle  $\hat{\Omega}'$ , this is

$$\bar{\mu}_0 \equiv \langle \hat{\Omega} \cdot \hat{\Omega}' \rangle = \left( \frac{1}{4\pi\Sigma_s} \right) \int_{4\pi} d\hat{\Omega} \int_{4\pi} d\hat{\Omega}' \hat{\Omega} \cdot \hat{\Omega}' \Sigma_s(\hat{\Omega} \cdot \hat{\Omega}')$$

In the center of mass system, the probability that a particle scatters in any direction is roughly uniform,  $\Sigma_{\text{CM}}(\theta_C) = \frac{\Sigma_s}{4\pi}$ ,

$$\bar{\mu}_0 = \frac{1}{\Sigma_s} \int_{4\pi} d\hat{\Omega} \int_{4\pi} d\hat{\Omega}' \hat{\Omega} \cdot \hat{\Omega}' \Sigma_{\text{CM}}(\theta_C).$$

## Problem 5

A critical reactor has a multiplication factor of  $k = 1$ . The multiplication factor (for an infinite reactor) can be defined as

$$k_{\infty} \equiv \frac{\# \text{ neutrons produced}}{\# \text{ neutrons absorbed}}$$

Mathematically, the number of neutrons produced is  $\int_0^{\infty} \nu \Sigma_f(E) \phi(E) dE$  and the number of neutrons absorbed is  $\int_0^{\infty} \Sigma_a(E) \phi(E) dE$ . Altogether, we can mathematically describe a critical reactor as

$$1 = \frac{\int_0^{\infty} \nu \Sigma_f(E) \phi(E) dE}{\int_0^{\infty} \Sigma_a(E) \phi(E) dE}$$

or equivalently

$$\int_0^{\infty} \nu \Sigma_f(E) \phi(E) dE = \int_0^{\infty} \Sigma_a(E) \phi(E) dE.$$

Since we are considering only thermal cross sections, we will let  $\Sigma_X(E) = \Sigma_X(0.025 \text{ eV}) = \Sigma_{X,T}$  and we find

$$\nu \Sigma_{f,T} \int_0^{\infty} \phi(E) dE = \Sigma_{a,T} \int_0^{\infty} \phi(E) dE.$$

The integrals over flux cancel, and so

$$\nu \Sigma_{f,T} = \Sigma_{a,T}.$$

The macroscopic cross sections can be rewritten as  $\Sigma_{f,T} = \Sigma_{f,T,f}$  and  $\Sigma_{a,T} = \Sigma_{a,T,f} + \Sigma_{a,T,m}$  where subscripts f and m denote fuel and moderator, respectively. Furthermore, each macroscopic cross section for each material can be expressed in terms of the material's number density and microscopic cross section,  $\Sigma = n\sigma$ . In total

$$\nu n_f \sigma_{f,T,f} = n_f \sigma_{a,T,f} + n_m \sigma_{a,T,m}.$$

The fuel-to-moderator density at criticality can then be expressed as

$$\frac{n_f}{n_m} = \frac{\sigma_{a,T,m}}{\nu \sigma_{f,T,f} - \sigma_{a,T,f}}.$$

**a.) Graphite**

$$\frac{n_f}{n_m} = 4.55 \times 10^{-6}$$

**b.) Beryllium**

$$\frac{n_f}{n_m} = 1.36 \times 10^{-5}$$

**c.) Water**

$$\frac{n_f}{n_m} = 8.99 \times 10^{-4}$$

**d.) Graphite**

$$\frac{n_f}{n_m} = 1.64 \times 10^{-6}$$

(see attached Jupyter notebook for full calculations)

## Problem 6

**a.)**

Like in Problem 5,

$$k_{\infty} = \frac{\int_0^{\infty} \nu \Sigma_f(E) \phi(E) dE}{\int_0^{\infty} \Sigma_a(E) \phi(E) dE}.$$

Again, since we are only considering cross sections averaged over the fast spectrum, we can state that  $\Sigma_X(E) = \Sigma_{X,F}$ , and then

$$k_{\infty} = \frac{\nu \Sigma_{f,F} \int_0^{\infty} \phi(E) dE}{\Sigma_{a,F} \int_0^{\infty} \phi(E) dE} = \frac{\nu \Sigma_{f,F}}{\Sigma_{a,F}}.$$

Next, we repeat our decomposition of the numerator and denominator, leaving

$$k_{\infty} = \frac{\nu_{\text{PuO}_2} n_{\text{PuO}_2} \sigma_{F,\text{PuO}_2} + \nu_{\text{UO}_2} n_{\text{UO}_2} \sigma_{F,\text{UO}_2}}{n_{\text{PuO}_2} \sigma_{a,F,\text{PuO}_2} + n_{\text{UO}_2} \sigma_{a,F,\text{UO}_2} + n_{\text{Fe}} \sigma_{a,F,\text{Fe}} + n_{\text{Na}} \sigma_{a,F,\text{Na}}}$$

After solving this equation with the given values, we find

$$k_{\infty} = 1.332$$

**b.)**

We can use the densities and volume fractions to determine the fraction of the core mass that is fuel.

$$61.6\% \text{ is fuel}$$

**c.)**

If  $k = 1$  and the non-leakage probability is  $P_{NL} = 0.90$ , then we can reexpress our earlier equation as

$$k = 1 = P_{NL} \frac{\nu \Sigma_{f,F} \int_0^{\infty} \phi(E) dE}{\Sigma_{a,F} \int_0^{\infty} \phi(E) dE} = P_{NL} \frac{\nu \Sigma_{f,F}}{\Sigma_{a,F}},$$

or

$$P_{NL} \nu \Sigma_{f,F} = \Sigma_{a,F}.$$

When separated into individual materials, this is

$$P_{NL} \sum_i \nu_i n_i \sigma_{f,i} = \sum_j n_j \sigma_{a,j}.$$

or explicitly

$$P_{NL} (\nu_{\text{PuO}_2} n_{\text{PuO}_2} \sigma_{F,\text{PuO}_2} + \nu_{\text{UO}_2} n_{\text{UO}_2} \sigma_{F,\text{UO}_2}) = n_{\text{PuO}_2} \sigma_{a,F,\text{PuO}_2} + n_{\text{UO}_2} \sigma_{a,F,\text{UO}_2} + n_{\text{Fe}} \sigma_{a,F,\text{Fe}} + n_{\text{Na}} \sigma_{a,F,\text{Na}}$$

We recall that  $n_{\text{PuO}_2} = f_{\text{f,PuO}_2} (0.3) \frac{\rho_{\text{PuO}_2} N_A}{m_{\text{PuO}_2}}$  and  $n_{\text{UO}_2} = (1 - f_{\text{f,PuO}_2}) (0.3) \frac{\rho_{\text{UO}_2} N_A}{m_{\text{UO}_2}}$ , while  $n_{\text{Na}} = (0.5) \frac{\rho_{\text{Na}} N_A}{m_{\text{Na}}}$  and  $n_{\text{Fe}} = (0.2) \frac{\rho_{\text{Fe}} N_A}{m_{\text{Fe}}}$ . We can substitute these expressions into the equation above and solve for  $f_{\text{f,PuO}_2}$ .

$$\text{The fuel must be 10.322\% PuO}_2 \text{ and 89.678\% UO}_2 \text{ for a critical reactor with } P_{NL} = 0.9$$

(see attached Jupyter notebook for full calculations)

## Problem 10

Answer the following questions as true or false, provide a one sentence justification for your answer

1. The integro-differential form of the transport equation expresses a local balance between neutron production and losses.
2. A vacuum boundary condition for the integro-differential transport equation implies a zero outgoing angular flux.
3. In the transport equation in curvilinear coordinates, the redistribution term allows neutrons to migrate between the directions as they move along a straight line.
4. A nuclear system is subcritical if its eigenvalues satisfy  $\max(\text{Re}(j)) < 1$ .
5. The energy spectrum of the fundamental eigenmode of the eigenvalue problem is skewed as though a 1 absorber is present.

# NE250\_HW02\_mnegus-prob5

October 5, 2017

## 1 NE 250 – Homework 2

### 1.1 Problem 5

10/6/2017

A critical reactor has a multiplication factor of  $k = 1$ . The multiplication factor (for an infinite reactor) can be defined as

$$k_{\infty} \equiv \frac{\# \text{ neutrons produced}}{\# \text{ neutrons absorbed}}$$

Mathematically, the number of neutrons produced is  $\int_0^{\infty} \nu \Sigma_f(E) \phi(E) dE$  and the number of neutrons absorbed is  $\int_0^{\infty} \Sigma_a(E) \phi(E) dE$ . Altogether, we can mathematically describe a critical reactor as

$$1 = \frac{\int_0^{\infty} \nu \Sigma_f(E) \phi(E) dE}{\int_0^{\infty} \Sigma_a(E) \phi(E) dE}$$

or equivalently

$$\int_0^{\infty} \nu \Sigma_f(E) \phi(E) dE = \int_0^{\infty} \Sigma_a(E) \phi(E) dE.$$

Since we are considering only thermal cross sections, we will let  $\Sigma_X(E) = \Sigma_X(0.025 \text{ eV}) = \Sigma_{X,T}$  and we find

$$\nu \Sigma_{f,T} \int_0^{\infty} \phi(E) dE = \Sigma_{a,T} \int_0^{\infty} \phi(E) dE.$$

The integrals over flux cancel, and so

$$\nu \Sigma_{f,T} = \Sigma_{a,T}.$$

The macroscopic cross sections can be rewritten as  $\Sigma_{f,T} = \Sigma_{f,T,f}$  and  $\Sigma_{a,T} = \Sigma_{a,T,f} + \Sigma_{a,T,m}$  where subscripts f and m denote fuel and moderator, respectively. Furthermore, each macroscopic cross section for each material can be expressed in terms of the material's number density and microscopic cross section,  $\Sigma = n\sigma$ . In total

$$\nu n_f \sigma_{f,T,f} = n_f \sigma_{a,T,f} + n_m \sigma_{a,T,m}.$$

The fuel-to-moderator (number) density ratio at criticality can then be expressed as

$$\frac{n_f}{n_m} = \frac{\sigma_{a,T,m}}{\nu \sigma_{f,T,f} - \sigma_{a,T,f}}.$$

```
In [1]: def FMratio(absorbXS_mod,absorbXS_fuel,fissionXS_fuel,nu):
        return absorbXS_mod/(nu*fissionXS_fuel - absorbXS_fuel)
```

We can define our thermal cross sections as follows (from ENDF/B-VII.1)

```
In [2]: fissionXS = {'U235': 585.086}
        absorbXS = {'C12': 0.00336,
                    'Be9': 0.01004,
                    'H1': 0.33201,
                    'H2': 0.00051,
                    'O16': 0.00019,
                    'U235': 585.086 + 98.6864
                    }
        absorbXS['graphite'] = absorbXS['C12']
        absorbXS['water'] = 2*absorbXS['H1'] + absorbXS['O16']
        absorbXS['heavy water'] = 2*absorbXS['H2'] + absorbXS['O16']
```

#### *a) Graphite*

```
In [3]: absorbXS_mod=absorbXS['graphite']
        print('Critical Fuel-to-Moderator Ratio: ',FMratio(absorbXS_mod,absorbXS['U
```

Critical Fuel-to-Moderator Ratio: 4.5457204996214396e-06

#### *b) Beryllium*

```
In [4]: absorbXS_mod=absorbXS['Be9']
        print('Critical Fuel-to-Moderator Ratio: ',FMratio(absorbXS_mod,absorbXS['U
```

Critical Fuel-to-Moderator Ratio: 1.358304577863073e-05

#### *c) Water*

```
In [5]: absorbXS_mod=absorbXS['water']
        print('Critical Fuel-to-Moderator Ratio: ',FMratio(absorbXS_mod,absorbXS['U
```

Critical Fuel-to-Moderator Ratio: 0.0008986050634087967

#### *d) Heavy Water*

```
In [6]: absorbXS_mod=absorbXS['heavy water']
        print('Critical Fuel-to-Moderator Ratio: ',FMratio(absorbXS_mod,absorbXS['U
```

Critical Fuel-to-Moderator Ratio: 1.6370005370660543e-06



# NE250\_HW02\_mnegus-prob6

October 5, 2017

## 1 NE 250 – Homework 2

### 1.1 Problem 6

10/6/2017

a) Like in Problem 5,

$$k_{\infty} = \frac{\int_0^{\infty} \nu \Sigma_f(E) \phi(E) dE}{\int_0^{\infty} \Sigma_a(E) \phi(E) dE}.$$

Again, since we are only considering cross sections averaged over the fast spectrum, we can state that  $\Sigma_X(E) = \Sigma_{X,F}$ , and then

$$k_{\infty} = \frac{\nu \Sigma_{f,F} \int_0^{\infty} \phi(E) dE}{\Sigma_{a,F} \int_0^{\infty} \phi(E) dE} = \frac{\nu \Sigma_{f,F}}{\Sigma_{a,F}}.$$

Next, we repeat our decomposition of the numerator and denominator. For neutron production, we find

$$\nu \Sigma_f = \sum_i \nu_i n_i \sigma_{f,i}$$

where  $i$  represents the  $i^{\text{th}}$  fuel.

```
In [1]: def production(fuels, nu, n, xsf):  
        p = [nu[i]*n[i]*xsf[i] for i in fuels]  
        return sum(p)
```

For neutron absorption,

$$\Sigma_a = \sum_j n_j \sigma_{a,j}$$

and  $j$  represents the  $j^{\text{th}}$  material in the reactor.

```
In [2]: def absorption(materials, n, xsa):  
        a = [n[j]*xsa[j] for j in materials]  
        return sum(a)
```

We substitute these into our equation for  $k_{\infty}$  and solve.

```
In [3]: def k_infinity(fuels,materials,nu,n,xsf,xsa):
        k_inf = production(fuels,nu,n,xsf)/absorption(materials,n,xsa)
        return k_inf
```

We are given the material densities in [g/cm<sup>3</sup>], the volume fractions of the materials, and the microscopic cross sections [1/cm<sup>2</sup>].

```
In [4]: densities = {'PuO2': 11.0,
                    'UO2': 11.0,
                    'Na': 0.97,
                    'Fe': 7.87
                  }
        vol_fracs = {'PuO2': 0.15*0.30,
                    'UO2': 0.85*0.30,
                    'Na': 0.500,
                    'Fe': 0.200
                  }
        xsf = {'PuO2': 1.95e-24,
              'UO2': 0.05e-24,
              }
        xsa = {'PuO2': 2.4000e-24,
              'UO2': 0.4040e-24,
              'Na': 0.0018e-24,
              'Fe': 0.0087e-24
              }
```

The number densities can be found by using the molar masses of the materials [g/mol] and the formula,  $n_j = f_j \frac{\rho_j N_A}{m_j}$ .

```
In [5]: m = {'PuO2': 244+15.999*2,
            'UO2': 238.03+15.999*2,
            'Na': 22.990,
            'Fe': 55.845
          }
        n = {material: vol_fracs[material]*densities[material]*6.022e23/m[material]}
```

We are also given  $\nu_{\text{PuO}_2}$  and  $\nu_{\text{UO}_2}$ .

```
In [6]: nu = {'PuO2': 2.98,
             'UO2': 2.47
           }
```

Now we can solve for  $k_\infty$

```
In [7]: fuels = ['PuO2', 'UO2']
        materials = ['PuO2', 'UO2', 'Na', 'Fe']
        print('k_infinity =', k_infinity(fuels,materials,nu,n,xsf,xsa))
```

```
k_infinity = 1.3324856823514228
```

b) The masses of the total core and fuel are given by  $\sum_k f_k \rho_k$ , where  $k$  represents the  $k^{\text{th}}$  material or fuel.

```
In [8]: def mass(components,vol_fracs,densities):
        return sum([vol_fracs[k]*densities[k] for k in components])

In [9]: total_mass = mass(materials,vol_fracs,densities)
        fuel_mass = mass(fuels,vol_fracs,densities)
        frac = fuel_mass/total_mass
        print('The fraction of the core which is fuel is {}%'.format(round(100*frac,2)))
```

The fraction of the core which is fuel is 61.579%

c) If  $k = 1$  and the non-leakage probability is 0.90, then we can reexpress our earlier equation as

$$k = 1 = P_{NL} \frac{\nu \Sigma_{f,F} \int_0^\infty \phi(E) dE}{\Sigma_{a,F} \int_0^\infty \phi(E) dE} = P_{NL} \frac{\nu \Sigma_{f,F}}{\Sigma_{a,F}},$$

or

$$P_{NL} \nu \Sigma_{f,F} = \Sigma_{a,F}.$$

When separated into individual materials, this is

$$P_{NL} \sum_i \nu_i n_i \sigma_{f,i} = \sum_j n_j \sigma_{a,j}.$$

or explicitly

$$P_{NL} (\nu_{\text{PuO}_2} n_{\text{PuO}_2} \sigma_{F,\text{PuO}_2} + \nu_{\text{UO}_2} n_{\text{UO}_2} \sigma_{F,\text{UO}_2}) = n_{\text{PuO}_2} \sigma_{a,F,\text{PuO}_2} + n_{\text{UO}_2} \sigma_{a,F,\text{UO}_2} + n_{\text{Fe}} \sigma_{a,F,\text{Fe}} + n_{\text{Na}} \sigma_{a,F,\text{Na}}$$

We recall that  $n_{\text{PuO}_2} = f_{f,\text{PuO}_2} (0.3) \frac{\rho_{\text{PuO}_2} N_A}{m_{\text{PuO}_2}}$  and  $n_{\text{UO}_2} = (1 - f_{f,\text{PuO}_2}) (0.3) \frac{\rho_{\text{UO}_2} N_A}{m_{\text{UO}_2}}$ , while  $n_{\text{Na}} = (0.5) \frac{\rho_{\text{Na}} N_A}{m_{\text{Na}}}$  and  $n_{\text{Fe}} = (0.2) \frac{\rho_{\text{Fe}} N_A}{m_{\text{Fe}}}$ .

Then,

$$P_{NL} \left( \nu_{\text{PuO}_2} f_{f,\text{PuO}_2} (0.3) \frac{\rho_{\text{PuO}_2} N_A}{m_{\text{PuO}_2}} \sigma_{F,\text{PuO}_2} + \nu_{\text{UO}_2} (1 - f_{f,\text{PuO}_2}) (0.3) \frac{\rho_{\text{UO}_2} N_A}{m_{\text{UO}_2}} \sigma_{F,\text{UO}_2} \right) = f_{f,\text{PuO}_2} (0.3) \frac{\rho_{\text{PuO}_2} N_A}{m_{\text{PuO}_2}} \sigma_{a,F,\text{PuO}_2} + (1 - f_{f,\text{PuO}_2}) (0.3) \frac{\rho_{\text{UO}_2} N_A}{m_{\text{UO}_2}} \sigma_{a,F,\text{UO}_2} + n_{\text{Fe}} \sigma_{a,F,\text{Fe}} + n_{\text{Na}} \sigma_{a,F,\text{Na}}$$

which can be expanded to

$$f_{f,\text{PuO}_2} P_{NL} \nu_{\text{PuO}_2} (0.3) \frac{\rho_{\text{PuO}_2}}{m_{\text{PuO}_2}} \sigma_{F,\text{PuO}_2} + P_{NL} \nu_{\text{UO}_2} (0.3) \frac{\rho_{\text{UO}_2}}{m_{\text{UO}_2}} \sigma_{F,\text{UO}_2} - f_{f,\text{PuO}_2} P_{NL} \nu_{\text{UO}_2} (0.3) \frac{\rho_{\text{UO}_2}}{m_{\text{UO}_2}} \sigma_{F,\text{UO}_2} = f_{f,\text{PuO}_2} (0.3) \frac{\rho_{\text{PuO}_2}}{m_{\text{PuO}_2}} \sigma_{a,F,\text{PuO}_2} + (1 - f_{f,\text{PuO}_2}) (0.3) \frac{\rho_{\text{UO}_2}}{m_{\text{UO}_2}} \sigma_{a,F,\text{UO}_2} + n_{\text{Fe}} \sigma_{a,F,\text{Fe}} + n_{\text{Na}} \sigma_{a,F,\text{Na}}$$

and simplified to

$$f_{f,\text{PuO}_2} \left( P_{NL} \nu_{\text{PuO}_2} (0.3) \frac{\rho_{\text{PuO}_2}}{m_{\text{PuO}_2}} \sigma_{F,\text{PuO}_2} - P_{NL} \nu_{\text{UO}_2} (0.3) \frac{\rho_{\text{UO}_2}}{m_{\text{UO}_2}} \sigma_{F,\text{UO}_2} - (0.3) \frac{\rho_{\text{PuO}_2}}{m_{\text{PuO}_2}} \sigma_{a,F,\text{PuO}_2} + (0.3) \frac{\rho_{\text{UO}_2}}{m_{\text{UO}_2}} \sigma_{a,F,\text{UO}_2} \right) = \frac{(0.3) \frac{\rho_{\text{UO}_2}}{m_{\text{UO}_2}} \sigma_{a,F,\text{UO}_2} + (0.2) \frac{\rho_{\text{Fe}}}{m_{\text{Fe}}} \sigma_{a,F,\text{Fe}} + (0.5) \frac{\rho_{\text{Na}}}{m_{\text{Na}}} \sigma_{a,F,\text{Na}} - P_{NL} \nu_{\text{UO}_2} (0.3) \frac{\rho_{\text{UO}_2}}{m_{\text{UO}_2}} \sigma_{F,\text{UO}_2}}{0.3 \left( P_{NL} \nu_{\text{PuO}_2} \frac{\rho_{\text{PuO}_2}}{m_{\text{PuO}_2}} \sigma_{F,\text{PuO}_2} - P_{NL} \nu_{\text{UO}_2} \frac{\rho_{\text{UO}_2}}{m_{\text{UO}_2}} \sigma_{F,\text{UO}_2} - \frac{\rho_{\text{PuO}_2}}{m_{\text{PuO}_2}} \sigma_{a,F,\text{PuO}_2} + \frac{\rho_{\text{UO}_2}}{m_{\text{UO}_2}} \sigma_{a,F,\text{UO}_2} \right)}$$

```

In [10]: P_NL = 0.90
          numerator = 0.3*xsa['UO2']*densities['UO2']/m['UO2'] + 0.2*xsa['Fe']*densi
          denominator = 0.3*(P_NL*nu['PuO2']*xsf['PuO2']*densities['PuO2']/m['PuO2'])
          f = numerator/denominator
          print('The reactor will be critical if {}% of the fuel volume is PuO2 ({}%

```

The reactor will be critical if 10.322% of the fuel volume is PuO2 (3.097% of the c