NE 250 Fall 2017
Homework #4 Due November 6th, 2017

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1. (10 points) Consider the streaming operator in spherical geometry,

   a. Verify that the conservation form is equal to the form derived in class, namely:

   $$\frac{\mu}{r^2}\frac{\partial(r^2\psi)}{\partial r} + \frac{\partial}{\partial\mu}\left[\frac{(1-\mu^2)\psi}{r}\right] = \mu\frac{\partial\psi}{\partial r} + \frac{(1-\mu^2)}{r}\frac{\partial\psi}{\partial\mu}$$

   b. Integrate the conservation form of the streaming operator over $-1 \leq \mu \leq 1$ and write the resulting expression in terms of the neutron current density, $\vec{J}$.

   c. Integrate the expression from b over the volume of a spherical shell $r_1 \leq r \leq r_2$.

2. (15 points) Streaming in cylindrical geometry:

   a. Using the technique demonstrated in section 1.3 of Lewis and Miller, derive the form of the streaming operator ($\hat{\Omega}\cdot\nabla$) given in Table 1-3 for one-dimensional cylindrical geometry.

   b. Integrate the transport equation in one-dimensional cylindrical geometry over all angles and find the resulting form of the neutron balance equation.

3. (10 points) Solving the Transport Equation for the Uncollided Flux:

   Consider a homogeneous, source-free slab of thickness a whose total cross section is Σ_t. A beam of monoenergetic (E_0) neutrons with intensity I is incident on the left face of the slab oriented at an angle cosine of $\tilde{\mu} > 0$. A vacuum boundary condition applies on the right face of the slab.

   Write the transport equation for the uncollided flux, $\psi_0(x, \mu, E)$, and the appropriate boundary conditions. Solve the resulting **ordinary** differential equation [Note: ignore fission].

4. (10 points) The Once Collided Flux:

   Use the solution of the previous problem to determine the first collision source, then write (but do *not* solve) the transport equation and boundary conditions for the once collided flux [Note: ignore fission].

5. (10 points) Solving the Integral Transport Equation:

   Determine the scalar flux cause by an isotropic point source $S(E)\delta(\vec{r})$ in an infinite, homogeneous, purely absorbing medium with total cross section $\Sigma_t(E) = \Sigma_t$. Use the form of the integral transport equation for isotropic sources and scattering that we derived in class.

6. (10 points) There is a strong motivation to obtain a power distribution as flat as possible in a reactor core. One way in which this may be accomplished is to load a reactor with a non-uniform fuel enrichment. To model such a scheme, consider a bare, critical slab reactor as described by the one-group diffusion theory. Determine the fuel distribution $N_{fuel}(x)$ which will yield a flat power distribution, i.e. $P(x) = E_f\Sigma_f(x)\phi(x) = \text{constant}$. For convenience, assume that the fuel only absorbs neutrons and that it does not significantly scatter them. Also assume that all other materials in the core are uniformly distributed.

7. (10 points) "Reflector saving" is the difference between the dimensions of a bare and a reflected core of the same composition. Determine the criticality condition and the reflector saving for an infinite cylindrical core of radius R surrounded by a reflector of thickness a.

8. (10 points) A one-dimensional slab reactor system consists of three regions: vacuum for $x < 0$, a multiplying core for $0 < x < a$, and an infinite non-multiplying reflector for $x > a$. Suppose the reactor is subcritical and an external source S_0 is uniformly distributed throughout the reflector region. Determine the flux at all points.

9. (15 points) A bare spherical reactor is to be constructed of a homogeneous mixture of D2O and 235U. The composition is such that for every uranium atom there are 2000 heavy water molecules. Calculate:

   a. the critical radius of the reactor;

   b. the mean number of scattering collisions made by a neutron during its lifetime in this reactor.

   Data: $\eta_{U235} = 2.06$, $D_{D2O} = 0.87$ cm, $\Sigma_a^{D2O} = 3.3 \times 10^{-5}$ cm^{-1}, $\sigma_a^{D2O} = 0.001$ b, and $\sigma_a^{U235} = 678$ b.