NE 250 Fall 2017
Homework #6 Due December 1st, 2017

1. (10 points) Calculate the mean, variance, and the cumulative distribution function for each of the following probability density functions:

   a.
$$f(x) = \begin{cases} \frac{1}{a}\,, & 0 < x < a \\ 0\,, & x < 0,\ x > a \end{cases}$$

   b.
$$f(x) = \lambda e^{-\lambda x}\,, \quad x > 0$$

2. (20 points) Write a Monte Carlo program (using a programming language of your choice) to sample the probability density function $f(x) = \exp(-x)$, $x > 0$.

   a. Estimate the mean and variance of $f(x)$ using 10, 40, and 160 histories and compare the results to the values that you obtain analytically.

   b. After running 100 or more batches of 100 histories each, make a histogram like the figure below for the distribution of the batch averages. Plot the Gaussian distribution that is predicted for the batch averages from the central limit theorem.

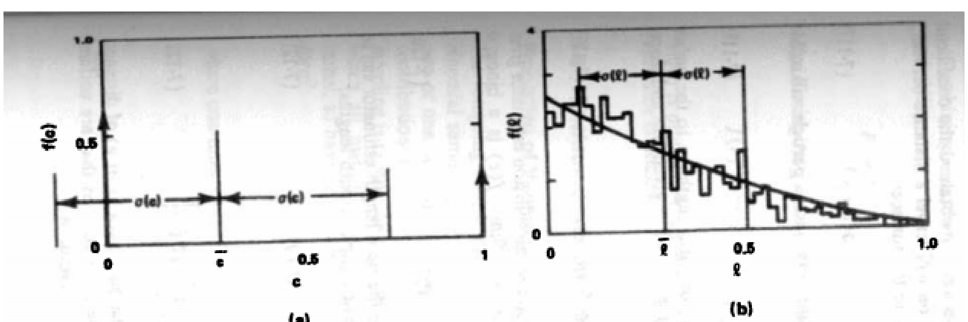

Figure 7-6 Characteristics of flux estimators for sample problem: (*a*) collision (binomial) estimator; (*b*) track length estimator.

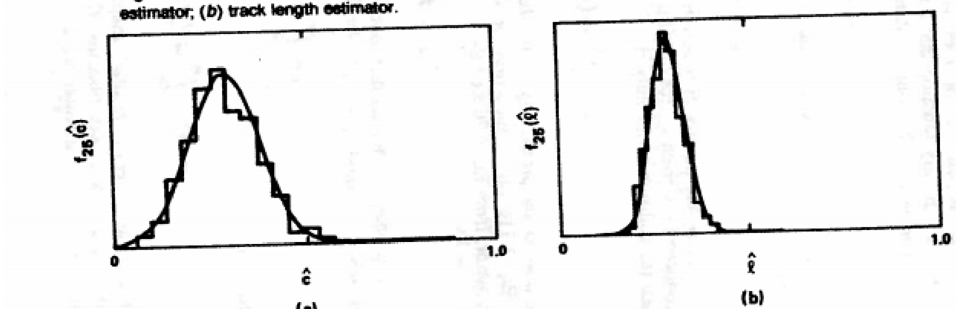

Figure 7-7 Distribution of average flux estimates with 25 histories / calculation: (*a*) collision estimator; (*b*) track length estimator.

Submit your code along with your results.

3. (15 points) Answer the following short answer questions. If true or false, provide a one sentence justification for your answer.

   a. True or False: we do not worry about normalizing PDFs for Monte Carlo, we can just sample from them directly.
   b. Compare and contrast: list three strengths and three weaknesses each for Monte Carlo and Deterministic methods.
   c. In *analog* Monte Carlo, the number of particles simulated, N, governs the solution accuracy. What is the relationship between N and relative error?
   d. What theorem allows us define confidence intervals about the expected value of our PDF using the sample mean and sample standard deviation? What is a crucial condition of that theorem?
   e. Why do we have to stop particles and boundary crossing when tracking their movement?
   f. When would you expect a collision estimator to be more accurate? When would you expect a track length estimator to be more accurate? Why?
   g. What is the consistent part in the CADIS method?

4. (25 points) Consider an infinite, steady-state, monoenergetic, two-region Monte Carlo problem with these characteristics:

   - treat the problem as 1-D
   - Region 1 has $\Sigma_s = 0.5$ and $\Sigma_t = 1.0$ (in both regions the only interactions are scattering or absorption).
   - Region 2 has $\Sigma_s = 0.75$ and $\Sigma_t = 0.9$.
   - Region 1 has $w_{nom} = 1$ and Region 2 has $w_{nom} = 2$. $w_{\max}$ and $w_{\min}$ can be found as $w_{nom} * 2.5$ or $w_{nom}/2.5$, respectively.
   - All particles are born in Region 1 with weight 1 at a location that is 1 cm to the left of the interface between Regions 1 and 2. The source is isotropic.
   - Isotropic scattering.

   Using this information:

   a. Write the algorithm for a Monte Carlo code to solve this specific problem. Include the PDFs required for sampling as well as algorithms for conducting sampling. Use a collision estimator to tally the flux. Include implicit capture, rouletting, and splitting.

b. Explicitly (write out all of the steps by hand or very explicitly in a short code) follow one particle through the geometry using your algorithm. Tally all collisions in each region and report the region tallies.
   Use the following random number sequence (generated by Python's `random.random()` when `random.seed(1)` is used) for this problem (this does not imply you need all of these random numbers):
   0.134364244112
   0.847433736937
   0.763774618977
   0.255069025739
   0.495435087092
   0.449491064789
   0.651592972723
   0.788723351136
   0.0938595867742
   0.028347476522
   0.83576510392
   0.432767067905
   0.762280082458
   0.00210605335111
   0.445387194055
   0.721540032341
   0.22876222127
   0.945270695554
   0.901427457611
   0.0305899830336