NE250 – Reactor Theory

8/24/2017

Nuclear Reactions

Two types

1. Spontaneous *(decay)*

- α: ${}^{A}_{Z}X \rightarrow^{A-4}_{Z-2} X +^{4}_{2} \alpha$
- β: ${}^{A}_{Z}X \rightarrow^{A}_{Z+1} X + \beta + \bar{\nu}$
- $\gamma\ X^* \ \rightarrow X+\gamma$

Decay Equations

$t \rightarrow N(t)$
$t + dt \rightarrow N(t + dt)$

$dN(t) = N(t + dt) - N(t)$
$dN(t) = -\lambda N(t)dt$ where λ is the decay constant.

... working through, with B.C. $N(t = 0) = N_0$ ...

$$N(t) = N_0 e^{-\lambda t}$$

Mean Lifetime:

$\frac{dN(t)}{N_0} = \lambda e^{-\lambda t} dt = p_d(t)$ where $p_d(t)dt$ is the probability of decay in time dt

$$\bar{t} = \int_0^\infty tp(t)dt = \frac{1}{\lambda}$$

Half Life:

$T_{1/2}$ is defined as the time s.t. $N(T_{1/2}) = \frac{N_0}{2}$;

$$T_{1/2} = \frac{ln2}{\lambda}$$

2. Induced *(projectile/target)*

this will be the emphasis of NE 250

Neutron-Nucleus

- elastic scattering
- inelastic scattering/compound nuclear reaction

$$n +_Z^A X \rightarrow_Z^{A+1} X^*$$

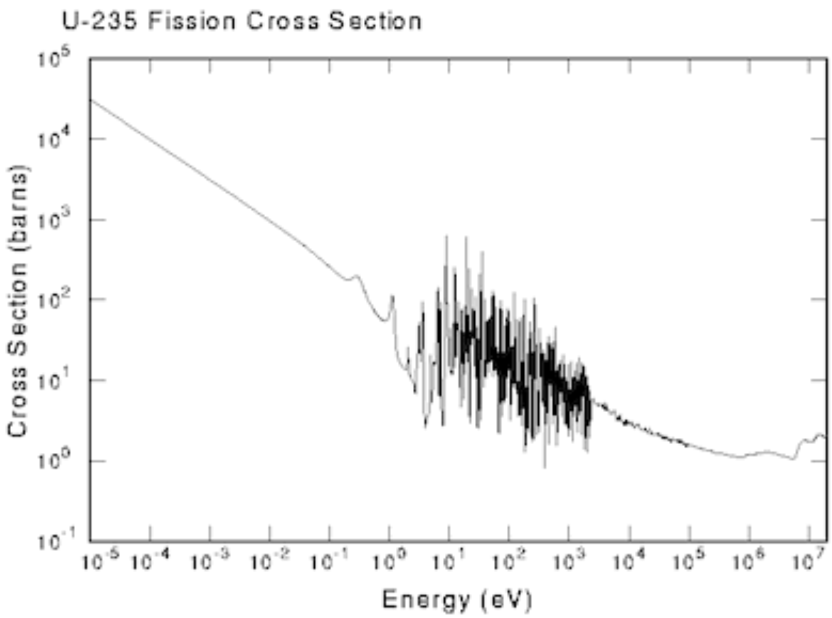
(could result in production of a γ or α, emission of a n, or fission)

Capture is a subset of absorption!

Absorption: $(n, \alpha), (n, \gamma), (n, f)$
Capture: (n, γ)

Cross Sections (σ)

- A property of the isotope and reaction
- A function of the isotope temperature (vibrational motion) and neutron speed (linear motion)
- Tabulated in XSec libraries *(3 common formats)*
    - ENDF (USA)
    - JEFF (Europe/NEA)
    - JENDL (Japan)



- Resonances in X-Sec plots due to excited energy levels that can be reached; nuclei only all excitation to these levels, and so only neutrons with this energy amount will be absorbed
- Cross sections measured at 300K (room temp); calculated using

$$E = k_B T$$

where k_B is Boltzmann's constant and $T = 300 \text{ K} \Rightarrow E = 0.0253 \text{ eV}$
- Higher temperatures cause resonant peak widths to broaden (less time spent near center of vibrational trajectory) → **Doppler Broadening**

Units

$1 \text{ barn} = 10^{24} \text{ cm}^2$
$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$

8/25/2017

Nuclear Reactions (cont.)

Fission

Can be spontaneous or induced

$$n +^{235} \text{U} \rightarrow X + Y + \bar{\nu} n + E$$

$\bar{\nu}$ is the average number of neutrons produced in a given fission event.

$E_f \approx 200\ \mathrm{MeV}$ (this is much higher than chemical reactions which are on the order of eV!)

Fissile Isotopes

$E_b > E_{\text{threshold}}$

These neutrons could (almost) be considered as "able to fission from 0 KE neutrons."

Includes $^{235}\mathrm{U}$, $^{233}\mathrm{U}$, $^{239}\mathrm{Pu}$, $^{241}\mathrm{Pu}$

Fissionable Isotopes

Fission requires collision with high E neutrons.

For $^{235}\mathrm{U}$ this is empirically given by

$$\chi(E) = 0.453e^{-1.036E}\sinh(\sqrt{2.29E})$$

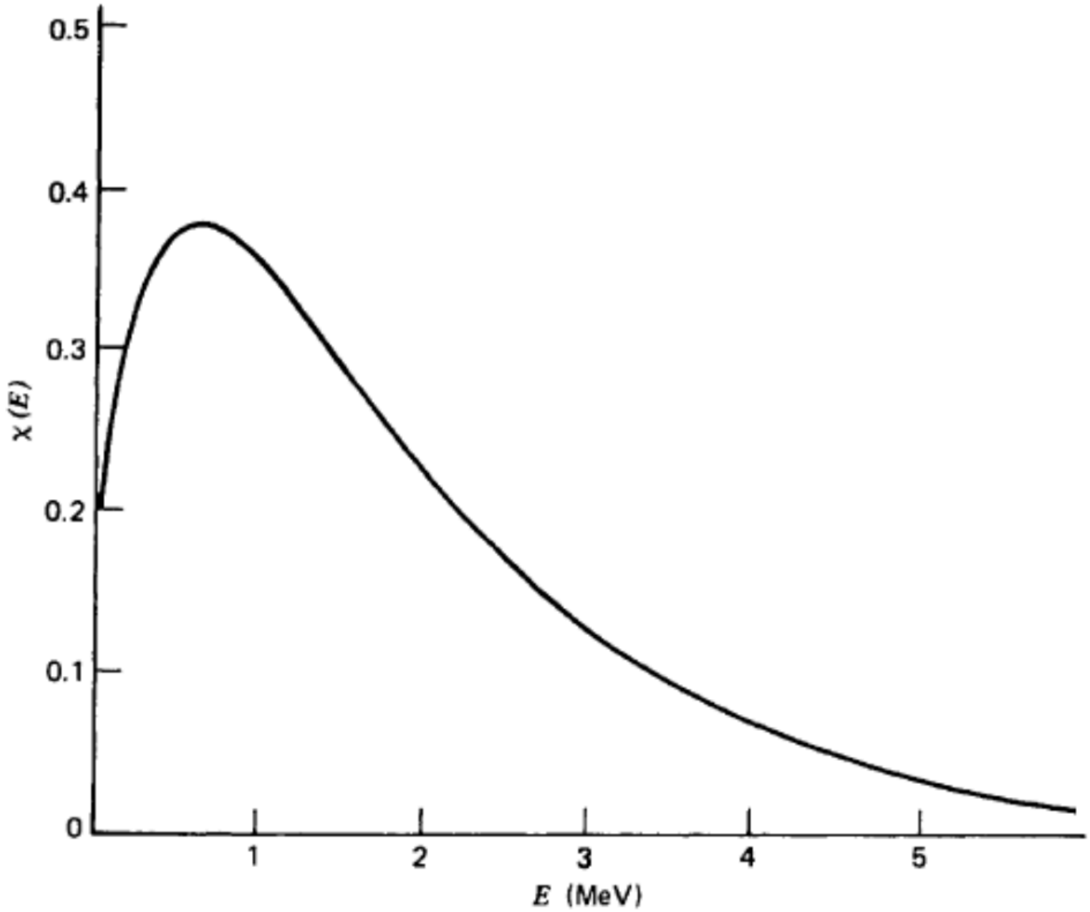


Figure 4: Fission spectrum for thermal neutron induced fission in $^2 35$U

Also, note that $\bar{\nu}$ depends on the isotope. Below is a plot of $\bar{\nu}$ for ^{239}Pu, ^{233}U and ^{235}U:

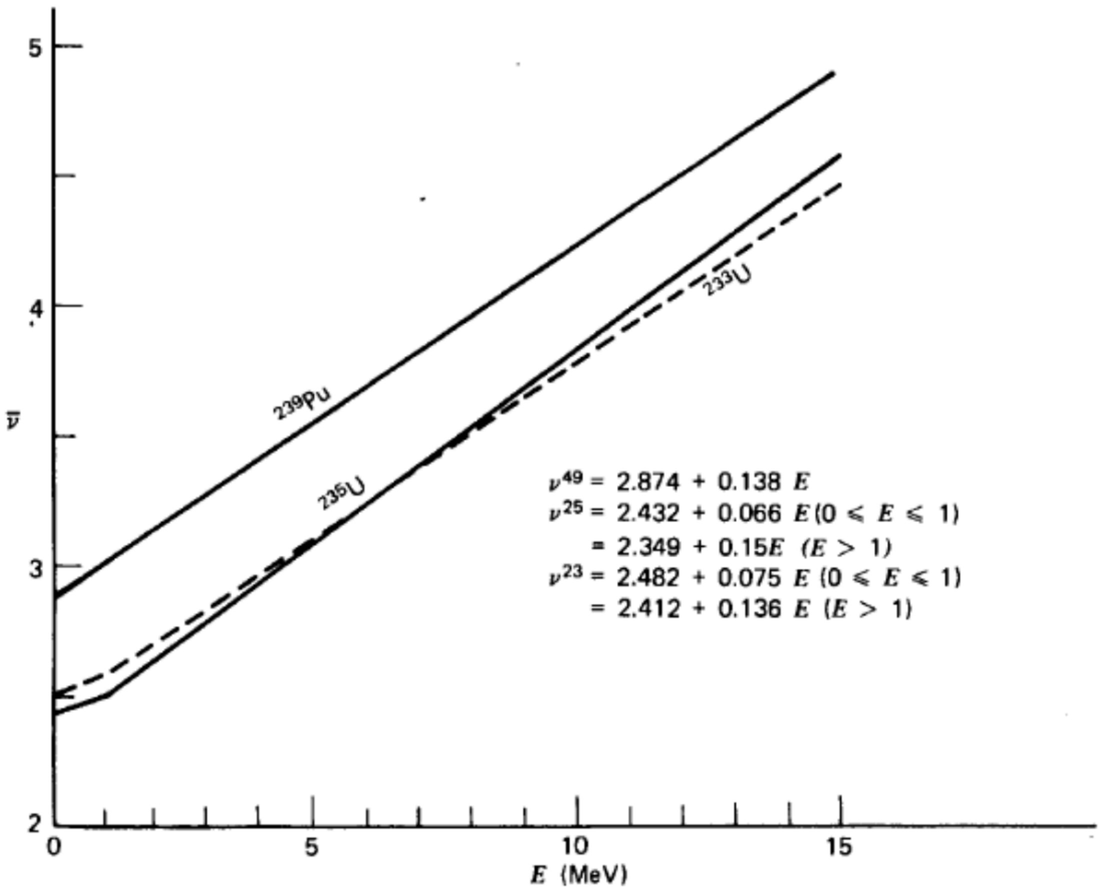


Figure 3: average # of neutrons released per fission as a function of energy

Fertile Isotopes

Isotopes which either undergo neutron capture (and subsequent decay) to become fissile isotopes.

Energy breakdown of fission outputs

- ~ 180 MeV in the KE of fission products
- ~ 5 MeV in the kinetic energy of neutrons
- ~ 7 MeV in prompt γs
- ~ 8 MeV in β^- decay of fission products
- ~ 7 MeV in delayed γs
- ~ 12 MeV in neutrinos

The energy from all outputs can be captured except for neutrinos.

Criticality

Multiplication Factor, k

$$k = \frac{\text{\# neutrons generated}}{\text{\# neutrons lost}}$$

$\text{\# neutrons generated} = \text{neutrons fission}$
$\text{\# neutrons lost} = \text{\# neutrons absorbed} + \text{\# neutrons leaked}$

$k = 1$: the reaction is critical; the chain reaction is controlled (reactor)
$k < 1$: the reactor is subcritcal; boring
$k > 1$: the reaction is supercritical; this is a bomb

Need to know:

1. n: neutron density $[n/\text{cm}^3]$
2. N: atom/nuclide density $[\text{nuclei/cm}^3]$
3. σ: microscopic cross section $[\text{cm}^2]$

Reaction rate: $R = nN\sigma$

Macroscopic cross section $[1/\text{cm}]$**:** $\Sigma = N\sigma$

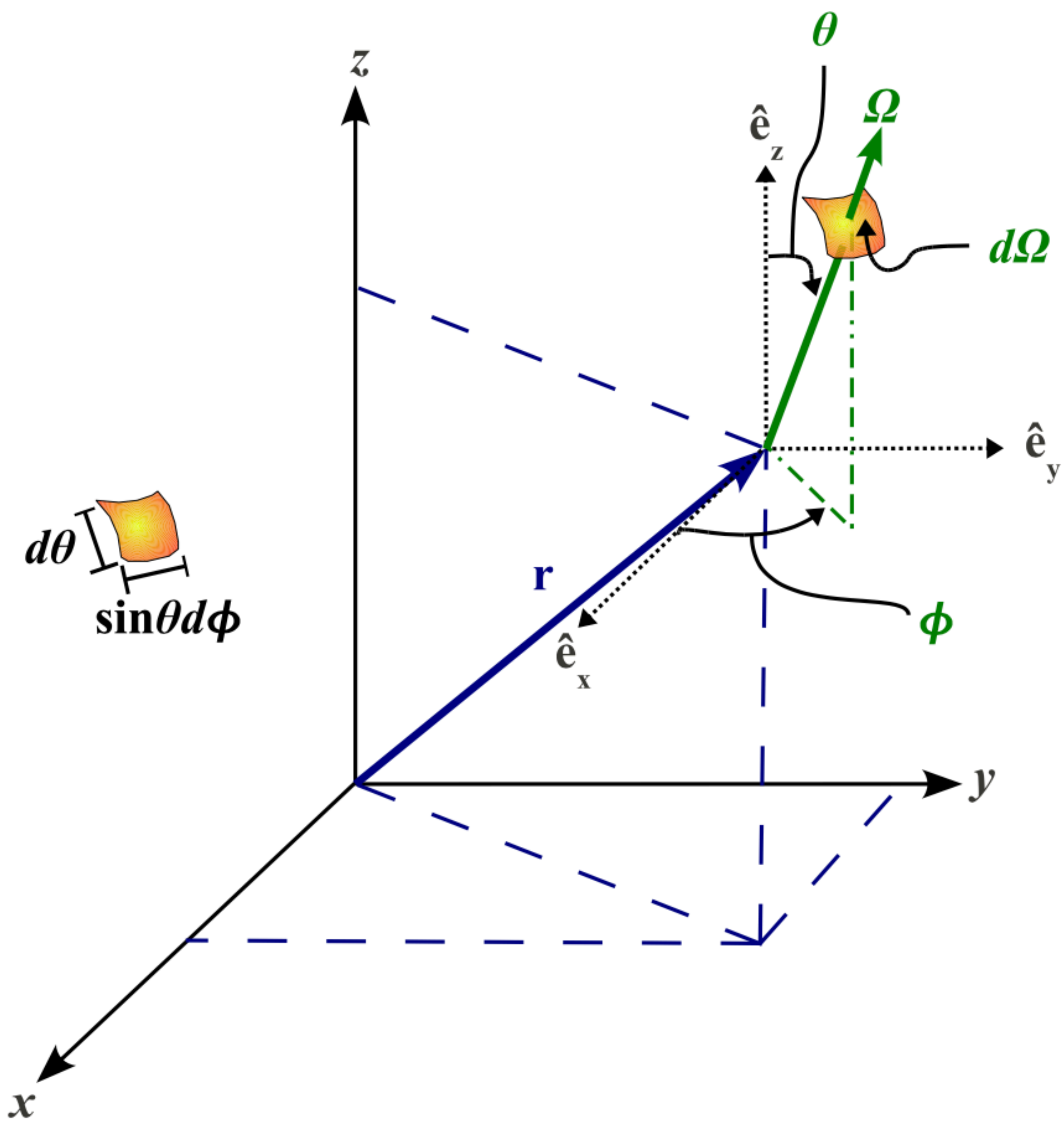
Angular neutron density $[\frac{n}{\text{cm}^3 \cdot \text{sr}}]$**:** $n(\vec{r}, E, \hat{\Omega}, t)$

$\vec{v} = v\,\hat{\Omega},\ \ |\hat{\Omega}| = 1$ (describes a sphere, formed by θ and ϕ)

$dr^3 = dx\ dy\ dz$

dE

$d\hat{\Omega} = \sin\theta\ d\theta\ d\phi$; $d\hat{\Omega}$ is a scalar, about the original position defined by vector $\hat{\Omega}$.



Altogether, $n(\vec{r}, E, \hat{\Omega}, t)\ d^3r\ dE\ d\hat{\Omega}$, gives the # of neutrons in the small volume about $\vec{r}$ with energy, E, and moving in direction $d\hat{\Omega}$ about $\hat{\Omega}$ at time t.

Angular neutron flux (scalar): $\phi(\vec{r}, E, \hat{\Omega}, t) = vn(\vec{r}, E, \hat{\Omega}, t)$
Angular neturon current (vector): $\vec{J}(\vec{r}, E, \hat{\Omega}, t) = \hat{\Omega}\phi(\vec{r}, E, \hat{\Omega}, t)$

We can find the number of neutrons in a volume V using

$$\int_V n(\vec{r}, E, \hat{\Omega}, t)\ d^3r$$

Change with time is then

$$\frac{\partial}{\partial t}\left[\int_V n(\vec{r}, E, \hat{\Omega}, t)\ d^3r\right] dE\ d\hat{\Omega} = \text{\# neutrons gained - \# neutrons lost}$$

neutrons gained: source (fission), in-scattering ($E', \hat{\Omega}' \rightarrow E, \hat{\Omega}$) # neutrons lost: absorption, scattering ($E, \hat{\Omega} \rightarrow E', \hat{\Omega}'$)

We also add a streaming term, to quantify neutrons leaking out (and in) to the system.

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