NE250_HW02_mnegus-prob5

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1 NE 250 – Homework 2

1.1 Problem 5

10/6/2017

A critical reactor has a multiplication factor of $k = 1$. The multiplication factor (for an infinite reactor) can be defined as

$$k_\infty \equiv \frac{\text{\# neutrons produced}}{\text{\# neutrons absorbed}}$$

Mathematically, the number of neutrons produced is $\int_0^\infty \nu\Sigma_f(E)\phi(E)\,dE$ and the number of neutrons absorbed is $\int_0^\infty \Sigma_a(E)\phi(E)\,dE$. Altogether, we can mathematically describe a critical reactor as

$$1 = \frac{\int_0^\infty \nu\Sigma_f(E)\phi(E)\,dE}{\int_0^\infty \Sigma_a(E)\phi(E)\,dE}$$

or equivalently

$$\int_0^\infty \nu\Sigma_f(E)\phi(E)\,dE = \int_0^\infty \Sigma_a(E)\phi(E)\,dE.$$

Since we are considering only thermal cross sections, we will let $\Sigma_X(E) = \Sigma_X(0.025\,\text{eV}) = \Sigma_{X,T}$ and we find

$$\nu\Sigma_{f,T}\int_0^\infty \phi(E)\,dE = \Sigma_{a,T}\int_0^\infty \phi(E)\,dE.$$

The integrals over flux cancel, and so

$$\nu\Sigma_{f,T} = \Sigma_{a,T}.$$

The macroscopic cross sections can be rewritten as $\Sigma_{f,T} = \Sigma_{f,T,\text{f}}$ and $\Sigma_{a,T} = \Sigma_{a,T,\text{f}} + \Sigma_{a,T,\text{m}}$ where subscripts f and m denote fuel and moderator, respectively. Furthermore, each macroscopic cross section for each material can be expressed in terms of the material's number density and microscopic cross section, $\Sigma = n\sigma$. In total

$$\nu n_\text{f}\sigma_{f,T,\text{f}} = n_\text{f}\sigma_{a,T,\text{f}} + n_\text{m}\sigma_{a,T,\text{m}}.$$

The fuel-to-moderator (number) density ratio at criticality can then be expressed as

$$\frac{n_\text{f}}{n_\text{m}} = \frac{\sigma_{a,T,\text{m}}}{\nu\sigma_{f,T,\text{f}} - \sigma_{a,T,\text{f}}}.$$

```
In [1]: def FMratio(xsa_mod,xsa_fuel,xsf_fuel,nu):
            return xsa_mod/(nu*xsf_fuel - xsa_fuel)
```

We can define our thermal cross sections as follows (from ENDF/B-VII.1)

```
In [2]: xsf = {'U235': 585.086}
        xsa = {'C12':   0.00336,
                    'Be9':   0.01004,
                    'H1':    0.33201,
                    'H2':    0.00051,
                    'O16':   0.00019,
                    'U235':  585.086 + 98.6864
                   }
        xsa['graphite'] = xsa['C12']
        xsa['water'] = 2*xsa['H1'] + xsa['O16']
        xsa['heavy water'] = 2*xsa['H2'] + xsa['O16']
```

a) Graphite

```
In [3]: xsa_mod=xsa['graphite']
        print('Critical Fuel-to-Moderator Ratio: ',FMratio(xsa_mod,xsa['U235'],xsf[

Critical Fuel-to-Moderator Ratio:  4.5457204996214396e-06
```

b) Beryllium

```
In [4]: xsa_mod=xsa['Be9']
        print('Critical Fuel-to-Moderator Ratio: ',FMratio(xsa_mod,xsa['U235'],xsf[

Critical Fuel-to-Moderator Ratio:  1.358304577863073e-05
```

c) Water

```
In [5]: xsa_mod=xsa['water']
        print('Critical Fuel-to-Moderator Ratio: ',FMratio(xsa_mod,xsa['U235'],xsf[

Critical Fuel-to-Moderator Ratio:  0.0008986050634087967
```

d) Heavy Water

```
In [6]: xsa_mod=xsa['heavy water']
        print('Critical Fuel-to-Moderator Ratio: ',FMratio(xsa_mod,xsa['U235'],xsf[

Critical Fuel-to-Moderator Ratio:  1.6370005370660543e-06
```