NE250 Homework 2

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Problem 1

If the energy distribution for fission neutrons from ^{235}U follows the functional approximation (for energy in MeV)

$$\chi(E) = 0.453e^{-1.036E}\sinh(\sqrt{2.29E}),$$

then the most probable energy of a neutron corresponds to the maximum of the function. A maximum value will be found at a critical point of the function, which can be found via differentiation (specifically when $\frac{d\chi}{dE} = 0$):

$$\begin{aligned}
\frac{d\chi}{dE} = 0 &= \frac{d}{dE}\left(0.453e^{-1.036E_{\max}}\sinh(\sqrt{2.29E_{\max}})\right) \\
0 &= 0.453\frac{d}{dE}\left(e^{-1.036E_{\max}}\sinh(\sqrt{2.29E_{\max}})\right) \\
0 &= 0.453\left[e^{-1.036E}\frac{d}{dE}\sinh(\sqrt{2.29E_{\max}}) + \frac{d}{dE}\left(e^{-1.036E_{\max}}\right)\sinh(\sqrt{2.29E_{\max}})\right] \\
0 &= 0.453\left[e^{-1.036E}\cosh(\sqrt{2.29E})\frac{d}{dE}\left(\sqrt{2.29E_{\max}}\right) - 1.036e^{-1.036E}\sinh(\sqrt{2.29E})\right] \\
0 &= 0.453\left[e^{-1.036E}\cosh(\sqrt{2.29E})\frac{\sqrt{2.29}}{2\sqrt{E_{\max}}} - 1.036e^{-1.036E}\sinh(\sqrt{2.29E})\right] \\
0 &= \frac{\sqrt{2.29}\cosh(\sqrt{2.29E})}{2\sqrt{E_{\max}}} - 1.036\sinh(\sqrt{2.29E}) \\
0 &= 1 - 1.369\sqrt{E_{\max}}\tanh(\sqrt{2.29E_{\max}}) \\
1 &= 1.369\sqrt{E_{\max}}\tanh(\sqrt{2.29E_{\max}})
\end{aligned}$$

$$\boxed{E_{\max} = 0.724 \text{ MeV}}$$

The average energy can be found by finding the expected value of the function on the domain $[0, \infty)$.

$$\begin{aligned}
E_{\text{ave}} &= \int_0^\infty E\,\chi(E)\,dE \\
&= \int_0^\infty E\left(0.453e^{-1.036E}\sinh(\sqrt{2.29E})\right)\,dE \\
&= 0.453\int_0^\infty E\,e^{-1.036E}\sinh(\sqrt{2.29E})\,dE
\end{aligned}$$

This integral cannot be solved analytically. Solving numerically (with Wolram Alpha),

$$\boxed{E_{\text{ave}} = 1.98 \text{ MeV}}$$

Problem 2

A general reaction rate for process x as a function of energy can be defined as

$$R_x(E) = \Sigma_x(E)\phi(E)$$

where $\Sigma_x(E)$ is the macroscopic cross section for reaction x and ϕ is the neutron flux, both at energy E. The macroscopic cross section can be further decomposed, so that

$$R_x(E) = n_x\sigma_x(E)\phi(E).$$

Comparing neutron-neutron reactions with all neutron-nuclei reactions, the ratio of reaction rates is

$$\frac{R_{nn}(E)}{R_{tot}(E)} = \frac{n_n\sigma_{nn}(E)\phi(E)}{n_{\mathrm{UO_2}}\sigma_{tot}(E)\phi(E)}.$$

We can know that the neutron flux is $\phi(0.025\text{ eV} = 10^{16}\text{ neutrons}/(\text{cm}^2\cdot\text{s})$ and they are at thermal energies ($E = 0.025$ eV and traveling at $v = \sqrt{\frac{2(0.025\text{eV})}{m_n}} = 2.190\times10^5$ cm/s). The neutrons that are then in a 1 cm^3 volume at any given second is

$$n_n = \frac{\phi(0.025\text{ eV})}{v} = \frac{10^{16}\text{ neutrons}/(\text{cm}^2\cdot\text{s})}{2.190\times10^5\text{ cm/s}} = 4.566\times10^{10}\text{ neutrons/cm}^3$$

For UO_2, $\rho = 10.97$ g/cm^3, $m_{\mathrm{O}} = 16.0$ g/mol, $m_{\mathrm{U8}} = 238.05$ g/mol, and $m_{\mathrm{U5}} = 235.04$ g/mol. If we use an enrichment of 5% (atom percent), then $m_{\mathrm{UO_2}} = 0.95(238.05) + 0.05(235.04) + 2(16.0) = 269.90$ g/mol. For number density, we find

$$n_{\mathrm{UO_2}} = \frac{\rho N_A}{m_{\mathrm{UO_2}}} = \frac{(10.97\text{ g/cm}^3)(6.022\times10^{23})}{269.90\text{ g/mol}} = 2.448\times10^{22}\text{ molecules/cm}^3$$

Additionally, we're given that $\sigma_{nn} = 10$ b, and we can determine the microscopic cross section for UO_2 from tabulated data. (From ENDF/B-VII.1 at 0.025 eV: $\sigma_{tot,\mathrm{U8}} = 11.962$ b, $\sigma_{tot,\mathrm{U5}} = 698.856$ b, and $\sigma_{tot,\mathrm{O}} = 3.852$ b)

$$\sigma_{tot} = 0.95\sigma_{tot,\mathrm{U8}} + 0.05\sigma_{tot,\mathrm{U5}} + 2\sigma_{tot,\mathrm{O}}$$

$$\sigma_{tot} = 0.95(11.962\text{ b}) + 0.05(698.856\text{ b}) + 2(3.852\text{ b})$$

$$\sigma_{tot} = 54.011\text{ b}$$

We can now sove for the ratio of reaction rates (noting that $\phi(E)$ cancels in the numerator and denominator)

$$\frac{R_{nn}(E)}{R_{tot}(E)} = \frac{n_n\sigma_{nn}(E)}{n_{\mathrm{UO_2}}\sigma_{tot}(E)} = \frac{(4.566\times10^{10}\text{ neutrons/cm}^3)(10\text{ b})}{(2.448\times10^{22}\text{ molecules/cm}^3)(54.011\text{ b})}$$

$$\frac{R_{nn}(E)}{R_{tot}(E)} = 3.40\times10^{-13}$$

The rate of neutron-neutron collisions is 13 orders of magnitudes less than the rate of neutron-UO_2 collisions.

Problem 3

Problem 4

First, we define the average scattering cosine $\bar{\mu}_0$ as the average dot product, $\langle \hat{\Omega} \cdot \hat{\Omega}' \rangle$. When normalized by $4\pi\Sigma_s$, the total of cross sections for scattering from any angle $\hat{\Omega}$ to any other angle $\hat{\Omega}'$, this is

$$\bar{\mu}_0 \equiv \langle \hat{\Omega} \cdot \hat{\Omega}' \rangle = \left(\frac{1}{4\pi\Sigma_s} \right) \int_{4\pi} d\hat{\Omega} \int_{4\pi} d\hat{\Omega}' \, \hat{\Omega} \cdot \hat{\Omega}' \Sigma_s(\hat{\Omega} \cdot \hat{\Omega}')$$

In the center of mass system, the probability that a particle scatters in any direction is roughly uniform, $\Sigma_{\text{CM}}(\theta_C) = \frac{\Sigma_s}{4\pi}$,

$$\bar{\mu}_0 = \frac{1}{\Sigma_s} \int_{4\pi} d\hat{\Omega} \int_{4\pi} d\hat{\Omega}' \, \hat{\Omega} \cdot \hat{\Omega}' \Sigma_{\text{CM}}(\theta_C).$$

Problem 5

A critical reactor has a multiplication factor of $k = 1$. The multiplication factor (for an infinite reactor) can be defined as

$$k_\infty \equiv \frac{\text{\# neutrons produced}}{\text{\# neutrons absorbed}}$$

Mathematically, the number of neutrons produced is $\int_0^\infty \nu\Sigma_f(E)\phi(E)\,dE$ and the number of neutrons absorbed is $\int_0^\infty \Sigma_a(E)\phi(E)\,dE$. Altogether, we can mathematically describe a critical reactor as

$$1 = \frac{\int_0^\infty \nu\Sigma_f(E)\phi(E)\,dE}{\int_0^\infty \Sigma_a(E)\phi(E)\,dE}$$

or equivalently

$$\int_0^\infty \nu\Sigma_f(E)\phi(E)\,dE = \int_0^\infty \Sigma_a(E)\phi(E)\,dE.$$

Since we are considering only thermal cross sections, we will let $\Sigma_X(E) = \Sigma_X(0.025\text{ eV}) = \Sigma_{X,T}$ and we find

$$\nu\Sigma_{f,T}\int_0^\infty \phi(E)\,dE = \Sigma_{a,T}\int_0^\infty \phi(E)\,dE.$$

The integrals over flux cancel, and so

$$\nu\Sigma_{f,T} = \Sigma_{a,T}.$$

The macroscopic cross sections can be rewritten as $\Sigma_{f,T} = \Sigma_{f,T,\mathrm{f}}$ and $\Sigma_{a,T} = \Sigma_{a,T,\mathrm{f}} + \Sigma_{a,T,\mathrm{m}}$ where subscripts f and m denote fuel and moderator, respectively. Furthermore, each macroscopic cross section for each material can be expressed in terms of the material's number density and microscopic cross section, $\Sigma = n\sigma$. In total

$$\nu n_\mathrm{f}\sigma_{f,T,\mathrm{f}} = n_\mathrm{f}\sigma_{a,T,\mathrm{f}} + n_\mathrm{m}\sigma_{a,T,\mathrm{m}}.$$

The fuel-to-moderator density at criticality can then be expressed as

$$\frac{n_\mathrm{f}}{n_\mathrm{m}} = \frac{\sigma_{a,T,\mathrm{m}}}{\nu\sigma_{f,T,\mathrm{f}} - \sigma_{a,T,\mathrm{f}}}.$$

a.) **Graphite**

$$\frac{n_\mathrm{f}}{n_\mathrm{m}} = 4.55\times10^{-6}$$

b.) **Beryllium**

$$\frac{n_\mathrm{f}}{n_\mathrm{m}} = 1.36\times10^{-5}$$

c.) **Water**

$$\frac{n_\mathrm{f}}{n_\mathrm{m}} = 8.99\times10^{-4}$$

d.) **Graphite**

$$\frac{n_\mathrm{f}}{n_\mathrm{m}} = 1.64\times10^{-6}$$

(see attached Jupyter notebook for full calculations)

Problem 6

a.)

Like in Problem 5,

$$k_\infty = \frac{\int_0^\infty \nu\Sigma_f(E)\phi(E)\,dE}{\int_0^\infty \Sigma_a(E)\phi(E)\,dE}.$$

Again, since we are only considering cross sections averaged over the fast spectrum, we can state that $\Sigma_X(E) = \Sigma_{X,F}$, and then

$$k_\infty = \frac{\nu\Sigma_{f,F}\int_0^\infty \phi(E)\,dE}{\Sigma_{a,F}\int_0^\infty \phi(E)\,dE} = \frac{\nu\Sigma_{f,F}}{\Sigma_{a,F}}.$$

Next, we repeat our decomposition of the numerator and denominator, leaving

$$k_\infty = \frac{\nu_{\mathrm{PuO_2}} n_{\mathrm{PuO_2}} \sigma_{F,\mathrm{PuO_2}} + \nu_{\mathrm{UO_2}} n_{\mathrm{UO_2}} \sigma_{F,\mathrm{UO_2}}}{n_{\mathrm{PuO_2}}\sigma_{a,F,\mathrm{PuO_2}} + n_{\mathrm{UO_2}}\sigma_{a,F,\mathrm{UO_2}} + n_{\mathrm{Fe}}\sigma_{a,F,\mathrm{Fe}} + n_{\mathrm{Na}}\sigma_{a,F,\mathrm{Na}}}$$

After solving this equation with the given values, we find

$$\boxed{k_\infty = 1.332}$$

b.)

We can use the densities and volume fractions to determine the fraction of the core mass that is fuel.

$$\boxed{61.6\% \text{ is fuel}}$$

c.)

If $k = 1$ and the non-leakage probability is $P_{NL} = 0.90$, then we can reexpress our earlier equation as

$$k = 1 = P_{NL}\frac{\nu\Sigma_{f,F}\int_0^\infty \phi(E)\,dE}{\Sigma_{a,F}\int_0^\infty \phi(E)\,dE} = P_{NL}\frac{\nu\Sigma_{f,F}}{\Sigma_{a,F}},$$

or

$$P_{NL}\nu\Sigma_{f,F} = \Sigma_{a,F}.$$

When separated into individual materials, this is

$$P_{NL}\sum_i \nu_i n_i \sigma_{f,i} = \sum_j n_j \sigma_{a,j}.$$

or explicitly

$$P_{NL}\left(\nu_{\mathrm{PuO_2}} n_{\mathrm{PuO_2}} \sigma_{F,\mathrm{PuO_2}} + \nu_{\mathrm{UO_2}} n_{\mathrm{UO_2}} \sigma_{F,\mathrm{UO_2}}\right) = n_{\mathrm{PuO_2}}\sigma_{a,F,\mathrm{PuO_2}} + n_{\mathrm{UO_2}}\sigma_{a,F,\mathrm{UO_2}} + n_{\mathrm{Fe}}\sigma_{a,F,\mathrm{Fe}} + n_{\mathrm{Na}}\sigma_{a,F,\mathrm{Na}}$$

We recall that $n_{\mathrm{PuO_2}} = f_{\mathrm{f,PuO_2}}(0.3)\frac{\rho_{\mathrm{PuO_2}} N_A}{m_{\mathrm{PuO_2}}}$ and $n_{\mathrm{UO_2}} = (1 - f_{\mathrm{f,PuO_2}})(0.3)\frac{\rho_{\mathrm{UO_2}} N_A}{m_{\mathrm{UO_2}}}$, while $n_{\mathrm{Na}} = (0.5)\frac{\rho_{\mathrm{Na}} N_A}{m_{\mathrm{Na}}}$and $n_{\mathrm{Fe}} = (0.2)\frac{\rho_{\mathrm{Fe}} N_A}{m_{\mathrm{Fe}}}$. We can substitute these expressions into the equation above and solve for $f_{\mathrm{f,PuO_2}}$.

$$\boxed{\text{The fuel must be } 10.322\% \ \mathrm{PuO_2} \text{ and } 89.678\% \ \mathrm{UO_2} \text{ for a critical reactor with } P_{NL} = 0.9}$$

(see attached Jupyter notebook for full calculations)

Problem 7

The general one-group diffusion equation is

$$\frac{1}{v}\frac{\partial\phi(\vec{r},t)}{\partial t} = S(\vec{r},t) - \Sigma_{a,1}(\vec{r})\phi(\vec{r},t) + \nabla\cdot D(\vec{r})\nabla\phi(\vec{r},t).$$

If we assume that our reflected system is steady state, then $\frac{\partial\phi(\vec{r},t)}{\partial t} = 0$. Also, we assume that our system is homogenous in each region (slab and reflector) so that $\Sigma_a(\vec{r}) = \Sigma_{a,\mathrm{X}}$ and $D(\vec{r}) = D_{\mathrm{X}}$ for each region X in the system. Then, we rearrange to find

$$-D_{\mathrm{X}}\nabla^2\phi(\vec{r}) + \Sigma_{a,\mathrm{X}}\phi(\vec{r}) = S(\vec{r}).$$

Where we are considering a slab geometry, the Laplacian of the flux simplifies to a second derivative, and $\vec{r}$ is replaced by x.

$$\frac{d\phi(\vec{r})}{dx} - \frac{\Sigma_{a,\mathrm{X}}}{D_{\mathrm{X}}}\phi(\vec{r}) = -\frac{S(\vec{r})}{D_{\mathrm{X}}}.$$

We let $L_{\mathrm{X}} = \sqrt{\frac{\Sigma_{a,\mathrm{X}}}{D_{\mathrm{X}}}}$, and so

$$\frac{d^2\phi(x)}{dx^2} - \frac{1}{L_{\mathrm{X}}^2}\phi(x) = -\frac{S(x)}{D_{\mathrm{X}}}.$$

Noting the symmetry of our system (a slab of width $2b$ with the midplane at the origin, bounded by a reflector of width a on each side), we can give the one-speed diffusion equation in the slab as

$$\frac{d^2\phi(x)}{dx^2} - \frac{1}{L_{\mathrm{S}}^2}\phi(x) = -\frac{S_0}{D_{\mathrm{S}}}$$

and in the reflector as

$$\frac{d^2\phi(x)}{dx^2} - \frac{1}{L_{\mathrm{R}}^2}\phi(x) = 0.$$

The boundary conditions on our system are that

$$\phi_S(b) = \phi_R(b) \qquad \phi_S(a+\tilde{b}) = 0$$
$$\vec{J}_S(b) = \vec{J}_R(b) \qquad \vec{J}_S(a+b) = 0$$

Problem 10

Answer the following questions as true or false, provide a one sentence justification for your answer

1. The integro-differential form of the transport equation expresses a local balance between neutron production and losses.
2. A vacuum boundary condition for the integro-differential transport equation implies a zero outgoing angular flux.
3. In the transport equation in curvilinear coordinates, the redistribution term allows neutrons to migrate between the directions as they move along a straight line.
4. A nuclear system is subcritical if its eigenvalues satisfy max(Re(j)) ¡ 1.
5. The energy spectrum of the fundamental eigenmode of the eigenvalue problem is skewed as though a 1 absorber is present.

NE250_HW02_mnegus-prob5

October 5, 2017

1 NE 250 – Homework 2

1.1 Problem 5

10/6/2017

A critical reactor has a multiplication factor of $k = 1$. The multiplication factor (for an infinite reactor) can be defined as

$$k_\infty \equiv \frac{\text{\# neutrons produced}}{\text{\# neutrons absorbed}}$$

Mathematically, the number of neutrons produced is $\int_0^\infty \nu\Sigma_f(E)\phi(E)\,dE$ and the number of neutrons absorbed is $\int_0^\infty \Sigma_a(E)\phi(E)\,dE$. Altogether, we can mathematically describe a critical reactor as

$$1 = \frac{\int_0^\infty \nu\Sigma_f(E)\phi(E)\,dE}{\int_0^\infty \Sigma_a(E)\phi(E)\,dE}$$

or equivalently

$$\int_0^\infty \nu\Sigma_f(E)\phi(E)\,dE = \int_0^\infty \Sigma_a(E)\phi(E)\,dE.$$

Since we are considering only thermal cross sections, we will let $\Sigma_X(E) = \Sigma_X(0.025\,\text{eV}) = \Sigma_{X,T}$ and we find

$$\nu\Sigma_{f,T}\int_0^\infty \phi(E)\,dE = \Sigma_{a,T}\int_0^\infty \phi(E)\,dE.$$

The integrals over flux cancel, and so

$$\nu\Sigma_{f,T} = \Sigma_{a,T}.$$

The macroscopic cross sections can be rewritten as $\Sigma_{f,T} = \Sigma_{f,T,\text{f}}$ and $\Sigma_{a,T} = \Sigma_{a,T,\text{f}} + \Sigma_{a,T,\text{m}}$ where subscripts f and m denote fuel and moderator, respectively. Furthermore, each macroscopic cross section for each material can be expressed in terms of the material's number density and microscopic cross section, $\Sigma = n\sigma$. In total

$$\nu n_\text{f}\sigma_{f,T,\text{f}} = n_\text{f}\sigma_{a,T,\text{f}} + n_\text{m}\sigma_{a,T,\text{m}}.$$

The fuel-to-moderator (number) density ratio at criticality can then be expressed as

$$\frac{n_\text{f}}{n_\text{m}} = \frac{\sigma_{a,T,\text{m}}}{\nu\sigma_{f,T,\text{f}} - \sigma_{a,T,\text{f}}}.$$

```
In [1]: def FMratio(absorbXS_mod,absorbXS_fuel,fissionXS_fuel,nu):
            return absorbXS_mod/(nu*fissionXS_fuel - absorbXS_fuel)
```

We can define our thermal cross sections as follows (from ENDF/B-VII.1)

```
In [2]: fissionXS = {'U235': 585.086}
        absorbXS = {'C12':   0.00336,
                    'Be9':   0.01004,
                    'H1':    0.33201,
                    'H2':    0.00051,
                    'O16':   0.00019,
                    'U235':  585.086 + 98.6864
                   }
        absorbXS['graphite'] = absorbXS['C12']
        absorbXS['water'] = 2*absorbXS['H1'] + absorbXS['O16']
        absorbXS['heavy water'] = 2*absorbXS['H2'] + absorbXS['O16']
```

a) Graphite

```
In [3]: absorbXS_mod=absorbXS['graphite']
        print('Critical Fuel-to-Moderator Ratio: ',FMratio(absorbXS_mod,absorbXS['U

Critical Fuel-to-Moderator Ratio:  4.5457204996214396e-06
```

b) Beryllium

```
In [4]: absorbXS_mod=absorbXS['Be9']
        print('Critical Fuel-to-Moderator Ratio: ',FMratio(absorbXS_mod,absorbXS['U

Critical Fuel-to-Moderator Ratio:  1.358304577863073e-05
```

c) Water

```
In [5]: absorbXS_mod=absorbXS['water']
        print('Critical Fuel-to-Moderator Ratio: ',FMratio(absorbXS_mod,absorbXS['U

Critical Fuel-to-Moderator Ratio:  0.0008986050634087967
```

d) Heavy Water

```
In [6]: absorbXS_mod=absorbXS['heavy water']
        print('Critical Fuel-to-Moderator Ratio: ',FMratio(absorbXS_mod,absorbXS['U

Critical Fuel-to-Moderator Ratio:  1.6370005370660543e-06
```

NE250_HW02_mnegus-prob6

October 5, 2017

1 NE 250 – Homework 2

1.1 Problem 6

10/6/2017

a) Like in Problem 5,

$$k_\infty = \frac{\int_0^\infty \nu\Sigma_f(E)\phi(E)\,dE}{\int_0^\infty \Sigma_a(E)\phi(E)\,dE}.$$

Again, since we are only considering cross sections averaged over the fast spectrum, we can state that $\Sigma_X(E) = \Sigma_{X,F}$, and then

$$k_\infty = \frac{\nu\Sigma_{f,F}\int_0^\infty \phi(E)\,dE}{\Sigma_{a,F}\int_0^\infty \phi(E)\,dE} = \frac{\nu\Sigma_{f,F}}{\Sigma_{a,F}}.$$

Next, we repeat our decomposition of the numerator and denominator. For neutron production, we find

$$\nu\Sigma_f = \sum_i \nu_i n_i \sigma_{f,i}$$

where i represents the i^{th} fuel.

```
In [1]: def production(fuels,nu,n,xsf):
            p = [nu[i]*n[i]*xsf[i] for i in fuels]
            return sum(p)
```

For neutron absorption,

$$\Sigma_a = \sum_j n_j \sigma_{a,j}$$

and j represents the j^{th} material in the reactor.

```
In [2]: def absorption(materials,n,xsa):
            a = [n[j]*xsa[j] for j in materials]
            return sum(a)
```

We substitute these into our equation for k_∞ and solve.

```
In [3]: def k_infinity(fuels,materials,nu,n,xsf,xsa):
            k_inf = production(fuels,nu,n,xsf)/absorption(materials,n,xsa)
            return k_inf
```

We are given the material densities in [g/cm^3], the volume fractions of the materials, and the microscopic cross sections [1/cm^2].

```
In [4]: densities = {'PuO2': 11.0,
                     'UO2':  11.0,
                     'Na':   0.97,
                     'Fe':   7.87
                    }
        vol_fracs = {'PuO2': 0.15*0.30,
                     'UO2':  0.85*0.30,
                     'Na':   0.500,
                     'Fe':   0.200
                    }
        xsf = {'PuO2': 1.95e-24,
               'UO2':  0.05e-24,
              }
        xsa = {'PuO2': 2.4000e-24,
               'UO2':  0.4040e-24,
               'Na':   0.0018e-24,
               'Fe':   0.0087e-24
              }
```

The number densities can be found by using the molar masses of the materials [g/mol] and the formula, $n_j = f_j \frac{\rho_j N_A}{m_j}$.

```
In [5]: m = {'PuO2': 244+15.999*2,
             'UO2':  238.03+15.999*2,
             'Na':   22.990,
             'Fe':   55.845
            }
        n = {material: vol_fracs[material]*densities[material]*6.022e23/m[material]
```

We are also given ν_{PuO_2} and ν_{UO_2}.

```
In [6]: nu = {'PuO2': 2.98,
              'UO2':  2.47
             }
```

Now we can solve for k_∞

```
In [7]: fuels = ['PuO2','UO2']
        materials = ['PuO2','UO2','Na','Fe']
        print('k_infinity =',k_infinity(fuels,materials,nu,n,xsf,xsa))
```

```
k_infinity = 1.3324856823514228
```

b) The masses of the total core and fuel are given by $\sum_k f_k \rho_k$, where k represents the k^{th} material or fuel.

```
In [8]: def mass(components,vol_fracs,densities):
            return sum([vol_fracs[k]*densities[k] for k in components])
```

```
In [9]: total_mass = mass(materials,vol_fracs,densities)
        fuel_mass = mass(fuels,vol_fracs,densities)
        frac = fuel_mass/total_mass
        print('The fraction of the core which is fuel is {}%'.format(round(100*frac
```

```
The fraction of the core which is fuel is 61.579%
```

c) If $k = 1$ and the non-leakage probability is 0.90, then we can reexpress our earlier equation as

$$k = 1 = P_{NL} \frac{\nu \Sigma_{f,F} \int_0^\infty \phi(E)\, dE}{\Sigma_{a,F} \int_0^\infty \phi(E)\, dE} = P_{NL} \frac{\nu \Sigma_{f,F}}{\Sigma_{a,F}},$$

or

$$P_{NL} \nu \Sigma_{f,F} = \Sigma_{a,F}.$$

When separated into individual materials, this is

$$P_{NL} \sum_i \nu_i n_i \sigma_{f,i} = \sum_j n_j \sigma_{a,j}.$$

or explicitly

$$P_{NL} \left(\nu_{\text{PuO}_2} n_{\text{PuO}_2} \sigma_{F,\text{PuO}_2} + \nu_{\text{UO}_2} n_{\text{UO}_2} \sigma_{F,\text{UO}_2} \right) = n_{\text{PuO}_2} \sigma_{a,F,\text{PuO}_2} + n_{\text{UO}_2} \sigma_{a,F,\text{UO}_2} + n_{\text{Fe}} \sigma_{a,F,\text{Fe}} + n_{\text{Na}} \sigma_{a,F,\text{Na}}$$

We recall that $n_{\text{PuO}_2} = f_{\text{f,PuO}_2}(0.3) \frac{\rho_{\text{PuO}_2} N_A}{m_{\text{PuO}_2}}$ and $n_{\text{UO}_2} = (1 - f_{\text{f,PuO}_2})(0.3) \frac{\rho_{\text{UO2}} N_A}{m_{\text{UO}_2}}$, while $n_{\text{Na}} = (0.5) \frac{\rho_{\text{Na}} N_A}{m_{\text{Na}}}$ and $n_{\text{Fe}} = (0.2) \frac{\rho_{\text{Fe}} N_A}{m_{\text{Fe}}}$.

Then,

$$P_{NL} \left(\nu_{\text{PuO}_2} f_{\text{f,PuO}_2}(0.3) \frac{\rho_{\text{PuO}_2} N_A}{m_{\text{PuO}_2}} \sigma_{F,\text{PuO}_2} + \nu_{\text{UO}_2} (1 - f_{\text{f,PuO}_2})(0.3) \frac{\rho_{\text{UO2}} N_A}{m_{\text{UO}_2}} \sigma_{F,\text{UO}_2} \right) = f_{\text{f,PuO}_2}(0.3) \frac{\rho_{\text{PuO}_2} N_A}{m_{\text{PuO}_2}} \sigma_{a,F,\text{PuO}_2}$$

which can be expanded to

$$f_{\text{f,PuO}_2} P_{NL} \nu_{\text{PuO}_2} (0.3) \frac{\rho_{\text{PuO}_2}}{m_{\text{PuO}_2}} \sigma_{F,\text{PuO}_2} + P_{NL} \nu_{\text{UO}_2} (0.3) \frac{\rho_{\text{UO2}}}{m_{\text{UO}_2}} \sigma_{F,\text{UO}_2} - f_{\text{f,PuO}_2} P_{NL} \nu_{\text{UO}_2} (0.3) \frac{\rho_{\text{UO2}}}{m_{\text{UO}_2}} \sigma_{F,\text{UO}_2} = f_{\text{f,PuO}_2}(0.3) \frac{\rho_{\text{PuO}_2}}{m_{\text{PuO}_2}} \sigma_{a,F,\text{PuO}_2}$$

and simplified to

$$f_{\text{f,PuO}_2} \left(P_{NL} \nu_{\text{PuO}_2} (0.3) \frac{\rho_{\text{PuO}_2}}{m_{\text{PuO}_2}} \sigma_{F,\text{PuO}_2} - P_{NL} \nu_{\text{UO}_2} (0.3) \frac{\rho_{\text{UO2}}}{m_{\text{UO}_2}} \sigma_{F,\text{UO}_2} - (0.3) \frac{\rho_{\text{PuO}_2}}{m_{\text{PuO}_2}} \sigma_{a,F,\text{PuO}_2} + (0.3) \frac{\rho_{\text{UO2}}}{m_{\text{UO}_2}} \sigma_{a,F,\text{UO}_2} \right)$$

$$f_{\text{f,PuO}_2} = \frac{(0.3) \frac{\rho_{\text{UO2}}}{m_{\text{UO}_2}} \sigma_{a,F,\text{UO}_2} + (0.2) \frac{\rho_{\text{Fe}}}{m_{\text{Fe}}} \sigma_{a,F,\text{Fe}} + (0.5) \frac{\rho_{\text{Na}}}{m_{\text{Na}}} \sigma_{a,F,\text{Na}} - P_{NL} \nu_{\text{UO}_2} (0.3) \frac{\rho_{\text{UO2}}}{m_{\text{UO}_2}} \sigma_{F,\text{UO}_2}}{0.3 \left(P_{NL} \nu_{\text{PuO}_2} \frac{\rho_{\text{PuO}_2}}{m_{\text{PuO}_2}} \sigma_{F,\text{PuO}_2} - P_{NL} \nu_{\text{UO}_2} \frac{\rho_{\text{UO2}}}{m_{\text{UO}_2}} \sigma_{F,\text{UO}_2} - \frac{\rho_{\text{PuO}_2}}{m_{\text{PuO}_2}} \sigma_{a,F,\text{PuO}_2} + \frac{\rho_{\text{UO2}}}{m_{\text{UO}_2}} \sigma_{a,F,\text{UO}_2} \right)}$$

```
In [10]: P_NL = 0.90
         numerator = 0.3*xsa['UO2']*densities['UO2']/m['UO2'] + 0.2*xsa['Fe']*densi
         denominator = 0.3*(P_NL*nu['PuO2']*xsf['PuO2']*densities['PuO2']/m['PuO2']
         f = numerator/denominator
         print('The reactor will be critical if {}% of the fuel volume is PuO2 ({}%
```

```
The reactor will be critical if 10.322% of the fuel volume is PuO2 (3.097% of the c
```