

Problem 1

If the energy distribution for fission neutrons from ^{235}U follows the functional approximation (for energy in MeV)

$$\chi(E) = 0.453e^{-1.036E} \sinh(\sqrt{2.29E}),$$

then the most probable energy of a neutron corresponds to the maximum of the function. A maximum value will be found at a critical point of the function, which can be found via differentiation (specifically when $\frac{d\chi}{dE} = 0$):

$$\begin{aligned} \frac{d\chi}{dE} = 0 &= \frac{d}{dE} \left(0.453e^{-1.036E_{\max}} \sinh(\sqrt{2.29E_{\max}}) \right) \\ 0 &= 0.453 \frac{d}{dE} \left(e^{-1.036E_{\max}} \sinh(\sqrt{2.29E_{\max}}) \right) \\ 0 &= 0.453 \left[e^{-1.036E} \frac{d}{dE} \sinh(\sqrt{2.29E_{\max}}) + \frac{d}{dE} (e^{-1.036E_{\max}}) \sinh(\sqrt{2.29E_{\max}}) \right] \\ 0 &= 0.453 \left[e^{-1.036E} \cosh(\sqrt{2.29E}) \frac{d}{dE} (\sqrt{2.29E_{\max}}) - 1.036e^{-1.036E} \sinh(\sqrt{2.29E}) \right] \\ 0 &= 0.453 \left[e^{-1.036E} \cosh(\sqrt{2.29E}) \frac{\sqrt{2.29}}{2\sqrt{E_{\max}}} - 1.036e^{-1.036E} \sinh(\sqrt{2.29E}) \right] \\ 0 &= \frac{\sqrt{2.29} \cosh(\sqrt{2.29E})}{2\sqrt{E_{\max}}} - 1.036 \sinh(\sqrt{2.29E}) \\ 0 &= 1 - 1.369\sqrt{E_{\max}} \tanh(\sqrt{2.29E_{\max}}) \\ 1 &= 1.369\sqrt{E_{\max}} \tanh(\sqrt{2.29E_{\max}}) \end{aligned}$$

$$\boxed{E_{\max} = 0.724 \text{ MeV}}$$

The average energy can be found by finding the expected value of the function on the domain $[0, \infty)$.

$$\begin{aligned} E_{\text{ave}} &= \int_0^{\infty} E \chi(E) dE \\ &= \int_0^{\infty} E \left(0.453e^{-1.036E} \sinh(\sqrt{2.29E}) \right) dE \\ &= 0.453 \int_0^{\infty} E e^{-1.036E} \sinh(\sqrt{2.29E}) dE \end{aligned}$$

$$\boxed{E_{\text{ave}} = 1.98 \text{ MeV}}$$

Problem 2

A general reaction rate for process x as a function of energy can be defined as

$$R_x(E) = \Sigma_x(E)\phi(E)$$

where $\Sigma_x(E)$ is the macroscopic cross section for reaction x and ϕ is the neutron flux, both at energy E . The macroscopic cross section can be further decomposed, so that

$$R_x(E) = n\sigma_x(E)\phi(E).$$

Comparing neutron-neutron reactions with all neutron-nuclei reactions

$$\frac{R_{nn}(E)}{R_{tot}(E)} = \frac{n\sigma_{nn}(E)\phi(E)}{n\sigma_{tot}(E)\phi(E)}.$$