NE 250 Fall 2017
Homework #3 Due October 20th, 2017

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1. (10 points) In the following equation, assume that scattering is isotropic and that the cross sections are space independent.

$$\big[\hat{\Omega}\cdot\nabla+\Sigma_t(E)\big]\psi(\vec{r},E,\hat{\Omega}) = \int_{4\pi} d\hat{\Omega}' \int_0^\infty dE'\, \Sigma_s(E',\hat{\Omega}'\to E,\hat{\Omega})\psi(\vec{r},E',\hat{\Omega}')$$
$$+\frac{1}{k}\frac{\chi(E)}{4\pi}\int_0^\infty dE'\,\nu(E')\Sigma_f(E')\int_{4\pi} d\hat{\Omega}'\,\psi(\vec{r},E',\hat{\Omega}')$$

Show that if $\psi(\vec{r},\hat{\Omega},E)=\psi_0(\hat{\Omega},E)\exp(\imath\,\hat{\Omega}\cdot\vec{B})$, the multiplication factor can be written as

$$k=\frac{\int dE\,\nu(E)\Sigma_f(E)\phi_0(E)}{\int dE\,\big[\Sigma_t(E)\gamma(E)^{-1}-\Sigma_s(E)\big]\phi_0(E)}\,,$$

where

$$\gamma(E)=\frac{\Sigma_t(E)}{|\vec{B}|}\tan^{-1}\left(\frac{|\vec{B}|}{\Sigma_t(E)}\right).$$

Note that $\vec{B}$ is a generic spatial direction and $\hat{\Omega}\cdot\vec{B}=\mu|\vec{B}|$.

Then, show that for small $|\vec{B}|$, the foregoing expression reduces to the diffusion form:

$$k\approx\frac{k_\infty}{1+L^2|\vec{B}|^2}\,.$$

2. (10 points) Using the one-group diffusion approximation, the effective multiplication factor is typically calculated as follows:

$$k_{eff}=\frac{\int_V \nu\Sigma_f\phi(\vec{r})\mathrm{d}V}{\int_V\left[\Sigma_a\phi(\vec{r})-D\nabla^2\phi(\vec{r})\right]\mathrm{d}V}$$

but (n,xn) reactions do not appear explicitly. Suggest how you would modify this expression to account for neutron gain/loss from such reactions.

3. (10 points) In a fusion-fission hybrid neutrons generated by D-T fusions drive a subcritical fission blanket. The fission blanket is often referred as an energy multiplier and is characterized by the "energy gain" that is the ratio of total power generated to fusion power only. What is the relation between the energy gain and the fission blanket effective multiplication factor? Assumptions: (1) energy contributions, positive or negative, from non-fission reactions are negligible; (2) before entering the fission blanket, neutrons are multiplied by means of (n,2n) reactions on beryllium with 80% efficiency; (3) no fusion-source neutron is lost.

4. (10 points) Linear Algebra refresher:

   a. Use the cofactor method (http://www.mathsisfun.com/algebra/matrix-inverse-minors-cofactors-adjugate.html) to determine the inverse of this matrix:

$$\begin{pmatrix} 1 & 1 & -1 & 3 \\ 1 & 2 & -4 & -2 \\ 2 & 1 & 1 & 5 \\ -1 & 0 & -2 & -4 \end{pmatrix}$$

   b. Determine the eigenvalues and associated eigenvectors of the following matrix:

$$\begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix}$$

5. (10 points) The transpose of the matrix of cofactors is also known as the **adjoint** matrix (a.k.a. adjudicate). Using this information, come up with a 3×3 matrix that is self-adjoint ($A = A^{\dagger}$) and demonstrate that this is so.

6. (10 points) Consider the space of real functions, $f(x)$, on the interval $[-a, a]$ that

   - have a derivative and
   - are symmetric about $x = 0$.

   Show that the one-dimensional Laplacian operator, $L \equiv \frac{d^2}{dx^2}$, with Dirichlet boundary conditions, $f(\pm a) = 0$, is self-adjoint.

   [Hint: use integration by parts twice and carefully select the adjoint BCs].

7. (10 points) In reactor analysis it is frequently of interest to determine the neutron flux in a so-called unit fuel cell of the reactor, that is, a fuel element surrounded by a moderator. For reasons that will be explained later on, one assumes that the fission neutrons that slow down to thermal energies appear as a source uniformly distributed over the moderator, but not directly in the fuel. Furthermore, it is assumed that there is no net transfer of neutrons from cell to cell, that is, the neutron current vanishes on the boundary of the cell, although the neutron flux will not vanish there.

   Consider a one-dimensional slab model of a fuel cell in which the center region consists of the fuel (thickness a), and the outer regions consist of a moderating material (thickness b) in which neutrons slow down to yield an effective uniformly distributed source of thermal neutrons S_0 neutrons/(cm^3 s). Determine the neutron flux in this cell. In particular, compute the so-called self-shielding factor f_s defined as the ratio between the average flux in the fuel to the average flux in the cell.

8. (10 points) By representing a plane source as a superposition of isotropic point sources, construct the plane source kernel

$$G_{pl}(x, x') = \frac{L}{2D} \exp\left(-\frac{|x - x'|}{L}\right)$$

by using the point source kernel

$$G_{pt}(\vec{r}, \vec{r'}) = \frac{\exp\left(-\frac{|\vec{r}-\vec{r'}|}{L}\right)}{4\pi D|\vec{r} - \vec{r'}|} \ .$$

9. (10 points) Consider a slab of nonmultiplying material containing a uniformly distributed neutron source. Determine the neutron flux in the slab

   a. by directly solving the diffusion equation and

   b. using eigenfunction expansions.

   Demonstrate that these solutions are indeed equivalent.

10. (20 points) Determine the geometric bucking B_g^2 and critical flux profile in the following bare reactor geometries:

    a sphere,

    b infinite cyllinder, and

    c parallepiped. [Hint: look at table 5-1 in Duderstadt to confirm your solutions.]

11. (10 points) What is the Kord Smith Challenge? What are the parameters and details of the original challenge? What happened in 2010?

    Given this context, what do you think about the future of transport and Monte Carlo methods for reactor design and analysis?