NE250 Homework 1 **Mitch Negus**
9/22/2017

Problem 1

The number of molecules, N, found in a sample of a compound with mass M is

$$N(\cdot) = \frac{M(\cdot)N_A}{m(\cdot)}$$

where m is the molar mass of the compound, and N_A is Avogadro's number ($N_A = 6.022 \times 10^{23}$ molecules per mole). Where each actinide isotope is found only once in a molecule of its respective oxide, N also gives the number of atoms of an isotope in the sample.

To find N, we begin by finding the masses of the compounds in the fuel. We can decompose the total mass of the fuel, M_f, into its mixed oxide components:

$$M_f = M(\mathrm{UO_2}) + M(\mathrm{PuO_2}).$$

Given a weight percent for plutonium, w_P, we note that $w_\mathrm{U} = 1 - w_\mathrm{P}$ and so calculate the total masses of both the $\mathrm{UO_2}$ and the $\mathrm{PuO_2}$ to be

$$M(\mathrm{UO_2}) = (1 - w_\mathrm{P})M_f$$
$$M(\mathrm{PuO_2}) = w_\mathrm{P} M_f.$$

We are told that the uranium is all $^{238}\mathrm{U}$, so

$$M(^{238}\mathrm{UO_2}) = M(\mathrm{UO_2}) = (1 - w_\mathrm{P})M_f.$$

¡¡¡¡¡¡¡ HEAD The weight percents of the plutonium isotope oxides are also given (as a fraction of total plutonium oxide). Using $^{239}\mathrm{PuO_2}$ as an example

$$M(^{239}\mathrm{PuO_2}) = w_\mathrm{P9} M(\mathrm{PuO_2}) = w_\mathrm{P9} w_\mathrm{P} M_f.$$

Next, we must determine the molar masses of the various oxides. For uranium,

$$m(^{238}\mathrm{UO_2}) = m(^{238}\mathrm{U}) + 2m(\mathrm{O}),$$

and similarly for plutonium.

When we combine the total and molar masses to determine the total number of atoms for each isotope, we find:
6.69×10^{20} atoms of $^{238}\mathrm{U}$
4.65×10^{20} atoms of $^{239}\mathrm{Pu}$
1.45×10^{20} atoms of $^{240}\mathrm{Pu}$
3.84×10^{19} atoms of $^{241}\mathrm{Pu}$
1.78×10^{19} atoms of $^{242}\mathrm{Pu}$

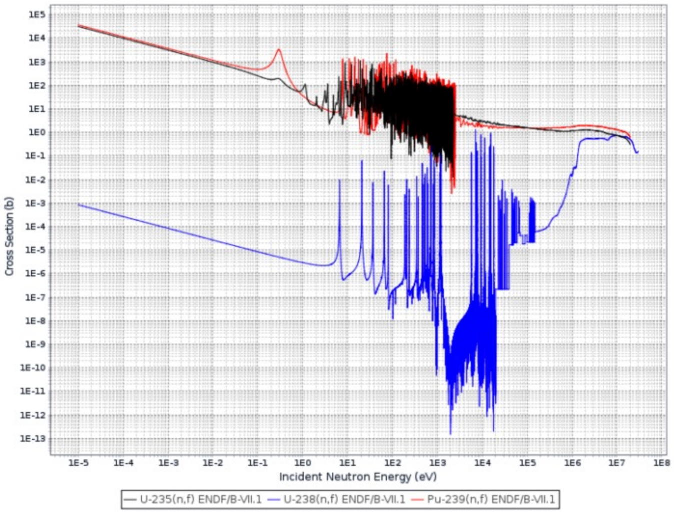
(for full calculation, see Jupyter notebook, attached)

Problem 2

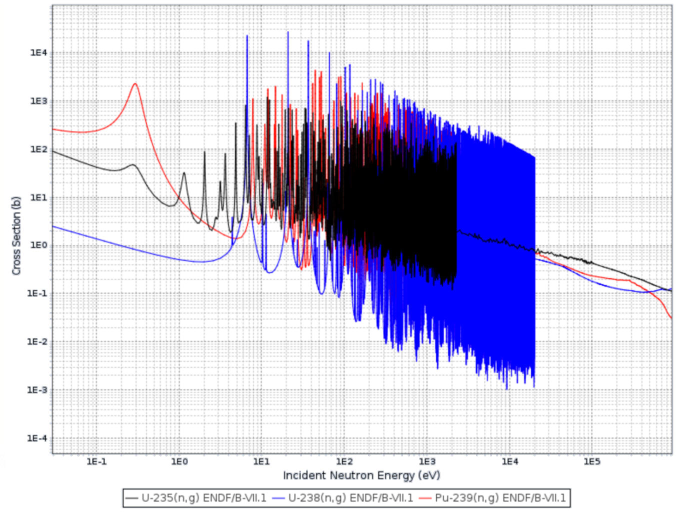
Problem 3

Cross sections plotted using ENDF/B-VII.1 from KAERI.

Fission Cross Sections



Capture Cross Sections



Capture-to-fission ratios at:

0.0253 eV

^{238}U: $\frac{3\text{ b}}{0.00003\text{ b}} = 100000$

^{239}Pu: $\frac{300\text{ b}}{800\text{ b}} = 0.38$

^{235}U: $\frac{100\text{ b}}{700\text{ b}} = 0.14$

0.73 MeV

^{238}U: $\frac{0.13\text{ b}}{0.004\text{ b}} = 32.5$

^{235}U: $\frac{0.13\text{ b}}{1\text{ b}} = 0.13$

^{239}Pu: $\frac{0.07\text{ b}}{2\text{ b}} = 0.04$

Problem 9

a.) If rapidly compressed to half volume, the reactor's density will dramatically double. Cross sections vary directly with density, and so the reactor's cross sections will increase. Greater cross sections mean more chances for fission to be induced, and the multiplication factor will increase. A critical reactor would then become supercritical.

b.) If squashed, into an ellipsoidal shape, the reactor's surface area would increase, with no change in volume. Greater surface area means that more neutrons would leak from the reactor, and the critical reactor would become subcritical (the streaming term in the TE will grow, k must decrease).

c.) Wrapping a sheet of cadmium around the outside of the reactor will cause neutrons to be reflected back into the reactor volume (assuming it replaces void). This will increase the neutrons able to cause fission, and the reactor will become supercritcal (the streaming term in the TE will diminish, k must increase).

d.) Again, neutrons will be reflected back into the reactor. A greater neutron density increases the neutron flux and rate of fission. The multiplication factor will increase and the reactor will become supercritical (the streaming term in the TE will diminish, k must increase).

e.) Adding an external neutron source to the reactor will also trigger an increase in the multiplication factor. There will be more neutrons in the system to cause fission events.

f.) Placing an identical reactor nearby the original could be considered as if adding another source. As in that case, the multiplication factor will increase.

g.) Over time, the fissionable nuclei in the reactor will be consumed in the fission reaction. With less fissionable nuclei, the fission cross sections of the reactor fuel material will decrease. The reactor will become subcritical and the multiplication factor will decrease.

Problem 10

Assumptions of the neutron transport equation:

1. *Neutrons are pointlike.* A reasonable assumption because the de Broglie wavelength of a neutron is significantlly less than the diameter of an atom. This assumption allows us to neglect rotation and quantum effects and write the transport equation as a function of energy. We need only being concerned with the particle's translational kinetic energy.

2. *Neutral particles travel in straight lines.* Neutron trajectories will not bend between collisions, and we can make the assumption that $\frac{\partial\theta}{\partial t} = 0$ and $\frac{\partial\varphi}{\partial t} = 0$.

3. *Particle-particle collisions are negligible.* Neutrons are generally very unlikely to collide/otherwise interact with other neutrons, allowing us to express the transport equation as a linear differential equation.

4. *Material properties are isotropic.* Valid when neutrons are moving with an appreciable velocity, this condition allows us to establish cross sections simply as functions of $\vec{r}$ and E (not $\hat{\Omega}$.

5. *Material composition is independent of time.* Similar to assumption 5, our cross sections become dependent only on $\vec{r}$ and E. Though material does change over long time scales (*i.e.* burnup) this assumption is valid for short term neutronics calculations.

6. *Quantities are expected values.* We may be unable to properly predict fluctuations in our results for cases where we are dealing with low density media.

NE250_HW01_mnegus-notebook

September 16, 2017

1 NE 250 – Homework 1

9/22/2017

1.1 Problem 1

The number of molecules, N, found in a sample of a compound with mass M is

$$N(\cdot) = \frac{M(\cdot)N_A}{m(\cdot)}$$

where m is the molar mass of the compound, and N_A is Avogadro's number ($N_A = 6.022 \times 10^{23}$ molecules per mole). Since each actinide isotope is found only once in a molecule of its respective oxide, N also gives the number of atoms of an isotope in the sample.

```
In [1]: def molecule_count(M,m):
            N_A = 6.022e23
            N = M*N_A/m
            return N
```

To find N, we begin by finding the masses of the compounds in the fuel. We can decompose the total mass of the fuel, M_f, into its mixed oxide components:

$$M_f = M(\text{UO}_2) + M(\text{PuO}_2).$$

Given a weight percent for plutonium, w_P, we note that $w_\text{U} = 1 - w_\text{P}$ and so calculate the total masses of both the UO_2 and the PuO_2 to be

$$M(\text{UO}_2) = (1 - w_\text{P})M_f$$

$$M(\text{PuO}_2) = w_\text{P} M_f.$$

```
In [2]: w_p = 0.3
```

```
In [3]: def mass_oxide(w,M_f):
            M_oxide = w*M_f
            return M_oxide
```

The mass of the oxide component for a given isotope is given by

$$M({}^{i}\mathrm{XO}_2) = w_i M(\mathrm{XO}_2)$$

where w_i is the weight percent of the oxide of isotope ${}^{i}X$ out of element X.

```
In [4]: def mass_isotope_oxide(w_i,M_oxide):
            M_i_oxide = w_i*M_oxide
            return M_i_oxide
```

```
In [5]: # Provided weight percents
        w = {'U238': 1, # all U is U238
             'Pu239': 0.697,
             'Pu240': 0.218,
             'Pu241': 0.058,
             'Pu242': 0.027
            }
```

Next, we must determine the molar masses of the various oxides. In general,

$$m({}^{i}\mathrm{XO}_2) = m({}^{i}\mathrm{X}) + 2m(\mathrm{O}),$$

```
In [6]: def molar_mass_isotope_oxide(m_i,m_o):
            m_i_oxide = m_i + 2*m_o
            return m_i_oxide
```

Finally, we use both the total mass of a compound with its molar mass in the original formula, using provided or tabulated values:

```
In [9]: # Tabulated molar masses [g/mol]
        m = {'U238':  238.051,
             'Pu239': 239.052,
             'Pu240': 240.054,
             'Pu241': 241.057,
             'Pu242': 242.059,
             'O16': 15.995
            }
```

```
In [10]: # Assume 1 gram of total fuel
         M_f = 1

         isotopes = ['U238','Pu239','Pu240','Pu241','Pu242']
         for i in isotopes:
             M_i_ox = mass_isotope_oxide(w[i],mass_oxide(w_p,M_f))
             m_i_ox = molar_mass_isotope_oxide(m[i],m['O16'])
             N = molecule_count(M_i_ox,m_i_ox)
             print(i,': ',N)
```

```
U238 :  6.690095207764747e+20
Pu239 :  4.645775193512445e+20
Pu240 :  1.4477025775242242e+20
Pu241 :  3.837537127307754e+19
Pu242 :  1.7799079726618233e+19
```