NUCENG 255 Homework 3 **Mitchell Negus**
10/20/2016

Problem 1

The slab geometry P_N equations give

$$\left(\frac{l'+1}{2l'+1}\right)\frac{d}{dx}\phi_{l'+1}(x) + \left(\frac{l'}{2l'+1}\right)\frac{d}{dx}\phi_{l'-1}(x) + \Sigma_t(x)\phi_{l'} = \Sigma_{sl'}(x)\phi_{l'}(x) + s_{l'}(x).$$

We also include the conditions that $\phi_{-1} = 0$, and either $\phi_{N+1} = 0$ or $\frac{d}{dx}\phi_{N+1} = 0$.

Following the example for the formal 3D conversion of the second-order form of the planar geometry P_1 equations, we convert the P_N equation to the SP_N equations. $\phi_{l'}$ becomes $\vec{phi}_{l'}$ and we separate even l' and odd l' equations such that the first order SP_N equations are given by

$$\nabla \cdot \vec{\phi}_1 + \Sigma_a \phi_0 = s_0$$

$$\left(\frac{l'+1}{2l'+1}\right)\nabla\phi_{l'+1} + \left(\frac{l'}{2l'+1}\right)\nabla\phi_{l'-1} + \Sigma_t\vec{\phi}_{l'} = \Sigma_{sl'}\vec{\phi}_{l'} + s_{l'}, \;\; l' = \text{odd, and}$$

$$\left(\frac{l'+1}{2l'+1}\right)\nabla\cdot\vec{\phi}_{l'+1} + \left(\frac{l'}{2l'+1}\right)\nabla\cdot\vec{\phi}_{l'-1} + \Sigma_t\phi_{l'} = \Sigma_{sl'}\phi_{l'} + s_{l'}, \;\; l' = \text{even and } l' > 0.$$

We can use these general equations to find SP_5.

$$\nabla \cdot \vec{\phi}_1 + \Sigma_a \phi_0 = s_0$$

$$\frac{2}{3}\nabla\phi_2 + \frac{1}{3}\nabla\phi_0 + [\Sigma_t - \Sigma_{s1}]\vec{\phi}_1 = 0$$

$$\frac{3}{5}\nabla\cdot\vec{\phi}_3 + \frac{2}{5}\nabla\cdot\vec{\phi}_1 + [\Sigma_t - \Sigma_{s2}]\phi_2 = 0$$

$$\frac{4}{7}\nabla\phi_4 + \frac{3}{7}\nabla\phi_2 + [\Sigma_t - \Sigma_{s3}]\vec{\phi 3} = 0$$

$$\frac{5}{9}\nabla\cdot\vec{\phi}_5 + \frac{4}{9}\nabla\cdot\vec{\phi}_3 + [\Sigma_t - \Sigma_{s4}]\phi_4 = 0$$

$$\frac{5}{11}\nabla\phi_4 + [\Sigma_t - \Sigma_{s5}]\vec{\phi}_5 = 0$$

Problem 2

a.)

In general, the area of the surface of a unit sphere is

$$\int_{4\pi} d\hat{\Omega} = \int_0^{2\pi}\int_0^{\pi} \sin\theta \; d\theta \; d\phi = 4\pi. \tag{1}$$

Using Level-Symmetric quadrature

$$\int_{4\pi} d\hat{\Omega} \cdot |\hat{\Omega}| = \sum_{a=1}^{N} w_a |\hat{\Omega}_a|. \tag{2}$$

Since we have designated that $\sum_{a=1}^{n} w_a = 1$ for n quadrature points in an octant. We must introduce a normalization factor to satisfy equation (1), A, so that $\sum_{a=1}^{N} Aw_a = 4\pi$, where N is the total number of quadrature points, $n = N/8$. We find

$$\sum_{a=1}^{N} Aw_a = 8\sum_{a=1}^{n} Aw_a = 8A\sum_{a=1}^{n} w_a = 4\pi$$

From this we find $8A = 4\pi$, and $A = \frac{\pi}{2}$.

Executing the integral from equation (2) using S_4 LQ_N quadrature gives

$$\int_{4\pi} d\hat{\Omega} \cdot |\hat{\Omega}| = \sum_{a=1}^{4(4+2)} Aw_a|\hat{\Omega}_a|.$$

This can be simplified and solved (using L&M figure 4-3, table 4-1, and $|\hat{\Omega}| \equiv 1$) as follows:

$$\int_{4\pi} d\hat{\Omega} \cdot |\hat{\Omega}| = 8A\sum_{a=1}^{3} w_a = 8A(w_{(\mu 1,\eta 1,\xi 2)} + w_{(\mu 1,\eta 2,\xi 1)} + w_{(\mu 2,\eta 1,\xi 1)})$$

$$\int_{4\pi} d\hat{\Omega} \cdot |\hat{\Omega}| = 8(\frac{\pi}{2})(0.3333333 + 0.3333333 + 0.3333333)$$

$$\int_{4\pi} d\hat{\Omega} \cdot |\hat{\Omega}| = 8(\frac{\pi}{2})(0.9999999)$$

$$\boxed{\int_{4\pi} d\hat{\Omega} \cdot |\hat{\Omega}| = 3.9999996\pi}$$

b.)

Now, using S_4 LQ_N quadrature , we find

$$\int_{4\pi} d\hat{\Omega} \cdot |\hat{\Omega}| = \sum_{a=1}^{6(6+2)} Aw_a|\hat{\Omega}_a|,$$

which can be similarly simplified to

$$\int_{4\pi} d\hat{\Omega} \cdot |\hat{\Omega}| = 8A\sum_{a=1}^{6} w_a = 8A(w_{(\mu 1,\eta 1,\xi 3)} + w_{(\mu 1,\eta 3,\xi 1)} + w_{(\mu 3,\eta 1,\xi 1)} + w_{(\mu 1,\eta 2,\xi 2)} + w_{(\mu 2,\eta 1,\xi 2)} + w_{(\mu 2,\eta 2,\xi 1)})$$

$$\int_{4\pi} d\hat{\Omega} \cdot |\hat{\Omega}| = 8(\frac{\pi}{2})(0.1761263 + 0.1761263 + 0.1761263 + 0.1572071 + 0.1572071 + 0.1572071)$$

$$\int_{4\pi} d\hat{\Omega} \cdot |\hat{\Omega}| = 8(\frac{\pi}{2})(1.0000002)$$

$$\boxed{\int_{4\pi} d\hat{\Omega} \cdot |\hat{\Omega}| = 4.0000008\pi}$$

c.)

[SEE CODE - LQ_Nquadrature.py on Github. Use with input files L&M_LQ_N_EqualWeights.txt, L&M_LQ_N˙QuadSets.txt]

Problem 3

a.)

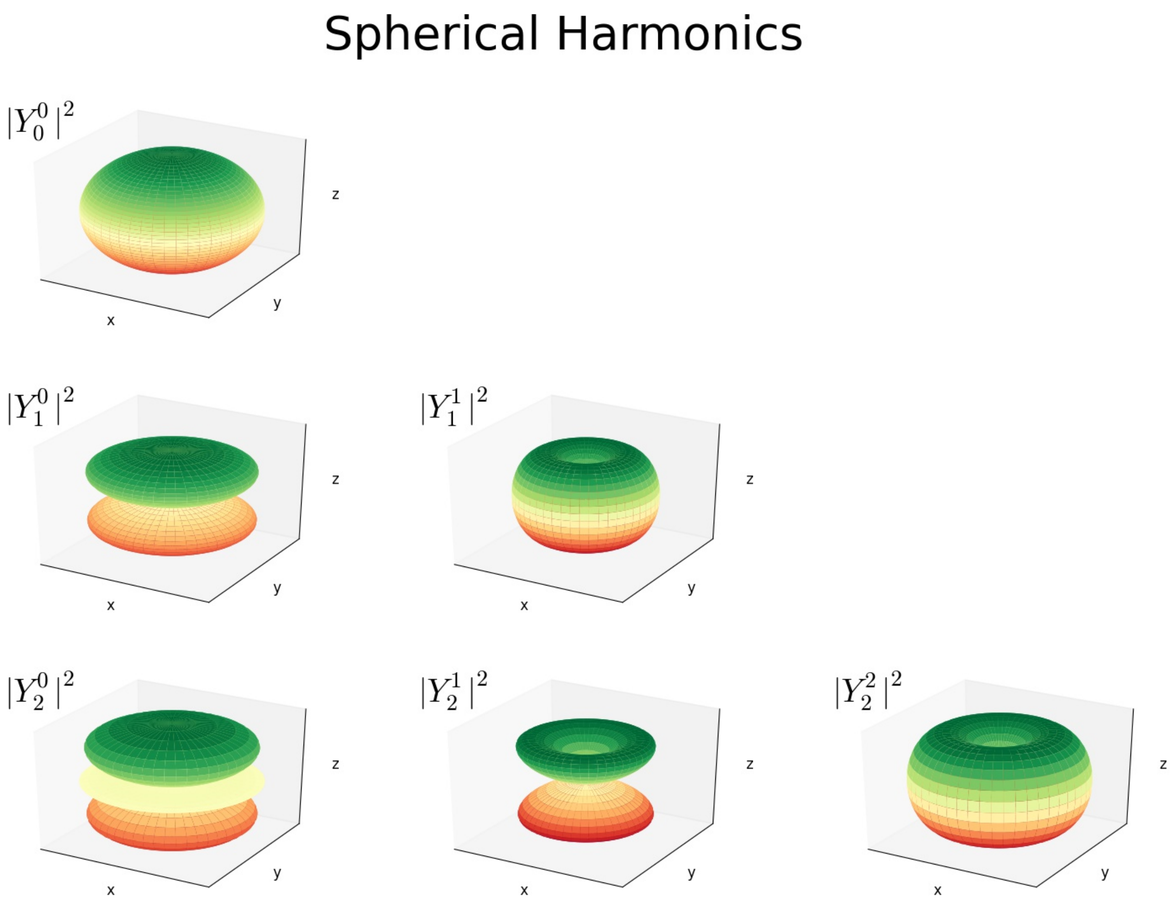
In terms of complexity, the diffusion equation is the least complex of the three methods, with fewer energy groups and a discretized as well as homogenized spacial component. Deterministic codes, on the other hand, often include many energy groups, discretize space, and handle the velocity direction unit vector through either discretization in S_N calculations, or expansions in P_N methods. In general, these deterministic codes are much faster than Monte Carlo codes, which feature continuous spatial resolutions, velocity directions, and energies. These characteristics make Monte Carlo codes much more parallelizable, though also generally slower due to the large sample sizes needed to gather accurate statistics.

b.)

Deterministic codes have the advantage that they are both fast and accurate. Unlike Monte Carlo codes, which may "waste" computation time on calculating null results, deterministic codes fully utilize their computations. Unfortunately, deterministic codes are much more challenging to parallelize as the effects of each subsequent calculation has bearing on many others. (This likely explains why MC methods have gained in popularity with increasing parallelization in supercomputing). Deterministic codes also suffer from ray effects with poor discretization in angle, and in general, finer discretization improves the quality of deterministic results.

Problem 4

a.)



[SEE CODE - SphericalHarmonics.py on Github]

b.)

The external source is given by

$$q_e^g(\vec{r},\hat{\Omega}) = \sum_{l=0}^{N}\left[Y_{l0}^e(\hat{\Omega})q_{l0}^g(\vec{r}) + \sum_{m=1}^{l}\left(Y_{lm}^e(\hat{\Omega})q_{lm}^g(\vec{r}) + Y_{lm}^o(\hat{\Omega})s_{lm}^g(\vec{r})\right)\right],$$

where the even and odd moments are given (respectively) by

$$q_{lm}^g = \int_{4\pi} Y_{lm}^e(\hat{\Omega})\; q_e^g(\hat{\Omega})\; d\hat{\Omega}, \quad m \geq 0$$

$$s_{lm}^g = \int_{4\pi} Y_{lm}^o(\hat{\Omega})\; q_e^g(\hat{\Omega})\; d\hat{\Omega}, \quad m > 0.$$

If we construct an additional code to solve for the external source, we can perform the S_4 quadrature integration over these moments by including the code developed in 2(c), noting that $q_e = 1$, and using

$$Y^e_{lm} = (-1)^m \sqrt{(2-\delta_{m0})\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}} P_{lm}\cos(m\varphi), \text{ and}$$

$$Y^o_{lm} = (-1)^m \sqrt{(2-\delta_{m0})\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}} P_{lm}\sin(m\varphi).$$

[IN PROGRESS]

Problem 5

NUCLEAR DATA LIBRARIES:
Joint Evaluated Fission and Fusion file (JEFF) - managed by NEA Data Bank member countries (mostly European countries)
Evaluated Nuclear Data File (ENDF/B) - managed by the Cross Section Evaluation Working Group in the United States & Canada
Japanese Evaluated Nuclear Data Library (JENDL) - managed by the Japanese Nuclear Data Committee (Japan)