NUCEN3 255 Homework 5 **Mitchell Negus**
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Problem 1

a.)

3iven the discretizations in the problem, we know that:

$3 = 3$ (energy groups)
$N = (2+1)^2 = 9$ (moments)
$n = 2(2+2) = 8$ (angular unknowns)
$c = 4 \times 4 \times 4 = 64$ (cells)
$u = 1$ (unknowns per cell; diamond difference)

If we define

$$\alpha = 3 \times n \times c \times u$$
$$\alpha = 3 \times 8 \times 64 \times 1$$
$$\alpha = 1,536$$

and

$$\beta = 3 \times N \times c \times u$$
$$\beta = 3 \times 9 \times 64 \times 1$$
$$\beta = 1,728$$

then we can use equation (4) in notes 05-01 and properties of matrix multiplication to say that operator sizes are:

$$\mathbf{L} = (\alpha \times \alpha) = 1,536 \times 1,536$$
$$\mathbf{M} = (\alpha \times \beta) = 1,536 \times 1,728$$
$$\mathbf{S} = (\beta \times \beta) = 1,728 \times 1,728$$

b.)

$$[\mathbf{M}]_{gg} = \begin{pmatrix} Y^e_{00}(\hat{\Omega}_1) & Y^e_{10}(\hat{\Omega}_1) & Y^o_{11}(\hat{\Omega}_1) & Y^e_{11}(\hat{\Omega}_1) & Y^e_{20}(\hat{\Omega}_1) & \cdots & Y^o_{99}(\hat{\Omega}_1) & Y^e_{99}(\hat{\Omega}_1) \\ Y^e_{00}(\hat{\Omega}_2) & Y^e_{10}(\hat{\Omega}_2) & Y^o_{11}(\hat{\Omega}_2) & Y^e_{11}(\hat{\Omega}_2) & Y^e_{20}(\hat{\Omega}_2) & \cdots & Y^o_{99}(\hat{\Omega}_2) & Y^e_{99}(\hat{\Omega}_2) \\ Y^e_{00}(\hat{\Omega}_3) & Y^e_{10}(\hat{\Omega}_3) & Y^o_{11}(\hat{\Omega}_3) & Y^e_{11}(\hat{\Omega}_3) & Y^e_{20}(\hat{\Omega}_3) & \cdots & Y^o_{99}(\hat{\Omega}_3) & Y^e_{99}(\hat{\Omega}_3) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ Y^e_{00}(\hat{\Omega}_8) & Y^e_{10}(\hat{\Omega}_8) & Y^o_{11}(\hat{\Omega}_8) & Y^e_{11}(\hat{\Omega}_8) & Y^e_{20}(\hat{\Omega}_8) & \cdots & Y^o_{99}(\hat{\Omega}_8) & Y^e_{99}(\hat{\Omega}_8) \end{pmatrix}.$$

$$\mathbf{S} = \begin{pmatrix} [\mathbf{S}]_{11} & [\mathbf{S}]_{12} & [\mathbf{S}]_{13} \\ [\mathbf{S}]_{21} & [\mathbf{S}]_{22} & [\mathbf{S}]_{23} \\ [\mathbf{S}]_{31} & [\mathbf{S}]_{32} & [\mathbf{S}]_{33} \end{pmatrix}$$

$$[\mathbf{S}]_{21} = \begin{pmatrix} \Sigma^{21}_{s\,1} & 0 & \cdots & 0 \\ 0 & \Sigma^{21}_{s\,2} & \cdots & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & \Sigma^{21}_{s\,9} \end{pmatrix},$$

$$\psi = \begin{pmatrix} [\psi]_1 & [\psi]_2 & [\psi]_3 \end{pmatrix}^T, \quad \text{and}$$

$$[\psi]_1 = \begin{pmatrix} \psi^1_1 & \psi^1_2 & \psi^1_3 & \cdots \psi^1_8 \end{pmatrix}^T.$$

$$[\phi]_1 = \begin{pmatrix} \phi^1_{00} & \phi^1_{10} & \vartheta^1_{11} & \phi^1_{11} & \phi^1_{20} & \cdots & \vartheta^1_{99} & \phi^1_{99} \end{pmatrix}^T$$

c.)

$\mathbf{D}$ is the discrete-to-moment operator. $\mathbf{D} = \mathbf{M^T W}$ where $\mathbf{W}$ is a matrix of quadrature weights.

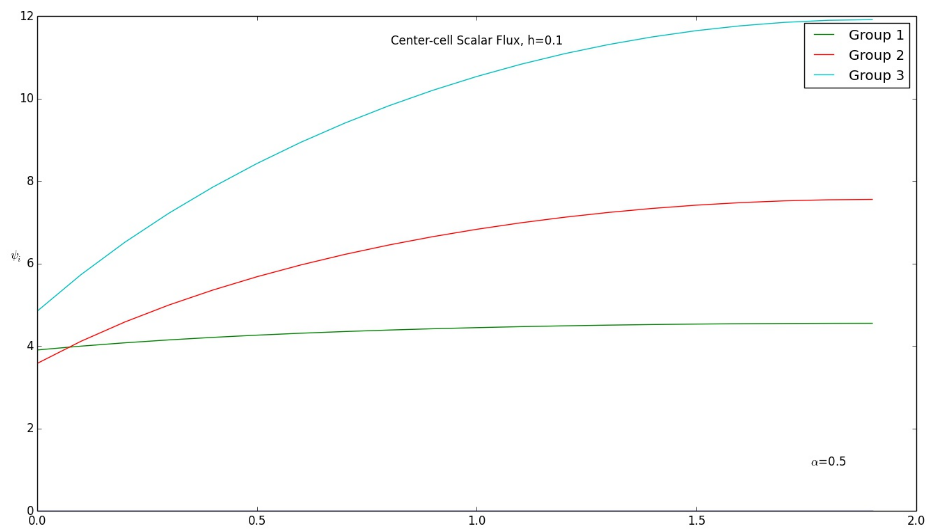
d.)

We do not usually form $\mathbf{L}$ as it is usually an unnecessary step in determining the vector quantities of interest (the fluxes or flux moments). Forming the matrix consumes significant amounts of memory which can impede the efficiency of calculations.

e.)

$$\mathbf{L}\psi = \mathbf{MS}\phi + \mathbf{M}q_e, \text{ and } \phi = \mathbf{D}\psi$$
$$\text{Let } \mathbf{D}^{-1}\phi = \psi,$$
$$\mathbf{L}(\mathbf{D}^{-1}\phi) = \mathbf{MS}\phi + \mathbf{M}q_e$$
$$\text{If } Q = \mathbf{DL}^{-1}\mathbf{M}q_e,$$
$$\mathbf{L}(\mathbf{D}^{-1}\phi) = \mathbf{MS}\phi + \mathbf{LD}^{-1}Q$$
$$\phi = \mathbf{DL}^{-1}\mathbf{MS}\phi + Q$$
$$\phi - \mathbf{DL}^{-1}\mathbf{MS}\phi = Q$$
$$(1 - \mathbf{DL}^{-1}\mathbf{MS})\phi = Q \text{ is similar to } \mathbf{A}x = b$$

Problem 2



(Code on GitHub)