NUCENG 255 Homework 6 **Mitchell Negus**
12/01/2016

Problem 1

a.)

Since the neutron source is isotropic, we can express its probability density function as uniform for both θ and ϕ. Call these unnormalized PDFs $g(\theta)$ and $h(\phi)$.

$g(\theta)\,d\theta = C\,d\theta, \quad 0 < \theta < \pi$
$G(\theta) = \int_{-\infty}^{\theta} g(\theta')d\theta'$
$G(\theta) = C\int_0^{\theta} d\theta' = C[\theta']_0^{\theta}$
$G(\infty) = G(\pi) = C(\pi - 0) = C\pi$

$h(\phi)\,d\phi = D\,d\phi, \quad 0 < \phi < 2\pi$
$H(\phi) = \int_{-\infty}^{\phi} g(\phi')d\phi'$
$H(\phi) = D\int_0^{\phi} d\phi' = D[\phi']_0^{\phi}$
$H(\infty) = H(2\pi) = D(2\pi - 0) = 2D\pi$

The normalized PDFs are then

$p_\theta(\theta) = \frac{g(\theta)}{G(\infty)} = \frac{C}{C\pi} = \frac{1}{\pi}\,, \quad 0 < \theta < \pi$

$p_\phi(\phi) = \frac{h(\phi)}{H(\infty)} = \frac{D}{2D\pi} = \frac{1}{2\pi}\,, \quad 0 < \phi < 2\pi$

and the normalized CDFs are

$P_\theta(\theta) = \int_{-\infty}^{\theta} p_\theta(\theta')d\theta' = \frac{1}{\pi}\int_0^{\theta} d\theta' = \frac{\theta}{\pi}$

$P_\phi(\phi) = \int_{-\infty}^{\phi} p_\phi(\phi')d\phi' = \frac{1}{2\pi}\int_0^{\phi} d\phi' = \frac{\phi}{2\pi}.$

Inverting, we can sample θ and ϕ using random numbers ξ and η respectively.

$\theta = P_\theta^{-1}(\xi) = \pi\xi$

$\phi = P_\phi^{-1}(\eta) = 2\pi\eta$

b.)

We know that the neutron collision probability along a given path of length s is given by $p_c(s)\,ds = \Sigma_t(s)e^{-\Sigma_t(s)s}ds$. If we represent the distance s in units of mean free path n, then we find $n = \Sigma_t(s)\,s$. Taking the derivative $\frac{dn}{ds}$ and knowing that Σ_t is piecewise constant, then $dn = \Sigma_t(s)ds$ and we can reexpress the collision probability at any n as $p_c(n)\,dn = e^n\,dn$. The CDF can be found through integration

$$P_c(n) = \int_0^n e^{-n'}dn' = -e^{-n}|_0^n = 1 - e^{-n}.$$

We can again invert, and sample to randomly determine the number of mean free paths to a collision

$$n_c = P_c^{-1}(\xi) = -\ln(1-\xi) = -\ln(\xi)$$

Next, calculate the distance from the particles initial location to the surface boundary in units of mean free path

$$n_{b1} = R_1\Sigma_{t1}$$

If $n_{c1} < n_{b1}$ then the collision occurs. n_{c1} is the distance to the collision. If $n_{c1} > n_{b1}$ then a boundary is reached and we must adjust the number of mean free paths to the decision accordingly: $n_{c2} = n_{c1} - n_{b1}$.

We repeat this process so that now $n_{b2} = (R_2 - R_1)\Sigma_{t2}$. If $n_{c2} < n_{b2}$ then the collision occurs and $n_{b1} + n_{c2}$ is the distance to the collision. If $n_{c2} > n_{b2}$ then a boundary is reached and adjust again so $n_{c3} = n_{c2} - n_{b2}$.

Again we repeat one more time so $n_{b3} = (R_3 - R2)\Sigma_{t3}$. If $n_{c3} < n_{b3}$ then the collision occurs and $n_{b1} + n_{b2} + n_{c3}$ is the distance to the collision. If $n_{c2} > n_{b2}$ then the particle escapes.

c.)

In the event of a collision, given a set of reactions $\{R = 1, 2, 3, 4\}$ (1 = elastic scattering; 2 = inelastic scattering; 3 = neutron capture in fission; 4 = (n,γ) neutron capture), the PDF is given by $p_R = \frac{\Sigma_R}{\Sigma_t}$. Since the reactions are discrete, the CDF is represented by the summation

$$P_R = \sum_r^R \frac{\Sigma_r}{\Sigma_t} = \sum_r^R p_r.$$

Taking a random sample ξ from the inverted CDF, the collision type R is $\sum_r^{R-1} p_r < \xi \le \sum_r^R p_r$ We find for

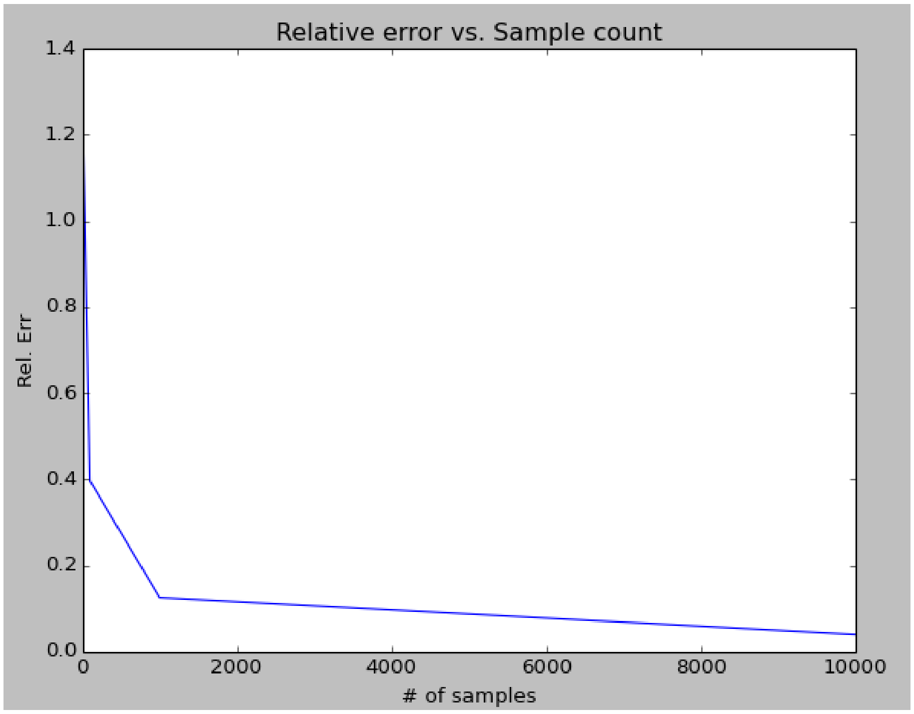
1, elastic scattering: $0 < \xi < p_1$
2, inelastic scattering: $p_1 < \xi < p_1 + p_2$
3, neutron capture in fission: $p_1 + p_2 < \xi < p_1 + p_2 + p_3$
4, (n,γ) neutron capture: $p_1 + p_2 + p_3 < \xi < p_1 + p_2 + p_3 + p_4$

Problem 2

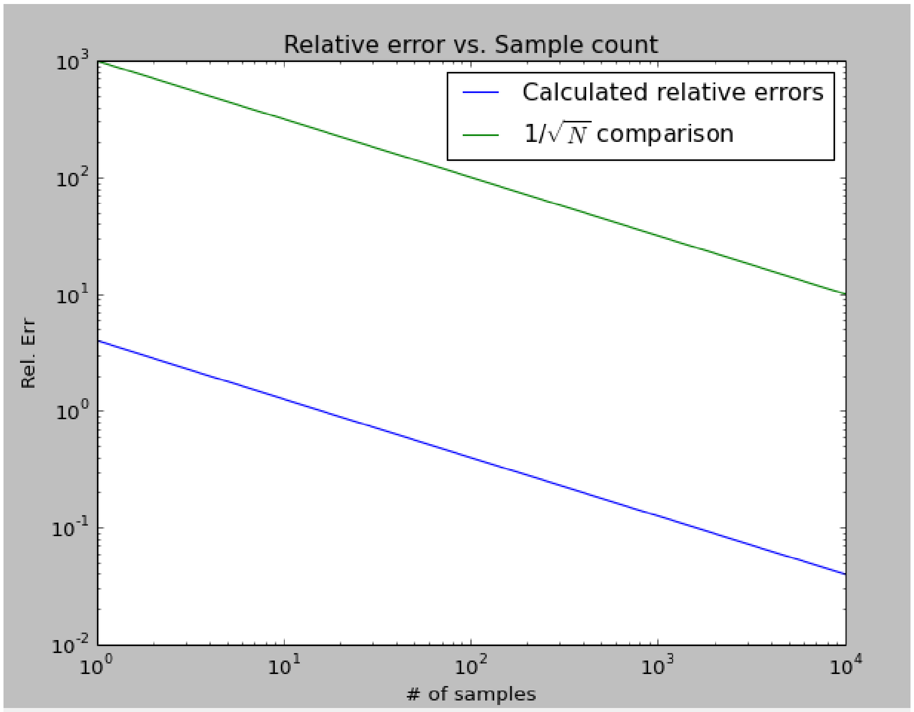
(Code on GitHub)

Samples	$\sqrt{1-x^2}$		$\frac{1}{1-x^2}$	
	Calculated π	Rel. Err	Calculated π	Rel. Err
10	2.8	1.258	3.2	1.256
100	3.24	0.395	3.24	0.397
1000	3.104	0.125	3.212	0.125
10000	3.1352	0.04	3.132	0.04

Relative Errors plotted as a function of number of samples used:



Relative Errors plotted on a log-log plot as a function of number of samples used. Note that the slope matches a plot of $f(x) = \frac{1}{\sqrt{x}}$ and so we can deduce that the relative errors goes like $\frac{1}{\sqrt{N}}$ where N is the number of samples.



(Code on GitHub)