NUCENG 255 Homework 4 **Mitchell Negus**
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Problem 1

a.)

In 1D and for 1 angle, the system of equations derived from the cell balance equation with closure is

$$\psi_i = \frac{s_i + \frac{2}{1+\alpha_i}\frac{|\mu|}{\Delta_i}\bar{\psi}_{i-1/2}}{\Sigma_i + \frac{2}{1+\alpha_i}\frac{|\mu|}{\Delta_i}} \quad \text{and} \quad \psi_{i+1/2} = \frac{2}{1+\alpha_i}\psi_i - \frac{1-\alpha_i}{1+\alpha_i}\bar{\psi}_{i-1/2}$$

for $\mu > 0$ ($\psi_{i-1/2}$ is the incoming flux). Given a flux at 1 boundary, we set this value this the first incoming flux $\bar{\psi} = \bar{\psi}_{i-1/2}$. Using the system of equations above it is possible to determine the cell-center flux ψ_i and the outgoing flux $\psi_{i+1/2}$ (so long as $\mu > 0$).

b.)

The process is nearly identical more $\mu < 0$ except that now $\bar{\psi} = \bar{\psi}_{i+1/2}$ (incoming), $\psi = \psi_{i-1/2}$ (outgoing), and the system is given by

$$\psi_i = \frac{s_i + \frac{2}{1-\alpha_i}\frac{|\mu|}{\Delta_i}\bar{\psi}_{i+1/2}}{\Sigma_i + \frac{2}{1-\alpha_i}\frac{|\mu|}{\Delta_i}} \quad \text{and} \quad \psi_{i-1/2} = \frac{2}{1-\alpha_i}\psi_i - \frac{1+\alpha_i}{1-\alpha_i}\bar{\psi}_{i+1/2}.$$

c.)

At the reflective right boundary we change from $\mu < 0$ to $\mu < 0$. Since when $\mu > 0$ the outgoing wave is $\psi = \psi_{i+1/2}$ and when $\mu < 0$ the incoming wave is $\bar{\psi} = \bar{\psi}_{i+1/2}$, we can simply set these 2 values equal and say $\bar{\psi}^{n+1}_{i+1/2} = \psi^n_{i+1/2}$ where n represents the n^{th} (final) step in the iteration across 1 dimension.

d.)

If we are performing the sweeping process to calculate the flux moments, we need to calculate the angular flux and then use it to calculate the flux moment in that cell. Then we must store the flux moment for that cell.

Problem 2

a.)

We can solve the given equation exactly to find the angular flux as a function of x.

$$\mu_a \frac{d\psi_a}{dx} + \Sigma_t \psi_a = 0$$

$$\frac{d\psi_a}{dx} = -\frac{\Sigma_t}{\mu_a}\psi_a$$

$$\int_x^{x'} \frac{1}{\psi_a} d\psi_a = -\int_x^{x'} \frac{\Sigma_t}{\mu_a} dx$$

$$\ln \psi_a(x') - \ln \psi_a(x) = -\frac{\Sigma_t(x'-x)}{\mu_a}$$

$$\ln \frac{\psi_a(x')}{\psi_a(x)} = -\frac{\Sigma_t(x'-x)}{\mu_a}$$

$$\frac{\psi_a(x')}{\psi_a(x)} = e^{-\frac{\Sigma_t(x'-x)}{\mu_a}}$$

$$\boxed{\psi_a(x') = \psi_a(x) e^{-\frac{\Sigma_t(x'-x)}{\mu_a}}}$$

b.)

Now if we let $x' = x_{i+1/2}$ and $x = x_{i-1/2}$ then we can rewrite our expression in (a) as

$$\psi_{a,i+1/2} = \psi_{a,i-1/2} e^{-\frac{\Sigma_t \Delta_i}{\mu_a}}$$

where $\Delta_i = x' - x = x_{i+1/2} - x_{i-1/2}$. If we define $h \equiv \frac{\Sigma_t \Delta_i}{2|\mu_a|}$, then

$$\boxed{\psi_{a,i+1/2} = \psi_{a,i-1/2} e^{-2h}}$$

c.)

Manipulating the given equation

$$\mu_a \frac{d\psi_a}{dx} + \Sigma_t \psi_a = 0$$

$$\mu_a \frac{(\psi_{a,i+1/2} - \psi_{a,i-1/2})}{\Delta_i} + \Sigma_t \psi_{a,i} = 0$$

$$\psi_{a,i} = -\mu_a \frac{(\psi_{a,i+1/2} - \psi_{a,i-1/2})}{\Sigma_t \Delta_i} = -\frac{1}{2h}(\psi_{a,i+1/2} - \psi_{a,i-1/2})$$

and setting it equal to the diamond difference equation

$$\psi_{a,i} = \frac{1}{2}(\psi_{a,i+1/2} + \psi_{a,i-1/2})$$

we find

$$\frac{1}{2}(\psi_{a,i+1/2} + \psi_{a,i-1/2}) = -\frac{1}{2h}(\psi_{a,i+1/2} - \psi_{a,i-1/2})$$

$$(1+h)\psi_{a,i+1/2} = (1-h)\psi_{a,i-1/2}$$

$$\boxed{\psi_{a,i+1/2} = \frac{(1-h)}{(1+h)}\psi_{a,i-1/2}}$$

d.)

The expansion of our answer from (b) through h^2:

$$\psi_{a,i+1/2} = \psi_{a,i-1/2}e^{-2h}$$

$$\psi_{a,i+1/2} = \psi_{a,i-1/2}\left[1 - 2h + \frac{(-2h)^2}{2}\right]$$

$$\psi_{a,i+1/2} = \psi_{a,i-1/2}\left[1 - 2h + 2h^2\right]$$

Similarly, the expansion of our answer from (b) through h^2:

$$\psi_{a,i+1/2} = \frac{(1-h)}{(1+h)}\psi_{a,i-1/2}$$

$$\psi_{a,i+1/2} = \psi_{a,i-1/2}\left[\frac{1-0}{1+0}h^0 + \frac{(1+0)(-1)-(1-0)}{(1+0)^2}h^1 + \frac{-(-2)(2)(1+0)}{(1+0)^4}\frac{h^2}{2}\right]$$

$$\psi_{a,i+1/2} = \psi_{a,i-1/2}\left[1 - 2h + 2h^2\right]$$

These expansions are both equivalent (through h^2).

e.)

To guarantee a positive flux, we can draw the conclusion that $h < 1$. We defined $h \equiv \frac{\Sigma_t \Delta_i}{2|\mu_a|}$, so for a fixed Σ_t, this condition implies $\frac{\Sigma_t \Delta_i}{2|\mu_a|} < 1$ or

$$\boxed{\Delta_i < 2\frac{|\mu|}{\Sigma_t}}$$

.

Problem 3

The form of the transport equation which we are modeling reduces to

$$\hat{\Omega}\cdot\nabla\psi(r,\hat{\Omega}) + \Sigma_t\psi(r,\hat{\Omega}) = q_e + \int_{4\pi} d\hat{\Omega}'\Sigma_s\psi(r,\hat{\Omega}')$$

under the conditions provided. When discretized with WDD, this becomes

$$|\mu|\frac{(\psi_{i+1/2} - \psi_{i-1/2})}{\Delta_i} + \Sigma_t\psi_i = q_e + \frac{1}{A}\sum_{a=1}^{A}\Sigma_s w_a\psi_{a,i}.$$

To find the cell-center flux we can rearrange this (with $h = \Delta_i$) as

$$|\mu|\frac{\left(\frac{2}{1+\alpha}\psi_i - \frac{1-\alpha}{1+\alpha}\psi_{i-1/2} - \psi_{i-1/2}\right)}{\Delta_i} + \Sigma_t\psi_i = q_e + \sum_{a=1}^{A}\Sigma_s w_a\psi_{a,i}$$

$$\frac{|\mu|}{h}\left(\frac{2}{1+\alpha}\psi_i - \frac{2}{1+\alpha}\psi_{i-1/2}\right) + \Sigma_t\psi_i = q_e + \frac{1}{A}\sum_{a=1}^{A}\Sigma_s w_a\psi_{a,i}$$

$$\frac{|\mu|}{h}\left(\frac{2}{1+\alpha}\psi_i\right) + \Sigma_t\psi_i = q_e + \frac{|\mu|}{h}\frac{2}{1+\alpha}\psi_{i-1/2} + \frac{1}{A}\sum_{a=1}^{A}\Sigma_s w_a\psi_{a,i}, \quad w_a = 1$$

$$\psi_i = \frac{q_e + \frac{|\mu|}{h}\frac{2}{1+\alpha}\psi_{i-1/2} + \frac{1}{A}\sum_{a=1}^{A}\Sigma_s\psi_{a,i}}{\frac{|\mu|}{h}\frac{2}{1+\alpha} + \Sigma_t}$$

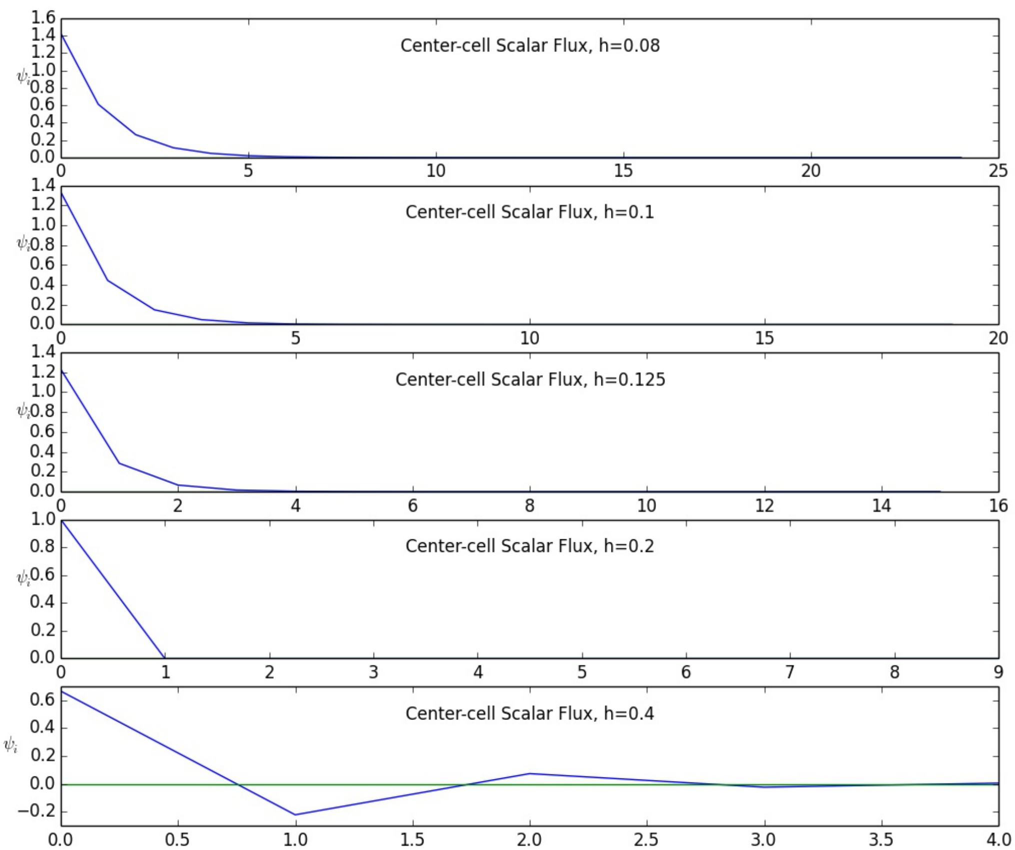
We can then use this equation, along with the coupled equation

$$\psi_{i+1/2} = \frac{2}{1+\alpha_i}\psi_i - \frac{1-\alpha_i}{1+\alpha_i}\bar{\psi}_{i-1/2}$$

in the code.
(Note: on our first pass through we calculate $\psi_{a=1,i}$ however we do not have $\psi_{a>1,i}$ so our scattering term will be incorrect. To remedy this, we iterate several times. We are looking for our answer (the scalar flux, ϕ) to converge, *i.e.* $||\phi^{(m)} - \phi^{(m+1)}||_2 < \varepsilon$)

a.)

(Code on GitHub) The code yields the following plots for the given mesh spacings:



It is obvious that the cell-center scalar flux begins having negative values at $h > 0.2$. This corresponds exactly to our previous problem, where we said that negative flux would be avoided if we ensure that the spacing (previously Δ_i, now h) obeys $h < 2\frac{|\mu|}{\Sigma_t}$. We have $|\mu| = 0.1$ and $\Sigma_t = 1$, so $h = 0.4 \not< 0.2$.

b.)

(Plots on GitHub)
For the various values of α we find the scalar flux has different behavior. It seems quite erratic for negative values of α (-0.9 and -0.5), especially when the step size is large. Furthermore, for positive values of α it reduces the

moderation of the flux. This makes sense when we consider how the weighting changes the dependence of the outgoing flux of any given cell on the cell-center flux or incoming flux.

c.)

(Plots on GitHub)

d.)

(Plots on GitHub)
When $\alpha = 0$ and $\Sigma_s = 0.9$ we see little/no decrease in the flux. This is not too surprising seeing as almost all of the neutrons are being scattered rather than absorbed.

Problem 4

General System of Equations

$$\frac{\mu_a}{h_i}\left(\psi^g_{a,i+1/2} - \psi^g_{a,i-1/2}\right) + \Sigma^g_{t,i}\psi^g_{a,i} = 2\pi\sum_{a=1}^{N} w_a \sum_{g'=1}^{G} \Sigma^{gg'}_{s,i}(a' \to a)\psi^{g'}_{a',i} + \frac{\chi_g}{2}\sum_{g'=1}^{G}\nu_{g'}\Sigma^{g'}_{f,i}\phi^{g'}_i + \frac{1}{2}Q^g_i$$

Coupled Equations for 5 Groups

(I have assumed that inscattering is possible - though it is not explicitly stated in the problem)

$$\frac{\mu_a}{h_i}\left(\psi^{(1)}_{a,i+1/2} - \psi^{(1)}_{a,i-1/2}\right) + \Sigma^{(1)}_{t,i}\psi^{(1)}_{a,i} = 2\pi\sum_{a=1}^{N} w_a \Sigma^{(1)(1)}_{s,i}(a' \to a)\psi^{(1)}_{a',i} + \frac{\chi_{(1)}}{2}\sum_{g'=1}^{5}\nu_{g'}\Sigma^{g'}_{f,i}\phi^{g'}_i + \frac{1}{2}Q^{(1)}_i \tag{1}$$

$$\frac{\mu_a}{h_i}\left(\psi^{(2)}_{a,i+1/2} - \psi^{(2)}_{a,i-1/2}\right) + \Sigma^{(2)}_{t,i}\psi^{(2)}_{a,i} = 2\pi\sum_{a=1}^{N} w_a \Sigma^{(2)(2)}_{s,i}(a' \to a)\psi^{(2)}_{a',i} + \frac{\chi_{(2)}}{2}\sum_{g'=1}^{5}\nu_{g'}\Sigma^{g'}_{f,i}\phi^{g'}_i + \frac{1}{2}Q^{(2)}_i \tag{2}$$

$$\frac{\mu_a}{h_i}\left(\psi^{(3)}_{a,i+1/2} - \psi^{(3)}_{a,i-1/2}\right) + \Sigma^{(3)}_{t,i}\psi^{(3)}_{a,i} = 2\pi\sum_{a=1}^{N} w_a \sum_{g'=1}^{5} \Sigma^{(3)g'}_{s,i}(a' \to a)\psi^{g'}_{a',i} + \frac{\chi_{(3)}}{2}\sum_{g'=1}^{5}\nu_{g'}\Sigma^{g'}_{f,i}\phi^{g'}_i + \frac{1}{2}Q^{(3)}_i \tag{3}$$

$$\frac{\mu_a}{h_i}\left(\psi^{(4)}_{a,i+1/2} - \psi^{(4)}_{a,i-1/2}\right) + \Sigma^{(4)}_{t,i}\psi^{(4)}_{a,i} = 2\pi\sum_{a=1}^{N} w_a \sum_{g'=1}^{5} \Sigma^{(4)g'}_{s,i}(a' \to a)\psi^{g'}_{a',i} + \frac{\chi_{(4)}}{2}\sum_{g'=1}^{5}\nu_{g'}\Sigma^{g'}_{f,i}\phi^{g'}_i + \frac{1}{2}Q^{(4)}_i \tag{4}$$

$$\frac{\mu_a}{h_i}\left(\psi^{(5)}_{a,i+1/2} - \psi^{(5)}_{a,i-1/2}\right) + \Sigma^{(5)}_{t,i}\psi^{(5)}_{a,i} = 2\pi\sum_{a=1}^{N} w_a \sum_{g'=1}^{5} \Sigma^{(5)g'}_{s,i}(a' \to a)\psi^{g'}_{a',i} + \frac{\chi_{(5)}}{2}\sum_{g'=1}^{5}\nu_{g'}\Sigma^{g'}_{f,i}\phi^{g'}_i + \frac{1}{2}Q^{(5)}_i \tag{5}$$