A surface-scope analysis of authoritative readings of modified numerals

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September 5, 2016

Introducing authoritative readings

According to the rules of the game . . .

(1) You're allowed to draw at most 3 cards.

Two kinds of inference:

- Upper bound (UB): You're not allowed to draw 4 or more cards.
- Free choice (FC): You're allowed to draw any number of cards in the range [0, 3].

First puzzle: Why an authoritative reading of *at most*?

On standard assumptions about the meanings of *allow* and *at most 3*, a surface-scope analysis of (1) predicts a weak literal meaning:

'There is a permissible world in which you draw 3 cards or fewer.'

Notation henceforth: $\diamond[\leq 3]$

Neither UB ($\neg\diamond[\geq 4]$) nor FC ($\diamond[=3] \wedge \diamond[=2] \wedge \cdots$) logically follow from this.

First puzzle: Why an authoritative reading of *at most*?

An inverse-scope analysis derives an ignorance reading:

'The maximum n such that you're allowed to draw n (or more) cards is 3 or fewer, and the speaker is ignorant about whether that number is 3 or less than 3.'

(cf. *Ann drew at most 3 cards* $\rightsquigarrow$ 'The maximum n such that Ann drew n cards (or more) is 3 or fewer, and the speaker is ignorant about whether that number is 3 or less than 3.')

Second puzzle: Why no authoritative reading of *at least*?

According to the rules of the game . . .

(2) ??You're allowed to draw at least 3 cards.

Cannot be used to convey that you're allowed to draw 3 or more cards, with FC in the range [3, . . .), and a lower bound (LB) that prohibits drawing 2 cards or fewer.

A previous solution: Penka 2014

Penka (2014) takes *at most* to be the 'oddity' in this puzzle.

She solves it by decomposing *at most* into a negative component, *ANT*, plus *at least*, and giving (1) a split-scope analysis.

A previous solution: Penka 2014

at most 3 $\rightsquigarrow$ *ANT* + *3* + *at least*

ANT 3 [λn [allowed [at least n [λm you draw m cards]]]]

$$\forall n[n > 3 \rightarrow \neg \diamond [\geq n]] \quad \equiv \quad \neg \diamond [\geq 4]$$

- UB is an entailment.
- FC follows from neo-Gricean reasoning, given certain assumptions about the Horn scales responsible for generating alternatives. (Actually, she derives weaker, non-total FC—by design.)

New observations

A variety of expressions, beyond just *at most*, give rise to authoritative readings—including antonym pairs.

From Google/Wikipedia corpus . . . . . .

New observations

(3) The catering premises may open at the earliest at 5:00 AM.

LB and FC inferences:

- They may not open earlier than 5:00 AM.
- They may open at any time later than 5:00 AM.

New observations

(4) Deductions may occur at the latest at the time of submission.

UB and FC inferences:

- They may not occur later than the time of submission.
- They may occur any time earlier than the time of submission.

New observations

(5) The Speaker is allowed to appoint between three and seven senior MPs [...] when he is out of the country [...].

LB, UB, and FC inferences:

- He is not allowed to appoint fewer than three MPs.
- He is not allowed to appoint more than seven MPs.
- He is allowed to appoint any number of MPs in the range [3, 7].

New observations

In addition, *at least* actually can sometimes give rise to a LB authoritative reading, e.g. when conjoined with *at most* under an existential modal. From Google:

(6) a. You're allowed to nominate at least 2 and at most 4 authors this week!

b. Each bidder is allowed to bid for at least 5 lots and at most 15 lots.

c. The guild may have at least 3 and at most 100 members.

(*allowed . . . at least m and at most n* seems to be synonymous with *allowed . . . between m and n*.)

New observations

We also note the following robust contrast:

(7) According to the rules on the syllabus ...

a. ??You're allowed to write at least three pages.

b. You're allowed to (either) give a presentation or (else) write at least three pages.

In particular, *at least* in (7b) has a LB authoritative reading: if you choose to write a paper, then you're not allowed to write fewer than three pages.

Against decomposition

The LB authoritative uses of *at least* that we found are prima facie evidence against decomposition.

Moreover, the availability of authorative readings for antonym pairs like *at the earliest/latest* casts doubt on the explanation of the asymmetry between *at least/most*.

- *at the earliest* $\rightsquigarrow$ 'not earlier than'
- *at the latest* $\rightsquigarrow$ 'not later than'
- *at most* $\rightsquigarrow$ 'not more than'
- *at least* $\not\rightsquigarrow$ 'not fewer than' (why not?)

Finally, extending the decomposition account to non-superlative expressions would require some rather *ad hoc* syntactic assumptions:

- *between m and n* $\rightsquigarrow$ 'not fewer than m and not more than n'

Against inverse scope

An inverse-scope analysis would predict ignorance inferences across the board.

This is because when these expressions take widest scope, ignorance readings obligatorily emerge (Penka 2010).

(8) a. Bill arrived {at the earliest/latest} at 8:00 AM.

b. The Speaker appointed between 3 and 7 MPs to exercise his powers to issue recess writs when he is out of the country.

c. Cindy nominated at least two authors.

Take-home message

It is not *at most* which is the oddity in this puzzle.

Nor is it *at least*, per se.

Rather, it is more specifically *at least* in just some sentences.

We therefore want to try to analyze all the above cases uniformly, without proposing that *at most* or any other expression has any special (non-standard) morphosyntax.

Our starting point: free-choice disjunction

Disjunction in the scope of an existential deontic modal licenses similar FC and 'bound' inferences.

Context: The relevant desserts are cake, gelato, and pie.

(9) You're allowed to have cake or gelato.

Three inferences:

- You're not allowed to have pie.
- You're allowed to have cake, and you're allowed to have gelato.
- You're not allowed to have both cake and gelato.
  (We ignore this 'exclusivity' inference henceforth.)

A surface-scope analysis of free-choice disjunction

Fox (2007) proposes that the literal meaning of (9), $\Diamond[c \vee g]$, is strengthened via recursive exhaustification.

$$[\![\text{exh}_B\ [\text{exh}_A\ [\text{allowed [you have cake or gelato]]]}]\!]$$

$$= \underbrace{\Diamond[c \vee g]}_{\text{prejacent}} \wedge \underbrace{\neg \Diamond p}_{\text{`bound'}} \wedge \underbrace{\Diamond c \wedge \Diamond g}_{\text{FC}} \wedge \underbrace{\neg \Diamond [c \wedge g]}_{\text{exclusivity}}$$

- Assuming that *You're allowed to have pie* is an alternative, then the first (inner) *exh* derives $\neg \Diamond p$.
- The second (outer, recursive) round of *exh* derives FC: $\Diamond c \wedge \Diamond g$. (The second *exh* negates the alternatives *You're allowed to have cake but not gelato* and *You're allowed to have gelato but not cake*.)

Our goal

Given the inferential (and syntactic) similarity between (1) (*allowed . . . at most*), henceforth S, and (9) (*allowed . . . or*), our goal is to provide a surface-scope analysis of S, in which the literal meaning $\Diamond[\leq 3]$ is strengthened via recursive exhaustification into:

$$[\![\text{exh}_B\ [\text{exh}_A\ S]]\!] = \underbrace{\Diamond[\leq 3]}_{\text{prejacent}} \wedge \underbrace{\neg\Diamond[\geq 4]}_{\text{UB}} \wedge \underbrace{\Diamond[= 3] \wedge \Diamond[= 2] \wedge \cdots}_{\text{FC}}$$

What are the alternatives to S?

We assume that the alternatives to S, $\text{alt}(S)$, are all those obtained by replacing *at most* with *exactly* and *3* with any numeral.

$$\text{alt}(S) = \{\diamonddot[\leq 3]\} \cup \{\diamonddot[= n] : n \in \mathbb{N}_0\} \qquad =: A$$

(We could also include *at most n* and *at least n*, or bare numeral *n*, alternatives: $\{\diamonddot[\leq n] : n \in \mathbb{N}_0\} \cup \{\diamonddot[\geq n] : n \in \mathbb{N}_0\}$. This would be more in line with a Katzir-style view of alternatives. However, it turns out that including these does not result in any additional inferences, so for simplicity we ignore them.)

First (inner) *exh*

$\text{exh}(B)(\text{exh}(A)(\Diamond[\leq 3]))$

Focusing just on the first (inner) *exh*, all alternatives of the form $\Diamond[= n]$, for $n \geq 4$, are innocently excludable (IE).

As a result, we negate all such alternatives, which derives an UB.

$$\begin{aligned}\text{exh}(A)(\Diamond[\leq 3]) &= \Diamond[\leq 3] \wedge \neg \Diamond [= 4] \wedge \neg \Diamond [= 5] \wedge \cdots \\ &\equiv \underbrace{\Diamond[\leq 3]}_{\text{prejacent}} \wedge \underbrace{\neg \Diamond [\geq 4]}_{\text{UB}}\end{aligned}$$

(This part is analogous to excluding *pie* earlier.)

Alternatives that are not logically independent from $\Diamond[\leq 3]$, e.g. $\Diamond[= 2]$, are not IE.

Second (outer, recursive) *exh*, first attempt

$\text{exh}(B)(\text{exh}(A)(\diamond[\leq 3]))$

Focusing now on the recursive *exh*, Fox (2007) assumes that alt(exh S) is the set of all strengthened alternatives to S:

$$\text{alt}(\text{exh } S) = \{\text{exh}(\text{alt}(S))(p) : p \in \text{alt}(S)\} \qquad =: B$$

A problem: derived FC is too weak

If we make this assumption, then the FC we derive is too weak.

Here's why . . .

A problem: derived FC is too weak

B (= alt(exh S)) would include the set of all strengthened *exactly* alternatives.

$B = \{\text{exh}(A)(\diamond[\leq 3])\} \cup \{\text{exh}(A)(\diamond[= n]) : n \in \mathbb{N}_0\}$

A problem: derived FC is too weak

The strengthened meaning of an *exactly* alternative $\diamond[= n]$ is:

$$\begin{aligned}
\mathrm{exh}(A)(\diamond[= n]) \quad &= \quad \diamond[= n] \\
&\qquad \wedge \neg \diamond [= (n-1)] \wedge \neg \diamond [= (n-2)] \wedge \cdots \\
&\qquad \wedge \neg \diamond [= (n+1)] \wedge \neg \diamond [= (n+2)] \wedge \cdots \\
&\equiv \quad \diamond[= n] \wedge \neg \diamond [< n] \wedge \neg \diamond [> n]
\end{aligned}$$

A problem: derived FC is too weak

The set of all such strengthened *exactly* alternatives is thus:

$\{\Diamond[=n] \wedge \neg\Diamond[<n] \wedge \neg\Diamond[>n] : n \in \mathbb{N}_0\}$

A problem: derived FC is too weak

Everything in this set is IE, so we exclude everything, yielding, for the strengthened meaning of S:

$$\underbrace{\diamond[\leq 3]}_{\text{prejacent}} \wedge \underbrace{\neg\diamond[\geq 4]}_{\text{UB}} \wedge \underbrace{\bigwedge\{\neg(\diamond[=n] \wedge \neg\diamond[<n] \wedge \neg\diamond[>n]) : n \in \mathbb{N}_0\}}_{\text{FC??}}$$

A problem: derived FC is too weak

This last big conjunction is equivalent to:

$$\underbrace{\bigwedge\{\Diamond[=n] \rightarrow (\Diamond[<n] \vee \Diamond[>n]) : n \in \mathbb{N}_0\}}_{\text{FC??}}$$

The problem: This 'FC' condition is satisfied even if, say, exactly 1 and exactly 3 are allowed, but exactly 2 is forbidden, or exactly 1 and exactly 2 are allowed, but exactly 3 is forbidden.

More generally: We derive the inference that there is free choice between (at least) two indeterminate numbers in the range [0, 3], but not every number in that range.

An amendment to the theory of *exh*

Our idea: The set of alternatives for the second *exh* includes not just all strengthened propositions taken from alt(S).

Rather, it includes all strengthened alternatives taken from the disjunctive closure of alt(S).

$$\text{alt}(\text{exh } S) \quad = \quad \{\text{exh}(\text{alt}(S))(p) : p \in \text{alt}(S)^{\vee}\}$$

Intuition behind the amendment

The effect of our amendment is to introduce weaker propositions into the alternative set, so that their exclusion results in stronger inferences overall, i.e. total FC.

Second (outer, recursive) *exh*, second attempt

For example, $p = \Diamond[=0] \vee \Diamond[=1] \vee \Diamond[=3]$ is in $\mathrm{alt}(S)^{\vee}$.

$\mathrm{exh}(\mathrm{alt}(S))(p)$, which is now in $\mathrm{alt}(\mathrm{exh}\ S)$, is
$p \wedge \neg \Diamond [=2] \wedge \neg \Diamond [\geq 4]$.

Negating $p \wedge \neg \Diamond [=2] \wedge \neg \Diamond [\geq 4]$ is equivalent to
$p \rightarrow (\Diamond[=2] \vee \Diamond[\geq 4])$.

And this, together with the strengthened assertion
$\mathrm{exh}(\Diamond[\leq 3]) = \Diamond[\leq 3] \wedge \neg \Diamond [\geq 4]$, entails $\Diamond[=2]$.

Second (outer, recursive) *exh*, second attempt

Thus, the overall meaning derived for S is:

$$\begin{aligned} & \text{exh}(B)(\text{exh}(A)(\diamond[\leq 3])) \\ = \; & \underbrace{\diamond[\leq 3]}_{\text{prejacent}} \wedge \underbrace{\neg \diamond [\geq 4]}_{\text{1st exh: UB}} \wedge \underbrace{\diamond[= 3] \wedge \diamond[= 2] \wedge \cdots}_{\text{2nd (recursive) exh: FC}} \end{aligned}$$

where:

- $A = \{\diamond[\leq 3]\} \cup \{\diamond[= n] : n \in \mathbb{N}_0\}$
- $B = \{\text{exh}(A)(p) : p \in A^{\vee}\}$

Generalizing the amendment

We don't have to assume that there is anything special about the second (recursive) *exh*.

Instead, we can just say that the strengthened meaning of a sentence S is not $\text{exh}(\text{alt}(S))(\llbracket S \rrbracket)$.

Rather, the strengthened meaning of any S is $\text{exh}(\text{alt}(S)^{\vee})(\llbracket S \rrbracket)$.

For non-recursive *exh*, this amendment makes no difference: excluding $p \vee q$ is equivalent to excluding p and excluding q.

The effect only surfaces for recursive *exh*.

Generalizing the amendment

Then the overall meaning derived for S is still:

$$
\begin{aligned}
&\text{exh}(B)(\text{exh}(A)(\diamondsuit[\leq 3])) \\
= {}&\underbrace{\diamondsuit[\leq 3]}_{\text{prejacent}} \wedge \underbrace{\neg \diamondsuit [\geq 4]}_{\text{1st exh: UB}} \wedge \underbrace{\diamondsuit[= 3] \wedge \diamondsuit[= 2] \wedge \cdots}_{\text{2nd (recursive) exh: FC}}
\end{aligned}
$$

where now:

- $A = \left(\{\diamondsuit[\leq 3]\} \cup \{\diamondsuit[= n] : n \in \mathbb{N}_0\}\right)^{\vee}$
- $B = \{\text{exh}(A)(p) : p \in A\}^{\vee}$

Free-choice disjunction, revisited

Importantly, our amendment does not disrupt the analysis of FC disjunction like (9) (*allowed . . . cake or gelato*).

In this case, the alternative set is already closed under disjunction.

New observations, revisited

Our account extends naturally to the new data from earlier.

We just have to assume that the relevant 'exactly' alternatives are available. For example:

(10) a. The catering premises may open at (exactly) {. . . , 4:59 AM, 5:00 AM, 5:01 AM, . . . }.

b. The Speaker is allowed to appoint exactly {. . . , 2, 3, 4, 5, 6, 7, 8, . . . } MPs [. . .] when he is out of the country [. . .].

Open problem: *at least*, revisited

Our proposal also predicts a LB authoritative reading for *allowed . . . at least 3*:

$$
\begin{aligned}
& \text{exh}(B)(\text{exh}(A)(\diamond[\geq 3])) \\
= & \underbrace{\diamond[\geq 3]}_{\text{prejacent}} \wedge \underbrace{\neg \diamond [\leq 2]}_{\text{1st exh: LB}} \wedge \underbrace{\diamond[=3] \wedge \diamond[=4] \wedge \cdots}_{\text{2nd exh: FC}}
\end{aligned}
$$

This prediction is only partially correct: Recall the contrast . . .

(11) a. ??You're allowed to write at least 3 pages.

b. You're allowed to (either) give a presentation or (else) write at least 3 pages.

Unfortunately, we still have no explanation for this contrast, or more generally why *at least* only sometimes has a LB authoritative reading.

Conclusion

Solving the puzzle of authoritative readings:

- Decomposing *at most* and positing split scope is not fully general, since *at the earliest/latest*, *between*, and even (sometimes) *at least* give rise to authoritative readings.
- An inverse-scope analysis would only predict ignorance readings.
- A surface-scope account based on recursive *exh* is possible, as long as we slightly modify the notion of 'strengthened meaning' so that the alternative set is closed under disjunction.
- Open problem: authoritative readings with *at least* should always be available, but are not.

Appendix: Semantics of *exh*

$[\![\text{exh}]\!](A)(p) = p \wedge \bigwedge\{\neg q : q \in \text{IE}(p, A)\}$

$\text{IE}(p, A) = \bigcap\{A' : A'$ is a maximal subset of A such that $\{p\} \cup \{\neg q : q \in A'\}$ is consistent$\}$

Informal notation: 'q is IE' is shorthand for '$q \in \text{IE}(p, A)$', for some contextually relevant p and A.