

Quotation as a modality*

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Abstract

The aim of this paper is to propose a new theory of quotation. I argue that quotes are essentially modal operators that shift the model of interpretation from the actual to an unactualised yet accessible one. This approach is supported by two independent observations. First, I present novel data from overt modals which, when taking scope over quotational expressions (QEs), obtain a special reading providing possible languages. Second, I argue that the account of QEs according to which the interpretation of quoted expressions is relativised to the quoted source (Maier 2014a, a.o.) must be extended in order to capture this effect. A natural solution comes from adapting this general approach to the modal account of reported speech presented by Kratzer (2013, 2016). The proposed modal framework captures mixed, pure and direct QEs, appearing with and without overt modals. Accordingly, it shows that quotation, as well as the whole reported speech, is part of modality and does not require dedicated mechanisms.

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1 Introduction

The semantic literature on quotation clearly ranges between two opposing ideas. On the one hand, quotational expressions (QEs) are treated as bare strings of symbols (cf. Gaskin & Hill 2013; Geach 1957; Maier 2014b; Pafel 2011; Potts 2007; Tarski 1933/1983; Werning 2005). This strategy has been mainly applied to the so-called pure and direct quotation exemplified in (1):

- (1) a. ‘Cicero is smart’ is a sentence.
- b. Alfred said ‘Cicero is smart’.

According to this approach, the QE in (1) is equivalent to the string of letters $C\hat{\cap}i\hat{\cap}c\hat{\cap}\dots\hat{\cap}a\hat{\cap}r\hat{\cap}t$, being devoid of formal properties responsible for its internal morphosyntactic and semantic complexity. In this sense QEs are “syntactically simple expressions” as understood by Tarski (1933/1983:159).

In the literature going in the opposite direction, QEs are treated as formally transparent expressions. Their properties give rise to special effects, but do not turn them into bare strings of symbols. This approach has been mainly applied, not unexpectedly, to the so-called mixed quotation (cf. Benbaji 2004; Maier 2014a; Shan 2010) exemplified in (2a):

- (2) a. According to Alfred, Cicero is ‘smart’.
- b. $\llbracket \text{‘smart’} \rrbracket \approx \lambda x_e. \text{‘smart’}(x) = 1$ iff x has the property expressed by Alfred as $\ulcorner \text{smart} \urcorner$

According to this approach, the meaning of the QE in (2a), sketched in (2b) as an adaptation of the account of Maier (2014a), is rigid w.r.t. the form of the quoting expression ($\ulcorner \text{smart} \urcorner$). Nevertheless, it retains morphosyntactic and semantic properties of the originally quoted expression (*smart*).

The middle-earth situated between those two approaches is divided among various accounts varying from general-conceptual to strictly computational ones.¹ All of them aim at bridging two conflicting properties of QEs, i.e. their complex and atomic character. On the one hand, the material appearing between quotes is not a formally atomic and syntactically unstructured string of letters. Apart from various technical problems, e.g. with the disquotational theory, there are tests showing that parts of QEs are sensitive to formal relations. Perhaps the simplest one is the effect of mixed quotation as in (2); others can be made by means of ellipsis and reporting verbs, as in (3)–(5):

- (3) a. Alfred said ‘Rudolf_i is smart’, but I don’t think he_i really is Δ .
- b. Russell’s ‘All movements_i go too far’ didn’t let me sleep, thought I don’t think they_i really do Δ .
- (4) #Alfred asked ‘Rudolf is smart’.
- (5) a. Alfred explained ‘That’s because the greenhouse gas emission is too high’
- b. #Alfred explained ‘greenhouse’.

Had QEs been mere strings of letters, neither ellipsis, nor coindexing would have been easily computable in both direct and pure QEs as in (3). It would also remain unclear how to explain relations holding between such QEs and root verbs, e.g. selection of the [INTERROGATIVE] feature as in (4) or content/discourse-related dependencies as in (5).²

On the other hand, there are a number of effects suggesting that the material flanked by quotes is indeed insensitive to formal semantics relations. Not only are parts of QEs blocked for the straightforward substitution version of compositionality, as in (6). What is especially interesting is that quotes are

¹In the demonstrative theory (Ajdukiewicz 1967; Davidson 1979), QEs work as an ostension pointing to the quoted expression. The proper name theory (Carnap 1947; Quine 1960) takes QEs to be the proper names of quoted expressions. According to the demonstration theory (Clark & Gerrig 1990; Davidson 2015; Härtl 2018, though the idea is rooted in the logic inquiry by Reach 1938 and subsequently Geach 1957; Read 1997), QEs display the quoted expression in the form of the material flanked by quotes. The identity theory (Harth 2011; Saka 1998, 2011; Tajtelbaum 1957; Washington 1992, but see also Bazzoni 2016 for a model-theoretic approach) takes every expression of natural language to be ambiguous, covering, a.o., its extension, intension, or form. Quotes disambiguate such an expression by specifying that the meaning of a QE ‘ α ’ is the form $\ulcorner \alpha \urcorner$ used for uttering the expression α . A related approach is taken by Pagin & Westerståhl (2010c), for whom quoting expressions are linked with quoted expressions by occurring in the particular type of context, i.e. the quotational context. Accordingly, the quoted expression can be retrieved by computational mechanisms, provided the right context-sensitive compositionality principles are defined. Finally, Ginzburg & Cooper (2014) encode the meaning of QEs making use of dependent types and developing the way they are arranged in the computation process. I send the reader to Capellen & Lepore (2007), Maier (2014b, to appear[a]), and Saka (2013) for surveys.

²Observations from ellipsis date back at least to Partee (1973); for some comments concerning the problem of the verb-QE relations, see Fabricius-Hansen & Sæbø (2011) and Oshima (2006). For a discussion on a pragmatic approach to quotation, see De Brabanter (2017) and Gutzmann & Stei (2011a,b).

the only tool in natural languages incorporating into grammar any material, including ungrammatical expressions and gibberish, as in (7):³

- (6) a. Alfred said (that) ‘Cicero is smart’. \nRightarrow Alfred said (that) ‘Tully is smart’.
- (7) a. Alfred ‘goed’ home/said ‘I goed home’.
- b. Alfred said ‘bishbadcach’.

As pointed out by Ginzburg & Cooper (2014), partially due to this property for years linguists were prone to taking quotation as lying outside the grammar *proper*. This state of affairs was challenged most notably from two directions. Maier (2014a) and Shan (2010) take mixed quotation as shifting the language in which the quoted expression is interpreted; hence the general semantics outlined in (2b). Focusing on direct quotation, Ginzburg & Cooper (2014) take a QE as incorporating the grammatical resource by which the quoted expression is generated. In each account quotes are treated as special, belonging to none of standardly recognized grammatical category.

In the present paper I extend these achievements, proposing a semantic theory of quotation that meets the two following demands. First, it must account for the data captured within these three frameworks, as well as for so far not discussed data arising for modals taking scope over QEs. Second, if QEs are, under certain conditions, grammatically transparent and complex, the theory must semanticise quotation by specifying, which formal grammatical category it represents. These two objectives motivate the structure of this paper. In section 2 I show that QEs appearing within the scope of overt modals give rise to a new reading. Rather than providing worlds corresponding to scenarios a world might be, they encode possible ways a language used to describe those worlds might be. Showing that these data require extending the existing solutions, in section 3 I sketch a modal system adapting Chomsky’s idea of I-languages to a model theoretic framework. Then in section 4 I show how the proposed theory captures the semantics of QEs. Section 5 concludes the discussion and suggests paths for future research. The formal system and sample computations are laid out in Appendix in section 6. Overall, the paper provides a generalized modal account of quotation, arguing that the metalinguistic discourse is part of a more general grammatical category, i.e. modality (Giannakidou & Yoon 2011). Though I primarily focus on the most complex and subtle problem of mixed QEs, the discussion covers pure and direct mode, with and without overt modals. In this sense it unifies the three basic types of QEs within a single theory (Ludwig & Ray 2017; Werning 2012).

2 Data: overt modals and quotation

In this section I present novel data showing that modals taking scope over QEs can receive a metalinguistic reading. The discussion splits into simple modals and modal comparatives presented in subsection 2.1 and 2.2, respectively.

2.1 Simple epistemic modality

One of the most puzzling concerns in the literature on quotation is that in mixed QEs expressions are both used and mentioned. The exact formal sense of being both used and mentioned is not crystal clear and thus has been a topic of debate (Capellen & Lepore 2007; Davidson 1979; Garcia-Carpintero 2017; Gómez-Torrente 2003, 2013, 2017; Maier 2007, 2008, 2014a, to appear(a); Schlenker 2017; Simchen 1999, a.o.). Perhaps the most explicit distinction is given by Maier (2014a). He argues that the quoted expression is used in the sense that it has the referential properties of the quoted expression; thus the QE in (8) is a transitive verb, just like the original Bushism. On the other hand, it is mentioned in the sense that it is presupposed that the expression of the particular form has been used by the contextually salient speaker. The argument for taking this piece of information to be presupposed is that it is sensitive to the *hey-wait-a-minute-test* (Fintel 2004; Shanon 1976), as illustrated in (8):

- (8) A: Haha, looks like Palin herself “misunderestimated” this thing.
- B: Hey wait a minute, I didn’t know Palin uses the verb ‘misunderestimate’ too. I thought that was a Bushism. [Maier 2014a:38]

³For a discussion on compositionality, also in connection with QEs, see Pagin (2009) and Pagin & Westerståhl (2010a,b, 2011). These problems also apply to hyperintensional contexts in the sense of Berto & Jago (2019).

However, to my knowledge it has never been observed that this effect does not arise for modal contexts. To see this, assume the context in (9) and consider the dialogue in (10)–(12):

- (9) **Context:** Bush coined a verb *misunderestimate* which is a Bushism mixture of *misunderstand* and *underestimate*. There is no evidence that Bush has ever coined a verb *misunderappreciate*, purely potentially conceivable as a Bushism mixture of *misunderstand* and *underappreciate*.
- (10) A: Bush told me he had misbehaved, but I don’t remember how did he call it.
- (11) B: He ‘misunderappreciated’ his guests.
- (12) B: He might have ‘misunderappreciated’ his guests.
- (13) He might have said ‘misunderappreciate’ when referring to what he did with his guests.

B’s answer in (11) gives rise to presupposition failure. If there is no evidence that Bush has ever used this word, it fails the test illustrated in (8). By contrast, (12) is fine, though not under every reading. Interpreted along the lines of standard Kratzerian modality, (12) is true iff there is an R -accessible world $w_1 : w_1 \in \text{Bush did with his guests what he called ‘misunderappreciate’}$. For this reading the fact that Bush used the relevant word is not secured, hence the presupposition failure. Still, there is another, metalinguistic reading, such that (12) is true iff for what Bush did with his guests, it is possible that he called it *misunderappreciate*. Under this reading, the QE placed in the modal scope quotes a purely hypothetical expression defined in Bush’s possible idiolect. Thus presupposition introduced by Maier is secured by Bush’s possible idiolect in which the verb *misunderappreciate* is defined. The presence of modal lifts the requirement of the existence of salient quoted source assumed by Maier (2014a) and Shan (2010) for bare mixed QEs. This metalinguistic reading approximates the meaning of (13), paraphrasing it by making use of direct quotation.

The effect sketched above arises not only for hypothetical expressions, but also for those whose usage is documented only in some languages. An interesting example comes from modals providing possible historic stages of languages. To see this, assume the context in (14) and consider the example in (15):

- (14) **Context:** There is strong evidence that: (i) approximately in the late 3rd c. Japan was ruled by the Emperor Ojin; (ii) approximately by the end of the 3rd c. Japan was called *Yamatai*.
- (15) Ojin might have ruled ‘Yamatai’.
- (16) It is possible that the name of Japan used by Ojin was ‘Yamatai’.

Note that (15) has two possible types of causal continuation, suggesting different readings of modality:

- (17) a. Ojin might have ruled ‘Yamatai’ because....
 b. ...he was born in the imperial family. [\leadsto a property of Ojin]
 c. ...because he spoke the old Japanese. [\leadsto a property of Ojin’s language]

In the first reading the proposition restricts possible worlds to those in which Ojin ruled whatever he called *Yamatai*. Accordingly, (17b) can be falsified by a context in which, e.g., Ojin necessarily did not become an emperor or ruled a different land. In the second reading modality imposes restrictions following from the metalinguistic dimension introduced by the QE and concerning possible languages used by Ojin. Then (17c) roughly means that whatever Ojin ruled, it is possible that he called it *Yamatai*. It can be falsified by a context in which, e.g., the name *Yamatai* necessarily appeared after Ojin’s death, so that it is not possible that it was defined in his idiolect. The second, metalinguistic reading arising for mixed QE approximates the meaning of (16), paraphrasing it by making use of pure quotation.

These preliminary observations are even more interesting in the face of data showing that the metalinguistic reading of modals can be overtly marked. The simplest evidence comes from modal structures containing additional restrictions, such as biscuit conditionals (Siegel 2006) or *must* involving indirect inference (Fintel & Gillies 2010, 2018) as in (18) and (19), respectively:

- (18) Ojin might have ruled ‘Yamatai’, if you are interested in his language.
- (19) He used the old Japanese, so he must have ruled ‘Yamatai’.

However, more significant data come from Japanese. While in English this and other metalinguistic readings can be marked by means of prosodic focus (Li 2017), in Japanese this option is not available. Thus the straightforward translation of (15) in (20) provides only one, non-metalinguistic reading. The second reading becomes fine if the QE is topicalized, e.g. by a cleft structure as in (21):

- (20) Kare-wa ‘Yamatai’-o tôti sita kamosirenai.
 He-TOP ‘Yamatai’-ACC ruled might
 ✓ It is possible that he ruled whatever he called *Yamatai*
 # Whatever he ruled, it is possible that he called it *Yamatai*
- (21) Kare-ga tôti sita-no-wa ‘Yamatai’ kamosirenai.
 He-NOM ruled-NMLZ-TOP ‘Yamatai’ might
 ✓ It is possible that what he ruled was *Yamatai*
 ✓ Whatever he ruled, it is possible that he called it *Yamatai*

Let us pause here and take stock. Overt modals taking scope over mixed quotation were shown to change the interpretation of QEs. They relativise the interpretation of QEs to a set of possible languages, as in (12) and (15), rather than a contextually salient one, and cancel the effect of presupposition failure, as in (11). This effect becomes natural if such constructions are taken as involving the metalinguistic dimension to the interpretation of modals, i.e. as providing possible languages, rather than worlds. The fact that this is not a mere intuition is confirmed by Japanese as in (20)–(21), where the metalinguistic reading requires overt marking.

Now the question arises whether the same effect can be observed for pure and direct quotation, as paraphrasing in (13)–(16) the examples in (12) and (15). Put differently, would it be of benefit to the semantic theory to propose two possible interpretations of (22):

- (22) He might have said ‘misunderappreciated’.
 \rightsquigarrow there is an accessible world w_1 : in w_1 he said *misunderappreciated*
 \rightsquigarrow he said f : in some possible language, c is understood as *misunderappreciated*

I argue that the answer is in the affirmative, but for the sake of coherence of the paper I will not go into detailed discussion. The general line of reasoning is as follows. It has been already observed (Lyons 1977) that, contrary to mixed quotation, pure and direct QEs can be interpreted in two ways, i.e. as denoting just strings of letters or meaningful expressions whose interpretation is relativised to the quoted source. Note that it is only the second type of interpretation that secures the second reading in (22). This goes in hand with the observation going from the opposite direction. Unless metalinguistic elements like languages are introduced to the interpretation, they can appear neither in biscuit conditionals nor in indirect inference as in (23)–(24):

- (23) #Ojin might have ruled Yamatai, if you are interested in his language.
 (24) #He used the old Japanese, so he must have ruled Yamatai.

This suggests that the metalinguistic reading of modals taking scope over QEs involves a different type of modality. In the next subsection I show that the relation between the possible worlds modality and the one involving possible languages becomes clearer and more challenging for comparative structures.

2.2 Modal comparatives

The complexity of the effect rooted in the ambiguity of modals as discussed in the previous subsection naturally increases together with the complexity of modal structure. This can be observed for modal comparatives that not only involve the effect observed above, but also give rise to a hybrid mode of comparison. To see this, assume, first, the context in (25) and consider the example in (26):

- (25) **Context:** There is strong evidence that: (i) approximately in the late 3rd c. Japan was ruled by the Emperor Ojin; (ii) approximately by the end of the 3rd c. Japan was called *Yamatai*; (iii) from the 8th c. onwards Japan has been called *Nihon*.
 (26) It is more likely that Ojin ruled ‘Yamatai’ than that he ruled ‘Nihon’.
 (27) It is more likely that the name of Japan used by Ojin was ‘Yamatai’ than that it was ‘Nihon’.

(26) allows two readings. First, assuming the Kratzerian approach to *better possibility* (Katz et al. 2012) and Maier’s (2014) semantics of mixed QEs, (26) is true iff some R -accessible world w_1 : $w_1 \in$ *Ojin ruled what he called ‘Yamatai’* is more highly ranked than every R -accessible world w_2 : $w_2 \in$ *Ojin ruled what he called ‘Nihon’*. Second, it can be interpreted as involving the metalinguistic type of modality. Then (26) is roughly interpreted as true iff for c : Ojin ruled c , the probability that Ojin called c *Yamatai* is higher than the probability that he called c *Nihon*. The second reading arising for mixed QEs approximates the meaning of (27) paraphrasing it by making use of pure QEs.

In this sense the type of ambiguity observed for modals is the same as in (12) and (15). What makes comparatives more significant is that they involve important additional effects. First, for the non-metalinguistic reading, in the two sorts of scenarios undergoing comparison the preferred interpretation is the one where Ojin ruled two different areas. Otherwise comparison is pointless, relating two identical situations. By contrast, the preferred interpretation for the metalinguistic reading is the one where in the two sorts of scenarios Ojin ruled the same area. The reason is that what undergoes comparison is the way the particular land is called. Accordingly, had each name picked out a different area, the comparison would have been trivial. This shows the type of modality affects the identification of referents of QEs.

Another important observation for modal comparatives concerns markers of metalinguistic modality. Just as in the case of (20), the straightforward translation of (26) into Japanese blocks the metalinguistic reading of modals. Still, what makes Japanese even more interesting than the cleft structure in (21) is the special marker *to iu* placed directly after QEs:

- (28) Kare-wa ‘Nihon’ \emptyset /to-*iu* yori ‘Yamatai’-o tôti sita kanôsei-ga takai.
 He-TOP ‘Nihon’ \emptyset /COMP-say rather than ‘Yamatai’-ACC ruled possibility-NOM high
 \checkmark / $?$ # It is more likely that he ruled what he called *Yamatai* than that he ruled what he called *Nihon*.
 #/ \checkmark Whatever he ruled, it is more likely that he called it *Yamatai* than that he called it *Nihon*.

The *to-*iu** marker is optional, but without it the metalinguistic reading of modal comparatives is not available.⁴ This observation pushes the one made for (21) a step further. It shows that the metalinguistic reading of modals has its lexical marker, attached independently from that of comparatives.

An important comment. Both the metalinguistic effect and the data from Japanese might suggest that the discussed effect is an instantiation of metalinguistic comparatives (Giannakidou & Yoon 2011; Herburger & Rubinstein 2019; Morzycki 2011). These are indeed marked in Japanese by *to iu*. Nevertheless, the two categories are substantially different. Contrary to the effects discussed above, metalinguistic comparatives exemplified in (29) involve judgements about the aptness of using a given term:

- (29) He is more an engineer than a mathematician.

They neither relativise the interpretation to that of salient speaker, nor do they allow ungrammatical material, nor do they involve a form-rigid interpretation of expressions undergoing comparison.

Viewed from that angle, effects observed for comparatives are to much extent similar to those discussed in the previous subsection. What makes them special is that they give rise to a puzzle that has no corresponding effects in simple modal structures. Having a more complex structure, comparatives are not only ambiguous in the way discussed above, but also allow hybrid comparisons as in (30):

- (30) It is more likely that Ojin ruled ‘Yamatai’ than that he was born in the 4th c.

Under the metalinguistic reading, (30) roughly means that for *c*: Ojin ruled *c*, it is more likely that Ojin called *c* *Yamatai* than that he was born in the 4th century. Significantly, (30) is different from the so-called comparison of deviation (Kennedy 1997; Kubota 2012), as in (31):

- (31) He is taller than she is happy.

In (31) comparison involves two different scales (hence the deviant character), but both are defined for objects existing in the actual world. The comparison in (30) relates neither likelihoods concerning two sorts of possible worlds, nor likelihoods concerning two sorts of idiolects. Rather, scenarios concerning Ojin’s idiolects (those where the land he rules is called *Yamatai*) are compared with scenarios concerning his life (those where he is born in the 4th century).

This observation is very important. The discussion so far showed that there is a metalinguistic type of modality involving possible languages or idiolects rather than worlds. Hybrid structures as in (30) prove that the possible worlds modality and the metalinguistic one interact with each other. In this sense they instantiate parts of a more general modality, rather than two disconnected categories.

2.3 Interim summary No. 1

In this section I presented novel data observed for QEs appearing within the scope of modals. The upshot of these observations can be summarized in three points. First, modals taking scope over all basic

⁴A more delicate problem arises for the non-metalinguistic reading of structures with *to iu*. While it is dispreferred in the out-of-the-blue context, it is also more sensitive to the assumed background knowledge, hence the $?$ # sign for the first reading. I am indebted to Shin’ya Okano for numerous discussions on this and other aspects of Japanese modals.

types of QEs provide scenarios for possible languages, empirically distinct from scenarios for possible worlds. Second, the metalinguistic reading of modals can be marked by both syntactic configurations and dedicated items. Third, possible languages can undergo modal comparison with other languages or possible worlds. These observations show that quotation provides a systematic modal way of shifting languages of interpretation. In the next section I propose a model-theoretic framework specifying both the sense and way in which that kind of language shifting can be captured in formal systems.

3 Modelling the plurality of languages

The data from section 2 show that QEs appearing within the scope of modals can pick out hypothetical expressions as defined in possible languages. These observations contribute much to the understanding of semantics of QEs. First, they are coherent with the unique property of QEs mentioned in section 1, i.e. incorporating into grammar expressions that are not interpreted in the language of the whole sentence. If quoted expressions are understood as defined in a different language, then this effect is not surprising. Second, they suggest the general direction in drawing a formal account of quotation. The interpretation of quoted expressions has been shown to involve language shifting, with the language of quoted source assumed as contextually salient. The present data prove such theories of quotation correct and specify the way they should be extended in order to capture more complex effects.

Following these hints, I argue that the operation of enquotation is essentially modal in that it makes use of mechanisms analogous to those defined for possible worlds. That is, we cannot truthfully refer to van Gogh who died in the actual world in 1930, but we can do that w.r.t. his counterpart representing van Gogh in some accessible world. In the same vein, we cannot meaningfully use the expression *misunderestimate* as interpreted in the actual model (standard English), but we can do that by quoting an expression defined in Bush’s idiolect. Moreover, van Gogh and Gauguin can be world-mates but van Gogh who died in 1890 and his counterpart who died in 1930 cannot. In the same vein *misunderstand* and *underestimate* can be defined in the same model, but an ill-formed *misunderestimate* and a well-formed *misunderestimate* cannot. Put more generally, in much the same way an expression E can be evaluated in this (actual in the sense of Lewis 1986) or in another (non-actual) world, so does E can be interpreted in this (actual) or another (non-actual) model.

In this section I formalize these intuitions, implementing them to a model theoretic semantic framework. In subsection 3.1 I show how the data from section 2 specify the way existing theories of quotation should be extended. Then in subsection 3.2 I conceptualise the plurality of models using Chomsky’s idea of I-languages. Finally, in subsection 3.2 I sketch basic elements of metalinguistic modality.

3.1 Problems

The problem arising for language shifting involved by the $[\text{Mod}^0 \dots [\text{QE} \dots]]$ structure is that it requires combining two categories, modality and quotation, that traditionally have not been framed within a common mechanism. Defining that kind of shifting as a fragment of a general world shifting device would require going much beyond standard solutions, e.g. various types of ordering over worlds (Pasternak 2016; Uegaki & Roelofsen 2018). Rather, worlds must have been selected by a different type of accessibility relation, call it R_{ling} , roughly as being compatible with the language of quoted source. Domains of such worlds would have to contain metalinguistic items, i.e. expressions, languages or interpretation functions, responsible for setting up the R_{ling} relation. This, however, is untenable for both conceptual and empirical reasons. First, worlds are parts of models; these are formal constructs specifying the meaning of expressions generated by particular languages. Had languages been parts of worlds (in the sense of Lewis 1986), models of languages would have contained worlds which in turn would have contained languages. Second, such an account not only overgenerates, but also fails to secure the metalinguistic reading discussed in section 2. To see this, recall the example (26), repeated as (32):

- (32) It is more likely that Ojin ruled ‘Yamatai’ than that he ruled ‘Nihon’.
 \rightsquigarrow for c : Ojin ruled c , the probability that he called c *Yamatai* is higher than the probability that he called c *Nihon*

Assuming the accessibility relation R_{ling} , (32) is true iff some R_{ling} -accessible $w_1 : w_1 \in \text{Ojin ruled ‘Yamatai’}$ is more highly ranked than every R_{ling} -accessible $w_1 : w_1 \in \text{Ojin ruled ‘Nihon’}$. Note, however, that R_{ling} selects all those worlds that are compatible with Ojin’s use of language. Thus it does not rule out those in which he called Japan *Yamatai*, but, e.g., had never become its ruler. Moreover, this account does not yield the metalinguistic reading given in (32). R_{ling} does not distinguish two types of likelihood, i.e. the

one concerning scenarios of Ojin’s life and the one concerning scenarios of his language. Accordingly, the metalinguistic modality is not rooted in world shifting devices (for conceptual reasons) and requires a more detailed account (for empirical reasons).

By contrast, the idea of language shifting has been proposed in various versions in the literature on QEs. In the most formalized account, Ginzburg & Cooper (2014) propose that a QE incorporates the grammatical resource from the language or dialect of the quoted expression. The grammatical resource responsible for formal properties of quoted expression is encoded as a contextual parameter within the formal representation of QE. Shan (2010) provides language shifting piggybacking on functions defined on grammatical categories of various languages. Quotes turn the relevant grammatical category of the salient quoted language to the corresponding category of the quoting language. Finally, Maier (2014a) secures the relevant shifting by relativising the interpretation of the quoted expression to that of the quoted speaker. Thus QEs provide the particular content (represented by a variable bound by iota operator) as understood by the contextually salient individual. To this extent, the strength of such accounts lies in that they secure in the formal representation of QEs a place for metalinguistic elements like languages. Still, in order to capture effects arising for modal contexts, the above proposals must be extended. As they stand, neither of them encodes sets of possible languages⁵ or formal mechanisms, in particular quantification, responsible for modal effects.

Viewed from that angle the data in 2 show that shifting the language in which the quoted expression is defined applies on a larger scale than parametrization assuming salient languages. Put more precisely, there are two technical points following from the foregoing discussion. First, effects observed for modals taking scope over QEs show that that kind of shifting must capture possible, sometimes purely hypothetical languages. Second, the formal mechanism of language shifting must be developed in order to secure a more flexible and detailed computation.

3.2 Towards quantification over models

The first step in building a modal theory of quotation is securing the plurality of languages that undergo shifting. This requires solving two problems. First, semantics of QEs must account for the fact that QEs are rigid w.r.t. the utterance form, but not necessarily copy the quoted form, as in (33). I take this, together with further arguments presented, a.o., by Davidson (2015), De Brabanter (2017), and Maier (2018), to justify the demonstrative approach to the meaning of QEs:

- (33) John said ‘hello’ in all East Asian languages.
 \rightsquigarrow John said what is demonstrated in every East Asian language by the string ‘hello’

Second, the set of languages relative to which the quoted expression is interpreted must comprise both individual idiolects, as in (26), as well as generalized languages, as in (33). The well-known concept underlying that kind of approach is that of I-/E-languages proposed by Chomsky (1986), the core idea going back at least to Chomsky (1980, 1982). The former stands for the individual, intensional aspect of languages. It is a system of procedures which corresponds to a part of mind/brain of every individual speaker and allows him/her to generate and parse linguistic expressions. The latter is a cover term for the construct “understood independently of the properties of mind/brain” (Chomsky 1986:20) or, as put by Pietroski (2018:52), “for anything that can be called a language but *isn’t* an I-language”.⁶

This approach, with I-languages corresponding to individual idiolects, provides a conceptual ground for the required plurality of languages. Still, I-languages understood as procedures of syntactic derivations are not an adequate candidate for an element of semantic system. What is required is a formal object providing interpretation for such procedures. A natural candidate is a model, understood as a scientific representation linking semantic features of expressions with human representational capacities (Yalcin 2018). Accordingly, I assume that for every I-language \mathcal{L}_i there is at least one model of interpretation \mathcal{M}_i . Every model of that sort is an ordered pair $\mathcal{M}_i = \langle F, \llbracket \cdot \rrbracket_i \rangle$ where F is a frame and $\llbracket \cdot \rrbracket_i$ is the interpretation function assigned to \mathcal{M}_i .⁷

The next problem is how to define languages like English or Japanese that are more general constructs than I-languages of individual speakers. Standardly (cf. Boeckx & Uriagereka 2007; Tancredi 2007a) these are treated as E-languages which are externalizations of I-languages. They are products of generative procedures, just as elements of the set $\{2, 4, 6, \dots\}$ are products of the function $\lambda x \in \mathbb{N}. 2x$ (cf. Baker 2001; Isac & Reiss 2008; Pietroski 2011, 2017, 2018). However, it would be incoherent to treat

⁵Ginzburg & Cooper (2014) assume a set of grammatical resources, but do not go into their detailed semantics.

⁶See Asoulin [forthcoming](#) for an in-depth discussion on the I-/E-language distinction.

⁷For a detailed definition, see subsection 6.2: i..

models providing interpretation for generative procedures (I-languages) on a par with models providing interpretation for products of such procedures (E-languages). Therefore, I follow Pietroski (2018) in taking such languages as English or Japanese to be generalizations over I-languages, defined in terms of language-external criteria.⁸ Viewed from that angle, the idea of language conventionally fixed w.r.t. interpretation so that it becomes a “shared social object” (Ludlow 2017:75), can be naturally accounted for in terms of generalizations over interpretations. If $M = \{\mathcal{M}_1, \dots, \mathcal{M}_n\}$ is a set of models, each with the interpretation function $\llbracket \cdot \rrbracket_1, \dots, \llbracket \cdot \rrbracket_n$, then there is a convention f_c which maps a subset $\{\mathcal{M}_1, \dots, \mathcal{M}_k\}$ of M onto a model \mathcal{M}_c such that $\llbracket \cdot \rrbracket_c$ is the intersection of all the interpretations $\llbracket \cdot \rrbracket_1, \dots, \llbracket \cdot \rrbracket_k$.⁹

So, the first step towards a formal system providing language shifting is adapting Chomsky’s idea of I-languages to a model theoretic approach by providing the set of models of interpretation. Its elements are either models of idiolects (I-languages) or generalizations over such models (labelled as *English*, *Dutch*, etc.). Thanks to this step the grammar is equipped with the set of model theoretic objects relative to which quoted expression are interpreted. In the next step I provide formal grounds for possibly standard modal mechanisms shifting models of interpretation, shown in subsection 3.1 to be required by QEs.

3.3 Accessible models and quotation

Apart from the accounts of quotation discussed in subsection 3.1, operations shifting languages or models of interpretation have been proposed in the literature, in various forms and with various aims. Quer (2001)¹⁰ takes such shifts to be connected with mood shifts responsible for differences between indicative and subjunctive. Investigating believes as held for particular languages, Tancredi & Sharvit (2019) propose a transformation function T_p defined for a perspective p and a language L . The function parametrizes the interpretation $\llbracket \cdot \rrbracket^{T_p(L),p}$, securing that it is consistent with p ’s presumed I-language. Working on generalizations over effects observed for doxastics, Tancredi (2007a,b) provides quantification over models, each having a different set of worlds serving as a modal base. Models can be shifted by metalinguistic operators. Finally, building an account of logical necessity, Lindström (2006) introduces an operator L defined as follows (I skip irrelevant terminological differences):

- (34) $L\phi$ is true at a world w in a model \mathcal{M} iff ϕ is true at the actual world $w_{@}$ in every model \mathcal{N}

Nevertheless, the question arises whether as heavy mechanism as model shifting is justified for QEs. I argue that the relevant arguments are rooted in both empirical and methodological facts. As for the former, the line of reasoning was presented in subsection 3.1. The metalinguistic reading of modals gives rise to interpretations that are substantially different from and not reducible to those obtained for possible worlds. They provide not ways a world might be, but a language used to describe worlds might be, as well as probabilities not that certain situation is the case, but that an expression has certain form. These observations are coherent with the foundations of model theoretic accounts of modality. Standardly, worlds are formal objects relative to which propositions are evaluated as true or false. Such worlds, together with their domains and an accessibility relation, are “turned into a modal model by specifying which propositional letters are true at which world” (Fitting & Mendelsohn 1998:12).¹¹ Thus, providing the relevant properties of propositional letters is part of defining a model, not a world where the proposition as such is evaluated as true or false. Accordingly, it is model shifting that is the right domain for assigning various properties to strings of letters representing possible expressions. This is especially significant in the case of expressions that are purely hypothetical, ill-formed in the speaker’s model, or ill-formed at all, as in (35)–(36):

- (35) Knowing Bushisms, he might have ‘misunderappreciated’ his guests.

- (36) ‘gwllch’ is not a word of any natural language.

⁸“We can use ‘English’ to group certain I-languages together, perhaps in terms of paradigmatic examples . . . We can use ‘Norwegian’ similarly, and classify both Norwegian and English I-languages as Germanic, without supposing that Germanic is a language shared by speakers of Norwegian and English. Analogies between linguistic and biological taxonomy can be preserved, whatever their worth, by thinking of specific I-languages as the analogs of the individual animals that get taxonomized” (Pietroski 2018:56). I take this idea to be supported by the following pragmatic factor. Any theory assuming the plurality of I-languages must explain why it is not accidental that speakers, having their own models of interpretation, can understand each other. I take this to be driven by a sort of Gricean maxim of manner. Given a speaker S , a hearer H , the corresponding models of interpretation \mathcal{M}_S and \mathcal{M}_H , and an expression E it is assumed that unless marked otherwise (mostly by means of quotes), $\llbracket \cdot \rrbracket_S$ and $\llbracket \cdot \rrbracket_H$ yield the same interpretation for E in the context at hand.

⁹For a detailed definition, see section 6.2.iv..

¹⁰See also the related work by Giannakidou and Farkas discussed in Portner (2018)

¹¹Significantly, in their type theoretic approach to quotation, Ginzburg & Cooper (2014) assume a similar role for grammatical resources (incorporated as a part of enquotation), letting them assign sign types to speech events.

The two sentences state not properties of the action of misunderappreciating or of the object *gwłch*, but of the corresponding expressions. Accordingly, possible models are natural candidates for an element of grammar responsible not for evaluating propositions as true or false but defining expressions as having various properties.

Now in order to secure a model shifting mechanism, the grammar must be equipped with accessibility relation over models. The set of models was introduced in subsection 3.2. As for the accessibility relation, a modal approach to reported speech was proposed for indirect discourse by Kratzer (2013, 2016) who lets such reports involve shifting between accessible worlds defined as follows:

- (37) $w' \in \mathcal{C}_{content}(x) : \mathcal{C}_{content}(x)$ is the set of worlds that are compatible with the content of x ,
undefined if x does not have intentional content

The content argument x is saturated by DPs denoting whatever can be believed or reported (stories, lies, etc.). Skipping irrelevant details, the indirect report in (38) is true iff for a situation in which Bush makes a report, he misunderstood Europeans in every world that is compatible with the content of report. However, the problem of quotation is more complex and, especially in the case of mixed QEs, more subtle. To see this, consider the examples in (39):

- (38) Bush said that he had misunderstood Europeans.
(39) a. Bush ‘misunderestimated’ Europeans.
b. According to Bush, he ‘misunderestimated’ Europeans.

The Bushism verb *misunderestimate* is interpreted as understood in Bush’s idiolect. Put more precisely, it is interpreted for the particular form (\ulcorner misunderestimate \urcorner) in the model \mathcal{M}_{Bush} of Bush’s I-language. However, the question arises what is the world of evaluation of this expression. Following Lindström’s (2006) approach in (34), unless marked otherwise, it should be the actual model of \mathcal{M}_{Bush} . But this is not true for every reading. (39a) can be interpreted as a partial report of Bush’s utterance, as specified in (39b). Then *miunderestimated* is evaluated at a world compatible with the content of Bush’s report. Moreover, it can also be given an underspecified interpretation, meaning roughly that Bush did with Europeans what he generally understands as *miunderestimated*. Then the world of evaluation remains open, perhaps belonging to some salient set of worlds, or a majority of worlds in \mathcal{M}_{Bush} . This shows that while every quoted expression E is interpreted in a non-actual model, the world of evaluation of E is not determined by the operation of enquotation and can be specified at various levels of computation, including the level of discourse.¹² This motivates the present proposal. Making use of the notion of quotational context (formalised in Pagin & Westerståhl 2010c) the accessibility relation adapted to the properties of QEs is defined as follows:

Definition 1 (Reportative accessibility relation for quotation) *If $M : \mathcal{M}, \mathcal{M}_1 \in M$ is a set of models and v an assignment function such that $w_{v(i)} =_v w \in W$ and $w_{1,v(i)}$ is a world of \mathcal{M}_1 , then \mathcal{R} is an accessibility relation such that $\mathcal{M}_1 \in \mathcal{R}(\mathcal{M})$ iff every expression E defined in \mathcal{M}_1 for $w_{1,v(i)}$ is defined for the quotational context as ‘ E ’ in \mathcal{M} , where $\ulcorner E \urcorner$ demonstrates E .*

The accessibility relation selects those models in which the quoted expression is defined as having certain form and formal features. Accordingly, it secures quotability, shifted readings and the plurality of models. This extends the proposal of Ginzburg & Cooper (2014), Maier (2014a), and Shan (2010), opening up paths for the proper analysis of at least two issues that are problematic for these accounts. All of them assume that the particular quoted source is contextually salient and thus parametrized. As discussed in subsection 3.1, they hardly account for structures with overt modals involving quantification over possible models. Moreover, such a simple parametrization is at odds with effects observed for QEs with a plural/group quoted source, as in (40):

- (40) Both Russian and Japanese diplomats felt ‘embarrassed’.

(40) allows at least two interpretations. According to one, for the salient model \mathcal{M}_1 (perhaps English), both Russian and Japanese diplomats felt the way interpreted in \mathcal{M}_1 as *embarrassed*. According to another, for two models \mathcal{M}_R and \mathcal{M}_J , Russian diplomats felt the way interpreted in \mathcal{M}_R as *embarrassed*, Japanese diplomats felt the way interpreted in \mathcal{M}_J as *embarrassed*, and the two differ from each other. These problems are known from the literature on plural/group attitude holders (Bar-Lev 2015;

¹²This also shows why QEs must involve shifting a model, and not just interpretation function. Shifting must be definable for various worlds, including those covered by various accessibility relations. These, however, are specified by models, not interpretation functions alone.

Szabó 2010). What it shows here is that identification of quoted speakers exceeds the scope of simple parametrization yielding the unique, contextually salient object (for other problems, see Sauerland & Yatsushiro 2014). Rather, it is regulated by more complex formal mechanisms. I let enquotation be a modal operation by definition assuming the plurality of accessible models. This strategy paves the way to a more flexible account. The way such an account works is scrutinized in the next section.

3.4 Interim summary No. 2

In this section I built basic elements of a modal framework which is in a position to account for the data presented in section 2. The line of reasoning is based on two observations. First, possible worlds are not, for empirical and conceptual reasons, proper formal objects for capturing the data at hand. Second, a proper approach can be formulated by making use of model shifting, an idea whose elements have their predecessors in I-languages, shifting languages or grammatical resources, and the reportative modality. As a result, at the current stage the proposed framework provides a set of models together with the accessibility relation over them. In this sense it secures a kind of frame in the model theoretic sense. The last required element is a model-shifting device, together with its interpretation. This will be the subject of the next, final section.

4 Modal semantics of quotation

In this section I put together the empirical observations from section 2 and the technical discussion from section 3 to provide a semantic account of QEs appearing with and without overt modals. The crucial element of the proposed account is the operator that corresponds to quotes and shifts models of interpretation. There are two general approaches to modal operators, originating from the common tradition of modal logic (Kaufmann & Kaufmann 2015). One, applied to model shifting by Tancredi (2007a,b), takes them to be complex λ -terms (M, M' are models):

$$(41) \quad [\text{Op}_{\text{circ}}]^M = \lambda p. \text{circumstantial}(M') \& R(M, M') \& [p]^{M'} = 1$$

The second approach, applied to model shifting by Lindström (2006) as in (34), takes them to be simple elements of modal system, most standardly \Diamond and \Box , taking scope over propositions or parts of λ -terms. Note, however, that applying the first approach to QEs is problematic. The reason is that quotes take any material, e.g. a nominal, a verb, or an untyped gibberish as in (42), and return well-formed QEs:

- (42) a. Ojin might have ruled ‘Yamatai’.
b. Bush might have ‘misunderappreciated’ his guests.
c. ‘gwllch’ might have been a word.

Accordingly, applying an approach as in (41) would require assuming at least as many type-variants of the operator as there are different types of quotable expressions (here e , $\langle e, \langle e, st \rangle \rangle$ and an untyped gibberish). Another way would be to assume a coercive or polymorphic operator (Maier to appear(b)). For the sake of parsimony, I stick to the one taken by Lindström (2006) and let quotes be a modal operator as used in modal predicate logic. To show how it works, I split the discussion into two parts. In subsection 4.1 I argue that QEs appearing without overt modals involve model shifting subject to rigorous restrictions. Next, in subsection 4.2, I show that QEs appearing within the scope of overt modals involve both model shifting and measure functions returning probabilities.

4.1 Quotes as modal operators

Drawing an account of indirect reports, Kratzer (2013, 2016) takes Mood in (43) to be a head providing world shifting sketched in (44):

- (43) [Subj... [reporting verb [_{CP} SAY [Mood [Left Periphery...]]]]
where SAY is a covert head coercing verbs into verbs of speaking

- (44) $\lambda p. \lambda x. [\forall w' (w' \in f(x) \rightarrow p(w'))]$
where x is a modal anchor and f a content-related domain projection defined in (37)

This approach is supported by its application to evidentials, as proposed by Korotkova (2020). Still, no related approach has been proposed for the second category of reported speech, i.e. quotation.

These two types of reported speech are not, of course, identical. Differences that are perhaps the most important in light of (43)–(44) is the lack of justified SAY head and the propositional argument. As for the former, contrary to indirect discourse, where reporting verbs are obligatory (hence covert SAY proposed by Grimshaw 2015; Major & Torrence 2020; Shimamura 2018, 2019 in various structures), QEs can provide reports without making use of reporting verbs, e.g. in the mixed mode:

- (45) John was ‘fed up’ with these questions.
 \rightsquigarrow John used the expression *fed up* talking about these questions

This automatically requires a different approach to Left Periphery involved by QEs (Wiślicki 2020). As for the latter, quotation applies not only to expressions that do not represent propositions, but also to strings that are not well-formed expressions at all. Nevertheless, empirical and formal effects as well as the cross-linguistic regularity clearly show that both types of reported speech form a coherent category (Bary & Maier 2019; Spronck & Nikitina 2019). From this point of view giving the reported speech at least partially unified treatment is empirically and methodologically expected.

The present framework makes a step towards this kind of unification. On the one hand, it follows the general approach taken by Korotkova (2020) and Kratzer (2013, 2016) in taking reported speech as a modality. On the other, it accounts for the special properties of QEs by relativising their semantics to the accessible model in which the quoted expression is interpreted. I let quotes by an operator Q sharing various properties with Lindström’s operator L in (34) and Kratzer’s Mood in (44). Q takes scope over the whole term merged as its sister, but does not change its type, as exemplified in (47).¹³ Accordingly, the interpretation of the QE in (46) reads as in (48):

- (46) Ojin ruled ‘Yamatai’.

$$(47) \quad \llbracket \text{‘Yamatai’} \rrbracket^{\mathcal{M}, w_{\otimes}, v} = Q \text{Yamatai} : e$$

$$\swarrow \quad \searrow$$

$$\llbracket \text{‘...’} \rrbracket^{\mathcal{M}, w_{\otimes}, v} = Q \quad \llbracket \text{Yamatai} \rrbracket^{\mathcal{M}, w_{\otimes}, v} = \text{Yamatai} : e$$

- (48) $\llbracket \text{‘Yamatai’} \rrbracket^{\mathcal{M}, w_{\otimes}, v} = Q \text{Yamatai} = c :$
 $\forall \mathcal{M}_1 (\mathcal{M}_1 \in \mathcal{R}(\mathcal{M}) \rightarrow c \text{ as defined in } \mathcal{M}_1 \text{ for } w_{1, v(i)} \text{ has the utterance form demonstrated by the form } \ulcorner \text{Yamatai} \urcorner)$

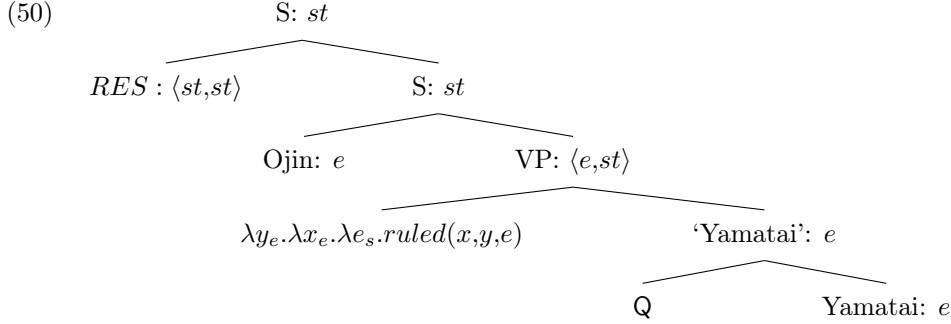
The operation of enquotation is to much extent just like standard modal operations. It neither involves single-branching, nor does it assume dedicated mechanisms of computation, both present in previous accounts (Ginzburg & Cooper 2014; Maier 2014a; Shan 2010). The quotational character is rooted in the domain of shifting. The operator shifts the actual model of interpretation to an \mathcal{R} -accessible model \mathcal{M}_1 . Thus the quoted expression receives the interpretation as specified by \mathcal{M}_1 for some world $w_{1, v(i)}$ of \mathcal{M}_1 , as discussed in the context of Definition 1. In this sense the interpretation of QE is relativised to the way it is understood by the quoted source for the particular form.

Before we move to more detailed comments, it is worth pointing out that the simple account in (47)–(48) overgenerates. (46) means that Ojin ruled what he or other contextually salient speaker called *Yamatai*. The universal quantification, even restricted by the accessibility relation, provides too many possible models of interpretation. As discussed in subsection 3.1 and 3.3, Ginzburg & Cooper (2014), Maier (2014a), and Shan (2010) assume that the particular source relative to which the quoted expression is interpreted is contextually salient, hence parametrized. This, however, blocks the proper interpretation of modals providing possible idiolects, as discussed in section 2. In order to secure the selection of proper objects returned by shifting, I use a function $RES : \langle st, st \rangle$ restricting the set of accessible objects. It behaves in much the same way as the modal choice function proposed by Rullmann et al. (2008), with the proviso that it can be applied to accessible worlds or models. It takes as its argument a proposition involving a modal base $f(x)$ and returns the same proposition with $f(x)$ restricted to its subset $r(f(x))$. The function RES searches through the surrounding context for candidates counting as the relevant possible worlds or models. These can be selected on the basis of various criteria, e.g. models of dialectical variants of someone’s idiolect, as in (49):

- (49) When in Santa Cruz, Peter orders “[eɪ]pricots” at the local market; when in Amherst, Peter orders “[æ]pricots” at the local market. [from Potts 2007]

¹³See subsection 6.2:vi.–vii. for a detailed definition of the operator and subsection 6.3, example (71a) for a complete computation of (46).

For simple cases as in (46) taken out-of-the-blue, *RES* restricts accessible models to the particular model of Ojin's idiolect. Utterances (type u in the sense of Potts 2007) of expressions generated by this I-language and defined in this model are elements of the domain of actual world. I abbreviate this as $r_{\textcircled{a}}$. Accordingly, (46) has the simplified structure as in (50) and interpretation as in (51)



(51) $\llbracket \text{Ojin ruled 'Yamatai'} \rrbracket^{\mathcal{M}, w_{\textcircled{a}}, v} = 1$ iff
 $\forall \mathcal{M}_{Ojin} (\mathcal{M}_{Ojin} \in r_{\textcircled{a}}(\mathcal{R}(\mathcal{M})) \rightarrow \langle \text{Ojin}, c, e \rangle \in \llbracket ruled \rrbracket^{\mathcal{M}, w_{\textcircled{a}}, v}),$
 where c as defined in \mathcal{M}_{Ojin} for $w_{Ojin, v(i)}$ has the utterance form demonstrated by the form
 $\lceil \text{'Yamatai'} \rceil$

The operation of enquotation yields an NP whose meaning is relativised to the way the quoted expression of the particular form is interpreted in the model of Ojin's idiolect.

The strength of the proposed approach is that, making use of standard operator-variable structures, it is flexible enough to account for more complex examples. To see this, consider (52) below:

(52) Bush 'misunderestimated' Europeans.

The technical difference between (46) and (52) is that here enquotation is defined not for a simple term of type e , but a verbal predicate. The complication concerns the scope of Q. Given that it is attached directly to its sister λ -term, it gives rise to the interpretation as in (53):

(53) $\llbracket [\text{Q misunderstood}] \rrbracket^{\mathcal{M}, w_{\textcircled{a}}} = \text{Q} \lambda y_e. \lambda x_e. \lambda e_s. [misunderestimated(x, y, e)]$

Accordingly, all the variables appear within the scope of Q (they are *de dicto* relative to Q). This means that the two arguments, i.e. *Bush* and *Europeans* substituting x and y , receive the shifted (i.e. quotational) reading. The result is the interpretation in which enquotation, taking the wide scope, yields the quoted sentence '*Bush misunderstood Europeans*', not just the quoted verb '*misunderestimated*'. To get the right reading, I follow a fairly standard (Keshet & Schwartz 2019) way of deriving *de re*, i.e. via movement and variable binding taking place over the operator:

(54) For a term $\lambda x.f(x)$, the operator Op, the argument c and its trace t_c the following holds:

$$\begin{array}{lll}
 [c [\text{Op } \lambda x.f(x)]] & \rightsquigarrow & \text{Op} \lambda x.[f(x)](c) \quad x \text{ is bound } de \text{ dicto relative to Op} \\
 [c [\lambda_c [t_c [\text{Op } \lambda x.f(x)]]]] & \rightsquigarrow & \lambda x.[\text{Op} f(x)](c) \quad x \text{ is bound } de \text{ re relative to Op}
 \end{array}$$

Thus, just as in the standard modal predicate logic, where arguments substituting variables bound *de dicto* receive the shifted reading, so do arguments bound *de dicto* relative to Q receive the quotational reading. By contrast, arguments substituting variables bound *de re* relative to Q are not part of QEs, but they undergo standard formal operations with QEs. This partially corresponds to what Maier (2014a) calls *transparent opacity* and underlies the semantics of mixed quotation. Accordingly, what gives rise to the effect of mixed quoted verb as in (52) is movement and *de re* binding as in (55):

(55) $[\text{Bush}_{2,e} [\lambda_{2,e} [\text{Europeans}_{1,e} [\lambda_{1,e} [\text{S } t_2 [\text{VP } [\text{V } \text{Q } \lambda y_e. \lambda x_e. \lambda e_s. misunderstood(x, y, e)] t_1]]]]]]]]$
 $\uparrow \quad \uparrow$
 binding *de re* relative to Q

Let us go through the part of derivation shown in (55) step-by-step. First, it builds the mixed quoted transitive verb '*misunderestimate*'. Next, it provides the two arguments of type e , *Europeans* and *Bush*, securing the complete argument structure. Still, the variables x and y are bound *de dicto* relative to Q. In order to avoid the quotational reading, the two arguments raise, leaving traces t_1, t_2 . Next, λ -operators provide binding above Q, so that both variables are bound in *de re* positions. Thus, the two arguments do not receive the quotational reading. The result is the mixed quoted verb '*misunderestimate*' with two non-quotational arguments, as expected. Then (52) is interpreted is in (56) below:

- (56) $\llbracket \text{Bush 'misunderestimated' Europeans} \rrbracket^{\mathcal{M}, w_{\otimes}, v} = 1$ iff
 $\forall \mathcal{M}_{Bush} (\mathcal{M}_{Bush} \in r_{\otimes}(\mathcal{R}(\mathcal{M})) \rightarrow \langle \text{Bush}, \text{Europeans}, e \rangle \in \llbracket f \rrbracket^{\mathcal{M}_{Bush}, w_{Bush, v(i)}})$,
 where f as defined in \mathcal{M}_{Bush} for $w_{Bush, v(i)}$ has the utterance form demonstrated by the form
 ‘misunderestimated’

This example is important for at least two reasons. First, it derives a mixed quoted predicate making use of standard machinery, i.e. forming *de re* by means of raising. Second, it computes a mixed quoted verb which is not interpretable outside the quotational context, i.e. as not relativised to the way it is understood by Bush.

To close this subsection, let us have a look at the pure QE in (57):

- (57) ‘gwllch’ is not a word.

Two comments are in order. First, taken out-of-the-blue, the QE in (57) does not pick out a string ‘gwllch’ as interpreted is some salient language. (57) states that the string does not represent a word in any language. This supports the presence of quantification over models. Second, pure QEs are not morphosyntactically transparent in the way mixed QEs are. They denote strings of symbols, rather than whatever is picked out by those strings and relativised to the utterance form. Bearing this in mind, I take the difference between pure and mixed QE to arise in the domain of word-formation mechanism as understood in Distributed Morphology. Pure QEs involve nominalization by little n^0 (Marantz 1996 *et seq.*) yielding a term of type u , as in (58).¹⁴ Then (57) is interpreted as in (59):

- (58) $\llbracket [_{nP} n^0 [\text{Q gwllch}]] \rrbracket^{\mathcal{M}, w_{\otimes}, v} = c : u$
 (59) $\llbracket \text{'gwllch' is not a word} \rrbracket^{\mathcal{M}, w_{\otimes}, v} = 1$ iff
 $\forall \mathcal{M}_1 (\mathcal{M}_1 \in r_{\otimes}(\mathcal{R}(\mathcal{M})) \rightarrow \langle c, e \rangle \notin \llbracket \text{word} \rrbracket^{\mathcal{M}, w_{\otimes}})$,
 where c as defined in \mathcal{M}_1 for $w_{1, v(i)}$ has the utterance form demonstrated by the form ‘gwllch’

Accordingly, nominalization aside, the crucial modal mechanism remains unchanged.

The general approach presented in this section is inspired by the Lewisian thinking about actuality, according to which it is an indexical term. The actual model is the present model of interpretation, the one in which one makes the present computation. Quotation shifts the model of interpretation from the actual one to another, non-actual one. Thanks to model shifting, the proposed framework accounts for QEs where quoted expressions are not interpretable in the actual model, like (52) and (57). Thanks to quantification, it captures examples like (57) that are more complex than those providing salient quoted speakers. In the next subsection I show how the modal framework accounts for QEs involving not only non-actual, but also purely hypothetical models.

4.2 Overt modals and quotation

In the previous subsection I sketched a modal semantics of QEs, complementary to Kratzer’s (2013; 2016) modal account of indirect reports within the category of reported speech. Equipped with such tools, it is now possible to tackle the data discussed in section 2, as in (60):

- (60) It is more likely that Ojin ruled ‘Yamatai’ than that he ruled ‘Nihon’.
 \rightsquigarrow for c : Ojin ruled c , it is more likely that he called c *Yamatai* than that he called c *Nihon*

Under one reading, what undergoes modal comparison is not two scenarios of Ojin’s life, but two scenarios of his language. The difference is not purely conceptual. As discussed in subsection 2.2, it affects the interpretation of quoted nominals and it can be marked, both structurally and lexically. This effect makes quotation non-trivially different from indirect reports. To see this, consider the examples in (61):

- (61) a. He might have said that p . \rightsquigarrow it is possible that he said p
 b. He might have ruled ‘ σ ’. \rightsquigarrow for what he ruled, it is possible that he call it σ

The straightforward application of modal predicate logic approach would involve recursive shifting and two accessibility relations. (61a) would be roughly interpreted as true iff there is an R_{epist} -accessible world w_2 such that every world w_3 R_{report} -accessible from w_2 is a p -world, as depicted in Figure 1:

¹⁴See Maier (2014b, to appear[b]) and Potts (2007) for a general outline and Wiślicki (2020) for a possible formalization of the morphosyntactic word-formation mechanism.

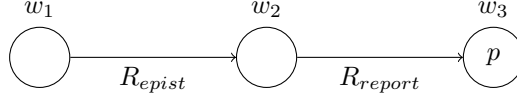


Figure 1: Recursive shifting for indirect reports under overt modals

This pattern does not necessarily hold for quotational reports. It does account for the non-metalinguistic reading, for which (61b) roughly means that there is an R_{epist} -accessible world in which he rules whatever he calls σ . Still, it does not yield the metalinguistic reading in which the modal extends the range of languages of quoted expression to the purely hypothetical ones.

This observation shows that overt modals taking scope over QEs, rather than involving recursive shifting as in Figure 1, affect the set of accessible models. To this extent they resemble the type of modality developed by Kratzer (2006, 2013) and Moulton (2015), where the embedding modals are predicates restricting the set of accessible worlds. Still, there is an important difference. This approach has been applied to structures with embedding attitude verbs and embedded modals like *must* or *probably* (Bogal-Allbritten 2015, 2016), resulting in the general pattern as in (62):

$$(62) \quad \llbracket [\text{att. verb} [\text{Mod}\phi]] \rrbracket = \lambda s. \lambda w. \text{mental-att.}(s)(w) \& \forall w' [w' \in \text{compatible}(s)(w) \rightarrow \phi(w')]$$

However, examples like (61b) involve a converse situation, where it is the modal *might* that takes scope over the reportative modal. Since modals like *might* can hardly be interpreted on a par with attitude verbs, the same approach cannot be applied. The question that arises, then, is how such modals can be interpreted in a way securing the kind of effect given in (62).

An all-important hint comes from the effect of hybrid comparatives as in (30), repeated below:

- (63) It is more likely that Ojin ruled ‘Yamatai’ than that he was born in the 4th c.
 \rightsquigarrow it is more likely that for c : Ojin ruled c , Ojin called c *Yamatai* than that he was born in the 4th c.

As noted before, the Kratzerian account of better possibility (Katz et al. 2012) runs into trouble. (63) would roughly be interpreted as being true iff some \mathcal{R} -accessible model \mathcal{M}_1 such that Ojin ruled what is defined in \mathcal{M}_1 as *Yamatai* is more highly ranked than every R -accessible world w_1 such that Ojin was born in the 4th c. in w_1 . However, such a comparison is both technically and conceptually problematic. Worlds and models are ordered by two different accessibility relations and thus cannot be compared with each other. The conceptual reason is that the sense of a possible world being more highly ranked than a possible of language used to describe worlds is at least unclear. By contrast, it does make sense to compare two probability values. (63) is perfectly intelligible if it is roughly interpreted as true iff the probability that, for c such that Ojin ruled c , Ojin called c *Yamatai* is higher than the probability that he was born in the 4th c. Note that apart from avoiding the conceptual complication, it is coherent with the observation made for (61b), i.e. that modals taking scope over QEs do not involve recursive shifting. If such modals provide probabilities, and not another shift, then the lack of recursive shifting is natural. The problem is how to derive probabilities from the two different qualitative orderings.

The solution I propose involves a so called hybrid/blended scalar-quantificational approach to modality, whose various versions have been gaining growing attention (Herburger & Rubinstein 2019; Klecha 2014; Swanson 2016, a.o.). In order to secure comparison between outputs of metalinguistic and non-metalinguistic type of modality, I let (at least some) modals be interpreted as items providing measure functions (Lassiter 2017). Such measure functions $\mu : \{p\} \rightarrow [0, 1]$ take propositions as arguments and return numbers within the interval $[0, 1]$ in \mathbb{R} corresponding to probability values. Probabilities are derived from qualitative orderings $\langle W, \succ_{\mathbf{g}} \rangle, \langle \mathcal{M}, \succ_{\mathbf{g}'} \rangle$ over worlds or models, defined as follows:

- (64) for $w_1, w_2 \in W$ and $\mathcal{M}_i, \mathcal{M}_j \in \mathcal{M}$, orderings $\langle W, \succ_{\mathbf{g}} \rangle$ and $\langle \mathcal{M}, \succ_{\mathbf{g}'} \rangle$, a proposition p and a quotational expression ‘ σ ’ occurring in the sentence representing p :

$$\begin{aligned} w_1 \succ_{\mathbf{g}} w_2 & \quad \text{iff the chance that } p \text{ holds in } w_1 \text{ is better than the chance that it holds in } w_2 \\ \mathcal{M}_1 \succ_{\mathbf{g}'} \mathcal{M}_2 & \quad \text{iff the chance that the quoted expression as defined in } \mathcal{M}_1 \text{ for } w_{1,v(i)} \\ & \quad \text{has the utterance form demonstrated by the form } \lceil \sigma \rceil \text{ is better than} \\ & \quad \text{the chance that the quoted expression has this form as defined in } \mathcal{M}_2 \text{ for } w_{2,v(i)} \end{aligned}$$

To illustrate with toy examples, the chance that a man drove a car in a world where cars are common is better than the chance that a man drove a car in a world where cars are rare. In a similar vein, the chance

that the name of Japan was *Yamatai* is better for a model of the 3rd century Japanese, when this word was common, than for a model of the 4th century Japanese, when the term was gradually disappearing. In this sense, functions yielding probability values have two, i.e. metalinguistic and non-metalinguistic, variants, resembling the comparative MORE as proposed by Giannakidou & Yoon (2011).

Let us now have a look at how this framework computes QEs appearing within the scope of modals. Let us start from simple cases as in (65). First of all, I let the *RES* function take as argument the whole sentence, as in (66):

(65) Ojin might have ruled ‘Yamatai’.

(66) $[RES_{best} [S \dots [Mod^0 \dots]]]$

This approach has very concrete advantages. Providing the structure and general semantics typical for modals, it saves a lot of computational burden. First, the preajcent *Ojin ruled ‘Yamatai’* is derived as shown in subsection 4.1, with the output as in (51) except the *RES* function. As discussed in the context of (49), *RES* searches through the surrounding context for candidates counting as the relevant possible worlds or models. In the case of bare QEs, *RES* selects models of those idiolects that generate expressions whose utterances belong to the domain of $w_{\textcircled{a}}$. Modals taking scope over QEs extend the domain of quantification to possible models of the idiolect at hand. In the present framework this means that *RES* merged over modals selects best possible models such that counterparts of utterances of expressions generated by the corresponding idiolects belong to the domain of $w_{\textcircled{a}}$. Put differently, such utterances are possible utterances as judged from the point of view of idiolects by which they are generated. Best models, delivered by r_{best} , are in turn selected as those being sufficiently highly ranked relative to the ordering $\succ_{\mathbf{g}'}$. The overt modal *might* is then interpreted as a function taking as arguments propositions and returning inequalities. These relate the value of measure function $\mu_{might}(p)$ to the contextually salient threshold value θ (Lassiter 2017):¹⁵

(67) $\llbracket \text{Ojin might have ruled ‘Yamatai’} \rrbracket^{\mathcal{M}, w_{\textcircled{a}}}$:

$$\begin{aligned} & \llbracket \text{might} \rrbracket^{\mathcal{M}, w_{\textcircled{a}}, v} (\llbracket \text{Ojin has ruled ‘Yamatai’} \rrbracket^{\mathcal{M}, w_{\textcircled{a}}, v}) = & \text{[by Lassiter 2017]} \\ & \lambda p_{\langle st \rangle} \cdot \lambda e'_s \cdot [\mu_{might}(p) > \theta \text{ in } e'] (Ojin \text{ ruled ‘Yamatai’}) & \text{[by FA]} \\ & \lambda e'_s \cdot [\mu_{might}(Ojin \text{ ruled ‘Yamatai’}) > \theta \text{ in } e'] \\ & \llbracket RES_{best} \rrbracket^{\mathcal{M}, w_{\textcircled{a}}} (\lambda e'_s \cdot [\mu_{might}(Ojin \text{ ruled ‘Yamatai’}) > \theta \text{ in } e']) = & \text{[by FA]} \\ & \lambda e'_s \cdot [\mu_{might}(Ojin \text{ ruled ‘Yamatai’}) > \theta \text{ in } e'] = & \text{[by (64)]} \end{aligned}$$

1 iff for $c : \langle \text{Ojin}, c, e \rangle \in \llbracket \text{ruled} \rrbracket^{\mathcal{M}, w_{\textcircled{a}}, v}$, the probability that for all models $\mathcal{M}_{Ojin} \in r_{best}(\mathcal{R}(\mathcal{M}))$, c as defined in \mathcal{M}_{Ojin} for $w_{Ojin, v(i)}$ has the utterance form demonstrated by the form ‘Yamatai’ is higher in e' than θ

Let us pause for a moment and check the path leading to the output of (67). First, the set of all models is ordered by $\succ_{\mathbf{g}'}$ relative to the content of (65). Basing on this ordering, the modal *might* delivers probability of the land at hand being called in these models *Yamatai*, as given in (64). The probability is next stated as being higher than the threshold value θ . This value is made salient by the ordering $\succ_{\mathbf{g}'}$, the lexical properties of *might* and the context. Finally, the function r_{best} selects the best candidates for models of Ojin’s idiolect where the land he ruled was called *Yamatai*.

Importantly, the proposed framework computes modal comparatives making use of the same mechanism. An important difference lies in the account of better possibility. According to Katz et al. (2012), the notion assumes comparison between some p -worlds and all p' -worlds, thus involving both existential and universal quantification. However, the Hintikkian approach assumed by Kratzer (2013, 2016) takes reported speech to involve only universal quantification. I solve this problem by means of r_{best} function. If $\mathcal{M}_y, \mathcal{M}_n$ are possible models of Ojin idiolect, then all (and at least one) $\succ_{\mathbf{g}'}$ -best models \mathcal{M}_y are more highly ranked than all $\succ_{\mathbf{g}'}$ -best models \mathcal{M}_n . Assuming the abbreviations $\phi_{\mathcal{M}_y}$ and $\psi_{\mathcal{M}_n}$ of the two clauses, the computation proceeds as in (69):¹⁶

(68) It is more likely that Ojin ruled ‘Yamatai’ than that he ruled ‘Nihon’.

\rightsquigarrow for $c : \text{Ojin ruled } c$, it is more likely that he called c *Yamatai* than that he called c *Nihon*

¹⁵For a complete computation, see 6.3, example (71b).

¹⁶For a complete computation, see subsection 6.3, example (71c).

(69) $\llbracket \text{It is more likely that } [\phi_{\mathcal{M}_y} \text{ Ojin ruled 'Yamatai'}] \text{ than that } [\psi_{\mathcal{M}_n} \text{ he ruled 'Yamatai'}] \rrbracket^{\mathcal{M}, w_{\otimes}, v} :$

$$\begin{aligned}
& \llbracket \text{it is more likely that } \rrbracket^{\mathcal{M}, w_{\otimes}, v} (\llbracket \phi_{\mathcal{M}_y} \rrbracket^{\mathcal{M}, w_{\otimes}, v}) = & [\text{by Lassiter 2017}] \\
& \lambda p'_{\langle s, t \rangle} \cdot \lambda p'_{\langle s, t \rangle} \cdot \lambda e'_s \cdot [\mu_{\text{likely}}(p) > \mu_{\text{likely}}(p') \text{ in } e'](\phi_{\mathcal{M}_y}) = & [\text{by FA}] \\
& \lambda p'_{\langle s, t \rangle} \cdot \lambda e'_s \cdot [\mu_{\text{likely}}(\phi_{\mathcal{M}_y}) > \mu_{\text{likely}}(p') \text{ in } e'] \\
& \llbracket \text{it is more likely that } \phi_{\mathcal{M}_y} \text{ than that } \psi_{\mathcal{M}_n} \rrbracket^{\mathcal{M}, w_{\otimes}, v} = & [\text{by Lassiter 2017}] \\
& \lambda p'_{\langle s, t \rangle} \cdot \lambda e'_s \cdot [\mu_{\text{likely}}(\phi_{\mathcal{M}_y}) > \mu_{\text{likely}}(p') \text{ in } e'](\psi_{\mathcal{M}_n}) = & [\text{by FA}] \\
& \lambda e'_s \cdot [\mu_{\text{likely}}(\phi_{\mathcal{M}_y}) > \mu_{\text{likely}}(\psi_{\mathcal{M}_n}) \text{ in } e'] \\
& \llbracket RES_{\text{best}} \rrbracket^{\mathcal{M}, w_{\otimes}} (\lambda e'_s \cdot [\mu_{\text{likely}}(\phi_{\mathcal{M}_y}) > \mu_{\text{likely}}(\psi_{\mathcal{M}_n}) \text{ in } e']) = & [\text{by FA}] \\
& \lambda e'_s \cdot [\mu_{\text{likely}}(\phi_{\mathcal{M}_y}) > \mu_{\text{likely}}(\psi_{\mathcal{M}_n}) \text{ in } e'] = & [\text{by (64)}]
\end{aligned}$$

1 iff for $c : \langle \text{Ojin}, c, e \rangle \in \llbracket \text{ruled} \rrbracket^{\mathcal{M}, w_{\otimes}, v}$, the probability that for all models $\mathcal{M}_y \in r_{\text{best}}(\mathcal{R}(\mathcal{M}))$, c as defined in \mathcal{M}_y for $w_{y, v(i)}$ has the utterance form demonstrated by the form $\ulcorner \text{Yamatai} \urcorner$ is higher in e' than the probability that
for $c : \langle \text{Ojin}, c, e \rangle \in \llbracket \text{ruled} \rrbracket^{\mathcal{M}, w_{\otimes}, v}$ for all models $\mathcal{M}_n \in r_{\text{best}}(\mathcal{R}(\mathcal{M}))$,
 c as defined in \mathcal{M}_n for $w_{n, v(i)}$ has the utterance form demonstrated by the form $\ulcorner \text{Nihon} \urcorner$

The computation is similar to the one in (67). The difference is that rather than relating one probability to the contextually salient threshold, it relates two probabilities derived from the ordering $\succ_{\mathbf{g}'}$ over possible models of Ojin idiolect. Moreover, the assignment function v over world indices provides a way of accounting for the effect discussed in the context of (26) in subsection 2.2. Recall that a pragmatically preferred reading is the one where in the two scenarios undergoing comparison, Ojin ruled the same area. This is captured by a discourse-level condition saying that $w_{y, v(i)} =_v w_{n, v(i)}$.

Finally, the last problem to be solved is the hybrid comparison as in (70). While the general mechanism remains the same as in (69), the new aspect is the qualitative ordering serving as the point of departure for deriving the relevant probability values. In the case of hybrid comparatives, the two probabilities are derived from different orderings $\succ_{\mathbf{g}'}$ and $\succ_{\mathbf{g}}$, defined for models and worlds, respectively. As for the first ordering, it must be assumed that there is a threshold such that for all $\succ_{\mathbf{g}'}$ -best models of Ojin's idiolect, the name of Japan as defined in those models has the utterance form demonstrated by the form *Yamatai*. As for the second ordering, it must be assumed that there is a threshold such that in all $\succ_{\mathbf{g}}$ -best worlds Ojin was born in the 4th century. Given this, the comparative straightforwardly relates probabilities derived from the two orderings:¹⁷

(70) $\llbracket \text{It is more likely that } [\phi_{\mathcal{M}_y} \text{ Ojin ruled 'Yamatai'}] \text{ than that } [\psi_{w_4} \text{ he was born in the 4}^{th} \text{ c}] \rrbracket :$

$$\begin{aligned}
& \llbracket \text{it is more likely that } \rrbracket^{\mathcal{M}, w_{\otimes}} (\llbracket \phi_{\mathcal{M}_y} \rrbracket^{\mathcal{M}, w_{\otimes}, v}) = & [\text{by Lassiter 2017, (64)}] \\
& \lambda p'_{\langle s, t \rangle} \cdot \lambda p'_{\langle s, t \rangle} \cdot \lambda e''_s \cdot [\mu_{\text{likely}}(p) > \mu_{\text{likely}}(p') \text{ in } e''](\phi_{\mathcal{M}_y}) = & [\text{by FA in } e''] \\
& \lambda p'_{\langle s, t \rangle} \cdot \lambda e''_s \cdot [\mu_{\text{likely}}(\phi_{\mathcal{M}_y}) > \mu_{\text{likely}}(p') \text{ in } e''] \\
& \llbracket \text{it is more likely that } \phi_{\mathcal{M}_y} \text{ than that } \psi_{w_4} \rrbracket^{\mathcal{M}, w_{\otimes}, v} = & [\text{by Lassiter 2017, (64)}] \\
& \lambda p'_{\langle s, t \rangle} \cdot \lambda e''_s \cdot [\mu_{\text{likely}}(\phi_{\mathcal{M}_y}) > \mu_{\text{likely}}(p') \text{ in } e''](\psi_{w_4}) = & [\text{by FA}] \\
& \lambda e''_s \cdot \mu_{\text{likely}}(\phi_{\mathcal{M}_y}) > \mu_{\text{likely}}(\psi_{w_4}) \text{ in } e'' \\
& \llbracket RES_{\text{best}} \rrbracket^{\mathcal{M}, w_{\otimes}} (\lambda e''_s \cdot \mu_{\text{likely}}(\phi_{\mathcal{M}_y}) > \mu_{\text{likely}}(\psi_{w_4}) \text{ in } e'') = & [\text{by FA}] \\
& \lambda e''_s \cdot \mu_{\text{likely}}(\phi_{\mathcal{M}_y}) > \mu_{\text{likely}}(\psi_{w_4}) \text{ in } e''
\end{aligned}$$

= 1 iff for $c : \langle \text{Ojin}, c, e \rangle \in \llbracket \text{ruled} \rrbracket^{\mathcal{M}, w_{\otimes}, v}$, the probability that for all models $\mathcal{M}_y \in r_{\text{best}}(\mathcal{R}(\mathcal{M}))$, c as defined in \mathcal{M}_y for $w_{y, v(i)}$ has the form demonstrated by the form $\ulcorner \text{Yamatai} \urcorner$ is higher in e'' than the probability that
for all worlds $w_4 \in r_{\text{best}}(\mathcal{R}(w_{\otimes}))$, in w_4 Ojin was born in the 4th c. in e'

Accordingly, hybrid modal comparatives are derived from two distinct orderings. Though they give rise to even more serious problems than comparatives of deviation, they can be naturally accounted for in terms of two probability values, rather than qualitative orderings alone.

¹⁷For a complete computation, see subsection 6.3, example (71d).

4.3 Interim summary No. 3

In this section I showed how the modal framework, whose conceptual grounds were laid out in section 3, captures QEs appearing with and without overt modals. The present approach extends the accounts proposed by Ginzburg & Cooper (2014), Maier (2014a), and Shan (2010) by means of modal tools combining those worked out by Kratzer (2013, 2016) and Lassiter (2017). By doing so it improves two aspects of the semantics of quotation. First, it captures novel data with QEs providing quantification over possible idiolects, rather than just assuming contextually salient quoted sources. Second, it specifies and deepens the present understanding of the nature of shifting items of metalinguistic dimension like languages. Rather than taking it as a special mechanism, the present model theoretic framework shows that, together with independently motivated measure functions, it is part of more general modal machinery. Moreover, the fact that parts of such a complex modality interact with each other in the form of hybrid comparatives proves that the system is internally coherent.

5 Conclusion and future prospects

In this paper I showed so far not discussed data from modals taking scope over QEs. Such structures, at least under one reading, provide possibilities concerning the metalinguistic dimension, i.e. ways not a world, but a language used to describe worlds might be. Moreover, this reading has its markers, like Japanese *to iu*, showing that it is straightforwardly encoded in the grammar. These data contribute to the present understanding of quotation in two ways. First, they prove correct the idea of encoding into semantics of QEs elements relative to which the quoted expression is interpreted, i.e. the quoted speaker (Maier 2014a), the language (Shan 2010) or the grammatical resource (Ginzburg & Cooper 2014) of quoted expressions. Second, modal contexts involving a full-scale shifting and quantification over such elements indicate their right place in the formal system. Putting these two together, the data show that the mechanism underlying the effect of quotation is essentially modal. This line of reasoning is in harmony with Kratzer’s (2013; 2016) approach to reported speech, where indirect reports instantiate a type of modality. I extended the existing theories of quotation by proposing a modal framework where quotes are operators shifting the actual model of interpretation.

The contribution of present proposal is of both empirical and methodological character. First, it accounts for old data with simple QEs (captured by existing theories) and with plural/group quoted sources (problematic for existing theories), as well as for new data with overt modals (exceeding existing theories). Second, it semanticizes the metalinguistic discourse by showing that QEs are computed within a general modal framework, and thus do not require dedicated mechanisms. In this sense it pushes one step further a claim made by Giannakidou & Yoon (2011) and saying that metalinguistic phenomena are not isolated effects lying outside the grammar *proper* (see also Ginzburg & Cooper 2014). Both quotational and indirect instantiations of reported speech are part of a more general category of modality.

Apart from semantics of quotation, the paper provides fresh observations concerning modality and opens up new paths for future inquiry. First, it shows that the blended scalar-quantificational version of modality is not only more flexible than any of the two competing theories, but also empirically supported. It solves the problem of modal embedding which does not involve recursive shifting, and of hybrid comparatives which relate the metalinguistic and non-metalinguistic scales. Second, given the essentially modal account of metalinguistic comparatives proposed by Giannakidou & Yoon (2011), it invites questions concerning the extent to which metalinguistic effects in general are driven by modal mechanisms. In particular, it provides a precise framework for asking whether different types of modality explain and categorize cross-linguistically different metalinguistic effects (Greenberg 2017; Shimamura 2018) and to what extent these belong to linguistic universals discovered for modals.

6 Appendix: formal system and computations

6.1 Outline

I propose a modal framework for quotation based on the following basic ideas:

- i. model theoretic account of modality (Fitting & Mendelsohn 1998)
- ii. enriched modality with operators shifting models of interpretation (Lindström 2006)
- iii. hybrid scalar-quantificational modal tools (Herburger & Rubinstein 2019)

- iv. general idea of quotation demonstrating the form of the quoted expression (Davidson 2015)
- v. concept of I-languages (Chomsky 1986)

6.2 Formal system

A fragment of formal system allowing computations as in (6.3) is defined as follows:

i. Let $F = \langle W, R, D, \mathcal{D} \rangle$ be a frame such that:

- W is a set of worlds;
- R is an accessibility relation such that for $w, w' \in W$, $w' \in R(w)$ iff w' is R -accessible from w ;
- D is a set of objects (a domain); D contains at least individuals (type e), utterances (type u), situations (type s) and truth values (type $t = \{0, 1\}$);
- $\mathcal{D} : W \rightarrow \wp(D)$ is a function from worlds to sets of objects existing in those worlds, so that $\mathcal{D}(w)$ is the set of objects existing in w .

ii. Let an I-language \mathcal{L}_i be a generative procedure of syntactic derivation. Then, for the interpretation function $\llbracket \cdot \rrbracket$, $\mathcal{M}_i = \langle F, \llbracket \cdot \rrbracket \rangle$ is a model of interpretation for \mathcal{L}_i . Assuming an assignment function $g : \text{variables} \rightarrow D$, the interpretation function $\llbracket \cdot \rrbracket^{\mathcal{M}, w, g}$ is defined as follows:

$$\begin{aligned} \llbracket x \rrbracket^{\mathcal{M}, w, g} &= g(x) : g(x) \in \mathcal{D}(w) && \text{iff } x \text{ is a variable} \\ \llbracket c \rrbracket^{\mathcal{M}, w, g} &= \llbracket c \rrbracket^{\mathcal{M}, w, g} : \llbracket c \rrbracket^{\mathcal{M}, w, g} \in \mathcal{D}(w) && \text{iff } c \text{ is a constant} \\ \llbracket f \rrbracket^{\mathcal{M}, w, g} &= \lambda y. \dots \lambda x. f(x, \dots, y) && \text{iff } f \text{ is a predicate} \end{aligned}$$

iii. If t_i is a variable, a constant or a predicate and \mathcal{M} is a model, then truth in \mathcal{M} is defined as follows:

$$\begin{aligned} \llbracket f(t_1, \dots, t_n) \rrbracket^{\mathcal{M}, w, g} &= 1 && \text{iff } \langle \llbracket t_1 \rrbracket^{\mathcal{M}, w, g}, \dots, \llbracket t_n \rrbracket^{\mathcal{M}, w, g} \rangle \in \llbracket f \rrbracket^{\mathcal{M}, w, g} \\ \llbracket \neg f(t_1, \dots, t_n) \rrbracket^{\mathcal{M}, w, g} &= 1 && \text{iff } \langle \llbracket t_1 \rrbracket^{\mathcal{M}, w, g}, \dots, \llbracket t_n \rrbracket^{\mathcal{M}, w, g} \rangle \notin \llbracket f \rrbracket^{\mathcal{M}, w, g} \\ \llbracket f(t_1, \dots, t_n) \wedge f'(t'_1, \dots, t'_n) \rrbracket^{\mathcal{M}, w, g} &= 1 && \text{iff } \llbracket f(t_1, \dots, t_n) \rrbracket^{\mathcal{M}, w, g} = 1 \text{ and } \\ &&& \llbracket f'(t'_1, \dots, t'_n) \rrbracket^{\mathcal{M}, w, g} = 1 \\ \Diamond \llbracket f(t_1, \dots, t_n) \rrbracket^{\mathcal{M}, w, g} &= && \\ \Diamond \lambda y. \dots \lambda x. f(x, \dots, y) (\llbracket t_1 \rrbracket^{\mathcal{M}, w', g}, \dots, \llbracket t_n \rrbracket^{\mathcal{M}, w', g}) &&& \text{iff } \exists w' (w' \in R(w) \ \& \\ = 1 &&& \langle \llbracket t_1 \rrbracket^{\mathcal{M}, w', g}, \dots, \llbracket t_n \rrbracket^{\mathcal{M}, w', g} \rangle \in \llbracket f \rrbracket^{\mathcal{M}, w', g}) \\ \Box \llbracket f(t_1, \dots, t_n) \rrbracket_i^{\mathcal{M}, w, g} &= \neg \Diamond \llbracket \neg f(t_1, \dots, t_n) \rrbracket^{\mathcal{M}, w, g} \end{aligned}$$

iv. Let $M = \{\mathcal{M}_1, \dots, \mathcal{M}_n\}$ be a set of all models of interpretation for I-languages such that if $\mathcal{M}_i \in M$, $1 \leq i \leq n$, then $\mathcal{M}_i = \langle F, \llbracket \cdot \rrbracket_i \rangle$. Then there is a conventional function $f_c : M' \mapsto \mathcal{M}_c$ such that:

$$\begin{aligned} M' &\subseteq M \text{ and} \\ M' &= \{\mathcal{M}_1, \dots, \mathcal{M}_k\}, \text{ such that } \mathcal{M}_i \in M', 1 \leq i \leq k, \text{ and} \\ \mathcal{M}_c &= \langle F, \llbracket \cdot \rrbracket_c \mid \llbracket \cdot \rrbracket_c = \bigcap_{i=1}^k \llbracket \cdot \rrbracket_i \rangle \end{aligned}$$

\mathcal{M}_c is a model of interpretation for a generalization over I-languages.

v. Let the triple $\langle \mathcal{M}, \mathcal{F}_c, \mathcal{R} \rangle$ be defined as follows::

- \mathcal{M} is a set of models such that every $\mathcal{M}_i \in \mathcal{M}$ is a model of interpretation either for an I-language or a generalization over I-languages;
- \mathcal{F}_c is a set of conventional functions f_c such that for every $\mathcal{M}_c \in \mathcal{M}$ there is an $f_c \in \mathcal{F}_c$;

- \mathcal{R} is an accessibility relation such that if v is an assignment function such that $w_{v(i)} =_v w \in W$ and $w_{1,v(i)}$ is a world of \mathcal{M}_1 , then for $\mathcal{M}, \mathcal{M}_1 \in \mathcal{M}$, $\mathcal{M}_1 \in \mathcal{R}(\mathcal{M})$ iff every expression E defined in \mathcal{M}_1 for $w_{1,v(i)}$ is defined for the quotational context as ‘ E ’ in \mathcal{M} , where ‘ E ’ demonstrates E .
- vi. If ‘ \dots ’ stands for any conventional exponent of quotation, then $\llbracket \dots \rrbracket^{\mathcal{M}, w_{\otimes}} = \mathbf{Q}$ is a modal operator. If \mathcal{M} is the actual model of interpretation, then the interpretation function is defined as in [ii.](#) and as follows:
- $$\begin{aligned} \mathbf{Q}\llbracket x \rrbracket^{\mathcal{M}, w_{\otimes}} &= g'(x) & \text{s.t. } \forall \mathcal{M}_1 (\mathcal{M}_1 \in \mathcal{R}(\mathcal{M}) \rightarrow g'(x) &= \llbracket c \rrbracket^{\mathcal{M}_1, w_{1,v(i)}, g'}) \\ & & \text{where } c \text{ as defined in } \mathcal{M}_1 \text{ for } w_{1,v(i)} \text{ has the utterance form} \\ & & \text{demonstrated by the form } \ulcorner c \urcorner \\ \mathbf{Q}\llbracket c \rrbracket^{\mathcal{M}, w_{\otimes}} &= \llbracket c \rrbracket^{\mathcal{M}_1, w_{1,v(i)}} & \text{s.t. } \forall \mathcal{M}_1 (\mathcal{M}_1 \in \mathcal{R}(\mathcal{M}) \rightarrow c \text{ as defined in } \mathcal{M}_1 \text{ for } w_{1,v(i)} \\ & & \text{has the utterance form demonstrated by the form } \ulcorner c \urcorner) \\ \mathbf{Q}\llbracket f \rrbracket^{\mathcal{M}, w_{\otimes}} &= \mathbf{Q}\lambda y. \dots \lambda x. f(x, \dots, y) & \text{s.t. } \forall \mathcal{M}_1 (\mathcal{M}_1 \in \mathcal{R}(\mathcal{M}) \rightarrow f(x, \dots, y) \text{ as defined in } \mathcal{M}_1 \text{ for } w_{1,v(i)} \\ & & \text{has the utterance form demonstrated by the form } \ulcorner f \urcorner) \end{aligned}$$
- vii. If ϕ of type $\langle st \rangle$ is a proposition, then truth in \mathcal{M} is defined as in [iii.](#) and as follows:
- $$\begin{aligned} \mathbf{Q}\llbracket \phi \rrbracket^{\mathcal{M}, w_{\otimes}} &= 1 & \text{iff } \forall \mathcal{M}_1 (\mathcal{M}_1 \in \mathcal{R}(\mathcal{M}) \rightarrow \llbracket \phi \rrbracket^{\mathcal{M}_1, w_{1,v(i)}, v} = 1) \\ & & \text{where } \phi \text{ as defined in } \mathcal{M}_1 \text{ for } w_{1,v(i)} \text{ has the utterance form demonstrated} \\ & & \text{by the form } \ulcorner \phi \urcorner, \\ \neg \mathbf{Q}\llbracket \phi \rrbracket^{\mathcal{M}, w_{\otimes}} &= 1 & \text{iff } \forall \mathcal{M}_1 (\mathcal{M}_1 \in \mathcal{R}(\mathcal{M}) \rightarrow \llbracket \phi \rrbracket^{\mathcal{M}_1, w_{1,v(i)}, v} = 0) \\ & & \text{where } \phi \text{ as defined in } \mathcal{M}_1 \text{ for } w_{1,v(i)} \text{ has the utterance form demonstrated} \\ & & \text{by the form } \ulcorner \phi \urcorner, \end{aligned}$$
- viii. If f is an accessibility relation and ϕ is a proposition such that $\llbracket \phi \rrbracket = \exists x / \forall x (f(x) \dots)$, then RES is a function of type $\langle st, st \rangle$ restricting f , so that $\llbracket RES \rrbracket(\llbracket \phi \rrbracket) = \exists x / \forall x (r(f(x)) \dots)$. The set $r(f(x))$ contains all and only those accessible objects that are compatible with properties of contextually salient candidates.
- ix. There is a set $\{\mathbf{g}, \mathbf{g}', \dots\}$ of ordering relations over worlds and over models such that the following pairs $\langle W, \succ_{\mathbf{g}} \rangle, \langle \mathcal{M}, \succ_{\mathbf{g}'} \rangle, \dots$ are defined.
- x. There is a set of measure functions $\{\mu_i : \{\phi_k : \langle st \rangle\} \rightarrow [0, 1]\}$ such that the following is true:
- $[0, 1]$ is an interval in \mathbb{R} corresponding to the probability scale;
 - every μ_i preserves the relevant ordering defined in [ix.](#).

The image of μ_i is a set defined as follows:

- $\mu(RES(\Diamond/\Box\phi))$ is the probability that for some/all $w_i \in r_{best}(R(w))$, ϕ holds in w_i , if μ preserves the ordering $\succ_{\mathbf{g}} : \langle W, \succ_{\mathbf{g}} \rangle$;
- $\mu(RES(\mathbf{Q}\phi))$ is the probability that for all $\mathcal{M}_1 \in r_{best}(\mathcal{R}(\mathcal{M}))$ the quoted expression as defined in \mathcal{M}_1 for $w_{1,v(i)}$ has the utterance form demonstrated by the form $\ulcorner \sigma \urcorner$, if μ preserves the ordering $\succ_{\mathbf{g}'} : \langle \mathcal{M}, \succ_{\mathbf{g}'} \rangle$ and the quoted expression is picked out by the quotation ‘ σ ’ occurring in the sentence representing ϕ .

6.3 Sample computations

I present four basic computations for a simple mixed quotation, mixed quotation with an overt modal, a modal comparative with mixed quotation and a hybrid modal comparative, as in [\(71\)](#):

- (71) a. Ojin ruled ‘Yamatai’.
b. Ojin might have ruled ‘Yamatai’.
c. It is more likely that Ojin ruled ‘Yamatai’ than that he ruled ‘Nihon’

d. It is more likely that Ojin ruled ‘Yamatai’ than that he was born in the 4th century.

For all these examples I assume the standard syntactic structure of modals (Hacquard 2006) with the *RES* function taking as its argument the whole sentence, i.e.:

$$[RES [S \dots [Mod^0 \dots]]]$$

Then the examples in (71) are computed as shown below.

(71a) Bare mixed quotation. The interpretation of bare mixed quotation, repeated below in (72), takes the quoted expression to be interpreted relative to the particular model of Ojin’s I-language.

(72) Ojin ruled ‘Yamatai’.

\rightsquigarrow Ojin ruled c : in his I-language Ojin referred to c with the word *Yamatai*

Basic terms:

$$\llbracket \text{Ojin} \rrbracket^{\mathcal{M}, w_{\textcircled{a}}} = \text{Ojin} : \text{Ojin} \in \mathcal{D}_e(w_{\textcircled{a}}) \quad [\text{by ii.}]$$

$$\llbracket \text{ruled} \rrbracket^{\mathcal{M}, w_{\textcircled{a}}} = \lambda y_e. \lambda x_e. \lambda e_s. [\text{ruled}(x, y, e)] \quad [\text{by ii.}]$$

$$\llbracket \text{‘Yamatai’} \rrbracket^{\mathcal{M}, w_{\textcircled{a}}, v} = \quad [\text{by vi.}]$$

$c : \forall \mathcal{M}_{Ojin} [\mathcal{M}_{Ojin} \in \mathcal{R}(\mathcal{M}) \rightarrow c \text{ as defined in } \mathcal{M}_{Ojin} \text{ for } w_{Ojin, v(i)} \text{ has the utterance form demonstrated by the form ‘Yamatai’}]$

$$\llbracket RES \rrbracket^{\mathcal{M}, w_{\textcircled{a}}} = RES : \phi \mapsto \phi' \quad [\text{by viii.}]$$

where ϕ' is just like ϕ except that the accessibility relation $\mathcal{R}(\mathcal{M})$ occurring in $\llbracket \phi \rrbracket$

is restricted to the singleton $r_{\textcircled{a}}(\mathcal{R}(\mathcal{M})) = \{\mathcal{M}_{Ojin}\}$

s.t. utterances of expressions defined in \mathcal{M}_{Ojin} are elements of $\mathcal{D}(w_{\textcircled{a}})$

Computation:

$$\begin{aligned} \llbracket \text{ruled} \rrbracket^{\mathcal{M}, w_{\textcircled{a}}} (\llbracket \text{‘Yamatai’} \rrbracket^{\mathcal{M}, w_{\textcircled{a}}, v}) &= \\ \lambda y_e. \lambda x_e. \lambda e_s. [\text{ruled}(x, y, e)](c) &= \end{aligned} \quad [\text{by FA}]$$

$$\begin{aligned} \lambda x_e. \lambda e_s. [\text{ruled}(x, c, e)] &= \\ \llbracket \text{ruled ‘Yamatai’} \rrbracket^{\mathcal{M}, w_{\textcircled{a}}, v} (\llbracket \text{Ojin} \rrbracket^{\mathcal{M}, w_{\textcircled{a}}}) &= \\ \lambda x_e. \lambda e_s. [\text{ruled}(x, c, e)](\text{Ojin}) &= \end{aligned} \quad [\text{by FA}]$$

$$\llbracket RES \rrbracket^{\mathcal{M}, w_{\textcircled{a}}} (\llbracket \text{Ojin ruled ‘Yamatai’} \rrbracket^{\mathcal{M}, w_{\textcircled{a}}, v}) = \quad [\text{by FA}]$$

$$1 \text{ iff } \forall \mathcal{M}_{Ojin} (\mathcal{M}_{Ojin} \in r_{\textcircled{a}}\mathcal{R}(\mathcal{M}) \rightarrow \langle \text{Ojin}, c, e \rangle \in \llbracket \text{ruled} \rrbracket^{\mathcal{M}, w_{\textcircled{a}}, v})$$

where c as defined in \mathcal{M}_{Ojin} for $w_{Ojin, v(i)}$ has the utterance form

demonstrated by the form ‘Yamatai’

Output:

The computation delivers a proposition where the interpretation of mixed quotation is relativized to the only selected model of Ojin’s idiolect. In this sense the modal computation is trivial w.r.t. possible models of interpretation, but not trivial w.r.t. relativising the interpretation to the one provided by the idiolect of the quoted speaker.

(71b) Simple modality. The interpretation of mixed quotation with an overt modal, repeated below in (73), takes the modal to deliver a probability value on the basis of the qualitative ordering over possible models (for the sake of simplicity, I neglect the problem of aspect).

(73) Ojin might have ruled ‘Yamatai’.

\rightsquigarrow Ojin ruled c : it is possible that in his I-language Ojin referred to c with the word *Yamatai*

Basic terms:

$$\llbracket \text{Ojin} \rrbracket^{\mathcal{M}, w_{\textcircled{a}}} = \text{Ojin} : \text{Ojin} \in \mathcal{D}_e(w_{\textcircled{a}}) \quad [\text{by ii.}]$$

$$\llbracket \text{ruled} \rrbracket^{\mathcal{M}, w_{\textcircled{a}}} = \lambda y_e. \lambda x_e. \lambda e_s. [\text{ruled}(x, y, e)] \quad [\text{by ii.}]$$

$\llbracket \text{'Yamatai'} \rrbracket^{\mathcal{M}, w_{\text{@}}, v} =$ [by vi.]

$c : \forall \mathcal{M}_{Ojin} (\mathcal{M}_{Ojin} \in \mathcal{R}(\mathcal{M}) \rightarrow c$ as defined in \mathcal{M}_{Ojin} for $w_{Ojin, v(i)}$ has the utterance form demonstrated by the form $\lceil \text{'Yamatai'} \rceil$)

$\llbracket \text{might} \rrbracket^{\mathcal{M}, w_{\text{@}}, \mathbf{g}'} =$ [by ix., x.,]

$\lambda p_{\langle s, t \rangle} . \lambda e'_s . [\mu_{\text{might}}(p) > \theta \text{ in } e']$, where $\langle \mathcal{M}, \succ_{\mathbf{g}'} \mid \mathcal{M}_1 \in \mathcal{M} \rangle$ & $\mu_{\text{might}}(p)$ preserves $\succ_{\mathbf{g}'}$ & every $\mathcal{M}_{Ojin} \in r_{\text{best}}(\mathcal{R}(\mathcal{M}))$ is $\succ_{\mathbf{g}'}$ -higher than the contextually salient threshold t & $\mu(t) = \theta$

$\llbracket \text{RES} \rrbracket^{\mathcal{M}, w_{\text{@}}} = \text{RES} : \phi \mapsto \phi'$ [by viii.]

where ϕ' is just like ϕ except that the accessibility relation $\mathcal{R}(\mathcal{M})$ occurring in $\llbracket \phi \rrbracket$ is restricted to the subset $r_{\text{best}}(\mathcal{R}(\mathcal{M})) = \{\mathcal{M}_{Ojin}, \mathcal{M}'_{Ojin}, \dots\}$ of $\succ_{\mathbf{g}'}$ -best possible candidates s.t. counterparts of utterances of expressions defined in every $\mathcal{M}_{Ojin} \in r_{\text{best}}(\mathcal{R}(\mathcal{M}))$ are elements of $\mathcal{D}(w_{\text{@}})$

Computation:

$\llbracket \text{ruled} \rrbracket^{\mathcal{M}, w_{\text{@}}} (\llbracket \text{'Yamatai'} \rrbracket^{\mathcal{M}, w_{\text{@}}, v}) =$

$\lambda y_e . \lambda x_e . \lambda e_s . [\text{ruled}(x, y, e)](c) =$ [by FA]

$\lambda x_e . \lambda e_s . [\text{ruled}(x, c, e)]$

$\llbracket \text{ruled 'Yamatai'} \rrbracket^{\mathcal{M}, w_{\text{@}}, v} (\llbracket \text{Ojin} \rrbracket^{\mathcal{M}, w_{\text{@}}}) =$

$\lambda x_e . \lambda e_s . [\text{ruled}(x, c, e)](\text{Ojin}) =$ [by FA]

$\lambda e_s . [\text{ruled}(\text{Ojin}, c, e)]$

$\llbracket \text{might} \rrbracket^{\mathcal{M}, w_{\text{@}}, \mathbf{g}'} (\llbracket \text{Ojin ruled 'Yamatai'} \rrbracket^{\mathcal{M}, w_{\text{@}}, v}) =$

$\lambda p_{\langle s, t \rangle} . \lambda e'_s . [\mu_{\text{might}}(p) > \theta \text{ in } e'](\text{Ojin ruled 'Yamatai'})$ [by FA]

$\lambda e'_s . [\mu_{\text{might}}(\text{Ojin ruled 'Yamatai'}) > \theta \text{ in } e']$

$\llbracket \text{RES} \rrbracket^{\mathcal{M}, w_{\text{@}}} (\llbracket \text{Ojin ruled 'Yamatai'} \rrbracket^{\mathcal{M}, w_{\text{@}}, v}) =$ [by FA]

1 iff for $c : \langle \text{Ojin}, c, e \rangle \in \llbracket \text{ruled} \rrbracket^{\mathcal{M}, w_{\text{@}}, v}$ the probability that for all models $\mathcal{M}_{Ojin} \in r_{\text{best}}(\mathcal{R}(\mathcal{M}))$,

c as defined in \mathcal{M}_{Ojin} for $w_{Ojin, v(i)}$ has the utterance form

demonstrated by the form $\lceil \text{'Yamatai'} \rceil$, is higher in e' than the contextually salient threshold value θ

Output:

The computation delivers the probability of the expression defined in best candidates for possible models of Ojin's idiolect having the particular form. It is derived from the qualitative ordering \mathbf{g}' over possible models.

(71c) Simple modal comparatives. The interpretation of simple modal comparatives with mixed quotation repeated below in (74) takes modals to provide two probability values derived from a qualitative ordering over models:

(74) It is more likely that Ojin ruled 'Yamatai' than that he ruled 'Nihon'.

\rightsquigarrow For $c : \text{Ojin ruled } c$, it is more likely that Ojin referred to c with the word *Yamatai* than that he referred to c with the word *Nihon*

Basic terms:

$\llbracket \text{Ojin} \rrbracket^{\mathcal{M}, w_{\text{@}}} = \text{Ojin} : \text{Ojin} \in \mathcal{D}_e(w_{\text{@}})$ [by ii.]

$\llbracket \text{ruled} \rrbracket^{\mathcal{M}, w_{\text{@}}} = \lambda y_e . \lambda x_e . \lambda e_s . [\text{ruled}(x, y, e)]$ [by ii.]

$\llbracket \text{'Yamatai'} \rrbracket^{\mathcal{M}, w_{\text{@}}, v} =$ [by vi.]

$c : \forall \mathcal{M}_y (\mathcal{M}_y \in \mathcal{R}(\mathcal{M}) \rightarrow c$ as defined in \mathcal{M}_y for $w_{y, v(i)}$ has the utterance form

demonstrated by the form $\lceil \text{'Yamatai'} \rceil$)

$\llbracket \text{'Nihon'} \rrbracket^{\mathcal{M}, w_{\text{@}}, v} =$ [by vi.]

$c : \forall \mathcal{M}_n (\mathcal{M}_n \in \mathcal{R}(\mathcal{M}) \rightarrow c$ as defined in \mathcal{M}_n for $w_{n, v(i)}$ has the utterance form

demonstrated by the form $\lceil \text{'Nihon'} \rceil$)

$\llbracket \text{he}_k \rrbracket^{\mathcal{M}, w_{\otimes}, g} = g(k) =_g \text{Ojin} : \text{Ojin} \in \mathcal{D}_e(w_{\otimes})$

[by ii.]

$\llbracket \text{it is more likely that } p \text{ than that } p' \rrbracket^{\mathcal{M}, w_{\otimes}, \mathbf{g}'} =$

[by ix., x.]

$\lambda p_{\langle s, t \rangle} \cdot \lambda p'_{\langle s, t \rangle} \cdot \lambda e'_s \cdot [\mu_{\text{likely}}(p) > \mu_{\text{likely}}(p') \text{ in } e']$

where $\langle \mathcal{M}, \succ_{\mathbf{g}'} \mid \mathcal{M}_y, \mathcal{M}_n \in \mathcal{M} \rangle$ & $\mu_{\text{likely}}(p)$ preserves $\succ_{\mathbf{g}'}$ &

$\mu_{\text{likely}}(\text{Ojin ruled 'Yamatai'}) > \theta_1$ & $\mu_{\text{likely}}(\text{Ojin ruled 'Nihon'}) > \theta_2$ &

every $\mathcal{M}_y \in r_{\text{best}}(\mathcal{R}(\mathcal{M}))$ is $\succ_{\mathbf{g}'}$ -higher than the contextually salient threshold t_1 &

every $\mathcal{M}_n \in r_{\text{best}}(\mathcal{R}(\mathcal{M}))$ is $\succ_{\mathbf{g}'}$ -higher than the contextually salient threshold t_2 &

t_i is $\succ_{\mathbf{g}'}$ -higher than t_2

$\llbracket RES \rrbracket^{\mathcal{M}, w_{\otimes}} = RES : \phi \mapsto \phi'$

[by viii.]

where ϕ' is just like ϕ except that the accessibility relation $\mathcal{R}(\mathcal{M})$ occurring in $\llbracket \phi \rrbracket$

is restricted to the subset $r_{\text{best}}(\mathcal{R}(\mathcal{M})) = \{\mathcal{M}_{\text{Ojin}}, \mathcal{M}'_{\text{Ojin}}, \dots\}$ of $\succ_{\mathbf{g}'}$ -best possible candidates

s.t. counterparts of utterances of expressions defined in every $\mathcal{M}_{\text{Ojin}} \in r_{\text{best}}(\mathcal{R}(\mathcal{M}))$

are elements of $\mathcal{D}(w_{\otimes})$

Computation:

$\llbracket \text{ruled} \rrbracket^{\mathcal{M}, w_{\otimes}} (\llbracket \text{'Nihon'} \rrbracket^{\mathcal{M}, w_{\otimes}, v}) =$

$\lambda y_e \cdot \lambda x_e \cdot \lambda e_s \cdot [\text{ruled}(x, y, e)](c) =$

[by FA]

$\lambda x_e \cdot \lambda e_s \cdot [\text{ruled}(x, c, e)]$

$\llbracket \text{ruled 'Nihon'} \rrbracket^{\mathcal{M}, w_{\otimes}, v} (\llbracket \text{he}_k \rrbracket^{\mathcal{M}, w_{\otimes}, g}) =$

$\lambda x_e \cdot \lambda e_s \cdot [\text{ruled}(x, c, e)](g(k)) =$

[by FA]

$\lambda e_s \cdot [\text{ruled}(g(k), c, e)] = \lambda e_s \cdot [\text{ruled}(\text{Ojin}, c, e)]$

[abbr. as $\psi_{\mathcal{M}_n}$]

$\llbracket \text{ruled} \rrbracket^{\mathcal{M}, w_{\otimes}} (\llbracket \text{'Yamatai'} \rrbracket^{\mathcal{M}, w_{\otimes}, v}) =$

$\lambda y_e \cdot \lambda x_e \cdot \lambda e_s \cdot [\text{ruled}(x, y, e)](c) =$

[by FA]

$\lambda x_e \cdot \lambda e_s \cdot [\text{ruled}(x, c, e)]$

$\llbracket \text{ruled 'Yamatai'} \rrbracket^{\mathcal{M}, w_{\otimes}, v} (\llbracket \text{Ojin} \rrbracket^{\mathcal{M}, w_{\otimes}}) =$

$\lambda x_e \cdot \lambda e_s \cdot [\text{ruled}(x, c, e)](\text{Ojin}) =$

[by FA]

$\lambda e_s \cdot [\text{ruled}(\text{Ojin}, c, e)]$

[abbr. as $\phi_{\mathcal{M}_y}$]

$\llbracket \text{It is more likely that } \phi_{\mathcal{M}_y} \text{ than that } p' \rrbracket^{\mathcal{M}, w_{\otimes}, \mathbf{g}'} =$

$\lambda p_{\langle s, t \rangle} \cdot \lambda p'_{\langle s, t \rangle} \cdot \lambda e'_s \cdot [\mu_{\text{likely}}(p) > \mu_{\text{likely}}(p') \text{ in } e'](\phi_{\mathcal{M}_y}) =$

[by FA]

$\lambda p'_{\langle s, t \rangle} \cdot \lambda e'_s \cdot [\mu_{\text{likely}}(\phi_{\mathcal{M}_y}) > \mu_{\text{likely}}(p') \text{ in } e']$

$\llbracket \text{It is more likely that } \phi_{\mathcal{M}_y} \text{ than that } \psi_{\mathcal{M}_n} \rrbracket^{\mathcal{M}, w_{\otimes}, \mathbf{g}'} =$

$\lambda p'_{\langle s, t \rangle} \cdot \lambda e'_s \cdot [\mu_{\text{likely}}(\phi_{\mathcal{M}_y}) > \mu_{\text{likely}}(p) \text{ in } e'](\psi_{\mathcal{M}_n}) =$

[by FA]

$\lambda e'_s \cdot [\mu_{\text{likely}}(\phi_{\mathcal{M}_y}) > \mu_{\text{likely}}(\psi_{\mathcal{M}_n}) \text{ in } e']$

$\llbracket RES \rrbracket^{\mathcal{M}, w_{\otimes}} (\lambda e'_s \cdot [\mu_{\text{likely}}(\phi_{\mathcal{M}_y}) > \mu_{\text{likely}}(\psi_{\mathcal{M}_n}) \text{ in } e']) =$

[by FA]

1 iff for $c : \langle \text{Ojin}, c, e \rangle \in \llbracket \text{ruled} \rrbracket^{\mathcal{M}, w_{\otimes}, v}$ the probability that for all models $\mathcal{M}_y \in r_{\text{best}}(\mathcal{R}(\mathcal{M}))$

c as defined in \mathcal{M}_y for $w_{y, v(i)}$ has the utterance form demonstrated by the form $\lceil \text{'Yamatai'} \rceil$

is higher in e' than the probability that

for $c : \langle \text{Ojin}, c, e \rangle \in \llbracket \text{ruled} \rrbracket^{\mathcal{M}, w_{\otimes}, v}$ for all models $\mathcal{M}_n \in r_{\text{best}}(\mathcal{R}(\mathcal{M}))$,

c as defined in \mathcal{M}_n for $w_{n, v(i)}$ has the utterance form demonstrated by the form $\lceil \text{'Nihon'} \rceil$

Discourse-level condition: $w_{y, v(i)} =_v w_{n, v(i)}$

Output:

Enriched by the discourse-level condition, the computation relates two probabilities of the particular land being called in two different ways. The relation is derived from the qualitative ordering \mathbf{g}' over models.

(71d) Hybrid modal comparatives. The interpretation of hybrid modal comparatives with mixed

quotation, repeated below in (75), takes modals to provide two probability values derived from two different qualitative orderings:

- (75) It is more likely that Ojin ruled ‘Yamatai’ than that he was born in the 4th century.
 \rightsquigarrow For c : Ojin ruled c , it is more likely that Ojin referred to c with the word *Yamatai* than that he was born in the 4th century

Basic terms:

$$\begin{aligned}
\llbracket \text{Ojin} \rrbracket^{\mathcal{M}, w_{\text{@}}} &= \text{Ojin} : \text{Ojin} \in \mathcal{D}_e(w_{\text{@}}) && [\text{by ii.}] \\
\llbracket \text{ruled} \rrbracket^{\mathcal{M}, w_{\text{@}}} &= \lambda y_e. \lambda x_e. \lambda e_s. [\text{ruled}(x, y, e)] && [\text{by ii.}] \\
\llbracket \text{‘Yamatai’} \rrbracket^{\mathcal{M}, w_{\text{@}}, v} &= && [\text{by vi.}] \\
c : \forall \mathcal{M}_y (\mathcal{M}_y \in \mathcal{R}(\mathcal{M}) \rightarrow c \text{ as defined in } \mathcal{M}_y \text{ for } w_{y, v(i)} \text{ has the utterance form} \\
&\text{demonstrated by the form } \ulcorner \text{Yamatai} \urcorner) \\
\llbracket \text{he}_k \rrbracket^{\mathcal{M}, w_{\text{@}}, g} &= g(k) =_g \text{Ojin} : \text{Ojin} \in \mathcal{D}_e(w_{\text{@}}) && [\text{by ii.}] \\
\llbracket \text{was born} \rrbracket^{\mathcal{M}, w_{\text{@}}} &= && [\text{by ii.}] \\
&\lambda x_e. \lambda e'_s. [\text{was_born}(x, e')] \\
\llbracket \text{in the 4}^{th} \text{ c.} \rrbracket^{\mathcal{M}, w_{\text{@}}} &= && [\text{by ii.}] \\
&\lambda e'_s. [4^{th}(e')] \\
\llbracket \text{it is more likely that } p \text{ than that } p' \rrbracket^{\mathcal{M}, w_{\text{@}}, \mathbf{g}, \mathbf{g}'} &= && [\text{by ix., x.}] \\
&\lambda p_{\langle s, t \rangle} \lambda p'_{\langle s, t \rangle} \lambda e''_s. [\mu_{\text{likely}}(p) > \mu_{\text{likely}}(p') \text{ in } e''] \\
&\text{where } \langle W, \succ_{\mathbf{g}} \mid w_j \in W \rangle \ \& \ \langle \mathcal{M}, \succ_{\mathbf{g}'} \mid \mathcal{M}_i \in \mathcal{M} \rangle \ \& \ \mu_{\text{likely}}(p) \text{ preserves } \succ_{\mathbf{g}'}, \succ_{\mathbf{g}} \ \& \\
&\text{every } \mathcal{M}_y \in r_{\text{best}}(\mathcal{R}(\mathcal{M})) \text{ is } \succ_{\mathbf{g}'}\text{-higher than the contextually salient threshold } t_1 \ \& \\
&\text{every } w' \in r_{\text{best}}(R(w_{\text{@}})) \text{ is } \succ_{\mathbf{g}}\text{-higher than the contextually salient threshold } t_2 \ \& \\
&\mu(t_1) > \mu(t_2) \\
\Box &\text{ the necessity operator} && [\text{by iii.}] \\
\llbracket RES \rrbracket^{\mathcal{M}, w_{\text{@}}} &= RES : \phi \mapsto \phi' && [\text{by viii.}] \\
&\text{where } \phi' \text{ is just like } \phi \text{ except that the accessibility relation } \mathcal{R}(\mathcal{M}) \text{ occurring in } \llbracket \phi \rrbracket \\
&\text{is restricted to the subset } r_{\text{best}}(\mathcal{R}(\mathcal{M})) = \{\mathcal{M}_{\text{Ojin}}, \mathcal{M}'_{\text{Ojin}}, \dots\} \text{ of } \succ_{\mathbf{g}'}\text{-best possible candidates} \\
&\text{s.t. counterparts of utterances of expressions defined in every } \mathcal{M}_{\text{Ojin}} \in r_{\text{best}}(\mathcal{R}(\mathcal{M})) \\
&\text{are elements of } \mathcal{D}(w_{\text{@}}) \ \& \\
&\text{the accessibility relation } R(w) \text{ occurring in } \llbracket \psi \rrbracket \text{ is restricted to the subset} \\
&r_{\text{best}}(R(w)) \text{ of } \succ_{\mathbf{g}}\text{-best possible candidates}
\end{aligned}$$

Computation:

$$\begin{aligned}
\llbracket \text{was born} \rrbracket^{\mathcal{M}, w_{\text{@}}} (\llbracket \text{he} \rrbracket^{\mathcal{M}, w_{\text{@}}, g}) &= \\
&\lambda x_e. \lambda e'_s. [\text{was_born}(x, e')](g(k)) = && [\text{by FA}] \\
&\lambda e'_s. [\text{was_born}(g(k), e')] \\
\llbracket \text{he}_k \text{ was born} \rrbracket^{\mathcal{M}, w_{\text{@}}} (\llbracket \text{in the 4}^{th} \text{ c.} \rrbracket^{\mathcal{M}, w_{\text{@}}, g}) &= \\
&\lambda e'_s. [\text{was_born}(g(k), e')](\lambda e'_s. [4^{th}(e')]) = && [\text{by EI}] \\
&\lambda e'_s. [\text{was_born}(g(k), e') \ \& \ 4^{th}(e')] = && [\text{by } g] \\
&\lambda e'_s. [\text{was_born}(\text{Ojin}, e') \ \& \ 4^{th}(e')] \\
\Box (\lambda e'_s. [\text{was_born}(\text{Ojin}, e') \ \& \ 4^{th}(e')]) &= && [\text{by iii.}] \\
\Box \lambda e'_s. [\text{was_born}(\text{Ojin}, e') \ \& \ 4^{th}(e')] &= && [\text{abbr. as } \psi_{w_4}] \\
\llbracket \text{ruled} \rrbracket^{\mathcal{M}, w_{\text{@}}} (\llbracket \text{‘Yamatai’} \rrbracket^{\mathcal{M}, w_{\text{@}}, v}) &= \\
&\lambda y_e. \lambda x_e. \lambda e_s. [\text{ruled}(x, y, e)](c) = && [\text{by FA}] \\
&\lambda x_e. \lambda e_s. [\text{ruled}(x, c, e)] \\
\llbracket \text{ruled ‘Yamatai’} \rrbracket^{\mathcal{M}, w_{\text{@}}, v} (\llbracket \text{Ojin} \rrbracket^{\mathcal{M}, w_{\text{@}}}) &=
\end{aligned}$$

$\lambda x_e. \lambda e_s. [\text{ruled}(x, c, e)](\text{Ojin}) =$ [by FA]
 $\lambda e_s. [\text{ruled}(\text{Ojin}, c, e)]$ [abbr. as $\phi_{\mathcal{M}_y}$]
 $\llbracket \text{It is more likely that } \phi_{\mathcal{M}_y} \text{ than that } p' \rrbracket^{\mathcal{M}, w_{\otimes}, \mathbf{g}, \mathbf{g}'} =$
 $\lambda p_{\langle s, t \rangle}. \lambda p'_{\langle s, t \rangle}. \lambda e''_s. [\mu_{\text{likely}}(p) > \mu_{\text{likely}}(p') \text{ in } e''](\phi_{\mathcal{M}_y}) =$ [by FA]
 $\lambda p'_{\langle s, t \rangle}. \lambda e''_s. [\mu_{\text{likely}}(\phi_{\mathcal{M}_y}) > \mu_{\text{likely}}(p') \text{ in } e'']$
 $\llbracket \text{It is more likely that } \phi_{\mathcal{M}_y} \text{ than that } \psi_{w_4} \rrbracket^{\mathcal{M}, w_{\otimes}, \mathbf{g}, \mathbf{g}'} =$
 $\lambda p'_{\langle s, t \rangle}. \lambda e''_s. [\mu_{\text{likely}}(\phi_{\mathcal{M}_y}) > \mu_{\text{likely}}(p') \text{ in } e''](\psi_{w_4}) =$ [by FA]
 $\lambda e''_s. [\mu_{\text{likely}}(\phi_{\mathcal{M}_y}) > \mu_{\text{likely}}(\psi_{w_4}) \text{ in } e''] =$
1 iff for $c : \langle \text{Ojin}, c, e \rangle \in \llbracket \text{ruled} \rrbracket^{\mathcal{M}, w_{\otimes}, v}$ the probability that for all models $\mathcal{M}_y \in r_{\text{best}}(\mathcal{R}(\mathcal{M}))$,
 c as defined in \mathcal{M}_y for $w_{y, v(i)}$ has the utterance form demonstrated by the form ‘Yamatai’
is higher in e'' than the probability that for all worlds $w_4 \in r_{\text{best}}(R(w_{\otimes}))$,
in w_4 Ojin was born in the 4th c. in e'

Output:

The computation provides comparison between two probability values derived from the qualitative orderings \mathbf{g} and \mathbf{g}' over models and worlds, respectively. It relates the outputs of two types of modality within a uniform dimension, i.e. mathematical probability.

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