Presupposed free choice and the theory of scalar implicatures*

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Abstract A disjunctive sentence like *Olivia took Logic or Algebra* suggests that Olivia didn't take both classes ('EXCLUSIVITY' inference) and that the speaker is ignorant as to which of the classes she took ('IGNORANCE' inference). The corresponding sentence with a possibility modal, *Olivia can take Logic or Algebra*, conveys that she can take Logic and that she can take Algebra ('FREE CHOICE' inference). These EXCLUSIVITY, IGNORANCE and FREE CHOICE inferences are argued by many to be scalar implicatures. Recent work has looked at cases in which EXCLUSIVITY and IGNORANCE appears to be computed in the presupposition, in addition to or independently from the assertion. On the basis of those data, Spector & Sudo (2017) has argued for a hybrid account based on two distinct scalar strengthening mechanisms, namely a grammatical theory of implicatures for deriving implicatures in the assertion and a pragmatic principle for deriving implicatures in the presupposition. In this paper, we observe that a sentence like *Noah is unaware that Olivia can take Logic or Algebra* has a reading on which FREE CHOICE appears in the presupposition (i.e., it suggests that Olivia can choose between the two classes), but not in the assertion (i.e., it conveys that Noah doesn't believe that she can take either one). We show that deriving this presupposed FREE CHOICE reading is challenging on Spector & Sudo's (2017) hybrid account. Following the dialectic in Fox 2007, we argue against a pragmatic approach to presupposition-based implicatures on the ground that it is not able to account for presupposed free choice. In addition, we raise a novel challenge for Spector & Sudo's (2017) system coming from the conflicting presupposed IGNORANCE triggered by sentences like *#Noah is unaware that I have a son or a daughter*, which is odd even if it's not common knowledge whether the speaker has a son or a daughter. More generally, our set of data reveals a systematic parallelism between the assertion and presuppositional levels in terms of EXCLUSIVITY, IGNORANCE, and FREE CHOICE. We argue that such parallels call for a unified analysis of those inferences at the two different levels at which they can arise, and we sketch how a grammatical theory of implicatures where meaning strengthening operates in the same way at both levels (Gajewski & Sharvit 2012, Magri 2009 and Marty 2017) can account for such parallels.

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Contents

1 Introduction

A disjunctive sentence like (1) has long been observed to give rise to two sorts of inferences: the exclusivity inference in (1-a), conveying that the speaker doesn't believe that both disjuncts are true, and the ignorance inference in (1-b), conveying that the speaker is ignorant as to which of the them is true.

(1) Olivia took Logic or Algebra.
  a. $\rightsquigarrow$ *Olivia didn't take both Logic and Algebra* EXCLUSIVITY
  b. $\rightsquigarrow$ *The speaker ignores which of the two classes she took* IGNORANCE

The corresponding sentence with a possibility modal, illustrated in (2), gives rise to another sort of inference called 'free choice.' That is, it conveys that Olivia can take Logic and that she can take Algebra, and thus that she can choose between the two (Kamp 1974 and much subsequent work). The EXCLUSIVITY, IGNORANCE and FREE CHOICE inferences above have all been argued to be scalar implicatures.[1]

(2) Olivia can take Logic or Algebra.
  $\rightsquigarrow$ *Olivia can choose between the two classes* FREE CHOICE

There are two main approaches to scalar implicatures. The first, the 'Neo-Gricean' approach, builds on Grice's (1975) original characterisation of implicatures as inferences arising from implicit reasoning in rational interactions.[2] On this approach, implicatures arise from the hearer's reasoning about what the speaker said and could have said instead. This approach provides a unified analysis of the inferences in (1) as implicatures in that sense. The second approach, generally referred to as the 'Grammatical' approach, argues instead that implicatures arise from the compositional calculation of meaning.[3] A common implementation of this approach involves a covert exhaustivity operator in the syntax, the meaning of which gives rise to the exclusivity implicature in (1-a). Some versions of this approach maintains that the ignorance implicature in (1-b) arises through pragmatic reasoning (e.g., Fox 2007), while others derive them in the grammar as well by making use of an additional epistemic operator (e.g., Meyer 2013, Buccola & Haida to appear).

Both the pragmatic and grammatical approaches to scalar implicature can

1 While this is relatively uncontroversial in the case of EXCLUSIVITY and IGNORANCE inferences, it is much more matter of debate for FREE CHOICE. We will come back to this point below. For an implicature approach to FREE CHOICE and relevant discussion see Fox 2007, Chemla 2010, Klinedinst 2007, Franke 2011, Santorio & Romoli 2017, Bar-Lev & Fox 2017, Bar-Lev 2018 among others.

2 Horn 1972, Gazdar 1979, Sauerland 2004, Schulz & Van Rooij 2006, Spector 2006, Geurts 2010, Chemla 2010, Franke 2011 among others.

3 Chierchia 2004, Fox 2007, Chierchia et al. 2012, Magri 2009, Chierchia 2013, Romoli 2012, Meyer 2013, Marty 2017, Bar-Lev & Fox 2017, Bar-Lev 2018 among others.

readily derive EXCLUSIVITY and IGNORANCE inferences. However, as Fox (2007) has extensively discussed, deriving FREE CHOICE is challenging for the pragmatic approach, while it is relatively unproblematic for the grammatical one. On the basis of these observations, he has argued that, to the extent that free choice is treated as an implicature, it constitutes an argument in favour of the grammatical approach.[4]

With this background in mind, let us turn to a recent line of work that has investigated cases where implicatures interact with factive presuppositions like the one arising from (3).[5]

(3) Noah is unaware that Olivia took Logic.
  a. Noah does not believe that Olivia took Logic. ASSERTION
  b. Olivia took Logic. PRESUPPOSITION

Specifically, this line of research has looked at cases where implicatures appear to be computed in the presupposition but not in the assertion, a 'presupposed implicature' henceforth.[6] For instance, it has been observed that a sentence like (4) has a reading on which an implicature only appears in the presupposition. On this reading, (4) conveys that, according to Noah, it is not true that some students took Logic (i.e., according to Noah, none of the students took Logic), (4-a), while at the same time suggesting that *some but not all of them* took Logic, (4-b) with (4-c) (a.o., Gajewski & Sharvit 2012, Spector & Sudo 2017).

(4) Noah is unaware that some of the students took Logic.
  a. ASSERTION
     Noah doesn't believe that any of the students took logic.
  b. PRESUPPOSITION

4 We should note right away that two kinds of replies have been given to Fox's argument and that, in principle, both these replies could extend to our own extension of Fox's argument to the presuppositional level below. The first is to argue that free choice inferences are not implicatures after all (see Bar-Lev 2018, Goldstein 2018, Franke 2011, Aloni 2018, Santorio & Romoli 2017 a.o. for discussion). The second is to argue that the pragmatic approach can after all derive free choice as an implicature (Franke 2011; but see Fox & Katzir 2019 for critical discussion). We leave these analytical options aside for now to focus instead on the consequences of the presuppositional data above for an implicature approach to free choice. We will yet go back to discuss a non-implicature approach to free choice in Section 6 below.

5 The study of the interaction between implicatures and presuppositions goes back to pioneer work by Gazdar 1979, Soames 1982 and van der Sandt 1992 among others, but has received more and more attention in the recent literature (see Sharvit & Gajewski 2008, Gajewski & Sharvit 2012, Magri 2009, Chemla 2010, Romoli 2012, Spector & Sudo 2017, Marty 2017, 2019, among others).

6 Two other types of interactions have been discussed in the recent literature. The first is about how implicatures can influence presuppositions and related phenomena (see Mayr & Romoli 2016 for discussion). The second has to do with the role of implicatures in filtering presuppositions (see Romoli & Santorio 2018 for discussion). We will briefly discuss the later case in the Conclusion.

Some of the students took Logic.

c. PRESUPPOSED IMPLICATURE
  $\rightsquigarrow$ *Not all of the students took Logic*

Gajewski & Sharvit (2012) take (4) to be an argument for a unified grammatical account of implicatures at the presuppositional and the assertion levels. Spector & Sudo (2017) have criticised this unified approach based on two novel observations. First, they note that a sentence like (4) is infelicitous in a context in which it is common knowledge that all of the students took Logic, while its positive counterpart in (5-b) is not, an asymmetry that the unified account by Gajewski & Sharvit (2012) does not account for.

(5) CONTEXT: It is common knowledge that all of the students took Logic.
  a. #Noah is unaware that some of the students took Logic
  b. Noah is aware that some of the students took Logic.

Second, Spector & Sudo (2017) observe that a sentence like (6), involving a disjunctive sentence as a factive complement, gives rise not only to a presupposed implicature of EXCLUSIVITY, (6-c), but also to inferences of presupposed IGNORANCE, (6-d). Specifically, Spector & Sudo (2017) observe that (6) gives rise to the inference that the interlocutors do not commonly believe either of the embedded disjuncts to be true and that, because of these inferences of ignorance, an utterance of (6) is felicitous only if neither of these disjuncts is common knowledge among the interlocutors. Here again, it is *prima facie* unclear how to account for these ignorance inferences on Gajewski & Sharvit's (2012) approach.

(6) Noah is unaware that Olivia took Logic or Algebra.
  a. ASSERTION
    Noah doesn't believe that Olivia took Logic or Algebra
  b. PRESUPPOSITION
    Olivia took Logic or Algebra
  c. PRESUPPOSED EXCLUSIVITY
    $\rightsquigarrow$ *Olivia didn't take both Logic and Algebra*
  d. PRESUPPOSED IGNORANCE
    $\rightsquigarrow$ *It is not common knowledge that Olivia took Logic*
    $\rightsquigarrow$ *It is not common knowledge that Olivia took Algebra*

On the basis of the those two problematic cases, Spector & Sudo (2017) propose an alternative hybrid account based on two distinct scalar strengthening mechanisms: they adopt a grammatical theory of implicatures for deriving scalar implicatures at the assertion level but posit an independent pragmatic principle for deriving weaker implicatures about what is common knowledge. As we discuss below, the interplay between both these strengthening mechanisms can account for both the asymmetry in (5) and the presupposed IGNORANCE inferences in (6).

In this article, we add to the empirical landscape above two further observations. First, we observe that, in addition to EXCLUSIVITY and IGNORANCE, FREE CHOICE can also arise at the presuppositional level (and sometimes at the presuppositional level only). Thus for instance, a sentence like (7) has a reading conveying that Noah doesn't believe that Olivia can take either class, (7-a), while suggesting that Olivia has free choice between the two, (7-c). Hence, in parallel to the EXCLUSIVITY inference in (6), the FREE CHOICE inference in (7) appears in the presupposition but not in the assertion.

(7) Noah is unaware that Olivia can take Logic or Algebra.

a. ASSERTION
   Noah doesn't believe that Olivia can take either one

b. PRESUPPOSITION
   Olivia can take Logic or Algebra

c. PRESUPPOSED FREE CHOICE
   $\rightsquigarrow$ *Olivia can take Logic and she can take Algebra*

Second, we observe that the ignorance inferences arising at the presuppositional level are stronger than discussed in Spector & Sudo 2017, and in fact of similar strength as those arising from asserted disjunctions. That is, we observe that, in parallel to (1), an utterance of (6) doesn't merely convey that it is not common knowledge which of those two classes Olivia took, but rather that the *speaker* is ignorant as to which of the two classes she took. While this fine-grained difference is hard to detect in cases like (7), the speaker-orientation of those ignorance inferences becomes obvious in cases like (8)-(9). In the (a)-sentences, the ignorance inferences that the speaker doesn't know whether Olivia has two or more children are compatible with the speaker not knowing anything about Olivia's family. However, in the (b)-sentence, the corresponding ignorance inferences now contradict the common assumption that people are normally knowledgeable about their own families. The (b)-sentences are thus perceived as infelicitous, even in a context in which it is not common knowledge how many children the speaker actually has.

(8) a. Olivia has two or more children.
    b. #I have two or more children.

(9) a. Noah is unaware that Olivia has two or more children.
    b. #Noah is unaware that I have two or more children.

We show that both these cases are problematic for Spector & Sudo (2017). First, deriving the presupposed free choice reading of (7) is challenging in two ways. To begin with, allowing this inference to be derived from the working of their exhaustivity operator leads to over-generation issues for cases similar to (6). Next, deriving this inference with the pragmatic side of their system is challenging for the same reasons as deriving regular free-choice inferences is challenging for a pragmatic approach to assertion-based implicatures. In that respect, our dialectic closely follows that of Fox 2007: we argue against a pragmatic approach to presupposition-based implicatures on the ground that it is not able to account for presupposed free choice. Finally, we show that the presupposed ignorance inferences which can be derived in Spector & Sudo's (2017) system are too weak to account for cases like (9).

More generally, these novel data reveal a systematic parallelism between the assertion and presuppositional levels in terms of EXCLUSIVITY (10), FREE CHOICE (11), and IGNORANCE inferences (12).

(10) **Exclusivity and Presupposed Exclusivity**

a. Olivia took Logic or Algebra.
$\rightsquigarrow$ *Olivia didn't take both Logic and Algebra*

b. Noah is unaware that Olivia took Logic or Algebra.
$\rightsquigarrow$ *Olivia didn't take both Logic and Algebra*

(11) **Free Choice and Presupposed Free Choice**

a. Olivia is allowed to take Logic or Algebra.
$\rightsquigarrow$ *Olivia can take Logic and she can take Algebra*

b. Noah is unaware that Olivia is allowed to take Logic or Algebra.
$\rightsquigarrow$ *Olivia can take Logic and she can take Algebra*

(12) **Ignorance and Presupposed Ignorance**

a. Olivia took Logic or Algebra.
$\rightsquigarrow$ *The speaker ignores which of the two classes Olivia took*

b. Noah is unaware that Olivia took Logic or Algebra.
$\rightsquigarrow$ *The speaker ignores which of the two classes Olivia took*

We argue that a grammatical theory of implicatures where meaning strengthening operates in the same way at the assertion and presupposition levels (Gajewski & Sharvit 2012, Magri 2009 and Marty 2017) can account for this parallelism and provide a unified analysis of those inferences. In particular, we will show how this approach can directly account for presupposed EXCLUSIVITY and presupposed FREE CHOICE and, once combined with a grammatical account of ignorance inferences à la Meyer (2013), for presupposed IGNORANCE as well, while maintaining an account of the asymmetry in (5).

The rest of the paper is organised as follows. In Section 2, we outline the pragmatic and grammatical approaches to scalar implicatures and the challenge that free choice raises for the former. We then move in Section 3 to present the phenomenon of presupposed scalar implicatures and sketch the grammatical account by Gajewski & Sharvit (2012) and Magri (2009) as well as the hybrid account by Spector & Sudo (2017). In Section 4, we present the novel data points involving presupposed free choice and presupposed ignorance, and we discuss the challenges they raise for Spector & Sudo's (2017) account. In Section 5, we move to outline a unified grammatical approach to assertion-based and presupposition-based implicatures which can address these challenges. In Section 6, we outline some other directions one could take to account for the data above and the challenges we see for those. Section 7 concludes.

2 Free choice and scalar implicatures

2.1 Two main approaches to scalar implicatures

Consider again the two sorts of implicatures arising from disjunctions like (1). As mentioned, there are two main approaches to those implicatures in the literature: the Neo-Gricean pragmatic approach and the Grammatical approach. In the following, we sketch the gist of both approaches in turn.

(1) Olivia took Logic or Algebra.

a. EXCLUSIVITY
$\rightsquigarrow$ *Olivia didn't take both Logic and Algebra*

b. IGNORANCE
$\rightsquigarrow$ *The speaker ignores which of the two classes Olivia took*

Building on Grice's (1975) seminal work, the Neo-Gricean approach hypothesises that implicatures arise from the hearer reasoning about what the speaker said and could have said instead (Horn 1972, Gazdar 1979, Sauerland 2004, Schulz & Van Rooij 2006, Spector 2006, Geurts 2010, Chemla 2010, Franke 2011 among others). Specifically, for what is most relevant for us, the hypothesis is that the hearer reasons under the assumption that the speaker obeys the Maxim of Quantity (MQ) in (13), which favours more informative statements over less informative ones when they are equally relevant to the topic of the conversation.[7]

7 The formulation of (13) is adapted from Fox 2007. As Fox discusses, a crucial property of (13) is its reliance on alternatives to a sentence, which in turn requires a theory of alternatives. We leave this issue aside as it is tangential to our purposes. For discussion on theories of alternatives see Katzir 2007, Breheny et al. 2017 and references therein.

(13) **Maxim of Quantity (MQ)**
If ϕ and ψ are both relevant to the topic of conversation, ψ is more informative than ϕ, and ψ is among the alternatives of ϕ, then, if the speaker believes that both are true, the speaker should prefer ψ to ϕ.

To illustrate how Quantity-based reasoning can derive both the implicatures above, consider first the exclusivity implicature. Upon hearing (1), if the more informative *and*-alternative to (1) in (14) is relevant, the hearer will reason that the speaker should have preferred (14), had she believed it to be true. Therefore the hearer will conclude that it's not true that the speaker believes (14) on the basis of MQ.

(14) Olivia took Logic and Algebra.

Assuming further that the speaker is opinionated as to whether (14) is true or false, the Opinionated Speaker (OS) assumption stated in (15) (adapted from Fox 2007), the hearer will strengthen the conclusion above to the belief that, according to the speaker, (14) is not true, deriving then the exclusivity implicature in (1-a).

(15) **Opinionated Speaker (OS)**
When a speaker S utters a sentence, ϕ, the addressee, H, assumes that S's beliefs determine the truth value of every alternative of ϕ, unless this assumption leads to the conclusion that S's beliefs are contradictory.

The same reasoning also derives ignorance implicatures. Assuming that (1) has among its alternatives each of its disjuncts (a.o., Sauerland 2004, Katzir 2007), the hearer will conclude from MQ that (i) it's not true that the speaker believes that Olivia took Logic (i.e., the right disjunct) and that (ii) it's not true that the speaker believes that Olivia took Algebra (i.e., the left disjunct). Together, these two inferences give us the ignorance implicature in (1-b): the speaker knows that one of the two disjuncts is true, but she doesn't know which. Crucially, note that OS cannot apply in this case since strengthening further those inferences would otherwise lead to the conclusion that the speaker has contradictory beliefs, namely that the speaker believes (1) to be true but both of (1)'s disjuncts to be false.

In sum, the Neo-Gricean approach provides a unified analysis of the exclusivity and ignorance implicatures arising from disjunctions based on the interaction between MQ and the OS assumption. We turn now to a second and very different conception of implicatures coming from the Grammatical approach.

This second conception has its origins in early criticism of the Gricean notion of implicatures (e.g., Cohen 1971) and envisions these inferences more on the semantic side of the semantics-pragmatics interface. That is, on this approach, scalar implicatures arise from the compositional calculation of meaning (a.o., Chierchia

2004, Chierchia et al. 2012, Magri 2009, Chierchia 2013, Romoli 2012, Meyer 2013, Fox 2007, Bar-Lev & Fox 2017, Marty 2017). A common implementation of this approach hypothesises a covert exhaustivity operator in the syntax, generally referred to as 'EXH', the application of which directly gives rise in (1) to the exclusivity implicature in (1-a). For our initial purposes, EXH can be defined as in (16-a), where ϕ is any sentence and $\textsc{alt}(\phi)$ the set of alternatives to ϕ. In a nutshell, applying EXH to a sentence ϕ outputs ϕ and the negation of all of ϕ's alternatives that are 'innocently excludable', (16-b), i.e. those alternatives to ϕ that can be consistently negated together without contradicting ϕ or entailing the truth of other alternatives. Note that the definition of innocent exclusion in (16-b) is parallel to and effectively doing the work of MQ plus the OS assumption, including the non-contradiction clause.

(16) a. Exhaustivity operator (EXH)
$[\![\textsf{EXH}\ \phi]\!](w) = [\![\phi]\!](w) \wedge \forall\psi \in \mathsf{IE}(\phi, \textsc{alt}(\phi))[\neg[\![\psi]\!](w)]$

b. Innocently Excludable alternatives (IE)
$$\mathsf{IE}(\phi, C) := \bigcap \left\{ C' \;\middle|\; \begin{array}{l} C' \subseteq C \text{ and } C' \text{ is a maximal subset of } C \\ \text{such that } \{\neg\psi : \psi \in C'\} \cup \{\phi\} \text{ is consistent} \end{array} \right\}$$

Some versions of the grammatical approach maintains that, unlike the exclusivity implicature in (1-a) (i.e., a secondary implicature in Neo-Gricean terms), ignorance implicatures arise in the same way as before, through pragmatic reasoning based on MQ (e.g., Fox 2007). On this view, the different implicatures observed in (1) are thus obtained as follows:

(17) EXH [Olivia took Logic or Algebra]
IE-alternatives:={Olivia took Logic and Algebra}

a. By Exhaustification:
$\rightsquigarrow$ *Olivia didn't take both Logic and Algebra*

b. By MQ:
$\rightsquigarrow$ *the speaker ignores whether Olivia took Logic*
$\rightsquigarrow$ *the speaker ignores whether Olivia took Algebra*

Following Meyer (2013), more recent versions of the grammatical approach have proposed that ignorance inferences are also derived in the grammar through the interaction of EXH with another another covert operator representing the speaker's beliefs (e.g., Meyer 2013, Buccola & Haida to appear). At the core of Meyer's proposal is the assumption — called the Matrix K Axiom — that assertively used sentences contain a covert doxastic operator K which is adjoined at the matrix level at LF (cf. Chierchia 2006, Alonso-Ovalle & Menéndez-Benito 2010). Much like the attitude verb *believe*, the Matrix K operator universally quantifies over the speaker's doxastic alternatives, (18). The subscript x refers to the doxastic source, i.e. the individual whose beliefs K is quantifying over. In the cases that we will be concerned with, x will always be the speaker, hence the notation K_s.[8]

(18) $[[K_x\ \phi]] = \lambda w.\ \forall w' \in Dox(x)(w)[[[\phi]](w')]$

Meyer shows that the Matrix K Axiom, together with the possible adjunction of EXH at any propositional node (i.e., below and above K), permits to derive in a principled way the ignorance inferences previously attributed to the working of MQ. Setting aside the specifics of the computation here to focus on the outcome, applying Meyer's proposal to (1) provides us with the following desired result:[9]

(19) EXH [K_s EXH [Olivia took Logic or Algebra]]

a. By Exhaustification below K:
   (i) IE-alternatives:=$\{[L \wedge A]\}$
   (ii) Implicatures: $\neg[L \wedge A]$
b. By Exhaustification above K:
   (i) IE-alternatives:=$\{[K_s$ EXH L], [K_s EXH A], [K_s EXH $L \wedge A]\}$
   (ii) Implicatures: $\neg K_s[L \wedge \neg A]$, $\neg K_s[A \wedge \neg L]$, $\neg K_s[A \wedge L]$
c. Final representation:
   (19) $\Leftrightarrow K_s[L \vee A] \wedge K_s \neg [L \wedge A] \wedge I_s[L] \wedge I_s[A]$
   <u>In words:</u> (19) is true if and only if the speaker believes that Olivia took Logic or Algebra but not both, and the speaker ignores which of both classes Olivia took.

This concludes our overview of the two approaches. While we have skipped over a variety of details and subtleties, it is enough as a background for us to grasp the challenge of free choice, to which we now turn.

2.2 The challenge of free choice

Consider again the free choice inference in (2):

(2) Olivia can take Logic or Algebra.
   ⇝ *Olivia can choose between the two classes* FREE CHOICE

As many researchers have observed, free choice is puzzling from the perspective of a traditional approach to the meaning of modals and disjunction: the predicted

8 Here and throughout this paper, we will use K in the meta-language to abbreviate the denotation of the Matrix K operator, i.e., we adopt the following convention: $[[K_x\ \phi]]$ *iff* $K_x\ \phi$.

9 For simplicity, we use L and A as short forms for the sentences *Olivia took Logic* and *Olivia took Algebra*, respectively. As is customary, we write $I_s(\phi)$ for 'the speaker s is ignorant about ϕ', where $I_s(\phi)$ holds if and only if both $\neg K_s(\phi)$ and $\neg K_s(\neg\phi)$ hold.

meaning of (2) only entails that Olivia can take Logic or she can take Algebra. To complicate things further, free choice disappears under negation. That is, the negative counterpart of (2) in (20) doesn't merely convey that it's not true that Olivia doesn't have free choice, but rather that Olivia can't take either of the two classes. The negated meaning of (2) in (20) is sometimes called 'dual prohibition', a terminology which we will adopt here.

(20) Olivia cannot take Logic or Algebra.
$\rightsquigarrow$ *Olivia cannot take either one* DUAL PROHIBITION

In essence, the theoretical challenge is to account for free choice in positive contexts like (2) and, at the same time, for dual prohibition in negative ones like (20). A prominent line of explanation takes the free choice-dual prohibition alternation to teach us that we should treat free choice as implicatures, hence accounting immediately for their sensitivity to monotonicity. Additional support for this proposal comes from the observation that free choice inferences exhibit another characteristic feature of implicatures: they can be suspended or cancelled, as illustrated in (21).

(21) Olivia can take Logic or Algebra, but I don't remember which (one).
$\not\rightsquigarrow$ *Olivia can choose between the two*

Despite these striking similarities, we note that the implicature approach to free choice is by no means uncontroversial, and we will come back to some alternative accounts in the conclusion. For now, what is important for us is that, if one assumes an implicature approach to free choice, there are important consequences for the debate between the pragmatic and grammatical approaches to implicatures. In particular, Fox (2007) has shown that deriving free choice is problematic under the former, but not under the latter. The gist of the challenge for the pragmatic approach is as follows. By the same Quantity-based reasoning as the one described above, a sentence like (2) is to be compared by the hearer to its modalised alternatives corresponding to its independent disjuncts, (22-a) and (22-b) respectively.

(22) a. Olivia can take Logic.
b. Olivia can take Algebra.

Since these alternatives are more informative than (2), the hearer will conclude by MQ that it's not true that the speaker believes either. In other words, the hearer will conclude that the speaker is ignorant as to whether Olivia can take Logic and as to whether Olivia can take Algebra. While this outcome can account for suspension cases like (21), it does not account for genuine cases of free choice like (2), and it is unclear how to proceed from that point. In particular, the OS assumption cannot apply in this case as it would otherwise lead to a contradiction with the asserted meaning.[10] In sum, the pragmatic approach can only derive ignorance inferences about the disjuncts, regardless of whether the asserted disjunction is modalised as in (2) or a simple one as in (1). Therefore, if free choice inferences are scalar implicatures, deriving them is a challenge for this pragmatic approach.[11]

2.3 The grammatical approach to free choice

In contrast to the pragmatic approach, the grammatical approach can account for free choice. We sketch here a version of this approach based on Bar-Lev & Fox 2017, Bar-Lev 2018 but, as far as we can see, any version of this approach would be compatible with the following (see, among others, Fox 2007, Klinedinst 2007, Chemla 2010, Santorio & Romoli 2017).

As a starting point, the literal meaning of a sentence like (2) can be represented as shown in (23-a), where the symbol '$\Diamond$' is used to represent modal operators of possibility (the possibility modal 'can' in the present case). On this representation, (2) is true if and only if Olivia can take at least one of Logic and Algebra, as one would expect. While (23-a) does not account in itself for free choice, note that its negation directly corresponds to dual prohibition, (23-b).

(23) a. $\Diamond(\mathsf{L} \vee \mathsf{A}) \Leftrightarrow \Diamond\mathsf{L} \vee \Diamond\mathsf{A}$ LITERAL MEANING
b. $\neg\Diamond(\mathsf{L} \vee \mathsf{A}) \Leftrightarrow \neg\Diamond\mathsf{L} \wedge \neg\Diamond\mathsf{A}$ DUAL PROHIBITION

The approach by Bar-Lev & Fox (2017) accounts for the free choice-dual prohibition pattern on the basis of two main ingredients. First, it assumes an extra layer to the exhaustification process: in addition to excluding a subset of alternatives to its prejacent, EXH also 'includes' a subset of other ones. To select which alternatives are thus includable, Bar-Lev & Fox (2017) propose a notion analogous to innocent exclusion, which they call 'innocent inclusion,' defined as in (24-b) (alongside the notion of innocent exclusion in (24-a), repeated for convenience). In short, an alternative is innocently includable if it is in all maximally includable subsets of alternatives, which in turn are those alternatives which can be consistently included with the prejacent and the negation of all innocently excludable alternatives.

10 In addition, we note that any attempt to derive free choice for (2), for instance by considering different alternatives for implicature computation (e.g., Kratzer & Shimoyama 2002), should make sure that it does not unwarrantedly extend to simple disjunctive sentences like (1), which cannot have the corresponding conjunctive meaning that Olivia took Logic and Algebra.

11 There are more recent proposals in the literature for deriving free choice as an implicature from a pragmatic perspective, which respond to Fox's (2007) challenge (a.o., Chemla 2010, Geurts 2010, Franke 2011). We cannot go into the details of these proposals here; see Romoli 2014, Fox & Katzir 2019 for some critical discussion.

(24) a. Innocently Excludable alternatives (IE)

$$\mathsf{IE}(\phi,C) := \bigcap\left\{ C' \;\middle|\; \begin{array}{l} C' \subseteq C \text{ and } C' \text{ is a maximal subset of } C \text{ s.t.} \\ \{\neg\psi : \psi \in C')\} \cup \{\phi\} \text{ is consistent} \end{array} \right\}$$

b. Innocently Includable alternatives (II)

$$\mathsf{II}(\phi,C) := \bigcap\left\{ C'' \;\middle|\; \begin{array}{l} C'' \subseteq C \text{ and } C'' \text{ is a maximal subset of } C \text{ s.t.} \\ \{\chi : \chi \in C''\} \cup \{\neg\psi : \psi \in \mathsf{IE}(\phi,C)\} \cup \{\phi\} \text{ is consistent} \end{array} \right\}$$

On this approach, the working of EXH is thus twofold: it excludes all innocently excludable alternatives while including all innocently includable ones, (25). For clarity, we notate Bar-Lev & Fox's exhaustivity operator as EXH^{IE+II}.

(25) Exhaustivity operator for IE and II alternatives (EXH^{IE+II}):

$$[\![\text{EXH}^{IE+II}\ \phi]\!](w) =$$
$$[\![\phi]\!](w) \wedge \forall\psi \in \mathsf{IE}(\phi, \text{ALT}(\phi))[\neg[\![\psi]\!](w)] \wedge \forall\chi \in \mathsf{II}(\phi, \text{ALT}(\phi))[[\![\chi]\!](w)]$$

To illustrate how this operator works, consider again (2) and assume that this sentence is parsed as in (26) and that the formal alternatives to EXH's prejacent are those illustrated in (27).

(26) EXH^{IE+II} [Olivia can take Logic or Algebra]

(27) $$\text{ALT} = \left\{ \begin{array}{lr} \text{Olivia can take Logic or Algebra} & \Diamond(\mathsf{L} \vee \mathsf{A}) \\ \text{Olivia can take Logic} & \Diamond\mathsf{L} \\ \text{Olivia can take Algebra} & \Diamond\mathsf{A} \\ \text{Olivia can take Logic and Algebra} & \Diamond(\mathsf{L} \wedge \mathsf{A}) \end{array} \right\}$$

Only one of these alternatives is in all maximally excludable subsets, $\Diamond(\mathsf{L} \wedge \mathsf{A})$. Hence, if it is relevant, this alternative can be innocently excluded.[12] Crucially, the other three alternatives — $\Diamond(\mathsf{L} \vee \mathsf{A})$, $\Diamond\mathsf{L}$ and $\Diamond\mathsf{A}$ — are all innocently includable. Including those alternatives upon exhaustification of (2)'s meaning delivers the free choice reading we were interested in, as illustrated in (28).

(28) $[\![\text{EXH}^{IE+II}$ [Olivia can take Logic or Algebra]$]\!]$ =
$\Diamond(\mathsf{L} \vee \mathsf{A}) \wedge \neg\Diamond(\mathsf{L} \wedge \mathsf{A}) \wedge \Diamond\mathsf{L} \wedge \Diamond\mathsf{A}$

The second ingredient, which is relevant for negative cases like (20), is a constraint on EXH's distribution that disallows this operator to appear under negation or, equivalently, that prevents implicatures from being computed under negation to avoid meaning weakening (see, among others, Chierchia et al. 2012, Enguehard

12 As Bar-Lev & Fox (2017), Bar-Lev (2018) discuss, this alternative can also be pruned from the set of alternatives if not relevant. This, in turn, can account for the fact that the inference corresponding to the negation of the conjunctive alternative (i.e. Olivia cannot take both Logic and Algebra) is not always associated with sentences like (2), regardless of whether they give rise to free choice.

& Chemla 2018 for discussion). The application of this constraint in cases like (20) preserves the good result in (23-b), namely dual prohibition, by blocking exhaustification process such as (29), the outcome of which would be logically weaker than the literal meaning of the base sentence.

(29) $\neg(\text{EXH}(\Diamond(\mathsf{L} \vee \mathsf{A}))) \Leftrightarrow \neg\Diamond\mathsf{L} \vee \neg\Diamond\mathsf{A}$

In sum, the grammatical approach to implicatures can account for free choice in a relatively straightforward way and, to the extent that we treat free choice as an implicature, this result constitutes an argument for such an approach. In the following subsection, we move on to discuss the phenomenon of presupposion-based scalar implicatures.

3 Presupposed scalar implicatures

3.1 The phenomenon

Building on previous observations by Simons (2001b) and Russell (2006), Gajewski & Sharvit (2012) discuss sentences like (4) and (6), repeated from above, where a weak scalar item (e.g., *some*, *or*) is embedded under the scope of a negative or negated factive predicate (e.g., *unaware*). Gajewski & Sharvit observe that such sentences have a salient reading on which their assertion in (a) retains its literal meaning, while their presupposition in (b) gives rise to a scalar implicature, (c).

(4) Noah is unaware that some of the students took Logic.

a. ASSERTION
Noah doesn't believe that any students took Logic

b. PRESUPPOSITION
Some of the students took Logic

c. PRESUPPOSED IMPLICATURE
$\rightsquigarrow$ *Not all of the students took Logic*

(6) Noah is unaware that Olivia took Logic or Algebra.

a. ASSERTION
Noah doesn't believe that Olivia took Logic or Algebra

b. PRESUPPOSITION
Olivia too Logic or Algebra

c. PRESUPPOSED IMPLICATURE
$\rightsquigarrow$ *Olivia didn't take both Logic and Algebra*

Crucially, Gajewski & Sharvit show that these reading are distinct from the readings that would obtain if the target implicatures were to be computed at the level of the embedded clause, i.e. under the scope of *unaware*. To illustrate this point, assume the EXH-based grammatical approach to scalar implicatures sketched above, setting innocent inclusion aside for the moment (we will go back to it below). Applying EXH at the embedded level in (6) would deliver for instance the reading in (30):

(30) Noah is unaware that EXH [Olivia took Logic or Algebra]
IE-alternatives:={Olivia took Logic and Algebra}

a. ASSERTION
Noah doesn't believe the following: Olivia took Logic or Algebra *and not both Logic and Algebra*

b. PRESUPPOSITION
Olivia took Logic or Algebra *but not both Logic and Algebra*

On this reading, (6) has the (exclusivity) implicature we are looking for in its presupposition. The problem is that this implicature is also present in the assertion, leading to an unwarranted weakening of the asserted content of (6). In particular, on this reading, (6) should be true if the speaker takes for granted that Olivia took either Logic or Algebra, and Noah believes that Olivia took both classes. This reading is weaker than the one we are after, and it is definitely a marked reading of (6), if possible at all.[13] Similar observations extend to the example in (4). In other words, deriving the observed readings of (4) and (6) is challenging for theories of scalar implicatures and presuppositions, and simply embedding an implicature operator in the scope of the factive predicate is not a solution. In the next subsection, we turn to what is possibly the simplest way of addressing this challenge, namely to allow scalar implicatures to be computed separately in the assertion and in the presupposition.

3.2 A unified grammatical approach and its problems

In order to account for the puzzling readings above, Gajewski & Sharvit (2012) propose that scalar implicatures are computed in the assertion and in the presupposition of a sentence in a similar way but *separately* (see also Magri 2009 for a similar proposal). This proposal relies on the following two key components. First, it assumes a two-dimensional theory of meaning in the style of Karttunen & Peters (1979) on which a sentence ϕ denotes two propositions, its presupposition $[\![\phi]\!]^{prs}$

13 Native speakers we have consulted consistently judged (6) as false in such cases. The unavailability of this reading is not surprising. It directly relates to the above-mentioned general observation that scalar implicatures tend to not arise at the assertion level in downward-entailing environments (e.g., under negation and other scale-reversal contexts) as their computation would amount in such cases to weaken (rather than strengthen) the assertion of the sentence at the global level; see Chierchia et al. 2012 for discussion.

and its assertion $[\![\phi]\!]^{asr}$, that is $[\![\phi]\!] = \langle [\![\phi]\!]^{prs}, [\![\phi]\!]^{asr} \rangle$ (see also Mandelkern 2016 for a more recent bi-dimensional account of presuppositions). On this view, a sentence like (6) denotes for instance the following two propositions:

(31) a. $[\![$Noah is unaware that Olivia took Logic or Algebra$]\!]^{prs}$
$= \lambda w.$ Olivia took Logic or Algebra in w
b. $[\![$Noah is unaware that Olivia took Logic or Algebra$]\!]^{asr}$
$= \lambda w.$ Noah doesn't believe in w that Olivia took Logic or Algebra

Second, this proposal hypothesises that, when the meaning of a sentence is exhaustified, exhaustification is performed at both dimensions separately. For exposition reasons, we will follow the implementation of this idea offered in Magri 2009 and rendered in (32). This implementation directly builds upon the basic conceptualization of EXH introduced in (16) and, in particular, upon the notion of innocent exclusion defined in (16-b). In a nutshell, when EXH is applied to a sentence meaning, it applies separately to its assertion and to its presupposition.[14]

(32) a. $[\![\text{EXH}\ \phi]\!](w) = \langle [\![\text{EXH}\ \phi]\!]^{prs}(w), [\![\text{EXH}\ \phi]\!]^{asr}(w) \rangle$
b. $[\![\text{EXH}\ \phi]\!]^{prs}(w) = [\![\phi]\!]^{prs}(w) \wedge \forall \psi \in \text{IE}(\phi, \text{ALT}^{prs}(\phi))[\neg [\![\psi]\!]^{prs}(w)]$
c. $[\![\text{EXH}\ \phi]\!]^{asr}(w) = [\![\phi]\!]^{asr}(w) \wedge \forall \psi \in \text{IE}(\phi, \text{ALT}^{asr}(\phi))[\neg [\![\psi]\!]^{asr}(w)]$

At the assertion level, EXH operates the same way as before. Thus for instance, for the *some*-sentence in (33), it gives us the familiar *not-all* implicature and, for a simple disjunctive case like (34), the familiar exclusivity implicature.

(33) EXH $[_\phi$ Some of the students took Logic$]$
a. $[\![\phi]\!]^{asr}(w) = 1$ iff some of the students took Logic in w
b. $\text{IE}(\phi, \text{ALT}^{asr}(\phi)) = \{$all of the students took Logic$\}$
c. $[\![\text{EXH}\ \phi]\!]^{asr}(w) = 1$ iff some but not all of the students took Logic in w

(34) EXH $[_\phi$ Olivia took Logic or Algebra$]$
a. $[\![\phi]\!]^{asr}(w) = 1$ iff Noah took Logic or Algebra in w
b. $\text{IE}(\phi, \text{ALT}^{asr}(\phi)) = \{$Olivia took Logic and Algebra$\}$
c. $[\![\text{EXH}\ \phi]\!]^{asr}(w) = 1$ iff Noah took Logic or Algebra but not both in w

In addition, the strengthening operation now applies also at the presuppositional level: the exhaustified presupposition of a sentence ϕ corresponds to the conjunction of ϕ's presupposition together with the negation of the presuppositions of all of ϕ's alternatives which are innocently excludable in the presuppositional dimension.

14 Gajewski & Sharvit (2012) uses a slightly different, yet comparable meaning strengthening mechanism, the composition rule of STRONG APPLICATION postulated in Chierchia (2004). The superficial differences between EXH and STRONG APPLICATION are orthogonal to our present purposes.

This theoretical move offers a simple solution to the kind of challenge posed by sentences like (4) and (6): applying EXH at the matrix level of these sentences now delivers the readings we were after, as illustrated in (35) and (36) respectively.[15]

(35) EXH [$_\phi$ Noah is unaware that some of the students took Logic]

a. ASSERTION
$\mathsf{IE}(\phi, \text{ALT}^{asr}(\phi)) = \{\ \}$
$[[\text{EXH}\ \phi]]^{asr}(w) = 1$ iff N. doesn't believe in w that any students took Logic

b. PRESUPPOSITION
$\mathsf{IE}(\phi, \text{ALT}^{prs}(\phi))$={N. is unaware that all of the students took Logic}
$[[\text{EXH}\ \phi]]^{prs}(w) = 1$ iff some but not all of the students took Logic in w

(36) EXH [$_\phi$ Noah is unaware that Olivia took Logic or Algebra]

a. ASSERTION
$\mathsf{IE}(\phi, \text{ALT}^{asr}(\phi)) = \{\ \}$
$[[\text{EXH}\ \phi]]^{asr}(w) = 1$ iff N. doesn't believe in w that O. took either one

b. PRESUPPOSITION
$\mathsf{IE}(\phi, \text{ALT}^{prs}(\phi))$={N. is unaware that O. took Logic and Algebra}
$[[\text{EXH}\ \phi]]^{prs}(w) = 1$ iff O. took Logic or Algebra, but not both in w

Thus for instance, on the parse in (36), the sentence in (6) presupposes that Olivia took Logic or Algebra together with the exclusivity implicature that she didn't take both. Crucially, this implicature is only computed at the presuppositional level. At the assertion level, EXH leaves the meaning of (6) unaffected for there isn't any innocently excludable alternatives to the assertion. In other words, EXH is effective at the presuppositional level, but vacuous at the assertion level. This outcome gives us the desired reading of (6). Similar observations hold for (4) on the parse in (35).

While these results are promising, Spector & Sudo (2017) has raised two important challenges for such an approach to presupposed implicatures. First, they observe that, in contrast to (4) and (6), the positive counterparts of (4) and (6) can be felicitously uttered in a context in which the stronger presuppositions of their target alternatives are common knowledge. The contrast in (37) illustrates this observation using the case of *some*:[16]

(37) CONTEXT: The interlocutors know that all of the students took Logic; Noah, however, has no idea that Logic is this popular among the students.

15 In addition, this proposal also derives the stronger form of the so-called implicated presuppositions (Sauerland 2008) as genuine scalar implicatures. As it is not relevant to our purposes here, we set it aside; see Magri 2009 and Marty 2017, 2019 for discussion.

16 Spector & Sudo (2017) notes that sentences like (37-a) are more naturally produced in such contexts with a focal stress on the scalar item, and that the intended reading can be appropriately paraphrased as follows: *Noah only realized that* SOME *of the students took Logic.*

a. Noah is aware that some of the students took Logic.
b. #Noah is unaware that some of the students Logic.

As Spector & Sudo discuss, this contrast shows that, unlike the presupposition of (37-b), the one of (37-a) does not give rise to a *not-all* implicature in the suggested context for otherwise (37-a) would also be perceived as infelicitous. This difference between (37-b) and its positive counterpart (37-a) raises a direct issue for the unified approach by Gajewski & Sharvit (2012): since both these sentences have the same presuppositions (i.e., *some of the students took Logic*) and the same presuppositional competitors (i.e., those derived by replacing *some* with *all* in the embedded clause), both sentences are predicted on this approach to be strengthened in a parallel fashion and thus to be infelicitous in the same contexts, contrary to facts.

Second, Spector & Sudo (2017) point out another data point that is left unexplained by the unified approach above: an utterance of (6) is infelicitous in a context in which one of the embedded disjuncts is already common knowledge, as illustrated in (38) (modeled after example (53) in Spector & Sudo 2017). This observation is puzzling since the presupposition of (38) is satisfied in such contexts, and this regardless of the absence or presence of an exclusivity implicature.

(38) Context: The interlocutors know that Olivia only took Logic.
#Noah is unaware that Olivia took Logic or Algebra.

Hence, the grammatical approach to presupposed implicatures developed in Gajewski & Sharvit 2012 and Magri 2009 derives the desired readings of (4) and (6), but it also faces the new challenges identified by Spector & Sudo (2017). In the remainder of this section, we sketch the alternative proposal by Spector & Sudo (2017) to take up both these challenges.

3.3 A hybrid account

In contrast to the previous approach, Spector & Sudo (2017) propose to posit two distinct forms of scalar strengthening, one operating at the assertion level and the other at the presuppositional one. At the assertion level, Spector & Sudo assume that scalar strengthening is performed by application of the exhaustivity operator. Adopting a trivalent semantics for presuppositions, they propose to adjust the bivalent definition of exh in (16) to a trivalent setting so as to let exh pass up the presuppositions of the alternatives it excludes, just like negation passes up the presuppositions of the sentence it negates.[17] On this refinement, the meaning of a

17 We refer the reader to Spector & Sudo (2017) for the relevant definitions (see in particular Section 5.1, (25), (26) and (27)). In short, Spector & Sudo's adjustments are twofold: (i) exh is defined so as to behave as a 'presupposition hole' with respect to the presupposition of the alternatives, and (ii)

sentence like (37-a) can be now strengthened as shown below. In a nutshell, if the conjunctive alternative to (37-a) is negated by the working of EXH, this negated alternative will then pass up its (stronger) conjunctive presupposition to the whole sentence, which will thus presuppose that Olivia took both Logic and Algebra.

(37-a) EXH [Noah is aware that Olivia took Logic OR Algebra]
IE-alternatives:={Noah is aware that Olivia took Logic and Algebra}

a. ASSERTION
Noah believes that Olivia took Logic or Algebra, *but he doesn't believe that Olivia took Logic and Algebra*

b. PRESUPPOSITION
Olivia took Logic or Algebra, *and Olivia took Logic and Algebra*

At the presuppositional level, on the other hand, Spector & Sudo propose that strengthening follows from an independent pragmatic principle along the lines of *Maximize Presupposition!*, which they call *Presupposed Ignorance Principle* (henceforth, PIP). The PIP makes a sentence infelicitous if that sentence has an alternative with a logically stronger presupposition that is satisfied in context, (39).[18]

(39) **Presupposed Ignorance Principle** (Spector & Sudo 2017: (5)/(56))
Let p be the presupposition of sentence ϕ. If ϕ has an alternative ψ presupposing q and q asymmetrically entails p, ϕ is infelicitous in context c if q is satisfied in c, i.e. if it is Common Knowledge in the context, $\mathsf{CK}_c(q)$.

The PIP can straightforwardly account for the infelicity of examples like (38), which was left unexplained by the previous approach. In a nutshell, (38) competes with the alternative *Noah is unaware that Olivia took Logic*, the presupposition of which is logically stronger than that of (38) and is satisfied in the given context since $\mathsf{CK}_c(\textit{Olivia took Logic})$. Yet by application of the PIP, (38) is predicted to trigger the ignorance inference $\neg\mathsf{CK}_c(\textit{Olivia took Logic})$ which blatantly contradicts common knowledge in that context, accounting thus for the infelicity of (38).

Next, the interactions between both scalar strengthening mechanisms provides an account of the asymmetry observed in (37). As shown above, in positive cases like (37-a), exhaustification has two related effects: EXH strengthens the meaning of its prejacent by negating the assertion of the conjunctive alternative and subsequently by passing up the (stronger) presupposition of that alternative. Following

the notion of IE-alternatives in (16-b) is redefined by making use of *strong* negation.

18 Note that, in contrast to *Maximize Presupposition!*, the PIP leaves out the requirement that the presuppositional competitors to a given sentence be contextually equivalent to that sentence, allowing in effect more competitors than *Maximize Presupposition!*. See Spector & Sudo 2017, Anvari 2018 for discussion.

this strengthening, the working of the PIP becomes vacuous, for the resulting presupposition is already maximally strong, accounting thus for the felicity of (37-a) in the suggested context. In negative cases like (37-b), however, it goes the other way around: the working of EXH is now vacuous since the corresponding conjunctive alternative is not excludable, and so the PIP effectively applies, giving rise to a presupposed ignorance inference that contradicts common knowledge, exactly as in (38).

(37-b) EXH [Noah is unaware that some of the students took Logic]
IE-alternatives:={ }

a. ASSERTION
Noah doesn't believe that some of the students took Logic.

b. PRESUPPOSITION
Some of the students took Logic

c. PRESUPPOSED IGNORANCE:
$\rightsquigarrow \neg \mathsf{CK}_c$(*all of the students took Logic*) by the PIP

To summarize, Spector & Sudo (2017) propose a hybrid account to implicatures. This account relies on a grammatical theory of implicatures at the assertion level, but on a pragmatic one at the presupposition level. Crucially, both scalar strengthening mechanisms operate at one particular level and in a determined order: (i) at the assertion level, EXH negates the assertion of certain alternatives and passes up their presuppositions, and (ii) at the presuppositional level, the PIP derives weaker inferences about what is common knowledge on the basis of (i). The resulting proposal accounts for the asymmetry between (37-b) vs. (37-a) and for additional infelicity effects like (38). We turn now to present the challenges that presupposed free choice raises for their view and examine presupposed ignorance in more detail.

4 New challenges

4.1 Presupposed free choice

With these ingredients in place, we can now go back to sentences like (7) and explain the challenges they raise for Spector & Sudo's (2017) hybrid account.

(7) Noah is unaware that Olivia can take Logic or Algebra.

a. ASSERTION
Noah doesn't believe that Olivia can take either one

b. PRESUPPOSITION
Olivia can take Logic or Algebra

c. PRESUPPOSED FREE CHOICE

$\rightsquigarrow$ *Olivia can take Logic and she can take Algebra*

As we already mentioned, (7) has a salient reading suggesting that Olivia can take Logic and that she can take Algebra, while conveying that Noah doesn't believe that she can take either one. That is, in a way similar to the implicatures in (4) and (6), we observe that the free choice inference appears here in the presupposition but not in the assertion. As we have seen in section 3.1, this observation already teaches us that free choice cannot simply be derived at the level of the embedded clause in a regular way (or it would show up both in the presupposition *and* the assertion). Cases similar to (7) can be easily reproduced with other factive verbs and adjectives, as illustrated in (40) and (41).[19]

(40) Noah didn't realise that Olivia can take Logic or Algebra.

a. ASSERTION
Noah doesn't believe that Olivia can take either class

b. PRESUPPOSITION
Olivia can take Logic or Algebra

c. PRESUPPOSED FREE CHOICE
$\rightsquigarrow$ *Olivia can take Logic and she can take Algebra*

(41) Noah is is sorry that Olivia can take Logic or Algebra.

a. ASSERTION
Noah wants that Olivia cannot take either Logic or Algebra.

b. PRESUPPOSITION
Olivia can take Logic or Algebra

c. PRESUPPOSED FREE CHOICE
$\rightsquigarrow$ *Olivia can take Logic and she can take Algebra*

These novel cases are challenging for the system proposed in Spector & Sudo 2017. To begin with, the application of the PIP predicts an utterance of (7) to be

19 Alxatib (2014) and Bar-Lev & Fox (2017) discuss similar data involving *only* such as (i), which also exhibit a presupposed free choice reading. As Alxatib (2014) shows, the free choice inference in (i-b) survives embeddings just like other presuppositions. Alxatib (2014) and Bar-Lev & Fox (2017) take these data to teach us something about the semantics of *only* and EXH. However, their accounts do not generalise to the other cases of presupposed free choice discussed in this paper. Conversely, the account we explore below extends to data like (i).

(i) Olivia is only allowed to take Logic or Algebra

a. ASSERTION
Olivia is not allowed to take any other course

b. PRESUPPOSED FREE CHOICE
$\rightsquigarrow$ *Olivia can take Logic and she can take Algebra*

infelicitous if it is common knowledge that Olivia can take Logic and that she can take Algebra (i.e., that she has free choice between the two). This prediction obtains because (7) has the alternatives in (42), the presuppositions of which are logically stronger than the disjunctive presupposition of (7) and satisfied in the suggested context. This prediction is however incorrect: all the sentences above are intuitively felicitous in a context in which it is common knowledge that Olivia can choose between Logic and Algebra.

(42) a. Noah is unaware that Olivia can take Logic.
PRESUPPOSITION: Olivia can take Logic
b. Noah is unaware that Olivia can take Algebra.
PRESUPPOSITION: Olivia can take Algebra

This issue would of course disappear if we could derive the free choice inference associated with (7) as a presupposition of (7), since (7) would then presuppose that Olivia can take Logic and that she can take Algebra, rendering then the working of the PIP vacuous. The problem however is that accounting for this inference in the system of Spector & Sudo (2017) is problematic. First, deriving this inference with the pragmatic side of their system is challenging for the same reasons as deriving regular free choice inferences is challenging for a pragmatic approach to assertion-based implicatures. Second, allowing this inference to arise from the working of an exhaustivity operator operating solely at the assertion level would also lead to over-generation issues for cases similar to (6) above.

To illustrate both these points in turn, consider first what we can derive just by using the PIP. As exemplified below, in a run-of-the-mill context, a sentence like (7) gives rise by the PIP to the ignorance inferences that it is not common ground that Olivia can take Logic and that it is not common ground that she can take Algebra. These inferences, however, do not lead to presupposed free choice.

(7) Noah is unaware that Olivia can take Logic or Algebra.
a. $\neg$CK[Olivia can take Logic] by the PIP
b. $\neg$CK[Olivia can take Algebra] by the PIP

The present observation is in fact entirely parallel to the observation that the Maxim of Quantity cannot derive assertion-based free choice inferences in unembedded cases like (2). Rather, MQ can only derive the ignorance inferences that the speaker is not certain that Olivia can take Logic and not certain that Olivia can take Algebra.

(2) Olivia can take Logic or Algebra.
a. $\neg$K[Olivia can take Logic] by MQ
b. $\neg$K[Olivia can take Algebra] by MQ

Another option in Spector & Sudo's (2017) system would be to derive presupposed free choice as a presupposition arising through exhaustification of the assertion (see (37-a) for an example). This option would indeed give us the reading with the stronger free choice presupposition and subsequently prevent the PIP from being effective. The problem, however, is that this cannot be achieved without spoiling in turn Spector & Sudo's (2017) account of the asymmetry between (37-b) and (37-a). To see this, consider a minimal extension of Spector & Sudo's (2017) definition of EXH that leaves room for II-alternatives so that EXH^{IE+II} negates all IE-alternatives and passes up their presuppositions and, in addition, includes all II-alternatives and passes up their presuppositions. Going back to (7), let us see now what we can derive by applying this operator globally as in (43-a), which can be represented schematically as in (43-b) (where '$\Box$' represents the universal modal associated with Noah's doxastic alternatives and the presupposition is indicated as subscripted).[20]

20 We note once again that deriving free choice directly in the embedded clause, for instance by following Bar-Lev & Fox's (2017) explanation for assertion-based free choice, will not do:

(i) Noah is unaware that EXH^{IE+II} [Olivia can take Logic or Algebra]

a. ASSERTION
Noah doesn't believe that Olivia can take Logic and can take Algebra

b. PRESUPPOSITION
Olivia can take Logic and she can take Algebra

The sentence in (i) has the desired free choice inference in its presupposition. Yet the presence of this inference in the assertion under negation makes the asserted content too weak. In particular, (i) is compatible with a situation in which, in all of Noah's belief worlds, Olivia can only take Logic. This might be a possible reading of this sentence, but it is certainly not the preferred one and not the one we are after. While this reading is marked, it seems nonetheless more accessible than the corresponding one with the embedded scalar implicature in simple disjunctive cases, see (30) above. This might reflect a general difference between regular scalar implicatures and free choice in terms of embeddability under negation and downward entailing contexts (see Enguehard & Chemla 2018 for discussion). This point can be sharpened using cases with negative emotive factives like *sorry*, *regret* and *dislike*, as in (41). For instance, the reading of (41) we are after is one suggesting that Olivia can choose between both classes, while at the same time conveying that Noah dislikes her being allowed to take either one. As illustrated in (ii), the reading we would obtain with embedded free choice would be once again too weak: (ii) is compatible with Noah being okay with, or even liking, the fact that Olivia can take only one of the two classes, but disliking that she can choose between the two. Similar observations hold for cases involving factive adjectives like *surprising*.

(ii) Noah is sorry that EXH^{IE+II} [Olivia can take Logic and can take Algebra]

a. ASSERTION
Noah wants that it's not true that Olivia can take Logic and can take Algebra

b. PRESUPPOSITION
Olivia can take Logic and can take Algebra

(43) a. EXH^{IE+II} [Noah is unaware that Olivia can take Logic or Algebra]
b. $\text{EXH}^{IE+II}\ [\neg\Box[\Diamond[\mathsf{L}\vee\mathsf{A}]]_{\Diamond[\mathsf{L}\vee\mathsf{A}]}]$

Consider now the set of formal alternatives in (44) (we leave the conjunctive alternative out for simplicity, but the end result does not change if we leave it in).

(44) $$\text{ALT} := \left\{ \begin{array}{ll} \text{Noah is unaware that O can take L or A} & \neg\Box[\Diamond[\mathsf{L}\vee\mathsf{A}]]_{\Diamond[\mathsf{L}\vee\mathsf{A}]} \\ \text{Noah is unaware that O can take L} & \neg\Box[\Diamond\mathsf{L}]_{\Diamond\mathsf{L}} \\ \text{Noah is unaware that O can take A} & \neg\Box[\Diamond\mathsf{A}]_{\Diamond\mathsf{A}} \end{array} \right\}$$

None of these formal alternatives are innocently excludable. To see this, it is enough to note that the exclusion of one disjunct would necessarily lead to the inclusion of the other. On the other hand, as in the simple unembedded case, the alternatives corresponding to the independent (modalised) disjuncts are both innocently includable. Therefore, the result of exhaustification is as follows:

(45) $$[\![(43\text{-a})]\!] = \neg\Box[\Diamond[\mathsf{L}\vee\mathsf{A}]]_{\Diamond[\mathsf{L}\vee\mathsf{A}]} \wedge \neg\Box[\Diamond\mathsf{L}]_{\Diamond\mathsf{L}} \wedge \neg\Box[\Diamond\mathsf{A}]_{\Diamond\mathsf{A}}$$

Adding the II-alternatives $\neg\Box[\Diamond\mathsf{L}]_{\Diamond\mathsf{L}}$ and $\neg\Box[\Diamond\mathsf{A}]_{\Diamond\mathsf{A}}$ does not affect the assertion of (43-a). However, it adds to (43-a)'s plain presupposition two novel presuppositions, $\Diamond\mathsf{L}$ and $\Diamond\mathsf{A}$. Hence, we derive for (43-a) the desired presupposed free choice reading that Olivia can take Logic ($\Diamond$L) and that she can take Algebra ($\Diamond$A).

These first results are promising since they would allow us to address the challenge of presupposed free choice in Spector & Sudo's (2017) system. The problem however is that the very same reasoning and the very same derivation can be reproduced with sentences like (4), hence spoiling the prediction that (4) should be infelicitous in a context in which it is common ground that all the students took Logic. To illustrate consider the parse of (4) in (46-a) together with its schematic representation in (46-b) and the formal alternatives in (46-c).

(46) a. EXH^{IE+II} [Noah is unaware that some of the students took Logic]
b. $\text{EXH}^{IE+II}\ [\neg\Box[\exists]]_{\exists}$
c. $$\text{ALT} := \left\{ \begin{array}{ll} \text{N is unaware that some students took L} & \neg\Box[\exists]_{\exists} \\ \text{N is unaware that all students took L} & \neg\Box[\forall]_{\forall} \end{array} \right\}$$

It is easy to verify that neither alternatives are innocently excludable. However, both of them are innocently includable and therefore the result of exhaustification is as shown in (47). Here again, adding the II-alternative $\neg\Box[\forall]_{\forall}$ does not add anything to the assertion, yet it strengthens the plain presupposition of (46-a) by adding the novel presupposition that all of the students took Logic ($\forall$). This in turn incorrectly predicts that (4) should be felicitous in a context in which it is known that all of the students took Logic.

(47) $[\![\text{(46-a)}]\!] = \neg\Box[\exists]_{\exists} \wedge \neg\Box[\forall]_{\forall}$

In other words, if we allow the sort of derivation in (43-a) to derive presupposed free choice, we loose the original result from Spector & Sudo 2017, and thus one of the main motivations underlying the elaboration of their system against a unified approach. Arguably, there is a natural way to block the unwarranted result in (47). In particular, one can assume an economy condition regulating the distribution of EXH on which EXH cannot apply if it is vacuous at the assertion level. Such a condition is in fact proposed and discussed in Spector & Sudo 2017 to deal with certain cases unrelated to our concerns. It worth noting however that appealing to this condition in the present cases would reverse the issue without solving it: this condition would block representations like (46-a), hence preserving the good result from Spector & Sudo 2017, but it would also block representations like (43-a), leaving us then with no account of presupposed free choice.

To summarise, presupposed free choice is challenging for the approach by Spector & Sudo (2017), which is otherwise successful in accounting for the interaction between presuppositions and scalar implicatures. As we discussed, presupposed free choice is a problem for Spector & Sudo (2017) in the same way assertive free choice is a problem for a pragmatic approach to assertion-based implicatures: in both cases only ignorance is derived. Furthermore, deriving presupposed free choice as a side-effect of an exhaustification of the assertion leads to over-generation issues with related cases. These issues can be avoided by positing an additional economy condition on EXH's distribution, but we are then left with no account of presupposed free choice in Spector & Sudo's (2017) system. In the following, we move to another challenge coming from presupposed ignorance, which was the other central motivation for their account.

4.2 Presupposed (speaker's) ignorance

As we already mentioned, Spector & Sudo (2017) observe that a sentence like *Noah is unaware that Olivia took Logic or Algebra* cannot be felicitously uttered in a context in which it is common knowledge among the interlocutors that Olivia took Logic, or alternatively that Olivia took Algebra:

(38) CONTEXT: The interlocutors know that Olivia only took Logic.
#Noah is unaware that Olivia took Logic or Algebra.

As Spector & Sudo (2017) and Anvari (2018) discuss, this observation cannot be accounted for under the grammatical approach to presupposed implicatures that we presented in Section 3.2. The reason for that is that this approach is bound to the notion of innocent exclusion and therefore, in such disjunctive cases, can only give rise to a presupposed exclusivity implicature, i.e. the presupposed implicature that *Olivia didn't take both Logic or Algebra.* Yet the presence of this implicature in (38) cannot account for the infelicity of (38) since its contribution is contextually vacuous: it is already taken for granted that Olivia didn't take both Logic or Algebra. The datapoint in (38) thus offers, at least *prima facie*, an empirical argument in favour of the pragmatic side of Spector & Sudo's (2017) system which, by contrast, can account for the infelicity of (38) through the PIP: an utterance of (38) is infelicitous in the suggested context because, by application of the PIP, it gives rise to two inferences, $\neg$CK*(Olivia took Logic)* and $\neg$CK(*Olivia took Algebra*), one of which is contradictory with common knowledge.

We believe that Spector & Sudo's (2017) observation raises indeed an interesting challenge for any grammatical approach to presupposed implicatures. Yet we also believe that their description of the challenge surrounding presupposed ignorance is incomplete and that, in light of the broader empirical picture, postulating a principle like the PIP may not be a satisfying answer after all. Specifically, we make here the novel observation that similar oddity effects reproduce in cases like (48) even if neither of the embedded disjuncts are common knowledge in the context.

(48) a. #Noah is unaware that I have two or more children.
b. #Sue didn't realize that my wife is from France or Italy.
c. #Mary was sorry that Sue had lunch with Noah or me yesterday.

We further observe that the oddity effects found in these examples are in fact similar to those previously found in their non-embedded, non-presuppositional variants in (49) (a.o., Gazdar 1979, Fox 2007, Singh 2008, 2010, Fox & Katzir 2011).

(49) a. #I have two or more children.
b. #My wife is from France or Italy.
c. #Sue had lunch with Noah or me yesterday.

As it seems, all these examples are odd because they give rise to speaker-oriented ignorance inferences that stand in contradiction with common knowledge. Intuitively, a sentence like (48-a), just like its unembedded variant in (49-a), sounds odd because it conveys that the *speaker* is ignorant as to how many children she actually has, and this piece of information arguably conflicts with the common assumption that people are normally knowledgeable about such personal facts. As Singh (2010) discusses, the generation of these ignorance inferences in simple disjunctive cases like (49) appears to be mandatory in normal conversational situations. This point is empirically supported for instance by the contrast in (50): while the exclusivity implicature associated with disjunctive sentences can be suspended, (50-a), overt attempts to cancel ignorance inferences fail, (50-b).

(50) Jane speaks French or Italian.

a. In fact, she speaks both languages.

b. #In fact, she speaks French.

We note that the exact same observations hold of our presuppositional cases: while presupposed exclusivity can be suspended, (51-a), overt attempts to cancel presupposed ignorance inferences fail, (51-b).

(51) Noah is unaware that Jane speaks French or Italian.

a. In fact, she speaks both languages.

b. #In fact, she speaks French.

In the next section, we will argue that the existing parallels between all those cases call for a unified analysis and that a grammatical account of implicatures extending to ignorance inferences precisely offers such a unification. Yet for now we shall simply observe that the application of the PIP in cases like (48) cannot account by itself for the mandatory presence of those conflicting inferences. Thus for instance, the PIP generates for a sentence like (48-a) the presupposed ignorance inference that the exact number of children that the speaker has is not common ground. As one can verify, this inference should yet be unproblematic as long as this information is not mutually shared by the interlocutors (e.g., if this information is not known to the speaker's addressee). Consequently, the sole application of the PIP leaves the infelicity of the examples in (48) unaccounted for.

This issue would of course disappear if we could force in some way the outcome of the PIP to be narrowed down from common knowledge to the speaker's epistemic state, or alternatively if we could amend the PIP so as to generate primary implicatures targeting the speaker's epistemic state. In this eventuality, a sentence like (48-a) would now give rise to two speaker-oriented implicatures, $\neg\mathsf{K}_s$(*the speaker has two children*) and $\neg\mathsf{K}_s$(*the speaker has more than two children*), which together do conflict with common assumptions about s's epistemic state. The problem, however, is that achieving these results in a pragmatic framework on the basis of the PIP is far from straightforward. We will illustrate this difficulty by discussing in turn two ways to strengthen the results of the PIP which we take to be the most principled ones based on the previous literature.

A first option would be to allow the outcome of the PIP to be strengthened by means of auxiliary pragmatic assumptions, for instance by appealing to the Authority assumption originally proposed in Chemla (2008) for strengthening the outcome of *Maximize Presupposition!* (see Rouillard & Schwarz 2017, 2018 for discussion and refinements). Simplifying a bit, the idea would be that a PIP-generated inference of the form $\neg\mathsf{CK}(p)$, where p is the presupposition of an alternative ϕ_p to the utterance, can be narrowed down to the speaker's epistemic state if the addressee assumes that the speaker could have convinced her that p is true simply by presupposing p (Authority assumption) and concludes then that the reason why the speaker didn't presuppose p is because she didn't believe p to be true in the first place, i.e. $\neg \mathsf{K}_s(p)$. As exemplified below, adding such assumptions to the working of the PIP would permit us for instance to account for the fact that, in a run-of-the-mill context, a sentence like (52) can give rise to speaker-oriented ignorance inferences:

(52) Noah is unaware that Sue has two or more children.
  a. By Quality:
     $\rightsquigarrow \mathsf{K}_s(\textit{Sue has two or more children})$
  b. By application of the PIP:
     $\rightsquigarrow \neg\mathsf{CK}(\textit{Sue has two children})$
     $\rightsquigarrow \neg\mathsf{CK}(\textit{Sue has more than two children})$
  c. From (52-b) by Authority assumptions:
     $\rightsquigarrow \neg\mathsf{K}_s(\textit{Sue has two children})$
     $\rightsquigarrow \neg\mathsf{K}_s(\textit{Sue has more than two children})$
  d. Logical consequence from (52-a) and (52-c):
     $\rightsquigarrow \mathsf{I}_s(\textit{Sue has two children}) \wedge \mathsf{I}_s(\textit{Sue has more than two children})$

Yet this reasoning does not easily extend to our problematic cases. The reason for that is that, on standard assumptions, a strengthening process like the one in (52) is to be thought of as an optional pragmatic enrichment. As such, the realization of this process should be guided by the plausibility of its outcome. In the present cases, this condition boils down to checking the consistency of the speaker's resulting beliefs. That is, the auxiliary assumptions used to derive (52-c) from (52-b) can be entertained only if they do not lead to the conclusion that the speaker's beliefs are contradictory. Consequently, for the cases we are interested in such as (48-a), this line of analysis is a non-starter: if it is assumed that the speaker knows how many children she has, then any strengthening along the lines of (52-c) should be blocked and therefore (48-a) should be felicitous; alternatively, if such a strengthening is allowed, then the speaker-oriented inferences so derived should not be conflicting and therefore (48-a) should be felicitous.

A second option, which would at least overcome the previous issue, would be to restate the PIP along the lines of MQ so as to directly generate speaker-oriented inferences. For the sake of the argument, consider the following *ad hoc* principle:

(53) **Presupposed Quantity**
Let p be the presupposition of sentence ϕ_p. If ϕ_p has an alternative ψ_q presupposing q and q is more informative than p, then, if the speaker

believes that both p and q are true, the speaker should prefer ϕ_q to ϕ_p.

Unlike the PIP, Presupposed Quantity would generate in (48) the conflicting ignorance inferences we are after, accounting thus for the infelicity of these examples. Yet this result would come at the cost of spoiling the previous account of (38): in a context where it is common knowledge that Olivia only took Logic, the sentence *Noah is unaware that Olivia took Logic or Algebra* is presuppositionally equivalent and assertively more informative than its alternative *Noah is unaware that Olivia took Logic*. As a result, applying Presupposed Quantity would simply be vacuous in (38) and therefore would leave the infelicity of this sentence unaccounted for.[21]

To summarize, Spector & Sudo's (2017) proposal only offers a partial answer to the challenge of presupposed ignorance: it accounts for the infelicity effects in (38) but leaves those in (48) unexplained. We have further discussed two natural extensions of their proposal and found them to fall short of an explanation at one point or another. While these results need not mean that the present data may not receive a principled pragmatic explanation, they certainly suggest that such an explanation would require certain assumptions that are non-standard in the pragmatic literature. Overall, the set of data we discussed suggest that the sort of implicature-generating mechanism we are after should have at least the following key features: (i) it should establish competition on the basis of logical strength, (ii) it should account for the mandatory presence of certain speaker-oriented ignorance inferences, and (iii) it should be modular enough to allow the generation of inferences that stand in contradiction with common knowledge. Finally, based on the novel parallels we unveiled, a fourth and last desideratum shall be added to this list: (iv) this mechanism should be general enough to apply to the assertion and presupposition of an utterance.

4.3 Taking stock

The data that we have gathered so far reveal a systematic parallelism between the assertion and presuppositional levels in terms of EXCLUSIVITY, FREE CHOICE and IGNORANCE inferences. These parallels are illustrated and synthesised in (10)-(12).

(10) **Exclusivity and Presupposed Exclusivity**

a. Olivia took Logic or Algebra.

21 As the astute reader will note, Presupposed Quantity also fails to account for the most basic *Maximize Presupposition!* effects such as *#A sun is shining* since the competitor, i.e. *The sun is shining*, is contextually equivalent to the actual utterance in normal contexts. This shortcoming is to be rooted into the fact that, just like MQ, Presupposed Quantity establishes competition by considering informativeness rather than logical strength. Remedying this shortcoming is precisely what motivated Heim's (1991) formulation of *Maximize Presupposition!* in the first place.

$\rightsquigarrow$ *Olivia didn't take both Logic and Algebra*

b. Noah is unaware that Olivia took Logic or Algebra.
$\rightsquigarrow$ *Olivia didn't take both Logic and Algebra*

(11) **Free Choice and Presupposed Free Choice**

a. Olivia can take Logic or Algebra.
$\rightsquigarrow$ *Olivia can take Logic and she can take Algebra*

b. Noah is unaware that Olivia can take Logic or Algebra.
$\rightsquigarrow$ *Olivia can take Logic and she can take Algebra*

(12) **Ignorance and Presupposed Ignorance**

a. Olivia took Logic or Algebra.
$\rightsquigarrow$ *The speaker ignores which of the two classes Olivia took*

b. Noah is unaware that Olivia took Logic or Algebra.
$\rightsquigarrow$ *The speaker ignores which of the two classes Olivia took*

We take these parallels to call for an analysis on which the assertion-based and presupposition-based versions of those three types of inferences are derived by means of the same meaning strengthening mechanism. As we have seen, such an analysis should also account for the asymmetry between negative and positive sentences in (37) from Spector & Sudo (2017), repeated below for convenience.

(37) Context: The interlocutors know that all of the students took Logic; Noah, however, has no idea that Logic is this popular among the students.

a. Noah is aware that some of the students took Logic.

b. #Noah is unaware that some of the students Logic.

In this next section, we develop a grammatical theory of implicatures that builds upon Marty's (2017) characterization of assertion-based and presupposition-based implicatures. This characterization will permit us to maintain the original idea from Magri 2009 and Gajewski & Sharvit 2012 that exhaustification operates in the same way in the assertion and in the presupposition of a sentence while at the same time offering an account of the asymmetry in (5) along the lines of Spector & Sudo (2017). We will show that the resulting theory accounts for the parallels in (10)-(12) and provides a unified analysis of those inferences. In particular, we will show how this theory can directly account for presupposed free choice and, once combined with a grammatical account of ignorance inferences à la Meyer (2013), for presupposed ignorance as well.

5 Back to a unified grammatical approach

5.1 Ingredients of the account

5.1.1 Presupposed implicatures in the grammar

The first part of our system follows and extends the grammatical view on presupposed implicatures recently advocated for in Marty (2017). In the spirit of the grammatical approaches we presented in section 3.2, Marty's (2017) approach is one on which assertion-based and presupposition-based implicatures are derived in the grammar by exhaustification, i.e. through the application of EXH. In contrast to previous grammatical approaches, however, it is hypothesised that the distinction between assertion-based and presupposition-based implicatures directly follows from the status of the alternatives under consideration, which is determined by the logical relationship between those alternatives and their base sentence.[22]

In substance, Marty proposes that, when computing the implicatures of a sentence ϕ_p (i.e., a sentence with presupposition p), speakers entertain two sets of alternatives that are mutually exclusive and distinguished on the basis of Strawson-entailment: (i) a set of (innocently excludable) *assertive alternatives*, comprising those alternatives to ϕ_p that can consistently be false whenever ϕ_p is true, (54), and (ii) a set of (innocently excludable) *presuppositional alternatives*, comprising those alternatives to ϕ_p that can only be undefined whenever ϕ_p is true, (55). Said differently, the assertive alternatives are those alternatives that are neither logically entailed nor Strawson-entailed by the prejacent, while the presuppositional alternatives are those alternatives that are not logically entailed but are however Strawson-entailed by the prejacent.

(54) **Assertive alternatives ($\textsc{alt}_{asr}$) and Innocent Exclusion (IE_{asr})**

a. $\textsc{alt}_{asr}(\phi_p) := \{\psi_q \in \textsc{alt}(\phi_p) :\ [[\phi_p]] \nsubseteq [[\psi_q]] \wedge ([[\phi_p]] \cap q) \nsubseteq [[\psi_q]]\}$

b. $\mathsf{IE}_{asr}(\phi_p, C) := \bigcap \left\{ C' \;\middle|\; \begin{array}{l} C' \subseteq C \text{ and } C' \text{ is a maximal subset of } C \text{ s.t.} \\ \{\neg\psi_q : \psi_q \in C'\} \cup \{\phi_p\} \text{ is consistent} \end{array} \right\}$

(55) **Presuppositional alternatives ($\textsc{alt}_{prs}$) and Innocent Exclusion (IE_{prs})**

a. $\textsc{alt}_{prs}(\phi_p) := \{\chi_r \in \textsc{alt}(\phi_p) :\ [[\phi_p]] \nsubseteq [[\chi_r]] \wedge ([[\phi_p]] \cap r) \subseteq [[\chi_r]]\}$

b. $\mathsf{IE}_{prs}(\phi_p, C) := \bigcap \left\{ C' \;\middle|\; \begin{array}{l} C' \subseteq C \text{ and } C' \text{ is a maximal subset of } C \text{ s.t.} \\ \{\neg r : \chi_r \in C'\} \cup \{\phi_p\} \text{ is consistent} \end{array} \right\}$

22 As Marty (2017) discusses, in contrast to previous grammatical approaches, the core of this theory does not require the richness of a two-dimensional theory of meaning. We will follow here his implementation which assumes a uni-dimensional approach to meaning and uses partial semantics for presuppositions. This implementation is also shown in Marty (2017: Chapter 2) to address a number of over-generation issues encountered by a separation approach à la Magri (2009).

Following this characterization of alternatives, Marty (2017) proposes that the exhaustivity operator be defined as shown in (56). For clarity, we will notate this variant of the exhaustivity operator as '$\text{EXH}_{asr+prs}$'.[23]

(56) **Exhaustivity for IE_{asr} and IE_{prs} alternatives** (Marty 2017: (62), p. 37)

$$[\![\text{EXH}_{asr+prs}\ \phi_p]\!] = \lambda w \quad : \ p(w) \wedge \forall \psi_q \in \mathsf{IE}_{asr}\ [q(w)] \wedge \forall \chi_r \in \mathsf{IE}_{prs}\ [\neg r(w)]$$
$$.\ \ [\![\phi_p]\!](w) \wedge \forall \psi_q \in \mathsf{IE}_{asr}\ [\neg [\![\psi_q]\!](w)]$$

At the level of the assertion, $\text{EXH}_{asr+prs}$ achieves the same results as the classical exhaustivity operator (see (16)). However, $\text{EXH}_{asr+prs}$ further strengthens the definedness conditions (i.e., the presupposition) of its prejacent in two ways, achieving in effect similar results as Magri (2009) and Spector & Sudo (2017) combined. First, in line with Magri (2009), $\text{EXH}_{asr+prs}$ presupposes the falsity of the presuppositions of the IE_{prs}-alternatives to its prejacent. Second, in line with Spector & Sudo's (2017), $\text{EXH}_{asr+prs}$ passes up to the whole sentence the presuppositions of the negated IE_{asr}-alternatives. To illustrate these different aspects of the working of $\text{EXH}_{asr+prs}$, consider for instance the following sentence and imagine that it is uttered at the beginning of a trial (adapted from Marty 2017):

(57) A lawyer of the defendant is late. ϕ_p
p := the defendant has one or more lawyers

a. ALT_{asr} := {a lawyer of the plaintiff is late} ψ_q
q := the plaintiff has one or more lawyers

b. ALT_{prs} := {the defendant's lawyer is late} χ_r
r := the defendant has a unique lawyer

This sentence has two potentially relevant alternatives, (57-a) and (57-b), each of which has a distinct status relative to (57). On the one hand, (57-a) is an assertive alternative to (57) since it can be false whenever (57) is true. One the other hand, (57-b) is a presuppositional alternative to (57) since it can be undefined, yet never false whenever (57) is true. Applying $\text{EXH}_{asr+prs}$ to (57) will thus deliver the results in (58). In a nutschell, (57)'s presupposition is strengthened first by adding the presupposition that the defendant doesn't have a unique lawyer (i.e., the negation of the uniqueness presupposition of (57-b)) and next by passing up the presupposition that the plaintiff has at least one lawyer (i.e., the existential presupposition of (57-a)). Finally, (57)'s assertion is strengthened by adding the assertion that no lawyers of the plaintiff is late (i.e., the negation of (57-a)).

(58) $\text{EXH}_{asr+prs}$ [$_{\phi_p}$ a lawyer of the defendant is late]

23 As is customary, we use the notation '$\lambda\chi : \psi(\chi)\ .\ \phi$' to represent a function defined only for objects of which ψ is true (convention from Kratzer & Heim 1998).

a. PRESUPPOSITION
   (i) the defendant has one or more lawyers, and p
   (ii) the defendant doesn't have a unique lawyer, and $\neg r$
   (iii) the plaintiff has one or more lawyers. q

b. ASSERTION
   (i) a lawyer of the defendant is late, and ϕ_p
   (ii) no lawyers of the plaintiff is late. $\neg\psi_q$

Capitalizing on Marty's (2017) proposal, we propose to refine the above account by extending the notion of innocent inclusion (Bar-Lev & Fox 2017, Bar-Lev 2018, see section 2.3) to presuppositional alternatives as well. Concretely, we propose to keep Bar-Lev & Fox's (2017) characterization of II-alternatives and to use it to define in this system the set of innocently includable *assertive* alternatives, (59-a). We add to this definition the corresponding one for innocently includable *presuppositional* alternatives in (59-b): the II_{prs}-alternatives to a sentence ϕ_p are those presuppositional alternatives to ϕ_p whose presupposition can be added to ϕ_p's presupposition without conflicting with any of the potential presupposed implicatures arising from ϕ_p on the basis of the IE_{prs}-alternatives.

(59) **Innocently Includable alternatives ($\mathbf{II}_{asr}$ and $\mathbf{II}_{prs}$)**

a. $\mathsf{II}_{asr}(\phi_p, C) :=$
$$\bigcap\left\{ C'' \;\middle|\; \begin{array}{l} C'' \subseteq C \text{ and } C'' \text{ is a maximal subset of } C \text{ s.t.} \\ \{\chi_r : \chi_r \in C''\} \cup \{\neg\psi_q : \psi_q \in \mathsf{IE}_{asr}(\phi_p, C)\} \cup \{\phi_p\} \text{ is consistent} \end{array} \right\}$$

b. $\mathsf{II}_{prs}(\phi_p, C) :=$
$$\bigcap\left\{ C'' \;\middle|\; \begin{array}{l} C'' \subseteq C \text{ and } C'' \text{ is a maximal subset of } C \text{ s.t.} \\ \{r : \chi_r \in C''\} \cup \{\neg q : \psi_q \in \mathsf{IE}_{prs}(\phi_p, C)\} \cup \{\phi_p\} \text{ is consistent} \end{array} \right\}$$

The exhaustivity operator in (56) must be then minimally revised so as to integrate and specify the contribution of the II_{asr} and II_{prs} alternatives to the outcome of exhaustification. We propose that it be expanded as shown in (60).

(60) **Exhaustivity for $\mathbf{IE}_{asr}$, $\mathbf{II}_{asr}$, $\mathbf{IE}_{prs}$, and $\mathbf{II}_{prs}$ alternatives**

$[[\textsc{exh}^{IE+II}_{asr+prs}\ \phi_p]] =$
$$\begin{array}{ll} \lambda w & : p(w) \wedge \forall \chi_r \in (\mathsf{IE}_{asr} \cup \mathsf{II}_{asr} \cup \mathsf{II}_{prs})\ [r(w)] \wedge \forall \psi_q \in \mathsf{IE}_{prs}\ [\neg\, q(w)] \\ & .\ [[\phi_p]](w) \wedge \forall \chi_r \in \mathsf{II}_{asr}\ [[[\chi_r]](w)] \wedge \forall \psi_q \in \mathsf{IE}_{asr}\ [\neg[[\psi_q]](w)] \end{array}$$

At a general level, this exhaustivity operator can be thought of as the synthesis of the exhaustivity operator proposed in Marty (2017) and the one proposed in Bar-Lev & Fox (2017). In particular, it is easy to verify that, at the level of the assertion, $\textsc{exh}^{IE+II}_{asr+prs}$ reproduces the results of $\textsc{exh}^{IE+II}$ in (25) and that, at the level of the presupposition, it preserves the results of $\textsc{exh}_{asr+prs}$ in (56). The novel feature of this operator is therefore solely concerned with the contribution of the II-alternatives at the presuppositional level: (i) $\textsc{exh}^{IE+II}_{asr+prs}$ passes up to the whole sentence the presuppositions of the II_{asr} alternatives, just like it passes up the presupposition of the IE_{asr} alternatives, and (ii) $\textsc{exh}^{IE+II}_{asr+prs}$ presupposes the truth of the presuppositions of the II_{prs} alternatives, which we take to be the presuppositional analog of conveying the truth of the II_{asr} alternatives in the assertion. As we shall later see, this feature will play a critical role in accounting for the parallels observed between FREE CHOICE and PRESUPPOSED FREE CHOICE.

To complete the first part of our theory, we need to clarify our assumptions concerning the distribution of the exhaustivity operator and the contextual factors affecting implicature-computation. Following Magri (2009, 2011, 2014, 2017), we assume that EXH can be inserted at any propositional node and that it is syntactically mandatory at matrix scope.[24] In this framework, the domain of quantification of EXH is contextually modulated by relevance considerations and those modulations account in turn for the context-dependency of implicatures. In line with the previous literature, we will assume that relevance considerations impact on implicature-computation in at least three major ways.[25]

First of all, EXH requires by definition that its prejacent be relevant. Second, relevance is closed with respect to contextual equivalence. Hence, whenever the prejacent ϕ_p of EXH is relevant, any *assertive alternative* to ϕ_p that is contextually equivalent to ϕ_p is also relevant. In the case of presuppositional alternatives, we take the evaluation of relevance to be restricted to the only meaning component differentiating those alternatives from their base sentence, namely the presupposition. Thus, whenever the prejacent ϕ_p of EXH is relevant, any *presuppositional*

24 This assumption about the obligatory insertion of EXH is extended in Magri (2011) and Marty (2017) from the matrix level to any embedded propositional level. However, for the purposes of this paper, it is enough to assume that the exhaustivity operator is mandatory at matrix level since we will not be concerned with infelicity effects arising from the computation of conflicting embedded implicatures. We refer the reader to Marty (2017: Appendix A) for discussion and refinements of this assumption in the case of presupposed implicatures.

25 The role of relevance in implicature-computation can be modeled à la Magri by assuming that, just like other (universal) quantifiers, the domain of the exhaustivity operator is restricted by a contextually assigned relevance predicate, notated R. On this view, our exhaustivity operator is thus to be formulated as shown in (i). Note however that what is crucial to our account is only the idea that, in order for an assertive or presuppositional alternative to enter implicature-computation, it need not only be excludable or includable in the appropriate sense but also relevant (i.e., there is no point in excluding or including meaning components of irrelevant alternatives).

(i) $[\![[[\textsc{exh}^{IE+II}_{asr+prs}\ R]\ [\phi_p]]]\!] =$

$$\begin{aligned} \lambda w\ &: [\![\phi_p]\!] \in \mathsf{R} \wedge p(w) \wedge \forall \chi_r \in (\mathsf{R} \cap (\mathsf{IE}_{asr} \cup \mathsf{II}_{asr} \cup \mathsf{II}_{prs}))\ [r(w)] \wedge \forall \psi_q \in (\mathsf{R} \cap \mathsf{IE}_{prs})\ [\neg q(w)] \\ &.\ [\![\phi_p]\!](w) \wedge \forall \chi_r \in (\mathsf{R} \cap \mathsf{II}_{asr})\ [[\![\chi_r]\!](w)] \wedge \forall \psi_q \in (\mathsf{R} \cap \mathsf{IE}_{asr})\ [\neg [\![\psi_q]\!](w)] \end{aligned}$$

alternative to ϕ_p whose presupposition is contextually equivalent to ϕ_p's presupposition is also relevant. Crucially, if an alternative is deemed relevant in that sense, that alternative cannot be pruned from the domain of EXH and consequently the assertive or presupposed implicature associated with it becomes then mandatory (Magri 2009, 2011). Finally, an alternative that is allowed to be pruned from the domain of EXH must effectively be pruned if the implicature associated with that alternative would make the global meaning of the sentence weaker and thus decrease its overall informativeness. As it has already been pointed out in the literature, additional relevance considerations may affect implicature-computation in specific environments such as disjunctive sentences. These cases, and the refinements they call for, are discussed below in 5.1.3 and 5.2.3.

5.1.2 Presupposed ignorance in the grammar

The second part of our system builds on the grammatical approach to ignorance implicatures proposed in Meyer (2013) and extends it to presuppositions. In essence, this is done by refining the semantics of the Matrix K operator as shown in (61) so as to offer a proper treatment of presuppositions.[26] As far as the distribution of this operator is concerned, we adopt Meyer's (2013) Matrix K Axiom: all assertively used sentences are covertly modalised by the global K operator in (61). Following these assumptions, an utterance of a sentence ϕ_p by a speaker s must be parsed at LF as $[K_s\ \phi_p]$, the semantic outcome of which presupposes that s believes the presupposition p of ϕ_p and asserts that s believes ϕ_p

(61) $[\![K_x\ \phi_p]\!] = \lambda w : \forall w' \in Dox(x)(w)[p(w')]\ .\ \forall w' \in Dox(x)(w)[[\![\phi_p]\!](w')]$

With our two operators in place, an utterance of the surface form ϕ_p is now to be mapped onto an LF of the schematic form in (62), which leaves the hearer with the task of deciding which alternatives are to be taken into account for implicature computation at each of the two levels where exhaustification can be performed, i.e. above and/or below K.[27]

(62) $[\text{EXH}\ [K_s\ [\text{EXH}\ [\phi_p]]]]$

We have already shown in (19) how, on Meyer's proposal, the interactions between EXH and K account for the pattern of inferences associated with simple disjunctive sentences, i.e. for the exclusivity and ignorance implicatures that these sentences

26 Note that this refinement simply corresponds to what is predicted by standard accounts of presupposition projection under attitude predicates (Heim 1992 among others).

27 For simplicity, we will leave out the superscripts and subscripts of our exhaustivity operator $\text{EXH}^{IE+II}_{asr+prs}$ (see (60) and its refined version in footnote 25) and simply call it EXH in the following.

give rise to. In the next section, we will see how our extended system permits to reproduce these good results for the presuppositional counterparts of these implicatures. For the time being, we shall simply emphasize another important feature of this system which is directly inherited from Meyer's approach: the scopal interactions between EXH and K permit us to account for subtle contextual variations in the logical strength of the assertion-based and presupposition-based implicatures accessed by speakers. To illustrate this feature, consider first an utterance of *Some of the students smoke* which, based on our assumptions, is to be parsed at LF as follows:

(63) [EXH$_{①}$ [K_s [EXH$_{②}$ [some of the students smoke]]]]
  a. ALT_{asr}(②) = {[all of the students smoke]}
  b. ALT_{asr}(①) = {[K_s [EXH [all of the students smoke]]]}

As before, the hearer can exhaustify the meaning of (63) at the level of EXH$_{②}$ on the basis of the assertive alternative in (63-a), generating then a so-called *secondary* implicature, one that attributes to the speaker a state of certainty towards the alternative it is based on: the speaker *believes* that *not* all the students smoke, (64-a). But the hearer now has another interpretative option since she can instead exhaustify the meaning of (63) at the level of EXH$_{①}$ on the basis of the assertive alternative in (63-b). This exhaustification process generates a *primary* implicature, one that leaves open the possibility that the speaker be ignorant as to whether the target modalised alternative is true: the speaker *does not believe* that all the students smoke, (64-b).

(64) a. Secondary assertion-based implicature through EXH$_{②}$:
  $K_s(\textit{some of the students smoke}) \wedge K_s\neg(\textit{all of the students smoke})$
  b. Primary assertion-based implicature through EXH$_{①}$:
  $K_s(\textit{some of the students smoke}) \wedge \neg K_s(\textit{all of the students smoke})$

For what is most relevant here, these results naturally extend on our account to presupposition-based implicatures. Consider for instance an utterance of *A brother of John smokes*, the plain meaning of which merely presupposes that John has at least one brother. In our system, this utterance is parsed at LF as shown in (65).

(65) [EXH$_{①}$ [K_s [EXH$_{②}$ [a brother of John smokes]]]]
  a. ALT_{prs}(②) = {[John's brother smokes]}
  b. ALT_{prs}(①) = {[K_s [EXH$_{②}$ [John's brother smokes]]]}

As it has long been noted in the literature (a.o., Sauerland 2008, Chemla 2008, Heim 1991, Rouillard & Schwarz 2017, Elliott & Sauerland 2019), the non-uniqueness presupposed implicature associated with such an utterance can be more or less strong depending on the context and take one of two forms: this utterance can convey that the speaker takes for granted that John has more than just one brother (*secondary* implicature), or simply that the speaker cannot take for granted that John has just one brother (*primary* implicature), e.g. because she is ignorant about that matter. Our account straightforwardly captures these possible variations in the logical strength of presupposition-based implicatures: the stronger form of this inference is derived by exhaustifying the meaning of (65) at the level of EXH② on the basis of the presuppositional alternative in (65-a), as shown in (66-a), while its weaker form is derived by doing so only at the level of EXH① on the basis of the presuppositional alternative in (65-b), as shown in (66-b).[28]

(66) a. Secondary presupposition-based implicature through EXH②:
K_s(*John has a brother*) $\wedge K_s\neg$(*John has a unique brother*)
b. Primary presupposition-based implicature through EXH①:
K_s(*John has a brother*) $\wedge \neg K_s$(*John has a unique brother*)

Our system thus offers a uniform account of the commonly observed variations in the strength of assertion-based *and* presupposition-based implicatures by deriving both their primary and secondary forms in the grammar. Specifically, for any utterance ϕ_p produced by a speaker s, any alternative $\psi_q \in \mathsf{IE}_{asr}(\phi_p)$ and any alternative $\chi_r \in \mathsf{IE}_{prs}(\phi_p)$, our system is able to generate presupposition-based implicatures of the form $K_s(\neg r)$ or $\neg K_s(r)$, which are themselves presuppositions, in addition to assertion-based implicatures of the form $K_s(\neg\psi_q)$ or $\neg K_s(\psi_q)$, which also add their presupposition q. In this system, pragmatics is the sum of principles that guide the decision process to compute an implicature of a particular strength at a given context. As on other approaches to scalar implicatures, we take this process to be guided among other things by one's understanding of what is relevant in context and one's assumptions about the speaker's epistemic state. In the next subsection, we review the core conditions under which the computation of an implicature becomes mandatory and gives rise to representations that contradict common knowledge.

5.1.3 Mandatory implicatures and contextual mismatches

Before we turn to the empirical scope of our theory, we need to emphasize one last important feature of the system we devised: it is a conservative extension of the

28 We note here that a similar proposal for deriving secondary presupposition-based implicatures in the grammar has recently been made in Elliott & Sauerland (2019). These authors show how this proposal addresses Heim's (1991) original observation that the inferences derived from *Maximize Presupposition!* may have a weak epistemic status.

theory of oddness originally developed in Magri 2009, 2011, 2014, later pursued in Schlenker 2012 and fully extended to presuppositional effects in Marty 2017. On this theory, the infelicity of sentences like (67)-(68) results from the mandatory computation of a mismatching implicature, that is an obligatory implicature which contradicts common knowledge. The logic underlying this process is as follows. First, the domain of quantification of EXH is constrained by a contextually assigned relevance predicate R (see 5.1.1 and footnote 25). Second, the denotation of EXH requires that its prejacent ϕ_p be relevant, i.e. $[[\text{EXH R}]\ \phi_p]$ presupposes $\phi_p \in \mathrm{R}$. Third, since relevance is closed under contextual equivalence, it follows that any assertive alternative ψ_q to ϕ_p that is contextually equivalent to ϕ_p and any presuppositional alternative χ_r to ϕ_p whose presupposition r is contextually equivalent to p must also be relevant.[29] In sentences like (67)-(68), we have it that the implicatures associated with the alternatives to EXH's prejacent become mandatory, resulting in a contextual mismatch, hence the oddness of these examples.[30]

(67) #Some Italians come from a beautiful country.

a. Parse: $\text{EXH}_R\ [_{\phi_p}$ some Italians come from a beautiful country$]$
b. Alternative: $[_{\psi_q}$ all Italians come from a beautiful country$]$
c. Relevance: $\psi_q \Leftrightarrow_c \phi_p$ and so $\psi_q \in R$
d. Obligatory implicature: #Not all Italians come from a beautiful country

(68) #A sun is rising.

a. Parse: $\text{EXH}_R\ [_{\phi_p}$ a sun is rising$]$
b. Alternative: $[_{\chi_r}$ the sun is rising$]$
c. Relevance: $r \Leftrightarrow_c p$ and so $\chi_r \in R$
d. Obligatory implicature: #There isn't a unique sun

Here we aim at establishing two things: first, the above results are fully preserved in our extended exhaustivity-based framework that integrates the Matrix K operator

29 Suitable algorithms for computing the domain restriction R as well as evaluating contextual equivalence at embedded levels are discussed in Schlenker (2012: Sections 4.1 & 4.2.1) and further developed in Marty (2017: Appendix A) based on the original proposal in Singh (2011).

30 We take it that the severity of the oddity effects arising from these sentences comes partly from the computation of a mismatching implicature and partly from the fact that the surprising piece of information is conveyed by the implicit, strengthened meaning of these sentences as opposed to be explicitly expressed by overt materials and be flagged as a controversial contribution, one that may be debated and may or may not be accepted by all interlocutors (after all, accepting those sentence would require a substantial revision of the interlocutors' common assumptions about the world). In particular, we note that the overt counterparts of those implicit meanings, e.g. *Some but not all Italians come from a beautiful country*, albeit slightly odd, does not seem to give rise to such strong repulsion effects, arguably because the controversial piece of information is now put forward and left up for further conversational developments (explanation and disagreement).

(for similar suggestions see Meyer 2013, 2014 for assertion-based implicatures and Elliott & Sauerland 2019 for presupposition-based implicatures) and second, our system leaves room for further mismatching implicature to arise due to the extra work of the exhaustivity operator occurring above K. To establish the first point, consider the parses in (69) and (70) which correspond to the parses that are predicted by our system for the sentences in (67) and (68), respectively:

(69) [EXH① [K_s [EXH② [some Italians come from a beautiful country]]]]
  a. IE_{asr}(②) = {[all Italians come from a beautiful country]}
  b. IE_{asr}(①) = {[K_s [EXH② [all Italians come from a beautiful country]]]}

(70) [EXH① [K_s [EXH② [a sun is shining]]]]
  a. IE_{prs}(②) = {[the sun is shining]}
  b. IE_{prs}(①) = {[K_s [EXH② [the sun is shining]]]}

At the level of EXH②, a mismatching implicature is mandatorily computed, just as before, because EXH②'s prejacent is contextually equivalent to its target (assertive or presuppositional) alternative. In (69) for instance, on the shared assumption that all Italians come from the same country, that *the speaker believes that some Italians come from a beautiful country* contextually entails that *she believes that all Italians do* and vice-versa. Hence, at the level of the Matrix K operator, i.e. EXH①'s prejacent, we get the following semantic outcome: K_s(*some but not all Italians come from a beautiful country*). The same reasoning applies in (70) where, on the shared assumption that there is a unique sun, the application of EXH② mandatorily gives rise to a mismatching non-uniqueness implicature. Next, at the level of EXH①, these outcomes are left unchanged since the additional implicatures that could arise at that level are rendered optional and superfluous by the work of EXH②. In (69) for instance, EXH①'s prejacent entails K_s(*some but not all Italians come from a beautiful country*) and therefore it is not contextually equivalent to its modalised *all*-alternative, which can thus be pruned from the domain of EXH①. Note however that computing the optional implicature associated with that alternative would be harmless (yet redundant) since EXH①'s prejacent already entails $\neg K_s$(*all Italians come from a beautiful country*). The very same observations hold of the example in (70). In sum, for these cases, our approach simply reproduces the classical results from previous exhaustivity-based accounts.

Yet there are also cases where our approach makes novel predictions and captures oddity effects that are beyond the explanatory scope of those previous accounts. Among them are the oddity effects triggered by mismatching ignorance implicatures arising from disjunctive sentences. Recall from our discussion of the examples in (49) that a sentence like (71) is deemed deviant because it gives rise to ignorance implicatures that contradict the contextual assumption that one should be knowledgeable about the personal facts at hands:

(71) #My wife speaks French or Italian.

a. $\rightsquigarrow \#I_s(\textit{the speaker's wife speaks French})$

b. $\rightsquigarrow \#I_s(\textit{the speaker's wife speaks Italian})$

In a grammatical system without the Matrix K operator, the oddness of these sentences is left to additional pragmatic principles like MQ. This is because a disjunctive sentence like (71) has only one innocently excludable alternative of immediate interest, its conjunctive alternative, and computing the implicature associated with that alternative only gives us that the speaker's wife doesn't speak both French and Italian, a piece of information which is unproblematic and thus cannot account for the oddness of (71).

(72) $[\textsc{exh}_{②}$ [my wife speaks French or Italian]$]$

a. $\mathsf{IE}_{asr}(②) = \{[\textsc{French and Italian}]\}$

b. $\rightsquigarrow [\mathsf{F} \vee \mathsf{I}] \wedge \neg[\mathsf{F} \wedge \mathsf{I}]$

But consider now the parse of (71) that is predicted on our approach (for our purposes, we can set aside the modalized conjunctive alternative here):

(73) $[\textsc{exh}_{①}\ [K_s\ [\textsc{exh}_{②}$ [my wife speaks French or Italian]$]]]$

a. $\mathsf{IE}_{asr}(①) = \{[K_s\ [\textsc{exh}_{②}\ [\textsc{French}]]], [K_s\ [\textsc{exh}_{②}\ [\textsc{Italian}]]]\}$

b. $\rightsquigarrow K_s[\mathsf{F} \vee \mathsf{I}] \wedge I_s[\mathsf{F}] \wedge I_s[\mathsf{I}]$

At the outermost level of exhaustification, both modalised disjuncts are innocently excludable alternatives to $\textsc{exh}_{①}$'s prejacent, and this regardless of one's assumptions about the relevance-based restrictions on the domain of quantification of $\textsc{exh}_{②}$:

(74) Suppose $\mathsf{R}_{②} = \{\ \}$

a. $[\![[K_s\ [\textsc{exh}_{②}\ [\mathsf{French}]]]]\!] = K_s(\mathsf{F})$

b. $[\![[K_s\ [\textsc{exh}_{②}\ [\mathsf{Italian}]]]]\!] = K_s(\mathsf{I})$

(75) Suppose $\mathsf{R}_{②} = \{[\textsc{French}],\ [\textsc{Italian}]\}$

a. $[\![[K_s\ [\textsc{exh}_{②}\ [\mathsf{French}]]]]\!] = K_s(\mathsf{F} \wedge \neg \mathsf{I})$

b. $[\![[K_s\ [\textsc{exh}_{②}\ [\mathsf{Italian}]]]]\!] = K_s(\mathsf{I} \wedge \neg \mathsf{F})$

Computing the implicatures associated with either of those pairs of alternatives gives rise, together with the truth of $\textsc{exh}_{①}$'s prejacent, to the ignorance inferences we are after: if $K_s(\mathsf{F} \vee \mathsf{I})$, $\neg K_s(\mathsf{F})$ and $\neg K_s(\mathsf{I})$, then $I_s(\mathsf{F})$ and $I_s(\mathsf{I})$.[31] As we have already established, the mandatory computation of such mismatching implicatures

31 This is because if $K_s(\mathsf{F} \vee \mathsf{I})$, $\neg K_s(\mathsf{F} \wedge \neg \mathsf{I})$ and $\neg K_s(\mathsf{I} \wedge \neg \mathsf{F})$, then $I_s(\mathsf{F})$ and $I_s(\mathsf{I})$.

would account for the oddness of (71). One question yet remains to be addressed: since none of the alternatives in (74)-(75) is contextually equivalent to EXH$_{①}$'s prejacent, what forces then the computation of those implicatures?

Recall that, in our system, the only way an implicature can be avoided is if the alternative it is based on can be pruned from R, i.e. the set of relevant propositions. So far, we have only discussed one general constraint on pruning stemming from Magri (2009): assertive alternatives that are contextually equivalent to EXH's prejacent and presuppositional alternatives whose presuppositions are contextually equivalent to EXH's prejacent's presupposition cannot be pruned from R. Several authors have argued, however, that certain environments like disjunctions or conditionals impose additional relevance-based constraints by virtue of their intrinsic properties (see in particular Simons 2001a, Fox 2007, Singh 2008, Fox & Katzir 2011). In particular, it has been argued that, in disjunctions, neither of the disjuncts can independently be pruned without also pruning the other. Since at least one of the disjuncts has to be relevant for the whole disjunction to be relevant, it follows that neither can be pruned from R if the whole disjunction is itself in R. We adopt here this line of explanation to account for the observation that disjunctions give rise to obligatory ignorance inferences in normal conversational situations, and in particular to account for the mandatory computation of the mismatching implicatures we described above. We will argue in 5.2.3 that this restriction on pruning extends to disjunctive presuppositions, accounting for our novel cases of presupposed ignorance.

5.2 Empirical scope of the account

5.2.1 Factive presuppositions and the asymmetry

We start by showing that our proposal provides an empirically adequate account of the generation and distribution of presupposed implicatures in factive environments. For these purposes, consider again the case of EXCLUSIVITY implicatures:

(10) **Exclusivity and Presupposed Exclusivity**

a. Olivia took Logic or Algebra.
⇝ *Olivia didn't take both Logic and Algebra*

b. Noah is unaware that Olivia took Logic or Algebra.
⇝ *Olivia didn't take both Logic and Algebra*

As we explained, the EXCLUSIVITY implicatures in (10) are identical in all but one important aspect: this implicature arises in (10-a) in the assertion, while it arises in (10-b) in the presupposition, leaving the assertion unaffected. These similarities and differences are immediately accounted for on our proposal. Ignoring for now the K layer (which we go back to below), the EXCLUSIVITY implicatures in (10) are both derived in our system on the basis of their conjunctive alternatives. Crucially, however, these conjunctive alternatives have a different logical status in both cases and consequently, upon exhaustification, contribute to strengthen the meaning of their base sentence in different ways. As before, the conjunctive alternative to (10-a) is an assertive alternative to (10-a). Hence, upon exhaustification, (10)'s meaning is strengthened by adding to its plain assertion the falsity of the assertion of its assertive alternative.

(76) EXH [Olivia took Logic or Algebra]

a. IE_{asr} = {[Olivia took Logic and Algebra]}

b. STRENGTHENED ASSERTION:
(Olivia took Logic or Algebra) $\wedge\neg$ *(Olivia took Logic and Algebra)*

In the second case, however, the corresponding conjunctive alternative is a presuppositional alternative to EXH's prejacent. Indeed, *Noah is unaware that Olivia took Logic or Algebra* is logically weaker than and Strawson-entailed by *Noah is unaware that Olivia took Logic and Algebra*, consistent with Marty's (2017) characterization of presuppositional alternatives. As a result, upon exhaustification, (10-b)'s meaning is strengthened by adding to its plain presupposition the falsity of the presupposition of its presuppositional alternative. The resulting EXCLUSIVITY implicature is in essence the same as in (10-a), but it is now in the presupposition.

(77) EXH [Noah is unaware that Olivia took Logic or Algebra]

a. IE_{prs} = {[Noah is unaware that Olivia took Logic and Algebra]}

b. STRENGTHENED PRESUPPOSITION:
(Olivia took Logic or Algebra) $\wedge\neg$ *(Olivia took Logic and Algebra)*

For completeness, we mention two additional results pertaining to the generality of our system. First, the integration of Meyer's (2013) K-operator and its extension to presuppositions makes it possible to derive instead the weaker, primary implicatures associated with the above alternatives. This result is achieved as before by letting only the outermost occurence of EXH operate on these alternatives, as illustrated in (78) and (79).

(78) [EXH$_{①}$ [K_s [EXH$_{②}$ [Olivia took Logic or Algebra]

a. IE_{asr}(①) = {[K_s [EXH$_{②}$ [Olivia took Logic and Algebra]]]}

b. Assume $\mathsf{R}_{②}$ = { }

c. STRENGTHENED ASSERTION:
K_s*(Olivia took Logic or Algebra)* $\wedge\neg K_s$*(Olivia took Logic and Algebra)*

(79) [EXH$_{①}$ [K_s [EXH$_{②}$ [Noah is unaware that Olivia took Logic or Algebra]

a. $\mathsf{IE}_{asr}(①) = \{[K_s\,[\textsc{exh}_{②}\,[\text{Noah is unaware that Olivia took Logic and Algebra}]]]\}$
b. Assume $\mathsf{R}_{②} = \{\ \}$
c. STRENGTHENED PRESUPPOSITION:
K_s(Olivia took Logic or Algebra)$\wedge\neg K_s$(Olivia took Logic and Algebra)

Second, our proposal naturally extends to other factive environments and other embedded scalars. For instance, the *not-all* implicature associated with sentence of (80-a) follows from the exhaustification process we described in (76) while its presuppositional version in (80-b) follows from the one we described in (77).

(80) a. Some of the students took Logic.
STRENGTHENED ASSERTION:
$\rightsquigarrow$ *some but not all of the students took Logic*
b. Noah didn't realize that some of the students took Logic
STRENGTHENED PRESUPPOSITION:
$\rightsquigarrow$ *some but not all of the students took Logic*

In sum, our proposal replicates so far the good results of previous grammatical approaches in full generality. What we shall now see is that it improves upon those previous approaches by accounting for the puzzling contrast in (37) discussed in Spector & Sudo (2017):

(37) CONTEXT: The interlocutors know that all of the students took Logic; Noah, however, has no idea that Logic is this popular among the students.
a. Noah is aware that some of the students took Logic.
b. #Noah is unaware that some of the students Logic.

In our system, the source of this contrast is to be found in the different status of the target *all*-alternatives in both cases. In (37-b), on the one hand, the target *all*-alternative is a presuppositional alternative to EXH's prejacent: upon exhaustification, the negation of its (stronger) presupposition can be added to the plain presupposition of EXH's prejacent. In the present case, since the presupposition of EXH's prejacent is contextually equivalent to that of its presuppositional *all*-alternative in the suggested context, this strengthening is mandatory and results in a contextual contradiction, as shown in (81), hence the infelicity of (37-b).[32]

32 For presentation purposes, we focus here on the outputs of the innermost level of exhaustification. On our assumptions, however, the full LFs of the sentences in (37) should be as shown in (i):

(i) a. $[\textsc{exh}_{①}\,[K_s\,[\textsc{exh}_{②}\,[\text{Noah aware that some of the students took Logic}]$
$\mathsf{IE}_{asr}(①):\quad [K_s\,[\textsc{exh}_{②}\,[\text{Noah is aware that all of the students took Logic}]$
b. $[\textsc{exh}_{①}\,[K_s\,[\textsc{exh}_{②}\,[\text{Noah is unaware that some of the students took Logic}]$
$\mathsf{IE}_{prs}(①):\quad [K_s\,[\textsc{exh}_{②}\,[\text{Noah is unaware that all of the students took Logic}]$

(81) #Noah is unaware that some of the students took Logic.

a. Parse: EXH$_R$ [$_{\phi_p}$ Noah is unaware that some of the students took Logic]

b. IE_{prs}: [$_{\psi_q}$ Noah is unaware that all of the students took Logic]

c. Relevance: $q \Leftrightarrow_c p$ and so $\psi_q \in R$

d. Obligatory implicature: #Not all the students took Logic

In (37-a), on the other hand, the corresponding *all*-alternative is an assertive alternative to EXH's prejacent: upon exhaustification, its presupposition and the negation of its (stronger) assertion can be added all together to the plain meaning of EXH's prejacent. In the present case, since EXH's prejacent is not contextually equivalent to its *all*-alternative, this strengthening remains yet optional.

(82) Noah is aware that some of the students took Logic.

a. Parse: EXH$_R$ [$_{\phi_p}$ Noah is aware that some of the students took Logic]

b. IE_{asr}: [$_{\psi_q}$ Noah is aware that all of the students took Logic]

c. Relevance: $\phi_q \not\Leftrightarrow_c \phi_p$ and so pruning from R is possible

d. Possible implicature: John is not aware that all the students took Logic

Our account of the contrast in (37) is thus very similar to Spector & Sudo's (2017). In particular, both accounts predict the sentence in (37-b) to be odd due to the mandatory generation of some conflicting inference (attributed to the working of the PIP in one case, and to the working of EXH in the other), while no such conflict need to arise in (37-a). Interestingly, however, both accounts make distinct predictions regarding the felicity conditions of (37-a). As we explained, Spector & Sudo's (2017) account predicts that, in order for (37-a) to be felicitous in the context at hand, the implicature in (82-d) must be computed so as to avoid the PIP from generating a contextual contradiction (see 3.3 for refinements). By contrast, our account predicts (37-a) to be felicitous in that same context independent from the optional strengthening process in (82-d). The example in (83) provides us with a test case to compare and evaluate these predictions:

(83) CONTEXT: The interlocutors know that all of the students took Logic.
I really don't know whether Noah is aware that all the students took Logic.
But *he is aware that some of them did.*

Those richer LFs deliver yet the same core results as the impoverished ones we use as working examples. In particular, we note that the implicatures generated at the level of EXH$_{①}$ can account by themselves for the contrast in (37): (i-a) may only give rise to the assertion-based implicature $\neg K_s$(*Noah is aware that all of the students took Logic*), consistent with common knowledge, while (i-b) must give rise to the presupposition-based implicature $\neg K_s$(*all of the students took Logic*), in conflict with common knowledge. While this refinement may make a difference for (i-a) regarding the strength of the derived implicature, it doesn't make any for (i-b) since, as we show below, the conflicting implicature at hand is already entailed by the exhaustified complement of K_s in (i-b).

$\not\rightsquigarrow \neg$(*Noah is aware that all the students took Logic*)

As before, the above context is one in which it is common ground that *all of the students took Logic*. However, the speaker is now explicitly stating that he is ignorant as to whether Noah is aware that all the students took Logic, and this information subsequently leads one to suspend the implicature previously associated with (37-a) (emphasized in italics in (83)). Crucially, we observe that the suspension of that implicature leaves the felicity of (37-a) unaffected. While this novel observation is in line with our predictions, it is problematic for Spector & Sudo (2017): in the absence of the target implicature, the PIP should apply just like in (37-b), and therefore the discourse in (83) should be perceived as infelicitous, contra speakers' intuitions. We conclude then that the test case in (83) supports our account, while it calls for further explanation on Spector & Sudo's (2017).

To summarize, our proposal improves upon past grammatical approaches in offering a satisfying solution to the challenge raised by the contrast in (37). In contrast to Spector & Sudo's (2017) proposal, these results are achieved here in a fully grammatical system, without need for a separate principle like the PIP. Finally, the present account makes novel predictions regarding the felicity of sentences like (37-a) which we have shown to be empirically supported and to improve upon Spector & Sudo's (2017) original account.

5.2.2 Presupposed free choice

Let us now go back to presupposed free choice, repeated in (84-a) and schematically represented in (84-b), and let us illustrate how it is accounted for on our proposal.

(84) a. Noah is unaware that Olivia can take Logic or Algebra.
b. $\neg\Box[\Diamond[\mathsf{L}\vee\mathsf{A}]]_{\Diamond[\mathsf{L}\vee\mathsf{A}]}$

Recall that, in our system, the logically non-weaker alternatives to a given sentences are divided into two sets: its assertive alternatives and its presuppositional ones. It is easy to see that among the formal alternatives to (84-a) already considered above in (44), those that are logically non-weaker than (84-a) are also Strawson-entailed by (84-a) and therefore qualify as presuppositional alternatives, (85).[33]

33 For simplicity, we will ignore again the conjunctive alternative to (84-a) in (i). As the reader can verify, if we were to add it in, the resulting presuppositional inference would be stronger, conveying further that *Olivia cannot take both Logic and Algebra*.

(i) Noah is unaware that Olivia can take Logic and Algebra.

The situation here is again parallel to what we see at the assertion level: the conjunctive alternative to (ii-a) in (ii-b) gives rise to the inference that *Olivia cannot take both Logic and Algebra*. The

(85) $\textsc{alt}_{prs}((84\text{-a})) := \left\{ \begin{array}{ll} \text{Noah is unaware that O can take L} & \neg\Box[\Diamond \mathsf{L}]_{\Diamond \mathsf{L}} \\ \text{Noah is unaware that O can take A} & \neg\Box[\Diamond \mathsf{A}]_{\Diamond \mathsf{A}} \end{array} \right\}$

If we now apply our exhaustivity operator to (84-a), we obtain the following LF (ignoring for now the K layer, to which we go back to below):

(86) EXH [Noah is unaware that Olivia can take Logic or Algebra]

What is the outcome of (86)? Or, to put it differently, how is (84-a)'s plain meaning to be strengthened on our approach? Since (84-a) has no assertive alternatives, the plain assertion of (84-a) will remain as it is. However, (84-a) has the presuppositional alternatives in (85). Those alternatives are not innocently excludable, but they are both innocently includable: it is possible to consistently add their presuppositions to the presupposition of EXH's prejacent.

(87) $\mathrm{II}_{prs}((84\text{-a})) := \left\{ \begin{array}{ll} \text{Noah is unaware that O can take L} & \neg\Box[\Diamond \mathsf{L}]_{\Diamond \mathsf{L}} \\ \text{Noah is unaware that O can take A} & \neg\Box[\Diamond \mathsf{A}]_{\Diamond \mathsf{A}} \end{array} \right\}$

Conjoining the presuppositions of those alternatives with the plain presupposition of (84-a) delivers the presupposed free choice reading we were after: Noah is unaware that Olivia can take Logic or Algebra, and Olivia can take Logic and she can take Algebra, (88).

(88) $\neg\Box[\Diamond[\mathsf{L} \vee \mathsf{A}]]_{\Diamond[\mathsf{L}\vee\mathsf{A}]\wedge\Diamond\mathsf{L}\wedge\Diamond\mathsf{A}}$

For completeness, we note that, given our assumption about the distribution of the Matrix K operator, the LF for (84-a) should actually be as follows:

(89) [EXH$_{①}$ [K_s [EXH$_{②}$ [Noah is unaware that Olivia can take Logic or Algebra]

Yet adding the Matrix K operator together with an extra layer of exhaustification does not change the main result above. To establish this point, consider the formal alternatives to EXH$_{①}$'s prejacent:

(90) $\textsc{alt}(①) := \left\{ \begin{array}{l} [K_s\ [\textsc{exh}_{②}\ [\text{N is unaware that Olivia can take L or A}]]] \\ [K_s\ [\textsc{exh}_{②}\ [\text{N is unaware that Olivia can take L}]]] \\ [K_s\ [\textsc{exh}_{②}\ [\text{N is unaware that Olivia can take A}]]] \end{array} \right\}$

prediction in both cases is that this inference should only arise when the conjunctive alternative is relevant; when this alternative is instead irrelevant, it will be pruned from the domain of EXH and the corresponding inference will not arise; see Fox 2007, Bar-Lev 2018 among others for discussion.

(ii) a. Olivia can take Logic or Algebra.
b. Olivia can take Logic and Algebra.

Unpacking exhaustification, the meaning of the formal alternatives in (90) can be schematically represented as shown in (91).

(91) $\textsc{alt}(①) := \left\{ \begin{array}{l} K_s\,[\neg\Box[\Diamond[\mathsf{L} \vee \mathsf{A}]_{\Diamond\mathsf{L}\wedge\Diamond\mathsf{A}}] \\ K_s\,[\neg\Box[\Diamond\mathsf{L}]_{\Diamond\mathsf{L}} \wedge \Box[\mathsf{A}]_{\Diamond\mathsf{A}}] \\ K_s\,[\neg\Box[\Diamond\mathsf{A}]_{\Diamond\mathsf{A}} \wedge \Box[\mathsf{L}]_{\Diamond\mathsf{L}}] \end{array} \right\}$

At this level of exhaustification, the alternatives corresponding to the independent disjuncts qualify as innocently excludable assertive alternatives: it is possible to consistently add the negation of their assertion to the assertion of $\textsc{exh}_{①}$'s prejacent, while letting their presupposition project into K_s .

(92) $\mathrm{IE}_{asr}(①) := \left\{ \begin{array}{l} K_s\,[\neg\Box[\Diamond\mathsf{L}]_{\Diamond\mathsf{L}} \wedge \Box[\mathsf{A}]_{\Diamond\mathsf{A}}] \\ K_s\,[\neg\Box[\Diamond\mathsf{A}]_{\Diamond\mathsf{A}} \wedge \Box[\mathsf{L}]_{\Diamond\mathsf{L}}] \end{array} \right\}$

Excluding those alternatives gives rise to the inferences in (93-a) and (93-b), respectively. That is, the speaker doesn't believe that Noah is unaware that Olivia can take Logic, but he is aware that she can take Algebra and vice-versa. In addition, since their presuppositions project into K_s, those inferences also add the presuppositions that the speaker believes that Olivia can take Logic and that she can take Algebra.

(93) a. $\neg K_s[\neg\Box[\Diamond\mathsf{L}] \wedge \Box[\mathsf{A}]]_{K_s[\Diamond\mathsf{L}]\wedge K_s[\Diamond\mathsf{A}]}$
b. $\neg K_s[\neg\Box[\Diamond\mathsf{A}] \wedge \Box[\mathsf{L}]]_{K_s[\Diamond\mathsf{L}]\wedge K_s[\Diamond\mathsf{A}]}$

It is easy to verify that both those inferences are in fact already entailed by the prejacent of $\textsc{exh}_{①}$ since the complement of K_s in the prejacent, $[\neg\Box[\Diamond[\mathsf{L} \vee \mathsf{A}]_{\Diamond\mathsf{L}\wedge\Diamond\mathsf{A}}]$, is incompatible with the complement of K_s in those inferences. Therefore, if the speaker believes the former, this already entails that it's not true that she believes the latter (assuming that the speaker does not have contradictory beliefs).

In sum, in extending the notion of innocent inclusion to presuppositional alternatives and presupposed implicatures, the system presented here can derive presupposed free choice rather straightforwardly. In the next subsection, we move to presupposed ignorance and we show how that phenomenon is accounted for given the ingredients above.

5.2.3 Presupposed (speaker's) ignorance

We now go back to the novel oddity effects in (48), which we observed to be parallel to those arising in their non-embedded, non-presuppositional variants in (49).

(48) a. #Noah is unaware that I have two or more children.
b. #Sue didn't realize that my wife is from France or Italy.

c. #Mary was sorry that Sue had lunch with Noah or me yesterday.

(49) a. #I have two or more children.
b. #My wife is from France or Italy.
c. #Sue had lunch with Noah or me yesterday.

Following the previous literature, we attributed the oddity of the sentences in (49) to the obligatory presence of speaker-oriented ignorance inferences about the disjuncts. Specifically, we argued that a sentence like (49-b) is odd because it has the LF in (73), which conveys that the speaker believes that his wife is from France or Italy but that he is ignorant as to which of these two countries she is from (an information that is generally odd for one not to have about a close relative).

(73) $[\textsc{exh}_{①}\ [K_s\ [\textsc{exh}_{②}\ [\text{my wife speaks French or Italian}]]]]$
a. $\mathsf{IE}_{asr}(①) = \{[K_s\ [\textsc{exh}_{②}\ [\textsc{French}]]], [K_s\ [\textsc{exh}_{②}\ [\textsc{Italian}]]]\}$
b. $\rightsquigarrow K_s[\mathsf{F} \vee \mathsf{I}] \wedge I_s[\mathsf{F}] \wedge I_s[\mathsf{I}]$

In our system, this line of explanation now straightforwardly extends to the presuppositional variants of these sentences. Thus for instance, we can extend the explanation for (49-b) to (48-b) by assuming that it is parsed at LF as follows:[34]

(94) $[\textsc{exh}_{①}\ [K_s\ [\textsc{exh}_{②}\ [\text{Sue didn't realise that my wife speaks French or Italian}]]]]$

To illustrate, consider first the logically non-weaker alternatives to $\textsc{exh}_{②}$'s prejacent schematised below: these alternatives are all Strawson-entailed by $\textsc{exh}_{②}$'s prejacent, $[\neg\Box[\mathsf{F}\vee\mathsf{I}]]_{\mathsf{F}\vee\mathsf{I}}$, and therefore are presuppositional alternatives.

(95) $$\textsc{alt}_{prs}(②) := \left\{ \begin{array}{l} \neg\Box[\mathsf{F}]_{\mathsf{F}} \\ \neg\Box[\mathsf{I}]_{\mathsf{I}} \\ \neg\Box[\mathsf{F}\wedge\mathsf{I}]]_{\mathsf{F}\wedge\mathsf{I}} \end{array} \right\}$$

The only alternative in the set of innocently excludable presuppositional alternatives is the conjunctive one, and neither of the remaining ones are innocently includable. The result of the exhaustification process is thus as shown in (96): it is asserted that Sue didn't realise that my wife is from France or Italy and it is presupposed that she is from France or Italy but not both.

(96) a. $\mathsf{IE}_{prs}(②) = \{\neg\Box[\mathsf{F}\wedge\mathsf{I}]]_{\mathsf{F}\wedge\mathsf{I}}\}$
b. $\mathsf{II}_{prs}(②) = \{\ \}$
c. $\rightsquigarrow \neg\Box[\mathsf{F}\vee\mathsf{I}]]_{(\mathsf{F}\vee\mathsf{I})\wedge\neg(\mathsf{F}\wedge\mathsf{I})}$

34 For simplicity, we are ignoring here another occurence of EXH in the complement of the factive predicate. We leave it to the reader to verify that nothing changes if it were added in.

As before, this first result does not account by itself for the oddness of (48-b). However, once we put it together with the rest of the LF in (94) we derive the presupposed ignorance inferences we are after. To see this, consider now the presuppositional alternatives to $\textsc{exh}_{①}$'s prejacent (again setting aside the modalised conjunctive alternative, but nothing hinges on this):

(97) $\textsc{alt}_{prs}(①) := \left\{ \begin{array}{l} K_s\, [\textsc{exh}_{②}[\neg \Box [\mathsf{F}]_{\mathsf{F}}]] \\ K_s\, [\textsc{exh}_{②}[\neg \Box [\mathsf{I}]_{\mathsf{I}}]] \end{array} \right\}$

Given the semantics of our K operator and the projection rules we assumed above, the presuppositions of the disjunctive alternatives are $K[\mathsf{F}]$ and $K[\mathsf{I}]$, respectively. Since both these presuppositional alternatives are innocently excludable, the negation of their presuppositions is added to the presupposition of $\textsc{exh}_{①}$'s prejacent: it is now presupposed that the speaker believes that his wife is from France or Italy and that he is ignorant as to whether she is from France and as to whether she is from Italy, (98).[35]

(98)
a. $\mathsf{IE}_{prs}(①) = \{K_s\, [\textsc{exh}_{②}\, [\neg \Box [\mathsf{F}]_{\mathsf{F}}]], [K_s\, [\textsc{exh}_{②}\, [[\neg \Box [\mathsf{I}]_{\mathsf{I}}]]]]]\}$
b. $\mathsf{II}_{prs}(①) = \{\ \}$
c. $\rightsquigarrow K_s[\neg \Box [\mathsf{F} \vee \mathsf{I}]]_{K_s[\mathsf{F}\vee\mathsf{I}] \wedge I_s[\mathsf{F}] \wedge I_s[\mathsf{I}]}$

As before, our assumptions about the calculation of relevance for disjunctive sentences make it so that, since the presupposition $\mathsf{F} \vee \mathsf{I}$ has to be relevant, the presuppositional alternatives presupposing the independent (embedded) disjuncts also have to be relevant and consequently cannot be pruned from the domain of quantification of $\textsc{exh}_{①}$, which makes the corresponding ignorance inferences obligatory (see 5.1.3 for discussion). In other words, our proposal derives presupposed ignorance in a completely analogous way as assertive ignorance, and we argue that this is the source of the oddness of the sentences in (48).

In sum, the system presented here provides a unified analysis of presupposed implicatures, including presupposed free choice and presupposed speaker's ignorance. More generally, it accounts for the parallelism between implicatures arising in the assertion and those arising only in the presupposition. Before concluding the paper, we briefly discuss in the next section other directions and issues to explore.

6 Other directions

Having presented the challenges at stake and explained how our proposal addresses them, we turn to two alternative directions and identify the issues which we

35 It is easy to verify that the derivation in (98) holds regardless of one's assumptions about the relevance-based restrictions on the domain of quantification of $\textsc{exh}_{②}$.

believe they would need to overcome. The first direction is based on a recent descriptive generalization by Anvari 2018, 2019, called *Logical Integrity*, which aims at subsuming under one roof a broad class of unacceptable sentences. We will not be able to make full justice to the intricacies of his proposal in the scope of this paper, but we will focus on our two main challenges, presupposed free choice and presupposed ignorance, and show that this proposal has troubles with both of them. The second direction is to consider an approach to free choice and ignorance different from the implicature one pursued in this paper. As we discuss, this approach addresses the challenge raised by presupposed free choice. Yet various issues carry over from the assertion to the presuppositional level, and it is less clear how this direction could extend to the presupposed ignorance cases.

6.1 Logical Integrity

Logical Integrity (LI henceforth) is a generalisation from Anvari 2018, 2019, the aim of which is to capture the unacceptability of a variety of sentences, part of which are the ones that Spector & Sudo's (2017) PIP was originally designed to capture. The gist of this generalisation is that a sentence ϕ is deemed infelicitous if it has an alternative ψ that is logically non-weaker, yet contextually entailed by ϕ. In other words, LI forces the logical relation between a sentence and its alternatives to be preserved once contextual information is considered, hence the name of 'logical integrity.' We will consider the formulation of this principle in (99) and assume that a sentence is odd if any part of it violates (99).[36]

(99) **Logical Integrity** (from Anvari 2019: (5))
A sentence ϕ must not be uttered in context c if it has an alternative ψ s.t. (i) ϕ contextually entails ψ in c, but (ii) ϕ does not logically entail ψ.

Consider first how LI fares with respect to presupposed free choice, (100).

(100) Noah is unaware that Olivia is allowed to take Logic or Algebra.
$\rightsquigarrow$ *Olivia can take Logic and she can take Algebra*

For these cases, LI encounters two issues that are similar to those encounters by Spector & Sudo's (2017) account. First, in a context in which it is common knowledge that Olivia has free choice between Logic and Algebra, (100) is predicted to be odd since the following alternatives are contextually but not logically entailed:

(101) a. Noah is unaware that Olivia is allowed to take Logic.

36 This is not the final version of the principle, which is associated with an additional 'projection principle' for local applications, but it is enough for our purposes; see Anvari 2019 for discussion.

b. Noah is unaware that Olivia is allowed to take Algebra.

Second, it is unclear how LI would derive the presupposed free choice reading of interest: in a run-of-the-mill context, reasoning with LI over (100) can at best give rise to the inferences that it's not common knowledge that Olivia can take Logic and that she can take Algebra. Of course, deriving presupposed free choice independently, e.g. with Spector & Sudo's (2017) EXH operator, would solve both these issues at once. However, as we have shown in 4.1, this move comes at the cost of losing the account of other cases we discussed, e.g. the asymmetry in (5).

Let us turn now to the presupposed ignorance cases and consider first the case discussed in Spector & Sudo's (2017) and repeated below.

(38) CONTEXT: The interlocutors know that Olivia only took Logic.
#Noah is unaware that Olivia took Logic or Algebra.

As Anvari (2018, 2019) discusses, LI captures the infelicity of this utterance since it contextually entails one of its logically non-weaker alternatives, namely (102):

(102) Noah is unaware that Olivia took Logic.

Despite this good result, the challenge we presented in this paper extends to Anvari's proposal. Specifically, just like the PIP, LI does not account for the infelicity effects in (48): as long as it is not common knowledge how many children the speaker has, the offending alternatives to an utterance of (48-a) are not contextually entailed by that utterance, and therefore (48-a) and similar examples are not predicted to be infelicitous by LI.

(48) a. #Noah is unaware that I have two or more children.
b. #Sue did not realize that my wife is from France or Italy.
c. #Mary was sorry that Sue had lunch with Noah or me yesterday.

In sum, the Logical Integrity approach does not, at least in its current form, constitute a solution for our presupposed free choice or the ignorance issues. In the next subsection, we turn to non-implicature approaches to free choice and ignorance.

6.2 Non-implicature approaches to free choice and ignorance

As we already mentioned, one common response to Fox's (2007) original argument against a pragmatic approach to free choice is to argue that free choice is not an implicature to begin with. And indeed, the status of free choice is still controversial in the current literature, with some arguments in favour and others against an implicature approach (see, a.o., Bar-Lev 2018, Goldstein 2018, Franke 2011, Aloni

2018, Santorio & Romoli 2017, Romoli & Santorio 2018 for discussion). Without going into the details of the different non-implicature accounts, we show in the following that a non-implicature approach to free choice can indeed be extended to our presupposed free choice cases. There is still of course the partly independent question as to which of these two approaches to free choice ultimately handles free choice best. We will not be able to address this more general issue here, but we will show that some of the problems of the non-implicature approach arising at the assertion level carry over to the presuppositional one.

A variety of non-implicature approaches to free choice have recently been defended in various forms (a.o., Aloni 2018, Willer 2017, Goldstein 2018). For concreteness, we will focus here on the account in Goldstein 2018 (see Rothschild & Yablo 2018 for a similar proposal). Most of the considerations we make below, however, apply to other non-implicature accounts. The gist of Goldstein's (2018) account relies on two main assumptions. First, it assumes that a sentence like (103), repeated from above, directly asserts FREE CHOICE, as schematised in (103-a). Second, it assumes that the modal introduces a homogeneity presupposition requiring that either both disjuncts are possible or neither of them is, (103-b).[37]

(103) Olivia can take Logic or Algebra.
  a. $\Diamond(\mathsf{L} \vee \mathsf{A}) = \Diamond\mathsf{L} \wedge \Diamond\mathsf{A}$ FREE CHOICE
  b. $\Diamond\mathsf{L} \leftrightarrow \Diamond\mathsf{A}$ HOMOGENEITY PRESUPPOSITION

In the positive case, the asserted meaning in (103-a) directly accounts for free choice, while the homogeneity inference plays no role, i.e. it is entailed by (103-a). Under negation, the asserted meaning becomes the negation of FREE CHOICE:

(104) $\neg(\Diamond(\mathsf{L} \vee \mathsf{A})) = \neg\Diamond\mathsf{L} \vee \neg\Diamond\mathsf{A}$ NEGATED FREE CHOICE

In combination with the homogeneity presupposition in (103-b), which projects through negation, (104) gives rise to the desired DUAL PROHIBITION reading. In sum, the assertion in (103-a) together with the homogeneity presupposition in (103-b) capture the attested pattern in the basic case of free choice and dual prohibition.

(105) $(\neg\Diamond\mathsf{L} \vee \neg\Diamond\mathsf{A}) \wedge (\Diamond\mathsf{L} \leftrightarrow \Diamond\mathsf{A})$
  $= \neg\Diamond\mathsf{L} \wedge \neg\Diamond\mathsf{A}$ DUAL PROHIBITION

Consider now how this approach to free choice can account for the presupposed free choice readings. We will focus here on the case in (7), repeated below:

(7) Noah is unaware that Olivia can take Logic or Algebra.

37 We set aside here how this result is obtained compositionally; see Goldstein 2018 for discussion.

This sentence can be schematically represented as in (106) (as before, we use '$\Box$' to represent Noah's belief worlds in the evaluation world).

(106) $\neg\Box[\Diamond(\mathsf{L}\vee\mathsf{A})]_{\Diamond(\mathsf{L}\vee\mathsf{A})}$

Crucially, on this account, the meaning of the embedded clause, $\Diamond(\mathsf{L}\vee\mathsf{A})$, is now directly encoding free choice, $\Diamond\mathsf{L}\wedge\Diamond\mathsf{A}$. In addition, (106) presupposes its embedded clause, which directly accounts for the free choice inference observed in the presupposition, as indicated in (107-a). Finally, we assume that the homogeneity presupposition projects universally through *unaware*, (107-b).[38]

(107) a. $\Diamond(\mathsf{L}\vee\mathsf{A}) = \Diamond\mathsf{L}\wedge\Diamond\mathsf{A}$ PRESUPPOSED FREE CHOICE
b. $\Box[\Diamond\mathsf{L}\leftrightarrow\Diamond\mathsf{A}]$ HOMOGENEITY PRESUPPOSITION

The representation resulting from the combination of these ingredients gives rise to the intuitively correct reading, (108): it asserts that it is possible for Noah that Olivia can't take either of the two classes, i.e. DUAL PROHIBITION is possible according to Noah, and presupposes at the same time that she can take one and that she can take the other, i.e. she actually has FREE CHOICE between the two.[39]

(108) a. $\Diamond[\neg\Diamond\mathsf{L}\wedge\neg\Diamond\mathsf{A}]$ ASSERTION
b. $\Diamond\mathsf{L}\wedge\Diamond\mathsf{A}$ PRESUPPOSITION

This non-implicature approach to free choice, combined with the system of Spector & Sudo (2017), accounts for the challenge of presupposed free choice. In addition to deriving presupposed free choice, it correctly predicts that, under the intended presupposed free choice reading, a sentence like (7) should be felicitous in the previously problematic contexts for its presupposition (i.e., that Olivia can choose between Logic and Algebra) is now stronger than those of any of its alternatives.

We note, however, that some of the issues identified in the literature for this approach at the assertion level, reproduce at the presuppositional one (see Romoli & Santorio 2018 for discussion). We briefly mention two of them here. The first one pertains to the assumption (at least in Goldstein's implementation) that dual prohibition is derived through a combination of the negated literal meaning and the homogeneity inference. We can, therefore, reproduce the problem of presupposed

38 A theory of how the homogeneity presupposition is to project in complex sentences is a central feature of this approach. The assumption we make here regarding universal projection through attitude predicates is based on the universal projection predictions for universal quantifiers; see Križ 2015 and Goldstein 2018 for discussion.

39 Cases involving factive adjectives and emotive factives can be derived in a similar way, though they require discussions of different assumptions about their meaning components and presupposition projection, which we cannot discuss here for reasons of space; see a.o. von Fintel 1999, Heim 1992.

free choice with presupposed dual prohibition. Consider (109) for instance:

(109) Noah is unaware that Olivia cannot take Algebra or Logic.
$\rightsquigarrow$ *Olivia cannot take Algebra and she cannot take Logic*
PRESUPPOSED DUAL PROHIBITION

Intuitively, (109) conveys that Olivia cannot take either of the two classes and that Noah believes that she can take at least one of the two. Yet the reading predicted under Goldstein's approach is now too weak. In particular, it predicts that the presupposition triggered by the complement of *unaware* in (110-a), $\neg(\Diamond(\mathsf{L} \vee \mathsf{A}))$, should merely express negated free choice rather than dual prohibition, (110-b).

(110) a. $\neg\Box(\neg(\Diamond(\mathsf{L} \vee \mathsf{A})))$
b. $\neg(\Diamond(\mathsf{L} \vee \mathsf{A})) = \neg\Diamond\mathsf{L} \vee \neg\Diamond\mathsf{A}$

The problem is that, in contrast to the unembedded case in (108), there is no obvious way here to strengthen the result in (110-b) to dual prohibition thanks to the homogeneity inference projecting through negation. Indeed, assuming as before that homogeneity projects universally under attitude predicates, the resulting presupposition of (110-a) should be as follows: according to Noah's beliefs, Olivia can take Logic if and only if she can take Algebra, (111).

(111) $\Box[\Diamond\mathsf{L} \leftrightarrow \Diamond\mathsf{A}]$

From this, we can conclude that, *according to Noah*, Olivia cannot take either of the two classes. In other words, we obtain dual prohibition but only relative to Noah's beliefs. Finally, note that there is no way to strengthen the presupposition of negated free choice in (110-b) to obtain the presupposed dual prohibition, $(\neg\Diamond\mathsf{A} \wedge \neg\Diamond\mathsf{L})$, which would entail that Olivia cannot take either of the two classes.[40]

A second problem is that non-implicature accounts like the above do not derive the so-called 'negative free choice' inferences (Fox 2007, Chemla 2009; see also Ciardelli et al. 2018, Romoli & Santorio 2018 for discussion). That is, they cannot account for the inference below arising from sentences like (112):

(112) Olivia is not required to take Logic and Algebra.

40 Note that this issue would be solved if we were to stipulate that homogeneity projects out of attitude predicates (Heim 1992, Geurts 1998 for discussion). It is clear, however, that we do not want homogeneity to project through attitudes in general, e.g. from (i) we do not conclude that Olivia can take Logic if and only if she can take Algebra.

(i) Noah believes that Oliva cannot take Logic or Algebra.
$\not\rightsquigarrow$ *Olivia can take one if and only if she can take the other*

$\rightsquigarrow$ *Olivia is not required to take Logic and she is not required to take Algebra*
NEGATIVE FREE CHOICE

We observe here that the very same inference is found at the presuppositional level, thus extending the problem above to 'presupposed negative free choice' inferences.

(113) Noah is unaware that Olivia is not required to take Logic and Algebra.
$\rightsquigarrow$ *Olivia is not required to take Logic and she is not required to take Algebra*
PRESUPPOSED NEGATIVE FREE CHOICE

In sum, while the non-implicature approach to free choice can account for the presupposed free choice challenge, there remains a variety of issues with this approach which carry over from the assertion level.

Finally, it remains an open question whether ignorance inferences can receive a non-implicature account in the same spirit and, if so, whether such an account would adequately extend to presupposed ignorance. One could hypothesize for instance that a disjunction like (114) literally presupposes that the speaker is ignorant as to whether Olivia took logic and as to whether she took Algebra.

(114) Olivia took Logic or Algebra.
a. $(\mathsf{L} \vee \mathsf{A})$ ASSERTION
b. $I_s(\mathsf{L}) \wedge I_s(\mathsf{A})$ IGNORANCE PRESUPPOSITION

Beside its stipulative nature, this hypothesis would also not address the presupposed ignorance challenge: for a sentence like (115), it would only predict presuppositions of the form '$\Box(I_s(\mathsf{L}) \wedge I_s(\mathsf{A}))$', where the ignorance presupposition of the embedded disjunction has universally projected into the attitude predicate. The presupposition could be thus paraphrased as follows: *according to Noah*, the speaker is ignorant as to whether Olivia took Logic or Algebra. This is obviously an unwarranted result.

(115) Noah is unaware that Olivia took Logic or Algebra.
a. $\neg\Box(\mathsf{L} \vee \mathsf{A})$ ASSERTION
b. $\Box(I_s(\mathsf{L}) \wedge I_s(\mathsf{A}))$ IGNORANCE PRESUPPOSITION

More dramatically perhaps, this hypothesis would make incorrect predictions for basic negated cases like (116): it would predict (116) to presuppose that it is possible, according to the speaker, that Olivia took Logic and Algebra, in contradiction with the asserted content that she didn't take either one.[41]

41 Note that locally accommodating the ignorance presupposition under negation would not help here as it would weaken the meaning of (116). In particular, it would make (116) true if, according to the speaker, it is possible that Olivia took Logic.

(116) Olivia didn't take Logic or Algebra.

a. $\neg(\mathsf{L} \vee \mathsf{A}) = \neg\mathsf{L} \wedge \neg\mathsf{A}$ ASSERTION

b. $I_s(\mathsf{L}) \wedge I_s(\mathsf{A})$ IGNORANCE PRESUPPOSITION

7 Conclusion

Previous work have investigated cases in which genuine implicatures appear to be computed only in the presupposition of a sentence. Some researchers have taken those cases to be evidence that implicatures can arise at the presuppositional level, and propose a unified grammatical account of assertion-based and 'presupposition-based' implicatures (Gajewski & Sharvit 2012, Magri 2009, Marty 2017). Spector & Sudo (2017) recently challenged this proposal based on the observation that ignorance inferences also arise at the presuppositional level, and propose instead a hybrid account relying on two distinct scalar strengthening mechanisms: a grammatical theory of implicatures for deriving implicatures in the assertion, and an independent pragmatic principle for deriving implicatures in the presupposition.

Our goal in this paper was to contribute to this ongoing debate by putting forward two novel empirical observations. First, we have observed that FREE CHOICE inferences can also arise at the presuppositional level, and sometimes only at the presuppositional level. Thus for instance, a sentence like (7), repeated below, has a reading conveying that Noah doesn't believe that Olivia can take either class, while suggesting that Olivia has free choice between the two. Hence, in parallel to the other cases of presupposed implicatures, the FREE CHOICE inference in (7) appears in the presupposition but not in the assertion.

(7) Noah is unaware that Olivia can take Logic or Algebra.

a. ASSERTION
Noah doesn't believe that Olivia can take either one

b. PRESUPPOSITION
Olivia can take Logic or Algebra

c. PRESUPPOSED FREE CHOICE
$\rightsquigarrow$ *Olivia can take Logic and she can take Algebra*

Second, we have observed that presupposition-based ignorance inferences are stronger than discussed in Spector & Sudo 2017, and in fact of similar strength as the well-known assertion-based ones. Thus for instance, an utterance of (117-a) doesn't merely convey that that it is not common knowledge whether Olivia has two or more children, but rather that the *speaker* is ignorant as to how many children Olivia has. While the speaker-orientation of those ignorance inferences is harder to detect in cases like (117-a), it is made obvious in cases like (117-b): this utterance is perceived as infelicitous, even in a context in which it is not common knowledge how many children the speaker actually has.

(117) a. Noah is unaware that Olivia has two or more children.
b. #Noah is unaware that I have two or more children.

We showed that both these cases are problematic for Spector & Sudo (2017). First, we showed that deriving the presupposed free choice reading of (7) is challenging in at least two ways. To begin with, allowing this inference to be derived from the working of their exhaustivity operator leads to over-generation issues. Next, deriving this inference with the pragmatic side of their system is challenging for the same reasons as deriving regular free-choice inferences is challenging for a pragmatic approach to assertion-based implicatures. In that respect, our dialectic closely follows that of Fox 2007: we have argued against a pragmatic approach to presupposition-based implicatures on the ground that it is not able to account for presupposed free choice. Second, we showed that the kind of presupposed ignorance inferences that can be derived in Spector & Sudo's (2017) system are too weak to account for cases like (9).

More generally, these novel data have unveiled a systematic parallelism between the assertion and presuppositional levels in terms of EXCLUSIVITY (10), FREE CHOICE (11), and IGNORANCE inferences (12). We have argued that a grammatical theory of implicatures where meaning strengthening operates in the same way at the assertion and presupposition levels (Gajewski & Sharvit 2012, Magri 2009 and Marty 2017), combined with a grammatical account of ignorance inferences à la Meyer (2013), can account for this parallelism and provide a unified analysis of those inferences.

(118) **Exclusivity and Presupposed Exclusivity**
a. Olivia took Logic or Algebra.
$\rightsquigarrow$ *Olivia didn't take both Logic and Algebra*
b. Noah is unaware that Olivia took Logic or Algebra.$\rightsquigarrow$ *Olivia didn't take both Logic and Algebra*

(119) **Free Choice and Presupposed Free Choice**
a. Olivia is allowed to take Logic or Algebra.
$\rightsquigarrow$ *Olivia can take Logic and she can take Algebra*
b. Noah is unaware that Olivia is allowed to take Logic or Algebra.
$\rightsquigarrow$ *Olivia can take Logic and she can take Algebra*

(120) **Ignorance and Presupposed Ignorance**
a. Olivia took Logic or Algebra.
$\rightsquigarrow$ *The speaker ignores which of the two classes Olivia took*

b. Noah is unaware that Olivia took Logic or Algebra.
$\rightsquigarrow$ *The speaker ignores which of the two classes Olivia took*

Finally, in the last part of this paper, we have discussed two alternatives directions, the first one based on the recent account by Anvari (2018, 2019) and the other based on non-implicature accounts of free choice and ignorance. We have presented the gist of those accounts and pointed out the issues they would have to overcome in order to achieve the same empirical coverage as the one offered by our proposal.

Before closing this paper, we would like to emphasise two more points, which we think can help contextualise our contribution in the general discussion of the interactions between presuppositions, free choice and implicatures. First, the parallelism between assertion and presuppositional levels that we found appears to extend to other inferences which have been claimed to be implicatures. For instance, the multiplicity inference associated with plural sentences like (121), which has been given an implicature account (see Spector 2007, Zweig 2009, Ivlieva 2013, Mayr 2015 among others), appears to reproduce at the presuppositional level, as shown in (122).

(121) There are students around.
$\rightsquigarrow$ *There is more than one student around* MULTIPLICITY

(122) Noah didn't realise that there are students around.

a. ASSERTION
Noah doesn't believe that there is any student around.

b. PRESUPPOSITION
There is one or more students around.

c. PRESUPPOSED MULTIPLICITY
$\rightsquigarrow$ *There is more than one student around*

As it seems, similar data can be reproduced for other inferences which have been treated as scalar implicatures like the homogeneity effects associated with plural definites (Magri 2014, Bar-Lev 2018) or the inference of neg-raising predicates (Romoli 2013). We take those parallels as further evidence that a unified analysis like the one offered in this paper is desirable, but leave an exploration of how such an approach would extend to these cases for further research.

Second, let us clarify how our presupposed free choice cases relate to and differ from the cases recently discussed in Romoli & Santorio 2018, cases in which a free choice inference appears to filter a presupposition. Some of these cases are conditional sentences like (123), which involve a presuppositional phrase in the consequent, the presupposition of which (i.e., that Olivia can go study in Japan) appears to be filtered by the free choice inference of the antecedent (i.e. that Olivia can go study in Tokyo and can go study in Boston).

(123) If Olivia can go study in Tokyo or Boston, she is the first in our school who can go study in Japan.
$\not\rightsquigarrow$ *Olivia can go study in Japan*

Our case, on the other hand, has to do with free choice inferences arising at the presuppositional level, independently from the assertion level. Both these cases are thus concerned with the interaction between free choice and presuppositions, but in different ways: their cases evidence the ability of free choice to *filter* presuppositions, while ours its potential to *enrich* presuppositions. We take both these cases to show, in their own ways, that looking at free choice in the context of the interaction between implicatures and presuppositions is a fruitful ground to help us further improve our understanding of such interactions and of free choice itself.

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