Comparatives bring a degree-based NPI licenser*

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Abstract. Comparatives license the use of negative polarity items (NPIs) within their *than*-clause. What exactly constitutes the NPI licenser in comparatives? In this paper, I argue that it is the very status of being the standard in a comparison that constitutes the NPI licenser. Based on Zhang and Ling (2020)'s interval-subtraction-based theory on comparatives, I show that by serving as the standard in a comparison and playing the role of subtrahend in a subtraction equation, a *than*-clause is inherently downward-entailing. Moreover, it demonstrates strong negativity like the classical negation operator *not* does. Therefore, a *than*-clause licenses both weak and strong NPIs. Crucially, this NPI licenser is due to monotonicity projection based on degree semantics (implemented with intervals), not due to a set-operation-based negation operator.

Keywords: Comparatives · *Than*-clauses · Negative polarity items · Degree semantics · Interval subtraction · Subtrahend · Monotonicity · Downward-entailingness · Hierarchy of negativity · Informativeness

1 Introduction

Within the formal semantics literature on comparatives, there have been debates on whether and how *than*-clauses/phrases provide a licensing environment for negative polarity items (NPIs) (see e.g., Hoeksema 1983, von Stechow 1984, Heim 2006, Giannakidou and Yoon 2010, Alrenga and Kennedy 2014).[1]

Empirically, as shown in (1)–(5), typical **weak NPIs** (e.g., *any*), **emphatic NPIs** (or **minimizers**, e.g., *give a penny*, *could help*), and some **strong NPIs** (e.g., *yet*, *in weeks*) are licensed within *than*-clauses. Strong NPIs generally require the licensing from strongly negative-flavored expressions like *not* or *without*.

(1) a. Roxy ran <u>faster than</u> **any** boy did.
b. (i) Roxy did<u>n't</u> see **any** boy.
(ii) *Roxy saw **any** boy.

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[1] I only focus on clausal comparatives and *than*-clauses in this paper.

(2) a. John would sooner roast in hell than **give a penny** to the charity.
b. (i) John left the world without **giving a penny** to his son.
(ii) *John left the world with **giving a penny** to his son.

(3) a. My urge to steal was stronger than I **could help**.
b. (i) I **could**n't **help** being so eager to steal.
(ii) *I **could help** being so eager to steal.

(4) a. It requires better performance than I've seen **yet**.
b. (i) I haven't read the book **yet**.
(ii) *I have read the book **yet**.

(5) a. He made me feel happier than I felt **in years**.
b. (i) He wasn't happy **in years**.
(ii) *He was happy **in years**.

One prevailing hypothesis is that a *than*-clause brings a silent negation operator (e.g., Alrenga and Kennedy 2014). As illustrated in (6), under the canonical '*A*-not-*A*' analysis for comparatives (see Schwarzschild 2008 for a review), this sentence includes a hidden negation, meaning that there exists a degree d such that Mary is d-tall but John is not d-tall. With this proposal of a hidden negation operator for a *than*-clause, it seems a natural consequence that this negation operator constitutes the NPI licenser for licensing *than*-clause-internal NPIs.

(6) Mary is taller than John is.
$\exists d.$[Mary is d-tall $\wedge\neg$ John is d-tall]
$\rightsquigarrow$ There exists a degree d such that Mary's height meets or exceeds d and John's height doesn't meet d.

However, this proposal of a silent negation operator is problematic for a few reasons. First, as pointed out by Giannakidou and Yoon (2010), strong NPIs like *either* cannot be licensed within a *than*-clause, as shown in (7).

(7) a. *Kevin is not tall, and John is taller than Bill is **either**.
b. (i) Bill is not tall, and I know that John isn't tall, **either**.
(ii) *Bill is tall, and I know that John is tall, **either**.

Moreover, the presence of a hidden negation should lead to scopal ambiguity. However, as illustrated by (8), no scopal ambiguity between negation and universal quantifier *every boy* is attested.

(8) Mary is taller than every boy is.
a. #$\exists d$[Mary is d-tall $\wedge\neg\forall x[\mathsf{boy}(x) \rightarrow x$ is d-tall]] $\neg > \forall$: unattested
b. $\exists d$[Mary is d-tall $\wedge\forall x[\mathsf{boy}(x) \rightarrow \neg\, x$ is d-tall]] $\forall > \neg$: ✓

Furthermore, whether a *than*-clause is inherently monotonic (i.e., downward- or upward-entailing) seems not fully settled, and empirical evidence seems mixed, against the prediction of those advocating a hidden negation for a *than*-clause. As noted by Larson (1988), Schwarzschild and Wilkinson (2002), and Giannakidou and Yoon (2010), though the downward-entailing (DE) pattern is observed for

(9), (10) shows a clear upward-entailing (UE) pattern. It seems likely that the monotonicity hinges rather on the kind of quantifiers within a *than*-clause.

(9) Downward entailment

a. X is taller than every **boy** is $\models$ X is taller than every **blond boy** is
b. X is taller than every **blond boy** is $\not\models$ X is taller than every **boy** is

(10) Upward entailment

a. X is taller than some **boy** is $\not\models$ X is taller than some **blond boy** is
b. X is taller than some **blond boy** is $\models$ X is taller than some **boy** is

However, though the 'hidden negation' hypothesis is not empirically favored, this does not entirely rule out the possibility that a *than*-clause is still inherently monotonic and provides an NPI licensing environment (see also Hoeksema 1983). After all, strong NPIs like *in years* are licensed within a *than*-clause (see (5)).

In this paper, I argue that a *than*-clause indeed creates a DE environment and thus contributes an NPI licenser. Crucially, it is not a negation operator, but a degree-based one. Following Zhang and Ling (2020)'s **interval-subtraction-based** approach to comparatives, I show that it is the very status of being the **standard in a comparison**, i.e., the **subtrahend in a subtraction equation**, that makes a *than*-clause an NPI licenser. The negativity of the subtrahend is as strong as the negation operator *not*, allowing a *than*-clause to license both weak and strong NPIs (see Zwarts 1981, Hoeksema 1983).

The paper is organized as follows. Section 2 presents Zhang and Ling (2020)'s interval-subtraction-based approach to comparatives. Section 3 and 5 demonstrates, respectively, the inherent DE-ness and the strong negativity of the standard – the subtrahend – in comparatives. Between them, Section 4, an interlude, shows the interplay between a *than*-clause and its internal quantifiers on monotonicity projection. Then Section 6 explains how various NPIs are licensed within a *than*-clause. Section 7 provides a further discussion. Section 8 concludes.

2 An interval-subtraction-based analysis of comparatives

Zhang and Ling (2020) (see also Zhang and Ling 2015) is a recent development of **interval-based** approaches to comparatives (cf. **degree-based** approaches, see Kennedy 1999, Schwarzschild 2008, and Beck 2011 for reviews; see Schwarzschild and Wilkinson 2002 and Beck 2010 for earlier development of interval-based approaches to comparatives).

According to Zhang and Ling (2020), comparatives are analyzed as a **subtraction relation** among three **definite descriptions** (see (11)): **two positions** along a scale – representing (i) the **standard** of comparison (here *3 o'clock*) and (ii) the measurement associated with the matrix subject (here *5 o'clock*) – and the **distance** (or difference) between them (here *two hours*).

(11) 5 o'clock is two hours later than 3 o'clock is.

$\underbrace{\text{5 o'clock}}_{\textbf{Position 1}} - \underbrace{\text{3 o'clock}}_{\textbf{Position 2: the standard}} = \underbrace{\text{2 hours}}_{\textbf{the distance}}$ (along a scale of time)

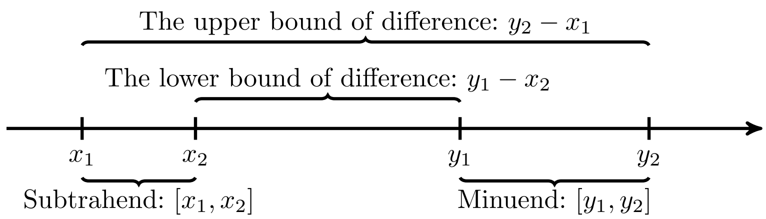

Fig. 1. The subtraction between two intervals. Here $[y_1, y_2]$ means the minuend, $[x_1, x_2]$ the subtrahend, and the difference between these two intervals is the largest range of possible differences between any two points in these two intervals, i.e., $[y_1 - x_2, y_2 - x_1]$.

Crucially, within the new development of Zhang and Ling (2020), these three definite descriptions are represented in terms of **intervals** (i.e., **convex sets of degrees**),[2] and the relation among them is represented as **interval subtraction** (see (12)). The use of intervals and interval arithmetic allows for characterizing the positions and distance in a **generalized** way, supporting the expression of **potentially not-very-precise measurements** (i.e., positions) on a scale.

As illustrated in Fig. 1, here $[y_1, y_2]$ and $[x_1, x_2]$ represent two not-very-precise positions along the scale, and thus, the shortest distance between these two positions is the value of $y_1 - x_2$, while the longest distance between these two positions is the value of $y_2 - x_1$ (see Moore 1979 for details of interval arithmetic).

(12) $$\underbrace{[y_1, y_2]}_{\text{Position 1: } \textbf{minuend}} - \underbrace{[x_1, x_2]}_{\text{Position 2: the standard, i.e., } \textbf{subtrahend}} = \underbrace{[y_1 - x_2, y_2 - x_1]}_{\text{the distance: } \textbf{difference}}$$

Some examples of interval subtraction are shown in (13). In (13a), the lower bound of the difference, 2, means the minimum distance between positions $[4, 8]$ and $[1, 2]$, while the upper bound of the difference, 7, means the maximum distance between these two positions. $[2, 7]$ stands for an interval of distance (i.e., a difference) in (13a), but an interval of position (i.e., a subtrahend) in (13b). Interval subtraction can be generalized to intervals involving open and/or unbounded end points (e.g., (13c)).

(13)
a. $[4, 8] - [1, 2] = [2, 7]$
b. $[4, 8] - [2, 7] = [-3, 6]$ ((13a) vs. (13b): $X - Y = Z \not\models X - Z = Y$)
c. $(5, +\infty) - [1, 3] = (2, +\infty)$

[2] A convex totally ordered set P is a totally ordered set such that for any two random elements a and b belonging to this set P (suppose $a \leq b$), any element x such that $a \leq x \leq b$ also belongs to this set P. For example, $\{x \mid x > 3\}$ and $\{x \mid 3 < x \leq 5\}$ are convex sets, i.e., intervals; $\{x \mid x < 3 \vee x > 5\}$ is not a convex set.

Since an interval is a convex set of degrees, an interval like $\{x \mid a \leq x < b\}$ can be written as $[a, b)$, with a **closed lower bound** '[' and an **open upper bound** ')'. Intervals like $\{x \mid x > a\}$ and $\{x \mid x \leq b\}$ are written as $(a, +\infty)$ and $(-\infty, b]$, where $+\infty$ and $-\infty$ mean positive and negative infinity.

Zhang and Ling (2020)'s interval-subtraction-based approach is particularly suitable for analyzing **clausal comparatives** that contain both *than*-clause-internal quantifiers and numerical differentials, as illustrated by (14).

(14) The giraffe is between 3 and 5 feet taller than every tree is.
$\rightsquigarrow$ The height of the giraffe falls within the interval I such that
$\underbrace{I}_{\textbf{Minuend}} - \underbrace{[\![\text{than every tree is } \cancel{\text{tall}}]\!]}_{\textbf{Subtrahend}} = \underbrace{[3', 5']}_{\textbf{Difference}}$

Intuitively, the standard of comparison here, i.e., $[\![$than every tree is$]\!]$, cannot be reduced to a single degree. However, a *than*-clause is a scope island, so that the embedded universal quantifier *every tree* cannot go through quantifier raising, disallowing the conduction of comparisons between the height of the giraffe and that of each tree (see e.g., Larson 1988, Schwarzschild and Wilkinson 2002). Under the interval-subtraction-based approach, a *than*-clause means a potentially not-very-precise position on a scale. Thus, for (14), $[\![$than every tree is$]\!]$ means the interval ranging from the height of the shortest to that of the tallest tree(s). Based on the formula of interval subtraction (see (12)), the sentence meaning of a comparative can be derived from the semantics of its *than*-clause and the differential. Eventually, only one comparison is performed, but both the lower and upper bounds of the comparison standard contribute to this comparison.

Specifically, gradable adjective *tall* means a relation between an interval I and an atomic entity x, meaning that the height measure of x falls at the position represented as interval I along a scale of height (see (15) – (17)). Since an interval is a convex set of degrees (of type d), the type of intervals is $\langle dt \rangle$.

(15) $[\![\text{tall}]\!]_{\langle dt,et\rangle} \overset{\text{def}}{=} \lambda I_{\langle dt\rangle}.\lambda x.\textsc{height}(x) \subseteq I$
(HEIGHT is a measure function of type $\langle e, dt\rangle$, taking an atomic entity as input and returning its measurement along a scale of height, i.e., the range of markings closest to the top of x.)

(16) **Measurement constructions**

a. My giraffe is between 19 and 20 feet tall.
$\textsc{height}(\textsf{my giraffe}) \subseteq [19', 20']$

b. I am 6 feet tall. $\textsc{height}(\textsf{I}) \subseteq [6', 6']$, or $\textsc{height}(\textsf{I}) \subseteq [6', +\infty)$
(*6 feet* can have an 'at least' reading or an 'exactly' reading.)

(17) **Positive use of adjectives** (see e.g., Bartsch and Vennemann 1972)
My giraffe is tall: $\textsc{height}(\textsf{my giraffe}) \subseteq I^C_{\textsc{pos}}$ ($I^C_{\textsc{pos}}$: the context-dependent interval of being tall for a relevant comparison class)

Comparative morpheme *-er/more* denotes a positive increase, i.e., the default, most general, positive interval $(0, +\infty)$ (see (18)). Like other additive particles (e.g., *another*, *also*), it carries a requirement of additivity: there is a discourse salient scalar value serving as the base of increase (i.e., standard).

(18) $[\![\text{-er/more}]\!]_{\langle dt\rangle} \overset{\text{def}}{=} (0, +\infty)$ **Requirement of additivity:** there is a discourse-salient value serving as the base of increase.

A *than*-clause is considered a short answer to its corresponding degree question. It is derived via (i) a lambda abstraction, which generates a set of intervals, and (ii) the application of an informativeness-based maximality operator, $[\![\text{than}]\!]$, which picks out the most informative definite interval (see (19) and (20)).[3]

(19) $[\![\text{than every tree is } \cancel{\text{tall}}]\!]$

a. Generating a degree question: $\lambda I.\forall x[\mathsf{tree}(x) \rightarrow \textsc{height}(x) \subseteq I]$

b. Deriving its most informative fragment answer:
$\iota I[\forall x[\mathsf{tree}(x) \rightarrow \textsc{height}(x) \subseteq I]$

(20) $[\![\text{than}]\!]_{\langle\langle dt,t\rangle\rangle,dt} \overset{\text{def}}{=} \lambda p_{\langle dt,t\rangle}.\iota I[p(I) \wedge \forall I'[[p(I') \wedge I' \neq I] \rightarrow I \subset I']]$,
$[\![\text{than}]\!]$ is defined when $\exists I[p(I) \wedge \forall I'[[p(I') \wedge I' \neq I] \rightarrow I \subset I']]$

Obviously, $[\![\text{than Bill is } \cancel{\text{tall}}]\!]$ addresses how tall Bill is, thus amounting to the height measurement of Bill.[4] $[\![\text{than every tree is } \cancel{\text{tall}}]\!]$ addresses how tall every tree is, thus amounting to the most informative (i.e., narrowest) interval ranging from the height of the shortest to the tallest tree(s). Suppose there are three trees in our context, measuring $[3', 5']$, $[6', 10']$, and $[11', 13']$, respectively. Then $[\![\text{than every tree is } \cancel{\text{tall}}]\!]$ amounts to the interval $[3', 13']$.

A silent operator is assumed to perform interval subtraction (see (21)). The inputs are two intervals: $I_{\textsc{stdd}}$ and $I_{\textsc{diff}}$, representing the subtrahend and the difference. The output is a third interval, the one representing the minuend.

(21) $[\![\ominus]\!]_{\langle dt,\langle dt,dt\rangle\rangle} \overset{\text{def}}{=} \lambda I_{\textsc{stdd}}.\lambda I_{\textsc{diff}}.\iota I[I - I_{\textsc{stdd}} = I_{\textsc{diff}}]$

Thus, for a clausal comparative like (14) (repeated here in (22)), its *than*-clause serves as the standard of comparison and plays the role of $I_{\textsc{stdd}}$ (see (22a)). A numerical differential (here *between 3 and 5 feet*) restricts the default positive differential *-er* (see (22b)). Eventually, matrix-level semantics is derived via interval subtraction (see (22c)). According to the formula of interval subtraction (see (12)), (22c) means that the height of my giraffe falls into an interval I' such that (i) the lower bound of I' minus the height of the tallest tree(s) is 3 feet, and (ii) the upper bound of I' minus the height of the shortest tree(s) is 5 feet.

(22) The giraffe is between 3 and 5 feet taller than every tree is. (= (14))
LF of (14): The giraffe is $\underbrace{[3', 5'] \ldots \text{-er}}_{I_{\textsc{diff}}} \ominus \underbrace{\text{than every tree is } \cancel{\text{tall}}}_{I_{\textsc{stdd}}}$ tall

a. $I_{\textsc{stdd}} = [\![\text{than every tree is } \cancel{\text{tall}}]\!] = \iota I[\forall x[\mathsf{tree}(x) \rightarrow \textsc{height}(x) \subseteq I]]$
(Roughly, this is an interval from the height of the shortest to that of the tallest tree(s): $[\textsc{height}(\mathsf{shortest\text{-}tree}), \textsc{height}(\mathsf{tallest\text{-}tree})]$.)[5]

b. $I_{\textsc{diff}} = [3', 5'] \cap (0, +\infty) = [3', 5']$

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[3] See also Zhang and Ling (2020) (especially footnote 21 in that paper) for a brief discussion on the short-answer (or free-relative) view of *than*-clauses.

[4] Evidently, the meaning of a *than*-clause is distinct from the positive use of gradable adjectives (see (17)). *Mary is taller than Bill is ~~tall~~* does not entail that Bill is tall.

[5] To facilitate notations, I avoid writing endpoints of $\textsc{height}(x)$ in this kind of cases.

c. $[\![(14)]\!] \Leftrightarrow \textsc{height}(\textsf{my-giraffe}) \subseteq \iota I'[I' - I_{\textsc{stdd}} = I_{\textsc{diff}}]$
$\Leftrightarrow \textsc{hght}(\textsf{grf}) \subseteq \iota I'[I' - \iota I[\forall x[\textsf{tree}(x) \rightarrow \textsc{hght}(x) \subseteq I]] = [3', 5']]$
$\Leftrightarrow \textsc{hght}(\textsf{grf}) \subseteq \iota I'[I' - [\textsc{hght}(\textsf{shortest}), \textsc{hght}(\textsf{tallest})] = [3', 5']]$
$\rightsquigarrow$ (i) the lower bound of I' minus the height of the tallest tree(s) is 3 feet, and (ii) the upper bound of I' minus the height of the shortest tree(s) is 5 feet (see (12)).

3 The downward-entailingness of a *than*-clause

The formula of interval subtraction (see (12), repeated in (23)) crucially underlies Zhang and Ling (2020)'s interval-subtraction-based approach to comparatives.

The three definite scalar values (in terms of intervals) in a subtraction equation constrain each other. Thus we can compute the value of the minuend from the given values of the subtrahend and the difference. In fact, this is how the matrix-level semantics of a comparative is derived (see (22)): sentence-level semantics is derived from the meaning of the *than*-clause and the differential.

Then as shown in (24), we cannot directly apply interval addition to the subtrahend and the difference to compute the value of the minuend (see Moore 1979 and the illustration in (25)).[6] Instead, we need to follow the formula of interval subtraction. Therefore, as shown in (24b), it is the **upper bound of the subtrahend** that contributes to the computation of the **lower bound of the minuend**, and it is the **lower bound of the subtrahend** that contributes to the computation of the **upper bound of the minuend**.

(23) $[y_1, y_2] - [x_1, x_2] = [y_1 - x_2, y_2 - x_1]$ **Interval subtraction** (= (12))

(24) $X - [a, b] = [c, d]$. Generally speaking, $X \neq [a + c, b + d]$

a. X is undefined if $b + c > a + d$. (i.e., for X to be defined, the lower bound of X cannot exceed the upper bound of X.)

b. When defined, $X = [b + c, a + d]$.
the **lower** bound of X = the **upper** bound of the subtrahend $[a, b]$
+ the **lower** bound of the difference $[c, d]$
the **upper** bound of X = the **lower** bound of the subtrahend $[a, b]$
+ the **upper** bound of the difference $[c, d]$

(25) a. $[6, 8] - [3, 4] = [2, 5]$ **Interval subtraction**
b. $[3, 4] + [2, 5] = [5, 9]$ **Interval addition**

An interval means a range of possible values of degrees. Thus, for a given interval, it becomes less informative (i.e., including more possibilities) if we lower

[6] Applying an operation on two intervals results in a third interval that represents the largest possible range of values (see Moore 1979). Here is a general recipe for basic operations – addition, subtraction, and multiplication:

(i) $[x_1, x_2]\langle\text{op}\rangle[y_1, y_2] = [\alpha, \beta]$
The lower bound of $\alpha = \textsc{min}(x_1\langle\text{op}\rangle y_1, x_1\langle\text{op}\rangle y_2, x_2\langle\text{op}\rangle y_1, x_2\langle\text{op}\rangle y_2)$
The upper bound of $\alpha = \textsc{max}(x_1\langle\text{op}\rangle y_1, x_1\langle\text{op}\rangle y_2, x_2\langle\text{op}\rangle y_1, x_2\langle\text{op}\rangle y_2)$

its lower bound or raise its upper bound, and it becomes more informative (i.e., including fewer possibilities) if we lower its upper bound or raise its lower bound.

As a consequence of (24b), raising the upper bound of the subtrahend leads to a higher lower bound for the minuend, thus decreasing the informativeness of the subtrahend but increasing the informativeness of the minuend. More generally, changing an endpoint of the subtrahend always makes the informativeness of the subtrahend and the minuend change in opposite directions. When the subtrahend becomes more informative, the minuend becomes less informative, and vice versa.

In this sense, the informativeness of a *than*-clause (i.e., a subtrahend) always projects to the matrix-level informativeness (which corresponds to the minuend) in a reverse way, demonstrating the hallmark of DE-ness (see Fauconnier 1978, Ladusaw 1979, 1980), as shown in (26) and (27):

(26) An expression f is downward-entailing iff $\forall x \forall y[x \subseteq y \rightarrow f(y) \subseteq f(x)]$.

(27) If $I_{\text{STDD}} \subseteq I'_{\text{STDD}}$, then $\iota I'[I' - I'_{\text{STDD}} = I_{\text{DIFF}}] \subseteq \iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]$.

It is worth noting that this DE-ness is due to the application of interval subtraction. It is by being the **standard of a comparison** and playing the role of **subtrahend in interval subtraction** that makes a *than*-clause – the subtrahend interval I_{STDD} – inherently DE.

Another remark is that the monotonicity and the polarity of the differential (i.e., I_{DIFF}) in a comparative never interfere with the monotonicity projection from a *than*-clause to matrix-level semantics (see (27)).

According to Zhang and Ling (2020), the differential of *more-than* comparatives is positive, i.e., a subset of $(0, +\infty)$ (see (28a)–(28c)), while the differential of *less-than* comparatives is negative, i.e., a subset of $(-\infty, 0)$ (see (28d) and (28e)). These positive and negative differentials are all definite descriptions of scalar values, i.e., similar to *the value of 4* (or *-4*). The notion of intervals is to generalize and include both precise and potentially not-very-precise values.[7]

Both I_{STDD} and I_{DIFF} are definite descriptions of intervals, each making independent contribution to the derivation of matrix-level semantics. The monotonicity projection from I_{STDD} to the minuend is entirely irrelevant to I_{DIFF} (see (27)). In particular, it is entirely irrelevant to the direction of inequalities – whether its the minuend or I_{STDD} that meets or exceeds more degrees along a scale (cf. (6)). The direction of inequalities actually amounts to the polarity of I_{DIFF} in this analysis. Thus, as illustrated in (28) and (29), the pattern of monotonicity projection is always the same for both *more-than* and *less-than* comparatives, regardless of the monotonicity or polarity of I_{DIFF}.

The contrast between (28) and (29) is due to the interplay between the subtrahend status of a *than*-clause and *than*-clause-internal quantifiers (universal vs. existential). Details of this interplay will be shown in Section 4.

(28) Downward entailment for comparatives with various differentials

[7] Even for a sentence like *Sue is a few inches taller than Tom is*, *a few inches* represents **the** measurement of **the** distance between two positions on a height scale, i.e., the measurement is a **definite item** which has a **potentially not very precise** value.

a. X is more than 2 inches taller than every **boy** is
$\models$ X is more than 2 inches taller than every **fat boy** is
(here $I_{\text{DIFF}} = (2, +\infty)$, a positive UE differential:
more than 2 **fat boys** ran $\models$ more than 2 **boys** ran)

b. X is at most 3 inches taller than every **boy** is
$\models$ X is at most 3 inches taller than every **fat boy** is
(here $I_{\text{DIFF}} = (0, 3]$, a positive DE differential:
at most 3 **boys** ran $\models$ at most 3 **fat boys** ran)

c. X is between 5 and 10 inches taller than every **boy** is
$\models$ X is between 5 and 10 inches taller than every **fat boy** is
(here $I_{\text{DIFF}} = [5', 10']$, a positive non-monotonic differential:
between 5 and 10 **fat boys** ran $\not\models$ between 5 and 10 **boys** ran
between 5 and 10 **boys** ran $\not\models$ between 5 and 10 **fat boys** ran)

d. X is less tall than every **boy** is
$\models$ X is less tall than every **fat boy** is
(here $I_{\text{DIFF}} = (-\infty, 0)$, a negative differential.)

e. X is between 5 and 10 inches less tall than every **boy** is
$\models$ X is between 5 and 10 inches less tall than every **fat boy** is
(here $I_{\text{DIFF}} = [-10', -5']$, a negative non-monotonic differential.)

(29) Upward entailment for comparatives with various differentials

a. X is more than 2 inches taller than some **fat boy** is
$\models$ X is more than 2 inches taller than some **boy** is

b. X is at most 3 inches taller than some **fat boy** is
$\models$ X is at most 3 inches taller than some **boy** is

c. X is between 5 and 10 inches taller than some **fat boy** is
$\models$ X is between 5 and 10 inches taller than some **boy** is

d. X is less tall than some **fat boy** is
$\models$ X is less tall than some **boy** is

e. X is between 5 and 10 inches less tall than some **fat boy** is
$\models$ X is between 5 and 10 inches less tall than some **boy** is

4 Monotonicity projection patterns from a *than*-clause

As illustrated in (30), the restrictor of universal quantifiers is DE (see (30a)), and so is the scope of *not* (see (30b). The interplay between them leads to two reverses in monotonicity projection and eventually an UE pattern (see (30c)).

(30) a. every **dog** is cute $\models$ every **black dog** is cute **DE**
$\because \lambda x.\mathsf{black\text{-}dog}(x) \subseteq \lambda x.\mathsf{dog}(x)$ (i.e., $[\![$black dog$]\!]$ entails $[\![$dog$]\!]$.)
$\therefore \lambda P.\forall x[\mathsf{black\text{-}dog}(x) \to P(x)] \supseteq \lambda P.\forall x[\mathsf{dog}(x) \to P(x)]$
(i.e., **Reverse** – $[\![$every dog$]\!]$ entails $[\![$every black dog$]\!]$.)

b. Bill did not **run** $\models$ Bill did not **run fast** **DE**
$\because \lambda x.\mathsf{run\text{-}fast}(x) \subseteq \lambda x.\mathsf{run}(x)$ (i.e., $[\![$run fast$]\!]$ entails $[\![$run$]\!]$.)
$\therefore \lambda x.\neg\mathsf{run\text{-}fast}(x) \supseteq \lambda x.\neg\mathsf{run}(x)$
(i.e., **Reverse** – $[\![$not running$]\!]$ entails $[\![$not running fast$]\!]$.)

c. not every **black dog** is cute $\models$ not every **dog** is cute **UE**
$\lambda P.\neg\forall x[\mathsf{black\text{-}dog}(x) \rightarrow P(x)] \subseteq \lambda P.\neg\forall x[\mathsf{dog}(x) \rightarrow P(x)]$
(i.e., $[\![$not every black dog$]\!]$ entails $[\![$not every dog$]\!]$.)

Similarly, the DE and UE patterns in (9) and (10) are due to the interplay between the subtrahend status of a *than*-clause and its internal quantifiers.

In (31), there is a *than*-clause-internal universal quantifier. Thus the monotonicity projection involves **three reverses**: (i) from the meaning of a noun phrase NP to that of *every NP*; (ii) from *every NP* to I_{STDD}, i.e., the most informative interval including the measurement of every NP; (iii) finally, from I_{STDD}, the subtrahend, to the matrix-level semantics. Eventually, these three reverses lead to the DE pattern in (9).

(31) This tree is taller than every animal/giraffe is.

a. **Reverse 1**: the projection from $[\![$NP$]\!]$ to $[\![$every NP$]\!]$
$\because \lambda x.\mathsf{giraffe}(x) \subseteq \lambda x.\mathsf{animal}(x)$ (i.e., $[\![$giraffe$]\!]$ entails $[\![$animal$]\!]$.)
$\therefore \lambda P.\forall x[\mathsf{giraffe}(x) \rightarrow P(x)] \supseteq \lambda P.\forall x[\mathsf{animal}(x) \rightarrow P(x)]$
(i.e., any property P such that $\forall x[\mathsf{animal}(x) \rightarrow P(x)]$
also makes $\forall x[\mathsf{giraffe}(x) \rightarrow P(x)]$ hold true.)
(i.e., **Reverse 1** – $[\![$every animal$]\!]$ entails $[\![$every giraffe$]\!]$.)

b. **Reverse 2**: the projection from $[\![$every NP$]\!]$ to the *than*-clause
$\because \lambda I.\forall x[\mathsf{grf}(x) \rightarrow \textsc{hght}(x) \subseteq I] \supseteq \lambda I.\forall x[\mathsf{anm}(x) \rightarrow \textsc{hght}(x) \subseteq I]$
(i.e., any interval I such that $\forall x[\mathsf{animal}(x) \rightarrow \textsc{height}(x) \subseteq I]$
also makes $\forall x[\mathsf{giraffe}(x) \rightarrow \textsc{height}(x) \subseteq I]$ hold true.)
$\therefore \iota I[\forall x[\mathsf{grf}(x) \rightarrow \textsc{hght}(x) \subseteq I]] \subseteq \iota I'[\forall x[\mathsf{anm}(x) \rightarrow \textsc{hght}(x) \subseteq I']]$
(i.e., the most informative interval I s.t. $\forall x[\mathsf{grf}(x) \rightarrow \textsc{hght}(x) \subseteq I]$
is not less informative than the most informative interval I' s.t.
$\forall x[\mathsf{animal}(x) \rightarrow \textsc{height}(x) \subseteq I']$.)
(i.e., **Reverse 2** –$[\![$than every giraffe is (tall)$]\!]$ entails
$[\![$than every animal is (tall)$]\!]$.)

c. **Reverse 3**: the projection from I_{STDD} to sentence meaning
$\because [\![$than every giraffe is (tall)$]\!] \subseteq [\![$than every animal is (tall)$]\!]$
$\therefore \iota I_{\text{MINUEND}}[I_{\text{MINUEND}} - \iota I[\forall x[\mathsf{giraffe}(x) \rightarrow \textsc{height}(x) \subseteq I]] = I_{\text{DIFF}}] \supseteq$
$\iota I'_{\text{MINUEND}}[I'_{\text{MINUEND}} - \iota I'[\forall x[\mathsf{animal}(x) \rightarrow \textsc{height}(x) \subseteq I']] = I_{\text{DIFF}}]$
(i.e., **Reverse 3** – $[\![$taller than every animal is$]\!]$ entails
$[\![$taller than every giraffe is$]\!]$.)

In (32), there is a *than*-clause-internal existential quantifier. The monotonicity projection from NP to *some NP* is straightforward. Then the projection involves two reverses: (i) from *some NP* to I_{STDD}; (ii) from I_{STDD} to the matrix-level semantics. Eventually, these two reverses lead to the UE pattern in (10).

(32) This tree is taller than some animal/giraffe is.

a. the projection from $[\![$NP$]\!]$ to $[\![$some NP$]\!]$
$\because \lambda x.\mathsf{giraffe}(x) \subseteq \lambda x.\mathsf{animal}(x)$
(i.e., $[\![$giraffe$]\!]$ entails $[\![$animal$]\!]$.)

$\therefore \lambda P.\exists x[\mathsf{giraffe}(x) \wedge P(x)] \subseteq \lambda P.\exists x[\mathsf{animal}(x) \wedge P(x)]$
(i.e., any property P such that $\exists x[\mathsf{giraffe}(x) \wedge P(x)]$
also makes $\exists x[\mathsf{animal}(x) \wedge P(x)]$ hold true.)
(i.e., ⟦some giraffe⟧ entails ⟦some animal⟧.)

b. **Reverse 1**: the projection from ⟦some NP⟧ to the *than*-clause
$\because \lambda P.\exists x[\mathsf{giraffe}(x) \wedge P(x)] \subseteq \lambda P.\exists x[\mathsf{animal}(x) \wedge P(x)]$
$\therefore$ for each most informative interval I s.t. $\exists x[\mathsf{grf}(x) \wedge \textsc{hght}(x) \subseteq I]$,
there must exist an interval I' s.t. $\exists x[\mathsf{anm}(x) \wedge \textsc{hght}(x) \subseteq I']$
and I' is not less informative than I.
(i.e., **Reverse 1** – ⟦than some animal is (tall)⟧ entails
⟦than some giraffe is (tall)⟧.)

c. **Reverse 2**: the projection from $I_{\textsc{stdd}}$ to sentence meaning
$\because$ ⟦than some animal is (tall)⟧ $\subseteq$ ⟦than some giraffe is (tall)⟧
$\therefore \iota I_{\textsc{minuend}}[I_{\textsc{minuend}} - \iota I[\exists x[\mathsf{giraffe}(x) \wedge \textsc{height}(x) \subseteq I]] = I_{\textsc{diff}}] \subseteq$
$\iota I'_{\textsc{minuend}}[I'_{\textsc{minuend}} - \iota I'[\exists x[\mathsf{animal}(x) \wedge \textsc{height}(x) \subseteq I']] = I_{\textsc{diff}}]$
(i.e., **Reverse 2** – ⟦taller than some giraffe is⟧ entails
⟦taller than some animal is⟧.)

5 The strong negativity of a *than*-clause

Within the literature on NPIs, it has been widely acknowledged since Zwarts (1981) that not all NPIs have the same requirement for their licensing environment. Zwarts (1981) (see also Zwarts 1998) classifies negative-flavored environments into three levels – **downward-entailing**, **anti-additive**, and **anti-morphic** (see (33) and (34)) – and proposes that the licensing of strong NPIs (cf. weak NPIs) requires an environment that is higher on this hierarchy.

Section 3 shows that due to its subtrahend status in a subtraction equation, a *than*-clause is by nature DE. Here I show that a subtrahend also satisfies the requirements in (33) and (34). Thus a *than*-clause is anti-morphic, demonstrating strong negativity like classical negation operator *not* does.

(33) An expression f is anti-additive iff $\forall x \forall y[f(x \vee y) \leftrightarrow f(x) \wedge f(y)]$.

(34) An expression f is anti-morphic iff it is anti-additive and anti-multiplicative.
An expression f is anti-multiplicative iff $\forall x \forall y[f(x \wedge y) \leftrightarrow f(x) \vee f(y)]$.

To show that the subtrahend status of a *than*-clause is anti-additive, I follow the recipe of interval subtraction (see (12)) to prove the equivalence in (35).

(35)
$$\underbrace{X \subseteq \iota I[I - [a_1, b_1] \cup [a_2, b_2] = [c, d]]}_{f(x \vee y)} \leftrightarrow$$
$$\underbrace{X \subseteq \iota I[I - [a_1, b_1] = [c, d]] \wedge X \subseteq \iota I[I - [a_2, b_2] = [c, d]]}_{f(x) \wedge f(y)}$$
(Suppose all these intervals are defined, i.e., $a_1 < b_1$, $a_2 < b_2$, and $c < d$.)

I adopt Moore (1979)'s definition for the **intesection** and **union** operations on two intervals. As shown in (36), for two intervals $[a_1, b_1]$ and $[a_2, b_2]$, if their intersection interval is non-empty (i.e., not the case that $a_1 > b_2$ or $a_2 > b_1$), then their intersection is again an interval – essentially the overlap between the two input intervals. Similarly, as shown in (37), if there is overlap between two intervals, then the union of the two intervals is also an interval – essentially the entire interval including all the elements in the two input intervals. Evidently, these two operations on intervals are parallel to those defined on sets.

(36) $[a_1, b_1] \cap [a_2, b_2] = [\textsc{max}(a_1, a_2), \textsc{min}(b_1, b_2)]$ **Interval intersection**
(Defined when their intersection is non-empty.)

(37) $[a_1, b_1] \cup [a_2, b_2] = [\textsc{min}(a_1, a_2), \textsc{max}(b_1, b_2)]$ **Interval union**
(Defined when their intersection is non-empty.)

Thus, (38) and (39) show the derivation for the left and right part of (35), respectively. Together, they prove the anti-additivity of the subtrahend status.

(38) $X \subseteq \iota I[I - [a_1, b_1] \cup [a_2, b_2] = [c, d]]$
$\Leftrightarrow X \subseteq \iota I[I - [\textsc{min}(a_1, a_2), \textsc{max}(b_1, b_2)] = [c, d]]$
$\Leftrightarrow X \subseteq [\textsc{max}(b_1, b_2) + c, \textsc{min}(a_1, a_2) + d]$
(defined when $\textsc{max}(b_1, b_2) + c < \textsc{min}(a_1, a_2) + d$.)

(39) $X \subseteq \iota I[I - [a_1, b_1] = [c, d]] \wedge X \subseteq \iota I[I - [a_2, b_2] = [c, d]]$
$\Leftrightarrow X \subseteq [b_1 + c, a_1 + d] \wedge X \subseteq [b_2 + c, a_2 + d]$
$\Leftrightarrow X \subseteq [\textsc{max}(b_1, b_2) + c, \textsc{min}(a_1, a_2) + d]$
(defined when $\textsc{max}(b_1, b_2) + c < \textsc{min}(a_1, a_2) + d$.)[8]

To show that the subtrahend status of a *than*-clause is also anti-multiplicative (see (34)), I also use interval subtraction to prove the equivalence in (40).

(40)
$$\underbrace{X \subseteq \iota I[I - [a_1, b_1] \cap [a_2, b_2] = [c, d]]}_{f(x \wedge y)} \leftrightarrow$$
$$\underbrace{X \subseteq \iota I[I - [a_1, b_1] = [c, d]] \vee X \subseteq \iota I[I - [a_2, b_2] = [c, d]]}_{f(x) \vee f(y)}$$

(41) and (42) show the derivation for the left and right part of (40), respectively. Together, they prove the anti-multiplicativity of the subtrahend status.

(41) $X \subseteq \iota I[I - [a_1, b_1] \cap [a_2, b_2] = [c, d]]$
$\Leftrightarrow X \subseteq \iota I[I - [\textsc{max}(a_1, a_2), \textsc{min}(b_1, b_2)] = [c, d]]$
$\Leftrightarrow X \subseteq [\textsc{min}(b_1, b_2) + c, \textsc{max}(a_1, a_2) + d]$
(defined when $\textsc{min}(b_1, b_2) + c < \textsc{max}(a_1, a_2) + d$.)

(42) $X \subseteq \iota I[I - [a_1, b_1] = [c, d]] \vee X \subseteq \iota I[I - [a_2, b_2] = [c, d]]$
$\Leftrightarrow X \subseteq [b_1 + c, a_1 + d] \vee X \subseteq [b_2 + c, a_2 + d]$

[8] Obviously, as far as $[\textsc{max}(b_1, b_2) + c, \textsc{min}(a_1, a_2) + d]$ is defined, i.e., $\textsc{max}(b_1, b_2) + c < \textsc{min}(a_1, a_2) + d$, then it must be the case that $b_1 + c < a_1 + d$, and $b_2 + c < a_2 + d$, i.e., $[b_1 + c, a_1 + d]$ and $[b_2 + c, a_2 + d]$ are defined.
Moreover, it must be the case that $b_2 + c < a_1 + d$ and $b_1 + c < a_2 + d$, i.e., the intersection between the intervals $[b_1 + c, a_1 + d]$ and $[b_2 + c, a_2 + d]$ is non-empty.

$\Leftrightarrow X \subseteq [\textsc{min}(b_1, b_2) + c, \textsc{max}(a_1, a_2) + d]$
(defined when $b_1 + c < a_1 + d$, and $b_2 + c < a_2 + d$.)[9]

(35) and (40) both hold true, indicating that the subtrahend in an interval subtraction equation is both anti-additive and anti-multiplicative.[10] Thus the subtrahend status is anti-morphic, demonstrating a negativity as strong as the classical negation operator *not*.[11] Therefore, by playing the role of subtrahend in an interval subtraction, a *than*-clause is by nature strongly negative-flavored.

Just like the inherent DE-ness of a *than*-clause is due to interval subtraction, its anti-additivity and anti-multiplicativity are also based on degree semantics implemented with interval arithmetic. The inference patterns with regard to *than*-clause-internal DPs are distinct from (35) and (40).

As shown in (43) and (44), it seems that the interpretation of comparatives is anti-additive, but not anti-multiplicative (see also Hoeksema 1983). These patterns are due to both (i) the subtrahend status of a *than*-clause and (ii) the analysis of a *than*-clause as the short answer to its corresponding degree question (see (19)). Suppose the most informative intervals standing for the heights of A and B are $[a_1, b_1]$ and $[a_2, b_2]$, respectively. As shown in (45), both *than A or B is* (*tall*) and *than A and B are* (*tall*) are analyzed as the interval $[\textsc{min}(a_1, a_2), \textsc{max}(b_1, b_2)]$.[12] For (45b), since the individual variable of a gradable adjective is an atomic entity (see (15)), I assume a distributivity operator, DIST,

[9] As fas as $b_1 + c < a_1 + d$ and $b_2 + c < a_2 + d$ (i.e., $[b_1 + c, a_1 + d]$ and $[b_2 + c, a_2 + d]$ are both defined), it must be the case that $\textsc{min}(b_1, b_2) + c < \textsc{max}(a_1, a_2) + d$.

[10] Here are two concrete examples illustrating (35) and (40). Suppose $[a_1, b_1] = [1, 3]$; $[a_2, b_2] = [2, 4]$; $[c, d] = [1, 7]$, then $[a_1, b_1] \cup [a_2, b_2] = [1, 4]$; $[a_1, b_1] \cap [a_2, b_2] = [2, 3]$.

(i) For the left of (35), $\iota I[I - [a_1, b_1] \cup [a_2, b_2] = [c, d]]$. Thus the unique I is $[5, 8]$. For the right of (35), the intersection between $\iota I[I - [a_1, b_1] = [c, d]]$ and $\iota I[I - [a_2, b_2] = [c, d]]$ amounts to intersecting $[4, 8]$ and $[5, 9]$, which is also $[5, 8]$.

(ii) For the left of (40), $\iota I[I - [a_1, b_1] \cap [a_2, b_2] = [c, d]]$. Thus the unique I is $[4, 9]$. For the right of (40), the union of $\iota I[I - [a_1, b_1] = [c, d]]$ and $\iota I[I - [a_2, b_2] = [c, d]]$ amounts to the union of $[4, 8]$ and $[5, 9]$, which is also $[4, 9]$.

[11] *Not* is also anti-morphic, as illustrated by (i):

(i) a. Mary didn't run → Mary didn't run fast
b. Mary didn't sing or dance ↔ Mary didn't sing ∧ Mary didn't dance
c. Mary didn't sing and dance ↔ Mary didn't sing ∨ Mary didn't dance

[12] The equivalence between ⟦than A or B is ~~tall~~⟧ and ⟦than A and B are ~~tall~~⟧ means that degree questions *how tall is A or B* and *how tall are A and B* have the same short answer. This is intuitively right, as suggested by analogous examples in (i):

(i) Context: A ate an orange. B ate an apple. C ate a peach.
a. – What did A, B, or C eat? – A piece of fruit (⇝ a range of items)
b. – What did (each of) A, B, and C eat? – A piece of fruit (⇝ a range)

in deriving ⟦than A and B are ~~tall~~⟧. Eventually, this analysis of *than A and B are* (*tall*) makes the left part of (44a) equal to 'X is taller than A is $\wedge$ X is taller than B is' (see (43)) and thus more informative than the right part of (44a).

(43) X is taller than A or B is $\leftrightarrow$ X is taller than A is $\wedge$ X is taller than B is

(44) a. X is taller than A and B are ($\leftrightarrow$ X is taller than A or B is) $\rightarrow$ X is taller than A is $\vee$ X is taller than B is
b. X is taller than A is $\vee$ X is taller than B is $\not\rightarrow$ X is taller than A and B are

(45) ⟦than A is ~~tall~~⟧ $= [a_1, b_1]$, and ⟦than B is ~~tall~~⟧ $= [a_2, b_2]$
a. ⟦than A or B is ~~tall~~⟧ $= [\textsc{min}(a_1, a_2), \textsc{max}(b_1, b_2)]$
b. ⟦than A and B are DIST ~~tall~~⟧ $= [\textsc{min}(a_1, a_2), \textsc{max}(b_1, b_2)]$
($\textsc{dist} \overset{\text{def}}{=} \lambda X_e.\lambda P_{\langle et\rangle}.\forall x[x \sqsubseteq_{\textsc{atom}} X \rightarrow P(x)]$)
$\rightsquigarrow \forall x[x \sqsubseteq_{\textsc{atom}} \mathsf{A} \oplus \mathsf{B} \rightarrow \textsc{height}(\mathsf{x}) \subseteq [\textsc{min}(a_1, a_2), \textsc{max}(b_1, b_2)]]$[13]

The DE-ness and anti-additivity of clausal comparatives have previously been demonstrated by Hoeksema (1983). Here based on Zhang and Ling (2020)'s interval-subtraction-based analysis of comparatives, I further pin down the source of the DE-ness and anti-additivity in clausal comparatives: it is the subtrahend status of their *than*-clause. Moreover, I show that the negativity of the subtrahend status is actually as strong as that of classical negation operator *not*, reaching the highest level of Zwarts' hierarchy.

6 NPI licensing by a *than*-clause

How are weak and strong NPIs licensed within a *than*-clause? The brief answer is that as a subtrahend, a *than*-clause is strongly negative-flavored, naturally creating an NPI-licensing environment. NPIs are thus licensed in both *more-than* and *less-than* comparatives (see naturally occurring examples of *less-than* comparatives in (46) and (47) and *more-than* comparatives in (1)–(5)).

(46) Millennials have <u>less</u> money <u>than</u> **any** other generation did at their age.[14]

(47) . . . , executives' views on the current global economy and expectations of future global growth are <u>less favorable than</u> they have been **in years**.[15]

[13] With the use of this distributivity operator, DIST, evidently, for measurement constructions and the positive use of gradable adjectives (see (16) and (17)), the following inference patterns hold, which are consistent with our intuition: (i) John and Bill are between 5.9 and 6.2 feet tall $\models$ John is between 5.9 and 6.2 feet tall; (ii) John and Bill are tall $\models$ John is tall. Nevertheless, the interval ⟦than A is ~~tall~~⟧ entails (i.e., is a subset of) the interval ⟦than A and B are DIST ~~tall~~⟧ (see (45b)).

[14] https://www.businessinsider.in/millennials-have-less-money-than-any-other-generation-did-at-their-age-but-youd-never-guess-it-from-the-way-theyre-flaunting-their-money-on-dating-apps/articleshow/69379306.cms

[15] https://www.mckinsey.com/business-functions/strategy-and-corporate-finance/our-insights/economic-conditions-snapshot-september-2019-mckinsey-global-survey-results

Specifically, as illustrated in (48), weak NPI *any* is analyzed as a narrow-scope, non-deictic indefinite (see also Giannakidou 2011). It is distinct from a genuine deictic indefinite (e.g., *some boy*) in the sense that its narrow-scope reading is compulsory (see Barker 2018 on the scoping behavior of NPIs), so that a dynamic update with this non-deictic indefinite cannot be non-deterministic. Roughly, *any boy* means a **random**, **very vague or low informative** boy conceptualized from the contextually relevant set of individuals.[16]

Thus as shown in (48a), the *than*-clause amounts to addressing the speed of a **random** boy in the context, denoting the most informative interval I' such that the speed of a random boy (among X, Y, and Z) falls within I': i.e., the interval of speed ranging from the slowest to the fastest boy's speed, which is the interval $[6.7\text{ m/s}, 7.8\text{ m/s}]$. The *than*-clause serves as the standard of comparison. Then with the value of I_{DIFF} (here $[0.1\text{ m/s}, +\infty)$), the matrix-level meaning can be thus derived via interval subtraction.

(48) (Context: Roxy ran at a speed of 8 ± 0.1 m/s, and the boys – X, Y, and Z – ran at a speed of 6.7 m/s, 7.2 m/s, and 7.8 m/s, respectively.)
Roxy ran (at least 0.1 m/s) faster than **any** boy did. (= (1a))
LF: Roxy ran $\underbrace{\text{at least 0.1 m/s} \ldots\text{-er}}_{I_{\text{DIFF}}} \ominus \underbrace{\text{than } \textbf{any} \text{ boy did } \cancel{\text{run fast}}}_{I_{\text{STDD}}}$ fast

a. $I_{\text{STDD}} = [\![\text{than } \textbf{any} \text{ boy did } \cancel{\text{run fast}}]\!]$
$= [\![\text{than } \textbf{a random boy} \text{ (among X, Y, and Z) did } \cancel{\text{run fast}}]\!]$
i.e., the interval ranging from the slowest to the fastest boy's speed (see also (45a)), which is $[6.7\text{ m/s}, 7.8\text{ m/s}]$ under the given context.

b. $I_{\text{DIFF}} = [0.1\text{ m/s}, +\infty) \cap (0, +\infty) = [0.1\text{ m/s}, +\infty)$

c. $\textsc{speed}(\mathsf{Roxy}) \subseteq \iota I[\underbrace{I - \underbrace{\iota I'[\textsc{speed}(\mathsf{a\text{-}random\text{-}boy}) \subseteq I']}_{=[6.7\text{ m/s},7.8\text{ m/s}]} = [0.1\text{ m/s}, +\infty)]}_{=[7.9\text{ m/s},+\infty)\text{ (see (12))}}$

The licensing and interpretation of emphatic and strong NPIs are similar, as sketched below. Emphatic NPIs contribute a narrow-scope, non-deictic, scalar-related item: i.e., they can be interpreted as **a random item** conceptualized from an ordered set (of actions, times, etc). Then in interpreting a *than*-clause, an interval – a range of measures – is yielded from the use of such an NPI.

In (49), *give a penny*, a minimizer (or emphatic NPI), can be considered **a random action**, a notion abstracted from an ordered set of actions (along a contextually relevant scale such as effort amount, generosity, willingness, etc), and a (lower or upper) bound of this ordered set is *give a penny*.

[16] In terms of dynamic semantics, we can consider *any boy* an introduced variable that (i) only exists very locally, taking narrow scope, and (ii) is vague in the sense that it only carries non-distinctive restrictions that hold true for all and each specific individual in the relevant set (e.g., here $\mathsf{boy}(x)$ and $\textsc{speed}(x) \subseteq [6.7\text{ m/s}, 7.8\text{ m/s}]$).

In this *would sooner ... than* sentence, the comparison is performed along a scale of willingness. Thus, the *than*-clause means a right-bounded interval, i.e., $(\ldots, \textsc{willingness}(\textsf{give-a-penny})]$, and serves as $I_{\textsc{stdd}}$ in this comparative.[17]

(49) He would sooner roast in hell than **give a penny** to others. (≈ (2a))

a. ⟦give a penny⟧
$\rightsquigarrow$ a random action abstracted from a set of actions (ordered along a certain scale, e.g., effort amount, generosity, willingness), 'give a penny' representing a (lower or upper) bound of this set
(i.e., any action that is at least/most like 'give a penny')

b. ⟦than ~~he would like to~~ **give a penny** to others⟧
$= (\ldots, \textsc{willingness}(\textsf{give-a-penny})]$

Similarly, in (50), *could help* can be considered a **random action** abstracted from an ordered set of actions (along a scale of self-control strength, or a scale of difficulty for resisting an urge). Eventually, the comparison here is performed along a scale of self-control strength, and the use of *could help* leads to a right-bounded interval in interpreting *than I could help*.[18]

(50) My urge to steal was stronger than I **could help**. (= (3a))

a. ⟦could help⟧ $\rightsquigarrow$ a random action from a set of actions (ordered along a certain scale, e.g., self-control strength)

b. ⟦than ~~the urge~~ I **could help** ~~is strong~~⟧
$= (\ldots,$ the largest value of my self-control strength$]$

For (51) and (52), strong NPIs *yet* and *in years* express a very vague range of time. From the semantics of *yet*, we only know that this range of time is right-bounded (see (51a)). From the semantics of *in years*, we only know that this range of time is measured with the unit of years (see (52a)). Intuitively, both *yet* and *in years* suggest a long time. The use of *yet* or *in years* presumably rules out the existence of some deictic time point/interval. The *than*-clauses convey a range of performance quality or happiness within these vague ranges of time.

(51) It requires better performance than I've seen **yet**. (= (4a))

a. ⟦yet⟧ $\rightsquigarrow$ a vague range of time: $(\ldots,$ an unspecified reference time$]$

[17] Why does *give a penny* correspond to the upper bound of an interval of willingness? I assume this is due to the meaning postulate of this idiomatic expression. This expression should also correspond to the lower bound of an interval of effort amount or generosity (e.g., *John didn't give a penny* means that John didn't even make the least effort or show the least generosity). In our world knowledge, larger effort should correlate with less willingness and more generosity.

[18] According to the interval-subtraction-based analysis, $I_{\textsc{stdd}}$ in *more-than* comparatives needs to be right-bounded, but $I_{\textsc{stdd}}$ in *less-than* comparatives needs to be left-bounded. Therefore, for *more-than* comparatives in (49) and (50), the two $I_{\textsc{stdd}}$ (along the scales of willingness and self-control strength) should be right-bounded. For a *less-than* comparative like *he did less than give a penny to his son*, $I_{\textsc{stdd}}$ has to be left-bounded (e.g., along a scale of effort amount).

b. ⟦than ~~the performances~~ I've seen **yet** ~~are good~~⟧
≈ [the lowest quality of all performances I've seen,
the highest quality of all performances I've seen]

(52) He made me feel happier than I felt **in years**. (= (5a))

a. ⟦in years⟧
⇝ a vague range of time measured with the unit of years: (. . . , . . .)

b. ⟦than I felt ~~happy~~ **in years**⟧
≈ [the lowest degree of my happiness over a long time,
the highest degree of my happiness over a long time]

For the cases of NPIs licensed by classical negation operator *not* (see (53)), the low informativeness of NPIs is directly flipped by the operation of negation. As shown in (48)–(52), for the cases of *than*-clause-internal NPIs, the low informativeness of these NPIs leads to low informative intervals that serve as comparison standard, and then it is during interval subtraction that low informativeness gets flipped into high informativeness at the matrix level.

(53) a. Roxy didn't see **any** boy.
⇝ No boy was seen by Roxy.

b. He left the world without **giving a penny** to his son.
⇝ No action, not even the least effort-demanding one, accompanied his leaving the world.

c. I **could**n't **help** laughing.
⇝ Laughing was beyond my self-control.

d. I haven't read the book **yet**.
⇝ At no time have I read the book.

e. He wasn't happy **in years**.
⇝ At no time was he happy.

In sum, NPIs convey a random, low informative, non-deictic item, which can be a deficient indefinite or a very uninformative range of time (see Giannakidou 2011 on the deficiency of NPIs). NPI licensers make use of them in a way that flips informativeness, i.e., projecting the low informativeness of NPIs to sentential-level meaning and, meanwhile, flipping low informativeness into high informativeness. The subtrahend status of a *than*-clause plays exactly this role in flipping informativeness, thus licensing NPIs.

7 Discussion

The current paper is innovative in addressing the monotonicity projection resulted from the operation of interval subtraction. Thus, the subtrahend status of a *than*-clause makes it a degree-semantics-based NPI licenser. As mentioned earlier, the basic view of Hoeksema (1983) is maintained: i.e., comparatives are DE and anti-additive. The current paper further strengthens and pinpoints this view, showing that due to its subtrahend status, the negativity of the comparison standard is actually as strong as that of classical negation operator *not*.

Previously, Giannakidou and Yoon (2010) argues that comparatives do not contain a DE operator that can license NPIs. Their analysis is problematic in a few respects. First, as I have shown throughout the paper, comparatives do contain a DE operator. It is the subtrahend status of the *than*-clause. However, distinct from the classical, set-operation-based, negation operator, the subtrahend status gets its negative flavor from the operation of **interval subtraction**.

Second, according to Giannakidou and Yoon (2010), only weak NPIs, but not strong NPIs, can be licensed in a non-DE environment (such as comparatives) via a rescuing mechanism. They also analyze English minimizers like *give a penny* as weak NPIs. However, empirical data like (4), (5), and (47) (a naturally occurring example) show that English strong NPIs like *yet* and *in years* are also licensed within a *than*-clause. Thus even if weak NPIs might not rely on a DE environment for licensing, we still need to explain why some strong NPIs are nevertheless licensed within a *than*-clause.

Third, Giannakidou and Yoon (2010) suggests that *than*-clause-internal *any* is likely to be a free choice item (FCI), not an NPI, and as a consequence, *than*-clause-internal *any* does not need a DE environment for licensing. This is suspicious for two reasons (see also Aloni and Roelofsen 2014 for discussion).

(i) First, FCI *any* is ill-formed in both positive and negative episodic sentences, and FCI *any* has its own licensing environments, such as modal statements (see (54)). Then it becomes puzzling why *any* is grammatical in an embedded episodic *than*-clause, as shown in (1a) (repeated here as (55)). If, as claimed by Giannakidou and Yoon (2010), the *than*-clause is not negative-flavored, then *any* should simply be ruled out in (55), no matter it is an NPI or an FCI.

(54) a. *Anyone ate. $\rightsquigarrow$ FCI *any*: ill-formed in positive episodic sentences
b. *Anyone didn't eat.
$\rightsquigarrow$ FCI *any*: ill-formed in negative episodic sentences
c. Anyone can eat. $\rightsquigarrow$ FCI *any*: licensed in modal statements

(55) a. Roxy ran faster than any boy did (yesterday). (= (1a))

(ii) Second, according to Giannakidou and Yoon (2010), *than*-clause-internal *any* can be modified by *almost*, suggesting that it is FCI *any*, not NPI *any* (see the contrast in (56)). However, it is questionable whether the use of *almost* is a great test for distinguishing FCI and NPI *any*, and the empirical evidence is not as clear-cut as shown in (56) (which repeat Giannakidou and Yoon 2010's (51)). On the one hand, naturally occurring examples from *Corpus of Contemporary American English* (CoCA, Davies 2008) show that NPI *any* can be compatible with the modification of *almost* (see (57)). On the other hand, Kadmon and Landman (1993) argue for a unified account for NPI and FCI *any*.

(56) a. Mary wrote more articles than **almost any** professor suggested.
b. ??Mary didn't buy **almost any** book.

(57) a. BA and BS aren't worth **almost anything** now ...
b. These people, they don't have **almost anything**.
c. ...they didn't get **almost anything** that they wanted.

Taken together, these provide evidence showing that it is questionable to analyze *than*-clause-internal *any* (see (1a)/(55)) as FCI.

A further issue raised by the analysis of Giannakidou and Yoon (2010) is on *either*. According to Giannakidou and Yoon (2010), *either* is a genuine strong NPI in English, and it cannot be licensed within a *than*-clause (see (7a), repeated here as (58)). Indeed, *either* can only appear in sentences containing classical negative words like *not*, *no one*, *never*, etc. However, I tend to think that the semantics of *either* is largely different from NPIs like *any*, *give a penny*, *could help*, *yet*, *in years*, etc. Intuitively, the ungrammatical use of *either* in positive sentences (see (7b-ii), repeated here as (59)) is much more similar to the ungrammatical use of *too* in negative sentences (see (60b)) than to an unlicensed NPI. If *too* is not analyzed as a positive polarity item (PPI), why do we need to analyze *either* as an NPI? After all, the interpretation of other NPI phenomena involves monotonicity projection and downward inferences, introducing narrow-scope, non-deictic variables, or triggering strengthening implications, but the interpretation of *either* does not involve any of these.

(58) *Kevin is not tall, and John is taller than Bill is **either**. (= (7a))

(59) *Bill is tall, and I know that John is tall, **either**. (= (7b-ii))

(60) a. Mary came. I know that Bill came, **too**.
b. *Mary didn't came. I know that Bill didn't came, **too**.

The current analysis on NPI licensing in comparatives is rooted in Ladusaw's and Zwarts' theories on DE-ness and negativity: NPI phenomena mark downward inferences. The current analysis is also compatible with three other influential theories of NPI phenomena.

Specifically, my sketched analysis of NPIs as narrow-scope, low informative, non-deictic items captures the essence of Giannakidou's non-veridicality theory of NPIs (see Giannakidou 2011 for a review): NPIs are distinct from genuine indefinites in that there is no projectable existential force.

Then the communicative value of NPIs in my analysis is consistent with Kadmon and Landman (1993)'s view that NPI licensing triggers strengthening implications: NPIs convey locally low informativeness, but this low informativeness is eventually flipped into high informativeness by DE operators.

Finally, according to Barker (2018)'s scope-marking theory, NPIs signal that an indefinite is taking narrow scope, and the narrow-scope reading is more informative than a wide-scope reading. This view captures our intuition that NPIs seem to be interpreted as locally existential, but globally universal (see (61)). Therefore, Barker (2018) provides a generalized view for the universal flavor of NPIs. My analysis of *than*-clause-internal *any* as NPI *any* is thus a special case. There is no need to attribute this universal flavor to an FCI-*any* account.

(61) Mary didn't see any cat. (cf. $\exists x[\mathsf{cat}(x) \wedge \neg\mathsf{see}(\mathsf{Mary}, x)]$ – $\exists > \neg$)
a. $\neg\exists x[\mathsf{cat}(x) \wedge \mathsf{see}(\mathsf{Mary}, x)]$ $\neg > \exists$
b. $\forall x[\mathsf{cat}(x) \rightarrow \neg\mathsf{see}(\mathsf{Mary}, x)]$ $\forall > \neg$

Among the core issues on NPIs, compositionality has not been much addressed in the current paper. I analyze the meaning of a *than*-clause as a definite, most informative scalar value (in terms of an interval) that is the short answer to a corresponding degree question. However, I haven't gone into the compositional details of a comparative containing *than*-clause-internal NPIs. Strong NPIs cannot be used in *wh*-questions or degree questions. Thus, a plausible derivation scheme should involve a delayed evaluation mechanism in interpreting a *than*-clause that contains NPIs (see Barker and Shan 2014, Zhang 2020 for relevant discussions on the evaluation order in NPI licensing and the compositional issue of *than*-clause-internal quantifiers). This is left for future research.

Another issue worth mentioning is how the current analysis can be extended to account for NPI licensing in phrasal comparatives (e.g., phrasal comparatives in Greek/English, Japanese *yori*-comparatives, Chinese *bǐ*-comparatives). Cross-linguistically, these constructions do not necessarily demonstrate the same pattern with regard to licensing *than*-phrase-internal NPIs. Besides, I suspect that emphatic and strong NPIs like *give a penny* and *in years* simply cannot be used in phrasal comparatives, due to syntactic reasons. A full investigation is also left for another occasion.

8 Conclusion

With the use of an existing, independently motivated analysis of comparatives (i.e., Zhang and Ling 2020's interval-subtraction-based analysis), I have shown that by serving as the standard in a comparison and playing the role of subtrahend in a subtraction equation, a *than*-clause is by nature strongly negative-flavored. The subtrahend status is downward-entailing, anti-additive, and anti-morphic, flipping the informativeness of an interval standing for the subtrahend. Therefore, a *than*-clause is a natural NPI licenser.

The current analysis has profound implications for theories of NPIs and NPI licensing, especially with regard to how NPIs are composed and evaluated with other parts of a sentence. There is still much left for future research.

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