The ups and downs of ignorance

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Abstract Plain disjunctive sentences, such as *The mystery box contains a blue ball or a yellow ball*, typically imply that the speaker does not know which of the two disjuncts is true. This is known as an IGNORANCE inference. We can distinguish between two aspects of this inference: the negated universal upper bound part (i.e., the speaker is uncertain about each disjunct), which we call UNCERTAINTY, and the existential lower bound part (i.e., the speaker considers each disjunct possible), which we call POSSIBILITY. In the traditional approach, UNCERTAINTY is derived as a primary implicature, from which POSSIBILITY follows. In this paper, we report on three experiments using a sentence-picture verification task based on the mystery box. Our findings show that POSSIBILITY can arise without UNCERTAINTY, and thus call for a re-evaluation of the traditional view of disjunction and IGNORANCE inferences. Our experimental findings are related to similar results involving disjunction in embedded contexts and pave the way for alternative theories that can account for the observed patterns of inference derivation in a unified fashion. We will discuss how recent implicature and non-implicature theories can account for the derivation of existential lower bound inferences without the presence of negated universal upper bound inferences.

Keywords: Disjunction, Ignorance inference, Implicature, Experimental Semantics & Pragmatics

Contents

1 Introduction

It is well-known that sentences with a plain disjunction like (1) typically give rise to IGNORANCE and EXCLUSIVITY inferences, indicated in (1a) and (1b):

(1) Sue went to the park or the cinema. $A \vee B$

a. $\rightsquigarrow$ *The speaker doesn't know which of the two is true* IGNORANCE

b. $\rightsquigarrow$ *Sue didn't go to the park and to the cinema* EXCLUSIVITY

In this work, we will mainly focus on IGNORANCE inferences, though we will also consider the role of EXCLUSIVITY when relevant. A natural way to think about IGNORANCE is as follows: a speaker s is ignorant about A when s does not believe that A is the case and does not believe that $\neg A$ is the case. In other words, both $\neg A$ and A are live possibilities in the speaker's doxastic state (Gazdar 1979, Hintikka 1962). We can thus distinguish between two aspects of an ignorance inference of a plain disjunction like (1): the upper bound UNCERTAINTY part in (2a) and the lower bound POSSIBILITY part in (2b), where $\Box_s$ and $\Diamond_s$ quantify universally and existentially over the speaker's doxastic state.

(2) $\Diamond_s A \wedge \neg \Box_s A \wedge \Diamond_s B \wedge \neg \Box_s B$ IGNORANCE INFERENCE
The speaker doesn't know which of the two is true

a. $\neg \Box_s A \wedge \neg \Box_s B$ UNCERTAINTY
The speaker is not certain that Sue went to the park and the speaker is not certain that Sue went to the cinema.

b. $\Diamond_s A \wedge \Diamond_s B$ POSSIBILITY
The speaker deems possible that Sue went to the park and the speaker deems possible that Sue went to the cinema.

The traditional approach to ignorance inferences derives UNCERTAINTY as an implicature of (1) through Gricean quantity reasoning about each individual disjunct. From this, POSSIBILITY follows, given also the quality assumption that the speaker believes (1) (among others, Sauerland 2004b, Fox 2007). As such, this approach predicts that POSSIBILITY *cannot* arise in the absence of UNCERTAINTY.[1]

Recent non-implicature proposals, however, suggest that POSSIBILITY should be derived independently of UNCERTAINTY, either through a presupposition (Goldstein 2019) or as so-called neglect-zero pragmatic inference (Aloni 2022). According to these theories, a sentence like (1) can give rise to POSSIBILITY in the absence of UNCERTAINTY. Consequently, these theories, unlike the traditional implicature

1 Grammatical versions of the implicature approach make the same prediction, but they derive the corresponding inferences by means of a covert exhaustivity operator, interacting with a covert doxastic operator (among others, Meyer 2013, Buccola & Haida 2019).

approach, predict that a disjunctive sentence like (1) should be judged as felicitous when the speaker is certain about one of the disjuncts, while only considering the other possible.

In this study, we report on three experiments testing these divergent predictions. All experiments consisted of a sentence-picture verification task based on the mystery box paradigm (Noveck 2001, Marty et al. 2023, to appear). The results of our experiments show that POSSIBILITY can indeed arise without UNCERTAINTY. Specifically, we found that participants judged sentences where UNCERTAINTY inferences were false as good as true controls, while more than half of them judged the sentence false when POSSIBILITY inferences were also false. These results pose a challenge for the traditional approach, which cannot predict POSSIBILITY in the absence of UNCERTAINTY.

Our experiments focused on cases of plain disjunctive sentences. Sentences involving disjunction in embedded contexts give rise to similar inference patterns and have sparked a similar debate (Marty et al. 2023, Crnič et al. 2015). We will examine the relationship between our findings and previous experimental research on embedded disjunction. In particular, we will interpret our experimental results from three different perspectives.

Firstly, we will explore how a recent grammatical account (Bar-Lev & Fox 2023), originally developed to capture the behavior of disjunction in embedded contexts, can be extended to deal with plain disjunctive sentences and what the consequences of this extension are. Secondly, our experimental results are in line with the recent non-implicature accounts of POSSIBILITY mentioned above (Aloni 2022, Goldstein 2019). We will focus on the account of Aloni (2022) and discuss how it derives POSSIBILITY. Thirdly, we will comment on the possible linking hypotheses behind our experiments and consider how our results fare with game-theoretic approaches to disjunction and modal inferences (Franke 2009). We will conclude with potential follow-ups and a discussion on the role of different experimental tasks and protocols for the same types of inferences, emphasizing the importance of taking a holistic experimental approach when studying pragmatic phenomena.

The rest of the paper is structured as follows. Section 2 summarizes the traditional approach and its predictions. Section 3 contains an overview of the experiments, followed by a report of Experiment 1, Experiment 2 and Experiment 3 in Section 4, Section 5 and Section 6, respectively. Section 7 discusses the results of our experiments, focusing on the connection with distributive inferences and the grammatical approach in Section 7.1, non-implicature accounts of disjunction in Section 7.3, and game-theoretic approaches in Section 7.4. Section 8 concludes.

2 The traditional approach and its predictions

The traditional Gricean approach (Grice 1975, 1989) derives IGNORANCE and EXCLUSIVITY inferences based on a general process of conversation that takes into account the information available to the speakers and the assumption that language users behave cooperatively. The most relevant principles underlying these derivations are the maxim of quantity, which roughly says that speakers should convey all and only the most informative statement given what they know; and the maxim of quality, which roughly says that speakers should be always truthful.

These maxims can be made formal in different ways and a number of accounts have been proposed stemming from the original Gricean insights (among others, Horn 1972, Gazdar 1979, Levinson 1983, Gamut 1991). In what follows, we will rely on the following formalization of inference derivation:

(3) For all relevant alternatives ψ of an utterance ϕ:

$$\text{if } \psi \vDash \phi \text{ and } \phi \nvDash \psi \text{, then } \neg \Box_S (\psi)$$

In words, when speakers choose to make a weaker statement (ϕ) instead of a stronger alternative (ψ), it implies that they do not believe that the stronger statement is true ($\neg \Box_S \psi$). The principle above refers to the notion of *relevant* alternatives. An important question for this approach is to determine what counts as an alternative, and what ultimately counts as a relevant alternative. We will assume, as proposed in Sauerland (2004b) and maintained in subsequent literature, that a disjunction has also the individual disjuncts as alternatives. And these alternatives will end up being relevant in most cases.

(4)
$$\left\{ \begin{array}{ll} \text{Sue went to the park or the cinema.} & A \vee B \\ \text{Sue went to the park.} & A \\ \text{Sue went to the cinema.} & B \\ \text{Sue went to the park and to the cinema.} & A \wedge B \end{array} \right\}$$

The Gricean account just outlined provides a systematic way to derive both IGNORANCE and EXCLUSIVITY inferences associated with a plain disjunction. As previously stated, IGNORANCE inferences are comprised of two elements: UNCERTAINTY and POSSIBILITY. The standard procedure is to derive UNCERTAINTY first, and then use quality to derive POSSIBILITY.[2]

For instance, $\neg \Box_S A$ would be derived as in (5). A similar process with the

2 Another option is to close the alternatives under negation, so that the negative counterparts of the positive alternatives count as relevant (Fox 2007). Even with this approach, the negative alternatives, responsible for POSSIBILITY, are present due to the positive alternatives, responsible for UNCERTAINTY.

alternative B and with the alternative $A \wedge B$ would give us $\neg \Box_s B$ and $\neg \Box_s (A \wedge B)$, respectively. Together, these are known as the primary implicatures of a plain disjunctive sentence.

(5) Derivation of UNCERTAINTY ($\neg \Box_s A$)
  a. Assertion: $A \vee B$
  b. A is a relevant alternative to $A \vee B$
  c. A is stronger than $A \vee B$
  d. From (5b) and (5c) above together with (3), $\neg \Box_s A$

POSSIBILITY is then derived from UNCERTAINTY together with the Quality assumption that speakers believe $A \vee B$. As a result, this method of deriving POSSIBILITY makes this inference dependent on UNCERTAINTY.

(6) Derivation of POSSIBILITY
  a. Assertion: $A \vee B$
  b. Quality: $\Box_s(A \vee B)$
  c. UNCERTAINTY: $\neg \Box_s A \wedge \neg \Box_s B$
  d. From (6b) and (6c) above, $\Diamond_s A \wedge \Diamond_s B$

As for the derivation of the EXCLUSIVITY inference in (1b), a similar argument to the derivation in (5) for the alternative $A \wedge B$ yields $\neg \Box_s (A \wedge B)$. However, to obtain the EXCLUSIVITY inference $\Box_s \neg(A \wedge B)$, the standard approach relies on an opinionatedness principle of the form $\Box_s \phi \vee \Box_s \neg \phi$. This principle, also known as *competence assumption* or *epistemic step*, states that the speaker knows the relevant facts (Sauerland 2004b, 2005, Chierchia et al. 2012, Geurts 2010). Applying this principle to $\neg \Box_s (A \wedge B)$ gives us the (secondary) implicature $\Box_s \neg(A \wedge B)$.

It is important to note here is that the derivation of POSSIBILITY in (6) does not rely on the opinionatedness principle just outlined. In fact, Sauerland (2004b) claims that individual disjuncts cannot be subject to opinionatedness, as generating inferences of the form $\Box_s \neg \psi$ from $\neg \Box_s \psi$ would, in combination with the Quality assumption about the assertion, contradict previously obtained implicatures (i.e. $\neg \Box_s \psi'$).[3]

3 Relatedly, we note that one of the driving factors behind the richer set of alternatives in (4) was the observation that UNCERTAINTY and POSSIBILITY, unlike standard scalar implicatures, are not cancellable (Sauerland 2004b,a). However, we observe that while POSSIBILITY is indeed not cancellable, UNCERTAINTY seems to pattern with EXCLUSIVITY in terms of its cancellability:

(i) a. #I saw the Queen or the Princess of the Netherlands at the airport.
  In fact, it cannot be that I saw the Queen.
  b. I saw the Queen or the Princess of the Netherlands at the airport.
  In fact, I saw both.

3 Overview of the experiments

As discussed in the previous section, the derivation of POSSIBILITY relies on UNCERTAINTY according to the standard approach. In the following, we report on three experiments testing this hypothesis, which we outline here.

All experiments were built upon Marty et al. (to appear)'s elaboration of Noveck (2001)'s mystery box paradigm (see also Marty et al. 2023). In all experiments, participants were presented with sentence-picture items like those in Figure 1. Each item depicted a set of four boxes, three open and one covered, displayed just above an utterance produced by one of two characters. Participants were instructed that the characters could see what's inside the first three boxes but not what's inside the covered one, the so-called *mystery box*. They were also instructed that the characters had been taught the rule that the mystery box always had the same contents as one of the three open boxes (see Section 4.3 and Appendix A for the experimental instructions). The task was to decide if the character's utterance was right given the information available to them and the rule that they had learned about how the mystery box works. Participants reported their response by clicking on one of two response buttons, labelled 'Good' and 'Bad', respectively. Example (7) illustrates the disjunctive test sentences studied in the experiments and its related inferences.

(7) 'The mystery box contains a yellow ball or a blue ball' $\Box_s(A \vee B)$

  a. $\neg \Box_s A \wedge \neg \Box_s B$ UNCERTAINTY

  b. $\Diamond_s A \wedge \Diamond_s B$ POSSIBILITY

The two critical conditions were TARGET-1 and TARGET-2. In the TARGET-1 conditions, test sentences were paired with pictures that make UNCERTAINTY false, but POSSIBILITY true; in the TARGET-2 conditions, they were paired with pictures that make both these inferences false. We hypothesized that, if POSSIBILITY is derived from UNCERTAINTY, no difference in participant's responses should be observed between both target conditions. On the other hand, if POSSIBILITY is derived independently from UNCERTAINTY, then participants should reject the test sentences to a greater extent in the TARGET-2 than in the TARGET-1 conditions.

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c. I saw the Queen or the Princess of the Netherlands at the airport.
In fact, I am pretty sure I saw the Queen and possibly the Princess.

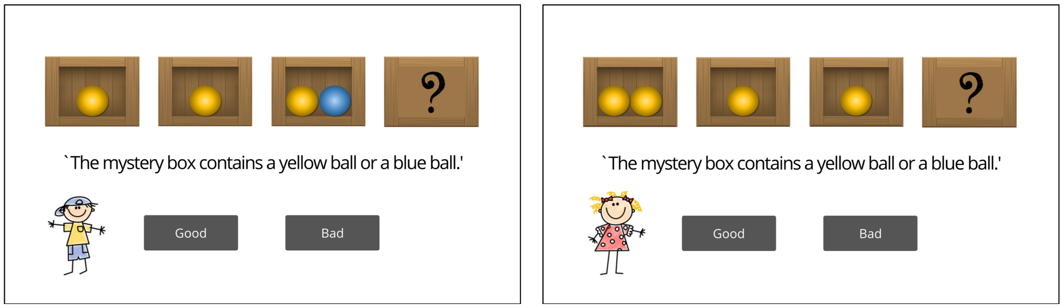


Figure 1 Example items illustrating the items' layout and task in Experiment 1, 2 and 3. These examples are instances of the TARGET-1 (on the left) and TARGET-2 (on the right) conditions in Experiment 2. UNCERTAINTY is false in both the TARGET-1 and TARGET-2 conditions while POSSIBILITY is false only in the TARGET-2 conditions.

4 Experiment 1

4.1 Participants

101 native speakers of English participated in this study (mean age 39 yrs; 51 female). Participants were recruited online through Prolific (https://www.prolific.co; see Palan & Schitter for an overview) using a suitable set of prescreening criteria (first language: English; nationality: UK/US; country of birth: UK/US; minimum approval rate: $\geq 90\%$). Participants were paid £1.20, and median completion time was about 8 minutes (hourly rate: £9/hr). All participants gave written informed consent. Data were collected and stored in accordance with the provisions of Data Protection Act 2018. The study was approved by the Research Ethics Committee at University College London and at University of Amsterdam.

4.2 Materials and Design

The experiment was based on the materials and method from Marty et al. (to appear: Exp.4-6) (see also see also Marty et al. 2023). Each item involved a sentence displayed just below a set of boxes and right above the picture of one of two characters, as exemplified in Figure 1. Sentences were constructed using the sentence frames in Table 1. The color adjectives used in the sentences, indicated by the [A] and [B] terms in Table 1, were picked at random from a list of four color terms – *yellow*, *blue*, *green* and *gray* – with replacement across items. Each sentence was placed inside simple quotation marks to show direct speech and to make it explicit that the sentence was uttered by whatever character was depicted on the item.

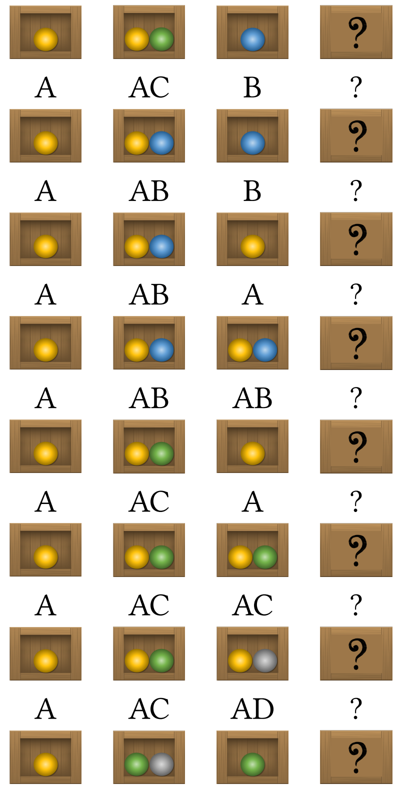
TEST		'The mystery box contains a [A] ball or a [B] ball.'
CONTROL	C1	'The mystery box contains a [A] ball.'
	C2	'The mystery box does not contain a [A] ball.'
	C3	'The mystery box contains a [A] ball and a [B] ball.'
	C4	'The mystery box does not contain a [A] ball or a [B] ball.'

Table 1 Schematic description of the sentences tested in Experiment 1, where [A] and [B] are placeholders for different colour adjectives; for a more concrete illustration, you may read [A] as *yellow* and [B] as *blue*.

Every item displayed a set of four boxes horizontally arranged, each of which was made of three open boxes, containing one or two balls, and a covered one, marked with the symbol '?' and referred to as *the mystery box*. The position of the boxes in the row was pseudo-randomly assigned so that the mystery box always appeared at the rightmost position. The contents of the open boxes in each quadruplet were experimentally manipulated to create different picture types corresponding to the experimental conditions of the study. The test sentences under investigation were paired with five different picture types, which are described in Table 2: the colors of A-balls and B-balls depicted in the open boxes always matched the [A] and [B] color terms used in the sentences (e.g., yellow and blue) whereas the colors of the C-balls and D-balls were randomly chosen from our list of color terms by excluding the color(s) of the matching balls (e.g., green and gray).

Target pictures were designed to make UNCERTAINTY always false, but POSSIBILITY either true or false. On the TARGET-1 pictures, each of the three open boxes contained an A-ball and at least one of them also contained a B-ball, making one of the UNCERTAINTY inferences of the test sentences false, but their POSSIBILITY inferences true. TARGET-2 pictures were obtained from the TARGET-1 pictures by replacing the B-ball(s) with balls of a non-matching color (i.e., a C-ball or a D-ball), thus making one of the UNCERTAINTY and one of the POSSIBILITY inferences false. Different variants of the TARGET-1 and TARGET-2 pictures were constructed by varying the number of matching B-balls for the former (1 B-ball vs. 2 B-balls) and by varying both the number and color of non-matching balls for the latter (1 C-ball vs. 2 C-balls vs. 1 C-ball and 1 D-ball). For the purposes of experimental design, variants of the TARGET-1 and TARGET-2 pictures were treated as sub-conditions of the TARGET-1 and TARGET-2 conditions.[4]

4 As reported in Section 4.5, no contrast in participants' responses was found between the variants of



Condition		Example picture			
TRUE-ADHOC		A	AC	B	?
TRUE-EXCL		A	AB	B	?
TARGET-1	i.	A	AB	A	?
	ii.	A	AB	AB	?
TARGET-2	i.	A	AC	A	?
	ii.	A	AC	AC	?
	iii.	A	AC	AD	?
FALSE		A	CD	C	?

Test sentence: *'The mystery box contains a yellow ball or a blue ball.'* $(A \vee B)$

Table 2 Schematic description and illustration of the picture types paired with the test sentences in Experiment 1. The color of the A-balls and B-balls always matched the color adjectives used in the sentence (e.g., *yellow* and *blue*) while the color of the C-balls and D-balls never did (e.g., *green* and *gray*). Picture types are illustrated here using the following color assignment: A=yellow, B=blue, C=green and D=gray.

FALSE, TRUE-EXCL and TRUE-ADHOC pictures were control pictures, each of which served a different experimental purpose. FALSE pictures were designed to provide a clear baseline for rejection. On these pictures, one of the open boxes contained an

the TARGET-1 pictures, nor between those of the TARGET-2 pictures, allowing us to aggregate the responses to the TARGET-1 and the TARGET-2 trials across sub-conditions without loss of information.

A-ball while the other two contained balls of a non-matching color, making the test sentences unambiguously false. TRUE-EXCL and TRUE-ADHOC pictures were designed to probe for the strength of two other inference types that the test sentences could give rise to: (i) 'not-both' exclusivity implicatures, e.g., *the mystery box does not contain both a [A] ball and a [B] ball* and (ii) ad-hoc exhaustivity inferences, e.g., *the mystery box does not contain a [C] ball or a [D] ball.* Crucially, the derivation of such inferences can affect participants' responses in the target conditions: the first one makes the test sentences false in the TARGET-1 conditions, where one or more boxes contain two matching balls, while the second makes them false in the TARGET-2 conditions, where one or more boxes contain a non-matching ball. The goal of the TRUE-EXCL and TRUE-ADHOC conditions was to assess the extent to which participants derived these inferences so as to factor out their potential effects from the comparison of primary interest between TARGET-1 and TARGET-2 conditions. On the TRUE-EXCL pictures, one of the open boxes contained both an A-ball and a B-ball, as on the TARGET-1 pictures. These pictures made the test sentences true unless an exclusivity inference is derived (i.e., *the mystery box contains a yellow ball or a blue, but not both*). On the TRUE-ADHOC pictures, one of the open boxes contained an A-ball and a C-ball, as on the TARGET-2 pictures. These pictures made the test sentence true unless an ad-hoc exhaustivity inference is derived (i.e., *the mystery box contains a yellow ball or a blue, and nothing else*).

In addition to the test sentences, there were four different types of control sentences: two positive sentences (C1 and C3) and two negative ones (C2 and C4), involving either one color adjective (C1 and C2) or two (C3 and C4). Each of these sentences were paired with pictures that made them either clearly true (GOOD conditions) or clearly false (BAD conditions), as described and illustrated in Figure 3. These control items were added to the experiment to identify low-effort responses as well as to control for certain low-level response strategies in the data treatment. Specifically, we worried that some participants may perform the task superficially, simply by checking whether or not the colors mentioned in the sentence match those of the balls depicted on the pictures. We reasoned that, if a participant follows such a strategy, then they should perform relatively poorly on these items, especially in those involving negative sentences (C2 and C4).

Pairing the test and control sentences with the relevant picture types gave rise to 5 test and 8 control conditions. Each control condition was instantiated 3 times, giving rise to 24 control trials. As for the test conditions, the TARGET-1, TARGET-2 and FALSE conditions were instantiated 6 times each, and the TRUE-EXCL and TRUE-ADHOC conditions 3 times each, giving rise to 24 test trials. Instances of the TARGET-1 and TARGET-2 conditions were evenly distributed across their respective sub-conditions. Thus, each survey included 48 trials in total.

Sentence	Condition	Example picture			
C1: A	GOOD	A	A	A	?
	BAD	A	CD	C	?
C2: $\neg A$	GOOD	CD	CD	C	?
	BAD	A	CD	A	?
C3: $A \wedge B$	GOOD	AB	AB	AB	?
	BAD	AB	CD	C	?
C4: $\neg(A \vee B)$	GOOD	C	CD	C	?
	BAD	A	CD	B	?

Table 3 Schematic description and illustration of the picture types paired with the control sentences in Experiment 1. Picture types are illustrated using the same color assignment as before.

4.3 Procedure

The experiment was run as an online survey using Gorilla Experiment Builder (Anwyl-Irvine et al. 2020). In the instructions, participants were introduced to two characters, Sam and Mia, and they were presented with a short cover story. The cover story went in substance as follows (see Appendix A for details): Sam and Mia are looking at quadruplets of boxes containing balls of various colors. Each time, they can only see what's inside the first three boxes. However, they have been taught the rule that the fourth box, known as 'the mystery box', always has the same contents as one of the three open boxes and, therefore, they can make certain inferences about what's inside the mystery box. Participants were then shown two examples illustrating what the characters can and cannot infer thanks to this rule. Participants were told that the characters would be presented with many quadruplets, each of which would be followed by an utterance from either Sam or Mia about what the mystery box contains, and that their task was to decide if this utterance was right given the information available to the characters and the rule that they learned. They were instructed to click on 'Good' if they consider that the characters got it right and, otherwise, to click on 'Bad'.

Following the instructions, participants completed a short training devised to consolidate their understanding of the cover story. The training phase included one instance of each control condition, hence 8 trials. During this phase, participants received feedback on the accuracy of their responses, together with a short explanation as to why the character got it right or wrong (e.g., *Sam got it right: since every open box contains a yellow ball, one can be certain that the mystery box does too.*). After the training, the study continued with a block of 48 experimental trials. Trials were presented in random order, with a 1000 ms inter-stimulus interval. Participants reported their responses by clicking with the mouse one of two response buttons labelled 'Good' and 'Bad', respectively. The position of the response buttons (i.e., on the left or on the right) was counterbalanced across participants. Items remained on the screen until participants gave their response.

4.4 Data availability

Stimuli, data and analysis code associated with Experiment 2 are available open access on the OSF platform at https://osf.io/4ut2c/?view_only=9fd1ae1db3bd466bb9ade688d362b241.

4.5 Data treatment and analysis

Data treatment and analysis were carried out in the R statistical environment (R Core Team 2021) using the Hmisc (Harrell Jr 2021), lme4 (Bates et al. 2015), car (Fox & Weisberg 2019) and sjPlot (Lüdecke 2023) packages for the R statistics program.

Responses from 15 participants were removed prior to analyses because their performance in the control trials did not reach the pre-established threshold of 80% accuracy.[5] The performance of the remaining subjects (n=86) was uniformly

5 The responses of the excluded participants were investigated to identify the possible sources of their low performance. We found little evidence for the low-level response strategy mentioned in Section 4.2. Rather, we found a discrepancy between the GOOD and BAD conditions (86% vs. 55% accuracy), showing that these participants generally (incorrectly) accepted the control sentences in the BAD conditions. Given the contents of the pictures in these conditions, it suggests that these

high both in the BAD conditions (M=94.7%, 95% CI[93.2, 95.9]) and the GOOD conditions (M=95.9%, 95% CI [94.5, 96.9]), with little variation among sentence types (all Ms>92.2% across all control conditions). Responses to the TARGET-1 and TARGET-2 trials were next inspected to check for potential discrepancies among the different variants of the TARGET-1 and TARGET-2 pictures (see Table 2). For each target condition, we fitted a mixed effect logistic regression model,predicting responses from the fixed effect of picture sub-type. In both cases, the maximal converging model only included a random intercept for participants. Both models were compared to a null model missing the fixed effect of interest. Neither model was significantly different from the null model (TARGET-1: $\chi^2(1) = 1.52$, $p = .21$; TARGET-2: $\chi^2(2) = 4.05$, $p = .13$), meaning that picture sub-type had no detectable effect on participants' judgements in the target conditions. Based on these results, responses to the TARGET-1 and TARGET-2 trials were aggregated across sub-conditions for the purpose of the main analysis.

Responses to the test trials were analysed by carrying out pairwise comparisons between each target condition and all other conditions. For each comparison, we fitted a mixed logistic regression model, predicting responses from the fixed effect of condition (2 levels; sum-coded). All models had the maximal random effect structure justifiable by the experimental design, i.e., they included random intercepts for participants and items, and random slopes for condition by participant and by item, as well as the correlation between the random intercept and random slope for both participant and item. The χ^2 and p-values we report on in the results were obtained by performing likelihood ratio tests in which the deviance of the models containing the fixed effect of condition was compared to another model without the relevant effect, but with the same random effect structure. p-values were adjusted using the Bonferroni correction method for multiple testing. Concretely, because 7 comparisons were tested, only p-values below 0.007 were treated as significant.

4.6 Results

Figure 2 shows the mean acceptance rate (i.e., the proportion of 'Good' responses) for the test sentences as a function of the experimental condition. The outputs of the statistical models and analyses are summarized in Table 4.

Acceptance rates were uniformly high in the TARGET-1 (M=94.7%, 95% CI[92.4, 96.3]) and TRUE-EXCL conditions (M=96.1%, 95% CI[93.0, 97.8]), with no significant difference between the two ($\chi^2(1) = 1.48$, adjusted $p = 1$). These results show that

participants generally considered an utterance as right as long as it was true of at least one box, that is, if it was describing a possible outcome (rather than a certainty). Whether this bias toward acceptance stems from an application of the Charity Principle or simply from a poor understanding of the instructions, it justifies further the conservative criterion we used for inclusion in the analysis.

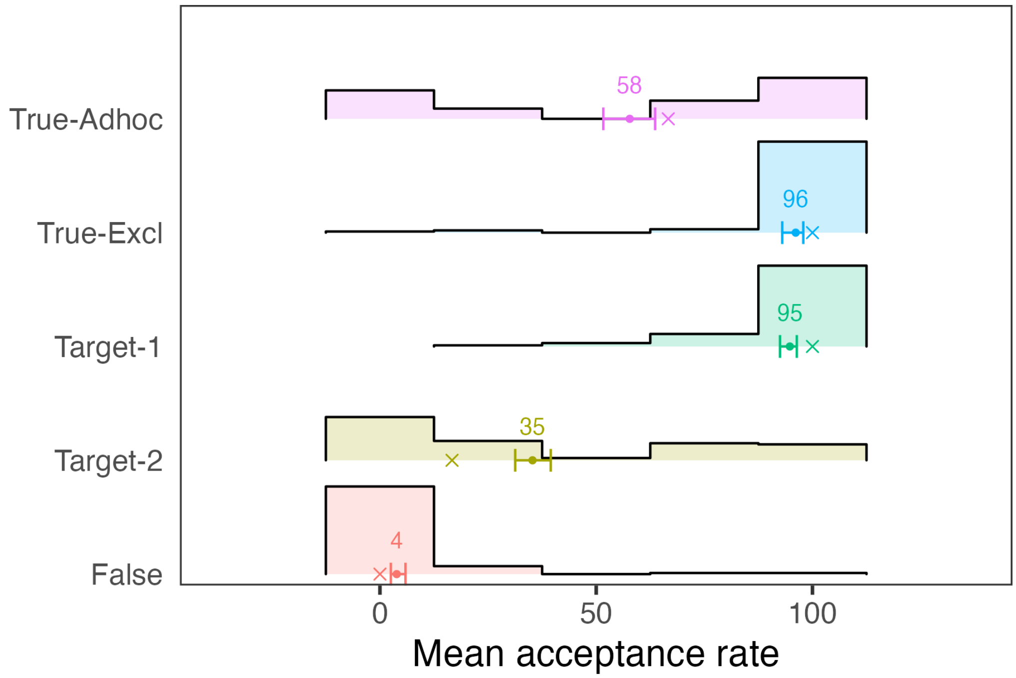


Figure 2 Mean acceptance rate (i.e., proportion of 'Good' responses) to test sentences by experimental condition in Experiment 1. For each condition, the distribution of by-participant mean rates is visualised by a histogram, the grand mean by a dot with its rounded value on top and the 95% CI around it, and the median by a cross.

the participants in our study did not derive the UNCERTAINTY inferences, nor the exclusivity implicatures associated with the test sentences. By contrast, the mean acceptance rate for the TARGET-2 and TRUE-ADHOC conditions were somewhat intermediate (M=35.2%, 95% CI[31.2, 39.4] and M=57.7%, 95% CI[51.6, 63.6], respectively), between the high(est) rates observed in the TARGET-1 and TRUE-EXCL conditions and the low(est) rate observed in the FALSE conditions (M=3.8%, 95% CI[2.5, 5.9]), with the TARGET-2 conditions yielding significantly lower rates than the TRUE-ADHOC conditions ($\chi^2 = 13.93$, adjusted $p < .001$).[6] The results from the

6 A visual inspection of the histograms in Fig. 2 suggest that, unlike in the TARGET-1 conditions, subjects' responses in the TARGET-2 and TRUE-ADHOC conditions were not uniform. To verify this impression, we carried out a post-hoc exploration of individual subjects' means by testing for unimodality of their distribution via Hartigans' dip tests in these three conditions, which indicated non-unimodality for TARGET-2 and TARGET-ADHOC but not for TARGET-1 (TARGET-1: $D = 0.04$, $p = 0.18$; TARGET-2: $D = 0.09$, $p < .001$; TRUE-ADHOC: $D = 0.14$, $p < .001$). For completeness, a Pearson correlation coefficient was computed to assess the linear relationship between subjects' mean rates in the TARGET-2 and TRUE-ADHOC conditions. The results show a low to moderate,

TRUE-ADHOC conditions indicate that participants derived the ad-hoc exhaustivity inferences associated with the test sentences to a noticeable extent. Since these inferences were false both in the TRUE-ADHOC and TARGET-2 conditions, it is possible that their derivation also affected to some extent participants' judgements in the TARGET-2 conditions. Thus, the absolute rates of acceptance for the TARGET-2 conditions in this experiment should be regarded with caution. Crucially, however, the contrast between the TARGET-2 and TRUE-ADHOC conditions show that, everything else being equal, participants rejected the test sentences significantly more often, when their POSSIBILITY inferences were also false. This contrast is explained if, in contrast to the UNCERTAINTY inferences, participants sometimes derived the POSSIBILITY inferences associated with the test sentences, possibly in addition to deriving ad-hoc exhaustivity inferences.

Comparison	*Estimate*	*95% CI*	χ^2	*p*	*Adjusted p*
TARGET-1 VS. TRUE-EXCL	-1.44	[-3.99, 1.10]	1.48	0.22	1
TARGET-1 VS. TRUE-ADHOC	3.12	[1.57, 4.68]	17.30	<.001	**<.001**
TARGET-1 VS. FALSE	8.58	[6.40, 10.76]	53.26	<.001	**<.001**
TARGET-1 VS. TARGET-2	3.55	[2.41, 4.70]	37.80	<.001	**<.001**
TARGET-2 VS. TRUE-EXCL	-5.73	[-7.68, -3.77]	35.01	<.001	**<.001**
TARGET-2 VS. TRUE-ADHOC	-1.24	[-1.72, -0.76]	13.93	<.001	**<.001**
TARGET-2 VS. FALSE	4.33	[2.19, 6.47]	35.42	<.001	**<.001**

Table 4 Summary of the outputs of the statistical models and analyses. For each pairwise comparison, the model included condition as a fixed effect (level 1 vs. level 2; sum coded: 1=level 1 and -1=level 2) and a maximal random effect structure. χ^2 and *p*-values were obtained via model comparison. *p*-values were adjusted for all comparisons tested.

4.7 Discussion

The results of this experiment show that participants uniformly accepted simple disjunctive sentences of the form 'A or B' in cases where UNCERTAINTY was false, but POSSIBILITY was true (TARGET-1 conditions), whereas they rejected these same sentences 65% of the time in cases where both UNCERTAINTY and POSSIBILITY were false (TARGET-2 conditions). Taken at face value, these findings go against the traditional approach to IGNORANCE inferences which predicts POSSIBILITY inferences to arise from and, thus to be licensed by, the derivation of UNCERTAINTY inferences. They suggest instead that POSSIBILITY inferences remain available to speakers even

positive correlation ($r(84) = 0.49$, 95% CI[0.32, 0.64], $p < .001$).

in the absence of the associated UNCERTAINTY inferences and, consequently, that the former can be derived independently of the latter.

As we discussed, however, the present results also suggest that the derivation of POSSIBILITY inferences may not be the only driver behind the contrast between the TARGET-1 and TARGET-2 conditions and, specifically, behind the low acceptance rates observed in the TARGET-2 conditions. We also found that the test sentences were rejected 42% of the time in the TRUE-ADHOC conditions, showing that participants also derived their ad-hoc exhaustivity inferences (e.g., 'A or B, and nothing else') to a noticeable extent. Since deriving these inferences would also lead to rejecting the test sentences in the TARGET-2 conditions, the interpretation of the contrast between the TARGET-1 and TARGET-2 conditions is conditional on that of the further contrast between the TRUE-ADHOC and TARGET-2 conditions. For the time being, we propose that, everything else being equal, the fact that the TARGET-2 conditions gave rise to significantly lower acceptance rates than the TRUE-ADHOC conditions supports the idea that the results to the TARGET-2 conditions cannot be entirely explained by the putative effect of ad-hoc exhaustivity inferences, that is, without also taking into account the falsity of the POSSIBILITY inferences in these conditions.

One potential worry with the interpretation above is that it relies on the assumption that the robustness and the frequency at which ad-hoc exhaustivity inferences were derived remained relatively constant across experimental conditions. While this assumption is certainly not implausible, the present results offer no independent evidence that would support it. In particular, we cannot exclude the possibility that (i) ad-hoc exhaustivity inferences were derived in the TARGET-2 conditions and (ii) these inferences were more robust, or simply derived more often in the TARGET-2 than in the TRUE-ADHOC conditions. As it is easy to see, making the alternative assumptions in (i) and (ii) would allow one to similarly account for the contrasts at hand without any need to refer to POSSIBILITY inferences at all.

In the following section, we report on a follow-up experiment testing this alternative explanation of the data. For these purposes, the test items from Experiment 1 were minimally modified to nullify the effect of ad-hoc exhaustivity inferences on participants' judgments so as to obtain more pristine TARGET-2 conditions. The results of this experiment show that the key contrast between TARGET-1 and TARGET-2 reproduces once the potential effects of ad-hoc exhaustivity inferences are factored out from the comparison, thus confirming our original interpretation of the data.

5 Experiment 2: factoring out ad-hoc exhaustivity inferences

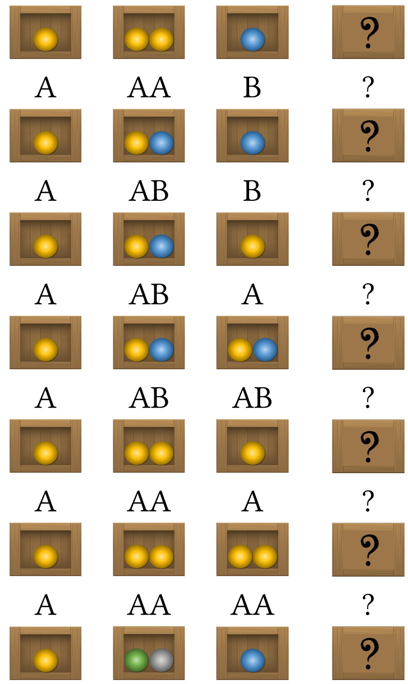
5.1 Participants

100 novel participants took part in this study (mean age 42 yrs; 50 female). Participants were recruited online through Prolific using the same prescreening criteria as in Experiment 1. Participants were paid £1.20, and median completion time was about 8 minutes (hourly rate: £9/hr). The consent and data collection procedures were the same as in Experiment 1.

5.2 Materials and Design

The materials were the same as in Experiment 1, except for the TARGET-2, TRUE-ADHOC and FALSE pictures, which were minimally modified to address specific issues left open by the results of Experiment 1. For simplicity, the novel conditions resulting from these picture modifications were labelled TARGET-2*, TRUE* and FALSE*, respectively. The picture types (and subtypes) used in the test trials of Experiment 2 are described and illustrated in Table 5.

First, the TARGET-2 pictures were modified to make the ad-hoc exhaustivity inferences associated with the test sentences true, while still making both UNCERTAINTY and POSSIBILITY false. The motivation was to create a more pristine version of these conditions enabling us to factor out the potential effect of ad-hoc exhaustivity inferences from the comparison of primary interest. The novel TARGET-2* pictures were obtained from the TARGET-2 pictures by replacing each ball of a non-matching color with an A-ball. As a result, all balls depicted on the TARGET-2* pictures were in effect A-balls. These modifications were applied to all variants of the TARGET-2 pictures resulting in two TARGET-2* sub-types, differing from each other only in terms of the number of A-balls they depicted. In the same vein, the TRUE-ADHOC pictures were modified to create genuinely 'true' pictures. The novel TRUE* pictures were obtained from the TRUE-ADHOC pictures through the same procedure as above – i.e., by replacing any ball of a non-matching color with an A-ball – thus making the test sentences true irrespective of the pragmatic inferences they may give to. Therefore, participants were expected to robustly and uniformly accept the test sentences in these conditions. Finally, the FALSE pictures were modified to create more challenging false controls that can also be used to probe for low-effort responses in trials involving the test sentences (as opposed to other control sentences). On the novel FALSE* conditions, one of the open boxes contained an A-ball, another one contained a B-ball while the last one contained balls of a non-matching color. Thus, if participants do not pay attention to all the open boxes in such cases (e.g., if they restrict their attention to boxes containing A-balls or B-balls), they should incorrectly accept these false controls



Condition		Example picture			
TRUE*		A	AA	B	?
TRUE-EXCL		A	AB	B	?
TARGET-1	i.	A	AB	A	?
	ii.	A	AB	AB	?
TARGET-2*	i.	A	AA	A	?
	ii.	A	AA	AA	?
FALSE*		A	CD	B	?

Test sentence: *'The mystery box contains a yellow ball or a blue ball.'* $(A \vee B)$

Table 5 Schematic description and illustration of the picture types paired with the test sentences in Experiment 2. The materials were the same as in Experiment 1, except for the pictures used in the novel TRUE*, TARGET-2* and FALSE* conditions. Picture types are illustrated using the same color assignment as before (A=yellow, B=blue, C=green and D=gray).

and, therefore, we should be able to detect and quantify this behavior in our data.

The rest of the design was identical to that of Experiment 1 in all respects. Thus, in particular, Table 3 also stands as a summary of the GOOD and BAD control conditions used in Experiment 2. Each control condition was instantiated 3 times, giving rise to 24 control trials. The TARGET-1, TARGET-2* and FALSE* conditions were instantiated 6 times each, and the TRUE* and TRUE-EXCL conditions 3 times each, giving rise to 24 test trials. As before, instances of the two target conditions were evenly distributed across their respective sub-conditions. Thus, each survey included 48 trials in total, exactly as in Experiment 1.

5.3 Procedure

The procedure was identical to the one used in Experiment 1 (see Section 4.3 for details).

5.4 Data availability

Stimuli, data and analysis code associated with Experiment 2 are available open access on the OSF platform at https://osf.io/4ut2c/?view_only=9fd1ae1db3bd466bb9ade688d362b241.

5.5 Data treatment and analysis

Data treatment was the same as in Experiment 1. Responses from 13 participants were removed because their performance in the control trials did not reach the pre-established threshold of 80% accuracy.[7] The performance of the remaining subjects (n=87) was uniformly high both in the BAD conditions (M=95.8%, 95% CI[94.4, 96.9]) and the GOOD conditions (M=97.0%, 95% CI [95.8, 97.9]), with little variation among sentence types (all Ms>93.6% across all control conditions). As before, for each target condition (i.e., TARGET-1 and TARGET-2*), we fitted a mixed effect logistic regression model, predicting responses from the fixed effect of picture sub-type (see Table 5). For both models, the maximal converging only included a random intercept for participants. Neither model was significantly different from the null model (TARGET-1: $\chi^2(1) = 0.54$, $p = .46$; TARGET-2*: $\chi^2(2) = 3.44$, $p = .06$). Responses to the TARGET-1 and TARGET-2* trials were thus aggregated across picture sub-conditions for the purpose of the main analysis. Responses to the test trials were analysed using the data analysis pipelines from Experiment 1. The models were the same as in Experiment 1 except in three instances where the random effect structure was simplified by removing the by-participant random slope for condition so as to avoid convergence or over-fitting issues.

5.6 Results

Figure 3 shows the mean acceptance rate (i.e., the proportion of 'Good' responses) for the test sentences as a function of the experimental condition. The outputs of the statistical models and analyses are summarized in Table 6.

7 For completeness, a descriptive analysis of the responses of the excluded participants was, here again, carried out (see fn. 5). As in Experiment 1, there was a discrepancy between the GOOD and BAD conditions across all control trials (81% vs. 57% accuracy). In this data set, however, the relevant discrepancy was found to be mainly driven by higher rates of incorrect responses in the BAD conditions of the negative sentences C2 and C4 (48% and 33% accuracy, respectively). These errors are those predicted by the low-level response strategy that we mentioned in Section 4.2.

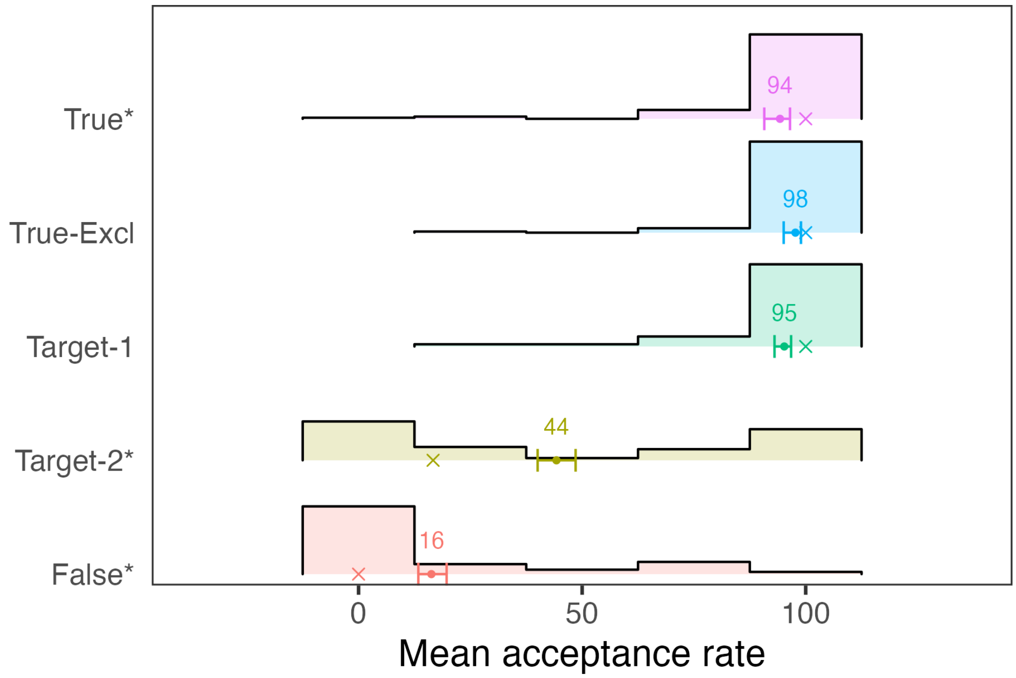


Figure 3 Mean acceptance rate (i.e., proportion of 'Good' responses) to target sentences by experimental condition in Experiment 2. For each condition, the distribution of by-participant mean rates is visualised by a histogram, the grand mean by a dot with its rounded value on top and the 95% CI around it, and the median by a cross.

The mean acceptance rate in the TARGET-1 conditions was very high (M=95.2%, 95% CI[93.0, 96.7]), similar to those found in the TRUE-EXCL conditions (M=97.7%, 95% CI[95.0, 98.9], $\chi^2(1) = 0.40$, adjusted $p = 1$) and in the novel TRUE* conditions (M=94.2%, 95% CI[90.7, 96.4], $\chi^2(1) = 0.86$, adjusted $p = 1$).[8] Thus, as in Experiment 1, there is no evidence that participants in this experiment ever derived the UNCERTAINTY inferences or exclusivity implicatures associated with the test sentences. By contrast, the mean acceptance rate in the novel TARGET-2* conditions was in the mid-range (M=44.2%, 95% CI[40.0, 48.5], between the high(est) rates observed in the TARGET-1, TRUE-EXCL and TRUE* conditions (all $\chi^2(1)$s> 19.78, all adjusted ps< .001) and the low(est) rate observed in the novel FALSE* conditions (M=16.2%, 95% CI[13.3, 19.6], $\chi^2(1) = 28.75$, adjusted $p < .001$). In sum, these

8 Similar to what we observed in the TARGET-2 conditions of Experiment 1 (see fn. 6), the by-participant mean rates in the TARGET-2* conditions was not distributed unimodally ($D = 0.16$, $p < .001$), as shown in Fig. 3 by the presence of two peaks in their distribution. Unlike in Experiment 1, however, the distribution of the by-participant mean rates was otherwise unimodal in all other conditions.

results show that the key contrast observed in Experiment 1 remains once the potential effects of ad-hoc exhaustivity inferences are factored out.

Comparison	*Estimate*	*95% CI*	χ^2	p	*Adjusted p*
TARGET-1 VS. TRUE-EXCL	-0.58	[-3.10, 1.94]	0.40	0.52	1
TARGET-1 VS. TRUE*	-0.16	[-2.03, 1.69]	0.03	0.86	1
TARGET-1 VS. FALSE	7.37	[5.60, 9.14]	48.99	<.001	**<.001**
TARGET-1 VS. TARGET-2	3.99	[2.29, 5.70]	31.09	<.001	**<.001**
TARGET-2 VS. TRUE-EXCL	-4.67	[-5.65, -3.68]	22.40	<.001	**<.001**
TARGET-2 VS. TRUE*	-3.46	[-4.12, -2.80]	19.78	<.001	**<.001**
TARGET-2 VS. FALSE*	1.49	[1.23, 1.76]	28.75	<.001	**<.001**

Table 6 Summary of the outputs of the statistical models and analyses. For each pairwise comparison, the model included condition as a fixed effect (level 1 vs. level 2; sum coded: 1=level 1 and -1=level 2). The random structure was maximal for the first four comparisons, while the by-Subject random slope for Condition was removed in the last three. χ^2 and p-values were obtained via model comparison. p-values were adjusted for all comparisons tested.

5.7 Discussion

We designed Experiment 2 as a minimal variant of Experiment 1 with the aim of factoring out the potential effects of ad-hoc exhaustivity inferences from the comparison between the TARGET-1 and TARGET-2 conditions. The results that we obtained show that this aim was achieved and confirm the main findings from Experiment 1.

First, in line with our expectations, the novel TRUE* conditions were uniformly accepted with a mean acceptance rate of 94%, contra 57% for the TRUE-ADHOC conditions in Experiment 1. These results show that the picture modifications that we have made in Experiment 2 were successful at preventing conflicting ad-hoc exhaustivity inferences from affecting participants' judgments. This, in turn, secures our interpretation of the comparison between the TARGET-1 and TARGET-2* conditions as assessing, specifically, the effect of false POSSIBILITY inferences on participants' judgments. Second, consistent with the results of Experiment 1, participants uniformly accepted the test sentences in the TARGET-1 conditions whereas they robustly rejected them in the novel TARGET-2* conditions. Taken together, these results confirm the main findings of Experiment 1.

Finally, we note that the novel FALSE* conditions gave rise to slightly higher rates of errors compared to the original FALSE conditions (16% contra 3%). These errors suggest that, on some occasions, some participants only verified whether or not balls of the matching colors were present somewhere on the pictures, i.e., disregarding whether or not these balls were present in every box. Assuming that this verification strategy was used at times, it may have magnified the contrast between the TARGET-1 and TARGET-2(*) conditions by pushing the participants further towards acceptance in the TARGET-1 conditions. The low frequency of these errors, however, indicates that the effect of this low-effort strategy was, at best, marginal among the participants and cannot account in any reasonable way for the large contrast in acceptance between the TARGET-1 and TARGET-2(*) conditions.

Before discussing the consequences of these results for current approaches to IGNORANCE inferences, let us consider yet another possible explanation for the contrasts observed between the TARGET-1 and TARGET-2(*) conditions, which was suggested to us by two anonymous reviewers.

This explanation starts from the observation that, in theory, sentences like *The mystery box contains a yellow ball or a blue ball* may allow for an interpretation on which the meaning of the first disjunct is strengthened locally. For expository purposes, we can think of this interpretation as the one delivered by embedding a covert instance of 'only' (notated $\mathcal{O}$ here) in the first disjunct, as illustrated in (8).[9]

(8) a. The mystery box contains ⟨only⟩ a yellow ball or a blue ball.
  b. $\mathcal{O}(A) \vee B \equiv (A \wedge \neg B) \vee B$

On the traditional Gricean approach, the critical alternatives for calculating UNCERTAINTY and POSSIBILITY in this case are thus as follows:

(9) $\left\{ \begin{array}{ll} \text{The mystery box contains only a yellow ball} & \mathcal{O}(A) \\ \text{The mystery box contains a blue ball} & B \end{array} \right\}$

Assuming that the stronger alternatives above are relevant, the Gricean account predicts for (8) the inferences in (10), by the same reasoning as the one presented in Section 2. As before, the UNCERTAINTY inferences in (10a) are derived as primary implicatures by Quantity; the POSSIBILITY inferences in (10b) are then derived from the former and from the Quality assumption that the speaker believes $\mathcal{O}(A) \vee B$.

(10) a. $\neg \Box_s \mathcal{O}(A) \wedge \neg \Box_s B$ UNCERTAINTY
  b. $\Diamond_s \mathcal{O}(A) \wedge \Diamond_s B$ POSSIBILITY

9 While this interpretation may be difficult to access upon introspection, it becomes more readily available if an additive particle is used in the second conjunct, e.g., 'The mystery box contains a yellow ball or *also/even* a blue ball'. For our purposes, we can think of such examples as variants of the example in (8).

The resulting inferences are thus somewhat different from those previously discussed for $A \vee B$. In particular, as far as UNCERTAINTY goes, in place of $\neg \Box_s A$ and $\neg \Box_s B$, we now have $\neg \Box_s \mathcal{O}(A)$ and $\neg \Box_s B$. Crucially, these UNCERTAINTY inferences, unlike the former, are compatible with the type of situations depicted in the TARGET-1 conditions. The reason is that, in these conditions, the color mentioned in the first disjunct (A) was consistently presented in all three visible boxes while the color mentioned in the second disjunct (B) was consistently presented in only one or two of them (see Table 2 and 5 for examples). Thus, in these conditions, the speaker was always certain that A was true but uncertain as to whether B also was, consistent with the UNCERTAINTY (and POSSIBILITY) inferences in (10). This observation is important because it means that, if participants in our studies parsed the test sentences as shown in (8), then the TARGET-1 conditions, as we designed them in Experiment 1 and 2, did not in fact discriminate between UNCERTAINTY and POSSIBILITY. Furthermore, note that, on this alternative parse, both UNCERTAINTY and POSSIBILITY would be true in the TARGET-1 conditions whereas, as before, both inference types would be false in the TARGET-2 conditions. As a result, the availability of a parse along the lines of (8) would also account for the contrasts of interest, and this independently of whether or not POSSIBILITY lives on UNCERTAINTY.

In the following, we report on a third and final experiment testing this alternative explanation of the data. The TARGET conditions in this experiment were designed so that, if participants parse the test sentences as initially assumed, the key contrasts from Experiment 1 and 2 should reproduce once again whereas, if they parse these sentences as shown in (8), they should now reject them to the same extent in the TARGET-1 and TARGET-2 conditions and, therefore, the key contrasts should disappear. The results of this experiment align in every respect with those of Experiment 1 and 2, in support of the first hypothesis and against the second.

6 Experiment 3: controlling for alternative parses

Experiment 3 was designed so as to investigate the availability of the alternative parsing strategy that we described at the end of the previous section. For these purposes, we modified the test items in Experiment 2 by reversing the order of the disjuncts in the test sentences, changing it from 'A or B' to 'B or A'. These sentences were paired with the same picture types as before, thus resulting in novel TARGET conditions in which the color mentioned in the *second* disjunct (as opposed to the first one) was always represented in all three visible boxes. As illustrated in Table 7, these modifications allowed us to discriminate between the two parses of interest, namely $B \vee A$ and $\mathcal{O}(B) \vee A$, on the basis of the POSSIBILITY inferences they are predicted to give rise to on the standard implicature approach.

Specifically, on the first parse (without $\mathcal{O}$), the predictions were exactly the

Parse	Inference		TARGET-1 A - AB - A(B)	TARGET-2 A - AA - A(A)
	QUALITY:	$\Box_s(B \vee A)$	✓	✓
$B \vee A$	UNCERTAINTY:	$\neg \Box_s B \wedge \neg \Box_s A$	✗	✗
	POSSIBILITY:	$\Diamond_s B \wedge \Diamond_s A$	✓	✗
	QUALITY:	$\Box_s(\mathcal{O}(B) \vee A)$	✓	✓
$\mathcal{O}(B) \vee A$	UNCERTAINTY:	$\neg \Box_s (\mathcal{O}(B) \wedge \neg \Box_s A$	✗	✗
	POSSIBILITY:	$\Diamond_s \mathcal{O}(B) \wedge \Diamond_s A$	✗	✗

Table 7 QUALITY, UNCERTAINTY and POSSIBILITY inferences for 'B or A', as predicted by the standard implicature approach, according to the two parses of interest. $\mathcal{O}$ indicates a covert instance of *only*. The last two columns show, for each parse, whether the predicted inferences are true (✓) or false (✗) in the TARGET conditions of Experiment 3.

same as before; in particular, one the UNCERTAINTY inferences, namely $\neg \Box_s A$, was false in both TARGET conditions whereas one of the POSSIBILITY inference, namely $\Diamond_s B$, was false only in the TARGET-2 conditions. By contrast, on the second parse (with $\mathcal{O}$), the corresponding inferences, namely $\neg \Box_s A$ and $\Diamond_s \mathcal{O}(B)$, were both false in both TARGET conditions; in particular, the POSSIBILITY inference $\Diamond_s \mathcal{O}(B)$ was false in the TARGET-1 conditions because, in these conditions, the speaker was always certain that A was true (i.e., $B \wedge \neg A$ is impossible). Thus, we reasoned that, if participants generally opt for the first parse, the key contrasts observed in Experiment 1 and 2 should reproduce in this experiment. On the other hand, if participants opt instead for the second parse, these contrasts should disappear, or at least shrink in a noticeable way. The first case would support our current interpretation of the results whereas the second would challenge it and favor of the alternative explanation discussed at the end of the previous section.

(11) a. **Current interpretation**
Participants opt for the first parse. The contrasts from Exp.1-2 obtain because POSSIBILITY may arise independently of CERTAINTY.
Prediction: similar contrasts should reproduce in Exp.3

b. **Alternative interpretation**
The contrasts from Exp.1-2 are consistent with POSSIBILITY living on CERTAINTY. They obtain because participants opt for the second parse.
Prediction: no such contrasts should be found in Exp.3

Before proceeding, let us briefly discuss two alternative parses for our test sentences, which can be set aside here due to their low plausibility. First, the meaning of each individual disjunct could be strengthened via a covert instance of ‘only’, as shown in (12). The reading associated with this parse amounts to an exclusive reading of the disjunction which, in our experiments, would be false in the TARGET-1 conditions and true in the TARGET-2 conditions. Thus, should participants opt for this strategy, we would see a contrast between the TARGET-1 and TARGET-2 conditions in the opposite direction to that we presently expect and previously observed. The results so far and those to come are incompatible with participants opting for this parse.

(12) a. The mystery box contains ⟨only⟩ a blue ball or ⟨only⟩ a yellow ball.
b. $\mathcal{O}(B) \vee \mathcal{O}(A) \equiv (B \wedge \neg A) \vee (A \wedge \neg B)$

Second, in place of the first disjunct, the meaning of the second disjunct could be strengthened, as shown in (13). If this sort of parse were available, it would be problematic for our present purposes. In particular, for the same reasons as those discussed for $\mathcal{O}(A) \vee B$, the availability of this parse would cast doubt on the ability of our novel TARGET-1 conditions to discriminate between UNCERTAINTY and POSSIBILITY inferences, as all of them would be true in these conditions.

(13) a. The mystery box contains a blue ball or ⟨only⟩ a yellow ball.
b. $B \vee \mathcal{O}(A) \equiv B \vee (A \wedge \neg B)$

However, evidence from Singh (2008) suggests that, unlike its symmetric variant, a parse like (13) is not readily available, even in cases where it would prevent infelicity from arising. To illustrate this point, consider the minimal pair in (14):[10]

(14) a. The mystery box contains a yellow ball, or both a yellow ball and a blue ball.
b. #The mystery box contains both a yellow ball and a blue ball, or a yellow ball.

These sentences are examples of so-called ‘Hurford’ disjunctions: in both cases, one of the disjuncts entails the other, at least superficially. Crucially, as Singh (2008) observes, a sentence like (14a), of the form ‘A or (A and B)’, is fully felicitous whereas its variant in (14b), of the form ‘(A and B) or A’, isn’t. This asymmetry is accounted for if we assume that (14a) genuinely allows for a parse that breaks the relation of entailment between the two disjuncts, namely $\mathcal{O}(A) \vee (A \wedge B)$, whereas (14b) doesn’t, i.e., $(A \wedge B) \vee \mathcal{O}(A)$ is not available or at least not to the same extent. We take these observations to suggest that, while an ‘only’-reading of the first disjunct is possible for our test sentences, an ‘only’-reading of the second disjunct

10 We thank one of our anonymous reviewers for pointing this out to us.

of the sort illustrated in (13) is very unlikely.

6.1 Participants

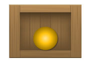
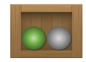
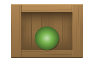
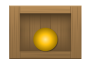
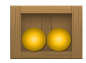
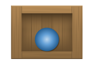
100 novel participants took part in this study (mean age 40 yrs; 49 female). Participants were recruited online through Prolific using the same prescreening criteria as in Experiment 1 and 2. Participants were paid £1.20, and median completion time was about 8 minutes (hourly rate: £9/hr). The consent and data collection procedures were the same as in Experiment 1 and 2.

6.2 Materials and Design

The materials from Experiment 2 were modified in two ways. First of all, as explained above, we reversed the order of the disjuncts in the test sentences, changing it from $A \vee B$ to $B \vee A$. These sentences were associated with the same picture types and subtypes as in Experiment 2. As a result, Table 5 also stands as an illustrative summary of the test conditions in Experiment 3, provided the modification of the test sentences that we just described. Concretely, the example sentence in Table 5, *'The mystery box contains a yellow ball or a blue ball*, should now read as *'The mystery box contains a blue ball or a yellow ball'*.

Second, we modified the pictures used in the BAD conditions for the $A \wedge B$ controls (C3) to further control for low-level response strategies of the sort previously discussed. Specifically, the relevant pictures were modified so as to depict A-balls and B-balls only, as shown in Table 8. We reasoned that, if a participant primarily focuses their attention on whether the colors of the balls depicted on the picture match the color terms used in the sentence, they should incorrectly accept these cases. While this color-matching strategy appears unlikely given the previous results to the C4-GOOD controls, the use of negation in the C4 sentences introduces an extra layer of complexity which may refrain participants from adopting this response strategy. These novel cases allowed us to probe again for this strategy in trials involving positive sentences structurally similar to our test sentences.

The rest of the design was identical to that of Experiments 1 and 2 in all respects. As in the previous experiments, each control condition was instantiated 3 times, resulting in 24 control trials. The TARGET-1, TARGET-2*, and FALSE* conditions were instantiated 6 times each, while the TRUE* and TRUE-EXCL conditions were instantiated 3 times each, resulting in 24 test trials. Instances of the two target conditions were evenly distributed across their respective sub-conditions. Thus, the experiment included a total of 48 trials.

Sentence	Condition	Experiment	Example picture			
C3: $A \wedge B$	BAD	Exp.1-2				
			A	CD	C	?
		Exp.3				
			A	AA	B	?

Sentence: *'The mystery box contains a yellow ball and a blue ball.'*

Table 8 Schematic description and illustration of the pictures types used in the BAD conditions for the C3 controls in Exp.1-2 and in Exp.3.

6.3 Procedure

The procedure was identical to the one used in Experiment 1 and 2 (see Section 4.3 for details).

6.4 Data availability

Stimuli, data and analysis code associated with Experiment 3 are available open access on the OSF platform at https://osf.io/4ut2c/?view_only=9fd1ae1db3bd466bb9ade688d362b241.

6.5 Data treatment and analysis

Data was treated and analysed using the data analysis pipelines from Experiment 1 and 2. Responses from 18 participants were removed because their performance in the control trials did not reach the pre-established threshold of 80% accuracy. The performance of the remaining subjects (n=82) was high both in the BAD conditions (M=92.8%, 95% CI[91.0, 94.2]) and the GOOD conditions (M=97.7%, 95% CI [96.5, 98.4]). We note however that, while there was little variation among the GOOD controls (all Ms>95%), the mean accuracy rate was slightly lower for the C3-BAD controls (M=84.1%, 95% CI [79.1, 88.2]) than the other BAD controls (all Ms>92%).

As explained above, a participant performing poorly on the C3-BAD controls is suggestive that they relied on low-level response strategies, at least on occasion. For this reason, participants' responses in the test trials were also analyzed by including only those participants who performed perfectly on all C3 control trials ($n = 63$). This analysis is reported in the analysis script associated with Experiment 3. The results show that the mean acceptance rates per condition were similar to those observed in the main analysis that we report on below. We take these results to rule out any explanation of the following results in terms of color-matching strategy.

6.6 Results

Figure 4 shows the mean acceptance rate (i.e., the proportion of 'Good' responses) for the test sentences as a function of the experimental condition. The outputs of the statistical models and analyses are summarized in Table 9.

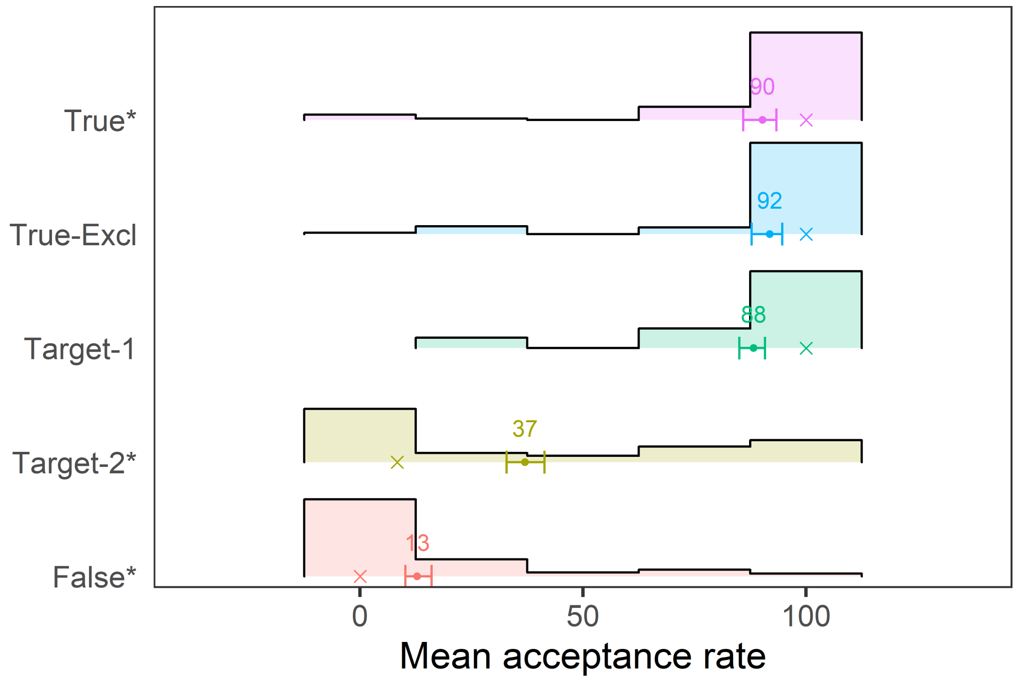


Figure 4 Mean acceptance rate (i.e., proportion of 'Good' responses) to target sentences by experimental condition in Experiment 3. For each condition, the distribution of by-participant mean rates is visualised by a histogram, the grand mean by a dot with its rounded value on top and the 95% CI around it, and the median by a cross.

The results were similar to those from Experiment 2. The mean acceptance rate in the TARGET-1 conditions was very high (M=88.21%, 95% CI[85.1, 90.8]), similar to those found in the TRUE-EXCL conditions (M=91.9%, 95% CI[87.8, 94.7], $\chi^2(1)$=0.40, adjusted p=1) and the TRUE* conditions (M=90.2%, 95% CI[85.9, 93.4], $\chi^2(1)$=0.03, adjusted p=1).[11] By contrast, the mean acceptance rate in the TARGET-2* conditions

11 As in Experiment 1 and 2, the by-participant mean rates in the TARGET-2(*) conditions were not distributed unimodally (D = 0.12, p < .001), as shown in Fig. 4 by the two peaks in the distribution.

was in the lower mid-range (M=37.0%, 95% CI[32.8, 41.3]), falling between the highest rates observed observed in the TARGET-1, TRUE-EXCL and TRUE* conditions (all $\chi^2(1)s$>19.5, all adjusted ps<.001) and the lowest rate observed in the FALSE* conditions (M=12.8%, 95% CI[10.1, 16.0], $\chi^2(1)$=27.14, adjusted p<.001). In sum, these results reproduce the key contrasts observed in the previous experiments.

Comparison	*Estimate*	*95% CI*	χ^2	*p*	*Adjusted p*
TARGET-1 VS. TRUE-EXCL	-0.61	[-1.97, 0.76]	0.85	0.36	1
TARGET-1 VS. TRUE*	-0.13	[-1.26, 1.00]	0.06	0.80	1
TARGET-1 VS. FALSE	6.09	[4.32, 7.85]	48.59	<.001	**<.001**
TARGET-1 VS. TARGET-2	3.78	[2.38, 5.18]	34.92	<.001	**<.001**
TARGET-2 VS. TRUE-EXCL	-3.89	[-3.90, -3.89]	19.5	<.001	**<.001**
TARGET-2 VS. TRUE*	-3.05	[-3.60, -2.50]	21.90	<.001	**<.001**
TARGET-2 VS. FALSE*	1.30	[1.05, 1.55]	27.14	<.001	**<.001**

Table 9 Summary of the outputs of the statistical models and analyses. For each pairwise comparison, the model included condition as a fixed effect (level 1 vs. level 2; sum coded: 1=level 1 and -1=level 2). The random structure was maximal for the first four comparisons, while the by-Subject random slope for Condition was removed in the last three. χ^2 and p-values were obtained via model comparison. p-values were adjusted for all comparisons tested.

6.7 Discussion

Experiment 3 aimed to control for another possible parse of the test sentences on which the first disjunct receives an 'only'-interpretation. For these purposes, we modified the materials used in Experiment 2 by swapping the position of the disjuncts in the test sentences, changing it from 'A or B' to 'B or A', while keeping the sentence-picture combinations the same as before. As we explained, this minimal change in design made it possible to discriminate between the two parses of interest, $B \vee A$ and $\mathcal{O}(B) \vee A$, and distinguish the predictions of two competing interpretations for the contrasts observed in Exp.1-2 (see (11) for a summary).

The results that we obtained were essentially similar to those of Exp.1-2, showing that reversing the order of the disjuncts did not affect participants' responses in any significant way. In particular, participants consistently accepted the novel 'B or A' sentences in the TARGET-1 conditions while they robustly rejected them in the TARGET-2* conditions. These findings are incompatible with participants opting for the alternative parse that we discussed and, in turn, undermine any explanation of the contrasts from Exp.1-2 based on this parsing strategy. These findings, on the other hand, align with our initial interpretation of the data and suggest that the main conclusion from the previous experiments should be maintained: disjunctive sentences can give rise to POSSIBILITY in the absence of UNCERTAINTY.

7 General discussion

The experiments discussed in Section 4, Section 5 and Section 6 pose a clear challenge to the traditional implicature approach: simple disjunctive sentences can give rise to POSSIBILITY inferences, even in cases where UNCERTAINTY inferences are not derived. For the sake of clarity, we repeat below an example with the relevant inferences.

(15) The mystery box contains a yellow or a blue ball. $A \vee B$

a. $\neg \Box_s A \wedge \neg \Box_s B$ UNCERTAINTY

b. $\Diamond_s A \wedge \Diamond_s B$ POSSIBILITY

To begin our discussion, we will first draw a connection between the findings presented in this work and similar observations that have been discussed in relation to disjunction in embedded contexts.

7.1 The connection with distributive inferences

In this work, we focused on cases on plain disjunction and its related inferences. However, embedded disjunction displays similar patterns. We illustrate this by means of a disjunction embedded under a universal modal.[12]

(16) The mystery box must contain a blue ball or a yellow ball. $\Box(A \vee B)$

a. $\neg \Box A \wedge \neg \Box B$ NEGATED UNIVERSAL

b. $\Diamond A \wedge \Diamond B$ DISTRIBUTIVE

---

12 Note that distributive inferences can also arise in the case of nominal quantifiers, as in (i):

(i) Every visible box contains a blue ball or a yellow ball. $\forall x(P(x) \vee Q(x))$

a. $\neg \forall x P(x) \wedge \neg \forall x Q(x)$ NEGATED UNIVERSAL

b. $\exists x P(x) \wedge \exists x Q(x)$ DISTRIBUTIVE

Crnič et al. (2015) showed that DISTRIBUTIVE inferences do indeed arise for nominal quantifier in the absence of NEGATED UNIVERSAL. They also argued that DISTRIBUTIVE inferences do not arise for the case of modals. Marty et al. (2023) replicated the results of Crnič et al. (2015) for the nominal case, but they also showed that distributive inferences do indeed arise for the modal case, as discussed at the beginning of this section.

The example in (16) gives rise to the NEGATED UNIVERSAL inference in (16a), corresponding to the UNCERTAINTY inferences discussed here for plain disjunction and illustrated in (15a). Likewise, (16) is also associated with the existential inference in (16b), often called DISTRIBUTIVE inference, corresponding to our POSSIBILITY inferences in (15b).

In the case of embedded disjunction, the traditional implicature approach predicts that the lower bound DISTRIBUTIVE inferences are dependent on the upper bound NEGATED UNIVERSAL part, similarly to the predictions for plain disjunction. However, experimental research (Crnič et al. 2015, Marty et al. 2023) found that DISTRIBUTIVE inferences can be derived even in the absence of NEGATED UNIVERSAL ones.

In light of the generality of the phenomena, we would want a theory that comprehensively captures the independent derivation of lower bound distributive/possibility inferences in a unified fashion.

We will start by looking at recent implicature accounts (Crnič et al. 2015, Bar-Lev & Fox 2023) which have been designed to predict DISTRIBUTIVE inferences in the absence of NEGATED UNIVERSAL ones. We will then consider whether these accounts can be extended to IGNORANCE inferences. Subsequently, we will shift our focus to non-implicature accounts, which do not derive POSSIBILITY as an implicature. To make the discussion concrete, we will focus on a recent proposal by Aloni (2022).

7.2 Implicature accounts

7.2.1 Distributive Inferences and Existential Alternatives

In this section, we outline how a recent account by Bar-Lev & Fox (2023) in the grammatical approach can derive DISTRIBUTIVE inferences without the need of deriving NEGATED UNIVERSAL ones. The core of the proposal in Bar-Lev & Fox (2023) is the application of recursive exhaustification, building on previous work by Bar-Lev & Fox (2020), Fox (2007). In this approach implicatures are computed by applying a covert exhaustivity operator, labeled as EXH[13], which negates innocently excludable (IE) alternatives (i.e., those alternatives to ϕ that can be negated simultaneously without contradicting ϕ or entailing the other alternatives):

(17) a. $IE(\phi,S) := \bigcap \left\{ S' \;\middle|\; \begin{array}{l} S' \subseteq S \text{ and } S' \text{ is a maximal subset of } S \\ \text{such that } \{\neg\psi : \psi \in S\} \cup \{\phi\} \text{ is consistent} \end{array} \right\}$

b. $[\![\textsc{exh}(\phi)]\!](w) = [\![\phi]\!](w) \wedge \forall \psi \in IE(\phi, ALT(\phi)) : \neg[\![\psi]\!](w)$

13 The EXH operator can be viewed as a more elaborate version of the simple $\mathcal{O}$ operator discussed at the end of Section 5.

In the following, we illustrate the derivation of DISTRIBUTIVE inferences for the modal case in example (16). The alternatives associated with $\Box(A \vee B)$ are the full set of alternatives in (18). Note that the existential alternatives in blue ($\Diamond A$ and $\Diamond B$) are essential for the derivation of POSSIBILITY.

(18) $\quad Alt(\Box(A \vee B)) = \{\Box A, \Box B, \Diamond A, \Diamond B, \Box(A \wedge B), \Box(A \vee B), \Diamond(A \vee B), \Diamond(A \wedge B)\}$

According to Bar-Lev & Fox (2023), the derivation of DISTRIBUTIVE inferences is achieved by using a double application of the exhaustivity operator EXH, along with the relevant pruning of alternatives, which we will discuss below.

In particular, the alternative in red in (18) needs to be pruned to avoid the derivation of EXCLUSIVITY, which, as we discuss in more detail in Section 7.2.4, would also derive NEGATED UNIVERSAL in combination with DISTRIBUTIVE.[14] As a result, the pruned set of alternatives, on which EXH operates, looks as in (19)[15]:

(19) $\quad Alt_{pruned}(\Box(A \vee B)) = \{\Box A, \Box B, \Diamond A, \Diamond B, \Box(A \vee B), \Diamond(A \vee B), \Box(A \wedge B)\}$

Distributive inferences are then obtained by recursive exhaustification, as in (20). On the first application of EXH, only the $\Box(A \wedge B)$ alternative is excludable. With the subsequent application of EXH, thanks to the presence of the individual existential alternatives, DISTRIBUTIVE inferences are then generated:

(20)
$$\begin{aligned}
&\textsc{exh}(\textsc{exh}\,\Box\,(A \vee B)) \\
&\equiv \Box(A \vee B) \wedge \neg\textsc{exh}(\Box A) \wedge \neg\textsc{exh}(\Box B) \wedge \neg\Box\,(A \wedge B) \\
&\equiv \Box(A \vee B) \wedge \neg(\Box A \wedge \neg\Box B \wedge \neg\Diamond B) \wedge \neg(\Box B \wedge \neg\Box A \wedge \neg\Diamond A) \wedge \neg\Box\,(A \wedge B) \\
&\equiv \Box(A \vee B) \wedge \Diamond A \wedge \Diamond B \wedge \neg\Box\,(A \wedge B) \\
&\Rightarrow \Box(A \vee B) \wedge \Diamond A \wedge \Diamond B
\end{aligned}$$

14 The pruning of these alternatives is allowed, as it does not break symmetry. Two alternative ψ_1 and ψ_2 of ϕ are symmetric when $\psi_1 \vee \psi_2 \equiv \phi$ and $\psi_1 \wedge \psi_2 \equiv \bot$.
An alternative explanation could link the pruning mechanism to the questions under discussion, instead of/in addition to symmetry breaking. For instance, it might be possible that conjunctive alternatives of the form $A \wedge B$ were not relevant to the QuD, but the individual alternatives A and B were.

15 To also avoid weaker exclusivity inferences of the form $\neg\Box\,(A \wedge B)$, also the alternative $\Box(A \wedge B)$ should be pruned. While this was not tested in our experiments, the absence of such inference has been observed in previous work by Crnič et al. (2015), which we expect to align with the current experimental paradigm. One reviewer appropriately points out that the set of alternatives with both conjunctive alternatives being pruned is not problematic for innocent exclusion, which we are assuming here, but it would be problematic if we also assume innocent inclusion (Bar-Lev & Fox 2020). Since no alternative would be excludable and the conjunction of all alternatives is consistent, EXH with innocent inclusion would give us the conjunction of all alternatives and thus $\Box A$ and $\Box B$ would end up being entailed. This implies that while innocent exclusion allows for deriving DISTRIBUTIVE without any form of exclusivity by pruning both conjunctive alternatives, innocent inclusion always derives weak exclusivity $\neg\Box\,(A \wedge B)$ together with DISTRIBUTIVE.

We have seen how an implicature approach in the grammatical tradition can account for the independent derivation of DISTRIBUTIVE inferences. What remains to determine is how this account can be extended to the case of plain disjunctive sentences and POSSIBILITY inferences discussed in our experimental studies. Before moving to that, we discuss another recent variant of the implicature approach by Crnič et al. (2015).

7.2.2 Distributive Inferences and Embedded EXH

We now outline a different implicature approach (Crnič et al. 2015) which does not rely on the presence of existential alternatives[16], but on the possibility of embedding the EXH operator, as in (21):

(21) a. The mystery box must contain A or B.
b. $\text{EXH}_1(\Box(\text{EXH}_2(A \vee B)))$

In (21b), we introduce two indices for the EXH operator to distinguish between the set of relevant alternatives on which the two EXH operate. In particular, the set of alternatives relevant for EXH_1 is the following:

(22) $Alt_{\text{EXH}_1}(\Box(\text{EXH}_2(A \vee B))) = \{\Box(\text{EXH}_2(A)), \Box(\text{EXH}_2(B)), \Box(\text{EXH}_2(A \wedge B))\}$
a. $\Box(\text{EXH}_2(A)) \equiv \Box(A \wedge \neg B)$
b. $\Box(\text{EXH}_2(B)) \equiv \Box(B \wedge \neg A)$
c. $\Box(\text{EXH}_2(A \wedge B)) \equiv \Box(A \wedge B)$

The crucial assumption to obtain the equivalences in (22) is that EXH_2 operates on the restricted set of alternatives in (23) where the conjunctive alternative is absent:

(23) $Alt_{\text{EXH}_2}(\psi) = \{A, B\}$

Given these assumptions, we can obtain POSSIBILITY as desired:

(24)
$$
\begin{aligned}
& \text{EXH}_1(\Box(\text{EXH}_2(A \vee B))) \\
& \equiv \Box(A \vee B) \wedge \neg\Box(A \wedge \neg B) \wedge \neg\Box(B \wedge \neg A) \wedge \neg\Box(A \wedge B) \\
& \equiv \Box(A \vee B) \wedge \Diamond(\neg A \vee B) \wedge \Diamond(\neg B \vee A) \wedge \neg\Box(A \wedge B) \\
& \equiv \Box(A \vee B) \wedge \Diamond A \wedge \Diamond B \wedge \neg\Box(A \wedge B) \\
& \Rightarrow \Box(A \vee B) \wedge \Diamond A \wedge \Diamond B
\end{aligned}
$$

Similarly to the recursive EXH approach, pruning the conjunctive alternative from (22) would remove the weaker exclusivity inference derived above.

16 We refer to Bar-Lev & Fox (2023) for criticism of Crnič et al. (2015) and the independent need of existential alternatives.

7.2.3 Extending the implicature approach to IGNORANCE

A natural way to extend the implicature approach to plain disjunctive sentences is the use of a silent doxastic operator (among others, Meyer 2013, Buccola & Haida 2019), which we indicate with $\Box_s$. The fundamental assumption is that every assertively used sentence is associated with a covert doxastic operator $\Box_s$, where the subscript refers to the doxastic source (in this case s stands for the speaker). For instance, an assertion like 'It is raining' will be rendered as $\Box_s p$, meaning that in all the doxastic possibilities of the speaker, it is the case that it is raining.

We first discuss the results and derivation for the recursive approach outlined in Section 7.2.1, and we then consider the embedded EXH account discussed in Section 7.2.2.

Under the extension of the first account, a plain disjunctive sentence is associated with the alternatives in (25), where $\Diamond_s$ corresponds to the existential counterpart of $\Box_s$. As discussed for the case of distributive inferences, we have pruned the alternative responsible for EXCLUSIVITY.

(25) $Alt_{pruned}(\Box_s(A \vee B)) = \{\Box_s A, \Box_s B, \Diamond_s A, \Diamond_s B, \Box_s(A \vee B), \Diamond_s(A \vee B), \Box_s(A \wedge B)\}$

By virtue of this silent operator, the derivation of the POSSIBILITY inferences $\Diamond_s A$ and $\Diamond_s B$ parallels the case discussed in (20), thus capturing our results.

One main question for the extension of the account by Bar-Lev & Fox (2023) to IGNORANCE inferences concerns the nature of the silent operator involved in the alternatives. While it is not particularly controversial to assume a silent belief operator $\Box_s$ for assertively used sentences, the use of $\Diamond_s$ as an implicit or covert operator that scopes over sentences is considerably less established. One way to make sense of $\Diamond_s$ is as a weak form of assertion (Incurvati & Schlöder 2019, Dorst & Mandelkern 2022, Mandelkern & Dorst 2022), the idea that language users can (weakly) assert something even when they have low rational credence in it. However, assuming that $\Diamond_s$ is also silent, the existence of such an operator would allow us to utter a sentence like (26) also when it is only compatible with our belief that it is raining ($\Diamond_s p$), which is arguably not the case.

(26) It is raining. $\Diamond_s / \Box_s\, p$

The use of the existential alternative $\Diamond_s$ thus poses a challenge for the recursive approach.[17]

The embedded approach discussed Section 7.2.2 fares better in this regard since

17 One possibility to explore for this approach in relation to this challenge is the observation that strong assertions are often unmarked, whereas weak assertions require an overt marker like 'perhaps', as in the sentence 'Perhaps it is raining' ($\Diamond_s p$), which can indeed be uttered when it is only compatible with our belief that it is raining.

it does not rely on existential alternatives. Analogously to the case with an overt modal discussed in Section 7.2.2, a plain disjunctive sentence like (27a) is associated with two EXH operators: one applied before the covert doxastic operator $\Box_s$, and one applied after.

(27) a. The mystery box contains A or B.
b. $\text{EXH}_1(\Box_s(\text{EXH}_2(A \vee B)))$

The derivation is then parallel to the one outlined in Section 7.2.2, with the crucial assumption of pruning the conjunctive alternative from the domain of the embedded EXH_2.[18]

In conclusion, the extension of the implicature approach to IGNORANCE relies on a silent doxastic operator $\Box_s$ and leads to significant theoretical consequences. On one hand, the recursive EXH approach would lead us to posit a weak covert doxastic operator $\Diamond_s$. On the other hand, the embedded EXH approach would lead us to require EXH operators to be adjoined before and after a silent operator, which has been previously considered in the literature (Meyer 2013).

7.2.4 The role of EXCLUSIVITY

In the previous discussion, we pruned the alternatives to prevent the generation of EXCLUSIVITY. The reason behind this decision was that EXCLUSIVITY, when combined with POSSIBILITY or DISTRIBUTIVE inferences, leads to UNCERTAINTY or NEGATED UNIVERSAL inferences, as in (28). This would have been undesirable because we aim to generate POSSIBILITY without also generating UNCERTAINTY.

(28) The role of EXCLUSIVITY
a. $\Diamond A \wedge \Diamond B$
b. $\Box\neg(A \wedge B)$
c. $\rightsquigarrow \neg\Box A \wedge \neg\Box B$

We illustrate this for the first approach illustrated above, which assumes existential alternatives. The derivation with the full set of alternatives in (18) is given in (29). In this case, on the first application of EXH, $\Diamond(A \wedge B)$ and $\Box(A \wedge B)$ are excludable, generating EXCLUSIVITY. As before, the subsequent application of EXH yields distributive inferences:

(29) $\text{EXH}(\text{EXH}\,\Box(A \vee B))$
$= \Box(A \vee B) \wedge \neg\Box(A \wedge B) \wedge \neg\Diamond(A \wedge B) \wedge \neg\text{EXH}(\Box A) \wedge \neg\text{EXH}(\Box B)$

18 Importantly, including the conjunctive alternative in the embedded exhaustification operator would yield UNCERTAINTY, as well as EXCLUSIVITY (see e.g., Meyer 2013: pp. 52–54).

$= \Box(A \vee B) \wedge \neg\Box(A \wedge B) \wedge \neg\Diamond(A \wedge B) \wedge \neg(\Box A \wedge \neg\Box B \wedge \neg\Diamond B) \wedge \neg(\Box B \wedge \neg\Box A \wedge \neg\Diamond A)$
$= \Box(A \vee B) \wedge \neg\Diamond(A \wedge B) \wedge \Diamond A \wedge \Diamond B$

As such, this approach predicts that UNCERTAINTY follows from EXCLUSIVITY (i.e., UNCERTAINTY cannot arise without the presence of EXCLUSIVITY). As regards our experiments, we found no evidence of either UNCERTAINTY or EXCLUSIVITY.[19] An important question, which we will explore in Section 7.5, is whether there can be cases in which UNCERTAINTY is derived independently of EXCLUSIVITY.

It is worth mentioning that the mechanism behind the derivation of both EXCLUSIVITY and POSSIBILITY is the same, in contrast to the non-implicature accounts presented in Section 7.3. In those cases, the derivation of EXCLUSIVITY is regarded as a type of implicature and is generally separated from distributive inferences.

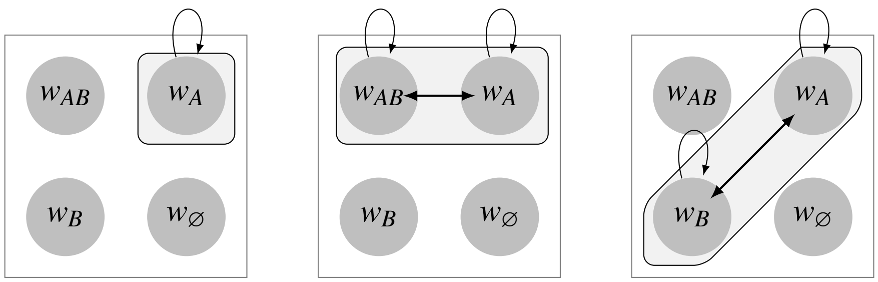
We will now outline a recent non-implicature approach (Aloni 2022) to IGNORANCE inferences which accounts independently for POSSIBILITY and related inferences. We will also explore how such a system can be enriched with a mechanism of implicature computation which is able to derive UNCERTAINTY via EXCLUSIVITY. It is worth noting that while our experimental data seem to suggest an independent mechanism to generate POSSIBILITY, the choice to take Aloni (2022)'s account as an example of a non-implicature approach is not crucial, and similar considerations can be explored using alternative accounts, such as the one discussed in Goldstein (2019).

7.3 A non-implicature account

On Aloni (2022)'s account, POSSIBILITY and related inferences do not arise from Gricean reasoning or the application of a covert exhaustification operator. Instead, these inferences are the result of another pragmatic factor referred to as neglect-zero. In particular, Aloni (2022) puts forward the idea that language users, when interpreting a sentence, construct structures that represent reality and they systematically disregard structures that vacuously satisfy the sentence due to an empty configuration, also referred to as zero-models. Neglect-zero relates to a tendency that has been observed in number cognition and language development with regard to the number zero, as discussed in Nieder (2016).

These structures are built upon the notion of a state, a set of possible worlds, which reflects the speaker's information state and upon which formulas are interpreted. For instance, Figure 5 depicts three states: $s_1 = \{w_A\}$, $s_2 = \{w_{AB}, w_A\}$

19 As discussed in fn. 15, we found no evidence of exclusivity inferences $\Box_s \neg(A \wedge B)$. In previous experiments on distributive inferences, Crnič et al. (2015) also found no evidence of weak exclusivity inferences $\neg\Box_s(A \wedge B)$, meaning that the configuration AB-AB-AB-? was judged as good.



(a) $s_1 \vDash A \vee B$
$s_1 \nvDash (A \vee B)^+$

(b) $s_2 \vDash A \vee B$
$s_2 \vDash (A \vee B)^+$

(c) $s_3 \vDash A \vee B$
$s_3 \vDash (A \vee B)^+$

Figure 5 Illustrations. A state s is a set of possible worlds (in w_{AB} both A and B hold, in w_A only A holds and so on). The accessibility relation, indicated with an arrow, is state-based/epistemic, since each world has access to the entire state.

and $s_3 = \{w_A, w_B\}$. As said, states are meant to encode the epistemic state of the speaker. For instance, in s_3, the speaker considers possible that only A holds or that only B holds, while in s_2 and s_1, the speaker is certain that A holds. Note that this notion of states provides a natural correspondence to the epistemic state of the child's character in our mystery box experiments. The three states in Figure 5 could correspond to the box configurations '$A - A - A -$?'; '$A - AB - A -$?' and '$A - B - B -$?', respectively.

Aloni (2022) formalises this in a Bilateral State-based Modal Logic, which we will refer as BSML. We will focus on the fundamental tenets relevant to us, and refer the reader to Aloni (2022) for further details.

Atomic formulas are true in a state when all worlds in the state make the formula true. For instance, in Figure 5, A holds in s_1 and s_2, but not in s_3, since A is not true in w_B. BSML employs a split notion of disjunction, which is supported in a state s when the state can be split into two substates (i.e., s is the union of two substates), each supporting one of the disjuncts. All the states in Figure 5 support a disjunction of the form $A \vee B$. For the state s_1 in Figure 5(a), the two substates would be $\{w_A\}$ supporting A and the empty state $\varnothing$ supporting B.[20]

Lastly, we note that each world can see/access other worlds according to the accessibility relation specified in the model, as represented in Figure 5 by means of an arrow. Roughly, a universal modal $\Box A$ is true when all worlds in the state see

20 Note that $\{w_A\} \cup \varnothing = \{w_A\} = s_1$.

A-worlds. A possibility modal $\Diamond A$ is true when all worlds in the state see some A-world.

An important notion is *state-basedness*: the accessibility relation is state-based when each world in the state sees all the other worlds in the state. This aligns with the view that speakers are aware of their epistemic state, and allows us to capture the fact that A is possible in a given epistemic state by means of $\Diamond A$, since A will be true in some world in the state. In the following, we will indicate that A is possible in the epistemic state of the speaker by means of $\Diamond_s A$, where we assume that the accessibility relation is state-based and s stands for the speaker as discussed in the previous sections.

7.3.1 Plain Disjunction and POSSIBILITY

As said above, Aloni (2022) proposes that language users neglect zero-models and therefore rule out the possibility of verifying a sentence by means of empty configurations, as in the last example discussed in the previous paragraph. Aloni (2022) implements this neglect-zero effect by means of an enrichment function $(\cdot)^+$ which excludes the empty set as a possible verifier. Formally, this can be defined recursively on the complexity of the formulas, but here we will focus on the case of disjunction.

An enriched disjunction $(A \vee B)^+$ is supported in a state when the state can be split into two *non-empty* substates, each supporting one of the disjuncts. For instance, the disjunction $A \vee B$ is no longer supported in s_1 in Figure 5 when enriched (i.e., $(A \vee B)^+$), since as we have discussed above one of the supporting substates is the empty set.

As a result, an enriched disjunction $(A \vee B)^+$ entails POSSIBILITY, but not UNCERTAINTY, where the entailment $\phi \vDash \psi$ roughly means that if a state in a model makes ϕ true, it also makes ψ true:

(30) a. $(A \vee B)^+ \vDash \Diamond_s A \wedge \Diamond_s B$
b. $(A \vee B)^+ \nvDash \neg \Box_s A \wedge \neg \Box_s B$

(30a) can be formally proved in BSML when the model is state-based. The idea is that a pragmatically enriched disjunction is supported only in states which include both A-worlds and B-worlds, and this ensure that $\Diamond_s A$ and $\Diamond_s B$ hold, as discussed above when introducing the notion of state-basedness. For (30b), a pragmatically enriched disjunction is also supported in states like Figure 5(b), which do not support $\neg \Box_s A$, since A is true in all words in the state.

Therefore, BSML offers a systematic explanation for the experimental findings presented in Section 4, Section 5 and Section 6, namely that POSSIBILITY inferences are derived as neglect-zero effects without UNCERTAINTY inferences.

7.3.2 Embedded Disjunction and DISTRIBUTIVE Inferences

In Section 7.1, we observed that there is a strong parallelism between inferences related to plain disjunction and those in an embedded context. Therefore, a uniform theory that captures both types of inferences is desirable. BSML readily accounts for DISTRIBUTIVE inferences, since $[\Box_{(s)}(A \vee B)]^+$ entails $\Diamond_{(s)}A$ and $\Diamond_{(s)}B$ both for epistemic and other modalities.[21] The reason is that when $[\Box_{(s)}(A \vee B)]^+$ holds, all worlds in the state must access both A-worlds and B-worlds, due to the pragmatic enrichment affecting the disjunction, making thus also $\Diamond_{(s)}A$ and $\Diamond_{(s)}B$ satisfied (Aloni 2022):[22]

(31) a. $[\Box(A \vee B)]^+ \vDash \Diamond A \wedge \Diamond B$
b. $[\Box(A \vee B)]^+ \nvDash \neg\Box A \wedge \neg\Box B$

Therefore BSML can provide a unified account of ignorance and distributive inferences.

7.3.3 The role of EXCLUSIVITY

The non-implicature framework explored so far accounted for the derivation of POSSIBILITY and related inferences in the absence of the corresponding negated upper bound inferences and EXCLUSIVITY. While this approach makes the correct predictions for our experimental paradigm, there are instances in which EXCLUSIVITY and UNCERTAINTY inferences are derived. We will return to this issue in Section 7.5.

For the moment, we observe that adding EXCLUSIVITY to either POSSIBILITY or DISTRIBUTIVE inferences leads to the derivation of UNCERTAINTY or NEGATED UNIVERSAL inferences. This aligns with our earlier observations in Section 7.2.4 within the context of the grammatical approach. Aloni (2022) does not provide an account of scalar implicatures. An important next step is extend non-implicature accounts by incorporating a scalarity operator $(\cdot)^\sigma$, which enables the generation of scalar implicatures (such as EXCLUSIVITY for a plain disjunction). In this way, the interplay between the neglect-zero enrichment $(\cdot)^+$ and the scalar enrichment $(\cdot)^\sigma$ could be adequately studied.

In the subsequent section, we will delve into an alternative account in the context of game-theoretic approaches to pragmatics, which builds upon Grice's original insights regarding cooperation between the speaker and hearer. This theory will offer an additional viewpoint on the behavior of disjunction and its

21 In this case, we are using $\Box_s$ for an overt epistemic modal and $\Box$ for an overt deontic modal.

22 Note also that first-order extensions of Aloni (2022)'s system, such as Aloni & van Ormondt (2021), can capture DISTRIBUTIVE inferences that arise in the context of nominal quantifiers.

associated inferences.

7.4 The Speaker-Hearer Perspective and Linking Hypotheses

A pivotal aspect of experimental research lies in identifying the appropriate linking hypotheses for an experimental paradigm. These hypotheses establish the connection between the measured response and the theoretical frameworks that we aim to enhance our understanding of. In this section, we will return to our experimental design and try to reflect on why certain inferences are derived, while others are not. We will also emphasize the potential value of recent game-theoretic approaches to pragmatics in addressing this question.

In the current experimental paradigm, participants consider what is being said and conveyed by the child character's utterance and then assess whether or not the result aligns with what can be assumed about the character's epistemic state.

A reviewer notes that, from a classical perspective on Gricean implicatures, implicatures involve pragmatic inferences about the speaker's epistemic state. Consequently, one might argue that there is no reason to detect implicatures in the current experimental design, since the listener (i.e., the participant) is fully aware of the speaker's epistemic state (i.e., the fictional character). Therefore, it should not be surprising that UNCERTAINTY inferences are not derived. Firstly, if this argument holds, we would also expect not to find POSSIBILITY inferences (unless such inferences have another non-Gricean source). However, the mean acceptance rate of our TARGET-2 condition clearly goes against this expectation. Secondly, the majority of verification tasks in the literature assumes that full information is available to both speaker and listener. For instance, in the case of scalar items like *some*, it has been consistently observed that in contexts where the stronger alternative *all* is true, sentences with *some* are still associated with lower judgments of acceptability in a truth-value task, which we would not expect if no scalar inference is generated.

Related to this, we have seen that traditional implicature accounts (e.g., Sauerland 2005) posit the presence of an opinitionatedeness assumption (also called 'epistemic step'), which leads to a distinction between primary implicatures, which do not require the speaker to be opinionated, and secondary implicatures, which arise from primary implicatures together with opinitionatedeness. In the current experiments, speakers were not always opinionated about the relevant propositions, but we only looked at primary implicatures (UNCERTAINTY), showing that they are not necessary to derive POSSIBILITY.[23] Further evidence for the alleged 'precedence' of primary implicatures comes from the work of Dieuleveut et al. (2019), who

23 As said, we stress again that POSSIBILITY is not a secondary implicature. It derives from the primary UNCERTAINTY implicature together with the assertion and quality, and not by opinionatedness.

showed that in contexts where the speaker is not opinionated, secondary implicatures of scalar items like *some* are derived more robustly than primary implicatures, challenging again the 'primary' status of such implicatures.

One possible avenue to situate the current experimental results comes from game-theoretic approaches to pragmatic inferences, which distinguish between speaker-oriented and listener-oriented inferences. In recent years, game-theoretic approaches have become an increasingly popular tool for modeling linguistic phenomena and understanding the dynamics of communication (among others, Dekker & Van Rooy 2000, Benz et al. 2005, Franke 2009, Frank & Goodman 2012). These models seek to capture the strategic interactions between speakers and listeners, and to provide a formal framework for analyzing how linguistic conventions and norms emerge.

While we will not provide a comprehensive alternative account, as we did for the implicature and non-implicature approaches discussed earlier, we will consider a previous account that has been developed and discussed in the literature for plain disjunction and assess its compatibility with our experimental results.[24]

In what follows, we will rely on the lifted Iterated Best Response (IBR) model discussed in Franke (2009), which allows us to model modal inferences of the kind examined in the present work. While more recent models could also be appropriately extended and implemented, we will rely on the account put forth by Franke (2009), as it offers an interesting parallelism with the notion of state discussed in the context of BSML in Section 7.3.

The lifted IBR model includes a set of states, corresponding to a set of worlds and fully parallel with the BSML notion discussed before, and a set of possible messages that speakers may utter. The sender/speaker is responsible for choosing a message (i.e., an utterance), while the receiver/listener is responsible for interpreting the message and inferring the intended meaning. We take the set of possible messages to be A, B, the disjunction $A \vee B$ and the conjunction $A \wedge B$. The sender and receiver are assumed to be rational agents who are trying to behave optimally.

Figure 6 displays the optimal strategies for the sender (S^*) and the receiver (R^*). The sender sends a message based on a certain state, and in the optimal configuration we observe that the states where a disjunction $A \vee B$ is possible pattern with the results discussed in Section 4 and Section 5: only states where POSSIBILITY is true are admitted. The receiver interprets the message of the sender. For the optimal receiver R^*, we observe that a disjunction is associated with a state

24 We acknowledge that it would be beneficial to thoroughly discuss all the relevant assumptions underlying these models and examine the different predictions they make. However, we believe that such an undertaking should be accompanied by empirical research. Therefore, for the purposes of this work, we will rely on previous theoretical work, and determine how it fares with our result.

$$S^* = \left\{ \begin{array}{ll} \{w_A\} & \mapsto A \\ \{w_B\} & \mapsto B \\ \{w_{AB}\} & \mapsto A \wedge B \\ \{w_A, w_{AB}\} & \mapsto A, A \vee B \\ \{w_B, w_{AB}\} & \mapsto B, A \vee B \\ \{w_A, w_B\} & \mapsto A \vee B \\ \{w_A, w_B, w_{AB}\} & \mapsto A \vee B \end{array} \right\} \quad R^* = \left\{ \begin{array}{lll} A & \mapsto & \{w_A\} \\ B & \mapsto & \{w_B\} \\ A \wedge B & \mapsto & \{w_{AB}\} \\ A \vee B & \mapsto & \{w_A, w_B\} \end{array} \right\}$$

.

Figure 6 Optimal strategies for sender and receiver in a lifted IBR model in Franke (2009)

where both UNCERTAINTY and POSSIBILITY need to hold.[25]

This discussion leads to two observations. First, the sender/speaker is not taking disjunction in its literal meaning, as a disjunction $A \vee B$ is not an optimal message for a state like $\{w_A\}$. As said, this aligns with treatment of disjunction in non-implicature frameworks, like in the neglect-zero enriched disjunction $(A \vee B)^+$ in BSML, which as discussed gives rise to POSSIBILITY. Interestingly, this squares with the fact that the logical system behind BSML has been developed to model the *assertability* conditions of a sentence, rather than its truth conditions. The pragmatic neglect-zero enrichment affects this assertability dimension and is as such *speaker*-oriented, as in the game-theoretic model just outlined.[26]

Second, the distinction between speaker and hearer might shed some light on the results of our experiments. In particular, we may associate the speaker-oriented behavioural profile of our experiments with the observation that sentence-picture verification tasks, like the ones used in our experiments, have been claimed to be a measure of production, rather than interpretation (Degen & Goodman 2014). Consequently, although further behavioral data is needed, we conjecture that in a sentence-picture verification paradigm, subjects reason from the point of view of the

25 There is however a notable difference with the results of our experiments. The state $\{w_{AB}\}$ is associated with the conjunction $A \wedge B$. While our experiments did not directly test this case, previous experimental studies (Crnič et al. 2015) suggest that, at least for some subjects, that state can also be associated with a disjunction. $A \vee B$ We also note that Franke (2009) relies on two fundamental assumptions. First, speakers are competent (i.e., they consider more uncertain states less likely). Second, speakers will select only the most likely messages in a state of maximal information (i.e., not a singleton set), while they will be indifferent when the state is not of maximal information. We leave to further research a more comprehensive discussion of these modelling assumptions.

26 Note that in principle this effect can be blocked in some instances (e.g., in mathematical reasoning). An interesting research question is to determine which contexts and experimental tasks are tied to the different pragmatic enrichments of neglect-zero or scalarity, and their interaction.

sender/speaker: given what they know (i.e., their epistemic state), they determine whether a certain statement can be asserted. There are of course cases where UNCERTAINTY and EXCLUSIVITY should be drawn, and we conjecture those cases emerge when language users adopt the point of view of the receiver/listener: given an utterance, they determine the most optimal interpretation for that utterance. In the next section, we briefly overview when and why such inferences could be generated.

7.5 EXCLUSIVITY and UNCERTAINTY

Our experiments showed that disjunction could give rise to POSSIBILITY, while UNCERTAINTY and EXCLUSIVITY were not derived. Still, it is natural to assume that in certain contexts those inferences which we typically associate with disjunction are generated. In line with what was observed at the end of the last section, we conjecture that an experiment aimed at measuring interpretation rather than production could provide valuable insights. In the current mystery box experiments, participants had to judge if a sentence was good or bad based on the information available to them. A variant of these experiments would consist in asking participants to correctly place the balls inside the boxes given a truthful sentence, thereby reflecting the listener/interpretation viewpoint.

Another case in which EXCLUSIVITY and UNCERTAINTY seem to play a role are conversational contexts like (32) and (33). In such contexts, a disjunction results in oddity. A preliminary explanation could attribute the oddness of such continuations to the fact that EXCLUSIVITY and UNCERTAINTY inferences clash with the information available in the context. In this regard, an important question is whether UNCERTAINTY inferences always correlate with EXCLUSIVITY. The seeming oddness of (33b) suggests a negative answer.

(32) Context: We all agree that Mary is here and John is here as well.
  a. ?Mary or John are here.

(33) Context: We all agree that Mary is here, and John might be too.
  a. ?Mary or John are here.
  b. (?)I don't know whether they are both here, but either one or the other is here.

An alternative explanation for the oddness observed in cases such as (32) and (33) is that we are bringing attention to the alternatives in a linguistic form. For example, in (33), we are highlighting the alternative that 'It is certain that Mary is here and it is possible John is here'. This alternative is generally more complex, and it is typically not considered unless we emphasize it linguistically, as suggested by structure-sensitive characterizations of alternatives like Katzir (2007).

We leave the investigation of cases like the above for future work.

8 Conclusion and next steps

In this work, we have reported the results of two experiments that challenge the traditional implicature account of IGNORANCE inferences associated with plain disjunctive sentences. Our findings indicate that participants did derive POSSIBILITY inferences even in the absence of UNCERTAINTY inferences. This study thus contributes to the ongoing debate on the nature of inferences associated with disjunction. Our results call for a revaluation of the classical view of IGNORANCE inferences and pave the way for alternative theories that can account for the observed patterns of inference derivation.

To contextualize our findings, we discussed three different perspectives: grammatical, non-implicature, and game-theoretic approaches. Furthermore, we situated our discussion within the broader context of disjunction in embedded contexts.

Further experiments could probe the robustness of our results using different experimental methodologies. For instance, experiments eliciting indirect judgments of felicity could provide converging evidence. Another potential follow-up would involve the cases we discussed in Section 7.5, where UNCERTAINTY and EXCLUSIVITY seem to play a role.

Additionally, studying the role of contextual factors in the derivation of these inferences could provide a more comprehensive understanding of the interplay between semantics and pragmatics in the interpretation of disjunctive sentences. For instance, we might conjecture that disjunction will be interpreted literally in mathematical reasoning, where even POSSIBILITY inferences are not derived. Similarly, our results showed some variation among participants, suggesting that some individuals were more prone to deriving POSSIBILITY inferences from disjunctive sentences than others. We leave exploring what drives these differences in future work.

A Instructions for Experiment 1 and 2

General Instructions

In this study, we will ask for your intuitions about certain kinds of sentences in English. These sentences will be uttered by two characters, Sam and Mia. Here they are:

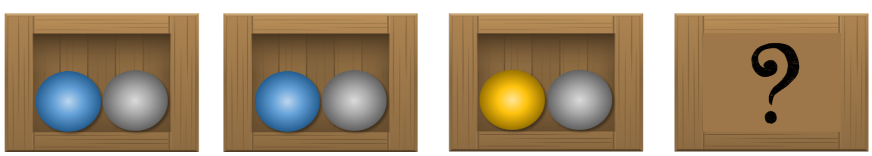
Sam

Mia

[continue]

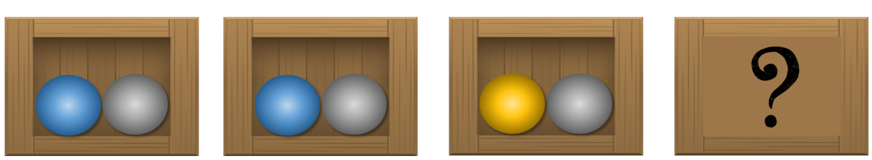
What do Sam and Mia know about the mystery box?

Sam and Mia will be presenting with quadruplets of boxes containing balls of various colors. Each time, they can see what's inside the first three boxes, but not what's inside the fourth one, the so-called 'mystery box'. However, they have been taught the rule that **the mystery box always has the same contents as one of the three open boxes.** Thanks to this rule, they can make certain inferences about what the mystery box contains and does not contain.



[try an example]

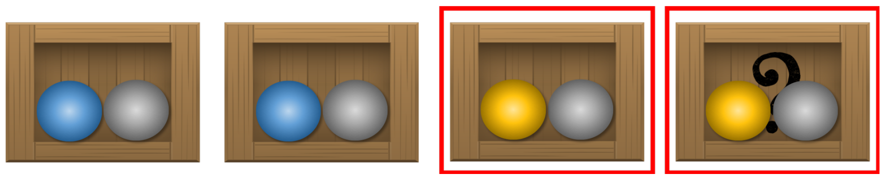
In this example, Sam knows that **every open box** contains a gray ball. Similarly, he knows that one open box also contains a yellow ball while the other two also contain a blue one.



Thanks to the rule, Sam should be certain that **the mystery box** contains a gray ball. He should also be certain that this box does not contain a green ball. However, he cannot be certain that the mystery contains a blue ball, nor can he be certain that it contains a yellow ball.

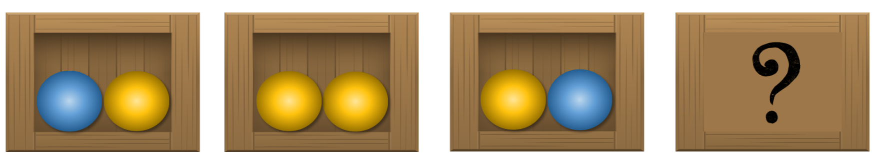
[Reveal the mystery box’s contents]

In this case, the contents of the mystery box were identical to those of the third box.



[Try another example]

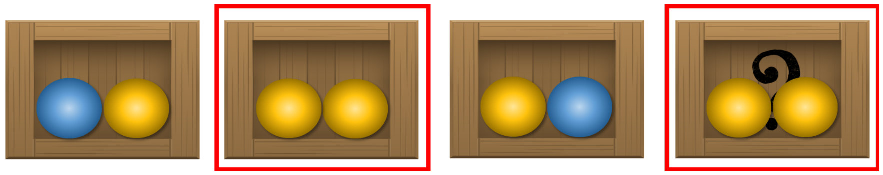
In this second example, Mia knows that **every open box** contains a yellow ball. Similarly, she knows that one open box contains another yellow ball while the other two contain a blue one.



Thanks to the rule, Mia should be certain that **the mystery box** contains a yellow ball. She should also be certain that it does not contain a gray or a green ball. However, she cannot be certain that the mystery box contains two yellow balls, nor can she be certain that it contains a blue ball.

[Reveal the mystery box’s contents]

In this case, the contents of the mystery box were identical to those of the second box.



[continue]

Sam and Mia will see many quadruplets like these ones, each of which will be followed by an utterance from one of them about what the mystery box contains or does not contain. Your task is to decide if this utterance is right given the information available to the characters and the rule that they have learned about how the mystery box works. You can click on 'Good' if you consider that they got it right; otherwise click on 'Bad'.

Training Phase

You will start with a short training to get you familiar with the task. During this training, you will receive **feedback** on your responses. If you answered correctly, your will see a **green smiley face**; otherwise, you will see a **red frowning face** and be asked to try again. Use this feedback wisely to improve your answers.

We are interested in your spontaneous responses, so don't think too long before answering.

Test Phase

As in the training, you will decide whether or not the character's utterances are right given the information available to them and the rule that they have learned about how the mystery box works. From now on, however, **you will no longer receive feedback on your responses**.

Recall that we are interested in your spontaneous responses, so don't think too long before answering.

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