The semantics of comparatives: A difference-based approach

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Abstract

Degree semantics has been developed to study how the meanings of measurement and comparison are encoded in natural language. Within degree semantics, this paper proposes a **difference-based** (or **subtraction-based**) approach to analyze the semantics of comparatives. The motivation is the measurability and comparability of differences involved in comparatives. The main claim is that comparatives encode a subtraction equation among three scalar values: two measurements along an interval scale and the difference between them. We contribute two innovations: (i) using interval arithmetic to implement subtraction, and (ii) analyzing comparative morpheme *-er*/*more* as an additive particle, denoting the default, most general, positive difference. Our analysis inherits existing insights in the literature. Moreover, the innovations bring new conceptual and empirical advantages. In particular, we address the interpretation of comparatives containing *than*-clause-internal quantifiers and various kinds of numerical differentials. We also account for three puzzles with regard to the scope island issue, the monotonicity of *than*-clauses, and the discourse status of the standard in comparison.

Keywords: measurement, comparison, gradable adjectives, comparatives, differentials/differences, comparative morpheme *-er*/*more*, measurement constructions, positive use of gradable adjectives, degrees, scales, intervals, units, orderings, interval arithmetic, interval subtraction, degree questions, definite descriptions, downward-entailing operator, additivity, anaphoricity.

1 Introduction

Humans measure objects along some dimension or scale and make comparisons among measurements. As illustrated in (1), we can compare how tall a giraffe is to a certain tree; we can compare some soup and coffee in terms of their temperature; and we can compare a train's arrival with the time it's supposed to arrive on a temporal scale.

(1) a. My giraffe is (5 inches) **taller than** that tree is.
$\rightsquigarrow$ On a scale of **height**: the measurement of my giraffe vs. the measurement of that tree

b. This soup is (much) **hotter than** that coffee seems to be.
$\rightsquigarrow$ On a scale of **temperature**: the measurement of this soup vs. the seeming measurement of that coffee

c. The train arrived (one hour) **later than** it should have.
$\rightsquigarrow$ On the scale of **time**: the actual arrival time vs. the scheduled arrival time

Natural language typically uses **comparatives** to express **comparisons yielding differences** (cf. **equatives**, which typically express **comparisons yielding no differences**). The notion of differences is obviously a gist of the meaning encoded in comparatives. Thus, starting with the view that differences constitute an indispensable central component in comparatives, this paper furthers our understanding of the semantics of comparatives and develops a new **difference-based** approach.

This introduction addresses the ontology of differences as involved in comparatives and lays out our basic assumption and motivation, paving the way for our proposal.

1.1 The ontology of differences in comparatives

We address the ontology of differences and their formal properties within a general view on measurement and comparison. In his influential paper on the theory of scales of measurement, Stevens (1946) paraphrases N. R. Campbell and points out that 'measurement, in the broadest sense, is defined as the assignment of numerals to objects or events according to rules'. Thus, measurement is a mapping function from items under measurement to values on a certain scale.

Stevens (1946) presents a four-level distinction of measurement and their related scales: **nominal scales**, **ordinal scales**, **interval scales**, and **ratio scales**. This four-level distinction is according to (i) the way of assigning values in measurement, (ii) the formal

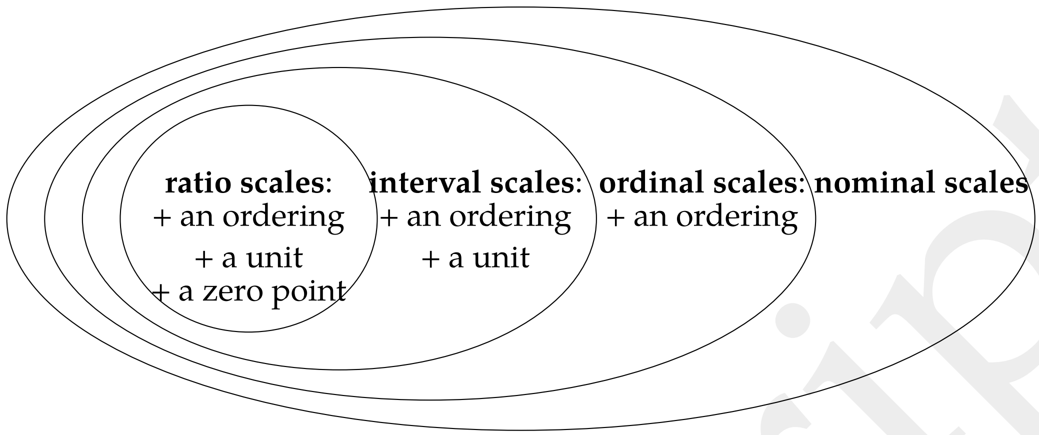

Figure 1: Four levels of scales, their entailment relationships (represented by the Venn diagram), and their defining attributes.

properties of the resulting scales, and (iii) the mathematical operations applicable to measurement values. The entailment relations among these four levels of scales are shown in the Venn diagram in Fig. 1.

Nominal scales do not even involve ordering. For example, if we assign a postal code to each address, the postal codes constitute a nominal scale. For distinct values on a nominal scale (e.g., distinct postal codes), all that matters is their distinctness, and further comparison is not mathematically meaningful.

Ordinal scales have **orderings**. For example, the ranking of my favorite soda brands forms an ordinal scale. Comparisons between two ranking values are characterized by inequality relations like '$>$', '$\leq$', etc., but beyond ordering relations, it is not meaningful to address to what extent a certain ranking exceeds another one.

Interval scales have both orderings and **units**. On an interval scale, if one value is positioned higher than another one, we can use units to measure the **distance** (i.e., the **difference**) between the two **positions**. Therefore, comparisons between measurement values on an interval scale yield **measurable and comparable differences**, allowing for addressing **to what extent one value exceeds another**.

Ratio scales are interval scales with a meaningful, absolute **zero point**. For example, a scale of temperature lacks an absolute zero point in the sense that 0 °C does not mean 'no heat', and thus this is not a ratio scale. In contrast, for a scale of spatial length, 0 m does mean 'no length'. Thus this scale has an absolute zero point and it is a ratio scale.[1]

[1]Based on the distinction between interval scales and ratio scales, Sassoon (2010) explains why only certain gradable adjectives are accepted in forming measurement constructions like *This movie is 4 hours long*

Then what kind of scales are involved in the meaning of comparatives? Empirically, English comparatives allow for addressing the 'to what extent' issue with regard to differences yielded from comparisons. As illustrated in (1), this is evidenced by the use of modifiers like *much* (see (1b)) or numerical differentials (e.g., *5 inches* in (1a) and *one hour* (1c)). These examples indicate that comparisons as encoded in comparatives are performed between measurements on interval scales and yield **measurable differences**.[2]

The comparison of deviations shown in (2) is another linguistic construction showing how differences yielded from comparisons on **a base measurement scale** can be further measured and compared on **a scale of differences**.[3] Here the scales of happiness and sadness are two base measurement scales. It is the differences yielded from comparisons along these two base measurement scales that constitute the measurements along the scale involved in the third comparison, i.e., a scale of differences. In this sense, **measurement** yields markings of **positions** along a base measurement scale, while **comparison** is actually the measurement of differences/**distances** between positions. The values of these differences/distances can again be considered positions along a scale of differences. Base measurement scales and scales of differences are both interval scales.

(2) Mona is more happy than Jude is sad. (Kennedy 1999: Chapter 1, (89))

a. **Comparison 1** – along a scale of happiness:
Mona's happiness vs. the standard of happiness

b. **Comparison 2** – along a scale of sadness:
Jude's sadness vs. the standard of sadness

c. **Comparison 3** – along a scale of deviation size (i.e., a scale of differences):
difference from Comparison 1 vs. difference from Comparison 2

The necessary role played by units in measuring and comparing differences is most evidently manifested by the measurement and comparison of times. For the case in (1c), ordering only tells which one between the scheduled and the actual arrival times

(cf. #*This cup of coffee is 30 °C warm*). Evidently, Stevens (1946)'s theory captures some crucial aspects of our conceptualization of measurement and comparison and their linguistic encoding.

[2]We do not claim that cross-linguistically, all comparatives or comparison-related meanings must be based on interval scales. Presumably, there might be comparison-related linguistic phenomena based on ordinal scales or even nominal scales (see our more recent work). It is also not unlikely that in a certain language, both interval-scale-based and ordinal-scale-based comparative constructions co-exist. However, we do claim that interval scales must be assumed for linguistic phenomena like English comparatives. We also predict that comparatives based on ordinal scales cannot support the expression of the size of differences.

[3]We thank an anonymous reviewer for suggesting this pair of terms.

occurred first, and it is units (e.g., hours, minutes) that measure time differences. Obviously, units like *hours* can by no means be derived just from the ordering of equivalence classes like {the scheduled arrival time of a train, 12 o'clock, ...} or {the actual arrival time of a train, 1 o'clock, ...}.[4]

In brief, based on Stevens (1946)'s theory on the levels of measurement and scales, we have shown that the notions of interval scales and the measurability and comparability of differences fundamentally underlie the meaning of comparatives.

1.2 Our assumption and motivation

Within the literature on the semantics of comparatives, the major assumption is that comparisons are performed between **degrees**, i.e., points that mark positions and represent scalar values on a relevant abstract scale (i.a., Seuren 1973, Cresswell 1976, Hellan 1981, Hoeksema 1983, von Stechow 1984, Heim 1985, Bierwisch 1989, Lerner and Pinkal 1992, 1995, Moltmann 1992, Gawron 1995, Izvorski 1995, Rullmann 1995). Kennedy (1999) provides a review and a convincing defense on this assumption.

Under this assumption, for our examples in (1), items undergoing comparisons are not entities (e.g., my giraffe, this soup) or events (e.g., a train's arrival) per se, but rather their heights, temperatures, or times. This assumption is not specific on the formal properties of degrees or scales involved in comparatives.[5]

A less explicit assumption is that degrees involved in comparatives are number-like values, so that the operations of addition or subtraction are applicable. This assumption is reflected in the analysis of comparatives containing numerical differentials – the parenthesized part in the examples in (1) (i.a., Hellan 1981, von Stechow 1984).

Within both **the 'A-not-A' analysis** (see Schwarzschild 2008 for a review) and **the '>' analysis** (see Beck 2011 for a review), for cases with no explicit numerical differentials, analyses are based on set operations and orderings (see (3a) and (4a)). Addition or subtraction is used to deal with explicit numerical differentials (see (3b) and (4b)).

[4]In English, *o'clock* is used to mark positions on a scale of time, while unit expressions like *hour* are used to measure differences on a scale of temporal differences (or temporal length). For many other dimensions (e.g., temperature, spatial length, weight), base measurement scales and scales of differences share unit expressions (e.g., $^{\circ}C$, *meter*, *kilo*), delusively blurring the distinction between conceptually distinct scales. E.g., a base measurement scale of temperature is not a ratio scale given that 0 °C does not mean 'no heat', but a scale of temperature differences is a ratio scale given that 0 °C means 'no temperature difference'.

[5]For Cresswell (1976), this abstract scale can be derived from the orderings among equivalence classes, but as shown in Section 1.1, mere orderings are insufficient for characterizing the semantics of comparatives. The abstract scale involved must be an interval scale. See also Kennedy (1999)'s discussion.

(3) My giraffe is (5 inches) taller than that tree is. **the 'A-not-A' analysis**

a. The difference set between $\{d \mid \text{the height of my giraffe} \geq d\}$ and $\{d \mid \text{the height of that tree} \geq d\}$ is non-empty.

b. For the difference set D between $\{d \mid \text{the height of my giraffe} \geq d\}$ and $\{d \mid \text{the height of that tree} \geq d\}$, $\textsc{max}(D) - \textsc{min}(D) \geq 5''$.

(4) My giraffe is (5 inches) taller than that tree is. **the '>' analysis**

a. $\textsc{max}(\{d \mid \text{the height of my giraffe} \geq d\}) > \textsc{max}(\{d \mid \text{the height of that tree} \geq d\})$

b. $\textsc{max}(\{d \mid \text{the height of my giraffe} \geq d\}) \geq$
$\textsc{max}(\{d \mid \text{the height of that tree} \geq d\}) + 5''$

The analyses shown in (3a) and (4a) only need to assume ordinal scales for measurement and comparison, while the analyses shown in (3b) and (4b) have to assume number-like degrees and thus interval scales. This discrepancy in their underlying assumptions has been largely unnoticed and under-discussed.

In the current paper, we explicitly assume that the semantics of comparatives is based on scalar values on interval scales. As we have shown, this assumption is empirically warranted by natural language phenomena involving comparatives. Making this assumption explicit will help with exploring the formal properties of degrees in comparatives and what operations to apply on them.

Therefore, this paper aims to push the existing semantic analyses of comparatives towards a full exploitation of this underlying assumption. Based on the measurability and comparability of differences, we will take maximum advantage of the operation of subtraction to build a uniform analysis for both comparatives with and without explicit numerical differentials. The main idea is that comparatives mean a **subtraction equation** among three scalar values: two measurements and the difference between them.

Specifically, we will propose (i) the use of interval subtraction and (ii) an additivity-based view for the semantic contribution of comparative morpheme *-er*/*more*. **Interval arithmetic** provides a convenient technique for characterizing differences in a generalized way, allowing for implementing equations with potentially not-very-precise scalar values.[6] Then *-er*/*more* essentially contributes the meaning of **increase**, which

[6] Here are some clarifications on terminology. Following Stevens (1946), we use **interval scales** to refer to scales equipped with both orderings and units. **Scalar values** mean positions on an interval scale: they can be represented as degrees or intervals. **Degrees** are points (i.e., elements) on an interval scale. **Intervals** are convex sets of degrees (e.g., $\{d \mid 3 \leq d \leq 5\}$). **Interval arithmetic** refers to operations on **intervals**. In particular, we focus on the operation of **interval subtraction**. See Section 3.1 for details.

turns out to be another way to convey the idea of differences yielded from comparisons. Both of our innovations are actually further development of existing insights or observations from the literature of degree semantics.

To assess our proposal, we will show how it brings new conceptual and empirical advantages. In particular, we will demonstrate that the interpretation of comparatives containing *than*-clause-internal quantifiers and all kinds of numerical differentials can be derived in a most natural and uniform way, with all details well taken care of. Moreover, the proposed interval-subtraction-based analysis accounts for three long-existing puzzles in the literature of comparatives: (i) How does a *than*-clause project information as a scope island? (ii) How does a *than*-clause contribute a downward-entailing operator? (iii) If comparison is involved in all uses of gradable adjectives, why is the comparative form (e.g., *taller*) still morphologically more complex than other uses (e.g., *tall*) (Klein 1980's puzzle)? We will show that interval subtraction and an additivity-based view for comparative morpheme *-er*/*more* provide the exact ingredients to solve these issues.

1.3 Outline of the paper

The paper is organized as follows. Section 2 presents our core innovations and their precursors in the existing literature. Section 3 develops a difference-based analysis of the semantics of comparatives, with a detailed formalism implemented in terms of interval subtraction. Section 4 shows the semantic derivation of complex cases: *more-than* and *less-than* comparatives containing numerical differentials and *than*-clause-internal quantifiers. Section 5 accounts for three puzzles, with regard to the scope island issue, the monotonicity of *than*-clauses, and Klein (1980)'s puzzle on the semantic contribution of *-er*/*more*. Section 6 further compares the current analysis with existing studies and ideas on the topic of comparatives. Section 7 concludes. Below, for simplicity, we often use 'scale' to mean 'interval scale' in addressing the semantics of comparatives.

2 The core innovations and their precursors

This section starts with the canonical analysis of comparatives. Against this background, we present the most direct precursors to the current proposal and then our core innovations. An informal sketch of our proposal is given at the end of this section.

2.1 The canonical analysis of comparatives

We follow mainly the review articles by Schwarzschild (2008) and Beck (2011) in sketching out the essence of the canonical analysis of comparatives. Many widely accepted ideas of the canonical analysis have already been established back to von Stechow (1984) and thoroughly discussed by Kennedy (1999). Our presentation glosses over compositional orders and technical details. It is by no means comprehensive. This presentation simply aims to set up the background for the discussion later.

Based on a degree-theoretic view for comparison (i.e., things undergoing comparison are degrees, not entities or events), the canonical analysis consists of three key components: (i) analyzing gradable adjectives as relations of type $\langle d, et \rangle$, instead of characteristic functions of type $\langle et \rangle$; (ii) analyzing the matrix and *than*-clauses as sets of degrees; and (iii) analyzing *-er*/*more* as a relation between sets of degrees.

As illustrated by (5), a gradable adjective denotes a relation between a degree (i.e., a point on a relevant scale) and an individual (see, e.g., Cresswell 1976, Hellan 1981, von Stechow 1984, Heim 1985, Beck 2011, cf. Kennedy 1999). Here HEIGHT means a measure function, mapping an individual to a degree on a relevant scale (here height).

(5) $[\![\text{tall}]\!]_{\langle d,et \rangle} \stackrel{\text{def}}{=} \lambda d_d.\lambda x_e.\textsc{height}_{\langle e,d \rangle}(x) \geq d$ (i.e., x is d-tall; x is tall to degree d)

The semantics of **measurement constructions** and the **positive use** of gradable adjectives can be thus derived straightforwardly, as illustrated by (6). In (6b), POS means a silent context-dependent degree threshold of tallness for a relevant comparison class (see Bartsch and Vennemann 1972a, Cresswell 1976, von Stechow 1984, Kennedy 1999).

(6) a. $[\![\text{Mary is 6 feet tall}]\!] = \textsc{height}(\text{Mary}) \geq 6'$ **Measurement construction**
b. $[\![\text{Mary is tall}]\!] = \textsc{height}(\text{Mary}) \geq \textsc{pos}$ **Positive use**

With the abstraction over a degree variable, both the matrix and *than*-clauses are considered representing sets of degrees (see (7)), including all degrees some entity meets or exceeds (i.e., totally ordered sets ranging from 0 to the measurement of something).

(7) The bathtub is wider than the door is tall.
LF: [-er [λd.the door is d-tall]][$\lambda d'$.the bathtub is d'-wide]
a. ***than*-clause**: λd. the door is d-tall $= \{d \mid 0 \leq d \leq \textsc{height}(\text{the-door})\}$
b. **matrix clause**: $\lambda d'$. the bathtub is d'-wide $= \{d' \mid 0 \leq d' \leq \textsc{width}(\text{the-bathtub})\}$

Comparative morpheme *-er/more* works like a quantificational determiner of type $\langle\langle et\rangle, \langle et, t\rangle\rangle$ (e.g., *every*) and relates two sets of degrees. Different implementations have been proposed. **The 'A-not-A' analysis** in (8) assumes a silent negation operator for the *than*-clause (see Ross 1969, Lewis 1970, Seuren 1973, 1984, McConnell-Ginet 1973, Kamp 1975, Klein 1980 for this idea, see Schwarzschild 2008 for a summary, and see Alrenga and Kennedy 2014 for a recent development). Heim (2006b) proposes a less widely used variation (see (9)). **The '>' analysis** in (10) assumes the use of maximality operators for both the matrix and *than*-clauses (see, e.g., Cresswell 1976, von Stechow 1984, Heim 1985, Rullmann 1995, and see Beck 2011 for a summary). Under this '>' analysis, *-er/more* actually amounts to relating two definite descriptions of degrees (see Russell 1905). With the analysis in (7) for the matrix and *than*-clauses, these implementations all result in the same truth condition for this kind of simplest case of comparatives.

(8) $[\![\text{-er/more}]\!]_{\langle\langle dt\rangle,\langle dt,t\rangle\rangle} \overset{\text{def}}{=} \lambda D_1.\lambda D_2.\exists d[d \in D_2 \wedge \neg[d \in D_1]]$ **the 'A-not-A' analysis**

(9) $[\![\text{-er/more}]\!]_{\langle\langle dt\rangle,\langle dt,t\rangle\rangle} \overset{\text{def}}{=} \lambda D_1.\lambda D_2.D_2 \supset D_1$ **the '$\supset$' analysis**

(10) $[\![\text{-er/more}]\!]_{\langle\langle dt\rangle,\langle dt,t\rangle\rangle} \overset{\text{def}}{=} \lambda D_1.\lambda D_2.\textsc{max}(D_2) > \textsc{max}(D_1)$ **the '>' analysis**
$(\textsc{max} \overset{\text{def}}{=} \lambda D.\iota d[d \in D \wedge \forall d'[d' \in D \to d' \leq d]])$

Within the literature, there is ample discussion on the distinction between **clausal comparatives** and **phrasal comparatives**. Our proposal focuses on the semantics of clausal comparatives. However, we will also address the contrast between clausal and phrasal comparatives with regard to the scope island issue in Section 5.1.

2.2 Precursors to our proposal

There are two lines of precursors to our proposed analysis. Schwarzchild and Wilkinson (2002) adopt an **interval**-based (cf. degree-based) semantics of comparatives (see also Landman 2010). This interval-based approach has later been developed by Beck (2010). On the other hand, Brasoveanu (2008), Greenberg (2010), and Thomas (2010) invite us to reconsider the semantic contribution of comparative morpheme *-er/more*.

2.2.1 The move from degrees to intervals

In Section 1.1, we have shown that comparison along a scale conceptually means the measurement of distances between positions. The canonical analysis uses **degrees** –

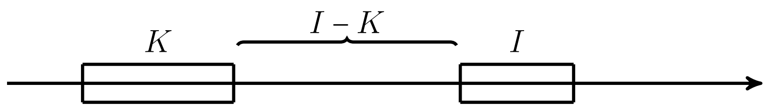

Figure 2: Intervals of Schwarzchild and Wilkinson (2002): I and K are two intervals representing positions under comparison (here K is below I, i.e., $K < I$). DIFF, the size of the interval $[I - K]$ (i.e., the interval that is below I and above K), represents the differential between the positions I and K. In a comparative, the default value of DIFF is SOME.

points – to represent positions on a scale (see Section 2.1). This analysis becomes problematic when the *than*-clause of a comparative contains a universal quantifier. For example, in (11), the canonical analysis amounts to comparing the height of Mary with that of the shortest boy, contradicting our intuitive interpretation of the sentence.

(11) Mary is taller than every boy is. **the canonical analysis**

a. ***than*-clause**: λd. every boy is d-tall. $= \{d \mid 0 \leq d \leq \textsc{height}(\text{shortest-boy})\}$

b. **matrix clause**: $\lambda d'$. Mary is d'-tall. $= \{d' \mid 0 \leq d' \leq \textsc{height}(\text{Mary})\}$

Schwarzchild and Wilkinson (2002) argue that if the price of the shirts ranges from \$20 to \$100 and the dress costs \$150, the dress is surely more expensive than the shirts are, but there is no single point on the scale of price that stands for the price of the shirts. Thus, they propose to use **intervals**, construed as **potentially non-convex, mass-like, homogeneous objects**, to characterize positions on a scale. Consequently, (i) adjectives relate an individual and an interval (see (12)), and this relation satisfies the **Persistence Principle** (see (13), '$\sqsubset$' means a proper part-of relation); (ii) the matrix and *than*-clauses are considered predicates of intervals, instead of predicates of degrees (see (14) vs. (11)).

(12) $[\![\text{tall}]\!] \stackrel{\text{def}}{=} \lambda I.\lambda x.\textsc{height}(x, I)$ (i.e., the height interval I covers the individual x.)

(13) $P(x, I) \rightarrow \forall I'[I \sqsubset I' \rightarrow P(x, I')]$ **Persistence Principle**

(14) Mary is taller than every boy is. **Schwarzchild and Wilkinson (2002)**

a. ***than*-clause**: $\lambda K.\forall x[\text{boy}(x) \rightarrow \textsc{height}(x, K)]$

b. **matrix clause**: $\lambda I.\textsc{height}(\text{Mary}, I)$

Maximality operator μ picks out the largest interval that a predicate of intervals holds for (see (15), '$\sqsubseteq$' means a part-of relation): e.g., the maximal interval that covers a given group of individuals homogeneously. μ is not a mereological sum operator.

(15) $\mu K[\phi(K)]$ picks out the largest interval all of whose non-empty parts are ϕ:
$$\mu K[\phi(K)] = K \text{ iff } \forall K'[[K' \neq 0 \wedge K' \sqsubseteq K] \rightarrow \phi(K')] \wedge$$
$$\forall K''[K \sqsubset K'' \rightarrow [\exists K'[K' \sqsubset K'' \wedge \neg\phi(K')]]]$$

As shown in Fig. 2, a **subtraction operation** '−' is used to implement comparison. For intervals I and K (suppose K is below I), $[I - K]$ picks out the interval that is below I and above K. DIFF, a predicate of intervals applicable to $[I - K]$, addresses the size of $[I - K]$. The value of DIFF can be SOME, a default one, or a numerical differential.

The derived sentential semantics of a comparative is shown in (16): Mary is covered by the maximal interval I such that I is away from the maximal interval K by a default differential SOME and K covers every boy. In this formula, the meaning of comparison, i.e., the part 'DIFF$(I - K)$', is embedded within the semantics of the *than*-clause.

(16) MATRIX-CLAUSE$(\mu I[$THAN-CLAUSE$(\mu K[$DIFF$(I - K)])])$ sentential semantics
[[Mary is taller than every boy is]]
$\Leftrightarrow$ HEIGHT(Mary, $\mu I[\forall x[\text{boy}(x) \rightarrow$ HEIGHT$(x, \mu K[$SOME$(I - K)])]])$

Compositional details aside, the truth condition characterized by (16) is too weak, due to the assumed homogeneity behind the notion of interval. Given our scenario in (17), intuitively, the sentence, which contains a downward-entailing numerical differential *up to $60*, is false. However, the derived sentential semantics is predicted to be true, since the size of $[I - K]$ is $50, which is below $60.

(17) Context: the dress is $150, and the price of the shirts varies between $20 and $100.
[[The dress is up to $60 more expensive than every shirt is]]
$\Leftrightarrow$ PRICE(the-dress, $\mu I[\forall x[\text{shirt}(x) \rightarrow$ PRICE$(x, \mu K[\text{up-to-\$60}(I - K)])]])$

The homogeneity assumption has two consequences. First, if an interval is considered covering multiple individuals homogeneously (see (13) and (15)), distinct values with regard to the measurement of each interval-internal individual are invisible. For the scenario in (17), our natural interpretation is that some shirts cost $20, and others cost $100, but the homogeneity assumption leads to an interpretation that the shirts all have a uniform price between $20 and $100. Second, intervals under comparison (i.e., K and I in Fig. 2) are idealized as abstract whole items that have no size but just mark a position on a scale. Thus neither the size of these intervals (i.e., the size of K and I) nor their lower or upper bound (e.g., the lowest shirt price, $20, in our example) matters in computing sentential semantics. Eventually, only the size of $[I - K]$ is involved in derivation, contrary to our intuition in interpreting (17) in this context.

Beck (2010) develops another interval-based analysis with a different ontology of intervals and a tighter connection to the traditions of degree semantics.

Beck (2010) considers intervals **sets of degrees**. A gradable adjective relates an individual and an interval (see (18)). With the abstraction over an interval variable, the matrix and *than*-clauses are first analyzed as sets of intervals (of type $\langle dt, t\rangle$). Then an informativeness-based maximality operator $\mathrm{M}_{\mathrm{inf}}$ picks out the most informative interval from a set of intervals (see (19)). Thus, the semantics of the *than*-clause in (19a) amounts to an interval ranging from the height of the shortest boy(s) to that of the tallest one(s), while the semantics of the matrix means a singleton set, only containing HEIGHT(Mary).

(18) $[\![\text{tall}]\!]_{\langle dt,et\rangle} \stackrel{\text{def}}{=} \lambda D_{\langle dt\rangle}.\lambda x_e.\textsc{height}(x) \in D$ (i.e., the height of x is a point in interval D.)

(19) Mary is taller than every boy is.
$\mathrm{M}_{\mathrm{inf}\langle\langle dt,t\rangle,dt\rangle} \stackrel{\text{def}}{=} \lambda p_{\langle dt,t\rangle}.\iota D[p(D) \wedge \neg\exists D'[p(D') \wedge D' \subset D]]$ (Beck 2010: p. 28, (82))
a. ***than*-clause**: $\mathrm{M}_{\mathrm{inf}}(\lambda D.\forall x[\text{boy}(x) \to \textsc{height}(x) \in D])$
b. **matrix clause**: $\mathrm{M}_{\mathrm{inf}}(\lambda D.\textsc{height}(\text{Mary}) \in D)$

A MAX operator picks out the largest degree of an interval (see (20)), and [[-er/more]] relates two degrees and implements comparison (see (21)). The derived truth condition is that the height of Mary exceeds that of the tallest boy(s) (see (22)).

(20) $\textsc{max} \stackrel{\text{def}}{=} \lambda D.\iota d[d \in D \wedge \forall d'[d' \in D \to d' \leq d]]$ Beck (2010)

(21) $[\![\text{-er/more}]\!]_{\langle d,dt\rangle} \stackrel{\text{def}}{=} \lambda d.\lambda d'.d' > d$ Beck (2010)

(22) [[Mary is taller than every boy is]] ⇔
$\textsc{max}(\mathrm{M}_{\mathrm{inf}}(\lambda D.\textsc{height}(\text{Mary}) \in D)) > \textsc{max}(\mathrm{M}_{\mathrm{inf}}(\lambda D.\forall x[\text{boy}(x) \to \textsc{height}(x) \in D]))$

Downward-entailing numerical differentials like *up to 3 inches* still challenge this analysis. The sentence in (23) is intuitively false, because under the given context, the height of Mary exceeds that of the shortest boy by more than 3 inches. However, the derived semantics predicts the sentence to be true and thus is too weak. In the same spirit as the homogeneity assumption made by Schwarzchild and Wilkinson (2002), Beck (2010) suggests some *ad hoc* mechanism that construes the interval associated with the *than*-clause as a size-less item: e.g., for (23), all boys are considered of the same height.

Fleisher (2016) points out that this might only work for comparatives containing a differential like *exactly 3 inches*, but cannot be extended to account for cases like (23). For (23), the height information of the shortest boy needs to be taken into consideration.

(23) Context: Mary is $6'1''$ tall, and the height of boys varies between $5'5''$ and $6'$.
[[Mary is up to 3 inches taller than every boy is.]]
$\Leftrightarrow \exists d[[\textsc{height}(\mathsf{Mary}) \geq \textsc{height}(\mathsf{tallest\text{-}boy(s)}) + d] \wedge [0 < d \leq 3'']]$

2.2.2 The additivity of *-er/more*

English morpheme *-er/more* is not exclusively used in comparatives. It also appears in additive constructions (see Greenberg 2010, Thomas 2010) and comparative correlatives.

Additive constructions are distinct from comparatives. The most natural interpretation of (24a) is that the amount of chocolate Mary ate after feeling full is above zero. In other words, the amount she ate at a later time does not necessarily exceed the amount she ate previously. Rather the amount Mary ate later is an **increase** on the **base** of the amount she ate before. It can be a large or small increase. This additive reading of *-er/more* becomes more evident when weak NPI *any* is used along with *more* (see (24b)).

(24) a. Mary ate chocolate until she felt full. Then she ate **more**. **Additive**
b. Mary refused to eat any **more**. **Additive**

The **comparative correlative** in (25) means that the **increase** of my knowledge about my dog (from one time to another) correlates with the **increase** of my fondness for her (between these two times). This sentence does not tell to what extent I know about my dog or how much I like her at these times. What the sentence conveys is the correlation between two **increases**, i.e., two positive **differentials** (see Brasoveanu 2008).

(25) The **more** I know about my dog, the **better** I like her. **Comparative correlative**

Based on Romanian data, Brasoveanu (2008) analyzes the phenomenon of comparative correlatives as an anaphora to differentials. For additive constructions, Greenberg (2010) analyzes *more* as an additive measure function (see also Thomas 2010). Given that addition and subtraction are inverse operations, increases are conceptually the same as (positive) differentials. Thus, taken together, these studies indicate a common semantic contribution of *-er/more* in distinct linguistic constructions, namely **additivity**.[7]

[7] The additivity of *-er/more* is also reflected in the meaning of additive connectives like *moreover*: *War*

Kennedy and McNally (2005) and Kennedy and Levin (2008) also suggest that the semantics of *-er/more* in comparatives can be developed along the notion of differentials.

An additivity/differential-based view is promising for a unified account for various uses of *-er/more*: *-er/more* denotes (i) the increase from a part to a whole in additive constructions and (ii) the difference between a lower and a higher scalar value in comparatives. However, a fully worked-out analysis along this additivity-based view of *-er/more* is still missing in the existing literature on comparatives.

2.3 The core innovations

In this paper, we further develop (i) the idea of using intervals to mark positions on a scale and operating on them and (ii) an additivity-based view for *-er/more*. The proposed difference-based analysis of comparatives results from a marriage of these two.

2.3.1 Interval subtraction

Schwarzchild and Wilkinson (2002) and Beck (2010) propose to use **intervals**, instead of **degrees**, to mark positions along a scale. The lesson from the cases like (17) and (23) is that in our intuitive interpretation of comparatives, intervals that mark positions and undergo comparison cannot be size-less, homogeneous objects. In particular, when a comparative contains downward-entailing numerical differentials like *up to $60/3 inches*, the lower bound of the interval associated with the *than*-clause matters in derivation.

If the lower and upper bound information of an interval needs to be visible and an interval cannot be reduced to an item directly applicable for inequalities (e.g., $>$, $\leq$), how to perform comparison? **Interval subtraction** is the answer we need.

As illustrated in Fig. 3, suppose a group of professors and students arrived individually. The professors arrived between 1 o'clock and 2 o'clock, while the students between 3:30 and 4:30. The arrival times of professors and students can be considered two **intervals** – two **convex sets** of time points – that mark two (ranges of) **positions** on a scale of time. The **distance** (or difference) between these two not-very-precise positions can be as short as 1.5 hours and as long as 3.5 hours. Thus, just like we can use *between 6 feet and 6 feet 2 inches* – a range of degrees – to address the height of a certain person, *between 1.5 and 3 hours* – a range of time differences – provides the information to address to what extent the professors arrived earlier than the students did. As sketched out in

brings depression. Moreover, it brings chaos. The use of *moreover* means that chaos is added on top of depression.

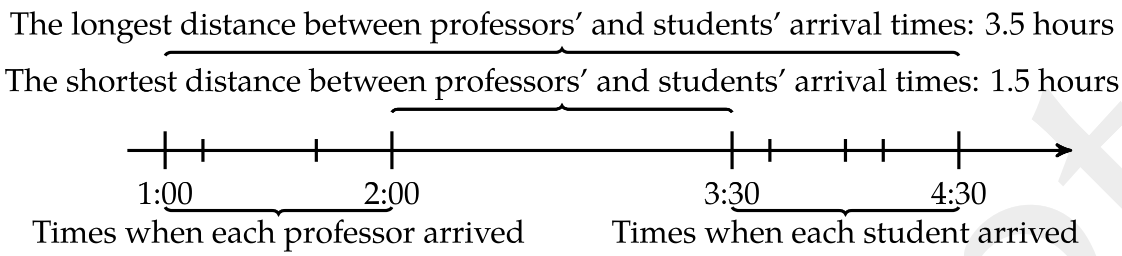

Figure 3: To what extent did the professors arrive earlier than the students did? The distance between the two positions representing the professors' and the students' arrival times can be as short as 1.5 hours and as long as 3.5 hours.

(26), the subtraction between two intervals (that mark positions on a scale of time) results in **a third interval** representing the time difference between the two positions.

(26) Given the context in Fig. 3, to what extent did the professors arrive earlier?
The professors arrived K-**earlier** than the students did.
$\underbrace{\textbf{the interval between 3:30 and 4:30}}_{\text{times when each student arrived}} - \underbrace{\textbf{the interval between 1:00 and 2:00}}_{\text{times when each professor arrived}} =$
$\underbrace{K\textbf{: the interval between 1.5 and 3.5 hours}}_{\text{to what extent did the professors arrive earlier}}$

The computation of this third interval – K in (26) – relies on the information of both the upper and lower bounds of the two intervals representing positions. The lower bound of K is the difference between the last professor's and the first student's arrival times (i.e., 2:00 and 3:30). The upper bound of K is the difference between the first professor's and the last student's arrival times (i.e., 1:00 and 4:30). Thus, in the conduction of comparison, intervals representing positions are not compressed into size-less items. Instead, their endpoint information is made use of and gets projected.

As we will show with details later, interval subtraction provides a generalized implementation for comparing two scalar values, and comparatives involving all kinds of numerical differentials (e.g., (17) and (23)) are analyzed in a natural and principled way.

2.3.2 Comparative morpheme *-er/more* as an additive particle

Based on the idea that the core semantic contribution of *-er*/*more* is **additivity**, we analyze *-er*/*more* as an **additive particle** similar to words like *other* or *another*. In the domain of intervals $D_{\langle dt \rangle}$, *-er*/*more* **asserts an increase** (of type $\langle dt \rangle$) **on a contextually salient scalar value** (of interval type $\langle dt \rangle$), just like in the domain of entities D_e, (*a*)*other*

asserts the existence of some entity (of type e) **in addition to a contextually salient one** (of type e). (27) shows the parallelism between the domains of entities and intervals.[8]

(27) The parallelism between the domains of entities and intervals

Domain	Indefinites	Definites	Additive words	Additivity+Restriction
D_e	*someone*	*Mary*	*(a)other*	*another girl, Mary*
$D_{\langle dt \rangle}$	*some (amount)*	*3 feet*	*-er / more*	*3 feet ... -er / more*

(28) shows existential assertions conveyed by the use of indefinites, which introduce a non-specified entity or scalar value.[9] (29) and (30) show that *another* and *-er / more* bring additivity. The contextually salient entity/value serving as the base for additivity can but does not necessarily occur in the same sentence as additive items do. In (29), the base for additivity (here *Mary* and *between 3 and 4 feet*) occurs in a previous sentence to the one hosting additive particles (here *another* and *-er*). In (30), the base for additivity (here *a girl* and the *than*-clause) and additive words occur in the same sentence. (30) also shows that additive expressions like *another girl* and *taller* can be further restricted.

(28) **Indefinites**: *someone* vs. *some* (*amount*)

a. Mary saw **someone**.

b. The height of these triangles differs from those by **some amount**.

(29) **Definites and additive items**: *Mary* vs. *between 3 and 4 feet*; *another* vs. *-er / more*

a. **Mary** is my friend. I have **another friend**.

b. This triangle is **between 3 and 4 feet** tall. That triangle is **taller**.

(30) **Additivity+Restriction**: *another girl*, *Junko* vs. *2 feet ... -er*

a. A girl, Hanako, saw **another girl, Junko**.

b. This triangle is **2 feet taller** than that triangle is.

[8] Since *-er / more* is used in both comparatives and additive constructions, its contribution of increase should be in both domains of entities and scalar values. We focus on its role as an increase in $D_{\langle dt \rangle}$, but include examples of additive constructions to illustrate its domain-general contribution of additivity.

[9] The notion of discourse salience for scalar values is also parallel to that in the domain of individuals. The introduction of a scalar value as discourse referent picks out some scalar value (from the immense set of scalar values) and grants it discourse salience.

The introduction of a scalar value as discourse referent does not necessarily hinge on an individual (i.e., introducing a scalar value as the measurement of some individual). In a sentence like *John is taller than 6 feet*, *6 feet* is introduced directly as a discourse-salient value. Actually, we consider $[\![\text{6 feet}]\!]$ parallel to a definite description, e.g., *the sun* (in *Everyone saw the sun*, see also the table in (27)).

Additivity is not a typical kind of presupposition. Though *another* and *-er/more* pass the classical tests for presuppositional triggers (see (31) and (32)), the base item for additivity is not always **pre**supposed in a discourse (see (30)). Moreover, a sentence like *Mary is taller* is not felicitous out of blue, though its presupposition (i.e., there is a certain height) can be easily accommodated (see Kripke 2009's discussion on additive *too*).[10]

(31) **Tests of projection**

a. It is possible that **another** girl came.

b. It is possible that **more** alcohol was consumed. **Additive construction**

c. It is possible that Sue is tall**er**. **Comparative**

(32) **Tests of local satisfaction**

a. Either Mary was not there, or **another** linguist gave the talk on comparatives.

b. Either they didn't even have a beer, or **more** alcohol was consumed. **Additive construction**

c. Either Sue is not even 5 feet tall, or she is tall**er**. **Comparative**

Following Beaver and Clark (2009)'s theory on anaphoricity and Thomas (2011)'s analysis of *another*, we consider additivity a phenomenon of QUD-based **anaphoricity** (Question Under Discussion, see Roberts 1996, Büring 2003, Zeevat 2004, Zeevat and Jasinskaja 2007). *-er/more* is an anaphora to a QUD and requires that there is a **discourse-salient, positive, non-overlap partial answer to the Current Question**. This requirement can be satisfied by accommodation, antecedents, or *than*-expressions.

As sketched out in (33) and (34), for additive constructions, *-er/more* is associated with the difference between the complete answer to the Current Question and a discourse-salient partial answer. For comparatives, *-er/more* denotes the difference between the total value addressing the Current Question and a discourse-salient value.

[10] We thank an anonymous reviewer for pointing out that (30b) challenges the presuppositional view for *-er/more*. This reviewer also asks whether felicitous comparative *Mary is taller* contains an elided *than*-clause. Analogous examples involving additive particles in (i) suggest that the role of a *than*-clause is more similar to an antecedent (i.e., the underlined part) than to an ellipsis (i.e., the stricken-through part). Elided content is irrelevant to the requirement of additive particles, but the meaning of a *than*-clause can satisfy the felicity condition of *-er/more*. Therefore, we do not pursue an ellipsis analysis for *Mary is taller*.

(i) a. (I saw a cat.) She saw **another** ~~one~~.

b. (Kate will come.) Jane will ~~come~~, **too**.

c. (This door is only 5 feet tall.) Mary is tall**er**.

Without a discourse-salient value to satisfy the requirement of *-er/more*, comparatives like *Mary is taller* would sound weird out of blue.

(33) Current Question: **What** happened? **Additive constructions**

a. **Something more** happened. (Something that is salient happened.)
$\rightsquigarrow$ something more = 'what happened' minus 'something that is salient'

b. **Something more** happened than what they knew.
$\rightsquigarrow$ something more = 'what happened' minus 'what they knew'

(34) Current Question: **How tall** is Mary? **Comparatives**

a. Mary is tall**er**. (There is a salient height value.)
$\rightsquigarrow$ $[\![\text{-er}]\!]$ = 'how tall Mary is' minus 'the salient height value'

b. Mary is tall**er** than 6 feet.
$\rightsquigarrow$ $[\![\text{-er}]\!]$ = 'how tall Mary is' minus '6 feet'

The non-overlap requirement is illustrated by (35) and (36). For (35), the two joint papers by Mary and Sue provide the salient partial answer, and *more* is associated with the one single-authored book by Mary. For (36), the height value $19'10''$ serves as the salient base value, and *-er* is associated with the difference, i.e., 2 inches. Thus, for both additive and comparative constructions, there cannot be overlap between (i) the entity or value serving as the base and and (ii) the additional part. This non-overlap requirement supports the use of subtraction equations to characterize the relation among (i) the base for an increase, (ii) the increase, and (iii) the complete answer to the Current Question.

(35) Context: Mary published a book. Mary and Sue published two papers together.
Current Question: What did Mary publish?

a. (Mary and Sue published two papers.) Mary had **one more** publication.

b. #(Mary and Sue published two papers.) Mary had **three more** publications.

(36) Context: This tree is 19 feet 10 inches tall. My giraffe is 20 feet tall.
Current Question: How tall is my giraffe?

a. (i) My giraffe is **2 inches taller** than this tree is.
(ii) (This tree is 19 feet 10 inches tall.) My giraffe is **2 inches taller**.

b. (i) #My giraffe is **20 feet taller** than this tree is.
(ii) #(This tree is 19 feet 10 inches tall.) My giraffe is **20 inches taller**.

To sum up, we propose (37) as the lexical entry of *-er/more* in comparatives. *-er/more* denotes the most general positive scalar value, i.e., the interval $\{d \mid d > 0\}$, and for felicitous uses, it requires that there is a salient scalar value serving as the base for an increase, providing discourse-salient partial information to a Current (Degree) Question. Thus, *-er/more* serves as the default differential in comparatives. This proposal captures the additivity (and anaphoricity) of *-er/more* within the domain of intervals.

(37) $[\![\text{-er/more}]\!]_{\langle dt\rangle} \stackrel{\text{def}}{=} \{d \mid d > 0\}$ (i.e., the most general positive interval)
Requirement: there is a salient scalar value serving as the base for an increase.

(37) is distinct from Schwarzchild and Wilkinson (2002)'s default differential SOME in two crucial ways. In (37), *-er/more* denotes an interval, instead of a single number, and the role of *-er/more* in comparatives is built on its discourse-level semantic contribution.

2.4 An informal sketch of our proposal

Our proposal consists of three core components:[11] (i) using intervals to represent all scalar values and analyzing a gradable adjective as a relation between an interval and an individual (see (38)); (ii) analyzing the matrix and *than*-clauses as definite descriptions of intervals (with the abstraction over an interval variable and an informativity-based maximality operator – Beck 2010's operator M_{inf} shown in (19), see (39)); (iii) using interval subtraction to implement comparison between definite intervals (see (40)).

(38) $[\![\text{tall}]\!]_{\langle dt,et\rangle} \stackrel{\text{def}}{=} \lambda I_{\langle dt\rangle}.\lambda x_e.\textsc{height}_{\langle e,dt\rangle}(x) \subseteq I$ (cf. (5), (12), (18))
(I.e., the height of x is an interval that is a subset of interval I.)

(39) Mary is taller than every boy is. (cf. (11), (14), (19))
a. ***than*-clause**: $\text{M}_{\text{inf}}(\lambda I.\forall x[\text{boy}(x) \rightarrow \textsc{height}(x) \subseteq I])$
b. **matrix clause**: $\text{M}_{\text{inf}}(\lambda I.\textsc{height}(\text{Mary}) \subseteq I)$

(40) [[Mary is (up to 3 inches) taller than every boy is.]] (cf. (16)/(17), (23))
$\Leftrightarrow [\![\textsc{matrix-clause}]\!] - [\![\textsc{than-clause}]\!] = [\![\text{up to 3 inches} \ldots\text{-er}]\!]$
a. The lower bound of $[[\![\textsc{matrix-clause}]\!] - [\![\textsc{than-clause}]\!]]$: basically the difference between Mary's height and the height of the tallest boy(s);

[11] Again, compositional orders and technical details are ignored here. The main point of this subsection is to show how our proposal inherits and improves on the predecessors.

The upper bound of [[[MATRIX-CLAUSE]] – [[THAN-CLAUSE]]]: basically the difference between Mary's height and the height of the shortest boy(s).

b. [[up to 3 inches …-er]]
$= [\![\text{-er}]\!] \cap [\![\text{up to 3 inches}]\!] = \{d \mid d > 0\} \cap \{d \mid d \leq 3''\} = \{d \mid 0 < d \leq 3''\}$

The first two of these three components are in the same spirit as those of the canonical analysis, but with an interval-based implementation similar to the approaches adopted by Schwarzchild and Wilkinson (2002) and Beck (2010). The third component combines our two innovations: the technique of interval subtraction and an additivity-based view of *-er*/*more*. Below we address the details of our proposal.

3 The semantics of comparatives

This section first presents the technical details of interval subtraction. Then we show the formal implementation of our proposed analysis step by step for the simplest cases.[12]

3.1 The technique of interval subtraction

3.1.1 The definition and notation of intervals

Degrees are points on an interval scale. Thus, a **scale** is a totally ordered set of degrees (e.g., the set of real numbers $\mathbb{R}$ is a scale). **Intervals** are **convex** subsets of a scale. According to the definition of convex sets (see (41)), sets such as $\{x \mid x > 0\}$, $\{x \mid x \leq 4\}$, and $\{x \mid 4 \leq x \leq 8\}$ are all convex sets, while sets like $\{x \mid x > 10 \vee x \leq 3\}$ are not convex. Degrees are of type d, and thus intervals are of type $\langle dt \rangle$.

(41) **The definition of a convex set**:
A totally ordered set P is **convex** iff for any elements a and b in the set (suppose $a \leq b$), any element x such that $a \leq x \leq b$ is also in the set P.

Since intervals are convex sets of degrees, we can rewrite an interval with its lower and upper bounds. As shown in (42), we use square brackets '[' and ']' for **closed** lower and upper bounds and round parentheses '(' and ')' for **open** lower and upper bounds.

(42) Interval notation:

[12] Readers who are familiar with interval arithmetic can skip Section 3.1.

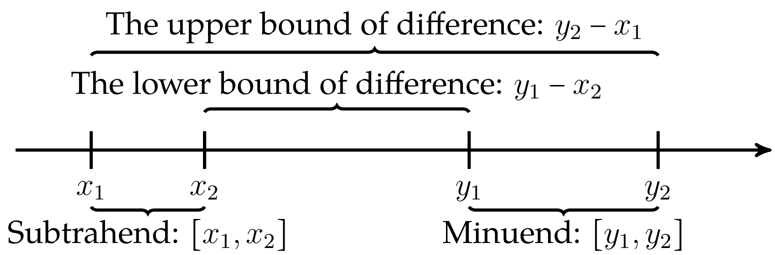

Figure 4: The subtraction between two intervals. Here $[y_1, y_2]$ means the minuend, $[x_1, x_2]$ the subtrahend, and the difference between these two intervals is the largest range of possible differences between any two random points in these two intervals, i.e., $[y_1 - x_2, y_2 - x_1]$.

$$\{x \mid I_{\text{min}} \leq x \leq I_{\text{max}}\} = [I_{\text{min}}, I_{\text{max}}] \qquad \text{A left- and right-closed interval}$$
$$\{x \mid I_{\text{min}} < x \leq I_{\text{max}}\} = (I_{\text{min}}, I_{\text{max}}] \qquad \text{A left-open and right-closed interval}$$
$$\{x \mid I_{\text{min}} \leq x < I_{\text{max}}\} = [I_{\text{min}}, I_{\text{max}}) \qquad \text{A left-closed and right-open interval}$$
$$\{x \mid I_{\text{min}} < x < I_{\text{max}}\} = (I_{\text{min}}, I_{\text{max}}) \qquad \text{A left- and right-open interval}$$

A singleton set like $\{x \mid x = 3''\}$ can be written as $[3'', 3'']$, the lower and upper bounds of which are equal. We write positive and negative infinity as '$+\infty$' and '$-\infty$'. Thus an interval like $\{x \mid x \geq 4\}$ (i.e., a **left-bounded and right-unbounded** interval) can be written as $[4, +\infty)$, and an interval like $\{x \mid x < 3\}$ (i.e., a **left-unbounded and right-bounded** interval) can be written as $(-\infty, 3)$.

3.1.2 Details of interval subtraction

An interval means a range of possible values of degrees. Applying an operation on two intervals results in a third interval that represents the largest possible range of values.[13] As shown in (43) and Fig. 4, the result of **subtraction**, i.e., the **difference**, is the largest range of possible differences between any two random points in two intervals.

(43) **Interval subtraction**: (see Moore 1979)

[13] To help reason about the notion of intervals, (i) shows a general recipe on the operations (e.g., addition, subtraction, multiplication) between intervals. The results are defined in terms of their upper and lower bounds. The operations can be extendable to cases with unbounded and/or open endpoints.

(i) Basic interval operations (see Moore 1979):
$[x_1, x_2]\langle\text{op}\rangle[y_1, y_2] = [\alpha, \beta]$
The lower bound $\alpha = \textsc{min}(x_1\langle\text{op}\rangle y_1, x_1\langle\text{op}\rangle y_2, x_2\langle\text{op}\rangle y_1, x_2\langle\text{op}\rangle y_2)$
The upper bound $\beta = \textsc{max}(x_1\langle\text{op}\rangle y_1, x_1\langle\text{op}\rangle y_2, x_2\langle\text{op}\rangle y_1, x_2\langle\text{op}\rangle y_2)$

$$\underbrace{[y_1, y_2]}_{\textbf{minuend: matrix subject's measurement}} - \underbrace{[x_1, x_2]}_{\textbf{subtrahend: comparative standard}} = \underbrace{[y_1 - x_2, y_2 - x_1]}_{\textbf{difference: differential}}$$

a. Example 1: $[5, 8] - [1, 3] = [2, 7]$ (2 and 7 are the minimum and maximum distances between the positions $[5, 8]$ and $[1, 3]$ respectively.)

b. Example 2: $(4, +\infty) - [2, 3] = (1, +\infty)$ (This subtraction operation can be generalized to intervals with open and/or unbounded ends.)

The subtraction between two intervals results in a third interval, but as mentioned before, these three intervals are not of the same kind. In (43), the **minuend** and **subtrahend** intervals (i.e., $[y_1, y_2]$ and $[x_1, x_2]$) represent two not-very-precise **positions** on a scale (i.e., each position is in terms of a range), while the difference, i.e., $[y_1 - x_2, y_2 - x_1]$, represents the **distance** between the minuend and the subtrahend. For the positions $[y_1, y_2]$ and $[x_1, x_2]$ on a **base measurement scale**, the distance between them – $[y_1 - x_2, y_2 - x_1]$ – can be considered a measurement on a **scale of differences**.[14]

Some numerical examples of interval subtraction are given in (44):

(44) a. $[5, 8] - [1, 2] = [3, 7]$

b. $[5, 8] - [3, 7] = [-2, 5]$

c. $[1, 2] - [5, 8] = [-7, -3]$

As shown in (44a) and (44c), when the minuend and the subtrahend are flipped, applying subtraction results in the **inverse** of the original difference (see (45) for details). Thus, the **direction in applying subtraction** is reflected by the **polarity of difference**.

(45) Flipping the direction of subtraction: (see (43))

a. $[y_1, y_2] - [x_1, x_2] = [y_1 - x_2, y_2 - x_1]$

b. $[x_1, x_2] - [y_1, y_2] = [x_1 - y_2, x_2 - y_1] = [-(y_2 - x_1), -(y_1 - x_2)] = [0, 0] - [y_1 - x_2, y_2 - x_1]$

The examples (44a) and (44b) show a crucial difference between the operation of subtraction defined in **interval arithmetic** and **number arithmetic**. In number arithmetic (i.e., when X, Y and Z represent numbers), if $X - Y = Z$, it follows necessarily that $X - Z = Y$ (see (46a)). However, in interval arithmetic (i.e., when X, Y and Z represent intervals), if $X - Y = Z$, generally speaking, it is not the case that $X - Z = Y$ (see (46b)).

[14] The conceptual distinction between interval-as-position vs. interval-as-distance is more visible in the dimension of time: e.g., [1:00, 1:30] and [4:00, 4:30] are intervals-as-position on a scale of time, while [2.5 hours, 3.5 hours] is an interval-as-distance between the above two intervals-as-positions (cf. $[5°C, 10°C]$ is ambiguous between (i) an interval-as-temperature and (ii) an interval-as-temperature-difference.)

(46) a. Number arithmetic: $X - Y = Z \vDash X - Z = Y$ (e.g., $5 - 2 = 3 \vDash 5 - 3 = 2$)

b. Interval arithmetic: $X - Y = Z \nvDash X - Z = Y$ (see (44a) vs. (44b))

Consequently, in interval arithmetic, given $X - Y = Z$ and given the values of the subtrahend Y and the difference Z, to compute the value of the minuend X, we cannot perform interval addition on Y and Z (see (47)).

(47) If $X - [a, b] = [c, d]$, then generally speaking, $X \neq [a + c, b + d]$.

Instead, we need to follow the formula (43) to derive the value of the minuend. As shown in (48), the minuend X is defined only when its lower bound does not exceed its upper bound. When the minuend is defined, as shown in (48), the **upper** bound of the **subtrahend** (here b) contributes to the computation of the **lower** bound of the **minuend** X, while the **lower** bound of the **subtrahend** (here a) contributes to the computation of the **upper** bound of the **minuend** X.

(48) If $X - [a, b] = [c, d]$,

a. X is undefined when $b + c > a + d$;
(i.e., undefined when the lower bound of X exceeds the upper bound of X.)

b. When defined, $X = [b + c, a + d]$. (see (43))
The **lower** bound of the **minuend** X
= the **lower** bound of the **difference** + the **upper** bound of the **subtrahend**;
($b + c$ meanings moving from the precise position b by a distance of c.)
the **upper** bound of the **minuend** X
= the **upper** bound of the **difference** + the **lower** bound of the **subtrahend**.
($a + d$ meanings moving from the precise position a by a distance of d.)

With the use of interval subtraction, we can now characterize **a generalized comparison between two not-very-precise positions on a scale** and precisely compute the distance (i.e., difference) between them. In particular, inequalities are represented by subtraction equations, and information with regard to the endpoints of positions and distances – including values, closedness, and boundedness – is fully taken care of with the use of this technique. Thus, interval subtraction is an ideal tool for compositionally deriving the semantics of various kinds of comparatives, especially for those complex cases involving numeral differentials and/or *than*-clause internal quantifiers.

3.2 The step-by-step derivation for the simplest cases of comparatives

Step 0: The semantics of measure function. We use intervals – ranges of values – to represent scalar values in a generalized way. A **measure function** maps a **single entity** to an interval, which represents the position corresponding to the measurement of the entity along a relevant scale (see (49)). Measurements are always subject to uncertainty. An informative interpretation of a measure function involves vagueness.

(49) **Measure function**: $\textsc{height}_{\langle e,dt\rangle} \stackrel{\text{def}}{=} \lambda x.\textsc{height}(x)$

For a given entity, what exact position range on a scale of height corresponds to its height measurement depends on contextual factors, such as measurement tools, environment, acceptable criteria of precision, etc. For example, vernier scales provide better precision in measuring along a linear scale than most rulers do. The notion of comparison class (i.e., 'objects deemed somehow similar to the target of predication', Kennedy 2011: Section 3.1, p. 514) is often relevant to contextually informative precision level of measurement. The precision level to 1 meter is fine-grained and informative in addressing the height of mountains, but way too coarse-grained for humans.

Suppose we use a scale to measure the height of my giraffe. Along this scale, the closest marking to the top of my giraffe is 20 feet with an error range of 1 foot. Then $\textsc{height}$(my-giraffe) is $20' \pm 1'$, i.e., $[19', 21']$. With idealized measurement in which the error is negligible, the interval $\textsc{height}$(my-giraffe) is a singleton set of degrees, and we write its unique item (of type d) as $\textsc{precise-height}$(my-giraffe).

Step 1: The analysis of gradable adjectives. We analyze the semantics of a gradable adjective as a relation between an individual x and an interval I (see (50)), meaning that the measurement of x falls at the position I on a scale associated with the dimension of the adjective (e.g., *tall* and *short* are associated with the same dimension of height, but with scales of opposite orderings; *early* and *late* are associated with time, but with scales of opposite orderings as well). This relational view of gradable adjectives inherits the spirit of the canonical analysis (see Section 2.1, cf. Kennedy 1999).[15]

[15] Since a measure function measures a single entity (see Step 0), the individual variable of a gradable adjective should not be a plurality. For a plurality, we assume that there is a distributivity operator $\textsc{dist}$:

(i) $\textsc{Dist} \stackrel{\text{def}}{=} \lambda X_e.\lambda P_{\langle et\rangle}.\forall x[x \sqsubseteq_{\textsc{atom}} X \to P(x)]$
i.e., for each atomic part x of the plural individual X, predicate P holds for x.
e.g., $[\![$the trees are $\textsc{Dist}$ I tall$]\!] = \forall x[x \sqsubseteq_{\textsc{atom}} \oplus\text{tree} \to \textsc{height}(x) \subseteq I]$

(50) $[\![\text{tall}]\!]_{\langle dt,et\rangle} \overset{\text{def}}{=} \lambda I_{\langle dt\rangle}.\lambda x_e.\textsc{height}_{\langle e,dt\rangle}(x) \subseteq I$ (= (38))
i.e., the measurement of x falls at the position I on the scale of height.

The semantics of **measurement constructions** is straightforwardly derived (see (51)). Bare numerals like *19 feet* are ambiguous between an 'exactly' reading and an 'at least reading' (see Spector 2013 for a review on this issue). The projection of this ambiguity leads to the two interpretations shown in (51a).[16] Modified numerals like *between 19 and 20 feet* naturally denote an interval and serve as the interval argument of *tall* (see (51b)).

(51) **Measurement constructions**

a. My giraffe is **19 feet** tall.
LF: [[my giraffe] is [[19 feet] tall]]
(i) The 'exactly' reading of *19 feet*: $[\![(51a)]\!] \Leftrightarrow \textsc{height}(\text{my-giraffe}) \subseteq [19', 19']$
(ii) The 'at least' reading of *19 feet*: $[\![(51a)]\!] \Leftrightarrow \textsc{height}(\text{my-giraffe}) \subseteq [19', +\infty)$

b. My giraffe is **between 19 and 20 feet** tall.
LF: [[my giraffe] is [[between 19 and 20 feet] tall]]
$[\![(51b)]\!] \Leftrightarrow \textsc{height}(\text{my-giraffe}) \subseteq [19', 20']$

The **positive** use of gradable adjectives assumes a silent free interval variable I^C_{POS} (see (52)). I^C_{POS} denotes the context-dependent interval of being tall for a relevant comparison class (see also Bartsch and Vennemann 1972a, Cresswell 1976, von Stechow 1984, Bierwisch 1989, Kennedy 1999), e.g., above 18 feet for a giraffe.[17]

(52) My giraffe is tall. **Positive use**
LF: [[my giraffe] is [I^C_{POS} tall]]
$[\![(51a)]\!] \Leftrightarrow \textsc{height}(\text{my-giraffe}) \subseteq I^C_{\text{POS}}$

Step 2: The analysis of comparative standard. *Than*-clauses/phrases play the role of **standard** (i.e., subtrahend) in comparatives. We focus on the semantics of *than*-clauses.

[16] When the 'at least' reading is adopted for interpreting a bare numeral in a measurement construction, obviously, the analysis in (51) captures the following familiar inference pattern:

(i) My giraffe is 19 feet tall. ⊨ My giraffe is 18 feet tall.
$\textsc{height}(\text{my-giraffe}) \subseteq [19', +\infty) \vDash \textsc{height}(\text{my-giraffe}) \subseteq [18', +\infty)$

[17] Formal properties (especially the boundedness) of this I^C_{POS} are subject to the structure of a scale associated with a gradable predicate (see Kennedy and McNally 2005).

The analysis of *than*-clauses involves two sub-steps: (i) lambda abstraction over an interval variable, and (ii) the use of an informativeness-based maximality operator.

Following the canonical analysis (see Bresnan 1973, 1975, Chomsky 1977), we assume that syntactically, a *than*-clause contains an elided gradable adjective – the same as the one used in the matrix clause – and a *wh*-movement (see (53)). Semantically, this amounts to a lambda abstraction over an interval variable, resulting in a set of intervals such that each represents a position where the measurement of an individual falls at.

(53) (My giraffe is taller) than that tree is ~~tall~~
LF: [than [λI. that tree is I tall]]

We propose that *than* contributes an informativeness-based maximality operator, similar to the operator M_{inf} proposed by Beck (2010) (see (19)). As shown in (54), for a set of intervals, [[than]] is defined when there is a unique interval entailing all other intervals in the set, and when defined, [[than]] returns this unique maximally informative interval.

(54) [[than]] is defined for a set of intervals p such that
$\exists I[p(I) \wedge \forall I'[[p(I') \wedge I' \neq I] \rightarrow I \subset I']]$
When defined, $[\![\text{than}]\!]_{\langle\langle dt,t\rangle,dt\rangle} \stackrel{\text{def}}{=} \lambda p_{\langle dt,t\rangle}.\iota I[p(I) \wedge \forall I'[[p(I') \wedge I' \neq I] \rightarrow I \subset I']]$

A *than*-clause is semantically the same as a free relative, which looks like a *wh*-clause but functions as a nominal phrase bearing definiteness (see Bresnan and Grimshaw 1978, Jacobson 1995, Caponigro 2003).[18] The semantic derivation of a *than*-clause (or free relative in general) can also be considered involving (i) the formation of a degree question under the categorial approach to questions (see Hausser and Zaefferer 1978 and Krifka 2011 for a review on question semantics), e.g., *how tall is that tree* for (53), and (ii) the generation of its fragment answer (i.e., short answer, see Chierchia and Caponigro 2013). Thus a *than*-clause is essentially a definite description of scalar value (see Russell 1905), providing a most informative, exhaustive answer to a degree question.[19]

[18] Actually all English words starting with *th* (pronounced as /ð/, not as /θ/) express definiteness: e.g., *the*, *they*, *that*, *then*, *there*, *these*, *thus*, *though* (which means 'in spite of the fact that' according to its dictionary definition, *Merriam-Webster's Collegiate Dictionary*, 11[e]). It is reasonable to assume that *than* contributes definiteness as well. A thorough investigation of definiteness and these ð-words is for another occasion.

[19] The fragment-answer view for free relatives is empirically advantageous, directly accounting for parallels between *wh*-questions and their answerhood on the one hand, and free relatives (including *than*-clauses) on the other hand. For example, just like its corresponding *wh*-question, free relative *where he can buy a coffee* has a mention-some interpretation (see Chierchia and Caponigro 2013). For comparatives containing a permission-related existential modal in their *than*-clause (e.g., *Lucinda is driving less fast than allowed*, see Beck 2013), their ambiguity is also likely rooted in the ambiguous answerhood for corresponding degree ques-

Specifically, our analysis means that given an entity or a group of entities as the target of predication, a *than*-clause denotes the most informative interval that the measurement of each entity falls into. For simplicity, assume that measurement yields very precise values, i.e., singleton sets of degrees. Then when the target of predication is a single entity, the meaning of a *than*-clause is equivalent to a singleton set of degrees, which is simply the measurement of the given entity (see (55a)). When the target of predication is a group of entities, the derived meaning of a *than*-clause is an interval ranging from the measurement of the least ADJ entity to the most ADJ one, e.g., a height interval ranging from that of the shortest to the tallest tree in (55b).[20]

(55) a. $[\![$than that tree is ~~tall~~$]\!]$ = $\textsc{height}$(that-tree)
$= [\textsc{precise-height}(\text{that-tree}), \textsc{precise-height}(\text{that-tree})]$

b. $[\![$than every tree is ~~tall~~$]\!]$ = $\iota I[\forall x[\text{tree}(x) \to \textsc{height}(x) \subseteq I]]$
$= [\textsc{precise-height}(\text{shortest-tree}), \textsc{precise-height}(\text{tallest-tree})]$

Step 3: The analysis of differentials. Comparative morpheme *-er*/*more* is considered the default positive differential. Thus, as shown in (56), it denotes the most general positive interval: $(0, +\infty)$. We will address numerical differentials in Section 4.

(56) $[\![\text{-er/more}]\!]_{\langle dt \rangle} \overset{\text{def}}{=} (0, +\infty)$ i.e., the most general positive interval (= (37))
Requirement: there is a salient scalar value serving as the base for an increase.

Step 4: The semantic derivation of comparatives. Interval subtraction is performed by a silent operator MINUS. It takes two intervals as inputs: the subtrahend, I_{stdd}, and the difference, I_{diff}. The output is the unique interval I representing the minuend (see (57)).

(57) $[\![\textsc{minus}]\!]_{\langle dt, \langle dt, dt \rangle\rangle} \overset{\text{def}}{=} \lambda I_{\textsc{stdd}}.\lambda I_{\textsc{diff}}.\iota I[I - I_{\textsc{stdd}} = I_{\textsc{diff}}]$

Now we are ready to derive the sentential semantics of a *more-than* comparative that contains no numerical differential. As shown by the LF in (58), at the matrix level, $[\![$tall$]\!]$

tions (e.g., *how fast is Lucinda allowed to drive*). A thorough investigation of this phenomenon is for another occasion (see our other works). There is also parallelism between ungrammatical degree question **how tall is no one?* and ungrammatical clausal comparative **Mary is taller than no one is*. Presumably, there is no non-trivial informative answer to address 'no one's height' in either case (see Abrusán 2014).

Our view is slightly distinct from Fleisher (2018, 2019), which analyze a *than*-clause as a degree question.

[20] We will continue making this assumption for simplicity below. Without this assumption, (55b) would be an interval starting from the lower bound of HEIGHT(shortest-tree) to the upper bound of HEIGHT(tallest-tree).

relates an entity, $[\![$my giraffe$]\!]$, and an interval – the minuend. The minuend is computed from a subtraction equation and known interval values for I_{DIFF} and I_{STDD}.

The interval I_{STDD} (i.e., comparative standard) represents the subtrahend in the equation and is contributed by the semantics of the *than*-clause – the height of that tree in this example (see (58a)). The interval I_{DIFF} represents the difference in the equation and is contributed by $[\![$-er$]\!]$ (see (58b)). Thus based on the intervals I_{STDD} and I_{DIFF}, the minuend – the interval serving as the interval variable for $[\![$tall$]\!]$ at the matrix level – can be computed (see (58c)). Finally, in (58d), with the assumption for an ideally precise measurement (i.e., the height of that tree is a singleton set of degrees) and the application of interval arithmetic (see (48)), the formula can be simplified: the lower bound of the minuend results from the addition of the upper bound of I_{STDD} and the lower bound of I_{DIFF}, while the upper bound of the minuend results from the addition of the lower bound of I_{STDD} and the upper bound of I_{DIFF}. Eventually, sentence (58) means that the height of my giraffe falls within the interval starting from the height of that tree, i.e., the height of my giraffe exceeds that of that tree.

(58) My giraffe is taller than that tree is.

$$\text{LF: [[my giraffe] is } \underbrace{[\ [\ \underbrace{\text{-er}}_{\textbf{difference: } I_{\text{DIFF}}}\ \ \textsc{minus}\ \underbrace{\text{than } [\lambda I.\ \text{that tree is } I \text{ (tall)}]}_{\textbf{subtrahend: } I_{\text{STDD}}}\ ]}_{\textbf{minuend: } \iota I'[I' - I_{\text{STDD}} = I_{\text{DIFF}}]} \text{ tall]]}$$

a. **Subtrahend**: $I_{\text{STDD}} = [\![\text{than}]\!][\![\lambda I.\text{that tree is } I \text{ (tall)}]\!]$
$= \iota I[\textsc{height}(\text{that-tree}) \subseteq I] = \textsc{height}(\text{that-tree})$
$= [\textsc{precise-height}(\text{that-tree}), \textsc{precise-height}(\text{that-tree})]$

b. **Difference**: $I_{\text{DIFF}} = [\![\text{-er}]\!] = (0, +\infty)$

c. **Minuend**: $\iota I'[I' - I_{\text{STDD}} = I_{\text{DIFF}}]$

d. $[\![(58)]\!] \Leftrightarrow \textsc{height}(\text{my-giraffe}) \subseteq \iota I'[I' - I_{\text{STDD}} = I_{\text{DIFF}}]$
$\Leftrightarrow \textsc{height}(\text{my-giraffe}) \subseteq \iota I'[I' - \textsc{height}(\text{that-tree}) = (0, +\infty)]$
$\Leftrightarrow \textsc{height}(\text{my-giraffe}) \subseteq$
$\iota I'[I' - [\textsc{precise-height}(\text{that-tree}), \textsc{precise-height}(\text{that-tree})] = (0, +\infty)]$
$\Leftrightarrow \textsc{height}(\text{my-giraffe}) \subseteq (\textsc{precise-height}(\text{that-tree}), +\infty)$ (see (48))

As illustrated by (59), when a *than*-clause contains a universal quantifier (here *every tree*), the derivation of the sentential semantics is exactly the same as that shown in (58), except that I_{STDD} is not a singleton set of degrees in this case, but an interval ranging from the height of the shortest tree(s) to that of the tallest. Eventually, after simplification with the recipe of (48) (see the last step of (59d)), we arrive at the truth condition consistent with our intuition: the height of my giraffe exceeds that of the tallest tree(s).

(59) My giraffe is taller than every tree is.

LF: $[\ [\text{my giraffe}]\ \text{is}\ [\ \underbrace{[\ \underbrace{\text{-er}}_{\textbf{difference: } I_{\text{DIFF}}}\ \ \textsc{minus}\ \ \underbrace{\text{than}\ [\lambda I.\ \text{every tree is}\ I\ (\text{tall})]}_{\textbf{subtrahend: } I_{\text{stdd}}}\ ]}_{\textbf{minuend: } \iota I'[I' - I_{\text{stdd}} = I_{\text{diff}}]}\ \text{tall}\ ]\ ]$

a. **Subtrahend**: $I_{\text{STDD}} = [\![\text{than}]\!][\![\lambda I.\text{every tree is } I \text{ (tall)}]\!]$
$= \iota I[\forall x[\text{tree}(x) \to \textsc{height}(x) \subseteq I]]$
$= [\textsc{precise-height}(\text{shortest-tree}), \textsc{precise-height}(\text{tallest-tree})]$

b. **Difference**: $I_{\text{DIFF}} = [\![\text{-er}]\!] = (0, +\infty)$

c. **Minuend**: $\iota I'[I' - I_{\text{STDD}} = I_{\text{DIFF}}]$

d. $[\![(59)]\!] \Leftrightarrow \textsc{height}(\text{my-giraffe}) \subseteq \iota I'[I' - I_{\text{STDD}} = I_{\text{DIFF}}]$
$\Leftrightarrow \textsc{height}(\text{my-giraffe}) \subseteq \iota I'[I' - \iota I[\forall x[\text{tree}(x) \to \textsc{height}(x) \subseteq I]] = (0, +\infty)]$
$\Leftrightarrow \textsc{height}(\text{my-giraffe}) \subseteq$
$\iota I'[I' - [\textsc{precise-height}(\text{shortest-tree}), \textsc{precise-height}(\text{tallest-tree})] = (0, +\infty)]$
$\Leftrightarrow \textsc{height}(\text{my-giraffe}) \subseteq (\textsc{precise-height}(\text{tallest-tree}), +\infty)$ (see (48))

4 Comparatives with numerical differentials

This section addresses comparatives containing numerical differentials and *than*-clause-internal universal quantifiers. We aim to show how I_{STDD} that are non-singleton sets of degrees interact with I_{DIFF}, and how the endpoint information of these intervals projects to sentential semantics.[21] In particular, we propose an **interval inverse operator *little***, using it to account for the semantics of *less-than* comparatives and analyzing its distinctions from the familiar negation operator.

4.1 *More-than* comparatives with numerical differentials

Suppose that we compare the height of my giraffe with that of a certain group of trees. According to the context in (60), $[\![\text{than every tree is (tall)}]\!]$ is equivalent to $[18', 21']$.

[21] For this purpose, we only choose *than*-clauses containing universal quantifiers to illustrate semantic derivation. The analysis of comparatives containing other types of quantifiers in their *than*-clause often requires extra mechanisms. A full discussion is beyond the scope of this paper. The case of non-monotonic quantifiers in *than*-clauses (e.g., *Balloon A is higher than exactly two of the others are*, see Schwarzschild 2008) is analyzed in our other work.

(60) Context: the trees are between 18 and 21 feet tall.

I_{STDD} : $[\![$than every tree is (tall)$]\!]$

$= [\![\text{than}]\!][\![\lambda I.\text{every tree is } I \text{ tall}]\!]$

$= \iota I[\forall x[\text{tree}(x) \to \textsc{height}(x) \subseteq I]]$

$= [\textsc{precise-height}(\text{shortest-tree}), \textsc{precise-height}(\text{tallest-tree})] = [18', 21']$

The sentences in (61) and (62) contain **upward-entailing** (e.g., *at least 5 feet*), **downward-entailing** (e.g., *at most 5 feet*), or **non-monotonic** numerical differentials (e.g., *between 5 and 10 feet*), and differ with regard to the direction of inequalities (i.e., *more than* vs. *less than*). (63) sketches out their uniform LF under our analysis: these sentences differ only in terms of the value of I_{DIFF}.

(61) a. My giraffe is **at least 5 feet taller** than every tree is.
b. My giraffe is **at most 5 feet taller** than every tree is.
c. My giraffe is **between 5 and 10 feet taller** than every tree is.

(62) a. My giraffe is **at least 5 feet less tall** than every tree is.
b. My giraffe is **at most 5 feet less tall** than every tree is.
c. My giraffe is **between 5 and 10 feet less tall** than every tree is.

(63) LF for all the sentences in (61) and (62):

$$[\ [\text{my giraffe}]\ \text{is}\ [\left\{ \begin{array}{l} \text{at least 5 feet} \ldots\text{-er} \\ \text{at most 5 feet} \ldots\text{-er} \\ \text{between 5 and 10 feet} \ldots\text{-er} \\ \text{at least 5 feet less} \\ \text{at most 5 feet less} \\ \text{between 5 and 10 feet less} \end{array} \right\} \textsc{minus}\ \text{than}\ [\lambda I.\text{every tree is } I \text{ (tall)}\ ]\ ]\ \text{tall}\ ]$$

$[\![(61)/(62)]\!] \Leftrightarrow \textsc{height}(\text{my-giraffe}) \subseteq \iota I'[I' - \iota I[\forall x[\text{tree}(x) \to \textsc{height}(x) \subseteq I]] = I_{\text{DIFF}}]$

Under the context in (60),

$[\![(61)/(62)]\!] \Leftrightarrow \textsc{height}(\text{my-giraffe}) \subseteq \iota I'[I' - [18', 21'] = I_{\text{DIFF}}]$

Numerical differentials are analyzed as additional restrictions on the default positive differential $(0, +\infty)$, yielding a more restricted value for I_{DIFF}.[22]

[22]Similar ideas have been developed in the analysis of quantity words like *many*, *much*, *few*, and *little* by Rett (see Rett 2007, 2008, 2014, 2018): the core semantic contribution of these words is to modify and restrict

Given the values of I_{DIFF} and I_{STDD} (which is $[18', 21']$ under the context in (60)), we can always use the same recipe of interval subtraction (see (48)) to simplify the formula of the minuend and thus that of sentential semantics (see (64)–(66)).

(64) a. $I_{\text{DIFF}} = [\![\text{at least 5 feet} \ldots\text{-er}]\!] = [5', +\infty) \cap (0, +\infty) = [5', +\infty)$

b. **Minuend**: $\iota I'[I' - I_{\text{STDD}} = [\![\text{at least 5 feet} \ldots\text{-er}]\!]$
$= \iota I'[I' - [18', 21'] = [5', +\infty)] = [26', +\infty)$

(65) a. $I_{\text{DIFF}} = [\![\text{at most 5 feet} \ldots\text{-er}]\!] = (-\infty, 5'] \cap (0, +\infty) = (0, 5']$

b. **Minuend**: $\iota I'[I' - I_{\text{STDD}} = [\![\text{at most 5 feet} \ldots\text{-er}]\!]$
$= \iota I'[I' - [18', 21'] = (0, 5']] = (21', 23']$

(66) a. $I_{\text{DIFF}} = [\![\text{between 5 and 10 feet} \ldots\text{-er}]\!] = [5', 10'] \cap (0, +\infty) = [5', 10']$

b. **Minuend**: $\iota I'[I' - I_{\text{STDD}} = [\![\text{between 5 and 10 feet} \ldots\text{-er}]\!]$
$= \iota I'[I' - [18', 21'] = [5', 10']] = [26', 28']$

Our analysis brings interesting consequences on (i) the projection of the endpoint information of I_{DIFF} and (ii) the definedness for the minuend and sentential semantics.

The projection of the endpoint information of I_{DIFF}. In (64)–(66), since I_{STDD} has both closed and bounded lower and upper bounds, the minuend directly inherits the closedness and boundedness of I_{DIFF}. For example, if the differential is left-closed, left-bounded, and right-unbounded (see (64)), then so is the minuend.

This explains why comparatives with no numerical differential express a strict inequality – because their differential is $(0, +\infty)$ (i.e., with an open lower bound), while comparatives containing numerical differentials often express non-strict inequalities – because a restricted differential can have a closed lower bound.

This also naturally explains the two observations raised by Fleisher (2016). First, *more-than* comparatives with an upward-entailing numerical differential have a MAX-reading (see (64)), in the sense that only the upper bound of I_{STDD} seems to get projected to sentential level. Second, in contrast, those with a downward-entailing or non-monotonic numerical differential have a MIN-&-MAX-reading (see (65) and (66)), in the sense that both the upper and lower bounds of I_{STDD} get projected to sentential level.

an interval. In comparatives, *much* and *a little* can also be used to restrict the default differential $(0, +\infty)$, yielding expressions like *much taller*, *a little shorter*. A thorough analysis of these expressions needs to be based on a detailed investigation on quantity words and is thus beyond the scope of our paper.

Our analysis shows that for the cases of upward-entailing numerical differentials like (64), I_{DIFF} is right-unbounded, so that the sum of this upper bound and the lower bound of I_{STDD} is still '$+\infty$', giving the impression that only the upper bound of I_{STDD} is eventually reflected in the computation of the minuend and sentential semantics.[23]

The definedness for the minuend and sentential semantics. We define the **width** of an interval as the difference between its upper and lower bounds (see (67)).

(67) The **width** of an interval I is the difference between its upper and lower bounds.

In the semantic derivation of a comparative, the minuend needs to be well-defined: i.e., its lower bound needs to be lower than its upper bound (see (48)). Consequently, the definedness condition shown in (68) needs to be met (see (69) for a proof). This definedness condition explains our intuitive inference in understanding a comparative.

(68) **Definedness condition for the minuend**: I_{STDD} needs to be less wide than I_{DIFF}.

[23] An anonymous reviewer raises the issue that comparatives like (i) (or (61b)) seem to have a MIN-reading, in the sense that only the lower bound of I_{STDD} projects. For the sentence to be true, John's height cannot exceed that of the shortest girl by more than 6 inches – John can even be shorter than the girls are.

(i) John is at most six inches taller than every girl is.

Following Krifka (1999) (see Szabolcsi 2010 (Chapter 10) for a review), we consider *at least* and *at most* focus sensitive items: their interpretation can be structurally ambiguous. For example, we assume that *at most* turns a singleton set of degrees into a left-unbounded interval (see (ii)), which basically means creating an alternative set. When associated with a larger structure (see (iiia)), *at most* modifies the derived value of [[six inches -er than every girl is]]. When associated simply with a number (see (iiib)), *at most* is part of the numerical differential *at most six inches* and modifies [[six inches]], giving rise to a MIN-&-MAX-reading.

Then according to (i) the input requirement of *at most* (see (ii)) and (ii) the definedness condition for comparatives (see (68) in this subsection), the interpretation of (iiia) suggests that the girls are of the same height. Thus the seeming MIN-reading of (iiia) is actually also a MIN-&-MAX-reading, i.e., (iiia) is true if the height of John does not exceed the girls' height by more than 6 inches – he can even be shorter than the girls.

(ii) $[\![\text{at most}]\!]_{\langle dt,dt\rangle} \overset{\text{def}}{=} \lambda I.(-\infty, \iota d[d \in I]]$
(The input of [[at most]] needs to be a singleton set of degrees.)

(iii) a. John is [[at most] six inches taller than every girl is].
LF: John is tall [[at most][six inches -er than every girl is]]
(To meet the requirement of [[at most]], [[six inches -er than every girl is]] is interpreted as 'exactly six inches -er than every girl is'. See also (70).)
$[\![(\text{iiia})]\!] \Leftrightarrow$ HEIGHT(JOHN) $\subseteq (-\infty,$ PRECISE-HEIGHT(tallest/shortest-girl) + 6″]

b. John is [[at most six inches] taller than every girl is] (see also (65))
LF: John is tall [[[at most six inches] -er][than every girl is]]
$[\![(\text{iiib})]\!] \Leftrightarrow$ HEIGHT(JOHN) $\subseteq$ (PRECISE-HEIGHT(tallest-girl), PRECISE-HEIGHT(shortest-girl) + 6″]

(69) For the minuend $\iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]$ to be well defined,
the **lower** bound of I_{DIFF} + the **upper** bound of of I_{STDD} <
the **upper** bound of I_{DIFF} + the **lower** bound of of I_{STDD} (see (48))
$\therefore$ the **upper** bound of I_{STDD} − the **lower** bound of I_{STDD} <
the **upper** bound of I_{DIFF} − the **lower** bound of I_{DIFF}
$\therefore$ I_{STDD} needs to be less wide than I_{DIFF}.

For upward-entailing differentials (e.g., (64)), I_{DIFF} is right-unbounded, i.e., $+\infty$. The definedness condition can always be met.

For downward-entailing and non-monotonic differentials (e.g., (65) and (66)), I_{DIFF} is right-bounded. Thus the definedness condition bears a consequence on inference. Sentences (61b) and (61c) are felicitous under the context in (60), because their I_{DIFF} (i.e., $(0, 5']$ and $[5', 10']$, respectively) is wider than the relevant I_{STDD} (i.e., $[18', 21']$).

This definedness condition explains why for sentences like (70), in which I_{DIFF} is a singleton set of degrees (here $[10', 10']$), our intuition is that it suggests that every tree should be of the same height, i.e., I_{DIFF} is also a singleton set of degrees. The technique of interval subtraction naturally captures this intuition, and there is no need to introduce other mechanisms to deal with this inference (see also Beck 2010, Alrenga and Kennedy 2014, Fleisher 2016 for more discussion).

(70) My giraffe is exactly 10 feet taller than every tree is.
$\rightsquigarrow$ Inference: every tree should be of the same height.

4.2 Inverse operator *little* and *less-than* comparatives

The LF in (63) shows that *less-than* comparatives with numerical differentials can be analyzed in exactly same way. Following previous studies (e.g., Rullmann 1995, Heim 2006b, Buring 2007, Büring 2007), we analyze *less* as the composition of *little* and *-er/more*. [[little]] takes a positive interval as input and returns its inverse as output (see (71)). Thus it can be considered an interval modifier, changing the polarity of a positive interval.

(71) $[\![\text{little}]\!]_{\langle dt,dt\rangle} \stackrel{\text{def}}{=} \lambda I \subseteq (0, +\infty).[[0, 0] - I]$ (see (45))

When [[little]] takes [[-er/more]] as input, the output is the most general negative differential, i.e., $(-\infty, 0)$. Similar to [[-er/more]], [[less]] also brings a felicity requirement: there is a salient scalar value serving as the base for a decrease (or a negative increase).

(72) $[\![\text{less}]\!]_{\langle dt\rangle} \overset{\text{def}}{=} [\![\text{little}]\!][\![\text{-er/more}]\!] = (-\infty, 0)$ (i.e., the most general negative interval)
Requirement: there is a salient scalar value serving as the base for a decrease.

The semantic derivation of a *less-than* comparative is parallel to that of a *more-than* comparative. (73) shows the step-by-step derivation (see also (59)).

(73) My giraffe is less tall than every tree is.
LF: $[\ [\text{my giraffe}]\ \text{is}\ [\ [\ \underbrace{\underbrace{\text{less}}_{\textbf{difference: } I_{\text{DIFF}}}\ \text{MINUS}\ \underbrace{\text{than}\ [\lambda I.\ \text{every tree is } I \text{ (tall)}]}_{\textbf{subtrahend: } I_{\text{stdd}}}}_{\textbf{minuend: } \iota I'[I'-I_{\text{stdd}}=I_{\text{diff}}]}\ ]\ \text{tall}\ ]\ ]$

a. **Subtrahend**: $I_{\text{STDD}} = [\![\text{than}]\!][\![\lambda I.\text{every tree is } I \text{ (tall)}]\!]$
$= \iota I[\forall x[\text{tree}(x) \to \text{HEIGHT}(x) \subseteq I]]$
$= [\text{PRECISE-HEIGHT}(\text{shortest-tree}), \text{PRECISE-HEIGHT}(\text{tallest-tree})]$

b. **Difference**: $I_{\text{DIFF}} = [\![\text{less}]\!] = (-\infty, 0)$

c. **Minuend**: $\iota I'[I' - I_{\text{STDD}} = I_{\text{DIFF}}]$

d. $[\![(59)]\!] \Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq \iota I'[I' - I_{\text{STDD}} = I_{\text{DIFF}}]$
$\Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq \iota I'[I' - \iota I[\forall x[\text{tree}(x) \to \text{HEIGHT}(x) \subseteq I]] = (-\infty, 0)]$
$\Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq$
$\iota I'[I' - [\text{PRECISE-HEIGHT}(\text{shortest-tree}), \text{PRECISE-HEIGHT}(\text{tallest-tree})] = (-\infty, 0)]$
$\Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq (-\infty, \text{PRECISE-HEIGHT}(\text{tallest-tree}))$ (see (48))
i.e., my giraffe's height falls within the interval between negative infinity and the height of the shortest tree.[24]

The only difference between the *more-than* comparative in (59) and the *less-than* comparative in (73) consists in the polarity of I_{DIFF}. By changing the polarity of the I_{DIFF}, $[\![\text{less}]\!]$ (or rather $[\![\text{little}]\!]$) changes the direction of an inequality. Thus, a *more-than* comparatives expresses a '$>$/$\geq$' relation, while a *less-than* comparative a '$<$/$\leq$' relation.

Similarly, as shown in (74)–(76), we use the same recipe of interval subtraction (see (48)) to compute the semantics of *less-than* comparatives containing upward-entailing, downward-entailing, or non-monotonic numerical differentials. In these *less-than* comparatives, we assume that a numerical differential first combines with *more* and restricts this positive interval, and then *little* operates on this restricted positive interval

[24] The non-existence of negative heights should be considered a world knowledge fact. A negative height is physically impossible in our actual world, but not linguistically or logically nonsensical. We can easily imagine some possible worlds with negative heights in fantasy stories. For some scales like temperature, negative scalar values are both linguistically and physically possible.

and returns its inverse. The projection pattern of the endpoint information of I_{DIFF} as well as the definedness condition for the minuend (see (68)) apply to *less-than* comparatives just like they apply to *more-than* comparatives.

(74) a. $I_{\text{DIFF}} = [\![\text{at least 5 feet less}]\!] = [\![\text{little}]\!][\![\text{at least 5 feet} \ldots\text{-er}]\!]$
$= [\![\text{little}]\!][5', +\infty) = (-\infty, -5']$

b. **Minuend**: $\iota I'[I' - I_{\text{STDD}} = [\![\text{at least 5 feet less}]\!]]$
$= \iota I'[I' - [18', 21'] = (-\infty, -5']] = (-\infty, 13']$

(75) a. $I_{\text{DIFF}} = [\![\text{at most 5 feet less}]\!] = [\![\text{little}]\!][\![\text{at most 5 feet} \ldots\text{-er}]\!]$
$= [\![\text{little}]\!](0, 5'] = [-5', 0)$

b. **Minuend**: $\iota I'[I' - I_{\text{STDD}} = [\![\text{at most 5 feet less}]\!]]$
$= \iota I'[I' - [18', 21'] = [-5', 0)] = [16', 18')$

(76) a. $I_{\text{DIFF}} = [\![\text{between 5 and 10 feet less}]\!] = [\![\text{little}]\!][\![\text{between 5 and feet} \ldots\text{-er}]\!]$
$= [\![\text{little}]\!][5', 10''] = [-10', -5']$

b. **Minuend**: $\iota I'[I' - I_{\text{STDD}} = [\![\text{between 5 and 10 feet less}]\!]]$
$= \iota I'[I' - [18', 21'] = [-10', -5']] = [11', 13']$

4.3 Inverse operator vs. negation operator

An interval is a convex set of degrees. Naturally, negation operator $[\![\text{no}]\!]$ can compose with and modify an interval. $[\![\text{little}]\!]$ and $[\![\text{no}]\!]$ are two distinct operators on intervals. $[\![\text{little}]\!]$ turns an interval into its inverse, while $[\![\text{no}]\!]$ negates an interval (i.e., it returns the complement of an interval). Therefore, *no more* and *no less* are different from *less* and *more*: the upper bound of *no more* and the lower bound of *no less* are closed, while the upper bound of *less* and the lower bound of *more* are open (see (77)).[25]

(77) a. $[\![\text{more}]\!] = (0, +\infty)$

b. $[\![\text{no more}]\!] = U \backslash (0, +\infty) = (-\infty, 0]$ $\qquad U = (-\infty, +\infty)$

c. $[\![\text{less}]\!] = [\![\text{little}]\!][\![\text{-er/more}]\!] = [0, 0] - (0, +\infty) = (-\infty, 0)$

d. $[\![\text{no less}]\!] = U \backslash (-\infty, 0) = [0, +\infty)$

[25] $[\![\text{little}]\!]$ can compose with all positive differentials (e.g., *at most 5 inches ... -er*), but, intriguingly, *no* only composes with the default positive and negative intervals $[\![\text{more}]\!]$ and $[\![\text{less}]\!]$. When taking a convex interval as its input, $[\![\text{little}]\!]$ returns its inverse as output – another convex interval. However, $[\![\text{no}]\!]$ potentially returns a set of degrees that is not a convex interval (e.g., the complement of $[0, 5']$ is $\{x \mid x < 0 \vee x > 5'\}$). Presumably, this explains the limited use of negation operator *no* in modifying intervals.

Based on our analysis of *no more*, (78) illustrates how to derive the meaning of a complex numerical differential: *no more than 5 feet less*. With the use of interval subtraction in analyzing comparatives and the proposed lexical entries for interval modifiers *little* and *no*, complex numerical differentials receive a uniform and principled treatment that naturally and precisely capture our intuitive interpretation for them.

(78) no more than 5 feet less
$[-5', 0)$

- little
  $\lambda I.[[0,0] - I]$
- no more than 5 feet …-er
  $(-\infty, 5'] \cap (0, +\infty)$
  i.e., $(0, 5']$
  - $\underbrace{\text{no more}}_{\textbf{differential:}(-\infty,0]}$ MINUS $\underbrace{\text{than 5 feet}}_{\textbf{subtrahend: }[5',5']}$
    minuend : $\iota I[I - [5', 5'] = (-\infty, 0]]$
    i.e., $(-\infty, 5']$
  - -er
    $(0, +\infty)$

4.4 A remark on bare numerals as differentials

For comparatives containing a bare numeral differential with an 'at least' reading, they demonstrate the inference patterns shown in (79) and (81). Given that the minuend directly inherits the endpoint information of I_{DIFF}, these inference patterns naturally follow the interpretation pattern of I_{DIFF}: given $x > y$, for a *more-than* comparative, $[x, +\infty)$ entails $[y, +\infty)$ (see (80)); for a *less-than* comparative, $(-\infty, -x]$ entails $(-\infty, -y]$ (see (82)). Overall, parallel inference patterns are observed for *more-than* and *less-than* comparatives.

(79) a. I am 3 cm taller than every boy is. $\vDash$ I am 2 cm taller than every boy is.
b. I am 3 cm taller than every boy is. $\nvDash$ I am 4 cm taller than every boy is.

(80) [[I am (at least) 3 inches taller than every boy is]] $\quad I_{\text{DIFF}} = [3\text{ cm}, +\infty)$
$\Leftrightarrow$ HEIGHT(I) $\subseteq$ [PRECISE-HEIGHT(tallest-boy) + 3 cm, $+\infty$)
Let PRECISE-HEIGHT(tallest-boy) $= x$, then $[x + 3\text{ cm}, +\infty) \subset [x + 2\text{ cm}, +\infty)$

(81) a. I am 3 cm less tall than every boy is. $\vDash$ I am 2 cm less tall than every boy is.
b. I am 3 cm less tall than every boy is. $\nvDash$ I am 4 cm less tall than every boy is.

(82) $[\![$I am (at least) 3 inches less tall than every boy is$]\!]$ $\qquad I_{\text{DIFF}} = (-\infty, -3\text{ cm}]$
$\Leftrightarrow$ $\textsc{height}(\mathsf{I}) \subseteq (-\infty, \textsc{precise-height}(\mathsf{shortest\text{-}boy}) - 3\text{ cm}]$
Let $\textsc{precise-height}(\mathsf{shortest\text{-}boy}) = y$, then $(-\infty, y - 3\text{ cm}] \subset (-\infty, y - 2\text{ cm}]$

5 Our solutions to three puzzles

Based on our proposed difference-based analysis implemented with interval subtraction, this section accounts for three puzzles. The first two involve the semantics of the *than*-clause. The third one involves the role of *-er*/*more* at the discourse level.

5.1 Information projection of the *than*-clause as a scope island

It has long been acknowledged that in a clausal comparative (cf. phrasal comparative), its *than*-clause is a scope island (Hankamer 1973, Larson 1988). Thus *than*-clause-internal quantifiers cannot scope out their hosting *than*-clause via Quantifier Raising (QR), a vanilla mechanism for scope-taking.[26] This scope island issue raises the question of how measurement information of multiple entities is used as comparison standard and gets projected to sentential-level semantics (see e.g., Gajewski 2008, van Rooij 2008).

Technically, QR generally faces the same constraints as *wh*-movement. (83) and (84) illustrate that overt and covert *wh*-movements from within a *than*-clause are ungrammatical (see Larson 1988, Schwarzchild and Wilkinson 2002). Therefore, QR is also unavailable for *than*-clause-internal quantifiers to take scope.

(83) a. [Which tree]$_i$ is my giraffe taller than? **Phrasal comparative**
b. *[Which tree]$_i$ is my giraffe taller than t_i is? **Clausal comparative**

(84) a. She wants to know who was taller than who else. **Phrasal comparative**
b. *She wants to know who was taller than who else is. **Clausal comparative**

Empirically, a series of contrasts between clausal and phrasal comparatives provide evidence for the scope island status of *than*-clauses. In (85), the phrasal comparative (85a)

[26] We mainly focus on whether/how universal quantifiers in a *than*-clause take scope. Indefinites and modified numerals contained within a scope island can still take exceptional scope, though not via a QR-style mechanism (see Brasoveanu 2013, Charlow 2014, Bumford 2017, and our more recent work).

is ambiguous between a surface scope reading '$\exists > \forall$' and an inverse scope reading '$\forall > \exists$', while the clausal comparative (85b) has only a surface scope reading '$\exists > \forall$' (see Larson 1988, p. 4, (12)). This contrast shows that *than*-phrase-internal universal quantifiers can take scope, but *than*-clausal-internal universal quantifiers cannot.[27]

(85) a. Someone is smarter than everyone. **Phrasal comparative**
$\rightsquigarrow$ ambiguous: $\checkmark \exists > \forall$, $\checkmark \forall > \exists$
b. Someone is smarter than everyone is. **Clausal comparative**
$\rightsquigarrow$ unambiguous: $\checkmark \exists > \forall$, $\# \forall > \exists$

As shown in (86) and (87), if *than*-clause-internal downward-entailing quantifiers *no tree* and *few trees* can take scope outside the *than*-clause, (86b) and (87b) would be grammatical and yield the same reading as (86a) and (87a) do. The ungrammaticality of (86b) and (87b) again shows that (i) phrasal comparatives and clausal comparatives are distinct language phenomena (see Hankamer 1973, Hoeksema 1983, Pinkal 1990, Kennedy 1999, Pancheva 2006) and (ii) the *than*-clause is a scope island.

[27] *Smart* is a gradable adjective showing dimension indeterminacy. Thus both sentences in (i) can be true without contradicting each other (see Kennedy 1999, Section 1.1.2). For (85a), its inverse scope reading is true in a context in which for everyone x, there is a person y smarter than x in a certain way, but there is no one that is smarter than everyone else. The surface scope reading of (85a) is false in this context.

(i) a. My dog is smarter than I am. $\rightsquigarrow$ In terms of behaving in a cute way
b. I am smarter than my dog is. $\rightsquigarrow$ In terms of working on mathematical problem sets

On the other hand, gradable adjectives like *long* show no dimension indeterminacy. Thus if the differential is upward-entailing (e.g., *-er*, or an 'at least' reading for *2 inches . . . -er*), the surface and inverse scope readings of phrasal comparative (iia) have the same truth conditions and cannot be teased apart from each other.

(ii) a. Some giraffe is (2 inches) taller than every tree. **Phrasal comparative**
b. Some giraffe is (2 inches) taller than every tree is. **Clausal comparative**

It is also worth noting that for clausal comparatives like (iii), our intuitive judgment on the availability of an inverse scope reading is not reliable enough, because of garden-path effects (i.e., corresponding phrasal comparatives have an inverse scope reading). Experiments (especially with the use of an eye-tracker or EEG) are needed for satisfactorily resolving this issue. This is beyond the scope of our current work. Our analysis only generates one reading for (iii), which suggests that every tree is of the same height (see the definedness condition in Section 4.1). Here we consider *an even number* a modified numeral (which does not have an 'at least' interpretation, see Szabolcsi 1997, Krifka 1999, de Swart 1999, Umbach 2005), and the checking of this cardinality requirement is based on a post-suppositional mechanism (see Brasoveanu 2013).

We thank an anonymous reviewer for raising these issues.

(iii) Some giraffe or other is an even number of inches taller than every tree is. (by Uli Sauerland)
$\exists x[\mathsf{giraffe}(x) \wedge \textsc{height}(x) \subseteq \iota I[I - \iota I'[\forall y[\mathsf{tree}(y) \rightarrow \textsc{height}(y) \subseteq I']] = [d'', d'']]]^{\wedge d \text{ modulo } 2=0}$
$\rightsquigarrow$ every tree is of the same height I', and some giraffe's height exceeds I' by d'' , and d modulo $2 = 0$.

(86) a. My giraffe is taller than no tree. **Phrasal comparative**
b. *My giraffe is taller than no tree is. **Clausal comparative**

(87) a. My giraffe is taller than few trees. **Phrasal comparative**
b. *My giraffe is taller than few trees are. **Clausal comparative**

Schwarzchild and Wilkinson (2002) also argue that our natural interpretation for (88) does not need to involve an individual prediction for each tree's height. In other words, (88) calls for an analysis that supports the *in situ* interpretation of *most trees*.

(88) My giraffe is taller than Bill predicted most trees are.

Given the scope island status and interpretation limitations of a *than*-clause, when its target of predication is a group of entities (e.g., *than every tree is* (*tall*), *than the trees are* (*tall*)), the possibility of projecting the measurement information of each involved individual to sentential level is basically ruled out. For the sentence in (89), not only the universal quantifier *every tree* has to be interpreted *in situ*, but also the scope-taking of each measurement for individual trees (see the discussion on 'degree plurality' in Section 6.4) cannot be workable, as evidenced by the lack of inverse scope reading for (85b).

Therefore, as summarized in (89), the interpretation of this clausal comparative cannot involve multiple comparisons (see (89a)). However, if the semantics of the *than*-clause is reduced to a single degree, as proposed by the canonical analysis (see (11)) or Beck (2010) (see (22)), the derived truth condition is too weak (see (89b)).

(89) My giraffe is between 5 and 10 feet taller than every tree is. (= (66))
a. #There are multiple comparisons – one for each tree.
$\rightsquigarrow$ Violating scope island constraints
b. #There is only one comparison – just for the shortest/tallest tree.
$\rightsquigarrow$ Too weak truth condition (see (11), (22), and the discussion in Section 2)

Then what information eventually gets projected from a *than*-clause for conducting comparison(s) at the sentential level? According to Beck (2010),

> 'I want to come out of the calculation of the semantics of the *than*-clause holding in my hand *the* degree we will be comparing things to.' (Beck 2010)

Our interval-based analysis responds to this challenge with a new and more generalized view.

We come out of the calculation of the semantics of the *than*-clause holding in our hand *the* scalar value we will be comparing things to, and this scalar value is represented as an interval, i.e., a potentially not-very-precise scalar value. Thus, as shown in (90), there is only one comparison, but both the upper and lower bounds of I_{STDD} – the interval serving as comparison standard (here the height of the shortest and the tallest trees) – are involved in this comparison.

(90) My giraffe is between 5 and 10 feet taller than every tree is.
$[\![\text{than every tree is } \cancel{\text{tall}}]\!] = \iota I[\forall x[\mathsf{tree}(x) \to \textsc{height}(x) \subseteq I]]$ (= (55b))
$= [\textsc{precise-height}(\mathsf{shortest\text{-}tree}), \textsc{precise-height}(\mathsf{tallest\text{-}tree})]$
$\rightsquigarrow$ There is only one comparison – for the interval ranging over the trees' height.

Our view is compatible with all those works that analyze the semantics of a *than*-clause as a definite description (e.g., Russell 1905, Heim 1985, Beck 2010). Our view also accounts for the cases of *than*-clause-internal downward-entailing quantifiers (see (86) and (87)) and the most natural interpretation of (88). (86b) and (87b) are ungrammatical because their *than*-clause is uninterpretable – there is no non-trivial convex interval I such that no one is I (or few trees are I tall) (see also Abrusán 2014). For (88), Bill's prediction can be a single, potentially not-very-precise value represented as an interval, and at sentential level, comparison is conducted with this interval.

In short, by using an interval to represent the standard of comparison and only projecting endpoint information from the *than*-clause, our interval-based implementation yields intuitively correct truth conditions without violating any island constraints.

5.2 The creation of a downward-entailing operator with intervals

Whether and how a *than*-clause contributes a downward-entailing (DE) operator and creates an NPI-licensing environment has been a debatable issue. According to theories proposing the inclusion of a covert negation operator inside a *than*-clause (e.g., Marques 2003, Schwarzschild 2008, Gajewski 2008, Alrenga and Kennedy 2014, and other adopters of the 'A-not-A' approach), a *than*-clause naturally becomes a DE environment.

However, empirical evidence is not fully compatible with this view. On the one hand, different from a negation operator, *than*-clauses only license minimizers (e.g., *give a penny* in (91a)) and weak NPIs that also work as Free Choice Items (FCI, e.g., *anyone* in (91b)), but not strong NPI *either* (see (91c); see Giannakidou and Yoon 2010).

(91) a. John would sooner roast in hell than **give a penny** to the charity.
b. Roxy ran faster than **anyone** had expected.
c. *John is taller than Bill is **either**. (Giannakidou and Yoon 2010: (42))

On the other hand, sometimes, the interpretation of *than*-clauses leads to an upward entailment, not a downward entailment, as illustrated by the contrast between (93) and (92) (see Larson 1988, Schwarzchild and Wilkinson 2002, Giannakidou and Yoon 2010).

(92) Downward entailment:
a. The tree is taller than **every animal** is $\vDash$ the tree is taller than **every giraffe** is.
b. The tree is taller than **any animal** is $\vDash$ the tree is taller than **any giraffe** is.

(93) Upward entailment:
a. The tree is taller than **some animal** is $\nvDash$ the tree is taller than **some giraffe** is.
b. The tree is taller than **some giraffe** is $\vDash$ the tree is taller than **some animal** is.

Following our interval-subtraction-based analysis, we show that interval subtraction naturally makes the subtrahend (i.e., I_{STDD}, or the semantics of a *than*-clause) a DE operator. There is no need to assume a covert negation operator within a *than*-clause.

As already addressed in Section 3.1, within interval arithmetic, given the values of a difference and a subtrahend, we need to follow the formula of interval subtraction (see (43)) to compute the value of the minuend. Specifically, as shown in (94) (which repeats (48)), in computing the value of the minuend, it is the **upper** bound of the subtrahend that contributes to the **lower** bound of the minuend, and it is the **lower** bound of the subtrahend that contributes to the **upper** bound of the minuend.

(94) If $X - [a, b] = [c, d]$, when defined, $X = [b + c, a + d]$.
a. The **lower** bound of the **minuend** X
= the **lower** bound of the **difference** + the **upper** bound of the **subtrahend**;
b. the **upper** bound of the **minuend** X
= the **upper** bound of the **difference** + the **lower** bound of the **subtrahend**.

An interval is a convex set of degrees. Thus, an interval becomes less informative if we raise its upper bound or lower its lower bound, and it becomes more informative if we lower its upper bound or raise its lower bound. Given (94), lowering the lower bound of the subtrahend leads to a lower upper bound for the minuend, thus decreasing the informativeness of the subtrahend (i.e., the interval standing for the subtrahend includes more possibilities) but increasing the informativeness of the minuend (i.e., the interval standing for the minuend includes fewer possibilities). Thus, generally, lowering or raising an endpoint of the subtrahend always causes the informativeness of the subtrahend and the minuend to change in opposite directions. When the subtrahend becomes more informative, the minuend becomes less informative, and vice versa.

Therefore, the informativeness of a *than*-clause (i.e., I_{STDD}, which plays the role of subtrahend) projects to sentential-level informativeness (i.e., the informativeness of the minuend) in a reverse way, demonstrating exactly the defining property of a typical DE operator (e.g., a negation operator, as shown in (95)) in reversing the relation of entailment (see Fauconnier 1978, Ladusaw 1979, 1980).

(95) $\because \lambda x.\mathsf{lizard}(x) \subseteq \lambda x.\mathsf{reptile}(x)$ (i.e., [[lizard]] entails [[reptile]].)
$\therefore \lambda x.\neg\mathsf{lizard}(x) \supseteq \lambda x.\neg\mathsf{reptile}(x)$ (i.e., [[not a reptile]] entails [[not a lizard]].)
$\rightsquigarrow$ [[not]] reverses the relation of entailment and works as a DE operator.
E.g., Roo is not a reptile $\vDash$ Roo is not a lizard.

Under the current analysis, the DE-ness of a *than*-clause is due to its role of subtrahend in interval subtraction. Thus, this DE-ness is with regard to the projection of informativeness for the interval I_{STDD}. The projection of informativeness for *than*-clause-internal expressions like *every giraffe* or *some giraffe* in (93)/(92) is subject to an interplay among several operators that affect the projection of informativeness.

For comparatives that contain a *than*-clause-internal universal quantifier (e.g., *every giraffe* in (92a), or *any giraffe* in (92b) – an FCI with a universal flavor), the relation of entailment gets reversed three times along the derivation of sentential semantics.

As shown in (96), we start with the lexical semantics of *giraffe* and *animal*. (i) From these nouns (or NPs) to their embedding DP '*every* NP', the relation of entailment is reversed. (ii) From '*every* NP' to I_{STDD} (i.e., the most informative interval serving as the standard of comparison), the relation of entailment is reversed a second time. (iii) From I_{STDD} to the value of minuend, as argued before, the relation of entailment is reversed a third time. Eventually, we obtain the entailment pattern shown in (92).

(96) $\because \lambda x.\mathsf{giraffe}(x) \subseteq \lambda x.\mathsf{animal}(x)$ (i.e., [[giraffe]] entails [[animal]].)
$\therefore \lambda P.\forall x[\mathsf{giraffe}(x) \to P(x)] \supseteq \lambda P.\forall x[\mathsf{animal}(x) \to P(x)]$
$\rightsquigarrow$ any property P such that $\forall x[\mathsf{animal}(x) \to P(x)]$ also makes $\forall x[\mathsf{giraffe}(x) \to P(x)]$

hold true. (i.e., **Reverse 1** – [[every animal]] entails [[every giraffe]].)
$\therefore \lambda I.\forall x[\text{giraffe}(x) \rightarrow \textsc{height}(x) \subseteq I] \supseteq \lambda I.\forall x[\text{animal}(x) \rightarrow \textsc{height}(x) \subseteq I]$
$\rightsquigarrow$ any interval I such that $\forall x[\text{animal}(x) \rightarrow \textsc{height}(x) \subseteq I]$ also makes $\forall x[\text{giraffe}(x) \rightarrow \textsc{height}(x) \subseteq I]$ hold true.
$\therefore \iota I[\forall x[\text{giraffe}(x) \rightarrow \textsc{height}(x) \subseteq I]] \subseteq \iota I'[\forall x[\text{animal}(x) \rightarrow \textsc{height}(x) \subseteq I']]$
$\rightsquigarrow$ the most informative interval I such that $\forall x[\text{giraffe}(x) \rightarrow \textsc{height}(x) \subseteq I]$ is not less informative than the most informative interval I' such that $\forall x[\text{animal}(x) \rightarrow \textsc{height}(x) \subseteq I']$.
(i.e., **Reverse 2** – 'the most informative interval I such that every giraffe is I tall' entails 'the most informative interval I' such that every animal is I' tall'.)
$\therefore \iota I_{\textsc{minuend}}[I_{\textsc{minuend}} - \iota I[\forall x[\text{giraffe}(x) \rightarrow \textsc{height}(x) \subseteq I]] = I_{\textsc{diff}}] \supseteq$
$\iota I'_{\textsc{minuend}}[I'_{\textsc{minuend}} - \iota I'[\forall x[\text{animal}(x) \rightarrow \textsc{height}(x) \subseteq I']] = I_{\textsc{diff}}]$
(i.e., **Reverse 3** – [[taller than every animal is]] entails [[taller than every giraffe is]].)

For comparatives that contain a *than*-clause-internal existential quantifier (e.g., *some giraffe* in (93)), the relation of entailment gets reversed twice along the derivation of sentential semantics. As shown in (97), we also start with the lexical semantics of *giraffe* and *animal*. From these NPs to their hosting DP '*some* NP', the relation of entailment is straightforward. (i) It is from '*some* NP' to $I_{\textsc{stdd}}$, the relation of entailment is reversed for the first time. (ii) Then from $I_{\textsc{stdd}}$ to the value of minuend, the relation of entailment is reversed a second time. Eventually, we obtain the entailment pattern shown in (93).

(97) $\because \lambda x.\text{giraffe}(x) \subseteq \lambda x.\text{animal}(x)$ (i.e., [[giraffe]] entails [[animal]].)
$\therefore \lambda P.\exists x[\text{giraffe}(x) \wedge P(x)] \subseteq \lambda P.\exists x[\text{animal}(x) \wedge P(x)]$
$\rightsquigarrow$ any property P such that $\exists x[\text{giraffe}(x) \wedge P(x)]$ also makes $\exists x[\text{animal}(x) \wedge P(x)]$ hold true. (i.e., [[some giraffe]] entails [[some animal]].)
$\therefore$ for each most informative interval I such that $\exists x[\text{giraffe}(x) \wedge \textsc{height}(x) \subseteq I]$, it follows that there exists an interval I' such that $\exists x[\text{animal}(x) \wedge \textsc{height}(x) \subseteq I']$ and I' is not less informative than I.
(i.e., **Reverse 1** – 'the most informative interval I' such that some animal is I' tall' entails 'the most informative interval I such that some giraffe is I tall'.)
$\therefore \iota I_{\textsc{minuend}}[I_{\textsc{minuend}} - \iota I[\exists x[\text{giraffe}(x) \wedge \textsc{height}(x) \subseteq I]] = I_{\textsc{diff}}] \subseteq$
$\iota I'_{\textsc{minuend}}[I'_{\textsc{minuend}} - \iota I'[\exists x[\text{animal}(x) \wedge \textsc{height}(x) \subseteq I']] = I_{\textsc{diff}}]$
(i.e., **Reverse 2** – [[taller than some giraffe is]] entails

[[taller than some animal is]].)

(96) and (97) demonstrate the interplay among operators that work together on informativeness projection, but after all, the informativeness of I_{STDD} always projects to sentential semantics in the same reverse way. Its subtrahend status is a DE operator.[28]

Since it is the subtrahend status that actually contributes the DE operator, this DE operator is performed outside the *than*-clause and never interferes with any *than*-clause-internal quantifiers (cf. Alrenga and Kennedy 2014). This correctly predicts that clausal comparatives are generally unambiguous, no matter whether there are universal/existential nominal/modal quantifiers in their *than*-clause (see (98)–(101)).[29]

(98) **Universal nominal quantifier**: *every boy*
Context: The height of boys is between 5 feet 5 inches and 6 feet.
 a. Mary is taller than every boy is. $\checkmark > 6'$; $\# > 5'5''$
 b. Mary is less tall than every boy is. $\checkmark < 5'5''$; $\# < 6'$

(99) **Existential nominal quantifier**: *some boys*
Context: The height of boys is between 5 feet 5 inches and 6 feet.
 a. Mary is taller than some boys are. $\checkmark > 5'5''$; $\# > 6'$
 b. Mary is less tall than some boys are. $\checkmark < 6'$; $\# < 5'5''$

(100) **Universal epistemic modal**: *be supposed to*
Context: the temperature of X is supposed to be between 83°C and 98°C.
 a. X reached a temperature higher than supposed to be. $\checkmark > 98$°C; $\# > 83$°C
 b. X reached a temperature less high than supposed to be. $\checkmark < 83$°C; $\# < 98$°C

[28] A general discussion on the licensing conditions of various kinds of NPIs is beyond the scope of this paper. For minimizers and weak NPIs (which arguably work as FCIs within a *than*-clause), their semantics is relevant to informativeness projection. Thus their licensing conditions should be informativeness-based. For words like *either*, presumably, their semantics is irrelevant to informativeness projection, and their licensing conditions should be related to other factors such as non-veridicality (see Giannakidou and Yoon 2010).

[29] We further predict that, when ambiguity does arise (see (i), which contains a *than*-clause-internal existential deontic (permission-related) modal), this ambiguity cannot be due to scopal interaction between a modal and some kind of negation-like quantifier built with a *than*-clause (see Rullmann 1995, Heim 2006b, Beck 2013, Alrenga and Kennedy 2014, Fleisher 2019 for discussion; see also footnote 19 on page 26).

(i) Context: This highway has a required minimum speed of 35 mph and a speed limit of 50 mph.
Lucinda was driving less fast than allowed. (Beck 2013: (1), (2))
 a. Lucinda was driving below the speed limit – 50 mph.
 b. Lucinda was driving below the required minimum – 35 mph.

(101) **Existential epistemic quantifiers**: *likely*

Context: the price of X is likely to be between \$8 000 to \$ 10 000 next year.

a. The price of X is higher than it's likely to be next year. ✓ > \$10K; # > \$8K

b. The price of X is less high than it's likely to be next year. ✓ < \$8K; # < \$10K

To sum up, in an equation of interval subtraction, a subtrahend naturally projects informativeness in a reverse way. Downward-entailing-ness is in the nature of the standard in a comparison (i.e., a *than*-clause) and does not need to resort to any additional operators or mechanisms.

5.3 Klein (1980)'s puzzle and the core contribution of *-er/more*

The third puzzle is raised by Klein (1980). Cross-linguistically, why is the positive form of gradable adjectives (e.g., *tall*) morphologically simpler than the comparative form (e.g., *taller*)? If gradable adjectives have an inherently relative meaning and always encode comparison (e.g., the meaning of *my giraffe is tall* is analyzed as a comparison between the height of my giraffe and the average height of giraffes), shouldn't the comparative use be more basic and have a morphologically simpler form?

Under our proposed difference-based analysis, *-er*/*more* contributes to the semantics of comparatives by playing the role of the default differential. The default positive value $(0, +\infty)$ aside, the differential status of *-er*/*more* is due to its additivity, a kind of anaphoricity. In this sense, what *-er*/*more* marks is actually the discourse salience of the value serving as the standard of comparison. Compared to other uses of gradable adjectives, comparatives are special in involving standards that have discourse salience.[30]

Empirical evidence is illustrated by the contrast shown in (102). The implicit standard for the interpretation of the positive form *tall* has no discourse salience, while the accommodated standard for the interpretation of the comparative form *taller* must have discourse salience. Without the marker *-er*, the *then*-clause in (102a) shares the same implicit standard with the *if*-clause. In contrast, with the salience marker *-er*, the

[30] In response to this puzzle he raises, Klein (1980) abandons the relativity inherent to the semantics of gradable adjectives and develops a delineation approach. Within this approach, gradable adjectives (e.g., *tall*) are like non-gradable ones (e.g., *red*) and denote sets of individuals, but the extension of a gradable adjective can change in evaluations, depending on the set of individuals that it is being compared with (see McConnell-Ginet 1973, Kamp 1975, Lewis 1979, Klein 1980, and see Burnett 2017 for a recent development). Degrees are not conceptual primitives within this approach, and the semantics of gradable adjectives does not involve comparison per se. Kennedy (1999) convincingly challenges this approach. In this paper, we follow Kennedy (1999) and adopt a degree-based semantics for gradable adjectives (see Section 1.2).

then-clause in (102b) requires a standard that has discourse salience, here the height of John, not the implicit standard involved in the interpretation of *if John is tall*.[31]

(102) a. If John is tall, then Bill is tall.
⤳ The heights of John and Bill are compared with the same context-relevant standard.

b. If John is tall, then Bill is taller.
⤳ The height of John is compared with a context-relevant standard, while the height of Bill is compared with the height of John.
(Here the height of John has discourse salience.)

Following the view that all uses of gradable adjectives involve comparison, we can further zoom into the interval argument of a gradable adjective (see (103a)), splitting it into two – I_{STDD} and I_{DIFF} – and hardwire the operation of subtraction into the lexical semantics of a gradable adjective (see (103)). For *-er/more* (i.e., the default, non-restricted, positive value, $(0, +\infty)$, that carries the requirement for a discourse-salient base, see (104a)), we also assume that there is a silent counterpart, POSITIVE-VALUE (i.e., $(0, +\infty)$), that carries no such requirement (see (104b)). The distinction between *-er/more* and POSITIVE-VALUE is parallel to that between *a* and *another* in the domain of entities.

(103) a. $[\![\text{tall}]\!]_{\langle dt,et\rangle} \stackrel{\text{def}}{=} \lambda I_{\langle dt\rangle}.\lambda x_e.\text{HEIGHT}_{\langle e,dt\rangle}(x) \subseteq I$ (= (38) = (50))

b. $[\![\text{tall}]\!]_{\langle dt,\langle dt,et\rangle\rangle} \stackrel{\text{def}}{=} \lambda I_{\text{STDD}}.\lambda I_{\text{DIFF}}.\lambda x_e.\text{HEIGHT}(x) \subseteq \iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]$

(104) a. $[\![\text{-er/more}]\!] \stackrel{\text{def}}{=} (0, +\infty)$ (i.e., the most general positive interval (= (37) = (56)))
Requirement: there is a salient scalar value serving as comparison standard (i.e., the base for increase).

b. $[\![\text{POSITIVE-VALUE}]\!] \stackrel{\text{def}}{=} (0, +\infty)$ **No additional requirement**

As shown in (105), different uses of gradable adjectives differ in (i) their selection of I_{STDD} and (ii) whether the default value of I_{DIFF} can be further restricted. Further numerical restriction for the default value of I_{DIFF} is obligatory for measurement constructions,

[31] Contextual manipulation helps to resolve uncertainty for interpreting the implicit standard for the positive use of a gradable adjective (see the notion of 'sharpening' in Barker 2002). This is analogous to the kind of contextual manipulation in the interpretation of other predicates. For example, the predicate *girl* in a sentence like *every girl is here* needs to be restricted and enriched by context. Under a specific context, this predicate cannot hold for any entity that is a girl in the universe. However, it is rather discourse salience, not contextual manipulation, that forms the base for the standard status of the value serving as the standard in a comparative. We thank an anonymous reviewer for raising this issue.

optional for comparatives, and impossible for the positive use.[32] Standards with no discourse salience (i.e., those for the positive use and measurement constructions) and POSITIVE-VALUE are silent. Thus these three uses of gradable adjectives are distinguishable by (i) the presence/absence of numerical restriction and (ii) the marker of discourse salience for their standard of comparison.

(105) The standard and differential involved in comparison:
(Only the marker of discourse salience and numerals are pronounced.)

Linguistic construction	Standard: I_{STDD}	Differential: I_{DIFF}
Comparative	*than*-clause/phrase or accommodated **(with discourse salience)**	*-er*/*more*; **optional** numerical restriction for $(0, +\infty)$
Measurement construction	absolute zero point $[0, 0]$ **(no discourse salience)**	POSITIVE-VALUE **with** numerical restriction
Positive use	the relevant average **(no discourse salience)**	POSITIVE-VALUE **with no** numerical restriction

Compared to the analysis shown in Section 3.2, (103)–(105) do not offer a new one, but rather a zoomed-in version. This zoomed-in version highlights the inherent relativity of the semantics of gradable adjectives: each use of gradable adjectives involves a comparison relative to a reference, i.e., standard. Within this zoomed-in version, the semantics of measurement constructions and the positive use can again be derived directly (see (106) and (107)). In both constructions, I_{STDD} has no discourse salience, so that the positive form (here *tall*) is used. Whether a sentence is interpreted as a measurement construction or the positive use depends on the presence of numerical restriction.

(106) My giraffe is exactly 20 feet tall. **Measurement construction** (see also (51))

LF: My giraffe is $[\,[\underbrace{[[20', 20'] \cap (0, +\infty)]}_{I_{\text{DIFF}}}\ \text{MINUS}\ \underbrace{[0, 0]}_{I_{\text{STDD}}}\,]\ \text{tall}\,]$

$\text{HEIGHT}(\text{my-giraffe}) \subseteq \iota I[I - [0, 0] = [20', 20']]$

(107) My giraffe is tall. **Positive use** (see also (52))

LF: My giraffe is $[\,[\underbrace{[(0, +\infty)]}_{I_{\text{DIFF}}}\ \text{MINUS}\ \underbrace{I_{\text{AVERAGE-HEIGHT-OF-GIRAFFES}}}_{I_{\text{STDD}}}\,]\ \text{tall}\,]$

[32]When numerical restriction of I_{DIFF} is absent (for the comparative or positive use), degree modifiers like *very*, *slightly*, and *much* can be used to modify I_{DIFF}, yielding *slightly tall*, *much taller*, etc (see also Rett 2018).

$$\textsc{height}(\text{my-giraffe}) \subseteq \iota I[I - I_{\textsc{average-height-of-giraffes}} = (0, +\infty)]$$

The analysis shown in (106) immediately implements Sassoon (2010)'s account for the limited distribution of gradable adjectives in measurement constructions. According to Sassoon (2010), only those gradable adjectives associated with ratio scales (i.e., scales with a meaningful, absolute zero point, see Fig. 1) can be used to form measurement constructions (see also the discussion in Schwarzschild 2005). In our analysis, measurement constructions require the existence of an absolute zero point to play the role of $I_{\textsc{stdd}}$. This requirement is met for a scale of temporal length (see (108a)), but not met for scales of temporal shortness, warmth, or earliness/lateness (see (108b)–(108d)).

(108) a. This tennis match was 1.5 hours **long**. — Temporal length: a ratio scale
$\rightsquigarrow$ On a scale of temporal length: 0 hours means 'no temporal length'.

b. *This tennis match was 1.5 hours **short**. — Temporal shortness
$\rightsquigarrow$ On a scale of temporal shortness, there is no absolute zero point.

c. *New York is now 70 degrees **warm**. — Warmth
$\rightsquigarrow$ On a scale of warmth, there is no absolute zero point.

d. *Our meeting time was 11 AM **early** / **late**. — Earliness, lateness
$\rightsquigarrow$ On a scale of earliness/lateness, there is no absolute zero point.

A degree-question addresses the position that some entity's measurement falls at on a scale. As shown in (109), different choices of $I_{\textsc{stdd}}$ lead to different ways of answering a degree question. Essentially, they mean that the position under discussion (here the height of my giraffe) can be considered relative to a certain reference position (e.g., a zero point, a relevant average for a comparison class, or a discourse-salient position).

(109) $[\![\text{How tall is my giraffe}]\!] = \lambda I.\textsc{height}(\text{my-giraffe}) \subseteq I$ (see (103a))

a. It is 20 feet tall. $\rightsquigarrow I_{\textsc{stdd}} = [0, 0]$

b. It is very tall. $\rightsquigarrow I_{\textsc{stdd}} = I_{\textsc{average-height-of-giraffes}}$

c. It is taller than that tree is. $\rightsquigarrow I_{\textsc{stdd}} = [\![\text{than that tree is}]\!]$

In Section 3.2, we analyze the semantics of a *than*-clause as a position on a scale – a short answer to its corresponding degree question. (109) shows that a position under discussion can be characterized relative to different reference positions. Thus for a comparative like (110), the semantics of its *than*-clause can be analyzed as relative to different reference positions within the *than*-clause, but at the matrix-clause level, it doesn't matter it is relative to which reference position that we address the height of the tree, i.e., $[\![$than the tree is (tall)$]\!]$. What this sentence conveys is that it is relative to the height of the tree – a discourse-salient I_{STDD} – that we address the height of the giraffe (and that the distance between these two positions on a scale of height is at least 2 feet).

(110) My giraffe is (at least 2 feet) taller than the tree is. **Comparative** (see also (58))

LF: My giraffe is [[$\underbrace{[\![[2', +\infty) \cap (0, +\infty)]\!]}_{I_{\text{DIFF}}}$ MINUS $\underbrace{[\text{than the tree is (tall)}]}_{I_{\text{STDD}}}$] tall]

$\textsc{height}(\text{my-giraffe}) \subseteq \iota I[I - \textsc{height}(\text{the tree}) = [2', +\infty)]$

With this detailed understanding of the *than*-clause, we can explain why gradable adjectives that are not associated with ratio scales (e.g., *short*, *warm*, *early*, *late*) can still be used in comparatives (see (108) vs. (111)). For (111b), the availability of a zero point on a scale of tempoeral shortness, or more generally, how to choose a reference position for addressing *how short is that movie*, does not matter at the matrix-clause level.

(111)
a. This tennis match was 1.5 hours **longer** (than that movie is).
b. This tennis match was 1.5 hours **shorter** (than that movie is).
c. New York is now 70 degrees **warmer** (than Antarctic is).
d. Our meeting time was 11 hours **earlier**/**later** (than I expected).

The zoomed-in version offers a slowed-down way to consider the semantics of a degree question and its answerhood. As shown in (112), this degree question is analyzed as addressing how far away the height of the tree is relative to a given reference position I_{STDD}. I_{DIFF}, the information sought for here, is the midway towards a full resolution of the position standing for the measurement of the tree on a scale of height.

(112) $[\![$How tall is the tree$]\!] = \lambda I_{\text{DIFF}}.\textsc{height}(\text{the-tree}) \subseteq \iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]$ (see (103b))

a. 20 feet. $\rightsquigarrow I_{\text{STDD}} = [0, 0]$
b. Slightly. $\rightsquigarrow I_{\text{STDD}} = I_{\text{AVERAGE-HEIGHT-OF-GIRAFFES}}$
c. 2 feet taller. $\rightsquigarrow I_{\text{STDD}}$ is an accommodated, discourse-salient value.

Naturally occurring examples like the **comparison of deviations** in (2) (repeated here in (113)) provide empirical support for this slowed-down view on degree questions.

The proposed LF in (113) involves three comparisons, i.e., three uses of gradable adjectives. (i) **The use of *sad*** in the *than*-clause: $[\![$than Jude is sad$]\!]$ denotes a short answer to the question how far away Jude's sadness is relative to average sadness, providing a discourse-salient value for further comparison. (ii) **The use of a hidden *much***: Based on the assumption that *more* is composed from gradable adjective *much* and the discourse salience marker *-er*,[33] the use of *more* involves a comparison with the salient value provided by the semantics of the *than*-clause, i.e., the difference between Jude's sadness and $I_{\text{AVE.-SAD}}$. (iii) **The use of *happy***: The derived meaning of [[more ... than Jude is sad]] plays the role of I_{DIFF} for the use of *happy*, addressing how far away Mona's happiness is relative to average happiness. Among these three comparisons, only the one performed along the scale of differences involves a discourse-salient I_{STDD}. Therefore, the comparison of deviations eventually bears only one discourse salience marker *-er*.[34]

(113) Mona is more happy than Jude is sad. (Kennedy 1999: Chapter 1, (89))

LF: Mona is $\underbrace{\underbrace{(0,+\infty)}_{I_{\text{DIFF-MUCH}}} \text{ MINUS } \underbrace{[\iota I_{\text{DIFF}}[\text{Jude is } I_{\text{DIFF}} \text{ MINUS } \underbrace{I_{\text{AVE.-SAD}}}_{I_{\text{STDD-SAD}}} \textbf{ sad}]]}_{I_{\text{STDD-MUCH}}} \textbf{ much}}_{I_{\text{DIFF-HAPPY}}}$ MINUS $\underbrace{I_{\text{AVE.-HAPPY}}}_{I_{\text{STDD-HAPPY}}}$ **happy**

For the comparison of deviations in (113), the two comparisons along the scales of sadness and happiness are conducted with relevant averages of sadness and happiness, yielding two positive uses of gradable adjectives (see (114a)). This entailment pattern in (114a) is distinct from the pattern for usual comparatives (see (114b)), because usual comparatives do not particularly involve comparisons with relevant averages.

[33] This decompositional analysis of *more* explains why *more happy* in (113) cannot be replaced by *happier*. (i) is ungrammatical, due to cross-polar anomaly (see Kennedy 1999).

(i) *Mona is happier than Jude is sad. **Cross-polar anomaly**

[34] It is worth noting that the comparison of deviations that we discuss here is distinct from two other special types of comparatives illustrated in (i) (see Bartsch and Vennemann 1972b, McCawley 1976, Embick 2007, Bale 2008, Wellwood 2019).

In particular, (ib) does not have the same entailment pattern as sentences of 'comparison of deviations' do (see (114a). (ib) does not entail that Esme is pretty and Einstein is clever (see Bale 2008).

A thorough comparison of all these types of comparatives within our theory is left for another occasion.

(i) a. Ann is more tall than Bill is wide **Metalinguistic comparison**
$\rightsquigarrow$ It's more accurate (to say) that Ann is tall than that Bill is wide.

b. Esme is prettier than Einstein is clever. **Indirect comparison**
$\rightsquigarrow$ Esme's prettiness (if there's any) exceeds Einstein's cleverness (if there's any).

(114) a. Mona is more happy than Jude is sad $\vDash$ Mona is happy $\wedge$ Jude is sad
b. Mona is happier than Jude is. $\nvDash$ Mona is happy $\vee$ Jude is sad/happy

In brief, within our analysis, the core semantic contribution of *-er*/*more* is additivity. All uses of gradable adjectives involve comparison (or relativity), and comparison does not beed to be marked (cf. Klein 1980). *-er*/*more* is rather a discourse-salience marker.

With this unified comparison-based understanding for the uses of gradable adjectives, issues such as the limited distribution of gradable adjectives in measurement constructions and the compositional details of Mona-sentences can be naturally accounted for. Klein (1980)'s puzzle is also resolved.[35]

6 Comparing our analysis with the existing literature

We started our paper with a discussion on the fundamental assumption underlying comparatives. We explicitly assume that comparison is not only performed between scalar values (instead of entities or events), but also these are values on interval scales.

Compared with the canonical analysis sketched out in Section 2.1, our proposed analysis makes a similar move on the analysis of adjectives, i.e., as a relation between a scalar value and an entity (see the first key component of the canonical analysis in Section 2.1). However, our analysis takes a different and more degree-semantics-based way in addressing (i) formal items as involved in comparison and (ii) the implementation of comparison itself (cf. the second and third key components of the canonical analysis in Section 2.1). In addition, our analysis is distinct in terms of (iii) its choice of subtraction (cf. addition) in equations and (iv) its explicit support for 'encapsulation' theories (cf. 'entanglement' theories, of which the approach of 'degree plurality' is a recent representative). Below we address each of these four issues and justify our view.

[35]We are not exhaustive on the uses of gradable adjectives here. Unaddressed uses include *enough*/*too*-constructions, equatives, and superlatives. The current comparison-based view can be immediately extended to account for the semantics of *enough*/*too*-constructions (see our recent work for details). Equatives and superlatives do not necessarily assume interval scales and thus require a different line of analysis (see Anderson and Morzycki 2015, Solt 2016 and our other recent work). A thorough investigation across all these uses of (gradable) adjectives is left for future research.

6.1 Formal items as involved in comparison: $(0, 6']$ vs. $[6', 6']$

The meaning of a *than*-clause contributes the standard of a comparison, i.e., a formal item that undergoes comparison. Thus, the semantic derivation of a *than*-clause reflects how the notion of formal-items-under-comparison is approached in theories on comparison.

Within the canonical analysis (see (7a)), a *than*-clause addresses the set of all degrees such that the measurement of its target of predication meets or exceeds. As illustrated in (115a), suppose the height of Mary is exactly 6 feet, then this *than*-clause is analyzed as a set of degrees ranging from 0 to the height of Mary, i.e., $(0, 6']$. In contrast, within our current analysis (see (55)), a *than*-clause essentially just means the position on a scale that represents the measurement of the target of predication. As illustrated in (115b), here this *than*-clause amounts to an interval, i.e., $[6', 6']$.

(115) [[than Mary is (tall)]]

a. **Canonical analysis**: $\lambda d.$ the height of Mary meets or exceeds d i.e., $(0, 6']$

b. **Our analysis**: $[\textsc{precise-height}(\mathsf{Mary}), \textsc{precise-height}(\mathsf{Mary})]$ i.e., $[6', 6']$

The idea of involving '$(0, 6']$' – the set of all degrees that Mary's height meets or exceeds – in comparison is conceptually problematic in two aspects.

The first issue is manifested in the contrast between *tall* and *hot*. For $(0, 6']$ in (115a), the choice of '0' as the lower bound of this formal-item-under-comparison assumes an absolute zero point. This choice cannot be generalized to all interval scales, and it actually never matters in a comparison. Gradable adjective *hot* is associated with a scale of temperature, a non-ratio interval scale lacking a meaningful, absolute zero point. Thus, for a *than*-clause like *than the coffee is* (*hot*), the set of all degrees that the temperature of the coffee meets or exceeds should be a set like, say, $(-\infty, 85°C]$, instead of $(0, 85°C]$. For formal items like $(0, 6']$ in (115) and $(-\infty, 85°C]$, if used in a comparison, they would be compared with sets such as $(0, x']$ and $(-\infty, y°C]$, and the eventual comparisons would be performed between $6'$ and x' and between $85°C$ and $y°C$. In other words, for $(0, 6']$ and $(-\infty, 85°C]$, the information of their lower bound makes no contribution in a comparison. Therefore, the adoption of a MAX operator (see (10)) in the '>' analysis (cf. the 'A-not-A' analysis) to reduce the set $(0, 6']$ into a single degree, $6'$, is conceptually more warranted (see, e.g., Rullmann 1995 for discussion on maximality).

The second issue is manifested in the contrast between *tall* and *short*. The reasoning behind the analysis in (115a) implicitly assumes that the semantics of a *than*-clause is based on a measurement construction (e.g., *Mary is 6 feet tall*, see (116)). However, the same reasoning cannot work for a *than*-clause like *than Mary is short*, because gradable adjectives like *short* are not associated with ratio scales and cannot be used to form a measurement construction (see Sassoon 2010 and the discussion in Section 5.3).

(116) $\lambda d.$ the height of Mary $\geq d$ = $\lambda d.$Mary is tall to degree d = $\lambda d.$ Mary is d-tall

Given that both *tall* and *short* can be used in comparatives and appear as an elided part in their *than*-clause, the reasoning behind the semantic derivation of a *than*-clause should not be based on a measurement construction in the first place. Our current analysis avoids this pitfall by analyzing a *than*-clause as the short answer to its corresponding degree question (e.g., *how tall is Mary*, *how short is Mary*). Thus, [[than Mary is tall/short]] only means the position that represents the measurement of Mary on a scale of height (or shortness), not including anything else (like other measurements that the measurement of Mary meets or exceeds). In this sense, it is also conceptually problematic to start with a set like $(0, 6']$ or $(-\infty, 85°C]$ in analyzing a *than*-clause and apply a MAX operator later.

Based on this discussion on the semantic derivation of *than*-clauses, our conclusion is that formal items involved in comparison should be directly considered measurements themselves, instead of sets of degrees that some measurements meet or exceed.

6.2 Implementing comparison: set operation vs. subtraction

By arguing against the view that formal items involved in comparison are sets of degrees that some measurements meet or exceed (see Section 6.1), we also have to rule out the possibility that comparison can be implemented as performing a set operation (e.g., set difference) between two such sets of degrees.

As advocated from the beginning of this paper (see Section 1.2), we explicitly assume interval scales in analyzing the semantics of comparatives and make use of the formal properties of interval scales by adopting subtraction in implementing comparison. Throughout the paper (from Section 3 to Section 5), we have shown that the use of interval subtraction in implementing comparison is empirically advantageous, naturally accounting for the semantic derivation of *more-than* and *less-than* comparatives containing various kinds of numerical differentials as well as the information projection from a *than*-clause, which is a scope island and plays the role of subtrahend.

As a consequence of this switch from set operation to subtraction, the parallelism between **generalized quantifiers** in the domains of individuals and degrees is discarded (cf. e.g., Heim 2006a). As illustrated in (117), under the 'A-not-A' approach, *-er* seems to behave like *every*, and *-er … than Mary is* is similar to a universal quantifier.

(117) a. Every giraffe is from Africa. $\rightsquigarrow$ ***every giraffe*: a generalized quantifier**
[[every]] relates two sets of individuals:
$\{x \mid x \text{ is a giraffe}\} \subseteq \{x' \mid x' \text{ is from Africa}\}$
b. Bill is taller than Mary is. $\rightsquigarrow$ ***-er than Mary is*: a degree quantifier**
In the 'A-not-A' approach, [[-er]] relates two sets of degrees:
$\{d \mid \text{Mary is } d\text{-tall}\} \subset \{d' \mid \text{Bill is } d'\text{-tall}\}$

By discarding this parallelism, we predict that comparatives are not subject to any scopal interaction that a true generalized quantifier (e.g., *every giraffe*) should be subject to. This prediction is borne out, as shown by the contrast in (118).

(118) a. Every giraffe is not from Antarctica. **Scopal ambiguity**
(i) *every giraffe* > *not*: $\{x \mid x \text{ is a giraffe}\} \subseteq \{x' \mid x' \text{ is not from Antarctica}\}$
(ii) *not* > *every giraffe*: $\{x \mid x \text{ is a giraffe}\} \nsubseteq \{x' \mid x' \text{ is from Antarctica}\}$
b. Bill is not taller than Kate is. **No scopal ambiguity**
(i) #*-er than Kate is* > *not*: $\{d \mid \text{Mary is } d\text{-tall}\} \subset \{d' \mid \text{Bill is not } d'\text{-tall}\}$
(ii) *not* > *-er than Kate is*: $\{d \mid \text{Mary is } d\text{-tall}\} \not\subset \{d' \mid \text{Bill is } d'\text{-tall}\}$

On the other hand, by analyzing formal items involved in comparison as measurements themselves (e.g., *the* height of Mary) and using subtraction to implement comparison, we actually advocate a parallelism between **definite descriptions** in the domains of individuals and scalar values (see also Russell 1905, Heim 1985, Rullmann 1995, Beck 2010). Therefore, the interpretation of a comparative is reminiscent of a cumulative-reading sentence (see Brasoveanu 2013): both involve several definite descriptions, and there is no scopal interaction among them.

(119) a. My giraffe is 2 feet taller than the tree is. **Comparative**
$\rightsquigarrow$ **the height of my giraffe** exceeds **the height of the tree** by 2 feet
a′. My giraffe is taller than 20 feet. **Comparative**
$\rightsquigarrow$ **the height of my giraffe** exceeds **the definite value of 20 feet**.
b. Exactly three boys saw exactly five movies. **Cumulative reading**

$\rightsquigarrow$ **the maximal sum of boys**, the cardinality of which is 3, saw **the maximal sum of movies**, the cardinality of which is 5.

Thus, with regard to the implementation of comparison, our analysis is closer to the '>' approach than to the 'A-not-A' approach. Comparison is considered a relation between definite descriptions of measurements, characterized as definite descriptions of degrees in the '>' approach, and definite descriptions of intervals in ours.

6.3 Addition vs. subtraction

In this paper, we use the notion of interval to characterize definite descriptions of positions (i.e., measurements) on a scale in a generalized way, allowing for not-very-precise positions. Then interval subtraction provides a convenient technique to analyze the distance between two not-very-precise positions.

Interval arithmetic is developed to compute on not-very-precise scalar values and handle measurement errors. Thus, as illustrated in (120), interval addition and interval subtraction are not inverse operations. Only interval subtraction, but not interval addition, is suitable for analyzing the distance between two not-very-precise positions.

(120) a. $[2,3] + [4,5] = [6,8]$ **Interval addition**
b. (i) $[6,8] - [2,3] = [3,6]$ **Interval subtraction**
(ii) $[6,8] - [4,5] = [1,4]$ **Interval subtraction**

However, even for analyzing the distance between two precise measurements, the operation of subtraction is more suitable for compositional derivation than addition.

As illustrated in (121), with the use of addition (see e.g., Hellan 1981, von Stechow 1984 and analyses with the use of inequalities '>/≥/</≤', see also Beck 2011 for a summary), the numerical differential is constantly added to the lower measurement between the two under comparison (here Mary's height). Thus, in *more-than* comparatives, addition is performed on the differential and the measurement associated with the *than*-clause (see (121a-ii)), but in *less-than* comparatives, addition is performed on the differential and the measurement associated with the matrix clause (see (121b-ii)). This imbalance potentially creates an additional compositional issue.

In contrast, subtraction is always performed between the two measurements under comparison, the one associated with the matrix clause constantly playing the role of minuend and the one associated with the *than*-clause constantly playing the role of subtrahend (see (121a-i) and (121b-i)). Therefore, subtraction allows for a uniform compositional derivation for both *more-than* and *less-than* comparatives.

(121) Context: Kate is precisely 6 feet 2 inches tall, and Mary is precisely 6 feet tall.

a. Kate is exactly 2 inches **taller** than Mary is.

(i) $\underbrace{\textsc{precise-measure}(\mathsf{Kate})}_{\text{Minuend}} - \underbrace{\textsc{precise-measure}(\mathsf{Mary})}_{\text{Subtrahend}} = 2''$ **Subtraction**

(ii) $\textsc{precise-measure}(\mathsf{Kate}) = \textsc{precise-measure}(\mathsf{Mary}) + 2''$ **Addition**

b. Mary is exactly 2 inches **less tall** than Kate is.

(i) $\underbrace{\textsc{precise-measure}(\mathsf{Mary})}_{\text{Minuend}} - \underbrace{\textsc{precise-measure}(\mathsf{Kate})}_{\text{Subtrahend}} = -2''$ **Subtraction**

(ii) $\textsc{precise-measure}(\mathsf{Mary}) + 2'' = \textsc{precise-measure}(\mathsf{Kate})$ **Addition**

6.4 Entanglement vs. encapsulation: comparison with the approach of 'degree plurality'

Fleisher (2016) divides semantic theories on comparatives into two camps: '**entanglement**' theories vs. '**encapsulation**' theories. Essentially, 'encapsulation' theories conform to the ideal of Beck (2010): at the end of the calculation of a *than*-clause, we hold in our hand *the* unique value that will serve as the standard for comparison. Thus at the matrix-clause level, a comparative encodes only one comparison, the one with this unique standard. Our interval-subtraction-based theory is a typical encapsulation theory. As illustrated in (122a), the derived semantics of the *than*-clause is a unique measurement represented in terms of an interval: $[16', 20']$. This sentence expresses the comparison between the height of my giraffe and this interval.

In contrast, 'entanglement' theories hold the view that the derivation of a *than*-clause potentially generates multiple scalar values (e.g., multiple degrees), so that at the matrix-clause level, a comparative can express multiple comparisons, each involving one of those scalar values (from the derivation of the *than*-clause) as its standard.

The '**degree plurality**' theory is a typical entanglement theory (see Beck 2014, Dotlačil and Nouwen 2016 and a similar idea in Heim 2006a). Within the 'degree plurality' theory, as illustrated in (122b), the derived semantics of the *than*-clause is a sum of degrees: $16' \oplus 18' \oplus 20'$. Then with the use of a distributivity operator, this sum of degrees is distributed at the matrix-clause level, leading to multiple comparisons.

(122) Context: My giraffe is 21 feet tall. There are three trees, which are 16 feet, 18

feet, and 20 feet tall, respectively.

My giraffe is taller than every tree is.

a. $[\![$than every tree is (tall)$]\!] = [16', 20']$ **our 'interval subtraction' theory**
$[\![$(122)$]\!] \Leftrightarrow$ HEIGHT(my giraffe) $\subseteq \iota I'[I' - [16', 20'] = (0, +\infty)]$

b. $[\![$than every tree is (tall)$]\!] = 16' \oplus 18' \oplus 20'$ **the 'degree plurality' theory**
$[\![$(122)$]\!] \Leftrightarrow \forall d \sqsubseteq_{\text{ATOM}} 16' \oplus 18' \oplus 20'$[the height of my giraffe $> d$]
(DIST $\overset{\text{def}}{=} \lambda D_d.\lambda P_{\langle dt \rangle} \forall d[d \sqsubseteq_{\text{ATOM}} D \rightarrow P(d)]$)

The 'degree plurality' theory is dubious for a few reasons. First, the 'degree plurality' theory still faces the issue of unattesed scopal interaction (see the discussions in Section 5.1). In (123) (which repeats (85b)), the 'degree plurality' theory generates two readings for this sentence, but the '$\forall > \exists$' reading is actually unattested (see (123b)).

(123) Someone is smarter than everyone is. **Clausal comparative: unambiguous**
Suppose $D =$ SMARTNESS$(x_1) \oplus$ SMARTNESS$(x_2) \oplus \ldots \oplus$ SMARTNESS(x_n)

a. $\exists x[\text{human}(x) \wedge \forall d \sqsubseteq_{\text{ATOM}} D[\text{SMARTNESS}(x) > d]]$ $\exists > \forall$: attested reading

b. $\forall d \sqsubseteq_{\text{ATOM}} D[\exists x[\text{human}(x) \wedge \text{SMARTNESS}(x) > d]]$ $\forall > \exists$: unattested reading

Second, the interpretation of a negative comparative is not parallel with that of a negative sentence containing a plural definite in the domain of entities, undermining the plausibility of analyzing a *than*-clause as a degree plurality.

Due to homogeneity effects (see Križ 2016 for a recent discussion), the interpretation of a negative sentence containing a plural definite like *the books* demonstrates a three-way distinction pattern, as illustrated in (124). In particular, the sentence is considered neither true nor false in a context where Mary read some, but not all of the books. However, the interpretation of a negative comparative is not subject to this kind of homogeneity effects, as illustrated in (125). The contrast between (124) and (125) suggests that even if the use of DIST and the issue of scopal interaction can be somehow circumvented, it is still problematic to consider a *than*-clause a degree plurality.

(124) Mary didn't read the books. **Subject to homogeneity effects**
True if Mary read none of the books.
False if Mary read all of the books.
Neither true nor false if Mary read some, but not all of the books.

(125) My giraffe is not taller than every tree is. **Not subject to homogeneity effects**

My giraffe is not taller than all trees are. **Not subject to homogeneity effects**
True if my giraffe is taller than no trees.
False if my giraffe is taller than all trees.
True if my giraffe is taller than some, but not all trees.

Third, a distinction on the answerhood to *wh*-questions containing universal quantifiers (e.g., *every boy*) vs. definite plurals (e.g., *the boys*) also questions the 'degree plurality' analysis for *than*-clauses.

In (126), the degree question *how tall are the boys* (which contains a plural DP) can be answered by a fragment answer like *5 feet, 5 feet 6 inches, and 6 feet* (*respectively*), while such a fragment answer sounds degraded for a degree question like *how tall is every boy* (which contains a universal quantifier). This contrast suggests that even if *5 feet, 5 feet 6 inches, and 6 feet* is indeed a degree plurality (i.e., a sum of degrees) and expressions like *than the boys are* (*tall*) indeed denote degree pluralities, it is unlikely that *than every boy is* (*tall*) also denotes a degree plurality. Instead, *between 5 and 6 feet*, which indicates an interval, is a good fragment answer here. Similar observations are available for other *wh*-questions. As illustrated in (127), while the sum *Madame Bovary, Jane Eyre, and Emma* is a felicitous fragment answer to *what did the boys read* (which contains a plural DP), it cannot be used to answer *what did every boy read* (which contains a universal quantifier). However, *a novel* is a good fragment answer to *what did every boy read* in this case. We do not delve into the details of fragment answerhood here, but the upshot is clear. For a *than*-clause containing a universal quantifier (e.g., *than every tree is* (*tall*)) instead of a plural DP, it is unlikely that the whole *than*-clause denotes a degree plurality.

(126) Context: Al, Bill, and Cal are 5 feet, 5 feet 6 inches, and 6 feet tall respectively.

a. – How tall are **the boys**? ✓ 5 feet, 5 feet 6 inches, and 6 feet (respectively)
b. – How tall is **every boy**? ? 5 feet, 5 feet 6 inches, and 6 feet (respectively)
b′. – How tall is **every boy**? ✓ between 5 and 6 feet

(127) Context: Al read *Madame Bovary*, Bill read *Jane Eyre*, and Cal read *Emma*.

a. – What did **the boys** read? ✓ *Madame Bovary*, *Jane Eyre*, and *Emma*
b. – What did **every boy** read? # *Madame Bovary*, *Jane Eyre*, and *Emma*
b′. – What did **every boy** read? ✓ a novel

Finally, we would like to cautiously point out that the very notion of 'degree plurality' might lack enough empirical support. The example in (128a) seems to give evidence that the notion of degree plurality is independently needed in natural language, since this sentence seems to have a cumulative reading (see Dotlačil and Nouwen 2016). However, it is likely that there is a silent *respectively* in this case (see (128b)). If it is so, then as a *respectively*-sentence, it is distinct from a typical cumulative-reading sentence. (129) and (130) show that in *respectively*-sentences, the order among the items conjoined by *and* matters, suggesting that in these cases, the use of *and* does not lead to sums of items as involved in typical cumulative-reading sentences.

(128) a. These three trees are 16 feet, 18 feet, and 20 feet tall. **cumulative?**
b. These three trees are 16 feet, 18 feet, and 20 feet tall, respectively. **not truly cumulative**

(129) John and Bill married Susan and Kate (respectively). **not truly cumulative**
$\rightsquigarrow$ John married Susan, and Bill married Kate. **order matters**

(130) The newborn's weight, length, and head circumference are 3.4 kg, 49.7 cm, and 33.6 cm, (respectively). **not truly cumulative**

Among these challenges to the 'degree plurality' theory, the issue on unattested scopal ambiguity (i.e., the first issue) is presumably carried over to other entanglement theories, because entanglement theories, by definition, involve the derivation and distribution of multiple distinct scalar values from a *than*-clause. Given that clausal comparative lack scopal ambiguity, entanglement theories are less suitable than encapsulation theories in the analysis of *than*-clauses and clausal comparatives.

That being said, natural language degree-related phenomena beyond English clausal comparatives might still call for entanglement theories. As shown in (131), the degree question *how tall is every boy* can have a fragment answer that denotes a single, not-very-precise measurement, but it can also have a pair-list answer that involves multiple measurements. Thus a sufficiently good characterization of degree questions (and other phenomena like phrasal comparatives in (85a)) has to go beyond encapsulation theories alone.

(131) Context: Al, Bill, and Cal are 5 feet, 5 feet 6 inches, and 6 feet tall respectively.
How tall is every boy?
a. Between 5 and 6 feet. **Fragment answer**
b. Al is 5 feet tall; Bill is 5 feet 6 inches tall; Cal is 6 feet tall. **Pair-list answer**

As a typical encapsulation theory, our derivation for the sentential semantics of a clausal comparative is eventually only based on the upper and lower bounds of the interval associated with the *than*-clause, which is exactly the information encoded in a most informative fragment answer to the corresponding degree question (see (131a)). There seems to be information loss in this fragment answer, but we believe that this information loss reflects the actual semantics of English clausal comparatives that native speakers have access to. After all, English clausal comparatives are not as expressive as phrasal comparatives (see also Kennedy 1999).

7 Conclusion

In this paper, we have presented a difference-based approach to the semantics of comparatives. Comparatives encode a subtraction relation among three scalar values: two measurements along a relevant interval scale and the difference between them.

In implementing this difference-based approach, we have innovated (i) the interval-based technique of characterizing scalar values and differences for natural language phenomena and (ii) the view on the semantic contribution of comparative morpheme *-er/more*. The technique of interval subtraction allows us to deal with subtraction equations that involve generalized, potentially not-very-precise scalar values. Comparative morpheme *-er/more* is considered an additive particle that contributes additivity by expressing a positive increase on a discourse-salient standard. The combination of these two ideas leads to our interval-subtraction-based analysis.

We have shown that our proposed analysis of comparatives naturally accounts for complex cases involving numerical differentials and *than*-clause-internal quantifiers, deriving their truth conditions in a most natural, precise, and uniform way. The proposed analysis also accounts for the scope island status and the monotonicity of *than*-clauses. Furthermore, our analysis accounts for Klein's puzzle within degree semantics plus a unified comparison-based picture for various uses of gradable adjectives. Instead of encoding or marking comparison per se, *-er/more* rather marks the discourse status of the scalar value serving as the standard in comparison.

Our work makes good use of existing mathematical tools (i.e., interval subtraction) and is based on the background assumption that the theory on measurement contributes to our understanding of human conceptualization and their linguistic encoding. We believe that our work will inspire future development on degree semantics.

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