Compounded Scales*

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1   Introduction

Most semantic analyses of gradable adjectives have assumed that there is a direct link between adjectives and degrees, or alternatively between adjectives and delineations, extents, or intervals which exhibit many of the same formal properties as degrees (see Bartsch and Vennemann 1972; Seuren 1973, 1978; Cresswell 1976; Klein 1980, 1982, 1991; Hellan 1981; von Stechow 1984a,b; Heim 1985, 2000; Bierwisch 1987; Kennedy 1999; Hackl 2000; Kennedy and McNally 2005; Schwarzschild and Wilkinson 2002; Fox and Hackl 2006 among others — a notable exception is Wheeler III 1972). However, there are significant problems with this hypothesis when it comes to providing a compositional interpretation for comparative sentences that involve conjunction like those in (1).

(1)   a.   Seymour is more handsome and talented than Patrick is.
       b.   This floorboard is less long and wide than that floorboard is.

The problems are two-fold. First, adjectives like *handsome* and *talented* are not commensurable—they involve different types of degrees. This incommensurability becomes problematic when the two adjectives are combined by *and*. Second, even with adjectives that are commensurable (such as *long* and *wide*), interpreting the two adjectives as directly involving degrees yields truth conditions—as I will show in section 2—that are dependent on only one of the adjectives. In (1b), such truth conditions would be dependent solely on measurements of width—measurements of length would be inconsequential. Obviously such an analysis is not empirically supported.

   One alternative to linking adjectives directly to degrees is to interpret them as more primitive building blocks from which scales and degrees can be constructed (see Bale 2006, 2011). This

paper outlines such an alternative. It proposes that gradable adjectives should be treated as binary relations between individuals, not between individuals and degrees. As noted by Cresswell (1976) and Klein (1991) among others, such relations can be converted into scales as long as they are transitive and asymmetric. Furthermore, this conversion can be incorporated into the interpretation of the comparative morphemes (*more* and *less*) independent of the adjectives. Unlike the traditional degree analysis, this type of interpretation provides adequate truth conditions for the sentences in (1). Given the boolean interpretation of *and* as intersection, the conjunction of the two binary relations is itself a binary relation. This compounded binary relation can be converted into a scale by the comparative morphemes in much the same way that the non-compounded relations are converted. The result is a comparison that involves a compounded scale (a scale that encodes two gradable properties). Such comparisons accurately account for the truth conditions of the sentences in (1).

The outline of this paper is as follows. In section 2, I outline why the traditional degree analysis cannot straightforwardly account for sentences like those in (1). In section 3, I discuss whether gapping might be able to explain the interpretation of conjoined adjectives. Ultimately, I demonstrate that gapping does not make the correct empirical predictions. In section 4, I show how interpreting adjectives as binary relations provides a better account of conjoined adjectives. However, there are problems for this account as well, specifically when it comes to data with differentials and disjunction. In section 5, I discuss this type of data as well as a possible alternative solution which would involve interpreting *and* as a non-boolean operator (see Winter 1995). I further demonstrate that even if this alternative is viable, it cannot account for all of the data and thus, derived compounded-scale might be needed nonetheless. Finally, section 6 concludes this paper.

2   The degree approach to conjoined adjectives

Broadly speaking (and abstracting away from many important details), there are two main hypotheses about how adjectives connect degrees to entities like people, tables and couches: the measurement-function hypothesis (Bartsch and Vennemann, 1972; Kennedy, 1999) and the degree-relation hypothesis (Cresswell, 1976). As demonstrated in this section, both approaches run into problems when trying to account for sentences with conjoined adjectives. However, before discussing such difficulties, let me first give a rough outline of both of these theories.

Kennedy (1999), following Bartsch and Vennemann (1972), suggests that adjectives should be interpreted as functions from entities to degrees. With such an interpretation, an adjective like *long* can apply directly to an entity like a couch yielding a measurement of length. In this type of theory, sentences like (2) would have truth conditions like those in (3), where t is the table, c is the couch, MAX is a function that picks out the greatest degree from a set, and $>_m$ and $\leq_m$ are the ordering relations.[1]

(2)   This table is longer than the couch is.

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[1]The truth conditions of this simple sentence could also be represented by comparing the results of the measure functions directly. In fact this is what Bartsch and Vennemann (1972) do. However, as discussed in Kennedy (1999) abstracting a degree variable and forming a set is needed to account for more complex sentences.

(3) $\mathrm{MAX}(\{d : d \leq_m [\![long]\!](t)\}) \; >_m \mathrm{MAX}(\{d : d \leq_m [\![long]\!](c)\})$
       $= [\![long]\!](t) \; >_m \; [\![long]\!](c)$

These truth conditions state that the table is longer than the couch if and only if the measurement of the table's length is greater than the couch's.

In a slight variation of a similar approach, Cresswell (1976) proposes that adjectives should be interpreted as relations between individuals and degrees. For example, an adjective such as *long* can be interpreted as a set of ordered pairs where the first member is an entity and the second is a degree. With *long*, an entity is related to a degree if and only if the the measurement of the entity's length is equal to or greater than that degree.[2] Given this interpretation, a sentence like (2) can be assigned the truth conditions in (4).

(4) $\mathrm{MAX}(\{d : [\![long]\!](t, d)\}) \; >_m \mathrm{MAX}(\{d : [\![long]\!](c, d)\})$

Similar to (3), these truth conditions state that the table is longer than the couch if and only if the largest degree representing the table's length is greater than the largest degree representing the couch's length.

For the remainder of this section, the notation 'x *is* d-ADJ' will be used to represent both the measurement function and degree-relation hypotheses. Thus, when considering the measure function approach 'x *is* d-ADJ' will translate as $d \leq_{adj} [\![\mathrm{ADJ}]\!](x)$. On the other hand, when considering the degree relation approach, 'x *is* d-ADJ' will translate as $[\![\mathrm{ADJ}]\!](x, d)$.

With this notational convention in mind, reconsider the two comparatives in (1), repeated in (5) below.

(5)   a.   Seymour is more handsome and talented than Patrick is.
       b.   This floorboard is less long and wide than that floorboard is.

The sentence in (5a) can be roughly paraphrased as stating that Seymour is more handsome than Patrick and more talented. The sentence in (5b) can be roughly paraphrased as stating that "this floorboard" is less long than "that floorboard" and also less wide. Applying the degree analysis to these sentences in the same way that we applied it to the sentences without conjoined adjectives would yield truth-conditions like the ones given in (6a) and (6b). (Note, for the sake of simplicity, we will represent the truth conditions of *less* using the "less-than" symbol instead of the "greater-than" symbol.)

(6)   a. $\mathrm{MAX}(\{d : \text{Seymour is } d\text{-handsome and } d\text{-talented}\}) > \mathrm{MAX}(\{d : \text{Patrick is } d\text{-handsome and } d\text{-talented}\})$
       b. $\mathrm{MAX}(\{d : \text{this floorboard is } d\text{-long and } d\text{-wide}\}) < \mathrm{MAX}(\{d : \text{that floorboard is } d\text{-long and } d\text{-wide}\})$

However these truth conditions do not accurately reflect speaker intuitions. The problem for (6a) is that the set of degrees would be empty. To put it intuitively, there are no degrees such that Seymour

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[2]Cresswell actually proposes that the relation of entities to degrees is one to one. An entity is related to a degree if and only if the individual's length is equal to that degree. However, others who adopt this type of interpretation usually adopt the relation specified here (see von von Stechow 1984a).

is handsome to that degree and talented to that degree. Degrees of handsomeness and talent are assumed to belong to different scales. To put it more formally, if *and* is interpreted as intersection (Boolean meet) then when the two adjectives are conjoined the result would be an empty set. The interpretations of the adjectives would have disjoint co-domains and hence the ordered pairs that represent the relation or function would also be disjoint. The emptiness of the conjoined adjectives would trivially entail that Seymour is not related to any degree within the conjoined adjectives. Hence, the function that picks out the maximal degree would have nothing to pick-out. As a result the sentence should be odd or undefined. Yet the sentence in (5a) is perfectly interpretable.

The problem becomes even more interesting when we consider adjectives that are commensurable such as the ones in (5b). Although we no longer necessarily have a problem with the empty set as we did with (5a),[3] the interpretation assigned to (5b) is nonetheless infelicitous. Consider the formula in (6b). Unlike the previous formula, there are degrees that are both degrees of length and width. Thus there are degrees d where "this floorboard" is d-long and d-wide. In fact, if we assume that, like all floorboards, "this floorboard" is longer than it is wide, it follows that for any degree d such that "this floorboard" is d-wide, it is also d-long (although the opposite does not hold). Hence, the set $\{d :$ this floorboard is d-long and d-wide$\}$ is equivalent to the set $\{d :$ this floorboard is d-wide$\}$. By similar reasoning, the set $\{d :$ that floorboard is d-long and d-wide$\}$ is equivalent to the set $\{d :$ that floorboard is d-wide$\}$. To put this in more formal terms, the intersection of the relations *wide* and *long* when restricted to floorboards is identical to the relation *wide* when restricted to floorboards. As a result of these two equivalences, one can replace the truth conditions in (6b) with the equivalent formula in (7).

(7)   MAX$(\{d :$ this floorboard is d-wide$\}) <$ MAX$(\{d :$ that floorboard is d-wide$\})$

However, the formula in (7) also represents the truth conditions for (8).

(8)   This floorboard is less wide than that floorboard is.

In other words, both the formulae in (6b) and (7) describe truth conditions that are based on a comparison of width only. Length is not relevant. This is obviously a problem. Intuitively, (5b) expresses a comparison based on both length and width.

In summary, the problem for the degree approach to adjectives is two-fold. Combining two adjectives directly with Boolean *and* either predicts that sentences should be anomalous (when they are intuitively perfectly well-formed) or that the truth conditions should be based on only one of the gradable properties denoted by the adjectives.

3   Reasons to think there is no ellipsis.

One potential solution to the problem described in section 2 is to hypothesize that the conjunction is not conjoining two adjectives but rather is conjoining two comparative expressions. For example,

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[3]This is not quite true. If the relation between individuals and degrees is one to one (instead of one to many) then the result of intersecting *long* and *wide* would still be empty, unless there are individuals that are equally as wide as they are long. However, for the sake of argument, I will focus on the *one-to-many* interpretations of the adjectives.

a sentence such as (9a) might only differ from (9b) in that the second instance of *more* in the latter sentence is unpronounced due to some kind of gapping, otherwise the two sentences are syntactically and semantically identical.[4]

(9)   a.   Seymour is more handsome and more talented than Patrick is.
       b.   Seymour is more handsome and talented than Patrick is.

However, there are problems with a gapping analysis. In other types of gapping constructions, the addition of adjuncts and other material after the gapped and antecedent segments does not affect the acceptability of the sentences (as long as parallelism is maintained between the two conjoined phrases). Consider the sentences in (10).

(10)   a.   John fought with Betty and Fred with Suzan.
       b.   John fought with Betty on Tuesday and Fred with Suzan on Wednesday.
       c.   John fought with Betty in the mountain park on Tuesday and Fred with Suzan in the fountain park on Wednesday.

The sentence in (10a) is a typical example of gapping. The sentences in (10b) and (10c) are identical to (10a) except for the addition of time and/or location adverbials after the antecedent segment and the gapped segment respectively. This additional material does not affect the acceptability of the sentences. In fact, for some English speakers who find gapped-constructions slightly anomalous, such additions improve acceptability.

Other potential examples of gapping demonstrate similar kinds of effects. For example, consider the sentences in (11).

(11)   a.   This fork and knife are gifts from my mother.
       b.   This fork and this knife are gifts from my mother.

As pointed out to me by Brendan Gillon (p.c.), there is some evidence supporting the hypothesis that (11a) is syntactically and semantically identical to (11b) except for an unpronounced determiner in the first sentence. The conjoined phrase in (11a) yields plural agreement when combined with an auxiliary: in (11a) the auxiliary is *are*. This is a common property of conjoined noun phrases, as shown in (11b). Furthermore the determiner in (11a) is singular. If the phrase *knife and fork* were responsible for the plural agreement marking on the auxiliary, then one would expect this conjoined noun phrase to combine with a plural determiner like *these*. The singular marking on the determiner in (11a) suggests that the determiner *this* is combining with the singular noun *fork* rather than a conjoined phrase *fork and knife*. All of this evidence is consistent with the hypothesis that the sentence in (11a) has two demonstrative determiners underlyingly (for an alternative analysis, see Link 1983).

Given these facts, consider the sentences in (12).

(12)   a.   This fork and knife that I used this morning are gifts from my mother.

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[4]For exposition purposes, gapping will be discussed as if it involves elision, as hypothesized by Ross (1970); Hartmann (2001) among others. However, the criticisms discussed in this section are theory independent. They are equally applicable to a theory such as Johnson's, 2009, which relies on movement and variable extraction.

b. This fork that I used yesterday and knife that I used this morning are gifts from my mother.

In (12a), the relative clause restrictor appears after the second noun in the conjunction. It is understood as restricting both of the nouns that precede it. This is similar to the *than*-clause in (5) which appears after the two adjectives. This similarity is even more striking given other parallels between *than*-clauses and relative clause restrictors (see Bhatt and Pancheva 2004). Notice that the elision of the determiner in (12b) remains acceptable when another relative clause restrictor is inserted immediately after the first noun.

These facts about gapping constructions contrast with the comparative constructions discussed earlier. If sentences such as (1) above—repeated in (13)—contain an elided comparative in the second half of the conjunction, then it would be expected that the addition of other material between the antecedent and the remnant would not affect the acceptability of the sentences.

(13)   a.   Seymour is more handsome and talented than Patrick is.
           b.   This floorboard is less long and wide than that floorboard is.

This prediction is not borne out. Although the sentences in (13a) and (14a) are completely acceptable, the sentence in (14b) is not as acceptable.

(14)   a.   Seymour is more handsome than Dan is and more talented than Patrick is.
           b. ?Seymour is more handsome than Dan is and talented than Patrick is.

Most speakers find the sentence in (14b) to be odd. A few speakers accept the sentence only with a marked intonational pattern (a slight pause before the second adjective followed by heavy emphasis). By *marked* I mean that this intonational pattern is not required for the acceptability of (13a). This is an important point and bears repeating. The sentences in (13) do not require a pause before pronouncing the second adjective nor do they require that any intonational emphasis be put on the second adjective. Insofar as speakers accept (14b), a pause is required as well as intonational emphasis.

Similar to the sentence in (14b), speakers find the sentence in (15b) odd despite the acceptability of (13b) and (15a).

(15)   a.   This floorboard is less long than the floorboard by the table is and less wide than the floorboard by the chair is
           b. ??This floorboard is less long than the floorboard by the table is and wide than the floorboard by the chair is

Unlike in (14b), a change in the intonational pattern does not have an effect on the acceptability of (15b). In fact, this seems to be the case in general with *less*-comparatives. Consider the sentences in (16a).

(16)   a.   Jen is less intelligent and beautiful than Morag is.
           b.   Jen is less intelligent than Betty is and less beautiful than Morag is.
           c. ??Jen is less intelligent than Betty is and beautiful than Morag is.

As with the sentences in (15), the sentences in (16) demonstrate that it is possible to have an extra *than*-clause inserted after the first adjective, but only when the comparative morpheme *less* is repeated in the second clause (hence the contrast between (16b) and (16c)).

The degraded acceptability of (14b), (15b) and (16c) is unexpected if gapping were involved in these types of constructions. One might object at this point that comparing *than*-clauses to time adverbials and relative clauses is a bit misleading. Clearly there is a difference between these types of constructions. One might even object that adverbials and relative clauses are truly optional unlike *than*-clauses. However as those of us who work on comparatives know well, this is not a completely accurate description. *Than*-clauses are in fact syntactically optional. For example, consider the second sentences in (17).

(17)   a.   Morag is beautiful. However, Jen is more beautiful.
        b.   Morag is tall. However, Jen is taller.

These sentences have comparatives without any *than*-clause. The object of comparison is provided contextually via the preceding sentence. *Than*-clauses are more like adjuncts than it would first appear. However, the point about gapping and comparatives can be made even without using *than*-clauses. If we use gradable properties that can vary over time or space (e.g., aggression and carefulness), then we can construct examples using the same kind of adjuncts that were used in prototypical gapping constructions. Consider the sentences in (18).

(18)   a.   Bob is an inexperienced skier. He is not very careful on the sharp turns and not very comfortable on the moguls but . . .
        b.   . . . Jen is less careful on the sharp turns and less comfortable on the moguls.
        c.   ??. . . Jen is less careful on the sharp turns and comfortable on the moguls.

Unlike (18b), it is extremely difficult to get a reading of (18c) that implies that Jen is less comfortable than Bob on the Moguls. As with the previous examples, this oddity would not be expected if gapping or ellipsis were involved.

In addition to the evidence above, there is at least one other piece of evidence that speaks against a gapping analysis. In prototypical gapping constructions (constructions that involve the elision of an auxiliary), comparative morphemes cannot be elided without also eliding the adjectives they modify. Consider the sentences in (19) where '_' marks the gap.

(19)   a.   Donald and Patrick are quite handsome and talented but [Seymour is more handsome than Donald] and [Bill _ more talented than Patrick].
        b.   Donald and Patrick are quite talented but [Seymour is more talented than Donald] and [Bill _ than Patrick].
        c.   * Donald and Patrick are quite handsome and talented but [Seymour is more handsome than Donald] and [Bill _ talented than Patrick].

The sentence in (19b), where the comparative morpheme is elided along with the adjective it modifies, is as acceptable as the sentence in (19a), where only the copula is elided. However, the sentence (19c), where only the comparative morpheme is elided, is unacceptable. It appears as if canonical gapping constructions do not permit the elision of the comparative morpheme without

also eliding the adjective. A gapping analysis of the sentence in (13) would have to hypothesize that such an elision would be permissible.

In summary, there are two challenges for a gapping analysis of sentences with conjoined gradable adjectives as in (1). First, unlike other gapping constructions the addition of adjuncts and other material between the two conjoined adjectives decreases the acceptability of the sentence. Second, in canonical gapping constructions, the comparative morphemes cannot be elided without also eliding the adjective.

4   Adjectives as binary relations

One alternative to interpreting adjectives as directly involving degrees is to interpret them as relations between two individuals. This alternative is foreshadowed in the work of Cresswell (1976), Klein (1991) and Bale (2006, 2008), and explicitly adopted in Bale (2011). Certain types of relations (namely transitive, asymmetric relations)[5] can be used to create scales. These scales, in turn, can be used as a basis for comparison in comparative sentences. In the current context, the importance of this alternative is that it provides a straightforward account of sentences with conjoined adjectives. The result of intersecting two transitive, asymmetric relations is another transitive, asymmetric relation. The intersected relation can be used to create a scale that provides adequate truth conditions for sentences with conjoined adjectives.[6]

4.1   The basic idea

Cresswell (1976), basing much of his discussion on measurement theory in mathematics (see, for example, Krantz et al. 1971), demonstrates that quotient structures—partial orders of sets—can be built from more basic relations and, furthermore that these quotient structures can serve as scales in the analysis of degree constructions.[7] The basic idea can be best demonstrated with an example.

Cresswell proposed that gradable adjectives are associated with an underlying binary relation, such as the following relation which expresses a strict weak ordering based on beauty (*strict weak orders* are transitive and asymmetric).

$$\{\langle x, y \rangle : x \text{ has more beauty than } y\}.$$

For convenience, let's label this relation as β.[8] By taking the quotient structure of this basic binary

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[5]Alternatively, the relations could be transitive and reflexive as discussed in Cresswell 1976 and Klein 1991 and proposed in Bale 2011.

[6]It should be noted that much of the discussion in this section is non-coincidentally similar to the discussion of multidimensional adjectives like *clever* in Klein 1980.

[7]An alternative way of deriving scalar meanings based on the same ideas expressed in this section—modulo the incorporation of possible worlds and the idea of a possible individual (pairings between worlds and entities)—is discussed in Schwarzschild, this volume. See also Schwarzschild 2013.

[8]A quick note is required here to dispel a potential source of confusion. On the surface, from the way the relation β is stated and describe, it might seem a bit circular to use the language of comparison (words like *more*) to describe a relation that eventually will be used to provide a semantics for comparison. However, it should be kept in mind that the

relation, a linear scale can be formed. Forming such a scale (aka, quotient structure) involves at least three steps.

1. **Form equivalence classes based on the original binary relation.** This can be done by associating each member in the domain with a set of elements that are indistinguishable from it, relative to the binary relation. Two elements x and y are indistinguishable if and only if there is no z that bears a relation to x but not y or vice versa (i.e., given a binary relation β, x and y are indistinguishable iff $\neg \exists z.(\langle x, z \rangle \in \beta \wedge \langle y, z \rangle \notin \beta) \vee (\langle z, x \rangle \in \beta \wedge \langle z, y \rangle \notin \beta) \vee (\langle y, z \rangle \in \beta \wedge \langle x, z \rangle \notin \beta) \vee (\langle z, y \rangle \in \beta \wedge \langle z, x \rangle \notin \beta))$.

2. **Use the set of equivalence classes to form the domain of the scale/quotient structure.** The domain of the quotient structure is the set of all equivalence classes that can be formed from the members of the original binary relation.

3. **Rank the equivalence classes in a way that is congruent to the original binary relation.** For any two equivalence classes X and Y, X is ranked above Y in the quotient structure iff for every member z of X and w of Y, $\langle z, w \rangle$ is a member of the original binary relation.

Consider the following example. Let's begin by outlining a particular transitive and asymmetric relation. To save time and space, such a relation can be specified graphically, as it is in the far left hand graph labelled as $>_\beta$ in (20).

(20)

$$>_\beta \qquad\qquad >_{\beta/\sim} \qquad\qquad >_{D_\beta}$$
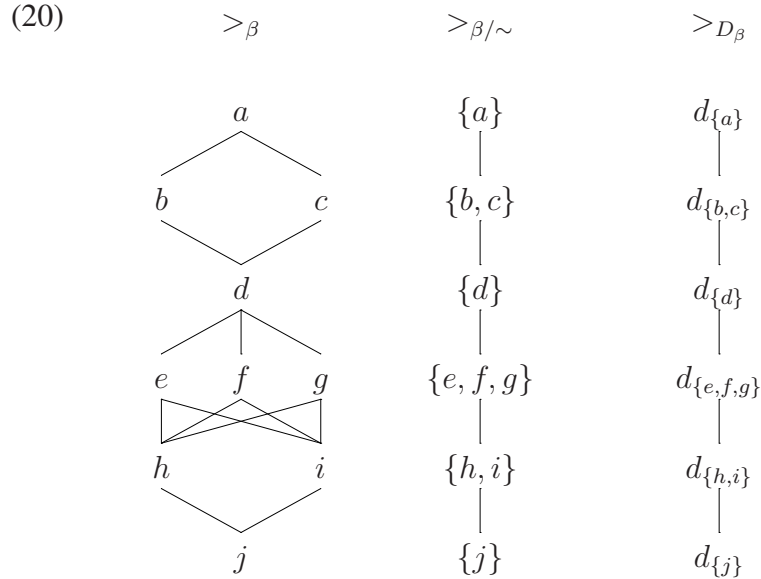
comparative language is being used to describe a non-linguistic concept. Although the comparative-language is useful in describing this concept, such a language is by no means necessary for having this concept. Even those without language (monkeys, cats, dogs) are able to compare two objects or individuals in terms of a certain property (to tell which food bowl has more, or which potential mates are more suitable/beautiful). The relation between individuals is formed from this language-independent concept of comparison rather than from the language dependent phrase *has more beauty than*. There is no circularity in basing a scale used to interpret comparative sentences on such an underlying concept (see Bale 2008 for a discussion).

The graph $>_\beta$ in (20) can be read in the following way. For any two members of the graph, call them x and y, if x appears above y and there is a downward path of lines from x to y, then x has as much beauty as y but not vice versa (i.e., $\langle x, y \rangle \in \beta$ but $\langle y, x \rangle \notin \beta$). For example, according to the graph in $>_\beta$, f has more beauty than h, i and j, (i.e., $\langle f, h \rangle \in \beta$, $\langle f, i \rangle \in \beta$, and $\langle f, j \rangle \in \beta$), but not more beauty than a, b, c, d, e, and g. However, only a, b, c and d have more beauty than f. e and g are not related to f (i.e., $\langle f, g \rangle \notin \beta$, $\langle g, f \rangle \notin \beta$, $\langle f, e \rangle \notin \beta$ and $\langle e, f \rangle \notin \beta$).

To construct a quotient structure from β (represented as $>_{\beta/\sim}$ above), one first needs to determine what the relevant set of equivalence classes is. As specified in step one, this is done by forming sets that contain all the members of the domain that are indistinguishable from one another. For example, e, f and g are indistinguishable since they relate to all other members of the domain in the same way in terms of beauty (only a, b, c and d have more beauty and only h, i and j have less beauty). Thus, they are put into one set $\{e, f, g\}$. Similarly, b and c are related to all other members of the domain in the same way and hence they are also put into one set $\{b, c\}$. However, a is the only element that that has more beauty than b or c (i.e., there is no $z \neq a$ such that $\langle z, b \rangle \in \beta$ and $\langle z, c \rangle \in \beta$), hence a is put into a set by itself (i.e., $\{a\}$ is an equivalence class). With respect to β, the set of equivalence classes would be $\{\{a\}, \{b, c\}, \{d\}, \{e, f, g\}, \{h, i\}, \{j\}\}$.

Once the set of equivalence classes is determined, the next step is to order these equivalence classes so that they are congruent to the original relation. This can be achieved by ordering the equivalence classes according to how their members relate to one another. If the members of one equivalence class A have more beauty than the members of another equivalence class B, then the set A is ordered above the set B. For example, since $\langle b, e \rangle$, $\langle b, f \rangle$, $\langle b, g \rangle$, $\langle c, e \rangle$, $\langle c, f \rangle$, and $\langle c, g \rangle$ are all members of β, the equivalence class $\{b, c\}$ is ranked above (is greater than) the equivalence class $\{e, f, g\}$. This resulting order of equivalence classes, one above the other, creates a scale which characterizes the quotient structure. For example, the relation β creates the quotient structure under the label $>_{\beta/\sim}$ in (20). These sets (or equivalence classes) can serve as degrees in this linear scale of beauty. For ease of notation, one can replace the set representation of the equivalence classes with d's and subscripts just to make the scalar use of these quotient structures clear. Klein (1991) sometimes follows this convention and I will do so here, as shown by the scale under the label $>_{D_\beta}$ in (20). Comparisons can be made through this scale using a semantics involving the manipulation of degrees without necessarily having to hypothesize language independent scales. Although the underlying relations are independent of language, the scales are derived from these simpler concepts.

Although Cresswell hypothesizes that these types of scales are only associated with adjectives that are not linked to an independently justified measurement system (the vast majority of adjectives as it so happens), it is clear that his derivation can be extended to all adjectives. Just as one can have relations based on underlying concepts of who/what has more beauty than another and who/what has more intelligence than another, one can also have underlying relations based upon who/what has more length than another or who/what has more width than another. The only difference between the former types of relations and the latter types is that relations based on width and height might have measurements in their domain as well as people and things, especially since one can be taller than six feet and six feet can be taller than five feet etc. In fact, one can manipulate the participation of measurements in the relation (and hence the equivalence classes) to

provide a semantics for direct comparisons (*John is taller than Bill is wide*) and differentials (*John is two feet taller than Fred*). I will forgo the details for now, but the curious reader should read the discussion in Bale (2006, 2008) for some details on direct comparisons (see also the discussion in Klein 1991).[9]

4.2 Compounded scales

Cresswell's (1976) idea about forming scales from relations can also be used to form scales from the intersection of two relations. For the purpose of this discussion, let's call such scales *compounded scales*. The key to forming compounded scales is that intersected relations preserve asymmetry and transitivity. Such intersected relations can then be converted into quotient structures just like the non-intersected relations. In this section, I first discuss the meaning of intersected relations before discussing how compounded scales can be built from such intersections.

To simplify matters somewhat, in this section and the rest of the paper I will assume contexts where all individuals are distinct from others in terms of the basic gradable properties being considered. For example, every one will be distinct in length, width, beauty and intelligence. This will simplify the representations of the relations slightly and will also simplify the quotient structures. In fact, since every individual forms its own equivalence class (due to its distinctness), the quotient structures will be isomorphic to the underlying relation. I believe that this simplification will allow for a clearer understanding of what the intersection of two relations looks like.

To begin, let's consider the two relations η and τ as defined in (21).

(21)   a. $\eta = \{\langle x, y \rangle : x \text{ is more handsome than } y\}$
       b. $\tau = \{\langle x, y \rangle : x \text{ has more talent than } y\}$

Since these two relations have similar domains (whatever has handsomeness to a certain extent also has talent to a certain extent), they can be non-trivially intersected. The result of this intersection can be characterized as follows.

(22) $\langle x, y \rangle \in (\eta \cap \tau) \text{ iff } (\langle x, y \rangle \in \eta) \wedge (\langle x, y \rangle \in \tau)$

In other words, x is related to y through the intersected relation $(\eta \cap \tau)$ if and only if (i) x is more handsome than y and (ii) x has more talent than y. If either of these conditions fail then $\langle x, y \rangle$ is not a member of the intersection $(\eta \cap \tau)$.

It is important to note that since η and τ are transitive and asymmetric relations, so is their intersection. This can be shown by the following reasoning:

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[9]The fundamental concept of such a manipulation is simple. One can provide a semantics for differentials by calculating the difference between two measurements that are in two different equivalence classes, the equivalence classes containing the comparative subject and the comparative object respectively. One can provide a semantics for direct comparison by manipulating the fact that adjectives like *long* and *wide* are associated with the same measurement scale. The equivalence classes for height and width will be ordered with respect to the same measurement scale. Thus the derived scales will be isomorphic to one another as well as to the measurement scale. This isomorphism allows for a comparison by measurement. See Bale 2006 for details. For an alternative approach to differentials that is also compatible with interpreting adjectives as binary relations, see the vector analysis discussed by Schwarzschild in this volume.

(23)   a.   **The intersection is asymmetric**: If x and y are both in the domain of η and τ then, by definition of asymmetry, it cannot be the case that both $\langle x, y \rangle$ and $\langle y, x \rangle$ are members of η. (Likewise for τ.) Hence, it cannot be the case that both $\langle x, y \rangle$ and $\langle y, x \rangle$ are in the intersection of η and τ. Thus, the intersection is asymmetric.

           b.   **The intersection is transitive**: Suppose that $\langle x, y \rangle$ and $\langle y, z \rangle$ are members of the intersected relation. By definition of intersection it follows that $\langle x, y \rangle$ and $\langle y, z \rangle$ are members of both η and τ. Since η and τ are transitive, $\langle x, z \rangle$ must be members of both η and τ. As a consequence $\langle x, z \rangle$ must be a member of the intersected relation. Hence the intersected relation is transitive.
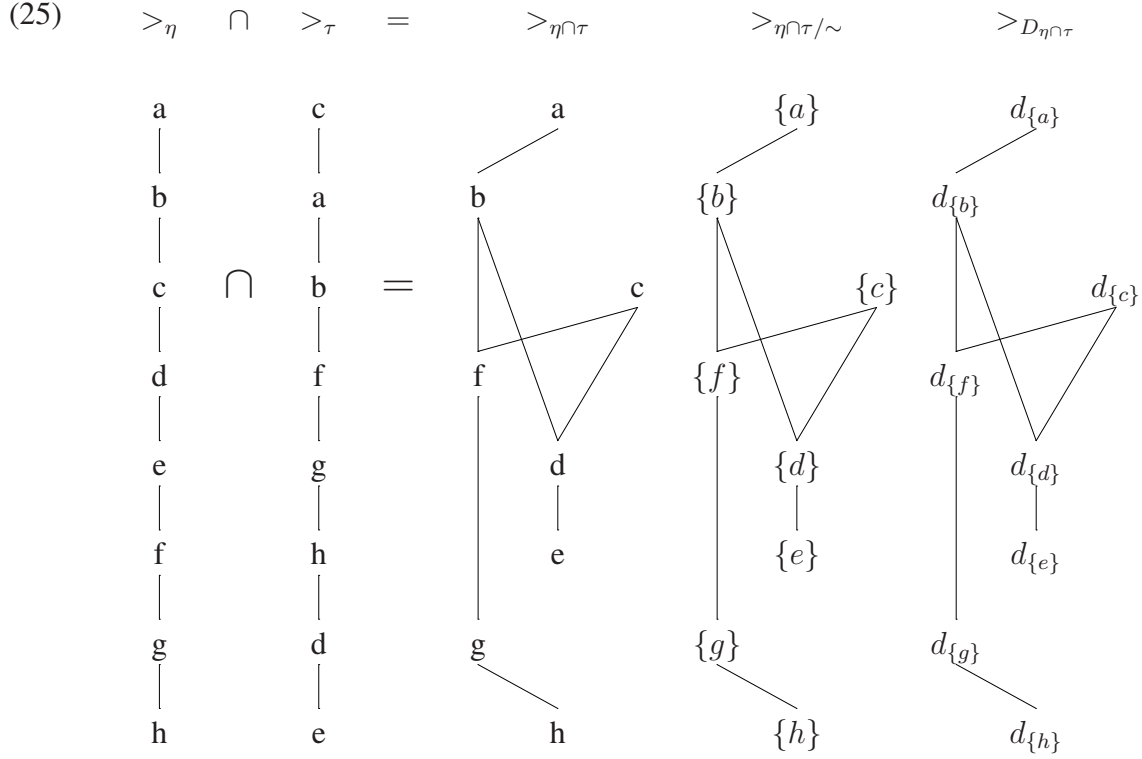
Since taking the quotient structure of this relation preserves asymmetry and transitivity, it has all the properties necessary for forming a scale (although critically, the intersected relation need not be connected, more on this below).

Having described the general outline, let's see how this works with a specific example. Consider a context where there are eight individuals represented by the letters a through h. Now their ranking in terms of handsomeness follows their ranking in terms of the alphabet: a is more handsome than b, b is more handsome than c and so on and so forth. Their ranking in terms of talent is a little more arbitrary. Using arrows to represent the relation of being more talented, consider the following ranking.

(24) $c \rightarrow a \rightarrow b \rightarrow f \rightarrow g \rightarrow h \rightarrow d \rightarrow e$

Thus, in this context, c has more talent than a, a has more talent than b and so on and so forth.

In this context, the relations η and τ can be represented by the graphs in (25) labeled $>_\eta$ and $>_\tau$. These diagrams are to be read similar to the previous diagrams where if an individual x is placed above another individual y and connected by a path, then $\langle x, y \rangle$ is a member of the relation. For example, according to the diagram, b is more handsome than c and c is more handsome than h but c is not more handsome than b nor is h more handsome than c.

(25)

$>_\eta$ $\cap$ $>_\tau$ $=$ $>_{\eta\cap\tau}$ $>_{\eta\cap\tau/\sim}$ $>_{D_{\eta\cap\tau}}$

$>_\eta$		$>_\tau$		$>_{\eta\cap\tau}$	$>_{\eta\cap\tau/\sim}$	$>_{D_{\eta\cap\tau}}$
a		c		a	$\{a\}$	$d_{\{a\}}$
b		a		b	$\{b\}$	$d_{\{b\}}$
c	$\cap$	b	$=$	c	$\{c\}$	$d_{\{c\}}$
d		f		f	$\{f\}$	$d_{\{f\}}$
e		g		d	$\{d\}$	$d_{\{d\}}$
f		h		e	$\{e\}$	$d_{\{e\}}$
g		d		g	$\{g\}$	$d_{\{g\}}$
h		e		h	$\{h\}$	$d_{\{h\}}$

The diagram labeled as $>_{\eta\cap\tau}$ in (25) represents the intersection of η and τ. Note that for $\langle a, b\rangle$ to be a member of the intersected relation (or any other ordered pair), it has to be a member of both η and τ. Hence, a must be both more handsome than b and more talented than b in the relevant context. In other words, membership in the intersected relation encodes both the properties of talent and handsomeness in relating the two individuals.

This intersected relation can then be converted into the quotient structure represented under the label $>_{\eta\cap\tau/\sim}$. Unlike our previous quotient structure in (20), each element is part of its own equivalence class. This is because each element behaves differently with respect to the intersected relation. For example, although f and d are not related to each other, they are also not equivalent. This is because, within the intersected relation, f is related to g (i.e., $\langle f, g\rangle \in \eta \cap \tau$) but d is not related to g (i.e., $\langle d, g\rangle \notin \eta\cap\tau$). Hence, f and d are distinct despite not being ranked in comparison to one another. Unlike the quotient structure in (20), $>_{\eta\cap\tau/\sim}$ is not fully connected (i.e., it is not a linear order). There are some equivalency classes that are not related to one another (e.g., $\{a\}$ and $\{c\}$, $\{c\}$ and $\{b\}$, etc.).

Note, this is an empirical advantage, rather than a flaw. In terms of entailment relations, conjoined adjectives do not behave as if they are connected. Consider the difference between (26) and (27).

(26)   a.   John is more handsome than Patrick is.
         b.   Patrick is more handsome than John is.
         c.   John and Patrick are equally as handsome as one another.

(27) a. John is more handsome and talented than Patrick is.
      b. Patrick is more handsome and talented than John is.
      c. John and Patrick are equally as handsome and talented as one another.

If neither (26a) nor (26b) are true, then it seems reasonable to conclude that (26c) must hold. This is because the scale derived from *handsome* is connected. If for any two individuals, one is not more handsome than the other, then they must be equally as handsome. In contrast, this does not hold for the sentences in (27). All three sentences can be false, if, for example, John is more handsome than Patrick but Patrick is more talented John.

4.3   Details of the potential analysis

Having explained how Boolean conjunction can lead to the creation of compounded scales, let me now fill in the details of a semantic theory that can use these scales. The interpretation of *more* and *less* can be defined in a way that derives a scale from an underlying binary relation. (Recall that gradable adjectives, under this analysis, are interpreted as binary relations, c.f., Bale 2011.)

(28) a. $[\![more]\!] = \lambda P. \lambda d. \lambda x. (x_{P/\sim} >_{P/\sim} d)$
      b. $[\![less]\!] = \lambda P. \lambda d. \lambda x. (d >_{P/\sim} x_{P/\sim})$

The interpretations in (28) have three arguments. The P argument is the adjective which is a binary relation. This argument is converted into a quotient structure which serves to determine the ordering relation in the scale, represented as $>_{P/\sim}$. The x argument is the subject of the comparative predicate. It is mapped to its equivalency class within the quotient structure, represented as $x_{P/\sim}$. The d argument is the degree typically derived from the than-clause.

As in most analyses of comparatives (e.g., see Kennedy 1999), I will assume that the than-clause contains a copy of the adjectival predicate from the matrix clause. For example, a clause such as [*than Patrick is*] in a sentence such as [*Seymour is more talented than Patrick is*] contains a copy of the adjective *talented* (i.e., [*than Patrick is*] = [*than Patrick is talented*]). In contrast, in the sentence [*Seymour is more handsome and talented than Patrick is*], the than-clause contains a copy of the conjoined adjective [*handsome and talented*] (i.e., [*than Patrick is*] = [*than Patrick is handsome and talented*]). As is standard in many analyses, I will assume that the than-clause contains a phonologically null degree argument (see the arguments in Bresnan 1975) as well as a phonologically null operator that relates degrees to individuals, however, this operator will (non-standardly) serve to convert the adjectival phrase to a scale. For simplicity, I will label this operator OP. The interpretation of this operator is given in (29).

(29) $[\![OP]\!] = \lambda P. \lambda d. \lambda x. (x_{P/\sim} >_{P/\sim} d) \lor (x_{P/\sim} = d)$

Essentially, this operator relates individuals to degrees such that the equivalency class associated with the individual in the scale formed from the quotient structure of the predicate P is either greater than d (in the quotient structure) or is equal to it. With this interpretation in mind, the template for interpreting a than-clause is given in (30), where P represents the interpretation of the elided adjectival phrase and x the interpretation of the subject.

14

(30) $[\![\textit{than } x \textit{ is } d \textit{ OP } P]\!]$
        $= \text{MAX}\{d : ((([\![\textit{OP}]\!] \ P) \ d) \ x)\}$
        $= \text{MAX}\{d : (x_{P/\sim} >_{P/\sim} d) \ \vee \ (x_{P/\sim} = d)\}$
        $= x_{P/\sim}$

As a result, in most cases, the maximal element of the set of degrees in the than-clause ends up being the degree/equivalence-class associated with the than-clause subject in the scale derived from the adjectival predicate.

Given this interpretation of the than-clause, we not only can interpret simple comparatives, but also comparatives with compounded scales. Let's first consider a simple comparative, as in (31).

(31)   Where a is Seymour, b is Patrick, τ is the binary relation associated with talent:
        $[\![\textit{Seymour is more talented than Patrick is}]\!]$
        $= (((((\lambda P. \ \lambda d. \ \lambda x. \ (x_{P/\sim} >_{P/\sim} d)) \ [\![\textit{talented}]\!]) \ [\![\textit{than Patrick is } d \textit{ OP talented}]\!]) \ [\![\textit{Seymour}]\!])$
        $= (a_{\tau/\sim} >_{\tau/\sim} b_{\tau/\sim})$

Given the relation of τ defined in (25), where a has more talent than b, the sentence would end up being true. Next, consider the interpretation of the more complex comparative in (32).

(32)   Where a is Seymour, b is Patrick, τ is the binary relation associated with talent, and η is the binary relation associated with handsomeness:
        $[\![\textit{Seymour is more handsome and talented than Patrick is}]\!]$
        $= (((((\lambda P. \ \lambda d. \ \lambda x. \ (x_{P/\sim} >_{P/\sim} d)) \ [\![\textit{handsome and talented}]\!])$
        $[\![\textit{than Patrick is } d \textit{ OP handsome and talented}]\!]) \ [\![\textit{Seymour}]\!])$
        $= (a_{(\eta \cap \tau)/\sim} >_{(\eta \cap \tau)/\sim} b_{(\eta \cap \tau)/\sim})$

Given the intersected relation defined in (25), the sentence would end up being true. More importantly, it is only true if Seymour is both more handsome and more talented than Patrick. If Seymour didn't exceed Patrick in either one of the qualities, then the ordered pair $\langle a, b \rangle$ would not be a member of the intersection and hence the degree associated with Seymour in the compounded scale (his equivalence class in terms of both qualities) would not be greater than the degree associated with Patrick.

Similar reasoning follows for sentences like *This floorboard is less tall and wide than that floorboard is*. The intersection of the two basic binary relations can be used to create a compounded scale. The sentence will be true if the degree associated with "this floorboard" on the compounded scale is ordered below the degree associated with "that floorboard". However, this is only possible if "this floorboard" is both shorter and narrower than "that floorboard". If "this floorboard" exceeds or equals "that floorboard" in either quality, the sentence will be false. For time and space reasons, I will forego the details.

5 Remaining problems and an alternative solution

The main advantage of the solution sketched out in section 4 is that it maintains a standard boolean interpretation of conjunction while keeping the main ingredients of a degree analysis of compara-

tives. Importantly, when the adjectival phrase does not contain any conjunctions, the derived scale is isomorphic to the linear order of degrees that is at the heart of most degree analyses, even those that map individuals to measurements. This is important to emphasize and often overlooked. As discussed in Klein 1991 and Bale 2006, derived scales support all of the operations that are associated with numerical scales (including differentials that involve addition and/or multiplication), as long as the underlying binary relations include measurements.

However, despite the theoretical appeal of such an approach, compounded scales are not free of empirical difficulties. There are two main problems, one involving differentials and the other disjunction. The problem with differentials arises when one considers potential analyses for sentences like the one in (33).

(33)   This floorboard is five centimetres less long and wide than that floorboard is.

If we assume that measurements are individuals, and that things can be longer than measurements and measurements can be longer than things, then both quotient structures derived from *long* and *wide* will be isomorphic to the linear order of measurements. Each individual will belong to the an equivalence class that contains its measurement and the order of equivalence-classes will mirror the order of measurements. Hence, the operations of $-$, $+$ and $\times$ can be defined for these scales.

However, the intersection of *long* and *wide* does not maintain this isomorphism. For example, let's suppose that that the floorboard f is five metres long and 10 centimetres wide. Thus, $\langle f, 4m \rangle$ would be in the relation defined by *long* but not in the one defined by *wide* (i.e., f has more length than four metres but not more width). In contrast, $\langle 5m, 4m \rangle$ would be in both relations (i.e., five meters is longer than 4 meters and five meters is wider than four meters). As a result, $\langle 5m, 4m \rangle$ would be in the intersected relation but $\langle f, 4m \rangle$ would not. Hence, f and $5m$ would no longer be associated with the same equivalence class. If the compounded-scale analysis is to be maintained, one would need to provide a more complicated analysis of differentials: perhaps one that would manipulate the binary relations prior to conjunction. It might be possible to do this with movement and across the board extraction of a variable, but for reasons of time and space, I will not explore such an analysis here.[10] It might also be possible to address this problem through a vector analysis of differentials, such as the one discussed in Schwarzschild (this volume). However, once again due to limitation of time and space, I will not pursue this line of inquiry here.

The other empirical problem involves the distribution of disjunction. In general, disjunction and conjunction are intersubstitutable for one another, and this intersubstitutability holds for constructions that have conjoined adjectives as in (1). For example, the sentences in (34) are identical to (1) except that *and* has been replaced by *or*. The resulting sentences are perfectly acceptable.

(34)   a.   Seymour is more handsome or talented than Patrick is.
         b.   This floorboard is less long or wide than that floorboard is.

As with the sentences containing conjunction, the truth conditions of the sentences in (34) can be paraphrase by repeating the comparative morpheme.

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[10]The basic idea would be for the phrase *two inches* to influence the granularity of the binary relations before they are intersected. For example, if ω is the binary relation $\{\langle x, y \rangle : x \text{ has more width than } y\}$, then *two inches* would alter this relation to $\{\langle x, y \rangle : x \text{ has at least two inches more width than } y\}$.

(35)   a.   Seymour is more handsome or more talented than Patrick is.
       b.   This floorboard is less long or less wide than that floorboard is.

Furthermore, as with conjunction, it is unlikely that gapping could explain the interpretation of the sentences in (36). Unlike typical gapping, the insertion of material after the first disjunct leads to a decline in acceptability.

(36)   a. ??Seymour is more handsome than Bill is or talented than Patrick is.
       b. ??This floorboard is less long than the one by the table is or wide than the one by the chair is.

The problem with these disjoined adjectives is that they cannot receive an analysis like the one outlined in section 4. Disjunction is usually analyzed as Boolean union, but the union of two binary relations neither preserves transitivity nor asymmetry.[11] Hence, disjoined binary relations do not have a corresponding quotient structure and cannot be associated with any kind of scale.

However, the idea that *or* is directly interpreted as Boolean union has long been questioned in the literature. It has been widely noted that the semantic effects of disjunction seems to take place at a point that is not directly associated with the surface syntactic position of *or* (see Alonso-Ovalle 2006, 2008; Larson 1985; Schwarz 1999; Simons 2005b,a; Zimmermann 2000; Geurts 2005 among others). To explain these effects, it has been hypothesized that *or*, at it's surface position, forms a set where each member corresponds to the semantic value of one of the disjuncts (i.e., $[\![A \text{ or } B]\!] = \{ [\![A]\!], [\![B]\!] \}$). Further functions/elements combine with this set through point-wise composition (e.g., where X and Y are of type α and Z is of type $\langle \alpha, \beta \rangle$, Z({X,Y}) = {Z(X), Z(Y)}, and where X and Y are of type $\langle \alpha, \beta \rangle$ and Z is of type α, {X,Y}(Z) = {X(Z), Y(Z)}). At some point latter in the derivation, the two members of the set are combined with boolean union (e.g., at some point in the derivation an operator applies such that $OP_\cup(\{X, Y\}) = X \cup Y$). It is easy to see that the data in (34), with the paraphrases in (35), fall out naturally from these types of hypotheses. I will not go over the details here, but it is sufficient to note that under this type of interpretation of disjunction, a sentence like [*Seymour is more handsome or talented than Patrick is*] will have a meaning that mirrors the sentence [*Seymour is more handsome than Patrick is or Seymour is more talented than Patrick is*], without hypothesizing any type of ellipsis or gapping.

However, if one adopts this type of analysis for disjunction, why not conjunction? Why couldn't *and* trigger set-formation and why couldn't point-wise composition apply in the same way it does for disjunction? One could hypothesize an operator that eventually combines two members of a set using Boolean intersection (e.g., $OP_\cap(\{X, Y\}) = X \cap Y$). Indeed, this hypothesis is explicitly adopted by Winter (1995), although it should be noted that such a hypothesis has often been rejected in the literature due to differences between the interpretation of *and* and *or* (i.e., *or* seems to be able to take wider scope than *and*, see in particular Hulsey 2006). This possibility also opens up a whole new can of worms. For example, one would have to think of constraints on when the operator $OP_\cap$ can apply. An unconstrained theory would predict that sentences like

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[11]For example, $\{\langle a, b \rangle, \langle b, c \rangle, \langle a, c \rangle\} \cup \{\langle b, a \rangle, \langle a, c \rangle, \langle b, c \rangle\} = \{\langle a, b \rangle, \langle b, c \rangle, \langle a, c \rangle, \langle b, a \rangle\}$. The resulting union is not transitive since it does not contain the pair $\langle a, a \rangle$ and it is not asymmetric since it contains the pairs $\langle a, b \rangle$ and $\langle b, a \rangle$.

those in (37) could have an interpretation that could be paraphrased by the sentences in (38). This is not the case. For example, the sentence in (37a) is only true if John is more handsome and more intelligent than one and the same younger man. In contrast, (38a) can be true if John is more handsome than one younger man and more intelligent than another.

(37) a. John is more handsome and intelligent than a man who is ten years younger than him.
  b. A man from Italy is less handsome and intelligent than John is.
  c. John is less scared and proud of a dog that Bill owns than Mary is.

(38) a. John is more handsome than a man who is ten years younger than him and more intelligent than a man who is ten years younger than him.
  b. A man from Italy is less handsome than John is and a man from Italy is less intelligent than John is.
  c. John is less scared of a dog that Bill owns than Mary is and less proud of a dog that Bill owns than Mary is.

Similar observations hold for the contrast between the other sentences. The sentences in (38) have weaker truth conditions than the sentences in (37).

However, even if the proper constraints can be sketched out, and even if we accept that something like Winter's (1995) theory is on the right track, we still might need compounded scales. Consider the contrast between the sentences in (39a) and (40a).

(39) a. Seymour is more handsome and talented than how handsome and talented Patrick is.
  b. Seymour is more handsome and talented than Patrick is.

(40) a. ??Seymour is more handsome or talented than how handsome or talented Patrick is.
  b. Seymour is more handsome or talented than Patrick is.

The sentence in (39a) has the same truth conditions as the sentence in (39b). This follows straightforwardly from the compounded scale analysis: the phrase [*handsome and talented*] is treated as a single binary relation. In fact, the only difference between (39a) and (39b) is that (39a) overtly expresses the degree abstraction operator.

In contast, (39a) cannot be readily analyzed by using set formation and point-wise composition. There are two instances of conjoined phrases, one contained in the matrix clause and the other contained in the than-clause. The two instances would create two sets and one would have to arbitrarily stipulate how these two sets should be combined in order to get the correct truth conditions. Furthermore, even if one makes such a stipulation, then it would be predicted that the sentence in (40a) should have the same truth conditions as the sentence in (40b). This is not the case. First of all, (40a) sounds quite odd. Second of all, in so far as one can assign an interpretation to (40a), its truth conditions cannot be paraphrased by (40b).

In summary, even if one adopts an interpretation of *and* that parallels *or*, there is still some evidence that compounded scales might be needed nonetheless. Furthermore, adopting an interpretation of *and* where the semantic realization of conjunction (i.e., Boolean intersection) is displaced opens up a whole new set of problems and requires a new theory of constraints.

6 Conclusion

The data outlined in the paper minimally demonstrates that conjoined adjectives are a serious challenge to the traditional degree analysis of gradable adjectives. These challenges could not be adequately addressed through gapping or any other type of ellipsis. Although some of the problematic data could be accounted for by adopting a non-Boolean interpretation of *and*, certain problems still remain. At least on the surface—in many types of constructions—conjoined adjectives appear to function as a single unit. The advantage of the compounded-scale analysis is that it treats such constructions as units in that intersected binary relations serve as the building blocks for a single (potentially non-connected) quotient structure. Certain empirical difficulties still remain (such as differentials), but the empirical advantages of the compounded scale analysis suggest that this potential solution to conjoined adjectives is at least worth pursuing.

More generally, it is philosophically appealing to derive scales from more basic relations rather than having scales as semantic primitives. From a cognitive perspective, binary relations are simpler and, in principle, independent of language (in that even dogs and cats have the conceptual resources to rank individuals in terms of who is bigger or which food tastes better). Furthermore, as discussed in Cresswell 1976, even though mapping individuals to measurements seems intuitive with respect to heights and lengths, it is definitely unintuitive when it comes to properties such as beauty and talent. These philosophical considerations make it worth-while to push, as far as it can go, the idea that scales are derived rather than primitive.

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