Trivalence and Transparency: a non-dynamic approach to anaphora*

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Abstract

This paper offers a new theory of donkey anaphora that does not include any *dynamic* component. Even if the approach is not dynamic, it retains a key aspect of the dynamic tradition, namely the view that information states include not just factual information about the world, but also information about *discourse referents*, e.g., variables. It also makes crucial use of *plural assignment functions* (sets of standard assigments, cf. van der Berg 1996; Nouwen 2003; Brasoveanu 2008). Unlike dynamic approaches, sentences are evaluated as true or false relative to a pair (w, G), where w is a possible world and G is a plural assignment, with no reference to contexts or information states, and compositional semantics does not refer in any way to context update. In order to predict adequate meanings and felicity conditions, I combine two ingredients that have been used to account for presupposition projection, namely *Trivalence* (Peters 1979; Beaver and Krahmer 2001) and Schlenker's *Transparency Principle*(Schlenker 2007, 2008).

Two ideas play a crucial role in the proposal. First, a sentence such as 'She$_x$ came' comes with the presupposition that the variable x is 'valued' by the assignment function G, which means that every atomic assignment in G maps x to the same value. Second I adopt Mandelkern's (2002) *witness condition*: an existential statement such as 'Someone$_x$ came' is undefined in (w, G) if it is classically true in w but G does not map x to a witness of the existential statement. Importantly, undefinedness is not equated with Presupposition Failure (e.g., even though 'Someone$_x$ came' can be undefined, it is in fact never a presupposition failure). Rather, presupposition projection is governed by Schlenker's (2007,2008) Transparency Principle: the presupposition 'x is valued' should be redundant in the syntactic position in which 'She$_x$ came' occurs.

In the end of the paper, I discuss well-known ambiguities with donkey sentences (weak vs. strong, existential vs. universal readings) and show how they can be addressed in my system.

The theory is presented here as a non-standard semantics for first-order logic, rather than a fragment of a natural language. Free variables are the counterparts of syntactically unbound pronouns, and existential quantifiers those of singular indefinites.

Keywords: Anaphora, Presupposition, Dynamic Semantics, Trivalent Logic

1 Goals and background

In this paper, I develop a new theory of singular anaphora. This theory does not include any *dynamic* component. It extends to anaphora, by combining them, two non-dynamic approaches to presupposition, the trivalent approach (Peters 1979; Beaver and Krahmer 2001) and the Transparency approach (Schlenker 2007, 2008). In the proposed theory, sentences are evaluated as true or false relative to a pair (w, G), where w is a possible world and G is a plural assignment function (i.e. a set of standard assignmments, cf. van der Berg 1996; Nouwen 2003; Brasoveanu 2008, a.o.). While non-dynamic, the approach preserves a key insight of dynamic approaches to anaphora, namely the idea that information states include not just factual

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information about the world, but also information about *discourse referents*, namely, in my setting, variables. The theory is here presented as a non-standard semantics for first-order logic, rather than a fragment of a natural language - in the spirit, say, of Dynamic Predicate Logic (Groenendijk and Stokhof 1991). Free variables are the counterparts of unbound pronouns, and existential quantifiers those of singular indefinites.

Let me at the outset sketch the key conceptual ingredients of my proposal, which I present here in a simplified form, one which does not involve plural assignments (essentially the version developed in sections 2 and 3, which is not my 'official' proposal but still contains its central ideas).

1. I adopt a key idea from Mandelkern (Mandelkern 2022), the *witness condition*. Unlike what happens in classical logic, in my system an existential statement $\exists x P(x)$ not only asserts that the predicate P has a non-empty extension, but also that the value of the variable x (provided by the assignment function g) is a witness of the existential claim, i.e. that the value of x is in the extension of P. Specifically, relative to a pair (w, g), such a sentence is false if P has an empty extension in w, true if $g(x)$ is in the extension of P in w, undefined otherwise.

2. Assignment functions can be partial, so some variables fail to have a value in certain pairs (w, g). Free variables *presuppose* that they are *valued* (i.e. are assigned a value by the assignment function).[1]

3. The presupposition of free variables gives rise to the same projection pattern as other presuppositions, and presupposition projection is governed by Schlenker's (2007) Transparency Condition. Roughly speaking, for a sentence to be felicitous, the presuppositional content of an expression that occurs in it must be semantically 'redundant' in the position where the expression occurs.

4. The meaning of propositional connectives is given by the so-called Middle-Kleene truth tables (Peters 1979; Beaver and Krahmer 2001). For instance, $A \wedge B$ is false when either A is false or A is true and B is false, undefined when either A is undefined or A is true and B is undefined, true when A and B are both true.

5. A sentence is considered true in a world w if there an assignment function g such that it is true in (w, g).

Let me illustrate how things work for a simple case of anaphora across conjunction.

(1) a. A woman is in the room and she is smiling.
b. $\exists x(W(x) \wedge R(x)) \wedge S(x)$

We can paraphrase (1b), which is meant to be a first-order logic translation of (1a), as follows:

(2) There is a woman in the room, x has a value and its value is a woman in the room, and the value of x smiles.

The first conjunct of (2b), namely $\exists x(W(x) \wedge R(x))$ is true in (w, g) if there is a woman in the room in w and $g(x)$ is defined and is such a woman; the second conjunct is true if $g(x)$ is smiling in w. Overall, the sentence is true in (w, g) just in case $g(x)$ is a woman in the room that is smiling in w, hence true simpliciter in w if there is a woman in the room that is smiling in w. Furthemore, the presupposition of the free variable that occurs in the second conjunct ($S(x)$), namely the proposition that x is valued, happens to be redundant at the point at which $S(x)$ occurs, which makes it felicitous; let us see why. First, if the first conjunct is true, this ensures that x has a value, so the presupposition that x is valued triggered by the second conjunct is vacuous once we know that the first conjunct is true. Second, if the first conjunct is false (resp. undefined), then the whole sentence is false (resp. undefined) given the truth-table we assume for conjunction, hence whether x is valued or not plays not role in the final truth-value, which makes again the presupposition that x is valued vacuous.

[1] When we move to plural assignments in section 4, we will dispense with partiality, and the notion of a variable being valued will be defined differently.

It should be clear that this analysis has none of the ingredients that characterize dynamic semantics systems. No notion of context update or transition between assignments plays any role; the existential quantifier does not 'extend' or modify an input assignment, conjunction is not defined as sequential update, etc. Rather, we characterize a sentence as true, false or undefined relative to a pair (w, g), without any reference to information states or contexts (typically viewed as sets of assignments, or sets of world-assignment pairs).

In the remainder of this section, I first introduce the general project, then discuss what makes a system dynamic or static, and I finally explain in what sense my proposal preserves the idea that sentences convey information about discourse referents.

1.1 Extending to anaphora non-dynamic approaches to presupposition

More than forty years ago, dynamic semantics was developed as a framework to account for both presupposition projection and anaphora (Kamp 1981; Heim 1982, 1983 are the most important early references). A striking parallelism was observed regarding the syntactic configurations in which a presupposition could be satisfied by preceding material in a complex sentence, and those in which a pronoun could refer back to a preceding indefinite. For instance, in (3a), a pronoun (*it*) occurs in the second conjunct of a conjunctive sentence, and can be interpreted as anaphoric to an indefinite in the first conjunct (*a violin*); in a parallel way, in (3b), a presupposition trigger occurs in the second conjunct of a conjunctive sentence (*know*), and the triggered presupposition (namely the presupposition that Mary owns a musical instrument) is 'satisfied' by the first conjunct, as a result of which the sentence as whole does not inherit this presupposition.[2]

(3) a. Mary owns a violin, and it is expensive.
    b. Mary owns a violin, and Peter knows that Mary owns a musical instrument.

This parallelism between presupposition and anaphora is in fact systematic, i.e. extends to more complex configurations, and dynamic semantics provided a conceptual and formal framework within which this parallelism was naturally captured, though various approaches captured it differently (van der Sandt 1992; Geurts 1999 offered a theory in which presupposition is a special case of anaphora, which is quite different from Heim's 1983 initial view).

Now, in the domain of presupposition projection, several alternatives to dynamic semantics have been offered, such as *static trivalent approaches* (Peters 1979; Beaver and Krahmer 2001) or purely pragmatic theories, esp. Schlenker's (2007; 2008) *Transparency Theory* (see also Chemla 2007 for a different proposal). A natural question that arises is whether these other approaches to presupposition can in principle be extended to anaphora. If not, this would provide a *prima facie* argument for a dynamic theory of presupposition, since only such a theory would hold hope of unifying presupposition with anaphora. In this paper, however, I combine the two major non-dynamic approaches to presupposition (static trivalence and Schlenker's Transparency-based approach) to offer a new theory of anaphora, suggesting that presupposition and anaphora can be unified in a static semantics framework.

To my knowledge, nobody has proposed an extension of the Transparency theory to anaphora. As to the static trivalent approach to presupposition, I know of only one attempt to extend it to anaphora, namely Rothschild (2017). Rothschild's approach relies in a crucial way on certain syntactic copying operations which allow a pronoun to find an antecedent in some constituent that is not present in the overt syntax. Everything else being equal, it is clear that it would be better to dispense with syntactically unmotivated covert syntactic operations. Two other relevant recent works should be mentioned: Mandelkern (2022) offers a bidimensional theory of anaphora in which sentences receive 'classical' truth-conditions, but the felicity conditions and 'dynamics' of anaphora are dealt with in a second dimension of meaning which includes a dynamic component (but see footnote 3). As already mentioned, I adopt one of Mandelkern's key ideas, namely the 'witness condition' mentioned at the very beginning of this paper. Elliott (2020, 2023) proposes a dynamic theory of anaphora in which the dynamic effect of connectors are fully *deducible* from their 'static'

[2] (3b) entails but does not presuppose that Mary owns a musical instrument, as is seen from the fact that someone asking the corresponding polar question (*Is it true that Mary owns a violin and that Peter knows that Mary owns a musical instrument?*) is not understood to necessarily believe that Mary owns a musical instrument.

classical meanings, on the basis of a general recipe that derives dynamic lexical entries from static lexical entries. Elliott also adopts a version of Mandelkern's witness condition. While my proposal is quite different from Mandelkern's and Elliott's, and is first and foremost motivated by the unification of anaphora and presupposition in a non-dynamic system, it will turn out to make similar predictions.

The proposal I develop in this paper is presented as a semantics for first-order logic rather than for a fragment of English - the idea being that 'free-variables' corresponds to pronouns and existential quantifiers to indefinites. There would be no difficulty to adapt it to a formalized fragment of English. The semantics by itself will correctly capture the truth-conditions of sentences with indefinites and pronouns (i.e., in the first-order logic context, existential quantifiers and free variables). Free variables will trigger a *presupposition* that they are 'valued', which will be responsible for predicting the felicity condition of pronouns, while indefinites will trigger an *anti-presupposition* whose effect will be similar to Heim's (1982) novelty condition on indefinites. Despite the fact that I will use trivalent semantics, presupposition failure will not be cashed out as corresponding to the 'third', undefined truth-value. Rather, I will use the Transparency Theory as a theory for presupposition projection. In other words, my static approach to anaphora borrows from the two main static approaches to presupposition projection, usually viewed as competing rather than complementary proposals, namely trivalence and transparency. I will only discuss singular indefinites and singular pronouns.

In the next subsections, I briefly explain what I mean when I characterize my proposal as 'static'. In sections 2 and 3, I introduce a simplified version of my system, and will then point out a major problem it faces, before moving to the final version, which makes crucial use of the notion of plural assignments used in plural dynamic logic (section 4). Section 5 discusses how my proposal can deal with potential ambiguities between existential and universal, weak and strong readings, which are much discussed in connection with donkey anaphora.

1.2 Static vs. Dynamic Semantics

What do I mean by 'static', as opposed to 'dynamic'? There is no well-established definition of what makes a given semantics static or dynamic. Rothschild and Yalcin (2016, 2017) have proposed various definitions on the basis of which they classify different systems in terms of their 'degree' of dynamicness.

Let me explain at a quite informal level what I mean when using these notions, following the spirit but not the letter of Rothschild and Yalcin (2016, 2017).

Any compositional semantic system for natural language, if meant to play a role in explaining anything about how language is used in thought and communication, has to be paired with some (possibly highly schematic and implicit) theory of how the formal objects that the semantics assigns to sentences interact with a listener's beliefs. For instance, we need to have an idea of what it means for a listener to accept a sentence, to agree with someone else who used the sentence, etc. In particular, one thing we want to characterize is how, when I trust the speaker and accept her sentence, I gain information. Following Rothschild & Yalcin, let me call a 'conversation system' the combination of a semantics which recursively assigns certain formal objects to sentences, a notion of information state, and an update rule which specifies how information states are updated when a sentence is accepted.

Within a classical semantics approach, the most simple picture is the following (see, e.g., Stalnaker 1974): an information state is a set of *indices* (typically possible worlds), and a sentence is true or false at an individual index. When, starting from an information state $\mathcal{I}$, we accept a sentence S, the resulting information state $\mathcal{I}_s$ is the intersection of $\mathcal{I}$ and the set of indices at which S is true (see however Goldstein and Kirk-Giannini (2022) for challenges to this approach). Whenever a proposed conversational system works in this way, I will consider the proposal *static*.[3]

[3]Mandelkern's (2022) system, as far as I can see, could be stated in such a way as to make it static in this sense. In his system, sentences are evaluated in two dimensions; in one dimension, they are true or false relative to a world-assignment pair; in the second dimension, they come with 'bounds' which are satisfied ('satt') or not relative to a *triple* consisting of a context, a world, and an assignment, and a context it itself a set of world-assignment pairs. When a context C is updated with a sentence S, the resulting context is the subset of C that contains all pairs (w, g) such that S is true at (w, g) and satt at (C, w, g). Since the parameter C seems to play a role only in defining the felicity conditions of pronouns and definite descriptions, I suspect that the system can be made quasi-intersective. That being said, the semantic rules that define satisfaction for bounds clearly

In contrast with a static system, a dynamic system is typically one where sentences are not evaluated for truth at individual indices, but, rather, denote a function from information states to information states, and where compositional semantics involves the construction of update functions out of the update functions denoted by the expressions being composed.[4]

Let me a bit more precise. I want to count a proposal such as Beaver and Krahmer (2001), in which context update sometimes fails (in case of presupposition failure), as static. In this approach, sentences can be true, false or undefined. When interpreted in an information state $\mathcal{I}$, a sentence is infelicitous if at some index of $\mathcal{I}$ it is undefined. When felicitous and accepted, a sentence leads to a new information state which consists of the initial information state $\mathcal{I}$ intersected with the indices at which the sentence is true. Let me call a conversational system 'quasi-intersective' if a) sentences receive a truth-value (possibly non-classical) at each index (with no need for extra parameters), b) an information state is a set of indices, and c) updating an information state with a sentence either fails (presupposition or referential failure) or gives rise to a new information state which is the intersection of of the initial information state with the set of indices at which the sentence receives a certain designated truth-value (typically, 'true'). I consider a conversational system static if it is quasi-intersective. Beaver & Krahmer's proposal for presupposition is in this sense static.

This characterization differs from Rothschild and Yalcin's (2016). In their system, Beaver & Krahmer's proposal does not count as 'strongly static' because of the possibility of presuppositional failure. It follows from their definitions that no system that captures felicity conditions in terms of presuppositional failure can be counted as strongly static in their sense. It seems to me, however, that on conceptual ground we should of think Beaver & Krahmer's approach to presupposition as being static.

The system I will propose in this paper is static in the same sense as that of Beaver & Krahmer's for presupposition. Evaluation indices, instead of consisting of possible worlds, are world-assignment pairs (in the final version, assignments will be *plural* assignments, i.e. sets of standard assignments). Information states are sets of world-assignment pairs. At a given index, a sentence can be true, false or undefined, but undefinedness plays no direct role in determining felicity conditions. Relative to an information state, a sentence will be felicitous if each occurrence of a free variable meets a certain condition relative to the information state (the *Transparency Condition*), and if every occurrence of an indefinite meets another specific condition (the *Anti-Transparency Condition*). When felicitous and accepted, a sentence leads to a new information state which results from intersecting the initial information state with the set of indices at which the sentence is true.

As shown by Rothschild and Yalcin, though, a dynamic system can be, sometimes, isomorphic to a static system (in a sense they make precise), making it uninterestingly dynamic.[5] While it is not trivial to diagnose 'essential' dynamicity (one has to determine whether an isomorphism to a static system exists), one can see immediately that a system is static, when it has the properties discussed in the previous paragraphs. File Change Semantics is is clearly not quasi-intersective, and it is not known whether it is isomorphic to a quasi-intersective system. The proposal I make in this paper is, in contrast, clearly quasi-intersective.[6]

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mimic standard dynamic approaches. For instance, a sentence $A \vee B$ is satt in (C, w, g) if A is satt in (C, w, g) and B is satt in $(C^{\neg A}, w, g)$, where C^S is the result of updating C with S. So the proposed semantics clearly has a dynamic component, even if this is not properly captured by the characterization discussed here in terms of (quasi)-intersectiveness.

[4]in Dynamic Predicate Logic, for instance, the natural notion of information state makes them sets of assignments (Groenendijk and Stokhof 1988), but sentences are not evaluated for truth at individual assignments; rather they are evaluated with respect to *pairs* of assignment. The meaning of a sentence S characterizes the pairs (g, h) such that h is a possible output when g is 'updated' with S. One could say that S is 'true' at such a pair (g, h), but there is no notion of a sentence being true at a single index – where an index, in this case, is simply an assignment.

[5]Even when a static system $\mathcal{S}$ happens to be isomorphic to an intersective system $\mathcal{S}'$, $\mathcal{S}'$ often involves objects of a much higher semantic type (for instance, information states in $\mathcal{S}'$ might consist of sets of information states as defined in $\mathcal{S}$). One could try to argue that a system is 'essentially dynamic' when the static systems it is isomorphic to (if any) define information states as sets of objects which are of a higher semantic type than in the initial system. But this is pure speculation at this point.

[6]In Rothschild & Yalcin's typology, both Heim's file change semantics approach to anaphora and Beaver & Krahmer's approach to presupposition belong to the same type of 'weakly stative systems', which means that there are both isomorphic to what they call an 'eliminative system', but are not isomorphic to an 'intersective system' (which in their typology excludes the possibility of update failure). If however, we define a system as being static if it is quasi-intersective or isomorphic to a quasi-intersective system, it is clear that Beaver & Krahmer's is then static, while we do not know whether this is the case of File Change Semantics.

It is worth noticing that the E-type approach to donkey anaphora (see, a.o., Elbourne 2002, Heim 1990), which is based on Situation Semantics, is not a static approach. While E-type proposals are typically not explicit about the discourse update rule they assume, a little bit of thought shows that the E-type approach cannot posit an intersective or quasi-intersective update rule. Consider the following discourse:

(4) A woman was in the room. She smiled.

In the E-type approach, the pronoun *she* is an elided definite description, i.e. is semantically equivalent to *the woman that is the room*. In order to satisfy the uniqueness presupposition associated with the definite description, the second sentence has to be evaluated relative to the *minimal* situations making the first sentence true (in which there is exactly one woman). Specifically, the situations at which the second sentence is true are situations that contain exactly one woman. Therefore, if we want the update rule to be intersective, the information state that results from updating some initial information state with the two sentences will contain only situations in which there is exactly one woman. But then suppose that after these two sentences, a third sentence is added, as in (5):

(5) A woman was in the room. She smiled. Another woman was in the room.

Now we have a problem. If information states only consist of minimal situations (in the sense that they cannot contain two situations s_1 and s_2 such that s_1 is a subsituation of s_2), then the third sentence will lead to an empty information state. So we need the information state that results from the second sentence to include non-minimal situations, or the third sentence to be able to *add* situations to its input context. However, this is only possible if either the update from the first sentence to the second one is itself not intersective (since the second sentence is true only in situations that contain exactly one woman in the room), or if the update from the second sentence to the third is non-intersective (since the third sentence must then add situations to its input context).[7]

1.3 Discourse referents and fine-grained semantic content without Dynamic Semantics

One core aspect of standard dynamic semantics for anaphora, which is retained in the system developed in this paper, is the idea that information states are more fine-grained than mere classical propositions (sets of possible worlds). Information states construed as sets of world-assignment pairs (instead of just assignments) specify not only factual beliefs about the world, but also 'bind' various pieces of information to specific variables, also known as 'Discourse Referents', which, in a certain sense, carry the relevant information. For instance, an information state representing the belief that there is a car in the street might distinguish a certain variable which, in all pairs (w, g) that make up the information state, denotes a car in the street. It is in fact plausible that, quite independently of communication, humans store information in such a way, i.e. by tying specific propositional information to 'mental files' (Recanati 2012), which can be viewed as the mental counterpart of variables.[8]

What makes dynamic semantics dynamic, however, is a second core property, namely the fact that the semantic value of a sentence is a function from information states to information states, and that semantic composition involves building information update functions out of the information update functions denoted by the expressions being composed. In the system developed here, I eliminate this second ingredient, but keep the first one. Just like in standard classical semantics, sentences receive a truth-value relative to a world and an assignment (except that in my final proposal, I use *plural* assignments). Note however that in the standard semantics for first-order logic, assignments play only a technical role in the clauses that define the meaning of quantifiers, and a sentence with no free variable receives the same truth-value across all assignments. Such sentences can then be said to be true or false at a world, and the propositions they

[7]See Mandelkern and Rothschild (2020) for a related argument that conjunction cannot be defined as intersection in the E-type approach to anaphora.

[8]The connexion between mental files and DRT discourse referents, Heim's files, or variables in other dynamic semantics frameworks is explicit in Récanati's notion of mental file.

express can be identified as sets of worlds.[9] In my system, in contrast with the classical treatment of variables, assignments will be relevant even to the truth of sentences with no free variables, and sentences are viewed as providing not only factual information about the world, but also information about variables, and this information is part of the cognitive content being communicated.

2 Preliminary proposal

In this section, I spell out a relatively simple static semantics for anaphora. I consider the language of first-order logic without the universal quantifier and material implication, and I provide a non-standard, but static trivalent semantics for it . Then I state a version of Schlenker's Transparency ondition to accont for presuppositions. I then illustrate in section 3 how the resulting system correctly captures the truth-conditions and presuppositions of a number of specific sentences, and I finally point out a major defect of this system, which, among others, prevents it from being enriched with a well-behaved universal quantifier. This will motivate the more sophisticated proposal offered in section 4.

2.1 Semantics

The language is that of predicate logic with no universal quantifier, no material implication and no constant. Predicates are denoted by capital letters, variables by lowercase letters. For convenience, I assume that the language only includes 1-place and 2-place predicates, but nothing important hinges on this, and extending the proposal to n-ary predicates, with $n > 2$, is straightforward. Finally, I enrich the syntax of 1st-order logic as follows: free variables (in the standard syntactic sense of 1st order logic) are *underlined*. Whether a variable is underlined or not does not play any role in the semantics, but will play a role when we move to the Transparency Condition in section 2.2. Specifically, free-variables will trigger a specific presupposition. When stating the semantics, I do not underline any variable, since as just said this plays no role in the semantics proper.

The semantics of the language is expressed relative to a simplified Krikpe model: a triplet $\{D, W, I\}$, where D is a set of individuals, W a set of possible worlds and I is an interpretation function. If P is a 1-place predicate, and w is world, $I(P, w)$ is a set of members of D. If R is a 2-place predicate and w is a world, $I(R, w)$ is a set of ordered pairs of members of D. The semantics is trivalent and truth-conditional: each sentence, relative to a world-assignment pair, is assigned a truth-value in $\{0, \#, 1\}$. Importantly, assignments can be partial. If g is an assignment, $g(x) = \#$ means that g is not defined for x. Furthermore, if a is a member of D, $g^{x\to a}$ denotes the assignment function which agrees with g on every variable except possibly x, and maps x to a. For a given domain D, we use a 'canonical' model, i.e. such that every combination of possible denotations for 1-place predicates and 2-place predicates is instantiated in some world of W. That is, in practice, each member of W corresponds to a standard 1st-order model based on domain D, and every standard 1st order model based on domain D corresponds to a member of W. We will systematically omit the model superscript when stating the semantics, i.e. we state the semantics for a single model $\{D, W, I\}$, which is fixed once and for all.

The interpretation of predicates. If P is a 1-place predicate, R a 2-place predicate, and x and y variables:

$$[\![P(x)]\!]^{w,g} = \begin{cases} 1 & \text{if } g(x) \in I(P, w) \\ 0 & \text{if } g(x) \neq \# \text{ and } g(x) \notin I(P, w) \\ \# & \text{if } g(x) = \# \end{cases}$$

[9] If free variables are viewed as the counterpart of referential pronouns in language, the most natural view, within the classical logic framework, would be to consider that sentences with free variables do not express propositions by themselves, but that they express a proposition in specific occasions of use, in which case the context provides a reference for the free variables, i.e., a specific assignment function. On this view, assignment functions are relevant to determine the truth-conditions of sentences with free variables (hence their truth-value in a given world), but we still view information states as consisting of classical propositions, i.e. sets of worlds.

$$[\![R(x,y)]\!]^{w,g} = \begin{cases} 1 & \text{if } (g(x), g(y)) \in I(R,w) \\ 0 & \text{if } g(x) \neq \# \text{ and } g(y) \neq \# \text{ and } (g(x), g(y)) \notin I(P,w) \\ \# & \text{if } g(x) = \# \text{ or } g(y) = \# \end{cases}$$

The existential quantifier. We adopt a version of Mandelkern's (2022) *witness condition.* Informally speaking, a statement such as $\exists x\phi$ is undefined relative to a pair (w, g) in case it is classically true in w but $g(x)$ is not a witness of the relevant existential claim: that is, we require the assignment function to give us a witness for the existential statement. When the statement is classically true and $g(x)$ is a witness, then it counts as true. Finally the statement the falsity condition is the same as in the classical semantics for first order logic.

$$[\![\exists x\phi]\!]^{w,g} = \begin{cases} 1 & \text{if } [\![\phi]\!]^{w,g} = 1 \\ 0 & \text{if for every } a \text{ in } D, [\![\phi]\!]^{w,g^{x\to a}} = 0 \\ \# & \text{otherwise .} \end{cases}$$

Connectives: conjunction, disjunction and negation. We use the Middle Kleene (Peters 1979; Beaver and Krahmer 2001) truth-tables for connectives. Negation maps 1 to 0, 0 to 1, and # to itself. If ϕ is undefined, then $\phi \vee \psi$ and $\phi \wedge \psi$ are undefined. If ϕ is not undefined, then the truth-values of $\phi \vee \psi$ and $\phi \wedge \psi$ are the same as in the Strong Kleene truth-tables:

Table 1: Middle Kleene connectives

$[\![\phi]\!]^{w,g}$	$[\![\psi]\!]^{w,g}$	$[\![\phi \wedge \psi]\!]^{w,g}$
1	1	1
1	#	#
1	0	0
#	1	#
#	#	#
#	0	#
0	1	0
0	#	0
0	0	0

$[\![\phi]\!]^{w,g}$	$[\![\psi]\!]^{w,g}$	$[\![\phi \vee \psi]\!]^{w,g}$
1	1	1
1	#	1
1	0	1
#	1	#
#	#	#
#	0	#
0	1	1
0	#	#
0	0	0

$[\![\phi]\!]^{w,g}$	$[\![\neg\phi]\!]^{w,g}$
1	0
#	#
0	1

Remark. Note that when ϕ is true, the truth-value of $\phi \wedge \psi$ is just the truth-value of ψ, and that when ϕ is false, the truth-value of $\phi \vee \psi$ is (again) just the truth-value of ψ – this will prove useful later on.

Truth at a world. Having fully specified a (non-standard) semantics, how shall we assess its adequacy? To have a full grasp of the system, we need to wait until the Transparency Condition is introduced. We can nevertheless start asking whether the system delivers adequate truth-conditions, especially for the cases that have historically motivated the move to dynamic semantics. To so do, we need, minimally, a criterion for truth relative to a world rather than just a world-assignment pair. I posit the following definition of truth relative to a world: a sentence ϕ is true at a world w if and only if there is an assignment g such that $[\![\phi]\!]^{w,g} = 1$. Based on such a definition, the sentences $P(x)$ and $\exists xP(x)$ have the same truth-condition (but not the same falsity-condition), and end up as true at a given world if there is an individual which has property P. We will want free variables to be interpreted as referential pronouns, hence to trigger presupposition failure if the context (viewed as a set of world-assignment pairs) does not ensure that the variable x is valued by every assignment function of the context, but this will be taken care of by the Transparency Condition introduced in section 2.2.

Starting with a classical case of 'dynamic' anaphora, a sentence such as $\exists xP(x) \wedge Q(x)$ comes out true at a world w if for some g it is true relative to (w, g), namely if for some g, $g(x)$ belongs to $I(P, w)$ and to $I(Q, w)$, which in turn will be the case if and only if there exists an individual d which belongs to the intersection of $I(P, w)$ and $I(Q, w)$, i.e. just in case $\exists x(P(x) \wedge Q(x))$ is classically true in w - one of the key desiderata for a theory of anaphora.

Consider now the first-order logic translation of the famous 'bathoom sentence': *Either there is no bathroom, or it is on the second floor*:[10]

(6) $\neg\exists x B(x) \vee F(x)$

We will show that (6) is true at a world w if and only if (7) is classically true at w, making the corresponding English sentence true just in case either there is no bathroom, or there is a bathroom on the second floor.[11]

(7) $\neg\exists x B(x) \vee \exists x(B(x) \wedge F(x))$

Proof:

- Let us first prove that if (7) is classically true at w, then there is a g such that (6) is true (according to our semantics) at (w, g).

Assume that (7) is classically true at w. We distinguish two cases:

1. The first disjunct is classically true at w, i.e. $\exists x B(x)$ is classically false at w.
2. The second disjunct is classically true at w (note that the two disjuncts are classically inconsistent with each other)

First case: since $\exists x B(x)$ is classically false at w, it follows that for every g, it is also false *according to our semantics* at (w, g) (since our falsity condition for the existential quantifier is classical). Given our semantics for negation, then, $\neg\exists x B(x)$ comes out true at (w, g) for every g, and (6) also comes out true at (w, g) for every g (given the Middle Kleene semantics for disjunction), hence true simpliciter at w.

Second case: $\exists x(B(x) \wedge F(x))$ is classically true at w. Let us pick a individual a which is a witness, in w, of $\exists x(B(x) \wedge F(x))$, and let us consider an assignment function g such that $g(x) = a$. (6) comes out true (according to our semantics) at (w, g). This is so because $\exists x B(x)$ is true at (w, g), therefore $\neg\exists x B(x)$ is false at (w, g) (hence, crucially, not undefined), and the second disjunct, $F(x)$ is true at (w, g), since $g(x) = a$ and $a \in I(F, w)$. So (7) comes out true at (w, g), hence true simpliciter at w.

- In the other direction, we now show that if (6) is true at some pair (w, g) relative to our semantics (hence true simpliciter at w), then (7) is classically true at w.

Assume that (6) is true at some pair (w, g) according to our semantics. Given the Middle-Kleene connectives, either $\neg\exists x B(x)$ is true at (w, g) (first case) or $\neg\exists x B(x)$ is false at (w, g) and $F(x)$ is true at (w, g) (second case). In the first case, given the meaning of negation, $\exists x B(x)$ is false at (w, g), hence classically false at w, which makes (7) classically true at w. In the second case, since $\neg\exists x B(x)$ is false at (w, g), $\exists x B(x)$ is true at (w, g), and $F(x)$ true at (w, g). So $g(x) \in I(B, w)$ and $g(x) \in I(F, w)$, hence $g(x)$ is a witness, in w, for the existential statement $\exists x(B(x) \wedge F(x))$, interpreted classically, and therefore $\exists x(B(x) \wedge F(x))$ is classically true at w, which makes (7) classically true at w.

2.2 Contexts, Discourse Dynamics, and the Transparency Condition

We will see in section 3.5 that the semantics just introduced is seriously inadequate for more complex sentences involving several existential quantifiers and variables, which will motivate a more sophisticated

[10] First generation dynamic theories failed to account for bathroom sentences. See Krahmer and Muskens (1996) and, more recently, Hofmann (2019); Elliott (2020), a.o., for dynamic approaches that correctly capture the interactions between anaphora, negation and disjunction.

[11] When we write 'true at a world', we mean true according to the above definition, i.e. there is a g that makes the relevant sentence true at (w, g) according to our semantics. We write 'classically true' to refer to the standard truth-conditions of first-order logic.

approach, based on plural assignment functions (section 4).

But even if we put this problem aside, the semantics we have proposed is clearly incomplete if viewed as providing a theory of the interpretation of indefinites and pronouns. To begin with, given our definition of 'truth at a world', a sentence with a free variable, e.g. $P(x)$, ends up expressing an existential statement. But since we want such sentences to translate English sentences that include a referential, non-bound pronouns ('It is on the first floor'), this is by itself inadequate.

Secondly, and relatedly, since sentences such as $\exists x P(x)$ and $P(x)$ end up with identical truth-conditions (though different falsity conditions), this semantics by itself does not capture the fact that indefinites in natural languages (which we translate into existential quantifiers) and pronouns have very different usage conditions. In a way, this is not unlike Heim's (1982) file change semantics, in which both pronouns and indefinites are treated as variables (which have the same semantics), but associated with different felicity conditions (the novelty condition and the familiarity condition.)

Another issue is the fact that a sentence such as $\exists x P(x) \wedge \exists x Q(x)$ also ends up true relative to (w, g) just in case $P(x) \wedge Q(x)$ is, hence true simpliciter, again, only if $\exists x(P(x) \wedge Q(x))$ is classically true, but this can never be the intuitive reading of a sentence such as *Someone is singing and someone is standing* (which can never *mean* 'Someone is singing and standing', even though situations where someone is singing and standing are among those that make the sentence intuitively true).

In this section, we introduce the idea that free variables introduce a *presupposition* that they are 'valued' (which can be viewed as a counterpart of Heim's familiarity condition), and we give a theory of presupposition satisfaction based on Schlenker's Transparency Principle (which, crucially, allows us to stick to a static semantics). A second principle will require that an existential quantifier introducing a variable x is infelicitous if the variable x could have been left free in the same position, i.e. would have met the Transparency principle, capturing the intuition that indefinites must be, in a certain sense, 'new', and making a sentence such as $\exists x P(x) \wedge \exists x Q(x)$ always infelicitous (this will be the counterpart of Heim's novelty condition on indefinites).

We will see these assumptions are able to solve the various issues we have just mentioned.

2.2.1 Contexts and Discourse Update

As is standard in theories of presuppositions, presuppositions will be viewed as introducing a condition on the context of utterance. We thus need to introduce a notion of context, and also of context update. While our semantics is static, i.e. not stated in terms of context change potentials, still we need to characterize how, given the semantic value of a sentence, a speaker who accepts the sentence changes her beliefs.

We view a *context* C as a set of world-assignment pairs (w, g). We assume a very simple, Stalnakerien update rule: if a sentence S is accepted as true in context C, then the resulting context is simply C intersected with the set of world-assignment pairs where S is true. Of particular interest to us is the *null context* which contains all possible world-assignment pairs.

Within this classical, Stalnakerian framework, it is tempting to want to capture the use conditions of free variables by imposing the following condition, akin to Stalnaker's bridge principle: a sentence S is infelicitous in context C if there is a world assignment pair (w, g) in C such that $[\![S]\!]^{w,g} = \#$. In the case of a simple sentence such as $P(x)$, this ensures that it is not felicitous in the null context, because for some assignments in the null context, $g(x) = \#$. This, however, will not do, since in the null context a sentence such as $\exists x P(x)$ is also undefined in some world-assignment pairs (w, g) (namely when it is classically true at w but $g(x)$ is not a witness), but we don't want such sentences to be infelicitous in the null context. So this is not the strategy we will pursue. Importantly, then, the third, undefined truth-value # should not be viewed, in the system described here, as corresponding to presupposition failure. Instead, we will impose another condition on context (the Transparency Condition), whose effect will be that, as desired, $P(x)$ is predicted to be infelicitous in the null context (in fact any context where x is not valued throughout the entire context), but $\exists x P(x)$ will be felicitous in the null context. We will also show that the Transparency Condition ensures that $\exists x P(x) \wedge Q(x)$ and $\neg \exists x P(x) \vee Q(x)$ are felicitous in the null context. Finally, we will restrict the use of the existential quantifiers through an 'Anti-Transparency Condition' for existential quantifiers, which will essentially play the role of Heim's 'novelty condition' on indefinites (Heim 1982).

2.2.2 The Transparency and Anti-Transparency Conditions

I posit that any occurrence of a free variable *presupposes*, in the sense of Schlenker's (2007; 2008) Transparency Principle, that it is valued. I adopt here the symmetric version of Transparency, which is not sensitive to linear order (cf. footnote 13).

To state the Transparency and Anti-Transparency Principles in the specific case of free variables, we need to enrich our language with a one place predicate, U, whose denotation is not fixed by the interpretation function, but by the following rule – note that this predicate is bivalent, i.e. it can only yield 'true' or 'false':

$$[\![U(x)]\!]^{w,g} = \begin{cases} 1 & \text{if } g(x) \neq \# \\ 0 & \text{if } g(x) \ = \# \end{cases}$$

$U(x)$ essentially means 'x is valued'.

Transparency Principle For any two variables x and y, any sentence S, any one-place predicate P and any two-place predicate R [ϕ is a metalanguage propositional variable which stands for an arbitrary formula]:

1. If $P(\underline{x})$, $R(\underline{x}, y)$ or $R(y, \underline{x})$ occurs in S, consider the sentence $S1$ that is obtained by replacing this occurrence by $U(x) \wedge \phi$ and the sentence $S2$ that is obtained by replacing that very same occurrence with ϕ.

2. if $R(\underline{x}, \underline{y})$ occurs in S, consider the sentence $S1$ that is obtained by replacing this occurrence by $(U(x) \wedge U(y)) \wedge \phi$ and the sentence $S2$ that is obtained by replacing that same occurrence with ϕ.

3. Transparency is satisfied in context C (a set of pairs (w, g)) relative to such an occurrence if and only if $S1$ and $S2$ have the same truth-value throughout C, for every possible instantiation of ϕ.

4. For S to be felicitous in C, Transparency must be satisfied in C relative to every occurrence of every free (underlined) variable.

Existential quantifiers are subject to their own felicity conditions, namely *Anti-Transparency.* The goal is to rule out sentences such as $\exists x P(x)$ in a context where x is already valued. To this end, I introduce an additional condition, Anti-Transparency, which might be derivable as a special case of the more general constraint known as *Maximize Presupposition* (Heim 1991), and hence be better motivated (though we do not offer a formal derivation of this constraint from *Maximize Presupposition*)

Anti-Transparency says that an existential quantifier binding a variable x can be used at a certain position in a sentence only if the variable x, used as a free variable in that position, would meet the Transparency Condition. The possible connection with *Maximize Presupposition!* is that, at least in the most simple cases, Anti-Transparency requires the use of a free variable rather than that of an existentially bound variable whenever the presupposition of the free variable is satisfied.

Anti-Transparency Principle: If a formula F of the form $\exists x\psi$ has an occurence in a sentence S, construct the two formula $S1$ and $S2$ obtained by replacing this occurrence in F with, respectively, ϕ and $U(x) \wedge \phi$. If, for every possible instantiation of ϕ, $S1$ and $S2$ have the same truth-value at every pair (w, g) of some context C, then S is not felicitous in C.

3 Applying Transparency and Anti-Transparency

I now show that the Transparency Condition predicts the right felicity conditions for pronouns which are not syntactically bound, and that Anti-Transparency correctly rules out certain occurrences of existentially bound variables when they have already occurred earlier, ensuring that existential quantifiers always contribute 'new' variables.

3.1 Free variables without an indefinite antecedent

Consider a simple sentence $P(\underline{x})$ (x is underlined, because it is a free variable, hence subject to the Transparency principle)[12]. For Transparency to be met in a certain context C, it has to be the case that for every sentence ϕ, $U(x) \wedge \phi$ and ϕ receive the same truth-value throughout C. In the 'null context' which contains all possible world-assignment pairs, Transparency is obviously not satisfied. Consider indeed a pair (w, g) such that $g(x) = \#$. Then $U(x) \wedge \phi$ is false relative at (w, g), irrespective of ϕ's truth-value (since $U(x)$ is false at (w, g)), but ϕ might of course be undefined or true. Transparency is therefore not satisfied. This captures the fact that a sentence such as *It is purple* is infelicitous out of the blue.

Suppose now that $P(\underline{x})$ is used after $\exists x Q(x)$ has been asserted and accepted in previous discourse. The context C that results from accepting $\exists x Q(x)$ contains all the pairs (w, g) at which $\exists x Q(x)$ is true, hence $g(x)$ is defined for all such pairs. As a result, $U(x)$ is true throught C', and given the Middle-Kleene truth-table for conjunction, for every ϕ, the truth-value of $U(x) \wedge \phi$ is the same as that of ϕ, throughout the context. This captures the fact that a discourse such as '*A table is in the room. It is purple.*' is felicitous.

Note that even though there is a kind of dynamics at play at the level of discourse, our treatement of intrasentential conjunction (discussed in the next subsection) does not involve any dynamicity.

3.2 Anaphora in conjunctive sentences

Consider now a sentence such as *A table is in the room and it is purple*, translated as $\exists x T(x) \wedge P(\underline{x})$. Our goal is to prove that Transparency is satisfied in any possible context. This amounts to prove that across all possible pairs (w, g), for any sentence ϕ, (8a) and (8b) have the same truth-value.

(8) a. $\exists x T(x) \wedge \phi$
    b. $\exists x T(x) \wedge (U(x) \wedge \phi)$

Consider a pair (w, g).

Suppose first that $\exists x T(x)$ is false in (w, g) (i.e. that T has an empty extension in w). Then both sentences are false and therefore have the same truth-value.

Next, suppose that $\exists x T(x)$ is undefined in (w, g) (which will be the case if the existential statement is classically true but $g(x)$ is not a witness of it). Then, given the Middle Kleene truth-table for conjunction, the two sentences are undefined, hence have the same truth-value.

Finally, assume that $\exists x T(x)$ is true in (w, g). Then g values x and therefore $[\![U(x)]\!]^{w,g} = 1$, from which it follows (from the truth-table for conjunction) that $[\![U(x) \wedge \phi]\!]^{w,g} = [\![\phi]\!]^{w,g}$, and therefore (8a) and (8b), have the same truth-value. This concludes the proof.

Suppose we now reverse the sentence, i.e. we consider $P(\underline{x}) \wedge \exists x T(x)$. For Transparency to be satisfied in the null context, (9a) and (9b) must have the same truth-value, for every ϕ, relative to every pair (w, g):

(9) a. $\phi \wedge \exists x T(x)$
    b. $(U(x) \wedge \phi) \wedge \exists x T(x)$

Take $\phi = P(x)$. If g does not value x, then (9a) comes out undefined at (w, g) (for any w), but (9b) comes out false at (w, g) (for any w), so Transparency is not satisfied – which is the desired result.

3.3 Anaphora in disjunctive sentences: bathroom sentences

Consider now a 'bathroom sentence', *There is no bathroom in this house, or it is hidden*, translated as $\neg\exists x B(x) \vee H(\underline{x})$. We have already seen that our semantics delivers the right truth-conditions for such a sentence. Namely, it is true in (w, g) just in case either there is no bathroom in the house in w, or there is at least one which is well hidden, and $g(x)$ is a bathroom in the house that is well hidden.

[12] When checking whether Transparency is met, I do not underline any variable in the sentences whose equivalence is to be checked, since underlining is only there to signal which variables are subject to Transparency, but does not affect semantic values.

We now set out to prove that this sentence satisfies Transparency in every possible context, i.e. that across all pairs (w, g), for every ϕ, (10a) and (10b) have the same truth-value.

(10) a. $\neg\exists x B(x) \vee (U(x) \wedge \phi)$
b. $\neg\exists x B(x) \vee \phi$

Let (w, g) be a world assignment pair. Given the Middle Kleene truth-table for disjunction, if $\neg\exists x B(x)$ is true or undefined in (w, g), both sentences in (10) are, respectively, true or undefined, hence have the same truth-value. If $\neg\exists x B(x)$ is false in (w, g), then $\exists x B(x)$ is true in (w, g), and therefore g values x. Therefore $U(x)$ is true in (w, g), as a result of which the truth-value of $U(x) \wedge \phi$ and that of ϕ are the same in (w, g), and therefore (10a) and (10b) have the same truth value in (w, g).

A reverse bathroom sentences such as 'It is well hidden, or there is no bathroom in this house', translated as $H(\underline{x}) \vee \neg\exists x B(x)$, is not predicted to meet Transparency in the null context (or any context in which the variable x has not been previously introduced by an existential quantifier in preceding discourse). That is, the sentences in (11) do not have the same truth-value across all pairs (w, g) and for every ϕ:

(11) $H(\underline{x}) \vee \neg\exists x B(x)$:
a. $\phi \vee \neg\exists x B(x)$:
b. $(U(x) \wedge \phi) \vee \neg\exists x B(x)$

To see that Transparency fails, pick a tautological sentence as the value for ϕ (e.g. $U(x) \wedge \neg U(x)$). Consider a g that does not value x. Then (11a) is true, and (11b) is undefined, so Transparency fails.

Now, it is in fact not empirically clear whether we want to rule out reverse bathroom sentences. There is quite strong evidence that, be it in relation with presupposition or anaphora, disjunction behaves more symmetrically than conjunction, and presupposition satisfaction as well as anaphora licensing 'from right to left', so to speak, should not be categorically excluded (see, a.o., Kalomoiros and Schwarz, to appear). The current system implements a strict left-to-right bias, through the use of Middle Kleene Truth Tables, which are asymmetric.[13]

3.4 Anti-Transparency and the Novelty Condition on indefinites

Consider a discourse such as $\exists x P(x).\exists x Q(x)$. Without Anti-Transparency, this discourse would convey the same information as $\exists x(P(x) \wedge Q(x))$. But this is not a possible interpretation for something like *Someone smokes. Someone drinks.* So we need to rule out such a sequence, and this is what Anti-Transparency does. When the first sentence is accepted, the initial context is intersected with the set of all pairs (w, g) at which $\exists x P(x)$ is true. The resulting context is the one with respect to which the second sentence is evaluated, and in this new context x is valued at every pair. Therefore, $U(x)$ is true at every pair, and so for any possible ϕ, $U(x) \wedge \phi$ has the same truth-value as ϕ at every pair of the context. The second sentence thus fails to meet Anti-Transparency. As a result, a natural language discourse such as *Someone*

[13]It would be natural to investigate a system based on the Strong Kleene truth table for disjunction, as well as one with Strong Kleene truth tables for both disjunction and conjunction, with or without an *incremental* version of the Transparency Principle, as discussed in Schlenker (2007, 2008). It turns out that adopting a Strong Kleeene disjunction gives rise to a non-trivial complication. Namely, under an SK disjunction, (10a) and (10b) are no longer equivalent: relative to a pair (w, g) such that g does not value x and $\exists x B(x)$ is undefined, if ϕ is a tautology, then (10b) is true but (10a) is undefined (under the SK truth table for disjunction). To solve this problem, we may modify the Transparency Principle so that the notion of equivalence that is involved in comparing the two relevant sentences (in this case, (10a) and (10b)) is weaker than the one we use in the official system: we would say that S_1 and S_2 are contextually equivalent (for the purpose of checking Transparency) if there does not exist a pair (w, g) in the context and relative to which one is true and the other is false, i.e. we would ignore all indices at which one of the two sentences is undefined. I have not explored the consequences of such a modified version of my system. In general, it would make sense to systematically investigate different variants of my proposal based on a) choice between Middle Kleene and Strong Kleene, b) choice between an incremental vs. global version of the Transparency Principle, c) choice of a notion of equivalence for trivalent propositions (see, e.g., Chemla et al. 2017 for a discussion of various notions of logical entailment in Trivalent Logic). It would also make sense to consider replacing the Transparency Principle with a related proposal ('Limited Symmetry') due to Kalomoiros (2022), which is designed to predict a left-to-right bias for conjunction but not for disjunction. I leave these ramifications to further research.

smokes. Someone drinks. will be translated into two sentences involving different variables, and will not have any reading amounting to *Someone smokes and drinks*. Here we seem to reason a bit like in a dynamic semantics approach, since we check whether Anti-Transparency is met for the second sentence after the context has been updated with the first sentence. This is because even though our semantics is static at the level of sentences (unlike what happens in genuinely dynamic approaches, where context update plays a role sentence-internally), we still view discourse as dynamic and as involving sequential updates. Importantly though, we also make the same prediction (violation of Anti-Transparency) when we move to a single sentence where the two existential statements are conjoined, with no reference to context update. That is, consider the sentence $\exists x P(x) \wedge \exists x Q(x)$. Again, we want to rule out such a sentence, since it would mean the same as $\exists x(P(x) \wedge Q(x))$, but we don't want this to be a possible meaning for *Someone smokes and someone drinks*. To check Anti-Transparency, we need to check whether, for every ϕ, the two sentences in (12) have the same truth-value across all pairs (w, g) of the context. But we already know that this is the case, since the two sentences in (12) have exactly the same form as those in (8).

(12) a. $\exists x P(x) \wedge (U(x) \wedge \phi)$
b. $\exists x P(x) \wedge \phi$

This result ensures that a sentence such as *Someone smokes and someone drinks* cannot be translated by $\exists x P(x) \wedge \exists x Q(x)$ (since this would violate Anti-Transparency), and cannot be interpreted as equivalent to *Someone smokes and drinks*.

We now consider a somewhat more complex case, namely $\exists x P(x) \wedge (\neg \exists x Q(x) \vee R(\underline{x}))$, which in principle could translate *A French woman is in the house, and either there is no female linguist in the house, or she is upstairs*. First, the Transparency Condition for $R(\underline{x})$ is satisfied since we already know that it is satisfied in the subconstituent $\neg \exists x Q(x) \vee R(\underline{x})$. Ignoring Anti-Transparency, this sentence comes out true at a pair (w, g) if $g(x)$ is in the extension of P at w, and either nothing is in the extension of Q at w, or $g(x)$ is is in the extension of both Q and R. That is, the corresponding English sentence would mean, under this translation, *A French woman is in the house, and either there is no female linguist in the house, or the French woman in the house (or at least one of them if there are several French women) is a linguist and is upstairs.* This is, quite obviously, not a possible reading for this complex sentence. Fortunately, Anti-Transparency rules out this formula (which therefore cannot be used to translate the corresponding English sentence), because Anti-Transparency is not met for the subformula $\neg \exists x Q(x)$. Let us see why. We need to compare the two following sentences, for any arbitrary ϕ:

(13) a. $\exists x P(x) \wedge (\neg(U(x) \wedge \phi) \vee R(\underline{x}))$
b. $\exists x P(x) \wedge (\neg \phi \vee R(\underline{x}))$

Consider an arbitrary pair (w, g). If $\exists x P(x)$ is false (resp. undefined) at (w, g), then both sentences are false (resp. undefined) at (w, g), hence have the same truth-value at (w, g). If $\exists x P(x)$ is true at (w, g), $U(x)$ is true at (w, g), hence $U(x) \wedge \phi$ and ϕ have the same truth-value at (w, g), hence (13a) and (13b) have the same truth-value at (w, g) as well. Therefore Anti-Transparency can never be satisfied.

3.5 A problem: failure of co-variation

The current system faces a critical problem, as was pointed out to me by Amir Anvari, in relation with universal quantification. It fails to predict a very salient interpretation of sentences such as those in (14):

(14) a. There isn't anyone who didn't speak to someone.
b. Not a single person failed to speak to someone.

The reading we are interested is the one where *someone* takes narrow-scope, i.e. the one that corresponds to the surface constituent structure of these sentences. Under this construal, both sentences are equivalent to *Everybody talked to someone*, with a narrow-scope interpretation of the indefinite – but as we shall see we do not predict this.

The straigthforward way to translate such sentences into first logic is to render them as $\neg\exists x \neg\exists y S(x,y)$ (assuming for simplicity that the domain over which variables range only consist of people). And of course, according to the classical semantics, this formula indeed means that for every x, there is a y such that $S(x,y)$, as desired.

However, the system we have developed fails to predict this reading. It in fact makes this formula true at a world w if and only if there is a specific individual a such that everyone is in the relation S to a, i.e. it derives a *wide-scope* interpretation for the most embedded existential quantifier, despite the fact that this quantifier is syntactically in the scope of all other operators. Let us indeed compute the truth-value of the relevant sentence relative to an arbitrary pair (w,g)

(15) $[\![\neg\exists x \neg\exists y S(x,y)]\!]^{w,g} = 1$ if:
$[\![\exists x \neg\exists y S(x,y)]\!]^{w,g} = 0$, i.e:
for all a, $[\![\neg\exists y S(x,y)]\!]^{w,g^{x\to a}} = 0$, i.e:
for all a, $[\![\exists y S(x,y)]\!]^{w,g^{x\to a}} = 1$, i.e:
for all a, $[\![S(x,y)]\!]^{w,g^{x\to a}} = 1$, i.e:
for all a, $(a, g(y)) \in I(S,w)$.

So, interpreting S as *spoke to*, the formula is true at (w,g) if and only if every individual spoke to $g(y)$ in w, and so it is true simpliciter at w if and only if there exists an individual b sucht hat every individual spoke to b.

This 'failure of covariation' is a major problem of the current system. There also does not seem to be a way to add a universal quantifier to language and to assign it a plausible meaning without running into the same issue for a sentence of the form $\forall x \exists y S(x,y)$.

4 A static semantics with Plural assignments

There are compelling arguments in the dynamic semantics literature that information states should contain not only information about individual variables, but also about their relationship. For instance, a sentence such as *[Every student]$_s$ read [a book]$_b$* must have the effect that the resulting information state records, at the level of assignment functions, that whoever s is, b is a book that s read. This is how Quantificational Subordination is accounted for: if the sentence is followed by *[Every student]$_s$ liked it$_b$*, this is understood to mean that every student liked the book they read. Technically, this is achieved by working with *plural assignments* instead of standard assignments (van der Berg 1996; Nouwen 2003; Brasoveanu 2008, a.o.). That is, instead of viewing a context as a set of pairs (w,g) where g is a single assignment function, contexts are viewed as pairs (w,G), where G is a set of assignments, a *plural assignment* for short. I adopt this view of information states. As a result, sentences will be evaluated not with respect to a pair (w,g), but to a pair (w,G), where G is a plural assignment. The system will remain static in the sense that a sentence is true, false, or undefined at a single index (w,G), while information states and contexts are viewed as sets of indices.

4.1 Background: plural assignments and quantificational subordination

Before presenting the system, let us have a look at the way dynamic plural semantics uses plural assignments. Consider the sentence *[Every student]$_s$ read [a book]$_b$*, translated into first order logic as in (16) (in a simplified form):

(16) $\forall s \exists b R(s,b)$

In plural dynamic semantics, when applied to the null context C which contains all possible pairs (w,G), where G is a set of atomic assignments, (16) returns the context C' which contains all pairs (w,G) such that (16) is classically true in w, and for every individual d of the domain, there is an individual assignment in

G, call it g_d such that $g_d(s) = d$, and furthermore, for every assignment g in G, $g(s)$ and $g(b)$ stand in the relation R in w.

The dynamic interpretation of a sentence like (16) can be grapsed more clearly by representing plural assignments as tables. Suppose there are exactly two students s_1 and s_2 and two books b_1 and b_2, and that in world w_1, s_1 read b_1 and s_2 read b_2, while in world w_2, s_1 read b_2 and s_2 read b_1. Suppose furthemore (for simplicity), that only in these two worlds is (16) classically true. After updating with (16), we end up with a sets of pairs which are either of the type (w_1, G_1) or of the type (w_2, G_2), where G_1 and G_2 are plural assignment functions that can be represented by the following tables:

Table 2: Plural Assignment G_1, associated with world w_1 where s_1 read b_1 and s_2 read b_2

Variables	s	b	x	y	...
Atomic Assignment 1	s_1	b_1	a	b	...
Atomic Assignment 2	s_1	b_1	c	d	...
Atomic Assignment 3	s_2	b_2	a	b	...
Atomic Assignment 4	s_2	b_2	c	d	...
...	s_2	b_2	...	...	...
...	s_1	b_1	...	...	...

Table 3: Plural Assignment G_2, associated with world w_2 where s_1 read b_2 and s_2 read b_1

Variables	s	b	x	y	...
Atomic Assignment 1	s_1	b_2	a	b	...
Atomic Assignment 2	s_1	b_2	c	d	...
Atomic Assignment 3	s_2	b_1	a	b	...
Atomic Assignment 4	s_2	b_1	c	d	...
...	s_2	b_1	...	...	...
...	s_1	b_2	...	...	...

Quantificational Subordination is now easy to capture. Suppose that the sentence *[Every student]$_s$ read [a book]$_b$* is followed with *[Every student]$_s$ liked it$_b$*, translated as:

(17) $\quad \forall s L(s, b)$

When applied to the context that results from updating with (16), this sentence filters out of the context all the pairs (w, G) where it is not the case that, for every atomic g in G, $g(s)$ liked $g(b)$. As a result, the pairs that will remain in the updated context contain only worlds where every student read a book and liked the book they read, paired with plural assignments which represent the dependency between students and the books they read and liked: every atomic assignment g in a surviving plural assignment G associated with world w is such that $g(s)$ read $g(b)$ and liked $g(b)$ in w.

4.2 A static semantics with plural assignments

We now add to the language the universal quantifier, with its standard syntax. An atomic assignment is a **total** assignment function in the standard sense, i.e. a total function from variables to individuals in the domain (note that **we now do away with partial assignments**). A plural assignment is a set of atomic assignments. A model is defined exactly as before, i.e. a triplet consisting of a set of worlds W, a domain D of individuals, and an interpretation function I). In order to state the semantics for the language, we need some auxiliary notions.

- A plural assignment G is said to *value* a variable x if for any g, g' in G, $g(x) = g'(x)$. When G values x, $G(x)$ denotes the unique value that x gets throughout G.[14]

[14]In Plural Dynamic Semantics, $G(x)$ would typically denote the sum of all objects d such that there is a g in G such that

- Notation: given a plural assignment G, a sequence of individuals $(a_i)_{i\in\mathbb{N}}$ in the domain and a sequence of variables $(x_i)_{i\in\mathbb{N}}$, $G_{x_1=a_1,x_2=a_2,\ldots,x_i=a_i,\ldots}$ denotes the subset of G that consists of the set of all atomic assignments g in G such that for every i, $g(x_i)=a_i$. For instance, $G_{x=a}$ is the subset of G that consists of all atomic assignments g in G that map x to a. Of course this set can be empty.

The truth-value of a sentence at a given pair (w,G) is recursively determined by the following set of semantic rules:

1. The truth-tables for connectives ($\wedge$, $\vee$, $\neg$) are exactly as before, i.e. are the Middle Kleene Truth-Tables.
2. If P is a one-place predicate and x is a variable, $[\![P(x)]\!]^{w,G}=\begin{cases}1 & \text{if } G(x) \text{ values x and } G(x)\in I(P,w)\\ 0 & \text{if } G(x) \text{ values x and } G(x)\notin I(P,w)\\ \# & \text{if } G(x) \text{ does not value x}\end{cases}$
3. if R is a two-place predicate and x and y are variables, $[\![R(x,y)]\!]^{w,G}=\begin{cases}1 & \text{if } G \text{ values both } x \text{ and } y \text{ and } (G(x),G(y))\in I(R,w)\\ 0 & \text{if } G \text{ values both } x \text{ and } y \text{ and } (G(x),G(y))\notin I(R,w)\\ \# & \text{if } G \text{ does not value } x \text{ or does not value } y\end{cases}$
4. If ϕ is a formula and x is a variable, $[\![\exists x\phi]\!]^{w,G}=\begin{cases}1 & \text{if } [\![\phi]\!]^{w,G}=1\\ 0 & \text{if for every } a \text{ in } D, G_{x=a}\neq\emptyset \text{ and } [\![\phi]\!]^{w,G_{x=a}}=0\\ \# & \text{otherwise}\end{cases}$
5. If ϕ is a formula and x is a variable, $[\![\forall x\phi]\!]^{w,G}=\begin{cases}1 & \text{if for every } a \text{ in } D, G_{x=a}\neq\emptyset \text{ and } [\![\phi]\!]^{w,G_{x=a}}=1\\ 0 & \text{if for some } a \text{ in } D, G_{x=a}\neq\emptyset \text{ and } [\![\phi]\!]^{w,G_{x=a}}=0\\ \# & \text{otherwise}\end{cases}$

I will say that a sentence S is true at a world w if there is a plural assignment G such that S is true at (w,G).

Remarks. Some comments are in order. First, where, in the previous system, undefinedness for simple, predicative sentences, stemmed from the possibility that assignment functions be partial, in the current system, atomic assignments are always total functions, and undefinedness results from a plural assignment function not assigning the same value to the relevant variable across all its atomic assignments.[15] Second, the falsity condition for the existential quantifier is no longer the same as the classical falsity condition. For an existentially quantified statement to count as false in (w,G), G must now map the relevant variable to all individuals in the domain. Third, the universal quantifier and the existential quantifiers are not duals of each other. While the truth-condition for $\forall x\phi$ is exactly the same as the truth-condition for $\neg\exists x\neg\phi$, their falsity conditions differ. For $\neg\exists x\neg\phi$ to be false at (w,G), $G(x)$ must be a witness of the existential statement $\exists x\neg\phi$, but no such requirement obtains for the falsity condition of $\forall x\phi$. For this last statement to count as false in (w,G), it is not required that G values x, but only that G contains an assignment g such that $g(x)$ falsifies the universal statement (i.e. makes ϕ false). The motivation for this non-duality is empirical: we do not want $\neg\forall xP(x)$ to license a subsequent use of x as referring to an individual that falsifies the universal statement, because the following discourse is not felicitous in English under a reading where the pronoun *he* would refer to a male student who didn't come: *Not every male student came. He stayed home.*[16]

$g(x)=d$. We will only use the notation $G(x)$ when G values x, i.e. assigns it a unique value across all its atomic assignments.

[15] The requirement that the relevant variable takes exactly one value across all assignments in G can be viewed as coming from singular morphology. In plural dynamic semantics, we can think of the 'reference' of a variable x relative to a plural assignment G as being the sum of all atomic individuals d such that there is an atomic assignment g in G such that $g(x)=d$ – cf. footnote 14. Singular morphology on a pronoun or an indefinite imposes the condition that this sum boils down to just one atomic individual, which is equivalent to requiring that across all atomic assignments in G, x is mapped to the same value, i.e., in our terms, that G values x.

[16] Of course, the following discourse is much better: *Not every male student came. They stayed home*, where *they* refers to the students who didn't come. Plural anaphora, however, is outside the scope of this paper.

4.3 Transparency and Anti-Transparency, Replicating previous good results

The Transparency and Anti-Transparency conditions are exactly as before, except that (w, g) must be systematically replaced with (w, G) in the clauses that define them and that the special predicate U needs to be redefined as follows:

$$[\![U(x)]\!]^{w,G} = \begin{cases} 1 & \text{if } G \text{ values } x \\ 0 & \text{if } G \text{ does not value } x \end{cases}$$

Furthermore, Anti-Transparency is extended to the universal quantifier, i.e. becomes:

Anti-Transparency Principle: If a formula F of the form $\exists x\psi$ or $\forall x\psi$ has an occurence in a sentence S, construct the two formula $S1$ and $S2$ obtained by replacing this occurrence in F with, respectively, ϕ and $U(x) \wedge \phi$. If, for every possible instantiation of ϕ, $S1$ and $S2$ have the same truth-value at every pair (w, G) of some context C, then S is not felicitous in C.

The cases of 'dynamic anaphora' discussed in section 2 and 3 can be shown to satisfy Transparency in the same way as before. Consider for instance again $\exists xT(x) \wedge P(\underline{x})$. Transparency is again satisfied with respect to any possible context, because, relative to any arbitrary pair (w, G), and any formula ϕ, $\exists xT(x)\wedge(U(x)\wedge\phi)$ and $\exists xT(x) \wedge \phi$ have the same truth-value. If $\exists xT(x)$ is false (resp. undefined) at (w, G), then both sentences are false (resp. undefined) at (w, G); if $\exists xT(x)$ is true at (w, G), then G values x, therefore $U(x)$ is true at (w, G), and the truth-value of $U(x) \wedge \phi$ at (w, G) is necessarily the same as that of ϕ, as a result of which the two sentences have the same truth-value at (w, G). As to bathroom sentences of the form $\neg\exists xB(x) \vee F(\underline{x})$, Transparency is again satisfied with respect to every possible context. That is, for any ϕ, $\neg\exists xB(x) \vee (U(x) \wedge \phi)$ and $\neg\exists xB(x) \vee \phi$ have the same truth-value with respect to any pair (w, G). Indeed, if $\neg\exists xB(x)$ is true (resp. undefined) at (w, G), then both sentences are true (resp. undefined); if $\neg\exists xB(x)$ is false at (w, G), then $\exists xB(x)$ is true at (w, G), hence G values x, and therefore $U(x)$ is true at (w, G), from which it follows that $(U(x) \wedge \phi)$ and ϕ have the same truth-value, and therefore the two sentences have the same truth-value as well. Likewise, Anti-Transparency works exactly as before for the cases we have already discussed.

Note also that a universally quantifier over some variable x does not license the use of x as a free variable in subsequent discourse, capturing the fact that a singular universal quantifier like *every* cannot serve as the antecedent of a pronoun it does not c-command. Consider for instance the following sentence:

(18) $\forall xP(x) \wedge Q(\underline{x})$

The Transparency Condition is not met in the null context for $Q(\underline{x})$. To see this, we need to compare the two following sentences:

(19) a. $\forall xP(x) \wedge (U(x) \wedge \phi)$
b. $\forall xP(x) \wedge \phi$

The key observation here is that, assuming that the domain D of individuals includes at least two entities, (19a) can never be true. Indeed, if the first conjunct $\forall xP(x)$ is true, then for every a in the domain, there is a g in G that maps x to a, hence G does not map x to the same individual throughout all its atomic assignments and therefore G does not value x, hence $U(x)$ is false, and therefore (19a) is not true. But (19b) is true at some pair (w, G): pick (w, G) so as to make the first conjunct true, and take $\phi = U(z) \vee \neg U(z)$, which is true at every pair (w, G).

Finally, the need to extend Anti-Transparency to the universal quantifier can be illustrated with the following sentence:

(20) $\exists xP(x) \wedge \neg\forall xQ(x)$

(20) is true in (w, G) if G values x and $G(x) \in I(P, w)$ and for some a, $G_{x=a} \neq \emptyset$ and $a \notin I(Q, w)$. For this to be the case, G has to map x to a unique individual d, and this individual d is then the only one that can

instantiate the existential statement 'for some a, $G_{x=a} \neq \emptyset$', hence that very same individual d must not be in $I(Q, w)$. So the sentence entails that there is an individual that has property P and not property Q. But there is no reading for the English sentence *Someone slept and not everyone snored* that would entail that someone slept and did not snore. It is therefore neceesary to rule out (20) as a possible translation – which, fortunately, the Anti-Transparency Condition, once extended to the universal quantifier, does. This is because $\exists x P(x) \wedge \neg(U(x) \wedge \phi)$ and $\exists x P(x) \wedge \neg\phi$ have the same truth-value relative to every (w, G), for every possible ϕ. Suppose first that $\exists x P(x)$ is false (resp. undefined) at (w, G). Then both sentences are false (resp. undefined) at (w, G). Then suppose that $\exists x P(x)$ is true at (w, G). Then G values x, and therefore $U(x) \wedge \phi$ and ϕ have the same truth-value at (w, G), from which it follows that both sentences have the same truth-value.

Before moving on, let us check that the truth-conditions of a bathroom sentence such as $\neg\exists x B(x) \vee F(x)$ are still the desired ones, where a sentence is said to be true at w if there is a G such that it is true at (w, G). We again want to show that this sentence is true (in this sense) at w. That is, we need to show that (21) is true in our sense at w just in case (22) is classically true:

(21) $\neg\exists x B(x) \vee F(x)$

(22) $\neg\exists x B(x) \vee \exists x(B(x) \wedge F(x))$

Proof:

- Let us first prove that if (22) is classically true at w, then there is a G such that (21) is true (according to our semantics) at (w, G).

Assume that (22) is classically true at w. We distinguish two cases:

1. The first disjunct is classically true at w, i.e. $\exists x B(x)$ is classically false at w.

2. The second disjunct is classically true at w (note that the two disjuncts are classically inconsistent with each other)

First case: since $\exists x B(x)$ is classically false at w, for every a in D, $a \notin I(B, w)$. Consider then a plural assignment G such that for every a, there is a g in G such that $g(x) = a$. It follows from the falsity condition of $\exists$ that $\exists x B(x)$ is false at (w, G), hence $\neg\exists x B(x)$ is true at (w, G), hence (21) is true at (w, G), and therefore true simpliciter at w.

Second case: $\exists x(B(x) \wedge F(x))$ is classically true at w. Let us pick a individual a which is a witness, in w, of $\exists x(B(x) \wedge F(x))$, and let us consider a plural assignment function G which values x and such that $G(x) = a$. (21) comes out true (according to our semantics) at (w, G). This is so because $\exists x B(x)$ is true at (w, G), therefore $\neg\exists x B(x)$ is false at (w, G) (hence, crucially, not undefined), and the second disjunct, $F(x)$ is true at (w, G) (since $G(x) \in I(F, w)$). So (22) comes out true at (w, G), hence true simpliciter at w.

- In the other direction, we now show that if (21) is true at some pair (w, G) relative to our semantics (hence true simpliciter at w), then (22) is classically true at w.

Assume that (21) is true at some pair (w, G) according to our semantics. Given the Middle-Kleene connectives, either $\neg\exists x B(x)$ is true at (w, G) (first case) or $\neg\exists x B(x)$ is false at (w, G) and $F(x)$ is true at (w, G) (second case). In the first case, given the meaning of negation, $\exists x B(x)$ is false at (w, G). In this case, for every a, there is a g in G such that $g(x) = a$ and $g(x)$ (i.e. a) is not in $I(B, w)$, hence for every a , $a \notin I(B, w)$, which makes $\neg\exists x B(x)$ is classically true at w, hence (22) is classically true at w. In the second case, since $\neg\exists x B(x)$ is false at (w, G), $\exists x B(x)$ is true at (w, G), so G values x and $G(x) \in I(B, w)$, and we also have $F(x)$ true at (w, G), hence $G(x) \in I(F, w)$. Therefore $G(x)$ is a witness, in w, of the existential statement $\exists x(B(x) \vee F(x))$, interpreted classically, and therefore $\exists x(B(x) \wedge F(x))$ is classically true at w, which makes (22) classically true at w.

4.4 Co-variation

We now proceed to show that the failure-of-covariation problem noted in section 3.5 for a sentence such as *Not a single person failed to speak to someone* is now solved, and also that the system delivers the correct meaning for the classically equivalent *Everybody spoke to someone.* So let us consider now compute the truth-conditions of $\neg\exists x\neg\exists y S(x,y)$.

(23) $[\![\neg\exists x\neg\exists y S(x,y)]\!]^{w,G} = 1$ if:
$[\![\exists x\neg\exists y S(x,y)]\!]^{w,G} = 0$, i.e.:
for all a in D, $G_{x=a} \neq \emptyset$ and $[\![\neg\exists y S(x,y)]\!]^{w,G_{x=a}} = 0$, i.e.:
for all a in D, $G_{x=a} \neq \emptyset$ and $[\![\exists y S(x,y)]\!]^{w,G_{x=a}} = 1$, i.e.:
for all a in D, $G_{x=a} \neq \emptyset$ and $[\![S(x,y)]\!]^{w,G_{x=a}} = 1$, i.e.:
for all a in D, $G_{x=a} \neq \emptyset$ and $G_{x=a}$ values y and $(a, G_{x=a}(y)) \in I(S,w)$ i.e.:
for all a in D, there is a b in D such that $G_{x=a} \neq \emptyset$ and $G_{x=a}(y) = b$ and $(a,b) \in I(S,w)$.

There is no failure of covariation anymore, because the most embedded existential quantifier is now evaluated not with respect to the initial G, but to $G_{x=a}$, for each a in the domain. For this sentence to be true in (w,G), it has to be true classically in w: the above truth-condition entails that for all a in D, there is a b in D such that (a,b) is in the denotation of S. Conversely, if the sentence is true classically in w, we can find a G such that it is true in the current system with respect to (w,G). Assume that the sentence is classically true. Then it is possible, for each a, to build an atomic assignment g that maps x to a and y to an individual b such that (a,b) is in the relation denoted by S in w (i.e. is in $I(w,P)$). Then take G to be the set of all these atomic assignments.

Let us now show that $\neg\exists x\neg\exists y S(x,y)$ meets the Anti-Transparency condition for both occurrences of the existential quantifier in a null context. For $\exists x$, we need to check that $\neg(U(x) \wedge \phi)$ does not necessarily have the same truth-condition as $\neg\phi$, i.e. that $U(x) \wedge \phi$ and ϕ do not necessarily have the same truth-value. This is straightforward: consider a case where G does not value x but ϕ is true in (w,G) (one can pick $\phi = U(z) \vee \neg U(z)$, which is always true). For $\exists y$, we need to compare $\neg\exists x\neg(U(y) \wedge \phi)$ and $\neg\exists x\neg\phi$. Consider a pair (w,G) such that G values x but not y and ϕ is tautological (e.g., $\phi = U(z) \vee \neg U(z)$). Then $\exists x\neg\phi$ cannot be true at (w,G) (it is in fact undefined, because G values x, hence $G_{x=a}$ is empty for some a), but $\exists x\neg(U(y) \wedge \phi)$ is true at (w,G). Hence $\neg\exists x\neg(U(y) \wedge \phi)$ and $\neg\exists x\neg\phi$ do not have the same truth-value at (w,G) (the first one is false, the second one is undefined). Anti-Transparency is therefore met for $\exists y$ in the null context.

Consider now *Everybody spoke to someone*, translated as $\forall x\exists y S(x,y)$. Looking at the truth-condition for the universal quantifier and the falsity-condition for the existential quantifier, we see that in general a sentence of the form $\forall x\phi$ is true at some pair (w,G) if and only if $\exists x\neg\phi$ is false at (w,G).[17] It follows immediately that $\forall x\exists y S(x,y)$ is true if and only if $\neg\exists x\neg\exists y S(x,y)$ is, which is the desired result (on the other hand, it is possible for the latter sentence to be undefined when the former is false). I leave it to the reader to check that Anti-Transparency is satisfied for both quantifiers in the null context.

4.5 Quantificational Subordination

Consider a sentence exhibiting Quantificational Subordination, such as *Everyone read something and everyone liked it*, translated as in (24):

(24) $\forall x\exists y R(x,y) \wedge \forall x L(x,\underline{y})$

We know, from the previous section, that the first conjunct is true at (w,G) just in case it is true classically at w and for every a in the domain, there is a g in G such that $g(x) = a$ and $g(y)$ is something that a read at w. Assume that the first conjunct is true at (w,G). The second conjunct is true at (w,G) if for all a, there is a g in G such that $g(x) = a$ (which is necessarily the case given the truth of the first conjunct) and a liked

[17] $\exists$ and $\forall$ are not dual of each other because the falsity condition for $\forall$ does not mirror the truth-condition for $\exists$, but, in the other direction, the truth-condition for $\forall$ mirrors the falsity-condition for $\exists$.

$g(y)$, with $g(y)$ being something that a read (given the truth of the first conjunct). The whole sentence thus ends up as true at w simpliciter if everyone read something and liked it, which is the desired outcome.[18]

Let me now show that the Transparency condition for y and the Anti-Transparency condition for x are satisfied, in the null context, in the second conjunct.[19]

To check the Transparency Condition for y in the second conjunct in the null context, we must check that, for any ϕ, the two sentences in (25) have the same truth-value with respect to every pair (w, G)

(25) a. $\forall x \exists y R(x, y) \wedge \forall x(U(y) \wedge \phi)$
b. $\forall x \exists y R(x, y) \wedge \forall x \phi$

Pick an arbitrary pair (w, G). First, if $\forall x \exists y R(x, y)$ is false (resp. undefined) in (w, G), both sentences are false (resp. undefined) in (w, G).

Suppose now that $\forall x \exists y R(x, y)$ is true in (w, G). As we showed before (6th line of (23)), this entails that for every a, $G_{x=a}$ is not empty and values y. Hence $U(y)$ is true, for every a, in $(w, G_{x=a})$. Therefore, for every a, $(U(y) \wedge \phi)$ and ϕ have the same truth-value relative to $(w, G_{x=a})$. $\forall x(U(y) \wedge \phi)$ is true in (w, G) if for every a, $(U(y) \wedge \phi)$ is true in $(w, G_{x=a})$, which is then the case if and only if $\forall x \phi$ is true in $(w, G_{x=a})$. And $\forall x(U(y) \wedge \phi)$ is false in (w, G) is for some a, $(U(y) \wedge \phi)$ is false relative to $(w, G_{x=a})$, which is again equivalent to ϕ being false, for some a, at $(w, G_{x=a})$.

Therefore, (25a) and (25b) necessarily have the same truth-value relative to every possible pair (w, G), and Transparency is satisfied in the null context (and every possible context).

Let me now establish that Anti-Transparency is satisfied in the null context for x in the second conjunct of (24). What we need to show is that at some pair (w, G), for some sentence ϕ, the two following sentences do not have the same truth-value:

(26) a. $\forall x \exists y R(x, y) \wedge (U(x) \wedge \phi)$
b. $\forall x \exists y R(x, y) \wedge \phi$

This is straightforward. (26a) can never be true (on the assumption that the domain contains at least two individuals), because if the first conjunct $\forall x \exists y R(x, y)$ is true at (w, G), then G does not value x (since the atomic assignments of G have to map x to different individuals, given the entry for $\forall$), making $U(x)$ false and hence (26a) not true. But (26b) can be true: pick a pair (w, G) where the first conjunct is true and take $\phi = U(z) \vee \neg U(z)$, which is always true.

4.6 A note on restrictors

So far we have translated universal statements as if they did not include a restrictor. For instance, we translated *Everyone read something* as $\forall x \exists y R(x, y)$, ignoring the fact that *everyone* is in fact restricted to humans, and *something* to inanimate objects. And of course we want to provide an account that extends to sentences such as *Every student read a book*. Following standard practice, we translate such a sentence as follows:

(27) $\forall x(S(x) \rightarrow \exists y(B(y) \wedge R(x, y)))$

We don't have the symbol '$\rightarrow$' in our language, so we simply view a clause '$\phi \rightarrow \psi$' as abbreviating '$\neg \phi \vee \psi$'. Hence, our official translation for *Every student read a book* becomes:

(28) $\forall x(\neg S(x) \vee \exists y(B(y) \wedge R(x, y)))$

I will check that a) this delivers correct truth-conditions, b) that quantificational subordination is still predicted to work if this sentence is conjoined with (the translation of) *Every student liked it*, and, importantly,

[18] If instead of a conjunctive sentence we consider a discourse made up of the two conjuncts as separate sentences, i.e. $\forall x \exists y R(x, y) . \forall x L(x, y)$, things work essentially in the same way, given a simple Stalnakarian context-update rule. After accepting the first sentence, we end up with a context throughout which the first sentence is true, and the second sentence then shrinks this context to the set of pairs (w, G) such that both sentences are true, i.e. their conjunction is true.

[19] I leave it to the reader to check Anti-Transparency for the first conjunct.

c) not if it is conjoined with *Everyone liked it*.

Regarding a), (28) is true in (w, G) if for every a in D, there is a g in G that maps x to a such that either a is not $I(S, w)$ or $G_{x=a}$ values y and $G_{x=a}(y) \in I(B, w)$ and $(a, G_{x=a}(y)) \in I(R, w)$. It is clear that, given a world w, one can find a G such that (28) is true at (w, G) just in case (28) is classically true in w.

Regarding b), when we conjoin the sentence $\forall x(\neg S(x) \vee L(x, y))$, as in (29), the resulting sentence is true at (w, G) if the former condition holds, and, furthermore, for every student a, $(a, G_{x=a}(y)) \in I(L, w)$.

(29) $\forall x(\neg S(x) \vee \exists y(B(y) \wedge R(x, y))) \wedge \forall x(\neg S(x) \vee L(x, \underline{y})))$

Again the sentence will be true *simpliciter* at w just in case every student in read a book that they liked in w. We need to check that Transparency is always satisfied for the free occurrence of y, i.e. that the two following sentences always have the same truth-value:

(30) a. $\forall x(\neg S(x) \vee \exists y(B(y) \wedge R(x, y))) \wedge \forall x(\neg S(x) \vee (U(y) \wedge \phi))$
b. $\forall x(\neg S(x) \vee \exists y(B(y) \wedge R(x, y))) \wedge \forall x(\neg S(x) \vee \phi)$

Pick an arbitrary pair (w, G). If the first big conjunct is false (resp. undefined) at (w, G), then both sentences in (30) are false (resp. undefined) at (w, G). If the first conjunct is true, then for every a, there is a g in G such that $g(x) = a$ and either a is not in $I(S, w)$ or $G_{x=a}$ values y and $(a, G_{x=a}(y)) \in I(R, w)$. We now need to check that for every a, $\neg S(x) \vee (U(y) \wedge \phi)$ and $\neg S \vee \phi$ have the same truth value at $(w, G_{x=a})$. Let pick an arbitrary a. If $S(x)$ is false (resp. undefined) at $(w, G_{x=a})$ (i.e. if a is not a student in w), then these two subformula are true (resp. undefined) at $(w, G_{x=a})$. If $S(x)$ is true, then we know thanks to the truth of the first conjunct that $G_{x=a}$ values y, ensuring that the two subformula $U(y) \wedge \phi$ and ϕ have the same truth-value at $(w, G_{x=a})$, and therefore (30a) and (30b) as well.

Regarding c), we need to show that Transparency is not satisfied in the null context for the last occurrence of y in the following sentence.

(31) $\forall x(\neg S(x) \vee \exists y(B(y) \wedge R(x, y))) \wedge \forall x L(x, \underline{y})$

That is, we need to show that, for some choice of ϕ, the following two sentences do not always have the same truth-value:

(32) a. $\forall x(\neg S(x) \vee \exists y(B(y) \wedge R(x, y))) \wedge \forall x(U(y) \wedge \phi)$
b. $\forall x(\neg S(x) \vee \exists y(B(y) \wedge R(x, y))) \wedge \forall x \phi$

This is relatively straightforward. Consider a pair (w, G) such that the first big conjunct is true at (w, G), i.e. for every a in D, there is a g in G that maps x to a, and either a is not in $I(S, w)$, or $G_{x=a}$ values y, $G_{x=a}(y) \in I(B, w)$ and $(a, G_{x=a}(y)) \in I(R, w)$. For some non-student a (i.e. an individual a which is not in $I(S, w)$),[20] pick a G such that $G_{x=a}$ does not value y (i.e., there exist g, g' in G which both map x to a but such that $g(y) \neq g'(y)$). Then, relative to $(w, G_{x=a})$, with $\phi = U(z) \vee \neg U(z)$, $U(y) \wedge \phi$ is false at $(w, G_{x=a})$ but ϕ is true at $(w, G_{x=a})$, and so $\forall x(U(y) \wedge \phi)$ is false at (w, G) but $\forall x \phi$ is true at (w, G), hence (32a) is not true at (w, G) but (32b) is true at (w, G).

4.7 Donkey sentences

I now show how my proposal handles the classical donkey sentence, *Every farmer who owns a donkey beats it*. This sentence gets translated as:

(33) $\forall x(\neg(F(x) \wedge \exists y(D(y) \wedge O(x, y))) \vee B(x, \underline{y}))$
'For every x, either it's not the case that x is a farmer and that there is a donkey y that x owns, or x beats y'

[20] This assumes that there are non-students in w. If in fact it is presupposed that every individual is a student, then (31) is predicted to be fine, which I believe is not a wrong result.

I will show that a) my proposal, like Mandelkern (2022); Elliott (2020), generates the so-called weak existential reading ('Every farmer who owns a donkey beats a donkey they own', as opposed to the strong universal reading, paraphrasable as 'Every farmer who owns a donkey beats every donkey they own'), and b) that the Transparency Condition is met in every context for the syntactically free pronoun *it* (i.e. the second occurrence of the variable y in (33)), and that Anti-Transparency for the two quantifiers is met in the null context.

4.7.1 Deriving the weak existential reading for donkey-sentences

Focusing on truth-conditions, we show that (33) is true in w (in the sense that for some G it is true in (w, G)) if and only if the weak existential reading is true in w, i.e. if for every individual a such that a if a farmer in w (i.e. $a \in I(F, w)$) and a owns a donkey d in w (i.e. $d \in I(D, w)$ and $(a, d) \in I(O, w)$), there is a donkey d' owned by a such that a beats d' in w (i.e. $d' \in I(D, w)$, $(a, d') \in I(O, w)$, and $(a, d') \in I(B, w)$). We first prove the first direction, i.e. if (33) is true in (w, G) then the weak existential reading is true in w, and then the other direction, namely if the weak existential reading is true in w, then for some G, (33) is true in (w, G)

First Direction: (33) true in (w, G) $\Rightarrow$ Weak Existential Reading is true.
Let $\mathcal{Z}$ be the subformula $\neg(F(x) \wedge \exists y(D(y) \wedge O(x, y)) \vee B(x, y)$. Assume that (33) is true in (w, G). Then for every individual a, $G_{x=a} \neq \emptyset$ and $\mathcal{Z}$ is true in $(w, G_{x=a})$. Let us fix a certain a. Since $\mathcal{Z}$ is true in $(w, G_{x=a})$, either $\neg(F(x) \wedge \exists y(D(y) \wedge O(x, y))$ is true in $(w, G_{x=a})$, or it is false in $(w, G_{x=a})$ and $B(x, y)$ is true in $(w, G_{x=a})$. In the former case, $F(x) \wedge \exists y(D(y) \wedge O(x, y))$ is false in $(w, G_{x=a})$, so either a is not a farmer in w, or a is a farmer in w, but $\exists y(D(y) \wedge O(x, y))$ is false in $(w, G_{x=a})$, i.e. for every b, $G_{x=a,y=b} \neq \emptyset$, and b is not a donkey owned by a in w. To sum up, one way for $\mathcal{Z}$ to be true in $(w, G_{x=a})$ is if a is not a farmer, or a is a farmer but for every b, b is not a donkey that a owns (and $G_{x=a,y=b}$ is not empty). The other way for $\mathcal{Z}$ to be true in $(w, G_{x=a})$ is if $\neg(F(x) \wedge \exists y(D(y) \wedge O(x, y))$ is false in $(w, G_{x=a})$ and $B(x, y)$ is true in $(w, G_{x=a})$. In this case, $(F(x) \wedge \exists y(D(y) \wedge O(x, y))$ and $B(x, y)$ are both true in $(w, G_{x=a})$, which means that a is a farmer and that $G_{x=a}$ values y and $G(y)$ is a donkey owned and beaten by a in w. Therefore, if (33) is true in (w, G), then for every a, either a is not a farmer in w, or a is farmer who does not own any donkey in w, or a is a farmer who owns and beats a donkey b in w. Therefore if (33) is true in (w, G), then the weak reading of the donkey sentence is true in w ('Every farmer who owns a donkey beats a donkey they own').

Second Direction: Weak Existential Reading is true in w $\Rightarrow$ For some plural assignment G, (33) true in (w, G).
Let us assume that the weak existential reading of the donkey sentence is true in some world w, i.e. for every individual farmer f in w (i.e. $f \in I(F, w)$) such that f owns a donkey d in w (i.e. $d \in I(D, w)$ and $(f, d) \in I(O, w)$), there is a donkey d' owned by f such that f beats d' in w (i.e. $d' \in I(D, w)$, $(f, d') \in I(O, w)$, and $(f, d') \in I(B, w)$). Let us build a plural assignment G such that (33) is true in (w, G), according to our semantics. First, for each individual d in the domain, we must include in G at least one atomic assignment g that maps x to d. If d is not a farmer in w, we need only one such assignment in G, and it can map y to any individual in the domain; if d is a farmer who does not own a donkey in w, then we must make sure that for every individual e in the domain, there is a g in G such that $g(x) = d$ and $g(y) = e$. If d is farmer who owns a donkey (and therefore beats at least one donkey), we include an atomic assignment g such that $g(x) = d$ and $g(y)$ is a donkey that d owns and beats in w, and we include no other assigmnent mapping x to d. To give a concrete view of this construction, consider a world w based on a domain with 7 individuals $\{a, f_1, f_2, f_3, d_1, d_2, d_3,\}$ where a is neither a donkey nor a farmer, f_1, f_2, are f_3 are farmers, d_1, d_2 and d_3 are donkeys, f_1 does not own anything, f_2 owns d_1 and nothing else and beats d_1, f_3 owns d_2 and d_3 and nothing else and beats d_2 but not d_3. This world can be described by the following table:

Table 4: A world w making the weak existential reading of (33) true

Individual	Farmer?	Donkey?	Donkeys owned by Individual	Donkeys owned and beaten by Individual
a	No	No	None	None
f_1	Yes	No	None	None
f_2	Yes	No	d_1	d_1
f_3	Yes	No	d_2, d_3	d_2
d_1	No	Yes	None	None
d_2	No	Yes	None	None
d_3	No	Yes	None	None

A plural assignment G built as described in the previous paragraph is given in Table 5:

Table 5: A plural assignment G such that (33) is true in (w, G)

Variables	x	y	...
Atomic Assignment 1	a	a	...
Atomic Assignment 2	d_1	a	...
Atomic Assignment 3	d_2	a	...
Atomic Assignment 4	d_3	a	...
Atomic Assignment 5	f_1	a	...
Atomic Assignment 6	f_1	d_1	...
Atomic Assignment 7	f_1	d_2	...
Atomic Assignment 8	f_1	d_3	...
Atomic Assignment 9	f_1	f_1	...
Atomic Assignment 10	f_1	f_2	...
Atomic Assignment 11	f_1	f_3	...
Atomic Assignment 12	f_2	d_1	...
Atomic Assignment 13	f_3	d_2	...

Let us check that (33) comes out true in (w, G). First, it is clear that for every individual d of the domain, $G_{x=d}$ is not empty. Let d be a certain individual in the domain. Let us now evaluate $\mathcal{Z}$ relative to $(w, G_{x=d})$ (recall that $\mathcal{Z}$ is the formula $\neg(F(x) \wedge \exists y(D(y) \wedge O(x,y)) \vee B(x,y))$. If d is a non-farmer, then $F(x)$ is false relative to $(w, G_{x=d})$, so is $F(x) \wedge \exists y(D(y) \wedge O(x,y)$, and therefore the first disjunct is true, which is enough to make $\mathcal{Z}$ true in $(w, G_{x=d})$. If d is a farmer who does not own any donkey (so for instance, in the above table, $d = f_1$), then $F(x)$ is true in $(w, G_{x=d})$ but $\exists y(D(y) \wedge O(x,y))$ is false in $(w, G_{x=d})$, since, by construction, for every b, there is a unique g in $G_{x=d}$ such that $g(y) = b$, ans b is not a donkey owned by d, i.e. $D(y) \wedge O(x,y)$ is false, for every b, relative to $(w, G_{x=d,y=b})$. For instance, in Table 5, there are 7 atomic assignments such that $g(x) = f_1$ (from Assignment 5 to Assignment 11), and these 7 assignments cover all possible values for $g(y)$ (there are 7 individuals in the model), none of which is a donkey owned by f_1, and therefore, for every j in the domain, $\exists y(D(y) \wedge O(x,y))$ is false in $(w, G_{x=f_1,y=j})$. Therefore the conjunctive statement $F(x) \wedge \exists y(D(y) \wedge O(x,y)$ comes out false in $(w, G_{x=d})$, hence its negation is true in $(w, G_{x=d})$, and therefore $\mathcal{Z}$ is true in $(w, G_{x=d})$. Finally, if d is a farmer who owns a donkey, then $G_{x=d}$ consists of a unique assignment g that maps $g(y)$ to a donkey owned and beaten by d (either Assignment 12 or Assignment 13 in Table 5), so that $F(x) \wedge \exists y(D(y) \wedge O(x,y)$ is true in $(w, G_{x=d})$, hence the first disjunct of $\mathcal{Z}$ is false in $(w, G_{x=d})$, but the second disjunct, namely $B(x,y)$, is true in $(w, G_{x=d})$, making again $\mathcal{Z}$ true in $(w, G_{x=d})$. Therefore, for every d in the domain, $G_{x=d} \neq \emptyset$ and $\mathcal{Z}$ is true relative to $(w, G_{x=d})$, from which it follows that (33) is true in (w, G).

4.7.2 Transparency and Anti-Transparency for donkey sentences

The last occurrence of y is free in (33), repeated in (34), which is why it is underlined:

(34) $\forall x(\neg(F(x) \wedge \exists y(D(y) \wedge O(x,y))) \vee B(x,\underline{y}))$

In order to check that Transparency is satisfied in the null context, we need to show that, for any ϕ, the two following sentences have the same truth-value in every pair (w,G).

(35) a. $\forall x(\neg(F(x) \wedge \exists y(D(y) \wedge O(x,y))) \vee (U(y) \wedge \phi)$
b. $\forall x(\neg(F(x) \wedge \exists y(D(y) \wedge O(x,y))) \vee \phi)$

Let us call $\mathcal{Z}_1$ the formula $\neg(F(x) \wedge \exists y(D(y) \wedge O(x,y))) \vee (U(y) \wedge \phi)$ and $\mathcal{Z}_2$ the formula $\neg(F(x) \wedge \exists y (D(y) \wedge O(x,y))) \vee \phi$. We first prove that for any (w,G), $\mathcal{Z}_1$ and $\mathcal{Z}_2$ have the same truth-value in (w,G). Let us pick an arbitrary pair (w,G). First, if $\neg(F(x) \wedge \exists y(D(y) \wedge O(x,y)))$ is undefined in (w,G), then both $\mathcal{Z}_1$ and $\mathcal{Z}_2$ are undefined in (w,G). Likewise, if $\neg(F(x) \wedge \exists y(D(y) \wedge O(x,y)))$ is true in (w,G), so are both $\mathcal{Z}_1$ and $\mathcal{Z}_2$. Finally, if $\neg(F(x) \wedge \exists y(D(y) \wedge O(x,y)))$ is false in (w,G), then $F(x) \wedge \exists y(D(y) \wedge O(x,y))$ is true in (w,G), and therefore G values y. Therefore $U(y)$ is true in (w,G), and so the truth-value of $U(y) \wedge \phi$ in (w,G) is the same as that of ϕ, from which it follows that both $\mathcal{Z}_1$ and $\mathcal{Z}_2$ have the same truth-value as ϕ in (w,G). Hence, for every (w,G), $\mathcal{Z}_1$ and $\mathcal{Z}_2$ have the same truth-value in (w,G). It follows straightforwardly, given the lexical entry for the universal quantifier, that (35a) and (35b) have the same truth-value in (w,G), for every pair (w,G).

We now need to check that Anti-Transparency is satisfied for $\forall$ and $\exists$ in (34) in the null context. In the case of the universal quantifier, since it takes scope over the whole sentence, Anti-Transparency is obviously satisfied (since in general, ϕ and $U(x) \wedge \phi$ are not equivalent). In the case of $\exists$, we need to check that the two following sentences are not in general equivalent in the null context.

(36) a. $\forall x(\neg(F(x) \wedge U(y) \wedge \phi) \vee B(x,y))$
b. $\forall x(\neg(F(x) \wedge \phi) \vee B(x,y))$

Let ϕ be the tautology (say $U(z) \vee \neg U(z))$). Let us pick a pair (w,G) such that:

1. For every d in the domain, $d \in I(F,w)$ (i.e. everything is a farmer in w)
2. For every d in the domain, $G_{x=d} \neq \emptyset$ and $G_{x=d}$ does not value y.

Let us call $\mathcal{Z}_1$ the formula $\neg(F(x) \wedge U(y) \wedge \phi) \vee B(x,y)$ and $\mathcal{Z}_2$ the formula $\neg(F(x) \wedge \phi) \vee B(x,y)$. For every d, $F(x)$ and ϕ are true in $(w,G_{x=d})$, and $U(y)$ is false in $(w,G_{x=d})$. Therefore, for every d, $F(x) \wedge \phi$ is true in $(w,G_{x=d})$, but $F(x) \wedge U(y) \wedge \phi$ is false in $(w,G_{x=d})$. Hence, for every d, the first disjunct of $\mathcal{Z}_1$ is true in $(w,G_{x=d})$, and the first disjunct of $\mathcal{Z}_2$ if false in $(w,G_{x=d})$. So, for every d, $\mathcal{Z}_1$ is true in $(w,G_{x=d})$, hence (36a) is true in (w,G). On the other hand, for every d, since the first disjunct of $\mathcal{Z}_2$ is false in $(w,G_{x=d})$ and the second disjunct ($B(x,y)$) is undefined in $(w,G_{x=d})$ (since $G_{x=d}$ does not value y), $\mathcal{Z}_2$ is undefined in $(w,G_{x=d})$, from which it follows that (36b) is undefined in (w,G). Therefore, (36a) and (36b) are not equivalent in the null context, which ensures that Anti-Transparency is satisfied for $\exists$ in (34).

5 Weak and Strong Readings

My proposal only generates *weak* readings. For instance, the canonical donkey sentence is predicted to mean that every farmer who owns at least one donkey beats a donkey they have. While such weak, existential readings are attested (e.g. *Every customer who had a credit card used it to pay*, see, e.g., Chierchia 1995), such sentences tend to give rise to a universal reading ('Every farmer who owns a donkey beats all the donkeys they have'). It is also relevant to look at sentences such as the following:

(37) It is not true that Sue has a donkey and that she beats it

(37) gets translated as:

(38) $\neg(\exists x(D(x) \wedge O(s,x)) \wedge B(s,\underline{x}))$

In my system, this sentence happens to be true relative to a pair (w, G) in which Sue owns two donkeys, and she beats one and not the other, and G maps x to the donkey that Sue owns and does not beat. This is so because in this case $\exists x(D(x) \wedge O(s,x))$ is true in (w, G), but $B(s,x)$ is false at (w, G), which is enough to make the conjunction false, and its negation true. This seems to be an unfortunate state of affairs. Since I view a sentence as true in w if it is true in some (w, G), I predict (38) to be essentially equivalent to 'Either Sue has no donkey, or she has a donkey she doesn't beat', i.e. 'It is not true that Sue has a donkey and beats every donkey she has'. As soon as Sue has, in a certain world w, a donkey d she does not beat, then if G values x and maps it to d, (38) is true in (w, G). But such a reading does not seem available. Rather, (38) seems to mean that it is not true that Sue beats any donkey she has. This problem is shared with some other approaches, in particular Mandelkern (2022); Elliott (2020). Let me note, however, that this prediction might be less dramatically wrong than it seems when one considers sentences such as the following (inspired by an 'umbrella' example in Chierchia 1995).[21].

(39) It is not true that Sue has an umbrella and that she left it home.

In a situation where Sue has two umbrellas, and she took one with her and left the other one at home, it seems that (39) could be used, as opposed to (40)

(40) It is not true that Sue has an umbrella that she left home.

Similar remarks can be made regarding the nuclear scope of *No*:

(41) No farmer who owns a donkey beats it.

(41) gets translated as:

(42) $\neg\exists x \exists y(F(x) \wedge D(y) \wedge O(x,y)) \wedge B(\underline{x}, \underline{y}))$

This sentence is true at a pair (w, G) in which there is a farmer who owns two donkeys and beats exactly one them, if G maps x to that farmer and y to the donkey that this farmer does not beat. Again, this amounts to a universal reading, equivalent to 'No farmer who owns a donkey beats all the donkeys they have', but it seems that the most natural reading of (42) is rather something like 'No farmer who owns a donkey beats any donkey they have'. That being said, the universal reading appears to be sometimes available, as in (43) (Chierchia 1995):

(43) No student who has an umbrella left it home.
$\rightsquigarrow$ No student who has an umbrella left every umbrella they have at home.

Generally speaking, in all these cases where the pronoun could be, informally speaking, interpreted either existentially ('…beats a donkey they own') or universally ('…beats every donkey they own'), the current proposal, along with Mandelkern's and Elliot's, predicts the *weaker* reading (existential when the pronoun is an upward-entailing context, universal if it is a downward entailing context). This is problematic since we do seem to access other readings, sometimes preferentially.

My strategy to address this problem includes two components. First, while so far I have considered that a sentence is true at a world w if it is true at some pair (w, G), I will introduce a second definition of truth that will be more demanding. The idea is that there are two modes of intepretation, one of which gives rise to the weak reading, while the other one gives rise to the strong reading. The second component consists in adopting a principle due to Kanazawa (1994) regarding how the resulting ambiguity is typically resolved in the absence of contextual biases in favor of one or the other reading.

The two notions of 'truth at a world' that I will use are given in (44). *Weak Truth* is the notion we have used so far, while *Strong Truth* is a stronger notion.

(44) Truth at a world

[21] Thanks to Matt Mandelkern for drawing my attention to this kind of examples and to Keny Chatain for extensive discussions

a. Weak Truth: A sentence S is weakly true at a world w if there exists a plural assignment G such that S is true at (w, G).
b. Strong Truth: A sentence S is strongly true at a world w is there exists a plural assignment G such that S is true at (w, G) and there does not exist a plural assigment G' such that S is false at (w, G').

The 'Strong Truth' interpretation is very similar to the strengthened reading derived by Elliott (2023) as a form of exhaustivity inference. Elliott's basic system (which is dynamic) generates the same kind of weak readings as mine, and the strong reading is essentially obtained by conjoining the weak reading of a sentence with the proposition that states that the weak reading of its negation is false; at a very informal level, this amounts to saying that S is true in the strong sense if it is weakly true and $\neg S$ is itself not weakly true, i.e. in our own terms, that there is no choice of assignment that makes S false.[22]

In the case of a simple existential sentence $\exists x P(x)$, weak and strong truth coincide. This is because when $\exists x P(x)$ is true at (w, G), then, given our semantics, it is also classically true at w, and when it is false at (w, G), it is also classically false at w. Since no sentence can be both classically true and classically false, the second conjunct in the definition of 'Strong Truth' is entailed by the first conjunct in this case.

But the two notions diverge in cases such as the following:

(45) a. There is a bathroom and it is upstairs.
$\exists x B(x) \wedge F(\underline{x})$
b. Either there isn't a bathroom, or it is upstairs.
$\neg\exists x B(x) \vee F(\underline{x})$
c. It is not true that there is a bathroom and that is it is upstairs.
$\neg(\exists x B(x) \wedge F(\underline{x}))$
d. Every farmer who owns a donkey beats it.
$\forall x(\neg(F(x) \wedge \exists y(D(y) \wedge O(x, y))) \vee B(x, \underline{y}))$
e. Some farmer who owns a donkey beats it.
$\exists x(\exists y(F(x) \wedge D(y) \wedge O(x, y)) \wedge B(\underline{x}, \underline{y}))$

Consider first (45a). In a world w where there are two bathrooms, only one of which is upstairs, then if G values x and $G(x)$ is the bathroom upstairs, then (45a) is true at (w, G), but if G values x but $G(x)$ is the bathroom that is not upstairs, then (45a) is false at (w, G). Therefore, (45a) is weakly true at w but is not strongly true at w. For (45a) to be strongly true at w, it must be the case that there is a bathroom in w and that for every G that values x and maps it to a bathroom, $G(x)$ is upstairs, hence it must be the case that every bathroom is upstairs. If (45a) is interpreted as strongly true, then, it ends up meaning that there is at least one bathroom and that all bathrooms are upstairs - that is, under a Strong Truth interpretation, we get a universal reading (which is in this particular case does not seem to be attested, but see our discussion below).

As to (45b), in a world w where there is in fact no bathroom, the sentence comes out strongly true. Indeed, if there is no bathroom in w, (45b) is true with respect to (w, G) if for every entity d, there is a G that maps x to d, and it cannot be false since the first disjunct will come out either true or undefined - for it to be false, there would have to be no bathroom in w. If there is more than one bathroom in w, and exacly one bathroom is upstairs, then the sentence in true in (w, G) if G values x and maps it to a bathroom that is upstairs, but false in (w, G) if G values x and maps it to a bathroom that is not upstairs. So for (45b) to be strongly true in w, either there is no bathroom in w, or all the bathrooms are upstairs in w. The Strong Truth requirement generates again a universal reading.

Regarding (45c), I discussed above the fact that it comes out weakly true in a world where there is a bathroom that is not upstairs, even if another bathroom is upstairs. In such a situation, however, (45c) would not be strongly true. This is because in a world with two bathrooms, one of which is upstairs but the other is not, then if G values x and maps it to a bathroom upstairs, (45c) is false at (w, G) . For (45c) to be

[22] The 'Strong Truth' approach is also somewhat similar in spirit to Champollion et al.'s (2019) homogeneity-based approach to ambiguities in donkey sentences. See also Chatain (2018) for a very related proposal.

strongly true at w, either there should be no bathroom, or no bathroom should be upstairs. That is, Strong Truth now generates an *existential* reading, in the sense that the reading in question can be paraphrased as *It is not true that there is a bathroom upstairs.* In general, Strong Truth generates, out of the existential and the universal readings, the stronger one.

Consider now the donkey sentence in (45d). In a world w where every farmer f who owns a donkey beats one of the donkeys they have, but at least one farmer f^* does not beat everydonkey they have, the sentence is true in (w, G) if for every farmer f there is a g in G that maps x to f and and such g's map y to a donkey that f beats. If however we change G minimally into a plural assignment G' such that for every g in G' that maps x to f^*, we make sure that g maps y to a donkey that f^* does not beat, the sentence if false in (w, G'). For the sentence to be strongly true in w, it must be the case that, in w, every farmer who owns a donkey beats all the donkeys they have.

As to (45e), it is weakly true if some farmer who owns a donkey beats a donkey they own, while it is strongly true is some farmer who owns a donkey beats every donkey they own.

Now, the strong and weak readings seem to be favored for different sentence types. As far as *preferences* are concerned, they happen to be correctly predicted by a generalization proposed by Kanazawa (1994). The idea is that the preferred reading is the one that makes the relevant reading consistent with the monotonicity property of the relevant quantifier in the absence of anaphoric dependencies. For instance, *every* is downward-entailing with respect to its restrictor and upward-entailing with respect to its scope. Under a universal reading, a sentence such as *Every farmer who owns a donkey beats it* entails *Every farmer who owns a blue donkey beats it* (If every farmer who owns a donkey beats every donkey they have, then every farmer who owns a blue donkey beats every blue donkey they have), but that would not be so under an existential reading (the fact that every farmer who owns a donkey beats one of their donkeys does not guarantee that every farmer who owns a blue donkey beats a blue donkey they have). So this generalization predicts a universal reading for this particular sentence and its negation. In the case of *some* and *no* the situation of the opposite: only the existential reading preserves the monotonicity properties of the quantifier. Under the existential reading, a sentence such as *Some farmer who owns a blue donkey beats it* entails *Some farmer who owns a donkey beats it* (if some farmer who owns a blue donkey beats a blue donkey they have, then some farmer who owns a donkey beats a donkey they have), but not under the universal reading (the fact that some farmer who owns a blue donkey beats all the blue donkeys they have does not guarantee that some farmer who owns a donkey beats all the donkeys they have). This generalization can be extended to sentences without quantifiers such as (45a) and (45b). Assuming the monotonicity property of the environment for the indefinite must be the same as if there were no pronoun, an existential reading is predicted for (45a) and (45c), but a universal reading is predicted for (45b), which seems to be in line with intuitions as well as with experimental evidence (Chatain et al. 2024).

The current system, based on two interpretative rules ('weak' and 'strong'), if supplemented with a principle of disambiguation based on Kanazawa's generalization, appears to make overall correct predictions. That being said, the experimental literature (Geurts 2002; Foppolo 2008, a.o.) suggests an asymmetry between existential and universal readings. It suggests that while it is possible to access an existential reading with sentence types that prefer a universal reading, the reverse is not really the case. That is, for sentences using the quantifier *some* (cf. (45e)) and *no* (cf.(41)), only existential readings are observed, even when interpreted in a context that should favor a universal reading for pragmatic reasons. Despite these experimental findings, it is in fact possible to create examples in which a universal reading is perceived for a sentence type which Kanazawa predicts should give rise to an existential reading. One such example was already given in (39) (negation of a conjunction) and (43) (an example with *no*), but can also be constructed with simple conjunction or the quantifier some:

(46) [Context: It is rainng] Sue has an umbrella but she left it home.
$\rightsquigarrow$ Sue left every umbrella she has home.

(47) [Context: it's raining] Some students who have an umbrella left it home
$\rightsquigarrow$ Some students left all their umbrellas home.

One might have the impression that in such cases a uniqueness inference is generated ('Sue has exactly one umbrella', 'students who have an umbrella have only one'), but it seems to me that we still access a universal reading when the potential uniqueness inference is blocked by *at least*], as in the following variations on the same examples:

(48) Sue has at least one umbrella, but she left it home.

(49) Some students who have at least one umbrella left it home.

It is also interesting to compare the weak and strong truth interpretations in the following case:

(50) Gloria has a credit card but did not pay with it.

a. Weak Truth interpretation: Gloria has a credit card that she did not pay with

b. Strong Truth interpretation: Gloria has a credit card and every credit card she has is such that she did not pay with it.

In this case, we seem to go for the Strong Truth interpretation (i.e., again, a universal reading), and as far as I can tell the sentence is not understood to entail that Gloria has only one credit card. In fact, the following is felicitous and gives rise, as far as I can see, to a universal intepretation as well:

(51) Gloria has a credit card (and probably more than one), but she did not pay with it.

So it seems that we end up with a decent proposal, especially given that no current approach seems to do better in terms of predicting patterns of preferences between various readings in a fully explanatory and predictively adequate way.

I would like to note, however, a significant problem for the approach sketched here: we can construct complex cases where the perceived reading is neither the weak nor the strong reading, but a kind of mixtures of both. Thus consider:

(52) Either it's not the case that Sue has a credit card and bought a cake with it, or she also used it to buy a book.

a. Weak Truth interpretation: Either it's not the case that Sue has a credit card and bought a cake with every card she has, or she has a credit card that she bought a cake with and she also used it to buy a book.

b. Strong Truth inteprretation: Either it's not the case that Sue has a credit card that she used it to buy a cake, or she has a credit card that she used to buy a cake, and she used every credit card with which she bought a cake to buy a book.

c. Most natural reading: Either it's not the case that Sue has a credit d card with which she bought a cake, or she has a credit card with which she bought a cake and with which she also bought a book.

One possibility in order to be able to generate this reading is to change perspective, and view the choice between strong and weak truth not as global choice of *mode of interpretation* but as corresponding to the presence or absence of a *strong truth* operator in the syntactic form of the sentence - an operator that takes the ordinary meaning and turns it into a strengthened meaning corresponding to the strong interpretation. We thus introduce the following strong truth operator $\mathcal{O}$ operator, associated with the following meaning:

(53)
$$[\![\mathcal{O}(S)]\!]^{w,G} = \begin{cases} 1 & \text{if } [\![S]\!]^{w,G} = 1 \text{ and there is no } G' \text{ such that } [\![S]\!]^{w,G'} = 0 \\ 0 & \text{if } [\![S]\!]^{w,G} = 0 \text{ and there is no } G' \text{ such that } [\![S]\!]^{w,G'} = 1 \\ \# & \text{otherwise} \end{cases}$$

On this view, the notion of *truth at a world* remains the same as before (a sentence S is true at w if for some G it is true at (w, G)), but when $\mathcal{O}$ is appended the truth-conditions become strong enough so that the end result is equivalent to Strong Truth.

Such a view also makes it possible to keep to a very simple intersective discourse update rule as discussed in section 1.2, while this would not be possible if we simply viewed 'Strong Truth' as a rule of interpretation.[23].

We now have a lot of difference parses for some complex sentences, and the choice between different parses in practice must involve complex pragmatic reasoning, together with a pressure to generate readings that comply with Kanazawa's generalization.

The most natural reading for (52), namely (52c), would then be generated by the following parse, where the Strong Truth operator applies only to the first disjunct.

(54) $\mathcal{O}(\neg(\exists x(\text{CREDIT-CARD}(x) \wedge \text{HAS}(s,x)) \wedge \exists y(\text{CAKE}(y) \wedge \text{BOUGHT-WITH}(s,y,\underline{x}))) \vee \exists z(\text{BOOK}(z) \wedge \text{BOUGHT-WITH}(s,z,\underline{x})))$

Once we make this move, it is not guaranteed that structures that satisfy Transparency and Anti-Transparency still do when embedded under $\mathcal{O}$, nor that structures that were ruled out by these conditions do not become 'legal' when embedded under $\mathcal{O}$. All the cases we considered in the paper where Transparency was satistifed were due to logical equivalence between the sentences involved in checking Transparency. In all such cases, it is guaranteed that appending $\mathcal{O}$ is not going to change anything, due to the following general fact, which directly follows from the definition of $\mathcal{O}$.

(55) If two sentences S_1 and S_2 are logically equivalent, i.e. have the same truth-value across all possible pairs (w, G), then $\mathcal{O}(S_1)$ and $\mathcal{O}(S_2)$ are equivalent as well.

Likewise, in all cases where Anti-Transparency were violated in structures without $\mathcal{O}$, this was due to logical equivalence between the relevant sentences to be checked for equivalence, and given (55), Anti-Transparency remains satisfied when $\mathcal{O}$ is appended.

The reciprocal version of the statement in (55), which would state that if $\mathcal{O}(S_1)$ and $\mathcal{O}(S_2)$ are logically equivalent, then S_1 and S_2 are, is not obviously valid, even though I have not found any counterexample (i.e. a case where $\mathcal{O}(S_1)$ and $\mathcal{O}(S_2)$ are logically equivalent but S_1 and S_2 are not). In the absence of such a result, one cannot exclude that there are sentences which, in a null context, fail to satisfy Transparency, but do when $\mathcal{O}$ is adjoined to them, or sentences which satisfy Anti-Transparency without $\mathcal{O}$ but fail to satisfy it with $\mathcal{O}$ is appended.

Whatever is in fact the case when $\mathcal{O}$ is inserted at matrix level in a null context, there are examples where embedding $\mathcal{O}$ in an embedded position causes a violation of Transparency, or turn a sentence that violates Anti-Transparency into one that doesn't. As we will see, this gives rise in some cases to problematic predictions.

Let us start with a case where appending $\mathcal{O}$ in an embedded position creates a violation of Transparency. Thus consider the following sentence in (56a), and the potential parse in (56b), with S standing for 'being an umbrella She has' and H for 'having been left home by Sue'.

(56) a. Sue has an umbrella and she left it home.
b. $\exists x S(x) \wedge \mathcal{O}(H(\underline{x}))$

Before checking if Transparency is satisfied for (56b) (it is not), let us consider its meaning. $\exists x S(x)$ is true in (w, G) if G values x and $G(x)$ has property S in w. $\mathcal{O}(H(x))$ is itself true in pairs (w, G) in which G values x and $G(x)$ has property H in w, and furthermore every individual has property H in w. (56b) ends up being true simpliciter at a world w if there is an entity that has property S and furthermore if everything has property H, making the sentence equivalent to *She has an umbrella and she left everything home* , obviously not a possible reading (since *everything* here ranges not just on umbrellas but on everything there is). It turns out that the parse in (56) is ruled out by the Transparency Condition, thanks to which we do

[23] In the absence of such an 'internalization' of Strong Truth in the object language, the mode of interpretation corresponding to Strong Truth could not correspond to an intersective update rule at the discourse level, because in order to update a context so as to keep only worlds where a sentence is strongly true, one has to eliminate some world-assignment pairs such that the sentence is true at (w, G), but only weakly true at w. If however the strong interpretation is derived by $\mathcal{O}$, this problem is solved, since on the relevant parses there will simply be no such pair.

not predict that such a reading is available. To check Transparency, we need to compare the two following sentences, for an arbitrary ϕ

(57) a. $\exists x S(x) \wedge \mathcal{O}(\phi)$
b. $\exists x S(x) \wedge \mathcal{O}(U(x) \wedge \phi)$

Let ϕ be a tautology (say $U(x) \wedge \neg U(x)$). Note that $\mathcal{O}$ applied to a tautology returns a tautology. If ϕ is a tautology, the two sentences in (57) become equivalent to:

(58) a. $\exists x S(x)$
b. $\exists x S(x) \wedge \mathcal{O}(U(x))$

Now, $\mathcal{O}(U(x))$ is necessarily undefined, since for a given w, we can always find G and G' such that $U(x)$ is true in (w, G) and false in (w, G'). Therefore, when (58a) is true, (58b) is undefined, and therefore (58a) and (58b) are not equivalent, hence Transparency is violated for (56b). This is a desirable result, since the corresponding reading does not appear to exist.[24]

However, another side of this observation is that *Anti-Transparency* is going to be satisfied for the second occurrence of the existential quantifier in a structure such as the following, since in general, if Transparency is violated for a certain occurrence of $D(\underline{x})$, then Anti-Transparency is satisfied when this occurrence is replaced with $\exists x D(x)$.

(59) $\exists x S(x) \wedge \mathcal{O}(\exists x D(x))$

This is not a desirable result. In this structure, $\mathcal{O}$ is vacuous, and, as we already discussed in section 3.4, the sentence is true in (w, G) if and only if $S(x) \wedge D(x)$ is true in (w, G), hence the sentence is true simpliciter if and only if the intersection of the denotations of S and D is not empty. This predicts that a sentence such as *Someone smokes and someone drinks* could be interpreted as equivalent to *Someone smokes and drinks*, contrary to fact. Without $\mathcal{O}$, the sentence is ruled out by Anti-Transparency, but this is no longer the case in the case of (59).

Faced with this difficulty, I propose that the Transparency and Anti-Transparency conditions need to be satisfied for both a) the sentence S under consideration, and b) the sentences obtained from S by deleting any number of occurrences of $\mathcal{O}$ in S. Given such a constraint, (56b) is ruled out because it does not satisfy Transparency, and (59) is ruled out because its variant without $\mathcal{O}$ violates Anti-Transparency. But this is still tentative at this point.

Generally speaking, the goal of this section was to show that it is possible in my framework to generate all attested readings, and that in this respect my approach is on a par with other proposals - none of which, as far as I can tell, provides an explanatory theory of why certain readings are deeply dispreferred, and why Kanazawa's generalization appears to hold.[25]

[24] A completely similar result holds for the 'discourse' version of (56b), as in (i):

(i) a. Sue has an umbrella. She left it home
b. $\exists x S(x). H(\underline{x})$.
c. $\exists x S(x). \mathcal{O}(H(\underline{x}))$.

Consider the context C that results from updating the null context with the first sentence. It contains all pairs (w, G) such that G values x and $G(x)$ has property S in w. Relative to such a context, Transparency is satisfied for the second sentence in (ib) (as discussed in section 3.1), but not in (ic). This is because Transparency for (ic) requires (among others) that the two following sentences be contextually equivalent, where $\top$ stands for a tautology (say $U(x) \vee \neg U(x)$)

(ii) a. $\mathcal{O}(\top)$
b. $\mathcal{O}(U(x))$

But while (iia) is itself a tautology, (iib) cannot be true at any (w, G) since it requires that for any G', $U(x)$ be true in G', which is obviously absurd. Therefore the parse in (ic) is ruled out by the Transparency condition.

[25] Champollion et al.'s (2019) proposal correctly predicts existential readings for donkey sentences involving existential quantifiers, such as (45e), and the availability of strong readings with other quantifiers, but does not predict Kanazawa's generalization in full generality, which has recently been confirmed with non-monotonic quantifiers (Denić and Sudo 2022).

6 Conclusion

My main goal in this paper was to show that it is possible to preserve the core intuitions of dynamic semantics, esp. the notion of 'discourse referent', within a non-dynamic system. Just like in dynamics semantics, information states can be viewed as structured 'files', to use Heim's (1982) terminology, in which information is attached to variables ('file cards'). For instance, the information state representing the belief that results from accepting *A* man_x *is in the room and a* $woman_y$ *is looking at* him_x can be viewed as a file containing two cards, named x, and y, and card x has the following information written on it: *is a man, is in the room, is beeing looked at by* y, while card y contains the following: *is a woman, is looking at* x. Formally, this 'file' can be identified to the set of world assignment pairs that (w, G) such that $G(x)$ is a man in w, $G(y)$ is a woman in w, and $G(y)$ is looking at $G(x)$ in w. In Heim's terminology, such pairs (w, G) are those that *satisfy* the file. But instead of viewing a sentence as denoting a function from files to files (i.e. from sets of worlds-assignment pairs to sets of world-assignment pairs), the meaning of a sentence is itself a file (i.e. the set of world-assignment pairs at which it is true). For instance, the sentence *A* man_x *is in the room and a* $woman_y$ *is looking at* him_x denotes the file we have just described - more properly, the set of world-assignment pairs that satisfy it. When, in a given information state, we accept a new sentence, the change that is made to the information state is very simple: the new information state, viewed as a set of world-assignment pairs, is the result of intersecting the input information state with the one denoted by the sentence, i.e. the set of world-assignment pairs at which the sentence is true. At no point did we refer to information update when stating compositional semantic rules, or to transitions from an assignment to a new one (as in Dynamic Predicate Logic); rather, we kept to a simple and classical picture where semantics characterizes *truth-conditions*, not information update, and this is what makes this system conceptually very different from dynamic semantics systems.

Because it retains the view that information is structured and attached to variable, the system differs from classical logic in the role it gives to assignment functions. While in classical logic assignment functions play no role in determining the semantic content of sentence without free variables, in my system the truth-value of all sentences depend on a choice of assignment. A statement of the form $\exists x P(x)$ imposes that the variable x be mapped to an object that has property P (Mandelkern's 2022 witness condition). In order to predict not just the desired readings, but also the felicity conditions of pronouns, I combined the Middle Kleene connectives with a general theory of presupposition which is itself not dynamic, the Transparency Theory. The felicity conditions of pronouns is thus reduced to a presupposition: free variables come with a presupposition that they are valued. Technically, a key ingredient is the use of plural assignment functions, which appear to be crucial in order to provide a viable account of universal quantification and of dependencies between variables - this is again a case where an innovation that originated in dynamic semantics is used but is decoupled from dynamic semantics.

Further work will aim to investigate different variants of the system in relation with left-to-right asymmetries or lack thereof (as discussed in footnote 13), and, most importantly, to extend the system to plural anaphora.

Conflict of interest statement

I declare that I have no conflict of interest related to this work.

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