Communicative stability and the typology of logical operators*

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Abstract

The typology of the logical vocabulary in natural language is highly skewed. In the domain of logical connectives, AND and OR are often lexicalized, lexicalizations of NOR are less common and tend to be structurally complex, and no other logical connective is ever lexicalized. Existing accounts fail to fully derive this major cross-linguistic pattern, and moreover resort to otherwise unwarranted assumptions. The goal of this paper is to provide an account which is less stipulative and has wider empirical coverage than previous accounts, based on a novel notion of communicative stability. Using a model of a rational speaker we observe that attested languages are stable languages, i.e., languages in which the optimal message for a speaker to choose when they want to convey a particular state they are in is not affected by which states they take to be more likely and which ones less. We argue that Stability can account both for why AND and OR are the only simple connectives lexicalized and for why NOR is the only complex connective lexicalized.

	Commutative						Trivial		Non-commutative							
	AND	OR	NOR	NAND	XOR	IFF	TAU	CONT	L	R	NOTL	NOTR	ONLYL	ONLYR	$\rightarrow$	$\leftarrow$
$p \wedge q$	1	1	0	0	0	1	1	0	1	1	0	0	0	0	1	1
$p \wedge \neg q$	0	1	0	1	1	0	1	0	1	0	0	1	1	0	0	1
$q \wedge \neg p$	0	1	0	1	1	0	1	0	0	1	1	0	0	1	1	0
$\neg p \wedge \neg q$	0	0	1	1	0	1	1	0	0	0	1	1	0	0	1	1

Table 1: Truth table for all binary logical connectives taking P and Q as arguments.

1 Introduction

Languages of the world make very different choices regarding what meanings they lexicalize. Yet there are some meanings that are never lexicalized, notably in the domain of the logical vocabulary (Horn 1972; Barwise and Cooper 1981; von Fintel and Matthewson 2008). In this paper we focus on the lexicalization pattern in the domain of logical connectives, and propose an account for this pattern based on a novel notion of communicative stability.

There are 16 logically possible binary connectives, whose truth tables are represented in table 1.[1] Among these connectives, AND and OR are often lexicalized, for example in English (*and* and *or*) and Hebrew (*ve* and *o*). NOR is also sometimes lexicalized as a complex connective, for example in English (*neither…nor*) which has no Hebrew counterpart. Surprisingly, no other connective is lexicalized in any known language. Focusing for now on the lexicalization pattern found within the 6 commutative connectives which are non-trivial (see table 1), we observe the following pattern (we will discuss the absence of trivial and non-commutative connectives in §5.2):

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[1]In this table and throughout the paper we sometimes use a connective (e.g., AND) for representing the result of this connective taking P and Q as arguments (e.g., P AND Q). We also use p as the proposition denoted by the message P, and likewise for Q and q.

(1)   **Typological pattern:**

    a.   Lexicalization of AND and OR is pervasive and seems to be morphologically simplex.

    b.   NOR is more rarely lexicalized; when lexicalized, it is morphologically complex and arguably syntactically and semantically complex.[2]

    c.   NAND, XOR and IFF are never lexicalized (Horn's puzzle, extended).

There are two ingredients that are common to most existing accounts of the typological pattern (Horn 1972; Katzir and Singh 2013; Uegaki 2022; Züfle and Katzir 2022), both due to Horn. The first ingredient is Strengthening, namely the fact that implicatures are computed.

(2)   **Strengthening**: Implicature computation plays a role in the analysis of the typological pattern.

Horn relies on Strengthening to explain why languages do not lexicalize both OR and NAND, based on the observation that sentences which express OR and sentences which express NAND end up having the same meaning (XOR) once implicatures are computed. For example, (3a) and (3b) both have the inference in (3c). It would then be redundant for a language to lexicalize both.

(3)   a.   Mary dreams of elephants or cats

    b.   Mary doesn't dream of both elephants and cats.

    c.   ⤳ Mary dreams of elephants or cats but not both.

The second ingredient is what we will refer to as Positivity:

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[2]This claim has been made mainly for quantifiers (see Sauerland 2000; Penka 2011; Zeijlstra 2011); similar considerations to those motivating this claim apply in the domain of connectives.

(4)   **Positivity**: 'Positive' connectives (e.g., AND and OR) are more basic than 'nega-
tive' ones (e.g., NAND and NOR).

How Positivity is cashed out varies between accounts. For Uegaki (2022); Züfle and Katzir
(2022), for instance, it is cognitively more costly to lexicalize negative connectives than
positive ones. Katzir and Singh (2013), in contrast, stipulate that the only connectives
which are lexicalizable as simple are AND and OR (and other connectives can be derived
by adding negation; see §4). Positivity can explain why languages would prefer to lexi-
calize OR over NAND (given that both yield the same strengthened meaning, as in Horn's
reasoning above): since NAND is cognitively more costly, lexicalizing OR is preferred.
As both Enguehard and Spector (2021) and Incurvati and Sbardolini (2020) recently
pointed out, however, unlike the Strengthening component which is needed indepen-
dently, Positivity relies on stipulation: There is no compelling evidence that positive sen-
tences are indeed cognitively simpler than negative ones. In a way, assuming Positivity
amounts to assuming part of what needs to be explained, that is, that AND and OR are the
only simple connectives.[3] Enguehard and Spector (2021); Incurvati and Sbardolini (2020)
made the only attempts we are aware of to explain the typology without having to assume
Positivity. Enguehard and Spector in their account of the typology of quantificational de-
terminers suggested replacing Positivity with assumptions about how the prior probabili-
ties of states (namely what situations are more likely than others) affect communication.[4]
Specifically, they argue that in most situations it is more beneficial for speakers to use OR
than to use NAND; because it is more likely that a sentence conveying OR is false than
that a sentence conveying NAND is false (an assumption they give some justification for,

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[3]One might consider justifying Positivity based on findings showing that sentences containing
monotone-decreasing operators have higher cognitive cost (Geurts and van der Slik 2005). As Enguehard
and Spector point out, however, such effects can be attributed to the syntactic complexity of such sentences.

[4]While Enguehard and Spector focus on the typological pattern with quantificational determiners rather
than with connectives, their proposal translates straightforwardly to connectives; we will discuss quantifica-
tional determiners in §5.4.

though see Kuhn and Pasalskaya 2022 for evidence to the contrary), it is more informative to use OR than to use NAND. Languages then lexicalize the more communicatively beneficial meaning. As Enguehard and Spector themselves point out, however (and as has been further demonstrated by Züfle and Katzir 2022), while their account excludes some languages containing NAND, it does not completely rule out lexicalization of NAND. This is since, just like in Horn's reasoning above, their account only helps explain why a language with OR but not NAND is preferred over a language with NAND but not OR, all else being equal; it does not explain for instance why a language like {AND, OR, NOR, NAND} does not exist (nor does it explain the absence of other connectives such as XOR and IFF), and as simulations by Züfle and Katzir (2022) suggest, their account faces problems when smaller inventories are considered.[5] In order to avoid these problems they would seem to have to assume Positivity after all (which would make their own theory redundant). Incurvati and Sbardolini (2020) take a different approach and aim to explain Positivity by relying on specific assumptions about the dynamics of conversation. However, as far as we can tell these assumptions are stipulated in order to account for the typology and are not otherwise motivated, which raises the question whether their account is less stipulative than ones assuming Positivity.[6]

On top of this, a major challenge for previous accounts (including Enguehard and Spector; Incurvati and Sbardolini) is the pattern concerning lexiclaization of NOR: Recall from (1a) and (1b) that NOR is more complex than AND and OR when it is lexicalized. This complexity is visible for example when comparing the lexiclizations of NOR and OR in English, i.e., *neither. . . nor* vs. *(either. . .)or*: Morphologically, the lexicalization of NOR seems to be composed from the lexicalization of OR and a negation morpheme *n-* (and as

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[5]In §4 we will mention that our theory also wrongly predicts {OR, NOR} to be a possible inventory (though only if NOR is lexicalized as complex); however, unlike Enguehard and Spector, we do not predict it to be preferred over {AND, OR}.

[6]Furthermore, their theory relies on the assumption that rejecting a conjunctive sentence amounts to rejecting both conjuncts, an assumption which we do not think is supported.

mentioned in (1b), it has been argued that this morphological complexity reflects syntactic and semantic complexity). There is no known exception to the complexity of NOR, yet this fact is left unexplained on mst accounts of the typology. The sole exception to this is Katzir and Singh (2013), but their ability to account for it comes at the cost of an exceptionally strong version of Positivity. And as discussed by Uegaki (2022), their account does not fully explain the typological pattern; for instance, it doesn't rule out a language with an inventory like {AND, NAND}.

The typology of logical connectives then presents a major cross-linguistic puzzle, which can be summarized with the following two questions:

(5)   a.   Why are AND and OR the only **simple** connectives attested in natural language?

        b.   Why is NOR the only **complex** connective attested in natural language?[7]

However, as we have seen, all existing accounts are stipulative (specifically, most of them have to rely on Positivity), and no existing account captures the full typological pattern in (1) (especially the fact that NOR is complex when lexicalized). Our goal in this paper is to provide an account of the typology which is less stipulative and has wider empirical coverage than all previous accounts, based on a novel notion of communicative stability. As we will demonstrate, using this notion of stability we can derive Positivity rather than stipulate it, namely provide an explanatory answer to (5a), and the same notion will also provide a natural answer to (5b).

The rest of the paper is structured as follows: In §2 we present the notion of communicative stability and discuss the role of prior probabilities in communication. We then first show in §3 how stability can explain why AND and OR are the only simple connectives

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[7]To simplify the discussion, we ignore complex connectives which can also be lexicalized as simple, for instance *both . . . and* and *either. . . or*. For more on this see fn. 20.

(answering (5a)), and then move on to show in §4 how stability can also provide the basis for explaining why NOR is always complex when it is lexicalized, and why no other complex connectives are lexicalized (answering (5b)). §5 is concerned with extending the account by considering epistemic states, with the reasons for the absence of non-commutative and trivial connectives, and with the assumptions needed for extending the theory to the very similar typological pattern found in the domains of quantificational determiners and modals.

2 Stability

We follow Enguehard and Spector in abandoning Positivity and attempting to explain the typological pattern using considerations having to do with prior probabilities of states. Unlike Enguehard and Spector, however, we do not make any specific assumptions about priors. Instead, we consider how different state priors may affect speakers' message choices given a model of communication which incorporates Strengthening; specifically, we will be using an Iterated Rationality Model (IRM; Franke 2009, 2011; Frank and Goodman 2012; Bergen et al. 2016). Furthermore, our account will derive Positivity and will address the morpho-syntactic puzzle in (5b) concerning the complexity of NOR. The key to the current proposal is the following claim:

(6)   **Stability**: Attested languages are **stable** languages, i.e., languages in which the optimal message for a speaker to choose when they want to convey a particular state is not affected by which states they take to be more likely and which ones less.

Attested inventories of connectives are indeed stable in the sense of (6); for instance, a speaker would never utter a sentence of the form P OR Q when they want to convey

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that P AND Q is true, regardless of priors (that is, when the language lexicalizes both; cf. Bowler 2014). As we will argue, this is not an automatic property of inventories in general. The essence of our proposal is that the stability of the attested inventories is not accidental: Attested languages are attested because they are stable.

A reasonable concern at this point is that replacing Positivity with Stability may seem to amount to replacing one stipulation with another, namely stipulating that languages must be stable. We would first like to highlight that even if Stability is a stipulation, it is one worth considering given the good empirical coverage it provides; specifically, as we will demonstrate in detail in the next two sections, it derives Positivity and it goes beyond existing accounts in offering a handle on the complexity of NOR, a major challenge for earlier proposals. Furthermore, our hope is that, rather than being a mere stipulation, the connection between Stability and the typology would turn out to be an outcome of independent considerations of communicative success and language evolution. While providing a detailed theory of how stability determines attestability is beyond the scope of this paper, we will offer an intuitive outline for why this might be so. It is reasonable to think that a stable language would be better for communication than an unstable one, because a hearer would be more likely to successfully retrieve a speaker's intended meaning if they don't also have to know the prior probabilities assumed by the speaker, which may well be different from those assumed by the hearer. We hypothesize that this, in turn, can affect language change: unstable languages will be filtered out when languages evolve because they are not efficient. In the following sections we will use speaker Stability as a proxy for communicative success; we leave the investigation of how Stability translates into a communicative advantage and how such advantage may shape the typology to future work.

Note that an account based on Stability leads to a nuanced view on the role of probabilities in communication: On the one hand, a Stability-based analysis must assume a system

of communication where prior probabilities play a role, otherwise it will be impossible for unattested languages to be unstable. On the other hand, this analysis also assumes that languages aim to minimize the effect that priors have on communication, which means it might be difficult to detect. As we just mentioned and as has been argued in several recent works (Fox and Katzir 2021; Asherov et al. 2022; Cremers et al. 2022), the effect that priors have on Strengthening indeed looks very restricted, a fact that has been brought up in these works as an argument against IRMs which have access to prior probabilities. Since our proposal relies on having access to priors, we should clarify our view regarding both what role prior probabilities have in communication and how Strengthening is carried out. First, we believe that there is justification for having a communication model with access to prior probabilities, based on disambiguation: Unlike Strengthening, ambiguity resolution seems sensitive to priors. One important kind of disambiguation where probabilities have been argued to play role is determining what the Question Under Discussion is (Cremers et al. 2022). We will make the simplifying assumption that the QUD is fixed, and consider the effect of priors given some QUD.[8] Second, we believe there is as of yet no

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[8]An illutsration of the possible effect of priors on the QUD comes from an observation by Enguehard and Spector, who argue that (ia) is preferred over (ib) in a situation in which it is expected that all the talks will be in English:

(i)   a.   Not every talk will be in English.
           b. ??Some talks will be in English.

Enguehard and Spector present (i) as evidence that prior probabilities affect communication, which may seem at odds with Stability. However, even if priors are indeed responsible for the effect in (i), this may be an indirect effect caused by the effect of priors on the QUD, in line with the claims made by Cremers et al. (2022). (ib) might be odd in this situation because it cannot be conceived as an answer to a reasonable question: In the given context the question of whether (ib) is true or not is an odd question, presumably because it presupposes that it is possible that no talk will be in English, which is highly unlikely in this context (for discussion of the idea that felicity is determined by whether a sentence can be conceived as a good answer to a good question see Katzir and Singh 2015). Note that if the possibility that no talk will be in English becomes slightly more likely, but still the possibility that every talk will be in English is the most likely one, (ib) becomes fine. Consider for example an annual conference where normally speakers must give their talks in English. However, there is a small chance that all the speakers selected this year will be Hebrew speakers, in which case an exception would be made and every speaker will decide if they want to give their talk in English or in Hebrew. We happen to know that Miriam is going to give her talk in English anyway

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viable alternative to grammatical strengthening. In this sense our view is in line with recent work which argues that both rational models of communication like the one we will rely on here and grammatical strengthening are needed (Champollion et al. 2019; Franke and Bergen 2020; Cremers et al. 2022).

For simplicity, however, our illustrations below will be done without assuming grammatical strengthening. As a result of this choice, the model of communication we will introduce in the next section (just like all current models) suffers from the problems discussed by Fox and Katzir (2021); Asherov et al. (2022); Cremers et al. (2022); specifically, in some cases it wrongly predicts attested languages to be unstable. In order to keep things simple, we restrict our discussion in this paper in several ways: First, we will use a model of communication (inspired by various IRMs in the literature) which minimizes these wrong predictions (see fn. 12). Second, we focus on demonstrating Stability by looking at simple cases where the problems for the model we use do not arise (though see fn. 15). Third, we only focus on Stability from the speaker's perspective and do not provide a full model which aims to capture rational hearers' behavior, where some of these problems pop up. As far as we know these problems are not solved without incorporating grammatical strengthening, which is a direction we intend to pursue in future work.

3 Simple Connectives

3.1 Communication Model

The rational speaker we assume is defined in (7b).[9] When this speaker wants to convey some state, the message they choose is the one which has the best chances of leading a naive hearer to choose that state (unless all messages are false in that state, in which case

(even though she is a Hebrew speaker), so we can confidently say that some talks will be in English.

[9]The code we used for our simulations below is available at https://github.com/moysh/Stability.

none can be chosen).[10] The naive hearer, in turn, attributes a probability to a state upon hearing a message based on the prior probability of that state (given by the function PR) and whether the message is true in that state (that is, if the message is false in that state the assigned probability is 0).[11] As mentioned above, we focus on stability with (first-level) rational speakers, as defined in (7b); we set aside how rational hearers behave.[12]

(7)   a.   A naive hearer NH: $NH(s|m) \propto PR(s) \cdot [\![\mathrm{m}]\!](s)$

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[10]We loosely talk here as if for each state it is guaranteed that there would be one unique optimal message, because that is by far the most common situation we encounter in our simulations below. Given the definition in (7b), in case of a tie between messages the probabilities will be evenly divided between the tied up messages if they don't all have probability zero of leading the naive hearer to choose the desired state. If they do, namely if s is s.t. $\sum_m NH(s|m) = 0$, then $RS(m|s)$ will be undefined (because it will involve division by zero). This means that RS is not technically speaking a probability distribution. This issue can be solved if the possibility of remaining silent is considered. To avoid clutter, we simply take $RS(m|s)$ to equal zero in such cases.

[11]We assume throughout the paper that every state has a positive prior probability.

[12]IRMs come in many shapes and forms. Our model (which is similar to Enguehard and Spector's) is based on early models like van Rooij and Benz (2007); Franke (2009), and as mentioned above aims to minimize wrong predictions made by some IRMs pertaining to the effect of priors. As far as we can tell, first-level rational speakers in more involved versions of IRMs than ours, such as (i) which uses a softmax function and a rationality parameter λ, make the same predictions about Stability of first-level rational speakers as the ones we have shown in §3 when we check the *optimal* choices for a speaker to make in a given state, namely the function in (ii), against random priors (regardless of the value chosen for λ).

(i) $RS(m|s) \propto \exp(\lambda \cdot \log NH(s|m))$

(ii) $OPT_{RS}(m|s) = \begin{cases} 1 & if\, RS(m|s) = \max\limits_{m'} RS(m'|s) \text{ and } RS(m|s) \neq 0 \\ 0 & otherwise \end{cases}$

These results however do not carry over if we incorporate message costs as in (iii), as often done within the RSA framework:

(iii) $RS(m|s) \propto \exp(\lambda \cdot \log NH(s|m) - c(m))$ (where $c(m)$ is the cost assigned to message m)

The definition in (iii) leads all languages to be speaker-unstable under the reasonable cost assignments $c(\mathrm{P}) = c(\mathrm{Q}) = 1$ and $c(\mathrm{P\ CON\ Q}) = 2$ (we thank Milica Denić for pointing out the potential relevance of message costs to our theory). The reason for this is what has been discussed by Cremers et al. (2022) under the term anti-exhaustivity: This model predicts that given some prior probabilities, P could be a better choice than P AND Q in the state $p \wedge q$. As Cremers et al. (2022) point out, however, anti-exhaustivity is a bad prediction of this kind of model. The model we assume here avoids this particular manifestation of anti-exhaustivity; it is however vulnerable to others, as we discuss in fn. 15.

b.   A rational speaker RS:

$$RS(m|s) \propto \begin{cases} 1 & \text{if } m \in \underset{m'}{\text{argmax}} NH(s|m') \text{ and } \underset{m'}{\max} NH(s|m') \neq 0 \\ \\ 0 & \text{otherwise} \end{cases}$$

Let us illustrate how this system gives rise to strengthening-like effects. Suppose that P OR Q and P AND Q are two alternative messages. The set of states we consider is given by the partition of logical space induced by the set of alternatives; in this case we get 3 states: one where both alternatives are true, one where P OR Q is true but P AND Q is false, and one where both are false. For illustration, suppose that all states are exactly as likely ('flat' state priors), that is $PR(s) = 1/3$ for any s. NH then assigns probabilities as follows:

NH	$p \wedge q$	$(p \vee q) \wedge \neg(p \wedge q)$	$\neg p \wedge \neg q$
AND	1	0	0
OR	1/2	1/2	0

Table 2: Naive hearer behavior with flat priors and the set of alternatives {P AND Q, P OR Q}

An optimal message for a speaker to use when they want to convey some state is one which has maximal utility, that is one that maximizes the probability that the naive hearer will choose that state. When they want to convey the state $p \wedge q$, the probability that this state will be chosen by the hearer is higher if they choose the message P AND Q than if they use P OR Q. The optimal choice for the rational speaker to make in this case is then P AND Q. In state $(p \vee q) \wedge \neg(p \wedge q)$ the message P OR Q is the optimal one, and in the state $\neg p \wedge \neg q$ all messages have zero probability of leading the hearer to choose that state and so none of them is an optimal choice (see fn. 10). Importantly, P OR Q will only be used in the state $(p \vee q) \wedge \neg(p \wedge q)$; that is, P OR Q will only be used in the state in which P XOR Q is true.

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RS	AND	OR
$p \wedge q$	1	0
$p \vee q \wedge \neg(p \wedge q)$	0	1
$\neg p \wedge \neg q$	0	0

Table 3: Rational speaker behavior with flat priors and the set of alternatives {P AND Q, P OR Q}

3.2 Deletion Alternatives

There is something missing though in the tables above: P OR Q and P AND Q sentences don't just have each other as alternatives. There is ample evidence that the two deletion alternatives P and Q are also alternatives to sentences of the form P CON Q, based on various phenomena such as ignorance inferences, distributive inferences, free choice inferences and simplification of disjunctive antecedents (see Sauerland 2004; Fox 2007; Franke 2011; Bar-Lev and Fox 2020). When these two alternatives are considered, our set of states (resulting again from the partition induced by the set of alternatives) also changes: the state $(p \vee q) \wedge \neg(p \wedge q)$ is now divided into two different states: $p \wedge \neg q$ and $q \wedge \neg p$. For example, the naive hearer table we had above should look like this (assuming flat priors as above, which given that we have 4 states now means that $PR(s) = 1/4$ for any s):[13]

NH	$p \wedge q$	$p \wedge \neg q$	$q \wedge \neg p$	$\neg p \wedge \neg q$
AND	1	0	0	0
OR	1/3	1/3	1/3	0
P	1/2	1/2	0	0
Q	1/2	0	1/2	0

Table 4: Naive hearer behavior with flat priors and the set of alternatives {P AND Q, P OR Q, P, Q}

Unlike previous accounts of the typology, we make crucial use of deletion alternatives.

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[13]The reader may notice that having deletion alternatives makes P OR Q a message that will never be chosen by the rational speaker. We will return to this issue in §5.1.

Specifically, based on the evidence for deletion alternatives mentioned above, we make the assumption in (8) (which also follows from the structural view of alternative generation in Katzir 2007; Fox and Katzir 2011).[14] As we will show, this assumption turns out to be the single most important factor determining which languages are stable.[15]

(8)   **The universal availability of deletion alternatives**:
        Utterances of the form P CON Q have both P and Q as alternatives.

Having our model for rational speaker behavior and assumptions about the alternatives generated in place, we can move on to test Stability. In §3.3 we discuss the stability of inventories in simple positive sentences of the form P CON Q, and in §3.4 we check Stability in negative sentences of the form NEG(P CON Q), with the upshot being that the only languages which are stable in both cases are the attested ones.

3.3   Stability in Positive Sentences

For a language to be stable, the choices that speakers make in that language must remain the same no matter what their beliefs are regarding the prior probabilities. As a first step, we consider here whether languages are stable when positive sentences are concerned. That is, we want to check stability for each language L (a set of connectives) based on the set of messages M_L^+ as defined in (9), i.e., the set of positive sentences containing all

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[14]Sauerland (2004) postulates the connectives L and R (cf. Table 1) in order to derive a similar effect to that of deletion alternatives in the computation of scalar implicatures. On the structural view of alternative generation deletion alternatives are instead derived with no need to assume that such unattested connectives are available as alternatives.

[15]Other alternatives are derivable as well for P CON Q on the structural view of alternative generation, for instance a logically independent alternative like R can be derived. We avoid discussing alternatives of this sort because the communication model we use here makes wrong predictions for such cases: Depending on the priors, P AND Q will be able to be the optimal choice in state $p \wedge q \wedge r$ when the set of alternatives is {P AND Q, P, Q, R}. As a result, considering logically independent alternatives leads to instability for a language like {AND}. This result is another manifestation of the anti-exhaustivity problem discussed by Cremers et al. (2022) (see fn. 12); we hope that this issue can be solved if one can come up with a model which avoids anti-exhaustivity altogether.

connectives in the language taking P and Q as arguments, together with the deletion alternatives P and Q. Since P and Q are always in M_L^+, the set of states S_L derived for any M_L^+ will be $\{p \wedge q, p \wedge \neg q, q \wedge \neg p, \neg p \wedge \neg q\}$, which is the partition induced by the set $\{P,Q\}$. S_L will not vary between languages because adding any sentence of the form P CON Q to $\{P,Q\}$ still induces the same partition. Based on these assumptions, we can ask the question in (10) for every language L. Since we are not directly interested in the stability of P and Q, only in how having them as alternatives in M_L^+ affects the stability of sentences with logical connectives, we check Stability in (10) ignoring choices made by RS for P and Q (which are anyway not expected to have the more complex P CON Q sentences as alternatives given the structural theory of alternative generation).

(9) $M_L^+ = \{\text{P CON Q} : \text{CON} \in L\} \cup \{\text{P}, \text{Q}\}$

(10)   Given M_L^+ and S_L, is there any message-state pair in the set $\{\langle m, s \rangle \: : \: m \in M_L^+ \setminus \{P, Q\}$ and $s \in S_L\}$ s.t. the value of $RS(m|s)$ varies given different prior probability distributions PR?

To check this we generated all 63 possible languages containing non-trivial commutative connectives (recall that there are 6 such connectives; ignoring the empty language, we get $2^6 - 1 = 63$ possible languages).[16] Each language was assigned random prior probabilities 1000 times.[17] A language was considered stable if all RS tables resulting from all 1000 prior probabilities turned out to be identical, namely when the answer to (10) is negative. Before we see what languages turn out to be stable, let us consider two examples which may help get an intuition of what makes a language stable in positive sentences.

1. $\{\text{AND}\}$ is stable: No matter what the prior probabilities are, $RS(\text{AND}|s)$ looks like

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[16]We discuss what happens if trivial and non-commutative connectives are considered in §5.2.

[17]Random priors were generated by drawing a sequence whose length is $|S_L| = 4$ from a flat Dirichlet distribution (using `numpy.random.dirichlet`).

this:

	AND
$p \wedge q$	1
$p \wedge \neg q$	0
$q \wedge \neg p$	0
$\neg p \wedge \neg q$	0

Table 5: Rational speaker behavior for P AND Q given the set of alternatives {P AND Q, P, Q}, regardless of what the priors are.

This is not very surprising: since AND is only compatible with the state $p \wedge q$, it can only be an optimal choice in that state. And since the other alternatives which are true in that state (that is P and Q) are also true in other states, AND ends up as the optimal choice for RS no matter what the priors are.

2. {NAND} is unstable: To illustrate, the following table shows 4 different results for $RS(\text{NAND}|s)$ which depend on the prior probabilities (for clarity, here and below the different priors detailed in each column also appear as indices on the messages).

	NAND$_1$	NAND$_2$	NAND$_3$	NAND$_4$
$p \wedge q$	0 $(PR_1(s) = 0.4)$	0 $(PR_2(s) = 0.3)$	0 $(PR_3(s) = 0.4)$	0 $(PR_4(s) = 0.4)$
$p \wedge \neg q$	1 $(PR_1(s) = 0.2)$	0 $(PR_2(s) = 0.3)$	0 $(PR_3(s) = 0.1)$	1 $(PR_4(s) = 0.4)$
$q \wedge \neg p$	1 $(PR_1(s) = 0.2)$	0 $(PR_2(s) = 0.3)$	1 $(PR_3(s) = 0.4)$	0 $(PR_4(s) = 0.1)$
$\neg p \wedge \neg q$	1 $(PR_1(s) = 0.2)$	1 $(PR_2(s) = 0.1)$	1 $(PR_3(s) = 0.1)$	1 $(PR_4(s) = 0.1)$

Table 6: 4 different prior-dependent patterns of rational speaker behavior for P NAND Q given the set of alternatives {P NAND Q, P, Q}.

To understand why there is a difference between NAND$_3$ and NAND$_4$, for example, consider the state $p \wedge \neg q$, in which the only true messages are P and NAND. Given PR_3, $NH(p \wedge \neg q|\text{NAND}) = 1/6$ while $NH(p \wedge \neg q|\text{P}) = 1/5$, which is why P would be the optimal choice for RS in that state. In contrast, given PR_4, $NH(p \wedge \neg q|\text{NAND}) = 2/3$ while $NH(p \wedge \neg q|\text{P}) = 1/2$, which is why NAND would be the optimal choice for RS in that state.

Considering all 63 languages, the 11 languages in (11) are the only ones which turn out to be stable. Among them we find the attested languages, namely those which only lexicalize AND and OR as simplex (these languages appear in boldface in (11)). However, many unattested languages are stable as well. In the next section we check stability in negative sentences and show that the only languages which are stable in both positive and negative sentences are those which only lexicalize AND and OR. Recall that at this stage we want to distinguish AND and OR from all other connectives, because they are the only ones that are lexicalized as simple. Hence finding that {AND, OR, NOR} is not stable in negative sentences will be a good result; however, when we discuss complex connectives in §4 we will make crucial use of the fact that this language is stable in positive sentences.

(11)   **Languages that are stable in positive sentences:**

    a.   {**AND**}

    b.   {**OR**}

    c.   {NOR}

    d.   {**AND, OR**}

    e.   {AND, NOR}

    f.   {AND, IFF}

    g.   {OR, NOR}

    h.   {AND, OR, NOR}

    i.   {AND, OR, IFF}

    j.   {AND, NOR, IFF}

    k.   {AND, OR, NOR, IFF}

3.4 Stability in Negative Sentences

We do not only use connectives in positive sentences. Since every language has means by which it can express negation (Horn 2010), it makes sense to check whether a language is stable when the connectives it lexicalizes are embedded under negation. In parallel to what we have done in the previous section, we can ask the question in (13), now considering the set of messages M_L^- as defined in (12), i.e., the set containing all connectives in the language taking P and Q as arguments when embedded under negation, together with the deletion alternatives P and Q which are also embedded under negation. This set of messages is again what is predicted based on the structural view of alternative generation, with the auxiliary assumption that negation is not deleted.[18] As before, the choices made for the deletion alternatives are omitted for the purposes of testing stability. (Note that the set of states S_L derived from the partition induced by M_L^- does not differ from the one derived in positive sentences given M_L^+ and again does not vary between languages.)

(12) $M_L^- = \{\text{NEG(P CON Q)} : \text{CON} \in L\} \cup \{\text{NEG(P)}, \text{NEG(Q)}\}$

(13)   Given M_L^- and S_L, is there any message-state pair in the set $\{\langle m, s \rangle : m \in M_L^- \setminus \{\text{NEG(P)}, \text{NEG(Q)}\}$ and $s \in S_L\}$ s.t. the value of $RS(m|s)$ varies given different prior probability distributions PR?

Before we look at the full list of stable languages, let us consider some examples of stable and unstable inventories (which are essentially mirror images of the examples we looked at in the previous section).

1. {OR} is stable in negative sentences: No matter what the prior probabilities are, the

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[18]If negation was deleted, NEG(P CON Q) would have P and Q as alternatives on top of NEG(P) and NEG(Q); this would result in unattested languages being stable in both positive and negative sentences, for instance {NOR}. We assume here that negation is not deleted based on arguments coming from the so-called symmetry problem which arises with operators embedded under negation; see Romoli (2012); Breheny et al. (2018) for discussion.

column for NEG(OR) in RS looks like this:

	NEG(OR)
$p \wedge q$	0
$p \wedge \neg q$	0
$q \wedge \neg p$	0
$\neg p \wedge \neg q$	1

Table 7: Rational speaker behavior for NEG(P OR Q) given the set of alternatives {NEG(P OR Q), NEG(P), NEG(Q)}, regardless of what the priors are.

Just like the stability of {AND} in positive sentences, this is not very surprising: since NEG(OR) is only compatible with the state $\neg p \wedge \neg q$, it can only be an optimal choice in that state. And since the other alternatives which are true in that state (that is NEG(P) and NEG(Q)) are also true in other states, $NH(\neg p \wedge \neg q | \text{NEG(OR)})$ will always be larger than $NH(\neg p \wedge \neg q | \text{NEG(P)})$ and $NH(\neg p \wedge \neg q | \text{NEG(Q)})$, rendering NEG(OR) the optimal choice for RS no matter what the priors are.

2. {NOR} is unstable in negative sentences: To illustrate, the following table shows 4 different results for $RS(\text{NEG(NOR)}|s)$ which depend on the prior probabilities.

	NEG(NOR)$_1$	NEG(NOR)$_2$	NEG(NOR)$_3$	NEG(NOR)$_4$
$p \wedge q$	1 ($PR_1(s) = 0.2$)	1 ($PR_2(s) = 0.1$)	1 ($PR_3(s) = 0.1$)	1 $PR_4(s) = 0.1$)
$p \wedge \neg q$	1 ($PR_1(s) = 0.2$)	0 ($PR_2(s) = 0.3$)	1 ($PR_3(s) = 0.4$)	0 $PR_4(s) = 0.1$)
$q \wedge \neg p$	1 ($PR_1(s) = 0.2$)	0 ($PR_2(s) = 0.3$)	0 ($PR_3(s) = 0.1$)	1 $PR_4(s) = 0.4$)
$\neg p \wedge \neg q$	0 ($PR_1(s) = 0.4$)	0 ($PR_2(s) = 0.3$)	0 ($PR_3(s) = 0.4$)	0 $PR_4(s) = 0.4$)

Table 8: 4 different prior-dependent patterns of rational speaker behavior for NEG(P NOR Q) given the set of alternatives {NEG(P NOR Q), NEG(P), NEG(Q)}.

To understand why there is a difference between NEG(NOR)$_3$ and NEG(NOR)$_4$, for example, consider the state $q \wedge \neg p$, in which the only true messages are NEG(P) and NEG(NOR). Given PR_3, $NH(q \wedge \neg p | \text{NEG(NOR)}) = 1/6$ while $NH(q \wedge \neg p | \text{NEG(P)}) = 1/5$, which is why NEG(P) would be the optimal choice for RS in that state. In contrast, given PR_4, $NH(q \wedge \neg p | \text{NEG(NOR)}) = 2/3$ while $NH(q \wedge$

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$\neg p | \text{NEG(P)}) = 1/2$, which is why NEG(NOR) would be the optimal choice for RS in that state.

Considering all 63 languages, the 11 languages in (14) are the only ones which turn out to be stable in negative sentences. Among them we find the attested languages, namely those which only lexicalize AND and OR as simplex (in boldface), but once again many unattested languages are stable as well.

(14) **Languages that are stable in negative sentences:**

    a. {**AND**}

    b. {**OR**}

    c. {NAND}

    d. {**AND, OR**}

    e. {AND, NAND}

    f. {OR, NAND}

    g. {OR, XOR}

    h. {AND, OR, NAND}

    i. {AND, OR, XOR}

    j. {OR, NAND, XOR}

    k. {AND, OR, NAND, XOR}

Crucially, however, the only languages that are stable in both contexts are those which lexicalize AND, OR and no other connective, (15). These are exactly the attested languages, at least as long as we only consider simple connectives.[19]

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[19]It is clear that there are languages which only lexicalize one connective, for instance Warlpiri (see Bowler 2014). It is not always clear though whether these languages lexicalize AND or OR (for discussion see Bowler 2014; Haslinger and Schmitt 2019).

(15)   **Languages that are stable in both positive and negative sentences:**

    a.   {AND}

    b.   {OR}

    c.   {AND, OR}

Note the crucial role of the universal availability of deletion alternatives in (8) in achieving these results: If no deletion alternatives are derived, then almost half of the languages end up as stable (28 out of 63). This includes languages like {NOR, NAND} and {AND, OR, NOR, NAND, IFF, XOR}. And what happens if we take sentences of the form P CON Q to have NEG(P) and NEG(Q) as alternatives, instead of P and Q, is perhaps even more striking. The languages that end up stable in both positive and negative sentences under this assumption are the following:

(16)   a.   {NOR}

    b.   {NAND}

    c.   {NOR, NAND}

Stability can then explain why AND and OR are the only simple connectives to be lexicalized: They are the only connectives which interact with the deletion alternatives in a way that makes speaker choices stable. Other connectives fail to do that, either in positive or in negative sentences. In other words, Stability offers us a way to derive Positivity rather than stipulating it.

3.5   Interim Summary

At the beginning of this paper we presented the following typological pattern, repeated from (1):

(17) **Typological pattern:**

a. Lexicalization of AND and OR is pervasive and seems to be morphologically simplex.

b. NOR is more rarely lexicalized; when lexicalized, it is morphologically complex and arguably syntactically and semantically complex.

c. NAND, XOR and IFF are never lexicalized (Horn's puzzle, extended).

In this section we demonstrated how stability can help us answer the first question we set out to answer about this pattern, repeated in (18a), based on the observation that the only languages that are stable in both positive and negative sentences are {AND}, {OR}, and {AND, OR}. In the next section we will argue that Stability can further help us answer the second question, repeated in (18b).

(18)   a. Why are AND and OR the only **simple** connectives attested in natural language?

b. Why is NOR the only **complex** connective attested in natural language?

4   Complex Connectives

How are complex connectives derived? Following Katzir and Singh (2013), we make the following assumption:[20]

(19)   Complex connectives in a language L can only be formed by combining an existing simple connective in L with negation.

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[20]To keep things simple, (19) is stronger than it needs to be, and it rules out complex connectives such as *both ... and* and *either ... or*. Weakening (19) so that it enables forming complex connectives by combining any two simple operators in the language would rule such complex connectives in and would not significantly change the picture we draw here.

Since the only simple connectives in natural language are AND and OR (which we proposed above can be explained by Stability), given (19) the only complex connectives that can be formed are NAND and NOR. Our goal now is to explain the absence of NAND and the existence of NOR in terms of Stability, proposing a new resolution of Horn's puzzle.

As our starting point, we observe that languages with NOR (alone or together with AND or OR or both) are stable in positive sentences, while languages with NAND are not (see (11)). This situation might be expected given that NOR is only compatible with one state ($\neg p \wedge \neg q$); in contrast, NAND is compatible with three states.[21] As we have seen, of course, this situation flips once we move to negative sentences, where languages with NOR are unstable and some languages with NAND are. However, we think it is reasonable to assume that when complex connectives are at stake what matters is only stability in positive sentences. This is because given (19), any language which lexicalizes NAND as a complex connective must also have AND as a simple one; and any language which lexicalizes NOR as a complex connective must also have OR as a simple one. As a result of this, NEG(P NOR/NAND Q) would be redundant given the availability of the equivalent and less complex P OR/AND Q. In other words, an utterance of NEG(P NOR/NAND Q) would be a systematic violation of Gricean Brevity. And indeed, considering the complex lexicalization of NOR in English, we observe that sentences of the form NEG(P NOR Q) are rather odd (at least barring special intonation indicating a 'meta-linguistic

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[21]An implicit and not entirely innocent assumption we make here is that the alternatives derived for P NAND/NOR Q are the same regardless of whether NAND and NOR are simple or complex. While our assumption that P NAND/NOR Q has P and Q (but not NEG(P) and NEG(Q)) as alternatives seems unobjectionable when NOR and NAND are simple, it becomes less obvious when these operators are complex and are thought of as having the underlying structure of NEG(P AND/OR Q). This might look especially suspicious given that we assumed that sentences of the form NEG(P CON Q) have NEG(P) and NEG(Q) as alternatives (but not P and Q; see fn. 18). We have to commit then to the claim that complex connectives don't have alternatives which are derived by replacing only some of their parts, a claim we cannot justify on independent grounds at the moment but one that looks unavoidable if the account we propose here is adopted. We thank Danny Fox for pointing out this issue.

negation' use):

(20)   ?It's not the case that Mary solved neither problem A nor problem B.

This is not to say that sentences with complex operators under negation are entirely non-existent, but the fact that they are marginal makes it likely that they do not significantly affect the stability of a language. We propose then that what underlies the fact that NOR is attested and NAND is not is that NOR is stable in positive sentences and NAND is not. Finally, we would like to point out a weakness of the analysis proposed here, which is that it does not rule out languages with OR and (complex) NOR but without AND. The reason for this is that the only prerequisite for having complex NOR in a language, based on (19), is that it have OR as a simple connective; nothing requires AND to be present as well. However, as far as we know no such language has been reported.

5   Further Issues

5.1   Epistemic States

Table 9 shows the stable rational speaker behavior for {AND, OR} in positive sentences. Note that OR looks redundant: It will never be chosen by a speaker. This happens because there is always a better message than P OR Q in states in which it is true. The message P AND Q, the message P, and the message Q are all better than P OR Q in the state $p \wedge q$; P is better than P OR Q in the state $p \wedge \neg q$; and Q is better than P OR Q in the state $q \wedge \neg p$. This is not just a particular problem with OR: a parallel issue arises for AND in negative sentences.[22] Indeed, though, a speaker will never choose OR if they know they are in one of these states. They will only ever choose it if they don't know whether P is true or Q is

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[22]The same issue arises also in the singleton languages {AND} and {OR}.

true, but we haven't considered such epistemic states so far.

RS	AND	OR	P	Q
$p \wedge q$	1	0	0	0
$p \wedge \neg q$	0	0	1	0
$q \wedge \neg p$	0	0	0	1
$\neg p \wedge \neg q$	0	0	0	0

Table 9: Rational speaker behavior with flat priors and the set of alternatives {P AND Q, P OR Q, P, Q}

In order to amend this we follow Franke (2011) in assuming that the set of states at least can be the set of epistemic states rather than the set of basic states we looked at so far; the set of epistemic states is the closure under union of the set of states S_L, that is $S_L^{\cup}$. Importantly, moving from S_L to its closure under union maintains the stability results we found above. This enrichment of the system further makes OR and NEG(AND) non-redundant: As table 10 shows, OR is now the optimal message for a speaker to choose when they want to convey that both $p \wedge \neg q$ and $q \wedge \neg p$ are compatible with their knowledge; parallel results obtain with NEG(AND).[23]

5.2 Trivial and Non-commutative Connectives

So far we only considered commutative and non-trivial connectives as potentially lexicalizable. Of course, a complete theory of the typology should also explain why trivial connectives and non-commutative ones are never lexicalized. Stability is of no help here: Checking (speaker) stability for all commutative connectives including the trivial ones TAU and CONT, we find that every combination of AND, OR, TAU and CONT is stable. And once we consider lexicalizability of non-commutative connectives, many other unattested languages also turn out to be stable; here is a sample of some languages containing

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[23]Note that P OR Q is the optimal choice for the rational speaker not only in the epistemic state $(p \wedge \neg q) \cup (q \wedge \neg p)$, but also in the epistemic state $(p \wedge q) \cup (p \wedge \neg q) \cup (q \wedge \neg p)$. Sentences of the form P OR Q are however preferably interpreted as conveying the former epistemic state (i.e., as exclusive OR). We leave this as an open issue for now.

RS	AND	OR	P	Q
$p \wedge q$	1	0	0	0
$p \wedge \neg q$	0	0	1	0
$q \wedge \neg p$	0	0	0	1
$\neg p \wedge \neg q$	0	0	0	0
$(p \wedge q) \cup (p \wedge \neg q)$	0	0	1	0
$(p \wedge q) \cup (q \wedge \neg p)$	0	0	0	1
$(p \wedge q) \cup (\neg p \wedge \neg q)$	0	0	0	0
$(p \wedge \neg q) \cup (q \wedge \neg p)$	0	1	0	0
$(p \wedge \neg q) \cup (\neg p \wedge \neg q)$	0	0	0	0
$(q \wedge \neg p) \cup (\neg p \wedge \neg q)$	0	0	0	0
$(p \wedge q) \cup (p \wedge \neg q) \cup (q \wedge \neg p)$	0	1	0	0
$(p \wedge q) \cup (p \wedge \neg q) \cup (\neg p \wedge \neg q)$	0	0	0	0
$(p \wedge q) \cup (q \wedge \neg p) \cup (\neg p \wedge \neg q)$	0	0	0	0
$(p \wedge \neg q) \cup (q \wedge \neg p) \cup (\neg p \wedge \neg q)$	0	0	0	0
$(p \wedge q) \cup (p \wedge \neg q) \cup (q \wedge \neg p) \cup (\neg p \wedge \neg q)$	0	0	0	0

Table 10: Rational speaker behavior with flat priors, the set of alternatives {P AND Q, P OR Q, P, Q}, and epistemic states.

non-commutative connectives which are speaker stable:

(21) a. {AND, IFF, $\rightarrow$}

  b. {AND, NAND, ONLYP}

  c. {NAND, NOR, ONLYP, $\rightarrow$}

  d. {AND, OR, NOR, NAND, IFF, XOR, $\rightarrow$, $\leftarrow$, ONLYP, ONLYQ, NOTP, NOTQ}

  e. …

A similar issue is discussed in Uegaki (2022), who also aims to explore the full space of possible lexicalizations of binary connectives.[24] We follow Uegaki's proposal that trivial connectives are ruled out due to a ban on the use of trivial expressions (see Gajewski

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[24]Even though his account does not predict all combinations of AND, OR, TAU and CONT to be optimal like we do, and the number of languages containing non-commutative connectives his system predicts to be optimal is smaller than in our system, his account of the typology and ours share the need for auxiliary assumptions in order to block trivial and non-commutative connectives.

2002). Note however that in order to entirely block any use of trivial connectives, this ban would have to apply before strengthening does, since the connectives TAU and CONT do not end up entirely trivial in our system (as well as in Uegaki's): TAU can be strengthened in positive sentences and CONT can be strengthened in negative sentences.[25]

As for non-commutative connectives, lacking any usage-based account for their absence, an account where universal grammar does not allow for their lexicalizability seems to be needed.[26] In order to explain the absence of non-commutative connectives we then follow Gazdar (1979, p. 75) in assuming that connectives must be defined based on functions from *sets* of truth values, rather than n-tuples (Gazdar traces the origins of this idea to McCawley 1972). This leaves no room for the notion of order of arguments to play a role (since {T,F}={F,T}, while <T,F>≠<F,T>), and consequently rules out non-commutative connectives.

This move however raises the question in what sense our account is to be preferred over previous accounts like Gazdar and Pullum (1976); Gazdar (1979); Katzir and Singh (2013) which stipulate that AND and OR are the only lexicalizable connectives, given that we anyway have to stipulate that only commutative connectives are lexicalizable. The account we offer here has however two significant advantages. The first advantage is that our only stipulation is a matter of making a specific choice in setting up the underlying logical system, one which is no less parsimonious than the alternative, that is, that connectives are functions from n-tuples of truth values (if anything, defining functions from

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[25]Note that, even without assuming a ban against triviality, a language with TAU or CONT is one where there is a connective which is unusable either in positive sentences (in the case of CONT) or in negative sentences (in the case of TAU). An alternative direction to the one proposed by Uegaki could then be that storing a connective which is only usable in one of these contexts is not worth it (assuming that there is some pressure for languages to have small inventories). Taking this route will however raise the question of why complex NOR can be lexicalized, given that we have argued in §4 that it is redundant in negative sentences.

[26]Uegaki (2022) entertains the idea that lexicalization of non-commutative connectives is more costly than commutative ones and that is why they are not lexicalized; this however looks stipulative and it is not clear to us how it should be implemented.

sets can be thought of as simpler). In contrast, the constraints on lexicalizability imposed by the authors mentioned above in order to ensure that only AND and OR are lexicalizable are only needed in order to account for the typology, and can be dropped with no need for replacement once the typology is not concerned.[27] The second advantage is that, as we have argued at the beginning of the paper, previous accounts failed to cover the full typological pattern, which we have shown is possible when relying on Stability. In other words, a theory which assumes Stability together with the assumption that connectives are functions from sets of truth values is simpler and has better empirical coverage than the alternatives.

5.3 More Than Two Arguments

The stability results we have shown above only partially extend to connectives taking three arguments or more, under the assumption that a sentence of the form P, Q CON R (e.g., *Mary had an apple, a banana or a pear*) should be translated as CON(P, Q, R) where the connective takes an n-ary set of arguments and has the following deletion alternatives (which are the ones expected by the structural theory of alternative generation):[28]

(22)   {CON(P, Q, R), CON(P, Q), CON(P, R), CON(Q, R), P, Q, R}

For example, with this set of alternatives in place, NAND(P, Q, R) ends up as stable in both positive and negative sentences.[29] One might suspect that the disappearance of the

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[27]Furthermore, while the stipulation about connectives being functions from sets concerns form, and only indirectly affects what content is possible for a connective, the other stipulations are directly aimed at restricting the content that can be expressed by a connective.

[28]For commutative connectives (which are the focus of this paper, see §5.2) there is a natural definition generalizaing to any number of arguments. The definition for NAND, for example, looks like this:

(i) $\llbracket \text{NAND} \rrbracket (P_1, \ldots, P_n) = 1$ *iff* $\neg (P_1 \wedge \ldots \wedge P_n)$

[29]The issue may not arise if instead of assuming that a sentence of the form P, Q CON R is translated as CON(P, Q, R) we consider instead interpreting it as (P CON Q) CON R, where the connective is always

difference in stability between {AND} and {NAND} when they take three connectives makes Stability an insufficient account of the typology. We however do not think this is the case. Importantly, while some unattested languages (e.g., {NAND}) become stable when three arguments are concerned, no attested language becomes unstable as a result of this change. In other words, attested languages are always stable (regardless of the number of arguments), while unattested languages are not always stable: some of them are unstable with two arguments and stable with more arguments, and some of them are unstable regardless of the number of arguments. Recall furthermore that our hypothesis is that Stability affects the typology because it has a communicative advantage. Since the usage of connectives with two arguments is rather common (and perhaps it is their most common use), the communicative advantage that a stable language would have over an unstable language would remain intact (even if the difference is slightly diminished).[30]

5.4   Extension to Quantifiers

Parallel patterns to the typological pattern in (1) have been observed with quantificational determiners and modals. While we do not intend to provide here a full account of the typology of quantificational determiners and modals, we would like to provide a preliminary discussion of what a Stability-based account of them could look like, focusing mainly on quantificational determiners. The most important ingredient needed for an extension to quantifiers is having a parallel set of alternatives to the one we have for connectives: Recall that what sets the positive connectives AND and OR apart from the other connectives is that the deletion alternatives P and Q are positive; so a necessary con-

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defined as a binary one (we thank a reviewer for pointing out the importance of this choice). As it turns out, under this assumption a language like {NAND} ends up as unstable even for a sentence like (P NAND Q) NAND R.

[30]Even with small differences, amplification through transmission over generations can lead to robust typological asymmetries. For a recent demonstration of this effect see Züfle and Katzir (2022).

dition for an extension to quantifiers is having positive alternatives parallel to P and Q with quantifiers as well. A sentence like *Det kid smiled* should then have the alternatives {*Mary smiled*, *John smiled*} if the kids are Mary and John. For concreteness, we point out one way in which such alternatives might be generated, which involves deleting the quantifier phrase as well as the (highest) lambda abstraction over the variable in its scope, and replacing the index on that variable with all possible indices, as in (23).

(23) $Alt(\text{Det A } [\lambda 1 \, [\ldots t_1 \ldots]]) \supseteq \{[\ldots t_i \ldots] : i \in \mathbb{N}\}$

Under the assumption that the assignment function g is surjective (i.e., for every individual x in the domain there is an index i such that $g(i) = x$), the alternative propositions for *Det kid smiled* end up being all propositions of the form *x smiled* where x is an individual in the domain. If John and Mary are in the domain, the desired alternatives are generated, as in (24).

(24) $Alt(\text{Det kids smiled}) \supseteq \{Mary\ smiled, John\ smiled\}$

Note that for existential and universal quantifiers, (23) results in what is sometimes called subdomain alternatives, which have been utilized for instance in the analysis of NPIs (Chierchia 2006, 2013; Crnič 2011, a.o.). Chierchia explicitly utilizes subdomain alternatives as the parallel to deletion alternatives with quantifiers, but does not derive them as in (23) but rather by replacing one domain variable with another, as in (25).[31]

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[31]There are several differences between subdomain alternatives as derived in Chierchia (2013) and in (23). Most importantly for our purposes, they predict different alternatives for (right) downward entailing determiners. While (23) derives the same alternatives for *EVERY boy smiled* and *NEVERY boy smiled*, (25) does not: It derives alternatives of the form *x smiled* for the EVERY sentence and alternatives of the form *NEG[x smiled]* for the NEVERY sentence. Since our account of Stability crucially relies on deletion alternatives always being positive, extending it to quantifiers requires diverging from (25). There are also some differences between the two derivations when (right) upward entailing determiners are concerned. First, (23) only derives singleton subdomain alternatives. Second, (23) derives alternatives of the form *x smiled* even if x is not a kid, which are not derived by (25). As far as we can tell, neither of these differences affects the theory of NPIs in any way: Singleton subdomain alternatives suffice for deriving the

(25) $Alt(\text{Det}_D \text{ A } [\lambda 1 [\ldots t_1 \ldots]]) \supseteq \{[\![\text{Det}_{D'} \text{ A } [\lambda 1 [\ldots t_1 \ldots]]]\!] : D' \subseteq D\}$

If there are just two individuals in $[\![A]\!]$ and in the domain, the resulting picture is the same as what we have seen with connectives taking two arguments in §3, at least as long as we consider the parallel quantifiers to the 6 commutative and non-trivial connectives we considered above (EVERY, SOME, NEVERY, NONE, SOME-BUT-NOT-ALL, NONE-OR-ALL).

One might worry that extending the account to quantifiers will suffer from the issue with connectives taking three arguments discussed in the previous section (which would be more problematic in the context of quantifiers since the usage of quantifiers with two individuals is arguably very restricted). Note, however, that our assumption about how deletion alternatives are generated for quantifiers in (23) does not derive a completely parallel set of alternatives for NAND(P, Q, R) and for *NEVERY kid smiled* when there are three kids (for elaboration on this see fn. 31). While the former has alternatives of the form NAND(P, Q) where the connective takes two arguments, the latter only has singleton subdomain alternatives of the form *x smiled*. As a result of this difference, the Stability results from section 3 are replicated with quantifiers with a domain of three individuals as well.

It is worth noting though that the correlation between attested languages and stable languages weakens as the number of individuals in the domain increases. When we consider quantifiers with a domain of five individuals some unattested languages become stable (e.g., {AND, NAND, XOR}). This situation raises a similar issue to the one we discussed in the previous section: How can Stability explain the typology of quantifiers if it is vul-

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results Chierchia is after, and as long as the determiner is conservative (and all lexicalized determiners are) the non-kids alternatives are logically independent from the prejacent, and as a result don't do any harm. As for the effect of these differences on Stability, see our discussion below for why deriving only singleton subdomain alternatives is crucial for our account. And as discussed in fn. 15, introducing logically independent alternatives has undesirable effects on Stability; for the reasons we mention there, we ignore logically independent alternatives in this paper.

nerable to domain size? Our response to this follows the same line of thought as in the previous section: The fact that attested languages are sometimes more stable than unattested ones (but never the other way around) may suffice in order for stability to shape the typology.[32,33] We leave a more in-depth discussion of the viability of our account for quantificational determiners and modals for future work.

6   Summary

We have argued that the novel notion of communicative stability provides a natural analysis for the typology of natural language connectives (simple and complex alike). On this analysis, the reason why AND and OR are the only simple (commutative) connectives attested in natural language is that the only languages that are stable in both positive and negative sentences are {AND}, {OR}, and {AND, OR}; and the reason why NOR is the only complex (commutative) connective attested in natural language is that complex connectives can only be formed by combining simple connectives with negation (generating only NAND and NOR), and while adding NAND to a language destabilizies it in positive sentences, adding NOR does not. Our stability-based theory of the typology has the advantage of deriving Positivity (which in most previous work has been stipulated) as well as accounting for the complexity of NOR (which has been left unexplained on most accounts).

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[32]Another potentially relevant observation (pointed out to us by Itai Bassi) is that it is conceivable that during language acquisition the domain size used with quantifiers is usually rather small. It is reasonable to imagine that higher chances for miscommunication at this stage would have a particularly devastating effect on the fitness of a language.

[33]The applicability of this line of thought to the typology of modals is questionable, since domain sizes for modals are possibly rather big. At this point we can only speculate that this might have something to do with the puzzling fact that the parallel of NAND in the modal domain, while relatively rare, is not entirely missing (e.g., English *unnecessary*, *needn't*; see Kuhn and Pasalskaya 2022).

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