Free Choice and Divisiveness

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The semantic contribution of *any* in existential modal sentences, as in *Mary is allowed to read any book*, is famuously stronger than that of simple existential quantification. This can be captured by assuming that the existential import of *any* can be strengthened by exhaustification (e.g., Chierchia 2013, Crnič 2019 for different implementations). We derive a new prediction from this assumption and the premise that some expressions in natural language have divisive denotations (Link 1983, Bunt 1985, Landman 1989, among others). Prospects for explaining the variation among NPIs by recourse solely to their semantic properties are discussed in light of our conclusions.

Contents

1 Free choice *any*

1.1 *Any*-DPs as existential quantifiers

The occurrences of *any* in existential modal sentences, which have been dubbed 'free choice occurrences' of *any* (Vendler 1967), pose a challenge for the view that *any*-DPs uniformly denote existential quantifiers that are subject to a single condition on their distribution. An example of a free choice occurrence of *any* is in (1), which has the meaning paraphrased in (2) (e.g., Horn 1972; see Menéndez-Benito 2010, Chierchia 2013 for qualifications).

(1) John is allowed to read any book.

(2) **Paraphrase of the meaning of (1):**
Every book is such that John is allowed to read it.

The meaning that *any* appears to contribute in (1) is stronger than existential quantification. One prominent response to these data has been to assume a distinct semantics for such occurrences of *any* as well as a distinct condition on their distribution (e.g., Ladusaw 1979, Dayal 1998, Sæbø 2001).[1] This response can be avoided, however.

Strengthening is needed. There have been several approaches to free choice *any*-DPs that do not require a departure from the assumption that they denote existential quantifiers. They all recruit some external mechanism or other in order to derive the universal-like interpretation of *any*-DPs in existential modal sentences (see, e.g., Kadmon & Landman 1993, Lahiri 1998, Aloni 2007b, Menéndez-Benito 2010, and Chierchia 2013). The recruited mechanisms range from an unselectively binding generic operator to modals that have a meaning sensitive to indefinites and disjunction. The LFs adopted on such approaches are schematically represented in the first line of (3), where OP stands for the different strengthening mechanisms associating with *any*-DPs (we are abstracting away from specific syntactic differences between them). Their interpretation corresponds roughly to that of LFs with a wide-scoping universal quantifier, schematically represented in the second line of (3).

(3) $[\underbrace{\text{OP } [\Diamond \text{ ... } [\text{any NP}]}\text{ ...}]]$

$\underset{\text{approx.}}{\Leftrightarrow}$ [every NP_{x} [$\Diamond$ ... x ...]]

In addition to their assumptions about the nature of the OP, the approaches also differ in their assumptions about what governs the distribution of *any*-DPs. On some approaches, the distribution is supposed to follow from the nature of the OP alone (e.g., Menéndez-Benito 2010, Chierchia 2013). On others, a separate licensing condition, or operator, is taken to govern their distribution (e.g., Kadmon & Landman 1993, Lahiri 1998, Aloni 2007b, Crnič 2017). This note focuses on predictions arising from the assumption that the OP in (3) is an exhaustification device, *exh*. (Our conclusions are compatible with all the approaches that employ *exh* in their account of the distribution of *any*-DPs in existential modal sentences, esp., Chierchia 2006, 2013, Dayal 2013, Crnič 2017, 2019, 2020b – that is, they are independent of the question whether further operators are needed to account for the licensing of *any*-DPs. Potential further operators are thus left out of our representations for readability.)

1.2 Exhaustification in grammar

We begin by describing how exhaustification can be utilized to capture the behavior of *any*-DPs in existential modal sentences (see Chierchia 2006, 2013 for the first account of free choice occurrences of *any* utilizing *exh*). The characterization of the exhaustification device that we adopt, proposed in Bar-Lev & Fox (2019), has two components that we will discuss in

[1] Another response has been to assume a universal quantifier semantics for all occurrences of *any* (e.g., Reichenbach 1947, Quine 1960). See Gajewski 2008 for a discussion of some problems for this position.

turn below: it negates all relevant alternatives that can be negated (that is, the alternatives that are 'Innocently Excludable' and in the resource domain of *exh*, R), and it asserts all alternatives that can be asserted (that is, the alternatives that are 'Innocently Includable').

(4) $[\![\text{exh}_R\ S]\!](w) = 1$ iff
   a. $\forall S' \in \text{Excl}(S) \cap R$: $\neg[\![S']\!](w)$, and
   b. $\forall S' \in \text{Incl}(S)$: $[\![S']\!](w)$.

The sets of Innocently Excludable and Innocently Includable alternatives are defined on the basis of the sister of *exh* and its formal alternatives. The formal alternatives of a sentence S are all other sentences that can be derived from S by replacing constituents of S with other linguistic material, in particular, with a lexical item or a subconstituent of a replacement target (which amounts to deleting constituents of S), as defined in (5) (see Katzir 2007, Fox & Katzir 2011, Trinh & Haida 2015 for a more thorough discussion of formal alternatives).

(5) $\text{ALT}(S) = \{S' \mid S'$ is derived from S by substitution of its constituents with their subconstituents or with lexical items$\}$

The Innocently Excludable alternatives of a sentence are defined in two steps. First: One identifies all the maximal subsets of the formal alternatives of the sentence such that the conjunction of the negations of all the alternatives in that set is compatible with the sentence; a maximal set of such alternatives is thereby one where adding a further alternative would result in the conjunction of all the negated alternatives being incompatible with the sentence. Second: The Innocently Excludable alternatives are those in the intersection of all such maximal sets. This is summarized in (6).

(6) $\text{Excl}(S) = \bigcap\{M \mid M$ is a maximal subset of ALT(S) such that $\{\neg[\![S']\!] \mid S' \in M\} \cup \{[\![S]\!]\}$ is consistent$\}$

The Innocently Includable alternatives of a sentence are defined on the basis of the Innocently Excludable ones. First: One identifies all the maximal subsets of the formal alternatives such that the conjunction of all the alternatives in them is compatible with the negation of all the Innocently Excludable alternatives. Second: The Innocently Includable alternatives are those in the intersection of all such maximal sets. This is summarized in (7).

(7) $\text{Incl}(S) = \bigcap\{M \mid M$ is a maximal subset of ALT(S) such that $\{[\![S']\!] \mid S' \in M\} \cup \{\neg[\![S']\!] \mid S' \in \text{Excl}(S)\}$ is consistent$\}$

We are now in a position to turn to free choice occurrences of *any*.

Free choice *any*. While the approaches that rely on exhaustification to deal with free choice occurrences of *any* differ along several dimensions, we ignore them in the following (our conclusions extend to all implementations). For concreteness, we adopt the implementation on which the *any*-DP is interpreted in the scope of the modal, while *exh* takes matrix scope (Crnič 2017, 2019, 2020b). (Note that on this approach *exh* enters the derivation merely as a rescue mechanism that induces an appropriate environment; see Chierchia 2006, 2013, Dayal 2013 for an analysis on which *exh* plays a more pivotal role.)

(8) a. John is allowed to read any book.
b. [exh_{R} [◇ [any_{D} book_{x} [John read x]]]]

The pertinent formal alternatives to the sister of *exh* are provided in (9): they differ from it solely in the domain or in the force of the quantifier (or both). (We are ignoring, for example, the alternatives induced by *John* and *allow*, not least because all of these are Innocently Excludable and can be pruned. We are also restricting our attention to so-called subdomain alternatives of the domain of *any*, in line with Krifka 1995, Chierchia 2013.)

(9) $\text{ALT}([\Diamond\ [\text{any}_{\text{D}}\ \text{book}_{\text{x}}\ [\text{John read x}]]]) =$
$\{[\Diamond\ [\text{any}_{\text{D}'}\ \text{book}_{\text{x}}\ [\text{John read x}]]], [\Diamond\ [\text{every}_{\text{D}'}\ \text{book}_{\text{x}}\ [\text{John read x}]]] \mid [\![\text{D}']\!] \subseteq [\![\text{D}]\!]\}$

Given this set of formal alternatives, the Innocently Excludable ones are those where *any* is replaced with *every* and the domain of *any* is replaced with subdomains that consists of at least two books (that is, no alternatives based on *any* are in all the maximal sets of alternatives that can all be negated consistently with the sister of *exh*). Let us illustrate this for the case of *any* quantifying over three books, as sketched in (10). One maximal set of excludable alternatives is in (11): negating an alternative besides those in (11) results in a meaning that is inconsistent with (10); (10) together with the negation of the alternatives in (11) entails that John is allowed to read book c. There are two other such maximal sets, differing in the alternatives built on the existential quantifier (the first three alternatives in the set in (11)): these, respectively, live on {a}, {c}, {a, c} (yielding one maximal set), and on {b}, {c}, and {b, c} (yielding the other maximal set).

(10) $[\Diamond\ [\text{any}_{\{\text{a,b,c}\}}\ \text{book}_{\text{x}}\ [\text{John read x}]]]$

(11) **One maximal set of excludable alternatives:**
$\{[\Diamond\ [\text{any}_{\{\text{a}\}}\ \text{book}_{\text{x}}\ [\text{John read x}]]], [\Diamond\ [\text{any}_{\{\text{b}\}}\ \text{book}_{\text{x}}\ [\text{John read x}]]],$
$[\Diamond\ [\text{any}_{\{\text{a,b}\}}\ \text{book}_{\text{x}}\ [\text{John read x}]]],$
$[\Diamond\ [\text{every}_{\{\text{a,b,c}\}}\ \text{book}_{\text{x}}\ [\text{John read x}]]], [\Diamond\ [\text{every}_{\{\text{a,b}\}}\ \text{book}_{\text{x}}\ [\text{John read x}]]],$
$[\Diamond\ [\text{every}_{\{\text{a,c}\}}\ \text{book}_{\text{x}}\ [\text{John read x}]]], [\Diamond\ [\text{every}_{\{\text{b,c}\}}\ \text{book}_{\text{x}}\ [\text{John read x}]]]\}$

The intersection of the maximal sets described above contains all the alternatives based on universal quantifiers whose domain contains at least two books form the set {a, b, c}.

(12) **Innocently Excludable alternatives:**
$\{[\Diamond\ [\text{every}_{\{\text{a,b,c}\}}\ \text{book}_{\text{x}}\ [\text{John read x}]]], [\Diamond\ [\text{every}_{\{\text{a,b}\}}\ \text{book}_{\text{x}}\ [\text{John read x}]]],$
$[\Diamond\ [\text{every}_{\{\text{a,c}\}}\ \text{book}_{\text{x}}\ [\text{John read x}]]], [\Diamond\ [\text{every}_{\{\text{b,c}\}}\ \text{book}_{\text{x}}\ [\text{John read x}]]]\}$

If we generalize this conclusion to domains of *any* of any size and any member make-up, we obtain the following set of Innocently Excludable alternatives:

(13) $\text{Excl}([\Diamond\ [\text{any}_{\text{D}}\ \text{book}_{\text{x}}\ [\text{John read x}]]]) =$
$\{[\Diamond\ [\text{every}_{\text{D}'}\ \text{book}_{\text{x}}\ [\text{John read x}]] \mid \text{card}([\![\text{D}']\!] \cap [\![\text{book}]\!]) \geq 2 \wedge [\![\text{D}']\!] \subseteq [\![\text{D}]\!]\}$

The Innocently Includable alternatives are all the alternatives based on *any* and its subdomains, namely, all of these alternatives are compatible with the negation of all the In-

nocently Excludable alternatives (the alternatives based on *every* and singleton subdomains are also Innocently Includable, though these are equivalent to their *any* counterparts).

(14) $\text{Incl}([\Diamond\ [\text{any}_{\text{D}}\ \text{book}_{\text{x}}\ [\text{John read x}]]]) =$
$\{[\Diamond\ [\text{any}_{\text{D}'}\ \text{book}_{\text{x}}\ [\text{John read x}]] \mid [\![\text{D}']\!] \cap [\![\text{book}]\!] \neq \emptyset \wedge [\![\text{D}']\!] \subseteq [\![\text{D}]\!]\}$

The assertion of all the Innocently Includable alternatives, and the negation of the Innocently Excludable alternatives (if all are deemed relevant), yields the meaning in (15). (The former inferences have been dubbed 'free choice inferences'.)

(15) $\Diamond$(John read a book in D) $\wedge$
$\forall \text{D}' \subseteq \text{D}\ (\text{D}' \cap \text{book} \neq \emptyset \rightarrow \Diamond(\text{John read a book in D}')$
$\Big(\wedge\ \forall \text{D}' \subseteq \text{D}\ (\text{card}(\text{D}' \cap \text{book}) \geq 2 \rightarrow \neg\Diamond(\text{John read every book in D}'))\Big)$

If we take none of the Innocently Excludable alternatives to be relevant, as indicated with the parantheses in (15), we obtain a meaning that corresponds to the paraphrase in the introduction, as stated in (16). Since this is the desired interpretation of the sentence, we conclude that we successfully derived the apparent universal quantification over books without abandoning the assumption that *any* denotes an existential quantifier.[2]

(16) Every subdomain of books in D is such that John is allowed to read a book in it.
$\Leftrightarrow$ Every book in D is such that John is allowed to read it.

1.3 The puzzle

Not all occurrences of *any* in existential modal sentences are acceptable, however. In particular, consider occurrences of *any* that have a mass NP complement, such as (17) (cf. Chemla et al. 2011, fn. 10). These are marked unless a 'portion' or 'kind' reading is coerced.[3]

(17) a. #John is allowed to drink any water.
b. #Mary is allowed to donate any blood.

On the face of it, this is puzzling for any theory that has the contours sketched out above, that is, any theory on which there is a single lexical entry for *any*: namely, the occurrences of

[2]All the approaches that rely on exhaustification to capture the distribution of *any*-DPs in (at least) existential modal sentences are able to account for their acceptability. On the one hand, the intepretation in (14) is consistent, and thus the *any*-DP is acceptable (Chierchia 2013, Dayal 2013, though their representations differ from those in the main text). On the other hand, the domain of the *any*-DP is on the construal in (8) dominated by a sentence that is Strawson downward-entailing with respect to it, as suggested by the paraphrase with *every*, thus satisfying the condition on its distribution (Crnič 2017, 2019, 2020b).

[3]Parallel observations hold for other modalized environments in which *any* is arguably rescued by exhaustification, such as the imperatives in (i). We do not discuss these explicitly here, though our proposal in the main text should extend to them straightforwardly as well (cf. Crnič 2019).

(i) a. Go ahead, take any apple.
b. #Go ahead, drink any water.

any in environments other than unembedded existential modal sentences are not sensitive to whether their complement is mass or count, as exemplified in (18), so it is not immediately obvious why there should be sensitivity to this difference in existential modal environments.

(18) a. John isn't allowed to drink any water.
b. Mary isn't allowed donate any blood.

In this note, we show that this state of affairs falls out straightforwardly from (i) the uniform approach to *any* coupled with the definition of exhaustification provided above and (ii) certain assumptions about mass nouns that have been argued for independently.

1.4 Preview

Exhaustification yields strengthened meanings of sentences to which it applies – in the cases discussed above, the sentences conjoined with the free choice inferences. However, the strengthened meanings can sometimes be too strong – contradictory. This has been observed by Fox & Hackl (2006), who study several sentence types where exhaustification fails to apply consistently (see also Fox 2007, Gajewski 2009). For example, they show that on the standard assumptions about the meaning of comparative expressions like *weigh more than 80 kg* (incl. that the weight scale is dense) and on the assumption that grammar is at some level encapsulated from some contextual information, exhaustification necessarily fails in (19-a). This explains, then, why the sentence cannot convey the scalar implicature in (19-b). (Although it has been observed that in appropriate contexts, scalar implicatures can be observed for measure phrase comparatives, say, that John does not weigh more than 90 kg for (19) – see, e.g., Fox & Hackl 2006, Cummins et al. 2012 –, the way contextual information is utilized to derive these scalar implicatures will be shown to be of no avail in rescuing the unacceptable occurrences of free choice *any*-DPs we discuss in this note.)

(19) a. John weighs more than 80 kg.
b. $\not\rightsquigarrow$ John weighs exactly 81 kg (80.1 kg, etc).

We describe a related prediction, one that involves free choice occurrences of *any*. In particular, we show that the generation of free choice inferences accompanying *any* in existential modal sentences leads to a contradiction if the arguments of *any* are divisive (that is, non-atomic and homogeneous). Upon describing and deriving this prediction (Section 2), we put forward, perhaps controversially, a candidate for its instantiation: *any* with mass noun complements (Section 3; see, e.g., Link 1983, Bunt 1985, Landman 1989). Subsequently, we discuss cases in which *any* ranges over intervals and show that they suffer from a similar problem (Section 4). The failure of exhaustification in these configurations has catastrophic consequences for the licensing of *any*. Finally, we study how the logic underlying our proposal may be extended to account for the variation among so-called weak NPIs (*any, ever*) with respect to their distribution in existential modal sentences (Section 5).

2 Failure of exhaustification

We introduce a new case of exhaustification failure. Before doing so, we recapitulate the logic behind exhaustification failure with measure phrase comparatives (Fox & Hackl 2006).

2.1 Measure phrase comparatives

Measure phrase comparatives cannot be exhaustified, as indicated in (19) (cf. Krifka 1999). It has been argued that the reason for this is that the application of exhaustification to sentences like (19) leads to an illicit outcome (Fox & Hackl 2006; see Gajewski 2009 for an elaboration), primarily because one cannot find a maximal set of alternatives to the sentence whose joint negation is consistent with the sentence (that is, of excludable alternatives). More precisely, consider the alternatives to sentence *John weighs more than 80 kg*, given in (20): all of these entail that John weighs more than 80 kg. (We are adopting a very simple analysis of measure phrase comparatives, see, e.g., Hackl 2000 for details. We ignore the alternatives entailed by the sentence for brevity.)

(20) ALT([-er than 80 kg]$_{\mathrm{d}}$ [John weighs d-much]) =
$\{$[-er than d$'$]$_{\mathrm{d}}$ [John weighs d-much] | $[\![\mathrm{d}']\!] \geq 80$ kg$\}$

(21) $[\![$[-er than 80 kg]$_{\mathrm{d}}$ [John weighs d-much]$]\!] = 1$ iff weight(John) $>$ 80 kg

The set of alternatives in (20) is 'dense', that is, it satisifies the condition in (22). This state of affairs is parasitic on the density of the weight scale: for every two distinct degrees on a weight scale, there is a degree properly between them.

(22) For every p, r $\in$ ALT([-er than 80 kg]$_{\mathrm{d}}$ [John weighs d-much]): if p $\overrightarrow{\not\in}$ r,
there exists q $\in$ ALT([-er than 80 kg]$_{\mathrm{d}}$ [John weighs d-much]): p $\overrightarrow{\not\in}$ q $\overrightarrow{\not\in}$ r.

We sketch how failure of exhaustification follows from this fact.

No maximal sets of excludable alternatives. Due to density, *John weighs more than 80 kg* entails there is a degree greater than 80 kg such that John weighs more than it (if, say, John is 80 kg+d heavy, for some degree of weight d, he is also heavier than 80 kg+$\frac{\mathrm{d}}{2}$).

(23) John weighs more than 80 kg.
$\Rightarrow$ There exists d$>$80 kg such that John weighs more than d.

This means that set (24) cannot be a set of excludable alternatives given sentence *John weighs more than 80 kg*: excluding all the alternatives in it contradicts the entailment of *John weighs more than 80 kg* that there is a degree greater than 80 kg that John weighs more than.

(24) $\{$[-er than d$'$]$_{\mathrm{d}}$ [John weighs d-much] | $[\![\mathrm{d}']\!] > 80$ kg$\}$

A set E of excludable alternatives given *John weighs more than 80 kg* would thus have to be a proper subset of (24), as stated in (25). We show now that every candidate for such an E fails to be maximal: one can always expand it by further alternatives.

(25) $E \subset \{[\text{-er than } d']_d \text{ [John weighs d-much]} \mid [\![d']\!] > 80 \text{ kg}\}$

Let E be any proper subset of (24) and a^* an alternative in (24) that is not in E. If there is an alternative a^w in E that is entailed by a^*, the negation of a^* is entailed by the negation of a^w so that excluding all elements of $E \cup \{a^*\}$ is equivalent to excluding all elements of E. Otherwise, because of density, there is an alternative a^m that is stronger than *John weighs more than 80 kg* but weaker than a^*. This means that the negation of a^* doesn't entail the negation of a^m so that excluding all elements of $E \cup \{a^*\}$ is consistent with a^m. Consequently, excluding all elements of $E \cup \{a^*\}$ does not contradict the entailment of *John weighs more than 80 kg* that there is a degree greater than 80 kg such that John weighs more than it. Hence, in both cases, $E \cup \{a^*\}$ is a set of excludable alternatives given sentence *John weighs more than 80 kg*. We thus conclude that there cannot be any maximal subsets of excludable alternatives in ALT([-er than 80 kg]$_d$ [John weighs d-much]) given sentence *John weighs more than 80 kg*.

Exhaustification issue. Given that there are no maximal sets of excludable alternatives, every alternative to the sentence (and more) is in the intersection over the set of maximal sets, as given in (26), and negating all of them is incompatible with the sentence, as stated in (27) (Gajewski 2009). While eliminating some alternatives in exhaustification (pruning) may rescue an otherwise illicit exhaustification, Fox & Hackl propose that this is not possible for the alternatives under discussion: at some level of grammatical representation, there is encapsulation of grammar from at least some contextual information (see Fox & Hackl 2006, Section 5, for a precise statement), meaning that contradictions at that level of representation cannot be avoided, and thus lead to ungrammaticality (see also Fox 2000, Gajewski 2002, Chierchia 2013, among others).[4]

(26) Excl([-er than 80 kg]$_d$ [John weighs d-much])

$= \bigcap\{M \mid M \text{ is a maximal subset of ALT}([\text{-er than } d']_d \text{ [John weighs d-much]}) \text{ such that } \{\neg[\![S']\!] \mid S' \in M\} \cup \{[\![S]\!]\} \text{ is consistent}\}$

$= \bigcap \emptyset$

$\supseteq$ ALT([-er than 80 kg]$_d$ [John weighs d-much])

(27) $[\![[\text{exh}_R \text{ [-er than 80 kg]}_d \text{ [John weighs d-much]}]]\!] = 1$ only if

$[\![[\text{-er than 80 kg}]_d \text{ [John weighs d-much]}]\!] = 1 \wedge \forall S \in \bigcap\emptyset \cap R$: $[\![S]\!] = 0$ iff $\bot$

Consequently, sentence *John weighs more than 80 kg* cannot generate scalar implicatures: if the sentence were exhaustified, it would not only have a contradictory meaning, it would

[4]Furthermore, it holds that if all alternatives are Innocently Excludable, all alternatives are also Innocently Includable by the definition in (7). Accordingly, a contradictory inclusion is predicted. However, since there are independent reasons for wanting to avoid this prediction, requiring a revision of inclusion, we avoid this aspect of exhaustification here (see Crnič 2020a for discussion). See footnote 6 for some further discussion.

even be ungrammatical. This is one instance of a failure of exhaustification.

2.2 Divisive predicates

We identify a similar failure of exhaustification in a different domain: existential quantification over divisive predicates, that is, predicates whose every element consists of (proper) parts that are in the predicate themselves. We show that in this case, as in the case of measure phrase comparatives discussed above, it is impossible to find maximal sets of excludable alternatives – and thus exhaustification is bound to fail. The underlying logic is similar to the one in the preceding subsection: while every set of excludable alternatives can be expanded by further excludable alternatives, all alternatives cannot be jointly negated.

Divisiveness. We begin by locking in some terminology. We say that a predicate P is divisive if every P element has a proper part (non-atomicity), for every proper part of a P element there is another proper part such that the two proper parts do not overlap (supplementation), and every proper part of a P element is a P element as well (homogeneity)[5] (see, e.g., Cheng 1973, Bunt 1985, Krifka 1989, Moltmann 1997, Rothstein 2010 for discussion.)

(28) P is divisive if and only if

$\forall x(Px \rightarrow \exists y.\, y \sqsubset x)$, (non-atomicity)

$\forall x, y(Px \wedge y \sqsubset x \rightarrow \exists z(z \sqsubset x \wedge y \sqcap z = \bot))$, and (supplementation)

$\forall x, y(Px \wedge y \sqsubset x \rightarrow Py)$. (homogeneity)

Divisive predicates and existential quantification over them have a host of intriguing properties. For example, in preparation for the following, every element in a divisive predicate generates a divisive predicate, stated in (29), and every divisive predicate may be split into non-intersecting divisive predicates that make-up the original predicate, stated in (30).

(29) For any divisive predicate P and every x in P: $[\lambda y.y \sqsubseteq x]$ is a divisive subset of P.

(30) For any divisive predicate P, there exist non-empty divisive subsets P^* and P^{**} of P such that $P^* \cap P^{**} = \emptyset$.

Hypothetical setup. With these definitions in hand, let us now turn to a hypothetical (modal) case of existential quantification over a divisive predicate – let's call the sentence containing such quantification 'S' for concreteness. We assume that the domain of the existential quantification in S is divisive and that the main predicate is homogeneous, as given in (31). We further assume that the alternatives to the sentence are the subdomain alternatives represented in (32). (We employ predicate logical formulas for LF representations for simplicity. We revert to less simplified representations once we turn to concrete examples.)

(31) $[\![S]\!] = 1$ iff $\Diamond(\exists x(Px \wedge Qx))$ *(where P divisive, Q homogeneous)*

(32) $ALT(S) = \{\Diamond(\exists x(P'x \wedge Qx)), \Diamond(\forall x(P'x \rightarrow Qx)) \mid P' \subseteq P\}$

[5]Note that together non-atomicity, supplementation, and homogeneity entail that, if P is divisive, every P element has two non-overlapping proper parts that are also P elements.

We sketch how exhaustification of S given the alternatives in (32) results in failure, specifically, in all of the alternatives in (32) being Innocently Excludable. In the following sections, we then turn to what we argue are natural language instantiations of this schema.

No maximal sets of excludable alternatives. Take an arbitrary set E of excludable alternatives given S. Now consider the sum of all the elements in the domains of the existential quantifier alternatives in E. It holds that this sum must be a proper part of the sum over the domain P, $\bigsqcup$P: if this were not the case we would obtain a contradiction – the existential quantification over P would be true (that is, S is true), but existential quantification over subdomains of P that span P would be false.

(33) $$\underbrace{\bigsqcup\bigcup\{P'|\Diamond(\exists x(P'x \wedge Qx)) \in E\}}_{y} \sqsubset \bigsqcup P$$

Given this fact, there must exist a divisive subdomain of P that does not intersect with any of the subdomains in E: namely, because of (33) and supplementation, $\bigsqcup$P must have a proper part z that does not overlap with y; and this element z generates a divisive subset of P, as stated in (29). Thus, we conclude that (34) holds.

(34) There exists a divisive $P^* \subset P$ such that
$\bigsqcup P^*$ does not overlap with $\bigsqcup\bigcup \{P'|\Diamond(\exists x(P'x \wedge Qx)) \in E\}$.

Take such a divisive P^*. As stated in (30), P^* has two non-empty disjoint subsets P^*_1 and P^*_2 that are divisive. Since P^*_1 and P^*_2 are disjoint, it is possible that existential quantification over one of them yields a true meaning, and a false meaning over the other, as stated in (35).

(35) There exists non-empty divisive P^*_1, $P^*_2 \subset P^*$ such that
$[\Diamond(\exists x(P^*_1 x \wedge Qx)) \wedge \neg\Diamond(\exists x(P^*_2 x \wedge Qx))]$ is consistent.

Since neither of these subsets intersects with the subdomains in the existential alternatives in E, we conclude that E cannot be maximal: E excludes an alternative that can be negated with the sentence S remaining true. Since E was chosen arbitrarily, we conclude that there are no maximal sets of excludable alternatives in ALT(S) given S.

Exhaustification issue. Given that there are no maximal sets of excludable alternatives, every alternative to the sentence is in the intersection over the set of maximal sets, as given in (36). Exhaustification of the sentence, as in (37), consequently yields a contradiction if the relevant alternatives are not restricted by contextual information (e.g., $\bigcap\emptyset \cap R = \emptyset$).

(36) $\text{Excl}(S) = \bigcap\emptyset \supseteq \text{ALT}(S)$

(37) $[\![\text{exh}_R\ S]\!] = 1$ only if $\Diamond(\exists x(Px \wedge Qx)) \wedge (\forall S' \in \bigcap\emptyset \cap R : \neg[\![S']\!])$ iff $\bot$

Moreover, since all alternatives are Innocently Excludable, it also holds that all alternatives are Innocently Includable (namely, there are no maximal sets of alternatives consistent with the negation of all Innocently Excludable alternatives). Since the inclusion of all alternatives also generates a contradiction (say, that it is and is not raining), an assumption of

encapsulation of grammar from certain contextual information is, in actual fact, unnecessary. Exhaustification thus not only fails to generate free choice inferences (that is, a consistent inclusion), it cannot even apply in the type of sentence under discussion.[6]

(38) $\text{Incl(S)} = \bigcap \emptyset \supseteq \text{ALT(S)}$

(39) $[\![\text{exh}_{\text{R}}\ \text{S}]\!] = 1$ iff $\bot$

Summary. We conclude that exhaustification over the subdomains of (modalized) existential quantification fails if (i) its domain is divisive and (ii) the main predicate is homogeneous: since one cannot identify maximal subsets of excludable subdomain alternatives in such cases, exhaustification generates contradictory inferences. In the following, we argue that mass *any*-DPs in existential modal sentences instantiate this hypothetical setup.

3 Mass nouns

If (i) mass nouns denote divisive predicates and (ii) quantification over them requires a homogeneous main predicate, exhaustification over the domain of such quantification is predicted to fail in existential modal sentences (or fail to generate free choice inferences). This holds in particular for *any*-DPs with mass noun complements, accounting for their unacceptability.

3.1 Instantiation

Divisiveness. The semantics of mass expressions has attracted considerable attention over the years (see already Jespersen 1924; Lasersohn 2011, Lima 2018 for recent reviews). We propose here that mass expressions denote divisive predicates:

(40) **Divisiveness Hypothesis**
Mass nouns in natural language denote divisive predicates in the sense of (28).

Our notion of divisiveness is composed of three conditions: (i) non-atomicity, (ii) supplementation, and (iii) homogeneity. While there does not seem to be a consensus about what the most adequate analysis of mass noun meanings is, the three conditions feature prominently in the pertinent literature on mass nouns and the theory of parthood relations. Specifically, the conjunction of (i) and (iii), i.e. the assumption that mass noun denotations do not consist of discrete parts but are homogeneous masses, is most prominently found in Bunt's Ensemble Theory, which he argues to be the foundational theory of mass noun denotations (Bunt 1979, 1985; for similar views, see ter Meulen 1980, Link 1983, Lønning 1987, Landman 1991, Higginbotham 1994). Condition (ii), i.e. the assumption that mass noun denotations do not contain elements that can be decomposed into a single proper part, is a widely assumed (although sometimes contested) ingredient of mereological theory (see Simons 1987,

[6] As discussed in footnote 4, one should revise the notion of inclusion so that it does not generate contradictions – in cases in which all alternatives are Innocently Excludable, only the prejacent may be included (see Crnič 2020a). In the examples in the next section, the fact that a contradictory inclusion can be avoided by not including any alternatives except the prejacent will be of no avail since inclusion of all subdomain alternatives is necessary for free choice occurrences of *any*-DPs to be acceptable.

Varzi 2019 for discussion). Overall, adopting (i)-(iii) as conditions governing the denotation of mass nouns amounts to assuming that all elements of the denotation of, say, water have two non-overlapping proper parts that are also (elements of the denotation of) water (see footnote 5).

Homogeneity. Independently, mass quantifiers are subject to an ill-understood constraint: they may only combine with homogeneous predicates, as stated in (41) (e.g., Bunt 1979, Lønning 1987, Higginbotham 1994, Moltmann 1997, among others).

(41) **Homogeneity Constraint** (Lønning 1987)
Mass quantifiers may only combine with homogeneous main predicates.

The data captured by this constraint are exemplified in (42)-(43). It holds that every element that boils or is wet is such that all its parts boil or are wet (homogeneity) – which accounts for the acceptability of the sentences in (42) – while no elements that weigh 2 grams or are heavy are such that all their parts weigh 2 grams or are heavy (no homogeneity) – which is responsible for the unacceptability of the sentences in (43).

(42) a. Some water boiled.
b. Most water is wet.

(43) a. #Some water weighs 2 grams.
b. #Most water is heavy.

Consequence. Given the assumption in (40) and the fact in (41), *any*-DPs with mass noun complements instantiate the prediction described in the preceding section: exhaustification responsible for generating free choice inferences is predicted to yield a contradictory interpretation of existential modal sentences with mass *any*-DPs, and is thus unavailable:

(44) ***Any* and divisive predicates**
An existential modal sentence that contains an occurrence of *any* with a mass noun complement cannot generate free choice inferences (and would, in fact, instantiate exhaustification failure absent appropriate pruning).

Now, if exhaustification cannot apply in existential modal sentences to generate free choice inferences, the *any*-DP is predicted to be unacceptable on all theories of *any*-DPs that rely on exhaustification: the sentence is either ill-formed (due to the uninterpretable exhaustification feature of the *any*-DP not being checked, as argued by Chierchia 2013) or the *any*-DP violates its licensing conditions (the domain of *any* is not in a Strawson downward-entailing environment absent free choice inferences, as argued by Crnič 2017, 2019, 2020b).

(45) If an *any*-DP is not accompanied by (consistent) free choice inferences in existential modal sentences, it is predicted to be ungrammatical.

In contrast to existential modal sentences, no similar issues arise with occurrences of *any*-DPs in (Strawson) downward-entailing environments, say, in the scope of negation. Namely, in these configurations, either exhaustification of *any*-DPs yields a consistent interpretation

(it is vacuous since all the subdomain alternatives are entailed by the sentence, as argued by Chierchia 2013), or is unnecessary (as argued by Crnič 2017, 2019, 2020b). We conclude that the distribution of mass *any*-DPs in existential modal sentences is correctly captured on the assumption that mass nouns denote divisive predicates, lending new support for it.

3.2 Obviation by coercion

Mass nouns are known to allow for a shifted non-divisive interpretation. This is illustrated with a pluralized example in (46), which has two construals: we drank some servings of beer and, perhaps less saliently, we drank different kinds of beer (see, e.g., Pelletier 1975, Bunt 1981 for the mechanisms involved).

(46) We drank some beers.

If a mass *any*-DP is construed as quantifying over servings (containers) or over subkinds, we lose divisiveness. Accordingly, one finds maximal sets of excludable alternatives in the application of exhaustification, just as one did in the count cases. For instance, consider sentence (47) on such construals. We present the two possible parses of (47) yielding the servings and the subkinds readings in (48): in (48-a), we assume that PCKG takes a non-atomic (in particular, divisive) predicate and maps it to an atomic one; in (48-b), we assume that $\cup$ maps properties to kinds and DKP is a derived kind predication operator that allows properties to apply to kinds, with PART mapping kinds to sets of subkinds (cf. Chierchia 1998b, Mendia 2017).

(47) We are allowed to drink any beer.

(48) a. $[\text{exh}_{\text{R}}\ [\Diamond\,[[\text{any}_{\text{D}}\ [\text{PCKG beer}]]\ [\lambda \text{x we drink x}]]]]$
b. $[\text{exh}_{\text{R}}\ [\Diamond\,[[\text{any}_{\text{D}}\ [\text{PART}\ [\cup \text{beer}]]]_{\text{x}}\ [\text{DKP}\ \lambda \text{x we drink x}]]]]$

3.3 Consequences for grammar

The analysis of mass nouns from which the behavior of *any* in modal contexts was derived is not uncontroversial, however (see, e.g., Chierchia 2010 for a review of issues). Following primarily Bunt (1985), we indicate how some pervasive intuitions and empirical phenomena going counter this proposal can be accommodated. Crucially, it suffices for our purposes that divisiveness obtains at a level of representation that is blind to certain contextual factors (such as "information about the world" as conceived by Bunt), and that subsequent recourse to pragmatics may introduce non-divisive individuation.[7] In this respect, the proposed treatment of the misgivings about divisiveness mirrors that of intuitively discrete scales (say, number of children) in Fox & Hackl (2006).

[7] On the theories of NPIs under discussion here, it is assumed that if an NPI leads to a pathological meaning at the level of representation that is blind to non-logical (incl. contextual) information (which is the case if free choice inferences are not generated for NPIs in existential modal sentences), the NPI is unacceptable (cf. Gajewski 2002, Chierchia 2013 for details). Thus, subsequent introduction of "information about the world" cannot rescue the occurrences of *any*-DPs in existential modal sentences.

Neat mass nouns. A particularly troubling case are the so-called neat mass nouns like *furniture* and *silverware* (also dubbed object mass nouns), which intuitively have atoms in their denotation. One may thus expect that they will exhibit a distinct distribution in combination with *any* than other mass nouns. This is not what we observe – they are as marked as other mass nouns in combination with *any*:

(49) a. #Mary is allowed to buy any furniture.
b. #John may put away any silverware.

There are some other configurations, however, in which neat mass nouns part ways from other mass nouns in that their intuitive atoms appear to become accessible to grammar (see Bale & Barner 2009, Rothstein 2010, among others, for discussion). Consider the comparative sentence in (50). It allows for an interpretation on which it is true if John moved more (but altogether smaller) pieces of furniture than Bill.

(50) John moved more furniture than Bill.

The pertinent question now is how the facts in (49) (exhibiting inaccessibility of intuitive atoms, and thus failure of free choice inclusion) can be reconciled with the availability of a count reading of (50) (exhibiting accessibility of intuitive atoms, allowing for their counting).

Grammar of mass nouns. We assume that at least at some level of representation in grammar, all mass noun phrases are divisive. This would follow from there being a specific mass feature in syntax, as part of the functional vocabulary (see Borer 2005; also Sharvy 1978). A uniform treatment of *any* would, then, be wedded to the assumption that it selects for both count and mass nouns given its distribution in non-modal contexts (similar to *the, all,* and *some*). On this view, *any water* may have the structural representation in (51).

(51) a. any water: [any [MASS water]]
b. $[\![\text{MASS}]\!] = \lambda$P: P is divisive. P

There may then be a contextual parameter that makes atoms accessible at some other level (a level at which the exhaustification described above would have already failed to rescue the *any*-DP) and thus allows to capture the fact, illustrated in (50), that two bodies of furniture can be compared on the basis of the number of furniture pieces that they are comprised of. Specifically, if PCKG is a contextually provided packaging function of the type assumed in the previous section (mapping non-atomic predicates to atomic ones) then we can define the following partial order on mass noun denotations P on the basis of the cardinality of subsets of PCKG(P).

(52) For all $x, y \subseteq P$, $x \leq_{\text{PCKG}} y$ iff $|\{z \in \text{PCKG}(P) : z \sqsubseteq x\}| \leq |\{z \in \text{PCKG}(P) : z \sqsubseteq y\}|$.

This partial order could then be employed to derive for (50) the observed truth conditions in (53), where PCKG_C is the function that maps the denotation of *furniture* to the denotation of *piece of furniture* and $<_{\text{PCKG}_C}$ the strict counterpart of $\leq_{\text{PCKG}_C}$.

(53) $[\![\text{John moved more furniture than Bill}]\!]^C$

$$= 1 \quad \text{iff} \quad \bigsqcup\{x \in [\![\textsc{Mass furniture}]\!] : \text{Bill moved x}\} <_{\text{PCKG}_C} \bigsqcup\{x \in [\![\textsc{Mass furniture}]\!] : \text{John moved x}\}$$

$$= 1 \quad \text{iff} \quad |\{x \in \text{PCKG}_C([\![\textsc{Mass furniture}]\!]) : \text{Bill moved x}\}| < |\{x \in \text{PCKG}_C([\![\textsc{Mass furniture}]\!]) : \text{John moved x}\}|$$

$$= 1 \quad \text{iff} \quad |\{x \in [\![\text{piece of furniture}]\!] : \text{Bill moved x}\}| < |\{x \in [\![\text{piece of furniture}]\!] : \text{John moved x}\}|$$

To summarize, we put forward that the distribution of mass *any*-DPs can be captured on the assumption that they contain a functional element that imposes divisiveness. While this divisiveness may subsequently be obscured in the context, this may only happen after the NPI has been deemed unacceptable by grammar. More precisely, the contextual factors that could bring about the obviation of the exhaustification failure cannot do so since they are not incorporated at the pertinent level or representation (cf. Fox & Hackl 2006 on the inaccessibility of the granularity parameter). While we could only hint at a mass/count system that is compatible with our assumptions, our hope is that such a system can be adequately fleshed out. One of its desirable side-effects would be accounting for the distribution of *any*.[8]

4 Intervals of degrees

We discuss a class of further occurrences of *any* whose exhaustification fails to yield free choice in existential modal sentences: *any* differentials in comparative constructions. We source the issue to the underlying interval semantics of the comparative, specifically, to the domain of intervals being divisive similarly to the denotation of a mass noun.

4.1 Comparatives and differential quantifiers

Consider the comparative sentence in (54-a). It conveys that there is an interval of degrees between the height of John and the height of Mary – more specifically, an interval of degrees whose minimal element is at least as great as Mary's height and whose maximal element is at most as great as John's height (cf., e.g., Schwarzschild 2005, 2013).[9]

(54) a. John is taller than Mary.
b. $\exists I$ (height(Mary) $\leq$ Min(I) $\wedge$ Max(I) $\leq$ height(John))

The intervals quantified over in the comparative can be further specified with measure phrase differentials like *2cm*, as in (55-a), which conveys that there is an interval between Mary's and John's height that measures 2cm.

(55) a. John is 2cm taller than Mary.
b. $\exists I$ (height(Mary) $\leq$ Min(I) $\wedge$ Max(I) $\leq$ height(John) $\wedge$ μ(I) = 2cm)

[8] A prominent alternative view of the semantics of mass nouns takes them to have denotations structurally parallel to those of adjectives, with it being vague/indeterminate as to what the atoms in the domain are (e.g., Chierchia 1998b, 2010, Rothstein 2010). One may explore whether the logic described in the main text can be extended to this view, that is, whether there are grounds for exhaustification not to be able to apply to sentences whose alternatives exhibit this type of vagueness; we cannot explore this possibility here.

[9] Intervals are convex sets of degrees with more than one member (i.e., intervals are non-degenerate).

Furthermore, one also finds overt quantificational expressions in place of measure phrase differentials, such as *no*. The sentence in (56-a) conveys that there is no interval between Mary's and John's heights whose minimum is at least as great as Mary's height and maximum at most as great as John's height, as given in (56-b). This is also the meaning conveyed by sentence (57), where the comparative combines with *any* in the scope of negation.

(56) a. John is no_{D} taller than Mary.
b. $\neg\exists \text{I} \in \text{D}$ (height(Mary) $\leq$ Min(I) $\wedge$ Max(I) $\leq$ height(John))

(57) John isn't any_{D} taller than Mary.

A possible compositional implementation of the semantics sketched above is provided in (58) and (59) for concreteness (cf., e.g., Schwarzschild 2005, 2013 for more detailed formulations). The LF of the sentence in (58-a) is given in (58-b).

(58) a. John is taller than Mary.
b. [$\exists$ [[er than Max_{d} Mary is d-tall] [Max_{d} John is d-tall]]]

As indicated, the comparative operator *er* combines with two degrees, provided by its arguments, viz. the maximal degree d of Mary's height and the maximal degree d' of John's height. Then $[\![\text{er}]\!](\text{d})(\text{d}')$ denotes the (possible empty) set of intervals that are subintervals of the interval $[\text{d}, \text{d}']$, which characterizes the difference in height between Mary and John, see (59).[10]

(59) a. $[\![\text{er}]\!](\text{d})(\text{d}') = \{\text{I} : \text{I} \subseteq [\text{d}, \text{d}']\}$ $\big(= \{\text{I} : \text{d} \leq \text{Min(I)} \wedge \text{Max(I)} \leq \text{d}'\}\big)$

Consequently, the LF structure in (58-b) is assigned the truth conditions in (54-b), where an existential closure operator over the set of intervaltervals conveys that it is not empty.

4.2 *Any* differential in existential modal sentences

While *any* is acceptable in downward-entailing environments, as in (57), it is not in existential modal sentences, no matter whether one is dealing with dense (e.g., height, (60-a)) or apparently non-dense scales (e.g., number of children, (60-b)). Differential *any* thus has the same distributional signature as mass *any* discussed in the preceding section.

(60) a. #John is allowed to be any taller than Mary is.
b. #John is allowed to have any more children than Mary is/does.

We show that this behavior of *any* differentials can be derived as a consequence of failure of generating free choice inferences (exhaustification failure).

Divisiveness and homogeneity. In Section 2.2, we showed that exhaustification of a sentence S necessarily fails if S has a truth condition of the form in (61) and exhaustification is performed relative to subdomain alternatives of S.

[10] A subinterval of an interval I is a subset of I that is an interval. The condition '$\text{I} \subseteq [\text{d}, \text{d}']$' in (59) is met by an interval I only if $\text{d} < \text{d}'$.

(61) $[\![S]\!] = 1$ iff $\Diamond(\exists x(Px \wedge Qx))$ *(where P divisive, Q homogeneous)*

We assume here that in the absence of an *exh* operator, (60-a) has the LF structure in (62).

(62) [$\Diamond$ [any_D [er Max_d Mary is d-tall] [Max_d John is d-tall]]]

The status of (60-a) follows naturally as a consequence of exhaustification failure if the domain D of *any* is divisive and the meaning of its sister is homogeneous. The scope of *any* in comparative constructions like the above ones is the set of intervals $[\![er]\!](d)(d')$ (where d and d' are the heights of different individuals, the sizes of different groups of children, etc). If the measurement scale is dense (which is uncontroversial for the scale of height), the scope of *any* is divisive. That is, every interval I in D has non-overlapping subintervals: if I is in an interval ranging from d_1 to d_2, density guarantees that there is a pair of degrees d', d'' such that $d_1 < d' < d'' < d_2$ so that the interval ranging from d_1 to d' and the interval ranging from d'' to d_2 are non-overlapping subintervals of I. This property delivers divisiveness, as defined in (28). To be precise, the domain of *any* in comparative constructions, i.e. the set of intervals $[\![er]\!](d)(d')$ defined in (59), is divisive as it satisfies the three conditions in (63). (In the case of scales that are intuitively not dense, which is the case for cardinality scales, we follow Fox & Hackl 2006 in assuming that these are underlyingly dense as well.)

(63) For every interval I in $[\![er]\!](d)(d')$, it holds that
- a. there is a subinterval I' of I, (non-atomicity)
- b. for every subinterval I' of I, there is a subinterval I'' of I such that I' and I'' do not overlap, and (supplementation)
- c. every subinterval I' of I is an element of $[\![er]\!](d)(d')$. (homogeneity)

Failure of exhaustification. The LF of the sentence in (60-a) is provided in (64). The alternatives to the sister of *exh* are provided in (65) (we assume that there is no universal quantifier alternative to *any* in this configuration, though this is not crucial).

(64) [exh_R [$\Diamond$ [any_D [er Max_d Mary is d-tall] [Max_d John is d-tall]]]]

(65) ALT([$\Diamond$ [any_D [er Max_d Mary is d-tall] [Max_d John is d-tall]]]) =
$\{[\Diamond\ [\text{any}_{D'}$ [er Max_d Mary is d-tall] [Max_d John is d-tall]]] $\mid [\![D']\!] \subseteq [\![D]\!]\}$

As in the case of mass *any*-DPs, we assume that the domain of *any* is divisive, as provided in (66) (it is non-atomic due to the intervals having to be non-degenerate, and homogeneity and supplementation hold due to density).

(66) $D = \{I \mid I \text{ is an interval}\}$

Accordingly, it follows that there are no maximal sets of excludable alternatives in (65). The reasoning is identical as in Section 3: Assume that such a maximal set of excludable alternatives, E, exists. There must exist an alternative, A, whose domain of *any* contains only intervals that do not overlap with any of the intervals in the domains of *any* in the alternatives in E (the sentence can only be true if there is an interval for which it is possible that it is between Mary's and John's heights). However, if such an alternative exists, E cannot be maximal: E can be expanded with an alternative, A', whose domain of *any*

consists of an interval, I, that partially overlaps with an interval, I′, in the domain of *any* in A. Negating this alternative, together with all the alternatives in E, is still compatible with it being possible that there exists an interval between Mary's and John's heights (in particular, the complement of the interval I′ in I). Consequently, since no maximal sets of excludable alternatives exists in (66), all the alternatives are Innocently Excludable – and thus no alternatives are Innocently Includable. Since free choice inferences cannot be generated, sentence (57) is unacceptable.

Summary. To summarize, we identified another occurrence of *any* for which we argued that its domain of quantification is divisive and that combines with a homogeneous predicate: differential *any* in comparative constructions. In this case, the domain consisted of (non-degenerate) intervals on a dense set of degrees. Accordingly, exhaustification that was meant to yield free choice failed, resulting in the *any* being unacceptable.

5 Variation

NPIs differ in what distributional patterns they exhibit. Relevantly for the purposes of this note, there are expressions like *ever* whose distribution matches that of count *any*-DPs in non-modal environments, as in (67), but not in existential modal environments, as in (68).

(67) a. #(I doubt that) John read any book.
b. #(I doubt that) John ever read a book.

(68) a. John is allowed to read any book.
b. #John is allowed to ever read a book.

On the other hand, its distribution matches that of mass *any*-DPs across the board. (The exception is that *ever* does not allow for a coerced construal described in Section 3.2.)

(69) a. #(I doubt that) John drank any water.
b. #John is allowed to drink any water.

Chierchia (2013) attributes this state of affairs to what kind of exhaustification operator associates with *any*-DPs vs. *ever* (recursive weak *exh* vs. non-recursive weak *exh*, which is determined by the feature specifications of the NPIs). A different way of fleshing out Chierchia's underlying intuition becomes possible, however, in light of the above discussion, a way that capitalizes more directly on the parallelism between *ever* and mass *any*-DPs: the unacceptability of *ever* in (69) may be analyzed as a consequence of how the properties of the domain of *ever*, which resemble those of mass *any*-DPs, affect exhaustification.

Interval semantics. Following Hamblin (1969, 1972) and others (see van Benthem 1983, Landman 1991 for reviews), we adopt interval semantics for tense, and assume that the domain of intervals is divisive (in particular, non-atomic).[11] *Ever* is an existential quantifier

[11]Note that this assumption has a number of implications. First, it implies that time is dense. Second, the domain of intervals cannot contain degenerate intervals, that is, intervals $[t, t]$ that contain only a single

over a domain of such intervals. For example, sentence (70-a) has the meaning in (70-b), which can be derived by defining *ever* as in (71). (For simplicity, we assume that *ever* applies above tense, and we do not represent tense explicitly in the LFs in the following.)

(70) a. John didn't ever_{D} arrive.
b. $\neg\exists$I (I$\in$D $\wedge$ I<now $\wedge$ John arrived at I)

(71) $[\![\text{ever}_{\text{D}}]\!](\text{P}_{((\text{it})\text{t})}) = 1$ iff $\exists$I (I$\in$D $\wedge$ P(I))

The domain of *ever* is provided in (72). (The parentheses in (72) indicate that it may be restricted to a salient interval, though see Krifka 1995 for reasons against such a restriction.)

(72) $\text{D} = \{\text{I} \mid \text{I is an interval } (\wedge\ \text{I} \subseteq \text{I}_{\text{c}})\}$

An existential modal sentence with *ever* may have the structure in (73), where the set of the subdomain alternatives to the sentence is provided in (74).

(73) a. #John is allowed to ever arrive.
b. $[\text{exh}_{\text{R}}\ [\Diamond\ [\text{ever}_{\text{D}}\ [\text{John arrive}]]]]$

(74) $\text{ALT}([\Diamond\ [\text{ever}_{\text{D}}\ [\text{John arrive}]]]) =$
$\{[\Diamond\ [\text{ever}_{\text{D}'}\ [\text{John arrive}]]], [\Diamond\ [\text{always}_{\text{D}'}\ [\text{John arrive}]]] \mid [\![\text{D}']\!] \subseteq [\![\text{D}]\!]\}$

No maximal sets of excludable alternatives. There are no maximal subsets of excludable alternatives in (74). The reason behind this parallels the one behind why there can be no maximal sets of excludable alternatives in examples with mass *any*-DPs in existential modal sentences. For any time interval of which a (modalized) predicate holds, the predicate also holds of all the subintervals of that interval. And it is possible that the predicate hold of a proper subinterval of that interval, while not holding of its complement in that interval. Accordingly, since not all the alternatives in (74) distinct from the sentence may be excluded, there must be a interval that is in none of the domains of *ever* in the subdomain alternatives – with none of its subintervals being in them either. But one can identify a subinterval of this interval such that it is possible that the predicate holds of it, but not its complement. This means that every potential maximal set of excludable alternatives can always be expanded by another alternative. Thus, all alternatives are Innocently Excludable, resulting in the failure of exhaustification, and so in the occurrence of *ever* being unacceptable.

Semantics and variation. We discussed a narrow range of NPIs: mass and count *any*-DPs, *any* differentials, and *ever* – that is, so-called weak NPIs. We showed that the variation among them in distribution can be accounted for without making any assumptions other than independently needed (and argued for) ones about the nature of their domains of quantification. The fact that the distribution of NPIs is adequately captured on them accordingly

point of time t, since degenerate intervals don't have proper parts. This in turn means that the domain of intervals cannot contain pairs of open and closed intervals with the same boundaries: any such pair, e.g. the pair $[\text{t}, \text{t}']$ and $[\text{t}, \text{t}')$, would violate supplementation. All of this is in accord with our previous assumptions about intervals. See footnote 9.

provides tentative support for them. This is summarized in (75): weak NPIs can be licensed, or rescued, by *exh* (depending on the specific theory of NPI licensing, cf. Chierchia 2013, Crnič 2019) – unless the NPIs have a divisive domain.

(75) **Environments and weak NPIs (predictions)**

a. Domains non-divisive – downward-entailing env. ✓ existential modal env. ✓

b. Domains divisive – downward-entailing env. ✓ existential modal env. ✗

6 Conclusion and outlook

Any-DPs with mass noun complements and *any* differentials are unacceptable in existential modal sentences, in contrast to their counterparts with count noun complements. We showed that this behavior can be derived from, and thus supports, (i) the assumption that *any*-DPs are exhaustified in these sentences and (ii) the assumptions that mass nouns are divisive (e.g., Bunt 1979, 1985, Link 1983, Landman 1991) and that measure predicates form a dense domain (cf., e.g., Fox & Hackl 2006). Subsequently, we showed how one can approach the variation in distribution among (weak) NPIs: the differences in the meanings of NPIs, together with how these interact with exhaustification, yield the distribution profiles we observe, specifically, the differences in acceptability in existential modal sentences (see Chierchia 2013, Crnič 2019 for a review of some other cases). We conclude by pointing to three issues that we leave for future research. We also leave comparison of the above proposal with potential alternatives that would rely on different assumptions about mass, degree, and tense semantics to future work (Crnič 2020b).

6.1 Obviation by universal modals

While one is not able to strengthen unembbeded measure phrase comparatives, as repeated in (76), strengthening is possible when these are embedded in the scope of a universal modal (though not an existential one). For example, the sentence in (77) may convey that for no weight greater than 80 kg, John is required to weigh more than it.

(76) John weighs more than 80 kg.

$\not\rightsquigarrow$ ¬John weighs more than 81 kg.

(77) John is required to weigh more than 80 kg to compete in this category.

$\rightsquigarrow$ ¬John is required to weigh more than 81kg to compete in this category.

Fox & Hackl (2006) observe that in cases like these, exhaustification is predicted to yield a consistent interpretation: namely, the negation of all stronger alternatives to the sentence is consistent with the sentence, given in (78).[12] Thus, failure of exhaustification is obviated.

[12]It is crucial to assume that either (i) existential modal sentences do not count as alternatives in these cases (see Bar-Lev & Fox 2019 for that assumption) or (ii) that they can be considered irrelevant at the level that should be blind to at least some contextual information. Otherwise we fail to obtain maximal sets of excludable alternatives. What contextual information can enter into exhaustification at different levels of representation remains to be studied. The reader is referred to Fox & Hackl 2006 for a discussion.

(78) a. [exh$_{\mathrm{R}}$ [□ [-er than 80 kg] [λd John weighs d-much]]]
b. □(John weighs more than 80 kg) ∧ ∀d>80 kg: ¬□(John weighs more than d)

This may *prima facie* give rise to an expectation that a similar obviation effect should be observable with mass *any*-DPs and *any* differentials in universal modal environments. There are independent reasons, however, why such an obviation cannot be observed.

Universal modals and *any*. *Any*-DPs are unacceptable in the scope of universal modals. This is expected on the extant approaches to *any*-DPs utilizing exhaustification (cf. Chierchia 2013, Crnič 2017, 2019 for two types of derivations).

(79) #John is required to read any book.

Accordingly, even if exhaustification would lead to obviation of exhaustification failure in examples with mass *any*-DPs and *any* differentials, the respective structures would yield pathological interpretations, leaving *any*-DPs unacceptable.

Possible exception. There is a class of attitude predicates in the scope of which *any*-DPs are judged to be acceptable, though only in very specific contexts and with specific intonational patterns. These include desire predicates like *wish, want,* and *hope* (see Kadmon & Landman 1993, Giannakidou 1999, among others). An example from Kadmon & Landman is provided in (80): it is felicitious if used in a context in which getting a ticket, any ticket, is desirable but perceived to have been unlikely. It turns out that mass *any*-DPs, *any* differentials, and *ever* have a similar distribution in these environments and are subject to similar contextual constraints. Two examples are provided in (81). (We signal the variation in acceptability of the NPIs across speakers and contexts with '%'.)

(80) %I am glad we got any tickets.

(81) a. %I hope there is any food left in the fridge.
b. %We hoped that we would be any better off after the manouever.
c. %The driver would be happy to ever hit 100 mph.

If these predicates indeed have a Strawson monotone semantics (e.g., von Fintel 1999), the pattern would fit naturally in the paradigm observed for measure phrase comparatives; it would, however, require a tweaked free choice analysis of *any*-DPs under universal modals. Rather than pursuing such an analysis here, we merely point out that the acceptability of *any*-DPs in (80) and (81) may be due to construals other than free choice ones. Namely, as discussed by Crnič (2013), NPIs that do not admit a free choice construal are acceptable in these environments as well, and they give rise to similar inferences about the context; two examples are provided in (82). Accordingly, whatever is responsible for the acceptability of the sentences in (82) may also be responsible for the acceptability of the sentences in (80)-(81) – and it might not crucially depend on a free choice inferences. Unfortunately, a further pursuit of these issues is beyond the scope of the current note.

(82) a. %John hopes that he will ever have time to read *War and Peace*.
b. %I am glad that even ONE person came to my party.

6.2 Partitives

Partitive mass *any*-DPs are acceptable in existential modal sentences:

(83) You are allowed to drink any of the beer.

This is not expected on what we presented above, all else equal. We point to two interrelated facts that may feature in the explanation of this fact. One is that partitives have been claimed to allow for a covert head, as the first argument of the quantificational determiner. The nature of this head, which is perhaps underspecified with respect to mass/count, may allow for the exhaustification to yield a consistent output (see, e.g., Chierchia 1998a).

(84) a. any of the water
b. [any [PART [of the MASS water]]]

The other fact is that the Homogeneity Constraint, which played a crucial role in deriving exhaustification failures, appears to hold less strictly when it comes to partitives. A pertinent example found online is provided in (85), in which a mass quantifier combines with a non-homogeneous predicate (cf. also Crnič 2010). While the obviation of the Homogeneity Constraint might be very restricted, the possibility of a consistent parse on one choice of lexical material would in principle suffice for acceptability in the architectural setup underlying the above discussion (cf. Gajewski 2002, Fox & Hackl 2006, Chierchia 2013).

(85) Analyses indicate that some of the water contains more than 1.5 ppm of fluoride.

6.3 Subtrigging

Another area in which one finds obviation effects are mass *any*-DPs that are modified by a relative clause. This is illustrated in (86). The episodic counterpart of (86) in which the modal is dropped is also acceptable, as shown in (87).

(86) ?John was allowed to drink any water that was on the counter.

(87) ?John drank any water that was on the counter.

We submit that the occurrences of *any* in (86)-(87) are of the same type, namely, the so-called 'subtrigged' occurrences of *any* (e.g., Dayal 2004, 2009, Aloni 2007a, Chierchia 2013). The acceptability of subbtrigged *any* is still ill-understood, though it is clear that the ingredients involved in its licensing are somewhat different from those involved in the free choice examples above (for instance, there is no existential modality in that allows for a free choice construal of disjunction). We hope that a better understanding of subtrigging will introduce paths towards a better understanding of the facts exemplified in (86)-(87) as well.

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