# Mereology

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## 1 Notions of parthood

**Mereology** (from Greek *μερος* 'part') is the study of **parthood**, an important notion in ontology, natural-language semantics and cognitive sciences. Examples include rice as part of paella, a thumb as part of a hand, John Lennon as part of the Beatles, the Adriatic Sea as part of the Mediterranean, the battle of Issos as part of Alexander's war against Persia, November 1989 as part of the year 1989, or *hood* as part of the word *parthood*. These examples illustrate that the relation may hold between masses, objects, objects and groups, locations, events, times, and abstract entities (see Simons 1987, Winston e.a. 1986 for classification of parthood relations).

One important distinction is between what we may call **structured** and **unstructured** parthood. Structured parts are cognitively salient parts of the whole. The thumb is a structured part of a hand: it is one of *the parts* of the hand, besides the fingers and the palm. The hand is itself a cognitively salient, integrated whole and not just a random collection of parts. Similarly, *hood* is a structured part of the word *parthood*: it is a suffix with a meaning of its own, attached to the stem *part*, and contributing to the overall meaning of the word. The notion of unstructured parthood, in contrast, can be illustrated with the water in the left half of a cup as part of the water in the entire cup, or the letter sequence *artho* as part of the sequence *parthood*. Unstructured parts need not be cognitively salient parts of the whole, but may slice up the whole in arbitrary ways. For discussion of related philosophical issues, see Simons (1987), Moltmann (1997, 2005) and Varzi (2010).

The two part relations have different structural properties. Structured parthood is not transitive. The thumb is a structured part of the hand, and the hand is a structured part of the arm, but the thumb is not a structured part of the arm. Linguistic evidence for this comes from the fact that we can say *the thumb of this hand*, or *a hand without thumb*, in contrast to *\*the thumb of this arm*, or *\*an arm without thumb*. In contrast, unstructured parthood is transitive: e.g., the sequence *rth* is part of *artho*, and *artho* is part of *parthood*, hence *rth* is part of *parthood*. For unstructured parts, we can use *part* as a mass noun (x is *part* of y); for structured parts, as a count noun (x is *a part* of y). For unstructured parts of rigid entities that hang together, we may also use the word *piece* (cf. Cruse 1986). As a consequence of transitivity, unstructured parts may overlap, in contrast to structured parts. For example, *art* and *tho* are both unstructured parts of *parthood*.

In this article, we will focus on unstructured part relations. This is not to deny that structured part relations are of great importance in natural-language semantics. There are two such structured part relations: **meronymy** (the relation between meronyms like *hand* and holonyms like *arm*), and **hyponymy** (the relation between hyponyms like *poodle* and hyperonyms like *dog*, in English expressed by words like *kind* or *type*). These relations structure large parts of the lexicons of natural languages, and they have certain general properties (cf. e.g. Kay 1975 for taxonomies, based on the

hyponymy relation, and Cruse 1986 for a textbook treatment). But they interact less with compositional semantics than the unstructured parthood relation, on which we concentrate here. In this, we make no claim about the semantics of the English word *part* (see Rescher 1955 for differences between different notions of parthood).

We will take the parthood relation to be reflexive, which is at variance with how English *part* is used. We will write ≤ for parthood, and distinguish it from irreflexive <, which will be called **proper** parthood.

If we consider parthood for entities extended in space or time, we may restrict the notion of part to **contiguous** parts, e.g. strings like *art* or *ood,* or we may allow for non-contiguous parts, such as discontinuous strings like *pa---od* or *p-r-h-o*, as parts of *parthood.* Contiguity is a topological notion, and from topology we also can derive notions like **borders** or **interior parts** of objects. The formal notion of parthood typically is understood in a way as to allow for non-contiguous parts. Even structured parts can be non-contiguous; for example, the fingers are considered part of the hand, and circumfixes and infixes lead to non-contiguous morphological parts of words. (For combinations of mereology and topology, cf. Smith 1996, Casati and Varzi 1999, Forrest 2010).

Parthood relations have played an important role in philosophy, especially ontology (cf. Simons 1987, Varzi 2010 for overviews). Yet the formal study of parthood relations took off in the foundations of mathematics, as an alternative to set theory. Set theory is based on two parthood relations, element ∈ and subset ⊆, and a corresponding type-theoretic distinction (if α ⊆ β, then α and β must be sets; if α ∈ β, then just β is required to be a set). Consequently, set theory distinguishes between singleton sets and their elements (a ≠ {a}), and assumes an empty set. Mereology was proposed by Leśniewski (1916, see Simons 2011) and Leonard & Goodman (1940) as a simpler alternative without such assumptions. It does not distinguish between elementhood and subsethood, and it does not assume abstract entities like sets. Consequently, it does not distinguish between singleton sets and their elements, and does not entertain the notion of the empty set.

This article will be concerned specifically with the applications that mereology has found in natural-language semantics. After a section on essential formal properties of the (unstructured) parthood relations and their axiomatization we will deal with linguistic applications in the nominal domain, in the expression of measurement functions and in the verbal domain.

# 2   Axiomatizations of Mereology

The notion of parthood has been captured with various axioms. The most congenial approach to mereology is to take the notion of **part** as central; this gives us an order-theoretic perspective. There is another approach that starts with the notion of **sum**; this is the lattice-theoretic perspective.

## 2.1  The order-theoretic perspective

We present an axiomatization of the parthood relation known as classical extensional mereology (CEM). This system is the one most commonly used in natural language semantics, and comes closest to a standard in mereology.[2] We take reflexive, unstructured parthood (≤) as the primitive relation and formulate axioms that impose constraints on it. The following axioms constrain parthood to be a partial order:

(1)     Axiom of reflexivity: $\forall x[x \leq x]$

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2   The discussion in this section is based on Champollion (2010) and on the excellent surveys of mereology in Simons (1987), Casati and Varzi (1999) and Varzi (2010).

(2)    Axiom of transitivity: $\forall x \forall y \forall z[x \leq y \wedge y \leq z \rightarrow x \leq z]$

(3)    Axiom of antisymmetry: $\forall x \forall y[x \leq y \wedge y \leq x \rightarrow x = y]$

We often want to talk of objects that share parts. For this purpose, we introduce the auxiliary concept of **overlap**, which we write as $\circ$, following Simons (1987) and Link (1998).

(4)    Definition: Overlap: $x \circ y \Leftrightarrow \exists z[z \leq x \wedge z \leq y]$

With the part relation, we can define the notion of **sum**, also called **fusion**. Sums formally capture the pretheoretical concept of collection — that which you get when you put several parts together. In natural-language semantics, two common applications of sums are conjoined terms and definite descriptions. For example, Link (1983) proposes to represent the denotation of the conjoined term *John and Mary* as the sum of the individual John with the individual Mary.[3] Sharvy (1980) suggests representing the denotation of a definite description like *the water* as the sum of all water. We come back to definite descriptions in Section 3.2.

As discussed in Hovda (2009), there are several ways to define sum, and while they are equivalent given the CEM axiom system, they are not logically equivalent. This means that when putting together the axioms, it is important to choose the definition carefully to make sure that the axioms that use it lead to CEM as intended. The definitions below are metalanguage statements about shorthand expansions, that is, they indicate what formula "sum(x,P)" is a shorthand for. Intuitively, this shorthand stands for "x is a sum of (the things in) P".[4]

The first definition of sum is due to Tarski (1929).

(5)    Definition: Sum (1): $\text{sum}(x, P) =_{\text{def}} \forall y[P(y) \rightarrow y \leq x] \wedge \forall z[z \leq x \rightarrow \exists z' [P(z') \wedge z \circ z']]$
       (A sum of a set P is a thing that consists of everything in P and whose parts each overlap with something in P.)

The second definition appears in Simons (1987, p. 37) and Casati and Varzi (1999, p. 46):

(6)    Definition: Sum (2): $\text{sum}(x, P) =_{\text{def}} \forall y[y \circ x \leftrightarrow \exists z[y \circ z \wedge P(z)]]$
       (A sum of a set P is a thing such that everything which overlaps with it also overlaps with something in P , and vice versa.)

The following fact follows from either definition of sum:

(7)    Fact: $\forall x[\text{sum}(x, \{x\})]$

The proof is immediate if sum is defined as in (5). If sum is defined as in (6), we can rewrite (7) as $x \leq x \wedge \forall z[z \leq x \rightarrow z \circ x]$. The first conjunct follows from reflexivity (1); the second follows from the fact that parthood is a special case of overlap, which in turn follows from reflexivity.

Different mereology systems disagree on what kinds of collections have a sum, and whether it is possible for one and the same collection to have more than one sum. In CEM, sums are unique, therefore two things composed of the same parts are identical. This is expressed by the following axiom:

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3   See Winter (2001) for a dissenting view. For Winter, *and* is always translated as generalized conjunction, in this case as conjunction of two generalized quantifiers, and the equivalent of sum formation enters the system through a collectivization operator on the resulting quantifier.

4   If we formulate our theory in first-order logic, whichever one of these two definitions we pick must be interpreted as an axiom schema, or a list of axioms in each of which P(y) is instantiated by an arbitrary first-order predicate that may contain y as a free variable. If we formulate our theory in second-order logic, we can instead understand the definition we pick as quantifying over a predicate variable P, which may be interpreted as ranging over sets, as suggested by our paraphrases. The set interpretation makes the theory more powerful (Pontow and Schubert, 2006; Varzi, 2010). The main issue here is that there are only countably many formulas, but uncountably many sets. In the following, we will talk about sets rather than axiom schemas.

(8)    Axiom of uniqueness of sums $\forall P[P \neq \varnothing \rightarrow \exists! z\ sum(z,P)]$

The operators in (9) and (10) give us a way to refer explicitly to the sum of two things, and to the sum of an arbitrary set.

(9)    Definition Binary sum: $x \oplus y = \iota z\ sum(z, \{x, y\})$.

For example, the meaning of the term *John and Mary* can be written as $j \oplus m$.

(10)   Definition Generalized sum: For any nonempty set P, its sum $\oplus P$ is defined as $\iota z\ sum(z,P)$.

For example, the meaning of the term *the water* can be written as $\oplus water$.[5]

If Tarski's definition of sum (5) is used, CEM is defined by the axioms of reflexivity (1), transitivity (2) and antisymmetry (3) taken together with uniqueness of sums (8). In fact, this setup makes axioms (1) and (3) redundant, because any transitive relation that satisfies axiom (8) is provably reflexive and antisymmetric. If the definition of sum (6) is used,  transitivity and uniqueness of sums are not sufficient to define CEM, as there is a model of these axioms in which we have one element that is not part of itself. Adding reflexivity and transitivity rules out this model but still does not exclude certain more complex models which are not models of CEM. Tarski's definition of sum is therefore to be preferred. For discussion and proofs of these facts, see Hovda (2009).

The properties of  parthood described by CEM are very similar to those of  subsethood  in standard set theory. More specifically, one can prove that the powerset of any given set, with the empty set removed, and with the partial order given by the subset relation, satisfies the axioms of CEM. The empty set must be removed because it is a subset of every other set (a "bottom element"), but CEM precludes anything from being part of everything. For practical purposes one can therefore often regard sums as sets, parthood as subsethood, and sum formation as union. Some correspondences between CEM and set theory are listed in Table (11). Readers who are unfamiliar with mereology might find this table useful to strengthen their intuitions about the properties of the parthood relation and of the other operations in CEM.

(11)   Correspondences between CEM and set theory

|   | Property | CEM | Set Theory |
|---|---|---|---|
| 1 | Reflexivity | $x \leq x$ | $x \subseteq x$ |
| 2 | Transivity | $x \leq y \wedge y \leq z \rightarrow x \leq z$ | $x \subseteq y \wedge y \subseteq z \rightarrow x \subseteq z$ |
| 3 | Antisymmetry | $x \leq y \wedge y \leq x \rightarrow x = y$ | $x \subseteq y \wedge y \subseteq x \rightarrow x = y$ |
| 4 | Interdefinability | $x \leq y \leftrightarrow x \oplus y = y$ | $x \subseteq y \leftrightarrow x \cup y = y$ |
| 5 | Unique sum/union | $P \neq \varnothing \rightarrow \exists! z\ sum(z,P)$ | $\exists! z\ z = \cup P$ |
| 6 | Associativity | $x \oplus (y \oplus z) = (x \oplus y) \oplus z$ | $x \cup (y \cup z) = (x \cup y) \cup z$ |
| 7 | Commutativity | $x \oplus y = y \oplus x$ | $x \cup y = y \cup x$ |
| 8 | Idempotence | $x \oplus x = x$ | $x \cup x = x$ |
| 9 | Unique separation | $x < y \rightarrow \exists! z[x \oplus z = y \wedge \neg x \circ z]$ | $x \subset y \rightarrow \exists! z[z = y - x]$ |

## 2.2  The algebraic (lattice-theoretical) perspective

Above, we have developed the notion of mereology starting out from the notion of parthood, deriving from it the notion of sum, or fusion. In the most succinct formulation, Properties 2 (transitivity) and 5 (unique sum) in Table (11) are considered axioms, and the other properties follow from them

5   The semantic literature following Sharvy (1980) often uses the equivalent notation $\sigma x\ P(x)$ rather than $\oplus P$. We use $\oplus$ to make the connection between binary and generalized sum clearer.

as theorems. Alternatively, we may start out with the sum operation, and derive from that the part relation.

That is, we may start out with the notion of a **lattice**, or to be specific, a **join semi-lattice**. This is defined as a structure ⟨L, ⊕⟩, where L is a set, and ⊕ is a two-place operation on this set called **join** (full lattices also have an operation of **meet**, ⊗). For each two elements x, y ∈ L, the join x⊕y is defined, and x⊕y ∈ L. By definition, the join operation of a lattice meets properties 6 (associativity), 7 (commutativity) and 8 (idempotence). **(**For full lattices with a meet operation, the law of **absorption** holds: x⊕(x⊗y) = x, x⊗(x⊕y) = x). If we furthermore impose property 9 (unique separation), the join operation has the same properties as the sum operation in mereology. A lattice in which unique separation holds is called **complemented.** Given unique separation, if we define parthood in terms of join as in property 4, then the properties of reflexivity (1), transitivity (2) and antisymmetry (3) follow as theorems.

In order to qualify for a mereology, the join semi-lattice must not have a bottom element. This entails that there is no general meet operation. But we still can define a restricted meet of x⊗y = z as the largest element such that $z \leq x$ and $z \leq y$, provided that there is such an element.

Unique separation plays an important role in excluding non-intended models of parthood such as in (12)i below. (The models in (12) show the parthood relation exhaustively. The elements that form the leaves of these models – a, b, c in (12)i, for example – are atoms, and they are pairwise disjoint.) Intuitively, we want a⊕b to consist of a and b and nothing else which is not already part of a and b. To see that unique separation is violated in (12)i, notice that a is a proper part of d, and that there is more than one thing x disjoint from a such that a⊕x=d. Specifically, a⊕b = a⊕c = d. Structures like (12)i have "too many" parts. Structures like (12)ii and (12)iii, on the other hand, have "too few" parts; in both of them, b is a proper part of c, but there is no other proper part of c. Such structures are excluded by unique separation as well, as we have b < c, but there is no x disjoint from b such that b⊕x=c. If we have three atomic elements, a, b, and c, then the only lattice with three atomic elements a, b, c satisfying commutativity, associativity, idempotence, and unique separation is (12)iv. Note that (12)iv is isomorphic to the powerset of the set {a, b, c} minus the empty set, where parthood corresponds to the subset relation. This isomorphism is the reason for the correspondences in Table (11) above.
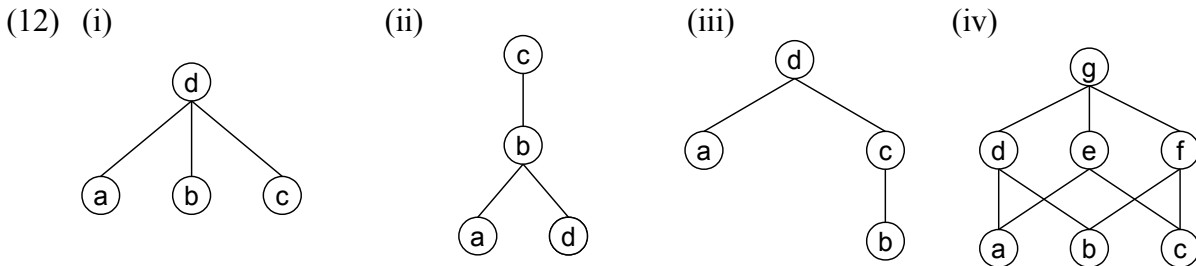
(12) (i)　　　　　　　(ii)　　　　　(iii)　　　　　(iv)



The role that unique separation plays in eliminating (12)i, (12)ii, (12)iii is played by distributivity in full lattices, i.e. lattices with a bottom element (where distributivity, here for sets, states that x∪(y∩z) = (x∪y)∩(x∪z)). .

Structure (12)iv would also be identified as the lattice that is generated by the atoms a, b, c, where the join operation satisfies commutativity, associativity, and idempotence, does not express equalities that are not required by these laws (such as a⊕b = a⊕c in (12)i) and does not entertain any differences not required by these laws (such as c≠b in (12)ii and (12)iii). Such structures are called **free lattices** that are **generated** by a set of atoms (see Landman 1991).

The notion of join as introduced above entails that the join of any finite subset of L is uniquely defined. That is, for any non-empty finite subset L′ ⊆ L, the elements of L′ form a unique join,

which is the minimal upper bound of L′ (the smallest x such that for all a∈L′, a ≤ x). This is because by commutativity, associativity, and idempotence, the order of joining two elements, of more than two elements, and of joining the same element repeatedly does not matter. We can write ⊕L′ for this minimal upper bound, corresponding to the generalized sum operation in the previous section. As a consequence, every finite lattice is **bounded**, in the sense that there is an y ∈ L such that for all x ∈ L it holds that x ≤ y. However, this does not guarantee that join is defined for infinite sets. The existence of a unique minimal upper bound for infinite sets would have to be stipulated explicitly. The resulting structure is often called a **complete join semilattice**, where completeness implies that all subsets of L have a join; but recall that the empty set has no join, as this would require a bottom element. The structure can also be described as a complete lattice with the bottom element removed, or equivalently, as a complete Boolean algebra with the bottom element removed.

The algebraic conception of a lattice generates similar structures as the order-theoretic conception; a fundamental theorem of lattice theory shows that the two coincide (cf. Grätzer 1967, Landman 1991). The algebraic perspective is motivated by the fact that natural language allows for a conjunction of terms, expressed by *and*, which is to be modeled by the join operation. As this conjunction can only generate phrases of finite length, we might not even want to require that is infinite. However, if the join operation is used to define the part relation, and the part relation in turn is used to render the meaning of definite descriptions, then a finite join operation is not sufficient to cover terms like *the natural numbers*, whose nouns apply to an infinite number of entities.

Other comparable systems are the logic of plurality defined by Link (1983, 1998), the part-of structures of Landman (1989, 1991, 2000), and the lattice sorts and part structures of Krifka (1990, 1998). These systems are clearly intended to describe CEM. For example, Link (1983) and Landman (1989) explicitly argue that modeling reference to plurals requires systems with the power of a complete join semilattice. However, careful review shows that many of these axiomatizations contain errors or ambiguities and therefore fail to characterize CEM as intended (Hovda, 2009).

## 2.3  Mereology versus set theory, and the issue of atomicity

In the wake of Massey (1976) and Link (1983), most theories of plurals and mass nouns have been formulated in a mereological framework. Given the correspondences in Table (11), working directly with set theory instead of mereology might look like a better choice. After all, set theory is better known and more generally accepted than mereology. Indeed, early approaches to plural semantics adopted set theory (Hausser, 1974; Bennett, 1974).

So what are the reasons for preferring mereology? Some authors, such as Link (1983), have advanced philosophical reasons: Following the philosophical doctrine of "composition as identity", they take an ontological commitment to sums to be less substantive than a commitment to sets, because a sum is taken to be nothing over and above its parts, as opposed to a set. Related to this, sets are abstract, but sums are concrete if their parts are concrete. This corresponds to how we conceive of sum individuals: the plural individual consisting of these three apples has a certain weight, and is situated in space – properties that sets, as abstract entities, arguably lack.

However, others, like Landman (1989), have argued that these reasons are independent of linguistic considerations, and that sets can be used as a **model** of sum individuals. In order to do so, mereological atoms need to be identified with the singleton sets that contain them (Scha, 1981; Schwarzschild, 1996).

With ordinary set theory, we are forced to assume that everything is ultimately composed of **atoms**, represented by singleton sets:[6]

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6   Alternatively, one can provide room for non-atomic structures in set theory by modifying the standard axioms so as

(13)  Definition Atom: $atom(x) \Leftrightarrow \neg\exists y[y < x]$
      (An atom is something which has no proper parts.)

However, atomistic mereologies may cause problems for the modeling of events, mass entities, and spatiotemporal intervals, where one does not want to be forced to assume the existence of atoms.

By themselves, the axioms of CEM do not specify whether atoms exist or not: among the models they describe, there are some in which everything is made up of atoms (in particular, this includes all models that contain finitely many elements), some in which there are no atoms at all, and intermediate cases. Either (14) or (15) can be added to CEM to constrain it to one of the two limiting cases. For example, many authors assume that the count domain is constrained to be atomic. The referents of proper names and the entities in the denotations of singular count nouns are then taken to be mereological atoms.

(14)  Optional axiom Atomicity: $\forall y\exists x[atom(x) \wedge x \leq y]$
      (All things are made up of atoms.)

(15)  Optional axiom Atomlessness: $\forall x\exists y[y < x]$
      (Everything is infinitely divisible.)

When neither of these axioms is added, the system remains underdetermined with respect to whether or not atoms exist. This underdetermination is one of the advantages of mereology: When we describe the domains of space, time, and mass substances, we do not need to decide whether atomic events and atomic instants of time exist or whether mass substances can be infinitely subdivided.

A different kind of atoms, called impure atoms or groups, is used in Landman (1989, 2000) and elsewhere to model certain instances of collective predication and group nouns like *committee* or *team*. Groups are atomic entities which are derived from sums via a special group formation relation or operator. Group formation introduces a distinction between the sum a⊕b whose proper parts are the individuals a and b, and the group that corresponds to this sum. This group has a and b as "members", but it has no proper parts. Groups can be seen as adding a notion of structured parthood into mereology that is otherwise absent, and formally speaking, the group formation operation is not a part of CEM but added on to it. Landman's view on group nouns is influential but not uncontested; other authors assume that group nouns involve reference to set- or sum-like pluralities (Bennett 1974), that they involve sum formation of discourse referents instead of objects (Krifka 1991), or that it is not possible to recover the membership of a group from the denotation of a group noun (Barker 1992). Pearson (2011) points out that group nouns as a class are heterogeneous: some of them (e.g. *bunch, collection*) behave less like *committee* and more like measure nouns (Section 3.2).


# 3   Linguistic Applications


## 3.1  Nominal Domain

One of the principal applications of mereology in semantics is the characterization of oppositions such as **count-mass** and **singular-plural** in terms of higher-order properties. For example, various authors have identified the properties of mass and plural terms like *gold* and *horses* with the notions of **cumulative reference**: if two things are gold then their sum is also gold (Quine, 1960), and if you add some horses to some other horses then you again get some horses (Link, 1983).

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to allow infinitely descending sequences of sets, as in the ensemble theory of Bunt (1985).

(16)  Definition Cumulative reference: $CUM(P) \Leftrightarrow \forall x \forall y[P(x) \wedge P(y) \rightarrow P(x \oplus y)]$
       (A predicate P is cumulative if and only if whenever it holds of two things, it also holds of their sum.)

As an example, consider diagram (17)(A); the gray circles form a cumulative set.

(17)



Diagram A                          Diagram B
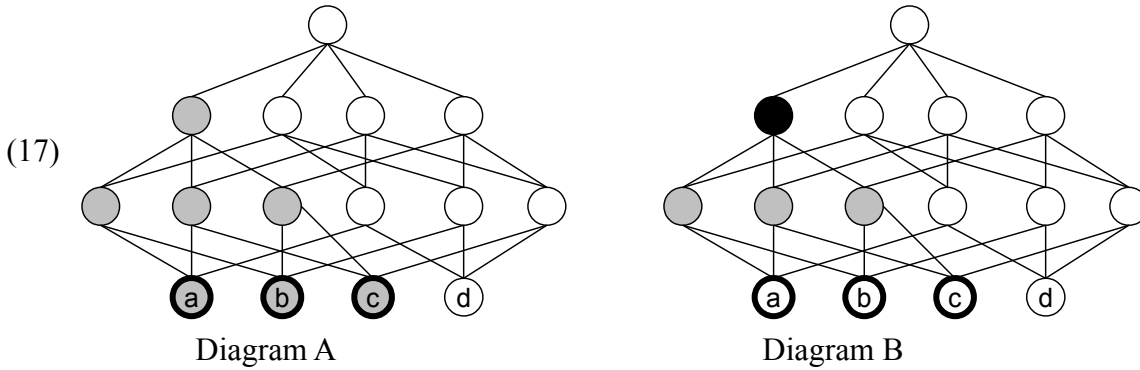
Cumulativity can be generalized for infinite sets as follows:

(18)  Definition Cumulative Reference: $CUM(P) \Leftrightarrow \forall P'[P' \neq \varnothing \wedge P' \subseteq P \rightarrow \oplus P' \in P]$
       (A set is cumulative iff it contains the sums of all of its nonempty subsets.)

The dual of cumulative reference, **divisive reference**, has also been proposed to describe mass terms (Cheng, 1973). While cumulative reference "looks upward" from the part to the sum, divisive reference "looks downward" from the sum to the parts. On the view that mass term like *gold* have divisive reference, this means that any part of anything which is gold is also gold.

(19)  Definition Divisive reference: $DIV(P) \Leftrightarrow \forall x[P(x) \rightarrow \forall y[y < x \rightarrow P(y)]]$
       (A predicate P is divisive if and only if whenever it holds of something, it also holds of each of its proper parts.)

The assumption that divisive reference holds of mass denotations runs into the **minimal-parts problem**. If we want it to apply to mass nouns like *water*, divisivity is at odds with chemistry, as a hydrogen atom as part of an $H_2O$ molecule does not count as water. The problem of divisivity is even more obvious with heterogeneous mass terms like *fruit cake* (e.g. Taylor, 1977): a portion of fruit cake may contain sultanas, but these sultanas do not themselves qualify as fruit cake. Most semanticists accept that mass nouns do not in general have divisive reference (Gillon, 1992). Many authors have modified the definition of divisive reference to avoid the minimal-parts problem, e.g. by adding a "granularity parameter" that prevents it from applying from parts that are lower than a certain threshold (Champollion, 2010). Landman (2011) proposed a further distinction beyond atomicity: **mess nouns** like *lemonade* have overlapping atoms, while the atoms of neat nouns like *furniture* do not overlap; this cuts across the grammatical mass/count distinction, as e.g. *fence* is a messy count noun.

There is another property that has been proposed to model semantic properties of nominals, **quantized reference** (cf. Krifka 1989):

(20)  Definition Quantized reference: $QUA(P) \Leftrightarrow \forall x[P(x) \rightarrow \forall y[y < x \rightarrow \neg P(y)]]$
       (A predicate P is quantized if and only if whenever it holds of something, it does not hold of any of its proper parts.)

For example, the fat circles in (17)(B) form a quantized set, and so do the gray circles. Singular count nouns are interpreted as quantized sets. If we consider count nouns as applying to atoms, quantization follows vacuously; if not, quantization seems to be an essential property (as e.g. a proper part of a chair is not a chair). However, there are count nouns like *twig* or *sequence* for

which quantization is a problematic assumption (Zucchi & White 2001): A part of a twig may also be a twig, and a part of a sequence may also be a sequence. One possible line of response is that the meaning of these nouns is partially specified by context, and that when the context is fixed, each of them denotes a quantized set (Chierchia 2010, Rothstein 2010).

Given the denotation of a singular count noun, the denotation of the corresponding plural count noun can be described by **algebraic closure**, often represented by the star operator (Link, 1983). Algebraic closure extends a predicate P so that whenever it applies to a set of things individually, it also applies to their sum.

(21)   Definition Algebraic closure: If set P is nonempty, $*P = \{x | \exists P' \subseteq P[x = \oplus P']\}$.
        (The algebraic closure of a set P is the set that contains any sum of things taken from P.)

As an example, suppose the set $C = \{a, b, c\}$ is the set of all cats. Then C represents the meaning of the singular count noun *cat*. The algebraic closure of C is written $*C$ and is the set $\{a, b, c, a \oplus b, b \oplus c, a \oplus c, a \oplus b \oplus c\}$. This set contains everything that is a cat or a sum consisting of two or more cats, so it is a superset of C. Also, it is a cumulative set; whenever $x, y \in C$, then $x \oplus y \in C$. In Diagram (17)(A), the set of gray circles is the algebraic closure of the set of fat circles.

Other definitions of algebraic closure are also sometimes found in the literature. For example, Sternefeld (1998) renders Link's definition as follows:

(22)   Alternative definition Algebraic closure:
        For any set P, $*P$ is the smallest set such that
        a) $P \subseteq *P$
        b) If $a \in *P$ and $b \in *P$, then $a \oplus b \in *P$.

This reformulation is equivalent to the one in (21) for finite P, but the equivalence breaks down when P has countably infinite cardinality. Definition (21) amounts to powerset formation; by Cantor's diagonal argument, the powerset of a countably infinite set is uncountably infinite. Definition (22), by contrast, amounts to forming the union of countably many countably infinite sets. Given the axiom of choice, this union is itself only countably infinite. So given a countably infinite P such as *natural number*, the cardinality of $*P$ will vary depending on the definition.

In most mereological approaches to the semantics of count nouns, a plural count noun denotes the algebraic closure of the set denoted by its singular form. There is disagreement on whether the denotation of a plural noun also contains entities denoted by its singular form (Farkas and de Swart 2010). On the **exclusive** view, singular and plural forms of a count noun denote disjoint sets (Link, 1983; Chierchia, 1998a). For example, if in (17)(A) the set $\{a, b, c\}$ forms the denotation of *cat*, then the four gray circles without bold lining form the denotation of *cats*. The plural form *cats* essentially means *two or more cats*. On the **inclusive** view, the plural form of a count noun denotes its algebraic closure (Krifka, 1986; Sauerland, 2003; Sauerland et al., 2005; Chierchia, 2010). In (17)(A) this is the set of all gray circles. The plural form essentially means *one or more cats*. The inclusive view assumes that when singular reference is intended, singular and plural forms are in pragmatic competition, and the more specific singular form blocks the the plural form. There are good arguments against the exclusive view (cf. Schwarzschild 1996, page 5). For example, suppose that there is only one doctor. On the exclusive view, the plural form *doctors* denotes the empty set. Therefore, the exclusive view cannot derive the meanings of sentences like (23) compositionally without resorting to devices such as intensionality.

(23)   No doctors are in the room.

A more tricky problem is presented by dependent plurals, i.e. bare plurals which are c-commanded by a plural coargument (de Mey, 1981).

(24) Boys / Some boys / Several boys / Five boys flew kites.

These sentences have a reading which entails that each of the boys in question flew one or more kites (which is a problem for the exclusive view), and that the total number of kites flown was two or more (which is unaccounted for by the inclusive view). A possible solution, which is argued for by Zweig (2009), may be to take the inclusive view and to treat the inference that at least two kites were flown in total as a grammaticalized scalar implicature.

Mass nouns generally do not show a singular/plural contrast, except if coerced to count nouns, as in *we ordered three beers*. In English, they typically occur in the singular, although some plural nouns have been argued to be mass (e.g. *clothes,* cf. Ojeda 2005). If mass nouns are intrinsically cumulative, then plural formation has no function, as it does not change their denotation – if CUM(P), then P = *P (Chierchia, 1998b). Proof: Assume CUM(P); as it always holds that P ⊆ *P we just have to show that *P ⊆ P. Assume to the contrary that *P ⊄ P, that is, there is an a, a ∈ *P and a ∉ P. Then there is, following (21), a P′ with P′ ⊆ P and a = ⊕P′. But as P is cumulative, we have ⊕P′ ∈ P, following (18). Hence a ∈ P, contrary to assumption. — However, there are cases in which mass nouns can be pluralized with special meaning effects, e.g. in Greek for expressing large quantities (Tsoulas 2008).

We end this section by pointing out that bare plural and mass terms also can be used to refer to kinds, as in *Potatoes were first cultivated in South America*, or *Nylon was invented in 1935.* For further reading, see Krifka et al. 1995; Chierchia 1998b; Krifka 2004; Delfitto 2005; Cohen 2007; Lasersohn 2010; and Doetjes 2012.

## 3.2 Measure constructions

We now turn to constructions like *three liters of milk* or *two cats.* Such expressions refer to quantized sets; no proper part of an entity that falls under *three liters of milk* also falls under *three liters of milk*, and no proper part of an entity that falls under *two cats* should fall under *two cats.* We can derive the quantized status of *two cats* from the properties of its parts. We introduce the following operator:

(25) Definition Atomic number: atoms(x) = card({y | y ≤ x ∧ atom(y)})
     (The atomic number of x is the cardinality of the set of atoms that are part of x.)

With this, we can interpret *two cats* as {x | *cat(x) ∧ atoms(x) = 2}, which can be shown to be a quantized set, assuming that *cat* only applies to atoms. This set is represented by the gray circles in Diagram (17)(B), if the set of cats is {a, b, c}. However, this kind of representation is not useful for pseudopartitive expressions based on measure nouns and mass nouns, like *three liters of water*. Here, we assume a **measure function** that stands in a systematic relation to parthood. Measure functions can be thought as mapping substances to positive real numbers (Krifka, 1989), in which case *three liters of water* may be represented as {x | water(x) ∧ liters(x) = 3}. Alternatively, measure functions map substances to degrees which are in turn mapped to numbers by what we may call **unit functions**, in which case *three liters of water* may be represented as {x | water(x) ∧ liters(volume(x)) = 3} (Lønning, 1987). While we will use the former representation for convenience, the latter one has advantages: for example, *three inches of water* may be represented either using inches(depth(x)) or inches(diameter(x)) depending on context. For other advantages, see Schwarzschild (2002).

Not every measure function is permissible in a pseudopartitive; while *three liters of water* is admissible, *\*three degrees Celsius of water* is not. This constraint is related to the distinction between extensive and intensive measure functions (Krifka 1998). In physics and measurement theory, an extensive measure function is one whose magnitude is **additive** (Krantz et al. 1971; Mills et al.

2007). For example, when one considers the water in a tank, *liters* is an extensive measure function because the water as a whole measures more liters than the volume of any of its proper parts. But *degrees* is not extensive because the temperature in degrees of the water as a whole is no different from the temperature of its proper parts.

One way of making this notion precise is the following:

(26)   A measure function μ is extensive on a set S ⊆ dom(μ) iff:
$$\forall x[x \in S \rightarrow \forall y[y < x \land y \in \text{dom}(\mu) \rightarrow \mu(y) < \mu(x)]]$$
(For every proper part of every element in the set, the function returns a lower value than for that element.)

Not all measure functions used in pseudopartitives are extensive in this strict sense. If two feet of snow fell on West Berlin and two feet of snow fell on East Berlin, then two feet, and not four feet, fell on Berlin as a whole. So the notion of extensivity needs to be appropriately relativized. Intuitively, the underlying measure function *height* is still extensive as long as we restrict ourselves to layers of snow piled on one another rather than adjacent layers. (Schwarzschild, 2006; Champollion, 2010).

Given that the measure function *liters* is extensive on the set of all water quantities, we can show that the denotation of *three liters of water,* {x | water(x) ∧ liters(x) = 3} denotes a quantized set. Proof: Take an x in the set and a proper part y, y < x. Then either ¬water(y), in which case y cannot be in the set; or water(y), in which case, since *liters* is extensive, we have liters(y) < 3, so y cannot be in the set.

We find the definite article in expressions like *the two cats*, or *the three liters of water.* It has been suggested to interpret the definite article as generalized sum (e.g., Sharvy, 1980). This works well for cumulative sets; e.g. in the model above, *the cats* would refer to ⊕cat = a⊕b⊕c, the sum of all cats, represented by the black circle in Diagram (17)(B). However, this would not work in the case of quantized sets; for example, *the two cats* should not be defined in the model above, but ⊕{x | x∈*cat ∧ atoms(x) = 2} is defined, and would also refer to the black circle in Diagram (17)(B). Following a proposal of Montague (1973), we can alternatively interpret the definite article as referring to the supremum of a set (cf. Krifka 1986):

(27)   Definition Supremum of a set P: sup(P) = ιz [P(z) ∧ ∀x[P(x) → x ≤ z]]
(The supremum of a set P is the single element z in P that is greater or equal than any of its elements)

Many sets will not have a supremum. For a cumulative set like *C, the supremum sup(*C) exists, and is identical to ⊕*C. For a non-cumulative set like {x | x∈*C ∧ atoms(x) = 2}, the supremum exists only if it is a singleton set, and then refers to the element in this set – hence, only if there are exactly two cats. This captures the intuitive meaning of *the two cats*, and the proposal can be extended to *the cat* as well, if *cat* applies to atomic cat entities; the expression will be defined only if there is exactly one cat, and will refer to that cat.

The notion of measure function in a mereological setting has been fruitful to explain interesting linguistic phenomena. Krifka (1986) has suggested that count nouns in English like *cat* do not refer to atoms, but contain a built-in measure function, called "natural unit". In this they differ from count nouns in languages like Chinese, which require a classifier. Krifka (1990) and Doetjes and Honcoop (1997) have analyzed sentences like *4000 ships passed through the lock*, on the reading where any given ship may pass through the lock many times, as involving the construction of a complex additive measure function for events.

## 3.3 Verbal Domain

Mereological notions have been applied to predicates to model certain cases of conjunction, as in *the girls sang and danced*, which is true if some of the girls sang and the other girls danced. Link (1983) proposed to lift sum formation to predicates. Take *S and *D to be the set of singers and dancers; then *sang and danced* denotes the set $\{x \oplus y \mid x \in {}^*S \land y \in {}^*D\}$, the set of individuals consisting of singers x and dancers y. Krifka (1990) has extended this to other types, for example to prepositions, to handle cases like *the planes flew above and below the clouds*.

The notion of parthood has been used to formally reconstruct the telic-atelic opposition. Mereology makes it possible to formally relate this opposition to the singular-plural and count-mass oppositions (Krifka, 1986; Bach, 1986; Champollion, 2010). Stating generalizations across nouns and verbs is easiest if these categories both denote sets, so it is convenient to assume that verbs denote sets of events (the neo-Davidsonian position, Parsons, 1990). A reformulation of mereological applications to the verbal domain into eventless semantics is possible in most cases, though not always conspicuous in practice (Bayer, 1997).

Divisive reference has been used to formally capture the notion of atelicity. A verbal predicate like *run* is **atelic**, or according to Vendler (1957), an activity; it has the property that "any part of the process is of the same nature as the whole". In a neo-Davidsonian event semantics, *run* will be interpreted as a set of events R. Now Vendler's characterization can be restated by claiming that activities, and other atelic predicates like states (e.g. *be happy*), have divisive reference: Let $e \in R$ be an event of John running from his house to the store. Now e will have parts, e.g. a part $e' \leq e$ in which John runs from his house halfway to the store, and a part $e'' \leq e$ in which he runs from the halfway point all the way to the store. These parts $e'$ and $e''$ will themselves be running events, and hence be elements of R. As with mass nouns, divisivity is limited by the minimal parts problem; there may be certain parts of e that are too small to count as running. A notion that is similar to divisive reference is the subinterval property, which holds of any predicate P just in case whenever P holds at a temporal interval, it also holds at every subinterval of it (Bennett and Partee, 1978, Dowty, 1979). Cumulativity and nonquantization are also occasionally invoked to model atelicity. The differences between these various notions are investigated in Champollion (2010).

Constructions such as *run for two hours* can be seen as the verbal equivalent of pseudopartitives (cf. *two hours of running)*. Restricting our attention to events that cannot be executed simultaneously, like runnings by one agent, we find that $\{e \mid e \in R \land hours(e) = 2\}$ is a quantized set, if *hours* is an extensive measure function. The verbal predicate *run for two hours* then is **telic**, an accomplishment; other examples of telic predicates are accomplishments like *run a mile* and *build a house* and achievements like *reach the summit*. Telic predicates imply, according to Vendler (1957), "definite and unique time periods" and may be characterized using quantized reference and related notions (Krifka, 1998). Filip (2008) analyses perfectivity as a grammatical operation that expresses telicity, and involves maximalization, similar to the definite article in the nominal domain.

There is a correspondence between the parts of an event and the parts of the participants of an event, which has been expressed in Krifka (1986, 1992) in terms of a homomorphism-like relation. For example, let e be the event in which John (j) lifts a certain box b, hence j is the agent and b is the theme of e, which we note as $AG(e) = j$ and $TH(e) = b$. Similarly, assume that $e'$ is the event in which Mary (m) lifts a certain table t. We assume that AG and TH have the homomorphic property called "summativity" or "cumulativity" in Krifka 1986, 1989; Landman 2000:

(28)   Definition: cumulative thematic relation $\Theta$: $\Theta(e \oplus e') = \Theta(e) \oplus \Theta(e')$
       (The $\Theta$-participant of the sum of two events is the sum of the $\Theta$-participants of the two events.)

Let L be the set of lifting events; we then have $e \in L \wedge AG(e) = j \wedge TH(e) = b$ (*John lift the box*) and $e' \in L \wedge AG(e') = m \wedge TH(e') = t$ (*Mary lift the table*). We then also have $e \oplus e' \in L \wedge AG(e \oplus e') = j \oplus m \wedge TH(e \oplus e') = b \oplus t$, i.e., we can derive the truth of *John and Mary lifted the box and the table*. One condition, of course, is that L is cumulative. Such inferences motivate the assumption of **lexical cumulativity** (called "lexical" because it is taken to apply to all verbs, but not to all verb phrases): Whenever two events are in the denotation of a verb, so is their sum (cf. Scha, 1981; Schein, 1986, 1993; Lasersohn, 1989; Krifka, 1986, 1992; Landman, 1996, 2000; Kratzer, 2007). In this property, verbs are similar to mass nouns or bare plurals.

But there are problems with cumulativity of thematic relations. Suppose that there are three events $e_1$, $e_2$, $e_3$ in which Al dug a hole, Bill inserted a rosebush in it, and Carl covered the rosebush with soil. Then one can say that there is also an event $e_4$ in which Al, Bill, and Charles planted a rosebush. Do we consider $e_4$ equal to the proper sum event $e_1 \oplus e_2 \oplus e_3$? If we do, this scenario is a counterexample to the cumulativity assumption (Kratzer, 2003). The themes of $e_1$, $e_2$, $e_3$ are the hole, the rosebush, and the soil, and the theme of $e_4$ is just the rosebush. The theme of $e_4$ is not the sum of the themes of $e_1$, $e_2$, and $e_3$, violating cumulativity. A possible objection is that $e_4$ is not actually the sum of $e_1$, $e_2$, and $e_3$. Even though the existence of $e_4$ can be traced back to the occurrence of $e_1$, $e_2$, and $e_3$, nothing forces us to assume that these three events are mereological parts of $e_4$ (for general discussion and other solutions, see also Williams 2009 and Piñón 2011).

It has been observed (e.g., Verkuyl 1972) that mereological properties of the arguments or adjuncts of verbs have an impact on mereological properties of complex verbal constructions. For example, *drink milk* is atelic, whereas *drink two liters of milk* is telic. This can be explained on the basis of general properties of thematic relations. We show that *drink milk* is cumulative, and therefore atelic. Assume that D and M are cumulative; then the set $\{e \mid D(e) \wedge \exists x[M(x) \wedge TH(e)=x]\}$ is cumulative, too. Proof: Assume that $e_1$, $e_2$ are two events in this set; this means that there are $x_1$, $x_2$ such that $M(x_1)$, $M(x_2)$ and $TH(e_1) = x_1$, $TH(e_2) = x_2$. As M, D and TH are cumulative, $e_1 \oplus e_2$ is also in this set. Hence *drink milk* denotes a cumulative set. To show that *drink two liters of milk* is quantized, and therefore telic, we have to assume that certain thematic roles have the additional property we may call Distinctiveness (cf. also the notion of incremental theme in Dowty 1991):

(29)  Definition Distinctiveness of $\Theta$: If $e \neq e'$, then $\Theta(e) \neq \Theta(e')$
      'Distinct events have distinct $\Theta$-participants.'

This applies, for example, to the theme of *drink*: Two distinct drinking events will have two distinct entities that are drunk. Proof of quantization of *drink two liters of milk*: Assume that 2LM is quantized, and assume that $e_1$ is an event in $\{e \mid D(e) \wedge \exists x[2LM(x) \wedge TH(e)=x]\}$. Hence there is an $x_1$ such that $2LM(x_1)$ and $TH(e_1)=x_1$. Assume now a proper part $e_2$ of $e_1$, that is, $e_2 < e_1$, hence $e_2 \neq e_1$. By distinctiveness, there is an $x_2$ such that $TH(e_2) = x_2$ and $x_2 \neq x_1$, and by cumulativity, $x_2 \leq x_1$, hence $x_2 < x_1$. As 2LM is quantized, we have $\neg 2LM(x_2)$, but then also $e_2 \notin e_1$; hence the set $\{e \mid D(e) \wedge \exists x[2LM(x) \wedge TH(e)=x]\}$ is quantized, too.

The notion of homomorphism inherent in lexical cumulativity, (28), also applies to **trace functions**, which map events to those entities (e.g. intervals) that represent their temporal and spatial locations (Krifka, 1986). Trace functions have been used for many purposes; among others, they relate event semantics to interval semantics; they are involved in the denotation of adverbials like *for an hour* and *to the store*; and they play the same role as measure functions in the analysis of pseudopartitives such as *three hours of running*. Commonly, two such functions are assumed: temporal trace or runtime ($\tau$) and spatial trace or location ($\sigma$). For example, if $e_1$ and $e_2$ are two events, the runtime of their sum, $\tau(e_1 \oplus e_2)$, is the sum of their runtimes, $\tau(e_1) \oplus \tau(e_2)$. For more on temporal and spatial trace functions, see Hinrichs (1985), Lasersohn (1988), Krifka (1998), Link (1998), as well as Zwarts (2005, 2006), where the spatial and temporal trace functions are combined into one.

The telic-atelic opposition, as well as the singular-plural and count-mass oppositions, can be formally related to the collective-distributive opposition (Champollion, 2010). Distributive predicates require any event in their denotation to be divisible into events that are atomic with respect to the appropriate thematic role. For example, any plural event in the denotation of the distributive predicate *smile* must be divisible into parts that have atomic agents and that belong to the denotation of the same predicate. This captures the distributive entailment from *John and Mary smiled* to *John smiled*. By contrast, the subjects of collective predicates like *be numerous* or *gather* can be plural entities whose parts do not themselves satisfy the predicate. Formally, the subinterval property can be modified and generalized so that it covers both the property of being atelic and the property of being distributive. Instead of requiring a predicate that holds at a certain interval to hold at every subinterval, we can require a predicate that holds of a certain plural individual to hold of every atomic part of that plural individual.

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