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Non-Convex Sets are Needed for Comparatives: A Note on Schwarzschild and Wilkinson (2002)*

Uli Sauerland
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Abstract Schwarzschild and Wilkinson (2002, *Natural Language Semantics*) propose a semantics of degree based on intervals. We show that degrees are predicted to be non-convex sets for two classes of examples by Schwarzschild and Wilkinson's semantics.

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1 Introduction

Schwarzschild and Wilkinson (2002) propose a new semantics for comparatives based on what they refer to as a *semantics of degree based on* ***intervals*** [my emphasis] in their paper. Intervals are usually understood as convex subsets of a totally ordered set where convex is defined as follows:

(1) S is *convex* iff. $\forall x, y \in S\colon \forall z\colon (x \leq z \leq y \rightarrow z \in S)$

In fact, a number of researchers have understood Schwarzschild and Wilkinson's proposal to entail that degrees are convex sets (Heim 2006, Beck 2007, van Rooij 2007). My point in this note is in some sense terminological: I point out that Schwarzschild and Wilkinson should have more properly spoken of a semantics of degree based on **sets** rather than intervals. To do this, I show that there are two kinds of examples of comparatives where the proposal of Schwarzschild and Wilkinson requires degree expressions to denote non-convex sets, one of which Schwarzschild and Wilkinson already point out in their paper. The relevant examples make this point more generally rather than just for the specific system of Schwarzschild and Wilkinson: if one assumes that quantifiers cannot scope out of comparative clauses, one must also allow sets that are not convex in degree-semantics. This in turn argues that Schwarzschild and Wilkinson's proposal cannot be reduced to granularity in scales as I discuss in Section 3 below in more detail.

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2 Non-Convex Sets Instead of Intervals

Schwarzschild and Wilkinson propose that the degrees are subsets of some ordered set that satisfies their persistence and overlap principles (p. 17–18) and possibly additional axioms. For concreteness, I assume that the relevant scale for heights would be the set of non-negative real numbers $\mathbb{R}_0^+$, though nothing hinges on this. If all degrees are convex, any comparative clause must denote a convex subset of $\mathbb{R}_0^+$, i.e. there need to be two values l and u from the non-negative reals numbers or ∞ such that the denotation of the comparative clause is one of the intervals [l,u], [l,u), (l,u], and (l,u).

Schwarzschild and Wilkinson (page 28, figure 3) point out that the following example requires non-convex sets as a degree. This point probably generalizes

(2) Two inches (tall)er than one of the boys is.[1]

Now, I show that also the examples (3) and (4) (the literal German translation of (3)) can only receive a satisfactory account within Schwarzschild and Wilkinson's theory if the set of degrees also includes non-convex sets. Consider the following scenarios: Ekaterina is on a volleyball team. She is 206 cm tall. If her teammates are 193cm, 187cm, 181cm, 177cm, and 163cm tall, (3) and (4) are true. But if her teammates are 193cm, 186cm, 181cm, 177cm, and 163cm tall, both examples are false since one of her teammates is an even number of centimeters shorter than her. Similarly, (3) and (4) are false if her teammates are 194cm, 187cm, 181cm, 177cm, and 163cm tall or 193cm, 187cm, 180cm, 177cm, and 163cm or 193cm, 187cm, 181cm, 178cm, and 163cm, or 193cm, 187cm, 181cm, 177cm, and 162cm. We see that the exact height of each of Ekaterina's teammates is important for the truth of (3) and (4), not just that of the shortest or tallest of them.

(3) Ekaterina is an odd number of centimeters taller than each of her teammates.

(4) Ekaterina ist eine ungerade Zahl von Zentimetern größer als jede
Ekaterina is an odd number of centimeters taller than each
ihrer Mannschaftskamerad-in-nen (GERMAN)
her teammate-FEM-GEN.PL

Schwarzschild and Wilkinson's account of (3) assumes an LF-structure like that in (5):

(5) $\text{more}(\underbrace{\text{an odd number of centimeters}}_{\text{Diff}})$

$(\underbrace{\lambda i\ .\ \text{than each of her teammates is } i\text{-tall}}_{\text{Q}})(\underbrace{\lambda d\ .\ \text{Ekaterina is } i\text{-tall}}_{\text{P}})$

1 In the published paper, the example reads *Two indices (tall)er than one of the boys is.* However, the word *indices* is clearly a typographical error here.

In the following, I use the abbreviations indicated in (6): P for the matrix clause, Q for the subordinate clause, and Diff for the differential phrase. Schwarzschild and Wilkinson's (p. 26) lexical entry for *more* is the following:

(6) $[\![\text{more}]\!]$(Diff)(Q)(P) = 1 iff. P(μi . (Q(μk . Diff($i-k$))))

This formula relies on the following set of additional definitions. To define $i-k$, we make use of the induced partial order for sets of real numbers defined by the following:

$$i \leq j \text{ iff. } \forall d \in i \colon \forall d' \in j \colon d \leq d'$$

With this, $i-k$ can be defined as the following (cf. Schwarzschild and Wilkinson, p. 16):

$$i - k = \{d \mid k < \{d\} < i\}$$

For the μ operator, Heim (p. 25) argues that one should use the following definition, rather than the one Schwarzschild and Wilkinson provide.[2]

$$\begin{aligned} \mu i.\phi(i) \quad &= \iota i. \quad \forall i'(i' \neq \emptyset \wedge i' \subseteq i \Rightarrow \phi(i')) \\ &\qquad \wedge \forall i''(i \subset i'' \Rightarrow \exists i'(i' \neq \emptyset \wedge i' \subset i'' \wedge \neg\phi(i'))) \end{aligned}$$

The lexical entry for *tall* is given in (7):

(7) $[\![\text{tall}]\!]$(i)(x) = 1 iff. the exact height of x is an element of i

Now consider again the first scenario introduced above: Ekaterina is 206 cm tall, her teammates are 193cm, 187cm, 181cm, 177cm, and 163cm tall. In this case, we have the following equivalences for P and Q:

(8) P(i) = 1 iff. $206 \in i$

(9) Q(i) = 1 iff. $\{193, 187, 181, 177, 163\} \subseteq i$

Finally, look at the contribution of the differential. Before looking at *an odd number of centimeters*, consider the two easier examples in (10):

(10) a. Ekaterina is taller than each of her teammates.
b. Ekaterina is exactly 23cm taller than each of her teammates.

Following Schwarzschild and Wilkinson, I assume that a silent SOME fills the differential position in (10a) so that in both examples in (10) a differential is present. For the differentials, I assume the definitions in (11).[3]

2 For my present purposes, the result actually isn't affected by the choice of Heim's version of the μ-operator. The version of Schwarzschild and Wilkinson derives exactly the same result.

3 Schwarzschild and Wilkinson (p. 16) build a notion of granularity into the definition of SOME. As far as I can see, nothing in the following hinges on my use of a simplified entry of SOME.

(11) a. SOME(j) = 1 iff. $j \neq \emptyset$

b. $[\![$exactly 23cm$]\!]$(j) = 1 iff. $\sup(j) - \inf(j) = 23$

Recall for (11b) that the *infimum* $\inf(j)$ (also called *greatest lower bound* of j) is the biggest real number d with $\{d\} \leq j$ and the *supremum* $\sup(j)$ (also called *least upper bound*) the smallest real number d with $j \leq \{d\}$.

Example (10a) then is true in the above scenario, as shown by the following calculation:

$$\begin{aligned} \mathrm{P}(\mu i.(\mathrm{Q}(\mu k.\mathrm{SOME}(i-k)))) &= \mathrm{P}(\mu i.(\mathrm{Q}(\mu k.i > k))) \\ &= \mathrm{P}(\mu i.(\mathrm{Q}([0, \inf(i)] \setminus i))) \\ &= \mathrm{P}((193, \infty)) \qquad = 1 \end{aligned}$$

And the following calculation illustrates how Schwarzschild and Wilkinson predict (10b) to be false in the given scenario. Here it is important that μ requires all non-empty subsets of its argument to also satisfy its scope.

$$\begin{aligned} \mathrm{P}(\mu i \quad . &\quad (\mathrm{Q}(\mu k.[\![\text{exactly } 23]\!](i-k)))) \\ = &\quad \mathrm{P}(\mu i.(\mathrm{Q}(\mu k.\sup(j) - \inf(k) = 23))) \\ = &\quad \mathrm{P}(\mu i.(\mathrm{Q}(\{\sup(j) - 23\}))) \\ = &\quad 0 \end{aligned}$$

Now consider again example (3) repeated here in (12).

(12) Ekaterina is an odd number of centimeters taller than each of her teammates.

We need to define a lexical meaning for the differential *an odd number of centimeters* such that (5) (repeated in (13)) receives the right interpretation?

(13) $[\![$more$]\!]$(Diff)(Q)(P) = 1 iff. P(μi . (Q(μk . Diff($i-k$))))

In analogy to (10b), we define the following:

(14) $[\![$an odd number of centimeters$]\!]$(j) = 1 iff. $\sup(j) - \inf(j)$ is an odd integer

I use the shorthand ODD for this property. ODD is incompatible with a restriction to intervals because the μ-operator is not defined in many cases of i and j we need to consider to compute (13). For example, the expression μk . ODD($\{206\} - k$) is not defined because both $k = \{193\}$ and $k = \{191\}$ satisfy the conditions of the μ. But, clearly this is an important case where μ should result in a value. This difficulty is solved, however, if we do not restrict μ to intervals, but allow all sets. Then neither of the two k just mentioned satisfies the conditions μ-imposes, since their union $k' = \{191, 193\}$ also satisfies the requirement of μ that each subset k'' of k' satisfy $\mathrm{ODD}(\{206\} - k)$. Importantly, k' is not an interval, but a non-convex set.

To see that ODD results in the desired meaning for (13) once the interval restriction is given up, first observe that (15) holds.

(15) $\mu k.\mathrm{ODD}(i - k) = \{\inf(i) - 1, \inf(i) - 3, \inf(i) - 5, \ldots\} \cap \mathbb{R}_0^+$

The value of Q, we are considering has the property that there is a supreme point that satisfies Q; the height of the second tallest player on the team. We define:

$$S = \sup(\{d \mid Q(\{d\}))$$

For cases where such an S exists,[4] we can now compute:

$$\begin{aligned} \mathrm{P}(\mu i. \quad & (\mathrm{Q}(\mu k.\mathrm{ODD}(i - k)))) \\ = \quad & \mathrm{P}(\mu i.(\mathrm{Q}(\{\inf(i) - 1, \inf(i) - 3, \inf(i) - 5, \ldots\} \cap \mathbb{R}_0^+ \\ = \quad & \begin{cases} 0 & \text{if } \mathrm{Q}(\{S, S-2, S-4, \ldots\}) = 0 \\ \mathrm{P}(\{S+1, S+3, S+5, \ldots\}) & \text{otherwise} \end{cases} \end{aligned}$$

This result shows that a set-based semantics of degrees successfully predicts the desired truth-conditions for (3): For (3) to be true, on the one hand all the other teammates must be an even number of centimeters shorter than the tallest teammate, since otherwise $\mathrm{Q}(\{S, S-2, S-4, \ldots\}) = 0$, and on the other hand, Ekaterina must be an odd number of centimeters taller than her tallest teammate since otherwise $\mathrm{P}(\{S+1, S+3, S+5, \ldots\}) = 0$.

3 Conclusion

In conclusion, I have shown that Schwarzschild and Wilkinson's semantics of comparatives makes the right prediction for (3) if non-convex sets are possible degrees as Schwarzschild and Wilkinson assume. If, however, degrees must be convex two classes of examples do not receive the right semantics: examples with an indefinite in the comparative clause like (2) and examples with an indefinite differential like (3). Once any set is allowed as a degree, however, the system of Schwarzschild and Wilkinson makes the right prediction for both types of examples.

This result takes on new importance in the light of recent attempts to extend and/or simplify Schwarzschild and Wilkinson's analysis. Specifically, both Beck (2007) and van Rooij (2007) have explored in talks the idea that granularity introduces intervals into the semantics of degrees. Granularity allows us to lump together

4 And I leave the other cases for future research.

an interval of a scale and regard it essentially as one point (see also Krifka 2007, Sauerland and Stateva 2007). The granularity intervals can be compared with others by simply looking at their infimum and supremum points. However, this approach will almost certainly make wrong predictions for examples like (3) where it is clear that more than two points must be taken into consideration – the heights of each of the teammates. I conclude, therefore, that appeal to granularity alone is unlikely to be sufficient to explain the use of sets in degree semantics.

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