The Human Capacity for Measurement with a Unit

1. Introduction

Is Frodo a hobbit or a human? Compared to Frodo, is Sam tall? How much taller is Gandalf than Sam? Is he more than twice as tall as the adventurous hobbit? An animal that can ponder these questions is an animal full of measurement. This paper is about the mental life of one such animal, the human animal. What is involved in mentally representing thoughts that require measurement? What is unique to humans in the kind of measurement-invoking thoughts that they have? To address these question, we need some understanding of what measurement is.

In general terms, measurement is about finding structure in a set of objects (or events). It is about associating a set of objects whose structure we wish to understand with a set of formal objects, typically numbers, whose structure we understand. Underlying each measurement is a scale and measurement scales differ in how much information they provide about the objects being measured. Stevens (1946) distinguishes between four types of measurement scales: nominal, ordinal, interval and ratio. These scale types can be classified into those that involve a unit of measurement (interval and ratio) and those that do not (nominal and ordinal). In this paper, we rely on this distinction in order to explicate a hypothesis about the nature of human concepts. More specifically, we suggest that *humans and only humans mentally represent concepts that involve measurement scales that make use of a unit (i.e. interval and ratio scales).*

In what follows, we first provide evidence for nominal and ordinal concepts in non-human animals (henceforth: animals) and in humans. We then show that the capacity for measurement with a unit presupposes having internalized a quantificational grammar. If, as Bolender and colleagues (Bolender, 2016; Bolender et al., 2008) suggest, quantificational grammar is a distinctive aspect of human cognition, then it follows as a corollary that the cognitive capacity for measurement with a unit is also a uniquely human trait. We provide linguistic evidence for the significance of scales with units (i.e. ordinal and ratio scales) for human concepts.

We finally turn to social cognition in order to test this uniqueness claim. Fiske (1991; 1992) has developed a theory of human social-relational cognition that has measurement scales as its formal core. There is a one-to-one relation between Steven's (1946) measurement scales and Fiske's relational models. If mentally representing scales with units is a uniquely human trait, as we claim it is, then those relational models that require measurement with a unit (i.e. *Equality Matching* and *Market Pricing*, which presuppose an interval scale and a ratio scale respectively) must be limited to human sociality. This conjecture is supported by what has been reported in the relevant literature. In other words, relational social cognition in humans provides independent evidence for the claim that measurement with a unit is a distinguishing aspect of human cognition.

2. Nominal Concepts in Apes and in Humans

In the simplest case, measurement is about sorting; it is about partitioning a set of entities into exclusive classes. Looking at various living organisms, we might need to decide for each of them whether it is an animal, a plant, or neither. Looking at various animals, we might need to decide for each of them whether it is cold-blooded or warm-blooded. Each such decision involves the implementation of a type of measurement scale called *a nominal scale*. The object under measurement is an X or it is not an X; this is what such a scale is designed to tell us.

There is experimental and anecdotal evidence that animals can categorize objects into classes. For some animals, this takes the form of stimulus generalization, whereby a type of behavior towards a stimulus is generalized to objects that have physical/perceptual similarity to the stimulus. In a famous experiment, Herrnstein et al. (1976) showed that pigeons can be trained to respond selectively to novel pictures containing trees, bodies of water, or a specific woman in different settings. Dretske (1999) discusses some birds that distinguish between different kinds of butterflies and learn to avoid eating the members of the Monarch (sub-)family, which are typically poisonous to them. Some otherwise edible species of butterflies (e.g. Viceroy) have developed a type of deceptive coloration called mimicry, thanks to which they show a striking resemblance to Monarch and avoid being eaten. In other words, some butterflies have taken advantage of the fact that stimulus generalization by birds relies on physical similarity.

It seems that non-human great apes (henceforth: apes) can classify entities in ways that go beyond physical/perceptual similarity. There is experimental evidence that apes can group similar-looking objects into distinct classes. Hanus et al. (2011) tested whether apes could spontaneously solve *the floating peanut test*, in which they were expected to retrieve a peanut from inside a Plexiglas tube by raising the level of water inside the tube. To do so, they had to collect water from the water dispenser placed within one meter from the tube and spat the water they filled their mouths with into the tube so that they could bring the floating peanut within their reach. While the apes (19 chimpanzees and 5 gorillas) located in the Leipzig Zoo all failed in the task, five of the 25 chimpanzees tested in Ngamba Island Chimpanzee Sanctuary in Uganda completed the task successfully. Four of the chimpanzees in the latter experiment ended up adding water into the tube but not enough to obtain the peanut. In an attempt to understand the discrepancy between these experimental results, Hanus et al. (2011) hypothesized that the apes in the Leipzig group were fixated in treating the water dispenser as a tool for quenching their thirst, a restriction on cognition called *functional fixedness*. Consequently, they failed to see that the water from the dispenser could be put into another use. In a third experiment with 19 chimpanzees in the Leipzig Zoo, 16 of whom were also the participants of the first experiment, a new dispenser was added to the experimental setting, which the chimpanzees had never used before. In this case, two of the chimpanzees obtained the reward and five of them spat into the tube but failed to complete task. In other words, as would be expected from functional-fixedness interpretation of the result in the first experiment, adding a new dispenser to the experiment did bring the success of the chimpanzees in the Leipzig group closer to that of the Ngamba group.

What is important for us is that some of the chimpanzees were able to associate distinct functions (eliminating thirst and collecting water) with similar-looking objects (water dispensers). We suggest that they interpreted these two objects as distinct kinds of things. We can formally represent this idea by making use of a measure relation, μ, which we shall index with the word "kind", i.e. μ_{kind}, to express the fact that it relates objects to their kinds.[1] This relation holds between objects, here WD1 = Water_Dispenser-1 and WD2 = Water_Dispenser-2, and the kind specifications associated with these objects. The symbolic thoughts represented by the successful Leipzig apes can be shown as in (1).

(1) a. μ_{kind}(WD1, kind_7), where kind_7 is a "DRINK WATER"-kind[2]

b. μ_{kind}(WD2, kind_{12}), where kind_{12} is a "COLLECT WATER"-kind

The numerals on the kind terms are only classificatory. In other words, these numerals do not have any number-like properties except to distinguish kinds from each other, which means that the relation μ_{kind} exploits a nominal scale.

The concept of *kind* plays an explanatory role in linguistic semantics, too. Consider the relative acceptability status of (2a) and (2b). McNally & Boleda (2004) argue that the adjective *legal* functions as a modifier of kinds so that its merger with the noun *adviser* denotes a new kind via the rule of kind composition. This kind-level entity can be turned into a set of object-level entities that can be modified by the adjective *clever,* which is understood to be a property of object tokens. However, if we first merge the adjective *clever* with the noun *adviser*, we end up with a set of object-level entities, which cannot undergo kind composition when later merged with the adjective *legal*. This accounts for the relative unacceptability of (2b) when compared to (2a), an explanation that crucially requires giving kind terms a role in semantic grammar (the hash symbol "#" is intended to convey semantic anomaly).

(2) a. a clever legal adviser

b. #a legal clever adviser

We have seen examples of nominal concepts in apes and in humans. Concepts that involve kind attribution to objects rely on a nominal scale, a scale of classification. In what follows, we shall look at another type of scale, the ordinal scale, and the role it plays in animal and human mental life.

[1] "μ_{kind}" is a relation (and not a function) in that an object can, in principle, be associated with more than one kind specification, a property that will be become important in Section 3 in the context of ordinal measurement. One way to think about *functional fixedness* discussed above is that for great apes, "μ_{kind}" is more likely to be employed as a function, relating each object to exactly one kind. In other words, treating "μ_{kind}" as a relation might turn out to be a cognitive feat, indicative of flexibility. This interesting issue is orthogonal to our concerns, however.

[2] To say that a kind is an X-kind (e.g. "DRINK WATER"-kind) is to say that an object of this kind enables one to X (e.g. drink water). This is intended to be a way of taking about mental representation of kinds in apes, remaining as unspecific about their content as possible.

3. Ordinal Concepts

In a nominal measurement, the only formal relation of interest concerning the results of measurement is that they are distinct (e.g. $\text{kind}_7 \neq \text{kind}_{12}$). The outputs of an ordinal measurement, on the other hand, are ordered with respect to each other (e.g. $\text{kind}_7 \ll \text{kind}_{12}$). When we are told about the results of Women's 100m at the Paris 2024 Olympics, we learn about the order in which the competitors completed the race. After this measurement, the runners end up being linearly ordered with respect to each other: the first, the second, the third, ... and the last. In short, what an ordinal scale does is order objects in some dimension of interest.

3.1. Ordinal Concepts in Animals

Social hierarchy among baboons provides a striking example of the implementation of an ordinal scale in the context of social cognition (Bergman et al., 2003). At the center of some baboon troops are matrilines, kinship groups traced through the female line. Females within a family are ranked among each other (say, b_1 « b_2 « b_3). This is not, however, the only kind of social dominance hierarchy concerning baboon families: "[A]ll female members of one matriline outrank or are outranked by all female members of another" (Bergman et al., 2003, p. 1234). This means social dominance relations among female baboons can be represented as in (3), which can be thought as involving embedding of an ordinal relation (ranking within a family) within another ordinal relation (ranking among families), a potential case of recursion (Bolender, 2010)

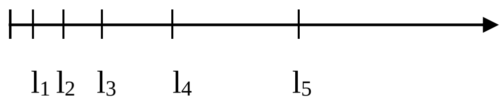
(3) $$\ldots \ll \underset{\text{Martiline}_1}{\boxed{b_1 \text{ « } b_2 \text{ « } b_3}} \ll \underset{\text{Martiline}_2}{\boxed{b_4 \text{ « } b_5 \text{ « } b_6}} \ll \ldots$$

There is experimental evidence that the representation in (3) is not just a way of summarizing our observations about social hierarchy among baboon families but it is also an aspect of cognitive representations entertained by baboons in interpreting the social relations they are embedded in. In a playback experiment, Bergman et al. (2003) got female Hamadyras baboons to listen to calls signaling a fight between two females, which involved threat grunts indicating dominance and screams indicating submission, where the submissive scream of the higher-ranked female is taken to be indicative of a rank reversal. Bergman et al. (2003) found that "[s]ubjects looked toward the speaker for significantly longer durations when hearing sequences that mimicked a between-family rank reversal than when hearing both within-family rank reversal and no-reversal control sequences" (p. 1235). This is likely because a rank reversal among martilines has wider consequences for the social structure of baboon troops than a rank reversal within a single martiline. What this experiment suggests is that the embedded hierarchical social structure shown in (3) is something that Hamadyras baboons are spontaneously sensitive to, an aspect of their social cognition.

Humans share with other animals the Approximate Number System (ANS), one of the core number systems, which enables us to distinguish between large quantities in the environment without analyzing these quantities into their discrete constituents (Cantlon & Brannon, 2005; Feigenson et al., 2004). This capacity obeys Weber's Law, which says that discriminability between two magnitudes depends on the ratio between them and not, for instance, on a constant factor. If an animal can distinguish between 10 and 20 objects, this does not guarantee that this animal will also be capable of distinguishing between 30 and 40 objects even though there is a constant factor of 10 objects (40 – 30 = 20 – 10 = 10) between these two pairs of sets of stimuli. Rather, the minimal ratio needed to distinguish between the first two sets of stimuli (say, 1 to 2) must be preserved in the second pair (e.g. 30 vs 60 objects) for these quantities to be distinguishable.

ANS imposes a *more-than* relation on distinguishable magnitudes, which implies that this system relies on an ordinal scale. As an example, let us take the concept of length and assume that there are different *kinds* of lengths that are ordered on a line (say, $l_1 \ll l_2 \ll l_3 \ll l_4$). Here the numerals on the kind terms associated with length do more than just express distinctness of kinds; they express order among magnitudes. If $k > j$, then an object whose maximal length is of the kind l_k is longer than an object whose maximal length is of the kind l_j. One way to think about an object with maximal length l_j is to say that it has a length that starts at the bottom and stretches through the line and ends somewhere between l_j and l_{j+1}. Every object whose maximal length is of the same kind will be measured as having the same length; they will be indistinguishable in terms of their length. Each such object with maximal length of l_j is understood to be longer than any object with maximal length of l_{j-1}, l_{j-2}, l_{j-3}, etc. The asymmetric nature of relations between kinds of length makes it possible to represent them on a line, as in (4).

(4)



The distance between kinds of lengths on the line increases as the numeral on the kind term increases. This is intended to capture Weber's Law, which requires that the proportion between the consecutive lengths remain constant (i.e. $l_1/l_2 = l_2/l_3 = l_3/ l_4 = \ldots = c$).

Imagine an ape coming across two sticks that differ in the kind of length they have. Suppose that the first stick (S1) is l_4 kind of long and the second stick (S2) is not, meaning that S1 is strictly longer than S2.[3] These thoughts can be symbolically represented using the ordinal measure relation $\mu_{length,O}$, where the index "length, O" means that this is an Ordinal measure relation from entities to the kinds of length they have.[4]

(5) $\mu_{length,O}(S1, l_4) \ \& \ \neg(\mu_{length,O}(S2, l_4))$

In a set of experiments on future-oriented tool use by a zoo-housed Sumatran orangutan with the name of Riau, Mulcahy (2018) observed that Riau secured a long stick, which he had just used for obtaining food, by wedging it into the gaps of the fences in his closure. He showed this behavior precisely when he could envision further use for the tool within a short time. "The possible advantages of securing the tool were that Riau did not have to keep holding the heavy

[3] Why does this entail that the first stick is strictly longer than the second stick? Following Heim (2000), we assume that the ordinal measurement is downward monotone. If an object comes out as being of kind_j out of an ordinal measurement, then for any $k < j$, this object is also of kind_k, If the shorter stick is not of the kind l_4, then for any $m > 4$, it is not of kind l_m, either. Since the first stick has (at least) the lenght associated with the kind l_4, it follows that the first stick is strictly longer than the second stick.

[4] These formulas can be read as follows: "l_4 is a kind of lenght that S1 has and l_4 is not a kind of lenght S2 has."

tool or he did not have to keep climbing down to pick to the tool if he dropped it to the floor after raking-in the rewards" (Mulcahy, 2018, p. 1). This experiment was repeated with a shorter and lighter stick to test the hypothesis that Riau secured the tool due to its length and weight. Indeed, he exhibited no securing behavior involving this new (shorter and lighter) tool, which suggests that he distinguished between objects in terms of their sizes, a capacity that relies on ordinal measurement, and acted accordingly.[5]

This is our summary of animal concepts that implement an ordinal scale. Some of these appear to be domain-specific (e.g. social dominance hierarchy among baboons, which is possibly the result of an innate faculty organizing social cognition, see Fiske, 1991) while other implementations of this measurement scale exhibit a higher degree of cognitive flexibility (e.g. selective tool use by Riau). Regardless, the capacity for ordinal measurement appears to be a noteworthy aspect of animal mental life.

3.2. Ordinal Concepts in Humans

Inferences associated with comparative constructions provide linguistic evidence that ordinal measurement plays an important role in accounting for the structure of some aspects of human thought as expressed in natural languages.

Kennedy (2007) distinguishes between two types of constructions of comparison: implicit (6a) and explicit (6b).

(6) a. Compared to Lee, Kim is tall.

b. Kim is taller than Lee.

A comparative expression is said to show crisp-judgment effects just in case small differences between the entities being measured have consequences for the truth condition of this expression. Explicit comparison (6b) licenses crisp (i.e. exact) judgments in that the smallest measureable difference between the heights of Kim and Lee renders (6b) true so long as it is to Kim's advantage. Implicit comparison, on the other hand, does not allow crisp judgments. For (6a) to be felicitous in a context of utterance, there must be some notable height difference between Lee and Kim. We may hypothesize that the reason why (6a) does not license crisp judgments is that implicit comparison involves the use of an ordinal scale, which does not make use of exact measurement of the kind we find in explicit comparison. Suppose that Lee and Kim have the same kind of height (say, "$\mu_{height,O}(Lee, h_4)$" and "$\mu_{height,O}(Kim, h_4)$"). All we can conclude from this is that each has a height that contains the segment $[0, h_4]$ (and is possibly contained in the segment $[0, h_5]$). We cannot conclude that they are of the same exact height. In other words, an ordinal scale does not lead to exact measurement or exact comparison.

Let us consider a state of affairs that makes (6a) true, where Kim is h_5 kind of tall and Lee is not (i.e. "$\mu_{height,O}(Kim, h_5)$" and "$\neg(\mu_{height,O}(Lee, h_5))$"). Observe that, so long as we rely on an ordinal scale to measure height, we cannot tell exactly how much taller Kim is than Lee. In other words,

[5] See Boysen & Berntson (1995) for evidence concerning the symbolic representation of quantities in ape cognition. In fact, this symbolic capacity enables them to solve some problems that would, otherwise, remain beyond their reach.

we cannot precisely calculate the difference between their heights. This observation accounts for the fact that implicit comparison is incompatible with exact differential measurement (Kennedy, 2007).

(7) a. #Compared to Lee, Kim is 10cm tall.

b. Kim is 10 cm taller than Lee.

There are languages in which comparative constructions rely exclusively on ordinal scales. In Washo, for instance, the only way of expressing comparison is via conjunction of sentences with predicates that are negatives (or antonyms) of each other. Bochnak (2015) notes that comparatives in Washo are infelicitous in crisp-judgment contexts (AOR = aorist, COP = copula, IMPF = imperfective, NMZL = nominalizer, NEG = negation)

Context: Comparing two ladders, where one is only slightly taller than the other

(8) #wí:diʔ ʔitmáŋa delkáykayiʔ k'éʔi

this ladder NMLZ.tall cop.IMPF

wí:diʔ delkáykayiʔ-e:s k'aʔaš

this NMLZ.tall-NEG cop.AOR

Intended: 'This ladder is taller than that one.'

Literal: 'This ladder is tall; that one is not tall.'

Comparison in Washo is always implicit, which, in the context of this paper, implies that this language only makes use ordinal scales in interpreting comparative constructions. As expected, the lack of explicit comparison in Washo means that "differential measure phrases are also not found in this language" (Bochnak, 2015, p. 23).

How should we analyze explicit comparison, which licenses crisp judgments (6b) and exact differential measurement (7b)? Before we can answer this question, we need to take a look at another human capacity: quantification over kinds.

4. Quantifying over kinds

Throughout the paper we have seen various instances of thoughts, in the form of sentences, involving nominal and ordinal measurement. The sentences corresponding to such measurement-invoking thoughts had the following syntactic schema:

(9) The Syntactic Rule for Basic Sentences with Measurement

If "a" is a name for an object,

"μ_d" is a measurement relation with the dimension d,

and "k_j" a kind term (with $j \in \mathbb{N}$)

Then "$\mu_d(a, k_j)$" is a sentence, meaning $(a, k_j) \in \mu_d$

Given the rule in (9), let us see how one might express the logical form of the comparative sentence in (10), repeated from (7).

(10) Compared to Lee, Kim is tall.

We have already seen that the logical form of (10) involves a measurement function that relies on an ordinal scale. Using the syntactic rule above, we can describe various scenarios that make (10) true, two of which are given below: [6]

(11) a. $\mu_{height,O}(Kim, h_5)$ & $\neg(\mu_{height,O}(Lee, h_5))$

b. $\mu_{height,O}(Kim, h_3)$ & $\neg(\mu_{height,O}(Lee, h_3))$

Listing a set of situations in which a sentence comes out as true does not count as providing a proper analysis of its logical form. The syntactic rule in (9) forces us to be explicit about the kind of height Kim has. However, we need not have such a specification at hand in order to entertain the thought expressed in (10). Suppose that all we know is that Lee is a baby and Kim is not. Our knowledge about babies and non-babies may lead us to believe that Kim has a kind of height that Lee lacks (This is, in essence, the *A-not-A analysis* of comparatives discussed in Schwarzschild, 2008). In other words, *there is a kind of height* such that Kim has it but Lee does not, with no specification as to the kind of height each might have. The phrase "there is a kind of height such that" here suggests that we are in the arena of quantificational thoughts. In other words, to get a better approximation to the meaning of the sentence in (10), we must understand kind terms as *variables* and let quantifiers targeting kind variables into our syntax.

Bolender and colleagues (Bolender, 2016; Bolender et al., 2008) claim that mental representation of quantificational thoughts is an aspect of human uniqueness. Quantificational thoughts enable humans to entertain the capacity for *Cognition by Description*, a naturalistic/mentalist analogue of Russell's Knowledge by Description (1905; 1911), which is "the ability to talk and think about entities which one has never perceived and which, in some cases, do not exist" (Bolender et al., 2008, p. 130). As you are having a nice bowl of soup for dinner, you might find yourself wondering about the inventor of the spoon. Building on Russell's (1905) work on definite descriptions, Bolender suggests that conceptualizing the inventor of the spoon without having any direct perception of this individual requires mentally representing the quantificational thought that there is exactly one individual who invented the spoon. Taking the quantifier "$\exists!x$" to stand for "there is one and only one x such that..", we can formally express this thought as in (12). The logical forms with the quantifier "$\exists!x$" are said to involve uniqueness quantification.[7]

(12) $\exists!x.$ invented(x, spoon)

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[6] Throughout the paper we presuppose that the primate mind can represent negative and conjunctive thoughts, the syntactic rules underlying which are given below:
If φ and ψ are sentences, so are $\neg\varphi$, φ & ψ etc.

[7] Uniqueness quantification is existential quantification with universal quantification within its scope. "$\exists!x. P(x)$" is equivalent to "$\exists x. (P(x) \& \forall y. (P(y) \rightarrow x = y))$". In a sense, it is a kind of existential quantification that runs universal quantification as a subroutine.

Similarly, mental representation of quantificational thoughts grants us with the capacity to conceptualize measurements that we cannot explicitly specify as being of a certain kind. The thought associated with the sentence (10), for instance, can be represented as an instance of existential quantification over kinds, i.e. *there is a kind of height such that Kim has it but Lee does not*,[8] without being explicit about exactly which kinds of height they have.

(13) $\exists k.\ (\mu_{height,O}(Kim, k)\ \&\ \neg(\mu_{height,O}(Lee,k)))$

We might entertain the thought that K2, which lies in the Karakorum range, is the second largest mountain, which means that there is exactly one mountain that is higher than K2 (and, by assumption, has a different kind of height considering the height of K2). This thought, which brings together uniqueness quantification over entities and existential quantification over kinds, can be represented as in (14).

(14) $\exists!x.\ (mountain(x)\ \&\ \exists k.\ (\mu_{height,O}(x, k)\ \&\ \neg(\mu_{height,O}(K2, k))))$

In (13) and (14) we see examples of sentences with quantification over kinds. We conclude that humans can mentally manipulate syntactic rules defining sentences with quantification. The rules in question, which we take to be unique to human cognition, are given in (15).

(15) The Syntactic Rule for Quantificational Sentences with Measurement

If "φ" is a sentence containing (tokens of) free kind variable "k", i.e. "$\varphi[\ldots k\ldots]$",

Then "$\exists k.\ \varphi[\ldots k\ldots]$" is a sentence where (tokens of) "k" is bound,

with the meaning $\{k: \varphi[\ldots k\ldots]\} \neq \emptyset$

and "$\forall k.\ \varphi[\ldots k\ldots]$" is a sentence where (tokens of) "k" is bound,

with the meaning $\{k: \varphi[\ldots k\ldots]\} = U_k$ (the domain of all kinds)

What we have seen so far is that a quantificational grammar that involves ordinal scales gives us a way of approximating the logical form of comparative sentences such as (10). While doing measurement with an ordinal scale is more informative than what a nominal scale can provide, it has its limitations. The ordinal scale does not let us measure *distances* between kinds of heights. As we shall see, however, there are some human thoughts that rely on measuring such distances. We need new types of scales to characterize such thoughts. This is the topic of the next section.

[8] This entails that Kim is taller than Lee. As noted before, we take it that ordinal scales are downward monotone. If an object comes out as being of $kind_j$ out of the ordinal measurement, then for any $k < j$, this object is also of $kind_k$ (Heim, 2000). If Lee is not of $height_k$, then he is at most of $height_{k-1}$.

5. Measurement with a unit

Explicit comparison differs from implicit comparison in (at least) two respects: (1) crisp judgments and (2) exact differential measurement. In what follows, we show that these two properties can be captured once we let more sophisticated forms of measurement scales into our semantic analysis (Sassoon, 2010; Zhang, 2020; Zhang & Ling 2021), scales that invoke a *unit* of measurement.

The interval scale differs from the nominal or ordinal scale in that it allows for the measurement of distances between kinds associated with objects. This type of scale stipulates that consecutive kind terms (such as k_3 and k_2 or, more generally, k_{j+1} and k_j) be equidistant from each other by some specific distance *u*, short for *unit* (with respect to the dimension d). In other words, the ordinal scale yields comparable exact differences in terms of units.

(16) Measurement with a Unit (Interval)

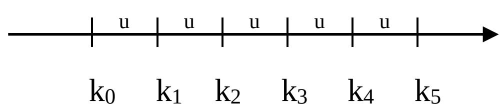
A measurement relation "μ_d" employs an interval scale iff

it is an ordinal scale and $\exists$u. distance(k_{j+1}, k_j) = u

e.g. distance(k_5, k_3) = 2 · u , distance(k_3, k_3) = 0, distance (k_3, k_4) is undefined

We can represent this idea using a line with equal distances between consecutive kind terms.

(17)



Before we show how explicit comparison can be modelled with the help of measurement scales with units, let us note that there is a sense in which the capacity for measurement with a unit depends on having internalized a quantificational grammar: The definition in (16) involves existential quantification over distances. We have already noted that internalizing a quantificational grammar is a unique aspect of human cognition (Bolender, 2016; Bolender et al., 2008). Since the capacity for measurement with a unit presupposes an internalized quantificational grammar, we expect this capacity to be unique to humans as well.

Premise 1. Quantificational grammar is unique to human cognition.

Premise 2. The cognitive capacity for measurement with a unit requires having internalized a quantificational grammar.

Conclusion. Measurement with a unit is unique to human cognition.

In Section 6, we will test this hypothesis in the context of social cognition and conclude that it provides an account of some unique aspects of human sociality as well. Before we do that, however, we need to see how interval and ratio scales are employed inside human grammars.

As we have noted before, using an interval scale as part of the logical form of natural language sentences gives us a way of expressing explicit comparison. The sentence in (18a), repeated from (6b), can be given the logical translation in (18b), where the index "height, I" indicates that we are working with an interval scale of height (see Zhang & Ling, 2021 for a difference-based analysis of comparatives that makes crucial use of interval scales).

(18) a. Kim is taller than Lee.

b. $\exists k. (\mu_{height,I}(Kim, k) \,\&\, \neg(\mu_{height,I}(Lee,k)))$

The use of an interval scale presupposes a specific unit of distance between consecutive terms. We can choose as small a unit as we want: one centimeter, one millimeter, one micrometer (a unit which is 1000 smaller than a millimeter) or whatever small unit one might imagine. Consider a situation in which Kim is just one unit taller than Lee. Letting this unit be as small as possible, we find that the smallest measurable height difference between Kim and Lee can be made to be consequential for the truth conditions of (18a). This is how we get crisp-judgment effects associated with explicit measurement

Similarly, we can represent the thoughts expressed by differential measurement constructions. In (19), this requires representing the thought that, when measured via the Interval Celsius scale, the distance between the maximal temperature of the sea and the maximal temperature of the pool is at least two units.

(19) The sea is (at least) 2°C warmer than the pool.

We first specify a temperature kind t such that for any other temperature kind t' that the sea/the pool has, the distance from t to t' is non-zero, which means that t is the maximal temperature measureable via the Celsius scale that the sea/the pool has.

(20) a. $\exists t.\ \mu_{Celsius,I}(the\_sea, t) \,\&\, \forall t' \neq t\ (\mu_{Celsius,I}(the\_sea, t') \rightarrow distance(t, t') > 0)$

b. $\exists t''.\ \mu_{Celsius,I}(the\_pool, t) \,\&\, \forall t''' \neq t''\ (\mu_{Celsius,I}(the\_pool, t''') \rightarrow distance(t'', t''') > 0)$

We then indicate that the distance between these two maximal temperature specifications comes out as (at least) two units, which gives us (21) as the logical form of (19).

(21) $\exists t.\ [\mu_{Celsius,I}(the\_sea, t) \,\&\, \forall t' \neq t\ (\mu_{Celsius,I}(the\_sea, t') \rightarrow distance(t, t') > 0)$

$\&\ \exists t''.\ [\mu_{Celsius,I}(the\_pool, t) \,\&\, \forall t''' \neq t''\ (\mu_{Celsius,I}(the\_pool, t''') \rightarrow distance(t'', t''') > 0)$

$\&\ distance(t, t'') \geq 2 \cdot u\ ]]$

There is an important difference between the Celsius scale and the Kelvin scale. When the temperature of an object is measured as *0°C*, that is not due to the absence of kinetic energy among the particles in the relevant object. At *0°K*, on the other hand, the average kinetic energy inside the object equals to zero: Zero Kelvin is Absolute zero. Similarly, if an object has a length of 0, then this object has no length. If the Moon has a population of 0, then there is no one living on the Moon. A ratio scale is an ordinal scale that has Absolute zero as the value of the lowest kind term "k_0". With this kind of scale, we can measure the distance of any kind term to absolute zero using a function that we denote as "distance$_0$", which can be read as "the distance function to absolute zero". The distance of 20°K to zero is twice as the distance of 10°K to zero. (The same cannot be said about 20°C and 10°C.) Thus, the ratio scale enables us to work with meaningful proportions.

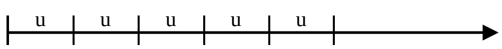
(22) Measurement with a Unit (Ratio)

A measurement relation "μ_d" employs the ratio scale iff

it is an interval scale and $\text{distance}_0(k_0) = 0$

e.g. $\text{distance}_0(k_4) = 2 \cdot \text{distance}_0(k_2)$

We can represent this idea using a line of equal distances that starts at zero.

(23) 

k_0 k_1 k_2 k_3 k_4 k_5

There are natural language expressions that suggest a role for ratio scales in the logical form of sentences. The equative construction in (24a), for instance, can be given the logical translation in (24b), which makes use of the ratio scale enabling the representation of multiplicative relations among distances.

(24) a. John is (at least) twice as tall as Mary.

b. $\exists h.\ [\mu_{height,R}(\text{John}, h)\ \&\ \forall h' \neq h\ (\mu_{height,R}\ (\text{John}, h') \rightarrow \text{distance}(h, h') > 0)$

$\&\ \exists h''.\ [\mu_{height,R}(\text{Mary}, h'')\ \&\ \forall h''' \neq h''\ (\mu_{height,R}(\text{Mary}, h''') \rightarrow \text{distance}(h'', h''') > 0)$

$\&\ \text{distance}_0(h) \geq 2 \cdot \text{distance}_0(h'')]]$

Similarly, constructions that involve exact (maximal) measurement that has zero at its starting point require the use of a ratio scale. The sentence in (25a), for instance, says that when measured with the ratio scale for height with one centimeter as the unit, John comes out as h_{175} kind of tall, whose distance to absolute zero is 175 units of centimeter and h_{175} is the maximum kind of height measureable with $\mu_{centimeter,R}$ that John has.

(25) a. John is 175 cm tall.

b. $\mu_{centimeter,R}(\text{John}, h_{175})$, where $\text{distance}_0(h_{175}) = 175 \cdot \text{cm}$

$\&\ \forall h' \neq h_{175}\ \mu_{centimeter,R}(\text{John}, h') \rightarrow \text{distance}(h_{175}, h') > 0$

Sassoon (2010) claims that there are natural language expressions that trace the distinction between the interval scale and the ratio scale. We have seen the adjective *tall* can be used in constructions that involve multiplication as in (24) and exact measurement as in (25), which presuppose ratio scales. Sassoon (2010) observes that the negative adjective *short* is not licensed in similar contexts ((26a) and (26b)). In fact, unlike most positive adjectives, negative adjectives (e.g. *short*, *small*, *narrow* etc.) are not acceptable in constructions that require the implementation of ratio scales. Negative adjectives implement an interval scale, which is why (27c), which is obtained from (27b) by turning it into a construction of explicit comparison, is acceptable with the adjective *short*.

(26) a. The table is twice as tall/ #short as the chair.

b. John is 175 cm tall/ #short

c. John is 175 cm taller/shorter than Bill

In Section 2 and Section 3, we have seen examples of linguistic constructions that are sensitive to the distinction between scales with and without a unit (implicit vs explicit comparison, see Zhang, 2020 for further discussion of the significance of units in grammar). Repeating some of Sassoon's (2010) arguments, this section has provided evidence for the linguistic reality of the distinction between ratio scales and interval scales (positive vs negative adjectives). These observations suggest that measurement scales are integral to linguistic grammar. They play an explanatory role in our theories of why certain constructions have the semantic properties that they do.

6. A note on Social Cognition

Fiske (1991, 1992) has developed a theory of social cognition that takes the formal properties of scales of measurement as its starting point. Since we claim that the capacity for measurement with a unit is uniquely human, we expect the relational models that presuppose scales with a unit (i.e. *Equality Matching* and *Market Pricing*, as we elaborate below) to be social forms specific to human communities. In this section, we first give an explicit formulation of this claim and then provide evidence for this predication.

Fiske's Relational Models Theory (see Fiske & Haslam, 2005 for an overview) is about how people interpret and experience social relations (and obligations associated with them) within groups, which plays a crucial role in how they produce moral judgments concerning social behaviors of individuals in their group (themselves included). There are four basic relational models implemented by social cognition: Community Sharing, Authority Ranking, Equality Matching and Market Pricing. The complexity of social life is hypothesized to be a consequence of these four fundamental relational models being combined in novel ways to generate forms that characterize distinct facets of human sociality (Fiske, 1991; 1992; Bolender, 2010). (This is similar to the complexity of physical universe being a combination of four fundamental forces in physics.)

Communal Sharing (CS) is typically associated with strong symmetrical bonds connecting individuals inside a group. People who are related to each other by Communal Sharing have something in common that make them indistinguishable from each other in some respects (such as access to food resources). The nominal scale underlies Communal Sharing in that the individuals inside this relation are understood to be of the same kind to the exclusion of others who are of a different kind. Some hearth-warming example of CS would be partners immersed in romance or intimate parental love. A disturbing example of CS would be ethnic racism as manifested in undifferentiated hatred against a group of people due to their ethnicity, i.e. due to their "kind". "We are all one" and "One for all/ All for one" is how members of CS-groups tend to feel/think. CS-bonds are reinforced by physical contact, thanks to which people's bodies come closer (hugging, caressing, petting etc), and rhythmic synchronous movement, whereby the distinction between bodies are blurred. Connor (2007) observes that alliance formation among bottlenose dolphins relies on contact behaviors such as petting as well as movement in unison, which are indicative of CS-bonds (Bolender, 2011).

Authority Ranking (AR) is associated with asymmetrical bonds that hierarchically order individuals inside a group. In AR groups, there are those who have the right to make the decisions and give the orders and there are those who are obliged to accept these decisions and obey these orders. In a (simplified) patriarchal nuclear family, for instance, the father typically assumes the role of the ultimate leader and this role is likely to be assumed by the oldest son in his absence. The men in such a family can make decisions concerning the women, who are considered to be subordinates, but they are also expected to protect them and provide for them. There is no exact measure as to how much higher-ranked men are than women: All that can be said is that they are. As one might guess, Authority Ranking is an implementation of an ordinal scale. The asymmetry between superiors and subordinates may be reflected in the sitting arrangements (e.g. Father sits at the head of the table), in the kinds and the quantities of the food served, or in order in which food is served (e.g. Men are served first). The hierarchical structure of such a strictly patriarchal family, where all men are ranked higher than all women and the hierarchy within each group is determined by age, can be represented as in (27). Observe that this is structurally quite similar to what we have observed with baboon troops in (3). That is, AR is a model of relational social cognition that is implemented both by humans and by (non-human) animals.

(27) $\underbrace{\boxed{\text{daughter} \ll \text{mother}}}_{\text{WOMAN}} \lll \underbrace{\boxed{\text{son}_{2nd} \ll \text{son}_{1st} \ll \text{father}}}_{\text{MAN}}$

Equality Matching (EM) is about recognizing specifiable imbalances among the participants of a social interaction and eliminating these imbalances in order to achieve the kind of justice associated with equality.[9] EM is "based on a model of even balance and one-for-one correspondence, as in turn taking, egalitarian distributive justice, in-kind reciprocity, tit-for-tat retaliation, eye-for-an-eye revenge" (Fiske, 1992, p. 691) among other reciprocal acts. In a fair game, opposing teams must have equal numbers of players. In a fair work environment, equal work deserves equal pay. In a fair election, each citizen must have exactly one vote. If some citizen happens to cast more than one vote, this creates a sense of injustice caused by inequality. The good news is that we know how to restore the balance. We either cancel all the extra votes

[9] One can also implement Equality Matching (EM) to sustain exact imbalances in a social situation. If a tenured professor is to get exactly 100 dollars more than a non-tenured professor, then this is a type of inequality that can be expressed in an Equality Matching model among professors. Authority Ranking would not work here since the ordinal scale cannot be used to measure the degree of difference. It is more typical, however, to think of Equality Matching as a social model intended to achieve absolute equality, i.e. to eliminate imbalances. Why so? We may speculate that EM and CS share the basic property of being about similarity. CS is concerned with qualitative similarity, which is, in effect, just sameness, and EM is about quantitative similarity. In the same spirit, it might be that there are some commonalities between AR and MP. AR relies on qualitative differences while MP is all about quantifiable distinctions. Such interrelations between scales remain to be explored.

by the fraudulent citizen or we let every other citizen cast as many votes as he did. Equality Matching presupposes an interval scale, thanks to which we can measure the degree of imbalance in a social event and decide what it takes to remedy the situation.

In Market Pricing (MP), social relations are typically coordinated and evaluated on the basis of the proportion of the measureable gains obtained in the relationship to the measureable losses caused by the relationship. The value of the relevant relationship depends on the ratio that comes out of the benefit-cost analysis, which may be quite variable depending on the circumstances. MP presupposes the implementation of a ratio scale, which is the only scale that licenses calculation with meaningful proportions. The calculative mode underlying decisions in MP-relations may give one a sense of a lack of emotional commitment, which is why such relations can sometimes be classified as being cold and perhaps weak, but they also tend to result in a relatively low sunk cost when the relationship fails. (Note that losing a CS-bond might feel similar to losing a body part). Typical MP-desires are maximizing the gains and minimalizing the losses and making a contract concerning a social relation may be one way of securing an optimum of these desires. Indeed, a prenuptial contract can be thought of as an MP-structure that is superimposed on a social relation that is typically formed in a CS and/or an AR mode. MP enables comparison between distinct objects in terms of proportional equivalences (e.g. an equivalence relation between 1 banana, 2 oranges and 3 apples). We can then use money as the common metric to express all such proportions (e.g. a banana = $0.6, an orange = $0.3, an apple = $0.2). Those who has spent enough time within MP-dominant relationships may find themselves assigning proportionally expressible significance values to people in their social environment (Consider: My mother is twice as important to me as my father. So, I prefer my mother living for five more years to my father living for eight more years).

Communal Sharing and Authority Ranking "correspond to social structures that are widely observable in other mammalian genera and vertebrate families" (Fiske, 1992, p. 716) We have already seen an instance of CS in the alliance formation behavior of male bottlenose dolphins and an instance of AR in the complex embedded social hierarchy among Hamadyras baboons. How about Equality Matching and Market Pricing? Fiske (1992) writes:

> "There is no evidence for any other species actually making precisely balanced one-for-one exchanges or equal distributions, or using subtraction to keep track of the magnitude of imbalance conferred. As regards MP, it is an even clearer and more striking fact that no other species uses money, … produces things just to exchange for other commodities, or even makes social exchanges or distributions at fixed rates." (pp. 716 – 717)

This paper provides a principled explanation for the absence of EM and MP in the social life of (non-human) animals. Since the capacity for measurement with a unit is uniquely human, any social-relational model that presupposes scales with units must also be uniquely human. The reasoning goes as follows:

Premise 1. The cognitive capacity for measurement with a unit is unique to humans.

Premise 2. EM and MP presuppose measurement scales with units.

Conclusion. Implementing EM and MP in a social relation is unique to humans.

The absence of EM in the animal kingdom is striking if Gouldner (1960) is right in claiming that "[c]ontrary to some cultural relativists, it can be hypothesized that a norm of reciprocity is universal" (p. 171) to human societies as an aid to the stability of social systems. The absence of MP can be attributed to the fact that this model relies on what Fiske & Haslam (2005) call *Abstract Symbolism*: "MP relations are organized with reference to ratios, an utterly abstract concept." (p. 287). Indeed, proportional quantifiers such as *most* and *one-third* are so abstract that they cannot be defined within standard First Order Logic (see Barwise & Cooper, 1981 for a proof), a system in which numerals such as *at least ten*, *exactly ten*, *at most ten* can be defined. One needs logical systems of higher order to express the denotation of a proportional quantifier, which is the gist of Generalized Quantifier Theory in formal semantics.

7. Conclusion

The significance of measurement scales for semantic analysis is being recognized (Sassoon, 2010; Solt 2015; Zhang, 2020). The more general relevance of these scales to the study of human cognition is yet to be appreciated. This paper has been an attempt in this direction. We hope to have shown that measurement scales are more than just tools to organize and analyze experimental data. They help us formulate explicit hypotheses about mental objects and processes. For instance, measurement scales can be used to measure relative complexity of thoughts. We know that scales are in a proper containment relation, where Nominal ⊂ Ordinal ⊂ Interval ⊂ Ratio. Let T and T' be two thoughts that differ only in the measurement scale involved in them (say, $T = \mu_{height,R}(John, h_4)$ and $T' = \mu_{height,O}(John, h_4)$). Then we can say that T is strictly more complex than T' if the scale in T contains the scale in T'.

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