# Numerals, Numbers and Cognition: Towards a Localist Theory of the Mind

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## 0. INTRODUCTION TO THE PROPOSAL

- We attempt to show that the bases of cognition are *locative*.
- In other words: the ***foundations of complex thought*** are to be found in the spatial relations between entities.
  - They must be ***restricted*** enough to account for the representations licensed by the phenomenological world, and at the same time ***flexible*** enough to allow for the inclusion of further layers of information as required by determined faculties (all interpretative components of the mind, with neurological basis, respond to localist principles).
  - All variation results from the interpretative systems, since generation can be subsumed to a single concatenation algorithm to be applied in n-dimensional mental workspaces. In language, generation is driven by the need to derive significant effects on the output, in particular, to ***increase the informational load***.
- We intend to show that our proposal finds support in the characteristic properties of the three generative systems represented by ***language***, ***mathematics*** (geometry) and ***music***, i.e. we intend to show how all three are interconnected and ***locative in nature***.
- We use ***numerals*** as the bridge between language and the musical and arithmetical capacities.

## 1. NUMERALS AS LOCATIONS

- Numerals are ***locative*** predicators affecting ***sortal*** (***perfective***) entities.
  - ***Perfectivity*** allows conceptual iteration of an entity, which is linguistically expressed by a numeral .
  - With ***imperfectivity*** numerals cannot be functors, as there is not a conceptually defined (i.e. limited and seen as a whole) entity over which to have logical scope).
- ***Location*** is expressed by means of an *n*-plet of coordinates in an *n*-dimensional space, such that a figure-ground dynamics can be formalized as a pair ($F_{X, Y, Z \ldots n}$, $G_{X, Y, Z \ldots n}$), where F is the ***figure*** (the object that moves towards or stays in a location) and F is the ***ground*** (the relevant concrete or abstract location, i.e., properties, states, places). X, Y, Z…*n* are the relevant dimensions in a certain workspace.

This theory allows us to work with a very restricted set of primitives, since relations are decomposed into locative primitives, linguistically represented by the procedural element P.

- Two kinds of elements are present in syntax: ***roots*** and ***procedural elements***, distinguished ***at the semantic interface*** :
  - ***Roots***: pre-categorial linguistic instantiations of a-categorial generic concepts. Generic concepts are "severely underspecified", since they are used by many faculties, and therefore cannot have any property readable by only some of them. Roots' potential extension is maximal (expressible by the superset that properly contains all referential sets), given their semantic underspecification: bare roots have no (spatio-temporal) anchor ($\surd$ = S, where S = {α1...αn}).
  - ***Procedural elements*** convey procedural instructions to the post-syntactic semantic parser as to how to manipulate a given semantic substance. Instructions play two main roles:
    - Restrict reference in terms of a proper subset of the root. Each element restricts the set in different ways, say: (a) X = {α, β, γ}, (b) Y = {γ, λ, δ}
    - Provide instructions as to: Where to retrieve information? What kind of information to retrieve?
    - Therefore, ***procedural elements convey locative meaning*** in the sense that they ***relate a figure*** (i.e., the root) ***to a ground*** (a set of intensional properties), and they are thus ***predicators*** (i.e, functors).
- Formally, we have the following situation in the case of a lexical item LI:
  - A lexical item LI is a structure {X...α...$\surd$} ∈ $W_x$, where X is a procedural category (D, T, P), α is an *n* number of non-intervenient nodes for category recognition purposes at the semantic interface, and $\surd$ is a root.
  - f($\surd$) = X ∩ Y (in our case, f($\surd$) = {γ})

  If the root within the nominal domain is an element to be read off at the semantic interface as N, the semantics of D and Num (the nodes focus on) is predication. They have to have semantic scope over their (logical) argument.

## 2. COMPRESSION IN NUMBERS AND MUSIC

- Although used mainly in arithmetics, numbers are deeply embedded in language (Hurford 1987, Dehaene 1997, 2011).
- Human perception of numbers appears to share crucial characteristics with the Scale of Natural Harmonics (SNH), in particular, the effect of ***compression*** that we observe in both, and which follows a logarithmic function (Fig. 1 and Fig. 2).

**Fig. 1** The mental compression of numbers representing a logarithmic function (based on Dehaene 2011: 265; Viarouge, Hubbard, Dehaene & Sackur 2010: 449, 454) (the continuous line shows ratio 1:2, the dotted line shows ratio 1:3).

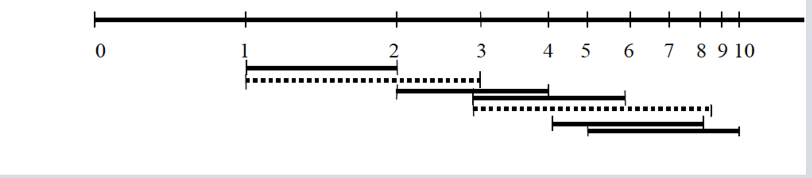
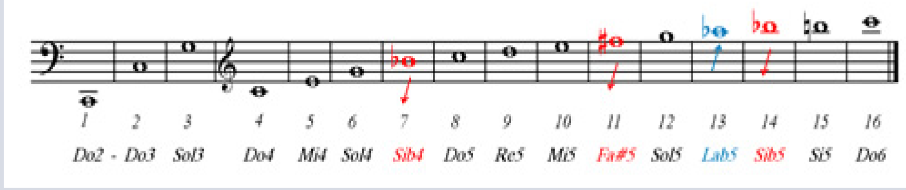



**Fig. 2** The scale of natural harmonics (SNH), starting from C2 (the reader can transport the scale to other keys maintaining the intervals)




Correlations between musical capacity and linguistic numerals are neurologically plausible. If we analyze the SNH, we see that each position is defined as a ratio:

**(1)** $f(x) = x/(x-1)$

Such that the scale can be ordered following a series of rational numbers:

**(2)** 1/2, 2/3, 3/4, 4/5, 5/6... = C3/C2, G3/C3, C4/G3, E4/C4, G4/E4...

Notice that the ratios between the numbers in Fig. 1, obtained from the very same procedure as in the SNH (i.e., dividing each element by the immediate precedent) form the following sequence:

**(3)** 2 — 1,5 — 1,3 — 1,25 — 1,2 — 1,17 —1,14 — 1,125

- The difference *asymptotically* tends to 0 as we increase the values of both the numerator and the denominator. As the frequency in Hz. of the sounds in the SNH also increases and, moreover, the interval between adjacent elements of the scale reduces as pitch increases, the results are the same: an asymptotical tendency to 0.
- A PROVISIONAL CONCLUSION*: **Compression*** never results in total neutralization: if our brains were capable of performing incredibly complex computations with very high numbers following the ratio in (2), we would get an infinite series of numbers the difference between which would decrease until the infinite, but never reach 0.
- ***Compression*** is not only a cognitive tool, but part of the design of a mathematical structure. We will show how the research conducted on the locative nature of cognition can also open the door for understanding the structure of the phenomenological world, and how it is apprehended by the human mind using the available resources and fitting the information in a locative template.

## 3. GEOMETRICAL CONCEPTUALIZATION OF MENTAL SPACE

Pylyshyn (2007: 156-158) proposes a series of problems that arise in a geometrical conceptualization of the mental space. These are:

- *If we represent the fact that A is further from C than from B (AC > AB), then there would be a greater quantity of represented space (as distinct from a representation of more space, which makes no commitment about "amount of represented space") between A and C than between A and B. In other words, distance is represented in some form (perhaps in an analog form) so that each point in empty space is somehow explicitly represented.*
- *If we represent A, B, and C as being ordered and collinear, then there would be an explicit representation of B as being between A and C (where by an "explicit representation" I mean that the relation "between" need not be inferred, but can be "read off" by some non inferential means such as by pattern matching).*
- *If we represent three objects A, B, and C, then it would always be the case that the distance from A to B plus the distance from B to C would never be less than the distance from A to C (i.e., the triangle inequality of measure theory would hold so that AB + BC ≥ AC).*
- *If we represent three objects A, B, and C so that AB is orthogonal to BC, then for short distances AB, BC, and CD it would be the case that $AC^2 + AB^2 = BC^2$ (i.e., distances would be locally Euclidean so that the Pythagorean theorem would hold for short distances).*

These problems dissolve as epiphenomenal if we consider that it is a non-euclidean geometry that describes mental spaces. OUR THESIS**:** several geometrical models coexist in our minds and their frames are activated whenever necessary.

- The performance of multitask computations is a known fact in human brain, controlling both conscious and unconscious bodily functions, and sometimes interchanging. In the light of such a state of affairs, we will circumscribe ourselves to the best known geometrical models, each of which is understood as a model of apprehension of the phenomenological world, and "activates" in a workspace W according to a scale of simplicity:

  **(4)** Fractal >> Elliptical >> Hyperbolic >> Euclidean
- To conceive a Euclidean space requires less use of working capacity because its basic tenets are the default case for our perception. This means that if a phenomenon can be conceptualized using Euclidean geometry, then there is no need to activate (say) hyperbolic working spaces.
- If a Euclidean workspace is not enough, i.e., the interface effects we get from activating a hyperbolic W outnumber those we would get from a Euclidean W, then another frame is activated by interpretative interface requirement .
- The human mind can conceptualize what it has never perceived (like hypercubes), but, apparently, there is a limit determined by interface conditions .
- Our model of the mind, therefore, has more promising properties than that of Pylyshyn, due to the fact that we do not restrict its capacities a priori: multidimensional workspaces are a mathematical reality, and if the physical world is in itself a mathematical structure, our model has a great potential to arise as a plausible theory of the mind-brain as a physical system, its properties being determined by more general requirements of the structure of reality. Taking a very basic concatenation operation as a primitive, the rest of the system is developed taking into account the specific properties of the relevant interpretative systems, physically and biologically constrained.

## SELECTED REFERENCES

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