

Scalarity and additivity in natural language: (II) comparatives

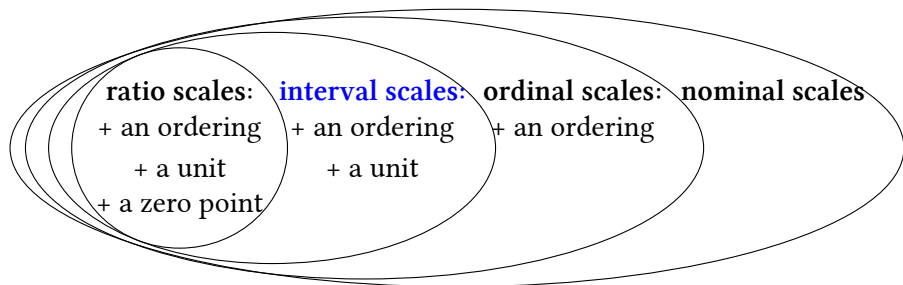
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Slides are available on lingbuzz:
<https://lingbuzz.net/lingbuzz/008301>

Recapitulation

- Measurement means mapping an entity (or an event) to a value along a scale according to rules (see [Stevens 1946](#)).
- A **scale** is a set that includes all potential values resulting from a certain way of taking measurement.
- There are **4 levels of scales**: nominal scales, ordinal scales, interval scales, and ratio scales.
- **Comparatives assume interval scales** (i.e., with an **ordering** and a **unit** that supports the measurement of differences).



Today

- Day 1: Basics of scales and degrees; how they are relevant to natural language
 - What are scales? What are their formal properties? What operators do they support?
- Day 2 and Day 3: Comparatives and *-er/more*
 - How an additivity-based perspective improve our understanding of scalarity-related phenomena?
 - What is additivity?
- Day 4 and Day 5: *Even* and its cross-linguistic siblings
 - How a scalarity-based perspective improve our understanding of additivity-related phenomena?

Today: English Comparatives and the use of *-er/more*



Mephisto is talking about poor farmer Faustus:

'He always wanted *more* ...
more free time and *more* wealth ...
fewer struggles, *better* health ...
work *less* and eat all day ...'

(From *Sapiens: A Graphic History*, Vol.2,
by David Vandermeulen, David Casanave, Yuval Noah Harari, 2021)

Outline

- 1 English clausal comparatives and the classical analysis (to be revisited)
- 2 Empirical and conceptual challenges to the classical analysis
- 3 A new perspective on *-er/more*
- 4 Comparatives: from an inequality-based view to a difference-based view

Ontological assumptions in analyzing comparatives

- From now on, we focus on **comparatives** and use ‘scales’ to mean interval scales (scales with ordering and units, supporting the measurement of differences).
- **Measurement** means mapping an entity / event to a degree along a relevant (interval) scale (e.g., height, time line, time length).
- **Degrees** are like real numbers (see also **Fox and Hackl 2006**: the universal density of measurement)

$$(1) \quad \{d \mid -\infty < d < +\infty\}$$










\leadsto a scale including all potential degree values

$$(2) \quad \{d \mid 0 < d \leq 7'\}$$

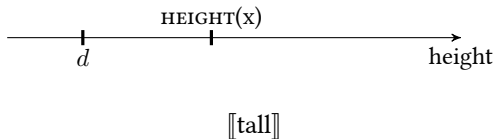
\leadsto a part of a scale of height / length, including degrees between 0 and 7 feet (the upper bound is included; the lower bound is not included)

- Most works on comparatives are explicitly or implicitly based on these assumptions: e.g., **Seuren (1973)**, **von Stechow (1984)**, **Heim (1985)**, **Kennedy (1999)**, **Schwarzchild and Wilkinson (2002)**, **Zhang and Ling (2021)** (cf. **Cresswell 1976** does not consider ‘degree’ a primitive type; the delineation approach adopted by e.g., **Klein 1980**, **Burnett 2017**)

Illustrations: scales and parts of scales

(a, b)	$\{x : a < x < b\}$	
$[a, b]$	$\{x : a \leq x \leq b\}$	
$(a, b]$	$\{x : a < x \leq b\}$	
$[a, b)$	$\{x : a \leq x < b\}$	
(a, ∞)	$\{x : x > a\}$	
$[a, \infty)$	$\{x : x \geq a\}$	
$(-\infty, b)$	$\{x : x < b\}$	
$(-\infty, b]$	$\{x : x \leq b\}$	
$(-\infty, \infty)$	\mathbb{R}	

The meaning of gradable adjectives



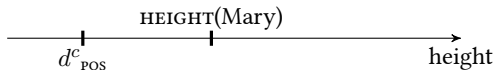
[[tall]]: a relation between an individual and a degree

- (3) $[[\text{tall}]]_{\langle d, et \rangle} \stackrel{\text{def}}{=} \lambda d_d. \lambda x_e. \text{HEIGHT}_{\langle e, d \rangle}(x) \geq d$ (i.e., x is d -tall)
On the scale of height, the position of x **meets or reaches degree d** .

- There are two pieces in this lexical entry
 - A measure function of type $\langle ed \rangle$: $\text{HEIGHT}_{\langle e, d \rangle}(x)$
 - Indicating the direction (of comparison): $\geq d$ (cf. Kennedy 1999)

(See e.g., Cresswell 1976, Hellan 1981, von Stechow 1984, Heim 1985, Schwarzschild 2008, Beck 2011)

Major uses of gradable adjectives: Positive use



[[Mary is POS tall]]

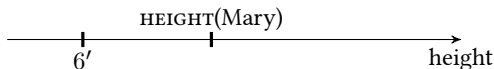
[[tall]]: a relation between an individual and a degree

- (3) $[[\text{tall}]]_{\langle d, et \rangle} \stackrel{\text{def}}{=} \lambda d_d. \lambda x_e. \text{HEIGHT}_{\langle e, d \rangle}(x) \geq d$ (i.e., x is d -tall)
On the scale of height, the position of x **meets or reaches degree d** .

- (4) $[[\text{Mary is POS tall}]] \Leftrightarrow \text{HEIGHT}(\text{Mary}) \geq d^c_{\text{pos}}$ **Positive use**
(pos: the context-dependent threshold for tallness)

(See e.g., Cresswell 1976, Hellan 1981, von Stechow 1984, Heim 1985, Kennedy 1999, Schwarzschild 2008, Beck 2011)

Major uses of gradable adjectives: Measurement sentence



[[Mary is 6 feet tall]]

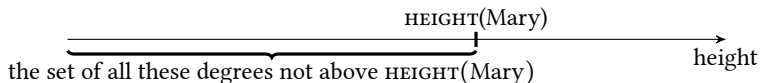
[[tall]]: a relation between an individual and a degree

- (3) $[[\text{tall}]]_{\langle d, et \rangle} \stackrel{\text{def}}{=} \lambda d_d. \lambda x_e. \text{HEIGHT}_{\langle e, d \rangle}(x) \geq d$ (i.e., x is d -tall)
On the scale of height, the position of x **meets or reaches degree d** .

- (5) $[[\text{Mary is 6 feet tall}]] \Leftrightarrow \text{HEIGHT}(\text{Mary}) \geq 6'$ **Measurement**

(See e.g., Cresswell 1976, Hellan 1981, von Stechow 1984, Heim 1985, Kennedy 1999, Schwarzschild 2008, Beck 2011)

Major uses of gradable adjectives: Degree question



$\llbracket \text{how tall is Mary} \rrbracket$

$\llbracket \text{tall} \rrbracket$: a relation between an individual and a degree

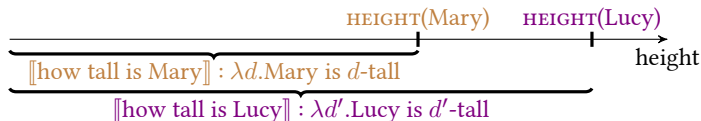
- (3) $\llbracket \text{tall} \rrbracket_{\langle d, et \rangle} \stackrel{\text{def}}{=} \lambda d_d. \lambda x_e. \text{HEIGHT}_{\langle e, d \rangle}(x) \geq d$ (i.e., x is d -tall)
On the scale of height, the position of x **meets or reaches degree d** .

- (6) $\llbracket \text{how tall is Mary} \rrbracket \Leftrightarrow \lambda d. \text{Mary is } d\text{-tall}$
 $\Leftrightarrow \lambda d. \text{HEIGHT}(\text{Mary}) \geq d$

Degree question

(See the categorial approach to questions represented by Hausser and Zaefferer 1978.)

Major uses of gradable adjectives: Clausal comparative



[[Lucy is taller than Mary is tall]]

(7) The bathtub is **wider** than the door is **tall**. subcomparative

(8) [[**Lucy is tall** er **than Mary is tall**]]

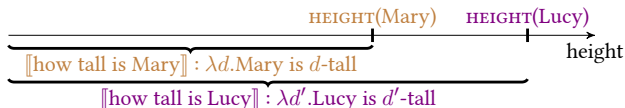
how tall Lucy is

how tall Mary is

LF: [-er [λd .Mary is d -tall]] [$\lambda d'$.Lucy is d' -tall]

(See e.g., von Stechow 1984, Kennedy 1999, Schwarzschild 2008, Beck 2011 for a review)

The role of *-er/more* in the canonical analysis



[[Lucy is taller than Mary is tall]]

(8) [[Lucy is tall er than Mary is tall]]

how tall Lucy is

how tall Mary is

LF: [-er [λd .Mary is d -tall]] [$\lambda d'$.Lucy is d' -tall]

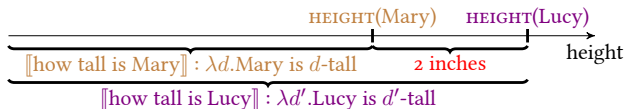
[-er/more] performs comparison by encoding an inequality

(9) $[-\text{er/more}]_{\langle \langle dt \rangle, \langle dt, t \rangle \rangle} \stackrel{\text{def}}{=} \lambda D_{\text{than}}. \lambda D_{\text{matrix}}. \exists d [d \in D_{\text{matrix}} \wedge \neg [d \in D_{\text{than}}]]$

(10) $[-\text{er/more}]_{\langle \langle dt \rangle, \langle dt, t \rangle \rangle} \stackrel{\text{def}}{=} \lambda D_{\text{than}}. \lambda D_{\text{matrix}}. \text{MAX}(D_{\text{matrix}}) > \text{MAX}(D_{\text{than}})$

(See e.g., von Stechow 1984, Kennedy 1999, Schwarzschild 2008, Beck 2011 for a review)

Numerical differentials in the canonical analysis



$\llbracket \text{Lucy is 2 inches taller than Mary is tall} \rrbracket$

$$(11) \quad \llbracket \text{Lucy is 2 inches taller than Mary is tall} \rrbracket$$

$$\text{LF: } [-\text{er } 2'' [\lambda d.\text{Mary is } d\text{-tall}]] [\lambda d'.\text{Lucy is } d'\text{-tall}]$$

- Without a numerical differential: $\llbracket -\text{er/more} \rrbracket$ is of type $\langle \langle dt \rangle, \langle dt, t \rangle \rangle$

$$(10) \quad \llbracket -\text{er/more} \rrbracket \stackrel{\text{def}}{=} \lambda D_{\text{than}}. \lambda D_{\text{matrix}}. \text{MAX}(D_{\text{matrix}}) > \text{MAX}(D_{\text{than}})$$

- With a numerical differential: $\llbracket -\text{er/more} \rrbracket$ is of type $\langle d, \langle \langle dt \rangle, \langle dt, t \rangle \rangle \rangle$

$$(12) \quad \llbracket -\text{er/more} \rrbracket \stackrel{\text{def}}{=} \lambda d_{\text{diff}}. \lambda D_{\text{than}}. \lambda D_{\text{matrix}}. \text{MAX}(D_{\text{matrix}}) \geq \text{MAX}(D_{\text{than}}) + d_{\text{diff}}$$

(See e.g., von Stechow 1984, Kennedy 1999, Schwarzschild 2008, Beck 2011 for a review)

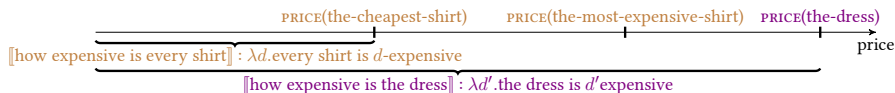
Interim summary of the canonical view

	Without a numerical differential	With a numerical differential
Assumption	(Ordinal/interval) scales	Interval scales
Comparison	Inequality: $\text{measurement}_1 > \text{measurement}_2$	Inequality: $\text{measurement}_1 \geq \text{measurement}_2 + d$
Representations of & operations on scalar values	Degree points & ordering between degree points	Degree points & a combination of ordering and addition
The semantics of -er/more	Ordering: $\lambda m_2. \lambda m_1. m_1 > m_2$	A combination of ordering and addition: $\lambda d. \lambda m_1. \lambda m_2. m_2 \geq m_1 + d$

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When the *than*-clause has a universal quantifier



[[The dress is more expensive than every shirt is expensive]]

- (13) Scenario: the price of the shirts ranges from \$20 to \$100 and the dress costs \$150.

[[the dress is more expensive than every shirt is expensive]]

how expensive the dress is

how expensive every shirt is

LF: [more [λd .every shirt is d -expensive]] [$\lambda d'$.the dress is d' -expensive]

$= \{ d \mid 0 \leq d \leq \text{PRICE}(\text{the-cheapest-shirt}) \}$

$= \{ d' \mid 0 \leq d' \leq \text{HEIGHT}(\text{the-dress}) \}$

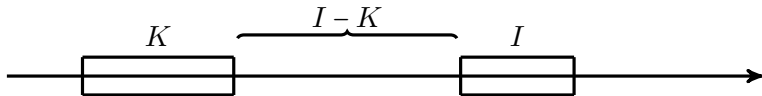
- **Our intuition:**

the dress is more expensive than **the most expensive** shirt is.

- **The analysis under the canonical view:**

the dress is more expensive than **the least expensive** shirt is.

Schwarzchild and Wilkinson (2002)'s solution: intervals



(14) $\llbracket \text{expensive} \rrbracket \stackrel{\text{def}}{=} \lambda I. \lambda x. \text{PRICE}(x, I)$ (i.e., Interval I covers individual x .)

(15) The dress is more expensive than every shirt is.

a. **than-clause:** $\lambda K. \forall x [\text{shirt}(x) \rightarrow \text{PRICE}(x, K)]$

b. **matrix clause:** $\lambda I. \text{HEIGHT}(\text{the-dress}, I)$

The semantics of comparison

(16) $\text{MATRIX-CLAUSE}(\mu I [\text{THAN-CLAUSE}(\mu K [\text{DIFF}(I - K)])])$

The differential predicate **DIFF** holds for each gap between any subpart of the interval $I_{\text{main-clause}}$ and any subpart of the $K_{\text{than-clause}}$.

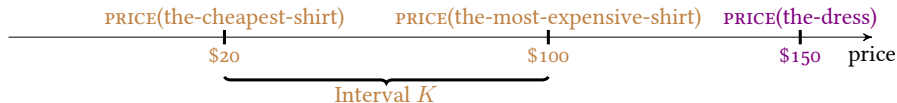
Beck (2010)'s comment

- $\text{MATRIX-CLAUSE}(\mu I[\text{THAN-CLAUSE}(\mu K[\text{DIFF}(I - K)]))$
Schwarzchild and Wilkinson (2002): embedding ‘ $\text{DIFF}(I - K)$ ’ within the scope of two maximality operators μ
 - ▶ The standard of comparison K is no longer a scalar value independent of comparison.
 - ▶ K is eventually yielded as the largest interval that makes DIFF hold for all the gaps involved in the numerous sub-interval-level comparisons.

Beck (2010)

‘I want to come out of the calculation of the semantics of the *than*-clause holding in my hand *the* degree we will be comparing things to.’

Beck (2010)'s solution



[[The dress is (up to \$60) more expensive than every shirt is expensive]]

- Beck (2010): using the largest value in the interval K for comparison.

- (15) The dress is more expensive than every shirt is. ✓
- (17) The dress is up to \$60 more expensive than every shirt is.
↪ False under our scenario that the dress costs \$150 and shirts vary between \$20 and \$100, but true under the analysis of Beck (2010)

(See Fleisher 2016)

Issues to be solved

(17) The dress is **up to \$60** more expensive than every shirt is.

- The standard of comparison, i.e., the meaning of the *than*-clause, should be able to represent a **range of values**.
 - Beck (2010): ‘I want to come out of the calculation of the semantics of the *than*-clause holding in my hand *the* degree we will be comparing things to.’ (cf. Schwarzschild and Wilkinson 2002)
 - This range of values should not be reduced to a single degree point. (cf. Beck 2010)

Conceptual challenge: what does *-er/more* do?



Mephisto is talking about poor farmer Faustus:

'He always wanted *more* ...
more free time and *more* wealth ...
fewer struggles, *better* health ...
work *less* and eat all day ...'

(From *Sapiens: A Graphic History*, Vol.2,
by David Vandermeulen, David Casanave, Yuval Noah Harari, 2021)

Conceptual challenge: what does *-er/more* do?

- *-er/more* can be preceded by definite determiner *the*:

- (18)
- a. The more you read, the more you learn.
 - b. The taller you are, the less mobile and quick you are.

- *-er/more* can be used repeatedly:

- (19)
- a. Lucy is tall er and tall er and taller.
 - b. We will have more and more money.

- These data are unexpected if *-er/more* essentially encodes an inequality, meaning ' $\lambda m_2. \lambda m_1. m_1 > m_2$ ' or ' $\lambda d. \lambda m_1. \lambda m_2. m_2 \geq m_1 + d$ '.

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Parallelism between *-er/more* and *another*

- *-er/more* has an **additive use** similar to *another* (see also Greenberg 2010 and Thomas 2010):

(20) Increase in the domain of entities: Additive use

- a. I ate $\underbrace{\text{an}^x \text{ apple}}_{\text{base item}}$. Then I ate $\underbrace{\text{another}^y \text{ (apple)}}_{\text{increase}}$.
- b. $\underbrace{\text{A}^x \text{ girl}}_{\text{base item}}$, Sue, met $\underbrace{\text{another}^y \text{ girl}}_{\text{increase}}$, Mary.
- c. I ate $\underbrace{\text{two}^x \text{ bars of chocolate}}_{\text{base item}}$. Then I ate $\underbrace{\text{(a bit) more}^y}_{\text{increase}}$.

From the additive use to the comparative use of *-er/more*

- Additive use of *more*: in the domain of **entities**
- Comparative use of *-er/more*: in the domain of **scalar values** (i.e., degrees)

(20c) Increase in the domain of entities: Additive use

I ate $\underbrace{\text{two}^x \text{ bars of chocolate}}_{\text{base item}}$. Then I ate $\underbrace{(\text{a bit}) \text{ more}^y}_{\text{increase}}$.

(21) Increase in the domain of scalar values: Comparative use

a. Mary is tall. Sue is tall er . Across sentences
 base item: HEIGHT(Mary) increase

b. Sue is tall **er** than Mary is tall. Within the same sentence

increase base item:
HEIGHT(Mary)

More uses of *-er/more* and *another*

- *-er/more* and *another*

- denotes an increase in the domain of entities or scalar values
- presuppose there is a salient base that the increase is anaphoric to

(22) Repetitive use of *-er/more* and *another*

- Lucy is becoming taller and taller and taller.
- Janice had a little lamb and another and another and another.

(23) Accumulating increases along with a universal quantifier

- Every year Mary wrote a more interesting book.
- Everyday there is another story to write.

(24) They can all be preceded by definite determiner *the*:

- The more you read, the more you learn.
- The taller you are, the less mobile and quick you are.
- I've brought two books: one is *Le Petit Prince*, and the other is *Exhalation*.

What gets ‘increased’? What does additivity mean?

- **Additivity** should be considered a phenomenon of **QUD-based anaphoricity**, indicating an extension of a previous salient answer in addressing the QUD
 - ▶ For the **additive use** in the domain of entities, *more* / *another* indicates an **increase from a part to a whole**.

(25) Additive use of *more*

Current question (CQ): What did you eat?

- a. I ate two bars of chocolate. Then I ate (a bit) more.

base item:
a partial answer to the CQ

increase

- b. #I didn't eat a bar of chocolate. Then I ate more. (see Li 2023)


(e.g., Roberts 1996/2012, Zeevat 2004, Zeevat and Jasinskaja 2007, Beaver and Clark 2009, Thomas 2011, Zhang and Ling 2021)

The anaphoricity of *-er/more*

- **Additivity** should be considered a phenomenon of **QUD-based anaphoricity**, indicating an extension of a previous salient answer in addressing the QUD
 - For the **comparative use** in the domain of scalar values, *-er/more* indicates an **increase from a lower to a higher scalar value**.

(26) **Comparative use of *-er/more***
Current question (CQ): How tall is Sue?

Mary is not tall. Sue is tall **er** .


base item – a partial answer
to the CQ: HEIGHT(Mary)


increase

(e.g., Roberts 1996/2012, Zeevat 2004, Zeevat and Jasinskaja 2007, Beaver and Clark 2009, Thomas 2011, Zhang and Ling 2021)

The semantics of English *-er/more*

- The canonical view:

	Without a numerical differential	With a numerical differential
Comparison	Inequality: $\text{measurement}_1 > \text{measurement}_2$	Inequality: $\text{measurement}_1 \geq \text{measurement}_2 + d$
The semantics of <i>-er/more</i>	Ordering: $\lambda m_2. \lambda m_1. m_1 > m_2$	A combination of ordering and addition: $\lambda d. \lambda m_1. \lambda m_2. m_2 \geq m_1 + d$

- In contrast to this canonical view, English *-er/more* works like *another* in being an **additive particle**, denoting an **increase on a discourse-salient base**, extending this existing base in addressing the Current Question.

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Comparatives: A difference-based view



[[Lucy is taller_{er} than Mary is tall]]

(27) [[Lucy is tall _{er} than Mary is tall]]

how tall Lucy is

how tall Mary is

$\leadsto \text{HEIGHT}(\text{Lucy}) - \text{HEIGHT}(\text{Mary}) = [-\text{er}]$

- [-er] denotes an increase, i.e., a positive value.

Comparatives: A difference-based view

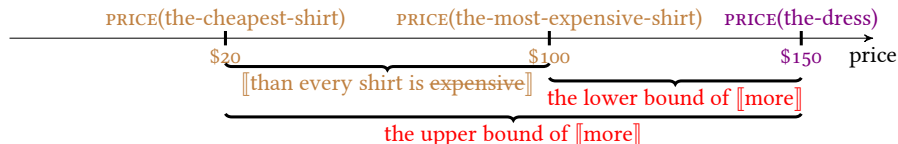


[[Lucy is 2 inches taller than Mary is tall]]

(28) $\llbracket \underbrace{\text{Lucy is 2 inches tall}}_{\text{how tall Lucy is}} \text{er} \underbrace{\text{than Mary is tall}}_{\text{how tall Mary is}} \rrbracket$
 $\leadsto \text{HEIGHT}(\text{Lucy}) - \text{HEIGHT}(\text{Mary}) = \llbracket \text{2 inches ...-er} \rrbracket$

- **“2 inches ...-er”** denotes an increase **with a specified size**, i.e., 2 inches.

Comparatives: A difference-based view



[[The dress is **more** expensive than every shirt is expensive]]

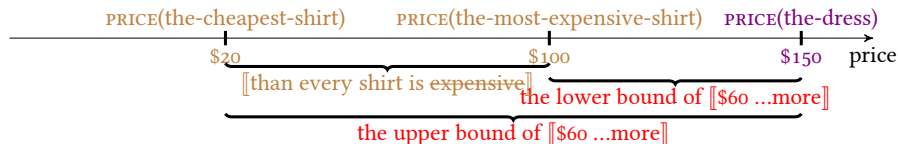
(29) [[The dress is **more** expensive than every shirt is expensive]]

how expensive the dress is how expensive every shirt is

\leadsto PRICE(the dress) – the interval that ranges from
 PRICE(the-cheapest-shirt) to PRICE(the-most-expensive shirt) = [[more]]

- [[more]] denotes an increase, i.e., a positive value.

Comparatives: A difference-based view



[[The dress is **up to \$60 more** expensive than every shirt is expensive]] (false under this scenario)

(30)

[[The dress is **up to \$60 more** expensive than every shirt is expensive]]

how expensive the dress is how expensive every shirt is

\leadsto PRICE(the dress) – the interval that ranges from










PRICE(the-cheapest-shirt) to PRICE(the-most-expensive shirt) = [[**up to \$60 more**]]

- [[**up to \$60 more**]] denotes an increase with a specific size, i.e., $(-\infty, \$60]$
 \leadsto the upper bound of the increase is \$60

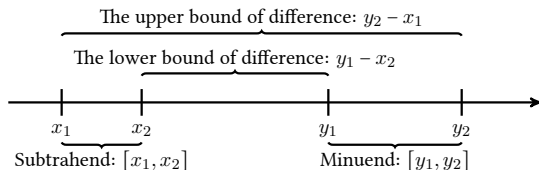
From degrees to intervals

- **Degrees:** points on a scale (an interval scale à la [Stevens 1946](#))
- **Intervals:** convex sets of degrees
 - **Convex set:** A totally ordered set P is **convex** iff for any elements a and b in the set (suppose $a \leq b$), any element x such that $a \leq x \leq b$ is also in the set P .
 - E.g., $\{x \mid x > 0\}$, $\{x \mid x \leq 4\}$, and $\{x \mid 4 \leq x \leq 8\}$ are all convex sets.
 - Sets like $\{x \mid x > 10 \vee x \leq 3\}$ are not convex.
- **Notation of intervals:**
 - $\{x \mid x > 0\} = (0, +\infty)$
 - $\{x \mid x \leq 4\} = (-\infty, 4]$
 - $\{x \mid 4 \leq x \leq 8\} = [4, 8]$

Intervals

(a, b)	$\{x : a < x < b\}$	
$[a, b]$	$\{x : a \leq x \leq b\}$	
$(a, b]$	$\{x : a < x \leq b\}$	
$[a, b)$	$\{x : a \leq x < b\}$	
(a, ∞)	$\{x : x > a\}$	
$[a, \infty)$	$\{x : x \geq a\}$	
$(-\infty, b)$	$\{x : x < b\}$	
$(-\infty, b]$	$\{x : x \leq b\}$	
$(-\infty, \infty)$	\mathbb{R}	

Subtraction between intervals



$$(31) \quad \underbrace{[y_1, y_2]}_{\text{minuend: matrix}} - \underbrace{[x_1, x_2]}_{\text{subtrahend: comparative standard}} = \underbrace{[y_1 - x_2, y_2 - x_1]}_{\text{difference: differential}}$$

a. Example 1: $[5, 8] - [1, 3] = [2, 7]$

b. Example 2: $(4, +\infty) - [2, 3] = (1, +\infty)$

- We are now ready to use **intervals** and **interval subtraction** to provide a systematic, generalized formal analysis for all kinds of comparatives.
 - ▶ With *than*-internal quantifiers and/or various numerical differentials
 - ▶ Crucially based on an **additivity/increase**-based view of *-er/more*

(See Moore 1979)

Today's take-home messages

- Day 2 and Day 3: Comparatives and *-er/more*
 - How an additivity-based perspective improve our understanding of scalarity-related phenomena?
 - What is additivity?
- Additivity is a phenomenon of QUD-based anaphoricity, indicating an extension of a previous salient answer in addressing the QUD.

	The canonical view	The new difference-based view
Assumption	(Ordinal/interval) scales	Interval scales
Comparison	Inequality: $M_1 > M_2$	Subtraction: $M_1 - M_2 = D$
Representations of ⌚ operations on scalar values	Degree points ⌚ ordering between degree points	Intervals (i.e., set of degrees) ⌚ interval subtraction
The semantics of <i>-er/more</i>	Ordering: >	Additivity a default positive difference: $(0, +\infty)$

Tomorrow

- Day 2 and Day 3: Comparatives and *-er/more*
 - How an additivity-based perspective improve our understanding of scalarity-related phenomena?
 - What is additivity?
- Tomorrow
 - Formal implementation (see Zhang and Ling 2021 and Zhang and Zhang 2024)
 - Antonyms
 - Cross-linguistic phenomena
 - etc.

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