

Does Montague’s PTQ provide a fragment of English?

Patrick Muñoz

1 Introduction

Richard Montague begins ‘The proper treatment of quantification in ordinary English’ (PTQ)¹ as follows:

“The aim of this paper is to present in a rigorous way the syntax and semantics of a certain fragment of a certain dialect of **English**.”
– Montague (1973: 221) [my emphasis]

Crucially, he claims to be providing a fragment of English, a natural language, and not of some auxiliary formal language, or of an English-like language supplemented with non-English expressions similar to those typical of the artificial languages of mathematical logic.

This qualification to the Montagovian project is important, because at the time Montague, along with certain other mathematicians, was making claims about how the tools used to treat formal languages were equally suitable to the treatment of natural languages. For example, Montague opens ‘English as a formal language’ as follows:²

“I reject the contention that an important theoretical difference exists between formal and natural languages.”
– Montague (1970: 188)

This statement echoes one made by Alonzo Church, in ‘The need for abstract entities in semantic analysis:’

“Although the foregoing account has been concerned with the case of a formalized language, I would go on to say that in my opinion there is no difference in principle between this case and that of one of the natural languages [...] The difference of

¹I’ll use ‘PTQ’ to refer both to the paper and to the fragment presented in it.

²‘English as a formal language’ also includes a claim to the effect that the fragment provided there at least approximates a fragment of English: “In the present paper I shall accordingly present a precise treatment, culminating in a theory of truth, of a formal language that I believe may be reasonably regarded as a fragment of ordinary English” (Montague 1971: 188). I don’t know if it’s significant that the programmatic claim here is more hedged than the one in PTQ, but it should be noted that the fragment in ‘English as a formal language’ can’t reasonably be regarded as a fragment of English, since it employs non-English expressions in the form of variables, and so includes non-English sentences like ‘ v_0 loves v_1 .’ This is effectively the same thing that PTQ does; the reason that the non-English nature of PTQ is less obvious is that the variables in PTQ are superficially made to look like English pronouns, which they are not (see below).

a formalized language from a natural language lies not in any matter of principle, but in the degree of completeness that has been attained in the laying down of explicit syntactical and semantical rules and the extent to which vaguenesses and uncertainties have been removed from them. For this reason the English language itself may be used as a convenient though makeshift illustration of a language for which syntactical and semantical rules are to be given.”

– Church (1951: 106-107)

What exactly it means for there to be no ‘important theoretical difference,’ or no ‘difference in principle,’ between formal and natural languages isn’t obvious, but presumably it at least means that one can build fragments of natural languages using tools familiar from formal logic. If reasonably large and accurate fragments of natural languages can’t be built in this way, then the Montagovian hunch is wrong. And so it’s worth asking whether Montague himself makes good on his claim: that is, does Montague actually provide a fragment of English in PTQ?

The fragment provided in PTQ is not a fragment of English. It’s rather a fragment of an English-like formal language, which resembles a small fragment of English, but which also includes non-English auxiliary expressions that have no analogue in English or, to my knowledge, in any natural language. In particular, it makes use of certain variable expressions, made superficially to look like English pronouns, which are analogous to the variables of certain formal languages, but which have no counterpart in the natural language that Montague claims to give a fragment of.

What’s more, these non-English expressions are integral to the functioning of the fragment, since they’re necessary for deriving the key results that Montague emphasizes in PTQ. The core virtues of PTQ that Montague touts therefore depend crucially on the very materials that make it not a fragment of English, and so it’s precisely the constructions in English that he is most intent on capturing that **fail** to be treated by the formal tools he uses: these constructions require him to introduce non-English materials, and so to apply his tools to a formal language that contains constructions that simply don’t exist in English.

Does it matter that PTQ doesn’t provide a fragment of English? It does to the extent that semanticists share the Montagovian impulse to use the tools inherited from mathematical logic to provide fragments of natural languages: Montague’s own work doesn’t justify this impulse, and so to substantiate it, some reasonably large and accurate fragment of a natural language using the tools of mathematical logic must be provided elsewhere. It’s not obvious that this has ever been done.

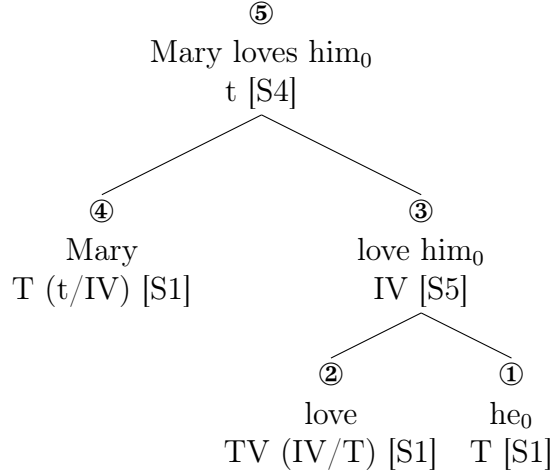
And so Montague’s PTQ serves as a case study to reopen the methodological issue in natural language semantics: are the tools of mathematical logic suitable to providing fragments of natural languages? Whatever the case, we have to put up or shut up. Until a fragment in the spirit of Montague can demonstrably be built, we ought to remain open to the possibility that the inheritance semantics has taken over from mathematical logic is not in general suitable to its subject matter.³

³Of course, one can always render the Montagovian claim difficult to falsify by expanding the tools available to the mathematical logician, or insisting that it’s unclear which tools fall within the domain of mathematical logic. But then it falls to the one insisting on a Montagovian picture to clarify what tools are actually needed to write fragments of natural languages, and why they can reasonably be claimed to belong

2 Montague's indexed pronouns aren't in English

The set of basic expressions of category T ('terms') in PTQ includes a denumerably infinite number of pronouns subscripted with numerical indices: he_0 , he_1 , he_2 , etc. (these are members of the set B_T : cf. Montague 1973: 223). These pronouns act as variable expressions, similar to the variables found in various formal languages; but as Montague presents them, they are not expressions of English, and so neither are any expressions that contain them.⁴

To illustrate: given the syntactic rules of PTQ (cf. *ibid.* §2.1), the fragment includes sentences like *Mary loves him₀*:⁵



This sentence is then given the following interpretation, which isn't the right interpretation for 'Mary loves him' in English:⁶

to mathematical logic as well. My only goal here is to show that the tools of mathematical logic as Montague uses them don't bear out his claim.

⁴There are a number of other expressions in PTQ that arguably are not in English. For example, the fragment treats verb phrase modifiers as iterable operators, yielding sentences like *Mary loves John slowly slowly rapidly slowly*, which insofar as it's interpretable in English, wouldn't have the interpretation assigned to it by PTQ (its true interpretation being not some combination of redundant and contradictory, as the grammar would predict, but rather inappropriate for an academic setting). Even a modalization of one of the examples Montague spends some time on, *Necessarily the price rises*, strikes me as at best only marginal English: even without the somewhat odd preposing of the adverb, i.e. even with *The price necessarily rises*, I'd need a pretty rich context to give it a plausible interpretation, and even then, it doesn't plausibly mean what PTQ says it does (and a more natural universally quantified sentence, like *The price always rises* or *The price has to rise* are even less plausibly captured by the necessity operator appealed to by Montague). These sorts of sentences involve what we might call 'para-English,' which is to say, English that is not at all 'ordinary,' but is rather translated from some formal language that contains things like iterable operators and sentence-scoping all-purpose necessity modals. It's difficult to know what to make of claims about 'para-English,' since it doesn't really exist outside of a stipulatory context: I leave this sort of thing to the side, because I think the foundational issue of the status of indexed pronouns is more important.

⁵These analysis trees are meant to resemble the ones that Montague himself provides in PTQ. Each labeled node lists an expression of the language, alongside its syntactic category and the syntactic rule taken from *ibid.* §2.1 that justifies its derivation from the expression(s) it immediately dominates in the tree. There's a slight discrepancy with Montague's own trees, which list not the rule justifying the derivation of each expression, but rather the syntactic operation that the rule licenses: since multiple rules cause the same operation to be applied, listing the number of the rule instead is clearer.

⁶Each numbered entry gives the interpretation of the corresponding numbered node in the analysis tree above. I use the translations of the expressions into Montague's intensional logic as a shorthand for this

- ① $\hat{P}P\{x_0\}$
[T1e]
- ② love'
[T1a]
- ③ $\text{love}'(\hat{P}P\{x_0\})$
[T5]
- ④ m^*
[T1d]
- ⑤ $m^*(\wedge \text{love}'(\hat{P}P\{x_0\}))$
 $= \hat{P}[P\{\wedge m\}](\wedge \text{love}'(\hat{P}P\{x_0\}))$
 $= \wedge \text{love}'(\hat{P}P\{x_0\})\{\wedge m\}$
 $= \text{love}'(\hat{P}P\{x_0\})(\wedge m)$
 $= \text{love}'(\wedge m, \hat{P}P\{x_0\})$
 $= \text{love}'_*(m, \vee x_0)$
[T4]

That is, *Mary loves him₀* is true just in case Mary loves whatever the variable x_0 maps to. Before commenting on why this interpretation is wrong, a couple of preliminary comments are in order about this sentence. First, since its translation into Montague's intensional logic (IL) contains a free variable x_0 , it receives a truth value only relative to a variable assignment, and not relative to an interpretation and point of evaluation *simpliciter*.⁷ In logicians' terms, the translation of *Mary loves him₀* is an open formula, not a sentence.

It's not clear if this has any significance, i.e. if it means that sentences like *Mary loves him₀* are supposed to be somehow deficient as interpretable expressions: if so, maybe it's beside the point to show that the interpretation assigned to it isn't right for English, because PTQ doesn't intend to treat it as a normally interpretable expression in some sense (maybe such free-variable-introducing expressions are to have their variables bound off to yield expressions that translate as sentences of IL, and only these are supposed to be intuitively judged for their truth values as English sentences).

But Montague makes no comment to this effect, and the translation of *Mary loves him₀* is included in the set of meaningful expressions of IL, so I'll assume that expressions that introduce free variables into their translations are supposed to be meaningful in the ordinary

interpretation, and retain Montague's own conventions in writing the expressions of the intensional logic. The interpretation of each node is followed by the rule of translation in *ibid.* §4.1 that justifies it.

⁷Cf. *ibid.* 231: the interpretation rule (2) guarantees that the variable (and so the formula as a whole) has an extension relative to an arbitrary interpretation, point of evaluation, and assignment, but the extensions of a formula at a point of evaluation must agree on every assignment for that formula to receive a truth value (or at least, the value of *true*) relative to an interpretation and point of evaluation *simpliciter*: "If ϕ is a *formula* (that is, a member of ME_t), then ϕ is *true* with respect to \mathfrak{U}, i, j if and only if $\phi^{\mathfrak{U}, i, j, g}$ is 1 for every \mathfrak{U} -assignment g ." This won't apply to the translation of *Mary loves him₀*, since its extension will change depending on the assignment chosen, according to what that assignment maps x_0 to.

way, and evaluated for truth the way an actual English sentence would be.⁸ If they're not, then there's an immediate problem, in that PTQ isn't be able to provide working interpretations for all of its expressions (which are meant to be, I remind the reader, expressions of **English**: *Mary loves him₀* is a well-formed sentence in the fragment, and the fragment is meant to be one of **English**), meaning that it doesn't present an interpreted fragment of English.

Second, there's an obvious sense in which this sentence isn't an expression of English: it contains an indexed pronoun *him₀*, and this pronoun is not an expression of English, since English pronouns aren't indexed. While this is a serious point in its own right, I'll leave this issue to the side, and take for granted that the indexation is somehow a technical convenience that PTQ could in principle be modified to eliminate without loss of expressive power or empirical adequacy.⁹

The issue with this sentence is that the above interpretation is not the interpretation of 'Mary loves him' in English. The reason for this is that the English pronoun 'him' (at least when free) carries various features that restrict its reference, while *him₀* in PTQ does not. 'Him' is for instance a masculine pronoun, and so typically requires its referent to be non-grammatically masculine (usually in the world of the context of utterance) for its use to be felicitous.¹⁰

That the Montagovian pronoun has no such restriction can be seen from the translation of *he₀* in node 1: it introduces the variable x_0 , which according to the rules of IL receives a value relative to an assignment function (rule (2) in *ibid.* 231). Since it's a variable of type $\langle s, e \rangle$, its extension on an assignment is some individual concept or other (*ibid.* 232); however, there's no provision in the way variable assignments or the variables x_n introduced by the pronouns *he_n* are characterized that would place any restrictions on what sort of individual concept this is, and so there's no guarantee that it maps to an object that has to

⁸But it's not obvious what this means, since PTQ doesn't contain a recipe for correlating the interpretations provided for English sentences with the sorts of intuitive truth judgments those sentences are supposed to yield, and so what the difference between the interpretation of a sentence whose extension varies on the assignment and one whose extension doesn't so vary isn't spelled out. My point here is just that on any reasonable way of spelling this out, the interpretation for sentences with free pronouns isn't right.

⁹In PTQ itself, the indices on pronouns are not just a technical convenience: without the indices, the grammar simply doesn't work, since having distinctly-indexed pronouns is necessary for treating instances of homophonous free pronouns that receive distinct interpretations on the same assignment (and it's necessary that this be allowed, to capture the banal fact that homophonous free English pronouns contained in the same expression can have distinct referents). It's further not clear how PTQ could be modified to get rid of the indices, and obvious attempts to cast their presence as insignificant tend to have bizarre, unwelcome consequences: for instance, insisting that the indices are intended to have no phonological content, so that PTQ doesn't imply that English **overtly** indexes its pronouns (though what it would mean for English to **covertly** index its pronouns, I don't know), results in the claim that English contains an infinity of homophonous pronouns pronounced 'he' (or 'him'), which, insofar as it's an understandable claim (I'm not sure it is), is presumably false. In general, formal semantics is surprisingly lax about the use of such indexation devices, given that natural languages don't make use of them, although artificial formal languages do. Their inclusion thus seems to go against the Montagovian spirit of providing fragments of **natural** languages.

¹⁰'Him' is of course restricted in its reference in other ways as well: its felicitous use typically requires that its referent be animate (though maybe this is entailed by its being masculine), atomic, the demonstratum (where used demonstratively), and neither the speaker nor the addressee in most default discourse conditions. Any or all of these features of the pronoun could be used to make the same point: I use non-grammatical gender here just as a perspicuous example.

be masculine at a point of evaluation.

In short, x_0 is a featureless variable in IL, which effectively means that him_0 is a featureless variable in PTQ. This in turn means roughly that the interpretation of *Mary loves him_0* is that it's true (at a point of evaluation and on an assignment) just in case Mary loves the relevant individual, which need not be masculine. This is not what 'Mary loves him' means, and the discrepancy comes from the fact that him_0 (or he_0) does not mean what 'him' means.

There are no featureless pronouns, unrestricted in their reference when free, in English. All free English pronouns are restricted in some way by features of some combination (in)animacy, person, number, or non-grammatical gender, and all free pronouns in every natural language I'm aware of are restricted in their reference by similar features (sometimes not including features present in English, such as non-grammatical gender, and other times including features not present in [contemporary] English, such as honorific or obviational status). There are, in short, no pronouns in natural languages that act like variables in the way that PTQ's he_0 does.

Further, expressions like he_0 are not only not natural language pronouns, they are not expressions of a sort that exist in English at all, or in any natural language that I'm aware of.¹¹ PTQ therefore contains non-English expressions, and so as it stands, it's a fragment of a language containing certain artificial expressions that serve a technical convenience, and not a fragment of English.

3 The indexed pronouns are required

The question is then whether this use of artificial variables in PTQ is superfluous or not – that is, whether their inclusion is just a convenient artifice (like we assumed that indexation was), which could in principle be removed without damage to the spirit of the fragment. Is it possible, in other words, to achieve the main results touted in PTQ without crucially resorting to these artificial expressions?

No, it is not – in fact, the indexed pronouns are obligatory in order to derive most of the major results that Montague is concerned to present in PTQ. These same results could not be achieved if these artificial expressions were excluded, unless the grammar were totally rewritten. The reason for this is that the indexed pronouns of PTQ don't just appear as free, assignment-relative expressions. They also serve another crucial function: they act as expressions that are to be replaced during one of two kinds of quantificational operation. These quantificational operations are needed in order to derive a number of interpretations of English sentences that Montague is concerned to account for, and they in turn require the presence of indexed pronouns to function non-vacuously. It'll help to show this by example, after explaining the operations themselves.

The first kind of quantificational operation in PTQ is 'quantifying in,' which (i) replaces the leftmost indexed pronoun in a phrase that bears a certain index with a quantifier (determiner + common noun), and (ii) replaces all other indexed pronouns in that phrase bearing

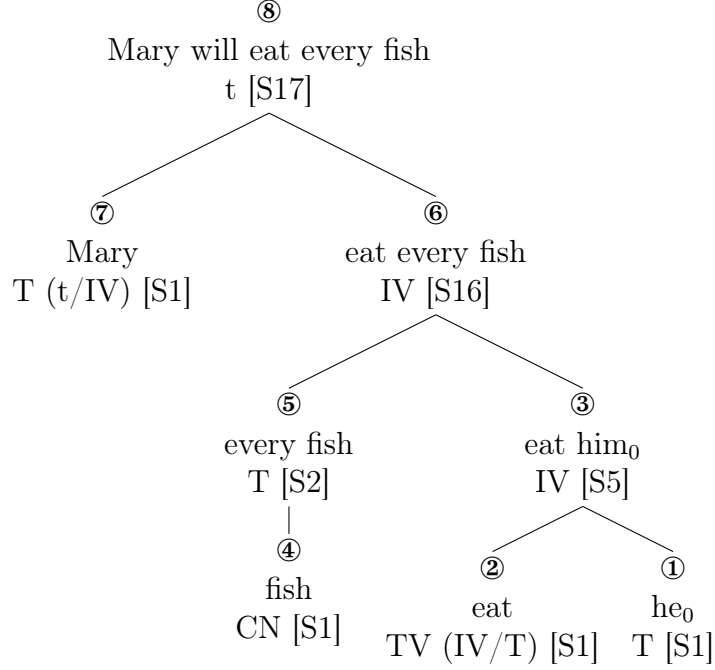
¹¹Are there any plausible candidates for such expressions? Resumptive pronouns and pronouns bound by a commanding quantifier won't cut it, since we need expressions appearing free (and in any case, natural language pronouns often [always?] have their reference restricted in some way, even when bound).

the same index with appropriate non-indexed pronouns (S14-16, *ibid.* 225). The second kind of quantificational operation is the formation of ‘such-that’ relative clauses, which replaces all indexed pronouns in a sentence bearing a certain index with appropriate non-indexed pronouns (S3, *ibid.* 224).¹²

These operations treat expressions containing free pronouns with a certain index as ‘stepping stones’ to other expressions. The quantificational operations that work on these expressions serve to eliminate expressions with a certain index, and at the same time to bind off that index, which has the interpretive effect of removing sensitivity to the variable assignment on that index. The result of operating on the relevant index in this way is therefore an expression that (i) does not contain the indexed expression, and (ii) doesn’t contain any free, unrestricted reference in its interpretation due to the presence of a featureless free variable.

For example, take the operation of quantifying into a verb phrase, represented by rule S16. PTQ includes sentences like *Mary will eat every fish*, and there are in general multiple ways to derive sentences like this, with quantifiers like *every fish* in direct object position. One way is just to place everything *in situ* during the derivation, and so to allow the verb *eat* to take the quantifier *every fish* immediately as its direct object. But another way to generate the sentence is to have the verb compose with an indexed pronoun like he_0 , in order to generate an intermediate intransitive verb *eat him₀* containing (the accusative variant of) that pronoun, then to ‘lower’ the quantifier *every fish* into the direct object slot, so that it replaces *him₀*, and finally to compose the result with the subject, *Mary*. This quantifying in strategy works as follows.

¹²The use of ‘such-that’ relative clauses, rather than ordinary English relative clauses with gaps, is of course more para-English (cf. fn. 4). ‘Such-that’ relative clauses (it’s not clear that they actually are relative clauses) are at best extremely marginal in ordinary English; to my own ear, they are not useable at all in most discourse conditions, and competence with them comes primarily from their use as an idiom in formal contexts, where it’s not obvious that the way they’re to be read is not stipulated as a term of art. Some of them are worse than others: it’s questionable if, for example, something like *a fish such that John ate it* is English at all (I’m tempted to say that it’s not in my idiolect). The reason these expressions are used is because their overt structure more closely resembles how quantification works in formal languages (since formal languages bind overt variables in sentence-like formulas that they take scope over, and don’t make use of gaps, like regular English relative clauses do). And so they exist in PTQ because they have been ‘translated back’ from formal languages into an English-like idiom, which while technically existent, is so marginal that the attention paid to it can’t possibly be justified on empirical grounds.



① $\hat{P}P\{x_0\}$
[T1e]

② **eat'**
[T1a]

③ **eat'**($\hat{P}P\{x_0\}$)
[T5]

④ **fish'**
[T1a]

⑤ $\hat{P} \wedge x[\mathbf{fish}'(x) \rightarrow P\{x\}]$
[T2]

⑥ $\hat{y}[\hat{P} \wedge x[\mathbf{fish}'(x) \rightarrow P\{x\}]](\hat{x}_0[\mathbf{eat}'(\hat{P}P\{x_0\})(y)])$
 $= \hat{y}[\hat{P} \wedge x[\mathbf{fish}'(x) \rightarrow P\{x\}]](\hat{x}_0[\mathbf{eat}'(y, \hat{P}P\{x_0\})])$
 $= \hat{y}[\hat{P} \wedge x[\mathbf{fish}'(x) \rightarrow P\{x\}]](\hat{x}_0 \mathbf{eat}'_*(\vee y, \vee x_0))$
 $= \hat{y} \wedge x[\mathbf{fish}'(x) \rightarrow [\hat{x}_0 \mathbf{eat}'_*(\vee y, \vee x_0)\{x\}]]$
 $= \hat{y} \wedge x[\mathbf{fish}'(x) \rightarrow [\hat{x}_0 \mathbf{eat}'_*(\vee y, \vee x_0)(x)]]$
 $= \hat{y} \wedge x[\mathbf{fish}'(x) \rightarrow \mathbf{eat}'_*(\vee y, \vee x)]$
 $= \hat{y} \wedge u[\mathbf{fish}'_*(u) \rightarrow \mathbf{eat}'_*(\vee y, u)]$
[T16]

⑦ m^*
[T1d]

⑧ $Wm^*(\hat{y} \wedge u[\mathbf{fish}'_*(u) \rightarrow \mathbf{eat}'_*(\vee y, u)])$
 $= W[\hat{P}P\{\wedge m\}](\hat{y} \wedge u[\mathbf{fish}'_*(u) \rightarrow \mathbf{eat}'_*(\vee y, u)])$

$$\begin{aligned}
&= W[\hat{y} \wedge u[\mathbf{fish}'_*(u) \rightarrow \mathbf{eat}'_*(^{\vee}y, u)]]\{\wedge m\} \\
&= W[\hat{y} \wedge u[\mathbf{fish}'_*(u) \rightarrow \mathbf{eat}'_*(^{\vee}y, u)]](\wedge m) \\
&= W \wedge u[\mathbf{fish}'_*(u) \rightarrow \mathbf{eat}'_*(m, u)] \\
&[\text{T17}]
\end{aligned}$$

These quantifying-in operations are somewhat bizarre, and it's not clear what status they're supposed to have. They combine expressions in ways that the category notation suggests they shouldn't be combined (in the above case, S16 composes something of category T with something of category IV to produce another expression of category IV , which the system of categories gives us no reason to expect is possible).¹³ They further don't involve structure-building, like the other syntactic rules, but rather the replacement of existing material in an already-composed expression. They therefore resemble the transformations of classical generative syntax.¹⁴ The best sense I can make of what such operations are doing is to say that they record a certain systematic relation between distinct expressions of English: whenever a certain kind of expression appears in the language, so does an expression of another kind, whose overt form and interpretation are in some way systematically related those of the former (Is this a kind of 'composition,' though? Not in any obvious way).

But putting aside any qualms about such operations, what's crucial to note is that they (where non-vacuous)¹⁵ crucially depend on the presence of indexed pronouns in the expressions that they quantify into. This means that insofar as these operations are necessary to achieve certain readings of English sentences in PTQ, the derivation of those readings is dependent on the existence of non-English expressions in PTQ. That is, quantifying in is

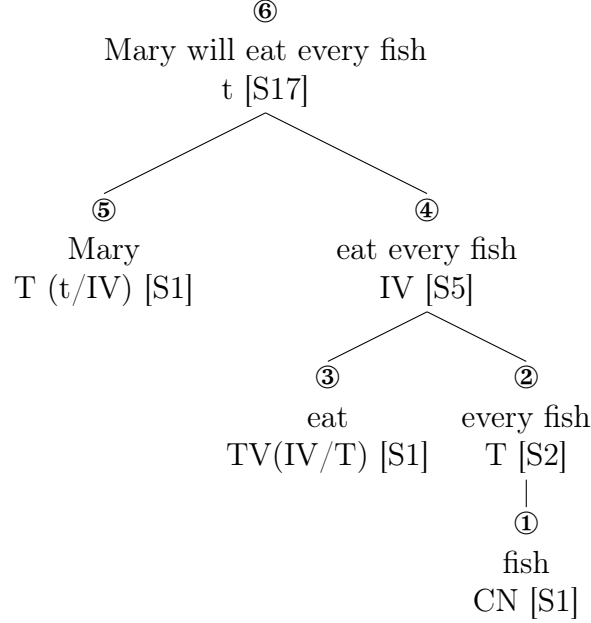
¹³In PTQ, the syntactic operations are given rule-by-rule, so technically every syntactic operation is categorically arbitrary in this way: while Montague appeals to Ajdukiewicz's categorial grammar, in truth the slash notation for the syntactic categories in PTQ is purely mnemonic, and the categories could be called anything. I take it, though, that that spirit of the fragment is to have the slash notation reflect which syntactic combinations are possible, and the quantifying-in rules are in violation of that spirit. One might remedy this by taking quantifiers to be of a different syntactic type (for the above example, say, mapping intransitive verbs to intransitive verbs), but this creates the oddity of claiming that expressions which overtly appear as arguments (quantified noun phrases) are actually verbal modifiers; and even this won't be adequate for PTQ, because quantifiers would also effectively have to serve as sentence modifiers and noun modifiers (in accord with rules S14 and S15). My own suspicion is that something has gone very wrong with all of this.

¹⁴Actually, the transformations of classical generative syntax are less suspect than the Montagovian quantificational rules, since transformations at least perform operations on natural language expressions, whereas the Montagovian rules are meant to be applied to non-natural expressions. And so while there are issues with understanding what exactly a transformation is supposed to be, those issues least don't involve appealing to expressions not actually in the language. The Montagovian quantificational rules more closely resemble the rules governing movement and traces of a slightly later generative syntax: traces, much like Montagovian indexed pronouns, are arguably not expressions of any natural language either (here I think the point is at least debatable, though I'm inclined to say there are no traces in natural language, and so much of generative syntax is committed to analyzing artificial languages, too). The closest generative analogue to Montague's actual rules is the quantifier lowering of the generative semanticists.

¹⁵The reason for the 'while non-vacuous' qualifier here is that PTQ allows vacuous quantification in the absence of an appropriate indexed variable, which has no phonological or semantic effects, into all sentences, verb phrases, and common nouns. Vacuous quantification in can happen any number of times in principle: this is presumably why Montague says that "it can be shown that every declarative sentence of our fragment has infinitely many analysis trees" (*ibid.* 227). It's not obvious what to make of such a possibility in the grammar.

parasitic on the non-English portion of PTQ, and therefore so are any readings obligatorily achieved in PTQ using quantifying in. Yet the most important results delivered by Montague using the fragment crucially require these operations.

If one sticks to sentences like *Mary will eat every fish*, given the interpretation above, then this dependence isn't obvious, since there is no need to make use of these operations to derive such interpretations. This is because *Mary will eat every fish*, as noted above, can also be derived with the quantifier appearing *in situ* in object position, with the same semantic result:



① **fish'**
[T1a]

② $\hat{P} \wedge x[\mathbf{fish}'(x) \rightarrow P\{x\}]$
[T2]

③ **eat'**
[T1a]

④ $\mathbf{eat}'(\hat{P} \wedge x[\mathbf{fish}'(x) \rightarrow P\{x\}])$
[T5]

⑤ m^*
[T1d]

⑥ $Wm^*(\wedge \mathbf{eat}'(\hat{P} \wedge x[\mathbf{fish}'(x) \rightarrow P\{x\}])))$
 $= W[\hat{P}P\{\wedge m\}](\wedge \mathbf{eat}'(\hat{P} \wedge x[\mathbf{fish}'(x) \rightarrow P\{x\}])))$
 $= W\wedge \mathbf{eat}'(\hat{P} \wedge x[\mathbf{fish}'(x) \rightarrow P\{x\}])\{\wedge m\}$
 $= W\mathbf{eat}'(\hat{P} \wedge x[\mathbf{fish}'(x) \rightarrow P\{x\}])\{\wedge m\}$
 $= W\mathbf{eat}'(\wedge m, \hat{P} \wedge x[\mathbf{fish}'(x) \rightarrow P\{x\}])$

$$\begin{aligned}
&= W \wedge x[\mathbf{fish}'(x) \rightarrow \mathbf{eat}'_*(m, {}^\vee x)]^{16} \\
&= W \wedge u[\mathbf{fish}'_*(u) \rightarrow \mathbf{eat}'_*(m, u)]
\end{aligned}$$

As far as such examples go, then, the ubiquitous presence of the possibility of quantifying in might be taken to be harmless, though superfluous: the relevant interpretations are achievable without these operations, and so there is in principle no dependence on a non-English portion of the grammar to derive them.^{17,18}

But this is not true for many of the interpretations that PTQ derives: a number of them do require (non-vacuous) quantifying in, and so do depend on non-English constructions. Further, the interpretations that depend on quantifying in are crucially involved in the main results that Montague presents PTQ as achieving. Among these results are treatments of: (i) *de re* / *de dicto* ambiguities beneath intensional verbs; (ii) scopal ambiguities within sentences containing multiple quantifiers; (iii) preservation of reference between a quantifier and pronoun beneath intensional verbs. In each of these three cases, the use of quantifying in is needed to derive certain readings crucial for the result. These results are therefore all dependent on the existence of indexed pronouns in PTQ, and so on the features of PTQ that make it not an English fragment.

To take the example of *de re* / *de dicto* ambiguities with intensional verbs first: Montague is concerned to account for the different interpretations of sentences like (1).¹⁹

(1) John seeks a unicorn.

a. John's trying to make it so that there's a unicorn he finds.

¹⁶The move to this line from the previous one is justified by the principle involving transitive verbs at the top of *ibid.* 237, plus the relevant simplifications. Montague (in *ibid.* 236-237) claims that this principle falls out as a result of the postulates in *ibid.* 235, but so far as I can see, this isn't true: in the case of transitive verbs, an additional assumption is required, that what satisfies the existential by standing in for *S* in postulate (4) is the intension of the starred variant of δ . Nothing states that this has to be the case, but I assume that's what's intended.

¹⁷Although the availability in principle of such operations does mean that, for sentences like *Mary will eat every fish*, there are multiple ways to derive them that yield identical truth conditions. Is this meant to be an empirical claim of any sort? It would seem not – while there's no datum that can contravene this result, it strikes me as methodologically unwholesome.

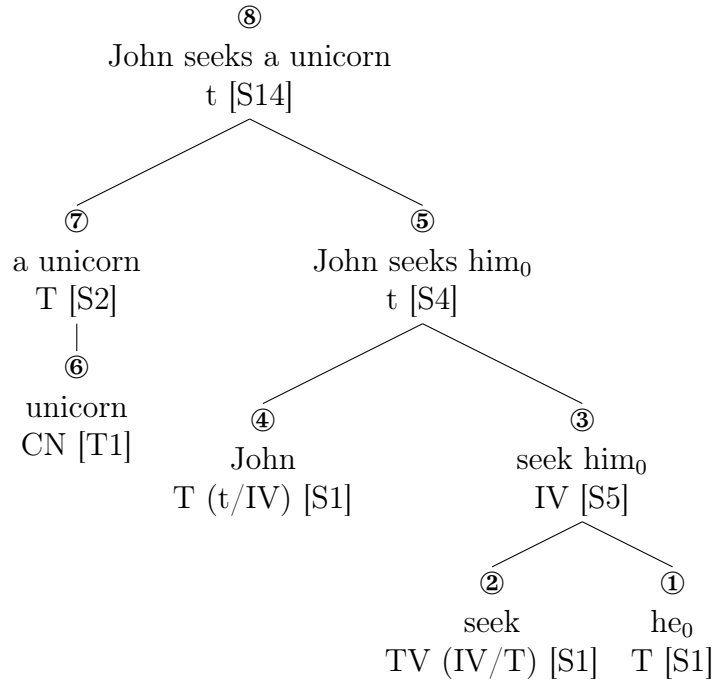
¹⁸Well, this isn't quite true either. Even for sentences like *Mary will eat every fish*, there are yet further potential interpretations according to PTQ that do require quantifying into indexed variables. In this case, if one first constructs the sentence *Mary will eat him₀*, and then lowers the quantifier into object position (so that the logical form is $[[\text{every fish}] [\text{Mary} [\text{eat he}_0]]]$), the resulting interpretation is $\wedge u[\mathbf{fish}'_*(u) \rightarrow \mathbf{Weat}'_*(m, u)]$, as the reader can confirm. On this reading, every fish will be eaten by Mary at some future time or other (not all necessarily at the same time), whereas the interpretations in the text say that there is some future time at which every fish will be eaten by Mary. Is the English sentence really ambiguous in this way? I don't know – as with many judgments of ambiguity, the problem is that one interpretation entails the other, and so it's difficult to disentangle them. I leave such cases to the side, because they pertain to the empirical adequacy of the fragment.

¹⁹Again, *John seeks a unicorn* smacks of para-English: it's not a great sentence, at least on the interpretation that seems to be intended for it in PTQ, unless some sort of slight archaism is intended. For present purposes, I'll assume that its interpretation is not relevantly different from a more natural English sentence, like 'John's looking for a unicorn.' In this case, the para-English is presumably intended to avoid two delicate topics in English semantics: (i) the contribution of progressive aspect, and (ii) the contribution of the preposition in the phrasal verb 'look for' (as opposed e.g. to 'look up,' 'look over,' 'look through,' 'look after,' etc., on their non-directional readings).

- b. There's a unicorn that John's trying to find.

A classical position states that there are at least two interpretations of (1), which are roughly and awkwardly paraphrasable as (1a) and (1b). The first interpretation is the *de dicto*: it states that John's search is over when he finds some unicorn or other – it doesn't matter which – and its truth conditions make reference to no one unicorn in particular. The second interpretation is the *de re*: it states that John's search is over when he finds some particular individual, and that this individual is a unicorn. I'll assume that the sentence is indeed at least two-ways ambiguous, and so has at least these two readings.

PTQ derives the *de re* reading, but to do so, it must quantify into an expression that contains an indexed pronoun. For example, the *de re* reading is achieved (as Montague himself emphasizes: cf. the second tree in *ibid.* 228, and the second interpretation for the sentence given in *ibid.* 238) when one (i) constructs a sentence with an indexed pronoun in object position, like *John seeks him₀*, and then (ii) lowers the quantifier *a unicorn* into the object slot, as follows:



- ① $\hat{P}P\{x_0\}$
[T1e]
- ② **seek'**
[T1a]
- ③ **seek'**($\hat{P}P\{x_0\}$)
[T5]
- ④ j^*
[T1d]

$$\begin{aligned}
\textcircled{5} \quad & j^*(\wedge \mathbf{seek}'(\hat{P}P\{x_0\})) \\
& = [\hat{P}P\{\wedge j\}](\wedge \mathbf{seek}'(\hat{P}P\{x_0\})) \\
& = [\wedge \mathbf{seek}'(\hat{P}P\{x_0\})]\{\wedge j\} \\
& = \mathbf{seek}'(\hat{P}P\{x_0\})(\wedge j) \\
& = \mathbf{seek}'(\wedge j, \hat{P}P\{x_0\}) \\
& = \mathbf{seek}'_*(j, \vee x_0) \\
& \text{[T4]}
\end{aligned}$$

$$\begin{aligned}
\textcircled{6} \quad & \mathbf{unicorn}' \\
& \text{[T1a]}
\end{aligned}$$

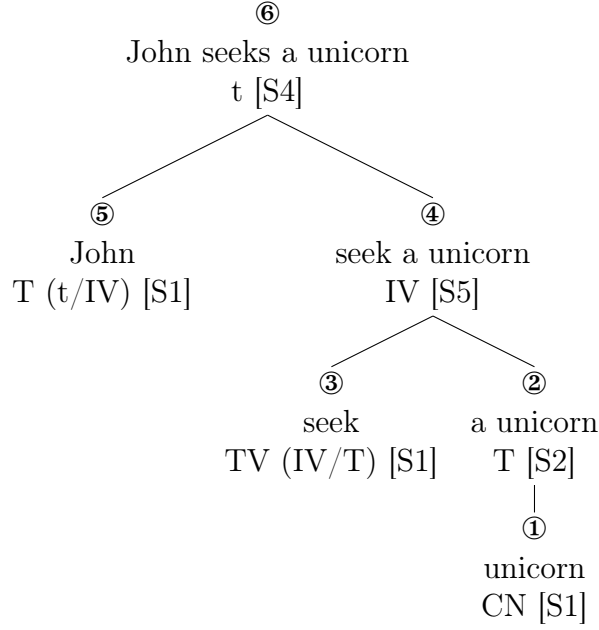
$$\begin{aligned}
\textcircled{7} \quad & \hat{P} \vee x[\mathbf{unicorn}'(x) \wedge P\{x\}] \\
& \text{[T2]}
\end{aligned}$$

$$\begin{aligned}
\textcircled{8} \quad & [\hat{P} \vee x[\mathbf{unicorn}'(x) \wedge P\{x\}](\hat{x}_0 \mathbf{seek}'_*(j, \vee x_0))] \\
& = \vee x[\mathbf{unicorn}'(x) \wedge [\hat{x}_0 \mathbf{seek}'_*(j, \vee x_0)]\{x\}] \\
& = \vee x[\mathbf{unicorn}'(x) \wedge [\hat{x}_0 \mathbf{seek}'_*(j, \vee x_0)](x)] \\
& = \vee x[\mathbf{unicorn}'(x) \wedge \mathbf{seek}'_*(j, \vee x)] \\
& = \vee u[\mathbf{unicorn}'_*(u) \wedge \mathbf{seek}'_*(j, u)] \\
& = \vee u[\mathbf{unicorn}'_*(u) \wedge \mathbf{try-to}'(\wedge j, \hat{y} \mathbf{find}'_*(\vee y, u))]^{20} \\
& \text{[T14]}
\end{aligned}$$

There is no way to derive this reading without the device of quantifying in: if one puts everything *in situ*, then the result is obligatorily the *de dicto* reading (cf. the first tree in *ibid.* 228, and the first interpretation for the sentence given in *ibid.* 238):²¹

²⁰This final line is derived from postulate (9) in *ibid.* 235, which makes *seek* and *try to find* intertranslatable in a certain way (cf. *ibid.* 238). What exactly it means for John's individual concept to 'try-to' the property of finding *u* isn't spelled out, but presumably it means that John tries to bring it about (*de se*) that he has that property (i.e., the property of finding *u*, which is a specific unicorn).

²¹Crucially, we see these differences in the *de re* versus *de dicto* readings because *seek* is an intensional verb, which means the extraposition of the quantifier used for the *in situ* derivation of the interpretation of *Mary will eat every fish* above isn't licensed. Note that the final step in the derivation does make use of the extraposition of the quantifier for the interpretation of *find*, because *find* is an extensional verb. The resulting interpretation here ends up being that John tries to bring it about that he has the property of finding some unicorn or other, not that he tries to bring it about that he has the property of finding some specific individual *u*, which is a unicorn.



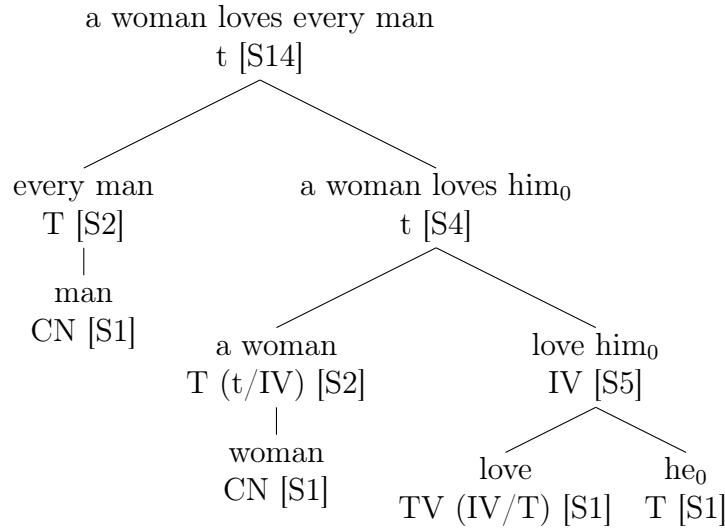
- ① **unicorn'**
[T1a]
- ② $\hat{P} \vee x[\mathbf{unicorn}'(x) \wedge P\{x\}]$
[T2]
- ③ **seek'**
[T1a]
- ④ $\mathbf{seek}'(\hat{P} \vee x[\mathbf{unicorn}'(x) \wedge P\{x\}])$
[T5]
- ⑤ j^*
[T1d]
- ⑥ $j^*(\wedge \mathbf{seek}'(\hat{P} \vee x[\mathbf{unicorn}'(x) \wedge P\{x\}]))$
 $= [\hat{P}P\{\wedge j\}](\wedge \mathbf{seek}'(\hat{P} \vee x[\mathbf{unicorn}'(x) \wedge P\{x\}]))$
 $= [\wedge \mathbf{seek}'(\hat{P} \vee x[\mathbf{unicorn}'(x) \wedge P\{x\}])]\{\wedge j\}$
 $= \mathbf{seek}'(\hat{P} \vee x[\mathbf{unicorn}'(x) \wedge P\{x\}]) (\wedge j)$
 $= \mathbf{seek}'(\wedge j, \hat{P} \vee x[\mathbf{unicorn}'(x) \wedge P\{x\}])$
 $= \mathbf{try-to}'(\wedge j, \wedge [\mathbf{find}'(\hat{P} \vee x[\mathbf{unicorn}'(x) \wedge P\{x\}])])$
 $= \mathbf{try-to}'(\wedge j, \hat{y} \vee u[\mathbf{unicorn}'_*(u) \wedge \mathbf{find}'_*(\vee y, u)])$

In other words, there is no capturing the ambiguity, because there is no capturing the *de re* reading, without quantifying in, and so without indexed variables, and so without an intermediate non-English expression like *John seeks him₀*. This result, then, crucially depends on PTQ not resembling English in the way it treats quantification: it needs an intermediate step through a non-English expression, which is to say that the formal tools of PTQ derive the *de re* reading only by operating on a non-English portion of the fragment.

This pattern of requiring the existence of indexed pronouns in order to achieve the key results of PTQ is typical of the cases that Montague spends the most time illustrating in the article, and which presumably constitute “all the more puzzling cases of quantification and reference” that he mentions in *ibid.* 221. This can be illustrated with the other two examples mentioned above, of inverse-scope readings of sentences with multiple quantifiers, and cases of pronouns inheriting the reference of a quantifier in intensional contexts. In the first case, Montague is concerned to account for examples like the following (cf. *ibid.* 240):

- (2) A woman loves every man.
 a. There is a woman, and she loves every man.
 b. Every man has a woman that loves him.

There is a classical view, which we can again take for granted, that a sentence like (2) has at least two readings, paraphrased as above. The surface scope reading, represented by (2a), says that there is some specific woman who loves every man, while the inverse scope reading, represented by (2b), says that for every man, there’s some woman or other who loves him (maybe different woman for each man). Crucially, the inverse scope reading can only be derived in PTQ using quantifying in: as the reader can confirm, deriving *A woman loves every man* with everything generated in surface position obligatorily results in the surface scope reading. If one wants the inverse scope reading, one must lower the quantifier *every man* into object position, like so, which yields the following interpretation (again, as the reader can confirm):

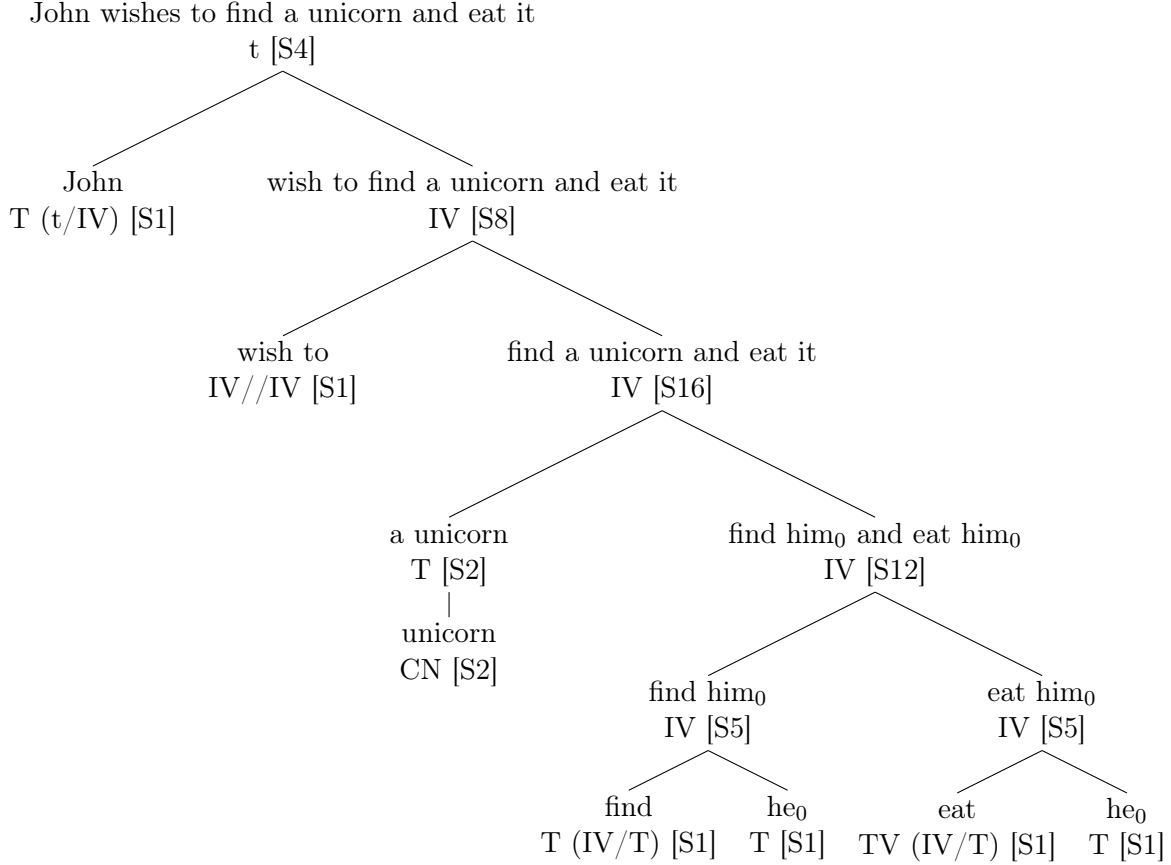


$$\bigwedge v[\mathbf{man}'_*(v) \rightarrow \bigvee u[\mathbf{woman}'_*(u) \wedge \mathbf{love}'_*(u, v)]]$$

Again, there is no achievement of this result without the use of indexed pronouns and quantifying in, and so it relies crucially on the treatment of the non-English portions of the fragment.

Likewise for cases where a pronoun inherits the reference of a quantifier in an intensional context: sentences like *John wishes to find a unicorn and eat it* (cf. *ibid.* 240) can only be derived by generating two indexed pronouns with the same index, and then quantifying

into the first and simultaneously binding off the second. This sort of construction therefore requires the use of multiple indexed pronouns, and so again is dependent on non-English expressions. Here, for instance, is one way of deriving *John wishes to find a unicorn and eat it*, on an interpretation for which the non-specific unicorn is the very thing that John wants to eat (and so there is a preservation of reference, internal to an intensional context, between *a unicorn* and *it*):²²



wish-to'($\wedge j, \hat{y} \vee u[\mathbf{unicorn}'_*(u) \wedge \mathbf{find}'_*(\vee y, u) \wedge \mathbf{eat}'_*(\vee y, u)]$)

Getting rid of these indexed pronouns is not an option, unless the grammar is to be substantially rewritten, if the core results of PTQ are to be preserved. Their inclusion is therefore crucial, which means that the inclusion of non-English material in PTQ is crucial. It further means that the extent to which PTQ captures these results is precisely the extent to which it alters English to fit the mold of a formal language, which treats quantification in two steps, first by introducing featureless variables, and second by introducing a quantifier that scopes over it. English, however, does not do this.

²²In fact, it's transparent that such a sentence requires indexed variables to construct, because the only way to produce pronouns like *it* in PTQ is by replacing indexed pronouns. Any sentence that occurs with non-indexed pronouns at all in PTQ therefore requires the indexed pronouns as intermediaries that're replaced. I assume that the interpretation of *wish* is as follows: an individual concept wishes a property where the relevant individual's desires are fulfilled just in case that property is true of it.

4 The indexed pronouns can't be fixed

At this point, we might wonder whether the indexed pronouns of PTQ could be altered to be more like their English counterparts. For instance, one might change their semantics, to include appropriate restrictions on their reference of the kind that genuine free English pronouns have. This might be possible if the current lack of restrictions on their reference is just a kind of technical convenience, as we've assumed that the presence of indexation is, or if it's just an oversight.

It could be, for example, that there is no formal obstacle to adding gender restrictions to the reference of the pronouns he_n , but that Montague just neglected to do this. His presentation is, after all, not primarily concerned with non-grammatical gender, but with issues like quantification and scopal ambiguities in intensional contexts, and so appropriately restricting the reference of free pronouns wasn't on his agenda. There are a number of *ad hoc* treatments in PTQ of English phenomena that are not Montague's focus, such as the assignment of accusative case (rule S5, *ibid.* 224), subject-verb agreement (rule S4, *ibid.*), and even the gendering of pronouns to agree with the head nouns of 'such-that' relative clauses (rule S3, *ibid.*) and binding quantifiers (rule S14, *ibid.* 225). Maybe some similar treatment, even if presently *ad hoc*, could be implemented for the gender requirements on free pronouns, to await a fuller treatment in a spirit that doesn't damage the architecture of PTQ.

As it turns out, this is not possible – one cannot, for instance, alter the pronouns he_n to refer only to masculine individuals in PTQ, while preserving the integrity of the grammar. There's an important reason why this can't be done, but it will help for expository purposes first to point out a less serious technical reason that it's not possible in PTQ as it stands, in order to leave it to the side.

The less serious technical issue is that PTQ has no notion of a context of utterance as distinct from a point of evaluation, and so cannot differentiate between features of interpretation that depend on the circumstances in which an utterance is made, as opposed to those that depend on the circumstance relative to which it is evaluated. This means that there is no possible treatment of indexicality in the fragment, because it is not 'double-indexed' to track features of the context and the point of evaluation separately. Since the non-grammatical gender features of pronouns are often indexical in this way – e.g., since English 'him' often requires its referent to be masculine at the context of utterance, even when it appears in intensional contexts that shift the point of evaluation – their reference can't be appropriately captured in PTQ.²³

But I'll leave that aside, on the assumption that there's no barrier to supplementing PTQ to include double-indexing of the sort that can treat indexicality. As a half-measure, then, suppose that the fragment is modified so that the variables x_n , which the pronouns he_n introduce via translation into IL, must map to individual concepts that are masculine

²³I have in mind examples like 'Mary thinks he's a woman,' where 'he' refers to a man, say John. To my ears, there is typically no reading of this sentence where the gender features of the pronoun can scope beneath the attitude verb, and so it must be that the referent is masculine in the context of utterance, and not in the point(s) of evaluation tracked by Mary's belief. That is, the sentence cannot report that Mary thinks that some individual that *she* thinks is masculine is a woman; it can only report that there's some individual that's *actually* masculine, and Mary thinks that this individual is a woman.

at their points of evaluation. There are a lot of ways to do this in principle, but it helps to pick one for concreteness: suppose then that variable assignments are characterized so that they map variables x_n (i.e., those variables introduced by indexed masculine pronouns) only to masculine individual concepts.²⁴ That is:

- For any assignment g , variable x_n , world i , and time j :
 $g(x_n)(i, j)$ is masculine in i at j .

As noted above, this isn't quite right, since it only guarantees that the referent of the pronoun is masculine at the point of evaluation, and so this principle would introduce incorrect interpretations of English sentences with pronouns appearing free in intensional contexts. But setting this to the side, the principle does allow the interpretation of sentences like *Mary loves him₀* to more closely approximate the real English interpretation of 'Mary loves him,' by requiring that *him₀* refer to a masculine individual at the point of evaluation: the interpretation is the same as what's given for node 5 above, but now the restriction on assignments requires that the extension of $g(x_0)$ be masculine at any index.

The reason that this restriction doesn't solve the problem is that it corrects the interpretation of the indexed pronouns when they're free, at the cost of giving them the wrong interpretation when they're bound using the quantificational operations of PTQ. In the case of this gender restriction in particular, making the pronouns he_n refer only to masculine individuals in this way causes quantification over individuals when they are replaced by non-indexed pronouns or quantifiers to be only over masculine individuals as well. In other words, a masculine free variable retains its masculine specification as a bound variable, and as a result, if the indexed pronouns are intrinsically specified as masculine (like the English pronoun 'he' is), then the fragment only allows quantification over masculine individuals.

To see this, it helps to return to the interpretation of an example like the *de re* reading of *John seeks a unicorn* discussed in §3 above. The third line showing the interpretation of the topmost node, node 8, reads as follows there:

$$\bullet \bigvee x[\mathbf{unicorn}'(x) \wedge [\hat{x}_0\mathbf{seek}'_*(j,^\vee x_0)](x)]$$

The simplification of this line to the following fourth line depends on plugging the individual concept x into the function $\hat{x}_0\mathbf{seek}'_*(j,^\vee x_0)$. However, note that the variable over which the functor abstracts in this function-denoting expression is x_0 , which per the above refers on all assignments to masculine individual concepts. The semantics of abstraction in IL (*ibid.* 231, rule (3)) requires that the value of the expression $[\hat{x}_0\mathbf{seek}'_*(j,^\vee x_0)](x)$ as a whole, on some assignment, is the value of $\mathbf{seek}'_*(j,^\vee x_0)$ on an assignment identical to this one except for the fact that the variable x_0 maps to whatever x maps to. But per the above, there are no variable assignments that map x_0 to anything but masculine individual concepts. It follows that the expression $[\hat{x}_0\mathbf{seek}'_*(j,^\vee x_0)](x)$ as a whole is well-defined only if x also maps to a

²⁴This principle can be thought of as a postulate governing which assignments are to be used in the interpretation of IL (and so of English), similar to the postulates in *ibid.* 235, which govern which interpretations of IL are to be used. There's also no requirement in PTQ that the individual concepts that these variables introduce be rigid, and so in principle this still allows a variable to map to distinct masculine individuals at distinct points of evaluation. I assume this too isn't a serious shortcoming, and could be remedied with a postulate like the one for individual constants (postulate (1) in *ibid.*).

masculine individual concept: otherwise, the truth value of $\mathbf{seek}'_*(j, {}^\vee x_0)$ is undefined, since the relevant assignment crucial to determining its truth value is non-existent. The function $\hat{x}_0\mathbf{seek}'_*(j, {}^\vee x_0)$ is therefore effectively a partial function, from masculine individual concepts to true just in case John seeks the relevant individual, to which that masculine concept maps at the point of evaluation. The final denotation, then, which is written above as follows:

$$\bullet \bigvee u[\mathbf{unicorn}'_*(u) \wedge \mathbf{try-to}'(\wedge j, \hat{y}\mathbf{find}'_*(\vee y, u)]$$

...now effectively contains an implicit restriction on the individual variable u , such that whatever individual satisfies the existential (that is, the specific unicorn that John's looking for) must be masculine. That is, if the sentence as a whole is to be true, the individual that John seeks must be one whose individual concept is masculine, and therefore defined as an argument of the above function (a non-masculine unicorn won't fulfill the truth conditions, since when 'plugged in,' it will cause the calculation of the truth conditions to crash). What *John seeks a unicorn* on this reading effectively means, then, is that there is some unicorn, 'presupposed' to be masculine, that John is trying to find. And the masculinity of the sought-for unicorn is enforced by the masculinity of the initial indexed variable that the quantifier *a unicorn* quantified into.

A similar result will obtain for any reasonable way of restricting the variable introduced by a pronoun like he_0 to masculine individuals: wherever the variable can refer only to masculine individuals when free, by that fact it allows quantification only over masculine individuals when quantified into. The restrictions on reference that attend the free pronoun, in other words, must be inherited by the quantifier. Since they are in fact not so inherited in English – that is, since *John seeks a unicorn* cannot, on its *de re* reading, mean that the unicorn that John's looking for is masculine – the pronoun he_0 cannot be treated in this way, simultaneously as an expression that can occur free to refer to masculine individuals, and one that can be quantified into to quantify over all individuals, masculine or not.

In other words, if the indexed pronouns of PTQ are to function correctly and be genuine English expressions, they have to play two roles: first, they must appear as free pronouns, with the appropriate restrictions on their reference, and second, they must appear as variable expressions to be bound off, with no restrictions on their reference. They cannot, using the quantificational operations of PTQ, occupy both these roles at once, since there is no way to specify the restrictions on their reference correctly, so that they enact these restrictions in some constructions, but not in others. Either the indexed pronouns are free variables meant to be replaced in quantification, in which case they must carry no restrictions on their reference, or they are expressions meant to act like free pronouns, in which case they must have such restrictions. There is no one class of expression in PTQ that can do both of these things.

Note further that the problem is not simply that Montague has erroneously assigned a single class of expressions two roles: this might suggest that all one has to do to fix the issue is create a second class of expression, and then divide the two roles up among this new class of expression and the old indexed pronouns. This misses the point: for instance, it wouldn't help to add a new class of genuine free pronouns, appropriately restricted in their reference by non-grammatical gender, etc., and then to keep the indexed pronouns without any such

restrictions to act as free variables for use in quantification. Then one would still have the unrestricted free variables, which still do **not** exist in English.²⁵

PTQ in this case would have interpretable free pronouns, but all of its quantificational mechanisms would still depend on non-English expressions in the same way. The problem, in other words, is still the indexed pronouns that act as variables, which must have their reference unrestricted in order to serve their quantificational function, but which by that very fact cannot be expressions of English. Quantification in PTQ intrinsically depends on a class of expressions – free, unrestricted variables appearing in term positions – that don’t exist in natural languages. The only way to solve the issue is therefore to include the indexed variables to be bound off in PTQ, but also to make them actual expressions of English, like pronouns, which have restricted reference – and this is exactly what can’t be done.

But, one might object, isn’t the above problem only an artifact of the decision to include just one class of indexed pronouns, he_n , in the grammar? It’s true that the *de re* reading of *John seeks a unicorn* can’t mean that the unicorn in question must be masculine, but might this not be remedied if we include other indexed pronouns, that don’t have to refer to masculine individuals, and which do on their own resemble other real pronouns of English? What if, for example, we added two more classes of indexed pronoun, she_m and it_o , which had their reference restricted to feminine and inanimate individuals, respectively, like so?²⁶

- For any expressions she_m , he_n , it_o , $m \neq n \neq o$.
- For any assignment g , variable x_m , world i , and time j :
 $g(x_m)(i, j)$ is feminine in i at j .
- For any assignment g , variable x_o , world i , and time j :
 $g(x_o)(i, j)$ is inanimate in i at j .

Then we’d be able to derive the right *de re* reading of *John seeks a unicorn*: we just generate the sentence with the following logical form, putting a pronoun like it_2 in the slot into which *a unicorn* lowers:²⁷

- $[[[a\ unicorn] [John\ [seek\ it_2]]]]$

²⁵It should also be noted that it also doesn’t help to insist that pronouns in English are composite expressions, which contain a referential ‘core’ that acts as an unrestricted variable, and which when not quantified into are supplemented by ϕ -features that restrict reference when interpreted (and presumably cause the pronoun to be spelled out as *he*, etc.). If this were so, one might be tempted to claim that the restrictions on reference, e.g. from non-grammatical gender, only attend the ‘full’ pronoun with all of its ϕ -features, while it is the pronominal core, without those features, and without those restrictions on reference, that is quantified into. One would thus have a choice either between free variable pronouns ‘naked’ of the features that restrict their reference, which (for some reason) cannot appear free, but must be quantified into, and full-blooded pronouns including their restricted reference, which (for some reason) cannot be quantified into, but must occur where free pronouns occur in English. This doesn’t solve the problem, since there is no more reason to believe that these ‘pronominal cores’ are expressions of English than what we started with!

²⁶Assume also that the translation rule T1e holds for all indexed pronouns, translating $\hat{P}P\{x_i\}$ for any indexed pronoun p_i .

²⁷The idea for the indexing of the pronouns here is that 0 is an index appropriate to masculine pronouns, 1 to feminine ones, and 2 to inanimate ones. This technically runs slightly counter to the way indexing works in PTQ as written, but I take that it doesn’t matter.

If we presume, following Montague, that all unicorns are appropriately referred to using the pronoun ‘it,’²⁸ then the interpretation here works out: the quantifier quantifies only over inanimate individuals, which unicorns are presumed to be, and so the gender of the pronoun is redundant with the gendered denotation of the noun, and the presumption that the specific unicorn is an it is harmless. Does this solve the problem?

No, it does not. Note first that on such a view, the grammar still allows the generation of this sentence with a *de re* reading using masculine and feminine pronouns like *he*₀ and *she*₁. This means that the *de re* reading of the sentence is expected to be three-ways ambiguous, and to have an all-masculine reading, on which the quantifier quantifies only over masculine individuals, and a corresponding all-feminine reading, both of which exist alongside the all-inanimate reading gotten by the logical form above. If we assume that unicorns can also be appropriate referents of ‘he’ or ‘she’ as well as ‘it,’ this means there should be a reading on which the unicorn must be masculine, and a reading on which it must be feminine, as well as the inanimate (or inclusive) reading. If instead we assume that all unicorns are only properly referred to with ‘it,’ then this means there should be two semantically contradictory readings available for the sentence.

Neither of these is true, but we might imagine in either case that supplementary principles of either the semantics or the pragmatics rule out the deviant readings. If one supposes that the noun *unicorn* covers only individuals only to be referred to with ‘it’ to begin with, then the all-masculine and all-feminine readings are semantically deviant, because they are contradictory, since their truth conditions require that the relevant unicorn be both masculine or feminine and not masculine or feminine. If, on the other hand, unicorns can also be *hes* and *shes*, we might say (wave hands above head here) that the pragmatics rules out the all-masculine and all-feminine readings as being impossible to detect as opposed to the more inclusive *it*-reading, which is selected as the only reasonable interpretation of the surface form.

Even such appeals to supplementary principles will not work in all cases, though. For instance, if you take a gender-neutral noun denoting individuals usually not referable to using ‘it,’ like *teacher*, it’s not possible on this picture to get a right reading of sentences like the *de re* reading of *John seeks a teacher*.²⁹ There would be three ways of deriving it, from the following logical forms:

- [[a teacher] [John [seek it₂]]]
- [[a teacher] [John [seek she₁]]]
- [[a teacher] [John [seek he₀]]]

That is, the relevant quantification would have to be over only masculine, only feminine, or only inanimate individuals. None of the resulting readings is appropriate – the *de re* reading of the sentence simply cannot mean that John is looking for a masculine teacher or for a feminine teacher: it is utterly neutral as to the non-grammatical gender of the teacher (and,

²⁸This isn’t a safe assumption: Peter Beagle’s famed last unicorn, for example, is referred to only with ‘she.’

²⁹PTQ doesn’t actually have any such nouns, but presumably it would have to be expanded to include them, to have any hope of being a reasonably large fragment of English.

we can assume, the *it*-reading is ruled out by the above considerations, too).³⁰ And so there are no appropriate readings for the actual English sentence provided by the grammar in this case.

I can't, of course, claim to have exhausted all possibilities of accounting for such apparently bad predictions with supplementary pragmatic principles. For instance, one might try to account for the above readings by saying that ambiguous sentences which mutually exhaust some domain of interpretation are interpreted pragmatically as the disjunction of the possible interpretations (roughly, since the hearer can't tell which of the three readings is intended, and they only serve to mutually exhaust the options of the non-grammatical gender of the teacher, the hearer is obliged to interpret the speaker as saying some one of the options, regardless of the unicorn's gender, and so the communicative effect of such an assertion is to say that John's looking for a specific unicorn, which is either masculine, or feminine, or inanimate).³¹ But trying to gerrymander the results to correct for an apparent error in predictions in this way reveals a deeper problem, that such reparatory principles don't address.³²

The basic problem is that treating the variable expressions that allow quantification in PTQ as pronouns gives us some reason to expect the interpretation of quantified expressions in some way to mirror the pronominal inventory of the language, and yield readings consonant with the restrictions on reference that attend pronouns when they're free. And there's simply no reason to think this is true – what is quantified over by quantified expressions simply has nothing to do with the inventory of pronouns that a language possesses, and so the link between the two is spurious. Even if one somehow managed to rig things correctly for a language, to make the pronouns play both the above-mentioned roles, it would be only as the result of postulating a large number of principles whose only purpose is to correct for bad predictions.³³

³⁰Of course the teacher John is looking for *de re* might be masculine or feminine, and the conversational participants, or John himself, might know this. The point is rather that the sentence cannot *mean that* who John is looking for is of any gender, while it must mean that whoever he's looking for is a teacher.

³¹I'm not actually sure I can make sense of any such account as this, since the proposed ambiguity doesn't function the way ambiguity normally does. Note that the problem isn't that the grammar generates multiple readings of the same sentence, some of which are more plausible than others – it's that *none* of the derived readings are correct, and the pragmatics must somehow turn ambiguity between readings, all of which are incorrect, into a disjunctive reading that is correct. Maybe some such transformation does fall out of an independently reasonable pragmatic bridge principle dealing with certain kinds of ambiguity, but I'm not sure exactly how it would work. For instance, speakers can typically intend one reading over another of an ambiguous sentence, and it doesn't seem to me that a speaker can coherently intend a sentence like *John seeks a teacher*, on its *de re* reading, to *mean* that the teacher is of a certain gender. Again, maybe this is because of the kind of ambiguity present, and how it interacts with the pragmatics, but here things are getting mighty sketchy. If anyone were interested in such an account (I'm not, obviously), I'd leave it to them.

³²This is not to mention the multiplication of bizarre spurious readings that the grammar would generate if this pronoun-variable equation were taken seriously: for instance, in PTQ it would imply that there were all-masculine and all-feminine readings of quantified sentences like *Mary ate every fish*, with logical forms like $[[\text{every fish}] [\text{Mary} [\text{eat she}_1]]]$ – this would require all the fish that Mary ate to be feminine. Of course, such readings might be ruled out by the sort of auxiliary principles mentioned above, but the proliferation of such readings that exist only to be ruled out is a sign that something is wrong.

³³One wonders how this sort of picture would be carried out cross-linguistically. Indeed, one would expect that if it were true, one could make predictions about the sorts of individuals that can be quantified over in the

The point of this song and dance isn't to pretend as though the above assimilation between genuine pronouns and variables required for quantification is a hypothesis taken seriously by any linguists. It's rather to show that the reason it's not taken seriously is because everyone knows that the variables required for quantification aren't really pronouns – they aren't natural language expressions of any kind, and they could not in any reasonable way be modified to be so.

5 What is PTQ?

Given the above, the status of the most important results of PTQ are in an odd position. If the purpose of PTQ is to show that the tools of mathematical logic can be successfully applied to natural languages, then the fact that the bulk of the interesting results shown off in the article require the use of non-English expressions makes it unclear what if anything the fragment has actually succeeded in doing, where English is concerned.

It's no vindication of these formal tools as applied to natural languages to demonstrate that if English is **modified** to contain non-English expressions, and in particular to contain expressions that bring English into line with the structure of various formal languages, which introduce featureless free variables and then have quantifiers scope over them, then it can be treated with those tools. If the claim is that the tools of formal logic work on languages that look like formal logical languages, then there's little to dispute – but what this has to do with English is mysterious. If English **were** more like these formal languages – that is, if, contrary to fact, it contained these sorts of featureless variables and quantificational structures – then it **would** be treatable in such a way. But it is not that way, so what has been demonstrated?

At the crux of these questions lie PTQ's indexed pronouns, which raise the question: if these pronouns are not expressions of English, what are they? It would seem that they're not supposed to be much of anything, other than a technical contrivance on the way to the quantificational mechanisms of the language: they exist only as 'in-between' expressions that are needed in order to derive certain English expressions correctly, but which on their own are meant to be somehow 'invisible' to our own evaluation of the grammar, and not taken seriously as expressions of the language.

There are a couple hints in PTQ that this is how the indexed pronouns are supposed to be thought of. First, as noted in §2 above, they're indexed, which English pronouns are not, while the quantifiers that replace the indexed pronouns by quantifying in, along with the replacement pronouns that acts as bound variables, are not indexed, and so do look more like actual English expressions (with the replacement pronouns taking the forms *he*, *she*, *it*, *him*, or *her*). Second, the 'replacement' pronouns that supplant the indexed ones have masculine, feminine, and inanimate forms, and so are meant to explicitly encode the sorts

language (at least, in constructions that require quantifying in), based on which pronoun(s) in principle the argument slot that contains the quantifier could have been derived from, and this would vary from language to language as the pronoun inventory varied! Thus, for instance, *de re* readings requiring quantifying in, in a language without inanimate personal pronouns, should only be able to quantify over animate individuals in these constructions. This sort of thing, to my knowledge, never happens – and so the task would remain to explain via supplementary principles why we never see these semantic effects.

of animacy and non-grammatical gender restrictions that the indexed pronouns don't.

This all suggests that the pronouns he_n are just 'stop-gap' expressions: that they look like masculine pronouns in English is an illusion of sorts, and they could just as easily be written she_n , or it_n , or x_n . They record only the indexing of a variable (as well as the case of the pronoun), and exist only as intermediaries to be replaced by true, non-indexed English expressions through quantifying in or the formation of a 'such-that' relative clause. As noted above, this leaves the fragment without a treatment of free pronouns, which the indexed pronouns don't provide. And even if such free pronouns were added, the fragment would still be crucially stuck with these indexed variable expressions, which would still not be expressions of English.

In short, the indexed pronouns of PTQ are not really pronouns of any natural language at all, and the choice of the form he_n to make them look that way is just a coincidence, or a sleight of hand.³⁴ In reality, they are completely novel, non-English expressions whose sole purpose is to provide 'dummy variables' that can serve as a stop-gap for the quantificational expressions of PTQ. Since these expressions don't exist in natural languages, this means that the quantificational mechanisms of PTQ, which are the primary focus of the paper, don't record the way that quantification actually works in English.

What did Montague expect us to make of these technical expressions? My own hunch is that there's just no answer to this question: insofar as PTQ really is meant to be a grammar of English, the indexed variables are just not supposed to be there. Their inclusion is very deliberate, of course, but in thinking of PTQ as a fragment of English, we're just not supposed to think about the implications of including them. One might, of course, say that they are not 'proper' expressions of the fragment, or that their interpretation is deficient because it's relative to an assignment (again, cf. §2 above), but this doesn't solve the issue – then there would just be English expressions that can't be interpreted, or we would arbitrarily insist that certain expressions contained in a fragment of **English** aren't real expressions of **English**, which is a claim I can't make sense of.³⁵

And this in turn leads us to the final question of what it is that PTQ is actually doing.

³⁴Of course I have no idea if Montague intentionally labeled them to look like pronouns of English in order to give the impression that they really were pronouns. As noted in fn. 2 above, he doesn't do this in 'English as a formal language,' so perhaps there was some reason for moving between obviously non-English variables to ones that look superficially like English pronouns. Whatever the intentions behind the move, though, it doesn't solve the problem: the indexed pronouns are still featureless variables, not natural language pronouns.

³⁵Something like this latter move seems to be happening in latter-day Montague-inspired grammars, where one finds comments to the effect that an expression is not well-formed unless all its traces / gaps / featureless indexed variables that serve the same function as Montague's are bound off or discharged in some way. But what could this possibly mean? The grammar generates them, and crucially relies on them compositionally in the formation of English expressions – what does the *ad hoc* insistence that they aren't 'well-formed' do? Is it that they *are* English expressions, but of a category that isn't uttered, because not well-formed? But there is no such thing as an unutterable category of English expressions (barring occasional cases of phonological or semantic ineffability, which isn't at issue here). Is the idea that they are *not* English expressions, and the grammar only documents the 'well-formed' expressions, i.e. the ones without such variables, as true expressions of English? But then since the grammar includes these expressions, it is of an auxiliary formal language, which then bears some secondary relationship to English, but does not show us how English actually compositionally interprets quantification (instead, we point to a non-English language, and say, 'the end result is like what would happen if we interpreted this *other* language compositionally, like so').

It is indeed a fragment, but the language for which it provides a semantics is not English. It is therefore a fragment of an auxiliary formal language, which contains featureless variables. It is, in other words, a fragment of an English-like language that is supplemented with non-English tools from mathematical logic, a language that treats quantification as these auxiliary languages do, and not as English does. PTQ is therefore a document of formal interest, but its claim to be a contribution to natural language semantics is made hollower by two conclusions that follow from the above.

First, regarding the methodological question of whether the tools of formal logic are capable of providing fragments of natural languages: it means that PTQ doesn't demonstrate this, and so leaves the question open (as do any number of contemporary fragments and treatments in linguistic semantics, which also modify the natural languages they work on to include artificial expressions and operations). If the Montagovian claim is to be an interesting one, and not one that we insist is true *a priori*, it has to be given substance by showing how it is right or wrong, and insofar as PTQ is taken to be a proof of concept, it doesn't accomplish what it sets out to do, because it has not treated a fragment of English.

Second, regarding the empirical questions of concern to the natural language semanticist, as to how phenomena like quantification, intensionality, and scopal ambiguity actually work in natural languages, and which PTQ is presumably intended to provide insight into: it's not clear that the fragment tells us anything about how English or any other natural language treats these phenomena, since it isn't operating on English. That one can **modify** English to look more like a formal language, and that one can apply the tools of formal logic to treat this **modified** English, which includes featureless variables, shows us nothing in principle about **English**.

As I said in the introduction, the fact that PTQ is not a fragment of English matters for semanticists insofar as they're tempted to make use of tools borrowed from mathematical logic that resemble Montague's. The charge leveled at Montague can also be leveled at any semanticist who makes use of such tools, insofar as those tools resemble nothing actually found in natural languages: this goes for the standard treatment of variables in contemporary semantics, which has been inherited from mathematical logic despite having no natural language counterpart, but it's also fairly rampant in other formal tools adopted by the discipline.

For mathematical logic, the question of whether the tools used to treat formal languages are also applicable to natural languages is not integral to the subject: it might be so, or it might not be, but in either case mathematical logic is what it is, and can exist independently of what the linguist does with natural languages. But for the linguistic semanticist, who at least nominally is involved in the treatment of natural and not artificial languages, that same question is crucial in determining what tools are appropriate for the discipline. If the tools used to treat formal languages are not in general appropriate for the treatment of natural languages, this means that any semanticist successfully building a fragment using those tools is effectively treating not a natural language, but an artificial one.

I suspect that this is what's typically happening in formal semantics: the formal semanticist treats not natural languages, but formal languages that resemble natural languages in certain respects, but which are supplemented with non-natural expressions and operations

familiar from mathematical logic.³⁶ If this is right, then the subject matter of formal semantics is miscast, and it's not obvious what interest this hybrid formal discipline has, which treats artificial languages that resemble natural languages only partially. Presumably, we ought to just study natural languages instead.

Supposing that semanticists typically make use of formal devices that don't correspond to anything in natural language, does this show that formal semantics as a whole is on the wrong track, if it wants to stay true to the mission statement of treating natural languages? Not necessarily, but as noted above, we have to put up or shut up. If formal tools resembling those of mathematical logic really can be successfully applied to natural languages, we ought to take the program seriously and try to show just that, rather than admitting the existence of artifices that are orthogonal to that program.

At the end of the day, the question is whether formal semantics is an empirical discipline or not. The answer that's been given since about 1970 or so, corresponding roughly with the release of PTQ, is 'sort of:' it remains in a hybrid limbo, where one addresses natural language phenomena, but by performing the kind of regimentation to fit the mold of formal languages that was common practice before 1970. Formal semantics has never really shaken off its origins as a hybrid formal-empirical discipline, and it's not really clear what the value is in a discipline that's sometimes empirical, sometimes not, and that treats genuine natural language phenomena essentially by speculating on what they would look like if they were different, and so looked more like the phenomena of formal languages instead. Eyes on the prize: the formal tools may be fun, but the phenomena are real!

References

- [1] Church, A. (1951). The need for abstract entities in semantic analysis. *Proceedings of the American Academy of Arts and Sciences* 80(1): 100-112.
- [2] Montague, R. (1970). English as a formal language. In B. Visentini, ed., *Linguaggi nella Società e nella Tecnica*, 188-221. Milan: Edizioni di Comunità.
- [3] Montague, R. (1973). The proper treatment of quantification in ordinary English. In K. J. J. Hintikka, J. M. E. Moravcsik, & P. Suppes, eds., *Approaches to Natural Language: Proceedings of the 1970 Stanford Workshop on Grammar and Semantics*, 221-242. Dordrecht: D. Reidel.

³⁶I don't have a ready list of all the tools in formal semantics that would seem to be vulnerable to this charge, but there are a lot of them, some inherited from mathematical logic, and some homegrown. In addition to the difficulty with variables (and their counterparts, like traces or gaps), for instance, there's no obvious reason why natural languages should be thought to contain things like indexation, covert abstraction, covert movement, type shifting, a distinct syntactic level of 'logical form,' and so on. It's true that positing that a language has these things makes the formal tools that semanticists favor work properly in treating that language; but this doesn't suffice to show that the language that the semanticist is successfully treating with these tools is a natural language, rather than an artificial language supplemented with all the relevant posited machinery.