Towards a Geometrical Syntax: a Formalization of Radical Minimalism[1]

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Abstract:

In this paper we attempt a logic-mathematical formalization of the program for (linguistic) investigation that we have called Radical Minimalism, in order to make the model fully explicit and reveal its potential to be applied not only sub-personally and to the language domain exclusively, but to *any physical system one considers*. For those purposes, we have devised a logical model of syntax (understood as a general-purpose free and unbounded generative algorithm), with which we capture the properties of mental workspaces and the cognitive organization of the phenomenological reality. We will combine our *Strong Radically Minimalist Thesis* with Tegmark's (2007) *Mathematical Universe Hypothesis*, and analyze how both strengthen in interaction gaining in empirical coverage and theoretical weight. The aim is to formalize a possibility for the study of language as a physical system from an interdisciplinary perspective: a joint work between Formal Linguistics, Logic, Physics and Mathematics.

Keywords: formal syntax, Radical Minimalism, mathematics, interfaces in language design

1) INTRODUCTION

[1] I am indebted to Juan Uriagereka, Pierre Pica, Roland Friederich, Gennadi Sardanashvily, Peter Kosta, Katarzyna Mathiasen and Mike Putnam for comments and opinions. I am also indebted to Chris Collins and Edward Stabler's (2012) paper as it was an inspiration for this formalization. Cool figures (1) and (4) are entirely Juan Cruz Aramburu Blanch's merit. All mistakes, needless to say, are entirely my responsibility.

Our goal in this paper is to provide tools for the formalization of a model of syntax with bases on geometry and set theory, following the proposals made in Krivochen (2011a, b) and Krivochen & Miechowicz-Mathiasen (2012). Using formal tools devised within mathematical logic, we will first provide definitions of the substantive primitives we will work with and the operations that apply to such primitives, in order to generate structures of a higher order of complexity. The result, we expect, will constitute the prolegomenon to further studies of language within a mathematical framework, in which ambiguities in the formulation of principles and concepts disappear by adopting a universal notation traceable back to Frege's *Begriffsschrift* and Russell & Whitehead's *Principia Mathematica*, adapted to current demands of simplicity and economy on derivation and representation.

We will begin by reviewing the main tenets of the (linguistic) theoretical framework we will adopt, Radical Minimalism (Krivochen, 2011a)

(1) Postulates:

a) Language is part of the "natural world"; therefore, it is fundamentally a physical system.

b) As a consequence of (a), it shares the basic properties of physical systems and the same principles can be applied, the only difference being the properties of the elements that are manipulated in the relevant system.

c) The operations are taken to be very basic, simple and universal, as well as the constraints upon them, which are determined only by the interaction with other systems.

d) (b) and (c) can be summarized as follows:

e) **Strong Radically Minimalist Thesis (SRMT):**

All differences between physical systems are "superficial" and rely only on the characteristics of their basic units [i.e., the elements that are manipulated]*, which require minimal adjustments in the* ***formulation*** *of operations and constraints* [that is, only notational issues]. *At a* ***principled level****, all physical systems are identical, make use of the same operations and respond to the same principles.*

SRMT licenses the possibility of looking for biological, physical and mathematical properties of mental computations (i.e., syntax), ***without its being a metaphor*** but a *true account in three levels: description, explanation* and *justification*. The description is the *what*, the explanation is the *how*, and finally, the justification is the *why*. The latter has been either taken for granted or done in a truly *non-minimalist* way both substantively and methodologically. Our effort, then, will focus on trying to set a *radically minimalist* alternative of justification, taking into account that a theory of the physical universe must address all three. Attempting *justification* is what we understand as the ultimate goal of going "beyond explanatory adequacy". We have chosen geometry because it licenses the possibility of *n*-dimensional representations, unlike logics, whose representations, although hierarchical, are bi-dimensional, and therefore inaccurate to represent processes in the quantum human mind we work with (Krivochen, 2011d).

2) SRMT and MUH:

In this section we spell out how our hypotheses are influenced by Tegmark's conceptions of the mathematical structure of the physical reality. We will develop our framework and then tackle the problems he finds in mathematical structures. Our work is intimately related to Tegmark (2007) and his *Mathematical Universe Hypothesis*, stated as follows (Tegmark, 2007: 1):

1. **Mathematical Universe Hypothesis (MUH):**

Our external physical reality is a mathematical structure.

A (mathematical) structure is defined as a set S of abstract entities and relations $\{R_1, R_2 \ldots R_n\}$ between them.

The "physical reality" of the MUH is identical to the "physical systems" of SRMT, and the postulation of a subjacent mathematical structure is made explicit in SRMT with the mention of Universal *operations* and *principles*, which we have formulated in past works (Krivochen, 2011a, b, 2012; Kosta & Krivochen, 2011). Our Radical Minimalism provides principles and operations that go in consonance with both MUH and its stronger version, the *Computable Universe Hypothesis* (Tegmark, 2007: 20):

2. **Computable Universe Hypothesis (CUH):**

The mathematical structure that is our external physical reality is defined by computable functions.

Tegmark explains this hypothesis as meaning that "(…) *the relations (functions) that define the mathematical structure* […] *can all be implemented as computations that are guaranteed to halt* *after a finite number of steps* (…)" (Tegmark, 2007: 20). This is clearly explained in the Radically Minimalist derivational dynamics, which we will formalize below: we only need one computational operation: *Merge*, and it halts always and only when a fully interpretable object is recognized by the relevant interface level, what we have called a *phase*, and *transfer* to an interpretative system occurs.

What linguistics can provide a mathematical approach like Tegmark's is not only empirical basis, but also strong predictions regarding a limited domain of the physical reality, in which those predictions, that follow from general hypothesis, can be put to test. At the same time, a mathematical approach to the "natural world" results in a formalization that is most necessary for deepening our understanding of the very structure of reality, in our case, the structure and functioning of the mind-brain. This, in our opinion, can and should be expressed in unambiguous mathematical terms. In this work, we attempt to formalize Radical Minimalism in such a way that we can base syntax, understood as a generative mechanism operating there where complexity is necessary, in mathematics, following a geometrical model. This framework also reinforces Izard, Pica et. al. (2011)'s theory of geometry as a Universal mental constructions, upon Euclidean basis[2]. After having presented our formalized model in section 3, we will analyze to what extent RM can contribute to answer some questions that have arisen within the MUH framework and make the system more powerful.

3) Towards a Geometrical Syntax:

In this section we will provide some concepts and definitions that will configure our formal model. Afterwards, we will circumscribe ourselves to language, and analyze the consequences the adoption of this model has for the study of syntax[3].

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2 In our framework, the mathematical workspace can be Euclidean or not (hyperbolic or elliptic) depending on interface requirements, see Krivochen & Miechowicz-Mathiasen (2012). What is innate, in the case of human mind, is the quantum processor that allows multiple derivations simultaneously. Interface requirements derive from the interaction between interpretative components and the phenomenological world.

3 The reader may find it useful to confront our formalization with that of Collins & Stabler (2012), which formalizes orthodox minimalism following Chomsky's (1995 et. seq.) proposals.

Definition 1 (non-final): a point is an *n*-plet (x, y, z…*n*), *n* = number of dimensions of a workspace W.

Definition 2: W is an *n*-dimensional *generative* workspace. Taking two distinct workspaces W_X and W_Y, either

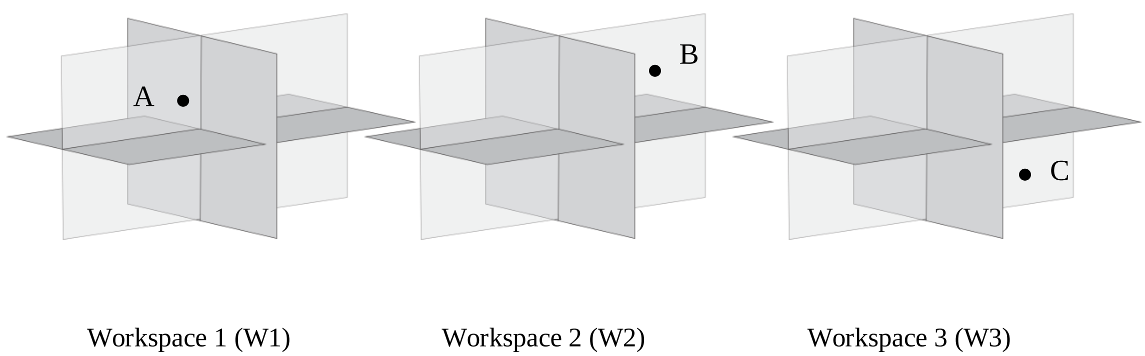
a) $W_X \equiv W_Y$ *iff* $\forall(x) \mid x \in W_X, x \in W_Y \wedge \nexists(x), x \in W_X, x \notin W_Y$

b) $W_X \neq W_Y$ *iff* $\nexists$ $(x) \mid x \in W_X, x \in W_Y$

c) $W_X \cong W_Y$ *iff* $\exists(x) \mid x \in W_X, x \in W_Y$

This allows us to define the relations of *identity*, *difference* and *similarity* in set-theoretical terms. *Identity* holds between W_X and W_Y if and only if every element of W_X is also an element of W_Y and vice versa (formalized in (a)). If this condition obtains, there is also *logical equivalence* between W_X and W_Y in all relevant contexts. *Logical equivalence* is entailed by *identity*, as it is to be expected. *Difference* holds if and only if there is no common element between W_X and W_Y, which means that they are not set-theoretically related (formalized in (b)). *Similarity* is a relation in which there are common elements *at least* -but not necessarily *only*- one, between W_X and W_Y (formalized in (c)). Let us consider Figure (1) to exemplify the generative operation we are to formalize:

(1)



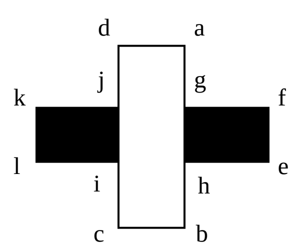
a) *Merge* (A ∈ W1, B ∈ W2, C ∈ W3) by *unrestricted distinct Merge*

b) Triangle α = {A ∈ W1, B ∈ W2, C ∈ W3}

Of course, α is a *mental construct* in W_4, a workspace in which concatenation is interpreted. $\{W_1 \ldots W_4\}$ must be isodimensional for interface purposes, so that the coordinates can be joined in a single W (our W_4). If we are working with the mind-brain, W is always 3-D, and, according to recent research (Gómez Urgellés, 2010), it should be *hyperbolic* since human perception tends to construct the outer physical reality as a hyperbolic space. In humans, Ws emerge from the interaction between the prefrontal cortex and relevant areas of the brain and is ***tri*-dimensional**. It is worth noticing that our mind-brain, which works in a tri-dimensional phenomenological world, can conceive the fourth dimension, but there is no account for dimensions > 4, while they are theoretically possible. A legitimate question would be if the characteristics of the phenomenological world in which an *interpretative system* matures determine its dimensional limitations. We believe they do, in the same way that the flow of Time, which can be defined as a *concatenation* of coordinates in 3-D workspaces, affects temporal schemes known as *Aktionsarten* (Vendler, 1957): in a lightspeed world there would be no *achievements* (i.e., telic-non durative events).

Not only these kinds of abstract figures require more than one W and a separate W to put the information together. Consider figure (2):

(2)



In (2) what we can *objectively* see is a white rectangle and two black rectangles at the sides. It is likely that each is perceived and processed in a separate W, such that rectangle {a, b, c, d} is processed, say, in W_1, {e, f, g, h}, in W_2 and {i, j, k, l} in W_3. However, the reader most likely constructed a single rectangle {e, f, k, l}, assuming that {g, h, i, j} are the points of intersection between a white rectangle and a single black rectangle. This operation of "reconstruction" must be performed in a further W, such that it is *isodimensional* to the others, which are in turn *isodimensional*: this means, not only having the same number of dimensions, but also *the same dimensions* (e.g., a Calabi-Yau figure has six specific dimensions, therefore, only a W with those very six dimensions will be taken as *isodimensional* to a Calabi-Yau figure). This procedure only makes sense if there is an *interpretative system* that has to read the information, and it is driven by economy: perceiving *two* figures is simpler than perceiving *three*: two figures allows the mind to activate the well-known *figure-ground* dynamics, and so we can say "there is a white rectangle above the black one", even if it only occurs in our conceptual space, and is ultimately an *inference*.

Definition 3: *concatenation* defines a *chain* of coordinates in *n*-dimensional generative workspaces W of the form $\{(x, y, z\ldots n) \subset W_X \ldots (x, y, z\ldots n) \subset W_Y\ldots(x, y, z\ldots n) \subset W_n\}$ where $W_Y \equiv W_X \equiv W_n$ or $W_Y \neq W_X \neq W_n$. If $W_X \neq W_Y$, they must be isodimensional in the sense specified above.

The *concatenation* function is called *Merge*, defined as follows:

***Merge** is a **free unbounded operation** that applies to an x number of objects sharing format, either ontological or structural.*

Definition 4: geometrical "figures", sentences, and other "observable" objects which constitute the *phenomenological world* are epiphenomenal results of *concatenation* in one or several W read off at an interpretative system.

This definition, which follows clearly from what we have been saying, can be formulated in a stronger way: *there is no physical reality beyond the interpretation of the concatenation function applied to an n-number of objects sharing format.* This is another way to express Tegmark's MUH; the so-called physical reality *is* a mathematical structure. We accept MUH without necessarily accepting ERH (the External Reality Hypothesis) as it is formulated: in any case, the notion of *external* and *reality* should be redefined. This must not be interpreted as a plea for solipsism: the concatenation function applies to objects that are *external* to the human mind and perception plays no role in generation. Moreover, there is no need to resort to a *human* mind: an automaton with the algorithm incorporated and interpretative routines (based on Relevance Principles) could serve as well.

Definition 5: an *Interface Level* IL is an *interpretative system* that has access to a W in which *concatenate* applies and establishes legibility conditions regarding its output.

Definition 6: all operations within Ws are interface-required in order to satisfy *Dynamic (Full) Interpretation*.

<u>Principle 1:</u> ***Dynamic (Full) Interpretation***: *any derivational step is justified only insofar as it increases the information and/or it generates an interpretable object*.

In this point, it is crucial to say that, if our view of syntax is that of a generative component that manipulates objects regardless their nature, then it can be applied to any so-called "complex object" insofar as complexity can be decomposed in layers of simple, atomic elements somehow concatenated for interpretation purposes. If this view is correct, all physical systems would have "derivations", in the sense of "successive applications of *concatenation* and subsequent interpretation".

<u>On Information and Objects:</u>

<u>Principle 2:</u> ***Conservation Principle****: information cannot be eliminated in the course of a derivation, but it must be instantiated in the relevant system in such a way that it can be read and it is fully preserved.*

Definition 7: *Entropy* in a derivation occurs only when there is a *generative-interpretative* interaction. If there is no *interpretative system* to read a structure, there is no point in introducing *entropy*.

Definition 8: ($W1_0 \ldots W1_n$) indicate successive derivational steps *within* a W.

Definition 9: LEX_S is the full set of *type*-symbols that can be manipulated by a computational system S, which is a generative W.

Definition 10: an *array* is a set of *types* drawn from LEX_S.

Definition 11: a *type* is an abstract element in a physical system Φ.

<u>Corollary:</u> there are two kinds of *types* in a linguistic derivation: those that convey *conceptual* meaning (i.e., *roots*) and those that convey instructions as to how to interpret the *relation* between conceptual types (*procedural* types). Determiner, Time and Preposition are *procedural types* (just like operators like [+], [-], etc. are procedural in Mathematics). The procedural or conceptual character of a node is of no importance to syntax, it is read at the semantic interface, and only there is it of any relevance.

Definition 12: a *token* is an occurrence of a *type* within W_X. There are no *a priori* limits to the times a *type* can be instantiated as a *token* but those required by Interface Conditions IC.

Definition 13: a *token* is never fully interpretable at the relevant Interface Level IL unless within a larger structure.

Within Linguistics, this definition is the RM way to formalize Distributed Morphology's "*categorization assumption*", adding empirical and theoretical coverage. This has been expressed as the *Conceptual-procedural interface symmetry* (Krivochen, 2011d)*:*

There cannot be bare roots without having been merged with a procedural node or procedural nodes without having been merged with a root in the syntax-semantics interface.

Corollary: an element in W_X is always a *token*. Its interpretation depends on its own content and the structural position of its occurrence (i.e., local relations with other tokens, *conceptual* or *procedural*).

Definition 1' (final): A point $p = (\{x, y, z\ldots n\} \in W_X)$ is a *token* of a single-*type* LEX_S.

Definition 1'': A lexical item LI is a structure $\{X\ldots\alpha\ldots\surd\} \in W_X$, where X is a procedural category (Determiner, Tense, Preposition), α is an *n* number of non-intervenient nodes for category recognition purposes at the semantic interface, and √ is a *root*.

***Roots** are **pre-categorial** linguistic instantiations of **a-categorial generic concepts** from the Conceptual-Intentional system. Generic concepts are "severely underspecified", since they are used by many faculties, and therefore cannot have any property readable by only some of them; otherwise, the derivation would crash in whatever faculty we are considering. Roots convey **conceptual** instructions, whereas **functional nodes** convey **procedural** instructions to the post-syntactic semantic parser.*

In formal terms, to be refined below, a root √ is a semantic *genotype* (i.e., establishes potentials and limits for variation)[4]. For all natural languages NL, it is the case that $\forall(x)$, $x = \surd$, $NL \ni x$. LI

4 This could be interpreted as a metaphor, in the sense of Teso (2002): "El significado es una especie de sustancia química, lista para reaccionar con el contexto que le propongamos (Teso, 2002 p. 44)" [meaning is a sort of chemical substance, ready to react with the context we propose]. Following the line of Kosta, Krivochen & Peters (2011), if syntax is a biological system and there is no such a thing as a sub-categorial / categorial distinction as in traditional lexicalist models, then the very same formalization we have proposed for syntactic structures understood in the categorial level must apply also at the sub-categorial level. As a consequence, the entropic biological model we have argued in favor of in Krivochen (in preparation) applies to "lexical" derivations as well, since there

in particular NL are *phenotypic* instantiations, and it can be the case that $\exists(\text{NL}) \wedge \text{NL} \ni x \wedge \text{NL} \not\ni \text{LI}$.

√ are always *types*. LIs are always *tokens* as they are interface readings of a specific syntactic configuration as depicted in **Definition 1''**.

For all NL, and for the set of all procedural elements $P = \{P_1,\ldots, P_n\}$, it is the case that $\text{NL} \ni P$.

Given a *specific* structured set $p = \{P_1 \{P_2 \{\ldots P_n\}\}\}$, it may be the case that $\text{NL}_X \not\ni p$.

Definition 14: A *sentence* is an epiphenomenal result of *concatenation* of LIs between isodimensional Ws following interface requirements.

Definition 15: *format* is either *structural* or *ontological*. Ontological format is the nature of the elements involved in an operation, whereas structural format is the way in which these elements are organized.

<u>On Operations:</u>

Definition 15: *Select* instantiates a *type* in a W_X following **Principle 2**.

Definition 16: *Merge* concatenates LI-tokens in a W_X driven by the interfaces' constraint expressed in **Definition 13**.

Definition 17: *Analyze* evaluates the objects built via *Merge* in W_X in order to verify full interpretability in IL_X.

Definition 18: *Transfer* is the operation via which an Interface Level IL_X takes a fully interpretable object from W to proceed with further computations.

<u>Corollary:</u> if W_X interfaces with more than one IL, *Transfer* applies for each IL *separately*.

Definition 19: *Merge, Analyze* and *Transfer* are both interface-driven and interface required.

Definition 20: 15, 16, 17 and 18, occurring cyclically, determine the derivational dynamics in W_X.

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are only "derivations" in Radical Minimalism, without any further distinction. If this is so, then there is no "metaphor" at all, as in Teso's quote.

Definition 21: *category recognition* requires X and √ to be in the same workspace at the point in the derivation in which category is read off. X and √ may, however, have entered the derivation in different but isodimensional Ws.

On Linguistic Derivations:

Definition 22: a *Derivation* is the finite set of computational steps that define a Syntactic Object SO, projecting a *type-Array* onto a legible object for IL via *token-Merge*.

A note is in order here: the finite character of the linguistic derivation is determined by memory issues and computational capacity of interface systems, which is limited by the biological substrate. Computations *in abstracto* are not finite by principle, and do not need to be. As we will see below, the presence of a "halting algorithm" in the form of IC only applies when there is *generation-interpretation* interaction. If a function is not interpreted (say, for example, Peano's axioms), there is no need to stop the derivation at any point.

Definition 23: a syntactic object *Converges* if and only if all of its components are interpretable by the relevant IL/s.

On Dependencies:

Definition 24: if α and β are interface-associated via their coordinates in W, there exists a *Dependency* between α and β.

Definition 25: if $\alpha \in W_X$ and $\beta \in W_Y$ and either $W_X \equiv W_Y$ or $W_X \not\equiv W_Y$ and α and β are defined by the same *n*-plet of coordinates in their respective *isodimensional* Ws, α and β are *bound* and the *dependency* is called *co-referentiality*.

Definition 26: *Reference* is location of a symbolic object in the conceptual multi-dimensional space via LI's coordinates, which are interpretable by the conceptual system C-I.

Definition 27: *Dependencies* are read off in IL, not in W.

On Locality:

Definition 28: A *dependency* is *Local* if and only if there is no intervenient object γ (of arbitrary complexity) such that: (i) the relation between α and γ is equivalent to that between α and β for interface purposes (ii) α, β and γ belong to the same W and (iii) γ is structurally closer to α than β

This definition, combined with definition (24), has the consequence that *no dependencies can be established in a generative W*, but at Interface levels, which we have expressed in definition (27). This also means that if there is no interpretative system involved, there are *no dependencies*. For example, in the structures generated by a mathematical algorithm that are never interpreted (see example (22) below) no dependencies can be posited.

Definition 29: P_W is a *phase* in a W if and only if it is the minimal object fully interpretable in IL.

Definition 30: *Transfer* applies only to *phases*.

Once we have defined the units we will work with, the operations that apply, and the conditions under which those operations apply to certain symbolic objects[5], we will devote the rest of the paper to the consequences the adoption of our model would have for the theory of natural language structure. Our hope, however, is that the model hitherto devised is not limited to natural languages, but can also serve mathematical or physical purposes, since we do not limit the power of our generative algorithm to just one domain in the natural world. Let us now consider some central questions in formal linguistics under this light.

4) Some Applications in Formal Linguistics

4.1 Merge and Phrase Structure

This framework allows us to manage symbolic representations in a way in which we can best capture their essential property, *hierarchy*, while accounting for epiphenomena in terms of interface conditions. Let us first review the classic X-bar theory axioms, which characterized phrase structure in the Government and Binding model (Chomsky, 1981 et. seq.) and underlie most current models of phrase structure to different extents:

1. <u>Endocentricity</u>: every projection has a head

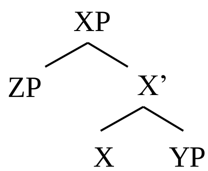
2. <u>Projection</u>: every head projects a phrase

3. <u>Binary branching</u>: every non-terminal node is binarily-branched

[5] The mental entity of these objects is not at stake here. Either if the symbolic structures our system generates exist only in the mind, or as external objects (assume, for example, that a mathematical structure has entity independently of a perceiving mind), our system holds because it makes no prediction at this respect. Thus, the Postal-Chomsky controversy regarding realism or conceptualism is completely tangential to this proposal: it could work in a conceptualist theory, in a realist one or in a third party without falling in contradictions.

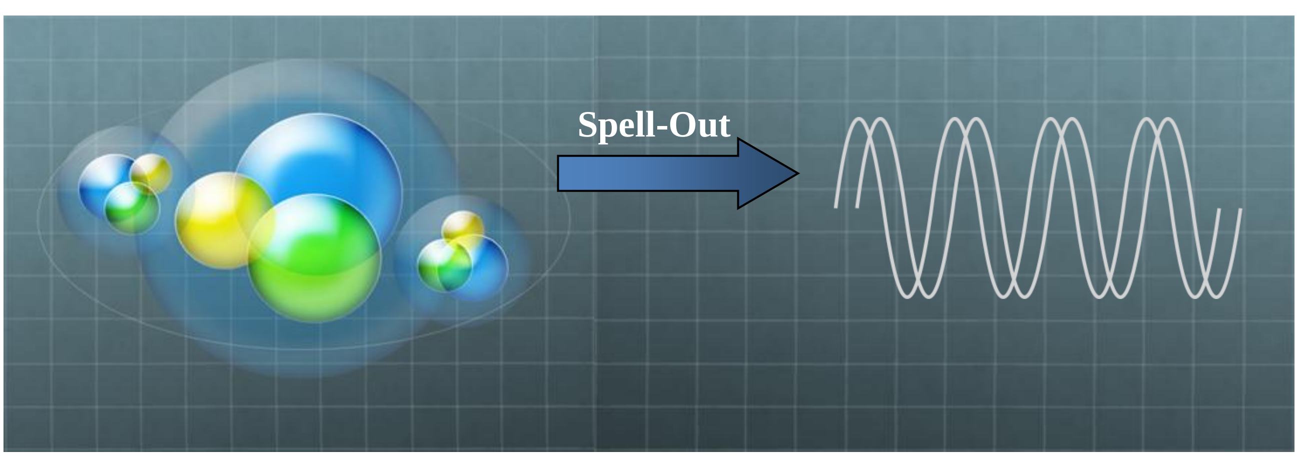
The kind of structures generated by this system is as follows:

(3)



These are bi-dimensional representations within the syntactic workspace, afterwards Spelled-Out. It is commonly assumed that Spell-Out implies a "flattening" of the hierarchical structure onto a linear representation: our claim is that there is no *essential* difference between a tree-like representation and its linearized form. Syntactic derivations have been claimed to be either *bottom-up* (Chomsky, 1995 and other orthodox works) or *top-down* (Zwart, 2009, in a different framework, Uriagereka, 1999), but always maintaining the underlying assumption that the *up-down* opposition is defining for syntactic processes: c-command is *top-down*, whereas m-command is *bottom-up*; Agree is also a *directional* operation (although there is no general agreement with regard to whether it is top-down or bottom-up, see Zeijlstra, 2011 and Putnam, Van Koopen & Dickers, 2011) and, given the fact that syntactic operations in orthodox Minimalist syntax are driven by the need to check features via Agree, directionality is centrally embedded within the theory. Syntactic structures, then, are *bi-dimensional*, only differing from phonological representations in epiphenomenal characteristics, like *headedness* or *binarity* which, as we have shown here and in past works, are C-I *interface requirements*. If a true qualitative difference is to be found between syntax and phonological externalized structures, then the nature of syntax must be revisited, as we have done. In our system, there is a difference between *syntax* and *phonology* regarding the dimensions of each domain: syntax is *tri-dimensional*, and its conversion to phonology implies flattening the structure to have *two dimensions* (we will analyze phonological representations below). The representations would look as follows:

(4)



In figure (4) we show how a syntactic structure -whose more exact representation, in our opinion, is that of an atom, with a *nucleus* (but no *head*) and peripheral elements (so-called *specifiers*, *complements* and *adjuncts*, all derived from the assumption of headedness) - is converted into externalized sound waves, after inserting Vocabulary Items in the syntactic terminals: semantic information has a phonological correlate. The materialization follows this hierarchical nucleus-periphery dynamics, as we have posited in Krivochen (2011d). Embick & Noyer (2004) propose a linearization procedure LIN (X, Y) = (X*Y) / (Y*X) (where * means phonological precedence), but the possibilities grow when we have more than two nodes to linearized, giving us an *n!* situation, which is not what we would want since there is no interface condition that can orient us towards the preferred option(s). The optimal scenario would be that in which interface relations are based upon *pre-existing relations*, namely, those created in the syntactic configuration. *Ceteris paribus*[6], *the nodes are spelled out mirroring the interface relation they maintain with the root, from the closest to the most detached*. From this claim, we derive that procedural nodes always have a closer relation with the root than peripheral nodes like Agreement, as they generate categorial interpretations in the semantic interface. Let us now assume that a language L allows for an X number of combinations of dimensions in terminal nodes, i.e., *morphemes*. That language has a Y number of Vocabulary Items to Spell Out those dimensions specified to some extent regarding their distribution. The feasible scenarios (but not the logically possible ones, which are more) are the following:

i) X = Y
ii) X > Y

Of (i) we would say it is a morphologically very rich language, and it is possible that there is a language in which this occurs. Of such a language we will say that it is *maximally specified*. The normal situation for any natural language is (ii), in which the number of Vocabulary Items is inferior to the number of terminal nodes. This situation is called *Underspecification*. Note that the phonological interface SM cannot read terminal nodes, but phonological matrices, so Vocabulary Insertion is a step that we cannot miss. This operation puts a VI in a terminal node; in a 1-to-1 relation in the simplest cases (we will see that this is not always the case, which makes the syntax-morphophonology interface a very complex object). In order for a VI to be inserted into a terminal node, it must match *some* of the features of that terminal node, there being a competence

6 This ceteris paribus condition refers to the availability of VI.

among several possible VI as to which one is more specified. In this way, universal constraints are replaced, *prima facie*, by a simple *subset principle*:

> ***Subset Principle:*** *The phonological exponent of a Vocabulary Item is inserted into a position if the item matches all or a subset of the features specified in that position. Insertion does not take place if the Vocabulary Item contains features not present in the morpheme. Where several Vocabulary Items meet the conditions for insertion, the item matching the greatest number of features specified in the terminal morpheme must be chosen. (Halle 1997).*

Our model can account for the syntax-phonology interface more elegantly than orthodox Minimalism, in terms of dimensional relations between one domain and the other. This way, we capture the true properties of the Spell-Out operation and establish a more general principle for Transfer operations:

(5) **Dimensional Impoverishment:**

Transfer ***may*** *imply the "flattening" of an X-dimensional structure to an X-n dimensional structure, while respecting Conservation Principle.*

An example of "dimensional flattening" occurs in Spell-Out (i.e., Transfer to S-M), whereas this is not the case in the syntax-semantics interface. We will analyze these cases below.

4.1.2 Some considerations about "Simplest Merge" and the Economy of Derivations and Representations

Our definition of *Merge* (definition 3 and corollary) entails the existence of a free and unbounded generative engine, which is available to any mental faculty that requires combination of atomic objects to form complex structures. Our hypothesis is that that very generative operation is available in the natural world, and is the principle of all complexity found in the Universe. If, as MUH entails, the Universe is not only mathematical but also simple (and describable by a finite and optimally reduced number of principles), the combinatory operation must follow not only from conceptual necessity but also economy principles. The notion of *simplest Merge* (Zwart, 2009) enters the scene. Zwart's formulation of Simplest Merge is as follows (2009: 2):

(6) Simplicity required in the derivational procedure:

a) Merge manipulates a single element of N [a Numeration] *at each step of the derivation*
b) Merge manipulates each element from N only once

However, the requirements do not follow from interface conditions, and are thus stipulations. It is also fundamental not to confuse economy in the theory with economy in the physical world. Even though Zwart's arguments in favor of his proposal have advantages over the version he presents of orthodox Minimalism (which is not quite accurate, to our understanding), the "requirement" for simplicity in the derivational procedure is stipulated in a syntacticocentric way. As a consequence, those principles have no interface relevance, and can be dismissed. Even if *Merge* is actually simpler applying to only *one* element, its interface value is null:

(7) Merge (α) = {α}
Merge ({α}) = {{α}}
Merge ({{α}}) = {{{α}}}

And so on. We see that this is a violation of our *Principle 2,* DFI. Economy in the theory must not be stated as a principle, but follow from third factor-like constraints, related to the architecture of the relevant object and the systems that read the output.
At this point, arguments for top-down (Zwart, 2009, Uriagereka, 1999 –although in a different way-) and bottom-up (Chomsky, 1998, 1999, 2005) derivations are equally implausible, since they both assume what we will call "the 2-D fallacy", which implies that tree-like representations have mental reality. Our multi-dimensional geometry has made a strong case against 2-D syntax, and argued strongly in favor of *n*-dimensional Ws in which operations are constrained only by the interpretative interfaces. Our revised version of "simplest Merge" is formulated as follows:

(8) *n*-dimensional Merge:

a) Merge manipulates an x number of elements at each step, being x interface-determined. If {{α}} is taken to be trivial, as we do[7]*, then x > 1, being 2 the minimal non-trivial number of objects.*

b) Merge manipulates each ***token*** *only once in a derivation* $\subset W_X$*, but each* ***type*** *as many times as needed*.

This definition of Merge allows us to dispense with Movement in terms of *displacement*, but reinterpret it as *token-Merge* and *dependency* establishment between *tokens* at the relevant IL, with basis on their coordinates in isodimensional W. As it allows multiple n-dimensional W to be working at once, our system provides a better account of both monotonic and non-monotonic merge than most orthodox theories, as we will explore in the next section.

4.1.3 Complex-unit Merge

Let us assume the following transitive sentence:

(9) The man saw a woman

According to Juan Uriagereka, parallel derivational spaces are a conceptual necessity (Uriagereka, 1999: 121) when Merge applies to complex objects, already output of a Merge operation: for example, in the derivation of [$_{\text{saw}}$ the man saw a woman], [$_{\text{the}}$ the man] and [$_{\text{saw}}$ saw [$_{\text{a}}$ a woman]] are generated separately and then assembled externally. The derivation in his terms would proceed as follows[8]:

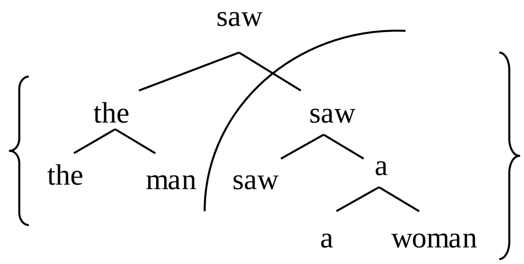
---

7 Boban Arsenijevic (p.c) claims that "{{a}} is non-trivial in at least one faculty: the arithmetic capacity. Hence, output conditions can't be that bare to favor a binary merge". However, our position is that if Merge is considered to be an operation and we assume also DFI, that is, the assumption that application of any operation must either lead to a legible object or increase the informational load, to apply Merge to a single object is trivial in any faculty. If {a} is already legible in the relevant interface level, then why apply Merge in the first place? It would be computationally redundant, and therefore far from Minimalist. We maintain that binary Merge is the minimal-maximal non-trivial option, and therefore reject any proposal of unitary Merge on interface grounds.

8 Nunes (2004) analyzes parasitic gaps using a mechanism very similar to the derivational Cascades model by Uriagereka (1999).

(10)

Derivation 1, *Merged* in W_1.

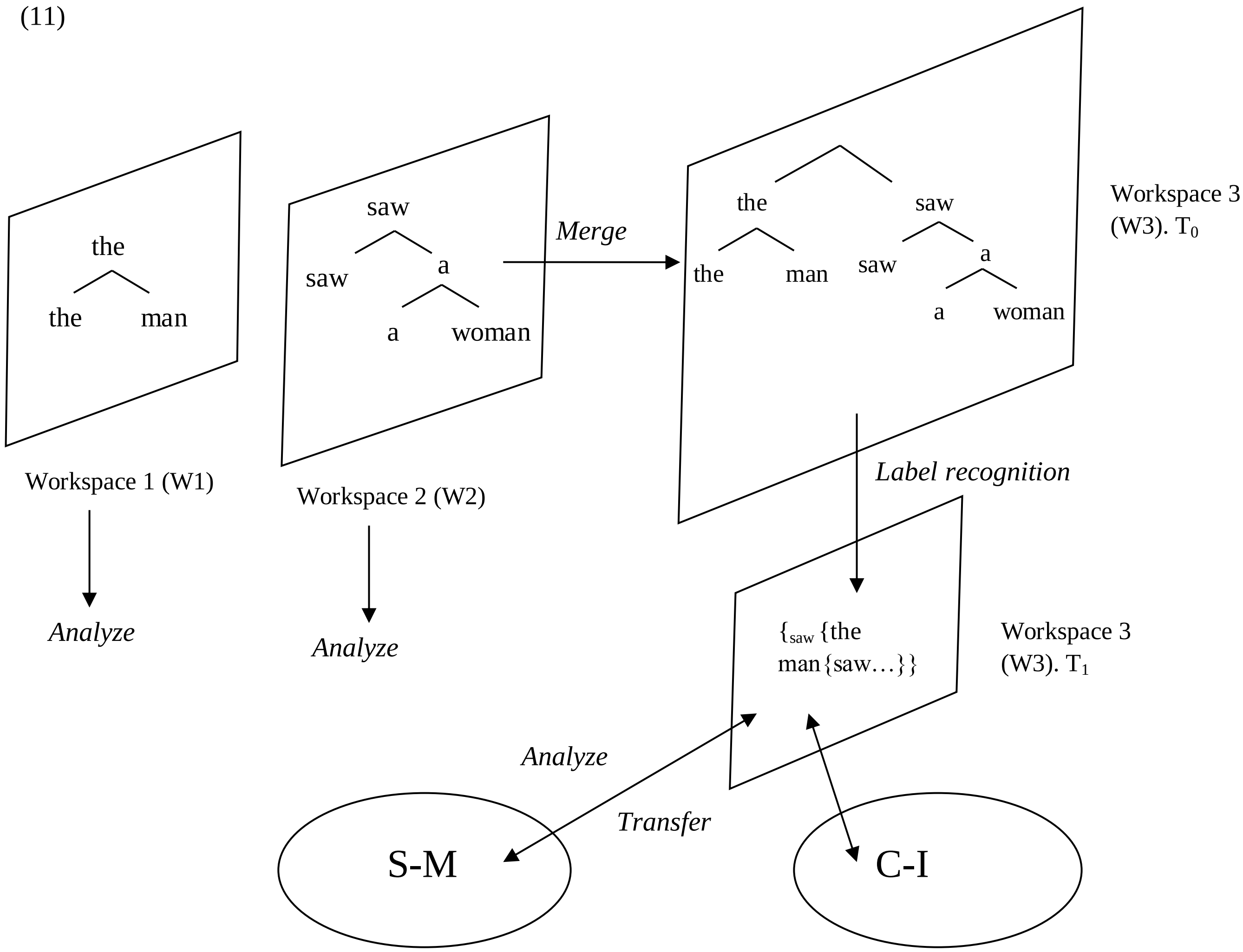


Derivation 2, *Merged* in W_2.

If the derivation occurred in just one space derivational space, then the units [the] and [man] would be merged to [saw a woman] sequentially (i.e., monotonically) and not as a fully-fledged constituent [the man]. That is, if we assume that [saw] would project a label indicating the properties of the complex object as a unit for the purposes of future computations, the final phrase marker would be the ill formed sequence: *$[_{\text{saw}}$ the $[_{\text{saw}}$ man $[_{\text{saw}}$ saw $[_{\text{a}}$ a woman]]]].

Within our newly-developed framework, its (summarized) derivation would be represented, in terms of *command units* (Uriagereka, 1999/2002), as follows (we beg the reader to think of it in 3-D terms, like in (1)):

(11)



We see that each "derivational cascade" (Uriagereka, 2002) is assembled in a separate tri-dimensional workspace W, and then Merge applies to tokens of both objects in a third W, to which the interfaces have access and therefore *evaluate* the objects created.

The only characteristic Ws must have is that it must be *n*-D, *n* > 2 and interacting Ws for *non-monotonic Merge* purposes must be isodimensional. 2-D W would result in a 2-D model of the mind, which, even if quantum, could not account for the multiple tasks the brain carries simultaneously, many of which *require* at least 3 dimensions (like symbolic structures –syntax as the first example-, figure-ground dynamics, among others). Of course, a *sine qua non* condition for *concatenation* effects at interpretative interfaces is that *all Ws involved* are *isodimensional* (Definition 3).

4.1.4 Syntactic relations: Coordination and Subordination revisited:

In this brief section we will revisit the traditional concepts of hypotaxis and parataxis in geometrical terms, following the definitions above. We took a sentence to be an epiphenomenal object, an *interface reading* of a set of coordinates, which allows us to define any syntactic relation between sentences in set-theoretical terms, thus maintaining the number of primitive concepts in the theory to the minimum. We will now express the aforementioned relations in mathematical terms:

- *Coordination*:

Considering a Syntactic Object S1 and a Syntactic Object S2, both set-theoretically defined, S2 and S2 are coordinated to form the "matrix" S3 *iff*:

a) $S1 \subset S3 \land \nexists(x), x \in S1 \land x \notin S3$

b) $S2 \subset S3 \land \nexists(x), x \in S2 \land x \notin S3$

c) $S1 \not\subset S2 \land S2 \not\subset S1$

- *Subordination*:

Considering a Syntactic Object S1 and a Syntactic Object S2, S2 is subordinated to S1 *iff*:

a) $S2 \subset S1 \land \nexists(x), x \in S2 \land x \notin S1$

b) $\exists(x), x \in S1 \land x \notin S2$

4.1.5 How do we derive?

In this section we will analyze the functioning of our framework in linguistic derivations, so as to make differences with orthodox approaches clear for the reader. The generator function we have defined in **Definition 3** dispenses with featural requirements for Merge, such as Pesetsky & Torrego's (2007) *Vehicle Requirement on Merge*. Free Merge has already been argued for in, for example, Chomsky (2004), but in a different venue. Moreover, our derivational model is closer to that proposed by Putnam (2010) insofar as it is only interface-constrained: as we will see below, we propose an extremely local evaluation mechanism (OT's EVAL) *Analyze*$_{\text{IL}}$ (that is, *Analyze* from the interface level IL) that applies after each instance of *Concatenate* and determines whether the object is fully legible by the relevant interface. Echoing Boeckx (2010), *Analyze* can be seen as the interpretative systems (whichever they turn out to be) "peering into the syntax", and taking the minimal unit they can read. Merge, as we take it, is a completely free operation that can apply as long as the objects to which it applies have the same format, motivated by interface conditions (this is, $\{\alpha\}$ is trivial at the interface levels, while $\{\alpha, \beta\}$ is not[9]). In the linguistic W (bear in mind that *linguistic* does not assume any differences as to W, but merely as to the interfaces it interacts with), we have atomic elements (either roots or procedural elements), and we can say that they have the same format since they share a nature, they are linguistic instantiations of elements that, *per se*, are not manipulable by $C_{(HL)}$. The only attribute of Merge would be putting things together, without any restriction by principle as regards the nature or number of objects, since it would be a stipulation. An example of the derivational dynamics we are proposing is the following:

5) Concatenate $(\alpha, \beta) = \{\alpha, \beta\}$
   Analyze_{IL} $\{\alpha, \beta\}$ [is $\{\alpha, \beta\}$ fully interpretable by IL?]
   (Transfer $\{\alpha, \beta]\}$ if Analyze_{IL} results in convergence at IL)

The idea of "invasive interfaces" is a natural result of separating interpretative and generative systems, following an OT architecture in which GEN and EVAL functions are clearly delimited. If generation is restricted to a single operation *concatenate*, which is the optimal scenario (since there are no *a priori* Agreement restrictions, Cf. Pesetsky & Torrego 2004) and it occurs in an *n*-dimensional workspace (another null hypothesis: restricting dimensions to 2, as in traditional Kynean tree diagrams, is stipulative, derived from considerations of "unambiguous paths" and LCA), it is only natural that the operation cannot *read* or evaluate what it has built. On the other hand, it is also only natural that the EVAL function is not separated from the interfaces but in

[9] Boban Arsenijevic (p.c) claims that "*{{a}} is non-trivial in at least one faculty: the arithmetic capacity. Hence, output conditions can't be that bare to favor a binary merge*". However, our position is that if Merge is considered to be an operation and we assume also a *dynamic* version of Full Interpretation that states that *any derivational step must be interface-justified*, that is, the application of any operation must lead to a legible object the application of Merge to a single object is trivial in any faculty, as it does not contribute in any way to legibility. If {{a}} is legible for the interface system the arithmetic faculty has to interact with, then why apply Merge in the first place, to apply Merge to a single object is trivial in any faculty. If {a} is already legible in the relevant interface level, then why apply Merge in the first place? It would be computationally redundant, and therefore far from Minimalist. We maintain that binary Merge is the minimal-maximal non-trivial option. We therefore reject any proposal of *unitary Merge* on interface grounds.

itself be the set of Bare Output Conditions that each interface has. Then, if we depart from considerations of computational simplicity like "maintain as few structure at once in W", that is, "transfer as soon as you can" (bearing an obvious resemblance to Pesetsky's 1995 *Earliness Principle*), this "as soon as you can" is determined not by internal syntactic conditions (as they do not exist in our proposal), but by *Analyze*. In this way, the generative workspace is wiped clean several times along a derivation thus liberating working memory without the concomitant problems of defining, for example, endocentric transfer domains (i.e., chomskyan *phases*).

4.2 The Quantum Human Computer Hypothesis: RM as a model of Quantum Linguistics

We must say at this point that, despite what it may seem[10], there is no *reductionism* in treating the so-called Faculty of Language (in case there is one, see below) as a physical system if we consider that a ***physical system*** is merely *the portion of the universe taken for analysis*. If we consider that universe to be the so-called "natural world", then, our thesis follows naturally. That is, we are not making a reduction of biology to physics, but simply analyzing a *biological* phenomenon in physical terms, as a physical system (in which there is no contradiction whatsoever) and, as such, applying the *tools* that have been devised in physics in the degree that it is possible, and without confusing the *methodological* tools with *substantive* elements. Of course, looking for exact correlates between *any* two fields would be irrational in the *substantive* level (i.e, units of analysis, as Poppel & Embick, 2005 correctly point out), but we put forth that the *methodological level* has much to tell us, as we are all working with "parcels" of the same Universe that, we will try to show, are identical in a principled level of abstraction, which is the main thesis of Radical Minimalism. This opens many possibilities for research, in multidisciplinary contexts. So far, there have been important works in this field, like Kosta, Krivochen & Peters (2011), Kosta & Krivochen (2011) and Krivochen & Kosta (in press). We will exemplify such attempts in the following section, where the notion of "quantum linguistics" will be introduced, following Salmani Nodoushan (2008) and Krivochen (2011e, in press)'s Quantum Human Computer Hypothesis. We will show that the linguistic mechanism is better thought of as a quantum computer than as a (Turing-like) digital computer, and the crucial relevance this has for theoretical linguistics, particularly of cognitive-biological orientation.

10 We thank Phoevos Panagiotidis (p.c.) for making this objection, and other valuable comments as well.

The Quantum Human Computer Hypothesis, presented in Salmani-Nodoushan (2008) and developed and formalized in Krivochen (2011d), distinguishes two types of computers: *digital* and *quantum*. The *digital computer* (DC) works by transforming an input in a series of, say, 1 and 0 regardless the input's nature or inner complexity. Once in its binary form, the information can be manipulated by the computer in a simpler way, since there is a unification of the format. The states 1 and 0 are, crucially, mutually exclusive. The digital computer can be analyzed as a non-recursive manipulator of discrete units, consisting mainly of an algorithm for transduction, a working area for the performance of mathematical operations and an active operation that performs those computations. The operations are strictly linear and sequential, and all steps have to be fulfilled in order to have a successful derivation. The formal expression of this operation is a rewriting rule of the form:

(12) $X \longrightarrow Y$

This type of generation rules is *crash-proof* in the sense that the output strings are limited and there is no chance to generate illegible objects. Early Generative Grammar (Standard Theory -ST- See Chomsky, 1965) based its Phrase Structure Component on this rule format. This is an example of highly restricted, constructivist system, incompatible with our definition of the *concatenation* function. The generative procedure is a fixed algorithm, one of whose forms is a sequence of divisions by 2 of any natural number, until the reminder in each division equals 0 or 1. The result is again divided by 2, as many times as needed, until it becomes zero. This is a simple *base component generative rule*, which operates with a reduced lexicon, namely, LEX = {1, 0}. Unlike ST, however, there is no *transformational component*, which makes this binary grammar very simple but descriptively and explanatorily inadequate for natural languages.

A more abstract sense in which we can think of a DC is a system that does not allow an object X to comprise several states at once. This can be seen as a simplification of the system, since it allows only 1-to-1 correlations between objects, but it also means that the characteristics of the relevant objects must be determined beforehand in a stipulative way. Besides, as we will see below, this system fails to capture many properties of natural languages.

The main characteristic of *Quantum Computers* is that they are not limited by the nature of binary *bits*, *either 1 or 0,* but they can work with 1 *and* 0 simultaneously. This means that there is no discretion in the definition of the units, which can be expressed as a "wave function" in isolation: prior to observation, the *quantum bit* or *qbit* is *both* 1 and 0, an undetermined state of a quantum field. Observation, according to Heisenberg (1999), collapses the state to one of the possible outcomes, in our particular case, 1 and 0. Of course, the possible outcomes of the quantum function can be more than 2, in which case the isolated element would comprise as many states as necessary. With this in mind, the QHC hypothesis states that *the human mind works like a quantum computer*. The advantages of the QHC hypothesis are clear: the system can perform many more operations in less time than a DC. DCs are constrained by what has been called "*the hundred step rule*" (see Carreiras, 1997), which establishes that, at most, a computer can perform a hundred sequential derivational steps per second, and many tasks require more than this. A QHC could perform "*billions of computations in no time*" (Salmani-Nodoushan, 2008: 30). QHC allows the mind to process information coming from faculties directly related with the phenomenological world (e.g., the visual faculty) and logic units (presumably *procedural*, taking into account that according to Leonetti & Escandell Vidal (2000), procedural categories do not have an *encyclopedic* entry, but they do activate a *logical* entry that determines the relation that the procedural element will establish between an *n* number of conceptual elements) in a *multidimensional* space. A disadvantage we find in this point is that there is no principled way to determine the path information follows. This could easily lead to a chaotic space, rather than a quantum field, in the sense that everything is possible, and if the mind is optimally designed, there must be some principled, third-factor constraints (in the sense of Chomsky, 2005: factors nonspecific to a mental faculty). With our definition of *analyze-driven Transfer*, however, we make sure that there is a halt to the computations as soon as a fully legible object is assembled: we have described and explained a self-regulating system in which the interfaces drive the derivation, and there is no possibility for the generative function (the *generator*, in Optimality-Theoretic terms) not to stop after a finite number of steps. We will come back to this below, when revisiting the MUH under the RM light.

4.3 Features and derivations:

Orthodox Minimalism has posited that there are two "apparent imperfections" in the Faculty of Language: uninterpretable features and Movement, understood as literal displacement of constituents (Chomsky, 1995, 1998; cf. Krivochen & Kosta, in preparation). Chomsky solves this apparent problem for the Strong Minimalist Thesis by saying that it is the need to eliminate those features that are uninterpretable at the interfaces that triggers movement. Notice that, elegant though this theoretical move may seem, it presupposes the existence of:

a) A Faculty of Language

b) Features (which, in turn, imply both a *Dimension* and *n* number of *values*, or, in our terms, *outcomes*)

c) An operation to relate Features (i.e., Agree)

d) Displacement

All three represent a substantive complication for the theory, and imply the isolation of the study of language from the study of other mental faculties, since the "imperfections" are not architectural constraints of the mind-brain but intra-theoretical stipulations: Minimalism has *created* both a problem and its solution. Radical Minimalism is all about *eliminating* problems, and impoverishing the theoretical apparatus to the minimum, so that only conceptual necessity dictates what is required in the model, while maintaining empirical coverage. We will now introduce our *Quantum Dimension Theory*, and apply it to the analysis of Case.
Imagine we have an electron in a tridimensional space, and we want to know its location. In order to do so, we need to see it, projecting some kind of light on it. This light is projected in the form of a *photon*, a particle with mass. The "problem" is that when the photon crashes with the electron, there is a change in the original location, which remains unknown. That original location (we have taken this magnitude just for the sake of the example, but we could have also worked with speed or trajectory) is taken to be a "superposition" of all possible locations, expressed in the form of a "*wave function*" (in de Broglie's terms). Therefore, there will always be a magnitude whose real value will remain unknown to us. In this kind of physical systems, it is the ***observation*** that makes the relevant dimension *collapse* to one of the possible states[11].
Uncertainty is a natural characteristic of physical systems, and by no means an instrumental problem, taking physical system in its technical sense, that is, any portion of the physical universe chosen for analysis. We take "physical universe" to be equivalent to "natural world", and we will use one or the other indistinctly. Magnitudes (or *dimensions*, to maintain a term more closely related to linguistics, since we are not dealing with measurable elements) are *not* necessarily binary; what is more, *in abstracto* they can comprise as many states as the system requires, which, as we will show later, leads us to a much simpler form of minimalism. ***We will express it by using this notation: for any dimension D, [D_X] expresses its quantum state.***

11 See, for example, the well-known EPR (Einstein-Podolsky-Rosen) paradox, which inspired Schrödinger's (1935) paper.

Let us assume the framework outlined so far and the following quantum dimension: [$Case_X$]. Following the idea presented in Krivochen (2010a, 2011a), this dimension comprises three possible "outcomes": NOM sphere (φ), ACC sphere (θ) and DAT sphere (λ). All three are possible final states of the system, and therefore the linear combination must also be considered a legitimate state of the system. The dimension *in abstracto* could then be expressed as follows, using SE:

(13) $N\varphi + A\theta + D\lambda$

As we have said before, this only holds if no "measurement" takes place, in Schrödinger´s terms. We will not speak of "measurement", since Case is not a magnitude, but we will consider that the factor that makes the relevant dimension collapse is ***the merger of a functional / procedural node***. What we must take into account is that not only do we have DPs with [Case] and functional heads in interaction in the computational system, but the output (i.e, the resultant state) must also converge at the interface levels, so our problem is a bit more complicated. As usual, we will focus ourselves in the C-I component. What we want to do now is derive the relations P-DAT; *v*-ACC and T-NOM from interface conditions, apart from the argumentation we have made in Krivochen (2012) in relation with θ-roles and Case, to which we refer the reader. Anything else would be stipulative, and that is something we cannot accept in Radical Minimalism.

Epistemologically, we have the advantage over theoretical physics that we have observable stretches of language where to test our hypothesis. So, let us take a ditransitive *Prepositional Indirect Object Construction* (PIOC):

(14) I_{NOM} gave a $book_{ACC}$ to $Mary_{DAT}$
(Yo)$_{NOM}$ di un $libro_{ACC}$ a $María_{DAT}$
(Ego)$_{NOM}$ $librum_{ACC}$ $Mariae_{DAT}$ dedi

By looking at the same construction in other languages with rich casual morphology (like Latin, Sanskrit or Greek) it has been established that the {D} [I] has Nominative Case, [the book] has Accusative Case and {P} [to Mary] has Dative Case. But this is nothing more than a description, with no explanation as to why things are the way they are. We will not review the classic attempts of explanation (see Chomsky, 1981, 1995, among others), but go directly to our point.

The weak hypothesis would be that each nominal construction will bear a dimension [$Case_X$], whose value when entering the derivation will be a "ψ-function", or a complex vector [$N\varphi + A\theta + D\lambda$]. That is, the quantum dimension will comprise all three possible values, as all three are possible states of the Case system for a particular DP, and since "syntax" is reduced to *concatenate* (a purely generative function), there is no problem in manipulating elements in their ψ-state as there is no "peeping into the box" as far as generation is concerned. However, [D_X] cannot be read by the interface levels, so the quantum dimension must "collapse" to one of the possible states. Here, it is not "measurement" but the reading off a configuration built by *concatenation* that does the work. Let us take the {P} structure as an example. We have the merger of {$to_{[TO]}$, $Mary_{[CaseX]}$} -and the subsequent "labeling" of the structure as {P} in Logical Form for semantic interpretability, since in a dynamic model interface-interpretation is in *real-time*-. At this point in the derivation, we are already in condition of collapsing the quantum dimension in [Mary], since we have a *procedural node* in a minimal configuration that can license that dimension in an XP (adapting the idea from Rizzi, 2004). This minimal configuration is defined in our terms *within phase boundaries*, following the definition given in **29**. Since the procedural information conveyed by P is essentially *locative*, the quantum case dimension on [Mary] will collapse to the *locative* sphere, i.e., *Dative*. A stronger hypothesis, perhaps more plausible, is that *the mere configuration is enough to license an interpretation*, without positing that there is a [D_X] beyond expositive purposes. The licensing takes place only if there is no closer functional / procedural head that can license the relevant interpretation, in order to respect Minimality. To account for this, we will draw another principle: ***Locality***, but as defined in **28** and **29**. Just like a particle cannot influence any other than its surrounding particles, within field theory, any object has a certain area of influence where it can license operations / interpretations if necessary. It is interesting that influence can be indirect, that is, a particle α may not be able to influence particle γ since particle β is "in the middle", but by influencing β, α will have an effect on γ, and thus we have a flexible definition of *Locality*, revisiting Rizzi's 2004 approach. That is what we call *compositionality*, in linguistics. We have to pay attention to the whole derivation (in *phase-level* terms) to understand, for example, the interpretation that a certain syntactic object receives at the interfaces.

4.4 Figure-Ground revisited:

Given the fact that figure-ground dynamics depend on the interaction between the prefrontal cortex and the temporal and parietal lobes respectively, our representation in terms of multiple workspaces geometrically defined working in parallel can provide further insight not only on language but on general perception and organization of phenomenological / perceptual information. Cognitive Linguistics has argued that entities are located within a conceptual space, which we have formally expressed as W. Our advantage is that the explicit case for multiple Ws implies optimization of computational efficiency, thus following from general economy principles. Regarding the specific case of Figure-Ground dynamics, which appears to be a fundamental opposition in the apprehension and organization of experience, the situation would be formally represented as follows:

(15) $W_X \cong W_Y \wedge \exists(x) \mid x \in W_X, x \in W_Y$

Representation in which there is a partial "overlapping" of spaces, an intersection, in set-theoretical terms. So, having a Figure *F*, and a Ground *G*, we need two workspaces to account for the biological basis of the complex structure:

(16)
$$F \in W_1$$
$$G \in W_2$$
$$\exists(x) \mid x = W_1 \cap W_2$$

In grammatical terms, *x* = *preposition* (P), as it is a procedural element that relates F and G in terms of *central* or *terminal* coincidence (Hale & Keyser, 1993 et. seq.). We now have a *logical* definition of P.

This definition allows us to establish some conditions on predication, assuming the localist hypothesis that all predication is ultimately *locative*, and relates a *figure* and a *ground*. The constraint we would like to posit is the following:

(17) *Given two conceptual entities α and β and an unspecified locative procedural element p, the entities can enter a locative relation iff p = α ∩ β. If there is no intersection between α and β, no locative relation can be established.*

This is a semantic restriction upon interpretation, but *by no means* combination: relations are read off configurations, not the other way around. Therefore, the freedom of *concatenation* is maintained.

4.4.1 Deixis and topological spaces:

In this section we will devote our attention to how deictic units fall within the figure-ground dynamics we have argued in favor of above. Deictic elements are linguistic units whose referential behaviour is traditionally said to vary according to elements of the context of situation, more specifically, the participants and the spatio-temporal context[12]. This characterization, which underlies the whole French tradition (e.g., Benveniste, 1977) and is partly based on Jakobson and Jespersen's work, draws a line between two kinds of nominal constructions: in Russellian terms, those that can be replaced by a definite description *salva veritate* (common names, quantified expressions) and those that cannot[13]. Such a characterization introduces an asymmetry in the linguistic system that is not desirable, if a general theory is an alternative. However, the semantic particularities of deictic items were completely overlooked by Generative Grammar, after the interest that had arisen within structuralism due to Jakobson's works. Since to this day there is not a Generative theory of reference, deixis is "no man's land" for formal studies. Our perspective is completely different from those that have been mentioned: we will try to formalize deixis mathematically, in an attempt to show that there is no fundamental difference between structures with deictic elements and without them, when it comes to syntax and, even, semantics.
To this end, deictic units are taken here to be *vectors*, whose elements are the same that are commonly assumed in physics and mathematics:

a) <u>Point of Origin</u>: array of coordinates {x, y, z,...n} in an *n*-dimensional space such that it is the starting point of the vector.
b) <u>Module</u>: the so-called "intensity" of the vector, the length of the segment that defines it.
c) <u>Direction</u> (or "line of action"): the straight line in which the vector is inscribed.
d) <u>Sense</u>: indicates to which side of the line of action the vector points. It is represented by an

12 We are taking only this definition, not even considering those that claim that the semantics of deictic elements change because it is simply impossible: if the semantics change, there is no possible reference. However, if the semantics are underspecified enough (but stable), referential changes are perfectly possible.

13 The first versions of Russell's theory of Definite Descriptions (1905) grouped demonstratives and personal deictic elements as "logically proper names", whose meaning is in itself the referred object. Nevertheless, in later writings Russell denied the personal deictic units the status of logically proper names and reserved the label for demonstratives. In any case, our theory makes no claim about the descriptionalism / direct reference debate, since we simply do not work with the same categories. Moreover, we crucially do not have any interest on the "truth conditions" question, since it falls out of the scope of linguistics altogether.

arrowhead.

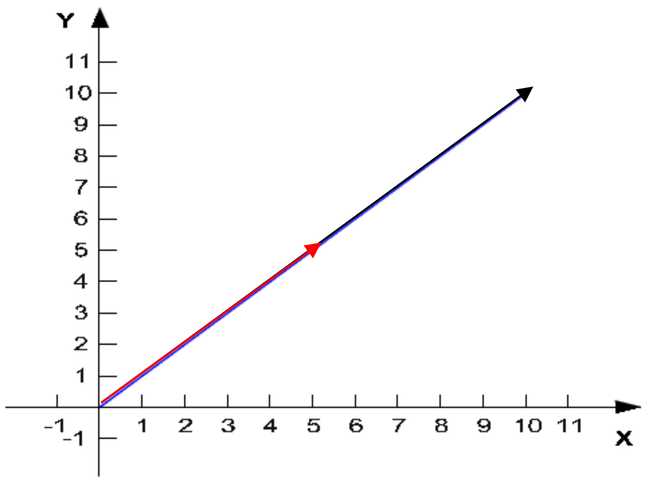
We consider that the unit lexically encodes only *one* of the aforementioned elements: the *module*. Take, for example, the following opposition:

(18) *This-That*

Here-There

Now-Then

(19)



Our hypothesis will be the following: deictic units have an invariant *locative* nature, the only variants being the kind of information that has to be retrieved. In any case, the relation between a deictic item and the element it modifies is such that the deictic item *anchors* the reference of the modified element, which is semantically underspecified, in a (conceptual) spatial continuum. Accordingly, the simplest possible structure for any referential expression is the following:

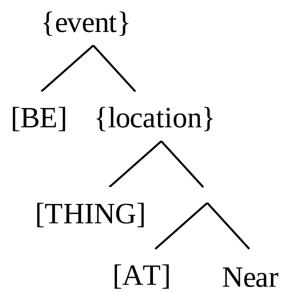
(20) $(e_X, a_Y \ldots x_N)$

This is, an ordinate pair including an *entity* and an *anchor*, which we identify with either D or T[14]. In any case, the examples in (18) are *unaccusative* structures, where the {event} domain denotes

14 The motivation for the inclusion of an xN variable is straightforward: in an n-dimensional workspace, we cannot stipulate a priori how many coordinates will be needed for identification in particular cases. It is

a generic *stative atelic event*, categorial interpretation at the C-I interface being determined by the local Merge of D or T. Roughly, the categorically underspecified structure (i.e., before the merger of D) would be as follows:

(21)



However, it would be inaccurate to pretend that the reference of a deictic item is strictly a point, since, crucially; their referential scope can either *widen* or *narrow*. See, for example, (22-24):

(22) It is very easy to have internet access *now* (Ref: past 15 years including utterance moment)

(23) There are no more dinosaurs *now* (Ref: last 60 million years)

(24) Do it *now* (Ref: a T immediate *after* utterance)

How can we explain these facts with our geometrical model of *deictics-as-vectors*? The solution comes from the mathematical concept of *topological space*. This means that the referent of a DP containing a deictic item (or, more generally, any DP) is not exactly a point, but a potentiality, just like it would be in Heisenberg's theory. A *root*, underspecified though it might be, determines a topological area in the conceptual space, possibly purely mathematical through

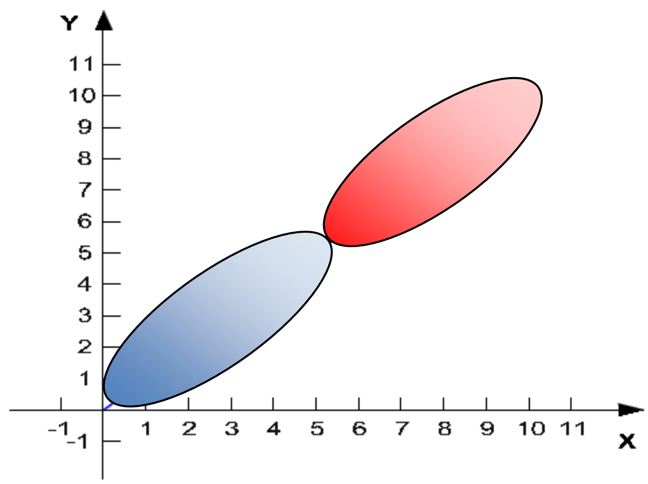
possible, and currently under research, that aY is in itself complex, thus we are left with a representation like the following:

i) (eX , (pY, qZ))

In (i), the second coordinate is complex. This may be required to formalize different localist systems, for example, left-to-right and bottom-up models for the numerical capacity.

resources like general memory capacities. A "functional" layer restricts that space, but the potentiality we mentioned above is built within the system. The direct consequence of the adoption of such a theory (i.e., the *uncertainty principle*) is that no amount of further specification, for example, via recursive use of relative clauses, can provide a mathematically exact coordinate. Therefore, we must reformulate (19) as (25):

(25)



Therefore, we say that a structure containing an element like "here" is to be conceptually located within the blue area, without the possibility of having absolute certainty about the coordinate matrix in the *n*-dimensional space.

A question at this point is whether our model applies to personal deixis, since *a priori* it seems to be more difficult to express it in locative terms. The fact that temporal deixis is subsumed to locative deixis is clear, but personal deixis somehow seems to resist the locative theory. Nevertheless, the so-called "projection" cases can help us in our inquiry:

(26) A. (Doctor to patient) How are *we* feeling today?

B. Cómo nos senti*mos* hoy? (Spanish)

C. (Waiter to customer) Does *the sir* need anything else?

D. Necesita algo más *el señor*? (Spanish)

In these cases, we see that there is a displacement of the relevant coordinates. In (A-B), the topological space that is supposed to include the coordinates of both the speaker and the hearer is inferentially *narrowed* to include only the hearer, whereas in (C-D) the topological space that is supposed to include a third-person party, neither the speaker nor the hearer, is simply *shifted*[15] to the hearer's coordinates (there is no *widening* because the third person is not included as a referent, but *only* the second). Can our model account for these facts? Yes, but not without the addition of some extra assumptions about the syntactic structure of constructions including deictics.

There is a curious difference between English and Spanish regarding ellipsis in certain contexts:

(27)

a) I want those two blue toys. (pre-nominal Adj)
b) Quiero esos dos juguetes azules. (post-nominal Adj)

Let us ask the question "Which toys did you say you want?" (or something of the sort). The answer could be (d) in Spanish, but (c) is banned in English:

(28)

c) * I want those two blue Ø.
d) Quiero esos dos Ø azules.

Is there an inter-linguistic difference regarding the relative position of the root and other nodes (Num, Deg, Gen, etc.) within the {D} structure? Certainly, that would *not* be the optimal scenario, as it would require positing some sort of "*parameter*", and we have argued against parameters in previous works (Krivochen, 2010d), somehow following the line of Boeckx (2010b). Besides, linearization via LCA (Kayne, 1994) does not work in a 3-D model of syntax like the one we propose. We can explain that post-N Adj act in Spanish as *abridged restrictive*

15 It is crucial to notice that our use of the Word "shift" does not convey the same meaning as in Jakobson (1990) as a unit via which the code refers to the message.

relative clauses (ARRC), whereas pre-N Adj are just qualifying Adj, that do not restrict reference[16]. Thus:

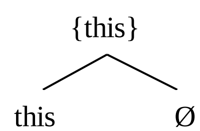
(29)

e) (? In my variety, but it depends on the register) Azules juguetes (the set of relevant blue things and the set of relevant toys are identical)
f) Juguetes azules (of the whole set of existent toys, just the blue ones: juguetes [que son] azules)

ARRC are enough to restrict the reference of the phonologically null root, so an explicature can be built. English ARRC are commonly PPs or Present Participle non finite clauses, informationally heavy structures that go at the end of the nominal construction. Thus, pre-N (Num, Deg, Quality, etc.) elements cannot assure C-I convergence / legibility, as they *qualify* but do not restrict enough for C-I to identify a referent.

Coming back to demonstratives, their *pronominal* use, as we have analyzed it, show a structure in which all the information is conserved. Consider what would happen if we had posited something like:

(30)



We would have a null element merged with the bundle of vectorial dimensions: *initial point* (0:0), *sense*, *magnitude* and *direction*, which compositionally with Num$_0$ and Gen$_0$ give us *this, that, these, those*. In Spanish, Gender plays a role along with these dimensions, giving *esto/s, esta/s, eso/s, esa/s, aquello/s, aquélla/s*. This configuration, however, reminds that of De Belder & van Craenenbroeck (2011), which we have criticized: if we accept that there is a RSS underlying these kind of elements, then, erasing all trace of the root would be a violation of the *Conservation*

16 Interestingly, Czech exhibits the same alternance regarding the position of the Adj as Spanish, but it is an exception among Slavic languages.

Principle. We return to our previous thesis: instead of [Ø] we have a generic root √THING, whose Spell-Out is *irrelevant* unless further specification is provided[17]. The presence of a root in these constructions is essential, since if there is a root, then there is malleable content, following Escandell Vidal & Leonetti (2011: 4):

> "(…) *In the cognitive pragmatic tradition, it is common to assume that conceptual representations are flexible and malleable, which means that they can be enriched, elaborated on and adjusted in different ways to meet the expectations of relevance. All the interpretive phenomena that are usually considered as instances of meaning modulation and ad hoc concept formation stem from this basic property (Wilson 2003, Wilson and Carston 2007). We claim that instructions, on the contrary, are rigid[18]: they cannot enter into the mutual adjustment processes, nor can they be modulated to comply with the requirements of conceptual representations, either linguistically communicated or not.* (…)"

The variations in the "size" of the topological space and the processes of narrowing, widening or shifting of the reference can be only accounted for if we assume the presence of a malleable element within the structure. These considerations leave us with a desirable result: the syntax of deixis is *always locative*. Moreover, we have a proposal for the syntax of the minimal referential structure:

(31)

p √THING

In this representation, *p* is a procedural node (say, D or T), conveying vectorial procedural instructions, that anchors the reference of the generic root (the *figure* above) and makes it interpretable by the post-syntactic semantic parser, licensing the "referent assignment" process in the construction of a full propositional form or *explicature* (Wilson & Sperber, 2003). ***The root determines a certain topographic space within our conceptual system, which the procedural node delimits and specifies***. Moreover, as Radical Minimalism works with a free-generator, our

17 This proposal is reminiscent of that of Panagiotidis (2002), but the foundations of each approach differ greatly.

18 Notice the parallel with early DM (Noyer, 1998): f-morphemes' Spell-Out was said to be "deterministic", whereas l-morphemes' Spell-Out was free.

minimal proposal does not exclude cartographic approaches, but only include the relevant nodes when interface conditions require so. Thus, we do not have a fixed functional skeleton for syntactic structures, but nodes that are freely-merged following C-I requirements.

4.5 Two different kinds of systems:

We would like to make a distinction that we consider essential when building a theory about the mind and analogous systems: the distinction between *Generative systems* and *Interpretative systems*. This distinction is not only terminological, but has major consequences to the theory of Quantum Human Computer (Salmani-Nodoushan, 2008, Krivochen, 2011d) since we will demonstrate that only *certain systems* allow elements in their *quantum state* (i.e., comprising all possible outcomes), which, following Schrödinger (1935), we will call the *ψ-state*.

a) *Generative Systems*: Generation equals *Merge*, a free, unbounded, blind operation that takes elements sharing either *ontological* or *structural* format and puts them together. For example, the syntactic component, the arithmetical capacity, the musical capacity and the pre-syntactic instance of the conceptual-intentional system.

b) *Interpretative Systems*: Interface systems, they have to read structural configuration build up by generative systems. For example, the sensory-motor system and Relevance Theory's inferential module (the post-syntactic instance of the conceptual-intentional system).

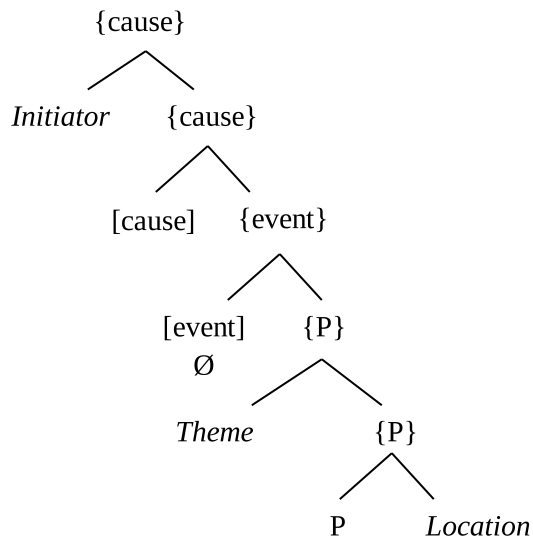
An essential difference is that, as Generative systems are blind to anything but format, they can manipulate objects in their ψ-state and transfer them to the interface systems (to which we will come back later). ***Transfer and interpretation is Heisenberg's observation or opening "Schrödinger's box": a structural relation between an element in its ψ-state and a procedural element / logical unit with specific characteristics collapses the quantum state onto one of the possible outcomes.*** In the syntactic working area W, elements enter in their ψ-state and are blindly manipulated, merged together and transferred to the interface levels, where structural configurations determine unambiguous interpretations in terms of outcomes. The interfaces can "peer into" the syntactic W, to make sure a syntactic object (i.e., *any symbolic representation*) is transferrable: this is what we call "*Analyze*". That is, *collapse occurs as a result of the generative-interpretative tension reinterpreted as a tension between quantum systems and unambiguity requirements of legibility*.

4.6 On the nature of Conceptual Representations:

In this section we will define some features of conceptual representations. First of all, we will make it clear that we will discuss the properties of *conceptual* and not *semantic* representations, since we have a definition of *concept*, whereas the definition, scope and object of *semantics* remain unclear. Conceptual representations are mental representations of our apprehension of the world via sensorial stimuli, which are organized syntactically in order to have a structured perception of the physical world. As the raw material for conceptual representations comes from different faculties, those in closer relation to the physical universe (e.g., the visual faculty, the auditory systems, etc.), the requirements that the corresponding module must be underspecified enough to deal with different kinds of data. Conceptual representations, *a priori*, are *n*-dimensional, with the possibility of *computing* 1-D, 2-D and 3-D stimuli, but, as far as modern neuropsychology tells us, there is no way to *perceive* the 4th dimension unless converted to 3-D representations (e.g., drawings of *hypercubes*). However, we can perfectly *conceive* a 4th dimension, even if there is no way to access it empirically. A possible explanation for this is that 4-D stimuli do not form part of the input we receive, and thus considerations of brain maturation with basis on sensitive data would come into play against a mature brain having the capacity to perceive 4-D objects, but not to dismiss it as a theoretical possibility. Conceptual representations are fundamentally *locative*, and built syntactically between the *temporal* lobes (*Figure*) and the *parietal* lobe (*Ground*), once the prefrontal cortex is activated, originating Ws whose objects are later assembled in a third W, as we have claimed above. Any *n*-dimensional W needs some operation like Merge to organize the material into hierarchical representations, so we are dealing with a very complex system, possibly analogous to Fodor's (1983) *Central Processor*, a system that is interpretative and generative. This suggests that the mind-brain is dynamic enough to have systems that can be both generative *and* interpretative as required by the input. This conception of the mind-brain follows the line of the *Quantum Human Computer Hypothesis* explained above: systems are dynamic, and an element comprises all of its possible outcomes (i.e., the interpretation it receives at some point in the derivation) until *Transferred*. An essential point to make is that the weight of concepts and phonology in language is not symmetrical: this is, there could be no "language" without concepts, as they provide the objects that are manipulated by Merge, but phonology is by no means a necessary requirement. Generic concepts are instantiated in *roots*, which are semantically underspecified and thus have to be Merged to procedural elements to be interpreted. For example, a root like √TABLE (we use English words, but roots are universal and thus language-neutral) denotes a generic entity (which is not a Universal Quantification interpretation: in a UQ interpretation, we have a set and the extension of a predicate is the whole set. In genericity as we are using the concept here, we are dealing with the abstract idea of "table", which is pre-categorial). Generic entities, either eventive or sortal, cannot be read by the semantic interface, and so Determiner (for sortal entities) and Time (for eventive entities) must come into play. This gives rise to a *conceptual-procedural* derivational dynamics that is interface-driven, and therefore *principled*, non-stipulative. This path leads to a highly desirable conclusion: a dynamic, interface-driven derivation with biological plausibility instead of feature-driven operations whose justification lies only within the theory. We have built a case for Relevance Principles as third factor principles in our previous papers, and we maintain that position here. *Optimal Relevance,* understood as a biological adaptation (Manuel Leonetti, p.c.) seems to us to be a plausible universal rector of mental operations.

Information enters the mind chaotically, but it is organized in very much a Kantian way, into categories that exist as *a priori* forms: mainly, *space* (if we accept that *time* is conceptualized as a metaphor of space). However, the existence of more complex conceptual templates must be posited, as our awareness and understanding of the phenomenological world around us does not limit to *things* in a *location* (be it concrete or abstract). In previous works, we have depicted a theory of semantic primitives, which we will sum up here. Ontogenetically (and, perhaps, phylogenetically), the most primitive category is the *noun*, denoting *things*. Things, however, are not isolated, but related in various ways, the most basic of which is a purely *spatial* relation in terms of *central* or *terminal* coincidence (Hale & Keyser, 1997, Mateu, 2000). We have, then, two categories so far, one conceptual, the other, procedural: N and P respectively. Further up on the structure, a spatial relation between two entities is a static *event*, and we have thus derived *uncaused verbs* (i.e., *Unaccusative V*). Different kinds of unaccusative Vs arise when varying the nature of the P node: telic and atelic, static and dynamic unaccusative Vs. The most complex structures appear when the *event* has an external initiator, which requires the presence of a *cause* primitive. We have now *caused events,* which may or may not include a spatial relation: (Di)*Transitive* and *Unergative* verbs. Having this conceptual (pre-linguistic) skeleton, we can fill the places with the available information, participants, time, place, etc. Let us illustrate the aforementioned structures:

(32)



4.7 On the nature of Phonological Representations:

We will now turn to the other relevant interface: the so-called S-M system, responsible for sound perception and articulation of the vocal organs. Phonological representations define a 2-D system, which implies a necessary *dimensional impoverishment*, from $n > 2$ to 2. It may seem that the linearity of the sentence would define a 1-D system, but sound externalization systems require 2 dimensions: wave *frequency* and *longitude*. We can thus describe any string of sounds (e.g., an utterance) as a set of coordinates $S = \{(X_1, Y_1), (X_2, Y_2) \ldots (X_n, Y_n)\}$, where the X axis is longitude and the Y axis is frequency. If the Y axis corresponds to dB and X to time, then, for example, sound intensity in a given time interval (t_1, t_2) can be described as the integral $\int(t_1\ t_2)$ of the function sin(x); pitch level, as the relation between peaks and sines in an interval (t_1, t_2) such that the more peaks, the higher the pitch. Pitch movement, as the reader may assume, is the result of applying *concatenation* to pitch levels at different t_n, consequently, it is not a primitive notion in the theory. The phonological interface is *not quantum*, it can only be *interpretative*: Merge does not apply in any form. However, it constraints the grouping of primitives in terminal nodes since it is a characteristic of languages that they do not group dimensions in a single node if there is no single VI to spell that node out. This feature is a key to understanding language typology: Anglo-Saxon and Germanic roots cannot express both *motion* and *direction*, so direction is materialized in a satellital position:

(33) John went$_{[MOTION]}$ into$_{[DIRECTION]}$ the room[19]

On the other hand, Romance languages cannot Spell-Out *motion* and *manner* in the same way Germanic languages can, in the form of a *Path-of-Motion* construction:

(34) a. John run into the kitchen

b. *Juan corrió dentro de la cocina (impossible with the POM reading, possible if the whole event of running took place *inside* the kitchen)

c. Juan entró a la cocina corriendo (*manner* is Spelled-Out in a satellite position)

In past works we have used a *descriptive* generalization to account for this situation, the *Morpheme Formation Constraint*:

(35) ***Morpheme formation constraint***:

Dimensions cannot be grouped in a terminal node (i.e., a morpheme) if there is no Vocabulary Item specified enough to materialize that node.

The fundamental operation is Vocabulary Insertion, also known as Spell-Out, in which the sensory-motor S-M system "grabs" a part of the derivation in W that has been *analyzed* and determined to be fully interpretable and provides means for externalization. This operation, as we have seen, implies *dimensional impoverishment*. The *syntax-phonology interface* has been represented graphically in (3), so that the reader can have a mental image of what *dimensional impoverishment* is.

4.8 On "Format":

a) *Ontological format* refers to the nature of the entities involved. For example, Merge can apply to conceptual addresses (i.e., roots) and procedural elements because they are all *linguistic* instantiations of conceptual information. It is especially useful if we want to

19 Of course, there exists the possibility of using "enter", but that is a Latin borrowing (derived from "in" + "eo"), not a pure English root.

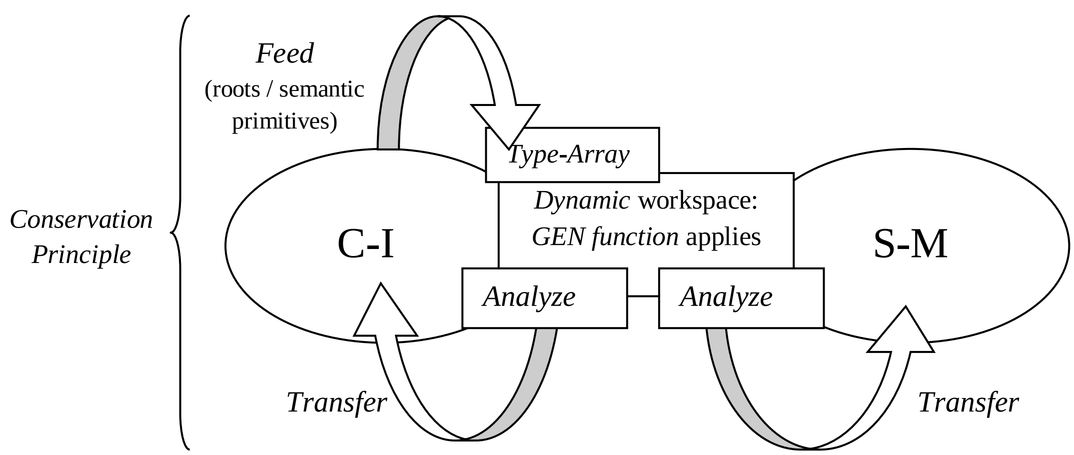
explain in simple terms why Merge cannot apply cross-modularly: ontological format is part of the legibility conditions of individual modules.

b) *Structural format*, on the other hand, refers to the way in which elements are organized. If what we have said so far is correct, then only binary-branched hierarchical structures are allowed in human mind. Merge optimally operates with the smallest non-trivial number of objects. Needless to say, given the fact that ontological format is a necessary condition for Merge to apply (principled because of interface conditions, whatever module we want to consider), the resultant structures will always consist on formally identical objects

4.9 The architecture of the (linguistic) system:

The system we have argued in favor of above has the following graphic representation:

(36)



What we have described above is the functioning of *any generative system* in the mind-brain, and its interaction with the corresponding interfaces. In the specific case of Language, there is a pre-linguistic conceptual structure, built via *Merge* with generic concepts, which determines what information will be conveyed (following the *Conservation Principle*). This structure can be linguistically instantiated or not, if it is not, it can become an *action* (the mind pictures a state of affairs and the brain sends electric impulses to the corresponding muscles to act upon an object, for example, and get to the aforementioned state of affairs in the phenomenological world). If there is linguistic instantiation, generic concepts must become *roots*: as C-I interfaces with many faculties; its units must be underspecified enough such that there is no feature that is not common to all of those interfaces, of DFI would be violated by the presence of an uninterpretable element. This is why *a-categorial* concepts are instantiated as *roots*¸ *pre-categorial* elements that, once in a local relation with a procedural element, are ready to be interpreted in the semantic interface. The cycle between C-I and the computational system, absent in the S-M interface is in concordance with our claim that there cannot be language without semantics, but the S-M interface only provides means for externalization, apart from constraining the grouping of dimensions in terminal nodes as expressed by our MFC. However, that constraint is epiphenomenal for both the semantic component and the Generative algorithm.

5. CONSEQUENCES OF ADOPTING *RM* FOR *MUH / CUH*

Radical Minimalism can provide provisional answers for some questions and problems that can arise within a MUH-approach: considering that *mathematical structures, formal systems* and *computations* are all part of the same "*transcendent structure*" (Tegmark, 2007: 20), three potential problems arise:

(1) Mathematical structures may have relations that are undefined

(2) Formal systems may contain undecidable statements

(3) Computations may not halt after a finite number of steps

In our system, as we have formalized it, there are not undefined mathematical relations, which are limited to a very small number, basically *Merge*. All other operations and dependencies are defined upon it (in the natural world / physical reality as a whole); consequently, if our system is consistent –as we think it is- there will be no undefined relations.

Regarding the second point, the notion of consistency is also essential. If all statements of a formal system are mathematical statements, and all mathematical statements describing the physical world, which refer to *Merge* and the relations thus created, give rise to no contradiction or ambiguity, then a consistent mathematical structure will be a model of a fully decidable formal system. In our system, given a formula generated via *Merge* and an interpretative system (i.e., an IL); there is a procedure to determine whether the formula is well- or ill-formed, and this procedure is *Analyze*.

Analyze and *Transfer* are the answer to the third problem: if the mind-brain works on strict economy principles (as it would be the optimal scenario, and the most plausible biologically speaking), then something like Pesetsky's (1995) *Earliness Principle,* stating that *if the conditions for the application of an operation obtain, it **must** apply*, applies, the system regulates its own economy. Assume *Merge* applies an *n* number of times. Our derivational dynamics state that *Analyze* evaluates the output of every instance of Merge, and if an object is indeed fully interpretable, it *Transfers*. Now, we strengthen this argument, by saying that the aforementioned object ***must*** undergo *Transfer*, and the relevant interface "takes it away" from the workspace and gives it an interpretation. The number of derivational steps is necessarily finite since no interpretative system can read objects of infinite length and complexity. A generative mechanism, however, can generate such objects, and as it is blind to the characteristics of the elements it manipulates, there is no need to introduce a *halting algorithm* (Tegmark, 2007: 20) to stop a computation, as long as the output of the computation is not interpreted. If there is an interpretative interface, then halting is automatically determined by legibility conditions. Thus, for the predicate $T(n)$, defined by:

(37) $T(n) \equiv (\exists a)\ [(a > n) \wedge \mathrm{P}(a) \wedge \mathrm{P}(a + 2)]$

There is no need to *stipulate* a halting algorithm, since this formula generates an infinite series of twin prime numbers larger than *n,* but without the need for interpretation at any point. This is consistent with our characterization of *generative systems*, and represents no problem for the MUH / CUH under our RM light.

6. ON THE CONCEPT OF "TIME" IN LANGUAGE, PHYSICS AND COGNITION:

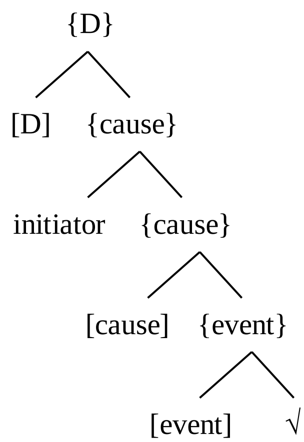
We will now analyze the impact our geometrical theory of syntax has for the definition of one of the fundamental concepts in the study of the Natural World: the concept of *Time,* focusing on the consequences for language. We will introduce a distinction between three fundamental senses in which *time* is interpreted:

- *Zeit*: a purely conceptual dimension
- *Time*: a physical magnitude
- *Tense*: an inflectional linguistic category

The most important thing is that they are independent but interact in cognition, as they all have mental existence. The question is: do they have any other existence? This is, does any of them exist independently of a mind?

The *Zeit* / *Tense* distinction has already been introduced in previous works (Krivochen, 2012) with reference to *derived nominals*: let us consider the case of [the robbery (of the bank)]. There is a *root* and eventive and causative nodes, since information cannot be lost in the course of the derivation (ConsP) and the Relational Semantic Structure of the LI includes *both* eventive and causative information. The categorial interpretation is created by the local [D…α…√] relation at the semantic interface. The relevant structure is as follows:

(38)



We see that there is no {Tense} node here (completely devoided of all Aspectual or Modal meaning, as we are assuming Krivochen's 2012 *Split TP hypothesis*), or the categorial interpretation would have been *verbal* (i.e., extending-into-time-perspective): in simpler terms, *nouns* do not inflect for [± present]. However, we are dealing with an event, in conceptual terms, and events are located within the "flow of time": otherwise, there is a genericity effect that impedes *referent assignment* at the semantic interface, as there is maximal extension, and minimal relevance. Events must have a *time frame* (see Vendler, 1957, for an attempt to explicit such frames in *aktionsarten* terms), which is provided by *both* {Tense} *and Zeit* in case we are dealing with *verbs* and only by *Zeit* in case the final categorial interpretation at the semantic interface turns out to be nominal (i.e., sortal entity). Tense is morphologically manifested in V in morphologically rich languages, sometimes fused with Mood and Aspect. It is an artificial, deictic category, whose interpretation depends on a reference point that is either the utterance moment (exophoric reference, so-called *absolute tense*) or anaphorically determined by a hierarchically higher T node (*relative tense*), as in the case of infinitival subordinate clauses. In more exact terms, all Tenses are *relative* to a certain point of reference: ***T_X in W_1 and T_Y in W_2 are relative either to each other or to T_Z in W_3, and all respond to the same general principles***.

Now, we must turn to a mode delicate matter, which is the formalization of *Time*, the physical magnitude. Instead of reviewing other conceptions (for example, take Time to be the "direction towards entropy increases", following García Mayoraz, 1989), we will directly introduce our own, in the light of Radical Minimalism. Physical *Time* is ***the interpretation of the concatenation function applied to a set of sets of coordinates which are not identical, and each of which can be said to describe a "snapshot" of the Universe***[20]. Such set S may include sets from one or more Ws, as in the following example:

(39) $S = \{(X \in W_1, Y \in W_1 \ldots n \in W_1), (P \in W_2, Q \in W_2 \ldots n \in W_2) \ldots (R \in W_n, T \in W_n \ldots nW_n)\}$

The interpretation of the concatenation function requires, essentially, that the sets share some of the coordinates and that some others, at least one, changes. This change represents a change in the state of affairs S can be said to describe, over the underlying continuity of those coordinates that allow the relevant interpretative system to put the snapshots together to form a single complex object. Physical *Time*, then, is not a primitive (therefore, it cannot be a dimension), but can be expressed in the form of simpler units, *spatial* coordinates. This general conception of *Time* as an interpretation of successive *spatial* coordinates has a very strong biological plausibility, since the procedural node P is the first procedural element acquired, relating a *figure* and a *ground*. The acquisition of P is the acquisition of the very notion of *Time* and *Zeit*, even before it can be expressed grammatically via *Tense*.

7. CONCLUSION

In this work we have formalized the framework we have called "Radical Minimalism", which is the result of applying the principles of substantive and methodological parsimony to the

20 This definition makes the "slower than light vs. near lightspeed" issue irrelevant, as "lightspeed" itself is an epiphenomenon of interpretation, not something that has real entity.

maximum departing from the assumption that Language is an epiphenomenon of the interaction between a generative system that is in principle the same for the whole "physical world" and an *n* number of interpretative systems, which impose constraints on the generator via *legibility conditions* (the characteristics that an object must have to be computable). In order to have a fully explicit model of the theory we have attempted to derive "syntax" in a very narrow sense (understood as the formal characteristics of natural languages) from conceptual necessity in a wider mathematical model, with concomitant effects on physics. We have tried to account for natural language as a physical system adopting SRMT and a strong version of MUH, which we understand as complementary hypotheses. Our ultimate goal is to simplify communication between different disciplines, by formalizing linguistic syntactic theory in such a way that it can be understood and enriched by mathematics and physics. To summarize, we will present some advantages of the Radically Minimalist model we have outlined so far:

a. Provides an account of multiple workspaces for parallel derivations (Chomsky, 2001; Uriagereka, 1999, 2002)
b. Deeper explanatory adequacy, as we take into account both the biological and computational implications of our claims with the perspective of the interface systems. Includes biological aspects in the formalization (Cf. Collins & Stabler, 2012).
c. Solves theoretical problems in MUH (Tegmark, 2007: 20 ff.) while supporting it empirically.
d. Generative systems are unified under Merge: there is no *Faculty of Language* in the traditional sense, mental architecture becomes simpler and so does its formalization.
e. Only conceptually needed operations, optimally, just *Merge* and *Transfer*. Thus, relations R in a mathematical structure are only two, making both a mathematical theory of reality and a theory of the formal representation of geometrical knowledge in the mind-brain much more attainable[21].

8. REFERENCES

21 Tegmark (2007: 27) claims that "*A mathematical structure typically has a countably infinite number of relations*". We have provided principled explanations for halting algorithms and reduced the possible relations to two. Whether this model applies successfully to all cases, only time can tell.

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