

Diego Krivochen, UNLP

e-mail: [diegokrivochen@hotmail.com](mailto:diegokrivochen@hotmail.com)

0. Introduction:

In this paper we attempt a mathematical formalization of Radical Minimalism, in order to make the model fully explicit and reveal its potential to be applied not only sub-personally, but to any physical system one thinks of. For those purposes, we have devised a geometrical model of syntax (understood as a general-purpose free generative mechanism), with which we capture the properties of mental workspaces and the organization of the physical reality. We will combine our *Strong Radically Minimalist Thesis* (Krivochen, 2011a, b, c, d, e) with Tegmark's (2007) *Mathematical Universe Hypothesis*, and analyze how both strengthen gaining empirical coverage and theoretical weight. The aim is to generate a window to the Theory of Everything from an interdisciplinary perspective: a joint work between Formal Linguistics and Mathematics.

1. Postulates:

- a) Language is part of the “natural world”; therefore, it is fundamentally a physical system.
- b) As a consequence of (a), it shares the basic properties of physical systems and the same principles can be applied, the only difference being the properties of the elements that are manipulated in the relevant system.
- c) The operations are taken to be very basic, simple and universal, as well as the constraints upon them, which are determined only by the interaction with other systems.
- d) (b) and (c) can be summarized as follows:
- e) **Strong Radically Minimalist Thesis (SRMT):**

*All differences between physical systems are “superficial” and rely only on the characteristics of their basic units [i.e., the elements that are manipulated], which require minimal adjustments in the **formulation** of operations and constraints [that is, only notational issues]. At a **principled level**, all physical systems are identical, make use of the same operations and respond to the same principles.*

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<sup>1</sup> I am indebted to Pierre Pica, Roland Friederich, Peter Kosta and Mike Putnam for comments and opinions. I am also indebted to Chris Collins and Edward Stabler's (2011) paper as it was an inspiration for this formalization. Cool figures (1) and (4) are entirely Juan Cruz Aramburu Blanch's merit. All mistakes, needless to say, are entirely my responsibility.

SRMT licenses the possibility of looking for biological, physical and mathematical properties of mental computations (i.e., syntax), *without its being a metaphor* but a *true account in three levels: description, explanation and justification*. The description is the *what*, the explanation is the *how*, and finally, the justification is the *why*. The latter has been either taken for granted or done in a truly *non-minimalist* way both substantively and methodologically. Our effort, then, will focus on trying to set a *radically minimalist* alternative of justification, taking into account that a theory of the physical universe must address all three. Attempting *justification* is what we understand as the ultimate goal of going “beyond explanatory adequacy”. We have chosen geometry because it licenses the possibility of *n*-dimensional representations, unlike logics, whose representations, although hierarchical, are bi-dimensional, and therefore inaccurate to represent processes in the quantum human mind (Krivochen, 2011e).

## 2. SRMT and MUH: advances towards the Theory of Everything?

Our work is intimately related to Tegmark (2007) and his *Mathematical Universe Hypothesis*, stated as follows (Tegmark, 2007: 1):

### 1. **Mathematical Universe Hypothesis (MUH):**

*Our external physical reality is a mathematical structure.*

A (mathematical) structure is defined as a set *S* of abstract entities and relations  $\{R_1, R_2 \dots R_n\}$  between them.

The “physical reality” of the MUH is identical to the “physical systems” of SRMT, and the postulation of a subjacent mathematical structure is explicited in SRMT with the mention of *Universal operations and principles*, which we have formulated in past works (Krivochen, 2011a, b, c; Kosta & Krivochen, 2011). Our Radical Minimalism provides principles and operations that go in consonance with both MUH and its stronger version, the *Computable Universe Hypothesis* (Tegmark, 2007: 20):

### 2. **Computable Universe Hypothesis (CUH):**

*The mathematical structure that is our external physical reality is defined by computable functions.*

Tegmark explains this hypothesis as meaning that “(...) *the relations (functions) that define the mathematical structure [...] can all be implemented as computations that are guaranteed to halt after a finite number of steps* (...)” (Tegmark, 2007, 20). This is clearly explained in the Radically Minimalist derivational dynamics, which we will formalize below: we only need one computational operation: *Merge*, and it halts always and only when a fully interpretable object is recognized by the relevant interface level, what we have called a *phase*, and *transfer* occurs.

What linguistics can provide a mathematical approach like Tegmark's is not only empirical basis, but also strong predictions regarding a limited domain of the physical reality, in which those predictions, that follow from general hypothesis, can be put to test. At the same time, a mathematical approach to the "natural world" results in a formalization that is most necessary for deepening our understanding of the very structure of reality, in our case, the structure and functioning of the mind-brain. This, in our opinion, can and should be expressed in unambiguous mathematical terms. In this work, we attempt to formalize Radical Minimalism in such a way that we can base syntax, understood as a generative mechanism operating there where complexity is necessary, in mathematics, following a geometrical model. This framework also reinforces Izard, Pica et. al. (2011)'s theory of geometry as a Universal mental constructions, upon Euclidean basis<sup>2</sup>. After having presented our formalized model in section 3, we will analyze to what extent RM can contribute to answer some questions that have arisen within the MUH framework and make the system more powerful.

### 3. Towards a Geometrical Syntax:

**Definition 1 (non-final):** a point is a  $n$ -plet  $(x, y, z \dots n)$ ,  $n$  = number of dimensions of a workspace  $W$ .

**Definition 2:**  $W$  is an  $n$ -dimensional *generative* workspace. Taking two distinct workspaces  $W_X$  and  $W_Y$ , either

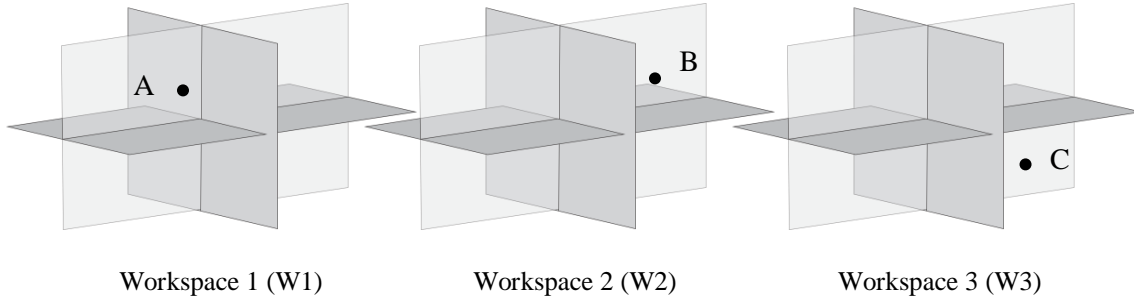
- a)  $W_X \equiv W_Y \text{ iff } \forall (x), x \in W_X, x \in W_Y \ \& \ \nexists (x), x \in W_X, x \notin W_Y$
- b)  $W_X \neq W_Y \text{ iff } \nexists (x), x \in W_X, x \in W_Y$
- c)  $W_X \cong W_Y \ \& \text{iff } \exists (x), x \in W_X, x \in W_Y$

This allows us to define the relations of *identity*, *difference* and *similarity* in set-theoretical terms. *Identity* holds between  $W_X$  and  $W_Y$  if and only if every element of  $W_X$  is also an element of  $W_Y$  and vice versa (formalized in (a)). If this condition obtains, there is also *logical equivalence* between  $W_X$  and  $W_Y$  in all relevant contexts. *Logical equivalence* is entailed by *identity*, as it is to be expected. *Difference* holds if and only if there is no common element between  $W_X$  and  $W_Y$ , which means that they are not set-theoretically related (formalized in (b)). *Similarity* is a relation in which there are common elements *at least* -but not necessarily *only*- one, between  $W_X$  and  $W_Y$  (formalized in (c)).

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<sup>2</sup> In our framework, the mathematical workspace can be Euclidean or not (hyperbolic or elliptic) depending on interface requirements. What is innate, in the case of human mind, is the quantum processor that allows multiple derivations simultaneously. Interface requirements derive from the interaction between *interpretative components* and the phenomenological world.

(1)



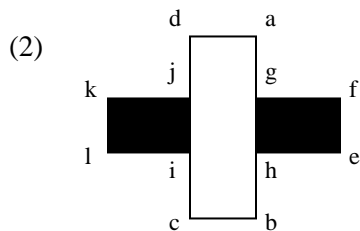
a) *Merge* ( $A_{W1}, B_{W2}, C_{W3}$ ) by *Unrestricted distinct Merge*

i.e.,  $A \in W1, B \in W2, C \in W3$

b) Triangle  $\alpha = \{A_{W1}, B_{W2}, C_{W3}\}$

Of course,  $\alpha$  is a *mental construct* in  $W_4$ , a workspace in which concatenation is interpreted.  $\{W_1 \dots W_4\}$  must be isodimensional for interface purposes, so that the coordinates can be joined in a single  $W$  (our  $W_4$ ). If we are working with the mind-brain,  $W$  is always 3-D, and, according to recent research (Gómez Urgellés, 2010), it should be *hyperbolic* since human perception tends to construct the outer physical reality as a hyperbolic space. In humans,  $W$ s emerge from the interaction between the prefrontal cortex and relevant areas of the brain and is **tri-dimensional**. It is worth noticing that our mind-brain, which works in a tri-dimensional phenomenological world, can conceive the fourth dimension, but there is no account for dimensions  $> 4$ , while they are theoretically possible. A legitimate question would be if the characteristics of the phenomenological world in which an *interpretative system* matures determine its dimensional limitations. We believe they do, in the same way that the flow of Time, which can be defined as a *concatenation* of coordinates in 3-D workspaces, affects temporal schemes known as *Aktionsarten* (Vendler, 1957): in a lightspeed world there would be no *achievements* (for detailed discussion, see Krivochen, 2010b).

Not only these kind of abstract figures require more than one  $W$  and a separate  $W$  to put the information together. Consider figure (2):



In (2) what we can *objectively* see is a white rectangle and two black rectangles at the sides. It is likely that each is perceived and processed in a separate W, such that rectangle {a, b, c, d} is processed, say, in  $W_1$ , {e, f, g, h}, in  $W_2$  and {i, j, k, l} in  $W_3$ . However, the reader most likely constructed a single rectangle {e, f, k, l}, assuming that {g, h, i, j} are the points of intersection between a white rectangle and a single black rectangle. This operation of “reconstruction” must be performed in a further W, such that it is *isodimensional* to the others, which are in turn *isodimensional*. This procedure only makes sense if there is an *interpretative system* that has to read the information, and it is driven by economy: perceiving *two* figures is simpler than perceiving *three*: two figures allows the mind to activate the well-known *figure-ground* dynamics, and so we can say “there is a white rectangle above the black one”, even if it only occurs in our conceptual space, and is ultimately an *inference*.

**Definition 3:** *concatenation* defines a *chain* of coordinates  $\{(x, y, z \dots n)W_X \dots (x, y, z \dots n)W_Y \dots (x, y, z \dots n)W_n\}$  where  $W_Y \equiv W_X \equiv W_n$  or  $W_Y \neq W_X \neq W_n$ . If  $W_X \neq W_Y$ , they must be *isodimensional*.

The *concatenation* function is called *Merge*, defined as follows:

***Merge*** is a **free unbounded operation** that applies to an *n-number* of objects sharing format, either *ontological* or *structural*.

**Definition 4:** geometrical “figures”, sentences, and other “observable” objects which constitute the *phenomenological world* are epiphenomenic results of *concatenation* in one or several W read off at an interpretative system.

**Definition 5:** an *Interface Level* IL is an *interpretative system* that has access to a W and establishes legibility conditions regarding its output.

**Definition 6:** all operations within W are interface-required in order to satisfy *Dynamic (Full) Interpretation*.

**Principle 1: Dynamic (Full) Interpretation:** any derivational step is justified only insofar as it increases the information and/or it generates an interpretable object.

Information and Objects:

**Principle 2: Conservation Principle:** information cannot be eliminated in the course of a derivation, but it must be instantiated in the relevant system in such a way that it can be read and it is fully preserved.

**Definition 7:** *Entropy* occurs only when there is a *generative-interpretative* interaction. If there is no *interpretative system* to read a structure, there is no point in introducing *entropy*.

**Definition 8:**  $(W1_0 \dots W1_n)$  indicate successive derivational steps *within* a W.

**Definition 9:**  $LEX_S$  is the full set of *type*-symbols that can be manipulated by a computational system  $S$ , which is a generative  $W$ .

**Definition 10:** an *array* is a set of *types* drawn from  $LEX_S$ .

**Definition 11:** a *type* is an abstract element in a physical system  $\Phi$ .

Corollary: there are two kinds of *types* in a linguistic derivation: those that convey *conceptual* meaning (i.e., *roots*) and those that convey instructions as to how to interpret the *relation* between conceptual types (*procedural* types). Determiner, Time and Preposition are *procedural types* (just like operators like [+], [-], etc. are procedural in Mathematics). The procedural or conceptual character of a node is of no importance to syntax, it is read at the semantic interface, and only there is it of any relevance.

**Definition 12:** a *token* is an occurrence of a *type* within  $W_X$ . There are no *a priori* limits to the times a *type* can be instantiated as a *token* but those required by Interface Conditions IC.

**Definition 13:** a *token* is never fully interpretable at the relevant Interface Level IL unless within a larger structure.

Within Linguistics, this definition is the RM way to formalize Distributed Morphology's "*categorization assumption*", adding empirical and theoretical coverage. This has been expressed as the *Conceptual-procedural interface symmetry* (Krivochen, 2011d):

***There cannot be bare roots without having been merged with a procedural node or procedural nodes without having been merged with a root in the syntax-semantics interface.***

Corollary: an element in  $W_X$  is always a *token*. Its interpretation depends on its own content and the structural position of its occurrence (i.e., local relations with other tokens, *conceptual* or *procedural*).

**Definition 1' (final):** A point  $p = [\{x, y, z...n\} \in W_X]$  is a *token* of a single-type  $LEX_S$ .

**Definition 1'':** A lexical item LI is a structure  $\{X...a...\sqrt{\phantom{x}}\} \in W_X$ , where  $X$  is a procedural category (D, T, P),  $a$  is a  $n$  number of non-intervient nodes for category recognition purposes at the semantic interface, and  $\sqrt{\phantom{x}}$  is a *root*.

***Roots are pre-categorial linguistic instantiations of a-categorial generic concepts from C-I. Generic concepts are "severely underspecified", since they are used by many faculties, and therefore cannot have any property readable by only some of them; otherwise, the derivation would crash in whatever faculty we are considering. Roots convey conceptual instructions, whereas functional nodes convey procedural instructions to the post-syntactic semantic parser.***

**Definition 14:** A *sentence* is an epiphenomenic result of *concatenation* of LIs between isodimensional Ws following interface requirements.

**Definition 15:** *format* is either *structural* or *ontological*. Ontological format is the nature of the elements involved in an operation, whereas structural format is the way in which these elements are organized.

Operations:

**Definition 15:** *Select* instantiates a *type* in  $W_X$  following **Principle 2**.

**Definition 16:** *Merge* concatenates LI-tokens in  $W_X$  driven by the interfaces' constraint expressed in **Definition 13**.

**Definition 17:** *Analyze* evaluates the objects built via *Merge* in  $W_X$  in order to verify full interpretability in  $IL_X$ .

**Definition 18:** *Transfer* is the operation via which an Interface Level  $IL_X$  takes a fully interpretable object from  $W$  to proceed with further computations.

Corollary: if  $W_X$  interfaces with more than one IL, *Transfer* applies for each IL *separately*.

**Definition 19:** *Merge*, *Analyze* and *Transfer* are both interface-driven and interface required.

**Definition 20:** 15, 16, 17 and 18, occurring cyclically, determine the derivational dynamics in  $W_X$ .

**Definition 21:** *category recognition* requires  $X$  and  $\sqrt{\phantom{x}}$  to be in the same workspace at the point in the derivation in which category is read off.  $X$  and  $\sqrt{\phantom{x}}$  may, however, have entered the derivation in different but isodimensional Ws.

Linguistic Derivations:

**Definition 22:** a *Derivation* is the set of computational steps that define a Syntactic Object SO, projecting a *type-Array* onto a legible object for IL via *token-Merge*.

**Definition 23:** a syntactic object *Converges* if and only if all of its components are interpretable by the relevant IL/s.

Dependencies:

**Definition 24:** if  $\alpha$  and  $\beta$  are interface-associated via their coordinates in  $W$ , there exists a *Dependency* between  $\alpha$  and  $\beta$ .

**Definition 25:** if  $\alpha \in W_X$  &  $\beta \in W_Y$  and either  $W_X \equiv W_Y$  or  $W_X \not\equiv W_Y$  and  $\alpha$  and  $\beta$  are defined by the same  $n$ -plet of coordinates in their respective *isodimensional*  $W$ s,  $\alpha$  and  $\beta$  are *bound* and the *dependency* is called *co-referentiality*.

**Definition 26:** *Reference* is location of a symbolic object in the conceptual multi-dimensional space via LI's coordinates, which are interpretable by the conceptual system C-I.

**Definition 27:** *Dependencies* are read off in IL, not in  $W$ .

Locality:

**Definition 28:** A *dependency* is *Local* if and only if there is no intervenient object  $\gamma$  (of arbitrary complexity) such that: (i) the relation between  $\alpha$  and  $\gamma$  is equivalent to that between  $\alpha$  and  $\beta$  for interface purposes (ii)  $\alpha$ ,  $\beta$  and  $\gamma$  belong to the same  $W$  and (iii)  $\gamma$  is structurally closer to  $\alpha$  than  $\beta$

This definition, combined with definition (24), has the consequence that *no dependencies can be established in a generative  $W$* , but at Interface levels, which we have expressed in definition (27). This also means that if there is no interpretative system involved, there are *no dependencies*. For example, in the structures generated by a mathematical algorithm that are never interpreted (see example (21) below) no dependencies can be posited.

**Definition 29:**  $P_W$  is a *phase* in  $W$  if and only if it is the minimal object fully interpretable in IL.

**Definition 30:** *Transfer* applies only to *phases*.

#### 4. Some Applications in Formal Linguistics:

##### 4.1 Merge and Phrase Structure

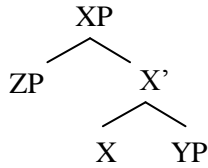
This framework allows us to manage symbolic representations in a way in which we can best capture their essential property, *hierarchy*, while accounting for epiphenomena in terms of interface conditions. Let us first review the classic X-bar theory axioms, which characterized phrase structure in the GB model and underlie most current models of phrase structure to different extents:

1. Endocentricity: every projection has a head
2. Projection: every head projects a phrase
3. Binary branching: every non-terminal node is binarily-branched

The kind of structures generated by this system is as follows:

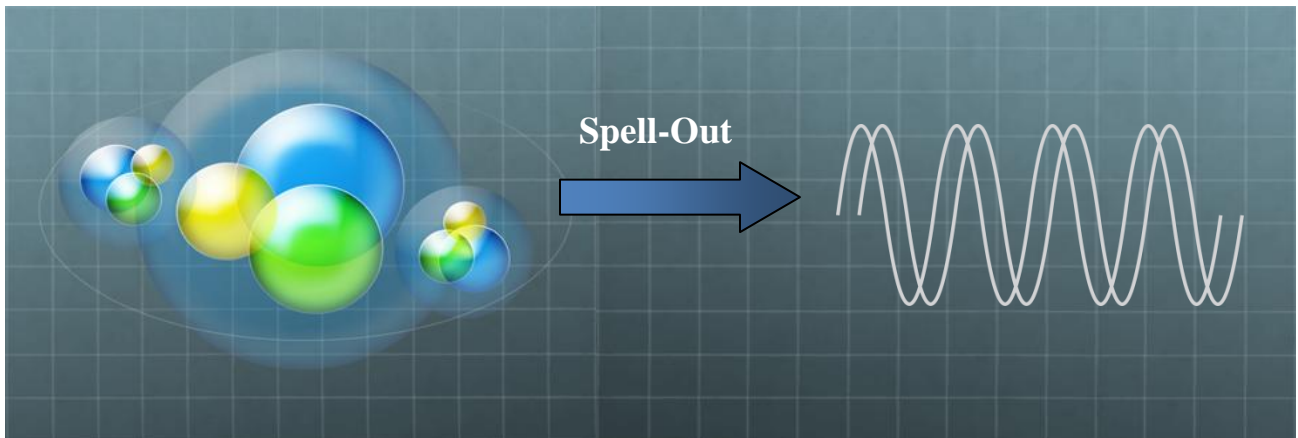


(3)



These are bi-dimensional representations within the syntactic workspace, then Spelled-Out. It is commonly assumed that Spell-Out implies a “flattening” of the hierarchical structure onto a linear representation: our claim is that there is no *essential* difference between a tree-like representation and its linearized form. Syntactic derivations have been claimed to be either *bottom-up* (Chomsky, 1995 and other orthodox works) or *top-down* (Zwart, 2009, in a different framework, Uriagereka, 1999), but always maintaining the underlying assumption that the *up-down* opposition is defining for syntactic processes: c-command is *top-down*, whereas m-command is *bottom-up*; Agree is also a *directional* operation (although there is no general agreement with regard to whether it is top-down or bottom-up, see Zeijlstra, 2011 and Putnam, Van Koopen & Dickers, 2011) and, given the fact that syntactic operations in orthodox Minimalist syntax are driven by the need to check features via Agree, directionality is centrally embedded within the theory. Syntactic structures, then, are *bi-dimensional*, only differing from phonological representations in epiphenomenic characteristics, like *headedness* or *binarity* which, as we have shown here and in past works, are C-I *interface requirements*. If a true qualitative difference is to be found between syntax and phonological externalized structures, then the nature of syntax must be revisited, as we have done. In our system, there is a difference between *syntax* and *phonology* regarding the dimensions of each domain: syntax is *tri-dimensional*, and its conversion to phonology implies flattening the structure to have *two dimensions* (we will analyze phonological representations below). The representations would look as follows:

(4)



In figure (4) we show how a syntactic structure -whose more exact representation, in our opinion, is that of an atom, with a *nucleus* (but no *head*) and peripheral elements (so-called *specifiers*, *complements* and *adjuncts*, all derived from the assumption of headedness) - is converted into externalized sound waves, after inserting Vocabulary Items in the syntactic terminals: semantic information has a phonological correlate. The materialization follows this hierarchical nucleus-periphery dynamics, as we have posited in Krivochen (2011d). Embick & Noyer (2004) propose a linearization procedure  $LIN(X, Y) = (X*Y) / (Y*X)$  (where \* means phonological precedence), but the possibilities grow when we have more than two nodes to linearized, giving us an  $n!$  situation, which is not what we would want since there is no interface condition that can orient us towards the preferred option(s). The optimal scenario would be that in which interface relations are based upon *pre-existing relations*, namely, those created in the syntactic configuration. *Ceteris paribus*<sup>3</sup>, the nodes are spelled out mirroring the interface relation they maintain with the root, from the closest to the most detached. From this claim, we derive that procedural nodes always have a closer relation with the root than peripheral nodes like Agreement, as they generate categorial interpretations in the semantic interface. Let us now assume that a language L allows for an X number of combinations of dimensions in terminal nodes, i.e., *morphemes*. That language has a Y number of Vocabulary Items to Spell Out those dimensions, specified to some extent regarding their distribution. The feasible scenarios (but not the logically possible ones, which are more) are the following:

- i)  $X = Y$
- ii)  $X > Y$

Of (i) we would say it is a morphologically very rich language, and it is possible that there is a language in which this occurs. Of such a language we will say that it is *maximally specified*. The normal situation for any natural language is (ii), in which the number of Vocabulary Items is inferior to the number of terminal nodes. This situation is called *Underspecification*. Note that the phonological interface SM cannot read terminal nodes, but phonological matrices, so Vocabulary Insertion is a step that we cannot miss. This operation puts a VI in a terminal node, in a 1-to-1 relation in the simplest cases (we will see that this is not always the case, which makes the syntax-morphophonology interface a very complex object). In order for a VI to be inserted into a terminal node, it must match *some* of the features of that terminal node, there being a competence among several possible VI as to which one is more specified. In this way, universal constraints are replaced, *prima facie*, by a simple *subset principle*:

**Subset Principle:** *The phonological exponent of a Vocabulary Item is inserted into a position if the item matches all or a subset of the features specified in that position. Insertion does not take place if the Vocabulary Item contains features not present in the morpheme. Where several Vocabulary Items*

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<sup>3</sup> This *ceteris paribus* condition refers to the availability of VI.

*meet the conditions for insertion, the item matching the greatest number of features specified in the terminal morpheme must be chosen. (Halle 1997).*

Our model can account for the syntax-phonology interface more elegantly than orthodox Minimalism, in terms of dimensional relations between one domain and the other. This way, we capture the true properties of the Spell-Out operation and establish a more general principle for Transfer operations:

**(5) Dimensional Impoverishment:**

*Transfer **may** imply the “flattening” of an  $X$ -dimensional structure to an  $X$ - $n$  dimensional structure, while respecting Conservation Principle.*

An example of “dimensional flattening” occurs in Spell-Out (i.e., Transfer to S-M), whereas this is not the case in the syntax-semantics interface. We will analyze these cases below.

4.1.2 Some considerations about “Simplest Merge” and the Economy of Derivations and Representations

Our definition of *Merge* (definition 3 and corollary) entails the existence of a free and unbounded generative engine, which is available to any mental faculty that requires combination of atomic objects to form complex structures. Our hypothesis is that that very generative operation is available in the natural world, and is the principle of all complexity found in the Universe. If, as MUH entails, the Universe is not only mathematical but also simple (and describable by a finite and optimally reduced number of principles), the combinatory operation must follow not only from conceptual necessity but also economy principles. The notion of *simplest Merge* (Zwart, 2009) enters the scene. Zwart’s formulation of Simplest Merge is as follows (2009: 2):

**(6) Simplicity required in the derivational procedure:**

- a) Merge manipulates a single element of  $N$  [a Numeration] at each step of the derivation*
- b) Merge manipulates each element from  $N$  only once*

However, the requirements do not follow from interface conditions, and are thus stipulations. It is also fundamental not to confuse economy in the theory with economy in the physical world. Even though Zwart’s arguments in favor of his proposal have advantages over the version he presents of orthodox Minimalism (which is not quite accurate, to our understanding), the “requirement” for simplicity in the derivational procedure is

stipulated in a syntactocentric way. As a consequence, those principles have no interface relevance, and can be dismissed. Even if *Merge* is actually simpler applying to only *one* element, its interface value is null:

- (7)  $\text{Merge}(\alpha) = \{\alpha\}$   
 $\text{Merge}(\{\alpha\}) = \{\{\alpha\}\}$   
 $\text{Merge}(\{\{\alpha\}\}) = \{\{\{\alpha\}\}\}$

And so on. We see that this is a violation of our *Principle 2*, DFI. Economy in the theory must not be stated as a principle, but follow from third factor-like constraints, related to the architecture of the relevant object and the systems that read the output.

At this point, arguments for top-down (Zwart, 2009, Uriagereka, 1999 –although in a different way-) and bottom-up (Chomsky, 1998, 1999, 2005) derivations are equally implausible, since they both assume what we will call “the 2-D fallacy”, which implies that tree-like representations have mental reality. Our multi-dimensional geometry has made a strong case against 2-D syntax, and argued strongly in favor of *n*-dimensional Ws in which operations are constrained only by the interpretative interfaces. Our revised version of “simplest Merge” is formulated as follows:

(8) *n*-dimensional Merge:

- a) *Merge manipulates an  $n$  number of elements at each step, being  $n$  interface-determined. If  $\{\{\alpha\}\}$  is taken to be superfluous, as we do<sup>4</sup>, then  $n > 1$ , being 2 the minimal non-trivial number of objects.*
- b) *Merge manipulates each **token** only once in a derivation  $\subset W_x$ , but each **type** as many times as needed.*

This definition of Merge allows us to dispense with Movement in terms of *displacement*, but reinterpret it as *token-Merge* and *dependency* establishment between *tokens* at the relevant IL, with basis on their coordinates in isodimensional W.

#### 4.1.3 Complex-unit Merge

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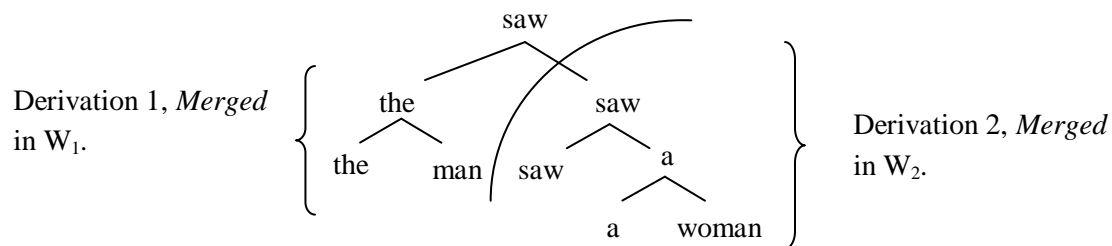
<sup>4</sup> Boban Arsenijevic (p.c) claims that “ $\{\{a\}\}$  is non-trivial in at least one faculty: the arithmetic capacity. Hence, output conditions can't be that bare to favor a binary merge”. However, our position is that if Merge is considered to be an operation and we assume also DFI, that is, the assumption that application of any operation must either lead to a legible object or increase the informational load, to apply Merge to a single object is trivial in any faculty. If  $\{a\}$  is already legible in the relevant interface level, then why apply Merge in the first place? It would be computationally redundant, and therefore far from Minimalist. We maintain that binary Merge is the minimal-maximal non-trivial option, and therefore reject any proposal of *unitary Merge* on interface grounds.

Let us assume the following transitive sentence:

(9) The man saw a woman

According to Uriagereka, parallel derivational spaces are a conceptual necessity (Uriagereka, 1999: 121) when Merge applies to complex objects, already output of a Merge operation: for example, in the derivation of [<sub>saw</sub> the man saw a woman], [<sub>the</sub> the man] and [<sub>saw</sub> saw [<sub>a</sub> a woman]] are generated separately and then assembled externally. The derivation in his terms would proceed as follows<sup>5</sup>:

(10)

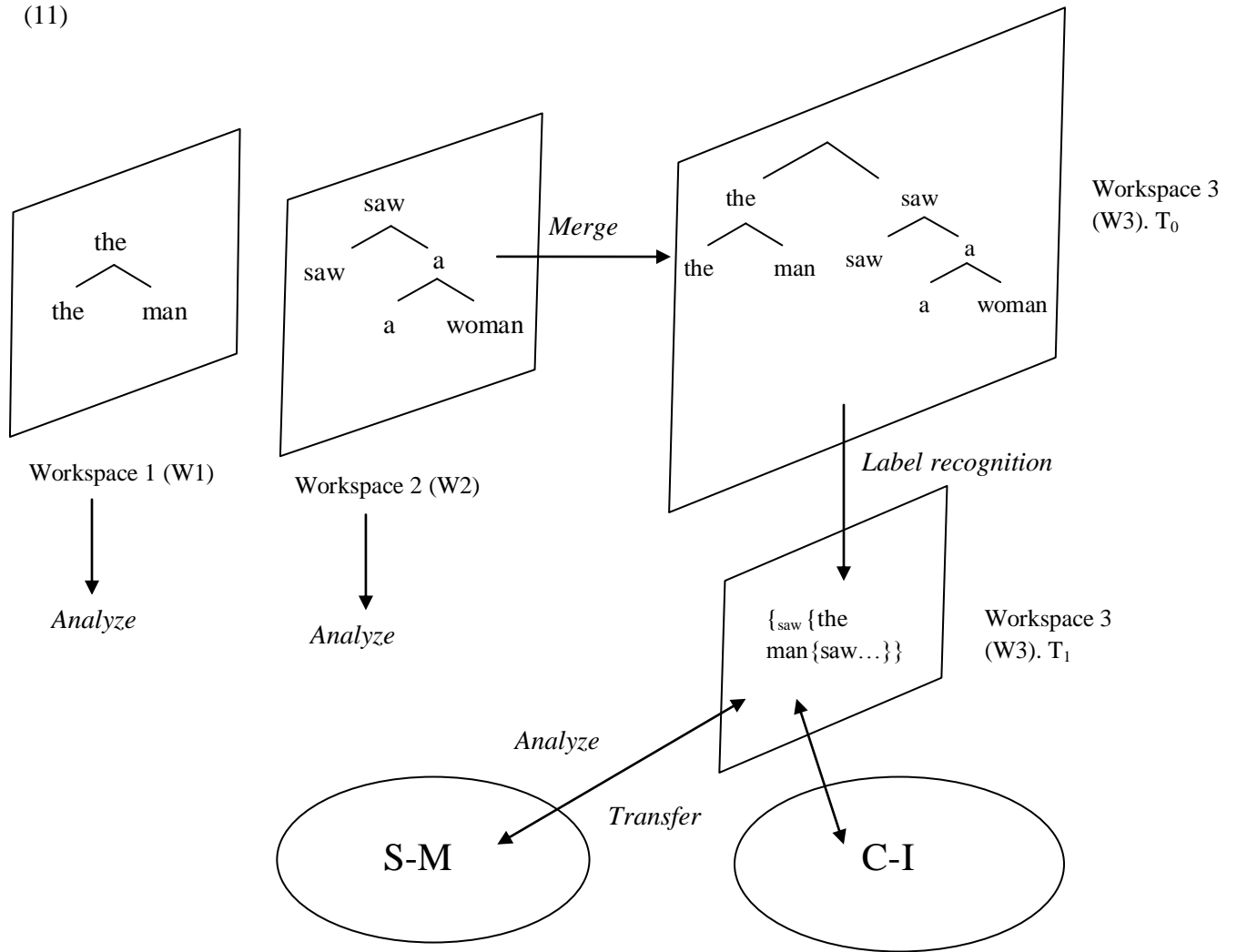


If the derivation occurred in just one space derivational space, then the units [the] and [man] would be merged to [saw a woman] sequentially (i.e., monotonically) and not as a fully-fledged constituent [the man]. That is, if we assume that [saw] would project a label indicating the properties of the complex object as a unit for the purposes of future computations, the final phrase marker would be the ill formed sequence: \*<sub>saw</sub> the <sub>saw</sub> man <sub>saw</sub> saw [<sub>a</sub> a woman]]].

Within our newly-developed framework, its (summarized) derivation would be represented, in terms of *command units* (Uriagereka, 1999/2002), as follows (we beg the reader to think of it in 3-D terms, like in (1)):

<sup>5</sup> Nunes (2004) analyzes parasitic gaps using a mechanism very similar to the derivational Cascades model by Uriagereka (1999).

(11)



We see that each “derivational cascade” (Uriagereka, 2002) is assembled in a separate tri-dimensional workspace  $W$ , and then Merge applies to tokens of both objects in a third  $W$ , to which the interfaces have access and therefore *evaluate* the objects created.

The only characteristic  $W$ s must have is that it must be  $n$ -D,  $n > 2$  and interacting  $W$ s for *non-monotonic Merge* purposes must be isodimensional. 2-D  $W$  would result in a 2-D model of the mind, which, even if quantum, could not account for the multiple tasks the brain carries simultaneously, many of which *require* at least 3 dimensions (like symbolic structures –syntax as the first example-, figure-ground dynamics, among others). Of course, a *sine qua non* condition for *concatenation* effects at interpretative interfaces is that *all Ws involved* are *isodimensional* (Definition 3).

#### 4.2 The Quantum Human Computer Hypothesis:

The Quantum Human Computer Hypothesis, presented in Salmani-Nodoushan (2008) and developed and formalized in Krivochen (2011d), distinguishes two types of computers: *digital* and *quantum*. The *digital computer* (DC) works by transforming an input in a series of, say, 1 and 0 regardless the input's nature or inner complexity. Once in its binary form, the information can be manipulated by the computer in a simpler way, since there is a unification of the format. The states 1 and 0 are, crucially, mutually exclusive. The digital computer can be analyzed as a non-recursive manipulator of discrete units, consisting mainly of an algorithm for transduction, a working area for the performance of mathematical operations and an active operation that performs those computations. The operations are strictly linear and sequential, and all steps have to be fulfilled in order to have a successful derivation. The formal expression of this operation is a rewriting rule of the form:

$$(12) \quad X \longrightarrow Y$$

This type of generation rules is *crash-proof* in the sense that the output strings are limited and there is no chance to generate illegible objects. Early Generative Grammar (Standard Theory -ST- See Chomsky, 1965) based its Phrase Structure Component on this rule format. This is an example of highly restricted, constructivist system, incompatible with our definition of the *concatenation* function. The generative procedure is a fixed algorithm, one of whose forms is a sequence of divisions by 2 of any natural number, until the remainder in each division equals 0 or 1. The result is again divided by 2, as many times as needed, until it becomes zero. This is a simple *base component generative rule*, which operates with a reduced lexicon, namely,  $LEX = \{1, 0\}$ . Unlike ST, however, there is no *transformational component*, which makes this binary grammar very simple but descriptively and explanatorily inadequate for natural languages.

A more abstract sense in which we can think of a DC is a system that does not allow an object X to comprise several states at once. This can be seen as a simplification of the system, since it allows only 1-to-1 correlations between objects, but it also means that the characteristics of the relevant objects must be determined beforehand in a stipulative way. Besides, as we will see below, this system fails to capture many properties of natural languages.

The main characteristic of *Quantum Computers* is that they are not limited by the nature of binary *bits*, either 1 or 0, but they can work with 1 and 0 simultaneously. This means that there is no discretion in the definition of the units, which can be expressed as a “wave function” in isolation: prior to observation, the *quantum bit* or *qbit* is *both* 1 and 0, an undetermined state of a quantum field. Observation, according to Heisemberg (1999), collapses the state to one of the possible outcomes, in our particular case, 1 and 0. Of course, the possible outcomes of the quantum function can be more than 2, in which case the isolated element would comprise as many states as necessary. With this in mind, the QHC hypothesis states that *the human mind works like a quantum computer*. The advantages of the QHC hypothesis are clear: the system can perform many more operations in less time than a DC. DCs are constrained by what has been called “*the hundred step rule*” (see

Carreiras, 1997), which establishes that, at most, a computer can perform a hundred sequential derivational steps per second, and many tasks require more than this. A QHC could perform “*billions of computations in no time*” (Salmani-Nodoushan, 2008: 30). QHC allows the mind to process information coming from faculties directly related with the phenomenological world (e.g., the visual faculty) and logic units (presumably *procedural*, taking into account that according to Leonetti & Escandell Vidal (2000), procedural categories do not have an *encyclopedic* entry, but they do activate a *logical* entry that determines the relation that the procedural element will establish between an *n* number of conceptual elements) in a *multidimensional* space. A disadvantage we find in this point is that there is no principled way to determine the path information follows. This could easily lead to a chaotic space, rather than a quantum field, in the sense that everything is possible, and if the mind is optimally designed, there must be some principled, third-factor constraints (in the sense of Chomsky, 2005: factors non specific to a mental faculty). With our definition of *analyze-driven Transfer*, however, we make sure that there is a halt to the computations as soon as a fully legible object is assembled: we have described and explained a self-regulating system in which the interfaces drive the derivation, and there is no possibility for the generative function (the *generator*, in Optimality-Theoretic terms) not to stop after a finite number of steps. We will come back to this below, when revisiting the MUH under the RM light.

#### 4.3 Features and derivations:

Orthodox Minimalism has posited that there are two “apparent imperfections” in the Faculty of Language: uninterpretable features and Movement, understood as literal displacement of constituents (Chomsky, 1998, cf. Krivochen & Kosta, in preparation). Chomsky solves this apparent problem for the Strong Minimalist Thesis by saying that it is the need to eliminate those features that are uninterpretable at the interfaces that triggers movement. Notice that, elegant though this theoretical move may seem, it presupposes the existence of:

- a) A Faculty of Language
- b) Features (which, in turn, imply both a *Dimension* and *n* number of *values*, or, in our terms, *outcomes*)
- c) An operation to relate Features (i.e., Agree)
- d) Displacement

All three represent a substantive complication for the theory, and imply the isolation of the study of language from the study of other mental faculties, since the “imperfections” are not architectural constraints of the mind-brain but intra-theoretical stipulations: Minimalism has *created* both a problem and its solution. Radical Minimalism is all about *eliminating* problems, and impoverishing the theoretical apparatus to the minimum, so that only conceptual necessity dictates what is required in the model, while maintaining empirical coverage. We will now introduce our *Quantum Dimension Theory*, and apply it to the analysis of Case.



Imagine we have an electron in a tridimensional space, and we want to know its location. In order to do so, we need to see it, projecting some kind of light on it. This light is projected in the form of a *photon*, a particle with mass. The “problem” is that when the photon crashes with the electron, there is a change in the original location, which remains unknown. That original location (we have taken this magnitude just for the sake of the example, but we could have also worked with speed or trajectory) is taken to be a “superposition” of all possible locations, expressed in the form of a “*wave function*” (in de Broglie’s terms). Therefore, there will always be a magnitude whose real value will remain unknown to us. In this kind of physical systems, it is the **observation** that makes the relevant dimension *collapse* to one of the possible states<sup>6</sup>. *Uncertainty is a natural characteristic of physical systems, and by no means an instrumental problem*, taking physical system in its technical sense, that is, any portion of the physical universe chosen for analysis. We take “physical universe” to be equivalent to “natural world”, and we will use one or the other indistinctly. Magnitudes (or *dimensions*, to maintain a term more closely related to linguistics, since we are not dealing with measurable elements) are *not* necessarily binary; what is more, *in abstracto* they can comprise as many states as the system requires, which, as we will show later, leads us to a much simpler form of minimalism. ***We will express it by using this notation: for any dimension D, [D<sub>x</sub>] expresses its quantum state.***

Let us assume the framework outlined so far and the following quantum dimension: [Case<sub>x</sub>]. Following the idea presented in Krivochen (2010a, 2011a), this dimension comprises three possible “outcomes”: NOM sphere (φ), ACC sphere (θ) and DAT sphere (λ). All three are possible final states of the system, and therefore the linear combination must also be considered a legitimate state of the system. The dimension *in abstracto* could then be expressed as follows, using SE:

$$(13) \quad N\phi + A\theta + D\lambda$$

As we have said before, this only holds if no “measurement” takes place, in Schrödinger’s terms. We will not speak of “measurement”, since Case is not a magnitude, but we will consider that the factor that makes the relevant dimension collapse is ***the merger of a functional / procedural node***. What we must take into account is that not only do we have DPs with [Case] and functional heads in interaction in the computational system, but the output (i.e, the resultant state) must also converge at the interface levels, so our problem is a bit more complicated. As usual, we will focus ourselves in the C-I component. What we want to do now is derive the relations P-DAT; v-ACC and T-NOM from interface conditions, apart from the argumentation we have made in Krivochen (2010a) in relation with θ-roles and Case, to which we refer the reader. Anything else would be stipulative, and that is something we cannot accept in Radical Minimalism.

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<sup>6</sup> See, for example, the well-known EPR (Einstein-Podolsky-Rosen) paradox, which inspired Schrödinger (1935) paper.

Epistemologically, we have the advantage over theoretical physics that we have observable stretches of language where to test our hypothesis. So, let us take a ditransitive *Prepositional Indirect Object Construction* (PIOC):

- (14) I<sub>NOM</sub> gave a book<sub>ACC</sub> to Mary<sub>DAT</sub>  
 (Yo)<sub>NOM</sub> di un libro<sub>ACC</sub> a María<sub>DAT</sub>  
 (Ego)<sub>NOM</sub> librum<sub>ACC</sub> Mariae<sub>DAT</sub> dedi

By looking at the same construction in other languages with rich casual morphology (like Latin, Sanskrit or Greek) it has been established that the {D} [I] has Nominative Case, [the book] has Accusative Case and {P} [to Mary] has Dative Case. But this is nothing more than a description, with no explanation as to why things are the way they are. We will not review the classic attempts of explanation (see Chomsky, 1981, 1995, among others), but go directly to our point.

The weak hypothesis would be that each nominal construction will bear a dimension [Case<sub>x</sub>], whose value when entering the derivation will be a “ $\psi$ -function”, or a complex vector  $[N\phi + A\theta + D\lambda]$ . That is, the quantum dimension will comprise all three possible values, as all three are possible states of the Case system for a particular DP. However, [D<sub>x</sub>] cannot be read by the interface levels, so the quantum dimension must “collapse” to one of the possible states. Here, it is not “measurement” but *Merge* that does the work. Let us take the {P} structure as an example. We have the merger of {to<sub>[TO]</sub>, Mary<sub>[Case<sub>x</sub>]</sub>} -and the subsequent “labeling” of the structure as {P} in Logical Form for semantic interpretability, since in a dynamic model interface-interpretation is in *real-time*-. At this point in the derivation, we are already in condition of collapsing the quantum dimension in [Mary], since we have a *procedural node* in a minimal configuration that can license that dimension in an XP (adapting the idea from Rizzi, 2004). This minimal configuration is defined in our terms *within phase boundaries*, following the definition given in 29. Since the procedural information conveyed by P is essentially *locative*, the quantum case dimension on [Mary] will collapse to the *locative* sphere, i.e., *Dative*. A stronger hypothesis, perhaps more plausible, is that *the mere configuration is enough to license an interpretation*, without positing that there is a [D<sub>x</sub>] beyond expositive purposes. The licensing takes place only if there is no closer functional / procedural head that can license the relevant interpretation, in order to respect Minimality. To account for this, we will draw another principle: **Locality**, but as defined in 28 and 29. Just like a particle cannot influence any other than its surrounding particles, within field theory, any object has a certain area of influence where it can license operations / interpretations if necessary. It is interesting that influence can be indirect, that is, a particle  $\alpha$  may not be able to influence particle  $\gamma$  since particle  $\beta$  is “in the middle”, but by influencing  $\beta$ ,  $\alpha$  will have an effect on  $\gamma$ , and thus we have a flexible definition of *Locality*, revisiting Rizzi’s 2004 approach. That is what we call *compositionality*, in linguistics. We have to pay attention to the whole derivation (in *phase-level* terms) to understand, for example, the interpretation that a certain syntactic object receives at the interfaces.

#### 4.4 Figure-Ground revisited:

Given the fact that figure-ground dynamics depend on the interaction between the prefrontal cortex and the temporal and parietal lobes respectively, our representation in terms of multiple workspaces geometrically defined working in parallel can provide further insight not only on language but on general perception and organization of phenomenological / perceptual information. Cognitive Linguistics has argued that entities are located within a conceptual space, which we have formally expressed as  $W$ . Our advantage is that the explicit case for multiple  $W$ s imply optimization of computational efficiency, thus following from general economy principles. Regarding the specific case of Figure-Ground dynamics, which appears to be a fundamental opposition in the apprehension and organization of experience, the situation would be formally represented as follows:

$$(15) \quad W_X \cong W_Y \ \& \ \exists(x), x \in W_X, x \in W_Y$$

Representation in which there is a partial “overlapping” of spaces, an intersection, in set-theoretical terms. So, having a Figure  $F$ , and a Ground  $G$ , we need two workspaces to account for the biological basis of the complex structure:

$$(16) \quad \begin{aligned} F &\in W1 \\ G &\in W2 \\ \exists(x) \ \& \ x &= W1 \cap W2 \end{aligned}$$

In grammatical terms,  $x = \textit{preposition}$  ( $P$ ), as it is a procedural element that relates  $F$  and  $G$  in terms of *central* or *terminal* coincidence (Hale & Keyser, 1993 et. seq.). We now have a *logical* definition of  $P$ .

#### 4.5 Two different kinds of systems:

We would like to make a distinction that we consider essential when building a theory about the mind and analogous systems: the distinction between *Generative systems* and *Interpretative systems*. This distinction is not only terminological, but has major consequences to the theory of Quantum Human Computer (Salmani-Nodoushan, 2008, Krivochen, 2011d) since we will demonstrate that only *certain systems* allow elements in their *quantum state* (i.e., comprising all possible outcomes), which, following Schrödinger (1935), we will call the  $\psi$ -state.

- a) *Generative Systems*: Generation equals *Merge*, a free, unbounded, blind operation that takes elements sharing either *ontological* or *structural* format and puts them together. For example, the syntactic

component, the arithmetical capacity, the musical capacity and the pre-syntactic instance of the conceptual-intentional system.

- b) *Interpretative Systems*: Interface systems, they have to read structural configuration build up by generative systems. For example, the sensory-motor system and Relevance Theory's inferential module (the post-syntactic instance of the conceptual-intentional system).

An essential difference is that, as Generative systems are blind to anything but format, they can manipulate objects in their  $\psi$ -state and transfer them to the interface systems (to which we will come back later). ***Transfer and interpretation is Heisenberg's observation or opening "Schrödinger's box": a structural relation between an element in its  $\psi$ -state and a procedural element / logical unit with specific characteristics collapses the quantum state onto one of the possible outcomes.*** In the syntactic working area W, elements enter in their  $\psi$ -state and are blindly manipulated, merged together and transferred to the interface levels, where structural configurations determine unambiguous interpretations in terms of outcomes. The interfaces can "peer into" the syntactic W, to make sure a syntactic object (i.e., *any symbolic representation*) is transferrable: this is what we call "Analyze".

#### 4.6 On the nature of Conceptual Representations:

In this section we will define some features of conceptual representations. First of all, we will make it clear that we will discuss the properties of *conceptual* and not *semantic* representations, since we have a definition of *concept*, whereas the definition, scope and object of *semantics* remain unclear. Conceptual representations are mental representations of our apprehension of the world via sensorial stimuli, which are organized syntactically in order to have a structured perception of the physical world. As the raw material for conceptual representations comes from different faculties, those in closer relation to the physical universe (e.g., the visual faculty, the auditory systems, etc.), the requirements that the corresponding module must be underspecified enough to deal with different kinds of data. Conceptual representations, *a priori*, are *n*-dimensional, with the possibility of *computing* 1-D, 2-D and 3-D stimuli (i.e., points, lines / planes and solids in Euclid's terms), but, as far as modern neuropsychology tells us, there is no way to *perceive* the 4<sup>th</sup> dimension unless converted to 3-D representations (e.g., drawings of *hypercubes*). However, we can perfectly *conceive* a 4<sup>th</sup> dimension, even if there is no way to access it empirically. A possible explanation for this is that 4-D stimuli do not form part of the input we receive, and thus considerations of brain maturation with basis on sensitive data would come into play against a mature brain having the capacity to perceive 4-D objects, but not to dismiss it as a theoretical possibility. Conceptual representations are fundamentally *locative*, and built syntactically between the *temporal*

lobes (*Figure*) and the *parietal* lobe (*Ground*), once the prefrontal cortex is activated, originating Ws whose objects are later assembled in a third W, as we have claimed above. Any  $n$ -dimensional W needs some operation like Merge to organize the material into hierarchical representations, so we are dealing with a very complex system, possibly analogous to Fodor's (1983) *Central Processor*, a system that is interpretative and generative. This suggests that the mind-brain is dynamic enough to have systems that can be both generative *and* interpretative as required by the input. This conception of the mind-brain follows the line of the *Quantum Human Computer Hypothesis* explained above: systems are dynamic, and an element comprises all of its possible outcomes (i.e., the interpretation it receives at some point in the derivation) until *Transferred*. An essential point to make is that the weight of concepts and phonology in language is not symmetrical: this is, there could be no "language" without concepts, as they provide the objects that are manipulated by Merge, but phonology is by no means a necessary requirement. Generic concepts are instantiated in *roots*, which are semantically underspecified and thus have to be Merged to procedural elements to be interpreted. For example, a root like  $\sqrt{\text{TABLE}}$  (we use English words, but roots are universal and thus language-neutral) denotes a generic entity (which is not a Universal Quantification interpretation: in a UQ interpretation, we have a set and the extension of a predicate is the whole set. In genericity as we are manipulating the concept here, we are dealing with the abstract idea of "table", which is pre-categorical). Generic entities, either eventive or sortal, cannot be read by the semantic interface, and so Determiner (for sortal entities) and Time (for eventive entities) must come into play. This gives rise to a *conceptual-procedural* derivational dynamics that is interface-driven, and therefore *principled*, non-stipulative. This path leads to a highly desirable conclusion: a dynamic, interface-driven derivation with biological plausibility instead of feature-driven operations whose justification lies only within the theory.

#### 4.7 On the nature of Phonological Representations:

We will now turn to the other relevant interface: the so-called S-M system, responsible for sound perception and articulation of the vocal organs. Phonological representations define a 2-D system, which implies a necessary *dimensional impoverishment*, from  $n > 2$  to 2. It may seem that the linearity of the sentence would define a 1-D system, but sound externalization systems require 2 dimensions: wave *frequency* and *longitude*. We can thus describe any string of sounds (e.g., an utterance) as a set of coordinates  $S = \{(X_1, Y_1), (X_2, Y_2) \dots (X_n, Y_n)\}$ , where the X axis is longitude and the Y axis is frequency. The phonological interface is *not quantum*, it can only be *interpretative*: Merge does not apply in any form. However, it constraints the grouping of primitives in terminal nodes since it is a characteristic of languages that they do not group dimensions in a single node if there is no single VI to spell that node out. This feature is a key to understanding language typology: Anglo-Saxon and Germanic roots cannot express both *motion* and *direction*, so direction is materialized in a satellital position:

(17) John went<sub>[MOTION]</sub> into<sub>[DIRECTION]</sub> the room<sup>7</sup>

On the other hand, Romance languages cannot Spell-Out *motion* and *manner* in the same way Germanic languages can, in the form of a *Path-of-Motion* construction:

(18) a. John run into the kitchen

b. \*Juan corrió dentro de la cocina (impossible with the POM reading, possible if the whole event of running took place *inside* the kitchen)

c. Juan entró a la cocina corriendo (*manner* is Spelled-Out in a satellite position)

In past works we have used a *descriptive* generalization to account for this situation, the *Morpheme Formation Constraint*:

(19) ***Morpheme formation constraint:***

*Dimensions cannot be grouped in a terminal node (i.e., a morpheme) if there is no Vocabulary Item specified enough to materialize that node.*

The fundamental operation is Vocabulary Insertion, also known as Spell-Out, in which the sensory-motor S-M system “grabs” a part of the derivation in W that has been *analyzed* and determined to be fully interpretable and provides means for externalization. This operation, as we have seen, implies *dimensional impoverishment*. The *syntax-phonology interface* has been represented graphically in (3), so that the reader can have a mental image of what *dimensional impoverishment* is.

#### 4.8 On “Format”:

- a) *Ontological format* refers to the nature of the entities involved. For example, Merge can apply to conceptual addresses (i.e., roots) and procedural elements because they are all *linguistic* instantiations of conceptual information. It is specially useful if we want to explain in simple terms why Merge cannot apply cross-modularly: ontological format is part of the legibility conditions of individual modules.

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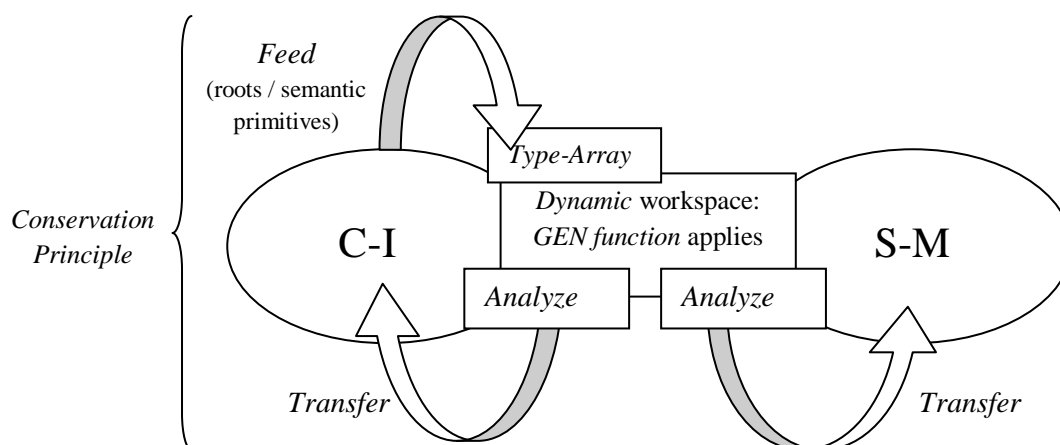
<sup>7</sup> Of course, there exists the possibility of using “enter”, but that is a Latin borrowing (derived from “in” + “eo”), not a pure English root.

- b) *Structural format*, on the other hand, refers to the way in which elements are organized. If what we have said so far is correct, then only binary-branched hierarchical structures are allowed in human mind. Merge optimally operates with the smallest non-trivial number of objects. Needless to say, given the fact that ontological format is a necessary condition for Merge to apply (principled because of interface conditions, whatever module we want to consider), the resultant structures will always consist on formally identical objects

#### 4.9 The architecture of the (linguistic) system: (from Krivochen & Kosta, in preparation)

The system we have argued in favor of above has the following graphic representation:

(20)



What we have described above is the functioning of *any generative system* in the mind-brain, and its interaction with the corresponding interfaces. In the specific case of Language, there is a pre-linguistic conceptual structure, built via *Merge* with generic concepts, which determines what information will be conveyed (following the *Conservation Principle*). This structure can be linguistically instantiated or not, if it is not, it can become an *action* (the mind pictures a state of affairs and the brain sends electric impulses to the corresponding muscles to act upon an object, for example, and get to the aforementioned state of affairs in the phenomenological world). If there is linguistic instantiation, generic concepts must become *roots*: as C-I interfaces with many faculties, its units must be underspecified enough such that there is no feature that is not common to all of those interfaces, of DFI would be violated by the presence of an uninterpretable element. This is why *a-categorical* concepts are instantiated as *roots*, *pre-categorical* elements that, once in a local relation with a procedural element, are ready to be interpreted in the semantic interface. The cycle between C-I and the computational system, absent in the S-

M interface is in concordance with our claim that there cannot be language without semantics, but the S-M interface only provides means for externalization, apart from constraining the grouping of dimensions in terminal nodes as expressed by our MFC. However, that constraint is epiphenomenic for both the semantic component and the Generative algorithm.

#### 5. Consequences of adopting RM for MUH / CUH:

Radical Minimalism can provide answers for some questions and problems that can arise within a MUH-approach: considering that *mathematical structures*, *formal systems* and *computations* are all part of the same “*transcendent structure*” (Tegmark, 2007: 20), three potential problems arise:

- (1) Mathematical structures may have relations that are undefined
- (2) Formal systems may contain undecidable statements
- (3) Computations may not halt after a finite number of steps

In our system, as we have formalized it, there are not undefined mathematical relations, which are limited to a very small number, basically *Merge*. All other operations and dependencies are defined upon it (in the natural world / physical reality as a whole), consequently, if our system is consistent –as we think it is- there will be no undefined relations.

Regarding the second point, the notion of consistency is also essential. If all statements of a formal system are mathematical statements, and all mathematical statements describing the physical world, which refer to *Merge* and the relations thus created, give rise to no contradiction or ambiguity, then a consistent mathematical structure will be a model of a fully decidable formal system. In our system, given a formula generated via *Merge* and an interpretative system (i.e., an IL), there is a procedure to determine whether the formula is well- or ill-formed, and this procedure is *Analyze*.

*Analyze* and *Transfer* are the answer to the third problem: if the mind-brain works on strict economy principles (as it would be the optimal scenario, and the most plausible biologically speaking), then something like Pesetsky’s (1995) *Earliness Principle*, stating that *if the conditions for the application of an operation obtain, it must apply*, applies, the system regulates its own economy. Assume *Merge* applies an *n* number of times. Our derivational dynamics state that *Analyze* evaluates the output of every instance of *Merge*, and if an object is indeed fully interpretable, it *Transfers*. Now, we strengthen this argument, by saying that the aforementioned object *must* undergo *Transfer*, and the relevant interface “takes it away” from the workspace and gives it an interpretation. The number of derivational steps is necessarily finite since no interpretative system can read objects of infinite length and complexity. A generative mechanism, however, can generate such objects, and as it



is blind to the characteristics of the elements it manipulates, there is no need to introduce a *halting algorithm* (Tegmark, 2007: 20) to stop a computation, as long as the output of the computation is not interpreted. If there is an interpretative interface, then halting is automatically determined by legibility conditions. Thus, for the predicate  $T(n)$ , defined by:

$$(21) \quad T(n) \equiv (\exists a) [(a > n) \ \& \ P(a) \ \& \ P(a + 2)]$$

There is no need to *stipulate* a halting algorithm, since this formula generates an infinite series of twin prime numbers larger than  $n$ , but without the need for interpretation at any point. This is consistent with our characterization of *generative systems*, and represents no problem for the MUH / CUH under our RM light.

#### 6. On the concept of “time” in Language, Physics and Cognition:

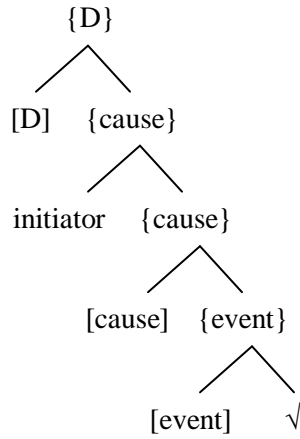
Following the steps of Einstein (1965 -2005-), we will analyze the impact our geometrical theory of syntax has for the definition of one of the fundamental concepts in the study of the Natural World: the concept of *Time*. We will introduce a distinction between three fundamental senses in which *time* is interpreted:

- *Zeit*: a purely conceptual dimension
- *Time*: a physical magnitude
- *Tense*: an inflectional linguistic category

The most important thing is that they are independent but interact in cognition, as they all have mental existence. The question is: do they have any other existence? This is, does any of them exist independently of a mind?

The *Zeit* / *Tense* distinction has already been introduced in previous papers (Krivochen, 2011c) with reference to *derived nominals*: let us consider the case of [the robbery (of the bank)]. There is a *root* and eventive and causative nodes, since information cannot be lost in the course of the derivation (CP) and the Relational Semantic Structure of the LI includes *both* eventive and causative information. The categorial interpretation is created by the local [D...α...√] relation at the semantic interface. The relevant structure is as follows:

(22)



We see that there is no {Tense} node here (completely devoided of all Aspectual or Modal meaning, as we are assuming Krivochen’s 2010a *Split TP hypothesis*), or the categorial interpretation would have been *verbal* (i.e., extending-into-time-perspective): in simpler terms, *nouns* do not inflect for [ $\pm$  present]. However, we are dealing with an event, in conceptual terms, and events are located within the “flow of time”: otherwise, there is a genericity effect that impedes *referent assignment* at the semantic interface, as there is maximal extension, and minimal relevance. Events must have a *time frame* (see Vendler, 1957, for an attempt to explicit such frames in *aktionsarten* terms), which is provided by *both* {Tense} *and* *Zeit* in case we are dealing with *verbs* and only by *Zeit* in case the final categorial interpretation at the semantic interface turns out to be nominal (i.e., sortal entity). Tense is morphologically manifested in V in morphologically rich languages, sometimes fused with Mood and Aspect. It is an artificial, deictic category, whose interpretation depends on a reference point that is either the utterance moment (exophoric reference, so-called *absolute tense*) or anaphorically determined by a hierarchically higher T node (*relative tense*), as in the case of infinitival subordinate clauses. In more exact terms, all Tenses are *relative* to a certain point of reference:  ***$T_X$  in  $W_1$  and  $T_Y$  in  $W_2$  are relative either to each other or to  $T_Z$  in  $W_3$ , and all respond to the same general principles.*** This is the result of the application of the special relativity principle to the analysis of linguistic Tense.

Now, we must turn to a more delicate matter, which is the formalization of *Time*, the physical magnitude. Instead of reviewing other conceptions, we will directly introduce our own, in the light of Radical Minimalism. Physical *Time* is ***the interpretation of the concatenation function applied to a set of sets of coordinates which are not identical, and each of which can be said to describe a “snapshot” of the Universe.*** Such set S may include sets from one or more  $W$ s, as in the following example:

$$(23) \quad S = \{(X_{W1}, Y_{W1} \dots n_{W1}), (X_{W2}, Z_{W2} \dots n_{W2}) \dots (X_{Wn}, K_{Wn} \dots n_{Wn})\}$$

The interpretation of the concatenation function requires, essentially, that the sets share some of the coordinates and that some others, at least one, changes. This change represents a change in the state of affairs S can be said

to describe, over the underlying continuity of those coordinates that allow the relevant interpretative system to put the snapshots together to form a single complex object. Physical *Time*, then, is not a primitive (therefore, it cannot be a dimension), but can be expressed in the form of simpler units, *spatial* coordinates. This general conception of *Time* as an interpretation of successive *spatial* coordinates has a very strong biological plausibility, since the procedural node P is the first procedural element acquired, relating a *figure* and a *ground*. The acquisition of P is the acquisition of the very notion of *Time* and *Zeit*, even before it can be expressed grammatically via *Tense*.

## 7. Conclusion:

In this work we have formalized the framework we have called “Radical Minimalism”, which is the result of applying the principles of substantive and methodological parsimony to the maximum departing from the assumption that Language is an epiphenomenon of the interaction between a generative system that is in principle the same for the whole “physical world” and an *n* number of interpretative systems, which impose constraints on the generator via *legibility conditions* (the characteristics that an object must have to be computable). In order to have a fully explicit model of the theory we have attempted to derive “syntax” in a very narrow sense (understood as the formal characteristics of natural languages) from conceptual necessity in a mathematical model. We have tried to account for natural language as well as any other physical system in the natural world adopting SRMT and a strong version of MUH, which we understand as complementary hypothesis.

### 7.1 Advantages of the Radically Minimalist model:

- a. Provides an account of multiple workspaces for parallel derivations (Chomsky, 2001, Uriagereka, 1999)
- b. Follows the line of the SRMT, which, if proved correct, is a very powerful generalization about the structure of the Universe.
- c. Accounts for complex syntactic structures (in a wide sense, either linguistic or mathematical) in a simple and elegant way.
- d. Deeper explanatory adequacy, as we take into account both the biological and computational implications of our claims with the perspective of the interface systems. Includes biological aspects in the formalization (Cf. Collins & Stabler, 2011).
- e. Solves theoretical problems in MUH (Tegmark, 2007: 20 ff.) while supporting it empirically.
- f. Generative systems are unified under Merge: there is no *Faculty of Language* in the traditional sense.

- g. Only conceptually needed operations, optimally, just *Merge* and *Transfer*. Thus, relations R in a mathematical structure are only two, making both a mathematical theory of reality and a theory of the formal representation of geometrical knowledge in the mind-brain much more attainable<sup>8</sup>.

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<sup>8</sup> Tegmark (2007: 27) claims that “A mathematical structure typically has a countably infinite number of relations”. We have provided principled explanations for halting algorithms and reduced the possible relations to two. Whether this model applies successfully to all cases, only time can tell.

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