Scalarity and additivity in natural language: (II) comparatives

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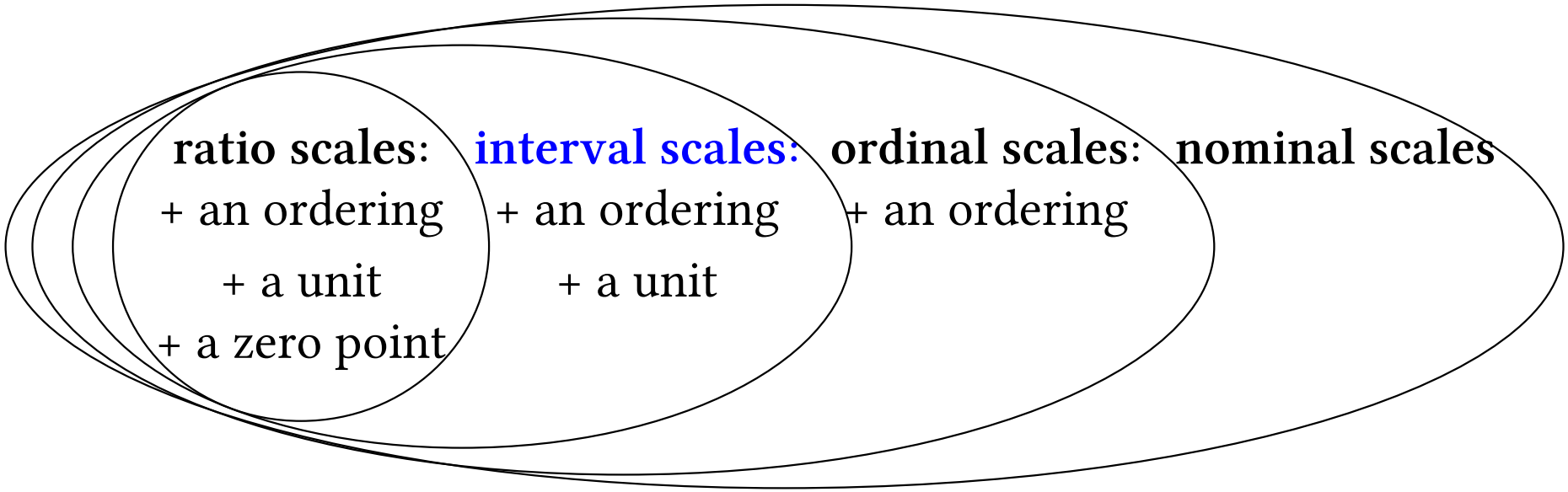
Recapitulation

- Measurement means mapping an entity (or an event) to a value along a scale according to rules (see Stevens 1946).
- A scale is a set that includes all potential values resulting from a certain way of taking measurement.
- There are 4 levels of scales: nominal scales, ordinal scales, interval scales, and ratio scales.
- Comparatives assume interval scales (i.e., with an ordering and a unit that supports the measurement of differences).



Today

- Day 1: Basics of scales and degrees; how they are relevant to natural language
  - What are scales? What are their formal properties? What operators do they support?
- Day 2 and Day 3: Comparatives and *-er*/*more*
  - How an additivity-based perspective improve our understanding of scalarity-related phenomena?
  - What is additivity?
- Day 4 and Day 5: *Even* and its cross-linguistic siblings
  - How a scalarity-based perspective improve our understanding of additivity-related phenomena?

Today: English Comparatives and the use of *-er*/*more*

Mephisto is talking about poor farmer Faustus:

'He always wanted more ...
more free time and more wealth ...
fewer struggles, better health ...
work less and eat all day ...'

(From *Sapiens: A Graphic History*, Vol.2,
by David Vandermeulen, David Casanave, Yuval Noah Harari, 2021)

Outline

Ontological assumptions in analyzing comparatives

- From now on, we focus on comparatives and use 'scales' to mean interval scales (scales with ordering and units, supporting the measurement of differences).
- Measurement means mapping an entity / event to a degree along a relevant (interval) scale (e.g., height, time line, time length).
- Degrees are like real numbers (see also Fox and Hackl 2006: the universal density of measurement)

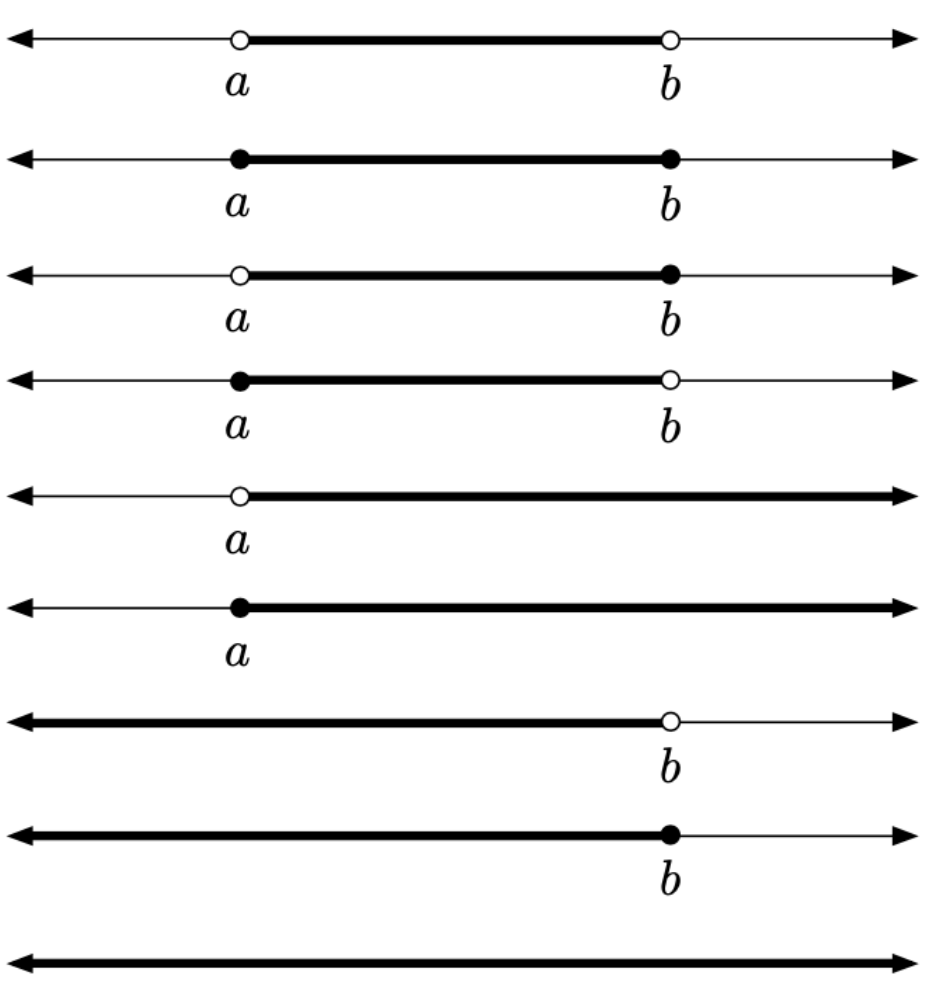
(1) $\{d \mid -\infty < d < +\infty\}$
$\rightsquigarrow$ a scale including all potential degree values

(2) $\{d \mid 0 < d \leq 7'\}$
$\rightsquigarrow$ a part of a scale of height / length, including degrees between 0 and 7 feet (the upper bound is included; the lower bound is not included)

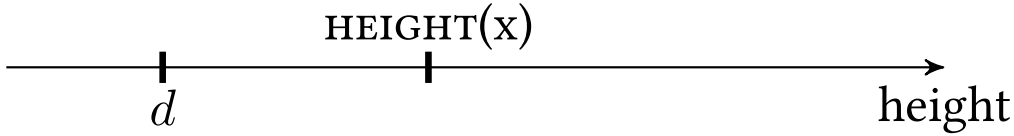
- Most works on comparatives are explicitly or implicitly based on these assumptions: e.g., Seuren (1973), von Stechow (1984), Heim (1985), Kennedy (1999), Schwarzchild and Wilkinson (2002), Zhang and Ling (2021) (cf. Cresswell 1976 does not consider 'degree' a primitive type; the delineation approach adopted by e.g., Klein 1980, Burnett 2017)

Illustrations: scales and parts of scales

(a, b)	$\{x : a < x < b\}$
$[a, b]$	$\{x : a \leq x \leq b\}$
$(a, b]$	$\{x : a < x \leq b\}$
$[a, b)$	$\{x : a \leq x < b\}$
(a, ∞)	$\{x : x > a\}$
$[a, \infty)$	$\{x : x \geq a\}$
$(-\infty, b)$	$\{x : x < b\}$
$(-\infty, b]$	$\{x : x \leq b\}$
$(-\infty, \infty)$	$\mathbb{R}$



The meaning of gradable adjectives



⟦tall⟧

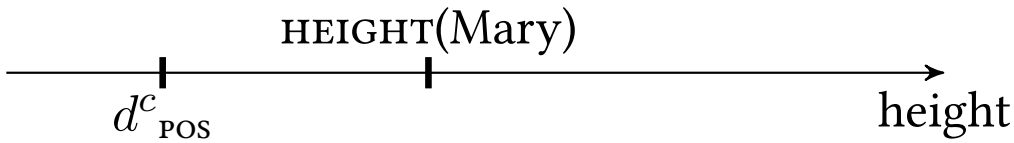
⟦tall⟧: a relation between an individual and a degree

(3) $[\![\text{tall}]\!]_{\langle d,et\rangle} \stackrel{\text{def}}{=} \lambda d_d.\lambda x_e.\textsc{height}_{\langle e,d\rangle}(x) \geq d$ (i.e., x is d-tall)
On the scale of height, the position of x meets or reaches degree d.

- There are two pieces in this lexical entry
  - A measure function of type $\langle ed\rangle$: $\textsc{height}_{\langle e,d\rangle}(x)$
  - Indicating the direction (of comparison): $\geq d$ (cf. Kennedy 1999)

(See e.g., Cresswell 1976, Hellan 1981, von Stechow 1984, Heim 1985, Schwarzschild 2008, Beck 2011)

Major uses of gradable adjectives: Positive use



⟦Mary is POS tall⟧

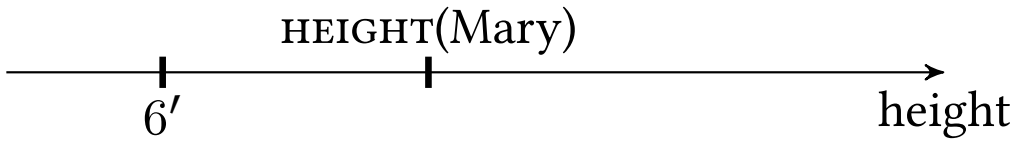
⟦tall⟧: a relation between an individual and a degree

(3) $[\![\text{tall}]\!]_{\langle d,et\rangle} \overset{\text{def}}{=} \lambda d_d.\lambda x_e.\text{HEIGHT}_{\langle e,d\rangle}(x) \geq d$ (i.e., x is d-tall)
On the scale of height, the position of x meets or reaches degree d.

(4) $[\![\text{Mary is POS tall}]\!] \Leftrightarrow \text{HEIGHT}(\text{Mary}) \geq d^c_{\text{POS}}$ Positive use
(POS: the context-dependent threshold for tallness)

(See e.g., Cresswell 1976, Hellan 1981, von Stechow 1984, Heim 1985, Kennedy 1999, Schwarzschild 2008, Beck 2011)

Major uses of gradable adjectives: Measurement sentence



⟦Mary is 6 feet tall⟧

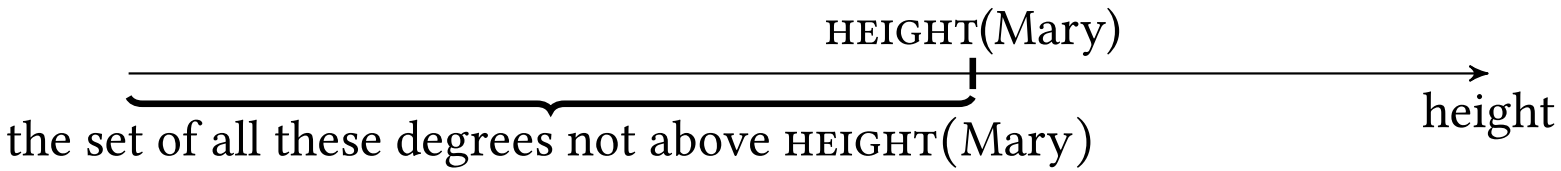
⟦tall⟧: a relation between an individual and a degree

(3) $[\![\text{tall}]\!]_{\langle d,et\rangle} \stackrel{\text{def}}{=} \lambda d_d.\lambda x_e.\textsc{height}_{\langle e,d\rangle}(x) \geq d$ (i.e., x is d-tall)
On the scale of height, the position of x meets or reaches degree d.

(5) $[\![\text{Mary is 6 feet tall}]\!] \Leftrightarrow \textsc{height}(\text{Mary}) \geq 6'$ Measurement

(See e.g., Cresswell 1976, Hellan 1981, von Stechow 1984, Heim 1985, Kennedy 1999, Schwarzschild 2008, Beck 2011)

Major uses of gradable adjectives: Degree question



⟦how tall is Mary⟧

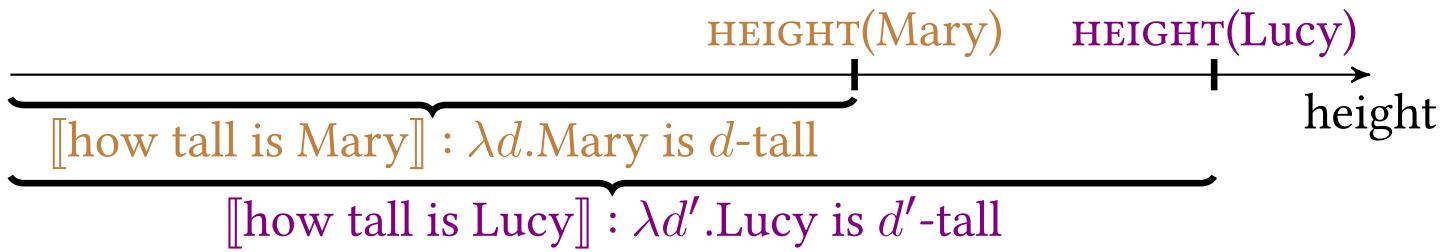
⟦tall⟧: a relation between an individual and a degree

(3) $[\![\text{tall}]\!]_{\langle d,et\rangle} \stackrel{\text{def}}{=} \lambda d_d.\lambda x_e.\textsc{height}_{\langle e,d\rangle}(x) \geq d$ (i.e., x is d-tall)
On the scale of height, the position of x meets or reaches degree d.

(6) $[\![\text{how tall is Mary}]\!] \Leftrightarrow \lambda d$. Mary is d-tall
$\Leftrightarrow \lambda d.\textsc{height}(\text{Mary}) \geq d$ Degree question

(See the categorial approach to questions represented by Hausser and Zaefferer 1978.)

Major uses of gradable adjectives: Clausal comparative



⟦Lucy is taller than Mary is ~~tall~~⟧

(7) The bathtub is wider than the door is tall. subcomparative

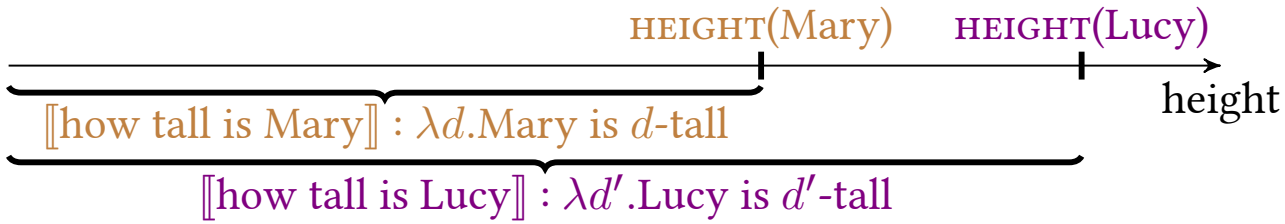
(8) ⟦ Lucy is tall er than Mary is ~~tall~~ ⟧

how tall Lucy is — how tall Mary is

LF: [-er [λd.Mary is d-tall]][λd′.Lucy is d′-tall]

(See e.g., von Stechow 1984, Kennedy 1999, Schwarzschild 2008, Beck 2011 for a review)

The role of *-er/more* in the canonical analysis



⟦Lucy is taller than Mary is ~~tall~~⟧

(8) ⟦ Lucy is tall er than Mary is ~~tall~~ ⟧

how tall Lucy is — how tall Mary is

LF: [-er [λd.Mary is d-tall]][$\lambda d'$.Lucy is d'-tall]

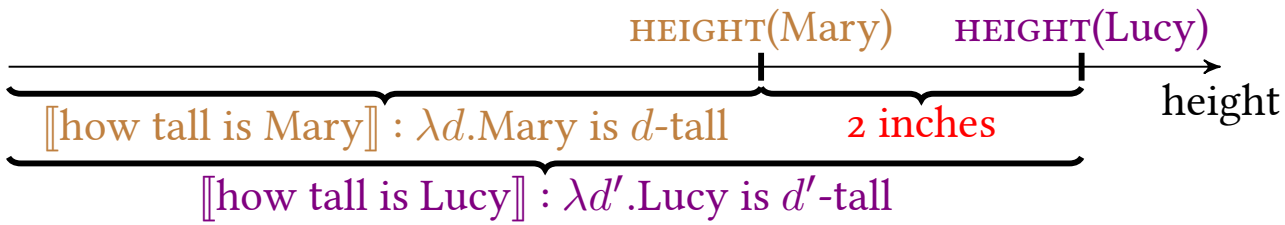
⟦-er/more⟧ performs comparison by encoding an inequality

(9) $[\![\text{-er/more}]\!]_{\langle\langle dt\rangle,\langle dt,t\rangle\rangle} \stackrel{\text{def}}{=} \lambda D_{\text{than}}.\lambda D_{\text{matrix}}.\exists d[d \in D_{\text{matrix}} \wedge \neg[d \in D_{\text{than}}]]$

(10) $[\![\text{-er/more}]\!]_{\langle\langle dt\rangle,\langle dt,t\rangle\rangle} \stackrel{\text{def}}{=} \lambda D_{\text{than}}.\lambda D_{\text{matrix}}.\textsc{max}(D_{\text{matrix}}) > \textsc{max}(D_{\text{than}})$

(See e.g., von Stechow 1984, Kennedy 1999, Schwarzschild 2008, Beck 2011 for a review)

Numerical differentials in the canonical analysis



⟦Lucy is 2 inches taller than Mary is ~~tall~~⟧

(11) ⟦ Lucy is 2 inches taller than Mary is ~~tall~~ ⟧
LF: [-er 2″ [λd.Mary is d-tall]][λd′.Lucy is d′-tall]

- Without a numerical differential: ⟦-er/more⟧ is of type $\langle\langle dt\rangle, \langle dt, t\rangle\rangle$

(10) $[\![\text{-er/more}]\!] \stackrel{\text{def}}{=} \lambda D_{\text{than}}.\lambda D_{\text{matrix}}.\textsc{max}(D_{\text{matrix}}) > \textsc{max}(D_{\text{than}})$

- With a numerical differential: ⟦-er/more⟧ is of type $\langle d, \langle\langle dt\rangle, \langle dt, t\rangle\rangle\rangle$

(12) $[\![\text{-er/more}]\!] \stackrel{\text{def}}{=} \lambda d_{\text{diff}}.\lambda D_{\text{than}}.\lambda D_{\text{matrix}}.\textsc{max}(D_{\text{matrix}}) \geq \textsc{max}(D_{\text{than}}) + d_{\text{diff}}$

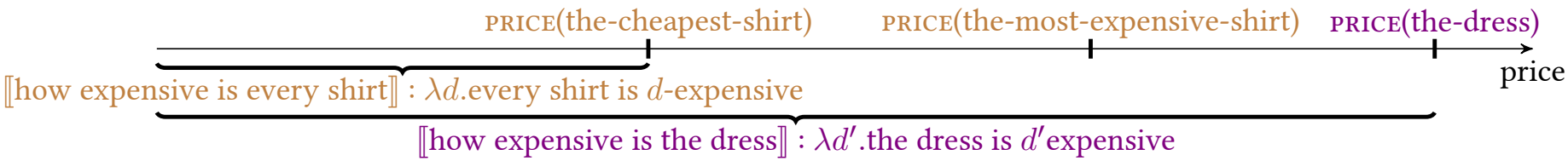
(See e.g., von Stechow 1984, Kennedy 1999, Schwarzschild 2008, Beck 2011 for a review)

Interim summary of the canonical view

	Without a numerical differential	**With a numerical differential**
Assumption	(Ordinal/interval) scales	Interval scales
Comparison	Inequality: $\text{measurement}_1 > \text{measurement}_2$	Inequality: $\text{measurement}_1 \geq \text{measurement}_2 + d$
Representations of & operations on scalar values	Degree points & ordering between degree points	Degree points & a combination of ordering and addition
The semantics of *-er/more*	Ordering: $\lambda m_2.\lambda m_1.m_1 > m_2$	A combination of ordering and addition: $\lambda d.\lambda m_1.\lambda m_2.m_2 \geq m_1 + d$

Outline

When the *than*-clause has a universal quantifier



⟦The dress is more expensive than every shirt is ~~expensive~~⟧

(13) Scenario: the price of the shirts ranges from $20 to $100 and the dress costs $150.

⟦ the dress is more expensive than every shirt is ~~expensive~~ ⟧

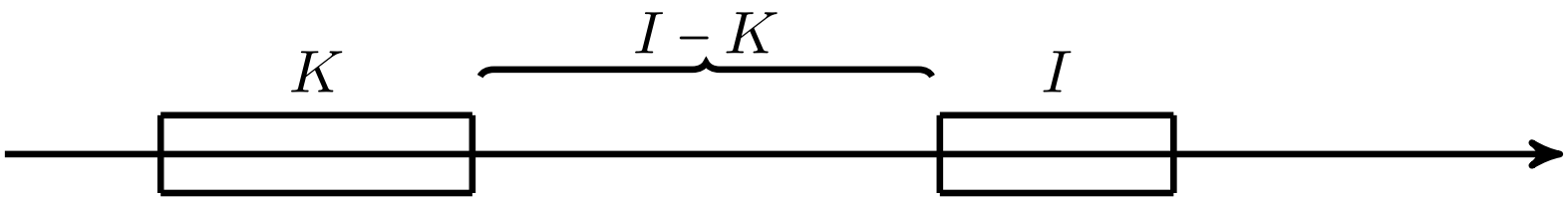
how expensive the dress is — how expensive every shirt is

LF: [more [λd.every shirt is d-expensive]][λd′.the dress is d′-expensive]

$=\{d \mid 0 \leq d \leq \text{PRICE}(\text{the-cheapest-shirt})\}$ $=\{d' \mid 0 \leq d' \leq \text{HEIGHT}(\text{the-dress})\}$

- **Our intuition**:
  the dress is more expensive than the most expensive shirt is.
- **The analysis under the canonical view** – too weak!:
  the dress is more expensive than the least expensive shirt is.

Schwarzchild and Wilkinson (2002)'s solution: intervals



(14) $[\![\text{expensive}]\!] \stackrel{\text{def}}{=} \lambda I.\lambda x.\textsc{price}(x, I)$ (i.e., Interval I covers individual x.)

(15) The dress is more expensive than every shirt is.

a. ***than*-clause**: $\lambda K.\forall x[\text{shirt}(x) \rightarrow \textsc{price}(x, K)]$

b. **matrix clause**: $\lambda I.\textsc{height}(\text{the-dress}, I)$

The semantics of comparison

(16) $\textsc{matrix-clause}(\mu I[\textsc{than-clause}(\mu K[\textsc{diff}(I - K)])])$

The differential predicate DIFF holds for each gap between any subpart of the interval $I_{\text{main-clause}}$ and any subpart of the $K_{\text{than-clause}}$.

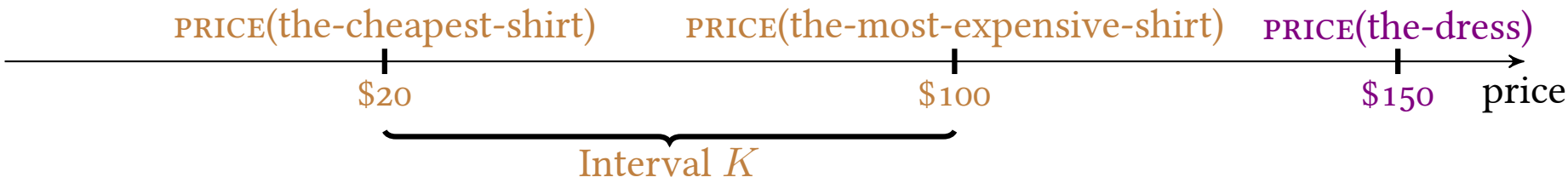
Beck (2010)'s comment

- MATRIX-CLAUSE(μI[THAN-CLAUSE(μK[DIFF($I - K$)])])
  Schwarzchild and Wilkinson (2002): embedding 'DIFF($I - K$)' within the scope of two maximality operators μ
  - The standard of comparison K is no longer a scalar value independent of comparison.
  - K is eventually yielded as the largest interval that makes DIFF hold for all the gaps involved in the numerous sub-interval-level comparisons.

Beck (2010)

'I want to come out of the calculation of the semantics of the *than*-clause holding in my hand *the* degree we will be comparing things to.'

Beck (2010)'s solution



⟦The dress is (up to $60) more expensive than every shirt is ~~expensive~~⟧

- Beck (2010): using the largest value in the interval K for comparison.

(15) The dress is more expensive than every shirt is. ✓

(17) The dress is up to $60 more expensive than every shirt is.
⇝ False under our scenario that the dress costs $150 and shirts vary between $20 and $100, but true under the analysis of Beck (2010)
(this example will be discussed later)

(See Fleisher 2016)

Issues to be solved

(17) The dress is up to $60 more expensive than every shirt is.

- The standard of comparison, i.e., the meaning of the *than*-clause, should be able to represent **a range of values**.
  - Beck (2010): 'I want to come out of the calculation of the semantics of the *than*-clause holding in my hand *the* degree we will be comparing things to.' (cf. Schwarzchild and Wilkinson 2002)
  - This range of values should not be reduced to a single degree point. (cf. Beck 2010)

Conceptual challenge: what does *-er*/*more* do?

Mephisto is talking about poor farmer Faustus:

'He always wanted more ...
more free time and more wealth ...
fewer struggles, better health ...
work less and eat all day ...'

(From *Sapiens: A Graphic History*, Vol.2,
by David Vandermeulen, David Casanave, Yuval Noah Harari, 2021)

Conceptual challenge: what does *-er/more* do?

- *-er/more* can be preceded by definite determiner *the*:

(18) a. The more you read, the more you learn.
b. The taller you are, the less mobile and quick you are.

- *-er/more* can be used repeatedly:

(19) a. Lucy is taller and taller and taller.
b. We will have more and more money.

- These data are unexpected if *-er/more* essentially encodes an inequality, meaning '$\lambda m_2.\lambda m_1.m_1 > m_2$' or '$\lambda d.\lambda m_1.\lambda m_2.m_2 \geq m_1 + d$'.

Outline

Parallelism between *-er/more* and *another*

- *-er/more* has an additive use similar to *another* (see also Greenberg 2010 and Thomas 2010):

(20) **Increase in the domain of entities: Additive use**

a. I ate an^{x} apple. Then I ate **another**^{y} (apple).

base item: an^{x} apple; increase: **another**^{y} (apple)

b. A^{x} girl, Sue, met **another**^{y} girl, Mary.

base item: A^{x} girl; increase: **another**^{y} girl

c. I ate two^{x} bars of chocolate. Then I ate (a bit) **more**^{y}.

base item: two^{x} bars of chocolate; increase: (a bit) **more**^{y}

From the additive use to the comparative use of *-er/more*

- Additive use of *more*: in the domain of entities
- Comparative use of *-er/more*: in the domain of scalar values (i.e., degrees)

(20c) **Increase in the domain of entities: Additive use**
I ate two^{x} bars of chocolate. Then I ate (a bit) **more**^{y}.
base item: two^{x} bars of chocolate; increase: (a bit) **more**^{y}

(21) **Increase in the domain of scalar values: Comparative use**

a. Mary is tall. Sue is tall **er** . Across sentences
base item: HEIGHT(Mary); increase: **er**

b. Sue is tall **er** than Mary is ~~tall~~. Within the same sentence
increase: **er**; base item: HEIGHT(Mary)

More uses of *-er/more* and *another*

- *-er/more* and *another*
  - denotes an increase in the domain of entities or scalar values
  - presuppose there is a salient base that the increase is anaphoric to

(22) **Repetitive use of *-er/more* and *another***

a. Lucy is becoming taller and taller and taller.
b. Janice had a little lamb and another and another and another.

(23) **Accumulating increases along with a universal quantifier**

a. Every year Mary wrote a more interesting book.
b. Everyday there is another story to write.

(24) They can all be preceded by definite determiner *the*:

a. The more you read, the more you learn.
b. The taller you are, the less mobile and quick you are.
c. I've brought two books: one is *Le Petit Prince*, and the other is *Exhalation*.

Beyond English

- French *plus* and German *mehr*, which are similar to English *-er*/*more* in being an obligatory item in comparatives, share similar uses:

(25) *more and more*

a. de plus en plus de gens
'more and more people' French

b. Mehr und mehr geschätzte regionale Erzeuger im zentralen Hochland.
'more and more valued regional producers in central highland' German

(26) *the more ...the more*

a. Le plus tu travailles, le plus tu réussis.
'The more you work, the more you achieves.' French

b. Je mehr Sie arbeiten, desto mehr mentale und körperliche Energie müssen Sie aufwenden.
'The more you work, the more mental and physical energy you have to spend.' German

What gets 'increased'? What does additivity mean?

- Additivity should be considered a phenomenon of QUD-based anaphoricity, indicating an extension of a previous salient answer in addressing the QUD
  - For the additive use in the domain of entities, *more* / *another* indicates an increase from a part to a whole.

(27) **Additive use of *more***

Current question (CQ): What did you eat?

a. I ate two bars of chocolate. Then I ate (a bit) more.

base item: a partial answer to the CQ

increase

b. #I didn't eat a bar of chocolate. Then I ate more. (see Li 2023)

(e.g., Roberts 1996/2012, Zeevat 2004, Zeevat and Jasinskaja 2007, Beaver and Clark 2009, Thomas 2011, Zhang and Ling 2021)

The anaphoricity of *-er/more*

- Additivity should be considered a phenomenon of QUD-based anaphoricity, indicating an extension of a previous salient answer in addressing the QUD
  - For the comparative use in the domain of scalar values, *-er/more* indicates an increase from a lower to a higher scalar value.

(28) **Comparative use of *-er/more***

Current question (CQ): How tall is Sue?

Mary is not tall. Sue is tall er .

base item – a partial answer to the CQ: HEIGHT(Mary)

increase

Feedback from the audience: this might not be a good example, and we need to pragmatically enhance how the information of Mary's height contributes to addressing the CQ. E.g., 'Mary is below 6 feet. Sue is taller.' ⇝ The height of Sue is an increase based on HEIGHT(Mary) (here Sue is not necessarily taller than 6 feet).

(e.g., Roberts 1996/2012, Zeevat 2004, Zeevat and Jasinskaja 2007, Beaver and Clark 2009, Thomas 2011, Zhang and Ling 2021)

The semantics of English *-er/more*

- The canonical view:

	Without a numerical differential	**With a numerical differential**
Comparison	Inequality: $\text{measurement}_1 > \text{measurement}_2$	Inequality: $\text{measurement}_1 \geq \text{measurement}_2 + d$
The semantics of *-er/more*	Ordering: $\lambda m_2.\lambda m_1.m_1 > m_2$	A combination of ordering and addition: $\lambda d.\lambda m_1.\lambda m_2.m_2 \geq m_1 + d$

- In contrast to this canonical view, English *-er/more* works like *another* in being an additive particle, denoting an increase on a discourse-salient base, extending this existing base in addressing the Current Question.

Outline

Comparatives: A difference-based view

⟦Lucy is taller than Mary is ~~tall~~⟧

(29) ⟦ Lucy is tall er than Mary is ~~tall~~ ⟧

how tall Lucy is — how tall Mary is

$\rightsquigarrow$ HEIGHT(Lucy) − HEIGHT(Mary) = ⟦-er⟧

- ⟦-er⟧ denotes an increase, i.e., a positive value. $\rightsquigarrow (0, +\infty)$

Comparatives: A difference-based view

⟦Lucy is 2 inches taller than Mary is ~~tall~~⟧

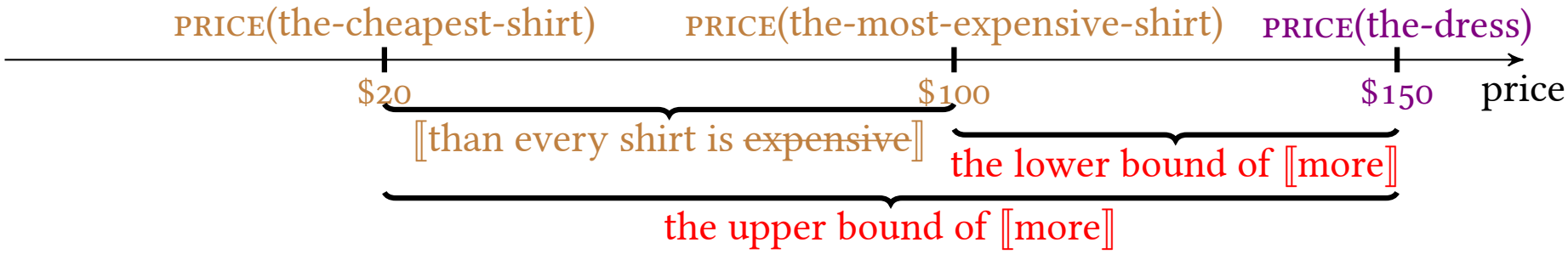
(30) ⟦ Lucy is 2 inches tall er than Mary is ~~tall~~ ⟧

how tall Lucy is — how tall Mary is

$\rightsquigarrow$ HEIGHT(Lucy) − HEIGHT(Mary) = ⟦2 inches …-er⟧

- ⟦2 inches …-er⟧ denotes an increase with a specified size, i.e., 2 inches.

Comparatives: A difference-based view



⟦The dress is more expensive than every shirt is ~~expensive~~⟧

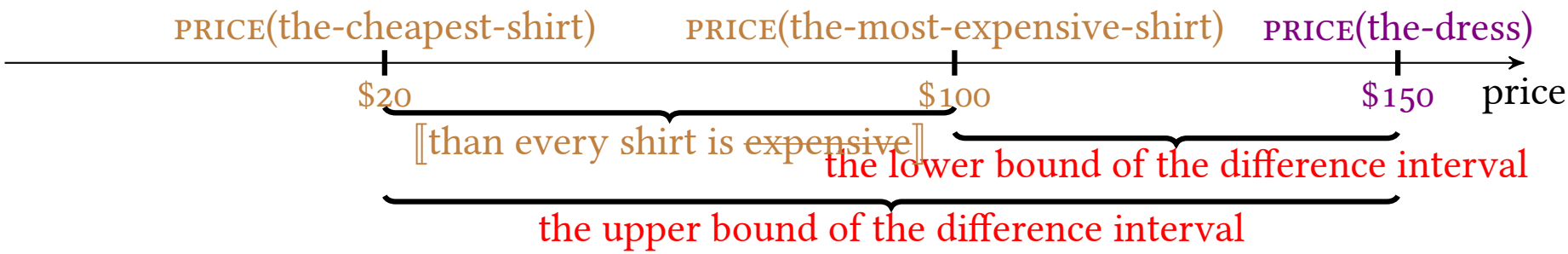
(31) ⟦ The dress is more expensive than every shirt is ~~expensive~~ ⟧

how expensive the dress is — how expensive every shirt is

$\rightsquigarrow$ PRICE(the dress) – the interval that ranges from PRICE(the-cheapest-shirt) to PRICE(the-most-expensive shirt) = ⟦more⟧

- ⟦more⟧ denotes an increase, i.e., a positive value.

Comparatives: A difference-based view



⟦The dress is up to $60 more expensive than every shirt is ~~expensive~~⟧ (false under this scenario)

(32)

⟦ The dress is up to $60 more expensive than every shirt is ~~expensive~~ ⟧

how expensive the dress is — how expensive every shirt is

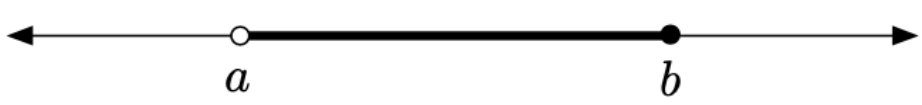
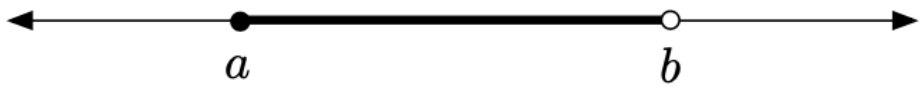
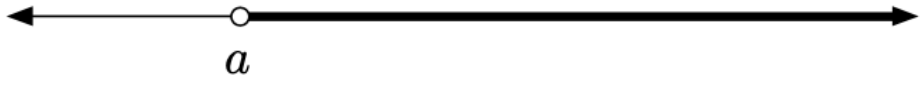
$\rightsquigarrow$ PRICE(the dress) – the interval that ranges from PRICE(the-cheapest-shirt) to PRICE(the-most-expensive shirt) = ⟦up to $60 more⟧

- ⟦up to $60 more⟧ denotes an increase with a specific size, i.e., $(-\infty, \$60] \cap (0, +\infty) = (0, \$60] \rightsquigarrow$ the upper bound of the increase is $60

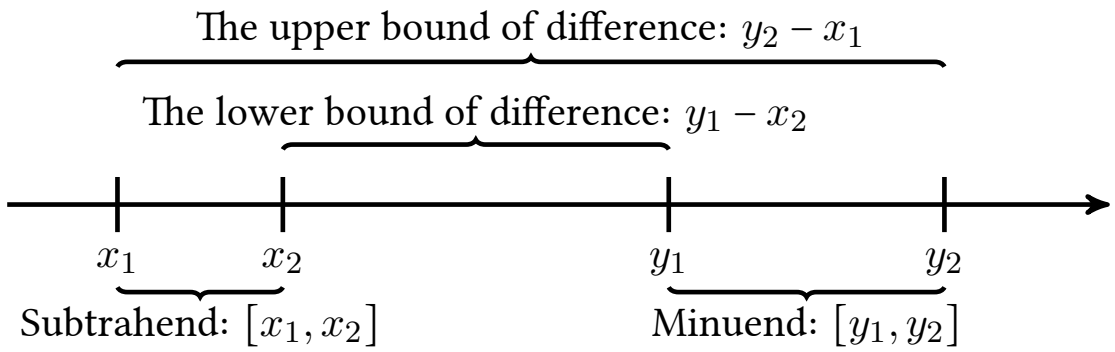
From degrees to intervals

- **Degrees**: points on a scale (an interval scale à la Stevens 1946)
- **Intervals**: convex sets of degrees
  - **Convex set**: A totally ordered set P is **convex** iff for any elements a and b in the set (suppose $a \leq b$), any element x such that $a \leq x \leq b$ is also in the set P.
  - E.g., $\{x \mid x > 0\}$, $\{x \mid x \leq 4\}$, and $\{x \mid 4 \leq x \leq 8\}$ are all convex sets.
  - Sets like $\{x \mid x > 10 \vee x \leq 3\}$ are not convex.
- Notation of intervals:
  - $\{x \mid x > 0\} = (0, +\infty)$
    $\{x \mid x \leq 4\} = (-\infty, 4]$
    $\{x \mid 4 \leq x \leq 8\} = [4, 8]$

Intervals

(a, b)	$\{x : a < x < b\}$	
$[a, b]$	$\{x : a \leq x \leq b\}$	
$(a, b]$	$\{x : a < x \leq b\}$	
$[a, b)$	$\{x : a \leq x < b\}$	
(a, ∞)	$\{x : x > a\}$	
$[a, \infty)$	$\{x : x \geq a\}$	
$(-\infty, b)$	$\{x : x < b\}$	
$(-\infty, b]$	$\{x : x \leq b\}$	
$(-\infty, \infty)$	$\mathbb{R}$	

Subtraction between intervals



(33) $\underbrace{[y_1, y_2]}$ – $\underbrace{[x_1, x_2]}$ = $\underbrace{[y_1 - x_2, y_2 - x_1]}$

minuend: matrix **subtrahend: comparative standard** **difference: differential**

a. Example 1: $[5, 8] - [1, 3] = [2, 7]$

b. Example 2: $(4, +\infty) - [2, 3] = (1, +\infty)$

- We are now ready to use intervals and interval subtraction to provide a systematic, generalized formal analysis for all kinds of comparatives.
  - With *than*-internal quantifiers and/or various numerical differentials
  - Crucially based on an additivity/increase-based view of *-er*/*more*

(See Moore 1979)

Today's take-home messages

- Day 2 and Day 3: Comparatives and *-er/more*
  - How an additivity-based perspective improve our understanding of scalarity-related phenomena?
  - What is additivity?
- Additivity is a phenomenon of QUD-based anaphoricity, indicating an extension of a previous salient answer in addressing the QUD.

	The canonical view	**The new difference-based view**
Assumption	(Ordinal/interval) scales	**Interval scales**
Comparison	Inequality: $M_1 > M_2$	**Subtraction**: $M_1 - M_2 = D$
Representations of & operations on scalar values	Degree points & ordering between degree points	**Intervals (i.e., set of degrees) & interval subtraction**
The semantics of *-er/more*	Ordering: $>$	**Additivity a default positive difference**: $(0, +\infty)$

Tomorrow

- Day 2 and Day 3: Comparatives and *-er*/*more*
  - How an additivity-based perspective improve our understanding of scalarity-related phenomena?
  - What is additivity?
- Tomorrow
  - Formal implementation (see Zhang and Ling 2021 and Zhang and Zhang 2024)
  - Antonyms
  - Cross-linguistic phenomena
  - etc.

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