

Sets, Rules and Natural Classes: $\{\}$ vs. $[\]$

Alan Bale, Charles Reiss & David Ta-Chun Shen

1 Introduction

Developing ideas introduced in Bale et al. (2014), this paper explores some of the consequences of analyzing segments as sets of features and, as a corollary, analyzing natural classes as sets of sets. We adopt the view that features, not segments are the primitives of phonological representation: the “alphabetic symbols that we use freely [are] nothing more than convenient ad hoc abbreviations for feature bundles, introduced for ease of printing and reading but without systematic import” (Chomsky and Halle, 1968, 64). Taking these feature bundles to be sets allows us to apply ideas from set theory to phonology. In this paper, we explore some of the theoretical consequences of using set-theory to represent underspecification. In particular, we discuss the logical possibility that some segments may be fully underspecified and thus are represented by the empty-set. We also discuss how the empty-set can be used to define a natural class over all segments.

In section 2, we review some of the proposals in Bale et al. (2014) and show how they vitiate the need for a feature $[\pm\text{SEGMENT}]$. In section 3, we identify three different sources for underspecified segments in a derivation, using a fully underspecified segment $/\{\ }/$ for illustration. We then demonstrate how to indirectly target only underspecified segments using rules that explicitly target *all* segments.

2 Features and Natural Classes

2.1 Interpreting Phonological Rules

Bale et al. (2014) introduce an innovation to the notation used in phonological rules to better reflect the type-theoretic nature of the components of rules, based on the idea that segments consist of sets of features.¹ The basic idea behind this innovation is best demonstrated by providing an example of a prototypical phonological transformation.

Suppose we have a language with a five-vowel inventory that raises its mid vowels to their high counterparts before a nasal consonant. Informally, we might indicate such a process as in (1):

(1) **A Simple Rule:**

o,e	→	u,i	/	___	n, m, ŋ
Target (SD)		Change			Environment

The targetted vowels *o,e* each turn into the corresponding high vowel before any one of the nasals *n,m,ŋ* that define the environment. Traditional formalizations of such a process might represent the target set, the set of mid vowels in the language, as $[-\text{HIGH}, -\text{LOW}]$.²

(2) **Traditional Rule:**

$[-\text{HIGH}, -\text{LOW}] \rightarrow [+ \text{HIGH}] / \text{___} [+ \text{NASAL}]$

¹The following discussion does not address the issue of contour segments, such as prenasalized stops and affricates, which appear to contain conflicting tokens or feature values within a single segment (e.g., $[\text{+NASAL}]$ and $[\text{-NASAL}]$). We think the logical issues raised in this paper and by Bale et al. (2014) would survive extension to structures more complex than sets, a topic of our current research.

²Or else the target set might be represented as just $[-\text{LOW}]$, if such a process is assumed to apply vacuously to vowels specified $+\text{HIGH}$.

This traditional notation refers to the set of targets *via* a superset relation: Segments are sets of valued features, and a segment is a target of the rule if and only if it is a superset of the set $\{-\text{HIGH}, -\text{LOW}\}$.³ The target of the process is thus a set of segments; that is, a set of sets of valued features, as in (3):

$$(3) \quad \text{Target—natural class of segments } X = \{x : x \supseteq \{-\text{HIGH}, -\text{LOW}\}\}$$

The characterization of the environment of our rule must also be a set of segments, namely the set of segments that are a superset of the set of features $\{+\text{NASAL}\}$, as in (4):

$$(4) \quad \text{Environment—natural class of segments } Y = \{y : y \supseteq \{+\text{NASAL}\}\}$$

The square brackets in the target and the environment in traditional rules like (2) are used to symbolize these natural classes.

$$(5) \quad \begin{array}{ll} \text{a. } \{x : x \supseteq \{-\text{HIGH}, -\text{LOW}\}\} = [-\text{HIGH}, -\text{LOW}] \\ \text{b. } \{y : y \supseteq \{+\text{NASAL}\}\} = [+ \text{NASAL}] \end{array}$$

Although traditional notation also uses square brackets to refer to the structural change of a rule, $[+\text{HIGH}]$ in our example, the change does *not* refer to a set of segments—the interpretation of the change is not parallel to the “natural class” interpretation of the target and environment. The structural change lists the features that are changed or added to the target segments when they appear in the rule environment.

Since the square brackets do not have the same set-theoretical meaning for the structural change as they do for the target and environment, we replace the square brackets with standard set braces. The set-brace notation, as in (6), makes clear the difference between natural classes (i.e., sets of sets of features) and mere sets of features.

$$(6) \quad \text{Structural Change—set of features: } \{+\text{HIGH}\}$$

With this set-theoretically consistent notation, rule (2) is expressed instead as (7).

$$(7) \quad \text{Rule with consistent bracketing:} \\ [-\text{HIGH}, -\text{LOW}] \rightarrow \{+\text{HIGH}\} / ___ [+ \text{NASAL}]$$

- a. $\{\dots\}$ denotes sets of features
- b. $[\dots]$ denotes sets of sets of features (= sets of segments)

Our revised notation consistently distinguishes a natural class of segments from a set of features.⁴

Two points follow from our interpretation of natural classes in rules. First, the notation works fine with underspecification. If a language has a voiced /d/, a voiceless /t/ and a coronal stop unspecified for voicing /D/, as Inkelas (1995) proposes for Turkish, then the representation in (8) describes the natural class that contains all three of these segments.

- $$(8) \quad \begin{array}{l} \bullet \text{ Description of class:} \\ \quad [+ \text{CORONAL}, - \text{CONTINUANT}] = \\ \quad \{x : x \supseteq \{+\text{CORONAL}, -\text{CONTINUANT}\}\} \\ \bullet \text{ Members of the class} \\ \quad \text{a. } /t/ = \{-\text{VOICE}, +\text{CORONAL}, -\text{CONTINUANT} \dots\} \\ \quad \text{b. } /d/ = \{+\text{VOICE}, +\text{CORONAL}, -\text{CONTINUANT} \dots\} \\ \quad \text{c. } /D/ = \{+\text{CORONAL}, -\text{CONTINUANT} \dots\} \text{ (no VOICE feature)} \end{array}$$

³See discussion in Bale et al. (2014), especially fn. 6, about how this traditional interpretation differs from that of *SPE* (Chomsky and Halle, 1968).

⁴In Bale et al. (2014), we applied this notation in our analysis of the ‘ \rightarrow ’ symbol, suggesting that the interpretation of the arrow depends on whether the process in question is feature-changing or feature filling. We treated feature-filling as unification, and feature-changing as deletion (via set subtraction) followed by unification, essentially, deletion then insertion. In our framework, then, the deletion and unification components of a feature-changing process constitute two separate rules.

Second, it is impossible to have a rule that targets a segment unspecified with respect to a feature F without also targeting segments specified with respect to F. No rule, given our system, can target /D/ without targeting /t/ and /d/ as well. This is because the description of /D/ subsumes the descriptions of /t/ and /d/. Since we do not allow reference to [0voiced], it is impossible for a rule to target /D/ without targeting /t/ and /d/.

It follows, then, that no rule can target a *fully* underspecified segment, which we will denote /{ }/, without targeting every possible segment. For example, consider the rule in (9), which is the only possible rule that could target a fully underspecified segment.

(9) **Rule with an empty target:**

$$[\quad] \rightarrow \{+HIGH\} / ___ [+NASAL]$$

The notation [] in (9) cannot refer to a segment without features to the exclusion of all other segments. Rather it refers to the set of all segments (*qua* feature sets), one of which may be the empty set. (The notation logically refers to the set of all segments, including /{ }/, but a particular language may not have this segment.)

(10) Empty Target—natural class of segments $X = \{x : x \supseteq \{\}\}$

As a consequence, this means that there can be no rule that only targets an X-slot associated with an empty feature set. We return to this issue in Section 3 where we show how to *indirectly* target underspecified segments.

2.2 Getting rid of the feature [SEGMENT]

Chomsky and Halle (1968, p.64) introduce a feature [\pm SEGMENT] to distinguish segments from morpheme and word boundary symbols. The use of such a feature was subsequently discussed in various contexts, such as (Hyman, 1985), and sometimes rejected. Independent of the merits of using this feature, it should be pointed out that a set theoretical representation of targets where natural classes are represented as sets of sets vitiates the need of a +SEGMENT feature. The natural class based on the empty set, e.g., [], targets all possible segments in a given language without the need to have a feature that is shared by all segments.

In Bale and Reiss (Forthcoming) we use the empty set symbol \emptyset to refer to morphemes that lack phonological content. For insertion and deletion rules we use a *null* segment symbol ϵ , as in (11):

- (11) a. $j \rightarrow \epsilon / i ___ i$ “DELETE *j* between two *i*’s”
b. $\epsilon \rightarrow j / i ___ i$ “INSERT *j* between two *i*’s”

We use # and % to represent initial and final word boundaries, respectively, following Raimy (2000). Using this notation, a rule that deletes any word-final segment can be represented as in (12).

(12) **Segment Deletion:**

$$[\quad] \rightarrow \epsilon / ___ \%$$

Note that in rules like (12), the ϵ , # and % symbols are best treated syncategorematically rather than categorically. In other words, unlike [−HIGH, −LOW] which represents a set of segments and {−HIGH, −LOW} which can represent an individual segment, ϵ , # and % do not correspond to a specific type of mental representation. The symbols ϵ , # and % are not segments but rather are notational symbols that linguists use to write phonological rules that affect the way the function that is associated with the phonological rule is interpreted. Let’s take the symbol % as an example, leaving the complexity of ϵ aside for now. The rule specified in (13) does not symbolize that mid vowels become high vowels when they appear before a percent sign.

(13) **Rule with Boundaries:**

$$[-HIGH, -LOW] \rightarrow \{+HIGH\} / ___ \%$$

Instead, this rule represents the function that changes mid vowels to high vowels in a string when no other segment appears after the vowel in the string. The end of the string is represented by the last segment in the string not having any other segments after it rather than by a mental representation of the % symbol. Below, we’ll return briefly to discussion of ϵ , and fuller discussion of such syncategorematic symbols is provided in Bale and Reiss (Forthcoming).

3 How to use $\{\}$ in rules

In this section, we illustrate the applicability of the empty feature set segment $/\{\}/$ in phonology, drawing on three widely discussed types of phenomena. Two issues must be distinguished, the source of the segment $/\{\}/$ in a derivation and the question of how the phonology can target $/\{\}/$ (regardless of its origin) and assign features to the segment.

3.1 Three sources of $/\{\}/$

The segment $/\{\}/$ can enter into a derivation in three ways. First, the segment can be part of the phonological representation of a morpheme. In this case, the segment $/\{\}/$ is present in the *lexicon*. Second, the segment can be derived by deleting the features of another segment. Such a token of $/\{\}/$ is introduced by a *phonological* process that deletes features from a segment, a process we model via set subtraction. Third, the segment $/\{\}/$ can be introduced by a *phonological* insertion rule, analogous to any other segment epenthesis rule.

3.1.1 $/\{\}/$ in the lexicon

There is a large literature proposing various kinds of ‘empty’ segments, including moras, CV-slots, X-slots or syllable constituents unassociated to a segmental melody. In some cases, such as the “moraic affixes” discussed most recently by Trommer (2015), we can analyze the segmental phonological content of a morpheme as consisting exclusively of $/\{\}/$ (along with mechanisms for combining this morpheme with others). Bendjaballah and Haiden (2008) present a scale of eight levels of ‘emptiness’ relevant to templatic morphology systems alone. Some kind of lexically present empty timing slot has also been proposed in various analyses for the segments that realize the doubled consonants of Italian *raddoppiamento sintattico* in some Italian dialects (Borrelli (2013)), the *h-aspiré* of French (see Côté (2008) for discussion), and similar phenomena.

While we cannot provide an analysis of each case where empty or “ghost” segments have been proposed, by providing a formalization of one kind of empty element it will be easier to evaluate such proposals and examine the extent to which there is unity of behavior among these elements in terms of the well-understood set-theoretic notions invoked in Bale et al. (2014). We propose that generally the empty segment is precisely $\{\}$. The phonology of a morpheme may consist exclusively of the segment $/\{\}/$ (realized, perhaps as lengthening of the root vowel), or this segment may be part of a morpheme that contains other segments, too, such as the words that trigger *raddoppiamento sintattico* in Italian.

3.1.2 Phonological derivation of $\{\}$ from another segment

Instead of entering a derivation directly from the lexicon, the segment $\{\}$ can also be derived from other segments. In Bale et al. (2014), we argue that feature changing rules must be analyzed as deletion of features *via* set subtraction followed by insertion *via* unification. In this context, rules of total assimilation and compensatory lengthening (which is just a form of total assimilation) can be modelled by deriving $\{\}$ from another segment, then unifying this $\{\}$ with the complete feature set of another segment.

For concreteness, let’s consider a language that deletes nasal consonants in codas and lengthens the preceding vowel. We can model this by first deleting all the features of the nasal, then unifying the output with the features of a preceding vowel. In fairly traditional notation this would be expressed something like the rule in (14), although it should be kept in mind that in our system (14) represents an “abuse” of notation.

$$(14) \quad [+NASAL] \rightarrow \{\} / \text{ in CODA}$$

To accurately represent the kind of phonological process that (14) is supposed to capture, the interpretation of the traditional arrow “ \rightarrow ” here has to be the operation of set subtraction applied as follows—each segment in the natural class of targets is a set, and we subtract all the valued features from each such a set. Let’s formalize this.

Since the nasal consonants features are deleted whatever they are, we need a way to refer to variables in our rule. We need to target a segment that is specified $+NASAL$ and subtract all the features from that segment. If we let \mathcal{F} be the set of all features F , and each α be an element of $\{+, -\}$, we can express the rule that copies the value of a following segment onto $\{\}$ as follows, adopting and extending the use of quantification in rules advocated by Reiss (2003):

$$(15) \quad \forall F \in \mathcal{F}. \forall \alpha \in \{+, -\}. [+NASAL] - \{\alpha F\} / \text{in CODA}$$

Equivalently, we could represent the process as the subtraction of all possible features, as in (16).⁵

$$(16) \quad [+NASAL] - \{x : \exists F \in \mathcal{F}. \exists \alpha \in \{+, -\}. x = \alpha F\} / \text{in CODA}$$

The input to this rule is the natural class of nasals, say, $\{m, n, \eta\}$, occurring in coda position. The output via set subtraction is the same for any input nasal—it is always the segment $\{ \}$, because set subtraction removes all the features of the input segments. This is why the formulation in (14) is useful—it reflects the neutralization effected by the rule.⁶

3.1.3 Phonological epenthesis of $\{ \}$

A third source of $\{ \}$ in a derivation is by phonological insertion. Epenthesis of default vowels that end up as copies of other vowels in the word can be analyzed as phonological insertion of $\{ \}$. Lengthening of a vowel under particular syllable structure or stress conditions can also be analyzed as insertion of $\{ \}$. Note that our suggestions are not in conflict with analyses that involve merely inserting a mora or a CV or X timing slot. Such proposals have to be enhanced by a model of how such positions end up associated with segmental (featural) content, and the appeal to the segment $\{ \}$ may be applicable in a wide array of such cases. In other words, models that involve insertion of, say, a mora in some phonological condition either implicitly assume that the inserted mora is prelinked to $\{ \}$, or else they need to be combined with a mechanism (like autosegmental association) to provide the inserted mora with valued features. Our proposals concerning $\{ \}$ are potentially relevant to either of these two scenarios.

3.2 A digression on one more kind of nothing

As mentioned above, we use a *null* segment symbol ϵ to express insertion and deletion rules. Our syncategorematic semantics for this symbol treats it differently from the segment symbols, which denote sets of features. It is important to note that the null segment symbol ϵ is thus clearly distinct from the empty set segment $\{ \}$ in our model. The latter is a set (and thus a segment), the former is the absence of a segment (and yet distinct from $\#$ and $\%$). The segment $\{ \}$ can be associated with timing slots and be the target of rules that, say, insert valued features, but ϵ cannot be. Thus, our discussion of natural classes of segments and sets of features denoted in rules is not relevant to ϵ .

3.3 Targetting $\{ \}$

Now that we have identified three distinct sources of $\{ \}$ in derivations, we need to explain how such a segment can end up on the surface with featural content and how we can target this segment without targetting *all* segments. We reiterate that the analysis generalizes to any case of feature-filling into a (partially) underspecified segment, but we illustrate with the extreme case of the fully underspecified $\{ \}$.

The critical idea, presented in Bale et al. (2014), is that feature-filling is the result of unification, which is a partial operation. This idea is probably best explained by contrasting unification with set-union. Set-union is not partial: for any two sets, A and B , the union of the two, $A \cup B$, is always defined (as the set that includes all the elements of A and all the elements of B , but no other elements). Unification, in contrast, is defined only when its output is *consistent*. For phonological purposes, we define *consistency* as follows:

$$(17) \quad \text{CONSISTENCY: A set of features } \rho \text{ is consistent if and only if there is no feature } F \in \mathcal{F} \text{ such that } +F \in \rho \text{ and } -F \in \rho.$$

When the consistency requirement is met, the output of set unification is identical to the output of set union:

⁵Note that set subtraction does not care if the subtracted set contains elements that are not in the set to be subtracted. For example, $(\{b, c, d\} - \{b, c\}) = (\{b, c, d\} - \{a, b, c, e, f, g\}) = \{d\}$.

⁶We use the subtraction symbol to represent the function that maps segments that fit the structural description to the same segment minus the structural change. This is a slight abuse of notation, but allows us to remind the reader about the nature of the mapping. To be clear, we are *not* using it to represent the subtraction of the structural change from the structural description, but rather the subtraction of the structural change from each segment in the class denoted by the structural description.

- (18) UNIFICATION: For any two sets, A and B , the unification of A and B , $A \sqcup B$, is defined iff $A \cup B$ is consistent. When defined, $A \sqcup B = A \cup B$.

By treating feature-filling through unification, we can avoid performing over-application of our feature-filling rules.

Consider the following concrete example. Let's suppose there is a language which has a phonological process that deletes all the features in a nasal segment that appears in the coda position and then lengthens the preceding vowel. Such a language would map underlying forms like /tan-so/ and /tak-so/ to the surface forms in (19):

- (19) a. tan-so \rightarrow ta:so
b. tak-so \rightarrow takso

In our system, the derivation of such forms would involve two steps. First, the rule in (15), repeated in (20), would apply:

- (20) $\forall F \in \mathcal{F}. \forall \alpha \in \{+, -\}. [+NASAL] - \{\alpha F\} / \text{in CODA}$

The output of (20) applied to /tanso/ is $ta\{ \}so$, whereas the output of the rule applied to /takso/ is $takso$ since the underlying form does not contain any nasals in coda position.

In the next step in the derivation, we don't need to target $\{ \}$ to the exclusion of other segments since we can take advantage of the idea that feature filling involves unification. Consider the rule in (21) which represents the function defined in (22)

- (21) $\forall F \in \mathcal{F}. \forall \alpha \in \{+, -\}. [] \sqcup \{\alpha F\} / [\alpha F] \text{ ______}$

- (22) Let f be a function from strings $x_1x_2 \dots x_n$ to strings $y_1y_2 \dots y_n$ such that for each index i , if $x_{i-1} \sqcup x_i$ is defined, then $y_i = x_{i-1} \sqcup x_i$, otherwise $y_i = x_i$.

Recall from the discussion above that $[]$ denotes “the natural class of those segments that are supersets of the empty set”, $\{x : x \supseteq \{ \} \}$. So, the target class is the set of *all* segments. However, in our examples derived from /tan-so/ and /tak-so/, the empty segment $\{ \}$ derived from the /n/ of underlying /tanso/ will unify with the features of the preceding vowel /a/. However, unification will fail for every segment that contains any feature that is inconsistent with the features in the previous segment, the /a/ in our example. When unification is undefined, there is no change to the segment. When unification is defined, two things can happen: 1) either the result of unification does not alter the form of the segment since the relevant segment already contains all the features that are in the previous segment (so-called vacuous unification) or 2) the target segment is (fully or partially) underspecified with respect to the features in the previous segment and hence feature-filling occurs. As a result, the system indirectly targets $\{ \}$ because, *for everything other than $\{ \}$* , unification either fails or vacuously applies. In brief, even though we use $[]$ to target the set of all segments, including $\{ \}$, feature filling will only occur when the target segment is underspecified with respect to the features in the preceding segment.

4 Conclusions

Our set theoretical treatment of segments provides mechanisms to address the problem of referring to underspecified segments. First, the empty set is not ‘nothing’, and so referring to the empty set does not give rise to the problem of referring to nothing. Second the nature of unification provides another formal mechanism for referring to absence—we can target underspecified segments for unification with a representation ρ , by virtue of the failure of unification with ρ of any element that is not consistent with ρ . In this paper, we have suggested that these simple formal techniques are applicable to a range of widely discussed phonological phenomena, including underlying and epenthetic empty segments, as well as those derived by rules that delete features by set subtraction. In phonological terms these processes correspond to phenomena that go by the names of assimilation, compensatory lengthening, copy vowels and templatic morphology. By attempting to unify these various phenomena we further the goal of linguistic theory “to abstract from the welter of descriptive complexity certain general principles governing computation that would allow the rules of a particular language to be given in very simple forms” (Chomsky, 2000).

References

- Bale, Alan, Maxime Papillon, and Charles Reiss. 2014. Targeting underspecified segments: A formal analysis of feature changing and feature filling rules. *Lingua* 148:240–253.
- Bale, Alan, and Charles Reiss. Forthcoming. *Phonology: A formal introduction*. MIT Press.
- Bendjaballah, Sabrina, and Martin Haiden. 2008. A typology of emptiness in templates. sounds of silence: Empty elements in syntax and phonology, edited by jutta hartmann, veronika hegedüs & henk van riemsdijk, 23-59.
- Borrelli, Doris Angel. 2013. *Raddoppiamento sintattico in italian: a synchronic and diachronic cross-dialectal study*. Routledge.
- Chomsky, Noam. 2000. Language as a natural object. In *New horizons in the study of language and mind*, 106–133. Cambridge, MA, US: Cambridge University Press.
- Chomsky, Noam, and Morris Halle. 1968. *The sound pattern of English*. New York: Harper & Row, Publishers.
- Côté, Marie-Hélène. 2008. Empty elements in schwa, liaison and h-aspiré: The french holy trinity revisited. *Sounds of Silence: Empty Elements in Syntax and Phonology*, Amsterdam: Elsevier 61–103.
- Hyman, Larry. 1985. *A theory of phonological weight*. Dordrecht: Foris.
- Inkelas, Sharon. 1995. The consequences of optimization for underspecification. In *Proceedings of the North East Linguistic Society 25*, ed. Jill Beckman, 287–302. University of Pennsylvania: Graduate Linguistic Student Association.
- Raimy, Eric. 2000. *The phonology and morphology of reduplication*, volume 52. Berlin: M. de Gruyter.
- Reiss, Charles. 2003. Quantification in structural descriptions: Attested and unattested patterns. *The Linguistic Review* 20:305–338.
- Trommer, Jochen. 2015. Moraic affixes and morphological colors in dinka. *Linguistic Inquiry* 46:77–112.