# Multiplicity and modifiers*

Jacopo Romoli
*Ulster University*

Agata Renans
*Ulster University*

**Abstract** As is well known, a sentence with an adverbial modifier under negation like *Mike didn't wash the window with soap* gives rise to a robust inference that Mike did wash the window (Simons 2001, Katzir 2007, Schlenker 2008). It is equally well known that a sentence with a plural noun like *Mike washed windows* gives rise to a so-called 'multiplicity' inference that Mike washed more than one window. This inference however disappears under negation: *Mike didn't wash windows* conveys that Mike didn't wash any window, not that he didn't wash more than one window (Krifka 1989, Sauerland et al. 2005b a.o.). In this paper, we focus on the interaction between these two inferences in sentences containing both an adverbial modifier and a plural noun under negation, like *This morning, Mike didn't wash windows with soap*. We observe that this sentence has a reading conveying that this morning, Mike didn't wash any window with soap but that he did wash multiple windows (albeit not with soap). In other words, the multiplicity inference seems to arise on top of the inference of the adverbial modifier. As we discuss, this reading is not predicted by any version of the (otherwise pretty successful) implicature approach to the multiplicity inference (Sauerland et al. 2005b, Spector 2007, Mayr 2015 a.o.) in combination with the implicature treatment of the inference of adverbial modifiers (Katzir 2007 a.o.). We sketch two solutions for this problem. The first keeps the implicature approach to the multiplicity inference but treats the inference of adverbial modifiers as a presupposition, building on proposals by Simons (2001) and Schlenker (2008). In addition, it adopts the idea that presuppositions can be strengthened via implicatures, as proposed by Spector & Sudo (2017). The second solution keeps instead the implicature approach to adverbial modifiers but adopts a non-implicature approach to multiplicity, the recent homogeneity-based proposal by Križ (2017). Either way, the interaction between multiplicity and the inference of adverbial modifiers suggests that we cannot treat both as implicatures: if we want to treat either one as an implicature, we need to do something different for the other. We end the paper discussing related issues, and comparing the case above to analogous cases involving different scalar inferences. Finally, we briefly show that the alternative non-implicature ambiguity approach to the multiplicity inference (Farkas & de Swart 2010, Marti 2017 a.o.) doesn't, as it stands, provide a solution to our problem.

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## 1 Introduction

A sentence with an adverbial modifier under negation like (1-a) gives rise to the inference in (1-b) (Simons 2001, Katzir 2007, Schlenker 2008; see also Chemla 2007 and Cremers et al. 2017 for experimental evidence of this inference):[1]

(1)    a.    This morning, Mike didn't wash the window with soap.
        b.    ⤳ *This morning, Mike did wash the window*

It is equally well known that a sentence with a plural noun like (2-a) gives rise to the inference in (2-b), generally called 'multiplicity inference:'

(2)    a.    This morning, Mike washed windows.
        b.    ⤳ *This morning, Mike washed more than one window*
                                    MULTIPLICITY INFERENCE

Moreover, this multiplicity inference disappears under negation: (3-a) doesn't convey (3-b) bur rather (3-c) (Krifka 1989, Sauerland et al. 2005b among others; for discussion about experimental evidence related to this pattern see Tieu & Romoli To appear).

(3)    a.    This morning, Mike didn't wash windows.
        b.    ⇸ *This morning, Mike didn't wash more than one window*
        c.    ⤳ *This morning, Mike didn't wash any window*

A successful analysis of the multiplicity inference in the literature treats it as an implicature, arising in positive cases like in (2-a) and disappearing – as implicatures generally do – under negation in cases like (3-a) (Sauerland 2003, Sauerland et al. 2005b, Mayr 2015, Spector 2007, Ivlieva 2013, Zweig 2009).

In this paper, we focus on the interaction between these two inferences in sentences containing both an adverbial modifier and a plural noun under negation, as in (4-a).

(4)    a.    This morning, Mike didn't wash windows with soap.
        b.    ⤳ *This morning, Mike did wash more than one window*

We observe that the sentence in (4-a) has a reading conveying that Mike didn't wash any window with soap but, at the same time, that he did wash multiple windows (albeit not with soap).[2] In other words, (4-a) has a reading in which a multiplicity

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1 This inference is sensitive to focus; see Simons (2001) for discussion. For this reason, we use sentence final adverbs and the sentences are intended to be read with default intonation.

2 We examined the interpretation of (4-a) and similar examples with several English native speakers.

inference appears to arise on top of the inference of adverbial modifiers.[3] As we discuss, this reading is not predicted by any version of the implicature approach to the multiplicity inference of plural nouns in combination with the implicature treatment of the inference of adverbial modifiers (Katzir 2007 among others).

We sketch two solutions for this problem. The first keeps the implicature approach to the multiplicity inference but treats the inference of adverbial modifiers as a presupposition, building on proposals by Simons (2001) and Schlenker (2008). Moreover, it adopts the idea that presuppositions can be strengthened via implicatures, as proposed by Gajewski & Sharvit (2011) and Spector & Sudo (2017). The second keeps instead the implicature approach to adverbial modifiers but adopts a non-implicature approach to multiplicity, the recent homogeneity-based proposal by Križ (2017). We also discuss related issues to each solution and the implications for the alternative non-implicature ambiguity approach to multiplicity (Farkas & de Swart 2010, Grimm 2013, Marti 2017) and show that, as it stands, it doesn't provide a solution to our problem. In sum, the interaction between multiplicity and the inference of adverbial modifiers suggests that we cannot treat both as implicatures: if we want to treat either one as an implicature, we need to do something different for the other.

The rest of the paper is organised as follows. In section 2, we sketch the implicature theory of the inference of adverbial modifiers and the main theories within the implicature approach to the multiplicity inference. In section 3, we discuss the problem coming from the combination of these two inferences in more detail. In section 4, we sketch the two solutions mentioned above. The first keeps the implicature approach to the multiplicity inference but abandons the implicature approach to the inference of adverbial modifiers in favour of a presuppositional analysis. The second, on the other hand, keeps the implicature approach to adverbial modifiers, but adopts a non-implicature approach to the multiplicity inference. As we show, both of these solutions can account for the problematic reading above. We discuss some open and related issues for each of the two solutions in section 5 and we conclude in section 6.

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3 Note that there is a scopal interaction between negation and the adverb here, so (4-a) could also have a reading in which the latter scopes above the former. This interpretation, which we could paraphrase as 'by using soap, Mike didn't wash windows,' is however pragmatically implausible. The intended interpretation we are focusing on here is the one in which the adverb scopes below negation.

## 2 Multiplicity, modifiers, and implicatures

### 2.1 The implicature theory of the inference of adverbial modifiers

A successful explanation of the inference in (1-b) above, repeated below in (5-a), is to analyse it as a scalar implicature arising from the competition between (5-a) and a version of this sentence without the adverbial modifier in (6), as proposed by Katzir (2007).

(5)    a.    This morning, Mike didn't wash the window with soap.
          b.    ⇝ *This morning, Mike did wash the window*

(6)    This morning, Mike didn't wash the window.

Now, given a theory of scalar implicatures according to which a hearer tends to conclude that any relevant and more informative alternative sentence that the speaker might have said and didn't say is false (Grice 1975 and much subsequent work), the inference in (5-b) is straightforwardly derived.[4] That is, since (6) is a stronger alternative, upon hearing (5-a) the hearer would conclude that (6) is false, giving rise to the inference in (5-b).

    This account of (5-b) is simple and not only connects very well to theories of implicatures but it also arises nicely from a general theory of alternatives. That is, rather than being an isolated claim about the inference of adverbial modifiers, Katzir's (2007) proposal is a general principled theory of alternatives according to which (6) is predicted to be an alternative for (5-a). In addition, this approach can account for cases beyond negation. For example, (7-a) suggests (7-b), which can be accounted for by the negation of its alternative without the modifier i.e., *every candidate was elected*. Same for (8-a) and (8-b) (both examples are from Katzir 2007).

(7)    a.    Every candidate who sang was elected.
          b.    ⇝ *Not every candidate was elected*

(8)    a.    If any tall man comes to the party, he'll be disappointed.
          b.    ⇝ *It's not true that if any man comes to the party, he will be disappointed*

In sum, there is a simple implicature-based theory of the inference of adverbial modifiers, which can account for the basic case and related more complex ones. In

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4 We keep the assumptions about the underlying theory of implicatures as minimal as possible here, given that the above for now suffices for our purposes. Some of cases that we discuss below will require that we move to a theory of implicatures where the latter arise also from alternatives that aren't more informative that the asserted sentence but logically independent to it. We will mention this where needed.

the next section, we show that, however, this theory combined with any implicature theory of multiplicity inferences cannot account for the reading of the sentence containing both an adverbial modifier and a plural noun that we saw above.

## 2.2   The implicature approach to the multiplicity inference

The main issue that any theory of multiplicity has to explain is why the multiplicity inference arises in positive sentences, such as (9), but disappears in negative ones, such as (10). That is, (10) does not tend to convey the meaning that Mike didn't wash more than one window, the negation of (9), but rather that he didn't wash any window at all.[5]

(9)     Mike washed windows.
         ⤳ *Mike washed more than one window*

(10)     Mike didn't wash windows.

         a.     ⤳̸ *Mike didn't wash more than one window*
         b.     ⤳ *Mike didn't wash any window*

The general consensus in the literature is that in order to account for the contrast between (9) vs. (10), the former has to have a reading that is compatible with Mike washing just one window. Under this reading, (9) can be paraphrased as in (11):

(11)     Mike washed one or more windows.

This immediately accounts for the interpretation of (10) in (10-b) by simply analysing (10) as the negation of (11). However, what of course still remains to be explained is how the multiplicity inference arises in positive cases like (9).

   As mentioned, a successful approach to the pattern above is to analyse the multiplicity inference as a scalar implicature. The main idea is that the literal meaning of a plural sentence like (11) is compared with a singular competitor which we could paraphrase as in (12):

(12)     Mike washed exactly one window.

   There are three main types of implicature accounts in the literature and they differ on how they derive the competitor in (12). In the following we sketched them

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5 The reading that Mike didn't wash more than one window can be forced by marked intonation as in (i). We put this reading aside here, as it is not relevant and can anyway be accounted for by any of the approaches discussed below; see Tieu & Romoli To appear for discussion.

(i)     Mike didn't wash any winDOWS, he washed only one!

in turn.

The first one is based on enriching the sentence locally, at the predicate level, where the plural and the singular nouns are assumed to differ (Mayr 2015, Ivlieva 2013, Zweig 2009). While a singular noun, as in (13), has only atomic elements in its denotation (assuming all the relevant windows in the context are $a$, $b$ and $c$), its corresponding plural form denotes both atoms and sums thereof, as in (14) (see Schwarzschild 1996 among many others):

(13)   $[[\text{window}]] = \{a, b, c\}$

(14)   $[[\text{windows}]] = \{a, b, c, \{a, b\}, \{a, c\}, \{c, b\}, \{a, b, c\}\}$

Thus when combined with the rest of the sentence, (14) gives rise to the meaning in (11) that Mike washed one or more windows. Notice that the singular predicate is stronger than the plural one.[6] It is conceivable, therefore, that the latter can be strengthened with a local implicature at the predicate level (provided a theory of implicature which can apply at that level).[7] That is, the predicate in (14) can be compared to the predicate in (13). Since (13) is stronger, it will be negated and therefore (14) will be enriched with its negation, i.e., it will effectively become equivalent to (15):

(15)   $[[\text{windows}]] = \{\{a, b\}, \{a, c\}, \{c, b\}, \{a, b, c\}\}$

When (15) is combined with the rest of the sentence, it gives rise to the multiplicity inference in (9) that Mike washed more than one window.

Moreover, when plural predicates appear in downward entailing environments, such as in the scope of negation, no implicature is computed as the competitor is not stronger anymore.[8] Therefore this approach correctly captures the contrast between (9) vs. (10).

A second type of implicature account is based on higher order implicatures:

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6 Strength at predicate level is based on a standard notion of generalised entailment:

(i)   **Generalised entailment**:

    a.   For any $p, q$ of type $\langle s, t \rangle$, $p$ entails $q$ iff for any world $w$ in which $p(w) = 1$ then $q(w) = 1$

    b.   For any $P, Q$ of type $\langle \delta, \langle s, t \rangle \rangle$, $P$ entails $Q$ if for any world $w$ and any $a$ of type $\delta$ such that $P(a)(w) = 1$, then $Q(a)(w) = 1$

7 Zweig (2009) and Ivlieva (2013) actually make use of an event semantics and have the strengthening apply to a predicate of events. This detail is not important for our purposes so we put it aside here.

8 The sensitivity to monotonicity exhibited by scalar implicatures is generally derived by appealing to a principle which prevents them from arising if they weaken the literal meaning of the sentence. This captures the difference between the contribution of implicatures in upward versus downward entailing environments; see Chierchia et al. 2012 among many others for discussion.

implicatures arising on alternatives and the implicatures that such alternatives would have had, had they been asserted (Spector 2007). The starting point is that that a singular and a plural sentence are equivalent and both mean (11). The different interpretations they give rise to - and in particular the multiplicity inference - follows, in this approach, from the assumption that they are compared to different alternatives. In particular, the singular sentence in (16) has as alternative the sentence in (17). This, in turn, makes it so that when (16) is enriched with the negation of the stronger alternative in (17), it becomes equivalent to (12), i.e., that Mike washed exactly one window.

(16)    Mike washed a window.

(17)    Mike washed at least two windows.

A plural sentence is then compared to the corresponding singular sentence already enriched by its implicature, e.g., (16) is compared to (12). Since (12) is stronger, it will end up being negated by the implicature reasoning giving rise to the multiplicity inference: if Mike washed one or more windows, but it's not true that he washed exactly one window, then he washed more than one window. Again, as implicatures tend not to appear in the scope of downward entailing operators (see fn. 8), this approach as well can account for the contrast between (9) vs. (10).

Finally, a third type of implicature accounts, proposed by Sauerland (2003) and Sauerland et al. (2005b), is based on the comparison between the singular and the plural at the level of presuppositions, via a Maximise Presupposition reasoning (e.g., Heim 1991, Percus 1998, Singh 2008, Chemla 2008). The assumption is that while (9) and (16) truth-conditionally convey the same meaning, they differ in the presuppositions they give rise to. That is, while the singular sentence in (16) presupposes that Mike washed exactly one window, so that its overall meaning ends up equivalent to (12), (9) doesn't presuppose anything. Therefore, upon hearing (9), Maximise Presupposition makes the hearer conclude that the exactly-one presupposition of the singular is not met in the context. As a result, the hearer concludes that Mike washed more than one window.[9] The sensitivity to monotonicity, the contrast between (9) vs. (10), is derived under this approach by the assumption that Maximise Presupposition can only apply if it strengthens the entire sentence. This immediately predicts that it will not apply under negation, as it would make the sentence weaker than it's literal meaning.[10] Therefore, the alternation between

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9 This is generally derived in two steps: first a weaker inference that is not common ground that Mike washed exactly one window is derived; an inference that can be further strengthened to the conclusion that it is common ground that Mike didn't wash exactly one window (and that therefore he washed more than one); see Chemla 2008 for discussion.

10 The formulation of the principle is as in (i) (Sauerland et al. 2005b):

positive and negative contexts in relation to the multiplicity inference is captured also by this type of implicature account.

In sum, there are three main types of theories within the implicature approach and they can account for the sensitivity to monotonicity and a variety of complex data in relation to the multiplicity inference which we didn't review here (see Spector 2007, Zweig 2009, Križ 2017 and Ivlieva 2013 for discussion). We turn now to show, however, that none of these theories can account for the problematic case discussed above, if combined with an implicature approach to the inference of adverbial modifiers.

## 3 The problem

### 3.1 The problematic case again

Consider again the inference of adverbial modifiers in the case above and similar cases as in (18), (19) and (20).

(18)    This morning, Mike didn't wash the window with soap.
        ↝ *Mike did wash the window*

(19)    Conor didn't do the homework by himself.
        ↝ *Conor did do the homework*

(20)    Sarah didn't organise the database manually.
        ↝ *Sarah did organise the database*

And consider now some more cases of multiplicity inferences arising in positive cases and disappearing under negation:

(21)    a.    This morning, Mike washed windows.
              ↝ *This morning, Mike washed more than one window*
        b.    This morning, Mike didn't wash windows.
              ↝ *This morning, Mike didn't wash any window*

(22)    a.    Yesterday, Conor solved difficult problems.
              ↝ *Yesterday, Conor solved more than one difficult problem*
        b.    Yesterday, Conor didn't solve difficult problems.
              ↝ *Yesterday, Conor didn't solve any difficult problem*

(23)    a.    Yesterday Sarah organised files.
              ↝ *Yesterday, Sarah organised more than one file*

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(i)     Maximize Presuppositions is enforced in the scope of an existential if this strengthens the entire sentence.

   b.   Yesterday Sarah didn't organise files.
        ⤳ *Yesterday, Sarah didn't organise any file*

Now, when we combine the two, we observe that each of the sentence below has a reading in which the inference of the adverbial modifier appears to be combined with a multiplicity inference. In other words, a sentence like (24) not only tend to suggest that Mike washed one or more windows, but that he washed multiple windows. Same for the other cases.

(24)   This morning, Mike didn't wash windows with soap.
       ⤳ *This morning, Mike did wash more than one window (albeit not with soap)*

(25)   Yesterday, Conor didn't solve difficult problems by himself.
       ⤳ *Yesterday, Conor did solve more than one difficult problem (albeit not by himself)*

(26)   Yesterday, Sarah didn't organise files manually.
       ⤳ *Yesterday, Sarah organised more than one file (albeit not manually)*

We turn now to see how these cases are problematic for each of the implicature theory of the multiplicity inference discussed above.

## 3.2   Why the problematic case is problematic

Let us go back to the sentence in (24), and consider the predictions of the implicature approach to adverbial modifiers combined with different versions of the implicature approach to the multiplicity inference, starting from the local-strengthening theories by Mayr (2015), Ivlieva (2013), and Zweig (2009). Under this approach, the plural noun can be strengthened locally, giving rise to the multiplicity inference, or it can be interpreted literally.[11] Either way however the reading we are after is not predicted. For illustration, consider first the non-strengthened case: the predicted reading can be paraphrased as in (27). As is easy to see, (27) doesn't capture the intuition about the multiplicity inference arising from the inference of the adverbial modifier. That is, it is compatible with Mike washing only one window. While this may be a possible reading of the sentence, it doesn't account for the reading described above.

(27)   Mike didn't wash any window with soap but he did wash one or more

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11 Some of these local implicature proposals assume that the strengthening of plural is obligatory. It can however happen globally, rather than locally, which in the relevant cases discussed here would be vacuous and therefore equivalent to not strengthening the predicate at all; see Mayr 2015 and Ivlieva 2013 for discussion.

windows.

If, on the other hand, the predicate is strengthened locally, the predicted reading is in (28). The multiplicity inference arising from the inference of the adverbial modifier is now correctly captured, but the literal meaning of the sentence (the first conjunct in (28)) appears too weak: it is compatible with Mike washing one window with soap, contra intuitions.

(28)    Mike didn't wash more than one window with soap but he did wash more than one window.

Let us now consider the global approach with higher-order implicatures by Spector (2007). This approach also fails to account for the reading we are after, but since this is slightly more complicated to show, we do it in detail in Appendix B. What is however worse for this approach, is that not only does it fail to predict the reading we want, but, as it stands, it actually predicts the intuitively wrong reading paraphrased in (29). We also discuss the details of this prediction in Appendix B.

(29)    Mike didn't wash any window with soap but he did wash exactly one window.

Finally, consider the presupposed implicature approach by Sauerland (2003) and Sauerland et al. (2005b). The problem for this account is that given the way Maximise Presupposition is restricted, it is predicted not to apply in the case of (24), as it would weaken the meaning of the sentence. Therefore, the predicted reading is again (27). If we lift the restriction and nonetheless apply Maximise Presupposition, the predicted reading is (28), which is also intuitively incorrect.

In sum, none of the implicature theories to the multiplicity inference in combination with an implicature-based analysis of the inference of adverbial modifiers can account for the problematic reading above. In the next section, we sketch two solutions. The first one is a presuppositional theory of the inference of adverbial modifiers, in combination with a theory of presupposition strengthening, while the second one is based on the homogeneity approach to the multiplicity inference by Križ (2017) combined with the implicature approach to the adverbial modifiers.

## 4 Two solutions

### 4.1 The first solution: The inference of modifiers is not an implicature

#### 4.1.1 The inferences of modifiers as presuppositions

A possible solution to the puzzle is to abandon the theory of the inference of adverbial modifiers as scalar implicatures as sketched in Section 2.1 and analyse them as presuppositions instead, as proposed by Simons (2001) and Schlenker (2008). Under this approach, the sentence in (30) not only would entail that Mike washed windows, but also presuppose it.[12]

(30)    Mike washed windows with soap.
        ↝ *Mike washed windows*                      PRESUPPOSITION

This idea seems to be supported by the projection data in (31-a)-(31-d) which suggest that Mike washed windows, i.e., this inference project out of the scope of the operators below and in that it resembles closely the behaviour of presuppositions.

(31)    a.   Mike didn't wash windows with soap.
        b.   Did Mike wash windows with soap?
        c.   Perhaps Mike washed windows with soap.
        d.   If Mike washed windows with soap, he will probably to it again.
             ↝ *Mike washed windows*

Going back to our problematic case in (32), the idea would be to assume that (32) presupposes that Mike washed windows as in (32-a). Of course this alone does not solve the puzzle, because (32-a) just means that Mike washed one or more windows, not that he washed more than one window. What we need is the multiplicity inference arising on top of (32-a). We turn to this in the next subsection.

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12 A question that arises for a presuppositional treatment of the inference of adverbial modifiers is how to encode the presupposition in these constructions. For illustration, compare the sentence in (i), where the general assumption is that the presupposition is encoded in the word *stop*, with the case in (ii). Here it would have to be assumed that *with soap* encodes somehow the presupposition that Mike washed the window.

(i)     Mike stopped smoking. ↝ *Mike used to smoke*

(ii)    Mike washed the window with soap ↝ *Mike washed the window*

Without going into details here, we would need a theory of presuppositions which allows them to arise compositionally from the composition of adverbial modifiers with the rest of the sentence. See Simons 2001 and Schlenker 2008 for discussion.

(32)    Mike didn't wash windows with soap.

      a.    ↝ *Mike did wash windows*          PRESUPPOSITION

### 4.1.2    Presupposition strengthening

The remaining ingredient is adopting the theory of presupposition strengthening by Spector & Sudo (2017), see also Gajewski & Sharvit 2011 and Sudo & Romoli 2017. The gist of the idea is that implicatures can be calculated both at the level of the assertion and at the level of presuppositions. Therefore, by adopting the presuppositional theory of the adverbial modifiers, a theory of multiplicity inferences as implicatures can be kept. In the case of (32), the multiplicity inference arising on top of the presupposition in (32-a) would be strengthened to (33):

(33)    Mike did wash more than one window.

In other words, the problematic reading of (34), repeated from above, would be then predicted as a combination of its assertive and presuppositional components, where the latter is additionally strengthened via the multiplicity inference analysed as an implicature.

(34)    Mike didn't wash windows with soap.

      a.    ↝ Mike didn't wash any window.          ASSERTION
      b.    ↝ Mike did wash one or more windows.          PRESUPPOSITION
      c.    ↝ *Mike did wash more than one window*
                PRESUPPOSITION STRENGTHENED BY IMPLICATURE

Therefore this account correctly predicts that the sentence in (35) asserts that Mike didn't wash any window with soap and presupposes that Mark did wash more than one window (albeit no with soap). In sum, this approach can account for the problematic reading we are after and –to the extent that this is correct – it can be seen as an argument for a presuppositional treatment of the inference of adverbial modifiers, an implicature approach to multiplicity inferences, and the idea that presuppositions can be strengthened via implicatures. The formal details of this analysis are presented in Appendix A.

## 4.2    The second solution: the multiplicity inference is not an implicature

### 4.2.1    The homogeneity approach to the multiplicity inference

Križ (2017), building on his account of plural definite descriptions (Križ 2015), proposes an alternative approach to the multiplicity inference in terms of homogeneity.

The main idea is that most predicates are undefined under certain conditions when they apply to pluralities. In particular, homogeneity arises, under this approach, as a lexical property of predicates in combination with a general homogeneity principle.[13] Together these two ingredients give the trivalent truth conditions of sentences involving predicates applying to pluralities. For example, the nominal predicate *windows* has the following meaning:

(35) $[[\text{windows}]] = \lambda x \begin{cases} 1 \text{ iff } x \text{ is a plurality of windows} \\ 0 \text{ iff } x \text{ doesn't contain any window} \\ \star \text{ otherwise} \end{cases}$

As Križ (2017) shows, when a predicate like (35) appears in an episodic sentence such as (36), it gives rise to the following trivalent truth-conditions: it is true when both (37-a) and (37-b) are true, false when both are false, and undefined otherwise. This gives us the intuitively correct reading of the sentence in (36), in particular the fact that is true if and only if Mike washed more than one window.

(36) Mike washed windows.

(37) a. Mike washed one or more windows.
b. Mike washed multiple windows.

When (37) is negated, negation leaves undefinedness untouched, so that the conditions for (38) are as follows: (38) is true when both (39-a) and (39-b) are true, false when both are false, and undefined otherwise. These appear the correct conditions for (38) and in particular they capture the intuition that the sentence is true if and only if Mike didn't wash any window.[14]

(38) Mike didn't wash windows.

(39) a. Mike didn't wash one or more windows.
b. Mike didn't wash multiple windows.

In sum, the homogeneity approach can account for the alternation between positive and negative cases. In the next subsection, we demonstrate that combined with the implicature approach to the adverbial modifiers it provides a straightforward solution

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13 The principle is in (i):

(i)  GENERALISED HOMOGENEITY: A homogeneous predicate *P* is undefined of a plurality *a* if it is not true but there is a plurality *b* that overlaps with *a* (i.e. has a constituent individuals in common) such that *P* is true of *b*.                    (Križ 2015)

14 Note that the negation assumed here is standard negation, making a true sentence false, a false sentence true and keeping the undefinedness untouched.

to the problematic cases.

### 4.2.2 Applying homogeneity to our case

Coming back to the problematic case in (40), its meaning strengthened with the implicature of the adverbial modifier *á la* Katzir (2007) is as in (41):

(40)    Mike didn't wash windows with soap.

(41)    Mike didn't wash windows with soap and he did wash windows.

The prediction for the homogeneity approach is that the first conjunct of (41) is true whenever both (37-a) and (37-b) are true, false when both are false and undefined otherwise. The second conjunct, on the other hand, is true when both (39-a) and (39-b) are true, false if both are false and undefined otherwise. Putting these together, it follows that the conjunction in (41) is true if and only if all of (37-a), (37-b), (39-a) and (39-b) are true.[15] This reading can be paraphrased as in (42) and –as one can see – this is precisely the reading we are after:

(42)    Mike washed no window with soap but he did wash multiple windows.

Thus adopting the homogeneity approach allows us to account of the problematic case by keeping the implicature approach to the inference of adverbial modifiers. free choice inferences, and the implications for alternative approaches to the multiplicity inference.

### 5    Discussion

The two solutions we have sketched above entail that we either drop the implicature approach for the inference of the adverbial modifiers or we we drop it for the multiplicity inference of plural nouns. The interaction between these two inferences, therefore, suggests that we cannot treat both as an implicature.

In this section, we discuss some related and open issues for each solution and we briefly consider the alternative ambiguity approach to the multiplicity inference by Farkas & de Swart (2010) and others and we show that, as it stands, it doesn't provide a solution to our cases.

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15 Notice that the implicit assumption here is that the negation of the alternatives is also the standard negation defined above in fn. 14; see Spector & Sudo 2017 for discussion.

## 5.1   Issues for the presuppositional solution

The presuppositional solution integrates a presuppositional account of the inference of adverbial modifiers with an implicature treatment of the multiplicity inference. As we discuss in this subsection, however, not all implicature account of the multiplicity inference fare equally well in this approach. There are two issues, in particular, to consider: first, how straightforwardly an implicature account can be assumed to apply at the presuppositional level, as necessary in the proposal above. Second, whether the result of deriving the inference of adverbial modifiers as strengthened presuppositions is compatible with maintaining the implicature approach to the inference of adverbial modifiers, which arises from the general theory of alternatives by Katzir (2007) (which we might want to retain).

Starting from the first issue, let's consider briefly how each of the type of implicature accounts of the multiplicity inference can be applied at the level of presuppositions. As a start, it is not entirely clear to us how to combine the presuppositional approach to the inference of adverbial modifiers with Sauerland et al.'s (2005a) and Sauerland et al.'s (2005b) approach to the multiplicity inference. To illustrate, recall that in this type of account the singular sentence such as *Mike washed a window* presupposes that *Mike washed exactly one window*. Maximise Presupposition would regulate then the use of the plural sentence *Mike washed windows* in such a way that it would give rise to the inference that Mike washed more than one window. Combining this approach with the presuppositional account of adverbial modifiers would require a competition between the presupposition of (43-a) in (43-b) with the presupposition of the singular counterpart (44-a) in (44-b). While this gives the correct result, it is conceptually unclear however why the hearer should compare the presuppositions of (43-b) to that of (44-a) via Maximise Presupposition given that (43-b) is itself a presupposition.

(43)   a.   Mike didn't wash windows with soap.
       b.   Mike washed windows.                          PRESUPPOSITION

(44)   a.   Mike washed a window.
       b.   Mike washed exactly one window.               PRESUPPOSITION

By contrast, combining the presuppositional approach to the inference of the adverbial modifiers with the implicature approach to the multiplicity inference is more straightforward under the local account proposed by Mayr (2015), Ivlieva (2013) and Zweig (2009) or the higher order implicatures approach by Spector (2007). In particular, if we allow implicatures to arise from presuppositions, we can calculate an implicature locally on the plural predicate of the presupposition or globally over the strengthened alternative of the singular counterpart, which would then result in

the strengthened presupposition that Mike washed more than one window.

As for the second issue, we notice that the local approach by Mayr (2015), Ivlieva (2013) and Zweig (2009) is compatible with retaining Katzir's (2007) general theory of alternatives. This is because the latter in the case of adverbial modifiers would generate weak implicatures which are entailed by the strengthened presupposition of the sentence i.e., for (43-a) it would derive the implicature that Mike washed one or more windows, which is entailed by the strengthened presupposition that Mike washed more than one window.

The situation is different, however, if one adopts Spector's (2007) approach to the multiplicity inference. As noted above, combining Spector's account of the multiplicity inference with Katzir's approach to the adverbial modifiers leads to the clearly unattested reading, repeated below in (45), which is also in contradiction with the correct reading that we obtain via the strengthened presupposition paraphrased in (46). The question for this approach is therefore how to block the wrong reading in (45). We sketch the formal details of this issue in Appendix B.

(45)    Mike didn't wash any window with soap but he did wash exactly one window.

(46)    Mike didn't wash any any window with soap but he did wash exactly one window.

In sum, not all implicature accounts are straightforwardly compatible with the presuppositional solution sketched above and not all of them are compatible with retaining Katzir's (2007) theory of alternatives as well. In the next subsection, we will consider the inference of adverbial modifiers combined with other scalar inferences. In particular, we will show that the implicature approach to adverbial modifiers seems to give wrong results in the case of scalar adjectives but gives intuitively correct results in the case of the inference of plain disjunction and disjunction in the scope of possibility modals. The resulting picture of the interaction between scalar inferences and modifiers under negation appears therefore mixed and needs to be investigated further.

## 5.2   A comparison with other scalar terms

In this subsection, we enlarge the focus from multiplicity inferences to scalar terms more generally and how they interact with modifiers under negation.[16]

As a start, consider a sentence like (47-a) in which a scalar adjective appears in the type of structures considered above with a modifier under negation. Intuitively

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16 Many thanks to Benjamin Spector (p.c.) for very helpful discussion on the issues touched upon in this section.

(47-a), if anything, suggests (47-b). However, the implicature approach to the inference of adverbial modifiers predicts the opposite inference, namely that Patricia gave Peter an excellent grade. The same considerations as before makes it so that we cannot derive this implicature locally, or we would weaken the literal meaning of the sentence as well, contra intuitions.

(47)   a.   Patricia didn't give Peter a good grade gladly.
       b.   ↝ *Patricia gave Peter a good grade (but not an excellent one).*

To illustrate, consider the alternatives of (47-a) in (48) and assume a theory of implicatures in which the latter not only arise from alternatives that are stronger than the asserted sentence, but also from alternatives that are logically independent from it.[17]

(48)   a.   Patricia didn't give Peter a good grade.
       b.   Patricia didn't give Peter an excellent grade.
       c.   Patricia didn't give an excellent grade gladly.
       d.   Patricia didn't give a good grade gladly.

Since the alternatives in (48-a) and (48-b) are logically independent than the prejacent in (47-a), they will be negated leading to the inference that Patricia did give Peter an excellent grade. This might suggest that the problem we discuss above actually lies in Katzir's (2007) approach to adverbial modifiers and we should abandon or modify it independently from the case of multiplicity inferences.

On the other hand, when we look at other scalar terms, like disjunction, the implicature approach to adverbial modifiers predicts readings which appear intuitively attested. Consider first the case of the free choice inference, triggered by a disjunction embedded in the scope of a possibility modal, analysed by many as an implicature (Fox 2007, Chemla 2010, Klinedinst 2007, Franke 2011, Santorio & Romoli 2017, Bar-lev & Fox 2017 among others). (49-a) gives rise to the inference that each of the disjunct is possible: Miriam can play with her computer and she can play with her i-Pad (though possibly not with both).

(49)   Miriam can play with her computer or her i-Pad.
       ↝ *Miriam can play with her computer and she can play with her i-Pad*

Now let's consider the analogous case of the one we focused on above, involving a free choice inference and the inference of an adverbial modifier under negation, as

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17 This will need to be refined further so that the excluded alternatives are only the ones 'innocently excludable' in the sense of Fox (2007). See Fox 2007, Magri 2010 and Romoli 2012 for discussion and arguments for a theory of implicatures of this sort.

in (50):[18] While judgments are more subtle in this case, the sentence in (49) has a reading suggesting that Miriam can't play with her computer and that she cannot play with her i-Pad after dinner, but before dinner, she has free choice between the two: she can play with one and she can play with the other. That is, the free choice inference seems to arise on top of the inference of the adverbial modifiers, similarly to the reading above in which the multiplicity inference arises on top of the inference of adverbial modifiers.

(50)     Miriam can't play with her computer or her i-Pad after dinner.
         ⤳ *Miriam can't play with her computer and she can't play with her i-Pad after dinner, but before dinner she can play with one and she can play with the other*

If this is true, surprisingly, combining the implicature approach to the inference of the adverbial modifier with an implicature account of the free choice inference, predicts the reading in (50) straightforwardly. We show it in details in Appendix C using Fox's (2007) account of free choice.

Even more surprisingly, the implicature approach to the inference of adverbial modifiers predicts a conjunctive inference with a simple disjunction; an inference which is at least not obviously wrong, if not clearly intuitively attested. In other words, a sentence like (51) seems to have a reading in which Mirian did go to Paris and she did go to Berlin.

(51)     Miriam didn't go to Paris or Berlin by train.
         ⤳ *Miriam did go to Paris and she did go to Berlin (not by train).*

If this is correct, the inference above can be taken as another argument for the implicature approach to adverbial modifiers as it straightforwardly predicts this reading, as we show in Appendix C.

In sum, this subsection was more tentative and admittedly the judgments for the sentences above are delicate and need to be tested further. However, the cases involving disjunction above suggest that if we want an implicature approach to the multiplicity inference, we should model it as being more similar to the implicature approach to free choice or a disjunction. On the other hand, the case of adjectives like *good* seems to push in the opposite direction, suggesting that we should modify or abandon the implicature approach to the inference of adverbial modifiers, which doesn't make the right prediction in that case. The picture is therefore mixed and the general interaction between modifiers under negation and scalar terms needs to be explored further, this, however, goes beyond the scope of this short paper, so we

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18 Again, as in the case of the multiplicity inference and the inference of the adverbial modifiers, the intended reading is one in which *after dinner* scopes under negation.

have to leave it for future work.

## 5.3   A note on the ambiguity approach to the multiplicity inference

Another non-implicature approach to the multiplicity inference is the ambiguity approach defended in Farkas & de Swart 2010, Grimm 2013, Marti 2017. In this section, we briefly show that this approach does not provide a solution to the problem and it is not clear how to make it compatible with our first solution either.

The main idea of this approach is that a plural noun like *windows* is ambiguous between an inclusive, number-neutral interpretation which includes atomic elements and sums thereof, and an exclusive interpretation which only includes sums, as illustrated in (52) vs (53) (assuming that the set of relevant windows is $\{a,b,c\}$).

(52)    $[[\text{windows}_1]] = \{a,b,c,\{a,b\},\{a,c\},\{c,b\},\{a,b,c\}\}$    INCLUSIVE

(53)    $[[\text{windows}_2]] = \{\{a,b\},\{a,c\},\{c,b\},\{a,b,c\}\}$    EXCLUSIVE

When the denotation in (52) is selected, the reading it gives rise to is the inclusive one and can be paraphrased as in (54), while when the interpretation in (53) is selected, the exclusive reading in (55) arises. In addition, this approach assumes that a preference for the strongest reading, the one that asymmetrically entails the other, guides us in choosing between the meanings in (52) and (53).[19] Since (53) is stronger in positive cases, while (52) in negative ones, the ambiguity approach can account for the pattern of heterogeneous behaviour of the multiplicity inference across positive and negative sentences seen above.

(54)    Mike washed one or more windows.

(55)    Mike washed more than one window.

It is easy to see, however, that this approach – combined with either the implicature or presuppositional approach to adverbial modifiers – cannot account for the problematic case we are focusing on in this paper. To illustrate this, let us first assume the implicature approach to adverbial modifiers. If the number neutral, inclusive meaning of plural in (52) is selected — as it gives rise to the stronger reading under negation — then the resulting reading of (56) would be (57). The interpretation in (57) however doesn't account for the multiplicity in the inference of

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19 The Strongest Meaning Hypothesis for plural is formulated as in (i) (Farkas & de Swart 2010):

(i)    **The Strongest Meaning Hypothesis for Plurals**: for a sentence involving a plural nominal, prefer that interpretation of [Pl] which leads to the stronger overall interpretation for the sentence as a whole, unless this interpretation conflicts with the context of utterance.

the adverbial modifier. Namely, it is compatible with Mike washing just one window. While this is a possible reading of (56), it doesn't account for the reading described above.

(56)    Mike didn't wash windows with soap.

(57)    Mike didn't wash any window with soap but he did wash one or more window.

On the other hand, if nevertheless the exclusive reading of the plural in (53) is selected, the resulting reading of (56) is (58). The multiplicity inference in the inference of the adverbial modifier is now correct, but the literal meaning is too weak. Namely, it is compatible with Mike washing one window with soap which does not correspond to the native speakers' intuitions.

(58)    Mike didn't wash more than one window with soap but he did wash more than one window.

Let us now consider combining the ambiguity approach with the presuppositional analysis of the inference of adverbial modifiers. Recall that the intended reading of (56) is (59):

(59)    Mike didn't wash any window with soap but he did wash more than one window (albeit not with soap).

In order to obtain this reading, we would have to be able to select the inclusive meaning of *windows* in the asserted part and the exclusive one in the presupposed part. It is unclear to us how to do it compositionally.

More importantly, it would predict that in case of lexical ambiguity, the meaning selected for one word could be different in the assertion and the presupposition. For instance, (60) should be able to obtain the reading paraphrased in (61), in which *bank* obtains a different interpretation in the assertion and the presupposition (as the land near a river and the financial institution, respectively). This reading is however clearly unattested.

(60)    Patricia knows that the fisherman went to the bank.

(61)    Patricia believes that the fisherman went to the bank$_1$ [*as the land near the river*] and the fisherman did go to the bank$_2$ [*as the financial institution*].

In sum, the ambiguity approach cannot account for the interpretation we are after and we do not see any obvious amendment to this theory which would allow it to.[20]

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20 As discussed by Farkas & de Swart (2010), Spector (2007), Ivlieva (2013) and Križ (2017), the ambiguity approach also has problems with accounting for the multiplicity inference in sentences

# 6 Conclusion

In this paper, we have focused on the interaction between adverbial modifiers and multiplicity inferences in sentences containing a plural noun and an adverb in the scope of negation like *This morning, Mike didn't wash windows with soap*. We have shown that a reading of this sentence – entailing that Mike didn't wash any window with soap but that he did wash more than one window (albeit not with soap) – is problematic for the implicature approach to the multiplicity inference (Spector 2007, Mayr 2015, Ivlieva 2013, Zweig 2009, Sauerland et al. 2005b) in combination with the implicature treatment of the inference of adverbial modifiers (Katzir 2007). We have sketched two solutions for this problem. The first was to keep the implicature approach to the multiplicity inference but treat the inference of adverbial modifiers as a presupposition, building on proposals by Simons (2001) and Schlenker (2008), in combination with the idea that presuppositions can be strengthened via implicatures, as proposed by Spector & Sudo (2017). The second solution was keeping instead the implicature approach to adverbial modifiers but adopting a non-implicature approach to multiplicity, the homogeneity-based proposal by Križ (2017). Either way, the interaction between multiplicity and the inference of adverbial modifiers suggests that we cannot treat both as implicatures: if we want to treat either one as an implicature, we need to do something different for the other.

# 7 Appendix A: The presuppositional solution in more detail

Following Simons (2001) and Schlenker (2008), let us analyse the inference of the adverbial modifier not as an implicature but a presupposition so that (62-a) would presuppose (62-b):

(62)  a.  Mike washed windows with soap.
      b.  Mike washed one or more windows.                    PRESUPPOSITION

Since presuppositions project out of the scope of negation, (63-a) would presuppose (62-b), repeated below in (63-b):

(63)  a.  Mike didn't wash windows with soap.
      b.  Max washed one or more windows.                     PRESUPPOSITION

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with non-monotonic quantifiers, as in (i):

(i)    Exactly two students solved problems.
       → *two students solved more than one problem and all of the others didn't solve any problems*

This case is similar at a certain level of abstraction to the ones discussed in this paper.

Further, let us adopt the idea that presuppositions, as well as assertions, can come with their own implicatures (Gajewski & Sharvit 2011, Spector & Sudo 2017, Sudo & Romoli 2017). Namely, (63-b) would come with its implicature in (64), which in turn accounts for the multiplicity in the inference of adverbial modifiers.

(64)    Max washed more than one window.

In informal terms, by uttering (63-a), the speaker conveys also (63-b) as a presupposition. Since the speaker didn't convey the meaning that *Mike washed exactly one window*, which is stronger than (63-b), the listener will conclude that the speaker must believe that it is false. If the alternative is false and (63-b) is true, the result is the multiplicity inference (within the inference of the adverbial modifier):

(65)    Mike washed one more or more windows and it is not true that Mike washed exactly one window = Mike washed more than one window.

In order to capture this intuition formally, following Sudo & Romoli (2017), we could define an operator, call it $\text{EXH}_p$, which computes implicatures on the alternatives of the presupposed meaning component, as in (66) (where $\langle\langle\phi\rangle\rangle$ stands for the presupposition of $\phi$, "$Alt(\langle\langle\phi\rangle\rangle)$" is the set of alternatives of the presupposition of $\phi$ and $\phi \nsqsubseteq \psi$ means that $\psi$ is not entailed by $\phi$). The operator $\text{EXH}_p$ combines with the alternatives of the presupposition of $\phi$ and excludes those that are not entailed by the presupposition of $\phi$:[21]

(66)    a.    $[\![\text{EXH}_p(\phi)]\!]^w = [\![\phi]\!]^w$
        b.    $\langle\langle\text{EXH}_p(\phi)\rangle\rangle^w = \langle\langle\phi\rangle\rangle^w \wedge \forall \psi \in Alt(\langle\langle\phi\rangle\rangle)[\langle\langle\phi\rangle\rangle \nsqsubseteq \psi \rightarrow \neg\psi(w)]$

Let us now come back to the example in (63-a). The meaning of 'Mike didn't wash windows with soap' can be rendered formally as in (67-a) and its presupposition in (67-b).

(67)    a.    $[\![(63\text{-a})]\!] = \text{not}[(\text{wash})(\text{one or more windows with soap})(\text{Mike})]$

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21 Note that the operator we are proposing here differs slightly from the scalar presuppositional strengthening in Sudo & Romoli (2017), which in turn builds on Spector & Sudo 2017. In particular, while the operator they define operates on the alternatives of the sentence in (63-a) enriched with their presuppositions, we define an operator that works directly on the alternatives of the presupposition of (63-a). As is easy to show, while the operator in (66) can account for the original examples in Gajewski & Sharvit (2011) and Spector & Sudo (2017), the one defined in Sudo & Romoli (2017) cannot account for the multiplicity in the inference of the adverbial modifier. The definition of the operator we adopt here, however, raises a question about the nature of alternatives: if presuppositions are propositions, we are bound to consider propositional alternatives rather than alternatives as sentences, as more generally assumed. While this can be done, it gives rise to the so-called 'Symmetry' problem (see Katzir 2007 for discussion). We leave this open here, thanks to Yasu Sudo (p.c.) for discussion on this point.

  b.  $\langle\langle(63\text{-}a)\rangle\rangle = (\text{wash})(\text{one or more windows})(\text{Mike})$

The alternative to the presupposition in (67-b) is 'Mike washed exactly one window' (remaining neutral here on how this exactly-one-alternative is derived. See discussion in section 4.1.2 above).

(68)  $Alt(\langle\langle\langle\phi\rangle\rangle\rangle) =$
       {Mike washed one more more windows, Mike washed exactly one window}

Now, if we apply $\text{EXH}_p$ to (63-a) we get (69):

(69)  a.  $[\![\text{EXH}_p(63\text{-}a)]\!] =$
          $\text{not}[(\text{wash})(\text{one or more window with soap})(\text{Mike})]$
      b.  $\langle\langle\text{EXH}_p(63\text{-}a)\rangle\rangle = [(\text{wash})(\text{one or more windows})(\text{Mike})]$
          $\wedge\,\text{not}[(\text{wash})(\text{exactly one window})(\text{Mike})]$
          $= (\text{wash})(\text{more than one window})(\text{Mike})]$

This correctly derives the intended meaning that Mike didn't wash any window with soap but he did wash more than one window.

## 8  Appendix B: An over-generation problem

As mentioned, combining Spector's (2007) theory of the multiplicity inference with Katzir's (2007) theory of the inference of adverbial modifiers predicts a not attested reading for the sentences we have focused on in this paper. We turn to illustrate this in detail, starting by summarising the gist of Spector's (2007) proposal.

   Consider how in Spector 2007, the multiplicity inference of a basic case like (70) in (71) arises.

(70)  Mike washed windows.

(71)  ↝ *Mike washed more than one window*

Spector's idea is that (71) comes about once (70) is compared to its singular counterpart in (72) together with its own scalar implicature that Mike didn't wash two windows.

(72)  Mike washed a window.

In other words, (70) gives rise to a multiplicity inference as a higher-order/recursive scalar implicature through competing with (72) as enriched with its scalar implicatures, which effectively makes it equivalent to (73).

(73)  Mike washed exactly one window.

More schematically, the idea is that (72) is compared to (74), giving rise to the enriched meaning in (75)

(74)    Mike washed (at least) two windows.

(75)    Mike washed a window and it's not true that he washed two windows = Mike washed exactly one window.

Then (70) is compared to (75) and (75) ends up being negated, with the resulting meaning being (76).

(76)    Mike washed windows and it's not true that he washed exactly one = Mike washed more than one window.

More schematically let $\boxed{\text{SG}}$ stand for (72), $\boxed{\text{PL}}$ for (70) and $\boxed{\text{TWO}}$ for (74). The basic assumptions are outlined below:

(77)    $\text{SG} = \text{PL}$                                   plural and singular are equivalent

(78)    $\text{TWO} \Rightarrow \text{SG/PL}$                         (at least) two entails both

(79)    $\langle \text{SG}, \text{TWO} \rangle$             (at least) two is an alternative of the singular

(80)    $\langle \text{SG}, \text{PL} \rangle$                    the singular and the plural are alternatives

Also assume for concreteness that implicatures come about through the result of an exhaustification process EXH (Chierchia, Fox & Spector 2012, Fox 2007 among many others). Note though that any theory that allows implicatures to be recursive would work here. The operator EXH can be defined as in (81)— it asserts a proposition $\phi$ and negates all of its alternatives that are not entailed by it.[22]

(81)    $[[\text{EXH } \phi]]^w = [[\phi]]^w \wedge \forall \psi \in \text{EXCL}(\phi, Alt(\phi))[\phi \not\subseteq \psi \rightarrow \neg[[\psi]]^w]$

We can now show the following:

(82)    $\text{EXH}(\text{PL}) = \text{PL}$                          the alternative SG can't be negated

(83)    $\text{EXH}(\text{SG}) = \text{SG} \wedge \neg\text{TWO}$              the enriched meaning of the singular

(84)    $\text{EXH}(\text{EXH}(\text{PL})) = \text{PL} \wedge \neg(\text{SG} \wedge \neg\text{TWO}) = \text{TWO}$
        the plural and the negation of the enriched singular = multiplicity inference

In sum, the derivation of the basic case in (85), over the set of alternatives in (86) and (87) is as follows:[23]

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22 This definition of EXH needs to be refined with the notion of 'innocent exclusion', but it is sufficient as it is for our purposes; see Fox 2007 for discussion.

23 Where the alternatives for EXH(SG) are {SG, TWO}.

(85)     $\text{EXH}_1\big[\text{EXH}_2[\text{PL}]\big] = \text{PL} \wedge \neg(\text{SG} \wedge \neg\text{TWO}) = \text{TWO}$

(86)     $\text{Alt}_2 = \{\text{PL}, \text{SG}\}$

(87)     $\text{Alt}_1 = \left\{ \begin{array}{l} \text{EXH}_2(\text{PL}) = \text{PL} \\ \text{EXH}_2(\text{SG}) = \text{SG} \wedge \neg\text{TWO} \end{array} \right\}$

Now let's go back to the sentence in (88) with the reading in which it implies that he did wash more than one window (not with soap).

(88)     Mike didn't wash windows with soap.

Consider now what happens if we recursively exhaustify above negation (where ADV is the adverbial modifier, and the rest is as above).

(89)     $\text{EXH}_1\big[\text{EXH}_2[\neg[\text{PL ADV}]]\big]$

(90)     $\text{Alt}_2 = \{\neg[\text{PL ADV}], \neg[\text{SG ADV}], \neg[\text{PL}], \neg[\text{SG}]\}$

(91)     $\text{EXH}_2\big[\neg[\text{PL ADV}]\big] =$
         $\neg[\text{PL ADV}] \wedge \text{PL} \wedge \text{SG} =$
         $\neg[\text{PL ADV}] \wedge \text{PL}$

So after the first EXH we only obtain the weak inference that he did wash one or more windows (not with soap). Consider what happens with the second EXH over the exhaustified alternatives below in (92).

(92)     $\text{Alt}_1 = \left\{ \begin{array}{l} \text{EXH}_2(\neg[\text{PL ADV}]) = \neg[\text{PL ADV}] \wedge \text{PL} \\ \text{EXH}_2(\neg[\text{SG ADV}]) = \neg[\text{SG ADV}] \wedge \text{TWO} \\ \text{EXH}_2(\text{PL}) = \text{PL} \\ \text{EXH}_2(\text{SG}) = \text{SG} \end{array} \right\}$

The second alternatives is excludable, so we negate it and we have:[24]

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24 Where the exhaustified singular alternative is obtained as below:

(i)      $\text{EXH}_2(\neg[\text{SG ADV}]) = \neg[\text{SG ADV}] \wedge \text{TWO}$

(ii)     $\text{Alt} = \left\{ \begin{array}{l} \neg[\text{SG ADV}] \\ \neg[\text{TWO ADV}] \\ \neg[\text{SG}] \\ \neg[\text{TWO}] \end{array} \right\}$

This by itself also doesn't seem right: a sentence with the singular like (iii), doesn't seem to ever give rise to the inference that Mike did wash at least two windows (not with soap).

(iii)    Mike didn't Wash a window with soap.

(93)  $\text{EXH}_1\big[\text{EXH}_2[\neg[\text{PL ADV}]]\big] =$
$\neg[\text{PL ADV}] \wedge \text{PL} \wedge \neg\big[\neg[\text{SG ADV}] \wedge \text{TWO}\big] = \neg[\text{PL ADV}] \wedge \text{PL} \wedge \neg\text{TWO}$

That is we obtain the reading that Mike didn't wash windows with soap and he did wash exactly one window(!) (not with soap). In other words, not only do we not obtain the inference we want but we obtain an opposite unattested reading.

## 9   Appendix C: Comparison with disjunction

Consider first the derivation of free choice in Fox 2007. The basic assumptions are EXH, defined as above, and the alternatives of a disjunction under a possibility modal $\Diamond(A \vee B)$ to be as in (94).

(94)  $\text{Alt} = \left\{ \begin{array}{l} \Diamond(A \vee B) \\ \Diamond A \\ \Diamond B \\ \Diamond(A \wedge B) \end{array} \right\}$

The basic free choice case from a sentence like (95) is derived as in (96), over the alternatives in (97) and (98).

(95)  Miriam can play computer or i-Pad.
$\rightsquigarrow$ *Miriam can play computer and can play i-Pad*

(96)  $\text{EXH}_1\big[\text{EXH}_2[\Diamond[A \vee B]]\big] =$
$\Diamond(A \vee B) \wedge \neg \Diamond(A \wedge B) \wedge \neg(\Diamond A \wedge \neg \Diamond B) \wedge \neg(\Diamond B \wedge \neg \Diamond A) =$
$\Diamond(A \vee B) \wedge \neg \Diamond(A \wedge B) \wedge \Diamond A \wedge \Diamond B)$

(97)  $\text{Alt}_2 = \left\{ \begin{array}{l} \Diamond(A \vee B) \\ \Diamond A \\ \Diamond B \\ \Diamond(A \wedge B) \end{array} \right\}$

(98)  $\text{Alt}_2 = \left\{ \begin{array}{l} \text{EXH}_2(\Diamond(A \vee B)) = \Diamond(A \vee B) \wedge \neg \Diamond(A \wedge B) \\ \text{EXH}_2(\Diamond A = \Diamond A \wedge \neg \Diamond B \\ \text{EXH}_2(\Diamond B) = \Diamond B \wedge \neg \Diamond A \\ \text{EXH}_2(\Diamond(A \wedge B)) = \Diamond(A \wedge B) \end{array} \right\}$

Now, let us go back the sentence in (99) schematised as in (100).

(99)  Miriam can't play computer or i-Pad after dinner.

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(iii) might be taken as a further problem for Spector (2007) or it might be taken to suggest that we should rethink what the inferences of scalar terms interacting with modifiers under negation are. See section 5.2 for discussion.

⤳ *Miriam can play computer and can play i-Pad before dinner*

(100)     $\neg[\diamond(A \vee B)\ \text{ADV}]$

The desired reading can now be obtained just one round of exhaustification as shown in (101) over the alternatives in (102) (ignoring the conjunctive alternatives here just for simplicity).[25]

(101)     $\text{EXH}_1[\neg[\diamond(A \vee B)\ \text{ADV}]] = \neg[\diamond(A \vee B)\ \text{ADV}] \wedge \diamond A \wedge \diamond B$

(102)     $\text{Alt}_1 = \left\{ \begin{array}{l} \neg[\diamond(A \vee B)\ \text{ADV}] \\ \neg[\diamond(A)\ \text{ADV}] \\ \neg[\diamond(B)\ \text{ADV}] \\ \neg[\diamond(A \vee B)] \\ \neg[\diamond(A)] \\ \neg[\diamond(B)] \end{array} \right\}$

Analogously in the case of simple disjunction, (103-a) obtains the reading in (103-b):

(103)     a.     Miriam didn't go to Paris or Berlin by train.
          b.     ⤳ *Mirian didn't go to Paris by train and she didn't go to Berlin by train but she did go to Paris and she did go to Berlin*

As before, the reading we are after is derived after one round of exhaustification, as demonstrated in (104), where both ¬A and ¬B are excludable.

(104)     $\text{EXH}_1[\neg[(A \vee B)\ \text{ADV}]] = \neg[(A \vee B)\ \text{ADV}] \wedge A \wedge B$

(105)     $\text{Alt}_1 = \left\{ \begin{array}{l} \neg[(A \vee B)\ \text{ADV}] \\ \neg[A\ \text{ADV}] \\ \neg[B\ \text{ADV}] \\ \neg[A \vee B] \\ \neg[A] \\ \neg[B] \end{array} \right\}$

## 10  Appendix D: Combining alternatives

Given the cases discussed above, one may think that doing the same in the multiplicity inference case, would be a solution. That is, one should allow the alternatives of the modifier, the singular and the plural to combine as we did for those of the disjunction and those of the modifier above.

And it is in fact true that this would be a solution under Spector's (2007) ap-

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25 In fact, adding the conjunctive alternatives give rise to a even stronger reading that Miriam can play both computer and i-Pad before dinner.

proach. However, we would lose an account of the basic case. To illustrate, consider first that if we were to revise the assumptions about alternatives in Spector 2007, so that now the plural has also TWO among its alternatives as in (106), we could then derive the desired reading.

(106)    ⟨SG, PL, TWO⟩                          the singular and the plural are alternatives

The derivation on the set of alternatives below, which combine those of the plural and those of the singular, that we are now assuming are altogether is shown below: now after only one round of exhaustification we would obtain the desired reading.

(107)    $\text{EXH}_1\big[\neg[\text{PL ADV}]\big] = \neg[\text{PL ADV}] \wedge \text{PL} \wedge \text{SG} \wedge \text{TWO} = \neg[\text{PL ADV}] \wedge \text{TWO}$

(108)    $\text{Alt}_1 = \left\{ \begin{array}{c} \neg[\text{PL ADV}] \\ \neg[\text{SG ADV}] \\ \neg[\text{TWO ADV}] \\ \neg[\text{PL}] \\ \neg[\text{SG}] \\ \neg[\text{TWO}] \end{array} \right\}$

However, if we were to allow that, in the normal positive case, we would predict the opposite inference of the multiplicity inference: That is, we would predict that (109) would give rise to the inference that Mike washed exactly one window, rather than the desired inference that he washed more than one window.

(109)    Mike washed windows.

This is of course the reason why Spector (2007) divided the alternatives of the plural and the singular in the first place. To illustrate, consider the basic case again but now combining the alternatives as above:

(110)    $\text{EXH}_1\big[\text{PL}\big] = \text{PL} \wedge \neg \text{TWO}$

(111)    $\text{Alt}_1 = \left\{ \begin{array}{c} \text{PL} \\ \text{SG} \\ \text{TWO} \end{array} \right\}$

The challenge for this strategy would then be to explain why the alternatives of the plural and the singular can be combined in the negated adverbial case but not in the normal basic case.

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