Partially Ordered Categories: Optionality, Scope, and Licensing COMMENTS WELCOME

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Abstract

This paper uses a partially ordered set of syntactic categories to accommodate optionality and licensing in natural language syntax. A complex but well-studied data set pertaining to the syntax of quantifier scope and negative polarity licensing in Hungarian is used to illustrate the proposal. The presentation is geared towards both linguists and logicians. The main ideas can be implemented in different grammar formalisms; in this paper the partial ordering on categories is given by the derivability relation of a calculus with residuated and galois-connected unary operators.

1 Introduction

Among the basic issues that all syntactic theories have to deal with are the following:

(1) Expressions often specify not only the broad categorial status of the expressions that they combine with, but also a particular subcategory. E.g., while *will be hungry* combines with any noun phrase subject, *are hungry* requires one in the plural.

(2) Some expressions have a fixed position in the sentence, but their presence or absence is optional; negation is an example. Negation is not iterable, and so *not walk* must be categorially distinct from *walk*. It is remarkable that despite this fact the English modal auxiliaries apparently recognize that they are getting the desired complement whether it is of the form *walk* or of the form *not walk*.

(3) Some expressions that have a sentential status are nevertheless ungrammatical and need a licensor for some of their components. E.g., *Mary drank any more wine* is a sentence as far as theta-role assignment and

inflection are concerned, but it requires the addition of a decreasing expression to be grammatical, e.g. *Not that Mary drank any more wine*, *whether Mary drank any more wine*.

The problem in (1) requires fine-tuning the system of syntactic categories. Bernardi (2002) proposes to assimilate (2) and (3) to (1). In a nutshell, the basic idea is that each syntactic category is a partially ordered set of subcategories. Some subcategories go back to traditionally recognized features, such as [+singular] and [+plural]. The ordering relation $\leq$ will specify, then, that *Singular Noun Phrase* $\leq$ *Noun Phrase* (i.e. every expression in the category Singular Noun Phrase is also in the category Noun Phrase) and *Plural Noun Phrase* $\leq$ *Noun Phrase* (i.e. every expression in the category Plural Noun Phrase is also in the category Noun Phrase). Singular Noun Phrase and Plural Noun Phrase are not ordered with respect to each other, that is, $\leq$ may indeed be a genuinely partial (not total) ordering. The expression *they* belongs to the category Plural Noun Phrase. This categorization immediately explains that *they* can be the subject of *are hungry*, and together with the ordering *Plural Noun Phrase* $\leq$ *Noun Phrase* it explains that *they* can also be the subject of *will be hungry*.

To extend this method to the problem of how complement selection deals with optionality, Bernardi makes use of non-traditional subcategories. In this introductory section we invent ad hoc speaking names. Assume a partially ordered set

$$\langle\{\text{Verb Phrase, Unmodified Verb Phrase, Negated Verb Phrase}\}, \leq\rangle$$

such that *Unmodified Verb Phrase* $\leq$ *Verb Phrase* and *Negated Verb Phrase* $\leq$ *Verb Phrase*. English *not* selects for an Unmodified Verb Phrase as its complement and yields a Negated Verb Phrase. The modal *may* selects for a Verb Phrase. If *walk* is an Unmodified Verb Phrase then these categories, together with the ordering relation, explain that *not* occurs right before *walk*, and *may* takes either *walk* or *not walk* as its complement.

Turning to the licensing issue, any expression in which the theta-criterion and inflectional requirements are satisfied is a kind of sentence (say, it is of type t). However, such sentences may still be lacking in some crucial respect. Assume the following partially ordered set (the category labels are, again, ad hoc speaking names):

$$\langle\{\text{Complete Sentence, Incomplete Sentence, Good-enough Sentence}\}, \leq\rangle$$

with the following ordering relation: *Good-enough Sentence* $\leq$ *Incomplete Sentence* and *Good-enough Sentence* $\leq$ *Complete Sentence*. Crucially, *Incomplete Sentence* $\not\leq$ *Complete Sentence*. The rest of the grammar will ensure that *Mary drank a glass of wine* is a Good-enough Sentence, and *Mary drank any more wine* is an Incomplete Sentence. *Whether* and *not that* select for an Incomplete Sentence and yield a Complete one. Notice that they are free to combine with either one of our two specimens, given the ordering *Good-enough Sentence* $\leq$

Incomplete Sentence. On the other hand, whereas the ordering relation tells us that *Mary drank a glass of wine*, by itself, is not only a Good-enough Sentence but also a Complete one, *Mary drank any more wine* must combine with *whether* or *not that* to be part of a Complete Sentence. *Whether* and *not that* act as licensors precisely because they can take a complement that is not Good-enough on its own and turn it into a Complete Sentence. – This type of account generalizes to other licensing relations.

To summarize, the same tool – a partially ordered set of subcategories – accounts for fine-grained complement selection, the viability of optional elements, and licensing. This is the core of Bernardi's proposal, which can be compared with and implemented in many different grammar formalisms.

The type-logical approach views grammar as a logic: its axioms are the representations of lexical items and its theorems are the representations of sentences. Bernardi (2002) develops her proposal in this setting, relying specifically on innovations in Kurtonina and Moortgat (1995). In type-logical grammar, categories are labeled with logical formulae. Then, each partially ordered set of subcategories is a set of formulae with a derivability relation on it. In other words, the exotic-looking (4) is traded for the more austere (5).

(4) $\langle\{\text{Complete Sentence, Incomplete Sentence, Good-enough Sentence}\}, \leq\rangle$

(5) $\langle\{\psi_1, \psi_2, \psi_3\}, \vdash\rangle$, where $\vdash$ is the derivability relation of a particular calculus

With category labels as formulae, the ordering sheds its ad hoc character.

This paper has three goals.

(A) Present Bernardi's theory in a way that is accessible to linguists whose home theory is not type-logical grammar. This joint project has also led to some extensions of the logical aspects of the theory; these will be folded in.

(B) Argue in detail that the use of a partially ordered set of categories offers a natural solution to the problems of optionality and licensing.

(C) Illustrate and test the working of the theory with a fairly complex set of empirical data pertaining to Hungarian quantifiers. The surface left-to-right order of quantifier phrases in Hungarian largely mirrors their scope order and thus the language makes the syntax of scope directly observable. Another important property of Hungarian is that linear order is determined not by grammatical function but by quantifier class (group denoting, distributive, counting, negative concord, etc. quantifiers). In terms of Minimalism, in this case study quantifier scoping represents the checking of interpretable features in overt syntax (Chomsky 1995).

2 The grammar

2.1 Proof theoretical approach

As was mentioned in the introduction, the central idea to be explored here is that the set of syntactic categories is partially ordered. The ordering could be simply stipulated and would still have an advantage over theories that assume no such ordering. However, Bernardi (2002) also showed that certain commonalities of the patterns can be elegantly captured if the ordering is given by the derivability relation of a particular calculus. This is why we present the proposal in a proof theoretical format.

The proof theoretical approach to syntax presents syntax as a calculus, where the syntactic category labels assigned to lexical expressions are the axioms and the syntactic category labels derived for complex expressions – sentences among them – are the theorems.[1] Our case study focuses on the behavior of quantifiers and negation, so we do not need to talk about how simple sentences are built. We only work with two kinds of expressions: sentences and sentential operators. The discussion will be framed within a novel version of the Lambek calculus. Lambek's (1958) idea was to take syntactic category labels to be formulae of a propositional calculus with just material implication $\rightarrow$, notated in categories as $/$. The pertinent inference rules of this simple calculus are the ones corresponding to modus ponens (the elimination of $\rightarrow$) and conditionalization (the introduction of $\rightarrow$).

An expression of category y/x followed by an expression of category x forms an expression of category y. Compare:

$$\begin{array}{rl} & x \rightarrow y \\ & x \\ \therefore & \mathrm{y} \end{array}$$

If an expression is of some category z such that when followed by another expression of category x, they form an expression of category y, then the category z derives y/x. Compare:

$$\text{if} \quad \begin{array}{rl} & x \\ & z \\ \therefore & y \end{array} \quad \text{then} \quad \begin{array}{rl} & z \\ \therefore & x \rightarrow y \end{array}$$

The category of sentences will be informally notated as s and that of sentential operators as s/s. The argument category is under the slash; i.e. s/s is $s_{value}/s_{argument}$. The following Hungarian examples show the working of negation, *nem* as s/s:

[1] Syntactic categories are sets of expressions: those expressions that belong to the given category. VP, $e\backslash t$, etc. are category labels: names for such sets. This distinction is important to bear in mind when one talks about categories as formulae, although the literature is often sloppy about it and we will also take the liberty to sometimes use the term "category" to refer to a category label.

(6) a. nem 'not' is an expression of category s/s.
b. ég a ház is an expression of category s.
burns the house
'The house is on fire'
c. nem ég a ház is an expression of category s.
not burns the house
'The house is not on fire'

This framework easily accommodates subcategories and their relations, which are at the heart of the enterprise. We need multiple subcategories of s. Following Kurtonina and Moortgat (1995), we consider an extension of this propositional calculus. The different subcategories of the same category will be labeled with different tensed modal logical formulae. E.g., p and $\Box\Diamond p$, which are both propositions, will be used to label two distinct subcategories of sentences (s2 and s3 of Figure 1 below). The fact that p derives $\Box\Diamond p$ will correspond to the fact that every expression that belongs to the (sub)category p also belongs to the (sub)category $\Box\Diamond p$.

It is worth pausing here for a moment and stress that this syntax is a calculus in which derivability amounts to the following:

(7) If A, B are formulae serving as category labels, and in the calculus A derives B, then every expression that belongs to category A also belongs to category B.

That is, the derivability relation between category labels corresponds to a subset relation between the sets of expressions bearing those labels:

$$A \longrightarrow B \text{ iff } [\text{expressions labeled } A] \subseteq [\text{expressions labeled } B]$$

A logic is a calculus plus a model theory. Linguists are most accustomed to logics in which the models contain individuals, events, worlds, etc. In those cases if p derives q, then every world in which p is true is also one in which q is true. Our calculus has an associated model theory and so questions of its soundness and completeness can be raised, but the models contain linguistic expressions, and the derivability relation between category labels says something about the syntactic behavior of the expressions, as in (7), not about their meanings.[2]

2.2 Functional Application, Scope, and Intervention

Recall problem (1) mentioned in the introduction: the verb phrase *will be hungry* combines with any noun phrase as its subject, but *are hungry* requires one in the plural, and *is hungry* requires one in the singular. This can be handled with the following assumptions:

she $\in$ Singular Noun Phrase, *they* $\in$ Plural Noun Phrase

[2] An overview of the framework more suitable to linguists is Moortgat (2002), whereas logicians are referred to Moortgat (1997).

will be hungry wants an argument of category Noun Phrase
are hungry wants an argument of category Plural Noun Phrase
is hungry wants an argument of category Singular Noun Phrase
Singular Noun Phrase $\leq$ Noun Phrase
Plural Noun Phrase $\leq$ Noun Phrase

Will be hungry can take either *she* or *they* as an argument. While these expressions do not receive the category label Noun Phrase in the lexicon, the grammar tells us that their lexical category labels derive Noun Phrase. In general,

(8) An expression of category A/C combines with an expression of category B as an argument iff B derives C.[3]

From our perspective scope taking can be reduced to functional application. A quantifier phrase like *every man* that denotes a generalized quantifier of type $\langle\langle e,t\rangle,t\rangle$ syntactically combines with its scope by Montague's Quantifying-in rule or some reincarnation thereof. Its interaction with other operators is determined by what sentential (sub)category it can be quantified into (its argument category), and what sentential (sub)category it feeds to higher operators (its value category).

We can schematically think of the category of operators, including quantifier phrases as s_{val}/s_{arg}.[4] As will be seen below, this is especially straightforward in Hungarian, where quantifier phrases in the preverbal field line up in their scopal order, rather than stay in subject, object, etc. position as in English. The generalization in (8) simply extends to quantification, as in (9).

(9) A quantifier phrase (operator) of category s_{val}/s_{arg} takes immediate scope over a syntactic domain of category s_x iff s_x derives s_{arg}.

Crucially to the solution of problems (2) and (3), recognizing the derivability (inclusion) relations between categories offers an account of when the intervention of some sentential operator OP_b between OP_a and OP_c is optional or obligatory.[5]

Given the total ordering and the category assignment as in (10), the left-to-right order of the operators follows:

(10) $s1 \longrightarrow s2 \longrightarrow s3$

$$\begin{array}{ccccc} OP_3 & > & OP_2 & > & OP_1 \\ s3/s3 & & s2/s2 & & s1/s1 \end{array}$$

[3] (8) amounts to saying that a function category is always order reversing with respect to the category selection (not the denotation!) of its argument: If $B \leq C$, i.e. every expression in B is also in C then, if a function category F combines with elements of C as an argument, it also combines with elements of B as an argument.

[4] The reader interested in an in-depth formal presentation of the treatment of QPs in categorial type logic is referred to Moortgat (1997).

[5] In what follows, $X > Y$ notates "X precedes and/or scopes over Y", $X \longrightarrow Y$ notates "X derives Y". In this paper the $\leq$ notation is reserved for the pretheoretical, informal notion of an ordering relation.

By transitivity, $s1 \longrightarrow s3$, viz. OP_3 can also scope over OP_1 directly. OP_2 is optional.

$$\begin{array}{lcl} OP_3 & > & OP_1 \\ s3/s3 & & s1/s1 \end{array}$$

If the ordering relation is not total but partial, as in Figure 1, then in (11) OP_7 may only scope over OP_5 if OP_6 intervenes and bridges between OP_5's value category and OP_7's argument category. For instance, given the derivability relations in Figure 1 and OP_7 and OP_5 of category $\cdot/s4$ and $s2/\cdot$, respectively, OP_7 can precede OP_5 only if operator OP_6 of category $s4/s2$ mediates, because $s2 \not\longrightarrow s4$.[6]

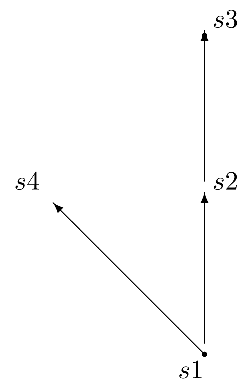


Figure 1: Partial order of sentential subcategories.

Hence,

$$(11) \quad \begin{array}{lclclcclcl} OP_7 & > & OP_6 & > & OP_5 & \quad \text{but} \quad & {}^*OP_7 & > & OP_5 \\ \cdot/s4 & & s4/s2 & & s2/\cdot & & \cdot/s4 & & s2/\cdot \end{array}$$

The bridging between two categories that the ordering relation does not relate to each other is crucial for the account of many phenomena, among others, NPI-licensing.

2.3 Syntactic Calculus

Our framework is a Categorial Grammar (CG) known as Categorial Type Logic (CTL).[7] It consists of (i) the logical rules of binary operators[8] and (ii) the logical rules of unary operators.

[6]The dot acts as a placeholder in the presence of syncategorematic operators.

[7]Alternative names are Type Logical Grammar, see for instance Morrill (1994), and Multimodal Categorial Grammar (Moortgat and Oehrle 1994).

[8]The binary operators are $\backslash, \bullet, /$. For ease of exposition we will focus only on $/$ and $\bullet$.

The rules of the binary operator / are the same as the introduction and elimination rules of the propositional calculus $\rightarrow$, see the standard deduction theorem:

$$\Gamma \bullet p \vdash q \text{ iff } \Gamma \vdash q/p$$

In words: Γ concatenated with p belongs to the category q if and only if Γ belongs to the category q/p. The $\bullet$ indicates the concatenation of structures.

The relation above between the $\bullet$ and the / is known in algebra as the residuation principle. The $\bullet$ and the / form a residuated pair in the same way as addition and subtraction or multiplication and division do. Recall how one solves an algebraic equation like $3 \times x \leq 5$ by isolating the unknown x using the law connecting $(\times, \div)$ and producing the solution $x \leq \frac{5}{3}$. The law connecting these two binary (residuated) operators says:

$$x \times y \leq z \text{ iff } x \leq \frac{z}{y}$$

In CTL, such a pair of operators is used to put together and take apart linguistic expressions as sketched in Section 2.1.

It follows from residuation that A/C is order reversing (with respect to category selection) in its argument position (C), and order preserving in its value position (A). If B derives C, then A/C is also happy with B as an argument; if A derives D, then A/B also counts as D/B. This is formally represented by the inferences below.

(12) $$\frac{B \longrightarrow C}{A/C \longrightarrow A/B} \qquad \frac{A \longrightarrow D}{A/B \longrightarrow D/B}$$

The residuated unary operators, to which we now turn, will serve to create a fine grained partial order of categories. We show that the partial order among the sentential subcategories required to control scope and word order can be encoded as the derivability relation driven by residuated unary operators.

Kurtonina and Moortgat (1995) further explored the space of the Lambek calculus by exploiting unary operators inspired by tense logic. The idea of this line of research is to take the *minimum logic*, i.e. a logic characterized by those properties that are at the core of any logic (namely, transitivity of the derivability relation and upward/downward monotonicity of operators) as a starting point to analyze linguistic universals, and then extend its language so as to increase its expressivity and analyze linguistic structures and cross-linguistic variation.[9]

The past possibility and future necessity operators of tensed modal logic have just the core properties. That is, they obey the algebraic principle of residuation introduced above:

$$PastPossA \longrightarrow B \text{ iff } A \longrightarrow FutNecB$$

[9]Of course a more basic question is the identification of **the** minimum logic.

Following Kurtonina and Moortgat (1995), we fashion our residuated pair of unary operators after these and notate them as $\Diamond$ and $\Box$. These symbols are hijacked for typographical convenience and must not be confused with the standard modal operators, which form a pair of duals and not a pair of residuals.[10] Thus, in the notation to be used below:[11]

(13) $\Diamond A \longrightarrow B$ iff $A \longrightarrow \Box B$

The properties below follow from (13); see details in Appendix A

1. $\Diamond\Box A \longrightarrow A$ [Unit]
2. $A \longrightarrow \Box\Diamond A$ [Co-unit]
3. $\Diamond A \longrightarrow \Diamond B$, if $A \longrightarrow B$ [$\Diamond$ upward monotonic]
4. $\Box A \longrightarrow \Box B$, if $A \longrightarrow B$ [$\Box$ upward monotonic]
5. $\Diamond\Box A \longrightarrow \Diamond\Box B$, if $A \longrightarrow B$ [$\Diamond\Box$ upward monotonic]
6. $\Box\Diamond A \longrightarrow \Box\Diamond B$, if $A \longrightarrow B$ [$\Box\Diamond$ upward monotonic]

In this paper, we use the $\Diamond, \Box$ operators as *decorations on sentential categories*. The derivability relation among decorated categories defines a partial order. As in Bernardi (2002), that partial order will be used to express the fine grained partial ordering among sentential categories that is necessary to capture the differential scoping abilities of quantifier phrases. Section 4 will illustrate this with Hungarian material.

The last feature of the calculus that needs to be introduced here is the availability of multiple modes for the unary operators. There are various linguistic applications of multimodality in CTL, some of them quite different from our own application.[12] In this paper we will use just two modes, one notated with empty $\Diamond, \Box$ and the other with filled $\blacklozenge, \blacksquare$. The two modes will add further flexibility to the logic whose derivability relation formalizes the partial ordering of sentential categories.

The consequences of residuation listed above hold for unary operators of the same mode. Distinct modes do not mix, i.e. there is no law that derives anything from $\Diamond\blacksquare A$. On the other hand, the same Co-unit property that gives $\Diamond\Box s \longrightarrow \Box\Diamond\Diamond\Box s$ also derives $\blacklozenge\blacksquare s \longrightarrow \Box\Diamond\blacklozenge\blacksquare s$. Likewise, the Unit propety that gives $\Diamond\Box\Box\Diamond s \longrightarrow \Box\Diamond s$ also produces $\blacklozenge\blacksquare\Box\Diamond s \longrightarrow \Box\Diamond s$. This means that several, so to speak, parallel paths may be constructed from one element of the partially ordered set to another: one involving only operators in the empty mode, another involving both empty and filled ones, etc. The role of the two modes will become clear in Section 4.

[10] Past possibility and past necessity (as well as future possibility and future necessity) are duals, whereas, past possibility and future necessity are residuals.

[11] Roughly, the $\Diamond$ is a unary $\bullet$ and the $\Box$ is a unary implication, viz. take "$\Diamond\cdot$" to be "$\cdot \bullet p$" and "$\Box\cdot$" to be "$\cdot/p$".

[12] See Heylen (1999) for a detailed study of the use of unary operators to encode feature structure information.

3 Quantifier order and scope in Hungarian, and some questions that they raise

3.1 A bird's eye view

The syntax of scope in Hungarian will serve as our testing ground. Our interest is not in the Hungarian operators per se, but rather in the fact that (i) they illustrate a case where the surface syntactic distribution of expressions depends on their interpretable features, and (ii) they are numerous enough to give rise to rather complex interactions.

To a significant extent, the syntax of scope is **the** syntax of Hungarian: the left-to-right order of operators in the preverbal field unambiguously determines their scopal order. Another remarkable property is that the possible orders are determined by quantifier class and not by grammatical function. Thus, the examples in (14) illustrate the fact that a distributive universal must precede a counting quantifier with *kevés* 'few', irrespective of which is the subject and which is the direct object and, given their order, the former inescapably outscopes the latter.[13]

(14) a. Minden doktor kevés filmet látott.
every doctor-nom few film-acc saw
'Every doctor saw few films', viz. $\text{every}_{\text{Subject}} > \text{few}_{\text{Object}}$

b. Minden filmet kevés doktor látott.
every film-acc few doctor-nom saw
'Few doctors saw every film', viz. $\text{every}_{\text{Object}} > \text{few}_{\text{Subject}}$

c. *Kevés doktor minden filmet látott.
few doctors-nom every film-acc saw

d. *Kevés filmet minden doktor látott.
few film-acc every doctor-nom saw

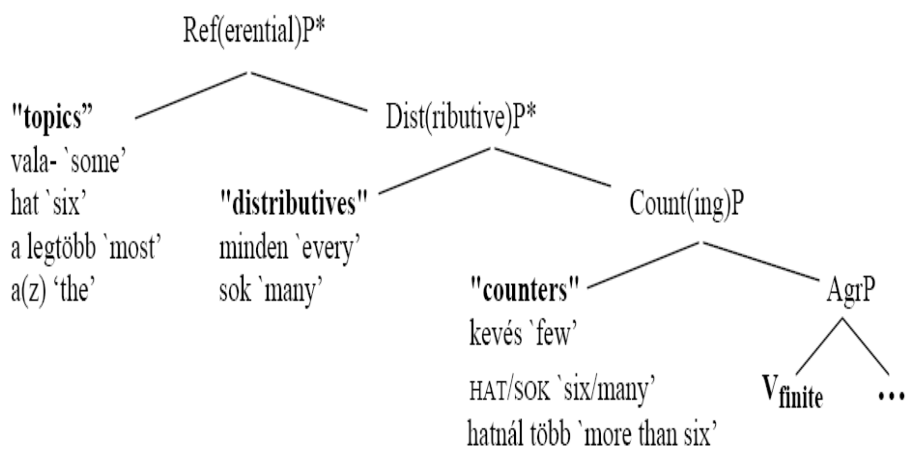
A common way to capture these facts has been to assume that operators move into designated positions in the manner of wh-movement, and their left-to-right order translates into a quantifying-in hierarchy. This assumption differs from Fox-Reinhart style Interface Economy, according to which quantifier scope is assigned by Quantifier Raising, an adjunction operation that applies only when it makes a truth conditional difference (Fox 1999; Reinhart 2006). Notice, however, that both Fox and Reinhart concern themselves with a case where scope assignment has no effect on surface constituent order: covert scope shifting in English. The case of Hungarian is different: quantifier phrases in Hungarian occur in the positions to be discussed below irrespective of whether this has a disambiguating effect. Even if one were to ignore the fact that left-to-right order determines interpretive order, the syntax of Hungarian would have to account

[13] We draw directly from the results of Szabolcsi (1997, 1981), Brody and Szabolcsi (2003), É. Kiss (1987, 1991, 1998, 2002), Puskas (2000), Horvath (2000, 2006), Hunyadi (1999), and Surányi (2003).

for the fact that certain word orders are grammatical and others are not.

The following diagram illustrates three of the relevant positions with their characteristic inhabitants. For space reasons only the determiners are included. Some though not all quantifier phrases may occur in more than one position and their interpretations vary accordingly. An example in (15) is *sok* ‘many’. When *sok ember* ‘many men’ occurs in the “counter” position, which is the only possible position of *hatnál több ember* ‘more than six men’, it supports both distributive and collective readings, but when it occurs in the ”distributive” position, which is the only possible position of *minden ember* ‘every man’, the collective interpretation is not available. Such matters are discussed in detail in Szabolcsi (1997).

(15)



The filling of each of these positions is optional; however, all the positions can be filled simultaneously. RefP and DistP are recursive (cf. the Kleene stars), subject to the same “left-to-right order determines scope” rule. Preverbal operators normally outscope all postverbal ones (those occur in the . . . part of (15)). Therefore, a counter gets a chance to outscope a distributive quantifier if the latter occupies a postverbal position.[14]

Postverbal quantifier order is virtually free. É. Kiss (1998) and Brody and Szabolcsi (2003) argue however that the sequence of operator positions observed preverbally reiterates itself in the postverbal field. The impression of postverbal order freedom is due to the fact that of the inflectional heads that separate the

[14]Inverse scope, i.e. one that does not match left-to-right order and where, specifically, a postverbal operator outscopes preverbal ones, is possible in two cases: (i) with a postverbal specific indefinite, and (ii) with a postverbal distributive that bears primary stress. Neither of these is assumed to involve overt or covert operator movement and will not be further discussed in this paper. The wide existential scope of indefinites may be attributed to existential closure over choice functions à la Reinhart (1997). As regards primary stressed postverbal distributives, both É. Kiss (1998) and Brody and Szabolcsi (2003) argue in detail that they effectively occur in the highest DistP projection and their postverbal ordering is obtained using permutation rules that do not affect c-command and scope relations.

operator sequences – Agr(eement), T(ense), etc. – only the highest is visible: the one that hosts the finite verb. Therefore two adjacent operators in the postverbal field need not belong to the same operator sequence and need not conform to the sequence-internal hierarchy. The overt or covert inflectional heads play the kind of beneficial mediating role that was described in Section 2.2.[15]

3.2 Total order?

An important fact about the operators reviewed above is that they can all co-occur. Adding focus and negation to the mix raises new questions about how expressions, and their categories, can be ordered.

First consider focus. Hungarian is one of those languages that have a reflex of focussing in surface syntax. Counting quantifiers and foci (emphatic focus, identificational focus, and phrases modified by *csak* 'only') are complementary in the immediately preverbal position. As they never co-occur, no left-to-right ordering can be established between them:

$$\text{topic} \quad > \quad \text{distributive} \quad > \quad \begin{matrix}\text{counter} \\ \\ \text{focus}\end{matrix} \quad (> \text{verb} \ldots)$$

Next consider negation. The preverbal field may contain two distinct instances of sentential negation (*nem*), to be dubbed as hi-neg and lo-neg when the distinction is necessary. The two happily co-occur and, naturally, do not cancel out, when an appropriate third party intervenes. The postverbal field houses no negation. See Koopman and Szabolcsi (2000, Appendix B).

Even putting aside negative polarity items, operators come in different flavors as regards their ordering constraints with respect to negation. Negation may follow a focus or a counter, and it may precede a focus, though not a counter:

(16) (*hi-neg) > counter > lo-neg (> verb . . .)
hi-neg > focus > lo-neg (> verb . . .)

Distributive universals cannot scope immediately above negation, nor for that matter immediately below it,[16]

[15] The analysis of the postverbal field is a matter of some disagreement, see Surányi (2003). The postverbal facts will play little role in this paper; they are mentioned only to enable us to provide a concrete sample derivation in Appendix C.

[16] Examples with *nem minden fiú* 'not every boy' contain phrase internal negation and not a hi-neg preceding the quantifier *minden fiú* in one of its otherwise legitimate positions. The critical data that show this involve order interaction with verbal particles. The verbal particle (*fel* 'up', etc.) precedes the verb unless the next element to the left is negation, or a focus, or a counter. With non-negated *minden*-phrases the only possibility is (i). However, *nem minden*-phrases require the verbal particle to follow the verb, as in (ii). Thus *nem minden*-phrases represent a separate quantifier class, cf. also (18).

(i) Minden fiú fel-ébredt.
every boy up-woke

(17) *neg $>$ distributive-$\forall >^*$ neg

whereas negative concord (NC) universals such as *senki* 'no one' come with something like the opposite restriction:[17]

(18) *NC-$\forall > OP$, unless OP = neg or *nem_minden* 'not_every' or NC-$\forall$

In contrast, topics and distributive existentials do not care whether or not there is negation in their immediate scope:

topic $>$ (neg)
distributive-$\exists >$ (neg)

It follows from (16) and (18) that counters and NC universals cannot co-occur in the preverbal field. Likewise, it follows from (17) and (18) that distributive and NC universals cannot co-occur in the preverbal field. Therefore there is no basis for ordering NC universals with respect to either counters or distributive universals within the same operator sequence.

Notice that these observations about co-occurrence and orderability pertain to (classes of) expressions. It is quite possible that if we turn to the syntactic categories of the same expressions, the conclusions will no longer hold.

The standard assumption in Minimalist syntax is that the ordering of functional categories is total. Could (15) be extended to the additional data, maintaining a total order? The answer is Yes, as long as the proposal can be supplemented with further assumptions or constraints. One contender would be as follows (only the head categories are listed):

(19) Ref* $>$ Dist* $>$ Hi-Neg $>$ Pred $>$ Lo-Neg $>$ AgrS...

One supplementary assumption will be that counters and foci are not two distinct categories in complementary distribution; instead both carry a [pred] feature and compete for the specifier position of a Pred head. This analysis follows the "focussing as predication" view recently advocated by É. Kiss (2001, 2006). The view is not uncontroversial (see Horvath (2000, 2006) for another view), but for present purposes it suffices that such a unification is imaginable. The fact that identificational foci and *csak* 'only' phrases can be preceded by Hi-Neg, but counters cannot (unless they have a contrastive component) is one argument for the distinct categories analysis. However, it could be accommodated in (19) by adding that Hi-Neg requires its complement to carry the feature [contrast], and not all [pred] phrases have [contrast].

Another assumption we need is that distributive universals, distributive existentials, and NC universals all have a [dist] feature and are thus headed for

(ii) Nem minden fiú ébredt fel.
not every boy woke up

For simplicity's sake the mini-grammar to be presented in Section 4 does not include verbal particles.

[17] Hungarian is a so-called strict negative concord language. Negative concord items (NC) are interpreted as universals, following Szabolcsi (1981), Giannakidou (2000), Puskas (2000).

the specifier of a Dist head, but they are marked differently as to what features their complements should carry. NC universals require that the closest head below them have [neg]; distributive universals require that the same head not have [neg]; distributive existentials and expressions with [topic] are not marked in this regard. Moreover, not only *nem* 'not' has [neg], but also *nem mindenki* 'not everyone', and *senki* 'no one, NC' come with a [neg] feature that they transmit by specifier-head agreement. The treatment of *nem mindenki* itself remains difficult. E.g. *mindenki* 'everyone' could be considered ambiguous: *mindenki*$_1$ would have [dist] and require its complement not to have [neg], as above, whereas *mindenki*$_2$ would have [pred] and [contrast] and require the head above it to be the specific item *nem* 'not'. Accommodating its distribution on the analysis that *nem mindenki* is a separate quantifier phrase would take comparably exotic assumptions.

This will suffice to show both that the total order in (19) could be maintained and what kind of cost it would incur. A description using a total order of categories is possible, but the result does not look very Minimalist.

3.3 Optionality

The Hungarian data highlight another fundamental question. As was noted in 3.1, the presence of all the operators discussed in this section is optional. Consider:

(20) Tudom, hogy [$_{\text{RefP}}$ az emberek [$_{\text{AgrSP}}$ láttak]].
know-1sg that the men saw-3pl.1sg
'I know that the men saw me'

(21) Tudom, hogy [$_{\text{DistP}}$ minden ember [$_{\text{AgrSP}}$ látott]].
know-1sg that every man saw-3sg.1sg
'I know that every man saw me'

(22) Tudom, hogy [$_{\text{AgrSP}}$ láttál]].
know-1sg that saw-2sg.1sg
'I know that you saw me'

These examples raise the optionality problem (2) of Section 1. The complementizer head *hogy* 'that' is apparently equally happy to recognize RefP, DistP, and AgrSP as suitable arguments. Likewise, Ref selects for DistP, but it is equally happy with AgrSP, and so is Dist, which selects for PredP. How are the complement selection requirements of these heads (functional categories) satisfied?

As the discussion of (2) indicated, the optionality problem is by no means specific for Hungarian; Hungarian just illustrates it on a large scale. The optionality problem does not seem to have received much attention in the Minimalist literature and we are not aware of a standard solution. In line with Cinque's (1999) influential proposal that the sequence of functional categories is invariant and universal, one hypothesis could be that the full sequence of categories in (19) is always present, but the individual categories need not host lexical items in every sentence. This hypothesis would have been easy to accommodate in earlier, phrase structure rule based versions of generative syntax, but it is not in Minimalism, the mainstream format of the theory for over a decade. The problem is that in Minimalist Theory categorial structure is projected from the lexical items that make up the sentence: no lexical item, no category. So, one would need to postulate that for each optional head category there exists a “dummy lexical item”, which has no ability to attract a phrase to its specifier but suffices to project the phrasal category that satisfies the complement selection requirements of the head above it. Moreover, to accommodate the constraints discussed in connection with the total order (19), one would need to ensure that phrasal categories headed by dummies inherit the features of the next phrasal category below them that is headed by a real lexical item.

The conclusion is the same as that of the previous subsection: a solution involving an invariant sequence of categories is in principle possible, but it does not look very Minimalist.

3.4 Partial order and derivability/inclusion relations: Two birds with one stone

As was pointed out in sections 1 and 2, the optionality problem receives a natural solution if expressions are not thought to have a unique category label but derivability/inclusion relations among categories are recognized. If the grammar recognizes the DistP $\longrightarrow$ RefP (DistP $\subseteq$ RefP) relation, and the complementizer head *hogy* ‘that’ does not look for a complement specifically labeled as RefP but accepts any category that derives RefP, then the grammaticality of (21) no longer comes as a surprise; and similarly for the other examples.

The derivability/subset relation that does this job is naturally chosen to be reflexive. Thus the solution to the optionality problem points to a partially, not totally, ordered set of categories. This means that the effort to create a total order in (19) and supplement it with an additional device, a set of featural constraints as detailed above, is unnecessary. Those same constraints can be expressed as finer details of the partial ordering. The next section demonstrates how this works.[18]

4 A mini-grammar of operators in Hungarian

Within the multi-modal categorial type logic introduced in Section 2, capturing the partial order of the most important Hungarian operators requires the use of

[18]The same conclusion that syntactic categories should be partially, rather than totally, ordered was reached in Nilsen (2002, 2004) on somewhat different grounds. Nilsen’s empirical arguments come from the distribution of adverbs in Norwegian. He observes, contra Cinque (1999), that Norwegian adverbs that co-occur in the same sentences are not ordered with respect to each other; whatever ordering restrictions one finds follow from the fact that the abverbs are often ordered with respect to negation.

the portion of the logical space exhibited in Figure 2.[19]

For easier reference each sentential category is given a number. The category $s1$ is assigned to sentences whose initial element is an inflected verb.

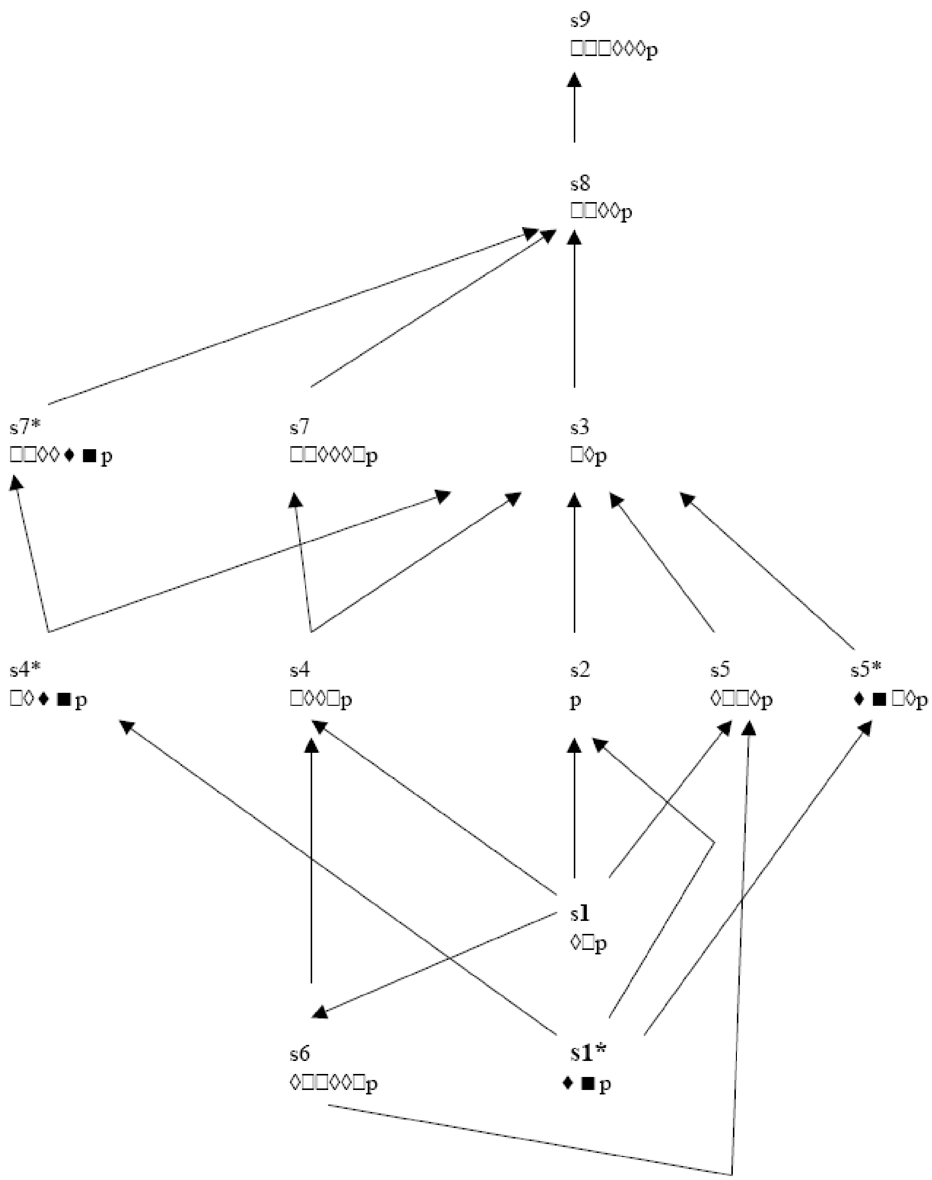


Figure 2: The space of Hungarian operators, derivability patterns only

[19]The categories s5 and s6 will come into play later.

Prior to locating the Hungarian operators in this space, let us draw attention to the two modes in Figure 2. The basic mode is represented with empty boxes and diamonds. The filled boxes and diamonds can be seen to add an alternative dimension to some parts of the system; to highlight this, the sentential categories that involve filled modes have the numbers of the corresponding categories in the empty mode plus an asterisk.[20] So, for example, parallel to $\Diamond\Box p$ $(= s1)$ is $\blacklozenge\blacksquare p$ $(= s1^*)$ and parallel to $\Box\Diamond\Diamond\Box p$ $(= s4)$ is $\Box\Diamond\blacklozenge\blacksquare p$ $(= s4^*)$. The two alternative dimensions merge where s_n and s_n^* derive the same category. $s1$ and $s1^*$ both derive p $(= s2)$, and $s4$ and $s4^*$ both derive $\Box\Diamond p$ $(= s3)$, etc. The categories based on the filled mode will be used to capture the behavior of those operators – negative concord items – that must scope immediately above negation or another negative concord item. So $s1$ is the category of basic affirmative sentences and $s1^*$ the category of basic negative sentences. The argument category of lo-neg is $s1$ and its value category is $s1^*$, yielding the functor category $s1^*/s1$.

Operator expressions have the category val/arg and are represented in Figure 3 as curved arrows pointing from the argument category to the value category. The curved arrows are labelled either with the informal names of the classes (topic, counting quantifier, focussed XP, hi-neg, lo-neg) or with a representative member (*who*, *no one-NC*, *no one neither-NC*, *XP neither-NC*, *everyone*, *XP too*, *many people*, *not everyone*).[21]

Expressions that do not care about scoping directly above negation have their argument categories on the $s1 - s2 - s3 - s8 - s9$ track; those that must not scope directly above negation on the $s1 - s4 - s7$ track; and those that must scope directly above negation or another operator of their own kind (negative condord items) on the $s1^* - s4^* - s7^*$ track.

To see how Figure 3 captures other data reviewed in subsection 3.1, recall that counters $(s7/s2)$ and foci $(s4/s2)$ do not co-occur and are therefore not ordered with respect to each other. Notice that neither $s4$ nor $s7$ derives $s2$. The reader is invited to find other examples.

[20] These asterisks are not to be confused with the Kleene stars that indicate iteration.

[21] For completeness, Figure 3 contains, in addition to the operators discussed above, a couple of negative concord items whose restrictions are slightly different from those of *no one-NC*, and a representative of question words (*who*). The consensus in the literature is that Hungarian question words are in focus position, but the fact is that they cannot be preceded by either negation or distributives; restrictions that a total ordering ought to also ensure in featural terms.

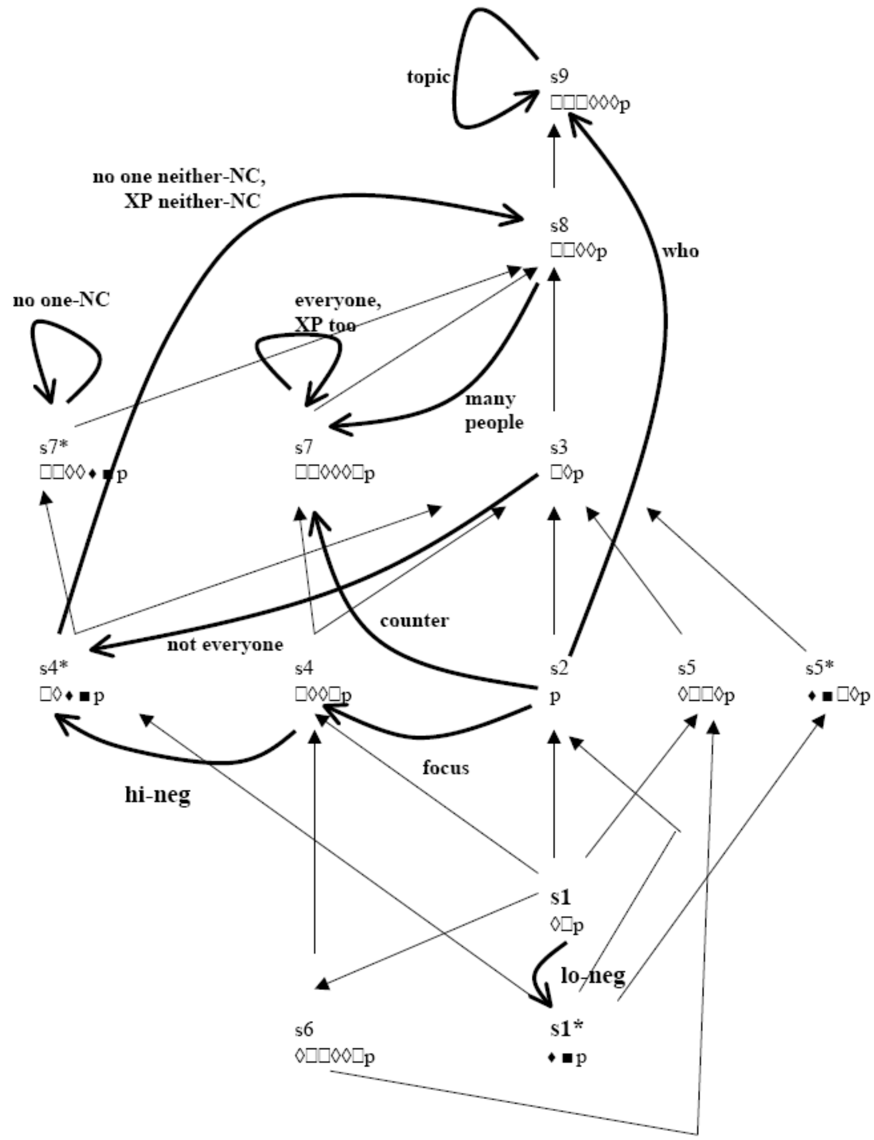


Figure 3: Hungarian operators

Whenever the value category of an expression derives the argument category of another, the predicted results are grammatical, although more than five or six operators preceding the verb may sound crowded. Consider just one example:

Kati	a legtöbb napon	mindenkivel	sok újságot
Kate	on most days	with everyone	many pieces of news.acc
$s9/s9$	$s9/s9$	$s7/s7$	$s7/s8$

rosszindulatból	nem	közölt.
out of malice	not	shared.3sg
$s4/s2$	$s1^*/s1$	$s1/\ldots$

'On most days for every person there were many pieces of news such that it was out of malice that Kate did not share those pieces of news with that person'

Although this paper is concerned only with the operator categories, for concreteness Appendix C spells out the complete Natural Deduction style derivation of a simple sentence, including inflectional categories. This derivation will show how the optional operators co-exist with obligatory elements in a grammar. Some comments are added pertaining to innovations in the treatment of syntactic phenomena, but the syntactic framework is not introduced there. Readers not familiar with Moortgat (1997) may want to skip Appendix C.

5 The logic of licensing

5.1 Salvaging ungrammatical categories

We now turn to the licensing problem discussed in (3) in Section 1. Bernardi (2002) develops a proposal for negative polarity item (NPI) licensing,[22] which we can take to be the representative of licensing in general. That is, from our perspective the following two structures are alike, and whatever we say about NPI-licensing carries over to the licensing of inversion, for example:

licensor	**licensee**
never	saw anything
never	would I do that

That the NPI needs to be licensed means that a structure containing the NPI is ungrammatical unless it is within the scope of an appropriate operator. Notating the category of an NPI as val_{NPI}/arg, val_{NPI} does not derive $s9$, the category of complete grammatical sentences: val_{NPI} is an "ungrammatical category". The fact that being within the immediate scope of a licensor

[22] Negative polarity items are expressions like *ever*, *any*, and others that must occur within the immediate scope of a monotonically decreasing operator called the licensor. See further details in the next section.

salvages the structure means that the licensor is capable of bringing it back to the grammatical fold: the value category of the licensor is a grammatical one. More surprising is the fact that the licensor can scope immediately above the structure containing the NPI: it shows that val_{NPI} derives arg_{lic}. Since an "ungrammatical category" by definition cannot derive a grammatical one, this means that arg_{lic} itself is an "ungrammatical category" too. For example:

seldom	>	anything	>	saw
val/arg_{lic}		val_{NPI}/arg		arg

On the other hand, the licensor does not require for there to be an NPI within its immediate scope. This means, in turn, that arg_{lic} is also derived by various grammatical categories:

seldom	>	everything	>	saw
val/arg_{lic}		val_{dist}/arg		arg

So, we have:

$$\text{(23)}\quad \begin{array}{l} val_{NPI} \text{ (ungrammatical)} \not\longrightarrow s_n \text{ (grammatical)} \\ arg_{lic} \text{ (ungrammatical)} \not\longrightarrow s_n \text{ (grammatical)} \\ val_{NPI} \text{ (ungrammatical)} \longrightarrow arg_{lic} \text{ (ungrammatical)} \\ val_{dist} \text{ (grammatical)} \longrightarrow arg_{lic} \text{ (ungrammatical)} \end{array}$$

It is in principle possible to set up the partial order of syntactic categories in such a way that ungrammatical categories reside in a "blind alley" that satisfies the requirements in (23). For example, $\Box\Box\Box\Box\Box\Diamond\Diamond\Diamond\Diamond\Diamond p$ is derivable from all the categories used in Figure 3 but does not derive any one of them. $\Box\Box\Box\Box\Box\Diamond\Diamond\Diamond\Diamond\Diamond p$ might be designated as "the ungrammatical category". Given however the variety of licensing relations the grammar has to accommodate, this solution would probably be ad hoc and unable to capture finer patterns.

5.2 The logic of ungrammatical categories

Bernardi (2002) proposes a systematic way to encode the kind of derivability relations described in (23) using unary Galois operators. These were first introduced into CTL in Areces and Bernardi (2001) inspired by Dunn (1991), Goré (1998). These works show that the realm of minimum logic (i.e. the logic characterized by just the core properties of the transitivity of derivability and the monotonicity of the logical operators) has space for operators that reverse the derivability relation among formulae. Recall

$$\Diamond A \longrightarrow B \;\text{ iff }\; A \longrightarrow \Box B$$

Let ${}^0\cdot$ and $\cdot^0$ be two unary operators. They are said to be Galois-connected if they obey the definition below.

$$B \longrightarrow {}^0A \;\text{ iff }\; A \longrightarrow B^0$$

These two operators behave exactly like $\Diamond$ and $\Box$, except that they are downward monotonic, cf. the fact that B occurs on the righthand side of the arrow in $\Diamond A \longrightarrow B$ but on the lefthand side in $B \longrightarrow^0 A$. The algebraic analogy now involves reciprocals: the greater a number, the smaller its reciprocal:[23]

$$x \times y \leq 2 \;\text{ iff }\; x \leq \tfrac{2}{y}$$

As in the case of $\Diamond$ and $\Box$, the properties regarding the composition and the monotonicity behavior of the Galois operators follow, namely:

1. $A \longrightarrow {}^0(A^0)$
2. $A \longrightarrow ({}^0A)^0$
3. ${}^0A \longrightarrow {}^0B$, if $B \longrightarrow A$ [${}^0\cdot$ downward monotonic]
4. $A^0 \longrightarrow B^0$, if $B \longrightarrow A$ [$\cdot^0$ downward monotonic]
5. ${}^0(A^0) \longrightarrow^0 (B^0)$, if $A \longrightarrow B$ [${}^0(\cdot^0)$ upward monotonic]
6. $({}^0A)^0 \longrightarrow ({}^0B)^0$, if $A \longrightarrow B$ [$({}^0\cdot)^0$ upward monotonic]

Notice that since the composition of two downward monotonic operators is upward monotonic, ${}^0(A^0)$ and $({}^0A)^0$ are upward monotonic in A. In what follows we will only use them in pairs, i.e. as (composite) upward monotonic operators.

Double-galois operators can be used to create additional layers of the poset given by $\Box$ and $\Diamond$. As (24) indicates, each s_a derives ${}^0(s_a^0)$, and if $s_a \longrightarrow s_b$, ${}^0(s_a^0) \longrightarrow {}^0(s_b^0)$. This means that the derivability relations within each double-galois layer are the same as those within the $\Box$ and $\Diamond$ layer of the poset. However, the paths are unidirectional: double-galois operators can only be added, not removed. This means that grammatical categories derive “ungrammatical” ones, but no “ungrammatical” category derives a grammatical one. This is precisely what ungrammaticality is.[24]

[23] As in the case of the unary residuated operators $\Diamond$ and $\Box$, the Galois-connected unary operators can be seen as binary operators with a fixed argument:

$$x \times 2 \leq z \;\text{ iff }\; x \leq \frac{z}{2} \qquad \text{let } \Diamond\cdot = \cdot \times 2 \text{ and } \Box = \tfrac{\cdot}{2}$$

$$y \leq \frac{2}{x} \;\text{ iff }\; x \leq \frac{2}{y} \qquad \text{let } {}^0\cdot = \tfrac{2}{\cdot} = \cdot^0$$

If we take the two directional implication $\backslash$ and $/$ in the place of the undirectional reciprocal $\div$, we obtain ${}^0\cdot \neq \cdot^0$

[24] The pairs ${}^0(\cdot^0)$ and $({}^0\cdot)^0$ are closure operators, therefore the iteration of the same pairs of Galois produces equalities, viz. $({}^0({}^0A)^0)^0 \longleftrightarrow ({}^0A)^0$ and similarly for the other pair. On the other hand, the iteration of different pairs, i.e. ${}^0(\cdot^0)$ followed by $({}^0\cdot)^0$ and conversely $({}^0\cdot)^0$ followed by ${}^0(\cdot^0)$ produces inequalities, $({}^0A)^0 \longrightarrow {}^0((({}^0A)^0)^0)$ but ${}^0((({}^0A)^0)^0) \not\longrightarrow ({}^0A)^0$ and similarly for the other combination (see more details in Appendix B). Turning back to our application, the iterations of different pairs of Galois gives us the possibility to express many “ungrammatical” sentential layers.

$$
\text{(24)}\qquad
\begin{array}{ccccc}
\textbf{grammatical} & & \textbf{ungrammatical} & & \textbf{ungrammatical} \\
s_b & \longrightarrow & {}^0(s_b^0) & \longrightarrow & ({}^0({}^0(s_b^0)))^0 \\
\uparrow & & \uparrow & & \uparrow \\
s_a & \longrightarrow & {}^0(s_a^0) & \longrightarrow & ({}^0({}^0(s_a^0)))^0
\end{array}
$$

An advantage of using the double-galois operators to encode ungrammaticality is that we now have a systematic solution, rather than an ad hoc "blind alley" for ungrammatical categories.

6 Negative polarity item licensing

6.1 The monotonicity of licensing

This section will examine whether and how Bernardi's theory of licensing can be implemented in a realistic setting. But an empirical property of NPI-licensing has to be introduced first. Different negative polarity items require different licensors. Zwarts (1983) proposed that the relevant distinctions can be made in terms of the "negative strength" of the licensors, characterizable with how many of the de Morgan implications each bears out.

f is anti-morphic (AM) e.g., *not*	iff	$f(a \vee b) = fa \wedge fb$ and $f(a \wedge b) = fa \vee fb$
f is anti-additive (AA) e.g., *never*, *nobody*	iff	$f(a \vee b) = fa \wedge fb$
f is decreasing (DE) e.g., *seldom*, *at most five men*	iff	$f(a \vee b) \longrightarrow fa \wedge fb$

Thus we have the following subset relations:

(25) anti-morphic $\subseteq$ anti-additive $\subseteq$ decreasing

van Wouden (1997) provides a detailed discussion of the Dutch NPI-licensing data in these terms. To use examples from other languages, Nam (1994) argues that the Korean exceptive *pakkey* 'only' is an NPI that requires an antimorphic licensor. English *yet* requires an antiadditive one, and *ever* is satisfied with one that is (roughly) decreasing:

a. We haven't been there yet.
b. Nobody has been there yet.
c. *At most five men have been there yet.

a. We haven't ever been there.
b. Nobody has ever been there.
c. At most five men have ever been there.

These properties play a role in other licensing relations as well. Roughly decreasing adjuncts undergo negative inversion in English (Büring 2004):

(26) Under no / few / *some circumstances would I do this.

These data sets exhibit what we may call "the monotonicity of licensing":

Monotonicity of Licensing:

1. A weak NPI is licensed by an operator that is decreasing **or stronger**.
2. A medium NPI is licensed by an operator that is anti-additive **or stronger**.
3. Negative inversion involves adjuncts that are decreasing **or stronger**.

We expect the syntax of licensing to conform to this generalization (where it indeed holds). How could this be done? One possibility is for *nobody*, for instance, to be tagged separately as decreasing and as anti-additive. But one hopes that it is not necessary to resort to such brute force methods, and the monotonicity of licensing can be captured in the form of derivability (inclusion) relations.

At first blush one might think that this requires incorporating the inclusion relations in (25) into the syntax, but that is not the case.[25] It suffices if the following derivability relations hold between the categories:

$$\begin{array}{lcl} \text{arg. of strong, antimorphic licensor} & \longleftarrow & \text{val. of strong licensee} \\ & & \uparrow \\ \text{arg. of medium, antiadditive licensor} & \longleftarrow & \text{val. of medium licensee} \\ & & \uparrow \\ \text{arg. of weak, decreasing licensor} & \longleftarrow & \text{val. of weak licensee} \end{array}$$

It is easy to see that if these relations hold, a weak licensee like *ever* for example can be licensed by any of the three kinds of licensors – without the syntax incorporating any derivability relations between the categories of the licensors.

6.2 Is it logically viable?

Does our calculus in general and the logical space explored in Figure 2 in particular make it possible to pick categories in the desired way? The following assignment of value categories to licensees and argument categories to licensors will do. No arrow between two categories means no derivability.[26]

[25] In subsection 6.4 we come back to the question whether incorporating (25) into the syntax would be possible at all.

[26] The downward monotonic nature of both the licensors of NPI and the galois operators is a pure coincidence. Notice that we use galois operators always in a pair, i.e. as upward monotonic operators, and moreover, the same application of galois operators could be used to model other sorts of licensing relations that do not involve downward monotonicity of the licensors.

$$
(27)\quad
\begin{array}{llcl}
\text{arg. of strong lic-or} & {}^0(s_4^0) \longleftarrow & {}^0(s_4^0) & \text{val. of strong lic-ee} \\
 & & \uparrow & \\
\text{arg. of medium lic-or} & {}^0(s_5^0) \longleftarrow & {}^0(s_6^0) & \text{val. of medium lice-ee} \\
 & & \uparrow & \\
\text{arg. of weak lic-or} & {}^0(s_2^0) \longleftarrow & {}^0(s_1^0) & \text{val. of weak lic-ee}
\end{array}
$$

Figure 4 below shows that this is indeed viable. Figure 4 exhibits double-galois categories and their derivability relations. Recall from Section 5 that ${}^0(s_a^0) \longrightarrow^0 (s_b^0)$ iff $s_a \longrightarrow s_b$. Therefore the patterns of the derivability in Figure 4 are familiar; they are exactly the same as the ones in galois-free Figure 2. Because no double-galois category derives a galois-free category, relations between the galois-free ("grammatical") categories that are not part of the diagram cannot cause trouble. The derivability relations relevant in (27) are highlighted with double lines in Figure 4. It is easy to see that all the relations required in (27) hold. On the other hand, ${}^0(s4^0)$, ${}^0(s2^0)$, and ${}^0(s5^0)$ are independent.

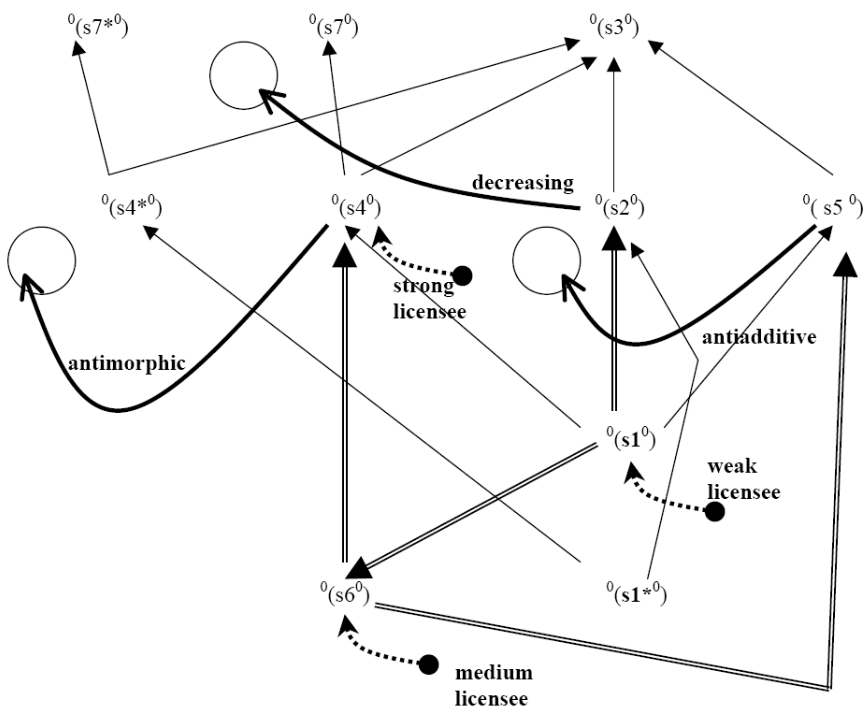


Figure 4: Licensors and licensees with derivability relations among "ungrammatical" categories

The diagram also contains curved arrows corresponding to the categories of linguistic expressions. The argument categories of licensees (with dotted lines) are replaced by bullets, since they are irrelevant from the present perspective and will vary with the licensees under consideration. The strong, medium and weak licensors are supposed to be antimorphic, antiadditive, and decreasing functors, respectively, which, following Section 5, point from double-galois (ungrammatical) to galois-free (grammatical) categories. What their concrete value categories are is irrelevant from the general logical perspective, and so Figure 4 indicates them with empty circles. They are however absolutely relevant from an empirical perspective, to which we now turn; the reader is invited to fill in the circles in due course.

6.3 Is it empirically viable?

What we have seen demonstrates that it is possible in our calculus to assign categories to licensees and licensors in the manner envisaged in Section 5. The empirical question is whether natural language expressions can be matched up with these possibilities. In this paper we only discuss the empirical properties of licensors in detail. We simply assume that the licensees can be assigned to categories in accordance with Figure 4.

We take the Hungarian operator poset in Figure 3 as a point of departure. A quick glance at Figure 3 reveals that Hungarian has suitable decreasing operators. Almost all the merely decreasing quantifiers in Hungarian are counters, assigned to $s7/s2$ in Figure 3; this is now revised to $s7/^0(s2^0)$. The revision does not affect the word order behavior of counters, since all and only those galois-free categories that derive $s2$ derive $^0(s2^0)$.[27]

Nem mindenki 'not_everyone' is decreasing but its word order behavior slightly differs from that of counters; its category in Figure 3 is $s4^*/s3$. If *nem mindenki* is a good NPI-licensor, then its category should be revised to $s4^*/^0(s3^0)$. It turns out that negated universals are cross-linguistically poor licensors of even weak NPIs, compare:

(28) *Not everyone saw anything / has ever been there.

Why this is so is something of a mystery. One possibility is to attribute the unacceptability of (28) to the intervention of *everyone* between *not* and *anything/ever*, cf. **I don't think that everyone saw anything* (Linebarger 1987). If however there is reason to analyze *not everyone* as one complex quantifier, then the intervention account becomes less obvious. Indeed, the complex quantifier analysis was motivated for Hungarian in footnote 16. Since the judgment in (28) is replicated in Hungarian, there is no reason to assign the licensor category $s4^*/^0(s3^0)$ to *nem mindenki.*

Hungarian is a strict negative concord language and as such it has no antiadditive quantifier phrases comparable to English *no one*. For the purposes of the present investigation we may however contemplate a closely related imaginary language Hungarian′ that has no strict negative concord but has an antiaddi-

[27]Although both 'more than six men' and 'few men' are counting quantifiers, their categories are now distinguished: 'more than six men' is $s7/s2$ but 'few men' is $s7/^0(s2^0)$. Thus while their word order behavior is otherwise the same, only the latter is a licensor.

tive. This imaginary item is comparable to *no one* in its scope behavior: it does not scope immediately above negation but can be immediately outscoped by a decreasing counter, cf.

> *No one didn't laugh.
>
> Few men saw no one.

These properties are guaranteed by assigning it to the category $s2/s5$, to be revised as $s2/^0(s5^0)$ because it is an NPI-licensor.

Hungarian has even two anti-morphic operators: lo-neg and hi-neg. Are they both strong licensors? If not, which of the two is? It turns out that the choice of lo-neg is simply incompatible with our most basic assumptions. If it were a strong licensor, its category would be $s1^*/^0(s1^0)$. But $^0(s1^0)$ is the bottom element of our small set of categories, and indeed the "center" of the whole (infinite) set of categories defined by our calculus. If the value category of strong licensees (or medium licensees, for that matter) derived $^0(s1^0)$, then it would derive the argument categories of all licensors. That move would wipe out all the strong/weak distinctions we are trying to accommodate. Therefore lo-neg is not in the game. Fortunately, we can resort to the hi-neg version of *nem*, previously assigned to category $s4^*/s4$. This is now revised as $s4^*/^0(s4^0)$. $^0(s4^0)$ is the right value category for strong licensees, because it does not derive the argument categories of either weak or medium licensors, and of course it derives itself in the capacity of being the argument category of the strong licensors.

The assumption that hi-neg is a licensor but lo-neg is not is empirically less strange than it may initially sound. In (29), where the finite verb is preceded by just one negation and by no focus or counter, this negation could be an instance of either lo-neg or hi-neg. The category of the inflected verb, $s1$ derives the argument categories of both $s1$ and $s4$.

(29)	**Nem**	hiszem,	hogy	**valaki**	**is**	hallotta	volna	a hírt.
	not	think.1sg	that	someone	even	heard	aux	the news
	'I don't think that anyone heard the news'							

Since (29) contains the NPI *valaki is* 'someone even', we will simply take its *nem* to be hi-neg. The one case where hi-neg and lo-neg are distinguishable is where a focus or counter precedes the negation. Our analysis makes the prediction that (30), which cannot but involve lo-neg, is unacceptable. As linguists often say, the judgment is subtle, but the example is certainly less natural than (29):

(30)	??**ÉN**	**nem**	hiszem,	hogy	**valaki**	**is**	hallotta	volna	a hírt.
	I	not	think.1sg	that	someone	even	heard	aux	the news
	'It is me who doesn't think that anyone heard the news'								

Hi-neg licenses an NPI only in the absence of an intervening focus, cf. (31). Since Linebarger (1987) NPI-licensing has been known to be sensitive to operator intervention. Whatever technique is employed to capture this, it will rule out (31):

(31)

***Nem**	**én**	hiszem,	hogy	**valaki**	**is**	hallotta	volna	a	hírt.
not	I	think.1sg	that	someone	even	heard	aux	the	news

'It is not me who thinks that anyone heard the news'

All in all, it is not unreasonable to assume that in this licensing domain hi-neg, but not lo-neg, represents the strong, antimorphic licensor and, given the fact that lo-neg is at the bottom of our category set, this is indeed the only option.

To summarize, the following licensors fit the recipe and work for the reincarnation of Hungarian dubbed Hungarian′:

Strong NPI-licensor:	$s4^*/^0(s4^0)$	– example: hi-neg *nem*
Medium NPI-licensor:	$s2/^0(s5^0)$	– example: imaginary 'no one'
Weak NPI-licensor:	$s7/^0(s2^0)$	– example: any decreasing counter

Notice that in this theory not only NPI-licensing is a licensing relation. Any structure that must be immediately outscoped by a particular kind of operator is a "licensee" – one whose value category is an "ungrammatical category", wherefore its superstructure can only derive $s9$, the category of grammatical sentences if it is brought back to the "grammatical plane". Does our theory predict that no licensee might call for, or allow for, lo-neg as a licensor in Hungarian (or in Hungarian′)? It does not. Recall from Section 5 footnote 24 that infinitely many distinct "ungrammatical planes" can be formed by adding new pairs of galois-operators:

(32)
$$\begin{array}{ccccccc}
\textbf{grammatical} & & \textbf{ungrammatical} & & \textbf{ungrammatical} & & \\
s_b & \longrightarrow & {}^0(s_b^0) & \longrightarrow & ({}^0({}^0(s_b^0)))^0 & \longrightarrow & \ldots \\
\uparrow & & \uparrow & & \uparrow & & \\
s_a & \longrightarrow & {}^0(s_a^0) & \longrightarrow & ({}^0({}^0(s_a^0)))^0 & \longrightarrow & \ldots
\end{array}$$

Suppose that some licensee has the value category $({}^0({}^0(s_1^0)))^0$, and that is the argument category of lo-neg. Since $({}^0({}^0(s_1^0)))^0$ does not derive ${}^0(s_1^0)$, this addition does not interfere with (32). Essentially, each kind of licensing relation may be associated with a different "ungrammatical plane".

6.4 Semantics in the syntax? Is licensing truly monotonic?

In (25) it was observed that licensors exhibit a semantic inclusion relation: anti-morphic $\subseteq$ anti-additive $\subseteq$ decreasing. Perhaps the most appealing way to accommodate of the monotonicity of licensing would be to import the corresponding derivability relations between argument and value categories into the syntax.[28]

[28]This section summarizes research done during the one-month visit of R. Bernardi and Ø. Nilsen in the Department of Linguistics at New York University in Spring 2005, sponsored by NYU's International Visitors Program.

Would that be possible? We have already seen some empirical reasons why it would not be. First and foremost, the semantic approach would force us to treat hi-neg and lo-neg alike. Or, if they can be semantically distinguished, it is probably lo-neg that would end up as "the" anti-morphic operator. But lo-neg is at the bottom of both the semantic inclusion hierarchy and the syntactic category hierarchy. Therefore, as was observed above, if the value category of strong licensees derived the argument category of lo-neg, it would inescapably derive the argument categories of medium and weak licensors as well, and all the licensing distinctions would be lost.

Secondly, notice that for the sake of the argument we considered a Hungarian′ which is not a strict negative concord language (and thus has anti-additive generalized quantifiers) and whose NPIs are exactly like NPIs in English or Dutch. These two related properties of Hungarian′ do not hold of plain Hungarian. Progovac (1994) observed that the distribution of English *anything* is covered by two complementary items in Serbo-Croatian: *ništa* in the context of clause-mate negation and *išta* elsewhere. *Ništa* is a strict negative concord item in our sense, and *išta* a NPI. But the licensing of *išta* is non-monotonic: while it is licensed by clause-mate 'few men', it is not licensed by clause-mate 'not'. The same holds for Hungarian: *senki* is the equivalent of *ništa* and *valaki is* of *išta*. (The latter would have the value category ${}^{0}(s2^{0})$, not ${}^{0}(s1^{0})$ when its licensor is clausemate.) If derivability relations corresponding to semantic inclusion were part of the syntax, licensing should always be monotonic, which Serbo-Croatian and Hungarian show is not the case.[29]

Finally, if in some language other than Hungarian we somehow managed to pick categories for licensors and licensees in such a way that neither of the two problems just discussed arose, we would still likely make dubious predictions about the iterability of operators like negation. When the categories of functions are represented as *val/arg*, the following observation relates semantic inclusion to syntactic derivability:[30]

> Inclusion and derivability:
> If $F_3 \subseteq F_4$, viz. $F_3 \longrightarrow F_4$, then $val_3 \longrightarrow val_4$ and $arg_4 \longrightarrow arg_3$

$$F_3 \subseteq F_4 \qquad \text{iff} \qquad \mathrm{val}_3 \,/\, \mathrm{arg}_3 \qquad\qquad \mathrm{val}_4 \,/\, \mathrm{arg}_4$$

Likewise recall our observations about scope and derivability:

> Scope and derivability:
> An operator OP_1 can scope over OP_2 iff the value category of OP_2 derives the argument category of $OP_1 : val_2 \longrightarrow arg_1$.

[29] Empirically even the English data are more complicated, see De Decker et al. (2005). In this paper we investigated the idealization that forms the basis of the consensus in the literature.

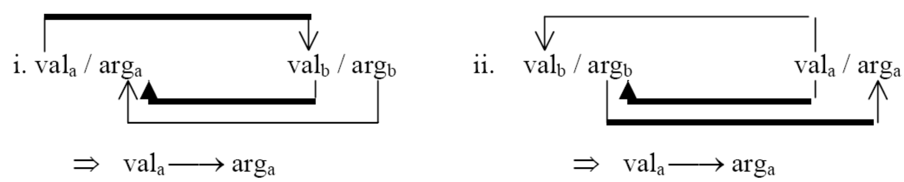
[30] Recall example (12) for the observation that for any function, the value is a categorially upward monotonic and the argument is a categorially downward monotonic position.

$$OP_1 > OP_2 \quad \text{iff} \quad val_1 / arg_1 \qquad val_2 / arg_2$$

Putting these together, we get a prediction. In (33) the arrows involved in making the iterability prediction are highlighted.

(33) Prediction:

> If the class of operators that OP_a belongs to is included in the class that OP_b belongs to, i.e. if $cat(OP_a) \subseteq cat(OP_b)$, and the corresponding derivability relations between value and argument categories are part of the syntax then, if either OP_a or OP_b can directly outscope the other, the stronger operator, OP_a is predicted to directly outscope itself, i.e. to be iterable.



This prediction is entirely independent of what kind of calculus underlies the derivability relations between categories. But this prediction as it stands is incorrect. Decreasing operators can outscope (and in Hungarian, directly precede) anti-morphic negation:

(34) a. Few men didn't read. [DE > AM]

b. Kevés ember nem olvasott [DE > AM]
few men not read

The prediction is that negation itself is iterable. But while (35) is acceptable,

(35) I didn't NOT read. [AM > AM]

it requires contrastive stress on the lower *not*. This indicates that focussing is involved, i.e. the first AM operator is not really scoping directly over the second one. Moreover, further iterations are clearly not possible. Unless it can be convincingly shown that the impossibility of such iterations is due to a processing problem, it drives the last nail into the coffin of the attempt to import semantic inclusion relations into syntax.

The conclusion is that semantic inclusion does not amount to syntactic inclusion. It is not true that a semantically stronger operator can do everything in syntax that a semantically weaker one can. It may have restrictions of its own that the weaker one lacks. Therefore, simply importing semantic inclusion relations into the syntax is not possible.

The natural explanation of the mismatch between semantic properties and word order behavior is that each expression has many semantic properties, whereas our syntax builds all word order properties into the syntactic category of the expression. (This is indeed the basic idea of categorial grammar. If I know your category, I know how you behave.) But then we cannot expect one particular semantic property to correspond to a syntactic category. Our syntax differs from the Minimalist syntax employed in Stabler (1997), for example, where each lexical item is a bundle of syntactic features, including [determiner], [decreasing], [singular], etc. Stabler's idea is to couple that syntax with a Natural Logic, whose inference schemata are anchored to some of the features, e.g. [decreasing]. Stabler's framework would probably lend itself more easily to studying whether a thorough-going match between genuine semantic properties and syntactic behavior can be found.

7 Conclusion

This paper has argued that using a partially ordered set of categories offers a unified theory for solving the problem of complement selection in the presence of optional categories and accommodating licensing relations. Categories in this theory form a "multi-layered" set, with ordering (derivability) relations inside each layer and between the layers.

Let us recall some examples.

(i) The categories $s1$, $s2$, and $s3$ are members of the same basic layer and are ordered as $s1 \longrightarrow s2 \longrightarrow s3$. This models the situation where expressions of category $s1$ or $s2$ can satisfy a higher head that selects for a complement of category $s3$, i.e. where $s2$ and $s3$ are optional.

(ii) Such a layer, or parts of such a layer, can be multiplied by the use of different modes. The categories $s1$ and $s1^*$ belong to two distinct modes and are not ordered with respect to each other. However, just as $s1 \longrightarrow s4 \longrightarrow s3, s1^* \longrightarrow s4^* \longrightarrow s3$. Therefore there are two minimally distinct ways to derive $s3$. This models the situation where $s1$ and $s1^*$ differ from each other in one respect and some other categories are sensitive to the distinction; in all other respects however $s1$ and $s1^*$ as well as those other categories behave identically. (We used two modes to capture some quantifiers' constraints with respect to negation in their immediate scope.)

(iii) The basic layer (together with its distinct modes) is fully replicated by arbitrarily many other layers unidirectionally derived from it: $s1' \longrightarrow s2' \longrightarrow s3', s1'' \longrightarrow s2'' \longrightarrow s3''$, etc. Fully replicated means that the exact same derivability relations obtain in each layer: $s1 \longrightarrow s2$ iff $s1' \longrightarrow s2'$, and unidirectionally derived means that $s1 \longrightarrow s1' \longrightarrow s1''$ but never the other way around. Given this unidirectionality, an expression whose category belongs to one of the "primed" layers can only be part of a grammatical sentence (whose category is on the basic layer) if a wider scoping operator maps it back to the basic layer. This models the situation where an otherwise well-formed expression is "ungrammatical" in that it requires licensing by a particular wider scoping operator; each "primed" layer corresponds to one kind of licensing need. (We used such an ungrammatical layer to assign categories to expressions containing an unlicensed NPI.)

The fact that the different modes and layers have identical internal derivability relations ensures that "other things" are always kept equal.

The grammar outlined this paper was formulated using a version of the Lambek calculus. The ordering relation on the basic layer was given by the derivability relation between propositions decorated with $\Box$ and $\Diamond$, and the ordering relation between this layer and the "ungrammatical" layers was given by the derivability relation between the former propositions and ones additionally decorated with the galois operators ${}^{0}\cdot$ and $\cdot^{0}$. However, the ideas are independent both of the Lambek calculus and of these particular additions. The same ideas pertaining to the role of partial ordering can be formulated in theories that do not use these particular techniques.

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Appendix A: Properties of Unary Residuated Operators

The proofs of the properties of residuated unary operators are given below. All the derivability relations among two formulae decorated with $\Diamond$ and $\Box$ are due to these properties. The arrows in the Figure 2 are the results of different orders of application of these properties.

(a) Unit: $\Diamond\Box A \longrightarrow A$

  i. $\Box A \longrightarrow \Box A$ [Axiom]
  ii. $\Diamond\Box A \longrightarrow A$ [Residuation]

(b) Co-unit: $A \longrightarrow \Box\Diamond A$

  i. $\Diamond A \longrightarrow \Diamond A$ [Axiom]
  ii. $A \longrightarrow \Box\Diamond A$ [Residuation]

(c) Monotonicity of $\Box$

  i. $\Diamond\Box A \longrightarrow A$ [by Unit]
  ii. $A \longrightarrow B$ [*Hypothesis*]
  iii. $\Diamond\Box A \longrightarrow B$ [from i. and ii. by trans]
  iv. $\Box A \longrightarrow \Box B$ [from iii. by Residuation]

(d) similarly for the Monotonicity of $\Diamond$

Not all iterations of unary operators patterns produce formulae that are not equivalent to simpler ones. In particular, the iteration of $\Box\Diamond$ is unproductive, i.e. we obtain equalities of formulae: $\Box\Diamond\Box\Diamond A \longleftrightarrow \Box\Diamond A$; similarly for $\Diamond\Box$, $\Diamond\Box A \longleftrightarrow \Diamond\Box\Diamond\Box A$.

On the other hand, if we compose $\Box\Diamond$ with the other pair $\Diamond\Box$ we obtain inequalities, viz. $\Diamond\Box A \longrightarrow \Box\Diamond\Diamond\Box A$ and similarly for $\Diamond\Box A \longrightarrow \Diamond\Box\Box\Diamond A$. But neither $\Box\Diamond\Diamond\Box A$ nor $\Diamond\Box\Box\Diamond A$ derive $\Diamond\Box A$, though they derive $\Box\Diamond A$. Other productive patterns of unary operators are what we call *center embeddings*. In particular, if we plug the pair $\Diamond\Box$ into the middle of $\Diamond\Box$ we obtain an inequality among the formulae, namely $\Diamond\Diamond\Box\Box A \longrightarrow \Diamond\Box A$ but not the other way around. Similarly, if we plug $\Box\Diamond$ into the middle of $\Box\Diamond$ we obtain a new formula, viz. $\Box\Diamond A \longrightarrow \Box\Box\Diamond\Diamond A$. In the following, we highlight the embedded pairs by underlining them.

Unproductive Iteration Unproductive iterations are due to the fact that both $\Diamond\Box\cdot$ and $\Box\Diamond\cdot$ are closure operators.

(a) $\Box\Diamond A \longleftrightarrow \Box\Diamond\Box\Diamond A$

  i. $\Diamond A \longrightarrow \Diamond A$ [Axiom]

ii. $\Diamond\Box\Diamond A \longrightarrow \Diamond A$ [from i. by Unit]

iii. $\Box\Diamond\Box\Diamond A \longrightarrow \Box\Diamond A$ [from ii. by Mon. of $\Box$]

i'. $\Box\Diamond A \longrightarrow \Box\Diamond\Box\Diamond A$ [by Co-unit]

(b) $\Diamond\Box A \longleftrightarrow \Diamond\Box\Diamond\Box A$

i. $\Box A \longrightarrow \Box A$ [Axiom]

ii. $\Box A \longrightarrow \Box\Diamond\Box A$ [from i. by Co-unit]

iii. $\Diamond\Box A \longrightarrow \Diamond\Box\Diamond\Box A$ [from ii. by Mon. of $\Diamond$]

i'. $\Diamond\Box A \longleftarrow \Diamond\Box\Diamond\Box A$ [by Unit]

Productive Iterations Productive iterations are obtained in two ways, (I) by combining different pairs of unary operators, namely $\Diamond\Box$ with $\Box\Diamond$ and conversely, and (II) by center embeddings.[31] The derivations are spelled out below.

(I) i. $A \longrightarrow A$

ii. $\Diamond\Box A \longrightarrow \Diamond\Box A$ [by Mon. of $\Box$ and Mon. of $\Diamond$]

iii. $\Diamond\Box A \longrightarrow \Box\Diamond\Diamond\Box A$ [by Co-Unit]

i. $A \longrightarrow A$

ii. $A \longrightarrow \Box\Diamond A$ [Co-Unit]

iii. $\Diamond\Box A \longrightarrow \Diamond\Box\Box\Diamond A$ [Mon. of $\Diamond$ and Mon. of $\Box$]

(II) (a) $\Diamond\underline{\Diamond\Box}\Box A \longrightarrow \Diamond\Box A$

i. $\Box A \longrightarrow \Box A$ [Axiom]

ii. $\underline{\Diamond\Box}\Box A \longrightarrow \Box A$ [from i. by Unit]

iii. $\Diamond\underline{\Diamond\Box}\Box A \longrightarrow \Diamond\Box A$ [from ii. by Mon. of $\Diamond$]

whereas, $\Diamond\Box A \not\longrightarrow \Diamond\Diamond\Box\Box A$

(b) $\Box\Diamond A \longrightarrow \Box\underline{\Box\Diamond}\Diamond A$

i. $\Diamond A \longrightarrow \Diamond A$ [Axiom]

ii. $\Diamond A \longrightarrow \underline{\Box\Diamond}\Diamond A$ [from i. by Co-unit]

iii. $\Box\Diamond A \longrightarrow \Box\underline{\Box\Diamond}\Diamond A$ [from ii. by Mon. of $\Box$]

whereas, $\Box\Box\Diamond\Diamond A \not\longrightarrow \Box\Diamond A$

[31] These patterns have been pointed out to us by Eytan Zweig during the visit of the first author to the NYU Linguistics Department.

Appendix B: Properties of the Galois Operators

Similar observations hold for the Galois operators. In this case as well we have to consider the two different pairs, namely, $({}^{0}\cdot)^{0}$ and ${}^{0}(\cdot^{0})$. For the sake of transparency we notate the second pair as ${}^{\bullet}(A^{\bullet})$. See Goré (1998) for the modal theoretical interpretation of these operators, and Areces et al. (2003) for the prove of soundness and completness of the full Lambek calculus with Galois and Residuated unary operators.

As in the case of the residuated operators, iteration yields an equality, $({}^{0}A)^{0} \longleftrightarrow ({}^{0}(({}^{0}A)^{0}))^{0}$ and ${}^{\bullet}(A^{\bullet}) \longleftrightarrow {}^{\bullet}(({}^{\bullet}(A^{\bullet}))^{\bullet})$. On the other hand, the composition of different pairs produces inequalities, namely $({}^{0}A)^{0} \longrightarrow ({}^{0}({}^{\bullet}(A^{\bullet})))^{0}$, $({}^{0}A)^{0} \longrightarrow^{\bullet} ((({}^{0}A)^{0})^{\bullet})$, and the same holds for the formula ${}^{\bullet}(A^{\bullet})$.

Furthermore, in this case as well productive patterns are obtained by means of center embeddings. We can embed ${}^{0}(\cdot^{0})$ within another pair of the same sort ${}^{0}(\cdot^{0})$ obtaining ${}^{0}({}^{0}((A^{0})^{0})) \longrightarrow^{0} (A^{0})$ and similarly for the other pair.

(a) Co-unit': $A \longrightarrow ({}^{0}A)^{0}$

i. ${}^{0}A \longrightarrow {}^{0}A$ [Axiom]

ii. $A \longrightarrow ({}^{0}A)^{0}$ [Galois]

(b) similarly for the other pair

(c) Monotonicity of $\cdot^{0}$

i. $A \longrightarrow ({}^{0}A)^{0}$ [Co-unit']

ii. $B \longrightarrow A$ [Axiom]

iii. $B \longrightarrow ({}^{0}A)^{0}$ [from i. and ii. by trans.]

iv. ${}^{0}A \longrightarrow^{0} B$ [from iii. by Galois def.]

(d) similarly for ${}^{0}\cdot$

Unproductive Iterations

(a) $({}^{0}A)^{0} \longleftrightarrow ({}^{0}(({}^{0}A)^{0}))^{0}$

i. ${}^{0}A \longrightarrow^{0} A$ [Axiom]

ii. ${}^{0}A \longrightarrow^{0} (({}^{0}A)^{0})$ [from i. by Co-Unit']

iii. $({}^{0}(({}^{0}A)^{0}))^{0} \longrightarrow ({}^{0}A)^{0}$ [from ii. by Mon. of $\cdot^{0}$]

i'. $({}^{0}A)^{0} \longrightarrow ({}^{0}(({}^{0}A)^{0}))^{0}$ [by Co-unit']

(b) similarly for the other pair.

Productive Iterations

(I) $({}^0A)^0 \longrightarrow^{\bullet} ((({}^0A)^0)^{\bullet})$

simply by Co-unit. Whereas, ${}^{\bullet}((({}^0A)^0)^{\bullet}) \not\longrightarrow ({}^0A)^0$

(II) (a) i. $A^0 \longrightarrow A^0$ [Axiom]

ii. $A^0 \longrightarrow^0 ((A^0)^0)$ [from i. by Co-Unit']

iii. ${}^0({}^0((A^0)^0)) \longrightarrow^0 (A^0)$ [from ii. by Mon. of ${}^0\cdot$]

(b) similarly with the other pair.

For the application in this paper it is important to pay particular attention to the following difference between Galois-connected and residuated operators: while the pair of residuated operators $\Diamond\Box$ can disappear from a formula by means of the Unit $\Diamond\Box A \longrightarrow A$, there is not such possibility for the pairs of Galois, there is neither $({}^0A)^0 \longrightarrow A$ nor ${}^0(A^0) \longrightarrow A$. The failure of both these derivability relations is easily checked: in the definition of Galois operators both ${}^0\cdot$ and $\cdot^0$ are on the right side of the $\longrightarrow$ and hence they cannot be brought on the left as it happens for the $\Diamond$ (see the derivation of the Unit above 7.)

This fact is relevant for us, since we use pairs of Galois operators to mark "ungrammatical" expressions, and of course, they should not have the power of becoming grammatical by themselves, but rather only when a proper operators (a licensor) take scope over them.

Appendix C: A sample Natural Deduction derivation

(36) Kati nem látott mindenkit.
Kate not saw.3sg everyone-acc
'Kate did not see everyone'

The analytical assumptions in Figure 5 follow strictly what is argued for in Brody and Szabolcsi (2003) and merely recapture it in a different framework. These assumptions are as follows. Inflectional heads are obligatory; operator expressions are optional. Figure 5 contains three inflectional heads, C(complementizer), Agr(eement), and T(ense). Morphology spells out the finite verb in Agr but the verb does not move there in syntax. Negation is specified to occur only in the operator sequence of Agr; a name and a distributive universal may occur in any of the operator sequences. Although within a single sequence the universal is ordered before negation, negation is capable of scoping over it in (36) because the universal occurs in the operator sequence of T. The intervening Agr head mediates between the two sequences. The topic *Kati* occurs in the Agr-sequence. The topic and the universal bind traces of category *dp*.

Undecorated *s* serves as the category of uninflected sentences. The obligatoriness of inflectional heads is captured by assigning them categories decorated with an indexed box (see Moortgat (1999) for a detailed description of this use of unary operators). T(ense) for example has the category $\Box_T(s1/s)$. [$\Box$E] moves the decoration over to T in the form of $\langle\ldots\rangle_T$, and the structural rules abbreviated as [Pxxx] pass it back to the whole chunk containing T, right before Agr should enter the picture. Agr now has the category $\Box_A(s1/\Box_T s9)$, which crucially differs from that of T in that the argument it seeks is not uninflected S but a sentence already containing T. [$\Box$I] allows $(everyone\ \circ\ (T \circ (see\ \circ \Diamond\Box dp)))$ to be recognized as such. The same holds for C requiring an argument that contains Agr. C closes off the clause.

The value categories of all operators derive $s9$. Both Agr and C have $s9$ for their argument categories. This allows a full set of operator expressions – or no operator expression at all – to occur right below C and Agr. The categories of operators that freely occur in any sequence (i.e. either preverbally or postverbally) are not tagged for inflectional heads. [/I] is interpreted as λ-abstraction and thus allows the operators to bind their dp traces. Negation however occurs only in the preverbal field. To ensure this its argument category is decorated with $\Box_A$. The $\Box_A$ decoration on its whole functor category plays the same role as it does with Agr.

$$
\begin{prooftree}
\AxiomC{$C \vdash c/\Box_A s_9$}
\AxiomC{$\mathrm{Kati} \vdash s_9/(s_9/\Diamond\Box dp)$}
\AxiomC{$\mathrm{not} \vdash \Box_A(s_1^*/\Box_A s_1)$}
\RightLabel{[$\Box$E]}
\UnaryInfC{$\langle \mathrm{not}\rangle_A \vdash s_1^*/\Box_A s_1$}
\AxiomC{$\mathrm{Agr} \vdash \Box_A(s_1/\Box_T s_9)$}
\RightLabel{[$\Box$E]}
\UnaryInfC{$\langle \mathrm{Agr}\rangle_A \vdash s_1/\Box_T s_9$}
\AxiomC{$\mathrm{everyone} \vdash s_7/(s_7/\Diamond\Box dp)$}
\AxiomC{$\mathrm{T} \vdash \Box_T(s_1/s)$}
\RightLabel{[$\Box$E]}
\UnaryInfC{$\langle \mathrm{T}\rangle_T \vdash s_1/s$}
\AxiomC{$\mathrm{see} \vdash (s/dp)/dp$}
\AxiomC{$\Diamond\Box dp \vdash dp$}
\RightLabel{[/E]}
\BinaryInfC{$(\mathrm{see} \circ \Diamond\Box dp) \vdash s/dp$}
\RightLabel{[/E]}
\UnaryInfC{$((\mathrm{see} \circ \Diamond\Box dp) \circ \Diamond\Box dp) \vdash s$}
\RightLabel{[/E]}
\BinaryInfC{$\langle \mathrm{T}\rangle_T \circ ((\mathrm{see} \circ \Diamond\Box dp) \circ \Diamond\Box dp) \vdash s_1$}
\RightLabel{[2P$_{ass}$]}
\UnaryInfC{$(\langle \mathrm{T}\rangle_T \circ (\mathrm{see} \circ \Diamond\Box dp)) \circ \Diamond\Box dp \vdash s_1$}
\RightLabel{[$s_1 \longrightarrow s_7$]}
\UnaryInfC{$(\langle \mathrm{T}\rangle_T \circ (\mathrm{see} \circ \Diamond\Box dp)) \circ \Diamond\Box dp \vdash s_7$}
\RightLabel{[/I]}
\UnaryInfC{$(\langle \mathrm{T}\rangle_T \circ (\mathrm{see} \circ \Diamond\Box dp)) \vdash s_7/\Diamond\Box dp$}
\RightLabel{[/E]}
\BinaryInfC{$\mathrm{everyone} \circ (\langle \mathrm{T}\rangle_T \circ (\mathrm{see} \circ \Diamond\Box dp)) \vdash s_7$}
\RightLabel{[$s_7 \longrightarrow s_9$]}
\UnaryInfC{$(\mathrm{everyone} \circ (\langle \mathrm{T}\rangle_T \circ (\mathrm{see} \circ \Diamond\Box dp))) \vdash s_9$}
\RightLabel{[P$_{xxx}$]}
\UnaryInfC{$\langle(\mathrm{everyone} \circ (\mathrm{T} \circ (\mathrm{see} \circ \Diamond\Box dp)))\rangle_T \vdash s_9$}
\RightLabel{[$\Box$I]}
\UnaryInfC{$(\mathrm{everyone} \circ (\mathrm{T} \circ \mathrm{see} \circ \Diamond\Box dp)) \vdash \Box_T s_9$}
\RightLabel{[/E]}
\BinaryInfC{$\langle \mathrm{Agr}\rangle_A \circ (\mathrm{everyone} \circ (\mathrm{T} \circ \mathrm{see} \circ \Diamond\Box dp)) \vdash s_1$}
\RightLabel{[P$_{xxx}$]}
\UnaryInfC{$\langle(\mathrm{Agr} \circ (\mathrm{everyone} \circ (\mathrm{T} \circ \mathrm{see})) \circ \Diamond\Box dp)\rangle_A \vdash s_1$}
\RightLabel{[$\Box$I]}
\UnaryInfC{$(\mathrm{Agr} \circ (\mathrm{everyone} \circ (\mathrm{T} \circ \mathrm{see})) \circ \Diamond\Box dp) \vdash \Box_A s_1$}
\RightLabel{[/E]}
\BinaryInfC{$\langle \mathrm{not}\rangle_A \circ (\mathrm{Agr} \circ (\mathrm{everyone} \circ (\mathrm{T} \circ \mathrm{see})) \circ \Diamond\Box dp) \vdash s_1^*$}
\RightLabel{[2P$_{ass}$]}
\UnaryInfC{$(\langle \mathrm{not}\rangle_A \circ (\mathrm{Agr} \circ (\mathrm{everyone} \circ (\mathrm{T} \circ \mathrm{see})))) \circ \Diamond\Box dp \vdash s_1^*$}
\RightLabel{[$s_1^* \longrightarrow s_9$]}
\UnaryInfC{$(\langle \mathrm{not}\rangle_A \circ (\mathrm{Agr} \circ (\mathrm{everyone} \circ (\mathrm{T} \circ \mathrm{see})))) \circ \Diamond\Box dp \vdash s_9$}
\RightLabel{[/I]}
\UnaryInfC{$(\langle \mathrm{not}\rangle_A \circ (\mathrm{Agr} \circ (\mathrm{everyone} \circ (\mathrm{T} \circ \mathrm{see})))) \vdash s_9/\Diamond\Box dp$}
\BinaryInfC{$\mathrm{Kati} \circ (\langle \mathrm{not}\rangle_A \circ (\mathrm{Agr} \circ (\mathrm{everyone} \circ (\mathrm{T} \circ \mathrm{see})))) \vdash s_9$}
\RightLabel{[P$_{xxx}$]}
\UnaryInfC{$\langle \mathrm{Kati} \circ (\mathrm{not} \circ (\mathrm{Agr} \circ (\mathrm{everyone} \circ (\mathrm{T} \circ \mathrm{see}))))\rangle_A \vdash s_9$}
\RightLabel{[$\Box$I]}
\UnaryInfC{$\mathrm{Kati} \circ (\mathrm{not} \circ (\mathrm{Agr} \circ (\mathrm{everyone} \circ (\mathrm{T} \circ \mathrm{see})))) \vdash \Box_A s_9$}
\RightLabel{[/E]}
\BinaryInfC{$C \circ (\mathrm{Kati} \circ (\mathrm{not} \circ (\mathrm{Agr} \circ (\mathrm{everyone} \circ (\mathrm{T} \circ \mathrm{see}))))) \vdash c$}
\end{prooftree}
$$

Figure 5: Sample derivation