Agreeing Minimalist Grammars

Marina Ermolaeva Gregory M. Kobele

October 2023

1 Introduction

Minimalist grammars (MGs), introduced by Stabler (1997), represent a formalization of the foundational principles of the minimalist program (Chomsky 1995). A notable disparity has existed between the practical use of agreement in minimalist linguistics and its treatment in formal linguistic frameworks. Within the linguistic community, Agree is widely recognized as a fundamental grammatical operation, while in the formal community, it has been largely overlooked.

Agreement obtains when morpho-syntactic features of one item are influenced by those of another, as exemplified by subject-verb agreement.

(1) a. He *loves* waffles;
    b. They *love* waffles.

It is intuitive to treat the verbs in sentences (1a) and (1b) as being the same underlying lexical item, whose surface realization is determined by the person and number features it obtains from the subject.

Ermolaeva and Kobele (2022) propose an extension of MGs that incorporates a mechanism for agreement between lexical items. Their approach, which is similar to post-syntactic interpretations of Agree such as (Bobaljik 2008), decouples agreement from the syntax proper and mediates it through annotations on lexical items. To illustrate this concept metaphorically, one can envision morphological information as a liquid, syntactic dependencies established between lexical items as channels, and lexical annotations as sluices controlling the flow of liquid through channels. However, the formal characteristics of MGs augmented with agreement have yet to be thoroughly examined. Building upon and extending (Ermolaeva and Kobele 2023), this paper seeks to fill the gap.

2 Agreeing minimalist grammars

Standard minimalist grammars (Stabler 1997) define lexical items (LIs) as strings annotated with syntactic feature bundles. We use a version of this formalism where the structure-building operations Merge and Move are triggered by the same feature types. It checks matching features of opposite polarities — *positive*, of the form `+x`, and *negative*, of the form `-x`. Merge and Move can only target the first unchecked feature in any LI's bundle.

If an expression starts with a positive feature and contains a sub-expression with a matching negative one, the operation that applies is Move. Otherwise two expressions are combined via Merge, and the one with the positive feature becomes the head of the new expression. A *complete expression* has no unchecked features left other than `-t` on its head.

An example MG is shown in Figure 1, along with the derivation tree of the complete expression *the boy is jumping* generated by it. Without additional machinery, the subject-verb agreement in this example is handled by having multiple lexical items for determiners and auxiliaries and using distinct syntactic features to ensure that only compatible LIs can merge.

the :: `+n3s -d -k3s`

the :: `+n3p -d -k3p`

boy :: `-n3s`

boys :: `-n3p`

I :: `-d -k1s`

they :: `-d -k3p`

jumping :: `+d -g`

am :: `+g +k1s -t`

is :: `+g +k3s -t`

are :: `+g +k3p -t`

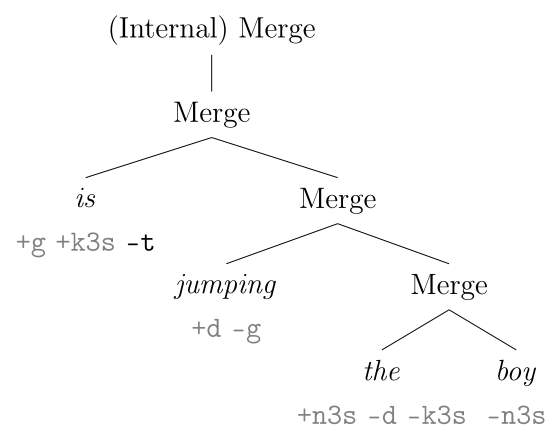

Figure 1: A toy minimalist grammar

More formally, and using the chain notation of (Stabler and Keenan 2003), a minimalist grammar G is defined as a 5-tuple $\langle \Sigma, \mathit{Syn}, \mathit{Types}, \mathit{Lex}, \mathit{Merge} \rangle$, where:

- Σ is a finite set of pronounced segments;
- Let *Base* be a finite set of *categories* such that $\mathtt{t} \in \mathit{Base}$. $\mathit{Syn} = \{\mathtt{+f} \mid \mathtt{f} \in \mathit{Base}\} \cup \{\mathtt{-f} \mid \mathtt{f} \in \mathit{Base}\}$ is a set of syntactic features;

- $Types = \{::, :\}$ (lexical, derived);

- Let the set of *initial chains* $IC = \Sigma^*\ Types\ Syn^*$, and the set of *non-initial chains* $NC = \Sigma^*\ \{:\}\ Syn^*$. The lexicon $Lex \subset \Sigma^*\ \{::\}\ Syn^*$, a subset of IC, is a finite set of lexical items;

- *Merge* is the union of the following five functions, for $s, t \in \Sigma^*$; $\cdot \in Types$; $\texttt{+f}, \texttt{-f} \in Syn$; $\gamma, \zeta \in Syn^*$; $\delta \in Syn^+$; and for $\alpha_1, ..., \alpha_k,\ \beta_1, ..., \beta_k\ \in NC$ ($k \geq 0$) such that none of $\alpha_1, ..., \alpha_k$ have $\texttt{-f}$ as their first feature:

$$\frac{s\ ::\ \texttt{+f}\gamma \qquad t\ \cdot\ \texttt{-f},\ \beta_1, ..., \beta_l}{st\ :\ \gamma,\ \beta_1, ..., \beta_l}\ \text{(right Merge)}$$

$$\frac{s\ :\ \texttt{+f}\gamma,\ \alpha_1, ..., \alpha_k \qquad t\ \cdot\ \texttt{-f},\ \beta_1, ..., \beta_l}{ts\ :\ \gamma,\ \alpha_1, ..., \alpha_k,\ \beta_1, ..., \beta_l}\ \text{(left Merge)}$$

$$\frac{s\ \cdot\ \texttt{+f}\gamma,\ \alpha_1, ..., \alpha_k \qquad t\ \cdot\ \texttt{-f}\delta,\ \beta_1, ..., \beta_l}{s\ :\ \gamma,\ \alpha_1, ..., \alpha_k,\ t\ :\ \delta,\ \beta_1, ..., \beta_l}\ \text{(non-final Merge)}$$

$$\frac{s\ \cdot\ \texttt{+f}\gamma,\ \alpha_1, ..., \alpha_{i-1},\ t\ \texttt{-f},\ \alpha_{i+1}, ..., \alpha_k}{ts\ :\ \gamma,\ \alpha_1, ..., \alpha_{i-1},\ \alpha_{i+1}, ..., \alpha_k}\ \text{(final internal Merge)}$$

$$\frac{s\ \cdot\ \texttt{+f}\gamma,\ \alpha_1, ..., \alpha_{i-1},\ t\ \texttt{-f}\delta,\ \alpha_{i+1}, ..., \alpha_k}{s\ :\ \gamma,\ \alpha_1, ..., \alpha_{i-1},\ t\ :\ \delta,\ \alpha_{i+1}, ..., \alpha_k}\ \text{(non-final internal Merge)}$$

- An expression is a member of $IC\ NC^*$. A complete expression has the form $s\ \cdot\ \texttt{-t}$, where $s \in \Sigma^*, \cdot \in Types$.

Using this formalism as a starting point, we introduce *agreeing minimalist grammars*. In order to define an agreeing MG, one has to specify two components:

- An annotated lexicon;

- A set of morphological equations.

The lexicon differs from standard MGs in two ways. First, string components of LIs are replaced with a unique identifier of the lexeme (indicated by smallcaps) and a finite set of morphological features; the latter can either have values or be unvalued ($\perp$).

Second, directions of exchange of morphological information are encoded as *channels* annotated on syntactic features. Emitting channels, which allow information to flow out, are denoted by $^{\rightarrow}$ and can be annotated with lexically specified morphological features. Receiving channels, which accept information, are denoted by $_{\leftarrow}$.

These lexical items are generally *underspecified* (in a sense similar to underspecification of a segment in phonology). Each underspecified LI can be thought of as a shorthand for a finite set of LIs whose morphological features are fully valued, and channels annotated with all information they emit or receive. Figure 2 shows an example lexicon, where only $[\textsc{jumping}] :: \texttt{+d -g}$ is fully specified, since it has no unvalued features or agreement channels. The rest of LIs can be instantiated in different ways, depending on how their morphological features are valued and what information gets transmitted through their channels in a given derivation.

$$[\textsc{the}] :: \texttt{+n}^{\rightarrow}_{\leftarrow}\ \texttt{-d}\ \texttt{-k}^{\rightarrow}_{\leftarrow}$$
$$\left[\textsc{boy}, \substack{\phi:3\textsc{s}\\ c:\bot}\right] :: \texttt{-n}^{\rightarrow\phi:3\textsc{s}}_{\leftarrow}$$
$$\left[\textsc{boy}, \substack{\phi:3\textsc{p}\\ c:\bot}\right] :: \texttt{-n}^{\rightarrow\phi:3\textsc{p}}_{\leftarrow}$$
$$\left[\textsc{i}, \substack{\phi:1\textsc{s}\\ c:\bot}\right] :: \texttt{-d}\ \texttt{-k}^{\rightarrow\phi:1\textsc{s}}_{\leftarrow}$$
$$\left[\textsc{they}, \substack{\phi:3\textsc{p}\\ c:\bot}\right] :: \texttt{-d}\ \texttt{-k}^{\rightarrow\phi:3\textsc{p}}_{\leftarrow}$$
$$\left[\textsc{be}, \phi:\bot\right] :: \texttt{+g}\ \texttt{+k}^{\rightarrow c:\textsc{nom}}_{\leftarrow}\ \texttt{-t}$$
$$[\textsc{jumping}] :: \texttt{+d}\ \texttt{-g}$$

Figure 2: Agreeing MG

The string components are decoupled from the LIs and stored as a list of morphological equations that map lexeme identifiers and morphological features to strings. The list of morphological equations associated with the agreeing MG in Figure 2 is given in Figure 3.

$$[\textsc{the}] = \textit{the}$$
$$\left[\textsc{boy}, \substack{\phi:3\textsc{s}\\ c:\textsc{nom}}\right] = \textit{boy}$$
$$\left[\textsc{boy}, \substack{\phi:3\textsc{p}\\ c:\textsc{nom}}\right] = \textit{boys}$$
$$\left[\textsc{i}, c:\textsc{nom}\right] = \textit{I}$$
$$\left[\textsc{they}, c:\textsc{nom}\right] = \textit{they}$$
$$\left[\textsc{be}, \phi:1\textsc{s}\right] = \textit{am}$$
$$\left[\textsc{be}, \phi:3\textsc{s}\right] = \textit{is}$$
$$\left[\textsc{be}, \phi:3\textsc{p}\right] = \textit{are}$$
$$[\textsc{jumping}] = \textit{jumping}$$

Figure 3: Morphological equations

Morphological equations function in a way similar to Distributed Morphology's (Halle and Marantz 1993) vocabulary insertion. This allows for competition between equations and conflict resolution through the Subset Principle, where the equation matching the greatest number of the LI's features is chosen.

Agreement is defined as transmission of morphological feature values between LIs at each Merge step, when (at least) one of the syntactic features being checked carries an emitting channel, and the other a receiving channel. In the example lexicon in Figure 2, ϕ-features (person and number) and c(ase) values are transmitted across the {`+n`, `-n`} and {`+k`, `-k`} dependencies, enforcing subject-verb agreement. Agreement assumptions (2) specify how morphological features pass over channels and how conflicts are resolved.

(2) **Agreement assumptions**

(i) Lexically specified feature values supersede those received from other LIs;

(ii) Emitting channels send out the last version of all feature values received through any receiving channel of the LI, modified by (i) and (iii);

(iii) No information is sent back along the same syntactic dependency that it was received through.

The condition (ii) is the overwriting principle used in (Ermolaeva and Kobele 2022) to ensure that features received along later channels take priority. Additionally, (i) prevents values specified in the lexicon from being overwritten, and (iii) avoids breaking agreement in cases when a syntactic feature can both emit and receive information. Taken together, these assumptions ensure that each emitting channel sends out either the lexically determined value of each morphological feature (if present) or the value received through the latest channel that is not attached to the same syntactic feature.

Agreement updates not only feature values associated with emitting channels, as per (ii), but also those in the lexical items' morphological feature bundles, with the same assumptions in effect. Namely, each morphological feature associated with an LI gets either the original lexically specified value or, if initially unvalued, the latest value provided by the LI's receiving channels.

For a specific example, consider the derivation of *the boy is jumping*. The syntactic dependencies between the LIs and the paths taken by transmitted morphological features are schematically represented in Figure 4.

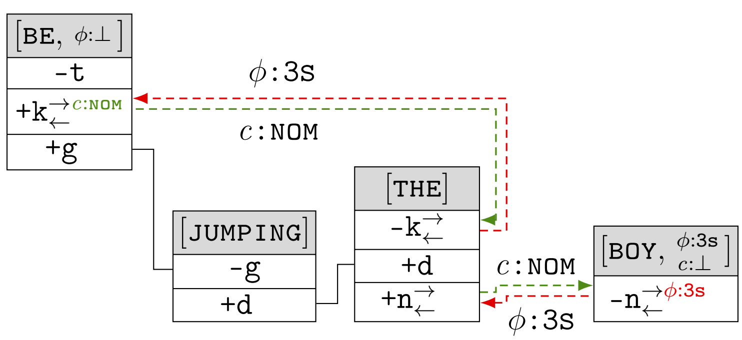

Figure 4: Dependency graph of *the boy is jumping*

In this derivation, BOY transmits its ϕ:3S feature to THE via the {`+n`, `-n`} channel. In its turn, the emitting channel on THE's `-k` feature picks up these values, following the agreement assumptions (ii) and (iii), to be transmitted via BE s `+k` receiving channel. Since each syntactic feature in the {+n, -n} and {+k, -k} pairs has both an emitting and a receiving channel, the same syntactic dependencies also transmit c:NOM from BE to BOY.

3 Relation to attribute grammars

The MG implementation of agreement can be viewed from the perspective of attribute grammars (Knuth 1968, AGs), which were originally developed as a means of semantically interpreting context-free grammars. An AG consists of a context-free grammar, where each node (terminal and non-terminal) has a set of *attributes*, the values of which are determined by *attribute equations* attached to the rules.

The derivation trees of any MG are given by a context-free grammar whose non-terminals are tuples of feature bundles (Michaelis 2001). An alternative but equivalent representation for MG derivations (Kobele 2012), reminiscent of TAG derivation trees, is more useful for our present purposes. These can be obtained from the graphs above by removing all edges between movement features (at which point they become trees). In general, lexical items will be associated with a family of context-free rules in Greibach normal form (GNF), where the lexical item itself is the terminal element.

For example, in the grammar discussed above the lexical item BE is associated with the following context-free rule in GNF:

(3) $\langle \texttt{-t} \rangle \rightarrow \text{BE}\ \langle \texttt{-g; -k} \rangle$

This rule mirrors the node BE in the dependency structure of Figure 4, minus the movement edges. In that structure, the node BE has a single dependent, namely the node JUMPING. This node, upon being connected to BE heads a tree whose open features are -g and -k. This corresponds to the non-terminal $\langle \texttt{-g}, \texttt{-k} \rangle$ in the rule. After satisfying its positive features, the tree rooted in the node BE in Figure 4 has open features -t. This corresponds to the left-hand side non-terminal $\langle \texttt{-t} \rangle$. Rules for all nodes of this tree are given below.

$$\langle \texttt{-t} \rangle \rightarrow \text{BE}\ \langle \texttt{-g; -k} \rangle$$
$$\langle \texttt{-g; -k} \rangle \rightarrow \text{JUMPING}\ \langle \texttt{-d -k} \rangle$$
$$\langle \texttt{-d -k} \rangle \rightarrow \text{THE}\ \langle \texttt{-n} \rangle$$
$$\langle \texttt{-n} \rangle \rightarrow \text{BOY}$$

Figure 5: Context-free rules

To systematize the translation, each node has as attributes a number of copies of the morphological features of the grammar, one copy per emitting or receiving channel. The attribute equations fall into the following three groups:

- **Lexical:** represent lexically determined morphological feature values. Each instance of a morphological feature value specified in the grammar corresponds to one lexical attibute equation;
- **Agreement:** determined by the channel annotations and represent exchange of information between lexical items. Each pair of channels on matched syntactic features that allows agreement information to be passed through it corresponds to a set of agreement attribute equations, one per morphological feature in the grammar;
- **Consistency:** represent exchange of information within a lexical item. Consistency attribute equations are an implementation of the agreement assumptions in (2).

Any instance of a morphological feature that does not appear on the left-hand side of an attribute equation remains unvalued. In order to minimize notational clutter, we suppress unvalued features in channel annotations.

In the dependency structure in Figure 4 the lexical item BOY has only one emitting and receiving channel. Labeling the copies of morphological features associated with its only emitting channel as ϕ_1, c_1, and those associated with its receiving channel as ϕ_2, c_2, we get a single lexical attribute equation:[1]

(4) $\quad \text{BOY}.\phi_1 := 3\text{S}$

Now consider THE, which has two pairs of channels. We could give the following labels to the associated morphological features:

(5) $\quad [\text{THE}]\ \texttt{+n}^{\rightarrow c_1,\phi_1}_{\leftarrow c_2,\phi_2}\ \texttt{-d}\ \texttt{-k}^{\rightarrow c_3,\phi_3}_{\leftarrow c_4,\phi_4}$

THE agrees with BOY across the {+n, -n} feature. This agreement transmits THE's ϕ_1 and c_1 to BOY's ϕ_2 and c_2, and BOY's ϕ_1 and c_1 to THE's ϕ_2 and c_2. Within this derivation, BOY transmits only its ϕ value, and THE only its c value; however, nothing in the system would prevent them from exchanging (different) values of the same feature. This translates into attribute equations as follows:

(6)
$$\begin{array}{ll} \text{BOY}.\phi_2 := \text{THE}.\phi_1 & \text{THE}.\phi_2 := \text{BOY}.\phi_1 \\ \text{BOY}.c_2 := \text{THE}.c_1 & \text{THE}.c_2 := \text{BOY}.c_1 \end{array}$$

[1]The attributes are here written using record notation instead of function notation ($A.f$ instead of $f(A)$).

Finally, consistency requires that whatever morphological features THE receives across its `-k` channel be emitted via `+n`. Similarly, features received via its `+n` dependency must be sent out by the `-k` channel. In both cases, as required by the agremeent assumptions, the emitting channel sends out the last version of all features it receives, with the exception that no information is sent back across the same syntactic dependency it was received. Thus, the only consistency equations involving THE are the following:

$$(7) \qquad \begin{array}{ll} \text{THE}.\phi_1 := \text{THE}.\phi_4 & \text{THE}.\phi_3 := \text{THE}.\phi_2 \\ \text{THE}.c_1 := \text{THE}.c_4 & \text{THE}.c_3 := \text{THE}.c_2 \end{array}$$

The GNF context-free derivation tree for *the boy is jumping*, along with all attribute equations, is given in Figure 6. Note that some copies of morphological features (such as $\text{THE}.\phi_1$) never receive a value.

Lexical:

$\text{BOY}.\phi_1 := 3\text{S}$

$\text{BE}.c_1 := \text{NOM}$

Agreement:

$\text{BOY}.\phi_2 := \text{THE}.\phi_1$

$\text{BOY}.c_2 := \text{THE}.c_1$

$\text{THE}.\phi_2 := \text{BOY}.\phi_1$

$\text{THE}.c_2 := \text{BOY}.c_1$

$\text{BE}.\phi_2 := \text{THE}.\phi_3$

$\text{BE}.c_2 := \text{THE}.c_3$

$\text{THE}.\phi_4 := \text{BE}.\phi_1$

$\text{THE}.c_4 := \text{BE}.c_1$

Consistency:

$\text{THE}.\phi_1 := \text{THE}.\phi_4$

$\text{THE}.c_1 := \text{THE}.c_4$

$\text{THE}.\phi_3 := \text{THE}.\phi_2$

$\text{THE}.c_3 := \text{THE}.c_2$

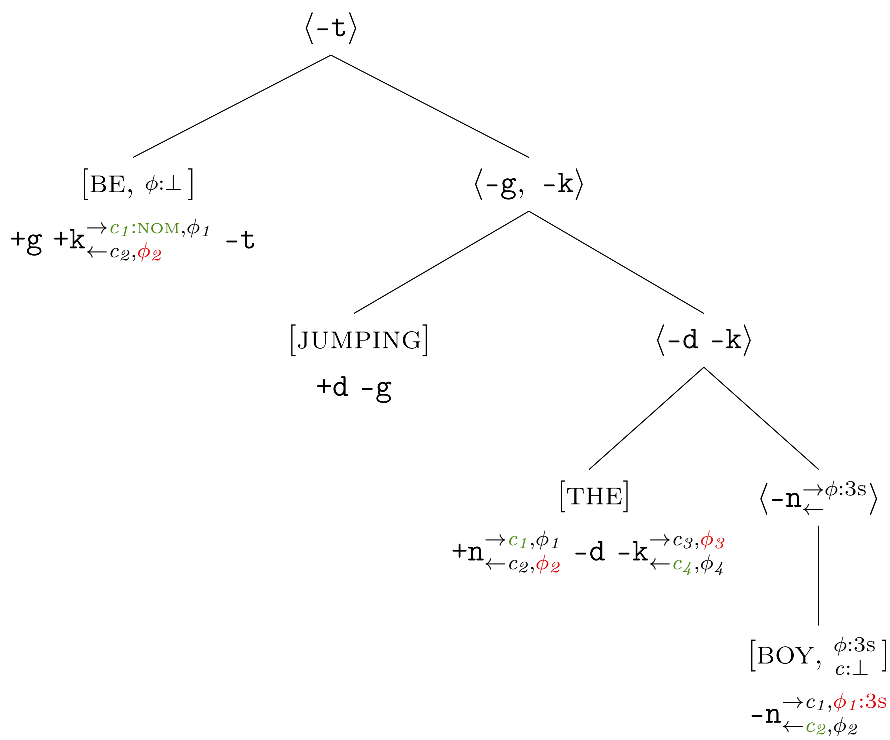

Figure 6: Attribute equations of *the boy is jumping*

Solving these equations produces channel annotations — morphological feature values transmitted through each channel within this specific derivation. At this point, the morphological feature bundles of all LIs are valued based on information received through their channels; the resulting set of specific instantiations of the LIs is given in Figure 7. Finally, we replace each LI's lexeme identifier and morphological features with a string realization obtained from morphological equations and subscript each syntactic feature with channel annotations; by convention, the channel sending information to a positive-polarity syntactic feature is listed first. This step yields a set of fully specified lexical items compatible with the formalism of standard MGs (Figure 8).

$[\text{THE}] :: \texttt{+n}^{\to c:\text{NOM}}_{\leftarrow \phi:3\text{S}}\ \texttt{-d}\ \texttt{-k}^{\to \phi:3\text{S}}_{\leftarrow c:\text{NOM}}$

$[\text{BOY}, {}^{\phi:3\text{S}}_{c:\text{NOM}}] :: \texttt{-n}^{\to \phi:3\text{S}}_{\leftarrow c:\text{NOM}}$

$[\text{BE}, {}^{\phi:3\text{S}}] :: \texttt{+g}\ \texttt{+k}^{\to c:\text{NOM}}_{\leftarrow \phi:3\text{S}}\ \texttt{-t}$

$[\text{JUMPING}] :: \texttt{+d}\ \texttt{-g}$

Figure 7: Annotated LIs of Figure 6

the $:: \texttt{-n}_{[\phi:3\text{S}][c:\text{NOM}]}\ \texttt{-d}\ \texttt{-k}_{[\phi:3\text{S}][c:\text{NOM}]}$

boy $:: \texttt{-n}_{[\phi:3\text{S}][c:\text{NOM}]}$

is $:: \texttt{+g}\ \texttt{-k}_{[\phi:3\text{S}][c:\text{NOM}]}\ \texttt{-t}$

jumping $:: \texttt{+d}\ \texttt{-g}$

Figure 8: Subscripted LIs of Figure 6

4 Unpacking a lexicon

As mentioned earlier, LIs in an annotated lexicon are underspecified, in the sense that each of them represents a set of LIs where the morphological feature bundle and the information transmitted across agreement channels is fully instantiated. An underspecified lexicon can be considered a shorthand for an *unpacked* lexicon of fully specified LIs, where all feature bundles are valued, and each channel carries exactly the set of features it actually transmits or receives. The entire lexicon can be unpacked and then straightforwardly converted into a standard MG via the following unpacking algorithm.

Step 1. For each underspecified LI, generate its *augments* — the set of LIs with all possible combinations of morphological feature annotations for each available channel.

Step 2. Check each augment against the agreement assumptions (2), which serve as a lexical filter. The filter is defined locally, strictly in terms of properties of individual LIs. This allows each augment to be inspected for internal consistency in isolation, without considering its context in a derivation. LIs that fail the check are removed from the unpacked lexicon. For example, (8) and (9) are valid augments of THE $:: \texttt{+n}^{\to}_{\leftarrow}\ \texttt{-d}\ \texttt{-k}^{\to}_{\leftarrow}$, whereas (10)–(12) are not.

(8) $[\text{THE}] :: \texttt{+n}^{\to c:\text{NOM}}_{\leftarrow \phi:3\text{S}}\ \texttt{-d}\ \texttt{-k}^{\to \phi:3\text{S}}_{\leftarrow c:\text{NOM}}$

(9) $[\text{THE}] :: \texttt{+n}^{\to c:\text{NOM}}_{\leftarrow \phi:3\text{P}}\ \texttt{-d}\ \texttt{-k}^{\to \phi:3\text{P}}_{\leftarrow c:\text{NOM}}$

(10) $[\text{THE}]$ :: $\texttt{+n}^{\rightarrow c:\text{ACC}}_{\leftarrow \phi:3\text{S}}$ $\texttt{-d}$ $\texttt{-k}^{\rightarrow \phi:1\text{S}}_{\leftarrow c:\text{NOM}}$

(emits features it did not receive)

(11) $[\text{THE}]$ :: $\texttt{+n}^{\rightarrow c:\text{NOM}}_{\leftarrow \phi:3\text{S}}$ $\texttt{-d}$ $\texttt{-k}^{\rightarrow}_{\leftarrow c:\text{NOM}}$

(does not emit all received features)

(12) $[\text{THE}]$ :: $\texttt{+n}^{\rightarrow c:\text{NOM},\phi:3\text{S}}_{\leftarrow c:\text{NOM},\phi:3\text{S}}$ $\texttt{-d}$ $\texttt{-k}^{\rightarrow c:\text{NOM},\phi:3\text{S}}_{\leftarrow c:\text{NOM},\phi:3\text{S}}$

(sends features back along the same dependency)

Step 3. For each remaining augment, obtain its string realization from the morphological equations based on its lexeme identifier, the last value of each morphological feature it received, and lexically specified features. Syntactic features are subscripted with their channel annotations. For clarity, we also remove any useless LIs (that cannot be part of any complete expression) by converting the grammar into a context-free grammar, as discussed in the previous section, and removing non-generating and unreachable rules. The standard MG obtained from Figure 2 in this way is given in Figure 9.

the :: $\texttt{-n}_{[\phi:3\text{S}][c:\text{NOM}]}$ $\texttt{-d}$ $\texttt{-k}_{[\phi:3\text{S}][c:\text{NOM}]}$	*they* :: $\texttt{-d}$ $\texttt{-k}_{[\phi:3\text{P}][c:\text{NOM}]}$
the :: $\texttt{-n}_{[\phi:3\text{P}][c:\text{NOM}]}$ $\texttt{-d}$ $\texttt{-k}_{[\phi:3\text{P}][c:\text{NOM}]}$	*jumping* :: $\texttt{+d}$ $\texttt{-g}$
boy :: $\texttt{-n}_{[\phi:3\text{S}][c:\text{NOM}]}$	*am* :: $\texttt{+g}$ $\texttt{-k}_{[\phi:1\text{S}][c:\text{NOM}]}$ $\texttt{-t}$
boys :: $\texttt{-n}_{[\phi:3\text{P}][c:\text{NOM}]}$	*is* :: $\texttt{+g}$ $\texttt{-k}_{[\phi:3\text{S}][c:\text{NOM}]}$ $\texttt{-t}$
I :: $\texttt{-d}$ $\texttt{-k}_{[\phi:1\text{S}][c:\text{NOM}]}$	*are* :: $\texttt{+g}$ $\texttt{-k}_{[\phi:3\text{P}][c:\text{NOM}]}$ $\texttt{-t}$

Figure 9: Standard MG counterpart of Figure 2

5 An abstract example

As a further illustration of agreement as channels and its implementation as the unpacking procedures outlined above, consider an abstract example from (Ermolaeva 2018) presented in Figure 10.

$$[\text{A},\ m{:}\bot] :: \texttt{-a}_{\leftarrow}\ \texttt{-h}_{\leftarrow}$$
$$[\text{B},\ m{:}\bot] :: \texttt{+a}^{\rightarrow}\ \texttt{-b}_{\leftarrow}\ \texttt{-g}_{\leftarrow}$$
$$[\text{C},\ m{:}\bot] :: \texttt{+b}^{\rightarrow}\ \texttt{-c}\ \texttt{-f}_{\leftarrow}$$
$$[\text{X},\ m{:}\text{V}] :: \texttt{+c}\ \texttt{+g}^{\rightarrow m:\text{V}}\ \texttt{-x}$$

Figure 10: An abstract agreeing MG

The dependency graph in Figure 11 represents the derivation of an incomplete expression. The lexical item C still has its `-f`, and A has an unchecked `-h`.

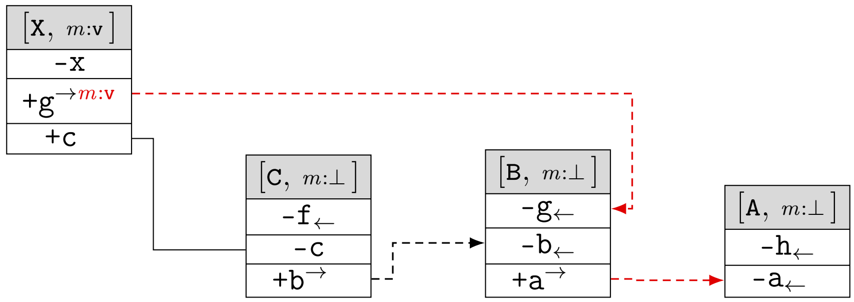

Figure 11: Dependency graph for a derivation in Figure 10

Just before the latest step of this incomplete derivation — Move via {`+g`, `-g`} — there are three moving non-initial chains in the structure. They are headed by A, B, and C and carry `-h`, `-g`, and `-f` respectively. At the Move step, B receives the value m:V from X and transmits it to A, whereas the feature m on C remains unvalued.

If X did not transmit m:V to B, and C received some value of m later in the derivation (via its `-f`), it would be overwritten on all three non-initial chains: C, B (which is connected to C via {`+b`, `-b`}), and A (which is connected to B via {`+a`, `-a`}). On the other hand, regardless of information transmitted across other channels, any agreement received by A via `-h` would rewrite only its own value of m.

Lexical:

X.m_1 := V

Agreement:

A.m_1 := B.m_1

B.m_2 := C.m_1

B.m_3 := X.m_1

Consistency:

C.m_1 := C.m_2

B.m_1 := B.m_2 < B.m_3

A.m_1 := A.m_2

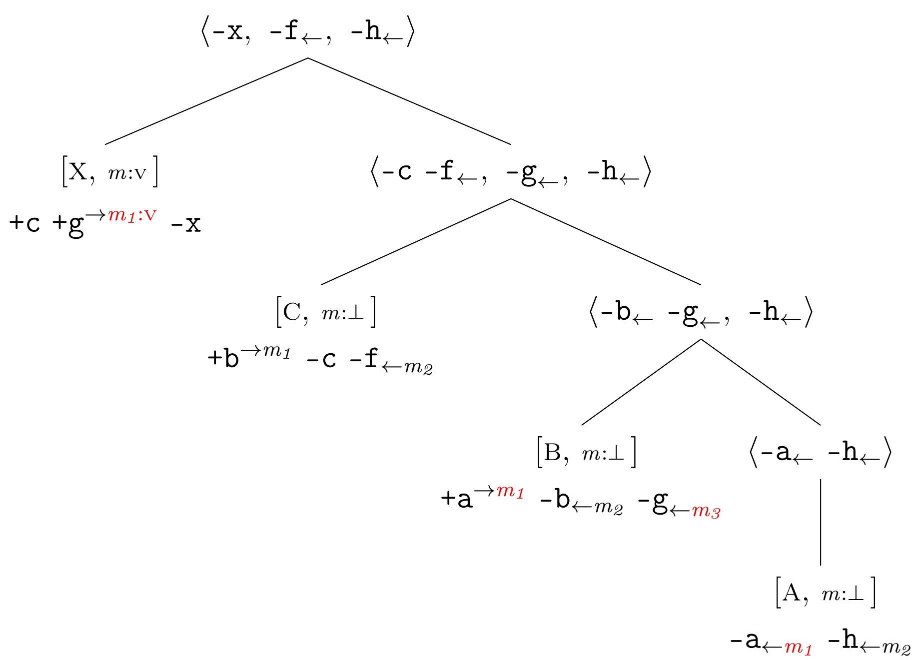

Figure 12: Attribute equations for Figure 11

As discussed by Ermolaeva (2018), encoding this directly in chain-based MGs would require keeping track of which chains inherit features from which. This is possible (since the number of chains in an expression is guaranteed to be finite) but challenging and bookkeeping-heavy. Alternatively, the passing of information between these lexical items can be represented as the attribute equations in Figure 12. Note that overwriting of values is implemented as a consistency equation: $\textsc{b}.m_1$ receives a value from $\textsc{b}.m_2$ and $\textsc{b}.m_3$, prioritizing the latter. This ensures that B's value of m does not get overwritten, even if another value is transmitted later in the derivation over {+b, -b}.

Solving the equations yields a set of LIs that are partially annotated, since the derivation is incomplete. These LIs are given in Figure 13. In particular, A is underspecified with respect to its -h channel, B has no annotation on its -b channel, C has no annotations, and X is fully annotated.

$$[\text{A},\ m{:}\text{v}]\ ::\ \texttt{-a}_{\leftarrow m:\text{v}}\ \texttt{-h}_{\leftarrow}$$
$$[\text{B},\ m{:}\text{v}]\ ::\ \texttt{+a}^{\rightarrow m:\text{v}}\ \texttt{-b}_{\leftarrow}\ \texttt{-g}_{\leftarrow m:\text{v}}$$
$$[\text{C},\ m{:}\bot]\ ::\ \texttt{+b}^{\rightarrow}\ \texttt{-c}\ \texttt{-f}_{\leftarrow}$$
$$[\text{X},\ m{:}\text{v}]\ ::\ \texttt{+c}\ \texttt{+g}^{\rightarrow m:\text{v}}\ \texttt{-x}$$

Figure 13: Partially annotated LIs of Figure 10

To unpack this grammar, as decribed in Section 4, we first construct all augments of the underspecified LIs (Figure 14) and then filter out those that are not consistent with the lexical filter of agreement assumptions. The resulting unpacked lexicon in Figure 15 contains multiple augments compatible with the partially annotated LIs above, highlighted in red.

$$[\text{A},\ m{:}\bot]\ ::\ \texttt{-a}_{\leftarrow}\ \texttt{-h}_{\leftarrow}$$
$$[\text{A},\ m{:}\text{v}]\ ::\ \texttt{-a}_{\leftarrow}\ \texttt{-h}_{\leftarrow m:\text{v}}$$
$$[\text{A},\ m{:}\text{v}]\ ::\ \texttt{-a}_{\leftarrow m:\text{v}}\ \texttt{-h}_{\leftarrow}$$
$$[\text{A},\ m{:}\text{v}]\ ::\ \texttt{-a}_{\leftarrow m:\text{v}}\ \texttt{-h}_{\leftarrow m:\text{v}}$$

$$[\text{B},\ m{:}\bot]\ ::\ \texttt{+a}^{\rightarrow}\ \texttt{-b}_{\leftarrow}\ \texttt{-g}_{\leftarrow}$$
$$[\text{B},\ m{:}\text{v}]\ ::\ \texttt{+a}^{\rightarrow}\ \texttt{-b}_{\leftarrow}\ \texttt{-g}_{\leftarrow m:\text{v}}$$
$$[\text{B},\ m{:}\text{v}]\ ::\ \texttt{+a}^{\rightarrow}\ \texttt{-b}_{\leftarrow m:\text{v}}\ \texttt{-g}_{\leftarrow}$$
$$[\text{B},\ m{:}\text{v}]\ ::\ \texttt{+a}^{\rightarrow}\ \texttt{-b}_{\leftarrow m:\text{v}}\ \texttt{-g}_{\leftarrow m:\text{v}}$$
$$[\text{B},\ m{:}\bot]\ ::\ \texttt{+a}^{\rightarrow m:\text{v}}\ \texttt{-b}_{\leftarrow}\ \texttt{-g}_{\leftarrow}$$
$$[\text{B},\ m{:}\text{v}]\ ::\ \texttt{+a}^{\rightarrow m:\text{v}}\ \texttt{-b}_{\leftarrow}\ \texttt{-g}_{\leftarrow m:\text{v}}$$
$$[\text{B},\ m{:}\text{v}]\ ::\ \texttt{+a}^{\rightarrow m:\text{v}}\ \texttt{-b}_{\leftarrow m:\text{v}}\ \texttt{-g}_{\leftarrow}$$
$$[\text{B},\ m{:}\text{v}]\ ::\ \texttt{+a}^{\rightarrow m:\text{v}}\ \texttt{-b}_{\leftarrow m:\text{v}}\ \texttt{-g}_{\leftarrow m:\text{v}}$$

$$[\text{C},\ m{:}\bot]\ ::\ \texttt{+b}^{\rightarrow}\ \texttt{-c}\ \texttt{-f}_{\leftarrow}$$
$$[\text{C},\ m{:}\text{v}]\ ::\ \texttt{+b}^{\rightarrow}\ \texttt{-c}\ \texttt{-f}_{\leftarrow m:\text{v}}$$
$$[\text{C},\ m{:}\bot]\ ::\ \texttt{+b}^{\rightarrow m:\text{v}}\ \texttt{-c}\ \texttt{-f}_{\leftarrow}$$
$$[\text{C},\ m{:}\text{v}]\ ::\ \texttt{+b}^{\rightarrow m:\text{v}}\ \texttt{-c}\ \texttt{-f}_{\leftarrow m:\text{v}}$$

$$[\text{X},\ m{:}\text{v}]\ ::\ \texttt{+c}\ \texttt{+g}^{\rightarrow m:\text{v}}\ \texttt{-x}$$

Figure 14: All augments of Figure 10

$[\mathrm{A},\ m{:}\bot] :: \texttt{-a}_{\leftarrow}\ \texttt{-h}_{\leftarrow}$

$[\mathrm{A},\ m{:}\mathrm{v}] :: \texttt{-a}_{\leftarrow}\ \texttt{-h}_{\leftarrow m:\mathrm{v}}$

$[\mathrm{A},\ m{:}\mathrm{v}] :: \texttt{-a}_{\leftarrow m:\mathrm{v}}\ \texttt{-h}_{\leftarrow}$

$[\mathrm{A},\ m{:}\mathrm{v}] :: \texttt{-a}_{\leftarrow m:\mathrm{v}}\ \texttt{-h}_{\leftarrow m:\mathrm{v}}$

$[\mathrm{B},\ m{:}\bot] :: \texttt{+a}^{\rightarrow}\ \texttt{-b}_{\leftarrow}\ \texttt{-g}_{\leftarrow}$

$[\mathrm{B},\ m{:}\mathrm{v}] :: \texttt{+a}^{\rightarrow m:\mathrm{v}}\ \texttt{-b}_{\leftarrow}\ \texttt{-g}_{\leftarrow m:\mathrm{v}}$

$[\mathrm{B},\ m{:}\mathrm{v}] :: \texttt{+a}^{\rightarrow m:\mathrm{v}}\ \texttt{-b}_{\leftarrow m:\mathrm{v}}\ \texttt{-g}_{\leftarrow}$

$[\mathrm{B},\ m{:}\mathrm{v}] :: \texttt{+a}^{\rightarrow m:\mathrm{v}}\ \texttt{-b}_{\leftarrow m:\mathrm{v}}\ \texttt{-g}_{\leftarrow m:\mathrm{v}}$

$[\mathrm{C},\ m{:}\bot] :: \texttt{+b}^{\rightarrow}\ \texttt{-c}\ \texttt{-f}_{\leftarrow}$

$[\mathrm{C},\ m{:}\mathrm{v}] :: \texttt{+b}^{\rightarrow m:\mathrm{v}}\ \texttt{-c}\ \texttt{-f}_{\leftarrow m:\mathrm{v}}$

$[\mathrm{X},\ m{:}\mathrm{v}] :: \texttt{+c}\ \texttt{+g}^{\rightarrow m:\mathrm{v}}\ \texttt{-x}$

Figure 15: Augments of Figure 10 after lexical filter

6 Discussion and future work

In this paper, we have sketched a formalization of the channel-based agreement system of (Ermolaeva and Kobele 2022), which we call agreeing MGs. Underspecified lexical items carry morphological features, which may start out valued or unvalued and can be transmitted to other LIs across syntactic dependencies. Pronounced strings are decoupled from lexical items and placed into morphological equations. This implementation is modular; in particular, the agreement assumptions and the way lexeme identifiers and morphological features are mapped to strings via morphological equations can be modified without affecting the rest of the system.

Agreeing MGs are naturally formalized in terms of AGs. As morphological features are finite-valued, the attributes can be unpacked into the lexicon. Normally AGs are used to obtain a value from the entire derivation tree, but here we are just sending information to leaves. The appropriate interpretation of a derivation tree in this setting is as a sentence (a string of words). It is well-known that minimalist derivation trees can be mapped to sentences with a single synthesized (bottom-up) attribute being a tuple of strings (Michaelis 2001). It is simple to overlay this onto our present system, so that the words used depend on the morphological features inherited. By separating agreement and word order, we can see the distinct nature of both.

One interesting avenue for future work involves *agglomeration of information*. Our current agreement assumptions is based on overwriting; each emitting channel must send out either lexically specified values or, lacking those, the rightmost set of values they received. This handles basic cases including agreement in English but appears insufficient for some phenomena that involve interaction between values obtained through different channels. A well-known example is feature resolution in coordinate noun phrases, in which which the features (such as gender and number) of multiple individual conjuncts contribute to those of a coordinate structure (Corbett 1983).

In general, agreement assumptions can be considered a function that computes an output from multiple morphological feature values. Overwriting is a special case of such a function that returns the priority union of its arguments; and a different function would be needed to deal with feature resolution in coordinate structures.

Another issue is the possibility of *cyclicity* in agreement. A major topic in the AG literature is detecting and avoiding cyclic dependencies between attributes. In the general case, detecting cyclic dependencies can require time exponential in the size of the grammar (Knuth 1968; Jazayeri et al. 1975). Some static restrictions on such grammars have been proposed to guarantee non-cyclicity (such as synthesized attributes only, or left-to-right evaluation, etc). However, agreeing MGs do not implement these by virtue of their architecture. Indeed, it is simple to construct an agreeing MG which gives rise to cyclic dependencies, although it does not seem linguistically natural.

Consider the lexicon in Figure 16 and a dependency graph involving its lexical items in Figure 17. This derivation produces a complete expression. None of the lexical items carry any lexically determined morphological feature values, so there are no lexical attribute equations. At the same time, this configuration of channels, expressed as agreement and consistency equations, ensures that for any morphological feature α its values would be transmitted from X to Y via {+x, -x}, Y to T via {+y, -y}, and T to X via {+k, -k}.

[X] :: $\texttt{-x}^{\rightarrow}$ $\texttt{-f}_{\leftarrow}$

[Y] :: $\texttt{+x}_{\leftarrow}$ $\texttt{-y}^{\rightarrow}$

[T] :: $\texttt{+y}_{\leftarrow}$ $\texttt{+f}^{\rightarrow}$ $\texttt{-t}$

Figure 16: Cyclicity in an agreeing MG

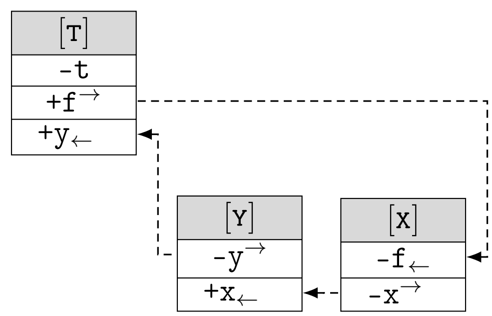

Figure 17: A cyclic derivation

Agreement:

Y.α_1 := X.α_1

T.α_1 := Y.α_2

X.α_2 := T.α_2

Consistency:

X.α_1 := X.α_2

Y.α_2 := Y.α_1

T.α_2 := T.α_1

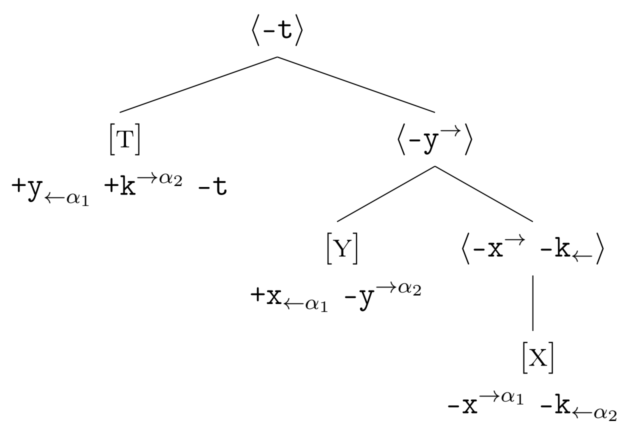

Figure 18: Attribute equations for Figure 17

If agreement information is transmitted in a loop, as in the abstract example above, the lexical filter will allow any agreement without requiring that the values be lexically specified on any LI. It is unclear how to require that every morphological feature in a derivation come from the lexical specification of some LI, as that cannot be determined by considering LIs in isolation.

Although it is not clear whether there is a structural property relevant to minimalist grammars which would ban circularity, Jones (1990) proves that there is a semantic condition which applies in our current setting, and which allows for the efficient evaluation of even circular dependencies. As there are only finitely many distinct morphological feature bundles, a least fixed-point computation over the system of equations specified by a minimalist derivation will terminate after a finite number of steps.

References

J. D. Bobaljik. Where's phi? Agreement as a post-syntactic operation. *Phi-Theory: Phi features across interfaces and modules*, 4410:295–328, 2008.

N. Chomsky. *The Minimalist Program.* MIT Press, Cambridge, MA, 1995.

G. Corbett. Resolution rules: agreement in person, number, and gender. In G. Gazdar, E. Klein, and G. K. Pullum, editors, *Order, Concord, and Constituency*, chapter 8, pages 175–206. Walter de Gruyter, 1983.

M. Ermolaeva. Morphological agreement in minimalist grammars. In *Formal Grammar: 22nd International Conference, FG 2017, Toulouse, France, July 22-23, 2017, Revised Selected Papers*, pages 20–36, Berlin, Germany, 2018. Springer.

M. Ermolaeva and G. M. Kobele. Agree as information transmission over dependencies. *Syntax*, 25(4):466–507, 2022.

M. Ermolaeva and G. M. Kobele. Formal properties of agreeing minimalist grammars. *Proceedings of the Society for Computation in Linguistics*, 6(1):385–388, 2023.

M. Halle and A. Marantz. Distributed morphology and the pieces of inflection. In K. Hale and S. Keyser, editors, *The view from building 20*, pages 111–176. The MIT Press, 1993.

M. Jazayeri, W. F. Ogden, and W. C. Rounds. The intrinsically exponential complexity of the circularity problem for attribute grammars. *Communications of the ACM*, 18(12): 697–706, 1975.

L. G. Jones. Efficient evaluation of circular attribute grammars. *ACM Transactions on Programming Languages and Systems*, 12(3):429–462, 1990.

D. E. Knuth. Semantics of context-free languages. *Mathematical systems theory*, 2(2):127–145, 1968.

G. M. Kobele. Idioms and extended transducers. In *Proceedings of the Eleventh International Workshop on Tree Adjoining Grammars and Related Frameworks (TAG+11),* Paris, pages 153–161, 2012.

J. Michaelis. *On formal properties of minimalist grammars.* PhD thesis, University of Potsdam, 2001.

E. P. Stabler. Derivational minimalism. In C. Retoré, editor, *Logical Aspects of Computational Linguistics: First International Conference, LACL '96 Nancy, France, September 23–25, 1996 Selected Papers*, pages 68–95. Springer, Berlin, Germany, 1997.

E. P. Stabler and E. L. Keenan. Structural similarity within and among languages. *Theoretical Computer Science*, 293(2):345–363, 2003.