Free choice and presuppositional exhaustification

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Abstract Sentences such as *Olivia can take Logic or Algebra* (‘$\Diamond\vee$-sentences’) are typically assigned the ‘Free Choice’ (FC) reading that Olivia can take Logic and can take Algebra. Given a standard semantics for modals and disjunction, such FC readings are not predicted from the surface form of $\Diamond\vee$-sentences. An influential approach treats FC readings as a kind of scalar enrichment generated by a covert exhaustification operator. This approach can also account for the ‘double prohibition’ readings of $\neg\Diamond\vee$-sentences like *Olivia can't take Logic or Algebra* via general principles of implicature cancellation in downward entailing environments. Marty & Romoli (2020) and Romoli & Santorio (2019) examine the projection and filtering behavior of embedded $\Diamond\vee$ and $\neg\Diamond\vee$-sentences, focusing on two kinds of cases which challenge this influential approach. First, $\Diamond\vee$-sentences under negative factives: e.g., *Noah is unaware that Olivia can take Logic or Algebra*, which in its most salient reading presupposes that Olivia has free choice and yet attributes to Noah ignorance not just concerning whether Olivia has free choice but also whether she can take even one of the classes. Second, $\neg\Diamond\vee$-sentences embedded under disjunction: e.g., *Either Maria can't study in Tokyo or Boston, or she is the first in our family who can study in Japan and the second who can study in the States*, which in its most salient reading filters out the FC presupposition of the second disjunct while the first disjunct gets the standard double prohibition reading. These sentences present a serious challenge to extant accounts of FC. In this paper, we present a novel exhaustification-based account of FC that issues in a uniform solution to Marty, Romoli and Santorio's puzzles concerning the presuppositional and filtering behavior of embedded $\Diamond\vee$, $\neg\Diamond\vee$, and related FC sentences. Our account builds on the proposal—advanced in Bassi et al. (2021) and Del Pinal (2021) as a general theory of scalar implicatures—that covert exhaustification is a presupposition trigger such that the prejacent forms the assertive content while any excludable (or includable) alternatives are incorporated at the non-at issue, presuppositional level.

Keywords: free choice, scalar implicatures, exhaustification, presuppositions, presupposed free choice, filtering free choice, accommodation, pragmatics. **Words:** 11,337

Contents

1 Introduction

According to 'Grammatical' theories, the scalar implicatures (SIs) of a sentence ϕ in context c are the result of adjoining to ϕ a covert exhaustification operator, **exh**, which outputs as its assertive content both ϕ and the negation of each excludable alternative of ϕ that is relevant in c (see Chierchia et al. 2012, a.o.). Theories differ on how to define the set of excludable alternatives (Katzir 2007, Fox & Katzir 2011), on whether to also 'include' certain alternatives (Bar-Lev & Fox 2020), and on specific assumptions about the distribution of **exh** (Magri 2011, Chierchia 2013). In this paper, we propose a high level constraint on Grammatical theories which we argue greatly improves their descriptive coverage. The basic idea is that covert exhaustification—like similar overt exhaustification operators such as *only*—is a presupposition trigger. According to standard formulations, the entire output of **exh**—i.e., the prejacent and any excludable and includable alternatives—goes into its assertive content. In contrast, we propose to model covert exhaustification via an operator which, relative to how it structures assertive vs. presupposed content, is roughly the mirror image of its overt counterpart *only*:[1] its prejacent is part of its assertive content, but any excludable or includable alternatives go into the non-at issue, presuppositional level. In Bassi et al. (2021) and Del Pinal (2021), we show that our presuppositional exhaustification-based theory solves various empirical challenges faced by current Grammatical theories of SIs. These include the oddness-inducing SIs studied by Magri (2009b, 2011), and several puzzles about the way SIs project from embedded environments, including in *some*-under-*some* sentences and various kinds of DE positions. This paper extends our argument by developing a novel presuppositional exhaustification-based theory of free choice (FC) and related phenomena, which we will argue provides a uniform solution to various extant puzzles faced by current Grammatical (and other competing) theories of FC.

FC phenomena have played a central role in the argument for **exh**-based theories of SIs over both neo-Gricean (or 'pragmatic') and Lexicalist (also called 'semantic') accounts (Zimmerman 2000, Fox 2007). Recently, FC phenomena have also been at the center of debates amongst specific Grammatical theories (Chierchia 2013, Bar-Lev & Fox 2020). To set the stage for our discussion, it will be useful to recap why FC inferences have been so important for the development of theories of SIs.

First, '$\Diamond\vee$-sentences' with disjunction in the scope of a possibility modal, such as (1), give rise to the conjunctive inference that both disjuncts are possible. Yet those 'free choice' (FC) inferences don't follow from the surface form of $\Diamond\vee$-sentences, given standard accounts of existential modals and disjunction.

(1) Maria is allowed to eat cake or ice-cream. $\Diamond(C \vee IC) \Leftrightarrow (\Diamond C \vee \Diamond IC)$

a. $\rightsquigarrow$ *Maria is allowed to eat cake*
$\rightsquigarrow$ *Maria is allowed to eat ice-cream* $\Diamond C \wedge \Diamond IC$

In contrast, the FC reading of (1) can be derived via recursive exhaustification, a type of operation that is expected on an **exh**-based theory, but harder to capture with a genuine pragmatic account of SIs. Given suitable alternatives at each **exh** site, Fox established the result in (2a).

(2) a. $\mathbf{exh}[\mathbf{exh}[\Diamond(C \vee IC)]] = (\Diamond C \leftrightarrow \Diamond IC) \wedge \Diamond(C \vee IC) = \Diamond C \wedge \Diamond IC$
b. $\mathbf{exh}^{IE+II}[\Diamond(C \vee IC)] = \Diamond C \wedge \Diamond IC$

More recent Grammatical theorists hold that **exh** can not just exclude but also include certain alternatives (Bar-Lev & Fox 2020). As a result, the FC reading of (1) can be derived without recursive exhaustification, as in (2b). Still, the revised operator, $\mathbf{exh}^{IE+II}$, doesn't have any (obvious) pragmatic analogues, and has the

1 Assuming here the standard presuppositional analysis of *only*, as defended by e.g. Horn (1969).

property—distinctive of syntactically 'real' covert operators—that it can be inserted in embedded, non-asserted environments.

Secondly, FC readings, like SIs in general, tend to be cancelled in downward entailing (DE) environments. This is illustrated by the default 'double prohibition' reading of '$\neg\Diamond\vee$-sentences' such as (3), which conveys not merely the negation of FC, but the stronger claim that Maria can't study in either one of Tokyo or Boston.

(3) Maria is not allowed to eat cake or ice-cream. $\neg\Diamond(C \vee IC)$

a. $\rightsquigarrow$ *Maria is not allowed to eat cake*
$\rightsquigarrow$ *Maria is not allowed to eat ice-cream* $\neg\Diamond(C \vee IC) \Leftrightarrow \neg\Diamond C \wedge \neg\Diamond IC$

The preference for the double prohibition reading of (3) is hard to explain for standard 'Lexicalist' accounts of the FC reading of (1), which instead of appealing to SIs, 'hard-wire' such readings via a non-standard semantics for possibility modals and/or disjunction. For suppose that (1), based on its surface form, semantically entails FC, namely, $\Diamond C \wedge \Diamond IC$. When (1) is embedded under negation, as in (3), one would then expect a reading that corresponds to the negation of FC—i.e., $\neg(\Diamond C \wedge \Diamond IC) \Leftrightarrow \neg\Diamond C \vee \neg\Diamond IC$—which is weaker than the observed double prohibition reading. In contrast, in **exh**-based accounts, the cancellation of FC readings in DE environments is captured by appealing to a general preference for parses with strong meanings. Roughly, when a parse with **exh** occurs in an environment that leads to a weaker meaning relative to a corresponding parse without **exh**, as in (4a) vs. (4b), the latter parse is treated as the default, non-marked option.

(4) a. $\neg\mathbf{exh}^{IE+II}[\Diamond(C \vee IC)]$ $\Leftrightarrow \neg\Diamond C \vee \neg\Diamond IC$

b. $\neg\Diamond(C \vee IC)$ $\Leftrightarrow \neg\Diamond C \wedge \neg\Diamond IC$

Importantly, **exh**-based accounts of FC readings of $\Diamond\vee$-sentences can be extended to structurally analogous FC readings observed in other kinds of configurations, including sentences with disjunctions under other existential quantifiers, not just possibility modals, sentences with conjunctions under the scope of negation and universal quantifiers, and indefinites under universal quantifiers (Fox 2007, Chierchia 2013, Meyer 2020). Despite their initial success, however, recent work suggests that standard **exh**-based Grammatical accounts of FC have important shortcomings. The challenge which is the main focus of this paper, due to Marty & Romoli (2020) and Romoli & Santorio (2019), concerns the way in which sentences like (1) and (3) behave when embedded in two kinds of environments.

Consider first an occurrence of a $\Diamond\vee$-sentence like (1) embedded under a negative factive, as in (5) (Marty & Romoli 2020). In its most natural reading, (5) presupposes that Maria has free choice. This is what we would predict—given the factive presupposition of *unaware*—if the embedded sentence is parsed with $\mathbf{exh}^{IE+II}$, i.e., parallel to the parse that supports the FC reading of (1). Yet this assumption generates the wrong prediction for the content of Sam's beliefs, on this natural reading: (5) conveys that what Sam doesn't believe is that Maria can study in either city, not just that Sam doesn't believe that Maria has free choice.

(5) Sam is unaware that Maria can study in Tokyo or Boston.

a. $\rightsquigarrow$ *Maria can study in Tokyo*
   $\rightsquigarrow$ *Maria can study Boston*

b. $\rightsquigarrow$ *Sam doesn't believe that Maria can study in Tokyo*
   $\rightsquigarrow$ *Sam doesn't believe that Maria can study in Boston*

Consider next an embedded occurrence of a $\neg\Diamond\vee$-sentence like (3) in (6) (Romoli & Santorio 2019). While the scope/consequent of *unless* in (6) (= *Maria is the first ... that can study in Japan and the second ... that can study in the US*) presupposes that Maria can study in Japan and in the US, (6) doesn't seem to inherit that FC presupposition, as captured in (6a). The reason for that, it is natural to think, is that the FC presupposition is somehow filtered out by (the negation of) the restrictor of *unless* in (6) (= *Maria can't study in Tokyo or Boston*). Yet the embedded *Maria can't study in Tokyo or Boston* in the restrictor has the double prohibition reading, which calls for a parse without $\mathbf{exh}^{IE+II}$ under negation. Yet given such a parse, the (negated) restrictor wouldn't help filter out the FC presupposition triggered in the scope/consequent of *unless*.

(6) Unless Maria can't study in Tokyo or Boston, she is the first in our family that can study in Japan and the second that can study in the US.

a. $\not\rightsquigarrow$ *Maria can study in Japan*
   $\not\rightsquigarrow$ *Maria can study in the US*

Marty & Romoli (2020) and Romoli & Santorio (2019) show that standard **exh**-based accounts of the FC reading of $\Diamond\vee$-sentences like (1) and the double prohibition reading of $\neg\Diamond\vee$-sentences like (3) don't predict, in a principled way, the target readings for sentences like (5) and (6). This holds whether **exh** is modeled as asserting just the prejacent and the negated excludable alternatives, or whether it also asserts alternatives that can be included without inconsistency, as proposed by Bar-Lev & Fox (2020). Call the challenge of giving a principled account of sentences like (1), (3), (5), and (6) the 'presupposed & filtering FC puzzles'.

There have been two kinds of responses to the presupposed & filtering FC puzzles. Grammatical theorists have proposed that exhaustification applies in analogous ways to both the assertive and the presuppositional content of the prejacent (Gajewski & Sharvit 2012, Marty & Romoli 2020). While such accounts can deal with part of the puzzle, we will argue that they don't provide a general solution to the challenge, i.e., they can't explain (1), (3), (5), and (6) taken together. The other response has been to revise Lexicalist accounts, i.e., accounts which tweak or abandon the standard semantics for modals and/or disjunction (Aloni 2018, Starr 2016, Willer 2017, Goldstein 2018). Yet these accounts do not issue in a fully general solution to the puzzle. The main reason for this, emphasized in Marty & Romoli (2020) and Romoli & Santorio (2019), is that the puzzle can be restated with (negative) FC readings of sentences with conjunction under negation and a universal quantifier, as in *Maria isn't required to study in Tokyo and Boston*. Yet such negative FC readings are not directly derivable on most traditional and remodeled Lexicalist accounts. In short, neither standard nor recent accounts of FC provide a principled, general solution to the presupposed & filtering FC puzzles.

The goal of this paper is to show that if the Grammatical approach to FC is refined based on our hypothesis that covert exhaustification is a presupposition trigger—such that the prejacent is part of the assertive content and any excludable or includable alternatives are part of the non-at issue/presupposed content—we get a uniform solution to the presupposed & filtering FC puzzles. The plan is simple. In §2, we present and defend a presuppositional exhaustification-based analysis of the FC reading of basic $\Diamond\vee$-sentences like (1) and the double prohibition reading of $\neg\Diamond\vee$-sentences like(3), and present preliminary evidence for its unique components. In §3-§4, we show that, unlike competing Grammatical and Lexicalist accounts, our theory supports a uniform account of presupposed FC cases like (5), filtering FC cases like (6), and analogous variants with embedded (negative) free choice conjunctions.

2 Presuppositional exhaustification and basic free choice effects

Our central hypothesis is that covert exhaustification divides its output into an assertive and a non-at issue/presupposed component. Concerning its core operations, the standard Grammatical view is that exhaustification asserts its prejacent and the negation of any (relevant) excludable alternatives. Yet Bar-Lev & Fox show that adding an inclusion function simplifies the derivation of FC, while preserving (and in some cases improving) the predictions of exhaustification for simpler kinds of SIs. We will also formulate our presuppositional exhaustification operator, which we call '**pex**', with both an exclusion and an inclusion function. In §2.1 we propose a conservative way of adding an inclusion function to **pex**, and show that this modification leaves intact the main predictions for basic SIs which we used in Bassi et al. (2021) and Del Pinal (2021) to solve various puzzles for current theories of SIs. In §2.2 we present a **pex**-based account of basic FC and double prohibition readings, and present some initial evidence for our unique predictions concerning the presupposed vs. the asserted components of those readings.

2.1 Presuppositional exhaustification with innocent inclusion

Let us begin by defining the sets from which exhaustification picks the excludable and includable alternatives of its prejacent ϕ. Following Fox (2007), we assume that the negated alternatives are selected from the set of 'innocently excludable' alternatives, defined as follows:

(7) Innocently Excludable alternatives of the prejacent ϕ:
  a. Take all maximal sets of alternatives of ϕ that can be assigned 'false' consistently when conjoined with ϕ.
  b. Those alternatives that are members in all such sets form the set of the 'innocently excludable' (*IE*) alternatives of ϕ.

For the inclusion part, we follow Bar-Lev & Fox (2020) and assume that the set consists of those alternatives which are consistent with the conjunction of the prejacent ϕ and the negation of any *IE* alternatives. For our purposes, we subtract ϕ from *II* (the reason for this will be clear once we divide the output of exhaustification into presupposed/non-at issue and assertive components):

(8) Innocently Includable alternatives of the prejacent ϕ:
  a. Take all maximal sets of alternatives of ϕ that can be assigned 'true' consistently with ϕ and the falsity of all *IE* alternatives of ϕ.
  b. The set of alternatives that are members in all such sets, minus the set which includes just the prejacent ϕ, is the set of 'innocently includable' (*II*) alternatives of ϕ.

How should a presuppositional exhaustification operator with both *IE* and *II* be formulated? We still follow our original proposal that only the prejacent, ϕ, should be included in its assertive content (Bassi et al. 2021, Del Pinal 2021). Accordingly, the prejacent goes into the assertive and the negated *IE* alternatives into the presupposed content, as captured in (9a) and (9b-i). It also follows that the *II* alternatives should go into the presupposed content. But we incorporate them as follows: instead of simply including each *II* alternative, we include the subtly weaker homogeneity proposition that the *II* alternatives have the same truth value, as captured in (9b-ii). The former option might seem like a more direct implementation of Bar-Lev & Fox (2020)'s proposal—which is that exhaustification of ϕ asserts the falsity of its *IE* alternatives and the truth of its *II* alternatives—but we will show, based on the FC puzzles, that it is descriptively inferior to our homogeneity-based suggestion. Call this version of presuppositional exhaustification '$\mathbf{pex}^{IE+II}$':

(9) For a structure ϕ of propositional type and a local context c, $[\![\mathbf{pex}^{IE+II}(\phi)]\!]$:

a. **asserts:** $[\![\phi]\!]$

b. **presupposes:**

(i) $\bigwedge \neg [\![\psi]\!] : \psi \in IE(\phi) \wedge [\![\psi]\!] \in R$

(ii) $\forall \alpha (\alpha \in II(\phi) \wedge [\![\alpha]\!] \in R \rightarrow [\![\alpha]\!] = 1) \vee$
$\forall \alpha (\alpha \in II(\phi) \wedge [\![\alpha]\!] \in R \rightarrow [\![\alpha]\!] = 0)$

where R = a contextually assigned 'relevance' predicate which minimally satisfies the following two conditions: (i) the prejacent, ϕ, is relevant (i.e., $[\![\phi]\!] \in R$), and (ii) any proposition that is contextually equivalent to the prejacent is also in R (i.e., if $[\![\phi]\!] \cap c \equiv [\![\psi]\!] \cap c$, then $[\![\psi]\!] \in R$).

Except for sensitivity to II, the other details of our implementation of $\mathbf{pex}^{IE+II}$ follow Bassi et al. (2021) and Del Pinal et al. (2021). In particular, propositional clauses are parsed with $\mathbf{pex}^{IE+II}$ by default. Accordingly, the role of a relevance parameter R is crucial: for many cases in which an SI is 'cancelled' are handled by showing that the alternative responsible for the potential SI is not in R (see Magri 2009b, Bassi et al. 2021).[2] In addition, since it is likely that there are various kinds of 'non-at issue'/'presuppositional' contents (Tonhauser et al. 2013), what specific properties are we attributing to the SIs triggered by $\mathbf{pex}^{IE+II}$? In terms of their global constraints on the common ground, we assume that when the presuppositions triggered by $\mathbf{pex}^{IE+II}$ are consistent yet not entailed by the common ground, they are globally accommodated by default. Still, like non-at issue content in general, the SIs triggered by $\mathbf{pex}^{IE+II}$ should not be inconsistent with the common ground, and more generally, are unsuitable for proposing revisions (as opposed to just updates) of the common ground. Yet what is most important for us here is that, when the SIs triggered by $\mathbf{pex}^{IE+II}$ appear in embedded positions, they behave, with respect to projection and licensing conditions for local accommodation, like typical presuppositions.

To begin to illustrate how $\mathbf{pex}^{IE+II}$ works, we first consider its effect on simple (non-FC) scalar sentences, and show that its predictions match those obtained with **pex** without II. Then in §2.2 we use $\mathbf{pex}^{IE+II}$ to derive basic FC and double prohibition readings. As we will see, those readings are predicted to be structured into non-at issue and at issue components. That structuring, we will show in §3-§4, is the key to resolving the presupposed & filtering FC puzzles.

According to our $\mathbf{pex}^{IE+II}$-based approach, a simple scalar sentence like (10) is parsed by default as in (10a). Given the alternatives in (10b), it is trivial to check that the only alternative in IE is $\forall$ and that the set of II alternatives is empty, as captured

2 As captured in (9), R prunes alternatives only after the set of IE and II alternatives is determined based on the formal alternatives of the prejacent.

in (10c)-(10d). Assuming that the $\forall$-alternative is relevant, the interpretation of (10a) is then as in (10e).

(10) Some students passed the exam.

a. $\mathbf{pex}^{IE+II}$[some students passed] $= \mathbf{pex}^{IE+II}(\exists)$

b. Alt($\exists$) = $\{\exists, \forall\}$

c. IE($\exists$) = $\{\forall\}$

d. II($\exists$) = $\{\ \}$

e. $[\![(10a)]\!] = \begin{cases} \textbf{ps: } \neg\text{all students passed} & (= \neg\forall) \\ \textbf{asserts: } \text{some students passed} & (= \exists) \end{cases}$

Since II is inert in this case, $\mathbf{pex}^{IE+II}$ has the same effect as our original $\mathbf{pex}^{IE}$ without II. This also holds for other basic (non-FC) scalar sentences. For example, it is easy to check that for exhaustification of disjunctions, such as *Mary had cake or ice-cream* (= $\vee$), $[\![\mathbf{pex}^{IE+II}(\vee)]\!] = \vee_{\neg\wedge}$.[3] That is equivalent to the overall reading and structuring into presupposed vs. asserted components predicted by $[\![\mathbf{pex}^{IE}(\vee)]\!]$. Those correspondences are schematically captured in (11a)-(11b):

(11) a. $[\![\mathbf{pex}^{IE+II}(\vee)]\!] = [\![\mathbf{pex}^{IE}(\vee)]\!] = \vee_{\neg\wedge}$

b. $[\![\mathbf{pex}^{IE+II}(\exists)]\!] = [\![\mathbf{pex}^{IE}(\exists)]\!] = \exists_{\neg\forall}$

(11a)-(11b) suggest that moving from $\mathbf{pex}^{IE}$ to $\mathbf{pex}^{IE+II}$ preserves the empirical advantages of adopting a **pex**-based account of basic (non-FC) SIs. For it is precisely that structuring into non-at issue/presupposed and asserted components that we used to solve various extant puzzles concerning (i) how SIs project from various embedded positions, including the conditions under which they are locally accommodated, and (ii) why SIs tend to generate oddness (and are hard to globally accommodate) when they conflict with the common ground (see Bassi et al. 2021, Del Pinal 2021).

2.2 Derivation of basic free choice and double prohibition effects

The next step is to use our $\mathbf{pex}^{IE+II}$-based theory to derive the FC reading of $\Diamond\vee$-sentences and the double prohibition reading of $\neg\Diamond\vee$-sentences. We will also begin to highlight the subtle but important ways in which the resulting predictions are different from those obtained with a standard, flat $\mathbf{exh}^{IE+II}$ operator.

$\mathbf{Pex}^{IE+II}$ allows for a simple derivation of the FC readings of $\Diamond\vee$-sentences, as shown in (12a)-(12d). Given the prejacent, $\Diamond(p \vee q)$, and its formal alternatives in (12b), there is only one IE alternative, $\Diamond(p \wedge q)$, as captured in (12c). In addition, since the disjunctive alternatives, $\Diamond p$ and $\Diamond q$, can be simultaneously and consistently conjoined with the prejacent and the negation of the IE alternatives—i.e., with

3 We use subscripts to formulas, as in p_q to indicate that q is a presupposition of p.

$\Diamond(p \vee q) \wedge \neg\Diamond(p \wedge q)$—they are in II, as captured in (12d). Based on our formulation of $\mathbf{pex}^{IE+II}$, we thus have to add, as a presupposition, the negation of each IE alternative and the homogeneity presupposition that the II alternatives get the same truth-value, as captured in the **ps** part of (12e).

(12)
a. $\mathbf{pex}^{IE+II}[\Diamond[p \vee q]$
b. $\text{Alt}(\Diamond[p \vee q]) = \{\Diamond[p \vee q], \Diamond p, \Diamond q, \Diamond[p \wedge q]\}$
c. $\text{IE}(\Diamond[p \vee q]) = \{\Diamond[p \wedge q]\}$
d. $\text{II}(\Diamond[p \vee q]) = \{\Diamond p, \Diamond q\}$
e. $[\![\text{(12a)}]\!] = \begin{cases} \textbf{ps: } (\Diamond p \leftrightarrow \Diamond q) \wedge \neg\Diamond(p \wedge q) \\ \textbf{asserts: } \Diamond(p \vee q) \end{cases}$

$[\![\mathbf{pex}^{IE+II}[\Diamond[p \vee q]]\!]$, as in (12e), entails the FC inference $\Diamond p \wedge \Diamond q$. Since the excluded conjunctive alternative, $\neg\Diamond(p \wedge q)$, won't play an important role in our target FC puzzles, we will from now on assume that it is irrelevant and drop it from the **ps** part of $[\![\mathbf{pex}^{IE+II}[\Diamond[p \vee q]]\!]$.[4] Finally, it should now be obvious that, relative to the FC readings of $\Diamond\vee$-sentences, $\mathbf{exh}^{IE+II}$-based accounts and our $\mathbf{pex}^{IE+II}$-based account predict the same overall entailments, as captured in (13)-(14):

(13) $[\![\mathbf{exh}^{IE+II}[\Diamond[p \vee q]]]\!] = \Diamond(p \vee q) \wedge \Diamond p \wedge \Diamond q$

(14) $[\![\mathbf{pex}^{IE+II}[\Diamond[p \vee q]]]\!] = \Diamond(p \vee q)_{\Diamond p \leftrightarrow \Diamond q} \qquad \models \Diamond p \wedge \Diamond q$

Despite having the same overall entailments, we show in §3-4 that, relative to the presupposed and filtering FC puzzles, theories that output uniformly flat, assertive interpretations, as in (13), make incorrect predictions avoided by theories which output structured presupposed vs. assertive interpretations as in (14).[5]

Based on this basic analysis, we can easily derive the FC reading for $\Diamond\vee$-sentences, such as (1), and the double prohibition reading for $\neg\Diamond\vee$-sentences, such as (3). For (1), repeated in (15), simply note that its default parse, in (15a), generates

4 As noted by Simons (2005a), among others, $\Diamond\vee$-sentences like (1)/(15) have two subtly different FC readings, one in which Maria can choose cake, can choose ice-cream, and also can have both, and one in which she can choose between cake and ice-cream but cannot have both. The first reading is captured by (12e) and the second by (14). Again, since this difference doesn't affect our puzzles, we will focus on the simpler interpretation in (14).

5 Importantly, we can also derive the FC reading for $\Diamond\vee$-sentences with a purely IE-based version of **pex**. The derivation with $\mathbf{pex}^{IE}$ is more complicated, however, since it requires recursive exhaustification and some subtlety concerning projection and local accommodation. Still, the predicted FC readings have exactly the same assertive vs. non-at issue components as (14). That is a good result because, as we will see, that structuring is precisely what resolves the puzzles concerning the behavior of embedded $\Diamond\vee$ and related sentences (we will provide a link to alternative derivations of basic FC effects with $\mathbf{pex}^{IE}$; interested readers can also consult the first version of this paper, which can be retrieved at https://ling.auf.net/lingbuzz/006122).

the FC reading. Again, the key difference with **exh**-based accounts, is that the entailments are now divided into a presupposed part, $\Diamond C \leftrightarrow \Diamond IC$, and an assertive part, $\Diamond[C \vee IC]$. Bracketing for now whether the homogeneity presupposition is attested, we get the desired FC inferences for basic $\Diamond\vee$-sentences.

(15) Maria is allowed to eat cake or ice-cream.

a. $\mathbf{pex}^{IE+II}[\Diamond[C \vee IC]]$

b. $[\![(15a)]\!] = \Diamond[C \vee IC]_{\Diamond C \leftrightarrow \Diamond IC}$ $\qquad \models \Diamond C \wedge \Diamond IC$

Consider next $\neg\Diamond\vee$-sentences, such as (3), repeated in (16). On standard Grammatical accounts, recall, the double prohibition (DP) reading for (16) follows from economy. A parse with **exh** is dispreferred if it generates a weaker reading than a parallel parse without **exh**. The situation changes subtly if we adopt a $\mathbf{pex}^{IE+II}$-based account. Since we take parses with $\mathbf{pex}^{IE+II}$ as the default, we predict that the preferred/unmarked parse for (16) is as in (16a). Yet due to the presuppositional structure of the FC effect, the negated part only directly affects the assertive output of $\mathbf{pex}^{IE+II}$, as captured in (16b) (as a result, (16a) doesn't violate economy).

(16) Maria is not allowed to eat cake or ice-cream.

a. $\neg[\mathbf{pex}^{IE+II}[\Diamond[C \vee IC]]$

b. $[\![(16a)]\!] = \neg[\Diamond[C \vee IC]_{\Diamond C \leftrightarrow \Diamond IC}] = \neg\Diamond[C \vee IC]_{\Diamond C \leftrightarrow \Diamond IC}$
$\models \neg\Diamond C \wedge \neg\Diamond IC$

This gets us precisely the DP reading. Yet there are two important differences relative to standard **exh**-based analyses: (i) we can derive the DP inferences despite having a $\mathbf{pex}^{IE+II}$ operator below the negation, and (ii) just as for the FC readings of $\Diamond\vee$-sentences, we also get a homogeneity presupposition for the DP reading of $\neg\Diamond\vee$-sentences.

In §3-4, we show that the unique elements of our $\mathbf{pex}^{IE+II}$-based account of FC and DP readings—i.e., the uniform underlying parses with $\mathbf{pex}^{IE+II}$ and the precise presupposed vs. assertive structuring of the FC reading of $\Diamond\vee$-sentences and DP reading of $\neg\Diamond\vee$-sentences—are the key to solve the presupposed & filtering FC puzzles. Yet before showing that, we will present independent evidence for those unique components of our theory.

In our account, we use uniform parses with $\mathbf{pex}^{IE+II}$ to derive both the FC reading of $\Diamond\vee$ and the DP reading $\neg\Diamond\vee$-sentences. In unembedded cases, we thus predict a homogeneity presupposition for both kinds of sentences, which can be (automatically) globally accommodated, but only if it is consistent with the common ground. Tieu, Romoli & Bill (2019) report a series of experiments that test those predictions. Suppose that (15) and (16) are evaluated against a situation that is explicitly *inconsistent* with the homogeneity presupposition $\Diamond C \leftrightarrow \Diamond IC$ (Maria is allowed to eat ice cream if and only if she is allowed to eat cake), e.g., a situation s_1 which entails that Maria can have cake but not ice-cream. **Exh**-based theories predict that, in s_1, (15) should be judged as true but having a false implicature (for s_1 conflicts with the exhaustified but not with the bare content of the prejacent), whereas (16) should be judged as straightforwardly false (since from this perspective the DP reading doesn't involve exhaustification). In contrast, in our $\mathbf{pex}^{IE+II}$-based analysis *both* the FC reading of (15) and the DP reading of (16) presuppose $\Diamond C \leftrightarrow \Diamond IC$, so we predict that, in s_1, both (15) and (16) should be judged as presupposition failures. Accordingly, only the $\mathbf{pex}^{IE+II}$ view predicts that (15) and (16) should elicit parallel and high ratings relative to a reasonable measure of presupposition failure. Tieu et al. (2019)'s results support the latter prediction.[6]

Moving next to embedded cases, the uniform parses used by our $\mathbf{pex}^{IE+II}$-based theory to get the FC reading of $\Diamond\vee$-sentences and DP reading $\neg\Diamond\vee$ sentences are the key to derive certain readings that resist an **exh**-based analyses.[7] Gotzner, Romoli & Santorio (2020) present experimental evidence that (17) is read, by default, as saying that just one student has FC, while all other students have DP, as captured in (17a) (the 'all others DP' reading).

(17) Exactly one student can take Spanish or Calculus.

a. $\rightsquigarrow$ *Exactly one student can take Spanish and can take Calculus*
$\rightsquigarrow$ *Each of the other students can take neither Spanish nor Calculus*

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6 A reviewer points out that although the results of Tieu et al. (2019) support our $\mathbf{pex}^{IE+II}$-based account of FC phenomena, they do not seem to support some of the predictions of the $\mathbf{pex}^{IE+II}$-based account of ordinary SIs which we defended in Bassi et al. (2021) and Del Pinal (2021). In principle, one could adopt a $\mathbf{pex}^{IE+II}$-based theory for FC while remaining agnostic about its application to SIs in general. The reviewer suggests that this non-uniform approach may be motivated by recent psycholinguistic work which explores whether SIs and presuppositions have similar effects on behavioral measures such as response times. Yet we think that those results are too preliminary. For example, only a small number of different kinds of presupposition triggers and SIs have been compared, and while some results show uniform behavior others point to key differences (Bill et al. 2018, Romoli & Schwarz 2015). In addition, the implications of our theory for reaction times are rather subtle. Such measures might be affected, in particular, by the sensitivity of exclusion and inclusion computations to relevance, which makes direct comparisons hard since, typically, standard presuppositions triggers are not sensitive to relevance. In addition, we would have to compare $\mathbf{pex}^{IE+II}$ with both soft or hard triggers. Although we hope to discuss implications for behavioral measures in future work, in this paper we will focus on predicting the final readings/truth-conditions of the target sentences.

7 This is part of a more general result, discussed in Bassi et al. (2021), where **exh**-based accounts appeal to different parses, depending on monotonicity, while $\mathbf{pex}^{IE+II}$ allows for more uniformity. In various cases where scalar items appear under DE and non-monotonic operators, the parses with $\mathbf{pex}^{IE+II}$ lead to better predictions.

They also present evidence that (18) is read, by default, as saying that just one student has DP, while all other students have FC, as captured in (18a) (the 'all others FC' reading):

(18) Exactly one student can't take Spanish or Calculus.

a. $\rightsquigarrow$ *Exactly one student can take neither Spanish nor Calculus*
$\rightsquigarrow$ *Each of the other students can take Spanish and can take Calculus*

Interestingly, Gotzner et al. (2020) show that **exh**-based theories predict the 'all others DP' reading of (17), but not the 'all others FC' reading of (18). In contrast, our $\mathbf{pex}^{IE+II}$-based account captures the 'all others DP' reading of (17) via the LF in (19), and the 'all others FC' reading of (18) via the LF in (20) (note that $\textsc{one}!(P)(Q) = 1 \Leftrightarrow |P \cap Q| = 1$). Those LFs for the embedded $\Diamond\vee$ and $\neg\Diamond\vee$-sentences correspond to the ones we used to capture their default FC and DP readings in unembedded cases.

(19) $\textsc{one}!(\llbracket student \rrbracket)\ \lambda x.\ \mathbf{pex}^{IE+II}[\Diamond[s(x) \vee c(x)]]$

(20) $\textsc{one}!(\llbracket student \rrbracket)\ \lambda x.\ \neg[\mathbf{pex}^{IE+II}[\Diamond[s(x) \vee c(x)]]]$

$\mathbf{pex}^{IE+II}[\Diamond[s(x) \vee c(x)]]$ triggers the homogeneity presupposition $\Diamond s(x) \leftrightarrow \Diamond c(x)$ in the scope of the non-monotonic quantifier in both (19) and (20). Given standard assumptions about projection from the scope of quantifiers (Chemla 2009a, Fox 2013, Mayr & Sauerland 2015), (19) and (20) presuppose the universally quantified homogeneity proposition, $\forall x \in \llbracket student \rrbracket(\Diamond s(x) \leftrightarrow \Diamond c(x))$ (each student can take both or neither of Spanish and Calculus). In (19), its assertive part entails that only one student is allowed to take both Spanish and Calculus—and when conjoined with universal homogeneity, that entails that all the others can't take either one:

$$\begin{aligned}
&\forall x \in \llbracket student \rrbracket(\Diamond s(x) \leftrightarrow \Diamond c(x))\\
&\wedge(|\{x \in \llbracket student \rrbracket : \Diamond s(x) \wedge \Diamond c(x)\}| = 1)\\
&\qquad\qquad\models \forall x \in \llbracket student \rrbracket(\neg(\Diamond s(x) \wedge \Diamond c(x)) \rightarrow (\neg\Diamond s(x) \wedge \neg\Diamond c(x)))
\end{aligned}$$

This captures exactly the target 'all others DP' reading. In the case of (20), its assertive part entails that only one student can take neither Spanish nor Calculus—and when conjoined with universal homogeneity, that entails that all the others can take Spanish and can take Calculus:

$$\begin{aligned}
&\forall x \in \llbracket student \rrbracket(\Diamond s(x) \leftrightarrow \Diamond c(x))\\
&\wedge(|\{x \in \llbracket student \rrbracket : \neg\Diamond s(x) \wedge \neg\Diamond c(x)\}| = 1)\\
&\qquad\qquad\models \forall x \in \llbracket student \rrbracket((\Diamond s(x) \vee \Diamond c(x)) \rightarrow (\Diamond s(x) \wedge \Diamond c(x)))
\end{aligned}$$

This captures exactly the target ‘all others FC’ reading. Finally, it is easy to check that this analysis predicts the target ‘all others DP’ reading for any *n* in sentences of the form *Exactly n student/s can take Spanish or Calculus*, and also the the target ‘all others FC’ reading for any *n* in sentences of the form *Exactly n student/s can’t take Spanish or Calculus*.

3 Free choice under (negative) factives

3.1 The challenge

The first of the presupposed & filtering FC puzzles, due to Marty & Romoli (2020), concerns the behavior of FC when embedded under certain factive attitude verbs. Consider the pattern in (21)-(23). We begin with a simple example of a FC-$\Diamond\vee$-sentence, in (21), and of DP-$\neg\Diamond\vee$-sentence, in (22). The challenge is to have an account of those basic cases that can simultaneously capture the default reading of (23), which presupposes FC, as captured in (23a), yet asserts that Noah doesn’t believe Olivia can take either one of Logic or Algebra, as captured in (23b).

(21) Olivia is allowed to take Logic or Algebra.

a. $\rightsquigarrow$ *Olivia can take Logic*
$\rightsquigarrow$ *Olivia can take Algebra*

(22) Olivia isn’t allowed to take Logic or Algebra.

a. $\rightsquigarrow$ *Olivia can’t take Logic*
$\rightsquigarrow$ *Olivia can’t take Algebra*

(23) Noah is unaware that Olivia is allowed to take Logic or Algebra.

a. $\rightsquigarrow$ *Olivia can take Logic*
$\rightsquigarrow$ *Olivia can take Algebra*

b. $\rightsquigarrow$ $\neg$*Noah believes that Olivia can take Logic*
$\rightsquigarrow$ $\neg$*Noah believes that Olivia can take Algebra*

The pattern in (21)-(23) poses a serious problem for standard Grammatical accounts of FC. Recall that, on **exh**-based accounts, the DP reading of $\neg\Diamond\vee$-sentences is obtained by applying an economy constraint which prevents **exh** from being inserted in positions where it leads to an overall weakening of meaning. Accordingly, the parse in (24a) supports the FC reading of (21), and the one in (24b) the DP reading of (22):

(24) a. $\textbf{exh}^{IE+II}[\Diamond[L\vee A]] \quad \Leftrightarrow \Diamond L\wedge\Diamond A$

b. $\neg[\Diamond[L\vee A]] \quad \Leftrightarrow \neg\Diamond L\wedge\neg\Diamond A$

Why is (23) problematic for this kind of **exh**-based account? Notice that while (23) presupposes FC—namely, that Olivia can take Logic and can take Algebra—what Noah doesn't believe is that Olivia can take either class (as opposed to that Olivia doesn't have FC, which is compatible with believing that she can take, say, Logic but not Algebra, or vice-versa). Assume an interpretation for the target propositional attribution in (25), according to which 'x is unaware that p' presupposes that p and asserts that it is not the case that x believes that p:

(25) $[\![x \text{ is unaware that } p]\!] = \neg B_x(p)_p$

To try to capture the default reading of (23), there are two natural candidate parses, presented in (26) and (27). If we parse (23) as in (26), with $\textbf{exh}^{IE+II}$ over the embedded *Olivia is allowed to take Logic or Algebra*, we predict the correct presuppositions for the entire sentence. For the factivity of *unaware* will guarantee that the FC content projects, as captured in the **ps** part of (26a). However, for the assertive part we predict that Noah doesn't believe that Olivia has FC, as captured in the **asserts** part of (26a). Yet the target reading, recall, is that Noah doesn't believe Olivia can take even one of the classes.

(26) Noah is unaware $[\textbf{exh}^{IE+II}[\Diamond(L \vee A)]]$

a. $[\![(26)]\!] = \begin{cases} \textbf{ps: exh}^{IE+II}[\Diamond(L \vee A)] = \Diamond L \wedge \Diamond A \\ \textbf{asserts: } \neg B_N(\textbf{exh}^{IE+II}[\Diamond(L \vee A)]) = \neg B_N(\Diamond L \wedge \Diamond A) \end{cases}$

If we parse (23) as in (27), where the embedded *Olivia is allowed to take Logic or Algebra* isn't exhaustified, we predict the correct content for Noah's doxastic state, namely, that he doesn't believe Olivia can take even one of Logic or Algebra, as captured in the **asserts** part of (27a). However, as captured in the **ps** part of (27a), we also derive a presupposition that is too weak, namely, that Olivia is allowed to take one class but not necessarily the other one, whereas what we wanted was to get the entailment that Olivia has FC.

(27) Noah is unaware $[\Diamond(L \vee A)]$

a. $[\![(27)]\!] = \begin{cases} \textbf{ps: } \Diamond(L \vee A) \\ \textbf{asserts: } \neg B_N(\Diamond(L \vee A)) \end{cases}$

Importantly, the standard **exh**-based account of FC is not the only Grammatical account of SIs that has trouble deriving the default readings of sentences with presupposed FC under negative factives such as (23). As Marty & Romoli (2020) show, (23) also presents a serious challenge to the Grammatical account with multidimensional exhaustification developed by Gajewski & Sharvit (2012).

3.2 A solution based on presuppositional exhaustification

We will now show that the default reading of sentences with presupposed FC under negative factives, such as (23), is directly captured given our $\textbf{pex}^{IE+II}$-based account of the FC reading of $\Diamond\vee$-sentences like (21) and the DP reading of $\neg\Diamond\vee$-sentences like (22). In §2.2 we showed that the parse in (28a) supports the FC reading of (21), and the one in (29a) supports the DP reading of (22):

(28) a. $\textbf{pex}^{IE+II}[\Diamond(L \vee A)]$
b. $[\![(28a)]\!] = \Diamond(L \vee A)_{\Diamond L \leftrightarrow \Diamond A}$

(29) a. $\neg[\textbf{pex}^{IE+II}[\Diamond(L \vee A)]]$
b. $[\![(29a)]\!] = \neg\Diamond(L \vee A)_{\Diamond L \leftrightarrow \Diamond A}$

Since, on our account, covert exhaustification is a presupposition trigger, to deal with FC under negative factives, in sentences like (23), we will need an independent account of presupposition projection from the complement of belief operators. Two well-known options, each defended on independent grounds, are (30a) and (30b):

(30) a. $B_x(p'_p) = B_x(p')_{B_x(p)}$ — Heim (1992)/Schlenker
b. $B_x(p'_p) = B_x(p')_p$ — Geurts (1999)/DRT

(23) is repeated in (31). Since the target reading presupposes the FC reading of the embedded *Olivia is allowed to take Logic or Algebra*, it is natural to consider the parse in (31a) (ignore for simplicity a matrix $\textbf{pex}^{IE+II}$, which could associate with *Olivia* or *Noah*, among other options which are not relevant here):

(31) Noah is unaware that Olivia is allowed take Logic or Algebra.
a. Noah is unaware $[\textbf{pex}^{IE+II}[\Diamond(L \vee A)]]$
b. $[\![(31a)]\!] = \begin{cases} \textbf{ps: pex}^{IE+II}[\Diamond(L \vee A)] \\ \textbf{asserts: } \neg B_N(\textbf{pex}^{IE+II}[\Diamond(L \vee A)]) \end{cases}$
c. $\textbf{pex}^{IE+II}[\Diamond(L \vee A)] = \Diamond(L \vee A)_{\Diamond L \leftrightarrow \Diamond A}$
d. $\neg B_N(\textbf{pex}^{IE+II}[\Diamond(L \vee A)]) =$
$\neg[B_N(\Diamond(L \vee A)_{\Diamond L \leftrightarrow \Diamond A})] =$ (given Heim's (30a))
$\neg[B_N(\Diamond(L \vee A))_{B_N(\Diamond L \leftrightarrow \Diamond A)}] =$ (ps projects under $\neg$)
$\neg B_N(\Diamond(L \vee A))_{B_N(\Diamond L \leftrightarrow \Diamond A)}$

From the **ps** part of (31b), and the equivalence in (31c), we get the target result that (31) presupposes the FC proposition that Olivia is allowed to take Logic and is allowed to take Algebra. We next need to check whether the **asserts** part of (31b), $\neg B_N(\textbf{pex}^{IE+II}[\Diamond(L \vee A)])$, captures the target content for Noah's doxastic state. Given the presupposed and assertive components resulting from the embedded $\textbf{pex}^{IE+II}$, and the Heim/Schlenker assumption in (30a) for how belief operators interact with presupposed content in their complements, we derive the target content, as shown in (31d). What Noah doesn't believe, on this analysis, is that Olivia can take either one of Logic or Algebra (as opposed to Noah not believing merely that she has FC, which, recall, is compatible with Noah believing that Olivia can take Algebra but not Logic, or vice-versa). The parse in (31a), then, makes the correct predictions for the default interpretation of (31) concerning both its presuppositions and the content of the doxastic attribution.

Now, as captured by the last equivalence of (31d), we also predict the additional presupposition that Noah believes the homogeneity proposition that if Olivia can take either one of the classes, she can take the other one. This specific presupposition is perhaps unattested in cases specifically like (31). Yet we can avoid that prediction, without affecting the other desired parts of the derivation, by adopting, instead of (30a), the DRT rule in (30b) concerning how presuppositions project from the complement of doxastic operators. In this case, the derivation in (31d) should be replaced with this one:

(31) d′. $\neg B_N(\mathbf{pex}^{IE+II}[\Diamond(L \vee A)]) =$
$\neg[B_N(\Diamond(L \vee A)_{\Diamond L \leftrightarrow \Diamond A})] =$ (given DRT's (30b))
$\neg[B_N(\Diamond(L \vee A))_{\Diamond L \leftrightarrow \Diamond A}] =$ (ps projects under $\neg$)
$\neg B_N(\Diamond(L \vee A))_{\Diamond L \leftrightarrow \Diamond A}$

We now predict that $\neg\mathbf{pex}^{IE+II}[B_N(\Diamond(L \vee A)_{\Diamond L \leftrightarrow \Diamond A})] \Leftrightarrow \neg B_N(\Diamond(L \vee A))_{\Diamond L \leftrightarrow \Diamond A}$. Yet the homogeneity presupposition, $\Diamond L \leftrightarrow \Diamond A$, doesn't add any presuppositional constraints to (31), since it is entailed by the presuppositions triggered by the factivity of *unaware*, spelled out in (31c). So the DRT-style route captures exactly the target reading singled out by Marty & Romoli (2020). For our purposes, however, we can remain neutral between these different views concerning the interaction between belief operators and the projection behavior of presuppositions triggered in their complement.[8]

3.3 Extension to presupposed negative free choice

As Marty & Romoli (2020) point out, the phenomenon of presupposed FC extends to other types of embedded FC sentences, not just $\Diamond\vee$-sentences. This is important because recent Lexicalist accounts can explain the puzzle in the version illustrated

8 Indeed, consider simpler examples of $\Diamond\vee$-sentences embedded under belief and knowledge operators, such as *Noah believes that Olivia is allowed to take Logic or Algebra*, focusing on the reading which attributes to Noah the belief that Olivia has free choice vis-a-vis taking Logic or Algebra. It is easy to check that, given our $\mathbf{pex}^{IE+II}$-based account, a simple route to that reading is via a parse with embedded $\mathbf{pex}^{IE+II}$ immediately over the $\Diamond\vee$-sentence and using the Heim/Schlenker assumption in (30a). For Noah is then represented as believing both $\Diamond(L \vee A)$ and $\Diamond L \leftrightarrow \Diamond A$.

in (21)-(23), but have trouble with variants with 'negative' FC sentences embedded under negative factives.

A negative FC sentence, such as (32), consist of a conjunction under negation and a universal quantifier. In one of its salient readings, (32) suggests that Olivia is allowed to not take Logic and is also allowed to not take Algebra, as captured in (32a). Experimental evidence that this reading is in general available is presented in Marty, Romoli, Sudo & Breheny (2021). From now on, we refer to sentences of the surface form of (32) as '$\neg\Box\wedge$-sentences', and when needed use a 'FC' prefix to signal the target reading.

(32) Olivia is not required to take Logic and Algebra.

a. $\rightsquigarrow$ *Olivia is allowed to not take Logic*
$\rightsquigarrow$ *Olivia is allowed to not take Algebra*

Most Lexicalist accounts are not designed to handle these case, and it is not entirely obvious whether they can modified to handle them without negatively affecting other predictions. Yet as Marty & Romoli (2020) emphasize, the puzzle of presupposed FC under (negative) factives is also observed with embedded $\neg\Box\wedge$-sentences. Consider (33). In one of its most salient readings, it presupposes that Olivia is allowed to not take Logic and is also allowed to not take Algebra ('negative FC'), as captured in (33a), and asserts that Noah doesn't believe that Olivia can take either one (and not just that Olivia doesn't have negative FC), as captured in (33b):

(33) Noah is unaware that Olivia is not required to take Logic and Algebra.

a. $\rightsquigarrow$ *Olivia is allowed to not take Logic* [$= \Diamond\neg L$]
$\rightsquigarrow$ *Olivia is allowed to not take Algebra* [$= \Diamond\neg A$]

b. $\rightsquigarrow$ $\neg$*Noah believes Olivia is allowed to not take Logic*
$\rightsquigarrow$ $\neg$*Noah believes that Olivia is allowed to not take Algebra*

Standard **exh**-based theories do predict the negative FC reading of $\neg\Box\wedge$-sentences such as (32). Yet as Marty & Romoli (2020) show, they also fail to predict the target reading of $\neg\Box\wedge$-sentences under negative factives, such as (33), for reasons which exactly parallel their difficulties in deriving the default reading of $\Diamond\vee$-sentences embedded under negative factives (see §3.1 above).

In contrast, our **pex**$^{IE+II}$-based account also resolves this version of the presupposed FC puzzle. To show this, we first sketch the derivation of the negative FC reading for basic $\neg\Box\wedge$-sentences. We then show how that result, when combined with the same assumptions used in §3.2, gets us the target reading of $\neg\Box\wedge$-sentences embedded under (negative) factives.

The derivation of the negative FC reading of $\neg\Box\wedge$-sentences like (32) is straightforward. It is based on the parse in (34a), which is structurally analogous to the one used to derive the FC reading of $\Diamond\vee$-sentences (see §2.2). It is easy to check that the only *IE* alternative is $\neg\Box[p \vee q]$, as captured in (34c). In addition, since $\neg\Box p$ and $\neg\Box q$ can together be consistently conjoined with the conjunction of the prejacent and the negation of the *IE* alternative—i.e., with $\neg\Box[p \wedge q] \wedge \Box[p \vee q]$—they are in *II*, as captured in (34d). Given our formulation of $\mathbf{pex}^{IE+II}$, the negation of the *IE* alternative and the homogeneity presupposition that the *II* alternatives get the same truth-value go into the **ps** dimension, as in (34e).

(34)
a. $\mathbf{pex}^{IE+II}[\neg\Box[p \wedge q]]$
b. $\mathrm{Alt}(\neg\Box[p \wedge q]) = \{\neg\Box[p \wedge q], \neg\Box p, \neg\Box q, \neg\Box[p \vee q]\}$
c. $\mathrm{IE}(\neg\Box[p \wedge q]) = \{\neg\Box[p \vee q]\}$
d. $\mathrm{II}(\neg\Box[p \wedge q]) = \{\neg\Box p, \neg\Box q\}$
e. $[\![\text{(12a)}]\!] = \begin{cases} \textbf{ps: } \neg\Box p \leftrightarrow \neg\Box q \;\wedge\; \Box[p \vee q] \\ \textbf{asserts: } \neg\Box(p \wedge q) \end{cases}$

Since the *IE* alternative is again irrelevant for our target readings, we drop it for simplicity. Accordingly, we can abbreviate the negative FC reading as in (35), where we add an equivalence that is helpful when comparing the predictions of our account with intuitions about the target reading:

(35) $[\![\mathbf{pex}^{IE+II}[\neg\Box[p \wedge q]]]\!] =$
$\neg\Box(p \wedge q)_{\neg\Box p \leftrightarrow \neg\Box q} \Leftrightarrow (\Diamond\neg p \vee \Diamond\neg q)_{\Diamond\neg p \leftrightarrow \Diamond\neg q} \qquad \models \Diamond\neg p \wedge \Diamond\neg q$

Building on that result, consider next $\neg\Box\wedge$-sentences under negatives factives such as (33), repeated in (36). The parse in (36a) is structurally analogous to the one we used to derive the default reading of $\Diamond\vee$-sentences under (negative) factives. As shown in (36b)-(36c), this parse gets us the target prediction that (36) presupposes negative FC. This takes care of Marty & Romoli's first desiderata, captured in (33a). What is the prediction for Noah's doxastic state? On the target reading of (36), Noah doesn't believe that Olivia is allowed to not take Logic, and also doesn't believe that Olivia is allowed to not take Algebra. That doxastic state is different from one in which what Noah doesn't believe is that Olivia doesn't have the (negative) FC to not take either class. The latter, but not the former, is compatible with Noah believing that Olivia is required to take Logic but not Algebra (or vice versa). Crucially, that is precisely what we predict for the **asserts** part, captured in (36b), given the equivalences in (36d). As before, although the **asserts** part triggers the homegeneity presupposition $\Diamond\neg p \leftrightarrow \Diamond\neg q$, that proposition is already entailed by the **ps** part, as captured in (36c), so it doesn't strengthen the overall presuppositions of (36).

(36) Noah is unaware that Olivia is not required to take Logic and Algebra
a. Noah is unaware $\mathbf{pex}^{IE+II}[\neg\Box[L \wedge A]]$

b. $[\![(36a)]\!] = \begin{cases} \textbf{ps: } \mathbf{pex}^{IE+II}[\neg\Box[L \wedge A]] \\ \textbf{asserts: } \neg B_N(\mathbf{pex}^{IE+II}[\neg\Box[L \wedge A]]) \end{cases}$

c. $\mathbf{pex}^{IE+II}[\neg\Box[L \wedge A]] = \Diamond\neg L \vee \Diamond\neg A_{\Diamond\neg L \leftrightarrow \Diamond\neg A}$

d. $\neg B_N(\mathbf{pex}^{IE+II}[\neg\Box[L \wedge A]])$
$= \neg[B_N((\Diamond\neg L \vee \Diamond\neg A)_{\Diamond\neg L \leftrightarrow \Diamond\neg A})]$ (given DRT's (30b))
$= \neg[B_N(\Diamond\neg L \vee \Diamond\neg A)_{\Diamond\neg L \leftrightarrow \Diamond\neg A}]$ (ps projects under $\neg$)
$= \neg B_N(\Diamond\neg L \vee \Diamond\neg A)_{\Diamond\neg L \leftrightarrow \Diamond\neg A}$

3.4 Comparison with other revised Grammatical accounts

At this point, we should add that Marty & Romoli (2020) don't just introduce the presupposed FC puzzle, they also develop as a response a novel Grammatical account of SIs. Their account combines insights from Magri (2009b), Gajewski & Sharvit (2012) and Spector & Sudo (2017) concerning the effect of exhaustification on the assertive and presuppositional content in its scope, with Bar-Lev & Fox (2020)'s proposal that covert exhaustification has both an exclusion and an inclusion function. In this section, we introduce Marty & Romoli's theory, focusing on its derivation of FC readings, and highlighting the similarities and differences vis-à-vis our $\mathbf{pex}^{IE+II}$-based account. Marty & Romoli's revised Grammatical theory, we will see, resolves the puzzle of presupposed FC under negative factives, which is one of the class of cases for which it was designed. However, we will show in §4 that it doesn't help with the filtering FC part of the puzzle.

Marty & Romoli call their novel exhaustification operator '$\mathbf{exh}^{IE+II}_{asr+ps}$'. To highlight what is distinctive about $\mathbf{exh}^{IE+II}_{asr+ps}$, suppose that its prejacent, ϕ_p, triggers a non-trivial presupposition p. $\mathbf{exh}^{IE+II}_{asr+ps}$ is sensitive to a distinction between the 'assertive' (*asr*) and the 'presuppositional' (*psr*) formal alternatives to ϕ_p: *asr*-alternatives are neither logically nor Strawson-entailed by ϕ_p, while *psr*-alternatives are not logically but are Strawson-entailed by ϕ_p. Marty & Romoli (2020) then define sets of *IE* and *II* alternatives for each of the *asr* and *psr* alternatives, and propose that $\mathbf{exh}^{IE+II}_{asr+ps}$ performs the following operations on those sets:

(37) $$[\![\mathbf{exh}^{IE+II}_{asr+ps}(\phi_p)]\!](w) = \begin{cases} \textbf{ps: } p(w) \wedge \forall\chi_r \in (IE_{asr} \cup II_{asr} \cup II_{ps})[r(w)] \wedge \forall\psi_q \in IE_{ps}[\neg q(w)] \\ \textbf{asserts: } [\![\phi_p]\!](w) \wedge \forall\chi_r \in II_{asr}[[\![\chi_r]\!]] \wedge \forall\psi_q \in IE_{asr}[\neg[\![\psi_q]\!](w)] \end{cases}$$

Let us go over the main elements of this complex operator. At the assertive level, $\mathbf{exh}^{IE+II}_{asr+ps}$ basically mimics Bar-Lev & Fox (2020)'s $\mathbf{exh}^{IE+II}$: it asserts the prejacent, ϕ_p, each alternative in II_{asr}, and the negation of each alternative in IE_{asr}. At the presupposition level, $\mathbf{exh}^{IE+II}_{asr+ps}$ adds any presuppositions triggered by its prejacent or by any alternative in IE_{asr}, II_{asr}, and II_{psr}, and also the *negation* of any presuppositions triggered by any alternatives in IE_{psr}.

It is important to note that, unlike $\mathbf{pex}^{IE+II}$, $\mathbf{exh}^{IE+II}_{asr+ps}$ is not itself a presupposition trigger: specifically, if neither its prejacent nor any of its formal alternatives are presuppositional, then the output of $\mathbf{exh}^{IE+II}_{asr+ps}$ matches the fully assertive output of $\mathbf{exh}^{IE+II}$. To illustrate, consider the derivation of the FC reading of basic $\Diamond\vee$-sentences based on the parse in (38a). $\Diamond[L \wedge A]$ is the only IE_{asr}-alternative and $\Diamond L$ and $\Diamond A$ the only II_{asr}-alternatives. According to (37), $\mathbf{exh}^{IE+II}_{asr+ps}$ will then add to the assertive level $\Diamond L \wedge \Diamond A$ and the negation of $\Diamond[L \wedge A]$. This basically matches what we get when applying the standard $\mathbf{exh}^{IE+II}$ to a $\Diamond\vee$-sentence like $\Diamond[L \vee A]$

(38) Olivia can take Logic or Algebra.

a. $\mathbf{exh}^{IE+II}_{asr+ps}[\Diamond[L \vee A]]$

b. $Alt(\Diamond[L \vee A]) = \{\Diamond[L \vee A], \Diamond L, \Diamond A, \Diamond[L \wedge A]\}$

c. $IE_{asr}(\Diamond[L \vee A]) = \{\Diamond[L \wedge A]\}$

d. $II_{asr}(\Diamond[L \vee A]) = \{\Diamond L, \Diamond A\}$

e. $IE_{psr}(\Diamond[L \vee A]) = II_{psr}(\Diamond[L \vee A]) = \emptyset$

Next, note that since neither the prejacent, $\Diamond[L \vee A]$, nor any of its formal alternatives triggers any presuppositions, both sets of potential *psr* alternatives, IE_{psr} and II_{psr}, are empty. Generalizing, applying $\mathbf{exh}^{IE+II}_{asr+ps}$ to basic (non-presuppositional) $\Diamond\vee$-sentences affects only its assertive level output, by adding any II_{asr} alternatives and the negation of any IE_{asr} alternatives. Accordingly, exhaustification of basic $\Diamond\vee$-sentences with $\mathbf{exh}^{IE+II}_{asr+ps}$ has the same effect as with $\mathbf{exh}^{IE+II}$, and it has the same overall entailments, but different at-issue vs presupposed components, as with $\mathbf{pex}^{IE+II}$ (we ignore the conjunctive alternative for simplicity):

(39) a. $[\![\mathbf{exh}^{IE+II}_{asr+ps}[\Diamond(p \vee q)]]\!] = [\![\mathbf{exh}^{IE+II}[\Diamond(p \vee q)]]\!] = \Diamond(p \vee q) \wedge \Diamond p \wedge \Diamond q$

b. $[\![\mathbf{pex}^{IE+II}[\Diamond(p \vee q)]]\!] = \Diamond(p \vee q)_{\Diamond p \leftrightarrow \Diamond q}$

How does $\mathbf{exh}^{IE+II}_{asr+ps}$ help solve the presupposed FC puzzle? Recall that the task is to derive the reading of sentences like (40) captured in (40a)-(40b):

(40) Noah is unaware the Olivia can take Logic or Algebra.

a. $\rightsquigarrow$ *Olivia can take Logic*
$\rightsquigarrow$ *Olivia can take Algebra*

b. $\rightsquigarrow$ $\neg$*Noah believes that Olivia can take Logic*
$\rightsquigarrow$ $\neg$*Noah believes that Olivia can take Algebra*

Given the result in (39a), a parse as in (41)—parallel to the one that predicts the target reading with $\mathbf{pex}^{IE+II}$—predicts the reading in (41a), which doesn't fully capture the target reading. From the factivity of 'unaware' and the embedded $\mathbf{exh}^{IE+II}_{asr+psr}[\Diamond(L \vee A)]$ we get the desired FC entailment, but at the assertion level we predict the too weak reading that what Noah doesn't believe is that Olivia has FC.

(41) Noah is unaware $[\mathbf{exh}^{IE+II}_{asr+psr}[\Diamond[L \vee A]]]$

a. $[\![(41)]\!] = \begin{cases} \textbf{ps: } \mathbf{exh}^{IE+II}_{asr+psr}[\Diamond(L \vee A)] = \Diamond L \wedge \Diamond A \\ \textbf{asserts: } \neg B_N(\mathbf{exh}^{IE+II}_{asr+psr}[\Diamond(L \vee A)]) = \neg B_N(\Diamond L \wedge \Diamond A) \end{cases}$

The parse in (42), with matrix $\mathbf{exh}^{IE+II}_{asr+psr}$, is more promising. For due to the factive presupposition triggered by *unaware*, in this case the prejacent and all its formal alternatives, in (43), are presuppositional, so the novel operations of $\mathbf{exh}^{IE+II}_{asr+psr}$ can kick in.

(42) $\mathbf{exh}^{IE+II}_{asr+ps}$[Noah is unaware $[\Diamond(L \vee A)]$]

(43) $Alt(\text{Noah is unaware } [\Diamond(L \vee A)]) = \begin{cases} \text{Noah is unaware } [\Diamond[L \vee A]] \\ \text{Noah is unaware } [\Diamond[L \wedge A]] \\ \text{Noah is unaware } [\Diamond L] \\ \text{Noah is unaware } [\Diamond A] \end{cases}$

Again, the conjunctive alternative is irrelevant for the target reading, and we can safely ignore it without affecting the derivation. Of the remaining options in (43), none are *asr* alternatives. However, the two disjunctive alternatives are *psr* alternatives (they can each be undefined when the prejacent is true), and while neither is in IE_{psr}, they are both in II_{psr}:

(44) $II_{psr}(\text{Noah is unaware } [\Diamond(L \vee A)]) = \begin{cases} \text{Noah is unaware } [\Diamond L] \\ \text{Noah is unaware } [\Diamond A] \end{cases}$

Given (37), the effect of $\mathbf{exh}^{IE+II}_{asr+ps}$ in (42) is the following. At the assertive level, it outputs the assertive content of the prejacent, $\neg B_N[\Diamond[L \vee A]$, and at the presuppositional level, it outputs the presuppositions triggered by the prejacent, $\Diamond[L \vee A]$, and by each of the alternatives in II_{psr}, $\Diamond L$ and $\Diamond A$. This is captured in (45):

(45) $[\![\mathbf{exh}^{IE+II}_{asr+ps}[\text{Noah is unaware } [\Diamond(L \vee A)]]]\!] = \begin{cases} \textbf{ps: } \Diamond[L \vee A] \wedge \Diamond L \wedge \Diamond A \\ \textbf{asserts: } \neg B_N[\Diamond[L \vee A] \end{cases}$

This captures the target reading of (40): namely, that Noah doesn't believe that Olivia can take even one of Logic or Algebra, and also that Olivia has FC, i.e., can take Logic and can take Algebra.

As Marty & Romoli (2020) show, this account also predicts the target reading for negative FC $\neg\Box\wedge$-sentences under negative factives, like (33). We don't need to go into the details here, but note that the key, again, is to use a parse with matrix level $\mathbf{exh}^{IE+II}_{asr+psr}$ parallel to (42).

3.5 Summary

The first of the presupposed & filtering FC puzzles concerns the behavior of $\Diamond\vee$ and $\neg\Box\wedge$-sentences when embedded under (negative) factives. These sentences, taken together, present a serious challenge to many theories of FC, including various influential **exh**-based and recent Lexicalist theories. In contrast, we have shown that our $\mathbf{pex}^{IE+II}$-based theory of FC issues in a uniform solution to this part of the puzzle. We also saw that the recent Grammatical theory developed by Marty & Romoli (2020), based on their $\mathbf{exh}^{IE+II}_{asr+psr}$-operator, can also deal with presupposed FC under negative factives. These results are summarized in Table 1 below.

Accounts	**FC-$\Diamond\vee$**	**DP-$\neg\Diamond\vee$**	$\neg K > \Diamond\vee$	$\neg K > \neg\Box\wedge$
Lexicalist old	✓	✗	✗	✗
Lexicalist new	✓	✓	✓	✗
$\mathbf{exh}^{IE/IE+II}$	✓	✓	✗	✗
$\mathbf{exh}^{IE}_{asr+psr}$	✓	✓	✗	✗
$\mathbf{exh}^{IE+II}_{asr+psr}$	✓	✓	✓	✓
pex	✓	✓	✓	✓

Table 1 Predictions of theories of FC for presupposed FC under (negative) factives. '$\neg K > P$' stands for sentences where P is embedded under a negative factive. References for theories: Lexicalist old: Zimmerman (2000) and Simons (2005b). Lexicalist new: Willer (2017), Goldstein (2018), Aloni (2018), Rothschild & Yablo (2018). Grammatical with $\mathbf{exh}^{IE/IE+II}$: Fox (2007), Chierchia (2013), Bar-Lev & Fox (2020). Grammatical with $\mathbf{exh}^{IE}_{asr+psr}$ (allow for exhausitifcation of presuppositions triggered by the prejacent, but only sensitive to IE): Gajewski & Sharvit (2012), Spector & Sudo (2017), Magri (2009a). Grammatical with $\mathbf{exh}^{IE+II}_{asr+psr}$: Marty & Romoli (2020).

4 Filtering free choice

4.1 The challenge

The second of the presupposed & filtering FC puzzles, due to Romoli & Santorio (2019), concerns an intricate pattern of presupposition projection and filtering in certain complex sentences that involve FC effects. The basic pattern is presented in (46)-(48). Recall that $\Diamond\vee$-sentences like (46) have an FC reading which, however, tends to disappear under negation, as captured by the default DP reading of $\neg\Diamond\vee$-sentences like (47). With that in mind, consider the most salient reading of the key case (48). The first main disjunct (*Maria can't study in Tokyo or Boston*) has the DP reading. In addition, although the second main disjunct (*she is the first/second in our family who can go study in Japan/States*) triggers the FC presupposition that Maria can study in Japan and in the States, (48) as a whole doesn't inherit that presupposition, as captured in (48a). What seems to be going on is that the negation of the first main disjunct somehow filters out the FC presupposition triggered by the second main disjunct.

(46) Maria can go study in Tokyo or Boston.

  a. $\rightsquigarrow$ *Maria can study in Tokyo*
     $\rightsquigarrow$ *Maria can study in Boston*

(47) Maria can't go study in Tokyo or Boston.

  a. $\rightsquigarrow$ *Maria can't study in Tokyo*
     $\rightsquigarrow$ *Maria can't study in Boston*

(48) Either Maria can't go study in Tokyo or Boston, or she is the first in our family who can go study in Japan and the second who can go study in the States.

  a. $\not\rightsquigarrow$ *Maria can study in Japan*
     $\not\rightsquigarrow$ *Maria can study in the States*

To appreciate the challenge presented by this pattern, it will be helpful to introduce some notational conventions and background assumptions about presupposition projection. Following Romoli & Santorio (2019), we schematically represent the target example in (48) as in (49), where A^+ asymmetrically entails A, B^+ asymmetrically entails B and $C_{\Diamond A \wedge \Diamond B}$ says that C is asserted while $\Diamond A \wedge \Diamond B$ is presupposed (for readability, we represent the assertive part of the conjunction in the second main disjunct with just C).

(49) Either $\neg\Diamond(A^+ \vee B^+) \vee C_{\Diamond A \wedge \Diamond B}$ $\qquad$ $\not\rightsquigarrow \Diamond A; \not\rightsquigarrow \Diamond B$

We assume that disjunctions with a presupposition in the second disjunct, $p \vee q_r$, project a conditional presupposition, as in (50) (Heim 1982, Chierchia 1995, Beaver 2001), from which it follows that the presupposition r is filtered if $\neg p \models r$. That explains why a sentence like *Either Maria didn't study in Tokyo, or she is the first in our school who studied in Japan* doesn't presuppose that Maria studied in Japan. We also assume that presuppositions triggered in the first disjunct tend to project unconditionally, as in (51) (since $\neg p_r \rightarrow q$ presupposes r, e.g., *if John doesn't find out that Mary came, she will feel sad* presupposes that Mary came).[9]

(50) $$p \vee q_r = \begin{cases} \textbf{ps: } \neg p \rightarrow r \\ \textbf{asserts: } p \vee q \end{cases}$$

(51) $$p_r \vee q = \begin{cases} \textbf{ps: } r \\ \textbf{asserts: } p \vee q \end{cases}$$

With these conventions and assumptions in place, we can now see why the pattern in (46)-(48) presents a problem for **exh**-based accounts of FC (Romoli & Santorio 2019). On the one hand, to get the DP reading of the first main disjunct of (48) (*Maria can't go study in Tokyo or Boston*), we need a parse for that disjunct without $\textbf{exh}^{IE+II}$ under negation, as in (52a). Yet given that parse, we predict an incorrect presupposition, captured in the **ps** part of (52b). For as shown in (52c), the conditional presupposition in the **ps** part doesn't filter out $\Diamond A$ and $\Diamond B$ (i.e., that Maria can study in Japan and in the States), since $\Diamond(A^+ \vee B^+) \not\models \Diamond A \wedge \Diamond B$.

(52)
a. $\neg\Diamond(A^+ \vee B^+) \vee C_{\Diamond A \wedge \Diamond B}$

b. $$[\![(52a)]\!] = \begin{cases} \textbf{ps: } \neg\neg\Diamond(A^+ \vee B^+) \rightarrow (\Diamond A \wedge \Diamond B) \\ \textbf{asserts: } \neg\Diamond(A^+ \vee B^+) \vee C \end{cases}$$

c. $$\neg\neg\Diamond(A^+ \vee B^+) \rightarrow (\Diamond A \wedge \Diamond B) \\ = \Diamond(A^+ \vee B^+) \rightarrow (\Diamond A \wedge \Diamond B) \qquad \neq \top$$

On the other hand, consider a parse for (48) with $\textbf{exh}^{IE+II}$ under negation in the first main disjunct, as in (53a). As captured in the **ps** part of (53b), and given the simple equivalence in (53c), this would correctly filter out the presupposition, triggered in the second main disjunct, that Maria can study in Japan and in the States. However, we now loose the target DP reading for the first disjunct, *Maria can't go study in Tokyo or Boston*, and get instead the unattested and weaker ('negation of FC') reading, as captured in the **asserts** part of (53b).

(53)
a. $\neg\textbf{exh}^{IE+II}[\Diamond(A^+ \vee B^+)] \vee C_{\Diamond A \wedge \Diamond B}$

b. $$[\![(53a)]\!] = \begin{cases} \textbf{ps: } \neg\neg\textbf{exh}^{IE+II}[\Diamond(A^+ \vee B^+) \rightarrow (\Diamond A \wedge \Diamond B)] \\ \textbf{asserts: } \neg\textbf{exh}^{IE+II}[\Diamond(A^+ \vee B^+)] \vee C \end{cases}$$

c. $$\neg\neg\textbf{exh}^{IE+II}[\Diamond A^+ \vee \Diamond B^+] \rightarrow (\Diamond A \wedge \Diamond B) \\ = (\Diamond A^+ \wedge \Diamond B^+) \rightarrow (\Diamond A \wedge \Diamond B) \qquad = \top$$

9 The projection rules in (50) and (51) are not the only candidates. We selected those rules because they are fairly standard and, as Romoli & Santorio (2019) show, other reasonable candidates worsen the predictions of **exh**-based theories for sentences like (48).

A parse like (53a), except with $\mathbf{exh}^{IE+II}$ above the negation, also doesn't help. For it is easy to check that $\mathbf{exh}^{IE+II}[\neg\Diamond(A^+ \vee B^+)] = \neg\Diamond(A^+ \vee B^+)$, so we end up with the same result as with (52a).

Romoli & Santorio (2019) show that filtering FC cases like (48) also challenge various Grammatical theories which allow exhaustification to have an effect on the assertive and presuppositional content of its prejacent (e.g., Magri 2009b, Gajewski & Sharvit 2012, Spector & Sudo 2017). Now, as we saw in §3.4, Marty & Romoli (2020)'s $\mathbf{exh}^{IE+II}_{asr+psr}$-based theory can be seen as a way of combining exhaustification with both exclusion and inclusion functions with the main insights of multi-dimensional exhaustication. Since the $\mathbf{exh}^{IE+II}_{asr+psr}$-based theory improves the predictions of previous Grammatical accounts, and solves the presupposed FC puzzles, it is important to determine if it can also deal with the filtering FC puzzles and bypass the objections which Romoli & Santorio (2019) raise against previous Grammatical theories with multidimensional exhaustification.

It turns out that the $\mathbf{exh}^{IE+II}_{asr+psr}$-based theory of FC doesn't help with the filtering FC puzzles. To see why, recall two facts about $\mathbf{exh}^{IE+II}_{asr+ps}$, which we established in §3.4. The first is that, when applied to basic $\Diamond\vee$-sentences, $\mathbf{exh}^{IE+II}$ and $\mathbf{exh}^{IE+II}_{asr+ps}$ have the same effect, namely, FC at the assertive level and no presuppositions. For recall that, unlike $\mathbf{pex}^{IE+II}$, $\mathbf{exh}^{IE+II}_{asr+ps}$ is not a presupposition trigger, and when neither its prejacent nor any of its formal alternatives are presuppositional, $\mathbf{exh}^{IE+II}_{asr+ps}$ outputs the same fully assertive content as $\mathbf{exh}^{IE+II}$. The second fact is that, when its prejacent is presuppositional, $\mathbf{exh}^{IE+II}_{asr+ps}$ can only strengthen the presuppositions of its output: for it not only passes on the presuppositions of its prejacent, but also adds those of any alternatives in IE_{asr}, II_{asr}, and II_{psr}, plus the negation of any presuppositions in IE_{psr}. Just like the function of standard $\mathbf{exh}^{IE+II}$, at the assertive level, is to strengthen the content of its prejacent, $\mathbf{exh}^{IE+II}_{asr+ps}$ is designed to strengthen, not weaken, the presuppositional content of its prejacent.

With that in mind, let us consider possible parses, beginning with (54). Given the first fact, and in particular that $\mathbf{exh}^{IE+II}_{asr+ps}[\Diamond(A^+ \vee B^+)] = \Diamond A \wedge \Diamond B$, it is clear that (54) gets us the same reading as the one predicted by the parallel parse with $\mathbf{exh}^{IE+II}$. That is, we get the target filtering FC effect, but not the target DP reading of the first main disjunct, since $\neg\mathbf{exh}^{IE+II}_{asr+ps}[\Diamond(A^+ \vee B^+)]$ amounts to the denial of FC, rather than the stronger double prohibition reading.

(54) $\neg\mathbf{exh}^{IE+II}_{asr+ps}[\Diamond(A^+ \vee B^+)] \vee C_{\Diamond A \wedge \Diamond B}$

Again, dropping the $\mathbf{exh}^{IE+II}_{asr+ps}$ from under negation in (54) doesn't help: for as shown in (52a)-(52c) above, although that predicts the target DP reading of the first disjunct, it fails to filter out the FC presupposition triggered by the second main disjunct. Another possibility is to apply $\mathbf{exh}^{IE+II}_{asr+psr}$ over the entire disjunction, as in

(55). This might seem promising, since the expressive power of $\mathbf{exh}^{IE+II}_{asr+psr}$ really comes out when its prejacent or its alternatives are presuppositional.

(55) $\mathbf{exh}^{IE+II}_{asr+ps}[\neg\Diamond(A^+ \vee B^+) \vee C_{\Diamond A \wedge \Diamond B}]$

(55) captures the desired DP reading for the first main disjunct. Yet recall the second fact about $\mathbf{exh}^{IE+II}_{asr+psr}$, i.e., that it can only strengthen the presuppositional content of its prejacent. In (55), the prejacent of $\mathbf{exh}^{IE+II}_{asr+psr}$ is $\neg\Diamond(A^+ \vee B^+) \vee C_{\Diamond A \wedge \Diamond B}$, which we have shown triggers the FC presupposition $\Diamond A \wedge \Diamond B$. The FC presupposition is also triggered by the conjunctive alternative, $\neg\Diamond(A^+ \vee B^+) \wedge C_{\Diamond A \wedge \Diamond B}$, which is in $IE_{asr}(\neg\Diamond(A^+ \vee B^+) \vee C_{\Diamond A \wedge \Diamond B})$. For as shown in (56), the first conjunct doesn't entail the FC presupposition triggered by the second conjunct, hence the latter is not filtered out:

(56) $\neg\Diamond(A^+ \vee B^+) \wedge C_{\Diamond A \wedge \Diamond B}$ $\qquad \neg\Diamond A^+ \wedge \neg\Diamond B^+ \not\models \Diamond A \wedge \Diamond B$

It follows that the FC proposition is undoubtedly predicted to be part of the presuppositional level output of the matrix $\mathbf{exh}^{IE+II}_{asr+psr}$ in (55). Yet filtering FC sentences like (48), on the target reading, don't presuppose or more generally entail FC.

The filtering FC puzzle, then, presents a serious challenge to basically all extant **exh**-based theories of FC.

4.2 A solution based on presuppositional exhaustification

In contrast to **exh**-based accounts of FC, our $\mathbf{pex}^{IE+II}$-based account directly predicts the target readings for (46)-(48). A key difference between these accounts, recall, is that while **exh**-based accounts assign a flat, fully assertive structure to the FC reading of $\Diamond\vee$-sentences, as in (57), our **pex**-based account decomposes the FC reading into presupposed and assertive components, as in (58):

(57) $\mathbf{exh}^{IE+II}[\Diamond(p \vee q)] = \mathbf{exh}^{IE+II}_{asr+psr}[\Diamond(p \vee q)] = \Diamond p \wedge \Diamond q$

(58) $\mathbf{pex}^{IE+II}[\Diamond(p \vee q)] = \Diamond(p \vee q)_{\Diamond p \leftrightarrow \Diamond q}$ $\qquad \models \Diamond p \wedge \Diamond q$

Both accounts predict the FC reading for $\Diamond\vee$-sentences like (46) and the DP reading for $\neg\Diamond\vee$-sentences like (47). Yet due to its structuring of the FC interpretation into presupposed vs. asserted components, only the $\mathbf{pex}^{IE+II}$-based account can derive both readings while locally exhaustifying the embedded $\Diamond\vee$-sentence, i.e., using $\mathbf{pex}^{IE+II}[\Diamond(A^+ \vee B^+)]$ for (46) and $\neg\,\mathbf{pex}^{IE+II}[\Diamond(A^+ \vee B^+)]$ for (47) (see §2.2). This fact plays a crucial role when trying to derive both the DP reading for the first disjunct of (48) (= *Maria can't go study in Tokyo or Boston*), and the normal FC reading for the negation of that first disjunct to determine whether the presupposition of the second disjunct is filtered out (i.e., to determine whether 'Maria can study in Japan and in the States' is filtered out).

The key filtering FC sentence (48) is repeated in (59). Given our $\mathbf{pex}^{IE+II}$-based theory, the parse in (59a) is a natural choice. And as we now show, it generates the target reading. Applying (50) to (59a) to derive its presupposition, we get that (59) presupposes $\mathbf{pex}^{IE+II}[\Diamond(A^+ \vee B^+)] \rightarrow (\Diamond A \wedge \Diamond B)$, as shown in the **ps** part in (59b). The antecedent of this conditional is just an FC interpretation of a $\Diamond\vee$-sentence, as shown in (59c), which entails $\Diamond A^+ \wedge \Diamond B^+$ and thus correctly filters out $\Diamond A$ and $\Diamond B$. Consider next the content of the first main disjunct *Maria can't go study in Tokyo or Boston*, which is parsed as $\neg\,\mathbf{pex}^{IE+II}[\Diamond(A^+ \vee B^+)$. As shown in (59d), due to the presupposed vs. assertive structure generated by $\mathbf{pex}^{IE+II}$, the homogeneity presupposition projects from under negation. As a result, the negation applies directly to $\Diamond(A^+ \vee B^+)$, and we get the target DP reading of *Maria can't go study in Tokyo or Boston*.[10]

(59) Either Maria can't go study in Tokyo or Boston, or she is the first in our family who can go study in Japan and the second who can go study in the States.

a. $\neg\,\mathbf{pex}^{IE+II}[\Diamond(A^+ \vee B^+)] \vee C_{\Diamond A \wedge \Diamond B}$

b. $[\![(59a)]\!] = \begin{cases} \textbf{ps: } \mathbf{pex}^{IE+II}[\Diamond(A^+ \vee B^+)] \rightarrow (\Diamond A \wedge \Diamond B) \\ \textbf{asserts: } \neg\,\mathbf{pex}^{IE+II}[\Diamond(A^+ \vee B^+)] \vee C \end{cases}$

c. $\mathbf{pex}^{IE+II}[\Diamond(A^+ \vee B^+)] \rightarrow (\Diamond A \wedge \Diamond B)$
$= \Diamond(A^+ \vee B^+)_{\Diamond A^+ \leftrightarrow \Diamond B^+} \rightarrow (\Diamond A \wedge \Diamond B)$

d. $\neg\,\mathbf{pex}^{IE+II}[\Diamond(A^+ \vee B^+)]$
$= \neg[\Diamond(A^+ \vee B^+)_{\Diamond A^+ \leftrightarrow \Diamond B^+}]$ (ps projects under $\neg$)
$= \neg\Diamond(A^+ \vee B^+)_{\Diamond A^+ \leftrightarrow \Diamond B^+}$

Unlike **exh**-based accounts of FC, then, our $\mathbf{pex}^{IE+II}$-based account resolves the filtering FC puzzle posed by (46)-(48). Specifically, our account predicts that (48)/(59) doesn't presuppose that Maria can study in Japan and in the States, while simultaneously assigning to the embedded *Maria can't go study in Tokyo or Boston* the desired DP reading. We have shown that those predictions follow directly from our original $\mathbf{pex}^{IE+II}$-based account of FC-$\Diamond\vee$-sentences like (46) and DP-$\neg\Diamond\vee$-sentences like (47), given standard assumptions about presupposition projection and filtering in disjunctions. That meets the main desiderata, laid out by Romoli & Santorio (2019), to capture the default reading of sentences like (48)/(59).[11]

10 The first disjunct of the main disjunction, $\neg\,\mathbf{pex}^{IE+II}[\Diamond(A^+ \vee B^+)$, also presupposes $\Diamond A^+ \leftrightarrow \Diamond B^+$, as shown in (59d). Assuming (51), we would then predict that this homogeneity presupposition should be inherited by (59). For discussion of this prediction, see §4.4 below.

11 The filtering FC puzzle directly supports our proposal that $\mathbf{pex}^{IE+II}$ should presuppose homogeneity over, rather than the conjunction of, the *II*-alternatives. Presupposing the truth of each *II* alternative

4.3 Extension to filtering negative free choice

The filtering FC puzzle is also observed with embedded FC conjunctions, as illustrated in (60)-(62) below. Recall that (61), in one of its salient readings, has the negative FC entailment that Maria is allowed to not study in Japan and is also allowed to not study in the States. Next, notice that while the second main disjunct of (62) (*[Maria is] the first in her family who is not required to study in Tokyo and the second who's not required to study in Boston*) presupposes the negative FC proposition that Maria is allowed to not study in Tokyo and is also allowed to not study in Boston, that proposition doesn't project as a presupposition of (62), as captured in (62a). As a first guess, it seems that the negative FC proposition triggered by the second disjunct is entailed and hence filtered out by the (negative FC reading of) the negation of the first disjunct of (62).

(60) Maria is required to study in Japan and the States.

a. $\rightsquigarrow$ *Maria is required to study in Japan*
$\rightsquigarrow$ *Maria is required to study in the States*

(61) Maria is not required to study in Japan and the States.

a. $\rightsquigarrow$ *Maria is allowed to not study in Japan*
$\rightsquigarrow$ *Maria is allowed to not study in the States*

(62) Either Maria is required to study in Japan and the States, or she's the first in her family who is not required to study in Tokyo and the second who's not required to study in Boston.

a. $\not\rightsquigarrow$ *Maria is allowed to not study in Tokyo*
$\not\rightsquigarrow$ *Maria is allowed to not study in Boston*

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might seem superficially closer to Bar-Lev & Fox's proposal, but it fails to derive the target reading of (59). Recall: to filter out the presupposition triggered in the second disjunct of (59), we need a parse as in (59a). If $\mathbf{pex}^{IE+II}$ is defined so as to presuppose the conjunction of the *II* alternatives, $\mathbf{pex}^{IE+II}[\Diamond(A^+ \vee B^+)]$ would presuppose $\Diamond A^+ \wedge \Diamond B^+$. Due to projection from under negation, the first disjunct of (59), $\neg\mathbf{pex}^{IE+II}\,\Diamond(A^+ \vee B^+)$, would then entail $\Diamond A^+ \wedge \Diamond B^+$, which would directly clash with its assertive content, $\neg(\Diamond A^+ \vee \Diamond B^+)$, or project out as a presupposition of (59) itself. Either way, we fail to get the target reading. Local accommodation of the *II* alternatives under negation would not help us get the target reading, since we would then only get the negation of FC reading of the first disjunct of (59), which is too weak. Again, if we use a parse without $\mathbf{pex}^{IE+II}$ under negation in the first main disjunct, we fail to filter out the $\Diamond A \wedge \Diamond B$ presupposition triggered in the second main disjunct, since obviously $\neg\neg\Diamond(A^+ \vee B^+) \not\models \Diamond A \wedge \Diamond B$. In §4.3, we present further evidence to support our suggestion that, when modelling covert exhaustification as a presupposition trigger, the incorporation of *II*-alternatives should be via a homogeneity presupposition.

Following Romoli & Santorio (2019), let us schematically represent the target reading of (62)—our key example of filtering negative FC—as in (63), where A^+ asymmetrically entails A and B^+ asymmetrically entails B:

(63) Either $\Box(A \wedge B) \vee C_{\neg\Box A^+ \wedge \neg\Box B^+}$ $\qquad \not\rightsquigarrow \neg\Box A^+$, $\not\rightsquigarrow \neg\Box B^+$

Based on the projection rule for disjunctions in (50), the presupposition triggered in the second main disjunct of (62)/(63), $\neg\Box A^+ \wedge \neg\Box B^+$, is filtered out if it is entailed by the negation of the first main disjunct, $\neg\Box(A \wedge B)$. It follows that, given an LF which closely matches the surface form of (63), we would not predict the target filtering effect, since $\neg\Box(A \wedge B) \not\models \neg\Box A^+ \wedge \neg\Box B^+$.

As Romoli & Santorio (2019) emphasize, this version of the filtering FC puzzle is specially interesting because, while recent Lexicalist accounts such as Goldstein (2018) predict the main desiderata for the original filtering FC pattern in (46)-(48), they don't help with the analogous pattern in (60)-(62). The basic the problem parallels the one posed by presupposed (negative) FC under negative factives (see §3.3). Recall that the presupposition triggered in the second main disjunct of (62) is filtered out if it is entailed by the negation of the first disjunct. So we would get the target filtering effect if we could derive a negative FC reading for ¬*Maria is required to study in Japan and the States*. However, these Lexicalist accounts do not predict a negative FC reading for $\neg\Box\wedge$-sentences such as (61).

Going back to standard Grammatical accounts, and given the result in (63), one may hope that a parse as in (64a) somehow predicts the target filtering effect. However, the effect of $\mathbf{exh}^{IE+II}$ in the first disjunct is vacuous, since obviously no *IE* alternatives can be negated, and the *II* alternatives $\Box A$ and $\Box B$ are entailed by the prejacent. Since $\mathbf{exh}^{IE+II}[\Box(A \wedge B)] = \Box(A \wedge B)$, the effect is as if we had a parse without $\mathbf{exh}^{IE+II}$ in the first disjunct. As a result, the $\neg\Box A^+$ and $\neg\Box B^+$ presuppositions of the second disjunct are not filtered out, since $\neg\mathbf{exh}^{IE+II}[\Box(A \wedge B)]$ (= $\neg\Box(A \wedge B)$) entails neither $\neg\Box A^+$ nor $\neg\Box B^+$, as shown in (64c).

(64) a. $\mathbf{exh}^{IE+II}[\Box(A \wedge B)] \vee C_{\neg\Box A^+ \wedge \neg\Box B^+}$

b. $[\![(64a)]\!] = \begin{cases} \textbf{ps: } \neg\mathbf{exh}^{IE+II}[\Box(A \wedge B)] \rightarrow (\neg\Box A^+ \wedge \neg\Box B^+) \\ \textbf{asserts: } \Box(A \wedge B) \vee C \end{cases}$

c. $\neg\mathbf{exh}^{IE+II}[\Box(A \wedge B)] \rightarrow (\neg\Box A^+ \wedge \neg\Box B^+)$
$= \neg[\Box(A \wedge B)] \rightarrow (\neg\Box A^+ \wedge \neg\Box B^+)$
$= (\neg\Box A \vee \neg\Box B) \rightarrow (\neg\Box A^+ \wedge \neg\Box B^+)$ $\qquad \neq \top$

This result suggests that, to filter out the presuppositions of the second disjunct of (62), we need to somehow get—when calculating the presuppositions of (62)—$\mathbf{exh}^{IE+II}$ to scope over the negation of the first disjunct, since $\mathbf{exh}^{IE+II}[\neg\Box(A \wedge B)]$ entails both $\neg\Box A^+$ and $\neg\Box B^+$. The problem, however, is that we can't get that scoping effect, at least without appealing to some non-standard syntactic operation.[12]

In contrast, our $\mathbf{pex}^{IE+II}$-based account captures the pattern in (60)-(62) without any additional stipulations. Consider the parse in (65a), which is structurally analogous to the one in (64a). $\mathbf{pex}^{IE+II}[\Box(A \wedge B)]$ doesn't exclude anything. However, the conjunctive alternatives of the prejacent, $\Box A$ and $\Box B$, are in II, since taken together they can be consistently conjoined with the prejacent and negation of any (in this case none) IE alternatives. It follows that $\mathbf{pex}^{IE+II}[\Box(A \wedge B)] = [\Box(A \wedge B)]_{\Box A \leftrightarrow \Box B}$. Then when determining the presupposition of (65a), shown in (65b), the homogeneity presupposition will project out of the negation in the antecedent, as shown in (65c).

(65) a. $\mathbf{pex}^{IE+II}[\Box(A \wedge B)] \vee C_{\neg\Box A^+ \wedge \neg\Box B^+}$

b. $[\![(65a)]\!] = \begin{cases} \textbf{ps: } \neg\,\mathbf{pex}^{IE+II}[\Box(A \wedge B)] \rightarrow (\neg\Box A^+ \wedge \neg\Box B^+) \\ \textbf{asserts: } \Box(A \wedge B) \vee C \end{cases}$

c. $\neg\,\mathbf{pex}^{IE+II}[\Box(A \wedge B)] \rightarrow (\neg\Box A^+ \wedge \neg\Box B^+)$
$= \neg[\Box(A \wedge B)]_{\Box A \leftrightarrow \Box B} \rightarrow (\neg\Box A^+ \wedge \neg\Box B^+)$
$= (\neg\Box A \vee \neg\Box B)_{\Box A \leftrightarrow \Box B} \rightarrow (\neg\Box A^+ \wedge \neg\Box B^+) \qquad = \top$

It is easy to check that the antecedent of (65c) entails the consequent, since $(\neg\Box A \vee \neg\Box B)_{\Box A \leftrightarrow \Box B} \models \neg\Box A \wedge \neg\Box B$, and $\neg\Box A \wedge \neg\Box B \models \neg\Box A^+ \wedge \neg\Box B^+$ (i.e., that Maria is not required to study in Japan and not required to Study in the States entails that she is not required to study in Tokyo and not required to study in Boston). Accordingly, the $\neg\Box A^+ \wedge \neg\Box B^+$ presupposition triggered in the second disjunct of (65a) is filtered out (by the negation of the first main disjunct), so is not inherited as a presupposition of (65a) as a whole. This is precisely the target result.

Interestingly, the core result which we used to solve the filtering (negative) FC puzzle—that $\mathbf{pex}^{IE+II}$, when applied to $\Box\wedge$-sentences, triggers a homogeneity presupposition—helps solve other important puzzles for theories of FC. Consider

12 Romoli & Santorio (2019) consider a syntactic operation—inspired by related proposal in Rothschild (2017)—that licenses the free insertion and then exhaustification of redundant material at LF (but see Elliott (2020) for an alternative account of the original motivating data). The basic idea is that target structures like (ia) can be expanded to (ib), which would result in the desired filtering.

(i) a. $\Box(A \wedge B) \vee C_{\neg\Box A^+ \wedge \neg\Box B^+}$

b. $\Box(A \wedge B) \vee \underline{\mathbf{exh}^{IE+II}[\neg\Box(A \wedge B)] \wedge} C_{\neg\Box A^+ \wedge \neg\Box B^+}$

Romoli & Santorio (2019) convincingly argue that the required operation includes various unmotivated stipulations: e.g., it over-generates in certain simple cases where the insertion and exhaustification of redundant material doesn't seem to be available. Finally, even if this kind of operation were validated, standard **exh**-based theories wouldn't issue in a uniform account of the full presupposed & filtering FC puzzles. For free insertion of redundant material doesn't help with the presupposed FC under negative factives puzzles (see §3) the pattern in (66)-(67) with universal and negative universal FC, due to Chemla (2009b), which Bar-Lev & Fox (2020) use to motivate their proposal that **exh** has an inclusion function.

(66) Every student is allowed to eat cake or ice cream.

a. $\rightsquigarrow$ *Every student is allowed to eat cake* $\forall x \Diamond Cx$
$\rightsquigarrow$ *Every student is allowed to ice cream* $\forall x \Diamond ICx$

(67) No student is required to solve problem A and problem B.

a. $\rightsquigarrow$ *No student is required to solve problem A* $\neg\exists x \Box Ax$
$\rightsquigarrow$ *No student is required to solve problem B* $\neg\exists x \Box Bx$

Standard Grammatical theories with IE-based **exh** have difficulties with (66)-(67). The target FC reading of (66) can be derived from a parse with embedded recursive $\mathbf{exh}^{IE}$ over $\Diamond(Cx \vee ICx)$. However, a parallel parse for (67) with embedded recursive $\mathbf{exh}^{IE}$ doesn't predict its target reading: for $\mathbf{exh}^{IE}[\mathbf{exh}^{IE}[\Box(Ax \wedge Bx)]]$ is vacuous, since $\mathbf{exh}^{IE}[\Box(Ax \wedge Bx)]$ has no IE alternatives. In contrast, a $\mathbf{pex}^{IE+II}$-based account can directly predicts the target FC reading for both (66) and (67) based on the following structurally analogous LFs with embedded $\mathbf{pex}^{IE+II}$:

(68) a. $\text{EVERY}([\![student]\!])\ \lambda x.\ \mathbf{pex}^{IE+II}[\Diamond(Cx \vee ICx)]$
b. $\text{NO}([\![student]\!])\ \lambda x.\ \mathbf{pex}^{IE+II}[\Box(Ax \wedge Bx)]$

Consider first (66) given the LF in (68a). $\mathbf{pex}^{IE+II}[\Diamond(Cx \vee ICx)]$ triggers the homogeneity presupposition $\Diamond Cx \leftrightarrow \Diamond ICx$ in the scope of *every student*. Assuming as before that a presupposition in the scope of a quantifier projects universally, when it is combined with the assertive content we get the target FC inference:

$$\forall x \in [\![student]\!](\Diamond Cx \leftrightarrow \Diamond ICx) \wedge \forall x \in [\![student]\!]\Diamond(Cx \vee ICx)$$
$$\models \forall x \in [\![student]\!]\Diamond Cx \wedge \forall x \in [\![student]\!]\Diamond ICx$$

Consider next (67) given the LF in (68b). $\mathbf{pex}^{IE+II}[\Box(Ax \wedge Bx)]$ triggers the homogeneity presupposition $\Box Ax \leftrightarrow \Box Bx$ in the scope of *no student*. Assuming again universal projection of that presupposition, when it is combined with the assertive content we get the target (negative) FC inference:

$$\forall x \in [\![student]\!](\Box Ax \leftrightarrow \Box Bx) \wedge \neg\exists x \in [\![student]\!]\Box(Ax \wedge Bx)$$
$$\models \neg\exists x \in [\![student]\!]\Box Ax \wedge \neg\exists x \in [\![student]\!]\Box Bx$$

This result supports our proposal that we should implement the hypothesis that exhaustification has both an exclusion and an inclusion function using a $\mathbf{pex}^{IE+II}$-like operator. For not only do we resolve the presupposed and filtering FC puzzles, which we don't with a flat $\mathbf{exh}^{IE+II}$ operator, but we can also account, using parallel analyses and no additional stipulations, for core cases that motivated the adoption of $\mathbf{exh}^{IE+II}$ in the first place.[13]

Summing up, we have seen that cases of filtering negative FC, illustrated by (62), presents a serious challenge to current theories of FC. Various old and new Lexicalist theories fail because they don't predict negative FC, even in simple cases, while both $\mathbf{exh}^{IE+II}$ and $\mathbf{exh}^{IE+II}_{asr+psr}$-based Grammatical theories are forced to appeal to non-trivial syntactic stipulations. In contrast, our $\mathbf{pex}^{IE+II}$-based account of FC directly predicts, without the need of any additional stipulations, the target reading of filtering (negative) FC sentences.

4.4 Homogeneity in enemy territory

Our $\mathbf{pex}^{IE+II}$-based account of filtering FC sentences such as (69) uses the parse in (69a) to predict the target reading: i.e., a DP reading for the first main disjunct and filtering of the presupposition of the second main disjunct. Yet as we showed in (59a)-(59d), (69a) also predicts that (69) inherits the homogeneity presupposition $\Diamond A^+ \leftrightarrow \Diamond B^+$ (= Maria can study in Tokyo iff she can study in Boston).

13 Bar-Lev & Fox (2020) argue that there should be a non-embedded derivation of the FC reading even for (66). They appeal to cases like (i) in which a positive universal FC sentence like (66) licenses a VP ellipsis where the elided material is in a DE environment and gets the negative FC reading:

(i) Every student in section A is allowed to eat cake or ice cream on their birthday. Weirdly, no student in section B is ~~allowed to eat cake or ice cream on their birthday~~.

a. $\rightsquigarrow$ *Every student in A is allowed to eat cake*
$\rightsquigarrow$*Every student in A is allowed to eat ice cream*

b. $\rightsquigarrow$*No student in B is allowed to eat cake*
$\rightsquigarrow$*No student in B is allowed to eat ice cream*

Suppose that the material embedded under *every student in x* has to be part of the parallelism domain for ellipsis. Still, the target reading can be straightforwardly derived using embedded $\mathbf{pex}^{IE+II}$. At this point, it is easy to check that, using the same structure with embedded $\mathbf{pex}^{IE+II}$ in the scope of *every* and *no*—i.e., $\mathbf{pex}^{IE+II}[\Diamond(Cx \vee ICx)]$—we predict the FC reading for the first sentence and the DP reading for the second sentence with the ellided VP, which is determined by how the homogeneity presupposition interacts with the corresponding quantificational claim, as shown in (ii)-(iii):

(ii) $$\forall x \in [\![student\ in\ A]\!](\Diamond Cx \leftrightarrow \Diamond ICx) \wedge \forall x \in [\![student\ in\ A]\!]\Diamond(Cx \vee ICx) \models \forall x \in [\![student\ in\ A]\!]\Diamond Cx \wedge \forall x \in [\![student\ in\ A]\!]\Diamond ICx$$

(iii) $$\forall x \in [\![student\ in\ B]\!](\Diamond Cx \leftrightarrow \Diamond ICx) \wedge \neg\exists x \in [\![student\ in\ B]\!]\Diamond(Cx \vee ICx) \models \neg\exists x \in [\![student\ in\ B]\!]\Diamond Cx \wedge \neg\exists x \in [\![student\ in\ B]\!]\Diamond ICx$$

(69) Either Maria can't go study in Tokyo or Boston, or she is the first our family who can study in Japan (and the second who can study in the States).

a. $\neg\,\mathbf{pex}^{IE+II}[\Diamond(A^+ \vee B^+)] \vee C_{\Diamond A \wedge \Diamond B}$

Romoli & Santorio (2019) argue, however, that accounts which predict that homogeneity presupposition may face an independent problem: it seems that S can felicitously assert (69)—with the target reading—in a context that makes explicit that S doesn't believe the homogeneity proposition. Consider:

(70) Maria applied to Tokyo or Boston. I have no idea whether she was admitted to only one, both, or neither, but ...

a. Either she can't go study in Tokyo or Boston, or she is the first in our family who can study in Japan and the second who can study in the States.

Can our $\mathbf{pex}^{IE+II}$-based account handle these cases? As we said at the outset (see also Bassi et al. 2021, Del Pinal 2021), the presuppositions triggered by $\mathbf{pex}^{IE+II}$ tend to be globally accommodated when they are consistent with the common ground. A non-collapsed common ground which entails, say, the information in (70) can be consistently updated so as to entail the homogeneity proposition that Maria can study in either both or neither of Tokyo and Boston. So why not appeal, as in other superficially similar cases, to global accommodation? The problem is that, in (70), S explicitly acknowledges ignorance concerning the homogeneity proposition; and as the example is intended to be framed, S doesn't, in the middle of the discourse, acquire or remember any relevant new information. Accordingly, interlocutors can't reasonably globally accommodate the homogeneity proposition when they process (70a): for they would then represent S as simultaneously agnostic and believing in the homogeneity proposition.[14]

What we seem to need, then, is to block the projection of the homogeneity proposition from out of the first main disjunct of (70a), without affecting other components of the target reading. Could we just apply local accommodation (ACC) over the first main disjunct? Consider the parse in (71a), which is like the one that supports the target predictions (in neutral contexts) for our original filtering FC examples with DP $\neg\Diamond\vee$-sentences, except that we apply ACC to the first main disjunct to block the projection of homogeneity. Although we preserve the desired DP reading for the first disjunct, as captured in the **asserts** part in (71b), given the equivalence in (71b), the problem now is that, at the presuppositional level, we no longer filter out $\Diamond A$ and $\Diamond B$, as can captured in the **ps** part of (71b). For,

14 For similar reasons, we can't plausibly use matrix level ACC to block (70a) from presupposing the homogeneity proposition.

given (71c), $\neg\text{ACC}(\neg\,\mathbf{pex}^{IE+II}[\Diamond(A^+ \vee B^+)]) = \neg(\neg\Diamond A^+ \wedge \neg\Diamond B^+) = \Diamond A^+ \vee \Diamond B^+$, and obviously $\Diamond A^+ \vee \Diamond B^+ \not\models \Diamond A \wedge \Diamond B$.

(71) a. $\text{ACC}[\neg\,\mathbf{pex}^{IE+II}[\Diamond(A^+ \vee B^+)]] \vee C_{\Diamond A \wedge \Diamond B}$

b. $[\![(71a)]\!] = \begin{cases} \textbf{ps: } \neg\text{ACC}(\neg\,\mathbf{pex}^{IE+II}[\Diamond(A^+ \vee B^+)]) \to \Diamond A \wedge \Diamond B \\ \textbf{asserts: } \text{ACC}(\neg\,\mathbf{pex}^{IE+II}[\Diamond(A^+ \vee B^+)]) \vee C \end{cases}$

c. $\text{ACC}(\neg\,\mathbf{pex}^{IE+II}[\Diamond(A^+ \vee B^+)])$
$= (\Diamond A^+ \leftrightarrow \Diamond B^+) \wedge \neg\Diamond(A^+ \vee B^+)$
$= \neg\Diamond A^+ \wedge \neg\Diamond B^+$

Could we go for a 'direct' solution and apply ACC over each disjunct, as in (72)?

(72) $\text{ACC}_1[\neg\,\mathbf{pex}^{IE+II}[\Diamond(A^+ \vee B^+)]] \vee \text{ACC}_2[C_{\Diamond A \wedge \Diamond B}]$

ACC_1 cancels the projection of the homogeneity proposition from the first main disjunct, without altering its desired DP reading. In addition, ACC_2 cancels the projection of $\Diamond A$ and $\Diamond B$ from out of the second disjunct. Accordingly, the parse in (72) captures the two core elements of the target reading of (70a). The key question, then, is this: Are there independently motivated licensing conditions which permit, in special contexts like (70), the insertion of ACC so as to generate parses like (72)?

Local accommodation is typically thought to have strict licensing conditions. A standard hypothesis is that ACC is only licensed when it is marked with specific intonation patterns or the corresponding parse without ACC would result in incoherent or defective contents. This approach will work for our purposes, if understood as including the condition that avoidance of inter-sentential, speaker-anchored incoherence licenses ACC. Yet that extension is needed to apply the local accommodation-based account (of the coherence) of sentences like (73a) to parallel discourses like (73b), as seems natural:

(73) a. The king of France isn't bald, since there is no king of France!

b. The kind of France isn't bald. For there is no king of France!

Based on those licensing conditions for ACC, we can show that the parse in (72) is licensed when a speaker S asserts (70a) after (70)—or more generally, when (70a) is evaluated relative to a common ground which entails that S doesn't believe the homogeneity proposition. As we showed earlier, ACC_1 is required to avoid attributing to S the incoherent attitude of being both agnostic towards and believing in the homogeneity proposition $\Diamond A^+ \leftrightarrow \Diamond B^+$. ACC_2 is required to avoid uncharitably attributing to S a basic incapacity to draw the implications of S's own doxastic states. For without ACC_2, S would be represented as holding both of the following beliefs:

(B_1) Maria can study in Japan and in the States.

(B_2) (Only) if Maria can study in Tokyo and in Boston, she is the first in her family who can study in Japan and the second who can study in the States.[15]

Given B_1, it is hard to see why *S* would believe B_2. For given that Maria can study in Japan and the States, under what conditions could it follow that it is only if she can study specifically in Tokyo and Boston that she will be the first/second in her family that is allowed to study in Japan/the States?

- Option 1: we can try to reconcile B_1 and B_2 by interpreting them as implying—given the background that *S* believes that no one else in Maria's family prior to her can study in Japan, only one other family member prior to her can study in the States—that Maria can study in Tokyo and Boston, and hence is the first in her family who can study in Japan and second who can study in the States. An obvious problem with this analysis is that it clashes with the prior assertion that *S* is agnostic concerning whether Maria can study in Tokyo, Boston, both or neither.

- Option 2: *S* believes that Maria can study in Japan and in the States, and is agnostic about whether Maria can study in Tokyo or Boston. *S* also believes that only if Maria can study in Tokyo, but not elsewhere in Japan, and in the Boston, but not elsewhere in the States, will she then count as the first/second in her family who can study in Japan/States. This amounts to attributing to *S* the strange belief that whether anyone in Maria's family before her was allowed to study in Japan/States depends on whether she is now allowed to study specifically in Tokyo/Boston.

These results suggest that, in contexts that entail that *S* is agnostic with respect to the homogeneity proposition, both ACC operators in (72) are licensed to avoid attributing to *S* incoherent or strange beliefs. Importantly, this conclusion is compatible with holding that parallel ACC operators over each of the main disjuncts are not licensed in, say, out of the blue assertions of sentences like (69), or more specifically, when the common ground is at least compatible with (hence allows for global accommodation of) the homogeneity presupposition.[16]

15 Note that the conditional belief attributed to *S* is likely exhaustive (i.e., read as an *only if* conditional), because the main disjunction in sentences like (70a) is usually interpreted as exclusive disjunction. Indeed, this kind of enrichment is systematically triggered in configurations of the form *either p or q* and *p or else q*. This can be captured by adding a matrix $\mathbf{pex}^{IE+II}$ to the parse in (72), as in (i), and assuming that it is associated with the main $\vee$:

(i) $\mathbf{pex}^{IE+II}[\text{ACC}_1[\neg\mathbf{pex}^{IE+II}[\Diamond(A^+ \vee B^+)]] \vee \text{ACC}_2[C_{\Diamond A \wedge \Diamond B}]]$

16 For example, in light of our previous argument, one could try to deal with filtering FC sentences, in general, by appealing to the presence of ACC over the second main disjunct, as in (ia):

4.5 Summary

The second of the presupposed & filtering FC puzzles concerns the FC filtering effects of embedded $\neg\Diamond\vee$ and $\Box\wedge$-sentences such as (48) and (62). These cases present a serious challenge to various Grammatical theories, including versions with $\mathbf{exh}^{IE+II}$ and with even more powerful operators like $\mathbf{exh}^{IE+II}_{asr+psr}$ which have a non-trivial effect on both the assertive and presuppositional content of its prejacent. They are also challenging for various recent Lexicalist theories which lack an account of negative FC conjunctions, and can thus again only deal with half the puzzle. In contrast, our $\mathbf{pex}^{IE+II}$-based theory supports a uniform solution to the filtering FC puzzles, which follows directly from the default parses for the embedded $\neg\Diamond\vee$ and $\Box\wedge$-sentences, given standard assumptions about presupposition projection and accommodation. These results are summarized in Table 2.

5 Conclusion

FC-related phenomena present a fascinating and complex puzzle for semantic and pragmatic theories. In this paper, we focused on recent observations, due to Marty & Romoli (2020) and Romoli & Santorio (2019), concerning how $\Diamond\vee$, $\neg\Diamond\vee$, $\Box\wedge$ and $\neg\Box\wedge$ sentences, such as those in (74), behave with respect to their projection and filtering properties when embedded in environments such as those in (75) and (76). The 'presupposed & filtering FC puzzles' amount to the challenge of giving an account of the FC reading of (74a), the negative FC reading of (74d), the double prohibition reading of (74b) and the double requirement reading of (74c) that—when combined with independently justified assumptions about presupposition projection,

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(i) Either Maria can't go study in Tokyo or Boston, or she is the first in our family who can go study in Japan (and the second who can study in the States).

a. $\neg\Diamond(A^+ \vee B^+) \vee \textsc{acc}[C_{\Diamond A\wedge\Diamond B}]$

This predicts the DP reading for the first main disjunct and cancellation of the presuppositions triggered in the second main disjunct, as desired. It also has the advantage of being independent of specific accounts of FC. Romoli & Santorio (2019) reject this proposal on the grounds that it restss on licensing conditions for ACC which are too unconstrained and over-generate presupposition cancellation in other cases. Indeed, note that when (i) is asserted in neutral or homogeneity friendly contexts we get different licensing conditions for ACC compared to when (i) is asserted in homogeneity unfriendly contexts like (70). Specifically, when the common ground is compatible with S believing the homogeneity proposition, it is not incoherent to resolve the tension between B_1 and B_2 via Option 1, i.e., by attributing to S the belief that Maria can study in Tokyo and in Boston, and so is the first/second in her family who can study in Japan/the States. Granted, expressing that content via (i) seems gratuitously indirect, but the point is just that insertion of ACC is not required to avoid attributing to S incoherence or implausible beliefs. This discussion does highlight, however, that the filtering FC puzzles can be taken as a challenge to theories of FC specifically only if we hold that ACC has relatively strict licensing conditions, which is arguably the standard view.

Accounts	**FC-¬□∧**	**filt. FC w. DP-¬◊∨**	**filt. FC w. FC-□∧**
Lexicalist old	✗	✗	✗
Lexicalist new	✗	✓	✗
$\mathbf{exh}^{IE/IE+II}$	✓	✗	✗
$\mathbf{exh}^{IE}_{asr+psr}$	✓	✗	✗
$\mathbf{exh}^{IE+II}_{asr+psr}$	✓	✗	✗
$\mathbf{pex}^{IE+II}$	✓	✓	✓

Table 2 Predictions of theories of FC for filtering FC with embedded ¬◊∨ and □∧-sentences. To complete the results reported in Table 1, we include here predictions concerning the negative FC reading of simple ¬□∧-sentences, as well as the two main cases of filtering FC. References for theories: Lexicalist old: Zimmerman (2000) and Simons (2005b). Lexicalist new: Willer (2017), Goldstein (2018), Aloni (2018), Rothschild & Yablo (2018). Grammatical with $\mathbf{exh}^{IE/IE+II}$: Fox (2007), Chierchia (2013), Bar-Lev & Fox (2020). Grammatical with $\mathbf{exh}^{IE}_{asr+psr}$ (allow for exhausitfcation of presuppositions triggered by the prejacent, but only sensitive to *IE*): Gajewski & Sharvit (2012), Spector & Sudo (2017), Magri (2009a). Grammatical with $\mathbf{exh}^{IE+II}_{asr+psr}$: Marty & Romoli (2020).

accommodation and filtering—explains their complex embedded behavior in and the default reading of (75a)-(75b) and (76a)-(76b):

(74) **FC reading of ◊∨-sentences, DP reading of ¬◊∨-sentences, double requirement reading of □∧-sentences, and negative FC reading of ¬□∧-sentences** (for availability evidence, see Tieu et al. 2019, Marty et al. 2021):

a. Maria can study in Tokyo or Boston.
b. Maria can't study in Tokyo or Boston.
c. Olivia is required to take Logic and Algebra.
d. Olivia is not required to take Logic and Algebra.

(75) **FC and negative FC under (negative) factives** (Marty & Romoli 2020):

a. Noah is unaware that Olivia can take Logic or Algebra.
⇝ *Olivia can take Logic*
⇝ *Olivia can take Algebra*
⇝ *¬Noah believes that Olivia can take Logic*
⇝ *¬Noah believes that Olivia can take Algebra*

b. Pete is unaware that Olivia is not required to take Logic and Algebra.
⇝ *Olivia is not required to take Logic*
⇝ *Olivia is not required to take Algebra*
⇝ *¬Noah believes Olivia is allowed to not take Logic*
⇝ *¬Noah believes that Olivia is allowed to not take Algebra*

(76) **Filtering FC and negative FC conjunctions** (Romoli & Santorio 2019):

a. Either Maria can't go study in Tokyo or Boston, or she is the first in our family who can go study in Japan and the second who can go study in the States.
$\not\rightsquigarrow$ *Maria can study in Japan*
$\not\rightsquigarrow$ *Maria can study in the States*

b. Either Maria is required to go to Japan and the States, or she's the first in her family who is not required to go to Tokyo and the second who's not required to go to Boston.
$\not\rightsquigarrow$ *Maria is allowed to not study in Tokyo*
$\not\rightsquigarrow$ *Maria is allowed to not study in Boston*

Previous accounts of FC can deal with parts of the presupposed & filtering FC puzzles, but we have seen that none can deal—without additional complications and stipulations—with the full pattern in (74)-(76). This applies, in particular, to standard **exh**-based Grammatical accounts (Fox 2007, Bar-Lev & Fox 2020), and also to their more complex multi-dimensional variants (Gajewski & Sharvit 2012, Marty & Romoli 2020). We have proposed a novel Grammatical-account of FC, formulated in terms of an independently motivated exhaustification operator, $\mathbf{pex}^{IE+II}$, which asserts its prejacent but is a presupposition trigger with respect to any of its innocently excludable or includable alternatives. We showed that our $\mathbf{pex}^{IE+II}$-based account supports a uniform and descriptively adequate solution to the presupposed & filtering FC puzzles. In addition, although our $\mathbf{pex}^{IE+II}$-based theory shares some predictions with recent Lexicalist accounts of FC (e.g., Goldstein 2018), it has the comparative advantage that it also resolves versions of the puzzles with negative FC conjunctions.

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