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High and low uniqueness in singular *wh*-interrogatives*

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Abstract While simple singular *wh*-interrogatives carry a uniqueness presupposition, this is not so when they contain possibility modals. Hirsch & Schwarz (2019) account for this contrast by assuming (i) that questions can have multiple maximally informative true answers and (ii) that uniqueness is triggered lexically inside the scope of interrogative. We show that their proposal overgenerates on two accounts. Firstly, it predicts too weak a presupposition for modalized interrogatives. Secondly, it predicts unattested interpretations for interrogatives containing negation. We show that both issues can be solved using exhaustification operators. On the one hand, we obtain the desired presupposition for modalized interrogatives by assuming the lexical trigger for uniqueness to be a presuppositional variant of an exhaustification operator (Bassi, Del Pinal & Sauerland 2019). On the other, we show that unattested readings of negation can be blocked by assuming that questions presuppose that the pointwise exhaustification of their answers partitions the context of evaluation (Fox 2019). We argue that proper empirical coverage for singular *wh*-interrogatives requires the interaction of both exhaustification operations.

Keywords: singular wh-interrogatives, uniqueness, modality, exhaustification, higher-order quantification

1 Introduction

The number on the restrictor of English *wh*-interrogatives restricts what kinds of answers they admit. When this restrictor is singular, as in (1), only fragment answers consisting of a singular noun phrase are admissible (Higginbotham & May 1981; Dayal 1996). An answer consisting of a plural noun phrase, such as (1-b), seems at odds with the inference from (1) that exactly one student arrived.

(1) Which student arrived?
  a. Al.
  b. #Al and Beth.

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The standard account of this restriction is to treat the inference that exactly one student arrived in (1) as a presupposition of the interrogative. Dayal (1996) proposes that interrogatives presuppose the existence of a unique true answer which implies all true answers to the question, which she implements by taking matrix interrogatives to fall under the scope of an answerhood operator. This assumption in concert with standard assumptions on the semantics of number and interrogatives derives the infelicity of plural noun phrase answers to (1).

Though successful in deriving uniqueness for simple cases, Dayal's proposal is known to overgenerate. Her analysis predicts that interrogatives with possibility modals in their scope will carry a uniqueness presupposition, such that (2-a) should presuppose that a unique wine is such that it could be poisoned.

(2) a. Which wine could be poisoned?
 b. The Port or the Shiraz.

The availability of a mention-some interpretation for (2-a) as well as the free choice inference drawn from disjunctive answers like (2-b) (i.e., that the Port could be poisoned and the Shiraz could be poisoned) argue strongly that Dayal's account is overly restrictive. Hirsch & Schwarz (2019) propose to account for the contrast between (1) and (2-a) by making two simple assumptions. Firstly, they propose to include in the domain of the answerhood operator sets of propositions containing multiple maxima. This is achieved by weakening the notion of a maximally informative true answer, allowing for propositions to be maximal in a set so long as they include no other member of this set. Secondly, Hirsch and Schwarz propose a lexical trigger for uniqueness to be present in the scope of interrogatives, one which can crucially interact scopally with modal operators. This view unambiguously derives uniqueness for (1) while predicting a reading for (2-a) where the lexical trigger of uniqueness scopes below the possibility modal.

In this paper, we point out two problems with Hirsch and Schwarz's account. On the one hand, we argue that the presupposition they predict for interrogatives containing possibility modals is too weak. On the other, we show that the scopal interaction of their lexical trigger for uniqueness with operators in the syntax predicts unattested readings for interrogatives containing negation. These observations lead us into an examination of how exhaustification operators may help solve both issues. More specifically, we show that the correct presupposition of interrogatives like (1-a) can be derived if we assume the lexical trigger for uniqueness is a presuppositional variant of an exhaustification operator first proposed for independent reasons in Bassi et al. (2019). Moreover, we show how unattested readings for interrogatives containing negation can be blocked if we define our answerhood operator, following Fox (2019), as requiring sets of propositions to partition the context via a pointwise exhaustification of their members. We will conclude that the interaction of both exhaustification operators is sufficient to capture the contrast between (1) and (2-a) while avoiding the empirical challenges faced by Hirsch and Schwarz.

2 Background

This uniqueness presupposition of (1) can be seen as emerging from the interaction of the semantics of number and that of interrogatives. A singular nominal can be taken to denote a property of singularities (Sauerland, Anderssen & Yatsushiro 2005, a.o.). Suppose the set of singularities who are students is invariant across worlds, and contains only a, b, and c. Switching freely from function-talk to set-talk, we will say that the singular nominal "student" denotes the set in (3).

(3) $[\![\text{student}]\!] = \{\mathsf{a}, \mathsf{b}, \mathsf{c}\}$

It is commonplace to take interrogatives to denote, at least at some point in their derivation, a set of propositions representing their possible answers (Hamblin 1973; Karttunen 1977). Hence (2), at some point in its derivation, denotes the set of propositions of the form "arrv x", where the variable "x" is assigned an individual who is a student. Such sets will henceforth be referred to as *Hamblin sets*.

(4) $\{\lambda w.\ \mathsf{arrv}_w\ x \mid x \in \{\mathsf{a}, \mathsf{b}, \mathsf{c}\}\}$

The final and crucial assumption is that an interrogative is defined only if there is in its Hamblin set a maximally informative true answer, i.e. an answer true in the world of evaluation which implies all other true answers (Dayal 1996). The relevant notion of maximality is defined below in (5).

(5) $\mathsf{max}_1(w, p, Q) = 1$ iff $w \in p \in Q \wedge \forall q \in Q(w \in q \rightarrow p \subseteq q)$

Dayal implements this idea by assuming this presupposition is introduced by the answerhood operator ANS_D defined in (6). When combined with a set of propositions Q, this operator outputs a function from worlds to propositions. This function is defined for a world w only when there is in w a maximally informative true answer to Q. When defined, the output is that answer.

(6) $[\![\text{ANS}_D]\!](Q) = \lambda w\colon\ \exists! p(\mathsf{max}_1(w, p, Q)).\ \iota p(\mathsf{max}_1(w, p, Q))$

Applying ANS_D to (4) denotes a partial function from worlds to the maximally informative true answer to (4) in those worlds. No matter how one assumes an interrogative updates a context, we can think that it successfully does so only if the function is defined for every world in the context. Otherwise, it results in (pragmatic) presupposition failure. This offers a natural explanation for the uniqueness presupposition of singular *wh*-interrogatives. If a context were to contain a world w where multiple students arrived, in this case a and b, two propositions in (4) would be true in w. Because these propositions are logically independent, neither would qualify as the maximally informative true answer in w, ensuring that the function is undefined for that world and thus leading to a presupposition failure. Only in contexts presupposing that a unique student arrived will (1) be felicitous.

Although this account makes the right predictions for questions like (1), Hirsch & Schwarz (2019) provide examples which suggest that the presupposition resulting from this answerhood operator is overly restrictive. Consider the following scenario. You are in charge of security at a wine tasting event when suddenly you realize some of the guests are the owners of winery whose wines your event has snobbed. The guests in question have a history of poisoning one of the wines at events which do not feature their product. You alert your co-workers and one of them asks you the question in (7).

(7) Which wine could be poisoned?

Dayal's analysis predicts that (7) will presuppose that exactly one wine is such that it could be poisoned. If "wine" denotes the set of individuals in (8-a) invariably across worlds, the Hamblin set for (7) will be (8-b).

(8) a. $[\![\text{wine}]\!] = \{\mathsf{p}, \mathsf{s}\}$
 b. $\{\lambda w.\ \Diamond^u_w\ \mathsf{psn}_u\ x \mid x \in \{\mathsf{p}, \mathsf{s}\}\}$

The propositions in (8-b) are all logically independent of one another, meaning that ANS_D's definedness condition can only ever be met for worlds where exactly one of these answers is true, so we expect the presupposition that exactly one wine could be poisoned.

(9) $[\![(7)]\!] = \lambda w\colon\ \exists! x \in \{\mathsf{p}, \mathsf{s}\}(\Diamond^u_w\ \mathsf{psn}_u\ x)\ .\ \iota p \in \{\mathsf{p}, \mathsf{s}\}(\Diamond^u_w\ \mathsf{psn}_u\ x)$

Hirsch and Schwarz point out two main issues with this prediction. On the one hand, interrogatives like (7) admit so-called mention-some interpretations. Suppose you and your co-workers are placing bets on who at the event will drink the poisoned wine and get sick. In such a scenario, the answers in (10-a) and (10-b) are not necessarily interpreted as exhaustive. Rather, they can be seen as mentioning only one of the possible wines that could be poisoned.

(10) Q: Which wine could be poisoned?
 A: The Port.
 A′: The Shiraz.

Hirsch and Schwarz also point out that answering modalized interrogatives with disjunctive answers leads to free choice interpretations. Suppose you and your coworkers are of good moral character and would like to figure out the exhaustive list of wines which could be poisoned so you can rid the event of them. The disjunctive answer in (11) is most easily understood as saying that the Port could be poisoned and the Shiraz could be poisoned, while none of the other wines could be poisoned.

(11) Q: Which wine could be poisoned?
A: The Port or the Shiraz.

These data seem to directly contradict the presupposition predicted by Dayal's answerhood operator. To remedy this situation, Hirsch and Schwarz first propose to weaken the answerhood operator, adopting a variant of the operator proposed in (Fox 2013), which relies on a notion of maximal informativity which admits multiple maxima for a given set.

(12) $\mathsf{max}_2(w,p,Q) = 1$ iff $w \in p \in Q \wedge \neg\exists q \in Q(w \in q \wedge q \subset p)$

When applied to a Hamblin set Q, the operator $\textsc{ans}^1_F$ outputs a partial function from worlds to sets of propositions defined only for worlds in which there is at least one maximally informative true answer to Q. For each world for which it is defined, the function outputs the set of maximally informative true answers to Q in that world.

(13) $[\![\textsc{ans}^1_F]\!](Q) = \lambda w\colon \exists p(\mathsf{max}_2(w,p,Q)).\ \{p \mid \mathsf{max}_2(w,p,Q)\}$

While it is easy to see that applying $\textsc{ans}^1_F$ to (8-b) will not presuppose uniqueness, this is also true when it is applied to (4). Hirsch and Schwarz propose the uniqueness presupposition of singular *wh*-interrogatives is lexically triggered inside the scope of interrogatives, such that each answer in the Hamblin set of (1) presupposes that a unique student arrived.

(14) $\{\lambda w\colon \exists! y \in \{\mathsf{a},\mathsf{b},\mathsf{c}\}(\mathsf{arrv}_w\ y).\ \mathsf{arrv}_w\ x \mid x \in \{\mathsf{a},\mathsf{b},\mathsf{c}\}\}$

If we combine $\textsc{ans}^1_F$ with (14), we obtain a function defined only for worlds in which there is at least one maximally informative true answer in Q. This in turn can only be the case for worlds where the presupposition of the members of Q is defined, i.e. worlds where exactly one student left. The interrogative is thus expected to be licensed only for contexts where it is presupposed that exactly one student arrived.

In questions containing possibility modals, the trigger for uniqueness is assumed to have two available positions relative to the possibility modal. On the one hand, it can scope above the modal, as represented in (15-a), or below the modal as in (15-b).

(15) a. $[_{CP}\ [_{TP} \ldots \exists! \ldots \Diamond \ldots]\ ]$
b. $[_{CP}\ [_{TP} \ldots \Diamond \ldots \exists! \ldots]\ ]$

In the case where the trigger for uniqueness scopes above the modal, the Hamblin set for (7) will be (16), where each member is defined only for worlds where a unique wine is such that it could be poisoned. In combination with $\textsc{ans}^1_F$, this results in the presupposition that exactly one wine is such that it could be poisoned, which seems like an available reading for this interrogative.

(16) $\{\lambda w\colon \exists! y \in \{\mathsf{p},\mathsf{s}\}(\Diamond^u_w\ \mathsf{psn}_u\ y).\ \Diamond^u_w\ \mathsf{psn}_u\ x \mid x \in \{\mathsf{p},\mathsf{s}\}\}$

Hirsch and Schwarz assume that presuppositions project existentially from under possibility modals, such that the Hamblin set derived from assuming the trigger for uniqueness scopes under the modal is (17).

(17) $\{\lambda w\colon \Diamond^u_w\ \exists! y \in \{\mathsf{p},\mathsf{s}\}(\mathsf{psn}_u\ y).\ \Diamond^u_w\ \mathsf{psn}_u\ x \mid x \in \{\mathsf{p},\mathsf{s}\}\}$

Combining $\textsc{ans}^1_F$ with this set of alternatives will result in a partial function from worlds to sets of propositions. The function is defined only for worlds for which there is an accessible world where exactly one wine is poisoned. Under this reading, (7) will carry the very weak presupposition that it could be the case that exactly one wine bottle is poisoned. This is compatible with the fact that such questions are consistent with answers suggesting that multiple wines could be poisoned.

In the next two sections, we address two problems with the analysis proposed in Hirsch and Schwarz. We will firstly argue that the presupposition associated with the set of alternatives in (17) is too weak. We will propose that the correct presupposition can be captured by assuming universal projection under the modal while weakening the presupposition of the lexical trigger. We will secondly show that Hirsch and Schwarz's analysis overgenerates, predicting for *wh*-interrogatives containing negation unavailable scopal configurations. We will propose to restrict the available readings by assuming that the set of propositions in a Hamblin set must be able to partition the context through pointwise exhaustification (Fox 2019).

3 Problem 1: Too weak a presupposition

While Hirsch and Schwarz correctly predict the absence of a uniqueness presupposition for questions like (18), there is reason to believe the presupposition they predict for such interrogatives is too weak. As discussed in the section above, the interrogative under its weakest interpretation presupposes that there is an accessible world where exactly one wine is poisoned.

(18) Which wine could be poisoned?
Presupposes that $\Diamond^u\, \exists! x \in \{\mathsf{p}, \mathsf{s}\}(\mathsf{psn}_u\, x)$

This is too weak, however, as it is compatible with contexts where it could be that multiple wines were poisoned. To see that this is undesirable, consider a context where it is believed that one or more wines could have been poisoned by the guests. In such contexts, the singular interrogative in (19-a) is deviant, as opposed to its plural counterpart in (19-b).

(19) *Context: guests are suspected to have poisoned one of more wines.*
a. #Which wine could be poisoned.
b. Which wines could be poisoned.

One way to block the use of singular interrogatives in such contexts would simply be to assume that presuppositions project universally from below possibility modals. The Hamblin set for the weaker interpretation of (19) will then be (20).

(20) $\{\lambda w\colon \Box^u_w\, \exists! y \in \{\mathsf{p}, \mathsf{s}\}(\mathsf{psn}_u\, y).\ \Diamond^u_w\, \mathsf{psn}_u\, x \mid x \in \{\mathsf{p}, \mathsf{s}\}\}$

Combined with ANS^1_F, this results in the presupposition that in every accessible world, a unique wine bottle is poisoned. While this correctly predicts that (19) is unavailable in contexts where more than one wine could be poisoned, this presupposition itself appears to be too strong. Indeed, it seems clear that an utterance of (19) is compatible with context where it is possible no wine is poisoned.

We seem to want to derive the weaker presupposition that in all accessible worlds, if a wine is poisoned then only it is. Borrowing from work by Bassi et al. (2019) and Fox (2020), we can arrive at this presupposition by assuming the lexical trigger responsible for uniqueness is a presuppositional variant of an exhaustification operator, called "P-EXH". This operator is defined in terms of a more standard exhaustification operator "exh", which combines with a prejacent proposition p and set of alternatives C and outputs the set of worlds included in the set of innocently excludable and innocently includable alternatives to C (Fox 2007; Bar-Lev & Fox 2020).[1]

The set of innocently excludable alternatives contains the negations (=complements) of those alternatives which are in the generalized intersection of every maximal subset of C the negation of whose members is consistent with p. This will require some unpacking. We will say that a subset of C is compatible with p if the negations of its members is consistent with p.

1 We use of the phrase “innocently excludable alternative" in a non-standard way to simplify defining "exh". While the set of innocently excludable alternatives typically consists of alternatives which "exh" negates, we take this set to already contain the negations of alternatives which "exh" asserts.

(21) $\mathsf{comp}(p,C,A) = 1$ iff $A \subseteq C \wedge \exists w(w \in p \wedge \forall q \in A(w \in \overline{q}))$

We say that A is a maximal subset of C compatible with p whenever it is not the proper subset of any subset of C compatible with p.

(22) $\mathsf{maxc}(p,C,A) = 1$ iff $\mathsf{comp}(p,C,A) \wedge \neg\exists B(\mathsf{comp}(p,C,B) \wedge A \subset B)$

The set of innocently excludable alternatives of p is then defined as the negation of every member in the generalized intersection of those maximal subsets of C compatible with p.

(23) $\mathsf{IE}(p,C) := \{\overline{q} \mid q \in \bigcap\{A \mid \mathsf{maxc}(p,C,A)\}\}$

The set of innocently includable alternatives of p is defined as the members of C whose complement is not innocently excludable. Note that naturally, p itself is always naturally included.[2]

(24) $\mathsf{II}(p,C) := \{q \mid q \in C \wedge q \notin \bigcap\{A \mid \mathsf{maxc}(p,C,A)\}\}$

We can define "exh" as an operator which outputs the set of worlds contained in every innocently excludable and included alternative of its prejacent.

(25) $\mathsf{exh}(p,C) := \lambda w.\ w \in \bigcap\bigcup\{\mathsf{IE}(p,C), \mathsf{II}(p,C)\}$

The presuppositional variant of this operator we will consider as the lexical trigger for uniqueness, "P-EXH", is defined as a function which takes a proposition p and a set of alternatives C and outputs p with an added domain restriction. This amended proposition is a function restricted to worlds where if p is true, then the exhaustification of p relative to C is true.

(26) $[\![\text{P-EXH}]\!](C)(p) = \lambda w\!:\ w \in p \to w \in \mathsf{exh}(p,C).\ w \in p$

Now let's see how this operator applies in the case of basic singular *wh*-interrogatives. We assume that the syntactic locus of "P-EXH" is situated inside the interrogative's scope. The Hamblin set for (27-a) will be (27-b).

2 We are assuming a simplified version of innocent inclusion which can lead to contradictory inferences. Below is a definition for the set of innocently includable alternatives which prevents contradictions from arising.

$$\mathsf{II}(p,C) := \bigcap\left\{A \;\middle|\; \begin{array}{l} A \subseteq C \wedge \exists w(\forall q \in A(w \in q) \wedge \forall r(r \in IE(p,c) \to w \in r)) \wedge \\ \neg\exists B(\exists w(\forall s \in B(w \in s) \wedge \forall t(t \in IE(p,c) \to w \in t)) \wedge A \subset B) \end{array}\right\}$$

(27) a. Which student arrived?
b. $\{\lambda w\colon \mathsf{arrv}_w\, x \rightarrow w \in \mathsf{exh}(\lambda u.\ \mathsf{arrv}_u\, x, C).\ \mathsf{arrv}_w\, x \mid x \in \{\mathsf{a}, \mathsf{b}, \mathsf{c}\}\}$

We stipulate from hereon that in every example, the alternatives of the prejacent of P-EXH are defined by substituting for the assignment of the variable quantified over by the *wh*-phrase some assignment which satisfies the restrictor of the *wh*-item. The set of alternatives C in (27-b) consists of propositions of the form "$\mathsf{arrv}\ x$", where $x \in \{\mathsf{a}, \mathsf{b}, \mathsf{c}\}$. For clarity, let's take the member of (27-b) in (28-a), whose alternatives are in (28-b). The innocently excludable alternatives of (28-a) consist of the set in (28-c) and its innocently includable alternatives the set in (28-d).

(28) a. $\lambda w.\ \mathsf{arrv}_w\ \mathsf{a}$
b. $\{\lambda w.\ \mathsf{arrv}_w\ \mathsf{a}, \lambda w.\ \mathsf{arrv}_w\ \mathsf{b}, \lambda w.\ \mathsf{arrv}_w\ \mathsf{c}\}$
c. $\mathsf{IE}(\text{(28-a)}, \text{(28-b)}) = \{\lambda w.\ \neg\mathsf{arr}_w\ \mathsf{b}, \lambda w.\ \neg\mathsf{arr}_w\ \mathsf{c})\}$
d. $\mathsf{II}(\text{(28-a)}, \text{(28-b)}) = \{\lambda w.\ \mathsf{arrv}_w\ \mathsf{a}\}$

Applying "exh" to (28-a) given the set of alternatives in (28-b) will result in the set of worlds where a but not b or c arrived.

(29) $\mathsf{exh}(\text{(28-a)}, \text{(28-b)}) = \lambda w.\ \mathsf{arrv}_w\ \mathsf{a} \wedge \neg\mathsf{arrv}_w\ \mathsf{b} \wedge \neg\mathsf{arrv}_w\ \mathsf{c}$

It is easy to see that each member of (27-b) defined over some x will presuppose that if x arrived, then only x did. We can thus rewrite the Hamblin set in (27-b) as (30).

(30)
$$\left\{\begin{array}{l} \lambda w\colon \mathsf{arrv}_w\ \mathsf{a} \rightarrow \neg\exists x \in \{\mathsf{b}, \mathsf{c}\}(\mathsf{arrv}_w\ x).\ \mathsf{arrv}_w\ \mathsf{a} \\ \lambda w\colon \mathsf{arrv}_w\ \mathsf{b} \rightarrow \neg\exists x \in \{\mathsf{a}, \mathsf{c}\}(\mathsf{arrv}_w\ x).\ \mathsf{arrv}_w\ \mathsf{b} \\ \lambda w\colon \mathsf{arrv}_w\ \mathsf{c} \rightarrow \neg\exists x \in \{\mathsf{a}, \mathsf{b}\}(\mathsf{arrv}_w\ x).\ \mathsf{arrv}_w\ \mathsf{c} \end{array}\right\}$$

Combining this set with ANS^1_F results in a function defined only on worlds where there is at least one maximally informative true answer in (30). We make the assumption here that this definedness condition is met in a world so long as at least one of the answers is defined and is not entailed by any other defined answer. Only in worlds where exactly one student arrived can this condition be satisfied, in which case we predict the interrogative to presuppose uniqueness.

For questions with possibility modals, we once more have two available scopes for P-EXH. On the one hand it can scope above the modal, as in (31-b).

(31) a. Which wine could be poisoned?
b. $\{\lambda w\colon \Diamond_u\ \mathsf{psn}_u\ x \rightarrow w \in \mathsf{exh}(\lambda v.\ \Diamond^u_w\ \mathsf{psn}_u\ x, C).\ \Diamond_u\ \mathsf{psn}^u_w\ x \mid x \in \{\mathsf{p}, \mathsf{s}\}\}$

Here, we are assuming C contains all propositions of the form "$\Diamond^u\ x\ \mathsf{arrived}_u$", where $x \in \{\mathsf{p}, \mathsf{s}\}$. Following a reasoning similar to that of the case in (27-b), we can see that each proposition is (31-b) is defined only for worlds where only one wine bottle is such that it could be poisoned.

(32) $$\left\{ \begin{array}{l} \lambda w\colon \Diamond^u_w \ \mathsf{psn}_u \ \mathsf{a} \to \neg\Diamond^u_w \ \mathsf{psn}_u \ \mathsf{b}.\ \Diamond^u_w \ \mathsf{psn}_u \ \mathsf{a} \\ \lambda w\colon \Diamond^u_w \ \mathsf{psn}_u \ \mathsf{b} \to \neg\Diamond^u_w \ \mathsf{psn}_u \ \mathsf{a}.\ \Diamond^u_w \ \mathsf{psn}_u \ \mathsf{b} \end{array} \right\}$$

In combination with ANS^1_F, (32) will presuppose that only one wine bottle is such that could be poisoned. In addition to this reading, another possible reading is derived when P-EXH scopes below the modal, in which case we obtain the Hamblin set in (33), where we are again assuming presuppositions project universally from under possibility modals.

(33) $$\{\lambda w\colon \Box^u_w(\mathsf{psn}_u \ x \to w \in \mathsf{exh}(\lambda v.\ \mathsf{psn}_v \ x, C)).\ \Diamond^u_w \ \mathsf{psn}_u \ x \mid x \in \{\mathsf{p},\mathsf{s}\}\}$$

Alternatives in C are identical to those in (27-b), allowing us to rewrite the set of propositions in (33) as (34), where both propositions are defined only for worlds for which in every accessible world, if a wine is poisoned then only one is.

(34) $$\left\{ \begin{array}{l} \lambda w\colon \Box^u_w \ (\mathsf{psn}_u \ \mathsf{a} \to \neg\mathsf{psn}_u \ \mathsf{b}).\ \Diamond^u_w \ \mathsf{psn}_u \ \mathsf{a} \\ \lambda w\colon \Box^u_w \ (\mathsf{psn}_u \ \mathsf{b} \to \neg\mathsf{psn}_u \ \mathsf{a}).\ \Diamond^u_w \ \mathsf{psn}_u \ \mathsf{b} \end{array} \right\}$$

When finally combined with ANS^1_F, the interrogative presupposes that in all accessible worlds, if a wine is poisoned then exactly one wine is poisoned, which seems like the desired presupposition. It disallows for contexts where it is possible that multiple wines are poisoned, while allowing for the possibility that no wine is poisoned.

4 Problem 2: Unavailable readings with negation

We have seen that allowing uniqueness to be triggered by an operator local to the scope of interrogatives allows us to predict an ambiguity for singular interrogatives containing possibility modals. This is so because this operator can scope above or below the modal. Such scopal options are not however expected only between the trigger of uniqueness and modals.

Consider the interrogative in (35), which appears to unambiguously presuppose that a unique student didn't arrive.

(35) Which student didn't arrive?

This is the reading we expect if P-EXH takes scope above the negation, leading to the Hamblin set in (36).

(36) $$\{\lambda w\colon \neg\mathsf{arrv}_w \ x \to w \in \mathsf{exh}(\lambda u.\ \neg\mathsf{arrv}_u \ x, C).\ \neg\mathsf{arrv}_w \ x \mid x \in \{\mathsf{a},\mathsf{b},\mathsf{c}\}\}$$

C denotes the set of alternatives of the form "$\neg\mathsf{arrv}\ x$", where x is a student. Each member of (36) presupposes that x didn't arrive only if all other students arrived. We can show reasoning as we have in the previous section that under this interpretation (35) presupposes that a unique student didn't arrive.

However, another scopal configuration is expected to be available for (35), where P-EXH takes scope below negation, as in (37).

(37) $$\{\lambda w\colon\ \mathsf{arrv}_w\ x \to w \in \mathsf{exh}(\lambda u.\ \mathsf{arrv}_u\ x, C).\ \neg\mathsf{arrv}_w\ x \mid x \in \{\mathsf{a},\mathsf{b},\mathsf{c}\}\}$$

This produces for the interrogative the Hamblin set in (38), where each proposition is defined only for worlds where if a student arrived, only one did.

(38) $$\left\{\begin{array}{l} \lambda w\colon\ \mathsf{arrv}_w\ \mathsf{a} \to \neg\exists x \in \{\mathsf{b},\mathsf{c}\}(\mathsf{arrv}_w\ x).\ \neg\mathsf{arrv}_w\ \mathsf{a} \\ \lambda w\colon\ \mathsf{arrv}_w\ \mathsf{b} \to \neg\exists x \in \{\mathsf{a},\mathsf{c}\}(\mathsf{arrv}_w\ x).\ \neg\mathsf{arrv}_w\ \mathsf{b} \\ \lambda w\colon\ \mathsf{arrv}_w\ \mathsf{c} \to \neg\exists x \in \{\mathsf{a},\mathsf{b}\}(\mathsf{arrv}_w\ x).\ \neg\mathsf{arrv}_w\ \mathsf{c} \end{array}\right\}$$

This will predict a reading for singular *wh*-interrogatives carrying a very weak presupposition. In combination with ANS^1_F, the interrogative will presuppose that at most one student arrived.[3] This is very clearly an unattested interpretation for (35), which calls for a means of preventing P-EXH from scoping below negation.

The unavailability of such scopal configurations is predicted if we assume a variant of the answerhood operator proposed in (Fox 2019). This version of the operator imposes the restriction on the Hamblin set of an interrogative that the pointwise exhaustification of its members is a partition of the context. Once more let us do some unpacking to make this statement clearer. We say of a set A that it is a partition of a set B whenever A is a set of disjoint subsets of B excluding the empty set whose generalized union is B. In other words, a partition of a set B defines an equivalence relation on B such that each of its members is mapped to exactly one subset of B, where every such subset is called a cell.

(39) $$\mathsf{part}(A,B) = 1 \text{ iff } \emptyset \notin A \wedge \forall X,Y \in A(X \cap Y = \emptyset) \wedge \bigcup A = B$$

The pointwise exhaustification of a set Q is the set resulting from applying to each $p \in Q$ the operation $\mathsf{exh}(p,Q)$.

(40) $$\mathsf{EXH}(Q) = \{\mathsf{exh}(p,Q) \mid p \in Q\}$$

3 Though we assume here the revised uniqueness presupposition motivated in section 3, a comparable problem would arise assuming Hirsch and Schwarz's original low uniqueness presupposition as in (i).

(i) $$\{\lambda w\colon\ \exists! y \in \{\mathsf{a},\mathsf{b},\mathsf{c}\}(\mathsf{arrv}_w\ y).\ \neg\mathsf{arrv}_w\ x \mid x \in \{\mathsf{a},\mathsf{b},\mathsf{c}\}\}$$

Here, the interrogative would simply end up presupposing that a unique student arrived, which is also an unattested reading.

Fox's answerhood operator is interpreted relative to an information state(=context) I, such that it is defined only for Hamblin sets whose pointwise exhaustification partitions I. In other words, it is defined for a Hamblin set Q only if each member of $\{\mathsf{exh}(p,Q) \mid p \in Q\}$ intersected with I is mapped to a cell in a partition of I, and every cell in this partition is mapped to a member of $\{\mathsf{exh}(p,Q) \mid p \in Q\}$. When defined, it returns a function from a world w to those answers in Q which contain w and are included in the member of $\mathsf{EXH}(Q)$ which contains w.

(41) $$[\![\text{ANS}^2_F]\!]^I = \lambda Q \left[\begin{array}{l} :\ \mathsf{part}(\{q \cap I \mid q \in \mathsf{EXH}(Q)\}, I) \\ .\ \lambda w.\ \{p\ \mid w \in p \in Q \wedge p \subseteq \iota q(\mathsf{max}_1(w,q,\mathsf{EXH}(Q)))\} \end{array} \right]$$

To see how this works, let us consider how "exh" operates on one of the members of (38), in this case the proposition "$\lambda w\!:\ \mathsf{arrv}_w\ \mathsf{a} \rightarrow \neg\exists x \in \{\mathsf{b},\mathsf{c}\}(\mathsf{arrv}_w\ x).\ \neg\mathsf{arrv}_w\ \mathsf{a}$". A subset of (38) is compatible with this proposition whenever there is a world where the negation of each of its members is true while the prejacent is true. The crucial observation here is that the definedness condition of each member of (38) interacts with the exhaustification operation. For instance, any world where the prejacent of exh is true but where the alternative "$\lambda w\!:\ \mathsf{arrv}_w\ \mathsf{b} \rightarrow \neg\exists x \in \{\mathsf{a},\mathsf{c}\}(\mathsf{arrv}_w\ x).\ \neg\mathsf{arrv}_w\ \mathsf{b}$" is false must be defined for both propositions. Hence, any such world will presuppose that if b arrived, then c did not. As a result, it cannot be that for any such world the alternative "$\lambda w\!:\ \mathsf{arrv}_w\ \mathsf{c} \rightarrow \neg\exists x \in \{\mathsf{a},\mathsf{b}\}(\mathsf{arrv}_w\ x).\ \neg\mathsf{arrv}_w\ \mathsf{c}$" is false, for this would result in a contradiction. What we therefore learn is that the subsets of (38) compatible with the prejacent are all singletons. Indeed, the set of subsets of (38) compatible with the prejacent is present in (42).

(42) $$\left\{ \begin{array}{l} \{\lambda w\!:\ \mathsf{arrv}_w\ \mathsf{b} \rightarrow \neg\exists x \in \{\mathsf{a},\mathsf{c}\}(\mathsf{arrv}_w\ x).\ \neg\mathsf{arrv}_w\ \mathsf{b}\}, \\ \{\lambda w\!:\ \mathsf{arrv}_w\ \mathsf{c} \rightarrow \neg\exists x \in \{\mathsf{a},\mathsf{b}\}(\mathsf{arrv}_w\ x).\ \neg\mathsf{arrv}_w\ \mathsf{c}\} \end{array} \right\}$$

Neither of these sets includes the other, so it follows that both are maximal subsets of (38) compatible with the prejacent. The innocently excludable alteratives are the negations of every proposition in the generalized intersection of (42). However, this generalized intersection is empty, and so must be the set of innocently excludable alternatives of the prejacent.

(43) $$\mathsf{IE}(\lambda w\!:\ \mathsf{arrv}_w\ \mathsf{a} \rightarrow \neg\exists x \in \{\mathsf{b},\mathsf{c}\}(\mathsf{arrv}_w\ x).\ \neg\mathsf{arrv}_w\ \mathsf{a}, (38)) = \emptyset$$

The set of innocently includable alternatives of the prejacent being those whose negations are not innocently excludable, this set will simply be (38).

(44) $$\mathsf{II}(\lambda w\!:\ \mathsf{arrv}_w\ \mathsf{a} \rightarrow \neg\exists x \in \{\mathsf{b},\mathsf{c}\}(\mathsf{arrv}_w\ x).\ \neg\mathsf{arrv}_w\ \mathsf{a}, (38)) = (38)$$

The exhaustification of the prejacent proposition is the set of worlds where every innocently excludable and included alternative is true. This will be the set of worlds where no student arrived.

(45) $\mathsf{exh}(\lambda w\colon \mathsf{arrv}_w\ \mathsf{a} \to \neg\exists x \in \{\mathsf{b},\mathsf{c}\}(\mathsf{arrv}_w\ x).\ \neg\mathsf{arrv}_w, (38)) =$
$$\lambda w \left[\begin{array}{l} :\ \exists x \in \{\mathsf{a},\mathsf{b},\mathsf{c}\}(\mathsf{arrv}_w\ x) \to \exists! x \in \{\mathsf{a},\mathsf{b},\mathsf{c}\}(\mathsf{arrv}_w\ x) \\ .\ \neg\exists x \in \{\mathsf{a},\mathsf{b},\mathsf{c}\}(\mathsf{arrv}_w\ x) \end{array} \right]$$

We leave it to readers to convince themselves that the result of exhaustifying each proposition in (38) is the same, such that the pointwise exhaustification of this set results in the singleton set containing (45). The only way for this set to (trivially) partition a context *I* is if it presupposes that no student arrived. But then any context where this interrogative is defined will be one which implies that each answer in its Hamblin set is true. It seems natural to assume that a question is ill formed whenever the truth of each of its possible answers is presupposed, as the question will fail its pragmatic function as a request for information. We thus propose to rule out the interpretation of (35) where P-EXH scopes below negation on the grounds that this results in a degenerate question.

An immediate worry with defining the answerhood operator in these terms is that uniqueness is now predicted for interrogatives containing possibility modals. Assuming P-EXH is situated in the scope of the possibility modal, the Hamblin set for (46-a) is (46-b).

(46) a. Which wine could be poisoned?

b. $$\left\{ \begin{array}{l} \lambda w\colon \Box^u_w\ (\mathsf{psn}_u\ \mathsf{p} \to \neg\mathsf{psn}_u\ \mathsf{s}).\ \Diamond^u_w\ \mathsf{psn}_u\ \mathsf{p} \\ \lambda w\colon \Box^u_w\ (\mathsf{psn}_u\ \mathsf{s} \to \neg\mathsf{psn}_u\ \mathsf{p}).\ \Diamond^u_w\ \mathsf{psn}_u\ \mathsf{s} \end{array} \right\}$$

The pointwise exhaustification of (46-b) results in the set in (47). While in principle the definedness of any member in this set will be strengthened to the conjunction of the definedness conditions of all its members, this does not need to be indicated for this very limited case on account of the equivalence between the definedness condition of each member of (46-b).

(47) $$\left\{ \begin{array}{l} \lambda w\colon \Box^u_w\ (\mathsf{psn}_u\ \mathsf{p} \to \neg\mathsf{psn}_u\ \mathsf{s}).\ \Diamond^u_w\ \mathsf{psn}_u\ \mathsf{p} \wedge \neg\Diamond^u_w\ \mathsf{psn}_u\ \mathsf{s} \\ \lambda w\colon \Box^u_w\ (\mathsf{psn}_u\ \mathsf{s} \to \neg\mathsf{psn}_u\ \mathsf{p}).\ \Diamond^u_w\ \mathsf{psn}_u\ \mathsf{s} \wedge \neg\Diamond^u_w\ \mathsf{psn}_u\ \mathsf{p} \end{array} \right\}$$

The set in (47) can only partition contexts which can be split into two equivalence classes. On the one hand, one finds worlds from which can be accessed at least one world where p is poisoned but none from which s is poisoned, and on the other those from which is accessible a world where s is poisoned but none where p is. It follows from this that no world is such that it can access a world where p is poisoned and one where s is, and so our context must presuppose that exactly one wine is such that it could be poisoned. As we will discuss in the following section, we can have our cake and eat it by assuming assuming that *wh*-interrogatives allow for higher-order readings where the Hamblin set consists of a set of alternatives defined over not a first- but rather a third-order variable.

5 Higher-order readings of interrogatives

As mentioned in Section 1, singular interrogatives with possibility modals allow for complete disjunctive answers from which free choice is inferred.

(48) a. Which wine could be poisoned?
b. The Port or the Shiraz.

The answer in (48-b) is most naturally interpreted as saying that the Port could be poisoned and the Shiraz could be poisoned. It is unclear how with any of the assumptions made throughout the course of this paper an answer with the form of a disjunction could produce free choice. The only set of assumptions which do not predict a uniqueness presupposition for interrogatives containing modals are those of Hirsch and Schwarz, where the answerhood operator merely presupposes that one or more of the answers in the interrogative's Hamblin set is true and entailed by no other true answer. What we might expect given an answer of the form in (48-b) is the inference that at least one of these answers is true, not that both are.

The literature on interrogatives has produced increasing evidence that the members of a Hamblin set for a *wh*-interrogative can be defined over third-order variables which can be assigned at least the meanings of generalized disjunctions (Spector 2007,Xiang 2020, a.o.). The particularity of such generalized disjunctions is how they are assumed to interact scopally with modals. More specifically, (48-a) is thought to allow for an interpretation where the generalized quantifier in terms of which members of its Hamblin set are defined is found within the modal's scope. This is represented in (49), where we assume "P-EXH" scopes below the third-order variable "Π".

(49) $$\left\{ \lambda w \left[\begin{array}{l} : \Box^{u}_{w}(\Pi(\lambda x.\ \mathsf{psn}_{u}\ x \rightarrow w \in \mathsf{exh}(\lambda v.\ \mathsf{psn}_{v}\ x, C))) \\ .\ \Diamond^{u}_{w}\ \Pi(\lambda x.\ \mathsf{psn}_{u}\ x) \end{array} \right] \middle| \ \Pi \in \mathsf{G}(\{\mathsf{p},\mathsf{s}\}) \right\}$$

For the sake of simplicity, we will be assuming that the set of generalized quantifiers used to define this set of propositions contains only generalized disjunctions of the members of $\{\mathsf{p},\mathsf{s}\}$, as defined explicitly in (50).

(50) $$\mathsf{G}(A) = \{\lambda P.\ \exists x \in B(x \in P) \mid B \subseteq A\}$$

We can thus list out the members of (49) as in (51). To make things easier to read, we write out propositions with existential quantifiers ranging over two entities as simple disjunctions, and reduce those where they range over a single entity as simple statements about that entity. Once again for this simple case the domain restriction of each alternative is identical, though we assume that in general presuppositions project existentially from under the scope of an existential quantifier over individuals. In cases involving more than two entities, the domain restriction of disjunctive answers will also be a disjunction.

$$(51)\quad \left\{ \begin{array}{l} \lambda w\colon \Box^u_w(\mathsf{psn}_u\ \mathsf{p} \rightarrow \neg\mathsf{psn}_u\ \mathsf{s}).\ \Diamond^u_w\ \mathsf{psn}_u\ \mathsf{p} \\ \lambda w\colon \Box^u_w(\mathsf{psn}_u\ \mathsf{p} \rightarrow \neg\mathsf{psn}_u\ \mathsf{s}).\ \Diamond^u_w\ \mathsf{psn}_u\ \mathsf{s} \\ \lambda w\colon \Box^u_w(\mathsf{psn}_u\ \mathsf{p} \rightarrow \neg\mathsf{psn}_u\ \mathsf{s}).\ \Diamond^u_w\ (\mathsf{psn}_u\ \mathsf{p} \vee \mathsf{psn}_u\ \mathsf{s}) \end{array} \right\}$$

In the case of non-disjunctive members of (51), pointwise exhaustification will produce the same result it did when disjunctions were not in the set. To take an example, the proposition in (52-a) has (52-b) as its innocently excludable alternatives and (52-c) as its innocently includable alternatives, such that exhaustification of this proposition produces (52-d).

(52)
a. $\lambda w\colon \Box^u_w(\mathsf{psn}_u\ \mathsf{p} \rightarrow \neg\mathsf{psn}_u\ \mathsf{s}).\ \Diamond^u_w\ \mathsf{psn}_u\ \mathsf{p}$
b. $\mathsf{IE}((\text{52-a}),(51)) = \{\lambda w\colon \Box^u_w(\mathsf{psn}_u\ \mathsf{p} \rightarrow \neg\mathsf{psn}_u\ \mathsf{s}).\ \neg\Diamond^u_w\ \mathsf{psn}_u\ \mathsf{s}\}$
c. $\mathsf{II}((\text{52-a}),(51)) = \left\{ \begin{array}{l} \lambda w\colon \Box^u_w(\mathsf{psn}_u\ \mathsf{p} \rightarrow \neg\mathsf{psn}_u\ \mathsf{s}).\ \Diamond^u_w\ \mathsf{psn}_u\ \mathsf{p} \\ \lambda w\colon \Box^u_w(\mathsf{psn}_u\ \mathsf{p} \rightarrow \neg\mathsf{psn}_u\ \mathsf{s}).\ \Diamond^u_w\ (\mathsf{psn}_u\ \mathsf{p} \vee \mathsf{psn}_u\ \mathsf{s}) \end{array} \right\}$
d. $\mathsf{exh}((\text{52-a}),(51)) = \lambda w\colon \Box^u_w(\mathsf{psn}_u\ \mathsf{p} \rightarrow \neg\mathsf{psn}_u\ \mathsf{s}).\Diamond^u_w\ \mathsf{psn}_u\ \mathsf{p} \wedge \neg\Diamond^u_w\ \mathsf{psn}_u\ \mathsf{s}$

But consider now the case of the disjunctive proposition in (51). Since it can never be that this proposition is true while both alternatives are false, the maximal subsets of (51) compatible with it will each be singletons containing the non-disjunctive alternatives. As a result, the generalized intersection of this set will be empty, and so will be the set of innocently excludable alternatives of this proposition. Each proposition in (51) is innocently includable such that exhaustification strengthens the disjunctive alternative by asserting each proposition in (53), effectively transforming this disjunction into a conjunction.

(53)
a. $\lambda w\colon \Box^u_w(\mathsf{psn}_u\ \mathsf{p} \rightarrow \neg\mathsf{psn}_u\ \mathsf{s}).\ \Diamond^u_w\ (\mathsf{psn}_u\ \mathsf{p} \vee \mathsf{psn}_u\ \mathsf{s})$
b. $\mathsf{IE}((\text{53-a}),(51)) = \emptyset$
c. $\mathsf{II}((\text{53-a}),(51)) = (51)$
d. $\mathsf{exh}((\text{53-a}),(51)) = \lambda w\colon \Box^u_w(\mathsf{psn}_u\ \mathsf{p} \rightarrow \neg\mathsf{psn}_u\ \mathsf{s}).\ \Diamond^u_w\ \mathsf{psn}_u\ \mathsf{p} \wedge \Diamond^u_w\ \mathsf{psn}_u\ \mathsf{s}$

Having understood how exhaustification will operate on the members of (51), we see that pointwise exhaustification of this set return (54).

$$(54)\quad \left\{ \begin{array}{l} \lambda w\colon \Box^u_w(\mathsf{psn}_u\ \mathsf{p} \rightarrow \neg\mathsf{psn}_u\ \mathsf{s}).\ \Diamond^u_w\ \mathsf{psn}_u\ \mathsf{p} \wedge \neg\Diamond^u_w\ \mathsf{psn}_u\ \mathsf{s} \\ \lambda w\colon \Box^u_w(\mathsf{psn}_u\ \mathsf{p} \rightarrow \neg\mathsf{psn}_u\ \mathsf{s}).\ \Diamond^u_w\ \mathsf{psn}_u\ \mathsf{s} \wedge \neg\Diamond^u_w\mathsf{psn}_u\ \mathsf{p} \\ \lambda w\colon \Box^u_w(\mathsf{psn}_u\ \mathsf{p} \rightarrow \neg\mathsf{psn}_u\ \mathsf{s}).\ \Diamond^u_w\ \mathsf{psn}_u\ \mathsf{p} \wedge \Diamond^u_w\ \mathsf{psn}_u\ \mathsf{s} \end{array} \right\}$$

This set can partition a context so long as it can be split into three equivalence classes, the first consisting of worlds where only p could be poisoned, the second where only s could be poisoned, and the third where both p could be poisoned and s could be poisoned. Crucially for cell of the partition, each world accessible from a member of that cell is such that only one wine is poisoned in that world. Such a context clearly does not presuppose that there is exactly one wine such that it could be poisoned. We can thus suppose that when singular *wh*-interrogatives with possibility modals lack a uniqueness inference, this is a result of their having a higher-order interpretation.

It is worth pondering whether a trigger for uniqueness local to the scope of interrogatives is necessary to avoid uniqueness once we admit higher-order readings of interrogatives into our system. Indeed, were "P-EXH" dispensed with, we would simply obtain the Hamblin set in (55) instead of (51), which differs from it only insofar as the propositions themselves lack the definedness condition introduced by "p-exh".

(55) $$\left\{\begin{array}{l} \lambda w.\ \Diamond_w^u\ \mathsf{psn}_u\ \mathsf{p} \\ \lambda w.\ \Diamond_w^u\ \mathsf{psn}_u\ \mathsf{s} \\ \lambda w.\ \Diamond_w^u\ (\mathsf{psn}_u\ \mathsf{p} \vee \mathsf{psn}_u\ \mathsf{s}) \end{array}\right\}$$

The pointwise exhaustification of (55) is the set in (56). Once more, we see that the interrogative will be defined only for contexts which contain worlds where p could be poisoned and s could be poisoned. We therefore see that this analysis also allows for readings of singular interrogatives which do not presuppose uniqueness.

(56) $$\left\{\begin{array}{l} \lambda w.\ \Diamond_w^u\ \mathsf{psn}_u\ \mathsf{p} \wedge \neg\Diamond_w^u\ \mathsf{psn}_u\ \mathsf{s} \\ \lambda w.\ \Diamond_w^u\ \mathsf{psn}_u\ \mathsf{s} \wedge \neg\Diamond_w^u \mathsf{psn}_u\ \mathsf{p} \\ \lambda w.\ \Diamond_w^u\ \mathsf{psn}_u\ \mathsf{p} \wedge \Diamond_w^u\ \mathsf{psn}_u\ \mathsf{s} \end{array}\right\}$$

However, the same problem discussed for Hirsch and Schwarz's analysis in Section 3 is present here, namely that the presupposition of the modalized interrogative is too weak. Contrary to our linguistic intuitions, (48-a) would be expected to be admissible in a context where it could be the case that multiple wines are poisoned. We thus find evidence that a lexical trigger for uniqueness local to the scope of the interrogative remains necessary to avoid deriving readings where (48-a)'s presupposition is too weak.

Finally, we note that our current system does not incorrectly predict that simple cases such as (57-a) can lack a uniqueness presupposition. Suppose we were to define the Hamblin set for (57-a) over generalized disjunctions of members of $\{\mathsf{a}, \mathsf{b}, \mathsf{c}\}$, as in (57). To make things more legible, we will abbreviate a proposition such as "$\lambda w\colon\ \mathsf{arrv}_w\ \mathsf{a} \rightarrow \neg(\mathsf{arrv}_w\ \mathsf{b} \vee \mathsf{arrv}_w\ \mathsf{c}).\ \mathsf{arrv}_w\ \mathsf{a}$" as simply "$\mathsf{a} \rightarrow \neg(\mathsf{b} \vee \mathsf{c}).\ \mathsf{a}$".

(57) a. Which student arrived?

b. $$\left\{\begin{array}{l} \mathsf{a} \rightarrow \neg(\mathsf{b} \vee \mathsf{c}).\ \mathsf{a} \\ \mathsf{b} \rightarrow \neg(\mathsf{a} \vee \mathsf{c}).\ \mathsf{b} \\ \mathsf{c} \rightarrow \neg(\mathsf{a} \vee \mathsf{b}).\ \mathsf{c} \\ \mathsf{a} \rightarrow \neg(\mathsf{b} \vee \mathsf{c}) \vee \mathsf{b} \rightarrow \neg(\mathsf{a} \vee \mathsf{c}).\ \mathsf{a} \vee \mathsf{b} \\ \mathsf{a} \rightarrow \neg(\mathsf{b} \vee \mathsf{c}) \vee \mathsf{c} \rightarrow \neg(\mathsf{a} \vee \mathsf{b}).\ \mathsf{a} \vee \mathsf{c} \\ \mathsf{b} \rightarrow \neg(\mathsf{a} \vee \mathsf{c}) \vee \mathsf{c} \rightarrow \neg(\mathsf{a} \vee \mathsf{b}).\ \mathsf{b} \vee \mathsf{c} \\ \mathsf{a} \rightarrow \neg(\mathsf{b} \vee \mathsf{c}) \vee \mathsf{b} \rightarrow \neg(\mathsf{a} \vee \mathsf{c}) \vee \mathsf{c} \rightarrow \neg(\mathsf{a} \vee \mathsf{b}).\ \mathsf{a} \vee \mathsf{b} \vee \mathsf{c} \end{array}\right\}$$

The pointwise exhaustification of non-disjunctive sentences in (57-b) is sterile given their presuppositions. Take for example the proposition in (58-a). This proposition is already defined only for worlds where a and no one else arrived. Its sets of innocently excludable and included alternatives are listed in (58-b) and (58-c), respectively. We see that (58-a) is exhaustified to (58-d), which is equivalent to it.

(58) a. $\mathsf{a} \rightarrow \neg(\mathsf{b} \vee \mathsf{c}).\ \mathsf{a}$

b. $$\left\{\begin{array}{l} \mathsf{b} \rightarrow \neg(\mathsf{a} \vee \mathsf{c}).\ \neg\mathsf{b} \\ \mathsf{c} \rightarrow \neg(\mathsf{a} \vee \mathsf{b}).\ \neg\mathsf{c} \\ \mathsf{b} \rightarrow \neg(\mathsf{a} \vee \mathsf{c}) \vee \mathsf{c} \rightarrow \neg(\mathsf{a} \vee \mathsf{b}).\ \neg(\mathsf{b} \vee \mathsf{c}) \end{array}\right\}$$

c. $$\left\{\begin{array}{l} \mathsf{a} \rightarrow \neg(\mathsf{b} \vee \mathsf{c}).\ \mathsf{a} \\ \mathsf{a} \rightarrow \neg(\mathsf{b} \vee \mathsf{c}) \vee \mathsf{b} \rightarrow \neg(\mathsf{a} \vee \mathsf{c}).\ \mathsf{a} \vee \mathsf{b} \\ \mathsf{a} \rightarrow \neg(\mathsf{b} \vee \mathsf{c}) \vee \mathsf{c} \rightarrow \neg(\mathsf{a} \vee \mathsf{b}).\ \mathsf{a} \vee \mathsf{c} \\ \mathsf{a} \rightarrow \neg(\mathsf{b} \vee \mathsf{c}) \vee \mathsf{b} \rightarrow \neg(\mathsf{a} \vee \mathsf{c}) \vee \mathsf{c} \rightarrow \neg(\mathsf{a} \vee \mathsf{b}).\ \mathsf{a} \vee \mathsf{b} \vee \mathsf{c} \end{array}\right\}$$

d. $\mathsf{a} \rightarrow \neg(\mathsf{b} \vee \mathsf{c}) \wedge \mathsf{b} \rightarrow \neg(\mathsf{a} \vee \mathsf{c}) \wedge \mathsf{c} \rightarrow \neg(\mathsf{a} \vee \mathsf{b}).\ \mathsf{a} \wedge \neg\mathsf{b} \wedge \neg\mathsf{c}$

Pointwise exhaustification cannot strengthen disjunctive answers into conjunctions, resulting instead in a contradiction. If we take as an example (59-a), the set of innocently excludable alternatives is the singleton in (59-b). In turn, we will have the set of innocently includable alternatives in (59-c). The problem with this is that given their respective presuppositions, the alternatives in (59-c) cannot be consistently asserted, and hence exh yields a contradiction. Since contradictions by definition cannot be mapped onto a partition of the context, we obtain a presupposition failure.[4]

(59) a. $\mathsf{a} \rightarrow \neg(\mathsf{b} \vee \mathsf{c}) \vee \mathsf{b} \rightarrow \neg(\mathsf{a} \vee \mathsf{c}).\ \mathsf{a} \vee \mathsf{b}$

b. $\{\mathsf{c} \rightarrow \neg(\mathsf{a} \vee \mathsf{b}).\ \neg\mathsf{c}\}$

c. (57-b)$\setminus\{\mathsf{c} \rightarrow \neg(\mathsf{a} \vee \mathsf{b}).\ \neg\mathsf{c}\}$

4 This hinges on the fact we have defined innocently includable alternatives in a way which is not contradiction free. Adopting the definition in footnote 2, what we would obtain is that the disjunctive answers cannot be strengthened to conjunctions through pointwise exhaustification. This too fails to meet the requirement that the exhaustified Hamblin set partition the context, as we would obtain overlapping cells. For instance, the exhaustification of "$\mathsf{a} \rightarrow \neg(\mathsf{b} \vee \mathsf{c}).\ \mathsf{a}$" would overlap with any cell picked out by the exhaustification of a disjunctive answer where "a" is one of the disjuncts.

Note that here too that without the uniqueness presupposition triggered by "P-EXH", we would predict readings for (57-a) which do not presuppose uniqueness. Indeed without this presupposition, pointwise exhaustification would be able to strengthen disjunctions into conjunctions without leading to a contradiction. The Hamblin set in (60) would therefore be pointwise exhaustified into (61), which can partition any context where it is presupposed that at least one student arrived. This again provides good reason to believe singular *wh*-interrogatives in English require two loci of exhaustification, one local to the scope of the interrogative and one over the whole interrogative.[5]

(60) $$\left\{\begin{array}{l} \mathsf{a} \\ \mathsf{b} \\ \mathsf{c} \\ \mathsf{a} \vee \mathsf{b} \\ \mathsf{a} \vee \mathsf{c} \\ \mathsf{b} \vee \mathsf{c} \\ \mathsf{a} \vee \mathsf{b} \vee \mathsf{c} \end{array}\right\}$$

(61) $$\left\{\begin{array}{l} \mathsf{a} \wedge \neg\mathsf{b} \wedge \neg\mathsf{c} \\ \mathsf{b} \wedge \neg\mathsf{a} \wedge \neg\mathsf{c} \\ \mathsf{c} \wedge \neg\mathsf{a} \wedge \neg\mathsf{b} \\ \mathsf{a} \wedge \mathsf{b} \wedge \neg\mathsf{c} \\ \mathsf{a} \wedge \mathsf{c} \wedge \neg\mathsf{b} \\ \mathsf{b} \wedge \mathsf{c} \wedge \neg\mathsf{a} \\ \mathsf{a} \wedge \mathsf{b} \wedge \mathsf{c} \end{array}\right\}$$

6 Conclusion

Dayal's (1996) view that interrogatives presuppose the existence of a unique maximally informative true answer fails to account for why simple singular *wh*-interrogatives presuppose uniqueness but those with possibility modals do not. Hirsch & Schwarz (2019) propose to weaken Dayal's proposal by (i) weakening the notion of maximal informativity to allow a Hamblin set to contain multiple maximal elements and (ii) assuming that the scopes of *wh*-interrogatives contain a lexical item triggering a uniqueness presupposition which projects existentially from below modals. We have argued that this leads to a theory of singular *wh*-interrogatives which is too weak. We utilize a local and a global exhaustification procedure which together produce a theory of interrogatives which we believe satisfactorily accounts for the contrast between simple and modalized *wh*-interrogatives without the under- and overgeneration of Dayal and Hirsch and Schwarz's proposals.

5 Fox (2019) is able to avoid deriving readings of singular interrogatives like (57-a) which lack uniqueness without a local trigger for uniqueness. His strategy involves assuming that the Hamblin set for higher-order readings of the interrogative obligatorily contains generalized conjunctions in addition to the generalized disjunction. This moves also makes it impossible to strengthen the disjunctions into conjunctions such that pointwise exhaustification here too fails to define a partition on any context.

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