Deriving closed-scale adjectives from ratios of measure functions*

Ryan Walter Smith
University of Manchester

1 Introduction

In this paper, I propose an analysis of closed-scale adjectives, such as *full*, *open*, *visible*, and their scalar antonyms, as having *ratios of measure functions* as part of their meaning. This approach derives a scale of rational numbers between 0 and 1 for such adjectives, a totally closed-scale in the sense of Kennedy & McNally (2005). I show that the approach successfully accommodates the interaction of closed-scale adjectives with a full range of fractional modifiers, such as *half* and *three quarters*, but also *modified* fractions, such as *more than half* and *less than two thirds*. I analyze these fractional modifiers as fractions themselves, and thereby assimilate their analysis to that of ordinary (modified) numerals. I show that the analysis naturally extends to both color terms, such as *red*, *white*, etc., and denominal adjectives, such as *wooden* and *golden*, which are also compatible with fractional modifiers (Kennedy & McNally 2010; Paillé 2021). I further discuss data that calls into question the idea that fractions cannot modify open-scale adjectives, with implications for the basic meaning of adjectives as sets of states (Wellwood 2014; Wellwood 2015; Cariani *et al.* 2023).

The paper is structured as follows. §2 provides background on the distinction between absolute and relative adjectives, as diagnosed by their differential compatibility with proportional modifiers like *half*, and reviews Kennedy & McNally's (2005) approach to scales and the meaning of *half* that explains this differential compatibility. §3 demonstrates that closed-scale adjectives can be modified by a full range of fractional modifiers other than *half*, and shows that these fractional modifiers can be modified in the same way as numerals. §4 introduces the basic ingredients of my analysis, while §5 presents the full analysis and shows how the approach can be combined with a simple analysis of fractional modifiers as (modified) numerals to explain the facts in §3. §6 extends the approach to color and denominal adjectives. §7 discusses the contrast between absolute and relative adjectives on my approach, and presents new data that calls into question the idea that fractions cannot modify open-scale adjectives, with implications for the basic meaning of adjectives. §8 concludes with discussion of potential areas for future research.

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*ryan.smith-3@manchester.ac.uk. Many thanks to Vera Hohaus, Jens Hopperdietzel, Andrew Koontz-Garboden, the audience at the 59th Meeting of the Chicago Linguistic Society, especially Chris Kennedy and Johanna Alstott, and anonymous reviewers from CLS59 and WCCFL41 for their feedback and comments on this work.

2 Background

Proportional modifiers like *half* have been used as a diagnostic to distinguish absolute adjectives (*full,open*), from relative adjectives (*tall, wide*) (Kennedy & McNally 2005).

(1) a. The glass is half full.
 b. #The glass is half tall.

The generalization arrived at in previous literature is that *half* is compatible with adjectives that map individuals to degrees on a *totally closed scale*, for which both maximal and minimal degrees are defined. To capture this interaction of *half* with scale structure, Kennedy & McNally (2005) propose that *half* is defined in terms of the maximum and minimum degree on the scale associated with an adjective's underlying measure function (see also Bochnak (2010)).

(2) *half* $\rightsquigarrow \lambda G.\lambda x.\exists d[\textsc{diff}(\textsc{max}(S_G))(d) = \textsc{diff}(\textsc{min}(S_G))(d) \wedge G(d)(x)]$

In words, *half* is true if the degree to which x is G is equidistant from the minimum and maximum points of the scale associated with G. On this approach, the incompatibility of *half* with relative adjectives like *tall* and *wide*, among others, falls out from the use of MAX and MIN operators in its definition: relative adjectives, according to Kennedy & McNally and much subsequent work in degree semantics, are associated with (at laest partially) open scales, which lack either a maximum or a minimum degree (or both). For adjectives associated with such scales, either one or both of MAX and MIN are undefined, and are therefore expected to be incompatible with *half*.

3 Beyond *half*: interactions with fractional modifiers

In addition to *half*, closed scale adjectives are compatible with a full range of fractional modifiers, such as *three quarters*, *two thirds*, and *one eighth*(3).

(3) a. The bottle is three quarters full
 b. The window is two thirds open
 c. The iceberg is one eighth visible from this point

These phrases are difficult to accommodate on an approach that takes its point of departure from Kennedy & McNally's analysis of *half*, which is defined solely in terms of distances from the minimum and maximum degrees on a scale. While *half* can be easily formalized in this way, i.e. as a point on the scale equidistant from the scale's endpoints, other fractions are difficult to represent in terms of distances from the maximum and minimum without directly invoking fractions in their definition.

Nevertheless, fractional modifiers are like *half* in being incompatible with open-scale adjectives, as (4) demonstrates.

(4) a. #The bottle is three quarters tall
 b. #The window is two thirds wide

Ideally, then, an explanation for the incompatibility of *half* with such adjectives should extend to any other fraction.

3.1 Fractions can be modified

Fractional modifiers of closed-scale adjectives may be modified like any other numeral; any numeral modifier known from the literature on modified numerals may modify a fraction, as the examples in (5) below show.

(5) a. This glass is less than half full
    b. The cup must be at least four fifths full
    c. Her eyes are rarely more than three quarters open
    d. Windows should be closed or at most half open
    e. The constellation is between two thirds and three quarters visible.

(5) shows clearly that the analysis of *half* in (2) is inadequate: its degree argument is existentially quantified, and thus not available for modification. This incorrectly predicts that fractional modifiers cannot be modified, contrary to fact. While an approach building on (2) is likely not impossible, it also misses an important generalization emerging from (3) and (5): fractional modifiers behave just like (modified) numerals. It is therefore desirable to give fractional modifiers of adjectives an analysis that assimilates them to (modified) numerals.

4 Closed scale adjectives via ratios of measure functions

I propose that closed-scale adjectives contain *ratios of measure functions* as a core component of their meanings. The guiding intuition behind this is approach is that closed-scale adjectives take an individual and divide a measure of *part* of the individual by the measure of the entire individual. For example, in order to understand how*full* a glass is, one measures the volume of the largest part of the glass containing a substance or object(s) and then finds a ratio of that volume over the volume of the entire glass. Likewise, to find the degree to which a window is open, one may measure the part of the window associated with some *aperture*, and then divide that measure by that of the entire window. I will start by defining the formal ingredients necessary to make these intuitions precise, and then combine them in the full analysis.

4.1 Formal ingredients

4.1.1 State predicates

Building on many recent proposals (Wellwood 2014; Wellwood 2015; Cariani *et al.* 2023), I propose that a *predicate of states* lies at the core of an adjective's meaning. For example, at the core of the adjective *full* is a state predicate, FULLNESS, which I will take to be true of states of being *completely filled* with some substance or group of objects. Likewise, at the core of the adjective *open* is a state predicate APERTURE, which is true of states of being *completely open*. In other words, I assume here that state predicates are true of states that would, on a degree-based approach, meet the positive standard in the context.

(6) a. $\lambda s.\text{FULLNESS}(s)$
    b. $\lambda s.\text{APERTURE}(s)$

4.1.2 Mereological structure

I assume that the domain of individuals is *mereologically structured* (Link 1983; Krifka 1992; Champollion 2017). This amounts to the claim that individuals are partially ordered by a relation of *parthood*. Importantly for my purposes, the relevant notion of parthood is the *singular part relation*, notated $\leq_s$, which relates *atomic* individuals with the parts that make them up (Moltmann 1997; Morzycki 2002).

4.1.3 Measure functions

Finally, I make use of *extensive measure functions*, notated μ, which are functions from individuals to numbers such that the number returned by μ is understood as a measure of an individual along some dimension. I will assume here that numbers are of type *d*, and are thus *degrees*. Importantly, measure functions are required to be *monotonic* with respect to the singular parthood relation, with monotonicity defined as in (7) (Schwarzschild 2002; Wellwood 2015; Pasternak 2019).

(7) $\forall\mu$,x,y[x $<_s$ y $\rightarrow$ μ(x) $<$ μ(y)]

In words, this requires any part of an individual to have a strictly smaller measure than the individual of which it is a part.

4.2 Putting the ingredients together

Putting all of this together, the general form of the analysis of a closed-scale adjective is as a relation between degrees d and individuals x, such that the ratio of the measure μ of the *largest singular part* of x (encoded with Link's σ operator) related to a state determined by the adjective and the total measure μ of x itself is greater than or equal to d. Particular closed-scale adjectives differ with respect to the kind of state parts of individuals are related to, and which measures are used. *Full*, for example, relates degrees d and individuals for whom the *carrying capacity* of the largest part related to a *fullness* state divided by the carrying capacity of the entire individual is greater than or equal to d (8a). *Open* is analyzed the same way, but the measure function at issue involves *area* and the state one of *aperture* (8b).[1]

(8) a. *full* $\rightsquigarrow \lambda$x.λd.$\frac{\mu_{cc}(\sigma y[y \leq_s x \wedge \exists s[\text{FULLNESS}(s) \wedge Hd(s)=y]])}{\mu_{cc}(x)} \geq$ d

b. *open* $\rightsquigarrow \lambda$x.λd.$\frac{\mu_{area}(\sigma y[y \leq_s x \wedge \exists s[\text{APERTURE}(s) \wedge Hd(s)=y]])}{\mu_{area}(x)} \geq$ d

Because μ is monotonic with respect to the singular part relation, the measure of a part of an individual may be at most equal to that of th individual itself. In this case, the ratio will return a measure of 1. Likewise, if x is completely empty or closed, the measure of the part of the individual related to a state of fullness or aperture will be 0, making the measure returned by the ratio in turn 0.[2]

[1] Nothing crucial hinges on the particular choice of measure function, as long as it is spatial in nature. For example, if the aperture of a doorway is understood as the space between the door and its frame, one could instead measure the *volume* of this part of the doorway. In general, a measure of quantity will suffice for delivering the appropriate numerical values.

[2] Here I assume that individuals may have a null part with measure 0 (Bylinina & Nouwen 2018). This assumption is necessary, as otherwise the ratio will always return non-zero values at best, and be undefined at worst.

The upshot of this is that adjectives like *full*, by virtue of being defined in terms of ratios of measure functions, are associated with rational numbers in the closed interval [0,1]. Because this set has a minimum (0) and a maximum (1), we derive the fact that *full* and other adjectives like it are associated with a totally closed scale.

5 Modifying adjectives with fractions

With the analytical core in place, I can now turn to the analysis of modification of closed-scale adjectives by fractions. My analysis of fractional modifiers is simple: they denote rational numbers between 0 and 1, singular terms of type d (Buccola & Spector 2016).[3]

(9) a. *half* $\rightsquigarrow \frac{1}{2}$
b. *three quarters* $\rightsquigarrow \frac{3}{4}$
c. *two thirds* $\rightsquigarrow \frac{2}{3}$

Following Kennedy (2015) and Buccola & Spector (2016), I treat numeral modifiers as functions from numbers to degree quantifiers (type <<d,t>,t>). Here I will make use of Kennedy's analysis, in terms of MAX.

(10) a. *more than* $\rightsquigarrow \lambda$d.λD.MAX(D) $>$ d
b. *less than* $\rightsquigarrow \lambda$d.λD.MAX(D) $<$ d
c. *at least* $\rightsquigarrow \lambda$d.λD.MAX(D) $\geq$ d
d. *at most* $\rightsquigarrow \lambda$d.λD.MAX(D) $\leq$ d

Assuming the adjective composes first with its subject, bare fractional modifiers saturate the degree argument of the adjective with the numeral. This delivers (11) as the analysis of (1a).[4]

(11) $\frac{\mu_{cc}(\sigma y[y \leq_s the.glass \wedge \exists s[fullness(s) \wedge Hd(s)=y]])}{\mu_{cc}(the.glass)} \geq \frac{1}{2}$

Composition with a modified fraction is similarly straightforward: *full* delivers a predicate of degrees after composing with its individual argument, which serves as the argument of the modified fraction. (12) gives the analysis of the class of examples in (5) above.

(12) $\textsc{max}(\lambda d'.\frac{\mu_{cc}(\sigma y[y \leq_s the.glass \wedge \exists s[fullness(s) \wedge Hd(s)=y]])}{\mu_{cc}(the.glass)} \geq d') < \frac{1}{2}$

The analysis of closed-scale adjectives as incorporating ratios of measure functions, in combination with a simple analysis of fractional modifiers as (modified) numerals, thus assigns the correct truth conditions to the sentences discussed above, and explains the observed compatibility of closed-scale adjectives with a full range of fractional modifiers, both simplex and modified.

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[3] Alternatively, they may be given a quantificational analysis, as in (i), with the singular term meaning in (9) derived via the application of BE and IOTA type shifters (Kennedy 2015; Bylinina & Nouwen 2020).
(i) *half* $\rightsquigarrow \lambda$D.MAX(D) = $\frac{1}{2}$

[4] This delivers a weak "at least" reading of the fraction. A strong reading could be derived through scalar implicature, or through the degree quantifier analysis discussed in footnote 3 (Kennedy 2015).

6 Extending the analysis: color and denominal adjectives

Fractions, both simple and modified, also compose with color terms (13) and denominal adjectives like *wooden* (14).

(13) This flag is (less than) half white

(14) The table is (more than) three quarters wooden.

While Wellwood (2014) treats such adjectives as essentially non-gradable, other works have taken different approaches. For example, Kennedy & McNally (2010) argue that color terms and denominal adjectives are ambiguous between a *quality* and a *quantity* reading. The quality reading corresponds to the *intensity* of a color's hue, while the quantity reading corresponds to *how much* of an object has the color. From a different perspective, Paillé (2021) also argues that color terms are lexically *weak*, and require only that some part of an object has the color in question, with the *total* reading derived by exhaustification.

Quantity readings of color and denominal adjectives can be captured straightforwardly on my approach, using a ratio of measure functions encoding the proportion of an object related to a state of having some color or other property. For example, the quantity reading of *white* can be analyzed as incorporating a ratio of measure functions that measure the proportion of an object associated with a WHITENESS state. Likewise, the denominal adjective *wooden* measures the proportion of an object associated with a state of WOODENNESS.

(15) a. $white_{quantity} \rightsquigarrow \lambda\text{x}.\lambda\text{d}.\frac{\mu_{quantity}(\sigma y[y \leq_s x \wedge \exists s[whiteness(s) \wedge Hd(s) = y]])}{\mu_{quantity}(x)} \geq \text{d}$

b. $wooden_{quantity} \rightsquigarrow \lambda\text{x}.\lambda\text{d}.\frac{\mu_{quantity}(\sigma y[y \leq_s x \wedge \exists s[woodenness(s) \wedge Hd(s) = y]])}{\mu_{quantity}(x)} \geq \text{d}$

From this perspective, color and denominal adjectives, on their quantity readings, are expected to be able to compose with fractions in the same way as other closed-scale adjectives. The approach has additional advantages as well. First, it explains the total reading of color adjectives as an interaction of POS with an adjective associated with a totally closed scale; for such adjectives, POS returns the maximum on the scale. In this case, the maximum is the number 1, and thus the analysis correctly predicts that (13-14), in the absence of fractional modifiers, require that the object be totally white/wooden. Second, it permits for a principled approach to the quality/quantity ambiguity, such that the quantity version of the adjective can be derived from the quality version by factoring out the meaning of the adjective as a predicate of states. I elaborate on this further in the next section.

7 The open/closed distinction and the basic meaning of adjectives

The account allows for the observed difference between (1) and (2) above in one of two ways. From one perspective, one could argue that, while adjectives like *full* are associated with rational numbers in the interval [0,1], open-scale adjectives are not, and thus cannot compose with fractions directly. Open-scale adjectives must first compose with an appropriate unit in order to compose with such an adjective, e.g. *half an inch tall*.

The analysis allows for another perspective to be taken, however: because my analysis embeds Wellwood's analysis of adjective meanings, my approach is compatible with a view on which adjectives, rather than lexically denoting measure functions or degree relations, start out life denoting predicates of states, which may compose with a separate head encoding a ratio of measure functions. While treating adjectives as predicates of states generally may seem problematic for the interaction of fractions with open-scale adjectives, this actually turns out to be a boon for the analysis: notice that sentences like (16), in which a fraction modifies an open-scale adjective, are acceptable in the right context.

(16) a. This bridge is half wide and half narrow
b. The pool is three quarters deep and a quarter shallow

(16a) is true in a context in which half of the bridge in question counts as wide in the context, while the other half counts as narrow. Likewise, (16b) is true if three quarters of the pool is deep enough to exceed the contextual standard of depth, while the remaining quarter counts as shallow. In other words, the fractions in these sentences embed the *positive* form of the adjective. This can be handled directly on my approach: building on Cariani et al. (2023), we can treat the positive form of an adjective as basic, denoting a relation between individuals and states, where the state in question counts in the context as wide, deep, hot, etc.[5] The example below provides an entry for *wide*.

(17) $wide_c \rightsquigarrow \lambda s.wide_c(s)$

We can then extract the ratio of measure functions from the denotations proposed above for closed-scale adjectives. I will denote this measure function as FRAC, and define it in (18).

(18) $\textsc{frac} \rightsquigarrow \lambda S.\lambda x.\lambda d.\frac{\mu(\sigma y[y \leq_s x \wedge \exists s[S(s) \wedge Hd(s)=y]])}{\mu(x)} \geq d$

The analysis of a sentence like *the bridge is half wide* is then straightforward, and delivers the correct truth conditions as in 19.

(19) $\frac{\mu(\sigma y[y \leq_s the.bridge \wedge \exists s[wide_c(s) \wedge Hd(s)=y]])}{\mu(the.bridge)} \geq \frac{1}{2}$

This renders the analysis of fractional modification of open-scale adjectives entirely parallel to that of closed-scale adjectives; recall that on my approach, *full* has at its core a predicate of states of being completely full. In other words, the analysis of closed-scale adjectives also involves embedding of the positive form. Therefore, we can provide a unified analysis of fractional modification of adjectives as modification of the positive form of the adjective.

[5]On Cariani et al.'s approach, the domain of states is associated with a *domain* and a *background structure*, the latter of which is a pre-order on the domain of states. On their analysis, the comparative can be derived from the positive form by accessing the positive adjective's background structure. This eliminates positive entailments while allowing for a derivation of the comparative from the positive.

8 Conclusion

In this paper, I've proposed an approach to closed-scale adjectives on which they incorporate ratios of measure functions as part of their meaning. Combined with a simple approach to fractions, including a treatment of modified fractions as simply modified numerals, the analysis is able to deal with the interaction of fractions with closed-scale adjectives straightforwardly. What's more, since the approach embeds a Wellwood-style analysis of adjectives as predicates of states, the analysis is can be easily cast in such a way that ratios can be introduced compositionally on top of a state-characterizing adjectival core. This turns out to be a key advantage of the analysis: fractional modifiers *can* modify positive-form open-scale adjectives, and a compositionally introduced ratio of measure functions can compose with an open-scale adjective *qua* predicate of states to deliver the desired readings in the same way as in the analysis of closed-scale adjectives.

Several areas for future research remain. First, in addition to closed-scale adjectives, fractions may also occurs with *partitives*, as can be seen in (20).

(20) a. (More than) half of the ball is red.
b. (Less than) two-thirds of pie is covered in chocolate.

A natural extension of the analysis would involve treating partitives of the sort in (20) as encoding a ratio of the measure of part of an object over that of the entire object, and predicating that part of the nuclear scope property.

Second, I have primarily focused on partitive readings of the interaction between closed-scale adjectives and fractional modifiers thereof. This may be appropriate for many, if not most, cases of adjectival modification by fractions, but some examples elude a characterization in terms of partitivity. One such example is provided in (21). While this example *can* be interpreted as meaning that half of the meat was cooked, it can also mean that the *entirety* of the meat was cooked, but its *degree of cookedness* is halfway between totally raw and fully cooked, i.e. the meat may still be somewhat pink (Bochnak 2010).

(21) The meat is half cooked.

There are a number of options for treating this second reading of (21). One possibility is to posit further decomposition, such that FRAC may compose with different kinds of measures. Another option would be to invoke a distinct FRAC that accesses a distinct scale, defined on certain kinds of states, rather than a ratio of measures based on parthood. Still, avoiding a proliferation of silent operators is to be preferred, so I leave this issue to future research.

A final issue concerns the analysis of fractions. On my analysis, fractions are simple rational numbers of type *d*. The upshot of this analysis is that it can be adapted straightforwardly to extant analyses of modified numerals, which, in combination with the ratio-based analysis of closed-scale adjectives, delivers the desired readings without issue. However, if fraction phrases simply denote numbers, then it is strange that very similar terms, such as decimals, cannot modify adjective phrases in the same way as fractions, as (22) shows.[6] Note that modified versions

[6] Thanks to Johanna Alstott for pointing this out to me.

of decimals are no better, which is especially surprising given that, on the analysis I adopted here, composition of a number with a numeral modifier returns a degree quantifier, which should compose straightforwardly with an adjective phrase.

(22) a. *The glass is .5 full
b. *The constellation is .75 visible
c. *The door is more than .25 open

This difference in distribution extends beyond modification of adjective phrases. For instance, while fractions may appear in partitive constructions, decimals may not.

(23) a. Half/two thirds of the statue is red
b. *.5/.75 of the statue is red

On the other hand, decimals are acceptable in measure phrases, in which case they are compatible with numeral modifiers. Fractions, on the other hand, cannot directly compose with measure words. Instead, the two types of expression must interact via the partitive construction.

(24) a. This bug is (less than) .75 centimeters long
b. This bug is (less than) three quarters *(of a) centimeter(*s) long

Clearly there is more to be said about the different distributions of fractions and intuitively synonymous decimals. I leave an investigation of this to future research.

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