

# The semantics of comparatives: A difference-based approach

Linmin Zhang<sup>1, 2, \*</sup> and Jia Ling<sup>3</sup>

<sup>1</sup>NYU Shanghai

<sup>2</sup>NYU-ECNU Institute of Brain & Cognitive Science at NYU Shanghai

<sup>3</sup>Lewis-Sigler Institute for Integrative Genomics, Princeton University

\*Correspondence to: New York University Shanghai, 1555 Century Avenue, Shanghai 200122, China. E-mail address: [zhanglinmin@gmail.com](mailto:zhanglinmin@gmail.com)

Final version after proof-reading, Jan. 12, 2021. In press at *Journal of Semantics*.

## Abstract

Degree semantics has been developed to study how the meanings of measurement and comparison are encoded in natural language. Within degree semantics, this paper proposes a **difference-based** (or **subtraction-based**) approach to analyze the semantics of comparatives. The motivation is the measurability and comparability of differences involved in comparatives. The main claim is that comparatives encode a subtraction equation among three scalar values: two measurements along an interval scale and the difference between them. We contribute two innovations: (i) using interval arithmetic to implement subtraction, and (ii) analyzing comparative morpheme *-er/more* as an additive particle, denoting the default, most general, positive difference. Our analysis inherits existing insights in the literature. Moreover, the innovations bring new conceptual and empirical advantages. In particular, we address the interpretation of comparatives containing *than*-clause-internal quantifiers and various kinds of numerical differentials. We also account for three puzzles with regard to the scope island issue, the monotonicity of *than*-clauses, and the discourse status of the standard in comparison.

**Keywords:** measurement, comparison, gradable adjectives, comparatives, differentials/differences, comparative morpheme *-er/more*, measurement constructions, positive use of gradable adjectives, scales, degrees, intervals, units, orderings, interval arithmetic, interval subtraction, degree questions, definite descriptions, downward-entailing operators, additivity, anaphoricity.

# 1 Introduction

Humans measure objects along some dimension or scale and make comparisons among measurements. As illustrated in (1), we can compare how tall a giraffe is to a certain tree; we can compare some soup and coffee in terms of their temperature; and we can compare a train's arrival with the time it's supposed to arrive on a temporal scale.

- (1) a. My giraffe is (5 inches) **taller than** that tree is.  
    ↪ On a scale of **height**: the measurement of my giraffe vs. the measurement of that tree
- b. This soup is (much) **hotter than** that coffee seems to be.  
    ↪ On a scale of **temperature**: the measurement of this soup vs. the seeming measurement of that coffee
- c. The train arrived (one hour) **later than** it should have.  
    ↪ On the scale of **time**: the actual arrival time vs. the scheduled arrival time

Natural language typically uses **comparatives** to express **comparisons yielding differences** (cf. **equatives**, which typically express **comparisons yielding no differences**). The notion of differences is obviously a gist of the meaning encoded in comparatives. Thus, starting with the view that differences constitute an indispensable central component in comparatives, this paper furthers our understanding of the semantics of comparatives and develops a new **difference-based** approach.

This introduction addresses the ontology of differences as involved in comparatives and lays out our basic assumption and motivation, paving the way for our proposal.

## 1.1 The ontology of differences in comparatives

We address the ontology of differences and their formal properties within a general view on measurement and comparison. In his influential paper on the theory of scales of measurement, [Stevens \(1946\)](#) paraphrases N. R. Campbell and points out that 'measurement, in the broadest sense, is defined as the assignment of numerals to objects or events according to rules'. Thus, measurement is a mapping function from items under measurement to values on a certain scale.

[Stevens \(1946\)](#) presents a four-level distinction of measurement and their related scales: **nominal scales**, **ordinal scales**, **interval scales**, and **ratio scales**. This four-level distinction is according to (i) the way of assigning values in measurement, (ii) the

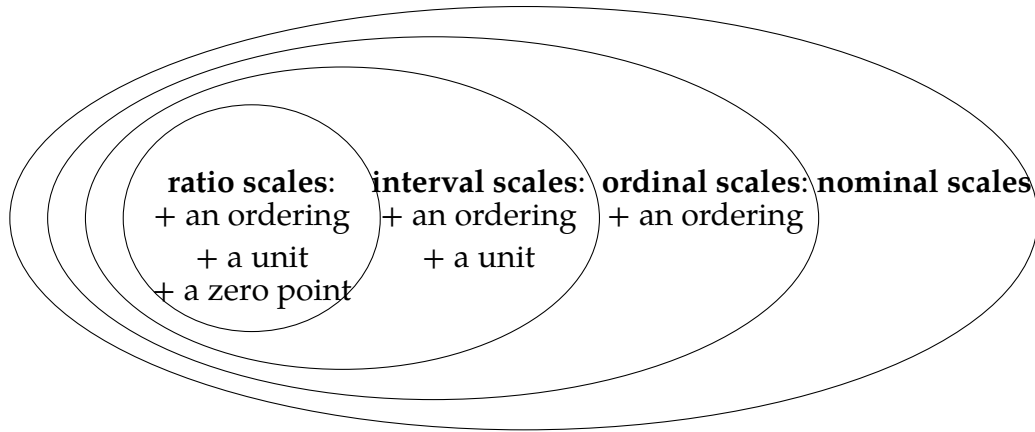


Figure 1: Four levels of scales, their entailment relationships (represented by the Venn diagram), and their defining attributes.

formal properties of the resulting scales, and (iii) the mathematical operations applicable to measurement values. The entailment relations among these four levels of scales are shown in the Venn diagram in Fig. 1.

**Nominal scales** do not even involve ordering. For example, if we assign a postal code to each address, the postal codes constitute a nominal scale. For distinct values on a nominal scale (e.g., distinct postal codes), all that matters is their distinctness, and further comparison is not mathematically meaningful.

**Ordinal scales** have **orderings**. For example, the ranking of my favorite soda brands forms an ordinal scale. Comparisons between two ranking values are characterized by inequality relations like '>', '<=', etc., but beyond ordering relations, it is not meaningful to address to what extent a certain ranking exceeds another one.

**Interval scales** have both orderings and **units**. On an interval scale, if one value is positioned higher than another one, we can use units to measure the **distance** (i.e., the **difference**) between the two **positions**. Therefore, comparisons between measurement values on an interval scale yield **measurable and comparable differences**, allowing for addressing **to what extent one value exceeds another**.

**Ratio scales** are interval scales with a meaningful, absolute **zero point**. For example, the Celsius scale of temperature lacks an absolute zero point in the sense that 0 °C does not mean 'no heat', and thus this is not a ratio scale. In contrast, for a scale of spatial length, 0 m does mean 'no length'. Thus this scale has an absolute zero point and it is a

ratio scale.<sup>1</sup>

Then what kind of scales are involved in the meaning of comparatives? Empirically, English comparatives allow for addressing the ‘to what extent’ issue with regard to differences yielded from comparisons. As illustrated in (1), this is evidenced by the use of modifiers like *much* (see (1b)) or numerical differentials (e.g., *5 inches* in (1a) and *one hour* (1c)). These examples indicate that comparisons as encoded in comparatives are performed between measurements on interval scales and yield **measurable differences**.<sup>2</sup>

The **comparison of deviations** shown in (2) is another linguistic construction showing how differences yielded from comparisons on a **base measurement scale** can be further measured and compared on a **scale of differences**.<sup>3</sup> Here the scales of happiness and sadness are two base measurement scales. It is the differences yielded from comparisons along these two base measurement scales that constitute the measurements along the scale involved in the third comparison, i.e., a scale of differences. In this sense, **measurement** yields markings of **positions** along a base measurement scale, while **comparison** is actually the measurement of differences/**distances** between positions. The values of these differences/distances can again be considered positions along a scale of differences. Base measurement scales and scales of differences are both interval scales.

(2) Mona is more happy than Jude is sad. (Kennedy 1999: Chapter 1, (89))<sup>4</sup>

a. **Comparison 1** – along a scale of happiness:

Mona’s happiness vs. the standard of happiness      ~> Mona is happy

b. **Comparison 2** – along a scale of sadness:

Jude’s sadness vs. the standard of sadness      ~> Jude is sad

<sup>1</sup>Based on the distinction between interval scales and ratio scales, Sassoon (2010) explains why only certain gradable adjectives are accepted in forming measurement constructions like *This movie is 4 hours long* (cf. *#This cup of coffee is 30 °C warm*). Evidently, Stevens (1946)’s theory captures some crucial aspects of our conceptualization of measurement and comparison and their linguistic encoding.

<sup>2</sup>We do not claim that cross-linguistically, all comparatives or comparison-related meanings must be based on interval scales. Presumably, there might be comparison-related linguistic phenomena based on ordinal scales or even nominal scales (see e.g., Solt 2016 and Zhang 2020b). It is also likely that in a certain language, both interval-scale-based and ordinal-scale-based comparative constructions co-exist. However, we do claim that interval scales must be assumed for linguistic phenomena like English comparatives. We also predict that comparatives simply based on ordinal scales cannot support the expression of the size of differences.

<sup>3</sup>We thank an anonymous reviewer for suggesting this pair of terms.

<sup>4</sup>See Kennedy (1999), Section 1.2.3.2, for a detailed discussion on the interpretation of this sentence, its positive entailments (i.e., Mona is happy, and Jude is sad), and how this sentence challenges the theory of Klein (1980), which generates a too weak truth condition. The analysis under our proposal is presented in Section 5.3.

- c. **Comparison 3** – along a scale of deviation size (i.e., a scale of differences):  
difference from Comparison 1 vs. difference from Comparison 2

The necessary role played by units in measuring and comparing differences is most evidently manifested by the measurement and comparison of times. For the case in (1c), ordering only tells which one between the scheduled and the actual arrival times occurred first, and it is units (e.g., hours, minutes) that measure time differences. Obviously, units like *hours* can by no means be derived just from the ordering of equivalence classes like {the scheduled arrival time of a train, 12 o'clock, ...} or {the actual arrival time of a train, 1 o'clock, ...}.<sup>5</sup>

In brief, based on Stevens (1946)'s theory on the levels of measurement and scales, we have shown that the notions of interval scales and the measurability and comparability of differences fundamentally underlie the meaning of comparatives.

## 1.2 Our assumption and motivation

Within the literature on the semantics of comparatives, the major assumption is that comparisons are performed between **degrees**, i.e., points that mark positions and represent scalar values on a relevant abstract scale (i.a., Seuren 1973, Cresswell 1976, Hellan 1981, Hoeksema 1983, von Stechow 1984, Heim 1985, Bierwisch 1989, Lerner and Pinkal 1992, 1995, Moltmann 1992, Gawron 1995, Izvorski 1995, Rullmann 1995). Kennedy (1999) provides a review and a convincing defense on this assumption.

Under this assumption, for our examples in (1), items undergoing comparisons are not entities (e.g., my giraffe, this soup) or events (e.g., a train's arrival) per se, but rather their heights, temperatures, or times. This assumption is not specific on the formal properties of degrees or scales involved in comparatives.<sup>6</sup>

---

<sup>5</sup>In English, *o'clock* is used to mark positions on a scale of time, while unit expressions like *hour* are used to measure differences on a scale of temporal differences (or temporal length). Of course, *minutes*, *days*, *years*, etc., are also units that can be used to measure temporal differences. For our current purpose, the actual choice among these units does not really matter. The upshot is that the measurability and comparability of differences relies on the notions of interval scales and units.

For many dimensions (e.g., temperature, spatial length, weight), base measurement scales and scales of differences share unit expressions (e.g., °C, *meter*, *kilo*), delusively blurring the distinction between conceptually distinct scales. For example, the Celsius scale, a base measurement scale of temperature, is not a ratio scale given that 0 °C (i.e., the freezing point of water) does not mean 'no heat', but a scale of temperature differences is a ratio scale given that 0 °C means 'no temperature difference'.

<sup>6</sup>For Cresswell (1976), this abstract scale can be derived from the orderings among equivalence classes, but as shown in Section 1.1, mere orderings are insufficient for characterizing the semantics of comparatives.

A less explicit assumption is that degrees involved in comparatives are number-like values, so that the operations of addition or subtraction are applicable. This assumption is reflected in the analysis of comparatives containing numerical differentials – the parenthesized part in the examples in (1) (i.e., Hellan 1981, von Stechow 1984).

Within both the ‘A-not-A’ analysis (see Schwarzschild 2008 for a review) and the ‘>’ analysis (see Beck 2011 for a review), for cases with no explicit numerical differentials, analyses are based on set operations and orderings (see (3a) and (4a)). Addition or subtraction is used to deal with explicit numerical differentials (see (3b) and (4b)).

(3) My giraffe is (5 inches) taller than that tree is. **the ‘A-not-A’ analysis**

- a. The difference set between  $\{d \mid \text{the height of my giraffe} \geq d\}$  and  $\{d \mid \text{the height of that tree} \geq d\}$  is non-empty.
- b. For the difference set  $D$  between  $\{d \mid \text{the height of my giraffe} \geq d\}$  and  $\{d \mid \text{the height of that tree} \geq d\}$ ,  $\text{MAX}(D) - \text{MIN}(D) \geq 5''$ .

(4) My giraffe is (5 inches) taller than that tree is. **the ‘>’ analysis**

- a.  $\text{MAX}(\{d \mid \text{the height of my giraffe} \geq d\}) > \text{MAX}(\{d \mid \text{the height of that tree} \geq d\})$
- b.  $\text{MAX}(\{d \mid \text{the height of my giraffe} \geq d\}) \geq \text{MAX}(\{d \mid \text{the height of that tree} \geq d\}) + 5''$

The analyses shown in (3a) and (4a) only need to assume ordinal scales for measurement and comparison, while the analyses shown in (3b) and (4b) have to assume number-like degrees and thus interval scales. This discrepancy in their underlying assumptions has been largely unnoticed and under-discussed.

In the current paper, we explicitly assume that the semantics of comparatives is based on scalar values on interval scales. As we have shown, this assumption is empirically warranted by natural language phenomena involving comparatives. Making this assumption explicit will help with exploring the formal properties of degrees in comparatives and what operations to apply on them.

Therefore, this paper aims to push the existing semantic analyses of comparatives towards a full exploitation of this underlying assumption. Based on the measurability and comparability of differences, we will take maximum advantage of the operation of

---

The abstract scale involved must be an interval scale. See also Kennedy (1999)’s discussion.

Solt and Gotzner (2012) uses experimental evidence to show that orderings are also insufficient for characterizing the positive use of gradable adjectives, i.e., the positive use should also be based on interval scales.

subtraction to build a uniform analysis for both comparatives with and without explicit numerical differentials. The main idea is that comparatives mean a **subtraction equation** among three scalar values: two measurements and the difference between them.

Specifically, we will propose (i) the use of interval subtraction and (ii) an additivity-based view for the semantic contribution of comparative morpheme *-er/more*. **Interval arithmetic** provides a convenient technique for characterizing differences in a generalized way, allowing for implementing equations with potentially not-very-precise scalar values.<sup>7</sup> Then *-er/more* essentially contributes the meaning of **increase**, which turns out to be another way to convey the idea of differences yielded from comparisons. Both of our innovations are actually further development of existing insights or observations from the literature of degree semantics.

To assess our proposal, we will show how it brings new conceptual and empirical advantages. In particular, we will demonstrate that the interpretation of comparatives containing *than*-clause-internal quantifiers and all kinds of numerical differentials can be derived in a natural and uniform way. Moreover, the proposed interval-subtraction-based analysis accounts for three long-existing puzzles in the literature of comparatives: (i) How does a *than*-clause project information as a scope island? (ii) How does a *than*-clause contribute a downward-entailing operator? (iii) If comparison is involved in all uses of gradable adjectives, why is the comparative form (e.g., *taller*) still morphologically more complex than other uses (e.g., *tall*) (Klein 1980's puzzle)? We will show that interval subtraction and an additivity-based view for comparative morpheme *-er/more* provide the exact ingredients to solve these issues.

### 1.3 Outline of the paper

The paper is organized as follows. Section 2 presents our core innovations and their precursors in the existing literature. Section 3 develops a difference-based analysis of the semantics of comparatives, with a detailed formalism implemented in terms of interval subtraction. Section 4 shows the semantic derivation of complex cases: *more-than* and *less-than* comparatives containing numerical differentials and *than*-clause-internal quantifiers. Section 5 accounts for three puzzles, with regard to the scope island issue,

---

<sup>7</sup>Here are some clarifications on terminology. Following Stevens (1946), we use **interval scales** to refer to scales equipped with both orderings and units. **Scalar values** mean positions on an interval scale: they can be represented as degrees or intervals. **Degrees** are points (i.e., elements) on an interval scale. **Intervals** are convex sets of degrees (e.g.,  $\{d \mid 3 \leq d \leq 5\}$ ). **Interval arithmetic** refers to operations on **intervals**. In particular, we focus on the operation of **interval subtraction**. See Section 3.1 for details.



the monotonicity of *than*-clauses, and Klein (1980)’s puzzle on the semantic contribution of *-er/more*. Section 6 further compares the current analysis with existing studies and ideas on the topic of comparatives. Section 7 concludes. Below, for simplicity, we often use ‘scale’ to mean ‘interval scale’ in addressing the semantics of comparatives.

## 2 The core innovations and their precursors

This section starts with the canonical analysis of comparatives. Against this background, we present the most direct precursors to the current proposal and then our core innovations. An informal sketch of our proposal is given at the end of this section.

### 2.1 The canonical analysis of comparatives

We follow mainly the review articles by Schwarzschild (2008) and Beck (2011) in sketching out the essence of the canonical analysis of comparatives. Many widely accepted ideas of the canonical analysis have already been established back to von Stechow (1984) and thoroughly discussed by Kennedy (1999). Our presentation glosses over technical details of composition. It is by no means comprehensive. This presentation simply aims to set up the background for the discussion later.

Based on a degree-theoretic view for comparison (i.e., things undergoing comparison are degrees, not entities or events), the canonical analysis consists of three key components: (i) analyzing gradable adjectives as relations of type  $\langle d, et \rangle$ , instead of characteristic functions of type  $\langle et \rangle$ ; (ii) analyzing the matrix and *than*-clauses as sets of degrees; and (iii) analyzing *-er/more* as a relation between sets of degrees.

As illustrated by (5), a gradable adjective denotes a relation between a degree (i.e., a point on a relevant scale) and an individual (see, e.g., Cresswell 1976, Hellan 1981, von Stechow 1984, Heim 1985, Beck 2011, cf. Kennedy 1999). Here HEIGHT means a measure function, mapping an individual to a degree on a relevant scale (here height).

$$(5) \quad [[\text{tall}]]_{\langle d, et \rangle} \stackrel{\text{def}}{=} \lambda d_d. \lambda x_e. \text{HEIGHT}_{\langle e, d \rangle}(x) \geq d \quad (\text{i.e., } x \text{ is } d\text{-tall; } x \text{ is tall to degree } d)$$

The semantics of **measurement constructions** and the **positive use** of gradable adjectives can be thus derived straightforwardly, as illustrated by (6). In (6b), POS means a silent context-dependent degree threshold of tallness for a relevant comparison class (see Bartsch and Vennemann 1972a, Cresswell 1976, von Stechow 1984, Kennedy 1999).



- 206 (6) a.  $[[\text{Mary is 6 feet tall}]] = \text{HEIGHT}(\text{Mary}) \geq 6'$  **Measurement construction**  
 207 b.  $[[\text{Mary is tall}]] = \text{HEIGHT}(\text{Mary}) \geq \text{POS}$  **Positive use**

208 With the abstraction over a degree variable, both the matrix and *than*-clauses are  
 209 considered representing sets of degrees (see (7)), including all degrees some entity  
 210 meets or exceeds (i.e., totally ordered sets ranging from 0 to the measurement of  
 211 something).

- 212 (7) The bathtub is wider than the door is tall.  
 213 LF:  $[-\text{er } [\lambda d. \text{the door is } d\text{-tall} ] ] [\lambda d'. \text{the bathtub is } d'\text{-wide} ]$   
 214 a. **than-clause:**  $\lambda d. \text{the door is } d\text{-tall} = \{d \mid 0 \leq d \leq \text{HEIGHT}(\text{the-door})\}$   
 215 b. **matrix clause:**  $\lambda d'. \text{the bathtub is } d'\text{-wide} = \{d' \mid 0 \leq d' \leq \text{WIDTH}(\text{the-bathtub})\}$

216 Comparative morpheme *-er/more* works like a quantificational determiner (e.g.,  
 217 *every*) of type  $\langle\langle et \rangle, \langle et, t \rangle\rangle$  and relates two sets of degrees. Different implementations  
 218 have been proposed. **The ‘A-not-A’ analysis** in (8) assumes a silent negation operator  
 219 for the *than*-clause (see Ross 1969, Lewis 1970, Seuren 1973, 1984, McConnell-Ginet 1973,  
 220 Kamp 1975, Klein 1980 for this idea, see Schwarzschild 2008 for a summary, and see  
 221 Alrenga and Kennedy 2014 for a recent development). Heim (2006b) proposes a less  
 222 widely used variation (see (9)). **The ‘>’ analysis** in (10) assumes the use of maximality  
 223 operators for both the matrix and *than*-clauses (see, e.g., Cresswell 1976, von Stechow  
 224 1984, Heim 1985, Rullmann 1995, and see Beck 2011 for a summary). Under this ‘>’  
 225 analysis, *-er/more* actually amounts to relating two definite descriptions of degrees (see  
 226 Russell 1905). With the analysis in (7) for the matrix and *than*-clauses, these  
 227 implementations all result in the same truth condition for this kind of simplest case of  
 228 comparatives.

- 229 (8)  $[[-\text{er/more}]]_{\langle\langle dt \rangle, \langle dt, t \rangle\rangle} \stackrel{\text{def}}{=} \lambda D_1. \lambda D_2. \exists d [d \in D_2 \wedge \neg [d \in D_1]]$  **the ‘A-not-A’ analysis**  
 230 (9)  $[[-\text{er/more}]]_{\langle\langle dt \rangle, \langle dt, t \rangle\rangle} \stackrel{\text{def}}{=} \lambda D_1. \lambda D_2. D_2 \supset D_1$  **the ‘ $\supset$ ’ analysis**  
 231 (10)  $[[-\text{er/more}]]_{\langle\langle dt \rangle, \langle dt, t \rangle\rangle} \stackrel{\text{def}}{=} \lambda D_1. \lambda D_2. \text{MAX}(D_2) > \text{MAX}(D_1)$  **the ‘>’ analysis**  
 232  $(\text{MAX} \stackrel{\text{def}}{=} \lambda D. \iota d [d \in D \wedge \forall d' [d' \in D \rightarrow d' \leq d]])$

233 Within the literature, there is ample discussion on the distinction between **clausal**  
 234 **comparatives** and **phrasal comparatives**. Our proposal focuses on the semantics of  
 235 clausal comparatives. However, we will also address the contrast between clausal and

phrasal comparatives with regard to the scope island issue in Section 5.1.

## 2.2 Precursors to our proposal

There are two lines of precursors to our proposed analysis. Schwarzschild and Wilkinson (2002) adopt an **interval**-based (cf. degree-based) semantics of comparatives (see also Landman 2010). This interval-based approach has later been developed by Beck (2010).<sup>8</sup> On the other hand, Brasoveanu (2008), Greenberg (2010), and Thomas (2010) invite us to reconsider the semantic contribution of comparative morpheme *-er/more*.

### 2.2.1 The move from degrees to intervals

In Section 1.1, we have shown that comparison along a scale conceptually means the measurement of distances between positions. The canonical analysis uses **degrees** – **points** – to represent positions on a scale (see Section 2.1). This analysis becomes problematic when the *than*-clause of a comparative contains a universal quantifier. For example, in (11), the canonical analysis amounts to comparing the height of Mary with that of the shortest boy, contradicting our intuitive interpretation of the sentence.

- (11) Mary is taller than every boy is. **the canonical analysis**
- a. **than-clause:**  $\lambda d$ . every boy is  $d$ -tall.  $= \{d \mid 0 \leq d \leq \text{HEIGHT}(\text{shortest-boy})\}$
- b. **matrix clause:**  $\lambda d'$ . Mary is  $d'$ -tall.  $= \{d' \mid 0 \leq d' \leq \text{HEIGHT}(\text{Mary})\}$

Schwarzschild and Wilkinson (2002) argue that if the price of the shirts ranges from \$20 to \$100 and the dress costs \$150, the dress is surely more expensive than the shirts are, but there is no single point on the scale of price that stands for the price of the shirts. Thus, they propose to use **intervals**, construed as **potentially non-convex, mass-like, homogeneous objects**, to characterize positions on a scale. Consequently, (i) adjectives

---

<sup>8</sup>In addition to this notion of **interval**, Schwarzschild also develops another related notion, **segment** (see Schwarzschild 2013). A segment is construed as a directed **vector**: it has a start and an end, encoding two scalar values. The semantics of a comparative is characterized as the existence of a segment such that its end (i.e., the value associated with the matrix subject) is larger than its start (i.e., the comparison standard).

The notion of interval that we will adopt in this paper is based on Schwarzschild and Wilkinson (2002) and Beck (2010). Thus we consider an interval a non-directed, potentially not-very-precise scalar value that represents one whole position on a scale (see Section 3.1). Of course, the subtraction technique that we will use in analyzing comparatives is essentially directed: the minuend minus the subtrahend.

A detailed comparison between (i) the use of a directed vector vs. (ii) ‘non-directed scalar values + a directed subtraction operation’ is beyond the scope of this paper and has to be left for future research. We thank an anonymous reviewer for referencing us to Schwarzschild (2013) and pointing out the relatedness.

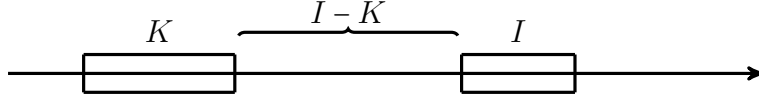


Figure 2: Intervals of [Schwarzchild and Wilkinson \(2002\)](#):  $I$  and  $K$  are two intervals representing positions under comparison (here  $K$  is below  $I$ , i.e.,  $K < I$ ).  $\text{DIFF}$ , the size of the interval  $[I - K]$  (i.e., the interval that is below  $I$  and above  $K$ ), represents the differential between the positions  $I$  and  $K$ . In a comparative, the default value of  $\text{DIFF}$  is  $\text{SOME}$ .

relate an individual and an interval (see (12)), and this relation satisfies the **Persistence Principle** (see (13), ‘ $\sqsubset$ ’ means a proper part-of relation); (ii) the matrix and *than*-clauses are considered predicates of intervals, instead of predicates of degrees (see (14) vs. (11)).

$$(12) \quad [[\text{tall}]] \stackrel{\text{def}}{=} \lambda I. \lambda x. \text{HEIGHT}(x, I) \quad (\text{i.e., the height interval } I \text{ covers the individual } x.)$$

$$(13) \quad P(x, I) \rightarrow \forall I' [I \sqsubset I' \rightarrow P(x, I')] \quad \text{Persistence Principle}$$

$$(14) \quad \text{Mary is taller than every boy is.} \quad \text{Schwarzchild and Wilkinson (2002)}$$

$$\text{a. } \textit{than-clause: } \lambda K. \forall x [\text{boy}(x) \rightarrow \text{HEIGHT}(x, K)]$$

$$\text{b. } \textit{matrix clause: } \lambda I. \text{HEIGHT}(\text{Mary}, I)$$

**Maximality operator**  $\mu$  picks out the largest interval that a predicate of intervals holds for (see (15), ‘ $\sqsubseteq$ ’ means a part-of relation): e.g., the maximal interval that covers a given group of individuals homogeneously.  $\mu$  is not a mereological sum operator.

$$(15) \quad \mu K [\phi(K)] \text{ picks out the largest interval all of whose non-empty parts are } \phi:$$

$$\mu K [\phi(K)] = K \text{ iff } \forall K' [K' \neq 0 \wedge K' \sqsubseteq K] \rightarrow \phi(K') \wedge$$

$$\forall K'' [K \sqsubset K'' \rightarrow \neg \exists K' [K' \sqsubset K'' \wedge \neg \phi(K')]]]$$

As shown in Fig. 2, a **subtraction operation** ‘ $-$ ’ is used to implement comparison. For intervals  $I$  and  $K$  (suppose  $K$  is below  $I$ ),  $[I - K]$  picks out the interval that is below  $I$  and above  $K$ .  $\text{DIFF}$ , a predicate of intervals applicable to  $[I - K]$ , addresses the size of  $[I - K]$ . The value of  $\text{DIFF}$  can be  $\text{SOME}$ , a default one, or a numerical differential.

The derived sentential semantics of a comparative is shown in (16): Mary is covered by the maximal interval  $I$  such that  $I$  is (between 2 and 4 inches) away from the maximal interval  $K$  that covers every boy. In this formula, the meaning of comparison, i.e., the part ‘ $\text{DIFF}(I - K)$ ’, is embedded within the semantics of the *than*-clause.

$$(16) \quad \text{MATRIX-CLAUSE}(\mu I [\text{THAN-CLAUSE}(\mu K [\text{DIFF}(I - K)])]) \quad \text{sentential semantics}$$

281  $[[\text{Mary is (between 2 and 4 inches) taller than every boy is}]]$   
 282  $\Leftrightarrow \text{HEIGHT}(\text{Mary}, \mu I[\forall x[\text{boy}(x) \rightarrow$   
 283  $\text{HEIGHT}(x, \mu K[\text{SOME/between-2''-and-4''}(I - K))]])]$

284 In (16), the two applications of the operator  $\mu$  guarantee that the differential  
 285 predicate (here *SOME* or *between-2''-and-4''*) holds for each gap between any subpart of  
 286 the main-clause-associated interval  $I$  and any subpart of the *than*-clause-associated  
 287 interval  $K$ . Therefore, the interval-based analysis of Schwarzschild and Wilkinson (2002)  
 288 successfully handles the semantic contribution of differentials (see also Fleisher 2016 for  
 289 a detailed discussion). For comparatives with *than*-clause-internal quantifiers like *every*  
 290 *boy*, correct truth conditions are derived, while the semantics of their *than*-clause is not  
 291 reduced to a single point, which is consistent with our intuition.

292 The analysis of Schwarzschild and Wilkinson (2002) is an important advancement in  
 293 the semantic research on comparatives. In particular, their use of intervals introduces a  
 294 more generalized notion of scalar values: a scalar value is not necessarily as precise as a  
 295 single-point position on a scale. However, by embedding the part ' $\text{DIFF}(I - K)$ ' within the  
 296 scope of two maximality operators  $\mu$ , this analysis actually turns the comparison  
 297 between two intervals into a series of comparisons performed on pairs of sub-intervals.  
 298 This embedding has a conceptual consequence: the standard of comparison (i.e., the  
 299 meaning of the *than*-clause in the canonical analysis, see Section 2.1) is no longer a scalar  
 300 value independent of comparison. Rather, the standard of comparison is eventually  
 301 yielded as the largest interval that makes the differential predicate hold for all the gaps  
 302 involved in the numerous sub-interval-level comparisons. Can we simply adopt the  
 303 notion of intervals but keep the classical view of first deriving the independent value of  
 304 standard before conducting comparison? This is a direction worth further exploration.<sup>9</sup>

305 Beck (2010) develops another interval-based analysis with a different ontology of  
 306 intervals and a tighter connection to the traditions of degree semantics.

307 Beck (2010) considers intervals **sets of degrees**. A gradable adjective relates an  
 308 individual and an interval (see (17)). With the abstraction over an interval variable, the  
 309 matrix and *than*-clauses are first analyzed as sets of intervals (of type  $\langle dt, t \rangle$ ). Then an  
 310 informativeness-based maximality operator  $M_{\text{inf}}$  picks out the most informative interval  
 311 from a set of intervals (see (18)). Thus, the semantics of the *than*-clause in (18a) amounts  
 312 to an interval ranging from the height of the shortest boy(s) to that of the tallest one(s),

<sup>9</sup>There are exceptions that require a delayed evaluation for the semantics of a *than*-clause: e.g., *Mary is taller than exactly two boys are* (see Zhang 2020c for details).

313 while the semantics of the matrix means a singleton set, only containing  $\text{HEIGHT}(\text{Mary})$ .

314 (17)  $\llbracket \text{tall} \rrbracket_{\langle dt, et \rangle} \stackrel{\text{def}}{=} \lambda D_{\langle dt \rangle} . \lambda x_e . \text{HEIGHT}(x) \in D$  (i.e., the height of  $x$  is a point in interval  
315  $D$ .)

316 (18) Mary is taller than every boy is.

317  $M_{\text{inf}}(\langle dt, t \rangle, dt) \stackrel{\text{def}}{=} \lambda p_{\langle dt, t \rangle} . \iota D [p(D) \wedge \neg \exists D' [p(D') \wedge D' \subset D]]$  (Beck 2010: p. 28, (82))

318 a. **than-clause:**  $M_{\text{inf}}(\lambda D . \forall x [\text{boy}(x) \rightarrow \text{HEIGHT}(x) \in D])$

319 b. **matrix clause:**  $M_{\text{inf}}(\lambda D . \text{HEIGHT}(\text{Mary}) \in D)$

320 A  $\text{MAX}$  operator picks out the largest degree of an interval (see (19)), and  
321  $\llbracket \text{-er/more} \rrbracket$  relates two degrees and implements comparison (see (20)). The derived  
322 truth condition is that the height of Mary exceeds that of the tallest boy(s) (see (21)).

323 (19)  $\text{MAX} \stackrel{\text{def}}{=} \lambda D . \iota d [d \in D \wedge \forall d' [d' \in D \rightarrow d' \leq d]]$  Beck (2010)

324 (20)  $\llbracket \text{-er/more} \rrbracket_{\langle d, dt \rangle} \stackrel{\text{def}}{=} \lambda d . \lambda d' . d' > d$  Beck (2010)

325 (21)  $\llbracket \text{Mary is taller than every boy is} \rrbracket \Leftrightarrow$

326  $\text{MAX}(M_{\text{inf}}(\lambda D . \text{HEIGHT}(\text{Mary}) \in D)) > \text{MAX}(M_{\text{inf}}(\lambda D . \forall x [\text{boy}(x) \rightarrow \text{HEIGHT}(x) \in D]))$

327 However, downward-entailing numerical differentials like *up to 3 inches* challenges  
328 this analysis. The sentence in (22) is intuitively false, because under the given context,  
329 the height of Mary exceeds that of the shortest boy by more than 3 inches. However, the  
330 derived semantics predicts the sentence to be true and thus is too weak. Beck (2010)  
331 suggests some *ad hoc* mechanism that construes the interval associated with the  
332 *than*-clause as a size-less item: e.g., for (22), all boys are considered of the same height.  
333 Fleisher (2016) points out that this might only work for comparatives containing a  
334 differential like *exactly 3 inches*, but cannot be extended to account for cases like (22). For  
335 (22), the height information of the shortest boy needs to be taken into consideration.

336 (22) Context: Mary is 6'1" tall, and the height of boys varies between 5'5" and 6'.

337  $\llbracket \text{Mary is up to 3 inches taller than every boy is} \rrbracket$

338  $\Leftrightarrow \exists d [\llbracket \text{HEIGHT}(\text{Mary}) \geq \text{HEIGHT}(\text{tallest-boy(s)}) + d \rrbracket \wedge [0 < d \leq 3'']]$

## 2.2.2 The additivity of *-er/more*

English morpheme *-er/more* is not exclusively used in comparatives. It also appears in additive constructions (see [Greenberg 2010](#), [Thomas 2010](#)) and comparative correlatives.

**Additive constructions** are distinct from comparatives. The most natural interpretation of (23a) is that the amount of chocolate Mary ate after feeling full is above zero. In other words, the amount she ate at a later time does not necessarily exceed the amount she ate previously. Rather the amount Mary ate later is an **increase** on the **base** of the amount she ate before. It can be a large or small increase. This additive reading of *-er/more* becomes more evident when weak NPI *any* is used along with *more* (see (23b)).

- (23) a. Mary ate chocolate until she felt full. Then she ate **more**. **Additive**  
b. Mary refused to eat any **more**. **Additive**

The **comparative correlative** in (24) means that the **increase** of my knowledge about my dog (from one time to another) correlates with the **increase** of my fondness for her (between these two times). This sentence does not tell to what extent I know about my dog or how much I like her at these times. What the sentence conveys is the correlation between two **increases**, i.e., two positive **differentials** (see [Brasoveanu 2008](#)).

- (24) The **more** I know about my dog, the **better** I like her. **Comparative correlative**

Based on Romanian data, [Brasoveanu \(2008\)](#) analyzes the phenomenon of comparative correlatives as an anaphora to differentials. For additive constructions, [Greenberg \(2010\)](#) analyzes *more* as an additive measure function (see also [Thomas 2010](#)). Given that addition and subtraction are inverse operations, increases are conceptually the same as (positive) differentials. Thus, taken together, these studies indicate a common semantic contribution of *-er/more* in distinct linguistic constructions, namely **additivity**.<sup>10</sup>

[Kennedy and McNally \(2005\)](#) and [Kennedy and Levin \(2008\)](#) also suggest that the semantics of *-er/more* in comparatives can be developed along the notion of differentials.

An additivity/differential-based view is promising for a unified account for various uses of *-er/more*: *-er/more* denotes (i) the increase from a part to a whole in additive constructions and (ii) the difference between a lower and a higher scalar value in comparatives. However, a fully worked-out analysis along this additivity-based view of *-er/more* is still missing in the existing literature on comparatives.

<sup>10</sup>The additivity of *-er/more* is also reflected in the meaning of additive connectives like *moreover*: *War brings depression. Moreover, it brings chaos*. The use of *moreover* means that chaos is added on top of depression.

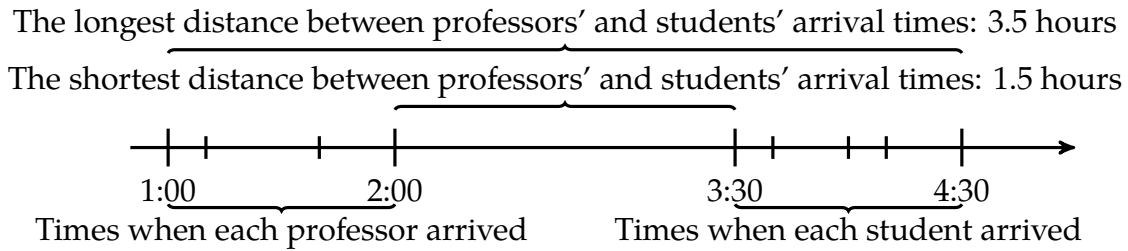


Figure 3: To what extent did the professors arrive earlier than the students did? The distance between the two positions representing the professors' and the students' arrival times can be as short as 1.5 hours and as long as 3.5 hours.

## 2.3 The core innovations

In this paper, we further develop (i) the idea of using intervals to mark positions on a scale and operating on them and (ii) an additivity-based view for *-er/more*. The proposed difference-based analysis of comparatives results from a marriage of these two.

### 2.3.1 Interval subtraction

Schwarzchild and Wilkinson (2002) and Beck (2010) propose to use **intervals**, instead of **degrees**, to mark positions along a scale. Our discussion on these works suggests that ideally, (i) the interval representing the standard in a comparison is derived independently from the comparison, and (ii) intervals are not reduced to single degrees for conducting comparison (cf. (22)). In particular, when a comparative contains downward-entailing numerical differentials like *up to 3 inches*, the lower bound of the interval associated with the *than*-clause matters in derivation.

If the lower and upper bound information of an interval needs to be visible and an interval cannot be reduced to an item directly applicable for inequalities (e.g.,  $>$ ,  $\leq$ ), how to perform comparison? **Interval subtraction** is the answer we need.

As illustrated in Fig. 3, suppose a group of professors and students arrived individually. The professors arrived between 1 o'clock and 2 o'clock, while the students between 3:30 and 4:30. The arrival times of professors and students can be considered two **intervals** – two **convex sets** of time points – that mark two (ranges of) **positions** on a scale of time. The **distance** (or difference) between these two not-very-precise positions can be as short as 1.5 hours and as long as 3.5 hours. Thus, just like we can use *between 6 feet and 6 feet 2 inches* – a range of degrees – to address the height of a certain person,



between 1.5 and 3 hours – a range of time differences – provides the information to address to what extent the professors arrived earlier than the students did.<sup>11</sup> As sketched out in (25), the subtraction between two intervals (that mark positions on a scale of time) results in a **third interval** representing the time difference between the two positions.

(25) Given the context in Fig. 3, to what extent did the professors arrive earlier?  
 The professors arrived *K*-**earlier** than the students did.  

$$\underbrace{\text{the interval between 3:30 and 4:30}}_{\text{times when each student arrived}} - \underbrace{\text{the interval between 1:00 and 2:00}}_{\text{times when each professor arrived}} =$$
  

$$\underbrace{K: \text{the interval between 1.5 and 3.5 hours}}_{\text{to what extent did the professors arrive earlier}}$$

The computation of this third interval – *K* in (25) – relies on the information of both the upper and lower bounds of the two intervals representing positions. The lower bound of *K* is the difference between the last professor's and the first student's arrival times (i.e., 2:00 and 3:30). The upper bound of *K* is the difference between the first professor's and the last student's arrival times (i.e., 1:00 and 4:30). Thus, the interval that represents a position that serves as the comparison standard is independent from the conduction of comparison, and intervals are not compressed into points for conducting comparison. The endpoint information of intervals is made use of and gets projected.

As we will show with details later, interval subtraction provides a generalized implementation for comparing two scalar values, and comparatives involving all kinds of numerical differentials are analyzed in a natural and principled way.

### 2.3.2 Comparative morpheme *-er/more* as an additive particle

Based on the idea that the core semantic contribution of *-er/more* is **additivity**, we analyze *-er/more* as an **additive particle** similar to words like *other* or *another*. In the domain of intervals  $D_{\langle dt \rangle}$ , *-er/more* **asserts an increase** (of type  $\langle dt \rangle$ ) **on a contextually**

<sup>11</sup>In common usage, we often naturally use expressions that indicate a rough estimate to answer degree questions. Here are some further examples:

- (i) a. Q: How tall is Mary?  
 A: Roughly around 5'5".
- b. Q: How long was the game?  
 A: Not very long, between 1.5 hours and 2 hours.
- c. Q: How early did the professors arrive?  
 A: About 2 or 3 hours earlier than the students did.

**salient scalar value** (of interval type  $\langle dt \rangle$ ), just like in the domain of entities  $D_e$ , *(a)other* asserts the existence of some entity (of type  $e$ ) in addition to a contextually salient one (of type  $e$ ). (26) shows the parallelism between the domains of entities and intervals.<sup>12</sup>

(26) The parallelism between the domains of entities and intervals

Domain	Indefinites	Definites	Additive words	Additivity+Restriction
$D_e$	<i>someone</i>	<i>Mary</i>	<i>(a)other</i>	<i>another girl, Mary</i>
$D_{\langle dt \rangle}$	<i>some (amount)</i>	<i>3 feet</i>	<i>-er / more</i>	<i>3 feet ... -er / more</i>

(27) shows existential assertions conveyed by the use of indefinites, which introduce a non-specified entity or scalar value.<sup>13</sup> (28) and (29) show that *another* and *-er / more* bring additivity. The contextually salient entity/value serving as the base for additivity can but does not necessarily occur in the same sentence as additive items do. In (28), the base for additivity (here *Mary* and *between 3 and 4 feet*) occurs in a previous sentence to the one hosting additive particles (here *another* and *-er*). In (29), the base for additivity (here *a girl* and the *than*-clause) and additive words occur in the same sentence. (29) also shows that additive expressions like *another girl* and *taller* can be further restricted.

(27) **Indefinites:** *someone* vs. *some (amount)*

- a. Mary saw **someone**.
- b. The height of these triangles differs from those by **some amount**.

(28) **Definites and additive items:** *Mary* vs. *between 3 and 4 feet*; *another* vs. *-er / more*

- a. **Mary** is my friend. I have **another friend**.
- b. This triangle is **between 3 and 4 feet** tall. That triangle is **taller**.

<sup>12</sup>Since *-er / more* is used in both comparatives and additive constructions, its contribution of increase should be in both domains of entities and scalar values. We focus on its role as an increase in  $D_{\langle dt \rangle}$ , but include examples of additive constructions to illustrate its domain-general contribution of additivity.

It is likely that the additivity in the domain of  $D_e$  (e.g., *what is more, war brings chaos*) does not involve the assumption of interval scales, but involves a part-whole relation. Thus it should require a distinct analysis (e.g., set difference). A full investigation of cross-domain additivity is left for future research.

<sup>13</sup>The notion of discourse salience for scalar values is also parallel to that in the domain of individuals. The introduction of a scalar value as discourse referent picks out some scalar value (from the immense set of scalar values) and grants it discourse salience.

The introduction of a scalar value as discourse referent does not necessarily hinge on an individual (i.e., introducing a scalar value as the measurement of some individual). In a sentence like *John is taller than 6 feet*, *6 feet* is introduced directly as a discourse-salient value. Actually, we consider *[[6 feet]]* parallel to a definite description, e.g., *the sun* (in *Everyone saw the sun*, see also the table in (26)).

(29) **Additivity+Restriction:** *another girl, Junko* vs. *2 feet ...-er*

- a. A girl, Hanako, saw **another girl, Junko**.
- b. This triangle is **2 feet taller** than that triangle is.

Additivity is not a typical kind of presupposition. Though *another* and *-er/more* pass the classical tests for presuppositional triggers (see (30) and (31)), the base item for additivity is not always presupposed in a discourse (see (29)). Moreover, a sentence like *Mary is taller* is not felicitous out of the blue, though its presupposition (i.e., there is a certain height) can be easily accommodated (see Kripke 2009's discussion on additive *too*).<sup>14</sup>

(30) **Tests of projection**

- a. It is possible that **another** girl came.
- b. It is possible that **more** alcohol was consumed. **Additive construction**
- c. It is possible that Sue is taller. **Comparative**

(31) **Tests of local satisfaction**

- a. Either Mary was not there, or **another** linguist gave the talk on comparatives.
- b. Either they didn't even have a beer, or **more** alcohol was consumed. **Additive construction**
- c. Either Sue is not even 5 feet tall, or she is taller. **Comparative**

Following Beaver and Clark (2009)'s theory on anaphoricity and Thomas (2011)'s analysis of *another*, we consider additivity a phenomenon of QUD-based **anaphoricity** (Question Under Discussion, see Roberts 1996, Büring 2003, Zeevat 2004, Zeevat and Jasinskaja 2007). *-er/more* is an anaphora to a QUD and requires that there is a **discourse-salient, positive, non-overlap partial answer to the Current Question**. This

<sup>14</sup>We thank an anonymous reviewer for pointing out that (29b) challenges the presuppositional view for *-er/more*. This reviewer also asks whether felicitous comparative *Mary is taller* contains an elided *than*-clause. Analogous examples involving additive particles in (i) suggest that the role of a *than*-clause is more similar to an antecedent (i.e., the underlined part) than to an ellipsis (i.e., the stricken-through part). Elided content is irrelevant to the requirement of additive particles, but the meaning of a *than*-clause can satisfy the felicity condition of *-er/more*. Therefore, we do not pursue an ellipsis analysis for *Mary is taller*.

- (i) a. (I saw a cat.) She saw **another** ~~one~~.
- b. (Kate will come.) Jane will ~~come~~, **too**.
- c. (This door is only 5 feet tall.) Mary is taller.

requirement can be satisfied by accommodation, antecedents, or *than*-expressions.

As sketched out in (32) and (33), for additive constructions, *-er/more* is associated with the difference between the complete answer to the Current Question and a discourse-salient partial answer. For comparatives, *-er/more* denotes the difference between the total value addressing the Current Question and a discourse-salient value. Without a discourse-salient value to satisfy the requirement of *-er/more*, comparatives like *Mary is taller* would sound weird out of blue.

(32) Current Question: **What** happened? **Additive constructions**

a. **Something more** happened. (Something that is salient happened.)

~ something more = 'what happened' minus 'something that is salient'

b. **Something more** happened than what they knew.

~ something more = 'what happened' minus 'what they knew'

(33) Current Question: **How tall** is Mary? **Comparatives**

a. Mary is **taller**. (There is a salient height value.)

~  $[[\text{-er}]]$  = 'how tall Mary is' minus 'the salient height value'

b. Mary is **taller** than 6 feet.

~  $[[\text{-er}]]$  = 'how tall Mary is' minus '6 feet'

The non-overlap requirement is illustrated by (34) and (35). For (34), the two joint papers by Mary and Sue provide the salient partial answer, and *more* is associated with the one single-authored book by Mary. For (35), the height value 19'10" serves as the salient base value, and *-er* is associated with the difference, i.e., 2 inches. Thus, for both additive and comparative constructions, there cannot be overlap between (i) the entity or value serving as the base and (ii) the additional part. This non-overlap requirement supports the use of subtraction equations to characterize the relation among (i) the base for an increase, (ii) the increase, and (iii) the complete answer to the Current Question.

(34) Context: Mary published a book. Mary and Sue published two papers together.  
Current Question: What did Mary publish?

a. (Mary and Sue published two papers.) Mary had **one more** publication.

b. #(Mary and Sue published two papers.) Mary had **three more** publications.

(35) Context: This tree is 19 feet 10 inches tall. My giraffe is 20 feet tall.

Current Question: How tall is my giraffe?

- 489 a. (i) My giraffe is **2 inches taller** than this tree is.  
 490 (ii) (This tree is 19 feet 10 inches tall.) My giraffe is **2 inches taller**.  
 491 b. (i) #My giraffe is **20 feet taller** than this tree is.  
 492 (ii) #(This tree is 19 feet 10 inches tall.) My giraffe is **20 inches taller**.

493 To sum up, we propose (36) as the lexical entry of *-er/more* in comparatives.  
 494 *-er/more* denotes the most general positive scalar value, i.e., the interval  $\{d \mid d > 0\}$ , and  
 495 for felicitous uses, it requires that there is a salient scalar value serving as the base for an  
 496 increase, providing discourse-salient partial information to a Current (Degree)  
 497 Question. Thus, *-er/more* serves as the default differential in comparatives. This proposal  
 498 captures the additivity (and anaphoricity) of *-er/more* within the domain of intervals.

499 (36)  $[[\text{-er/more}]]_{\langle dt \rangle} \stackrel{\text{def}}{=} \{d \mid d > 0\}$  (i.e., the most general positive interval)  
 500 Requirement: there is a salient scalar value serving as the base for an increase.

501 (36) is distinct from Schwarzchild and Wilkinson (2002)'s default differential *SOME*  
 502 in two crucial ways. In (36), *-er/more* denotes an interval along a scale of differences (see  
 503 the notion of scales of differences in Section 1.1 and the technical details in Section 3), and  
 504 the role of *-er/more* in comparatives is built on its discourse-level semantic contribution.

## 505 2.4 An informal sketch of our proposal

506 Our proposal consists of three core components:<sup>15</sup> (i) using intervals to represent all  
 507 scalar values and analyzing a gradable adjective as a relation between an interval and an  
 508 individual (see (37)); (ii) analyzing the matrix and *than*-clauses as definite descriptions  
 509 of intervals (with the abstraction over an interval variable and an informativity-based  
 510 maximality operator – Beck 2010's operator  $M_{\text{inf}}$  shown in (18), see (38)); (iii) using  
 511 interval subtraction to implement comparison between definite intervals (see (39)).

512 (37)  $[[\text{tall}]]_{\langle dt, et \rangle} \stackrel{\text{def}}{=} \lambda I_{\langle dt \rangle} . \lambda x_{e \cdot \text{HEIGHT}_{\langle e, dt \rangle}}(x) \subseteq I$  (cf. (5), (12), (17))  
 513 (I.e., the height of  $x$  is an interval that is a subset of interval  $I$ .)

514 (38) Mary is taller than every boy is. (cf. (11), (14), (18))

515 a. **than-clause:**  $M_{\text{inf}}(\lambda I . \forall x [\text{boy}(x) \rightarrow \text{HEIGHT}(x) \subseteq I])$

516 b. **matrix clause:**  $M_{\text{inf}}(\lambda I . \text{HEIGHT}(\text{Mary}) \subseteq I)$

<sup>15</sup>Again, technical details of composition are ignored here. The main point of this subsection is to show how our proposal inherits and improves on the predecessors.

(39)  $\llbracket \text{Mary is (up to 3 inches) taller than every boy is.} \rrbracket$  (cf. (16), (21)/(22))

$\Leftrightarrow \llbracket \text{MATRIX-CLAUSE} \rrbracket - \llbracket \text{THAN-CLAUSE} \rrbracket = \llbracket \text{up to 3 inches} \dots \text{-er} \rrbracket$

- a. The lower bound of  $\llbracket \text{MATRIX-CLAUSE} \rrbracket - \llbracket \text{THAN-CLAUSE} \rrbracket$ : basically the difference between Mary's height and the height of the tallest boy(s);  
The upper bound of  $\llbracket \text{MATRIX-CLAUSE} \rrbracket - \llbracket \text{THAN-CLAUSE} \rrbracket$ : basically the difference between Mary's height and the height of the shortest boy(s).

- b.  $\llbracket \text{up to 3 inches} \dots \text{-er} \rrbracket$

$$= \llbracket \text{-er} \rrbracket \cap \llbracket \text{up to 3 inches} \rrbracket = \{d \mid d > 0\} \cap \{d \mid d \leq 3''\} = \{d \mid 0 < d \leq 3''\}$$

The first two of these three components are in the same spirit as those of the canonical analysis, but with an interval-based implementation similar to the approaches adopted by Schwarzchild and Wilkinson (2002) and Beck (2010). The third component combines our two innovations: the technique of interval subtraction and an additivity-based view of *-er/more*. Below we address the details of our proposal.

### 3 The semantics of comparatives

This section first presents the technical details of interval subtraction. Then we show the formal implementation of our proposed analysis step by step for the simplest cases.<sup>16</sup>

#### 3.1 The technique of interval subtraction

##### 3.1.1 The definition and notation of intervals

**Degrees** are points on an interval scale. Thus, a **scale** is a totally ordered set of degrees (e.g., the set of real numbers  $\mathbb{R}$  is a scale). **Intervals** are **convex** subsets of a scale.

According to the definition of convex sets (see (40)), sets such as  $\{x \mid x > 0\}$ ,  $\{x \mid x \leq 4\}$ , and  $\{x \mid 4 \leq x \leq 8\}$  are all convex sets, while sets like  $\{x \mid x > 10 \vee x \leq 3\}$  are not convex.

Degrees are of type  $d$ , and thus intervals are of type  $\langle dt \rangle$ .

#### (40) The definition of a convex set:

A totally ordered set  $P$  is **convex** iff for any elements  $a$  and  $b$  in the set (suppose  $a \leq b$ ), any element  $x$  such that  $a \leq x \leq b$  is also in the set  $P$ .

<sup>16</sup>Readers who are familiar with interval arithmetic can skip Section 3.1.

Since intervals are convex sets of degrees, we can rewrite an interval with its lower and upper bounds. As shown in (41), we use square brackets '[' and ']' for **closed** lower and upper bounds and round parentheses '(' and ')' for **open** lower and upper bounds.

(41) Interval notation:

$\{x \mid I_{\min} \leq x \leq I_{\max}\} = [I_{\min}, I_{\max}]$	A left- and right-closed interval
$\{x \mid I_{\min} < x \leq I_{\max}\} = (I_{\min}, I_{\max}]$	A left-open and right-closed interval
$\{x \mid I_{\min} \leq x < I_{\max}\} = [I_{\min}, I_{\max})$	A left-closed and right-open interval
$\{x \mid I_{\min} < x < I_{\max}\} = (I_{\min}, I_{\max})$	A left- and right-open interval

A singleton set like  $\{x \mid x = 3''\}$  can be written as  $[3'', 3'']$ , the lower and upper bounds of which are equal. We write positive and negative infinity as '+∞' and '-∞'. Thus an interval like  $\{x \mid x \geq 4\}$  (i.e., a **left-bounded and right-unbounded** interval) can be written as  $[4, +\infty)$ , and an interval like  $\{x \mid x < 3\}$  (i.e., a **left-unbounded and right-bounded** interval) can be written as  $(-\infty, 3)$ .

### 3.1.2 Details of interval subtraction

An interval means a range of possible values of degrees. Applying an operation on two intervals results in a third interval that represents the largest possible range of values.<sup>17</sup> As shown in (42) and Fig. 4, the result of **subtraction**, i.e., the **difference**, is the largest range of possible differences between any two random points in two intervals.

(42) Interval subtraction: (see Moore 1979)

$$\underbrace{[y_1, y_2]}_{\text{minuend: matrix subject's measurement}} - \underbrace{[x_1, x_2]}_{\text{subtrahend: comparative standard}} = \underbrace{[y_1 - x_2, y_2 - x_1]}_{\text{difference: differential}}$$

- a. Example 1:  $[5, 8] - [1, 3] = [2, 7]$  (2 and 7 are the minimum and maximum distances between the positions  $[5, 8]$  and  $[1, 3]$  respectively.)
- b. Example 2:  $(4, +\infty) - [2, 3] = (1, +\infty)$  (This subtraction operation can be generalized to intervals with open and/or unbounded ends.)

<sup>17</sup>To help reason about the notion of intervals, (i) shows a general recipe of the operations (e.g., addition, subtraction, multiplication) between intervals. The results are defined in terms of their upper and lower bounds. The operations can be extendable to cases with unbounded and/or open endpoints.

- (i) Basic interval operations (see Moore 1979):  
 $[x_1, x_2] \langle \text{op} \rangle [y_1, y_2] = [\alpha, \beta]$   $\langle \text{op} \rangle$ : a binary operator on two intervals  
The lower bound  $\alpha = \min(x_1 \langle \text{op} \rangle y_1, x_1 \langle \text{op} \rangle y_2, x_2 \langle \text{op} \rangle y_1, x_2 \langle \text{op} \rangle y_2)$   
The upper bound  $\beta = \max(x_1 \langle \text{op} \rangle y_1, x_1 \langle \text{op} \rangle y_2, x_2 \langle \text{op} \rangle y_1, x_2 \langle \text{op} \rangle y_2)$



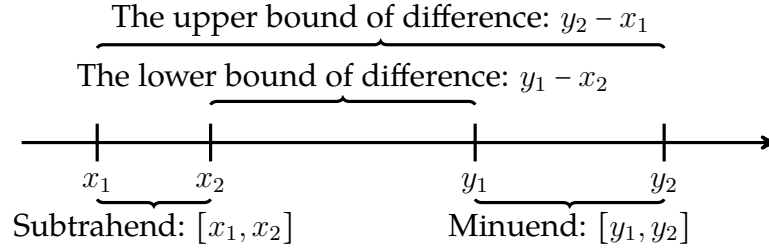


Figure 4: The subtraction between two intervals. Here  $[y_1, y_2]$  means the minuend,  $[x_1, x_2]$  the subtrahend, and the difference between these two intervals is the largest range of possible differences between any two random points in these two intervals, i.e.,  $[y_1 - x_2, y_2 - x_1]$ .

The subtraction between two intervals results in a third interval, but as mentioned before, these three intervals are not of the same kind. In (42), the **minuend** and **subtrahend** intervals (i.e.,  $[y_1, y_2]$  and  $[x_1, x_2]$ ) represent two not-very-precise **positions** on a scale (i.e., each position is in terms of a range), while the difference, i.e.,  $[y_1 - x_2, y_2 - x_1]$ , represents the **distance** between the minuend and the subtrahend. For the positions  $[y_1, y_2]$  and  $[x_1, x_2]$  on a **base measurement scale**, the distance between them, i.e.,  $[y_1 - x_2, y_2 - x_1]$ , can be considered a measurement on a **scale of differences**.<sup>18</sup>

Some numerical examples of interval subtraction are given in (43):

- (43) a.  $[5, 8] - [1, 2] = [3, 7]$   
 b.  $[5, 8] - [3, 7] = [-2, 5]$   
 c.  $[1, 2] - [5, 8] = [-7, -3]$

As shown in (43a) and (43c), when the minuend and the subtrahend are flipped, applying subtraction results in the **inverse** of the original difference (see (44) for details). Thus, the **direction in applying subtraction** is reflected by the **polarity of difference**.

- (44) Flipping the direction of subtraction: (see (42))  
 a.  $[y_1, y_2] - [x_1, x_2] = [y_1 - x_2, y_2 - x_1]$   
 b.  $[x_1, x_2] - [y_1, y_2] = [x_1 - y_2, x_2 - y_1] = [-(y_2 - x_1), -(y_1 - x_2)] = [0, 0] - [y_1 - x_2, y_2 - x_1]$

The examples (43a) and (43b) show a crucial difference between the operation of subtraction defined in **interval arithmetic** and **number arithmetic**. In number arithmetic

<sup>18</sup>The conceptual distinction between interval-as-position vs. interval-as-distance is more visible in the dimension of time: e.g.,  $[1:00, 1:30]$  and  $[4:00, 4:30]$  are intervals-as-position on a scale of time, while  $[2.5 \text{ hours}, 3.5 \text{ hours}]$  is an interval-as-distance between the above two intervals-as-positions (cf.  $[5^\circ\text{C}, 10^\circ\text{C}]$  is ambiguous between (i) an interval-as-temperature and (ii) an interval-as-temperature-difference.)

(i.e., when  $X$ ,  $Y$  and  $Z$  represent numbers), if  $X - Y = Z$ , it follows necessarily that  $X - Z = Y$  (see (45a)). However, in interval arithmetic (i.e., when  $X$ ,  $Y$  and  $Z$  represent intervals), if  $X - Y = Z$ , generally speaking, it is not the case that  $X - Z = Y$  (see (45b)).

- (45) a. Number arithmetic:  $X - Y = Z \models X - Z = Y$  (e.g.,  $5 - 2 = 3 \models 5 - 3 = 2$ )  
b. Interval arithmetic:  $X - Y = Z \not\models X - Z = Y$  (see (43a) vs. (43b))

Consequently, in interval arithmetic, given  $X - Y = Z$  and given the values of the subtrahend  $Y$  and the difference  $Z$ , to compute the value of the minuend  $X$ , we cannot perform interval addition on  $Y$  and  $Z$  (see (46)).

- (46) If  $X - [a, b] = [c, d]$ , then generally speaking,  $X \neq [a + c, b + d]$ .

Instead, we need to follow the formula (42) to derive the value of the minuend. As shown in (47), the minuend  $X$  is defined only when its lower bound does not exceed its upper bound. When the minuend is defined, as shown in (47), the **upper** bound of the **subtrahend** (here  $b$ ) contributes to the computation of the **lower** bound of the **minuend**  $X$ , while the **lower** bound of the **subtrahend** (here  $a$ ) contributes to the computation of the **upper** bound of the **minuend**  $X$ .

- (47) If  $X - [a, b] = [c, d]$ ,  
a.  $X$  is undefined when  $b + c > a + d$ ;  
(i.e., undefined when the lower bound of  $X$  exceeds the upper bound of  $X$ .)  
b. When defined,  $X = [b + c, a + d]$ . (see (42))  
The **lower** bound of the **minuend**  $X$   
= the **lower** bound of the **difference** + the **upper** bound of the **subtrahend**;  
( $b + c$  means moving from the precise position  $b$  by a distance of  $c$ .)  
the **upper** bound of the **minuend**  $X$   
= the **upper** bound of the **difference** + the **lower** bound of the **subtrahend**.  
( $a + d$  means moving from the precise position  $a$  by a distance of  $d$ .)

With the use of interval subtraction, we can now characterize a **generalized comparison between two not-very-precise positions on a scale** and precisely compute the distance (i.e., difference) between them. In particular, inequalities are represented by subtraction equations, and information with regard to the endpoints of positions and distances – including values, closedness, and boundedness – is fully taken care of with the use of this technique. Thus, interval subtraction is an ideal tool for compositionally

deriving the semantics of various kinds of comparatives, especially for those complex cases involving numeral differentials and/or *than*-clause internal quantifiers.

## 3.2 The step-by-step derivation for the simplest cases of comparatives

**Step 0: The semantics of measure function.** We use intervals – ranges of values – to represent scalar values in a generalized way. A **measure function** maps a **single entity** to an interval, which represents the position corresponding to the measurement of the entity along a relevant scale (see (48)). Measurements are always subject to uncertainty. An informative interpretation of a measure function involves vagueness.

(48) **Measure function:**  $\text{HEIGHT}_{\langle e, dt \rangle} \stackrel{\text{def}}{=} \lambda x. \text{HEIGHT}(x)$

For a given entity, what exact position range on a scale of height corresponds to its height measurement depends on contextual factors, such as measurement tools, environment, acceptable criteria of precision, etc. For example, vernier scales provide better precision in measuring along a linear scale than most rulers do. The notion of comparison class (i.e., ‘objects deemed somehow similar to the target of predication’, Kennedy 2011: Section 3.1, p. 514) is often relevant to contextually informative precision level of measurement. The precision level to 1 meter is fine-grained and informative in addressing the height of mountains, but way too coarse-grained for humans.

Suppose we use a scale to measure the height of my giraffe. Along this scale, the closest marking to the top of my giraffe is 20 feet with an error range of 1 foot. Then  $\text{HEIGHT}(\text{my-giraffe})$  is  $20' \pm 1'$ , i.e.,  $[19', 21']$ . With idealized measurement in which the error is negligible, the interval  $\text{HEIGHT}(\text{my-giraffe})$  is a singleton set of degrees, and we write its unique item (of type *d*) as  $\text{PRECISE-HEIGHT}(\text{my-giraffe})$ .

**Step 1: The analysis of gradable adjectives.** We analyze the semantics of a gradable adjective as a relation between an individual *x* and an interval *I* (see (49)), meaning that the measurement of *x* falls at the position *I* on a scale associated with the dimension of the adjective (e.g., *tall* and *short* are associated with the same dimension of height, but with scales of opposite orderings; *early* and *late* are associated with time, but with scales of opposite orderings as well). This relational view of gradable adjectives inherits the spirit of the canonical analysis (see Section 2.1, cf. Kennedy 1999).<sup>19</sup>

<sup>19</sup>Since a measure function measures a single entity (see Step 0), the individual variable of a gradable adjective should not be a plurality. For a plurality, we assume that there is a distributivity operator *DIST*:

646 (49)  $\llbracket \text{tall} \rrbracket_{\langle dt, et \rangle} \stackrel{\text{def}}{=} \lambda I_{\langle dt \rangle} . \lambda x_e . \text{HEIGHT}_{\langle e, dt \rangle}(x) \subseteq I$  (= (37))  
 647 i.e., the measurement of  $x$  falls at the position  $I$  on the scale of height.

648 The semantics of **measurement constructions** is straightforwardly derived (see  
 649 (50)). Bare numerals like *19 feet* are ambiguous between an ‘exactly’ reading and an ‘at  
 650 least reading’ (see Spector 2013 for a review on this issue). The projection of this  
 651 ambiguity leads to the two interpretations shown in (50a).<sup>20</sup> Modified numerals like  
 652 *between 19 and 20 feet* naturally denote an interval and serve as the interval argument of  
 653 *tall* (see (50b)).

- 654 (50) **Measurement constructions**
- 655 a. My giraffe is **19 feet** tall.
- 656 LF:  $\llbracket \text{my giraffe} \rrbracket$  is  $\llbracket \llbracket 19 \text{ feet} \rrbracket \text{ tall} \rrbracket$
- 657 (i) The ‘exactly’ reading of *19 feet*:
- 658  $\llbracket (50a) \rrbracket \Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq [19', 19']$
- 659 (ii) The ‘at least’ reading of *19 feet*:
- 660  $\llbracket (50a) \rrbracket \Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq [19', +\infty)$
- 661 b. My giraffe is **between 19 and 20 feet** tall.
- 662 LF:  $\llbracket \text{my giraffe} \rrbracket$  is  $\llbracket \llbracket \text{between 19 and 20 feet} \rrbracket \text{ tall} \rrbracket$
- 663  $\llbracket (50b) \rrbracket \Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq [19', 20']$

664 The **positive** use of gradable adjectives assumes a silent free interval variable  $I_{\text{POS}}^C$   
 665 (see (51)).  $I_{\text{POS}}^C$  denotes the context-dependent interval of being tall for a relevant  
 666 comparison class (see also Bartsch and Vennemann 1972a, Cresswell 1976, von Stechow  
 667 1984, Bierwisch 1989, Kennedy 1999), e.g., above 18 feet for a giraffe.<sup>21</sup>

668 (51) My giraffe is tall. **Positive use**

- 
- (i)  $\text{DIST} \stackrel{\text{def}}{=} \lambda X_e . \lambda P_{\langle et \rangle} . \forall x [x \sqsubseteq_{\text{ATOM}} X \rightarrow P(x)]$   
 i.e., for each atomic part  $x$  of the plural individual  $X$ , predicate  $P$  holds for  $x$ .  
 e.g.,  $\llbracket \text{the trees are DIST } I \text{ tall} \rrbracket = \forall x [x \sqsubseteq_{\text{ATOM}} \oplus \text{tree} \rightarrow \text{HEIGHT}(x) \subseteq I]$

<sup>20</sup>When the ‘at least’ reading is adopted for interpreting a bare numeral in a measurement construction, obviously, the analysis in (50) captures the following familiar inference pattern:

- (i) My giraffe is 19 feet tall.  $\models$  My giraffe is 18 feet tall.  
 $\text{HEIGHT}(\text{my-giraffe}) \subseteq [19', +\infty) \models \text{HEIGHT}(\text{my-giraffe}) \subseteq [18', +\infty)$

<sup>21</sup>Formal properties (especially the boundedness) of this  $I_{\text{POS}}^C$  are subject to the structure of a scale associated with a gradable predicate (see Kennedy and McNally 2005).

669 LF: [ [my giraffe] is [  $I_{\text{POS}}^C$  tall] ]  
 670  $\llbracket (51) \rrbracket \Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq I_{\text{POS}}^C$

671 **Step 2: The analysis of comparative standard.** *Than*-clauses/phrases play the role of  
 672 **standard** (i.e., subtrahend) in comparatives. We focus on the semantics of *than*-clauses.

673 The analysis of *than*-clauses involves two sub-steps: (i) lambda abstraction over an  
 674 interval variable, and (ii) the use of an informativeness-based maximality operator.

675 Following the canonical analysis (see [Bresnan 1973, 1975, Chomsky 1977](#)), we  
 676 assume that syntactically, a *than*-clause contains an elided gradable adjective – the same  
 677 as the one used in the matrix clause – and a *wh*-movement (see (52)). Semantically, this  
 678 amounts to a lambda abstraction over an interval variable, resulting in a set of intervals  
 679 such that each represents a position where the measurement of an individual falls at.

680 (52) (My giraffe is taller) than that tree is tall  
 681 LF: [than [  $\lambda I$ . that tree is  $I$  tall] ]

682 We propose that *than* contributes an informativeness-based maximality operator,  
 683 similar to the operator  $M_{\text{inf}}$  proposed by [Beck \(2010\)](#) (see (18)). As shown in (53), for a  
 684 set of intervals,  $\llbracket \text{than} \rrbracket$  is defined when there is a unique interval entailing all other  
 685 intervals in the set, and when defined,  $\llbracket \text{than} \rrbracket$  returns this unique maximally  
 686 informative interval.

687 (53)  $\llbracket \text{than} \rrbracket$  is defined for a set of intervals  $p$  such that  
 688  $\exists I[p(I) \wedge \forall I'[[p(I') \wedge I' \neq I] \rightarrow I \subset I']]$   
 689 When defined,  $\llbracket \text{than} \rrbracket_{\langle \langle dt, t \rangle, dt \rangle} \stackrel{\text{def}}{=} \lambda p_{\langle dt, t \rangle}. \iota I[p(I) \wedge \forall I'[[p(I') \wedge I' \neq I] \rightarrow I \subset I']]$

690 A *than*-clause is semantically the same as a free relative, which looks like a *wh*-clause  
 691 but functions as a nominal phrase bearing definiteness (see [Bresnan and Grimshaw 1978](#),  
 692 [Jacobson 1995, Caponigro 2003](#)).<sup>22</sup> The semantic derivation of a *than*-clause (or free  
 693 relative in general) can also be considered involving (i) the formation of a degree  
 694 question under the categorial approach to questions (see [Hausser and Zaefferer 1978](#) and  
 695 [Krifka 2011](#) for a review on question semantics), e.g., *how tall is that tree* for (52), and (ii)  
 696 the generation of its fragment answer (i.e., short answer, see [Chierchia and Caponigro](#)

<sup>22</sup>Actually all English words starting with *th* (pronounced as /ð/, not as /θ/) express definiteness: e.g., *the, they, that, then, there, these, thus, though* (which means ‘in spite of the fact that’ according to its dictionary definition, *Merriam-Webster’s Collegiate Dictionary*, 11<sup>e</sup>). It is reasonable to assume that *than* contributes definiteness as well. A thorough investigation of definiteness and these ð-words is for another occasion.

2013). Thus a *than*-clause is essentially a definite description of scalar value (see Russell 1905), providing a most informative, exhaustive answer to a degree question.<sup>23</sup>

Specifically, our analysis means that given an entity or a group of entities as the target of predication, a *than*-clause denotes the most informative interval that the measurement of each entity falls into. For simplicity, assume that measurement yields very precise values, i.e., singleton sets of degrees. Then when the target of predication is a single entity, the meaning of a *than*-clause is equivalent to a singleton set of degrees, which is simply the measurement of the given entity (see (54a)). When the target of predication is a group of entities, the derived meaning of a *than*-clause is an interval ranging from the measurement of the least ADJ entity to the most ADJ one, e.g., a height interval ranging from that of the shortest to the tallest tree in (54b).<sup>24</sup>

- (54) a.  $\llbracket \text{than that tree is tall} \rrbracket = \text{HEIGHT}(\text{that-tree})$   
 $= [\text{PRECISE-HEIGHT}(\text{that-tree}), \text{PRECISE-HEIGHT}(\text{that-tree})]$   
 b.  $\llbracket \text{than every tree is tall} \rrbracket = \iota I [\forall x [\text{tree}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]]$   
 $= [\text{PRECISE-HEIGHT}(\text{shortest-tree}), \text{PRECISE-HEIGHT}(\text{tallest-tree})]$

**Step 3: The analysis of differentials.** Comparative morpheme *-er/more* is considered the default positive differential. Thus, as shown in (55), it denotes the most general positive interval:  $(0, +\infty)$ . We will address numerical differentials in Section 4.

- (55)  $\llbracket \text{-er/more} \rrbracket_{(dt)} \stackrel{\text{def}}{=} (0, +\infty)$  i.e., the most general positive interval (= (36))  
 Requirement: there is a salient scalar value serving as the base for an increase.

<sup>23</sup>The fragment-answer view for free relatives is empirically advantageous, directly accounting for parallels between *wh*-questions and their answerhood on the one hand, and free relatives (including *than*-clauses) on the other hand. For example, just like its corresponding *wh*-question, free relative *where he can buy a coffee* has a mention-some interpretation (see Chierchia and Caponigro 2013). For comparatives containing a permission-related existential modal in their *than*-clause (e.g., *Lucinda is driving less fast than allowed*, see Beck 2013), their ambiguity is also likely rooted in the ambiguous answerhood for corresponding degree questions (e.g., *how fast is Lucinda allowed to drive*). A thorough investigation of this phenomenon is for another occasion (see also Zhang and Ling 2017a). There is also parallelism between ungrammatical degree question *\*how tall is no one?* and ungrammatical clausal comparative *\*Mary is taller than no one is*. Presumably, there is no non-trivial informative answer to address ‘no one’s height’ in either case (see Abrusán 2014).

Our view is slightly distinct from Fleisher (2018, 2020), which analyze a *than*-clause as a degree question.

<sup>24</sup>We will continue making this assumption for simplicity below. Without this assumption, (54b) would be an interval starting from the lower bound of  $\text{HEIGHT}(\text{shortest-tree})$  to the upper bound of  $\text{HEIGHT}(\text{tallest-tree})$ .

**Step 4: The semantic derivation of comparatives.** Interval subtraction is performed by a silent operator MINUS. It takes two intervals as inputs: the subtrahend,  $I_{\text{std}}$ , and the difference,  $I_{\text{diff}}$ . The output is the unique interval  $I$  representing the minuend (see (56)).

$$(56) \quad \llbracket \text{MINUS} \rrbracket_{\langle dt, \langle dt, dt \rangle \rangle} \stackrel{\text{def}}{=} \lambda I_{\text{STDD}}. \lambda I_{\text{DIFF}}. \iota I [I - I_{\text{STDD}} = I_{\text{DIFF}}]$$

Now we are ready to derive the sentential semantics of a *more-than* comparative that contains no numerical differential. As shown by the LF in (57), at the matrix level,  $\llbracket \text{tall} \rrbracket$  relates an entity,  $\llbracket \text{my giraffe} \rrbracket$ , and an interval – the minuend. The minuend is computed from a subtraction equation and known interval values for  $I_{\text{DIFF}}$  and  $I_{\text{STDD}}$ .

The interval  $I_{\text{STDD}}$  (i.e., comparative standard) represents the subtrahend in the equation and is contributed by the semantics of the *than*-clause – the height of that tree in this example (see (57a)). The interval  $I_{\text{DIFF}}$  represents the difference in the equation and is contributed by  $\llbracket \text{-er} \rrbracket$  (see (57b)). Thus based on the intervals  $I_{\text{STDD}}$  and  $I_{\text{DIFF}}$ , the minuend – the interval serving as the interval variable for  $\llbracket \text{tall} \rrbracket$  at the matrix level – can be computed (see (57c)). Finally, in (57d), with the assumption for an ideally precise measurement (i.e., the height of that tree is a singleton set of degrees) and the application of interval arithmetic (see (47)), the formula can be simplified: the lower bound of the minuend results from the addition of the upper bound of  $I_{\text{STDD}}$  and the lower bound of  $I_{\text{DIFF}}$ , while the upper bound of the minuend results from the addition of the lower bound of  $I_{\text{STDD}}$  and the upper bound of  $I_{\text{DIFF}}$ . Eventually, sentence (57) means that the height of my giraffe falls within the interval starting from the height of that tree, i.e., the height of my giraffe exceeds that of that tree.

(57) My giraffe is taller than that tree is.

$$\text{LF: } \llbracket \llbracket \text{my giraffe} \rrbracket \text{ is } \llbracket \underbrace{\llbracket \text{-er} \rrbracket \text{ MINUS } \llbracket \text{than } [\lambda I. \text{that tree is } I \text{ (tall)}] \rrbracket}_{\text{minuend: } \iota I' [I' - I_{\text{STDD}} = I_{\text{DIFF}}]} \rrbracket \text{ tall} \rrbracket \rrbracket$$

$\underbrace{\llbracket \text{-er} \rrbracket}_{\text{difference: } I_{\text{DIFF}}} \quad \underbrace{\llbracket \text{than } [\lambda I. \text{that tree is } I \text{ (tall)}] \rrbracket}_{\text{subtrahend: } I_{\text{STDD}}}$

a. **Subtrahend:**  $I_{\text{STDD}} = \llbracket \text{than} \rrbracket \llbracket \lambda I. \text{that tree is } I \text{ (tall)} \rrbracket$

$$= \iota I [\text{HEIGHT}(\text{that-tree}) \subseteq I] = \text{HEIGHT}(\text{that-tree})$$

$$= [\text{PRECISE-HEIGHT}(\text{that-tree}), \text{PRECISE-HEIGHT}(\text{that-tree})]$$

b. **Difference:**  $I_{\text{DIFF}} = \llbracket \text{-er} \rrbracket = (0, +\infty)$

c. **Minuend:**  $\iota I' [I' - I_{\text{STDD}} = I_{\text{DIFF}}]$

d.  $\llbracket (57) \rrbracket \Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq \iota I' [I' - I_{\text{STDD}} = I_{\text{DIFF}}]$

$$\Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq \iota I' [I' - \text{HEIGHT}(\text{that-tree}) = (0, +\infty)]$$



$$\begin{aligned}
&\Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq \\
&\iota I' [I' - [\text{PRECISE-HEIGHT}(\text{that-tree}), \text{PRECISE-HEIGHT}(\text{that-tree})] = (0, +\infty)] \\
&\Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq (\text{PRECISE-HEIGHT}(\text{that-tree}), +\infty) \quad (\text{see (47)})
\end{aligned}$$

As illustrated by (58), when a *than*-clause contains a universal quantifier (here *every tree*), the derivation of the sentential semantics is exactly the same as that shown in (57), except that  $I_{\text{STDD}}$  is not a singleton set of degrees in this case, but an interval ranging from the height of the shortest tree(s) to that of the tallest. Eventually, after simplification with the recipe of (47) (see the last step of (58d)), we arrive at the truth condition consistent with our intuition: the height of my giraffe exceeds that of the tallest tree(s).

(58) My giraffe is taller than every tree is.

LF:  $[[\text{my giraffe}] \text{ is } [\underbrace{[\text{-er}] \text{ MINUS } \underbrace{[\lambda I. \text{every tree is } I \text{ (tall)}]}_{\text{subtrahend: } I_{\text{std}}}}_{\text{difference: } I_{\text{diff}}}] \text{ tall} ]]$

minuend:  $\iota I' [I' - I_{\text{std}} = I_{\text{diff}}]$

a. **Subtrahend:**  $I_{\text{STDD}} = [[\text{than}]] [[\lambda I. \text{every tree is } I \text{ (tall)}]]$   
 $= \iota I [\forall x [\text{tree}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]]$   
 $= [\text{PRECISE-HEIGHT}(\text{shortest-tree}), \text{PRECISE-HEIGHT}(\text{tallest-tree})]$

b. **Difference:**  $I_{\text{DIFF}} = [[\text{-er}]] = (0, +\infty)$

c. **Minuend:**  $\iota I' [I' - I_{\text{STDD}} = I_{\text{DIFF}}]$

d.  $[(58)] \Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq \iota I' [I' - I_{\text{STDD}} = I_{\text{DIFF}}]$   
 $\Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq \iota I' [I' - \iota I [\forall x [\text{tree}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]] = (0, +\infty)]$   
 $\Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq$   
 $\iota I' [I' - [\text{PRECISE-HEIGHT}(\text{shortest-tree}), \text{PRECISE-HEIGHT}(\text{tallest-tree})] = (0, +\infty)]$   
 $\Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq (\text{PRECISE-HEIGHT}(\text{tallest-tree}), +\infty) \quad (\text{see (47)})$

## 4 Comparatives with numerical differentials

This section addresses comparatives containing numerical differentials and *than*-clause-internal universal quantifiers. We aim to show how  $I_{\text{STDD}}$  that are non-singleton sets of degrees interact with  $I_{\text{DIFF}}$ , and how the endpoint information of these intervals projects to sentential semantics.<sup>25</sup> In particular, we propose an **interval**

<sup>25</sup>For this purpose, we only choose *than*-clauses containing universal quantifiers to illustrate semantic derivation. The analysis of comparatives containing other types of quantifiers in their *than*-clause often requires extra mechanisms. A full discussion is beyond the scope of this paper. The case of non-monotonic

773 **inverse operator** *little*, using it to account for the semantics of *less-than* comparatives and  
 774 analyzing its distinctions from the familiar negation operator.

#### 775 4.1 *More-than* comparatives with numerical differentials

776 Suppose that we compare the height of my giraffe with that of a certain group of trees.  
 777 According to the context in (59),  $[[\text{than every tree is (tall)}]]$  is equivalent to  $[18', 21']$ .

778 (59) Context: the trees are between 18 and 21 feet tall.  
 779  $I_{\text{STDD}} : [[\text{than every tree is (tall)}]]$   
 780  $= [[\text{than}]] [[\lambda I. \text{every tree is } I \text{ tall}]]$   
 781  $= \iota I [\forall x [\text{tree}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]]$   
 782  $= [\text{PRECISE-HEIGHT}(\text{shortest-tree}), \text{PRECISE-HEIGHT}(\text{tallest-tree})] = [18', 21']$

783 The sentences in (60) and (61) contain **upward-entailing** (e.g., *at least 5 feet*),  
 784 **downward-entailing** (e.g., *at most 5 feet*), or **non-monotonic** numerical differentials  
 785 (e.g., *between 5 and 10 feet*), and differ with regard to the direction of inequalities (i.e.,  
 786 *more than* vs. *less than*). (62) sketches out their uniform LF under our analysis: these  
 787 sentences differ only in terms of the value of  $I_{\text{DIFF}}$ .

- 788 (60) a. My giraffe is **at least 5 feet taller** than every tree is.  
 789 b. My giraffe is **at most 5 feet taller** than every tree is.  
 790 c. My giraffe is **between 5 and 10 feet taller** than every tree is.
- 791 (61) a. My giraffe is **at least 5 feet less tall** than every tree is.  
 792 b. My giraffe is **at most 5 feet less tall** than every tree is.  
 793 c. My giraffe is **between 5 and 10 feet less tall** than every tree is.

---

quantifiers in *than*-clauses (e.g., *Balloon A is higher than exactly two of the others are*, see Schwarzschild 2008) has been analyzed in Zhang (2020c).

(62) LF for all the sentences in (60) and (61):

$$[ [\text{my giraffe}] \text{ is } \left[ \begin{array}{l} \text{at least 5 feet ...-er} \\ \text{at most 5 feet ...-er} \\ \text{between 5 and 10 feet ...-er} \\ \text{at least 5 feet less} \\ \text{at most 5 feet less} \\ \text{between 5 and 10 feet less} \end{array} \right] \text{ MINUS than } [\lambda I. \text{every tree is } I \text{ (tall)} ] ] \text{ tall } ]$$

$$[[ (60)/(61) ]] \Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq \iota I' [ I' - \iota I [ \forall x [ \text{tree}(x) \rightarrow \text{HEIGHT}(x) \subseteq I ] ] ] = I_{\text{DIFF}}$$

Under the context in (59),

$$[[ (60)/(61) ]] \Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq \iota I' [ I' - [18', 21'] ] = I_{\text{DIFF}}$$

Numerical differentials are analyzed as additional restrictions on the default positive differential  $(0, +\infty)$ , yielding a more restricted value for  $I_{\text{DIFF}}$ .<sup>26</sup>

Given the values of  $I_{\text{DIFF}}$  and  $I_{\text{STDD}}$  (which is  $[18', 21']$  under the context in (59)), we can always use the same recipe of interval subtraction (see (47)) to simplify the formula of the minuend and thus that of sentential semantics (see (63)–(65)).

$$(63) \quad \text{a. } I_{\text{DIFF}} = [[\text{at least 5 feet ...-er}]] = [5', +\infty) \cap (0, +\infty) = [5', +\infty)$$

$$\text{b. Minuend: } \iota I' [ I' - I_{\text{STDD}} ] = [[\text{at least 5 feet ...-er}]] \\ = \iota I' [ I' - [18', 21'] ] = [5', +\infty) ] = [26', +\infty)$$

$$(64) \quad \text{a. } I_{\text{DIFF}} = [[\text{at most 5 feet ...-er}]] = (-\infty, 5'] \cap (0, +\infty) = (0, 5']$$

$$\text{b. Minuend: } \iota I' [ I' - I_{\text{STDD}} ] = [[\text{at most 5 feet ...-er}]] \\ = \iota I' [ I' - [18', 21'] ] = (0, 5'] ] = (21', 23']$$

$$(65) \quad \text{a. } I_{\text{DIFF}} = [[\text{between 5 and 10 feet ...-er}]] = [5', 10'] \cap (0, +\infty) = [5', 10']$$

$$\text{b. Minuend: } \iota I' [ I' - I_{\text{STDD}} ] = [[\text{between 5 and 10 feet ...-er}]] \\ = \iota I' [ I' - [18', 21'] ] = [5', 10'] ] = [26', 28']$$

Our analysis brings interesting consequences on (i) the projection of the endpoint

<sup>26</sup>Similar ideas have been developed in the analysis of quantity words like *many*, *much*, *few*, and *little* by Rett (see Rett 2007, 2008, 2014, 2018): the core semantic contribution of these words is to modify and restrict an interval. In comparatives, *much* and *a little* can also be used to restrict the default differential  $(0, +\infty)$ , yielding expressions like *much taller*, *a little shorter*. A thorough analysis of these expressions needs to be based on a detailed investigation on quantity words and is thus beyond the scope of our paper.

813 information of  $I_{\text{DIFF}}$  and (ii) the definedness for the minuend and sentential semantics.

814 **The projection of the endpoint information of  $I_{\text{DIFF}}$ .** In (63)–(65), since  $I_{\text{STDD}}$  has both  
 815 closed and bounded lower and upper bounds, the minuend directly inherits the  
 816 closedness and boundedness of  $I_{\text{DIFF}}$ . For example, if the differential is left-closed,  
 817 left-bounded, and right-unbounded (see (63)), then so is the minuend.

818 This explains why comparatives with no numerical differential express a strict  
 819 inequality – because their differential is  $(0, +\infty)$  (i.e., with an open lower bound), while  
 820 comparatives containing numerical differentials often express non-strict inequalities –  
 821 because a restricted differential can have a closed lower bound.

822 This also naturally explains the two observations raised by Fleisher (2016). First,  
 823 *more-than* comparatives with an upward-entailing numerical differential have a  
 824 MAX-reading (see (63)), in the sense that only the upper bound of  $I_{\text{STDD}}$  seems to get  
 825 projected to sentential level. Second, in contrast, those with a downward-entailing or  
 826 non-monotonic numerical differential have a MIN-&-MAX-reading (see (64) and (65)), in  
 827 the sense that both the upper and lower bounds of  $I_{\text{STDD}}$  get projected to sentential level.  
 828 Our analysis shows that for the cases of upward-entailing numerical differentials like  
 829 (63),  $I_{\text{DIFF}}$  is right-unbounded, so that the sum of this upper bound and the lower bound  
 830 of  $I_{\text{STDD}}$  is still  $+\infty$ , giving the impression that only the upper bound of  $I_{\text{STDD}}$  is eventually  
 831 reflected in the computation of the minuend and sentential semantics.<sup>27</sup>

<sup>27</sup>An anonymous reviewer raises the issue that comparatives like (i) (or (60b)) seem to have a MIN-reading, in the sense that only the lower bound of  $I_{\text{STDD}}$  projects. For the sentence to be true, John's height cannot exceed that of the shortest girl by more than 6 inches – John can even be shorter than the girls are.

- (i) John is at most six inches taller than every girl is.

Following Krifka (1999) (see Szabolcsi 2010 (Chapter 10) for a review), we consider *at least* and *at most* focus sensitive items: their interpretation can be structurally ambiguous. For example, we assume that *at most* turns a singleton set of degrees into a left-unbounded interval (see (ii)), which basically means creating an alternative set. When associated with a larger structure (see (iiia)), *at most* modifies the derived value of  $\llbracket \text{six inches -er than every girl is} \rrbracket$ . When associated simply with a number (see (iiib)), *at most* is part of the numerical differential *at most six inches* and modifies  $\llbracket \text{six inches} \rrbracket$ , giving rise to a MIN-&-MAX-reading.

Then according to the input requirement of *at most* (see (ii)) and the definedness condition for comparatives (see (67) in this subsection), the interpretation of (iiia) suggests that the girls are of the same height. Thus the seeming MIN-reading of (iiia) is actually also a MIN-&-MAX-reading, i.e., (iiia) is true if the height of John does not exceed the girls' height by more than 6 inches – he can even be shorter than the girls.

- (ii)  $\llbracket \text{at most} \rrbracket_{\langle dt, dt \rangle} \stackrel{\text{def}}{=} \lambda I. (-\infty, \iota d [d \in I])$   
 (The input of  $\llbracket \text{at most} \rrbracket$  needs to be a singleton set of degrees.)

- (iii) a. John is  $\llbracket \text{at most} \rrbracket$  six inches taller than every girl is  $\llbracket \rrbracket$ .

**The definedness for the minuend and sentential semantics.** We define the **width** of an interval as the difference between its upper and lower bounds (see (66)).

(66) The **width** of an interval  $I$  is the difference between its upper and lower bounds.

In the semantic derivation of a comparative, the minuend needs to be well-defined: i.e., its lower bound needs to be lower than its upper bound (see (47)). Consequently, the definedness condition shown in (67) needs to be met (see (68) for a proof). This definedness condition explains our intuitive inference in understanding a comparative.

(67) **Definedness condition for the minuend:**  $I_{\text{STDD}}$  needs to be less wide than  $I_{\text{DIFF}}$ .

(68) For the minuend  $\iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]$  to be well defined,  
the **lower** bound of  $I_{\text{DIFF}}$  + the **upper** bound of  $I_{\text{STDD}}$  <  
the **upper** bound of  $I_{\text{DIFF}}$  + the **lower** bound of  $I_{\text{STDD}}$  (see (47))  
 $\therefore$  the **upper** bound of  $I_{\text{STDD}}$  – the **lower** bound of  $I_{\text{STDD}}$  <  
the **upper** bound of  $I_{\text{DIFF}}$  – the **lower** bound of  $I_{\text{DIFF}}$   
 $\therefore I_{\text{STDD}}$  needs to be less wide than  $I_{\text{DIFF}}$ .

For upward-entailing differentials (e.g., (63)),  $I_{\text{DIFF}}$  is right-unbounded, i.e.,  $+\infty$ . The definedness condition can always be met.

For downward-entailing and non-monotonic differentials (e.g., (64) and (65)),  $I_{\text{DIFF}}$  is right-bounded. Thus the definedness condition bears a consequence on inference.

Sentences (60b) and (60c) are felicitous under the context in (59), because their  $I_{\text{DIFF}}$  (i.e.,  $(0, 5']$  and  $[5', 10']$ , respectively) is wider than the relevant  $I_{\text{STDD}}$  (i.e.,  $[18', 21']$ ).

This definedness condition explains why for sentences like (69), in which  $I_{\text{DIFF}}$  is a singleton set of degrees (here  $[10', 10']$ ), our intuition is that it suggests that every tree

---

LF: John is tall [[ at most ] [ six inches -er than every girl is ]]

(To meet the requirement of [[at most]], [[six inches -er than every girl is]] is interpreted as ‘exactly six inches -er than every girl is’. See also (69).)

[[ (iiiia) ]  $\Leftrightarrow$  HEIGHT(JOHN)  $\subseteq (-\infty, \text{PRECISE-HEIGHT}(\text{tallest/shortest-girl}) + 6''$ ]

b. John is [[ at most six inches ] taller than every girl is ] (see also (64))

LF: John is tall [[[ at most six inches ] -er ] [ than every girl is ]]

[[ (iiib) ]  $\Leftrightarrow$  HEIGHT(JOHN)  $\subseteq (\text{PRECISE-HEIGHT}(\text{tallest-girl}), \text{PRECISE-HEIGHT}(\text{shortest-girl}) + 6''$ ]

However, according to the reviewer, the actual reading of the sentence (i) seems to be a mixture between (iiiia) and (iiib): i.e.,  $(-\infty, \text{PRECISE-HEIGHT}(\text{shortest-girl}) + 6'']$ . Can this be due to something like binocular rivalry effects in our language comprehension? We have no idea at this moment. We need neuropsychological experiments to investigate this issue in the future.

should be of the same height, i.e.,  $I_{\text{DIFF}}$  is also a singleton set of degrees. The technique of interval subtraction naturally captures this intuition, and there is no need to introduce other mechanisms to deal with this inference (see also Beck 2010, Alrenga and Kennedy 2014, Fleisher 2016 for more discussion).

- (69) My giraffe is exactly 10 feet taller than every tree is.  
 $\leadsto$  Inference: every tree should be of the same height.

## 4.2 Inverse operator *little* and *less-than* comparatives

The LF in (62) shows that *less-than* comparatives with numerical differentials can be analyzed in exactly the same way. Following previous studies (e.g., Rullmann 1995, Heim 2006b, Buring 2007a,b), we analyze *less* as the composition of *little* and *-er/more*.  $[[\text{little}]]$  takes a positive interval as input and returns its inverse as output (see (70)). Thus it can be considered an interval modifier, changing the polarity of a positive interval.

$$(70) \quad [[\text{little}]]_{\langle dt, dt \rangle} \stackrel{\text{def}}{=} \lambda I \subseteq (0, +\infty). [[0, 0] - I] \quad (\text{see (44)})$$

When  $[[\text{little}]]$  takes  $[-\text{er/more}]$  as input, the output is the most general negative differential, i.e.,  $(-\infty, 0)$ . Similar to  $[-\text{er/more}]$ ,  $[[\text{less}]]$  also brings a felicity requirement: there is a salient scalar value serving as the base for a decrease (or a negative increase).

$$(71) \quad [[\text{less}]]_{\langle dt \rangle} \stackrel{\text{def}}{=} [[\text{little}]] [[-\text{er/more}]] = (-\infty, 0) \quad (\text{i.e., the most general negative interval})$$

Requirement: there is a salient scalar value serving as the base for a decrease.

The semantic derivation of a *less-than* comparative is parallel to that of a *more-than* comparative. (72) shows the step-by-step derivation (see also (58)).

$$(72) \quad \text{My giraffe is less tall than every tree is.}$$

$$\text{LF: } [ [ \text{my giraffe} ] \text{ is } [ [ \underbrace{\text{less}}_{\text{difference: } I_{\text{DIFF}}} \text{ MINUS } \underbrace{\text{than } [\lambda I. \text{every tree is } I \text{ (tall)}]}_{\text{subtrahend: } I_{\text{std}}} ] \text{ tall} ] ] ]$$

$$\underbrace{\hspace{15em}}_{\text{minuend: } \iota I' [I' - I_{\text{std}} = I_{\text{diff}}]}$$

a. **Subtrahend:**  $I_{\text{STDD}} = [[\text{than}]] [[\lambda I. \text{every tree is } I \text{ (tall)}]]$   
 $= \iota I [ \forall x [ \text{tree}(x) \rightarrow \text{HEIGHT}(x) \subseteq I ] ]$   
 $= [\text{PRECISE-HEIGHT}(\text{shortest-tree}), \text{PRECISE-HEIGHT}(\text{tallest-tree})]$

- 881 b. **Difference:**  $I_{\text{DIFF}} = \llbracket \text{less} \rrbracket = (-\infty, 0)$   
 882 c. **Minuend:**  $\iota I' [I' - I_{\text{STDD}} = I_{\text{DIFF}}]$   
 883 d.  $\llbracket (58) \rrbracket \Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq \iota I' [I' - I_{\text{STDD}} = I_{\text{DIFF}}]$   
 884  $\Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq \iota I' [I' - \iota I [\forall x [\text{tree}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]] = (-\infty, 0)]$   
 885  $\Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq$   
 886  $\iota I' [I' - [\text{PRECISE-HEIGHT}(\text{shortest-tree}), \text{PRECISE-HEIGHT}(\text{tallest-tree})] = (-\infty, 0)]$   
 887  $\Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq (-\infty, \text{PRECISE-HEIGHT}(\text{shortest-tree}))$  (see (47))  
 888 i.e., my giraffe's height falls within the interval between negative infinity  
 889 and the height of the shortest tree.<sup>28</sup>

890 The only difference between the *more-than* comparative in (58) and the *less-than*  
 891 comparative in (72) consists in the polarity of  $I_{\text{DIFF}}$ . By changing the polarity of the  $I_{\text{DIFF}}$ ,  
 892  $\llbracket \text{less} \rrbracket$  (or rather  $\llbracket \text{little} \rrbracket$ ) changes the direction of an inequality. Thus, a *more-than*  
 893 comparatives expresses a ' $>/\geq$ ' relation, while a *less-than* comparative a ' $</\leq$ ' relation.

894 Similarly, as shown in (73)–(75), we use the same recipe of interval subtraction (see  
 895 (47)) to compute the semantics of *less-than* comparatives containing upward-entailing,  
 896 downward-entailing, or non-monotonic numerical differentials. In these *less-than*  
 897 comparatives, we assume that a numerical differential first combines with *more* and  
 898 restricts this positive interval, and then *little* operates on this restricted positive interval  
 899 and returns its inverse. The projection pattern of the endpoint information of  $I_{\text{DIFF}}$  as well  
 900 as the definedness condition for the minuend (see (67)) apply to *less-than* comparatives  
 901 just like they apply to *more-than* comparatives.

- 902 (73) a.  $I_{\text{DIFF}} = \llbracket \text{at least 5 feet less} \rrbracket = \llbracket \text{little} \rrbracket \llbracket \text{at least 5 feet ...-er} \rrbracket$   
 903  $= \llbracket \text{little} \rrbracket [5', +\infty) = (-\infty, -5']$   
 904 b. **Minuend:**  $\iota I' [I' - I_{\text{STDD}} = \llbracket \text{at least 5 feet less} \rrbracket]$   
 905  $= \iota I' [I' - [18', 21']] = (-\infty, -5'] = (-\infty, 13']$   
 906 (74) a.  $I_{\text{DIFF}} = \llbracket \text{at most 5 feet less} \rrbracket = \llbracket \text{little} \rrbracket \llbracket \text{at most 5 feet ...-er} \rrbracket$   
 907  $= \llbracket \text{little} \rrbracket (0, 5'] = [-5', 0)$   
 908 b. **Minuend:**  $\iota I' [I' - I_{\text{STDD}} = \llbracket \text{at most 5 feet less} \rrbracket]$   
 909  $= \iota I' [I' - [18', 21']] = [-5', 0) = [16', 18')$

<sup>28</sup>The non-existence of negative heights should be considered a world knowledge fact. A negative height is physically impossible in our actual world, but not linguistically or logically nonsensical. We can easily imagine some possible worlds with negative heights in fantasy stories. For some scales like temperature, negative scalar values are both linguistically and physically possible.



- 910 (75) a.  $I_{\text{DIFF}} = \llbracket \text{between 5 and 10 feet less} \rrbracket = \llbracket \text{little} \rrbracket \llbracket \text{between 5 and feet ...-er} \rrbracket$   
 911  $= \llbracket \text{little} \rrbracket [5', 10'] = [-10', -5']$   
 912 b. **Minuend:**  $\iota I' [I' - I_{\text{STDD}} = \llbracket \text{between 5 and 10 feet less} \rrbracket]$   
 913  $= \iota I' [I' - [18', 21']] = [-10', -5'] = [11', 13']$

### 914 4.3 Inverse operator vs. negation operator

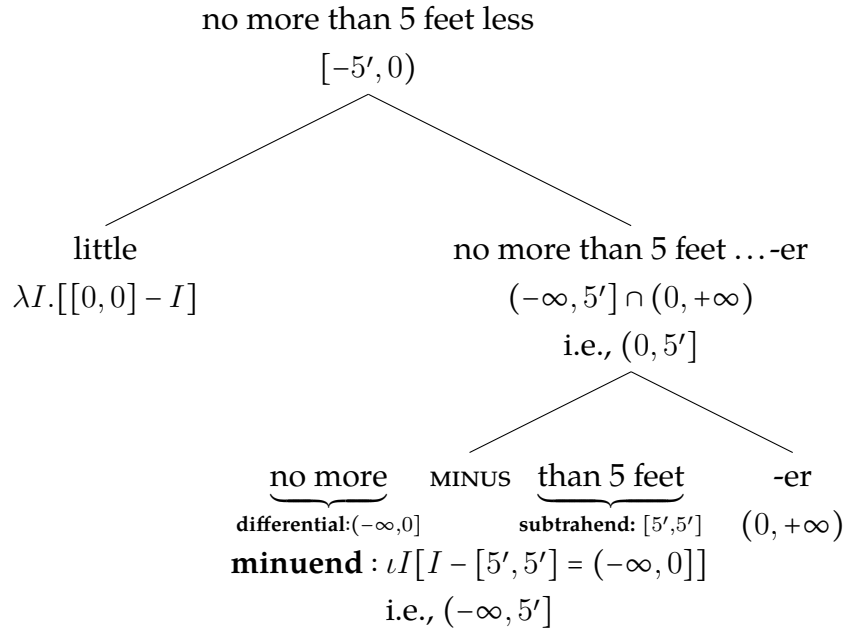
915 An interval is a convex set of degrees. Naturally, negation operator  $\llbracket \text{no} \rrbracket$  can compose  
 916 with and modify an interval.  $\llbracket \text{little} \rrbracket$  and  $\llbracket \text{no} \rrbracket$  are two distinct operators on intervals.  
 917  $\llbracket \text{little} \rrbracket$  turns an interval into its inverse, while  $\llbracket \text{no} \rrbracket$  negates an interval (i.e., it returns  
 918 the complement of an interval). Therefore, *no more* and *no less* are different from *less* and  
 919 *more*: the upper bound of *no more* and the lower bound of *no less* are closed, while the  
 920 upper bound of *less* and the lower bound of *more* are open (see (76)).<sup>29</sup>

- 921 (76) a.  $\llbracket \text{more} \rrbracket = (0, +\infty)$   
 922 b.  $\llbracket \text{no more} \rrbracket = U \setminus (0, +\infty) = (-\infty, 0]$   $U = (-\infty, +\infty)$   
 923 c.  $\llbracket \text{less} \rrbracket = \llbracket \text{little} \rrbracket \llbracket \text{-er/more} \rrbracket = [0, 0] - (0, +\infty) = (-\infty, 0)$   
 924 d.  $\llbracket \text{no less} \rrbracket = U \setminus (-\infty, 0) = [0, +\infty)$

925 Based on our analysis of *no more*, (77) illustrates how to derive the meaning of a  
 926 complex numerical differential: *no more than 5 feet less*. With the use of interval  
 927 subtraction in analyzing comparatives and the proposed lexical entries for interval  
 928 modifiers *little* and *no*, complex numerical differentials receive a uniform and principled  
 929 treatment that naturally and precisely capture our intuitive interpretation for them.

<sup>29</sup> $\llbracket \text{little} \rrbracket$  can compose with all positive differentials (e.g., *at most 5 inches ...-er*), but, intriguingly, *no* only composes with the default positive and negative intervals  $\llbracket \text{more} \rrbracket$  and  $\llbracket \text{less} \rrbracket$ . When taking a convex interval as its input,  $\llbracket \text{little} \rrbracket$  returns its inverse as output – another convex interval. However,  $\llbracket \text{no} \rrbracket$  potentially returns a set of degrees that is not a convex interval (e.g., the complement of  $[0, 5']$  is  $\{x \mid x < 0 \vee x > 5'\}$ ). Presumably, this explains the limited use of negation operator *no* in modifying intervals.

930 (77)



#### 931 4.4 A remark on bare numerals as differentials

932 For comparatives containing a bare numeral differential with an ‘at least’ reading, they  
 933 demonstrate the inference patterns shown in (78) and (80). Given that the minuend  
 934 directly inherits the endpoint information of  $I_{\text{DIFF}}$ , these inference patterns naturally  
 935 follow the interpretation pattern of  $I_{\text{DIFF}}$ : given  $x > y$ , for a *more-than* comparative,  $[x, +\infty)$   
 936 entails  $[y, +\infty)$  (see (79)); for a *less-than* comparative,  $(-\infty, -x]$  entails  $(-\infty, -y]$  (see  
 937 (81)). Overall, parallel inference patterns are observed for *more-than* and *less-than*  
 938 comparatives.

- 939 (78) a. I am 3 cm taller than every boy is.  $\models$  I am 2 cm taller than every boy is.  
 940 b. I am 3 cm taller than every boy is.  $\not\models$  I am 4 cm taller than every boy is.

941 (79)  $[[\text{I am (at least) 3 inches taller than every boy is}]] \quad I_{\text{DIFF}} = [3 \text{ cm}, +\infty)$   
 942  $\Leftrightarrow \text{HEIGHT}(I) \subseteq [\text{PRECISE-HEIGHT}(\text{tallest-boy}) + 3 \text{ cm}, +\infty)$   
 943 Let  $\text{PRECISE-HEIGHT}(\text{tallest-boy}) = x$ , then  $[x + 3 \text{ cm}, +\infty) \subseteq [x + 2 \text{ cm}, +\infty)$

- 944 (80) a. I am 3 cm less tall than every boy is.  $\models$  I am 2 cm less tall than every boy is.  
 945 b. I am 3 cm less tall than every boy is.  $\not\models$  I am 4 cm less tall than every boy is.

946 (81)  $[[\text{I am (at least) 3 inches less tall than every boy is}]] \quad I_{\text{DIFF}} = (-\infty, -3 \text{ cm}]$   
 947  $\Leftrightarrow \text{HEIGHT}(I) \subseteq (-\infty, \text{PRECISE-HEIGHT}(\text{shortest-boy}) - 3 \text{ cm}]$

Let  $\text{PRECISE-HEIGHT}(\text{shortest-boy}) = y$ , then  $(-\infty, y - 3 \text{ cm}] \subset (-\infty, y - 2 \text{ cm}]$

## 5 Our solutions to three puzzles

Based on our proposed difference-based analysis implemented with interval subtraction, this section accounts for three puzzles. The first two involve the semantics of the *than*-clause. The third one involves the role of *-er/more* at the discourse level.

### 5.1 Information projection of the *than*-clause as a scope island

It has long been acknowledged that in a clausal comparative (cf. phrasal comparative), its *than*-clause is a scope island (Hankamer 1973, Larson 1988). Thus *than*-clause-internal quantifiers cannot scope out their hosting *than*-clause via Quantifier Raising (QR), a vanilla mechanism for scope-taking.<sup>30</sup> This scope island issue raises the question of how measurement information of multiple entities is used as comparison standard and gets projected to sentential-level semantics (see e.g., Gajewski 2008, van Rooij 2008).

Technically, the availability of *wh*-movement provides a good test for the availability of QR. (82) and (83) illustrate that overt and covert *wh*-movements from within a *than*-clause are ungrammatical (see Larson 1988, Schwarzschild and Wilkinson 2002). Therefore, QR is also unavailable for *than*-clause-internal quantifiers to take scope.

- |      |    |   |                            |
|------|----|---|----------------------------|
| (82) | a. | [Which tree] <sub>i</sub> is my giraffe taller than?                          | <b>Phrasal comparative</b> |
|      | b. | *[Which tree] <sub>i</sub> is my giraffe taller than <i>t<sub>i</sub></i> is? | <b>Clausal comparative</b> |
| (83) | a. | She wants to know who was taller than who else.                               | <b>Phrasal comparative</b> |
|      | b. | *She wants to know who was taller than who else is.                           | <b>Clausal comparative</b> |

Empirically, a series of contrasts between clausal and phrasal comparatives provide evidence for the scope island status of *than*-clauses. In (84), the phrasal comparative (84a) is ambiguous between a surface scope reading ' $\exists > \forall$ ' and an inverse scope reading ' $\forall > \exists$ ', while the clausal comparative (84b) has only a surface scope reading ' $\exists > \forall$ ' (see Larson 1988, p. 4, (12)). This contrast shows that *than*-phrase-internal universal

<sup>30</sup>We mainly focus on whether/how universal quantifiers in a *than*-clause take scope. Indefinites and modified numerals contained within a scope island can still take exceptional scope, though not via a QR-style mechanism (see Brasoveanu 2013, Charlow 2014, Bumford 2017, and Zhang 2018b, 2020c).

973 quantifiers can take scope, but *than*-clausal-internal universal quantifiers cannot.<sup>31</sup>

- 974 (84) a. Someone is smarter than everyone. **Phrasal comparative**  
 975  $\leadsto$  ambiguous:  $\checkmark \exists > \forall, \checkmark \forall > \exists$   
 976 b. Someone is smarter than everyone is. **Clausal comparative**

<sup>31</sup> *Smart* is a gradable adjective showing dimension indeterminacy (see Kennedy 1999, Section 1.1.2). Thus both sentences in (i) can be true without contradicting each other.

- (i) a. My dog is smarter than I am.  $\leadsto$  In terms of behaving in a cute way  
 b. I am smarter than my dog is.  $\leadsto$  In terms of working on mathematical problem sets

For (84a) and (84b), if *someone* and *everyone* have the same domain, the surface-scope reading of the two sentences is contradictory (i.e., false in all models, because no one can be smarter than themselves). However, phrasal comparative (84a) also has a contingent reading: under a scenario in which there exists no one such that s/he is the smartest one in all ways, (84a) is true due to its inverse-scope reading, i.e., for each person  $x$ , there exists a person  $y$  such that  $y$  is smarter than  $x$  in a certain way. This contingent reading is unavailable for the clausal comparative (84b).

For (84a) and (84b), if *someone* and *everyone* have different domains, then as illustrated in (ii), under the given scenario, the inverse-scope reading is true, but the surface-scope reading is false, showing that the surface-scope and inverse-scope readings are distinct.

- (ii) Scenario: Ordering of smartness in terms of writing skills: Professor A > Student C > Professor B > Student D; Ordering of smartness in terms of talking skills: Professor B > Student D > Professor A > Student C.  
 a. Some student is smarter than every professor. **Phrasal comparative**

For clausal comparatives like (iii) and (iv), do they indeed lack an inverse-scope reading? There seems some discrepancy among reported judgments. Fleisher (2018) claims that the internal reading of *different* is available for (iv) (i.e., for each tree  $x$ , there is a distinct giraffe  $y$  such that  $y$  is exactly one foot taller than  $x$  is). However, among our informants on Facebook, many claim that this reading is only acceptable if the sentence-final *is* in (iv) is deleted. In other words, their judgments suggest that this '*every > different*' reading might only be available for phrasal comparatives, but not for clausal comparatives.

- (iii) Some giraffe or other is an even number of inches taller than every tree is. (by Uli Sauerland)  
 $\exists x[\text{giraffe}(x) \wedge \text{HEIGHT}(x) \subseteq \iota I[I - \iota I'[\forall y[\text{tree}(y) \rightarrow \text{HEIGHT}(y) \subseteq I']]] = [d'', d'']]^{\wedge d \text{ modulo } 2=0}$   
 $\leadsto$  every tree is of the same height  $I'$ , and some giraffe's height exceeds  $I'$  by  $d''$ , and  $d \text{ modulo } 2 = 0$ .  
 (Here we consider an *even number* a modified numeral (which does not have an 'at least' interpretation, see Szabolcsi 1997, Krifka 1999, de Swart 1999, Umbach 2005), and the checking of this cardinality requirement is based on a post-suppositional mechanism (see Brasoveanu 2013).)  
 (iv) A different giraffe is exactly a foot taller than every tree is.  
 $\checkmark$  the external reading of *different* (*different > every*);  
 $\#$  the internal reading of *different* (*every > different*) (cf. the judgment reported in Fleisher 2018)

Our intuitive judgment on the (un)availability of an inverse-scope reading for sentences (iii)/(iv) might not be fully reliable, due to garden-path effects (i.e., corresponding phrasal comparatives have an inverse scope reading). Therefore, we advocate the use of rigorous large-scale judgment elicitation or carefully designed experiments with the use of an eye-tracker or EEG to settle down this issue.

We thank an anonymous reviewer for raising these issues.

977  $\leadsto$  unambiguous:  $\checkmark \exists > \forall, \# \forall > \exists$

978 As shown in (85) and (86), if *than*-clause-internal downward-entailing quantifiers  
 979 *no tree* and *few trees* can take scope outside the *than*-clause, (85b) and (86b) would be  
 980 grammatical and yield the same reading as (85a) and (86a) do. The ungrammaticality of  
 981 (85b) and (86b) again shows that (i) phrasal comparatives and clausal comparatives are  
 982 distinct linguistic phenomena (see Hankamer 1973, Hoeksema 1983, Pinkal 1990,  
 983 Kennedy 1999, Pancheva 2006) and (ii) the *than*-clause is a scope island.

- |     |      |    |   |                            |
|-----|------|----|---|----------------------------|
| 984 | (85) | a. | My giraffe is taller than no tree.        | <b>Phrasal comparative</b> |
| 985 |      | b. | *My giraffe is taller than no tree is.    | <b>Clausal comparative</b> |
| 986 | (86) | a. | My giraffe is taller than few trees.      | <b>Phrasal comparative</b> |
| 987 |      | b. | *My giraffe is taller than few trees are. | <b>Clausal comparative</b> |

988 Schwarzchild and Wilkinson (2002) also argue that our natural interpretation for  
 989 (87) does not need to involve an individual prediction for each tree's height. In other  
 990 words, (87) calls for an analysis that supports the *in situ* interpretation of *most trees*.

- 991 (87) My giraffe is taller than Bill predicted most trees are.

992 Given the scope island status and interpretation limitations of a *than*-clause, when  
 993 its target of predication is a group of entities (e.g., *than every tree is (tall)*, *than the trees are*  
 994 *(tall)*), the possibility of projecting the measurement information of each involved  
 995 individual to sentential level is basically ruled out. For the sentence in (88), not only the  
 996 universal quantifier *every tree* has to be interpreted *in situ*, but also the scope-taking of  
 997 each measurement for individual trees (see the discussion on 'degree plurality' in Section  
 998 6.4) cannot be workable, as evidenced by the lack of inverse scope reading for (84b).

999 Therefore, as summarized in (88), the interpretation of this clausal comparative  
 1000 cannot involve multiple comparisons (see (88a)). However, if the semantics of the  
 1001 *than*-clause is reduced to a single degree, as proposed by the canonical analysis (see  
 1002 (11)) or Beck (2010) (see (21)), the derived truth condition is too weak (see (88b)).

- |      |      |  |   |
|------|------|--|---|
| 1003 | (88) | My giraffe is between 5 and 10 feet taller than every tree is. | (= (65))  |
| 1004 |      | a.   | #There are multiple comparisons – one for each tree.                |
| 1005 |      |  | $\leadsto$ Violating scope island constraints                       |
| 1006 |      | b.   | #There is only one comparison – just for the shortest/tallest tree. |

1007                    $\leadsto$  Too weak truth condition (see (11), (21), and the discussion in Section 2)

1008           Then what information eventually gets projected from a *than*-clause for conducting  
1009 comparison(s) at the sentential level? According to Beck (2010),

1010           ‘I want to come out of the calculation of the semantics of the *than*-clause  
1011 holding in my hand *the* degree we will be comparing things to.’ (Beck 2010)

1012           Our interval-based analysis responds to this challenge with a new and more  
1013 generalized view. We come out of the calculation of the semantics of the *than*-clause  
1014 holding in our hand *the* scalar value we will be comparing things to, and this scalar value  
1015 is represented as an interval, i.e., a potentially not-very-precise scalar value. Thus, as  
1016 shown in (89), there is only one comparison, but both the upper and lower bounds of  
1017  $I_{\text{STDD}}$  (i.e., the interval serving as comparison standard) – here the height of the shortest  
1018 and the tallest trees – are involved in this comparison.

1019 (89)   My giraffe is between 5 and 10 feet taller than every tree is.  
1020            $[[\text{than every tree is tall}]] = \iota I [\forall x [\text{tree}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]]$            (= (54b))  
1021           =  $[\text{PRECISE-HEIGHT}(\text{shortest-tree}), \text{PRECISE-HEIGHT}(\text{tallest-tree})]$   
1022            $\leadsto$  There is only one comparison – for the interval ranging over the trees’ height.

1023           Our view is compatible with all those works that analyze the semantics of a  
1024 *than*-clause as a definite description (e.g., Russell 1905, Heim 1985, Beck 2010). Our view  
1025 also accounts for the cases of *than*-clause-internal downward-entailing quantifiers (see  
1026 (85) and (86)) and the most natural interpretation of (87). (85b) and (86b) are  
1027 ungrammatical because their *than*-clause is uninterpretable – there is no non-trivial  
1028 convex interval  $I$  such that no tree is  $I$  tall (or few trees are  $I$  tall) (see also Abrusán  
1029 2014). For (87), Bill’s prediction can be a single, potentially not-very-precise value  
1030 represented as an interval, and at sentential level, comparison is conducted with this  
1031 interval.

1032           In short, by using an interval to represent the standard of comparison and only  
1033 projecting endpoint information from the *than*-clause, our interval-based implementation  
1034 yields intuitively correct truth conditions without violating any island constraints.

## 5.2 The creation of a downward-entailing operator with intervals

Whether and how a *than*-clause contributes a downward-entailing (DE) operator and creates an NPI-licensing environment has been a debatable issue. According to theories proposing the inclusion of a covert negation operator inside a *than*-clause (e.g., Marques 2003, Schwarzschild 2008, Gajewski 2008, Alrenga and Kennedy 2014, and other adopters of the ‘A-not-A’ approach), a *than*-clause naturally becomes a DE environment.

However, empirical evidence does not fully support this view. Different from a negation operator, *than*-clauses only license some NPIs, typically minimizers like *give a penny* (see (90a)) and weak NPIs that also work as Free Choice Items (FCI, e.g., *anyone* in (90b)), but not strong NPI *either* (see (90c); see Giannakidou and Yoon 2010).

- (90) a. John would sooner roast in hell than **give a penny** to the charity.  
b. Roxy ran faster than **anyone** had expected.  
c. \*John is taller than Bill is **either**. (Giannakidou and Yoon 2010: (42))

On the other hand, sometimes, the interpretation of *than*-clauses leads to an upward entailment, not a downward entailment, as illustrated by the contrast between (92) and (91) (see Larson 1988, Schwarzschild and Wilkinson 2002, Giannakidou and Yoon 2010).

(91) Downward entailment:

- a. The tree is taller than **every animal** is  $\models$  the tree is taller than **every giraffe** is.  
b. The tree is taller than **any animal** is  $\models$  the tree is taller than **any giraffe** is.

(92) Upward entailment:

- a. The tree is taller than **some animal** is  $\not\models$  the tree is taller than **some giraffe** is.  
b. The tree is taller than **some giraffe** is  $\models$  the tree is taller than **some animal** is.

Following our interval-subtraction-based analysis, we show that interval subtraction naturally makes the subtrahend (i.e.,  $I_{\text{STDD}}$ , or the semantics of a *than*-clause) a DE operator. There is no need to assume a covert negation operator within a *than*-clause.

As already addressed in Section 3.1, within interval arithmetic, given the values of a difference and a subtrahend, we need to follow the formula of interval subtraction (see (42)) to compute the value of the minuend. Specifically, as shown in (93) (which repeats



1066 (47)), in computing the value of the minuend, it is the **upper** bound of the subtrahend  
 1067 that contributes to the **lower** bound of the minuend, and it is the **lower** bound of the  
 1068 subtrahend that contributes to the **upper** bound of the minuend.

- 1069 (93) If  $X - [a, b] = [c, d]$ , when defined,  $X = [b + c, a + d]$ .
- 1070 a. The **lower** bound of the **minuend**  $X$   
 1071 = the **lower** bound of the **difference** + the **upper** bound of the **subtrahend**;  
 1072 b. the **upper** bound of the **minuend**  $X$   
 1073 = the **upper** bound of the **difference** + the **lower** bound of the **subtrahend**.

1074 An interval (of type  $\langle dt \rangle$ ) is a convex set of degrees. Thus, an interval becomes less  
 1075 informative if we raise its upper bound or lower its lower bound (e.g., changing from  
 1076  $[3, 5]$  to  $[3, 6]$  or  $[2, 5]$  leads to a decrease of informativeness), and it becomes more  
 1077 informative if we lower its upper bound or raise its lower bound (e.g., changing from  
 1078  $[3, 5]$  to  $[3, 4]$  or  $[4, 5]$  leads to an increase of informativeness).

1079 Given (93), lowering the lower bound of the subtrahend leads to a lower upper  
 1080 bound for the minuend, thus decreasing the informativeness of the subtrahend (i.e., the  
 1081 interval standing for the subtrahend includes more possibilities) but increasing the  
 1082 informativeness of the minuend (i.e., the interval standing for the minuend includes  
 1083 fewer possibilities). Thus, generally, lowering or raising an endpoint of the subtrahend  
 1084 always causes the informativeness of the subtrahend and the minuend to change in  
 1085 opposite directions. When the subtrahend becomes more informative, the minuend  
 1086 becomes less informative, and vice versa.

1087 Therefore, as shown in (94), the informativeness of a *than*-clause (i.e.,  $I_{\text{STDD}}$ , which  
 1088 plays the role of subtrahend) projects to sentential-level informativeness (i.e., the  
 1089 informativeness of the minuend) in a reverse way, demonstrating exactly the defining  
 1090 property of a typical DE operator (e.g., a negation operator, as shown in (95)) in  
 1091 reversing the relation of entailment (see Fauconnier 1978, Ladusaw 1979, 1980).

1092 (94) Function  $f$  is downward-entailing iff  $\forall x \forall y [x \text{ entails } y \rightarrow f(y) \text{ entails } f(x)]$ .

1093 Let  $f(K) = \iota I' [I' - K = I_{\text{DIFF}}]$  (here  $I_{\text{DIFF}}$  means a given free variable):

1094 If  $I_{\text{STDD}} \subseteq I'_{\text{STDD}}$ , then  $\iota I' [I' - I'_{\text{STDD}} = I_{\text{DIFF}}] \subseteq \iota I [I - I_{\text{STDD}} = I_{\text{DIFF}}]$ .<sup>32</sup>

1095 (95)  $\therefore \lambda x. \text{lizard}(x) \subseteq \lambda x. \text{reptile}(x)$  (i.e.,  $\llbracket \text{lizard} \rrbracket$  entails  $\llbracket \text{reptile} \rrbracket$ .)

<sup>32</sup>We'd like to emphasize that intervals are convex sets of degrees, and thus the entailment relation defined on intervals is the same as the entailment relation defined on sets.

$\therefore \lambda x. \neg \text{lizard}(x) \supseteq \lambda x. \neg \text{reptile}(x)$  (i.e.,  $\llbracket \text{not a reptile} \rrbracket$  entails  $\llbracket \text{not a lizard} \rrbracket$ .)  
 $\rightsquigarrow \llbracket \text{not} \rrbracket$  reverses the relation of entailment and works as a DE operator.  
 E.g., Roo is not a reptile  $\models$  Roo is not a lizard.

Under the current analysis, the DE-ness of a *than*-clause is due to its role of subtrahend in interval subtraction. Thus, this DE-ness is with regard to the projection of informativeness for the interval  $I_{\text{STDD}}$ . The projection of informativeness for *than*-clause-internal expressions like *every giraffe* or *some giraffe* in (92)/(91) is subject to an interplay among several operators that affect the projection of informativeness.

For comparatives that contain a *than*-clause-internal **universal** quantifier (e.g., *every giraffe* in (91a), or *any giraffe* in (91b) – an FCI with a universal flavor), the relation of entailment gets reversed three times along the derivation of sentential semantics.

As shown in (96), we start with the lexical semantics of *giraffe* and *animal*. (i) From these nouns (or NPs) to their embedding DP ‘*every NP*’, the relation of entailment is reversed. (ii) From ‘*every NP*’ to  $I_{\text{STDD}}$  (i.e., the most informative interval serving as the standard of comparison), the relation of entailment is reversed a second time. (iii) From  $I_{\text{STDD}}$  to the value of minuend, as argued before, the relation of entailment is reversed a third time. Eventually, we obtain the entailment pattern shown in (91).

(96) The tree is taller than every animal/giraffe is.

- a. **Reverse 1:** the projection from  $\llbracket \text{NP} \rrbracket$  to  $\llbracket \text{every NP} \rrbracket$ .  
 $\therefore \lambda x. \text{giraffe}(x) \subseteq \lambda x. \text{animal}(x)$  (i.e.,  $\llbracket \text{giraffe} \rrbracket$  entails  $\llbracket \text{animal} \rrbracket$ .)  
 $\therefore \lambda P. \forall x [\text{giraffe}(x) \rightarrow P(x)] \supseteq \lambda P. \forall x [\text{animal}(x) \rightarrow P(x)]$   
 (i.e.,  $\llbracket \text{every animal} \rrbracket$  entails  $\llbracket \text{every giraffe} \rrbracket$ : any property  $P$  such that  $\forall x [\text{animal}(x) \rightarrow P(x)]$  also makes  $\forall x [\text{giraffe}(x) \rightarrow P(x)]$  hold true.)
- b. **Reverse 2:** the projection from  $\llbracket \text{every NP} \rrbracket$  to  $I_{\text{STDD}}$ .  
 $\therefore \lambda I. \forall x [\text{giraffe}(x) \rightarrow \text{HEIGHT}(x) \subseteq I] \supseteq \lambda I. \forall x [\text{animal}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]$   
 (i.e., any interval  $I$  such that  $\forall x [\text{animal}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]$  also makes  $\forall x [\text{giraffe}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]$  hold true.)  
 $\therefore \iota I [\forall x [\text{giraffe}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]] \subseteq \iota I' [\forall x [\text{animal}(x) \rightarrow \text{HEIGHT}(x) \subseteq I']]$   
 (i.e.,  $\llbracket \text{than every giraffe is (tall)} \rrbracket$  entails  $\llbracket \text{than every animal is (tall)} \rrbracket$ : the most informative interval  $I$  such that  $\forall x [\text{giraffe}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]$  is not less informative than the most informative interval  $I'$  such that  $\forall x [\text{animal}(x) \rightarrow \text{HEIGHT}(x) \subseteq I']$ .)
- c. **Reverse 3:** the projection from  $I_{\text{STDD}}$  to sentence meaning.

1129  $\therefore \llbracket \text{than every giraffe is (tall)} \rrbracket \subseteq \llbracket \text{than every animal is (tall)} \rrbracket$   
 1130  $\therefore \iota I_{\text{MINUEND}} [I_{\text{MINUEND}} - \iota I [\forall x [\text{giraffe}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]] = I_{\text{DIFF}}] \supseteq$   
 1131  $\iota I'_{\text{MINUEND}} [I'_{\text{MINUEND}} - \iota I' [\forall x [\text{animal}(x) \rightarrow \text{HEIGHT}(x) \subseteq I']] = I_{\text{DIFF}}]$   
 1132 (i.e.,  $\llbracket \text{taller than every animal is} \rrbracket$  entails  $\llbracket \text{taller than every giraffe is} \rrbracket$ .)

1133 For comparatives that contain a *than*-clause-internal **existential** quantifier (e.g., *some*  
 1134 *giraffe* in (92)), the relation of entailment gets reversed twice along the derivation of  
 1135 sentential semantics. As shown in (97), we also start with the lexical semantics of *giraffe*  
 1136 and *animal*. From these NPs to their hosting DP ‘some NP’, the relation of entailment is  
 1137 straightforward. (i) It is from ‘some NP’ to  $I_{\text{STDD}}$ , the relation of entailment is reversed for  
 1138 the first time. (ii) Then from  $I_{\text{STDD}}$  to the value of minuend, the relation of entailment is  
 1139 reversed a second time. Eventually, we obtain the entailment pattern shown in (92).

1140 (97) The tree is taller than some animal/giraffe is.  
 1141 a. The projection from  $\llbracket \text{NP} \rrbracket$  to  $\llbracket \text{some NP} \rrbracket$ .  
 1142  $\therefore \lambda x. \text{giraffe}(x) \subseteq \lambda x. \text{animal}(x)$  (i.e.,  $\llbracket \text{giraffe} \rrbracket$  entails  $\llbracket \text{animal} \rrbracket$ .)  
 1143  $\therefore \lambda P. \exists x [\text{giraffe}(x) \wedge P(x)] \subseteq \lambda P. \exists x [\text{animal}(x) \wedge P(x)]$   
 1144 (i.e.,  $\llbracket \text{some giraffe} \rrbracket$  entails  $\llbracket \text{some animal} \rrbracket$ : any property  $P$  such that  
 1145  $\exists x [\text{giraffe}(x) \wedge P(x)]$  also makes  $\exists x [\text{animal}(x) \wedge P(x)]$  hold true.)  
 1146 b. **Reverse 1:** the projection from  $\llbracket \text{some NP} \rrbracket$  to  $I_{\text{STDD}}$ .  
 1147  $\therefore \lambda P. \exists x [\text{giraffe}(x) \wedge P(x)] \subseteq \lambda P. \exists x [\text{animal}(x) \wedge P(x)]$   
 1148  $\therefore$  for each most informative interval  $I$  such that  
 1149  $\exists x [\text{giraffe}(x) \wedge \text{HEIGHT}(x) \subseteq I]$ , it follows that there exists an interval  $I'$  such  
 1150 that  $\exists x [\text{animal}(x) \wedge \text{HEIGHT}(x) \subseteq I']$  and  $I'$  is not less informative than  $I$ .  
 1151 (i.e., ‘the most informative interval  $I'$  such that some animal is  $I'$  tall’  
 1152 entails ‘the most informative interval  $I$  such that some giraffe is  $I$  tall’.)<sup>33</sup>  
 1153 c. **Reverse 2:** the projection from  $I_{\text{STDD}}$  to sentence meaning.  
 1154  $\therefore \llbracket \text{than some animal is (tall)} \rrbracket \subseteq \llbracket \text{than some giraffe is (tall)} \rrbracket$   
 1155  $\therefore \iota I_{\text{MINUEND}} [I_{\text{MINUEND}} - \iota I [\exists x [\text{giraffe}(x) \wedge \text{HEIGHT}(x) \subseteq I]] = I_{\text{DIFF}}] \subseteq$   
 1156  $\iota I'_{\text{MINUEND}} [I'_{\text{MINUEND}} - \iota I' [\exists x [\text{animal}(x) \wedge \text{HEIGHT}(x) \subseteq I']] = I_{\text{DIFF}}]$

<sup>33</sup>Here’s a concrete illustration. Suppose that there are two giraffes and some other animals, and the height of the two giraffes are  $14' \pm 1'$  and  $15' \pm 1'$  (i.e., with an error range of 1 foot). Thus the most informative intervals that the height of some giraffe falls at are  $[13', 15']$  and  $[14', 16']$ . These two intervals are not less informative than themselves, but it’s likely that with a better precision level in measuring the height of other animals, there might be a more informative interval  $I'$ , e.g.,  $[13.5', 14.5']$ , such that the height of some other animal falls at  $I'$ .

1157 (i.e., [[taller than some giraffe is]] entails [[taller than some animal is]].)

1158 (96) and (97) demonstrate the interplay among operators that work together on  
 1159 informativeness projection, but after all, the informativeness of  $I_{\text{STDD}}$  always projects to  
 1160 sentential semantics in the same reverse way. Its subtrahend status is a DE operator.<sup>34</sup>

1161 Since it is the subtrahend status that actually contributes the DE operator, this DE  
 1162 operator is performed outside the *than*-clause and never interferes with any  
 1163 *than*-clause-internal quantifiers (cf. Alrenga and Kennedy 2014). This correctly predicts  
 1164 that clausal comparatives are generally unambiguous, no matter whether there are  
 1165 universal/existential nominal/modal quantifiers in their *than*-clause (see (98)–(101)).<sup>35</sup>

1166 (98) **Universal nominal quantifier:** *every boy*

1167 Context: The height of boys is between 5 feet 5 inches and 6 feet.

- 1168 a. Mary is taller than every boy is. ✓ > 6'; # > 5'5"  
 1169 b. Mary is less tall than every boy is. ✓ < 5'5"; # < 6'

1170 (99) **Existential nominal quantifier:** *some boys*

1171 Context: The height of boys is between 5 feet 5 inches and 6 feet.

- 1172 a. Mary is taller than some boys are. ✓ > 5'5"; # > 6'  
 1173 b. Mary is less tall than some boys are. ✓ < 6'; # < 5'5"

1174 (100) **Universal epistemic modal:** *be supposed to*

1175 Context: the temperature of X is supposed to be between 83°C and 98°C.

<sup>34</sup>A general discussion on the licensing mechanism of various kinds of NPIs is beyond the scope of this paper. For minimizers and weak NPIs (which arguably work as FCIs within a *than*-clause), their semantics is relevant to informativeness projection. Thus their licensing conditions should be informativeness-based. For words like *either*, presumably, their semantics is irrelevant to informativeness projection, and their licensing conditions should be related to other factors such as non-veridicality (see Giannakidou and Yoon 2010).

See also Zhang (2020a) for a further discussion on the strong negative flavor (i.e., anti-additivity (see also Hoeksema 1983) and anti-multiplicativity) of the subtrahend status.

<sup>35</sup>We further predict that, when ambiguity does arise (see (i), which contains a *than*-clause-internal existential deontic (permission-related) modal), this ambiguity cannot be due to scopal interaction between a modal and some kind of negation-like quantifier built with a *than*-clause (see Rullmann 1995, Heim 2006b, Beck 2013, Alrenga and Kennedy 2014, Fleisher 2020, and Zhang and Ling 2017a for discussion; see also footnote 21 on page 27).

- (i) Context: This highway has a required minimum speed of 35 mph and a speed limit of 50 mph.  
 Lucinda was driving less fast than allowed. (Beck 2013: (1), (2))  
 a. Lucinda was driving below the speed limit – 50 mph.  
 b. Lucinda was driving below the required minimum – 35 mph.

- 1176 a. X reached a temperature higher than supposed to be. ✓ > 98°C; # > 83°C  
 1177 b. X reached a temperature less high than supposed to be. ✓ < 83°C; # < 98°C

1178 (101) **Existential epistemic modal:** *likely*

1179 Context: the price of X is likely to be between \$8 000 to \$ 10 000 next year.

- 1180 a. The price of X is higher than it's likely to be next year. ✓ > \$10K; # > \$8K  
 1181 b. The price of X is less high than it's likely to be next year. ✓ < \$8K; # < \$10K

1182 To sum up, in an equation of interval subtraction, a subtrahend naturally projects  
 1183 informativeness in a reverse way. Downward-entailing-ness is in the nature of the  
 1184 standard in a comparison (i.e., a *than*-clause) and does not need to resort to any  
 1185 additional operators or mechanisms.

1186 **5.3 Klein (1980)'s puzzle and the core contribution of -er/more**

1187 The third puzzle is raised by Klein (1980). Cross-linguistically, why is the positive form  
 1188 of gradable adjectives (e.g., *tall*) morphologically simpler than the comparative form  
 1189 (e.g., *taller*)? If gradable adjectives involve an inherently relative meaning and always  
 1190 encode comparison (e.g., the meaning of *my giraffe is tall* is analyzed as a comparison  
 1191 between the height of my giraffe and the average height of giraffes), shouldn't the  
 1192 comparative use be more basic and have a morphologically simpler form?

1193 Under our proposed difference-based analysis, *-er/more* contributes to the semantics  
 1194 of comparatives by playing the role of the default differential. The default positive value  
 1195 (0, +∞) aside, the differential status of *-er/more* is due to its additivity, a kind of  
 1196 anaphoricity. In this sense, what *-er/more* marks is actually the discourse salience of the  
 1197 value serving as the standard of comparison. Compared to other uses of gradable  
 1198 adjectives, comparatives are special in involving standards that have discourse salience.<sup>36</sup>

1199 Empirical evidence is illustrated by the contrast shown in (102). The implicit  
 1200 standard for the interpretation of the positive form *tall* has no discourse salience, while

<sup>36</sup>In response to this puzzle he raises, Klein (1980) abandons the relativity inherent to the semantics of gradable adjectives and develops a delineation approach. Within this approach, gradable adjectives (e.g., *tall*) are like non-gradable ones (e.g., *red*) and denote sets of individuals, but the extension of a gradable adjective can change in evaluations, depending on the set of individuals that it is being compared with (see McConnell-Ginet 1973, Kamp 1975, Lewis 1979, Klein 1980, and see Burnett 2017 for a recent development). Degrees are not conceptual primitives within this approach, and the semantics of gradable adjectives does not involve comparison per se. Kennedy (1999) convincingly challenges this approach. In this paper, we follow Kennedy (1999) and adopt a degree-based semantics for gradable adjectives (see Section 1.2).

1201 the accommodated standard for the interpretation of the comparative form *taller* must  
 1202 have discourse salience. Without the marker *-er*, the *then*-clause in (102a) shares the  
 1203 same implicit standard with the *if*-clause. In contrast, with the salience marker *-er*, the  
 1204 *then*-clause in (102b) requires a standard that has discourse salience, here the height of  
 1205 John, not the implicit standard involved in the interpretation of *if John is tall*.<sup>37</sup>

- 1206 (102) a. If John is tall, then Bill is tall.  
 1207        $\leadsto$  The heights of John and Bill are compared with the same  
 1208       context-relevant standard.  
 1209       b. If John is tall, then Bill is taller.  
 1210        $\leadsto$  The height of John is compared with a context-relevant standard, while  
 1211       the height of Bill is compared with the height of John.  
 1212       (Here the height of John has discourse salience.)

1213 Following the view that the meaning of comparison is constantly involved in various  
 1214 uses of gradable adjectives, we can use a type-shifter COMPARE (see (103)) to characterize  
 1215 these uses in a uniform way. Essentially, COMPARE plays the role of MINUS (see (56)) and  
 1216 encodes the operation of interval subtraction. With this type shifter, as shown in (104),  
 1217 we actually zoom into the interval argument of a gradable adjective (see (49)) and  
 1218 consider this interval argument  $I$  always a value computed from  $I_{\text{STDD}}$  and  $I_{\text{DIFF}}$ . In other  
 1219 words, the use of this type shifter allows us to name and directly have access to the  
 1220 components of the interval variable of  $[[\text{tall}]]$ .<sup>38</sup>

1221 (103)  $[[\text{COMPARE}]]_{\langle\langle dt, et \rangle, \langle dt, \langle dt, et \rangle \rangle\rangle}$   
 1222  $\stackrel{\text{def}}{=} \lambda G_{\langle dt, et \rangle} \cdot \lambda I_{\text{STDD}} \cdot \lambda I_{\text{DIFF}} \cdot \lambda x_e \cdot G\text{-DIMENSION}(x) \subseteq \iota I [I - I_{\text{STDD}} = I_{\text{DIFF}}]$

1223 (104)  $[[\text{COMPARE tall}]]_{\langle dt, \langle dt, et \rangle \rangle} \stackrel{\text{def}}{=} \lambda I_{\text{STDD}} \cdot \lambda I_{\text{DIFF}} \cdot \lambda x_e \cdot \text{HEIGHT}_{\langle e, dt \rangle}(x) \subseteq \iota I [I - I_{\text{STDD}} = I_{\text{DIFF}}]$

1224 For *-er/more* (i.e., the default, non-restricted, positive value,  $(0, +\infty)$ ), that carries the

<sup>37</sup>Contextual manipulation helps to resolve uncertainty for interpreting the implicit standard for the positive use of a gradable adjective (see the notion of ‘sharpening’ in Barker 2002). This is analogous to the kind of contextual manipulation in the interpretation of other predicates. For example, the predicate *girl* in a sentence like *every girl is here* needs to be restricted and enriched by context. Under a specific context, this predicate cannot hold for any entity that is a girl in the universe. However, it is rather discourse salience, not contextual manipulation, that forms the base for the standard status of the value serving as the standard in a comparative. We thank an anonymous reviewer for raising this issue.

<sup>38</sup>This is reminiscent of the ‘as-pattern’ used in programming language syntax (e.g., the as-pattern of Haskell): it allows for the naming of a variable and at the same time, pattern-matching the underlying structure of the variable and possibly also naming the components in the underlying structure.

1225 requirement for a discourse-salient base, see (105a)), we also assume that there is a silent  
 1226 counterpart, POSITIVE-VALUE (i.e.,  $(0, +\infty)$ ), that carries no such requirement (see (105b)).  
 1227 The distinction between *-er/more* and POSITIVE-VALUE is parallel to that between *a* and  
 1228 *another* in the domain of entities.

1229 (105) a.  $\llbracket \text{-er/more} \rrbracket \stackrel{\text{def}}{=} (0, +\infty)$  (i.e., the most general positive interval (= (36) =  
 1230 (55)))

1231 **Requirement:** there is a discourse salient scalar value serving as  
 1232 comparison standard (i.e., the base for increase).

1233 b.  $\llbracket \text{POSITIVE-VALUE} \rrbracket \stackrel{\text{def}}{=} (0, +\infty)$  **No additional requirement**

1234 As shown in (106), different uses of gradable adjectives differ in (i) their selection of  
 1235  $I_{\text{STDD}}$  and (ii) whether the default value of  $I_{\text{DIFF}}$  can be further restricted. Further  
 1236 numerical restriction for the default value of  $I_{\text{DIFF}}$  is obligatory for measurement  
 1237 constructions, optional for comparatives, and impossible for the positive use.<sup>39</sup>  
 1238 Standards with no discourse salience (i.e., those for the positive use and measurement  
 1239 constructions) and POSITIVE-VALUE are silent. Thus these three uses of gradable adjectives  
 1240 are distinguishable by (i) the presence/absence of numerical restriction and (ii) the  
 1241 marker of discourse salience for their standard of comparison.

1242 (106) The standard and differential involved in comparison:  
 1243 (Only the marker of discourse salience and numerals are pronounced.)

Linguistic construction	Standard: $I_{\text{STDD}}$	Differential: $I_{\text{DIFF}}$
Comparative	<i>than</i> -clause/phrase or accommodated <b>(with discourse salience)</b>	<i>-er/more</i> ; <b>optional</b> numerical restriction for $(0, +\infty)$
Measurement construction	absolute zero point $[0, 0]$ <b>(no discourse salience)</b>	POSITIVE-VALUE <b>with</b> numerical restriction
Positive use	the relevant average <b>(no discourse salience)</b>	POSITIVE-VALUE <b>with no</b> numerical restriction

1245 Compared to the analysis of  $\llbracket \text{tall} \rrbracket$  shown in Section 3.2,  $\llbracket \text{COMPARE tall} \rrbracket$  does not  
 1246 offer a fundamentally new analysis, but rather highlight the inherent relativity of the

<sup>39</sup>When numerical restriction of  $I_{\text{DIFF}}$  is absent (for the comparative or positive use), degree modifiers like *very*, *slightly*, and *much* can be used to modify  $I_{\text{DIFF}}$ , yielding *slightly tall*, *much taller*, etc (see also Rett 2018).



semantics of gradable adjectives: their various uses all involve a comparison relative to a reference, i.e., standard. The semantics of measurement constructions and the positive use can again be derived directly (see (107) and (108)). In both constructions,  $I_{\text{STDD}}$  has no discourse salience, so that the positive form (here *tall*) is used. Whether a sentence is interpreted as a measurement construction or the positive use depends on the presence of numerical restriction.

(107) My giraffe is exactly 20 feet tall. **Measurement construction** (see also (50))

$$\text{LF: My giraffe is } \underbrace{[20', 20'] \cap (0, +\infty)}_{I_{\text{DIFF}}} \text{ COMPARE tall } \underbrace{[0, 0]}_{I_{\text{STDD}}}$$

$$\text{HEIGHT(my-giraffe)} \subseteq \iota I[I - [0, 0] = [20', 20']]$$

(108) My giraffe is tall. **Positive use** (see also (51))

$$\text{LF: My giraffe is } \underbrace{(0, +\infty)}_{I_{\text{DIFF}}} \text{ COMPARE tall } \underbrace{I_{\text{AVERAGE-HEIGHT-OF-GIRAFFES}}}_{I_{\text{STDD}}}$$

$$\text{HEIGHT(my-giraffe)} \subseteq \iota I[I - I_{\text{AVERAGE-HEIGHT-OF-GIRAFFES}} = (0, +\infty)]$$

The analysis shown in (107) immediately implements Sassoon (2010)'s account for the limited distribution of gradable adjectives in measurement constructions. According to Sassoon (2010), only those gradable adjectives associated with ratio scales (i.e., scales with a meaningful, absolute zero point, see Fig. 1) can be used to form measurement constructions (see also the discussion in Schwarzschild 2005). In our analysis, measurement constructions require the existence of an absolute zero point to play the role of  $I_{\text{STDD}}$ . This requirement is met for a scale of temporal length (see (109a)), but not met for scales of temporal shortness, warmth, or earliness/lateness (see (109b)–(109d)).

(109) a. This tennis match was 1.5 hours **long**. Temporal length: a ratio scale

→ On a scale of temporal length: 0 hours means 'no temporal length'.

b. \*This tennis match was 1.5 hours **short**. Temporal shortness

→ On a scale of temporal shortness, there is no absolute zero point.

c. \*New York is now 70 degrees **warm**. Warmth

→ On a scale of warmth, there is no absolute zero point.

d. \*Our meeting time was 11 AM **early** / **late**. Earliness, lateness

→ On a scale of earliness/lateness, there is no absolute zero point.

A degree-question addresses the position that some entity's measurement falls at on a scale. As shown in (110), different choices of  $I_{\text{STDD}}$  lead to different ways of answering a

1277 degree question. Essentially, they mean that the position under discussion (here the  
 1278 height of my giraffe) can be considered relative to a certain reference position (e.g., a  
 1279 zero point, a relevant average for a comparison class, or a discourse-salient position).

- 1280 (110)  $[[\text{How tall is my giraffe}]] = \lambda I. \text{HEIGHT}(\text{my-giraffe}) \subseteq I$  (see (49))
- 1281 a. It is 20 feet tall.  $\leadsto I_{\text{STDD}} = [0, 0]$
- 1282 b. It is very tall.  $\leadsto I_{\text{STDD}} = I_{\text{AVERAGE-HEIGHT-OF-GIRAFFES}}$
- 1283 c. It is taller than that tree is.  $\leadsto I_{\text{STDD}} = [[\text{than that tree is}]]$

1284 In Section 3.2, we analyze the semantics of a *than*-clause as a position on a scale – a  
 1285 short answer to its corresponding degree question. (110) shows that a position under  
 1286 discussion can be characterized relative to different reference positions. Thus for a  
 1287 comparative like (111), the semantics of its *than*-clause can be analyzed as relative to  
 1288 different reference positions within the *than*-clause, but at the matrix-clause level, it  
 1289 doesn't matter it is relative to which reference position that we address the height of the  
 1290 tree, i.e.,  $[[\text{than the tree is (tall)}]]$ . What this sentence conveys is that it is relative to the  
 1291 height of the tree – a discourse-salient  $I_{\text{STDD}}$  – that we address the height of the giraffe  
 1292 (and that the distance between these two positions on a scale of height is at least 2 feet).

- 1293 (111) My giraffe is (at least 2 feet) taller than the tree is. **Comparative** (see also (57))
- 1294 LF: My giraffe is  $\underbrace{[2', +\infty) \cap (0, +\infty)}_{I_{\text{DIFF}}} \text{ COMPARE } \underbrace{\text{tall than the tree is (tall)}}_{I_{\text{STDD}}}$
- 1295  $\text{HEIGHT}(\text{my-giraffe}) \subseteq \iota I [I - \text{HEIGHT}(\text{the tree}) = [2', +\infty)]$

1296 With this detailed understanding of the *than*-clause, we can explain why gradable  
 1297 adjectives that are not associated with ratio scales (e.g., *short*, *warm*, *early*, *late*) can still be  
 1298 used in comparatives (see (109) vs. (112)). For (112b), the availability of a zero point on  
 1299 a scale of temporal shortness, or more generally, how to choose a reference position for  
 1300 addressing *how short is that movie*, does not matter at the matrix-clause level.

- 1301 (112) a. This tennis match was 1.5 hours **longer** (than that movie is).  
 1302 b. This tennis match was 1.5 hours **shorter** (than that movie is).  
 1303 c. New York is now 70 degrees **warmer** (than Antarctica is).  
 1304 d. Our meeting time was 11 hours **earlier/later** (than I expected).

1305 The zoomed-in version offers a slowed-down way to consider the semantics of a  
 1306 degree question and its answerhood. As shown in (113), this degree question is

analyzed as addressing how far away the height of the tree is relative to a given reference position  $I_{\text{STDD}}$ .  $I_{\text{DIFF}}$ , the information sought for here, is the midway towards a full resolution of the position standing for the measurement of the tree on a scale of height.

- (113)  $[[\text{How tall is the tree}]] = \lambda I_{\text{DIFF}}. \text{HEIGHT}(\text{the-tree}) \subseteq \iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]$  (see (104))
- a. 20 feet.  $\leadsto I_{\text{STDD}} = [0, 0]$
- b. Slightly.  $\leadsto I_{\text{STDD}} = I_{\text{AVERAGE-HEIGHT-OF-GIRAFFES}}$
- c. 2 feet taller.  $\leadsto I_{\text{STDD}}$  is an accommodated, discourse-salient value.

Naturally occurring examples like the **comparison of deviations** in (2) (repeated here in (114)) provide empirical support for this slowed-down view on degree questions.

For the sentence *Mona is more happy than Jude is sad*, the proposed LF in (114) involves three comparisons, i.e., three uses of gradable adjectives. (i) **The use of *sad*** in the *than*-clause:  $[[\text{than Jude is sad}]]$  denotes the short answer to the question how far away Jude's sadness is relative to average sadness, providing a discourse-salient value for further comparison. (ii) **The use of *much***: here *more* is composed from gradable adjective *much* and the discourse salience marker *-er*.<sup>40</sup> Thus along the scale of amount of difference, i.e., the scale associated with *much*, there is a comparison with the salient value provided by the semantics of the *than*-clause, i.e., the difference between Jude's sadness and  $I_{\text{AVE.-SAD}}$ . (iii) **The use of *happy***: The derived meaning of  $[[\text{more ... than Jude is sad}]]$  plays the role of  $I_{\text{DIFF}}$  for the use of *happy*, addressing how far away Mona's happiness is relative to average happiness. Among these three

<sup>40</sup>In English, there are two distinct words *more*, and they bear different meanings. Throughout the paper, we have been focusing on the English comparative morpheme, which has two allomorphs: *-er* and *more*. The comparative form of monosyllabic gradable adjectives (e.g., *tall*) is formed with *-er* (e.g., yielding *taller*), and the comparative form of multisyllabic gradable adjectives (e.g., *beautiful*) is usually formed with *more* (e.g., yielding *more beautiful*). For some bisyllabic adjectives, both forms are acceptable: e.g., *cleverer* and *more clever* are both the comparative forms of *clever* (see also relevant discussion on StackExchange: <https://english.stackexchange.com/questions/145683/conundrum-cleverer-or-more-clever-simpler-or-more-simple-etc>).

However, in expressions like *more and more*, *more coffee*, *more animals*, etc., the word *more* is not an allomorph of the English comparative morpheme. Instead, it is the comparative form of *much*, i.e., the result of combining *much* with comparative morpheme *-er/more* (see also Bresnan 1973 and Wellwood 2019 for relevant discussion on *more*).

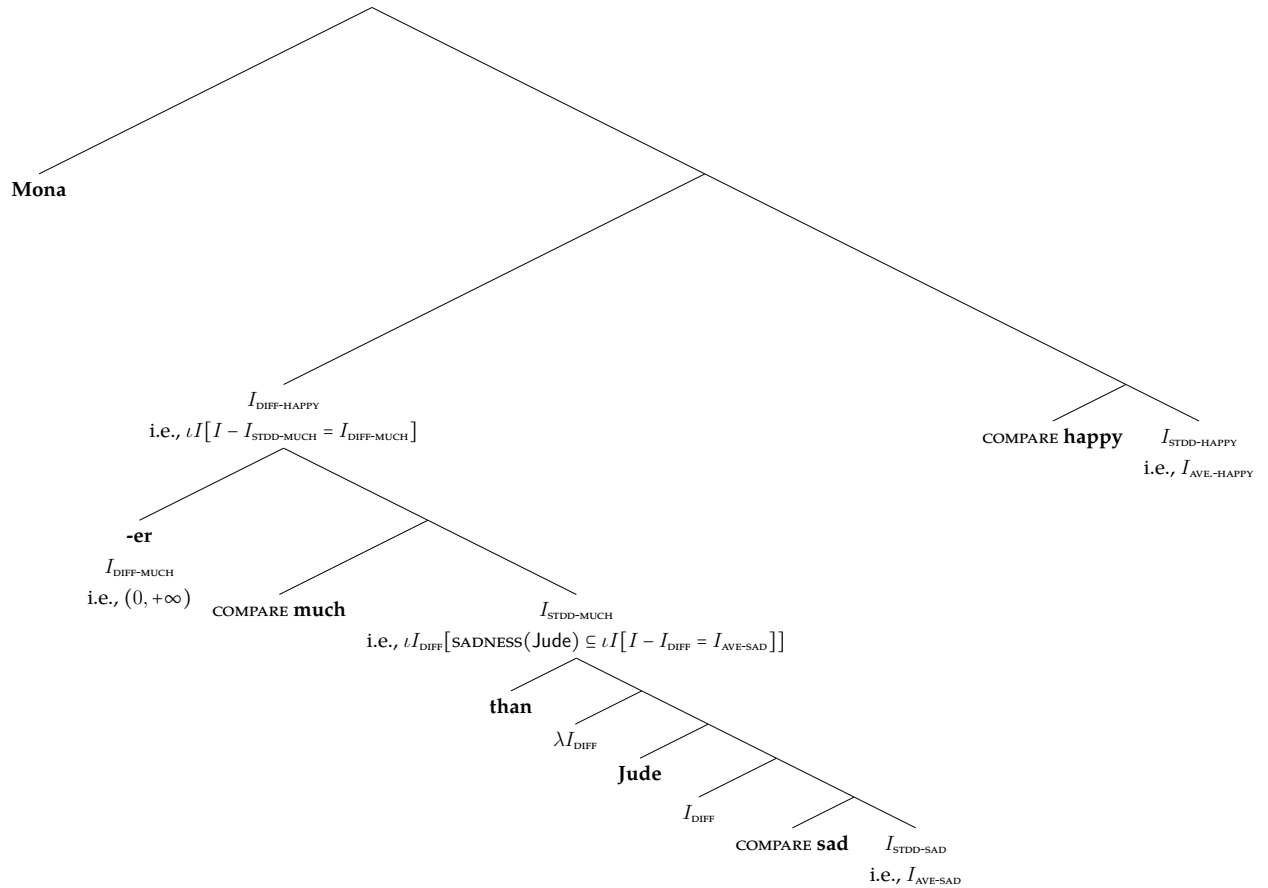
We believe that in the comparison of deviation, i.e., the Mona sentence (2)/(114), the use of *more* is actually the second case, i.e., *much+er/more*. This explains why the comparative form of *happy* is *happier*, but in (114), *more happy* is used, instead of *happier*. Actually, replacing *more happy* with *happier* results in ungrammaticality, due to cross-polar anomaly (see Kennedy 1999).

(i) \*Mona is happier than Jude is sad.

Cross-polar anomaly

comparisons, only the one performed along the scale of differences (i.e., the one associated with ‘COMPARE much’) involves a discourse-salient  $I_{STDD}$ . Therefore, the comparison of deviations eventually bears only one discourse salience marker *-er*.<sup>41</sup>

- (114) Mona is more happy than Jude is sad. (Kennedy 1999: Chapter 1, (89))  
 LF (the spelt-out part is in bold font, and to save space and improve readability, the semantically vacuous copula *is* is omitted):



<sup>41</sup>It is worth noting that the comparison of deviations that we discuss here is distinct from two other special types of comparatives illustrated in (i) (see Bartsch and Vennemann 1972b, McCawley 1976, Embick 2007, Bale 2008, Wellwood 2019).

In particular, (ib) does not have the same entailment pattern as sentences of ‘comparison of deviations’ do (see (115a)). (ib) does not entail that Esme is pretty and Einstein is clever (see Bale 2008).

A thorough comparison of all these types of comparatives within our theory is left for another occasion.

- (i) a. Ann is more tall than Bill is wide. **Metalinguistic comparison**  
 ~ It’s more accurate (to say) that Ann is tall than that Bill is wide.  
 b. Esme is prettier than Einstein is clever. **Indirect comparison**  
 ~ Esme’s prettiness (if there’s any) exceeds Einstein’s cleverness (if there’s any).

For the comparison of deviations in (114), the two comparisons along the scales of sadness and happiness are conducted with relevant averages of sadness and happiness, yielding two positive uses of gradable adjectives (see (115a)). This entailment pattern shown in (115a) is distinct from the pattern for usual comparatives (see (115b)), because usual comparatives do not particularly involve comparisons with relevant averages.

- (115)     a.    Mona is more happy than Jude is sad  $\models$  Mona is happy  $\wedge$  Jude is sad  
              b.    Mona is happier than Jude is.  $\not\models$  Mona is happy  $\vee$  Jude is sad/happy

In brief, within our analysis, the core semantic contribution of *-er/more* is additivity. All uses of gradable adjectives involve comparison (or relativity), and comparison does not need to be marked (cf. Klein 1980). *-er/more* is rather a discourse-salience marker.<sup>42</sup>

With this unified comparison-based understanding for the uses of gradable adjectives, issues such as the limited distribution of gradable adjectives in measurement constructions and the compositional details of Mona-sentences can be naturally accounted for. Klein (1980)'s puzzle is also resolved.<sup>43</sup>

<sup>42</sup>Some languages (e.g., Chinese, Japanese, Swahili) lack a comparative morpheme, i.e., the positive and gradable forms of a gradable adjective are morphologically the same. Presumably, in these languages, various uses of gradable adjectives still involve comparison, but even for comparatives, the reference (i.e., standard) of comparison does not require a discourse-salience marker. A thorough investigation of related phenomena in these languages is also left for another occasion (see also Zhang 2019 for a brief discussion).

<sup>43</sup>We are not exhaustive on the uses of gradable adjectives here. Unaddressed uses include *enough/too*-constructions, equatives, and superlatives.

The current comparison-based view can be immediately extended to account for the semantics of *enough/too*-constructions (see Zhang 2018a): essentially, *enough* means reaching the lower bound of the interval serving as the reference of comparison, and *too* means exceeding the upper bound of the interval serving as the reference of comparison.

Equatives and superlatives are not based on measurable differences and do not necessarily assume interval scales. Thus a different line of analysis can be more suitable for these constructions (see Anderson and Morzycki 2015, Solt 2016 and Zhang 2020b). A thorough investigation across all these uses of (gradable) adjectives is left for future research.

Morphologically, there is extensive evidence showing that superlative forms are constructed out of comparative forms (Bobaljik 2012). Presumably, the requirement for discourse-salient items serving as the reference of comparison underlies the semantics of both comparatives and superlatives, and superlatives additionally involve an ordinal-number-related component (i.e., *first*). A rigorous, detailed investigation is also left for future research.

## 6 Comparing our analysis with the existing literature

We started our paper with a discussion on the fundamental assumption underlying comparatives. We explicitly assume that comparison is not only performed between scalar values (instead of entities or events), but also these are values on interval scales.

Compared with the canonical analysis sketched out in Section 2.1, our proposed analysis makes a similar move on the analysis of adjectives, i.e., as a relation between a scalar value and an entity (see the first key component of the canonical analysis in Section 2.1). However, our analysis takes a different way in addressing (i) formal items as involved in comparison and (ii) the implementation of comparison itself (cf. the second and third key components of the canonical analysis in Section 2.1). In addition, our analysis is distinct in terms of (iii) its choice of subtraction (cf. addition) in equations and (iv) its explicit support for ‘encapsulation’ theories (cf. ‘entanglement’ theories, of which the approach of ‘degree plurality’ is a recent representative). Below we address each of these four issues and justify our view.

### 6.1 Formal items as involved in comparison: $(0, 6']$ vs. $[6', 6']$

The meaning of a *than*-clause contributes the standard of a comparison, i.e., a formal item that undergoes comparison. Thus, the semantic derivation of a *than*-clause reflects how the notion of formal-items-under-comparison is approached in theories on comparison.

Within the canonical analysis (see (7a)), a *than*-clause addresses the set of all degrees such that the measurement of its target of predication meets or exceeds. As illustrated in (116a), suppose the height of Mary is exactly 6 feet, then this *than*-clause is analyzed as a set of degrees ranging from 0 to the height of Mary, i.e.,  $(0, 6']$ . In contrast, within our current analysis (see (54)), a *than*-clause essentially just means the position on a scale that represents the measurement of the target of predication. As illustrated in (116b), here this *than*-clause amounts to an interval, i.e.,  $[6', 6']$ .

(116) [[*than* Mary is (tall)]]

- a. **Canonical analysis:**  $\lambda d.$  the height of Mary meets or exceeds  $d$  i.e.,  $(0, 6']$
- b. **Our analysis:**  $[\text{PRECISE-HEIGHT}(\text{Mary}), \text{PRECISE-HEIGHT}(\text{Mary})]$  i.e.,  $[6', 6']$

The idea of involving ‘ $(0, 6']$ ’ – the set of all degrees that Mary’s height meets or exceeds – in comparison is conceptually problematic in two aspects.

The first issue is manifested in the contrast between *tall* and *hot*. For  $(0, 6']$  in (116a), the choice of '0' as the lower bound of this formal-item-under-comparison assumes an absolute zero point. This choice cannot be generalized to all interval scales, and it actually never matters in a comparison. Gradable adjective *hot* is associated with a scale of temperature, a non-ratio interval scale lacking a meaningful, absolute zero point. Thus, for a *than*-clause like *than the coffee is (hot)*, the set of all degrees that the temperature of the coffee meets or exceeds should be a set like, say,  $(-\infty, 85^{\circ}C]$ , instead of  $(0, 85^{\circ}C]$ . For formal items like  $(0, 6']$  in (116) and  $(-\infty, 85^{\circ}C]$ , if used in a comparison, they would be compared with sets such as  $(0, x']$  and  $(-\infty, y^{\circ}C]$ , and the eventual comparisons would be performed between  $6'$  and  $x'$  and between  $85^{\circ}C$  and  $y^{\circ}C$ . In other words, for  $(0, 6']$  and  $(-\infty, 85^{\circ}C]$ , the information of their lower bound makes no contribution in a comparison. Therefore, the adoption of a MAX operator (see (10)) in the '>' analysis (cf. the 'A-not-A' analysis) to reduce the set  $(0, 6']$  into a single degree,  $6'$ , is conceptually more warranted (see, e.g., Rullmann 1995 for discussion on maximality).

The second issue is manifested in the contrast between *tall* and *short*. The reasoning behind the analysis in (116a) implicitly assumes that the semantics of a *than*-clause is based on a measurement construction (e.g., *Mary is 6 feet tall*, see (117)). However, the same reasoning cannot work for a *than*-clause like *than Mary is short*, because gradable adjectives like *short* are not associated with ratio scales and cannot be used to form a measurement construction (see Sassoon 2010 and the discussion in Section 5.3).

(117)  $\lambda d.$  the height of Mary  $\geq d = \lambda d.$  Mary is tall to degree  $d = \lambda d.$  Mary is  $d$ -tall

Given that both *tall* and *short* can be used in comparatives and appear as an elided part in their *than*-clause, the reasoning behind the semantic derivation of a *than*-clause should not be based on a measurement construction in the first place. Our current analysis avoids this pitfall by analyzing a *than*-clause as the short answer to its corresponding degree question (e.g., *how tall is Mary*, *how short is Mary*). Thus,  $[[\text{than Mary is tall/short}]]$  only means the position that represents the measurement of Mary on a scale of height (or shortness), not including anything else (like other measurements that the measurement of Mary meets or exceeds). In this sense, it is also conceptually problematic to start with a set like  $(0, 6']$  or  $(-\infty, 85^{\circ}C]$  in analyzing a *than*-clause and apply a MAX operator later.

Based on this discussion on the semantic derivation of *than*-clauses, our conclusion is that formal items involved in comparison should be directly considered measurements



themselves, instead of sets of degrees that some measurements meet or exceed.

## 6.2 Implementing comparison: set operation vs. subtraction

By arguing against the view that formal items involved in comparison are sets of degrees that some measurements meet or exceed (see Section 6.1), we also have to rule out the possibility that comparison can be implemented as performing a set operation (e.g., set difference) between two such sets of degrees.

As advocated from the beginning of this paper (see Section 1.2), we explicitly assume interval scales in analyzing the semantics of comparatives and make use of the formal properties of interval scales by adopting subtraction in implementing comparison. Throughout the paper (from Section 3 to Section 5), we have shown that the use of interval subtraction in implementing comparison is empirically advantageous, naturally accounting for the semantic derivation of *more-than* and *less-than* comparatives containing various kinds of numerical differentials as well as the information projection from a *than*-clause, which is a scope island and plays the role of subtrahend.

As a consequence of this switch from set operation to subtraction, the parallelism between **generalized quantifiers** in the domains of individuals and degrees is discarded (cf. e.g., Heim 2006a). As illustrated in (118), under the ‘*A-not-A*’ approach, *-er* seems to behave like *every*, and *-er ... than Mary is* is similar to a universal quantifier.

- (118) a. Every giraffe is from Africa.  $\leadsto$  ***every giraffe*: a generalized quantifier**  
 [[every]] relates two sets of individuals:  
 $\{x \mid x \text{ is a giraffe}\} \subseteq \{x' \mid x' \text{ is from Africa}\}$
- b. Bill is taller than Mary is.  $\leadsto$  ***-er than Mary is*: a degree quantifier**  
 In the ‘*A-not-A*’ approach, [[-er]] relates two sets of degrees:  
 $\{d \mid \text{Mary is } d\text{-tall}\} \subset \{d' \mid \text{Bill is } d'\text{-tall}\}$

By discarding this parallelism, we predict that comparatives are not subject to any scopal interaction that a true generalized quantifier (e.g., *every giraffe*) should be subject to. This prediction is borne out, as shown by the contrast in (119).

- (119) a. Every giraffe is not from Antarctica. **Scopal ambiguity**  
 (i) *every giraffe > not*:  $\{x \mid x \text{ is a giraffe}\} \subseteq \{x' \mid x' \text{ is not from Antarctica}\}$   
 (ii) *not > every giraffe*:  $\{x \mid x \text{ is a giraffe}\} \not\subseteq \{x' \mid x' \text{ is from Antarctica}\}$
- b. Bill is not taller than Kate is. **No scopal ambiguity**

- 1442 (i) *#-er than Kate is > not*:  $\{d \mid \text{Mary is } d\text{-tall}\} \subset \{d' \mid \text{Bill is not } d'\text{-tall}\}$   
 1443 (ii) *not > -er than Kate is*:  $\{d \mid \text{Mary is } d\text{-tall}\} \not\subset \{d' \mid \text{Bill is } d'\text{-tall}\}$

1444 On the other hand, by analyzing formal items involved in comparison as  
 1445 measurements themselves (e.g., *the height of Mary*) and using subtraction to implement  
 1446 comparison, we actually advocate a parallelism between **definite descriptions** in the  
 1447 domains of individuals and scalar values (see also [Russell 1905](#), [Heim 1985](#), [Rullmann](#)  
 1448 [1995](#), [Beck 2010](#)). Therefore, the interpretation of a comparative is reminiscent of a  
 1449 cumulative-reading sentence (see [Brasoveanu 2013](#)): both involve several definite  
 1450 descriptions, and there is no scopal interaction among them.

- 1451 (120) a. My giraffe is 2 feet taller than the tree is. **Comparative**  
 1452  $\leadsto$  **the height of my giraffe** exceeds **the height of the tree** by 2 feet  
 1453 a'. My giraffe is taller than 20 feet. **Comparative**  
 1454  $\leadsto$  **the height of my giraffe** exceeds **the definite value of 20 feet**.  
 1455 b. Exactly three boys saw exactly five movies. **Cumulative reading**  
 1456  $\leadsto$  **the maximal sum of boys**, the cardinality of which is 3, saw **the**  
 1457 **maximal sum of movies**, the cardinality of which is 5.

1458 Thus, with regard to the implementation of comparison, our analysis is closer to the  
 1459 ' $>$ ' approach than to the ' $A$ -not- $A$ ' approach. Comparison is considered a relation  
 1460 between definite descriptions of measurements, characterized as definite descriptions of  
 1461 degrees in the ' $>$ ' approach, and definite descriptions of intervals in ours.

### 1462 6.3 Addition vs. subtraction

1463 In this paper, we use the notion of interval to characterize definite descriptions of  
 1464 positions (i.e., measurements) on a scale in a generalized way, allowing for  
 1465 not-very-precise positions. Then interval subtraction provides a convenient technique to  
 1466 analyze the distance between two not-very-precise positions.

1467 Interval arithmetic is developed to compute on not-very-precise scalar values and  
 1468 handle measurement errors. Thus, as illustrated in (121), interval addition and interval  
 1469 subtraction are not inverse operations. Only interval subtraction, but not interval  
 1470 addition, is suitable for analyzing the distance between two not-very-precise positions.

- 1471 (121) a.  $[2, 3] + [4, 5] = [6, 8]$  **Interval addition**

b. (i)  $[6, 8] - [2, 3] = [3, 6]$

## Interval subtraction

(ii)  $[6, 8] - [4, 5] = [1, 4]$

## Interval subtraction

However, even for analyzing the distance between two precise measurements, the operation of subtraction is more suitable for compositional derivation than addition.

As illustrated in (122), with the use of addition (see e.g., [Hellan 1981](#), [von Stechow 1984](#) and analyses with the use of inequalities ' $>/\geq/</\leq$ ', see also [Beck 2011](#) for a summary), the numerical differential is constantly added to the lower measurement between the two under comparison (here Mary's height). Thus, in *more-than* comparatives, addition is performed on the differential and the measurement associated with the *than*-clause (see (122a-ii)), but in *less-than* comparatives, addition is performed on the differential and the measurement associated with the matrix clause (see (122b-ii)). This imbalance potentially creates an additional compositional issue.

In contrast, subtraction is always performed between the two measurements under comparison, the one associated with the matrix clause constantly playing the role of minuend and the one associated with the *than*-clause constantly playing the role of subtrahend (see (122a-i) and (122b-i)). Therefore, subtraction allows for a uniform compositional derivation for both *more-than* and *less-than* comparatives.

(122) Context: Kate is precisely 6 feet 2 inches tall, and Mary is precisely 6 feet tall.

a. Kate is exactly 2 inches **taller** than Mary is.

(i)  $\underbrace{\text{PRECISE-MEASURE}(\text{Kate})}_{\text{Minuend}} - \underbrace{\text{PRECISE-MEASURE}(\text{Mary})}_{\text{Subtrahend}} = 2''$  **Subtraction**

$$(ii) \quad \overset{\text{Minuend}}{\text{PRECISE-MEASURE(Kate)}} = \overset{\text{Subtrahend}}{\text{PRECISE-MEASURE(Mary)}} + 2'' \quad \textbf{Addition}$$

b. Mary is exactly 2 inches **less tall** than Kate is.

(i)  $\underbrace{\text{PRECISE-MEASURE}(\text{Mary})}_{\text{Minuend}} - \underbrace{\text{PRECISE-MEASURE}(\text{Kate})}_{\text{Subtrahend}} = -2''$  **Subtraction**

$$(ii) \quad \overset{\text{Minuend}}{\text{PRECISE-MEASURE(Mary)}} + \overset{\text{Subtrahend}}{2''} = \text{PRECISE-MEASURE(Kate)} \quad \textbf{Addition}$$

## 6.4 Entanglement vs. encapsulation: comparison with the approach of 'degree plurality'

Fleisher (2016) divides semantic theories on comparatives into two camps: ‘**entanglement**’ theories vs. ‘**encapsulation**’ theories. Essentially, ‘encapsulation’ theories conform to the ideal of Beck (2010): at the end of the calculation of a *than*-clause, we hold in our hand *the* unique value that will serve as the standard for

comparison. Thus at the matrix-clause level, a comparative encodes only one comparison, the one with this unique standard. Our interval-subtraction-based theory is a typical encapsulation theory. As illustrated in (123a), the derived semantics of the *than*-clause is a unique measurement represented in terms of an interval:  $[16', 20']$ . This sentence expresses the comparison between the height of my giraffe and this interval.

In contrast, ‘entanglement’ theories hold the view that the derivation of a *than*-clause potentially generates multiple scalar values (e.g., multiple degrees), so that at the matrix-clause level, a comparative can express multiple comparisons, each involving one of those scalar values (from the derivation of the *than*-clause) as its standard.

The ‘degree plurality’ theory is a typical entanglement theory (see Beck 2014, Dotlačil and Nouwen 2016 and a similar idea in Heim 2006a). Within the ‘degree plurality’ theory, as illustrated in (123b), the derived semantics of the *than*-clause is a sum of degrees:  $16' \oplus 18' \oplus 20'$ . Then with the use of a distributivity operator, this sum of degrees is distributed at the matrix-clause level, leading to multiple comparisons.

(123) Context: My giraffe is 21 feet tall. There are three trees, which are 16 feet, 18 feet, and 20 feet tall, respectively.

My giraffe is taller than every tree is.

- a.  $[[\text{than every tree is (tall)}]] = [16', 20']$       **our ‘interval subtraction’ theory**  
 $[[\text{(123)}]] \Leftrightarrow \text{HEIGHT}(\text{my giraffe}) \subseteq \iota I' [I' - [16', 20'] = (0, +\infty)]$
- b.  $[[\text{than every tree is (tall)}]] = 16' \oplus 18' \oplus 20'$       **the ‘degree plurality’ theory**  
 $[[\text{(123)}]] \Leftrightarrow \forall d \sqsubseteq_{\text{ATOM}} 16' \oplus 18' \oplus 20' [\text{the height of my giraffe} > d]$   
 $(\text{DIST} \stackrel{\text{def}}{=} \lambda D_d. \lambda P_{(dt)} \forall d [d \sqsubseteq_{\text{ATOM}} D \rightarrow P(d)])$

The ‘degree plurality’ theory is dubious for a few reasons. First, the ‘degree plurality’ theory still faces the issue of unattested scopal interaction (see the discussions in Section 5.1). In (124) (which repeats (84b)), the ‘degree plurality’ theory generates two readings for this sentence, but the ‘ $\forall > \exists$ ’ reading is actually unattested (see (124b)).

(124) Someone is smarter than everyone is.      **Clausal comparative: unambiguous**

Suppose  $D = \text{SMARTNESS}(x_1) \oplus \text{SMARTNESS}(x_2) \oplus \dots \oplus \text{SMARTNESS}(x_n)$

- a.  $\exists x [\text{human}(x) \wedge \forall d \sqsubseteq_{\text{ATOM}} D [\text{SMARTNESS}(x) > d]]$        $\exists > \forall$ : attested reading
- b.  $\forall d \sqsubseteq_{\text{ATOM}} D [\exists x [\text{human}(x) \wedge \text{SMARTNESS}(x) > d]]$        $\forall > \exists$ : unattested reading

Second, the interpretation of a negative comparative is not parallel with that of a negative sentence containing a plural definite in the domain of entities, undermining the

plausibility of analyzing a *than*-clause as a degree plurality.

Due to homogeneity effects (see [Križ 2016](#) for a recent discussion), the interpretation of a negative sentence containing a plural definite like *the books* demonstrates a three-way distinction pattern, as illustrated in (125). In particular, the sentence is considered neither true nor false in a context where Mary read some, but not all of the books. However, the interpretation of a negative comparative is not subject to this kind of homogeneity effects, as illustrated in (126). The contrast between (125) and (126) suggests that even if the use of DIST and the issue of scopal interaction can be somehow circumvented, it is still problematic to consider a *than*-clause a degree plurality.

- |       |  |   |
|-------|--|---|
| (125) | Mary didn't read the books.  | <b>Subject to homogeneity effects</b>     |
|       | <b>True</b> if Mary read none of the books.                                |   |
|       | <b>False</b> if Mary read all of the books.                                |   |
|       | <b>Neither true nor false</b> if Mary read some, but not all of the books. |   |
| (126) | My giraffe is not taller than every tree is.                               | <b>Not subject to homogeneity effects</b> |
|       | My giraffe is not taller than all trees are.                               | <b>Not subject to homogeneity effects</b> |
|       | <b>True</b> if my giraffe is taller than no trees.                         |   |
|       | <b>False</b> if my giraffe is taller than all trees.                       |   |
|       | <b>True</b> if my giraffe is taller than some, but not all trees.          |   |

Third, a distinction on the answerhood to *wh*-questions containing universal quantifiers (e.g., *every boy*) vs. definite plurals (e.g., *the boys*) also questions the 'degree plurality' analysis for *than*-clauses.

In (127), the degree question *how tall are the boys* (which contains a plural DP) can be answered by a fragment answer like *5 feet, 5 feet 6 inches, and 6 feet (respectively)*, while such a fragment answer sounds degraded for a degree question like *how tall is every boy* (which contains a universal quantifier). This contrast suggests that even if *5 feet, 5 feet 6 inches, and 6 feet* is indeed a degree plurality (i.e., a sum of degrees) and expressions like *than the boys are (tall)* indeed denote degree pluralities, it is unlikely that *than every boy is (tall)* also denotes a degree plurality. Instead, *between 5 and 6 feet*, which indicates an interval, is a good fragment answer here. Similar observations are available for other *wh*-questions. As illustrated in (128), while the sum *Madame Bovary, Jane Eyre, and Emma* is a felicitous fragment answer to *what did the boys read* (which contains a plural DP), it cannot be used to answer *what did every boy read* (which contains a universal quantifier).

1566 However, *a novel* is a good fragment answer to *what did every boy read* in this case. We do  
 1567 not delve into the details of fragment answerhood here, but the upshot is clear. For a  
 1568 *than*-clause containing a universal quantifier (e.g., *than every tree is (tall)*) instead of a  
 1569 plural DP, it is unlikely that this kind of *than*-clause denotes a degree plurality.

- 1570 (127) Context: Al, Bill, and Cal are 5 feet, 5 feet 6 inches, and 6 feet tall respectively.
- 1571 a. – How tall are **the boys**? ✓ 5 feet, 5 feet 6 inches, and 6 feet (respectively)
- 1572 b. – How tall is **every boy**? ? 5 feet, 5 feet 6 inches, and 6 feet (respectively)
- 1573 b'. – How tall is **every boy**? ✓ between 5 and 6 feet
- 1574 (128) Context: Al read *Madame Bovary*, Bill read *Jane Eyre*, and Cal read *Emma*.
- 1575 a. – What did **the boys** read? ✓ *Madame Bovary*, *Jane Eyre*, and *Emma*
- 1576 b. – What did **every boy** read? # *Madame Bovary*, *Jane Eyre*, and *Emma*
- 1577 b'. – What did **every boy** read? ✓ a novel

1578 Finally, we would like to cautiously point out that the very notion of ‘degree  
 1579 plurality’ might lack enough empirical support. The example in (129a) seems to give  
 1580 evidence that the notion of degree plurality is independently needed in natural  
 1581 language, since this sentence seems to have a cumulative reading (see Dotlačil and  
 1582 Nouwen 2016). However, it is likely that there is a silent *respectively* in this case (see  
 1583 (129b)). If it is so, then as a *respectively*-sentence, it is distinct from a typical  
 1584 cumulative-reading sentence. (130) and (131) show that in *respectively*-sentences, the  
 1585 order among the items conjoined by *and* matters, suggesting that in these cases, the use  
 1586 of *and* does not lead to sums of items as involved in typical cumulative-reading sentences  
 1587 (see e.g., Zhang 2015, Kubota and Levine 2015 for discussion).

- 1588 (129) a. These three trees are 16 feet, 18 feet, and 20 feet tall. **cumulative?**
- 1589 b. These three trees are 16 feet, 18 feet, and 20 feet tall, respectively.
- 1590 **not truly cumulative**
- 1591 (130) John and Bill married Susan and Kate (respectively). **not truly cumulative**
- 1592 ~ John married Susan, and Bill married Kate. **order matters**
- 1593 (131) The newborn’s weight, length, and head circumference are 3.4 kg, 49.7 cm, and
- 1594 33.6 cm, (respectively). **not truly cumulative**

1595 Among these challenges to the ‘degree plurality’ theory, the issue on unattested

scopal ambiguity (i.e., the first issue) is presumably carried over to other entanglement theories, because entanglement theories, by definition, involve the derivation and distribution of multiple distinct scalar values from a *than*-clause. Given that clausal comparatives lack scopal ambiguity, entanglement theories are less suitable than encapsulation theories in the analysis of *than*-clauses and clausal comparatives.

That being said, natural language degree-related phenomena beyond English clausal comparatives might still call for entanglement theories. As shown in (132), the degree question *how tall is every boy* can have a fragment answer that denotes a single, not-very-precise measurement, but it can also have a pair-list answer that involves multiple measurements. Thus a sufficiently good characterization of degree questions (and other phenomena like phrasal comparatives in (84a)) has to go beyond encapsulation theories alone.

(132) Context: Al, Bill, and Cal are 5 feet, 5 feet 6 inches, and 6 feet tall respectively.  
How tall is every boy?

a. Between 5 and 6 feet.

**Fragment answer**

b. Al is 5 feet tall; Bill is 5 feet 6 inches tall; Cal is 6 feet tall. **Pair-list answer**

As a typical encapsulation theory, our derivation for the sentential semantics of a clausal comparative is eventually only based on the upper and lower bounds of the interval associated with the *than*-clause, which is exactly the information encoded in a most informative fragment answer to the corresponding degree question (see (132a)). There seems to be information loss in this fragment answer, but we believe that this information loss reflects the actual semantics of English clausal comparatives that native speakers have access to. After all, English clausal comparatives are not as expressive as phrasal comparatives (see also Kennedy 1999).

## 7 Conclusion

In this paper, we have presented a difference-based approach to the semantics of comparatives. Comparatives encode a subtraction relation among three scalar values: two measurements along a relevant interval scale and the difference between them.

In implementing this difference-based approach, we have innovated (i) the interval-based technique of characterizing scalar values and differences for natural language phenomena and (ii) the view on the semantic contribution of comparative



morpheme *-er/more*. The technique of interval subtraction allows us to deal with subtraction equations that involve generalized, potentially not-very-precise scalar values. Comparative morpheme *-er/more* is considered an additive particle that contributes additivity by expressing a positive increase on a discourse-salient standard. The combination of these two ideas leads to our interval-subtraction-based analysis.

We have shown that our proposed analysis of comparatives naturally accounts for complex cases involving numerical differentials and *than*-clause-internal quantifiers, deriving their truth conditions in a most natural, precise, and uniform way. The proposed analysis also accounts for the scope island status and the monotonicity of *than*-clauses. Furthermore, our analysis accounts for Klein’s puzzle within degree semantics plus a unified comparison-based picture for various uses of gradable adjectives. Instead of encoding or marking comparison per se, *-er/more* rather marks the discourse status of the scalar value serving as the standard in comparison.

Our work makes good use of existing mathematical tools (i.e., interval subtraction) and is based on the background assumption that the theory on measurement contributes to our understanding of human conceptualization and their linguistic encoding. In this regard, our current work joins existing research (especially, [Fox and Hackl 2006](#)’s theory on the universal density of measurement in natural language, [Sassoon 2010](#)’s account for the limited distribution of measure phrases, and [Wellwood 2019](#)’s work on the structure-preserving of measure functions) in relating the formal computation and the intuitive cognition of measurement. We believe that our work will inspire future theoretical development and empirical investigation within degree semantics.

## Acknowledgments

[Zhang and Ling \(2015, 2017b\)](#) contain preliminary development of the main innovations of this paper. For comments and discussion on various parts and ideas of this work, we thank Duk-Ho An, Alan Bale, Chris Barker, Lucas Champollion, Nicholas Fleisher, Peter Hallman, Chris Kennedy, Haoze Li, Mingming Liu, Louise McNally, Rick Nouwen, Toshiko Oda, Colin Phillips, Roger Schwarzschild, Stephanie Solt, Anna Szabolcsi, Carla Umbach, and audiences at New York University, *the 20th Amsterdam Colloquium* (2015), *the 34th West Coast Conference on Formal Linguistics* (WCCFL 34, 2016), *Semantics and Linguistic Theory* (SALT 27, 2017), and the workshop on *Degrees & Grammar* at Nanjing University (2019). We are also very thankful to our editor, Benjamin Spector, and the

1659 three anonymous reviewers for their constructive and detailed comments. Linmin Zhang  
1660 gratefully acknowledges financial support from New York University, Concordia  
1661 University, and Shanghai Municipal Education Commission (the Program for Eastern  
1662 Young Scholar at Shanghai Institutions of Higher Learning). Inspired by the discussions  
1663 between the authors, most of the writing was happily undertaken by the first author  
1664 (Linmin Zhang). Any remaining errors are the first author's.

## References

- Abrusán, Márta. 2014. *Weak island semantics*. OUP Oxford.
- Alrenga, Peter, and Christopher Kennedy. 2014. *No more shall we part: Quantifiers in English comparatives*. *Natural Language Semantics* 22:1–53.
- Anderson, Curt, and Marcin Morzycki. 2015. Degrees as kinds. *Natural Language & Linguistic Theory* 33:791–828.
- Bale, Alan Clinton. 2008. A universal scale of comparison. *Linguistics and Philosophy* 31:1–55.
- Barker, Chris. 2002. The dynamics of vagueness. *Linguistics and philosophy* 1–36.
- Bartsch, Renate, and Theo Vennemann. 1972a. The grammar of relative adjectives and comparison. *Linguistische Berichte* 20:19–32.
- Bartsch, Renate, and Theo Vennemann. 1972b. *Semantic structures: A study in the relation between semantics and syntax*. Frankfurt am Main: Athenäum.
- Beaver, David I., and Brady Z. Clark. 2009. *Sense and sensitivity: How focus determines meaning*, volume 12. John Wiley & Sons.
- Beck, Sigrid. 2010. Quantifiers in *than*-clauses. *Semantics and Pragmatics* 3 (1):1–72.
- Beck, Sigrid. 2011. Comparative constructions. In *Semantics: An International Handbook of Natural Language Meaning*, ed. Claudia Maienborn, Klaus von Stechow, and Paul Portner, volume 2, 1341–1390. de Gruyter.
- Beck, Sigrid. 2013. Lucinda driving too fast again – the scalar properties of ambiguous *than*-clauses. *Journal of Semantics* 30:1–63.
- Beck, Sigrid. 2014. Plural predication and quantified ‘*than*’-clauses. In *The art and craft of semantics: A Festschrift for Irene Heim*, ed. Luka Crnić and Uli Sauerland, volume vol. 1, MITWPL 70, 91–115. MIT Working Papers in Linguistics.
- Bierwisch, Manfred. 1989. The semantics of gradation. In *Dimensional Adjectives: Grammatical Structure and Conceptual Interpretation*, ed. M. Bierwisch and E. Lang, 71–261. Berlin: Springer Verlag.

- 1692 Bobaljik, Jonathan David. 2012. *Universals in comparative morphology: Suppletion,*  
1693 *superlatives, and the structure of words*, volume 50. MIT Press.
- 1694 Brasoveanu, Adrian. 2008. Comparative correlatives as anaphora to differentials. In  
1695 *Semantics and Linguistic Theory*, volume 18, 126–143.
- 1696 Brasoveanu, Adrian. 2013. Modified Numerals as Post-Suppositions. *Journal of Semantics*  
1697 30:155 – 209.
- 1698 Bresnan, Joan. 1975. Comparative deletion and constraints on transformations. *Linguistic*  
1699 *Analysis* 1:25–74.
- 1700 Bresnan, Joan, and Jane Grimshaw. 1978. The syntax of free relatives in English.  
1701 *Linguistic Inquiry* 9:331–391.
- 1702 Bresnan, Joan W. 1973. Syntax of the comparative clause construction in English.  
1703 *Linguistic inquiry* 4:275–343.
- 1704 Bumford, Dylan. 2017. Split-scope effects in definite descriptions. Doctoral Dissertation,  
1705 New York University.
- 1706 Büring, Daniel. 2003. On D-trees, beans, and B-accent. *Linguistics and philosophy*  
1707 26:511–545.
- 1708 Büring, Daniel. 2007a. Cross-polar nomalies. In *Semantics and Linguistic Theory*, ed.  
1709 T. Friedman and M. Gibson, volume 17, 37–52. CLC Publications, Cornell University.
- 1710 Büring, Daniel. 2007b. More or less. In *Proceedings from the annual meeting of the Chicago*  
1711 *Linguistic Society*, volume 43 (2), 3–17. Chicago Linguistic Society.
- 1712 Burnett, Heather. 2017. *Gradability in Natural Language*. Oxford University Press.
- 1713 Caponigro, Ivano. 2003. Free not to ask: On the semantics of free relatives and wh-words  
1714 cross-linguistically. Doctoral Dissertation, University of California, Los Angeles.
- 1715 Charlow, Simon. 2014. On the semantics of exceptional scope. Doctoral Dissertation,  
1716 New York University.
- 1717 Chierchia, Gennaro, and Ivano Caponigro. 2013. Questions on questions and free  
1718 relatives. In *Proceedings of Sinn und Bedeutung*, volume 18.

- 1719 Chomsky, Noam. 1977. On *wh*-movement. In *Formal syntax*, ed. P. Culicover, T. Wasow,  
1720 and A. Akmajian, 71–132. Academic Press.
- 1721 Cresswell, Max J. 1976. The semantics of degree. In *Montague grammar*, ed. Barbara  
1722 Partee, 261–292. New York: Academy Press.
- 1723 Dotlačil, Jakub, and Rick Nouwen. 2016. The comparative and degree pluralities. *Natural*  
1724 *Language Semantics* 24:45–78.
- 1725 Embick, David. 2007. Blocking effects and analytic/synthetic alternations. *Natural*  
1726 *Language & Linguistic Theory* 25:1–37.
- 1727 Fauconnier, Gilles. 1978. Implication reversal in a natural language. In *Formal semantics*  
1728 *and pragmatics for natural languages*, 289–301. Springer.
- 1729 Fleisher, Nicholas. 2016. Comparing theories of quantifiers in *than* clauses: lessons from  
1730 downward-entailing differentials. *Semantics and Pragmatics* 9 (4):1–23.
- 1731 Fleisher, Nicholas. 2018. *Than* clauses as embedded questions. In *Semantics and Linguistic*  
1732 *Theory*, volume 28, 120–140.
- 1733 Fleisher, Nicholas. 2020. Nominal quantifiers in *than*-clauses and degree questions. In  
1734 *Syntax and Semantics Vol. 42: Interactions of Degree and Quantification*, 364–381. Brill.
- 1735 Fox, Danny, and Martin Hackl. 2006. The universal density of measurement. *Linguistics*  
1736 *and Philosophy* 29:537–586.
- 1737 Gajewski, Jon. 2008. More on quantifiers in comparative clauses. In *Semantics and*  
1738 *Linguistic Theory*, volume 18, 340–357.
- 1739 Gawron, Jean Mark. 1995. Comparatives, superlatives, and resolution. *Linguistics and*  
1740 *Philosophy* 333–380.
- 1741 Giannakidou, Anastasia, and Suwon Yoon. 2010. No NPI licensing in comparatives. In  
1742 *Proceedings of the 46th Meeting of the Chicago Linguistic Society*. Chicago, IL: Chicago  
1743 Linguistic Society.
- 1744 Greenberg, Yael. 2010. Additivity in the domain of eventualities (or: Oliver twist’s *more*).  
1745 In *Proceedings of Sinn und Bedeutung*, volume 14, 151–167.

- 1746 Hankamer, G. 1973. Why there are two *than*'s in English. In *Papers from the Ninth Regional*  
1747 *Meeting of the Chicago Linguistic Society*, ed. C. Corum, T. C. Smith-Stark, and S. Weisler.
- 1748 Hausser, Roland, and Dietmar Zaefferer. 1978. Questions and answers in a  
1749 context-dependent Montague grammar. In *Formal semantics and pragmatics for natural*  
1750 *languages*, 339–358. Springer.
- 1751 Heim, Irene. 1985. Notes on comparatives and related matters. Unpublished ms.,  
1752 University of Texas, Austin.
- 1753 Heim, Irene. 2006a. Remarks on comparative clauses as generalized quantifiers.  
1754 Unpublished ms., MIT.
- 1755 Heim, Irene. 2006b. *Little*. In *Semantics and Linguistic Theory*, ed. M. Gibson and  
1756 J. Howell, volume 16, 35–58. Ithaca, NY: Cornell University.
- 1757 Hellan, Lars. 1981. *Towards an integrated analysis of comparatives*. Tübingen: Narr.
- 1758 Hoeksema, Jack. 1983. Negative polarity and the comparative. *Natural Language &*  
1759 *Linguistic Theory* 1:403–434.
- 1760 Izvorski, Roumyana. 1995. A solution to the subcomparative paradox. In *Proceedings of*  
1761 *the 14th West Coast Conference on Formal Linguistics*, volume 14, 203–219. University of  
1762 Southern California.
- 1763 Jacobson, Pauline. 1995. On the quantificational force of English free relatives. In  
1764 *Quantification in natural languages*, ed. Emmon Bach, Eloise Jelinek, Angelika Kratzer,  
1765 and Barbara Partee, volume II, 451–486. Springer.
- 1766 Kamp, J. A. W. 1975. Two theories about adjectives. In *Formal semantics of natural*  
1767 *language*, ed. Edward Keenan, 123–155. Cambridge, UK: Cambridge University Press.
- 1768 Kennedy, Christopher. 1999. *Projecting the adjective: The syntax and semantics of gradability*  
1769 *and comparison*. Routledge.
- 1770 Kennedy, Christopher. 2011. Ambiguity and vagueness: An overview. In *Semantics: An*  
1771 *International Handbook of Natural Language Meaning*, ed. Claudia Maienborn, Klaus von  
1772 Heusinger, and Paul Portner, volume 1, 507–535. de Gruyter.

- 1773 Kennedy, Christopher, and Beth Levin. 2008. Measure of change: The adjectival core of  
1774 degree achievements. In *Adjectives and adverbs: Syntax, semantics and discourse*, 156–182.  
1775 Oxford University Press Oxford.
- 1776 Kennedy, Christopher, and Louise McNally. 2005. Scale structure, degree modification,  
1777 and the semantics of gradable predicates. *Language* 345–381.
- 1778 Klein, Ewan. 1980. A semantics for positive and comparative adjectives. *Linguistics and*  
1779 *philosophy* 4:1–45.
- 1780 Krifka, Manfred. 1999. At least some determiners aren't determiners. In *The*  
1781 *semantics/pragmatics interface from different points of view, volume 1 of current research in*  
1782 *the semantics/pragmatics interface*, ed. Ken Turner, 257–291. Elsevier Science B. V.
- 1783 Krifka, Manfred. 2011. Questions. In *Semantics: An international handbook of natural*  
1784 *language meaning*, ed. Klaus Heusinger, Claudia Maienborn, and Paul Portner,  
1785 volume 2, 1742–1785. Berlin: Mouton de Gruyter.
- 1786 Kripke, Saul A. 2009. Presupposition and anaphora: Remarks on the formulation of the  
1787 projection problem. *Linguistic Inquiry* 40:367–386.
- 1788 Križ, Manuel. 2016. Homogeneity, non-maximality, and *all*. *Journal of Semantics*  
1789 33:493–539.
- 1790 Kubota, Yusuke, and Robert Levine. 2015. Against ellipsis: Arguments for the direct  
1791 licensing of 'noncanonical' coordinations. *Linguistics and Philosophy* 38:521–576.
- 1792 Ladusaw, William. 1979. Negative polarity items as inherent scope relations. Doctoral  
1793 Dissertation, University of Texas at Austin.
- 1794 Ladusaw, William A. 1980. On the notion *affective* in the analysis of negative polarity  
1795 items. *Journal of Linguistic Research* 1:1–16.
- 1796 Landman, Fred. 2010. Internal and interval semantics for cp-comparatives. In *Logic,*  
1797 *language and meaning*, 133–142. Springer.
- 1798 Larson, Richard K. 1988. Scope and comparatives. *Linguistics and philosophy* 1–26.
- 1799 Lerner, Jan, and Manfred Pinkal. 1995. Comparative ellipsis and variable binding. In  
1800 *Semantics and Linguistic Theory*, volume 5, 222–236.



- 1801 Lerner, Jean-Yves, and Manfred Pinkal. 1992. Comparatives and nested quantification. In  
1802 *Computerlinguistik an der Universität des Saarlandes Report*, volume No.21.
- 1803 Lewis, David. 1979. Score-keeping in a language game. *Journal of Philosophical Logic*  
1804 8:339–359.
- 1805 Lewis, Davis. 1970. General semantics. *Synthese* 22:18–67.
- 1806 Marques, Rui. 2003. Licensing and interpretation of N-words in comparative clauses. In  
1807 *Proceedings of Sinn und Bedeutung*, ed. Matthias Weisgerber, volume 7, 199–212.
- 1808 McCawley, James D. 1976. Quantitative and qualitative comparison in English. In  
1809 *Grammar and Meaning*, 1–14.
- 1810 McConnell-Ginet, Sally. 1973. Comparative constructions in English: A syntactic and  
1811 semantic analysis. Doctoral Dissertation, University of Rochester.
- 1812 Moltmann, Friederike. 1992. Coordination and comparatives. Doctoral Dissertation, MIT.
- 1813 Moore, Ramon E. 1979. *Methods and Applications of Interval Analysis*. SIAM.
- 1814 Pancheva, Roumyana. 2006. Phrasal and clausal comparatives in Slavic. In *Formal*  
1815 *Approaches to Slavic Linguistics #14: The Princeton Meeting 2005*, 236–257. Michigan  
1816 Slavic Publications.
- 1817 Pinkal, Manfred. 1990. On the logical structure of comparatives. In *Natural language and*  
1818 *logic: Lecture notes in artificial intelligence*, ed. R. Studer, 146–167. Springer.
- 1819 Rett, Jessica. 2007. How *many* Maximizes in the Balkan Sprachbund. In *Semantics and*  
1820 *Linguistic Theory*, volume 16, 190–207.
- 1821 Rett, Jessica. 2008. Degree modification in natural language. Doctoral Dissertation,  
1822 Rutgers, the State University of New Jersey.
- 1823 Rett, Jessica. 2014. The polysemy of measurement. *Lingua* 143:242–266.
- 1824 Rett, Jessica. 2018. The semantics of *many*, *much*, *few*, and *little*. *Language and Linguistics*  
1825 *Compass* 12:e12269.

- 1826 Roberts, Craige. 1996. Information Structure in Discourse: Towards an Integrated Formal  
1827 Theory of Pragmatics. In *OSU Working Papers in Linguistics 49: Papers in Semantics*, ed.  
1828 J. H. Yoon and Andreas Kathol, 91–136.
- 1829 van Rooij, Robert. 2008. Comparatives and quantifiers. In *Empirical issues in syntax and*  
1830 *semantics: Papers from CSSP 2007*, ed. Olivier Bonami and Patricia Cabredo Hofherr,  
1831 volume 7, 423–444. Paris, France: CSSP, CNRS.
- 1832 Ross, John R. 1969. A proposed rule of tree-pruning. In *Modern studies in English:*  
1833 *Readings in transformational grammar*, 288–299. Englewood Cliffs NJ.
- 1834 Rullmann, Hotze. 1995. Maximality in the semantics of *wh*-constructions. Doctoral  
1835 Dissertation, University of Massachusetts Amherst.
- 1836 Russell, Bertrand. 1905. On denoting. *Mind* 14:479–493.
- 1837 Sassoön, Galit Weidman. 2010. Measurement theory in linguistics. *Synthese* 174:151–180.
- 1838 Schwarzschild, Roger, and Karina Wilkinson. 2002. Quantifiers in comparatives: A  
1839 semantics of degree based on intervals. *Natural language semantics* 10:1–41.
- 1840 Schwarzschild, Roger. 2005. Measure phrases as modifiers of adjectives. *Recherches*  
1841 *linguistiques de Vincennes* 34:207–228.
- 1842 Schwarzschild, Roger. 2008. The semantics of comparatives and other degree  
1843 constructions. *Language and Linguistics Compass* 2:308–331.
- 1844 Schwarzschild, Roger. 2013. Degrees and segments. In *Semantics and Linguistic Theory*,  
1845 volume 23, 212–238.
- 1846 Seuren, Pieter A. M. 1973. The comparative. In *Generative grammar in Europe*, ed. F. Kiefer  
1847 and N. Ruwet, 528–564. Springer.
- 1848 Seuren, Pieter A. M. 1984. The comparative revisited. *Journal of Semantics* 3:109–141.
- 1849 Solt, Stephanie. 2016. On measurement and quantification: The case of *most* and *more*  
1850 *than half*. *Language* 92:65–100.
- 1851 Solt, Stephanie, and Nicole Gotzner. 2012. Experimenting with degree. In *Semantics and*  
1852 *Linguistic Theory*, volume 22, 166–187.

- 1853 Spector, Benjamin. 2013. Bare numerals and scalar implicatures. *Language and Linguistics*  
1854 *Compass* 7:273–294.
- 1855 von Stechow, Arnim. 1984. Comparing semantic theories of comparison. *Journal of*  
1856 *semantics* 3:1–77.
- 1857 Stevens, S. S. 1946. On the theory of scales of measurement. *Science* 103:677–680.
- 1858 de Swart, Henriette. 1999. Indefinites between predication and reference. In *Semantics*  
1859 *and Linguistic Theory*, volume 9, 273–297.
- 1860 Szabolcsi, Anna. 1997. Strategies for scope taking. In *Ways of scope taking*, ed. Anna  
1861 Szabolcsi, 109–154. Springer.
- 1862 Szabolcsi, Anna. 2010. *Quantification*. Cambridge University Press.
- 1863 Thomas, Guillaume. 2010. Incremental *more*. In *Semantics and Linguistic Theory*,  
1864 volume 20, 233–250.
- 1865 Thomas, Guillaume. 2011. Another additive particle. In *Semantics and Linguistic Theory*,  
1866 volume 21, 634–651.
- 1867 Umbach, Carla. 2005. Why do modified numerals resist a referential interpretation? In  
1868 *Semantics and Linguistic Theory*, volume 15, 258–275.
- 1869 Wellwood, Alexis. 2019. *The Meaning of More*. Oxford University Press.
- 1870 Zeevat, Henk. 2004. Particles: Presupposition triggers, context markers or speech act  
1871 markers. In *Optimality theory and pragmatics*, 91–111. Springer.
- 1872 Zeevat, Henk, and Katja Jasinskaja. 2007. *And* as an additive particle. In *Language,*  
1873 *representation and reasoning. Memorial volume to Isabel Gómez Txurruka*, 315–340.  
1874 University of the Basque Country Press.
- 1875 Zhang, Linmin. 2015. Decomposing English particles *and* and *or*. In *NELS 45: Proceedings*  
1876 *of the 45th Meeting of the North East Linguistic Society*, volume 3, 261–270.
- 1877 Zhang, Linmin. 2018a. *Enough, too*, and causal dependence. In *Proceedings of Sinn und*  
1878 *Bedeutung*, ed. Uli Sauerland and Stephanie Solt, volume 22 (2), 481 – 498.

- 1879 Zhang, Linmin. 2018b. Modified numerals revisited: The cases of *fewer than 4* and *between*  
1880 *4 and 8*. In *Proceedings of Sinn und Bedeutung*, ed. Robert Truswell, Chris Cummins,  
1881 Caroline Heycock, Brian Rabern, and Hannah Rohde, volume 21, 1371 – 1388.
- 1882 Zhang, Linmin. 2019. The semantics of comparison in mandarin chinese. In *Proceedings*  
1883 *of GLOW in Asia XII & SICOOG XXI: Universal Grammar and Its Cross-linguistic*  
1884 *Instantiations*, ed. Sae-YounCho, 643–652.
- 1885 Zhang, Linmin. 2020a. Comparatives bring a degree-based NPI licenser. In *Proceedings of*  
1886 *TLLM 2020*, ed. Dag Westerståhl and Mingming Liu, TBA. Springer.
- 1887 Zhang, Linmin. 2020b. Degrees as kinds vs. degrees as numbers: Evidence from  
1888 equatives. In *Proceedings of Sinn und Bedeutung*, ed. M. Franke et al., volume 24 (2),  
1889 503–520.
- 1890 Zhang, Linmin. 2020c. Split semantics for non-monotonic quantifiers in *than*-clauses. In  
1891 *Syntax and Semantics Vol. 42: Interactions of Degree and Quantification*, ed. Peter Hallman,  
1892 332–363. Brill.
- 1893 Zhang, Linmin, and Jia Ling. 2015. Comparatives Revisited: Downward-Entailing  
1894 Differentials Do Not Threaten Encapsulation Theories. In *Proceedings of the 20th*  
1895 *Amsterdam Colloquium*, ed. Thomas Brochhagen, Floris Roelofsen, and Nadine Theiler,  
1896 478–487.
- 1897 Zhang, Linmin, and Jia Ling. 2017a. Ambiguous *than*-clauses and the mention-some  
1898 reading. In *Semantics and Linguistic Theory*, volume 27, 191–211.
- 1899 Zhang, Linmin, and Jia Ling. 2017b. LITTLE: Not a dichotomy-based negation operator,  
1900 but a trivalence-based polar opposition operator. In *Proceedings of the 34th West Coast*  
1901 *Conference on Formal Linguistics*, ed. Aaron Kaplan et al., 590–598. Cascadilla  
1902 Proceedings Project.