# From the Conservativity Constraint to the Witness Set Constraint*

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**Abstract**

This paper is initially concerned with a famous constraint on the class of possible determiners in natural languages: the so-called Conservativity Constraint. We shall briefly illustrate the force of this constraint and informally sketch Keenan and Stavi (1986)'s view according to which the Conservativity Constraint derives from the boolean structure of natural language semantics. We shall proceed to discuss two well-known linguistic categories that have been argued to be interpreted by non-conservative functions: *only* and the relative proportional *many*. We shall take the challenge posed by the existence of these categories in order to propose an alternative to the Conservativity Constraint. This alternative will be dubbed the Witness Set Constraint, which is inspired in Barwise and Cooper (1981)'s considerations on the semantic processing of generalized quantifiers. We shall defend that the proposed constraint does not suffer from the empirical shortcomings that have been attributed to the Conservativity Constraint, and indeed, we shall argue in detail that it correctly predicts (a) the existence of conservative determiners, (b) the non-existence of certain non-conservative determiners, such as inner negations, and most importantly, (c) the existence of the non-conservative functions denoted by *only* and the relative proportional *many*. This line of reasoning suggests that the class of generalized quantifiers permitted in natural languages is constrained by a principle that simplifies the semantic processing of sentences that contain generalized quantifiers.

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# 1 Introduction: the Conservativity Constraint

As has been clear since Montague's work on quantification in English Montague (1974), it is possible to characterize natural language determiners using the mathematical concept of *generalized quantifier* (Mostowski (1955), Sgro (1977)).

A generalized quantifier (and a natural language determiner more particularly) is a function that assigns to each set included in a given universe $U$ a family of subsets of $U$. For instance, if $A$ is a subset of $U$ denoted by a noun, the denotation of the quantified noun phrases $[\![all]\!]$(A), $[\![some]\!]$(A), and $[\![most]\!]$(A) would correspond to the following families of sets:

$$[\![All]\!](A) = \{X \subseteq E : A \subseteq X\}$$
$$[\![some]\!](A) = \{X \subseteq E : X \cap A \neq \emptyset\}$$
$$[\![Most]\!](A) = \left\{X \subseteq E : |A \cap X| > \left|A \cap \overline{X}\right|\right\}$$

Whereas $A$ is the property denoted by a noun, $[\![D]\!]$(A) is the set of properties denoted by a quantified phrase whose head is the determiner $D$.

Accordingly, given a particular Universe $U$ that includes the set $M$ of men and the set $R$ of runners, when we calculate the denotation of (1-a) we need to determine whether $R$ belongs to $[\![all]\!]$(M); or in other words, we need to check whether $M \subseteq R$. Similarly, in order to determine the denotation of (1-b) we must find out whether $R \cap M \neq \emptyset$. Finally, the sentence (1-c) will be true if and only if the intersection of $M$ and $R$ is larger than the intersection of $M$ and $\overline{R}$ (the set of non-runners).

(1) a. All men run
b. Some men run
c. Most men run

In principle, we could expect that, for any conceivable function from a subset of the universe $U$ to a family of subsets of $U$, there would exist a natural (language) determiner conveying such a function. If this were the case, natural languages determiners would realize all semantic possibilities, and consequently, one could argue, the class of natural determiners would have a maximal expressive power: it would be as rich as it would be logically possible.

However, Barwise and Cooper (1981)'s investigations of how Montague's treatment of quantification can be further developed in order to obtain important implications for a theory of natural languages suggested certain universals that constrain the set of natural determiners.

In this paper we shall be concerned with perhaps the most popular universal that constrains the set of natural determiners: the so called *Conservativity Constraint* or *Conservativity Universal*.

(2) *Conservativity Constraint* ((Keenan and Stavi, 1986, p.260))
Determiners in all languages are interpreted by *conservative* functions

As common, we shall understand that a determiner $D$ denotes a conservative function (or more concisely, that a determiner $D$ is conservative) iff:

(3) $D(A, B) \Leftrightarrow D(A, (A \cap B))$.

As is well known, the conservativity property has also been studied under the name *intersectivity* by Higginbotham and May (1981) and was originally conceived by Barwise and Cooper (1981) in terms of the property "lives on".

(4) *The property "lives on"* Barwise and Cooper (1981)
A quantifier $Q$ lives on a set $A \subseteq U$ if $Q$ is a set of subsets of $U$ with the property that for any $X \subseteq U$, $X \in Q$ iff $(X \cap A) \in Q$.

The intuition is that in quantified statements such as those in (1) the first argument of the determiner, namely, the set $M$, has a special status. $M$ provides the *restriction* or the *domain* of individuals relevant for the determiner. When we consider whether the relation denoted by the determiner holds between the domain provided by $M$ and the set $R$ (the second argument of the determiner), $M$ is *conserved*, i.e., it is *preserved* without being tampered with. It is in this sense that we can say, following Keenan and Stavi (1986)'s terminology, that a determiner is *conservative* on its first argument, and that the denotation of $[\![\text{D NP}]\!]$ (a quantifier, in Barwise and Cooper (1981)'s terminology), *lives on* the set denoted by the *NP* selected by *D* as its first argument.

Here we shall adopt the term *conservativity*, which is perhaps the most widely used since Keenan and Stavi (1986) coined it.

## 1.1 Basic illustrations

Let us briefly illustrate how the Conservativity Constraint bans certain determiners that would be otherwise expected. We shall focus our attention on the class of cardinal numerals.

To begin with, for any natural number $n \geq 1$, we can define a cardinal numeral $[\![n]\!]$ such that:

(5) $$[\![n]\!](A) = \{X \subseteq U : |X \cap A| \geq n\}.$$

Therefore the denotation $[\![n]\!](A)$ would be the set of all those subsets of $U$ whose intersection with $A$ has at least $n$ elements.

It is easy to see that all $[\![n]\!]$-numerals are conservative functions. Consider an arbitrary numeral $[\![n]\!]$ and let us check whether, for any two subsets $A$, $B$ of $U$, $[\![n]\!]$ satisfies the property of being conservative as defined in (3):

(6) $$[\![n]\!](A, B) \Leftrightarrow [\![n]\!](A, (A \cap B)).$$

If we now apply to (6) the general definition of $[\![n]\!]$-numeral provided in (5), we obtain the following biconditional:

(7) $|A \cap B| \geq n \Leftrightarrow |A \cap (A \cap B)| \geq n.$

Note that $A \cap A = A$, since intersection is an idempotent operation, whereby $A \cap B = A \cap (A \cap B)$. As a consequence, the biconditional (7) is necessarily true, and thus $[\![n]\!]$ is conservative.

The numeral *two* is an instance of conservative $[\![n]\!]$-numeral that denotes the set

$$[\![two]\!](D) = \{S \subseteq U : |S \cap D| \geq 2\}.$$

Accordingly, a sentence like *two doctors speak French* would be true iff the intersection of the set $D$ of doctors and the set $F$ of French speakers contains at least two members ($|S \cap D| \geq 2$).

We can easily check that *two* is interpreted by a conservative function by following the reasoning developed above for an arbitrary $[\![n]\!]$-numeral. The crucial observation is that $S \cap D = S \cap S \cap D$; as a consequence, it has to be the case that

(8)
$$|S \cap D| \geq 2 \Leftrightarrow |S \cap S \cap D| \geq 2,$$

which implies that $[\![two]\!]$ is conservative.

We can now continue by constructing, for any natural number $n \geq 1$, a cardinal $[\![\widehat{n}]\!]$ such that

(9)
$$[\![\widehat{n}]\!](A) = \left\{S \subseteq U : \left|S \cap \overline{A}\right| \geq n\right\}.$$

For $n = 2$, we obtain the $[\![\widehat{2}]\!]$-numeral –call it *owt*– that denotes the set

$$[\![owt]\!](A) = \left\{S \subseteq U : \left|S \cap \overline{A}\right| \geq 2\right\}.$$

In accordance with this definition, the sentence *owt doctors speak French* will be true iff there are at least two individuals in the universe who are not doctors and speak French.

Whereas $[\![n]\!]$-numerals are pervasive in natural languages, $[\![\widehat{n}]\!]$-numerals are plainly unattested. Nonetheless, both type of numerals seem to be logically legitimate; as a matter of fact, the intended meaning of a synthetic $[\![\widehat{n}]\!]$-numeral can be expressed by the combination of an $[\![n]\!]$-numeral and a negative marker *non-* prefixed to the noun.

(10) Two non-doctors speak French

But it is also true that $[\![n]\!]$-numerals are conservative, as we have just shown, whereas $[\![\widehat{n}]\!]$-numerals are non-conservative, since they do not fulfill the conservativity condition defined above (3). Indeed we can show that the following biconditional does not hold:

(11) $[\![\widehat{n}]\!](A, B) \Leftrightarrow [\![\widehat{n}]\!](A, (A \cap B))$.

Observe that the statement in the left hand of the biconditional (11), '$[\![\widehat{n}]\!](A, B)$', is a contingency: according to the definition of $[\![\widehat{n}]\!]$-determiner provided in (9), '$[\![\widehat{n}]\!](A, B)$' is true when the intersection of $\overline{A}$ and $B$ contains at least $n$ elements and false otherwise. The truth of this statement will depend on the particular model and the particular universe. For instance, the statement *owt doctors speak French* is true given a very realistic situation: we would only need to find at least two individuals who are not doctors and speak French.

However, if we apply the definition of $[\![\widehat{n}]\!]$-determiner given in (9) to the statement in the right hand of the biconditional (11), '$[\![\widehat{n}]\!](A, (A \cap B))$', we obtain the formula

(12)
$$\left|(A \cap B) \cap \overline{A}\right| \geq n,$$

which is necessarily false. In order to see this, note that it is always the case that $\left|A \cap \overline{A}\right| = 0$. As a consequence, $\left|(A \cap \overline{A}) \cap B\right| = 0$. And given that intersection is a commutative operation,

$$\left|(A \cap \overline{A}) \cap B\right| = \left|(A \cap B) \cap \overline{A}\right| = 0.$$

Therefore, the statement in (12) must be false, because $[\![\widehat{n}]\!]$-determiners –like $[\![n]\!]$-determiners– are defined for any natural number that is equal or larger to 1 and $\left|(A \cap B) \cap \overline{A}\right| = 0$. Given that the sentences in the left and the right hand of the biconditional (11), are respectively, a contingency and a contradiction, $[\![\widehat{n}]\!]$-numerals are non-conservative.[1]

[1]It is common to use not only in informal presentations but also in technical studies a particular type of paraphrase to informally verify whether or not a determiner is conservative (or whether or not a quantifier lives on). For instance, in order to verify that *two*, *some* and *all* are conservative, whereas *owt* is not, we can check whether the following biconditionals hold:

(i)
  a. Two doctors speak French ⇔ Two doctors are doctors that speak French
  b. Some doctors speak French ⇔ Some doctors are doctors that speak French
  c. All doctors speak French ⇔ All doctors are doctors that speak French
  d. Owt doctors speak French ⇔ Owt doctors are doctors that speak French

In examples (i-a), (i-b) and (i-c), if the sentences in the left hand of the biconditional are true, then the redundant sentences in the right hand must also be true, and vice versa. Therefore, their respective determiners are conservative. In example (i-d), the biconditional does not hold, for the first sentence (which is equivalent to *two non-doctors speak French*) is a contingency, whereas the second one (which is equivalent to *two non-doctors are doctors that speak French*) is a contradiction. Since the sentences that appear in the left side and in the right side of the biconditional symbol in examples (i) correspond, respectively, to $D(A, B)$ and $D(A, A \cap B)$, this intuitive procedure is used to verify the conservativity property defined in ((3)).

Here we articulate our arguments directly in set-theoretical terms, without resorting to natural language paraphrases, in order to attain the appropriate level of generality; this allows us to study not only particular determiners, such as *some*, *all*, *two* or *owt*, but crucially classes of determiners, such as $[\![n]\!]$-determiners and $[\![\widehat{n}]\!]$-determiners.

## 1.2 Motivation for the Conservativity Constraint

It is thus clear that languages instantiate only a portion of all logically possible determiners, and the Conservativity Constraint seems to have an important role in restricting the class of natural determiners. As has been argued by Keenan and Stavi (1986), the Conservativity Constraint is remarkably strong since it bans most ways in which the set $P$ of properties (i.e., of NP denotations) can be mapped into $P*$, the power set of $P$, the set of quantified NP denotations. According to Proposition 4 of (Keenan and Stavi, 1986, p.290),

> In a model with $n$ individuals there are $2^{2^{2n}}$ functions from P into P*. Provably only $2^{3n}$ of which are conservative.

For instance, as (Keenan and Stavi, 1986, p.290) observe, "in a model with only two individuals there are four (extensional) properties and $2^4 = 16$ sets of properties. So there are $16^4 = 65.536$ functions from P to P*. Provably only 512 of these are conservative!"

The Conservativity Constraint, (Keenan and Stavi, 1986, p.291) note, "may be interpreted in a way analogous to that in which linguists interpret syntactic constraints: the language learner does not have to seek the meaning of a novel determiner among all logically possible ways in which CNP denotations might be associated with NP denotations. He only has to choose from among those ways which satisfy conservativity". Thus, inasmuch as a syntactic constraint such as the Structure Dependence Principle (Chomsky (1972), Berwick et al. (2011)) restricts the class of possible internal merge operations, the Conservativity Constraint would narrow down the set of linguistically possible determiner denotations.

A further important question is why the possible determiner denotations of natural languages would have to be precisely conservative functions. Keenan and Stavi (1986)'s answer is that the Conservativity Constraint is a byproduct of the boolean structure of language (cfr. also van Benthem (1983)). Given a finite universe $U$, we can define an initial class of basic conservative determiner relations, say inclusion (*all*) and overlap (*some*). We can now apply to this set of basic determiners boolean operations (the conjunction, the negation and the disjunction of more basic determiners). Then the set of determiner relations that are generated is precisely the set of conservative determiners. In other words, "the set of conservative determiners is closed under the boolean operations and contains certain basic simple functions we need to interpret simple determiners (Keenan and Stavi, 1986, p.291)."

At this point we must cast doubt on this result for well-known empirical reasons (see also section 4): in spite of the power of the Conservativity Constraint in banning certain determiner relations, several instances of non-conservative relations have been attested in natural languages. In section 2 we shall succinctly present the case of *only* and the case of the relative proportional determiner *many*, two instances of linguistic categories that, according to the literature, are interpreted by non-conservative functions. In section 3 we shall present the Witness Set Constraint, which is inspired in Barwise and Cooper (1981)'s considerations on the semantic processing of quantified statements. We shall show how this constraint accounts not only for the remarkable absence of certain non-conservative determiners, such as the $[\![\widehat{n}]\!]$-numerals above defined, but also for the presence of attested non-conservative relations mentioned in section 2. In this sense, the Witness Set Constraint is more general and empirically more accurate than the Conservativity Constraint.

## 2 Attested non-conservative relations

In this section we shall mention two grammatical categories that have been argued to be interpreted by non-conservative functions.

The best-known instance of non-conservative relation denoted by a linguistic item is provided by *only*:

(13) Only Athenians think

If we assume the following simplified semantic definition of *only*,

(14) $[\![only]\!](A) = \{X : X \subseteq A\}\,,$

then the sentence in (13) will be true iff the set $T$ of thinkers is included in the set $A$ of Athenians.[2] For *only* to be conservative it must meet the following requirement:

(15) $[\![only]\!](A, B) \Leftrightarrow [\![only]\!](A, A \cap B)).$

If we apply to (15) the simplified semantics of *only* described in (14), we obtain the following biconditional:

$$B \subseteq A \Leftrightarrow A \cap B \subseteq B.$$

The sentence in the right hand of the biconditional is a set-theoretical truth: for any two sets $A$, $B$, $A \cap B \subseteq B$. But the truth of $A \cap B \subseteq B$ does not entail the truth of $B \subseteq A$. Therefore, *only* is not conservative.

It is common to consider that the non-conservative nature of *only* is not a problem for the Conservativity Constraint, since the syntactic distribution of *only* suggests that it is not a determiner but rather an adverb "of some type" (cfr., among others, van Benthem (1983); von Fintel (1997); von Fintel and Matthewson (2008)). For instance, *only* differs from canonical determiners in that it "can combine with pronouns or names", it "can occur on top of other determiners" and it can also "combine with categories other than noun phrases" (von Fintel and Matthewson, 2008, p.163).

(16) a. Only John/they came late
b. Only some guests came late
c. John only stayed for a couple of minutes

Thus, according to this view, the Conservativity Constraint applies solely to determiners; since *only* is not a determiner, it is not a counterexample to the Constraint.

[2]Cfr. von Fintel (1997) for a detailed study of the semantics of *only*.

However, in our view, even if the Conservativity Constraint is restricted to determiners and even if it is correct to claim that *only* cannot be a determiner, the non-conservativity of *only* poses an interesting and genuine problem for the Conservativity Constraint. If it is true that there can be no determiners interpreted by non-conservative functions, why is it that an alleged non-determiner can be interpreted by a non-conservative function? If the class of determiners must be semantically constrained in a particular way, why is it that the class of adverbs does not need to meet such a requirement? If it is true that the Conservativity Constraint is a byproduct of the boolean structure of language Keenan and Stavi (1986), should we stipulate that the adverb *only* does not reflect this deep semantic property whereas determiners must adhere to it?

There is a further observation that forces us to rethink the Conservativity Constraint. It has been known since Westerståhl (1985) that there is a rather uncontroversial determiner, *many*, that casts doubt on the Conservativity Constraint. As Westerståhl observes, sentence (17-a) is equivalent to sentence (17-b) if *many* has a relative proportional reading Cohen (2001).

(17) a. Many Scandinavians have won the Nobel Prize in literature
b. Many of the winners of the Nobel prize in literature were Scandinavians

Indeed, in 1984, by the time Westerståhl considered sentence (17-a), there were 14 winners of the Novel Prize in literature from Scandinavia out of a total of 81. This situation makes sentence (17-a) true under a relative proportional interpretation of *many*.[3]

Westerståhl's account for the relative proportional reading of (17-a) was

[3]The relative proportional interpretation of *many* is different from other interpretations that can be attributed to *many*: the so-called *cardinal interpretation* and *proportional interpretation* Partee (1988). Note, in this regard, that sentence (i) can mean either that the number of customers that bought the new product is considered large, or that a large proportion of customers bought the new product. In the former case we obtain a cardinal meaning of *many*, whereas in the latter we obtain a proportional reading.

(i) Many customers bought the new product based on the idea that the order of arguments of *many* is switched or reversed: although *many* forms a constituent with the NP to the exclusion of the VP in syntax, at the relevant level of semantic interpretation the VP and the NP would be respectively the first and the second argument of *many*. Crucially, *many* would not take its expected restriction, namely the set denoted by the NP *Scandinavians*, but rather the set denoted by the VP *have won the Nobel Prize in literature*. In this sense, the relative proportional reading of *many* would be non-conservative on the set denoted by the NP, but rather on the set denoted by the VP (cfr. also Keenan (1996) and Keenan (2002)).[4]

This reverse interpretation of the relative proportional reading of *many* is followed as well in Herburger (1997)'s proposal, according to which what triggers the reverse order of arguments of *many* is the focalization of *Scandinavians* (but see Cohen (2001) for discussion).

In the remaining of this article we shall argue for the Witness Set Constraint, an alternative to the Conservativity Constraint just reviewed, in order to account not only for the observation that certain non-conservative determiners are not attested, but also for the non-conservativity of *only* and *many*.

# 3 The Witness Set Constraint

In reading Barwise and Cooper (1981) it is clear that these authors were not concerned with deriving the property "lives on" (see above (4)) from deeper principles. They defined this property, which had gone unnoticed by semanticists, claimed that they were not aware of any natural language determiner that did not map a common noun denotation $A$ to a quantifier that did not live on $A$, and assumed the property in order to define certain linguistic universals and prove relevant propositions of their theory. The authors observed as well that the property is necessary in order to prove

[4] This is supported by the observation that sentence (17-a), under the reverse interpretation paraphrased in (17-b), is not equivalent to the sentence *many that are Scandinavian have won the Nobel prize in literature and are Scandinavian*. Cfr. footnote 1 for this type of paraphrase.

quantified statements such as (18), where the determiner *more than half* applies to an infinite set, the set of integers.

(18) More than half of the integers are not prime

The truth of this statement is not dependent "on an a priori logic", Barwise and Cooper note, but rather on "which underlying measure of infinite sets one is using" (Barwise and Cooper, 1981, p.163). This measure may live on different infinite sets: if the chosen metrics lives on the set of prime number, then the statement is false, but "more common measures which do not give special weight to primes will make [(18)] true" (Barwise and Cooper, 1981, footnote 3).

What is more relevant to our concerns is that the property of "lives on" (or equivalently, the conservativity property) may be related to the semantic processing of quantified statements. Let us focus our attention on this issue.

Barwise and Cooper mentioned an objection that could be leveled against Montague's treatment of NP's: in order to check the truth of a sentence like *John runs*, "we need to calculate the denotation of $[\text{John}]_{NP}$, namely, the family of all sets $X$ to which John belongs, and then see if the set of runners is one of these sets". This procedure for checking the truth of a simple sentence containing a proper noun seems "well nigh impossible", and "clearly corresponds in no way to the reasoning process actually used by a native speaker of English (Barwise and Cooper, 1981, p.191)". As a consequence, they suggest an intuitive checking procedure for simple NPs, that can be applied in general to quantified expressions, based on the notion of *witness set*. This notion incorporated the property of "lives on" in the following way ((Barwise and Cooper, 1981, p.191)):[5]

(19) *Witness set* (preliminary definition)
A witness set for a quantifier *D(A)* living on $A$ is any subset $w$ of A such that $w \in D(A)$.

[5]Cfr. Szabolcsi (1997) for an introduction to the notion of witness set and Beghelli et al (1997) for an application of this notion to the study of scope ambiguities.

The only witness set for $[\![John]\!]$ is the singleton $\{John\}$. A witness set for $[\![$a doctor$]\!]$ and for $[\![$most doctors$]\!]$ are, respectively, any non-empty subset of doctors and any subset of doctors that contains most doctors. And the only witness set for $[\![$no doctor$]\!]$ is $\emptyset$.

The notion of witness set simplifies the semantic processing of quantified statements by reducing the sets to be considered. In order to perceive this observation, note, for instance, that the set of presidents, the set of men and the set of pianists would all belong to the family $[\![$a doctor$]\!]$, for the intersection of any of those three sets with the set of doctors is non-empty: we may be able to find a president, a man or a pianist who is also a doctor. However, none of those sets would be a witness set for the family $[\![$a doctor$]\!]$, because none of them satisfies the condition of being a subset of the set of doctors, contrary to what the definition of witness set requires; indeed, in a very realistic situation, we may be able to find a president, a man and a pianist who is not a doctor. Consequently, those three sets would be irrelevant when, for instance, the statement *a doctor speaks French* is semantically processed.

Once a witness set $w$ for a quantified expression *D(A)* has been found, we can decide whether $X \in D(A)$ by following this procedure (Barwise and Cooper (1981), Szabolcsi (1997)):

(20) a. If $D$ is monotone increasing, check whether $w \subseteq X$
b. If $D$ is monotone decreasing, check whether $X \cap A \subseteq w$
c. If $D$ is non-monotonic, check whether $X \cap A = w$

Accordingly, in order to determine whether *a doctor speaks French* is true in a particular model we would need to find firstly a witness set $w$ for $[\![$a doctor$]\!]$, i.e., a non-empty subset of the set $D$ of doctors that, according to the model under consideration, belongs to $[\![$a doctor$]\!]$. Assume for instance that $w = \{Marie, David\}$, i.e., assume that Marie and David belong to $D$ and that $w$ belongs to $[\![$a doctor$]\!]$. Given that the determiner $a$ is monotone increasing, then we would need to follow step (20-a) and check whether $w$ is included in $F$, i.e., whether Marie and Peter, which belong to $D$, belong to $F$ as well.

The introduction of the property of "lives on" into the definition of witness set was quite natural in Barwise and Cooper (1981), especially because they had previously claimed that they knew "of no counterexamples in the world's languages to the following requirement":

> *U3. Determiner Universal* (Barwise and Cooper, 1981, p.177)
> Every natural language contains basic expressions (called determiners) whose semantic function is to assign to common noun denotations (i.e., sets) $A$ a quantifier that lives on $A$.

However, as we shall see, it is not necessary to bring the property of "lives on" (or equivalently, the conservativity property) into the definition of witness set, and indeed it is desirable to define the concept of witness set without appealing to it, in the following way:

(21) *Witness set* (final definition)
A set $w$ is a witness set for a generalized quantifier $D(A)$ iff $w$ is a subset of $A$ such that $w \in D(A)$.

This modification will allow us to argue for the following claims in the next subsections:

1. Attested conservative determiners have a witness set
2. Non-attested non-conservative determiners (such as $[\![\widehat{n}]\!]$-numerals) lack a witness set
3. Attested non-conservative functions (such as those expressed by *only* or the determiner *many* in a relative proportional reading) have a witness set

Thus, if we do not restrict the notion of witness set to conservative determiners, we can understand why certain non-conservative functions are attested in natural languages whereas others seem to be banned.

This leads us to a new understanding of conservativity. It is not a primitive principle of natural language semantics, but rather a byproduct of a truth calculation constraint that requires witness sets in order to develop a simpler model for how quantified statements are interpreted. If this is correct, the property that dictates what generalized quantifiers are possible in natural languages is not the Conservativity Constraint, but rather the Witness Set Constraint:

> *Witness set Constraint*
> All generalized quantifiers denoted by linguistic categories in natural languages need to have a witness set.

As we shall argue in detail, this provides us with a particularly interesting result: we can now account for the presence of certain categories that express relations that are non-conservative (on their first argument) with no further stipulation. It will be immaterial for our concerns whether *only* is a determiner or an adverb from a syntactic point of view, or whether certain determiners are conservative on their first argument, whereas others are conservative on their second argument. The crucial semantic requirement for generalized quantifiers in natural languages will be to have a witness set.

## 3.1 Attested conservative determiners and non-attested non-conservative determiners

**$[\![n]\!]$-numerals have a witness set.** The witness set for $[\![n]\!](A)$ is, by the definition of witness set (21), any subset of $A$ that belongs to $[\![n]\!](A)$. In other words, $w$ is a witness set for $[\![n]\!](A)$ iff $w \subseteq$ A and, by the definition of $[\![n]\!]$-determiner provided in (11), $|w \cap A| \geq n$.

This means, simply, that any subset of $A$ that contains at least $n$ elements will be a witness set for $[\![n]\!](A)$.

Consider, for concreteness, the $[\![n]\!]$-determiner *two*. In order to find out whether the statement *two doctors speak French* is true, i.e., whether $F \in [\![two]\!](D)$, we need to follow the next three steps. Firstly, we must find a witness set $w$ for $[\![two]\!](D)$; assume that $w = \{Pierre, John, Marie\}$. Secondly we must determine whether *two* denotes a monotone increasing function, a monotone decreasing function, or a non-monotonic function. As is well-known, $[\![two]\!]$ (and in general $[\![n]\!]$-determiners) are increasing monotonic, since

$$(X \in [\![two]\!](A) \wedge X \subseteq Y \subseteq U) \Rightarrow Y \in [\![two]\!](A).$$

Finally, according to the procedure indicated in (20-a), we must check whether $w = \{Pierre, John, Marie\}$ is included in $F$. Only if this is the case can we conclude that $F \in [\![two]\!](F)$.

**$[\![\widehat{n}]\!]$-numerals have no witness set.** Consider now $[\![\widehat{n}]\!]$-determiners. We shall see that they cannot have a witness set. Recall the semantic definition of $[\![\widehat{n}]\!]$-determiner previously provided in (9) and repeated below for clarity:

$$\text{For any } n \geq 1, X \in [\![\widehat{n}]\!](A) \text{ iff } \left|X \cap \overline{A}\right| \geq n.$$

According to this definition, for an arbitrary set $X \subseteq U$, $X$ belongs to $[\![\widehat{n}]\!](A)$ iff $\left|X \cap \overline{A}\right| \geq n$. Note, though, that for any $[\![\widehat{n}]\!]$-determiner, $X \cap \overline{A}$ must be non-empty, since $[\![\widehat{n}]\!]$-determiners are defined for any natural number $n$ that is equal or larger to 1. Consequently, if $X$ belongs to $[\![\widehat{n}]\!](A)$, then it has some elements that are not elements of $A$ (because they are elements of $\overline{A}$), whereby $X$ cannot be included in $A$. This leads us to the situation where

$$X \in [\![\widehat{n}]\!](A) \text{ iff } X \nsubseteq A,$$

which implies that there is no witness set for $[\![\widehat{n}]\!]$.

Note that $[\![\widehat{n}]\!]$-determiners are monotone increasing. For instance, the sentence *owt doctors arrived late* (i.e., "two individuals who are not doctors arrived late") entails the sentence *owt doctors arrived* ("two individuals who are not doctors arrived"), but not the other way around. However, $[\![owt]\!](D)$ lacks a witness set, since we cannot find a set $w$ such that:

$$w \subseteq D \wedge w \in [\![owt]\!](D).$$

Therefore, the procedure described in (20-a) to calculate the truth conditions of quantified statements yields no output: it cannot calculate whether $w \subseteq X$ because there is no witness set $w$. If witness sets are required, following Barwise and Cooper's insights, in order to provide a more feasible model for the computation of statements that contain generalized quantifiers, then the unavailability of witness sets for $[\![\widehat{n}]\!]$-determiners may be the source of the non-existence of this type of determiner.

**The non-attested 'allnon' determiner has no witness set.** The reasoning developed above can also be applied to account for the absence in the world's languages of a further non-conservative determiner, the *allnon* determiner discussed by (Chierchia and McConnell-Ginet, 2000, p. 426-427):

(22)
$$allnon(A) = \{X \subseteq U : (U - A) \subseteq B\}.$$

According to this definition of *allnon*, the following sentence would state that all the individuals of the universe $U$ under consideration who are not doctors speak French.

(23) Allnon doctors speak French

In this sentence *allnon* would take as its first argument the noun *doctors*, whose denotation is the set $D$ of doctors, and as its second argument the VP *speak French*, whose denotation is the set $F$ of French speakers.

Let us investigate whether there is any witness set $w$ for $[\![allnon]\!](D)$, i.e., any set $w$ such that:

$$w \subseteq D \wedge w \in allnon(D).$$

According to the definition of *allnon* provided in (22), a set $X$ belongs to $[\![allnon]\!]$(D) iff $(U - D) \subseteq X$. Therefore, $X$ belongs to $[\![allnon]\!]$(D) iff $X$ contains all the elements of the universe that are not contained in $D$, in which case $X$ is not included in $D$. This means that

$$X \in [\![allnon]\!](D) \text{ iff } X \not\subseteq D.$$

In other words, whereas $D$ contains all doctors of $U$, $[\![allnon]\!](D)$ contains all sets whose members are not doctors. For this reason, there can be no set $w$ that is both included in $D$ and contained in $[\![allnon]\!](D)$, whereby there is no witness set for *allnon*.

In sum, "inner negations" such as those denoted by *allnon* or by $[\![\widehat{n}]\!]$-determiners have no witness set. Note that $[\![allnon]\!](D)$ shares the property of being monotone increasing with $[\![\widehat{n}]\!]$-determiners, since *allnon doctors arrived late* entails the sentence *allnon doctors arrived*, but not the other way around.

## 3.2 Witness sets for attested non-conservative relations

The non-attested inner negations studied above can be banned by both the Conservativity Constraint and the Witness Set Constrain. With the aim of defending that the latter is empirically more adequate than the former we shall show that certain well-attested and non-conservative functions expressed by linguistic categories do have a witness set. This will strongly suggest that the Witness Set Constraint is a more likely candidate for a universal principle that constrains the set of functions that are expressed by generalized quantifiers in natural languages.

**'Only'.** The relation denoted by *only* is non-conservative but has a witness set. Recall that:

$$[\![only]\!](A) = \{X \subseteq E : X \subseteq A\}.$$

This means that $X \in [\![only]\!](A)$ iff $X \subseteq A$. Thus, any subset of $A$ will be a member of $X \in [\![only]\!](A)$, and any member of $[\![only]\!](A)$ will be a subset of $A$. Consequently, the existence of witness sets for *only* is ensured given the definition (21) of witness set.

Note that *only* is neither monotone increasing nor monotone decreasing. Consider, for instance, the two statements *only Athenians arrived late* and *only Athenians arrived*. Observe that the set of individuals who arrived late is included in the set of individuals who arrived. But neither the first statement entails the latter, nor the latter entails the former.

Therefore, *only* is non-monotonic; as a consequence, we shall semantically process the statement *only Athenians think* by checking whether the intersection of the set $T$ of thinkers and the set $A$ of Athenians is a witness set (see (20-c)), i.e., whether $T \cap A$ is both a subset of $A$ and a member of $[\![only]\!](A)$. Only if this is the case will the statement under consideration be true.

**The relative proportional reading of 'many'.** Let us finally consider the relative proportional reading of *many* in the following sentence, repeated from (17-a):

(24) Many Scandinavians have won the Nobel Prize in literature

We shall investigate whether the determiner *many*, in the relative proportional reading, has a witness set. We shall adapt Cohen (2001)'s characterization of *many*. We need to consider the following three basic sets:

1. the set $S$ of Scandinavians,
2. the set $L$ of Nobel laureates in literature, and
3. the set $\bigcup T$ of individuals belonging to the set $T$ of territorial identities.

We shall follow Cohen's criticism of the reverse interpretation, and thus we shall assume that in (24) $S$ is the first argument and $L$ the second argument of *many*.

Intuitively, the statement in (24) is true "iff the proportion of Scandinavians who won the Nobel Prize is greater than the proportion of Nobel laureates in literature in the world (or European) population as a whole (Cohen, 2001, p.54)". Formally, in order to calculate the denotation of (24) we need to check whether $L$ belongs to

(25)
$$[\![many]\!](S) = \left\{X : \frac{|S \cap X|}{|S|} > \frac{|\bigcup T \cap X|}{|\bigcup T|}\right\}.$$

We thus need to verify whether:

(26)
$$\frac{|S \cap L|}{|S|} > \frac{|\bigcup T \cap L|}{|\bigcup T|}.$$

We can show that, for any set $X \subseteq U$, $S \cap X$ is a valid witness set of $[\![many]\!](S)$, since

$$[(S \cap X) \subseteq S] \wedge [(S \cap X) \in [\![many]\!](S)].$$

We shall consider firstly the first component of this conjunction and secondly the second component. Thus, to begin with, note that it is necessary the case that $S \cap X$ is included in $S$, for the intersection of two arbitrary sets is included in both sets. Therefore, $(S \cap X) \subseteq S$ is always true.

Now we need to determine under what circumstances $S \cap X$ belongs to $[\![many]\!](S)$. According to the definition of the relative proportional determiner *many* provided in (25), the set $S \cap X$ will belong to the denotation of $[\![many]\!](S)$ iff:

(27)

$$\frac{|S \cap (S \cap X)|}{|S|} > \frac{|\bigcup T \cap (S \cap X)|}{|\bigcup T|}.$$

But note that $S \cap X = S \cap (S \cap X)$. Accordingly, the inequality (27) is equivalent to the following inequality:

$$\frac{|S \cap X|}{|S|} > \frac{|\bigcup T \cap X|}{|\bigcup T|}.$$

This implies that $S \cap X \in [\![many]\!](S)$ iff X $\in [\![many]\!](S)$. Therefore, we can find a witness set $w$ for $[\![many]\!](S)$, namely the intersection of $S$ with a subset $X$ of the Universe.

We can now use this witness set to study how the statement (24) is semantically processed. In this regard, note that the relative proportional *many* is non-monotonic: it is not monotone increasing because statement (24) does not entail the statement *Many Scandinavians have won the Nobel prize*, and it is not monotone decreasing because the latter statement does not entail either the former. Accordingly, for us to process statement (24) we need to follow step (20-c) and verify whether $S \cap L$ is a witness set, i.e., whether it is included in $S$ and belongs to $[\![many]\!](S)$.

# 4 Discussion

In this paper we have argued for the Witness Set Constraint, which requires that generalized quantifiers denoted by grammatical categories of natural languages possess a witness set. This has several interesting consequences for the understanding of what universal principles determine the set of possible generalized quantifiers in natural languages.

Firstly, we can understand why inner negations (such as the *allnon* determiner or the $[\![\widehat{n}]\!]$-determiners) are banned; these non-conservative relations are not permitted because they have no witness set. If witness sets are necessary to provide a feasible account for the semantic processing of sentences with generalized quantifiers, as suggested by Barwise and Cooper (1981), then we can understand the absence of these non-conservative relations as a result of a simplicity condition that is required in order to calculate the truth conditions of quantified statements in a simpler and more realistic way.

Secondly, two familiar categories that have been argued to denote non-conservative functions, namely *only* and the relative proportional *many*, do have a witness set. Therefore, their existence is anticipated by the Witness Set Constraint. We do not need to consider whether or not *only* is a determiner, whether the relative proportional reading of *many* is conservative on the second argument, or whether any other operations, such as focalization, are involved in order to allow determiners that are non-conservative on their first argument.

As a consequence, not only the absence of certain non-conservative categories but most interestingly the existence of certain non-conservative categories can be derived from a presumable constraint involved in the semantic processing of generalized quantifiers. This indicates the empirical superiority of the Witness Set Constraint compared to the Conservativity Constraint.

Finally, we would like to remark that the approach developed in these pages, according to which it is a constraint on semantic processability that restricts the set of generalized quantifiers permitted in natural languages, has a more restrictive and accurate power of definability than the argument developed by Keenan and Stavi (1986). According to this argument (informally sketched in section 1.2), natural language determiners would need to be conservative as a result of the boolean structure of natural language semantics. However, as noted by van Benthem (1983), every binary relation between subsets of a universe can be defined using boolean operations, "for every such relation may be viewed as a (finite) disjunction of singleton cases" (cfr. (van Benthem, 1983, p.455) for a detailed discussion). This "simple amendment" introduced by van Benthem allows us to generate inner negations, such as *allnon* –as van Benthem himself notes– and $[\![\widehat{n}]\!]$-determiners). This leads us "to excessive power of definability", since finite disjuntions of singletons generate non-attested determiners that indeed are non-conservative, such as inner negations.

Nonetheless, the Witness Set Constraint seems to predict in an accurate way not only the existence of conservative determiners and the non-existence of certain non-conservative determiners, such as inner negations, but also the existence of certain categories that are interpreted by non-conservative functions.

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