Cumulative reading, QUD, and maximal informativeness

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0. Overview. Sentence (1) has a distributive reading and a cumulative reading (see Brasoveanu 2013). This paper focuses on its **cumulative reading** and argues that **(i)** although cumulative reading involves multiple modified numerals, it does not involve multiple independent maximality operators, but only one, and **(ii)** this maximality operator is not mereology-based, but informativeness-based, with regard to a salient QUD.

(1) $\underline{\text{Exactly three}^u\text{ boys}}$ saw $\underline{\text{exactly five}^\nu\text{ movies}}$. Brasoveanu (2013)
Cumulative reading: The cardinality of all boys who saw any movies is 3, and the cardinality of all movies seen by any boys is 5.

1. Background. According to Brasoveanu (2013)'s account for the cumulative reading of (1), the semantic contribution of modified numerals, *exactly three* (*boys*) and *exactly five* (*movies*), is two-fold (or split, see also Bumford 2017): **(i)** First, (the indefinite component of) modified numerals introduce (plural) discourse referents (drefs), x and y; **(ii)** Then, after all the relevant restrictions, i.e., $\text{BOY}(x)$, $\text{MOVIE}(y)$, and $\text{SEE}(x, y)$, are added onto these drefs, (the definite component of) modified numerals impose maximality and cardinality tests, as delayed, post-suppositional evaluations (i.e., these tests are applied at a global sentential level, and in particular, after the restriction $\text{SEE}(x, y)$ is applied). The maximality tests pick out the maximal boy-sum and movie-sum, and the cardinality tests check the cardinalities of the maximal boy-sum and movie-sum (see (2)).

(2) $$[\![(1)]\!] \Leftrightarrow \underbrace{\sigma x \sigma y[\text{BOY}(x) \wedge \text{MOVIE}(y) \wedge \text{SEE}(x, y)]}_{\text{the mereologically maximal } x \text{ and } y \text{ satisfying these restrictions}} \wedge \underbrace{|y| = 5 \wedge |x| = 3}_{\text{cardinality tests}}$$

(σ: maximality operator; for notation simplicity, cumulative closure is assumed.)

Crucially, as pointed out by Brasoveanu (2013), the two maximality operators need to be applied **simultaneously** to guarantee that all 'boy-seeing-movie' events in the context are taken into consideration. Otherwise, the reading (3) would be derived. Intuitively, (1) is true in the context of Fig. 1, but false in the context of Fig. 2, indicating that the pseudo-cumulative reading in (3) (true for Fig. 2) is unattested and needs to be blocked.

(3) **Unattested pseudo-cumulative reading of (1)**:

$$\underbrace{\sigma x[\text{BOY}(x) \wedge \underbrace{\sigma y[\text{MOVIE}(y) \wedge \text{SEE}(x, y)]}_{\text{the mereologically maximal } y} \wedge |y| = 5]}_{\text{the mereologically maximal } x} \wedge |x| = 3$$

i.e., The maximal plural individual x satisfying the restrictions (i.e., atomic members of x are boys and each of them saw some movies, and x saw a total of 5 movies between them) has the cardinality of 3. $\rightsquigarrow$ **True** for Fig. 2! (see $b_2 \oplus b_3 \oplus b_4$ and $b_1 \oplus b_2 \oplus b_4$, and there is no larger boy-sum satisfying these restrictions.)

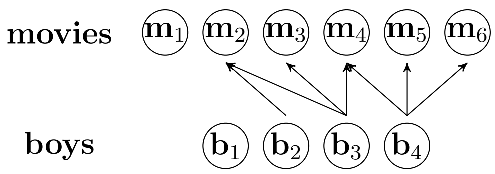

Figure 1: The genuine **cumulative** reading of (1) is **true** in this context.

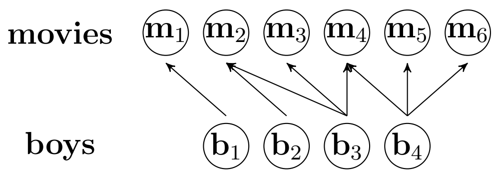

Figure 2: The genuine **cumulative** reading of (1) is **false** in this context.

2. An empirical challenge. As already pointed out by Krifka (1999), such a mereological-maximality-based analysis does not work for the case of (4). (4) is parallel with (1) in having a cumulative reading. However, Krifka (1999) argues that mereology-based maximality operators should pick out all land-owners and all the land, yielding an uninformative sentence like *In Guatemala, 100% of the population own 100% of the land*: 'What is peculiar with (4) is that it wants to give information about the bias of a statistical distribution . . . a picture of the skewing of the land distribution'. Evidently, this skewed picture is due to the contrast between a small part of population and a large part of land.

(4) In Guatemala, <u>at most 3%</u> of the population own <u>at least 70%</u> of the land.

Krifka (1999)'s discussion on (4) suggests that in accounting for the cumulative reading, (i) mereological maximality might only be a special case, and (ii) the multiple numerical expressions that together contribute to the cumulative reading should be inter-connected (see also Brasoveanu 2013's **simultaneity** in applying maximality operators).

3. Proposal. Obviously, sentences like (1) and (4) are used in different contexts, addressing different QUDs. Intuitively, (1) tells about an overall picture of film consumption among boys and can serve as a felicitous answer to questions like *how many boys saw how many movies*. Thus as analyzed by Brasoveanu (2013), when (1) is felicitously used, its interpretation is based on the cardinality measurement of the mereologically maximal relevant boy-sum and movie-sum (see the right-uppermost dot circled out in Fig. 3).

Then as argued by Krifka (1999), (4) addresses a degree QUD like *how skewed wealth distribution is in Guatemala*. Thus when (4) is felicitously used, its interpretation is based on the ratio between the percentage of owned land and the percentage of their owners in the population, and the extreme value of this ratio (or gradient) is achieved at the left-uppermost corner of the parallelogram (i.e., the point circled out) in Fig. 4.

The right-uppermost dot in Fig. 3 and the left-uppermost corner in Fig. 4 constitute extreme cases and represent maximal informativeness in addressing their respective QUD.

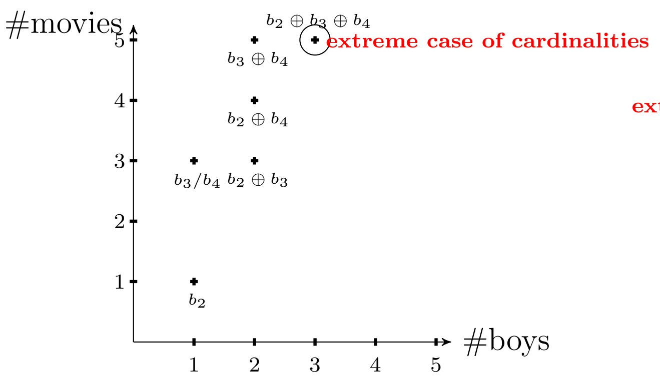

Figure 3: QUD: How much is the overall film consumption among boys? The cardinalities of some boy-sums and movie-sums in the context of Fig. 1 are plotted as dots.

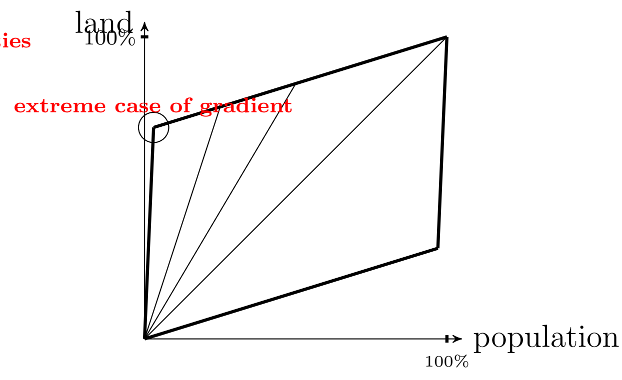

Figure 4: QUD: How skewed is wealth distribution in Guatemala? The percentages of the population and their owned land are within a parallelogram-like area.

Thus I propose a new **QUD-related, informativeness-based maximality operator** and implement it within a dynamics semantics framework:

(5) $\mathbf{M}_{u_1,u_2,\ldots} \stackrel{\text{def}}{=} \lambda m.\lambda g.\,\{h \in m(g) | \neg\exists h' \in m(g).\, G_{\text{QUD}}(h'(u_1,u_2,\ldots)) >_{\text{info}} G_{\text{QUD}}(h(u_1,u_2,\ldots))\}$
(Type of m: $g \rightarrow \{g\}$; Type of $\mathbf{M}$: $(g \rightarrow \{g\}) \rightarrow (g \rightarrow \{g\})$)

I assume meaning derivation to be a series of updates from an information state to another, and an information state is represented as a function from an input assignment function to an output set of assignment functions. As shown in (5), the operator $\mathbf{M}_{u_1,u_2,\ldots}$ works like a filter on information states. With the application of $\mathbf{M}_{u_1,u_2,\ldots}$, the discourse referents (drefs, which are assigned to $u_1, u_2, \ldots$) that lead to the maximal informativeness in resolving a QUD will be selected out. More specifically, to represent the resolution of a QUD, the operator G_{QUD} is applied on drefs and returns a value indicating informativeness. In this sense, G_{QUD} can be considered similar to a measurement function.

In addressing an overarching QUD like how high film consumption is among boys, higher informativeness means higher consumption (e.g., with $d_1 > d_2$, *the consumption level is d_1-high* entails (i.e., is more informative than) *the consumption level is d_2-high*). Thus the measurement of informativeness amounts to the measurement of cardinalities of plural drefs (see (6b)). Maximal informativeness is achieved when the mereologically maximal drefs (i.e., $b_2 \oplus b_3 \oplus b_4$ and $m_2 \oplus m_3 \oplus m_4 \oplus m_5 \oplus m_6$ in Fig. 1) are assigned (see (6c)). A step-by-step compositional analysis for (1) is thus shown in (6):

(6) a. $p = [\![\text{some}^u \text{ boys saw some}^\nu \text{ movies}]\!] = \lambda g.\ \left\{ g^{\nu \mapsto y}_{u \mapsto x} \,\middle|\, \textsc{movie}(y), \textsc{boy}(x), \textsc{saw}(x,y) \right\}$

b. $G_{\text{QUD}} = \lambda x.\lambda y.|x| + |y|$ (Simultaneously maximizing x and y is QUD-driven, not stipulated. cf. Brasoveanu 2013.)

c. $$\mathbf{M}_{u,\nu}(p) = \lambda g.\ \left\{ g^{\nu \mapsto y}_{u \mapsto x} \,\middle|\, \begin{array}{l} y = \iota y.[\textsc{movie}(y) \wedge \underbrace{\exists x[\textsc{boy}(x) \wedge \textsc{saw}(x,y)]}_{\text{some boys saw } y} \\ \wedge \underbrace{\forall y' \neq y[\textsc{movie}(y') \wedge \exists x[\textsc{boy}(x) \wedge \textsc{saw}(x,y')] \rightarrow y' \sqsubset y]}_{y \text{ is mereologically maximal}}\ ] \\ x = \iota x.[\textsc{boy}(x) \wedge \underbrace{\exists y[\textsc{movie}(y) \wedge \textsc{saw}(x,y)]}_{x \text{ saw some movies}} \\ \wedge \underbrace{\forall x' \neq x[\textsc{boy}(x) \wedge \exists y[\textsc{movie}(y) \wedge \textsc{saw}(x,y)] \rightarrow x' \sqsubset x]}_{x \text{ is mereologically maximal}}\ ] \end{array} \right\}$$

d. $[\![(1)]\!] = [\![\text{exact } 3^u \text{ boys saw exactly } 5^\nu \text{ movies}]\!] =$
$$\lambda g.\ \left\{ g^{\nu \mapsto y}_{u \mapsto x} \,\middle|\, \begin{array}{l} y = \iota y.[\textsc{movie}(y) \wedge \exists x[\textsc{boy}(x) \wedge \textsc{saw}(x,y)] \\ \wedge \forall y' \neq y[\textsc{movie}(y') \wedge \exists x[\textsc{boy}(x) \wedge \textsc{saw}(x,y')] \rightarrow y' \sqsubset y]\ ] \\ x = \iota x.[\textsc{boy}(x) \wedge \exists y[\textsc{movie}(y) \wedge \textsc{saw}(x,y)] \\ \wedge \forall x' \neq x[\textsc{boy}(x) \wedge \exists y[\textsc{movie}(y) \wedge \textsc{saw}(x,y)] \rightarrow x' \sqsubset x]\ ] \end{array} \right\}, \text{ if } \begin{array}{l} |x| = 3, \\ |y| = 5 \end{array}$$

In addressing how skewed wealth distribution is in Guatemala, higher informativeness corresponds to higher skewedness, which means higher gradient (see Fig. 4). Thus the measurement of informativeness amounts to the measurement of the ratio between the quantity of drefs (see (7b)). Maximal informativeness is achieved when the quantity of a dref y satisfying $\textsc{land}(y) \wedge \textsc{own}(x,y)$ divided by the quantity of a dref x satisfying $\textsc{human}(x) \wedge \textsc{own}(x,y)$ yields the maximal quotient (see (7c)). A step-by-step compositional analysis for (4) is thus shown in (7).

(7) a. $p = [\![\text{some}^u \text{ population own some}^\nu \text{ land}]\!]$
$= \lambda g.\ \left\{ g^{\nu \mapsto y}_{u \mapsto x} \,\middle|\, \textsc{land}(y), \textsc{human}(x), \textsc{own}(x,y) \right\}$

b. $G_{\text{QUD}} = \lambda x.\lambda y.|y| \div |x|$

c. $\mathbf{M}_{u,\nu}(p) = \lambda g.\left\{ g^{\nu\mapsto y}_{u\mapsto x} \middle| \begin{array}{l} \langle x, y\rangle = \langle \iota x, \iota y\rangle \text{ such that } \textsc{land}(y) \wedge \textsc{human}(x) \wedge \textsc{own}(x,y) \\ \wedge \neg \underbrace{\exists y' \sqsupset y[\textsc{land}(y') \wedge \textsc{own}(x,y')]}_{y \text{ is the maximal land owned by } x} \\ \wedge \underbrace{\neg\exists x' \sqsupset x[\textsc{human}(x') \wedge \textsc{own}(x', y)}_{x \text{ is the maximal owner of } y}] \\ \wedge \forall x'' \forall y''[\textsc{land}(y'') \wedge \textsc{human}(x'') \wedge \textsc{own}(x'', y'') \\ \wedge \underbrace{\neg\exists y''' \sqsupset y''[\textsc{land}(y''') \wedge \textsc{own}(x'', y''')]}_{y'' \text{ is the maximal land owned by } x''} \\ \wedge \underbrace{\neg\exists x''' \sqsupset x''[\textsc{human}(x''') \wedge \textsc{own}(x''', y'')]}_{x'' \text{ is the maximal owner of} y''} \rightarrow \frac{|y|}{|x|} \geq \frac{|y''|}{|x''|}] \end{array} \right\}$

d. $[\![(4)]\!] = [\![$at most $3\%^u$ of the population own at least $70\%^\nu$of the land$]\!] = \mathbf{M}_{u,\nu}(p)$, if $|x| \subseteq (0, 3\%], |y| \subseteq [70\%, 1]$

4. Discussion. Under the current analysis, it is the contextually salient QUD (i.e., what interlocutors care about, their ultimate motivation behind uttering sentences) that determines how informativeness is actually measured (see the implementation of G_{QUD} in (6b) vs. (7b)), which further determines how the informativeness-based maximality operator $\mathbf{M}_{u_1,u_2,...}$ filters on drefs (before the evaluation of the quantity of selected drefs).

The notion of informativeness-based maximality proposed here is in the same spirit as but more generalized than the one proposed by Fintel et al. (2014) (which primarily aims to account for the interpretation of *the*; see also Schlenker 2012). According to Fintel et al. (2014), informativeness ordering is based on entailment relation (see (8)).

(8) Fintel et al. (2014)'s notion of informativeness ordering: For all x, y of type α and property ϕ of type $\langle s, \langle \alpha, t\rangle\rangle$, $x \geq_\phi y$ iff $\lambda w.\phi(w)(x)$ entails $\lambda w.\phi(w)(y)$.

Thus as shown in (9), depending on the monotonicity of properties, maximal informativeness corresponds to maximum or minimum values.

(9) a. For upward monotone properties, maximal informativeness means maximum values: e.g., given that $6 > 5.5$, *Mary is* $6'$ *tall* entails *Mary is* $5.5'$ *tall.*

b. For downward monotone properties, maximal informativeness means minimum values: e.g., given that $m > n$, *n walnuts are sufficient to make a pan of baklava* entails *m walnuts are sufficient to make a pan of baklava.*

Compared to Fintel et al. (2014), the notion of QUD-based maximal informativeness developed in the current paper is more generalized in two aspects.

First, in cumulative-reading sentences where multiple numerical expressions can be involved, maximal informativeness does not directly correspond to whether the uttered numbers are considered maximum or minimum values. In example (4), as observed by Krifka (1999), each of the numerical expressions (i.e., *at most 3%* and *at least 70%*) alone cannot be maximum or minimum values. It is how the combination of these uttered numbers contributes to resolve an implicit, underlying QUD that leads to the achievement of maximal informativeness.

Second, it is worth noting that under the scenario of Fig. 1, although *exactly 3 boys saw exactly 5 movies* holds true, *exactly 1 boy saw exactly 4 movies* does not hold true (in Fig. 1, no boys saw more than 3 movies). Thus, it seems problematic to build informativeness ordering directly upon the entailment relation between uttered sentences and their alternatives (here derived by replacing uttered numbers with other numbers).

However *exactly 3 boys saw exactly 5 movies* does indicate a higher film consumption (or a more prosperous situation) than the consumption level indicated by *exactly 1 boy saw exactly 4 movies*, i.e., the uttered sentence indicates a higher informativeness in addressing an underlying QUD than its alternatives. In this sense, by resorting to QUD, the current proposal provides a more generalized perspective on informativeness.

5. More empirical coverage. An anonymous reviewer asks whether, beyond cumulative-reading sentences, there are other cases where informativeness is determined by the immediate QUD rather than semantic entailment relation. Here is another case, which I have discussed in Zhang (2022). As shown in (10) (this example is from Szabolcsi 2017), under the given scenario, the use of an *even*-sentence is perfectly natural, but it challenges the traditional analysis of *even*. First, the presuppositional requirement of additivity is not met, because Eeyore was the only one who took a bite of thistles and spit them out. In other words, the truth of the prejacent does not entail the truth of alternatives like X *spit thistles out* (X is a member in the domain and different from Eeyore). Second, if no one other than Eeyore took a bite of thistles, it seems also questionable to claim that the likelihood of the truth of the prejacent is lower than that of X *spit thistles out*.

In Zhang (2022), I propose a new degree-QUD-based analysis for the presupposition of *even*. The use of *even* is always based on a contextually salient degree QUD (for (10), *how prickly are those thistles*). The prejacent of *even* (here *Eeyore spit those thistles out*) provides information to resolve this degree QUD with an increasingly positive answer, and compared with alternatives, this prejacent is also considered maximally informative in resolving this degree QUD (i.e., here *Eeyore spit the thistles out* is maximally informative in resolving the degree question *how prickly are those thistles*).

(10) **Scenario**: Imagine Pooh and friends coming upon a bush of thistles. Eeyore (known to favor thistles) takes a bite but spits it out.

a. Those thistles must be really prickly! Even $[\text{Eeyore}]_F$ spit them out!
((10a) $\not\leadsto$ Someone other than Eeyore spit thistles out.)

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