# Decomposing logic: Modified numerals, polarity, and exhaustification

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## Abstract

This dissertation investigates differences with respect to ignorance and anti-negativity in modified numerals, using as a starting point similar variation among various types of disjunction and indefinites, and their analysis under an alternative-based approach.

In Ch. 1 I lay out the empirical patterns with respect to ignorance and anti-negativity for the English disjunction *or* and the English indefinite *some* $NP_{SG}$, and show that with respect to ignorance CMNs are like *some* $NP_{SG}$ and SMNs like *or*, while with respect to anti-negativity CMNs are like *or* and SMNs are like *some* $NP_{SG}$. Using insights from the existing literature, I show how these patterns can be derived for *or* and *some* $NP_{SG}$. The overarching goal of the thesis is to extend this approach to CMNs and SMNs.

The first step in extending the approach to CMNs and SMNs is to clarify their formal similarity to *or* and *some* $NP_{SG}$. I do this in Ch. 2, where I offer a new way to decompose their truth conditions and derive from them their alternatives.

A prediction is that CMNs and SMNs should give rise to scalar implicatures. This prediction goes against the received view. I defend it in Ch. 3.

The solution to ignorance in CMNs and SMNs is given in Ch. 4. It is based on obligatory exhaustification relative to pre-exhaustified subdomain alternatives, and a subdomain alternative pruning parameter. I also discuss predictions and general fit to the existing experimental evidence.

The solution to anti-negativity in CMNs and SMNs is given in Ch. 5. It is based on obligatory exhaustification relative to pre-exhaustified subdomain alternatives, and a proper strengthening parameter. New experimental evidence regarding anti-negativity patterns is also presented and discussed.

Ch. 6 offers a global summary and directions for future research.

# Contents

To all my teachers.

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# Chapter 1

# Introduction

Disjunction and indefinite expressions in episodic contexts typically give rise to an ignorance effect. Despite this commonality, they also often vary in (at least the following) two ways: First, they may vary with respect to the strength of this ignorance effect. Second, they may vary with respect to whether they are able to take scope under negative operators.

Modified numerals are strikingly similar – they also share the ability to give rise to ignorance and they also differ in terms of the strength of this effect and in their ability to take scope under negative operators.

The first, preliminary, goal of this dissertation is to find an approach to ignorance and anti-negativity (in the sense described above) that can capture similarity and variation with respect to these effects in disjunction and indefinites. The second, ultimate, goal is to extend this approach to modified numerals also. This second goal will also require developing a suitable theory of modified numerals that would integrate well with this theory of ignorance and anti-negativity.

In §1.1 I illustrate our patterns of interest for disjunction/indefinites and modified numerals. In §1.2 I show how an alternatives-and-exhaustification approach to ignorance and anti-negativity can capture the patterns in disjunction and indefinites. In §1.3 I lay out the plan for the rest of the thesis, which is to extend this approach to modified numerals also. The result is a new theory of (bare and) modified numerals.

## 1.1 Ignorance and anti-negativity

The English disjunction *or* and the English singular indefinite *some*, *some* $NP_{SG}$ , can both give rise to ignorance inferences. These inferences can be paraphrased as in (1).

(1) a. John called Teddy or Sue.

$\rightsquigarrow$ For all the speaker knows it could be Teddy and it could be Sue.

b. John called some student.

$\rightsquigarrow$ For all the speaker knows it could be $a$ and it could be $b$ and it could be $c$ etc.

However, *or* and *some* $NP_{SG}$ differ with respect to this ignorance effect. Using scenarios from the literature on epistemic indefinites (Chierchia 2013 and sources therein, among which Alonso-Ovalle & Menéndez-Benito 2010 for the bathroom scenario), we find that *or* is bad in a context such as the one in (2) where the speaker knows that one of the options is true (*John cheated*) and also in a context such as the one in (3) where the speaker knows that one of the options is false (*John is in the bathroom*); however, *some* $NP_{SG}$ is fine in both. We will summarize this by saying that *or* requires total ignorance but *some* $NP_{SG}$ is compatible with two types of partial ignorance / knowledge scenarios, both one where we had a 'winner' (knowledge that $p$) and one where we had a 'loser' (knowledge that $\neg p$).

(2) a. John cheated. #Therefore John or Bill is a cheater.

b. John cheated. Therefore some student in your class is a cheater.

(3) a. John is in the kitchen or the bathroom, #but not the bathroom.

b. John is in some room in that house, but not the bathroom.

In addition to this, *or* and *some* $NP_{SG}$ also differ with respect to their ability to embed in downward-entailing environmens: *or* is acceptable in a downward-entailing environment such as the scope of a negative operator, (4), the antecedent of a conditional, (5), or the restriction of a universal, (6); however, *some* $NP_{SG}$ is degraded in the scope of a negative operator (cf. also Szabolcsi 2004:414; Nicolae 2012 for *someone*, *something*) although it is fine in the antecedent of a conditional or the restriction of a universal. We will summarize this by saying that *or* does not exhibit anti-negativity but *some* $NP_{SG}$ does.

(4) a. John didn't call Teddy or Sue. ✓*not* $>$ *or*

b. #John didn't call some student. # *not* $>$ *some* $NP_{SG}$

(5) a. If John called Teddy or Sue, he won. ✓*if* $>$ *or*

b. If John called some student, he won. ✓*if* $>$ *some* $NP_{SG}$

(6) a. Everyone who called Teddy or Sue won. ✓*every* $>$ *or*

b. Everyone who called some student won. ✓ *every* $>$ *some* $NP_{SG}$

Table 1.1 summarizes these contrasts in terms of total ignorance and anti-negativity.

| | | total ignorance | |
|---|---|---|---|
| | | yes | no |
| anti-negativity | no | *or* | |
| | yes | | *some* $NP_{SG}$ |

Table 1.1: Ignorance and anti-negativity: *or* vs. *some* $NP_{SG}$ .

But why should we assume that these contrasts are somehow related, the way our presentation and summary suggest? After all, the fact that *some* $NP_{SG}$ is consistent with speaker certainty / lack of ignorance and the fact that *or* is fine under negation are simply non-effects. And the fact that *or* seems incompatible with speaker certainty / lack of ignorance and *some* $NP_{SG}$ is unable to take scope under clausemate negation could be due to independent factors.

A good reason to assume that these contrasts between *or* and *some* $NP_{SG}$ are related is because we can find variation along the same dimensions not only between a disjunction such as *or* and an indefinite such as *some* $NP_{SG}$ , but also between one disjunction and another, or between one type of indefinite and another. For example, the French disjunction *ou* is a plain disjunction, just like *or*, and it gives rise to the same total ignorance effect as *or*, (7)-(8) (cf. Aurore Gonzalez and Laurence B. Violette, p.c.); however, unlike *or*, it is bad under clausemate negation (cf. also Spector 2014, Nicolae 2017, (9).

(7) John a triché. #Donc, John ou Bill est un tricheur.
John cheated therefore John *ou* Bill is a cheater

(8) John est dans la cuisine ou la salle de bain, #mais pas dans la salle de bain.
John is in the kitchen or the bathroom but not in the bathroom

(9) #Marie va pas aller au cinéma lundi *ou* mardi.
Teddy will not go to.the cinema Monday *ou* Tuesday

*# not > ou*

Similarly, the German indefinite *irgendein* is an indefinite just like *some* $NP_{SG}$, and it is compatible with some partial ignorance scenarios (Aloni & Port 2011), (11) (the 'loser' case; judgment cf. also Niels Torben Kühlert, p.c.), just like *some* $NP_{SG}$, but, unlike *some* $NP_{SG}$, it is able to take scope under negative operators (Kratzer & Shimoyama 2017 [2002], Chierchia 2013) (where it acts essentially like English *any*), (12).

(10) John hat geschummelt. #Deshalb ist irgendein Student aus deiner Klasse ein Betrüger.
John cheated therefore is *irgendein* student in your class a cheater

(11) John ist in irgendeinem Zimmer im Haus aber nicht im Badezimmer.
John is in *irgendein* room in.the house but not in.the bathroom

(12) Niemand hat *irgendein* Buch mitgebracht.
no one has *irgendein* book brought along
'No one has brought along any book.' ✓*no one > irgendein*

But why should disjunction or indefinites in one language be exactly the same as in another? Couldn't the differences between *or*/*ou* and *someone*/*irgendein* be explained in terms of other peculiarities of those languages?

Language-specific facts should definitely be taken into account, and factored into our final understanding of these items. However, the same observations can be made between items within the same language also. The Italian indefinites *un qualsiasi/qualunque NP*, *un NP qualsiasi/qualunque*, and *un qualche NP* can all give rise to a modal effect in episodic contexts (although whether the flavor is epistemic, i.e., ignorance, may vary), but in *un qualsiasi/qualunque NP* and *un NP qualsiasi/qualunque* this effect is total while in *un qualche NP* it is partial, and *un qualsiasi/qualunque NP* is fine under negative operators but *un NP qualsiasi/qualunque*, and *un qualche NP* aren't (Chierchia 2013:260-70 and references therein). The Romanian indefinite determiners *vreun* and *un NP oarecare* are a similar pair, although they differ with respect to both the strength of the modal effect – anti-total for *vreun*, total for *un NP oarecare*, – as well as polarity sensitivity – *vreun* is fine under negative operators but *un NP oarecare* isn't (Fălăuş 2014). And so on.

In short, it seems that, if we want to gain a thorough understanding of ignorance effects and polarity sensitivity, it not only makes sense to consider English *or* to English *some* $NP_{SG}$ (including their apparent non-effects), but also to add to our dataset a host of other disjunctions and indefinites, to probe the limits of variation. This is precisely the approach taken by Chierchia (2013) for a host of indefinites.

In addition to that, I will argue that our dataset should include not just disjunction and indefinites but also modified numerals – by which, in what follows, I will specifically have in mind English comparative-modified numerals (CMNs; e.g., *more/less than three*) and English superlative-modified numerals (SMNs; e.g., *at least/most three*).

One reason for this is the usual reason: Just like all the other pairs that we have examined so far, CMNs and SMNs in episodic contexts can both give rise to ignorance inferences (Nouwen 2015, Westera & Brasoveanu 2014, Cremers, Coppock, Dotlacil, & Roelofsen 2017), just that, instead of ranging over individuals, these now range over numerals, (13). And CMNs and SMNs also contrast in the same two ways, with respect to whether the ignorance effect is total or not and with respect to whether they exhibit anti-negativity or not: CMNs are compatible with partial ignorance / knowledge scenarios (of both the 'winner' and the 'loser' type) but SMNs are not (Geurts & Nouwen 2007, Nouwen 2010, Geurts, Katsos, Cummins, Moons, & Noordman 2010, Cummins & Katsos 2010, Nouwen 2015, Cremers et al. 2017), (14)-(15), and CMNs can take scope under clausemate negation but SMNs can't, although they are both fine in the antecedent of a conditional or the restriction of a universal (Nilsen 2007, Geurts & Nouwen 2007, Cohen & Krifka 2014, Spector 2015, Mihoc & Davidson 2017), (16)-(18).

(13) a. John called more than two people.

$\rightsquigarrow$ For all the speaker knows it could be 3 and it could be 4 and it could be 5 etc.

b. John called at least three people.

$\rightsquigarrow$ For all the speaker knows it could be 3 and it could be 4 and it could be 5 etc.

(14) a. John called three people. Therefore, he called more than two people.

b. John called three people. #Therefore he called at least three people.

(15) a. John called more than two people, but not five.

b. John called at least three people, #but not five.

(16) a. John didn't call more than two people. ✓*not* > CMN

b. #John didn't call at least three people. # *not* > SMN

(17) a. If John called more than two people, he won. ✓*if* > CMN

b. If John called at least two people, he won. ✓*if* > SMN

(18) a. Everyone who called more than two people won. ✓*every* > CMN

b. Everyone who called at least three people won. ✓*every* > SMN

A second reason to add English CMNs and SMNs to our dataset is that they are a pair where we have a lot of reasons to expect uniformity: these items are pairwise truth-conditionally equivalent (e.g., *more than two* $\sim$ *at least three* and *less than four* $\sim$ *at most three*); they share the same morphological pieces (*much/little*, a numeral); and they are within the same lexical category of language (modified numerals) and the same language (English). Thus, if it is surprising to find ignorance and anti-negativity contrasts between, e.g., the English disjunction *or* and the German indefinite *irgendein* or the English disjunction *or* and the French disjunction *ou*, these differences between English CMNs and English SMNs are no less surprising.

A final reason to add English CMNs and SMNs to our dataset is because they reveal a remarkable uniformity of ignorance and polarity sensitivity effects across otherwise unrelated domains of language –

disjunction/indefinites and modified numerals. This uniformity further emphasizes the need for a general, uniform solution.

To sum up, in addition to *or* and *some* $NP_{SG}$, a variety of other pairs of disjunctions and indefinites, as well as modified numerals, give rise to ignorance in episodic contexts. Moreover, just like *or*/*some* $NP_{SG}$, they vary with respect to the strength of this ignorance effect and/or with respect to anti-negativity. Table 1.2 summarizes these facts. (We only include in this summary the items for which we reviewed concrete data above, namely, *or*, *some* $NP_{SG}$, *ou*, *irgendein*, CMNs, and SMNs.)

| | | total ignorance | |
|---|---|---|---|
| | | yes | no |
| anti-negativity | no | *or* | *irgendein*, CMNs |
| | yes | *ou*, SMNs | *some* $NP_{SG}$ |

Table 1.2: Ignorance and anti-negativity: *or*, *some* $NP_{SG}$, *ou*, *irgendein*, CMNs, SMNs.

In light of this more extended dataset, the parametric shape of our table summary no longer seems contrived. A unified theory of ignorance and polarity sensitivity will have to explain not only the striking commonality between all of *or*, *some* $NP_{SG}$, *irgendein*, *ou*, CMNs, and SMNs (and other items) – namely, the fact that they can all give rise to ignorance inferences – but also the non-trivial variation between these items with respect to the strength of the ignorance effect and with respect to polarity sensitivity – variation which makes it such that items that are superficially different end up being so similar (ignorance: *or*/*ou*/SMNs vs. *some* $NP_{SG}$/*irgendein*/CMNs;[1] polarity sensitivity: *or*/*irgendein*/CMNs vs. *some* $NP_{SG}$/*ou*/SMNs) while items that are superficially similar end up being so different (*or* vs. *ou*, *some* $NP_{SG}$ vs. *irgendein*, CMNs vs. SMNs).

[1]It might be tempting to say that all disjunctions have the same strong ignorance effect and all indefinites have the same weak ignorance effect. I am not sure if this can be maintained for disjunction in general, but for indefinites recall that the Italian pair *un NP qualsiasi*/*un qualche NP* and the Romanian pair *vreun* and *un NP oarecare* are both examples of indefinites that differ in strength. Thus, any generalizations of this sort must be made with respect to a much larger dataset, and not on the basis of a partial summary like Table 1.2, which, for reasons of transparency, we have limited to those items that we have also illustrated with at least some examples.

In the next section we will try to sketch such a theory of ignorance and anti-negativity for *or* and *some $NP_{SG}$* .

## 1.2 An alternatives-and-exhaustification solution for *or* and *some $NP_{SG}$*

We want to find a unified approach to ignorance and anti-negativity for *or* and *some $NP_{SG}$* .

There are many approaches to ignorance and anti-negativity in disjunction and indefinites in the literature. However, all the approaches that derive these phenomena, and variations with respect to them, in a *unified* way are approaches based on alternatives and exhaustification (some variant of the grammatical theory of scalar implicatures). In our search for a solution to ignorance and anti-negativity in *or* and *some $NP_{SG}$* we will try to play with alternatives and exhaustification also. The first step will thus be to define the alternatives of *or* and *some $NP_{SG}$* . The second – to define our formal tools, namely, exhaustification and how it operates. There are at least a couple of ways to implement these two steps; we will choose to implement them in the style of Chierchia (2013). The third and last step will be to articulate our concrete proposal for how ignorance and anti-negativity, and variations with respect to them, arise in *or* and *some $NP_{SG}$* ; here too we will follow the spirit of Chierchia (2013), though the letter will sometimes differ; we will make sure to signal any deviations.

At the end of the section we will also include a discussion of other theoretical choices that have been proposed in the literature in these regards, and where we stand with respect to them.

### 1.2.1 *Or* and *some $NP_{SG}$* activate alternatives ((Exh)DA , $\sigma$A )

We said that we would look for an alternatives-and-exhaustification solution for *or* and *some $NP_{SG}$* . But what are the alternatives of *or* and *some $NP_{SG}$* ?

Consider an *or* utterance, e.g., *John called Teddy or Sue*. Such an utterance is usually represented as $p \vee q$. And its alternatives are traditionally taken to include a scalar alternative derived by replacing the disjunction with a conjunction, $p \wedge q$, (this assumption goes back to Horn 1989's proposal that *or* and *and* form a scale, $\langle$ *or*, *and* $\rangle$) and also alternatives obtained by replacing the disjunction with its disjuncts, $p$ and $q$ (this assumption goes back to Sauerland 2004). And, since the more recent discussions of the Free Choice effect in disjunction (e.g., Fox 2007), the alternatives of a disjunction are also said to include a variant of the individual disjuncts on which they are interpreted exhaustively – *only* $p = p \wedge \neg q$, *only* $q$ $= q \wedge \neg p$.

Consider now a *some NP*$_{SG}$ utterance, e.g., *John called some student*. Such an utterance is usually represented as $\exists x \in D[P(x)]$, where $D = [\![\text{student}]\!]$. And its alternatives are traditionally taken to include a scalar alternative derived by replacing the existential with a universal, $\forall x \in D[P(x)]$ (this assumption goes back to Horn 1989's proposal that *some* and *every/all* form a scale, $\langle$ *some*, *every/all* $\rangle$). More recently (e.g., Chierchia 2013), this set has also been argued to include subdomain alternatives, i.e., obtained by replacing the domain of quantification of the existential quantifier with its subdomains, $\exists x \in D'[P(x)]$, where $D' \subset D$, and also a variant of these subdomain alternatives on which they are interpreted exhaustively, e.g., *only* $\exists x \in D'[P(x)]$, where $D' \subset D$, that is, $\exists x \in D'[P(x)] \wedge \forall D''[\neg\exists x \in D''[P(x)]]$, where $D', D'' \subset D \wedge D' \neq D''$ ($D'$ and $D''$ are different subsets of $D$).

Note now that, in a context where the set of students consists of Teddy and Sue, an *or* utterance such as *John called Teddy or Sue* and a *some NP*$_{SG}$ utterance such as *John called some student* are truth-conditionally equivalent, and their alternatives are also the same – *John called Teddy and Sue*, *John called Teddy*, *John called Sue*, *John only called Teddy*, *John only called Sue*. All these are spelled out below in both the *or* and the *some NP*$_{SG}$ form. From now on we will call the alternative based on *and*/$\forall$ the *scalar alternative* ($\sigma$A ), the alternatives based on the individual disjuncts/subdomains – the *subdomain alternatives* (DA ), and the exhaustively interpreted individual disjunct/subdomain alternatives – the *pre-exhaustified subdomain alternatives* (ExhDA ).

(19) a. John called Teddy or Sue / John called some student (assertion)

(i) $C(j,t) \vee C(j,s)$

(ii) $\exists x \in \{t,s\}[C(j,x)]$

b. {John called Teddy and Sue} ($\sigma$A )

(i) $\{C(j,t) \wedge C(j,s)\}$

(ii) $\{\forall x \in \{t,s\}[C(j,x)]\}$

c. {John called Teddy, John called Sue} (DA )

(i) $\{C(j,t),\ C(j,s)\}$

(ii) $\{\exists x \in \{t\}[C(j,x)],\ \exists x \in \{s\}[C(j,x)]\}$

d. {John only called Teddy, John only called Sue} (ExhDA )

(i) $\{C(j,t) \wedge \neg C(j,s),\ C(j,s) \wedge \neg C(j,t)\}$

(ii) $\{\exists x \in \{t\}[C(j,x)] \wedge \neg \exists x \in \{s\}[C(j,x)],\ \exists x \in \{s\}[C(j,x)] \wedge \neg \exists x \in \{t\}[C(j,x)]\}$

To sum up, given the same domain of reference, *or* and *some* $NP_{SG}$ are truth-conditionally equivalent. On either way of stating their truth-conditions – disjunctive or existential – their truth conditions make reference to both a scalar element and a domain, and alternatives can be obtained uniformly from them by replacing the scalar item with a scalemate alternative, and the domain with a subdomain alternative; moreover, the latter can also be interpreted exhaustively. From here onwards, for brevity, we will represent the truth conditions and alternatives for both in terms of the disjunctive form.

To keep our ensuing dicussion focused, we will now reveal another piece of our ultimate solution: We will assume that, of the various types of alternatives that they could in principle activate, both *or* and *some* $NP_{SG}$ activate scalar alternatives ($\sigma$A ) and pre-exhaustified subdomain alternatives (ExhDA ), and that both are factored in by default.

### 1.2.2 Active alternatives must be used by (silent) exhaustivity operators (O )

We said that *or* and *some* $NP_{SG}$ activate scalar and pre-exhaustified subdomain alternatives. But how are these alternatives factored into meaning?

The traditional view for how an utterance of the form *or* acquires its *not and* meaning or how *some* acquires its *not every* meaning is that this happens via scalar implicature.

The traditional view of scalar implicatures goes back to Grice (1975). Grice famously proposed that conversation is regulated by conventional rules – the Gricean conversational maxims. By these rules the speaker is assumed to say things that they believe to be true (Quality) and, in so doing, to be maximally informative (Quantity), maximally relevant (Relevance), and maximally brief/clear (Manner). If a speaker chooses to make a weak statement when a stronger statement would be equally relevant and more informative (and thus it should win by Quantity), that must be because s/he isn't convinced that the stronger statement is true, or maybe even believes that it is false (thus, it fails by Quality). Going back to our *or*/*some* $NP_{SG}$ utterance, if a speaker chose to make an utterance of that form although it would have been equally relevant and more informative to make an *and*/*all* utterance, that must be because the speaker isn't convinced that the *and*/*all* utterance is true, or maybe even believes it to be false.

But while the Gricean implicature calculation process described above is typically understood as a speech act level, pragmatic phenomenon, a more recent approach has been to say that the exhaustive interpretations of items like *or* and *some* $NP_{SG}$ arise in the grammar (hence the name 'the grammatical theory of scalar implicatures'), through the action of a silent exhaustivity operator with a meaning akin to *only* (Chierchia, Fox, & Spector 2012, Chierchia 2013). Below we review a few of the major arguments for this view, taken from (Chierchia 2013:109-10) (and references therein).

A first argument for the view that exhaustive meanings arise through silent exhaustivity operators comes from question-answer pairs. In the question-answer pair below B's answer is typically interpreted as saying not just that B saw Paul and Sue but also that s/he didn't see anyone else of his/her old friends – thus, that any non-weaker alternative to the utterance (e.g., *Paul and Teddy*, *Paul, Teddy, and Sue*, etc.) is false. This is an exhaustive intepretation that is fundamentally similar to the exhaustive interpretation of *or* or *some* $NP_{SG}$ , and so we would ideally want a common treatment. However, cases like this have long resisted a traditional analysis in terms of just the Gricean maxims. On the other hand, if exhaustive interpretations came from the presence of a silent *only*, the puzzle would be solved. Such an analysis is also supported by the fact that B's answer can easily be paraphrased as *only Paul*, with an overt *only*.

(20) A: Which of your old friends did you see at the party?

B: Paul.

A second argument for the view that exhaustive interpretations arise through silent exhaustivity operators comes from examples like the one below. The example below is interpreted as saying not just that the speaker's ex came to the party, but also that s/he was the least likely to show up – thus, that anyone else would have been more likely to come. This is again an exhaustive interpretation, so we would want to make sense of it in a similar way as for the other cases. However, note that it carries a different flavor – now we are saying that the assertion is the least likely among its alternatives. This new flavor is somewhat unexpected if in all cases we just had the Gricean maxims at play. However, it is unsurprising on the view that such examples contain a covert exhaustivity operator. Overt exhaustivity operators come in multiple flavors, for example, *only* and *even*. If before it was natural to say that we were dealing with a silent counterpart of *only*, here it is similarly natural to say that we are dealing with a silent counterpart of *even*.

(21) Really everybody came to my party. Imagine that MY EX came.

A third argument for the view that exhaustive interpretations arise through silent exhaustivity operators comes from plausibility. The example below has a reading on which it is interpreted as if it contained a silent counterpart of *each*. In the literature on distributivity the existence of this silent counterpart of *each* is commonly accepted, and it is even considered necessary – since the subject DP *John and Mary* is non-quantificational, in the absence of such an operator the scopal interaction between the subject DP and the quantificational object would be unexplained (Schwarzschild 1996). Thus it seems that the idea that an overt operator may have a covert counterpart is needed more generally, and represents a natural way to capture a wide range of phenomena.

(22) John and Mary hit a pole.

a. John hit a pole and Mary hit a pole.

b. There is a pole that John and Mary both hit.

Thus, it seems that in many cases exhaustive interpretations are better handled if we assume that they arise in the grammar, via silent exhaustivity operators.[2] We will assume that the *not and* and *not every* implicatures of *or* and *some* $NP_{SG}$ arise in this way also.

Now, the grammatical view of implicatures predicts that, if these items occur at an embedded level, they should be able to give rise to scalar implicatures at that level also. This seems to be a welcome prediction: In all the examples below (from Chierchia 2013:96-7, slightly adapted for expository reasons), the continuation/context forces the embedded disjunction to be interpreted as $(p \vee q) \wedge \neg(p \wedge q)$, and this interpretation seems to be able to arise at arbitrarily embedded levels. On the traditional Gricean view, where implicature calculation is assumed to be a speech act level, matrix phenomenon, these embedded implicatures would be completely unexpected. However, on the view on which they arise via a silent exhaustivity operator, these implicatures can be captured by assuming that in all the examples below the operator is inserted at the level of the antecedent of the conditional.

(23) If you get fruit or dessert, you won't have charges above the menu's prix fixe.
(But if you get fruit *and* dessert, there will be an extra charge.)

(24) If you assign a problem set or an experiment, students will love it.
(But if you assign a problem set *and* an experiment, they will hate it.)

(25) John is certain that Bill believes that if you get fruit or dessert, you won't have extra charges on the menu's prix fixe.
(But John is certain that Bill believes that if you get fruit *and* dessert, there will be an extra charge.)

We have seen both conceptual and empirical reasons to say that the *not and* and *not every* implicatures of *or* and *some* $NP_{SG}$ arise via a silent exhaustivity operator akin to a silent *only*. But how is this operator defined? The literature provides a number of answers (Chierchia et al. 2012, Chierchia 2013; see also Spector 2016 for a discussion of other options). We will follow the specific way in which it is used in Chierchia

[2]This is not to say that the traditional Gricean view must be altogether abandoned. It may well be that for highly context-dependent cases exhaustive meanings are still derived via the high-level Gricean method. This would include, for example, Grice's famous recommendation letter example – *John has beautiful handwriting* $\rightsquigarrow$ He has no research skills. See Chierchia (2013:104) for other examples and more discussion in this sense.

(2013), not only because it is syntactically more explicit but also because its semantics is the closest to the original Gricean implicature calculation reasoning. Like Chierchia, we will also abbreviate this operator as O . The syntax and semantics of O in Chierchia (2013) are as follows:

Syntactically, O is defined as a sentence-level alternative-sensitive operator which must be projected in a position where it is a sister to a node dominating an alternative-bearing item. The relation between O and the alternative-bearing item it c-commands is (tentatively) conceptualized as some sort of a syntactic agreement relation – the alternative-bearing item carries an unvalued alternative feature and this prompts the insertion of O to value it. Consider our items *or* and *some* $NP_{SG}$ . As we argued in the previous sections, these items have both scalar and pre-exhaustified subdomain alternatives. Thus, they come with a feature bundle consisting of a scale feature $\sigma$ and a domain feature D, $_{[\sigma,\mathrm{D}]}$ , (26). This prompts the insertion at a higher (c-commanding) position in the structure of exhaustivity operator(s) targeting those alternative features. Now, O is assumed to have multiple variants, specialized for various types of alternatives. $\mathrm{O}_{\sigma \mathrm{A}}$ looks for scalar alternatives; it assigns a plus value to the feature $\sigma$ (and a minus to the feature D) and thus activates the scalar alternatives, (27). $\mathrm{O}_{\mathrm{ExhDA}}$ looks for pre-exhaustified subdomain alternatives; it assigns a plus value to the feature D (and a minus to the feature $\sigma$) and thus activates the pre-exhaustified subdomain alternatives, (28). And $\mathrm{O}_{\mathrm{ExhDA}+\sigma \mathrm{A}}$ (in Chierchia 2013, $\mathrm{O}_{\text{Exh-ALT}}$) looks for both scalar and pre-exhaustified subdomain alternatives in one go, assigning a plus to both the feature D and the feature $\sigma$ and thus activating both types of alternatives, (29).

(26) John called Teddy or$_{[\sigma,\mathrm{D}]}$ Sue / John called some$_{[\sigma,\mathrm{D}]}$ student

(27) $\mathrm{O}_{\sigma \mathrm{A}}$ (John called Teddy or$_{[+\sigma,-\mathrm{D}]}$ Sue / John called some$_{[+\sigma,-\mathrm{D}]}$ student)

(28) $\mathrm{O}_{\mathrm{ExhDA}}$ (John called Teddy or$_{[-\sigma,+\mathrm{D}]}$ Sue / John called some$_{[-\sigma,+\mathrm{D}]}$ student)

(29) $\mathrm{O}_{\mathrm{ExhDA}+\sigma \mathrm{A}}$ (John called Teddy or$_{[+\sigma,+\mathrm{D}]}$ Sue / John called some$_{[+\sigma,+\mathrm{D}]}$ student)

As in Rooth (1985)-style alternative semantics, the alternatives thus activated grow (via Pointwise Functional Application) until they meet the exhaustivity operator, which factors them into the overall meaning. This brings us to the semantics of O .

Semantically, O is defined in analogy to overt *only*, modulo the fact that overt *only* presupposes the truth of its prejacent while O asserts it. More concretely, given $p$, a proposition, and $[\![\text{p}]\!]^{\text{C}}$, the set of alternatives C to $p$, an application of $\text{O}_{\text{C}}$ (a variant of O specifically looking for the alternatives C) to $p$ will assert $p$ and furthermore say that all the propositions in $[\![\text{p}]\!]^{\text{C}}$ that are true are already entailed by $p$, (30), i.e., that all of the non-entailed (stronger or logically independent) alternatives to $p$ are false, (30).

(30) $[\![\text{O}_{\text{C}}\ (p)]\!]^{g,w} = [\![\text{p}]\!]^{g,w} \wedge \forall q \in [\![\text{p}]\!]^{\text{C}}\ [[\![\text{q}]\!]^{g,w} \rightarrow \lambda w'\ .\ [\![\text{p}]\!]^{g,w'} \subseteq q]$ (Chierchia 2013:139)

By replacing C with $\sigma$A , ExhDA , or ExhDA+$\sigma$A we get the meanings of $\text{O}_{\sigma\text{A}}$ , $\text{O}_{\text{ExhDA}}$ , and $\text{O}_{\text{ExhDA+}\sigma\text{A}}$ – they have the same semantic core but look at different sets of alternatives. We are now ready to compute the meanings of (26)-(29).

Before we do that, some notes on notation. First, following the discussion related to (19) at the end of the previous section, we will represent both the *or* and the *some* $NP_{SG}$ utterance and alternatives together in terms of the disjunctive form: the assertion *John called Toby or Sue / John called some student* as $(p \vee q)$, the set of scalar alternatives {John called Toby and Sue} as $\{p \wedge q\}$, the set of subdomain alternatives {John called Toby, John called Sue} as $\{p, q\}$, and the set of pre-exhaustified subdomain alternatives {John only called Toby, John only called Sue} as $\{p \wedge \neg q, q \wedge \neg p\}$. Second, taking advantage of our newly defined O operator, we will abbreviate the pre-exhaustified subdomain alternative set {John only called Toby, John only called Sue} = $\{p \wedge \neg q, q \wedge \neg p\}$ as $\{\text{O}\ p, \text{O}\ q\}$, assuming (as in the pre-theoretical notion of pre-exhaustified subdomain alternatives) that the set of alternatives relevant for the pre-exhaustification of a subdomain alternative consists simply of all the other subdomain alternatives. (Thus, a subdomain alternative is pre-exhaustified relative to the DA set of the assertion; we will discuss this again later.)

These said, we are ready to tackle the meanings of (26)-(29). If no exhaustivity operator is inserted, no alternatives are activated, and we get no exhaustification, (31).

(31) $p \vee_{[\sigma,\text{D}]}\ q$ (no exhaustification)

$= p \vee q$

If an exhaustivity operator is inserted, the outcome will depend on the variant, i.e., on which alternatives are being targeted. First, $\mathrm{O}_{\sigma\mathrm{A}}$ will assert the prejacent and negate those of its scalar alternatives that are not entailed by it (here, all), (32). The result is the traditional scalar implicature *not and / not every/all.*

(32) $\mathrm{O}_{\sigma\mathrm{A}}\ (p \vee_{[+\sigma,-\mathrm{D}]}\ q)$

$= (p \vee q) \wedge \neg(p \wedge q)$ (*not and/not every* implicature)

Second, $\mathrm{O}_{\mathrm{ExhDA}}$ will assert the prejacent and negate those of its pre-exhaustified subdomain alternatives that are not entailed by it (here, all), (33). Unpacking the negations of the pre-exhaustified subdomain alternatives, we end up with a double implication that is true iff $p$ and $q$ are both false or both true. The first possibility leads to a contradiction of the G(grammatically)-trivial[3] kind; see Chierchia (2013:49-54) (and references therein) for a discussion of how such contradictions are different from ordinary contradictions, and how they are a cause for ungrammaticality, (33-a). The second possibility turns *or* into *and* and *some* into *every/all*, (33-b).

(33) $\mathrm{O}_{\mathrm{ExhDA}}\ (p \vee_{[-\sigma,+\mathrm{D}]}\ q)$

$= (p \vee q) \wedge \neg \mathrm{O}\, p \wedge \neg \mathrm{O}\, q$

$= (p \vee q) \wedge \neg(p \wedge \neg q) \wedge \neg(q \wedge \neg p)$

$= (p \vee q) \wedge (p \rightarrow q) \wedge (q \rightarrow p)$

$= (p \vee q) \wedge p \leftrightarrow q$

a. $(p \vee q) \wedge \neg p \wedge \neg q$

$= \bot$ (G-trivial)

b. $= (p \vee q) \wedge p \wedge q$

$= p \wedge q$ (*and/every* meaning)

And, third, $\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}$ will yield the intersection of the results of $\mathrm{O}_{\mathrm{ExhDA}}$ and $\mathrm{O}_{\sigma\mathrm{A}}$ , (34). The unique

[3]G(rammatical)-triviality: = A sentence $\phi$ is G-trivial iff for any situation $s$ and model $M$, $[\![\phi']\!]^{M,s}$ = same (where same is either 1 or 0 and $\phi'$ is obtained from $\phi$ by an *arbitrary* substitution of its lexical terminal nodes (Chierchia 2013:51).

viable option of $\text{O}_{\text{ExhDA}}$ that we saw before in (33-b) clashes with the result of $\text{O}_{\sigma\text{A}}$ that we saw before in (32), yielding again a G-trivial meaning, (34-b).

(34) $\text{O}_{\text{ExhDA+}\sigma\text{A}}\ (p \vee_{[+\sigma,+\text{D}]}\ q)$

$= (p \vee q) \wedge \underbrace{(p \leftrightarrow q)}_{=\neg p \wedge \neg q / p \wedge q} \wedge \neg(p \wedge q)$

a. $= \underbrace{(p \vee q) \wedge \neg p \wedge \neg q}_{=\bot} \wedge \neg(p \wedge q)$

$= \bot$ (G-trivial)

b. $= (p \vee q) \wedge \underbrace{p \wedge q \wedge \neg(p \wedge q)}_{=\bot}$

$= \bot$ (G-trivial)

Now, we proposed at the end of the previous section that *or* and *some $NP_{SG}$* not only have alternatives of the forms we considered above, namely, $\sigma$A and ExhDA , but these alternatives must also all be factored in by default. This means that, from all the options illustrated above, only the parse with $\text{O}_{\text{ExhDA+}\sigma\text{A}}$ in (34) is actually available to them. (A parse of the form $\text{O}_{\sigma\text{A}}\ \text{O}_{\text{ExhDA}}\ (p \vee q)$ would also be, but for this case it yields the same results as $\text{O}_{\text{ExhDA+}\sigma\text{A}}$ , which is why we did not discuss it separately.) As we just saw, this parse is not grammatical. But scalar implicatures are known to be cancelable. Can't we just cancel it, then, and obtain a consistent result? We could, but then we would wind up with a conjunctive meaning, which does not generally seem desirable. I will argue that $\sigma$A -implicatures are always computed as a default, and that they can be canceled, but only if forced by the context. We will discuss this again later. For now retain only that the default result for $\text{O}_{\text{ExhDA+}\sigma\text{A}}\ (p \vee q)$ is always a crash.

In the next few sections we will show how, in other configurations (e.g., with an intervening modal), a parse with $\text{O}_{\text{ExhDA+}\sigma\text{A}}$ is in fact grammatical, and yields precisely the results we want.

Before that, a comment on notation: From now on we will no longer mark the features on the scalar item explicitly. We will just assume that, if an exhaustivity operator marked for a certain type of alternatives is present (e.g., $\text{O}_{\text{ExhDA+}\sigma\text{A}}$ above), the corresponding alternative features on the relevant target item have been assigned a '$+$' value, and the corresponding alternatives have been activated.

### 1.2.3 $\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}$ $\Diamond/\Box$ *or/some $NP_{SG}$* yields a Free Choice (FC) effect

Exhaustification of an *or/some $NP_{SG}$* utterance relative to both their scalar and their pre-exhaustified subdomain alternatives failed for the case where we just had the utterance on its own. But what happens if we add in an overt modal?

Consider first the exhaustification of an *or/some $NP_{SG}$* utterance across a *possibility* modal. First, just like an utterance of the form $(p \vee q)$ had pre-exhaustified subdomain alternatives of the form O $p$, O $q$ and a scalar alternative $p \wedge q$, an utterance of the form $\Diamond(p \vee q)$ also has pre-exhaustified subdomain alternatives of the form O $\Diamond p = \Diamond p \wedge \neg\Diamond q$ and O $\Diamond q = \Diamond q \wedge \neg\Diamond p$ and a scalar alternative $\Diamond(p \wedge q)$. The results of exhaustification are then as below. As in the case without a modal, exhaustification relative to the ExhDA leads to a double implication which can be resolved by making both of $\Diamond p$, $\Diamond q$ false or by making both of them true. As before, the first option leads to a contradiction. However, the second option leads to a consistent meaning: the assertion says that there is a world where one of $p$ and $q$ is true; the ExhDA -implicatures add that there is a world where $p$ is true and there is a world where $q$ is true – the famous Free Choice effect of disjunction under possibility modals; and the $\sigma$A -implicatures add that there is no world where both $p$ and $q$ are true together. In concrete terms, *John may call Teddy or Sue/some student* says that John may call Teddy or Sue, and either one is an option, but he may not call both.

(35) John may call Teddy or Sue / John may call some student

$$\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}\ (\Diamond(p \vee q))$$
$$= \Diamond(p \vee q) \wedge \neg\mathrm{O}\ \Diamond p \wedge \neg\mathrm{O}\ \Diamond q \wedge \neg\Diamond(p \wedge q)$$
$$= \Diamond(p \vee q) \wedge \neg(\Diamond p \wedge \neg\Diamond q) \wedge \neg(\Diamond q \wedge \neg\Diamond p) \wedge \neg\Diamond(p \wedge q)$$
$$= \Diamond(p \vee q) \wedge (\Diamond p \rightarrow \Diamond q) \wedge (\Diamond q \rightarrow \Diamond p) \wedge \neg\Diamond(p \wedge q)$$
$$= \Diamond(p \vee q) \wedge (\Diamond p \leftrightarrow \Diamond q) \wedge \neg\Diamond(p \wedge q)$$

a. $$= \underbrace{\Diamond(p \vee q) \wedge \neg\Diamond p \wedge \neg\Diamond q}_{=\perp} \wedge \neg\Diamond(p \wedge q)$$
$$= \perp \qquad \text{(G-trivial)}$$

b. $= \Diamond(p \vee q) \wedge \Diamond p \wedge \Diamond q \wedge \neg\Diamond(p \wedge q)$ (Free Choice and strong scalar implicature)

'There is an accessible world where John calls Teddy or Sue and there is an accessible world where he calls Teddy, and there is an accessible world where he calls Sue, and there is no accessible world where he calls both.'

As we mentioned earlier, we will assume that $\sigma$A -implicatures are always computed by default, but can be suspended if they clash with the context. This is needed to capture the fact that an utterance like the above is compatible with a continuation of the form *in fact,, he may call both*. Regardless of that, the overall result of $\mathrm{O}_{\mathbf{ExhDA+}\sigma\mathbf{A}}\ \Diamond(p \vee q)$ is always Free Choice.

Consider now the case where exhaustification of an *or/some NP$_{SG}$* utterance happens across a necessity modal. Just like before, an utterance of the form $\Box(p \vee q)$ has pre-exhaustified subdomain alternatives of the form $\mathrm{O}\ \Box p = \Box p \wedge \neg\Box q$ and $\mathrm{O}\ \Box q = \Box q \wedge \neg\Box p$ and a scalar alternative of the form $\Box(p \wedge q)$. The results of exhaustification are then as below. As in all the previous cases we discussed, exhaustification relative to ExhDA leads to a double implication which can be resolved by making both of $\Box p$, $\Box q$ false or by making both of them true. The first option leads to a Free Choice effect similar to the one we were getting from the possibility modal case – the assertion together with the ExhDA -implicatures entails $\Diamond p \wedge \Diamond q$. On this option our *or/some NP$_{SG}$* utterances are interpreted as saying that John must call Teddy or Sue, and he is free to choose either one, but he must not call both. The second option, on the other hand, crashes, because the $\sigma$A -implicature contradicts the conjunction of the ExhDA -implicatures.

(36) John must call Teddy or Sue / John must call some student

$\mathrm{O}_{\mathbf{ExhDA+}\sigma\mathbf{A}}\ \Box(p \vee q)$

$= \Box(p \vee q) \wedge \neg\mathrm{O}\ \Box p \wedge \neg\mathrm{O}\ \Box q \wedge \neg\Box(p \wedge q)$

$= \Box(p \vee q) \wedge \neg(\Box p \wedge \neg\Box q) \wedge \neg(\Box q \wedge \neg\Box p) \wedge \neg\Box(p \wedge q)$

$= \Box(p \vee q) \wedge (\Box p \rightarrow \Box q) \wedge (\Box q \rightarrow \Box p) \wedge \neg\Box(p \wedge q)$

$= \Box(p \vee q) \wedge (\Box p \leftrightarrow \Box q) \wedge \neg\Box(p \wedge q)$

a. $= \underbrace{\Box(p \vee q) \wedge \neg\Box p \wedge \neg\Box q}_{\rightarrow\ \Diamond p \wedge \Diamond q} \wedge \neg\Box(p \wedge q)$ (Free Choice)

'In every accessible world John calls Teddy or Sue, and it is not the case that in every world he calls Teddy, and it is not the case that in every world he calls Sue, and it is not the case that in every world he calls both.'

b. $= \Box(p \vee q) \wedge \underbrace{\Box p \wedge \Box q \wedge \neg\Box(p \wedge q)}_{=\bot}$ (G-trivial)

As we mentioned earlier, we will assume that $\sigma$A -implicatures are always computed by default, but can be suspended if they clash with the context. This is needed to capture the fact that an utterance like the above is compatible with a continuation of the form *in fact, he must call both*. The overall result of $\mathrm{O}_{\mathrm{ExhDA+}\sigma\mathrm{A}}$ $\Diamond(p \vee q)$ is thus either total variation/Free Choice or no variation. Since the no variation result is always contingent on whether the context forces the suspension of the $\sigma$A -implicature or not, the only result that is always available by default is thus only total variation/Free Choice = ignorance.

What have we learned? In the default case (no suspension of the $\sigma$A -implicature), exhaustification via $\mathrm{O}_{\mathrm{ExhDA+}\sigma\mathrm{A}}$ of an *or/some* $NP_{SG}$ across an overt modal always seems to yield a Free Choice effect. In the next section we will see how that helps us derive ignorance.

### 1.2.4 Ignorance is a FC effect involving a null epistemic/doxastic necessity modal

Recall our original ignorance effect, shared between *or* and *some* $NP_{SG}$ :

(37) (= (1) on p. 2)

a. John called Teddy or Sue.

$\rightsquigarrow$ For all the speaker knows it could be Teddy and it could be Sue.

b. John called some student.

$\rightsquigarrow$ For all the speaker knows it could be $a$ and it could be $b$ and it could be $c$ etc.

All the pragmatic approaches to ignorance assume that this effect arises as an implicature obtained by reasoning about the epistemic state of the speaker. All thus rely on the notion that utterances like the above are prefixed by a null epistemic/doxastic necessity modal. We will assume that this is the case also, and we will abbreviate this modal as $\Box_S$ (where $\Box$ indicates that it is a necessity modal, the gray color indicates that it is silent, and the subscript $_S$ indicates that the modal base is indexed to the speaker).

The idea of a null modal is not new in the implicature literature – it is already present in traditional Gricean pragmatics, where we were reasoning about $\mathrm{Bel}_S$ 'the speaker believes that ...'. The idea of a null matrix level modal is also not confined to the implicature literature – as Chierchia (2013) notes, it is a key ingredient in the traditional analysis of generics or infinitives also. Thus, it is both traditional and general.[4]

At the same time, there are some differences in how this null epistemic/doxastic modal $\Box_S$ is conceptualized. In the Gricean literature, and more recently in Meyer (2013), it is argued that it is always there (Meyer calls it the $K$ operator, for 'knowledge). In other parts of the literature (e.g., Kratzer & Shimoyama 2017 [2002], Chierchia 2013) it is conceptualized as a last resort rescue mechanism that can be inserted in between an exhaustivity operator and its target when exhaustification would otherwise fail. I will adopt the latter stance and assume that $\Box_S$ is a last resort rescue mechanism for an exhaustification parse that would otherwise fail. This choice doesn't make any difference to our discussion of ignorance – since $\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}$ $(p \vee q)$ failed, we will assume that an *or/some* $NP_{SG}$ utterance in an episodic context must in fact always be interpreted as if it were prefixed by $\Box_S$, just as on the default $\Box_S$ view.

But if the *or/some* $NP_{SG}$ utterances above are prefixed by a $\Box_S$, then we expect exhaustification of these items relative to their pre-exhaustified subdomain and their scalar alternatives to proceed exactly as in the case of exhaustification across an overt necessity modal before – the only difference is that instead of quantifying over worlds compatible with some requirement, now we are quantifying over worlds compatible with what the speaker knows/believes. We show this below. Thus, as Chierchia (2013) notes, ignorance is just another manifestation of the Free Choice effect – it is an epistemic Free Choice effect.

(38) John called Teddy or Sue / John called some student

[4] In the same spirit of trying to understand this null modal better, Chierchia (2013) also points out that, if modals can carry a variety of modal flavors – doxastic/epistemic, bouletic, deontic – depending on what type of modal base they quantify over, then we might expect to see the same in this null modal also. Chierchia notes that this expectation is indeed attested – for some types of epistemic indefinites the assertion can only be understood as if it were prefixed by a null modal with, e.g., a bouletic flavor. While we will mostly be speaking of $\Box_S$, it is thus important to keep in mind that it is not the only type of null modal that may be available. We will discuss this again briefly in §4.6.

$$
\begin{aligned}
&\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}\ \Box_S\,(p \vee q)\\
&= \Box_S\,(p \vee q) \wedge \neg \mathrm{O}\,\Box_S\, p \wedge \neg \mathrm{O}\,\Box_S\, q \wedge \neg\Box_S\,(p \wedge q)\\
&= \Box_S\,(p \vee q) \wedge \neg(\Box_S\, p \wedge \neg\Box_S\, q) \wedge \neg(\Box_S\, q \wedge \neg\Box_S\, p) \wedge \neg\Box_S\,(p \wedge q)\\
&= \Box_S\,(p \vee q) \wedge (\Box_S\, p \rightarrow \Box_S\, q) \wedge (\Box_S\, q \rightarrow \Box_S\, p) \wedge \neg\Box_S\,(p \wedge q)\\
&= \Box_S\,(p \vee q) \wedge (\Box_S\, p \leftrightarrow \Box_S\, q) \wedge \neg\Box_S\,(p \wedge q)
\end{aligned}
$$

a. $= \underbrace{\Box_S\,(p \vee q) \wedge \neg\Box_S\, p \wedge \neg\Box_S\, q}_{\rightarrow\, \Diamond p \wedge \Diamond q} \wedge \neg\Box_S\,(p \wedge q)$ (Free Choice)

'In every accessible worlds John calls Teddy or Sue, and it is not the case that in every world he calls Teddy, and it is not the case that in every world he calls Sue, and it is not the case that in every world he calls both.'

b. $= \Box_S\,(p \vee q) \wedge \underbrace{\Box_S\, p \wedge \Box_S\, q \wedge \neg\Box_S\,(p \wedge q)}_{=\perp}$ (G-trivial)

As before, we will assume that $\sigma$A -implicatures are always computed by default, but can be suspended if they clash with the context. This is needed to capture the fact that an utterance like the above is compatible with a continuation of the form *in fact, he called both*.[5] The overall result of $\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}\ \Box_S\,(p \vee q)$ is thus either total variation/Free Choice=ignorance or no variation/ignorance. Since the no variation result is always contingent on the context, the result that is available by default is only total variation/Free Choice = ignorance.

But if *or* and *some* $NP_{SG}$ are the same in that they always give rise to total ignorance in a default context and are always compatible with no ignorance in a context where the scalar implicature is suspended, then why do they differ in their ability to be uttered in a context where we are certain that one particular domain

[5] Interestingly, a variety of studies, e.g., Singh, Wexler, Astle-Rahim, Kamawar, & Fox (2016), Hochstein, Bale, Fox, & Barner (2016), and Tieu et al. (2017), show that a conjunctive meaning for disjunction is precisely the meaning that young children tend to compute even in the default case (and Singh et al. derive it similarly to how we did, only using recursive exhaustification instead of exhaustification relative to pre-exhaustified subdomain alternatives). But why do children by default compute this meaning without the $\sigma$A -implicature? Singh et al. suggest this might be because children below a certain age might have not yet learned that *and* is a scalar alternative to *or*. This is an interesting possibility (and one that we will consider again later on, in Ch. 3, when we discuss differences in the strength of the scalar implicature in bare numerals vs. other scalars) in that it shows that, while subdomain alternatives are easy to consider because they are provided in a sense by the assertion, scalar alternatives are much more subject to variation due to external conditions such as how well-known/automatic the scale is, or how salient in a given context.

alternative is true or false? We get to this next.

### 1.2.5 (In)Compatibility with certainty comes from the (in)ability to prune DA

Recall our original strength of the ignorance effect puzzle, namely, the fact that *or* was bad in both a partial ignorance/knowledge context where the speaker was certain that a domain alternative was true (we dubbed this the 'winner' case, the case where the speaker has knowledge that $p$) as well as a in a partial ignorance/knowledge context where the speaker was certain that a domain alternative was false (we dubbed this the 'loser' case, the case where the speaker has knowledge that $\neg p$). We repeat these facts below.

(39) (= (2))

a. John cheated. #Therefore John or Bill is a cheater.

b. John cheated. Therefore some student in your class is a cheater.

(40) (= (3))

a. John is in the kitchen or the bathroom, #but not the bathroom.

b. John is in some room in that house, but not the bathroom.

If *or* and *some* $NP_{SG}$ both undergo $\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}$ across $\Box_S$ and the results are always the same, as we discussed in the previous section, then why do they differ this way?

Chierchia (2013) argues that total vs. partial effects in items that obligatorily activate alternatives can be derived by saying that some items are free to prune their DA set to some natural subset, e.g., just the singletons, or just the non-singletons, while others can't. The former are the items that end up with partial variation effects, while the latter are the items that gives rise to total variation effects. We will adopt this idea also, although we will implement it in a slightly different way than in Chierchia (2013). (Specifically, the alternatives we say must be pruned to derive various partial effects are different, and also the way pre-exhaustification of the DA for a case where the domain contains more than two elements happens.)

Let's take it slowly. Consider $\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}$ across $\Box_S$ of a disjunctive expression $(p \vee q)$ with two disjuncts, that is, for a domain with two elements. As we proposed in the previous sections, such an exhaustification will by default proceed relative to the prejacent, the pre-exhaustified versions of the alternatives in the DA set below, and the scalar alternatives in the $\sigma$A set below.

(41) $\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}\ \Box_S\,(p \vee q)$

a. $\Box_S\,(p \vee q)$ (prejacent)

b. $\Box_S\,p$ (singleton DA )

c. $\Box_S\,q$ (singleton DA )

d. $\Box_S\,(p \wedge q)$ ($\sigma$A )

Suppose now that we try to prune the set of DA . As we said, the assumption is that DA -pruning must be done in a principled way, that is, we can only remove a natural subclass of the DA . The only natural subclass of the DA set in this case is the class of singleton DA . But the DA set only has these singleton DA , so removing them would mean destroying the domain. I will assume that this option is not allowed.

The minimum relevant case we need to consider in order to see the effects of pruning the set of DA is thus a case where the domain has at least three elements.

Consider then the exhaustification $\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}$ across $\Box_S$ of a disjunctive expression $(p \vee q \vee r)$ with three disjuncts, that is, for a domain with three elements. This case is more complex then before, so it is good to be explicit about our assumptions.

First, what are the alternatives for $(p \vee q \vee r)$? Of course, by analogy with the two-disjunct case, this utterance too should have the singleton DA $p$, $q$, $r$ and the tripleton $\sigma$A $(p \wedge q \wedge r)$.

But, in addition to these, a variety of other alternatives suggest themselves. Here it helps to recall our original method for generating alternatives. First, we said that the truth conditions of an *or/some* $NP_{SG}$ utterance can be written either disjunctively or existentially; for our three-disjunct case, this would be either $P(a) \vee P(b) \vee P(c)$ or $\exists x \in \{a, b, c\}[P(x)]$. Then, we said that subdomain alternatives are uniformly generated by replacing the domain of reference, here $\{a, b, c\}$, with its subdomains. This yields not only the singleton DA above but also the doubleton DA $(p \vee q)$, $(q \vee r)$, $(p \vee r)$. Furthermore, we said that scalar alternatives are uniformly generated by replacing the scalar item $\vee/\exists$ with its scalemate, $\wedge/\forall$. There is one small wrinkle here: the disjunctive variant of the truth conditions contains multiple occurrences of the scalar item ($\vee$) while the existential only one ($\exists$). We will assume that for the case we are interested in (where *or* and *some* $NP_{SG}$ are the same, and where the disjunctive and the existential variant of their truth conditions are really equivalent) $\vee$ must really be understood as a generalized sort of $\vee$, $\vee(P(a), P(b), P(c))$, and thus the disjunctive variant of the truth conditions also has just one scalar element to be replaced, just like $\exists x \in \{a, b, c\}[P(x)]$.[6] This yields the tripleton $\sigma$A $(p \wedge q \wedge r)$. All the assumptions so far are completely classical and on a par with the two-element domain case. But the three-element domain case highlights a new possibility that was not obviously right in the two-domain case. If we have a $\sigma$A ranging over all the three elements of the domain – as we already labeled it, a tripleton $\sigma$A – then why don't we also have scalar alternatives ranging, i.e., over two elements of the domain – e.g., doubleton $\sigma$A ? Indeed, if alternatives are generated from the truth conditions by replacing the domain and the scalar element in $\vee(P(a), P(b), P(c))/\exists x \in \{a, b, c\}[P(x)]$ with subdomains and scalemates, respectively, then, if we do both at the same time, we also obtain the doubleton $\sigma$A $(p \wedge q)$, $(q \wedge r)$, $(p \wedge r)$, and in fact also the singleton $\sigma$A $p, q, r$ (obtained by replacing the domain with singleton subdomains, and $\vee/\exists$ with $\wedge/\forall$). The doubleton $\sigma$A thus obtained are natural enough and have already been used in the literature (Chierchia 2013). The singleton $\sigma$A are somewhat less natural as, at least for *or* and *some* $NP_{SG}$ , they are identical to the singleton DA . Still, their existence falls out of the same natural algorithm that yields the doubleton $\sigma$A that we said we wanted. We will thus assume that they are in principle available also, although for some cases they will have to be pruned.[7]

To sum up, we will assume that an exhaustification of the form $O_{ExhDA+\sigma A}$ $\Box_S$ $(p \vee q \vee r)$ will by

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[6]I believe that for cases like *John called Teddy or Sue, or Maggie*, where it is clear that we are dealing with a parse of the form $\vee(\vee(P(a), P(b))), P(c))$, scalar alternatives can also be obtained by replacing just one $\vee$ but not the other.

[7]This is not crucial to this $\Box_S$ case where the singleton $\sigma$A are essentially the same as the DA . But it will be crucial to cases where we want to compute $O_{ExhDA}$ above $\Box_S$ but $O_{\sigma A}$ below, $O_{ExhDA}$ $\Box_S$ $O_{\sigma A}$ $(p \vee q)$. We want this to yield the consistent result $\Box_S$ $((p \vee q) \wedge \neg(p \wedge q)) \wedge \neg\Box_S\, p \wedge \neg\Box_S\, q$ – corresponding to *John called Mary or Sue but not both, and I'm not sure which one* – instead of the crash $\Box_S$ $((p \vee q) \wedge \neg p \wedge \neg q \wedge \neg(p \wedge q)) \wedge \neg\Box_S\, p \wedge \neg\Box_S\, q$. Note that an obvious way to rule the second case out would be to say that the $\sigma$A must be pruned if their negation would contradict the prejacent. However, this is crucially different from what we generally allow for the DA of *or* and *some* $NP_{SG}$ . So I am not sure how to justify why in this case we can prune the singleton $\sigma$A . At any rate, the overall lesson might be that the singleton $\sigma$A are generated in a principled way, so we must let them in, but they yield undesirable results, so we must also find a principled way to rule them out.

default proceed relative to the prejacent in (42-a), the pre-exhaustified versions of the DA in (42-b)-(42-g), and the $\sigma$A in (42-h)-(42-n).

(42) $\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}\ \Box_S\ (p \vee q \vee r)$

a. $\Box_S\ (p \vee q \vee r)$ (prejacent)

b. $\Box_S\ p$ (singleton DA )

c. $\Box_S\ q$ (singleton DA )

d. $\Box_S\ r$ (singleton DA )

e. $\Box_S\ (p \vee q)$ (doubleton DA )

f. $\Box_S\ (q \vee r)$ (doubleton DA )

g. $\Box_S\ (p \vee r)$ (doubleton DA )

h. $\Box_S\ p$ (singleton $\sigma$A )

i. $\Box_S\ q$ (singleton $\sigma$A )

j. $\Box_S\ r$ (singleton $\sigma$A )

k. $\Box_S\ (p \wedge q)$ (doubleton $\sigma$A )

l. $\Box_S\ (q \wedge r)$ (doubleton $\sigma$A )

m. $\Box_S\ (p \wedge r)$ (doubleton $\sigma$A )

n. $\Box_S\ (p \wedge q \wedge r)$ (tripleton $\sigma$A )

We will discuss later on the results of this exhaustification done relative to the entire DA and $\sigma$A set that were generated from the truth conditions by default. Before we get to that, let us consider the easier cases where we have pruned the DA set.

Consider first the case where we have pruned all the non-singleton DA . In such a case exhaustification will proceed relative to the same prejacent but instead of the full set of alternatives above we will only have the singleton DA , the singleton $\sigma$A , and the tripleton $\sigma$A (because it is based on the assertion) (the doubleton $\sigma$A is based on the doubleton DA , and since those are pruned, these are presumably no longer available either).

(43) $O_{\text{ExhSgDA+}\sigma\text{A}}\ \Box_S\ (p \vee q \vee r)$

a. $\Box_S\ (p \vee q \vee r)$ (prejacent)

b. $\Box_S\ p$ (singleton DA )

c. $\Box_S\ q$ (singleton DA )

d. $\Box_S\ r$ (singleton DA )

e. $\Box_S\ p$ (singleton $\sigma$A )

f. $\Box_S\ q$ (singleton $\sigma$A )

g. $\Box_S\ r$ (singleton $\sigma$A )

h. $\Box_S\ (p \wedge q \wedge r)$ (tripleton $\sigma$A )

$O_{\text{ExhSgDA+}\sigma\text{A}}$ will assert the prejacent, (44-a), negate the pre-exhaustifications of the DA , (44-b), and negate the $\sigma$A , (44-c). The result is as below. In (44-b), the first underbrace spells out our assumptions about how the DA are pre-exhaustified – we assume that each singleton DA is exhaustified relative to the other singleton DA (consistent with a pre-exhaustification method where we either pre-exhaustify relative to whatever else there is in the DA set or relative to just other DA of the same size); the second underbrace spells out the logical result of negating the individual pre-exhaustifications (using the fact that a formula $\neg(a \wedge \neg b)$ is logically equivalent to $a \rightarrow b$).

(44) $O_{\text{ExhSgDA+}\sigma\text{A}}\ \Box_S\ (p \vee q \vee r)$

a. $\Box_S\ (p \vee q \vee r) \wedge$

b. $\underbrace{\neg \underbrace{O\,\Box_S\, p}_{\Box_S\, p \wedge \neg \Box_S\, q \wedge \neg \Box_S\, r}}_{\Box_S\, p \rightarrow \Box_S\, q \vee \Box_S\, r} \wedge \underbrace{\neg \underbrace{O\,\Box_S\, q}_{\Box_S\, q \wedge \neg \Box_S\, p \wedge \neg \Box_S\, r}}_{\Box_S\, q \rightarrow \Box_S\, p \vee \Box_S\, r} \wedge \underbrace{\neg \underbrace{O\,\Box_S\, r}_{\Box_S\, r \wedge \neg \Box_S\, p \wedge \neg \Box_S\, q}}_{\Box_S\, r \rightarrow \Box_S\, p \vee \Box_S\, q} \wedge$

c. $\neg \Box_S\ p \wedge \neg \Box_S\ q \wedge \neg \Box_S\ r \wedge \neg \Box_S\ (p \wedge q \wedge r)$

Consider now our scenarios of interest and whether or not such an exhaustification is compatible with them.

| scenario | possible? |
|---|---|
| No ignorance: $\Box_S\, p \wedge \Box_S\, q \wedge \Box_S\, r$ | ✗/✓ Clash with the $\sigma$A -implicature(s). Possible if they are suspended. |
| Partial ignorance, 'winner' type: $\Box_S\, p \wedge \neg\Box_S\, /\Box_S\, \neg q \wedge \neg\Box_S\, /\Box_S\, \neg r$ | ✗ $\Box_S\, p \wedge \neg\Box_S\, q \wedge \neg\Box_S\, r$ works for both the second and the third implication. However, if $\neg\Box_S\, q \wedge \neg\Box_S\, r$ is true, then the consequent of the first implication becomes false, and since $\Box_S\, p$ is not false, the implication overall becomes false. |
| Partial ignorance, 'loser' type: $\Box_S\, \neg p \wedge \neg\Box_S\, q \wedge \neg\Box_S\, r$ | ✓ |
| Total ignorance: $\neg\Box_S\, p \wedge \neg\Box_S\, q \wedge \neg\Box_S\, r$ | ✓ |

Table 1.3: Scenarios for $\mathrm{O}_{\mathrm{ExhSgDA}+\sigma\mathrm{A}}$ (pruning of non-singleton DA ).

To sum up, if we prune the DA set to just the singletons, the result is either no ignorance, partial ignorance of the 'loser' type, or total ignorance.

Consider now the case where we have pruned all the singleton DA . In such a case exhaustification will proceed relative to the same prejacent but instead of the full set of alternatives we will have just the doubleton DA , the doubleton $\sigma$A , and the tripleton $\sigma$A .

(45) $\mathrm{O}_{\mathrm{ExhNonSgDA}+\sigma\mathrm{A}}\ \Box_S\, (p \vee q \vee r)$

a. $\Box_S\, (p \vee q \vee r)$ (prejacent)

b. $\Box_S\, (p \vee q)$ (doubleton DA )

c. $\Box_S\, (q \vee r)$ (doubleton DA )

d. $\Box_S\, (p \vee r)$ (doubleton DA )

e. $\Box_S\, (p \wedge q)$ (doubleton $\sigma$A )

f. $\Box_S\,(q \wedge r)$ (doubleton $\sigma$A )

g. $\Box_S\,(p \wedge r)$ (doubleton $\sigma$A )

h. $\Box_S\,(p \wedge q \wedge r)$ (tripleton $\sigma$A )

$O_{\text{ExhNonSgDA+}\sigma\text{A}}$ will assert the prejacent, (46-a), negate the pre-exhaustifications of the DA , (46-b), and negate the $\sigma$A , (44-c). The result is as below. In (46-b), the first underbrace spells out our assumptions about how the DA are pre-exhaustified – we assume that each doubleton DA is exhaustified relative to the other doubleton DA (consistent with a pre-exhaustification method where we either pre-exhaustify relative to whatever else there is in the DA set or relative to just other DA of the same size); the second underbrace spells out the logical result of negating the individual pre-exhaustifications (again, using the fact that a formula $\neg(a \wedge \neg b)$ is logically equivalent to $a \rightarrow b$).

(46)

$O_{\text{ExhNonSgDA+}\sigma\text{A}}\ \Box_S\,(p \vee q \vee r)$

a. $\Box_S\,(p \vee q \vee r) \wedge$

b. $\underbrace{\neg \underbrace{O\,\Box_S\,(p \vee q)}_{\Box_S\,(p\vee q)\wedge\neg\Box_S\,(q\vee r)\wedge\neg\Box_S\,(p\vee r)}}_{\Box_S\,(p\vee q)\rightarrow\Box_S\,(q\vee r)\vee\Box_S\,(p\vee r)} \wedge \underbrace{\neg \underbrace{O\,\Box_S\,(q \vee r)}_{\Box_S\,(q\vee r)\wedge\neg\Box_S\,(p\vee q)\wedge\neg\Box_S\,(p\vee r)}}_{\Box_S\,(q\vee r)\rightarrow\Box_S\,(p\vee q)\vee\Box_S\,(p\vee r)} \wedge \underbrace{\neg \underbrace{O\,\Box_S\,(p \vee r)}_{\Box_S\,(p\vee r)\wedge\neg\Box_S\,(p\vee q)\wedge\neg\Box_S\,(q\vee r)}}_{\Box_S\,(p\vee r)\rightarrow\Box_S\,(p\vee q)\vee\Box_S\,(q\vee r)} \wedge$

c. $\neg\Box_S\,(p \wedge q) \wedge \neg\Box_S\,(q \wedge r) \wedge \neg\Box_S\,(p \wedge r) \wedge \neg\Box_S\,(p \wedge q \wedge r)$

Consider again our scenarios of interest and whether or not such an exhaustification is compatible with them.

| scenario | possible? |
|---|---|
| No ignorance:<br>$\Box_S\,p \wedge \Box_S\,q \wedge \Box_S\,r$ | ✗/✓ Clash with the $\sigma$A -implicature(s). Possible if they are suspended. |
| Partial ignorance, 'winner' type:<br>$\Box_S\,p \wedge \neg\Box_S\,/\Box_S\,\neg q \wedge \neg\Box_S\,/\Box_S\,\neg r$ | ✓ |

Table 1.4 (Continued)

| | |
|---|---|
| Partial ignorance, 'loser' type: $\Box_S \neg p \wedge \neg\Box_S q \wedge \neg\Box_S r$ | ✗ Consider, for example, the second implication (ExhDA -implicature). Suppose $\Box_S \neg p$ is true. Then, if $\neg\Box_S q \wedge \neg\Box_S r$ is also true, the whole consequent is false. This means that the implication can be true iff the antecedent $\Box_S (q \vee r)$ is also false. But this would contradict $\Box_S (p \vee q \vee r) \wedge \Box_S \neg p = \Box_S (q \vee r)$. |
| Total ignorance: $\neg\Box_S p \wedge \neg\Box_S q \wedge \neg\Box_S r$ | ✓ |

Table 1.4: Scenarios for $\mathrm{O}_{\text{ExhNonSgDA+}\sigma\text{A}}$ (pruning of singleton DA ).

To sum up, if we prune the DA set to just the non-singletons, the result is either no ignorance, partial ignorance of the 'winner' type, or total ignorance.

We are now ready to consider the case where we don't prune any DA . In this case exhaustification will proceed relative to the default set of DA generated based on the truth conditions, that is, the set we figured out earlier and which we are repeating below.

(47) $\mathrm{O}_{\text{ExhDA+}\sigma\text{A}}\ \Box_S (p \vee q \vee r)$

a. $\Box_S (p \vee q \vee r)$ (prejacent)

b. $\Box_S p$ (singleton DA )

c. $\Box_S q$ (singleton DA )

d. $\Box_S r$ (singleton DA )

e. $\Box_S (p \vee q)$ (doubleton DA )

f. $\Box_S (q \vee r)$ (doubleton DA )

g. $\Box_S (p \vee r)$ (doubleton DA )

h. $\Box_S p$ (singleton $\sigma$A )

i. $\Box_S\, q$ (singleton $\sigma$A )

j. $\Box_S\, r$ (singleton $\sigma$A )

k. $\Box_S\, (p \wedge q)$ (doubleton $\sigma$A )

l. $\Box_S\, (q \wedge r)$ (doubleton $\sigma$A )

m. $\Box_S\, (p \wedge r)$ (doubleton $\sigma$A )

n. $\Box_S\, (p \wedge q \wedge r)$ (tripleton $\sigma$A )

$\mathrm{O}_{\mathrm{ExhDA+}\sigma\mathrm{A}}$ will assert the prejacent, (48-a), negate the pre-exhaustifications of the DA , (48-b), and negate the $\sigma$A , (48-c). The result is as below. (48-b) brings together the negations of all the pre-exhaustified DA , both singleton and doubleton. The first underbrace again spells out our assumptions about how the DA are pre-exhaustified – we will assume as before that each singleton DA is exhaustified relative to the other singleton DA and each doubleton DA is exhaustified relative to the other doubleton DA (however, unlike before, where this was consistent with a pre-exhaustification method where we either pre-exhaustified relative to whatever else there was in the DA set or relative to just other DA of the same size, now we have essentially picked the latter method)[8]; the second underbrace spells out the logical result of negating the individual pre-exhaustifications (again, using the fact that a formula $\neg(a \wedge \neg b)$ is logically equivalent to

[8]The other method of saying that pre-exhaustification is done relative to all the DA that are available in the DA set might work also. However, while on the pre-exhaustification method we have chosen the intersection of the singleton and doubleton ExhDA -implicatures ensures incompatibility with either one of the partial variation case (and is only compatible with a total ignorance case), on this alternate pre-exhaustification method the intersection of the singleton and doubleton ExhDA -implicatures would also be compatible with the partial ignorance of the 'winner' type (as well as being compatible with total ignorance). This would be a direct consequence of the new method of pre-exhaustifying, which for a singleton DA such as, e.g., $\Box_S\, p$ would yield $\neg\mathrm{O}\,\Box_S\, p = \Box_S\, p \rightarrow \Box_S\, (q \vee r) \vee \Box_S\, q \vee \Box_S\, r$ – a meaning compatible with the 'winner' case. The 'winner' case predicted by this new way of pre-exhaustifying the singletons can still be ruled out by the singleton $\sigma$A -implicatures, but, given everything we have said about the $\sigma$A being prunable, we might prefer not to rely on the $\sigma$A to ensure total ignorance. Otherwise, if the $\sigma$A are suspended, we would predict that an item such as *or* could also always be compatible with the 'winner' case. (Note that this method of pre-exhaustification also requires us to use $\mathrm{O}_{\mathrm{IE}}$ , i.e., the variant of O that avoids contradiction by excluding only those alternatives that are Innocently Excludable, i.e., that can be excluded together while the prejacent – the DA being pre-exhaustified – stays true. This is because otherwise O $\Box_S\, p$ would also exclude doubleton DA such as $\Box_S\, (p \vee q)$ or $\Box_S\, (p \vee r)$, which would lead to contradiction. As Chierchia (2013) argues, even on approaches like ours that otherwise use the non-contradiction-avoiding O for assertions, we might still want to use the contradiction-avoiding $\mathrm{O}_{\mathrm{IE}}$ for pre-exhaustifications, the reason being simply that we never want pre-exhaustifications to be contradictory, otherwise they would never succeed in their goal of strengthening the alternatives.)

$a \rightarrow b$). (48-c) brings together the negations of all the $\sigma$A , both singleton and non-singleton.

(48)

$\mathrm{O}_{\mathbf{ExhDA+\sigma A}}\ \Box_S\,(p \vee q \vee r)$

a. $\Box_S\,(p \vee q \vee r)\wedge$

b. $\neg \underbrace{\underbrace{\mathrm{O}\,\Box_S\,p}_{\Box_S\,p\wedge\neg\Box_S\,q\wedge\neg\Box_S\,r}}_{\Box_S\,p\rightarrow\Box_S\,q\vee\Box_S\,r} \wedge \neg \underbrace{\underbrace{\mathrm{O}\,\Box_S\,q}_{\Box_S\,q\wedge\neg\Box_S\,p\wedge\neg\Box_S\,r}}_{\Box_S\,q\rightarrow\Box_S\,p\vee\Box_S\,r} \wedge \neg \underbrace{\underbrace{\mathrm{O}\,\Box_S\,r}_{\Box_S\,r\wedge\neg\Box_S\,p\wedge\neg\Box_S\,q}}_{\Box_S\,r\rightarrow\Box_S\,p\vee\Box_S\,q} \wedge$

$\neg \underbrace{\underbrace{\mathrm{O}\,\Box_S\,(p\vee q)}_{\Box_S\,(p\vee q)\wedge\neg\Box_S\,(q\vee r)\wedge\neg\Box_S\,(p\vee r)}}_{\Box_S\,(p\vee q)\rightarrow\Box_S\,(q\vee r)\vee\Box_S\,(p\vee r)} \wedge \neg \underbrace{\underbrace{\mathrm{O}\,\Box_S\,(q\vee r)}_{\Box_S\,(q\vee r)\wedge\neg\Box_S\,(p\vee q)\wedge\neg\Box_S\,(p\vee r)}}_{\Box_S\,(q\vee r)\rightarrow\Box_S\,(p\vee q)\vee\Box_S\,(p\vee r)} \wedge \neg \underbrace{\underbrace{\mathrm{O}\,\Box_S\,(p\vee r)}_{\Box_S\,(p\vee r)\wedge\neg\Box_S\,(p\vee q)\wedge\neg\Box_S\,(q\vee r)}}_{\Box_S\,(p\vee r)\rightarrow\Box_S\,(p\vee q)\vee\Box_S\,(q\vee r)} \wedge$

c. $\neg\Box_S\,p \wedge \neg\Box_S\,q \wedge \neg\Box_S\,r \wedge \neg\Box_S\,(p\wedge q) \wedge \neg\Box_S\,(q\wedge r) \wedge \neg\Box_S\,(p\wedge r) \wedge \neg\Box_S\,(p\wedge q\wedge r)$

Since for this exhaustification relative to all the DA we did nothing more than bring together the results from $\mathrm{O}_{\mathbf{ExhSgDA+\sigma A}}$ and $\mathrm{O}_{\mathbf{ExhNonSgDA+\sigma A}}$ , the result is also nothing more than the intersection of what we got in those cases. That is, it is compatible with either no ignorance at all or with total ignorance.

| scenario | possible? |
|---|---|
| No ignorance:<br>$\Box_S\,p \wedge \Box_S\,q \wedge \Box_S\,r$ | ✗/✓ Clash with the $\sigma$A -implicature(s). Possible if they are suspended. |
| Partial ignorance, 'winner' type:<br>$\Box_S\,p \wedge \neg\Box_S\,/\Box_S\,\neg q \wedge \neg\Box_S\,/\Box_S\,\neg r$ | ✗ |
| Partial ignorance, 'loser' type:<br>$\Box_S\,\neg p \wedge \neg\Box_S\,q \wedge \neg\Box_S\,r$ | ✗ |
| Total ignorance:<br>$\neg\Box_S\,p \wedge \neg\Box_S\,q \wedge \neg\Box_S\,r$ | ✓ |

Table 1.5: Scenarios for $\mathrm{O}_{\mathbf{ExhDA+\sigma A}}$ (no DA pruning).

The solution to the difference with respect to the strength of the ignorance effect in *or* and *some NP$_{SG}$*

that we will propose is then as follows: By default both *or* and *some $NP_{SG}$* have to be exhaustified relative to both ExhDA and $\sigma$A , and the result is as in the no pruning case, i.e., total ignorance. In the presence of a no ignorance continuation, both can prune their $\sigma$A to accommodate it. In the presence of a context of partial ignorance of the 'winner' or the 'loser' type, which is incompatible with exhaustifying relative to the full set of DA , *some $NP_{SG}$* but not *or* is able to also prune its DA set to either just the non-singleton DA – this accommodates the 'winner' case – or just the singleton DA – this accommodates the 'loser' case.

At this point it would be interesting to say a little bit more about the notion of pruning that comes out of this. Note that, in the way we described things just now, it is not only the pruning of $\sigma$A but (for items that allow this) also the pruning of DA that happens only if forced by the context. Thus, although in discussing the results of $O_{\text{ExhSgDA+}\sigma\text{A}}$ /$O_{\text{ExhNonSgDA+}\sigma\text{A}}$ we suggested that, in addition to the 'loser'/'winner' case, they were also compatible with no ignorance or total ignorance, if the pruning of the DA is triggered by a context of the 'loser'/'winner' type in the first place, then the no ignorance or total ignorance cases would already be filtered out, because they would be incompatible with the context. Thus, $O_{\text{ExhSgDA+}\sigma\text{A}}$ /$O_{\text{ExhNonSgDA+}\sigma\text{A}}$ really only yield the 'loser'/'winner' case. Nothing in our dataset so far forces us to adopt this view of pruning of the DA , but it seems desirable to constrain it somehow, just as we did for $\sigma$A , and constraining via context of the kind that we just described seems like a natural way to do so.

To sum up, we have proposed that, if the ignorance effect comes from obligatory $O_{\text{ExhDA+}\sigma\text{A}}$ across $\Box_S$ , variations in the strength of this effect come from the ability to prune DA .

But wouldn't a simpler account of ignorance in *or* vs. *some $NP_{SG}$* be to say that, while the total ignorance effect in *or* comes from obligatory $O_{\text{ExhDA}}$ and arises as shown above, the weaker effect in *some $NP_{SG}$* comes simply from an *optional* application of $O_{\text{ExhDA}}$ ? In the next section we will see a good reason to say that $O_{\text{ExhDA}}$ is obligatory for *some $NP_{SG}$* as well. This will provide additional support for the approach presented here.[9]

[9]Another reason to get partial variation effects from obligatory $O_{\text{ExhDA+}\sigma\text{A}}$ plus pruning of some subclass of DA again comes from looking at partial variation effects in items other than *or* and *some $NP_{SG}$* . For example, in (10)-(11) on p. 4 we saw that the German indefinite *irgendein* was felicitous in partial ignorance scenarios of the 'loser' but not the 'winner' type. On the present approach we could say that it must be exhaustified relative to

### 1.2.6 Anti-negativity comes from obligatory $O_{ExhDA}$ plus a requirement for P(roper) S(trengthening)

Recall that *or* and *some* $NP_{SG}$ contrast in their ability to take scope below downward-entailing operators – *some* $NP_{SG}$ is bad under negation although, like *or*, it is fine in the antecedent of a conditional or the restriction of a universal.

(49) (= (4))

a. John didn't call Teddy or Sue. ✓*not* > *or*

b. #John didn't call some student. # *not* > *some* $NP_{SG}$

(50) (= (5))

a. If John called Teddy or Sue, he won. ✓if > *or*

b. If John called some student, he won. ✓if > *some* $NP_{SG}$

(51) (= (6))

a. Everyone who called Teddy or Sue won. ✓every > *or*

b. Everyone who called some student won. ✓every > *some* $NP_{SG}$

How might we derive this distribution?

First let us focus on the negation case. Why is *or* fine under negation but *some* $NP_{SG}$ not?

The existing alternative-based approaches to polarity sensitivity provide a crucial insight. Chierchia (2013)'s solution to anti-positivity / Negative Polarity Items (NPIs) relies on the assumption that they come with a requirement for obligatory exhaustification relative to the DA , and the observation that in positive contexts this crashes (and for various reasons can't be rescued via $\Box_S$ as in the case of epistemic indefinites in general, or of our items *or* and *some* $NP_{SG}$ in particular). Then, Chierchia (2013), Spector (2014), or Nicolae (2017)'s solution to anti-negativity / Positive Polarity Items (PPIs) all rely on the as-

either all the DA or just the singleton DA . This would capture not only its default total ignorance effect but also the fact that it is compatible with partial ignorance contexts of the 'loser' but not the 'winner' type.

sumption that not only do some items come with a requirement for obligatory exhaustification relative to their DA , but some additionally reject a vacuous result.

Let's see how this insight can help us make sense of *or* and *some* $NP_{SG}$ .

First, recall that we assumed that both *or* and *some* $NP_{SG}$ come with obligatory $\mathrm{O}_{\mathrm{ExhDA}}$ . The challenge then is to formalize the ban on vacuous $\mathrm{O}_{\mathrm{ExhDA}}$ . We will do that as in Chierchia (2013), that is, we will argue that some items come with a requirement not simply for $\mathrm{O}_{\mathrm{C}}$ but rather for $\mathrm{O}_{\mathrm{C}}^{\mathrm{PS}}$ – a presuppositional variant of $\mathrm{O}_{\mathrm{C}}$ that is defined iff $\mathrm{O}_{\mathrm{C}}$ leads to a properly stronger (PS) meaning, and if defined it then behaves just like $\mathrm{O}_{\mathrm{C}}$ . (Below the proper strengthening requirement is marked as a superscript PS on O . The subscript C denotes the relevant alternatives, whatever they might be. For our cases it would refer to the ExhDA .)

(52) Exhaustification with Proper Strengthening, first version:

$[\![\mathrm{O}_{\mathrm{C}}^{\mathrm{PS}}\,(\mathrm{p})]\!]^{g,w}$ is defined iff $\lambda w\,.\ [\![\mathrm{O}_{\mathrm{C}}\,(\mathrm{p})]\!]^{g,w} \subset p$. Whenever defined, $[\![\mathrm{O}_{\mathrm{C}}^{\mathrm{PS}}\,(\mathrm{p})]\!]^{g,w} = [\![\mathrm{O}_{\mathrm{C}}\,(\mathrm{p})]\!]^{g,w}$.

Let's see how $\mathrm{O}^{\mathrm{PS}}$ can help us make sense of anti-negativity.

Consider the exhaustification $\mathrm{O}_{\mathrm{ExhDA}}^{\mathrm{PS}}$ of an *or/some* $NP_{SG}$ utterance embedded under negation. We can represent the $\mathrm{O}_{\mathrm{ExhDA}}^{\mathrm{PS}}$ parse, its prejacent, and its DA schematically as below. (We continue to assume that $\sigma$A are always factored in, but they don't matter for the present discussion, so we leave them out.)

(53) John didn't call Teddy or Sue / John didn't call some student

$\mathrm{O}_{\mathrm{ExhDA}}^{\mathrm{PS}}\, \neg(p \vee q)$

a. $\neg(p \vee q)$ (prejacent)

b. $\neg p$ (DA )

c. $\neg q$ (DA )

$\mathrm{O}_{\mathrm{DA}}^{\mathrm{PS}}$ is defined iff $\mathrm{O}_{\mathrm{ExhDA}}$ leads to proper strengthening, and if so, then it is the same as $\mathrm{O}_{\mathrm{ExhDA}}$ . So before we can discuss $\mathrm{O}_{\mathrm{DA}}^{\mathrm{PS}}$ we must first consider $\mathrm{O}_{\mathrm{ExhDA}}$ and its outcome. $\mathrm{O}_{\mathrm{ExhDA}}$ will as usual assert the prejacent, (54-a), and negate the pre-exhaustifications of the DA , (54-b).

(54) John didn't call Teddy or Sue / John didn't call some student

$O_{\text{ExhDA}}\ \neg(p \vee q)$

a. $\neg(p \vee q) \wedge$

b. $\underbrace{\neg \underbrace{O\,(\neg p)}_{\substack{\neg p \wedge \neg(\neg q),\ = \neg p \wedge q \\ \text{already excluded by the prejacent}}}}_{\text{can't satisfy PS}} \wedge \underbrace{\neg \underbrace{O\,(\neg q)}_{\substack{\neg q \wedge \neg(\neg p),\ = \neg q \wedge p \\ \text{already excluded by the prejacent}}}}_{\text{can't satisfy PS}}$

Note that each of the ExhDA is incompatible with the assertion, and thus already excluded by it. This means that their negation doesn't add anything. Thus, $O_{\text{ExhDA}}$ is vacuous, it does not lead to a stronger meaning, and so the PS presupposition of $O^{\text{PS}}_{\text{ExhDA}}$ cannot be satisfied. Note also that inserting a $\Box_S$ between $O_{\text{ExhDA}}$ and its target doesn't change anything.

Suppose now that *or* comes with obligatory $O_{\text{ExhDA}}$ but *some* $NP_{SG}$ comes with obligatory $O^{\text{PS}}_{\text{ExhDA}}$. This means that *or* can tolerate a result like the above but *some* $NP_{SG}$ can't. This captures why *or* is fine in the scope of negation while *some* $NP_{SG}$ is not.

But the solution that derived badness under negation above should also derive badness in any other downward-entailing environments. However, as we know, although bad in the scope of negation, *some* $NP_{SG}$ is acceptable in the antecedent of a conditional and the restriction of a universal. How can we capture this?

Von Fintel (1999) notes that a crucial distinction between a downward-entailing environment such as the scope of negation and downward-entailing environments such as the antecedent of a conditional and the restriction of a universal is that the latter carry an existential presupposition.

(55) John didn't call Teddy or Sue / #some student.

presupposes: nothing

(56) If John called Teddy or Sue / some student, he won.

presupposes: There is an accessible world where John called Teddy or Sue / some student.

(57) Everyone who called Teddy or Sue / some student won.

presupposes: Someone called Teddy or Sue / some student.

Building on this insight, the various alternative-based approaches to polarity sensitivity derive this split acceptability of certain items with respect to downward-entailing environments by saying that for some items exhaustification is sensitive to the presuppositions/implicatures in the prejacent, if any. Building on Gajewski (2011), Chierchia (2013) uses this assumption to explain why certain items that exhibit anti-positivity (items that want to be under negation/downward-entailing environments, i.e., NPIs) may still be bad in the antecedent of a conditional and the restriction of a universal (cf. strong NPIs). Spector (2014) and Nicolae (2017) also use this idea to derive why items that exhibit anti-negativity (like our *some* $NP_{SG}$ ) are always fine in the antecedent of a conditional / restriction of a universal (Spector notes that, because of this, the distribution of these items, which he calls PPIs – as we might also – seems to be the mirror image of that of strong NPIs).

Let's see how this insight can help us make sense of *or* and *some* $NP_{SG}$ .

First, let us formalize the notion that exhaustification may be sensitive to presuppositional content. We will do this as in Chierchia (2013). First, we will define $\pi(p)$ as the presupposition-enriched content of $p$ consisting of the conjunction of its assertive component and of its presuppositional component.

(58) $\pi(p) = {}^{\alpha}p \wedge {}^{\pi}p$ (Chierchia 2013:219)

Second, we will define strong exhaustification, achieved via $\mathrm{O}^{\mathrm{S}}$ 'O strong', as a variant of O that behaves exactly like O , just that instead of taking into account the assertive component of the prejacent and the assertive component of the alternatives, it rather targets their *presupposition-enriched* assertive content. ('Strong' exhaustification is marked below as a superscript on O .)

(59) $\left[\!\left[\mathrm{O}^{\mathrm{S}}_{\mathrm{C}}\,(p)\right]\!\right]^{g,w} = [\![\mathrm{p}]\!]^{g,w} \wedge \forall q \in [\![\mathrm{p}]\!]^{\mathrm{C}}\ [\pi([\![\mathrm{q}]\!])^{g,w} \rightarrow \pi(\lambda w'\,.\,[\![\mathrm{p}]\!]^{g,w'}) \subseteq \pi([\![\mathrm{q}]\!])]$

(Chierchia 2013:220)

We will assume that both *or* and *some* $NP_{SG}$ are in fact exhaustified not via plain $\mathrm{O}_{\mathrm{ExhDA}}$ but rather via $\mathrm{O}^{\mathrm{S}}_{\mathrm{ExhDA}}$ . (This doesn't change anything for our previous computations with $\mathrm{O}_{\mathrm{ExhDA}}$ , since this the ‘strong’ part matters only makes a difference for presuppositional environments.)

Finally, we will assume that $\mathrm{O}^{\mathrm{PS}}$ is in fact defined not in terms of O but rather relative to $\mathrm{O}^{\mathrm{S}}$ .

(60) Exhaustification with Proper Strengthening, final version:

$[\![\mathrm{O}^{\mathrm{PS}}_{\mathrm{C}}\,(\mathrm{p})]\!]^{g,w}$ is defined iff $\lambda w\,.\ [\![\mathrm{O}^{\mathrm{S}}_{\mathrm{C}}\,(\mathrm{p})]\!]^{g,w} \subset p$.

Whenever defined, $[\![\mathrm{O}^{\mathrm{PS}}_{\mathrm{C}}\,(\mathrm{p})]\!]^{g,w} = [\![\mathrm{O}^{\mathrm{S}}_{\mathrm{C}}\,(\mathrm{p})]\!]^{g,w}$.

(This update in the definition of $\mathrm{O}^{\mathrm{PS}}$ does not affect our previous results for the negation case since in that case there was no presupposition.)

We are now ready to get back to our discussion of *or/some* $NP_{SG}$ in the antecedent of a conditional or the restriction of a universal. Let’s consider the exhaustification via $\mathrm{O}^{\mathrm{S}}_{\mathrm{ExhDA}}$ of *or/some* $NP_{SG}$ in these environments. First, to discuss these environments together, we will abbreviate the world variable $w$ from the conditional and the individual variable $x$ from the universal as $v$; then, we will also assume, as usual, that *or* and *some* $NP_{SG}$ have the same domain and use $t, s$ to represent ‘John called Teddy/Sue’ in the first case and ‘called Teddy/Sue’ in the second. Then, since we are dealing with strong exhaustification, for both the prejacent and the DA we will consider the conjunction of the truth-conditional and the presuppositional component. The exhaustification parse, the prejacent, and the DA are then as below. In preparation for the computation to come, underneath each DA we also show the negation of its pre-exhaustification.

(61) If John called Teddy or Sue / some student, he won

Everyone who called Teddy or Sue / some student won

$\mathrm{O}^{\mathrm{S}}_{\mathrm{ExhDA}}\, \forall v[t_v \vee s_v \rightarrow W_v]$

a. $\forall v[t_v \vee s_v \rightarrow W_v] \wedge \exists v[t_v \vee s_v]$ (prejacent)

b. $\forall v[t_v \rightarrow W_v] \wedge \exists v[t_v]$ (DA )

(i) $\neg\mathrm{O}\ (\forall v[t_v \rightarrow W_v] \wedge \exists v[t_v])$ (negation of ExhDA )

$= \neg((\forall v[t_v \rightarrow W_v] \wedge \exists v[t_v]) \wedge \neg(\forall v[s_v \rightarrow W_v] \wedge \exists v[s_v]))$

$= (\forall v[t_v \rightarrow W_v] \wedge \exists v[t_v]) \rightarrow (\forall v[s_v \rightarrow W_v] \wedge \exists v[s_v])$

c. $\forall v[s_v \to W_v] \wedge \exists v[s_v]$ (DA )

(i) $\neg \mathrm{O}\ (\forall v[s_v \to W_v] \wedge \exists v[s_v])$ (negation of ExhDA )

$= \neg((\forall v[s_v \to W_v] \wedge \exists v[s_v]) \wedge \neg(\forall v[t_v \to W_v] \wedge \exists v[t_v]))$

$= (\forall v[s_v \to W_v] \wedge \exists v[s_v]) \to (\forall v[t_v \to W_v] \wedge \exists v[t_v])$

$\mathrm{O}^{\mathrm{S}}_{\mathrm{ExhDA}}$ will as usual assert the prejacent, (62-a), and the negations of the pre-exhaustifications of the DA , (62-b).

(62) If John called Teddy or Sue / some student, he won

Everyone who called Teddy or Sue / some student won

$\mathrm{O}^{\mathrm{S}}_{\mathrm{ExhDA}}\ \forall v[t_v \vee s_v \to W_v]$

a. $\forall v[t_v \vee s_v \to W_v] \wedge \exists v[t_v \vee s_v] \wedge$

b. $\underbrace{\neg \mathrm{O}\ (\forall v[t_v \to W_v] \wedge \exists v[t_v])}_{(\forall v[t_v \to W_v] \wedge \exists v[t_v]) \to (\forall v[s_v \to W_v] \wedge \exists v[s_v])} \quad \wedge \quad \underbrace{\neg \mathrm{O}\ (\forall v[s_v \to W_v] \wedge \exists v[s_v])}_{(\forall v[s_v \to W_v] \wedge \exists v[s_v]) \to (\forall v[t_v \to W_v] \wedge \exists v[t_v])}$

As usual, the negations of the ExhDA in (62-b) are equivalent to implications. Logically speaking, each implication can be true if both its terms are true, if both are false, or if the first term is false and the second is true. First note that each term consists of the conjunction of a universal term coming from the assertion and of an existential term coming from the presupposition, and the universal term is entailed by the assertion, but the existential term is not, so it is only this term essentially that has the potential to drive any strengthening. Let's consider now the various cases for the implication. The true-true case yields $\exists v[t_v] \wedge \exists v[s_v]$. This is a case that strengthens the utterance from a meaning where it presupposes that there is an accessible world where John called Teddy or Sue / that someone called Teddy or Sue to a meaning where it presupposes that there is an accessible world where John called Teddy and there is an accessible world where John called Sue / that someone called Teddy and someone called Sue. (The reader might have noticed that this is exactly like the Free Choice implicatures arising from $\mathrm{O}_{\mathrm{ExhDA}}\ \Diamond(p \vee q)$, which is unsurprising given the $\exists v$ intervening between $\mathrm{O}_{\mathrm{ExhDA}}$ and the disjunction.) The false-false case yields $\neg\exists v[t_v] \wedge \neg\exists v[s_v]$. (Recall that the universal component is entailed by the prejacent, so it cannot be false, so falsity in each term must come from the existential component.) This clashes with the prejacent $\exists v[t_v \vee s_v]$, so it is ruled out. It can be rescued by prefixing the assertion and presupposition (and, as a consequence, also their alternatives) with $\Box_S$. In that case the result will be $\neg\Box_S\ \exists v[t_v] \wedge \neg\Box_S\ \exists v[s_v]$. This is a case that strengthens the utterance from a meaning where it presupposes that the speaker is certain there is an accessible world where John called Teddy or Sue / that someone called Teddy or Sue to a meaning where it additionally presupposes that the speaker is ignorant whether there is an accessible world where John called Teddy and ignorant whether there is an accessible world where John called Sue. (The reader might have noticed that this is similar to the Free Choice implicatures arising from $\mathrm{O}_{\mathrm{ExhDA}}\ \Box_S\ (p \vee q)$.) Finally, the false-true case. This is actually not a possibility, since making the first implication true by false-true would yield $\neg\exists v[t_v] \wedge \exists v[s_v]$, which would however make the second implication false by forcing its consequent to be false.

All in all, these results show that $\mathrm{O}^{\mathrm{S}}_{\mathrm{ExhDA}}$ of *or*/*some* $NP_{SG}$ in the antecedent of a conditional or the restriction of a universal can lead to proper strengthening. This captures why not only *or* but also *some* $NP_{SG}$ is fine in these environments.

To sum up, if both *or* and *some* $NP_{SG}$ undergo obligatory $\mathrm{O}^{\mathrm{S}}_{\mathrm{ExhDA}}$ but for *or* it is of the plain kind while for *some* $NP_{SG}$ it is of the PS kind, then this explains why *or* is fine in the scope of negation while *some* $NP_{SG}$ is not, and also why they are both fine in the antecedent of a conditional or the restriction of a universal.

We have reached our goal to make sense of why *or* and *some* $NP_{SG}$ vary with respect to the scope of negation but not the antecedent of a conditional or the restriction of a universal. But the discussion above invites further questions also. Below we discuss a few of the more prominent issues in the literature on items with anti-negativity / PPIs.

First, let's consider embedding under other downward-entailing operators, for example, *few*. In the literature on PPIs it is often claimed that items that are bad in the scope of negation are fine in the scope of *few*. Below I illustrate with an example for the English indefinite *someone* from Szabolcsi (2004), Nicolae (2012).

(63) a. John didn't talk to someone. # *not* > *someone*

b. Few people talked to someone yesterday. ✓*few* > *someone*

Is *some* $NP_{SG}$ the same? An informal questionnaire reveals that native speakers tend to find *some* $NP_{SG}$ quite degraded under *few*.

(64) Few people called some student. ?? *few* > *some* $NP_{SG}$

Our account seems to straightforwardly capture the badness of *some* $NP_{SG}$ under *few* – $O^{PS}_{ExhDA}$ across a downward-entailing operator crashes, and *few* also doesn't carry any presuppositions that might make a difference. Why then might other items that are bad under negation be fine under *few*? Note that we said that $O^{PS}$ is based on $O^{S}$ , which takes into account not just the truth-conditional content of the prejacent but also non-truth-conditional content such as presuppositions. But what if $O^{S}$ was defined to include non-truth-conditional content of not just the presuppositional kind but also of the implicature kind? (This is in fact how it is used in Chierchia 2013 to handle strong NPIs.) Then we would actually be able to capture acceptability under *few* also, because although *few* carries no presuppositions, it does typically give rise to a positive implicature that its *some* alternative is true (*Some people did call Teddy or Sue*). The bottom line would be that we must distinguish between at least two varieties of $O^{S}$ , one that cares only about the presupposition-enriched assertion (relevant to *some* $NP_{SG}$ ) and one that cares about the presupposition-*and-implicature*-enriched assertion.

Second, let's consider embedding under a negative attitude as expressed by *not think* or *doubt*. In the literature on PPIs it is often claimed that items bad in the scope of a negative operator are fine if the negative operator is at a distance. Below we illustrate this with an example from Szabolcsi (2004).

(65) a. John didn't call someone. # *not* >*someone*

b. I don't think that John called someone. *not* > $[_{CP}$ *someone*

However, an informal survey (judgments courtesy of Shannon Bryant, Caitlin Keenan, Chantale Yunt) reveals that *some* $NP_{SG}$ is in fact quite degraded even under extra-clausal negation.

(66) I don't think that John called some student. ?? *not* > $[_{CP}$ *some* $NP_{SG}$

Spector (2014) shows that items that exhibit anti-negativity vary with respect to whether they are degraded in the scope of just a local negation or also in the scope of negation at any distance. He calls the former 'local' PPIs and the latter 'global PPIs' and argues that on an approach like ours, where unacceptability under negation comes from a requirement for exhaustification and a ban on vacuous exhaustification, the difference could be captured by saying that for some items $O^{PS}$ can be satisfied locally (e.g., by inserting $\Box$ at the level of the embedded clause and exhaustifying across it at that site) while for others it has to be satisfied globally (i.e., what matters is exhaustification at the matrix level). Why this should be the case however remains unexplained. I will argue then that *some* $NP_{SG}$ is like global PPIs, and I leave it to future research to explain why some PPIs are of the local kind while others of the global kind.

Third, let's consider embedding under not one, but two downward-entailing operators. In the literature on PPIs it is often argued that in such a configuration items bad in the scope of a negative operator improve if the negative operator is itself embedded in an additional downward-entailing environment. This effect is called 'rescuing'. Below we illustrate with two examples from Nicolae (2012:476; for *someone*) and Spector (2015:4; for *almost*), where the additional downward-entailing environment is *doubt* or the antecedent of a conditional.

(67) a. John didn't call someone. # *not* > *someone*

b. I doubt that John didn't call someone. ✓*doubt* > *not* > *someone*

(68) a. John didn't almost finish his homework. # *not* > *almost*

b. If John had not almost finished his homework, ... ✓*if* > *not* > *almost*

An informal survey (judgments again courtesy of Shannon Bryant, Caitlin Keenan, Chantale Yunt) reveals that *some* $NP_{SG}$ also exhibits rescuing, but also that the degree of rescuing differs depending on the higher downward-entailing element – # *not* > *some* $NP_{SG}$ improves both under *doubt* and in the antecedent of a conditional, but somewhat more in the second case.

(69) a. John didn't call some student. # *not* > *some* $NP_{SG}$

b. (i) I doubt John didn't call some student. ?*doubt* > *not* > *some* $NP_{SG}$

(ii) If John didn't call some student, I'd be surprised. ✓*if* $>$ *not* $>$ *some* $NP_{SG}$

As Spector (2014) argued, the rescuing effect itself can be explained by reasoning about the polarity / monotonicity of the overall environment of *some* $NP_{SG}$ in this configuration: If one downward-entailing operator creates a downward-entailing environment, two cancel out and give rise to an upward-entailing environment, so an item under two downward-entailing operators is the same as under none. I will adopt this line of thought for *some* $NP_{SG}$ also, although note that it doesn't yet account for why the nature of the additional downward-entailing environment seems to make a difference. I leave that to future research.

Fourth, let's consider the effect of an intervening universal operator. In the literature on PPIs (in analogy to the literature on NPIs) it has been noticed that an item bad in the immediate scope of negation improves if a universal operator comes in between. This effect is sometimes called 'shielding' (cf. Nicolae 2012). We illustrate it below with the English indefinite *something* and two universal operators, *always* and *everyone* (examples from Nicolae 2012:477).

(70) a. John didn't read something. # *not* $>$ *something*

b. (i) John didn't always read something. ✓*not* $>$ *always* $>$ *something*

(ii) Anna didn't tell everyone something. ✓*not* $>$ *everyone* $>$ *something*

An informal survey (judgments yet again courtesy of Shannon Bryant, Caitlin Keenan, Chantale Yunt) reveals that the same is true for *some* $NP_{SG}$ .

(71) John didn't give every book to some student. ✓*not* $>$ *every* $>$ *some* $NP_{SG}$

The explanation for this intervention effect by universal operators could be simply that, in this case, if we exhaustify directly below negation, the presence of the universal operator will ensure that exhaustification succeeds (just as in the case of embedding under a universal modal discussed in the section on ignorance and before).

Fifth, let's consider the case of an intervening factive. In the literature on PPIs it is known that an item bad in the scope of negation can improve in the presence of an intervening factive. Since an intervening factive means that the negation must be extrauclausal, to make this point we will need a global PPI. We will illustrate this with the French disjunction *soit …soit* (similar to English *either …or*), which is bad with a distant negation in the presence of an intervening non-factive such as *think* but improves in the presence of an intervening factive such as *know* (examples from Spector 2014).

(72) a. Je ne pense pas que Jacques ait invité soit Anne soit Paul à dîner.
I not think not that Jacques has.SBJV invited *soit* Anne *soit* Paul to dinner
\# *not* $>$ *think* $>$ *soit …soit*

b. Jacques ne sait pas que Marie étudie soit l'italien soit l'anglais.
Jacques not knows not that Mary studies *soit* Italian *soit* English
'John doesn't know that Mary studies either Italian or English.'

✓*not* $>$ *know* $>$ *soit …soit*

An informal survey (judgments courtesy of the same) reveals that *some* $NP_{SG}$ exhibits the same improvement with a factive.

(73) Mary doesn't know that John called some student. ✓*not* $>$ *know* $>$ *some* $NP_{SG}$

The explanation for this intervention effect is quite different than in the case of intervening universal quantifiers. Factives are known to introduce a factive presupposition, so Spector (2014) suggests this is similar to what happens in the antecedent of a conditional or the restriction of a universal – the presupposition is factored in and can help make exhaustification non-vacuous.

To sum up, anti-negativity can be derived from obligatory $O_{ExhDA}$ coupled with a proper strengthening requirement. If we assume that $O^{PS}$ is generally sensitive to presuppositions (thus, it relies on strong exhaustification, $O^{S}$ ), then this explains why items that exhibit anti-negativity also always seem to be fine in presuppositional downward-entailing environments such as the antecedent of a conditional or the restriction of a universal. In addition to negation and presuppositional downward-entailing environments, we also discussed a number of other issues that are typically addressed in the literature on items with anti-negativity / PPIs. First, we discussed embedding under *few*, a downward-entailing operator which is not presuppositional but gives rise to a positive implicature. Then, we discussed embedding under a downward-entailing operator at a distance. Third, we discussed rescuing effects in the presence of two downward-entailing environments. Fourth, we discussed cases of intervention of the shielding type, via intervening universal operators. Fifth, we considered cases of intervention of the presuppositional type, via intervening factives. We showed that the general approach we used here to explain why some items are bad in the scope of negation but not in the antecedent of a conditional or the restriction of a universal can help make sense of these new cases also, although, since research on PPIs is still quite young, a lot more investigation is needed, both on the empirical side (all the patterns reported above could benefit from experimental investigation) as well as on the theoretical side (in explaining the subtler points of variation, for which the explanations, if at all available, are still only at a preliminary, imprecise stage).

### 1.2.7 Comparison to existing accounts

#### 1.2.7.1 $O_{(Exh)DA}$ + PS vs. $O_{IE\text{-}(Exh)DA}$ + ban on vacuous O

In our solution to ignorance and anti-negativity we have referenced similar solutions for disjunction by Spector (2015) and Nicolae (2017). However, while also couched in an alternatives-and-exhaustification approach, these solutions use a different definition of the exhaustivity operator. On our approach O is defined such that it excludes all the non-entailed alternatives, regardless of whether that would lead to contradiction or not – the parses where the result is contradictory are simply ruled out as ungrammatical (cf. Chierchia 2013).[10] But Spector (2015) and Nicolae (2017) use a different definition of the exhaustivity operator on which it excludes only a subset of the non-entailed alternatives, namely, only those alternatives that can all be negated together while the prejacent remains true, i.e., the so-called 'Innocently Excludable' alternatives (cf. Fox 2007, Chierchia et al. 2012); we will write this operator as $O_{IE}$ .[11] In addition to this, they also derive ignorance and anti-negativity by exhaustifying relative to plain DA rather than pre-

[10]Because of its embrace of contradictory results – which it uses crucially to derive the ungrammaticality of NPIs in positive contexts, cf. Chierchia 2013 – this variant of the grammatical theory of implicatures is known as the 'contradiction-based' variant.

[11]Because of its avoidance of contradictory results, this variant of the grammatical theory of implicatures is known as the 'contradiction-free' variant.

exhaustified DA . We will discuss this choice of DA vs. ExhDA later. For now let's just ask: How would one deriving ignorance and anti-negativity with $O_{IE}$ as opposed to O ?

Consider first the result of doing $O_{DA}$ vs. $O_{IE\text{-}DA}$ in the positive case without $\Box_S$ , and then in the negative case. $O_{DA}$ comes out contradictory (the DA are stronger than the prejacent, so they are negated, but that leads to contradiction) in the positive case and vacuous in the second case (the DA are entailed), (74-a)-(74-b). $O_{IE\text{-}DA}$ , however, comes out vacuous on both counts (in the positive case because the alternatives are not IE, since negating them together with the assertion would lead to contradiction, and in the negative case because they are entailed), (75-a)-(75-b).

(74) Contradiction-based exhaustification via O :

a. $O_{DA}\ (p \vee q),\ = (p \vee q) \wedge \neg p \wedge \neg q,\ = \bot$ (G-trivial)

b. $O_{DA}\ (\neg(p \vee q)),\ = \neg(p \vee q)$ (exhaustification vacuous)

(75) Contradiction-free / Innocent-Exclusion-based exhaustification via $O_{IE}$ :

a. $O_{IE\text{-}DA}\ (p \vee q),\ = (p \vee q)$ (exhaustification vacuous)

b. $O_{IE\text{-}DA}\ (\neg(p \vee q)),\ = \neg(p \vee q)$ (exhaustification vacuous)

Now, we said that to derive ignorance and anti-negativity exhaustification relative to the (in our case, pre-exhaustified) DA is obligatory. This is an assumption that Spector (2015) and Nicolae (2017) make also. But, given the results of $O_{DA}$ vs. $O_{IE\text{-}DA}$ above, the way to derive ignorance and anti-negativity is slightly different. If we use O , we rule out the positive case without $\Box_S$ by pointing out that contradictions are never relevant, and the negative case – by invoking the item-specific proper strengthening requirement and showing that it is violated in (74-b). However, if we use $O_{IE}$ , both the positive and the negative case are derived in the same way, by invoking an across-the-board economy condition banning vacuous exhaustification and showing that it is violated in both the positive and the negative case.

But the bottom line seems to be that both approaches have a way to capture both why a parse with $\Box_S$ is obligatory – and therefore the default nature of the ignorance effect in the positive case – as well as anti-negativity. Is there then any difference between them at all, at least insofar as our data are concerned?

A difference might be the predicted correlation between ignorance and anti-negativity. On our O + PS approach the prediction is that ignorance may or may not be accompanied by anti-negativity, depending on whether we are dealing with an item that is lexically specified for proper strengthening or not. On the other hand, on the $O_{IE}$ + ban on vacuous exhaustification approach obligatory ignorance and anti-negativity are always predicted to co-occur, as they have the exact same source.

Recall now that our *or* came with a strong ignorance effect (it was even incompatible with partial ignorance) but no anti-negativity. And the same can be said of other items, e.g., the English complex disjunction *either...or*, which also carries a strong ignorance effect but no anti-negativity. An approach like ours, based on O + PS, can capture the interpretation (ignorance) and distribution (no anti-negativity) of all these items. However, the alternative approach based on $O_{IE}$ + ban on vacuous exhaustification cannot.

Of course, it is still possible to say that there are actually two versions of the economy constraint, one that says 'strengthen if you can', which would capture the behavior of items like *or*, and one that says 'strengthen always', which would capture the behavior of items like *some* $NP_{SG}$ . But this would ultimately amount to the same thing as our assumption that some items come with a requirement for Proper Strengthening and others don't.

At this point we might say that, at least for the data at hand, the two approaches both yield the same results. However, there is an additional point that we might want to take into account, one that comes from the fact that our O + PS recipe is borrowed from Chierchia (2013)'s system, which is designed to capture not just Free Choice (FC) / ignorance and anti-negativity / PPI-hood but also anti-positivity / NPI-hood. Can an $O_{IE}$ + some version of the economy condition achieve the same?

Consider the predictions of an O $\pm$ PS approach. Suppose an item comes with O , +PS, and can also get $\Box_S$ ; such an item will get ignorance and anti-negativity, and is essentially like our *some* $NP_{SG}$ . Suppose next that an item comes with O , $-$PS, and can still get $\Box_S$ ; such an item will get ignorance and no anti-negativity, and is essentially like our *or*. Suppose finally that an item comes with O , $-$PS, and for some reason can't get the $\Box_S$ parse; such an item will get anti-positivity (in the sense of bad in episodic contexts) and no anti-negativity, and is essentially the classic NPI (this is indeed Chierchia's analysis of NPIs).

Consider now the predictions of the $O_{IE}$ + economy condition approach. If an item comes with $O_{IE}$, 'strengthen always', and can get $\Box_S$, then it will get ignorance and anti-negativity, and it will be like our *some NP*$_{SG}$. Then, if an item comes with $O_{IE}$ + 'strengthen if you can', and can get $\Box_S$, it will come with ignorance and no-anti-negativity, and it will be like our *or*. So far we have obtained the same results as from O $\pm$PS. But consider also the following cases, where we consider an item that can't get $\Box_S$. If an item comes with $O_{IE}$, 'strengthen always', and no $\Box_S$, then it will have both anti-positivity (in the sense of bad in episodic contexts) and anti-negativity – thus, it will be an item that can only show up embedded in downward-entailing environments, and only if that leads to strengthening. Finally, if an item comes with $O_{IE}$, 'strengthen if you can', and no $\Box_S$, it will have anti-positivity (in the sense of bad in episodic contexts) but not anti-negativity – thus, it will be an item that can only show up embedded in downward-entailing environments, where, if strengthening is possible, it will also lead to implicatures, and that can embed under negation. I am not sure if the last two options are attested or not. The last one in particular would mean that this approach might also have a way to capture NPI-hood, although it would be of quite a peculiar kind.

To sum up, there are two main alternatives-and-exhaustification approaches to anti-negativity, one, like ours, articulated in terms of the contradiction-based variant of the grammatical theory of implicatures, and one articulated in terms of the contradiction-free variant of the grammatical theory of implicatures. Both ultimately seem to have a way to capture the data at hand. However, since the contradiction-based variant was from the get-go designed to capture not just FC effects and anti-negativity / PPI behavior but also anti-positivity / NPI behavior, we tried to determine whether the contradiction-free approach to polarity sensitivity can derive anti-negativity / NPI behavior also. At first glance that seems possible also, but it is much too early to draw any final conclusions. At any rate, our main point is that it seems desirable to have a unified approach to not just ignorance and anti-negativity, which was our main goal, but anti-positivity also, that is, to Free Choice and polarity sensitive phenomena more generally. It is an advantage of the present approach, then, that it is anchored in a system like that of Chierchia (2013) that already does that.

#### 1.2.7.2 ExhDA vs. DA

All the current alternative-based approaches to ignorance and anti-negativity in disjunction rely on the observation that an exhaustification relative to subdomain alternatives (not necessarily called so on every approach) crashes if the prejacent is directly $(p \vee q)$ but improves with an intervening $\Box_S$ , and is vacuous if the prejacent is $\neg(p \vee q)$ (e.g., Sauerland 2004 for an approach to ignorance couched in the neo-Gricean theory of implicatures and Spector 2015, Nicolae 2017 for an approach to both ignorance and anti-negativity couched in the grammatical theory of implicatures). However, in most cases these effects are derived from exhaustification relative to DA (plain, non-pre-exhaustified subdomain alternatives) rather than ExhDA (pre-exhaustified subdomain alternatives). We illustrate this below with both our O and $\mathrm{O}_{\text{IE-DA}}$ . (Since O essentially embodies the traditional Gricean reasoning, the O cases illustrate both how Sauerland 2004 on his neo-Gricean approach would compute the implicature as well as how we would. $\mathrm{O}_{\text{IE-DA}}$ illustrates how Spector 2015 or Nicolae 2017 on their contradiction-free approach would.)

(76) a. $\mathrm{O}_{\text{DA}}\ (p \vee q)$

$= (p \vee q) \wedge \neg p \wedge \neg q$ (⊥, G-trivial)

b. $\mathrm{O}_{\text{IE-DA}}\ (p \vee q)$

$= p \vee q$ ($\mathrm{O}_{\text{IE}}$ vacuous)

(77) $\mathrm{O}_{\text{DA}}\ /\mathrm{O}_{\text{IE-DA}}\ \Box_S\ (p \vee q)$

$= \Box_S\ (p \vee q) \wedge \neg\Box_S\ p \wedge \neg\Box_S\ q$ (ignorance)

(78) $\mathrm{O}_{\text{DA}}\ /\mathrm{O}_{\text{IE-DA}}\ \neg(p \vee q)$

$= \neg(p \vee q)$ (DA = $\{\neg p, \neg q\}$; DA already entailed, so $\mathrm{O}_{\text{DA}}\ /\mathrm{O}_{\text{IE-DA}}$ vacuous)

The computations with DA are much simpler, and seem to yield the same result. Why then did we choose to use ExhDA throughout?

The first reason is directly related to our dataset. The idea is simply that to account for all the patterns we have discussed so far we need to appeal at some point to pre-exhaustified subdomain alternatives. We will illustrate with two examples.

First, consider a prejacent of the form $\Diamond(p \vee q)$, where *or* or *some* $NP_{SG}$ appear under a possibility modal. As is well known from, e.g., the literature on Free Choice in disjunction (e.g., Fox 2007), if we exhaustify such a prejacent relative to the DA , we get a crash.

(79) $\mathrm{O_{DA}}\,\Diamond(p \vee q)$

$= \Diamond(p \vee q) \wedge \neg\Diamond p \wedge \neg\Diamond q$

$= \bot$ (G-trivial)

To fix this, the literature that works with DA has to assume some form of recursive exhaustification, as below, and also crucially needs to exhaustify with $\mathrm{O_{IE}}$ , otherwise the lower occurrence of the exhaustivity would crash. With both these assumptions in place, however, the result can be as desired: The lower exhaustification is vacuous insofar as the prejacent is concerned but has the effect that the DA relevant to the higher exhaustivity operator are enriched with O – that is, pre-exhaustified. The higher O ends up exhaustifying relative to pre-exhaustified subdomain alternatives, which yields the desired Free Choice effect. To sum up, to account for the Free Choice effect with possibility modals, one needs some notion of pre-exhaustified subdomain alternatives, just as on our account.

(80) $\mathrm{O_{IE}}\,(\underbrace{\mathrm{O_{IE}}\,\Diamond(p \vee q)}_{\text{vacuous, but } p,\, q \text{ become } \mathrm{O_{IE}}\, p,\, \mathrm{O_{IE}}\, q})$

$= \mathrm{O_{IE}}\,\Diamond(p \vee q) \wedge \neg\mathrm{O_{IE}}\,\Diamond p \wedge \neg\mathrm{O_{IE}}\,\Diamond q$

$= \Diamond(p \vee q) \wedge \Diamond p \leftrightarrow \Diamond q$

Second, consider a prejacent of the form $\Box(p \vee q)$, where *or* or *some* $NP_{SG}$ appear under a necessity modal. If we exhaustify relative to the DA , we get our familiar Free Choice effect that we were getting as one of the results obtained from exhaustifying relative to ExhDA . However, recall our discussion of the fact that the assertion of *or/some* $NP_{SG}$ under a necessity modal is consistent with a continuation of the form $\Box(p \wedge q)$ (*in fact, both*). The result below is incompatible with that – we would have to cancel the ExhDA -implicatures also, which would however mean that we can no longer say that $\mathrm{O_{ExhDA}}$ is obligatory, which is a conclusion that the theories that use DA might not want either (especially Spector 2014, 2015, Nicolae 2017, who also derive strong ignorance and anti-negativity from obligatory $\mathrm{O_{ExhDA}}$ ).

(81) $\mathrm{O}_{\mathrm{DA}} \,\Box(p \vee q)$

$= \Box(p \vee q) \wedge \neg\Box p \wedge \neg\Box q$

If, as we discussed in the earlier sections in this chapter, one used ExhDA , the result would be $\Box(p \vee q) \wedge \Box p \leftrightarrow \Box q$. Compatibility with the $\Box(p \wedge q)$ case would simply come from the true-true case of the implication (along with the suspension due to context of the $\sigma$A -implicature $\neg\Box(p \wedge q)$). On the other hand, the literature that uses DA has to again resort to some form of recursive exhaustification and $\mathrm{O}_{\mathrm{IE}}$ , and that will again essentially yield a form of pre-exhaustified subdomain alternatives. For example, Nicolae (2017) assumes two occurrences of $\mathrm{O}_{\mathrm{IE}}$ , one at matrix level and one below $\Box$. The lower $\mathrm{O}_{\mathrm{IE}}$ is again vacuous insofar as the prejacent is concerned, but strengthens the alternatives. To sum up, to derive the compatibility of an exhaustification relative to the DA with a continuation of the form $\Box(p \wedge q)$, one needs some notion of pre-exhaustified subdomain alternatives, just as on our account.

(82) $\mathrm{O}_{\mathrm{IE}} \,(\Box(\mathrm{O}_{\mathrm{IE}} \,(p \vee q)))$

$= \Box(\mathrm{O}_{\mathrm{IE}} \,(p \vee q)) \wedge \neg(\Box\mathrm{O}_{\mathrm{IE}}\, p) \wedge \neg(\Box\mathrm{O}_{\mathrm{IE}}\, q)$

$= \Box(p \vee q) \wedge \neg(\Box(p \wedge \neg q)) \wedge \neg(\Box(q \wedge \neg p))$

All in all, it seems that some form of pre-exhaustified subdomain alternatives is always needed. Insofar as we are concerned, one could obtain it either as we did, by directly assuming our own notion of O along with pre-exhaustified subdomain alternatives, or by using the $\mathrm{O}_{\mathrm{IE}}$ with recursive exhaustification. If we assume $\mathrm{O}_{\mathrm{IE}}$ , however, we would however also be inheriting the open issues outlined in the previous section. And if we assume recursive exhaustification, we would have to revise our assumptions about the syntax of O , because on our view the alternative features activated by an item can only be checked off once, so it wouldn't be possible for two separate operators to exploit the DA alternatives introduced by the same one $\vee$. But changing that might affect other issues for which the syntactic approach adopted here has proven fruitful (cf. Chierchia 2013's approach to intervention effects), so we might not want to do that for independent reasons.

A final question has to do with whether we have to say that *or* and *some* $NP_{SG}$ must be exhaustified relative to just ExhDA or tolerate exhaustification relative to DA also, where possible. Nothing in our story for ignorance / Free Choice behavior and anti-negativity / Positive Polarity commits us to just ExhDA . However, ExhDA helped us get everything we wanted, so at this point we could say that they activate either DA and (if needed) ExhDA , or just ExhDA . A reason to bite the bullet and go with the ExhDA option would be the fact that, as Chierchia (2013:195-204), if one wants a similarly unified treatment of Free Choice Items and Negative Polarity Items, then it helps to be able to say that some items can't activate ExhDA . The exact arguments for why this is so are subtle and not directly relevant to our story, but, as Chierchia shows, this small assumption can help us derive the minimally different distributions of items like Italian *alcun* vs. Italian *nessuno* or German *irgendein* vs. English *ever*. And if so, then this points to the conclusion that items may have grammaticalized the way they use their alternatives – some ExhDA , others DA . Note that on a recursive exhaustification approach one could say that some tolerate recursive exhaustification while others don't, but this again seems to come down to lexical specification.

### 1.2.8 Summary

We set out to find a theory of ignorance and polarity sensitivity that would capture the fact that a disjunction like *or* or an indefinite like *some* $NP_{SG}$ are both able to give rise to ignorance effects, but they differ in the strength of the effect and in whether or not they can take scope below negation.

We sketched an approach using insights from the alternatives-and-exhaustification approaches to ignorance and polarity sensitivity. We argued with Chierchia (2013) that *or* and *some* $NP_{SG}$ have the same basic logical shape – $\exists x \in D[P(x)]$; that this logical shape contains reference to both a scalar element, $\exists$, as well as to a domain, $D$; and that for this reason these items activate both scalar and subdomain alternatives. We then proposed that for both *or* and *some* $NP_{SG}$ these alternatives are activated by default (although the scalar ones can be pruned via context). We also adopted the idea that, for items with lexically activated alternatives, these alternatives are factored into meaning via a silent exhaustivity operator, and chose to use Chierchia (2013)'s contradiction-based variant of the grammatical theory of implicatures. We argued that crucial to the interpretations of *or* and *some* $NP_{SG}$ is exhaustification relative to their pre-exhaustified subdomain alternatives. We showed that, when this happens across a modal, this gives rise to a Free Choice effect. We also showed that, if we assume (following a consensus in the literature) that assertions are prefixed by a silent epistemic/doxastic necessity modal, ignorance can be captured as a FC effect arising from exhaustification across this silent modal. Adopting an insight from Chierchia (2013) that total vs. partial variation (for our case of interest, ignorance) effects can be obtained by assuming that some items can prune their subdomain alternative set to a natural subclass, we proposed that *or* cannot prune its subdomain alternative set while *some* $NP_{SG}$ can prune it down to either just singletons or just non-singletons, then showed how this predicts total ignorance in the first case but tolerance for partial ignorance of either the 'loser' or the 'winner' type in the second case. Finally, adopting insights from Chierchia (2013), Spector (2014), and Nicolae (2017) to the effect that anti-negativity comes from obligatory exhaustification relative to subdomain alternatives plus a requirement that it can't be vacuous, and that the acceptability of an item with anti-negativity in the antecedent of a conditional or the restriction of a universal comes from the fact that, if the presuppositions of these environments are factored in, they can help satisfy the non-vacuity requirement, we articulated an account that captured the distribution of *or* and *some* $NP_{SG}$ in these environments. We also showed how this approach to anti-negativity yields desirable preliminary results for other cases of embedding in downward-entailing environments also. Finally, we compared our theoretical choices in deriving ignorance and anti-negativity to two popular competing choices in the literature, namely the use of the contradiction-free variant of the grammatical theory of implicatures along with an economy condition banning vacuous exhaustification and the use of plain, non-pre-exhaustified subdomain alternatives. We noted that our choices to use pre-exhaustified subdomain alternatives and the contradiction-based variant of the grammatical theory of implicatures not only replicate the results of the competing approaches but are perhaps from the start anchored in a more general approach to Free Choice effects and polarity sensitivity.

Table 1.6 repeats our starting ignorance and anti-negativity puzzles for *or* and *some* $NP_{SG}$ (Table 1.1), only this time annotating the descriptions of the empirical patterns with the pieces of our analysis that derive them.

<table>
<tr><td colspan="2"></td><td colspan="2">total ignorance</td></tr>
<tr><td colspan="2"></td><td>yes<br>($\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}$ $\Box_S$ )</td><td>no<br>($\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}$ / $\mathrm{O}_{\mathrm{ExhSgDA}+\sigma\mathrm{A}}$ / $\mathrm{O}_{\mathrm{ExhNonSgDA}+\sigma\mathrm{A}}$ $\Box_S$ )</td></tr>
<tr><td rowspan="2">anti-negativity</td><td>no</td><td>or</td><td></td></tr>
<tr><td>yes (PS)</td><td></td><td>some $NP_{SG}$</td></tr>
</table>

Table 1.6: Ignorance and anti-negativity: *or* vs. *some* $NP_{SG}$ .

Recall that in terms of empirical patterns CMNs and SMNs occupied the cells above *some* $NP_{SG}$ and below *or*, respectively. Can we extend our solution for *or* and *some* $NP_{SG}$ to CMNs and SMNs also?

## 1.3 An alternatives-and-exhaustification solution for CMNs and SMNs also?

A crucial step in our analysis for *or* and *some* $NP_{SG}$ was to identify their shared logical form ($\exists x \in D[P(x)]$) and observe that it contained reference to both a scalar element ($\exists$) as well as a domain ($D$). This naturally gave us both scalar and subdomain alternatives, and helped us articulate the account we did. The first step in our analysis of CMNs and SMNs will thus be to find the relevant logical form that will give us that type of alternatives as naturally for them also. We will do this in Ch. 2. But once we find the scalar and subdomain alternatives of CMNs and SMNs, we expect them to give rise to scalar implicatures, ignorance/FC implicatures, and anti-negativity (or not), just as *or* and *some* $NP_{SG}$ did. However, none of these empirical patterns can be taken for granted, as the existing literature typically rejects that CMNs and SMNs give rise to scalar implicatures, doesn't acknowledge that CMNs give rise to ignorance, and doesn't tackle the infelicity of SMNs under negation. Thus, in each of Chs. 3, 4, and 5 we will discuss not just the predictions from theory but also the empirical work regarding the scalar implicatures, ignorance, and anti-negativity patterns of numerals. In Ch. 6 we conclude with a summary of the overall contribution of the thesis and a discussion of some of the new questions that emerge.

# Chapter 2

# The truth conditions and alternatives of bare and modified numerals

We want to make sense of ignorance and anti-negativity in modified numerals. We have seen how this can be done from truth conditions involving both a scalar item and a domain. The first step in extending this approach to modified numerals is thus to get a grasp on their truth conditions and their alternatives. Neither task is trivial: The existing literature provides many answers, and they often disagree. The goal of this chapter is to distill a principled solution for both. In particular, we will aim to propose a theory of the truth conditions of BNs, CMNs, and SMNs from which their alternatives follow. (Thus we add a first secondary desideratum to our main one.) At the end we will also consider a number of connections, extensions, and limitations.

## 2.1 The bounding entailments of BNs, CMNs, and SMNs

Truth conditions are about what is entailed, and for numerals this minimally[1] concerns their *bounding* entailments. This refers to the following: Utterances of the form *three P Q*, *more than three P Q*, and *at*

[1]Other theories of numerals also take other pieces of their meanings to be part of their entailments, e.g., ignorance for superlative-modified numerals, as discussed in Ch. 4.

*least three P Q*, (1-a), all entail a lower bound. This can be seen from the fact that they all trigger negative inferences about values below the asserted value, (1-b), and these negative inferences are non-cancelable, (1-c).

(1) a. Alice has three / more than three / at least three diamonds.

b. ¬ The number of diamonds that Alice has is two or less / three or less / two or less.

c. Alice has three / more than three / at least three diamonds, # if not less.

In contrast, utterances of the form *less than three P Q* and *at most three P Q*, (2-a), both entail an upper bound (van Benthem 1986, Krifka 1999, Geurts & Nouwen 2007, Buccola & Spector 2016, a.o.). We can see this from the fact that they both trigger negative inferences about values above the asserted value, (2-b), and that, just as before, these negative inferences are non-cancelable, (2-c).

(2) a. Alice has less than three / at most three diamonds.

b. ¬ The number of diamonds that Alice has is three or more / four or more.

c. Alice has less than three / at most three diamonds, # if not more.

How can we capture these entailments, and what is the view of numerals that comes with them?

## 2.2 *three*, *more/less than three*, *at least/most three* are type $\langle et, ett\rangle$

An early and influential proposal for the truth conditions of bare and modified numerals was given as part of Barwise & Cooper (1981)'s Generalized Quantifier Theory (GQT). GQT treats all NPs the same, namely, as generalized quantifiers, type $\langle et, t\rangle$.

Implicit in this view is the idea that traditional quantificational expressions such as *every*, *no*, *a*, on the one hand, and numerical quantificational expressions such as *three* (bare numerals, henceforth BNs), *more/less than three*, and *at least/most three*, on the other, are treated on a par with quantifiers, that is, as expressions of type $\langle et, ett\rangle$ denoting relations between a predicate with a nominal meaning $P$ and a predicate with a verbal meaning $Q$.

Thus, for example, non-numerical quantifiers such as *every*, *no*, or *a* say that the intersection of $P$ and $Q$ is equal to $P$ / null / non-null, respectively.

(3) $[\![\text{every}]\!] = \lambda P_{\langle e,t\rangle} \,.\, \lambda Q_{\langle e,t\rangle} \,.\, P \subseteq Q$ (equivalent to $P \cap Q = P$)

(4) $[\![\text{no}]\!] = \lambda P_{\langle e,t\rangle} \,.\, \lambda Q_{\langle e,t\rangle} \,.\, P \cap Q = \varnothing$

(5) $[\![\text{a}]\!] = \lambda P_{\langle e,t\rangle} \,.\, \lambda Q_{\langle e,t\rangle} \,.\, P \cap Q \neq \varnothing$

Similarly, numerical quantifiers say that the cardinality of the intersection of $P$ and $Q$ stands in a certain ordering relation to a numeral; for *three P Q*, *more than three P Q*, and *at least three P Q* this relation is $\geq$, $>$, and $\geq$, respectively, and for *less than three P Q* and *at most three P Q* it is $<$ and $\leq$.

(6) $[\![\text{three}]\!] = \lambda P_{\langle e,t\rangle} \,.\, \lambda Q_{\langle e,t\rangle} \,.\, |P \cap Q| \geq 3$

(7) $[\![\text{more than three}]\!] = \lambda P_{\langle e,t\rangle} \,.\, \lambda Q_{\langle e,t\rangle} \,.\, |P \cap Q| > 3$

(8) $[\![\text{less than three}]\!] = \lambda P_{\langle e,t\rangle} \,.\, \lambda Q_{\langle e,t\rangle} \,.\, |P \cap Q| < 3$

(9) $[\![\text{at least three}]\!] = \lambda P_{\langle e,t\rangle} \,.\, \lambda Q_{\langle e,t\rangle} \,.\, |P \cap Q| \geq 3$

(10) $[\![\text{at most three}]\!] = \lambda P_{\langle e,t\rangle} \,.\, \lambda Q_{\langle e,t\rangle} \,.\, |P \cap Q| \leq 3$

All of these go with a syntactic structure as in Fig. 2.1.

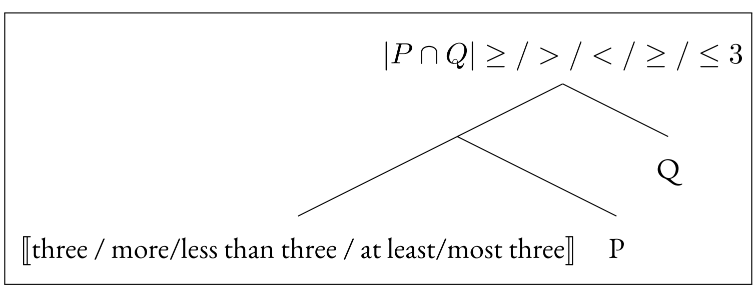


Figure 2.1: BNs, CMNs, and SMNs as quantifiers over individuals, type $\langle et, ett\rangle$.

Note that on these meanings the bounding entailments are straightforwardly captured. This is because the cardinality function picks out the total number of individuals with property $P$ and $Q$. As in *three*, *more than three*, and *at least three* this number is said to be greater than or equal to the bound given by the the numeral, we get the lower-bounding entailments (the inference that the number is not less than the numeral). And as in *less than three* and *at most three* this number is said to be less than or less than or equal to the bound imposed by the numeral, we get the upper-bounding entailments (the inference that the number is not more than the numeral).

To sum up, GQT offers not only a very attractive way to unify all these items as determiners, but also seems to capture our bounding entailments very straightforwardly.

However, the GQT truth conditions for BNs, CMNs, and SMNs have been challenged. Below we review a couple of the more influential challenges and the changes in this picture of numerals that have been proposed in response to that.

## 2.3 *three* is in fact type $d$ or $\langle e, t\rangle$

Krifka (1999) notes that the treatment of the numeral in GQT is unsatisfactory for a number of reasons. In BNs the numeral is treated as a quantifier, but in CMNs and SMNs it is used as a degree; we want a clarification of this status. Then, there are reasons to want the numeral to be able to have a predicative meaning (e.g., for the treatment of the singular/plural distinction, cumulative/collective readings). Finally, *three* and *at least three* have the exact same truth conditions, but they are very different, even if we only look at the morphology.[2]

We could address all these points if we say that a numeral such as *three* originally denotes a degree, (11-a), but can be typeshifted into a predicative meaning denoting the set of plural individuals whose atoms count is 3, (11-b). (The other way around is conceivable also.)

(11) a. $[\![\text{three}]\!] = 3$ (type $d$)

b. $[\![\texttt{isCard}]\!]\,([\![\text{three}]\!])$

[2] As Krifka (1999) and the literature since also points out, there are further good reasons to distinguish between these two, and they have to do with scalar implicatures, ignorance, and anti-negativity. We will discuss all of these later, in the next few chapters.

$= [\lambda d \,.\, \lambda x \,.\, |x| = d](3)$

$= \lambda x \,.\, |x| = 3$ (the typeshifter `isCard`, cf. Buccola & Spector 2016:162)

Then the syntax and semantics of a BN sentence such as *Three people quit* could be as in Fig. 2.2. First, a number head # with a singular/plural meaning SG/PL, which takes as a complement the NP with the meaning $P$. Semantically, SG checks for singularity ($[\![\text{SG}]\!] = \lambda P_{\langle e,t\rangle} : \forall x \in P[|x| = 1].P$) and PL applies when singularity is not satisfied ($[\![\text{PL}]\!] = \lambda P_{\langle e,t\rangle} \,.\, P$) (SG/PL cf. Scontras 2013). Second, the numeral phrase NumP with the meaning $[\![\text{three}]\!]$ is merged in the specifier position of #P. Semantically this corresponds to a typeshifting of $[\![\text{three}]\!]$ via `isCard` and then composition of the resulting $[\![\text{isCard}]\!]\,(3)$ and $P$ meanings via predicate modification (Heim & Kratzer 1998) to yield another predicative meaning. This meaning is then closed via a silent existential quantifier $\varnothing_\exists$ hosted in D. All of these are summarized in Fig. 2.2. The basic decomposition and the final truth conditions that arise from it are also spelled out in (12).[3]

(12) $[\![\text{Three people quit}]\!]$

$\varnothing_\exists(([\![\texttt{isCard}]\!]\,([\![\text{three}]\!]))([\![\text{PL}]\!]\,(\lambda x \,.\, \text{people}(x))))(\lambda x \,.\, \text{quit}(x))$

$= 1 \text{ iff } \exists x[|x| = 3 \wedge P(x) \wedge Q(x)]$

Thus, a BN utterance of the form *three P Q* says that there exists a plurality of individuals with cardinality 3 that has both property $P$ and $Q$. This is a lower-bounded meaning, since it is compatible with there also being a plurality with a cardinality larger than 3 with both property $P$ and $Q$. Thus, the above represents an 'at least' semantics for BNs, one on which *three* entails 'at least three'. We assume it to be the basic meaning of BNs, in line with Horn (1972). We will discuss and motivate this choice again in Ch. 3.

This view of BNs captures their lower-bounding entailment just as well as GQT. However, it improves on GQT because it clarifies the status of the numeral across BNs, CMNs, and SMNs – the numeral is

[3]These existential truth conditions for BNs are identical to, e.g., Krifka (1999), although one difference is that he gives numerals a modifier meaning, type $\langle et, et\rangle$, across BNs, CMNs, and SMNs. The end result for BNs is however the same.

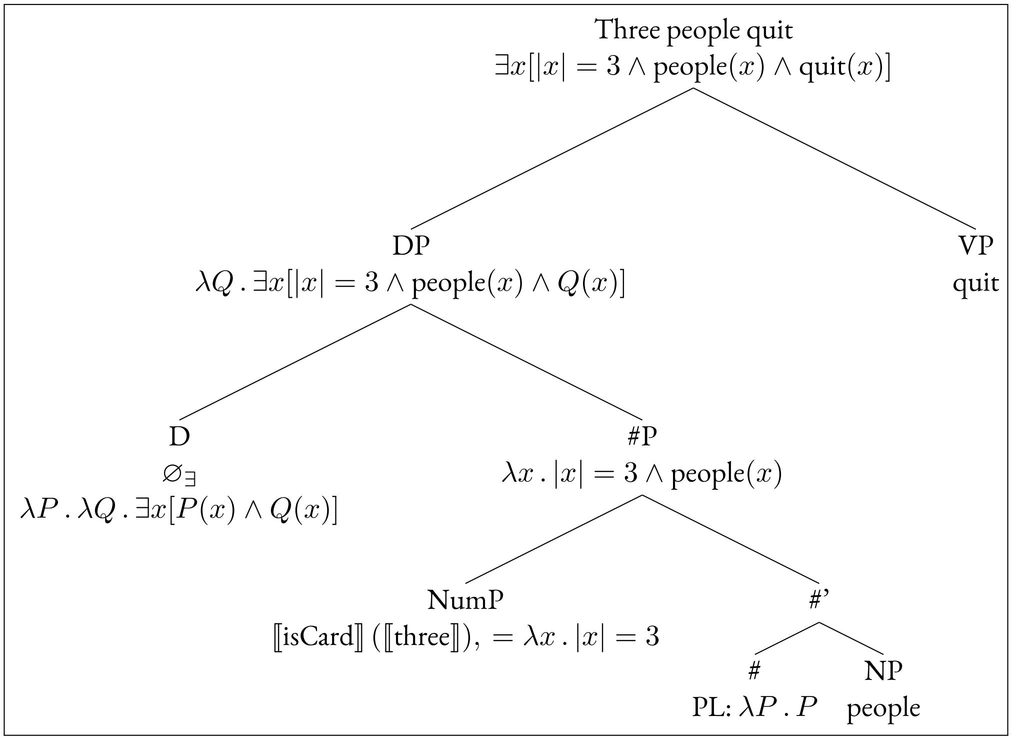


Figure 2.2: The syntax and semantics of BNs.

ambiguous between a degree and a predicate meaning, and it is the predicate meaning that goes into the truth conditions of BNs and the degree meaning that feeds into the truth conditions of CMNs and SMNs.

At the same time, if we want to keep the new view of BNs alongside the GQT view of CMNs and SMNs, then a clarification is in order. The GQT view of numerals assumed that $P$ and $Q$ only included atoms, and the cardinality function gave us directly the atom count of the intersection of $P$ and $Q$. However, the revised view of BNs reflects an updated view of $P$ (= the meaning of a plural NP) and $Q$ (= the meaning of a VP) on which they include pluralities, and the cardinality function gives us the atom count of a plurality that is in both $P$ and $Q$. Since the GQT notion of $P$ and $Q$ is the more dated one, the right move would be to update it and say as for BNs that they include pluralities also, (13-a)-(13-b). But if we make this change yet continue to apply the cardinality function as in GQT, we get the wrong result, because we end up counting the number of pluralities in the intersection of $^*P$ and $^*Q$ instead of the number of atoms, (13-c). This could be fixed if we assumed, for example, that what the cardinality function applies to is not directly the intersection of $^*P$ and $^*Q$ but rather the *infinitary union* of this intersection, (13-d). This collapses the set of pluralities to a single plurality, and we get the correct result.

(13) Less than three people quit.

a. $[\![^*\text{people}]\!] = \{\{a\}, \{b\}, \{a, b\}\}$ (updated $P$)

b. $[\![^*\text{quit}]\!] = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$ (updated $Q$)

c. $|\,[\![^*\text{people}]\!] \cap [\![^*\text{quit}]\!]\,|$

$= |\{\{a\}, \{b\}, \{a, b\}\} \cap \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}|$

$= |\{\{a\}, \{b\}, \{a, b\}\}|$

$= 3$ ✗

d. $|\bigcup([\![^*\text{people}]\!] \cap [\![^*\text{quit}]\!])|$

$= |\bigcup\{\{a\}, \{b\}, \{a, b\}\}|$

$= |\{a, b\}|$

$= 2$ ✓

Note moreover that BNs on their predicative meaning can also be stated in these terms. Take a plurality with 3 atoms, e.g., $x = \{a, b, c\}$. If atoms can be identified with their singleton sets (as we already did in the way we wrote the atoms in $[\![\text{people}]\!]$ or $[\![\text{quit}]\!]$, cf. Schwarzschild 1996, originally from Quine), then this can also be restated as $x = \{\{a\}, \{b\}, \{c\}\}$. Note now that the infinitary union of the latter is precisely $\{a, b, c\}$. So for a plural individual $x$, $x$ and $\bigcup x$ are exactly the same thing, and thus also $|x|$ and $|\bigcup x|$.

Thus, for the new view of BNs to blend well with the GQT view of CMNs and SMNs, we may have to smooth out certain wrinkles. This notwithstanding, the new view of BNs leaves the main proposal of GQT untouched – *three P / more/less than three P / at least/most three P* all continue to denote generalized quantifiers over individuals, type $\langle et, t\rangle$.

## 2.4 *more/less than*, *at least/most* are in fact type $\langle dt, dtt \rangle$

In GQT the comparative and superlative modifiers are treated as unanalyzed wholes, but they clearly bear similarity to comparative and superlative constructions elsewhere in language. Thus, a better analysis of CMNs and SMNs would be one where these connections are made transparent.

An influential proposal in this sense comes from Hackl (2000)'s discussion of comparative modifiers. Hackl argues that the comparative piece in their meaning (incorporated into the morphological form of *more* and *less*) can in general be analyzed as a quantifier over degrees encoding a strict comparison relation ($>$ for positive adjectives such as *much*, and $<$ for negative adjectives such as *little*) between the maxima of two sets of degrees, (14). This is a meaning that seems to give the right results across constructions as diverse as *The table is longer than the rug is wide*, (15), or *John is taller than 6ft*, (16), and can also be straightforwardly adopted for CMNs (assuming with Hackl that a degree like 3 can be mapped onto its singleton set), (17).

(14) $[\![ [\text{comp}^{+/-}] ]\!] = \lambda D_{\langle d,t \rangle} \,.\, \lambda D'_{\langle d,t \rangle} \,.\, \max(D') > / < \max(D)$ (Hackl 2000:50)

where $\max := \lambda D_{\langle d,t \rangle} \,.\, \iota d$ s.t. $D(d) = 1 \wedge \forall d' [D(d') = 1 \rightarrow d' \leq d]$

(15) $[\![$The table is longer than the rug is wide$]\!]$

$= \max(\lambda d \,.\,$ the table is $d$-long $) > \max(\lambda d \,.\,$ the rug is $d$-wide$)$

'The maximum degree to which the table is long is greater than the maximum degree to which the rug is wide.'

(16) $[\![$John is taller than 6 ft$]\!]$

$= \max(\lambda d \,.\,$ John is $d$-tall $) > \max(\lambda d \,.\, d = 6')$

$= \max(\lambda d \,.\,$ John is $d$-tall $) > 6'$

(assuming $d$ can be mapped into its singleton set)

'The maximum degree to which John is tall is greater than 6.'

(17) $[\![$More than three people quit$]\!]$

$= \max(\lambda d\,.\,\exists x[|x| = d \wedge \text{people}(x) \wedge \text{quit}(x)]) > \max(\lambda d\,.\,d = 3)$

$= \max(\lambda d\,.\,\exists x[|x| = d \wedge \text{people}(x) \wedge \text{quit}(x)]) > 3$

(assuming $d$ can be mapped into its singleton set)

'The maximum degree such that there is a plurality numerous to that degree of people who quit is greater than 3.'

While this analysis has the advantage of being more general with respect to comparative meanings than GQT, insofar as CMNs are concerned it simply seems to replace the GQT way of getting at the relevant cardinality (picking the cardinality of the set of atoms in the intersection of $P$ and $Q$) with a different, clumsier way (picking the maximum in the set of degrees such that there exists a plurality of individuals in the extensions of both $P$ and $Q$ whose cardinality (atom count) is that degree). Conceptually, also, it marks a significant departure from GQT – CMNs are no longer generalized quantifiers over individuals, type $\langle et, t\rangle$, like other quantifiers such as *some*, *every*, etc., but instead generalized quantifiers over *degrees*, type $\langle dt, t\rangle$, which in turn means that even for a basic CMN sentence such as (17) above we now have movement for type reasons, as illustrated in Fig. 2.3 (which also assumes Hackl's early gradable adjective meaning for *many*[4] as well as the idea that degrees can be mapped into their singleton set). So why should we embrace this proposal?

One reason to embrace it is, of course, the greater generality for the way the comparative function is defined for a variety of comparanda.

Another, even more compelling, reason is the fact that, although on the surface equivalent to GQT, this new meaning is however different from GQT, and in a way that we might need. To be more specific, the Hackl truth conditions for CMNs contain two quantificational elements, a degree quantifier (positive *-er than* in Fig. 2.3 / [comp$^{+/-}$] more generally, type $\langle dt, dtt\rangle$) and an individual quantifier ($\exists$, type $\langle et, ett\rangle$). This predicts that in the presence of an additional sentence-level operator Op, split scope configurations should be possible, that is, configurations where Op takes scope in between these

[4] As opposed to his later idea that *many* is a parametrized determiner, $[\![\text{many}]\!] = \lambda d\,.\,\lambda P\,.\,\lambda Q\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]$, which he adopts in order to ensure that $\exists x$ is scopally inert and thus avoid generating LFs where it could take wide scope with respect to the degree quantifier.

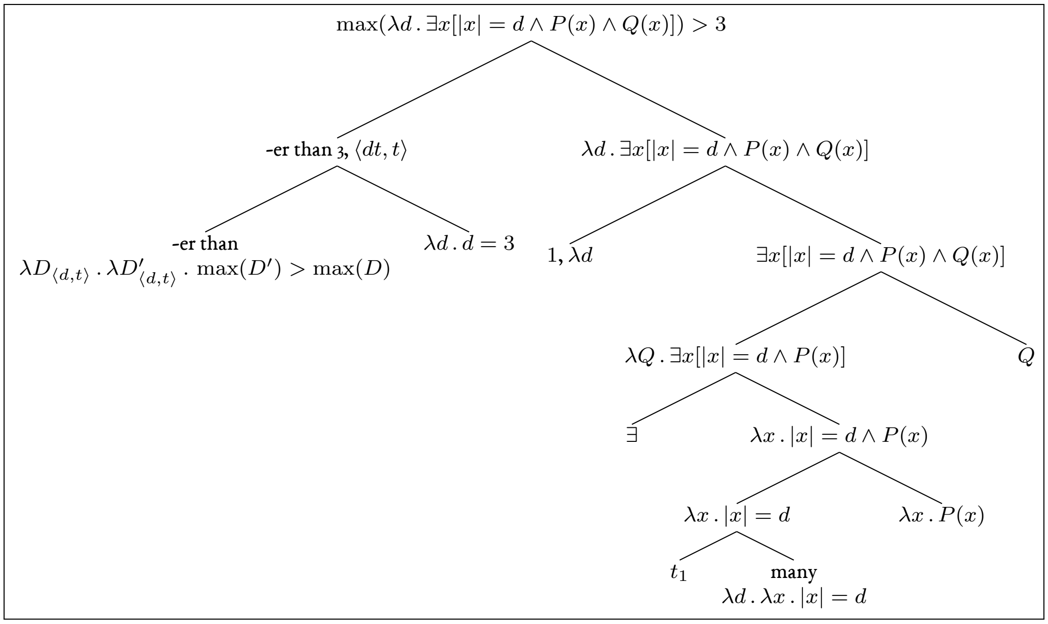


Figure 2.3: CMNs (and SMNs, see below) as quantifiers over degrees, with Hackl-style *many*.

two quantificational elements. Such configurations do indeed seem to be attested. For example, the sentence below involving a downward-monotone CMN (*fewer than 5*) and a modal operator has two attested readings, one according to which John is forbidden from reading 5 papers or more, and one according to which he can read more, if he wants, but all that is required of him is some number less than 5. The first reading corresponds to the case where the modal operator takes wide scope, (18-a), and the second arises when the degree quantifier takes wide scope, (18-b); the latter is the split scope reading.

(18) John is required to read fewer than 5 papers.

a. $\Box \max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) < 5$

'In every world the maximum degree $d$ such that there exists an $x$ which is a plurality with cardinality $d$ of papers that John reads is less than 5.'

(He is forbidden to read 5 or more.)

b. $\max(\lambda d\,.\,\Box\exists x[|x| = d \wedge P(x) \wedge Q(x)]) < 5$

'The maximum degree $d$ such that in every world there is an $x$ which is a plurality with cardinality $d$ of papers that John reads is less than 5.'

(He is also allowed to read 5 or more.)

Scope-splitting configurations like the above (where the degree quantifier takes scope above the operator) are often hard to detect.[5] However, to the extent that we can find them, they constitute an argument against GQT and for truth conditions of the Hackl sort. That is because, while GQT also allows for both scope readings, none of the scope readings that can be produced corresponds to the meaning yielded by the scope-splitting configuration in (18-b) – the GQT reading where the numeral takes wide scope is merely a *de re* reading, (19-b).

(19) John is required to read fewer than 5 papers.

a. $\Box|P \cap Q| < 5$

'In every world the number of papers that John reads is less than 5.'

b. $|P \cap \lambda x \,.\, \Box Q(x)| < 5$

'There is a set of specific papers (e.g., Heim 2000, Seuren 1984, etc.) such that in every world John reads them, and their number is less than 5.' (*de re* reading)

Indeed, the literature on modified numerals after Hackl (2000) generally abandons GQT and embraces the Hackl-style truth conditions for CMNs.

Some of the literature (e.g., Nouwen 2010, Kennedy 2015) in fact also extends it to SMNs – that is, they too are analyzed as degree quantifiers encoding a relation (this time, of *non*-strict comparison: $\geq$ for *at least n*, $\leq$ for *at most n*) between the maxima of two sets of degrees (the latter being a singleton set containing $n$). This is justified by the fact they too seem able to give rise to the same split scope readings as CMNs, (20-b), which suggests that their truth conditions might also rely on a degree quantifier and an

[5] Hackl notes that there are at least two reasons for this: (1) Scope-splitting configurations across a quantificational DP are generally unavailable. This (unexplained) generalization is known as Kennedy's Generalization and can be summarized as follows: If the scope of a quantificational DP contains the trace of a degree quantifier it also contains the degree quantifier itself. (2) Scope-splitting configurations are available with intervening modals, but, unless the degree+individual quantifier meaning comes from a non-monotonic quantifier or an *exactly* differential or a *less*-comparative, the truth conditions produced these way are indistinguishable from the truth conditions obtained via a non-scope-splitting configuration. See Heim (2000) and Hackl (2000:Ch. 3) for more discussion. Note also that both the scope-splitting examples we had above involved a modal operator and a downward-monotone modified numeral (*less than*, *at most*).

individual quantifier.

(20) John is required to read at most 5 papers.

a. $\Box \max(\lambda d \,.\, \exists x[|x| = d \wedge P(x) \wedge Q(x)]) \leq 5$

'In every world the maximum degree $d$ such that there exists an $x$ which is a plurality with cardinality $d$ of papers that John reads is less than or equal to 5.'

(He is forbidden to read 6 or more.)

b. $\max(\lambda d \,.\, \Box\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \leq 5$

'The maximum degree $d$ such that in every world there is an $x$ which is a plurality with cardinality $d$ of papers that John reads is less than or equal to 5.'

(He is allowed to read 6 or more.)

To sum up, in an attempt to give a unified account of comparatives, Hackl (2000) proposes that the truth conditions of *more/less than three P Q* are not $|P \cap Q| > / < 3$ but rather $\max(\lambda d \,.\, \exists x[|x| = d \wedge P(x) \wedge Q(x)]) > / < 3$ – thus, they rely on both a quantifier over degrees and a quantifier over individuals. Superficially this move looks like a mere restatement of the GQT way of getting at the total cardinality. However, conceptually it is quite different from GQT – we move from CMNs as type $\langle et, ett\rangle$ to type $\langle dt, dt\rangle$. Moreover, the presence of two quantificational elements offers an empirical advantage over GQT, because in the presence of an additional operator Op it predicts split scope readings like $\max(\lambda d \,.\, \text{Op}\; \exists d[|x| = d \ldots])$, and this prediction is attested, and in fact not only for CMNs, but also for SMNs. Thus, we will (like, e.g., Kennedy 2015) adopt the view that the GQT truth conditions for both CMNs and SMNs must (for a start) be stated as Hackl already did for CMNs.

(21) a. $[\![\text{more/less than three}]\!] = \lambda D \,.\, \max(\lambda d \,.\, D(d)) > / < 3$

b. $[\![\text{more/less than three P Q}]\!] = \max(\lambda d \,.\, \exists x[|x| = d \wedge P(x) \wedge Q(x)]) > / < 3$

(22) a. $[\![\text{at least/most three}]\!] = \lambda D \,.\, \max(\lambda d \,.\, D(d)) \geq / \leq 3$

b. $[\![\text{at least/most three P Q}]\!] = \max(\lambda d \,.\, \exists x[|x| = d \wedge P(x) \wedge Q(x)]) \geq / \leq 3$

## 2.5 *much*/*little* are extent indicators, type $\langle d, dt \rangle$; [comp], [at-sup]

The Hackl truth conditions for CMNs and the Hackl-style truth conditions for SMNs[6] improve on GQT. However, other than giving the comparative modifiers a logical form that works across comparative constructions more generally, they are not much more compositional than GQT. In particular, while, like GQT, they also associate CMNs with strict comparison and SMNs with non-strict comparison, and assume that *more than* and *at least* encode positive meanings ($>$, $\geq$) while *less than* and *at most* have negative meanings ($<$, $\leq$), also like GQT, they leave unexplained the presence of *much* and *little* in both CMNs and SMNs, and the fact that *much* leads to a strict comparison, lower-bounded meaning in CMNs (*more than*) but a non-strict comparison, upper-bounded meaning in SMNs (*at least*), while *little* leads to a strict comparison, upper-bound meaning in CMNs (*less than*) but a non-strict comparison upper-bounded meaning in SMNs (*at most*). Thus, in a sense, these truth conditions also fail to capture the comparative meaning that is common to *more than* and *less than* – let's call it [comp] – and the superlative meaning that is common to *at least* and *at most* – let's call it [at-sup]. Can we do better?

A useful starting point is Kennedy (1997, 2001)'s algebra of extents (itself inspired after Seuren 1984), which comes with the following definitions of a *scale*, *extent*, *proper extent*, and *positive extent* or *negative extent* (all from Kennedy 1997:51-2).

(23) A *scale* $S_\delta$ is a dense, linearly ordered set of points along a dimension $\delta$ which may have a minimal element but has no maximal element.

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[6] I call them 'Hackl-style' because these are not in fact the truth conditions Hackl proposes for SMNs. Those are in fact as below, and they look quite different for *at least* vs. *at most*.

(i) a. $[\![\text{at least three}]\!] = \lambda D_{\langle d,t \rangle} \, . \, D(3)$
b. $[\![\text{at least three P Q}]\!] = \exists x[|x| = 3 \wedge P(x) \wedge Q(x)]$

(ii) a. $[\![\text{at most three}]\!] = \lambda D_{\langle d,t \rangle} \, . \, \neg \exists d[d > 3 \wedge D(d)]$
b. $[\![\text{at most three P Q}]\!] = \neg \exists d[d > 3 \wedge \exists x[|x| = d \wedge P(x) \wedge Q(x)]$

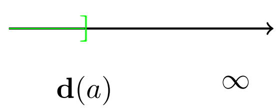
(24) An *extent* is a non-empty, convex subset of a scale $S_\delta$, i.e., a subset $E$ of $S_\delta$ with the following property: $\forall p_1, p_2 \in E \, \forall p_3 \in S_\delta [p_1 < p_3 < p_2 \rightarrow p_3 \in E]$ (essentially, an interval).

(25) A *proper extent* on a scale $S_\delta$ is a nonempty, convex proper subset of $S_\delta$.

(26) Assume that for any object $a$ which can be ordered according to some dimension $\delta$ there is a function $\mathbf{d}$ from $a$ to a unique point on the scale $S_\delta$. Then:

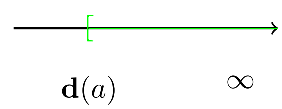
a. *the positive extent* of $a$ on $S_\delta$ is a proper extent that ranges from the lower end of the scale to $\mathbf{d}(a)$:

$\textsc{pos}_\delta(a) = \{p \in S_\delta \mid p \leq \mathbf{d}(a)\}$




b. the *negative extent* of $a$ on $S_\delta$ is a proper extent that ranges from $\mathbf{d}(a)$ to the upper end of the scale:

$\textsc{neg}_\delta(a) = \{p \in S_\delta \mid p \geq \mathbf{d}(a)\}$




Kennedy argues that positive gradable adjectives (e.g., *tall*) are like POS in (26-a) – they denote functions from objects into positive extents – and negative gradable adjectives (e.g., *short*) are like NEG in (26-b) – they denote functions from objects into negative extents. He shows how this assumption can help us make sense of important phenomena in adjectives such as the cross-polar anomaly.[7]

I propose that the *much*/*little* in modified numerals are similar to positive/negative gradable adjectives, except that the objects in their domain are degrees rather than entities, $S_\delta$ is a scale of cardinalities, and for any numeral $n$ denoting a degree $n$ the function $\mathbf{d}$ simply maps it to the same degree $n$ on $S_\delta$, (27).

(27) a. $[\![\text{much}]\!]_\delta\,(n)$
$= \{p \in S_\delta \mid p \leq \mathbf{d}(n)\}$
$= \{p \in S_\delta \mid p \leq n\}$

b. $[\![\text{little}]\!]_\delta\,(n)$
$= \{p \in S_\delta \mid p \geq \mathbf{d}(n)\}$
$= \{p \in S_\delta \mid p \geq n\}$

[7] In Kennedy's terminology, the cross-polar anomaly refers to the fact that comparative constructions formed out pairs of adjectives where one is positive and the other is negative are anomalous: *#The Brothers Karamazov is longer than The Idiot is short.*

Thus, omitting the explicit reference to $S_\delta$, *much* and *little* are functions type $\langle d, dt\rangle$ which, applied to a numeral $n$ (treated as a degree), yield the set of degrees below $n$ and including $n$ and the set of degrees above $n$ and including $n$, respectively, which in lambda notation is as in (29). (For most purposes we will assume that these degrees are points on a cardinality scale, thus, a scale of positive natural numbers with granularity 1.)[8]

(28) a. $[\![\text{much}]\!]\,(n) = \lambda d\,.\,d \leq n$ b. $[\![\text{little}]\!]\,(n) = \lambda d\,.\,d \geq n$

For example, if $n = 3$ and $S = \mathbb{N}$, then

(29) a. $[\![\text{much}]\!]\,(3)$
$= \lambda d\,.\,d \leq 3$
$= \{0, 1, 2, 3\}$

b. $[\![\text{little}]\!]\,(3)$
$= \lambda d\,.\,d \geq 3$
$= \{3, 4, 5, \ldots\}$

We are now ready to give the meanings of CMNs and SMNs.

I propose that the meanings of *more than n* and *less than n* are obtained as in (30). The comparative morpheme [comp] is a function type $\langle\langle d, dt\rangle, \langle d, \langle dt, t\rangle\rangle\rangle$ that takes in $[\![$much/little$]\!]$, $n$, and a degree predicate $D$, and yields true iff the maximum of the set of degrees in the extension of $D$ is a number in the complement of the positive/negative extent of $n$ (obtained by applying $[\![$much/little$]\!]$ to $n$).

(30) a. $[\![[\text{comp}]]\!] = \lambda f_{\langle d,dt\rangle}\,.\,\lambda n_d\,.\,\lambda D_{\langle d,t\rangle}\,.\,\max(\lambda d\,.\,D(d)) \in \overline{f(n)}$

b. $[\![\text{more/less than n}]\!]$
$= [\![[\text{comp}]]\!]\,([\![\text{much/little}]\!])(n)$
$= \lambda D_{\langle d,t\rangle}\,.\,\max(\lambda d\,.\,D(d)]) \in \overline{[\![\text{much/little}]\!]\,(n)}$

[8] This way of viewing the scale differentiates the present proposal from proposals such as that of Fox & Hackl (2006), where the scale is assumed to be universally dense. Our account does not rule it out that the scale could sometimes be infinitely dense. The point is simply that that is usually not the case, and that in out-of-the-blue contexts with bare and modified numerals the type and granularity of the scale seems to be determined implicitly by what we are counting. For example, when we count individuals (which will be the case for most of the examples we will discuss) the implicit scale is typically a scale of positive natural numbers with granularity 1. In other types of settings (e.g., in a lab, or when talking about the temperature, or in the context of a game where each scores come in multiples of three, etc.) the implicit range and granularity of the scale could very well be quite different. Our account allows for all these possibilities. We will come back to this again in Chapter 3.

Next, I propose that the meanings of *at least n* and *at most n* are obtained as in (31). The [at-sup] meaning common to both can be regarded as a function type $\langle\langle d, dt\rangle, \langle d, \langle dt, t\rangle\rangle\rangle$ that takes in [[much/little]], $n$, and a degree predicate $D$, and yields true iff the maximum of the set of degrees in the extension of $D$ is a number in the positive/negative extent of $n$ (obtained by applying [[much/little]] to $n$).

(31) a. $[\![\text{[at-sup]}]\!] = \lambda f_{\langle d,dt\rangle} \,.\, \lambda n_d \,.\, \lambda D_{\langle d,t\rangle} \,.\, \max(\lambda d \,.\, D(d)) \in f(n)$

b. $[\![\text{at most/least n}]\!]$

$= [\![\text{[at-sup]}]\!]\ ([\![\text{much/little}]\!])(n)$

$= \lambda D_{\langle d,t\rangle} \,.\, \max(\lambda d \,.\, D(d)]) \in [\![\text{much/little}]\!]\ (n)$

The syntactic and semantic derivation of a sentence with a CMN or a SMN is then as in Fig. 2.4. Just like in the case of a BN, the CMN *more/less than 3* / SMN *at least/most 3* is base generated as a phrase, let's call it Mod(ifier)P, in the specifier of #'. ModP consists of a complex Mod(ifier) head formed from [comp]/[at-sup] and *much/little*, and a complement Num(eral)P, familiar from BNs, which hosts the numeral $n$. Now, ModP is semantically type $\langle dt, t\rangle$ (generalized quantifier over degeree), so it can't be interpreted in its base position near #'. Thus, it moves out of the base clause, leaving behind a trace of type $d$. The syntactic and semantic derivation in the base clause then proceeds as described for BNs in §2.3, Fig. 2.2 (modulo the fact that in the base position for the numeral we now have a trace, which we will assume with Buccola & Spector 2016 is typeshifted via `isCard`). Getting back to ModP, at the site of movement it induces lambda abstraction over its trace type $d$, which gives rise to a predicate type $\langle d, t\rangle$. This predicate goes on to saturate the $\langle d, t\rangle$ argument slot of the meaning of ModP. All of these are summarized in Fig. 2.4. The basic decomposition and the resulting truth conditions are also spelled out in (32) and (33) below.

(32) $[\![\text{More/less than three people quit}]\!]$

$= 1 \text{ iff } [\![\text{[comp]}]\!]\ ([\![\text{much/little}]\!])([\![\text{three}]\!])$

$(\lambda d \,.\, \varnothing_{\exists}(([\![\texttt{isCard}]\!]\ ([\![\text{d}]\!]))(([\![\text{PL}]\!])(\lambda x \,.\, P(x))))(\lambda x \,.\, Q(x)))$

$= 1 \text{ iff } [\lambda D_{\langle d,t\rangle} \,.\, \max(\lambda d \,.\, D(d)]) \in \overline{[\![\text{much/little}]\!]\ (3)}](\lambda d \,.\, \exists x[|x| = d \wedge P(x) \wedge Q(x)])$

$= 1 \text{ iff } \max(\lambda d \,.\, \exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \overline{[\![\text{much/little}]\!]\ (3)}$

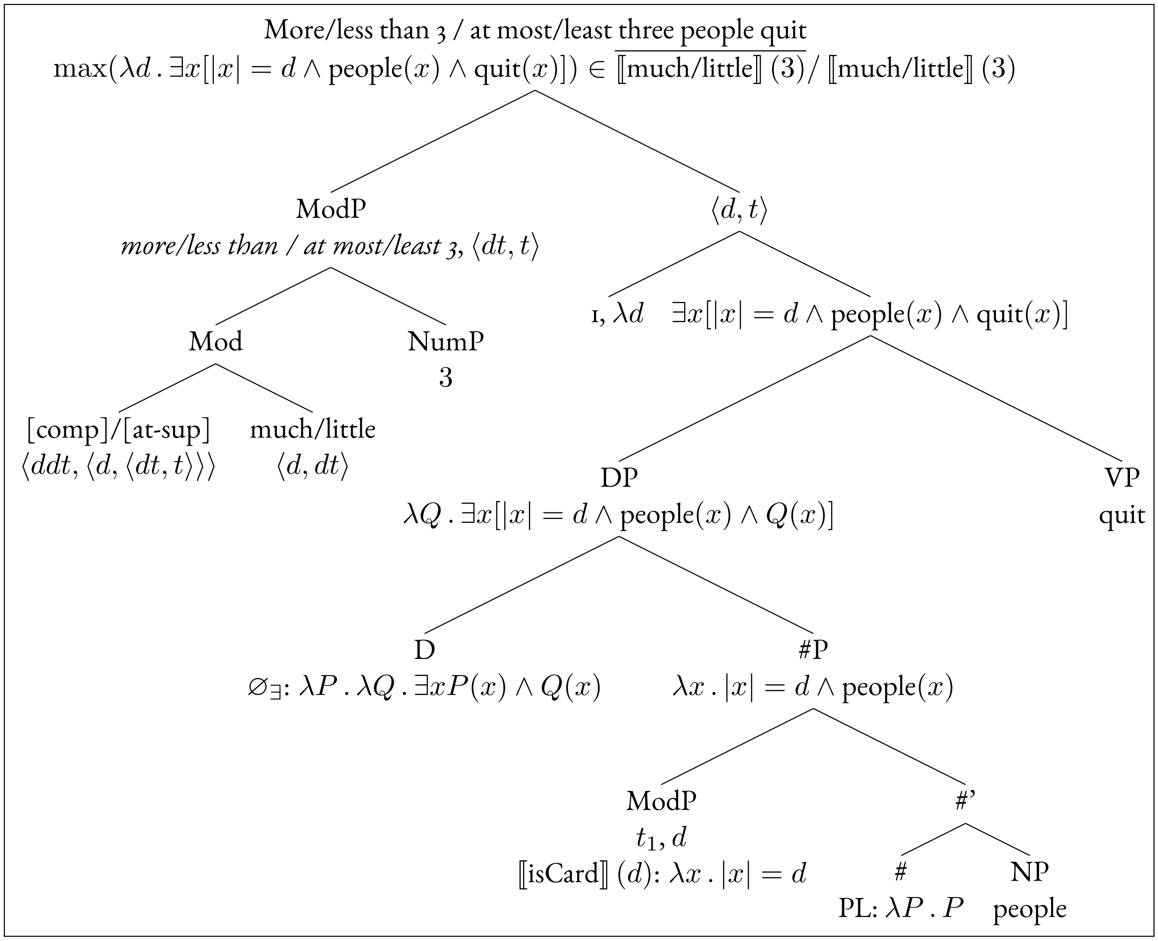



Figure 2.4: The syntax and semantics of CMNs and SMNs.

$$= 1 \text{ iff } \max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \overline{\{0,1,2,3\}}/\overline{\{3,4,5,\ldots\}}$$

$$= 1 \text{ iff } \max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \{4,5,\ldots\}/\{0,1,2\}$$

$$= 1 \text{ iff } \max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) > / < 3 \qquad (= (21))$$

(33) $[\![$At least/most three people quit$]\!]$

$$= 1 \text{ iff } [\![\text{[at-sup]}]\!]\,([\![\text{little/much}]\!])([\![\text{three}]\!])$$

$$(\lambda d\,.\,\varnothing_\exists(([\![\texttt{isCard}]\!]\,([\![\text{d}]\!]))(([\![\text{PL}]\!])(\lambda x\,.\,P(x))))(\lambda x\,.\,Q(x)))$$

$$= 1 \text{ iff } [\lambda D_{\langle d,t\rangle}\,.\,\max(\lambda d\,.\,D(d)]) \in [\![\text{little/much}]\!]\,(3)](\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)])$$

$$= 1 \text{ iff } \max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in [\![\text{little/much}]\!]\,(3)$$

$$= 1 \text{ iff } \max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \{3,4,5,\ldots\}/\{0,1,2,3\}$$

$= 1$ iff $\max(\lambda d \,.\, \exists x[|x| = d \wedge P(x) \wedge Q(x)]) \geq / \leq 3$ (= (22))

Note that the syntax and semantics of CMNs and SMNs given here is overall very similar to Hackl's, Fig. 2.4. The only major difference comes from our new treatment of *much/little* and [comp]/[at-sup], but after we factor those in, the resulting truth conditions (the last line of (32) and (33)) end up being the same as the Hackl/Hackl-style truth conditions in (21)-(22). What exactly have we achieved then?

First, we now have a more compositional view of CMNs and SMNs where the strict/non-strict comparison relation of the maximum of $D$ to the numeral is stated in a way that makes use of *much* and *little* and generalizes [comp] across *more than* and *less than* and [at-sup] across *at least* and *at most*.

Second, and more importantly for our main goal in this whole chapter, understanding how the relation of the maximum of $D$ to the numeral comes about will give us a crucial advantage in deriving the alternatives of CMNs and SMNs.

## 2.6 The truth conditions and alternatives of BNs, CMNs, and SMNs

We have come to the following overall picture of the truth conditions of BNs, CMNs, and SMNs.

(34) ⟦Three people quit⟧

$= 1$ iff $\exists x[|x| = 3 \wedge P(x) \wedge Q(x)]$ (cf. (12))

(35) ⟦More/less than three people quit⟧

$= 1$ iff $\max(\lambda d \,.\, \exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \overline{[\![\text{much/little}]\!]\,(3)}$ (cf. (32))

(36) ⟦At most/least three people quit⟧

$= 1$ iff $\max(\lambda d \,.\, \exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in [\![\text{much/little}]\!]\,(3)$ (cf. (33))

Note that the truth conditions of all of BNs, CMNs, and SMNs make reference to a scalar element – the numeral, here, 3.

Moreover, the truth conditions of CMNs and SMNs also make reference to a set of degrees based on the numeral, $\overline{[\![\text{much/little}]\!]\,(3)}$ and $[\![\text{much/little}]\!]\,(3)$, respectively. Note that this set of degrees is essentially a domain to which the cardinality of interest belongs.

This reminds us of the truth conditions of *or/some* $NP_{SG}$ , both of which contained both a scalar element and a domain. There we obtained scalar alternatives by replacing the scalar element in the truth conditions with another element from its alternative set, and subdomain alternatives by replacing the domain in the truth conditions with its subdomains (§1.2.1).

I propose, then, that the alternatives of BNs, CMNs and SMNs are obtained in the exact same way. In all of them, we can get scalar alternatives by replacing the numeral with some other numeral from a relevant scale $S$ (typically of natural numbers, but see Fn. 8 on p. 69). And in CMNs and SMNs we can also get subdomain alternatives by replacing the set of degrees that gives us the value of max – $\overline{[\![\text{much/little}]\!]\,(3)}$ for CMNs, $[\![\text{much/little}]\!]\,(3)$ for SMNs – with its subsets.

(37) Three people quit.

a. $\exists x[|x| = 3 \wedge P(x) \wedge Q(x)]$ (assertion)

b. $\{\exists x[|x| = n \wedge P(x) \wedge Q(x)] \mid n \in S\}$ ($\sigma$A )

c. – (no DA )

(38) More/less than three people quit.

a. $\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \overline{[\![\text{much/little}]\!]\,(3)}$ (assertion)

b. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \overline{[\![\text{much/little}]\!]\,(n)} \mid n \in S\}$ ($\sigma$A )

c. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in D' \mid D' \subset \overline{[\![\text{much/little}]\!]\,(3)}\}$ (DA )

(39) At most/least three people quit.

a. $\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in [\![\text{much/little}]\!]\,(3)$ (assertion)

b. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in [\![\text{much/little}]\!]\,(n) \mid n \in S\}$ ($\sigma$A )

c. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in D' \mid D' \subset [\![\text{much/little}]\!]\,(3)\}$ (DA )

Before we move on, one clarification. In our discussion of the alternatives of *or/some $NP_{SG}$* we also said that one could obtain alternatives by at the same time replacing both the scalar item and the domain. Let's recall that discussion here. For a domain with three elements, $a$, $b$, and $c$, *or/some $NP_{SG}$* had the truth conditions $\exists x \in \{a, b, c\}[P(x)]$. Then, if we replaced both the scalar element and the domain at the same time, we were able to obtain, e.g., a mixed-type alternative of the form $\forall x \in \{a, b\}[P(x)]$. The mixed-type alternatives thus obtained were distinct from the alternatives generated by replacing just the scalar element or just the domain. Moreover, for some of the crucial cases we discussed they were stronger than the assertion, so when they were included in the exhaustification (where the DA set was not pruned) they ended up being crucial to deriving precisely the results we wanted. Are there such additional alternatives for CMNs and SMNs also? First, note that in the case of CMNs/SMNs the relation between the scalar element and the domain is different: the domain is parasitic on the scalar element, so replacing the scalar element with a scalemate alternative already results in a replacement of the domain. Moreover, depending on the scalemate alternative, the domain thus obtained could be either smaller or larger than the original domain. In the first case this new alternative would be equivalent not only to an $\sigma$A from the original set, but also to a DA . In the second case this new alternative would be equivalent to an $\sigma$A from the original set. Either way, the alternatives thus obtained would be no different from the set predicted by replacing just the scalar element or just the domain. Thus, the alternatives summarized above are all the alternatives that are generated.

To sum up, our revision to the Hackl/Hackl-style truth conditions for CMNs and SMNs helped us derive in a natural way their alternatives. As we will see in the next couple of chapters, the question of what the alternatives of modified numerals should be is a major issue at the heart of every recent account of modified numerals. Thus, having a principled way to derive them is a most welcome result.

These truth conditions and alternatives are what we will use going forward. The reader curious about how this view of *much*, *little*, [comp], and [at-sup] fits with the rest of the literature on adjectives, comparatives, and superlatives will find some answers in the next few sections until the end of this chapter. The reader primarily interested in how we can use these truth conditions and alternatives to derive the scalar implicature, ignorance, and polarity sensitivity patterns of CMNs and SMNs can skip forward to

Chs. 3, 4, or 5, respectively.

## 2.7 Connections: [comp]

We started from Hackl truth conditions for CMNs and modified them. At the end of §2.5 we emphasized that the change for CMNs was minimal – we merely rewrote the relation to the numeral encoded by [comp] in a way that made use of our new meanings for *much* and *little*. However, the way we did that does have small consequences for how we look at comparative constructions more generally. Below we clarify a few.

### 2.7.1 The type of [comp]

Hackl (2000)'s type for [comp] was $\langle dt, \langle dt, t\rangle\rangle$, but on our approach it became $\langle ddt, \langle d, \langle dt, t\rangle\rangle\rangle$, because we treated *much/little* and $n$ as separate arguments to [comp] rather than one argument giving us the result of applying $[\![\text{much/little}]\!]$ to $n$. We did that to highlight the effect of *much/little* in giving rise to a domain of degrees based on $n$. However, note that nothing prevents us from going back to Hackl's type – the truth conditions remain the same. Fig. 2.5 below illustrates this.

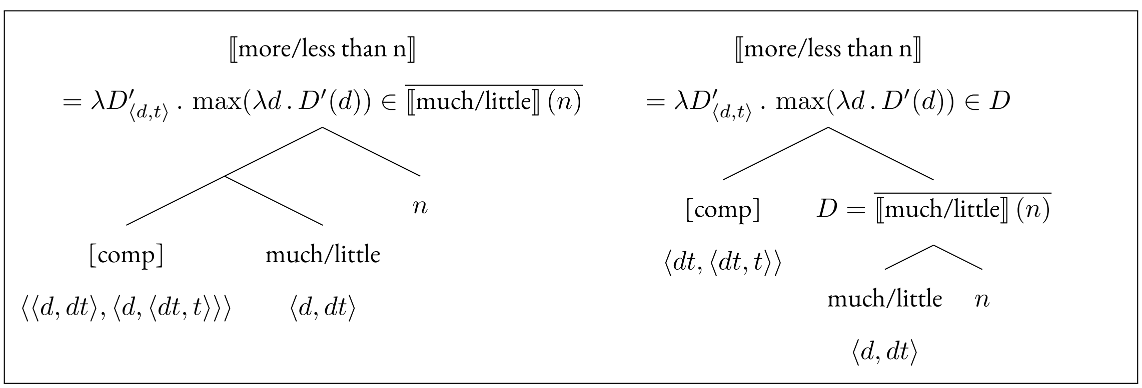



Figure 2.5: Types for [comp]: On the left is our proposal from Fig. 2.4. On the right is our proposal with types/compositionality modified to match Hackl's proposal from Fig 2.3.

## 2.7.2 The adjective in the base clause

We assumed that the modified numeral phrase, including *much/little* with the meaning of extent indicators, moves as a whole, leaving behind a trace of type $d$. We assumed the glue between this meaning and $P$ was provided via the typeshifter `isCard`. On the other hand, on Hackl's account illustrated in Fig. 2.3 the adjective doesn't move with the modifier; rather, it stays behind and behaves like a regular measure phrase. Fig. 2.6 below shows these side by side.

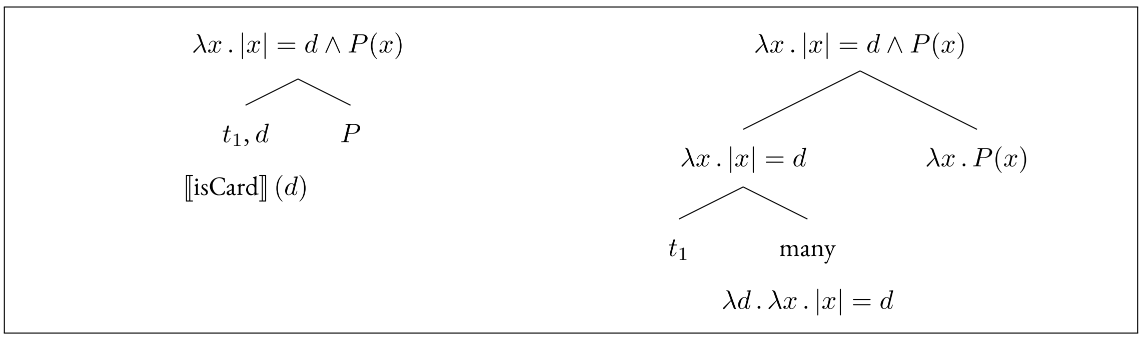



Figure 2.6: The adjective in the base clause: On the left is our proposal from Fig. 2.4. On the right is Hackl's proposal from Fig 2.3.

Note that, while our approach with typeshifting in the base clause worked fine for numerals, for comparative constructions in general we might want an actual adjective in that position, just like on Hackl's approach. That is because comparative constructions can have different adjectives in the base clause vs. the comparative clause, as in Hackl's example *The table is longer than the rug is wide*. Thus, for such an example we might want a representation as below, with *long* in the base clause (the max part of the truth conditions) and *wide* in the comparative clause (the complement set part of the truth conditions).

(40) $[\![\text{The table is longer than the rug is wide}]\!]$

$$= 1 \text{ iff } \max(\lambda d \,.\, [\![\text{long}]\!]_{\langle e,dt\rangle}\,(\iota x[\text{table}(x)])(d)) \in \overline{[\![\text{wide}]\!]_{\langle e,dt\rangle}\,(\iota y[\text{rug}(y)])}$$

$$= 1 \text{ iff } \max(\lambda d \,.\, d \leq \mu_{\text{length}}(\iota x[\text{table}(x)])) \in \overline{\lambda d \,.\, d \leq \mu_{\text{width}}(\iota y[\text{rug}(y)])}$$

$$= 1 \text{ iff } \mu_{\text{length}}(\iota x[\text{table}(x)]) \in \lambda d \,.\, d > \mu_{\text{width}}(\iota y[\text{rug}(y)])$$

$$= 1 \text{ iff } \mu_{\text{length}}(\iota x[\text{table}(x)]) > \mu_{\text{width}}(\iota y[\text{rug}(y)])$$

But Hackl treated this adjective in the base clause as a simple measure function, e.g., $[\![\text{long}]\!]\,(x) = \lambda d\,.\,d = \mu_{\text{length}}(x)$. However, just now (and on our approach more generally) we treated gradable adjectives as extent indicators, mapping an individual to their extent on a scale, e.g., $[\![\text{long}]\!]\,(x) = \lambda d\,.\,d \leq \mu_{\text{length}}(x)$. Which one is it?

Recall that in the general case it was important for the adjective in the comparative clause to be interpreted as extent-denoting, e.g., $[\![\text{wide}]\!]\,(x) = \lambda d\,.\,d \leq \mu_{\text{width}}(x)$ (after complementation, that helped us get the desired set of degrees). However, note that, also in the general case, we actually don't want this meaning for the adjective in the base clause, because an extent-type meaning would crash for negative adjectives, as the λ-abstract would be a set with a minimum but no maximum, so applying $\max$ to it later on, in keeping with our general truth conditions for comparatives, would fail.

(41) $[\![\text{Jim is shorter than Teddy (is short)}]\!]$

$= 1$ iff $\max(\lambda d\,.\,[\![\text{short}]\!]_{\langle e,dt\rangle}\,(j)(d)) \in \overline{\lambda d\,.\,[\![\text{short}]\!]_{\langle e,dt\rangle}\,(m)(d)}$

$= 1$ iff $\max(\lambda d\,.\,d \geq \mu_{\text{height}}(j)) \in \overline{\lambda d\,.\,[\![\text{short}]\!]_{\langle e,dt\rangle}\,(m)(d)}$

$= 1$ iff $\boxed{\text{crash!}} \in \overline{\lambda d\,.\,[\![\text{short}]\!]_{\langle e,dt\rangle}\,(m)(d)}$

However, things would work just fine if we had an extent-indicator meaning in the comparative clause, as before, but a measure-function meaning in the base clause (in the λ-abstract operated on by $\max$).

(42) $[\![\text{Jim is shorter than Teddy (is short)}]\!]$

$= 1$ iff $\max(\lambda d\,.\,[\![\text{short}]\!]_{\langle e,dt\rangle}\,(j)(d)) \in \overline{\lambda d\,.\,[\![\text{short}]\!]_{\langle e,dt\rangle}\,(m)(d)}$

$= 1$ iff $\max(\lambda d\,.\,d = \mu_{\text{height}}(j)) \in \overline{\lambda d\,.\,[\![\text{short}]\!]_{\langle e,dt\rangle}\,(m)(d)}$

To conclude, although assuming that adjectives in the comparative clause are extent-indicators gives us an advantage in stating the meaning of the comparative function, for the resulting meaning to make sense we still want the adjective in the base clause to behave like a regular measure function.

### 2.7.3 Quantifiers in the scope of the comparative

The issue of quantifiers in the scope of a comparative *than*-clause (e.g., *most* in *Irving was closer to me than he was to most of the others*) is not really an issue for our discussion of CMNs, as in CMNs we never have a quantifier in that position, we just have a numeral. However, it is an important issue in the literature on comparatives more generally (Schwarzchild & Wilkinson 2002, Heim 2006) and it would be interesting to see how our current take on [comp] fits with existing approaches to this.

The issue is as follows. Quantifiers in comparative-clauses are interpreted as if they scoped out. E.g., the example below only has the interpretation that the width of the desk exceeds the length of every single couch, it does not have the interpretation that the width of the desk exceeds the minimum length across couches.

(43) The desk is wider than every couch is long.
$\forall x[\text{couch}(x) \rightarrow \text{the desk's width} > x\text{'s length}]$

On our approach we could simply say that *every couch* can't directly compose with $[\![\text{long}]\!]_{\langle e,dt\rangle}$ in the *than*-clause, so it has to move out, and that's how this scope reading obtains.

However, as discussed in Heim (2006) (and references therein), this is puzzling, because, syntactically, *than*-clauses ought to be scope-islands, and semantically they should be definite descriptions of degrees, thus we don't expect movement of the QP to be possible out of them. Moreover, as shown in Schwarzchild & Wilkinson (2002), even if movement were in fact possible, it would give rise to the wrong interpretation.

Heim proposes a solution on which the QP doesn't move, rather, what moves is the entire *than*-clause that contains it, this clause being treated as a generalized quantifier over degrees.

The implementation of this solution makes crucial use of an invisible operator Π, with a meaning as below.

(44) $[\![\Pi]\!] = \lambda D_{\langle d,t\rangle} \,.\, \lambda D' \,.\, _{\langle d,t\rangle}\, \max(D') \in D$

Heim (2006) proposes that Π is generated in the degree argument position of an adjective, where it combines with whatever is traditionally generated in that slot, e.g., the comparative [comp]-*than*-clause. The resulting Π-phrase has to move for type reasons. Now assume that our [comp] meaning is actually such that it takes a set of degrees and returns its complement ($[\![\text{[comp]}]\!] = \lambda D_{\langle d,t\rangle} \,.\, \overline{D}$), and its sister is an extent obtained by applying an abstract POS/NEG type meaning (as in our definition of positive and negative extents in (26-a)-(26-b)) to the trace type $d$ of the *than*-clause type $\langle dt, t\rangle$. The abstract POS/NEG extent-indicator function can be regarded as a way to typeshift a meaning type $d$ such that it reflects the intrinsic polarity (positive or negative) of the adjectives on either side of the comparison (cf. Kennedy (2001)'s discussion of cross-polar anomaly, these polarities have to match, so it's always just POS or just NEG). Fig. 2.7 on p. 80 gives a full derivation blending Heim (2006:14-5)'s Larsonian view of the *than*-clause (with *wh* in the *than*-clause creating a $\lambda$ abstract type $\langle d,t\rangle$ binding a trace type $\langle d,t\rangle$) and her use of Π with our idea that the adjectival meaning in the comparative clause (although not in the base clause, for the reason discussed in the previous section) is an extent. Note that Heim's Π + our new [comp] essentially reconstruct what we were previously getting just from [comp] – Π takes over the degree quantifier and maximal quantification over degrees meaning.

The point of this otherwise tangential discussion was to show that, even if the best, most general meaning for [comp] may ultimately be different than the one we gave for CMNs, our basic view of how comparative constructions work, and of the adjectival meanings in the comparative clause as extent-indicating, can be maintained.

## 2.8 Connections: [at-sup]

### 2.8.1 [at-sup] + [numeral]

The meaning we obtained for [comp] in §2.5 connected quite naturally to comparative meanings more generally, but how does our meaning for [at-sup] connect to superlative meanings? In what sense is *At most/least three people quit* superlative?

Recall that our truth conditions for a sentence like this were as below.

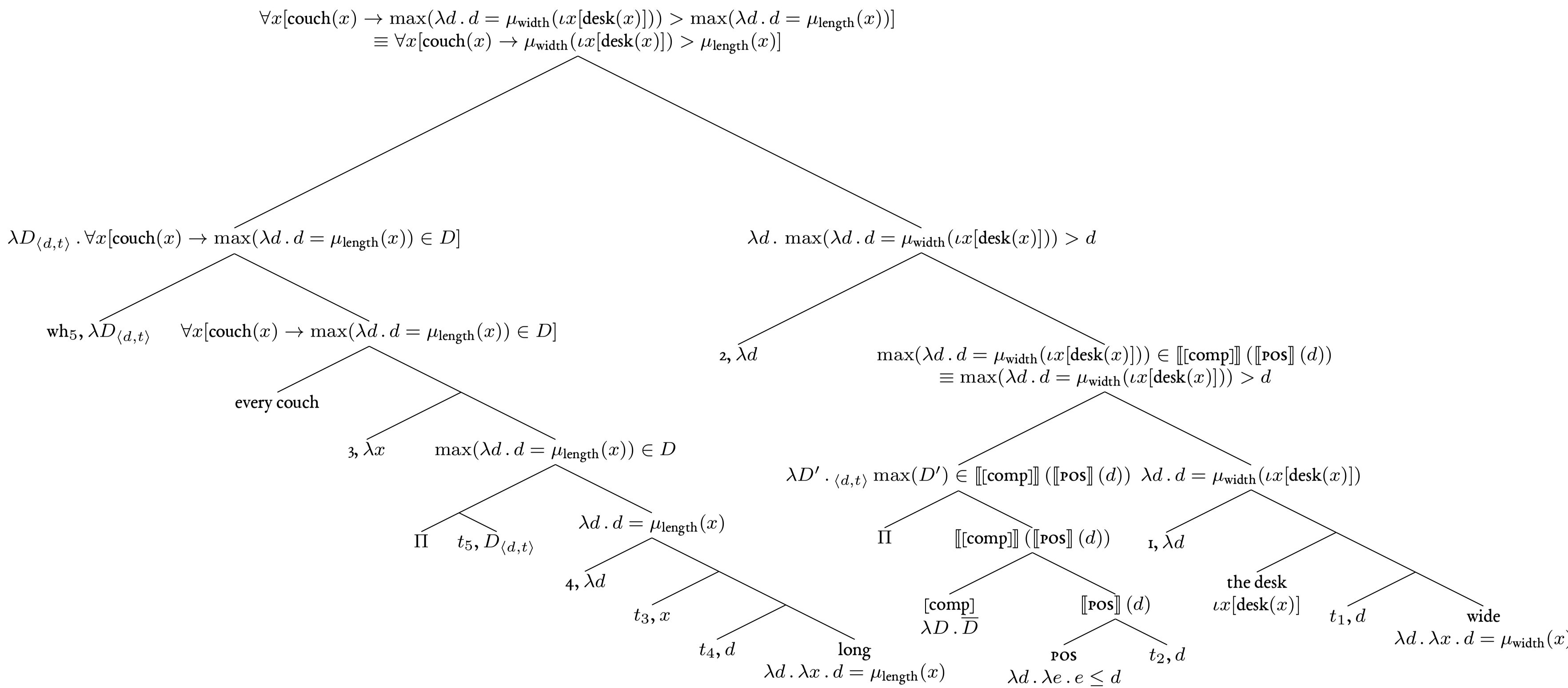



Figure 2.7: Quantifiers in the *than*-clause (blending Heim 2006 with our view of adjectives as extents).
$[\![\Pi]\!] = \lambda D_{\langle d,t\rangle}\,.\,\lambda D'\,._{\langle d,t\rangle} \max(D') \in D$

(45) $\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in$ ⟦much/little⟧ $(3)$

It appears that what [at-sup] does is tell us that some cardinality of interest (i.e., the one given by $\max$) is in a set whose superlative large/small member is 3 (in our notation, the set denoted by the positive/negative extent of $3$). But can we find a decomposition of [at-sup] that would make this thought more specific?

Chen (2018) offers us crucial clues towards a decomposition of precisely this sort. First, he argues that a superlative modifier such as *at most/at least* must be decomposed as in Fig. 2.8 – using Bobaljik (2012)'s Containment Hypothesis, i.e., the idea that the superlative is built atop a comparative, positing a separate function associated with *at*, and assuming that the resulting *at most/at least* meaning takes as its first argument a covert semantic variable $C$ that denotes a subset of the focus value of the focus associate of the superlative.

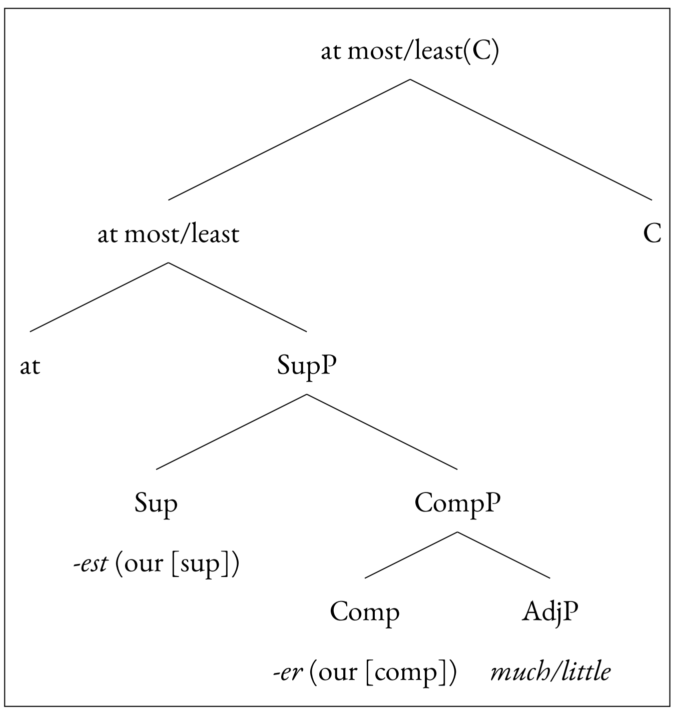



Figure 2.8: Chen (2018)'s decomposition of *at most*.

Semantically, Chen's decomposition up to SupP goes pretty much as in Heim (2000, 1999), Hackl

(2000, 2009). We will recast this part using our own semantics for *much/little* and [comp] and Heim (1999)/Hackl (2009)-style semantics for [sup], and adjusting everything for degrees (rather than individuals).

(46) a. *much / little*

$[\![\text{much/little}]\!] = \lambda d_d \,.\, \lambda e_d \,.\, e \leq / \geq d$

b. *more / less*

$[\![\text{[comp](much/little)}]\!]$

$= \lambda d_d \,.\, \lambda D_{\langle d,t\rangle} \,.\, \max(\lambda d' \,.\, D(d')) \in \overline{[\![\text{much/little}]\!]\,(d)}$

$= \lambda d_d \,.\, \lambda D_{\langle d,t\rangle} \,.\, \max(\lambda d' \,.\, D(d')) > / < d$

c. *most / least*

$[\![\text{[sup]([comp](much/little))}]\!]$

$= \lambda C_{\langle d,t\rangle} \,.\, \lambda e_d \,.\, \forall d[d \in C \wedge d \neq e \rightarrow [\![\text{[comp](much/little)}]\!]\,(d)(e)]$

$= \lambda C_{\langle d,t\rangle} \,.\, \lambda e_d \,.\, \forall d[d \in C \wedge d \neq e \rightarrow \max(\lambda d' \,.\, d' = e) > / < d]$

($[\![\text{[comp](much/little)}]\!]\,(d)$ needs an argument $D_{\langle d,t\rangle}$ but gets $e_d$. As Hackl (2000:50), we assume $e$ can be typeshifted into its corresponding singleton set $\lambda d' \,.\, d' = e$, which can go into the $\lambda D_{\langle d,t\rangle}$ slot of $[\![\text{[comp](much/little)}]\!]\,(d)$.)

$= \lambda C_{\langle d,t\rangle} \,.\, \lambda e_d \,.\, \forall d[d \in C \wedge d \neq e \rightarrow e > / < d]$

*Most/least* thus derived have a modifier type, $\langle dt, dt\rangle$. They are functions that take as an argument a degree predicate $C$ – a subset of the focus value of the focus associate of the superlative – and a degree $e$ – the focus associate of the superlative – and yield true iff any degree $d$ that is in $C$ and different from $e$ is smaller/larger than $e$.

Now, the part that turns *most/least* into *at most/at least* is *at*. Adapting Chen (2018:237-8)'s proposal for *at* (specifically the variant that captures the non-propositional use of superlative-modifiers) to our numeral types and to our assumption that the modifier in the end yields a quantifier over degrees, *at* is a function that takes the meaning of *most/least* thus derived and turns it into a function that continues to look for a degree predicate $C_{\langle d,t\rangle}$ – a subset of the focus value of the focus associate of the superlative –

and a degree $e_d$ – the focus associate of the superlative – but in addition to that it also takes as an argument another predicate of degrees $D$, and yields true iff there is a degree $f$ in $C$ such that the maximum degree in $D$ is equal to $f$ and $C$ is such that it verifies $[\![$most/least$]\!]$ $(C)(e)$. Finally, *at most/least C n* is a function that takes as an argument a degree predicate $D$ and yields true iff there is a degree $f$ in $C$ such that the maximum degree in $D$ is equal to $f$ and $C$ is such that it verifies $[\![$most/least$]\!]$ $(C)(e)$.

(46) cont'd:

d. *at most / at least*

$[\![$at$]\!]$ ($[\![$[sup]([comp](much/little))$]\!]$)

$= \lambda C_{\langle d,t\rangle} \,.\, \lambda e_d \,.\, \lambda D_{\langle d,t\rangle} \,.\, \exists f[f \in C \wedge \max(\lambda d \,.\, D(d)) = f \wedge$

$[\![$[sup]([comp](much/little))$]\!]$ $(C)(e)]$

$= \lambda C_{\langle d,t\rangle} \,.\, \lambda e_d \,.\, \lambda D_{\langle d,t\rangle} \,.\, \exists f[f \in C \wedge \max(\lambda d \,.\, D(d)) = f \wedge \forall d[d \in C \wedge d \neq e \rightarrow e >$

$/ < d]]$

e. *at most C n / at least C n*

$[\![$[at]([sup]([comp](much/little)))(C)(n)$]\!]$

$= \lambda D_{\langle d,t\rangle} \,.\, \exists f[f \in C \wedge \max(\lambda d \,.\, D(d)) = f \wedge \forall d[d \in C \wedge d \neq n \rightarrow n > / < d]],$

where $C \subset [\![\text{n}]\!]^{\text{f}}$ ($C$ is a subset of the focus value of the numeral).

$\Rightarrow \lambda D_{\langle d,t\rangle} \,.\, \max(\lambda d \,.\, D(d)) \in [\![\text{much/little}]\!]\,(n)$ (= (33))

As we can see, $[\![$at most/least C n$]\!]$ thus derived is a function that takes as an argument the predicate of degrees $D_{\langle d,t\rangle}$ and says that there is a number $f$ in $C$ such that it is the maximum of $D$ and $C$ is a subset of the focus value of the $n$ whose largest/smallest number is $n$ – that is, recalling out definition of positive/negative extents, $C$ is the positive/negative extent of $n$. The net result is exactly our own proposed meaning for SMNs in (33), only this time derived using Chen's more detailed syntax and semantics. Figure 2.9 illustrates the updated syntax that goes with the semantic composition above.

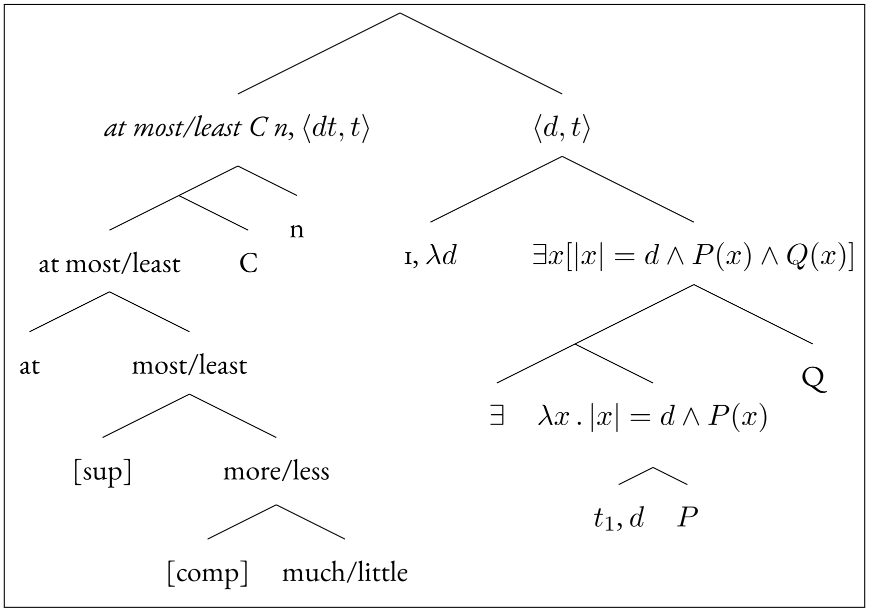



Figure 2.9: SMNs as quantifiers over degrees, with Chen-style *at most/least*.

Thus, although not an accurate decomposition of *at most/least C n*, our analysis of [at-sup](much/little)(n) in §2.5 still seems to have captured the correct idea.

### 2.8.2 [at-sup] + [non-numeral]

Krifka (1999) notes that the comparative and superlative modifiers can associate with phrases of different sizes via focus, for example with numerals, DPs, APs, VPs, or Det's. Below I illustrate this for *at least* (examples adapted from Krifka 1999, with some additions).

(47) a. At least [three]$_F$ boys left.

b. At least [three boys / John and Teddy / the first year students]$_F$ left.

c. Teddy was at least [satisfied]$_F$.

d. At least [some]$_F$ determiners aren't determiners.

This association is reflected in the sort of questions each of these examples can answer. (47-a) can answer a question of the form *How many boys left?*, with the modified numeral being able to act as a short answer: *At least three*. (47-b) answers a question of the form *Who left?* And so on.

This use of (especially *at least*) with associates of types and sizes other than a numeral exhibits similar ignorance and anti-negativity effects as SMNs: All the examples above convey some sense of speaker ignorance, and they also resist taking scope below negation – e.g., *Teddy wasn't at least satisfied* is degraded.

I propose that the explanation for this is the same as we will discuss in detail for SMNs in the next few chapters, and it has to do with the same decomposition of the superlative modifiers that we proposed for SMNs, namely, a decomposition where their truth conditions make reference to a domain. For example, *At most C John and Teddy left* could be interpreted as saying that $\max(\lambda d\,.\,\exists x[\mu_c(x) = d \wedge \text{left}(x)]) \in$ $[\![\text{much}]\!]\ (\mu_c(j \oplus m))$. Or, in Chen's more detailed decomposition, that there is a degree $f$ in $C$ (where $C$ is a subset of the focus value of the degree that is the measure of $[\![\text{John and Teddy}]\!] = j \oplus m$ on some scale) and $f$ is the maximum $d$ such that there exists an $x$ with measure $d$ that left, and by the way $C$ is a set of degrees such that if a degree $d$ in it is different from the measure of $j\oplus$ on the relevant scale, then the measure of $j \oplus m$ on the relevant scale is larger than $d$.

Thus, a decomposition of the kind we want seems to work for cases where the associate of the superlative modifier is not a numeral also. We will however not discuss cases like these any further here.

## 2.9 Connections: *many P Q*, *more/fewer P Q*, *most/*fewest P Q*

In §2.5 we figured out a way to analyze *three P Q*, *more/less than three P Q*, and *at most/least three P Q*. Now one may wonder, how do these meanings relate to sentences such as the ones below, with *many/few P Q*, *more/fewer P Q*, and *most/*fewest P Q*?

(48) Many/Few people quit.

(49) More/Fewer people quit.

(50) Most/*Fewest people quit.

I propose that *many/few P Q* relies on [pos](much/little)(s)(D), where $[\![\text{much/little}]\!]$ and $D$ are as in our analysis of CMNs and SMNs, $s$ is a contextual standard for the relevant measure, i.e., cardinality, and [pos] is a function that says that $s$ is in the positive/negative extent of the maximum of $D$. (Note: This is the [pos] of positive adjectives, not the Pos of positive extents that we discussed a few pages earlier.)

(51) $[\![\text{many/few s}]\!]$

$= [\![\text{[pos]}]\!] \, ([\![\text{much/little}]\!])(s)$

$= \lambda D_{\langle d,t\rangle} \,.\, s \in [\![\text{much/little}]\!] \, (\max(\lambda d \,.\, D(d)))$

$= \lambda D_{\langle d,t\rangle} \,.\, \max(\lambda d \,.\, D(d)) \geq / \leq s$

*More/fewer P Q* relies on [comp](much/little)(s)(D), where [comp], $[\![\text{much/little}]\!]$, and $D$ are as for CMNs and SMNs, and $s$ is again a contextual standard (presumably set anaphorically with respect to some number previously mentioned in the context).

(52) $[\![\text{more/fewer s}]\!]$

$= [\![\text{[comp]}]\!] \, ([\![\text{much/little}]\!])(s)$

$= \lambda D_{\langle d,t\rangle} \,.\, \max(\lambda d \,.\, D(d)) \in \overline{[\![\text{much/little}]\!]}(s)$

$= \lambda D_{\langle d,t\rangle} \,.\, \max(\lambda d \,.\, D(d)) > / < s$

Thus, both *many/few s* and *more/fewer s* can be analyzed as generalized quantifiers over degrees, just like CMNs and SMNs.

However, *most/*fewest P Q* is quite different. Traditional analyses (e.g., Hackl 2009, Gajewski 2010) argue that *most* involves a structure where the gradable adjective has a modifier type – it first composes with P and yields a relation between individuals $x$ and degrees $d$ such that they have the property of manyness to degree $d$. Let *many/few* then have meanings as in (53-a-i). Since by now we know that the superlative relies on a comparative meaning, and recalling our discussion in §2.7.2 about the fact that an adjectival meaning in the base part of a comparative construction cannot be an extent because maximization crashes for negative extents, let *many/few* also have the measure function meanings in (53-a-ii).[9] Then, *many/few*

[9]I am not saying adjectives have two types of basic meanings. It is possible, for example, that the extent type of meaning is really contributed by the comparative.

$P$ is as in (53-b). And, glossing over the details of how $[\![\text{[comp](many/little P)}]\!]$ inside of the superlative might be defined, *most/*fewest* $P$ could be obtained as in (53-c). (Modulo our treatment of *many/few*, these truth conditions are as in Hackl 2009.) Since, as Hackl (2009) argues, $y \neq x$ for pluralities must be understood as saying that $x$ and $y$ are two non-overlapping pluralities, the result says that '$\lambda x_e \,.\, x$ is more/less numerous than any other non-overlapping plurality $y$ with property $P$.' This meaning is then closed via a silent existential quantifier.

(53) a. (i) $[\![\text{many/few}]\!] = \lambda P_{\langle e,t\rangle} \,.\, \lambda x_e \,.\, \lambda d_d \,.\, |x| \leq / \geq d \wedge P(x)$ (extent indicator)

(ii) $[\![\text{many'/few'}]\!] = \lambda P_{\langle e,t\rangle} \,.\, \lambda x_e \,.\, \lambda d_d \,.\, |x| = d \wedge P(x)$ (measure function)

b. (i) $[\![\text{many/few people}]\!] = \lambda x_e \,.\, \lambda d_d \,.\, |x| \leq / \geq d \wedge \text{people}(x)$ (extent indicator)

(ii) $[\![\text{many'/few' people}]\!] = \lambda x_e \,.\, \lambda d_d \,.\, |x| = d \wedge \text{people}(x)$ (measure function)

c. $[\![\text{most/\#fewest people}]\!]$

$= [\![\text{[sup](C)([comp](many/little people))}]\!]$

$= \lambda x \,.\, \forall y[y \in C \wedge y \neq x \rightarrow \max(\lambda d \,.\, [\![\text{many'/few' people}]\!]\,(x)(d)) \in \overline{\lambda d \,.\, [\![\text{many/few people}]\!]\,(y)(d)}]$

$= \lambda x \,.\, \forall y[y \in C \wedge y \neq x \rightarrow \max(\lambda d \,.\, |x| = d \wedge \text{people}(x)) \in \overline{\lambda d \,.\, |y| \leq / \geq d \wedge \text{people}(y)}]$

$= \lambda x \,.\, \forall y[y \in C \wedge y \neq x \rightarrow \max(\lambda d \,.\, |x| = d \wedge \text{people}(x)) \in \lambda d \,.\, d > / < |y| \wedge \text{people}(y)]$

d. $[\![\text{most/\#fewest people quit}]\!]$

$= \exists x[\forall y[y \in C \wedge y \neq x \rightarrow \max(\lambda d \,.\, |x| = d \wedge \text{people}(x)) \in \lambda d \,.\, d > / < |y| \wedge \text{people}(y)] \wedge \text{quit}(x)]$

'There is a plurality $x$ such that for all pluralities $y$ in $C$ that are different from it (i.e., that don't overlap with it) the maximum degree such that $x$ has the property of being a plurality of people numerous to that degree is in the set of degrees larger/smaller than the degree such that $y$ is a plurality of people numerous to that degree, and $x$ quit.'

So far we have been marking *fewest P* as ungrammatical, without however explaining why this should be so. Hackl (2009:83) offers an explanation. *Fewest P* would say that the plurality $x$ is smaller than any disjoint plurality $y$ with property $P$. But this is never possible, because even if $x$ is a plurality with just one atom, so long as $P$ is not a singleton property (which is prevented anyway by the presupposition of [sup]), there will always be other non-overlapping pluralities that are not smaller – those would be other pluralities with just one atom.

At any rate, *many/few*, *more/fewer*, and *most/#fewest* are as in Fig. 2.10.

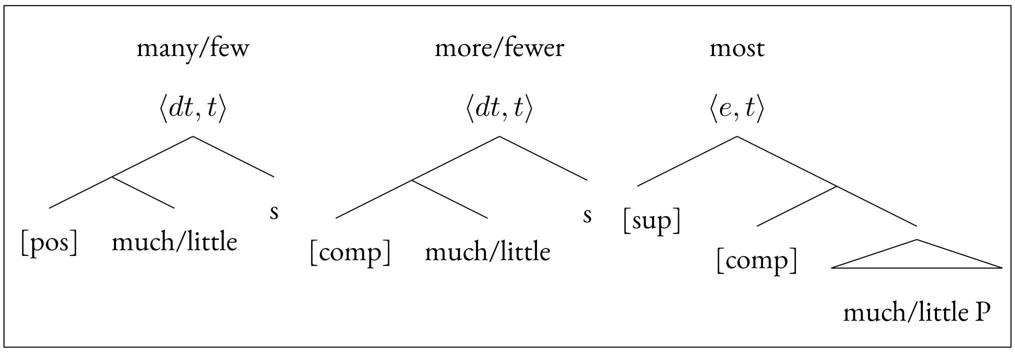



Figure 2.10: *Many/few P Q*, *more/fewer P Q*, *most/#fewest P Q*

To sum up, the semantics for *many/few P Q* and *more/fewer P Q* nicely resembles that of *three P Q* and *more/less than n P Q*, but the semantics of *most/#fewest P Q* is quite different from *at most/least n P Q*. Is this a bad result? Not necessarily. *At most/least n P Q* and *most/#fewest P Q* differ in a number of ways: The notion of superlativity is intuitively quite different – a range where the numeral is superlatively large/small vs. a majority of individuals; the former is not just limited to the superlative meaning, it contains other pieces also; the former is downward-entailing on both its NP and its VP argument, but, as Gajewski (2010) shows, *most P Q* is downward-entailing on its NP argument but not at the level of the whole DP, and not on its VP argument; etc. Moreover, this pattern where we find parallelism between *three P Q* and *many 3 P Q* or *more than 3 P Q* and *more P Q* but not between *at most 3 P Q* and *most P Q* is found in language after language, suggesting that there is indeed something else going on in this case.

## 2.10 A note on *less than* and *at most* with collective predicates

So far our meanings for CMNs and SMNs always involved a maximality degree. The GQT truth conditions that we started with picked out the total cardinality, and then the Hackl-style truth conditions made this even more visible with the use of $\max$.

(54) ⟦More/less than three people quit⟧

$$\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \overline{[\![\text{much/little}]\!]\ (3)}$$

(55) ⟦At least/most three people quit⟧

$$\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in [\![\text{much/little}]\!]\ (3)$$

Note that this piece of meaning was never really necessary for the upward-entailing modifiers. The truth conditions of *more than* and *at least* could just as well have been stated in terms of existential rather than maximal quantification.

(56) ⟦More than two / at least three people quit⟧

a. $\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \{3, 4, \dots\}$

b. $\exists d[d \in \{3, 4, \dots\} \wedge \exists x[|x| = d \wedge P(x) \wedge Q(x)]]$

Since we have also assumed with Hackl (2000) that comparative constructions more generally involve $\max$, note that the same comment can be made for comparative constructions involving positive adjectives. (Below we revert to Hackl's truth conditions, because we are only interested in the use of $\max$. This is also orthogonal to our previous comments about whether we want an extent-indicator or a measure-function type meaning for the adjective in the scope of $\max$.)

(57) ⟦John is taller than Teddy⟧

a. $\max(\lambda d\,.\,\text{tall}(j, d)) > \max(\lambda d\,.\,\text{tall}(t, d))$

b. $\exists d[d > \max(\lambda d\,.\,\text{tall}(j, d)) \wedge \neg\text{tall}(t, d)]$

However, note that for the downward-entailing modified numerals $\max$ does make a difference – only the $\max$ meaning derives the upper-bounding entailments we saw at the outset in §2.1. (The same is true for the comparative form of negative adjectives also – for *John is shorter than Teddy* we don't just want to say that there is a degree to which John is tall that is smaller than some/the maximum degree to which Teddy is tall, rather, it is crucial to talk about the *maximum* degree to which John is tall, his actual height.)

(58) ⟦Less than four / at most three people quit⟧

a. $\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \{0, 1, 2, 3\}$

b. $\exists d[d \in \{0, 1, 2, 3\} \wedge \exists x[|x| = d \wedge P(x) \wedge Q(x)]]$

So should we conclude that maximality is superfluous for the lower-bounded meanings but crucial to the upper-bounded meanings?

Addressing this exact issue in connection to *less than* and *at most*, Buccola & Spector (2016) bring up the fact that with collective predicates *less than* and *at most* in fact do seem to be able to take/require the $\exists d$ meaning. This is also known since, e.g., Krifka (1999)'s discussion of cases of statistical distribution such as the one below, which have a lower-bounded meaning.

(59) Fewer than 20 people own over 80% of the land in Guatemala.

Buccola & Spector (2016) discuss a number of ways in which this could be derived, starting from Hackl-style truth conditions. In particular, they explore a variety of solutions, some of which involve the individual quantifier in these truth conditions taking wide scope with respect to the degree quantifier. The ultimate solution remains elusive, and I have nothing to add to this discussion other than to say that it is possible that our assumptions about $\max$ so far might ultimately need to be revised / cast in a less strong or different form, although that shouldn't affect the core of our main story, which has to do with the alternatives of modified numerals and how they help us account for ignorance and anti-negativity.

## 2.11 A note on maximality and null individuals

Our truth conditions for *less than* and *at most* allow for the possibility that $\max$ could be $0$.

(60) $[\![$Less than four / at most three people quit$]\!]$

$$= 1 \text{ iff } \max(\lambda d \,.\, \exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \{0, 1, 2, 3\}$$

This is somewhat problematic in that it suggest the possibility of null individuals.

A way to avoid this would be to say that what we are measuring is not directly the cardinality of a plurality but rather its rank on some scale. Something along these lines was recently proposed by Bylinina & Nouwen (2018), who argue for allowing in our ontology an entity with zero quantity, and show how this might help us make sense of the interesting semantics of *zero*.

The issue of null individuals is however a complicated and long-standing issue in the literature, and I will not attempt to resolve it here. I will just assume that whatever turns out to be the best solution in the general case should account for this particular case also.

## 2.12 Summary

A crucial step in our analysis of ignorance and anti-negativity in *or* and *some* $NP_{SG}$ was finding the right way to state their truth conditions; once we did that, we were able to derive all their alternatives, both scalar and subdomain, and from them ignorance and anti-negativity. In order to be able to articulate our analysis of CMNs and SMNs we thus also had to first find the right shape to state their truth conditions. This turned out to be a highly non-trivial task, so to our original ignorance and anti-negativity desiderata we effectively added another one: to find a suitable theory of BNs, CMNs and SMNs. We started by observing that a minimum requirement for a theory of BNs, CMNs, and SMNs is to capture their bounding entailments (§2.1). GQT offered a straightforward way to do that but it suffered from shortcomings. We showed how they could be overcome by adopting $\exists x$ truth conditions for BNs (§2.3) and Hackl and Hackl-style/Kennedy truth conditions based on both a degree quantifier ($max(\lambda d \,.\, \dots)$ as well as an individual quantifier ($\exists x$) for CMNs and SMNs (§2.4). The resulting theory of numerals was still not fully compositional at the morphological level. We attempted to make it more so by uncovering the uniform contribution of *much*/*little*, [comp], and [at-sup] for CMNs and SMNs (§2.5). This last step crucially revealed that, while BNs, CMNs, and SMNs all make reference in their truth conditions to a scalar element – the numeral – CMNs and SMNs also make reference to a domain. In a manner analogous to what we did for *or* and *some* $NP_{SG}$ , this observation helped us derive in a natural way the alternatives of BNs, CMNs, and SMNs (§2.6). Last but not least, we showed that the novel pieces of our proposal that helped us get this welcome result connect naturally to the existing literature on comparatives (§2.7), superlatives and *at*-superlatives (§2.8), and *many/few*, *more/fewer*, *most/#fewest* (§2.9), although open issues, not specific to the present proposal, still remain (§2.10, §2.11).

In the next few chapters we will use our proposal regarding the truth conditions and alternatives of bare and modified numerals (§2.6) to resume our discussion of ignorance and anti-negativity in CMNs and SMNs.

Before we get to that, note that, just like for *or* and *some* $NP_{SG}$ , our proposal contains scalar alternatives for all of BNs, CMNs, and SMNs. Thus, we expect all of these items to give rise to scalar implicatures, just as *or* and *some* $NP_{SG}$ did. However, the idea that BNs, CMNs, and SMNs have scalar alternatives has been contested for all three. In what follows we will therefore begin by trying to clarify our take on this issue (Ch. 3). This will not only make the emerging theory of BNs, CMNs, and SMNs more complete but will also be relevant to our original and ultimate goal, since, as we will see right after (Chs. 4-5), scalar implicatures interact in interesting ways with both ignorance and negativity.

# Chapter 3

# Scalar implicatures

We want to make sense of ignorance and anti-negativity in modified numerals the way we did for *or* and *some*. A first step was to gain a good understanding of the truth conditions of modified numerals and to find a principled way to derive their alternatives from them. We found that all of BNs, CMNs, and SMNs contain reference to a scalar element, the numeral, and so we argued that, just like *or* and *some*, that must mean that they activate scalar alternatives obtained by replacing this scalar element in their truth conditions with a scalemate alternative – another numeral. This simple move on our part however has complicated consequences. The reason is because the idea that BNs, CMNs, and SMNs give rise to scalar implicatures has been challenged. Thus we owe a discussion of the issues that have prompted this, and a proposal of how we believe one can deal with them. In what follows we will consider the major challenges to the idea that BNs, CMNs, and SMNs give rise to scalar implicatures. We will show that the scalar implicature view can and (possibly even must) be defended. We will propose that the problem data are not a reason to abandon the scalar implicatures of BNs, CMNs, and SMNs, but rather a reason to better understand their interaction with other factors. (Thus, in addition to our earlier secondary desideratum of trying to find truth conditions for BNs, CMNs, and SMNs from which their alternatives would naturally follow, we now add another one, which is to make sense of some of the puzzles associated with their scalar implicatures.)

## 3.1 The predicted scalar implicatures of BNs

We said in our proposal in §2.6 that BNs have truth conditions and alternatives as below.

(1) Three people quit. (= (37) on p. 73)

a. $\exists x[|x| = 3 \wedge P(x) \wedge Q(x)]$ (assertion)

b. $\{\exists x[|x| = n \wedge P(x) \wedge Q(x)] \mid n \in S\}$ ($\sigma$A )

c. – (no DA )

The scalar alternatives in particular mean that an utterance as the above based on the numeral *three* will activate alternatives based on other numerals, e.g., *two* or *four*. Since in the example above the numeral *three* occurs in an upward-entailing environment, the stronger scalar alternatives will be alternatives based on higher-ranking numerals, i.e., *four*, *five*, etc. Exhaustification relative to the set of scalar alternatives will thus end up negating these alternatives. The result will be that *three* ends up being interpreted as 'exactly three'. All these are illustrated below.

(2) John called three people.

a. $\mathrm{O}_{\sigma\mathrm{A}}\,(\exists x[|x| = 3 \wedge P(x) \wedge C(j, x)])$

$= \exists x[|x| = 3 \wedge P(x) \wedge C(j, x)] \wedge \neg\exists x[|x| = 4 \wedge P(x) \wedge C(j, x)]$

'John called exactly three people.'

The idea that BNs have existential truth conditions, that the numeral in their meaning activates scalar alternatives, and that, through the usual implicature calculation process, this gives rise to negative inferences about the stronger scalar alternatives is entirely classical, and goes back to Horn (1972, 1989).

Horn's scalar implicature view of numerals is based on the observation that while, as we saw in Ch. 2-(1), the lower-bounding inference of BN utterances is non-cancelable, (3), the upper-bounding inference is, (4).

(3) a. John called three people.

b. ¬ The number of people that John called is two or less.

c. John called three people, # if not less.

(4) a. John called three people.

b. ¬ The number of people that John called has is four or more.

c. John called three people, if not more.

Horn's view is further supported by the fact that this cancelability of an inference negating a stronger compatible meaning is not confined to numerals but is true of items such as *or* or *some* also.

(5) a. John called Teddy or Sue.

b. ¬ John called Teddy and Sue.

c. John called Teddy or Sue, if not both.

(6) a. Some students failed.

b. ¬ All students failed.

c. Some students failed, if not all.

Thus, Horn's scalar implicature view of BNs not only captures the fact that one of their bounding inferences is obligatory while the other is optional – it also does so in a way that is elegant and general, and makes the behavior of BNs completely unexceptional and predictable based on the fact that they are just another type of scalar items.

However, especially in the last decade, this view has been challenged. Can the scalar implicature view of BNs be maintained?

## 3.2 Reasons to doubt the scalar implicature view of BNs

The scalar implicature view of BNs has been challenged.

First, it has been argued (most recently, by Kennedy 2013) that the traditional view of scalar implicatures predicts that a low scalar item should be able to lead to a scalar implicature about a higher ranking scalemate only in an upward-entailing environment (because in a downward-entailing environment the higher ranking scalemate would no longer be stronger), but in the sequence below the BN *three* can be interpreted as 'exactly three' (as required by the following discourse) in spite of the fact that it occurs in a downward-entailing environment.

(7) Neither of them read three of the articles on the syllabus. Kim read two and Lee read four.

But (as Kennedy himself notes) this is a serious issue for a scalar implicature account of BNs only if we adopt the traditional (neo-)Gricean view of scalar implicatures on which they are exclusively a root phenomenon. For the past decade or so it has become increasingly more recognized that scalar implicatures can occur at embedded levels also. It was data like these and more that have led to a shift from the traditional Gricean implicature calculation paradigm to the idea that we adopted already in Ch. 1 that implicatures are calculated via silent exhaustivity operators (Chierchia et al. 2012). Thus, using our familiar exhaustivity operator O , examples like the above can be handled as below, by simply inserting O below the negative quantifier. The only comment we might want to add is that such embedded implicature parses are dispreferred – as Chierchia (2013) suggests, possibly due to the fact that, when inserted below negation, O does not actually lead to strengthening, which goes against its prototypical use, which is to strengthen.

(8) $\neg\exists x \in \{k, l\}[\mathrm{O}_{\sigma\mathbf{A}}\,(\exists y[|y| = 3 \wedge A(y) \wedge R(x, y)])]$
$= \neg\exists x \in \{k, l\}[\exists y[|y| = 3 \wedge A(y) \wedge R(x, y)] \wedge \neg\exists z[|z| = 4 \wedge A(z) \wedge R(x, z)]]$

Still, the availability of 'exactly' meanings for BNs in downward-entailing environments is not the only challenge raised against the scalar implicature view of BNs. A more serious challenge comes from the fact that on this view BNs are expected to be just another type of scalar items, but both introspective judgments and experimental data seem to point to a contrast between BNs and other scalar items.

On the introspective judgments side, Horn (1992, 1996) shows that the strong meaning of BNs is more readily available than for other scalar items. This can be seen from the fact that a question about whether *three* can be easily answered with *no, four*, which means that it can be easily understood as *three but not four* / 'exactly three', but a question about whether, e.g., *many*, can't be easily answered with *no, all*, which means that *many* can't as easily be understood as *many but not all*, (9)-(10). (For *many*, the answer *no, all* is acceptable only with a special intonational prominence on *all*.) Similarly, while, as we saw earlier and are showing again below, a BN in the scope of a negative quantifier can easily be interpreted as 'exactly three' (as required by the following discourse), other scalars have a much harder time doing so (and so they fail to meet the requirement imposed by the following discourse, which is why the resulting sentences are degraded), (11)-(12).

(9) A: Did you read three of the articles on the syllabus?
B: No, I read four of them.

(10) A: Did you read many of the articles on the syllabus?
B: ?No, I read all of them.

(11) Neither of them read three of the articles on the syllabus. Kim read two and Lee read four.

(12) ??Neither of them read many of the articles on the syllabus. Kim read one and Lee read them all.

On the experimental side, a long series of psycholinguistic studies including Noveck (2001), Papafragou & Musolino (2003), Musolino (2004), Guasti et al. (2005), Pouscoulous, Noveck, Politzer, & Bastide (2007) or Huang & Snedeker (2009) show that in young children the 'exact' reading of numerals is much more available than the scalar implicature reading of other scalar items. In addition to this, Marty, Chemla, & Spector (2013) show that under memory load adult speakers err toward the strengthened ('exactly') reading for BNs, but not for another scalar item such as *some*. Similarly, Huang, Spelke, & Snedeker (2013) show that, when given an unconscious choice, both adults and children prefer the strengthened ('exactly') reading for BNs but not for *some*. All of these findings essentially make the same point that the strong interpretation is much more available / effortless / enforced for BNs than for other scalar items.

Thus, there is both introspective and experimental data suggesting a difference between BNs and other scalar items. Such data have been used to motivate a series of non-implicature approaches to BNs, including: (1) an underspecification view on which BNs can freely receive an 'at least' or an 'exactly' (or an 'at most') reading depending on which one is the most contextually relevant (Carston 1988, 1998); or (2) an 'exactly'-only view on which the literal meaning of BNs is the 'exactly' one, and the 'at least' (or 'at most' reading) arises through interaction with background, non-linguistic knowledge (Breheny 2008); or (3) an ambiguity view on which numerals are lexically ambiguous between an 'at least' and an 'exactly' reading (Geurts 2006, Nouwen 2010); or (4) an ambiguity view on which numerals have a basic 'exactly' meaning and can acquire an 'at least' meaning via typeshifting (Kennedy 2013, 2015). See Spector (2013) for a discussion of views (1-2) and why they might not be the right way to go. As for views (3) and (4), they are as below. On view (3) – the lexical ambiguity view – Geurts (and following him also Nouwen 2010) suggests that, while on one lexical meaning BNs have existential truth conditions (as on our account), on a minimally different lexical meaning they have existential truth conditions articulated in terms of unique existence, $!\exists$. The latter is an 'exactly' meaning, because to say that there exists a unique plurality with atom count 3 of people who quit is to say that the total number of people who quit is 3.

(13) Three people quit.

a. $\exists x[|x| = 3 \wedge P(x) \wedge Q(x)]$

b. $!\exists x[|x| = 3 \wedge P(x) \wedge Q(x)]$

On view (4) – the ambiguity via typeshifting from 'exactly' into 'at least' view – Kennedy (2015) suggests that numerals have a basic meaning modeled on Hackl (2000)'s meanings for modified numerals, that is, involving maximality. This is an 'exactly' meaning, because to say that 'the maximum degree such that there exists a plurality numerous to that degree of people who quit is equal to 3' is to say that the total number of people who quit is 3. Kennedy then argues that from this meaning a plain degree meaning can be derived by successive application of Partee (1987)'s *BE* and *iota*. This degree meaning is then factored into Hackl's determiner *many*, defined as a parametrized determiner. The result is existential truth conditions yielding the 'at least' meaning.

(14) Three people quit.

a. $\max(\lambda d \,.\, \exists x[|x| = d \wedge P(x) \wedge Q(x)]) = 3$

b. (i) $[\![BE]\!] = \lambda Q_{\langle\langle\alpha,t\rangle,t\rangle} \,.\, \lambda x_\alpha \,.\, Q(\lambda y_\alpha \,.\, y = x)$

$[\![BE]\!]\,([\![\text{three}]\!])$

$= \lambda x_d \,.\, x = 3$

(ii) $[\![\textit{iota}]\!] = \lambda P_{\langle\alpha,t\rangle} \,.\, \iota z_\alpha[P(z)]$

$[\![\textit{iota}]\!]\,(\lambda x_d \,.\, x = 3)$

$= 3$

(iii) $[\![\textit{many}]\!]\,(3) = \lambda P_{\langle e,t\rangle} \,.\, \lambda Q_{\langle e,t\rangle} \,.\, \exists x[|x| = 3 \wedge P(x) \wedge Q(x)]$

Should we then abandon the scalar implicature view of BNs and adopt a view like that of Geurts (2006) or Kennedy (2015)?

## 3.3 Reasons to reaffirm the scalar implicature view of BNs

Note that the challenges to a scalar implicature view of BNs presented just now, and which were used to motivate departures from this view, do not have to mean that BNs are *altogether* different from other types of scalar items – they just show that for BNs scalar implicatures are for some reason more of a default than for other scalar items. Indeed, Spector (2013) suggests that they might be intrinsically focused, an idea supported by the fact that they don't seem to require stress in contexts where, in order to be felicitous, other elements do (cf. the question-pair and the negative quantifier examples we saw just now). Or it may well be that the scale is much more automatic and salient for numerals than for other items.[1] This is of course no more than the sketch of an explanation. Still, it shows that, while the facts brought forward by the literature cited above must not be neglected, their conclusion does not have to lead to an abandonment of the idea that the 'exactly' meaning of BNs arises through implicature.

Moreover, while the debate on the meaning of BNs brought to light difficulties for a scalar implicature account, it also brought to light supporting evidence. Barner & Bachrach (2010) show that children

[1]Recall our discussion in Fn. 5 on p. 22 of how children interpret *or* as *and* as if they were not computing a $\sigma$A -implicature, and also the hypothesis advanced in the literature that this might be because they have not yet learned that *and* is a scalar alternative to *or*. This suggests that scales are learned, and some are more automatically or reliably recognized as such than others.

who interpret as exact numerals up to $n$ also have knowledge of $n+1$. This finding is consistent with the scalar implicature story, where consideration of higher ranked numerals is needed to obtain the 'exactly' meaning. Then, Panizza, Chierchia, & Clifton (2009) found via both off-line and on-line measures that: (1) adult speakers compute an 'exactly' meaning more frequently in upward-entailing rather than in a downward-entailing environment; (2) reading times of the numeral were longer in an upward-entailing environment, consistent with the idea that this meaning is obtained via implicature and that implicature calculation, no matter how automatic, is still costly; (3) with a following context requiring an 'exactly' reading of a preceding numeral, regressions to the numeral were more frequent if the numeral occurred in a downward-entailing rather than an upward-entailing environment, consistent with the idea that the 'exactly' reading is obtained via scalar implicature and with the general observation that scalar implicatures, whose primary purpose is to lead to strengthening, are computed more readily in contexts where they would lead to a stronger meaning, thus, in upward-entailing rather than downward-entailing contexts. These findings not only support the scalar implicature view of BNs but also pose a challenge to the non-implicature accounts of BNs, on which none of these subtle patterns are expected.

Last but not least, there are also introspective judgment data that pose a pretty serious challenge to non-implicature accounts. These come from BN utterances like the one below, where *if three* can easily lead to an indirect scalar implicature of the form *...but not if two*. (In the terminology of Chierchia 2004, a *direct* scalar implicature is one where the strength relation between a numeral and its alternatives is determined based on just the numeral's position on a scale – for BNs, one where *three* gives rise to implicatures about larger numerals – while an *indirect* scalar implicature is one where the strength relation between the numeral and its alternatives is determined based on the numeral in interaction with some negative operator, which reverses the natural direction of entailment of a scale – for BNs, one where *three*, in interaction with some negative operator, gives rise to implicatures about smaller numerals.)

(15) If John called three people, he won. $\rightsquigarrow$ But not if he called two.

On a scalar implicature view of BNs, this implicature is entirely predicted. The BN occurs in a downward-entailing environment, so if the implicature is computed at the top (not at an embedded level), the scalar alternatives that come out stronger than the BN prejacent are not alternatives based on larger numerals but rather on smaller numerals (because downward-entailing environments reverse the direction of entailment). The result of computing an implicature relative to the scalar alternatives is then as below.

(16) $\mathrm{O}_{\sigma\mathbf{A}}\,(\forall w \in \mathrm{Acc}[\exists x[|x|_w = 3 \land P_w(x) \land C_w(j,x)] \rightarrow W_w(j)])$
$= \forall w[\exists x[|x|_w = 3 \land P_w(x) \land C_w(j,x)] \rightarrow W(j)] \land \neg\forall w[\exists x[|x|_w = 2 \land P_w(x) \land C_w(j,x)] \rightarrow W_w(j)]$

However, on a non-implicature view of BNs this implicature is completely unexpected. That is because, while all these accounts offer a way for a numeral like *three* to have both an 'at least three' and an 'exactly three' meaning, none can account for this case where *three* in a downward-entailing environment can give rise to an inference that *not two*.

The issue is further amplified by the fact that cases like these, that are highly problematic for non-implicature accounts, are not limited to the antecedent of a conditional, but are generally true of cases where the BN occurs in a downward-entailing environment. As Spector (2013:279-80) shows, other such cases include (but are of course not limited to) the restriction of a universal quantifier or the scope of a possibility modal itself in the scope of negation. We copy his examples below.

(17) Every student who solved three problems passed. $\rightsquigarrow$ But not every student who solved two.

(18) In this country, one is not allowed to have three children. $\rightsquigarrow$ But one is allowed to have two.

To sum up, there has been a long and rich debate on the meaning of BNs. The main conclusions seem to be as follows. On the one hand, any scalar implicature account of BNs must explain why their scalar implicatures are more automatic / enforced in all environments than for other scalars. On the other hand, a non-implicature account of BNs is by design inadequate as it is unable to explain why their 'exactly' meaning is sensitive to the monotonicity of the environments where they appear or to capture their indirect scalar implicatures, namely, the fact that they can give rise to scalar implicatures not just about higher but also about lower ranking scalar alternatives.

Overall, it seems that our entirely classical idea that BNs give rise to scalar implicatures can, and perhaps should, be maintained. And, just like the classical account, it has the advantage that it captures a fundamental similarity between *or*, *some*, and BNs – they all entail a weak meaning compatible with a stronger meaning (*and*, *all*, *four/five/...*) but rule out that stronger meaning via implicature.

Before we continue, we should also mention one new difficulty for the scalar implicature view of BNs. (This is a difficulty that will reoccur in our discussion of scalar implicatures for CMNs and SMNs also, and we will eventually propose a unified solution.) This difficulty has to do with the fact that, while a scalar implicature account gave us the welcome indirect scalar implicature in (15), it also gives us the unwelcome indirect scalar implicature below.

(19) John didn't call three people. #$\leadsto$ He called two.

This arises from the fact that a prejacent such as *not three* has as a stronger scalar alternative *not two*, which the exhaustification process will have to deny. The negation of this alternative together with the assertion yields an 'exactly two' meaning.

(20) $\mathrm{O}_{\sigma\mathbf{A}}\,(\neg\exists x[|x| = 3 \wedge P(x) \wedge C(j,x)])$
$= \neg\exists x[|x| = 3 \wedge P(x) \wedge C(j,x)] \wedge \neg(\neg\exists x[|x| = 2 \wedge P(x) \wedge C(j,x)])$
$= \exists x[|x| = 2 \wedge P(x) \wedge C(j,x)] \wedge \neg\exists x[|x| = 3 \wedge P(x) \wedge C(j,x)]$
# 'John called exactly two people.'

This is a long-standing issue in the implicature approach to BNs, and one that we will discuss again in a bit. In the meantime, notice that this problem arises only for a scale with granularity 1. If instead of *not two* we assume that the next strongest scalar alternative to *not three* is *not one*, the implicature is again just as expected.

(21) $\mathrm{O}_{\sigma\mathbf{A}}\,(\neg\exists x[|x| = 3 \wedge P(x) \wedge C(j,x)])$
$= \neg\exists x[|x| = 3 \wedge P(x) \wedge C(j,x)] \wedge \neg(\neg\exists x[|x| = 1 \wedge P(x) \wedge C(j,x)])$
$= \neg\exists x[|x| = 3 \wedge P(x) \wedge C(j,x)] \wedge \exists x[|x| = 1 \wedge P(x) \wedge C(j,x)]$

'John didn't call three people, but he did call one.'

## 3.4 The predicted scalar implicatures of CMNs and SMNS

We argued that the classical scalar implicature view of BNs can be maintained.

But upholding the classical view opens one up to classical problems. As Krifka (1999) points out, if the presence of a numeral activates scalar alternatives in BNs, it should do so in CMNs and SMNs also – again, just as we argued in §2.6 and are repeating below.

(22) More/less than three people quit. (= (38) on p. 73)

a. $\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \overline{[\![\text{much/little}]\!]\,(3)}$ (assertion)

b. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \overline{[\![\text{much/little}]\!]\,(n)} \mid n \in S\}$ ($\sigma$A )

c. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in D' \mid D' \subset \overline{[\![\text{much/little}]\!]\,(3)}\}$ (DA )

(23) At least/most three people quit. (= (39) on p. 73)

a. $\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in [\![\text{much/little}]\!]\,(3)$ (assertion)

b. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in [\![\text{much/little}]\!]\,(n) \mid n \in S\}$ ($\sigma$A )

c. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in D' \mid D' \subset [\![\text{much/little}]\!]\,(3)\}$ (DA )

However, if CMNs and SMNs activate scalar alternatives, just like BNs, then they should give rise to scalar implicatures, also just like BNs. But this prediction seems to yield the wrong results because, as shown below, through entirely parallel reasoning as for BNs, all of our modified numerals should give rise to 'exactly' meanings also, yet this is not how we use them at all. (Note: *more than three* and *at least three* are upward-entailing, so in an upward-entailing environment as below their stronger scalar alternatives are based on numerals larger than three; *less than three* and *at most three* are downward-entailing, so in an upward-entailing environment as below their stronger scalar alternatives are based on numerals smaller than three.)

(24) $\mathrm{O}_{\sigma\mathrm{A}}$ (John called more than three people)

$= \max(\lambda d\,.\,\exists x[|x| = d \ldots]) \in \underbrace{\overline{[\![\text{much}]\!]\ (3)}}_{=\{4,5,\ldots\}} \wedge \neg \max(\lambda d\,.\,\exists x[|x| = d \ldots]) \in \underbrace{\overline{[\![\text{much}]\!]\ (4)}}_{=\{5,6,\ldots\}}$

# 'John called exactly four people.'

(25) $\text{O}_{\sigma\text{A}}$ (John called less than three people)

$= \max(\lambda d\,.\,\exists x[|x| = d \ldots]) \in \underbrace{\overline{[\![\text{little}]\!]\ (3)}}_{=\{0,1,2\}} \wedge \neg \max(\lambda d\,.\,\exists x[|x| = d \ldots]) \in \underbrace{\overline{[\![\text{little}]\!]\ (2)}}_{=\{0,1\}}$

# 'John called exactly two people.'

(26) $\text{O}_{\sigma\text{A}}$ (John called at least three people)

$= \max(\lambda d\,.\,\exists x[|x| = d \ldots]) \in \underbrace{[\![\text{little}]\!]\ (3)}_{=\{3,4,\ldots\}} \wedge \neg \max(\lambda d\,.\,\exists x[|x| = d \ldots]) \in \underbrace{[\![\text{little}]\!]\ (4)}_{=\{4,5,\ldots\}}$

# 'John called exactly three people.'

(27) $\text{O}_{\sigma\text{A}}$ (John called at most three people)

$= \max(\lambda d\,.\,\exists x[|x| = d \ldots]) \in \underbrace{[\![\text{much}]\!]\ (3)}_{=\{0,1,2,3\}} \wedge \neg \max(\lambda d\,.\,\exists x[|x| = d \ldots]) \in \underbrace{[\![\text{much}]\!]\ (2)}_{=\{0,1,2\}}$

#'John called exactly three people.'

How might we handle this?

## 3.5 Ways to get rid of the scalar implicatures of CMNs and SMNs

In response to data like those reviewed just now, there is a consensus in the literature that (at least in episodic contexts)[2] one must get rid of scalar implicatures for CMNs and SMNs. A variety of solutions have been proposed. We will review the main approaches below.

A first approach represented by Krifka (1999), Geurts & Nouwen (2007), and Coppock & Brochhagen (2013) says that the scalar implicatures of CMNs and SMNs are absent due to the modifier in their meaning. The idea is as follows: The modifier makes it such that the CMN or SMN utterance ends up denoting the union of those of the scalar alternatives to the assertion that stand in a particular ordering

[2]The more recent theories of modified numerals recognize the fact that some form of implicatures must be derived for modified numerals embedded under a universal modal, but except for Fox & Hackl (2006)'s analysis of CMNs, no theory actually attempts to derive this from the classical scalar alternatives.

relation to it (strict for CMNs, non-strict for SMNs). We can illustrate this as in Krifka.

(28) ($[\![\alpha_A]\!]$ = the alternative value of $\alpha$, defined in terms of a strength relation over alternatives)

$$
\begin{aligned}
[\![\text{more than } \alpha]\!] &= \bigcup\{\beta \mid \langle\alpha,\beta\rangle \in [\![\alpha_A]\!]\} \\
[\![\text{more than [three people]}]\!] &= \lambda x\,.\,[|x| > 3 \wedge \text{people}(x)]
\end{aligned}
$$

$$
\begin{aligned}
[\![\text{less than } \alpha]\!] &= \bigcup\{\beta \mid \langle\beta,\alpha\rangle \in [\![\alpha_A]\!]\} \\
[\![\text{less than [three people]}]\!] &= \lambda x\,.\,[|x| < 3 \wedge \text{people}(x)]
\end{aligned}
$$

$$
\begin{aligned}
[\![\text{at most } \alpha]\!] &= \bigcup\{\beta \mid \langle\beta,\alpha\rangle \in [\![\alpha_A]\!]\} \\
[\![\text{at most [three people]}]\!] &= \lambda x\,.\,[|x| \leq 3 \wedge \text{people}(x)]
\end{aligned}
$$

$$
\begin{aligned}
[\![\text{at least } \alpha]\!] &= \bigcup\{\beta \mid \langle\alpha,\beta\rangle \in [\![\alpha_A]\!]\} \\
[\![\text{at least [three people]}]\!] &= \lambda x\,.\,[|x| \geq 3 \wedge \text{people}(x)]
\end{aligned}
$$

The explanation for the missing scalar implicatures of CMNs and SMNs is then as follows: Krifka and Geurts & Nouwen use a classical notion of union as a result of which the set of scalar alternatives is collapsed to a simple property (as shown above); this means that the alternatives no longer exist, and thus no implicatures are derived. Coppock & Brochhagen use an Inquisitive Semantics notion of union as a result of which the alternatives are in fact preserved; to ensure that these alternatives do not give rise to scalar implicatures in episodic contexts, the authors also modify the definition of exhaustification such that none of the alternatives thus generated can be excluded, and the result is again no scalar implicatures.

A second approach represented by Fox & Hackl (2006) also says that the scalar implicatures of CMNs (their solution doesn't work for SMNs, see below) are absent due to the fact that the reference scale is always dense, obeying a principle called the Universal Density of Measurement (UDM): For any $n$ and $n + \epsilon$ there is a degree $n + \delta$ such that $n < n + \delta < n + \epsilon$ (Fox & Hackl 2006:542). This assumption indeed predicts that trying to compute a scalar implicature for a CMN in an episodic context will always lead to a crash, as shown below.

(29) ⟦John weighs more than 120 pounds⟧

a. $= 1$ iff John weighs $120 + \epsilon$ pounds.

b. If John weighs $120 + \epsilon$ pounds then he weighs more than $120 + \epsilon/2$ pounds. Thus, there is a degree $d > 120$ such that John weighs more than $d$ pounds.

c. UDM: The stronger scalar alternatives to the assertion are propositions of the form *John weighs more than n pounds*, with $n > 120$. These should give us scalar implicatures of the form *It is not the case that John weighs more than n pounds* for all $n > 120$. Among other things, we should get an implicature that *It is not the case that John weighs more than* $120 + \epsilon/2$ *pounds*. This would contradict the entailment in (b), and so it would be incompatible with the assertion.

However, as shown in Mayr (2013), the exact same reasoning would in fact yield scalar implicatures for SMNs, contrary to what we want.

(30) ⟦John weighs at least 120 pounds⟧

a. $= 1$ iff John weighs $120$ or $120 + \epsilon$ pounds.

b. If John weighs $120 + \epsilon$ pounds then he weighs at least $120 + \epsilon/2$ pounds. Thus, there is a degree $d > 120$ such that John weighs at least $d$ pounds.

c. UDM: The stronger scalar alternatives to the assertion are propositions of the form *John weighs at least n pounds*, with $n > 120$. These should give us scalar implicatures of the form *It is not the case that John weighs at least n pounds* for all $n > 120$. Among other things, we should get an implicature that *It is not the case that John weighs at least* $120 + \epsilon/2$ *pounds*. This would contradict (b) but it would not contradict the assertion – it would merely strengthen it to the meaning that John weighs exactly $120$ pounds.

Fox & Hackl (2006) are aware of this asymmetric prediction but suggest that for SMNs something else might be responsible for the absence of their scalar implicatures. (At the time what they had in mind was

Geurts & Nouwen 2007's analysis of SMNs.)

Finally, a third approach represented by Büring (2008), Mayr (2013), Kennedy (2015), Spector (2015), Schwarz (2016), Buccola & Haida (2017) says that CMNs and SMNs do not give rise to scalar implicatures because, contrary to the traditional expectation, they do not in fact activate scalar alternatives of the classical sort, but rather a different kind of alternatives. The exact alternatives that are proposed differ quite a bit from one theory to another, and the theories also vary with respect to whether they do this explicitly for just SMNs or (can be extended to) also CMNs (it is usually just SMNs). Nonetheless, these new alternatives are always designed to be symmetric, which in the jargon of alternative-based theories means that they are such that they cannot all be excluded at the same time without contradiction. To be more concrete, they are formally similar to the domain alternatives of a disjunction, whose concomitant exclusion led to contradiction: $\mathrm{O}_{\mathrm{DA}}\ (p \vee q),\ = \ (p \vee q) \wedge \neg p \wedge \neg q,\ = \ \bot$. There are multiple ways in which one can generate such alternatives for modified numerals, but a popular choice, especially for SMNs, has been to say that an SMN such as, e.g., *at least three* activates the alternatives *exactly three* and *more than three/at least four* (cf. all of Büring 2008, Kennedy 2013, 2015, Spector 2015, Schwarz 2016, Buccola & Haida 2017; theories differ, though, in how they justify these alternatives). The net result is as we just saw for disjunction: Trying to exclude these alternatives via implicature leads to a crash, so no scalar implicatures arise.

Although very different, all of these approaches uniformly predict that modified numerals in episodic contexts are unable to give rise to scalar implicatures. This seems to take care of the bad prediction we made before. Should we adopt one of these solutions as well? Could it be that the numeral activates scalar alternatives in all of BNs, CMNs, and SMNs, but in the latter two they are activated only for these to be systematically neutralized, or could it be that the numeral activates scalar alternatives in BNs but not in CMNs and SMNs? Should we abandon the scalar implicature view of CMNs and SMNs?

## 3.6 Reasons to reaffirm the scalar implicature view of CMNs and SMNs

If disjunction or existential quantifiers gave rise to no scalar implicatures, then an utterance of *John called Teddy or Sue* / *John called some student* should simply convey a disjunctive / existential meaning, with no hint that the conjunctive / universal meaning that it is compatible with might not in fact be the case. This is clearly not how we use *or* and *some* $NP_{SG}$ . For that reason it is commonly accepted that these items do in fact give rise to scalar implicatures. But, although suppressing the scalar implicatures doesn't seem to be a good idea for *or* or *some*, might it be so for CMNs and SMNs, as the data and literature review from the previous section would suggest?

Let us consider the empirical consequences of such a move through the same lens as for *or* or *some* $NP_{SG}$ . If an utterance such as *The reading assignments for this week total more than 100 / at least 100 pages* gave rise to no scalar implicatures, then we should only get from it its lower-bounding entailment, with no hint that the speaker probably didn't in fact mean, for example, 1,000 pages. And, completely analogously, if an utterance such as *The reading assignments for this week total less than 100 / at most 100 pages* gave rise to no scalar implicatures, it should only carry the meaning that the intended number is within the mentioned upper bound, but give us no clue that it is not, for example, zero. None of these consequences of removing scalar implicatures from the picture seem desirable. Thus we seem to want scalar implicatures for CMNs and SMNs no less than we wanted them for *or* or *some* $NP_{SG}$ .

But it is conceivable that these unwanted consequences for CMNs and SMNs can be filtered out in other ways. For example, it could be said that, for upward-entailing modified numerals, the 1,000 case can be ruled out simply by general considerations of relevance, without recourse to any particular scalar alternatives. And, for the downward-entailing modified numerals, the zero case can be ruled out by saying that they come with an existential presupposition (as suggested, e.g., by Krifka 1999) or some general sort of implicature unrelated to scalar alternatives.

However, while it is very likely that relevance (and, as we will see shortly, roundness, namely the fact that 100 is in the order of hundreds rather than thousands) plays a role in making 1,000 unlikely, this is not specific to *more than* or *at least* but presumably happens with BNs also – when we hear *three* we do not really wonder why the speaker didn't utter *a hundred*. Thus, it makes more sense to say that, just as for BNs, this happens not through some reasoning independent of the scalar alternatives but rather through reasoning directly related to them, that is, through pruning of the infinite set of scalar alternatives of numerals using considerations of relevance.

As for the *not zero* inference of *less than* and *at most*, it can't be a presupposition because it is defeasible: The *at most 20* assertion below can be followed by a *zero* continuation without giving rise to ungrammaticality (cf. Alrenga 2016 and references therein).

(31) a. LeBron scored at most 20 points in last night's game.
 b. $\neg$ LeBron scored zero points in last night's game.
 c. LeBron scored at most 20 points in last night's game, and it's even possible that he didn't score any points at all.

And, while it is conceivable that this inference that *not zero* could also be due to some other general form of implicature unrelated to scalar alternatives, it is much more economical to think that it is no different than the positive implicature of negative quantifiers such as *few*, illustrated below. *Few* comes with a Horn-scale of the form $\langle$ not all, few, no(ne) $\rangle$ (Horn 1989), so an utterance of *few* has as a stronger scalar alternative an utterance of *no(ne)*. Through the usual reasoning we conclude that this alternative must be false – *not none* = *some*. As is usually the case with implicatures, this positive inference is cancelable.

(32) a. Few students passed.
 b. $\neg$ No students passed. = Some did.
 c. Few students passed. In fact, none did.

To sum up, it makes sense to derive the *not 1,000* and the *not zero* inferences of the utterances above as scalar implicatures.

This makes sense all the more since, unlike what one might have suspected from the examples above, such implicatures are not just about eliminating extreme values. Spector (2014:42) shows that in a context like the one below an utterance of *John solved more than five problems* can easily give rise to an implicature that he didn't solve more than nine (so he gets a B). And the same observation can be replicated for the other modified numerals also.

(33) *Context: Grades are attributed on the basis of the number of problems solved. People who solve between one and five problems get a C. People who solve more than five problems but fewer than nine problems get a B, and people who solve 9 problems or more get an A.*
John solved more than five problems. Peter solved more than nine.
$\rightsquigarrow \neg$ John solved more than nine.

All in all, it would seem that CMNs and SMNs in episodic contexts are in fact able to give rise to scalar implicatures. The existing approaches to scalar implicatures in modified numerals reviewed earlier, which were explicitly designed to avoid such scalar implicatures from arising, have no way to capture this.

Neither the data above nor the conclusion that the existing theories of CMNs and SMNs fail to capture them are novel – they have already been pointed out in, e.g., Cummins, Sauerland, & Solt (2012).

Cummins et al. additionally provide experimental evidence that utterances with CMNs and SMNs do in fact involve scalar reasoning. They showed participants a statement involving a modified numeral, e.g., *More than 100 people attended the public meeting about the new highway construction project*, and asked them to answer the question, *Based on reading this, how many people do you think attended the meeting?* In a between-participants design, they collected responses of the form *Between __ and __ people attended* (what they called the 'range' condition) and __ *people attended* (what they called the 'single number' condition). In providing their answers participants were encouraged to give comments explaining their reasoning. These comments were generally of the form "I feel that if there was more than 150, the newspaper would say more than 150" or "I chose the above number because I felt had the numbers been higher the paper would have said more than 200." As Cummins et al. point out, this suggests that in picking their number they reasoned with stronger scalar alternatives.

Of course, it is possible that in making their choice participants did not in fact consider scalar alternatives in the strict grammatical sense, i.e., perhaps for *more than 100* they did not in fact consider alternatives of the form *more than* $n$ but simply $m$, some number larger than 100. However, Cummins et al. made another interesting finding. The numbers participants chose in their answers directly drew on some property of the actual modified numeral provided in the assertion. This was reflected in the fact that in guessing the intended number they often picked a number of the same roundness. For example, if the assertion was *more than 100*, they estimated values up to 150, as if computing an implicature of the form 'not more than 125' or 'not more than 150', picking on a scale that included the assertion and had some granularity relevant to the assertion, i.e., 25 or 50. Or, if *more than 110*, they often picked values up to 120, as if computing an implicature of the form 'not more than 115' or 'not more than 120', drawing on a scale with granularity 5 or 10, as suggested by the roundness of the numeral in the assertion, Or, if the assertion was *more than 93*, values up to 100, as if computing an implicature of the form 'not more than 95' or 'not more than 100', drawing on alternatives from a scale where the alternative immediately stronger than the assertion was at the next mid-decimal or decimal point. Thus, although participants varied in the granularities they were assuming for the underlying scale (i.e., 25 or 50 for an utterance involving 100), the roundness of the numeral always had some influence on the estimates, which suggests that the alternatives that participants considered were always modeled on the assertion, as we might expect if CMNs and SMNs had classical scalar alternatives.

Thus, it would seem that CMNs and SMNs can give rise to scalar implicatures in episodic contexts also.

## 3.7 The predicted scalar implicatures don't arise only when they would overgenerate 'exactly' meanings

Notice that a crucial difference between our examples of failed scalar implicatures at the outset and the successful scalar implicatures above was the fact that in the former case we considered scalar alternatives that were immediately stronger than the assertion on a scale with granularity 1 – e.g., ⟨ ..., more than three, more than four, ...⟩ – but in the latter case we considered scalar alternatives that were stronger on a scale with varying non-1 granularities – e.g., ⟨ ..., more than five, more than nine, ...⟩.

On the one hand, this observation is interesting because it reminds us of a similar case we saw with BNs, where a BN under negation was predicted to give rise to an unattested scalar implicature, cf. (20) above, and where we also noticed that changing the granularity to some other value seemed to make the scalar implicatures fine again, cf. (21) above. This reinforces our impression so far that the ultimate key to the scalar implicatures of CMNs and SMNs does not lie with abandoning the classical story but rather with understanding it better.

On the other hand, this observation also means that, if we want to have a proper understanding of the scalar implicatures of all of BNs, CMNs, and SMNs, we need to understand why they are sometimes restricted in cases where the scale has granularity 1.

But before we even try to sketch an explanation: Is it really a scale with granularity 1 that is the problem?

Consider the case of a bare or modified numeral in the scope of a universal operator. Here all the scalar implicatures based on a scale with granularity 1 seem fine. (These implicatures are for the most part acknowledged in the literature on numerals, but they are typically derived in other ways and not from the classical scalar alternatives. We will discuss this again in Ch. 4.)

(34) $O_{\sigma A}$ (John is required to call three / more than three / less than three / at least three / at most three people.)
$\rightsquigarrow \neg$ John is required to call four / more than four / less than two / at least four / at most two people.

(35) $O_{\sigma A}$ (Everyone called three / more than three / less than three / at least three / at most three people.)
$\rightsquigarrow \neg$ Everyone called four / more than four / less than two / at least four / at most two people.

The same is true of the (indirect) scalar implicatures of bare and modified numerals in the antecedent of a conditional or the restriction of a universal. (Note: These are downward-entailing environments.

Thus, the stronger scalar alternative to, e.g., *three* here is *two*.) (Also note: There are reports, e.g., cf. Cohen & Krifka 2014, that modified numerals in the antecedent of a conditional or the restriction of a universal may be sensitive to the positivity/negativity of the continuation. This is not crucial here, but, in order to make the sentences below as good as possible, we allow the reader to choose between a positive and a negative continuation. We will discuss this further in Ch. 5.)

(36) $O_{\sigma A}$ (If John called three / more than three / less than three / at least three / at most three people, he won/lost.)
$\rightsquigarrow \neg$ If John called two / more than two / less than four / at least two / at most four diamonds, he won/lost.

(37) $O_{\sigma A}$ (Everyone who called three / more than three / less than three / at least three / at most three people won/lost.)
$\rightsquigarrow \neg$ Everyone who called two / more than two / less than four / at least two / at most four people won/lost.

In all these cases we were able to obtain scalar implicatures in spite of the fact that the relevant scale had a granularity of 1, just like our original problem cases. What is different about these new examples?

Recall that in all the problem cases, including our original problem case for negated BNs in (20), the reason we didn't want the scalar implicatures that came out of a scale with granularity 1 was because they led to 'exactly' meanings that were intuitively wrong – e.g., *more than three* ended up implicating *not more than four* which together with the assertion gave rise to the meaning 'exactly four', or *not three* gave rise to an implicature that *two*, which together with the assertion ended up meaning 'exactly two'. But, due to the nature of the universal modal and of the conditional, in the contexts above this did not happen. It seems then that it is not having a scale with granularity 1 in itself that is the problem, but rather the 'exactly' meanings that it overgenerates.

That it is the overgeneration of 'exactly' meanings that underlies all the bad cases can be verified for other configurations also. If in any of the embedded numeral cases above we compute the implicature at an embedded level, we obtain the same 'exactly' meanings as in the unembedded case, and the same pattern where they are fine for a bare BN[3] but not for a CMN or SMN. (Note: This is unsurprising, since if we compute the implicature below the embedding operator, the prejacent for $O_{\sigma A}$ is the same as in the unembedded case.)

(38) John is required $O_{\sigma A}$ (to call three / more than three / less than three / at least three / at most three people.)
$\rightsquigarrow$ John is required $\neg$ to call four / more than four / less than two / at least four / at most two people.
'John is required to call exactly three / #exactly four / #exactly two / #exactly three / #exactly three people.'

(39) Everyone$_1$ $O_{\sigma A}$ ($t_1$ called three / more than three / less than three / at least three / at most three people.)
$\rightsquigarrow$ Everyone called $\neg$ four / more than four / less than two / at least four / at most two people.
'Everyone called exactly three / #exactly four / #exactly two / #exactly three / #exactly three people.'

(40) If $O_{\sigma A}$ (John called three / more than three / less than three / at least three / at most three people), he won/lost.
$\rightsquigarrow$ If $\neg$ John called four / more than four / less than two / at least four / at most two people, he won/lost.
'If John called exactly three / #exactly four / # exactly two / #exactly three / #exactly three people, he won/lost.'

(41) Everyone $O_{\sigma A}$ (who called three / more than three / less than three / at least three / at most three people) won/lost.
$\rightsquigarrow$ Everyone who $\neg$ called four / more than four / less than two / at least four / at most two people won/lost.

[3] Although perhaps less salient than the implicatures we obtained earlier by applying $O_{\sigma A}$ at matrix level.

'Everyone who called exactly three / #exactly four / # exactly two / #exactly three / #exactly three people won/lost.'

And the same is true of a bare or a modified numeral under negation – whether we exhaustify at the matrix level or at the embedded level, if we get an 'exactly' meaning for anything other than a plain BN on its own, that 'exactly' meaning is undesirable. (Note: As mentioned in Ch. 1, SMNs are degraded under negation. I am including them in the discussion all the same, for a complete picture of the predicted scalar implicatures.)

(42) $O_{\sigma A}$ (John didn't call three / more than three / less than three / #at least three / #at most three people.)
$\rightsquigarrow$ $\neg$ John didn't call two / more than two / less than four / at least two / at most four people.
'John called #exactly two / #exactly three / #exactly three / #exactly two / #exactly four people.'

(43) John didn't $O_{\sigma A}$ (call three / more than three / less than three / #at least three / #at most three people).
$\rightsquigarrow$ John didn't $\neg$ call four / more than four / less than two / at least four / at most two people wins/loses.
'John didn't call exactly three / #exactly four / # exactly two / #exactly three / #exactly three people.'

Thus, the real issue is not a scale with granularity 1 but rather the fact that it sometimes leads to unwanted 'exactly' meanings. And the reason why altering the granularity helps is because it helps avoid generating the unwanted 'exactly' meanings. We already saw that for negated BNs (cf. (21)) and for CMNs and SMNs in episodic contexts with granularity more than 1. We show it again below for all of BNs, CMNs, and SMNs, for the negative case. Sure enough, a scale with a coarser granularity helps us avoid the overgeneration of 'exactly' meanings, and the result is fine.

(44) $O_{\sigma A}$ (John didn't call three / more than three / less than three / #at least three / #at most three people.)

$\rightsquigarrow \neg$ John didn't call one / more than one / less than five / at least one / at most five people.

'John didn't call three / more than three / less than three / #at least three / #at most three diamonds, but he did call one / more than one / less than five / at least one / at most five.'

To conclude, CMNs and SMNs can in principle *always* give rise to the classical scalar implicatures, except when that would result in an 'exactly' meaning. This issue does not seem to be specific to modified numerals – it arises in BNs too, where we also find that an 'exactly' result is sometimes undesirable, namely when it arises from a negated BN rather than from a BN on its own. The real issue we need to tackle then is not why granularity 1 is sometimes bad but rather how the unwanted 'exactly' meanings are avoided. How might we do that?

## 3.8 Ways to explain why we sometimes don't get the predicted 'exactly' implicatures

A tempting possibility could be to say that there is some sort of competition going on between plain BNs on the one hand and negated BNs and CMNs and SMNs that would ban the generation of an 'exactly' meaning for anything other than a plain BN. However, it is not obvious how to state such a competition, and it also feels stipulative.

Another, more principled, possibility would be to recall the close interaction between ignorance and scalarity in *or* and *some* $NP_{SG}$ and suggest that the reason why we don't get the predicted 'exactly' meanings is because of some clash between ignorance and certainty. The interaction between ignorance and scalarity in *or* and *some* $NP_{SG}$ that we discussed in Ch. 1 was somewhat different than it will be in numerals, so the comparison is not necessarily straightforward. Still, consider this: We know that CMNs and SMNs give rise to ignorance. Now, if, e.g., *at least three* indicated speaker ignorance about, say, *three*, then this would clash with a scalar implicature that would yield certainty about *three*, and, in the absence of pruning, such a parse should fail. The same would be true of CMNs. This then looks like a promising solution. We will explore it further in §4.2.

But so far we haven't said anything about BNs giving rise to ignorance, so how could this approach work for them? Note that although a BN in a plain episodic context indeed does not give rise to ignorance, a BN under negation may well do. The most natural context in which I might say *Alice doesn't have three diamonds* is a context of the form *I'm not sure how many diamonds Alice has, but she doesn't have three*. Negation thus seems to automatically give rise to an ignorance effect. Note moreover that this seems to be true not just of numerals but of other cases also. For example, if I am asked *Who came to the party?* and I answer *Not John*, that again most naturally indicates that I'm not sure who did come (and also fails to give rise to a scalar implicature that everyone did, as we would predict if we reasoned with $O_{\sigma A}$ in the usual way and negated all the non-entailed alternatives). Thus it seems plausible that ignorance is in some way the problem in this negation case also, although it might be of a different sort than in the episodic case. We will explore this further in §5.2.

## 3.9 Summary

To sum up, our proposal in §2.6 that BNs, CMNs, and SMNs activate scalar alternatives of the classical sort predicts scalar implicatures for all of BNs, CMNs, and SMNs.

The literature on numerals has contested these implicatures for all of these items. We have reviewed some of the main reasons against these implicatures, and existing proposals in the literatures that do away with them, but also presented reasons why these classical scalar implicatures might in fact be welcome, or even necessary, just as predicted on our (otherwise entirely classical) account.

However, if we reaffirm the scalar implicatures of BNs, CMNs, and SMNs, certain old issues reappear. First, we have to explain why the strong, 'exactly', meaning of BNs is so much more available than the strong meaning of other scalar items. We suggested that a plausible hypothesis might be that numerals are intrinsically focused, or that the scale in their case is better known and therefore more automatic and salient. Second, we have to explain why all of BNs, CMNs, and SMNs sometimes seem to not give rise to scalar implicatures. We showed that all the problem cases are cases where an 'exactly' meaning is generated for prejacents other than a plain BN. We suggested that the reason these 'exactly' meanings are ruled out may have to do with ignorance effects in episodic or negative contexts. We will discuss this further in the next two chapters, where we tackle the main two goals of our thesis, namely ignorance and anti-negativity in CMNs and SMNs.

# Chapter 4

# Ignorance

## 4.1 Deriving ignorance

Recall our starting patterns regarding ignorance: CMNs and SMNs can both give rise to ignorance inferences, but CMNs are also compatible with partial ignorance scenarios of the 'winner' type (the speaker is certain that a subdomain alternative is true) or the 'loser' type (the speaker is certain that a subdomain alternative is false), while SMNs are not.

(1) (= (13) on p. 6)

a. John called more than two people.

$\rightsquigarrow$ For all the speaker knows it could be 3 and it could be 4 and it could be 5 etc.

b. John called at least three people.

$\rightsquigarrow$ For all the speaker knows it could be 3 and it could be 4 and it could be 5 etc.

(2) (= (14) on p. 6)

a. John called three people. Therefore, he called more than two people.

b. John called three people. #Therefore he called at least three people.

(3) (= (15) on p. 6)

a. John called more than two people, but not five.

b. John called at least three people, #but not five.

How might we analyze these effects in CMNs and SMNs?

As we noticed from the outset, in Ch. 1, the ignorance patterns in CMNs and SMNs are identical to those we saw in *some* $NP_{SG}$ and *or*. Ideally the same solution should work here also.

For *or* and *some* $NP_{SG}$ we started by figuring out their truth conditions and alternatives. We argued that in both cases the truth conditions made reference to both a domain and a scalar element, and that alternatives were obtained by replacing the domain with a subdomain and the scalar element with its scalemates. We showed how that yielded both the traditional scalar alternatives but also subdomain alternatives, and in fact, as we argued later on, also mixed-type alternatives obtained by replacing both the scalar item and the domain at the same time. The first step to extending our solution for ignorance to CMNs and SMNs is thus to figure out their truth conditions and alternatives. We did this in Ch. 2, §2.6. We argued that their truth conditions also make reference to both a domain and a scalar element, and so they too must have alternatives obtained by replacing the domain with a subdomain and the scalar element with its scalemates. We showed how that yielded not just the traditional scalar alternatives as well as subdomain alternatives (but no additional mixed-type alternatives). We copy both the truth conditions and the alternatives of CMNs and SMNs from §2.6 below. (To make our following discussion on ignorance in CMNs and SMNs visually more parallel to our discussion of ignorance in *or* and *some* $NP_{SG}$ in Ch. 1, we also rearrange the alternatives, listing the DA first, and the $\sigma$A second.)

(4) More/less than three people quit.

a. $\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \overline{[\![\text{much/little}]\!]\,(3)}$ (assertion)

b. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in D' \mid D' \subset \overline{[\![\text{much/little}]\!]\,(3)}\}$ (DA)

c. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \overline{[\![\text{much/little}]\!]\,(n)} \mid n \in S\}$ ($\sigma$A)

(5) At least/most three people quit.

a. $\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in [\![\text{much/little}]\!]\,(3)$ (assertion)

b. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in D' \mid D' \subset [\![\text{much/little}]\!]\,(3)\}$ (DA )

c. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in [\![\text{much/little}]\!]\,(n) \mid n \in S\}$ ($\sigma$A )

Next, we noticed that, given the same domain, *or* and *some NP<sub>SG</sub>* were equivalent in terms of both truth conditions and alternatives, so we were able to discuss them together. Note now that CMNs and SMNs are also pairwise equivalent. For example, the truth conditions and alternatives of *Less than three people quit* and of *At most two people quit* can be written the same as below.

(6) Less than three people quit / At most two people quit

a. $\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \underbrace{\{0,1,2\}}_{\text{from } \overline{[\![\text{little}]\!](3)}/[\![\text{much}]\!](2)}$ (assertion)

b. $\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \{0\}$ (DA )

$\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \{1\}$ (DA )

$\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \{2\}$ (DA )

$\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \{0,1\}$ (DA )

$\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \{1,2\}$ (DA )

$\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \{0,2\}$ (DA )

c. $\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \underbrace{\{0\}}_{\text{from } \overline{[\![\text{little}]\!](1)}/[\![\text{much}]\!](0)}$ ($\sigma$A )

$\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \underbrace{\{0,1\}}_{\text{from } \overline{[\![\text{little}]\!](2)}/[\![\text{much}]\!](1)}$ ($\sigma$A )

$\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \underbrace{\{0,1,2\}}_{\text{from } \overline{[\![\text{little}]\!](3)}/[\![\text{much}]\!](2)}$ ($\sigma$A )

$\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \underbrace{\{0,1,2,3\}}_{\text{from } \overline{[\![\text{little}]\!](4)}/[\![\text{much}]\!](3)}$ ($\sigma$A )

...

For a more compact notation, in all the cases where there is no danger of confusion we will abbreviate these truth conditions and alternatives as below.

(7) Less than three people quit / At most two people quit

a. $0 \vee 1 \vee 2$ (assertion)

b. $0$ (DA )

$1$ (DA )

$2$ (DA )

$0 \vee 1$ (DA )

$1 \vee 2$ (DA )

$0 \vee 2$ (DA )

c. $0$ ($\sigma$A )

$0 \vee 1$ ($\sigma$A )

$0 \vee 1 \vee 2$ ($\sigma$A )

$0 \vee 1 \vee 2 \vee 3$ ($\sigma$A )

$\ldots$

Once we figured out the truth conditions and alternatives of *or* and *some NP<sub>SG</sub>*, we noticed that exhaustification relative to their pre-exhaustified subdomain alternatives and scalar alternatives, $\mathrm{O}_{\mathrm{ExhDA+}\sigma\mathrm{A}}$, in a plain case, without an intervening modal, failed, but with an intervening overt possibility or necessity modal it succeeded and yielded a Free Choice effect. Is this true of CMNs and SMNs also? We check below for a simple case where the prejacent is $(0 \vee 1)$, the DA $= \{0, 1\}$, and the stronger $\sigma$A $= \{0\}$.

First, let us check the case with no intervening modal. After unpacking the negations of the ExhDA , we end up with a double implication that can be resolved either by making both 0 and 1 false or by making both of them true. On the first option the ExhDA -implicatures together contradict the assertion, so the result is out; the $\sigma$A -implicature is also entailed by the ExhDA -implicatures, so it adds nothing. On the second option the ExhDA -implicatures clash with the $\sigma$A -implicature and the result is a contradiction; this result is out also.

(8) John called less than two people / at most one person

$\mathrm{O}_{\mathrm{ExhDA+}\sigma\mathrm{A}}$ $(0 \vee 1)$

$$= (0 \vee 1) \wedge \neg \mathrm{O}\, 0 \wedge \neg \mathrm{O}\, 1 \wedge \neg 0$$

$$= (0 \vee 1) \wedge \neg(0 \wedge \neg 1) \wedge \neg(1 \wedge \neg 0) \wedge \neg 0$$

$$= (0 \vee 1) \wedge (0 \to 1) \wedge (1 \to 0) \wedge \neg 0$$

$$= (0 \vee 1) \wedge 0 \leftrightarrow 1 \wedge \neg 0$$

a. $$= \underbrace{(0 \vee 1) \wedge \neg 0 \wedge \neg 1}_{\perp} \wedge \neg 0$$
$$= \perp \qquad \text{(G-trivial)}$$

b. $$= (0 \vee 1) \wedge \underbrace{\underbrace{0 \wedge 1}_{\perp} \wedge \neg 0}_{\perp}$$
$$= \perp \qquad \text{(G-trivial)}$$

Both these results are the same as for *or/some* $NP_{SG}$ with the difference that for those items removing the $\sigma$A -implicature in the second case resulted in a consistent *and/every* meaning, while here, due to the nature of the truth conditions, the ExhDA -implicatures are also mutually incompatible – $\max(\lambda d\,.\,\exists x[|x| = d\ldots] \wedge \ldots)$ can't at the same time be both in $\{0\}$ and in $\{1\}$.

Second, let us check the case with an intervening possibility modal. On the first way to resolve the double implication the ExhDA -implicatures contradict the assertion; the $\sigma$A -implicature is also entailed by the ExhDA -implicatures, so it again adds nothing. On the second way to resolve the double implication the ExhDA -implicatures again clash with the $\sigma$A -implicatures and the result is a contradiction.

(9) John may call less than two people / at most one person

$$\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}\ \Diamond(0 \vee 1)$$

$$= \Diamond(0 \vee 1) \wedge \neg \mathrm{O}\, \Diamond 0 \wedge \neg \mathrm{O}\, \Diamond 1 \wedge \neg \Diamond 0$$

$$= \Diamond(0 \vee 1) \wedge \neg(\Diamond 0 \wedge \neg \Diamond 1) \wedge \neg(\Diamond 1 \wedge \neg \Diamond 0) \wedge \neg \Diamond 0$$

$$= \Diamond(0 \vee 1) \wedge (\Diamond 0 \to \Diamond 1) \wedge (\Diamond 1 \to \Diamond 0) \wedge \neg \Diamond 0$$

$$= \Diamond(0 \vee 1) \wedge (\Diamond 0 \leftrightarrow \Diamond 1) \wedge \neg \Diamond 0$$

a. $$\underbrace{\Diamond(0 \vee 1) \wedge \neg \Diamond 0 \wedge \neg \Diamond 1}_{\perp} \wedge \neg \Diamond 0$$
$$= \perp \qquad \text{(G-trivial)}$$

b. $\Diamond(0 \vee 1) \wedge \underbrace{\Diamond 0 \wedge \Diamond 1 \wedge \neg\Diamond 0}_{\perp}$

$= \perp$ (G-trivial)

Without the $\sigma$A -implicature, Free Choice effect:

'There is an accessible world where John calls 0 and there is an accessible world where he calls one.'

While the first result is as in the case of *or/some* $NP_{SG}$, the second one is however different –$\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}$ $\Diamond(p \vee q)$ yielded $\Diamond(p \vee q) \wedge \Diamond p \wedge \Diamond q \wedge \neg\Diamond(p \wedge q)$, that is, a Free Choice accompanied by a scalar implicature consistent with it. The issue of how to handle this undesirable $\sigma$A -implicature in CMNs and SMNs is interesting, and we will discuss it again later. For now we will retain just that the ExhDA -implicatures on their own do yield the usual Free Choice effect. (Note: This is not an exhaustive discussion of CMNs and SMNs under possibiliy modals. We will discuss this embedding again in §4.3.)

Third, and last, let us check the case with an intervening necessity modal. On the first interpretation of the ExhDA -implicatures we get the Free Choice effect. On the second interpretation of the ExhDA -implicatures we get a crash.

(10) John must call less than two people / at most one person

$\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}\ \Box(0 \vee 1)$

$= \Box(0 \vee 1) \wedge \neg\mathrm{O}\,\Box 0 \wedge \neg\mathrm{O}\,\Box 1 \wedge \neg\Box 0$

$= \Box(0 \vee 1) \wedge \neg(\Box 0 \wedge \neg\Box 1) \wedge \neg(\Box 1 \wedge \neg\Box 0) \wedge \neg\Box 0$

$= \Box(0 \vee 1) \wedge (\Box 0 \rightarrow \Box 1) \wedge (\Box 1 \rightarrow \Box 0) \wedge \neg\Box 0$

$= \Box(0 \vee 1) \wedge (\Box 0 \leftrightarrow \Box 1) \wedge \neg\Box 0$

a. $\Box(0 \vee 1) \wedge \neg\Box 0 \wedge \neg\Box 1 \wedge \neg\Box 0$ (Free Choice and weak $\sigma$A -implicature)

'In every accessible world John calls 0 or 1 people, and it is not the case that in every world he calls 0, and it is not the case that in every world he calls 1.'

b. $\Box(0 \vee 1) \wedge \underbrace{\underbrace{\Box 0 \wedge \Box 1}_{\perp} \wedge \neg\Box 0}_{\perp}$

$= \perp$ (G-trivial)

Both results are as for *or/some* $NP_{SG}$, but the second one is different – whereas for *or/some* $NP_{SG}$ the crash was due to a conflict with the $\sigma$A -implicature ruling out an *and/every* interpretation, here it is due simply to the fact that the ExhDA -implicatures are not compatible with each other, for the reasons we mentioned earlier. These differences aside, the bottom line is that the exhaustification of a CMN / SMN across a necessity modal also yields a Free Choice effect, just as in *or/some* $NP_{SG}$. (Note: This is not an exhaustive discussion of CMNs and SMNs under necessity modals. We will discuss this embedding again in §4.3.)

To sum up, in our discussion of *or/some* $NP_{SG}$, $O_{\text{ExhDA+}\sigma\text{A}}$ with no intervening modal yielded a crash, but with an intervening modal it yielded a Free Choice effect. In spite of some differences and complications, the same seems to be true of CMNs and SMNs.

The next step in our discussion of *or/some* $NP_{SG}$ was the observation that, if we assume (along with the rest of the literature) that assertions are prefixed by a null epistemic/doxastic necessity modal, $\Box_S$, and apply $O_{\text{ExhDA+}\sigma\text{A}}$ across this modal, we get an epistemic Free Choice effect – i.e., ignorance. The same is true in this case also. The result is as below – predictably, it looks identical to the overt necessity modal case, except that now the accessible worlds are no longer worlds compatible with some requirement but rather worlds compatible with what the speaker knows/believes.

(11) John called less than two people / at most one person.

$O_{\text{ExhDA+}\sigma\text{A}}\ \Box_S\,(0 \vee 1)$

$= \Box_S\,(0 \vee 1) \wedge \neg O\,\Box_S\,0 \wedge \neg O\,\Box_S\,1 \wedge \neg\Box_S\,0$

$= \Box_S\,(0 \vee 1) \wedge \neg(\Box_S\,0 \wedge \neg\Box_S\,1) \wedge \neg(\Box_S\,1 \wedge \neg\Box_S\,0) \wedge \neg\Box_S\,0$

$= \Box_S\,(0 \vee 1) \wedge (\Box_S\,0 \rightarrow \Box_S\,1) \wedge (\Box_S\,1 \rightarrow \Box_S\,0) \wedge \neg\Box_S\,0$

$= \Box_S\,(0 \vee 1) \wedge (\Box_S\,0 \leftrightarrow \Box_S\,1) \wedge \neg\Box_S\,0$

a. $\Box_S\,(0 \vee 1) \wedge \neg\Box_S\,0 \wedge \neg\Box_S\,1 \wedge \neg\Box_S\,0$ (Free Choice)

'In every accessible world John called 0 or 1 people, and it is not the case that in every world he called 0, and it is not the case that in every world he called 1.'

b. $\Box_S (0 \vee 1) \wedge \underbrace{\underbrace{\Box_S 0 \wedge \Box_S 1}_{\perp} \wedge \neg \Box_S 0}_{\perp}$

$= \perp$ (G-trivial)

At this step in our discussion of ignorance in *or/some* $NP_{SG}$ we noticed that this analysis predicts that in a default, seemingly episodic context they should always yield total ignorance. (We call it 'seemingly episodic' because we now know that, due to $\Box_S$ , it is in fact modal.) We showed however that if an item can prune its DA set to either just the singletons or just the non-singletons, it becomes compatible with partial ignorance of the 'loser'/'winner' type. We argued that *or* can't prune its DA , which is why it can't accommodate partial ignorance, while *some* $NP_{SG}$ can, which is why it is able to do so. Let's check this for CMNs and SMNs also. As in the case of *or/some* $NP_{SG}$ , we need to look at $O_{\text{ExhDA+}\sigma\text{A}}$ across $\Box_S$ for a domain with three elements, e.g., $O_{\text{ExhDA+}\sigma\text{A}}\ \Box_S (0 \vee 1 \vee 2)$ (corresponding to a CMN utterance such as *Less than three people quit* or an SMN utterance such as *At most two people quit*), and consider the cases where we have just the singleton DA or just the non-singleton DA or all of the DA from the set above. We will do this below.

First, consider the case where we have pruned all the non-singleton DA . The exhaustification parse along with its prejacent and alternatives will be as below. (Note: Unlike in the case of *or/some* $NP_{SG}$ , the larger $\sigma$A here are not contingent on the larger subdomain alternatives, so they are not affected by this pruning.)

(12) $O_{\text{ExhSgDA+}\sigma\text{A}}\ \Box_S (0 \vee 1 \vee 2)$

a. $\Box_S (0 \vee 1 \vee 2)$ (prejacent)

b. $\Box_S 0$ (DA )

$\Box_S 1$ (DA )

$\Box_S 2$ (DA )

c. $\Box_S 0$ ($\sigma$A )

$\Box_S (0 \vee 1)$ ($\sigma$A )

$\Box_S (0 \vee 1 \vee 2)$ ($\sigma$A )

$$\Box_S\,(0 \vee 1 \vee 2 \vee 3) \qquad (\sigma A\,)$$

$\ldots$

$O_{\text{ExhSgDA}+\sigma A}$ will assert the prejacent, (13-a), negate the pre-exhaustifications of the DA , (13-b), and negate the stronger alternatives from among the $\sigma$A , (13-c). The result is as below. In (13-b), the first underbrace spells out our assumptions about how the DA are pre-exhaustified – as for *or* and *some* $NP_{SG}$ , we assume that each singleton DA is exhaustified relative to the other singleton DA (consistent with a pre-exhaustification method where we either pre-exhaustify relative to whatever else there is in the DA set or relative to just other DA of the same size); the second underbrace spells out the logical result of negating the individual pre-exhaustifications (using the fact that a formula $\neg(a \wedge \neg b)$ is logically equivalent to $a \rightarrow b$).

(13) $O_{\text{ExhSgDA}+\sigma A}\ \Box_S\,(0 \vee 1 \vee 2)$

a. $\Box_S\,(0 \vee 1 \vee 2) \wedge$

b. $\underbrace{\neg \underbrace{O\,\Box_S\,0}_{\Box_S 0 \wedge \neg \Box_S 1 \wedge \neg \Box_S 2}}_{\Box_S 0 \rightarrow \Box_S 1 \vee \Box_S 2} \wedge \underbrace{\neg \underbrace{O\,\Box_S\,1}_{\Box_S 1 \wedge \neg \Box_S 0 \wedge \neg \Box_S 2}}_{\Box_S 1 \rightarrow \Box_S 0 \vee \Box_S 2} \wedge \underbrace{\neg \underbrace{O\,\Box_S\,2}_{\Box_S 2 \wedge \neg \Box_S 0 \wedge \neg \Box_S 1}}_{\Box_S 2 \rightarrow \Box_S 0 \vee \Box_S 1} \wedge$

c. $\neg \Box_S\,0 \wedge \neg \Box_S\,(0 \vee 1)$

Consider our scenarios of interest and whether they are compatible with this.

| scenario | possible? |
|---|---|
| No ignorance: $\Box_S\,0 \wedge \Box_S\,1 \wedge \Box_S\,2$ | ✗ Impossible due to the nature of the truth conditions and of the domain (e.g., max can't be at the same time in $\{0\}$ and $\{1\}$). |

Table 4.1 (Continued)

| | |
|---|---|
| Partial ignorance, 'winner' type: $\Box_S\, 0 \wedge \neg\Box_S\, /\Box_S\, \neg 1 \wedge \neg\Box_S\, /\Box_S\, \neg 1$ | ✗ Note first that, given the nature of the domain, the 'winner' scenario must actually have this shape: $\Box_S\, 0 \wedge \Box_S\, \neg 1 \wedge \Box_S\, \neg 2$. Regardless of this, it can't be verified by all the ExhDA - implicatures at the same time: $\Box_S\, 0 \wedge \Box_S\, \neg 1 \wedge \Box_S\, \neg 2$ works for both the second and the third implication. However, if $\Box_S\, \neg 1 \wedge \Box_S\, \neg 2$ is true, then the consequent of the first implication becomes false, and since $\Box_S\, 0$ is not false, the implication overall becomes false. |
| Partial ignorance, 'loser' type: $\Box_S\, \neg 0 \wedge \neg\Box_S\, 1 \wedge \neg\Box_S\, 2$ | ✗/✓ For $\Box_S\, \neg 2$ clash with the $\sigma$A - implicature(s). Possible if they are suspended. |
| Total ignorance: $\neg\Box_S\, 0 \wedge \neg\Box_S\, 1 \wedge \neg\Box_S\, 2$ | ✓ |

Table 4.1: Scenarios for $\mathrm{O}_{\mathrm{ExhSgDA}+\sigma\mathrm{A}}$ (pruning of non-singleton DA ).

To sum up, if we prune the DA set to just the singletons, the result we get is either partial ignorance of the 'loser' type or total ignorance.

Second, consider the case where we have pruned all the singleton DA . In such a case we will have the exhaustification parse, prejacent, and alternatives below. (Note: Unlike the discussion for *or/some* $NP_{SG}$ , here the singleton $\sigma$A is allowed because to derive it we don't need to rely on substitutions to both the scalar element and the domain – it already falls out of replacing the numeral in *less than three/at most two* with 1/0, which yields the $\sigma$A *less than one/at most zero*, both of which are schematically represented as 0.)

(14) $\mathrm{O}_{\mathrm{ExhNonSgDA}+\sigma\mathrm{A}}\ \Box_S\, (0 \vee 1 \vee 2)$

a. $\Box_S\,(0 \vee 1 \vee 2)$ (assertion)

b. $\Box_S\,(0 \vee 1)$ (DA )

$\Box_S\,(1 \vee 2)$ (DA )

$\Box_S\,(0 \vee 2)$ (DA )

c. $\Box_S\,0$ ($\sigma$A )

$\Box_S\,(0 \vee 1)$ ($\sigma$A )

$\Box_S\,(0 \vee 1 \vee 2)$ ($\sigma$A )

$\Box_S\,(0 \vee 1 \vee 2 \vee 3)$ ($\sigma$A )

...

$\mathrm{O}_{\mathrm{ExhNonSgDA}+\sigma\mathrm{A}}$ will assert the prejacent, (15-a), negate the pre-exhaustifications of the DA , (15-b), and negate the stronger $\sigma$A , (44-c). The result is as below. In (15-b), the first underbrace spells out our assumptions about how the DA are pre-exhaustified – as for *or/some* $NP_{SG}$ , we assume that each doubleton DA is exhaustified relative to the other doubleton DA (consistent with a pre-exhaustification method where we either pre-exhaustify relative to whatever else there is in the DA set or relative to just other DA of the same size); the second underbrace spells out the logical result of negating the individual pre-exhaustifications (again, using the fact that a formula $\neg(a \wedge \neg b)$ is logically equivalent to $a \rightarrow b$).

(15)

$\mathrm{O}_{\mathrm{ExhNonSgDA}+\sigma\mathrm{A}}\ \Box_S\,(0 \vee 1 \vee 2)$

a. $\Box_S\,(0 \vee 1 \vee 2) \wedge$

b. $\underbrace{\neg \underbrace{\mathrm{O}\,\Box_S\,(0 \vee 1)}_{\Box_S (0\vee1)\wedge\neg\Box_S (1\vee2)\wedge\neg\Box_S (0\vee2)}}_{\Box_S (0\vee1)\rightarrow\Box_S (1\vee2)\vee\Box_S (0\vee2)} \wedge \underbrace{\neg \underbrace{\mathrm{O}\,\Box_S\,(1 \vee 2)}_{\Box_S (1\vee2)\wedge\neg\Box_S (0\vee1)\wedge\neg\Box_S (0\vee2)}}_{\Box_S (1\vee2)\rightarrow\Box_S (0\vee1)\vee\Box_S (0\vee2)} \wedge \underbrace{\neg \underbrace{\mathrm{O}\,\Box_S\,(0 \vee 2)}_{\Box_S (0\vee2)\wedge\neg\Box_S (0\vee1)\wedge\neg\Box_S (1\vee2)}}_{\Box_S (0\vee2)\rightarrow\Box_S (0\vee1)\vee\Box_S (1\vee2)} \wedge$

c. $\neg\Box_S\,0 \wedge \neg\Box_S\,(0 \vee 1)$

Consider again our scenarios of interest and whether they are compatible with this exhaustification.

| scenario | possible? |
|---|---|
| No ignorance: $\Box_S\, 0 \wedge \Box_S\, 1 \wedge \Box_S\, 2$ | ✗ Impossible due to the nature of the truth conditions and of the domain (e.g., max can't be at the same time in $\{0\}$ and $\{1\}$). |
| Partial ignorance, 'winner' type: $\Box_S\, 0 \wedge \neg\Box_S\, /\Box_S\, \neg 1 \wedge \neg\Box_S\, /\Box_S\, \neg 1$ | ✗/✓ Note first that, given the nature of the domain, the 'winner' scenario must actually have this shape: $\Box_S\, 0 \wedge \Box_S\, \neg 1 \wedge \Box_S\, \neg 2$. Regardless of this, clash with the $\sigma$A -implicature(s). Possible if they are suspended. |
| Partial ignorance, 'loser' type: $\Box_S\, \neg 0 \wedge \neg\Box_S\, 1 \wedge \neg\Box_S\, 2$ | ✗ Consider, for example, the second implication (ExhDA -implicature). Suppose $\Box_S\, \neg 0$ is true. Then, if $\neg\Box_S\, 1 \wedge \neg\Box_S\, 2$ is also true, the whole consequent is false. This means that the implication can be true iff the antecedent $\Box_S\, (1 \vee 2)$ is also false. But this would contradict $\Box_S\, (0 \vee 1 \vee 2) \wedge \Box_S\, \neg 0 = \Box_S\, (1 \vee 2)$. |
| Total ignorance: $\neg\Box_S\, 0 \wedge \neg\Box_S\, 1 \wedge \neg\Box_S\, 2$ | ✓ |

Table 4.2: Scenarios for $\mathrm{O}_{\mathrm{ExhNonSgDA}+\sigma\mathrm{A}}$ (pruning of singleton DA ).

To sum up, if we prune the DA set to just the non-singletons, the result we get is either partial ignorance of the 'winner' type or total ignorance.

Last, consider the case where we don't prune any alternatives. In this case exhaustification will proceed relative to the full set of DA and $\sigma$A generated based on the truth conditions.

(16) $\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}\ \Box_S\, (0 \vee 1 \vee 2)$

a. $\Box_S\, (0 \vee 1 \vee 2)$ (assertion)

b. $\Box_S\, 0$ (DA )

$\Box_S\, 1$ (DA )

$\Box_S\, 2$ (DA )

$\Box_S\, (0 \vee 1)$ (DA )

$\Box_S\, (1 \vee 2)$ (DA )

$\Box_S\, (0 \vee 2)$ (DA )

c. $\Box_S\, 0$ ($\sigma$A )

$\Box_S\, (0 \vee 1)$ ($\sigma$A )

$\Box_S\, (0 \vee 1 \vee 2)$ ($\sigma$A )

$\Box_S\, (0 \vee 1 \vee 2 \vee 3)$ ($\sigma$A )

...

$O_{\text{ExhDA+}\sigma\text{A}}$ will assert the prejacent, (17-a), negate the pre-exhaustifications of the DA , (17-b), and negate the stronger $\sigma$A , (17-c). The result is as below. (17-b) brings together the negations of all the pre-exhaustified DA , both singleton and doubleton. The first underbrace again spells out our assumptions about how the DA are pre-exhaustified – as we did for *or/some* $NP_{SG}$ for the whole DA set, we will continue to assume that each singleton DA is exhaustified relative to the other singleton DA and each doubleton DA is exhaustified relative to the other doubleton DA (however, as for *or/some* $NP_{SG}$ , while in the just singleton DA case or the just non-singleton DA case this assumption was consistent with an underlying pre-exhaustification method where we either pre-exhaustified relative to whatever else there was in the DA set or just relative to other DA of the same size, here it means that our chosen overall pre-exhaustification method is to pre-exhaustify relative to DA of the same size); the second underbrace spells out the logical result of negating the individual pre-exhaustifications (again, using the fact that a formula $\neg(a \wedge \neg b)$ is logically equivalent to $a \rightarrow b$). (17-c) shows the negations of the stronger $\sigma$A .

(17)

$O_{\text{ExhDA+}\sigma\text{A}}\ \Box_S\, (0 \vee 1 \vee 2)$

a. $\Box_S\, (0 \vee 1 \vee 2) \wedge$

b. $\underbrace{\neg \underbrace{\mathrm{O}\,\Box_S\, 0}_{\Box_S 0 \wedge \neg\Box_S 1 \wedge \neg\Box_S 2}}_{\Box_S 0 \rightarrow \Box_S 1 \vee \Box_S 2} \wedge \underbrace{\neg \underbrace{\mathrm{O}\,\Box_S\, 1}_{\Box_S 1 \wedge \neg\Box_S 0 \wedge \neg\Box_S 2}}_{\Box_S 1 \rightarrow \Box_S 0 \vee \Box_S 2} \wedge \underbrace{\neg \underbrace{\mathrm{O}\,\Box_S\, 2}_{\Box_S 2 \wedge \neg\Box_S 0 \wedge \neg\Box_S 1}}_{\Box_S 2 \rightarrow \Box_S 0 \vee \Box_S 1} \wedge$

$\underbrace{\neg \underbrace{\mathrm{O}\,\Box_S\,(0 \vee 1)}_{\Box_S (0\vee 1) \wedge \neg\Box_S (1\vee 2) \wedge \neg\Box_S (0\vee 2)}}_{\Box_S (0\vee 1) \rightarrow \Box_S (1\vee 2) \vee \Box_S (0\vee 2)} \wedge \underbrace{\neg \underbrace{\mathrm{O}\,\Box_S\,(1 \vee 2)}_{\Box_S (1\vee 2) \wedge \neg\Box_S (0\vee 1) \wedge \neg\Box_S (0\vee 2)}}_{\Box_S (1\vee 2) \rightarrow \Box_S (0\vee 1) \vee \Box_S (0\vee 2)} \wedge \underbrace{\neg \underbrace{\mathrm{O}\,\Box_S\,(0 \vee 2)}_{\Box_S (0\vee 2) \wedge \neg\Box_S (0\vee 1) \wedge \neg\Box_S (1\vee 2)}}_{\Box_S (0\vee 2) \rightarrow \Box_S (0\vee 1) \vee \Box_S (1\vee 2)} \wedge$

c. $\neg\Box_S\, 0 \wedge \neg\Box_S\, (0 \vee 1)$

Since for this exhaustification relative to all the DA we did nothing more than bring together the results from $\mathrm{O}_{\mathrm{ExhSgDA}+\sigma\mathrm{A}}$ and $\mathrm{O}_{\mathrm{ExhNonSgDA}+\sigma\mathrm{A}}$, the result is also nothing more than the intersection of what we got in those cases.

| scenario | possible? |
|---|---|
| No ignorance:<br>$\Box_S\, 0 \wedge \Box_S\, 1 \wedge \Box_S\, 2$ | ✗ Impossible due to the nature of the truth conditions and of the domain (e.g., max can't be at the same time in $\{0\}$ and $\{1\}$). |
| Partial ignorance, 'winner' type:<br>$\Box_S\, 0 \wedge \neg\Box_S\, / \Box_S\, \neg 1 \wedge \neg\Box_S\, / \Box_S\, \neg 1$ | ✗ |
| Partial ignorance, 'loser' type:<br>$\Box_S\, \neg 0 \wedge \neg\Box_S\, 1 \wedge \neg\Box_S\, 2$ | ✗ |
| Total ignorance:<br>$\neg\Box_S\, 0 \wedge \neg\Box_S\, 1 \wedge \neg\Box_S\, 2$ | ✓ |

Table 4.3: Scenarios for $\mathrm{O}_{\mathrm{ExhSgDA}+\sigma\mathrm{A}}$ (no DA pruning).

To sum up, if we don't prune any DA / exhaustify relative to all the DA , the result we get is total ignorance.

Our solution for ignorance in CMNs and SMNs is then as follows: By default, both CMNs and SMNs have to be exhaustified relative to both ExhDA and $\sigma$A , and the result is total ignorance, $\neg\Box_S\, 0 \wedge \neg\Box_S\, 1 \wedge \neg\Box_S\, 2$. Then, a no ignorance context – in the sense from our discussion of *or*/*some* $NP_{SG}$ , where what we meant by it was a case where all the DA were true at the same time – is logically impossible, because, due to the nature of the domain and of the truth conditions, it is impossible for all the DA to be simultaneously true. A context of partial ignorance of the 'winner'/'loser' type, $\Box_S\ 0 \wedge \neg\Box_S\ 1 \wedge \neg\Box_S\ 2$ / $\Box_S\ \neg 0 \wedge \neg\Box_S\ 1 \wedge \neg\Box_S\ 2$, is however logically possible. At the same time, it is incompatible with exhaustifying relative to the full set of DA . In such a context CMNs can prune their DA (and $\sigma$A that would clash with such a context / such a pruning of the DA ), which is why they can accommodate such a context, but SMNs can't, which is why they are incompatible with either partial ignorance context and compatible only with total ignorance.

But, as for *or* and *some* $NP_{SG}$ , the proposal as phrased here seems to suggest that for CMNs DA -pruning also is contingent on the context. Thus, we seem to predict that in an out-of-the-blue case context both CMNs and SMNs should be on a par in the sense that they should both by default give rise to ignorance. However, a common example used in the literature to argue for a contrast between them is precisely for an out-of-the-blue case. This is the famous example of a person stating how many children they have, where a CMN is fine but an SMN is very odd because it suggests they don't know.

(18) a. I have more than two children.

b. ??I have at least three children.

Or the similarly famous example of a flight attendant stating how many emergency exits the plane s/he serves on has, where again an SMN gives rise to oddity because of an inescapable suggestion of ignorance.

(19) a. This plane has more than five emergency exits.

b. ??This plane has at least six emergency exits.

In both these cases there is no explicit context of certainty that the CMNs could accommodate and the SMNs clash with. How then do we capture this contrast between them? Note that in the example above the reason the SMN sentence is found to be odd is because it is implicitly assumed that a person knows how many children they have / that a flight attendant knows how many emergency exits the plan s/he serves on has. I will argue that oddness here comes from a clash of SMNs with an implicit assumption of knowledge, thus, with an implicit 'winner' context. Then the fact that CMNs can accommodate it but SMN can't is precisely what we would expect.

Let's take stock. We have used our insights from our analysis of total/partial ignorance effects in *or/some* $NP_{SG}$ to derive the same effects in CMNs/SMNs. Due to differences in the nature of the truth conditions and of the alternatives, there were also some differences in the way in which the various scenarios were ruled in or out for *or/some* $NP_{SG}$ vs. CMNs/SMNs (e.g., the no ignorance scenario was logically possible for *or/some* $NP_{SG}$ , and could be obtained by suspending the $\sigma$A -implicatures). However, the same basic insights worked in both cases – the truth conditions make reference to both a domain and a scalar element; alternatives are generated by replacing the domain with subdomains and the scalar element with its scalemates; ignorance is a Free Choice effect obtained by exhaustifying relative to pre-exhaustified subdomain alternatives and scalar alternatives across a null epistemic/doxastic necessity modal; partial ignorance comes from being able to prune DA (in addition to the usual ability to prune $\sigma$A ), and total ignorance comes from not being able to prune DA . In sum, we now have a theory of total vs. partial ignorance effects that works for both *or/some* $NP_{SG}$ and CMNs/SMNs.

## 4.2 Ignorance and scalar implicatures

In our discussion of ignorance we often mentioned $\sigma$A -implicatures, but they were all of the weak type, i.e., $\sigma$A -implicatures computed above $\Box_S$ . It is now time to get back to another type of scalar implicatures, scalar implicatures of the strong type, $\sigma$A -implicatures computed below $\Box_S$ – the type of implicatures we discussed in Ch. 3, without however using the terminology 'below $\Box_S$ '.

In our discussion of those strong scalar implicatures in Ch. 3 we showed that a CMN or SMN exhaustified via $O_{\sigma A}$ relative to all of its $\sigma$A yields an implausible meaning. We copy those examples below (cf. (25)-(27) on p. 104; the numeral in the SMN example is changed to allow us to discuss CMNs and SMNs together).

(20) $O_{\sigma A}$ (John called less than three people / John called at most two people)

$$= \max(\lambda d\,.\,\exists x[|x| = d \wedge \ldots]) \in \underbrace{\overline{[\![\text{little}]\!]\,(3)}}_{=\{0,1,2\}} / \underbrace{[\![\text{much}]\!]\,(2)}_{=\{0,1,2\}} \wedge$$

$$\neg \max(\lambda d\,.\,\exists x[|x| = d \wedge \ldots]) \in \underbrace{\overline{[\![\text{little}]\!]\,(2)}}_{=\{0,1\}} / \underbrace{[\![\text{much}]\!]\,(1)}_{=\{0,1\}}$$

# 'John called less than three / at most two but not less than two / at most one, i.e., he called exactly two people.'

At that point we suggested that this result might be ruled out due to a clash with the ignorance effect of these items. We are now ready to check this hypothesis. Let's consider an exhaustification where $\mathrm{O}_{\mathbf{ExhDA}}$ is above $\Box_S$ , as in the ignorance case, but $\mathrm{O}_{\sigma\mathbf{A}}$ below $\Box_S$ , as we were implicitly using it in Ch. 3. This exhaustification is as below. (Note: The DA (before pre-exhaustification) actually have the form $\Box_S\, \mathrm{O}_{\sigma\mathbf{A}}\, 0$, ..., with $\mathrm{O}_{\sigma\mathbf{A}}$ . However, a DA doesn't have its own $\sigma$A , so we will assume that $\mathrm{O}_{\sigma\mathbf{A}}$ applied to a DA is vacuous, and leave it out.)

(21) John called less than three people / at most two people

$\mathrm{O}_{\mathbf{ExhDA}}\, (\Box_S\, \mathrm{O}_{\sigma\mathbf{A}}\, (0 \vee 1 \vee 2))$

$$= \Box_S\, \mathrm{O}_{\sigma\mathbf{A}}\, (0 \vee 1 \vee 2) \wedge \neg\mathrm{O}\, \Box_S\, 0 \wedge \neg\mathrm{O}\, \Box_S\, 1 \wedge \neg\mathrm{O}\, \Box_S\, 2 \wedge \neg\mathrm{O}\, \Box_S\, (0 \vee 1) \wedge \neg\mathrm{O}\, \Box_S\, (1 \vee 2) \wedge \neg\mathrm{O}\, \Box_S\, (0 \vee 2)$$

$$= \underbrace{\Box_S\, ((0 \vee 1 \vee 2) \wedge \neg(0 \vee 1))}_{=\Box_S\, 2} \wedge$$

$$\underbrace{\neg\mathrm{O}\, \Box_S\, 0 \wedge \neg\mathrm{O}\, \Box_S\, 1 \wedge \neg\mathrm{O}\, \Box_S\, 2 \wedge \neg\mathrm{O}\, \Box_S\, (0 \vee 1) \wedge \neg\mathrm{O}\, \Box_S\, (1 \vee 2) \wedge \neg\mathrm{O}\, \Box_S\, (0 \vee 2)}_{\neg\Box_S\, 0 \wedge \neg\Box_S\, 1 \wedge \neg\Box_S\, 2}$$

$$= \underline{\Box_S\, 2} \wedge \neg\Box_S\, 0 \wedge \neg\Box_S\, 1 \wedge \underline{\neg\Box_S\, 2}$$

$$= \bot \qquad \text{(G-trivial)}$$

As we can see, the assertion yields certainty about the domain, $\Box_S\, (0 \vee 1 \vee 2)$. Then $\mathrm{O}_{\sigma\mathbf{A}}$ strengthens it to certainty about, essentially, a singleton DA , $\Box_S\, 2$. But $\mathrm{O}_{\mathbf{ExhDA}}$ yields total ignorance, including ignorance about this singleton DA , $\neg\Box_S\, 2$. The results contradict each other, so the result is a crash. This parse is thus bad for the exact same reason why the parses for *or/some $NP_{SG}$* below are under the interpretation of the ExhDA -implicatures as $\Box_S\, p \wedge \Box_S\, q$ – this interpretation of the ExhDA -implicatures would yield an *and/every* reading that would clash with the $\sigma$A -implicature which, whether we compute it at the top or not, always yields $\neg\Box_S\,(p \wedge q)$.

(22) a. $\mathrm{O}_{\mathbf{ExhDA+\sigma A}}\ \Box_S\,(p \vee q),\ = \Box_S\,(p \vee q) \wedge \Box_S\,p \wedge \Box_S\,q \wedge \neg\Box_S\,(p \wedge q),\ = \bot$

b. $\mathrm{O}_{\mathbf{ExhDA}}\ \Box_S\ \mathrm{O}_{\sigma\mathbf{A}}\ (p \vee q),\ = \Box_S\,((p \vee q) \wedge \neg(p \wedge q)) \wedge \Box_S\,p \wedge \Box_S\,q,\ = \bot$

Thus, while our original hunch in Ch. 3 that the reason why CMNs and SMNs do not give rise to the 'exactly'-inducing $\sigma$A -implicature is because of a clash between certainty and ignorance was right, what is happening here is in fact part of a more general set of cases where we get a crash due to a clash between $\mathrm{O}_{\mathbf{ExhDA}}$ and $\mathrm{O}_{\sigma\mathbf{A}}$ .

Now, recall that, in addition to showing that a CMN/SMN exhaustified relative to all its $\sigma$A yields an implausible meaning, in Ch. 3 we also showed that, if it is exhaustified relative to just a subset of its $\sigma$A , it does yield plausible $\sigma$A -implicatures.

(23) $\mathrm{O}_{\sigma\mathbf{A}}$ (John called less than three people / John called at most two people)

$= \max(\lambda d\,.\,\exists x[|x| = d \wedge \ldots]) \in \underbrace{\overline{[\![\text{little}]\!]\,(3)}}_{=\{0,1,2\}} / \underbrace{[\![\text{much}]\!]\,(2)}_{=\{0,1,2\}} \wedge$

$\neg \max(\lambda d\,.\,\exists x[|x| = d \wedge \ldots]) \in \underbrace{\overline{[\![\text{little}]\!]\,(1)}}_{=\{0\}} / \underbrace{[\![\text{much}]\!]\,(0)}_{=\{0\}}$

'John called less than three / at most two people but not less than one / at most zero, i.e., he called less than three / at most two but not none, i.e., he called one or two.'

Let us consider, then, an exhaustification as before, but where $\mathrm{O}_{\sigma\mathbf{A}}$ is done as above, relative to just the $\sigma$A 0 (without the $\sigma$A $(0 \vee 1)$).

(24) John called less than three people / at most two people

$\mathrm{O}_{\mathbf{ExhDA}}\ (\Box_S\ \mathrm{O}_{\sigma\mathbf{A}}\ (0 \vee 1 \vee 2))$

$= \Box_S\ \mathrm{O}_{\sigma\mathbf{A}}\ (0 \vee 1 \vee 2) \wedge \neg\mathrm{O}\ \Box_S\ 0 \wedge \neg\mathrm{O}\ \Box_S\ 1 \wedge \neg\mathrm{O}\ \Box_S\ 2 \wedge \neg\mathrm{O}\ \Box_S\ (0 \vee 1) \wedge \neg\mathrm{O}\ \Box_S\ (1 \vee 2) \wedge$
$\neg\mathrm{O}\ \Box_S\ (0 \vee 2)$

$= \underbrace{\Box_S\,((0 \vee 1 \vee 2) \wedge \neg 0)}_{=\Box_S\,(1 \vee 2)} \wedge$

$$\underbrace{\neg \mathrm{O}\,\Box_S\, 0 \wedge \neg \mathrm{O}\,\Box_S\, 1 \wedge \neg \mathrm{O}\,\Box_S\, 2 \wedge \neg \mathrm{O}\,\Box_S\,(0 \vee 1) \wedge \neg \mathrm{O}\,\Box_S\,(1 \vee 2) \wedge \neg \mathrm{O}\,\Box_S\,(0 \vee 2)}_{\neg\Box_S\, 0 \wedge \neg\Box_S\, 1 \wedge \neg\Box_S\, 2}$$

$$= \Box_S\,(1 \vee 2) \wedge \neg\Box_S\, 0 \wedge \neg\Box_S\, 1 \wedge \neg\Box_S\, 2 \qquad \text{(scalar implicature + ignorance)}$$

This time the assertion conveys certainty about the domain, $\Box_S\,(0 \vee 1 \vee 2)$, $\mathrm{O}_{\sigma\mathbf{A}}$ strengthens it to certainty about a *non*-singleton DA , $\Box_S\,(1 \vee 2)$, and this result is compatible with the ignorance about the singleton DA yielded by $\mathrm{O}_{\mathbf{ExhDA}}$ , $\neg\Box_S\, 0 \wedge \neg\Box_S\, 1 \wedge \neg\Box_S\, 2$. Since the result for this $\mathrm{O}_{\sigma\mathbf{A}}$ computed related to a pruned $\sigma$A set is not a contradiction, this explains why it is acceptable.

The discussion above shows how an interaction between $\mathrm{O}_{\sigma\mathbf{A}}$ and ExhDA can explain why we get a crash in the case without $\sigma$A -pruning but not a crash in the case with $\sigma$A -pruning. However, it doesn't explain how this form of $\sigma$A -pruning works. So far we have assumed that $\sigma$A -implicatures can be suspended if forced by the context. But here it seems that they can be suspended in a default setting also, merely to avoid a clash between $\mathrm{O}_{\sigma\mathbf{A}}$ and $\mathrm{O}_{\mathbf{ExhDA}}$ . And it seems that this doesn't have to happen wholesale (we don't have to get rid of $\mathrm{O}_{\sigma\mathbf{A}}$ altogether) – we are allowed to get rid of just the inconvenient $\sigma$A . So what regulates this $\sigma$A -pruning? And why isn't it available to *or/some* $NP_{SG}$ also, to rescue (i.e., burden us with) the (undesirable) *and/every* reading that we just saw again above?

An important observation at this point is that in the *or/some* $NP_{SG}$ case $\sigma$A -pruning would have as a result an *and/every* meaning that could have been expressed more easily by its conjunctive $\sigma$A , $\Box_S\,(p \wedge q)$. However, in the CMN/SMN case $\sigma$A -pruning would not have as a result a meaning that could have been expressed more easily by any of their $\sigma$A . For example, in our discussion above of *less than three/at most two*, pruning of the scalar alternative $(0 \vee 1)$ took us from $\Box_S\,(0 \vee 1 \vee 2)$ to $\Box_S\,(1 \vee 2)$ and, while there is a way to express this meaning more easily, e.g., *one or two*, this simpler expression is not part of the $\sigma$A set of *less than three/at most two*.

In the literature on disjunction that, either via $\mathrm{O}_{\mathbf{ExhDA}}$ (cf. Chierchia 2013 and the present account) or via recursive $\mathrm{O}_{\mathbf{DA}}$ (Fox 2007), generates this undesirable conjunctive meaning, there have been a number of ways to deal with it.

First, one can do as we did and invoke a clash with the $\sigma$A -implicature. While that seems like an explanation we may want to keep, we have seen, however, that it cannot be the end of the story for

CMNs/SMNs since it doesn't explain why they allow $\sigma$A -pruning while *or/some* $NP_{SG}$ don't.

Second, one can invoke some economy constraint. E.g., Chierchia (2013) invokes a Fox & Katzir (2011)-style exhaustification economy constraint, articulated as below.

(25) Exhaustification Economy: Avoid unnecessary exhaustification. Chierchia (2013:129)

a. $^*\mathrm{O}_{\mathrm{A}}[_{\mathrm{Y}} \ldots X_{[+\mathrm{A}]} \ldots]$, if the result is itself a member of $[\![\mathrm{Y}]\!]^{\mathrm{ALT}}$ different from $[\![\mathrm{X}]\!]$

b. $^*\mathrm{O}_{\mathrm{A}}[\ldots X_{[+\mathrm{A}]} \ldots]$, if logically equivalent to $[_{\mathrm{Y}} \ldots X_{[-\mathrm{A}]} \ldots]$

The first part of the constraint says: Don't exhaustify an expression $X$ if the result is identical to one of its formal alternatives. Since $\sigma$A -pruning would lead to a result identical to one of the formal $\sigma$A for *or/some* $NP_{SG}$ but not for CMNs/SMNs, this would correctly rule out $\sigma$A -pruning for the former but not the latter. The second part of the constraint says: Don't exhaustify an expression $X$ if exhaustifying it would be the same as not exhaustifying. This simply ensures that in the case of *or/some* $NP_{SG}$ the *and/every* alternative itself abides by exhaustification economy by not being obtained from a vacuous exhaustification of the form $\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}\ \Box_S\ (p \wedge q)$. All in all, exhaustification economy seems to get the job done. Should we adopt it also?

There are two main reasons not to adopt exhaustification economy. First, as we might recall, an exhaustification like $\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}\ \Box_S\ (p \vee q)$ doesn't just yield a meaning equivalent to the scalar alternative $\Box_S\, p \wedge \Box_S\, q = \Box_S\ (p \wedge q)$, it also yields the Free Choice effect we want, namely, $\neg\Box_S\, p \wedge \neg\Box_S\, q$. We wouldn't want to rule this parse out just because one of its readings fails to abide by exhaustification economy. A second reason not to adopt it would be because so far it has been crucial to our account to assume obligatory exhaustification, or exhaustification economy runs directly against that. So it seems that the solution for us might not be exhaustification economy after all.

What we seem to need instead is a third solution, also an economy constraint, but this time a *pruning* economy constraint. Something of this form is suggested in Meyer (2016), who calls it 'brevity-based' pruning and articulates it informally as below.

(26) Brevity-based pruning: Narrowing down ALT to A is allowed only if the enriched meaning that

this derives could not have been expressed in a simpler manner.

If by a 'simpler manner' we are allowed to specify, as on the previous economy constraint, only elements from among the formal $\sigma$A set of the prejacent, then we can get exactly what we want. Since $\sigma$A -pruning would lead to a result identical to one of the formal $\sigma$A for *or/some* $NP_{SG}$ but not for CMNs/SMNs, this predicts that $\sigma$A -pruning should be bad for *or/some* $NP_{SG}$ but fine for CMNs/SMNs.

Before we move on, we have to clarify one more thing. Do we want to say that $\sigma$A -pruning is always done strictly to the degree to which it would avoid a clash between $O_{\sigma A}$ and $O_{\mathrm{ExhDA}}$ ? If this were true, then, e.g., *less than three*, wouldn't be interpreted as *not less than two* = *exactly two*, but it would always, as a default, mean *not less than one* = *one or two*. This prediction doesn't seem too bad for this case – after all, we already said that *less than* and *at most* tend to carry an existential implicature (that can be canceled by context). However, it seems a little rigid for other cases where the domain is larger. For example, it would also mean that *John called less than 20 people* should always by default mean that he called *not less than 18* = *18 or 19*. This seems wrong, and in the general case we can interpret this statement in a variety of ways, e.g., that the speaker is sure that John called less than 20 but probably not less than 15, and ignorant about which number exactly in the range 15-19 he called. Thus, it seems that the result of $O_{\sigma A}$ is affected not just by a pressure to avoid the clash with the result of $O_{\mathrm{ExhDA}}$ , but also by considerations of relevance. That relevance plays a role in $O_{\sigma A}$ for CMNs/SMNs is unsurprising – that is what it tends to do in general. For example, *Some of us will be home* can strictly speaking give rise to inferences that all of its *many*, *most*, and *all* alternatives are false, but that happens most reliably for *all*, while the other $\sigma$A seem much more subject to relevance (Chierchia 2013:103). Also it is not just for CMNs/SMNs that $O_{\sigma A}$ is affected by relevance – that is true of BNs also. For example, as is well known, in a context where only *three* is relevant, you can say *I have three children* even if you actually have four. However, it is possible that relevance might play a somewhat stronger role for modified numerals than for bare numerals, and that all exhibit further interesting phenomena that have to do with roundness, etc. (cf. Cummins et al. 2012's discussion on granularity and roundness in modified numerals). We will be unable to discuss those issues here, but we believe that the story sketched here should be compatible with whatever the best solution for those other phenomena turns out to be.

To sum up, the reason why CMNs and SMNs don't give rise to 'exactly'-inducing $\sigma$A -implicatures is because that would lead to a clash between certainty and ignorance, more generally, between $O_{\sigma A}$ and $O_{ExhDA}$ . The reason they are however able to give rise to non-'exactly'-inducing $\sigma$A -implicatures is because, to avoid the clash between $O_{\sigma A}$ and $O_{ExhDA}$ , they are able to by default weaken $O_{\sigma A}$ by pruning alternatives from the $\sigma$A set. We have argued that default $\sigma$A -pruning is regulated by a pruning economy constraint that prevents $\sigma$A -pruning if the result of $O_{\sigma A}$ after pruning would be equivalent to one of the formal $\sigma$A of the prejacent. We have shown that this not only captures why CMNs/SMNs are able to prune this way, but also why *or/some* $NP_{SG}$ can't.

Looking beyond the technical details, note that the theory of ignorance presented in the previous section predicts a pretty strong ignorance effect where we derive ignorance about every element of the domain. Scalar implicatures however ensure that this effect does not in fact go unchecked, because they narrow down the domain to a relevant subset, and then ignorance will only range over a subpart of the domain (and relevance can also further decide whether every $\sigma$A is relevant, or only some). Thus scalar implicatures are crucial not just in ensuring that *more than two* doesn't usually mean a thousand, but also in ensuring that it doesn't usually trigger ignorance about a thousand either.

## 4.3 Prediction: CMNs/SMNs under overt possibility and necessity modals

In our discussion of *or/some* $NP_{SG}$ in §1.2 we first exhaustified them via $O_{ExhDA+\sigma A}$ across an overt modal, noticed that that always yielded a Free Choice effect, and then used that to motivate an analysis of ignorance as a Free Choice effect obtained by applying $O_{ExhDA+\sigma A}$ across $\Box_S$ . In our discussion of CMNs/SMNs in §4.1 we also went through these same steps, but only dwelt on the exhaustification via $O_{ExhDA+\sigma A}$ across an overt modal to the extent that we could conclude that it yielded the same basic Free Choice effect as *or/some* $NP_{SG}$ , such that we could feel justified in pursuing the same analysis of ignorance in CMNs/SMNs as for those items. However, there is much more than that to say about the interaction of CMNs/SMNs with overt modals. In what follows, then, we will take another look at those cases.

First, in §2.4 we noticed with Hackl (2000) that a CMN/SMN embedded under an overt modal gives rise not just to a surface, wide scope for the modal, reading, but also to an inverse scope reading of a special kind – a split scope for the CMN/SMN reading. At that point we only discussed overt necessity modals, but we expect the same to arise for possibility modals too. We spell out all the predicted scope readings for both possibility and necessity modals below.

(27) John may call less than two people / at most one person.

a. $\Diamond \max(\lambda d \,.\, \exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \underbrace{\overline{[\![\text{little}]\!]\ (2)}/\ [\![\text{much}]\!]\ (1)}_{\{0,1\}}$

'There is an accessible world where the maximum degree $d$ s.t. there exists an $x$ which is a plurality with cardinality $d$ of people that John calls is in $\{0, 1\}$.'

(He is allowed to call two or more.)

b. $\max(\lambda d \,.\, \Diamond \exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \underbrace{\overline{[\![\text{little}]\!]\ (2)}/\ [\![\text{much}]\!]\ (1)}_{\{0,1\}}$

'The maximum degree $d$ s.t. there is an accessible world where there exists an $x$ which is a plurality with cardinality $d$ of people that John calls is in $\{0, 1\}$.'

(He is forbidden to call 2 or more.)

(28) John must call less than two people / at most one person. (= (18) & (20)in Ch. 2)

a. $\Box \max(\lambda d \,.\, \exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \underbrace{\overline{[\![\text{little}]\!]\ (2)}/\ [\![\text{much}]\!]\ (1)}_{\{0,1\}}$

'In every accessible world the maximum degree $d$ s.t. there exists an $x$ which is a plurality with cardinality $d$ of people that John calls is in $\{0, 1\}$.'

(He is forbidden to call 2 or more.)

b. $\max(\lambda d \,.\, \Box \exists x[|x| = d \wedge P(x) \wedge Q(x)])\ \underbrace{\overline{[\![\text{little}]\!]\ (2)}/\ [\![\text{much}]\!]\ (1)}_{\{0,1\}}$

'The maximum degree $d$ s.t. in every accessible world there is an $x$ which is a plurality with cardinality $d$ of people that John calls is in $\{0, 1\}$.'

(He is allowed to call 2 or more.)

For convenience, we will abbreviate these readings as below.

(29) a. $\Diamond\,\mathrm{max}\ (0 \vee 1)$

b. $\mathrm{max}\,\Diamond\,(0 \vee 1$

(30) a. $\Box\,\mathrm{max}\ (0 \vee 1)$

b. $\mathrm{max}\,\Box\,(0 \vee 1)$

Second, at this point we will also notice with Büring (2008) that each of these scope readings corresponds to a distinct interpretation – the surface scope readings $\Diamond\,\mathrm{max}$ and $\Box\,\mathrm{max}$ is what Büring calls the 'authoritative speaker' readings, and the split scope readings $\mathrm{max}\,\Diamond$ and $\mathrm{max}\,\Box$ are what he calls the 'insecure speaker' readings. Below we will discuss each of these readings in turn.

Let's start from the easiest case, which is the 'insecure speaker' readings arising from both $\mathrm{max}\,\Diamond$ and $\mathrm{max}\,\Box$. In light of all our discussions so far regarding the outcome of exhaustifying a CMN/SMN utterance without an intervening modal as opposed to across an intervening modal, the fact that $\mathrm{max}\,\Diamond$ and $\mathrm{max}\,\Box$ should give rise to 'insecure speaker' readings is completely unsurprising: $\mathrm{O}_{\mathbf{ExhDA+}\sigma\mathbf{A}}\ \mathrm{max}(0 \vee 1)$ always crashed, and the way to rescue it was to insert the null doxastic/epistemic necessity modal; $\mathrm{O}_{\mathbf{ExhDA+}\sigma\mathbf{A}}\ \Box_S\ \mathrm{max}(0 \vee 1)$ yielded epistemic Free Choice/ignorance. Inserting a modal in the scope of max shouldn't make any difference, so $\mathrm{O}_{\mathbf{ExhDA+}\sigma\mathbf{A}}\ \mathrm{max}\,\Diamond/\Box\,(0 \vee 1)$ and $\mathrm{O}_{\mathbf{ExhDA+}\sigma\mathbf{A}}\ \Box_S\ \mathrm{max}\,\Diamond/\Box\,(0 \vee 1)$ should be bad and good for the same reasons. We illustrate the outcome of the latter, good, case below. (Since we have seen this type of computation a number of times before, we will however skip over some of the details.)

(31) John may/must call less than two people / at most one person

$\mathrm{O}_{\mathbf{ExhDA+}\sigma\mathbf{A}}\ \Box_S\ \mathrm{max}\,\Diamond/\Box\,(0 \vee 1)$

$= \Box_S\ \mathrm{max}\,\Diamond/\Box\,(0 \vee 1) \wedge \underbrace{\neg\mathrm{O}\,\Box_S\ \mathrm{max}\,\Diamond/\Box\,0 \wedge \neg\mathrm{O}\,\Box_S\ \mathrm{max}\,\Diamond/\Box\,1}_{\neg\Box_S\ \mathrm{max}\,\Diamond/\Box\,0 \wedge \neg\Box_S\ \mathrm{max}\,\Diamond/\Box\,1} \wedge \neg\Box_S\ \mathrm{max}\,\Diamond/\Box\,0$

$= \Box_S \max \Diamond/\Box\,(0 \vee 1) \wedge \neg\Box_S \max \Diamond/\Box\, 0 \wedge \neg\Box_S \max \Diamond/\Box\, 1$ (epistemic Free Choice)

'In every world compatible with what the speaker believes, John may/must call 0 or 1 people, and it is not the case that in every world he may/must call 0, and it is not the case that in every world he may/must call 1.'

Thus, other than the fact that the max statement now contains a modal, the 'insecure speaker' readings arising from $\max \Diamond/\Box$ are no different than the epistemic Free Choice effect/ignorance we were getting in the episodic case: The speaker indicates knowledge about the range such that the maximum that is possible/necessary is in that range, and ignorance about the exact value of this maximum.

Let's now move on to the next case, which, proceeding again in order of easiness, is the 'authoritative speaker' reading arising from $\Box \max$ . As can be seen from the exhaustification below, we are again applying $O_{ExhDA+\sigma A}$ across a necessity modal, so insofar as exhaustification is concerned, the results are precisely the same as in the previous case, and we can skip directly to the final result. Meaning-wise, the only difference is that now we are quantifying over worlds compatible with a non-epistemic modal flavor – e.g., deontic, bouletic, etc – and there is no longer a modal in the scope of max. The implicatures yield the familiar Free Choice effect.

(32) John must call less than two people / at most 1 person

$O_{ExhDA+\sigma A}\ \Box \max\ (0 \vee 1)$

$= \Box \max\ (0 \vee 1) \wedge \neg\Box \max\ 0 \wedge \neg\Box \max\ 0$ (deontic/bouletic/...Free Choice)

'In every world compatible with, e.g., what is required John calls 0 or 1 people, and it is not the case that in every world he calls 0, and it is not the case that in every world he calls 1. '

The 'authoritative speaker' reading arising from $\Box \max$ is thus also pretty straightforward: The speaker communicates what the required range is, and emphasizes that each part of the range falls within what is allowed. Note that the latter part is a total variation/deontic Free Choice effect completely analogous to the total variation/epistemic Free Choice/ignorance effect we were getting in the unembedded cases. There we noticed that CMNs are also compatible with a partial effect but SMNs weren't. Note that the same contrast between CMNs and SMNs with respect to partial variation can be observed here also.

(33) John must call two people. Therefore, he must call less than three / #at most two.

(34) John must call less than three / #at most two people, but not zero/none.

And the same analysis – i.e., pruning of DA – should be able to account for this contrast here as well.

Finally, let's discuss the 'authoritative speaker' reading arising from $\Diamond \max$ . As is well known in the literature on modified numerals (cf., e.g., the lengthy discussion in Kennedy 2015), the case of embedding under a possibility modal is trickier on a number of levels. First, the computation of $O_{\mathrm{ExhDA}+\sigma\mathrm{A}}$ itself is somewhat less straightforward. Second, the interpretation we may get from the most straightforward application of $O_{\mathrm{ExhDA}+\sigma\mathrm{A}}$ might not be the right one. We will tackle each of these in turn. First, as we already noticed in our cursory discussion of a CMN/SMN under a possibility modal in §4.1, although $O_{\mathrm{ExhDA}+\sigma\mathrm{A}}$ across $\Diamond$ yields ExhDA -implicatures that give rise to the familiar Free Choice effect, it also yields a $\sigma$A -implicature that clashes with one of the Free Choice implicatures. (If our assertion had been $\Diamond \max\ (0 \vee 1 \vee 2)$, the ExhDA -implicatures would have been $\Diamond \max\ 0 \wedge \Diamond \max\ 1 \wedge \Diamond \max\ 2$, and the $\sigma$A -implicature $\neg\Diamond \max\ (0 \vee 1)$ would have clashed with two of the Free Choice implicatures, both $\Diamond \max\ 0$ and $\Diamond \max\ 1$.)

(35) John may call less than two people / at most one person

$$O_{\mathrm{ExhDA}+\sigma\mathrm{A}}\ \Diamond \max\ (0 \vee 1)$$

$$= \Diamond \max\ (0 \vee 1) \wedge \neg \mathrm{O} \Diamond \max\ 0 \wedge \neg \mathrm{O} \Diamond \max\ 1 \wedge \neg\Diamond \max\ 0$$

$$= \Diamond \max\ (0 \vee 1) \wedge \neg(\Diamond \max\ 0 \wedge \neg\Diamond \max\ 1) \wedge \neg(\Diamond \max\ 1 \wedge \neg\Diamond \max\ 0) \wedge \neg\Diamond \max\ 0$$

$$= \Diamond \max\ (0 \vee 1) \wedge \Diamond \max\ 0 \leftrightarrow \Diamond \max\ 1 \wedge \neg\Diamond \max\ 0$$

$$= \Diamond \max\ (0 \vee 1) \wedge \Diamond \max\ 0 \wedge \Diamond \max\ 1 \wedge \neg\Diamond \max\ 0$$

$$= \bot$$

This is in fact an 'exactly'-inducing sort of $\sigma$A -implicature, so, using the results of our discussion in the previous section, we will assume that it can be fixed via $\sigma$A -pruning. Since here the $\sigma$A set only contains one $\sigma$A , we can just cross it out.

(36) John may call less than two people / at most one person

$O_{\text{ExhDA+}\sigma\text{A}}\ \Diamond \max\ (0 \vee 1)$

$= \Diamond \max\ (0 \vee 1) \wedge \Diamond \max\ 0 \wedge \Diamond \max\ 1$

$= \Diamond \max\ 0 \wedge \Diamond \max\ 1$ (deontic/bouletic/...Free Choice)

'There is a world compatible with, e.g., what is required where John calls 0 or 1 people, and there is a world where he calls 0, and there is a world where he calls 1.'

The meaning we obtain now seems fine but, despite the usual strengthening effect of the Free Choice implicatures, as pointed out by Geurts & Nouwen (2007) and the literature thereafter, it also seems exceedingly weak. A much more natural reading of the CMN/SMN sentence above is one where it forbids calling more than one person. How may we obtain such a reading? Consider an exhaustification as below where we exhaustify relative to $O_{\text{ExhDA}}$ and $O_{\sigma\text{A}}$ separately and in this order.

(37) John may call less than two people / at most one person

$O_{\sigma\text{A}}\ O_{\text{ExhDA}}\ \Diamond \max\ (0 \vee 1)$

a. $O_{\text{ExhDA}}\ \Diamond \max\ (0 \vee 1),\ = \Diamond \max\ 0 \wedge \Diamond \max\ 1$ (prejacent to $O_{\sigma\text{A}}$ )

b. $O_{\text{ExhDA}}\ \Diamond \max\ (0),\ = \Diamond \max\ 0$ ($\sigma$A -alternative)

$O_{\text{ExhDA}}\ \Diamond \max\ (0 \vee 1 \vee 2),\ = \Diamond \max\ 0 \wedge \Diamond \max\ 1 \wedge \Diamond \max\ 2$ ($\sigma$A -alternative)

... ...

$= \Diamond \max\ 0 \wedge \Diamond \max\ 1 \wedge \neg(\Diamond \max\ 0 \wedge \Diamond \max\ 1 \wedge \Diamond \max\ 2) \wedge \neg \ldots$

$= \Diamond \max\ 0 \wedge \Diamond \max\ 1 \wedge \neg\Diamond \max\ 2 \wedge \neg \ldots$

The prejacent to $O_{\sigma\text{A}}$ is now $O_{\text{ExhDA}}\ \Diamond \max\ (0 \vee 1)$ which, through the mechanism shown above, yields the Free Choice effect $\Diamond \max\ 0 \wedge \Diamond \max\ 1$. Now, the $\sigma$A to this prejacent are going to be expressions of the form $O_{\text{ExhDA}}\ \Diamond \max\ \overline{[\![\text{little}]\!]\ (n) / }\ [\![\text{much}]\!]\ (n)$, where $n$ is any positive number. If we replace $n$ with $0$ we get an $\sigma$A $O_{\text{ExhDA}}\ \Diamond \max\ 0$. If we replace it with $2$ we get an $\sigma$A $O_{\text{ExhDA}}\ \Diamond \max\ (0 \vee 1 \vee 2)$. And so on. Due to $O_{\text{ExhDA}}$ , each of these $\sigma$A will be enriched with Free Choice implicatures. Note now that the strength ordering of these $\sigma$A with respect to the prejacent are not what they were without $\mathrm{O}_{\mathrm{ExhDA}}$. In the usual case, the stronger $\sigma$A of $\Diamond$(*less than two/at most one*) are $\sigma$A based on smaller numbers. However, the stronger $\sigma$A of $\mathrm{O}_{\mathrm{ExhDA}}$ $\Diamond$(*less than two/at most one*) are now $\sigma$A based on larger numbers. Computing $\mathrm{O}_{\sigma\mathrm{A}}$ then results in the assertion $\mathrm{O}_{\mathrm{ExhDA}}$ $\Diamond$(*less than two/at most one*) and the negation of $\mathrm{O}_{\mathrm{ExhDA}}$ $\Diamond$(*less than n/at most n-1*) for all $n > 2$. That will result in asserting that John can call 0 and he can call 1 but he can't call 2 and he can't call 3 etc. people – thus, that he can't call more than one. We have thus derived the mysterious upper-bounding inference of downward-monotone CMNs/SMNs under a possibility modal – it is a $\sigma$A -implicature computed atop a Free Choice effect. Note that this meaning is much stronger than the previous result we got from $\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}$; this also explains why it might be the preferred way to interpret the 'authoritative speaker' scope-splitting configuration $\Diamond \max$.

With this we conclude our discussion of CMNs/SMNs under overt modals. For the most part the discussion for $\max \Box$, $\max \Diamond$, and $\Box \max$ just replicates results that are already available in the literature – the most noticeable innovation being perhaps the fact that, instead of confining this analysis to SMNs, we treat both CMNs and SMNs on a par. The discussion of $\Diamond \max$ is however quite new, both in the way the basic exhaustification is computed as well as in the fact that it provides a solution to the long-standing and puzzling issue of the upper-bounding inference of downward-monotone modified numerals under a possibility modal. There are two empirical claims that this result is still at odds with. First, it has been claimed (e.g., by Geurts & Nouwen 2007, and the literature thereafter) that there is a contrast between *less than* and *at most* with respect to the strength of this upper-bounding inference, whereas on our account they are predicted to be exactly the same. I illustrate both these claims below with examples from Geurts & Nouwen (2007) and Kennedy (2015).

(38) a. You may have fewer than three beers.

b. You may have at most two beers.

(39) a. Third-year students are allowed to register for (fewer than) three classes, but they may register for more if they want to.

b. Third-year students are allowed to register for at most three classes, #but they may register for more if they want to.

Second, it has been claimed that *at least* under a possibility modal does not give rise to an analogous lower-bounding inference, although on our account that is precisely what we would expect (as the reader can verify by running the reasoning above for *more than* and *at least*). I illustrate this claim with an example from Chen (2018:254).

(40) a. Adam is allowed to register for at least three courses.

b. Adam is allowed to register for at most three courses.

I am not sure that I share these judgments, but to the extent that they are supported that would mean that we are still missing something.

## 4.4 Prediction: CMNs/SMNs under universal quantifiers

Consider a CMN/SMN sentence as below, where the CMN/SMNs is embedded under a universal quantifier and takes scope below it.

(41) Everyone called less than two people / at most one person.

$$\forall x[P(x) \rightarrow \max(\lambda d\,.\,\exists y[|y| = d \wedge P(y) \wedge C(x,y)]) \in \{0,1\}]$$

For convenience, let's abbreviate these truth conditions as $\forall x(0 \vee 1)$.

If we apply $\mathrm{O}_{\mathrm{ExhDA+}\sigma\mathrm{A}}$ we obtain the following results. (Since we have already seen computations like these many times, I will skip some of the steps.)

(42) Everyone called less than two people / at most one person.

$= \mathrm{O}_{\mathrm{ExhDA+}\sigma\mathrm{A}}\ \forall x(0 \vee 1)$

$= \forall x(0 \vee 1) \wedge \neg\mathrm{O}\,\forall x0 \wedge \neg\mathrm{O}\,\forall x1 \wedge \neg\forall x0$

$= \forall x(0 \vee 1) \wedge \neg\forall x0 \wedge \neg\forall x1$ (quantificational variability/Free Choice)

'For every $x$ the number of people they called was 0 or 1, and it is not the case that for all it was 0 and it is not the case that for all it was 1.'

The result is a quantificational variability effect. This is also an effect that is known in the literature on SMNs, and the more recent literature points out that, as we predict, it also exists in CMNs (Alexandropoulou, Dotlacil, McNabb, & Nouwen 2015). We again derived it in a similar way for both CMNs and SMNs, by applying $O_{ExhDA}$ over an operator with a universal meaning. Note that here too we find the same contrast between CMNs and SMNs as in the case of ignorance or embedding under an over necessity modal – CMNs are compatible with partial variation over the numbers in the domain but SMNs are not.

(43) Everyone called two people. Therefore, everyone called less than three / #at most two.

(44) Everyone called less than three / #at most two people, but not zero/none.

This prediction that CMNs/SMNs under $\Box_S$ / $\Box$ / $\forall$ should be so similar is expected on any account which derives ignorance from some form of domain alternatives. Thus, it is not specific to the present account. What is, however, innovative in the present account is the fact that, while most of the literature confines this analysis to just SMNs, we consistently extend it to CMNs also, at the same time offering a way to explain why these effects seem weaker in CMNs and SMNs.

## 4.5 Comparison to existing accounts

There have been a lot of proposals for ignorance in CMNs and SMNs. It would be impossible and also perhaps unfruitful to discuss each one by one. What we will do instead is consider various major choices, how the literature generally handled them, and how we did.

To begin with, let's consider the data and the empirical generalization at the root of the various analyses. The starting data are often the same – CMNs are compatible with a context of exact knowledge while SMNs are not. But virtually every single account of ignorance in CMNs and SMNs takes that to mean that CMNs don't give rise to ignorance while SMNs do; the goals then become to (1) derive ignorance in

SMNs and (2) not derive it in CMNs. On the other hand, we used similar ignorance (and anti-negativity) patterns in disjunction and indefinites, and then insights from unified theoretical approaches to these patterns, to argue that both CMNs and SMNs give rise to ignorance, but for the former it can be either weak or strong while for the latter it is always strong; our goals then became to (1) derive ignorance in both CMNs and SMNs and (2) find a way to explain why it can be weak in CMNs. Moreover, on our account we expect to find completely similar effects in cases of embedding under an overt modal or a quantifier. That these effects exist and reveal a parallelism between CMNs and SMNs is beginning to be recognized in the more recent literature also, although the idea that the same conclusion should be drawn for ignorance is still rejected, such that even attempts that derive the patterns of CMNs and SMNs under modals in parallel ways still continue to look for ways to suppress ignorance for CMNs in episodic contexts (cf., e.g., Buccola & Haida 2017.

Now, let's consider the broad approach to ignorance. Earlier accounts such as Geurts & Nouwen (2007) or Nouwen (2010) derive it by saying that SMNs (but not CMNs) have an epistemic modal in their truth conditions. This semantic approach ran into multiple problems. The hard-wired modal led to complications with embedding, and required special compositionality rules to yield the right results; at the end of their attempt, Geurts & Nouwen (2007) themselves note that similar difficulties had arisen for the hard-wired modal analysis of the Free Choice effect of disjunction, and that whatever turned out to be the best analysis for disjunction should be the right approach for SMNs also. The hard-wired modal also predicted the ignorance effect in SMNs to be present and equally strong in embedded environments, whereas it has become increasingly better known that under an overt modal the effect becomes optional (recall the 'insecure speaker' vs. the 'authoritative speaker' of modified numerals under overt modals). In response to these issues, paralleling similar developments in the literature on disjunction, the literature beginning with Büring (2008) has moved towards pragmatic accounts of ignorance on which SMNs are viewed as in some sense disjunctive, where they activate alternatives similar to those of disjunction, and where ignorance inferences are derived as implicatures arising from these alternatives. Our account is part of this latter trend.

Next, let's consider the narrow approach to ignorance. While there are many alternative-based approaches to ignorance, there is also quite a bit of variety. First, the alternatives used to derive ignorance are often quite different. For an SMN such as *at least three* the alternatives that have been proposed include alternatives of the form *at least* $n$ for $n > 3$ (Coppock & Brochhagen 2013); or alternatives of the form *at least n* and *at most n* (Mayr 2013) (Mayr doesn't actually discuss ignorance but uses these alternatives to derive the Free Choice effect under overt necessity modals); or two alternatives, *exactly three* and *more than three/at least four* (Büring 2008, Kennedy 2015, Spector 2015); or two sets of alternatives of the form *at least* $n$ and *exactly* $n$ (Schwarz 2016, Buccola & Haida 2017). Second, the way these alternatives are used to derive ignorance is also quite different – the first proposal in this list, where the alternatives are not symmetric, was combined with an Inquisitive Semantics view of ignorance, while the remaining proposals, where the altenatives are symmetric, all draw on the implicature approaches to disjunction and implement their analysis using some version of the neo-Gricean or grammatical theory of scalar implicatures. Our account is part of the latter set of proposals also.

Moving on, let's consider the details of how ignorance is computed.

First, let's consider the way the alternatives are derived. On some accounts these are simply stipulated (Büring 2008, Spector 2015) (the focus of these accounts is primarily to show the how basic analysis would work if the alternatives were such) and, as illustrated above, typically assumed only for SMNs (and most often demonstrated just for *at least*). This of course begs the question as to why SMNs should have such alternatives (cf. Coppock & Brochhagen 2013: In what sense are SMNs disjunctive?) and why CMNs (or BNs) can't have them also (cf. Nouwen 2015: Why can't we say that *more than three* has as alternatives *four* and *more than four*, or, for that matter, why can't we do that for *three* also?). More recent accounts thus generally try to justify these alternatives by coming up with various generation mechanisms. On one account BNs, CMNs, and SMNs all have Hackl (2000)-style truth conditions, BNs carry a default 'exact' meaning ($\max(\lambda d\,.\,\exists x[|x| = d\ldots] \wedge \ldots)$= 3), and BNs, CMNs, and SMNs based on the same numeral are assumed to be each other's alternatives (i.e., the $\sigma$A set is $\{\max = 3, \max > 3, \max < 3, \max \geq 3, \max \leq 3\}$); this is designed to ensure not only that SMNs in an episodic context have as stronger alternatives Büring (2008)'s alternatives, but also that CMNs have no stronger alternatives at all (Kennedy 2015). On other accounts numerals have their usual Horn alternatives but, in addition to that, the superlative modifiers have their own alternative too – *exactly* (Schwarz 2016, Buccola & Haida 2017); this latter type of accounts also predicts that similar alternatives should be possible for CMNs also (as we mentioned, the authors, e.g., Buccola & Haida, sometimes embrace the consequences of these alternatives for cases of embedding under overt modals but still suppress them for ignorance, continuing to believe that CMNs don't give rise to ignorance). While all these assumptions about how the alternatives are derived are in principle possible, I believe ours is more principled: The sense in which SMNs are disjunctive/have disjunction-like alternatives is because their truth conditions make reference to a domain, just like that of actual disjunction. We also embrace the other consequences of such an assumption about how the alternatives are generated: The truth conditions of CMNs make reference to a domain too, so they have the same type of alternatives also. Moreover, just as disjunction has scalar alternatives because it makes reference in its truth conditions to a scalar element, CMNs and SMNs do also. We have seen the consequences of these assumptions in Ch. 3 and in the present chapter also – both CMNs and SMNs are predicted to give rise to parallel effects arising from those alternatives, and these alternatives also interact in interesting ways to yield crucial results for both plain and embedded contexts.

Second, let's discuss the implicature calculation system. The various alternative-based approaches differ in whether they assume the neo-Gricean theory of implicatures (Kennedy 2015) or some version of the grammatical theory of implicatures (Spector 2015) or a combination of the two, sometimes also along with other theoretical tools (e.g., Schwarz 2016 uses Sauerland 2004's neo-Gricean mechanism for computing primary and secondary implicatures, but adds to that the notion of Innocent Exclusion characteristic of some versions of the grammatical theory of implicatures; Buccola & Haida 2017 assume a variant of the grammatical theory of implicatures and also Fox & Hackl 2006's Universal Density of Measurement assumption; etc.). However, they all try to capitalize on the same idea that we did, namely, the fact that computing an implicature relative to symmetric alternatives without an intervening modal fails, but computing it relative to the epistemic state of the speaker yields ignorance implicatures. Thus, the basic mechanisms through which ignorance is obtained are quite similar, and all draw on the alternative-based approaches to disjunction. However, some differences in the discussion arise from the way implicature calculation is assumed to work. For example, on some approaches the same DA are used to derive both implicatures of the type $\neg\Box_S \ldots$ and $\Box_S \neg \ldots$, and these sometimes clash, so there is a discussion for how to handle that clash (cf. the discussion of consistency preservation between 'primary' and 'secondary' Quantity implicatures in Sauerland 2004, Kennedy 2015, Schwarz 2016). On our syntactic approach to the way the exhaustivity operator checks off alternatives, an operator can only use a certain type of alternatives once, so this problem never arises. Then, on many approaches the null epistemic operator is assumed to be always present at the matrix level, which gives rise to some odd configurations for cases with overt modals (e.g., $\Box_S \Diamond(p \vee q)$ in Kennedy 2015). On our account, however, $\Box_S$ is conceptualized as a last resort mechanism, and since $O_{ExhDA}$ is already successful when applied directly across the overt modal, such configurations never arise. Another point of difference comes from the fact that, on some approaches that discuss embedding under a possibility modal, reference is made to the solution for the Free Choice effect in disjunction, but the idea of pre-exhaustifying the alternatives is adopted without however adopting other pieces that come with that type of analysis, or the general insight that ignorance too is just another manifestation of the Free Choice effect. Of course, all these theoretical options can be explored, but if the general consensus is that ignorance in modified numerals is as in disjunction and we should derive it in the same way, then our account seems to have an advantage, since it articulates a completely uniform account of this effect (and others) in both. Last but not least, while some of these accounts leave it open whether the effect thus derived is optional or obligatory (e.g., Kennedy 2015), our predictions are in general very clear – $O_{ExhDA}$ is obligatory, so parses without it or parses with it that crash are thrown out; at the same time, $\sigma$A - and DA -pruning help qualify the results in again fairly well-regulated ways.

To sum up, our account of ignorance in modified numerals is fundamentally similar to the alternative-based approaches, but differs from them in offering an arguably more general way both to derive the alternatives and to compute the implicatures. This generality is not merely a matter philosophical difference but has clear empirical consequences: We predict parallel ignorance effects – and in fact also other types of Free Choice effects with overt modals or quantifiers – in both CMNs and SMNs, but also that CMNs should be able to accommodate contexts of partial ignorance/knowledge – and in fact also other types of contexts with partial variation. Our preliminary introspective judgments support these predictions. In the next section we will also discuss some experimental results.

## 4.6 Fit to experimental data

As we could see in the previous section, there is a strong consensus in the literature that SMNs give rise to ignorance but CMNs don't. However, there has been a shift in this view in the recent literature. On the theoretical side Nouwen (2015) comments that the correct generalization seems to be not that SMNs are compatible with ignorance and CMNs are not; rather, what seems to be the case is that they are both compatible ignorance but SMNs are incompatible with exact knowledge. On the experimental side Westera & Brasoveanu (2014) show that both CMNs and SMNs are judged ignorant, at least for a *how many?* QUD. More recently, intrigued by Westera & Brasoveanu's results, Cremers et al. (2017) also conducted a series of experiments to further test these claims. Their experiments are directly relevant to our claim that both CMNs and SMNs give rise to ignorance as a default but CMNs can also accommodate contexts of certainty; for this reason we review them below.

Cremers et al. (2017) conduct three experiments with the goal of testing for ignorance in CMNs and SMNs. In each of these experiments they in fact test for a variety of things. For simplicity but also for clarity, we will try to simplify the discussion just to the issues that immediately concern us, and, to the extent that it is possible, abstract away from other issues that do not (e.g., QUD manipulations, overinformative answers, etc.).

In Exp. 1 the context is as follows: Mary is a player in a card game. At various stages of the game she can see part/all of her cards. At some point her friend stops by and asks her how many cards of a certain type she has. Mary gives a CMN/SMN answer. The task is to evaluate if the CMN/SMN utterance is appropriate given Mary's information state (no ignorance, ignorance). A sample stimulus is shown in Figure 4.1.


Figure 4.1: Cremers et al. (2017) Exp. 1: Stimulus illustrating the combination Situation = Ignorance, Quantifier = SMN, QUD = *How many?*.

The results revealed that for the CMN sentences there was no significant difference between the no ignorance vs. ignorance situation, while for the SMN sentences there was a significant difference – SMNs were judged worse in the no ignorance context. As the authors note, this finding supports the results of previous experiments such as those conducted by Geurts & Nouwen (2007) or Geurts et al. (2010), which showed that SMNs are penalized in contexts of speaker certainty but CMNs are not. On our account this is to be expected – CMNs can accommodate a state of speaker certainty, while SMNs can't.

In Exp. 2 both the context and the task change. The context introduces some situation with a limit, e.g., a hospital that has to have a certain minimum physicians present at all times, or an elevator that can carry a certain maximum load of people. Then some incident occurs. An investigation follows where an investigator tries to establish whether the minimum or the maximum has been violated. A witness replies with a CMN/SMN utterance. The task for the participants is to say whether they would conclude that the witness had exact knowledge or not. A sample stimulus is shown in Figure 4.2. Since the authors are interested in many things other than our main question, the stimulus they provide here illustrates a polar QUD rather than the *how many?* QUD that we had before. They however also tested for the *how many?* QUD, where the difference was that the stimuli did not explicitly mention a threshold, and the question was of the form *How many physicians were there at the hospital on that day?* or *How many people where there in the elevator when the incident occurred?*

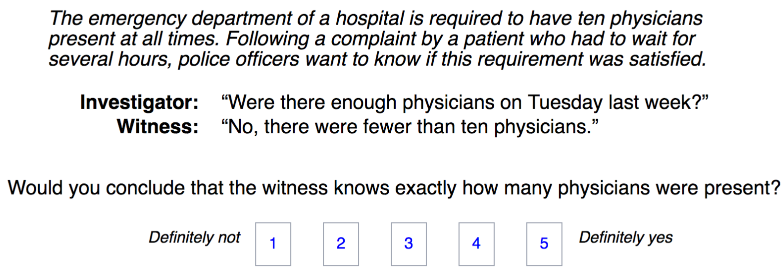



Figure 4.2: Cremers et al. (2017) Exp. 2: Stimulus illustrating the combination Quantifier = CMN, QUD = Polar, Context = Upward (minimum requirement).

The results showed no significant difference between CMNs and SMNs in how ignorant they were judged. However, they did show that the ignorance effect was in general sensitive to the QUD – both CMNs and SMNs were judged more ignorant with a *how many?* QUD than with a polar QUD. Let's discuss these results in turn. First, the fact that with a *how many?* QUD both CMNs and SMNs were judged on a par, and ignorant. Note that the *how many?* QUD context was one in which no threshold was mentioned at all. Thus, the context doesn't force us in any way to assume speaker knowledge, let alone certainty. As such, our prediction is that both CMNs and SMNs should give rise to ignorance, just as observed. Second, let's discuss the fact that with a polar QUD both CMNs and SMNs were judged on a par, and less ignorant. First, note that in this context the threshold is explicitly mentioned. Thus, the speaker is a speaker who must have knowledge about the number at least to the degree that s/he can answer whether the threshold has been violated. At the same time, the use of a CMN/SMN utterance doesn't have to indicate ignorance, it could simply be a way to maintain accuracy – that is, to give an indication that the speaker could be more precise, and might have been more specific, if it had been relevant. Thus, it might indicate a meaning of the form 'all the worlds compatible with what the speaker wants to convey are worlds where [range] is true and not all are worlds where [exact number 1] is true and not all are ...'. This would thus be a *bouletic* instead of an epistemic Free Choice effect. This is an effect that we alluded to briefly in Fn. 4 on p. 21. It can be captured by assuming that our familiar $\Box_S$ doesn't have to be epistemic, it can carry other modal flavors also. This would not be surprising at all but rather to be expected given similar effects in other epistemic indefinites of the type discussed in Chierchia (2013: Ch. 5), for some of which it has even been argued that the default null modal flavor is bouletic rather than epistemic. Thus an account along the lines that we have pursued could make sense of both these results of Exp. 2.

Since, depending on one's assumptions (e.g., if different from our own), the results of Exp. 1 and Exp. 2 might seem at odds with each other, Cremers et al. (2017) ran a third experiment. In Exp. 3 they combined the context of Exp. 2 with the task of Exp. 1. Thus, as in Exp. 2, the context again introduced some situation with a limit, e.g., a minimum or a maximum requirement – however, as in Exp. 1, there was variation in whether the knowledge of the witness as specified by the context was approximate or precise. Then there was an incident, an investigation trying to establish whether the limit has been violated – polar or *how many?* QUD – and a witness replying with a CMN/SMN utterance. However, as in Exp. 1, the task was for the participants to say whether the CMN/SMN answer of the witness was appropriate or not. A sample stimulus is shown in Fig. 4.3.

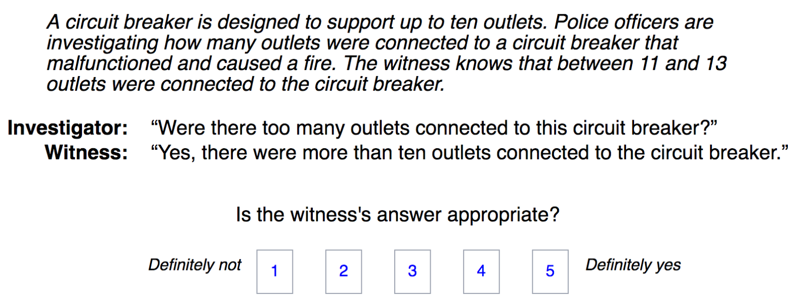



Figure 4.3: Cremers et al. (2017) Exp. 3: Stimulus illustrating the combination Knowledge = Approximate, Quantifier = CMN, QUD = Polar, Context = Downward (maximum).

The results were very similar to those found in Exp. 1: SMNs were judged significantly worse in the precise speaker knowledge condition, whereas no significant difference was found for CMNs between the precise and the approximate speaker knowledge conditions. On our account these results are expected for the same reason as they were for Exp. 1 – we expect both CMNs and SMNs to be compatible with speaker ignorance, but SMNs to be incompatible with speaker knowledge.

Cremers et al. (2017)'s interpretation of the results of these three experiments is somewhat different than our own. For example, in Exp. 3 they take the fact that there was a significant difference between SMNs but not CMNs in the approximate vs. precise condition as evidence of ignorance inferences in the former but not the latter, whereas on our approach that just means that CMNs can accommodate precise knowledge. In spite of the different interpretation, the experimental findings themselves provide support for our story. And Exp. 2, which, like Westera & Brasoveanu (2014), shows that in a context that doesn't force speaker certainty there is no significant difference between CMNs and SMNs, provides crucial support for our basic line than in a default setting both CMNs and SMNs give rise to parallel ignorance effects.

But, as we saw in our discussion of CMNs and SMNs under overt modals, our account predicts that CMNs and SMNs should give rise to parallel Free Choice effects in other contexts also. For experimental support for parallel effects for CMNs and SMNs with respect to quantificational variability effects see Alexandropoulou et al. (2015). More such experimental investigation would be welcome and timely. As Nouwen, Alexandropoulou, & McNabb (2018) conclude in their recent survey of experimental work on modified numerals: "What we originally thought to be about the absence or presence of inferences turns out to be about something much more subtle. The difference between superlatives and comparatives is due to the degree to which the various inferences are hard-wired in the lexical entry. If this kind of finding turns out to be robustly supported by future experimental evidence, then it is up to the theoretical frameworks to make sense of this. What could this hard-wiring be? [...and] Why is there no cross-linguistic variation in what kind of modifiers end up having stronger inferences?" While our account provides an answer to the what question, the why question at this point remains an open issue.

## 4.7 Summary

In this chapter we recalled our analysis of ignorance in *or* and *some* $NP_{SG}$ in Ch. 1 and extended it to CMNs and SMNs also; as we could see, the basic approach that worked for the former worked for the latter too, and the only small adjustments came from differences related to the different nature of the truth conditions and of the domain in one case versus the other. In deriving ignorance we made crucial use of the truth conditions and alternatives obtained in Ch. 2 to derive the ignorance patterns: We showed that exhaustification of CMNs/SMNs relative to pre-exhaustified subdomain alternatives across a modal yields a total Free Choice effect, which for episodic contexts, due to a null epistemic necessity modal, is ignorance; CMNs can prune their set of subdomain alternatives and that captures why they can also accommodate certainty. After that, we showed that the scalar implicatures of CMNs and SMNs discussed in Ch. 3 clash with ignorance and show how that captures the scalar implicature patterns of CMNs and SMNs in episodic contexts. Next, we looked at the predictions of our basic approach to ignorance for CMNs and SMNs under overt modals. In our comparison to the existing literature we noticed that our theory is fundamentally similar to the alternative-based approaches to modified numerals, but offers both conceptual and empirical advantages over the existing accounts. Finally, we reviewed some experimental evidence regarding our most controversial empirical claim, namely, that it is not just SMNs that exhibit ignorance/variability effects, but CMNs also, and showed that it supports our claim.

In the next chapter we turn to our next (and, for now, last) major goal for modified numerals, which is to provide an account for their behavior with respect to anti-negativity.

# Chapter 5

# Anti-negativity

## 5.1 Deriving anti-negativity

Recall our starting patterns regarding anti-negativity: CMNs are fine in the scope of clausemate negation but SMNs are not, and both are acceptable in downward-entailing environments such as the antecedent of a conditional and the restriction of a universal.

(1) (= (16))

a. John didn't call more than two people. ✓*not* > CMN

b. #John didn't call at least three people. # *not* > SMN

(2) (= (17))

a. If John called more than two people, he won. ✓*if* > CMN

b. If John called at least two people, he won. ✓*if* > SMN

(3) (= (18))

a. Everyone who called more than two people won. ✓*every* > CMN

b. Everyone who called at least three people won. ✓*every* > SMN

As we noticed already in Ch. 1, with respect to anti-negativity CMNs are like *or* and SMNs are like *some* $NP_{SG}$. As in the case of ignorance, the key for anti-negativity again had to do with the pre-exhaustified subdomain alternatives. Let's first remind ourselves of the truth conditions and alternatives of CMNs and SMNs (from §2.6; rearranged to list the DA first).

(4) More/less than three people quit.

a. $\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \overline{[\![\text{much/little}]\!]\,(3)}$ (assertion)

b. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in D' \mid D' \subset \overline{[\![\text{much/little}]\!]\,(3)}\}$ (DA)

c. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \overline{[\![\text{much/little}]\!]\,(n)} \mid n \in S\}$ ($\sigma$A)

(5) At least/most three people quit.

a. $\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in [\![\text{much/little}]\!]\,(3)$ (assertion)

b. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in D' \mid D' \subset [\![\text{much/little}]\!]\,(3)\}$ (DA)

c. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in [\![\text{much/little}]\!]\,(n) \mid n \in S\}$ ($\sigma$A)

Again, we will note that the truth conditions and alternatives of CMNs and SMNs are pairwise equivalent.

(6) Less than two people quit / At most one person quit

a. $\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \underbrace{\{0, 1\}}_{\text{from } \overline{[\![\text{little}]\!](2)}/[\![\text{much}]\!](1)}$ (assertion)

b. $\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \{0\}$ (DA)

$\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \{1\}$ (DA)

c. $\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \underbrace{\{0\}}_{\text{from } \overline{[\![\text{little}]\!](1)}/[\![\text{much}]\!](0)}$ ($\sigma$A)

$\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \underbrace{\{0, 1, 2\}}_{\text{from } \overline{[\![\text{little}]\!](3)}/[\![\text{much}]\!](2)}$ ($\sigma$A)

$\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \underbrace{\{0, 1, 2, 3\}}_{\text{from } \overline{[\![\text{little}]\!](4)}/[\![\text{much}]\!](3)}$ ($\sigma$A)

$\ldots$

As usual, wherever not confusing we will abbreviate these as below.

(7) Less than two people quit / At most one person quit

a. $0 \vee 1$ (assertion)

b. $0$ (DA )

$1$ (DA )

c. $0$ ($\sigma$A )

$0 \vee 1 \vee 2$ ($\sigma$A )

$0 \vee 1 \vee 2 \vee 3$ ($\sigma$A )

$\ldots$

For *or* and *some* $NP_{SG}$ we said that the overall solution to anti-negativity lay with the assumption that these items must be exhaustified via $\mathrm{O}^{\mathrm{S}}_{\mathrm{ExhDA}}$ (if any presuppositions are present, they must be factored in), but *some* $NP_{SG}$ additionally requires that the result must lead to Proper Strengthening. We then showed how this captured why *or* is fine under negation while *some* $NP_{SG}$ isn't, and also why both are fine in the antecedent of a conditional or the restriction of a universal. For CMNs and SMNs we will say the same: While both CMNs and SMNs must be exhaustified via $\mathrm{O}^{\mathrm{S}}_{\mathrm{ExhDA}}$ , SMNs must also satisfy Proper Strengthening. This captures their patterns. We will show this below.

First, consider the exhaustification $\mathrm{O}^{\mathrm{S}}_{\mathrm{ExhDA}}$ of a CMN/SMN under negation. (As for *or*/*some* $NP_{SG}$ , we continue to assume that $\sigma$A are always factored in, but they don't matter for the present discussion, so we leave them out.) There are no presuppositions, so $\mathrm{O}^{\mathrm{S}}_{\mathrm{ExhDA}}$ is actually plain $\mathrm{O}_{\mathrm{ExhDA}}$ . We can represent the $\mathrm{O}_{\mathrm{ExhDA}}$ parse, its prejacent, and its DA schematically as below.

(8) John didn't call less than two people / at most one person

$\mathrm{O}_{\mathrm{ExhDA}}\ \neg(0 \vee 1)$

a. $\neg(0 \vee 1)$ (prejacent)

b. $\neg 0$ (DA )

c. $\neg 1$ (DA )

As usual, $\mathrm{O}_{\text{ExhDA}}$ will assert the prejacent, (9-a), and negate the pre-exhaustifications of the DA , (9-b).

(9) $\mathrm{O}_{\text{ExhDA}}\ \neg(0 \vee 1)$

a. $\neg(0 \vee 1) \wedge$

b. $$\underbrace{\neg \underbrace{\mathrm{O}\,(\neg 0)}_{\substack{\neg 0 \wedge \neg(\neg 1),\ = \neg 0 \wedge 1 \\ \text{already excluded by the prejacent}}}}_{\text{can't satisfy PS}} \wedge \underbrace{\neg \underbrace{\mathrm{O}\,(\neg 1)}_{\substack{\neg 1 \wedge \neg(\neg 0),\ = \neg 1 \wedge 0 \\ \text{already excluded by the prejacent}}}}_{\text{can't satisfy PS}}$$

Each of the ExhDA is individually incompatible with the assertion and is therefore already excluded by it. Their negation thus doesn't add any new information. As such, $\mathrm{O}_{\text{ExhDA}}$ is vacuous. Now, if SMNs require PS but CMNs don't, that means that they can't tolerate this result, while CMNs can. In turn, this captures why SMNs are bad under negation while CMNs are fine. Note also that inserting a $\Box_S$ in between $\mathrm{O}_{\text{ExhDA}}$ and its target doesn't change anything.

Now, consider the exhaustification $\mathrm{O}^{\mathrm{S}}_{\text{ExhDA}}$ of a CMN/SMN embedded in the antecedent of a conditional or the restriction of a universal, (10). As for *or/some $NP_{SG}$* , to discuss the two environments together, we will abbreviate the world variable $w$ from the conditional and the individual variable $x$ from the universal as $v$, and also use $0, 1$ to represent 'John called 0/1 people' in the first case and 'called 0/1 people' in the second. Then, since these environments also contain an existential presupposition, $\mathrm{O}^{\mathrm{S}}_{\text{ExhDA}}$ cannot be reduced to $\mathrm{O}_{\text{ExhDA}}$ . Instead, its prejacent will consist of both the truth-conditional and the presuppositional component of the assertion, and the DA will have this shape also. The exhaustification parse, the prejacent, and the DA are then as below. In preparation for the computation to come, underneath each DA we also show the negation of its pre-exhaustification.

(10) If John called less than two people / at most one person, he lost.

Everyone who called less than two people / at most one person lost.

$\mathrm{O}^{\mathrm{S}}_{\text{ExhDA}}\ \forall v[0_v \vee 1_v \rightarrow L_v]$

a. $\forall v[0_v \vee 1_v \to L_v] \wedge \exists v[0_v \vee 1_v]$ (prejacent)

b. $\forall v[0_v \to L_v] \wedge \exists v[0_v]$ (DA )

(i) $\neg\text{O}\ (\forall v[0_v \to L_v] \wedge \exists v[0_v])$ (negation of ExhDA )

$$= \neg((\forall v[0_v \to L_v] \wedge \exists v[0_v]) \wedge \neg(\forall v[1_v \to L_v] \wedge \exists v[1_v]))$$

$$= (\forall v[0_v \to L_v] \wedge \exists v[0_v]) \to (\forall v[1_v \to L_v] \wedge \exists v[1_v])$$

c. $(\forall v[1_v \to L_v] \wedge \exists v[1_v])$ (DA )

(i) $\neg\text{O}\ (\forall v[1_v \to L_v] \wedge \exists v[1_v])$ (negation of ExhDA )

$$= \neg((\forall v[1_v \to L_v] \wedge \exists v[1_v]) \wedge \neg(\forall v[0_v \to L_v] \wedge \exists v[0_v]))$$

$$= (\forall v[1_v \to L_v] \wedge \exists v[1_v]) \to (\forall v[0_v \to L_v] \wedge \exists v[0_v])$$

$\text{O}^{\text{S}}_{\text{ExhDA}}$ will as usual assert the prejacent, (11-a), and the negations of the pre-exhaustifications of the DA , (11-b).

(11) If John called less than two people / at most one person, he lost.

Everyone who called less than two people / at most one person lost.

$\text{O}^{\text{S}}_{\text{ExhDA}}\ \forall v[0_v \vee 1_v \to L_v]$

a. $\forall v[0_v \vee 1_v \to L_v] \wedge \exists v[0_v \vee 1_v] \wedge$

b. $\underbrace{\neg\text{O}\ (\forall v[0_v \to L_v] \wedge \exists v[0_v])}_{(\forall v[0_v \to L_v] \wedge \exists v[0_v]) \to (\forall v[1_v \to L_v] \wedge \exists v[1_v])} \quad \wedge \quad \underbrace{\neg\text{O}\ (\forall v[1_v \to L_v] \wedge \exists v[1_v])}_{(\forall v[1_v \to L_v] \wedge \exists v[1_v]) \to (\forall v[0_v \to L_v] \wedge \exists v[0_v])}$

As usual, the negations of the ExhDA in (11-b) are equivalent to implications. Each implication can be true if both its terms are true, if both are false, or if the first term is false and the second is true. As for *or/some NP$_{SG}$* , each term consists of the conjunction of a universal term coming from the assertion and of an existential term coming from the presupposition, and the universal term is entailed by the assertion, so only the existential term has the potential to lead to strengthening. Going through the various possibilities for the implication to be true, we then find the following. The true-true case yields $\exists v[0_v] \wedge \exists v[1_v]$. This is a case that strengthens the utterance from a meaning where it presupposes that there is an accessible world where John called 0 or 1 people / that someone called 0 or 1 people to a meaning where it presupposes that there is an accessible world where John called 0 people and there is an accessible world where John called 1 person / that someone called 0 people and someone called 1 person. (Note that this is exactly like the Free Choice implicatures arising from $\mathrm{O}_{\mathrm{ExhDA}}\ \Diamond(0 \vee 1)$.) The false-false case yields $\neg\exists v[0_v] \wedge \neg\exists v[1_v]$. (Recall that the universal component is entailed by the prejacent, so it cannot be false, so falsity in each term must come from the existential component.) This clashes with the presuppositional component of the prejacent ($\exists v[0_v \vee 1_v]$), so it is ruled out. However, it can be rescued by prefixing the assertion and presupposition (and, as a consequence, also their alternatives) with $\Box_S$, in which case the result becomes $\neg\Box_S\ \exists v[t_v] \wedge \neg\Box_S\ \exists v[s_v]$. This is a case that strengthens the utterance from a meaning where it presupposes that the speaker is certain there is an accessible world where John called 0 or 1 people / that someone called 0 or 1 people to a meaning where it additionally presupposes that the speaker is ignorant whether there is an accessible world where John called 0 people and ignorant whether there is an accessible world where John called 1 person. (Note that this is similar to the Free Choice implicatures arising from $\mathrm{O}_{\mathrm{ExhDA}}\ \Box_S\ (0 \vee 1)$.) Finally, the false-true case is not possible, since false-true for, e.g., the first implication would yield $\neg\exists v[0_v] \wedge \exists v[1_v]$, which would however make the second implication false by forcing its consequent to be false.

Altogether, this discussion shows that $\mathrm{O}^{\mathrm{S}}_{\mathrm{ExhDA}}$ of a CMN/SMN in the antecedent of a conditional or the restriction of a universal can lead to proper strengthening. This captures why not only CMNs but also SMNs are fine in these environments.

To sum up, if both CMNs and SMNs undergo obligatory $\mathrm{O}^{\mathrm{S}}_{\mathrm{ExhDA}}$ but SMNs additionally require proper strengthening, then this captures why CMNs are fine in the scope of negation while SMNs are not, and also why they are both fine in the antecedent of a conditional or the restriction of a universal. Thus, the same approach that helped us capture anti-negativity in *or*/*some NP$_{SG}$* helped us capture it in CMNs/SMNs also.

Now, for *some NP$_{SG}$* we also discussed a variety of other configurations involving embedding in downward-entailing environments. Do SMNs show the same patterns as *some NP$_{SG}$* in these environments also?

First, we showed that *some NP$_{SG}$* also seemed to be degraded under a downward-entailing operator such as *few*; that it was also degraded under a negative attitude such as *not think* or *doubt* / extra-clausal negation; that it improved in the scope of a negation if the negation was itself embedded under an additional downward-entailing environment (rescuing), although the degree of improvement varied somewhat depending on the exact additional downward-entailing environment; that it improved in the scope of negation if a universal operator came in between (shielding). For convenience, we repeat those examples below.

(12) Few people called some student. ?? $few > some\ NP_{SG}$

(13) I don't think that John called some student. ?? $not > [_{CP}\ some\ NP_{SG}$

(14) a. I doubt John didn't call some student. ?$doubt > not > some\ NP_{SG}$

b. If John didn't call some student, I'd be surprised. ✓$if > not > some\ NP_{SG}$

(15) John didn't give every book to some student. ✓$not > every > some\ NP_{SG}$

(16) Mary doesn't know that John called some student. ✓$not > know > some\ NP_{SG}$

All these judgments were based on very limited and informal surveys, so there is certainly need for more investigation before we can take them for granted. However, at this point, what are the intuitions about SMNs? Pooling together claims from the literature for SMNs in these environments, we find the following patterns. (With the exception of the penultimate, shielding, example, all the examples are from Spector 2014, 2015.)

(17) Few people believe that Mary is at least 20 years old. ?? *few* > SMN

(18) John doesn't think Mary is at least 20 years old. ?? *not* > [$_{CP}$ SMN

(19) If Jack were not at least 40 years old, ... ✓*if* > *not* > SMN

(20) John didn't loan every book to at least three students. ✓*not* > *every* > SMN

(21) John doesn't know that Mary is at least 20 years old. ✓*not* > *know* > SMN

These patterns suggest that SMNs are essentially like *some* $NP_{SG}$ – they don't take into account implicatures to satisfy PS (which is why they are bad under *few*), the PS requirement must be satisfied globally (which is why they are bad even in the scope of an extraclausal negation), they improve if under an even number of downward-entailing operators (that makes the prejacent to global exhaustification positive, so $O^{PS}_{ExhDA}$ succeeds for the reasons it does in the episodic case), they improve in the presence of an intervening universal operator (for the same reasons as we saw for an intervening modal in Ch. 4), or in the presence of an intervening factive (due to the fact that factives carry a presupposition and proper strengthening can be met thanks to this presupposition).

To sum up, just as in the case of *or/some* $NP_{SG}$ , the key to anti-negativity lies with obligatory exhaustification relative to subdomain alternatives coupled with a proper strengthening requirement; this captures anti-negativity. Acceptability in the antecedent of a conditional or the restriction of a universal is due to the fact that those environments are presuppositional, and if exhaustification takes that into account, then proper strengthening can be satisfied. As for *or/some* $NP_{SG}$ , we also cursorily reviewed further cases of embedding under downward-entailing operators and noticed that the same general trends that have been noticed in the literature for other items with anti-negativity, often called PPIs (in particular, global PPIs), are true of SMNs also. While the data patterns reported in this segment could all benefit from further scrutiny, the general approach to anti-negativity that we have pursued here seems to offer us a solid starting point for capturing these further patterns also. We discuss both some of these other data patterns and some potential extensions to the present analysis again in §5.4.

## 5.2 Negation and scalar implicatures

At the end of Ch. 3 on scalar implicatures we suggested that the puzzle regarding the missing scalar implicatures of CMNs and SMNs in episodic contexts has to do with ignorance, and the puzzle regarding the missing scalar implicatures of all of BNs, CMNs, and SMNs also with ignorance, but of a different kind, specific to this environment. We already saw a solution to the first puzzle in §4.2. Now we are ready to discuss the negation case. In what way does it give rise to ignorance for all of BNs, CMNs, and SMNs, and how does it interact with the way in which CMNs and SMNs typically give rise to ignorance, that is, via their DA ?

First, let's remind ourselves of the puzzle. If BNs, CMNs, and SMNs activate scalar alternatives, then an exhaustification of, e.g., *three*, *more than three*, or *at least three* relative to these alternatives across negation should negate the scalar alternatives based on lower numerals and, via indirect scalar implicature, give rise to an 'exactly two' meaning. We illustrate this below.

(22) $\mathrm{O}_{\sigma \mathrm{A}}$ (John didn't call three / more than two / #at least three people)
= John didn't call three / more than two / #at least three people and it is not the case that he didn't call two / more than one / at least two
$\sim$ John called two / more than one / at least two and he didn't call three / more than two / at least three
# $\rightsquigarrow$ He called exactly two.

While indirect implicatures are generally attested, this particular 'exactly'-inducing implicature is not, so it is undesirable and must be ruled out just as we ruled out its undesirable counterpart for CMNs and SMNs in the positive episodic case. Recall also that this is not a problem specific to our account – any implicature-based theory of BNs will have to address it as well.

The solution I propose is as follows: If in the positive case we said that the 'exactly'-inducing implicature was generated but ruled out, I propose that in this case it is not generated in the first place. Specifically, I propose that an exhaustification above negation of a numeral / scalar item more generally in the scope of negation will proceed relative to two types of alternatives. First, the Horn-scale alternatives coupled with the negation that is part of the prejacent to the exhaustivity operator, as the ones that yielded the undesirable 'exactly' implicature above. This is the traditional notion of a scalar alternative under negation that we used above and which says that the alternatives to, e.g., *not three* are things of the form *not n*. Second, in addition to these, also alternatives obtained by deleting the negation. Thus, for every alternative of the form *not n*, also its positive counterpart *n*. I illustrate the result of exhaustifying across negation relative to a set of alternatives of this form below, showing how it would work out for *three* / *more than three* /

*at least three*. First, note that the alternatives that end up being excluded are still those based on smaller numerals – the prejacent already rules out (entails or is incompatible with) the ones based on larger numerals, whether they are of the form *not n* or *n*. (Of course, for *less than* and *at most* it would be those based on larger numerals.) But now, instead of just negative alternatives, we also have their positive counterparts. If we exhaustify like this without any intervening element, the result is a crash, since the newly added positive alternatives clash with the old negative ones.

(23) John didn't call three / more than two / #at least three people

$$O_{\sigma A}\, \neg(3 \vee 4 \vee \dots)$$

$$= \underbrace{\neg(3 \vee 4 \vee \dots)}_{\neg\geq 3/>2/\geq 3} \wedge \underbrace{\neg\underbrace{\neg(2 \vee 3 \vee 4 \vee \dots)}_{\sigma A\, =\neg\geq 2/\neg>1/\neg\geq 2} \wedge \neg\underbrace{(2 \vee 3 \vee 4 \vee \dots)}_{\sigma A\, =\geq 2/>1/\geq 2}}_{\perp} \wedge \underbrace{\neg\underbrace{\neg(1 \vee 2 \vee 3 \vee \dots)}_{\sigma A\, =\neg\geq 1/\neg>0/\neg\geq 1} \wedge \dots}_{\perp}$$

However, if we exhaustify across $\Box_S$ – our usual exhaustification rescuing operator – the result is consistent and yields ignorance.

(24) John didn't call three / more than two / #at least three people

$$O_{\sigma A}\, \Box_S\, \neg(3 \vee 4 \vee \dots)$$

$$= \Box_S\, \neg(3 \vee 4 \vee \dots) \wedge \neg\Box_S\, (2 \vee 3 \vee 4 \vee \dots) \wedge \neg\Box_S\, \neg(2 \vee 3 \vee 4 \vee \dots) \wedge \neg\Box_S\, (1 \vee 2 \vee 3 \vee 4 \vee \dots) \wedge \neg\Box_S\, \neg(1 \vee 2 \vee 3 \vee 4 \vee \dots) \wedge \dots$$

'In all the worlds compatible with what the speaker believes the relevant number is not three or more (so, not one of $3 \vee 4 \vee \dots$) but the speaker is not sure which one of the remaining numbers it is (so, not sure if it is one of $2 \vee 3 \vee 4 \vee \dots$ or one of $1 \vee 2 \vee 3 \vee 4 \vee \dots$ or one of $0 \vee 1 \vee 2 \vee 3 \vee 4 \vee \dots$.'

Moreover, a continuation of the form *but he did call one*, $\Box_S\, (1 \vee 2 \vee 3 \vee 4 \vee \dots)$, would force the pruning of the ignorance-inducing $\sigma$A that would clash with it – $\neg\Box_S\, (1 \vee 2 \vee 3 \vee 4 \vee \dots)$. It would also impose a lower boundary that would narrow down the original scale imposed by the assertion. As we saw for domain alternatives, after such narrowing down, the ignorance effect yielded via, this time, the scalar alternatives will simply range over the numerals that are left, again capturing the fact that *John didn't call three …but he did call one* still gives rise to ignorance about whether one or two.

(25) $\Box_S\,(1 \vee 2 \vee 3 \vee 4 \vee \ldots) \wedge \Box_S\,\neg(3 \vee 4 \vee \ldots) \wedge \neg\Box_S\,(2 \vee 3 \vee 4 \vee \ldots) \wedge \neg\Box_S\,\neg(2 \vee 3 \vee 4 \vee \ldots)$

'In all the worlds compatible with what the speaker believes John called 1 or more but not 3 or more (so, 1 or 2), and it is not the case that in all the worlds he called 2 or more (so, ignorance whether 2) and it is not the case that in all the worlds he didn't call 2 or more (so, whether exactly 1).'

Thus, saying that a scalar item under negation exhaustified above negation has both positive and negative alternatives not only does not predict the undesirable 'exactly' meaning but instead captures an effect that always seems to accompany the negation of scalar items, namely, ignorance.

But what about other types of downward-entailing environments? Do we want to say that we have both positive and negative alternatives there also? First, in those cases the shape of the downward-entailing operator is quite different, and two sets of alternatives, both negative and positive, cannot be justified as easily as for negation, where we could simply assume a Fox & Katzir (2011)-style deletion mechanism deriving from the negative set also a positive set. Second, in those cases we don't need anything beyond the Horn-alternatives that we have already, since those already gave us the right results. (E.g., the indirect scalar implicatures predicted for, e.g., the antecedent of a conditional or the restriction of a universal seemed just right.) For both these reasons I will assume that this addition to the traditional scalar alternatives of BNs, CMNs, and SMNs is restricted to negation.

Of course, for negative sentences an 'exactly' meaning can also be obtained by exhaustifying below negation. We show this below.

(26) John didn't call three / more than two / #at least three people

$\Box_S\,\neg O_{\sigma A}\,(3 \vee 4 \vee \ldots)$

$= \Box_S\,\neg((3 \vee 4 \vee \ldots) \wedge \neg(4 \vee 5 \vee \ldots))$

$= \Box_S\,\neg(= 3)$

How might we handle this?

First, for BNs this is a possibility we want. As we already discussed in Ch. 3 for examples like *Neither of them read three of the articles on the syllabus – Kim read two and Lee read four*, a BN like *three* under negation can be followed by a higher numeral statement, e.g., *four*. This would be contradictory if *not three* were interpreted as *not three or more*, but is fine if it is interpreted as *not* $O_{\sigma A}$ *(three) = not exactly three*.

(27) John didn't call three people – he called four.

Second, it is not obvious to me that the same isn't true for CMNs. The example below, parallel to the BN example, seems to work just fine. (There is some special prosody on *more than four*, but that is generally true of embedded scalar implicatures, as Chierchia 2013 argues for disjunction.)

(28) John didn't call more than three people – he called more than four.

However, as we also mentioned in Ch. 3 for BNs, implicatures computed below downward-entailing operators lead to weakening, so we expect these implicatures to be disprefered, and not just for CMNs, but for BNs also.

Last, what about SMNs? For all these cases, we predict the same results as for CMNs and SMNs. However, recall that CMNs and SMNs must also be exhaustified relative to their ExhDA , and recall also from the previous section that for all these configurations exhaustification fails to lead to a properly stronger meaning. Since CMNs tolerate this result, we expect them to be fine insofar as the ExhDA are concerned, and in addition to exhibit the ignorance result arising from exhaustifying relative to this new set of $\sigma$A , as shown above. However, since SMNs don't tolerate vacuous exhaustification relative to their ExhDA , they are out for independent reasons, although otherwise they might have given rise to the same effects as CMNs and SMNs. In fact, consider an SMN under negation, e.g., *John didn't call at least three people*; if we manage to get past the oddness of this embedding, what it means is arguably the same as its BN counterpart – the speaker knows that he didn't call three or more – and it gives rise to the same ignorance effect – i.e., the speaker doesn't know the exact number that he did call.

To sum up, if prejacents containing negation and a scalar item actually have as scalar alternatives both the usual negated Horn alternatives as well as positive Horn alternatives, without negation, then $O_{\sigma A}$ doesn't generate the bad 'exactly' meanings in the first place – instead, it yields ignorance. This effect doesn't actually interact with exhaustification relative to the pre-exhaustified subdomain alternatives – if the latter crashes for its own usual reasons, the only consequence is that this effect is rendered irrelevant also.

## 5.3 Comparison to existing accounts

Anti-negativity in SMNs has been noticed as early as ignorance (it is a major point of Nilsen 2007 and it is also mentioned in Geurts & Nouwen 2007, the paper that arguably started the recent debate on ignorance in modified numerals). However, while the vast majority of the existing theories of CMNs and SMNs engage with ignorance, they mostly don't engage with anti-negativity. Still, a survey of the literature still reveals three main existing approaches.

First, there is the modal approach. As the reader might recall from our literature review for ignorance, Geurts & Nouwen (2007) argue that SMNs contain an epistemic modal in their truth conditions. For example, they argue that an SMN utterance such as *Betty had at least three martinis* says that in every world compatible with what the speaker believes Betty had three martinis and there are worlds compatible with what the speaker believes where she had more. Although their main focus is on ignorance, Geurts & Nouwen also note the anti-negativity facts and suggest that their proposal that SMNs contain an epistemic modal in their meaning might also capture anti-negativity, since epistemic modals are known to resist embedding under negation. At the same time, they also note that, although hard to process, SMNs do have a perfectly intelligible meaning, and their modal analysis of SMNs doesn't capture it. They point out the same problem for embedding in the antecedent of a conditionals. We illustrate both below.

(29) Betty didn't have at least three martinis.

# Predicted meaning: It is not the case that it must be the case that Betty had three martinis and it may be that she had more.

(30) If Betty had at least three martinis, she must have been drunk.

# Predicted meaning: If it must be the case that Betty had three martinis and it may be that she had more than three, then she must have been drunk.

Thus, the modal approach doesn't really offer a solution to anti-negativity.

Second, there is the speech act approach. Cohen & Krifka (2011, 2014) propose that for SMNs falsity follows *semantically* – by saying *John petted at least three rabits* the speaker denies that John petted zero, one, or two rabbits – whereas truth follows *pragmatically* – for all the values that the speaker did not deny, the hearer concludes by implicature that the speaker *grants* that John petted that number of rabbits, i.e., that he petted three, four, five, ..., rabbits. Thus, to interpret SMNs, one must compute an implicature. But, since scalar implicatures tend to be cancelled in downward-entailing environments, this explains why SMNs are infelicitous under negation.

Of course, this raises the issue of why SMNs are felicitous in other downward-entailing environments such as the antecedent of conditionals or the restriction of universals. Following Kay (1992), Cohen & Krifka suggest that this is a different meaning of SMNs, a so-called 'evaluative' sense – an example such as *Everybody who donates at least 10 BGN will get a thank you card* suggests that donaitng 10 BGN is a good thing – and that this sense comes with its own constraints – e.g., *Everybody who donates at least 10 BGN #is a fool* is degraded. They give many examples in support of this 'evaluative' reading, and its sensitivity to the polarity of the property in the continuation. However, as Cohen & Krifka themselves note, their prediction is that, so long as they carry an evaluative flavor, SMNs should be able to occur in any downward-entailing environment, yet in a negative declarative they seem to be bad regardless of whether they are used evaluatively, no matter how hard we try to satisfy any additional restrictions on the polarity of the property that might come from evaluativity (e.g., by combining SMNs in this environment with a property that is generally understood as positive, e.g., *being centrally located*, or a property generally understood to be negative, e.g., *being far away*).

(31) a. ??This hotel isn't at least centrally located.

b. ??This hotel isn't at least far away.

The speech act + evaluativity approach thus does not seem to offer a solution to anti-negativity either. On the other hand, Cohen & Krifka's discussion of evaluativity does reveal interesting additional restrictions on the use of superlative modifiers in general and SMNs in particular. Our account so far does not acknowledge or capture them, so we are missing something. We will address this to some extent in §5.4.3.

Last but not least, there is the alternative-based approach. Building on the alternative-based approaches to ignorance and the literature on PPIs, Spector (2014) and in particular Spector (2015) proposes that SMNs have disjunctive alternatives, they must be exhaustified relative to these alternatives, and the result cannot be vacuous. Our analysis is thus fundamentally similar to that of Spector. Spector (2015)'s solution differs from ours in the details of the implementation – he uses Büring (2008)-style alternatives (*at least three* has as alternatives *exactly three*) and uses the contradiction-free variant of the grammatical theory of implicatures plus an economy condition banning vacuous exhaustification (recall our discussion of these theoretical choices in §1.2.7) – and the scope – it being merely a handout, he doesn't discuss SMNs in the antecedent of a conditional and the restriction of a universal, and also doesn't say anything about *at most* or CMNs. At the same time, Spector (2015) (and, earlier, §7.1 in Spector 2014) marks an important moment in the literature on anti-negativity in SMNs since it points out the fundamental similarity between modified numerals and polarity sensitive items and the need for a unified solution. As such, our account is an heir to Spector (2015) in that its main goal is to bridge the gap not only between the literature on epistemic indefinites / Free Choice items and discussions of ignorance in modified numerals, but also the literature on polarity sensitive items and discussions of anti-negativity in modified numerals.

## 5.4 Fit to experimental data

As we saw in §5.3, a virtue of the present approach to modified numerals is that it articulates a detailed account of CMNs and SMNs in at least some downward-entailing environments. But, given the general uncertainty regarding the empirical patterns of items with anti-negativity, is there any experimental evidence that could help bolster the empirical claims that we have been assuming and trying to derive?

Unlike ignorance, which has been tested in multiple experiments, to my knowledge there is no quantitative investigation of anti-negativity in CMNs and SMNs. To fill in this gap, in joint work with Kathryn Davidson, I conducted three experiments. The first experiment tests CMNs and SMNs in the scope of clausemate negation, in the antecedent of a conditional, and in the restriction of a universal. Due to the design, we were also able to test rescuing for the configuration where SMNs were in the scope of a negation that was further embedded in the antecedent of a conditional / restriction of a universal. We found support for badness under negation and acceptability in positive antecedents/restrictions, but not for rescuing in negative antecedents/restrictions. To probe this further, we tested rescuing in two follow-up experiments. Below I describe all of these experiments, and how they bear on the present account.

### 5.4.1 General notes on methodology

All the experiments were conducted online on Amazon Mechanical Turk. Participants were self-reported native speakers of English. They were paid $2 (Exp. 1) and $1 (Exps. 2 and 3) for their participants. For each experiment after Exp. 1 we prevented participation from people who had already been in one of our other experiments.

Participants were introduced to the task via a context describing a card game scenario modeled after Cremers & Chemla (2017). The reason we chose a scenario like this is because it would provide the most natural backdrop for testing sentences involving numerals. After the introduction, participants saw stimuli consisting of a picture, a sentence, and a question prompt, where the pictures all depicted a hand of cards with some cards covered, the sentence was a CMN/SMN sentence uttered by someone trying to truthfully describe this hand, and the question was for the participants to provide comprehensibility judgments. We chose this format for a number of reasons. First, we were interested to test CMNs/SMNs in settings where they appeared under negation, and we wanted to get judgments for the narrow scope reading. We moreover wanted to keep ignorance a constant – as we know by now, SMNs are incompatible with certainty, but both CMNs and SMNs are compatible with ignorance. The pictures helped enforce both the narrow scope reading and ignorance. Then, the nature of the contrast between CMNs and SMNs under negation is not obvious – sentences with the latter are also syntactically well-formed and their truth conditions are computable, even if they take more effort. Thus, it seemed that asking *Is this sentence good / grammatical / acceptable?* might not have elicited a response to our phenomenon of interest. Also, a lot of our target sentences are highly unnatural. Thus, asking *Is this sentence natural?* might again have elicited a response to something else. On the other hand, a lot of the preliminary reactions we got on SMN sentences under negation were of the sort 'It just melted my brain!' Thus, it seemed that asking a *comprehensibility* question, i.e., something of the form *Do you think x will understand what y said?*, might elicit reactions to precisely what we were interested in.

Participants only saw the target CMN/SMN sentences, obtained by crossing the factors of interest. It seemed to us that the contrasts between CMNs/SMNs in these conditions were fairly subtle, so, in order to bring to light any existing contrasts, we chose not to use any fillers. The questionnaires were prepared in Qualtrics (Qualtrics Labs 2016). The items were presented in a different, random order for each participant. The text in each item contained reference to some name, a CMN/SMN, and a suit of cards (diamonds, spades, hearts, clubs); the names were different for each item and the suits were counterbalanced across the items, but the numeral in the CMN/SMN was kept the same across items to minimize variations due to numeral complexity.

The results were analyzed in R (R Core Team 2015). For all the experiments I will first report the descriptive statistics in the form of a plot (*ggplot2*; Wickham 2009) showing the raw means and the binomial confidence intervals associated with them (calculated using the *binom* package in R, specifically, the function `binom.confint` with the `wilson` method; Dorai-Raj 2014). In the literature on modified numerals it is known that the downward-entailing modified numerals *less than* and *at most* sometimes behave differently than their upward-entailing counterparts. Each time we will thus also be careful to check for contrasts not just between CMNs and SMNs but also just between *at least* and *more than*, or *at most* and *less than*. For the statistical analyses we will fit logistic mixed effects models with fixed effects for all the factors of interest and all their interactions, and a random effects structure including an intercept for participant and random slopes for the maximal principled structure for which the model converged (*lme4* package; cf. Bates, Mächler, Bolker, & Walker 2015). (We don't add random effects for items – the names of the card suits, i.e., diamonds, hearts, clubs, and spades – because we don't expect them to interact with comprehensibility judgments.) After that in each case we will unpack the model further by extracting the predicted group mean probabilities and their associated measures (confidence intervals (CIs), $z$ values, $p$ values) (*effects*, Fox 2003) and/or the predicted ORs (OR) for the contrasts of interest, and their associated measures (confidence intervals, $z$ values, $p$ values) (*lsmeans*, Lenth 2016; the reported $p$ values are adjusted for the smaller set of comparisons of interest using the 'holm' method). In many cases CMNs will be degraded also; in that case we will just be interested in the relative difference between CMNs and SMNs.

### 5.4.2 Exp. 1: Anti-licensing, no-anti-licensing, and rescuing

#### 5.4.2.1 Question

The literature on SMNs in negative environments suggests that SMNs should exhibit the following patterns: (1: anti-licensing) degraded in the scope of clausemate negation; (2: no anti-licensing) fine in the antecedent of a conditional or the restriction of a conditional; and (3: rescuing) fine in the scope of clausemate negation if it is itself embedded in a downward-entailing environment. Can we find support for these claims?

#### 5.4.2.2 Methods

(Participants) 99.

(Task and instructions) Participants were introduced to the task as follows:

> In this survey you will answer questions about a group of friends playing a game. At the beginning of the game each player gets dealt a hand of seven cards. After taking a quick look at them, they must place the cards face down and try to remember their hands. Then they take turns giving clues about their hands to the other players in the form of statements describing their hands. You will see what a player remembers about his/her cards and the

statement s/he makes, then you will be asked if you think the other players will understand what s/he said.

Note: a  or  a means that the player doesn't remember if a particular card in his hand was a diamond or a heart, or a club or a spade, respectively.

(Stimuli) Participants then saw picture-sentence items presented as in Figure 5.1.

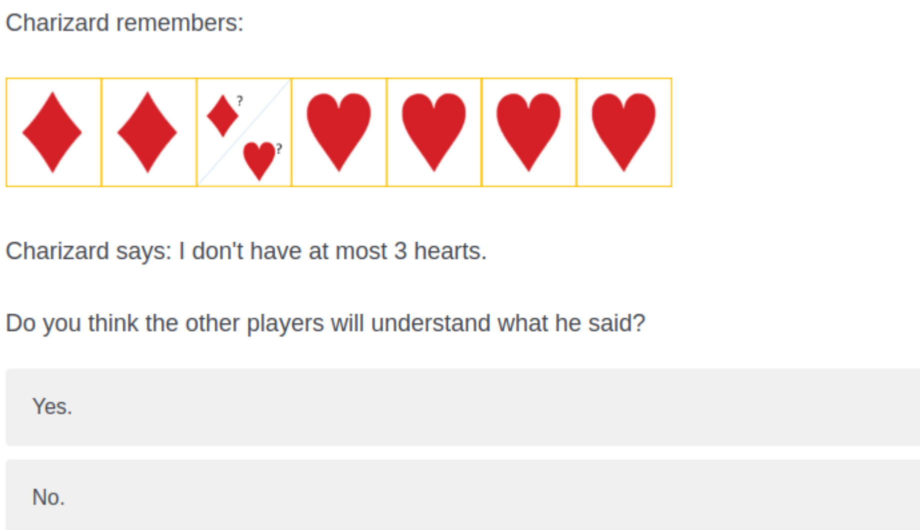



Figure 5.1: Example trial: SMN in a negative declarative.

(Design) Each participant saw 24 trials in total, obtained by crossing the following factors: (1) Modifier Monotonicity (levels: Upward-Entailing, Downward-Entailing) x Modifier Type (levels: Comparative, Superlative) / Modifier (levels: *more than*, *less than*, *at least*, *at most*); (2) Polarity (levels: Positive, Negative); and (3) embedding Environment (levels: Declarative (that is, no embedding), Antecedent of Conditional, Restriction of Universal). See Table 5.1 for a summary.

| Env | Pol | Schematic structure of item |
|---|---|---|
| Decl | Pos | I have Comp/Sup 3 Y |
| | Neg | I don't have Comp/Sup 3 Y |

Table 5.1 (Continued)

| | | |
|---|---|---|
| AntCond | Pos | If you have Comp/Sup 3 Y, then we have something in common |
| | Neg | If you don't have Comp/Sup 3 Y, then we have something in common |
| RestUniv | Pos | Everyone who has Comp/Sup 3 Y has something in common with me |
| | Neg | Everyone who doesn't have Comp/Sup 3 Y has something in common with me |

Table 5.1: Environment and polarity types, and the schematic structure of the sentence associated with them – where Comp ∈ {more than, less than} and Sup ∈ {at least, at most} and Y ∈ {diamonds, spades, hearts, clubs}.

#### 5.4.2.3 Predictions

SMNs should be much worse than CMNs in DECL-NEG (1: anti-licensing) but not in ANTCOND / RESTUNIV (2: no-anti-licensing) or in ANTCOND-NEG / RESTUNIV-NEG (3: rescuing).

#### 5.4.2.4 Results

The raw results by Modifier are as in Figure 5.2.

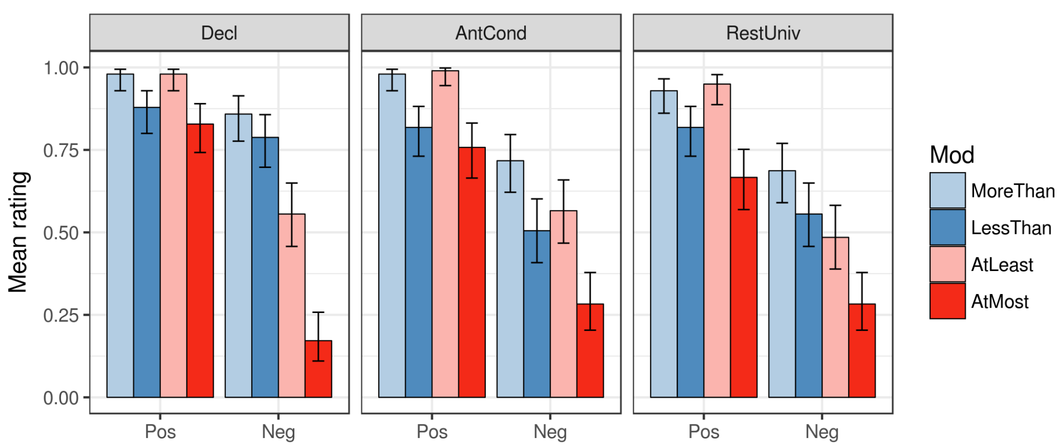



Figure 5.2: Exp 1 raw means, by Modifier. Bars represent 95% binomial confidence intervals.

Let's fit a logistic mixed-effects model with fixed effects for Modifier Monotonicity (UE, DE), Modifier Type (COMP, SUP), Polarity (POS, NEG), and Environment (DECL, ANTCOND, RESTUNIV), and all their interactions, and the maximal random effects structure for which the model converges. Taking the first level of each of these factors as the reference level, we find a significant effect of ModMon = DE ($\beta = -1.98,\ z = -2.532,\ p = 0.0113$ *) and of Pol = NEG ($\beta = 2.18,\ z = -2.805,\ p = 0.0050$ **).

Let's unpack the model further. First, pooling MORE/LESSTHAN and ATLEAST/MOST together by averaging over the levels of MODMON, for all Env types, we find no significant difference between COMP and SUP for Pol = POS but a highly significant difference between them for Pol = NEG. This is summarized in Table 5.2.

| Env | Pol | ModType | OR | CI | z | p |
|---|---|---|---|---|---|---|
| Decl | Pos | Comp - Sup | 1.25 | [0.34, 4.66] | 0.407 | 1.0000 |
| AntCond | Pos | Comp - Sup | 0.86 | [0.18, 4.02] | -0.234 | 1.0000 |
| RestUniv | Pos | Comp - Sup | 1.32 | [0.56, 3.08] | 0.773 | 1.0000 |
| Decl | Neg | Comp - Sup | 13.16 | [6.86, 25.26] | 9.468 | <.0001 |
| AntCond | Neg | Comp - Sup | 2.58 | [1.48, 4.47] | 4.111 | <.0001 |
| RestUniv | Neg | Comp - Sup | 3.24 | [1.87, 5.60] | 5.124 | <.0001 |

Table 5.2: Exp. 1 predicted contrasts for levels of Modifier Type.

However, judging by the shape of the raw data, it looks like Modifier Monotonicity might make a difference, especially in ANTCOND and RESTUNIV. So, to ensure that these results are not driven, e.g., just by ATLEAST or just by ATMOST, we also check these contrasts by comparing COMP and SUP just between the upward-entailing modifiers and then just between the downward-entailing ones. With one exception from ATMOST in RESTUNIV-POS which is significantly worse than LESSTHAN, the more detailed comparisons seem to support the same trends. This is summarized in Table 5.3.

| Env | Pol | ModMon | ModType | OR | CI | z | p |
|---|---|---|---|---|---|---|---|
| Decl | Pos | UE | Comp - Sup | 1.00 | [0.09, 11.30] | -0.000 | 1.0000 |
| AntCond | Pos | UE | Comp - Sup | 0.49 | [0.03, 9.42] | -0.577 | 1.0000 |
| RestUniv | Pos | UE | Comp - Sup | 0.69 | [0.16, 2.99] | -0.610 | 1.0000 |

Table 5.3 (Continued)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Decl | Pos | DE | Comp - Sup | 1.56 | [0.57, 4.32] | 1.055 | 0.5342 |
| AntCond | Pos | DE | Comp - Sup | 1.51 | [0.62, 3.66] | 1.110 | 0.5342 |
| RestUniv | Pos | DE | Comp - Sup | 2.52 | [1.07, 5.93] | 2.587 | 0.0291 |
| Decl | Neg | UE | Comp - Sup | 6.08 | [2.49, 14.87] | 4.839 | <.0001 |
| AntCond | Neg | UE | Comp - Sup | 2.17 | [1.00, 4.72] | 2.395 | 0.0166 |
| RestUniv | Neg | UE | Comp - Sup | 2.70 | [1.26, 5.79] | 3.108 | 0.0038 |
| Decl | Neg | DE | Comp - Sup | 28.48 | [11.17, 72.62] | 8.567 | <.0001 |
| AntCond | Neg | DE | Comp - Sup | 3.05 | [1.40, 6.66] | 3.426 | 0.0006 |
| RestUniv | Neg | DE | Comp - Sup | 3.88 | [1.77, 8.49] | 4.148 | 0.0001 |

Table 5.3: Exp. 1 predicted contrasts for levels of Modifier Type.

#### 5.4.2.5 Discussion

The results uniformly support (1) our anti-licensing expectation – SMNs were significantly worse than CMNs in the scope of clausemate negation. The results also for the most part support (2) our no-anti-licensing expectation – AtLeast was on a par with MoreThan in both AntCond-Pos and RestUniv-Pos, and AtMost was on a par with LessThan in AntCond-Pos (thought not in RestUniv), and when we averaged over modifier monotonicity Sup as a whole was on a par with Comp. This would bolster the empirical claims that we have assumed and derived in this chapter.

At the same time, the results do not support (3) our rescuing expectation – in the scope of a clausemate negation itself embedded in the antecedent of a conditional / restriction of a universal both CMNs and SMNs were degraded, but SMNs continued to be significantly worse than CMNs, and this is seen in both AtLeast and AtMost. This shows that a simple explanation along the lines of 'two negatives make a positive' is not sufficient to account for SMNs in these environments. The fact that the magnitude of the contrast (as judged based on the ORs) between Comp and Sup in the rescuing configuration was generally much smaller than in the anti-licensing case does however suggest that there is something else going on in these cases than in the case where SMNs were just in the scope of negation. Also, there seems to be a small trend for all modifiers to be rated less comprehensible in RestUniv as opposed to AntCond.

### 5.4.3 Exp. 2: Rescuing follow-up 1: The additional polarity

#### 5.4.3.1 Question

Citing Nilsen (2007), Cohen & Krifka (2014) note that SMNs in the antecedent of a conditional / restriction of a universal seem to be sensitive to whether there is positivity/negativity match between the antecedent / restriction and the continuation. In (32) below a SMN in a negative restriction of a universal (*Everyone who doesn't donate at least $10 ...*) is felicitous with a continuation that is pragmatically perceived as negative (*...is a fool*) but not with a continuation that is pragmatically perceived as positive (*...will get a thank-you card*).

(32) Everyone who doesn't donate at least $10 # will get a thank-you card / ✓is a fool.

We note that CMNs don't seem to be sensitive to this contrast, at least not as much.

(33) Everyone who doesn't donate more than $10 ✓will get a thank-you card / ✓is a fool.

Cohen & Krifka suggest that in fact SMNs are sensitive in this way not just when they are embedded in a negative antecedent/restriction, but also when the antecedent/restriction itself is positive but the continuation is negative. They note that CMNs don't seem to be sensitive to this contrast.

(34) Everyone who donates at least $10 ✓will get a thank-you card / # is a fool.

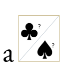
(35) Everyone who donates more than $10 ✓will get a thank-you card / ✓is a fool.

Now, remember that in Exp. 1 the continuations in the conditional and universal conditions were always neutral (*...then we have something in common* / *...has something in common with me*). This means that every time the SMN was in a negative antecedent / restriction, there was, if not a mismatch, then, at least, no match between the polarity of the antecedent / restriction and that of the continuation. Could this then be the reason why SMNs were worse than CMNs in those configurations? In this experiment we probe this question by testing CMNs and SMNs in positive and negative antecedents of conditionals / restrictions of universals, this time however also varying the polarity of the predicate in the continuation.

#### 5.4.3.2 Methods

(Participants) 45, of which 5 excluded prior to analysis (they rated all the sentences the same, which suggests that they didn't understand, or ignored, the task).

(Task and instructions) Because we wanted to be able to manipulate the polarity of the continuation, we modified the context from Exp. 1 as below.

> In this survey you will answer questions about a group of friends playing a game. At the beginning of the game each player gets dealt a hand of seven cards. They are not allowed to see their own cards but they are allowed to take a quick look at their neighbor's hand. They try to remember their neighbor's hand as well as they can because in the next step they have to come up with a rule that would make that neighbor (and possibly other players too) lose or win. You will see what a player remembers about their neighbor's hand and the rule they make up, then you will be asked if you think the other players will understand what they said. Note, we're not asking you if it is a good rule or a bad rule, but whether it is a rule that is going to be understandable for the other players to follow.
>
> Note: a  or  a means that the player doesn't remember if a particular card in his hand was a diamond or a heart, or a club or a spade, respectively.

(Stimuli) Participants saw picture-sentence items presented as in Figure 5.3.

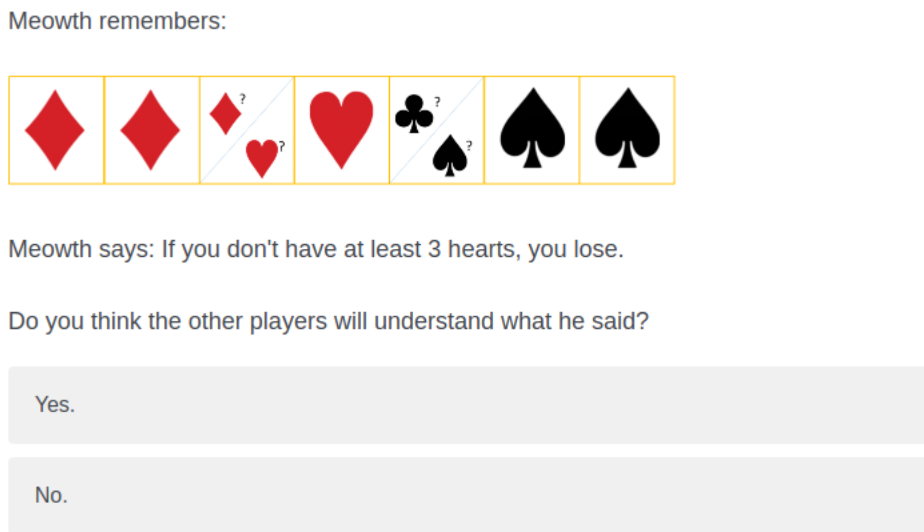



Figure 5.3: Example trial: SMN under negation, embedded in the antecedent of a conditional, with negative polarity in the continuation.

(Design) Each participant saw 32 trials in total, obtained by crossing the following factors factors: (1) Modifier Monotonicity (levels: Upward-Entailing, Downward-Entailing) x Modifier Type (levels: Comparative, Superlative) / Modifier (levels: *more than*, *less than*, *at least*, *at most*); (2) Polarity1 of the embedding environment (same as Polarity in Experiment 1; levels: Positive, Negative); (3) embedding Environment (levels: Antecedent of Conditional, Restriction of Universal); and (4) Polarity2 of the continuation (levels: Positive, Negative). See Table 5.4 for a summary.

| Env | Pol1 | Pol2 | Schematic structure of item |
|---|---|---|---|
| AntCond | Pos | Pos | If you have Comp/Sup 3 Y, you win |
| | | Neg | If you have Comp/Sup 3 Y, you lose |
| | Neg | Pos | If you don't have Comp/Sup3 Y, you win |
| | | Neg | If you don't have Comp/Sup 3 Y, you lose |
| RestUniv | Pos | Pos | Everyone who has Comp/Sup 3 Y wins |
| | | Neg | Everyone who has Comp/Sup 3 Y loses |
| | Neg | Pos | Everyone who doesn't have Comp/Sup 3 Y wins |

Table 5.4 (Continued)

| | | | |
|---|---|---|---|
| | | Neg | Everyone who doesn't have Comp/Sup 3 Y loses |

Table 5.4: Environment, polarity of the antecedent/restriction, polarity of the consequent/scope, and the schematic structure of the sentence associated with them – where Comp ∈ {more than, less than} and Sup ∈ {at least, at most}, and Y ∈ {diamonds, spades, hearts, clubs}.

#### 5.4.3.3 Predictions

Polarity mismatch should make a difference to SMNs but not CMNs.

#### 5.4.3.4 Results

The raw results by Modifier are as in Figure 5.4.

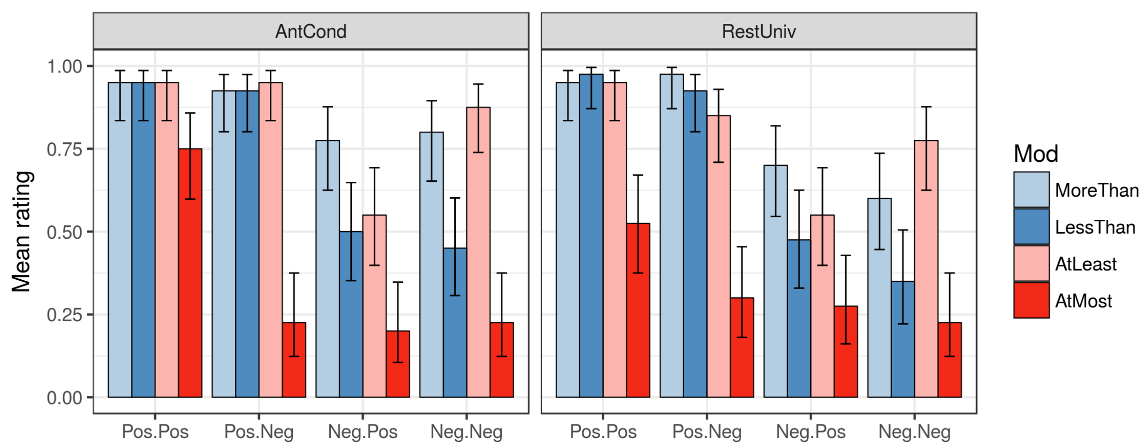



Figure 5.4: Exp 2 raw means, by Modifier. Bars represent 95% binomial confidence intervals.

Even more obviously than in Exp. 1, Modifier Monotonicity seems to make a difference in this case. Moreover, in SMNs, this difference is not of a parallel but rather a divergent kind.

Let's fit a logistic mixed-effects model with fixed effects for Modifier Monotonicity (UE, DE), Modifier Type (Comp, Sup), Polarity1 (Pos, Neg), Polarity2 (Pos, Neg), and Environment (AntCond, RestUniv), and all their interactions, and the maximal random effects structure for which the model converges. Taking the first level of each factor as the reference level, we find a significant effect of Pol1 = Neg ($\beta = -1.80,\ z = -2.167,\ p = 0.0303$ *).

Unpacking the model further, we find that the contrast between Comp and Sup depends very much on ModMon. There is in general no significant difference between AtLeast and MoreThan, while in every condition there is a significant difference between AtMost and LessThan, either just for AntCond or for both AntCond and RestUniv. All these are summarized in Table 5.5.

| Env | Pol1 | Pol2 | ModMon | ModType | OR | CI | z | p |
|---|---|---|---|---|---|---|---|---|
| AntCond | Pos | Pos | UE | Comp - Sup | 1.00 | [0.10, 10.20] | -0.000 | 1.0000 |
| RestUniv | Pos | Pos | UE | Comp - Sup | 1.00 | [0.10, 10.22] | 0.000 | 1.0000 |
| AntCond | Pos | Pos | DE | Comp - Sup | 7.08 | [1.11, 45.22] | 2.365 | 0.0180 |
| RestUniv | Pos | Pos | DE | Comp - Sup | 45.15 | [4.08, 500.05] | 3.551 | 0.0008 |
| AntCond | Pos | Neg | UE | Comp - Sup | 0.64 | [0.08, 5.43] | -0.468 | 0.6400 |
| RestUniv | Pos | Neg | UE | Comp - Sup | 7.38 | [0.61, 89.96] | 1.793 | 0.1461 |
| AntCond | Pos | Neg | DE | Comp - Sup | 62.48 | [12.05, 323.99] | 5.631 | <.0001 |
| RestUniv | Pos | Neg | DE | Comp - Sup | 40.53 | [8.14, 201.71] | 5.171 | <.0001 |
| AntCond | Neg | Pos | UE | Comp - Sup | 3.18 | [0.99, 10.23] | 2.222 | 0.0526 |
| RestUniv | Neg | Pos | UE | Comp - Sup | 2.07 | [0.68, 6.30] | 1.464 | 0.1431 |
| AntCond | Neg | Pos | DE | Comp - Sup | 4.74 | [1.43, 15.73] | 2.904 | 0.0074 |
| RestUniv | Neg | Pos | DE | Comp - Sup | 2.67 | [0.86, 8.26] | 1.944 | 0.0518 |
| AntCond | Neg | Neg | UE | Comp - Sup | 0.55 | [0.13, 2.31] | -0.938 | 0.3484 |
| RestUniv | Neg | Neg | UE | Comp - Sup | 0.40 | [0.12, 1.28] | -1.768 | 0.1542 |
| AntCond | Neg | Neg | DE | Comp - Sup | 3.20 | [0.99, 10.34] | 2.222 | 0.0526 |
| RestUniv | Neg | Neg | DE | Comp - Sup | 1.99 | [0.60, 6.56] | 1.296 | 0.1949 |

Table 5.5: Exp. 2 predicted contrasts for levels of Modifier Type.

Even more interesting is the way in which the polarity switch in the continuation affects individual modifiers based on both ModMon and ModType. Chances of comprehensibility are quite high for all the modifiers in Pos-Pos (with the notable exception of AtMost in RestUniv), but for AtMost they drop dramatically between Pos-Pos and Pos-Neg, while for the rest they stay at their Pos-Pos levels. And, chances of comprehensibility drop for all the modifiers in Neg-Pos, but for AtLeast they improve dramatically between Neg-Pos and Neg-Neg, while for the rest stay at their Neg-Pos levels. Tabel 5.6 summarizes all these.

| Env | Pol1 | Pol2 | ModMon | ModType | OR | CI | z | p |
|---|---|---|---|---|---|---|---|---|
| AntCond | Pos | Pos - Neg | UE | Comp | 1.56 | [0.18, 13.24] | 0.468 | 1.0000 |
| RestUniv | Pos | Pos - Neg | UE | Comp | 0.48 | [0.03, 7.98] | -0.585 | 1.0000 |
| AntCond | Pos | Pos - Neg | DE | Comp | 1.56 | [0.18, 13.26] | 0.468 | 0.6398 |
| RestUniv | Pos | Pos - Neg | DE | Comp | 3.25 | [0.23, 46.43] | 0.995 | 0.6397 |
| AntCond | Pos | Pos - Neg | UE | Sup | 1.00 | [0.10, 10.22] | 0.000 | 0.9999 |
| RestUniv | Pos | Pos - Neg | UE | Sup | 3.55 | [0.51, 24.63] | 1.464 | 0.2863 |
| AntCond | Pos | Pos - Neg | DE | Sup | 13.80 | [3.97, 47.99] | 4.719 | <.0001 |
| RestUniv | Pos | Pos - Neg | DE | Sup | 2.92 | [0.96, 8.92] | 2.153 | 0.0314 |
| AntCond | Neg | Pos - Neg | UE | Comp | 0.85 | [0.24, 3.05] | -0.286 | 0.7750 |
| RestUniv | Neg | Pos - Neg | UE | Comp | 1.64 | [0.54, 5.03] | 0.992 | 0.6423 |
| AntCond | Neg | Pos - Neg | DE | Comp | 1.26 | [0.43, 3.66] | 0.477 | 0.6332 |
| RestUniv | Neg | Pos - Neg | DE | Comp | 1.80 | [0.60, 5.37] | 1.205 | 0.4562 |
| AntCond | Neg | Pos - Neg | UE | Sup | 0.15 | [0.04, 0.56] | -3.214 | 0.0026 |
| RestUniv | Neg | Pos - Neg | UE | Sup | 0.31 | [0.10, 1.01] | -2.222 | 0.0263 |
| AntCond | Neg | Pos - Neg | DE | Sup | 0.85 | [0.23, 3.08] | -0.286 | 1.0000 |
| RestUniv | Neg | Pos - Neg | DE | Sup | 1.35 | [0.40, 4.58] | 0.542 | 1.0000 |

Table 5.6: Exp. 2 predicted contrasts for levels of Polarity2 (polarity of the continuation).

#### 5.4.3.5 Discussion

This experiment revealed that, while both CMNs and SMNs are affected by negative polarity in the antecedent/restriction, SMNs are also affected by the polarity of the continuation, although in different ways, depending on their monotonicity – in a positive antecedent/restriction, AtLeast is fine with a negative continuation but AtMost is not, and in a negative antecedent/restriction AtLeast improves if the continuation is negative also, while AtMost does not. We already know that the rescuing hypothesis doesn't help us capture these patterns. On the other hand, it seems that Cohen & Krifka (2014)'s polarity match requirement doesn't help us either, since it would predict AtLeast to be bad in Pos-Neg, and AtMost to improve in Neg-Neg, contrary to what we see. At the same time, our results, which so flatly go against Cohen & Krifka (2014)'s data, call for an explanation. What is going on?

An important part of our design in Exp. 2 was the fact that, in an attempt to isolate the interaction between the negation in the antecedent and the polarity of the predicate in the continuation, we made special efforts to ensure that the polarity of the predicate in the antecedent does not interfere. To be more concrete, we designed the context such that it could never be taken for granted that, e.g., having more cards of a particular suit is always bad and having fewer is always good. That might be precisely what's driving the difference between our data and Cohen & Krifka's data, where they were using predicates with highly stereotypical polarities (e.g., *donate* or *get a thank-you card* carry clear positive connotations, whereas *not donate* and *be a fool* carry clear negative connotations). So perhaps the right way to go about understanding our new data is to backtrack and first try to better understand the interaction between our modified numerals, the polarity of the predicate in the antecedent, and the polarity of the antecedent, and only then try to understand the interaction of all these pieces together with the polarity of the continuation. We probe more such configurations introspectively in the next section, and also sketch a possible way to capture this in an alternative-based approach.

### 5.4.3.6 Expanding the dataset, extending the theory

Cases where a scalar item seems to be sensitive to the polarity of the predicate have been discussed in the alternative-based literature in Crnič (2011, 2012). Crnič's idea is to say that they all contain a silent *even*,[1] and thus they presuppose that the prejacent is the least likely among its alternatives relative to some probability measure; badness arises when this presupposition is not met.

This analysis is in fact inspired by an analysis of sentences with overt *even*. First, consider a classical definition of *even* as below: Given a sentence $p$ and a set C of (relevant) alternatives to $p$, $[\![\text{even}]\!]_{\text{C}}\ (p)$ is defined iff all the alternatives in $C$ different from $p$ are more likely than $p$ with respect to some contextually relevant contextual probability measure $c$ (that is, iff $p$ is the least likely of all), and if defined it asserts $p$, (36).

(36) $[\![\text{even}]\!]_{\text{C}}\ (p)^{g,c,w}$

a. is defined iff $\forall q \in [\![\text{p}]\!]^{\text{C}}\ [[\![\text{q}]\!]^{g,w} \neq [\![\text{p}]\!]^{g,w} \rightarrow [\![\text{p}]\!]^{g,w} \prec_c [\![\text{q}]\!]^{g,w}]$

b. if defined, $[\![\text{even}]\!]_{\text{C}}\ (p)^{g,c,w} = [\![\text{p}]\!]^{g,w}$

Starting from this definition, Crnič (2011) sets out to explain grammaticality/ungrammaticality in a variety of cases. A low scalar such as *one* is bad under *even* in a positive context because, given the usual Horn scale ⟨ *one*, *two*, ...⟩, it is entailed by all its scalar alternatives, so it cannot possibly be less likely than them. On the other hand, it is fine under *even* in a negative context because in that context it entails all of its scalar alternatives, and so it is indeed able to verify the presupposition that it is less likely than all of them. (Below, and going forward, we will test this likelihood presupposition underneath each example, and use ✓ and ✗ to mark if the reasoning goes through or not.)

(37) #John read even one book.

that John read one book $\prec_c$ that John read two books ✗

[1] Recall that in Ch. 1 we already anticipated the possibility of a silent counterpart of *even* and suggested it might be what's giving rise to the 'the ex was the least likely to come' meaning of an utterance like *Really everybody came to my party; imagine that* MY EX *came*.

(38) John didn't read even one book.

that John didn't read one book $\prec_c$ that John read two books ✓

So far it seems that 'least likely' simply means 'logically strongest'. However, Crnič shows that this is not always true. In a downward-entailing environment such as the antecedent of a conditional we expect *one* to always be fine under *even*, just as it was in the negation case. What we find, though, is that sometimes it is fine, and sometimes it isn't. The polarity of the consequent seems to make a crucial difference – the *one* utterance is not assessed in a vacuum but rather relative to common assumptions about what type of situation correlates with what type outcome.

(39) Even if John read one book, he will (still) pass the exam.

that if John read one book, he will pass the exam $\prec_c$

that if John read two books, he will pass the exam. ✓

(40) Even if John read one book, #he will fail the exam.

that if John read one book he will fail the exam $\prec_c$

that if John read two books he will fail the exam. ✗

But how is it possible for likelihood to go against logical strength? More concretely, since in a downward-entailing context *one* entails *two*, how can it fail to be less likely than it? Crnič argues that what is going on is that the numeral and its alternatives are in fact not interpreted in a plain way but rather as if they were pre-exhaustified with (using our own notation) $O_{\sigma A}$ . Since $O_{\sigma A}$ (one) = exactly one, and $O_{\sigma A}$ (two) = exactly two, they are no longer in an entailment relation but rather logically independent. Then assessment can no longer be made based on logical strength but defaults to contextual assumptions about likelihood.[2]

[2] Crnič (2012) in fact also uses this assumption to account for why high scalars, e.g., *all*, can associate with *even* across a downward-entailing operator. This case is not directly relevant to us, but the solution is as below.

(i) Even if John read all of the books, he will (still) fail the exam.

a. that if John read all of the books, he will fail the exam $\prec_c$ that if John read some of the books, he will fail the exam ✗

(41) Even if John read $\mathrm{O}_{\sigma\mathrm{A}}$ (one) (= exactly one) book, he will (still) pass the exam.

that if John read $\mathrm{O}_{\sigma\mathrm{A}}$ (one) (= exactly one) book, he will pass the exam $\prec_c$

that if John read $\mathrm{O}_{\sigma\mathrm{A}}$ (two) (= exactly two) books, he will pass the exam ✓

(42) Even if John read $\mathrm{O}_{\sigma\mathrm{A}}$ (one) (= exactly one) book, #he will fail the exam.

that if John read $\mathrm{O}_{\sigma\mathrm{A}}$ (one) (= exactly one) book he will fail the exam $\prec_c$

that if John read $\mathrm{O}_{\sigma\mathrm{A}}$ (two) (= exactly two) books he will fail the exam ✗

I will argue that this is what's going on for our CMN/SMN cases also. But *one* was a low scalar, it only had alternatives in one direction, that is, either alternatives that it entailed, or alternatives that entailed it, whereas for our CMN/SMN cases that was never the case since they (e.g., *more than three*) were generally middle of the scale. What is the relevant scale in those cases?

Consider CMNs/SMNs in a plain, unembedded context. Note that the upward-entailing modifiers *more than* and *at least* suggest an expectation of less, while the downward-entailing modifiers suggest an expectation of more, and also that this seems to be the case regardless of the polarity of the predicate the CMN/SMN associates with (where *solve a problem* is assumed to carry positive polarity and *make a mistake* is assumed to carry negative polarity).

(43) a. John solved more than two / at least three problems. expected less

b. John made more than two / at least three mistakes. expected less

(44) a. John solved less than four / at most three problems. expected more

b. John made less than four / at most three mistakes. expected more

It seems then that *more than two* / *at least three* are pitched against alternatives based on lower numerals, while *less than four* and *at most three* are pitched against alternatives based on higher numerals. I will argue that the relevant scale is based on this expectation: for the upward-entailing modifiers we look at

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b. that if John read $\mathrm{O}_{\sigma\mathrm{A}}$ (all) (= all) of the books, he will fail the exam $\prec_c$
that if John read $\mathrm{O}_{\sigma\mathrm{A}}$ (some) (= some but not all) of the books, he will fail the exam ✓

alternatives smaller than them, while for the downward-entailing modifiers we look at alternatives larger than them. I will also argue (for now) that, in the absence of any further contextual information, likelihood is based on logical strength. With these assumptions in hand, let's see what happens if we add in *even*/E . (Both for brevity, and because we are ultimately interested to argue for a silent E , I will use E . However, the likelihood judgments in this case might be easier with an overt *even*.) As we can see, we predict that the presupposition of E should be satisfied in all these cases.

(45) a. E (John solved more than two / at least three problems)
that John solved more than two / at least three problems $\prec_c$
that John solved more than one / at least two problems ✓

b. E (John made more than two / at least three mistakes)
that John made more than two / at least three mistakes $\prec_c$
that John made more than one / at least two mistakes ✓

(46) a. E (John solved less than four / at most three problems)
that John solved less than four / at most three problems $\prec_c$
that John solved less than five / at most four problems ✓

b. E (John made less than four / at most three mistakes)
that John made less than four / at most three mistakes $\prec_c$
that John made less than five / at most four problems ✓

The reasoning here may seem a little circular. The basic idea is however simply that the relevant scale in all the cases consists of the assertion and the alternatives it entails – thus, a fairly natural idea that for middle-of-the-scale items *even*/E simply truncates the scale such that its application to this item might be valid, that is, such that it consists only of alternatives that the item entails and is thus less likely than. Then, that the likelihood scale should simply follow the logical strength scale is also very natural. All in all, the reasoning here is not different than Crnič's reasoning for *one* in the negative case. (If we are right, then his reasoning for *one* in the positive case might have to be revised, since, given our assumptions here, we would say that the relevant scale for *one* consists of itself and lower ranked alternatives, that is, *zero*. We would still predict a crash, because, due to the special nature of *zero*, the usual entailment relation that holds between numerals in an upward-entailing context is not verified: *one* does not entail *zero*.)

What is more interesting, though, is what this reasoning predicts for other types of environments. If truncation is always such as to make the assertion logically the strongest and therefore in the default case the least likely, then in a downward-entailing environment the scale should be reversed – for the upward-entailing modifiers we should now consider alternatives based on larger numerals and use a rationale of 'expected more', and for the downward-entailing modifiers we should consider alternatives based on smaller numerals and use a rationale of 'expected less'. Moreover, as we know from *one* in the antecedent of a conditional, the relevant scale is no longer an entailment scale and likelihood is assessed based on contextual/stereotypical assumptions. (Note that we might even say that, at least for our items, the relevant scale for *even*/E is never an entailment scale – we are always looking at items pre-exhaustified with $O_{\sigma A}$. In that case though we would have to revise what we said for the unembedded case. For example, we could say that likelihood is based on contextual assumptions, and they are stereotypically such that for the upward-entailing modifiers less is more likely, and for the downard-entailing ones more is more likely.) The results are as below. As we can see, the predictions from E (marked on the margins) pretty much match the introspective judgments (full acceptability not marked, rough acceptability marked with ?, unacceptability marked with #).

(47) a. E (If John solved $O_{\sigma A}$ (more than two / at least three problems), he passed / #failed)
that if John solved 3 problems, he passed / failed $\prec_c$
that if John solved 4 problems, he passed / failed ✓/✗

b. E (If John made $O_{\sigma A}$ (more than two / at least three mistakes), he #passed/ ?failed)
that if John made 3 mistakes, he passed / failed $\prec_c$
that if John made 4 mistakes, he passed / failed ✗/✓

(48) a. E (If John solved $O_{\sigma A}$ (less than four / at most three problems), he #passed / failed)
that if John solved 3 problems, he passed / failed $\prec_c$

that if John solved 2 problems, he passed / failed ✗/✓

b. E (If John made $O_{\sigma A}$ (less than four / at most three problems mistakes), he passed / #failed) that if John made 3 mistakes, he passed / failed $\prec_c$ that if John made 2 mistakes, he passed / failed ✓/✗

But how does this square with our experiment, namely, the fact that all our modifiers were fine in AntCond-Pos-Pos (positive antecedent with positive continuation) and all continued to be fine in AntCond-Pos-Neg (positive antecedent with negative continuation) except for *at most*, whose chances of being rated comprehensible dropped dramatically?

(49) If you have more than / less than / at least / at most three [diamonds], you win.

(50) If you have more than / less than / at least / # at most three [diamonds], you lose.

As we mentioned at some point earlier, one difference between the sentences in our experiment and those reported in Cohen & Krifka and analyzed above is that the ones in our experiment contained a predicate in the antecedent that was underspecified with respect to polarity – due to how we set up the context, *having [diamonds]* was never intrinsically good or bad. Suppose then that a positive continuation such as *win* doesn't force any type of color on the predicate of a positive antecedent, but a negative continuation such as *lose* does. Then in the *win* case *having [diamonds]* can be interpreted as a good thing for *more than* and *at least* – so this case is like *If John solved at least three problems, he passed*, which was ✓– and as a bad thing for *less than* and *at most* – so this case is like *If John made at most three mistakes, he passed*, which was again ✓. But in the *lose* case *having [diamonds]* has to be interpreted as negative for both *more than / at least* as well as *less than / at most*. This makes the *more than / at least* case similar to *If John made more than two / at least three mistakes, he #failed*, which was ✓, and the *less than / at most* case similar to *If John made less than four / at most three mistakes, he #failed*, which was ✗. This captures one of our puzzles – namely, why *at least* is fine with *lose* but *at most* is not. At the same time, it also predicts that *less than* should be bad with *lose*, contrary to what we found – *less than* was at ceiling in this condition also. A tentative conclusion would be that some part of this whole set of assumptions (whether it is E , or the way the predicate in the continuation should bias the predicate in the antecedent, etc.) applies just to SMNs, or applied more forcefully to SMNs than to CMNs.

We still have to discuss the case of negative antecedents. Due to embedding in two downward-entailing environments, the relevant scale for E is as in the unembedded case: the upward-entailing CMNs/SMNs are pitched against alternatives based on lower numerals, and the downward-entailing CMNs/SMNs – against alternatives based on larger numerals. However, here if we assume that the elements of the scale are pre-exhaustified with $O_{\sigma A}$ , we'd be forcing a very unnatural interpretation of the assertion, e.g., *If John didn't solve* $O_{\sigma A}$ *(more than two / at least three) problems, he failed* would be understood as saying that he failed if he didn't solve exactly three. (This interpretation is possible, but it's certainly not the most natural.) (Note that in fact the same is true of the previous cases of embedding in an antecedent that we discussed also.) So we want to keep the default Horn scale. At the same time, recall from our early discussion of *one* that if we base our likelihood judgments on logical strength, then we cannot explain why sometimes our items can fail to be less likely than the alternatives they entail. All in all, it seems that we might never want the likelihood relation to be based on logical strength but always on assumptions about how the world works. With these revised assumptions in mind – entailment scale but likelihood always assessed based on context – let's check the case of negative antecedents. As before, we notice that our reasoning with E pretty much captures the introspective judgments.

(51) a. E (If John didn't solve more than two / at least three problems, he #passed / failed)

that if John did$\underbrace{\text{n't solve more than two / at least three}}_{=\text{solved } 0 \vee 1 \vee 2}$ problems he passed / failed $\prec_c$

that if John did$\underbrace{\text{n't solve more than one / at least two}}_{=\text{solved } 0 \vee 1}$ problems he passed / failed ✗/✓

b. E (If John didn't make $O_{\sigma A}$ (more than two / at least three) mistakes, he ?passed / #failed)

that if John did$\underbrace{\text{n't make more than two / at least three}}_{=\text{made } 0 \vee 1 \vee 2}$ mistakes he passed / failed $\prec_c$

that if John did$\underbrace{\text{n't make more than one / at least two}}_{=\text{made } 0 \vee 1}$ mistakes he passed / failed ✓/✗

(52) a. E (If John didn't solve less than four / at most three problems, he ?passed / #failed)

that if John did$\underbrace{\text{n't solve less than four / at most three}}_{=\text{solved } 4 \vee 5 \vee \ldots}$ problems he passed / failed $\prec_c$

that if John did$\underbrace{\text{n't solve less than five / at most four}}_{=\text{solved } 5\vee\ldots}$ problems he passed / failed ✓/✗

b. E (If John didn't make less than four / at most three mistakes, he #passed / failed)

that if John did$\underbrace{\text{n't make less than four / at most three}}_{=\text{solved } 4\vee 5\vee\ldots}$ mistakes he passed / failed $\prec_c$

that if John did$\underbrace{\text{n't make less than five / at most four}}_{=\text{solved } 5\vee\ldots}$ mistakes he passed / failed ✗/✓

As before, though, we have to ask: How does this square with the results of our experiment? There the judgments were different. In particular, in Ant-Neg-Pos all the modifiers were degraded and all continued to be degraded in Ant-Neg-Neg except for *at least*, whose chances of being rated comprehensible improved significantly. If we tried to mark ungrammaticality for these patterns using the plot summaries, they might look as follows.

(53) If you don't have ?/✓more than / ??less than / ?at least / #at most three [diamonds], you win.

(54) If you don't have ?more than / ??less than / ?/✓at least / #at most three [diamonds], you lose.

As before, we will note that the polarity of the property in the antecedent, *having [diamonds]*, is underspecified. Suppose then that a positive continuation such as *win* forces the predicate of a negative antecedent to be interpreted as negative, and a negative continuation such as *lose* forces the predicate of a negative antecedent to be interpreted as positive. Then, the *win* case becomes just like the *make a mistake – pass* cases above, which were ✓for *more than* and *at least* (introspective judgment '?') but ✗ for *less than* and *at most*. On the other hand, the *lose* case becomes like the *solve a problem – fail* cases above, which were ✓for *more than* (introspective judgment '✓') and *at least* and ✗ for *less than* and *at most*. At this point we have captured all the patterns our experiment brought to light for *at most* in the antecedent of a conditional. (The paterns in the restriction of a universal are similar, although, for some mysterious reason, they seem to be in general worse.) And if we again assume that CMNs are not/less affected by all these mechanisms than SMNs, this also captures why *less than* was less bad in these cases also. However, since the predictions for *at least* were in both cases ✓, the story outlined here doesn't explain why *at least* in a negative antecedent improves from a positive to a negative continuation – that is, a contrast that in our introspective judgments above we marked with a '?' for *If John didn't make at least three mistakes, he ?passed* vs. nothing (perfect acceptability) for *If John didn't solve at least three problems, he failed*. I am not sure what the explanation for this contrast could be.

To conclude, in an attempt to make sense of the results of Experiment 2, in this section we reviewed an approach to sensitivity of scalar items to the the polarity of the predicates they combine with. This approach suggests that this has to do with an underlying reasoning with *even* over truncated entailment numeral scales and contextual likelihood scales. We have sketched an account of how this could be extended to CMNs and SMNs, focusing in particular on capturing the patterns for SMNs. The results so far seem promising, but the analysis remains, of course, at a very tentative stage. In particular, while we have suggested that *even*/E uses the same traditional scalar alternatives of CMNs/SMNs that we defended in Ch. 3, we have not clarified the relation between O and E . Given our syntactic assumptions which say that the scalar features can only be used by one operator at a time, we will want to say that CMNs/SMNs can be exhaustified with either O or E . However, given the fact that the patterns in our experiment look very similar to what we would obtain from E , we may also want to say that a use of E is preferred in contexts such as the one in our experiment where, due to the nature of predicates such as *win* and *lose*, likelihood-based reasoning becomes very salient. I will leave a more detailed investigation of these issues to future research.

Interestingly, Cohen & Krifka (2014) (citing Lakoff 1969) note that the same sensitivity that we found in SMNs can be found in other items with anti-negativity like *some* (what we might call PPIs) and also in items with anti-positivity like *any* (what are commonly called NPIs).

(55) If you eat ✓some / #any spinach, I will give you $10.

(56) If you eat #some / ✓any candy, I will whip you.

Citing Regine Eckardt (p.c.), they note this sensitivity might be stronger in NPIs such as *budge an inch*.[3]

[3]They call this a strong NPI. However, in Chierchia (2013) it is classified as a weak NPI, being less degraded in non-strongly negative environments than an NPI such as *sleep a wink*. Just like *sleep a wink*, however, it is analyzed as involving exhaustification via a silent *even*.

(57) If you budge an inch, I will ✓kill / # thank you.

The same is noticed by Crnič (2011) for *lift a finger*.

(58) Everyone that lifted a finger to help ✓was rewarded / # was wearing blue jeans.

Both *budge an inch* and *lift a finger* have been analyzed as involving exhaustification via a silent *even* (Crnič 2011, Chierchia 2013). This lends further credence to an approach along the lines we sketched above.

### 5.4.4 Exp. 3: Rescuing follow-up 2: The additional downward-entailing environment

#### 5.4.4.1 Question

In Exp. 1 we tested rescuing for the case where the immediate downward-entailing environment was the scope of clausemate negation and the next one up was the antecedent of a conditional / restriction of a universal. We found that SMNs were significantly worse than CMNs in this configuration, suggesting that rescuing did not happen as predicted. In Exp. 2 we probed SMNs in supposedly-rescuing configurations of the same type further, trying to figure out whether/how the presence of additional polarities in the environment affect them. In this experiment we again probe SMNs in supposedly-rescuing configurations, comparing embedding under a negation in the antecedent of a conditional to embedding under a negation itself in the scope of a matrix negation, in an attempt to figure out whether/how the nature of the additional environment affects them.

#### 5.4.4.2 Methods

(Participants) 45.

(Task and instructions) Because we wanted to test configurations with an embedded negation and a matrix negation, we modified our context from Exp. 1 to support clausal embedding.

In this survey you will consider a commentator for a televised card-playing game, and answer questions about how understandable the commentator is.

At the beginning of the game each player gets dealt seven cards, two of which are hidden. Then in each round some rule is issued, and players can choose whether or not to bet on their own hand. A commentator, who knows what the hidden cards are for each player, discusses the player's move.

You will see a player's hand and the commentator's comment, then you will be asked if you think the viewers will understand what the commentator said.

Note: In the hands that you will see, cards with a white background such as ♦ represent cards that are visible to the player, while cards with a grey background such as ♦ represent hidden cards, that is, cards that are not visible to the player but visible to the commentator.

**Stimuli** Participants saw picture-sentence pairs presented as in Figure 5.5.

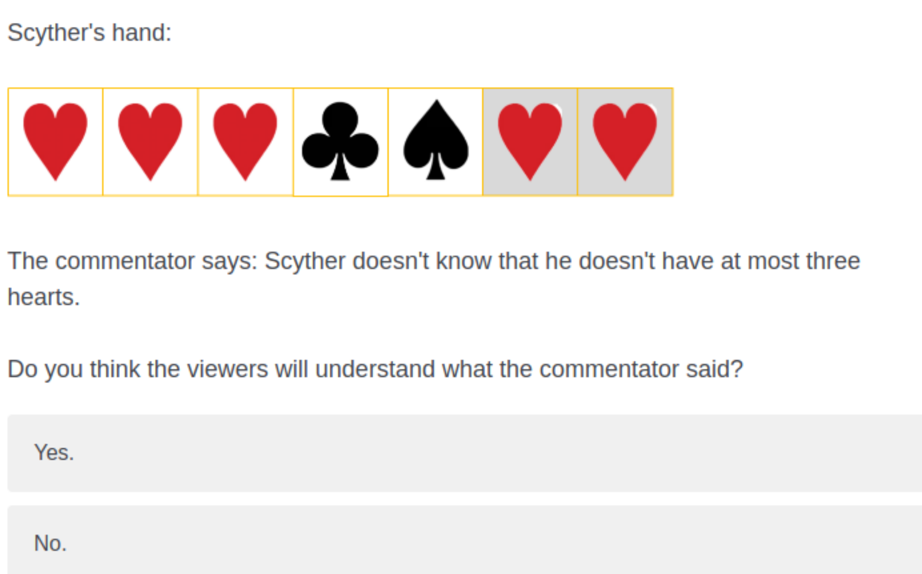


Figure 5.5: Example trial: SMN under a negation itself embedded in the scope of a matrix negation.

(Design) Participants saw 16 trials, obtained by crossing the following factors: (1) Modifier Monotonicity (levels: Upward-Entailing, Downward-Entailing) x Modifier Type (levels: Comparative, Superlative) / Modifier (levels: *more than*, *less than*, *at least*, *at most*); (2) Polarity of the embedded clause (levels: Positive, Negative); and (3) matrix embedding Environment (levels: Matrix Negation, Antecedent of a Conditional). (Note: AntCond-Pos/Neg here is similar to AntCond-Pos/Neg in Exp. 1 and Exp. 2 in that we are talking CMNs/SMNs in the scope of a negation that is itself embedded in the antecedent of a conditional. It is however also different in the sense that here the positive/negative clause does not make up the entire antecedent but is merely the complement of the verb *know*.) See Table 5.7 for a summary.

| Env | Pol | Schematic structure of item |
|---|---|---|
| MatrixNeg | Pos | X doesn't know that s/he has Comp/Sup 3 Y |
| | Neg | X doesn't know that s/he doesn't have Comp/Sup3 Y |
| AntCond | Pos | If X knew that s/he has Comp/Sup 3 Y, s/he would bet differently |
| | Neg | If X knew that s/he doesn't have Comp/Sup 3 Y, s/he would bet differently |

Table 5.7: Matrix embedding environment and polarity of the embedded clause, and the schematic structure of the sentence associated with them – where X ∈ {[Pokemon names]}, Comp ∈ {more than, less than} and Sup ∈ {at least, at most} and Y ∈ {diamonds, spades, hearts, clubs}.

Note that here we do not necessarily expect the polarity of the continuation to make a difference, as the modified numeral is embedded at a deeper level inside the antecedent, and the immediate clause containing the modified numeral doesn't seem directly related to the outcome.

#### 5.4.4.3 Predictions

SMNs under a negation itself embedded in an additional downward-entailing environment should be the same regardless of the nature of this additional environment.

#### 5.4.4.4 Results

The raw results by Modifier are as in Figure 5.6.

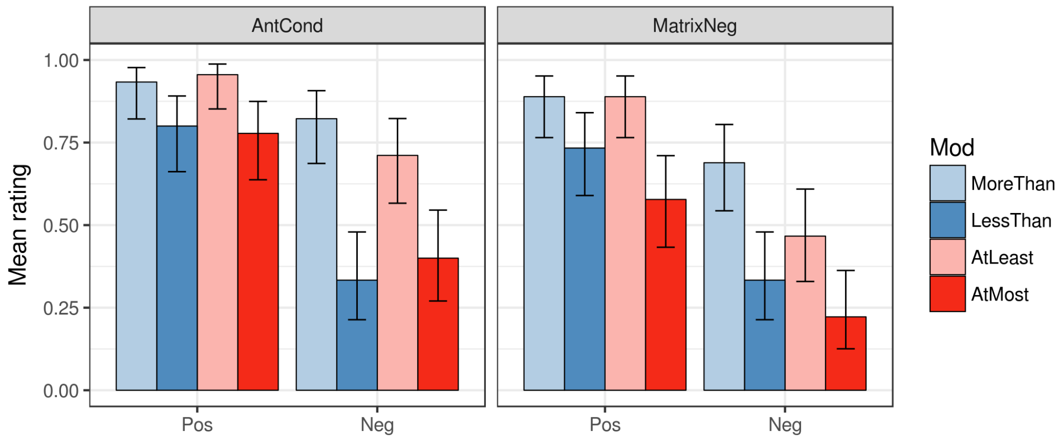


Figure 5.6: Exp 3. raw means, by Modifier. Bars represent 95% binomial confidence intervals.

Again, modifier monotonicity seems to make a difference – as usual, the downward-entailing modifiers are worse than their upward-entailing counterparts, and especially so in the cases with negative polarity. As we did before, it would be wise to study them both by monotonicity and by type.

Let's fit a logistic mixed-effects model with fixed effects for Modifier Monotonicity (UE, DE), Modifier Type (Comp, Sup), Polarity (Pos, Neg), and Environment (AntCond, MatrixNeg), and all their interactions, and the maximal random effects structure for which the model converges. Taking the first level of each factor as the reference level, we find a significant effect of ModMon = DE ($\beta = -2.34,\ z = -2.481,\ p = 0.0131$ *) and of Pol = Neg ($\beta = -2.15,\ z = -2.186,\ p = 0.0288$ *). Both are as we would already expect by now.

Unpacking the model further, we find that there is in general no significant difference between AtLeast and MoreThan / AtMost and LessThan except in MatrixNeg-Neg, where there was chances for AtLeast to be judged comprehensible were significantly worse than for MoreThan.

| Env | Pol | ModMon | ModType | OR | CI | z | p |
|---|---|---|---|---|---|---|---|
| AntCond | Pos | UE | Comp - Sup | 1.30 | [0.10, 16.05] | 0.233 | 0.8161 |
| MatrixNeg | Pos | UE | Comp - Sup | 2.47 | [0.34, 17.98] | 1.023 | 0.6131 |
| AntCond | Pos | DE | Comp - Sup | 1.33 | [0.30, 5.96] | 0.428 | 0.6687 |

Table 5.8 (Continued)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| MatrixNeg | Pos | DE | Comp - Sup | 3.13 | [0.84, 11.63] | 1.946 | 0.1033 |
| AntCond | Neg | UE | Comp - Sup | 2.96 | [0.69, 12.71] | 1.672 | 0.0946 |
| MatrixNeg | Neg | UE | Comp - Sup | 4.38 | [1.17, 16.40] | 2.505 | 0.0245 |
| AntCond | Neg | DE | Comp - Sup | 0.62 | [0.16, 2.37] | -0.793 | 0.5065 |
| MatrixNeg | Neg | DE | Comp - Sup | 2.09 | [0.49, 8.85] | 1.143 | 0.5065 |

Table 5.8: Exp. 3 predicted contrasts for levels of Modifier Type.

Even more interesting is the way in which each modifier compares to itself between environments. For the same levels of polarity, chances of being judged comprehensible for MoreThan and LessThan were not affected by the matrix embedding environment, whereas for AtLeast chances of being judged comprehensible dropped significantly from AntCond to MatrixNeg in cases with Pol = Neg, and for AtMost they dropped signficantly from AntCond to MatrixNeg for both Pol = Pos and Pol = Neg.

| Env | Pol | ModMon | ModType | OR | CI | z | p |
|---|---|---|---|---|---|---|---|
| AntCond - MatrixNeg | Pos | UE | Comp | 1.68 | [0.20, 14.37] | 0.539 | 0.5898 |
| AntCond - MatrixNeg | Neg | UE | Comp | 2.32 | [0.56, 9.64] | 1.320 | 0.3738 |
| AntCond - MatrixNeg | Pos | DE | Comp | 1.56 | [0.38, 6.38] | 0.706 | 0.9598 |
| AntCond - MatrixNeg | Neg | DE | Comp | 1.07 | [0.29, 3.89] | 0.111 | 0.9598 |
| AntCond - MatrixNeg | Pos | UE | Sup | 3.19 | [0.35, 29.04] | 1.179 | 0.2386 |
| AntCond - MatrixNeg | Neg | UE | Sup | 3.42 | [1.10, 10.66] | 2.426 | 0.0305 |
| AntCond - MatrixNeg | Pos | DE | Sup | 3.66 | [1.02, 13.11] | 2.282 | 0.0450 |
| AntCond - MatrixNeg | Neg | DE | Sup | 3.57 | [0.90, 14.23] | 2.065 | 0.0450 |

Table 5.9: Exp. 3 predicted contrasts for levels of Environment.

#### 5.4.4.5 Discussion

The fact that in the conditions with Pol = Neg *at least* was significantly different from (worse than) *more than* but *at most* was not significantly different from *less than* reveals a surprising degradation of *less than* in those conditions. This is outside of our main concern, but it is interesting and calls for an explanation.

The fact that both *at least* and *at most* were significantly worse under a negation further embedded under a matrix negation as opposed to a negation further embedded in the antecedent of a conditional reveals that felicity for SMNs embedded under a negation the scope of an additional downward-entailing environment is affected by the nature of this additional environment. Thus, this goes against the prediction that in such configurations rescuing should work the same. As such, more investigation is need, both empirical (comparison of other pairs of nested downward-entailing environments) and theoretical (to explain the sources of the difference).

Although our experiment had as its main goal to test rescuing, the particular design we adopted also allows us to discus further patterns. At the time when we designed it we did not aim to test long-distance anti-negativity or intervention by a factive – the additional patterns that our cursory discussion at the end of §5.1 revealed for SMNs. However, the design permits us to draw conclusions about those empirical claims also. The fact that there was no significant difference between SMNs and SMNs in MatrixNeg-Pos would seem to suggest that SMNs do not in fact have long-distance anti-negativity. This would be surprising, except for the fact that in this condition the proximity between matrix negation and the SMN was interrupted by an intervening factive – the verb *know*. Thus, in fact, instead of providing evidence against long-distance anti-negativity, this result provides evidence for rescuing through intervention by a factive. Could the intervening factive play a role in explaining the results for MatrixNeg-Neg? Perhaps, but recall that the same factive was present in AntCond-Neg also, so while it is likely that the factive played a role, one must still explain why there was still a small, but statistically significant difference between the two cases.

To sum up, this experiment provides support for some existing empirical claims such as intervention by a factive but also reveals the need to revise or refine other existing empirical claims. We will not attempt to do that here. We merely present these data to point out the need to probe further, both empirically and theoretically, not just for SMNs but also for other items that so far seemed to exhibit the same patterns.

## 5.5 Summary

Just as we did for ignorance, in this chapter we recalled our analysis of anti-negativity in *or* and *some* $NP_{SG}$ in Ch. 1 and extended it to CMNs and SMNs. Just as in the case of ignorance, the same approach we took for anti-negativity in *or* and *some* $NP_{SG}$ worked for CMNs and SMNs also: Exhaustification of CMNs/SMNs relative to their pre-exhaustified subdomain alternatives across a downward-entailing operator is vacuous for non-presuppositional downward-entailing environments but leads to proper strengthening for presuppositional downward-entailing environments; SMNs don't tolerate vacuous exhaustification, which captures their anti-negativity. In deriving anti-negativity we made crucial use of the truth conditions and alternatives obtained in Ch. 2. After that, we also used our discussion of scalar implicatures from Ch. 3 to investigate the interaction between the scalar alternatives of a BN, CMN, and SMN under negation and anti-negativity. In our comparison to the existing literature we noticed that our theory is fundamentally similar to the alternative-based solution to anti-negativity for SMNs provided in Spector (2015), just that we articulated it in greater detail and extended it to account for embedding in the antecedent of a conditional and the restriction of a universal. Just like Spector, our account connects anti-negativity in CMNs and SMNs to positive polarity sensitivity more generally. Finally, we presented experimental evidence that both supported our starting patterns and showed the need for further investigation, both empirical and theoretical.

We have reached all our main goals for this thesis. In the next chapter we present the global conclusions, and the new desiderata for future research that emerge from them.

# Chapter 6

# Conclusion

## 6.1 Global summary and conclusions

The main goal of this thesis was to offer a new theory of ignorance and anti-negativity in modified numerals, specifically to account for the fact that, while both comparative- and superlative-modified numerals seem to give rise to ignorance, the former are also compatible with certainty, and the fact that, while both are felicitous in downward-entailing environments such as the antecedent of a conditional or the restriction of a universal, the latter are bad in the scope of negation.

In Ch. 1 we started out by showing that these phenomena are not confined to modified numerals but have been studied extensively in the literature on disjunction and various types of indefinites under labels such as epistemic indefinites / Free Choice Items and polarity sensitive items (NPIs, PPIs). Drawing on that literature, we put together a formal account of ignorance and anti-negativity and showed how it captures the patterns for *or* and *some* $NP_{SG}$. Specifically, we showed how their behavior can be derived by noticing that they activate not only scalar but also subdomain alternatives, and by thinking through the consequences of making exhaustification relative to all their alternatives in general a default, and exhaustification relative to their pre-exhaustified subdomain alternatives in particular obligatory. More concretely, we showed that exhaustification relative to these alternatives across a modal always yields a Free

Choice effect, among which, assuming that assertions may be prefixed with a (last resort) null epistemic necessity modal, an epistemic Free Choice effect – what is more commonly known as ignorance. The fact that items vary with respect to the strength of ignorance can be derived by assuming that some but not others are able to prune their subdomain alternatives to a natural subset. We also showed that exhaustification of the pre-exhaustified subdomain alternatives across a downward-entailing operator is vacuous, unless the environment is presuppositional. The fact that items vary with respect to the anti-negativity can be derived by assuming that some tolerate vacuous exhaustification and others don't.

After sketching this solution for ignorance and anti-negativity for *or* and *some* $NP_{SG}$ , we set out to implement it in modified numerals also.

In Ch. 2 we showed that, given a certain principled decomposition of their truth conditions, both comparative- and superlative-modified numerals make reference in their truth conditions both to a scalar item and to a domain, and argued that for that reason they too activate scalar and subdomain alternatives, just like *or* and *some* $NP_{SG}$ .

Since saying that modified numerals activate scalar alternatives gives rise to known problems, in Ch. 3 we tackle this. We argued, against the received view, that, just like bare numerals, modified numerals give rise to scalar implicatures also, and provided evidence in support of this. We also argued that the traditional issues that arise from this should be handled not by departing from the view that they give rise to scalar implicatures but rather by trying to understand the interaction of these implicatures with other factors. In particular, I showed that the bad 'exactly' implicatures of CMNs and SMNs in episodic contexts can be ruled out by a clash with ignorance and a principled scalar alternative pruning mechanism, and the bad 'exactly' implicatures arising from a negated BN, CMN, or SMN can be handled by rethinking the types of alternatives that arise from a prejacent consisting of negation and a scalar item.

In Ch. 4 we turned to the first of our two main questions, namely, ignorance in CMNs and SMNs. We showed how, just as for *or/some* $NP_{SG}$ , exhaustification relative to their pre-exhaustified subdomain alternatives across a null epistemic necessity modal, or across an overt possibility or necessity modal, or across a universal quantifier predicts parallel ignorance / Free Choice effects for CMNs and SMNs. We also showed that, if we say that, like *some* $NP_{SG}$ , CMNs can prune down their set of subdomain alternatives but, like *or*, SMNs can't, that captures why the former can accommodate a context of speaker certainty while the latter can't. In our comparison to existing accounts section we saw that our account is similar to the alternative-based accounts of SMNs, but has conceptual advantages over them regarding the way the alternatives are derived and how they must be treated – we used general principles and assumptions that can handle not just modified numerals, but disjunction and indefinites in general also. But, aside from replicating the results of previous accounts for SMNs, as we mentioned, our theory also predicts parallel effects for CMNs. This very much goes against the received view, according to which CMNs don't give rise to ignorance. In our fit to experimental literature section we thus try to show that our view of how ignorance in CMNs and SMNs works can in fact help us make sense of otherwise puzzling findings from the latest experimental literature on ignorance, which show that both CMNs and SMNs can give rise to ignorance but CMNs are also compatible with certainty, while SMNs are not.

In Ch. 5 we turned to the second of our two main questions, which was anti-negativity in CMNs and SMNs. We showed how, just as for *or/some* $NP_{SG}$, exhaustification relative to their pre-exhaustified subdomain alternatives across a downward-entailing operator is vacuous for non-presuppositional downward-entailing environments but leads to proper strengthening for presuppositional downward-entailing environments, if we are allowed to take into account their presupposition. We also showed that, if we say that, like *or*, CMNs tolerate vacuous exhaustification, but, like *some* $NP_{SG}$, SMNs can't, that captures why only the former are fine under negation, although both are fine in the antecedent of a conditional or the restriction of a universal. In our comparison to existing accounts section we saw that our account is similar to the alternative-based account of anti-negativity in SMNs sketched in Spector (2015); although it differs in the letter, it is very much in the same spirit, which is to handle anti-negativity in modified numerals with the same tools that have already been used to capture it in the literature on polarity sensitive items. In our fit to experimental literature section we first noted that there is no existing experimental literature that we can refer to aside from a series of experiments I conducted myself in joint work with Kathryn Davidson. The first experiment tests CMNs and SMNs in three types of configurations – in the scope of negation, in the antecedent of a conditional or the restriction of a universal, and in the scope of a negation itself embedded in the antecedent of a conditional or the restriction of a universal. The first two configurations were as we assumed and derived, but the third went against general empirical claims for SMNs and items with anti-negativity more generally. For that reason we probed rescuing configurations in two more experiments. In one we varied the polarity of the predicate in the continuation; this revealed an interesting sensitivity of SMNs to this polarity, and prompted us to consider an analysis where we exhaustified with a silent *even* relative to alternatives from a truncated Horn scale relative to a contextual likelihood ordering. In the other we varied the type of higher downward-entailing environment, revealing an interesting sensitivity of SMNs to the nature of this environment; due to the design, we were also able to draw conclusions about intervention by factives. Overall, the experiments both provide support for the approach we have taken and reveal that much more work with respect to anti-negativity is still to be done, both empirically and theoretically, in regard to modified numerals in particular, and PPIs in general.

All in all, the theory of ignorance and anti-negativity in modified numerals presented here both replicates the results of the existing accounts and pushes the boundaries of the earlier discussions further in a number of ways, both small and large, the largest of which is perhaps that it shows that a variety of major topics in the literature on modified numerals are in fact major topics in the literature on disjunction and indefinites also, and can be analyzed in the exact same way.

## 6.2 Outlook: Expanding the dataset, extending the theory

In both our discussion of ignorance and in our discussion of anti-negativity we pointed out new empirical claims beyond our starting set. In relation to ignorance these were related to modified numerals under possibility modals, and in relation to anti-negativity these were related to modified numerals under other types of downward-entailing environments, or under downward-entailing operators at a distance vs. in the same clause, with or without other intervening operators, or in multiple nested downward-entailing environments.

Some of these empirical claims are well-established in the literature, might have also already been discussed, or else seem to be within the reach of the present approach; an example is the case of intervention by a universal operator. Other patterns are however much less well-established, have not yet received a formal treatment, and seem to require extensions to the present approach; an example is the case of SMNs in the antecedent of a conditional or the restriction of a universal, where felicity seemed to be very much affected by the polarity of the continuation in a way that requires us to consider additional theoretical tools such as exhaustification with a silent *even*, as we discussed at length in §5.4.3.

Thus, while the approach to ignorance and anti-negativity articulated here took us some way towards understanding these phenomena, a challenge for future work will be to investigate all these outlying patterns further, both empirically and theoretically, and not just in relation to modified numerals, but also in relation to these other items such as *or* and *some* $NP_{SG}$ / PPIs that we have argued they resemble.

## 6.3 Outlook: Predictions for the range of empirical variation

Suppose the approach to ignorance and anti-negativity articulated here is on the right track and can even be extended to handle new complications. At that point we may start to wonder about the predictions it makes for the range of empirical variation. That is because currently both differences in ignorance and in anti-negativity have been derived from minimal parametric switches – whether an item has the ability to prune natural subsets of its subdomain alternatives or not and whether it tolerates vacuous exhaustification or not. This type of parametric approach is certainly justified for the literature on disjunction and indefinites, where variations of precisely this minimal sort were what inspired it in the first place. (Recall our examples in Ch. 1 of minimally different indefinites such as English *some* $NP_{SG}$ vs. German *irgendein* or disjunctions such as English *or* and French *ou* or *soit …soit*.) However, it is not clear that it is justified for CMNs and SMNs, where, if all parametric options were possible, we might expect to find a language where a CMN and a SMN that can otherwise be analyzed as in English (i.e., they make reference in their truth conditions to both a scalar element and a domain, etc.) exhibit the opposite patterns – that is, a CMN that is incompatible with certainty and bad under negation and a SMN that is compatible with certainty and fine under negation – or patterns arising from other type of parametric switches – e.g., a CMN that is incompatible with certainty but fine under negation, like *or*, and a SMN that is compatible with certainty but bad under negation, like *some* $NP_{SG}$ . In the absence of evidence that such variants of CMNs and SMNs exist, instead of leaving the parametric settings for CMNs and SMNs to chance, we should probably find a way to derive them. This reminds us of Geurts & Nouwen (2007)'s distinction between numeral modifiers involving strict vs. non-strict comparison – in their terminology which has by now become well-known, Class A vs. Class B modifiers. But why should, e.g., non-strict comparison correlate with inability to prune subdomain alternatives and a requirement for proper strengthening? To sum up, while they seem to get the job done, it is likely that the parametric settings that we have assumed for CMNs and SMNs cannot be set at chance, and in that case an important next challenge for future work is to try and justify why they should be the way they are.

## 6.4 Outlook: Predictions for the nature of ungrammaticality

On our solution for ignorance in SMNs in episodic contexts this effect is as good as hard-wired – we claim it arises from obligatory exhaustification relative to the entire set of subdomain alternatives, and the only result this yields is total ignorance. Thus, our explanation for why SMNs are bad in a context of speaker certainty is to say that that is so because their assertion in those contexts would lead to contradiction.

Now, an early question in the experimental literature on ignorance in SMNs was whether it was actually semantic – as claimed by the hard-wired modal account of ignorance in SMNs due to Geurts & Nouwen (2007) – or pragmatic – as claimed by the neo-Gricean alternative-based account of ignorance in SMNs due to Büring (2008). Cummins & Katsos (2010) tested this by collecting graded judgments on sentences with CMNs/SMNs in certainty scenarios, e.g., *Jean has at least / more than n houses. Specifically/In fact, she has exactly n+1* and in addition to this also on sentences like *Jean has some houses* with continuations that either contradicted them (logical contradiction case) – *Specifically/In fact, she has none of the houses* – or entailed them (logical entailment case) – *Specifically/In fact, she has half of the houses* – or canceled the scalar implicature triggered by them (pragmatic infelicity case). On the one hand, they found that CMNs in the certainty condition scored as highly as logical entailments, while SMNs scored lower; this is expected on our view that CMNs can accommodate certainty but SMNs can't. On the other hand, they also found that SMNs were rated higher than both logical contradictions (this was a result also replicated by McNabb & Penka 2015) and pragmatic infelicities, suggesting both that ignorance in SMNs is pragmatic in nature and that it is easier to cancel than scalar implicatures.

Both these findings are interesting and, given our assumptions about the nature of ignorance in SMNs, call for some discussion. Let's tackle first the fact that SMNs in a context of certainty where their ignorance implicatures gave rise to contradiction were judged better than cases where contradiction arose semantically. I believe this could be explained by the fact that for SMNs to avoid contradiction one would simply have to ignore their implicatures, while for the other case one would have to ignore the truth conditions themselves, thus, the whole stimulus. The fact that SMNs were still rated lower than CMNs suffices to make our point that they don't usually accommodate certainty. Let's tackle now the fact that SMNs in the same context were judged better than pragmatic infelicities whose accommodation required the suspension of scalar implicatures. This is somewhat more surprising, since on our account we expect ignorance with SMNs to be very strong. At the same time, note that the item used for the pragmatic infelicity control was *some*, which arguably carries a very strong scalar implicature, whose cancelation might reasonably be expected to incur a fairly large penalty. Still, it shows the need for more empirical investigation into the severity of the penalties associated with one versus the other type of implicatures, and empirical variation along these dimensions.

And such questions about the nature of ungrammaticality are relevant for anti-negativity / PPIs also. To my knowledge there are currently no experimental investigations into the nature of ungrammaticality in anti-negativity. At the same time, the existing literature does make testable predictions. For example, on the present alternative-based analysis of anti-negativity / PPI-hood (which is also as in Chierchia 2013, Spector 2015, Nicolae 2017), anti-negativity arises because obligatory exhaustification relative to pre-exhaustified subdomain alternatives doesn't lead to proper strengthening. This doesn't seem like such a disastrous result, and could explain why even items that require proper strengthening and are thus usually bad under negation still carry a sensible meaning, as can be seen from the fact that SMNs in the scope of negation are difficult but we can still compute their truth conditions. This contrasts, for example, with similar alternative-based analyses of anti-positivity / NPI-hood (as in Chierchia 2013), on which ungrammaticality is said to arise because obligatory exhaustification relative to pre-exhaustified subdomain alternatives leads to logical contradiction. This seems like a pretty disastrous result and could explain why NPIs in an episodic context make no sense, cf. *#I make any sense*. A very welcome next step for future research would be to probe such predictions experimentally, with graded judgments.

To sum up, in this thesis we have tried to answer some questions related to ignorance and anti-negativity in modified numerals and beyond. As is always the case with research, (and must be why we love it so much,) a lot more questions remain.

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