The left-CONS2 Constraint

İsa Kerem BAYIRLI — *TOBB University of Economics and Technology*

Abstract. We propose a new generalization over determiner denotations. We claim that, for any determiner *Det*, *Det(P)(Q)* entails *Det(P ⋂ Q)(Q)* (*the left-CONS2 Constraint*, as we elaborate in the paper). We discuss potential counterexamples to the validity of this constraint including restricted universals (e.g. *every ... but John*, *approximately/almost all/every…*) and proportional expressions with an upper bound (e.g. *few*^{PROP}, *at most one-third…*, *fewer than seventy percent* …). Following earlier proposals, we provide evidence that problematic inferences associated with them arise not from their denotations but either from the operator *Exh* or from degree operators.

Keywords: determiners, conservativity, semantic universals

1. Introduction

One of the central tasks of formal semantics is to a provide a theory of natural language (NL) meanings that accounts for our intuitions about semantic relations between sentences. For instance, intuitively, (1a) entails (1b) just as (2a) entails (2b):

(1) a. Linda bought a yellow carpet.
    b. Linda bought a carpet.
(2) a. Every student read *Macbeth*
    b. Every American student read *Macbeth.*

Translating the sentences in (1) and (2) into the language of (standard) First Order Logic (FOL), we observe that the intuitive entailment relations between these sentences can be captured as formal (semantic) entailment within this logical system.

(3) a. $\exists x((Yx \land Cx) \land Blx)$
    b. $\exists x(Cx \land Blx)$
    c. $\{\exists x((Yx \land Cx) \land Blx)\} \vDash \exists x(Cx \land Blx)$
(4) a. $\forall x(Sx \rightarrow Rxm)$
    b. $\forall x((Sx \land Ax) \rightarrow Rxm)$
    c. $\{\forall x(Sx \rightarrow Rxm)\} \vDash \forall x((Sx \land Ax) \rightarrow Rxm)$

One might be tempted to think that translating NL-sentences into the sentences of FOL will suffice to model speaker intuitions about entailment relations between sentences. Such an optimism would be unwarranted, however. Barwise & Cooper (1981) show that proportional determiners such as *most, more than half, one-third* cannot be defined within standard FOL (p. 213 – 216, Theorem C12 and C13). That is, NL-sentences with proportional determiners, examples of which are shown in (5a) and (5b) cannot be paired with FOL-sentences. As a result, the intuitive entailment relation between (5a) and (5b), where (5a) ⇒ (5b), cannot be expressed using the language of (standard) FOL.

(5) a. Most students read both *Macbeth* and *Hamlet*.
b. Most students read *Macbeth*.

This observation, among others, has been one of the motivations behind the application of Generalized Quantifier Theory (GQT, Mostowsky, 1957) to natural language quantifiers (Barwise & Cooper, 1981; Keenan & Stavi, 1986 a.o.). Within this theory, a quantified nominal expression (i.e. a quantifier) is analyzed as denoting a set of subsets of E, the set of Entities provided by the model ($S \subseteq E$ is the set of students).

(6) $\|$ every student $\| = \{X \subseteq E: S \subseteq X\}$
$\|$ most students $\| = \{X \subseteq E: |S \cap X| > |S \setminus X|\}$
$\|$ at least one third of students $\| = \{X \subseteq E: |X \cap S| / |S| \geq 1 / 3\}$

We can also express the denotation of a quantifier as a function characterizing a set of sets. That is to say, a generalized quantifier can also be understood to be a function from properties to truth values.[1]

(7) $\|$ every student $\| = \lambda Q_{et}.S \subseteq Q^*$
$\|$ most students $\| = \lambda Q_{et}.|S \cap Q^*|>|S \setminus Q^*|$
$\|$ at least one third of students$\| = \lambda Q_{et}.|S \cap Q^*| / |S| \geq 1 / 3$

Determiners can, then, be represented as denoting functions from properties to generalized quantifiers:

(8) $\|$ every $\| = \lambda P_{et}.\lambda Q_{et}.P^* \subseteq Q^*$
$\|$ most $\| = \lambda P_{et}.\lambda Q_{et}.|P^* \cap Q^*| > |P^* \setminus Q^*|$
$\|$ at least one third $\| = \lambda P_{et}.\lambda Q_{et}.|P^* \cap Q^*| / |P^*| \geq 1 / 3$

One important observation about quantification in natural languages is that there are restrictions on what types of determiners exist in the world's languages. That is, not every possible determiner is a natural language determiner. For instance, there seems to be no natural language in which the pseudo-determiner *blah₁*, with the denotation given in (9), is to be found (Keenan, 1996, see Chierchia & McConnell-Ginet, 2000 for other examples):

(9) $\| \text{blah}_1 \| = \lambda P_{et}.\lambda Q_{et}.\ |P^*| = |Q^*|$

There are various mathematical properties that natural language determiners must satisfy (Barwise & Cooper, 1981; Keenan & Stavi, 1986; van Benthem, 1983, 1986 a.o.). A well-known and much discussed generalization about determiners is that they denote functions that are conservative on the first argument, a notion that can be defined as:

(10) A determiner denotation $Det \in D_{<et,<et,t>>}$ is conservative on its first argument (or is CONS1) iff for any $P, Q \in D_{<et>}$, $Det(P)(Q) \Leftrightarrow Det(P)(P \cap Q)$[2]
The CONS1 Constraint: NL-determiners denote CONS1 functions.

---

[1] Given a function *f* of type *<et>*, $f^* = \{x: f(x)=1\}$. That is, f^* is the set characterized by *f*.
[2] Given any two functions *P,Q* of type *<et>*, $P \cap Q = \lambda z.\ P(z) \wedge Q(z)$. Note that $(P \cap Q)^* = P^* \cap Q^*$.

This constraint provides an explanation for the absence of a determiner like *blah₁*.

(11) $\|\text{ blah}_1 \|(P)(Q) \not\Leftrightarrow \|\text{ blah}_1 \|(P)(P \cap Q)$
since $|P^*| = |Q^*| \not\Leftrightarrow |P^*| = |P^* \cap Q^*|$

Intuitively, the CONS1 Constraint entails that the truth value of an expression of the form *Det(P)(Q)* can be determined simply by looking at the properties of those entities which are in the set characterized by the first argument (i.e. P*) of the determiner.[3] For instance, to determine the truth value of a sentence like (2a) (in a model), we need not have any information as to whether any of the non-students (in this model) read Macbeth. The properties of the members of the complement set of students are irrelevant to the truth value of this sentence.

A natural question that arises at this point is whether NL-determiners can be said to be conservative on their second argument, too, which can be defined as:

(12) A determiner denotation Det $\in D_{<et,<et,t>>}$ is conservative on its second argument (or is CONS2) iff for any P, Q $\in D_{<et>}$, $Det(P)(Q) \Leftrightarrow Det(P \cap Q)(Q)$
The CONS2 Constraint: NL-determiners denote CONS2 functions

If NL-determiners are conservative on their second argument, then the truth value of an expression of the form *Det(P)(Q)* is only sensitive to the properties of the entities in the set characterized by the second argument (i.e. Q*) of the determiner. This would mean that a pseudo-determiner *blah₂*, with the denotation in (13), cannot be found in any natural language.

(13) $\|\text{ blah}_2 \|(P)(Q) \Leftrightarrow |P^* \setminus Q^*| = |P^* \cap Q^*|$

Keenan (2006:304) notes that "many natural classes of Dets fail CONS2." Universal determiners like *every*, *all* and proportional determiners like *most*, *at least one-third* do not denote CONS2 functions:

(14) a. $\|\text{every}\|(P)(Q) \not\Leftrightarrow \|\text{every}\|(P \cap Q)(Q)$
since $P^* \subseteq Q^* \not\Leftrightarrow P^* \cap Q^* \subseteq Q^*$
b. $\|\text{most}\|(P)(Q) \not\Leftrightarrow \|\text{most}\|(P \cap Q)(Q)$
since $|P^* \cap Q^*| > |P^* \setminus Q^*| \not\Leftrightarrow |(P^* \cap Q^*) \cap P^*| > |(P^* \cap Q^*) \setminus Q^*|$

The conclusion seems to be that NL-determiners obey the CONS1 Constraint but not the CONS2 Constraint.

2. The left-CONS2 Constraint

The presence of determiners such as *most* and *every* shows that NL-determiners can denote non-CONS2 functions. At this point, it is important to note that these two determiners fail to denote CONS2 functions in a *specific* way. To see this, let us first note that the definition of a CONS2 function contains a biconditional statement ($Det(P)(Q) \Rightarrow Det(P \cap Q)(Q)$ as well as $Det(P \cap Q)(Q) \Rightarrow Det(P)(Q)$). Separating each conditional that enters into the definition of

[3] To be precise, for *CONS1* to have this consequence two other conditions, *Extension* and *Quantity*, must be assumed (Barwise & Cooper, 1981; van Benthem, 1986; Keenan & Stavi, 1986). For discussion, see Clark (2001).

CONS2 functions, we can distinguish between two types of functions: left-CONS2 functions and right-CONS2 functions:

(15) Left-CONS2 functions
A determiner denotation Det $\in D_{<et,<et,t>>}$ is left-conservative on its second argument (or is left-CONS2) iff for any P,Q $\in D_{<et>}$ Det(P)(Q) $\Rightarrow$ Det(P $\cap$ Q)(Q).

(16) Right-CONS2 functions
A determiner denotation Det $\in D_{<et,<et,t>>}$ is right-conservative on its second argument (or is right-CONS2) iff for any P,Q $\in D_{<et>}$ Det(P $\cap$ Q)(Q) $\Rightarrow$ Det(P)(Q)

The universal determiner *every* and the proportional determiner *most* are not CONS2 functions since they do not denote right-CONS2 functions:

(17) a. $\|every\|(P \cap Q)(Q) \nRightarrow \|every\|(P)(Q)$
since $P^* \cap Q^* \subseteq Q^* \nRightarrow P^* \subseteq Q^*$
b. $\|most\|(P \cap Q)(Q) \nRightarrow \|most\|(P)(Q)$
since $|(P^* \cap Q^*) \cap P^*| > |(P^* \cap Q^*) \setminus Q^*| \nRightarrow |P^* \cap Q^*| > |P^* \setminus Q^*|$

Crucially both *every* and *most* denote left-CONS2 functions:

(18) a. $\|every\|(P)(Q) \Rightarrow every(P \cap Q)(Q)$
since $P^* \subseteq Q^* \Rightarrow P^* \cap Q^* \subseteq Q^*$
b. $\|most\|(P)(Q) \Rightarrow \|most\|(P \cap Q)(Q)$
since $|P^* \cap Q^*| > |P^* \setminus Q^*| \Rightarrow |(P^* \cap Q^*) \cap P^*| > |(P^* \cap Q^*) \setminus Q^*|$

In this paper we claim that being a left-CONS2 function is a mathematical property that natural language determiners must satisfy.

(19) The left-CONS2 Constraint
NL-determiners denote left-CONS2 functions.

In this way, we can provide an explanation for the absence of the determiner *blah*$_2$ in (13).

(20) $\| blah_2 \|(P)(Q) \nRightarrow \| blah_2 \|(P \cap Q)(Q)$
since $|P^* \setminus Q^*| = |P^* \cap Q^*| \nRightarrow |(P^* \cap Q^*) \setminus Q^*| = |(P^* \cap Q^*) \cap Q^*|$

At an intuitive level, what the left-CONS2 Constraint says is that the truth of an expression of the form *Det(P)(Q)* cannot be sensitive to the presence of entities in the set P* \ Q*. Note that the truth value of *Det(P)(Q)* can be sensitive to the absence of entities in P* \ Q* (Consider *every*, which requires that the set P* \ Q* be empty). There cannot, however, be a determiner *Det* that requires that P* \ Q* should be non-empty, or that it should have the cardinality n, for some $n \geq 1$.

3. Some potential counterexamples to the left-CONS2 Constraint

In what follows, we shall take a closer look at potential counterexamples to the left-CONS2 Constraint. These include restricted universals and proportional expressions with an upper bound. Following earlier proposals, we suggest that the semantic contribution of sentential operators (like *Exh*) and degree operators should be severed from the denotation of the determiners. Once this is done, we see that potential counterexamples to the left-CONS2 Constraint are spurious; NL-determiners denote left-CONS2 functions after all.

3.1. Restricted Universals

Consider (21):

(21) Every student but John read Macbeth.

In one of the earlier treatments of universal (connected) exceptive constructions[4] like (21) within GQT, Keenan & Stavi (1986) claim that the analysis in (22) captures the semantic properties of such constructions ($M^* = \{x \in E: x \text{ read Macbeth}\}$)

(22) $\|\text{every…but John}\|(S)(M) \Leftrightarrow S^* \setminus M^* = \{j\}$

It is not hard to see that the discontinuous determiner *every…but John* does not denote a left-CONS2 function under this analysis.

In a comprehensive analysis of exceptives, von Fintel (1993) suggests that the denotation of such constructions consists of two components: Subtraction and Exhaustivity (or Uniqueness). The subtraction component is responsible for subtracting the excepted entities from the domain of quantification of the determiner. The exhaustivity (or "uniqueness" in the terminology of von Fintel) component requires that the set that consists of the excepted entities be the smallest one whose exclusion renders the sentence true.[5]

(23) $\|\text{Det P but X Q}\| \Leftrightarrow \text{Det}(P \setminus X)(Q)$ (Subtraction)
$\wedge\ \forall X': X' \not\subseteq X \rightarrow \neg\text{Det}(P \setminus X')(Q)$ (Exhaustivity)

Under this analysis, a universal (connected) exceptive like (21) is given the following analysis:

[4] There is a second type of exceptive constructions, free exception constructions (Hoeksema, 1987), in which the excepted phrase can occupy various positions within the sentence:

(1) a. Except for you, I would not trust any dentist.
b. I would not, except for you, trust any dentist.
c. I would not trust any dentist except for you.

We ignore such constructions in this paper and focus on connected exceptives. For discussion of free exceptives, see von Fintel (1993) and Hoeksema (1995).

[5] One important consequence of this analysis is that it provides an explanation for certain co-occurence restrictions on exceptive constructions.

(24) $\|$every.. but John $\|$(S)(M) $\Leftrightarrow$ $S^* \setminus \{j\} \subseteq M^*$ (Subtraction)
$\wedge\ \forall X$: $j \notin X \rightarrow S^* \setminus X \not\subseteq M^*$ (Exhaustivity)

There are also analyses in which subtractive and exhaustive inferences associated with connected exceptives come from distinct sources (Gajewski, 2008, 2013; Hirsh, 2016). Under such analyses, subtractive inferences are a consequence of the denotation of exceptive determiners. That is, the overall effect of a *but*-phrase is to restrict the domain of the quantification of the determiner it is associated with.

(25) $\|$every…but John $\|$(S)(M) $\Leftrightarrow$ $S^* \setminus \{j\} \subseteq M^*$ (Exceptives as Subtraction)

Exhaustive inferences arise due to the presence of an exhaustivity operator (*Exh*), which is responsible for negating alternative sentences that are not entailed by the prejacent sentence (Hirsch, 2016; see also Gajewski, 2013 for an analysis based on an *Exh* operator that takes second-order alternatives into consideration).

(26) *Exh*[Every student but [John]$_F$ read Macbeth]

This operator acts on the alternative propositions that are obtained by replacing the focus-marked constituent (here, the complement of *but*) with its alternatives. Assuming a context with *Mary*, *Sue* and *John* as three distinct individuals, the alternatives to (21) are given in (27):

(27) ALT$_C$(21) =
{Every student but Mary read Macbeth, Every student but Sue read Macbeth}

Due to *Exh*, the alternatives that are not entailed by (21), i.e. the prejacent, are negated. As a result, we obtain the inference that John didn't read Macbeth.[6]

(28) $S^* \setminus \{j\} \subseteq M^*$ (prejacent)
$S^* \setminus \{s\} \not\subseteq M^*$ (negated alternative)
$S^* \setminus \{m\} \not\subseteq M^*$ (negated alternative)
$j \notin M^*$ (conclusion)

That is, under such decompositional approaches, the inferences associated with universal exceptives that are problematic for the left-CONS2 constraint do not come from the denotation of the determiner itself. If this is true, then universal (connected) exceptives, with the denotation given in (25), do not invalidate the left-CONS2 Constraint.

Crnič (2018) provides independent evidence that "[s]ubtraction, but not [e]xhausitivity, is encoded in the meaning of" exceptives (p. 744). The analysis relies on *Condition on VP-Ellipsis*, for which Crnič provides independent evidence.[7]

---

[6] We take it for granted that $j \in S^*$ holds. See von Fintel (1993) and Gajewski (2013) for discussion.
[7] Consider (1), discussed in Crnič (2018: 746 - 747), where strikethrough represents the elided material.

(1) a. John read exactly three books.
b. To get an A, he really had to ~~read exactly three books.~~

(29) Condition on VP Ellipsis
If a quantificational expression is interpreted in the antecedent VP, a semantically equivalent expression must be interpreted in a parallel position in the elided VP.

Crnič observes that VP-ellipsis construction like (30), where ~~strikethrough~~ represents the elided material, pose a challenge for approaches that take exhaustive inferences associated with exceptives to be internal and integral to the denotation of such determiners (e.g. Keenan & Stavi, 1986 and von Fintel, 1993):

(30) a. In the exam, John solved every exercise but the last one.
b. (To get an A), he really had to ~~solve every exercise but the last one~~.

The crucial observation is that (30b) is not understood to entail (31).

(31) (To get an A), John had to not solve the last exercise.

In a context with three exercises *{e1,e2, e3}*, where the third exercise is the last exercise, an integral approach predicts the denotation of (30b) to be as in (32a), in which case (30b) is predicted to entail (31) ($S_j = \lambda x.$ *John solves x*, E^* is the set of exercises)

(32) a. $\Box\ (E^* \setminus \{e3\} \subseteq S_j^* \wedge \forall X\text{: } e3 \notin X \rightarrow E^* \setminus X \not\subseteq S_j^*)$ (prejacent)
b. $\Box\ (\forall X\text{: } e3 \notin X,\ E^* \setminus X \not\subseteq S_j^*)$ (inference from (a))
c. $\Box\ (e3 \notin S_j^*)$ (letting $X = U \setminus \{e3\}$, U = the Universe)

The fact that such an inference is not obligatory can be accounted for within an approach to exceptives that take exhaustive inferences to arise due to an *Exh* operator at the sentence level. Under this analysis, the sentence in (30b) has the representation shown in (33) with the contextual alternatives given in (34).

(33) *Exh*[John had to solve every exercise but [the last one]$_F$]
(34) ALT_C(30b) =
{John had to solve every exercise but the first one,
John had to solve every exercise but the second one}

Negating all the alternatives that are not entailed by the prejacent of *Exh*, we obtain:

(35) a. $\Box\ (E^*/\{e3\} \subseteq S_j^*)$ (prejacent)
b. $\neg\Box\ (E^*/\{e1\} \subseteq S_j^*)$ (negated alternative)
c. $\neg\Box\ (E^*/\{e2\} \subseteq S_j^*)$ (negated alternative)

(1)b can only mean that John was required to read three books and not to read more than three books. That is "exactly" must be understood to be in the scope of the modal operator. The more plausible reading, in which John is required to read three books but is also allowed to read more than three books, could be derived by assuming that the quantifier "exactly three books" moves to a position higher than the modal operator. This would violate *Condition on VP Ellipsis*, which is why such (plausible) readings are not available. See Fiengo & May (1994) and Fox (2000) for further discussion and derivation of this condition.

This set of propositions is compatible with John not having to solve the last exercise. That is, the conjunction of the propositions in (35) does not contradict (36).

(36) $\Diamond$ (e3 $\in S_j$*)

Consider a model where w1 and w2 are accessible from the world of evaluation (say w0). In w1, John solves both exercise 1 and exercise 2; in w2, John solves all the three exercises (e1, e2 and e3). It can be seen that in such a scenario, each proposition in (35) and (36) is true. This means that (30b) does not entail (31), as expected.

Building upon a proposal by Spector (2014), Crnič (2018) argues that, similar to exceptives, approximative determiners also have subtractive analyses as in (37b), which involves the contextually determined inference that the subtracted set X is a small one (but, presumably, not empty. See Horn, 2002; Nouwen, 2006 for a discussion of this proximal component of *almost*).

(37) a. Almost every student read Macbeth.
b. $||\text{almost}_X \text{ every}||(P)(Q) \Leftrightarrow P^* \setminus X \subseteq Q^*$ (or $\exists X\colon P^* \setminus X \subseteq Q^*$)

This analysis raises the question of how to account for the negative inference in (38) associated with the approximative sentence in (37a). The negative inference is problematic for the left-CONS2 Constraint if it is to be encoded in the denotation of approximatives.[8]

(38) Not every student read Macbeth.

Crnič suggests that sentences containing approximate determiners are obligatorily parsed with an *Exh* operator. Replacing the (contextually determined) set X with the empty set, i.e. ∅, we obtain the alternative in (39a). Note that this alternative is stronger than (37a) given the analysis in (37b).

(39) a. Every student read Macbeth $\in$ ALT(37a)
b. Every student read Macbeth $\Rightarrow$ Almost every student read Macbeth

With these observations, we see that the negative inference we see in (38) is a result of negating the alternative in (39a) due to *Exh*. One piece of evidence for severing the contribution of the *Exh* operator from the denotation of approximative determiners comes, again, from VP-ellipsis contexts. Consider:

(40) a. In the exam, John solved almost every exercise.
b. (To get an A), he really had to ~~solve almost every exercise.~~

Crucially, Crnič observes, (40b) does not entail (41).

(41) To get an A, John had to not solve every exercise.

[8] Sadock (1981) and Ziegeler (2000) take this inference to be a conversational implicature. In Crnič's analysis, such inferences are conventional (i.e. grammatical) but this does not entail that they are integral to *almost every*. See Chierchia, Fox & Spector (2012) for an overview of the grammatical approach to scalar inferences.

Similar to what we have observed with exceptive constructions, the absence of this entailment can be explained on the assumption that the negative entailment of (37a) does not come from the determiner itself but from the *Exh* operator. That is, the sentence in (40b) has the LF-representation in (42).

(42) *Exh*[John had to solve almost-X_F every exercise]

This concludes our discussion of restricted universal determiners in the context of the left-CONS2 Constraint. We have reviewed the evidence indicating that inferences associated with such determiners that are problematic for the left-CONS2 Constraint (i.e. the negative inferences) come not from the denotation of determiners but from the obligatory presence of the operator *Exh*.[9] This suggests that restricted universal determiners do not invalidate the left-CONS2 Constraint.

3.2. many^{PROP} and few^{PROP}

Partee (1989) argues that the context-dependent determiner *many* and its antonym *few* are ambiguous between cardinal and proportional readings. In a scenario "where all the faculty children were at the picnic but there were few faculty children" in the first place, one can say:

(43) There were few faculty children at the 1980 picnic.

This interpretation of the sentence, in which "few faculty children" can be understood to correspond to "all faculty children", is captured with a cardinal analysis of this determiner.

(44) a. $||\text{few}^{CARD}||(P)(Q) \Leftrightarrow P^* \cap Q^*\,| < n$, a small number
b. $||\text{many}^{CARD}||(P)(Q) \Leftrightarrow |P^* \cap Q^*| > k$, a large number

There are, however, cases where a cardinal analysis is not sufficient to explain the interpretations available with *few* (and *many*). For instance, while stage-level predicates ("existence-asserting" ones) are compatible with the possibility of "few" being "all" as in (45a) individual level predicates do not allow such an interpretation (45b).

(45) a. Few egg-laying mammals turned up in our survey, perhaps because there are few.
b. #Few egg-laying mammals suckle their young, perhaps because there are few.

---

[9] Keenan & Stavi (1986) suggest that the string *not every* is a determiner and has the denotation in (2)

(1) Not every student came to the party.
(2) $||\text{not every}||(P)(Q) \Leftrightarrow P^* \setminus Q^* \neq \emptyset$

If *not every* were, indeed, a determiner, we would expect it to behave as a unit and take scope from a single position. However, Sternefeld (2006: 333 as discussed in Penka, 2011: 6) observes other operators can intervene between the negation operator and the universal determiner, which suggests that *not every* is not a determiner.

(3) Not every boy can be above average height. ($\neg$ » ◆ » $\forall$)

For further arguments against analyzing *not every* as a determiner, see Rothstein (1988).

(45b) is infelicitous because the continuation "because there are few" suggests a cardinal interpretation (which is compatible with the possibility of "few" being "all"). However, the determiner *few*, when used with an individual level predicate, has a proportional interpretation and "*few* can never be "all" on the proportional reading" (Partee, 1989).[10]

(46) a. $\|few^{PROP}\|(P)(Q) \Leftrightarrow |P^* \cap Q^*| / |P^*| < n$, a small proportion
b. $\|many^{PROP}\|(P)(Q) \Leftrightarrow |P^* \cap Q^*| / |P^*| > k$, a large proportion

Cardinal determiners denote symmetric functions, hence CONS2 functions and hence left-CONS2 functions. Unlike *many*^{PROP}, which denotes a left-CONS2 function, *few*^{PROP} does not denote a left-CONS2 function:

(47) $\| few^{PROP} \|(P)(Q) \not\Rightarrow \| few^{PROP} \|(P \cap Q)(Q)$
since $|P^* \cap Q^*| / |P^*| < n \not\Rightarrow |(P^* \cap Q^*) \cap Q^*| / |P^* \cap Q^*| < n$

There is reason to believe that the determiner *few* has a negative component that should be severed from the denotation of the determiner itself. In the presence of a modal operator, sentences with *few* has a (preferred) split-scope reading in which negation outscopes the modal operator and the quantifier is in interpreted in the scope of the modal operator (i.e. *negated de dicto readings*; see de Swart, 2000 for the example and discussion; see also Solt, 2006 and Penka, 2011 on split-scope readings).

(48) Ze hoeven weinig verpleegkundigen te ontslaan
They need few nurses to fire
"For a group Y consisting of few nurses y, it is necessary to fire each y" (de re)
"It is not necessary to fire more than a small number of nurses" (negated de dicto)

The first interpretation (i.e. the *de re* reading) corresponds to "few nurses" taking scope over the modal operator. Under this reading, there are a small number of specific nurses that must be fired. A more natural interpretation for (48) is the second interpretation (i.e. the *negated de*

[10] There is a third reading associated with *many* and *few* in which the interpretation is the reverse of the proportional readings (see Westerståhl, 1985 for a discussion of the reverse-proportional *many*. Herburger, 1997 makes similar observations for *few* and discusses the interaction of these readings with focus). Under these readings, a sentence like (1)a is understood to be truth-conditionally equivalent to (1)b:

(1) a. Many Scandinavians have won the Nobel Prize in literature.
b. Many winners of the Nobel Prize in literature are Scandinavians.

These readings could be captured with the following denotations for *many* and *few*.

(2) a. $\| many^{R.PROP}\|(P)(Q) \Leftrightarrow |P^* \cap Q^*| / |Q^*| > k$, a large proportion.
b. $\| few^{R.PROP} \|(P)(Q) \Leftrightarrow |P^* \cap Q^*| / |Q^*| < n$, a small proportion.

many$^{R.PROP}$ and *few*$^{R.PROP}$ are of interest because they seem to counterexemplify *the CONS1Constraint* we have discussed in *Introduction*. Interestingly, such determiners denote left-CONS2 functions, which is why we ignore them in this paper. For an analysis of these readings that makes them compatible with *the CONS1 Constraint*, see Romero (2015). For a recent general overview of the status of *the CONS1 Constraint*, see Zuber & Keenan (2019).

dicto reading), in which we learn that it is possible to not fire a large number of nurses.[11] These interpretations can be distinguished in a context in which it is necessary to fire large numbers of nurses but "there are only one or two nurses in particular for whom is it necessary that (s)he be fired." (de Swart, 2000: 114). In a context of this kind, the first reading comes out as true while the second reading is false.

In analyzing split readings, de Swart (2000) adopts a lexicalist approach in which the quantifier "few nurses" has a higher order interpretation in terms of quantification over properties.[12] Penka (2011), on the other hand, suggests that *few* can be equated with Degree Negation (i.e. $\lambda d.\ \lambda D.D(d) = 0$) similar to the analysis of *little* in Heim (2006). Romero (2015) claims that the (parametrized) determiner *few* consists of the (parametrized) determiner *many* and Degree Negation, which are not interpretable as such but they split in the course of derivation for interpretability. It seems fair to say that any approach to *few* that accounts for split-readings ends up suggesting that *few* is not a determiner as such.

We follow Romero (2015 see also Hackl, 2001) in assuming that $many^{PROP}$ and few^{PROP} are parametrized determiners with a degree argument. Following McNally (1998), we assume that *few* has the same denotation as *many*. The main difference between these two determiners is that *few* has an uninterpretable Degree Negation feature[13] (i.e. *uD.Neg*).

(49) $\|\text{many}^{\text{PROP}}\| = \|\text{few}^{\text{PROP}}{}_{[\text{uD.Neg}]}\| = \lambda d.\lambda P.\lambda Q.|P^* \cap Q^*| / |P^*| \geq d$

Due to the uninterpretable D.Neg feature, *few* must occur in the scope of the Degree Negation operator, whose function is to give the complement of a set of degrees.

(50) $\|\text{D.NEG}\| = \lambda D.D'$

Similar to adjectives, the determiners *many* and *few* are associated with a POS operator (Cresswell, 1976; Heim, 2006; von Stechow, 2009, a.o), which introduces a contextually determined *Neutral Segment* (an interval of degrees to be called N_c) such that each degree on this interval counts neither as many nor as few on the relevant scale. Consider an exam with 100 multiple choice questions, where the class average for correct answers is around 40%. The students who have answered less than 20% of the questions correctly can be said to have answered few questions correctly and the students who have answered more than 60% of the questions correctly can be said to have answered many questions correctly. The students who have solved between 20% and 60% of the questions have solved neither few nor many questions. The Neutral Segment (N_c) that is crucial for the interpretation of *many* and *few* can be encoded in the denotation of the operator POS_c.[14]

---

[11] There is a third possible reading in which the modal operator scopes over "few" and we obtain the interpretation that it is necessary that they fire few nurses. This reading is not available in (48) due to the fact that *hoeven* "need" in Dutch is a negative polarity item that must be in the scope of a negative operator. The absence of this reading is immaterial for our purposes.

[12] Under this approach, *few* is not a determiner as such; therefore, it is not in the scope of any generalization over determiner denotations. Other approaches that do not take *few* to have a determiner-like denotation include Solt, 2015 and Rett, 2018. If *few* is not a determiner, it does not constitute a problem for the left-CONS2 Constraint.

[13] This idea is reminiscent of analyzing the negative determiner "no" as an existential determiner with the additional syntactic requirement of occurring the scope of a (generally silent) Neg operator (Zeijlstra, 2004; Penka, 2011). Note that *few* must occur in the scope of the Degree negation and not the sentential negation operator.

[14] For a more principled approach to the demarcation of a Neutral Segment in the context of *few* and *many*, see Romero (2015).

(51) $POS_c = \lambda D_{<dt>}.\forall d \in N_c: D(d)$

Let us see now how this idea can be made to work in order to analyze split-scope readings of *few*^{PROP}. Consider (52) with the LF in (53), where the POS_c operator has moved for interpretability, leaving behind a degree trace. *D.NEG* merges with the constituent in which the variable in the tail of the movement chain is bound by a lambda operator.

(52) Few mammals can survive in the polar climate.
(53) $[POS_c$ [D.NEG $[\lambda d_1$ [can $[[d_1$-few$^{PROP}_{[UD.Neg]}$ mammals] [survive in polar climate]]]]]]

The denotation of (53) is given in (54), where $M_{w'} = \lambda x.$ *x is a mammal in w'*, $P_{w'} = \lambda x.$ *x survives in the polar climate in w'*.

(54) $\lambda w.\ \forall d \in N_c, \forall w' \in ACC_w, |M_{w'}{*} \cap P_{w'}{*}| / |M_{w'}{*}| < d$

This denotation suggests that, by nomological necessity, the proportion of mammals that survive in the polar climate to the totality of mammals is less than the average of the survival rates of mammals in different climates. This is the *negated de dicto* reading. Other readings associated with *few*^{PROP} can be represented by letting *D.NEG* scope below the modal operator (the narrow scope reading) or by moving the quantifier “few mammals” to a position higher than the modal operator (the *de re* reading).

We have shown that we can make sense of the idea that *few*^{PROP} is a type of determiner on the assumption that it is semantically equivalent to *many*^{PROP}. Recall that *many*^{PROP} is a parametrized determiner with an additional degree argument. Strictly speaking, *many*^{PROP} is not a left-CONS2 function given that its denotation is not a member of $D_{<et,<et,t>>}$. We can, however, say that, *many*^{PROP} *contains* a determiner. That is, once its degree argument is satisfied with any d, 0 < d < 1, *many*^{PROP}*(d)* is a determiner and denotes a left-CONS2 function.

(55) $\forall d, 0 < d < 1, \|$ many$^{PROP}\|(d)$ is a left-CONS2 function.

We speculate that no natural language expression contains a determiner that does not denote a left-CONS2 function. In the next section, we shall see that degree operators denote left-CONS2 functions, too.

3.3. Modified proportional numerals with an upper bound

Nouwen (2010) distinguishes between two types of numeral expressions. Class A contains positive (e.g. *more than 3*) and negative (e.g. *fewer than 1/3*, *less than 4)* variants of numeral comparatives. Class B consists of expressions that impose a minimal (e.g. *at least 80%*, *minimally 4*) or a maximal (e.g. *at most 50%*, *maximally* 50) bound on some degree. Within the Generalized Quantifier Theory (GQT, e.g. Keenan and Stavi, 1986) such expressions are analyzed as denoting relations between sets (i.e. as determiners). Under this approach, the determiners in (56a) have the denotations shown in (56b). While positive comparative expressions modifying a proportional numeral (e.g. *more than 1/3*) denote left-CONS2 functions, negative comparatives do not (56c).

(56) a. More than /Fewer than one third of the students came to the party.
b. $\|$more than/fewer than $1/3\|(P)(Q) \Leftrightarrow |P^* \cap Q^*| / |P^*| \{>, <\} 1/3$
c. $\|$fewer than $1/3\|(P)(Q) \not\Rightarrow \|$fewer than $1/3\|$ $(P \cap Q)(Q)$
since $|P^* \cap Q^*| / |P^*| < 1/3 \not\Rightarrow |(P^* \cap Q^*) \cap Q^*| / |(P^* \cap Q^*)| < 1/3$

Note also that, under the GQT approach, proportional determiners expressing a minimal bound denote left-CONS2 functions. Determiners of maximality, however, are potential counterexamples to the left-CONS2 Constraint, as can be seen in (57c).

(57) a. At most /at least seventy percent of the students came to the party.
b. $\|$at most / at least $70\%\|(P)(Q) \Leftrightarrow |P^* \cap Q^*| / |P^*| \{\leq, \geq\} 70/100$
c. $\|$ at most $70\%\|(P)(Q) \not\Rightarrow \|$ at most $70\%\|(P \cap Q)(Q)$
since $|P^* \cap Q^*| / |P^*| \leq 70\% \not\Rightarrow |(P^* \cap Q^*) \cap Q^*| / |(P^* \cap Q^*)| \leq 70\%$

GQT takes an idiomatic approach towards complex numerals (see Hackl, 2001 for discussion). The internal composition of a determiner is not expected to affect its semantic behavior. For instance, within GQT, the determiners *more than n* and *at least n+1* are expected to show similar semantic distribution when they are evaluated within a discrete domain (that is, within a domain where fractions do not matter). This is not, however, what we find. For instance, the semantic behavior of these determiners in the context of collective predicates such as *separate* and *form a triangle* is not uniform (Hackl, 2001):

(58) a. ??John separated more than one animal.
b. John separated at least two animals.
c. ??More than two students were forming a triangle.
d. At least three students were forming a triangle.

In this paper, we shall follow the line of research that pays close attention to the internal make-up of determiners (as in our analysis of few^{PROP} in Section 3.2) and their interactions with other operators in the sentence. We assume that (proportional) numerals denote degree segments (Takahashi, 2006; Solt, 2011) as in:

(59) $\|\ 70\%\ \| = \lambda d.\ 70/100 \geq d$

Hackl (2001) suggests that numerals are associated with the parametrized determiner $many^{CARD}$ (also Takahashi, 2006 and Nouwen, 2010). We shall assume that proportional numerals are associated with the parametrized determiner $many^{PROP}$.

(60) $[_{TP}[_{DP}$ [[70%][many^{PROP}]] [students]] $[_{VP}$ came to the party]]

Previously we have seen that the POS_c operator induces universal quantification that is restricted by the contextually provided Neutral Segment. In the absence of any restrictors, *POS* denotes subset relation between degree segments.

(61) $\|POS\|(D)(D') \Leftrightarrow D \subseteq D'$

That is, $POS_c = POS(NS_c)$, where NS_c is the neutral segment provided by the context. We assume that proportional numerals function as the restrictor of the POS operator.

(62) $[_{TP}\ [_{DP}\ [[\text{POS } 70\%][\text{many}^{PROP}]]\ [\text{students}]\ ]\ [_{VP}\ \text{came to the party}]]$

The constituent *POS 70%* denotes a degree quantifier, which means that it cannot be interpreted as the sister of *many^{PROP}*. *POS 70%* undergoes Quantifier Raising, leaving behing a degree trace as in (63). The overall interpretation is given in (64).

(63) $[_{TP2}\ [\text{POS } 70\%]\ \lambda d\ [_{TP1}\ [_{DP}\ [\text{d-many}^{PROP}]\ [\text{students}]\ ]\ [_{VP}\ \text{came to the party}]]]$
(64) $\lambda d.\ 70/100 \geq d \subseteq \lambda d.|S^* \cap C^*| / |S^*| \geq d$

This corresponds to the *at-least* interpretation of proportional numerals which we take to be the basic interpretation of such expressions. Let us now discuss the two types of modified proportional numerals that might be taken to be problematic for the left-CONS2 constraint. We will start with expressions involving *at most*. Following Hackl (2001) and Nouwen (2010), we assume that *at most n* denotes a quantifier over degrees. We claim that *AT_MOST* is simply the converse of *POS*.[15]

(65) a. $\text{Det}^{-1}(P)(Q) =_{def} \text{Det}(Q)(P)$
b. $\text{POS}^{-1} = \text{AT\_MOST}$

A sentence like (66a) has the structure in (66b) and the truth conditions in (66c).

(66) a. At most seventy percent of the students came to the party.
b. $[_{TP2}\ [\text{AT\_MOST } 70\%]\ \lambda d\ [_{TP1}\ [_{DP}\ [\text{d-many}^{PROP}]\ [\text{students}]\ ]\ [_{VP}\ \text{came to the party}]]]$
c. $\text{AT\_MOST}(\lambda d.\ 70/100 \geq d)(\ \lambda d.|S^* \cap C^*| / |S^*| \geq d) \Leftrightarrow$
$\lambda d.|S^* \cap C^*| / |S^*| \geq d \subseteq \lambda d.\ 70/100 \geq d$

In our definition of *left-CONS2*, we have assumed that such functions relate sets of *individuals*. We might more generally ask whether degree determiners (i.e. determiners that relate sets of *degrees*) can be said to denote left-CONS2 functions. It turns out that the degree determiner *AT_MOST* denotes a left-CONS2 function:

(67) $\|\text{AT\_MOST}\|(D)(D') \Rightarrow \|\text{AT\_MOST}\|(D \cap D')(D')$
since $D' \subseteq D \Rightarrow D' \subseteq D \cap D'$

[15] Beck (2012: 237) proposes a covert variant of *AT_MOST* with the following denotation:

(1) $\|\text{AT\_MOST}_{BECK}\|(d)(D') \Leftrightarrow \max(D') \leq d$.

The similarity between *AT_MOST* and *AT_MOST$_{BECK}$* can be appreciated by observing that $\|\text{AT\_MOST}\|(D)(D') \Rightarrow \max(D') \leq \max(D)$. See Penka (2010); Solt (2011) and Coppock (2016) for analyses that capitalize on the fact that *at most* has a superlative form.

All in all, every determiner that is involved in the interpretation of a sentence with a degree quantifier of the form *at most prop. num* denotes a left-CONS2 function.[16]

Let us finally discuss negative comparatives involving proportional numerals. Following Heim (2006), Alxatib (2014) and Romero (2015), among others, we assume that the COMP(arative) operator denotes proper subset relation between degree segments.

(68) COMP(D)(D') $\Leftrightarrow$ D $\subset$ D'

Negative comparative quantifiers (e.g. *fewer than 70%*) are built on $few^{PROP}_{[UD.Neg]}$, which is semantically equivalent to $many^{PROP}$ (recall (49)). We shall assume that the quantifier in the sister of few^{PROP} contains *D.NEG* (see Takahashi, 2006 on this assumption). That is, before the application of Quantifier Raising, the syntactic representation of (69a) is as in (69b).

(69) a. Fewer/Less than seventy percent of the students came to the party.
b. $[_{TP}[_{DP}$ [[er than [D.NEG 70%]][few^{PROP}]] [students]] $[_{VP}$ came to the party]]

The QR of *[er than [D.NEG 70%]]* leaves a degree variable in the tail of the movement chain. A second instance of *D.NEG* is then merged to a position c-commanding $few^{PROP}_{[UD.Neg]}$. The overall interpretation given in (70b).

(70) a. $[_{TP3}$ [COMP [D.NEG 70%]] $[_{TP2}$ D.NEG [λd $[_{TP1}$ $[_{DP}$ [d-few^{PROP}] [students]] $[_{VP}$ came to the party]]]]]
b. $\|COMP\|(\lambda d.d > 70\%)(\lambda d. |S^* \cap C^*|/|S^*| < d) \Leftrightarrow$
$\lambda d.70\% < d \subset \lambda d. |S^* \cap C^*|/|S^*| < d \Leftrightarrow \lambda d. |S^* \cap C^*|/|S^*| \geq d \subset \lambda d. 70\% \geq d$

The assumption that *D.NEG* must be interpreted in two positions is discussed in Takahashi (2006:83). As far as we are aware, there has not been much discussion of how this assumption can be independently motivated.

---

16 At this point, one might wonder about the relation between *AT_MOST* and *AT_LEAST*. We suggest that the main difference between these determiners is that *AT_LEAST* is derived from $few^{PROP}_{[uD.NEG]}$. The pre-movement representation of the sentence in (1) is given in (2).

(1) At least seventy percent of the students came to the party.
(2) $[_{TP}[_{DP}$ [[AT_MOST [D.NEG 70%]][few^{PROP}]] [students]] $[_{VP}$ came to the party]]

The QR of *[AT_MOST [D.NEG 70%]]* is followed by the merger of another *D.NEG* to a position c-commanding few^{PROP}. The overall interpretation of (1) is shown in (4)

(3) $[_{TP3}$ [AT_MOST [D.NEG 70%]] $[_{TP2}$ D.NEG λd $[_{TP1}$ $[_{DP}$ [d-few^{PROP}][students]] $[_{VP}$ came …]]]]
(4) $\|1\| \Leftrightarrow \lambda d.|S^* \cap C^*|/|S^*| < d \subseteq \lambda d.\ 70\% < d \Leftrightarrow \lambda d.\ 70\% \geq d \subseteq \lambda d.|S^* \cap C|/|S^*| \geq d$

This corresponds to the *at-least* reading interpretation for proportional numerals that we have seen before. The need for the double interpretation of negation is discussed in Takahashi (2006) but the independent motivation for this assumption (other than getting the truth conditions right) is not clear. See the discussion of *fewer than 70%* below. For an alternative, and actually the opposite, view on the relation between *AT_MOST* and *AT_LEAST*, see Penka (2014).

Observe that the *COMP* operator denotes a left-CONS2 function.

(71) $||COMP||(D)(D') \Rightarrow ||COMP||(D \cap D')(D')$
since $D \subset D' \Rightarrow D \cap D' \subset D'$

We have noted that both *AT_MOST* and *COMP* denote left-CONS2 functions. However, neither *AT_MOST* nor *COMP* denote CONS1 functions.

(72) $||COMP||(D)(D') \not\Leftrightarrow ||COMP||(D)(D \cap D')$
since $D \subset D' \not\Leftrightarrow D \subset D \cap D'$

(73) $||AT\_MOST||(D)(D') \not\Leftrightarrow ||AT\_MOST||(D)(D \cap D')$
since $D' \subseteq D \not\Leftrightarrow D \cap D' \subseteq D$

This might be taken to indicate that while the CONS1 Constraint puts a restriction only on relations between sets of individuals (see Romoli, 2015 for some discussion) the left-CONS2 Constraint has a wider domain of relevance.

4. Conclusion

The main claim of this paper is that natural language determiners denote left-CONS2 functions. To support this claim, we have taken a closer look at some of the determiners that seem to falsify the validity of this constraint and provided evidence that the problematic inferences associated with restricted universal determiners and (modified) proportional expressions arise from a source distinct from their denotations

Hackl (2001: 19) notes that, within the Generalized Quantifier Theory, finding out the denotation of a determiner in a language involves “identify[ing] the syntactic constituents that denote the restrictor and the scope” of the determiner and “attribut[ing] to the so found determiner the Boolean and comparative operations necessary to represent the truth conditions”. In this paper we have suggested that such a procedure does not do justice to the contribution of various sentential and degree operators that play a crucial role in the overall interpretation of sentences involving determiners. Once these factors are taken into consideration, we see that possible determiner denotations are more restricted than what we might have thought.

References

Alxatib, S. 2014. The Comparative as Subsethood: How degree-referring restrictors fit in. In L. Crnič and U. Sauerland (eds.), *The Art and Craft of Semantics: A Festschrift for Irene Heim*, pp. 61–68 vol. 1, MITWPL 70.

Barwise, J. & R. Cooper (1981). Generalized quantifiers and natural language. *Linguistics and Philosophy* 4: 159–219.

Beck, S. (2012). DegP scope revisited. *Natural Language Semantics* 20(3): 227 - 272.

van Benthem, J. (1983). Determiners and logic. *Linguistics and Philosophy* 6: 447–478

van Benthem, Johan (1986). *Essays in Logical Semantics*. Dordrecht: D. Reidel.

Chierchia, G. and S. McConnell-Ginet (2000). *Meaning and Grammar: An introduction to semantics*, 2nd edition. Cambridge, MA: MIT Press

Chierchia, G, D. Fox, and B. Spector (2012). Scalar implicature as a grammatical phenomenon. In C. Maienborn, P. Portner, and K. von Heusinger (Eds.), *Semantics: An International Handbook of Natural Language Meaning*, Volume 3 2297 - 2331. Berlin/Boston: de Gruyter.

Clark, R. (2001). *Generalized quantifiers and semantic automata: A tutorial*. Ms. University of Pennsylvania.

Coppock, E. (2016). Superlative modifiers as modified superlatives. *Semantics and Linguistic Theory*, 26: 471–488.

Cresswell, M. (1976). The semantics of degree. In B. Partee (ed.), *Montague Grammar*. Academic Press

Crnič, L. (2018). A note on connected exceptives and approximatives. *Journal of Semantics* 35: 741–756.

Fiengo, R. & R. May (1994). Indices and Identity *MIT Press.* USA.

von Fintel, K. (1993). Exceptive constructions. *Natural Language Semantics* 1: 123–48.

Fox, D. (2000). Economy and Semantic Interpretation. *MIT Press.* USA.

Gajewski, J. (2008). NPI any and connected exceptive phrases. *Natural Language Semantics* 16: 69–110.

Gajewski, J. (2013). An analogy between a connected exceptive phrase and polarity items. In E. Csipak, R. Eckardt, M. Liu and M. Saller. *Beyond Any and Ever: New Explorations in Negative Polarity Sensitivity* 183–212. De Gruyter Mouton, Berlin

Hackl, M. (2001). *Comparative quantifiers.* Cambridge, Mass.: MIT PhD dissertation.

Heim, I. (2006). Little. In M. Gibson and J. Howell (eds.), *Semantics and Linguistic Theory (SALT)* 14 pp. 35 – 58. Cornell University.

Herburger E. (1997) Focus and weak noun phrases. *Natural Language Semantics* 5:53–78

Hirsch, A.(2016). An unexceptional semantics for expressions of exception. In *University of Pennsylvania Working Papers in Linguistics*, vol. 221.

Hoeksema, J. (1987). The Logic of Exception. *ESCOL* 4:100-113.

Hoeksema, J. (1995). The Semantics of Exception Phrases. In J. van der Does and J. van Eijck (eds) *Quantifiers, Logic, and Language* pp. 145-177 Stanford: CSLI Publications

Horn, L. (2002). Assertoric inertia and NPI licensing. In M. Andronis, E. Debenport, A. Pycha and K. Yoshimura (eds) *CLS 38* pp. 55–82. Chicago: University of Chicago.

Keenan, E. (1996). The semantics of determiners. In S. Lappin (ed.) Handbook of contemporary semantics. Blackwell.

Keenan, E. (2006). Quantifiers: semantics. In K. Brown (ed.) *Encyclopaedia of language and linguistics.* Vol 10. 302 – 308. Amsterdam: Elsevier.

Keenan, E. and J. Stavi (1986). A semantic characterization of natural language determiners. *Linguistics and Philosophy* 9:253–326.
McNally, L. (1998). Existential sentences without existential quantification. *Linguistics and Philosophy*. 21(3): 353 – 392.
Mostowski, A. (1957). On a generalization of quantifiers. *Fundamenta Mathematicae* 44(1):12–36.
Nouwen, R. (2006). Remarks on the polar orientation of almost. In *Linguistics in the Netherlands*, vol. 231. pp. 162–173. John Benjamins Publishing, Amsterdam.
Nouwen, R. (2010). Two kinds of modified numerals. *Semantics and Pragmatics* 3(3): 1 - 41.
Partee, B. (1989). Many Quantifiers. *ESCOL 89: Proceedings of the Eastern States Conference on Linguistics*: 383–402
Penka, D. (2010). *A superlative analysis of superlative scalar modifiers.* Handout from Sinn und Bedeutung 15
Penka, D. (2011). Negative Indefinites. *Oxford University Press*. Oxford.
Penka, D. (2014). `At most' at last. In Sinn und Bedeutung 19, 463- 480.
Rett, J. (2018). The semantics of many, much, few and little. *Language and Linguistics Compass*. 12: 1 – 18.
Romero, Maribel (2015) The conservativity of many. In *Proceedings of the 20th Amsterdam Colloquium*, pp. 20–29 ILLC, Amsterdam
Romoli, Ja. (2015). A structural account of conservativity. *Semantics-Syntax Interface* 2(1):28–57
Rothstein, S. (1988). Conservativity and the syntax of determiners. *Linguistics* 26: 999-1019
Sadock, J. M (1981). Almost. In Peter Cole (ed) *Radical Pragmatics*. Pp. 257–271 New York: Academic Press.
Solt, S. (2006). Monotonicity, Closure and the Semantics of Few. In D. Baumer, D. Montero and M. Scanlon (eds) *Proceedings of the 25th West Coast Conference on Formal Linguistics*. 380 – 389.
Solt, S. (2011). How many most's? In I. Reich, E. Horsch and D. Pauly (eds.), Sinn und Bedeutung 15 pp. 565–580. Saarland Unversity Press
Solt, S. (2015). Q-adjectives and the semantics of quantity. *Journal of Semantics* 3: 1 – 77.
Spector, B. (2014). Global positive polarity items and obligatory exhaustivity. *Semantics and Pragmatics* 7: 1–61.
von Stechow, A. (2009). The temporal degree adjectives früh(er)/spät(er) 'early(er)'/'late(r)' and the semantics of the positive. In A. Giannakidou and M. Rathert (eds.) *Quantification, definiteness, and nominalization* Oxford: Oxford University Press
de Swart, H. (2000). Scope ambiguities with negative quantifiers. In K. von Heusinger and U. Egli (eds) Reference and Anaphoric Relations. pp. 109–132 Dordrecht: Kluwer Academic Publishers.
Takahashi, S. (2006). More than two quantifiers. *Natural Language Semantics* 14(1). 57–101
Westerståhl, D. (1985). Determiners and context sets. In J. van Benthem and A. ter Meulen (eds) *Generalized Quantifiers in Natural Language*. Pp. 45 – 71 Dordrecht: Foris.
Zeijlstra, H. (2004). *Sentential negation and negative concord*. Ph.D. dissertation, University of Amsterdam, Amsterdam.
Ziegeler, D. (2000). What almost can reveal about counterfactual inferences. *Journal of Pragmatics* 32. 1743–1776.
Zuber, R. & E. Keenan. (2019). A note on Conservativity. *Journal of Semantics* 36(4): 573 – 582.