BE FLEXIBLE, BUT NOT TOO FLEXIBLE: LIMITED VARIABLE-FORCE MODALS IN KINANDE AND THE TYPOLOGY OF MODAL FORCE

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ABSTRACT OF THE DISSERTATION

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This dissertation has two goals: First, to describe and analyze a new kind of variable-force modality in Kinande, a Bantu language spoken in the Democratic Republic of Congo and in Uganda, contextualizing it within the existing typology of variable-force modals and expanding that typology descriptively; second, to take the new information gained from examining Kinande and use it to refine our typology of modal force, re-examining previously attested cases of variable-force modals and creating a more refined typology of modal force, and opening new avenues of further research into modal force variation cross-linguistically.

Prior work in modal semantics typically divides modals according to a binary of possibility and necessity, formalized in terms of existential or universal quantification over possible worlds, respectively. While such systems cover the empirical ground of English and several other languages reasonably well, variable-force languages such as St'át'imcets (Rullmann et al., 2008), Nez Perce (Deal, 2011), and Ecuadorian Siona

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(Jeretič, 2021a), among others, bring this binary into question with modals that, depending on their context, are sometimes interpreted akin to English *can* and sometimes akin to English *must* or *have to*. Since these first attestations there have been numerous efforts to account for this variation in modal force.

In this work I present novel data from Kinande (Bantu J, DRC) illustrating a previously unattested kind of variable-force modal; Kinande *anga* varies between possibility and weak necessity interpretations (akin to English *should*) but never allows strong necessity interpretations. I call this Limited Variable-Force. The existence of such limited variability demonstrates that just as modal force itself is not merely a binary between possibility and necessity, there is not a simple binary between fixed and variable force either. Kinande expands the typology to include languages that have variable-force modals that are more constrained in terms of their available interpretations than the modals in Nez Perce (for example), but still less constrained than the modals in say, English.

I present an account of Kinande variable-force *anga* in terms of exhaustification, capitalizing on the mechanisms of deriving scalar implicatures to derive so-called "scaleless implicatures" as proposed by Jeretič (2021a). While scalar implicatures are derived when exhaustification applies and a there is a stronger scalar alternative present, scaleless implicatures are derived when exhaustification applies in the absence of a stronger scalar alternative. I show that the lack of a weak necessity modal in Kinande (and the presence of a strong necessity one) allows *anga* to be variable to weak necessity, but not to strong necessity, because the strong necessity interpretation is blocked by scalar implicature through the same mechanism that derives the scaleless implicature.

This account not only provides a satisfactory explanation for the behavior of Kinande modals, it also provides a unified theory of modal force in general, where variable-force modals are simply a side-effect of a sparse modal paradigm, and fixed-force languages are a side-effect of a dense modal paradigm. There are no special

mechanisms required to get the distinction between the two; the effects fall out of how exhaustification in general and the lexicon of the individual language. This further generates a useful typology of modal force as a whole: Languages with variable-force modals are predicted to be languages with sparse modal paradigms, while languages with dense modal paradigms are expected not to show variable-force behavior. I give a typology situating previously-described modal systems (both variable-force and not) and discuss the implications of this typology for our understanding of modal force as a whole.

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List of glossing abbreviations

1	first person	ERG	ergative
2	second person	FUT	future
3	third person	FV	final vowel
ACC	accusative	IMP	imperative
ADV	adverb	IND	indicative
ASSOC	associative	INF	infinitive
AUG	augment	IPFV	imperfective
AUX	auxiliary	IRR	irrealis
c#	class marker for class #	LEX	lexical particle
CAUS	causative	LNK	linker
CND/COND	conditional	MOD	modal
сомр/С	complementizer	NEG	negation
COMPL	completive aspect	NPST	non-past
CONJ	conjunctive subject	NOM	nominative
COP	copula	NSUBJ	non-subject
CUST	customary	ОВЈ	object
DEIC	deictic	OM	object marker
DEM	demonstrative	PERF	perfective
DEON	deontic	PL	plural
DET	determiner	PREP	preposition
DIR	directive transitivizer	PROG	progressive
DU	dual	PST	past
EPIS	epistemic	REL	relative clause marker

RELPRO	relative pronoun	TM	tense marker
RELPRO	ICIALIVE DIVITOUTI	1 /V1	TCHSC HIALKEL

sg singular topic

sm subject marker tv transitivizer

subjunctive

Chapter 1

Introduction

1.1 Dissertation summary

This dissertation has two goals: First, to describe and analyze a new kind of variable-force modality in Kinande, a Bantu J language spoken in the Democratic Republic of Congo,¹ contextualizing it within the existing typology of variable-force modals and expanding that typology descriptively; second, to take the new information gained from examining Kinande and use it to refine our typology of modal force, re-examining previously attested cases of variable-force modals and creating a more refined typology of modal force, and opening new avenues of further research into modal force variation cross-linguistically.

In general, modals (expressions of what is or is not possible, or what is or is not necessary, or allowed, or required, etc.) have been broadly assumed to come in two basic types: Possibility modals, like English *can*, and necessity modals, like English *must*. The distinction between the two is typically captured in terms of quantification over possible worlds: possibility modals denote that there is at least one reasonably-similar world where the proposition the modal modifies is true, and necessity modals

¹A very closely-related language called Konzo is also spoken in Uganda. The two groups consider themselves to be closely related historically, and reportedly the main difference between the two varieties is in where they source their loanwords: Kinande largely has loanwords from French and Swahili, while Konzo largely has loanwords from English and Rutooro. The languages are otherwise similar enough that the main dictionary and grammar source currently available (Mutaka & Kavutirwaki, 2011), is in fact a dictionary of Kinande-Konzo. In this dissertation I refer exclusively to Kinande, however, as my consultant speaks Kinande and I was unable to conduct parallel fieldwork with Konzo speakers to verify the data on the Konzo side.

denote that all of the reasonably-similar worlds are worlds where the modified proposition is true.

The simple binary of possibility and necessity has been challenged and modified on two empirical fronts: that of weak necessity modals and that of variable-force modals.

Weak necessity modals, like English *should*, seem to be somewhere between *must* and *can*. *Should* seems to denote an obligation (in the deontic space), not a permission, but it doesn't seem to be as strong of an obligation as *must*. Thus the term "weak necessity" for these modals, and some of the most influential analyze them as a sort of special case of necessity modals (von Fintel & Iatridou, 2008; Rubinstein, 2012).²

Variable-force modals, on the other hand, are typically modal expressions that, depending on their context, can be interpreted as either possibility or necessity (or likely anything in-between) with no change to the sentence they modify, like the following case from St'át'imcets (Rullmann et al., 2008, Lillooet Salish):

(1) St'át'imcets (Rullmann et al., 2008, p. 331):

Context 1: The cougar was on a rampage and was killing cats, dogs, and racoons, and it had a child cornered and was growling; it <u>would</u> have killed a child.

Context 2: You just know that sometimes cougars kill children when they venture into built-up areas; it could have killed a child.

zúqw-s-as ka ta sk'úk'wmi7t-a ti7 ku swúw'a lh-cw7áoz-as die-CAUS-3ERG IRR DET child-DET DEIC DET cougar COMP-NEG-3CONJ kw s-qus-cit-ítas DET NOM-shoot-IND-3PL.ERG

'That cougar could/would have killed a child if they hadn't shot it.'

Variable-force modals pose a serious theoretical concern for the simple binary

²Though see for example Lassiter (2017); Portner & Rubinstein (2014); Klecha (2014); Pasternak (2016) for accounts of weak necessity as a scalar notion, though in at least some cases still treating weak necessity as a subtype of necessity.

of possibility and necessity; when a single morpheme can be used for both, what quantifier does it have? How does it affect our understanding of modal force overall?

Further investigation into modal force variation has resulted in attestations of variable-force modals in Gitksan (Matthewson, 2016; Peterson, 2010, Tsimshianic), Washo (Bochnak, 2015, Isolate), Nez Perce (Deal, 2011, Sahaptian), Nsyilxcen (Menzies, 2013, Okanagan, Southern Interior Salish), and Ecuadorian Siona (Jeretič, 2021a). Analyses of these modals fall into roughly three camps: those that analyze variable-force modals roughly along the lines of weak necessity modals by taking one quantifier as the base and using contextual restrictions to weaken or strengthen its meaning, those who take the modal to have the weakest of its available readings (possibility) and rely on pragmatics to make it permissible in necessity contexts, and those who use a grammaticalized scalar implicature mechanism to derive modal strengthening.

This dissertation expands on this picture both empirically and theoretically. Empirically, I present data from Kinande demonstrating a previously-unattested type of variable-force modal, which I call a limited variable-force modal, because rather than varying between possibility and (strong) necessity, Kinande *anga* is variable but only between possibility and weak necessity, and never to strong necessity. Prior attestations of variable-force modals have only been cases where the same morpheme in a sentence can be used in contexts that support anywhere between possibility and necessity, but Kinande's *anga*-sentences are more restricted than that. The existence of limited variable-force modals emphasizes that not only is there not a simple binary distinction between possibility and necessity, but there is not a simple distinction in modal systems between being fully variable and being fully fixed; there are even more nuances in modal systems.

The expanded theory that I develop to account for Kinande in Chapter 3 does not merely carve out a place for Kinande-type modal systems, however. The implications of that analysis prompt a look at variable-force modal systems that have been previously attested and analyzed, putting those into a new light and creating

further questions to ask about those languages. Furthermore, the analysis I propose for variable-force modals not only accounts for how the variable modals are variable, but it also addresses why the non-variable modal systems are non-variable, creating a picture of a typology of modal force overall, and not simply a typology of variable modal force.

In general, languages with variable-force modals tend to be languages with relatively sparse modal inventories: they have a few modals that cover a large swath of the modal force options. On the other side, languages with fixed-force modals tend to be languages with relatively dense modal inventories: there are many modals that each have relatively restricted (if even still somewhat overlapping) contexts where they are felicitous. This may seem intuitively natural, but thus far there has been no theory-based explanation for this correlation. I propose an analysis and typology that accounts for the correlation and behavior together, without the need for any novel theoretical mechanisms.

My analysis of variable-force modals is based on exhaustification, which has been proposed by Chierchia (2004); Chierchia et al. (2012) and others as a means for grammaticalizing scalar implicatures—explaining why sentences like *Rachel or Taylor came* to the party lead us to draw the conclusions that one of them came, but not both. In cases where there is a stronger scalar alternative (in this example, the existence of and in English), we draw the conclusion that since the speaker did not use that stronger alternative, it must not hold. So in most contexts the conclusion we draw is that the strong interpretation of or (which could include both) does not hold, and we must stick with merely the weaker interpretation.³ While this kind of pragmatic reasoning has been discussed since at least Grice (1975), Chierchia's exhaustification is an operator that grammaticalizes this process in the semantics, which allows for scalar

³Perhaps counter-intuitively, this gives us an overall **stronger** interpretation of the sentence, in that we have more information that the non-exhaustified version. *Rachel or Taylor (or both) came to the party* leaves us more options and less knowledge than *Rachel or Taylor came to the party (but not both)*.

implicatures to be drawn in contexts they are otherwise unavailable (like certain embedded contexts).

When exhaustification is applied in cases where there is a lack of a stronger scalar alternative, an interesting effect becomes available: the exhaustified item is strengthened to the meaning of the (missing) stronger scalar alternative. In essence, *You may have ice cream or cake*, in a language without a word for *and*, might be interpreted as *You may have ice cream and cake* (See Bowler, 2014, for an analysis of Warlpiri *manu* "or/and" along these lines, as well as further explanation in the next sections). The same mechanism that gives us scalar implicatures (exhaustification) can also give us these "scaleless" implicatures (term from Jeretič, 2021a).⁴

The same process can strengthen a possibility modal to a necessity meaning, if the language does not have a necessity modal in its lexicon. This means that exhaustification captures both angles of the correlation between sparse modal paradigms and variable-force modals at once: *You can go to the party* in English draws a scalar implicature that you do not have to, derived by exhaustification negating the stronger option; the equivalent of this sentence in Ecuadorian Siona has no available "have to" alternative and exhaustification derives the necessity reading. Analyzing variable-force modals as possibility modals strengthened by exhaustification means, straightforwardly, that whether a modal is variable-force or not depends directly on whether the modal paradigm is dense or sparse. Modals will only vary when they have the space to, and otherwise the scalar implicature blocks strengthening.

The analysis of Kinande *anga* tells us that while scalar implicatures block scaleless implicatures, they can also coexist, within certain parameters. Kinande *anga*, is best

 $^{^4}$ The specifics of this process are rather involved and are explained fully in Chapter 3. The short version is: exhaustification typically negates stronger scalar alternatives, if they're present. Exhaustifying over subdomain alternatives $\{a,b\}$ of $(a \lor b)$ when there is no $(a \land b)$ in those alternatives will derive readings like "not only a, and not only b", and there is not negation of the conjunction to block those "not only" readings. The result is that exhaustification strengthens the meaning of "or" to the previously-missing "and". The process is the same when the alternatives are worlds, rather than propositions (as is the case with modals).

analyzed as a possibility modal that is strengthened to weak necessity through scaleless implicature. The reason *anga* can't be interpreted as a strong necessity modal through this method is because Kinande has a dedicated strong necessity modal, *paswa*. So while scaleless implicature can strengthen *anga* to weak necessity, the existence of *paswa* as a strong necessity scalemate to *anga* causes the same mechanism to draw a "not have to" scalar implicature, preventing strengthening to strong necessity.

The scalar/scaleless implicature mechanism thus creates a set of powerful typological generalizations: First, it predicts that languages with observedly variable-force modals necessarily have a commensurate gap in their modal paradigm; variable-force modals need space to operate. Languages with dense modal paradigms will not show variable-force modality, and languages with paradigms in-between (like Kinande) can show limited variable-force. Second, if exhaustification is (as I propose) the only available method of changing the interpretation of one quantifier to that of a another, and because exhaustification can only move one direction (from possibility to necessity, never the reverse), then I predict that regardless of whether the language also uses domain restriction for part of its variable force, any modal that is variable across both possibility and necessity interpretations must at its heart be a possibility modal, not a necessity one. This in turn has implications for the behavior of these modals in downward-entailing contexts, especially under clausemate negation. This prompts a re-examination of some existing accounts of variable-force modals, and will require additional fieldwork to gather the crucial test cases. In the meantime, however, the typology I propose for modal force as a whole is at once more fine-grained (incorporating weak necessity and limited variable-force) than e.g. those discussed by Nauze (2008); Matthewson (2016) and more constrained by limiting the mechanisms that can achieve fully variable force.

In the remainder of this chapter I briefly preview the empirical facts in Kinande,

give a brief version of the analysis I develop, and outline the remainder of the dissertation.

1.2 The Kinande facts

Kinande has at least three modal constructions that show variable force, but only one of them (the modal prefix *anga*) will be given an in-depth formal analysis here. This prefix in one of the TAM (Tense-Aspect-Mood) morphology varies between possibility and weak necessity interpretations, depending on its context:

(2) Context 1 (epistemic possibility): Kábunga is not a consistent student. The professor never knows if he will come to class or not. Today, it is time to start class and he has not shown up yet.

Context 2 (epistemic weak necessity): Kábunga is not a consistent student, and often skips class on Wednesdays, but he is usually there on Mondays. Today is Monday, and so the professor isn't entirely sure whether Kábunga will show up, but since it's Monday and Kábunga is relatively consistent on Mondays, he thinks Kábunga will probably be there.

Kabunga a-anga-na-sy-a oko kalhasi ko munabwire Kabunga SM.c1-MOD-TM-come-FV c17 class c17 today "Kabunga might/should be coming to class today"

While at first glance it may seem that *anga* is a possibility modal that is acceptable in weak necessity contexts (perhaps because there is no dedicated weak necessity modal in Kinande), I show that *anga* continues to be variable in syntactic contexts that are predicted to be unambiguous on such a permissibility analysis, like the restrictor of a universal quantifier:

(3) a. Context: Some celebrities published a list of "1,000 books that everyone should read before they die". Since then, Swera has been reading the books on that list, and only those books. She intends to read every single

book on that list whenever she can get her hands on a copy, but she refuses to spend her time reading anything else.

Swera a-ka-som-a obuli kitabu
Swera SM.c1-TM-read-FV every c7.book
e-kyo-a-anga-na-som-a
AUG-c7.REL-SM.c1-MOD-TM-read-FV
"Swera reads every book that she should read"

b. Context: Masika is a very avid reader; she always has a book in her hands. She doesn't particularly follow recommendations or anything, she just

reads every book she can get her hands on, as quickly as she can.

Masika a-ka-som-a obuli kitabu Masika SM.c1-TM-read-FV every c7.book

e-kyo-a-anga-na-som-a

AUG-c7.REL-SM.c1-MOD-TM-read-FV

"Masika reads every book that she can read."

As I will discuss in Chapter 3, the weak necessity interpretation of *anga* in (3) is expected to be unavailable if *anga* is always interpreted as a possibility modal. The fact that *anga* retains its ambiguity in downward-entailing contexts indicates that there is an additional semantic mechanisms at play.

This is interesting for a number of reasons: From a descriptive perspective, I provide the first attestation of a variable-force modal in a Bantu language, and in fact one of the first formal descriptions of a modal system in Bantu (preceded only by work by Gluckman et al., 2017). Typologically, this is also the first attestation of a variable-force modal that is not variable across the entire range of modal force, as shown below for a deontic context:

(4) Context: In Uganda, the law states that when you ride a motor bike...

u-anga-na-ambal-a e-sapu 2sg-MOD-TM-wear-FV c9/3-helmet Intended: "You must wear a helmet"

Using the novel facts from Kinande as a springboard, and building on the variableforce modal literature as well as recent developments by Jeretič (2021a); Staniszewski (2020), I propose an analysis of variable-force modals that accounts for the pattern in Kinande, and also accounts for the typology and spread of variable-force modals as previously described in a principled way.

The core of my analysis is exhaustification and domain restriction, both mechanisms motivated extensively elsewhere in the semantic literature. When those mechanisms are included in our perspective of modality, they interact predictably, deriving relatively complex patterns. The result, I will show, is not only an explanation of Kinande *anga*, but richer explanations of previously-described variable-force languages and a constrained and informative typology of modal systems, of both fixed and variable force.

1.3 Analysis in a nutshell

I propose that Kinande anga is underlyingly a possibility modal, akin to English can. The definitions I propose for anga and its scalemate paswa are given below. Paswa is a relatively straightforward necessity modal, taking a modal base (MB), which is a set of propositions, typically expressing relevant facts about the world; and an ordering source (OS₁), which is a function that takes the worlds that satisfy all the propositions of the modal base and orders them according to how well they conform to additional (contextually-supplied) constraints, and then returns only the highest-ranked of those worlds (Kratzer, 1977).

Anga, on the other hand, is a possibility modal that takes not only the modal base MB and ordering source OS_1 , but also a secondary ordering source OS_2 , which per analyses of weak necessity modals by von Fintel & Iatridou (2008); Rubinstein (2012) is a set of lower-priority propositions that are brought into consideration. OS_2 takes the highest-ranked worlds returned by OS_1 and orders them according to its own criteria, further winnowing down the set of worlds that are quantified over.

(5)
$$[anga(p)] = \exists w'[w' \in OS_2(OS_1(\bigcap(MB(w)))) \land p(w')]$$

READ: There is some world w' in the set of the most ideal worlds according to the modal base and two ordering sources such that p is true in w'.

(6) $[\![paswa(p)]\!] = \forall w'[w' \in OS_1(\bigcap(MB(w))) \rightarrow p(w')]$ READ: For all worlds w' such that w' is in the set of ideal worlds according to the modal base and ordering source, p is true in w'.

It's variable across modal flavor, but its lexical semantics is that of a possibility modal. However, unlike English *can*, I propose that *anga* is an inherently domain-restricted possibility modal, using the notion of domain-restriction developed for weak necessity modals in von Fintel & Iatridou (2008); Rubinstein (2012). When used as a possibility modal, this domain restriction is basically "invisible", in that it has little to no effect on the truth conditions or felicity conditions of *anga*.

Because anga and paswa use the same conversational backgrounds in a given context, except for the additional restriction that anga has through a secondary ordering source (per von Fintel & Iatridou, 2008), the quantificational domain of anga will always be a subset of that of paswa. The scalar implicature is drawn by negating paswa as an innocently excludable (IE) scalemate, giving us the meaning "p is possible, but not necessary". The further exhaustification step, using innocent inclusion (Bar-Lev & Fox, 2017, abbreviated II,), exhaustifies over the subdomain alternatives of anga, which are derived from the possible subsets of anga's quantificational domain. All of the consistently assertable such alternatives are then added to the assertion, and most crucially, this includes the propositions quantifying over the singleton sets of each world. Altogether, those propositions are equivalent to universal quantification over the union of those singletons (which is simply anga's original quantificational domain). In the end, we derive a "not have to" scalar implicature, and then by scaleless implicature we derive a weak necessity modal with an interpretation similar to English should.

The domain-restriction of *anga* (and the lack of restriction for *paswa*) comes into play when an *anga*-expression undergoes exhaustification for scalar implicatures.

In such conditions, the structure of the Kinande modal system (specifically a lack of a weak necessity scalar alternative to *anga*), together with *anga*'s definition as a domain-restricted possibility modal, results in exhaustification of *anga* causing a strengthening effect, rather than simply drawing a scalar implicature as in English. The result is that a scaleless implicature is drawn in addition to the scalar implicature, and *anga*'s exhaustified meaning is something like the following:

(7) a.
$$[\![exh^{IE}(anga(p))]\!] = \exists w'[w' \in \{w_1, w_2, w_3\} \& p(w')]$$

$$\land \neg \forall w'[w' \in \{w_1, w_2, w_3, w_4, w_5\} \rightarrow p(w')]$$
b. $[\![exh^{IE+II}(anga(p))]\!] : \exists w'[w' \in \{w_1, w_2, w_3\} \& p(w')]$

$$\land \neg \forall w'[w' \in \{w_1, w_2, w_3, w_4, w_5\} \rightarrow p(w')]$$

$$\land \exists w'[w' \in \{w_1, w_2, w_3\} \& p(w')]$$

$$\land \exists w'[w' \in \{w_1, w_2\} \& p(w')]$$

$$\land \exists w'[w' \in \{w_1, w_3\} \& p(w')]$$

$$\land \exists w'[w' \in \{w_2, w_3\} \& p(w')]$$

$$\land \exists w'[w' \in \{w_1\} \& p(w')]$$

$$\land \exists w'[w' \in \{w_2\} \& p(w')]$$

$$\land \exists w'[w' \in \{w_3\} \& p(w')]$$

$$\exists \forall w[w' \in \{w_3\} \& p(w')]$$

$$\equiv \forall w[w' \in \{w_1, w_2, w_3\} \rightarrow p(w')]$$

Specifically, it is well-known that in downward-entailing contexts, such as under negation, in the antecedent of a conditional, or in the restrictor of a universal quantifier like *every* or *all*, existential quantifiers are the stronger of the two basic quantifiers, rather than the weaker one. And so under negation, we expect exhaustification to have no effect, and for *anga* to be come unambiguous. This is indeed what we see:

(8) nga-oko reglema yi-ka-bug-a, si-u-anga-sat-a, COMP-COMP c9.rules SM.c9-TM-say-FV, NEG-SM.2sg-MOD-dance-FV kundi w-oyo u-kwa-ire oko ku-boko because 2sg?-REPLRO SM.2sg-hurt-TM c17 c15-arm "According to the rules, you can't play because your arm is injured."

(9) Context: Kabunga often plays football with his friends, but today he is tired and doesn't want to play. One of his friends tells him that it's ok, he doesn't have to play today if he doesn't want to:

```
# sí-u-anga-sat-a
NEG-SM.2sg-MOD-dance-FV
Intended: "you don't have to play"
```

Assuming (following Jeretič, 2021a) that exhaustification applies at clause boundaries, these implicatures can still be drawn in other downward-entailing contexts, so long as there is a clause boundary that intervenes. In fact we do see that *anga*'s ambiguity returns in these contexts:

(10) a. Context: You are shopping for books for school, and one of the workers at the bookstore is helping you find the books you need. As you are walking past a shelf of books, the worker picks up a book and explains that it frequently sells out, so if it's one of the books you are supposed to read, you should buy it today while they still have stock.

```
u-a-ma-bi-a
                       i-u-anga-na-som-a
                                                     e-kitabu
SM.2sg-TM-TM-be-FV
                       C-SM.2sg-MOD-TM-read-FV
                                                     AUG-c7.kitabu
eki
        kvo
                 ekalhasi,
                           kumbe
                                        i-u-a-ki-gul-a
        PREP
                 class.
                                        C-SM.2sg-TM-Om.c7-buy-FV
c7.this
                            better.that
munabwire
today
"if you should read this book for school, you should buy it today"
```

b. Context: You are at a bookstore looking for books that are interesting to you to read. The worker here is giving you recommendations. She picks up a book from a shelf that's in German, but she says that it's a very good book and you should get it if you can read German.

```
u-a-ma-bi-a i-u-anga-na-som-a e-kitabu SM.2sg-TM-TM-be-FV C-SM.2sg-MOD-TM-read-FV AUG-c7.kitabu eki, kumbe i-u-a-ki-gul-a munabwire c7.this, better.that C-SM.2sg-TM-Om.c7-buy-FV today "If you can read this book, you should buy it today."
```

The result is a very tidy explanation for why anga varies between possibility and

weak necessity, and why it doesn't achieve a strong necessity reading, as well as why *anga*'s variable-force vanishes under clausemate negation, but persists in other downward-entailing environments. All from the facts of Kinande's modal paradigm—what is and is not in the Kinande lexicon—and a few semantic mechanisms that are already operating in the grammar.

What's more, those mechanisms—exhaustification and domain restriction—then derive a complete typology of modal force, and the difference between fixed-force and variable-force systems falls out from the way exhaustification works in general and its interactions with the lexicons of the individual languages. Exhaustification cannot draw a scaleless implicature to derive a variable-force modal if there is already a lexical modal occupying the relevant part of the scale, because exhaustification will first draw a scalar implicature blocking the scaleless one. As a result, there are no special mechanisms required to account for variable-force modals as distinct from fixed-force ones; both general types (as well as limited variable-force systems in between the two) are simply products of the shape of the individual language's lexicon.

1.4 Outline of the dissertation

This dissertation begins and ends with typology, seeking to stay theoretically-motivated throughout. Chapter 2 gives a descriptive background of modality, especially variable-force modality, and then gives a brief systematic description of the expression of modality in Kinande; it seeks to be theory-light, and accessible to those with little-to-no semantic background, situating Kinande in the modal typology as we currently understand it, and presenting the descriptive challenges that it poses for that typology. Chapter 3 then focuses on *anga* in particular, the issues it poses for current theories of variable-force modality, and presents my own proposal for *anga*. Chapter 4 takes everything we have learned from the theoretical analysis and returns to typological concerns; there I review the implications of my analysis of Kinande for

other variable-force languages, discuss how we might better understand and investigate variable-force modals cross-linguistically, and lay out a typology for modal force as a whole. I also re-examine some previous accounts of variable-force modals.

This dissertation is written with a few different readers in mind. On the one hand, there may be a theoretical semanticist who has a grasp on formal theory but relatively little experience with cross-linguistic phenomena. On the other hand, there may be a Bantuist who likely has a grasp on the morphosyntax of this language family and an interest in furthering the description and analysis of Bantu languages, but little experience with formal semantic theory. I expect there are a number of readers somewhere between these points as well, but I have done my best to be clear about the data, the theoretical background I am employing, and the interpretation of any formalism. My hope is that this will provide better clarity and understanding for whoever reads this dissertation, and that it will not be too much of a slog. The most extensive background portions are cordoned off in their own sections and subsections in each chapter; they are there for whoever would benefit from that background, but can be skipped by those already familiar with their contents without worry. The data and analysis are complicated enough that the relevant data points and theoretical implications will be brought up repeatedly, as they make themselves relevant.

Chapter 2

Description of the Kinande modal system

2.1 About this chapter

In this chapter I aim to give a descriptively-oriented introduction to variable-force modality, and then describe and situate Kinande's modal system within that context.

Modal systems being highly complex, and Kinande generally being underdescribed (and its modal system so far completely undescribed), this description is bound to be incomplete. Nevertheless, this chapter lays out all that is currently known about the Kinande modal system, couched in as accessible and non-theory-dependent terms as I am able. That said, per Matthewson (2013b), theoretical concerns and analysis play an inextricable role in the investigation and description of any language, whether acknowledged or not. This is a feature, rather than a bug: by presenting my findings in the context of their analysis, rather than from a supposed theory-neutral position, we gain both a more complete description as well as a better understanding of the limitations of that description.

In that context, this chapter is focused on giving the reader an idea of the descriptive landscape for variable-force modals and for Kinande modality, without bogging down too much in the formal intricacies of the prior accounts or how the Kinande morphemes should be analyzed. Once we are sufficiently familiar with the basic properties of each morpheme, and have a general idea of how they interact, we can focus in Chapter 3 on the more intricate details that affect the formal analysis.

To that end, section 2.2 presents an overview of the typological framework I plan

to engage with, introduces the main modality-related terms I will use in my description, and gives the first overview of the Kinande modal system within that typology. After a brief methodological note in section 2.3, in section 2.5 I walk systematically through the syntax and basic semantics of modal expressions in Kinande, one by one. Finally in 2.6 I return to a bird's-eye view of Kinande's modal system, discussing how it affects our understanding of modal typology, where it fits with what we know so far, and where it brings elements of our system into question. Because this chapter doesn't present a formal analysis of the morphemes described, a full discussion of typological implications is deferred to Chapter 4, where I offer a more in-depth predicted typology given the existing analyses of (variable-force) modals and my analysis of Kinande *anga* in conjunction with the rest of Kinande's modal system.

2.2 Overview of modal typology

In anticipation of any potential confusion: There is sometimes a break between how morphologists and syntacticians, on the one hand, define "modal", and how semanticists define it on the other hand. In general if you are working on morphology or syntax and you say "modal", you are referring to a particular syntactic or morphological category with a particular distribution: modal words like English *must* or *might*, which tend not to inflect and are often analyzed as being somewhere near tense. Semanticists, on the other hand, tend to take a broader interest in both these morphosyntactic modals as well as modal verbs, or "pseudomodals" such as *have to*, *be able to*, and so on. So much so that in the semantics it's common to lump a number of syntactic categories together under the umbrella term "modal" by virtue of the commonalities in their meanings.

Languages differ both in the types of morphemes they use to express modal meanings as well as in the meanings that those morphemes express, but it seems that there is no correlation between the two. One language may use a modal particle to express

a modal meaning that in another language is expressed by a modal verb; conversely, two modal particles (within or between languages) might differ widely in their meaning.

Because of this, and because I am looking to give a rough description of the entire system of modal expression in Kinande, I will follow the semantic literature in using the term "modal" generally, when I refer to mechanisms of modal expression as a group; this includes modal particles, modal verbs, and occasionally other constructions that encode modality. When I need to differentiate, I will use the more specific terms like "modal particle" or "modal verb", as I have been doing. This will hopefully allow for general statements when warranted, as well as specific ones when needed, while avoiding any terminological confusion.

2.2.1 Descriptive modal terms

Modals are typically described in terms of "modal flavor" and "modal force". And although languages vary in how they divide up the modal space (see van der Auwera & Plungian, 1998), and formal analyses of modality certainly vary in how these intuitive concepts are accounted for, they remain useful descriptors for discussing modals in a (relatively) pre-theoretic way.

Modal flavor describes what basis the speaker is making their modal claim on: what the speaker knows and can infer about the world, what the law says, what their goals are, and many other concerns. In English (a useful example language because English modals are so well-studied, and because it is likely familiar to those reading this dissertation), modal flavor is largely left up to context. English *must* can have epistemic flavor (based on what I know about the world, X must be the case), or it can have deontic flavor (based on my obligations, I must do X), for example. Some modals have relatively strong tendencies towards one flavor over another (English *might*, for example, strongly favors epistemic interpetations), but most modal expressions in English have contextually-determined modal flavor:

(11) Context 1: We pass by Adelaide's house, and see that her lights are on. By way of commenting on this, I say:

Context 2: There is a curfew that states that everyone must be in their homes by 10:30pm, and we are discussing Adelaide's obligations. It's 10:45 now, so:

a. Adelaide must be at home.

English *must* is easily read epistemically, drawing inferences based on what is known about the world, as in context 1; or deontically, making statements about what is required of Adelaide considering the various rules currently in effect, as in context 2. Modal flavor has a significant effect on the truth conditions of a sentence; read epistemically, (11) gives a relatively strong inference that Adelaide is in fact at home in the world of evaluation. Read deontically, however, (11) does not carry this inference, and it only expresses that if Adelaide is not home, she is in violation of the rule and may face consequences.

There's a lot of variance in the terms for modal flavor used in the literature, as well as how they are grouped and analyzed. Some authors group them into epistemic and root modals (Hacquard, 2006), others split what I am labeling circumstantial modals into internal and external non-epistemic modals (van der Auwera & Plungian, 1998). Empirically, the landscape is no less heterogeneous: different languages seem to group their modals differently, and may even relegate some apparently-modal concepts to constructions that can be argued to be entirely non-modal. Kinande, for example, does not seem to use any kind of modal expression for circumstantial necessity, especially not "internal" necessity like having to sneeze. Instead, Kinande simply uses a declarative sentence:

(12) Context: You are riding the bus with a friend to Kampala. Yo have not had a chance to go to the toilet for 6 hours, and your bladder is very full. You

-

¹Tlingit (Na-Dene; Alaska, British Columbia, the Yukon) is reported in Cable (2017) to similarly lack a dedicated circumstantial necessity modal, instead using the future mode in contexts that are expected to license a circumstantial necessity modal.

tell your friend:

n-ka-ya-subal-a SM.1sg-TM-pee-FV "I am going to pee"

Modal force describes how strong the modal claim is (thus it's also known as "modal strength"). The difference between English *must* (on an epistemic reading) and *might* is one of modal force: While *must* expresses something that is necessarily the case, *might* expresses a possibility, instead:

- (13) Terry **must** be in her office (#but I am not sure one way or the other).
- (14) Terry might be in her office (but I am not sure one way or the other).

Other modal flavors in English show a similar distinction: when *must* is used as a deontic modal it express obligation, while English *may* in its deontic use expresses permission:

- (15) You must finish your vegetables before you get dessert.
- (16) You may play outside after you have finished your homework.

Obligation and permission can be understood as a very similar contrast to epistemic necessity and possibility: where obligation characterizes what is necessarily true if everyone's obligations are fulfilled, permission characterizes what is possible if those obligations are fulfilled; that is, what you can do without violating any of those obligations.

In slightly more formal terms, the strength of a modal is typically notated by its quantifier over possible worlds. In general, a proposition including *must* is true if all of the relevant possible worlds (which can be roughly characterized as situations that might somewhat reasonably come to pass) are worlds where the proposition is true, and a proposition containing *might* is true if there is at least one such world:

(17) $\llbracket \mathit{must}(p) \rrbracket = \forall w' [w' \in Acc(w) \rightarrow p(w')]$

READ AS: For all worlds w': if w' is in the set of worlds accessible from the

evaluation world w, then proposition p is true in w'

(18) $\llbracket might(p) \rrbracket = \exists w' [w' \in ACC(w) \land p(w')]$

READ As: There is at least one world w' such that w' is in the set of worlds accessible from the evaluation world w and p is true in w'

The meanings for these modals are typically taken to be the same, with the exception of the quantifier. A universal quantifier in (17) says that p is a necessity from our perspective: all possibilities lead to p being true. An existential quantifier in (18) on the other hand simply says that p is not ruled out: there's at least one possibility that will lead to p being true.

While most treatments of modality concern themselves just with possibility and necessity, there are other distinctions in modal force that can be made; one that will be useful to us in looking at Kinande is **weak necessity**. Weak necessity modals like English *should* or *ought* are somewhere between (strong) necessity *must* and possibility *can* or *may* in strength, and the stronger propositions entail the weaker ones. This is shown by whether you can assert one modal and felicitously negate the other (von Fintel & Iatridou, 2008):

- (19) You should wash the dishes, but you don't have to
- (20) # You have to wash the dishes, but it's not the case that you should

In (19), it's possible to assert the weak necessity claim while denying the strong one, indicating that there's no entailment from a *should* sentence to a parallel *have to* sentence. (20) conversely indicates that there is an entailment form the *have to* sentence to the *should* one, because you cannot consistently assert the strong necessity version of the sentence and deny the weak necessity version.

The use of modals like English *should* to express concepts like obligations that are not absolute necessities has earned them the name "weak necessity". In most analyses, they are taken to be a kind of strong necessity modal with an additional weakening mechanism applied to them, often a second ordering source on top of the

first (von Fintel & Iatridou, 2008; Rubinstein, 2012). Other distinctions such as weak possibility, strong possibility, or various degrees of probability are also sometimes used in the theoretical literature (Kratzer, 1991; Lassiter, 2017; Yalcin, 2007).

The vast majority of analyses of modals reckon with weak necessity modals by allowing context to alter what kinds of facts and preferences are considered relevant for a modal claim. As a result, the underlying system is still binary (in that all modals are considered either possibility or necessity modals, using either an existential or a universal quantifier), but there is a little "wiggle room" inside of those categories. We will shortly see, however, that some languages have modals that vary even beyond this expectation, and present an analytical challenge for these analyses. As much of that analytical challenge depends on a more formal discussion of modal systems, I will have to defer most of this discussion to Chapter 3. For now, suffice to say that our understanding of modals predicts a typology of inherently binary underlying logic, and any further distinctions in modal force, or lack of distinction, so far has to be built accommodating that underlying binary.

2.2.2 Framework of modal typology

With the above concepts of modal force and flavor in hand, we can now turn to a crosslinguistic typology of modals as previously described, and discuss where Kinande fits in that typology from a descriptive standpoint.

As discussed above, English modals tend to specify their modal force, but leave their modal flavor to context. Arranging English modals over the space of modal expression, we get something like this:

Other languages, like Paciran Javanese (Austronesian), can draw distinctions along both modal force and flavor, seen in Table 4.3.

Languages that distinguish both force and flavor are relatively easy to deal with in the modal system we have created to explain say, English modals; we already expect specification for modal force, and additional specifications for modal flavor are not

	Epistemic	Deontic	Circumstantial	Teleological
Strong Necessity	must,have to	must,have to	must,have to	must,have to
Weak necessity	should	should	should	should
Possibility	can,may	can,may	can	can

Table 2.1: English modal morphemes

	Epistemic	Deontic	Teleological	Circumstantial	Bouletic
Necessity	mesthi	kudu			
Weak necessity	mestine	kudune			-
Possibility	paleng	oleh	-	iso	-

Table 2.2: Paciran Javanese modal system (Vander Klok, 2012)

difficult to handle.

What is more puzzling for many current accounts of modality are languages with what we call "variable-force" modals. Descriptively speaking, these are modals that leave their modal force up to context, either instead of or in addition to their modal flavor. The first attestation of such modals in the literature is from Rullmann et al. (2008) in St'át'imcets (Lillooet Salish), which has modal particles that are unambiguous about their modal flavor, but can be used in either possibility or necessity contexts:

(21) St'át'imcets (Rullmann et al., 2008, p. 331):

Context 1: The cougar was on a rampage and was killing cats, dogs, and racoons, and it had a child cornered and was growling; it would have killed a child.

Context 2: You just know that sometimes cougars kill children when they venture into built-up areas; it could have killed a child.

zúqw-s-as ka ta sk'úk'wmi7t-a ti7 ku swúw'a lh-cw7áoz-as die-caus-3erg irr det child-det det cougar comp-neg-3conj

kw s-qus-cit-ítas det nom-shoot-ind-3pl.erg

'That cougar could/would have killed a child if they hadn't shot it.'

	Epistemic	Deontic	Circumstantial	Future
Strong Necessity	k'a	ka	ka	kelh
Weak necessity	k'a	ka	ka	kelh
Possibility	k'a	ka	ka	kelh

Table 2.3: St'át'imcets modal morphemes (Rullmann et al., 2008)

Gitksan (Tsimshianic; Matthewson, 2016; Peterson, 2010) similarly has modals with contextual modal strength:

(22) yugwimahl dim iixwt (Gitksan, Peterson, 2010, p. 63) yukw-ima-hl iixw-(t)=s John PROG=MOD=CND fish-3=CND John "John might be fishing." ≻ "John must be fishing."

And Washo -e? is argued to leave both its modal force and modal flavor to context (Isolate; Bochnak, 2015):

(23) a. Context: You are planning to drive over the mountains. It's started to snow, and you know that whenever it snows, the road over the mountains is closed.

dé?eš yeweš gum-beyec'ig-i-gi k'-e?-i (Bochnak, snow-good-ipfv-sr road refl-close-ipfv-rel 3-mod-ipfv 2015, p. 7)

"It's snowing a lot, so the road must be closed."

b. Context: You hear a knock at the door. You can't see who it is, but can see that the person looks about the same height as Beverly.

bevali k'-e**?**-hel-i-gi k'-e?-i (Bochnak, 2015, p.9) Beverly 3-cop-subj-ipfv-rel 3-mod-ipfv "It might be Beverly." (24) a. Context: You borrowed a pot from Beverly, and now you need to give it back to her.

```
bevali wi:di? le-išil k'-e?-i (Bochnak, 2015, p. 6)
Beverly this 1-give 3-mod-ipfv
"I need to give this to Beverly."
```

b. Context: At a school dance, you see a shy boy who wants to talk to a girl but isn't. You ask your friend if that boy is allowed to talk to that girl. Your friend responds: "Yes..."

```
mé:hu šawlamhu-hak'a wagayay?-i-gi k'-e?-i (Bochnak, 2015, p. 8) boy girl-with talk-ipfv-rel 3-mod-ipfv "The boy is allowed to talk to the girl."
```

Similarly, Nez Perce *o'qa* is compatible with a number of modal flavors, and in both necessity and possibility contexts (though these facts change under downward-entailing operators, see Deal, 2011).

	Epistemic	Deontic	Circumstantial	Teleological
Strong Necessity			o'qa	
Weak necessity			o'qa	
Possibility			o'qa	

Table 2.4: Nez Perce modal morphemes

From the view so far, it looks like in order to have variable-force modals, a language can only have one modal per flavor. But relatively recent findings from Nsyilxcen (Okanagan, Southern Interior Salish, Menzies, 2013) and Ecuadorian Siona (Jeretič, 2021a) demonstrate that it's possible to have a variable-force language with scalemate modals as well.

First, Nsyilxcen has two epistemic modals, *mat* and *cmay*. Both are compatible with possibility contexts (25), but only *mat* is felicitous in necessity contexts (26):

(25) (Menzies, 2013, p. 2)

Context: You know that Mary loves to go running and often goes on runs

randomly. She could also be at the store or at school. I ask you, where is Mary?

- a. Mary mat ac-qíc-əlx
 Mary epis.mod cust-run-lex
 'Mary might be running'
- b. Mary cmay ac-qíc-əlx
 Mary EPIS.MOD CUST-run-LEX
 'Mary might be running'

(26) (Menzies, 2013, pp. 21–22)

Context: Mary runs every day to train for a marathon. She usually runs at 6pm on Tuesdays. Today is Tuesday and it's 6pm. I ask you, where is Mary?

- a. Mary mat ac-qíc-əlx
 Mary EPIS.MOD CUST-run-LEX
 'Mary must have gone running'
- b. # Mary cmay ac-qíc-əlx
 Mary EPIS.MOD CUST-run-LEX
 'Mary might be running'

Consultant comment: "*cmay*. We are just guessing. Mary might be running."

Nsyilxcen *mat* is felicitous in both possibility and necessity contexts, while *cmay* is only felicitous in possibility contexts (both of these retricted to epistemic modal flavor). This is unusual in the context of the literature. From a pragmatic perspective, this is surprising. With two modals, if one can have both necessity and possibility interpretations, and the other only possibility interpretations, then necessity compatible one is expected to be used just in necessity contexts, and the possibility modal *cmay* should be used for possibility, exerting a kind of pressure for *mat* to be used as a necessity modal, instead. The pragmatic reasoning goes something like this: If a speaker uses *mat*, then it must be in a context where saying *cmay*, which is unambiguous, is infelicitous. So then, even if the lexical semantics of *mat* permit a

possibility reading, pragmatic reasoning ought to force it into being used as a necessity modal, at least in these positive simplex clauses. But *mat* appears to vary based on semantic/pragmatic context, much like the other modals described here.

Leffel (2012) describes a future marker -ti in Masalit (Nilo-Saharan, Maban, Sudan) which, in addition to indicating future, is also used for various possibility-modal readings:

- (27) tí sû tò-rón-tì (Masalit, Leffel, 2012, 11b) 3sg goat.ACC 3sg-buy-ti "He will buy a goat." or "He might buy a goat."
- (28) ámá kómò á-kál-tì (Masalit, ex. 15b)
 1sg mountain.ACC 1sg-see-ti
 "I will see the mountain" (e.g., tomorrow) or

 "I can see the mountain." (e.g., from where I'm standing)

Interestingly, *-ti* can also be used for epistemic necessity, but only if there is the particle *de* ('only') with it to get a necessity reading:

- (29) tí màsàrà tú-tì (Masalit, Leffel, 2012, 17c) 3sg Masalit 3sg-ti "He might be Masalit"
- (30) tí màsàrà **dè** tú-**tì** (Masalit, Leffel, 2012, 17d) 3sg Masalit only 3sg-ti "He must be Masalit" (lit. "He could only be Masalit.")

Leffel analyzes *-ti* as a possibility modal tied to a temporal base (allowing for future readings), and proposal that *de* operates just like *only* from Rooth (1985), asserting that all of the alternatives to the expression without *only* are false.

Ecuadorian Siona (Jeretič, 2020, 2021a,b) is another case of variable-force modals with other pragmatic options, though in Siona the pragmatic reasoning outlined above is in effect. Siona *ba'iji* is interpreted as a necessity modal in simplex clauses, and there is another modal *de'oji* for possibility interpretations:

(31) Ecuadorian Siona modals (Jeretič, 2021b, p. 2)

```
a. Sai-ye ba-'i-ji
go-INF be-IMP-3S
"We must go." (literally, "there is to go")
b. Sai-ye de'o-ji
go-INF good-3S
"We can go." (literally, "it is good to go")
```

But, though *ba'iji* is umambiguously a necessity modal in these simple positive contexts, it's instead unambiguously read as a negated possibility modal in negated sentences:

```
(32) Sai-ye beo-ji
go-INF NEG.be-3S
"We mustn't go."

# "We don't have to go."
```

And in other downward-entailing contexts, such as extra-clausal negation, *ba'iji* is ambiguous between possibility and necessity readings:

(33) Sai-ye ba'i-ji ca-ye ba-hu<u>ё</u> më'ë-re (Jeretič, 2021b, pp. 4–5) go-INF be-IPF-3s say-INF NEG.be-PST.ОТН 2S-ОВЈ "He didn't say that you could/had to go."

The pattern in Siona appears to explicitly take advantage of Gricean reasoning: Jeretic analyzes *ba'iji* as a possibility modal, which in simple positive clauses is pressured by *de'oji* to being interpreted as a necessity modal. In downward-entailing contexts, however, entailment relations are reversed, and so the same reasoning doesn't apply, so *ba'iji* is ambiguous, instead.

It's still somewhat unexpected from a purely pragmatic perspective, however. *De'oji* and *ba'iji* collapse in some contexts, and the potential for differentiation isn't enough. Something more than pragmatics is going on, and the implications for this typologically remain to be explored. I will return to this in Chapter 4.

Returning to the implications for these facts for semantic theory, the traditional analyses of modals are perfectly capable of dealing with variable modal flavor: since they were largely built around languages like English, there are already robust tools

for encoding contextually-sensitive modal flavor, or contextually-insensitive modal flavor, as needed. However, having designed our modal systems around English comes back to haunt us when we consider these languages where the modal force is extra variable: our modal systems have enough space in their definitions for a little bit of variance, as I discussed previously, but there are limits to how far a modal's force can stretch; limits that variable-force languages seem to defy. We have just enough vagueness in our definitions to adjust the logical strength of a modal claim a little bit one way or the other, but it's far from obvious how we are to achieve real gradience of modal force like we seem to have in variable-force languages. Largely speaking, analyses of variable-force languages have focused on attempting to account variable-force modals while preserving the underlying logical binary that is deeply coded in the semantic formalism. Some other accounts of modals (e.g., Lassiter, 2017; Yalcin, 2007) have instead shifted to treating all modals as scalar expressions, making them similar to adjectives like tall or good, which can be modified by adverbs of degree, like very tall or somewhat good, as well as placed into comparative constructions. Two people can be tall and it can still be true that one is taller than the other. This can circumvent the binarity of the logical operators. However, as I will discuss in Chapter 3, a simple scalar account will not be able to account for the behavior of Kinande anga. As such, my eventual analysis will at its roots be quantification over worlds, rather than inherently scalar items. The challenge that variable-force languages pose is undeniable, and will have to be addressed in any analysis of modality; we will return to this in Chapter 3.

2.3 Methodological note

All of the Kinande data in this dissertation, except where otherwise cited, is from my own fieldwork with Dr. Philip Mutaka, who has been very generous with his time to serve as a Kinande language expert on this project. Most of the examples were elicited via translation and acceptability judgements in questionnaires given over email, though a small amount was elicited in person as well. All of the semantic judgments (for example to determine modal flavor or whether a modal is compatible with a particular force) were elicited in a given context, and the sentences were initially obtained via translation tasks in context, with some follow-up questions and/or modifications to the sentence or context as appropriate (Matthewson, 2004).

Most of the initial contexts were derived from the helpful template questionnaire in Vander Klok (2014), which I adapted for Kinande. After initial elicitations some contexts were modified further, or I wrote new contexts to probe particular aspects of modals or answer theoretical questions.

You may notice that there are very few examples cited from other sources, though certainly prior work on Kinande exists, including a large number of example sentences in the Afranaph database (Mutaka, 2020). While some example sentences were derived from Afranaph or literature examples, they have all be re-elicited in order to find a semantic judgment in context.

This is a common complaint in semantic studies, in fact: the information given in grammars is a shaky lead at best, and almost never enough to make even a tentative conclusion about a morpheme's meaning without extensive elicitation specifically meeting the criteria of a semantic datapoint: a context, a sentence, and a felicity judgement of that sentence in that context. Some positive evidence can be found in texts, but it is difficult to find the edges of a morpheme's meaning. I hope that the description given here will provide sufficient systematic evidence to further modal studies in Kinande and elsewhere, and help serve as a template for further descriptive efforts in modality.

One final note: I don't always present contexts alongside the examples, and just give translations or note that it is unacceptable in a particular kind of context. This is largely to save space and make presentation easier. The full set of data with contexts and judgments will be added to the Afranaph project, and in the meantime is freely

available upon request.

2.4 Background on Kinande

Kinande is a Bantu J language spoken mostly in the Northern Democratic Republic of Congo (Ethnologue, ISO:nnb). It has about 900,000 speakers. In Uganda the language is known as Konzo, spoken by an additional 893,000 people, who very much consider themselves and the Nande people to be one people group (Ethnologue, ISO:koo). The major difference between them is in the source of their borrowed words: Kinande speakers tend to borrow words from Swahili and French, while Konzo speakers tend to borrow words from Rutooro and English.

Kinande/Konzo is an SVO language with highly agglutinative morphology, typical of the languages in its family. It has a graded tense system as partially described in Mutaka & Kavutirwaki (2011); Baker (2003), but the Afranaph database glossing largely does not distinguish between the different tense and aspect morphemes, and glosses them all as "TM". As such, the data here is a bit inconsistently glossed for tense and aspect, pending further adjustments and investigation into the TAM system.

(34) Kambale mó-á-mú-pípire (Afranaph ID: 77) Kambale TM-SM.c1-OM.c1-praised 'Kambale praised him.'

Kinande has agreeing subject and object marking, as well as agreeing linkers analyzed in Schneider-Zioga (2015):

(35) Kámbale á-gúl-ir-a ekitábú kyo Nadíne Kambale SM.c1-bought-APPL-FV c7.book c7.LINK Nadine 'Kambale bought a book for Nadine.'

(Schneider-Zioga, 2015, ex. 1b)

2.5 Kinande modal morphemes

Turning to Kinande, there are a number of comparisons to draw between Kinande's modal system and the systems attested elsewhere. To begin with, Kinande's modal expressions seem to be spread across a number of syntactic categories, including verbs, modal prefixes in a tense/aspect slot, and modal adverbs or other particles in the far left of the clause. Additionally, Kinande uses some constructions that are not overtly modal to cover some modal expressions, potentially as a kind of implicit modality. All of these constructions participate explicitly in the modal system, and seem to be available when a speaker is determining how to express various modal propositions. As such, I have included all of them in the Kinande paradigm below.

	Epistemic	Deontic	Circumstantial	Teleological
Strong Necessity	paswa, kumbe, tolere	paswa,kumbe, tolere		paswa,kumbe,tolere
Weak necessity	anga,kumbe, tolere	anga,kumbe	anga,kumbe, tolere	kumbe,tolere
Possibility	anga	anga	anga	anga

Table 2.5: Kinande modal morphemes

Another difference between my description here and many given in the literature is that I am giving a more fine-grained paradigm along the modal force axis than previous descriptions have in other languages; the reason for this is hopefully clear from what that additional distinction reveals: While Kinande does not have fully variable-force modals, in that no modal is compatible with both strong necessity and possibility readings, there is significant overlap in the weak necessity space. And rather surprisingly given the usual analyses of weak necessity, Kinande anga varies between possibility and weak necessity, but not to strong necessity. This poses some questions for both variable-force analyses of modals as well as for our analyses of weak necessity, which up to this point have built weak necessity modals from strong necessity ones, and in a way that cannot easily build weak necessity from possibility. As a result, Kinande provides not only a previously unattested modal system from a

descriptive and typological viewpoint, it also serves as an important test case for our current theories of modality.

Kinande uses a mix of constructions for communicating modality, across a variety of structure types and morphological categories. Some of these are more directly identifiable as explicitly modal than others. The modal prefix *anga*, and modal verbs *paswa* and *tolere* are all of the more obviously modal kinds of expressions. But other constructions, like subjunctive-marked clauses introduced by the complementizer *kumbe*, are either less obviously modal, or where it is harder to determine where the modal aspect of their meaning resides.

Some of these constructions will be better-described than others. But for each one, I attempt to give a general morphosyntactic description, a descriptive summary of the kinds of modal contexts it appears in, and anything else of note, or remaining issues to be explored. Some aspects of these meanings will be dealt with in more depth in Chapter 3; I will indicate those sections as relevant.

In general, Kinande modals are flavor-ambiguous; there are very few apparent restrictions on epistemic vs. teleological contexts, for example. For cases that I have found robust preferences not to use a particular modal, I will mark as such on that modal's overview table and discuss the relevant examples in that modal's section.

2.5.1 Strong necessity paswa

Kinande *paswa* is a loan from Swahili, where it is used as a strong necessity modal akin to English *must*:

(36) tu-me-pasw-a ku-enda (Swahili; Johnson, 1939, entry: *pata*)

1PL-PERF-must-FV INF-go

'It is our duty to go'

In Kinande it seems to have been borrowed with much the same meaning.

Morphosyntactically, *paswa* is a modal verb; it appears with tense/aspect morphology and subject marking, and its complement is an infinitival clause. Overall,

paswa is acceptable in strong necessity contexts spanning the range of modal flavors:

(37) Context: Kambale regularly eats dinner at Swera's restaurant. He never misses a day, even when he is sick. It's not obligatory for Kambale, he just goes for coffee there all the time. It's dinner time now, so...

Kambale a-na-pasw-a eri-bi-a e-ri-yi e-restora ya Swera Kambale 3SG-TM-must-FV INF-be-FV c24-c24-be c24-restaurant of Swera "Kambale must be at Swera's restaurant" (epistemic)

(38) Context: In Uganda, the law states that when you ride a motorbike...

```
u-na-pasu-a e-ri-ambal-a e-sapu (deontic)
2sg-TM-must-FV INF-wear-FV c3-helmet
"You must wear a helmet"
```

(39) Context: There is only one main road between Kampala and Kakiri: the Kampala-Hoima road.

```
u-a-ma-bi-a i-u-ka-ya e-Kampalá eri-lu-a e-Kakiri 2sg-TM-TM-TM-be-fv C-2sg-pres-go c24-Kampala INF-leave-fv c24-Kakiri u-na-pasw-a eri-im-i-a endakya e-yi (teleolgical) 2sg-TM-must inf-take-CAUS-FV c9.road c9-this "If you are going to Kampala from Kakiri, you have to take this road"
```

Circumstantial strong necessity is the only strong necessity context where I was not able to confirm whether *paswa* is acceptable, but for those strong circumstantial contexts, Kinande instead uses an indicative clause or an imperative to express these kinds of needs:

(40) Context: You are riding the bus with a friend to Kampala. Yo have not had a chance to go to the toilet for 6 hours, and your bladder is very full. You tell your friend:

```
n-ka-ya-subal-a
SM.1sG-тм-тм-рее-ғv
"I am going to pee"
```

(41) Context: Kábunga is the sort of person who needs plenty of sleep in order to be able to work efficiently the next day. He needs at least 8 hours a night or

he's completely useless in the morning.

Kabunga a-ka-anz-a i-na-bug-a nikiniki saha mu-nani Kabunga SM.c1-TM-like-FV C-TM-finish-FV at.least hours c.18-eight omotugotseri c13.sleep "Kambale likes to finish at least 8 hours in the sleep (i.e., Kabunga likes to sleep during 8 hours at least)"

The example above is specifically internal circumstantial modality, rather than external (van der Auwera & Plungian, 1998). Internal circumstances are those relating to a person's internal abilities and needs, thus things like needing to sleep, or sneeze, or being able to swim. External circumstances are instead about what is possible or necessary given the impositions placed on the agent by the outside world: essentially, deontic and teleological flavors. In order to properly distinguish internal from external necessity, I've treated circumstantial necessity specifically as this kind of internal necessity, and circumstantial possibility as an ability modal, in order to ensure that I give as fine-grained a description as I can. As far as Kinande's modal system is concerned, the strong circumstantial necessity cell is the only cell of the table where the internal/external distinction seems to make any difference; in the possibility space, for example, *anga* is available in both internal possibility (ability) readings and external ones (teleological, deontic).

Paswa is infelicitous in weak necessity and possibility contexts:

- (42) Context: Kanyerê is still learning how to ride a motor bike. She just started 1 month ago. Her friend tells her that... (deontic weak necessity)
 - # Kanyere a-na-pasw-a eri-gend-i-a bolobolo Kanyere SM.c1-TM-must-FV INF-go-TV-FV slowly "Kanyere must drive slowly."
- (43) Context: Kambale's parents told him that he is not allowed to go see his friend in Kibangya because it is too far away. You know that Kambale is leaving Kampala next week, but you don't know where he will go. Kambale is a daring type of guy that often does things that he is not permitted to do. You think:

(epistemic possibility)

- # Kambale a-na-pasw-a eri-gend-a e-Kibangya Kambale SM.c1-TM-must-FV INF-go-FV c24-Kibangya Intended: "Kambale may go to Kibangya"
- (44) Context: Másikâ's teacher told her class that it was ok to go swimming, but Másikâ doesn't want to go because she cannot swim! (deontic possibility)
 - # Masika a-na-pasw-a eri-ya-tsig-a Masika SM.c1-TM-must-FV INF-go-swim-FV Intended: "Masika may go swimming"

This lines up with *paswa* behaving as a fairly typical strong necessity modal with an unspecified modal flavor, much like English *must* or *have to*.

2.5.2 Weak necessity tolere

Tolere is a modal verb derived from the verb for "to fit". Like *paswa* it is used as a modal verb, but unlike *paswa*, it takes a CP complement, rather than an infinitival one, and it appears to be an optionally hyper-raising verb, in that *tolere* embeds a finite CP, but the subject of that embedded clause can nonetheless raise to the matrix clause (Carstens & Diercks, 2009):

- (45) Context: The market is close by, so taking a taxi or walking are both viable options to get there, especially on nice days. (Deontic)
 - a. u-tolere i-u-a-gend-a omo bisando e-ri-y-a omo soko SM.2sg-fit C-SM.2sg-TM-go-FV c24 feet AUG-c5-go-FV c24 market "You should walk to go to the market."
 - b. li-**tolere** i-u-gend-a omo bisando e-ri-y-a omo soko c5-fit C-SM.2sg-TM-go-FV c24 feet AUG-c5-go-FV c24 market "You should walk to go to the market"

There is an *i*- prefix on the embedded verb, often glossed in Afranaph as a complementizer, and while it may or may not be a C on its own, it seems to be some kind of morpheme (possibly a mood marker) that is very high in the clausal space, and can at least be taken to indicate a clause boundary to its left. In the examples above, it is

visibly above the second person subject marker *u*- on the lower verb. (45a) has both *tolere* and the lower verb *genda* ('go') agreeing with the second person subject (which is pro-dropped in these examples), even while the lower verb is clearly a finite clause, as indicated by the *i*- prefix as well as the tense marker *a*- on *genda*.

Most cases of *tolere* in my elicited dataset are in weak necessity contexts, across several modal flavors.

- Context: There are two ways to get from Entebbe to Kampala: You can take the Kampala-Entebbe expressway or the Kampala-Entebbe road. The expressway is faster, so according to Kambale: (Teleological) e-ri-hik-a u-a-ma-sond-a e-Kampálá vó lúbá, SM.2sg-TM-TM-want-FV AUG-c5-arrive-FV c24-Kampala LNK early ú-tólere í-u-ím-i-a ekisipirése SM.2sg-fit C-SM.2sg-take-CAUS-FV c9.expressway "If you want to arrive in Kampala early, you should take the expressway"
- (47) Context: Kámbére was in a moterbike accident 3 weeks ago, and he sprained his ankle. He is able to walk now, but it hurts to do so, and so Kámbére doesn't want to. The doctor says Kámbére ought to start walking to rehabilitate his foot, but he doesn't absolutely have to yet, if it hurts too much. (Deontic) Kámbére a-ná-tólere í-na-bung-a Kambere SM.c1-TM-fit C-SM.3sg-walk-FV "Kambere should walk"
- (48) Context: Kábunga is not a consistent student, and often skips class on Wednesdays, but he is usually there on Mondays. Today is Monday, and so the professor isn't entirely sure whether Kábunga will show up, but since it's Monday and Kábunga is relatively consistent on Mondays, he thinks Kábunga will probably be there.

 (Epistemic)

Kábunga a-ná-tólere í-ná-s-a oko kalhásí kó munábwíre Kabunga SM.c1-TM-fit C-TM-come-FV LNK class LNK today "Kabunga should be coming to class today (based on what we know)"

Tolere's modal force may be affected by the marking of its embedded clause. In the

examples above, *tolere* embeds a CP with *i*- marking, and expresses weak necessity, but it appears that when it embeds an indicative-marked CP (lacking the *i*- prefix), the resulting expression is one of strong necessity, not weak necessity:

(49) Context: Kambale regularly eats dinner at Swera's restaurant. He never misses a day, even when he is sick. It's not obligatory for Kambale, he just goes for dinner there every day. It's dinner time now, so... (Epistemic)

Kámbale á-má-toler-á nga a-ly-a é-yi-ry-a e-restorá Kambale SM.c1-TM-fit-FV C SM.c1-be-FV AUG-c24-be-FV c24-restaurant ya Swera ASSOC Swera

"Kambale must be at Swera's restaurant"

This is a single data point, and one that has proven difficult to replicate across other contexts. But it suggests that the marking on *tolere*'s complement, as well as *tolere*'s final vowel (49 has an indicative final vowel -a, the other examples I have elicited have the subjunctive final vowel -e) both (whether individually or in concert) affect the interpretation of *tolere*. Further investigation into the Kinande subjunctive and mood marking is necessary to draw more than the most tentative conclusions, however.

2.5.3 *Kumbe*

Kumbe is either a complementizer or a high adverb that participates consistently in deontic constructions in Kinande:

(50) Context: In Uganda, the law states that when you ride a motor bike...

kumbe i-u-a-ambal-a e-sapu (deontic necessity) better.that. C-SM.2sg-TM-wear-FV c9-helmet "you must wear a helmet"

(51) Context: Kanyerê is still learning how to ride a motor bike. She just started 1 month ago. Her friend tells her that...

Kumbe Kanyere i-na-bi-a a-ka-gend-i-a bolobolo better.that Kanyere C-TM-be-FV SM.c1-TM-go-TV-FV slowly "It is better for Kanyere to drive slowly"

In most of the elicited data, *kumbe* occurs with an *i*- prefix on the following verb, and expresses weak necessity. However, in at least one example, when the verb following *kumbe* occurs with the subjunctive final vowel -*e* instead of the mood prefix *i*-, the resulting construction is interpreted as strong necessity, rather than weak necessity:

- (52) Context: On behalf of your friend Masika, you ask the teacher if she should go swimming. The teacher replies:
 - a. ihiii, Masika a-anga-na-ya-tsig-a yes, Masika sm.c1-mod-tm-go-swim-fv "Yes, Masika can go swimming..."
 - b. ...kumbe a-ya-tsig-e ...better.that sm.c1-go-swim-subj "She really should swim (i.e., she must swim)"

The apparent strengthening of *kumbe* with a subjunctive-marked verb like this is similar to the behavior of *tolere* discussed previously. Like with *tolere*, the available data for this variance is scant, and further investigation is necessary.

2.5.4 Anga

The only modal that might live in a syntactically-modal slot, *anga* is a prefix in one of Kinande's TAM slots. It is preceded by negation, mood marking, and subject marking, and can be followed by some additional tense/aspect marking between *anga* and the verb.

Aside from its use in simplex clauses as a modal, *anga* appears in counterfactual conditionals, as seen in the paradigm below:

(53) Context: Kambale is a very good soccer player. In fact, he's the best player on his team. His team has a game tomorrow, but the opposing team is not particularly good. If Kambale plays tomorrow, his team will certainly win the game. (If he doesn't, it's not quite so certain.) (indicative conditional)

Kambale a-ma-bi-a i-na-ka-ndi-sya-pik-a oko mupira ko Kambale SM.c1-TM-be-FV C-SM.c1-TM-TM-play-FV c17 c3.ball c17 omungya, i-na-ka-ndi-sya-sing-a tomorrow, C-SM.c1-TM-TM-win-FV "If Kambale plays football tomorrow, he will win"

Context: Like before, Kambale is an excellent soccer player, and his team has (54) a game tomorrow. But unfortunately Kambale is traveling on other business, and probably won't be able to make it to play in the game (there's only a small chance he will make it in time). This is a big shame, because if he could make it, his team would definitely win the game. (future less vivid) Kambale a-ma-sya-pik-a oko mupira ko omungya, Kambale SM.c1-TM-TM-play C17 c3.ball c17 tomorrow, i-na-ka-ndi-sya-sing-a C-SM.c1-TM-TM-TM-win-FV "If Kambale plays football tomorrow, he will win"

(55) Context: Again just like before, but this time Kambale has broken his leg, and there's no way he'll be able to play in the game tomorrow. If he had been able to play, his team would have won for sure. (past counterfactual) Kambale nga-a-pik-a oko mupira ko omuligolo, Kambale COMP-SM.c1-play-FV c3.ball yesterday, C17 C17 a-anga-sing-ire

SM.c1-MOD-win-TM

"If Kambale had played football yesterday, he would have won"

The equivalents of indicative conditionals (53) and future less vivid conditionals (54) do not appear to have overt conditional marking in the prejacent, and differ largely in their tense and aspect marking; the indicative conditional has an auxiliary verb, and the main verb of the antecedent also hosts a number of tense markers associated with the present or immediate future as described in Mutaka & Kavutirwaki (2011). The counterfactual in (54) has no auxiliary on the antecedent's verb, and generally has less TAM morphology overall. Both of the first two conditionals in these examples have the same immediate future/present tense marking on the verb in the consequent, however.

The Kinande equivalent of the English past perfect counterfactual in (55), however, is marked distinctly from the others. Here the antecedent is marked with the prefixal complementizer *nga*-, discussed further below, and the consequent's verb is marked with *anga*, though it is unclear whether *anga* is serving simply as a conditional/counterfactual marker, or if it is also contributing the kind of modal meaning seen in other contexts. Further investigation into the semantics of Kinande conditionals and counterfactuals is needed.²

Anga also looks similar to a prefixal complementizer, nga, which shows up in a number of other contexts:

(56)Tu-a-lagan-a na a-ba-lwana nga oko AUG-c2-boy SM.c2.1st-TM-promise-fv with as on tu-kandisya-tsurum-ir-a oko punda (Afranaph ID 15035) SM.c2-FUT-become.wild-APPL-FV on donkeys "We promised the boys that we would yell at the donkeys."

(57) a-ka-nyunyu ka-ka-bi-a nga-ka-bi-a
IV-c12-bird SM.c12-TM-be-fv COMP-SM.c12-be-fv
i-ka-biri-ri-a e-sioN-buto
COMP-SM.c12-TM-eat-fv IV-c10-seed
"A bird appears to have eaten the seeds" (Afranaph ID 15074)

Outside of explicitly conditional contexts, *anga* functions as both possibility and weak necessity modal, but not as a strong necessity one:

(58) Context 1 (epistemic possibility): Kábunga is not a consistent student. The professor never knows if he will come to class or not. Today, it is time to start class and he has not shown up yet.

Context 2 (epistemic weak necessity): Kábunga is not a consistent student, and often skips class on Wednesdays, but he is usually there on Mondays. Today is Monday, and so the professor isn't entirely sure whether Kábunga will show up, but since it's Monday and Kábunga is relatively consistent on Mondays,

_

 $^{^{2}}$ While they may be related diachronically, the two show up in sufficiently different syntactic positions and in different slots in the templatic morphology that I do not believe their meanings to be synchronically related, and I will not be able to give an account of nga itself here.

he thinks Kábunga will probably be there.

Kabunga a-anga-na-sy-a oko kalhasi ko munabwire Kabunga SM.c1-MOD-TM-come-FV c17 class c17 today "Kabunga might/should be coming to class today"

(59) Context: In Uganda, the law states that when you ride a motor bike...

```
# u-anga-na-ambal-a e-sapu
2sg-MOD-TM-wear-FV c9/3-helmet
Intended: "You must wear a helmet"
```

Based on the semantic/pragmatic context, the same (or very similar) *anga*-expression is consistent with either a possibility context or a weak necessity one.

We then might ask what kind of modal *anga* is underlyingly. One way to narrow this down is to negate an *anga* expression, which, depending on both the underlying modal force and the particular mechanism of its variability, might strip the modal of its ambiguity. And with *anga* we find that it does:

(60) Epistemic:

Context: We pass by Kambere's house, and see that his lights are on. But we know that he's a very forgetful person and maybe he just left his lights on, so we don't know whether he is home or not.

```
# Kambere si-anga-bi-a eka yo lino
Kambere NEG-MOD-be-FV home PREP now
# "Kambere doesn't have to be at home now (for all we know)"
only available: "Kambere can't be home now."
```

(61) Deontic:

a. Context: A player has been hurt in a football match, and the ref is speaking to them, and saying:

```
nga-oko reglema yi-ka-bug-a, si-u-anga-sat-a COMP-COMP c9.rules SM.c9-TM-say-FV, NEG-SM.2sg-MOD-dance-FV "According to the rules, you can't play."
```

b. Context: Kabunga often plays football with his friends, but today he is tired and doesn't want to play. One of his friends tells him that it's ok, he

doesn't have to play today if he doesn't want to:

sí-u-anga-sat-a NEG-SM.2sg-MOD-dance-FV Intended: "you don't have to play"

So far then, *anga* looks rather like Nez Perce *o'qa*, variable in positive contexts, but when negated it can only be a negated existential modal, and can no longer receive a weak necessity interpretation.³ In that case, we expect *anga* to also be non-varying in other downward-entailing environments. But in all of those such contexts, *anga* in fact goes back to being variable. In the antecedent of a conditional, which licenses NPIs in Kinande (largely these occur as nominals without their augment, as in (62-63), *anga* show variability:

- (62) mo-nyi-te-ta-langir-a mukali TM-TM-TM.NEG-TM.NEG-see-FV c1.woman "I didn't see any woman."
- (63) O-mugalimu mo-a-te-ta-bug-a a-ti ha-li AUG.c1-teacher TM-SM.c1-TM.NEG-TM.NEG-say-FV SM.c1-C c16-be mukali w-a-ya-tsig-a c1.woman REL.c1-TM-go-swim-FV "The teacher didn't say that there is a woman who went swimming."
- (64) Yohani a-ma-anz-a **mukali**, inya kande-tsema (Progovac, 1993, John SM.c1-TM-like-FV c1.woman is FUT-happy ex.34)
 - "If John likes any woman, he/she will be happy"
- (65) a. Context: You are at a bookstore looking for books that are interesting to you to read. The worker here is giving you recommendations. She picks up a book from a shelf that's in German, but she says that it's a very good book and you should get it if you can read German.

u-a-ma-bi-a i-u-anga-na-som-a e-kitabu SM.2sg-TM-TM-be-FV C-SM.2sg-MOD-TM-read-FV AUG-c7.book

-

 $^{^3}$ See Chapter 3.2.2 for further discussion of the analysis of Nez Perce in Deal (2011), and its incompatibility with the Kinande data.

eki, kumbe i-u-a-ki-gul-a munabwire c7.this, better.that C-SM.2sg-TM-Om.c7-buy-FV today "If you can read this book, you should buy it today."

b. Context: You are shopping for books for school, and one of the workers at the bookstore is helping you find the books you need. As you are walking past a shelf of books, the worker picks up a book and explains that it frequently sells out, so if it's one of the books you are supposed to read, you should buy it today while they still have stock.

u-a-ma-bi-a i-u-anga-na-som-a e-kitabu SM.2sg-TM-TM-be-FV C-SM.2sg-MOD-TM-read-FV AUG-c7.book ekalhasi, kumbe eki kyo i-u-a-ki-gul-a **PREP** class, C-SM.2sg-TM-Om.c7-buy-FV c7.this better.that munabwire today If you should read this book for school, you should buy it today."

And similarly in the restrictor of a universal quantifier, *anga* remains variable:

- (66) obuli mwana o-yo -wa-bandan-a **mukali** every child AUG.c1-REL SM.c1-meet-FV c1.woman mo-a-mu-ha-ire e-ri-wa
 TM-SM.c1-OM.c1-give-TM AUG.c5-c5-flower
 "Every child who met a woman gave her a flower"
- (67) Context: Some of the children can read, and some cannot. The ones that can are all excited about reading books, and so they all bought books from the market. The ones who can't read, of course, did not spend their money on books.

obuli mwana o-yo wa-anga-na-som-a (e-kitabu) every child AUG.c1-REL SM.c1-MOD-TM-read-FV (AUG.c7-c7.book) mo-a-gul-ire kitabu ki-gima omo soko TM-SM.c1-buy-TM c7.book c7-one at c5.market "Every child who can read (a book) bought one book at the market"

(68) Context: The schoolchildren are all supposed to read a book sometime this month. Some of the children were good and did their reading last week. Some of them haven't done their reading yet, and so they still should read a

book, so those children went to the market and bought one to read.

```
obuli mwana o-yo wa-bi-a i-na-anga-na-som-a every c1.child AUG.c1-REL SM.c1-be-FV C-SM.c1-MOD-TM-read-FV mo-a-gul-ire kitabu ki-gima kyo omo soko TM-SM.c1-buy-TM c7.book c7-one c7.LNK at c5.market "Every child who should read bought a book at the market"
```

And even with negation, if a clause boundary intervenes between *anga* and negation, *anga*'s ambiguity returns:

(69) omugalimu mo-a-te-ta-bug-a a-ti Masika c1.teacher TM-SM.c1-TM.NEG-TM.NEG-say-FV SM.c1-that Masika a-anga-na-tsig-a SM.c1-MOD-TM-swim-FV "The teacher didn't say that Masika can/should swim"

This is rather unlike the behavior of *o'qa* as described in Deal (2011). *Anga* acts like a possibility modal with clausemate negation, but ambiguously everywhere else. Due to its behavior in (61), I propose that *anga* is underlyingly a possibility modal, and it is strengthened by some mechanism that can apply in all of the above examples except for the clausemate negation case. In Chapter 4 I will provide further argument for this, and argue that the strengthening mechanism is exhaustification, much like in Ecuadorian Siona (Jeretič, 2021a), although *anga* only strengthens to weak necessity, and not all the way to strong necessity as we see elsewhere.

2.6 Kinande within a typology of modals

Kinande's modal system is spread across morphosyntactic categories, and hardly any two morphemes even share a category. This stands in contrast with the way modality is described in many languages; it is much tidier to be able to sort modal morphemes into a common category, even if that category is verbs rather than syntactic modals. But it's not entirely unknown in broader descriptions of modal systems; in particular, van der Auwera & Plungian (1998) describe a number of ways that languages might develop modals from various other categories. On their framework, Kinande

developed modals out of potential markers, verbs with non-modal meanings, complementizers/adverbs, as well as borrowing modals from other languages. Since Kinande has taken all of these options, the system as a whole is relatively difficult to parse as far as interactions between modal morphemes are concerned.

Within descriptions of variable-force modals, Kinande differs by having a relatively full modal semantic space, while still having at least one variable-force modal. Most of the variable-force languages, as currently described, have modal semantic paradigms that are relatively sparse on the axis of modal force, and the density of modal morphemes seems to correlate inversely with the presence of variable-force modals. Kinande breaks this pattern by having a relatively densely-populated modal paradigm, but still having variable-force modals. At first glance it's not clear why this would be.

Overall, the typology of modals cross-linguistically suggests that it's precisely a sparse modal paradigm that allows both variable force and variable flavor; English modals are densely packed along the force axis, and so don't show much variable force, but do show variable flavor, as there are not as many modals with specified modal flavors within the same paradigm (leaving aside some of the more phrasal modal expressions such as *be able to*), there is a lot of "space" for variance in flavor. St'át'imcets, on the other hand, has a relatively densely populated modal flavor axis, but a sparsely populated modal force domain, so the variance is in modal force rather than flavor (Rullmann et al., 2008; Matthewson, 2016). Other languages yet, like Paciran Javanese (Vander Klok, 2008) are densely populated along both axes, and so show little to no variability on either axis.

So what do we do with Kinande, descriptively speaking? The variability along modal flavor is pervasive across the language, and not necessarily surprising. But the pattern of modal force is somewhat baffling. *Anga* is a variable-force modal, but a restricted one. It can only reach as high as weak necessity, and can't reach strong necessity. This itself is unattested elsewhere, and, if we follow the speculation from

above, would suggest that the strong necessity spot is taken, preventing *anga* from stretching that far.⁴ But the weak necessity portion of Kinande's paradigm is positively crowded. But it's crowded with variable-force morphemes. There's *anga*, variable on context, and then *kumbe* and *tolere* both vary based on their complement's mood marking. This suggests that none of these items are lexically specified as weak necessity modals, and perhaps the reason there are so many ways of deriving weak necessity meanings in Kinande is because it is an overlap point for various modals, but there's no dedicated lexical item. We will return to the implications of this for analyzing *anga* in Chapter 3, and to some further typological implications in Chapter 4.

Even after taking into account the possible theoretical explanations of why Kinande's weak necessity paradigm is the way it is, I would like to highlight some important facets of the description given here, and their implications for further descriptive work:

First, we see that investigating weak necessity is an important part of the modal paradigm; without eliciting weak necessity contexts, Kinande modals look much like English ones; there are distinct morphemes for strong necessity and possibilityc. But attention to finer details in Kinande modal contexts illustrates a number of interacting modal items, which will prove to be informative in developing our theory of Kinande modality, and our theory of modals cross-linguistically.

Second, we see the importance of further probing questions to distinguish types of variable-force modal systems from one another. Simple positive sentences don't suffice to give an adequate description of *anga*, and we will see in the next chapter that they certainly don't suffice for giving an accurate analysis. Description and theory are best hand-in-hand, and we shall see in the next chapter that most of these further descriptive facts are driven by theoretical questions, and further on in Chapter 4 we will see the implications of both the description here and the analysis of the Kinande

⁴See Chapter 3 for an implementation incorporating this intuition.

facts for a broader typology and understanding of modality.

Chapter 3

Analysis of anga

If Chapter 2 was the descriptive core of this dissertation, this chapter is the theoretical core. This is the chapter where I take those descriptive facts, put them in better relation with current modal theory, and show how they inform each other. Although the vast majority of the Kinande data brought up in this analysis has aleady been covered in Chapter 2, know that the chapter order certainly does not illustrate the investigatory order; the other analyses in the literature informed the initial investigation of Kinande's modal system, and the ongoing theoretical analysis refined the description as the description informed the analysis (Matthewson, 2013a).

In this chapter I argue that Kinande's modal system, and especially the behavior of the modal prefix *anga*, is best accounted for under an approach that combines traditional domain-restriction approaches to variable-force modality and more recent exhaustification-based approaches. Given the particulars of Kinande modality, and the limited variability of *anga*, none of the prior analyses alone are sufficient to capture *anga*'s behavior. However, by combining domain restriction of the modal with the same exhaustification mechanism that we use for scalar implicatures, *anga* is fully accounted for, not only in simple positive clauses, but in embedded clauses (whether downward-entailing or not), and we can also account for *anga*'s lack of variability with clausemate negation. At the same time, domain restriction and general pragmatic principles explain why *anga* is never interpreted as a strong necessity modal, but as weak necessity, at its strongest.

In the interests of keeping this chapter accessible regardless of your background

in formal semantics, the first part of this chapter provides background on theory of modals in Section 3.1, and a theoretically-oriented background for variable-force modals specifically in Section 3.2, along with discussion of where the accounts do and do not line up with the Kinande facts. If you are already comfortable with current theories of modality, feel free to skip directly to Section 3.2, where the actual analytical comparison begins with reference to Kinande. Then in Section 3.3 I dive into the nitty-gritty of Kinande *anga* as a variable-force modal, and to a lesser extent the semantics of other Kinande modal morphemes. Section 3.4 discusses the implications for the theories of exhaustification and modality. Section 3.5 briefly argues against some of the other, less-discussed, analytical options for variable-force modals, and finally Section 3.6 sets the stage for the typological discussion that will come in Chapter 4.

3.1 Background: Quantificational accounts of modals

The accounts of modality discussed in this chapter (as well as the account I shall offer) are all generally couched in a Kratzerian theory of modality (Kratzer, 1981;?). Not because this is the only theory of modals available, but because it is as of this writing the most-developed theory, and currently the best situated for application to less-studied languages. If you already have a working knowledge of this kind of framework, you can safely skip the first subsection. If, however, you are unfamiliar with the semantics of modals, or indeed with much formal semantics at all, that section lays out the basic assumptions and workings of the system. The following section then gives the background specifically for analyses of variable-force modals, and begins to explore the ways that Kinande's modal system can inform our understanding of those accounts.

Modals are traditionally taken to be quantifiers over possible worlds, much like *some* and *every* are quantifiers over individuals; for a proposition p, *possibly* p asserts

that there is some (relevant) world in which p is true, and *necessarily* p asserts that every (relevant) world is one where p is true.

However, the parenthetical *relevant* is doing a lot of work. Obviously when I say *Sarah must be home*, I don't mean that she's home in every single world, imaginable and unimaginable. I just mean that, considering all of the relevant facts that are true in our world, all of the possibilities that take those facts into account are worlds where Sarah is home. So we add some notion of accessible worlds, and each world has a (potentially distinct) set of worlds that it can access for a given modal flavor.¹

With what we have so far, we can give English *can* and *must* definitions like the following:

- (70) $[\![can\ p\]\!]^w = \exists w'[w' \in \mathrm{Acc}(w)\ \&\ p(w')]$ READ AS: There is some world w' that is accessible from world w such that p is true in w'.
- (71) $[\![must\ p\,]\!]^w = \forall w'[w' \in \mathrm{Acc}(w) \to p(w')]$ READ AS: For all worlds w' that are accessible from w,p is true in each of those worlds.

These definitions can get us through some productive conversations about modals, especially if we relativize the accessibility relation for modal flavor. That move is easy enough, you can just define a set of functions, and for each one, when you give it a possible world w, it will return a set of possible worlds that are related to what is known about w (for an epistemic modal), or perhaps what the speaker's goals are in w (for a teleological modal).

The different quantifiers give us a contrast in **modal force**; the different accessibility relations give us a contrast in **modal flavor**. We can get a fair distance with these relatively simple definitions: Defining modals as vague or ambiguous with regard to their modal flavor, we can explain why modals like *must* can be read as epistemic

¹The relation is rather more nuanced than this, but we will get to that.

(expressing what has to be true given what we know about the world), or as deontic (expressing obligations given the circumstances and laws of the world). We can explain how a parent can tell their child "If you don't have to go to the restroom, you don't have to go to the restroom" without being tautological. It's all just a matter of different accessibility relations on what's otherwise the same modal.

Facts from Kratzer (1977) illustrate that we need a more nuanced understanding of accessibility relations, however. Kratzer proposes that we split the accessibility relation into two parts, a modal base and an ordering source. The modal base is a function from a world to a set of propositions that encodes knowledge or beliefs about that world (in the case of epistemic modals), or facts about the circumstances of that world (for other kinds of modals, sometimes called root or circumstantial modals). The ordering source is similarly a function from a world to a set of propositions that encodes things like rules and regulations (for deontic modals), goals (for teleological modals), or facts and inferences that are normally true, though not universally (for epistemic modals and some others); the ordering source then uses that set of propositions to rank the worlds of the modal base according to how well they conform to that set. The modal base defines a set of worlds where every proposition in the modal base is true, and then the ordering source ranks those worlds according to how well they conform to the propositions in that ordering source. Finally, the modal itself only quantifies over the best (highest-ranked) worlds; they will all be worlds where every single proposition in the modal base is true, but it's entirely possible that no world in the modal base also satisfies every single proposition in the ordering source: the two may conflict with each other. More formal definitions are as follows, adapted from Kratzer (1981):

(72)
$$[\![MB_{circ}]\!] = \lambda w. \{p \mid p \text{ is true in } w\}$$
 $\langle w, \langle \langle w, t \rangle, t \rangle \rangle$

 $[[Best(OS_{deon})]] = \lambda w. \lambda W. \lambda w'. w' \text{ is among the highest-ranked worlds in } W,$ where for all worlds y and $z \in W$, y is higher-ranked than z iff $\{p \mid p \in OS(w) \& p \text{ is true in } z\} \subseteq \{p \mid p \in OS(w) \& p \text{ is true in } y\}$ $\langle w, \langle \langle w, t \rangle, \langle w, t \rangle \rangle \rangle$

Why use a circumstantial modal base for a deontic modal? Although deontic modals express what we're obligated or allowed to do, rather than what we in fact do, they are still beholden to the facts of the situation. For example: in the actual world (w_0) , I jaywalked one time. The law states that jaywalkers must pay a fine. It also states other things, like that no one should jaywalk, and no one should commit murder. Our modal base (MB) and ordering source (OS) look something like this, then:²

$$j = ext{Lydia jaywalked}$$
 $ext{MB}_{circ}(w_0) = \{j\}$ $f = ext{Jaywalkers pay a fine}$ $ext{OS}_{deon}(w_0) = \{\neg j, \neg m, f\}$ $m = ext{Murder occurs}$

The circumstances of the actual world include j, the fact that I jaywalked. The law insists that jaywalking shouldn't happen $(\neg j)$, neither should murder $(\neg m)$, and also that jaywalkers must pay a fine (f).

The modal base provides the set of all possible worlds where I jaywalked, excluding any worlds where I did not. So unfortunately for our ordering source, it cannot satisfy all of its propositions; it only has access to the worlds that the modal base gives it. But it can take all of these j-worlds, currently a mixed bag of whether or not I pay a fine, and whether or not there is any murder, and rank them according to how well they conform to the rest of the laws it cares about. So the highest-ranked worlds will still be j-worlds, but they will also be $\neg m$ -worlds and f-worlds: Worlds where nobody murders anyone, and where I pay a fine for jaywalking. The modal itself only quantifies over these top-ranked worlds, ignoring all of the lower-ranked ones. So we can truthfully say the following:³

(74) (Since Lydia jaywalked,) she must pay a fine.

²This example is adapted from Kratzer (1977) on inconsistent premise sets.

³The denotation in (74) incorporates the Best function from ?, which assumes that there is always at least one best-ranked world, an assumption that I also adopt here.

$$(74') \quad \forall w'[w' \in \text{Best}(OS_{deon}(w_0)(\bigcap(MB_{circ}(w_0)))(w')) \rightarrow f(w')]$$

READ: In all worlds w' such that w' is among the best-ranked worlds as defined by the deontic ordering source and circumstantial modal base, Lydia pays a fine in those worlds.

Splitting the accessibility relation like this gets us several nice benefits: For one, it resolves issues around conflicts in the modal base and ordering source (such as a claim that there should be no jaywalkers and a claim that all jaywalkers must pay a fine), while letting us prioritize some over others. The hard constraints on our interpretation go in the modal base, while the soft constraints go in the ordering source, and by and large things then work as expected: the hard constraints are always honored, and as many soft constraints are honored as well as possible.

If there are two worlds w_1, w_2 such that in both of them, all but one proposition in the ordering source is true, but they differ on which proposition in the ordering source they fail to satisfy, they will both be ranked equally, with resulting effects on the felicity of both possibility and necessity modals over that set: if w_1 and w_2 are the only two best-ranked worlds, then if one of them is a p-world and the other is not, we should be able to say "possibly p", but not "necessarily p". If both of them are p-worlds, "necessarily p" will be felicitous. And of course, if neither one is a p-world, then neither "necessarily p" nor "possibly p" will be felicitous. All of this is without regard to how many of the worlds in the rest of the modal base are p-worlds or not: The strength of our assumptions in an ordering source can pick out what otherwise seem like quite improbable candidates, if we were to look at pure proportion of the set of worlds. What really matters is what we believe and what our priorities are.

Another nice effect is that because the modal base and ordering source consist of propositions, we can see now why they tend to be relatively fuzzy and subject to disagreement between speakers; you can disagree on what propositions are relevant for a given modal claim, even among the true ones (Abusch, 2012). This extends naturally to our intuition that modal flavors are relatively vague, compared to modal force.

Even if a general kind of modal base or ordering source is specified in a lexical entry (such as English *might*, which strongly favors epistemic interpretations), the propositions that are actually selected and put into the modal base and ordering source are still dependent on the discourse and real-world context, which leaves significant room for vagueness.

Last but not least, splitting the modal base and ordering source like this gives us a way of analyzing weak necessity modals. Adding an ordering source already strengthens *can* and weakens *must*; *can* no longer says merely that some world somewhere is a *p*-world, it says that one closely related to our world is. And *must* no longer says that every world anywhere is a *p*-world, or even every world in the modal base. Instead it says that all of the best worlds are *p*-worlds, though non-*p*-worlds may still be possible, though lower-ranked.

Noting that domain restriction weakens a universal quantifier, von Fintel & Iatridou (2008) use further domain restriction to give an analysis of weak necessity modals. Using a weak necessity modal instead of a strong one indicates that you are bringing in some additional assumptions or considerations than were previously considered relevant, and using those additional assumptions to further narrow the set of worlds. Logically speaking, the result is a weaker modal statement. Linguistically, however, the main point is to signal those additional propositions coming into play. As analyzed by von Fintel & Iatridou, and further developed by Rubinstein (2012), weak necessity modals involve a second ordering source that uses the additional propositions it contains to take the best worlds according to the first ordering source, and rank them according to the new set of propositions. The result is a kind of "best of the best" set of worlds, sorted according to propositions that matter less or are less relevant (or agreed upon) than even the soft constraints of the first ordering source.

- (75) (Since Lydia jaywalked,) she should pay a fine.
- (75') $\forall w'[w' \in \text{Best}(OS_{deon2}(w')(\text{Best}((OS_{deon1}(w')(\bigcap(MB_{circ}(w_0)))(w')))))) \rightarrow f(w')]$ READ: In all worlds w' such that w' is among the best-ranked worlds as defined

by both deontic ordering sources and circumstantial modal base, Lydia pays a fine in w'.

The second ordering source, much like the first one, can't bring any new worlds into consideration; it's stuck with the best ones from the first ordering source, just like the first ordering source is stuck with the worlds the modal base gives it. But the second ordering source can break ties among those best worlds, by adding in some additional preferences of the speaker, or more assumptions about what is usually the case, or so on. On the analysis given in Rubinstein (2012), the first ordering source consists of propositions that are presupposed to be shared information among the discourse participants, and therefore relatively firm assumptions. The second ordering source, on the other hand, consists of propositions that are presupposed not to be shared information, either because the negation of that proposition is taken to be true (resulting in counterfactual interpretations), or because it's taken to not be a shared priority between the discourse participants. If that same consideration is considered shared by all the participants in the context, then the weak necessity version is infelicitous, and the strong necessity sentence instead is felicitous.

- (76) Context: Rachel is coming to the United States next summer. It is now illegal not to have health insurance in the US. (Rubinstein, 2012, p. 3)
 - a. (Rachel will be a graduate student in a respectable university.)

 She has to (/² should) get health insurance.
 - b. (Rachel will be employed illegally, selling cheap jewelry at a shopping mall.)

She should ($/^{\text{false}}$ has to) get health insurance.

In the case of (76a), the shared assumptions of the interlocutors include propositions that Rachel needs to follow all of the laws, and that her employer will ensure that she does so with regard to health insurance, as well as that it is generally good to get health insurance even if it is costly, etc. In the case of (76b) however, not all of those

assumptions are clearly shared by both interlocutors, and *should* is used because the speaker is bringing in some of those non-shared assumptions.

Note, though, that the actual set of worlds each modal quantifies over might be identical; the logical weakening is a side effect that may not persist across contexts. The morphological marking for weak necessity doesn't directly track that weakening; instead it tracks with the presence of non-shared assumptions and priorities. Maybe the domain of quantification is ultimately the same in (76a) and (76b); but the use of *should* is required by bringing up the non-shared priorities.

Some other analyses proposed for weak necessity modals include scalar analyses and probability-based analyses (e.g., Lassiter, 2017; Yalcin, 2007). I will not be able to address these analyses in depth, but will touch on them in Section 3.5.

To sum up the main points of this section: Modals are quantifiers over possible worlds, and the domain of that quantification is contextually restricted in a variety of ways, reflecting a speaker's priorities in their reasoning. Differences in modal force are mostly gotten through using an existential vs. a universal quantifier, although some finer gradations can be found through additional domain restriction on the modal.

In the next section we'll see how variable-force modals complicate this system further, and explore a few ways that the literature has tried to account for them.

3.2 Variable-force modals

An understanding of a theoretical framework for modals in hand, we can turn now to how variable-force modals have been analyzed, specifically. In this section I review those analyses, as well as the natural language data that motivated them. As we go I will also discuss how Kinande's modal system poses problems for each analysis, before I go on to present my own proposal in Section 3.3.

Descriptively speaking, a variable-force modal language poses immediate problems for the quantificational modal framework discussed above. If a language has a modal that sometimes acts like a necessity modal, and sometimes like a possibility modal, how do we decide which quantifier it has in its lexical entry? And if there is any systematicity to when that modal means what, how do we capture that in its semantics?

Generally speaking, there are three extant types of analysis for variable-force modals in the literature: A domain-restriction analysis, a pragmatic scaleless approach, and a grammaticalized scaleless approach. I'll explain each one in turn, illustrating where it can and cannot account for the Kinande facts as I go.

3.2.1 Domain-restriction approaches

Rullmann et al. (2008) provide the first analysis of variable-force modals in the literature, after establishing their existence in St'át'imcets (Salish, British Columbia). St'át'imcets modal clitics appear to be specified for their modal flavor, but unspecified for their modal force, making St'át'imcets the inverse of English in this regard:

	Epistemic	Deontic	Circumstantial	Future
Strong Necessity	k'a	ka	ka	kelh
Weak necessity	k'a	ka	ka	kelh
Possibility	k'a	ka	ka	kelh

Table 3.1: St'át'imcets modal morphemes

(77) St'át'imcets (Rullmann et al., 2008, p. 331):

Context 1: The cougar was on a rampage and was killing cats, dogs, and racoons, and it had a child cornered and was growling; it would have killed a child.

Context 2: You just know that sometimes cougars kill children when they venture into built-up areas; it could have killed a child.

zúqw-s-as ka ta sk'úk'wmi7t-a ti7 ku swúw'a lh-cw7áoz-as die-caus-3erg irr det child-det det cougar comp-neg-3conj kw s-qus-cit-ítas det nom-shoot-ind-3pl.erg 'That cougar could/would have killed a child if they hadn't shot it.'

Depending on the context, the same modal morpheme (in this example the modal particle ka) can be understood as either a possibility modal or as a necessity one. The variation is purely contextual, rather than triggered by any morphology around the modal itself. The question is how to achieve this extreme context-dependence without positing lexical ambiguity across the entire St'át'imcets lexicon.

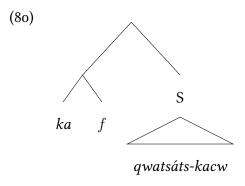
Rullmann et al. (2008), (and Peterson, 2010; Bochnak, 2015; Menzies, 2013, in the same vein) analyze the variance in variable-force modals as due to variable domain restriction. Capitalizing on the fact that restricting the domain of a universal quantifier creates a logically weaker proposition, and as Peterson (2010) notes, doing likewise for an existential quantifier creates a logically stronger one, Rullmann et al. (2008) propose that St'át'imcets modals are underlying universal quantifiers, but that they contain a subset function⁴ over their modal base which contextually selects a subset of the worlds provided by that modal base. The result is something similar to an ordering source, and in fact subsequent analyses (Peterson, 2010; ?) in this vein replace the subset function with an ordering source.

(78) Schematic modal definition: (Rullmann et al., 2008, ex. 53) $\| \text{ Modal } \|^{c,w} \text{ is only defined if } c \text{ provides a modal base } B.$ $\| \text{ Modal } \|^{c,w} = \lambda f_{\langle st,st \rangle}.\lambda p_{\langle s,t \rangle}.\forall w'[w' \in f(B(w)) \to p(w')]$

The subset function f occurs in the syntax as a complement to the modal, before the modal combines with the prejacent. An example of the syntax is below (from Rullmann et al., 2008, 51-52):

⁴Rullmann et al. (2008) call theirs a choice function.

(79) qwatsáts-kacw ka leave-2sg.subj deon '(Maybe) you should leave' (St'át'imcets)



The schematic definition in (78) is intended to be adjusted for each modal in St'át'imcets by specifying the kind of modal base that must be provided. This is to account straightforwardly for the fact that St'át'imcets modals are lexically specified for modal flavor, but the modal base could also be left unspecified for lexical entries that do not require any particular modal base. (This is what Bochnak, 2015, does to account for Washo variable-force modals.) However, one concern here (and with Peterson's analysis below) is that, as discussed in Section 3.1, modals cross-linguistically employ domain restriction, regardless of whether they are variable-force. I will return to this objection below.

Peterson (2010) takes a similar approach for Gitksan (Tsimshianic, British Columbia) epistemic modals =ima and $=\underline{k}at$, with two important differences: First, he takes the variable-force modals in Gitksan to be underlyingly possibility modals, rather than necessity modals, and so they must existentially quantify over possible worlds, rather than universally. Existential quantifiers are the inverse of universal ones in terms of strength and domain restriction: Where universal quantifiers get weaker as their domain is more and more restricted, existential quantifiers get stronger as their domain of quantification is restricted. Some linguistics students work hard is a stronger claim (more informative) than some students work hard. This means that straightforwardly applying the analysis from Rullmann et al. (2008) to Gitksan,

but switching the underlying quantification force, results in the subset function being able to strengthen the existential modal, rather than weaken it. This is not unwelcome; it means that if the subset function is "inactive", returning the exact set that it's given, the modal will get its default reading like we might expect a possibility modal in English to get. It's only if the subset function is "active" and selects a subset of the worlds it is given, that the modal force is weakened (in St'át'imcets) or strengthened (in Gitksan). But *modulo* the underlying force, the effect of this single mechanism is the same: we find a variable-force modal, affected by some context-dependent parameter.

Peterson's second difference from the original St'át'imcets analysis is that he uses a Kratzerian ordering source instead of a subset function to add the additional restriction. As ordering sources are already included in the modal system that Peterson uses, and they already serve the purpose of restricting the domain of modal quantification, this is arguably a more parsimonious method of obtaining variable force. In fact, Peterson argues that once you include an ordering source in the denotation of these modals, the subset function analysis in fact is equivalent to an ordering source analysis, except that the subset function is less constrained.

The division of labor between the modal base and ordering source is something like this: the inferential modal base provides evidence from the world, such as someone's truck missing from the driveway, or it being fishing season. The ordering source g(w) could contain additional assumptions or inferences about the world, such as the fact that John enjoys fishing, so you think it's likely that he went fishing. If the speaker chooses to bring in any of these additional assumptions, they go into the ordering source. If they are present, then the domain of quantification for the

existential quantifier is more restricted, and so stronger. However, if the speaker doesn't bring in any additional knowledge or assumptions, the modal remains weak, like an English possibility modal.

(82) yugwimahl dim iixwt (Gitksan, Peterson, 2010, p. 63) yukw-ima-hl iixw-(t)=s John PROG=MOD=CND fish-3=CND John "John might be fishing." ≻ "John must be fishing."

Using an ordering source rather than a subset function as the domain restrictor ties the variable-force mechanism to something familiar and more constrained: We already have ideas about what ordering sources do in modal constructions, and so we can attempt to draw on that literature to constrain what ordering sources do in variable-force contexts. Unfortunately, this brings its own problems. After all, languages without variable-force modals use a variety of ordering sources, that can have a variety of contents that are contextually supplied. These different ordering sources, in English as well as Gitksan (and throughout many other languages where modals have been studied) by nature provide variable amounts of domain restriction.

Once this change is made, variable-force modals in this analysis look precisely like English modals, just potentially with a lexically-specified modal base. Which raises the question: Why aren't English modals variable-force?

One option would be to say that they are, or would be, except that the English lexical space for modals is so crowded that it's difficult or impossible to see. If the accepted context doesn't support using *must* in its strong form, we tend to use *should* to overtly signal the weakening. And likewise, if we use domain restriction to weaken *must* (or *should*, for that matter) to the equivalent of a possibility modal, then using *can* is more informative, because it's overtly weak.

The problem is, it's not clear how to arrange the pragmatic scales for this to work; a necessity modal only properly collapses into a possibility one if its domain is restricted to a singleton set. If possibility modals draw a scalar implicature of "not necessarily" (and they seem to), then in possibility contexts where we might be able

to use a domain-restricted necessity modal, the possibility modal won't be able to draw that scalar implicature, because the possibility and necessity modals have become semantically equivalent for that domain. So we expect contexts where both are acceptable, without a change in meaning. So far this doesn't seem to be the case in English.

Another issue is that it doesn't seem like this extreme logical weakening and strengthening will actually get us the meanings we're after. If the kinds of context restrictions we see are based on human reasoning, we should expect to find some relatively natural assumptions or conditions to add to the ordering source to arrive at that interpretation; there should be a way of reasoning naturally to the level of restriction that we need. It's possible to reason around epistemic possibility and necessity being on a common scale of probability: if something is highly probable or certain, it's true in more worlds (i.e., you can truthfully state a universal with less stipulation), then you can use *must*. If there's at least one world in the domain, you can use might. It seems you could plausibly get from must to might by concatenating enough stipulations to arrive at a singleton set of worlds, or at least far enough that while every world in that restricted set is a p-world. However, that set is still far from all of the worlds in the less-restricted set. More worryingly for an epistemic interpretation, we run the risk of excluding the actual world from the domain of quantification with some of those additional restrictions, which seems to take us away from the meaning of *must* that we are trying to derive. In short, there are many ways to narrow down a set of worlds, but not all of them achieve the exact narrowed set that would make a *must* statement true and felicitous. In order to keep the actual world in the domain of quantification even as we add restrictions, all of the stipulations we add must be true, reliable information. Only then can we arrive at a set where the *must* statement would be true.

This intuition doesn't translate well to non-epistemic modals, though. Consider deontic necessity and possibility, or more naturally, obligation and permission. The

kinds of extra conditions we should add to restrict our domain are some facts of the world, perhaps, but mostly we're adding conditions on what the rules and regulations ask of us; if there is some world where we obey all the rules and regulations and still do a particular thing in at least one of those worlds, then we are allowed to do it: our obligations do not rule it out. If all worlds where we obey all those rules and regulations are worlds where we take a particular course of action, then we're obligated to take that course of action. But how do we translate from one to the other via domain restriction? Adding restrictions to a permission doesn't seem to turn it into an obligation; it just means that permission is very permissible. It's difficult to construct a reasonable deontic requirement for me not to blink, for example; I circumstantially have to blink as part of the functioning of my eyes, but no law of society says that I have to, either. It's an extremely permissible action for me to take, and it's possible that all of the worlds in which I fulfill my obligations are worlds in which I continue to blink regularly. Yet it's still odd to say that I am (deontically) obligated to blink, though it's perfectly felicitous (though obvious) to say that I am allowed to blink.

Conversely, we can try to go from an obligation to a permission, but the extra conditions we would have to add seem quite unnatural: If we want *Perry must go play a game with their friends* to be weakened from an obligation to a permission, what extra stipulations do we add? That Perry has to if they want to? That still doesn't seem true, they're still free to choose something else, whatever their desires. Perhaps if Perry wants to play a game with their friends and if they don't want to do anything else, then they have to? Perhaps, but it seems like Perry has permission to sit and do nothing, as well. We don't ever seem to arrive at a meaning that denotes permission rather than obligation.

The notions of permission and obligation, though related, apparently don't share a scale in quite the right way. Very weak obligations don't become permissions the way that weak probabilities become possibilities, and very persistent permissions don't become obligations, the way we might think persistent possibilities might become probabilities. Domain restriction can't get us fully variable-force modals. I will, however, make extensive use of domain restriction in the analysis I eventually provide, and will return to an account of these languages in Chapter 4. Domain restriction plays an important part in modal force variation, even though it cannot be the be-all and end-all.

3.2.2 Scaleless pragmatic accounts

Deal (2011) analyzes the Nez Perce (Sahaptian, Columbia River plateau) modal *o'qa*, which is used in both possibility and necessity contexts as well:

(83) 'inéhne-no'qa 'ee kii lepít cíikan. (Deal, 2011, ex. 1) take-MOD you DEM two blanket 'You can/should take these two blankets'

Rather than propose that o'qa uses domain restriction to achieve variable force, Deal analyzes o'qa as not being variable at all: It's simply a possibility modal in a language that has no necessity modals. Since there's no pragmatic competitor in the modal domain, o'qa doesn't draw a scalar implicature of "not necessarily", and so remains compatible with necessity contexts. After all, if I am obligated to do something, then I am also allowed to do that thing. Cross-linguistically, the usual infelicity of possibility modals in necessity contexts is due to a scalar implicature, which, however you derive it, depends on there being a stronger option (a necessity modal) to compare o'qa to.

Deal's analysis is supported by *o'qa*'s behavior in downward-entailing contexts. As soon as *o'qa* is embedded under negation, or in the antecedent of a conditional, or in the restrictor of a universal quantifier, its variability vanishes:

(84) Context: The referee is talking to an injured player.

tamáalwit-wecet wéet'u 'ee x̂eeleewi-yo'qa 'étke k'omáy'c 'ee wee-s-∅
rule-reason not you play-мор because hurt you be-P-PRES

'áatim.

arm

'According to the rules, you can't play, because your arm is injured.'

(Nez Perce, Deal, 2011, ex. 46)

(85) Context: You are explaining to someone who thinks they have to leave that they are not in fact required to do so. It's not necessary for them to leave.

#wéet'u 'ee kiy-ó'qa.

(Nez Perce, Deal, 2011, ex. 49)

not you go-MOD

Consultant: 'That's a different conversation, not this one. You're just saying wéet'u 'ee kiyô'qa, "you can't go".'

This behavior makes sense if *o'qa* is a possibility modal without any special domain restriction. Downward-entailing contexts reverse the relative strengths of existential and universal quantifiers; "not some (= no)" entails "not all", and parallel for other downward-entailing contexts. Since the negated possibility modal has such a strong interpretation, it's infelicitous in the (relatively weak) negated necessity contexts.

Deal's account works well enough for Nez Perce, and might even extend to some of the languages previously analyzed with domain restriction accounts, though it is difficult to test because the modals in St'át'imcets and Gitksan are resistant to being in downward-entailing contexts (Rullmann et al., 2008; Peterson, 2010). But unfortunately it will not extend to Kinande. *Anga*, Kinande's variable-force modal, is not only variable in simple positive sentences, but also in antecedents of conditionals:

(86) Context: You are at a bookstore looking for books that are interesting to you to read. The worker here is giving you recommendations. She picks up a book from a shelf that's in German, but she says that it's a very good book and you should get it if you can read German.

wamábyá íwanganásóm' ekitábw' ekí, u-a-ma-bi-a i-u-anga-na-som-a e-kitabu eki, SM.2sg-TM-TM-be-FV C-SM.2sg-MOD-TM-read-FV AUG-c7.book c7.this, kúmbé íwakígúlá munábwíre kumbe i-u-a-ki-gul-a munabwire better.that C-SM.2sg-TM-OM.c7-buy-FV today "If you can read this book, you should buy it today."

(87) Context: You are shopping for books for school, and one of the workers at the bookstore is helping you find the books you need. As you are walking past a shelf of books, the worker picks up a book and explains that it frequently sells out, so if it's one of the books you are supposed to read, you should buy it today while they still have stock. You could probably get by in the class without it, but you're more likely to do well if you have this book.

ekitábw' ekí, wamábyá íwanganásóm' u-a-ma-bi-a i-u-anga-na-som-a e-kitabu eki, SM.2sg-TM-TM-be-FV C-SM.2sg-MOD-TM-read-FV AUG-c7.book c7.this, kúmbé íwakígúlá munábwíre kumbe i-u-a-ki-gul-a munabwire better.that C-SM.2sg-TM-OM.c7-buy-FV today 'If you should read this book for school, you should buy it today."

It's also variable in the restrictor of a universal quantifier:

(88) Context: Masika is a very avid reader; she always has a book in her hands. She doesn't particularly follow recommendations or anything, she just reads every book she can get her hands on, as quickly as she can.

Masiká akásomá óbuli kitábu Masika a-ka-som-a obuli kitabu Masika SM.c1-TM-read-FV every c7.book ekyánganásóma

e-kyo-a-anga-na-som-a

AUG-c7.REL-SM.c1-MOD-TM-read-FV

"Masika reads every book that she can read."

(89) Context: Some celebrities published a list of "1,000 books that everyone should read before they die". Since then, Swera has been reading the books on that list, and only those books. She intends to read every single book on that list whenever she can get her hands on a copy, and she refuses to spend her time reading anything else.

Swera	akásomá	óbuli	kitábu			
Swera	a-ka-som-a	obuli	kitabu			
Swera	SM.c1-TM-read-FV	every	c7.book			
ekyánganásóma						
e-kyo-a-anga-na-som-a						
AUG-c7.REL-SM.c1-MOD-TM-read-FV						
"Swera reads every book that she should read"						

And even under negation, so long as that negation is not in the same clause as *anga*, we see variability:

Másiká (90) omugalímu mwátétábuga ati omugalimu mo-a-te-ta-bug-a a-ti Masika TM-SM.c1-TM.NEG-TM.NEG-say-FV c1.teacher SM.c1-that Masika anganátsíga a-anga-na-tsig-a SM.c1-MOD-TM-swim-FV "The teacher didn't say that Masika can/should swim"

In fact, the only construction that blocks *anga*'s variability is when it occurs with clausemate negation:

- (91) nga-oko reglema yi-ka-bug-a, si-u-anga-sat-a, COMP-COMP c9.rules SM.c9-TM-say-FV, NEG-SM.2sg-MOD-dance-FV kundi w-oyo u-kwa-ire oko ku-boko because 2sg-REPLRO SM.2sg-hurt-TM c17 c15-arm "According to the rules, you can't play because your arm is injured."
- (92) Context: Kabunga often plays football with his friends, but today he is tired and doesn't want to play. One of his friends tells him that it's ok, he doesn't have to play today if he doesn't want to:

```
# sí-u-anga-sat-a
NEG-SM.2sg-MOD-dance-FV
Intended: "you don't have to play"
```

The persistent variability of *anga* indicates that we will need a different type of analysis for Kinande's modal system than for Nez Perce's.

3.2.3 Exhaustification accounts

Jeretič (2021a) (and similarly Staniszewski, 2020, for english *should*) recently proposed an exhaustification-based analysis for deriving variable-force modals in Ecuadorian Siona, and Jeretič's analysis suggests a way forward for Kinande. Like Nez Perce, Siona has no necessity modal, only possibility modals. Unlike Nez Perce, but like Kinande, Siona *ba'iji* is variable in downward-entailing contexts:

```
(93) Sai-ye ba-'i-to, sa-si'i (Jeretič, 2021a, ex. 7-8) go-inf be-ipf-cond go-fut-oth 'If I can/must go, I will go'
```

But *ba'iji* is invariable in simplex clauses, where it has a strong necessity interpretation; for a possibility interpretation, *deoji* is used instead:

```
(94) Sai-ye ba-'i-ji (Jeretič, 2021a, ex. 1) go-INF be-IPF-3S 'We must go.' (lit., 'there is to go')
```

```
(95) Sai-ye de'o-ji (Jeretič, 2021a, ex. 2) go-INF good-3s 'We can go.' (lit., 'it is good to go')
```

And like Kinande, *ba'iji* is also invariable under clausemate negation:

```
(96) Sai-ye beo-ji
go-INF NEG.be-3s
'We mustn't go.'

# 'We don't have to go.'

(Jeretič, 2021a, ex. 5)
(Jeretič, 2021a, ex. 5)
```

Jeretič explains this peculiar pattern by analyzing Siona *ba'iji* as a possibility modal subject to exhaustification, drawing from Bowler (2014); Bar-Lev & Margulis (2014), among others to derive the strengthened meanings of *ba'iji* using the same mechanism that derives scalar implicatures when a stronger alternative is present in the grammar. Exhaustification is a semantic mechanism proposed (Fox, 2007; Bar-Lev & Fox, 2017; Chierchia et al., 2012) in order to account for scalar implicatures within the grammar, especially within embedded clauses. Essentially, the exhaustification

operator (exh) is a covert instance of $only^5$ that can occur on the left edge of a clause, where it contributes the meaning that while the proposition that it modifies is true, only that proposition is true, and none of its alternatives are true, so long as you can safely deny those alternatives without contradicting the original proposition.

- (97) Exhaustification operator with Innocent Exclusion (adapted from Fox, 2007):
 - a. $[exh](ALT)(p)(w) = p(w) \land \forall (q) \in IE(p, ALT)[\neg q(w)]$
 - b. $IE(p,ALT)=\bigcap\{C'\subseteq ALT:C' \text{ is a maximal subset of ALT, s.t. } \{\neg q:q\in C'\}\cup\{p\} \text{ is consistent.}\},$ where

ALT is the set of alternatives to p.

The exh operator takes a proposition, a world, and a contextually- and/or lexically-defined set of alternative propositions—other things the speaker could have said. Those alternatives can be derived, for example, by replacing specific lexical items with stronger versions of that item within the same category (e.g., replacing some with every). The exh operator then asserts p, passing along the original proposition, and then also asserts the negation of every proposition that can be innocently excluded. That is, for every proposition out of the alternative set that is not entailed by p, and also not entailed by p plus the negation of any combination of the other propositions in C. The innocently excludable alternatives are the alternatives that we can safely declare false without risking any contradiction with p.

Applying exh to English can thus derives a scalar implicature that negates the must version of the proposition, giving us the meaning "you can, but you don't have to", as expected. But because Siona ba'iji has no stronger scalemate, there is nothing for exh to negate when applied to a proposition with ba'iji. So no scalar implicature is derived.

 $^{^5}$ At least, the version of exh as originally proposed, using Innocent Exclusion but not Innocent Inclusion, is equivalent to a covert only.

That's not enough to strengthen *ba'iji* to a necessity modal, Jeretič needs the sub-domain alternatives of *ba'iji*, not merely its scalar alternatives. Subdomain alternatives (first proposed in Chierchia, 2013, for NPIs) are obligatorily triggered for some lexical items, and optionally for others, and are alternatives formed not by altering lexical items in a proposition, but by creating propositions corresponding to each possible subset of the domain of a lexical item. For *ba'iji*, the subdomain alternatives are the powerset of the set of worlds accessible to *ba'iji*, here with a toy model of three worlds:

(98)
$$[\![ba'iji]\!] = \exists w'[w' \in \{w_1, w_2, w_3\} \land p(w')]$$

(99) $ALT = \{\exists w'[w' \in \{w_1, w_2, w_3\} \land p(w')] \}$
 $\exists w'[w' \in \{w_1, w_2\} \land p(w')]$
 $\exists w'[w' \in \{w_2, w_3\} \land p(w')]$
 $\exists w'[w' \in \{w_1, w_3\} \land p(w')]$
 $\exists w'[w' \in \{w_1\} \land p(w')]$
 $\exists w'[w' \in \{w_2\} \land p(w')]$
 $\exists w'[w' \in \{w_3\} \land p(w')]$

If we add these to the alternative set for ba'iji and exhaustify, well, still nothing happens. None of the subdomain alternatives of ba'iji are innocently excludable (because while any single world could be excluded, you can't exclude them all at once while preserving the possibility claim, so none of them are in every possible consistent subset of alternatives). Jeretič follows Bowler (2014) and Fox (2007) in applying exh recursively to derive a strengthened meaning, but for the sake of clarity I will instead show the derivation using exhaustification with innocent inclusion from ?, which arrives at the same result with a modified exh operator:

 $^{^6}$ As far as I have been able to determine, there are no predictive differences between applying exh^{IE} twice and applying exh^{IE+II} once, at least for Siona or Kinande. I use Innocent Inclusion here for the tidier notation, but all these derivations will work with recursive exhaustification without

- (100) Exhaustification with Innocent Inclusion (adapted from ?):
 - a. $\llbracket exh \rrbracket (ALT)(p)(w) = \forall q \in IE(p,ALT)[\neg q(w)] \land \forall r \in II(p,ALT)[r(w)]$
 - b. $IE(p,ALT)=\bigcap\{C'\subseteq ALT:C' \text{ is a maximal subset of ALT, s.t. } \{\neg q:q\in C'\}\cup\{p\} \text{ is consistent.}\}$
 - c. $II(p)(ALT) = \bigcap \{C'' \subseteq ALT : C'' \text{ is a maximal subset of ALT, s.t. } \{r : r \in C''\} \cup \{p\} \cup \{\neg q : q \in IE(p)(ALT)\} \text{ is consistent} \}$

Innocent Inclusion is an addition to Innocent Exclusion, and the IE step has priority, preserving any scalar implicatures that would be drawn. However, as an extra step, exhaustification with Innocent Inclusion also asserts all of the alternatives that can be added without risking contradiction with p or with any of the innocently excluded alternatives. Bar-Lev & Fox (2020) use this operator to derive free choice implicatures for disjunctions under possibility modals.

For an alternative set that includes the individual disjuncts as well as the conjunction, Innocent Exclusion will negate the conjunction, but the individual disjuncts are not innocently excludable. They are, however, innocently includable, because they can both be asserted (specifically with the possibility modal applied to them) while preserving the truth of $\Diamond(p\vee q)$ and the truth of $\neg\Box(p\wedge q)$, resulting in $\Diamond p\wedge\Diamond q$, where the disjunction has been strengthened to a conjunction.

In the absence of a scalemate, Innocent Inclusion will derive strengthening of an existential to the equivalent of a universal. As a schematic, Warlpiri is analyzed by Bowler (2014) as a language that has a disjunction operator *manu*, but no conjunction operator. As a result, conjunction is missing from the set of *manu*'s scalar alternatives, and exhaustifying results in strengthening to conjunction:

(101) a.
$$[\![p \ manu \ q]\!] = p \lor q$$

b. ALT = $\{p, q, p \lor q\}$

c.
$$\llbracket exh^{IE}(p \text{ manu } q) \rrbracket = (p \lor q)$$

d.
$$[\![exh^{IE+II}(p \text{ manu } q)]\!] = (p \lor q) \land p \land q$$

 $\equiv p \land q$

Because there is no dedicated conjunction operator in Warlpiri, Innocent Exclusion doesn't negate the conjunction of p and q, like it does in English or other languages with dedicated words for and. As a result, Innocent Inclusion asserts the individual disjuncts, and can do so without running into contradiction with a negated conjunct. The result is simply asserting the conjunction that was missing from the language before.

In upward-entailing contexts, *manu* is interpreted as "and". But under negation, *manu* can only be interpreted as "or":

- (102) Cecilia manu Gloria=pala yanu tawunu-kurra Cecilia manu Gloria=3DU.SUBJ go.PST town-ALL "Cecilia and Gloria went to town."
- (103) Cecilia manu Gloria kula=pala yanu Lajamanu-kurra Cecilia manu Gloria NEG=3DU.SUBJ go.PST LajamanuALL "Neither Cecilia nor Gloria went to town."

The explanation for this is that a negated "or" is already exceedingly strong, and so exh can't draw a scalar implicature. It can draw a scaleless one, but only for implicatures that are already entailed by the original proposition anyway, and therefore no more informative than if exh had not applied at all:

(104) a.
$$\llbracket \neg (p \ manu \ q) \rrbracket = \neg (p \lor q)$$

b. ALT = $\{p, q, p \lor q\}$
c. $\llbracket exh^{IE}(p \ manu \ q) \rrbracket = \neg (p \lor q)$
d. $\llbracket exh^{IE+II}(p \ manu \ q) \rrbracket = \neg (p \lor q) \land \neg p \land \neg q$
 $\equiv \neg (p \lor q)$

In general, this should follow in any downward-entailing context. However, *manu* can be understood as either *or* or *and* in many such contexts, such as if-clauses

and restrictors of universal quantifiers:

- (105) Warlpiri (Bowler, 2014, ex. 9–10):
 - a. Kaji=npa kuyu manu mangarri ngarni ngula kapu=npa IRR=2SG.SUBJ meat manu food eat.NPST that AUX.FUT=2SG.SUBJ pirrjirdi-jarrimi. strong-become.NPST "If you eat meat and vegetables, you will become strong."
 - b. Kaji=npa jarntu pakarni manu window luwarni, ngula=ju IRR=2SG.SUBJ dog hit.NPST manu window shoot.NPST that=TOP Nungarrayi-rli kapi=ngki jirna-wangu-mani.
 Nungarrayi-erg Aux.fut=2SG.NSUBJ scold-NPST
 "If you hit the dog or break the window, then Nungarrayi will scold you."

However, this isn't surprising, given other evidence around scalar implicatures and exhaustification in the literature. A purely pragmatic approach predicts that scalar implicatures should be unavailable in downward-entailing contexts, because they do not, in general, strengthen the overall utterance. But, as established in Chierchia et al. (2012); Chierchia (2017) and others, exhaustification can take place rather readily in downward-entailing contexts, resulting in ambiguous sentences, especially when the context is right:

(106) Chierchia et al. (2012, p. 2305)

I don't expect that some students will do well, I expect that all students will.

Such contexts, in fact, are among the strongest arguments for an exhaustification operator in the syntax, rather than deriving scalar implicatures in a purely Gricean way (Grice, 1975), without reference to internal sentence structure. In order to derive the expected meaning for (106), a scalar implicature of "some but not all students" must be drawn under the matrix negation, or else the second half of the sentence will contradict the first.

Jeretič (2021a) provides further argumentation about the positions at which exh can apply. Jeretič observes that we do not obtain scalar implicatures or Free Choise implicatures under clausemate negation (examples from Jeretič, 2021a, exx. 59–60):

(107) Context: I am wondering who Zoe talked to out of Yann and Wynn. I then realize and that Yann and Wynn are inseparable, and Zoe couldn't have talked to only one.

"Zoe didn't talk to Yann or/OR Wynn." $\not\equiv \neg((y\vee w)\wedge \neg(y\wedge w))$...#She can only have talked to neither or both.

(108) Context: I think Zoe is allowed to talk to either director, Yann or Wynn. You disagree and say:

"Zoe is not allowed to talk to Yann or Wynn." $\not\equiv \neg(\lozenge y \land \lozenge w)$...#She is only allowed to talk to Yann.

The contexts in (107-108) should, respectively, permit a scalar implicature and a Free Choice implicature, which would then allow the follow-ups that are in fact infelicitous. If exh could apply under negation, then the scalar implicature could be drawn from the positive version of the sentence, and then negated, allowing a reading where the negation targets the scalar implicature, much like we saw in (106) above. But, as Jeretič shows, such readings are impossible. This is accounted for straightforwardly if the positions at which exh can take place as a covert operation are restricted. For Jeretič, exh applies at the TP level only, which is above where most languages place negation. These negated sentences are already as strong as possible, so if exh does apply, it has no effect. This being a structural constraint on the exhaustification operator, we expect that once negation is extra-clausal, the blocked readings become possible again, and this is indeed what we observe:

- (109) Jeretič (2021a, ex. 62):
 - a. Context: I am wondering who Zoe talked to out of Yann and Wynn. I then realize and that Yann and Wynn are inseparable, and Zoe couldn't have talked to only one.

"Zoe didn't talk to Yann or/OR Wynn."

...She can only have talked to neither or both.

b. Context: I think Zoe is allowed to talk to either director, Yann or Wynn.You disagree and say:

"Zoe is not allowed to talk to Yann or Wynn."

...She is only allowed to talk to Yann.

I follow Jeretič (2021a) in assuming there is a structural constraint on exh that prevents its application below clausemate negation, though to account for the Kinande data I must assume that it can apply slightly higher than TP. Kinande negation appears to be quite high in the syntax, above all of the other tense markers and even above subject marking, a position that Baker (2003) tentatively analyzes as a focus or polarity head:

(110) Si-nyi-ri-tog-ay-a ekiryato (Baker, 2003, ex. 11a)
NEG-1sS-TM-fall-CAUS-FV shoe
"I don't make the shoe fall."

Because of this, and because Kinande nevertheless patterns like English and Ecuadorian Siona with regard to scaleless implicatures with clausemate negation, I assume that exh, rather than applying at the TP level, applies at the CP level.

It's been noted, of course, that even embedded scalar implicatures seem to be dispreferred when they result in a sentence that is semantically weaker than if the implicature had not been calculated. But even then, it doesn't seem to be impossible, based on examples such as the following, from Chierchia (2017):

- (111) a. If some students in your class are having difficulties, talk to them
 - b. ...but if all of them do, do not talk to them; talk to me first.

On its own, perhaps (111a) tends toward an interpretation where you should always talk to any student having trouble, whether that's one out of the entire class or all of them. It seems that version where an embedded implicature of "some but not all students" is derived is dispreferred. But while that is certainly one possible interpretation of the sentence, and that interpretation requires the absence of an exh operator

in the if-clause, the follow-up in (111b) shows that exh must be possible in (111a), in order for the follow-up sentence to be non-contradictory. Thus the restriction on the occurrence of exh is something like the following:

(112) Parsing condition on exh (Chierchia, 2017): If using exh, do so in a way that does not lead to weakening (unless weakening is necessary to avoid a contradiction).

This parsing constraint should be understood not as a structural constraint on when or where exh can be used; rather, it's a statement of relatively vague pragmatic preference, a weaker approach even than the Strongest Meaning Hypothesis from Chierchia et al. (2012, p. 2327), which prefers exhaustification if it results in a stronger meaning. This sort of parsing constraint is much more of a suggestion than a rule, as befits the fuzziness of determining preferred interpretations out of context.

Given this discussion, I take the exh operator to be available at scope sites (clause boundaries), per Chierchia (2004); Chierchia et al. (2012), and freely so, as far as the syntax and semantics are concerned. Whether a given utterance appears to have an exhaustified meaning available or required, or neither, is a matter I take to be of pragmatics proper, and subject to considerations such as Gricean Implicature (Grice, 1975). But in principle, the semantics is capable of producing sentences either with or without an exh at any given CP layer.

These tools in our pockets, we can finally return to Jeretič's analysis for Siona, here recast in terms of Innocent Inclusion for consistency. Defining ba'iji as a possibility modal in (113) and then exhaustifying over its subdomain alternatives, we directly derive a strengthened reading of ba'iji, because Innocent Inclusion lets us assert the existential claim over all of the singleton sets of worlds; taken together, this is logically equivalent to universal quantification:

(113)
$$[\![ba'iji]\!] = \exists w'[w' \in \{w_1, w_2, w_3\} \land p(w')]$$

$$(114)$$
 ALT =

$$\exists w'[w' \in \{w_1, w_2, w_3\} \land p(w')]$$

$$\exists w'[w' \in \{w_1, w_2\} \land p(w')]$$

$$\exists w'[w' \in \{w_2, w_3\} \land p(w')]$$

$$\exists w'[w' \in \{w_1, w_3\} \land p(w')]$$

$$\exists w'[w' \in \{w_1\} \land p(w')]$$

$$\exists w'[w' \in \{w_2\} \land p(w')]$$

$$\exists w'[w' \in \{w_3\} \land p(w')]$$

$$\exists w'[w' \in \{w_1\} \land p(w')] \land$$

$$\exists w'[w' \in \{w_2\} \land p(w')] \land$$

$$\exists w'[w' \in \{w_3\} \land p(w')]$$

$$\exists w'[w' \in \{w_3\} \land p(w')]$$

$$\exists w'[w' \in \{w_3\} \land p(w')]$$

The exhaustification analysis accounts for the ambiguities in embedded downward-entailing contexts nicely, since exhaustification is optional for downward-entailing contexts (Chierchia et al., 2012).

In a somewhat similar vein, Leffel (2012) describes a future marker *-ti* in Masalit (Nilo-Saharan, Maban, Sudan) which, in addition to indicating future, is also used for various possibility-modal readings:

- (116) tísû tò-rón-tì (Masalit, Leffel, 2012, 11b) 3sg goat.Acc 3sg-buy-ti "He will buy a goat." or "He might buy a goat."
- (117) ámá kómò á-kál-tì (Masalit, ex. 15b) 1sg mountain.Acc 1sg-see-ti "I will see the mountain" (e.g., tomorrow) or

"I can see the mountain." (e.g., from where I'm standing)

Interestingly, *-ti* can also be used for epistemic necessity, but only if there is the particle *de* ('only') with it to get a necessity reading:

(118) tí màsàrà tú-tì (Masalit, Leffel, 2012, 17c) 3sg Masalit 3sg-ti "He might be Masalit" (119) tí màsàrà dè tú-tì (Masalit, Leffel, 2012, 17d) 3sg Masalit only 3sg-ti "He must be Masalit" (lit. "He could only be Masalit.")

Leffel analyzes *-ti* as a possibility modal tied to a temporal base (allowing for futurate readings), and proposal that *de* operates just like *only* from Rooth (1985), asserting that all of the alternatives to the expression without *only* are false (=118). In sum: It ensures that (118) is true, and none of the other possibilities are true. This amounts to what is essentially universal force, because there is only one possibility left. Masalit might be analyzed as overtly expressing the exhaustification operator that is covert in Siona.

The pattern for Kinande anga differs from Siona and Masalit in a few important ways: Most obviously, anga shows a limited kind of variable force, varying only between possibility and weak necessity, and never getting an interpretation as a strong necessity modal. For another, and likely connected to the first difference, anga has at least one scalemate, paswa. We will see that the presence of a universal quantifier in the alternative set poses some challenges for straightforwardly applying an exhaustification analysis. Finally, anga is variable in simple positive clauses as well as in downward entailing environments, distinct from Siona ba'iji. This likely comes from the lack of a fixed-force weak possibility modal in Kinande. In the next section I will address each of these problems while preserving the useful parts of the exhaustification approach, but here I close briefly listing my assumptions about the exhaustification operator.

(120) Assumptions about exh:

- a. exh can optionally apply at the CP layer, and nowhere else (similar to Jeretič, 2021a).
- b. A given lexical item might obligatorily or optionally trigger subdomain alternatives, or may idiosyncratically disallow subdomain alternatives to be triggered for it (following Chierchia, 2013).

c. Exhaustification uses Innocent Inclusion, (following Bar-Lev & Fox, 2020).

The optionality of exh is necessary to obtain cancellability of scalar implicatures given sufficient context, as we saw in (111) above, even if exhaustification is generally preferred. In Kinande, as we will see, anga's subdomain alternatives are only optionally triggered, because we can draw a scalar implicature without drawing a scaleless one. As discussed in Chierchia (2013), I assume that some lexical items obligatorily trigger subdomain alternatives (like English NPI any for Chierchia, and like Ecuadorian Siona ba'iji, here), while others only optionally do (like English some, or Kinande anga). Others yet might not allow subdomain alternatives at all, like Ecuadorian Siona de'oji, with the effect that such lexical items cannot draw scaleless implicatures, regardless of whether exhaustification applies or not. Finally, the assumption of Innocent Inclusion in (120c) is at this point one of convenience rather than empirical motivation; the examples to distinguish between Innocent Exclusion and Innocent Inclusion in Bar-Lev & Fox (2020) are unfortunately not possible in Kinande. I assume Innocent Inclusion for my own convenience, but note that very little in this analysis hangs on this: if you prefer to use recursive exhaustification without Innocent Inclusion, nothing will be lost from this analysis.

3.3 Exhaustifying anga

Recall this summary table of Kinande's modal system, here with anga highlighted:

	Epistemic	Deontic	Circumstantial	Teleological
Strong Necessity	paswa, kumbe, tolere	paswa,kumbe, tolere		paswa,kumbe,tolere
Weak necessity	anga,kumbe, tolere	anga,kumbe	anga,kumbe, tolere	kumbe,tolere
Possibility	anga	anga	anga	anga

Table 3.2: Kinande modal morphemes

Anga is felicitous across all modal flavors in both possibility and weak necessity

contexts. It is very specifically infelicitous in strong necessity contexts, where generally speaking paswa ('must') is used instead. I propose that we account for anga's variance, on the one hand, and its lack of compatibility with strong necessity contexts, on the other, with the same exhaustification operator. Anga draws a scalar implicature by the familiar methods we have seen, with the alternative paswa expression negated by Innocent Exclusion. At the same time, anga is a domain-restricted modal that optionally triggers subdomain alternatives, and since exhaustification also contains Innocent Inclusion, exh will assert as many of those subdomain alternatives as it can, resulting in a weak necessity reading.

This solution is nice and tidy; it lets us use the same mechanism to derive both where *anga* is felicitious and where it is not. It's also sensitive to the lexical inventory of Kinande, as suits our descriptive observations, that variable-force modality requires gaps in a modal paradigm, but can work in whatever size gap there may be. What distinguishes Kinande from Siona, St'át'imcets, or Nez Perce is that Kinande *anga* is limited in its variable force, and that Kinande has a fixed-force strong necessity modal. In this analysis here, those two facts are linked, and we expect them to be linked cross-linguistically. The result is an informative and constrained typology of variable and non-variable force.

The most important factors will be properly determining the set of alternatives for *anga*, based on Kinande's lexicon, and then the rest of the analysis will come about from domain restriction and exhaustification, both mechanisms well-motivated in the literature for modals and elsewhere. This will make the variable-force nature of these languages a function of a particular arrangement of a language's lexicon, requiring no other special machinery to derive.

3.3.1 A problem: determining our ALT set

Before we can even consider looking at exhaustifying *anga* to derive its variable force, we need to determine *anga*'s alternative set. This is easier said than done, especially

with Kinande's modals spread across syntactic categories. On its own, *anga* doesn't share a syntactic category with any other modal expression. If alternative sets for scalar implicature are derived structurally, as proposed by Katzir (2007); Fox & Katzir (2011), then *anga* should have no scalar alternatives at all.

Empirically, however, that doesn't seem to line up. *Anga* is interpreted as weaker than *paswa*, and robustly rejected in strong necessity environments, suggesting that it is not merely a matter of not being informative enough, but that there is a scalar implicature at work forcing contradiction with the strong necessity interpretation.

(121) Context: In Uganda, the law states that when you ride a motor bike...

```
#u-anga-na-ambal-a e-sapu
2sg-MOD-TM-wear-FV c9/3-helmet
(intended:) 'you have to wear a helmet'
```

Such evidence compels me to consider *paswa* a member of *anga*'s alternative set. Whether that set is lexically defined or structurally determined I do not know, but at the very least, they somehow stand in that relationship.

The next question is, what about *tolere*? Or *kumbe*? Both expressions are used for weak necessity, and *tolere* in particular is a modal verb, much like *paswa*. If scalar alternatives are defined structurally, it might be difficult to include *paswa* but not *tolere* in *anga*'s alternative set. Empirically speaking, it's less obvious that *tolere* and *kumbe* are alternatives to *anga*, and they both take CP complements rather than infinitivals or verb stems, but lacking an actual theory of deriving those alternatives, there's no good way of ruling them out for certain, so I may as well give preliminary definitions of all of these morphemes. These are all very naive, simplified definitions, and I have no doubt that there are greater complexities to be uncovered in future research.

Defining *paswa* is relatively simple; we saw in Chapter 2 that it acts like a strong necessity modal with variable modal flavor, so we can define it as such:

$$\texttt{(122)} \quad \llbracket \textit{paswa}(\textit{p}) \rrbracket = \forall w'[w' \in OS_1(\bigcap(MB(w))) \rightarrow p(w')]$$

The weak necessity expressions are less simple. Recall from Chapter 2 that tolere

and *kumbe* are also variable-force expressions, but based on the morphology of their complement, rather than on semantic/pragmatic context. This suggests that they are in fact strong necessity modals, and the mood marker *i*- that appears when they are used as weak necessity expressions functions as a kind of weakening (maybe via domain restriction but this is far from empirically settled). That leaves us with a question around their relationship with *anga*: what goes into the alternative set? Maybe we take a relatively free kind of method where we substitute the modal and then change other morphosyntax "accordingly", but what about when that modal is itself variable? Do both meanings get entered as alternatives? Or only the one that requires the least additional structure? The analysis I pursue is tidiest if the weak necessity interpretations of *tolere* and *kumbe* clauses are not included in *anga*'s alternative set. If *tolere* and/or *kumbe* are included in the alternative set and defined as inherently weak-necessity modals, then the exhaustification analysis will draw an unintuitive scalar implicature that does not seem to be part of *anga* expressions.

3.3.2 A first stab at formalization

So let's see what the exhaustification theory, as it stands, does with *anga* as a modal.

We can try defining *anga* as a straightforward possibility modal:

(123)
$$[anga(p)]^w = \exists w'[w' \in Best(OS(\bigcap MB(w))) \& p(w')]$$

Then we have to ask what *anga*'s alternatives are; in order to derive any strengthening we'll need *anga*'s subdomain alternatives. The next question is: what else goes in this set? In other words, does *anga* have scalar alternatives?

The answer to that question depends on how you want to determine scalar alternatives in general. If scalar alternatives are exclusively expressions of the same syntactic category (Katzir, 2007; Fox & Katzir, 2011), then *anga* has none. In that case, *anga* would behave just like Siona *ba'iji*. That won't work for us, so it seems we need *anga* to have *paswa* as a scalar alternative, even though they're different

syntactic categories with different selectional restrictions, because we need a scalar implicature to prevent *anga* from becoming too strong. And besides, *anga* shows a scalar implicature of "not necessarily", and we want to account for that.

What's slightly less obvious is whether the weak necessity clause-embedding verb tolere is a scalar alternative for anga. If paswa is, and we aren't limited to morphemes of the same syntactic category as anga, then it's difficult to exclude tolere from the alternative set in any principled way. On the one hand, I can say that we can derive anga strengthening to weak necessity whether or not tolere is a scalar alternative, and even if kumbe (a complementizer that also seems to contribute some weak necessity interpretations) is as well. Innocently excluding either modal (or both) will weaken anga by only one world out of the entire domain: the anga sentence will denote something like "all but one of the wolrds in the restricted domain are p-worlds". That's both a rather "unnatural" scalar implicature to draw, and a single world is unlikely to be relevant for modal reasoning. The mechanics will otherwise work just as we need them to, deriving the interpretations for anga that we see.

less syntactic difference between *anga* and *paswa* than between either of them and *tolere*, as well; *tolere* is the only one of the three that is bi-clausal in constructions: *anga* applies directly in the verbal spine, *paswa* takes an infinitival clause (which in Kinande might simply be a nominalization of a type), but *tolere* takes a fully-tensed, mood-marked CP. This might be enough to exclude *tolere* from competition with *anga*.

So our putative alternative set of anga looks something like this (for a very small toy domain of worlds $\{w_1, w_2, w_3\}$):

(124)
$$ALT(\llbracket anga(p) \rrbracket) = \{ \forall w'[w' \in \{w_1, w_2, w_3\} \rightarrow p(w')], \\ \exists w'[w' \in \{w_1, w_2, w_3\} \& p(w')], \\ \exists w'[w' \in \{w_1, w_2\} \& p(w')], \\ \exists w'[w' \in \{w_1, w_3\} \& p(w')], \\ \exists w'[w' \in \{w_2, w_3\} \& p(w')],$$

$$\exists w'[w' \in \{w_1\} \& p(w')],$$
$$\exists w'[w' \in \{w_2\} \& p(w')],$$
$$\exists w'[w' \in \{w_3\} \& p(w')]\}$$

Now we can try exhaustifying it. Unfortunately, doing so doesn't work out as we'd like. We draw a scalar implicature negating the universal claim, but we end up with something far too strong. *Anga* means that its complement is true in all but one of the worlds in the domain:

(125) a.
$$[\![exh^{IE}(anga(p))\!]\!] = \exists w'[w' \in \{w_1, w_2, w_3\} \& p(w')]$$

$$\land \neg \forall w'[w' \in \{w_1, w_2, w_3\} \rightarrow p(w')]$$
b. $[\![exh^{IE+II}(anga(p))\!]\!] : \exists w'[w' \in \{w_1, w_2, w_3\} \& p(w')]$

$$\land \neg \forall w'[w' \in \{w_1, w_2, w_3\} \rightarrow p(w')]$$

$$\land \exists w'[w' \in \{w_1, w_2, w_3\} \& p(w')]$$

$$\land \exists w'[w' \in \{w_1, w_2\} \& p(w')]$$

$$\land \exists w'[w' \in \{w_1, w_3\} \& p(w')]$$

$$\land \exists w'[w' \in \{w_2, w_3\} \& p(w')]$$

Innocent Exclusion negates the *paswa* alternative, but Innocent Inclusion asserts existential claims over all of the subsets of the domain that it can, and while that doesn't include the singleton sets, which would get us true universal quantification, it will include all of sets of size 2. The meaning we derive is that there is exactly one world in the domain where p is false, and that p is true in all of the other worlds. Call this the "all but one" meaning.

This is way too strong. This would make weak-necessity *anga* felicitous if and only if there's exactly one world where the prejacent is false. No more, no less. It's strange kind of modal reasoning to pick out a single world in a infinite set where the prejacent isn't true, and what's more it doesn't line up with the way weak necessity modals are used. English *should* can be followed up with multiple other options, and so can Kinande *anga* on a weak necessity interpretation:

(126) Context: In your opinion, it's best to walk to the marketplace, but riding a bike or driving a car are also options, so when giving advice on the matter, you say:

U-anga-génd-a bisando omo-sóko, náho omo e-ri-y-a 2SG-MOD-go-FV foot PREP-market, but **PREP** AUG-C5-go-FV u-anga-génd-y-a e-kínga kútse o-mú-toká 2SG-MOD-go-TV-FV AUG-c7.bike or **AUG-car** "You should walk to the market, but you also can ride a bike or drive a car."

In English it seems intuitively obvious that a weak necessity claim isn't narrowing you down to only two options, and the same seems to be true for Kinande: it can be true that you ought to walk to the market while it's still true that you can ride a bike or drive a car there, instead.

3.3.3 Weak necessity has a meaning

Naively exhaustifying over *anga*'s subdomain alternatives results in disaster; we don't derive a strengthened meaning of weak necessity at all, we get strong necessity, or something very like it. Not only does it essentially circumvent the scalar implicature with *paswa*, it results in a meaning for *anga* that is far stronger than we observe.

We can solve this problem by remembering that weak necessity is distinct from strong necessity by more than just number of worlds quantified over. On domain-restriction theories of weak necessity, additional assumptions come into play for evaluation of a modal. These assumptions have a side-effect of restricting the domain of quantification, but they serve a linguistic purpose as well.

Understanding that once *anga*'s meaning is strengthened it arrives at a weak necessity reading, we can solve our issue by simply incorporating this domain restriction into *anga* itself. Lexically, *anga* is a domain-restricted possibility modal:

(127)
$$[anga(p)] = \exists w'[w' \in OS_2(OS_1(\bigcap(MB(w)))) \land p(w')]$$

As discussed in Section 3.2.1, domain restriction of a possibility modal has very little, if any, discernible effect. If something is still possible given some additional (perhaps unshared) assumptions or conditions, then well, it's still possible. But we are a long way from likely, especially when those assumptions are not shared by everyone in the discourse, per Rubinstein (2012).

Making *anga* lexically domain-restricted while leaving *paswa* unrestricted puts a hard cap on *anga*'s strength, even when it is exhaustified into a necessity meaning. We can thus draw a scalar implicature negating the *paswa*-alternative, while drawing a scaleless implicature strengthening *anga* to weak necessity:

(128) a.
$$[\![exh^{IE}(anga(p))]\!] = \exists w'[w' \in \{w_1, w_2, w_3\} \& p(w')]$$

$$\land \neg \forall w'[w' \in \{w_1, w_2, w_3, w_4, w_5\} \rightarrow p(w')]$$
b. $[\![exh^{IE+II}(anga(p))]\!] : \exists w'[w' \in \{w_1, w_2, w_3\} \& p(w')]$

$$\land \neg \forall w'[w' \in \{w_1, w_2, w_3, w_4, w_5\} \rightarrow p(w')]$$

$$\land \exists w'[w' \in \{w_1, w_2, w_3\} \& p(w')]$$

$$\land \exists w'[w' \in \{w_1, w_2\} \& p(w')]$$

$$\land \exists w'[w' \in \{w_1, w_3\} \& p(w')]$$

$$\land \exists w'[w' \in \{w_2, w_3\} \& p(w')]$$

$$\land \exists w'[w' \in \{w_1\} \& p(w')]$$

$$\land \exists w'[w' \in \{w_2\} \& p(w')]$$

$$\land \exists w'[w' \in \{w_3\} \& p(w')]$$

$$\exists \forall w[w' \in \{w_3\} \& p(w')]$$

$$\equiv \forall w[w' \in \{w_1, w_2, w_3\} \rightarrow p(w')]$$

One might also ask whether anga is a degree modal, something like English possibly or probably, or even should on some analyses of English (Portner & Rubinstein, 2014; Pasternak, 2016; Lassiter, 2017). Syntactically, anga isn't modifiable by degree modifiers. Even if this is due to a syntactic quirk rather than a semantic one, it's not clear how we would derive anga's pattern under a scalar or degree analysis: if anga denotes that p exceeds some (underspecified) threshold on a modal scale, probably

it should be variable in all contexts, even under negation. Negating anga should say that p does not exceed that threshold, but that threshold is set by context and could plausibly be as high or low as usual, so we expect variability (that we do not see).

3.3.4 *Anga* in simplex clauses

With these tools, it's simple to derive the variability of *anga* in simple positive clauses. I assume, per (120b), that activating the subdomain alternatives on *anga* is optional; this allows us to draw the scalar implicature even when we do not draw the scaleless one. This is empirically-motivated, as *anga* sentences can be used as in possibility contexts, while reinforcing the scalar implicature:

(129) Context: The teacher says that Masika is allowed to go swimming, though she does not have to.

Masika angáyatsíga, aliwé sibabúgíre ambu Masika SM.c1-MOD-go-swim-FV, but NEG-SM.c2-say-TM that átanétatsíga Masika a-anga-ya-tsig-a, aliwe si-ba-bug-ire SM.c1-TM-TM-NEG-swim-FV ambu a-ta-ne-ta-tsig-a

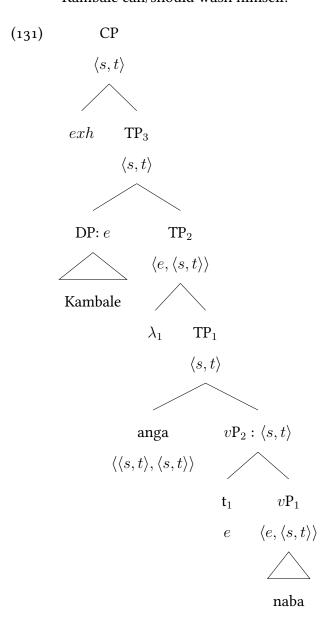
"Masika may go swimming, but she does not have to swim."

(lit. "...they do not say that she has to swim.")

I take examples like (129) to indicate that the scalar implicature of "not necessarily" is drawn even when the context doesn't license strengthening to weak necessity by scaleless implicature. Since this requires exhaustifying over anga's scalar alternatives without exhaustifying over its subdomain alternatives, I take this to show that the difference between strengthened and non-strengthened anga is one of subdomain alternative activiation, rather than solely the presence or absence of the exh operator in general.

Whether those subdomain alternatives are activated or not, I take the structure of an *anga*-expression to be something like the following:

(130) Kámbale a-angá-naba Kambale SM.c1-MOD-wash 'Kambale can/should wash himself.'



And as a reminder, we're working with these definitions for our modals:

(132)
$$\llbracket \operatorname{anga} \rrbracket = \lambda p. \lambda w. \exists w' [w' \in OS_2(OS_1(\bigcap (MB(w)))) \land p(w')]$$

(133)
$$\llbracket paswa \rrbracket = \lambda p.\lambda w. \forall w' [w' \in OS_1(\bigcap(MB(w))) \rightarrow p(w')]$$

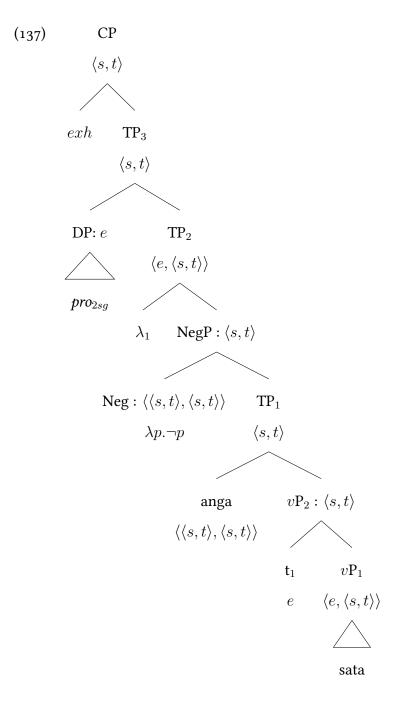
For possibility-interpreted *anga*, the subdomain alternatives for *anga* aren't used, and so the alternative set is just *paswa*, and the composition proceeds as follows:

Anga remains domain-restricted compared to the corresponding paswa sentence, even in this un-exhaustified derivation, but that doesn't mean much when there is no "weak possibility" modal to compare with anga. However, if the interpretation proceeds with anga's subdomain alternatives included in the alternative set, the composition goes as follows:

3.3.5 *Anga* with clausemate negation

Kinande negation is fairly high in the clause, and specifically outscopes *anga*, as well as preceding it in the modal template:

(136) nga-oko reglema yi-ka-bug-a, si-u-anga-sat-a, COMP-COMP c9.rules SM.c9-TM-say-FV, NEG-SM.2sg-MOD-dance-FV kundi w-oyo u-kwa-ire oko ku-boko because 2sg-REPLRO SM.2sg-hurt-TM c17 c15-arm "According to the rules, you can't play because your arm is injured."



Because the exhaustification operator is applied at the clause boundary rather than freely at any point in the syntax, there's no position between *anga* and negation to apply exhaustification. The result is a loss of variability for *anga*. It doesn't matter whether we include subdomain alternatives or not, nor even whether we exhaustify or not; the semantic value of negated *anga* is already stronger than any of its alternatives, and so exhaustification does nothing at all:

As an illustration of how this works, here is a schematic exhaustification derivation for negated anga with subdomain alternatives, and toy domains $D(paswa) = \{w_1, w_2, w_3, w_4, w_5\}$ and $D(anga) = \{w_1, w_2, w_3\}$:

$$\begin{array}{ll} \text{(139)} & \text{a. } ALT \llbracket \neg \textit{anga}(p) \, \rrbracket = \{ \neg \forall w'[w' \in \{w_1, w_2, w_3, w_4, w_5\} \rightarrow p(w')], \\ & \neg \exists w'[w' \in \{w_1, w_2\} \& \ p(w')] \\ & \neg \exists w'[w' \in \{w_1, w_3\} \& \ p(w')] \\ & \neg \exists w'[w' \in \{w_1, w_3\} \& \ p(w')] \\ & \neg \exists w'[w' \in \{w_2, w_3\} \& \ p(w')] \\ & \neg \exists w'[w' \in \{w_1\} \& \ p(w')] \\ & \neg \exists w'[w' \in \{w_2\} \& \ p(w')] \\ & \neg \exists w'[w' \in \{w_3\} \& \ p(w')] \} \\ & \text{b. } \ \llbracket exh^{IE}(\textit{anga}(p)) \rrbracket = \neg \exists w'[w' \in \{w_1, w_2, w_3\} \& \ p(w')] \\ & c. \ \llbracket exh^{IE+II}(\textit{anga}(p)) \rrbracket : \neg \exists w'[w' \in \{w_1, w_2, w_3\} \& \ p(w')] \\ & \wedge \neg \forall w'[w' \in \{w_1, w_2, w_3, w_4, w_5\} \rightarrow p(w')] \\ & \wedge \neg \exists w'[w' \in \{w_1, w_2\} \& \ p(w')] \\ & \wedge \neg \exists w'[w' \in \{w_1, w_3\} \& \ p(w')] \\ & \wedge \neg \exists w'[w' \in \{w_2, w_3\} \& \ p(w')] \\ & \wedge \neg \exists w'[w' \in \{w_1\} \& \ p(w')] \\ & \wedge \neg \exists w'[w' \in \{w_1\} \& \ p(w')] \\ & \wedge \neg \exists w'[w' \in \{w_2\} \& \ p(w')] \\ & \wedge \neg \exists w'[w' \in \{w_2\} \& \ p(w')] \\ & \wedge \neg \exists w'[w' \in \{w_2\} \& \ p(w')] \\ & \wedge \neg \exists w'[w' \in \{w_2\} \& \ p(w')] \\ & \wedge \neg \exists w'[w' \in \{w_2\} \& \ p(w')] \\ & \wedge \neg \exists w'[w' \in \{w_2\} \& \ p(w')] \\ & \end{pmatrix} \end{array}$$

Applying exhaustification with innocent exclusion does nothing at all, as there are no innocently-excludable alternatives. Exhaustification with innocent inclusion does allow the assertion of the *paswa* scalar alternative, as well as all of the subdomain alternatives, but as they are all already entailed by the negated *anga* sentence in the first place, the final meaning is unaffected. No weak necessity reading is possible, because *anga* is already maximally strong as a negated possibility modal.

Even just looking at the alternative set in 139a shows that exhaustification won't be able to adjust negated *anga*-sentences at all: the semantics of negated possibility already entail all of the alternatives, so there are none that are innocently excludable, and in fact all of them are innocently includable, though they don't alter the meaning of the original expression at all.

3.3.6 *Anga* in downward-entailing contexts with clause boundaries

When a clause boundary is introduced, we've seen that *anga* becomes variable once again, regardless of negation or directionality of entailment. This holds when negation is extra-clausal, if *anga* is in the antecedent of a conditional, or if it is in the restrictor of a universal quantifier:

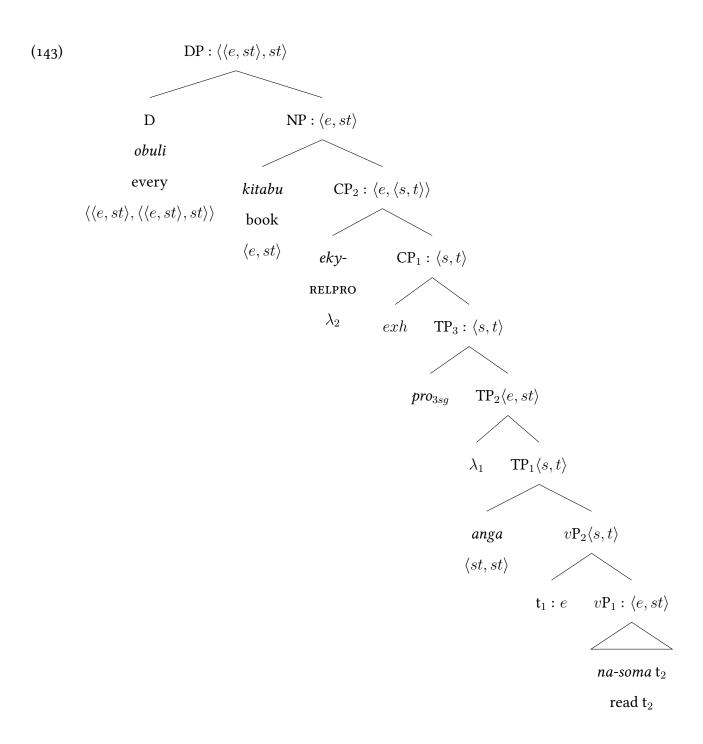
omugalímu Másiká mwátétábuga ati (140)omugalimu mo-a-te-ta-bug-a a-ti Masika c1.teacher TM-SM.c1-TM.NEG-TM.NEG-say-FV SM.c1-that Masika anganátsíga a-anga-na-tsig-a SM.c1-MOD-TM-swim-FV

"The teacher didn't say that Masika can/should swim"

ekitábw' (141) wamábyá íwanganásóm' ekí, e-kitabu eki, u-a-ma-bi-a i-u-anga-na-som-a SM.2sg-TM-TM-be-FV C-SM.2sg-MOD-TM-read-FV AUG-c7.kitabu c7.this, kúmbé íwakígúlá munábwíre kumbe i-u-a-ki-gul-a munabwire better.that C-SM.2sg-TM-Om.c7-buy-FV today "If you can/should read this book, you should buy it today."

kitábu (142) Swera akásomá óbuli obuli Swera a-ka-som-a kitabu SM.c1-TM-read-FV Swera c7.book every ekyánganásóma e-kyo-a-anga-na-som-a AUG-c7.REL-SM.c1-MOD-TM-read-FV "Swera reads every book that she can/should read"

These examples are in stark contrast with the clausemate negation examples, which stubbornly force anga into a possibility reading. But as discussed in Section 3.2.3, the presence of a clause boundary causes anga's variability to return, even though exh makes an ultimately weaker statement. Here is an example tree for the universal quantifier case:



$$\begin{split} & \llbracket \operatorname{CP}_1 \rrbracket = \lambda w. \forall w' [w' \in OS_2(OS_1(\bigcap(MB(w)))) \to \operatorname{read}(t_2)(\operatorname{pro}_{3sg})(w')] \\ & \wedge \neg \forall w' [w' \in OS_1(\bigcap(MB(w))) \to \operatorname{read}(t_2)(\operatorname{pro}_{3sg})(w')] \\ & \llbracket \operatorname{CP}_2 \rrbracket = \lambda y. \lambda w. \forall w' [w' \in OS_2(OS_1(\bigcap(MB(w)))) \to \operatorname{read}(y)(\operatorname{pro}_{3sg})(w')] \\ & \wedge \neg \forall w' [w' \in OS_1(\bigcap(MB(w))) \to \operatorname{read}(y)(\operatorname{pro}_{3sg})(w')] \\ & \llbracket \operatorname{NP} \rrbracket = \lambda y. \lambda w. \operatorname{book}(y) \wedge \forall w' [w' \in OS_2(OS_1(\bigcap(MB(w)))) \to \operatorname{read}(y)(\operatorname{pro}_{3sg})(w')] \\ & \wedge \neg \forall w' [w' \in OS_1(\bigcap(MB(w))) \to \operatorname{read}(y)(\operatorname{pro}_{3sg})(w')] \\ & \llbracket \operatorname{D} \rrbracket = \lambda P_{\langle e, st \rangle}. \lambda Q_{\langle e, st \rangle}. \lambda w. \forall y [P(y)(w) \to Q(y)(w)] \\ & \llbracket \operatorname{DP} \rrbracket = \lambda Q_{\langle e, st \rangle}. \lambda w. \forall y [(\operatorname{book}(y) \wedge \forall w' [w' \in OS_2(OS_1(\bigcap(MB(w)))) \to \operatorname{read}(y)(\operatorname{pro}_{3sg})(w')] \\ & \wedge \neg \forall w' [w' \in OS_1(\bigcap(MB(w))) \to \operatorname{read}(y)(\operatorname{pro}_{3sg})(w')] \to Q(y)(w)] \end{split}$$

The added universal quantifier adds more complexity to the derivation, but overall it proceeds much as before, but with subdomain alternatives exaustified over between anga and the *obuli* ('all'), the result is a weak necessity interpretation, even in this downward-entailing environment.

3.4 Implications and loose ends

3.4.1 A concern: recursive alternatives?

This system requires that *anga*'s alternative set crucially contains *anga*'s subdomain alternatives—existential statements over all of the possible subsets of *anga*'s domain, and most importantly the singleton sets—and also *anga*'s scalar alternatives (i.e., the *paswa* expression, with its universal quantifier), but it absolutely must not contain the subdomain alternatives of *anga*'s scalar alternatives. The result would be introducing universal statements over all of the possible subsets of *paswa*.

Most of those would be innocently excludable (specifically, all but the singleton subdomains). The result would be an "exactly one" implicature for *anga*, which forces a peculiar reading for a modal. Propositions that are true in precisely one world out

of an infinite set tend to be rather odd; they perfectly single out a particular possible world, and it's not clear human cognition reasons that to level of precision. And it certainly seems strange to think of a modal asserting that the proposition it modifies is true in exactly one world, even if we don't know which one.

One way out of this is to claim that *paswa* does not trigger obligatory subdomain alternatives, per Chierchia (2013) for English NPI *any* and English *some*, the former of which obligatorily triggers subdomain alternatives and the latter of which does not. It's hard to tell whether *paswa* triggers them or not, anyway. Another is that if it does, *anga* can't access them (but given that alternatives are often found recursively, this may not work). At this point I'm not picky about which.

3.4.2 Exhaustifying twice vs. Innocent Inclusion

Perhaps you aren't convinced that Innocent Inclusion is a good addition to our exhaustification operator, though you believe that exhaustification in general is still operational.

In that case, the same effects here can be derived with recursive exhaustification with just Innocent Exclusion. The formalism is somewhat different, but examples are in Bowler (2014) and Jeretič (2021a). In short, for Kinande, exhaustifying the first time will derive the scalar implicature, and exhaustifying the second time will then derive the strengthening. In that case you don't have to say that *anga* only optionally activates subdomain alternatives, since whether they are present or not, they will only be included by the second exhaustification operator. To my knowledge, there's no predicted difference in the analyses for variable-force modals between the two of them. I use a single operator with Innocent Inclusion because it makes the formalism easier to parse. But this is a mere formal convenience, and nothing hangs on it.

What my account cannot do without, however, is exhaustification in general, nor without exhaustification within embedded clauses and downward-entailing environments. The Kinande variable-force facts simply cannot be accounted for with

a global pragmatic approach to scalar implicature, nor by analyzing *anga* as merely permissable in stronger contexts, as opposed to semantically strengthened by exhaustification. In either of those cases, *anga* sentences should be non-variable in any downward-entailing context, contrary to fact. Take this as another argument for exhaustification operators existing in at least some languages, but see Chapter 4 for discussion of languages that might not have them, or at least not use them with their variable-force modals.

3.4.3 What happens if there's a weak necessity scalemate

Let's say for a moment that you are dissatisfied with my reasons several subsections ago for not including *tolere* as one of *anga*'s scalemates. Here's what happens if *tolere* is present as a scalemate:

First, *anga*'s alternative set now includes a universal modal over a restricted set of worlds. For now, assume the worst and make that a universal over the exact same domain as *anga*:

(145)
$$ALT(\llbracket anga(p) \rrbracket) = \{ \forall w'[w' \in \{w_1, w_2, w_3\} \rightarrow p(w')], \ \forall w'[w' \in \{w_1, w_2, w_3\} \rightarrow p(w')] \}$$

$$\exists w'[w' \in \{w_1, w_2, w_3\} \& p(w')], \ \exists w'[w' \in \{w_1, w_2\} \& p(w')], \ \exists w'[w' \in \{w_1, w_3\} \& p(w')], \ \exists w'[w' \in \{w_2, w_3\} \& p(w')], \ \exists w'[w' \in \{w_1\} \& p(w')], \ \exists w'[w' \in \{w_2\} \& p(w')], \ \exists w'[w' \in \{w_3\} \& p(w')] \}$$

Next, exhaustify with innocent exclusion, which will mean negating both the *paswa* claim and the *tolere* claim, both of which are innocently excludable:

(146)
$$[exh^{IE}(anga(p))] = \exists w'[w' \in \{w_1, w_2, w_3\} \& p(w')]$$

$$\land \neg \forall w'[w' \in \{w_1, w_2, w_3, w_4, w_5\} \to p(w')]$$
$$\land \neg \forall w'[w' \in \{w_1, w_2, w_3\}]$$

At this point we have a meaning for *anga* that's something along the lines of "You can, but you don't have to, and you aren't even weakly obligated to". But logically, we can still activate *anga*'s subdomain alternatives, and we can still innocently include most of them, just not the singleton alternatives::

$$\begin{aligned} & (147) \quad \llbracket exh^{IE+II}(anga(p)) \, \rrbracket \colon \exists w'[w' \in \{w_1, w_2, w_3\} \& \ p(w')] \\ & \wedge \neg \forall w'[w' \in \{w_1, w_2, w_3, w_4, w_5\} \to p(w')] \\ & \wedge \neg \forall w'[w' \in \{w_1, w_2, w_3\}] \\ & \wedge \exists w'[w' \in \{w_1, w_2, w_3\} \& \ p(w')] \\ & \wedge \exists w'[w' \in \{w_1, w_2\} \& \ p(w')] \\ & \wedge \exists w'[w' \in \{w_1, w_3\} \& \ p(w')] \\ & \wedge \exists w'[w' \in \{w_2, w_3\} \& \ p(w')] \end{aligned}$$

The result is a meaning that can exist, logically, but not one that seems very natural. It says that p is true in all but one of the *tolere* worlds. Innocent Exclusion prevents us from being able to innocently include the singleton sets over worlds, but it doesn't stop us from including all the sets of two, which will add up to an "all but one" meaning. This is an extremely awkward modal statement to make, and arguably one that no human can be in a position to truthfully say. It requires omniscience in a very strong sense, because it's not just that there's exactly one world that the speaker can differentiate amongst the many accessible worlds; no, there is precisely **one** world in the entire domain, between the worlds we have information to distinguish and the ones we cannot. This is functionally as strong as the original universal claim, but at the same time extremely unnatural and requiring a much stronger base of knowledge. Most likely something we would like to avoid.

⁷It is true that similarly odd meanings play a part in strengthening *anga* in the first place, because the alternative set includes singleton sets of worlds among the subdomain alternatives. However, I

The main way to avoid this is to suppose that *tolere* is not a scalemate to *anga*, as I've been assuming. Languages that have specific weak necessity modals in their paradigm probably shouldn't have exhaustification over the subdomain alternatives of their possibility modals, because the propositions you derive by doing so are so unlikely to be assertable.

3.4.4 Implications for weak necessity

Since *anga* is a domain-restricted possibility modal, we might ask: is it a weak possibility modal (as English *could* is sometimes called), or a strong possibility modal, as the logic of domain restriction would suggest? Vander Klok & Hohaus (2020) discuss the oddness of claiming domain-restriction for deriving weak possibility. And certainly if English conditional marking is directly restricting the domain of an existential quantifier, we expect to see at most strengthening, or perhaps nothing at all.

Unless the additional stipulations are not purely restrictive over the domain. English *could* and *would* (and less commonly, but tellingly, *should*) appear in counterfactual conditionals, which differ from indicative conditionals both in morphological marking and in their method of interpretation. Most notably, antecedents to counterfactual conditionals are presupposed not to be in the common ground—they may be known to be false, or they may simply be known to not have full commitment. Much like weak necessity modals for Rubinstein (2012). Because they are not in the common ground, there is no guarantee that the worlds under consideration (where you take that to be a common ground or a shared modal base, counterfactual antecedents have to be privileged when evaluating the conditional in ways that indicative conditional antecedents do not. Kratzer (1989) does this by allowing counterfactuals to

believe there to be a difference between using such meanings as an intermediate step in an exhaustification algorithm (which may or may not be the algorithm used in human modal reasoning when interpreting these propositions), on the one hand, and taking one of these meanings as the final semantic meaning of a proposition, on the other.

jettison any facts that don't line up with their antecedents, and potentially create a very different set of worlds than the original modal base. Starr (2014) accomplishes a similar effect in a dynamic framework by performing a non-eliminative update with the counterfactual antecedent, which rather than narrowing the context set (as indicative antecedents do), it shifts each world in the context set to one that is as close as possible, but where the antecedent is true. From there, both analyses work similarly: for a *could* counterfactual, you check to see if at least one of those worlds is a world in which the consequent is true. For a *would* counterfactual, you check to see if all of them are.

That works quite nicely for counterfactuals, but the telling facts from von Fintel & Iatridou (2008) indicate that we might just want to use this for weak necessity as well: cross-linguistically it's quite common for weak necessity modals to be formed from a strong necessity modal plus conditional/counterfactual marking, and weak possibility modals by a similar derivation from a regular possibility modal (cf. the parallel English should and could). In that case, the conditional marking is indicating that there is counterfactual reasoning going on: assumptions at play that are privileged, but not necessarily shared.

In that case, domain restriction isn't necessarily restricting the domain at all; it's signaling additional priorities that might be purely restrictive on the modal base, but may also shift or even expand the domain of quantification instead. If shifting is what's active in *could* as counterfactual-marked *can*, then it's much clearer why *could* is weaker: if the speaker knew that one of the worlds in *can*'s domain was a *p*-world, they would simply say that. Instead they chose *could*, which has this modification on it, and may just expand the set of worlds, or at least make them worlds that no longer include the world of evaluation.

Can we preserve the meaning we have for *should* with this? I think so. English

should is a separate lexical item for weak necessity, unlike many of the other languages in von Fintel & Iatridou (2008), but is argued to be the same as a strong necessity modal plus the additional modification of the modal base. Similar reasoning applies, however: if the speaker could make a necessity statement over the shared set of worlds, then they presumably would do so. The set of worlds must be shifted, or restricted, and for modalities like epistemic modals this brings into question whether the world of evaluation is in the domain or not. For non-alethic modalities like deontics, it indicates that the facts of the world and the deontic ordering source alone are not enough to conclude that p is an obligation.

One last bit in favor of this: let's go back to an alethic modality; epistemic modality. We walk into Terry's office and see that she's not there. We expected her to be there, she almost always is at this time. She even said she'd be here today. When you asked me whether she was in today, I could even say "She must be, her schedule says she is and she never deviates." Everything we know says we can expect her to be here. And yet she's not here.

- (148) # That's strange, Terry must be in her office
- (149) That's strange, Terry should be in her office

The only fact that's changed from my earlier *must* statement to this later (infelicitous) one is that now we know that she's not in her office. What's happened? Clearly one of our shared assumptions, that we accepted and were certain of, doesn't hold. Which one? We don't know. But if *should* is shifting rather than purely restricting, it doesn't matter; we just change the relation from one of basic intersection to one of non-eliminative update, or intersection favoring our asusmption(s). In the face of incontrovertible evidence that Terry is not where we thought she was, we can still acknowledge the expectation that she would be there, that something has gone wrong between our assumptions and their lining up with the facts of the world. The question is which of our assumptions was flawed, or perhaps which fact of the actual

world has prevented them from holding. What has befallen Terry?

For a purely domain-restrictive account of *should*, you would just say that given evidence that the shared assumptions do not in fact warrant a *must* claim (after all, Terry is not in her office), some or all of them moved into the secondary ordering source, no longer taken to be shared assumptions.

What's all this mean for Kinande? Well, maybe *anga* isn't a "strong possibility" modal, it's a weak one. Its domain isn't always restricted (though it can be, if the modification can be compatible with the common ground). And this might just explain why *anga* also acts as a counterfactual conditional marker:

(150) Context: Kambale is a very good soccer player. In fact, he's the best player on his team. His team has a game tomorrow, but the opposing team is not particularly good. If Kambale plays tomorrow, his team will certainly win the game. (If he doesn't, it's not quite so certain.)

Kambale a-ma-bya i-na-ka-ndi-sya-pik-a oko mupira ko omungya, Kambale 3sg-TM-be C-TM-TM-fut-fut-play-FV prep ball lnk tomorrow, i-na-ka-ndi-sya-sing-a C-TM-TM-fut-win-FV 'If Kambale plays football tomorrow, he will win'

(151) Context: Like before, Kambale is an excellent soccer player, and his team has a game tomorrow. But unfortunately Kambale is traveling on other business, and probably won't be able to make it to play in the game (there's only a small chance he will make it in time). This is a big shame, because if he could make it, his team would definitely win the game.

Kambale a-ma-sya-pik-a oko mupira ko omungya, Kambale 3sg-TM-fut-play-FV loc ball lnk tomorrow, i-na-ka-ndi-sya-sing-a C-TM-TM-fut-fut-win-FV 'If Kambale plays football tomorrow, he will win'

(152) Context: Again just like before, but this time Kambale has broken his leg, and there's no way he'll be able to play in the game tomorrow. If he had been able

to play, his team would have won for sure.

Kambale nga-pik-a oko mupira ko omuligolo, a-anga-sing-ir-e Kambale C-play-FV loc ball LNK yesterday, 3sg-MOD-win-compl-fv If Kambale had played football yesterday, he would have won'

Anga marks a shift in the modal base, and perhaps later grew from this potential/conditional marker to being a possibility modal, but when it can be exhaustified, because of the gaps in Kinande's modal paradigm, anga can be strengthened to necessity, but the inherent domain restriction/shift it introduces restrains it to weak necessity at most.

The next question to ask is whether this is true of all potential markers, or how it's possible to test it in other Bantu languages, at least.

And we might wonder, if this is indeed how we arrive at "weak possibility", why languages like Paciran Javanese have a marker that creates a weak necessity modal from a strong necessity one, but does not create a weak possibility modal. One answer is simply that such a marker does indeed do domain restriction, rather than this counterfactual-like modal base modification.

3.5 A brief aside on scalar accounts

Just like there have been other ways of capturing weak necessity in the literature, there have been other proposal for capturing variable-force modals than those I've discussed here. Mostly they've been elided because they did not specifically address variable-force, or any particular variable-force language, but as an example I will take a account of modals such as in Lassiter's work (Lassiter, 2017). On a degree-based account, modals are basically just another kind of degree expression, associated with scales and asserting that the prejacent exceeds some threshold according to that scale. The scale depends on the type of modality. For epistemic modals, it's probability, and a low threshold of probability licenses a *might* claim, while exceeding a high threshold licenses a *must* claim. English modals, not being variable-force, have lexical

conditions on those thresholds, ensuring that a *might* claim stays weaker than a *must* one. But as a sketch for applying it to variable-force modals, we can imagine *anga* having a much more vague threshold requirement, letting the context set it anywhere between what we usually associate with possibility to what we usually associate with weak necessity, and perhaps the context will disambiguate where that threshold was set.

Extending this to root modals runs into the same kind of problem as a domain restriction analysis; it's extremely difficult to find a scale that works for both permission and obligation, or for both teleological possibility and teleological necessity. Permissible things do not become obligatory by reaching a high threshold of permissibility. And Lassiter in fact puts most of his possibility and necessity modals each on their own scale, drawing entailment relations between them but removing the avenue of analyzing variable-force modals as vague degree expressions, at least if any of the readings include possibility interpretations in root modals. Now, this doesn't necessarily preclude degree-based analyses entirely, just from any language where the variable-force modal(s) include both possibility and necessity interpretations. Variable-force modals that only vary between weak and strong necessity could potentially be degree modals, especially if they are felicitous in degree constructions.

3.6 Stepping back and admiring our work

We've come a long way. By working through the theories previously proposed for variable-force modals, using them to further probe *anga*'s meaning, and find gaps in our understanding of both, eventually we developed an analysis for *anga* that captures its rather complex pattern, and points to further work to be done on Kinande weak necessity and scalar implicature. If this analysis is right, then we know that Kinande uses exhaustification over subdomain alternatives, maybe exhaustification

with Innocent Inclusion. It's beyond the scope of this work to examine all the implications of that, but it points to important parallels between the modal and nominal domains, for example.

The connection between gaps in the modal paradigm and variable-force modality, which seemed natural enough, now has a mechanical explanation, with exhaustification. Modals are variable-force by mechanisms of comparison on lexical scales. It's just that when there's nothing on that scale, you can get strengthening to fill that spot, and when there is something there, you strengthen the weaker terms to assert that competitor's negation. One is a scaleless implicature, the other a scalar implicature, but they come from the same operation, just on different scales. In the next chapter I will explore this further, and illustrate the typology of modal systems—variable and not—that we expect to see given these basic mechanisms of exhaustification and domain restriction, all constrained by the pressure scalemates can exert on one another in the pragmatics.

We might ask how *anga* came to be a domain-restricted modal in the first place. If it really is so difficult to actually see domain restriction in a possibility modal, then how does a language learner find out that *anga* is inherently domain restricted? I would speculate that *anga* has been a variable-force modal since before it was a domain-restricted one. *Paswa*, the scalar alternative to *anga*, is a relatively recent borrowing from Swahili. It's possible that *anga* used to be fully variable, like the modals in Siona or Nez Perce. Possibly it was even an exhaustified modal, but exhaustification was optional in order to allow for variable-force readings even in simplex sentences (unlike Siona *ba'iji*). But when *paswa* was borrowed into Kinande as an fixed-force strong necessity modal, pragmatic pressure on *anga* forced it to have a force ceiling at weak necessity, and the formal reflex for this is domain restriction. This means that if Kinande can be compared to its close relative Konzo, which shares a majority of its grammar and vocabulary, but rather than borrowing words via contact with French and Swahili, Konzo's lexical inventory is instead influenced by Rutooro

and English, we may see different results. If Konzo has borrowed a Rutooro (or even an English) modal for strong necessity, *anga* may have undergone a similar change. If Konzo speakers don't seem to have such a borrowing, then we might find *anga* as a fully variable modal.

Kinande's modal system, as observed in Chapter 2, has a very crowded weak necessity paradigm, but at the same time, none of the options for weak necessity belong exclusively there; *anga* is also a possibility modal, *tolere* and *kumbe* are both used for strong necessity contexts as well as weak necessity ones. But rather than varying simply on context, *tolere* and *anga* both seem to vary based on the mood marking of their complement.

- (153) Context: On behalf of your friend Masika, you ask the teacher if she should go swimming. The teacher replies that she can and must:
 - a. ihiii, Masika a-anga-na-ya-tsig-a... yes, Masika SM.c1-MOD-TM-go-swim-FV... "Yes, Masika can go swimming..."
 - b. ...kumbe a-ya-tsig-e. ...better.that SM.c1-go-swim-SUBJ "...She really should swim"
 - c. # ...kumbe i-na-ya-tsig-a ...better.that C-go-swim-FV "...She should go swimming."

Both of these modals get weak necessity readings when their complement has *i*-marking on the embedded verb. They achieve strong necessity readings when *i*- is absent, and perhaps some other marking is present (-*e* final vowel, or *nga*-prefixing).

Unfortunately I cannot give a complete analysis of these other weak necessity items, but we still might ask: what's their underlying modal force? It seems to be universal in some sense, since they vary between weak and strong necessity. I propose that they're underlyingly strong necessity modals that take CP complements, and the *i*- marker, somewhat like a subjunctive marker, weakens them from strong

necessity to weak necessity. Subjunctive marking tends to do this across many languages, but so far is still lacking an adequate explanation.

In the next chapter I will examine how the analysis of *anga* developed here extends to other attested variable-force languages and discuss the predicted typology of modal force that comes from domain restriction and exhaustification approaches.

Chapter 4

Towards a unified theory of modal force

In Chapter 3 I proposed an analysis of Kinande *anga* as a variable-force modal that accounts for *anga*'s pattern through a combination of domain restriction and optional exhaustification over subdomain alternatives that applies at clause boundaries. On that analysis, *anga* is a possibility modal that can be strengthened to a necessity modal via exhaustification, but lexical domain restriction, influenced by the presence of a stronger modal in the lexicon (*paswa*), means that the strength of *anga* is at most weak necessity. In this chapter I extend this bipartite method of deriving variable modal force to generate a typology of modal force. In doing so, I provide a unified theory of modal force variation that includes previously attested variable-force modals as well as modal systems that do not show variable modal force. My account furthermore explains the observation that variable-force languages tend to have relatively few modals, and specifically that gaps in the modal paradigm are required to see variable modal force.

The typology I propose here allows for variable-force modality to be more finegrained than a binary distinction between (fully) variable and not variable at all. Instead, there are a number of limited-variability options, as well as some subtle distinctions in the interpretation of other modals (for example, weak necessity) that fall within this typology.

The chapter is organized as follows: In Section 4.1 I briefly recapitulate the mechanisms of my analysis from Chapter 3, framing them as typological factors and pointing to potential ways they could vary cross-linguistically. In Section 4.2 I address

and explain the connection between a language having variable-force modals and that language having a relatively sparse modal paradigm. Then in Section 4.3 I lay out and discuss the actual typology of modal force that my analysis predicts, providing example attestations where they exist. Section 4.4 compares the typology here to previous modal typological work in Nauze (2008) and Jeretič (2021a), and discusses potential interactions between modal force and other aspects of linguistic systems, such as tense and aspect markers and conditionals.

4.1 The analysis pieces as typological factors

4.1.1 Lexical gaps and domain restriction

It's well-established that modals are domain restricted, often implicitly, and much of the weak necessity literature captures weak necessity via further domain restriction (e.g., Kratzer, 1991; von Fintel & Iatridou, 2005; Rubinstein, 2012). And of course, as discussed in the previous chapter, several analyses of variable-force modals use implicit domain restriction as their mechanism for variability. The analysis for Kinande, on the other hand, uses lexical domain restriction on the variable-force prefix *anga* in order to restrain *anga*'s variability.

We already see a number of different modal paradigms. There's English, with three modal force distinctions, each one a distinct lexical item; there are languages like Italian (von Fintel & Iatridou, 2008), which also have three-way distinctions in modal force, but one (weak necessity) is more transparently derived from another (strong necessity); there's Kinande, which seems to have two modal force distinctions, with a dedicated strong necessity modal and a modal that is used for both weak necessity and possibility; and then there are languages like St'át'imcets and Gitksan, where both types of necessity as well as possibility are all covered by a single lexical item.

Part of what Kinande shows us that we don't have a simple binary between variable and non-variable systems. Instead it hints at a more fine-grained typology based on the particulars of the lexicon of the language, and what (if any) gaps exist in its modal paradigm.

Conveniently, exhaustification predicts variability precisely where paradigmatic gaps exist. Scalar implicatures can't be drawn without a stronger scalar alternative, and scaleless implicatures only occur when we see these gaps. So we neatly derive the connection between variable-force modals and the language having fewer modal strength distinctions (by virtue of lacking scalar competitors).

The possible uses of a given modal, as discussed before, follow largely from scalar and scaleless implicatures. And in the more sparsely-populated paradigms, where there is no explicit lexical marking for strength differentiation, we might also expect contextual domain restriction to play a role in deriving weak versus strong readings, as well.

4.1.2 Deriving scalar alternatives

We should return to our discussion form Chapter 3 of how (scalar) alternatives are derived. The derivation of a scalar (or scaleless) implicature depends crucially on whether there is a stronger alternative in the alternative set. As such, we should have a theory that determines the right set of alternatives for those implicatures to be drawn (or not, as the case may be).

In general, scalar alternatives are taken to be propositions built by substituting individual lexical objects for objects of the same category (Horn, 1972; Chierchia, 2004; Katzir, 2007; Fox & Katzir, 2011). For example, replacing *some* with *all*, as in (154):

- (154) a. Some of the kids ate cake
 - b. All of the kids ate cake

The sentence (154a) has the negation of (154b) as a scalar implicature. In most contexts, saying that some of the kids ate cake will give the inference that not all of them did so. In this case simple lexical replacement achieves the desired result. This solves what Fox & Katzir (2011) and many others call the symmetry problem, where if a proposition like *some but not all of the kids ate cake* is permitted as an alternative to (154a), then we cannot draw (154b) as a scalar implicature, because (154b) is no longer innocently excludable (it is no longer in all of the maximal consistent subsets of negatable propositions for (154a)).

Things get more complicated when an inference can only be drawn by alternatives that are not pure lexical substitutions, such as the following from Katzir (2007):

- (155) a. Every candidate who sang was elected
 - b. Every candidate was elected

In a downward entailing context like (155), asserting (155a) gives the inference that not every candidate was elected. On a straightforward lexical substitution or generally scalar approach, this would require *candidate who sang* to be a scale-mate of *candidate* on its own. This creates problematic predictions for these expressions in upward-entailing contexts. To address this and several other problems, Katzir (2007) proposes that scalar alternatives are computable not only by lexical substitution, but also by deletion (allowing for simpler alternatives, but not more complex ones), as well as substitution of other subtrees that are already stated within the sentence (allowing for more complex alternatives under restrained circumstances where those alternatives are made explicit). However, replacement for all of these still follows syntactic category rules: if a DP is asserted, then the relevant alternatives will only replace that DP, and not simply add or change structure willy-nilly.

While these approaches make significant progress in providing a working theory for deriving scalar alternatives, they still run into issues around the quirks of a language's lexicon and syntax. For example, *some* and *all*, in English, are considered

scalar alternatives, allowing use to derive the implicature of *I did not eat all of the cookies* from a statement like *I ate some of the cookies*. However, there are still syntactic differences in when *some* versus *all* can be used. For instance, *all* can be used with a definite nominal without the aid of the preposition *of*, but *some* cannot:

- (156) All (of) my kids had cake
- (157) Some *(of) my kids had cake

For a structural approach like that of Katzir (2007), it's still possible to derive the notall implicature, because *all* is licit both with and without the preposition. The issue comes up in the case of the negated *all* sentence:

(158) Not all my kids had cake

IMPLICATURE: Some of my kids had cake

In order to derive the scalar implicature in (158), there needs to be a corresponding negated existential quantifier in the alternative set. If we are lexically replacing just all we can use some, but then we obtain the ungrammatical *not some my kids had cake. Perhaps instead the alternative is no or none. But none still requires of between the quantifier and determiner phrase, and while no kids had cake is grammatical, it cannot draw the implicature we need: (158) does ont simply implicate that some kids somewhere had cake, but that some of my kids had cake. The no sentence is insufficient to derive such an implicature. In order to obtain a grammatical and sensible alternative, a new node must be inserted into the structure, in violation of the method in Katzir (2007). But it appears that this kind of syntactic difference is ignored for the sake of calculating alternatives.

Examples like this have prompted Buccola et al. (2022) to propose that alternatives are not entirely structural or even necessarily from the language's lexicon. More specifically, they propose that the elements available for substitution include those that are not necessarily lexicalized in the relevant language, but may be nonetheless cognitively/psychologically available (and potentially lexicalized in another language

to prove this availability). Such abstract lexical items might be more difficult to access relative to those actually lexicalized in the language, and if the cost of an alternative is too high, then it will not be available.

While it is compelling to contend that alternatives go beyond strict category-matching lexical substitution, or even the simpler structural alternatives, I am not convinced that the alternatives should extend to abstract concepts outside of the language's lexicon. There are intriguing arguments that in most cases, the shape of the lexicon has a significant effect on the scalar alternatives for a language, even in cases of primitive logical concepts, like conjunctions. Warlpiri *manu*, for example, is analyzed convincingly by Bowler (2014) as the equivalent of *or* in a language that lacks an *and* to serve as a scalar alternative. As a result, *manu* is strengthened by exhaustification and interpreted as *and* in a number of contexts, but is interpreted as *or* under negation, for example.

(159) Warlpiri (Bowler, 2014, p. 138–139)

- a. Cecilia manu Gloria=pala yanu tawunu-kurra. Jirrama=juku. Cecilia manu Gloria=3DU.SUBJ go.PST town-ALL two=exactly 'Cecilia and Gloria went to town. Exactly two did.'
- b. Cecilia manu Gloria kula=pala yanu Lajamanu-kurra. Lawa. Cecilia manu Gloria NEG=3DU.SUBJ go.PST Lajamanu-ALL nothing 'Neither Cecilia nor Gloria went to Lajamanu. Neither one.'

This behavior is only explained if Warlpiri not only lacks a lexical realization of and as an alternative to manu, but it also must lack a conceptual alternative for and, otherwise Warlpiri manu would get the same interpretation as English or: exclusive disjunction in most contexts due to the conjunctive scalar alternative, and inclusive disjunction in others, but not conjunction under any circumstances. But this isn't what we observe; instead, it seems that neither the lexical nor conceptual alternative of and is available as an alternative for Warlpiri manu, which is precisely what gives rise to its chimeric behavior.

However, languages clearly vary in what goes into various paradigms, and what

they consider natural classes of expressions. English, for example, uses a combination of verbs and modal particles to form its modal paradigm, including *must*, *may*, *can*, and *have to* in a sort of natural class of modal items, even though they are not all the same syntactic category. In fact, even for the modal particles that are often considered the same syntactic category, there are visible scope differences explored in Nauze (2008) that indicate significantly different syntactic structures, beyond what would be allowed in alternatives under say, Katzir (2007).

A full explanation of how to determine the set of alternatives for a given expression and solve the symmetry problem is far beyond the scope of this dissertation. For our purposes here, I will assume that scalar alternatives are at least in part determined by what other expressions are in a given paradigm of a language, where those paradigms may occasionally cross category lines or ignore some syntactic differences. This is apparently the case in Kinande, where *paswa*, a modal verb, is treated as a robust alternative to *anga*, a modal prefix, which we see by *anga*'s infelicity in strong necessity contexts.

It is already commonplace to assume that when lexical substitution for the sake of alternatives takes place, inflectional differences are ignored. Person morphology, for example, is not considered when calculating alternatives, nor in many cases are singular/plural distinctions (Jacobson, 2012). I propose that other accommodations can be made as well; the presence or absence of a prepositional particle, or a difference in category across commonly-juxtaposed modals.

My working assumptions are as follows: lexical items that have scalar alternatives get them somehow, depending on the language's lexicon. The alternatives might include some items that aren't of exactly the same category, but I assume that those alternatives must be realizable in the language. The possible alternatives vary across languages, or else we wouldn't see the variety of scalar/scaleless implicatures that we find. These assumptions then amount to, roughly, claiming that lexical items have Horn scales. While Horn scales are not particularly explanatory, I use them

as a stopgap pending a solution to determining scalar alternatives that can address the symmetry problem and account for the variation in scalar implicatures that we see cross-linguistically.

4.1.3 Exhaustification over subdomain alternatives

The other crucial piece of the analysis is exhaustification, and in order to derive strengthened readings with scaleless implicatures, we need exhaustification over subdomain alternatives. Existential modals that project subdomain alternatives (in the form of the possible subsets of the worlds they quantify over) are then eligible to be strengthened under exhaustification, either by recursive exhaustification or by exhaustification with innocent inclusion, in a manner parallel with how free-choice effects are derived in e.g., Chierchia (2013) and Bar-Lev & Fox (2017).

However, it's not practical to assert that all modals, or even all variable-force modals, trigger subdomain alternatives. A blanket statement of all modals triggering subdomain alternatives runs into immediate problems in Ecuadorian Siona, where only one of the existentially-quantifying modals undergoes strengthening, and the other in non-variable. And even a weaker assertion that variable-force modals must trigger subdomain alternatives fails to explain languages like Nez Perce, which are variable in upward-entailing contexts but not in downward-entailing ones. There are two ways to derive Nez Perce in the typology I develop: one where Nez Perce modals do not trigger subdomain alternatives, and one where they do not undergo exhaustification at all.

Lacking further evidence, I prefer the former, mainly for reasons of parsimony. We can already see that not all lexical items trigger subdomain alternatives, even though they may be exhaustified. We have evidence for this from variable-force modals, but also previous work on scalar implicatures in the nominal domain, as proposed by Chierchia (2013) for English *any* and *some*.

Plus, the evidence for exhaustification is extensive enough across languages and

phenomena that we may well expect an exhaustification operator to be universal. Between stipulating that an otherwise-universal operation is not so universal, and stipulating that there may be lexical quirks across languages that interact with that operator, I will prefer to stipulate a lexical quirk. If subdomain alternatives are generated, then they have to be used up by exhaustification.

What's this mean for Nez Perce? Well, presumably it means that Nez Perce has exhaustification in some other domains, even if we don't see it with modals. And indeed, since Nez Perce has only a possibility modal without any stronger scalemates, exhaustification won't do anything if there aren't subdomain alternatives that exh can use to strengthen the existential to a universal. The analysis in Deal (2011) essentially stands as-is. If Nez Perce is further shown not to have exhaustification for scalar implicatures in other domains, then perhaps we will be justified in wondering whether the exh operator is so universal after all. But at this point I see no reason to suppose anything other than essentially an idiosyncratic lexical property of Nez Perce o'qa.

In sum, I believe it makes more sense to take exhaustification as a universal, at least across the typology I propose here. No harm seems to come of it. Instead the point of variation is whether any or all of the modals in the languages in question trigger subdomain alternatives to permit strengthening. If they do, and the lexical gaps line up, then we'll see variable-force modals that vary even in downward-entailing contexts (except clausemate negation). If the modal in question doesn't trigger those alternatives, then it will vary at most in upward-entailing contexts. If a modal has a stronger scalemate, then we expect to derive a scalar implicature, as usual.

4.2 Is a lack of scale required for variable force?

In Chapters 2 and 3, a question repeatedly came up around what's necessary for a language to show variable force. The preliminary typological evidence presented here suggests that semantics and pragmatics together conspire to determine whether a language will show evidence of variable-force modals or not. Taking into account the exhaustification and scalar implicature mechanics from Chapter 3, we can start to see why. Depending on the type of variable-force mechanism, there are different expectations. But in general, seeing what is descriptively known as variable-force will depend on a gap somewhere in the modal paradigm, as well as certain semantic factors. Depending on the shape of the paradigmatic gaps and the particular semantic mechanisms at play in the language's lexicon, we can expect to see a variety of different behaviors. And in fact, we might see a greater variety of interesting behaviors around modal force interpretations than just what we would descriptively consider variable-force.

4.2.1 Scalelessness is relative

The obvious contribution to modal typology that we see from Kinande is that the distinction between variable-force and non-variable modals is not binary; it's not a difference between a language having a scale for its modals or not, it's a matter of the size, shape, and location of the scalar gaps that affects the behavior of the modals in a language. Since descriptions of modal systems tend to differentiate at most three levels of modal force—strong necessity, weak necessity, and possibility—even while there are in principle four logical distinctions, we might find languages with some of the effects of variable force without the actual variability.

English is famously not a variable-force language, but nevertheless, there's a potential gap in the modal paradigm, somewhere adjacent to *should*, in the domain-restricted modal section. We only see three modals, where there could in principle be four. The question is: which potential meaning does *should* occupy, and which is a paradigmatic gap?

The traditional (though by no means the only) analysis is that *should* is a restricted universal modal. But there are also several observed properties of *should* that might

make us consider otherwise:

- (160) English (Staniszewski, 2020, p. 1):
 - a. Hana doesn't think that Tom should leave. $think > \square > NEG$
 - b. Hana doesn't think that Tom must leave. $think > NEG > \square$

English *should*, as seen in (160a), seems to prefer a reading where *should* outscopes negation, while *must* does not require such a reading. Iatridou & Zeijlstra (2013) and Homer (2015) keep the traditional neg-raising analysis, and analyze *should* as a Positive Polarity Item (PPI) that must outscope negation, leading to the stronger meanings under negation than otherwise expected. In essence, they assume that *should* is underlyingly still a universal quantifier, and propose that it obligatorily undergoes movement at LF in order to achieve the attested readings. However, Staniszewski (2020) shows that *should* can also receive existential-like readings, especially in downward-entailing environments, challenging the PPI/obligatory scope approach:

- (161) English (Staniszewski, 2020, p. 2):
 - a. Context: Walking through the tunnels to a talk on campus, we find ourselves in a basement area among potentially dangerous supplies (obviously not the best place to be).

Should we be down here?

b. Context: My housemate wants me to pick something up at the office, but...

I'm not sure that we should be on campus without a Covid test.

Both of the above examples are giving existential readings to *should*; (161a) is not asking whether the basement is a required or even optimal place to be (whether being in the basement is **required** by what is optimal or good), but rather whether it is permissible to be there (whether being in the basement is even **compatible** with what is optimal or good). Likewise (161b) is not indicating uncertainty about whether it's ideal to be on campus without a Covid test, but rather whether it is permissible.

The pattern for English *should* now begins to resemble the patterns for Ecuadorian Siona and Kinande: We have a modal item that is sometimes read as a necessity modal, but other times as an existential one, and the existential reading is the one that is forced under negation.

In a similar way to how Siona and Kinande were later analyzed in Jeretič (2021a) and this work respectively, Staniszewski in fact analyses *should* as a variable-force modal; he proposes that *should* is an underlying existential quantifier that undergoes exhaustification over its subdomain alternatives to give us a strengthened reading in most circumstances. When negated, however, only the weak reading obtains.

If we take this to be true, then English patterns like a variable-force language that has a relatively densely-populated modal paradigm of $\langle can_{\exists}, should_{\exists}, must_{\forall} \rangle$. This leaves a gap in the domain-restricted necessity modal position (the canonical weak necessity semantics). But there is also significant pragmatic pressure from both *can* and *must*. So, in upward-entailing contexts where the speaker intends to communicate mere possibility and not weak necessity, *can* will win out as the more appropriate expression for the situation. This is where English differs from Kinande, in the lexicon and therefore in observable behavior. English, then, illustrates what happens when a language uses the mechanisms identified in variable-force languages when the language lacks the usual lexical gaps.

So English, like Kinande, exploits a relatively small gap in the modal paradigm. But we don't typically consider *should* to have variable force; the pragmatics enforced by the English modal paradigm generally prevent it from showing up as variable in the right way. But patterns like *should* in downward-entailing environments are best explained with the same kind of mechanics as are used to account for variable-force modal systems, even though English would not necessarily be described as one of their number. The upshot is that this means that variable-force languages aren't a separate, parallel system of modality that uses different mechanisms from better-studied languages. A variable-force modal comes about whenever the mechanisms

for modality, exhaustification, and pragmatics line up properly. And there are a number of ways for them to do so, resulting in different kinds of variable-force languages.

4.3 A new typology

With these mechanisms in place, we can both find a typology of modal systems and make some predictions about what we should find elsewhere.

The three ingredients for a modal system boil down to the following:

- 1. The semantics of the individual modal items (including their relative strengths)
- 2. The use of covert domain restriction and overt domain-restricting modifiers
- 3. The use of exhaustification

This typology assumes that domain restriction alone is insufficient to achieve variable force across possibility and necessity. Given the previous discussions of this assumption, I will not rehash them here. I also set aside fully gradable accounts of modals (variable-force or not) as in Lassiter (2017) for the purposes of this typology, as none of the modals I consider are obviously capable of appearing in scalar structures.¹

4.3.1 Sparse inventories: variable-force languages

On the analysis presented here, the languages that are descriptively variable-force will be exactly those with sparsely-populated modal paradigms, and the sparser the paradigm, the more variable the force. Beyond that basic concept, however, there are further generalizations and nuances to explore.

¹This doesn't preclude scalar approaches from applying to certain modal adverbs, for example, regardless of whether the language has variable-force modals in the modal particle or verbal domain. Such radically scalar approaches face significant empirical challenges in languages like Kinande, Siona, and Nez Perce, where modals lose their variability under negation. As such, lacking any more explicitly worked-out application of scalarity to variable-force modals, I am excluding this type of analysis from the typology here.

Lexicon	$+ Alts_{sub}$	\mid -Alts $_{sub}$	Note
$\forall_s, \ \forall_w$			necessity only
\forall_s ,	Kinande	no weak nec in DE	
\exists_s			
\forall_s ,			
\exists_w			
\forall_w ,			
\exists_s			
$\forall_w,$			
\exists_w			
			no strong nec. (scalar
\exists_s, \exists_w			implicature blocks)
\forall_s			strong nec. only
\forall_w			not possible?
	no strong nec.		not possible?
\exists_s	_		_
	St'át'imcets, Washo,	Nez Perce	contextual dom.restr.
\exists_w	Gitksan		

Table 4.1: Typology of "sparse" modal systems (≤ 2 lexical force distinctions)

Nez Perce

On perhaps the simplest determination, we have Nez Perce, as already analyzed by Deal (2011). As the only modal item available, o'qa is an existential quantifier over possible worlds that is consistent across modal forces, simply by the fact that it has no scalar competitors:

(162) 'inéhne-no'qa 'ee kii lepít cíikan. (Deal, 2011, ex. 1) take-mod you dem two blanket 'You can/should take these two blankets'

So far this tidily settles the row Nez Perce will occupy in Table 4.3.1. The column is determined by the behavior of o'qa in downward-entailing environments, including embedded ones. As discussed previously, o'qa loses its variability in all these downward entailing environments:

- (163) Nez Perce (Deal, 2011, p. 574):
 - a. Context: You are explaining to someone who thinks that they have to leave that they are not in fact required to do so. It is not necessary for them to leave.
 - # wéet'u 'ee kiy-**ó**'qa. not you go-MOD
 - b. Context: I am watching people clean out a cooler and throw away various things.

hi-wqii-cix-Ø 'iléxni ke yox hi-pá-ap-o'qa. 3SUBJ-throw.away-IMPF.PL-PRES all food REL DEM

3SUBJ-S.PL-eat-MOD

'They are throwing away all the food that they could eat. They are throwing away all their food.'

- #'They are throwing away all the food that they should eat (but keeping some junk food)'
- c. c'alawí saykiptaw'atóo háamti'c páa-x-no'qa, simíinikem-x if doctor-erg quickly 3/3-see-mod Lewiston-to

ki-kiy-ó'qa. (Deal, 2011, p. 576) 3SUBJ-go-MOD

'If the doctor can see him in a hurry, then he should head over to Lewiston.'

'If the doctor needs to see him in a hurry, then he should head over to Lewiston.'

Rather straightforwardly, Nez Perce's modal system has a single existentiallyquantified modal and no others, which does not use exhaustification over subdomain alternatives to achieve variability, but simply on being permissible in stronger context.

Gitksan

In the same row, we might consider Gitksan, which Peterson (2010) analyzes as having only a possibility modal, but it is then strengthened under certain circumstances to be interpreted as a necessity modal:

(164) yugwimahl dim iixwt (Peterson, 2010, p. 63) yukw=ima=hl tim iixw-t PROG=MOD=CND FUT fish-3 "He might be going fishing"

"He must be going fishing" "He's probably going fishing"

"He's likely going fishing"

"Maybe/perhaps he's going fishing"

Peterson (2010) analyzes the variability in these modals as being from domain restriction. We can take it a step further though, adding an exhaustification operator to get true logical universal quantifiers when warranted. Combined with domain restriction, we get a fully-variable modal in any context except for under clausemate negation (where syntactially =ima cannot occur). Given the syntactic position of this modal, it seems to outscope negation anyway, making the one distinguishing syntactic context impossible to check in Gitksan; Peterson notes (Peterson, 2010, p. 123) that

this is due to the variable nature of =ima and "the fact that scopal interaction is often restricted for independent reasons", it is difficult to test the underlying quantifier of =ima. Peterson selects an existential quantifier for =ima's denotation based on its default interpretation. Coordination tests involving negation indicate that either =ima scopes above negation or it indeed must be a domain-restriction modal, because it is possible to coordinate two contradictory statements, but it is not possible to distinguish the two analyses lacking more data:

(165)guxwimahl gyuwadan, ii neeyimahl kuxw=ima=hl kyuwantan ii nee=ima=hl run.away=MOD=CND horse CONI NEG=MOD=CND guxwimahl gyuwadan (Peterson, 2010, ex. 4.19) kuxw-(t)=ima=hlkyuwatan run.away-3sg=MOD=CND horse "Maybe the horse ran away, and maybe it hasn't."

"The horse must've ran away, and might/must not have."

For parsimony's sake, and with attention to the qualms discussed previously around relying solely on domain restriction for variability in modals, I will follow Peterson in analyzing Gitksan modals as strengthened, but propose that the strengthening mechanism is (optional) exhaustification rather than simply domain restriction. This places them in the same row as Nez Perce, but the other column.

St'át'imcets

Like Gitksan, St'át'imcets (Lillooet Salish) has fully variable-force modals as first analyzed in Rullmann et al. (2008). Although it has more than one modal in the paradigm, they are specified for modal flavor, and variable in modal force:

(166) St'át'imcets (Rullmann et al., 2008, p. 331):

Context 1: The cougar was on a rampage and was killing cats, dogs, and racoons, and it had a child cornered and was growling; it <u>would</u> have killed a child.

Context 2: You just know that sometimes cougars kill children when they venture into built-up areas; it could have killed a child.

zúqw-s-as ka ta sk'úk'wmi7t-a ti7 ku swúw'a lh-cw7áoz-as die-CAUS-3ERG IRR DET child-DET DEIC DET cougar COMP-NEG-3CONJ kw s-qus-cit-ítas
DET NOM-shoot-IND-3PL.ERG
'That cougar could/would have killed a child if they hadn't shot it'

'That cougar could/would have killed a child if they hadn't shot it.'

When modal flavor is controlled for, each of the St'át'imcets modals make no apparent distinction in modal force, and so the St'át'imcets modal system qualifies as a sparse paradigm with regard to modal force.

Rullmann et al. (2008) analyze these modals as underlyingly universal quantifiers, largely on the basis that their default interpretations tend to be as necessity modals, although contextual evidence permits possibility readings quite easily. However, I argue that these modals are best analyzed as existential quantifiers over possible worlds, despite their apparent "default" interpretation.

The first motivation for considering that these might be existential modals is that St'át'imcets has adverbials that are unambiguously possibility expressions, which may be clausal scalar alternatives in the sense of Jeretič (2021a), in that they are weakly available in the context but are not lexical scalar alternatives that can be computed with exhaustification, clausal scalar alternatives use more global pragmatic mechanisms. The upshot is that there are ways of expressing modal possibility in St'át'imcets that avoid capitalizing solely on the variability of the modal clitic:

(167) St'át'imcets (Rullmann et al., 2008, exx. 42–43):

- a. lh-7ámh-as kw s-qwal'út-s-al'ap sxek
 COMP-GOOD-3CONJ DET NOM-talk-CAUS-2PL.ERG maybe
 um'-en-tumulh-ás kelh
 give-DIR-2PL.OBJ-3ERG FUT
 "If you talk to him nicely he might give you some"
- b. nilh ku cw7aoy-s kw s-k'úl'-em múta7 ku pála7 k'a sxek foc det neg-3poss det nom-make-mid again det one infer maybe

xetspásq'et week

"So she wouldn't have to make more for about a week maybe."

While these adverbs cannot act as scalar alternatives for the computation of scalar/scaleless implicatures, their existence may be enough to push the modal clitics towards a more necessity-like interpretation, regardless of their underlying quantifier.

The relevant test case for determining the underlying quantifier is, of course, downward-entailing contexts, especially clausemate negation. Unfortunately, while St'át'imcets modals are variable in negated sentences, the second-position clitics scope over negation, and have wide scope with respect to the clause (Rullmann et al., 2008). If this is true, then St'át'imcets modals cannot be in the scope of negation without an intervening clause boundary, and so cannot be tested for whether they are underlyingly universal or existential quantifiers.

If we assume that St'át'imcets modals are underlyingly existential quantifiers, however, we gain greater parsimony and stronger typological predictions. Universal quantifiers cannot be made into existential ones in upward-entailing contexts through exhaustification; the only potential method of achieving variable-force for a universal modal is through domain restriction, as originally analyzed by Rullmann et al. (2008). But even setting aside the theoretical issues with interpreting a highly-restricted universal as equivalent to an existential (as discussed in Chapter 3.2), allowing extreme domain restriction to amount to variable-force weakens the typological predictions considerably. Namely, we predict the possibility of variable-force under clausemate negation, which should be unattested if variable-force modals come about via exhaustification, but we also lose the correlation between sparseness of modal paradigm and variable modal force. While strengthening via exhaustification is subject to blocking by stronger scalar alternatives, domain restriction analyses have no ready mechanism to explain such blocking. Given that the standard analysis of

modals generally involves some level of domain restriction, it appears that we should see far more variability even in dense modal paradigms than we in fact observe.

Generally assuming that a variable-force modal that varies across possibility and necessity is underlyingly an existential quantifier over possible worlds limits domain restriction to its original function within modal analyses, giving subtle and contextual distinctions in modal reasoning, rather than radical changes in modal force. This constrains the possibility space for modal systems, as well, meaning that a child who hears a modal used in both possibility and necessity contexts does not need to then attempt to decipher whether it is a universal or existential quantifier, but can immediately determine that it is an existential quantifier subject to strengthening via exhaustification.

Domain restriction still plays a significant role in St'át'imcets modals, given that they are fully gradient across the spectrum of modality, receiving possibility, weak necessity, and strong necessity readings as appropriate within a context. On my re-analysis, this would be obtained by a combination of contextual domain restriction and exhaustification; if the modal is unexhaustified and relatively unrestricted, a possibility reading results. If it is exhaustified and relatively unrestricted, strong necessity obtains. And if it is exhaustified and relatively restricted, weak necessity will result. There will be as many shades of force distinction as domain restriction can permit, and exhaustification over its subdomain alternatives controls the interpretation of the quantifier itself.

Washo

Washo (Hokan/isolate), as analyzed in Bochnak (2015) behaves much like St'át'imcets, except that instead of having several modals specified for modal flavor, it has a single modal *-e?* that is variable along both force and flavor:

(168) a. Context: You are planning to drive over the mountains. It's started to snow, and you know that whenever it snows, the road over the mountains

is closed.

```
dé?eš yeweš gum-beyec'ig-i-gi k'-e?-i (Bochnak, snow-good-ipfv-sr road refl-close-ipfv-rel 3-mod-ipfv 2015, p. 7)
```

"It's snowing a lot, so the road must be closed."

b. Context: You hear a knock at the door. You can't see who it is, but can see that the person looks about the same height as Beverly.

```
bevali k'-e?-hel-i-gi k'-e?-i (Bochnak, 2015, p.9)
Beverly 3-cop-subj-ipfv-rel 3-mod-ipfv
"It might be Beverly."
```

(169) a. Context: You borrowed a pot from Beverly, and now you need to give it back to her.

```
bevali wi:di? le-išil k'-e?-i
Beverly this 1-give 3-MOD-IPFV
"I need to give this to Beverly."
```

b. Context: At a school dance, you see a shy boy who wants to talk to a girl but isn't. You ask you friend if that boy is allowed to to talk to that girl. Your friend responds: "Yes..."

```
mé:hu šawlamhu-hak'a wagayay?-i-gi k'-e?-i (Bochnak, 2015, p. 8) boy girl-with talk-ipfv-rel 3-mod-ipfv "The boy is allowed to talk to the girl."
```

Bochnak provides tests attempting to distinguish whether -e? is a universal or existential quantifier, but the relevant tests are difficult to apply in Washo, because -e? syntactically takes wide scope with respect to negation. As such, its variability in negated sentences is not indicative of its underlying quantifier:

- (170) Bochnak (2015, p. 12)
 - a. Context: You see someone trying to pick up a very heavy rock, but they can't lift it. You are not very strong, so you say that you can't pick up the rock either.

de?ek t'i:yel-i?-ŋa di-bips-e:s-i-š-gi k'-e**?**-i rock nmlz-big-attr-nc 1-pick.up-neg-ipfv-sr-rel 3-mod-ipfv "I can't pick up that big rock"

- b. * de?ek t'i:yel-i?-ŋa di-bips-i-š-gi k'-**e?-e**:s-i rock nmlz-big-attr-nc 1-pick.up-ipfv-sr-rel 3-mod-neg-ipfv
- (171) Bochnak (2015, p. 12)
 - a. Context: Someone offers you some candy, but your doctor says you shouldn't eat candy.

```
demuc'uc'u-ŋa le-i?iw-e:s-i-gi L-e?-i sweet-NC 1.eat-NEG-IPFV-REL 1-MOD-IPFV "I shouldn't eat candy"
```

b. * demuc'uc'u-ŋa le-i?iw-i-gi L-e?-e:s-i sweet-NC 1.eat-IPFV-REL 1-MOD-NEG-IPFV

Bochnak does not test -e?'s behavior in other downward-entailing contexts, but notes that the modal is difficult to embed in those as well. I note also that as we have seen with Kinande anga here and Warlpiri manu in Bowler (2014), the presence of a clause boundary in those contexts makes those unclear test cases for exhaustification analyses in any case.

Following the same logic for St'át'imcets then, I consider Washo -e? to also be an existential quantifier over possible worlds that undergoes strengthening by exhaustification. Washo differs from St'át'imcets in that St'át'imcets modals are specified for modal flavor and Washo modals are not, but as this distinction is not tracked in the typology as stated here, they are collapsed into the same status.

Ecuadorian Siona

Moving from the sparsest modal force systems (with only one modal covering the entire spectrum of modal force in a language), we next look at a language with two modal force distinctions, which nevertheless show variable-force behavior, although in more particular circumstances.

Ecuadorian Siona, as analyzed by Jeretič (2021a), has two modals, de'oji and ba'iji:

```
(172) Sai-ye ba-'i-ji (Jeretič, 2021a, ex. 1) go-INF be-IPF-3s 'We must go.' (lit., 'there is to go')
```

(173) Sai-ye de'o-ji (Jeretič, 2021a, ex. 2) go-INF good-3s 'We can go.' (lit., 'it is good to go')

While *de'oji* is unambiguously a possibility modal, *ba'iji* is in fact variable-force, though not in simple positive clauses. First, *ba'iji* is obligatorily interpreted as a negated possibility modal when in the scope of negation:

```
(174) Sai-ye beo-ji
go-INF NEG.be-3S
'We mustn't go.'

# 'We don't have to go.'
```

Furthermore, ba'iji is ambiguous between possibility and necessity interpretations in other downward-entailing environments, such as antecedents of conditionals:

Jeretič argues that this behavior is best explained if Siona is analyzed as having two possibility modals, rather than a possibility modal and a necessity modal. *Ba'iji* is a possibility modal that projects subdomain alternatives, and *de'oji* is a possibility modal that does not project those alternatives. When undergoing exhaustification in positive contexts, *ba'iji* is strengthened to a necessity interpretation. Under clausemate negation, exhaustification has no effect and *ba'iji* can only be read as a possibility modal, while in other downward-entailing contexts it is ambiguous because of the clause boundary providing leeway as to whether *ba'iji* is exhaustified before or after it is placed in a downward-entailing environment.

The result is that Siona has two modals with the same underlying modal force:

they are both possibility modals. They differ only that one of them projects subdomain alternatives and the other does not, and so when exhaustified, one is strengthened to a necessity interpretation while the other is not.

Nsyilxcen

Nsyilxcen (Interior Salish, British Columbia) is analyzed in Menzies (2013) as having two modals specified for epistemic modality, *mat* and *cmay*. While *cmay* is unambiguously an epistemic possibility modal, *mat* displays variable force:

- (176) Nsyilxcen (Menzies, 2013, p. 2):
 - a. Mary mat ac-qíc-lx
 Mary epis.mod cust-run-lex
 'Mary might be running'

'Mary must be running'

- b. Mary cmay ac-qíc-lx
 Mary EPIS.MOD CUST-run-LX
 'Mary might be running'
 - # 'Mary must be running'

Menzies analyzes Nsyilxcen *mat* as a universal quantifier over possible worlds, which can then undergo domain restriction to a possibility modal, essentially following the analysis in Rullmann et al. (2008), with the twist that Nsyilxcen also has a fixed-force epistemic modal.

Next to Siona, however, this pattern looks suspiciously familiar. The relevant test case is thus seeing whether the modal can be embedded under negation, and if so, what interpretation it receives. Unfortunately, it appears that the scope facts for Nsyilxcen negation do not allow for this to be checked, as Menzies reports that the linear order of negation and either of these modals in Nsyilxcen does not affect the interpretation:

(177) a. lut mat Xxwup i? Canucks (Menzies, 2013, exx. 24–25) NEG MOD win DET Canucks "The Canucks must not have won"

- b. mat lut Xxwup i? Canucks

 MOD NEG win DET Canucks

 "The Canucks must not have won"
- (178) a. cmay lut ks-qíc-əlx MOD NEG FUT-run-LEX "I might not run"
 - b. lut cmay ks-qíc-əlx NEG MOD FUT-run-LEX "I might not run"

While (177) could be taken to indicate that *mat* is variable even under negation, the facts in (178) throw this into doubt. *Cmay* is non-variable, but regardless of its linear order with regard to the negation particle *lut*, it receives an interpretation where it, as a possibility modal, scopes over negation. This suggests that the modal particles in Nsyilxcen scope consistently above negation, and so the clausemate negation test is unavailable for Nsyilxcen.

It is telling that the languages where a variable-force modal can be clearly embedded in the scope of negation are all also languages where the modal loses its variability under clausemate negation. Languages where a domain restriction analysis for a necessity modal have been proposed are all also (thus far) languages that cannot embed that modal under clausemate negation, and so are inadequate test cases for distinguishing between exhaustification and domain restriction analyses. Given the prior discussion, I place Nsyilxcen in the same category as Ecuadorian Siona, having two modals that are lexically the same force, but one of which can project subdomain alternatives and thus be strengthened to a necessity modal.

What of the great expanse of nothingness in the middle?

Currently the languages with variable-force modals are clustered in the bottom row of the table, with only possibility modals. Kinande is the only one that lexically distinguishes multiple modal forces, and as a result there are large gaps in the typology, especially in other potential combinations of two modal force distinctions.

For some of those potential modal paradigms, we might expect them to be relatively rare: a language with the means only of expressing strong necessity and no means of expressing possibility is likely not a particularly stable system. For similarly functional reasons it might be unexpected to find a language that only has lexically domain-restricted modals, one for weak necessity and one for so-called "strong possibility". While the mechanisms of the system may technically permit it, it is difficult to say how to usefully define "extra restricted" modals when there are no less-restricted versions to compare them to. Further research involving more detailed descriptions and analysis of modal systems is needed to refine this area of the typology, however.

I note also that two of the languages that descriptively have two modals with difference force specifications (Ecuadorian Siona and Nsyilxcen) are nonetheless clustered into having just a single modal force distinction, despite having two modals. This is an artifact of the table failing to reflect that a language may have one modal that projects subdomain alternatives and another of the same quantificational force that does not. This means there is variation even within the cells of the table, since languages may have some modals that project subdomain alternatives and some that do not, and those different modals will then have different behavior, to the point even of a language potentially masquerading as a non-variable-force language.

4.3.2 Dense inventories: non-variable-force languages

Exhaustification in a language that has a truly packed modal paradigm typically won't derive any strengthening apart from scalar implicatures, and domain restriction is unlikely to be an option if there are unambiguous competing morphemes that do not allow that restriction. Paciran Javanese as analyzed in Vander Klok (2008) might be a language that fits both of these criteria. Modals in Javanese are unambiguous for both force and flavor, including expressions for possibility, weak necessity, and strong necessity. The weak necessity modals *kudune* (deontic weak necessity) and

Lexicon	+Alts $_{sub}$	-Alts $_{sub}$	Note		
$\forall_s, \ \forall_w,$		doesn't matter	no variable force		
\exists_s, \exists_w			without pruning		
$\forall_s, \ \forall_w,$			no variable force		
\exists_s			without pruning		
$\forall_s, \ \forall_w,$		Paciran Javanese	no vf without prun-		
\exists_w			ing		
\forall_s ,	English?	no weak nec under	no apparent vf, weak		
\exists_s, \exists_w		neg	necessity apparently		
			requires wide scope		
			over neg.		
\forall_w ,			no distinction btw		
\exists_s, \exists_w			strong and weak nec		

Table 4.2: Typology of "dense" modal systems (≥ 3 lexical force distinctions)

mesthine (epistemic weak necessity), in particular, are derived from strong necessity modals *kudu* (deontic/teleological/circumstantial *must*) and *mesthi* (epistemic *must*) by adding a dedicated suffix -ne. The presence of an explicit marker for deriving weak necessity, as well as the presence of unambiguously strong necessity modals, means that there is very little space for any of the Javanese modal morphemes to be variable-force. For the same reason, I expect that if exhaustification operators are active in Javanese, they won't be seen strengthening any possibility modals to weak/strong necessity modals. Instead we'll just see a scalar implicature, as we always expected from our exhaustification operators.

Table 4.3 from Vander Klok (2012) summarizes the Paciran Javanese modal system. In general, it has a very densely-packed paradigm, both for modal force and modal flavor. As a result, there are very few and very narrow gaps to work within to observe variable-force modals, and so it is not surprising that we find that Paciran Javanese has a fixed-force modal system.

As argued by Vander Klok & Hohaus (2020), the *-ne* operator is likely a domain-restricting morpheme. What this means for weak necessity interpretations in Javanese is not yet entirely clear; is it purely domain-restricting, even though it be

	Epistemic	Deontic	Teleological	Circumstantial	Bouletic
Necessity	mesthi	kudu			
Weak necessity	mestine	kudune		-	
Possibility	paleng	oleh	-	iso	-

Table 4.3: Paciran Javanese modal system (Vander Klok, 2012)

used for counterfactual-like contexts as well? It's restricted from applying to possibility modals, but it's unclear whether that is a morphological issue or a semantic one.

This also means that variable-force modals, if their meaning includes possibility, are lexically existential modals. And, as discussed in Chapter 3.2.3, if exhaustification operators are available at CP boundaries and potentially where licensed by focus (like only), then we can make a prediction about variable-force modals and negation: In any language where a variable-force modal includes a possibility interpretation, any variance should vanish under clausemate negation, unless potentially special focus morphology or intonation applies. Of course, this is if the modal can be embedded under negation in the first place. In Gitksan, St'át'imcets, and Nsyilxcen, all languages claimed to have domain-restriction type variable-force modals, it's not actually possible to embed those modals under negation; they're all base-generated higher than negation and do not tolerate being embedded. As a result, it's quite difficult to apply most of the tests for quantificational force to those modals. But it is telling, I think, that all of the variable-force languages where modals can be under negation are shown to be underlying possibility modals.

Another type of language predicted for this system is, of course, the contextually domain-restricted necessity modal, which will be variable just as far as weak necessity, but won't be interpretable as a possibility modal. Because the modals in question are underlyingly universal quantifiers, exhaustification won't derive any scaleless implicatures; no strengthening is possible, whether in simple positive clauses or in

downward-entailing environments.

Some modal systems that are in this typology are difficult or impossible to differentiate: systems with lexical modals for all four of strong necessity, weak necessity, strong possibility, and weak possibility, crossing both quantifier types and overtly-marked domain restriction. A language of this type is very difficult to distinguish from a language that has two weak necessity modals, or two strong possibility modals plus exhaustification, for example. It's even difficult to distinguish the weak necessity language from the strong necessity one, since there is so much overlap in the middle of the scale, and especially if exhaustification can be involved. The telling context here will be under negation, and potentially other downward-entailing environments. The existential quantifier is expected to receive a strong interpretation (like English should does when negated), while the universal quantifier should behave more in line with the strong necessity modal of the language. But much hinges on the location and interpretation of negation and other downward-entailing operators.

Another difficult-to-establish generalization is whether there are languages without lexically-marked domain restricted modals, but that do not allow covert domain restriction to the extent to find a weak necessity interpretation. In the abstract, this is a language that has a variable-force modal that is interpreted as a weak possibility modal or as a strong necessity one, but never as a weak necessity modal. But given the nature of modals being domain-restricted regardless, and the difficulty of pinning down judgments on weak necessity modals even in languages that explicitly mark them, this generalization is difficult to test. My suggestion is that we likely won't find any such languages, that gaps in a scale correlate not only with exhaustification, but with contextually-permitted domain restriction as well, on at least some modals.

Swedish

There is, however, one attestation of a language with a dense modal paradigm that nonetheless has at least one (descriptively) variable-force modal. Jeretič (2021a) provides the data illustrating the variability of Swedish $f\mathring{a}$, as well as establishing that it does indeed have scalar competitors:

- (179) Swedish (Jeretič, 2021a, p. 98,159):
 - a. Han får äta sellerin innan han får lämna bordet.
 he far eats celery before he can leave table
 'He has to eat the celery before he can leave the table.'
 - b. Jag behövde gå. I need.pst go 'I had to go.'

Unlike English *should*, which on the surface appears to be a fixed-force weak necessity modal, but on further investigation can be analyzed as a subtle kind of variable-force modal, Swedish f_a^a is plainly variable force, as seen in (179a). Even though Swedish has a dedicated strong necessity modal, f_a^a is used in the same sentence as both a possibility modal and a necessity one.

Swedish defies expectations to this point. Broadly speaking, the typology I propose links density of a language's modal paradigm to the presence and type of variable-force modals in that language. Swedish has a dense modal paradigm, with separate lexical items for possibility, weak necessity, and strong necessity. We should expect, therefore, not to find any of those modals to be variable-force. There should not be room in the lexicon for any of them to vary.

And yet, Swedish $f\mathring{a}$ receives, depending on its context, interpretations ranging from possibility to strong necessity. This is unexpected, as the stronger modals should block $f\mathring{a}$ from strengthening through the lexical pressure they exert. But the stronger interpretations of $f\mathring{a}$ are not blocked, evidently.

From a pure mechanical standpoint, of course, it's entirely possible to derive a

strong interpretation of a modal that has one or more scalar competitors. There are two options: One, essentially that taken by Jeretič (2021a), is to prune the stronger scalar alternatives, at which point the scaleless implicature will be able to take place (and on her account, in fact be forced to take place). Another option is that strengthening remains optional regardless, and that the stronger modals in Swedish are, for whatever reason, not scalar alternatives to fa at all, in any circumstances.

Given that these modals seem to be at least as much in the same paradigm as Kinande *anga* and *paswa* do, I favor the pruning story in this case, but then we are met with a pressing question: Why is it possible to prune the stronger modals in Swedish, but not in Kinande? If this pruning is in principle possible, why do we not see variable-force behavior in other dense modal paradigms?

I don't have a completely satisfactory answer for why this kind of typological exception could occur. The is no clear generalization that seems to perfectly predict both the behavior of Swedish and Kinande: we expect something should prevent Kinande anga from being fully variable, but whatever mechanism does so needs to allow Swedish fa to be variable in very similar contexts. Add into the mix the facts of languages like English, which have dense paradigms and possibility modals that are not variable-force at all, and we seem to be stymied on all sides.

A possible way forward may be to say that Swedish f_a^a acts as a kind of superordinate term over Swedish modals in ways that none of the modals in Kinande and English do. An analogy could be the English word rain, which can be used to refer to any kind of liquid precipitation, even though there are a plethora of other related terms subsumed under the concept of rain: drizzle, sprinkle, downpour, among many others. Most of the time, though, no one will disagree with if you say that it's raining when technically the most precise term would be that it was drizzling; those that do, under some protestation that a drizzle is "not real rain", would typically still concede that a drizzle is, nevertheless, a type of rain. Perhaps Swedish f_a^a works in a similar way, acting as a superordinate term over the set of modal terms, and so the presence of those subordinate terms do not impinge on the possibility of using $f\mathring{a}$ in relatively strong contexts.

English and Kinande, and in fact all of the other languages examined here, on the other hand, have a relationship in their paradigm that resembles those in most functional spaces: Even if English *can* is definable as an elsewhere term, being the semantically weakest modal in its paradigm, it is unacceptable to use it in the contexts that warrant *must* and *should* precisely because those other modals are available. While *can* could potentially be superordinate, and could be used truthfully in necessity contexts, it is unable to be felicitous in those contexts because of the restrictions of the English lexicon. This is the arrangement that seems more common in the functional space.

This means, in the end, that the generalization that dense modal inventories do not allow variable-force modals is a relatively weak generalization. Given that it relies on the quirkier end of lexical organization, this is not as concerning as it might be. What's more, even as a weak generalization, lexical competition at least lets us state that languages like Swedish are unexpected, even though they are not impossible. Given the way that functional morphemes tend toward more specificity, Swedish's organization should be uncommon.

4.4 Comparisons and connections

There are two typologies of modality extant in the literature that the typology here can be usefully compared with the typology I propose here. I discuss typologies by Nauze (2008) and Jeretič (2021a), which have different typological goals, but it is worthwhile to consider how the generalizations might interact, and whether our typologies are compatible.

4.4.1 Nauze (2008)

Nauze (2008) gives a typological study of modal expressions across a number of languages, summarizing them in terms of their possible modal force and flavor uses, and further examines the possible scope relations between modal expressions of different flavors, giving a typology of modal flavor/scope relations.

The biggest and most obvious difference between the typology proposed here and the one in Nauze (2008) is that my typology is concerned exclusively with modal force, and Nauze's is concerned almost entirely with modal flavor and the scope relations between expressions of different modal flavors. Nauze does, however, draw one typological conclusion for modal force based on their typology (Nauze, 2008, p. 222):

Modal elements can only have more than one meaning along a unique axis of the semantic space: they either vary on the horizontal axis and thus are polyfunctional in the original sense of expressing different types of modality or they vary on the vertical axis and can express possibility and necessity, but they cannot vary on both axes.

At the time of its writing, this was a reasonable generalization to assert, as the only attestation in the literature for variable-force modals was St'át'imcets (also known as Lillooet Salish), which notably has modals that are variable-force, but fixed flavor. The other languages with adequate modal descriptions at the time largely had the inverse property, where they were variable-flavor, but fixed force. It might be reasonable to assume that, barring any attestations to the contrary, that languages only ever were variable on one axis or the other, but never both.

Subsequent research, however, has shown that this generalization doesn't quite hold. Various languages, including Washo (Bochnak, 2015), Siona (Jeretič, 2021a), and Kinande all have modals that are variable in both force and flavor. It can be shown now that all logical combinations of variable/fixed flavor and force can be attested,

with languages like English having modals that are fixed force and variable flavor, languages like St'át'imcets (Rullmann et al., 2008) with fixed flavor and variable force, languages like Paciran Javanese (Vander Klok, 2012) that are fixed for both force and flavor, and then languages like Washo which are variable in both force and flavor. Concerns that a language being variable on both axes causes more ambiguity and vagueness than a language can tolerate appear to be exaggerated.

Very little is lost from Nauze's typology if this generalization is dropped, however. The rest of the work is concerned with the scope relations between different modal flavors, where they find that, regardless of the syntactic status of any of the items, epistemic modal items outscope participant-external modal items (such as deontic modals), which in turn outscope participant-internal modal items (such as ability modals). Their conclusions are based not only on the core modal paradigms in each of the studied languages, but also on interactions with any modally-interpreted adverbs, mood markers, and other more complex lexical constructions.

This marks the other significant difference between our typologies: the typology I present here is concerned with modal force within a relatively constrained definition of a modal paradigm within a language, setting adverbs and other more peripheral members of a modal system aside. The reason for this is that by and large, the morphemes that appear to matter for drawing scalar and scaleless implicatures don't appear to include things like adverbs. Looking at St'át'imcets, the presence of an unambiguous possibility adverb in the epistemic space doesn't prevent the epistemic modal enclitic k'a from being variable:

- (180) St'át'imcets epistemic adverbs and enclitics (cited in Nauze, 2008, pp. 79–80):
 - a. sxək ^²ac'xən-kán-kɨ-tu k^wu-c'í^² (adverb, Van Eijk, 1997) perhaps see-(3SG-)1SG-kəɨ-tu^² DET-deer "Perhaps I might see a deer."
 - b. wá²-k'a k'wzúsəm (enclitic, necessity Van Eijk, 1997)
 IMPF-k'a work
 "He must be at work (that's why he's not here)"

c. wá[?]-k'a sə́na[?] q^wənúx^w (enclitic, possibility; Matthewson et al., 2005) IMPF-k'a ADV sick "He may be sick (maybe that's why he is not here)"

 $S\check{x}\partial k$ is capable of expressing epistemic possibility, and pragmatically we would think that k'a should leave the possibility contexts to $s\check{x}\partial k$ and occupy only the necessity contexts. This would be expected on any theory that sets $s\check{x}\partial k$ and k'a as alternatives to one another, regardless of whether those alternatives are resolved in a grammaticalized way (e.g., via exhaustification) or through global pragmatic means. We're forced to conclude that k'a does not have the adverb as a competitor.

This is hardly surprising: most theories of scalar alternatives require that scale-mates be of the same syntactic category, or otherwise derivable in some way that does not leave space for adverbs to be scalemates to "core" modal items. As discussed previously in this chapter, I argue that scalemates can at least sometimes be of different syntactic categories, as in Kinande. Even so, there must be a limit somewhere, and a line to draw is one that excludes adverbs from competition with modal verbs and modal particles.

Since such a distinction is so common cross-linguistically, I have not considered adverbs as part of the modal paradigms relevant for my typology. This is not due to any presumption that adverbs are not worth looking at, or that they might not themselves exhibit interesting characteristics with regard to modal force, only that they do not seem to participate as core members of a language's modal paradigm, and so are not clearly providing the kind of lexical pressure on the other modals of a language to potentially inhibit variable-force.

4.4.2 Comparison to Jeretič (2021a)

Another quite recent typology involving modals is that proposed in Jeretič (2021a), and like the typology I propose here, it is also concerned at least in part with accounting for some variable-force modals. Its scope differs from mine in that on the

one hand, her typology is not aiming to give a full account of modal force. On the other, it does aim to account for not only cases of modals strengthening from possibility to necessity, but also the phenomenon of wide-scope necessity modals, where a necessity modal, even when apparently under negation, yields a "necessarily not" meaning rather than a "not necessarily" one (English *should*, as discussed above, is one of these modals, though Jeretic takes a somewhat different approach to weak necessity than I do here). What's more, Jeretič is interested in accounting for when scaleless implicatures are obligatory and when they are optional. I on the other hand have focused more on when scaleless implicatures are possible or impossible. As a result our typologies look quite different, as they should, being concerned with different phenomena, even if they use the same core analysis.

The natural question to then ask is whether the typologies and the analyses are compatible with each other. In principle, one would think so, and hope we could reap the benefits of predicting the full span of where scaleless implicatures are obligatory, when they are optional, and when they are impossible. Unfortunately, it's here that we begin to run into points of friction.

Jeretič predicts an obligatory scaleless implicature when a modal that projects subdomain alternatives has no scalemate, and an optional one when such a modal does have a scalemate. The typology presented here, on the other hand, predicts that a scalemate for a modal will not make a scaleless implicature optional, but instead that it will block the implicature entirely.

Jeretič does in fact have an issue with her optional scaleless implicatures, however. She assumes, on the one hand, that exhaustification is obligatory on an utterance, and seems to take subdomain alternatives as consistently projected if an item is capable of doing so. The reason optional scaleless implicatures are optional, for her, is that the presence of a stronger scalemate, if not pruned from the alternatives, will prevent the strengthening of the modal, by drawing a scalar implicature.

However, as discussed in Chapter 3, the mere presence of a stronger scalemate

is insufficient to adequately prevent strengthening, if the subdomain alternatives are present. While it's true that the stronger scalemate will prevent the scaleless implicature from achieving actual full universal quantification, if exhaustification still has access to the subdomain alternatives, and applies until it does not have any further effect, then in every case of an optional scaleless implicature item where its stronger scalemate is not pruned, we expect to derive the implausible "all but one" meaning, a reading that we do not observe (per discussion in Chapter 3.3.2). And while it seems quite natural to say that the inclusion of the strong scalemate should block us from going even that far, there needs to be a mechanical method for ignoring the subdomain alternatives for optional scaleless implicature items.

However, if we assume, following Chierchia (2013), that some lexical items obligatorily project subdomain alternatives (such as *any* and *ever*), and others only optionally do (like *some*), then we can extend this to modals and derive optional and obligatory scaleless implicatures straightforwardly. Obligatory scaleless implicature items obligatorily project subdomain alternatives, and optional ones optionally do. When an optional scaleless implicature item (like Swedish *fa*) projects its subdomain alternatives, any scalemate it has will need to be pruned, in order to avoid the pragmatically grotesque "all but one" meaning. At this point it isn't a mechanical necessity, in that technically the unwanted reading can be derived, but we can expect that interpretation to be rare or even unattested, since it's far from likely that a language user would intend such a meaning (what does it mean to say something is true in every world in the domain but one?).

I cannot give a thorough treatment of wide-scope necessity modals here, but perhaps they could be incorporated as follows: for those that obligatorily take wide scope, they of course obligatorily project subdomain alternatives. Since they are already universal quantifiers, this isn't noticeable in positive contexts. When negated, however, those alternatives come into play, and potentially run into issues with a possibility scalemate to the necessity modal, as Jeretic observes. But rather than saying

that in cases of obligatory wide-scope necessity modals that the possibility modal is unable to act as a scalemate to the necessity one, we can say that the obligatory subdomain alternatives give two options: derive an all-but-one reading with the scalar implicature from the possibility modal, or prune the possibility modal and derive a wide-scope necessity reading. And for whatever reason the all-but-one reading must be avoided if at all possible, and so the possibility modal is pruned from the necessity modal's alternative set, and all is well.

The presence of a scalemate doesn't directly affect whether a scaleless implicature is obligatory or optional, in that case. The presence of a stronger scalemate is a distinct issue from whether a lexical item can, or has to, project subdomain alternatives. However, we might expect some tendencies to emerge from the more general pragmatic pressure it exerts. But these are the kinds of generalizations that have exceptions, because they are not enforced by the formal mechanisms, but rather by the relatively malleable facts that govern the choice of saying one thing over another.

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