A Semantic Theory of Redundancy

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Abstract

Theorists trying to model natural language have recently sought to explain a range of data by positing covert operators at logical form. For instance, many contemporary semanticists argue that the best way to capture scalar implicatures is through the use of such operators. We take inspiration from this literature by developing a novel operator that can account for a wide range of linguistic effects that until now have not received a uniform treatment. We focus on what we call *redundancy effects* which occur when attitude verbs and modals imply that certain bodies of information are unsettled about various claims. We explain three pieces of data, among others: diversity inferences, ignorance inferences, and free choice inferences. Our account yields an elegant model of redundancy effects, and has the potential to explain a wide range of puzzles and problems in philosophical semantics.

1 Introduction

Theorists trying to model natural language have recently sought to explain a range of data by positing covert operators at logical form. For instance, many contemporary semanticists argue that the best way to capture *scalar implicatures* is through the use of such operators.[1] To illustrate, consider a

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[1]See for example Fox 2007; Chierchia *et al.* 2009; Gajewski & Sharvit 2012; Chierchia *et al.* 2012; Crnič *et al.* 2015; Spector 2016; Bar-Lev & Fox 2020; Del Pinal forthcoming among many others.

canonical implicature-generating sentence such as (1-a), and its implicature in (1-b):

(1) a. Ann ate some of the cake.
  b. $\implies$ Ann didn't eat all of the cake.

The idea is that (1-a) is given the representation in (2), where O is the specified operator:

(2) $\mathsf{O}(\mathsf{x\ ate\ some\ cake})$

O is designed so that that (2) entails that Ann didn't eat all of the cake, which explains the inference in (1-b).

We take inspiration from this literature by developing a novel operator that can account for a wide range of linguistic effects that until now have not received a uniform treatment. We focus on what we call *redundancy effects* which occur when attitude verbs and modals imply that certain bodies of information are unsettled about various claims. We explain three pieces of data, among others: diversity inferences, ignorance inferences, and free choice inferences. §3 lays out each of these data points carefully. But before getting into details, we introduce some examples to give a general sense of the phenomenona we're interested in, and how our proposal works.

As many theorists have observed, attitude verbs such as 'hope' generally require that the subject's beliefs neither entail nor contradict the prejacent of the report, as in (3). That is, the subject's beliefs need to be "diverse" with respect to the prejacent.

(3) a. The detective hopes that Ann committed the crime.
  b. $\implies$ The detective thinks it's possible that Ann committed the crime, and the detective thinks it's possible that Ann didn't commit the crime.

And several authors have recently noted that attitude verbs with disjunctive complements characteristically generate an ignorance inference, as in (4):

(4) a. The detective believes that Ann or Bill committed the crime.
  b. $\implies$ The detective thinks it's possible that Ann committed the crime.

Finally, it is well known that disjunctions in the scope of deontic modals "distribute", as illustrated by (5). This is the "free choice" inference.

(5) a. Mary may read *Ulysses* or *Madame Bovary.*
b. $\Longrightarrow$ Mary may read *Ulysses* and Mary may read *Madame Bovary.*

In order to capture these effects, we posit a *redundancy operator* $\mathcal{R}$ in logical form. This operator makes use of the notion of an expression's *local context* which, roughly put, is the information relevant for the evaluation of that expression. $\mathcal{R}$ checks that neither its complement nor the negation of its complement is entailed by its local context. We explain redundancy effects by inserting $\mathcal{R}$ in the appropriate place. For instance, to a first approximation the local context of the complement of a hope report is the subject's beliefs. (3-a) is given the following representation:

(6) $\mathsf{x\ hopes\ }\mathcal{R}\mathsf{A}$

It follows that (6) is true only if 'Ann committed the crime' is non-redundant with respect to the detective's beliefs, i.e. only if there are some situations compatible with the detective's beliefs where Ann is guilty, and some situations compatible with the detective's beliefs where Ann is innocent. This accounts for the inference in (3-b). More generally, our account yields an elegant model of redundancy effects, and has the potential to explain a wide range of puzzles and problems in philosophical semantics.

The paper is structured as follows. §2 sets the stage by discussing sophisticated treatments of scalar implicatures. §3 presents a family of redundancy effects, and observes that these effects are local and optional. §4 develops our theory of redundancy. §5 compares the redundancy approach with the grammatical theory of scalar implicature. §6 discusses the distribution of the redundancy operator. §7 concludes.

2 A semantic theory of scalar implicatures

In this section we briefly review the way many contemporary semanticists model scalar implicatures. This discussion will provide helpful background for our positive proposal, since many of the considerations that shape the literature on scalar implicatures have analogues in the domain of redundancy effects.

To begin, we repeat example (1-a) featuring the implicature trigger 'some':

(1) a. Ann ate some of the cake.
b. $\Longrightarrow$ Ann didn't eat all of the cake.

An utterance of (1-a) usually gives rise to the inference in (1-b). This is surprising given that 'some' is usually taken to be equivalent to 'not none', and Ann failing to eat none of the cake is perfectly compatible with her eating all of it. Somehow the meaning of 'some' in (1-a) gets "strengthened" to 'some but not all'. This is the scalar implicature seen in (1-b).[2]

Theorists have drawn attention to two important features of scalar implicatures which serve to constrain possible explanations of these effects. The first crucial data point is that scalar implicatures are *local*: they can enter into interesting scopal relations with other expressions, and can arise in embedded environments. For instance, implicatures can take scope below operators such as quantifiers:[3]

(7) a. Every professor who fails some of the students will be put on probation; but every professor who fails all of the students will be fired.
b. = Every professor who fails *some but not all* of the students will be put on probation...

(7-a) is perfectly acceptable, and on its most natural reading is equivalent to (7-b) where the implicature triggered by 'some' scopes under the quantifier 'every', and occurs embedded in the restrictor of the quantifier. The implicature must occur embedded here, since restrictors of universal quantifiers are downward monotonic environments.[4] So, if 'some' simply meant 'not none' in (7-a), then the first sentence would entail 'Every professor who fails all of the students will be put on probation', which contradicts the second

[2]The basic observation here goes back at least to Grice 1975.

[3]The following example is adapted from Chierchia *et al.* 2012, 2307. See their paper for many other examples of embedded implicatures.

[4]An operator O is downward monotonic just in case $\mathsf{p} \models \mathsf{q}$ implies $\mathsf{Oq} \models \mathsf{Op}$. For instance, since being a boy who wears a hat entails being a boy, 'Every boy ran' entails 'Every boy wearing a hat ran'.

sentence. In other words, if the implicature didn't occur embedded in (7-a), then the sentence would be incoherent.

Locality is an important feature because it has convinced many semanticists that scalar implicatures have a semantic source. Although pragmatic accounts remain popular among philosophers, it is difficult to see how pragmatic explanations can account for embedded implicatures.[5] Pragmatic theories tend to operate at the level of whole clauses, and it is not straightforward to extend such accounts to subclausal constituents. More generally, the ability of an effect to enter into interesting scopal relations with other expressions is a signal that the relevant phenomenon has a semantic, rather than pragmatic, basis. Consequently, the locality data has compelled semanticists to develop semantic mechanisms which can explain implicature generation.[6]

This brings us to the second important property of implicatures: they are *optional*. That is, they tend to disappear in certain environments.[7] For instance, under negation:

(8) a. Ann didn't eat some of the cake.
 b. $\neq$ Ann ate all of the cake, or none of it.

An utterance of (8-a) would usually be taken to suggest that Ann didn't eat all of the cake. However, if the implicature was present and scoped below negation, then (8-a) would be equivalent to (8-b). And (8-b) does not entail that Ann didn't eat all of the cake, it only licenses a weaker inference.

Optionality is important because it constrains the choice of semantic mechanism responsible for implicatures. For instance, it rules out a flat-footed semantic account which simply builds implicatures into the meaning of the relevant trigger. Consider a theory on which 'some' simply means 'some but not all'. This straightforwardly explains examples such as (1-a) and the locality data in (7-a), but it doesn't explain optionality. For if scalar

[5]Perhaps the most influential pragmatic treatment of scalar implicatures stems from the work of Grice 1975 himself. For developments of the Gricean view, see for example Horn 1972; Fauconnier 1975; Sauerland 2004; Spector 2006.

[6]The problem that locality poses for pragmatic theories is clearly articulated by Chierchia *et al.* 2012. For a defense of the pragmatic approach to scalar implicatures from data involving embedded effects, see Geurts 2010.

[7]This observation goes back at least to Grice, who noted that implicatures can be "cancelled" in certain settings. Also see Chierchia *et al.* 2012; Fox & Spector 2018; Enguehard & Chemla 2019 for discussion.

implicatures are essentially built into the standing meaning of the relevant trigger, then it is mysterious how these effects could disappear in certain environments.

In response, theorists have tried to satisfy the twin constraints of locality and optionality by positing an *optionally insertable operator* at logical form.[8] The general idea is that since this operator is syntactically realized, it can enter into scopal relations with other expressions, and thus is able to explain locality. And since the operator is optional, and not mandatory, it straightforwardly explains optionality: the thought is simply that the operator will generally not appear in certain environments, e.g. under negation. This operator is known as the *exhaustification operator*, denoted EXH. Roughly put, $\mathsf{EXH(p)}$ is true iff p is true and every alternative to p is false.[9] To illustrate, (1-a) is given the representation in (9-a):

(9) a. $\mathsf{EXH(x\ ate\ some\ cake)}$
b. $\Longrightarrow \neg\mathsf{(x\ ate\ all\ cake)}$

Let us suppose that the only alternative to 'Ann at some of the cake' is 'Ann ate all of the cake'. Then (9-a) is true only if Ann didn't eat all of the cake, which is exactly the desired implicature in (9-b).

To illustrate locality, the idea is that the first conjunct in (7-a) receives the following form, where EXH scopes below the quantifier 'every':

(10) $\mathsf{Every\ [EXH(x\ fails\ some\ student)][x\ probation]}$

(10) is true just in case every professor who fails some but not all of the students will be put on probation, which is the target reading of the first conjunct in (7-a).

To summarize, semanticists have drawn attention to two important properties of scalar implicatures, namely locality and optionality. In order to capture both of these features, theorists have rejected the idea that scalar implicatures are pragmatic. Instead, these authors maintain that at least some scalar implicatures are generated in the grammar through the optional

[8]See for example Fox 2007; Chierchia *et al.* 2009; Gajewski & Sharvit 2012; Chierchia *et al.* 2012; Crnič *et al.* 2015; Spector 2016; Bar-Lev & Fox 2020; Del Pinal forthcoming among many others.

[9]There are several choice points in the precise articulation of EXH. See §5 for one popular way of making it more explicit.

insertion of an operator in logical form.[10]

3 Redundancy effects

Now that the reader has a sense of modern treatments of scalar implicatures, in this section we present the major data points for our theory. We are interested in three types of *redundancy effects*: diversity inferences, ignorance inferences, and free choice inferences. In each case, an attitude verb or modal operator implies that a claim is non-redundant with respect to some body of information. In §3.4, we observe that each effect is local and optional. These two properties motivate our theory, which posits a semantic redundancy operator that is inserted locally and optionally in logical form.

3.1 Diversity

Many theorists have observed that certain attitude verbs carry a diversity constraint. The prejacent must be possible but not necessary with respect to the relevant body of information. For instance, consider:

(11) a. [*Context*: The detective believes that Ann didn't commit the crime.]
✗ He hopes that/fears that/wonders whether she did.

b. [*Context*: The detective believes that Ann committed the crime.]
✗ He hopes that/fears that/wonders whether she did.

These sentences are infelicitous in context.[11] This provides evidence for a diversity constraint relative to the subject's beliefs: where V is a non-doxastic attitude verb, x Vs p is acceptable only if x neither believes p nor

[10]Although we are sympathetic to the grammatical theory of scalar implicatures, it is worth being clear that our positive proposal doesn't hang on the success of this theory. One could coherently endorse our account while maintaining that implicatures are best handled by a pragmatic mechanism. We have focused on the grammatical theory of scalar implicatures only because it offers a fruitful analogy to our own project.

[11]At least when we understand 'belief' to be "full belief" or surety. There is a use of 'believe' where one can say 'Wu believes he'll lose' even when he doesn't outright believe he'll lose (e.g. Wu is 60-40 confident he'll lose). When we're using 'believe' in this way, 'Wu believes he'll lose but he hopes he'll win' is acceptable. See Williamson 2020 for arguments to the effect that the default interpretation of 'believe' in natural language is that of full belief.

believes $\neg \mathsf{p}$.[12]

3.2 Ignorance inferences

Our second redundancy effect is that when logically complex sentences scope under attitude verbs, they give rise to ignorance inferences.[13] For example, (12-a) implies (12-b) and (12-c):

(12) a. The detective believes that/hopes that/fears that/wonders whether Ann or Bill committed the crime.
  b. $\Longrightarrow$ The detective thinks it's possible that Ann committed the crime.
  c. $\Longrightarrow$ The detective thinks it's possible that Bill committed the crime.

More generally, embedded disjunctions rule out belief that the disjuncts are false: x Vs $\mathsf{A} \vee \mathsf{B}$ implies that both A and B are doxastically possible for x.

Embedded conjunctions generate a similar ignorance inference:

(13) a. Mary hopes that/fears that/wonders whether Ann brought apple pie and Bill brought blueberry pie.
  b. $\Longrightarrow$ Mary thinks it's possible that Ann didn't bring apple pie.
  c. $\Longrightarrow$ Mary thinks it's possible that Bill didn't bring blueberry pie.

More generally, embedded conjunctions rule out belief that the conjuncts are true: x Vs $\mathsf{A} \wedge \mathsf{B}$ implies that both $\neg\mathsf{A}$ and $\neg\mathsf{B}$ are doxastically possible for x.

3.3 Free choice inferences

Our final collection of redundancy effects involve disjunctions in the scope of modals. First, there is the "free choice" inference exhibited by possibility modals: disjunctions in the scope of these modals distribute.[14]

[12] Belief constraints for attitudes are widely endorsed, e.g. Heim 1992; von Fintel 1999; Levinson 2003; Villalta 2008; Crnič 2011; Lassiter 2011; Rubinstein 2012; Condoravdi & Lauer 2016; Pearson 2016; Grano 2017; Phillips-Brown 2018; Blumberg & Holguín 2019; Jerzak 2019; Pasternak 2019; Phillips-Brown Forthcoming.

[13] See Roelofson & Uegaki 2016; Blumberg 2017; Cremers *et al.* 2019.

[14] See among others Kamp 1974, 1978; Zimmermann 2000; Kratzer & Shimoyama 2002; Asher & Bonevac 2005; Geurts 2005; Schulz 2005; Simons 2005; Alonso-Ovalle 2006; Aloni

(14) a. Mary may read *Ulysses* or *Madame Bovary.*
b. $\Longrightarrow$ Mary may read *Ulysses* and Mary may read *Madame Bovary.*

This is surprising, since the inference isn't truth preserving according to standard semantics for possibility modals.

Disjunctions under necessity modals generate a similar inference, which we will call "Ross's inference".[15] As with free choice, disjunctions in the scope of necessity modals suggest that each disjunct is possible.

(15) a. Mary is required to read *Ulysses* or *Madame Bovary.*
b. $\Longrightarrow$ Mary may read *Ulysses* and Mary may read *Madame Bovary.*

This is again surprising given standard semantics for necessity modals, since on these accounts (15-a) doesn't entail (15-b).

3.4 Locality and optionality

We now observe two features of redundancy effects that constrain the space of viable theories. These features have obvious analogues in the domain of scalar implicatures.

3.4.1 Locality

First, redundancy effects occur in embedded positions, taking scope below operators.

(16) [*Context*: There are three detectives and several suspects. All three detectives most desire that Ann committed the crime, since they already have her in custody. One detective is sure that Ann did it, but the others don't know anything yet.]
✓ Exactly two detectives hope that Ann committed the crime.

(16) sounds true. This requires that the diversity constraint takes scope below the quantifier 'exactly two detectives', so that (16) means something

2007; Fox 2007; Klinedinst 2007; Ciardelli *et al.* 2009; Chemla 2009; Barker 2010; Franke 2011; Aher 2012; Roelofsen 2013; Charlow 2015; Fusco 2015b; Starr 2016; Willer 2017; Romoli & Santorio 2017; Aloni 2018.

[15] See among others Ross 1941; Chierchia *et al.* 2009; Cariani 2013; Fusco 2015a.

like ‘There are exactly two detectives x such that x desires that Ann did it, and x thinks it’s possible Ann did it and x thinks it’s possible Ann didn’t do it’.

Ignorance inferences can also take local scope. Here are a few examples:

(17) [*Context*: There are three detectives, and two possible suspects: Ann and Bill. One detective has already ruled out Ann, but the others haven’t ruled out either Ann or Bill.]
✓ Exactly two detectives believe that Ann or Bill committed the crime.

(18) [*Context*: There are three detectives and three possible suspects: Ann, Bill, and Carol. The detectives have Ann and Bill in custody, but can’t find Carol. One detective is sure that Ann didn’t do it, but the others don’t know anything yet.]
✓ Exactly two detectives hope that Ann or Bill committed the crime.

(19) [*Context*: There are three detectives and three possible suspects: Ann, Bill, and Carol. One detective has already ruled out Ann, but the others haven’t ruled out anyone.]
✓ Exactly two detectives wonder whether Ann, Bill or Carol committed the crime.

(20) [*Context*: Three of Mary’s friends are at a potluck dinner. All three most prefer apple pie and blueberry pie to any other type of pie. One already knows that Ann brought apple pie, but the others don’t know anything about who brought what.]
✓ Exactly two of Mary’s friends hope that Ann brought apple pie and Bill brought blueberry pie.

On its most salient reading (17) means roughly ‘There are exactly two detectives x such that x thinks Ann or Bill did it, and x has ruled out neither Ann nor Bill’. And (20) means roughly ‘There are exactly two of Mary’s friends x such that x desires that Ann brought apple pie and Bill brought blueberry pie, and x thinks it’s possible both that Ann didn’t bring apple pie and that Bill didn’t bring blueberry pie’.[16]

[16] Cremers *et al.* (2019) experimentally tested and confirmed the locality of ignorance inferences for disjunctions embedded under ‘believe’ and ‘wonder’.

The locality of free choice is illustrated by (21-a):

(21) a. Every student may read *Ulysses* or *Madame Bovary.*
b. = Every student may read *Ulysses* and may read *Madame Bovary.*

On its most natural reading, (21-a) is equivalent to (21-b), where the free choice inference scopes below the quantifier 'every student'.[17]

Recall that the locality of scalar implicatures is an important feature because it has convinced semanticists that implicatures are a semantic phenomenon. Similarly, the locality of redundancy effects is a key data point for us because it militates against treating redundancy effects as purely pragmatic. For example, pragmatic principles generally explain why sentences that are predicted to be *true* on background semantics appear to be infelicitous. But it is more challenging to see how such principles could explain why sentences which are predicted to be *false* on background semantics sound true. For instance, note that (17) is predicted to be false on standard attitude semantics. For example, on a Hintikka-style entry for 'believe', *all three* detectives x are such that x satisfies the predicate 'believes Ann or Bill committed the crime'. It is not obvious how a pragmatic theory could account for why (17) is nevertheless acceptable.[18]

3.4.2 Optionality

The second important feature of redundancy effects is that they are optional. As with scalar implicatures, this means that the effects often disappear in certain environments, such as under negation. To see how diversity inferences are optional, consider the oddness of (22):

(22) ??The detective doesn't hope that Ann did it, because he knows she did.

[17] See Chemla 2009 for discussion.

[18] Roelofson & Uegaki 2016 also observe that the locality of ignorance effects pose a challenge for pragmatic theories of these inferences. We note that there are some effects whereby false sentences communicate true information, such as metaphor (Grice, 1989; Walton, 1993; Hoek, 2018) or so-called "loose talk" (Lasersohn, 1999; Lauer, 2016; Carter, 2021). But it is not clear that these effects are necessarily pragmatic in the relevant sense. If they are best explained by a pragmatic mechanism, then we leave as an open question how to marshall such resources to account for the locality data above.

But this sentence should be perfectly acceptable if the diversity constraint was operative and could be targeted by negation. For then the first conjunct would be equivalent to 'Either the detective doesn't desire that Ann did it, or he is sure that she did it, or he is sure that she didn't do it'.

Turning to ignorance inferences, observe that on the most natural reading of (23-a), it implies (23-b):

(23) a. The detective doesn't believe that Ann or Bill did it.
b. $\Longrightarrow$ The detective doesn't believe that Ann did it and the detective doesn't believe that Bill did it.

But if the ignorance effect generated by the embedded disjunction was present in (23-a), then this inference would not be licensed.[19]

As for free choice and optionality, note that (24-a) is equivalent to (24-b) and not (24-c):

(24) a. Mary may not read *Ulysses* or *Madame Bovary*.
b. = Mary may not read *Ulysses* and Mary may not read *Madame Bovary*
c. $\neq$ Mary may not read *Ulysses* or Mary may not read *Madame Bovary*

But again, (24-a) would be equivalent to (24-c) if the free choice effect was present and scoped below negation.[20]

Recall that optionality is an important feature of scalar implicatures because it puts constraints on the choice of semantic mechanism responsible for implicature generation. For instance, it rules out simple semantic accounts that build the implicature into the meaning of the relevant trigger. Similarly, optionality is a challenge for semantic accounts of redundancy effects that tie redundancy to the meaning of either the attitude verb/modal or the relevant connective. For if the attitude verb/modal or the connectives contribute a redundancy condition, then why should the effect disappear under certain operators, such as negation?[21]

[19] Cremers *et al.* (2019) experimentally confirm that ignorance effects are optional, disappearing under negative quantifiers.

[20] See Alonso-Ovalle 2006; Starr 2016; Romoli & Santorio 2019 among others for the disappearance of free choice under negation.

[21] We note that Aher 2012; Starr 2016; Willer 2017; Aloni 2018 develop semantic accounts of free choice that correctly predict it disappears under negation. However, it is difficult

In light of locality and optionality, what we propose parallels sophisticated treatments of scalar implicatures: we suggest that redundancy effects are generated by the optional insertion of operators in logical form. Before turning to our positive proposal, however, we pause to head off a tempting, but ultimately unsatisfactory response to the examples motivating optionality.

3.4.3 Presupposition

One response to optionality would treat redundancy effects as presuppositions triggered by the attitude verb or modal. Since presuppositions project through various operators, this would allow redundancy effects to escape the scope of negation. There is precedent for this approach in the case of diversity: many authors maintain that diversity arises through a presupposition triggered by attitude verbs.[22] For instance, the idea is that a report $\mathsf{x\ hopes\ p}$ is defined only if x neither believes p nor believes $\neg\mathsf{p}$.

However, none of the redundancy effects we are interested in project like presuppositions. To see this, consider the sentence 'Ann stopped smoking' which uncontroversially presupposes that Ann smoked in the past. Consequently, (25-b)-(25-d), just as much as (25-a), entail that Ann smoked in the past (Chierchia & McConnell-Ginet, 2000):

(25) a. Ann stopped smoking.
b. Ann didn't stop smoking.
c. Did Ann stop smoking?
d. If Ann stopped smoking, then I bet Bill did too.

By contrast, diversity constraints do not project at all. None of (26-a)-(26-c) suggest that the detective has no opinion about whether Ann committed the crime:

(26) a. The detective doesn't hope that Ann did it.
b. Does the detective hope that Ann did it?
c. If the detective hopes that Ann did it, then he's going to be disappointed.

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to see how to extend these treatments to handle diversity effects, or conjunctive ignorance inferences.

[22] See for example Heim 1992; von Fintel 1999; Levinson 2003 among many others.

Indeed, it is usually unacceptable to explicitly express ignorance about whether a presupposition holds and then continue by using an expression which triggers that same presupposition. For instance, 'I have no idea whether Ann smoked in the past, but if she's stopped smoking, then Bill has too' sounds strange. By contrast, it is perfectly felicitous to say something like 'I have no idea who the detective suspects of committing the crime, but if he hopes Ann did it, he's going to be disappointed'. The same points can be made about the other ignorance effects we are interested in: they do not project from embedded environments.

4 A semantic theory of redundancy

In this section, we present our positive account of redundancy effects in several steps. We begin by introducing our central idea, which is that grammar provides an operator $\mathcal{R}$ which checks that its complement is true and non-redundant in its local context (§4.1). We then use this operator to explain our target redundancy effects (§4.2). Finally, we sketch an extension of the account that explains further redundancy phenomena (§4.3).

4.1 The redundancy operator

To begin, consider a simple example of infelicity generated by redundancy:

(27) #Ann is pregnant and Ann is pregnant.

(27) is unacceptable because the second conjunct 'Ann is pregnant' is redundant. It is redundant because 'Ann is pregnant' is already entailed by the first conjunct. That is, as it appears in the second conjunct of (27), 'Ann is pregnant' doesn't provide us with any new information.

In order to explain the unacceptability of (27), theorists have put forward pragmatic redundancy principles.[23] A popular constraint appeals to the notion of *local contexts* (Stalnaker, 1974, 1978; Schlenker, 2009; Mandelkern & Romoli, 2018). The local context of an expression p aggregates information that is relevant for the interpretation of p.[24] More specifically,

[23]See for example van Der Sandt 1992; Singh 2008; Schlenker 2009; Meyer 2013; Katzir & Singh 2014; Mayr & Romoli 2016.

[24]Local contexts play an important role in the explanation of various linguistic phenomena. Most prominently, they have been used to explain patterns of presupposition

this information is contributed by the meaning of particular expressions in p's syntactic environment as well as the common ground. For instance, the local context of q in a conjunction $\mathsf{p} \wedge \mathsf{q}$ carries the information provided by the first conjunct p along with whatever is in the common ground. Then the idea is that (27) is bad because its second conjunct is *redundant* in its local context, i.e. this conjunct is either entailed or contradicted in its local context. That's because this local context already carries the information that Ann is pregnant.

We propose that our target effects involving attitudes and modals also arise because of a redundancy constraint. However, for reasons discussed in §3.4.1, a pragmatic ban on asserting redundant sentences cannot explain how redundancy effects can scope under higher operators. In order to handle the locality of our target data, we grammaticalize the redundancy requirement. Rather than implementing redundancy checks through a pragmatic mechanism, we introduce a covert *redundancy operator*, $\mathcal{R}$. $\mathcal{R}\mathsf{p}$ says that p is true and non-redundant in its local context.

To make this precise, we work within an information sensitive framework, where local contexts are a parameter of semantic evaluation, denoted s.[25] The interpretation function is $[\![\cdot]\!]^{g,s,w}$, where g is an assignment, s is a context, and w is a world. For simplicity, we suppress assignment relativity throughout.

$\mathcal{R}$ requires that each input is non-redundant in its local context. To model this precisely, we introduce a dedicated third truth value #, for "undefined due to redundancy". We then say that $\mathcal{R}\mathsf{p}$ is defined only when p is non-redundant in its local context; when defined, $\mathcal{R}\mathsf{p}$ asserts p.

(28) a. $$[\![\mathcal{R}\mathsf{p}]\!]^{s,w} = \begin{cases} 1 & \text{if } \exists v, v' \in s : [\![\mathsf{p}]\!]^{s,v} \neq [\![\mathsf{p}]\!]^{s,v'} \ \&\ [\![\mathsf{p}]\!]^{s,w} = 1 \\ 0 & \text{if } \exists v, v' \in s : [\![\mathsf{p}]\!]^{s,v} \neq [\![\mathsf{p}]\!]^{s,v'} \ \&\ [\![\mathsf{p}]\!]^{s,w} = 0 \\ \# & \text{if } \neg\exists v, v' \in s : [\![\mathsf{p}]\!]^{s,v} \neq [\![\mathsf{p}]\!]^{s,v'} \end{cases}$$

With our redundancy operator on the table, we now specify how various connectives and operators (i) control the projection of redundancy, and (ii) manipulate the local context parameter.

projection. See for example Kartunnen 1974; Stalnaker 1974; Heim 1983; Schlenker 2009 for discussion.

[25]See Heim 1992; Veltman 1996; Yalcin 2007; Gillies 2010; Mandelkern 2019 for various implementations of this general idea.

We assume that undefinedness of our redundancy operator proliferates upwards through Boolean connectives, as in a Weak Kleene theory of projection. Any connective containing an undefined input is itself undefined. These undefinedness conditions will then be captured and filtered by attitudes and modals. To implement this proposal, we introduce the usual Weak Kleene operations on the truth values $v \in \{1, 0, \#\}$:

(29) a. $\mathsf{neg}(x) = \begin{cases} 1 - x & \text{if } x \in \{1, 0\} \\ \# & \text{otherwise} \end{cases}$

b. $\mathsf{conj}(x, y) = \begin{cases} Min(x, y) & \text{if } x, y \in \{1, 0\} \\ \# & \text{otherwise} \end{cases}$

c. $\mathsf{disj}(x, y) = \begin{cases} Max(x, y) & \text{if } x, y \in \{1, 0\} \\ \# & \text{otherwise} \end{cases}$

Regarding local contexts, we assume a standard algorithm (Heim, 1983; Schlenker, 2009). The local context for the first conjunct and first disjunct of a complex claim, and for a negation, is the global context. The local context for the right disjunct of a disjunction is the global context narrowed down to the worlds where the first disjunct is false. And the local context for the right conjunct of a conjunction is the global context narrowed down to the worlds where the first conjunct is true. It is useful to introduce the abbreviation that $[\![\mathsf{p}]\!]^s = \{w \mid [\![\mathsf{p}]\!]^{s,w} = 1\}$. Then:

(30) a. $[\![\neg\mathsf{p}]\!]^{s,w} = \mathsf{neg}([\![\mathsf{p}]\!]^{s,w})$

b. $[\![\mathsf{p} \wedge \mathsf{q}]\!]^{s,w} = \mathsf{conj}([\![\mathsf{p}]\!]^{s,w}, [\![\mathsf{q}]\!]^{s \cap [\![\mathsf{p}]\!]^s, w})$

c. $[\![\mathsf{p} \vee \mathsf{q}]\!]^{s,w} = \mathsf{disj}([\![\mathsf{p}]\!]^{s,w}, [\![\mathsf{q}]\!]^{s \cap [\![\neg\mathsf{p}]\!]^s, w})$

To illustrate, we can explain why (27) is unacceptable if we assume that $\mathcal{R}$ appears on the second conjunct, as in (31):

(27) Ann is pregnant and Ann is pregnant.

(31) $\mathsf{P} \wedge \mathcal{R}\mathsf{P}$

The local context for the second conjunct will contain the information carried by the first conjunct, namely that Ann is pregnant, and the whole sentence will be undefined.

Attitude verbs and modal operators contribute a local context in addition to their usual quantificational force. We suppose throughout that what a subject x believes at world w is represented by their *belief set* Bxw—the set of worlds compatible with everything x believes at w (Hintikka, 1962). The truth conditions of x $\mathsf{believes}$ p are that Bxw implies p. In addition, believe shifts the information state parameter for p to the agent's belief worlds (Yalcin 2007). Similarly with hope and fear. $Best(Bxw)$ denotes the best worlds in Bxw, as determined by x's subjective preference ordering at w.[26] x hopes p says that $Best(Bxw)$ implies p. x fears p says that $Best(Bxw)$ excludes p (von Fintel, 1999). In addition, each operator shifts the local context parameter to Bxw.

(32) a. $[\![\mathsf{x\ believes\ p}]\!]^{s,w} = 1$ if $\forall v \in Bxw : [\![\mathsf{p}]\!]^{Bxw,v} = 1; 0$ otherwise
b. $[\![\mathsf{x\ hopes\ p}]\!]^{s,w} = 1$ if $\forall v \in Best(Bxw) : [\![\mathsf{p}]\!]^{Bxw,v} = 1; 0$ otherwise
c. $[\![\mathsf{x\ fears\ p}]\!]^{s,w} = 1$ if $\forall v \in Best(Bxw) : [\![\mathsf{p}]\!]^{Bxw,v} = 0; 0$ otherwise

We interpret deontic modals analogously to hope, quantifying over the best worlds in an information state. Following Kratzer (2012), we let this information state be a contextually supplied domain of quantification Mw, itself sensitive to the world of evaluation w. Crucially though, we let the local context supplied by modal claims be the best worlds in Mw, not Mw itself:

(33) a. $[\![\mathsf{may\ p}]\!]^{s,w} = 1$ if $\exists v \in Best(Mw) : [\![\mathsf{p}]\!]^{Best(Mw),v} = 1; 0$ otherwise
b. $[\![\mathsf{must\ p}]\!]^{s,w} = 1$ if $\forall v \in Best(Mw) : [\![\mathsf{p}]\!]^{Best(Mw),v} = 1; 0$ otherwise

It is worth bringing out an important feature of our semantics. Although undefinedness due to redundancy projects through the Boolean connectives, it does not project through attitudes and modals. This is ensured by the "0 otherwise" clauses in these entries. For instance, if $[\![\mathcal{R}\mathsf{p}]\!]^{Bxw,v} = \#$, for some v, then $[\![\mathsf{x\ believes}\ \mathcal{R}\mathsf{p}]\!]^{s,w} = 0$, for any s. As discussed in §3.4.2, this is exactly as it should be, since redundancy effects do not project like regular presuppositions.[27]

[26] This means that $Best$ itself is sensitive to x and w, but we suppress this complexity throughout.

[27] One could incorporate regular presuppositions into this framework by introducing an additional truth-value $\#_p$ for "undefined due to presupposition failure". The entries for the redundancy operator, attitude verbs and modals could then be enriched so that this sort of undefinedness projects through these expressions. Spector (2016) uses a similar technique to model the interaction between undefinedness from presupposition and indeterminacy from vagueness.

With our semantics in place, let's turn to our motivating data.

4.2 Applications

4.2.1 Diversity inferences

To explain diversity inferences involving attitudes, we let $\mathcal{R}$ scope underneath attitude verbs. For instance, we represent (34-a) as (34-b):

(34) a. The detective hopes that Ann committed the crime.
b. $\textsf{x hopes } \mathcal{R}\textsf{A}$

(34-b) entails that the detective is agnostic about whether Ann committed the crime, as the following derivation shows:

(35) a. $[\![\textsf{x hopes } \mathcal{R}\textsf{A}]\!]^{s,w} = 1$ iff
b. $\forall v \in Best(Bxw) : [\![\mathcal{R}\textsf{A}]\!]^{Bxw,v} = 1$ iff
c. $\forall v \in Best(Bxw) : [\![\textsf{A}]\!]^{Bxw,v} = 1 \ \& \ \exists u, u' \in Bxw : [\![\textsf{A}]\!]^{Bxw,u} \neq [\![\textsf{A}]\!]^{Bxw,u'}$ iff
d. $[\![\textsf{x hopes A} \wedge \neg(\textsf{x believes A}) \wedge \neg(\textsf{x believes } \neg\textsf{A})]\!]^{s,w} = 1$

In this derivation, the redundancy operator $\mathcal{R}$ applies to $\textsf{A}$ and requires that $\textsf{A}$ is non-redundant in the local context of $\mathcal{R}\textsf{A}$. Since $\mathcal{R}\textsf{A}$ occurs in the scope of $\textsf{hope}$, the local context of $\mathcal{R}\textsf{A}$ is the detective's belief worlds. So the detective must be agnostic about Ann having committed the crime.

This theory also explains the locality of diversity effects. Consider:

(36) a. Exactly two detectives hope that Ann committed the crime.
b. $[\textsf{Exactly two detectives}]_1 \ 1 \ \textsf{x}_1 \textsf{ hopes } \mathcal{R}\textsf{A}$

(36-b) is true just in case there are exactly two detectives x such that (i) x's best belief worlds imply that Ann committed the crime; and (ii) x is agnostic about whether Ann committed the crime.[28]

At this point, a natural question arises as to the distribution of $\mathcal{R}$. Why should (34-a) take the logical form in (34-b)? We will return to this issue in

[28] Strictly speaking, the local context of the scope is a function from individuals to the global context s (Schlenker, 2009). But we can ignore the first individual argument here, since this is guaranteed to be saturated during the course of semantic evaluation—see Mandelkern 2019 for a detailed account of how generalized quantifiers interact with the local context parameter.

§6. For now our primary aim is to show that $\mathcal{R}$ has the power and flexibility to capture our target phenomena.

4.2.2 Ignorance inferences

Let us turn to our second redundancy effect: the ignorance inferences that are generated by disjunctions and conjunctions under the scope of attitude verbs. We again let the redundancy operator scope below attitude verbs. But now the redundancy operator also scopes below connectives like $\vee$ and $\wedge$. For instance, we represent (37-a) as (37-b):

(37) a. The detective hopes that Ann or Bill committed the crime.

b. $\mathsf{x\ hopes\ } \mathcal{R}\mathsf{A} \vee \mathcal{R}\mathsf{B}$

(37-b) generates the desired ignorance inferences. Where **A** denotes the classical proposition associated with A (the worlds where A is true paired with some information state):

(38) a. $[\![\mathsf{x\ hopes\ } \mathcal{R}\mathsf{A} \vee \mathcal{R}\mathsf{B}]\!]^{s,w} = 1$ iff

b. $\forall v \in Best(Bxw) : [\![\mathcal{R}\mathsf{A} \vee \mathcal{R}\mathsf{B}]\!]^{Bxw,v} = 1$ iff

c. $\forall v \in Best(Bxw) : [\![\mathsf{A} \vee \mathsf{B}]\!]^{Bxw,v} = 1\ \&\ \exists u, u' \in Bxw : [\![\mathsf{A}]\!]^{Bxw,u} \neq [\![\mathsf{A}]\!]^{Bxw,u'}\ \&\ \exists u'', u''' \in Bxw \cap \neg\mathbf{A} : [\![\mathsf{B}]\!]^{Bxw\cap\neg\mathbf{A},u''} \neq [\![\mathsf{B}]\!]^{Bxw\cap\neg\mathbf{A},u'''}$ iff

d. $[\![\mathsf{x\ hopes\ (A \vee B) \wedge \neg(x\ bel\ A) \wedge \neg(x\ bel\ \neg A) \wedge \neg(x\ bel\ A \vee B) \wedge \neg(x\ bel\ A \vee \neg B)}]\!]^{s,w} = 1$

e. $\implies [\![\mathsf{x\ hopes\ (A \vee B) \wedge \neg(x\ bel\ \neg A) \wedge \neg(x\ bel\ \neg B)}]\!]^{s,w} = 1$

Three parts of this derivation are significant. First, $\mathcal{R}\mathsf{A} \vee \mathcal{R}\mathsf{B}$ is evaluated relative to the local context of hope, which is Bxw, the agent's belief worlds. Second, $\mathcal{R}\mathsf{A} \vee \mathcal{R}\mathsf{B}$ requires that the redundancy checks imposed by $\mathcal{R}\mathsf{A}$ and $\mathcal{R}\mathsf{B}$ are both passed. Third, $\mathcal{R}\mathsf{B}$ is evaluated at its own local context, which is $Bxw \cap \neg\mathbf{A}$, the belief worlds where the first disjunct is false.

We can also explain the locality of these effects. We give (39-a) the following LF:

(39) a. Exactly two detectives hope that Ann or Bill committed the crime.

b. $\mathsf{[Exactly\ two\ detectives]_1\ 1\ x_1\ hopes\ } \mathcal{R}\mathsf{A} \vee \mathcal{R}\mathsf{B}$

We predict that (39-b) is true only if there are exactly two detectives x such that (i) x's best belief worlds imply that Ann or Bill committed the crime; and (ii) x's beliefs are consistent with each of Ann and Bill committing the crime.

As for belief reports, we represent (40-a) as (40-b), since (40-c) inconsistently requires both that Bxw implies $\mathsf{A} \vee \mathsf{B}$, and that $Bxw \cap \neg\mathbf{A}$ contains a world where $\neg\mathsf{B}$ is true (we will return to this point in §6):

(40) a. The detective believes that Ann or Bill committed the crime.
b. $\mathsf{x\ believes}\ \mathcal{R}\mathsf{A} \vee \mathsf{B}$
c. #$\mathsf{x\ believes}\ \mathcal{R}\mathsf{A} \vee \mathcal{R}\mathsf{B}$

(40-b) still implies that the agent is ignorant about each of A and B. In particular, the detective's treating B as possible is implied by the twin requirements that the detective believes $\mathsf{A} \vee \mathsf{B}$ and that the detective does not believe A.

We generate ignorance inferences for conjunctions through forms such as (41-b):

(41) a. Mary hopes Ann brought apple pie and Bill brought blueberry pie.
b. $\mathsf{x\ hopes}\ \mathcal{R}\mathsf{A} \wedge \mathcal{R}\mathsf{B}$

The redundancy operator requires Mary's belief worlds are undecided about Ann bringing apple pie, and that Mary's belief worlds where Ann brings apple pie are undecided about Bill bringing blueberry pie.[29]

[29]The ignorance inference generated by conjunctions under attitudes can sometimes be suspended if 'and' is focused. For instance, although (i-a) is infelicitous, (i-b) sounds better:

(i) a. ??Mary knows that Ann brought apple pie, but she hopes that Ann brought apple pie and Bill brought blueberry pie.
b. Mary knows that Ann brought apple pie, but she hopes that Ann brought apple pie AND Bill brought blueberry pie.

To our ears, it is much harder to do this with asymmetrically entailing conjunctions. For instance, 'Mary hopes that Ann is pregnant and expecting a daughter' implies that Mary thinks it's possible Ann isn't pregnant. But 'Mary knows that Ann is pregnant, but she hopes that Ann is pregnant AND expecting a daughter' still sounds strange. We aren't sure what explains this contrast.

We also observe that although the ignorance effect arising from conjunctions embedded under non-doxastic attitudes can sometimes be suspended (as in (i-b)), it seems much harder to suspend the diversity constraint. One could potentially explain this by assuming that there is by default an $\mathcal{R}$ operator that appears at wide-scope to the complement of

4.2.3 Free choice inferences

As for modals, we can derive free choice effects through forms such as (42-b):

(42) a. Mary may have apples or bananas.
b. $\mathsf{may}\ \mathcal{R}\mathsf{A} \vee \mathcal{R}\mathsf{B}$

The derivation is as follows:

(43) a. $[\![\mathsf{may}\ \mathcal{R}\mathsf{A} \vee \mathcal{R}\mathsf{B}]\!]^{s,w} = 1$ iff
b. $\exists v \in Best(Mw) : [\![\mathcal{R}\mathsf{A} \vee \mathcal{R}\mathsf{B}]\!]^{Best(Mw),v} = 1$ iff
c. $\exists v \in Best(Mw) : [\![\mathsf{A} \vee \mathsf{B}]\!]^{Best(Mw),v} = 1\ \&\ \exists u, u' \in Best(Mw) : [\![\mathsf{A}]\!]^{Best(Mw),u} \neq [\![\mathsf{A}]\!]^{Best(Mw),u'}\ \&\ \exists u'', u''' \in Best(Mw) \cap \neg\mathbf{A} : [\![\mathsf{B}]\!]^{Best(Mw)\cap\neg\mathbf{A},u''} \neq [\![\mathsf{B}]\!]^{Best(Mw)\cap\neg\mathbf{A},u'''}$ iff
d. $[\![\mathsf{may\ A} \wedge \mathsf{may}\ \neg\mathsf{A} \wedge \mathsf{may}\ (\neg\mathsf{A} \wedge \mathsf{B}) \wedge \mathsf{may}\ (\neg\mathsf{A} \wedge \neg\mathsf{B})]\!]^{s,w} = 1$

Here, the key property is that the local context for deontic modals is the best worlds in Mw. For this reason, redundancy operators in the scope of deontic modals require that their input is undecided in the best worlds. This immediately generates the relevant permissions.[30]

Finally, consider an instance of Ross's inference:

(44) a. Mary must have apples or bananas.
b. $\mathsf{must}\ \mathcal{R}\mathsf{A} \vee \mathsf{B}$
c. $\#\mathsf{must}\ \mathcal{R}\mathsf{A} \vee \mathcal{R}\mathsf{B}$

Note that (44-c) is ruled out as a possible form because it inconsistently re-

the attitude verb. Moreover, unlike the operators that take atomic constituents, this wide-scope operator can't easily be removed through effects such as focus.

[30] One interesting feature of this derivation is its prediction that free choice constructions also grant the permission to do neither: $\mathsf{may}\ \neg\mathsf{A} \wedge \neg\mathsf{B}$ comes out true here. At this point, it is worth emphasizing that our account is parametric on a conception of local contexts. In particular, it is parametric on a choice of the local context for disjunction. As it happens, there is a fair deal of controversy about what this local context should be taken to be, and several approaches have been developed (for example, see Krahmer 1998; Geurts 1999; Rothschild 2008; Schlenker 2009; Rothschild 2011 for discussion.) For instance, Geurts (1999) argues that the local context for each disjunct is just the background context. In that case, our entry for disjunction would look as follows:

(i) $[\![\mathsf{p} \vee \mathsf{q}]\!]^{s,w} = \mathsf{disj}([\![\mathsf{p}]\!]^{s,w}, [\![\mathsf{q}]\!]^{s,w})$

With this entry we can still account for the free choice inference, but we would no longer predict that the subject is permitted to do neither disjunct. In short, some of the fine-grained predictions of our account are sensitive to the choice of local context algorithm.

quires that $Best(Mw)$ both implies $\mathsf{A} \vee \mathsf{B}$ and leaves room for $\neg(\mathsf{A} \vee \mathsf{B})$. Here, the form (44-b) correctly derives that Mary is permitted to have applies and permitted to have bananas. The key facts are that $\mathcal{R}\mathsf{A}$ generates a requirement that Mary is permitted not to have apples, which combined with her requirement to have either apples or bananas also generates a permission to have bananas.

This concludes the discussion of our core data.[31]

4.3 Further redundancy effects

We have now developed an operator $\mathcal{R}$ that tests its complement is non-redundant in its local context. In addition, we've seen that this operator explains all of the phenomena surveyed in §2. Before comparing our theory with grammatical accounts of scalar implicature, we explore a broader range of redundancy effects.

The account above required a different conception of local context for attitude verbs and for modals. We required the local context for the complement of an attitude verb like 'hope' to be the agent's belief worlds, rather than the best worlds in the agent's belief set. By contrast, we required the local context for the complement of deontic modals to be the best worlds in the modal base, rather than the modal base itself.

Each of these decisions appeared forced by the data we sought to derive. For deontic modals, we could not have derived free choice if the local context

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[31] The examples discussed above all involve sentences featuring at most a single connective. But more complex forms also give rise to redundancy effects. For instance, (i-a) implies (i-b)-(i-d):

(i) a. Mary hopes that Ann brought apple pie and Bill or Carol brought blueberry pie.
b. $\Longrightarrow$ Mary thinks it's possible that Ann didn't bring apple pie.
c. $\Longrightarrow$ Mary thinks it's possible that Bill brought blueberry pie.
d. $\Longrightarrow$ Mary thinks it's possible that Carol brought blueberry pie.

We can capture these effects through the following LF:

(ii) $\mathsf{x\ hopes\ } \mathcal{R}\mathsf{A} \wedge (\mathcal{R}\mathsf{B} \vee \mathcal{R}\mathsf{C})$

In this form $\mathcal{R}$ is attached to each atomic constituent. This follows from a more general result. A simple induction on our propositional fragment shows that the redundancy of atomics is sufficient for redundancy of every sentential constituent. Thus, so long as $\mathcal{R}$ adheres to the atomics, we will be guaranteed to capture all of the required ignorance effects. (In addition, this property guarantees that the forms above which generate ignorance effects also generate the relevant diversity effects.)

was simply the modal base. In that case, the redundancy operator would merely require that the relevant claims were circumstantially possible, rather than deontically permissible. Conversely, if the local context for 'hope' was the best of the belief worlds, then we could not have derived diversity. The form x hopes $\mathcal{R}\mathsf{A}$ would inconsistently require that A holds throughout the best of the belief worlds, and that A fails somewhere in the best of the belief worlds. Likewise with ignorance inferences. The form x hopes $\mathcal{R}\mathsf{A} \wedge \mathcal{R}\mathsf{B}$ would inconsistently require A and B to hold throughout the best belief worlds, while also requiring the best belief worlds to be agnostic about A and about B.

This raises an empirical question. Are there operators that simultaneously exemplify diversity, ignorance, and free choice inferences? Plausibly, the answer is yes. Attitude verbs like 'hope' go in for an analogue of Ross's inference:

(45) a. The detective hopes Ann or Bill committed the crime.
b. $\Longrightarrow$ The detective doesn't hope Ann didn't commit the crime.
c. $\Longrightarrow$ The detective doesn't hope Bill didn't commit the crime.

Conversely, deontic modals like 'must' potentially generate diversity inferences. Consider:

(46) Mary must read *Ulysses*.

Diversity requires that there be some world in Mw where Mary reads *Ulysses*, and some world in Mw where Mary doesn't read *Ulysses*.[32]

This all suggests that we need a richer conception of local context. In a sense, there are two different local contexts for the complement of 'hope': both the agent's belief worlds and the best of the agent's belief worlds seem to play a role. Similarly with deontic modals: both the modal base and the best worlds in the modal base operate as the local context for the complement.

An appendix to this paper provides a formal implementation of this idea. Building on Willer 2013, we introduce the notion of a *super context*, which is a set of local contexts. We suggest that attitudes and modals

[32]Indeed, theorists such as Condoravdi (2002) and Thomas (2014) use this diversity constraint to explain the distribution of perfect aspect in epistemic and metaphysical modals. Cf. Frank 1997; Zvolensky 2002.

supply super contexts for the evaluation of their complements. These super contexts themselves contain multiple different local contexts. Then we offer a theory on which the redundancy operator can select some or all of these local contexts, as constrained by the demands of consistency. The upshot is that the main ideas in §3 can be generalized to a richer setting in which a single operator can simultaneously generate diversity, ignorance, and free choice inferences.

5 Exhaustification

As discussed in §2, a prominent attempt to model embedded scalar implicatures appeals to an optional, covert exhaustification operator EXH. The contours of the debate around scalar implicatures has informed our own approach to redundancy effects, and motivated our development of the redundancy operator $\mathcal{R}$. However, one might wonder whether both EXH and $\mathcal{R}$ are needed. Couldn't one operator handle both implicatures and redundancy effects? In this section, we consider this question in detail, and conclude that both operators are required to adequately model natural language.

First, it should be fairly easy to see that $\mathcal{R}$ does not yield a general account of scalar implicatures. For example, it cannot explain why (1-a) implies (1-b):

(1) a. Ann ate some of the cake.
b. $\implies$ Ann didn't eat all of the cake.

A form such as $\mathcal{R}(\mathsf{x\ ate\ some\ cake})$ generates an implicature that is too weak: it only requires that there be some world where Ann ate some of the cake (this could be a world where she ate all of the cake), and some world where she has none of the cake. This clearly isn't equivalent to (1-b).

In the other direction, let us consider whether EXH yields an account of redundancy effects. To this end, it will be helpful to say a bit more about the configuration of EXH. $\mathsf{EXH}(\mathsf{p})$ says that p is true, and that any "innocently excludable" alternative to p is false (Fox, 2007; Chierchia *et al.*, 2012). Roughly, an alternative to p (in $\mathsf{Alt}(\mathsf{p})$) is innocently excludable when its falsity is compatible with the truth of p. More precisely:

(47) a. $\mathsf{Alt}(\mathsf{p})$ is the set of sentences q where p can be transformed into

q by deletions, contractions, and replacements of constituents in p with subtrees of p or items from the lexicon.[33]

b. $\mathsf{IE(p)} = \left\{ \mathsf{q} \in \mathsf{Alt(p)} \;\middle|\; \begin{array}{l} \mathsf{q} \text{ is contained in every maximal set } \mathsf{X} \subseteq \mathsf{Alt(p)} \\ \text{such that } \{\neg \mathsf{r} \mid \mathsf{r} \in \mathsf{X}\} \cup \{\mathsf{p}\} \text{ is consistent} \end{array} \right\}$

c. $[\![\mathsf{EXH(p)}]\!] = 1$ iff $[\![\mathsf{p}]\!] = 1$ and $\forall \mathsf{q} \in \mathsf{IE(p)} : [\![\mathsf{q}]\!] = 0$

As it happens, Cremers *et al.* (2019) use exhaustification to explain ignorance inferences generated by disjunctions in the scope of 'believe' and 'wonder'. The idea is that (48-a) is given the LF in (48-b):

(48) a. The detective believes that Ann or Bill committed the crime.
b. $\mathsf{EXH(x\ believes\ A \lor B)}$

Since 'The detective believes that Bill committed the crime' is an alternative to 'The detective believes that Ann or Bill committed the crime', and 'believe' is upward monotonic on standard attitude semantics, 'The detective believes that Bill committed the crime' is innocently excludable.[34] So, (48-b) is true only if 'The detective believes that Ann or Bill committed the crime' is true and 'The detective believes that Bill committed the crime' is false. But if the detective is sure that Ann didn't commit the crime, and 'The detective believes that Ann or Bill committed the crime' is true, 'The detective believes that Bill committed the crime' will be *true*. So, (48-b) can only be true if the detective thinks it's possible that Ann committed the crime.

However, there are at least four problems for trying to explain redundancy effects with EXH. First, this account cannot derive diversity. Diversity arises even with logically simple complements:

(49) The detective hopes that Ann committed the crime.

The only relevant LF is (50):

(50) $\mathsf{EXH(x\ hopes\ A)}$

But the only alternative to 'The detective hopes that Ann committed the crime' is just this sentence itself. There are no innocently excludable alterna-

[33]See Katzir (2007).

[34]An operator O is upward monotonic just in case $\mathsf{p} \models \mathsf{q}$ implies $\mathsf{Op} \models \mathsf{Oq}$.

tives, exhaustification has no effect, and no diversity constraint is derived.[35]

Second, the ability of exhaustification to capture ignorance inferences depends inappropriately on the logic of the embedded operator. For instance, this approach predicts that no ignorance effects are generated by conjunctions in the scope of upward monotonic operators. Such sentences entail all of their alternatives, so no alternatives will be innocently excludable. Similarly with disjunctions in the scope of downward monotonic operators. But it is plausible that 'hope' is upward monotonic (von Fintel, 1999; Crnič, 2011), and that 'fear' is downward monotonic.[36] This is reflected in our semantics above, where hope is about being implied by the best belief worlds, and fear is about being excluded by the best belief worlds. If that's correct, then exhaustification cannot explain the ignorance inferences generated by examples such as (51-a) and (51-b):

(51) a. Mary hopes that Ann brought apple pie and Bill brought blueberry pie.
b. The detective fears that Ann or Bill committed the crime.

Even if 'hope' and 'fear' don't ultimately have these monotonicity properties, the exhaustification account will still predict more generally that ignorance inferences are blocked when upward monotonic operators scope over conjunction, and when downward monotonic operators scope over disjunction.

[35]One proposal worth future investigation would be to let 'The detective believes that Ann committed the crime' be a logically stronger alternative to 'The detective hopes that Ann committed the crime'. Then hoping p could imply not believing p as a scalar implicature.

[36]Note that the (a)-examples below seem to entail the (b)-examples:

(i) a. I fear being served seafood.
b. $\Longrightarrow$ I fear being served prawns.

(ii) a. I fear being served prawns or lobster.
b. $\Longrightarrow$ I fear being served prawns.

Moreover, conjoining the (a)-examples with the negation of the (b)-examples leads to infelicity:

(iii) a. #I fear being served seafood, but I don't fear being served prawns.
b. #I fear being served prawns or lobster, but I don't fear being served prawns.

This is explained if 'fear' was downward monotonic. Also note that the downward monotonicity of 'fear' is no impediment to allowing $\mathcal{R}$ to scope under it, since this still results in a stronger overall meaning for the report.

A theory of ignorance inferences should not be hostage to fortune in this way.[37]

Third, the truth-conditions generated for conjunctions under non-upward monotonic operators like 'fear' and 'wonder' are too strong. Consider (52-a):

(52) a. Mary fears that Ann brought apple pie and Bill brought blueberry pie.
b. $\Longrightarrow$ $\neg(\mathsf{x\ believes\ A}) \wedge \neg(\mathsf{x\ believes\ B})$
c. $\not\Longrightarrow$ $\neg(\mathsf{x\ fears\ A}) \wedge \neg(\mathsf{x\ fears\ B})$

Intuitively, an utterance of (52-a) suggests that Mary doesn't believe Ann brought apple pie, and that Mary doesn't believe Bill brought blueberry pie, i.e. (52-b). The exhaustification approach can derive this implicature through the following LF:

(53) a. $\mathsf{EXH(x\ fears\ A} \wedge \mathsf{B)}$
b. $\Longrightarrow$ $\neg(\mathsf{x\ fears\ A}) \wedge \neg(\mathsf{x\ fears\ B})$

If fear is a matter of being excluded by the best belief worlds, then (53-a) is true only if Mary doesn't believe that Ann brought apple pie, and Mary doesn't believe that Bill brought blueberry pie. For instance, if Mary did believe that Ann brought apple pie, and 'Mary fears that Ann brought apple pie and Bill brought blueberry pie' was true, then 'Mary fears that Bill brought blueberry pie' would be true as well. But since 'fear' is not upward monotonic, (53-a) negates this last alternative, as in (53-b).

However, the entailment in (53-b) is too strong. An utterance of (52-a) does *not* suggest that Mary doesn't fear that Ann brought apple pie, or that Mary doesn't fear that Bill brought blueberry pie, i.e. (52-c). Indeed, an utterance of (52-a) made out-of-the-blue often implies just the opposite, that Mary fears that Ann brought apple pie, and that Mary fears that Bill brought blueberry pie. But as we have just seen, if one appeals to EXH, the inference in (52-b) is derived only if the inference in (52-c) is as well.

Finally, an approach that appeals to EXH can't explain the ignorance

[37] If diversity is captured as a presupposition triggered by attitude reports, choice points emerge as to how the exhaustification operator, and in particular the notion of an innocently excludable alternative, should be defined (Spector & Sudo, 2017). But this doesn't substantially impact our argument above. For instance, regardless of how these choice points are resolved, 'The detective fears that Ann committed the crime' fails to be an innocently excludable alternative for (51-b).

effects that arise from asymmetrically entailing conjunctions under 'wonder'. To illustrate, consider (54-a):

(54) a. Mary wonders whether Ann is pregnant and expecting a daughter.

b. $\mathsf{EXH}(\mathsf{x\ wonders\ P} \wedge \mathsf{P}^{+})$

On a prominent semantics for 'wonder', the diversity constraint on the subject's beliefs is part of the at-issue meaning of the verb (Ciardelli & Roelofsen, 2015). Thus, $\mathsf{x\ wonders\ p}$ is *false* if x believes p. This has two consequences. First, it makes wonder reports non-monotonic. So, since the second embedded conjunct entails the first in (54-a), the only innocently excludable alternative is 'Mary wonders whether Ann is pregnant'. And second, since the diversity constraint is part of the at-issue meaning of 'wonder', 'Mary wonders whether Ann is pregnant' will be false if Mary believes that Ann is pregnant. Thus, (54-b) can be *true* on Ciardelli & Roelofsen's semantics when Mary believes that Ann is pregnant. This means that no ignorance inference will be derived. To be clear, we aren't necessarily wedded to Ciardelli & Roelofsen's entry for 'wonder'. But we leave it to others to provide an alternative meaning for this verb which allows exhaustification to generate the target ignorance effect for examples such as (54-a).[38,39]

To summarize, $\mathcal{R}$ and EXH play distinct theoretical roles. Both are required to provide a satisfactory account of scalar implicatures on the one hand, and redundancy effects on the other.

[38]We did not present a semantics for 'wonder' in §4.1 because this verb takes inquisitive complements, and thereby raises issues that are orthogonal to our central concerns. But it can be shown that our general approach extends straightforwardly to these constructions, including those that feature asymmetrically entailing conjunctions such as (54-a). In short, if we suppose that inquisitive complements denote sets of propositions Γ, then we can derive ignorance effects so long as we assume that the local context for the complement of 'wonder' is the subject's belief set, and $\mathcal{R}$ operates pointwise over the members of Γ.

[39]For reasons of space, we defer to another day a critical comparison of redundancy and exhaustification regarding free choice inferences. We simply note that a single application of the exhaustification operator EXH does not generate free choice inferences. In order to derive free choice, theorists such as Fox (2007) and Bar-Lev & Fox (2020) have instead appealed to either iterated applications of EXH, or to more complicated variants of the EXH operator.

6 Distribution of $\mathcal{R}$

Before we conclude, we want to return to the issue of optionality discussed in §3.4.2: why do redundancy effects tend to disappear in certain environments, e.g. under negation? Given our framework, this question is equivalent to the following: what determines the distribution of $\mathcal{R}$? We won't be able to provide a complete answer here, but we will make some observations. We will also note that a similar question arises regarding the distribution of EXH, so the configuration of $\mathcal{R}$ is an instance of a broader set of issues concerning the appearance of operators at logical form.

We will orient our discussion around two candidate principles governing the distribution of covert operators. First, consider the *strongest meaning hypothesis*, which says that when a sentence is ambiguous, there is a preference for the strongest interpretation (Dalrymple *et al.*, 1998). Chierchia *et al.* (2012, 2327) precisify this constraint as follows, where O is an operator like $\mathcal{R}$:

(55) **Strongest Meaning Hypothesis.** Let S be a sentence of the form $[_{\mathsf{S}}...\mathsf{O}(\mathsf{X})...]$. Let S' be the sentence of the form $[_{\mathsf{S}'}...\mathsf{X}...]$, i.e. replacing $\mathsf{O}(\mathsf{X})$ in S with X. Then, all else equal, S' is preferred to S if S' is logically stronger than S.

The strongest meaning hypothesis explains why $\mathcal{R}$ tends to disappear under negation. Consider the logical forms in (56), where $\Box$ is a placeholder for either an attitude verb or a modal:

(56) a. $\neg\Box\mathsf{p}$
b. $\neg\Box\mathcal{R}\mathsf{p}$

Since negation is a downward monotonic operator, and attitude verbs and modals trap redundancy effects, the form in (56-b) will have a weaker meaning than that in (56-a). So, given the strongest meaning hypothesis, (56-a) will be preferred to (56-b). This accounts for our optionality observations in §3.4.2 that our target redundancy effects tend to disappear under operators like negation.

However, the strongest meaning hypothesis can't be quite right, since redundancy effects can appear in some downward monotonic environments. For instance, in §1 we noted that the restrictor of a universal quantifier is a downward monotonic environment, and yet (57-a) is naturally interpreted as (57-b):

(57) a. Every detective who hopes Ann or Bill committed the crime will be disappointed.

b. = Every detective who desires that Ann or Bill committed the crime *and is ignorant of whether Ann or Bill committed the crime* will be disappointed.

We will want explain the embedded ignorance effect in (57-b) by maintaining that $\mathcal{R}$ occurs embedded inside the restrictor of the quantifier. But the strongest meaning hypothesis predicts that such forms should be dispreferred compared to their $\mathcal{R}$-free analogues. In short, although the strongest meaning hypothesis provides an elegant account of why $\mathcal{R}$ can't scope under negation, it predicts that too many forms should be marked.

The second principle we will consider bans the insertion of optional operators when this results in contradictory meanings:

(58) **Contradictory Meaning Ban.** Let S be a sentence of the form $[_{\mathsf{S}}...\mathsf{O}(\mathsf{X})...]$. Let S' be the sentence of the form $[_{\mathsf{S}'}...\mathsf{X}...]$, i.e. replacing $\mathsf{O}(\mathsf{X})$ in S with X. Then, all else equal, S' is preferred to S if S is a logical contradiction.

The contradictory meaning ban explains our observation from §4.2.2 to the effect that the logical form of (40-a) is (40-b) rather than (40-c):

(40) a. The detective believes that Ann or Bill committed the crime.

b. x $\mathsf{believes}$ $\mathcal{R}\mathsf{A} \vee \mathsf{B}$

c. x $\mathsf{believes}$ $\mathcal{R}\mathsf{A} \vee \mathcal{R}\mathsf{B}$

As noted, (40-c) inconsistently requires both that Bxw implies $\mathsf{A} \vee \mathsf{B}$, and that $Bxw \cap \neg\mathbf{A}$ contains a world where $\neg\mathsf{B}$ is true.

However, the contradictory meaning ban is too restrictive. For instance, consider once again the simple case of infelicity generated by redundancy from §4.1:

(27) #Ann is pregnant and Ann is pregnant.

The candidate forms for this sentence are (59-a) and (59-b):

(59) a. $\mathcal{R}(\mathsf{P} \wedge \mathsf{P})$

b. $\mathsf{P} \wedge \mathcal{R}\mathsf{P}$

However, (59-a) is too weak since it simply requires that it not be settled in context that Ann is pregnant. And the contradictory meaning ban rules out (59-b) due to inconsistency, since the local context of the second conjunct already contains the information that Ann is pregnant.[40]

To sum up, although both the strongest meaning hypothesis and the contradictory meaning ban capture some facts about the distribution of $\mathcal{R}$, they are not quite adequate as they stand. At this point, it is worth observing that a similar situation arises with the exhaustification operator. As far as we are aware, theorists have yet to formulate acceptable principles governing the distribution of EXH. For instance, some authors have appealed to the strongest meaning hypothesis to explain the optionality of EXH, for example why it tends to disappear under negation.[41] However, recall the example of an embedded implicature from §2:

(7) a. Every professor who fails some of the students will be put on probation; but every professor who fails all of the students will be fired.

b. = Every professor who fails *some but not all* of the students will be put on probation...

As discussed, on the most natural reading of (7-a) the implicature triggered by 'some' in the first conjunct scopes under the quantifier, and is embedded in the restrictor. But the strongest meaning hypothesis predicts that the

[40]One possible response is to keep the contradictory meaning ban but supplement our proposal with a pragmatic redundancy principle. For instance, consider the following account from Schlenker (2009):

(i) **Pragmatic Redundancy.** p cannot be used in context s if there is any part q of p that is redundant, in that either q or $\neg\mathsf{q}$ is entailed in q's local context.

This would predict that (27) is unacceptable. More generally, Pragmatic Redundancy could be used for examples which involve contraventions of redundancy in the discourse context, while $\mathcal{R}$ could be used to handle our target effects, e.g. diversity, ignorance, etc. As far as we can tell, this bifurcated approach to redundancy is coherent, and we think it should be explored further. That said, one would ideally like to be able to explain the full range of redundancy effects through a single mechanism, so we are reluctant to officially endorse this response here.

[41]See for example Chierchia *et al.* 2012.

form corresponding to (7-b) should be marked, since restrictors of universal quantifiers are downward monotonic environments.

The contradictory meaning ban is also too restrictive for exhaustification: there are examples which require the presence of EXH, even though the resulting meanings are incoherent. Consider (60) from Chierchia *et al.* 2012:

(60) Every student, including Jack, solved either none of the problems or all of the problems, and # Jack solved some of them.

(60) is unacceptable. Theorists are inclined to explain its infelicity by maintaining that the second conjunct is exhaustified, which contradicts the first conjunct. But such forms would be ruled out by the contradictory meaning ban.[42]

So, the situation with $\mathcal{R}$ is not unique. Questions arise regarding the distribution of covert operators more generally. We leave this as an area for future work.

7 Conclusion

We end by highlighting what we take to be the key benefit of our approach. We are able to explain a broad swath of data using a single mechanism. This brings a significant explanatory advantage compared to standard theories of redundancy inferences which posit distinct mechanisms for each type of effect. Given our unified treatment of redundancy, our account makes a striking prediction: at the appropriate level of analysis, redundancy effects should all pattern together. As far as we can see, this prediction is borne out by the data, but further investigation could reveal differences between these effects.[43] Whatever the outcome, we hope that our discussion has opened up an interesting and fruitful line of inquiry into redundancy effects, and that our positive proposal will be of interest to researchers studying the

[42] More complex principles governing the distribution of EXH have been formulated by Fox & Spector (2018) and Enguehard & Chemla (2019). See Enguehard & Chemla 2019 for a critical discussion of Fox & Spector's principle. Enguehard & Chemla's principle has several shortcomings (some of which are identified by the authors themselves). For instance, it predicts that a perfectly acceptable sentence such as 'Either everyone passed or nobody did' should be marked. Suffice to say, much work remains on the distribution of EXH at logical form.

[43] Here, a natural place to look is recent research from Gotzner *et al.* 2020 about the behavior of free choice under non-monotonic quantifiers such as 'exactly two'.

patterns exhibited by natural language.[44]

[44]A further topic to explore involves different formulations of the redundancy operator. One alternative is to develop an operator that considers syntactic simplifications of its complement, in the style of the pragmatic redundancy accounts of Meyer 2013; Katzir & Singh 2014. For instance, one could develop an operator which checks that no syntactic simplification of its complement is truth conditionally equivalent to its complement in the local context. The idea would be to capture the target redundancy effects through forms such as x hopes $\mathcal{R}_2(\mathsf{A} \vee \mathsf{B})$, where $\mathcal{R}_2$ is the posited operator. This proposal deserves to be developed further, but we note one potential concern: forms such as x hopes $\mathcal{R}_2\mathsf{p}$ wouldn't generate diversity effects, given standard assumptions about the space of syntactic simplifications. To remedy this, it could be assumed that the tautology $\top$ is a simplification of any sentence. But then forms such as x $\mathsf{believes}$ $\mathcal{R}_2(\mathsf{A} \vee \mathsf{B})$ would be ruled out due to inconsistency, and it is unclear how disjunctive ignorance effects for belief could be captured. A similar concern arises with simple approaches that try to semanticize Pragmatic Redundancy. For example, consider an operator $\mathcal{R}_3$ which checks that no part of its complement is redundant in its local context. Forms such as x $\mathsf{believes}$ $\mathcal{R}_3(\mathsf{A} \vee \mathsf{B})$ would also be ruled out due to inconsistency, and disjunctive ignorance wouldn't be captured.

Appendix: super contexts

§3.3 argued that a single attitude verb or modal could simultaneously generate diversity, ignorance, and free choice inferences. We suggested this requires a more sophisticated notion of local context, according to which attitudes and modals can supply both an information state and the best worlds in that state as their local context. To implement this idea, we appeal to a richer conception of local context. Following Willer (2013), we let a "super context" S be a set of local contexts s, and relativize interpretation to this parameter.

Now expressions are evaluated relative to sets of local contexts, rather than individual local contexts. With this extra structure, we can let attitude verbs and modals provide two different local contexts for their complements' evaluation. For example, 'hope' and 'fear' evaluate their complements relative to the set $\{Bxw, Best(Bxw)\}$, containing both the belief worlds and the best belief worlds.[45]

(61) a. $[\![\textsf{x believes p}]\!]^{S,w} = 1$ iff $\forall v \in Bxw : [\![\textsf{p}]\!]^{\{Bxw\},v} = 1$

b. $[\![\textsf{x hopes p}]\!]^{S,w} = 1$ iff $\forall v \in Best(Bxw) : [\![\textsf{p}]\!]^{\{Bxw,Best(Bxw)\},v} = 1$

c. $[\![\textsf{may p}]\!]^{S,w} = 1$ iff $\exists v \in Best(Mw) : [\![\textsf{p}]\!]^{\{Mw,Best(Mw)\},v} = 1$

d. $[\![\textsf{must p}]\!]^{S,w} = 1$ iff $\forall v \in Best(Mw) : [\![\textsf{p}]\!]^{\{Mw,Best(Mw)\},v} = 1$

Now that various propositional operators supply a super context, we let our redundancy operator access this super context, and check for redundancy in various members of the super context.[46]

To do so, we enrich our representation of redundancy by supplying variables $\textsf{i}$ for super contexts in LF, which attach to the redundancy operator $\mathcal{R}$. Variables supply functions from worlds to super contexts. Some relevant values of $g(\textsf{i})$ are the following: $\lambda w.\{Bxw\}$, $\lambda w.\{Best(Bxw)\}$,

[45] This paper does not model presupposition projection. Nevertheless, it is worth noting that the switch from local contexts to super contexts is conservative with respect to this application. The key is that $Best(Bxw)$ is a subset of Bxw. So, whenever a presupposition is defined in Bxw, it is automatically defined in $Best(Bxw)$.

[46] We assume that Boolean connectives shift the super context parameter by operating pointwise over the elements in the super context. For instance, we assume the following entry for conjunction:

(i) $[\![\textsf{p} \wedge \textsf{q}]\!]^{S,w} = \textsf{conj}([\![\textsf{p}]\!]^{S,w}, [\![\textsf{q}]\!]^{\left\{s \cap [\![\textsf{p}]\!]^{S} \mid s \in S\right\},w})$

$\lambda w.\{Bxw, Best(Bxw)\}$, etc. We are primarily interested in $g(\mathsf{i})(w)$, the result of saturating $g(\mathsf{i})$ with the world of evaluation. This will derive super contexts like $\{Bxw\}$ and $\{Best(Bxw)\}$.

$\mathcal{R}\mathsf{i}$ takes the super context variable i, and requires that $g(\mathsf{i})(w)$ is a subset of the super context of evaluation. Then $\mathcal{R}\mathsf{i}\ \mathsf{p}$ requires that p is true in the super context of evaluation, and non-redundant in every member of $g(\mathsf{i})$.

(62) $$[\![\mathcal{R}\mathsf{i}\ \mathsf{p}]\!]^{g,S,w} = \begin{cases} 1 & \text{if } g(\mathsf{i})(w) \subseteq S \\ & \forall s \in g(\mathsf{i})(w) : \exists v, v' \in s : [\![\mathsf{p}]\!]^{g,\{s\},v} \neq [\![\mathsf{p}]\!]^{g,\{s\},v'} \\ & [\![\mathsf{p}]\!]^{g,S,w} = 1 \\ 0 & \text{if } g(\mathsf{i})(w) \subseteq S \\ & \forall s \in g(\mathsf{i})(w) : \exists v, v' \in s : [\![\mathsf{p}]\!]^{g,\{s\},v} \neq [\![\mathsf{p}]\!]^{g,\{s\},v'} \\ & [\![\mathsf{p}]\!]^{g,S,w} = 0 \\ \# & \text{otherwise} \end{cases}$$

Here is an example. Consider $[\![\mathcal{R}\mathsf{i}\ \mathsf{p}]\!]^{g,\{Bxw,Best(Bxw)\},w}$, where $g(\mathsf{i}) = \lambda w.\{Bxw\}$. $\mathcal{R}\mathsf{i}$ requires that $g(\mathsf{i})(w)$ is contained in the super context of evaluation. This condition is satisfied, since $\{Bxw\} \subseteq \{Bxw, Best(Bxw)\}$. Next, $\mathcal{R}\mathsf{i}\ \mathsf{p}$ requires that p is non-redundant in every context in $g(\mathsf{i})$. This requires that p is non-redundant in Bxw, but places no demand on $Best(Bxw)$.

This assignment function allows us to derive diversity inferences for 'hope':

(63) $\mathsf{x\ hopes}\ \mathcal{R}\mathsf{i}\ \mathsf{A}$

When $g(\mathsf{i}) = \lambda w.\{Bxw\}$, this sentence requires that A is entailed by $Best(Bxw)$ and that A is non-redundant in Bxw. Ignorance inferences for conjunctions embedded under 'hope' can be obtained in a similar fashion.[47]

Next consider the analogue of Ross's inference for 'hope'. In this case, we make use of two variables:

(64) $\mathsf{x\ hopes}\ \mathcal{R}\mathsf{i}\ \mathsf{A} \vee \mathcal{R}\mathsf{i}'\ \mathsf{B}$

[47]Assignments take variables to functions from worlds to supercontexts, and not just to supercontexts, in order to correctly capture counterfactual attitude reports and modal claims. For instance, suppose John says 'Mary hopes $\mathcal{R}\mathsf{i}$(Ann is pregnant)'. Then if i was simply assigned Mary's belief set at the actual world, we would incorrectly predict there to be true readings of 'Even if Mary was certain Ann was pregnant, what John said would still be true'.

This inference is derived when $g(\mathsf{i}) = \lambda w.\{Bxw, Best(Bw)\}$ and $g(\mathsf{i}') = \lambda w.\neg\mathbf{A} \cap \{Bxw\}$. Under such assignments, the sentence requires that $\mathsf{A} \vee \mathsf{B}$ is true throughout $Best(Bxw)$, that A is non-redundant in $Best(Bxw)$ and Bxw, and that B is non-redundant in Bxw. The result is free choice inferences for A and B, ignorance inferences for A and B, and a diversity inference for $\mathsf{A} \vee \mathsf{B}$. We assume that these assignments are selected through a generalization of the strongest meaning hypothesis. Speakers and hearers coordinate on variable assignments that produce stronger meanings.

Finally consider deontic modals. We derive diversity effects:

(65) $\mathsf{must}\ \mathcal{R}\mathsf{i}\ \mathsf{A}$

The key here is to let $g(\mathsf{i}) = \lambda w.\{Mw\}$ rather than $\lambda w.\{Mw, Best(Mw)\}$. While the latter produces absurdity, the former assignment produces the truth conditions that $Best(Mw)$ implies A while A is non-redundant in Mw.

Summarizing, this appendix offered a conservative extension of the theory in §3 that preserves our initial predictions while deriving new redundancy inferences for operators that simultaneously trigger diversity, ignorance, and free choice effects.

References

Aher, Martin. 2012. Free Choice in Deontic Inquisitive Semantics (DIS). *Lecture Notes in Computer Science*, **7218**, 22–31.

Aloni, Maria. 2007. Free Choice, Modals, and Imperatives. *Natural Language Semantics*, **15**(1), 65–94.

Aloni, Maria. 2018. *Free Choice Disjunction in State-Based Semantics.*

Alonso-Ovalle, Luis. 2006. *Disjunction in Alternative Semantics.* Ph.D. thesis, University of Massachusetts Amherst.

Asher, Nicholas, & Bonevac, Daniel. 2005. Free Choice Permission is Strong Permission. *Synthese*, **145**, 303–323.

Bar-Lev, Moshe E., & Fox, Danny. 2020. Free choice, simplification, and Innocent Inclusion. *Natural Language Semantics*, **28**(3), 175–223.

Barker, Chris. 2010. Free Choice Permission as Resource-Sensitive Reasoning. *Semantics and Pragmatics*, **3**(10), 1–38.

Blumberg, Kyle. 2017. Ignorance Implicatures and Non-doxastic Attitude Verbs. *Pages 135–145 of:* Alexandre Cremers, Thom van Gessel, & Roelofsen, Floris (eds), *Proceedings of the 21st Amsterdam Colloquium.*

Blumberg, Kyle, & Holguín, Ben. 2019. Embedded Attitudes. *Journal of Semantics*, **36**(3), 377–406.

Cariani, Fabrizio. 2013. ‘Ought’ and Resolution Semantics. *Noûs*, **47**(3), 534–558.

Carter, Sam. 2021. The Dynamics of Loose Talk. *Noûs*, **55**(1), 171–198.

Charlow, Nate. 2015. Prospects for an Expressivist Theory of Meaning. *Philosophers' Imprint*, **15**, 1–43.

Chemla, E. 2009. Universal Implicatures and Free Choice Effects: Experimental Data. *Semantics and Pragmatics*, **2**(2), 1–33.

Chierchia, Gennaro, & McConnell-Ginet, Sally. 2000. *Meaning and Grammar: An Introduction to Semantics*. MIT Press.

Chierchia, Gennaro, Fox, Danny, & Spector, Benjamin. 2009. *Presuppositions and implicatures. Proceedings of the MIT-Paris Workshop*. MIT. Chap. Hurford's constraint and the theory of scalar implicature, pages 47–62.

Chierchia, Gennaro, Fox, Danny, & Spector, Benjamin. 2012. *Semantics: An international handbook of natural language meaning, vol.3*. Berlin: Mouton de Gruyter. Chap. Scalar implicature as a grammatical phenomenon, pages 2297–2331.

Ciardelli, Ivano, Groenendijk, Jeroen, & Roelofsen, Floris. 2009. Attention! *Might* in Inquisitive Semantics. *Pages 91–108 of: Proceedings of SALT 29*.

Ciardelli, Ivano A., & Roelofsen, Floris. 2015. Inquisitive Dynamic Epistemic Logic. *Synthese*, **192**(6), 1643–1687.

Condoravdi, Cleo. 2002. Temporal interpretation of modals: Modals for the present and for the past. *Pages 59–88 of:* Beaver, David, Kaufmann, Stefan, & Martinez, L.C. (eds), *The Construction of Meaning*. CSLI Publications.

Condoravdi, Cleo, & Lauer, Sven. 2016. Anankastic conditionals are just conditionals. *Semantics and Pragmatics*, **9**(8), 1–69.

Cremers, Alexandre, Roelofsen, Floris, & Uegaki, Wataru. 2019. Distributive ignorance inferences with *wonder* and *believe*. *Semantics and Pragmatics*, **12**.

Crnič, Luka, Chemla, Emmanuel, & Fox, Danny. 2015. Scalar implicatures of embedded disjunction. *Natural Language Semantics*, **23**(4), 271–305.

Crnič, Luka. 2011. *Getting even*. Ph.D. thesis, MIT.

Dalrymple, Mary, Kanazawa, Makoto, Kim, Yookyung, McHombo, Sam, & Peters, Stanley. 1998. Reciprocal Expressions and the Concept of Reciprocity. *Linguistics and Philosophy*, **21**(2), 159–210.

Del Pinal, Guillermo. forthcoming. Oddness, Modularity, and Exhaustification. *Natural Language Semantics*, 1–44.

Enguehard, Émile, & Chemla, Emmanuel. 2019. Connectedness as a Constraint on Exhaustification. *Linguistics and Philosophy*, **44**(1), 1–34.

Fauconnier, Gilles. 1975. Pragmatic Scales and Logical Structure. *Linguistic Inquiry*, **6**(3), 353–375.

Fox, Danny. 2007. *Presupposition and Implicature in Compositional Semantics*. London: Palgrave Macmillan UK. Chap. Free Choice and the Theory of Scalar Implicatures, pages 71–120.

Fox, Danny, & Spector, Benjamin. 2018. Economy and Embedded Exhaustification. *Natural Language Semantics*, **26**(1), 1–50.

Frank, Annette. 1997. *Context Dependence in Modal Constructions*. Ph.D. thesis, Universität Stuttgart.

Franke, Michael. 2011. Quantity Implicatures, Exhaustive Interpretation, and Rational

Conversation. *Semantics and Pragmatics*, **4**(1), 1–82.

Fusco, Melissa. 2015a. Deontic Modality and the Semantics of Choice. *Philosophers' Imprint*, **15**.

Fusco, Melissa. 2015b. Deontic Modals and the Semantics of Choice. *Philosophers' Imprint*, **15**(28), 1–27.

Gajewski, Jon, & Sharvit, Yael. 2012. In defense of the grammatical approach to local implicatures. *Natural Language Semantics*, **20**(1), 31–57.

Geurts, Bart. 1999. *Presuppositions and Pronouns*. Elsevier.

Geurts, Bart. 2005. Entertaining Alternatives: Disjunctions as Modals. *Natural Language Semantics*, **13**, 383–410.

Geurts, Bart. 2010. *Quantity Implicatures*. Cambridge University Press.

Gillies, Anthony S. 2010. Iffiness. *Semantics and Pragmatics*, **3**(4), 1–42.

Gotzner, Nicole, Romoli, Jacopo, & Santorio, Paolo. 2020. Choice and Prohibition in Non-Monotonic Contexts. *Natural Language Semantics*, 141–174.

Grano, Thomas. 2017. The Logic of Intention Reports. *Journal of Semantics*, **34**(4), 587–632.

Grice, H. P. 1975. Logic and Conversation. *Pages 64–75 of:* Davidson, Donald, & Harman, Gilbert (eds), *The Logic of Grammar*.

Grice, H. P. 1989. *Studies in the Way of Words*. Cambridge: Harvard University Press.

Heim, Irene. 1983. On the projection problem for presuppositions. *In: WCCFL 2*. Stanford: CSLI Publications.

Heim, Irene. 1992. Presupposition Projection and the Semantics of Attitude Verbs. *Journal of Semantics*, **9**(3), 183–221.

Hintikka, Jaakko. 1962. *Knowledge and Belief*. Ithaca: Cornell University Press.

Hoek, Daniel. 2018. Conversational Exculpature. *Philosophical Review*, **127**(2), 151–196.

Horn, Laurence. 1972 (01). *On the Semantic Properties of Logical Operators in English*. Ph.D. thesis, UCLA.

Jerzak, Ethan. 2019. Two Ways to Want? *Journal of Philosophy*, **116**(2), 65–98.

Kamp, Hans. 1974. Free Choice Permission. *Proceedings of the Aristotelian Society*, 57–74.

Kamp, Hans. 1978. Semantics Versus Pragmatics. *Pages 255–287 of:* Gunthuer, F., & Schmidt, S.J. (eds), *Formal Semantics and Pragmatics for Natural Languages*. D. Deidel.

Kartunnen, Lauri. 1974. Presupposition and Linguistic Context. *Theoretical Linguistics*, **34**(1), 181–.

Katzir, Roni. 2007. Structurally-Defined Alternatives. *Linguistics and Philosophy*, **30**(6), 669–690.

Katzir, Roni, & Singh, Raj. 2014. Hurford disjunctions: embedded exhaustification and structural economy. *Pages 201–216 of:* Etxeberria, Urtzi, Fălăuş, Anamaria, Irurtzun, Aritz, & Leferman, Bryan (eds), *Proceedings of Sinn und Bedeutung 18*.

Klinedinst, Nathan. 2007. *Plurality and Possibility*. Ph.D. thesis, UCLA.

Krahmer, Emiel. 1998. *Presupposition and Anaphora*. CSLI Publications.

Kratzer, Angelika. 2012. *Modals and Conditionals*. Oxford: Oxford University Press.

Kratzer, Angelika, & Shimoyama, Junko. 2002. Indeterminate Pronouns: The View from Japanese. *In:* Otsu, Y. (ed), *The Proceedings of the Third Tokyo Conference on Psy-*

cholinguistics. Tokyo: Hituzi Syobo.

Lasersohn, Peter. 1999. Pragmatic Halos. *Language*, **75**(3), 522–551.

Lassiter, Daniel. 2011. *Measurement and Modality: The Scalar Basis of Modal Semantics*. Ph.D. thesis, New York University.

Lauer, Sven. 2016. On the pragmatics of pragmatic slack. *Proceedings of Sinn und Bedeutung*, **16**(2), 389–402.

Levinson, Dmitry. 2003. Probabilistic Model-theoretic Semantics for 'want'. *Semantics and Linguistic Theory*, **13**(0), 222–239.

Mandelkern, Matthew. 2019. Bounded Modality. *Philosophical Review*, **128**(1), 1–61.

Mandelkern, Matthew, & Romoli, Jacopo. 2018. Hurford Conditionals. *Journal of Semantics*, **35**(2), 357–367.

Mayr, Clemens, & Romoli, Jacopo. 2016. A puzzle for theories of redundancy: Exhaustification, incrementality, and the notion of local context. *Semantics and Pragmatics*, **9**(7), 1–48.

Meyer, Marie-Christine. 2013. *Ignorance and Grammar*. Ph.D. thesis, Massachusetts Institute of Technology.

Pasternak, Robert. 2019. A Lot of Hatred and a Ton of Desire: Intensity in the Mereology of Mental States. *Linguistics and Philosophy*, **42**(3), 267–316.

Pearson, Hazel. 2016. The semantics of partial control. *Natural Language & Linguistic Theory*, **34**(2), 691–738.

Phillips-Brown, Milo. 2018. I want to, but... *Proceedings of Sinn und Bedeutung 21 preprints*.

Phillips-Brown, Milo. Forthcoming. What does decision theory have to do with wanting? *Mind*.

Roelofsen, Floris. 2013. A Bare Bone Attentive Semantics for *Might*. *Pages 190–215 of:* Aloni, Maria, Franke, Michael, & Roelofsen, Floris (eds), *The Dynamic, Inquisitive, and Visionary Life of ϕ, ?ϕ, and $\Diamond\phi$*. Amsterdam: ILLC Publications.

Roelofson, Floris, & Uegaki, Wataru. 2016. The distributive ignorance puzzle. *In: Proceedings of Sinn und Bedeutung 21*.

Romoli, Jacopo, & Santorio, Paolo. 2017. Probability and implicatures: a unified account of the scalar effects of disjunction under modals. *Semantics and Pragmatics*.

Romoli, Jacopo, & Santorio, Paolo. 2019. Filtering free choice. *Semantics and Pragmatics*.

Ross, Alf. 1941. Imperatives and logic. *Theoria*, **7**, 53–71.

Rothschild, Daniel. 2008. Transparency Theory and its dynamic alternatives: Commentary on "Be Articulate". *Theoretical Linguistics*.

Rothschild, Daniel. 2011. Explaining Presupposition Projection with Dynamic Semantics. *Semantics and Pragmatics*, **4**(3), 1–43.

Rubinstein, Aynat. 2012. *Roots of Modality*. Ph.D. thesis, University of Massachusetts Amherst.

Sauerland, Uli. 2004. Scalar Implicatures in Complex Sentences. *Linguistics and Philosophy*, **27**(3), 367–391.

Schlenker, Philippe. 2009. Local Contexts. *Semantics and Pragmatics*, **2**(3), 1–78.

Schulz, Katrin. 2005. A Pragmatic Solution for the Paradox of Free Choice Permission. *Synthese*, **147**(2), 343–377.

Simons, Mandy. 2005. Dividing Things Up: The Semantics of *or* and the Modal/*or* Interaction. *Natural Language Semantics*, **13**, 271–316.

Singh, Raj. 2008. *Modularity and Locality in Interpretation.* Ph.D. thesis, Massachusetts Institute of Technology.

Spector, B. 2006. *Aspects de la pragmatique des opérateurs logiques.* Ph.D. thesis, Université Paris 7.

Spector, Benjamin. 2016. Multivalent Semantics for Vagueness and Presupposition. *Topoi*, **35**(1), 45–55.

Spector, Benjamin, & Sudo, Yasutada. 2017. Presupposed Ignorance and Exhaustification: How Scalar Implicatures and Presuppositions Interact. *Linguistics and Philosophy*, **40**(5), 473–517.

Stalnaker, Robert. 1974. Pragmatic Presuppositions. *Pages 47–62 of:* Stalnaker, Robert (ed), *Context and Content.* Oxford University Press.

Stalnaker, Robert. 1978. Assertion. *Syntax and Semantics (New York Academic Press)*, **9**, 315–332.

Starr, William B. 2016. Expressing Permission. *In: Proceedings of SALT 26.* CLC Publications, Ithaca, NY.

Thomas, Guillaume. 2014. Circumstantial modality and the diversity condition. *Proceedings of Sinn und Bedeutung*, **18**, 433–450.

van Der Sandt, Rob A. 1992. Presupposition Projection as Anaphora Resolution. *Journal of Semantics*, **9**(4), 333–377.

Veltman, Frank. 1996. Defaults in Update Semantics. *Journal of Philosophical Logic*, **25**(3), 221–261.

Villalta, Elisabeth. 2008. Mood and Gradability: An Investigation of the Subjunctive Mood in Spanish. *Linguistics and Philosophy*, **31**(4), 467–522.

von Fintel, Kai. 1999. NPI Licensing, Strawson Entailment, and Context Dependency. *Journal of Semantics*, **16**(2), 97–148.

Walton, Kendall L. 1993. Metaphor and Prop Oriented Make-Believe. *European Journal of Philosophy*, **1**(1), 39–57.

Willer, Malte. 2013. Dynamics of Epistemic Modality. *Philosophical Review*, **122**(1), 44–92.

Willer, Malte. 2017. Simplifying with Free Choice. *Topoi*, February, 1–14.

Williamson, Timothy. 2020. Knowledge, Credence, and the Strength of Belief. *In:* Flowerree, Amy, & Reed, Baron (eds), *Expansive Epistemology: Norms, Action, and the Social World*. Routledge: London.

Yalcin, Seth. 2007. Epistemic Modals. *Mind*, **116**(464), 983–1026.

Zimmermann, Thomas Ede. 2000. Free Choice Disjunction and Epistemic Possibility. *Natural Language Semantics*, **8**, 255–290.

Zvolensky, Zsófia. 2002. Is a Possible-Worlds Semantics of Modality Possible? A Problem for Kratzer's Semantics. *In:* Jackson, B (ed), *Proceedings of SALT 12*. Ithaca: Cornell.