

What part of 'no' don't you understand?

Some thoughts on Negative Quantifiers

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The goal of this short article is to present a novel idea that both Universal Quantifier and Existential Quantifier can be reduced to a Negative Quantifier.

In mathematics, only Universal Quantifiers and Existential Quantifiers are traditionally used. There are even special symbols for this: \forall (a vertically inverted "A") and \exists (a vertically inverted "E").

The letter **A** stands for **A**ll and the letter **E** stands for **E**xists.

When describing natural language, semanticists actually use more than just these two quantifiers. In English, quantifiers may compound with other words.

Universal Quantifier: everyone, everything, everywhere, ...

Existential Quantifier: someone, something, somewhere, ...

Negative Quantifier: noone, nothing, nowhere, ...

Interestingly, we can derive the other quantifiers from the negative one. But first we need a symbol for the Negative Quantifier. For this I suggest we use a vertically inverted **N**. (Luckily it isn't hard to find on the keyboard, since an inverted **N** is also written **N**.)

$\mathbf{N}x P(x)$ means "there is no x such that $P(x)$ ".

Both the Universal Quantifier $\forall x$ and the Existential Quantifier $\exists x$ can be defined in terms of $\mathbf{N}x$ in the following way:

$$(1) \forall x P(x) = \mathbf{N}y \mathbf{N}x P(x)$$

$$(2) \exists x P(x) = \mathbf{N}x \mathbf{N}y P(x)$$

Of course, the variable y must be free in $P(x)$.

$\forall x$ and $\exists x$ are no longer elementary.

A Regular Negation likewise can be expressed in terms of a Negative Quantifier **N** x :

$$(3) \neg P(x) = \mathbf{N}y P(x)$$

As is well known, all other logical connectives can be expressed in terms of Conjunction and Negation. Assuming that pre-linguistic intelligence (Margaliot, 2017) already has Conjunction, adding Negative Quantifier **N** immediately gives us all the other quantifiers as well as all of the logical connectives.

References

Margaliot, Sasson 2017 "Minimalist Semantics and Pre-Linguistic Intelligence", [lingbuzz/003409](#)