Negation and modality in unilateral truthmaker semantics[*]

Lucas Champollion
New York University

Timothée Bernard
Laboratoire de linguistique formelle, Université Paris Cité

Abstract Fine (2017a) develops a unilateral and a bilateral truthmaker semantics for propositional logic. The unilateral approach trades off the primitive exact falsification relation of the bilateral approach for a primitive exclusion relation between states, thereby raising the question if exclusion serves any purpose other than to avoid exact falsification. We argue that exclusion is motivated independently of its use in avoiding exact falsification, namely as a foundation for the reconstruction of modal notions such as possibility and necessity. This reconstruction in turn motivates what we call *emergent exclusion*: an atomic state can exclude a sum of atomic states collectively without excluding any of these atomic states individually. Emergent exclusion is banned in Fine (2017a) in order to maintain exact equivalence in de Morgan's Law $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$; we argue that the two sides of this law are not exactly equivalent and discuss a variety of state spaces that feature emergent exclusion. This paper aims to be accessible to linguists without prior exposure to truthmaker semantics. We highlight points of contact with natural language semantics, such as event semantics and algebraic semantics of plurals and conjunction.

Keywords: truthmaker semantics, exclusion, negation, possible worlds, formal semantics, hyperintensionality, de Morgan's Laws

1 Introduction

In possible-world semantics, the proposition denoted by a sentence is understood as the set of possible worlds at which it is true. Negation, conjunction and disjunction are interpreted in terms of complement, intersection, and union on these propositions. Entailment is truth preservation at each possible world. There is only one necessary proposition, the set of all worlds, and there is only one impossible proposition, the empty set.

* We are grateful to Chris Barker, John Harding, Matteo Plebani, Giuliano Rosella, Vita Saitta, and to audiences at NYU, at the Theoretical Linguistics at Keio (TaLK) 2022 conference in Tokyo, at Viola Schmitt's Typology of Cumulativity Project talk series, and at the 2023 NYU conference on truthmaker semantics and related approaches in Florence. Special thanks to Justin Bledin, Kit Fine and Zhuoye Zhao.

These features of possible-world semantics make it an inhospitable framework for expressing certain intuitive notions about logical operators, properties, and relations. These include the notions that negation is related to rejection and incompatibility; that conjunction expresses a summing up or compounding; that disjunction involves a choice between alternatives; that entailment requires more than mere truth preservation; and that there are many distinct necessary and impossible propositions.

Such notions find a natural home in truthmaker semantics, a framework which draws on many traditions including the philosophy of truthmaking (Mulligan, Simons & Smith 1984) and the logic of tautological entailment (van Fraassen 1969). Our work builds chiefly on Fine's recent formal and philosophical developments of truthmaker semantics (Fine 2014, 2017a,b,c).

In truthmaker semantics, the verifiers of a sentence are not possible worlds but entities called *states* or *truthmakers*. While possible worlds are taken to be both possible and maximal, truthmakers are not in general maximal, and on many versions of truthmaker semantics they need not even be possible. The leading intuition of truthmaker semantics is that a truthmaker in the denotation of a true formula is an "exact verifier" for this formula, in the sense that it is relevant in its entirety to the truth of this formula. In this they contrast not just with possible worlds but also with the "inexact verifiers" of frameworks such as situation semantics (Kratzer 2021).

A *prima facie* problem for truthmaker semantics arises from the need to formulate an adequate semantics for negation. Unlike in possible-world semantics, it is not in general appropriate in truthmaker semantics to assume that every state that fails to verify a given proposition verifies its negation. As Fine (2017a) puts it:

> It might be thought that falsification could be defined in terms of verification, since for a state to falsify a proposition is for it to verify the negation of the proposition. But, within the present framework, it is not clear which set of verifiers should be taken to correspond to the negation of a proposition …

One approach to this problem is to introduce falsification as an additional primitive relation between truthmakers and propositions, on a par with verification. This leads to bilateral semantics, which associates every sentence with a set of verifiers and a set of falsifiers. Negation, then, switches verifiers and falsifiers. Another approach is to introduce an incompatibility or exclusion relation between truthmakers as an additional primitive, and to use this relation to define the semantics of a negated proposition in terms of the verifiers of its prejacent. On this approach, one can dispense with the association of sentences with sets of falsifiers; accordingly, the approach is called unilateral semantics.[1,2]

Any bilateral semantics has the drawback that to the set of Tarski-style clauses that specify the verifiers of complex expressions there needs to be added another set of clauses that specify the falsifiers. The latter clauses cannot, in general, be obtained automatically from the former; and in the case of truthmaker semantics, this provides a degree of freedom that is unwanted at least to the extent that our intuitive grip on exact falsification is weaker than on exact verification. Moreover, since the semantics in common use for lambda calculi such as Ty2 (Gallin 1975) are unilateral, bilateral truthmaker semantics cannot straightforwardly be embedded into such frameworks.

For these reasons, the unilateral approach seems preferable. But it comes at a cost, namely the need to introduce exclusion between truthmakers as an additional primitive. It might seem that little is to be gained in this tradeoff if our intuitive grip on exclusion turns out to be just as unsteady as on the notion of exact falsification, or if exclusion serves no purpose other than to avoid exact falsification.[3]

The present paper is a study of the exclusion relation in unilateral truthmaker frameworks. We argue that exclusion can be motivated independently of its use in avoiding exact falsification in the definition of negation. Specifically, we motivate exclusion as providing a foundation for reconstructing modal notions such as possibility and necessity, which can be applied both to states and to propositions. This contrasts with the approach in Fine (2017a) and elsewhere which takes possibility to be a primitive property of states; however, this aspect of our proposal is compatible with Fine (2017a) and could be seen as a friendly amendment. As a matter of fact,

1 While it follows from plausible metaphysical assumptions that the truthmakers of true negative claims are positive existents, it is not obvious what these truthmakers should be (Molnar 2000). Proposals not reviewed here include the following: totality facts, understood as states of the sum of all occurring truthmakers being all there is (Armstrong 1997, 2004, 2010, Moltmann 2007, 2019); sums of all events occurring within a given time span (Krifka 1989); states of locations being as they actually are (Cheyne & Pigden 2006); negative facts (Russell 1918, 1919; see also Linsky 2018 for the historical context); negative propositions, understood either as descriptions of a contrary positive proposition (Demos 1917) or in terms of some entailment relation from positive propositions (Veber 2008); mirror images of positive facts (Beall 2000, Björnsson 2007, Barker & Jago 2011) or individuals (Bledin 2022); absences of individuals (Kukso 2006); and other strategies such as those reviewed in Hochberg (1969), Brownstein (1973) and Paolini Paoletti (2014).

2 While unilateral truthmaker semantics is due to Fine (2014, 2017a), bilateral truthmaker semantics goes back to van Fraassen (1969). Bilateral approaches to semantics more generally are also found, for example, in Schwarzschild (1994), Muskens (1995), L. K. Schubert (2000), and Veltman (1984, 1985).

3 This tradeoff is not specific to truthmaker semantics. For a general defense of the incompatibility-based, unilateral "Australian" plan for negation against the four-valued, bilateral "American" plan see Berto & Restall (2019). This terminology is due to Meyer & Martin (1986).

we are not the first to propose this amendment. After substantial work on this paper had been completed, we became aware that Plebani, Rosella & Saitta (2022) independently proposed using exclusion to define modal notions such as that of a possible state. In both cases, this move is presented as an improvement on Fine (2017a), where possible events are defined as a primitive notion.[4]

We furthermore argue that some applications of truthmaker semantics motivate what we call *emergent exclusion*, a property that is ruled out in Fine (2017a). A paradigmatic case of emergent exclusion obtains when an atomic state excludes a sum of atomic states as a whole without excluding any of these atomic states individually. Whether emergent exclusion should be countenanced is a metaphysical question which corresponds, on the semantic level, to the question whether a negated conjunction $\neg(P \wedge Q)$ has the same truthmakers as a disjunction of negations $\neg P \vee \neg Q$. Since bilateral semantics is flexible enough to assign either the same truthmakers or different truthmakers to these two formulas, and since one would like to know to what extent unilateral semantics can match this flexibility of bilateral semantics, this question bears on the relationship between unilateral and bilateral semantics. Fine (2017a) assumes that the two formulas do have the same truthmakers; we argue that this assumption is unwarranted and we present intuitively motivated state spaces which crucially involve emergent exclusion. This aspect of our proposal represents a more substantive break from the system in Fine (2017a), but it receives motivation from other parts of Fine's work on truthmaker semantics. As we will see, in other contexts Fine appeals to state spaces which, on inspection, turn out to involve emergent exclusion. Our work thus emphasizes a previously unnoticed fault line that runs through Finean truthmaker semantics.

As within any exact truthmaker semantics, a variety of logical notions of truth, entailment, equivalence, satisfiability and validity arise in our system. Among them we focus on the classical notions, and we show that they can be recovered from our system if desired. At the same time, like other truthmaker semantics and similar systems, our system is hyperintensional: it will in some cases assign different propositions to formulas that are true in the same possible worlds.

This paper aims to be accessible to linguists without prior exposure to truthmaker semantics. We highlight points of contact with natural language semantics,

4 Throughout this paper, we acknowledge conceptual and technical points of contact with Plebani, Rosella & Saitta (2022) in a series of footnotes. There are also significant differences between the papers. The formal system we develop is fully unilateral while the system in Plebani, Rosella & Saitta (2022) is unilateral only in the case of the clauses for propositional letters, and bilateral for all the other clauses. Furthermore, the argument for emergent exclusion previewed in the next paragraph and the concomitant break from Fine (2017a) are absent from Plebani, Rosella & Saitta (2022) and, to the best of our knowledge, are novel to the present paper.

such as event semantics (Davidson 1967) and algebraic semantics of plurals and conjunction (Link 1983).

The remainder of this paper is structured as follows. Section 2 reviews basic notions of lattice theory, which forms the algebraic underpinning of truthmaker semantics. Section 3 introduces the exclusion relation and the axioms that govern its behavior. Section 4 motivates exclusion as a means to recover modal properties of truthmakers and propositions. Section 5 wards off a possible circularity concern by arguing that modal properties do not attach to instances of the exclusion relation. Our argument for emergent exclusion is developed in Section 6. The consequences of our view for de Morgan's Law $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$ are discussed in Section 7. Section 8 defines possible worlds and gathers the constraints on exclusion in one definition. With this in place, we give the semantics of a propositional language in Section 9 and show that classical notions can be recovered. Section 10 concludes.

2 The lattice of truthmakers

We begin the formal part by recalling basic facts of lattice theory (Grätzer 1978, Landman 1991). We work within a complete distributive lattice $\langle E, \sqsubseteq \rangle$, the *state space*, where E is a set whose subsets play the role of propositions, and whose elements, the *states*, are taken to be fully determinate entities that play the role of truthmakers for these propositions (Jago 2022). The paradigm case of a state is an event in the sense of Davidson (1969); accordingly, we often write e_1, e_2, etc. for elements of E.[5] States may, perhaps, also include other entities, such as the attitudinal states of Moltmann (2020).

In virtue of being a complete lattice, our structure comes equipped with a partial order $\sqsubseteq$ which we understand as a parthood relation from "smaller" to "larger" truthmakers. When $e_1 \sqsubseteq e_2$, we say that e_1 is part of e_2 or, equivalently, that e_2 contains e_1. We write $e_1 \sqsubset e_2$ to indicate that e_1 is a "proper part" of e_2 or that e_2 "properly contains" e_1; that is, e_1 is part of but not identical to e_2.

Since our structure is a *lattice*, any $e_1, e_2 \in E$ have both a least upper bound ("sum", "fusion", or "join") $e_1 \sqcup e_2$ and a greatest lower bound ("product" or "meet") $e_1 \sqcap e_2$ with respect to $\sqsubseteq$; these operations are associative, commutative, and idempotent. E being *complete*, any $P \subseteq E$ has (unique) upper and lower bounds $\bigsqcup P$ and $\bigsqcap P$; in particular, E is *bounded* since it has both a greatest or "top" element $\bigsqcup E$ (the "full state", $\blacksquare$) and a least or "bottom" element $\bigsqcap E$ (the "null state", $\square$).[6]

5 The present work emerged from the development of a theory of negative events, reported in Bernard & Champollion (2018, 2024).

6 To avoid confusion in the context of our later application to propositional logic, we follow Fine (2017a) in using $\bigsqcup$, $\bigsqcap$, $\blacksquare$, and $\square$ instead of the more common symbols $\bigvee$, $\bigwedge$, $\top$, and $\bot$.

Thus, every $e \in E$ is part of the full state and contains the null state. We say that $e_1, e_2 \in E$ *overlap* whenever they have a nonnull part in common.

Since we think of events as truthmakers, it is worth mentioning that events are often thought to form a structure governed by the axioms of Classical Extensional Mereology (CEM, Tarski 1935, Pontow & R. Schubert 2006, Hovda 2009). For overviews of applications of CEM in linguistics, see Champollion & Krifka (2016) and Champollion (2017). We think of the $\sqsubseteq$ relation and $\sqcup$ operation as identical with the parthood and sum operations in CEM, modulo the presence of the null state, which is absent from models of CEM.

Compatibility with CEM is one reason we require the state space to be distributive. A lattice is *distributive* if for any e_1, e_2, e_3, we have $e_1 \sqcap (e_2 \sqcup e_3) = (e_1 \sqcap e_2) \sqcup (e_1 \sqcap e_3)$ and its dual. We discuss other reasons in Section 6.

Due to the absence of the null state, in CEM the product operation is only defined on overlapping entities. In contrast, we assume the null state so that we can maintain that the product operation $\sqcap$ is always defined, and so that the singleton of the null state is available as a proposition that is necessarily true.[7] We are thus broadly in agreement with semantic theories that explicitly embrace the null state (Bylinina & Nouwen 2018) or use it as a dummy individual (Link 1983).

3 The exclusion relation

Against this background of standard lattice theory, we now introduce elements that are specific to the metaphysics of truthmakers, in particular the exclusion relation (Fine 2017a).

Formally speaking, exclusion is an undefined primitive whose meaning will be constrained by a number of axioms but will not be given in terms of any other concept. Informally, we understand exclusion to represent incompatibility between two events; more specifically, it is a kind of incompatibility to which both events are wholly relevant. In other words, exclusion between two events is to be understood as indicating that something about these events in their entirety rules out their occurring together. We may think of one event as refuting, precluding, preventing, knocking out, contradicting, contravening, etc. the other event, so that at least one of them fails to occur. We write this relation as $e_1 \perp e_2$, to be read as "e_1 excludes e_2". Our relation is a variant of its namesake in the unilateral truthmaker semantics in Fine (2017a). While the relation in Fine (2017a) is exact on its first argument and inexact on its second, our variant is exact on both of its arguments. We come back to this point shortly.

7 In the framework developed in what follows, the singleton of the null state can serve as the negation of the empty proposition, which is necessarily false.

Our exclusion relation bears similarity to a relation used in orthologic (Goldblatt 1974, Dunn 1993, 1996, 1999, Dunn & Zhou 2005). The structures for orthologic include an orthogonality relation (also symbolized $\perp$) that is the inexact counterpart of our exclusion relation. Orthogonality compares to our exclusion relation in the same way as inexact truthmakers (such as situations) compare to exact truthmakers (such as events) in Fine (2017a). As we will see, an inexact counterpart of our exact exclusion relation (*conflict*) can also be defined in our system.[8]

In Section 2, we mentioned that we situate our theory within a complete lattice. A special case of this kind of structure is the powerset algebra of any given set (i.e., the Boolean algebra of its subsets). Some readers may find it helpful to work through the upcoming sections with a similar special case in mind, which we describe now.

Let us assume that a set A is the union of two disjoint sets of equal size $A_1 = \{p, q, r \dots\}$ and $A_2 = \{\overline{p}, \overline{q}, \overline{r} \dots\}$ whose elements stand in a one-to-one correspondence. That is to say, we assume that there is an involution that maps each $x \in A_1$ to its corresponding $\overline{x} \in A_2$ and vice versa. The powerset algebra of A, $\wp(A)$, is a complete distributive lattice and hence a state space. Its elements are sets such as $\emptyset$, $\{p\}$, $\{p, \overline{q}\}$, or $\{q, \overline{q}, \overline{r}\}$. With $S_1 = \{\{x\} \mid x \in A_1\}$ and $S_2 = \{\{\overline{x}\} \mid \overline{x} \in A_2\}$, let us define the involution m (for "mirror") between S_1 and S_2 such that $m(\{p\}) = \{\overline{p}\}$, $m(\{q\}) = \{\overline{q}\}$, and so on. In the following, we will not think of the elements of $\wp(A)$ as sets but as events, and we will write them as p rather than $\{p\}$, $p \sqcup \overline{p} \sqcup \overline{r}$ rather than $\{p, \overline{p}, \overline{r}\}$ and so on, $\square$ rather than $\emptyset$, and $\blacksquare$ rather than A. Finally, assume that each $x \in A_1$ excludes and is excluded by its corresponding $\overline{x} \in A_2$, and that no other exclusions hold.

We will refer to structures that are analogous to this example as *canonical frames*; they are special cases of *E-frames* (or simply "frames"; the "E" stands for "exact" as in exact truthmaking). In the case of E-frames in general, we do not make any assumption to the effect that every atomic event excludes one and only one event, nor do we assume that the set of events is atomic or nonatomic, finite or nonfinite. Formal definitions for E-frames and canonical frames are given at the end of Section 8.

In general, we assume that some events occur and some events do not occur; this is still an informal assumption that will also be given formal meaning in Section 8. When we say that two or more events co-occur, we mean that each of them occurs. We assume that no two events that stand in the exclusion relation co-occur. Thus in a canonical frame, for each x, either x or $\overline{x}$ fails to occur. As we will see, in general it may be the case that $e_1 \perp e_2$ and that neither e_1 nor e_2 occurs; in the

8 The relation between exact and inexact exclusion and the parallel to exact and inexact truthmaking are also pointed out in Plebani, Rosella & Saitta (2022: 214).

specific case of canonical frames, though, it will turn out that for any x, either x or $\overline{x}$ occurs.

Assumptions to the effect that some events do not occur, or that some states do not obtain, are implicit in many theories that resemble ours in some respects, such as Fine's truthmaker semantics or situation semantics (e.g., Kratzer 1989).

We will use the following historical scenario as a running example. On November 2, 1948, the incumbent president of the United States, Harry Truman, was reelected. The next day, the Chicago Daily Tribune erroneously ran a front-page article calling the election for his opponent Thomas Dewey. Under the headline "Dewey defeats Truman", the lead article went on to describe an event that did not in fact occur. Throughout this section, we assume that Truman's and Dewey's victories stand in the exclusion relation (we will revise this assumption in Section 5).

We assume that occurring and nonoccurring events alike are elements of E. We refer to the sum of all those events that occur as *actuality* or as the *actual world*. As will become clear later, we assume that the sum of any set of actual events is itself actual, and that all the parts of an actual event are actual; thus actuality is maximal among actual events. An event that does not occur may fail to overlap with actuality, or it may overlap with actuality but lie partly outside of it. For example, let e_1 be the (historical) event of Truman's victory over Dewey, and let e_2 be Dewey's victory over Truman. Then their sum $e_1 \sqcup e_2$ is a nonoccurring event even though it has a part that occurs (namely e_1). As another example, take a canonical frame and assume that p occurs but $\overline{p}$ does not. Then their sum $p \sqcup \overline{p}$ is a nonoccurring event even though it has a part that occurs (namely p).

When two events stand in the exclusion relation, both of them are understood to be wholly relevant to the excluding, in the same way that exact truthmaking between a truthmaker and a truthbearer is a relation in which the truthmaker is wholly relevant to the truthmaking. In the historical example, the occurrence of e_1 (Truman's victory over Dewey) is wholly relevant to the nonoccurrence of e_2 (Dewey's victory over Truman). There are many other events that are not relevant to the nonoccurrence of e_2 in any way, and others that are only partly and not wholly relevant. For example, another notable event in 1948 was Gandhi's assassination. Call this event f_1. Presumably f_1 is not related to the US elections and its occurrence is not even partly relevant to the occurrence of e_1 or e_2. Now consider the event $e_1 \sqcup f_1$ and its relation to e_2. In an intuitive sense, this event is just as incompatible with e_2 as e_1 itself is, since neither e_1 nor $e_1 \sqcup f_1$ could co-occur with e_2. But part of $e_1 \sqcup f_1$, namely f_1, is irrelevant to the nonoccurrence of e_2. For this reason the occurrence of $e_1 \sqcup f_1$ is not *wholly* relevant to the nonoccurrence of e_2. Accordingly, we assume that $e_1 \sqcup f_1$ does not exclude e_2 even though, as we assume in this section, e_1 excludes e_2. Similarly, in a canonical frame, $p \sqcup q$ does not exclude $\overline{p}$ even though p does.

Likewise, e_1 will not in general exclude the sum of e_2 and some unrelated event. Consider, for example, the sum $e_2 \sqcup f_1$. Here, e_1 by itself is, in an intuitive sense, incompatible with $e_2 \sqcup f_1$ simply in virtue of excluding e_2. But the occurrence of e_1 is not *wholly* relevant to the nonoccurrence of $e_2 \sqcup f_1$, because the latter contains a part which is not prevented by e_1 from occurring (and which indeed occurs).

The fact that f_1 is an occurring event is immaterial to this example and we could have equally well have used a nonoccurring event instead. For example, consider an event f_2 of Gandhi's dying of natural causes. It seems that $e_1 \sqcup f_2$ does not exclude e_2 and that e_1 does not exclude $e_2 \sqcup f_2$ either. In general, then, we do not assume that the exclusion relation is upward persistent on either its first or its second argument. That is to say, from $e_1 \perp e_2$, $e_1 \sqsubseteq e_1^+$ and $e_2 \sqsubseteq e_2^+$ alone one cannot infer either $e_1^+ \perp e_2$ or $e_1 \perp e_2^+$.

The points we have made with our historical example could have been made equally well if we had swapped e_1 with e_2 and f_1 with f_2. For example, it is reasonable to assume that either candidate's victory excluded the other's, irrespective of which one of them actually occurred. This brings us to our first point of divergence with Fine (2017a). We assume that the exclusion relation is symmetric, where for Fine it is not (in general) symmetric since it is exact on its first argument but inexact on its second. Symmetry clearly holds in canonical frames by construction, and we enforce it for all frames by stipulation. This is in line with Berto (2015) (and many others cited therein), where an inexact counterpart of exclusion (akin to the orthogonality relation in orthologic) is argued to be symmetric:

> [W]hatever ontological kinds a and b belong to, it appears that if a rules out b, then b has to rule out a; that if a's obtaining is incompatible with b's obtaining, then b's obtaining must also be incompatible with a's obtaining; etc. Alleged counterexamples, it seems to me, rely on equivocation, usually importing some asymmetry from causal relations or the temporal ordering of actions and processes.

Formally, symmetry is the first of a number of axioms we impose on the exclusion relation:

(1) **Axiom: Symmetry**
$\forall e_1 \forall e_2. [e_1 \perp e_2 \rightarrow e_2 \perp e_1]$
(If any event excludes another then the other excludes it too.)

Interestingly, the technical results we present further ahead do not appear to depend on the symmetry assumption. We adopt it to account for the intuition in Berto

(2015) and references cited therein; see also Section 4 for further motivation. The symmetry assumption seems to us to be in the spirit of exact truthmaker semantics. For Fine (2017a), the excluding state should be wholly relevant to the exclusion of the state that it excludes. This requirement is analogous to the idea that verifiers and falsifiers should be wholly relevant to their truthbearers. Since both relata of the exclusion relation have the same ontological status, it seems natural to adopt symmetry as a means to ensure that both states are wholly relevant to the excluding.

Our exclusion relation is not minimal, in the sense that while exclusion involves events that are wholly rather than partly relevant to each other's nonoccurrence, these events may have parts which are in turn also wholly relevant to each other's nonoccurrence (see Footnote 14 below for an example). So exclusion should not be understood as the result of restricting a more general "inexact" or partial incompatibility relation such as the orthogonality relation in orthologic by applying some kind of minimization operation. Nor can exclusion be gained from inexact incompatibility relations through exemplification as in Kratzer (2021). Whereas Kratzer and others attempt to define exact relevance out of partial relevance, we do the opposite, in agreement with Fine (2017c) in this respect. This mirrors a point made for exact vs. minimal truthmaking in Fine (2017a); Fine (2017c) even speaks of "minimalitis" in this connection.

4 Motivating exclusion

Let us now motivate the exclusion relation we have sketched. Recall that the unilateral approach simplifies the semantics by doing away with the falsitymaking relation, at the cost of introducing the exclusion relation into the metaphysics. The question now is how this theoretical cost should be assessed. Here we suggest that exclusion is independently motivated as a way to account for the modal properties of events and propositions. With Fine (2017a), we take the modal properties of propositions to be reducible to the modal properties of the events that make them true; that is, possibility and necessity are fundamentally properties of events and only derivatively properties of propositions. Going beyond Fine (2017a) but in line with his unilateral system, we show that these modal properties of events can be further reduced to the properties of the exclusion relation.

This approach contrasts with an exclusionless framework such as the bilateral system in Fine (2017a), in which only the reduction from the modal properties of propositions to those of events is available, and the modal properties of events themselves are settled by stipulation. Specifically, Fine assumes that each state space comes with a primitive partition of states (i.e., events in our parlance) into possible and impossible. The set of possible events is stipulated to be downward closed; that is, any part of a possible event is assumed to be possible. Such an approach does not illuminate what it is for an event to be possible, nor does it explain why possibility transfers from an event to its parts.

While Fine takes the partition between possible and impossible states to be primitive both in his unilateral and in his bilateral systems, we account for this partition and its downward-closedness by exploiting properties of the exclusion relation. This move and its use are not present in Fine's work, but they are compatible with his unilateral system and within its general spirit.[9] As we will see in the next section, though, developing our argument leads us to a conception of exclusion whose properties differ from those that Fine (2017a) imposes on it.

To start our account, we ask: What is it for an event to be possible? Let us use the analogy between events and bodies of information, such as might be given by testimonies or databases (Dunn 1996). Bodies of information can, in general, be coherent or incoherent. Consider an incoherent body of information, such as might be given by a testimony that contradicts itself. It is natural to think of its incoherence as arising from conflicting information within it. And if a testimony conflicts with itself, its content is ordinarily judged impossible. In the analogy, the possibility of an event's occurrence corresponds to the coherence of the corresponding body of information. This suggests a plausible answer to our question: For an event to be possible is for it to be free of conflicts.

The idea that events can be partitioned into conflicted and conflict-free naturally accounts for the assumption that the set of possible events is downward closed. For, if a testimony is conflict-free, then each of its parts is also conflict-free. Equivalently, if a testimony conflicts with itself, then the conflict involved will remain if more information is added: once witnesses contradict themselves, no amount of additional testimony they might add will change this. In other words, the notion of conflict is upward persistent while the notion of being conflict-free is downward persistent, just like the property of being a possible event is downward persistent.

How does the idea of a conflict within an event relate to exclusion between events? In the case of bodies of information, we think of conflicts as being grounded in specific pieces of information that are mutually incompatible in their entirety. Likewise in the case of events, we suggest that conflicts are grounded in a relation between one or more specific parts of the event that are incompatible in their

9 A partial precursor is part (i) in Fine (2017a: 668) of the definition of what he calls a classical exclusion relation. Part (i) states that if $s \perp t$ then the sum of s and t is impossible. This is only a partial precursor because it is not used to define impossibility and because it is a one-way implication; its converse is harder to state because decompositions of events into their parts are not unique and we do not require every such decomposition of an impossible event to involve exclusion. However, as already mentioned in Section 1, a proposal similar to ours has been independently made by Plebani, Rosella & Saitta (2022).

entirety, i.e., that exclude each other. This provides the desired motivation for the exclusion relation (and more specifically for an exclusion relation that is symmetric and exact on both its arguments). This motivation is unrelated to falsitymaking and hence independent of it.

If we are right that the possibility or impossibility of every truthmaker can be derived from the exclusion relation, this rules out many of what Fine (2021: §9.5) describes as

> modal 'monsters', impossible states whose impossibility is not attributable to any conflict between their possible parts.

The only modal monsters we recognize, though it is not clear if Fine would consider them as such, are states which stand in the exclusion relation to themselves. Fine speaks of a natural aversion to modal monsters but acknowledges that there may be reasons for wanting to allow them. While this paper concentrates on propositional languages, let us digress until the end of this paragraph and consider the case of identity, which does provide a reason to postulate monsters. In order to develop truthmakers for statements like $\neg(s = s)$ of a language with individuals and identity, one might wish to recognize not only the possible (and indeed necessary) state $s_=$ of Socrates' being self-identical, but also the impossible state $s_{\neq}$ of Socrates' being self-distinct. This latter state does not appear to have any distinct parts whose mutual exclusion might account for its impossibility. On our account, then, if a state like $s_{\neq}$ is recognized, we are led to the assumption that it excludes itself. As an anonymous reviewer suggests, this assumption might strike one as implausible (perhaps on the grounds that $s_{\neq}$ should be rejected as not being a fully determinate state), so we now turn to another example. As Fine notes, an alternative is to recognize impossible states such as the state of Socrates' and (for instance) Plato's being identical, and to take all such states to be truthmakers of $\neg(s = s)$. In the case of such a state it does seem plausible that its impossibility is due to self-exclusion. For in a language with individual constants, we may want to enrich our domain with individuals and allow states to have bearers (as well as agents, themes, and so on); and the state of Socrates' and Plato's being identical seems to place two mutually exclusive requirements on its bearer, just like any event of drawing a round square may be thought of as imposing two mutually exclusive requirements on its theme. This ends our digression.

We will say that e_1 *conflicts with* e_2 just in case some part of e_1 excludes some part of e_2. Given Symmetry of Exclusion, the conflict relation is symmetric as well. In other words, when an event conflicts with another, the latter also conflicts with the former, and we may thus simply say that the two events conflict. We write

$e_1 \not\sim e_2$ to indicate that e_1 conflicts with e_2 and reserve $\perp$ for the exclusion relation.

(2) **Definition: Conflict**
$e_1 \not\sim e_2 \overset{\text{def}}{=} \exists f_1 \exists f_2.\ f_1 \sqsubseteq e_1 \wedge f_2 \sqsubseteq e_2 \wedge f_1 \perp f_2$
(e_1 *conflicts with* e_2 just in case some part of e_1 excludes some part of e_2.)

It is important to keep in mind that when $e_1 \not\sim e_2$ in our system, there is no requirement for e_1 and e_2 to be distinct, nor for their excluding parts to be distinct. We also write $e_1 \sim e_2$, read "e_1 *coheres with* e_2", just in case they do not conflict:

(3) **Definition: Cohere**
$e_1 \sim e_2 \overset{\text{def}}{=} \neg(e_1 \not\sim e_2)$
(e_1 *coheres with* e_2 just in case e_1 does not conflict with e_2.)

For example, in a canonical frame the two events $p \sqcup q$ and $p \sqcup \overline{q}$ conflict even though they do not exclude each other. The events q and $\overline{q}$ conflict and exclude each other. The events p and q cohere.

As mentioned, the $\not\sim$ relation corresponds conceptually to the orthogonality relation in orthologic, but in our system it is derived rather than primitive. When two events stand in the $\not\sim$ relation, either one or both may contain extraneous material to the exclusion relation; in this sense we may say that the $\not\sim$ relation is inexact on both sides. The exclusion relation in Fine (2017a), being exact on its first argument and inexact on its second, thus lives in between our $\not\sim$ relation (inexact on both sides) and our $\perp$ relation (exact on both sides).

We are now in a position to define possibility in terms of being conflict-free. We call e_1 *possible* just in case it coheres with itself; otherwise it conflicts with itself and we call it *impossible*.[10]

(4) **Definition: Possible Event**
$\text{Poss}(e) \overset{\text{def}}{=} e \sim e$
(An event is possible just in case it coheres with itself.)

Since parthood is reflexive, any event that excludes itself is also impossible. For example, we might consider any event of squaring the circle, and any event of Socrates' being distinct from Socrates, to be self-excluding and therefore impossible. The converse, however, does not hold: An impossible event might merely conflict with itself without excluding itself. An example would be the sum of an event of Jones' squaring the circle and an event of Smith's buttering the sandwich.

10 This definition is also found in Plebani, Rosella & Saitta (2022: 223).

Clearly, according to our definitions, any part of a possible event is possible; and by contraposition, any event with an impossible part is impossible. This formalizes our account of the downward closure of possibility:[11]

Lemma 4.1 (Possibility is downward closed)**.**
$\forall e_1 \forall e_2.\ [[\mathrm{Poss}(e_1) \wedge e_2 \sqsubseteq e_1] \rightarrow \mathrm{Poss}(e_2)]$
(Any part of any possible event is also possible.)

See proof on page 40.
(Proofs of theorems in this paper can be found in the Appendix.)

Since the notion of impossibility that we adopt is based on the inexact notion of conflict, it is itself inexact. Various exact notions of impossibility could also be defined but they will not be used in what follows. For example, we might call an event thoroughly impossible just in case it excludes itself. An event that is not thoroughly impossible in this sense may nevertheless conflict with itself and therefore be impossible in our sense.

It is straightforward to extend possibility from events to propositions, as in Fine (2017a):

(5) **Definition: Possible Proposition**
A proposition P is possible iff P contains at least one possible event.

For completeness' sake, we also give the definitions for necessity here, taken from Fine (2018).[12]

(6) **Definition: Compossible Events**
Two events are compossible just in case their sum is possible.

(7) **Definition: Necessary Event**
An event is necessary just in case it is compossible with every possible event.

(8) **Definition: Necessary Proposition**
A proposition P is necessary just in case every possible proposition contains some event which is compossible with some event in P.

11 Within the framework in Plebani, Rosella & Saitta (2022), Lemma 4.1 follows from claim 2 on p. 224.

12 Our term *compossible* picks out the same concept as what Fine calls *compatible*. But the latter term is more naturally understood as denoting absence of mutual conflict between two entities, without implying possibility (Nelson 1930). To reduce the risk of confusion, we use the term *compossible* as a reminder that the notion in question is defined in terms of joint possibility. The word suggests Leibniz' notion of the compossible, though we do not use that notion here.

The intuition behind the definition of a necessary event is that if an event S is not necessary, it should be possible for it not to occur. But then this possibility should be "realized" in some event that can occur, but that cannot co-occur with S.

In canonical models, the only necessary event is the null state $\square$. Indeed, every possible nonnull state excludes (and is therefore not compossible with) its mirror image, which is a possible event; and every impossible event fails to be compossible with any possible event, such as $\square$. In noncanonical models, there may be nonnull necessary events, and it may even be the case that every event is necessary (if the exclusion relation is the null relation).[13]

To sum up this section, we have motivated the exclusion relation in terms of its ability to define what it is for an event to be possible and to explain why the set of possible events is downward closed. Definitions of possible propositions and of necessary events and propositions can likewise be given in terms of exclusion. The exclusion relation is thus motivated independently of the role it plays in the definition of negation in unilateral semantics.

5 Is exclusion contingent?

At this point one might object that contrary to what we have assumed, at least some events exclude each other only contingently, which would seem to preclude reconstructing modal notions from exclusion. This objection can be sharpened by relating exclusion to other merely possible properties of events. It is common to assume that events may keep, lose, or acquire properties and relations as circumstances vary (e.g., Davidson 1971). For example, Truman's victory could well have occurred a few hours earlier than it did (and in that case perhaps the Chicago Daily Tribune would not have produced its famous misprint). Likewise, at first glance it often seems to be the case that exclusion is just as contingent as temporal location, and that certain events that happen to exclude each other could have failed to do so. For example, we might assume that Truman's victory e_1 and Dewey's victory e_2 actually exclude each other, but that perhaps if history had taken a radically different turn, Truman and Dewey might have run on the same ticket, or the electoral system might have been different in such a way that the runner-up to the presidential election becomes vice president—as was indeed the case before the Twelfth Amendment to the United States Constitution was passed in 1804. These considerations might lead one to assume that in such historical or counterfactual scenarios, Truman's victory and Dewey's victory could have co-occurred and hence

13 Conversely, there are noncanonical models in which every event is impossible, except for the null state $\square$, whose possibility is a consequence of the Actuality axiom introduced further below in Section 8. The fact that $\square$ is possible also has the desirable consequence that every necessary event is possible.

failed to exclude each other even though they actually do, and that $e_1 \perp e_2$ holds only contingently.

This objection can be resisted in two ways. First, we can maintain that $e_1 \perp e_2$ holds necessarily, by placing limits on the extent to which the properties of e_1 and e_2 can vary under various counterfactual circumstances (van Inwagen 1978). In other words, we can assume that e_1 and e_2 have some of their properties essentially. Under this assumption, the event e_1 that actually plays the role of Truman's victory would not occur in the counterfactual scenarios in question; in that counterfactual scenario, another event, distinct from e_1, would play that role instead. This could be justified, for example, if we conceive of e_1 as the specific way in which Truman won, viz., against Dewey and in an election that does not permit ties. Absent these circumstances, Truman could not have won in the way he did, and so while he might still have achieved victory, those victories are not identical to e_1 and perhaps do not exclude e_2. Similar assumptions, mutatis mutandis, could be adopted for the event e_2 that actually plays the role of Dewey's nonoccurrent but reported victory.

The second way to resist the objection does not rely on postulating essential properties of events but, instead, denies that e_1 excludes e_2 in the first place. What we previously thought of as the exclusion relation between e_1 and e_2 can be reified into a third event e_3 that excludes the co-occurrence of e_1 and e_2. This amounts to dropping the assumption that $e_1 \perp e_2$ and replacing it by the assumption that there is at least one e_3 such that $e_3 \perp (e_1 \sqcup e_2)$.[14] We might think of e_3 as any occurring event that accounts intuitively for the incompatibility of Truman's victory e_1 and Dewey's victory e_2. For example, one such e_3 might be the ratification of the Twelfth Amendment while another might be Truman's and Dewey's running on different tickets. This response is completely general. As long as we avail ourselves of enough truthmakers, it appears that there is no substantive hurdle to restating instances of seemingly contingent exclusion in terms that do not require contingency, and thus no hurdle to reconstructing modal notions from exclusion.

14 We might even reasonably assume that the sum of any two of the three events e_1, e_2, and e_3 in this example excludes the third. However, this last assumption cannot be plausibly generalized to all cases. That is to say, from $e_1 \perp (e_2 \sqcup e_3)$ it does not in general follow that $(e_1 \sqcup e_2) \perp e_3$. For example, consider a scenario with two switches a and b of which each can be in either of two states—up or down—independently of the other. Let $e_1 = a\text{-up} \sqcup b\text{-up}$, $e_2 = a\text{-down}$ and $e_3 = b\text{-down}$. We might plausibly hold that $a\text{-up} \perp a\text{-down}$, that $b\text{-up} \perp b\text{-down}$, and also that $(a\text{-up} \sqcup b\text{-up}) \perp (a\text{-down} \sqcup b\text{-down})$. In that case, we have $e_1 \perp (e_2 \sqcup e_3)$. But it is plausible that the relationship between the impossible state $a\text{-up} \sqcup b\text{-up} \sqcup a\text{-down}$ and the state b-down is conflict rather than exclusion, because whether a-up occurs is not relevant to whether b-down occurs; so we do not have $(e_1 \sqcup e_2) \perp e_3$.

6 Emergent exclusion

Most of the technical development so far has closely followed and built on the unilateral semantics in Fine (2017a). But as it turns out, the previous example involves an important deviation from the underlying assumptions of Fine's unilateral semantics. Consider again e_1, e_2 and e_3 from our previous example. We have $e_3 \perp (e_1 \sqcup e_2)$, but there is no reason to believe that $e_3 \perp e_1$ or $e_3 \perp e_2$; nor is there reason to believe that e_3 has any parts that exclude e_1 or e_2. Indeed, in the case of e_3 being the ratification of the Twelfth Amendment, e_3 excludes neither e_1 nor e_2 and it does not have any parts that do. (Clearly, e_1 and e_3 do not stand in the exclusion relation, since they both occur. By analogy, e_2 and e_3 do not stand in the exclusion relation either.) When a relation $\perp$ instantiates this behavior, we say that it exhibits *emergent exclusion*. Our view is that emergent exclusion is legitimate and should not be ruled out. The intuition behind this term is that the exclusion is not targeting individual parts of the excluded state, but that state as a whole. In this section, we show that Fine (2017a) rules out emergent exclusion while Fine (2012, 2021) implicitly allows it.

To give a precise formulation of emergent exclusion, we rely on the notion of regular closure defined in Fine (2017a) and on two auxiliary definitions.

(9) **Definition: Regular Closure**
Let S be any set of states. The regular closure of S, written S^*_*, is the set $\{s \mid \exists r \in S.\, r \sqsubseteq s \sqsubseteq \bigsqcup S\}$.
(The regular closure of a set S is the set of those parts of the sum of S that contain an element of S.)

The intuition behind the following auxiliary definitions is that a state can exclude a set of states either collectively by excluding their sum, or individually by excluding one of its members; the motivation behind the precise way the definitions are formulated will become clear subsequently in this section.

(10) **Definition: Collective Excluder**
Let P be any set of states. A *collective excluder* of P is any s such that $s \perp \bigsqcup P$.
(A collective excluder of a set P is any state that excludes the sum of the states in P.)

(11) **Definition: Individual Excluder**
Let P be any set of states. An *individual excluder* of P is any element of $\{s \mid \exists p \in P.\, s \perp p\}^*_*$.
(An individual excluder of a set P is any element of the regular closure of the set of excluders of any state in P.)

By expanding the regular closure operator, the foregoing definition can be equivalently reformulated as follows:

(12) **Individual Excluder (expanded definition)**
An *individual excluder* of P is any s such that both of the following conditions hold:

a. $\exists s^{-} \sqsubseteq s.\ \exists p \in P.\ s^{-} \perp p.$
(Some part of s excludes a member of P.)

b. $s \sqsubseteq \bigsqcup\{r \mid \exists p \in P.\ r \perp p\}.$
(s is part of the sum of all excluders of members of P.)

It is worth noting that condition (12a) entails that s conflicts with a member of P; furthermore, when s is an atom, condition (12a) is equivalent to stating that s itself excludes a member of P. In what follows, we focus on (12a).

We combine the last two definitions to formalize the concept of an emergent excluder:

(13) **Emergent Excluder**
An *emergent excluder* of a set P is any collective excluder of P which is not an individual excluder of P.

In the rest of this section, we provide additional motivation for emergent exclusion as a natural property of many state spaces, and we point out the costs involved in ruling it out. A sympathetic reader might wonder why Fine (2017a) or anyone else would want to rule out emergent exclusion; we defer this question to the next section.

Fine (2012) and Fine (2021) contain examples which are meant to illustrate unrelated points but can be used to motivate emergent exclusion as well. These examples all involve blocks a, b, c stacked in alphabetical order on a table, with a at the bottom. We focus here on Fine (2012). There Fine develops a truthmaker-based model for the semantics of counterfactuals which he suggests is also relevant to the frame problem from AI, i.e., the problem of formally describing what aspects of the world stay the same when actions are performed (McCarthy & Hayes 1969). Fine uses his model to distinguish inertial from non-inertial states, changes, and propositions. Intuitively, an inertial state is a state which can be attained by carrying out an action that does not bring further actions in its train. Fine illustrates this with a state space whose occurring atomic states are a's being on the table, b's being on a, and c's being on b. We will represent these states as a-on-Table, b-on-a, and c-on-b respectively. In this example, if c were to be removed from the top of the stack and put directly on the table (next to a), the rest of the stack could remain unaffected; this illustrates that the state c-on-Table that would result from this action is an inertial state. If, instead, a were to be moved from the bottom of the stack to the top of c, this would require changes to the rest of the stack, and so the state a-on-c that would result from this action is not inertial in this sense.

To define inertial states, one needs to find a relevant distinction between these two cases. Fine observes that the first action results in a state c-on-Table which is compatible with a-on-Table, with b-on-a, and also with their sum. By contrast, the second action results in a state a-on-c which is compatible with b-on-a and with c-on-b individually but not with their sum. Generalizing from this example, Fine suggests that inertial states can be defined as states which are compatible with the sum of any set of occurring states with which they are individually compatible. It is natural to think of compatibility as coherence, i.e., absence of conflict between the two verifiers. In our terms, then, a state e_1 is inertial just in case the set $\{e_2 \mid e_2$ occurs and $e_1 \sim e_2\}$ is closed under sum. For example, the non-occurring state a-on-c is *non*-inertial because a-on-c coheres with both of the occurring states b-on-a and c-on-b, but not with their sum b-on-$a \sqcup c$-on-b. For the same reasons, a-on-c is also an emergent excluder of the set $\{b$-on-a, c-on-$b\}$: a-on-c is a collective excluder of this set, for a-on-c excludes the sum of both its members, and a-on-c is not an individual excluder of this set, for a-on-c does not conflict with either of its members and thus has no part that excludes either of them.

Now consider an opponent to our view who is committed to ruling out emergent exclusion. Clearly, this opponent must object to the legitimacy of the state space for the block world just described. This objection might be developed as follows: In addition to states such as a-on-Table, b-on-a, and c-on-b which encode objects being placed immediately on other objects, one should also recognize states such as b-above-Table, c-above-a, and c-above-Table which encode objects being located above but not necessarily on top of other objects. In that case, one might then deny a-on-$c \perp (c$-on-$b \sqcup b$-on-$a)$ and maintain that we have a-on-$c \perp c$-above-a instead. In order to account for the impossibility of a-on-$c \sqcup c$-on-$b \sqcup b$-on-a one would need to ensure that the nonoccurrence of c-above-a in some way forces the nonoccurrence of c-on-$b \sqcup b$-on-a despite the lack of exclusion between these states.

One way to do this would be to assume that c-above-a is a part of c-on-$b \sqcup b$-on-a; both on Fine's account an on our own, this assumption entails that if c-above-a fails to occur then so does c-on-$b \sqcup b$-on-a. We will argue that this assumption should be rejected for two reasons: first, it is in tension with a basic tenet of mereology, namely distributivity; second, it does not generalize to analogous examples. We develop both reasons in turn.

First, observe that c-above-a is neither a part of c-on-b nor of b-on-a, nor does it have parts in common with both c-on-b and b-on-a, since Fine assumes these states to be atomic. So, assuming that c-above-a is a part of c-on-$b \sqcup b$-on-a, it is what we might call an *emergent part* of c-on-$b \sqcup b$-on-a. This rules out distributivity, a property which is desirable insofar as it holds of any state space that satisfies Classical Extensional Mereology, a standard assumption in algebraic semantics of natural language (Link 1983, Champollion & Krifka 2016). Recall that a lattice is *distributive* if the sum and product operators distribute over each other. This means that for any e_1, e_2, e_3, we have $e_1 \sqcap (e_2 \sqcup e_3) = (e_1 \sqcap e_2) \sqcup (e_1 \sqcap e_3)$ and its dual; one consequence of distributivity is that any state that overlaps with a sum of states overlaps with at least one of them. Fine (2017a: 646) assumes distributivity (even though, as he notes, many of his results do not depend on this assumption).

To avoid giving up distributivity, our opponent might instead maintain that c-on-$b \sqcup b$-on-a does not contain c-above-a but still *necessitates* it in the sense of Fine (2018): every possible state that is incompatible with c-above-a is also incompatible with c-on-$b \sqcup b$-on-a. But it is plausible that a-on-c is incompatible with c-above-a. So a-on-c should then be incompatible with c-on-$b \sqcup b$-on-a. Yet a-on-c is clearly compatible with c-on-b and, separately, with b-on-a. This comes perilously close to emergent exclusion, which our opponent rejects; and given that we understand incompatibility as conflict and that these states are atomic, our opponent must deny distributivity once again. For if the atom a-on-c conflicts with c-on-$b \sqcup b$-on-a then a-on-c excludes some part of c-on-$b \sqcup b$-on-a; and if c-on-b and b-on-a are atoms then either that part is one of them, thereby giving rise to emergent exclusion, or it isn't, contrary to distributivity. We conclude that there is an irreconcilable tension between distributivity and a ban on emergent exclusion.

Our second reason for rejecting the assumption c-above-$a \sqsubseteq (c$-on-$b \sqcup b$-on-$a)$ is that this assumption fails to generalize to analogous examples. This is because it plays on the fact that the relation *above* is the transitive closure of the relation *on top of*. When we modify the example by replacing *on top of* with another relation, there is no longer guaranteed to be a natural counterpart of the relation *above*.

Suppose, for example, that we have a basket that has space for two eggs, and we have three eggs that we are trying to fit in our basket.[15] Let e_1 be the atomic state of the basket's containing the first egg, and similarly for e_2 and e_3. Here it is natural to assume that e_i excludes $e_j \sqcup e_k$ for any distinct eggs i, j, k but that no e_i excludes any e_j; so we have emergent exclusion. In this example, the instances of the containing relation are analogous to the instances of the "on top of" relation in the blocks example. But this time there is no analogue to the "above" relation.

The price to pay for our opponent who wishes to deny emergent exclusion is to postulate additional states that might not otherwise be needed and to deny distributivity. In the eggs scenario, for example, our opponent would need to enrich our scenario with states o_i of egg i being outside the basket. They could then

15 This is a variation on an example involving three sisters and a bed that has room for just two of them, considered by Veltman (2005) in the context of a theory of counterfactuals.

maintain that e_i (the state of i's being in the basket) does not exclude $e_j \sqcup e_k$ but rather excludes o_i, and that the nonoccurrence of o_i forces the nonoccurrence of $e_j \sqcup e_k$, either because $e_j \sqcup e_k$ contains o_i or because it necessitates o_i. But these options lead our opponent to deny distributivity for the same reasons as we have explained above, and in this sense the introduction of o_i has bought them nothing.

7 Exclusion and de Morgan's Law

We have seen that the benefits of tolerating emergent exclusion include the ability to maintain state spaces which are arguably intuitive, help account for the frame problem, and maintain compatibility with Classical Extensional Mereology by satisfying distributivity. Let us now turn to the question why one might want to prohibit emergent exclusion in the first place. It is instructive for this purpose to consider the use to which the ban on emergent exclusion is, in effect, put in Fine (2017a). Fine does not discuss this matter explicitly, so the relevant considerations must be determined by inspecting his unilateral framework.

The aspect of his framework that rules out emergent exclusion is the condition called Downward Exclusion. This condition relies on the notion of regular closure S^*_* of a set S given above in (9). Fine (2017a: 658) defines Downward Exclusion as follows:[16]

(14) **Downward Exclusion (Fine's formulation)**
If $q \perp p_1 \sqcup p_2 \sqcup \ldots$, then $q \in \{r \mid \text{for some } i, r \perp p_i\}^*_*$.
(If q excludes the sum of some states, then q is in the regular closure of the set of individual excluders of these states.)

When we expand this formulation by applying the definition of regular closure, it becomes apparent that Downward Exclusion really consists of two conditions that must jointly hold:

(15) **Downward Exclusion (expanded formulation)**
If q excludes the sum of a set S, then both of the following conditions hold:

a. $\exists q^- \sqsubseteq q.\ \exists s \in S.\ q^- \perp s$
(Some part of q excludes a member of S.)

b. $q \sqsubseteq \bigsqcup\{r \mid \exists s \in S.\ r \perp s\}$.
(q is part of the sum of all excluders of members of S.)

16 Due to a typo, the published version of Fine (2017a) omits the regular closure operator $^*_*$ at the end of this definition (K. Fine, p.c.).

In our terms, Downward Exclusion states that any collective excluder of a set S is also an individual excluder of S; in other words, Downward Exclusion rules out what we have called emergent excluders.

Let us now consider why Fine imposes Downward Exclusion on his unilateral system. The condition ensures that this system behaves like his bilateral system, in that the two sides of de Morgan's Law $\neg(P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$ have the same verifiers. Let us call this the exact version of de Morgan's Law.

In his unilateral system, (i) a verifier for $\phi \vee \psi$ is any state that is a verifier of ϕ or of ψ, (ii) a verifier for $\phi \wedge \psi$ is any state that is obtained by fusing a verifier of ϕ and a verifier of ψ, and (iii) a verifier for $\neg\phi$ is any state that is obtained by fusing an excluder of each verifier of ϕ.[17] For example, assume that p_i is the sole verifier for P and that q_i is the sole verifier for Q; then in Fine's system, both are verifiers for $P \vee Q$, and $p_i \sqcup q_i$ is the sole verifier for $P \wedge Q$. Any excluder of $p_i \sqcup q_i$ thus verifies $\neg(P \wedge Q)$, and any excluder of p_i or of q_i verifies $\neg P \vee \neg Q$.

Consider an excluder s of $p_i \sqcup q_i$; that is, assume that s is a collective excluder of the set $\{p_i, q_i\}$. Then s verifies $\neg(P \wedge Q)$. In the absence of Downward Exclusion, s may well cohere both with p_i and with q_i. In this case, s does not verify $\neg P \vee \neg Q$; furthermore, s is not an individual excluder but an emergent excluder of $\{p_i, q_i\}$. This shows that emergent exclusion can allow $\neg(P \wedge Q)$ to have more verifiers than $\neg P \vee \neg Q$ and break the exact version of this de Morgan's Law.

Consider again the version of our running example in Section 5: a state space in which the Twelfth Amendment excludes the sum of Truman's and Dewey's victories, but neither victory excludes the other by itself. In a version of Fine's unilateral system without Downward Exclusion, the disjunction of negations *Either Truman didn't win the election or Dewey didn't* has two verifiers in this state space: the sum of the Twelfth Amendment with Dewey's victory, and its sum with Truman's victory. The negated conjunction *Truman and Dewey did not both win the election* has these two verifiers and also a third which consists just of the Twelfth Amendment. Likewise, in our final example in Section 6, the negated conjunction *Egg 1 and egg 2 are not both in the basket* has among its verifiers e_3, the state of egg 3's being in the basket, since that basket can only hold two eggs; but the disjunction of negations *Either egg 1 or egg 2 is not in the basket* does not have that state among its verifiers.

We submit that this departure from the exact version de Morgan's Law has intuitive appeal. As we have mentioned in the introduction, it is a common notion that disjunction involves a choice between alternatives. This is naturally reflected

17 More specifically, these definitions apply in what Fine (2017a) calls the full domain of propositions. Fine also considers restrictions of his system in which propositions are subject to various closure conditions such as closure under sum; with these restrictions, the appropriate closures are also applied to the outputs of the definitions in the main text. See also Section 9 below.

by the assumption that a truthmaker for a disjunction is always also a truthmaker for one of its disjuncts; and so, in particular, a truthmaker for a disjunction of negations is always also a truthmaker for one of the negations. It is a natural thought that a negated conjunction does not force us to make this choice, at least in those cases where the domain makes a truthmaker available that rules out the co-occurrence of both conjuncts collectively but does not rule either of them out individually. By accepting emergent exclusion, we recognize such truthmakers, and thereby capture the intuition that the negation of a conjunction does not require us to make a choice between conjuncts (or between their negations) in the way disjunctions require us to choose among disjuncts.

In the following sections, we argue that this deviation from de Morgan's Law is benign in the sense that, while both sides of the law have different verifiers, they are still intensionally equivalent. We cash this out in a framework that uses possible worlds. We show that given a set of assumptions on the exclusion relation that is both natural and mirrored in Fine's work, in any possible world where either side of de Morgan's Law is true, so is the other.

8 Possible worlds and E-frames

This section defines a number of notions that will be useful in developing a truthmaker semantics with emergent exclusion (in Section 9). Many of these notions take the form of constraints on the exclusion relation, and are analogous to constraints imposed on what Fine (2017a) calls classical exclusion relations. We bring these constraints and concepts out in some detail because they seem to us to be of independent interest and are easily missed on a cursory reading of Fine (2017a). We indicate deviations from Fine where appropriate.

Possible worlds can be seen as maximally possible events. This kind of view is common in theories that build up possible worlds from primitives (e.g., Pollock 1967, Prior & Fine 1977, Plantinga 1978).[18] Here is our formal definition:

(16) **Definition: Possible World**

$\textsc{World}(e_1) \overset{\text{def}}{=} \text{Poss}(e_1) \wedge \neg\exists e_2.\ [e_1 \sqsubset e_2 \wedge \text{Poss}(e_2)]$

(A possible world is a possible event that is not a proper part of any other possible event.)

In a canonical frame based on the sets $A_1 = \{p, q, r\}$ and $A_2 = \{\overline{p}, \overline{q}, \overline{r}\}$, the possible worlds are just those combinations that contain exactly one of each pair of mirror

18 For related discussion, see also Plebani, Rosella & Saitta (2022: 224-228), in particular claim 5 on p. 228.

images: $p \sqcup q \sqcup r$, $p \sqcup q \sqcup \overline{r}$, $p \sqcup \overline{q} \sqcup r$, etc. In general, a canonical frame based on a set A_1 with cardinality n will contain 2^n possible worlds.

One can, alternatively, define a possible world as an event which contains any event with which it is compossible. Fine (2018) shows that these definitions are equivalent in a complete distributive lattice as long as possibility is downward closed. This is the case also in our setting due to Lemma 4.1 above ("Possibility is downward closed").

Recall that we distinguish between occurring and nonoccurring events and that we refer to the sum of all occurring events as *actuality* or as the *actual world*. Formally, we reserve the constant w_0 to refer to a distinguished possible world, which we call the actual world, and we define occurring events as the parts of w_0:

(17) **Axiom: Actuality**
$\textsc{World}(w_0)$

(18) **Definition: Occurring events**
$\textsc{Occurs}(e) \stackrel{\text{def}}{=} e \sqsubseteq w_0$
(An event occurs just in case it is part of the actual world.)

The two following theorems highlight two useful properties of occurring events.

Theorem 8.1 (Distributivity of occurrence)**.** Any part of any occurring event occurs.

Proof. Immediate, by transitivity of parthood. □

Theorem 8.2 (Cumulativity of Occurrence)**.** The sum of any set of occurring events itself occurs.

Proof. Consider any set E of occurring events. The sum of E is by definition its least upper bound, so it is part of any upper bound of E. Since w_0 contains every occurring event, it is an upper bound of E. The sum of E is then a part of w_0, which, by definition, means that it is possible. □

It is easy to see that when the state space is finite, every possible event is part of a possible world (Fine 2018). We will assume in what follows that this holds even in the case of an infinite state space. While this assumption is not required by our approach to negation and exclusion, we adopt it because it removes the need to generalize the classical definition of intensional equivalence as truth in the same possible worlds and because it makes our theory more readily compatible with the standard possible-world semantics framework and with theories expressed in that framework. The following axiom ensures it:

(19) **Axiom: Cosmopolitanism**
$\forall e_1.\ [\textsc{Poss}(e_1) \rightarrow \exists e_2.\ [\textsc{World}(e_2) \wedge e_1 \sqsubseteq e_2]]$
(Every possible event is part of a possible world.)

We will refer to a state space that satisfies Cosmopolitanism as a world space. This is called a W-space in Fine (2017a).[19]

Let us turn to the relation between possible worlds and impossible events. From Lemma 4.1 we know that no impossible event is part of a possible world. In canonical frames, it will furthermore hold that any impossible event conflicts with any possible world, because the impossible events are exactly those events which contain both some x and its mirror image $\overline{x}$, while the possible worlds are exactly those events which contain exactly one of x and $\overline{x}$ for any x. Whichever of these two events is contained in a given possible world will exclude its mirror image in the impossible event.

From the point of view of truthmaker semantics, this property of canonical frames turns out to be a desirable feature in relation with the semantics of negation, as we discuss in Section 9. But in the absence of further assumptions, there is no guarantee that this desirable property carries over from canonical frames to arbitrary settings. Consider again the sum of Truman's victory e_1 and Dewey's victory e_2. Assume that no possible world contains this event $e_1 \sqcup e_2$. By Cosmopolitanism, $e_1 \sqcup e_2$ is impossible. We would like it to be reflected within the actual world that it cannot contain $e_1 \sqcup e_2$. That is to say, the actual world should not just fail to contain this event but should moreover contain at least one e_3 (such as the Twelfth Amendment) that excludes this event; and similarly for every possible world and impossible event. But, again, in the absence of further assumptions, nothing forces any possible world to conflict with the impossible event $e_1 \sqcup e_2$. To illustrate, consider a model with just two atomic events e and w in which $e \perp e$ and no other exclusions hold. Then e and $e \sqcup w$ are impossible events, w is a possible world, and yet there is no conflict between e and w. To remedy this fact, we introduce the following axiom:[20]

---

19 Cosmopolitanism does not follow from the other assumptions we have made in previous sections. Consider for example a state space that is isomorphic to the set of all finite subsets of $\mathbb{N}$, with $\mathbb{N}$ itself added as the full state, and with the set-theoretic $\subseteq, \cup$, and $\cap$ as $\sqsubseteq, \sqcup$ and $\sqcap$. Now assume that $\mathbb{N}$ excludes itself and that there are no further exclusions. Then every finite subset of $\mathbb{N}$ is possible but $\mathbb{N}$ itself is not. Other than Cosmopolitanism and Actuality, this space conforms with all of the assumptions we have placed on it and yet there is no possible world, because there is no maximal finite subset of $\mathbb{N}$.

20 In the framework of Plebani, Rosella & Saitta (2022), this axiom follows from the definition of a possible world based on maximality with respect to compatibility, i.e., what we call coherence (p. 226), together with claim 2 (p. 224) which states that every part of a possible state must be compatible with itself.

(20) **Axiom: Harmony**
$\forall e_1 \forall e_2. [[\textsc{World}(e_1) \wedge e_1 \sim e_2] \rightarrow \textsc{Poss}(e_2)]$
(If a possible world coheres with an event, that event is possible.)

What this axiom says is that no possible world coheres with any event that conflicts with itself. (Figuratively speaking, such a world would lack harmony.) A counterpart of this axiom occurs in Fine (2017a) as part (iii) of the definition of what he calls classical exclusion: if a state t is possible then every possible state is compossible with some state t' that excludes t. This formulation is more general than ours in that it does not reference possible worlds explicitly.

Given the previous axioms, we can in fact strengthen the material conditional in the Axiom of Harmony to a biconditional:

Theorem 8.3 (Plenitude)**.** $\forall e_1. [\textsc{Poss}(e_1) \leftrightarrow \exists e_2. [e_1 \sim e_2 \wedge \textsc{World}(e_2)]]$
(The possible events are just those events that cohere with some possible world.)

See proof on page 40.

Plenitude essentially says that the universe of possible worlds is rich enough that for any possible event (but for no impossible event), some possible world coheres with it. Let us illustrate this in the canonical frame based on sets of atoms $A_1 = \{p,q,r\}$ and $A_2 = \{\overline{p},\overline{q},\overline{r}\}$. Recall that the possible worlds in this frame are the events that contain exactly one of each pair of mirror images ($p \sqcup q \sqcup r$, $p \sqcup q \sqcup \overline{r}$, $p \sqcup \overline{q} \sqcup \overline{r}$, etc.) and that the possible events are the ones that contain at most one of each pair of mirror images. Clearly, every possible event can be extended to a possible world by "filling in the gaps", i.e., by summing it with either x or $\overline{x}$ for any $x \in A_1$ whenever it does not yet contain either one. The resulting possible world coheres with the initial possible event. Conversely, the impossible events in a canonical frame are those events that contain both x and $\overline{x}$ for some $x \in A_1$. Any possible world will contain exactly one of these two atoms and will conflict with the other.

From what we have said so far, the sum of two events could in principle be impossible even if they are both possible and neither conflicts with the other, for example if their sum is or contains an event that excludes itself and that is not part of either of them. This sort of "emergent impossibility" is unintuitive: after all, if two testimonies are mutually compatible and each of them could be true in isolation, then it should be possible for both of them to be true together. The following principle ensures this by ruling out emergent impossibilities.[21]

21 The following axiom is identical to the principle PF in Plebani, Rosella & Saitta (2022: 228) and corresponds to part (ii) of the definition of a classical exclusion relation in Fine (2017a: 668): if two states s and t are possible but not compossible, then some part of s excludes t. The reason that the

(21) **Axiom: Rashōmon**
$\forall e_1 \forall e_2. \, [[\text{Poss}(e_1) \wedge \text{Poss}(e_2) \wedge e_1 \sim e_2] \rightarrow \text{Poss}(e_1 \sqcup e_2)]$
(If two possible events cohere, their sum is possible too.)

In a canonical frame, Rashōmon is satisfied because any two possible events e_1 and e_2 that cohere agree on each of the atoms they share. That is to say, if e_1 and e_2 cohere and some atom x is part of e_1, then its mirror image $\bar{x}$ cannot be part of e_2, and vice versa. So the sum of e_1 and e_2 cannot contain both x and $\bar{x}$. Since this is true for all atoms x, $e_1 \sqcup e_2$ is possible.

The following property is a consequence of Rashōmon:[22]

Theorem 8.4 (Manichaeism)**.** $\forall e_1. \, [\textsc{World}(e_1) \rightarrow \forall e_2. \, [e_2 \sqsubseteq e_1 \vee e_1 \not\sim e_2]]$
(Any possible world conflicts with any event it does not contain.)

See proof on page 40.

The preceding axioms are sufficient for establishing the technical results in the remainder of this paper. However, one may wish to impose further constraints on the exclusion relation.

One example of a natural constraint is Cumulativity: if e excludes e' and f excludes f', then $e \sqcup f$ excludes $e' \sqcup f'$. One may also wish to modify this constraint in various ways, e.g., by restricting it to the case where $e \neq f$ and $e' \neq f'$ or generalizing it from the binary case to the infinitary case.

Another example concerns the fact that in our system as presented so far, the truthmakers of some sentence P and their parts may lack excluders. Under Fine's semantics for negation, which we adopt with modifications in Section 9 below, this results in P being necessary and in $\neg P$ having no truthmakers. One may wish to rule this out, for example by imposing a condition such as Null Exclusion (Fine 2017a: 658), which (i) requires every nonnull event e to exclude some event and be excluded by some event and (ii) prevents the null state from taking part in the exclusion relation. The first part of this condition ensures that no sentence lacks excluders, so long as it has at least one nonnull truthmaker. The second part of this condition ensures that in the state space, impossibilities are attributed to "meaningful" exclusions. To see this, consider, by contrast, a frame in which the null state $\square$ excludes some event e. Since the null state is part of all events, e conflicts with every event whatsoever and in particular with itself, so it is impossible. If

last condition is given as "excludes t" and not "excludes some part of t", as one might expect, is that for Fine, this would be otiose, since he assumes that exclusion is inexact (upward persistent) on its second argument (as formalized in his Upward Exclusion condition, Fine 2017a: 658).

22 In the framework of Plebani, Rosella & Saitta (2022), the following principle follows from the definition of a possible world based on maximality with respect to compatibility (p. 226).

$e \perp \square$ is the only exclusion within e, no two positive pieces of information in e exclude each other; in this sense, e is contradiction-free. Yet it is impossible. This is clearly at odds with our approach to possibility.

Both Cumulativity and Null Exclusion are natural and consistent with the rest of our framework; in fact, our Actuality axiom entails one of the consequences of Null Exclusion, namely that the null state does not exclude itself.

We end this section by collecting the definitions of the preceding sections in the notions of an E-frame and a canonical frame.

(22) **Definition: E-frame**
An E-frame is a quadruple $\langle E, \sqsubseteq, \perp, w_0 \rangle$ where:

a. E, the *state space*, is a set (understood as containing events and possible worlds);
b. $\sqsubseteq$, the *parthood relation*, is a binary relation over E such that $\langle E, \sqsubseteq \rangle$ is a complete distributive lattice;
c. $\perp$, the *exclusion relation*, is a binary relation over E which satisfies Symmetry, Actuality, Cosmopolitanism, Harmony, and Rashōmon;
d. w_0, the *designated world*, is a possible world contained in E (understood as the actual world).

As described in Section 3, canonical frames are a specific kind of E-frame based on an atomistic algebra in which the exclusion relation can be "read off" the atoms.

(23) **Definition: Canonical frame**
A canonical frame is a quintuple $\langle E, \sqsubseteq, \perp, w_0, m \rangle$ such that:

a. $\langle E, \sqsubseteq, \perp, w_0 \rangle$ is an E-frame;
b. $\langle E, \sqsubseteq \rangle$ forms a complete atomic (and hence atomistic) Boolean lattice;
c. the set of atoms of $\langle E, \sqsubseteq \rangle$ can be partitioned into two sets A_1 and A_2;
d. m, the *mirror function*, is an involution between A_1 and A_2;
e. $\perp$ is the smallest relation such that for any $x \in A_1 \cup A_2$, $x \perp m(x)$.

Since m is an involution, the last point of this definition implies Symmetry. As we have seen, the other properties of $\perp$ that are stipulated by axiom in the general case likewise emerge as theorems in the case of a canonical frame.

With E-frames and canonical frames in place, we turn to a description of the semantics for a propositional language.

9 Logical connectives, truth and entailment

The purpose of this section is to argue that removing the ban on emergent exclusion from Fine's unilateral semantics results in a system which is still well-behaved, and in addition, to show that the two sides of de Morgan's Law $\neg(P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$ are intensionally equivalent in our system even though they may have different verifiers. To this end, we provide semantic clauses for the propositional connectives (negation, conjunction, and disjunction) that conform with exact truthmaker semantics. We also provide definitions for standard logical concepts such as truth in a model. We show how to capture classical entailment and various nonclassical entailment relations in our system, and we show that the connectives behave analogously to their classical counterparts as far as classical entailment and related notions such as classical equivalence are concerned. As in the previous section, our development largely follows Fine (2017a); we indicate deviations as appropriate.

Although the previous section has introduced possible worlds, we continue to view propositions as sets of events rather than sets of possible worlds. We call a proposition true just in case it contains at least one occurring event. We call a proposition false just in case it is not true; in this way, we ensure bivalence. Truth and falsity of sentences can be defined derivatively in terms of the truth and falsity of the propositions they denote. Our framework is hyperintensional in the sense that two distinct propositions or sentences (e.g., the two sides of the above-mentioned de Morgan's Law) can be true at the same possible worlds.

According to the above definition, the truth of a proposition depends on the occurrence of an event it contains. This absolute notion can be understood as derived from the more basic notion of truth relative to an event, or as we will also say, *verification* by an event (the *verifier*). As discussed below, verification can be understood in different ways, depending on the relation between the true propositions and their verifiers.

Below, we define negation as a function from propositions to propositions that is based on the exclusion relation. Thanks to the axioms on this relation introduced above, we account for the property that we will call *Classicality*: Negation turns any true sentence into a false sentence and vice versa. We take Classicality to be a constitutive property of negation. For the purposes of this paper, we set aside phenomena and systems that involve truth values other than True and False; while the system defined below does not involve such truth values, our formulation of Classicality is neutral on whether there are such truth values and how negation behaves when it is applied to a sentence with such a truth value.

In terms of possible worlds, Classicality is generally formalized as follows: for any possible world w, if a sentence is true in w then its negation is false in w, and vice versa. In standard possible-world semantics, where sentences denote sets of possible worlds, Classicality of Negation immediately follows from the assumption that negation denotes the set-theoretic complement operation on possible worlds. In the present framework, to prove Classicality of Negation we will need to show that for any world w and sentence ϕ, either ϕ or its negation, but not both, has a verifier that occurs in w.

We work within a simple propositional language $\mathscr{L}_{prop}$ whose formulas, or sentences, are built up in the usual way from a countable set of propositional letters $\mathscr{P}$ that includes at least P, Q, and R, with the help of the connective symbols $\neg$, $\wedge$, and $\vee$. The sentences of this language are interpreted in a model that encapsulates the assumptions stated in the previous sections. We call this an *E-model* to distinguish it from models of classical propositional logic, which we will refer to as *classical propositional models*. When we talk simply of models, we mean E-models.

(24) **Definition: E-model**
An E-model for a propositional language $\mathscr{L}_{prop}$ is a quintuple $\langle E, \sqsubseteq, \perp, w_0, I\rangle$ such that:

a. $\langle E, \sqsubseteq, \perp, w_0\rangle$ is an E-frame;
b. I, the *interpretation function*, is a function that maps each propositional letter of $\mathscr{L}_{prop}$ to a subset of E.

Just like a model is defined based on a frame, a *canonical model* is defined based on a canonical frame; see also Fine (2017a: 647) and Plebani, Rosella & Saitta (2022: 227). Canonical models are essentially built around canonical frames whose atomic events are defined in terms of lowercase versions of the literals of the propositional language. Recall that a literal is an expression that is either a propositional letter P or its negation $\neg P$. In the following definition, p stands for the lowercase version of a propositional letter P.

(25) **Canonical model**
A canonical model for a propositional language $\mathscr{L}_{prop}$ is a sextuple $\langle E, \sqsubseteq, \perp, w_0, m, I\rangle$ such that:

a. E is the powerset of $\bigcup_{P\in\mathscr{P}}\{p, \neg p\}$;
b. m, the mirror function, sends each $\{p\}$ to $\{\neg p\}$ and vice versa for each propositional letter $P \in \mathscr{P}$;
c. $\langle E, \sqsubseteq, \perp, w_0, m\rangle$ is a canonical frame whose atoms are the singletons of E;
d. I, the *interpretation function*, is a function that maps each propositional letter $P \in \mathscr{P}$ to $\{\{p\}\}$.

For example, suppose that the literals of $\mathscr{L}_{prop}$ are P, Q and their negations $\neg P, \neg Q$. Then a canonical model for $\mathscr{L}_{prop}$ has one null state $\square$ (i.e., $\emptyset$), four atomic events $\{p\}, \{q\}, \{\neg p\}, \{\neg q\}$, four possible worlds $\{p, q\}$, $\{p, \neg q\}$, $\{\neg p, q\}$, $\{\neg p, \neg q\}$, and seven impossible events $\{p, \neg p\}, \{q, \neg q\}$, $\{p, \neg p, q\}$, $\{p \neg p, \neg q\}$, $\{p, q, \neg q\}$, $\{\neg p, q, \neg q\}$ and $\{p, \neg p, q, \neg q\}$ (the full state $\blacksquare$). We have $I(P) = \{\{p\}\}$

and $I(Q) = \{\{q\}\}$ (each propositional letter is interpreted as having the singleton set of its lowercase version as its sole truthmaker) but as in previous sections we will write $\{p, \neg q\}$ as $p \sqcup \overline{q}$ and so on, and so we will write $I(P) = \{p\}$ and $I(Q) = \{q\}$. In each canonical model (as in noncanonical models), one of the possible worlds is designated as the actual world. This world will contain exactly one of p and $\neg p$ for each propositional letter $P \in \mathscr{P}$ and is therefore equivalent to a classical model for propositional logic.

For any propositional letter P, we understand $I(P)$ as the proposition expressed by P. In a canonical model, since $I(P) = \{p\}$, each propositional letter denotes a proposition that is verified by one and only one event. But in an arbitrary model this may not be the case.

As in Fine (2017c), we say that a proposition is *exactly verified by* an event e just in case it contains e. We call a proposition *inexactly verified* by an event e just in case it contains a part of e. We also apply these notions to formulas based on the propositions they denote. Then exact verification corresponds conceptually to wholly describing an event while inexact verification corresponds to describing a part of an event. In situation semantics (e.g., Kratzer 1989), inexact verification is taken to be the basic notion, while in our case exact verification is basic, as also in van Fraassen (1969), Yablo (2014), and others. Following Fine, we call a proposition *loosely verified* by an event e just in case any possible event that contains e is compossible with some event in the proposition. In the case of verification by a possible world, as Fine observes, inexact and loose verification coincide.[23]

We now associate each formula $\phi \in \mathscr{L}_{prop}$ with a proposition by recursively defining a *denotation function* $[\![\cdot]\!]^I$ which extends the interpretation function I from propositional letters to arbitrary formulas. Below, we drop the superscript I for conciseness. As we will see, the way we define this function here ensures that the same formulas are valid in E-models and in models of classical propositional logic. The clauses for conjunction and disjunction are standard in truthmaker semantics (e.g., van Fraassen 1969, Moltmann 2007, Fine 2017c). In the case of conjunction, the clause furthermore corresponds to proposals by Lasersohn (1995) and L. K. Schubert (2000) for event semantics, by Krifka (1990) for adjective coordination, by Heycock & Zamparelli (2005) for noun coordination, and more generally by Schmitt (2019, 2021). The idea behind this clause is that the conjunction of two propositions is verified by any sum of an event that verifies the first conjunct and an event that verifies the second conjunct:

23 Proof (see also Fine 2017a): By definition, a possible world w that inexactly verifies a proposition contains some event e in that proposition; since w (being a possible world) is the only possible event that contains w, and is compossible with e, it loosely verifies the proposition. Conversely, a possible world w that loosely verifies a proposition is compossible with some event e in that proposition; since w is a possible world, e is a part of w and w inexactly verifies the proposition.

(26) **Semantic rule for Conjunction**
$[\![\phi \wedge \psi]\!] = \{e_1 \sqcup e_2 \mid e_1 \in [\![\phi]\!], e_2 \in [\![\psi]\!]\}$
(The conjunction of ϕ and ψ denotes the set of all events that are the sum of an event in $[\![\phi]\!]$ and an event in $[\![\psi]\!]$.)

For example, in a canonical model, we have $[\![P \wedge Q]\!] = \{p \sqcup q\}$ and $[\![P \wedge P]\!] = \{p \sqcup p\} = \{p\} = [\![P]\!]$. In arbitrary models, however, it is not always the case that $[\![P \wedge P]\!] = [\![P]\!]$ since P may have more than one truthmaker (Fine 2017a).

We assume that the disjunction of two propositions is exactly verified by any event that exactly verifies either of them. Accordingly, our clause forms the union of the interpretation of the two disjuncts:

(27) **Semantic rule for Disjunction**
$[\![\phi \vee \psi]\!] = [\![\phi]\!] \cup [\![\psi]\!]$
(The disjunction of ϕ and ψ denotes the set of all events that are either in $[\![\phi]\!]$ or in $[\![\psi]\!]$.)

For example, in a canonical model, we have $[\![P \vee P]\!] = \{p\} \cup \{p\} = \{p\} = [\![P]\!]$, $[\![P \vee Q]\!] = \{p, q\}$, $[\![P \vee (Q \wedge R)]\!] = \{p, q \sqcup r\}$ and $[\![P \wedge (Q \vee R)]\!] = \{p \sqcup q, p \sqcup r\}$. In general, the truthmakers that canonical models assign to formulas correspond closely to their disjunctive normal forms, and one can identify the truthmakers for an arbitrary formula in a canonical model by applying the Quine-McCluskey algorithm for determining prime implicants (Quine 1952, McCluskey 1956).

Our semantic rule for disjunction is one of two variants for the disjunction rule found in the truthmaker literature. Fine (2017c) refers to it as the noninclusive semantics. The other variant, which Fine calls the inclusive semantics, is defined as $[\![\phi]\!] \cup [\![\psi]\!] \cup [\![\phi \wedge \psi]\!]$. That is, we have $[\![\phi \wedge \psi]\!] \subseteq [\![\phi \vee \psi]\!]$ on the inclusive semantics, but not on the noninclusive semantics.

The clause we will adopt for negation is inspired by the unilateral system in Fine (2017a) but adapted to reflect our differing assumptions about the exclusion relation (recall that unlike Fine's, our exclusion relation is exact on both its arguments). Since our goal is to ensure that the same sentences are valid in E-models and in classical propositional models, we need to ensure that the negation of any sentence is true if and only if the sentence is false. More precisely, since our definition of truth in a model given below is in fact inexact/loose truth relativized to its actual world, we need to provide a semantics for the connective $\neg$ such that in any E-model with actual world w_0, for any sentence ϕ, exactly one of ϕ and $\neg\phi$ is inexactly/loosely true in w_0.[24]

24 Of course, Fine's (2017a) bilateral semantics (without exclusion) would give this result more directly. This is the tradeoff for the advantage in unilateral semantics of not having to define separate falsity conditions for all of the connectives in the language.

The intuition behind our semantic rule for negation can best be appreciated by example. Consider first a formula ϕ that is verified by just one event e_1. How should we construct the set of events that verify $\neg\phi$? The events in this set are meant to be such that their occurrence would be fully relevant to the falsity of ϕ. Consider any event f_1 that excludes e_1; by definition, its occurrence would be fully relevant to the nonoccurrence of e_1. In an intuitive way, the occurrence of f_1 would then be fully relevant to the falsity of ϕ: f_1 must be in $[\![\neg\phi]\!]$. More generally, this reasoning applies to any event f_1 that excludes a part of e_1: Its occurrence would be fully relevant to the nonoccurrence of a part of e_1, and then to the nonoccurrence of e_1 itself, and then to the falsity of ϕ, so f_1 must be in $[\![\neg\phi]\!]$.

Now consider a disjunction $\phi \vee \psi$ where ϕ, as before, has e_1 as its sole exact verifier and ψ has a distinct event e_2 as its sole exact verifier. By the semantic rule for disjunction, the disjunction $\phi \vee \psi$ has two exact verifiers: e_1 and e_2. Which events should we take to verify its negation $\neg(\phi \vee \psi)$? Assume that de Morgan's Law $\neg(\phi \vee \psi) \Leftrightarrow (\neg\phi) \wedge (\neg\psi)$ is an exact equivalence, that is, the two sides have the same exact verifiers.[25] The reasoning applied to e_1 above has given us a set of verifiers for $\neg\phi$; an analogous reasoning gives us a set of verifiers for $\neg\psi$. By the semantic rule for conjunction, the sum of any verifier for $\neg\phi$ and $\neg\psi$ is a verifier for $\neg\phi \wedge \neg\psi$. By assumption, it also verifies $\neg(\phi \vee \psi)$.

Generalizing, this leads to the following notion. Given a set $S = \{e_1, e_2, \ldots\}$, we say that an event e *precludes* S just in case $e = f_1 \sqcup f_2 \sqcup \ldots$ in which f_1, f_2, …, are such that f_1 excludes some part of e_1, f_2 excludes some part of e_2, and so on. More formally:

(28) **Definition: Precluding a Set**
An event e precludes a set of events S just in case there is a function h from events to events such that $e = \bigsqcup\{h(f_i) \mid f_i \in S\}$ and for all events $f_i \in S$, $h(f_i)$ excludes some part of f_i.

The semantic rule we define for negation collects all of the events that preclude the prejacent into a set:

(29) **Semantic rule for Negation**
$[\![\neg\phi]\!] = \{e \mid e \text{ precludes } [\![\phi]\!]\}$
(The negation of ϕ denotes the set of all events that preclude $[\![\phi]\!]$.)

In the case of a canonical model, e precludes $[\![\phi]\!]$ if and only if there is a function h that maps each event in $[\![\phi]\!]$ to the mirror image of one of its atomic parts, and e is the sum of all of these mirror images. For example, assume that

25 This law should not be confused with the other de Morgan's Law $\neg(\phi \wedge \psi) \Leftrightarrow (\neg\phi) \vee (\neg\psi)$, for which we have argued that the left-hand side has some exact verifiers that the right-hand side lacks.

$[\![\phi]\!] = \{p, p \sqcup \overline{q}, q \sqcup r \sqcup \overline{r}\}$. Then one possible choice for h is the function that maps p to $\overline{p}$, $p \sqcup \overline{q}$ to q, and $q \sqcup r \sqcup \overline{r}$ to $\overline{r}$. The sum of these events is $\overline{p} \sqcup q \sqcup \overline{r}$. This, then, is an example of an event that precludes $[\![\phi]\!]$ and thus verifies $[\![\neg\phi]\!]$.

This semantic rule for negation does not collapse distinctions between intensionally equivalent formulas or propositions. For example, the two formulas $P \vee \neg P$ and $Q \vee \neg Q$ denote intensionally equivalent propositions (they are both true in every possible world) that may yet be distinct; in canonical models these propositions are $\{p, \overline{p}\}$ and $\{q, \overline{q}\}$. The negated formulas $\neg(P \vee \neg P)$ and $\neg(Q \vee \neg Q)$ also denote distinct propositions in canonical models: $\{p \sqcup \overline{p}\}$ and $\{q \sqcup \overline{q}\}$. Coming back to de Morgan's Law, while in a canonical frame $\neg(P \wedge Q)$ and $\neg P \vee \neg Q$ have the same set of truthmakers (namely $\{\overline{p}, \overline{q}\}$), this is not in general the case in all E-frames. That this is not the case is a necessary condition for emergent exclusion; but it is not a sufficient condition, as illustrated by Fine's (2017a) unilateral truthmaker semantics.[26]

Now that we have a notion of interpretation, we can specify what it means for a proposition and a formula to be true in a model:

(30) **Truth and falsity in a model**
A proposition is true in a model M whose designated world is w_0 just in case the proposition contains an event e such that $e \sqsubseteq w_0$, and false otherwise. A formula ϕ is true in a model M just in case $[\![\phi]\!]$ is true in M, and false otherwise.

For example, in a canonical model for a three-letter language whose designated world w_0 is $p \sqcup \overline{q} \sqcup r$, the proposition $\{p \sqcup \overline{q}, q \sqcup r\}$ is true but the proposition $\{p \sqcup q, q \sqcup r\}$ is false.

The notions of validity, satisfiability, and contradictoriness can be defined as usual as truth in all models, truth in some model, and truth in no model respectively. Truth in a model is just the notion of inexact truth that we have discussed earlier, relativized to the possible world which the model designates as actual. (One could equivalently characterize it as loose truth with respect to that world, given that inexact and loose truth coincide in the case of possible worlds.) Since we have defined falsity as absence of truth, we get bivalence:

26 We note in passing that this semantics is appropriate both for exact truthmaker semantics and for event semantics (Eckardt 1998, Bayer 1997, Champollion 2016) in that it avoids "leakage" or unwanted upward persistence: if ϕ is not upward persistent then $\neg\phi$ will not, in general, be upward persistent either. It is also worth noting that—at least in the absence of further constraints on the system—the formulas ϕ and $\neg\neg\phi$ will not, in general, have the same truthmakers, though in canonical frames they will.

Theorem 9.1 (Bivalence)**.** In any E-model, any given proposition is either true or false, but not both.

Proof. Immediate. □

Bivalence also extends to formulas since these denote propositions. It should not be confused with the following result, which we prove in short order:

Theorem 9.2 (Classicality of Negation)**.** In any E-model, the negation of a formula is true if and only if that formula is false.

If negation is classical, then the truth value (though not the denotation) of the negation of a formula in a given model is a function of the truth value of this formula in that model. Given Bivalence, Classicality of Negation is equivalent to stating that in any E-model M, for any formula ϕ, either ϕ or $\neg\phi$ is true, but not both.

Given the definition of truth in a model, the truth value of a formula ϕ depends only on the designated world w_0 of M and not on other possible worlds. Moreover, this distinguished world plays a purely honorary role in the sense that no operators in our language depend on it for their truth conditions. Accordingly, we now establish Classicality of Negation by proving a more general result: two formulas that are true in the same possible worlds in possible-world semantics are true in the same possible worlds also in our semantics. This result consists of two parts: No Gaps (no E-model contains any possible world in which neither ϕ nor $\neg\phi$ is inexactly true) and No Gluts (no E-model contains any possible world in which both of them are). Our terminology follows in part Fine (2017c).

Theorem 9.3 (No Gaps)**.** In any E-model M, for any formula ϕ, any possible world w contains either some member of $[\![\phi]\!]$ or some member of $[\![\neg\phi]\!]$.

See proof on page 41.

Theorem 9.4 (No Gluts)**.** In any E-model M, for any formula ϕ, no possible world w contains both some member of $[\![\phi]\!]$ and some member of $[\![\neg\phi]\!]$.

See proof on page 41.

No Gaps and No Gluts together entail Classicality of Negation. That is to say, from No Gaps and No Gluts it follows that in any model, any possible world (and in particular the designated world w_0) inexactly verifies either ϕ or $\neg\phi$ but not both.

Classicality of Negation has analogues for conjunction and disjunction:

Theorem 9.5 (Classicality of Conjunction)**.** In any E-model, a conjunction is true if and only if both of its conjuncts are true.

See proof on page 41.

Theorem 9.6 (Classicality of Disjunction)**.** In any E-model, a disjunction is true if and only if at least one of its disjuncts is true.

See proof on page 42.

We can summarize Classicality of Conjunction, Disjunction, and Negation in the following property:

Theorem 9.7 (Classicality for Propositional Logic)**.** In any E-model M, the truth value of any formula ϕ can be determined from the truth values that M assigns to the propositional letters of ϕ by recursively applying the truth tables of classical propositional logic.

Proof. By formula induction given Classicality of Conjunction, of Disjunction, and of Negation. $\square$

The results that follow correlate satisfiability and other logical notions in E-models with the corresponding notions in classical propositional logic. We speak of E-satisfiability and classical satisfiability respectively, and similarly for other notions.

Theorem 9.8 (Classical Satisfiability for Propositional Logic)**.** A formula ϕ is E-(un)satisfiable just in case ϕ is classically (un)satisfiable.

See proof on page 42.

Theorem 9.9 (Classical Validity for Propositional Logic)**.** ϕ is valid in classical propositional logic just in case ϕ is E-valid.

See proof on page 43.

As a corollary to Classical Validity, the two sides of de Morgan's Law $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$ are intensionally equivalent in the sense that they are true in the same possible worlds in every E-model. This is the case even though they are not hyperintensionally equivalent: in some E-models they have different verifiers and thus denote different propositions.

Given Classical Validity, the classical notion of logical consequence as truth preservation, either in a given model or in all models, carries over straightforwardly from classical propositional models to E-models. But we can also define various other, stricter notions of consequence that correspond at least roughly to notions such as first degree entailment, inexact or relevant entailment, analytical containment, and so on (Fine 2017a: p. 669). How close this correspondence is depends in part on the precise properties of negation, which differ slightly in this paper from those that Fine assumes.

This topic is of great importance for semantics since some of these alternative notions of entailment may well turn out to be more empirically adequate than classical entailment in a variety of domains. For example, both the inference from $P \wedge Q$ to P and the inference from P to $P \vee R$ are valid from the point of view of classical consequence, but only the former is valid from the point of view of analytic containment. The empirical distinction between these two cases can be observed in relation to, for instance, agreement between interlocutors, saying-that, commands, epistemic possibility, preconditions, explanation, confirmation, consequences of knowledge, and partial truth (Yablo 2014: p. 12-14). To take agreement as an example, if John holds that $P \wedge Q$ and Mary holds that P, then John and Mary agree on P. If John holds that P and Mary holds that R, however, Yablo argues that we do not usually think of them as agreeing on $P \vee R$ (as Yablo puts it, statements agree to the extent they have parts in common). Under this view, if both John and Mary hold that P but they do not both hold that R, then they agree on P but not on $P \vee R$: the inference from P to $P \vee R$ is not valid in the context of agreement.

Each of the following notions can be defined either relative to a given model or across all models. In Fine (2017a), exact consequence is also called disjunctive entailment because P exactly entails the disjunction $P \vee Q$; and analytic containment is also called conjunctive entailment because the conjunction $P \wedge Q$ analytically contains P. Inexact consequence corresponds to relevant first degree entailment (Dunn 1976, Belnap 1977).

(31) **Exact consequence**
ϕ exactly entails ψ just in case for all e, if $[\![\phi]\!]$ contains e then so does $[\![\psi]\!]$.
I.e., iff $[\![\phi]\!] \subseteq [\![\psi]\!]$.

(32) **Inexact consequence**
ϕ inexactly entails ψ just in case for all e, if $[\![\phi]\!]$ contains some part of e then $[\![\psi]\!]$ contains some part of e.
I.e., iff $\{e_1 \mid \exists e_2 \sqsubseteq e_1.\ e_2 \in [\![\phi]\!]\} \subseteq \{e_1 \mid \exists e_3 \sqsubseteq e_1.\ e_3 \in [\![\psi]\!]\}$.

(33) **Analytic containment**
ϕ analytically contains ψ just in case for all e, if $[\![\phi]\!]$ contains e then $[\![\psi]\!]$ contains some part of e, and if $[\![\psi]\!]$ contains e then e is a part of some member of $[\![\phi]\!]$.
I.e., iff $[\forall e_1 \in [\![\phi]\!].\ \exists e_2 \in [\![\psi]\!].\ e_2 \sqsubseteq e_1] \wedge [\forall e_2 \in [\![\psi]\!].\ \exists e_1 \in [\![\phi]\!].\ e_2 \sqsubseteq e_1]$.

To summarize, this section has shown that relaxing the ban on emergent exclusion in Fine (2017a) is consistent with obtaining a well-behaved unilateral truthmaker semantics. The semantics we have developed is based on a symmetric exclusion relation that is exact on both sides. In our system, negation is well-behaved in the sense that it satisfies a range of desirable properties: Theorem 9.2 ensures that negation flips truth values as in classical propositional logic, Theorems 9.8 and 9.9 ensure that $\neg\phi$ is valid (satisfiable) in E-models just in case $\neg\phi$ is valid (satisfiable) in classical models. At the same time, our semantics for negation does not collapse hyperintensional distinctions: when ϕ and ψ denote distinct but intensionally equivalent propositions, $\neg\phi$ and $\neg\psi$ may do so too. We have illustrated this with the case of $\phi = P \vee \neg P$ and $\psi = Q \vee \neg Q$. So like Fine's semantics, our semantics allows us to draw hyperintensional distinctions, but also makes it possible to relax these distinctions and recover classical notions such as intensional equivalence.

10 Conclusion

We have argued that the exclusion relation between truthmakers can be motivated beyond the primary purpose of providing truthmakers for negated sentences. Such motivation is provided by the ability to define modal notions such as possibility and necessity in terms of exclusion, which we conceive as a symmetric, non-upwards monotone, primitive relation between events. We have argued that exclusion itself is not a modal notion and apparent examples of contingent exclusion must therefore be recast. We have done so by assuming that exclusion is irreducibly collective or, as we have called it, *emergent*; that is to say, e_1 can exclude $e_2 \sqcup e_3$ without excluding (or conflicting with) e_2 or e_3 individually.

Our study of emergent exclusion has revealed a fault line within unilateral truthmaker semantics. On the one hand, a prohibition against emergent exclusion allows Fine (2017a) to show that the two sides of de Morgan's Law $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$ always have the same truthmakers, as they do on the standard bilateral truthmaker semantics. On the other hand, Fine (2012) provides motivation from counterfactuals and the frame problem for state spaces that crucially involve emergent exclusion; and we have argued that certain instances of $\neg(P \wedge Q)$ have truthmakers that leave open which of P and Q is false while no instances of $\neg P \vee \neg Q$ have such truthmakers.

We have developed a unilateral truthmaker metaphysics with symmetric emergent exclusion and shown that it provides a foundation for an exact semantics of a propositional language. This semantics shares many assumptions with the unilateral semantics in Fine (2017a) and makes the same variety of entailment and equivalence relations available. It makes the two sides of de Morgan's Law $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$ intensionally, though not exactly, equivalent.

Ours is not the first argument that the two sides of de Morgan's law $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$ have different semantic interpretations. Ciardelli, Zhang & Champollion (2018)—who include one of us—have argued based on experimental evidence that counterfactuals of the shape *If* $\neg(P \wedge Q)$ *then* C entail *If* $(\neg P \wedge \neg Q)$ *then* C, while counterfactuals of the shape *If* $\neg P \vee \neg Q$ *then* C lack that entailment. Consequently, they argue for a fine-grained notion of meaning that can distinguish between truth-conditionally equivalent antecedents. While they couch their account in terms of inquisitive semantics (Ciardelli, Groenendijk & Roelofsen 2018), they note that their account fits within truthmaker semantics—in particular, the intuitionistic truthmaker semantics of Fine (2014)—and other theories that can make such distinctions. In terms of truthmaker semantics, the empirical claim in Ciardelli, Zhang & Champollion (2018) can be recast theoretically as follows: the set of truthmakers of $\neg P \vee \neg Q$ is the union of the set of truthmakers for $\neg P$ with the set of truthmakers for $\neg Q$; the set of truthmakers for $\neg(P \wedge Q)$ also contains these truthmakers but furthermore includes any sum of a truthmaker for $\neg P$ with a truthmaker for $\neg Q$, that is, any state that precludes both P and Q rather than just one of them. Thus Ciardelli, Zhang & Champollion (2018) agree with the present paper in that $\neg(P \wedge Q)$ may have "additional truthmakers" compared with $\neg P \vee \neg Q$. However, it is worth noting that the "additional truthmakers" for $\neg(P \wedge Q)$ postulated in the present paper are different: what we postulate is that among the truthmakers of $\neg(P \wedge Q)$ there are some that preclude neither P nor Q but only their cooccurrence (they are emergent precluders).[27]

One possible avenue for further research concerns our identification of possible states and states that are free of conflict. A tempting application of truthmaker semantics is to the semantic paradoxes (Leitgeb 2019). In such a setting one may wish to have a truthmaker for a truth-teller sentence (e.g., "this sentence is true"). Since the truth-teller is not paradoxical, such a truthmaker should arguably not exclude itself, and in that case, it is considered in our theory to be a possible state. As a reviewer points out, however, one may wish to deny that this state is possible.

27 If desired, one can ensure the presence in our models of the additional truthmakers postulated by Ciardelli, Zhang & Champollion (2018) by adding the Cumulativity constraint discussed in Section 8.

Appendix

This Appendix repeats and proves those theorems whose proofs have been deferred in the main text.

Lemma 4.1 (Possibility is downward closed)**.**
$\forall e_1 \forall e_2.\ [[\text{Poss}(e_1) \wedge e_2 \sqsubseteq e_1] \rightarrow \text{Poss}(e_2)]$
(Any part of any possible event is also possible.)

Proof of Theorem 4.1. Let e_1 be a possible event and let e_2 be a part of e_1. Let us show that e_2 is then also possible. Since e_1 is possible, it coheres with itself; that is to say, no part of e_1 excludes any part of e_1. By transitivity of parthood, any part of e_2 is also a part of e_1. So *a fortiori*, no part of e_2 excludes any part of e_2. Hence e_2 coheres with itself and is possible. □

Theorem 8.3 (Plenitude)**.** $\forall e_1.\ [\text{Poss}(e_1) \leftrightarrow \exists e_2.\ [e_1 \sim e_2 \wedge \textsc{World}(e_2)]]$
(The possible events are just those events that cohere with some possible world.)

Proof of Theorem 8.3. The right-to-left direction is just the Axiom of Harmony: if a possible world e_2 coheres with an event e_1, that event is possible. To prove the left-to-right direction, let e be a possible event and let us show that it coheres with some possible world. Since e is possible, by Cosmopolitanism e is part of a possible world w. Assume for contradiction that e does not cohere with any possible world. Then e conflicts with w. So some e_1 that is part of e excludes some e_2 that is part of w. By transitivity of parthood, e_1 is also part of w. So w conflicts with itself and is not possible, and hence is not a possible world, contrary to assumption. □

Theorem 8.4 (Manichaeism)**.** $\forall e_1.\ [\textsc{World}(e_1) \rightarrow \forall e_2.\ [e_2 \sqsubseteq e_1 \vee e_1 \not\sim e_2]]$
(Any possible world conflicts with any event it does not contain.)

Proof of Theorem 8.4. Consider any possible world w and any event e that it does not contain. Let us show that w conflicts with e:

- Suppose e is possible. Since w does not contain e, $w \sqsubset (w \sqcup e)$, and since w is a possible world, $w \sqcup e$ is impossible. Since w and e are both possible, by Rashōmon some part of w excludes some part of e. So w conflicts with e.

- Suppose e is impossible. Then, by Harmony, either w is not a possible world or it conflicts with e. But w is a possible world by assumption. So w conflicts with e.

□

Theorem 9.3 (No Gaps)**.** In any E-model M, for any formula ϕ, any possible world w contains either some member of $[\![\phi]\!]$ or some member of $[\![\neg\phi]\!]$.

Proof of Theorem 9.3. Consider a E-model M, any possible world w, and any formula ϕ. We need to show that w contains (as a part) either some member of $[\![\phi]\!]$ or some member of $[\![\neg\phi]\!]$. Assume that no member of $[\![\phi]\!]$ is part of w and let us then show that some member of $[\![\neg\phi]\!]$ is part of w. Consider any $e \in [\![\phi]\!]$. By assumption, e is not part of w. By Manichaeism (which relies on Harmony and on Rashōmon), w conflicts with e, that is, w has a part that excludes some part of e. Thus, for all $e \in [\![\phi]\!]$, $C_{w,e} = \{e' \sqsubseteq w \mid e' \text{ excludes some part of } e\}$, the set of parts of w that exclude some part of e, is nonempty. Let us assume that we have a choice function g whose domain is $\{C_{w,e}\}_{e\in[\![\phi]\!]}$, the set of all these sets, and such that for any $e \in [\![\phi]\!]$, $g(C_{w,e}) \in C_{w,e}$ (the axiom of choice, as a last resort, ensures the existence of such a g). It is then the case that $\bigsqcup \mathrm{Img}(g) \in [\![\neg\phi]\!]$. Because $\bigsqcup \mathrm{Img}(g)$ is (by definition) the least upper bound of $\mathrm{Img}(g)$ and because for each $e \in [\![\phi]\!]$, $g(C_{w,e}) \sqsubseteq w$, $\bigsqcup \mathrm{Img}(g)$ is part of w. This shows that some part of w is a member of $[\![\neg\phi]\!]$. □

Theorem 9.4 (No Gluts)**.** In any E-model M, for any formula ϕ, no possible world w contains both some member of $[\![\phi]\!]$ and some member of $[\![\neg\phi]\!]$.

Proof of Theorem 9.4. Consider an E-model $M = \langle E, \sqsubseteq, \bot, w_0, I\rangle$, any possible world w, and any formula ϕ. We need to show that w does not contain both some member of $[\![\phi]\!]$ and some member of $[\![\neg\phi]\!]$. Assume that some member of $[\![\phi]\!]$, e_1, is a part of w and let us then show that no member of $[\![\neg\phi]\!]$ is part of w. Consider any $e_2 \in [\![\neg\phi]\!]$. By definition of $[\![\neg\phi]\!]$, there is some function $f : [\![\phi]\!] \to E$ such that $e_2 = \bigsqcup \mathrm{Img}(f)$ and $f(e_1)$ excludes some part of e_1. Now, since $e_2 = \bigsqcup \mathrm{Img}(f)$, $f(e_1)$ is part of e_2. Suppose for contradiction that e_2 is part of w. Then $f(e_1)$ is part of w as well by transitivity of parthood. By assumption, e_1 is part of w, so $f(e_1) \sqcup e_1$ is part of w. Since w is possible, so is $f(e_1) \sqcup e_1$. But $f(e_1)$ excludes some part of e_1, so $f(e_1) \sqcup e_1$ is impossible. By contradiction, e_2 is not part of w. Since this holds for any $e_2 \in [\![\neg\phi]\!]$, no member of $[\![\neg\phi]\!]$ is part of w. □

Theorem 9.5 (Classicality of Conjunction)**.** In any E-model, a conjunction is true if and only if both of its conjuncts are true.

Proof of Theorem 9.5. Consider a model M with w_0 as the distinguished actual world.

- Suppose that ϕ and ψ are both true in M. Then $[\![\phi]\!]$ contains some $e_\phi \sqsubseteq w_0$ and $[\![\psi]\!]$ contains some $e_\psi \sqsubseteq w_0$. By the definition of least upper bound, $e_\phi \sqcup e_\psi \sqsubseteq w_0$. By the semantic rule for conjunction, $[\![\phi \wedge \psi]\!]$ contains $e_\phi \sqcup e_\psi$ and $\phi \wedge \psi$ is therefore true in M.

- Conversely, suppose that ϕ is false in M (the case for ψ is analogous). By the semantic rule for conjunction, for every e in $[\![\phi \wedge \psi]\!]$, $[\![\phi]\!]$ contains some $e_\phi \sqsubseteq e$. Since ϕ is false in M, we have $e_\phi \not\sqsubseteq w_0$ and then, by transitivity of parthood, $e \not\sqsubseteq w_0$. Since this holds for every e in $[\![\phi \wedge \psi]\!]$, $\phi \wedge \psi$ is false in M.

□

Theorem 9.6 (Classicality of Disjunction)**.** In any E-model, a disjunction is true if and only if at least one of its disjuncts is true.

Proof of Theorem 9.6. Consider a model M with w_0 as the distinguished actual world.

- Suppose ϕ is true in M (the case for ψ is analogous). Then $[\![\phi]\!]$ contains some $e_\phi \sqsubseteq w_0$. By the semantic rule for disjunction, $[\![\phi \vee \psi]\!]$ contains e_ϕ as well and $\phi \vee \psi$ is therefore true in M.
- Conversely, suppose that ϕ and ψ are both false in M. By the semantic rule for disjunction, every e in $[\![\phi \vee \psi]\!]$ is also contained either in $[\![\phi]\!]$ or in $[\![\psi]\!]$. Since both ϕ and ψ are false in M, we have $e \not\sqsubseteq w_0$. Since this holds for every e in $[\![\phi \vee \psi]\!]$, $\phi \vee \psi$ is false in M.

□

Theorem 9.8 (Classical Satisfiability for Propositional Logic)**.** A formula ϕ is E-(un)satisfiable just in case ϕ is classically (un)satisfiable.

Proof of Theorem 9.8.

- Suppose ϕ is E-satisfiable. Then it is true in the distinguished world w_0 of some E-model M_e. We define a classical propositional model M_c (i.e., a complete assignment of truth values to propositional letters of the language) in the following way. For every propositional letter P in ϕ, if P is true in M_e then we set $M_c(P) = T$ (i.e., true), otherwise $M_c(P) = F$ (i.e., false). So every propositional letter in ϕ has the same truth value in both models. By Classicality, the same holds for ϕ itself.
- Conversely, suppose that ϕ is classically satisfiable. Then it is true in some classical propositional model M_c. Consider a canonical model M_e whose atoms are the literals of the language from which ϕ is taken and whose distinguished world w_0 contains among its atoms each literal l such that $M_c(l) = T$ (that is to say, these literals are true in w_0). By construction, for

every propositional letter P in ϕ, $\{p\} \subseteq w_0$ iff $M_c(P) = T$; thus every propositional letter in ϕ has the same truth value in both models. By Classicality, the same holds for ϕ itself.

□

Theorem 9.9 (Classical Validity for Propositional Logic)**.** ϕ is valid in classical propositional logic just in case ϕ is E-valid.

Proof of Theorem 9.9.

- Suppose ϕ is valid in classical propositional logic; then $\neg\phi$ is unsatisfiable in that sense; so by Classical Satisfiability, $\neg\phi$ is unsatisfiable in our sense too. Consider any E-model with distinguished world w_0. Then, $\neg\phi$ is false in w_0. By Classicality, ϕ is true in w_0. Since this holds for any E-model, ϕ is valid in our sense.

- Conversely, suppose ϕ is invalid in classical propositional logic; then $\neg\phi$ is satisfiable in that sense; so by Classical Satisfiability, $\neg\phi$ is satisfiable in our sense too. So there is an E-model M with distinguished world w_0 in which $\neg\phi$ is true. By Classicality, ϕ is false in w_0, so ϕ is false in M. Hence, ϕ is invalid in our sense.

□

References

Armstrong, David Malet. 1997. *A world of states of affairs* (Cambridge Studies in Philosophy). Cambridge, UK: Cambridge University Press. http://dx.doi.org/10.1017/cbo9780511583308.

Armstrong, David Malet. 2004. *Truth and truthmakers* (Cambridge Studies in Philosophy). Cambridge, UK: Cambridge University Press. http://dx.doi.org/10.1017/cbo9780511487552.

Armstrong, David Malet. 2010. *Sketch for a systematic metaphysics*. Oxford, UK: Oxford University Press. http://dx.doi.org/10.1093/acprof:oso/9780199590612.001.0001.

Barker, Stephen & Mark Jago. 2011. Being positive about negative facts. *Philosophy and Phenomenological Research* 85(1). 117–138. http://dx.doi.org/10.1111/j.1933-1592.2010.00479.x.

Bayer, Samuel L. 1997. *Confessions of a lapsed Neo-Davidsonian: Events and arguments in compositional semantics*. New York, NY: Garland. http://dx.doi.org/10.4324/9780203055106.

Beall, JC. 2000. On truthmakers for negative truths. *Australasian Journal of Philosophy* 78(2). 264–268. http://dx.doi.org/10.1080/00048400012349551.

Belnap, Nuel D. 1977. A useful four-valued logic. In J. Michael Dunn & George Epstein (eds.), *Modern uses of multiple-valued logic*, 5–37. Springer Netherlands. http://dx.doi.org/10.1007/978-94-010-1161-7_2.

Bernard, Timothée & Lucas Champollion. 2018. Negative events in compositional semantics. *28th Semantics and Linguistic Theory Conference (SALT 28)* 28. 512–532. http://dx.doi.org/10.3765/salt.v28i0.4429.

Bernard, Timothée & Lucas Champollion. 2024. *Negative events and compositional semantics*. Forthcoming in *Journal of Semantics*.

Berto, Francesco. 2015. A modality called 'negation'. *Mind* 124(495). 761–793. http://dx.doi.org/10.1093/mind/fzv026.

Berto, Francesco & Greg Restall. 2019. Negation on the Australian plan. *Journal of Philosophical Logic* 48(6). 1119–1144. http://dx.doi.org/10.1007/s10992-019-09510-2.

Björnsson, Gunnar. 2007. *If You Believe in Positive Facts, You Should Believe in Negative Facts*. Philosophical Papers Dedicated to Wlodek Rabinowicz: Hommage à Wlodek.

Bledin, Justin. 2022. *Composing truthmakers*. Manuscript.

Brownstein, Donald. 1973. Negative exemplification. *American Philosophical Quarterly* 10(1). 43–50. https://www.jstor.org/stable/20009473.

Bylinina, Lisa & Rick Nouwen. 2018. On "zero" and semantic plurality. *Glossa: a journal of general linguistics* 3(1). 98. http://dx.doi.org/10.5334/gjgl.441.

Champollion, Lucas. 2016. Covert distributivity in algebraic event semantics. *Semantics and Pragmatics* 9(15). 1–65. http://dx.doi.org/10.3765/sp.9.15.

Champollion, Lucas. 2017. *Parts of a whole: distributivity as a bridge between aspect and measurement*. Vol. 66 (Oxford Studies in Theoretical Linguistics). Oxford, UK: Oxford University Press. http://dx.doi.org/10.1093/oso/9780198755128.001.0001.

Champollion, Lucas & Manfred Krifka. 2016. Mereology. In Maria Aloni & Paul Dekker (eds.), *Cambridge handbook of formal semantics* (Cambridge Handbooks in Language and Linguistics), chap. 13, 369–388. Cambridge, UK: Cambridge University Press. http://dx.doi.org/10.1017/cbo9781139236157.014.

Cheyne, Colin & Charles Pigden. 2006. Negative truths from positive facts. *Australasian Journal of Philosophy* 84(2). 249–265. http://dx.doi.org/10.1080/00048400600759092.

Ciardelli, Ivano, Jeroen Groenendijk & Floris Roelofsen. 2018. *Inquisitive semantics*. Vol. 6 (Oxford surveys in semantics and pragmatics). Oxford, UK: Oxford University Press.

Ciardelli, Ivano, Linmin Zhang & Lucas Champollion. 2018. Two switches in the theory of counterfactuals: a study of truth conditionality and minimal change. *Linguistics and Philosophy* 41(6). 577–621. http://dx.doi.org/10.1007/s10988-018-9232-4.

Davidson, Donald. 1967. The logical form of action sentences. In Nicholas Rescher (ed.), *The logic of decision and action*, 81–95. Pittsburgh, PA: University of Pittsburgh Press. http://dx.doi.org/10.1093/0199246270.003.0006.

Davidson, Donald. 1969. The individuation of events. In Nicholas Rescher (ed.), *Essays in honor of Carl G. Hempel: A tribute on the occasion of his sixty-fifth birthday* (Synthese Library), 216–234. Dordrecht, Netherlands: Springer. http://dx.doi.org/10.1007/978-94-017-1466-2_11.

Davidson, Donald. 1971. Eternal vs. ephemeral events. *Noûs* 5(4). 335–349. http://dx.doi.org/10.2307/2214381.

Demos, Raphael. 1917. A discussion of a certain type of negative proposition. *Philosophical Inquiry* 40(3). 192–200. http://dx.doi.org/10.5840/philinquiry2016403/423.

Dunn, J. Michael. 1976. Intuitive semantics for first-degree entailments and 'coupled trees'. *Philosophical Studies* 29(3). 149–168. http://dx.doi.org/10.1007/bf00373152.

Dunn, J. Michael. 1993. Star and perp: two treatments of negation. *Philosophical Perspectives* 7. 331. http://dx.doi.org/10.2307/2214128. https://doi.org/10.2307%2F2214128.

Dunn, J. Michael. 1996. Generalized ortho negation. In Heinrich Wansing (ed.), *Negation*, vol. 7 (Perspektiven der Analytischen Philosophie / Perspectives in Analytical Philosophy), 3–26. de Gruyter. http://dx.doi.org/10.1515/9783110876802.3.

Dunn, J. Michael. 1999. A comparative study of various model-theoretic treatments of negation: a history of formal negation. In Dov M. Gabbay & Heinrich Wansing (eds.), *What is negation?*, vol. 13 (Applied Logic), 23–51. Springer. http://dx.doi.org/10.1007/978-94-015-9309-0_2.

Dunn, J. Michael & Chunlai Zhou. 2005. Negation in the context of gaggle theory. *Studia Logica* 80(2). 235–264. http://dx.doi.org/10.1007/s11225-005-8470-y.

Eckardt, Regine. 1998. *Adverbs, events, and other things*. Vol. 379 (Linguistische Arbeiten). Tübingen, Germany: Niemeyer. http://dx.doi.org/10.1515/9783110913781.

Fine, Kit. 2012. Counterfactuals without possible worlds. *The Journal of Philosophy* 109(3). 221–246. http://dx.doi.org/10.5840/jphil201210938.

Fine, Kit. 2014. Truth-maker semantics for intuitionistic logic. *Journal of Philosophical Logic* 43(2-3). 549–577. http://dx.doi.org/10.1007/s10992-013-9281-7.

Fine, Kit. 2017a. A theory of truthmaker content I: conjunction, disjunction and negation. *Journal of Philosophical Logic* 46(6). 625–674. http://dx.doi.org/10.1007/s10992-016-9413-y.

Fine, Kit. 2017b. A theory of truthmaker content II: subject-matter, common content, remainder and ground. *Journal of Philosophical Logic* 46(6). 675–702. http://dx.doi.org/10.1007/s10992-016-9419-5.

Fine, Kit. 2017c. Truthmaker semantics. In Bob Hale, Crispin Wright & Alexander Miller (eds.), *A companion to the philosophy of language*, 556–577. Wiley.

Fine, Kit. 2018. The world of truth-making. In Ivette Fred-Rivera & Jessica Leech (eds.), *Being necessary: themes of ontology and modality from the work of Bob Hale*, chap. 3, 36–59. Oxford, UK: Oxford University Press. http://dx.doi.org/10.1093/oso/9780198792161.003.0003.

Fine, Kit. 2021. Constructing the impossible. In Lee Walters & John Hawthorne (eds.), *Conditionals, paradox, and probability: themes from the philosophy of dorothy edgington*, chap. 9, 141–163. Oxford, UK: Oxford University Press. http://dx.doi.org/10.1093/oso/9780198712732.001.0001.

van Fraassen, Bas C. 1969. Facts and tautological entailments. *The Journal of Philosophy* 66(15). 477–487. http://dx.doi.org/10.2307/2024563.

Gallin, Daniel. 1975. *Intensional and higher-order modal logic, with applications to Montague semantics*. Vol. 19 (Mathematics Studies). Amsterdam, Netherlands & Oxford, UK: North Holland.

Goldblatt, Robert I. 1974. Semantic analysis of orthologic. *Journal of Philosophical Logic* 3(1-2). 19–35. http://dx.doi.org/10.1007/bf00652069. https://doi.org/10.1007/bf00652069.

Grätzer, George. 1978. *General lattice theory* (Mathematische Reihe Lehrbücher und Monographien aus dem Gebiete der Exakten Wissenschaften). Basel, Switzerland: Birkhäuser. http://dx.doi.org/10.1007/978-3-0348-7633-9.

Heycock, Caroline & Roberto Zamparelli. 2005. Friends and colleagues: Plurality, coordination, and the structure of DP. *Natural Language Semantics* 13(3). 201–270. http://dx.doi.org/10.1007/s11050-004-2442-z.

Hochberg, Herbert. 1969. Negation and generality. *Noûs* 3(3). 325–343. http://dx.doi.org/10.2307/2214554.

Hovda, Peter. 2009. What is classical mereology? *Journal of Philosophical Logic* 38(1). 55–82. http://dx.doi.org/10.1007/s10992-008-9092-4.

van Inwagen, Peter. 1978. Ability and responsibility. *The Philosophical Review* 87(2). 201–224. http://dx.doi.org/10.2307/2184752.

Jago, Mark. 2022. Truthmaker accounts of propositions. In Chris Tillman & Adam Murray (eds.), *Routledge handbook of propositions*, 231–243. Routledge. http://dx.doi.org/10.4324/9781315270500-16.

Kratzer, Angelika. 1989. An investigation of the lumps of thought. *Linguistics and Philosophy* 12(5). 607–653. http://dx.doi.org/10.1007/bf00627775.

Kratzer, Angelika. 2021. Situations in natural language semantics. In Edward N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*, Winter 2021. Stanford, CA: Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/win2021/entries/situations-semantics/.

Krifka, Manfred. 1989. Nominal reference, temporal constitution and quantification in event semantics. In Renate Bartsch, Johan van Benthem & Peter van Emde Boas (eds.), *Semantics and contextual expression*, 75–115. Dordrecht, Netherlands: Foris. http://amor.cms.hu-berlin.de/~h2816i3x/Publications/Krifka%201989%20Nominal%20Reference.pdf.

Krifka, Manfred. 1990. Boolean and non-boolean 'and'. In László Kálmán & László Pólos (eds.), *Papers from the 2nd Symposium on Logic and Language*, 161–188. Budapest, Hungary: Akadémiai Kiadó. http://amor.cms.hu-berlin.de/~h2816i3x/Boolean-and-Non-Boolean-And.pdf.

Kukso, Boris. 2006. The reality of absences. *Australasian Journal of Philosophy* 84(1). 21–37. http://dx.doi.org/10.1080/00048400600571679.

Landman, Fred. 1991. *Structures for semantics*. Kluwer.

Lasersohn, Peter. 1995. *Plurality, conjunction and events*. Dordrecht, Netherlands: Kluwer. http://dx.doi.org/10.1007/978-94-015-8581-1.

Leitgeb, Hannes. 2019. Hype: a system of hyperintensional logic (with an application to semantic paradoxes). *Journal of Philosophical Logic* 48(2). 305–405. http://dx.doi.org/10.1007/s10992-018-9467-0.

Link, Godehard. 1983. The logical analysis of plurals and mass terms: A lattice-theoretical approach. In Reiner Bäuerle, Christoph Schwarze & Arnim von Stechow (eds.), *Meaning, use and interpretation of language*, 303–323. Reprinted in Link 1998, Chapter 1. Also reprinted in , Portner & Partee 2002, Chapter 4. Berlin, Germany: de Gruyter. http://dx.doi.org/10.1002/9780470758335.ch4.

Link, Godehard. 1998. *Algebraic semantics in language and philosophy*. Stanford, CA: CSLI Publications.

Linsky, Bernard. 2018. The near riot over negative facts. In *The philosophy of logical atomism*, 181–197. Springer International Publishing. http://dx.doi.org/10.1007/978-3-319-94364-0_8.

McCarthy, John & Patrick J. Hayes. 1969. Some philosophical problems from the standpoint of artificial intelligence. In Bernard Meltzer & Donald Michie (eds.), *Machine intelligence*, 463–502. Edinburgh University Press. http://dx.doi.org/10.1016/b978-0-934613-03-3.50033-7.

McCluskey, Edward J. 1956. Minimization of Boolean functions. *The Bell System Technical Journal* 35(6). 1417–1444. http://dx.doi.org/10.1002/j.1538-7305.1956.tb03835.x.

Meyer, Robert K. & Errol P. Martin. 1986. Logic on the Australian plan. *Journal of Philosophical Logic* 15(3). http://dx.doi.org/10.1007/bf00248574.

Molnar, George. 2000. Truthmakers for negative truths. *Australasian Journal of Philosophy* 78(1). 72–86. http://dx.doi.org/10.1080/00048400012349361.

Moltmann, Friederike. 2007. Events, tropes, and truthmaking. *Philosophical Studies* 134(3). 363–403. http://dx.doi.org/10.1007/s11098-005-0898-4.

Moltmann, Friederike. 2019. Nominals and event structure. In Robert Truswell (ed.), *The Oxford handbook of event structure*, 341–367. Oxford University Press. http://dx.doi.org/10.1093/oxfordhb/9780199685318.013.21.

Moltmann, Friederike. 2020. Truthmaker semantics for natural language: attitude verbs, modals, and intensional transitive verbs. *Theoretical Linguistics* 46(3-4). 159–200. http://dx.doi.org/10.1515/tl-2020-0010.

Mulligan, Kevin, Peter Simons & Barry Smith. 1984. Truth-makers. *Philosophy and Phenomenological Research* 44(3). 287–321. http://dx.doi.org/10.2307/2107686.

Muskens, Reinhard. 1995. *Meaning and partiality*. Stanford, CA: CSLI Publications.

Nelson, Everett J. 1930. Intensional relations. *Mind* 39(156). 440–453. http://dx.doi.org/10.1093/mind/xxxix.156.440.

Paolini Paoletti, Michele. 2014. Falsemakers: something negative about facts. *Grazer Philosophische Studien* 90. 169–182.

Plantinga, Alvin. 1978. *The nature of necessity*. Oxford University Press. http://dx.doi.org/10.1093/0198244142.001.0001.

Plebani, Matteo, Giuliano Rosella & Vita Saitta. 2022. Truthmakers, incompatibility, and modality. *Australasian Journal of Logic* 5(4). 19.

Pollock, John L. 1967. The logic of logical necessity. *Logique et Analyse* 10(39/40). 307–323. https://www.jstor.org/stable/44084498.

Pontow, Carsten & Rainer Schubert. 2006. A mathematical analysis of theories of parthood. *Data & Knowledge Engineering* 59(1). 107–138. http://dx.doi.org/10.1016/j.datak.2005.07.010.

Portner, Paul & Barbara H. Partee (eds.). 2002. *Formal semantics: the essential readings*. Blackwell. http://dx.doi.org/10.1002/9780470758335.

Prior, Arthur N. & Kit Fine. 1977. *Worlds, times, and selves*. Duckworth.

Quine, Willard van Orman. 1952. The problem of simplifying truth functions. *The American Mathematical Monthly* 59(8). 521–531. http://dx.doi.org/10.2307/2308219.

Russell, Bertrand. 1918. The philosophy of logical atomism. I. Facts and propositions. *The Monist* 28(4). 495–527. http://dx.doi.org/10.5840/monist19182843.

Russell, Bertrand. 1919. The philosophy of logical atomism. III. Atomic and molecular propositions. *The Monist* 29(1). 32–63. http://dx.doi.org/10.5840/monist191929120.

Schmitt, Viola. 2019. Pluralities across categories and plural projection. *Semantics and Pragmatics* 12(17). 1–49. http://dx.doi.org/10.3765/sp.12.17.

Schmitt, Viola. 2021. Boolean and non-boolean conjunction. In Daniel Gutzmann, Lisa Matthewson, Cécile Meier, Hotze Rullmann & Thomas Ede Zimmermann (eds.), *The Wiley Blackwell companion to semantics*, 203–234. Hoboken, NJ: Wiley. http://dx.doi.org/10.1002/9781118788516.sem111.

Schubert, Lenhart K. 2000. The Situations We Talk About. en. In Jack Minker (ed.), *Logic-Based Artificial Intelligence* (The Springer International Series in Engineering and Computer Science), 407–439. Boston, MA: Springer US. http://dx.doi.org/10.1007/978-1-4615-1567-8_18.

Schwarzschild, Roger. 1994. Plurals, presuppositions and the sources of distributivity. *Natural Language Semantics* 2(3). 201–248. http://dx.doi.org/10.1007/bf01256743.

Tarski, Alfred. 1935. Zur Grundlegung der Booleschen Algebra. *Fundamenta Mathematicae* 24. Translated to English as Tarski 1956, 177–198. https://eudml.org/doc/212745.

Tarski, Alfred. 1956. On the foundations of the Boolean algebra. In Joseph Henry Woodger (ed.), *Logics, semantics, metamathematics. Papers from 1923 to 1938*, 320–341. Translation of Tarski 1935. Oxford, UK: Clarendon.

Veber, Michael. 2008. How to derive a 'not' from an 'is': a defense of the incompatibility view of negative truths. *Metaphysica* 9(1). 79–91. http://dx.doi.org/10.1007/s12133-008-0024-0.

Veltman, Frank. 1984. Data semantics. In *Truth, interpretation and information: selected papers from the third Amsterdam Colloquium*, vol. 2 (Groningen-Amsterdam Studies in Semantics (GRASS)), 43–63. Dordrecht, Netherlands: Foris. http://dx.doi.org/10.1515/9783110867602.43.

Veltman, Frank. 1985. *Logics for conditionals*. Amsterdam, Netherlands: University of Amsterdam dissertation.

Veltman, Frank. 2005. Making counterfactual assumptions. *Journal of Semantics* 22(2). 159–180. http://dx.doi.org/10.1093/jos/ffh022.

Yablo, Stephen. 2014. *Aboutness*. Princeton University Press. http://dx.doi.org/10.23943/princeton/9780691144955.001.0001.

Lucas Champollion
Department of Linguistics
New York University
10 Washington Place
New York, NY 10003
United States
champollion@nyu.edu

Timothée Bernard
Laboratoire de linguistique formelle,
Université Paris Cité
8 place Paul Ricœur
75205 Paris Cedex 13
France
timothee.bernard@u-paris.fr