Proportional readings of *many* and *few*: the case for an underspecified measure function*

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Abstract

In the so-called *reverse proportional reading* (Herburger 1997), the truth conditions of statements of the form *many/few* ϕ ψ appear to make reference to the ratio of the individuals that are in the extensions of both ϕ and ψ to the individuals that are in the extension of ψ. The analysis of such readings is controversial. One prominent approach assumes they are a symptom of *many* and *few* making reference to a context dependent standard of comparison. We observe that this initially attractive approach systematically undergenerates, failing to capture pervasive reverse proportionality in environments that remove context dependency of the standard. Instead, we propose that reverse proportionality in such cases can arise from the underspecification of the measure function underlying the meanings of *many* and *few*.

Keywords: proportional quantifiers, *many* and *few*, reverse proportionality, comparatives, degree constructions, measure functions, context dependency.

1 Introduction

Since Partee (1989), much work has assumed that *many* and *few* are lexically ambiguous between a *cardinal* and a *proportional* sense. Under the cardinal meaning, the truth of *many/few* ϕ ψ requires that the cardinality of $[\![\phi]\!] \cap [\![\psi]\!]$, the intersection of the extensions of ϕ and ψ, be above/below a contextually determined standard cardinality; under the proportional meaning, it requires that the ratio of individuals in $[\![\phi]\!] \cap [\![\psi]\!]$ to individuals in $[\![\phi]\!]$ be above/below a contextually determined standard proportion. This lexical ambiguity is posited to capture the range of interpretations that is illustrated for *few* by Partee's examples in (1).

(1) a. There were few faculty children at the 1980 picnic.
  b. Few egg-laying mammals suckle their young

*For comments and discussion, we would like to thank the audience at Sinn und Bedeutung 23 in Barcelona (in particular Ora Matushansky, Maribel Romero, Stephanie Solt, and Rob van Rooij), the members of the Semantics Research Group at McGill University (in particular Aron Hirsch), the members of SemanticsBabble at UCSD, three anonymous reviewers for *Linguistics and Philosophy*, and David Shanks. The authors acknowledge support from the Social Sciences and Humanities Research Council of Canada (SSHRC), Insight grant #435-2016-1376 (Bale, Schwarz), and Insight grants #435-2016-1448 and #435-2013-0592 (Schwarz).

Partee presents (1a) as illustrating the cardinal sense of *few*. The sentence can be judged true even if *all* of the faculty children were at the 1980 picnic, on the grounds that at the time there were only few faculty children to begin with. This suggests that the sentence portrays the cardinality of the intersection of $[\![$ *faculty children* $]\!]$ and $[\![$ *at the party* $]\!]$ as falling below a contextually determined standard. In contrast, Partee reports that truth conditions of (1b) do not impose similar requirements. Unlike the reading in (1a), the sentence in (1b) cannot be true if all the egg-laying mammals suckle their young, even if there are only a few egg-laying mammals existing in the world. Instead, the sentence is read as being about the ratio of individuals in the intersection of $[\![$ *egg-laying mammals* $]\!]$ and $[\![$ *suckle their young* $]\!]$ to individuals in $[\![$ *egg-laying mammals* $]\!]$, portraying that ratio as falling below a contextually determined standard. Hence Partee takes (1b) to illustrate the proportional sense of *few*.

In the proportional reading identified by Partee (1989), the proportion that *many/few* $\phi\ \psi$ refers to, namely $|\,[\![\,\phi\,]\!] \cap [\![\,\psi\,]\!]\,|/|\,[\![\,\phi\,]\!]\,|$, has a denominator determined by the nominal argument, the restrictor of *many/few*. Refining the standard terminology, we refer to this reading as the *forward* proportional reading. We use this terminology to distinguish it from the *reverse* proportional reading that is the focus of our investigation. The reverse proportional reading, first discussed in Westerståhl (1985b), is illustrated by the sentence in (2), taken from Herburger (1997).

(2) Few cooks applied.

Herburger reports that this sentence can be read as a statement about the ratio of the set of applicants that are cooks, the intersection of $[\![$ *cooks* $]\!]$ and $[\![$ *applied* $]\!]$, to the set of applicants, $[\![$ *applied* $]\!]$, stating that this ratio is below a contextually determined standard. In this reading, the proportion that *many/few* $\phi\ \psi$ refers to is $|\,[\![\,\phi\,]\!] \cap [\![\,\psi\,]\!]\,|/|\,[\![\,\psi\,]\!]\,|$, where the denominator is now determined by the *scope* of *many/few*, rather than by the noun phrase that serves as its restrictor.

The existence of reverse proportional readings appears to be beyond dispute. What is debated, however, is the analysis of such readings. Driven in part by considerations of theoretical parsimony, most authors reject Westerståhl's (1985b) assumption that reverse proportional readings are due to a reverse proportional lexical meaning of *many* and *few*. In one prominent school of thought, which we will refer to as the *standard-based* approach (Büring 1996; de Hoop and Solà 1996; Romero 2015, 2016; Solt 2009), reverse proportional readings are instead a symptom of *many* and *few* making reference to a context dependent standard of comparison, and are a natural consequence of this context dependency, under appropriate conditions, even in the absence of reverse proportional lexical entries.

However, the main objective of this paper is to demonstrate that the standard-based approach systematically undergenerates, as it fails to capture pervasive reverse proportionality in environments that remove context dependency of the standard of comparison (Section 3). Moreover, we offer an argument for the existence of an alternate source of reverse proportionality in such cases, proposing that it reflects the underspecification of the measure function underlying the meanings of *many* and *few* (Bale and Barner 2009; Wellwood 2014, 2015, 2018; Solt 2018; Sections 4 and 6). We also provide a brief discussion of constraints on reverse proportionality, specifically a prediction about constraints observable in cases that do not feature a context dependent standard of comparison (Section 5).

To set the stage for these arguments, we begin by spelling out in more detail the two analyses of reverse proportionality hinted at above, the lexical ambiguity analysis and the standard-based analysis (section 2).[1]

2 Reverse proportionality from context dependent standards

The literature develops the standard-based approach into different detailed analyses that diverge on important particulars (Büring 1996; de Hoop and Solà 1996; Romero 2015, 2016; Solt 2009). However, since our argument will apply to the standard-based approach as a whole, there is no need here for a comprehensive review of these different proposals. We will instead introduce this general approach by outlining one particular possible rendition. This rendition is discussed (although ultimately not endorsed) in Westerståhl (1985b), and it also follows closely the line of reasoning in Solt (2009).

As a baseline, we first define a family of lexical entries that captures the two types of readings associated with *many* and *few* that Partee (1989) argued for. Treating *many* and *few* as forming generalized quantifiers in the sense of Barwise and Cooper (1981), the cardinal and forward proprotional sense of *many* and *few* are given in (3) and (4), where n and p are contextually given standards of cardinality and proportion, respectively.

(3) a. $[\![\, many_1 \,]\!](\mathrm{X})(\mathrm{Y}) \Leftrightarrow |\mathrm{X} \cap \mathrm{Y}| > \mathrm{n}$
b. $[\![\, few_1 \,]\!](\mathrm{X})(\mathrm{Y}) \Leftrightarrow |\mathrm{X} \cap \mathrm{Y}| < \mathrm{n}$

(4) a. $[\![\, many_2 \,]\!](\mathrm{X})(\mathrm{Y}) \Leftrightarrow |\mathrm{X} \cap \mathrm{Y}|/|\mathrm{X}| > \mathrm{p}$
b. $[\![\, few_2 \,]\!](\mathrm{X})(\mathrm{Y}) \Leftrightarrow |\mathrm{X} \cap \mathrm{Y}|/|\mathrm{X}| < \mathrm{p}$

Following Westerståhl (1985b), reverse proportional readings could be captured in a straightforward way by positing the pair of lexical entries in (5), obtained from those in (4) by replacing the first set argument with the second in the denominator of the fraction.

(5) a. $[\![\, many_3 \,]\!](\mathrm{X})(\mathrm{Y}) \Leftrightarrow |\mathrm{X} \cap \mathrm{Y}|/|\mathrm{Y}| > \mathrm{p}$
b. $[\![\, few_3 \,]\!](\mathrm{X})(\mathrm{Y}) \Leftrightarrow |\mathrm{X} \cap \mathrm{Y}|/|\mathrm{Y}| < \mathrm{p}$

However, as also noted by Westerståhl (1985b), given that the standard proportion p in these meanings is context dependent, it can be argued that conventionally encoded reference to reverse proportions is dispensable. This is because the right sides of the equivalencies in (5) can be restated as in (6).

(6) a. $|\mathrm{X} \cap \mathrm{Y}|/|\mathrm{Y}| > \mathrm{p} \;\Leftrightarrow\; |\mathrm{X} \cap \mathrm{Y}| > \mathrm{n}$, where $\mathrm{n} := \mathrm{p} \times |\mathrm{Y}|$
b. $|\mathrm{X} \cap \mathrm{Y}|/|\mathrm{Y}| < \mathrm{p} \;\Leftrightarrow\; |\mathrm{X} \cap \mathrm{Y}| < \mathrm{n}$, where $\mathrm{n} := \mathrm{p} \times |\mathrm{Y}|$

[1] In this paper, apart from brief remarks in Section 5 and our concluding section (Section 7), we do not address the interactions between syntax, semantics and focus structure with regards to the interpretation of *many* and *few*. As far as we can see, the conclusions we reach in this paper stand regardless of how these issues are resolved. Given that we discuss readings previously unexplored in the literature, future work will have to examine how the new range of semantic interpretations interact with these factors. For a discussion of these interactions, see Büring (1996), de Hoop and Solà (1996), Cohen (2001), Herburger (1997), Partee (1989), Penka (2018), Romero (2015, 2016), among others.

Indeed, as Westerståhl (1985b) observes, the forward proportional readings, too, could be accounted for by manipulating the contextual standard, as shown in (7).

(7) a. $|X \cap Y|/|X| > p \Leftrightarrow |X \cap Y| > n$, where $n := p \times |X|$
b. $|X \cap Y|/|X| < p \Leftrightarrow |X \cap Y| < n$, where $n := p \times |X|$

Thus, *many* and *few* could have a univocal, cardinal meaning with polysemy rooted in an independently motivated contextually determined standard.[2]

This theoretically parsimonious option places the burden of proof on those wishing to argue for the existence of forward and reverse proportional lexical senses like those defined in (4) and (5).[3] Here we focus on the reverse proportional reading, though, which is also the one that is more commonly collapsed into a cardinal interpretation.[4]

Accepting this burden of proof, we will now proceed to establish that reverse proportionality is not in fact dependent on the presence of a contextually determined standard of comparison. While we do not know of any reasons, empirical or conceptual, for assuming that reverse proportionality can *never* be due to a particular setting of the standard, we will see that standard setting cannot be the only source of the relevant readings.[5]

3 Reverse proportionality without context dependent standards

Bresnan (1973) proposed that *many* and *few* function in the same way as gradable predicates. This proposal suggests itself for *few*, which combines with degree morphology in the characteristic way, in particular forming comparative and superlative forms *few+er* and *few+est*. Bresnan extends this type of analysis to *many* by analyzing *more* and *most* as

[2] Even in a language where cardinal and proportional meanings are lexicalized differently, the two expressions corresponding to *many* might have the same conventional meaning, differing merely in terms of how their syntax interacts with the mechanism of fixing the standard of comparison. Krasikova (2011) argues this very point for Russian *mnogie* and *mnogo*, which correspond to proportional and cardinal *many*, respectively.

[3] Westerståhl (1985b) warns against such an appeal to parsimony, noting that this would require enriching our semantics so that multiple contextual standards could be set within the same sentence. For example, Westerståhl (1985b) cites Barbara Partee's example *Many boys date many girls*, where it is apparent that the contextual standard of what counts as *many* in the first DP is much higher than what counts as *many* in the second. However, as Westerstahl notes in his work with respect to context sets (Westerståhl, 1985a), it seems to be a general property of language that contextually sensitive variables can receive distinct values for different DPs within the same sentence.

[4] There is some motivation in the literature to resist, in particular, having a reverse proportional lexical entry. For example, unlike the forward proportional lexical entry, a reverse proportional entry would not be *conservative* in the sense of van Benthem (1984). See the discussion in Westerståhl (1985b).

[5] Westerståhl (1985b) had initially detected reverse proportionality in the now famous example *Many Scandinavians have won the Nobel prize in literature*. However, subsequent authors argued that this sentence does not actually allow for the reverse proportional truth conditions of the sort derived by the lexical entry in (5a) (Cohen 2001, Romero 2015, 2016). Romero (2015, 2016) argues that the actual interpretation of Westerståhl's example crucially requires reference to the setting of the context dependent standard, which is to be calculated with reference to focus values in the sense of Rooth (1985). We are inclined to agree with Romero's assessment, which is compatible with the conclusions we draw in this paper. Again, it seems very plausible to us that the setting of a contextual standard can yield reverse proportionality or similar effects. What we deny is that standard setting is the only source of reverse proportionality. In Section 5 we briefly discuss the consequences of the view, invited by our findings, that there are two different routes to reverse proportionality.

many+er and *many+est*. Hackl (2000) further motivated this proposal by providing compelling semantic arguments that support decomposition.[6] In this section, we explore some of the consequences of this point of view, first reviewing a somewhat standard proposal for how to treat gradable adjectives before turning our attention back to cardinal and proportional interpretations of *many* and *few*, in particular interpretations that do not involve a comparison to a context-dependent standard.

In one prominent analysis of gradable adjectives (see Cresswell 1976 and von Stechow 1984, among others), gradable predicates—the elements which most commonly combine with comparative and superlative morphemes—are analyzed using measurements and degrees. For example, gradable adjectives like *tall* can be interpreted as comparing a measurement of height to a degree of some sort, e.g., $[\![\textit{tall}]\!] = \lambda d.\, \lambda x.\, \mu_{\text{ht}}(x) \geqslant d$, where μ_{ht} maps individuals to the degree of their height. The use of such predicates in constructions like *John is six feet tall* is rather straightforward ($[\![\textit{John is six feet tall}]\!] = \mu_{\text{ht}}([\![\textit{John}]\!]) \geqslant [\![\textit{six feet}]\!]$). However, the analysis of sentences without an overt degree argument requires a phonologically null operator, often called *POS* (see von Stechow 1984 and Kennedy 1999, among others), which takes an abstracted degree predicate as an argument. The *POS* operator compares the maximal value in the set given by the degree predicate to a contextually set standard. For example, let's suppose that the sentence in (8) has an LF structure like the one in (9a), where *POS* has a the meaning in (9b), where STND is a contextually set standard. Assuming (9a) and (9b), (8) will be assigned the truth conditions in (9c).

(8) John is tall.

(9) a. $POS\ \lambda d[\text{John is } d \text{ tall}]$
b. $[\![POS]\!] = \lambda D.\, \textsc{max}(D) > \textsc{stnd}$
c. $\textsc{max}(\{d : \mu_{\text{ht}}([\![\textit{John}]\!]) \geqslant d\}) > \textsc{stnd} \quad \Leftrightarrow$
$\mu_{\text{ht}}([\![\textit{John}]\!]) > \textsc{stnd}$

Criticially, the contextually set standard is not an integral part of the semantics of the degree predicate itself.[7] Not only is it absent when explicit measurement phrases are used (as with *six feet* in the example above), but it is also absent in comparative constructions. Although the details are not important for our purposes, for concreteness we will sketch a standard view on which the comparative morpheme *-er* denotes a function like (10), taking two degree properties as arguments, one obtained by abstraction in the *than*-clause and the other from the main clause after covert movement of the degree phrase formed by *-er* and the *than*-clause.[8]

[6]Some of the more compelling evidence that Hackl (2000) presents are instances of *split scope*. There are certain sentences that have a reading that is only compatible with truth conditions where the comparative morpheme scopes above an intensional operator while cardinal measurement function scopes below. For example, the sentence *A professor is required to write fewer than two books in order to get tenure* can be true in a context where a professor is only required to write at least one book to get tenure, although the professor is allowed to write more than one.

[7]For the sake of simplicity, we ignore the issue of vagueness in terms of setting a value for the standard. For an adequate discussion of vagueness with respect to a standard, see the discussions in Kennedy (2007), Klein (1980), Kamp (1975) and references therein.

[8]As argued by Heim (2000), there are two main facts that support a movement analysis of degree phrases

(10) $[\![\textit{-er}]\!] = \lambda D_2.\, \lambda D_1.\, \textsc{max}(D_1) > \textsc{max}(D_2)$

The argument D_2 and D_1 will be furnished by the *than*-clause and the main clause, respectively. To illustrate, a sentence like (11) would have an LF structure similar to the one in (12a), resulting in truth conditions like those represented in (12b).

(11) Mary is taller than Bill is.

(12) a. [-er λd[than Bill is d tall]] λd[Mary is d tall]
b. $\textsc{max}(\{d : \mu_{\text{ht}}([\![\textit{Mary}]\!]) \geqslant d\}) > \textsc{max}(\{d : \mu_{\text{ht}}([\![\textit{Bill}]\!]) \geqslant d\}) \Leftrightarrow$
$\mu_{\text{ht}}([\![\textit{Mary}]\!]) > \mu_{\text{ht}}([\![\textit{Bill}]\!])$

Such truth conditions compare two degrees that are explicitly determined by two clausal arguments, hence they do not make reference to a contextually set standard of comparison.

On this approach, the analysis of *many* and *few* as gradable expressions requires a revision of the lexical entries for *many* and *few* that separates the introduction of a contextually determined standard from the degree expression. Specifically, following Romero (2015), instead of the lexical entries for *many* in (3a) and (4a), we could now have those in (13).

(13) a. $[\![\, \textit{many}_1 \,]\!] = \lambda \mathrm{d}.\, \lambda \mathrm{X}.\, \lambda \mathrm{Y}.\ |\mathrm{X} \cap \mathrm{Y}| \geqslant \mathrm{d}$
b. $[\![\, \textit{many}_2 \,]\!] = \lambda \mathrm{d}.\, \lambda \mathrm{X}.\, \lambda \mathrm{Y}.\ |\mathrm{X} \cap \mathrm{Y}|/|\mathrm{X}| \geqslant \mathrm{d}$

Hackl (2000) called these types of meanings *parameterized determiners*. Note that for the sake of simplicity, we will limit our discussion here to *many*, but similar interpretations can be given for *few*.[9] Just like adjectives, these lexical entries compare a measurement (of cardinality or proportion) to a degree. In order to introduce some kind of contextually determined standard, the *POS* operator would need to be introduced. For example, the sentence in (14) has a logical form (15a) that gives rise to the cardinal interpretation in (16a), and also (15b), which yields the forward proportional interpretation in (16b).

(14) Many students cheated.

(15) a. POS λd[d many$_1$ students cheated]
b. POS λd[d many$_2$ students cheated]

headed by the comparative morpheme. One is that such movement can account for scope ambiguities with intensional operators. For example, there is a reading of *Mary read 5 pages and John is required to to read exactly 2 more pages than that*, which means that the number of pages that John is minimally required to read is exactly two pages more than what Mary read. The other main argument stems from Antecedent Contained Deletion with comparatives (see also Bresnan 1973, among others). For example, ACD is acceptable in sentences like *John was climbing taller buildings than Mary was.* However, it is unacceptable (or at least strained) in sentences like *John was climbing buildings that Mary was.* Movement of the Degree Phrase *-er [than Mary was]* out of the VP would create the right environment for VP ellipsis.

[9] The difference between *many* and *few* is akin to the difference between gradable antonyms like *short* and *tall*. Kennedy (1999), based off of a degree ontology introduced by von Stechow (1984), suggests that the difference between antonymous degrees is how they extend: positive degrees extend from zero to a measurement whereas negative degrees extend from a measurement to infinity. Such a solution can be adopted here for *few*. The details would closely follow Kennedy's analysis of the difference between *tall* and *short*.

(16) a. $\text{MAX}(\{d : |[\![\textit{students}]\!] \cap [\![\textit{cheated}]\!]| \geqslant d\}) > \text{STND} \Leftrightarrow$
$|[\![\textit{students}]\!] \cap [\![\textit{cheated}]\!]| > \text{STND}$

b. $\text{MAX}(\{d : |[\![\textit{students}]\!] \cap [\![\textit{cheated}]\!]|/|[\![\textit{students}]\!]| \geqslant d\}) > \text{STND} \Leftrightarrow$
$|[\![\textit{students}]\!] \cap [\![\textit{cheated}]\!]|/|[\![\textit{students}]\!]| > \text{STND}$

Moreover, as with regular gradable predicates like *tall*, it is predicted that reference to a contextually determined standard should be absent in comparative constructions. For example, the logical forms for the comparative in (17) are expected to be those in (18), which give rise to the non-standard dependent truth conditions in (19).

(17) More professors cheated than students.

(18) a. -er λd[than [[[d many_1] students] cheated]]
λd[[[d many_1] professors] cheated]

b. -er λd[than [[[d many_2] students] cheated]]
λd[[[d many_2] professors] cheated]

(19) a. $|[\![\textit{professors}]\!] \cap [\![\textit{cheated}]\!]| > |[\![\textit{students}]\!] \cap [\![\textit{cheated}]\!]|$

b. $|[\![\textit{profs}]\!] \cap [\![\textit{cheated}]\!]|/|[\![\textit{profs}]\!]| > |[\![\textit{students}]\!] \cap [\![\textit{cheated}]\!]|/|[\![\textit{students}]\!]|$

Thus, comparative constructions provide a natural testing ground for whether reverse proportional readings (and proportional readings in general for that matter) are always derived by manipulating a contextual standard. With this in mind, consider the example in (20), where the positive form of *few* in Herburger's (1997) classic example of a reverse proportional meaning (see (2) above) is replaced by the comparative form of *many*, accompanied by a *than*-phrase, with contrasting phrases *our program* and *yours*.

(20) More cooks applied to our program than to yours.

The sentence in (20) can be read as comparing two ratios, viz. the ratio of applicants to our program that are cooks relative to the total number of applicants to our program and the ratio of applicants to your program that are cooks relative to the total number of applicants to your program, stating that the former ratio is greater.[10] Such a comparison can explain why (20) can be judged as true in situations where the cardinal and forward proportional interpretation might well be false: the sentence can be judged true on the basis of no information about the sets of cooks and applicants to the two programs other than that cooks represent a greater proportion of the applicants to our program, say 20%, compared to the proportion of the applicants to yours, say 10%. We can state the truth conditions of (20) in the relevant reading as in (21).

(21) $|[\![\textit{cooks}]\!] \cap [\![\textit{applied to our program}]\!]|/|[\![\textit{applied to our program}]\!]| >$
$|[\![\textit{cooks}]\!] \cap [\![\textit{applied to your program}]\!]|/|[\![\textit{applied to your program}]\!]|$

[10] This reading becomes even more prominent once the cardinal reading is suppressed. One way to suppress such a reading is to explicitly signal that a proportional interpretation, as in *Proportionally speaking, more cooks applied to our program than to yours.*

Thus, (20) allows for a reading that is reverse proportional in the same sense as the relevant reading of (2) described by Herburger (1997)—a reading where both the main clause and the than-clause make reference to the ratio of the members of the set given by the intersection of the noun phrase and the scope to the members of the set given by the scope alone.

The existence of reverse proportional readings in comparatives seems uncontroversial. All the speakers we have consulted agree that the reading in question is indeed available for our constructed example (20). Moreover, the phenomenon can also be detected in naturally occurring examples. This is illustrated by (22).

(22) With the Class of 2021 containing more women than any in school history and female faculty members at the forefront of their fields, Tandon is fully committed to promoting and supporting women in engineering, science, technology, and math.[11]

In the original, this sentence is accompanied by a pie chart conveying that the proportion of females at NYU Tandon is 36% in the class of 2020, but 41% in the class of 2021. The text does not mention absolute numbers, so it seems clear that the comparative in the initial adjunct clause is indeed intended in the reverse proportional sense.[12]

We postpone until the next section the question how exactly reverse proportional truth conditions might arise in cases like (20) and (22). What is clear enough, however, is that in the absence of a contextually determined standard, reverse proportionality in such cases shows that reverse proportional readings are not dependent on the presence of a contextually determined standard after all, and therefore are not in general a symptom of the malleability of such a standard, contra the proposals in a whole branch of work on reverse proportionality (Büring 1996; de Hoop and Solà 1996; Romero 2015, 2016; Solt 2009).[13,14]

Comparatives are not the only type of degree construction that this conclusion can be

[11] http://archive.engineering.nyu.edu/about/year-review/women

[12] In the first argument of the comparative operator in (22), we can assume that covert raising of *many women* from object position forms the derived predicate λx*[the class of 2021 containing x]*. The sets in the first and the second argument of *many* are then the set of woman—the X restrictor argument—and the set of those contained in the class of 2021—the Y scope argument. Similarly for the comparative operator's second argument.

[13] Comments by Maribel Romero (p.c.) have alerted us to the possibility that the range of interpretations that comparatives are known to participate in could introduce a confound for our argument about the source of reverse proportionality. Indeed, the finding that reverse proportionality is attested in comparatives does not by itself constitute compelling evidence for this conclusion. Bartsch and Vennemann (1972) and Kennedy (1999) draw attention to so-called comparatives of deviation, such as *Frances is more reticent than Hilary is long-winded* (from Kennedy 1999). These authors argue that such comparatives do make reference to a standard of comparison, and possibly a different standard in the main clause and the *than*-clause. One could accordingly speculate that the reverse proportional readings in (20) and (22) are available in virtue of those comparatives being interpretable as comparatives of deviation. However, we do not see any independent support for this speculation, which is inconsistent with our characterization of the truth conditions of these sentences. Further support for our interpretation of (20) and (22) comes from additional observations that we will introduce shortly, in (23) and (24) below. Looking beyond comparatives, these observations suggest that the availability of reverse proportionality is never eliminated by removing the dependency on a contextually determined standard of comparison.

[14] We seem to be the first to observe that reverse proportionality, as we have characterized it, is attested in the absence of a contextually determined standard. However, presenting the example in (i), Partee (1989) in passing anticipated the point that cardinal and forward proportional readings do not exhaust the repertoire of available interpretations in comparatives.

based on. Reference to contextually determined standards is also known to be removed in, for example, degree questions, equatives, or cases with a demonstrative *that* used as a measure phrase whose reference is fixed by a proportion given in the discourse. In such constructions, too, reverse proportional readings can be detected, as the examples in (23) serve to illustrate.

(23) a. Julia found out how many cooks applied.
  b. That many cooks had never applied before.
  c. Twice as many cooks applied last year.

We take it, if this year, 10% of the applicants were cooks, (23a) could be judged as true in virtue of Julia having found out that that was the case, without implying that Julia found out about any other cardinalities or proportions, including the absolute number of applicant cooks; similarly, *that many* in (23b) can be read as picking out 10%, so the sentence can be judged true in virtue of the mere fact that in previous years the proportion of cooks among the applicants always remained below 10%, independently of any other cardinalities or proportions; and (23c) can be understood as conveying that last year 20% of the applicants were cooks, again without supporting inferences about other cardinalities or proportions.[15]

To be sure, the reverse proportional readings we have just described are not the only possible readings for the sentences in (23). In particular, they all allow for prominent cardinal readings, where they make reference to the cardinality of the set of cooks who applied (at various times), and do not make reference to proportions. In the absence of context, cardinal readings may well be easier to access than the reverse proportional readings we are interested in here, to the extent that cardinal readings might for some speakers mask the existence of

(i) There are more illiterate people in small rural towns than in large cities.

Partee alludes to a salient reading of (i) which conveys that the ratio of (adult) illiterate residents to the total adult population is greater in small rural towns than it is in large cities. This reading, which Partee labels *non-CN-based proportional*, is neither cardinal nor forward proportional. In parallel to our discussion of (20) and (22), Partee notes that in virtue of appearing in a comparative, the reading cannot be credited to the setting of a contextually determined standard. Interestingly, however, this reading of (i) does not cleanly fit the mould of a reverse proportional reading as we have characterized it. We will substantiate this claim in Section 4, where we take the existence of examples like (i) to provide an important clue about the source of reverse proportionality (and proportionality in general).

[15] The example in (23b) is similar to data discussed in Ahn and Sauerland (2015, 2017). Ahn and Sauerland note, for example, that a German sentence like (i) has a reverse proportional interpretation of sorts, conveying that 60% of those hired were women.

(i) Wir haben 60 Prozent Frauen eingestellt.
  we have 60 percent women hired

Ahn and Sauerland offer an analysis of such readings where proportions are manufactured by the lexical meaning of *Prozent* 'percent', and where the measure function referred to in the semantic composition merely measures cardinalities. However, Solt (2018) argues that such an analysis undergenerates interpretations that involve proportions, and that a broader empirical coverage requires that measure functions themselves can deliver proportions. Looking ahead (to Sections 4 and 6), our own conclusions are broadly aligned with Solt's. Her particular analysis, however, does not account for the data that we focus on here, nor was it designed to do so, so we refrain from reviewing the proposal in detail.

reverse proportional readings. However, the reality of such readings is confirmed by the naturally occurring examples in (24).

(24) a. How many women work in technology? According to the National Center for Women & Information Technology (NCWIT), 25% of the computing workforce was female in 2015.[16]

b. Women made up 50.4 percent of the 3,404 students admitted to its elite undergraduate program starting last October, according to the university's official journal, the Cambridge University Reporter. "There has never been this many women," Cambridge's head of admissions, Janet Graham, told The Associated Press on Monday. [...] Women accounted for 48 percent of new undergraduates in the previous two years. In 1999, they lagged behind at 45 percent.[17]

c. SOAS is a majority female employer – 52.1% female to 47.9% male – but doesn't employ as many women as the HEI average, 54% female to 46% male.[18]

Example (24a) features an unembedded *how many*-question of much the same form as the embedded *how many*-question in (23a). That this question has a reverse proportional reading is suggested by the fact that a proportion of the relevant sort is offered within the same text as an answer. Example (24b) is like (23b) in that it features a demonstrative measure phrase, here *this*. This measure phrase appears to refer to a reverse proportion mentioned in the preceding text, much like *that* in (23b) can be taken to refer to such a proportion. Also, we take example (24c) to confirm the availability of a reverse proportional reading in equatives like (23c), since the text furnishes a specific reverse proportion to undergird the claim conveyed by the equative statement.[19]

In view of the data surveyed here, the conclusion that there exists a source of reverse proportionality other than contextually determined standards of comparison seems inevitable. Again, we have no reason for doubting that reverse proportionality can in principle be due to the setting of a standard, but we have argued that such standard setting is insufficient to capture all instances of reverse proportionality.

What, then, is the nature of the non-standard based source of reverse proportionality? We will begin to address this question in the next section by surveying possible answers emerging from the literature.

4 Non-standard based sources of reverse proportionality

How might reverse proportionality arise in the absence of a context-dependent standard? To recap, analyzing *many* and *few* as gradable expressions, and applying the analysis of com-

[16]https://fairygodboss.com/articles/women-in-tech-facts-figures-and-percentages

[17]https://www.theintelligencer.com/news/article/Female-Cambridge-Undergraduates-Bridge-Gap-10468054.php

[18]https://www.soas.ac.uk/equality-diversity-and-inclusion-strategy/file117829.pdf

[19]In (24b), we take the set in the second argument of *many*—corresponding to Y1 in the schema in (21)—to be the set of those admitted to the undergraduate program in question, even though there is no overt reference to this set in the relevant sentence *There has never been this many women*. In (24c), the set in the second argument of *many* is the set of those that SOAS employs, which we can assume to be given by λx*[(SOAS) employ x]* after covert raising of *many women* (within the scope of negation).

paratives outlined in the last section, our example (20), repeated in (25) below, is assigned a logical form like (26).

(25) More cooks applied to our program than to yours.

(26) -er λd[than [[[d many] cooks] [applied to your program]]]
λd[[[d many] cooks] [applied to our program]]

Consider now the lexical entry for *many* in (27), which adapts Westerståhl's (1985b) reverse proportional entry proposed in (5a) to the assumed degree based semantics, in parallel to the entries for cardinal and forward proportional entries in (13).

(27) $[\![\textit{many}_3]\!] = \lambda \mathrm{d}.\, \lambda \mathrm{X}.\, \lambda \mathrm{Y}.\ |\mathrm{X} \cap \mathrm{Y}|/|\mathrm{Y}| \geqslant \mathrm{d}$

Applied to the structure in (26) (substituting $\textit{many}_3$ for *many*), this entry delivers the intended truth conditions in (28), which repeats (21).

(28) $|[\![\textit{cooks}]\!] \cap [\![\textit{applied to our program}]\!]|/|[\![\textit{applied to our program}]\!]| >$
$|[\![\textit{cooks}]\!] \cap [\![\textit{applied to your program}]\!]|/|[\![\textit{applied to your program}]\!]|$

So, while reverse proportional readings in comparatives are beyond the scope of the standard-based approach, their existence is correctly predicted on an a lexical analysis, where *many* and *few* are gradable expressions with reverse proportional lexical entries.

To be sure, the lexical analysis at best provides an adequate description of reverse proportional reading, and does little to elucidate the reasons for its existence or the conditions under which it is observed. This shortcoming has been among the motivations for pursuing a different non-standard based route to reverse proportional readings, one that also predicts such readings to exist in comparatives and other standard-fixing constructions. This approach, pursued in Herburger (1997) and Greer (2014), rejects the proliferation of lexical entries, and instead locates the added complexity in the syntax-semantics interface. To understand this strategy better, note that the forward proportional entry for *many* in (13b) above can be mapped to the reverse proportional entry in (27) simply by switching the order of two predicate arguments (i.e., the X and Y arguments). Exploring logical forms substantially different from (26), Herburger (1997) and Greer (2014) argue that this switch of the arguments can be accomplished by syntax, or at least focus marking at the syntactic level, rather than by hypothesizing multiple lexical entries.[20] We will refrain here from reviewing these accounts in detail—let's call them the *syntactic mapping* analyses. The point we wish to make for the present purposes, is that the syntactic mapping analyses of reverse proportionality are like the lexical ambiguity analyses in that they do not rely on the presence of a contextually determined standard of comparison. Therefore, such analyses, too, are not challenged by reverse proportionality in comparatives and other standard-fixing constructions.

Syntactic mapping and lexical ambiguity exhaust the non-standard based approaches

[20] The arguments are not actually "switched", but rather the restrictor argument is determined by context in interaction with a focus value in the sense of Rooth (1985).

offered in the literature on reverse proportionality. However, recent work by Wellwood (2014, 2015, 2018), while not itself concerned with proportional readings, presents an analysis of *many* that is also applicable to the problem of proportional readings. Like the standard-based approach, this proposal can make reverse proportionality fall out from a particular type of resolution of a context dependency; however, unlike the standard-based approach, it locates the relevant context dependency in *many* itself, and in this respect resembles the lexical ambiguity approach. Specifically, Wellwood proposes that the interpretation of *many* makes reference to a measure function whose identity is underspecified. That is, the measure function is assumed to not be completely fixed by grammar, and to vary with the linguistic or non-linguistic context. If so, it is conceivable that cardinal and proportional readings can be the effects of different settings of the value of the measure function.

To spell out this idea, we continue to confine attention to cases where *many*, or its comparative form *more*, combines with a noun phrase. We can then couch Wellwood's proposal in the form of the preliminary lexical entry in (29).

(29) $[\![\, \textit{many} \,]\!] = \lambda \mathrm{d}.\, \lambda \mathrm{X}.\, \lambda \mathrm{Y}.\ \mu(\sqcup(\mathrm{X} \cap \mathrm{Y})) \geqslant \mathrm{d}$

In (29), the function $\sqcup$ maps sets of elements to their supremum—i.e., the element that is equivalent to the join of all the members of the set. The symbol μ is a variable ranging over measure functions that map entities, in this case the supremum of $\mathrm{X} \cap \mathrm{Y}$, to measures. The assumption is that, subject to certain constraints that we will briefly discuss in a moment, the value of μ is fixed by the context of utterance.

Even though Wellwood did not introduce her proposal as a possible account of proportional meanings, it is apparent that particular values for occurrences of μ will produce the full range of readings discussed so far. We illustrate this with reference to example (25) and its logical form (26). For the purposes of continuity with Wellwood's proposal and other work on mass-count semantics, it would be best if we altered some of our assumptions slightly. In the mass-count literature (see Link 1983; Krifka 1989; Gillon 1992; Chierchia 1998; Bale and Barner 2009 and references therein), plural count nouns and other predicates are generally analyzed as a set consisting of not just the individuals that are true of the noun/predicate but also all the groups that can be formed from these individuals (e.g., if a, b and c are individual boys then $[\![\, \textit{boys} \,]\!] = \{a, b, c, ab, ac, bc, abc\}$). Thus, to calculate the number of individuals that are true of a given predicate, one needs to count the number of atoms rather than the cardinality of the set. This can be achieved by counting the number of atomic parts in the join of all the members of the predicate (i.e., the supremum). In (29), the application of the measure function to a supremum is already built into the lexical entry of *many.* Thus, to access the cardinal or forward-proportional meaning, we only need to specify that the relevant measure function either counts the number of atoms contained within an entity, or counts the number of atoms and then divides that number by the number of individuals in the restrictor. The cardinal reading of (25) can then be credited to the setting of μ in (30a), for both the *than*-clause and main clause, while the forward proportional reading will result from the setting in (30b), once again for both clauses.

(30) Where for any element $x \in D$, $|x| = |\{y : y \leqslant x \;\&\; \textsc{atom}(y)\}|$

a. $\mu = \lambda x.|x|$

b. $\mu = \lambda x. |x| / |\sqcup [\![\textit{cooks}]\!]|$

The reverse proportional reading will result from a pair of different settings for the than-clause and the main clause (hereon μ_1 and μ_2), as in (31a) and (31b), respectively.[21] In this case, then, the number of atoms in the input is divided by the number of atoms in the set given by the scope of *many* in the respective clause in (26).

(31) a. $\mu_1 = \lambda x.\ |x| / |\sqcup\{\text{x: x applied to your program}\}|$
b. $\mu_2 = \lambda x.\ |x| / |\sqcup\{\text{x: x applied to our program}\}|$

As noted, Wellwood herself is not actually concerned with proportional readings. Her own argument for positing an underspecified measure function in the semantics of *many* is indirect. Building on Bresnan (1973) and Chierchia (1998), Wellwood argues that *many* is the surface realization of an underlying morpheme MUCH. In its positive form, this underlying morpheme is taken to be pronounced as *much* when combining with a noun phrase headed by a mass noun (as in *much water*) and as *many* when in combination with a plural noun (as in *many cooks*), while its comparative form is pronounced as *more*. In a more faithful rendition of Wellwood's actual proposal, then, the lexical entry in (29) is replaced with (32). Under this rendition, the logical form in (26) would need to be updated accordingly, replacing both occurrences of *many* with MUCH.

(32) $[\![\text{MUCH}]\!] = \lambda \text{d}.\ \lambda \text{X}.\ \lambda \text{Y}.\ \mu(\sqcup(\text{X} \cap \text{Y})) \geqslant \text{d}$

On this analysis of *many* and *much*, evidence for an underspecified measure function now comes from comparatives with mass nouns. As discussed in Cresswell (1976) and Bale and Barner (2009), comparatives that modify mass nouns involve truth conditions that invoke fundamentally different types of measurements. For example, to judge the comparison in (33a), one normally needs to know the volume of water in the two buckets. In contrast, to adequately judge the comparisons in (33b) and (33c), one needs to know the length of the two strings and the surface areas of the pieces of land owned, respectively.

(33) a. John's bucket has more water than Mary's. (comparison of volume)
b. John has more string in his desk than Mary. (comparison of length)
c. John owns more land than Mary. (comparison of surface area)

These readings can be understood as reflecting different values of the underspecified measure function μ in (32). With this in mind, the readings attested in (33), can be understood as arising from the mappings listed in (34).

(34) a. $\mu = \lambda x.\ \text{VOLUME}(x)$
b. $\mu = \lambda x.\ \text{LENGTH}(x)$
c. $\mu = \lambda x.\ \text{AREA}(x)$

Under the hypothesis that *many* (or its comparative form *more*) is equivalent to *much* (or its

[21] Two contextual variables of the same type can often take on different values within the same sentence. See in particular Westerståhl's discussion of context set variables in Westerståhl 1985a.

comparative form *more*), then, comparatives with mass nouns provide an indirect argument for the assumption that the measure function in *many* is underspecified.

An urgent question raised by this proposal concerns the obvious semantic differences between *more* in combination with plural count nouns versus *more* in combination with mass nouns. In contrast to mass-noun restrictors, plural count noun restrictors do not permit comparisons by dimensions that are logically independent of cardinality, such as volume, length, or surface area, as illustrated for mass restrictors in (34). For example, *Sarah has more chairs than Ben* is understood as comparing numbers of chairs, and cannot be interpreted, for example, as a comparison of weight. To capture this property of *more*, Wellwood proposes that the range of permissible values of μ is constrained by the semantic properties of the noun phrase that MUCH combines with. The constraint that Wellwood formulates effectively guarantees that, when MUCH combines with a plural count noun like *cooks* or *chairs*, μ is *permutation invariant* as defined in (35).

(35) A measure function μ is *permutation invariant* if and only if for all entities α and β, and all atomic permutation functions π (i.e., functions that map each atom to a unique atomic substitute and that map groups accordingly),[22] if $\pi(\alpha) = \beta$, then $\mu(\alpha) = \mu(\beta)$.[23]

In effect, this means that measure functions with respect to count nouns are restricted to those that yield the same result for any two possible inputs α and β that have the same number of atoms. A measure function which simply counts the number of atoms contained within a given entity straightforwardly satisfies permutation invariance (i.e., if α and β contain the same number of atoms, then it trivially follows that $|\alpha| = |\beta|$). Furthermore, any proportional measure that is based on the number of atoms also satisfies permutation invariance (i.e, if α and β contain the same number of atoms, then it trivially follows that for any denominator n, $|\alpha|/n = |\beta|/n$). Thus, each of the values for μ in (30) and (31) is permutation invariant in this sense. Accordingly, an account of reverse proportionality in terms of an underspecified measure function is compatible with Wellwood's constraint.

In addition to the permutation-invariance constraint that is specific to count noun restrictors, Wellwood also posits a *monotonicity* constraint that applies to both count noun and mass noun restrictors alike. The monotonicity constraint, adopted from Schwarzschild (2006), requires that μ track the part-whole relation that structures the members of the restrictor set; that is, if $\alpha <_\rho \beta$ according to the partial ordering defined by the nominal restrictor ρ, then for any permissible measure function μ, $\mu(\alpha) < \mu(\beta)$. This constraint excludes interpretations of, for example, the sentences in (25) or (33) as comparisons of, say, temperature, talent, or beauty. For example, if we assume that μ measured temperature and we also assume that according to the part-whole relation defined by *water*, the water

[22] Given a set of atoms Y, a permutation function is any function that is a bijection (one-to-one mapping) from Y to Y. A permutation function can extend to groups in the following way: if π is a permutation function on Y then for any $x = \sqcup Z$, $Z \subseteq Y$, $\pi(x) = \sqcup\{w : \exists z.\ w = \pi(z)\ \&\ z \in Z\}$. In other words, π maps a group x to a group y by substituting the atoms that make up x.

[23] The text here adapts the constraint offered in Wellwood 2018 to the formalisms used earlier in the text. It differs from Wellwood 2018 in that it does not restrict the permutation functions to those that have a domain/range restricted to the noun. Adopting the more restricted definition should not matter with respect to the empirical points discussed in this paper.

aggregate α is a proper part of the water aggregate β, it does not follow that the temperature of α is strictly less than the temperature of β, thus illustrating that measurements of temperature do not track the part-whole relationship of water. The monotonicity constraint also has the added benefit of ruling out measure functions that trivially satisfy permutation invariance but do not track the part whole structure of the noun. For example, any measure function that maps any set to a constant number, let's say 3, trivially satisfies permutation invariance, since the output number is invariant. However, such a measure function does not satisfy monotonicity, since it does not track the part-whole relationship between different aggregates. Critically, the measure function that simply counts atoms as well as the proportional measure functions discussed above satisfy the monotonicity constraint. For example, let's say that there are two elements α and β such that $\alpha <_\rho \beta$ according to the part-whole structure imposed by some count noun restrictor ρ. Since ρ is a count noun, it follows that α has a fewer number of atoms than β and hence, $|\alpha| < |\beta|$. Furthermore, for any denominator value n greater than zero, it logically follows that $|\alpha|/n < |\beta|/n$.

However, even granting that permissible values of an underspecified measured function for *more* in combination with a count noun respect permutation invariance and monotonicity, the proposal based on (32) predicts a greater diversity of interpretations than an analysis of reverse proportionality in terms of lexical ambiguity or syntactic mapping. This is due to the fact that for any denominator value—i.e., the n value above, the resulting proportional measure function will satisfy all of Wellwood's constraints. In Section 6, we address this issue by presenting novel data showing that the manifestations of proportionality are indeed diverse in ways that have not been previously observed, but that are naturally expected under the underspecified measure function analysis. Critically, this new data is not predicted by either the lexical ambiguity approach or the syntactic mapping analysis.

Before discussing this new evidence in Section 6, we first briefly explore a prediction that emerges under each of the analyses surveyed in this section, a prediction about constraints on reverse proportionality in absolute versus standard-fixing constructions.

5 Constraints on reverse proportional readings

The last section has presented lexical ambiguity, syntactic mapping, and measure function underspecification as conceivable sources of reverse proportionality in comparatives and other standard-fixing constructions. While there are important differences between these three sources, they share the feature that reverse proportionality is not introduced by degree morphology itself. Instead, reverse proportionality arises from semantic or syntactic properties of *many*. This implies that whatever is responsible for reverse proportional readings in standard-fixing constructions should also be available in absolute constructions.

At the same time, as we have emphasized, nothing we have said excludes the possibility that in absolute constructions, the setting of a contextual standard introduced by POS provides a possible alternate route to reverse proportionality. We are thereby led to the hypothesis that the sources of reverse proportionality in standard-fixing constructions are a proper subset of the sources in absolute constructions. Absolute constructions should in principal permit the setting of a standard from POS *in addition* to whatever source or sources from *many* itself might be available in cases where the standard of comparison is conventionally determined.

This conclusion makes a prediction that we will call the *subset prediction*: any restrictions on reverse proportionality that may be observable in absolute constructions is predicted to also be observable in standard-fixing constructions (while the reverse need not necessarily hold). This is so because under the present hypothesis, restrictions with absolute constructions should reflect constraints limiting the meaning contributions associated with both *many* and POS, while restrictions with standard-fixing constructions should only reflect constraints pertaining to *many*.

Is the subset prediction correct? We must leave a thorough investigation to future research, but here we will briefly consider two restrictions on reverse proportional readings that have been proposed in previous literature: one concerning the prosodic realization of these readings and the other concerning the type of predicate in the scope of *many* or *few*.

Herburger (1997) portrays the reverse proportional reading of (2), repeated in (36a), as dependent on prosodic prominence falling on the restrictor *cook*, which she interprets as showing that the relevant reading depends on the restrictor being the semantic focus in the sense of Rooth (1985). Does Herburger's characterization of (36a) (which is endorsed in Büring 1996, Cohen 2001, and Romero 2015, 2016) carry over to our comparative example in (20), repeated here as (36b)?

(36) a. Few cooks applied.
b. More cooks applied to our program than to yours.

At least on the surface, it doesn't seem to. The natural realization of (36b), under a reverse proportional reading, places prosodic prominence on the contrasting expressions *our* and *yours*, with no obvious detectable prominence on *cooks*. But this is perhaps not so surprising. As is well-known (see, e.g., Rooth 1992, Gawron 1995), focus in comparatives influences the resolution of ellipsis in the *than*-clause. This known interaction between focus and ellipsis might well mask any observable influence of focus as an enabler of a reverse proportional reading. What about standard-fixing degree constructions other than comparatives? Consider again, for example, the reverse proportional *how many*-question in (23a), repeated here as (37).

(37) Julia found out how many cooks applied.

In this example, much like in (36a), the pronunciation of the sentence under a reverse proportional reading seems to call for prosodic prominence on the restrictor *cooks*. If this is correct, it suggests that after all, unless masked by independent factors, the prosodic signature of reverse proportionality in absolute cases is also found in standard-fixing cases. If so, the evidence from prosody is compatible with the subset prediction.

As a further restriction on reverse proportional readings in absolute constructions, Herburger (1997) suggests that such readings are unavailable in cases where the scope of *many* or *few* is an individual-level predicate in the sense of Carlson (1977). For example, Herburger reports that (38), which features the individual-level predicate *speak Spanish*, lacks a reverse proportional interpretation, which would convey that Salvadorean Spanish speakers constitute a small proportion of the world's Spanish speakers. Does the same constraint apply in a comparative construction such as (39a) or a *how many*-question like (39b)?

(38) Few Salvadoreans speak Spanish.

(39) a. More Canadians speak Icelandic than Spanish.
b. How many Canadians speak Icelandic?

This seems to be the case. It seems hard or impossible to read (39a) as reporting that the proportion of Canadians in the set of all Icelandic speakers is greater than the proportion of Canadians in the set of all Spanish speakers. It likewise seems hard or impossible to read (39b) as asking for the proportion of Canadians in the set of all Icelandic speakers. So the unavailability of reverse proportional readings with individual-level predicates like *speak Spanish* seems to extend to standard-fixing constructions. Like the evidence from prosody, then, this evidence is compatible with the subset prediction.[24]

In sum, as far as we can see, the evidence from restrictions on reverse proportional readings is compatible with the view that these readings can have two possible sources, and in particular with the subset prediction articulated above. The important unanswered question, of course, is why reverse proportionality might be subject to such restrictions. We will not attempt to answer this question here, but we will briefly return to the issue of constraints on reverse proportionality in our concluding section.

Before concluding, we will now return to the central question the we started addressing in Section 4, viz. what can account for reverse proportionality in standard-fixing constructions such as comparatives.

6 Reverse proportionality from underspecified measure functions

Elaborating on Wellwood (2014, 2015, 2018), we introduced in Section 4 the hypothesis that reverse proportionality could be a symptom of an underspecified measure function. This hypothesis invites us to consider data that extend the inventory of cases discussed above. In particular, any measure function that maps an input α to the output $|\alpha|/n$ (for some n greater than zero) will satisfy the permutation invariance and monotonicity constraints discussed in Section 4. Since underspecification has been most directly observed for comparatives with mass nouns, like those in (33) above, we are led to expect that such comparatives participate in interpretations similar to reverse proportional readings of cases with count nouns (modulo monotonicity and permutation invariance). This expectation is indeed borne out. Extending observations in Cresswell (1976), Bale and Barner (2009) discuss examples similar to (40), which features the mass noun *gold*.

(40) This ring has more gold in it than that necklace.

[24]Note, there is a difference between the sentences in (39) and the absolute construction in (38), namely that the constructions in (39) have a cardinal reading that is missing in the absolute construction. This seems to point to an interaction between the presence of a context dependent standard and and the setting of an underspecified measure function, such that in certain cases the standard-dependency appears to exclude a cardinal measure function. This interaction, while in need of explanation, is compatible with our subset prediction about the availability of reverse-proportional readings. (See Solt 2018 and Penka 2018 for a possible approach to this sort of interaction, which we will very briefly return to in our concluding section.)

This example can be understood as making reference to either weight or volume. The example thus again illustrates the underspecification of the measure function, and moreover demonstrates that the measure function associated with *more* is not fully determined by the noun phrase that *more* combines with.

Furthermore, whether (40) is understood to be about weight or volume, the sentence has an additional ambiguity. If we assume that the ring is small whereas the necklace is rather large and we further assume that the ring is slightly closer to being pure gold than the necklace, the sentence in (40) can be judged as both true and false. It can be false if the relevant measure is taken to be the absolute weight or volume of gold in the ring versus the weight or volume of gold in the necklace, but it can be true if the relevant measure is taken to be the proportion of gold in the ring versus the proportion of gold in the necklace. It is the latter, proportional readings that are of particular interest to us in the present context. Positing the logical form in (41), and given the lexical entry for *much* in (32) above, we can attribute these readings to the two settings of μ in (42) and (43), where (a) and (b) apply in the *than*-clause and the main clause, respectively.

(41) -er λd[than [[[d MUCH] gold] λx[this necklace has x in it]]]
λd[[[d MUCH] gold] λx[this ring has x in it]]

(42) a. $\mu_1 = \lambda x.\, \text{WEIGHT}(x)/\text{WEIGHT}(\text{this necklace})$
b. $\mu_2 = \lambda x.\, \text{WEIGHT}(x)/\text{WEIGHT}(\text{this ring})$

(43) a. $\mu_1 = \lambda x.\, \text{VOLUME}(x)/\text{VOLUME}(\text{this necklace})$
b. $\mu_2 = \lambda x.\, \text{VOLUME}(x)/\text{VOLUME}(\text{this ring})$

Cases with the same profile as (40) are easy to multiply. For example, *This bottle of wine has more alcohol in it than that bottle* permits much the same range of interpretations that we have described for (40), and is subject to the same analysis. By hypothesis, the interpretation of such examples, being relative to a contextually determined measure function, is not fully given by conventional meaning. We will therefore refer to such cases of proportional interpretations as instantiating *contextual proportionality*.

While Bale and Barner (2009) only considered instances of contextual proportionality with mass noun restrictors, the phenomenon is also attested with count nouns, that is, cases of much the same surface form as (25), our initial illustration of reverse proportionality in Section 3. Sentence (44) is a case in point. (Note that, while *bacteria* can be used as either a mass or count noun in English, the use of *are* in this example forces the plural count interpretation.)

(44) There are more Lactobacillus bacteria in my yoghurt than in yours.

This sentence has a prominent reading in which it can be judged true even if you have considerably more yoghurt than I do, so that the total number of Lactobacillus bacteria in your yoghurt might exceed the total number in mine, in which case (44) would be false under a cardinal interpretation. In the relevant reading, the sentence instead compares the proportions of Lactobacillus bacteria in the respective yoghurts, where proportion is most naturally understood as relative to volume. So the sentence can be judged true in virtue of the particle density of Lactobacillus bacteria being greater in my yoghurt (e.g., 5000 per ml) than it is in yours (e.g., 1000 per ml).

Hypothesizing an underspecified measure function, this reading is again unsurprising. Given the logical form in (45), we could interpret the underspecified measure function μ as taking on the values in (46), where again (a) and (b) determine the value of the measure function in the *than*-clause and the main clause, respectively.

(45) -er λd[than [[[d MUCH] Lactobacillus bacteria] [in your yoghurt]]]
λd[[[d MUCH] Lactobacillus bacteria] [in my yoghurt]]

(46) a. $\mu_1 = \lambda x. |x|/\textsc{volume}(\text{your yoghurt})$
b. $\mu_2 = \lambda x. |x|/\textsc{volume}(\text{my yoghurt})$

Note that the functions in (46) meet the constraints on permutation invariance and monotonicity posited in Wellwood (2018). First, both functions map all inputs with the same number of atoms to the same output. This is due to the fact that in both functions in (46), the input only determines the number of the numerator (i.e., the numerator is equal to the number of atoms in the input) whereas the denominator is fixed contextually. Second, for both functions, if β is a subgroup of α with respect to any count noun restrictor, then it follows that $|\beta| < |\alpha|$ (by definition of what it means to be a subgroup). Hence $|\beta|/n < |\alpha|/n$ for either $n = \textsc{volume}(\text{my yoghurt})$ or $n = \textsc{volume}(\text{your yoghurt})$.

It is not hard to multiply examples with profiles similar to that of (44). The comparative sentences in (47) are cases in point.

(47) a. There are more cars on Route 101 than on Route 104.
b. There are more molehills on my lawn than on yours.
c. Your manuscript has more typos than my manuscript.

Sentence (47a) can be read as conveying that the density or frequency of cars is greater on Route 101 than it is on Route 104; so the sentence can then be judged true even if Route 104 is much longer than Route 101, so that the total number of cars on the former might well be greater than the total number of cars on the latter. Similarly, sentence (47b) can be read as being about the density or frequency of molehills; it can then be judged true even if your lawn is several times larger than mine, so that the total number of molehills on my lawn is less than the total number on yours. And (47c) can be read as reporting that the density or frequency of typos is greater in your manuscript than in mine; in this reading, the sentence can be judged true even if the total number of typos in your manuscript might well be less than the total number of mine, in virtue of your manuscript being much shorter than mine.

Assuming underspecification of the underlying measure function, these cases are again unsurprising. The interpretations can be credited to the settings in (48), where in each case (i) and (ii) determine the values of the measure function in the *than*-clause and the main clause, respectively.

(48) a. (i) $\mu_1 = \lambda x. |x|/\textsc{length}(\text{Route 104})$
(ii) $\mu_2 = \lambda x. |x|/\textsc{length}(\text{Route 101})$

b. (i) $\mu_1 = \lambda x.\, |x|/\textsc{area}(\text{your lawn})$
(ii) $\mu_2 = \lambda x.\, |x|/\textsc{area}(\text{my lawn})$

c. (i) $\mu_1 = \lambda x.\, |x|/\textsc{wordcount}(\text{my manuscript})$
(ii) $\mu_2 = \lambda x.\, |x|/\textsc{wordcount}(\text{your manuscript})$

Echoing our earlier remarks about the reality of reverse proportional readings, we stress that the examples in (44) and (47) also allow for cardinal readings, and that without a concrete context specified, the availability of these cardinal readings could make it difficult for some speakers to detect the relevant contextually proportional readings.[25] However, naturally occurring texts confirm the existence of contextual proportionality of this sort. The examples in (49) are cases in point.

(49) a. Well if you're an immigrant from Poland you must be very Americanised . . . You're surprised by many normal things! Though I think it's much better that there are more road signs in Poland - I hate the road signs system in the US.[26]

b. Democratic supporters, Grammarly found, make 4.2 mistakes per 100 words, while Republican supporters made 8.7 mistakes per 100 words – and supporters of any Democrat made fewer mistakes than supporters of any Republican (with the exception of supporters of Hillary Rodham Clinton and Carly Fiorina, who were tied in their number of mistakes per 100 words, according to Grammarly).[27]

The comparative in (49a) is presumably not intended in a cardinal reading. After all, the total number of road signs in Poland is bound to be less, not more, than the total number of road signs in the US (given the two countries sizes and the concomitant lengths of their road systems). Instead the writer presumably meant to convey that the density or frequency of road signs is greater in Poland than it is in the US. This example is similar to (47a), then, referring to proportions that are relative to the lengths of the respective countries' road systems. Similarly, the comparative in (49b) is much like (47c) in that it compares texts in terms of the density or frequency of spelling or grammar mistakes, with proportions that are relative to the lengths of the relevant texts. Given that these comparatives do not contain constituents that semantically pick out the lengths of road networks or texts, these comparatives too instantiate contextual proportionality.

While we seem to be the first to pinpoint the phenomenon of contextual proportionality in comparatives with *more* plus count noun, we note that an example presented in Partee (1989), shown in (50) (and repeated from footnote 14), can also be taken to instantiate this type of interpretation.

(50) There are more illiterate people in small rural towns than in large cities.

Partee's example features an additional layer of complexity, due to the genericity that is

[25] As noted earlier for reverse proportional readings proper (see footnote 10), one can highlight these readings by adding text that suppresses the cardinal interpretation. For example, the sentence *Proportionally speaking, your manuscript has more typos than my manuscript* only has a contextually proportional reading.

[26] https://polishforums.com/travel/poland-vacation-trip-detailed-24388/

[27] https://www.washingtonpost.com/news/morning-mix/wp/2015/10/07/trump-supporters-have-the-worst-grammar-study-finds/?noredirect=on&utm_term=.a5078c87010d

introduced by the bare plurals *small rural towns* and *large cities*. Abstracting away from this complication, let us focus on the variant *There are more illiterate people in this small rural town than in that large city.* It seems clear that the salient truth conditions of this sentence depend on comparing the proportion of illiterates in the *adult population* in the small town to the proportion in the *adult population* of the large city. This is different from a reverse proportional reading simpliciter in that the denominator of the proportion is not just the total number of things in the small town/large city, but rather a more contextually restricted set (i.e., a set restricted to people over a certain age threshold). Thus, the relevant reading of Partee's example also instantiates contextual proportionality.

The existence of contextual proportionality in comparatives leads one to expect that the phenomenon extends to other degree constructions with *many* or *much*, given that it is those expressions that introduce the underlying measure function. This expectation is indeed borne out. For example, we judge the sentences in (51) to permit the relevant interpretations.

(51) a. Julia was shocked about how many typos her manuscript contained.
b. Why are there that many cars on Route 101?
c. There are twice as many Lactobacillus bacteria in my yoghurt than in yours.

These sentences can be read as making reference to the density or frequency of typos, cars, or bacteria in the relevant roads, manuscripts, or yoghurts. If Julia's manuscript contained an average of five typos per page, then (51a) can be read as reporting Julia's shock about this fact; if the cars on Route 101 are bumper to bumper, then (51b) can be interpreted as asking why that is the case; and (51c) has a reading which can be true in virtue of there being, say, 2000 Lactobacillus bacteria per ml in my yoghurt but only 1000 per ml in yours.

Once again, the relevant phenomena can also be identified in naturally occurring examples. As we invite the reader to verify, the attested examples in (52) have much the same profiles as the constructed cases in (51).

(52) a. How many typos are tolerable in a manuscript if a major publishing house is considering the work? Is there an expectation that the copy will be flawless, or are a few errors/typos tolerable every 20-50 pages or so?[28]
b. 4 straight years of 7 percent unemployment – and those numbers are only smoke and mirrors. Millions have just stopped looking and dropped off of the bloated 99 week Unemployment Rolls. There haven't been this many unemployed since World War II! Get used to it.[29]
c. After the fourth round of chemo, those getting mistletoe had three times as many white blood cells as the control group (3,000 count vs. 1,000 count).[30]

We conclude that the data presented in this section lend credibility to Wellwood's (2018) proposal that comparatives with *more* in combination with mass nouns or count nouns make reference to an underspecified measure function. The data make this case in virtue of es-

[28]https://www.quora.com/How-many-typos-are-tolerable-in-a-manuscript-if-a-major-publishing-house-is-considering-the-work-Is-there-an-expectation-that-the-copy-will-be-flawless-or-are-a-few-errors-typos-tolerable-every-20-50-pages-or-so

[29]http://americandigest.org/sidelines/2013/01/

[30]http://rexresearch.com/popp/popp.htm

tablishing that proportional interpretations in comparatives with *more*, and other standard-fixing constructions, show considerably more diversity than observed in previous work, indicating that at least some of the diversity that the proposal leads one to expect is indeed attested. Critically, these contextually proportional readings are not accounted for by any of the other approaches to the various readings of *many*.

This confirmation of the underspecified measure function hypothesis has far-reaching consequences. Since the hypothesis captures the full range of readings considered in previous work, viz. cardinal, forward proportional and reverse proportional readings, the burden of proof is placed on those who wish to argue, following Westerståhl (1985b), Herburger (1997), and Greer (2014), that reverse proportional readings are a matter of conventional meaning fixed by either lexical meaning alone (Westerståhl 1985b) or by the interaction of lexical meaning with the mapping of syntactic material to the argument positions of *many*/*few* (Herburger 1997; Greer 2014). In fact, more generally, we take the existence of contextual proportionality to present a new challenge to those wishing to argue, following Partee (1989), that *many* and *few* are lexically ambiguous.

7 Concluding remarks

In this paper, we have argued that the standard-based approach to reverse proportional readings is insufficient and must be supplemented with the assumption that such readings can also arise from the setting of an underspecified measure function. This "addition" to our theoretical toolbox has a welcome consequence, namely that the various readings of *many* and *much*, in both mass and count contexts, could all be accounted for by one, univocal, lexical interpretation. Thus, this "addition" ends up simplifying our grammatical theory considerably.

We will conclude by calling attention to two remaining issues: one a potentially fruitful avenue for future investigation—modifier phrases that explicitly mark comparisons of proportion—and the other a potential analysis concerning how certain readings with *many* and *few* are constrained by grammar, a question first broached in Section 5.

One benefit of our analysis is that it provides a basis for the study of phrases that explicitly indicate certain types of proportional comparisons. For example, the sentence in (53a) has truth conditions based on the average number of spelling errors per page, the one in (53b) on the ratio of bars to students, and the one in (53c) on the proportion of weeds relative to area.

(53) a. My manuscript contains more spelling errors **per page** than your manuscript.
b. My university has more bars **relative to the size of the student body** than your university.
c. **In terms of density**, my lawn has more weeds than your lawn.

We will not provide a full account of such phrases here, but we offer a few comments. First, the bolded phrases in (53) do not have a fixed syntactic position but rather appear to pattern like adverbial adjuncts. For example, (53a) can be paraphrased by any of the sentences below.

(54) a. **Per page** my manuscript contains more spelling errors than your manuscript.
b. My manuscript **per page** contains more spelling errors than your manuscript.
c. My manuscript contains more spelling errors than your manuscript **per page**.

Similar observations hold for the bolded phrases in (53b) and (53c). This suggests that such phrases may be compositionally flexible in a way that is similar to other types of adjuncts. Second, the bolded phrases can be completely omitted and the relevant readings are still possible, as long as there is some kind of contextual prime. For example, the information suggested by the bolded phrases can appear in separate sentences, as shown in (55).

(55) a. Your manuscript is much longer but let's consider certain properties relative to the number of pages. Taking this into consideration, my manuscript contains more spelling errors than your manuscript.
b. Your university is much larger than mine, but let's consider things relative to the size of the student body. Taking this into consideration, my university has more bars than your university.
c. Your lawn is much bigger than mine, but let's consider things relative to the size of the lawn. Taking this into consideration, my lawn has more weeds than your lawn.

Third, when the bolded text is omitted, the available readings are much broader. For example, if we alter (55a) in a way that leaves the specifics of the length measurement vague, we can access a variety of readings that are not available when the *per page* construction is present.

(56) Your manuscript is much longer but let's consider certain properties relative to the manuscripts' length. Taking this into consideration, my manuscript contains more spelling errors than your manuscript.

The second sentence in (56) can be true when the two manuscripts are both less than a page long, unlike (53a) or the second sentence in (55a). Given these observations, we think a good starting point in the analysis of such optional phrases is that they serve to help fix the contextual value of the measure function.

Among the various questions that our discussion has left open, one that is particularly urgent concerns the grammatical constraints on the accessibility of readings with *many* and *few*. In Section 5, we briefly addressed this issue with regard to reverse proportional readings, however cardinal readings too are in part regulated by grammar. For example, Partee (1989) reports that cardinal readings are excluded when *many* and *few* appear in partitives or as subjects of individual-level predicates, as shown by the examples in (57a) and (57b), respectively.

(57) a. Few of the philosophy majors (in the class) failed the exam.
b. Few philosophy majors (in the class) know calculus.

The sentences in (57) cannot be true in a context where the majority of the philosophy majors failed the exam or know calculus (respectively), even if the number of philosophy majors in the class is quite small. In other words, the sentences cannot be true in a context that only supports a cardinal reading.

Under our proposal, such restrictions must reflect limitations imposed by grammar on the available assignment of values to both the standard of comparison and the measure function. We conclude our paper by identifying a particular analysis of this sort, elaborating on the proposal in Penka (2018).

Modifying ideas in Romero (2015, 2016), Penka suggests that the value of the contextually determined standard is calculated with reference to a comparison class. For cases like those in (57), Penka proposes that grammar constrains the comparison class in a way that results in proportional readings. In more detail, Penka assumes that the silent operator POS in the absolute construction refers to a function θ which maps a comparison class to a standard cardinality (a threshold) based on the distribution of the measurements of members of the comparison class. Simplifying for concreteness and ease of exposition, suppose θ delivers the average cardinality of the members of C.[31] With regard to the comparison class C, Penka hypothesizes a grammatical constraint which in the cases in (57) has the effect of setting C to $[\![\textit{philosophy majors}]\!]$, the set of all (atomic or non-atomic) groups of philosophy majors. The average cardinality of the members of this comparison class is approximately half the cardinality of its supremum, the set of all atomic philosophy majors; that is, $\theta([\![\textit{philosophy majors}]\!]) \approx |\sqcup[\![\textit{philosophy majors}]\!]|/2$. In approximation, the sentences in (57) are then predicted to convey, not implausibly, that the cardinality of the set of atomic philosophy majors who failed the exam/know calculus is less than half of the cardinality of the set of atomic philosophy majors; that is, $|\sqcup[\![\textit{philosophy majors}]\!] \cap [\![\textit{failed the exam/know calculus}]\!]| < |\sqcup[\![\textit{philosophy majors}]\!]|/2$. (If the reader finds these truth conditions to be too weak, one could easily set the threshold to be some percentage of the average, perhaps even a vaguely determined percentage). As intended, these are forward proportional truth conditions.

The notable feature of this analysis, stressed by Penka, is that it delivers forward proportional truth conditions even though the measure function referred to in the semantic composition is cardinal. In the context of our proposal, however, the immediate question that arises is why the measure function associated with *few* would have to be cardinal, the concern being that unattested interpretations might be derived if different, non-cardinal, measure functions are taken to enter into the meaning of *few*. However, we note that the danger of overgeneration can be averted by making an assumption that strikes us as natural, viz. that μ_{few}, the measure function operative in the meaning of *few*, is the same as μ_θ, the measure function in θ that determines a distribution of measurements.

In Penka's analysis, these measure functions are indeed the same function, namely the one that maps an individual to its cardinality ($\mu_{\text{few}} = \mu_\theta = \lambda x.\, |x|$). Suppose now that the cardinality function was replaced throughout by a proportional measure function, one that meets the monotonicity and permutation invariance constraints introduced in Section 5. For example, suppose that $\mu_{\text{few}} = \mu_\theta = \lambda x.\, \frac{1}{3}|x|$. With this substitution, the above truth conditions for the sentences in (57) can be shown to become (approximately)

[31] In terms of the point are making here, the "threshold" could also be some percentage or multiple of this average, or, for that matter, any function that returns a value based on a certain distributional pattern. We use "average" since such a calculation is familiar to most readers.

$\frac{1}{3}|\sqcup[\![\textit{philosophy majors}]\!]\cap[\![\textit{failed the exam/know calculus}]\!]| < \frac{1}{3}|\sqcup[\![\textit{philosophy majors}]\!]|/2$, where $\frac{1}{3}|\sqcup[\![\textit{philosophy majors}]\!]|/2$ is approximately equal to the average for the comparison class with respect to the measure function $\lambda x.\, \frac{1}{3}|x|$. (Once again, perhaps the threshold is some vaguely specified percentage of this average. The following equivalencies will hold either way.)[32] But the statements $\frac{1}{3}n < \frac{1}{3}m/2$ and $n < m/2$ are equivalent for any cardinalities n and m, and this remains true when replacing 3 with any k greater than zero. So, under the assumption that μ_{few} and μ_{θ} are the same function, Penka's analysis of comparison classes predicts that the proposed underspecification would be obscured in examples like those in (57). In sum, then, the analysis we have advanced in this paper is compatible with the Penka's proposal about the effects of partitives and individual-level predicates.

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[32]For any n, m and k (greater than zero), and any percentage p, $\frac{1}{k}n < p(\frac{1}{k}m/2)$ iff $n < p(m/2)$.

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