Eager for Distinctness

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Abstract.
I provide evidence that the interpretation of disjunctive sentences is sensitive to the linear order of the disjuncts. I argue that the asymmetry is rooted in informational redundancies that are present in one order that are not present in the other. I propose a constraint that ensures informational distinctness between earlier and later disjuncts. The constraint is checked at a surprising point in the left-right interpretation of the sentence, hence making the dynamics of the interpretive process of crucial importance in accounting for the observed asymmetry.

1 The Puzzle

(Hurford 1974) observes that disjunctions $\ulcorner X$ or $Y \urcorner$ where one disjunct entails the other are infelicitous. Following (Simons 2000), call such sentences "entailing disjuncts."

(1) #John was born in Paris or in France

(2) #John was born in France or in Paris

Let us call whatever it is that rules out entailing disjuncts "Hurford's Constraint" (henceforth HC). Observe that although (3) and (4) are entailing disjuncts, they are nonetheless judged felicitous:

Question: Who (of John and Mary) came to the party?

(3) (John or Mary) or Both [came to the party]

(4) John or (John and Mary) [came to the party]

Hurford uses the felicity of (3) and (4) along with HC to argue that English *or* is ambiguous between an inclusive and an exclusive reading. For example, if the first disjunct in (3) is read exclusively, then there is no longer any entailment between the disjuncts. As such, HC is avoided, and the sentence is judged felicitous.

Against the conclusion that English *or* is ambiguous, (Gazdar 1979) and (Simons 2000) observe that *or* is always read exclusively in the scope of negation:

(5) I didn't eat beef or pork at the party (= I didn't eat beef and I didn't eat pork)

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If *or* had an exclusive meaning, (5) should be judged true if I ate both beef and pork at the party. Such a reading is unattested. Further, n-ary disjunctions $\ulcorner X_1$ or X_2 or ... or $X_n \urcorner$ are normally interpreted as 'only X_1 or ... or only X_n.' Let us call this "the only-one reading." Simons shows that an exclusive *or* has no way of generating the only-one reading.

Simons offers a solution to these puzzles. She develops a system whereby *or* is unambiguously inclusive (truth-conditionally). She derives the exclusive reading by exhaustiyfing each disjunct. Along with a pragmatically motivated constraint against entailing disjuncts, her system solves all the issues raised above: the infelicity of (1) and (2), the felicity of (3) and (4), the only-one reading of n-ary disjunctions, and the fact that *or* is unambiguously inclusive in the scope of negation. However, since disjunction is fully symmetric in Simons' system, she predicts that a sentence $\ulcorner X$ or $Y\urcorner$ should be felicitous iff $\ulcorner Y$ or $X\urcorner$ is. As such, she predicts that (6) and (7), which are the same as (3) and (4) but for the order of the disjuncts, should be felicitous. This prediction is incorrect.

(6) #(John and Mary) or (John or Mary) [came to the party]

(7) #(John and Mary) or John [came to the party]

The puzzle is: if exhaustifying disjuncts allows us to avoid HC in (3) and (4), why doesn't it allow us to do so in (6) and (7)?

2 Symmetric and Asymmetric Disjunction

How would standard theories of disjunction deal with the facts stated above? For purposes of exposition, it is useful to group the various theories into four classes, though I must insist that the grouping need not be read as forming a partition of the set of theories. One class is composed of what I call "global pragmatic systems," of which the state of the art representatives are (Spector 2003; van Rooy and Schulz 2004; Sauerland 2004). These systems compute strengthened meanings of sentences "globally," i.e. at the root S node. As such, the interpretation of sentences is insensitive to the gross syntactic form underlying it.[1] The truth-conditional output at the root node of sentence X, $M(X)$, is fed to a pragmatic component whose job is to locate some subset of $M(X)$, $SM(X)$, as the final interpretation of X. We call $SM(X)$ the "strengthened meaning of X."[2]

A second class of theories are what I call "list systems." I take (Simons 2000; Zimmerman 2000) as representative of this class of theories. The list system approach to

[1]Note that this insensitivity to syntactic structure means that these theories predict that truth-conditionally equivalent sentences differing in syntactic form should receive the same interpretation. For example, *John came to the party* is predicted to receive the same interpretation as (4), which is obviously incorrect when strengthening is involved.

[2]We use the term "strengthened meaning" without being committed to any particular procedure for strengthening meanings. The strengthening may come through the use of an exhaustive operator in the syntax/semantics, as an implicature using some form of Gricean reasoning, through default reasoning, or any other method of shrinking the meaning of X.

disjunction takes disjunctions to be lists of possibilities. A sentence $\ulcorner X_1$ or X_2 or ... or $X_n \urcorner$ will be interpreted as a list $\mathcal{L} = \{SM(X_1), SM(X_2), \ldots, SM(X_n)\}$.[3] The natural way of thinking of this list is as the set of answers to the question under discussion that are compatible with the speaker's epistemic state.

The third class of theories are what I call "syntactic approaches." (Chierchia 2004; Fox 2006) are the state of the art systems implementing this kind of approach.[4] Syntactic approaches posit the existence of a null morpheme in the syntax, *exh*, that can decorate the syntactic tree at any S node. This silent operator is used to strengthen meanings: by appending *exh* to any sentence X in the tree (global or embedded), one generates $SM(X)$ as the interpretation of X.

What is crucial to note here is that each of the above systems is fully symmetric with respect to the disjuncts: a sentence $\ulcorner X$ or $Y \urcorner$ is predicted to be felicitous iff $\ulcorner Y$ or $X \urcorner$ is. In a fuller version of this paper, I argue that without radical revisions or ad hoc stipulations, with the exception of the list systems approach the above mentioned frameworks are ill-suited for dealing with the asymmetries noted in (3), (4), (6) and (7). Space limitations prevent me from further discussion of this issue here. In this paper, I will simply propose a variant of the list systems approach that can account for the facts, and, unfortunately, leave for another occasion a more complete discussion of the nature of the mismatch between the facts of asymmetry and the theories briefly discussed above.

Before turning to my own proposal, I should like to mention the one theory that I am aware of that does posit an asymmetry between the disjuncts, namely, Lauri Karttunen's original dynamic proposal (Karttunen 1974). The theory developed there offers the following entry for disjunction: $\ulcorner X$ or $Y \urcorner$ means 'X or (Y and $\neg X$).' Unfortunately, the entry doesn't quite work. For instance, consider the question *Who of John and Mary came to the party?*, and imagine its answer *(John and Mary) or Sue.* Letting "*j*," "*m*", "*s*" be the obvious abbreviations, Karttunen predicts that *(j and m) or s* means '(j and m) or (s and (¬ j or ¬ m)).' As such, he predicts that the sentence could be judged true in a situation where John and Sue, but not Mary, came to the party. This is an incorrect prediction, for the only available reading of the sentence is that either only John and Mary came to the party or only Sue did.[5] Thus, even by explicitly encoding what seems, prima facie, to be the right kind of asymmetry given the above facts, we are still unable to arrive at the correct interpretations of disjunctive sentences.

[3]In Zimmermans's system, the members of $\mathcal{L}$ are epistemically modalized as "it is compatible with the speaker's knowledge that." This is irrelevant to the present discussion, however, where the concern is with symmetry.

[4]Fox's system is more explicit about the syntactic assumptions made, so I will restrict discussion to that work. Nonetheless, the conclusions I reach follow for Chierchia's system just the same.

[5]Allowing the system to strengthen the first disjunct does not help generate the correct meaning, as the reader can easily verify.

3 Analysis: Taking Dynamics Seriously

I believe that, despite the shortcomings of Karttunen's proposal, dynamic systems of interpretation provide the most natural framework for dealing with the facts of asymmetry. In this section, I will state my own dynamic proposal. I will attempt to keep the discussion as theory neutral as possible so as to convey the force of the idea itself. In Section 4, I will provide specific details of my particular way of implementing the ideas discussed here. This will necessarily involve additional assumptions I make about the dynamics of information flow in communication. Here I state, informally, the conceptual foundations of the theory, its content, and its intended range of application.

The main assumptions of the theory I wish to propose are:

Questions in Contexts Following (Collingwood 1940; Groenendijk and Stokhof 1997; Rescher 2000; Spector 2003; van Rooy and Schulz 2004), inter alia, I assume that any actual discourse always contains some (possibly implicit) question Q. In context c, I assume that Q partitions c into a set $c_Q = \{c_1, \ldots, c_r\}$.[6]

Local Strengthening Given the wealth of facts discussed in (Chierchia 2004; Fox 2006), and some of the facts alluded to above, I assume that meanings are strengthened locally at each disjunct.

Disjunctions as Lists I follow (Simons 2000; Zimmerman 2000) in assuming that disjunctions provide lists of (strengthened) answers to questions. More specifically, in answering a question Q in context c, the speaker has to provide a list $\mathcal{L}$ of possible answers to Q. Each disjunct provides an answer $\mathcal{D}_\rangle \in \wp(c_Q)$, the power set of c_q. The list $\mathcal{L} = \bigcup \mathcal{D}_\rangle$. [7]

Left-Right Asymmetry I assume that interpretation occurs in time. In partiular, I assume that propositions get added to the list following the L-R order of the disjuncts. As such, the construction of $\mathcal{L}$ through time becomes relevant.

Given these assumptions, I formulate a single constraint on the dynamics of list construction:

Constraint Enforcing Informational Distinctness In the L-R interpretation of a disjunction, information that has already been added to the list cannot be brought up as a candidate for list membership at later stages of interpretation. More specifically, if $\mathcal{L}$ is the list that's been constructed up to the current stage of interpretation, and the current disjunct provides the answer $\mathcal{D}$, $\mathcal{D} \in \wp(c_Q)$, then if $\mathcal{L} \cap \mathcal{D} = \emptyset$, one may continue constructing the list by combining the members of $\mathcal{L}$ and $\mathcal{D}$, $\mathcal{L} \cup \mathcal{D}$. If

[6] In this, I assume Groenendijk and Stokhof's partition semantics for questions. See (Groenendijk and Stokhof 1997) for a survey of theories of questions, including Groenendijk and Stokhof's own partition theory.

[7] This way of setting things up means that conversations follow "strong relevance," in the sense of (Spector 2003).

$\mathcal{L} \cap \mathcal{D} \neq \emptyset$ the sentence will be judged infelicitous. The constraint is checked at each disjunct in the L-R order in which the disjuncts appear.

I will now show that the resulting system derives HC (1),(2), its obviation in (3),(4), the inability to avoid HC in (6),(7), and the only-one reading of n-ary disjunctions, along with other new facts to be discussed in this section. As such, I claim that it is not quite entailing disjuncts that are the problem (cf. (Hurford 1974; Gazdar 1979; Simons 2000)), but rather, informational overlap between earlier and later stages of interpretation. I will make all of these ideas precise shortly, but let us first examine, informally, how the system is supposed to work.

Consider first the sentence in (1): *John was born in Paris or in France.* Imagine this is a response to the question *Where was John born?* We can assume without loss of generality that this is a "city-level" question, i.e. that the question is asking for which city it is that John was born in.[8] Imagine there are only three cities in each world in the common ground, Paris, Nancy, and Montreal. In such a context, then, c_Q = {[m], [n], [p]}, where $[x]$ = {w in c: John was born in x in w}. In interpreting this sentence, the list $\mathcal{L}$ is initially empty. The interpreter begins with the leftmost disjunct, and adds the information in each successive disjunct to the list. In this case, it begins by adding the information that John was born in Paris. Thus, at this stage, $\mathcal{L}$ = {[p]}. The next disjunct gives the information that John was born in France, i.e. $\mathcal{D}$ = {[n],[p]}. Since $\mathcal{D} \cap \mathcal{L} \neq \emptyset$, the constraint enforcing distinctness of information between earlier and later disjuncts is violated. Hence the infelicity. Note that the informational overlap in this case is invariant under order permutation, and so sentence (2), *John was born in France or in Paris*, will be ruled out for the same reason as (1).

Let us now consider the contrast between (4) and (7). We abbreviate (4) and (7) as *j or (j and m)* and *(j and m) or j*, respectively. Imagine these sentences uttered as answers to the question, *Who of John and Mary came to the party?* This question in context c results in partition c_Q = {[j,m], [j], [m], []}. This is a set of propositions where both John and Mary came to the party, only John came to the party, only Mary came to the party, and neither John nor Mary came to the party.[9]

Interpretation of (4) will proceed as follows. The strengthened meaning of the first disjunct, *j*,[10] will be $\mathcal{D}$ = {[j]}. Since $\mathcal{L}$ is empty at this point, we add [j] to $\mathcal{L}$, so that $\mathcal{L}$ = {[j]}. The information in the second disjunct, *j and m*, is a set of worlds [j,m] where both John and Mary came to the party. Since $\mathcal{L} \cap \{[j,m]\} = \emptyset$, the constraint against informational overlap is satisfied, and the sentence is accepted as felicitous.

In the other direction, at the point at which the interpreter is done with the first disjunct *j and m*, $\mathcal{L}$ = {[j,m]}. Next up is the sentence *j*. Observe that, truth-conditionally, *j* means {[j],[j,m]}. The strengthened meaning of *j*, SM(j) = {[j]}. This raises a question that has been lurking in the background of our discussion so far: does the interpreter check the constraint against informational overlap before the meaning of the disjunct is strengthened

[8]For more on "levels" in questions, see (Potts 2006).

[9]For formal definitions of how such partitions arise, see Section 4.

[10]However it is that strengthening is implemented

or after strengthening has already taken place? If the former, then then the observed infelicity will be predicted. If the latter, then we will still require some explanation as to why the sentence is judged inappropriate. Given the observed infelicity of this sentence, along with other facts to be discussed shortly, it seems that natural language opts to check the constraint before strengthening has a chance to apply. Note that this is a choice point that could have gone either way. I currently have no explanation for why the choice should have gone this way rather than the other. It is in fact this element of timing that forces us to take the L-R order of the disjuncts seriously. Thus, not only must distinctness be enforced, it seems it must be enforced at a particular point in the temporal evolution of the sentence's interpretation:

Timing Principle Ensure that the truth-conditional meaning (as a set of propositions) of the current disjunct has zero intersection with $\mathcal{L}$.

This suggests a certain sort of "eagerness" on the part of the constraint against overlap, for it seems to apply as soon as it can, not allowing strengthening to have a chance to potentially save the utterance. We may be justified in asking: why so eager? Why not allow for strengthening to take place before checking for distinctness? Whatever the answer to this question, the constraint really is one about local checking of distinctness, and not about global properties of the disjunction:

(8) #(((John and Mary) or John) or Sue) [came to the party]

Interestingly, no amount of contrastive focus can override the constraint:

(9) #((John and Mary) or [John]$_F$) or ((John and Mary) or [Sue]$_F$) [came to the party]

Note, additionally, that we predict that adding an *only* to the second disjunct of (7) should save it from the constraint:

(10) Either John and Mary came to the party or only John did [come to the party]

This is because *only j* means, truth-conditionally, that John came and no one else did. Since $\{[\mathrm{j}]\}$ has zero overlap with $\mathcal{L} = \{[\mathrm{j,m}]\}$, the sentence is (correctly) predicted to be felicitous.

Note that we not only derive HC as a subcase of informational overlap, we can actually show that HC is not in and of itself a correct generalization of the facts. Observe that (11) is infelicitous, despite the fact that neither disjunct entails the other:

(11) #John ate some of the cookies or not all of them

(Hurford 1974) and (Gazdar 1979) both predict that there should be at least one reading of (11) under which it becomes acceptable. Since strengthening is optional, by not strengthening the first disjunct to mean 'some but not all,' we can ensure that there is no entailment between the disjuncts. As such, Hurford and Gazdar both predict that (11) should be felicitous. However, under our analysis, whether you strengthen or not, there will be informational overlap. For imagine that we have a partition fine-grained enough to distinguish between worlds where John ate some but not all of the cookies and those where he ate all of them, so that c_Q = {[a], [sbna], [n]}.[11] Now, observe that if you do strengthen the first disjunct, $\mathcal{L}$ = {[sbna]}. The meaning of the second disjunct yields $\mathcal{D}$ = {[sbna], [n]}. Thus, there is overlap between $\mathcal{L}$ and $\mathcal{D}$, and we thereby incur a violation of the constraint on distinctness. If you don't strengthen the first disjunct, then $\mathcal{L}$ = {[sbna], [a]}, which again results in overlap with $\mathcal{D}$ = {[sbna], [n]}. Thus, under *any* way of interpreting the first disjunct, we predict the sentence to be infelicitous. This is the correct prediction. There is no way for this sentence to escape the constraint.

The analysis also connects with the theory of presupposition. Consider, for instance, the contrast between (12) and (13):

(12) Either there is no King of France or the King of France is in Paris

(13) #Either there is a King of France or the King of France is in Paris

Imagine these sentences as answers to the question *where is the King of France?* raised in a context in which it is not known whether or not there is a King of France.[12] Suppose that there are only two cities in France in each world in the common ground, Paris and Nancy. Then, in such a context, this "problematic question" will result in partition c_Q = {[-kf], [p], [n]}, a set of propositions where either there is no King of France, or there is and he's in Paris, or there is and he's in Nancy. In answer (12), interpretation of the first disjunct results in $\mathcal{L}$ = {[-kf]}. The content of the second disjunct yields the set $\mathcal{D}$ = {[p]}. Since $\mathcal{L} \cap \mathcal{D} = \emptyset$, the constraint is satisfied, and (12) is judged felicitous. In (13), on the other hand, the information conveyed by the first disjunct creates $\mathcal{L}$ = {[n], [p]}. The information in the second disjunct results in $\mathcal{D}$ = {[p]}. Thus, $\mathcal{L}$ and $\mathcal{D}$ have {[p]} as their intersection, and, because of this non-distinctness, the sentence is judged infelicitous.

Finally, we make a rather strong prediction about the interaction between meaning strengthening and presupposition. Imagine a sentence $\ulcorner X$ or $Y \urcorner$ where X has $\neg p$ as a scalar implicature and Y has p as a presupposition. Imagine further that the disjunction is uttered in a context compatible with both p and $\neg p$. Our constraint enforcing distinctness of information predicts that $\ulcorner X$ or $Y \urcorner$ should be felicitous, while $\ulcorner Y$ or $X \urcorner$ should be infelicitous. In interpreting $\ulcorner X$ or $Y \urcorner$, interpretation of X (including strengthening) of X

[11] One can imagine more fine-grained ways of partitioning the space. All that we require is that this be a fine enough partitioning of the space of possibilities. I assume it is, though the issue is not at all trivial when quantified expressions are used. Here, [a] = {w in c: John ate all of the cookies in w}, [sbna] = {w in c: John ate some but not all of the cookies in w}, [n] = {w in c: John ate none of the cookies in w}.

[12] For an illuminating discussion of such contexts, where a question has been raised whose presuppositions are not satisfied by the context, see (Collingwood 1940; Rescher 2000). These are admittedly highly artificial contexts, and probably not all the frequent in normal discourse. Indeed, Collingwood calls this the "fallacy of many questions," and Rescher calls such questions "problematic." They do create highly defective contexts, yet, as (12) shows, we can sometimes deal with them just fine, which is all that really matters here.

will result in $\mathcal{L} = \{[X, \neg p]\}$.[13] With local accommodation of p when interpreting Y, the truth-conditional output of the interpretation of Y will be $\mathcal{D} = \{[Y, p]\}$. Thus, distinctness will be met, and $\ulcorner X$ or $Y\urcorner$ will escape unscathed. Flipping the order to $\ulcorner Y$ or $X\urcorner$, however, should result in infelicity. Interpretation of Y, with local accommodation of p, will result in $\mathcal{L} = \{[Y, p]\}$. The truth-conditional meaning of X will result in $\mathcal{D} = \{[X, p], [X, \neg p]\}$. Then, if *for whatever reason*, the context licenses a further partitioning of $[X, p]$ into $[Y, p]$ and $[\neg Y, p]$, we will have the required infelicity. For in such a context, $\mathcal{L} = \{[Y,p]\}$, $\mathcal{D} = \{[Y, p],[\neg Y, p],[X, \neg p]\}$, $\mathcal{L} \cap \mathcal{D} = \{[\mathrm{Y, p}]\} \neq \emptyset$, and so we will have a violation of the constraint enforcing distincness. This is indeed what we find:[14]

(14) Either John ate some of the cookies or he regrets having eaten all of the cookies

(15) #Either John regrets having eaten all of the cookies or he ate some of the cookies

4 The Formal System

I assume familiarity with the context-change theory of (Heim 1983). I will revise this model along two dimensions. First, I will enrich contexts which, in (Heim 1983), are sets of worlds c (information taken for granted by speaker and hearer). In the system to be developed here, c will be partitioned by a question $Q = ?xPx$, resulting in a set of propositions $c_Q = \{c_1, \ldots, c_r\}$. I will add further structure to c_Q by ordering the cells of the partition by a measure on the extension of the question predicate P in the worlds in each cell, an idea inspired by proposals made in (van Benthem 1989; van Rooy and Schulz 2004). Second, I will modify the kinds of operations that can be performed on contexts by using variants of the "transformational" and "minimization" operators developed in (van Benthem 1989).

4.1 Enriching Contexts

Definition 1. *A question Q* **`partitions`** *c into $c_Q = \{c_1, \ldots, c_r\}$ by inducing an equivalence relation $\mathcal{R}_Q$ on c: $\forall w, w' \in c$, $w\mathcal{R}_Q w'$ iff $[[\mathrm{P}]]^w = [[\mathrm{P}]]^{w'}$.*

Example 1. *If* Who of John and Mary came to the party? *is raised in context c, then c_Q = {[j,m], [j], [m], []}, as seen earlier.*

[13] By $[X_1, \ldots, X_n]$ I simply mean that set of worlds where $X_1 \ldots X_n$ all hold.

[14] See the cautious words in Fn.12. A further difficulty is that it is not entirely clear how such partitions arise, which may point to a difficulty for question-based approaches, such as the one taken here. Nonetheless, imagining such a context is not all that difficult either, and, given such a context, the correct prediction is made. For instance, imagine we see John holding his stomach, moaning in pain and obviously distraught. I ask, *what's wrong with him?*, to which you respond with (14). Here, $[Y, p]$ is a set of worlds where he ate all of the cookies and regrets having done so, $[\neg Y, p]$ is a set of worlds where he ate all the cookies but doesn't regret having done so, and $[X, \neg p]$ is a set of worlds where he ate some but not all of the cookies.

Definition 2. *Let $c_Q = \{c_1, \ldots, c_r\}$. Then: $\forall c_i, cj \in c_Q, c_i \geq c_j$ iff for any $w_i \in c_i, w_j \in c_j$, $[[\mathrm{P}]]_i^w \cap [[\mathrm{P}]]_j^w = [[\mathrm{P}]]_j^w$.*

Example 2. *Let c_Q = {[j,m], [j], [m], []} as before. Then $\geq$ = {([j,m], [j]), ([j,m], [m]), ([j,m], []), ([j], []), ([m], [])}. We abbreviate this by writing $[j, m] \geq [j], [m] \geq []$.*

4.2 Operations on Contexts

In this section, I will define the basic operations from which we will develop compositional definitions of context change potentials (CCPs) for atomic, conjunctive, and disjunctive sentences. These CCPs are designed to exploit the enriched structure of contexts induced by the ordered partition. Recall that CCPs in this system are (partial) functions from c^* to a list $\mathcal{L} \subseteq c_Q$. They are constructed as combinations of two primitive operations. There is a transormational operator, τ, indexed by sentences of natural language. This operator, τ_ϕ, takes c^* as input and returns a set $\mathcal{D} \in \wp(c_Q)$. $\mathcal{D}$ is composed of those cells c_i of c_Q in which ϕ is true in all worlds in c_i. As such, τ_ϕ recaptures the effect of the $+\phi$ function of (Heim 1983). One can think of $\mathcal{D}$ as a set of candidates for the list $\mathcal{L}$. Given this set of candidates, a conversational minimization operator, μ, will select the minimal element (with respect to the order defined above) from this candidate set if such and such contextual conditions will be found to hold, otherwise it will select the entire candidate set. The output of minimization is then added to $\mathcal{L}$.

Definition 3. *Let k be an arbitrary set of worlds, ϕ an atomic sentence, and p ϕ's presupposition.*[15] *Then k **admits** ϕ just in case $p(w) = 1$ for all w in k.*

Example 3. *Suppose that all w in k are such that there is a King of France in w. Then k admits* The King of France is in Paris.

Now, let c^* be a structured context, ϕ an atomic sentence , and τ_ϕ the transformational operator τ indexed by ϕ. Then:

Definition 4. *Structured context $c^* \in dom(\tau_\phi)$ iff there is a c_i in c^* such that c_i admits ϕ. In such a case, we say that τ_ϕ is **well-defined** on c^*.*

Example 4. *Let $c^* = (c_Q, \geq)$, where c_Q = {[-kf], [n], [p]} as in Section 3. Then c^* admits* The King of France is in Paris.

Definition 5. *If τ_ϕ is well-defined on c^*, then $\tau_\phi(c^*) = \{c_i \in c^*$: c_i admits ϕ, and $\ulcorner\phi\urcorner$ in w for all w in $c_i\}$.*

Example 5. *Let ϕ = The King of France is in Paris, and let c^* be defined as above. Then $\tau_\phi(c^*)$ = {[p]}.*

[15] I say nothing about the source of presupposition in this paper. I am assuming that there is some function or other that assigns presuppositions to all atomic sentences of the language.

Example 6. *Let $c^* = (c_Q, \geq)$, where $c_Q = \{[j,m], [j], [m], []\}$, and consider ϕ = John came to the party. Then $\tau_\phi(c^*) = \tau_j(c^*) = \{[j, m], [j]\}$.*

With the transformational system now set up, we can define our conversational minimization operator. We first give a general definition of minimization on information states, making use of technical developments found in (van Benthem 1989).

Definition 6. *Let $K = \{k_1, \ldots, k_r\}$, where k_i is a set of worlds, and let $\geq$ be a partial order on K. Then $\mu^*(K) = \{k_i \in K$: there is no k_j in K such that $k_i \geq k_j\}$. Call μ^* a **minimization operator**.*

Definition 7. *Let K be as above. Then the speaker is **opinionated about** K if the speaker's believing the union of the propositions in K implies that she believes only one proposition in K.*

Definition 8. *Let K and μ^* be defined as above. Then our **conversational minimization operator**, μ, operates on K as follows:*

$$\mu(K) = \begin{cases} \mu^*(K) & \textit{if it is common ground that the speaker is opinionated about } K \\ K & \textit{otherwise} \end{cases}$$

Example 7. *Recall $c_Q = \{[j,m], [j], [m], []\}$, with order $[j,m] \geq [j], [m] \geq []$. Let $K = \tau_j(c^*) = \{[j,m], [j]\}$. Then, if it is common ground that the speaker is opinionated about K, then $\mu(K) = \{[j]\}$. Otherwise, $\mu(K) = K = \{[j,m], [j]\}$.*

For the remainder of this paper, we will assume that it is common ground that the speaker is opinionated at each point at which the choice comes up.

4.3 Revised Context Change Potentials

We associate with each sentence of the language ϕ a CCP π_ϕ. CCPs are partial functions from structured contexts to lists.

Definition 9. *Let ϕ be an atomic sentence uttered in structured context c^*. Then the **execution** of π_ϕ on c^* is: $\pi_\phi(c^*) := \mu(\tau_\phi(c^*))$.*

Example 8. *Imagine the question* Who of John and Mary came to the party? *is asked in context c. Then $c^* = (c_Q, \geq)$, where $c_Q = \{[j,m], [j], [m], []\}$. and $[j,m] \geq [j], [m] \geq []$. Recall from just above that $\tau_j(c^*) = \{[j,m], [j]\}$. Then $\pi_j(c*) = \mu(\tau_j(c*)) = \mu(\{[j, m], [j]\}) = \{[j]\}$. Before any conversational reasoning begins, our list $\mathcal{L}$ will be empty. Thus, in the atomic case, we simply identify $\mathcal{L}$ with the output of $\pi_\phi(c^*)$, so that, in this example, $\mathcal{L} = \{[j]\}$. Once the list has been completely specified, we can simply take the union of all the propositions in $\mathcal{L}$ to return a new unstructured context, $c' \subseteq c$, which will now be ready for a new question to come and partition it for another round of conversational reasoning. In this case, $c' = \bigcup \mathcal{L} = [j]$.*

Definition 10. *Let* $\phi = \ulcorner \alpha_1$ *and* α_2 *and* $\ldots$ *and* $\alpha_n \urcorner$.
Then $\pi_\phi(c^*) = \mu(\tau_\phi(c^*)) := \mu((\tau_{\alpha_1};\tau_{\alpha_2};\ldots;\tau_{\alpha_n})(c^*))$[16] $= \mu(\tau_{\alpha_n}(\tau_{\alpha_{n-1}}(\ldots(\tau_{\alpha_1}(c^*)))))$.

Example 9. *Suppose the answer to the question were* John and Mary came to the party*, which we abbreviate as* j and m. *Then* $\pi_{j\ and\ m}(c*) = \mu(\tau_{j\ and\ m}(c*)) = \mu((\tau_j;\tau_m)(c*)) = \mu(\tau_m(\tau_j(c*))) = \mu(\tau_m(\{[j,m],[j]\})) = \mu(\{[j,m]\}) = \{[j,m]\}$.
Minimization occurs after all the transformational operations have applied. Since it is minimization that determines which propositions go into $\mathcal{L}$*, we can, as in the atomic case, simply identify* $\mathcal{L}$ *with* $\pi_\phi(c^*)$ *when* ϕ *is a conjunctive sentence.*

Recall from our informal discussion in Section 3 that in disjunctive sentences, strengthening occurs locally at each disjunct. Thus, the decision about what goes into $\mathcal{L}$ is made at each disjunct, and the list is expanded at each disjunct. The list created by a disjunctive sentence in structured context c* will, in general, be the union of the output of the execution of the CCPs of each disjunct on c*. Thus, the recursion is on the CCPs of each disjunct. However, as opposed to the atomic and conjunctive cases, the list $\mathcal{L}$ changes as the interpretation of the disjunction proceeds. As such, the clearest way of representing the flow of information here is by writing out a high-level program incorporating both the CCP of disjunction and the constraint enforcing informational distinctness:

Definition 11. *Let* $\phi = \ulcorner \alpha_1$ *or* α_2 *or* $\ldots$ *or* $\alpha_n \urcorner$.
Then the execution of $\pi_\phi(c^*)$ *is represented in the following program:*
Initialize: $\mathcal{L} \longleftarrow \emptyset$

for i = 1, ..., n
  if $(\mathcal{L} \cap \tau_{\alpha_i}(c*)) = \emptyset$
    then $\mathcal{L} \longleftarrow (\mathcal{L} \cup \pi_{\alpha_i}(c))$
    else Output "#" and Halt
  end if
end for
Output $\mathcal{L}$

The choice point in the algorithm is at the minimization step; different selections by μ will result in different outputs. If all of μ's choices result in output "#," the sentence will be judged infelicitous.

4.4 Some Example Computations

We run through a few of the key examples discussed informally in Section 3. We focus on our running example, where the question is *Who of John and Mary came to the party?*. All the other examples will fall out in the same way – there is no practical or theoretical difference to speak of. We will proceed by making our way through the `for`-loop. We will

[16]The notation "X;Y" means first do X, then do Y. See (van Benthem 1989) for details.

call the result of the `if`-test *Step 1*. If the condition is satisfied, we will denote this with a `YES`; otherwise, we will say `NO`. Depending on the answer, we will either output "#" or add the result of applying μ to the current list $\mathcal{L}$. Call the result of this second stage of the computation *Step 2*.

Consider first the response *j or m*. Initially, $\mathcal{L} = \emptyset$. We begin with the first disjunct, j. Recall that $\tau_j(c^*) = \{[\mathrm{j,m}], [\mathrm{j}]\}$. Thus, Step 1 = `YES`. Since $\mu(\tau_j(c^*)) = \mu(\{[\mathrm{j,m}], [\mathrm{j}]\}) = \{[\mathrm{j}]\}$, Step 2 = $\mathcal{L} = \{[\mathrm{j}]\}$. Next, we get to m. $\tau_m(c^*) = \{[\mathrm{j,m}], [\mathrm{m}]\}$, which has zero intersection with $\mathcal{L} = \{[\mathrm{j}]\}$. Thus, Step 1 = `YES`. Since $\mu(\tau_m(c^*)) = \mu(\{[\mathrm{j,m}], [\mathrm{m}]\}) = \{[\mathrm{m}]\}$, Step 2 = $\mathcal{L} = \{[\mathrm{j}], [\mathrm{m}]\}$. We are at the end of the disjunction, and so we output $\mathcal{L}$. This is the correct result.

Now consider response (4), *j or (j and m)*. Begin with the first disjunct, and with an initially empty list. The result here will thus be the same as above, with Step 2 = $\mathcal{L} = \{[\mathrm{j}]\}$. At the second disjunct, *j and m*, $\tau_{j \text{ and } m}(c^*) = \{[\mathrm{j,m}]\}$. Thus, Step 1 = `YES`, and so Step 2 = $\mathcal{L} = \{[\mathrm{j}], [\mathrm{j,m}]\}$. This will be the output of the algorithm, which is the correct result.

We now consider response (7), which is (4) in reverse order. At the first disjunct, *j and m*, we have $\mathcal{L} = \{[\mathrm{j,m}]\}$. At the second disjunct, *j*, $\tau_j(c*) = \{[\mathrm{j,m}], [\mathrm{j}]\}$. Thus, Step 1 = `NO`, and so Step 2 = #. There is no other way to interpret this sentence, and so we have captured the observed infelicity.

5 Final Remarks

I would like to end with two short notes. First, the only-one reading follows immediately from the above system. The proof is by induction on the complexity of the disjunction. Second, if the approach developed here is on the right track, we have evidence that numerals should be given an *exactly* interpretation, for the following would be infelicitous if numerals came with an *at-least* interpretation:

(16) John has three sons or two. I forget which.

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