

Degrees as kinds vs. degrees as numbers: Evidence from equatives¹

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Abstract. Within formal semantics, there are two views with regard to the ontological status of degrees: the ‘**degree-as-number**’ view (e.g., Seuren, 1973; Hellan, 1981; von Stechow, 1984) and the ‘**degree-as-kind**’ view (e.g., Anderson and Morzycki, 2015). Based on (i) empirical distinctions between comparatives and equatives and (ii) Stevens’s (1946) theory on the four levels of measurements, I argue that both views are motivated and needed in accounting for measurement- and comparison-related meanings in natural language. Specifically, I argue that since the semantics of comparatives potentially involves measurable differences, **comparatives** need to be analyzed based on **scales with units**, on which degrees are like (**real**) **numbers**. In contrast, since equatives are typically used to convey the non-existence of differences, **equatives** can be based on **scales without units**, on which degrees can be considered **kinds**.

Keywords: (Gradable) adjectives, Comparatives, Equatives, Similatives, Levels of measurements, Measurements, Comparisons, Kinds, Degrees, Dimensions, Scales, Units, Differences.

1. Introduction

The notion of degrees plays a fundamental conceptual role in understanding measurements and comparisons. Within the literature of formal semantics, there are two major competing views with regard to the ontological status of degrees. For simplicity, I refer to these two ontologies as the ‘**degree-as-number**’ view and the ‘**degree-as-kind**’ view in this paper.

Under the more canonical ‘degree-as-number’ view, degrees are considered primitive objects (of type *d*). Degrees are points on abstract totally ordered scales (see Seuren, 1973; Hellan, 1981; von Stechow, 1984; Heim, 1985; Kennedy, 1999). For example, *6 feet (tall)*, *5 feet 5 inches (tall)*, . . . , are degrees, and they are elements of a totally ordered set called ‘height scale’. Thus, degrees are like (real) numbers, and the assumption of abstract totally ordered scales crucially underlies this ‘degree-as-number’ view. As the above-listed non-exhaustive references suggest, the ‘degree-as-number’ view is deeply rooted in the development of research on the semantics of comparatives.² Those interval-based analyses of comparatives (Schwarzchild and Wilkinson, 2002; Zhang and Ling, 2015, 2020) are also based on this ‘degree-as-number’ view.

The alternative view, which I dub as the ‘degree-as-kind’ view for simplicity, does not consider degrees primitive objects. Instead, degrees are rather complex objects derived from primitive

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²In cross-linguistic research, it has been found that the phenomenon of ‘degreeless comparatives’ is also widely attested (see e.g., Beck et al., 2009; Solt, 2015). The part ‘degree’ in this term ‘degreeless’ typically means ‘degree-as-number’. Thus, following this finding, we need to consider two possibilities: (i) some comparisons are better analyzed as performed between entities (of type *e*); (ii) some comparisons are performed between degrees, but presumably not number-like degrees. A thorough discussion of ‘degreeless comparatives’ is beyond this paper.

objects like entities or events. Within this view, degrees have been analyzed as **equivalence classes** (Cresswell, 1976), **tropes** (Moltmann, 2009), or **kinds** (Anderson and Morzycki, 2015; Scontras, 2017; Luo and Xie, 2018). For example, individuals of the same height are in the same equivalence class, and *6 feet (tall)* represents the set of individuals that are 6 feet tall (Cresswell, 1976). In the more subtle version proposed by Moltmann (2009), degrees are constructed out of spatio-temporal-specific instantiations of a property, say the redness of a certain box in front of me, and these instantiations are called tropes. Anderson and Morzycki (2015) draw parallels between individuals and events, using ‘kind’ as a generalized notion to cover properties of individuals (e.g., *being 6 feet tall*) and events (e.g., *crazily* (sing) – sing in a *crazy manner*). Thus, within the broad ‘degree-as-kind’ view, degrees are like properties and based on entities or events, and abstract totally ordered scales are not directly assumed.

Close to the end of their paper, Anderson and Morzycki (2015) ask whether the ‘degree-as-kind’ view can be taken as *the* ontology of degrees, or whether natural language semantics needs both ontologies for degrees:

Such a dual analysis may provide a way of coping with phenomena such as differential comparatives (*one inch taller*) and factor phrases (*three times taller*), where traditional degrees excel. But it would raise the question of why language might have these two systems existing side-by-side, different means to the essentially same end. (Anderson and Morzycki, 2015: Section 6, p. 821)

This paper argues that a dual ontology is indeed motivated. The ‘degree-as-number’ view and the ‘degree-as-kind’ view fundamentally assume different formal properties for measurements and scales, thus leading to different kinds of expressiveness. We need both views to fully account for measurement- and comparison-related meanings encoded in natural language.

More specifically, as summarized in (1), I argue that since **comparatives** typically encode comparisons resulting in differences and their semantics potentially involves measurable differences, comparatives need to be based on **scales with units**. For these scales with units, degrees are like (real) numbers. On the other hand, since **equatives** typically encode comparisons yielding no differences, their semantics can be based on **scales without units**. For these scales without units, degrees can be considered kinds.

(1) Proposal in a nutshell:

| Ontologies of degrees | Linguistic constructions | Meanings typically encoded |
|-----------------------|--------------------------|--------------------------------------|
| degrees as numbers | comparatives | comparisons resulting in differences |
| degrees as kinds | equatives | comparisons yielding no differences |

The current proposal is based on Stevens’s (1946) four levels of measurements and empirically motivated by English and Mandarin Chinese data of comparatives and equatives. Instead of discussing whether degrees are truly primitive objects, I follow Stevens (1946) and focus rather on **what formal properties degrees need to be equipped with**. At the heart of the proposed dichotomy is the issue of what kind of measurements requires what kind of scales.

In the following, Section 2 presents data of comparisons from English and Mandarin Chinese, focusing on the distinctions between comparatives and (literal and non-literal) equatives. Section 3 introduces Stevens’s (1946) theory on the four levels of measurements. Section 4 presents the proposal and explains why conceptually and empirically, both the ‘degree-as-number’ and the ‘degree-as-kind’ view are needed. Section 5 sketches a compositional analysis of (Chinese) literal and non-literal equatives. Section 6 concludes.

2. Empirical data of comparisons: comparatives vs. equatives

Comparatives typically express comparisons resulting in differences, while **equatives** typically express comparisons yielding no differences. Here I use empirical evidence from English and Mandarin Chinese to show further distinctions between them: (i) the compatibility between non-gradable adjectives and equatives is higher than that between non-gradable adjectives and comparatives; (ii) equatives include a subtype that is interpreted in a non-literal way. I will also briefly address an observation with regard to the standard of comparison in these constructions.

2.1. Comparatives vs. equatives: The compatibility with non-gradable adjectives

Within formal semantics, a **non-gradable adjective** like *red* is considered a set that only contains things that are red. Thus *red* is analyzed as a property of type $\langle et \rangle$, taking an individual x (of type e) as input and returning 1 if x is red (see (2a)). However, a **gradable adjective** like *tall* resists being characterized as a property (of type $\langle et \rangle$) that denotes a set of tall things. We can imagine that a tall man is still much shorter than a short giraffe, and the criterion of being a tall man is also context-dependent and vague. Thus, gradable adjective *tall* is instead analyzed as a relation between a degree d and an individual x , meaning that the measurement of x reaches the degree d on a relevant scale (here height, see (2b)) (see e.g., von Stechow, 1984; Beck, 2011). The positive (or **evaluative**) use of *tall* is derived based on a **context-dependent standard** for variable d , namely POS (e.g., the height threshold of tall men in a context) (see Bartsch and Vennemann, 1972; Cresswell, 1976; von Stechow, 1984; Kennedy, 1999).

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|-----|----|---|-------------------------------|
| (2) | a. | $\llbracket \text{red} \rrbracket_{\langle et \rangle} \stackrel{\text{def}}{=} \lambda x. \text{RED}(x)$ | Non-gradable adjective |
| | b. | $\llbracket \text{tall} \rrbracket_{\langle d, et \rangle} \stackrel{\text{def}}{=} \lambda d \lambda x. \text{HEIGHT}(x) \geq d$ | Gradable adjective |

It has been widely acknowledged that only gradable adjectives, but not non-gradable adjectives, are compatible with a series of constructions collectively called **degree constructions**: comparatives, superlatives, degree questions, degree modification, and *enough-/too*-constructions.³ Examples in (3) illustrate the use of gradable adjective *tall* in these degree constructions.

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|-----|----|---|----------------------------|
| (3) | a. | Brienne is probably taller than Jaime is. | Comparative |
| | b. | Brienne is the tallest of all girls. | Superlative |
| | c. | How tall is Brienne? | Degree question |
| | d. | Brienne is relatively tall , even compared with Jaime. | Degree modification |

³I use *degree constructions* to include these constructions and stay neutral on the ontology of ‘degree’ here. As will be shown later, the part ‘degree’ in ‘degree constructions’ is actually based on the ‘degree-as-number’ view.

- e. Brienne is **tall enough** to be a good knight.

Enough-construction

That being said, however, many non-gradable adjectives can be easily coerced into gradable ones (e.g., based on prototypicality) and thus used in degree constructions (see Rett, 2013; Morzycki, 2016), as illustrated by the naturally occurring examples found on Google in (4):

- (4) a. What could be **more non-atomic** than a policy involving a sequence of actions
...⁴ **Comparative**
b. The smile on your mouth was **the deadest** thing.⁵ **Superlative**
c. 75,000 - 100,000 cells are imaged per patch and images analyzed to determine the hexagonality score (**how hexagonal** is the cell) of each RPE cell.⁶
Degree question
d. “It does look **quite triangular**,” he said ...⁷ **Degree modification**
e. Modern cryptographic systems generate rather large prime numbers. After generating numbers of a specific length, they run through either the Fermat Primality Test or the Miller-Rabin Primality Test; that way, one knows their numbers are **prime enough** to be baffling.⁸ **Enough-construction**

These naturally occurring examples show that the use of typical non-gradable adjectives – even adjectives like *hexagonal* and *prime* – in degree constructions is attested. Thus the compatibility between a construction and an adjective is not a categorical issue, but rather a matter of degree.

Based on this understanding, I used distributional data from the *Corpus of Contemporary American English* (CoCA, Davies, 2008) to empirically test the generalization in (5). Specifically, I tested two hypotheses. First, as a canonical degree construction, comparatives are more compatible with gradable adjectives than with non-gradable ones (see the center column in Table (5)). Second, for non-gradable adjectives, they are better compatible with equatives than with comparatives (see the bottom row in Table (5)).

- (5) Generalization with regard to compatibility between an adjective and a construction:

| | Comparatives ADJ.+ <i>-er than</i> ; <i>more ADJ. than</i> | Equatives <i>as ADJ. as</i> |
|--------------------------------|---|---|
| Gradable adjectives | Compatibility: ++ (e.g., <i>taller than</i> , <i>more intelligent than</i>) | (e.g., <i>as tall as</i> , <i>as intelligent as</i>) |
| Non-gradable adjectives | Compatibility: + (e.g., <i>redder than</i> , <i>deader than</i>) | Compatibility: ++ (e.g., <i>as red as</i> , <i>as dead as</i>) |

⁴https://ftp.cs.ucla.edu/pub/stat_ser/r342.pdf

⁵*Neutral Tones* by Thomas Hardy: <https://www.poetryfoundation.org/poems/50364/neutral-tones>

⁶https://stm.sciencemag.org/content/scitransmed/suppl/2019/01/14/11.475.eaat5580.DC1/aat5580_SM.pdf

⁷<https://apnews.com/d5df36e52a49287657e174a06a5e862d>

⁸<https://webcache.googleusercontent.com/search?q=cache:0PfWq5rVrswJ:https://fly.io/articles/how-rsa-works-tls-foundations/+&cd=2&hl=en&ct=clnk&gl=hk>

I selected 16 typical gradable adjectives (*bad, big, broad, cheap, good, great, high, large, long, low, old, short, small, tall, wide, and young*) and 16 typical non-gradable ones (*American, anonymous, black, blue, brown, dead, French, gray, green, orange, pink, purple, red, round, white, and yellow*).⁹ Then I obtained their raw frequencies (by searching ‘[j*]’) as well as the raw counts of their occurrence in comparatives (by searching ‘*er than’ and ‘more * than’) and equatives (by searching ‘as * as’) from CoCA.¹⁰ For simplicity, the attributive use of comparative forms (e.g., *a better idea*) was not included, which would presumably lead to a lower estimation of the distribution of comparatives (particularly for those containing gradable adjectives), making the testing of the first hypothesis too conservative. However, intuitively, this practice would not affect the investigation of the overall compatibility pattern.

For each adjective, I summed up the counts of its occurrences in ‘*er than’ and ‘more * than’ and divided the sum by its raw frequency, yielding the value of $p(\text{com})$. For each adjective, I also divided the count of its occurrence in ‘as * as’ by the raw frequency of the adjective, yielding the value of $p(\text{eq})$ (see the Appendix). Thus for each adjective, the values $p(\text{com})$ and $p(\text{eq})$ provide a rough estimate of the proportion of a word’s occurrence in comparatives and equatives, without the contamination of raw frequency effects. All the data of raw counts and the values of $p(\text{com})$ and $p(\text{eq})$ are available in the Appendix. I conducted (I) two **two-tailed t -tests**: (i) $p_{\text{grad}}(\text{com})$ vs. $p_{\text{non-grad}}(\text{com})$, (ii) $p_{\text{grad}}(\text{eq})$ vs. $p_{\text{non-grad}}(\text{eq})$, and (II) two **two-tailed, paired sample t -tests**: (iii) $p_{\text{non-grad}}(\text{com})$ vs. $p_{\text{non-grad}}(\text{eq})$; and (iv) $p_{\text{grad}}(\text{com})$ with $p_{\text{grad}}(\text{eq})$.

The first two t -tests yielded highly significant differences (both $p < 0.0001$), showing that compared to non-gradable adjectives, gradable adjectives are overwhelmingly more likely – about 41 and 7 times more likely on average – to be used in either comparatives or equatives.

The crucial third t -test also yielded a significant difference ($p = 0.013$), showing that for non-gradable adjectives, they are more likely to be used in equatives ($p_{\text{non-grad}}(\text{eq}) = 0.00142$) than in comparatives ($p_{\text{non-grad}}(\text{com}) = 0.00099$). The fourth t -test also yielded a highly significant difference ($p < 0.0001$), showing that gradable adjectives demonstrate an opposite pattern: they are much more likely – about 4 times more likely on average – to be used in comparatives than in equatives. Thus, together, the results from these two tests suggest that non-gradable adjectives differ from gradable ones in that their compatibility with equatives is much higher (compared to their compatibility with comparatives).

Overall, these results provided preliminary but reliable evidence for the generalization pattern shown in (5). Thus, even though the use of non-gradable adjectives in degree constructions is attested, corpus data suggest that (i) non-gradable adjectives are less compatible with degree constructions than gradable ones are, but (ii) for non-gradable adjectives, their compatibility with equatives is higher than that with comparatives. To sum up, equatives contrast with comparatives with regard to their compatibility with non-gradable adjectives.

⁹It is worth noting that in English, among the adjectives of highest frequencies, most are gradable ones, and overall, there are far more gradable adjectives than non-gradable ones. Thus I had to include some non-gradable adjectives that are not as frequent as the rest of the words. For a more thorough investigation, we can consider using clustering techniques on all adjectives in a language. This is left for future research.

¹⁰See the search syntax of CoCA for more details: <https://www.english-corpora.org/coca/>.

2.2. Two subtypes of equatives

Distinct from comparatives, equatives include two subtypes: (i) **literal equatives** that are interpreted **literally**, and (ii) **non-literal equatives** that are interpreted **figuratively or metaphorically**. Intuitively, (6a) is interpreted **literally**, parallel to comparatives in that there is a literal comparison between two values (here Brienne's and Jaime's heights) along a single-dimensional scale (here height). Similarly, for (7a), the non-gradable adjective *red* is coercively interpreted as a gradable one, and two lipsticks' hues are compared along a single-dimensional scale of redness saturation. (6a) and (7a) are **not necessarily positive or evaluative**: i.e., (6a) does not entail that Brienne and Jaime are tall, and (7a) can be used to describe two nude lipsticks.¹¹

In contrast, (6b) and (7b) are **non-literal equatives**, and they are **necessarily evaluative**. (6b) does entail that the standard of comparison – mountains – and the sentence subject – Brienne – are both tall, and (7b) entails that both tomatoes and Sam's face are red. (6b) can be uttered felicitously in a context where Brienne measures 6 feet 6 inches tall, while the height of mountains is in general above 2000 feet. (6b) does not mean that the heights of Brienne and mountains are literally equal, but rather that their tallness gives a similar impression – in a similar **manner** (i.e., **qualitatively similar**) and to a similarly impressive **extent** (i.e., **quantitatively similar**). Similarly, (7b) does not mean that the redness saturation values of Sam's face and tomatoes are literally equal, but rather that their redness gives a similar impression, suggesting further that Sam's face was really red, perhaps due to fury or embarrassment.¹²

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|-----|----|---|-----------------------------|
| (6) | a. | Brienne is as tall as Jaime. | Literal equative |
| | | \leadsto Brienne's height \geq Jaime's height | (not evaluative) |
| | b. | Brienne is as tall as a mountain. | Non-literal equative |
| | | \leadsto Both mountains and Brienne are tall, in a similar manner and to a similar extent. | |
| | | | |
| (7) | a. | This lipstick is as red as that one. | Literal equative |
| | | \leadsto the redness saturation of this lipstick \geq the redness saturation of that lipstick | |
| | b. | Sam's face was as red as a ripe tomato. | Non-literal equative |
| | | \leadsto Both tomatoes and Sam's face are red, in a similar way and to a similar extent. | |

In this sense, non-literal equatives are distinct from genuine, single-dimension-based, degree constructions (e.g., comparatives, literal equatives). Non-literal equatives encode comparisons along some complex dimension, addressing similar extents and similar manners.

¹¹There is a distinction between *tall* and its antonym *short*. (6a) is not evaluative, but *Brienne is as short as Jaime* is evaluative – it entails that both Brienne and Jaime are short. See Rett (2007) for more discussion.

¹²The non-literal equatives (6b) and (7b) are reminiscent of **generic equatives**, a subtype of **similatives**. Structurally, equatives differ from similatives (e.g., (i)) in that equatives contain a **parameter marker** (i.e., the first *as* in *as tall/red as*), while similatives lack one. Semantically, similatives are evaluative. However, compared to literal and non-literal equatives, which always encode a quantitative similarity (e.g., being tall or red to the same **extent**), similatives seem to encode only a qualitative similarity. See Rett (2013) for more discussions on similatives.

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|-----|----|--|---|
| (i) | a. | Brienne is tall as a mountain. | Similative (or generic equative) |
| | | \leadsto Both mountains and Brienne are tall, in a similar manner, though probably not to the same extent. | |
| | b. | Sam's face was red as a ripe tomato. | Similative (or generic equative) |
| | | \leadsto Both tomatoes and Sam's face are red, in a similar way, though probably not to the same extent. | |

In Mandarin Chinese, the semantic distinction between literal vs. non-literal equatives is morphologically manifested. As illustrated by (8) and (9), these two Chinese equative constructions differ with regard to the standard marker (SM). The SM of the *gēn*-construction (8) is *gēn*, a morpheme meaning ‘along with’, while the SM of the *xiàng*-construction (9) is *xiàng*, a morpheme meaning ‘like, similar (to)’. Semantically, the interpretation of the *gēn*-construction (8) patterns with literal equatives, meaning that the redness saturation values of roses and blood are the same. In contrast, the interpretation of the *xiàng*-construction (9) is parallel to that of non-literal equatives, meaning that the redness of roses and blood gives a similar impression.

- (8) méi-guī **gēn** xuě yī-yàng hóng
 rose along-with blood same red/redder
 ‘Roses are as red as blood.’ **Chinese literal equative: *gēn*-construction**
 ∼ Roses and blood have the same value of redness saturation.

- (9) méi-guī **xiàng** xuě yī-yàng hóng
 rose similar/like blood same red/redder
 ‘Roses are as red as blood.’ **Chinese non-literal equative: *xiàng*-construction**
 ∼ Roses and blood share the same saturation and feeling (e.g., brightness) of redness.

In terms of evaluativity, *gēn*-constructions are not evaluative, while *xiàng*-constructions are evaluative, as illustrated by the contrast between (10) and (11). In (11), given that Hobbits are short – contrary to the evaluative meaning encoded in the sentence, this equative is infelicitous.

- (10) tā **gēn** hā-bǐ-rén yī-yàng gāo
 3SG. along-with Hobbits same tall/taller
 ‘He is as tall as Hobbits.’ ∼ Chinese literal equatives: not evaluative

- (11) # tā **xiàng** hā-bǐ-rén yī-yàng gāo
 3SG. similar/like Hobbits same tall/taller
 Intended: ‘He is as tall as Hobbits.’ ∼ Chinese non-literal equatives: evaluative

(12) sums up the empirical generalization here. Crucially, non-literal equatives differ with typical degree constructions like comparatives and literal equatives in a few ways.

- (12) Generalization with regard to the interpretation of comparison constructions:

| | Comparison constructions | | |
|--------------|---|---|--|
| | Degree constructions | | |
| | | Equatives (e.g., <i>as tall as</i> , <i>as red as</i>) | |
| | Comparatives | Literal equatives <i>gēn</i> -constructions | Non-literal equatives <i>xiàng</i> -constructions |
| Meaning | different extent | same extent | same manner & extent |
| Examples | <i>taller than</i> , <i>redder than</i> | (6a), (7a), (8) | (6b), (7b), (9) |
| Evaluativity | not evaluative | | evaluative |
| Adjectives | gradable or coerced non-gradable adjectives | | (non)-gradable |
| Dimension | single | | complex |

2.3. What can serve as the standard of comparison?

In some language, e.g., French and Mandarin Chinese, there is also a contrast between comparatives and equatives with regard to items acceptable as comparison standard. Due to space limit, I only present French data, but exactly the same pattern also exists in Mandarin Chinese.

In these languages, for comparatives (see (13)), both individuals (e.g., *Pierre*) and degree values (e.g., *1.8 meters*) can serve as comparison standard. However, for equatives (see (14) and (15)), degree values like *1.8 meters* or *dark red* cannot play the role of comparison standard.

For the examples with non-gradable adjectives in (15), this contrast between *blood* and *dark red* also suggests that they are different kinds of semantic objects. Presumably, *blood* patterns with expressions like *Pierre* and needs to be analyzed as individuals of type *e*, while *dark red* and measure phrase *1.8 meters* both belong to the notion of degrees in its broad sense.¹³

- (13) a. Marie est **plus grande que** Pierre.
 Mary be.3SG. more tall than Peter
 ‘Mary is taller than Peter.’ **French comparative**
- b. Marie est **plus grande qu’** un mètre 80.
 Mary be.3SG. more tall than 1.8 meters
 ‘Mary is taller than 1.8 meters.’ **French comparative**
- (14) a. Marie est **aussi grande que** Pierre.
 Mary be.3SG. also tall than Peter
 ‘Mary is as tall as Peter.’ **French equative**
- b. ?? Marie est **aussi grande qu’** un mètre 80.
 Mary be.3SG. also tall than 1.8 meters
 Intended meaning: ‘Mary is as tall as 1.8 meters.’ **French equative**
- (15) a. Ce rouge à lèvres est **aussi rouge que** le sang
 this lipstick be.3SG. also red than DET. blood
 ‘This lipstick is as red as blood’. **French equative**
- b. ?? Ce rouge à lèvres est **aussi rouge que** du rouge foncé.
 this lipstick be.3SG. also red than DET. red dark
 Intended meaning: ‘This lipstick is as red as dark red.’ **French equative**

Thus the empirical observations in this section raise three questions. (i) Why are equatives better compatible with non-gradable adjectives than comparatives are? (ii) Why and how are some equatives interpreted in a non-literal way? (iii) Why cannot degree values serve as comparison standard in equatives? Below I will mainly focus on the first two issues.

¹³Rett (2015) also points out that in English, equatives formed with a measure phrase (e.g., (ib)) are interpreted distinctly from those formed with a clause as the standard of comparison (e.g., (ia)):

- (i) a. John can dive as deep as Sue can. \leadsto the depth that John can dive \geq the depth that Sue can dive
 b. John can dive as deep as 500 m. \leadsto the maximal depth that John can dive is 500 m.

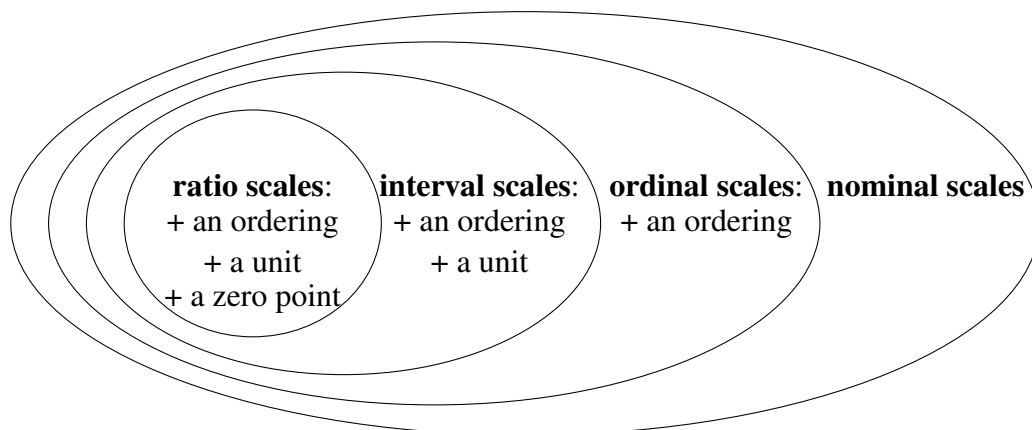


Figure 1: [Stevens's \(1946\)](#) theory on the four levels of scales, their entailment relationships (represented by the Venn diagram), and their defining attributes.

3. [Stevens \(1946\)](#): the four levels of measurements

Before answering the questions raised by the empirical findings and analyzing the ontology of degrees, here I introduce [Stevens's \(1946\)](#) theory on the four levels of measurements.

[Stevens \(1946\)](#) crucially points out that the notion of **measurement**, in its broad sense, involves different kinds of mappings between **items under measurement** and **values** assigned to them:

... we may say that measurement, in the broadest sense, is defined as the assignment of numerals to objects or events according to rules. The fact that numerals can be assigned under different rules leads to different kinds of scales and different kinds of measurement. The problem then becomes that of making explicit (a) the various rules for the assignment of numerals, (b) the mathematical properties (or group structure) of the resulting scales, and (c) the statistical operations applicable to measurements made with each type of scale. ([Stevens, 1946](#))

Thus, obviously, a measurement can be considered a mapping function. For a given mapping function, all the values that can potentially be assigned to items under measurement are **degrees**, in the broadest sense of this notion, and moreover, the range of degree values constitutes the **scale** for this measurement. Then based on the rules of measurement (i.e., types of mapping functions), and relatedly, the formal properties of scales and applicable operations on the degree values resulted from measurement, [Stevens \(1946\)](#) makes a four-level distinction of scales: **nominal scales**, **ordinal scales**, **interval scales**, and **ratio scales**. As shown by the Venn diagram in Fig. 1, nominal scales are the most general one of these four levels; ordinal scales are a subset of nominal scales; interval scales are a subset of ordinal scales; and ratio scales are a subset of interval scales. Here are some examples illustrating their formal properties.

Nominal scales result from measurements that do not even involve a meaningful ordering, e.g., assigning a postal code to each address. It is in general not meaningful to address the order between two different postal codes – all that matters is whether they are the same or different.

In contrast to nominal scales, ordinal scales have an **ordering**. For example, given the ranking of my favorite ice cream flavors, it is meaningful to address whether chocolate ranks higher than vanilla. However, beyond this ordering, the scale of ranking cannot address **to what extent** the 1st ranked flavor exceeds the 2nd ranked flavor or **whether the difference** between the 1st and the 2nd ranked flavor **is the same as** that between the 17th and 18th ranked flavor.

Interval scales have not only an ordering, but also **units**. For example, given a scale of time, due to the notion of **units**, it is meaningful to address not only that 2 o'clock is earlier than 3 o'clock, but also that the difference between 3 o'clock and 2 o'clock is the same as that between 5 o'clock and 4 o'clock. It is worth noting that *o'clock* and *hour* are conceptually different: *o'clock* is used to mark **positions** on a scale of time, while *hour* is used as a **unit to measure differences between positions** on the scale of time. Crucially, it is the notion of units that supports the further **measurement and comparison of differences** (e.g., 1 hour) between two positions (e.g., 5:30 and 5:00) along an interval scale (here time), so that we can know **to what extent** one measurement (here 5:30) exceeds the other (here 5:00) on this interval scale.

Finally, ratio scales are special interval scales that are further equipped with a meaningful, absolute **zero point**. For example, the temporal length has a meaningful zero point and is thus a ratio scale (cf. a scale of time, which lacks a meaningful zero point and is thus a non-ratio interval scale). On a scale of temporal length, not only we can compare the scalar values *4 hours* and *3 hours* and address to what extent *4 hours* exceeds *3 hours* – the difference here is 1 hour, but also it is meaningful to consider the ratio between *4 hours* and *3 hours* – the ratio here is 1.75, i.e., *4 hours* is 1.75 times as long as *3 hours*. Obviously, on a scale of time, where there is no meaningful zero point, the ratio between *4 o'clock* and *3 o'clock* is meaningless.

Previously, based on [Stevens's \(1946\)](#) theory on the four levels of scales, [Sassoon \(2010\)](#) explains which gradable adjectives can be used along with measure phrases in forming measurement constructions. As illustrated by the contrast in (16), *long* is compatible with the use of *2 meters*, forming a grammatical measurement construction, while *short* is incompatible with the use of a measure phrase, leading to an unacceptable sentence. The analysis of [Sassoon \(2010\)](#) is based on [Stevens's \(1946\)](#) distinction between interval scale and ratio scales. Essentially, for measurement constructions, their measure phrase refers to the difference between a measurement (e.g., the length of this rope) and the absolute zero point on the scale. Thus only gradable adjectives (e.g., *long*, *tall*) associated with ratio scales (i.e., interval scales equipped with a meaningful, absolute zero point) can be used to form measurement constructions.

- | | | |
|------|--|-----------------------|
| (16) | a. This rope is 2 meters long . | Spatial length |
| | b. *This rope is 2 meters short . | Shortness |

In this sense, [Stevens's \(1946\)](#) theory on the four levels of scales and their formal properties provides crucial insight on our intuitive knowledge (of mathematics and physics) and how this kind of intuitive knowledge is reflected in our use of natural language.

Following the work of [Stevens \(1946\)](#) and [Sassoon \(2010\)](#), in the following, I also base my proposal for the ontology of degrees on the levels of scales and the formal properties of scales.

4. Proposal: a dual ontology of degrees

As shown in (17), I propose a **dual ontology of degrees** based on the distinction between **scales with vs. without units**. These two types of scales have different formal properties, leading to a dual ontology of degrees: those belonging to **scales with units** and thus behaving like **(real) numbers**, and those belonging to **scales without units** and thus behaving like **kinds**.

As a consequence, for **degrees-as-numbers**, their scale – the set of degrees – is a totally ordered set with units, supporting the operation of subtraction on degrees along a single dimension (e.g., time, height). In natural language, gradable adjectives (and non-gradable adjectives coercively interpreted as gradable ones) are associated with this kind of ‘scales with units’. The formal properties of these scales and degrees are necessary in the semantic analysis of comparatives and measurement constructions. I will discuss in detail below.

For **degrees-as-kinds**, their scale is not equipped with units and cannot support the operation of subtraction on degrees. Thus on these ‘scales without units’, differences between degrees are not measurable. Some of these scales might involve a complex dimension, e.g., a combination of hue, saturation, and texture. Both gradable and non-gradable adjectives, when used in natural language constructions involving **no measurable differences** (e.g., equatives, superlatives (see Solt, 2016)), can be considered associated with ‘scales without units’, and formal properties specially belonging to ‘scales with units’ (for gradable adjectives) are not made use of.

(17) A dual ontology of degrees:

| Scales | scales with units (i.e., interval or ratio scales) | scales without units (i.e., nominal or ordinal scales) |
|------------------------------------|---|---|
| Degrees | like real numbers (e.g., <i>1 kg</i> , <i>3 o'clock</i>) | like kinds or manners (e.g., <i>dark red</i> , <i>boyishly tall</i>) |
| Dimension | single | single (ordinal scales), complex (nominal scales) |
| Differences along the dimension | measurable and comparable; the subtraction between degrees (e.g., <i>1 m</i> and <i>2 m</i>) is applicable | not measurable; subtraction is inapplicable |
| Adjectives | gradable or coercively gradable | not necessarily gradable |
| Constructions | comparatives; degree constructions with ratios or measure phrases | equatives (including literal & non-literal equatives); superlatives |

Comparatives containing numeral differentials (e.g., *2 inches* in (18a), *1 hour* in (18b), *3 cm* in (18c)), provide the most direct empirical motivation for the ‘degree-as-number’ view.

- (18)
- My giraffe is **2 inches** taller than this tree is.
 - The arrival of the train was **1 hour** later than scheduled.
 - Zhāng-Sān bǐ
Lǚ-Sì gāo
sān lí-mǐ.

Zhāng-Sān STANDARD-MARKER (SM)
Lǚ-Sì tall/taller three centimeter

‘Zhāng-Sān is 3 cm taller than Lǚ-Sì’.
Chinese comparative

In particular, as mentioned earlier, in (18b), *o'clock* is used in marking positions on a scale of time, while *hour* is used as a unit for measuring differences between such positions. Evidently, equivalence classes like {3 o'clock, ...} and {2:00, ...} can only establish ordering relations and constitute ordinal scales. Without the concept of units for measuring differences (e.g., *hour*), we can by no means address to what extent {3:00, ...} is posterior to {2:00, ...}. In this sense, languages able to express numeral differentials in comparatives all need the ‘degree-as-number’ view to fully account for measurement- and comparison-related phenomena.¹⁴

In addition to comparatives containing numeral differentials, **the comparison of deviations** also lends empirical support for the ‘degree-as-number’ view. As illustrated by the English and Chinese examples (19) and (20), the semantics of these sentences involves three comparisons. For (19), the first two comparisons result in evaluative meanings (i.e., Mona is happy, and Jude is sad), due to the existence of difference between a measurement (e.g., Mona’s happiness) and the context-dependent POS value. Then the third comparison is between these two differences resulted from the first two comparisons. Evidently, ordinal scales cannot support the third comparison here – i.e., the further measurement and comparison of the differences between measurements. The notion of interval scales, and together with it, the ‘degree-as-number’ view and the operation of subtraction, are indispensable in the semantic analysis of (19) and (20).

- (19) Mona is more happy than Jude is sad. (Kennedy, 1999)
 $\leadsto [\text{HAPPINESS}(\text{Mona}) - \text{POS}_{\text{HAPPINESS}}] > [\text{SADNESS}(\text{Jude}) - \text{POS}_{\text{SADNESS}}]$
Comparison 1: Mona’s happiness vs. $\text{POS}_{\text{HAPPINESS}}$ (along the scale of happiness)
Comparison 2: Jude’s sadness vs. $\text{POS}_{\text{SADNESS}}$ (along the scale of sadness)
Comparison 3: difference from Comparison 1 vs. difference from Comparison 2
 (along the scale of deviation size)

- (20) tā duō chī de bǐ wǒ shǎo chī de duō yī pán
 3SG. much/more eat PART. SM 1SG. little/less eat PART. much/more one dish (CL.)
 ‘The amount that he over-ate was 1 dish more than the amount that I under-ate.’

Finally, naturally occurring examples from COCA (see (21)) argue against the view that ordinal scales would be sufficient for analyzing **comparatives containing non-gradable adjectives**. Although measurements of redness cannot be naturally associated with number-like values, and numeral differentials can hardly be used along with *redder*, the use of degree modifiers *much* and *a bit* indicates that the notion of measurable differences is still indispensable. Thus, even for non-gradable adjectives, comparatives have to be based on ‘scales with units’, and non-gradable adjectives are necessarily coerced into gradable ones in comparatives.

- (21) a. Indeed, such ‘space weathering’ makes the lunar surface **much redder** than the color of pristine Moon rocks.
 b. ... change exposures and printing filters to make an image **a bit redder** ...

¹⁴Even for languages in which ‘degreeless comparatives’ are attested, e.g., Mandarin Chinese (see Li, 2015; Luo and Xie, 2018), as far as they also have comparative constructions involving measurable differences, the ‘degree-as-number’ view is still empirically motivated in these languages (see the Chinese examples (18c) and (20)).

In contrast to comparatives, **equatives** typically express comparisons yielding no differences. Therefore, to analyze equatives, we do not need to consider the further measurement or comparison of differences, because there are none. Consequently, the semantics of equatives does not require ‘scales with units’ or the ‘degree-as-number’ view. All scales, including **nominal scales and ordinal scales**, can be involved in the semantics of equatives.

This reasoning immediately answers the first question raised in Section 2. Since equatives do not require ‘scales with units’, gradable and non-gradable adjectives are both eligible. Thus, equatives are better compatible with non-gradable adjectives than comparatives are.

Among equatives, the distinction between *gēn*-constructions and *xiàng*-constructions in Mandarin Chinese means that in a certain language, there can still be additional requirements for the scales involved in some specific types of equatives. The interpretation of *gēn*-constructions in Chinese (see the discussion in Section 2.2) suggests that they are similar to comparatives in involving **single-dimensional scales with ordering**, i.e., **ordinal scales (w/o units)**. As a consequence, non-gradable adjectives used in *gēn*-constructions are coerced to be associated with ordinal scales, and similar to comparatives, *gēn*-constructions are not evaluative.¹⁵

Unlike comparatives or literal equatives (including *gēn*-constructions), non-literal equatives (including *xiàng*-constructions) are interpreted in an evaluative and non-literal way. These equatives are compatible with both gradable (see (22a)) and non-gradable adjectives (see (22b)). However, here even gradable adjectives like *tall* need to be interpreted as associated with a **nominal scale**, on which degrees are not number-like values, but rather kinds. For (22a), both Brienne and mountains are tall, in the same way and to the same impressive extent. In terms of Stevens's (1946) theory, the measurement value of Brienne's height is *mountainous tallness*, a degree(-as-kind) defined by the feeling of tallness in our conceptual knowledge of mountains.

- (22) a. Brienne is **as tall as** a mountain. (= (6b))
 b. Sam's face was **as red as** a ripe tomato. (= (7b))
 c. #A mountain is **as tall as** Brienne.

This analysis answers the second question raised in Section 2. Degrees on nominal scales are not necessarily as single-dimensional as numbers. The non-literal, figurative reading is due to invoking a complex dimension. In interpreting (22a), it is our conceptual knowledge of mountains that leads to the specific definition of a complex-dimensional degree value, which probably combines height, strength, and firmness. In this sense, degrees-as-kinds are distinct from equivalence classes defined as a set of items sharing the same value along a certain scale (see e.g., Cresswell, 1976). The ‘degree-as-equivalence-class’ view predicts that the semantics of equatives is symmetric, but this prediction is clearly not borne out: (22a) is felicitous, but intuitively, (22c) sounds weird. The sentences in (23) are also interpreted differently, because our conceptual knowledge of *blood* and *roses* leads to different **kinds** of redness.¹⁶

¹⁵Solt (2016) accounts for the distinction between *most* and *more than half* in a similar measurement-based way: the **superlative** form is based on ordinal scales w/o units, while the **comparative** form involves interval scales.

¹⁶See Percus and Sharvit (2014) and Zhang (2016) for more discussions on the semantic asymmetry and symmetry in related constructions, e.g., the use of *same* and sentences like *The morning star is the evening star*.

- (23) a. Roses are as red as blood. Degree(-as-kind) value: *bloody red*
 \leadsto Roses give an impression of dazzling, horrible redness as blood typically does.
 b. Blood is as red as roses. Degree(-as-kind) value: *rosy red*
 \leadsto Blood gives an impression of energetic, bright redness as roses typically do.

To sum up, I have argued that both the ‘degree-as-number’ and the ‘degree-as-kind’ view are conceptually necessary and empirically motivated. Based on [Stevens’s \(1946\)](#) theory of the four levels of measurements, this dual ontology of degrees explains both the core of the notion of degrees – they are values of a certain scale for measurement (in its broad sense) – and the various kinds of expressive potentials related to the uses of adjectives in natural language.

5. Analyzing equatives: Chinese *gēn*-constructions and *xiàng*-constructions

Based on the dual ontology of degrees, I sketch a formal compositional analysis of literal and non-literal equatives (or more generally, single- vs. complex-dimensional comparisons). Since Chinese makes a morphological distinction (i) between these two subtypes of equatives and (ii) between the parameter marker (PM – the first *as* in *as tall as X*) and standard marker (SM – the second *as* in *as tall as X*), the analysis is based on Chinese lexical items for clarity.

The basic idea is that an adjective provides a (single- or complex-dimensional) scale for measurement and can be analyzed as a **measure function**, mapping an item to its measurement (see also [Kennedy 1999](#)). The use of an adjective in different linguistic constructions makes it associated with different scales: domains like D_d , i.e., a **domain of number-like values** on which subtraction is applicable (see (24a)); or D_k , i.e., a **domain of kind-like items** (see (24b)).

- (24) $\llbracket \text{tall} / \text{gāo} \rrbracket_{\langle e, \delta \rangle} \stackrel{\text{def}}{=} \lambda x_e. \text{HEIGHT}(x)$ ($\delta \in \{d, k\}$)
 (Context: Brienne is 6’6” tall. Her tallness is reminiscent of strength and firmness.)
 a. On an interval scale: $\text{HEIGHT-NUMBER}_{\langle ed \rangle}(\text{Brienne}) = 6'6''$
 b. On a nominal scale: $\text{HEIGHT-TT-KIND}_{\langle ek \rangle}(\text{B}) = \text{BRIENNE’S KIND OF TALLNESS}$

In **positive, measurement, and comparative constructions**, *tall* is associated with an **interval scale** (see (25)). These constructions mean that the measurement of the sentence subject (i.e., the target of predication) reaches a certain degree along an interval scale. Thus I use a type-shifter ‘ $\lambda G_{\langle ed \rangle}. \lambda M_d. \lambda x_e. G(x) \geq M$ ’ to generate sentential semantics. In particular, the derivation of comparatives (see (25c)) is directly based on subtraction, an operation requiring interval scales (see [Zhang and Ling, 2020](#) for details on the implementation of subtraction).

- (25) a. $\llbracket \text{Brienne is tall} \rrbracket \Leftrightarrow [\lambda G. \lambda M. \lambda x. G(x) \geq M](\llbracket \text{tall} \rrbracket)(M_{\text{POS}}^c)(\llbracket \text{Brienne} \rrbracket)$
 $\Leftrightarrow \text{HT-NUMBER}(\text{Brienne}) \geq M_{\text{POS}}^c$ (M_{POS}^c : a contextual threshold for being tall)
 b. $\llbracket \text{Brienne is 6 feet 6 inches tall} \rrbracket \Leftrightarrow [\lambda G. \lambda M. \lambda x. G(x) \geq M](\llbracket \text{tall} \rrbracket)(6'6'')(\llbracket \text{Brienne} \rrbracket)$
 $\Leftrightarrow \text{HEIGHT-NUMBER}(\text{Brienne}) \geq 6'6''$
 c. $\llbracket \text{Brienne is 2 inches taller than Jaime is} \rrbracket$ ($\llbracket \text{-er} \rrbracket$: an unspecified positive value)
 $\Leftrightarrow [\lambda G. \lambda M. \lambda x. G(x) \geq M](\llbracket \text{tall} \rrbracket)(\llbracket 2 \text{ inches -er} \dots \text{ than Jaime is tall} \rrbracket)(\llbracket \text{Brienne} \rrbracket)$
 $\Leftrightarrow \text{HEIGHT-NUMBER}(\text{Brienne}) \geq \llbracket 2 \text{ inches -er} \dots \text{ than Jaime is tall} \rrbracket$
 $\Leftrightarrow \text{HEIGHT-NUMBER}(\text{Brienne}) \geq \iota M[M - \text{HEIGHT-NUMBER}(\text{Jaime}) = 2'']$

In expressions like *that tall* (see Anderson and Morzycki 2015) and **literal and non-literal equatives**, adjectives are associated with a **nominal scale** that is potentially complex-dimensional (an ordinal scale is considered a special case – a single-dimensional scale with an ordering (see Fig. 1 and the table in (17))). On a nominal scale, $\llbracket \text{tall} \rrbracket$, as a measure function, maps an entity to a degree-as-kind – a certain kind of tallness. Suppose Brienne’s tallness involves strength and firmness, and here *that* – a free variable of kind – denotes firmness. Then (26) is intuitively true. Thus I propose to use a type-shifter ‘ $\lambda G.\lambda M.\lambda x.G(x) \sqsubseteq_{\text{info}} M$ ’ to generate sentential semantics. I do not go into formal details of ‘ $\sqsubseteq_{\text{info}}$ ’ in the current paper. Informally, ‘ $\sqsubseteq_{\text{info}}$ ’ means that a kind (e.g., Brienne’s kind of tallness) entails (i.e., is more informative than) another.

$$(26) \quad \llbracket \text{Brienne is that}_j \text{ tall} \rrbracket \Leftrightarrow [\lambda G.\lambda M.\lambda x.G(x) \sqsubseteq_{\text{info}} M](\llbracket \text{tall} \rrbracket)(s_j)(\llbracket \text{Brienne} \rrbracket) \\ \Leftrightarrow \text{HGHT-KIND}(\text{Brienne}) \sqsubseteq_{\text{info}} s_j \quad (s_j: \text{a free variable meaning some kind of tallness})$$

The critical lexical items in equatives are (i) the **PM** *yī-yàng* and (ii) the **SMs** *xiàng* and *gēn*. Essentially, *gēn/xiàng* generates a definite degree-as-kind that serves as comparison standard in equatives. In this sense, the *gēn/xiàng*-phrase/clause addresses a *how* question, abstracting an entity or event into a degree-as-kind. The distinction between *gēn* and *xiàng* hinges on whether this degree-as-kind is along a single- or complex-dimensional scale (see (27)). Thus the semantics of a generic equative can be naturally derived: (28) expresses qualitative similarity.¹⁷

$$(27) \quad \llbracket \text{gēn/xiàng (SM)} \rrbracket_{\langle kt, k \rangle} \stackrel{\text{def}}{=} \lambda D_{\langle kt \rangle}.\iota d[D(d) \wedge \forall d'[D(d') \wedge d \neq d' \rightarrow d \sqsubseteq_{\text{info}} d']] \\ \text{a. } \llbracket \text{gēn} \rrbracket_{\langle kt, k \rangle}(\lambda d.\text{Jaime is } d\text{-tall}) = \text{Jaime's kind of tallness} \quad \leadsto \text{tallness} \\ \text{b. } \llbracket \text{xiàng} \rrbracket_{\langle kt, k \rangle}(\lambda d.\text{Jaime is } d\text{-tall}) = \text{Jaime's kind of tallness} \\ \leadsto \text{a combined kind of tallness, strength, masculinity, etc.}$$

$$(28) \quad \llbracket \text{Brienne is tall as}_{\text{SM}}/\text{xiàng a mountain} \rrbracket \quad \textbf{Simulative (generic equative)} \\ \Leftrightarrow [\lambda G.\lambda M.\lambda x.G(x) \sqsubseteq_{\text{info}} M](\llbracket \text{tall} \rrbracket)(\llbracket \text{as a mountain is tall} \rrbracket)(\llbracket \text{B} \rrbracket) \\ \Leftrightarrow \text{HEIGHT-KIND}(\text{Brienne}) \sqsubseteq_{\text{info}} \text{the kind of tallness that a mountain has}$$

On top of this qualitative similarity, *yī-yàng* further expresses similarity in terms of extent. I propose that kinds (of type k) can be further measured by measure functions of type $\langle kd \rangle$. In (literal and non-literal) equatives (see (30) and (31)), $\llbracket \text{yī-yàng} \rrbracket$ relates three items, an adjective G , a comparison standard M (which is a certain kind), and an entity x (see (29)), meaning that for any measure function P that can be used to measure M , P can also be used to measure $G(x)$ (i.e., in terms of kind, $G(x) \sqsubseteq_{\text{info}} M$), and when measured by P , $G(x)$ is to at least the same extent as M (i.e., $P(G(x)) \geq P(M)$). Obviously, when $G(x)$ and M are kinds on a single-dimensional scale (i.e., in literal equatives), the comparison results in just extent similarity (see (30)). (31) shows that a non-literal equative expresses similarities on both kinds and extents.

$$(29) \quad \llbracket \text{yī-yàng (Parameter marker)} \rrbracket_{\langle ek, \langle k, et \rangle \rangle} \stackrel{\text{def}}{=} \lambda G_{\langle ek \rangle}.\lambda M_k.\lambda x_e. \\ \forall P_{\langle kd \rangle}[P(M) \text{ is defined} \rightarrow P(G(x)) \text{ is defined} \wedge P(G(x)) \geq P(M)]$$

¹⁷As suggested by the example (27a), the use of *gēn* sounds somehow vacuous. Indeed, for Chinese literal equatives, sometimes *gēn* is optional: e.g., *Brienne (gēn) Jaime yī-yàng gāo*. Moreover, in Chinese, *gēn* cannot be used to form a generic equative (cf. (28)). Presumably, the semantics of a generic equative and the expression of qualitative similarity (e.g., sharing the same kind of tallness) must be based on complex-dimensional scales.

- (30) $\llbracket \text{Brienne is as}_{\text{PM}}/\text{yī-yàng tall as}_{\text{SM}}/\text{gēn Jaime is} \rrbracket$ ($\text{gēn} \leadsto$ single-dimensional scale)
 $\Leftrightarrow \llbracket \text{yī-yàng} \rrbracket(\llbracket \text{tall} \rrbracket)(\llbracket \text{as Jaime is tall} \rrbracket)(\llbracket \text{Brienne} \rrbracket)$
 $\leadsto \text{tallness}(\text{Brienne}) \geq \text{tallness}(\text{Jaime})$ (only ‘tallness’ is measured and compared)
- (31) $\llbracket \text{B is as}_{\text{PM}}/\text{yī-yàng tall as}_{\text{SM}}/\text{xiàng Everest is} \rrbracket$ ($\text{xiàng} \leadsto$ complex-dimensional scale)
 $\Leftrightarrow \llbracket \text{yī-yàng} \rrbracket(\llbracket \text{tall} \rrbracket)(\llbracket \text{as Everest is tall} \rrbracket)(\llbracket \text{Brienne} \rrbracket)$
 $\Leftrightarrow \forall P[P(\text{Everest's kind of tallness}) \text{ is defined} \rightarrow P(\text{Brienne's kind of tallness}) \text{ is defined} \wedge$
 $P(\text{Brienne's kind of tallness}) \geq P(\text{Everest's kind of tallness})]$

The current analysis also answers the third question raised in Section 2: directly inserting a degree value into the standard position in these equatives leads to a type-mismatch. This is not really a stipulation, because equatives essentially mean the sharing of kinds/extents between items, not just using degree values (kinds or numbers) to characterize the target of predication.

6. Conclusion

This paper argues for a dual ontology of degrees. Degrees, in the broadest sense of the term, are elements of a certain scale for measurement (Stevens, 1946). Therefore, degrees of ‘scales with units’ are like numbers and support the application of subtraction, while degrees of ‘scales without units’ can be considered kinds that potentially involve a complex-dimensional measurement or characterization. Empirically, this dual ontology of degrees is motivated by various kinds of natural language phenomena, especially comparatives containing numeral differentials (for the ‘degrees as numbers’ view) and non-literal equatives (for the ‘degrees as kinds’ view). The formal properties of relevant scales underline the expressiveness of these linguistic constructions. One issue on which I have not gone into detail is the formal description of a complex dimension and the informativeness relation between two kinds (e.g., Brienne’s tallness that involves firmness and strength vs. just firmness). This is related to the issue of ‘combined scales’ in the literature (see Bale, 2020). A thorough investigation on this is left for future research. More broadly, this project is related to the semantics of similatives and metaphors. The investigation of the whole spectrum from equatives to metaphors is also left for another occasion.

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| Grad. | *ER THAN | MORE * THAN | AS * AS | [J*] | $p_{gr}(com)$ | $p_{gr}(eq)$ | Non-grad. | *ER THAN | MORE * THAN | AS * AS | [J*] | $p_n(com)$ | $p_n(eq)$ |
|----------------|---------------------|------------------------|--------------------|-------------|----------------|----------------|------------------|---------------------|------------------------|--------------------|-------------|----------------|----------------|
| bad | 8555 | 15 | 2597 | 130432 | 0.0657 | 0.01991 | American | 0 | 92 | 177 | 304743 | 0.0003 | 0.00058 |
| big | 6196 | 2 | 2640 | 276062 | 0.02245 | 0.00956 | anonymous | 0 | 6 | 20 | 9632 | 0.00062 | 0.00208 |
| broad | 458 | 9 | 143 | 26893 | 0.01737 | 0.00532 | black | 116 | 21 | 215 | 216352 | 0.00063 | 0.00099 |
| cheap | 1334 | 0 | 108 | 17230 | 0.07742 | 0.00627 | blue | 57 | 18 | 108 | 72056 | 0.00104 | 0.0015 |
| good | 34226 | 139 | 6231 | 532774 | 0.0645 | 0.0117 | brown | 11 | 22 | 36 | 33387 | 0.00099 | 0.00108 |
| great | 8131 | 2 | 684 | 275728 | 0.0295 | 0.00248 | dead | 56 | 64 | 146 | 84746 | 0.00142 | 0.00172 |
| high | 11909 | 2 | 4403 | 267186 | 0.04458 | 0.01648 | French | 0 | 11 | 12 | 39774 | 0.00028 | 0.0003 |
| large | 6427 | 0 | 1441 | 158871 | 0.04045 | 0.00907 | gray | 41 | 36 | 66 | 26469 | 0.00291 | 0.00249 |
| long | 8459 | 0 | 46 | 196815 | 0.04298 | 0.00023 | green | 54 | 17 | 98 | 66715 | 0.00106 | 0.00147 |
| low | 5972 | 0 | 1629 | 89928 | 0.06641 | 0.01811 | orange | 0 | 8 | 12 | 18702 | 0.00043 | 0.00064 |
| old | 6859 | 0 | 1418 | 260903 | 0.02629 | 0.00543 | pink | 14 | 4 | 34 | 20815 | 0.00086 | 0.00163 |
| short | 1634 | 0 | 262 | 81506 | 0.02005 | 0.00321 | purple | 1 | 10 | 8 | 10995 | 0.001 | 0.00073 |
| small | 4227 | 0 | 945 | 225802 | 0.01872 | 0.00419 | red | 82 | 22 | 173 | 101249 | 0.00103 | 0.00171 |
| tall | 2191 | 0 | 807 | 31311 | 0.06998 | 0.02577 | round | 21 | 0 | 52 | 15623 | 0.00134 | 0.00333 |
| wide | 982 | 0 | 637 | 49042 | 0.02002 | 0.01299 | white | 144 | 33 | 309 | 182383 | 0.00097 | 0.00169 |
| young | 4575 | 3 | 1247 | 193370 | 0.02367 | 0.00645 | yellow | 9 | 21 | 26 | 31650 | 0.00095 | 0.00082 |
| Average | | | | | 0.04063 | 0.00982 | | | | | | 0.00099 | 0.00142 |
| SD | | | | | 0.02147 | 0.00716 | | | | | | 0.00061 | 0.00078 |

Appendix: Table of raw counts of strings in CoCA and the proportions of comparatives and equatives

$$p(com) = \frac{\text{Count}(*ER\ THAN) + \text{Count}(MORE\ *\ THAN)}{\text{Count}([J*])}; \quad p(eq) = \frac{\text{Count}(AS\ *\ AS)}{\text{Count}([J*])}$$

Test 1 – two-tailed *t*-test $p_{gr}(com)$ vs. $p_{n-gr}(com)$: $p < 0.0001$

Test 2 – two-tailed *t*-test $p_{gr}(eq)$ vs. $p_{n-gr}(eq)$: $p < 0.0001$

Test 3 – two-tailed paired-sample *t*-test $p_{n-gr}(com)$ vs. $p_{n-gr}(eq)$: $p = 0.013$

Test 4 – two-tailed paired-sample *t*-test $p_{gr}(com)$ vs. $p_{gr}(eq)$: $p < 0.0001$