Scalarity and additivity in natural language: (II) comparatives

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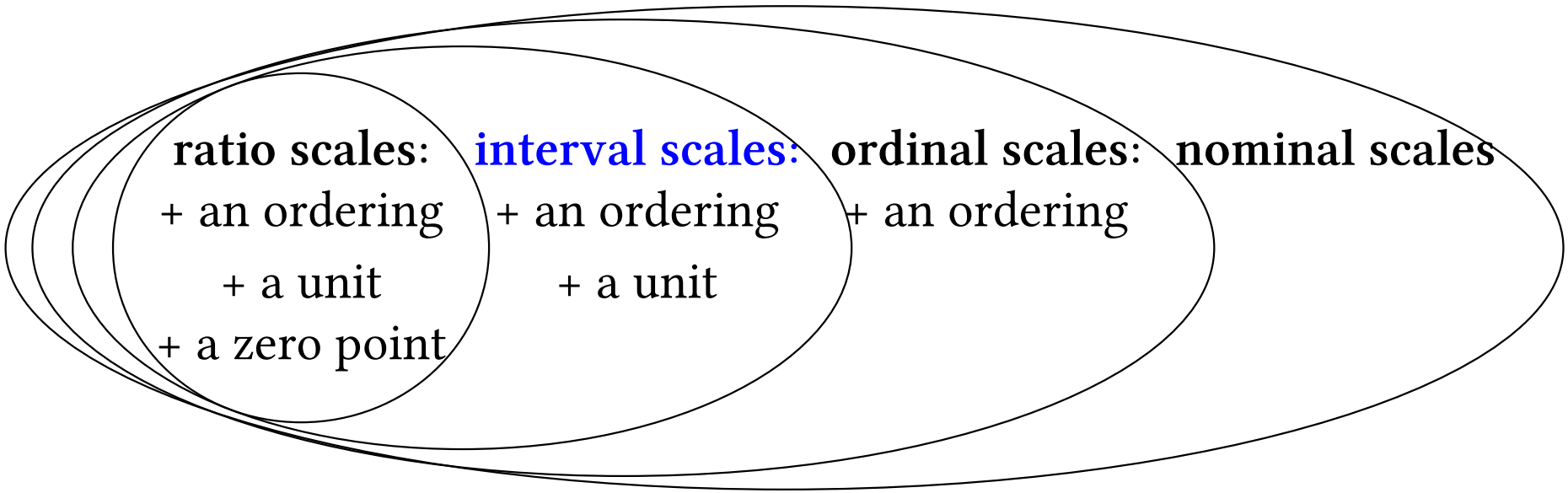
Slides are available on lingbuzz.

Recapitulation

- A scale is a set that includes all potential values resulting from a certain way of taking measurement.
- There are 4 levels of scales: nominal scales, ordinal scales, interval scales, and ratio scales.
- Comparatives assume interval scales (i.e., with an ordering and a unit that supports the measurement of differences).



Today

- Day 1: Basics of scales and degrees; how they are relevant to natural language
  - What are scales? What are their formal properties? What operators do they support?
- Day 2 and Day 3: Comparatives and *-er*/*more*
  - How an additivity-based perspective improve our understanding of scalarity-related phenomena?
  - What is additivity?
- Day 4 and Day 5: *Even* and its cross-linguistic siblings
  - How a scalarity-based perspective improve our understanding of additivity-related phenomena?

Today: English Comparatives and the use of *-er*/*more*

Mephisto is talking about poor farmer Faustus:

'He always wanted more ...
more free time and more wealth ...
fewer struggles, better health ...
work less and eat all day ...'

(From *Sapiens: A Graphic History*, Vol.2,
by David Vandermeulen, David Casanave, Yuval Noah Harari, 2021)

Outline

Ontological assumptions in analyzing comparatives

- A scale (i.e., an interval scale in Stevens 1946) is a totally ordered set of degrees:

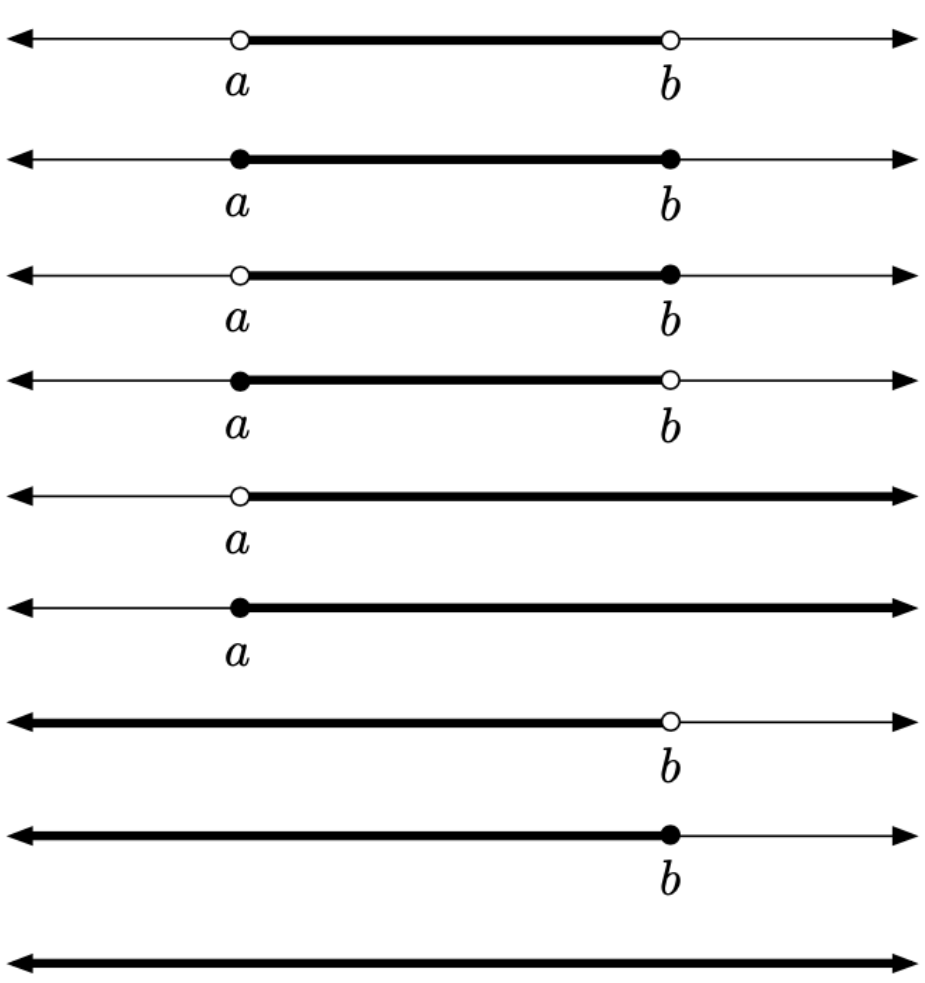
(1) $\{d \mid -\infty < d < +\infty\}$
$\rightsquigarrow$ a scale including all potential degree values

(2) $\{d \mid 0 < d \leq 7'\}$
$\rightsquigarrow$ a part of a scale of height / length, including degrees between 0 and 7 feet (the upper bound is included; the lower bound is not included)

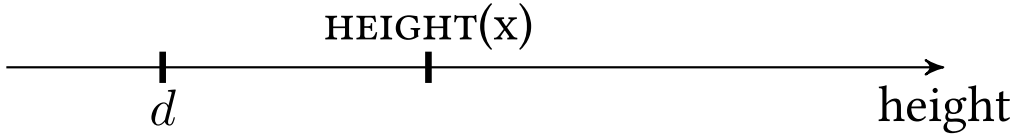
- Degrees are like real numbers (see also Fox and Hackl 2006: the universal density of measurement)
- Measurement means mapping an entity (or an event) to a degree along a relevant scale.
- Most works on comparatives are explicitly or implicitly based on these assumptions: e.g., Seuren (1973), von Stechow (1984), Heim (1985), Kennedy (1999), Schwarzchild and Wilkinson (2002), Zhang and Ling (2021) (cf. Cresswell 1976 does not consider 'degree' a primitive type; the delineation approach adopted by e.g., Klein 1980, Burnett 2017)

Illustrations: scales and parts of scales

(a, b)	$\{x : a < x < b\}$
$[a, b]$	$\{x : a \leq x \leq b\}$
$(a, b]$	$\{x : a < x \leq b\}$
$[a, b)$	$\{x : a \leq x < b\}$
(a, ∞)	$\{x : x > a\}$
$[a, \infty)$	$\{x : x \geq a\}$
$(-\infty, b)$	$\{x : x < b\}$
$(-\infty, b]$	$\{x : x \leq b\}$
$(-\infty, \infty)$	$\mathbb{R}$



The meaning of gradable adjectives



⟦tall⟧

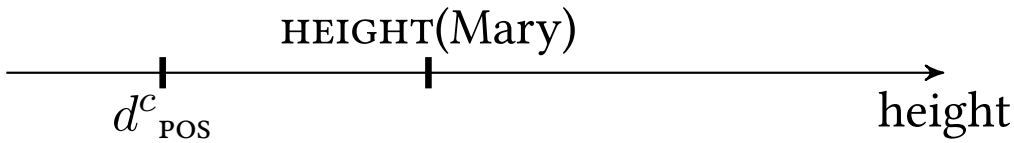
⟦tall⟧: a relation between an individual and a degree

(3) $[\![\text{tall}]\!]_{\langle d,et\rangle} \stackrel{\text{def}}{=} \lambda d_d.\lambda x_e.\textsc{height}_{\langle e,d\rangle}(x) \geq d$ (i.e., x is d-tall)
On the scale of height, the position of x meets or reaches degree d.

- There are two pieces in this lexical entry
  - A measure function of type $\langle ed\rangle$: $\textsc{height}_{\langle e,d\rangle}(x)$
  - Indicating the direction (of comparison): $\geq d$ (cf. Kennedy 1999)

(See e.g., Cresswell 1976, Hellan 1981, von Stechow 1984, Heim 1985, Schwarzschild 2008, Beck 2011)

Major uses of gradable adjectives: Positive use



⟦Mary is POS tall⟧

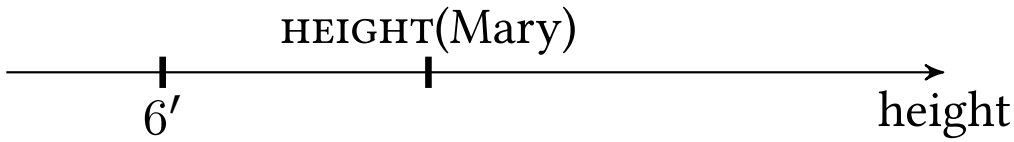
⟦tall⟧: a relation between an individual and a degree

(3) $[\![\text{tall}]\!]_{\langle d,et\rangle} \overset{\text{def}}{=} \lambda d_d.\lambda x_e.\text{HEIGHT}_{\langle e,d\rangle}(x) \geq d$ (i.e., x is d-tall)
On the scale of height, the position of x meets or reaches degree d.

(4) $[\![\text{Mary is POS tall}]\!] \Leftrightarrow \text{HEIGHT}(\text{Mary}) \geq d^c_{\text{POS}}$ Positive use
(POS: the context-dependent threshold for tallness)

(See e.g., Cresswell 1976, Hellan 1981, von Stechow 1984, Heim 1985, Kennedy 1999, Schwarzschild 2008, Beck 2011)

Major uses of gradable adjectives: Measurement sentence



⟦Mary is 6 feet tall⟧

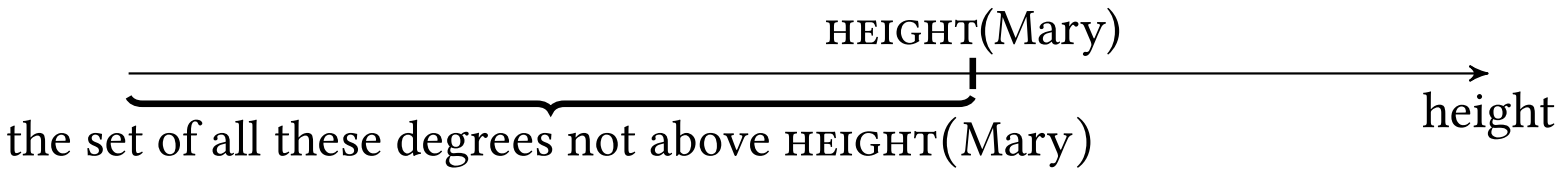
⟦tall⟧: a relation between an individual and a degree

(3) $[\![\text{tall}]\!]_{\langle d,et\rangle} \overset{\text{def}}{=} \lambda d_d.\lambda x_e.\textsc{height}_{\langle e,d\rangle}(x) \geq d$ (i.e., x is d-tall)
On the scale of height, the position of x meets or reaches degree d.

(5) $[\![\text{Mary is 6 feet tall}]\!] \Leftrightarrow \textsc{height}(\text{Mary}) \geq 6'$ Measurement

(See e.g., Cresswell 1976, Hellan 1981, von Stechow 1984, Heim 1985, Kennedy 1999, Schwarzschild 2008, Beck 2011)

Major uses of gradable adjectives: Degree question



⟦how tall is Mary⟧

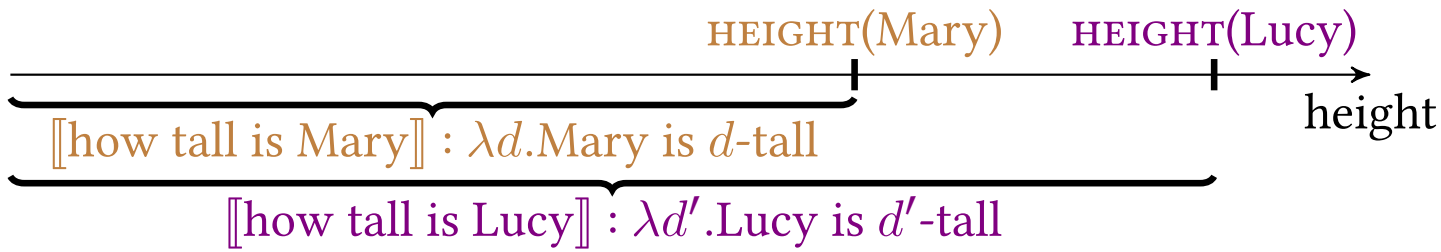
⟦tall⟧: a relation between an individual and a degree

(3) $[\![\text{tall}]\!]_{\langle d,et\rangle} \stackrel{\text{def}}{=} \lambda d_d.\lambda x_e.\textsc{height}_{\langle e,d\rangle}(x) \geq d$ (i.e., x is d-tall)
On the scale of height, the position of x meets or reaches degree d.

(6) $[\![\text{how tall is Mary}]\!] \Leftrightarrow \lambda d.$ Mary is d-tall
$\Leftrightarrow \lambda d.\textsc{height}(\text{Mary}) \geq d$ Degree question

(See the categorial approach to questions represented by Hausser and Zaefferer 1978.)

Major uses of gradable adjectives: Clausal comparative



⟦Lucy is taller than Mary is ~~tall~~⟧

(7) The bathtub is wider than the door is tall. subcomparative

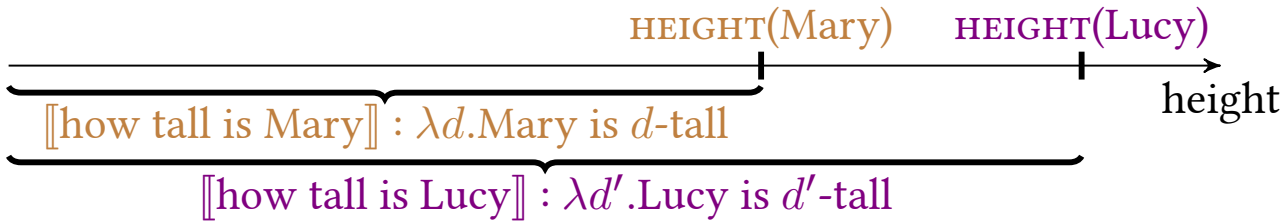
(8) ⟦ Lucy is tall er than Mary is ~~tall~~ ⟧

how tall Lucy is — how tall Mary is

LF: [-er [λd.Mary is d-tall]][λd′.Lucy is d′-tall]

(See e.g., von Stechow 1984, Kennedy 1999, Schwarzschild 2008, Beck 2011 for a review)

The role of *-er/more* in the canonical analysis



⟦Lucy is taller than Mary is ~~tall~~⟧

(8) ⟦ Lucy is tall er than Mary is ~~tall~~ ⟧

how tall Lucy is — how tall Mary is

LF: [-er [λd.Mary is d-tall]][$\lambda d'$.Lucy is d'-tall]

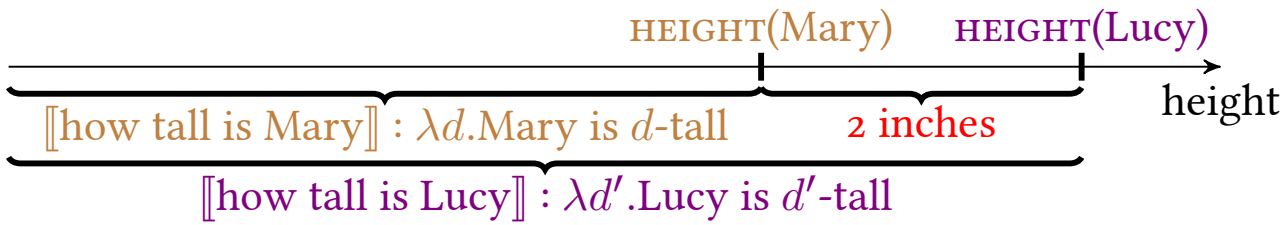
⟦-er/more⟧ performs comparison by encoding an inequality

(9) $[\![\text{-er/more}]\!]_{\langle\langle dt\rangle,\langle dt,t\rangle\rangle} \stackrel{\text{def}}{=} \lambda D_{\text{than}}.\lambda D_{\text{matrix}}.\exists d[d \in D_{\text{matrix}} \wedge \neg[d \in D_{\text{than}}]]$

(10) $[\![\text{-er/more}]\!]_{\langle\langle dt\rangle,\langle dt,t\rangle\rangle} \stackrel{\text{def}}{=} \lambda D_{\text{than}}.\lambda D_{\text{matrix}}.\text{MAX}(D_{\text{matrix}}) > \text{MAX}(D_{\text{than}})$

(See e.g., von Stechow 1984, Kennedy 1999, Schwarzschild 2008, Beck 2011 for a review)

Numerical differentials in the canonical analysis



⟦Lucy is 2 inches taller than Mary is ~~tall~~⟧

(11) ⟦ Lucy is 2 inches taller than Mary is ~~tall~~ ⟧
LF: [-er 2″ [λd.Mary is d-tall]][λd′.Lucy is d′-tall]

- Without a numerical differential: ⟦-er/more⟧ is of type $\langle\langle dt\rangle, \langle dt, t\rangle\rangle$

(10) $[\![\text{-er/more}]\!] \stackrel{\text{def}}{=} \lambda D_{\text{than}}.\lambda D_{\text{matrix}}.\text{MAX}(D_{\text{matrix}}) > \text{MAX}(D_{\text{than}})$

- With a numerical differential: ⟦-er/more⟧ is of type $\langle d, \langle\langle dt\rangle, \langle dt, t\rangle\rangle\rangle$

(12) $[\![\text{-er/more}]\!] \stackrel{\text{def}}{=} \lambda d_{\text{diff}}.\lambda D_{\text{than}}.\lambda D_{\text{matrix}}.\text{MAX}(D_{\text{matrix}}) \geq \text{MAX}(D_{\text{than}}) + d_{\text{diff}}$

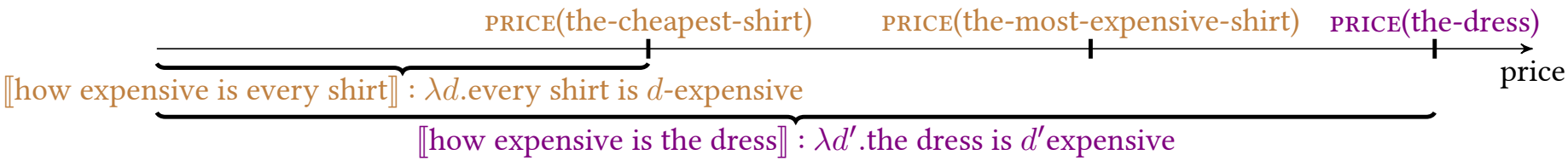
(See e.g., von Stechow 1984, Kennedy 1999, Schwarzschild 2008, Beck 2011 for a review)

Interim summary of the canonical view

	Without a numerical differential	**With a numerical differential**
Assumption	(Ordinal/interval) scales	Interval scales
Comparison	Inequality: $\text{measurement}_1 > \text{measurement}_2$	Inequality: $\text{measurement}_1 \geq \text{measurement}_2 + d$
Representations of & operations on scalar values	Degree points & ordering between degree points	Degree points & a combination of ordering and addition
The semantics of *-er/more*	Ordering: $\lambda m_2.\lambda m_1.m_1 > m_2$	A combination of ordering and addition: $\lambda d.\lambda m_1.\lambda m_2.m_2 \geq m_1 + d$

Outline

When the *than*-clause has a universal quantifier



⟦The dress is more expensive than every shirt is ~~expensive~~⟧

(13) Scenario: the price of the shirts ranges from $20 to $100 and the dress costs $150.

⟦ the dress is more expensive than every shirt is ~~expensive~~ ⟧

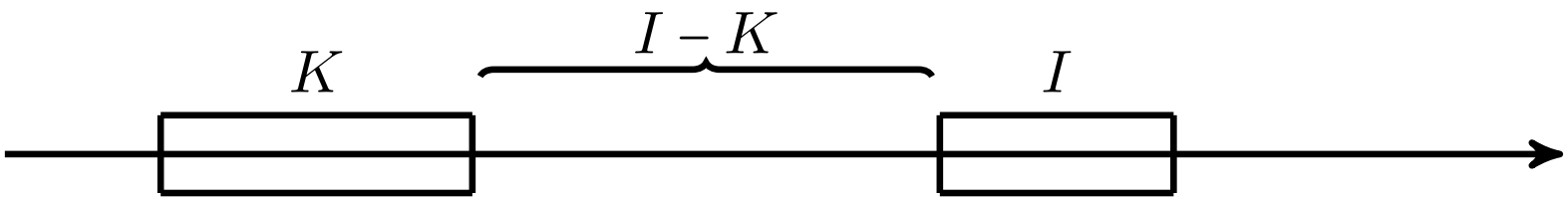
how expensive the dress is — how expensive every shirt is

LF: [more [λd.every shirt is d-expensive]][λd′.the dress is d′-expensive]

$=\{d \mid 0 \leq d \leq \text{PRICE}(\text{the-cheapest-shirt})\}$ $=\{d' \mid 0 \leq d' \leq \text{HEIGHT}(\text{the-dress})\}$

- **Our intuition**:
  the dress is more expensive than the most expensive shirt is.
- **The analysis under the canonical view**:
  the dress is more expensive than the least expensive shirt is.

Schwarzchild and Wilkinson (2002)'s solution: intervals



(14) $[\![\text{expensive}]\!] \stackrel{\text{def}}{=} \lambda I.\lambda x.\textsc{price}(x, I)$ (i.e., Interval I covers individual x.)

(15) The dress is more expensive than every shirt is.

a. ***than*-clause**: $\lambda K.\forall x[\text{shirt}(x) \rightarrow \textsc{price}(x, K)]$

b. **matrix clause**: $\lambda I.\textsc{height}(\text{the-dress}, I)$

The semantics of comparison

(16) $\textsc{matrix-clause}(\mu I[\textsc{than-clause}(\mu K[\textsc{diff}(I - K)])])$

The differential predicate DIFF holds for each gap between any subpart of the interval $I_{\text{main-clause}}$ and any subpart of the $K_{\text{than-clause}}$.

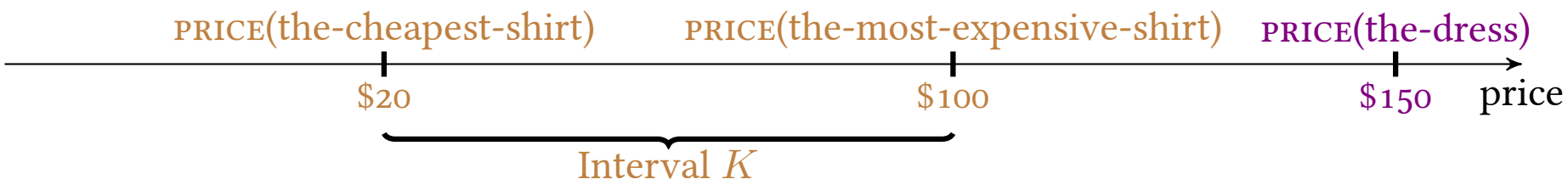
Beck (2010)'s comment

- MATRIX-CLAUSE(μI[THAN-CLAUSE(μK[DIFF($I - K$)])])
  Schwarzchild and Wilkinson (2002): embedding 'DIFF($I - K$)' within the scope of two maximality operators μ
  - The standard of comparison K is no longer a scalar value independent of comparison.
  - K is eventually yielded as the largest interval that makes DIFF hold for all the gaps involved in the numerous sub-interval-level comparisons.

Beck (2010)

'I want to come out of the calculation of the semantics of the *than*-clause holding in my hand *the* degree we will be comparing things to.'

Beck (2010)'s solution



⟦The dress is up to $60 more expensive than every shirt is ~~expensive~~⟧

- Beck (2010): using the largest value in the interval K for comparison.

(17) The dress is up to $60 more expensive than every shirt is.
⇝ False under our scenario that the dress costs $150 and shirts vary between $20 and $100, but true under the analysis of Beck (2010)

(See Fleisher 2016)

Issues to be solved

(17) The dress is up to $60 more expensive than every shirt is.

- The standard of comparison, i.e., the meaning of the *than*-clause, should be able to represent **a range of values**.
  - Beck (2010): 'I want to come out of the calculation of the semantics of the *than*-clause holding in my hand *the* degree we will be comparing things to.' (cf. Schwarzchild and Wilkinson 2002)
  - This range of values should not be reduced to a single degree point. (cf. Beck 2010)

Conceptual challenge: what does *-er*/*more* do?

Mephisto is talking about poor farmer Faustus:

'He always wanted more ...
more free time and more wealth ...
fewer struggles, better health ...
work less and eat all day ...'

(From *Sapiens: A Graphic History*, Vol.2,
by David Vandermeulen, David Casanave, Yuval Noah Harari, 2021)

Conceptual challenge: what does *-er/more* do?

- *-er/more* can be preceded by definite determiner *the*:

(18) a. The more you read, the more you learn.
b. The taller you are, the less mobile and quick you are.

- *-er/more* can be used repeatedly:

(19) a. Lucy is tall er and tall er and taller.
b. We will have more and more money.

- These data are unexpected if *-er/more* essentially encodes an inequality, meaning '$\lambda m_2.\lambda m_1.m_1 > m_2$' or '$\lambda d.\lambda m_1.\lambda m_2.m_2 \geq m_1 + d$'.

Outline

Parallelism between *-er/more* and *another*

- *-er/more* has an additive use similar to *another* (see also Greenberg 2010 and Thomas 2010):

(20) **Increase in the domain of entities: Additive use**

a. I ate an^{x} apple. Then I ate **another**^{y} (apple).

base item: an^{x} apple; increase: **another**^{y} (apple)

b. A^{x} girl, Sue, met **another**^{y} girl, Mary.

base item: A^{x} girl; increase: **another**^{y} girl

c. I ate two^{x} bars of chocolate. Then I ate (a bit) **more**^{y}.

base item: two^{x} bars of chocolate; increase: (a bit) **more**^{y}

From the additive use to the comparative use of *-er/more*

- Additive use of *more*: in the domain of entities
- Comparative use of *-er/more*: in the domain of scalar values (i.e., degrees)

(20c) **Increase in the domain of entities: Additive use**
I ate two^{x} bars of chocolate. Then I ate (a bit) **more**^{y}.
base item: two^{x} bars of chocolate; increase: (a bit) **more**^{y}

(21) **Increase in the domain of scalar values: Comparative use**

a. Mary is tall. Sue is tall **er** . Across sentences
base item: HEIGHT(Mary); increase: **er**

b. Sue is tall **er** than Mary is ~~tall~~. Within the same sentence
increase: **er**; base item: HEIGHT(Mary)

More uses of *-er/more* and *another*

- *-er/more* and *another*
  - denotes an increase in the domain of entities or scalar values
  - presuppose there is a salient base that the increase is anaphoric to

(22) **Repetitive use of *-er/more* and *another***

a. Lucy is becoming taller and taller and taller.
b. Janice had a little lamb and another and another and another.

(23) **Accumulating increases along with a universal quantifier**

a. Every year Mary wrote a more interesting book.
b. Everyday there is another story to write.

(24) They can all be preceded by definite determiner *the*:

a. The more you read, the more you learn.
b. The taller you are, the less mobile and quick you are.
c. I've brought two books: one is *Le Petit Prince*, and the other is *Exhalation*.

What gets 'increased'? What does additivity mean?

- Additivity should be considered a phenomenon of QUD-based anaphoricity, indicating an extension of a previous salient answer in addressing the QUD
  - For the additive use in the domain of entities, *more* / *another* indicates an increase from a part to a whole.

(25) **Additive use of *more***

Current question (CQ): What did you eat?

a. I ate two bars of chocolate. Then I ate (a bit) more.

base item: a partial answer to the CQ

increase

b. #I didn't eat a bar of chocolate. Then I ate more.

(e.g., Roberts 1996/2012, Zeevat 2004, Zeevat and Jasinskaja 2007, Beaver and Clark 2009, Thomas 2011, Zhang and Ling 2021)

The anaphoricity of *-er/more*

- Additivity should be considered a phenomenon of QUD-based anaphoricity, indicating an extension of a previous salient answer in addressing the QUD
  - For the comparative use in the domain of scalar values, *-er/more* indicates an increase from a lower to a higher scalar value.

(26) **Comparative use of *-er/more***

Current question (CQ): How tall is Sue?

Mary is not tall. Sue is tall er .

base item – a partial answer to the CQ: HEIGHT(Mary)

increase

(e.g., Roberts 1996/2012, Zeevat 2004, Zeevat and Jasinskaja 2007, Beaver and Clark 2009, Thomas 2011, Zhang and Ling 2021)

The semantics of English *-er/more*

- The canonical view:

	Without a numerical differential	**With a numerical differential**
Comparison	Inequality: $\text{measurement}_1 > \text{measurement}_2$	Inequality: $\text{measurement}_1 \geq \text{measurement}_2 + d$
The semantics of *-er/more*	Ordering: $\lambda m_2.\lambda m_1.m_1 > m_2$	A combination of ordering and addition: $\lambda d.\lambda m_1.\lambda m_2.m_2 \geq m_1 + d$

- In contrast to this canonical view, English *-er/more* works like *another* in being an additive particle, denoting an increase on a discourse-salient base, extending this existing base in addressing the Current Question.

Outline

Comparatives: A difference-based view

⟦Lucy is taller than Mary is ~~tall~~⟧

(27) ⟦ Lucy is tall er than Mary is ~~tall~~ ⟧

how tall Lucy is — how tall Mary is

$\rightsquigarrow$ HEIGHT(Lucy) − HEIGHT(Mary) = ⟦-er⟧

- ⟦-er⟧ denotes an increase, i.e., a positive value.

Comparatives: A difference-based view

⟦Lucy is 2 inches taller than Mary is ~~tall~~⟧

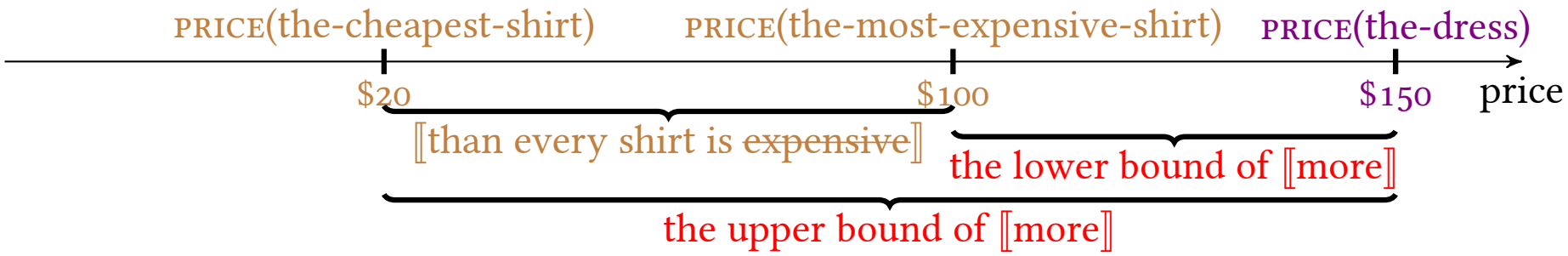
(28) ⟦ Lucy is 2 inches tall er than Mary is ~~tall~~ ⟧

how tall Lucy is — how tall Mary is

$\rightsquigarrow$ HEIGHT(Lucy) − HEIGHT(Mary) = ⟦2 inches ...-er⟧

- ⟦2 inches ...-er⟧ denotes an increase with a specified size, i.e., 2 inches.

Comparatives: A difference-based view



⟦The dress is more expensive than every shirt is ~~expensive~~⟧

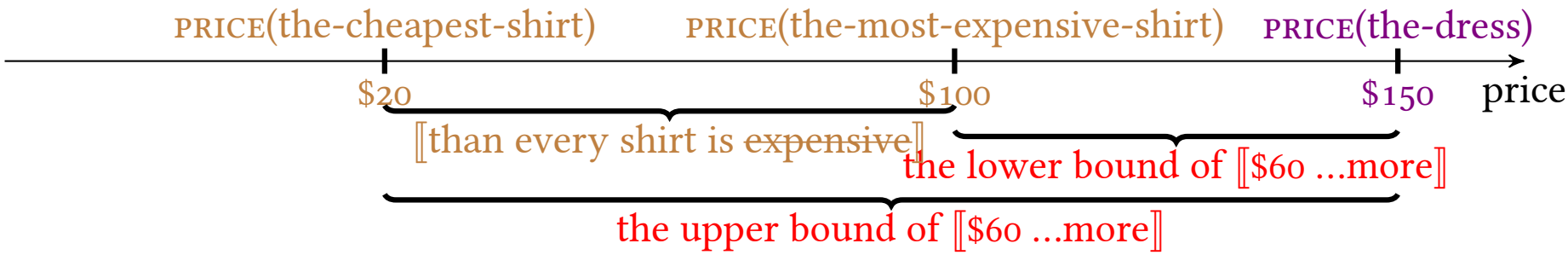
(29) ⟦ The dress is more expensive than every shirt is ~~expensive~~ ⟧

how expensive the dress is — how expensive every shirt is

$\rightsquigarrow$ PRICE(the dress) – the interval that ranges from PRICE(the-cheapest-shirt) to PRICE(the-most-expensive shirt) = ⟦more⟧

- ⟦more⟧ denotes an increase, i.e., a positive value.

Comparatives: A difference-based view



⟦The dress is up to $60 more expensive than every shirt is ~~expensive~~⟧ (false under this scenario)

(30)

⟦ The dress is up to $60 more expensive than every shirt is ~~expensive~~ ⟧

how expensive the dress is — how expensive every shirt is

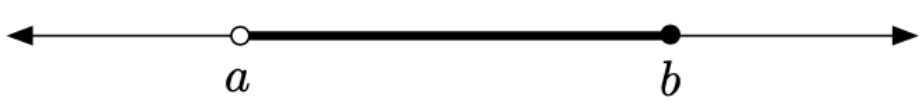
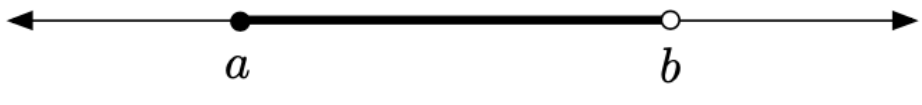
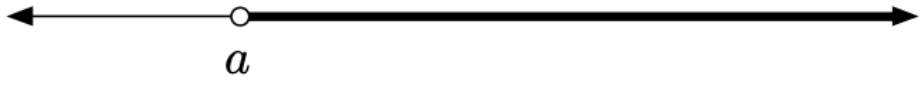
$\rightsquigarrow$ PRICE(the dress) – the interval that ranges from PRICE(the-cheapest-shirt) to PRICE(the-most-expensive shirt) = ⟦up to $60 more⟧

- ⟦up to $60 more⟧ denotes an increase with a specific size, i.e., $(-\infty, \$60]$
  $\rightsquigarrow$ the upper bound of the increase is $60

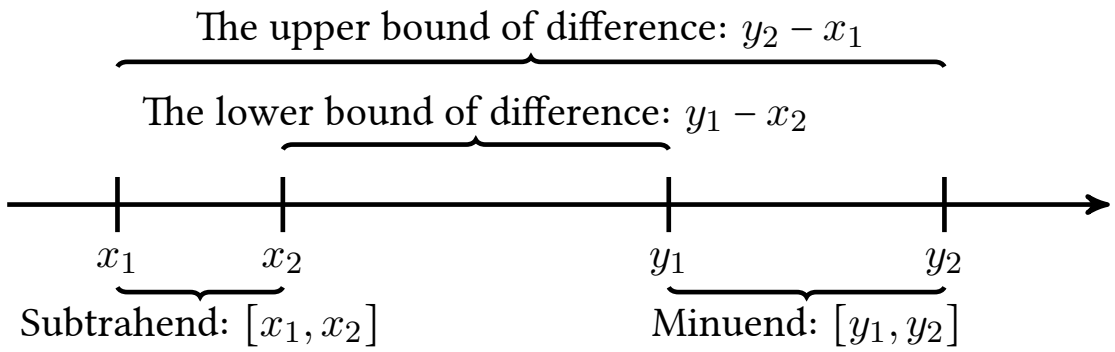
From degrees to intervals

- **Degrees**: points on a scale (an interval scale à la Stevens 1946)
- **Intervals**: convex sets of degrees
  - **Convex set**: A totally ordered set P is **convex** iff for any elements a and b in the set (suppose $a \leq b$), any element x such that $a \leq x \leq b$ is also in the set P.
  - E.g., $\{x \mid x > 0\}$, $\{x \mid x \leq 4\}$, and $\{x \mid 4 \leq x \leq 8\}$ are all convex sets.
  - Sets like $\{x \mid x > 10 \vee x \leq 3\}$ are not convex.
- Notation of intervals:
  - $\{x \mid x > 0\} = (0, +\infty)$
    $\{x \mid x \leq 4\} = (-\infty, 4]$
    $\{x \mid 4 \leq x \leq 8\} = [4, 8]$

Intervals

(a, b)	$\{x : a < x < b\}$	
$[a, b]$	$\{x : a \leq x \leq b\}$	
$(a, b]$	$\{x : a < x \leq b\}$	
$[a, b)$	$\{x : a \leq x < b\}$	
(a, ∞)	$\{x : x > a\}$	
$[a, \infty)$	$\{x : x \geq a\}$	
$(-\infty, b)$	$\{x : x < b\}$	
$(-\infty, b]$	$\{x : x \leq b\}$	
$(-\infty, \infty)$	$\mathbb{R}$	

Subtraction between intervals



(31) $\underbrace{[y_1, y_2]}$ – $\underbrace{[x_1, x_2]}$ = $\underbrace{[y_1 - x_2, y_2 - x_1]}$

minuend: matrix **subtrahend: comparative standard** **difference: differential**

a. Example 1: $[5, 8] - [1, 3] = [2, 7]$

b. Example 2: $(4, +\infty) - [2, 3] = (1, +\infty)$

- We are now ready to use intervals and interval subtraction to provide a systematic, generalized formal analysis for all kinds of comparatives.
  - With *than*-internal quantifiers and/or various numerical differentials
  - Crucially based on an additivity/increase-based view of *-er*/*more*

(See Moore 1979)

Today's take-home messages

- Day 2 and Day 3: Comparatives and *-er/more*
  - How an additivity-based perspective improve our understanding of scalarity-related phenomena?
  - What is additivity?
- Additivity is a phenomenon of QUD-based anaphoricity, indicating an extension of a previous salient answer in addressing the QUD.

	The canonical view	**The new difference-based view**
Assumption	(Ordinal/interval) scales	**Interval scales**
Comparison	Inequality: $M_1 > M_2$	**Subtraction**: $M_1 - M_2 = D$
Representations of & operations on scalar values	Degree points & ordering between degree points	**Intervals (i.e., set of degrees) & interval subtraction**
The semantics of *-er/more*	Ordering: $>$	**Additivity a default positive difference**: $(0, +\infty)$

Tomorrow

- Day 2 and Day 3: Comparatives and *-er*/*more*
  - How an additivity-based perspective improve our understanding of scalarity-related phenomena?
  - What is additivity?
- Tomorrow
  - Formal implementation (see Zhang and Ling 2021 and Zhang and Zhang 2024)
  - Antonyms
  - Cross-linguistic phenomena
  - etc.

Selected references I

Beaver, David I., and Brady Z. Clark. 2009. *Sense and sensitivity: How focus determines meaning*. John Wiley & Sons. https://doi.org/10.1002/9781444304176open_in_new.

Beck, Sigrid. 2010. Quantifiers in *than*-clauses. *Semantics and Pragmatics* 3 (1):1–72. https://doi.org/10.3765/sp.3.1.

Beck, Sigrid. 2011. Comparative constructions. In *Semantics*, ed. Claudia Maienborn, Klaus von Heusinger, and Paul Portner, volume 2, 1341–1390. Berlin, Boston: de Gruyter. https://doi.org/10.1515/9783110255072.1341.

Burnett, Heather. 2017. *Gradability in Natural Language*. Oxford, UK: Oxford University Press. https://doi.org/10.1093/acprof:oso/9780198724797.001.0001.

Cresswell, Max J. 1976. The semantics of degree. In *Montague grammar*, ed. Barbara Partee, 261–292. New York: Academy Press. https://doi.org/10.1016/B978-0-12-545850-4.50015-7.

Fleisher, Nicholas. 2016. Comparing theories of quantifiers in *than* clauses: lessons from downward-entailing differentials. *Semantics and Pragmatics* 9 (4):1–23. https://doi.org/10.3765/sp.9.4.

Fox, Danny, and Martin Hackl. 2006. The universal density of measurement. *Linguistics and Philosophy* 29:537–586. https://doi.org/10.1007/s10988-006-9004-4.

Greenberg, Yael. 2010. Additivity in the domain of eventualities (or: Oliver twist's *more*). In *Sinn und Bedeutung*, volume 14, 151–167. https://ojs.ub.uni-konstanz.de/sub/index.php/sub/article/view/465.

Selected references II

Hausser, Roland, and Dietmar Zaefferer. 1978. Questions and answers in a context-dependent Montague grammar. In *Formal semantics and pragmatics for natural languages*, 339–358. Springer. https://doi.org/10.1007/978-94-009-9775-2_12.

Heim, Irene. 1985. Notes on comparatives and related matters. https://semanticsarchive.net/Archive/zc0ZjY0M/Comparatives%2085.pdf, unpublished ms., University of Texas, Austin.

Hellan, Lars. 1981. *Towards an integrated analysis of comparatives*. Tübingen: Narr.

Kennedy, Christopher. 1999. *Projecting the adjective*. New York: Routledge. https://doi.org/10.4324/9780203055458.

Klein, Ewan. 1980. A semantics for positive and comparative adjectives. *Linguistics and philosophy* 4:1–45. https://link.springer.com/content/pdf/10.1007/BF00351812.pdf.

Moore, Ramon E. 1979. *Methods and applications of interval analysis*. SIAM. https://epubs.siam.org/doi/10.1137/1.9781611970906.

Roberts, Craige. 1996/2012. Information structure: Towards an integrated formal theory of pragmatics. *Semantics and pragmatics* 5:6:1–69. https://doi.org/10.3765/sp.5.6.

Schwarzchild, Roger, and Karina Wilkinson. 2002. Quantifiers in comparatives: A semantics of degree based on intervals. *Natural language semantics* 10:1–41. https://doi.org/10.1023/A:1015545424775.

Schwarzschild, Roger. 2008. The semantics of comparatives and other degree constructions. *Language and Linguistics Compass* 2:308–331. https://doi.org/10.1111/j.1749-818X.2007.00049.x.

Selected references III

Seuren, Pieter A. M. 1973. The comparative. In *Generative grammar in Europe*, ed. F. Kiefer and N. Ruwet, 528–564. Springer. https://doi.org/10.1007/978-94-010-2503-4_22.

von Stechow, Arnim. 1984. Comparing semantic theories of comparison. *Journal of semantics* 3:1–77. https://doi.org/10.1093/jos/3.1-2.1.

Stevens, Stanley Smith. 1946. On the theory of scales of measurement. *Science* 103:677–680. https://www.science.org/doi/pdf/10.1126/science.103.2684.677.

Thomas, Guillaume. 2010. Incremental *more*. In *Semantics and Linguistic Theory*, volume 20, 233–250. https://doi.org/10.3765/salt.v20i0.2558.

Thomas, Guillaume. 2011. Another additive particle. In *Semantics and Linguistic Theory*, volume 21, 634–651. https://doi.org/10.3765/salt.v21i0.2589.

Zeevat, Henk. 2004. Particles: Presupposition triggers, context markers or speech act markers. In *Optimality theory and pragmatics*, 91–111. Springer. https://doi.org/10.1057/9780230501409_5.

Zeevat, Henk, and Katja Jasinskaja. 2007. *And* as an additive particle. In *Language, representation and reasoning. Memorial volume to Isabel Gómez Txurruka*, 315–340. University of the Basque Country Press. https://idsl1.phil-fak.uni-koeln.de/sites/IDSLI/dozentenseiten/Jasinskaja/zeevat_jasinskaja_2007.pdf.

Zhang, Linmin, and Jia Ling. 2021. The semantics of comparatives: A difference-based approach. *Journal of Semantics* 38:249–303. https://doi.org/10.1093/jos/ffab003.

Zhang, Linmin, and Florence Zhang. 2024. Comparative morphemes are additive particles: English *-er*/*more* vs. Chinese *gèng*. https://ling.auf.net/lingbuzz/008122, manuscript.