Counting individuals and their halves*

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Abstract

Expressions like *two novels* are traditionally taken to convey information about cardinality and are analyzed using a cardinality function. Salmon (1997), Liebesman (2015, 2016, Forthcoming), and Haida and Trinh (2021) argue against this traditional account, claiming that it can't explain our use of expressions like *two and a half novels.* According to them, the proper analysis of such expressions requires a partiality measure, which maps entities to rational numbers such as 2.5. In this paper, we set out to defend the traditional account. To do so, we demonstrate that an analysis based on a partiality measure is inconsistent with the truth conditions of plural comparatives and equatives. We also show that such an account doesn't provide an adequate analysis for expressions like *two novels and a half*, nor for their French counterparts (e.g., *deux romans et demi*). Critically, such expressions contain no constituent that could refer to the number 2.5. We provide an alternative analysis for all these expressions, based on cardinality and an operation of non-overlapping summation.

Keywords: count nouns; counting; fractions; measure functions; cardinality; plurals

1 Introduction

Traditionally, numerals in phrases such as *two novels* have been analyzed using a cardinality function that maps inputs — whether sets or entities — to a natural number that represents the number of individuals contained within the input. Cardinality is closely tied to identity. Thus, to say that there are two things is to say that there are some x and y which are different from each other. Some researchers have challenged this traditional analysis (Salmon 1997; Fox and Hackl 2006; Liebesman 2015, 2016; Snyder and Barlew 2019; Haida and Trinh 2021). They've proposed that such expressions, in at least some contexts, require a measure more flexible than cardinality, one whose input domain includes partial entities (e.g., halves and quarters of novels) and whose range includes rational numbers (e.g., 2.5). One of the main motivations behind these challenges comes from phrases like *two and a half novels.* At least on the surface, such phrases are similar to constructions like *two and a half liters of water*, where *two and a half liters* is often

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interpreted as a degree argument within a densely ordered scale. By analogy, it's tempting to analyze *two and a half* in the phrase *two and a half novels* in the same way. Perhaps, *two and a half* denotes a degree argument within a densely ordered scale.

In this paper, while reviewing these challenges, we present new evidence in favor of the traditional account. Using judgments about plural comparatives and equatives (e.g., *more novels than, as many novels as*) and from numeral modification in French (e.g., *deux romans et demi*), we argue that the proper analysis of numeral expressions, when they're used to modify a count noun, doesn't involve any type of partiality measure function. Critically, such measure functions make the wrong predictions with respect to plural comparatives and equatives, namely that fractional differences should be able to influence judgments on what counts as more or less. In the end, we defend an analysis where *two and a half novels* is interpreted via the same compositional mechanisms that are employed in the interpretation of *two novels and a half* (see also the analyses in Ionin, Matushansky, and Ruys 2006; Snyder and Barlew 2019; Landman 2020).

Our paper is organized as follows. In Section 2, we review the proposals made by Salmon, Liebesman, and Haida and Trinh. In Section 3, we discuss judgments about comparative and equative constructions, suggesting that partiality measures don't play a significant role in their truth conditions. Section 4 presents some critical data from French, namely that nominal expressions like *deux romans et demi* ('two novels and a half') have the same interpretation as their English counterparts, despite having no constituent that could refer to the number 2.5. So, to account for the French data, one needs a different kind of analysis than what is proposed by Salmon, Liebesman, and Haida and Trinh. Sections 5 and 6 provide a unified analysis for English and French. Finally, in Section 7, we address some specific challenges put forward by Liebesman.

2 Challenges for a cardinality measure

Many of the challenges to cardinal measurement stem from the introduction of fractions within a numeral expression. Thus, Salmon (1997) argues that cardinality is inadequate once one considers phrases like *two and a half oranges*. He suggests that such constructions might require a form of measurement that yields rational values such as 2.5.[1] More recently, Liebesman (2015, 2016, Forthcoming) and Haida and Trinh (2021) have expanded on Salmon's original observations to develop a semantics involving a partiality measure. In this section, we review Salmon's observations together with Liebesman's and Haida and Trinh's proposals. The purpose of this section is to outline the alternatives to cardinality. Although we mention some of the arguments in favor of this alternative, we postpone a more thorough discussion until Section 7.

2.1 Salmon's observations

Salmon (1997) discusses the following puzzle. Imagine there are two whole oranges on the table plus an additional half orange along with some other piece of fruit — let's say a pear. Suppose someone asks the question in (1). Salmon notes that the most appropriate answer is the statement

1 Krifka (1989) independently hypothesizes a non-cardinal analysis of numeral modification. He provides a measurement-analysis in terms of 'natural units.' His primary motivation for doing so was to maintain a parallel between numeral modification and pseudo-partitive constructions that require a measurement analysis (e.g., *two apples* vs. *two pounds of apples*). We won't be focusing on this line of argument in this paper.

in (2a) as opposed to either (2b) or (2c), even though both (2b) and (2c) are technically true statements.

(1) Exactly how many oranges are there on the table?

(2) a. There are exactly two and a half oranges on the table.
 b. There are exactly two oranges on the table.
 c. There are exactly two and a half oranges and (exactly) one pear on the table.

Salmon argues that in order to maintain the hypothesis that the interpretation of numeral modification is based on cardinality, one would have to analyze (2a) in a way that parallels the paraphrase given in (3).

(3) There are exactly two oranges and (exactly) one half orange on the table.

In this paraphrase, the numeral *two* modifies the plural noun *oranges* and then this modified noun phrase combines via conjunction with a separate noun phrase, *one half orange*. Critically, a half orange isn't a whole orange; it's only a partial orange. On this analysis, (2a) can be broken down into two assertions: one about the cardinality of whole oranges and another about a quantity of partial oranges.

According to Salmon, such an interpretation can't easily account for the judgments observed. If the question in (1) were understood as a question about the exact cardinality of oranges, then one should prefer the answer in (2b) over the other two answers, contrary to fact. In contrast, if the question in (1) permitted an answer that stated the maximal cardinality of oranges along with some additional statement about the maximal quantity of things that aren't oranges, then one would expect (2c) to be as acceptable as (2a), once again contrary to fact.

According to Salmon, these observations would fall out naturally if there were a way to replace cardinal quantification by some kind of partiality measure: the question in (1) could then be construed as an inquiry about the exact quantity n such that n oranges are on the table, where 2.5 represents one possible value of n.

2.2 Liebesman's account

Liebesman (2015, 2016, Forthcoming) expands on Salmon's observations and provides a detailed implementation of Salmon's idea. Consider the sentences in (4), which don't overtly express exactness.

(4) a. Two bagels are on the table.
 b. Two and a half bagels are on the table.

As Liebesman notes, on the surface, both sentences have the same form: they share a predicate (*are on the table*) and their subjects consist of a plural noun *bagels* and a numeral expression (*two*, *two and a half*). Given their surface similarities, it seems reasonable to assume that the two sentences have a similar semantic analysis modulo the difference in the numeral expression. According to the traditional cardinal analysis, the sentence in (4a) is true under a non-exhaustified meaning if and only if there's a mereological sum x of bagels on the table whose cardinality, $\mathrm{card}(x)$, is 2, as represented by the truth conditions in (5).

(5) $\exists x.\ \text{bagels}(x) \wedge \text{card}(x) = 2 \wedge \text{on-the-table}(x)$

The exhaustified interpretation of (4a) is then derived by negating stronger alternative propositions that have the same basic form as (5), where 2 is replaced by stronger alternative values such as 3, as represented in (6).

(6) $\exists x.\ \text{bagels}(x) \wedge \text{card}(x) = n \wedge \text{on-the-table}(x)$, where n is a stronger alternative to 2.

However, (4b) can't be analyzed via this traditional approach. If we try to replace 2 with 2.5, as in (7), we end up with truth conditions that are non-sensical in two ways: first, the extension of a plural predicate like *bagels* normally doesn't contain any partial entities such as half slices of bagels, and second, 2.5 isn't a cardinal number.

(7) $\exists x.\ \text{bagels}(x) \wedge \text{card}(x) = 2.5 \wedge \text{on-the-table}(x)$

Moreover, the traditional analysis also makes a wrong prediction with respect to the exhaustified interpretation of (4a), as well as its paraphrase in (8).

(8) Exactly two bagels are on the table.

In the context considered, no natural number greater than 2 makes (6) true, so exactness is predicted to be satisfied, contrary to fact.

Given such difficulties, Liebesman argues that a better analysis of the sentences in (4) can be obtained by replacing the cardinality function with something that is more in line with the type of measure function used in sentences like (9a), as explicitly given in (9b), where the function 'kilo' can map something to a rational number:

(9) a. Two and a half kilos of bagels are on the table.
    b. $\exists x.\ \text{bagels}(x) \wedge \text{kilo}(x) = 2.5 \wedge \text{on-the-table}(x)$

To account for sentences like (4b), Liebesman (2016, sec. 2.1, 2.2, 3.3) proposes that the relevant measure function allows for partial bagels to be accessible for measurement. The details of Liebesman's analysis can be roughly outlined as follows.[2] Given a count noun P, a positive number n (natural or not), and a predicate Q, a sentence of the form *n Ps Q* would have the non-exhaustified truth conditions in (10).

(10) $\exists x.\ \mu(\langle x, P\rangle) = n \wedge Q(x)$

Here, μ is a measure function which applies to x relatively to the count noun P. (Formally, its argument is the ordered pair $\langle x, P\rangle$.) μ is the sum of two functions: a cardinality function 'card' and a 'partiality measure' function ψ:[3]

(11) $\mu(\langle x, P\rangle) = \text{card}(\langle x, P\rangle) + \psi(\langle x, P\rangle)$

The cardinality function returns the number of Ps within x. The partiality measure returns a number representing how much of a partial P is within x. Critically, a partial P can be measured even though it isn't a P. For instance, a half bagel counts as a partial bagel with a partiality measure of 0.5, while a quarter of a bagel counts as a partial bagel with a partiality measure of 0.25. Partiality measures can be added when their sum is strictly smaller than 1. For instance, if on the table there are two bagels and two quarter-slices from two different bagels, then the cardinality function would return 2, while the partiality measure function would return $0.25 + 0.25 = 0.5$ (Liebesman 2015, p. 7). The semantics thus expects (4b) to be true in this situation.

2 Liebesman's account is cashed out in plural logic. Here, we've adapted his proposal using mereology.

3 In later sections, in order to contrast this approach from one that only uses cardinality, we'll sometimes also refer to the sum of these two functions as a partiality measure.

When the sum is equal to or greater than 1, however, the partiality measure is undefined. For example, suppose there's one bagel plus three different halves of a bagel on the table. In such contexts, (4b) would fail to yield a value of true or false under Liebesman's analysis. Moreover, according to Liebesman, not every part of a P is a partial P. Thus, the wheel of a car doesn't count as partial car. Also, a partial P isn't always part of a P. Thus, if a factory has completed half of the process of building a car and stopped there, it has produced a partial car with partiality-measure 0.5, something which isn't part of any car.[4]

Given this measure function, Liebesman is able to provide the same type of analysis for both sentences in (4), one that looks very similar to the analysis of (9a). For example, Liebesman represents the truth conditions of (4a) (*Two bagels are on the table*) as in (12).

(12) $\exists x.\ \mu(\langle x, \text{bagel}\rangle) = 2 \wedge \text{on-the-table}(x)$
iff $\exists x.\ \text{card}(\langle x, \text{bagel}\rangle) + \psi(\langle x, \text{bagel}\rangle) = 2 \wedge \text{on-the-table}(x)$

In a context where there are no partial bagels, the truth conditions in (12) end up being equivalent to those in (13), which in turn are very close to the original truth conditions given in (5).

(13) $\exists x.\ \text{card}(\langle x, \text{bagel}\rangle) = 2 \wedge \text{on-the-table}(x)$

(5) $\exists x.\ \text{bagels}(x) \wedge \text{card}(x) = 2 \wedge \text{on-the-table}(x)$

However, Liebesman argues that the implications that can be derived from (12) are different from those that can be derived from (5). Unlike (5), the truth conditions in (12) include potential quantification over partial bagels as well as the potential for the measure function to yield fractional values. Thus, an upper-bound, exact reading of (12) would imply that there's no entity on the table consisting of bagels and partial-bagels whose μ-measurement is above 2. In particular, there are no entities consisting of two and a half bagels on the table.

With respect to the sentence in (4b) (*Two and a half bagels are on the table*), Liebesman can derive the plausible truth conditions in (14) by swapping out 2 for 2.5 in (12):

(14) $\exists x.\ \mu(\langle x, \text{bagel}\rangle) = 2.5 \wedge \text{on-the-table}(x)$
iff $\exists x.\ \text{card}(\langle x, \text{bagel}\rangle) + \psi(\langle x, \text{bagel}\rangle) = 2.5 \wedge \text{on-the-table}(x)$

Unlike (7), there's no requirement for the denotation of *bagel* to include partial entities, nor is the range of the measure function restricted to natural numbers.

Thus, by altering the way count nouns are used to measure, Liebesman solves two problems. He allows for a unified analysis of sentences with whole-number modifiers and fractional modifiers. He also explains why upper-bound 'exact' readings of whole-numbers imply that certain fractional alternatives are false.

In the next section, we discuss a similar account, that of Haida and Trinh (2021).

2.3 Haida and Trinh's account

Haida and Trinh (2021) discuss numeral modification with decimal expressions like 2.5 — see (15a). Thus, they focus on constructions that are different from those that interest us in this paper,

4 According to Liebesman (2016, p. 15), a P is a partial P whose partiality-measurement is 1. Correspondingly, it's specified that the partiality-measuring function should only measure partial Ps that aren't Ps. For simplicity of exposition, we follow everyday usage in our paper, taking 'partial P' to imply 'not P'. But nothing substantial hinges on this. The notion of partialhood is further clarified in Liebesman (Forthcoming).

e.g., (15b). We'll have more to say about decimal expressions in Section 8. However, for now, we're going to push this issue aside.

(15) a. John read 2.5 novels. (where '2.5'is pronounced 'two point five')
b. John read two and a half novels.

Although not discussed in their paper, the analysis that Haida and Trinh develop for decimals can also be used to account for fractional expressions. The resulting semantics is quite similar to Liebesman's. Under both proposals, a whole individual in the denotation of the noun is counted as one, while a part of such an individual is counted as less than one.

The main difference between the two approaches concerns when the partiality measure is defined. Unlike Liebesman, Haida and Trinh (2021, p. 281) assume that one can only count a partial individual relative to an existing whole. The measure is the proportion that the partial individual constitutes relative to this whole. For example, half of the novel *Crime and Punishment* yields a measure of 0.5, and a quarter of the novel *Emma* yields a measure of 0.25. However, the mereological sum of half of *Crime and Punishment* and a quarter of *Emma* can't be measured since there's no whole novel of which the merological sum is a part. According to Haida and Trinh (2021, p. 283), this explains why the conclusion in an argument like (16) doesn't follow from the premises, something which can't be explained under Liebesman's theory.

(16) John read half of the novel *Crime and Punishment* and then read a quarter of the novel *Emma*. # Therefore, John read 0.75 novels.

The use of decimal expressions isn't critical to the invalidity of the argument. A similar example can be constructed using only whole numbers in combination with fractions:

(17) John read the novel *The Catcher in the Rye*, a quarter of the novel *Crime and Punishment* and a quarter of the novel *Emma*. # Therefore, John read one and a half novels.

Like in (16), the premises fail to support the conclusion, despite the fact that 1+0.25+0.25=1.5.

Other than this difference in the way parts of individuals are measured, Liebesman's and Haida and Trinh's theories are equivalent. They both implement a version Salmon's general strategy. They both hypothesize that number phrases (e.g., *two and a half*, *two point five*) modify count nouns via a measure function and that this measure function is the sum of a cardinal measure (the number of whole individuals) and a partiality measure (a rational number indicating the proportion that a partial individual constitutes relative to a whole).

Crucially, Haida and Trinh make an empirical point that doesn't appear in Salmon (1997) or Liebesman (2015, 2016, Forthcoming). They report that the argument in (18) isn't valid.

(18) John read 2.5 novels and Mary read 2 novels. # Therefore, John read more novels than Mary. (Haida and Trinh 2021, p.287)

We have the same intuition about the variant in (19), which concerns us directly.

(19) John read two and a half novels and Mary read two novels. # Therefore, John read more novels than Mary.

According to them, the invalidity of such arguments stems from a general constraint on sentences with *more* when it is being used as the comparative counterpart to *many*, namely that the measure functions used in such constructions must be 'monotonic' (Schwarzschild 2006, Solt 2015a, Wellwood 2015, 2018, 2020). A measure function is monotonic if it tracks the part-whole relationship of the entities being measured: whenever x is a proper part of y, the measure of x must be strictly smaller than the measure of y.

(20) μ is **monotonic** $\equiv_{\mathrm{df}}$ $\forall x, y \in D.\ x <_p y \rightarrow \mu(x) < \mu(y)$,
where D is the domain and $<_p$ is the relation of proper part.

The monotonicity constraint is invoked to explain why only certain measures can be used to evaluate sentences like those in (21), and why only certain types of degree expressions are permitted when answering questions like (22).

(21) a. There's more water in this tub than in that tub.
b. There's as much water in this tub as in that tub.

(22) How much water is there in this tub?

The sentences in (21) can be evaluated in terms of the volume or weight of the water in the two tubs, but not in terms of their temperature. Similarly, the question in (22) can be answered with a statement about weight (e.g., *150 kilograms*) or volume (e.g., *150 liters*), but not temperature (e.g., **38 degrees*). A possible explanation of this restriction is that weight and volume are both monotonic: for any portion of water x which is a proper part of a portion of water y, the weight or volume of x is necessarily strictly less than the weight or volume of y. Temperature, on the other hand, isn't monotonic since the temperature of a proper part of a portion of water x is often the same as the temperature of the whole that it's part of.

According to Haida and Trinh, the invalidity of the argument in (16) shows that their measure function isn't monotonic. In this example, the mereological sum of half of the novel *Crime and Punishment* and a quarter of the novel *Emma* admits no measurement, since the sum isn't part of a single novel. If x is the relevant part of *Crime and Punishment* and y is the relevant part of *Emma*, then $x <_p x + y$. But, since $\mu(x + y)$ is undefined, it isn't the case that $\mu(x) < \mu(x + y)$.

However, it isn't clear what this example shows. Standard definitions of monotonicity assume that the domain of the parthood relation is the same as the domain of the entities that can be measured (i.e., the domain of μ). The universal statement in (20) is restricted to this shared domain, represented as D above. The literature on monotonicity doesn't discuss how the constraint should apply when the two domains aren't identical. In Haida and Trinh's account, the domain of the measure function is a proper subset of the domain of the parthood relation. They take this to imply that their measure function isn't monotonic. This means that they interpret the universal statement as being restricted to the domain of the parthood relation. However, another tack is possible: one could interpret the universal statement in (20) as being restricted to the domain of the measure function. If so, Haida and Trinh's measure function would end up satisfying monotonicity.[5] It's therefore difficult to draw any firm conclusion from their example.

In any case, Haida and Trinh's reasoning misunderstands the empirical scope of constraints like monotonicity put forward by Krifka (1989), Schwarzschild (2006), Wellwood (2015), and Champollion (2017). The goal of such constraints is to characterize which measure functions can appear in a variety of environments that include nominal comparatives, equatives, partitives, and quantified nouns — that is, in any type of quantity modification, mass or count, that occurs above the point where attributive adjectives combine (Schwarzschild 2006).[6] Crucially, on their

5 It would do so vacuously in terms of sums like $x + y$ above: this sum isn't in the domain of the measure function and therefore can't be used to establish a violation of monotonicity.

6 As acknowledged by Schwarzschild (2006) and Wellwood (2015, 2020), monotonicity is a necessary condition, but not a sufficient one.

view, quantified nouns include expressions like *two and a half Ns*. So, if Haida and Trinh's measure function really isn't monotonic, it shouldn't be usable in *any* of the environments of interest — comparatives, partitives, or quantified nouns.

To alleviate this conclusion, Haida and Trinh would need to put forward a novel, non-monotonic constraint — applying to all the environments mentioned above — and show that it's more successful than current proposals. However, such an account would no longer predict the invalidity of the conclusion in (18). The interpretation of *John read more novels than Mary* seems to require a measure of the novels read by John and a measure of the novels read by Mary. For Haida and Trinh, the measure of the first is 2.5, while the measure of the second is 2. Hence, the argument would be predicted to be valid after all.[7]

In the end, a lot rests on the observation that (18) — or its variant (19), the only one which concerns us directly — is an invalid argument. Thus, such examples require greater scrutiny. In the next section, we examine nominal comparatives and equatives in detail.

3 Comparatives and equatives in English

Although on the surface it might seem reasonable to hypothesize that a partiality measure mediates the relationship between *two and a half* and *novels* in a phrase like *two and a half novels*, there are two empirical hurdles that are difficult to explain under this type of analysis. The first concerns the interpretation of comparatives and equatives, which we discuss in this section. The second concerns how partial quantities are expressed in languages like French. We discuss this issue in Section 4.

3.1 The semantics of comparatives and equatives

We begin with a discussion of adjectival comparatives and equatives, and then consider their nominal counterparts. Ever since Bresnan (1975), adjectival comparative and equative sentences have been analyzed by breaking them down into three parts: the main clause, the comparative clause, and the comparative morpheme (see Cresswell 1976; Klein 1980, 1982; von Stechow 1984; Heim 1985; Kennedy 1999, among others).[8] For example, in (23a) and (23b), the main clause is *Esme is tall*, while the comparative clause is *Zoe is ~~tall~~*, with the adjective elided. In (23a), the comparative morpheme is *-er*, while in (23b), it's the first instance of *as*.

(23) a. Esme is taller than Zoe is.
     b. Esme is as tall as Zoe is.

For ease of exposition, we can interpret the main clause and comparative clause as degree predicates (see von Stechow 1984; Heim 1985; Hackl 2000). Under such an interpretation, the main clauses in (23) would denote the predicate D_{main} which is true of all the degrees d such that Esme is at least d tall, whereas the comparative clauses would denote the predicate D_{comp} which is true of all the degrees d such that Zoe is at least d tall.

7 These points also apply to Liebesman's theory.

8 This hypothesis is supported by several syntactic and semantic observations. In particular, several ellipsis tests suggest that the comparative clause is syntactically a complete sentence with varying degrees of ellipsis (Bresnan 1975; Kennedy 1999). Furthermore, certain types of scope ambiguities suggest that the morpheme of comparison can move independently of the main clause and the comparative clause (von Stechow 1984; Heim 1985; Hackl 2000).

(24) a. $D_{\text{main}} = \lambda d.\ \mu_{\text{height}}(\text{Esme}) \geq d$
b. $D_{\text{comp}} = \lambda d.\ \mu_{\text{height}}(\text{Zoe}) \geq d$

The morphemes *-er* and *as* would then be interpreted as relations between these predicates:

(25) a. $[\![\text{-er}]\!](D_{\text{main}}, D_{\text{comp}}) = 1$ iff $\textsc{max}(D_{\text{main}}) \ > \ \textsc{max}(D_{\text{comp}})$
b. $[\![\text{ as}]\!](D_{\text{main}}, D_{\text{comp}}) = 1$ iff $\textsc{max}(D_{\text{main}}) \ \geq \ \textsc{max}(D_{\text{comp}})$
where $\textsc{max}(D_{\text{main}})$ is the maximal degree satisfying D_{main}
and $\textsc{max}(D_{\text{comp}})$ is the maximal degree satisfying D_{comp}

This yields the truth conditions in (26).

(26) a. $\textsc{max}(\lambda d.\ \mu_{\text{height}}(\text{Esme}) \geq d) \ > \ \textsc{max}(\lambda d.\ \mu_{\text{height}}(\text{Zoe}) \geq d)$
b. $\textsc{max}(\lambda d.\ \mu_{\text{height}}(\text{Esme}) \geq d) \ \geq \ \textsc{max}(\lambda d.\ \mu_{\text{height}}(\text{Zoe}) \geq d)$

(26a) states that Esme's maximal degree of height is strictly greater than Zoe's maximal degree of height. (26b) states that Esme's maximal degree of height is greater than or equal to Zoe's maximal degree of height.[9] Because of competition with the comparative, (26b) is typically strengthened into a statement of equality.

Under this analysis, the adjective provides information about how the objects of comparison are measured, while the morphemes of comparison provide information about how the resulting measurements are compared. Such an analysis can easily be extended to nominal comparatives (Bresnan 1973; Cresswell 1976; Klein 1981; Hackl 2000; Schwarzschild 2006; Solt 2015b; Wellwood 2015). Consider the sentence in (27).

(27) Esme wrote as many novels as Zoe did.

Like its adjectival counterpart, this sentence can be broken down into its main clause (*Esme wrote x many novels*), comparative clause (*Zoe wrote x many novels*), and comparative morpheme (the first instance of *as*). The main clause and the comparative clause can be analyzed as degree predicates that serve as arguments to the equative morpheme. The compositional analysis can be specified as follows, where μ is the relevant measure function:[10]

(28) a. $[\![\text{many}]\!] = \lambda d.\lambda P.\lambda z.\ \mu(\langle z, P\rangle) = d$
b. $[\![\exists]\!] = \lambda P.\lambda Q.\exists x.\ P(x) \ \wedge \ Q(x)$
c. $[\![\exists\ d \text{ many } P]\!] = \lambda Q.\exists x.\ \mu(\langle x, P\rangle) = d \ \wedge \ Q(x)$

(29) a. **Main clause of (27)**:
$\lambda d.\ [\![\text{Esme wrote } \exists\ d \text{ many novels}]\!]$
$= \lambda d.\ \exists x.\ \mu(\langle x, \text{novels}\rangle) = d \ \wedge \ \text{wrote}(\text{Esme, x})$
b. **Comparative clause of (27)**:
$\lambda d.\ [\![\text{Zoe wrote } \exists\ d \text{ many novels}]\!]$
$= \lambda d.\ \exists x.\ \mu(\langle x, \text{novels}\rangle) = d \ \wedge \ \text{wrote}(\text{Zoe, x})$
c. $[\![(27)]\!] = \textsc{max}(\lambda d.\ [\![\text{Esme wrote } \exists\ d \text{ many novels}]\!])$

9 The ambiguity of sentences like *John is required to be taller than five feet* provides evidence that degree predicates are derived via movement (Bresnan 1975; von Stechow 1984; Heim 1985; Hackl 2000). The comparative morpheme moves out of the main clause, leaving a trace degree variable. Abstraction happens when the degree morpheme is re-attached after movement.

10 Hackl treats *many* as a parameterized determiner. Here, we treat *many* as a nominal modifier, much like the analysis of *many* given in Klein (1981). This is also similar to the analysis of *many/much* given by Solt (2015b) and Wellwood (2015, 2020).

$$\geq \ \textsc{max}(\lambda d.\ [\![\text{Zoe wrote } \exists\ d \text{ many novels}]\!])$$

The truth conditions in (29c) depend on how the measure function μ is defined. Traditionally, it's assumed that μ measures cardinality and thus the entity measured must be a whole member of the nominal predicate (Cresswell 1976; Klein 1981; Hackl 2000; Solt 2015b; Wellwood 2015). Liebesman (2015, 2016) and Haida and Trinh (2021) offer an alternative where a cardinal measure is combined with a partiality measure.

The analysis of the equative constructions has implications for the analysis of plural comparatives. Note that the negative counterpart of *many*, namely *few*, mirrors adjectives with respect to morphemes of comparison. Just as there are equative, comparative, and superlative versions of *tall* (*as tall*, *taller*, *tallest*), there are corresponding versions of *few* (*as few*, *fewer*, *fewest*). In principle, the grammar should be able to generate a similar paradigm for *many* (Bresnan 1973; Klein 1981; Hackl 2000). After all, the only difference between *few* and *many* is in terms of polarity. Furthermore, it's fairly common in English for paradigms of comparison to include irregularity in the comparative and superlative forms, but to be completely regular with respect to equatives. Thus, although the equative form of *good* is regular (*as good*), the comparative and superlative forms are irregular (*better* and *best*, not **gooder* or **goodest*). For this reason, Bresnan (1973) proposes that the counterparts of *fewer* and *fewest*, namely *more* and *most*, are irregular forms of the comparative and superlative version of *many* — i.e., *more* = *many+er* and *most* = *many+est*.[11] Given these observations, a nominal comparative sentence like (30a) should be analyzed in the same way as its equative counterpart, modulo the semantics of the comparative morpheme *-er*. So, (30a) is analyzed as (30b) and receives the truth conditions in (31).

(30) a. Esme wrote more novels than Zoe did.
 b. Esme wrote many+er novels than Zoe did.

(31) a. **Main clause of (30b)**:

$$\lambda d.\ [\![\text{Esme wrote } \exists\ d \text{ many novels}]\!]$$
$$= \lambda d.\ \exists x.\ \mu(\langle x, \text{novels}\rangle) = d\ \wedge\ \text{wrote(Esme, x)}$$

 b. **Comparative clause of (30b)**:

$$\lambda d.\ [\![\text{Zoe wrote } \exists\ d \text{ many novels}]\!]$$
$$= \lambda d.\ \exists x.\ \mu(\langle x, \text{novels}\rangle) = d\ \wedge\ \text{wrote(Zoe, x)}$$

 c. $[\![(30\text{b})]\!] = \textsc{max}(\lambda d.\ [\![\text{Esme wrote } \exists\ d \text{ many novels}]\!])$
$$> \ \textsc{max}(\lambda d.\ [\![\text{Zoe wrote } \exists\ d \text{ many novels}]\!])$$

As with equatives, the modifier *many* introduces the measure function, while the comparative morpheme *-er* specifies the method of comparison. Again, the truth conditions depend on how one defines the measure function μ. If one defines it in terms of cardinality, then the maximal degree is a natural number, such as 2. If one defines it in terms of Liebesman's or Haida and Trinh's partiality measure, then the maximal degree can be a rational number, such as 2.5.

As noted by Bresnan (1973) and Hackl (2000), the contribution that *many* makes to equatives and comparatives doesn't change when *many* is used outside of these constructions. Just as *many* introduces a measure function in (27) and (30a), it also introduces one in constructions like *so many novels*, *too many novels*, and *that many novels*. A sentence like (32a) should be analyzed as (32b), where *that* picks out a contextually salient degree that satisfies the degree argument of *many*.

11 A similar analysis can be developed for mass nouns, with *more* = *much+er* and *most* = *much+est*.

(32) a. Esme wrote that many novels.
b. $\exists x.\ \mu(\langle x, \text{novels}\rangle) = d \ \wedge\ \text{wrote}(\text{Esme}, x)$
where d is the demonstrated degree

Bresnan (1973) observes that overt instances of *many* are required in embedded and non-embedded questions about number:

(33) a. I know how many novels Esme wrote. Two.
b. How many novels did Esme write? Two.

To account for this, she hypothesizes that *many* always mediates the relationship between a degree and a count noun, but is phonologically elided when it appears adjacent to an overt numeral. Thus, (34a) can be analyzed as (34b) and receives the truth conditions in (34c).

(34) a. Esme wrote four novels.
b. Esme wrote four ~~many~~ novels.
c. $\exists x.\ \mu(\langle x, \text{novels}\rangle) = 4 \ \wedge\ \text{wrote}(\text{Esme}, x)$

As an added benefit, this type of analysis provides a simple explanation for why the same constraints on measurement apply to various types of nominal expressions: these constraints regulate the interpretation of *many*, which introduces the measure function that is used in comparatives, equatives and other types of nominal modification by degree-denoting phrases (Schwarzschild 2006; Solt 2015b; Wellwood 2015, 2020).

Judgments concerning arguments like (19) now become critical.

(19) John read two and a half novels and Mary read two novels. #Therefore, John read more novels than Mary.

If the measure function introduced by *many* is a partiality measure (as claimed by Liebesman and Haida and Trinh), then the novels read by John will be measured as 2.5 and those read by Mary as 2, so the argument is predicted to be *valid*. If the measure function is cardinality, the argument is predicted to be *invalid* – modulo an appropriate, alternative analysis of *two and a half novels*, as provided in Section 5. At this point, we'd like to stress the following. Bresnan's hypotheses are very attractive, so we've adopted them, following the literature. But it seems to us that *any* sufficiently plausible analysis of plural comparatives and equatives and of numeral modification will make the very same predictions.

In the next section, we discuss judgments about comparatives and equatives thoroughly.

3.2 Judgments about comparatives and equatives

Evaluating judgments about comparatives and equatives requires careful methodological controls. There are a couple of confounding factors. First, the choice of verb can interfere with speakers' judgments. Second, the choice of certain nouns (e.g., nouns denoting food[12]) can lead to an atypical *measure* reading. We discuss each point in turn, before presenting the results of consultation sessions with 15 English speakers. We expand on these initial results with an experiment in Section 3.3.

12 Although most of the examples given by Snyder (2021) and Snyder and Barlew (2019) involve food (particularly fruit), there are some examples of other types of nouns. For example, Snyder and Barlew (2019) provide a context where the noun *tire* can be coerced into a measure reading (p. 801-2). What seems to be critical for such a reading is whether it makes sense to talk about "*n* NOUNS worth of NOUN" (e.g., two apples-worth apple, two tires-worth of tire, etc.).

When collecting data involving partial objects, the choice of verb is critical. For example, take the verb *to see*. If somebody could see only half of a man (the other half being hidden by a desk, say), this would still count as seeing a man. In most contexts, seeing part of an object counts as seeing the object. Other types of verbs have similar implications. Take the verb *to listen*. At first blush, one might doubt whether listening to a half of a song would count as listening to a song. However, this implication does arise in certain contexts. To illustrate, suppose John listened to half of the song *Peace Train* by Cat Stevens while driving to work. In this context, (35) is true.

(35) John listened to a song about peace this morning.

These types of implications can make it difficult to assess whether comparatives are sensitive to partial measurement. For example, consider a context where John listened to two whole songs and three quarters of a third song, whereas Sue only listened to two whole songs. What can we conclude if a speaker judges (36) to be true in such a scenario?

(36) John listened to more songs than Sue did.

Unfortunately, not much. The speaker's judgment may reflect the use of partial measurement, but the speaker may also be treating the fact that John listened to three quarters of a third song as listening to an extra song (i.e., conclude that John listened to three songs).

Thus, to properly evaluate judgments about comparatives and equatives, test sentences should only include verbs that resist implications from parts to wholes — namely, verbs that clearly yield a false statement when inserted into the template in (37).

(37) If John VERB-ed half of a NOUN, then John VERB-ed a NOUN.[13]

Verbs of creation usually satisfy this requirement. For example, if John only wrote half of a novel, it's clearly false that he wrote a novel. If John only knitted half of a sweater, it's clearly false that he knitted a sweater.[14] For this reason, we'll use such verbs when assessing comparative and equative constructions.

The choice of noun is also important. As noted by Snyder and Barlew (2019) and Snyder (2021), certain count nouns permit an atypical interpretation in terms of volume/weight. Snyder and Barlew refer to this interpretation as the *measure reading*. Consider sentence (38) in the two contexts given in (39).

(38) There are two apples in the marmalade.

(39) a. **Individuating context**: Mary opens a big jar of orange marmalade and puts two apples in it. One is big and the other is small.
 b. **Measuring context**: Mary cuts four average-sized apples into small bits. She takes half of those bits and cooks them slowly with water and sugar to obtain some delicious apple marmalade.

13 A reviewer suggested that the template in (37) might simply be a way of testing whether a thematic relation connected to the verb has Krifka's (1992) "Uniqueness of objects" property (i.e., a thematic relation R has the uniqueness of objects property if and only if $\forall e, x, x'.[R(e,x) \wedge R(e,x') \rightarrow x = x']]$). Although (37) might test a formal semantic property such as this, it is also possible that the truth of sentences that fit the template in (37) arises from some kind of pragmatically lax interpretation or "loose talk" as often occurs with quantifiers (e.g., the acceptance of statements applying to "everyone" or "no one" in circumstances where there are clear counterexamples, as in "No one brings a lunch to school mom, everyone buys something at the cafeteria").

14 Not all verbs of creation render the template in (37) clearly false. Suppose John made half of a Caesar salad. In such a scenario, it isn't clear whether John made a Caesar salad or not.

The measure reading of (38) can be paraphrased as in (40a). Its more typical *individuated* reading, where whole apples are counted, can be paraphrased as in (40b).

(40) a. There are two apples worth of apple in the marmalade.
b. There are two whole apples in the marmalade.

Snyder and Barlew claim that, in the individuating context, (38) is true under the individuated reading, but false under the measure reading. Whereas in the measuring context, the sentence is true under the measure reading, but false under the individuated reading. Furthermore, they argue that this ambiguity carries over to expressions like *two and a half apples*. The individuating reading says that there are two apples and a half apple. The measure reading says that there are two and a half apples worth of apple stuff.

Crucially, the measure reading is only available for certain types of terms, such as those denoting food. Consider an artifact noun like *bike*. Although it's possible to disassemble four bikes and smash their parts into bits, one can't take an arbitrary half of this smashed up pile of bike bits and claim that it consists of two bikes. Nor does it make sense to refer to the half pile as two bikes worth of bike.

In our debate, Liebesman, Haida and Trinh and us are only concerned with the normal reading of count nouns, which Snyder and Barlew call the individuated reading. We only look at contexts in which, say, there are clearly two whole apples and half of an apple. The question is whether, on a normal/individuated reading of *apple*, the interpretation of *two and a half apples* involves a partiality measure or just cardinality. Still, the measure reading of nouns denoting food introduces a possible confound when assessing judgments about plural comparatives and equatives. In particular, contexts that satisfy the truth conditions of an individuated reading often also satisfy the truth conditions of a measure reading. For example, in contexts where it's true that there are two whole apples on the table and one half apple, it's also often true that there's two and a half apples worth of apple on the table since apples are often of a comparable size. Thus, it's better not to use such terms when collecting data about such constructions.

With these various controls in mind, we gathered data from 15 English speakers from Montréal, Canada (ages 18 to 56), using the creation verbs *built*, *drew*, *knitted*, and *wrote* in combination with the count nouns *car*, *circle*, *sweater*, and *novel*, respectively. Consultation sessions were done orally; however, critical test sentences with their contexts were also written down using a whiteboard or chalkboard. Each consultation session began with a pretest sentence, like the one in (41), to make sure that the consultant was unlikely to infer that interacting with a partial object implied interacting with the whole object. We only continued the consultation session for the particular verb under discussion if the consultant had strong intuitions that the pretest sentence was false.

(41) **Pretest sentence**: If Zoe knitted half of a sweater, then Zoe knitted a sweater.

After the pretest sentence, we presented consultants with a context in which one character (Zoe) did the verbal action to two and a half instances of the nominal kind (e.g., knitted two sweaters and half of a sweater), whereas the other character only did the verbal action to two instances of the nominal kind (e.g., knitted two sweaters). We also included other objects that didn't belong in the nominal kind in order to balance out the number of objects that the two characters interacted with in total (e.g., baby onesies with respect to knitted sweaters). An example of the context used for knitting sweaters is given in (42).

(42) **Context**: Zoe knitted a green sweater, a red sweater, half of a blue sweater, and a blue onesie. Esme knitted a purple sweater, a yellow sweater, a blue onesie, and a red onesie.

We then asked consultants to judge three sentences, with the possibility of responding *true*, *false*, or *unsure*. One sentence was a comparative statement where it was asserted that the character who interacted with the extra half had VERB-ed more NOUN-s. Another sentence was an equative statement where it was asserted that the character who didn't interact with any partial object had VERB-ed as many NOUNS-s as the character who interacted with the extra half-object. A third sentence asserted that one of the characters VERB-ed two and half NOUN-s. This third sentence established whether consultants were cognizant of the interaction between the characters and the partial object. Examples with knitting sweaters relative to the context in (42) are given in (43).

(43) a. **Comparative test sentence**: Zoe knitted more sweaters than Esme did.
b. **Equative test sentence**: Esme knitted as many sweaters as Zoe did.
c. **Partiality check**: Zoe knitted two and a half sweaters.

If consultants judged the comparative sentence to be true and the equative to be false while recognizing that the partiality check was true, then this would suggest that they were using a partiality measure when evaluating these sentences. In contrast, if they judged the comparative to be false and the equative to be true while still recognizing that the partiality check was true, then this would suggest that consultants were only considering cardinality measures. Any other pattern would be inconclusive.

Consultants agreed that all pretest sentences were false and that all partiality checks were true. Comparative test sentences were consistently judged to be false (true = 1, false = 49, unsure = 10) whereas equative test sentences were consistently judged to be true (true = 52, false = 0, unsure = 8).[15] Thus, these results suggest that partiality measures aren't at play in comparatives and equatives. We give a complete list of the pretest sentences, contexts, and test sentences in Appendix A.

In the next section, we expand on these results with an online experiment that tested how speakers judged comparatives relative to a picture that contains both whole and partial instances of a nominal kind.

3.3 Experiments with partial objects

In the consultation sessions described in the previous section, we presented our contexts orally without any visual stimuli. This was by design. As discussed in Winter (2022), English speakers sometimes coerce comparatives into a reading that evaluates volume or weight, even when the relevant comparison involves plural count nouns. Such readings are irrelevant with respect to the issue at hand. Both cardinality and partiality measures are independent of volume and weight (e.g., it's possible for John to have more apple-stuff than Sue while having fewer apples in number).

15 A reviewer questioned the robustness of these results given that there were 18 "unsure" judgments out of 120. Note, in any formal or informal experiment, there is always some degree of "experimental noise." Consultants usually answered several questions in sequence with fairly complex contexts that were relatively difficult to keep track of. We predicted, given previous experience with these types of consultation sessions, that some participants might experience some judgment fatigue, especially at the end of a consultation session. The "unsure" response was included as a way of not forcing the speaker into a true/false judgment, thus increasing the fidelity of the "true" response.

Without visual stimuli, consultants weren't able to easily estimate volume or weight, thus making it less likely that they based their assessment of comparatives or equatives on these measures.

However, in addition to consultation sessions, we conducted an online experiment using visual stimuli. To control for potential interference due to estimations of volume or weight, we used sentences that compared objects of vastly different sizes as in (44), although our test sentences were presented in a question format as in (45).

(44) a. There are more bikes than (there are) cars.
b. There are more chairs than (there are) tables.

(45) a. Are there more bikes than cars?
b. Are there more chairs than tables?

With objects of different sizes, we could readily detect if participants compared objects via weight or volume instead of number. Participants who use such dimensions should always judge the tables/cars as being *more* than the chairs/bikes independently of any differences in the number of objects.

Although the sentences in (44) and (45) involve two different plural nouns, the analysis of these sentences doesn't differ from the general analysis of comparatives discussed in Section 3.1. Thus, the sentence in (44a) can still be broken down into its main clause (*there are x many bikes*), comparative clause (*there are x many cars*), and comparative morpheme (*-er*). The resulting truth conditions are sketched out in (46).

(46) $\textsc{max}(\lambda d.\ [\![\text{there are } d \text{ many bikes}]\!]) > \textsc{max}(\lambda d.\ [\![\text{there are } d \text{ many cars}]\!])$
$= \textsc{max}(\lambda d.\ \exists x.\ \mu(\langle x, \text{bikes}\rangle) = d) > \textsc{max}(\lambda d.\ \exists x.\ \mu(\langle x, \text{cars}\rangle) = d)$

These truth conditions state that the greatest degree to which the bikes can be measured is higher than the greatest degree to which the cars can be measured. The goal of the experiment was to test whether partial objects influenced the comparison.

We tested 30 monolingual English speakers from Canada and United States (recruited through Prolific; www.prolific.com) using the online experimental platform *Gorilla Experiment Builder* (www.gorilla.sc). We had four test sentences: the two already given in (45), as well as their counterparts in (47).

(47) a. Are there more cars than bikes?
b. Are there more tables than chairs?

Each question was presented via audio and written text. The questions were accompanied by a picture and participants were asked to choose between the options "yes", "no," and "I don't know" given the information presented in the picture.

Each question was asked four times corresponding to four different trial types: control true, control false, half-less, and test. The two control trials essentially tested whether participants understood the experimental paradigm. In the control true trials, the picture depicted a scene where the answer to the question should be "yes" on both a cardinal interpretation and a partial measure interpretation, although critically not under a coerced volume/weight interpretation. See the example in Figure 1a. In contrast, in the control false trials, the picture depicted a scene where the answer to the question should be "no" on both a cardinal interpretation and a partial measure interpretation, although once again critically not under a coerced volume/weight interpretation. See the example in Figure 1b.

In the half-less trials, the cardinal measure of the main clause is 0.5 more than the maximal

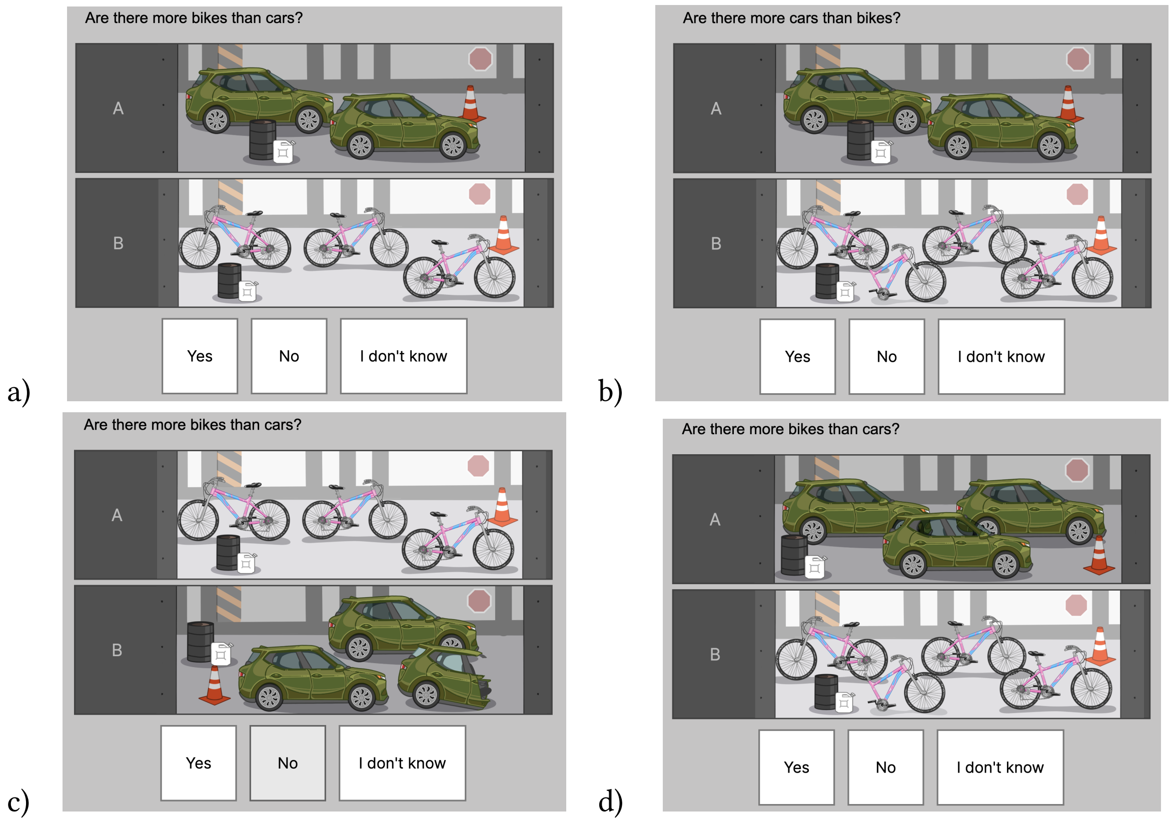


Figure 1: Examples of the four different trial types: a) Control true, b) Control false, c) Half-less, d) Test.

potential partial measure of the comparative clause (e.g., 3 bikes vs. 2.5 cars for a question like *Are there more bikes than cars?*). See the example in Figure 1c. This trial type was added to make sure that participants weren't counting halves as wholes. Like the control-true trials, participant are expected to answer "yes" whether they're using a cardinality measure or a partiality measure. However, they would be expected to answer "no" if they counted the half object as being equivalent to one whole object.

In the critical test trials, the potential maximal partial measure of the main clause is 0.5 more than the maximal cardinal measure of the comparative clause (e.g., 3.5 bikes vs. 3 cars for a question like *Are there more bikes than cars?*). Participants were expected to answer "yes" if they interpreted the sentence using a partiality measure but "no" if they interpreted the sentence using a cardinality measure. See the example in Figure 1d.

In total, there were 16 picture-question pairs per participant: 4 control true, 4 control false, 4 half-less, and 4 test. The trials were presented in a random order to each participant. The results are shown in Figure 2 in the bar graph on the left, labeled "No Prime" (data and R-scripts are available at https://osf.io/9vbta/). The control trials are near ceiling: 99% "yes" responses for the control-true trials and 99% "no" responses for the control-false trials. These results not only demonstrate that the participants understood the task, but also that they weren't comparing objects via weight or volume.

The test trials initially look like they're in favor of a partiality measure: 84% "yes" responses

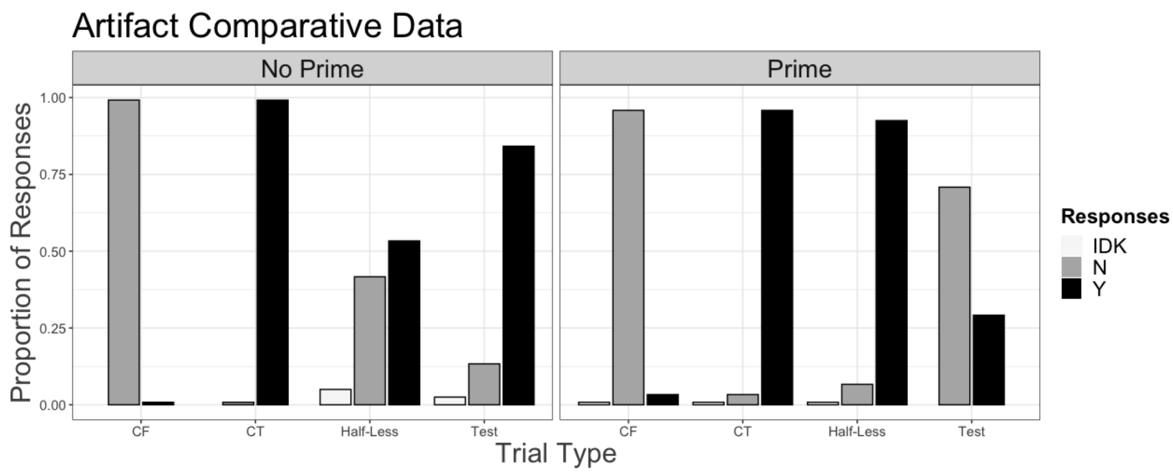


Figure 2: Percentage of responses in the No Prime and Prime conditions.

and only 13% "no" responses (3% "I don't know"). However, these results should be interpreted relative to the responses to the half-less trials. Both cardinality measures and partiality measures predict that participants should say "yes" in these trials, yet the number of "no" responses is quite high in this condition: 42% "no," 53% "yes," 5% "I don't know." Clearly, many of the participants are counting the half-object as if it were a whole object.

Given previous experiments with partial objects, these results aren't surprising. It's well known in the developmental literature that children (ages 3 to 6 years old) often count a fork broken in half as being "two forks" or a shoe broken into three pieces as being "three shoes", even when they previously counted the unbroken object as being "one" (Shipley and Shepperson 1990; Brooks, Pogue, and Barner 2011; Srinivasan et al. 2013; Syrett and Aravind 2022). This type of "mistake" even influences the way children evaluate comparatives with *more*. For example, if one character has a single shoe broken into three pieces but another character has two whole shoes, children will often say that the character with three broken pieces has "more shoes" than the person with two whole shoes (Brooks, Pogue, and Barner 2011).

As discussed in Syrett and Aravind (2022), although adults are less likely to count a broken fork as "two forks," there are other experimental tasks where they do treat partial objects as wholes. For example, when shown a picture with two characters, where one character has a whole object (e.g., a bowl) and another character has a partial object (e.g., half of a bowl), adults judged that both characters have a whole object (e.g., both characters have a bowl) around 66% of the time. Similarly, in a simple counting task, where adults were asked to count the number of instances of a given kind within a picture (e.g., being asked "Count the bowls" relative to a picture that contained instances of whole-bowls and partial bowls), adults counted the partial instances as wholes (e.g., halves of bowls as being one bowl) over 80% of the time (Syrett and Aravind 2022, p.257). Our experiments confirm that similar behaviour is exhibited when evaluating comparative statements.

Fortunately, there are experimental manipulations that reduce this tendency. For example, Syrett and Aravind (2022) found that if the experimenter made it clear that the speaker had a goal that involved distinguishing partial objects from whole objects (e.g., they needed a bowl to eat soup), then children were less likely to classify partial objects as instances of the nominal kind (e.g., partial bowls as being an instance of a bowl). Furthermore, Srinivasan et al. (2013, experiment 3) found that if children performed a task where they had to distinguish what was a whole instance of a kind vs. a partial instance, then they tended not to count partial instances as wholes in subsequent tasks. For example, in one of Srinivasan et al.'s trials, children were shown a picture of a whole sock beside a picture of a partial sock. The children were then asked "Can you point to the sock?" as well as "Can you point to the piece of a sock?". After doing this task, the children were then given a counting task where they were asked to "count the socks" given a picture with whole socks and pieces of socks. The children who did the pointing task before the counting task were much less likely to count partial instances of socks as "one sock" compared to children who only did the counting task.

With these manipulations in mind, we conducted a second experiment with a different group of 30 monolingual English speakers from Canada and the United States (ages 18 to 40, recruited via Prolific; www.prolific.com). This time we added a priming task before the comparative judgment task. In the priming task, participants were first introduced to a speaker (Farmer Brown). They were told that the speaker needed to buy some bikes and cars to travel around town. Critically, the bikes and cars needed to be in working order. As far as the speaker was concerned, half of a car wasn't a car and half of a bicycle wasn't a bicycle. Similarly, participants were informed that the speaker needed to buy some tables and chairs to use at his farm. Once again, the objects needed to be in working order. As far as the speaker was concerned, an incomplete chair wasn't a chair and half of a table wasn't a table.[16] (See Appendix B for Farmer Brown's full statement.)

Next, participants had to do a task where they were shown pictures of whole instances of cars, bikes, tables and chairs (6 trials), as well as partial instances of cars, bikes, tables and chairs (6 trials). The pictures used in this task were identical to the pictures used in the task with the comparative sentences. For each picture, participants were asked whether the picture was a whole or partial instance of the nominal kind (e.g., "Is this a picture of a car or half of a car?"). The participants were given the option of choosing a phrase that represented the whole-object (e.g., "a car") or choosing a phrase that represented the partial object (e.g., "half of a car"). The participants had to answer the question correctly before moving on to the next question (12 questions in total, see Appendix B for two examples). Feedback was given in terms of a red cross or a green check-mark.

After completing this priming task, participants did the exact same comparative judgment task from the previous experiment. The results are shown in the bar graph on the right in Figure 2, labelled "Prime". As can be seen, the priming manipulations made the half-less trials look much more like the control true trials (control true = 96% "yes", 3% "no", 1% "I don't know"; half-less = 92% "yes", 7% "no", 1% "I don't know"), thus demonstrating that participants were much less likely to count partial objects as whole objects. In other words, the experimental manipulations worked.

However, the priming task also had an effect on the test trials. Critically, the "no" answers are now much more common than the "yes" answers (71% "no" vs. 29% "yes"). To analyze the

16 A reviewer suggested that Farmer Brown's interest in purchasing whole objects rather than partial objects might lead participants to ignore partial objects in the rest of the experiment. However, this possibility seems unlikely given that the second priming task repeatedly asks participants to identify and classify partial objects as being partial (e.g., identify half of a car as "half of a car"). Furthermore, the critical test sentences did not involve any mention of "buying" nor Farmer Brown. Still, further experiments could control for this potential confound by testing to see, for example, if an interest in purchasing one sub-kind (e.g. purchasing a sports car vs. a sedan) would lead participants to not include other sub-kinds as members of the super-ordinate category (e.g. treating sedans as if they are not "cars").

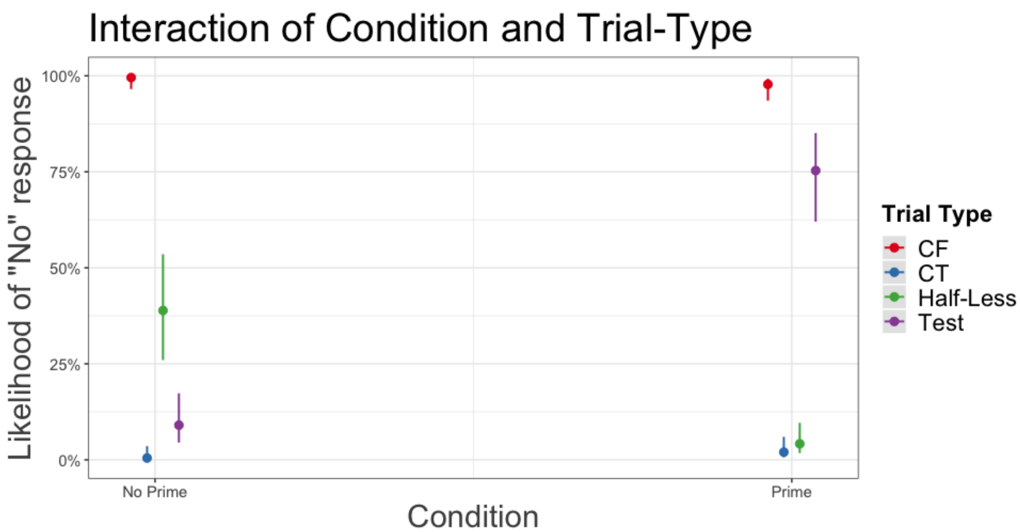


Figure 3: Model's prediction of "no" responses relative to trial type and condition. The "No Prime" condition is on the left and the "Prime" condition is on the right. Note that the likelihood of a "no" response lowers for the "Half-less" trial-type, but raises for the "Test" trial-type. All other trial types basically remain the same.

difference between these two experiments (No Prime vs. Prime), we constructed a generalized linear mixed-effects model with the glmer package (Bates et al. 2015) in R (R Core Team 2023). Our model predicted the proportion of "no" responses as a binary outcome of Condition (No Prime, Prime), Trial type (Control True, Control False, Half-less, Test), and their interaction. The model resulted in a singular fit when random intercepts of both participant and item were included, and failed to converge when we added random slopes of any sort. Thus, only the random intercepts for participants are included. This model revealed a significant interaction between the Prime condition and the Test trial-type in comparison to a model without any interaction (β = 5.03, SE = 1.21, $\chi^2(3)$ = 148.95, p < .001). As shown in Figure 3, this was primarily due to the increase in "no" responses in the Test trial-type from the No Prime condition to the Prime condition. This pattern of results is inconsistent with the hypothesis that participants readily use partiality measures. However, such a pattern is consistent with the hypothesis that participants primarily use cardinal measures whose detection in experimental paradigms is sometimes obscured by experimental noise.

These experimental results by themselves are not conclusive. Several questions remain. In particular, the priming condition did not completely remove all judgements of "yes" in the test condition. Does this mean that some small portion of the participants use partial measures? We suggested above that this is not the case, that the remaining "yes" responses are due to lingering experimental noise. We believe that this is not an unreasonable interpretation. It is unlikely that the priming condition would be able to remove all tendencies to treat partial objects as wholes (note, participants still responded "yes" 7% of the time even in the half-less trials in the Prime condition). However, these issues are ultimately empirical and thus should be teased apart by further experimentation.[17]

17 A reviewer pointed out that our experiment fails to test a critical condition, one where two pictures differ by a small

Still, the results from these experiments should be considered in combination with the results from the survey reported in Section 3.2, where potential noise factors were less likely given the lack of visual stimuli. Recall that out of 120 responses to comparative and equative statements, there was only 1 response that was consistent with a partial measure. Overall, the data suggests that partial measurements are not easily accessible in these types of constructions.

4 Fractions, comparatives and equatives in French

In this section, we consider fractions, comparatives and equatives in French. As we'll see, they mostly exhibit the same pattern as their English counterparts, but with one crucial difference: nominal expressions with fractions contain no constituent that could denote a rational number like 2.5.

As indicated by Nicolas (2016, pp. 111–112), in French, the literal translation of *two and a half apples* is ungrammatical, only the equivalent of *two apples and a half* is possible:

(48) a. *deux et demie pommes
`two and half apples`

b. deux pommes et demie
`two apples and half`
'two and a half apples'

A critical clue in analyzing such expressions is that, at least in written French, the word *demi(e)* receives the same grammatical gender as the noun preceding it — compare the case of the feminine noun *pomme* to that of the masculine noun *poireau* ('leek') in (49).

(49) a. deux pommes et demie (*demi)
`two apples and half-fem (*half-masc)`
'two and a half apples'

b. deux poireaux et demi (*demie)
`two leeks and half-masc (*half-fem)`
'two and a half leeks'

This suggests that *demi(e)* acts as an adjective modifying a silent copy of the noun (e.g., *deux pommes et demie ~~pomme~~*, *deux poireaux et demi ~~poireau~~*).[18]

vs. large partial measure. For example, consider one picture where there are two cars plus a quarter of a car, and another where there are two and a half bikes. In such cases, a partial-measure, unlike a cardinal measure, would predict a "true" response to the question "Are there more bikes than cars?" Critically, many potential factors that would lead to experimental noise (such as those discussed above) should not lead to a bias towards a "true" response. The only difficulty in building such an experimental condition is constructing pictures that reliably distinguish a half versus a quarter.

18 According to some native speakers, the same might hold for Italian as spoken in the south of the country. The interesting difference with French would then be that, in Italian, gender differences correspond to phonological differences (*mezza* versus *mezzo*).

(i) a. due mele e mezza
`two apples and half-fem`
'two and a half apples'

b. due limoni e mezzo
`two lemons and half-masc`
'two and a half lemons'

The construction *n Ps et demi(e)*, where *n* is a cardinal numeral, can be used with any count noun *P* whenever one can make sense of *un(e) demi-P* (‘a half P’) or *la moitié d’un(e) P* (‘half of a P’). Sometimes what counts as a half depends on there being a *P* that is divided into two, other times all that is required is that half of a *P* has been constructed even when the *P* doesn’t exist, as shown in (50).

(50) En une heure, ces ouvriers ont assemblé cinq voitures et demie.
In one hour, these workers have assembled five cars and half-FEM
‘In one hour, these workers assembled five and a half cars.’

The use of a fractional term other than *demi(e)* is extremely awkward, as shown by the unacceptable phrases with *quart* (‘a quarter’) and *tiers* (‘a third’).

(51) a. *deux pommes et quart
two apples and quarter

b. ??deux pommes et un quart
two apples and a quarter

c. *trois gâteaux et tiers
three cakes and third

d. ??trois gâteaux et un tiers
three cakes and a third

In fact, *quart* is acceptable only when it’s used with the noun *heure* (‘hour’), although such an exception doesn’t extend to *tiers*.[19]

(52) a. deux heures et quart
two hours and quarter
‘two hours and 15 minutes’

b. *deux heures et tiers
two hours and third

Thus, the distribution of fractional expressions in these types of counting constructions is very limited.

Critical to the issue at hand, the interpretation of expressions like *deux pommes et demie* is identical to that of *two and a half apples*. All the judgments pointed out by Liebesman and Haida and Trinh concerning English sentences also hold with respect to their French equivalents, such as (53).

(53) Il y a deux pommes et demie sur la table.
It there has two apples and half on the table
‘There are two and a half apples on the table.’

---

19 The fractional terms *quart* and *tiers* can’t be used with measurement terms such as *mètre* and *kilo*. This is demonstrated for *quart* below; similar judgments hold for *tiers*.

(i) a. *deux mètres et quart
two meters and quarter

b. *deux kilos et quart
two kilos and quarter

Moreover, the interpretation of plural comparatives and equatives in French seems to be the same as their English counterparts. Following the methodology described in Section 3.2, we tested six speakers from Montréal, all of them bilingual, but with French as a first language. The sentences we used employed four different verb-noun combinations, the translations of those given in Appendix A. Here's one example, where the verb is *dessiner* ('to draw') and the noun is *cercle* ('circle').

(54) **Pretest sentence**: Si Zoé a dessiné la moitié d'un cercle, alors Zoé a dessiné un cercle.
'If Zoe drew half of a circle, then Zoe drew a circle.'

(55) **Context**: Zoé a dessiné un cercle vert, un cercle rouge, un demi-cercle violet, et un triangle bleu. Esmé a dessiné un cercle violet, un cercle jaune, un triangle bleu, et un triangle rouge. 'Zoe drew a green circle, a red circle, a purple half-circle, and a blue triangle. Esme drew a purple circle, a yellow circle, a blue triangle, and a red triangle.'

a. **Comparative**: Zoé a dessiné plus de cercles qu'Esmé.
'Zoe drew more circles than Esme.'

b. **Equative**: Esmé a dessiné autant de cercles que Zoé.
'Esme drew as many circles as Zoe.'

c. **Partiality check**: Zoé a dessiné deux cercles et demi.
'Zoe drew two and a half circles.'

Similar to the results in English, speakers judged all of the pretest sentences to be false and all of the partiality check sentences to be true. The speakers also unanimously judged the equative sentences to be true. As for the comparative sentence, the judgment depended on noun-verb pairing. In the example given in (55a), all speakers judged it false. In two other cases (*a construit plus de voitures* 'constructed more cars', *a écrit plus de romans* 'wrote more novels'), it was judged false by three and four speakers, respectively. The other speakers were 'unsure' about the sentence's truth-value. In another case (*a tricoté plus de chandails* 'knitted more sweaters'), speakers perceived the comparative to be ambiguous between a substance/mass reading and a non-substance/count reading, but judged the sentence to be false under the non-substance/count reading. (Note, unlike English, there's no phonological difference between mass and count comparatives.)

In brief, French sentences containing a nominal expression like *deux cercles et demi*, a plural comparative like *plus de cercles que* or an equative like *autant de cercles que* seem to have the same interpretations as their English counterparts. But as shown in (48) above, the expressions *deux* and *demi(e)* don't combine to form a constituent that could denote the number 2.5. So, an account of the French data must differ from an analysis based on a partiality measure like Liebesman's and Haida and Trinh's proposals for the English construction *two and a half*.[20] In the next section, we provide an account of English. In Section 6, we'll show that it also applies to French, thereby providing a unified analysis of the two languages.

5 Conjunction and constituency in English

As we've seen, Liebesman (2015, 2016, Forthcoming) and Haida and Trinh (2021) argue that there's only one measure function in expressions such as *two and a half novels*, and that this function

20 At first glance, the same appears to be true of other romance languages like Italian and Romanian.

connects the nominal predicate *novels* to a degree argument expressed by *two and a half*, yielding a predicate that is true of sums consisting of two and a half novels, as shown in (56).

(56)

$$
\begin{forest}
[{$\lambda z.\mu(\langle z, \textsc{Novel}\rangle) = 2.5$ \\ $= \lambda z.\ (\mathrm{card}(\langle z, \textsc{Novel}\rangle) + \psi(\langle z, \textsc{Novel}\rangle)) = 2.5$}
  [{$\lambda P.\lambda z.\mu(\langle z, P\rangle) = 2.5$}
    [{$2{+}0.5$ \\ $= 2.5$}
      [{$[\![\text{two}]\!]$ \\ $= 2$}]
      [{$\lambda n.\ n{+}0.5$}
        [{$[\![\text{and}_{+}]\!]$ \\ $= \lambda m.\lambda n.\ n + m$}]
        [{0.5}
          [{$[\![\text{a half}]\!]$ \\ $= 0.5$}, roof]
        ]
      ]
    ]
    [{$[\![\cancel{\text{many}}]\!]$ \\ $= \lambda d.\lambda P.\lambda z.\mu(\langle z, P\rangle) = d$}]
  ]
  [{$[\![\text{novels}]\!]$ \\ $= \textsc{Novel}$}]
]
\end{forest}
$$

These authors suggest that *and* can be interpreted as numerical addition, as symbolized by ‘and$_+$’ above. Here, we argue for an analysis similar to those proposed by Ionin, Matushansky, and Ruys (2006), Snyder and Barlew (2019),[21] and Landman (2020). Like these authors, we hypothesize that *and* is interpreted as disjoint mereological summation. Under such an analysis, *two and a half* doesn’t denote a degree, and thus there is no need to abandon cardinality.

To begin, let’s first consider a different type of construction where the conjunctive morpheme, at least on the surface, can’t be interpreted as numerical addition. For example, the sentence in (57) contains a complex object composed of a modified noun phrase *two novels* in addition to a quantity expression *three chapters*.

(57) Esme read two novels and three chapters.

Such a construction is quite distinct from examples like *two and a half novels*. For starters, the numeral *two* and the quantity expression *three chapters* don’t form a constituent independent of the noun phrase *novels*. Furthermore, even if the numeral and the quantity phrase did form a constituent, it doesn’t seem plausible that they could be combined by numerical addition. Among other difficulties, quantity expressions such as *three chapters* don’t denote a degree that is on the same scale as numerals such as *two*. Thus, a different type of analyses is required.

Before providing such an analysis, we would first like to highlight two important empirical facts with regards to (57). First, quantity terms like *chapter* exhibits a certain degree of context sensitivity. For example, the meaning of the question in (59) changes depending on whether it’s preceded by the statement in (58a) or the one in (58b) — the first triggering a ‘chapter of a novel’ inquiry; the second a ‘chapter of a textbook’ inquiry.

(58) a. There are several novels on your bookshelf.
b. There are several textbooks on your bookshelf.

21 Snyder and Barlew (2019) provide two analyses of these types of constructions, one almost identical to the analysis given by Ionin, Matushansky, and Ruys (2006) and another that involves a type of coerced *grinding*. We discuss this distinction in more detail in Section 7.3.

(59) Could you read a chapter to me?

Plausibly, this context sensitivity could be due to ellipsis.[22] Indeed, the possible interpretations of (59) are identical to the two questions in (60), where *chapter* takes an overt partitive as a complement.

(60) a. Could you read a chapter of a novel to me?
b. Could you read a chapter of a textbook to me?

Second, the interpretation of (57) doesn't permit overlap. For (57) to be true, the three chapters that Esme read must be part of a third novel. Non-overlap is a common property of conjunction when it's used to combine mereological sums together to create larger sums, especially with respect to the formation of arguments for collective predicates. For example, consider the conjoined phrase *four lawyers and two doctors* in (61).

(61) Four lawyers and two doctors had dinner together.

Although it's possible for an individual to be both a doctor and a lawyer, the subject in (61) entails that six people had dinner together (instead of entailing a lower bound of four with the possibility of five). So when *and* is used to combine sums together to form (potentially) larger sums, such a combination seems to require a lack of overlap between the combined sums.

There are different ways one could capture this pattern. For example, Ionin and Matushansky (2006) pursue a pragmatic explanation based on Grice's Maxim of Manner. In contrast, Carlson (1987) discusses a more general semantic approach. How this ban on overlap is implemented isn't our main concern here. To keep the analysis simple and precise, we will follow Landman (2020, 2004) by defining a disjoint summation operator as in (62), where $\uplus$ represents the mereological sum of two disjoint entities.[23]

(62) $[\![\text{and}_{\uplus}]\!] = \lambda P.\lambda Q.\lambda x.\exists y.\exists z.\ \ x = y \uplus z \wedge Q(y) \wedge P(z)$

With such an operator, the phrase *four lawyers and two doctors* would have an interpretation similar to the one given in (63), where it's presupposed that if $\text{card}(\langle x, P\rangle) = n$ is true for any x, n and P, then $P(x)$ is true.

(63) $\lambda x.\ \exists y.\exists z.\ x = y \uplus z \wedge \text{card}(\langle y, \textsc{Lawyer}\rangle) = 4 \wedge \text{card}(\langle z, \textsc{Doctor}\rangle) = 2$

$\lambda x.\text{card}(\langle x, \textsc{Lawyer}\rangle) = 4$

$[\![\text{four lawyers}]\!]$

$\lambda Q.\lambda x.\exists y.\exists z.\ x = y \uplus z \wedge Q(y) \wedge \text{card}(\langle z, \textsc{Doctor}\rangle) = 2$

$[\![\text{and}_{\uplus}]\!] = \lambda P.\lambda Q.\lambda x.\ \exists y.\exists z.\ x = y \uplus z \wedge Q(y) \wedge P(z)$

$\lambda x.\ \text{card}(\langle x, \textsc{Doctor}\rangle) = 2$

$[\![\text{two doctors}]\!]$

Assuming existential quantification over this complex noun phrase (Winter 2002), one would derive the accurate truth conditions for (61) given in (64).

(64) $\exists x.\exists y.\exists z.\ x = y \uplus z \wedge \text{card}(\langle y, \textsc{Lawyer}\rangle) = 4 \wedge \text{card}(\langle z, \textsc{Doctor}\rangle) = 2 \wedge \text{had-dinner-together}(x)$

22 For a more thorough discussion of this type of phenomenon, see Gagnon (2013).

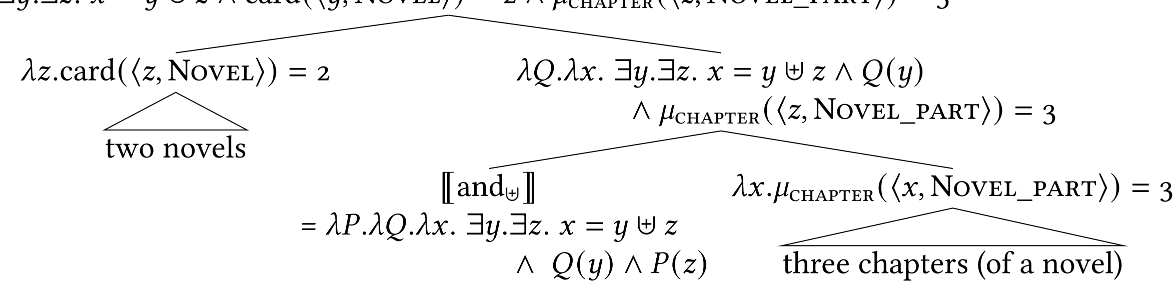
23 $y \uplus z$ is defined if and only if y and z are disjoint. When defined, $y \uplus z = y + z$, where $+$ is mereological summation.

Putting the anaphoric aspect of quantity terms like *chapter* together with the disjoint interpretation of conjunction, we now have a straightforward analysis of sentences like those in (57). For example, let's assume that *three chapters* in (57) has the same interpretation as *three chapters of a novel* (after anaphoric resolution, prior to ellipsis). For the sake of concreteness, we'll assign the denotation in (65) to this phrase, where NOVEL_PART represents the partitive interpretation of the phrase *of a novel*, i.e., the predicate that is true of parts of a novel. We'll assume that $\mu_{\text{CHAPTER}}(\langle x, \text{NOVEL\_PART}\rangle)$ is defined only if x is a member of the denotation of the partitive phrase. When defined, $\mu_{\text{CHAPTER}}(\langle x, \text{NOVEL\_PART}\rangle)$ yields the number of chapters in x (i.e., μ_{CHAPTER} measures the number of chapters). The details here don't matter, as long as the final denotation is a predicate that is true of parts of a novel that measure three chapters.

(65) $[\![\text{three chapters (of a novel)}]\!] = \lambda x.\ \mu_{\text{CHAPTER}}(\langle x, \text{NOVEL\_PART}\rangle) = 3$

Given this interpretation for *three chapters*, the complex noun phrase *two novels and three chapters* would have the following interpretation.

(66) $\lambda x.\ \exists y.\exists z.\ x = y \uplus z \wedge \text{card}(\langle y, \text{NOVEL}\rangle) = 2 \wedge \mu_{\text{CHAPTER}}(\langle z, \text{NOVEL\_PART}\rangle) = 3$



As a result, sentence (57) would have the appropriate truth conditions in (67).

(67) $\exists x.\exists y.\exists z.\ x = y \uplus z \wedge \text{card}(\langle y, \text{NOVEL}\rangle) = 2 \wedge \mu_{\text{CHAPTER}}(\langle z, \text{NOVEL\_PART}\rangle) = 3 \wedge \text{read(Esme,x)}$

Returning to fractional expressions, phrases like *a half* have many of the same distributional properties as quantity expressions like *three chapters*. For example, *a half* can take a partitive phrase as a complement (e.g., *a half of a novel*). It can also be used after a modified numeral expression, where the interpretation of *a half* is anaphorically dependent on the previous noun phrase, as shown in (68).[24]

(68) a. Esme read two novels and a half before the week was over.
b. Zoe built two houses and a half by the end of the month.

Moreover, there's evidence against analyzing the sentences in (68) as cases of rightward movement (i.e., against deriving *two novels and a half* from *two and a half novels* by displacing the phrase *and a half*). As shown in (69), *one and a half* triggers plural marking on the noun *novels* whereas *one novel and a half* requires singular marking.

24 A reviewer registered some doubt regarding the acceptability of the sentences in (68). Note, like all ellipsis, certain contextual factors may ease or inhibit recovery of the elided material. Thus, those who may initially judge the NP objects in (68) to be unacceptable or awkward may find that their acceptability greatly improves when they are used with more context (e.g., "Agatha Christie wrote 75 novels; Thus, a conscientious reader who wishes to become acquainted with all of them within a month would have to read more than two novels and a half every day, without stopping even on Sundays."). Still, there may be dialect differences, especially in terms of the distribution of ellipsis. Our analysis, as it does not depend on ellipsis, holds independent of these potential dialect differences.

(69) a. Esme read one and a half novels within a single afternoon!
 b. *Esme read one novels and a half within a single afternoon!
 c. Esme read one novel and a half within a single afternoon!

In a rightward movement analysis, the numeral modifier would still be complex, both semantically and syntactically (*one* in combination with the trace of *and a half*, with the interpretation of 1.5). It is difficult to see why this type of complex non-singular modifier would require a singular noun in contrast to all other complex modifiers with a similar interpretation.[25]

Given the parallels between sentences like those in (68) and (57), the same arguments that led us to analyze *two novels and three chapters* with disjoint union should also lead us to a similar analysis of *two novels and a half*. In other words, *two novels* would be interpreted as a predicate that is true of all the sums of novels with a cardinality of two, whereas *a half* *(~~of a novel~~)* would be interpreted as a predicate that is true of all the sums of novel parts that count as half of a novel (as determined by the measure function that counts halves, i.e., μ_{half}). The conjunctive morpheme, with its disjoint summation interpretation, combines these predicates to form a predicate that is true of sums that consist of two novels and an additional half novel, as shown in (70).

(70) $\lambda x.\ \exists y.\exists z.\ x = y \uplus z \wedge \text{card}(\langle y, \textsc{Novel}\rangle) = 2 \wedge \mu_{\textsc{half}}(\langle z, \textsc{Novel\_part}\rangle) = 1$

- $\lambda z.\text{card}(\langle z, \textsc{Novel}\rangle) = 2$
  - two novels
- $\lambda Q.\lambda x.\ \exists y.\exists z.\ x = y \uplus z \wedge Q(y) \wedge \mu_{\textsc{half}}(\langle z, \textsc{Novel\_part}\rangle) = 1$
  - $[\![\text{and}_{\uplus}]\!] = \lambda P.\lambda Q.\lambda x.\ \exists y.\exists z.\ x = y \uplus z \wedge Q(y) \wedge P(z)$
  - $\lambda x.\mu_{\textsc{half}}(\langle x, \textsc{Novel\_part}\rangle) = 1$
    - a half (of a novel)

With standard existential quantification over this conjoined predicate, (68a) receives the truth conditions in (71).

(71) $\exists x.\exists y.\exists z.\ x = y \uplus z \wedge \text{card}(\langle y, \textsc{Novel}\rangle) = 2 \wedge \mu_{\textsc{half}}(\langle z, \textsc{Novel\_part}\rangle) = 1 \wedge \text{read}(\text{Esme}, \text{x})$

Critically, in this analysis *two and a half* doesn't form a constituent, nor does it denote a degree. Furthermore, there are two measure functions: one that measures the number of novels and another that measures the number of half novels.

The plausibility of such an analysis for phrases like *two novels and a half* brings up the question of whether there's a similar analysis for constructions like *two and a half novels*, especially since the two types of noun phrases paraphrase one another.

One obvious way to implement a parallel analysis would be to hypothesize that phrases like *two and a half novels* are derived from *two novels and a half* via some kind of ellipsis or rightward movement of the noun *novels*, as proposed by Ionin, Matushansky, and Ruys (2006). However, the sentences in (69) above, which were used to provide evidence against rightward movement of the conjunct *and a half*, are also problematic for rightward movement/ellipsis of the noun phrase. Under the most straightforward implementation of this strategy, *one novel and a half* should lead to the derivation of *one and a half novel*. Similarly, *one and a half novels* would be derived from

25 Perhaps one could implement some type of number-agreement strategy to capture the pattern in (69), where agreement happens post movement, however the devil is in the details. What would the noun agree with to yield such a pattern? The numeral *one* and the noun are not in a c-command relationship. Any potential source of agreement higher than *one* would either dominate or c-command the constituent containing the trace of *and a half*, which has a non-singular interpretation.

one novels and a half. Yet both *one and a half novel* and *one novels and a half* are ungrammatical. Rescuing this analysis requires a more nuanced approach to number marking than is standardly assumed, an issue that lies beyond the scope of the current paper.[26]

Another way to equate the two types of constructions involves type shifting. For example, Landman (2020, 2004) hypothesizes that disjoint summation, which normally takes two arguments of type $\langle e,t\rangle$, can be shifted into an operator that takes two arguments of type $\langle\alpha,\langle e,t\rangle\rangle$, where α is a variable ranging over types.[27] Although Landman does not discuss the details of such a coercion, a general schema is given in (72), where subscripts represent variable types.

(72) $\lambda P_{\langle e,t\rangle}.\lambda Q_{\langle e,t\rangle}.\lambda x.\exists y.\exists z.\ x = y \uplus z \wedge Q(y) \wedge P(z)$
$\Rightarrow\ \lambda \mathcal{P}_{\langle\alpha,\langle e,t\rangle\rangle}.\lambda \mathcal{Q}_{\langle\alpha,\langle e,t\rangle\rangle}.\lambda R_{\alpha}.\lambda x.\exists y.\exists z.\ x = y \uplus z \wedge \mathcal{Q}(R)(y) \wedge \mathcal{P}(R)(z)$

As discussed in Landman (2020, ch. 3), such a strategy can be applied to phrases like *two and a half.*[28] In particular, $[\![\text{and}_{\uplus}]\!]$ can be lifted into an interpretation that coordinates nominal modifiers of type $\langle\langle e,t\rangle,\langle e,t\rangle\rangle$ when $\alpha = \langle e,t\rangle$. Such an interpretation is plausibly needed to coordinate subsective adjectives (e.g. *tall* and *short*) independent of any type of numeral modification (e.g., $[\![$*tall and short basketball players*$]\!]$ under the interpretation where it denotes a mixed group of tall and short).[29] However, it can also be extended to coordinate numeral modifiers with other types of measures. We have already introduced a modifier interpretation of *two* when it combines with a silent version of *many*, see (73a). Given phrases like *a half apple*, it seems plausible that *a half* can receive a similar interpretation, see (73b) where $P\_\textsc{part}$ denotes the set of all the parts of the members of P.

(73) a. $[\![\text{two (many)}]\!] = \lambda P.\lambda x.\ \text{card}(\langle x, P\rangle) = 2$
b. $[\![\text{a half}]\!] = \lambda P.\lambda x.\ \mu_{\textsc{half}}(\langle x, P\_\textsc{part}\rangle) = 1$

With these two interpretations in mind, the phrase *two and a half novels* could be decomposed as in (74). The final interpretation ends up being identical to the interpretation given to *two novels and a half.*

26 It might be possible to explain this kind of idiosyncratic pattern if 1) number marking on the noun was the result of an agreement operation, and 2) movement operations either fed plural agreement or bled singular agreement. For example, perhaps number agreement operations happen after movement operations. If so, then the rightward movement of the noun phrase could, in principle, move the noun phrase out of the domain of singular agreement, thus resulting in plural agreement. It is an empirical matter whether either of these assumptions are plausible.

27 Landman's type-shifting operations echo the compositional flexibility achieved by the Geach rule (Geach 1970), although to a more limited degree. The Geach rule has been used to account for a variety of different grammatical phenomena including binding, weak crossover, quantifier scope, paycheck sentences, non-sentential coordination, and category-flexible negation (Geach 1970; Jacobson 1999, 2000).

28 Landman (2004) labels the disjoint summation operator "sum pairing." Landman (2020, p. 73) uses type-shifting to analyze the English construction *two and a third* as well as its Dutch counterpart.

29 Let $\textsc{std}_P$ stand in for the standard height for category P. Suppose $[\![\textit{tall}]\!] = \lambda P.\lambda x.\mu_{\textsc{height}}(x) > \textsc{std}_P$, $[\![\textit{short}]\!] = \lambda P.\lambda x.\mu_{\textsc{height}}(x) < \textsc{std}_P$, and $[\![\text{and}_{\uplus}]\!]$ is lifted to $\lambda\mathcal{P}.\lambda\mathcal{Q}.\lambda R.\lambda x.\exists z.\exists y.\ x = z \uplus y \wedge \mathcal{Q}(R)(z) \wedge \mathcal{P}(R)(y)$, where $\mathcal{P}$ and $\mathcal{Q}$ are of type $\langle\langle e,t\rangle,\langle e,t\rangle\rangle$ and R is of type $\langle e,t\rangle$. With these denotations, $[\![\text{tall and short}]\!] = \lambda R.\lambda x.\exists z.\exists y.\ x = z \uplus y \wedge \mu_{\textsc{height}}(z) > \textsc{std}_R \wedge \mu_{\textsc{height}}(y) < \textsc{std}_R$.

(74)

$$\lambda x.\exists z.\exists y.\ x = z \uplus y \ \wedge\ \text{card}(\langle z, \textsc{novel}\rangle) = 2 \ \wedge\ \mu_{\textsc{half}}(\langle y, \textsc{novel\_part}\rangle) = 1$$

$$\lambda R.\lambda x.\exists z.\exists y.\ x = z \uplus y \ \wedge\ \text{card}(\langle z, R\rangle) = 2 \ \wedge\ \mu_{\textsc{half}}(\langle y, R\_\textsc{part}\rangle) = 1 \qquad [\![\text{novels}]\!] = \textsc{Novel}$$

$$\lambda P.\lambda x.\text{card}(\langle x, P\rangle) = 2 \qquad \lambda Q.\lambda R.\lambda x.\exists z.\exists y.\ x = z \uplus y \ \wedge\ Q(R)(z) \ \wedge\ \mu_{\textsc{half}}(\langle y, R\_\textsc{part}\rangle) = 1$$

two (many)

$$\lambda \mathcal{P}.\lambda Q.\lambda R.\lambda x.\exists z.\exists y.\ x = z \uplus y \wedge Q(R)(z) \wedge \mathcal{P}(R)(y) \qquad \lambda P.\lambda x.\mu_{\textsc{half}}(\langle x, P\_\textsc{part}\rangle) = 1$$

a half

(Type Shift)

$$[\![\text{and}_{\uplus}]\!] = \lambda P.\lambda Q.\lambda x.\exists y.\exists z.\ x = y \uplus z \wedge Q(y) \wedge P(z)$$

Of critical importance for the discussion at hand, there are two measure functions in (74): one that measures cardinality and another that measures the number of halves.[30]

In summary, there's a plausible compositional analysis of *two and a half novels* that builds on a general semantics for numerals in combination with quantity phrases. Although such an analysis isn't perfect, we submit that it isn't any worse off than those of Liebesman and Haida and Trinh. Moreover, our analysis is easily applied to French, as we explain below.

6 An analysis for French

In Section 4, we stressed two facts about French. On the one hand, sentences containing a nominal expression like *deux romans et demi*, a plural comparative like *plus de romans que* or an equative like *autant de romans que* have the same interpretations as their English counterparts. But on the other hand, a nominal expression like *deux romans et demi* contains no constituent that could

30 A possible advantage of the type-shifting analysis is that it might be able to explain why not all factions can coordinate with numerals. The theories of Liebesman and Haida and Trinh, where *and* is interpreted as numerical addition, predict that any fractional expression should be able to replace X in the template *two and X novels*. Yet, similar to the French data discussed in Section 4, the possible replacements for the X position are quite limited:

(i) a. Esme ate two and a half apples.
b. ?Esme ate two and a quarter apples.
c. ∗Esme ate two and three fifths apples.
d. ∗Esme ate two and five eighths apples.

The type-shifting analysis might be able to explain this distribution by appealing to a fraction's status as an NP modifier:

(ii) a. Esme ate a half apple.
b. ?Esme ate a quarter apple. (c.f., Esme ate a quarter of an apple.)
c. ∗Esme ate three fifths apple. (c.f., Esme ate three fifths of an apple.)
d. ∗Esme ate five eighths apple. (c.f., Esme ate five eighths of an apple.)

refer to the rational number 2.5. So, to account for the French data, one needs a different analysis than Liebesman's or Haida and Trinh's.

A unified analysis of English and French can be given by adapting the proposal from the previous section concerning *two novels and three chapters* and *two novels and a half*. The only difference is that, in French, *demi* doesn't combine with a partitive phrase (**un demi d'un roman*) but with a silent copy of the noun (*demi roman*). As pointed out by Gagnon (2013), French differs from English in that it allows for non-partitive NP ellipsis. For example, although in English, one can't easily elide the noun after an adjective, as shown by the awkward continuation in (75a), in French, one is free to do so, as shown by the preferred continuation in (75b).

(75) a. I saw the boys in the yard. ??The tall played with the short.

b. J'ai vu les garçons dans la cour. Les grands jouaient avec les petits.
`I have saw the boys in the yard The big play with the small`
'I saw the boys in the yard. The tall boys are playing with the short boys.'

Taking this small difference into account, we can analyze a sentence like the one in (53), repeated below, as in (76), where the conjunctive morpheme *et* combines the predicates *deux romans* and *demi roman*, requiring the sums denoted by each to be non-overlapping.

(53) Esmé a lu deux romans et demi.
`Esme has read two novels and half`
'Esme read two and a half novels.'

(76) $\lambda x.\ \exists y.\exists z.\ x = y \uplus z \wedge \text{card}(\langle y, \textsc{Novel}\rangle) = 2 \wedge \mu_{\text{HALF}}(\langle z, \textsc{Novel\_part}\rangle) = 1$

$\lambda z.\text{card}(\langle z, \textsc{Novel}\rangle) = 2$ — deux romans

$\lambda Q.\lambda x.\ \exists y.\exists z.\ x = y \uplus z \wedge Q(y) \wedge \mu_{\text{HALF}}(\langle z, \textsc{Novel\_part}\rangle) = 1$

$[\![\text{et}_{\uplus}]\!] = \lambda P.\lambda Q.\lambda x.\ \exists y.\exists z.\ x = y \uplus z \wedge Q(y) \wedge P(z)$

$\lambda x.\mu_{\text{HALF}}(\langle x, \textsc{Novel\_part}\rangle) = 1$ — demi (roman)

This is essentially the analysis put forward by Nicolas (2016). The main part of his account is that there's ellipsis of the noun. So (53) has the same interpretation as (77), where the conjunction *et* imposes a non-overlap condition.

(77) Esmé a lu deux romans et un demi-roman.
`Esme has read two novels and a half-novel`
'Esme read two novels and a half.'

Here, a puzzling set of facts about agreement should be mentioned. Consider the agreement pattern in (78) and (79).

(78) Un poème et demi est écrit / *sont écrits sur cette page.
`one poem and half is written / *are written on this page`

(79) Un poème et un demi-poème sont écrits / *est écrit sur cette page.
`one poem and one half-poem are written / *is written on this page`

Whereas *un poème et un demi-poème* triggers plural agreement on the verb, *un poème et demi* triggers singular agreement. On the ellipsis hypothesis, one would expect plural agreement in all cases.[31] This agreement pattern differs from what we see in English, where plural agreement is required whether or not *half* appears to the left or right of the noun.

(80) a. One and a half poems were / *was written (in her notebook).
b. One poem and a half were / *was written (in her notebook).

We don't have a good explanation of why this pattern exists in French but not in English. However, it's important to note that it can't be explained under Leibesman's or Haida and Trinh's analyses either.

In summary, in French, the expression *deux pommes et demie* contains no constituent that might refer to the number 2.5, so an analysis different from Liebesman's and Haida and Trinh's is required. To account for the facts, we adopt the main part of Nicolas' account, namely that *deux pommes et demie* is interpreted as *deux pommes et demie pomme*, and we note that the conjunction *et* requires non-overlap in nominal contexts.

7 Reconsidering fractional modification

Liebesman (2015, sec. 3) presents several arguments against analyses of the data based on ellipsis — such as the ones we put forward in Sections 5 and 6 — and in favor of an analysis with a partiality measure. We've labeled these the 'Exactly argument', the 'Anaphora argument', and the 'Quarters and Sliced arguments'. We show below how our analysis can answer them. Finally, we briefly consider an indirect argument in favor of Liebesman's or Haida and Trinh's account, based on the work of Fox and Hackl (2006).

7.1 The Exactly argument

As reviewed in Section 2, Liebesman uses upper-bound readings of numerals to motivate the introduction of a partiality measure. In particular, Liebesman argues that such a measure function could explain why the sentences in (4a) and (8), repeated below, imply that there aren't two and a half bagels on the table.

(4a) Two bagels are on the table.

(8) Exactly two bagels are on the table.

Liebesman's account of the data assumes that the upper-bound readings expressed in (4a) and (8) arise due to an upper limit placed on the degree argument (i.e., an upper limit of 2 implies not 2.5).

However, this isn't the only means by which one can achieve a 'precise' reading of *two bagels*. In fact, Gricean accounts of upper-bound readings typically invoke a competition between alternative utterances rather than hypothesizing a constraint on degrees (for the traditional pragmatic account see Grice 1975; Matsumoto 1995; Geurts 2010 among others; for accounts that rely on maximizing a degree interpretation, see Fox and Hackl 2006; Kennedy 2013). Given these Gricean accounts, all that is needed to derive a 'precise' upper-bound implication is the inclusion of (4b), repeated below, as a viable alternative to (4a).

31 This type of pattern isn't unique to French. The same facts hold for Romanian and Italian.

(4b) Two and a half bagels are on the table.

The fact that (4b) entails the weak, inclusive reading of (4a) means that any competent speaker who utters (4a) will implicate the falsity of (4b) via the Maxim of Quantity. This implication isn't dependent on whether the numeral expressions *two* and *two and a half* name degrees on the same scale. Critically, such quantity implicatures can be derived from our analysis of the sentences in (4) given in Section 5.[32]

A relevant observation with respect to a potential pragmatic account of the data is that there's independent evidence that upper-limits aren't always associated with degrees along the same scale. For example, consider the question in (81) and the potential answers in (82).

(81) How much fruit did Anouk eat?

(82) a. She ate two apples.
b. She ate two apples and one banana.

In a context where both the speaker and the listener know that Anouk has access to apples, bananas and oranges, the utterance in (82a) implies the falsity, not only of (82b), but of any other potential answer that entails (82a) and isn't entailed by it. Such implications hold even though the two answers aren't related to one another via a change to a degree argument. In fact, *two apples and one banana* relates to *two apples* in the same way that *two and a half bagels* relates to *two bagels* under the analysis we gave in Section 5: one expression denotes parts of the other.[33]

McNally (1998) independently argues that a similar explanation of upper-bound readings can be extended to the adverb *exactly*. According to her, *exactly* shouldn't be restricted to degree expressions, nor should it rely on a degree semantics. In support of this hypothesis, she notes that many sentences, like those in (83) — McNally's example (43) — contain an instance of the adverb *exactly* in the absence of any overt degree expressions.

(83) a. Stay exactly where you are.
b. How exactly do you intend to raise so much money?
c. I would have done exactly the same thing if I were in your position.
d. The hole needs to go exactly in the middle of the board.
e. You are exactly right.

To account for such examples in addition to the ones with degree terms, she suggests that the semantics of *exactly* "involves selecting from a set of contextually salient, ordered alternatives", where "the presence of *exactly* signals that the speaker is offering a maximally precise description of some situation" with respect to these alternatives (McNally 1998, p. 381). McNally assigns the

32 This type of explanation would require a morphologically more complex sentence to serve as an alternative to a morphologically simpler utterance, contrary to the spirit of Katzir (2007). However, even for him contextual salience can override restrictions imposed by simplicity. In fact, the requirement that *two and a half* be contextually salient might explain why the implication from (4a) to the falsity of (4b) is so weak compared to other types of quantity implicatures.

33 There are relevant differences between *two and a half bagels* on the one hand and *two apples and one banana* on the other, especially as it relates to Katzir's simplicity hypothesis (see footnote 32). It's possible to derive the relevant implicature from (82a) by considering *She ate one banana* as an alternative, as long as all non-weaker alternatives are negated by the strengthening process rather than just the stronger alternatives. Critically, such an alternative isn't morphologically more complex than the original utterance. In contrast, a parallel potential alternative to (4a), namely *A half bagel is on the table*, doesn't lead to an implicature that negates (4b) without making a critical assumption that a whole bagel being on a table doesn't entail a half bagel being on the table.

semantic interpretation in (84), under the assumption that *exactly*, much like the adverbs *only* and *at most*, moves covertly in order to operate at a sentential level.

(84) $[\![\text{exactly}]\!](\alpha) = \alpha \wedge \forall p.\ (p \in \textsc{alt} \wedge \textbf{true}(p)) \rightarrow \alpha \geq p$, where ALT is the set of contextually relevant alternatives and $\geq$ is an ordering of propositions in terms of their precision (i.e., $r \geq s$ iff r is as precise or more precise than s).[34]

Given this alternative, the upper-bound interpretation of numerals, with or without an overt expression of exactness, doesn't in itself provide evidence against the traditional analysis of cardinality.[35] The correct interpretation of exactness depends on how such an analysis fits in with other auxiliary data-points. Liebesman discusses two such points, namely evidence from degree anaphora and evidence from varying ranges of truth conditions. We address each of these in turn.

7.2 The Anaphora argument

Consider the sentences in (85), paying particular attention to the anaphoric pronoun *that*.

(85) Two and a half bagels are on the kitchen table. Twice as many onions as that are on the dining room table.

The second sentence conveys that five onions are on the dining room table, which in turn suggests that the demonstrative pronoun denotes the value 2.5. Liebesman proposes that this value is introduced into the discourse by virtue of fact that *two and a half* denotes the number 2.5.

However, as shown by Nicolas (2016, sec. 2.4), this isn't necessarily the case. Often, a single anaphoric value can be introduced by two separate phrases in the discourse. For example, consider the sentences in (86).

(86) Two boys and three girls were dancing. Twice as many adults as that were kissing.

Here, the pronoun *that* occurring in the second sentence can refer to the number five even though *five* isn't a constituent in the first sentence. We take it that the first sentence makes salient the sum of children together with its cardinality — five. The *that* in the second sentence is able to refer back to this cardinality. The point is then that the resources involved in the interpretation of (86), whatever they exactly are, may also be involved in the interpretation of (85).

Moreover, such independent resources are required for analog sequences of sentences involving the expression *two bagels and a half* or its French analog *deux bagels et demi*, since in such cases there's no constituent that could denote the number 2.5.

(87) Two bagels and a half are on the kitchen table. Twice as many onions as that are on the dining room table.

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34 We've adapted McNally's proposal to be more consistent with the accounts discussed in this paper.

35 A similar observation can be made about the modifiers *at most* and *at least*. For example, consider the sentences below.

(i) Context: John, Mary and Jill are roommates.
  a. Seymour invited at most John and Mary to his dinner party.
  b. Seymour invited at least John and Mary to his dinner party.

Neither (1a) nor (1b) contains a numeral, but both sentences express a boundary on the quantity of people invited to the dinner party. Since *at most* and *at least* can operate with respect to the conjoined phrase *John and Mary*, they should also be able to operate with respect to a conjoined interpretation of *two and a half apples*.

(88) Il y a deux bagels et demi sur la table de la cuisine. Il y a deux fois plus d'onions sur la table à manger.

This phenomenon is similar to other cases of split anaphora, as shown with the third and first person plural pronouns in (89).

(89) a. John met Mary at the library. They studied all night.
b. I met Mary at the library. We studied all night.

Here, *they* is most naturally read as referring back to the sum consisting of John and Mary, whereas *we* is most naturally read as referring back to the sum consisting of the speaker and Mary, even though there are no constituents that have such sums as their semantic value. For this type of split anaphora to work, discourse factors need only raise the salience of the parts that make up the sum in order to make the sum available as a possible anaphoric value.

It should be noted that given the interaction between context and demonstratives in general, this type of split anaphora is expected. Critically, the numerical value of *that* can be completely supplied by context. For example, if one had a piece of paper with the number 2 written on it, one could point to it and say (90).

(90) I have as many cars as that.

Similarly, if someone pointed to a set of two motorcycles saying *Look at how many bikes I own*, one could respond by saying (90), thus indicating that one has two cars. Clearly, when it comes to establishing a numerical value for a demonstrative pronoun, contextual salience alone is sufficient. It would be surprising if the contextual salience explicitly expressed in the discourse via the split values weren't sufficient.

In summary, due to its heavy reliance on contextual salience, anaphora isn't a determining factor in figuring out the semantic value of linguistic expressions like *two bagels and a half*.[36]

7.3 The Quarters and Sliced arguments

Liebesman offers two further arguments in favor of his analysis over one that hypothesizes an interpretation of *and* that is more akin to phrasal or sentential conjunction, as in our analysis in Section 5. We'll label these the 'Quarters' and 'Sliced' arguments, for reasons that will become obvious below. These arguments are specifically directed against an analysis based on ellipsis. Liebesman urges us to compare sentence (4b) to the sentences in (91).

(4b) Two and a half bagels are on the table.

36 A separate issue, not discussed here, concerns how the anaphoric term *that* relates to degrees in phrases like *that many bagels*. At first blush, it seem like *that* can combine directly with *many* even when it takes on the value of a partial measurement (e.g. 2.5). However, things are more complicated than they appear. *That*, when used as a measurement anaphor, appears to have a wider distribution than overt measurement phrases (e.g., *The box is that heavy* vs. **The box is 4 pounds heavy*). Also, unlike overt measure phrases, it takes on intensional values that cannot be direct outputs of the measure function it seemingly combines with. For example, if Mary is taller than all of her siblings, then we could say that "Bill would like to grow up to be that tall also" to mean that he would like to be taller than all of his siblings even if that height is not yet known or even determined.

(91) a. Two bagels and a half of a bagel are on the table.
b. Two bagels and a half bagel are on the table.
c. Two bagels are on the table and a half of a bagel is on the table.
d. Two bagels are on the table and a half bagel is on the table.

Liebesman's idea is that a semantics based on phrasal or sentential conjunction and ellipsis would predict that sentence (4b) should have truth conditions equivalent to at least one of the sentences in (91). Now, let's evaluate these sentences in two contexts, 'Quarters' and 'Sliced'.

(92) **Quarters**: On the table, there are two bagels, one quarter of a pumpernickel bagel and one quarter of a sesame bagel.

(93) **Sliced**: Two bagels are both sliced in half. The sliced bagels are placed on the table.

Liebesman observes that in the Quarters context, (4b) is true but (91b) and (91d) are false, while (91a) and (91c) are dubious. This suggests that (4b) doesn't have the same truth conditions as any of the sentences in (91). Hence a semantics based on ellipsis is bound to be wrong. In the Sliced context, Liebesman observes that there's no possible reading of (4b) in which it's true, yet there's a possible reading of each sentence in (91) where it's true (under a weak literal meaning). Again, ellipsis seems to make wrong predictions.

By and large, we share Liebesman's intuitions.[37] This suggests that at least one reading of each sentence in (91) has different truth conditions than (4b). However, this doesn't necessarily mean that the analysis presented in Section 5 is on the wrong track.

Let's address the Sliced context first. With respect to this context, the analysis presented in Section 5 makes the correct prediction. In particular, our analysis, like Landman's (2020), involves the combination of sums into larger sums through a disjoint summation operator, thus rendering (4b) false in the context in (93). In further support of this analysis, note that the sentence in (87), repeated below, patterns just like (4b) in terms of the Sliced context.

(87) Two bagels and a half are on the table.

This sentence can't be judged true when there are just two bagels sliced in half on the table. However, as with similar examples discussed in Section 5, the numeral *two* and the phrase *and a half* don't form a constituent in (87), and thus this sentence isn't compatible with Liebesman's analysis of (4b).

Why are the sentences in (91) different from those in (4b) and (87)? According to the semantic interpretation given in Section 5, nominal conjunction is interpreted in terms of summation and non-overlap. This suffices to explain the difference between (91c/d) and the sentences in (87) and (4b). The sentences in (91c/d) involve sentential conjunction, not nominal conjunction. Thus, the prohibition on overlap isn't semantically imposed. As for the sentences in (91a/b), we suspect that conjunction in these sentences can take wide scope and thus that there's a reading of these sentences that involves sentential conjunction as well. Our conjecture is that the null anaphora in

37 While we agree that (91a) and (91b) *can* be judged as true in the Sliced context, we nonetheless find it difficult to access the intended reading. Moreover, this reading disappears in other sentences, notably when the noun phrase is the direct object of the verb.

(i) a. John ate two bagels and a half of a bagel.
b. John ate two bagels and a half bagel.

Neither of these sentences can be judged true in a context similar to the Sliced context, namely if what John ate consisted of two bagels that had been cut in half.

(87), along with the compositional mechanisms needed to derive (4b) (e.g., type shifting), prevent conjunction from taking wide scope in these examples. Hence, there's no reading that allows for overlap.

The Quarters context is more difficult to explain. Although our analysis doesn't predict that there should be any difference between (4b) and the sentences in (91) in terms of two quarters counting as one half, it's worth noting that (87), which is incompatible with Liebesman's analysis, sounds much more natural in the Quarters context than any of the sentences in (91). Likewise, the French counterpart to (4b) given in (94), sounds just as natural as its English counterpart despite the fact that *deux et demi* doesn't form a constituent.

(94) Il y a deux bagels et demi sur la table.

Although we can't explain the awkwardness/falsity of the sentences in (91) in the Quarters context, Liebesman's analysis can't explain the lack of awkwardness for the sentences in (87) and (94).

It's also important to note that the facts aren't as straightforward as they may seem on the surface. Critical to the judgments above, one assumes that the two bagels and the two quarter pieces of bagel are approximately the same size in the Quarters context. However, when counting, the size of the objects usually doesn't matter. For example, let's suppose that there's a pumpernickel bagel on the table as well as a sesame bagel. Furthermore, let's suppose that the pumpernickel bagel is twice the size of the sesame bagel. In such a scenario, it's obviously true that there are two bagels on the table. Size isn't a factor.

Yet, size is a factor when it comes to counting quarters as one half. Let's suppose we have two bagels on the table, as mentioned above, but that there's also a quarter piece of another pumpernickel bagel and a quarter piece of another sesame bagel on the table. Once again, the pumpernickel quarter is twice the size of the sesame quarter. In this case, it's deceptive to say that there are two and a half bagels on the table – i.e., (4b) is no longer true. Even if we keep the quarters the same size, as implicitly assumed in the original context, but we let the sizes of the two bagels differ by a very large magnitude, (4b) is awkward at best.

Clearly, something else is going on in the Quarters context which isn't captured by any of the analyses presented thus far. In fact, such data are similar to the observations that led Snyder and Barlew (2019) and Snyder (2021) to hypothesize a difference between a measure reading and an individuated reading (see the discussion in Section 3.2). Perhaps the existence of a measure reading might be able to account for speakers' intuitions in these kinds of contexts.

In summary, the Quarters argument loses some of its force since the critical contexts needed for the argument to go through involve implicit assumptions about the size of the (partial) objects involved. As suggested by Snyder and Barlew, such assumptions may correspond to a certain type of coerced reading – a measure reading. Such a reading, which is commonly induced by food denoting terms but which otherwise is marginal, is beyond the scope of this paper.

7.4 Fox and Hackl (2006)

Before concluding, let us briefly discuss another indirect line of argument in favor of Liebesman's or Haida and Trinh's analysis. In an exploration of the nature of scales and degrees, Fox and Hackl (2006, sec. 2) discuss sentences like the ones in (95). (95a) doesn't imply any upper bound reading; in particular, it doesn't imply that there aren't four plates. Yet, its truth conditions are equivalent to the claim that there are three or more plates on the table, which essentially corresponds to the interpretation of (95b). Unlike (95a), (95b) does imply the upper bound reading that there aren't four plates (and hence, that there are exactly three plates).

(95) a. There are more than two plates on the table.
 b. There are three plates on the table.

As Fox and Hackl argue, the contrast between (95a) and (95b) is unexpected under a naive Gricean analysis. To compute the upper bound reading of (95b), the standard analysis assumes that numeral expressions combining with count nouns form a scale, corresponding to the integers. Thus, an alternative to (95b) is sentence (96).

(96) There are four plates on the table.

Negating this stronger alternative yields the upper bound reading of (95b), as expected.

Under the same assumptions, an alternative to (95a) is sentence (97).

(97) There are more than three plates on the table.

Negating this stronger alternative likewise yields an upper bound reading, implying that there aren't more than three plates, and thus that there are exactly three plates. Yet, empirically speaking, this implication is unattested.

Fox and Hackl suggest that the contrast can be explained if upper bound readings aren't computed relative to alternative numeral expressions, but rather relative to alternative degrees on a scale. For example, the upper bound reading of (95b) expresses the proposition that 3 is the maximal degree that can replace *d* in the template in (98).

(98) There are *d-many* plates on the table.

Fox and Hackl argue that degrees are always densely ordered and, in the case of the scale corresponding to numerals, that they include the rational numbers. This doesn't matter when computing the maximal degree with the template (98), but it makes a critical difference when one considers a similar template based on (95a).

(99) There are more than *d-many* plates on the table.

The alternatives to be considered correspond to instances of the template where *d* is any rational strictly greater than 2. By negating all of these alternatives, one would conclude that there are, in fact, exactly two plates, contradicting the literal content of (95a). Therefore, according to Fox and Hackl, this reading isn't generated.

If their analysis is correct, then it would indirectly support Liebesman's and Haida and Trinh's accounts of numeral modification. In their accounts, the measure function operative in numeral modification can sometimes map entities to rational numbers. So it would be unsurprising if the dimension that the measure function mapped into included the rational numbers. Thus, one would expect such a dimension to be densely ordered. In contrast, a densely ordered scale would be unexpected if the operative measure function were cardinality, as hypothesized in the traditional approach to numeral modification.

However, there are other possible analyses of the data discussed by Fox and Hackl that don't require one to hypothesize a densely ordered scale. In particular, under the traditional Gricean approach, it's critical that sentences with modified numerals like (100) aren't viable alternatives to (95b). If they were, then such sentences would block the derivation of an upper bound reading. In particular, since (100) is more informative than (95b), by asserting the latter the speaker would implicate that they don't believe (100) to be true.

(100) There are exactly three plates on the table.

To prevent such alternatives from being a part of a listener's pragmatic reasoning process, it's often stipulated that the alternatives to sentences with modified numerals can't compete with the alternatives to sentences without. However, in contrast to (95b), (95a) already contains a modified numeral (*more than two*). Its presence could lead the listener to consider other modified numeral expressions as possible alternatives. Hence, (100) could be a viable alternative to (95a), even though it isn't a viable alternative to (95b). It would then be expected that (100) would block the upper bound reading of (95a), but not the upper bound reading of (95b).

There are many nuances to this line of reasoning that need to be worked out. For example, as noted by Buccola and Haida (2021), if (100) were an alternative to (95a), then a standard Gricean analysis would expect the assertion of *more than two* to imply that the speaker is ignorant about the status of *exactly three*. Although such ignorance implications arise in certain circumstances (e.g., when (95a) is asserted as an answer to the question *How many plates are there on the table?*), they aren't as common place as one might expect under a typical Gricean analysis. Still, as discussed by Westera and Brasoveanu (2014), one might be able to explain the distribution of such ignorance inferences through a careful analysis of the relevant Questions Under Discussion. The details of such an explanation needn't concern us at this point. For our purposes, it's sufficient to note that the existence of dense scales with respect to sentences that express natural numbers isn't a settled issue. Instead, it's the subject of an on-going debate within the literature.

8 Conclusion

Count nouns combine with numerals to produce modified noun phrases like *two novels*. Prima facie, this combination seems to express a cardinality. Based on the interpretation of expressions like *two and a half novels*, Salmon, Liebesman and Haida and Trinh argue that this traditional account is wrong. According to them, the measure function that combines numerals with count nouns is a partiality measure.

However, as we stressed in Section 3, judgments about equatives and comparatives indicate that nominal comparisons are usually insensitive to partiality measurements, once proper controls are put into place. This is totally unexpected under theories that invoke partiality measures. Moreover, expressions like *two novels and a half*, as well as its French equivalent, *deux romans et demi*, are interpreted similarly to *two and a half novels*, even though they contain no constituent that could refer to the rational number 2.5. Accounting for their interpretations requires a different analysis, which we provided in Sections 5 and 6.

The most parsimonious theory should then adopt a similar analysis for *two and a half novels*, one that doesn't involve partiality measures. Under this view, cardinality is always used – except in special circumstances involving "food-like" categories as in the examples given by Snyder and Barlew (2019). For *two and a half novels* in English, we've suggested either rightward movement/ellipsis of the noun phrase, or type-shifting. For *deux romans et demi* in French, we've proposed ellipsis of the noun *roman* after *demi*. In all cases, the interpretation involves an operation of disjoint summation.

In section 7, we considered additional arguments in favor of partiality measures: the Exactly argument, the Anaphora argument and the Quarters and Sliced arguments. In each case, we demonstrated that there are explanations of the data that don't rely on such measures. All other things being equal, these explanations should be favored over ones that abandon cardinality.

There are a couple of issues that we didn't address in our paper. Consider the differential statement in (101).

(101) Zoe wrote two and a half more novels than Esme did.

Its truth conditions can be paraphrased as follows: the difference between the measure of the novels Zoe wrote and the measure of the novels Esme wrote is two and a half. This paraphrase mirrors some of the formal analyses of differentials offered in the literature (von Stechow 1984) and seems to require a partiality measure of the novels.

However, it isn't so clear that such a precise analysis of differentials is warranted. First, differential expressions often refer to units of measurement that can't be used to modify the relevant expressions in the main or comparative clauses. As shown in (102), although *pounds* can be used in a differential expression where the relevant adjective is *heavy*, it can't be used to express degrees that directly modify *heavy*.

(102) a. John is twenty pounds heavier than Mike.
b. *John is two hundred and twenty pounds heavy.

Second, differential expressions that seem to express precise differences when combined with some adjectives, as in (103a), specify vague differences when combined with other types of adjectives, as in (103b).

(103) a. Esme is two times taller than John.
b. Zoe is two times more talented than Sue.

Third, the availability of precise interpretations of differential expressions depends on specific world knowledge. For example, suppose that yesterday's temperature was −5°C, which is equivalent to 23°F. In such a context, someone with knowledge of the Fahrenheit scale would be able to interpret (104) meaningfully (e.g., expressing that today's temperature is 46°F). However, someone who only had knowledge of the Celsius scale would not be able to assign any precise interpretation.

(104) It is two times warmer today than yesterday.

This makes it tempting to develop a vague analysis of differentials — one relying on contextual cues and extra-linguistic knowledge about measurement systems in order to establish more precise interpretations. We won't do so here as it would bring us too far off topic.

Note that even if one pursued a precise semantics of differentials in the spirit of von Stechow (1984), this wouldn't change the judgments about comparatives and equatives reported in this paper and in Haida and Trinh (2021). In fact, such a semantics would make it particularly problematic to explain why partiality measurements are inaccessible when a differential expression isn't present. It would also render Haida and Trinh's general strategy for explaining such judgments untenable as a monotonicity constraint shouldn't depend on the presence or absence of a differential expression.

Another issue not addressed in this paper is decimal modification as in (105).[38]

(105) Zoe wrote 2.5 novels.

Here, we think there are good reasons to push such expressions aside. First, unlike fractional expressions like *half*, understanding decimal expressions requires explicit instruction, usually in the form of a math class. Furthermore, even after years of practice with such expressions, people

38 Haida and Trinh (2021) focus on such expressions.

are often still unable to assign precise interpretations. Second, once again unlike *half*, decimal expressions are often reserved for a type of elevated math-speak and aren't a regular part of spoken grammar. This is reminiscent of other types of speech affectation, such as using *whom* in wh-questions. Third, once again unlike *half*, decimal expressions have a very unusual syntactic and phonological signature. The numbers after the word *point* are often read like a list with a flat intonation. Furthermore, there's no evidence of any type of constituency within a decimal expression: parts of a decimal expression can't be fronted nor extraposed. Fourth, many if not most speakers don't interpret decimal expressions compositionally. For example, even if a speaker knew that *zero point one two five* represents one eighth, they would probably have no clue what meaning to assign to *zero point one three one*, at least not without using a calculator.

For these reasons, we think that decimal expressions aren't part of our innate grammar, but rather the product of an extra grammatical rule, much like the use of *whom* in affected speech. In this sense, these expressions are excellent candidates for what Morzycki (2017) calls *semantic viruses* — i.e., speech patterns that aren't technically part of the grammar but sometimes receive an interpretation via non-grammatical generalizations and ad hoc rules.

Finally, whatever analysis is assigned to decimals and differentials, the problems we raised in this paper for partiality measures still hold. On the whole, we think it's premature to abandon the traditional analysis of numeral modification in terms of cardinality. There are other theoretical explanations that can be invoked to explain the influence of partial objects.

Appendix A

Below is a complete list of the pretest, comparative, equative, and partiality test sentences that were used in the consultation session described in Section 3.2.

(41) **Pretest sentence**: If Zoe knitted half of a sweater, then Zoe knitted a sweater.

(42) **Context**: Zoe knitted a green sweater, a red sweater, half of a blue sweater, and a blue onesie. Esme knitted a purple sweater, a yellow sweater, a blue onesie, and a red onesie.

(43) a. **Comparative test sentence**: Zoe knitted more sweaters than Esme did.
b. **Equative test sentence**: Esme knitted as many sweaters as Zoe did.
c. **Partiality check**: Zoe knitted two and a half sweaters.

(106) **Pretest sentence**: If Zoe drew half of a circle, then Zoe drew a circle.

(107) **Context**: Zoe drew a green circle, a red circle, a purple half-circle, and a blue triangle. Esme drew a purple circle, a yellow circle, a blue triangle, and a red triangle.
a. **Comparative test sentence**: Zoe drew more circles than Esme did.
b. **Equative test sentence**: Esme drew as many circles as Zoe did.
c. **Partiality check**: Zoe drew two and a half circles.

(108) **Pretest sentence**: If Zoe built half of a car, then Zoe built a car.

(109) **Context**: Zoe built a sports car, a sedan (type of car), the front half of a second sports car (incomplete), and a canoe. Esme built a limo (type of car), a coupe (type of car), a canoe, and a sloop (type of boat).

a. **Comparative test sentence**: Zoe built more cars than Esme did.
b. **Equative test sentence**: Esme built as many cars as Zoe did.
c. **Partiality check**: Zoe built two and a half cars.

(110) **Pretest sentence**: If Zoe wrote half of a novel, then Zoe wrote a novel.

(111) **Context**: Zoe wrote a mystery novel, a romance novel, half of a second mystery novel (unfinished), and a magazine article. Esme wrote a fantasy novel, a Sci-Fi novel, a magazine article, and a newspaper article.
a. **Comparative test sentence**: Zoe wrote more novels than Esme did.
b. **Equative test sentence**: Esme wrote as many novels as Zoe did.
c. **Partiality check**: Zoe wrote two and a half novels.

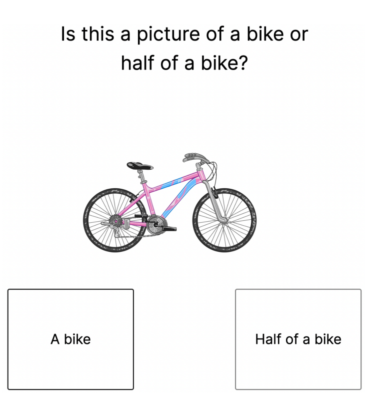
Appendix B

Instructions given to participants for experiment 2 (comparatives with priming). Note, instructions were presented via audio as well as appearing on the screen.

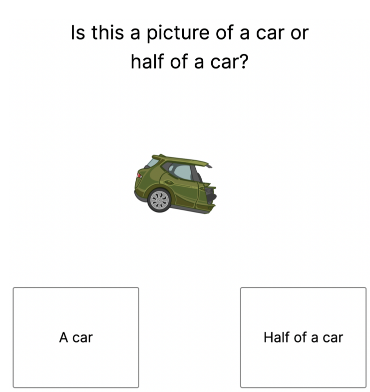
> [Picture of a cartoon farmer appears on screen.] Hello, I'm farmer Brown. I'll be asking you some questions today.
>
> I'm interested in buying some bikes and cars, so that I can use them to travel around my town. The bikes and cars must be in working order. As far as I'm concerned, half of a car is not a car. Likewise, half of a bike is not a bike.
>
> I'm also interested in buying some chairs and tables to use at my farm. The chairs and tables must also be in working order. As far as I'm concerned, an incomplete chair is not a chair. Half of a table is not a table.
>
> Let's get ready to answer some questions!
>
> Click on the best answer!

Example of two priming trials, one whole object and one partial object:

Example of a whole-object priming trial



Example of a partial-object priming trial



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