A formal investigation of Q-Theory in comparison to Autosegmental Representations*

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Abstract

We use model theory to rigorously evaluate Q-Theory as proposed in Shih and Inkelas (2019) as an alternative to Autosegmental Phonology. We find that Q-Theory is remarkably similar to AP, contra some of the claims in Shih and Inkelas. In particular, Q-Theory does not eschew the association relation, in Q-Theory the tone-bearing unit is the vowel, and Q-Theory and Autosegmental Phonology are equivalent in terms of the constraints they can express. However, this formal analysis clarifies the truly novel contribution of Q-Theory, which is the empirical claim that all segments are tripartite.

Keywords: phonology, representation, model theory, tone, autosegmental phonology, Q-Theory

1 Introduction

An important question when evaluating theories of phonological representation is whether two theories are substantially different in terms of their empirical predictions. This is related to the question of whether one theory states important generalizations in simpler terms than another. These are not easy questions to answer, as it

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requires a precise definition of representations in each theory and how the grammar operates over these representations. However, there are two related frameworks from mathematics and the theory of computation that provide tools for solving these problems: *model theory* (Libkin 2004) allows us to rigorously study phonological structure, and *mathematical logic* (Enderton 1972) provides constraint definition languages (Potts and Pullum 2002, de Lacy 2011, Jardine and Heinz 2016) that specify a range of constraints that operate over structures. These techniques have been applied to phonology before (Bird 1995, Scobbie et al. 1996, Potts and Pullum 2002, Graf 2010a,b, Jardine 2014), in particular with respect to comparing different theories of syllabic structure (Strother-Garcia and Heinz 2017, Strother-Garcia 2019).

In this paper, we use model theory to evaluate Q-Theory as proposed in Shih and Inkelas (2019, henceforth SI) as an alternative to Autosegmental Phonology (henceforth AP; Goldsmith 1976, Clements 1977). Q-Theory representations (henceforth QRs; see also Inkelas and Shih 2013, Shih and Inkelas 2014) capture classically 'autosegmental' tonal behavior, such as contours, by splitting each segment into exactly three subsegments. SI then analyze agreement and disagreement through surface correspondence, in an effort to incorporate tone into recent conceptions of phonological phenomena as mediated by surface correspondence (Hansson 2001, 2010, Rose and Walker 2004, Rhodes 2012, Bennett 2013).

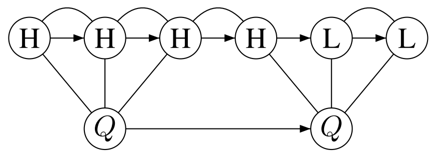
To give an example, SI give the following QR in (1a) for the Basaá word [hólôl] 'ripen' (Dimmendaal 1988, Hyman 2003), in which the first vowel is a level high tone and the second vowel is a falling tone. Each [o] vowel segment, or Q, is split into three sub-vowels, or qs, which each carry a tone. Indices on the qs represent correspondence between them. (The qs of the consonants have been abbreviated.)

(1) a. h(ó$_1$ ó$_{1,2}$ ó$_{2,3}$)l(ó$_3$ ò$_4$ ò$_4$)l b. H L
holol

In (1a), the first q of the second vowel is high-toned while the rest are low-toned; this thus represents the falling contour of the second vowel. Furthermore, the last q of the first vowel and the first q in the second are in correspondence (and both high-toned). This indicates that the falling contour on the second vowel is the result of partial agreement with the high-toned first vowel—a fact that in AP would be represented with spreading of a high tone from the first vowel to the second, as depicted in (1b).

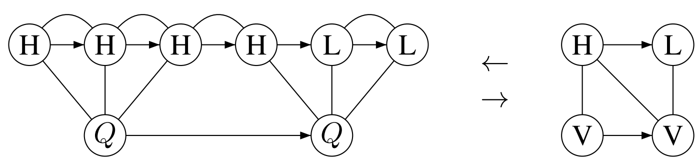
In comparing QRs to ARs, SI claim that 1) QRs dispense of the need for an association relation, thus simplifying the representation; 2) the q serves as the TBU, thus solving issues with ARs in determing what the relevant TBU is; and 3) QRs capture patterns that ARs cannot. In rigorously comparing QRs to autosegmental representations (henceforth ARs), we present several findings that weaken these arguments. The first is that, contra to SI's assertions, QRs do not dispose of the association relation in ARs.[1] Briefly, this is because there must be some relation connecting Qs to their composite qs in order for constraints to, for example, refer to the featural content of a particular Q. To illustrate, the following is a visual depiction of the model-theoretic representation of (1). (As above, the representation of consonants has been abbreviated.)

[1]SI: "[T]onal contours and tone assimilation can be straightforwardly captured in ABC+Q without recourse to the special representational machinery of autosegments and association lines" (p. 155).

(2) 

In (2), qs are depicted as the nodes labeled H and L, arrows indicate the order on the string of Qs and the string of qs, curved lines between qs depict correspondence, and straight lines depict the relation between Qs and their respective qs. As made clear by this visual representation (and will be shown formally in Section 2), this relation is virtually identical to AP's association relation between tone-bearing units and autosegments.

It is then possible to show that the similarities between the two representations run even further. SI assume that correspondence implies identity,[2] which allows us to define a *transformation* from QRs to ARs based on an equivalence between chains of corresponding qs and autosegments. An example of this transformation is given below in (3).

(3) 

This transformation clarifies several aspects of QRs. First, QRs are equivalent to ARs in which the segment is the tone-bearing unit (TBU). That is, qs are equivalent to autosegments, whereas Qs are equivalent to the vowels to which autosegments are associated. In other words, while SI claim that in QRs "q is the tone-bearing unit" (p. 152), our results show that it is Qs which are equivalent to the AP notion of a TBU. Thus, instead of solving the autosegmental problem of determining the correct TBU (SI, p. 155–6), QRs revert to the original autosegmental conception of the segment as the TBU (as in, e.g., Leben 1973, Goldsmith 1976). We discuss below how this weakens SI's claim that QRs better capture tone-consonant interaction.

Furthermore, a transformation between structures also implies a translation between constraints over these structures, and we establish that this shows two aspects of QRs to be equivalent to ARs. First, Q-correspondence—that is, correspondence between Q segments—cannot be derived from q-correspondence between q subsegments (or vice versa). SI argue that QRs are superior to ARs in their ability to capture patterns in which entire contours assimilate to each other. With ARs, assimilation of entire contours as units cannot be captured with autosegmental spreading, and thus must be captured with a separate mechanism checking the identity of TBUs. SI criticize ARs for this dual model of assimilation, but QRs also employ a dual model of assimilation: one based on q-correspondence and one based on Q-correspondence, which we show to be two distinct mechanisms. We illustrate this concretely with SI's analysis of Changzhi contour assimilation (Hou 1983, Duanmu 1994), which forms one of SI's

[2] "Our operating assumption is that GEN does not even produce candidates in which elements obey CORR but violate the associated IDENT-XX constraint." (p. 142)

main empirical examples of tone patterns they claim are better captured with QRs. We show that Q-correspondence is essentially analagous to vowel agreement in ARs.

Second, this translation between constraints shows that inter-vowel q-correspondence and intra-vowel q-correspondence are equivalent to constraints enforcing sharing of an autosegment and banning contours, respectively. Thus, the analyses using these constraints throughout SI's paper, including their analyses of Basaá and Mende, are no less possible using ARs.

In fact, while we focus on a small number of constraints, this transformation guarantees that for *any* surface constraint in Q-Theory, there is an equivalent constraint in AP that behaves exactly the same way and is no less complex. This result, which is based on established logical and model-theoretic techniques (Enderton 1972, Courcelle 1994, Courcelle et al. 2012), is given in detail in Danis and Jardine (2019). To summarize, Danis and Jardine give *first-order translations* from QRs to ARs and vice-versa. First-order logic is a weak predicate logic that is still able to express most, if not all, phonological constraints (Scobbie et al. 1996, Potts and Pullum 2002), and thus seems an appropriate upper bound for the complexity of phonological generalizations (Rogers et al. 2013). The translations of Danis and Jardine show for any constraint that can be written in the first-order logic of ARs, an equivalent constraint can be written in the logic of QRs, and vice versa. Thus, the two representations are equally expressive with respect to the kinds of constraints that one would write for phonological analyses. The purpose of this paper is to detail the consequences of this formal result for phonological theory.

While these results contradict some of SI's assertions, we do not claim that ARs and QRs are entirely identical, and we do not reject the utility of QRs. Our results show that ARs and QRs are equivalent with respect to their expressivity, but there are ways in which the two representational theories are conceptually distinct. In particular is the QR axiom that all segments are made up of exactly three parts. This claim is not usually implemented in ARs, and thus QRs allow phonologists to ask questions about phonology they might not otherwise. Rather than reject this distinction, the formal results we present highlight this as the true difference between QRs and ARs. Our paper thus follows in the spirit of Kornai and Pullum (1990), who formally analyze X-bar theory in order to distinguish its true novel theoretical contributions from specious differences to context-free grammars. We also find that interesting questions remain regarding the consequences of different assumptions regarding Q-Theory and correspondence, which we discuss in detail below. This thus highlights the value to phonological theory of the rigorous model-theoretic analysis of phonological structure.

It is worth noting that this paper focuses on constraints governing surface correspondence in QRs and their equivalent markedness constraints in ARs. This is because it is the nature of these surface constraints that form the bulk of SI's arguments. Additionally, the focus of this paper is on the structural similarities between ARs and QRs. We thus leave an exploration of the relevant input-output (IO) correspondence constraints to future work. The contribution of this work can be viewed as follows: *if* there is a difference in expressivity between Q-Theory and AP, then it is in IO faithfulness, and not in constraints on surface well-formedness (as SI argue).

This paper is structured as follows. §2 introduces model theory and uses it to define each representational theory. §3 shows how we can define a transformation from one representation to the other. §4 then gives examples of how constraints can be translated from one theory to another. §5 discusses the consequences and scope of these results, as well as the remaining questions, and §6 concludes.

2 Defining the representations

2.1 Model Theory

Model theory is the mathematical study of structure (Enderton 1972, Libkin 2004), and can be used to rigorously define phonological constraints and phonological theories (Bird 1995, Potts and Pullum 2002, Graf 2010a,b). In intuitive terms, a *model* of a structure explicitly describes the composite elements of a structure and the relationships between these elements. Formally, a *relational model* is a tuple

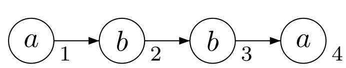
$$\langle D; R_1, R_2, ..., R_k \rangle$$

with a *domain* D and a finite set of k relations over D. Here we only need to consider models where each R_i is a *binary* relation $R_i \subseteq D \times D$ or a *unary* relation $R_i \subseteq D$. A *signature* is a fixed set of relations $\{R_1, R_2, ..., R_k\}$. For example, strings of as and bs (e.g., ab, ba, $babab$, $bbbb$, etc.) can be represented by relational models of the signature in (4).

(4) $\{\triangleleft, P_a, P_b\}$

In (4), D is the set of positions in a string, $\triangleleft$ is a binary *successor* relation indicating the order on positions, and P_a and P_b indicate the sets of positions occupied by an a and a b, respectively. For example, the string $abba$ can be represented by the model given in (5).

(5) 1. $\langle \; D = \{1,2,3,4\};$
$\triangleleft = \{(1,2),(2,3),(3,4)\},$
$P_a = \{1,4\},$
$P_b = \{2,3\} \; \rangle$

2. 

The string $abba$ contains four elements, enumerated by the domain set $D = \{1,2,3,4\}$. The relation $\triangleleft$ in (5a) explicitly shows that element 1 is succeeded by element 2, element 1 is succeeded by 3, element 2 is succeeded by element 3, and so forth. The unary relation P_a represents the set of elements labeled a, namely 1 and 4. Likewise, P_b represents the set of elements labeled b; namely, 2 and 3. The figure in (5b) depicts this model visually, with the elements in D depicted as circles, $\triangleleft$ as arrows and the elements belonging to P_a and P_b marked with the labels a and b, respectively.

We can represent any string of as and bs with a similar model using the signature $\{\triangleleft, P_a, P_b\}$. The converse, however, is only true if we assume some axioms on the model: a model $\langle D; \triangleleft, P_a, P_b \rangle$ represents a string if and only if $\triangleleft$ is a well-formed successor relation and the sets of P_a and P_b partition D into distinct sets.

In this way, models allow us to explicitly define classes of structures, and ultimately to define constraints which compute over these structures (see also Potts and Pullum 2002, Graf 2010a,b). The following uses model theory to explicitly define and compare ARs and QRs.

2.2 Autosegmental theory

In their own words, SI characterize Autosegmental Phonology (AP) as having the two defining properties:

(6)
1. "features exist autonomously, each on its own independent tier, organized by a central timing skeleton"
2. "the association between elements on featural tiers and elements on the timing tier can be one-to-one, one-to-many, many-to-one, or even zero-to-one in the case of floating features or featurally underspecified timing units" (p. 137)

For classical ARs, the timing tier consists of segments (or more properly, root nodes), with the signature below. We can follow standard formal definitions of ARs (Goldsmith 1976, Coleman and Local 1991, Kornai 1995, Jardine 2014). As we focus on tone, we make two assumptions: first, we focus on two-tier ARs in which the 'features' are high (H) tones or low (L) and second, the timing tier nodes are vowels. Treating vowels as the TBU is no longer standard (Yip 2002), but as we argue throughout the paper (in particular, §3.3), this is the closest AR equivalent to QRs.

(7) $\mathcal{S}_{\mathrm{AR}} = \{\triangleleft_{\mathrm{A}}, \mathcal{A}_{\mathrm{A}}, V_{\mathrm{A}}, H_{\mathrm{A}}, L_{\mathrm{A}}\}$

In this signature, $\triangleleft_{\mathrm{A}}$ and $\mathcal{A}_{\mathrm{A}}$ are binary relations refering to successor and autosegmental association, respectively. V_{A}, H_{A} and L_{A} are all unary relations that labels nodes as either vowels or H or L tones. For simplicity, but without loss of generalization, we omit the consonants from the representation.

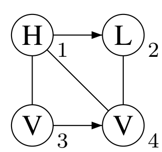


Figure 1: AR model for Basaá word [hólôl] 'ripen'

In this model, H_1 is associated to both V_3 and V_4 (i.e. these pairs are members of $\mathcal{A}_{\mathrm{A}}$), L_2 is associated to V_4, H_1 precedes L_2, and V_3 precedes V_4. This model also shows a one-to-many relation from tones to vowels (from H_1) and a one-to-many relation from vowels to tones (from V_4). However, there are additional axioms required to ensure a well-formed AR, such as the No-Crossing Constraint (Goldsmith 1976, Hammond 1988). We refrain from detailing these axioms here; for a recent discussion of axiomitizations of ARs see Jardine and Heinz (2015).

We further assume an axiom that a vowel can associate to at most three tonal autosegments. This is not often made explicit in the phonological literature, but recent formal work has shown how axioms limiting the length of vowel-to-tone associations can be naturally expressed in ARs (Yli-Jyrä 2013, Jardine 2014, Jardine and Heinz 2015).

2.3 Q-Theory

The foundational idea of Q-Theory is that representations consist of Q segments subdivided into three q segments each. Featural content—in this case, tone—is then part of the qs. Another assumption of Q-Theory is that QRs also include surface correspondence relations relating both Qs and qs. As qs carry featural information, agreement in tone is mediated through the qs.

However, the larger Q units are also essential to QRs. ("Qs are strings of qs which the grammar can refer to", SI p. 151; constraints on p. 153 indeed refer to Qs.) SI suggest that the Qs may be 'emergent' (SI p. 151) and not fundamental elements of the grammar. However, the much of the extant literature on Q theory makes extensive use of them (Inkelas and Shih 2013, Shih and Inkelas 2014). As an example, SI's analysis of the contour patterns in Changzhi (p. 160) and Tianjin (SI, p. 162) require correspondence at the Q level, and necessarily not at the q level. (In fact, on p. 152, SI state that "reference to the segment as a whole (Q) is crucial for capturing contour behavior.") We thus must accept Qs as an essential part of QRs under the current version of the theory.

Thus, for every vowel, a QR has a Q element representing the vowel at the segmental level and a series of three ordered q elements representing the vowel's featural information at the sub-segmental level. This is depicted explicitly for the QR from (1a) in figure. 2. (As with figure. 1, we abstract away from consonant Qs.) Elements 7 and 8 are Qs, and elements 1 through 6 are qs.

Crucially, because the featural content is not carried by the Qs, yet identity between Qs is based on the featural content of their constituent qs (SI, p. 153), there must be some relation connecting a particular Q to its composite qs. In model-theoretic terms, the QR signature must include a binary relation that relates Qs to qs. Let us call this $\mathcal{A}_{\mathrm{Q}}$, which is depicted as straight lines between Qs and qs in figure. 2.

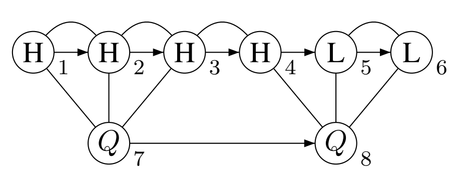


Figure 2: QR model for Basaá word [hólôl] 'ripen'

We need not necessarily call $\mathcal{A}_{\mathrm{Q}}$ 'association,' but as a binary relation associating elements on different tiers, it is essentially identical to the $\mathcal{A}_{\mathrm{A}}$ relation in ARs.

Finally, QRs also contain a correspondence relation; let us call this $\mathcal{R}_{\mathrm{Q}}$. This is depicted as curved lines between qs in figure. 2. Three things are worth mentioning about $\mathcal{R}_{\mathrm{Q}}$. First, while SI claim that their correspondence relation is non-transitive (SI, p. 140), their correspondence constraints are sensitive to *correspondence chains* of transitively connected chains of corresponding segments. SI state: "[A] sequence of three identical consecutive segments S in a grammar requiring that identical segments correspond would satisfy that constraint as follows: $S_1 S_{1,2} S_2$, where coindexation encodes correspondence" (p. 140). That is, in such a configuration the first S_1 and third S_2 satisfy any correspondence constraints. This is in direct contradiction to the statement that correspondence is intransitive—it is because S_1 corresponds to $S_{1,2}$, and that $S_{1,2}$ corresponds to S_2, that S_1 and S_2 satisfy, by transitivity, the constraint. For simplicity, we encode this transitivity in the relation itself.[3] However, in our depictions of correspondence (as in figure. 2), for visual clarity we only depict consecutive correspondence relations.

The second important assumption about correspondence is that it implies identity: "[o]ur operating assumption is that GEN does not even produce candidates in which elements obey CORR but violate the associated IDENT-XX constraint" (SI, p. 6). This is crucial for the transformations between structures discussed below. Third, correspondence can operate either between qs or between Qs; see §4.1.

Thus, in a QR we have:

(8) $\mathcal{S}_{\mathrm{QR}} = \{\triangleleft_{\mathrm{Q}}, \mathcal{R}_{\mathrm{Q}}, \mathcal{A}_{\mathrm{Q}}, Q_{\mathrm{Q}}, H_{\mathrm{Q}}, L_{\mathrm{Q}}\}.$

In $\mathcal{S}_{\mathrm{QR}}$, $\triangleleft_{\mathrm{Q}}$ is the ordering relation on the q tier and the Q tier (which functions identically to the $\triangleleft_{\mathrm{A}}$ relation in ARs), $\mathcal{R}_{\mathrm{Q}}$ is the correspondence relation $\mathcal{R}_{\mathrm{Q}}$, $\mathcal{A}_{\mathrm{Q}}$ is the 'association' relation between qs and Qs. There are also three unary relations: Q_{Q} identifying Qs, and H_{Q} and L_{Q} identifying H-toned and L-toned qs, respectively (as only qs carry featural information, these featural relations are enough to distinguish the qs from Qs).

3 From one representation to another

Visually, the similarities between Figs. 1 and 2 are clear: if we merge all of the qs in correspondence chains in figure. 2, (and relabel the Qs to Vs) then we obtain the AR in figure. 1.[4] In this section, we describe a step-by-step process that transforms one structure into the other, and show how this implies a translation of constraints in one theory to another. That this transformation is of limited computational complexity—

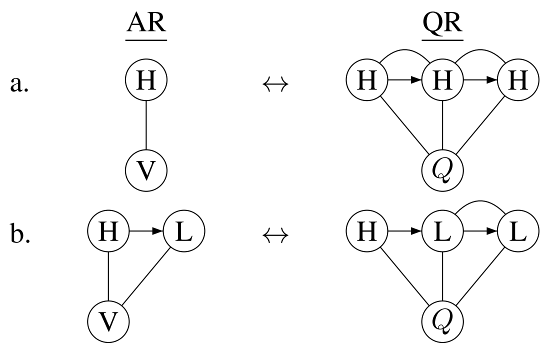
[3] If we were to not encode this transitivity in the representation, the correspondence constraints themselves must be endowed with the ability to detect an unbroken chain of local, intransitive correspondence relations. This is demonstratably beyond the expressive power normally attributed to phonological constraints. To relate this to the notion of first-order definability mentioned in the Introduction, detecting an unbroken path through an intransitive relation is taking its *transitive closure*, which is well-known to be not first-order definable (see, e.g., Libkin 2004).

[4] SI treat qs as feature bundles, so this description abstracts away from other features. To accommodate other features, we would 'pull out' tonal features from corresponding qs and put them on a tonal tier in the autosegmental representation.

and thus preserves the expressivity of the constraints in each theory—is shown in Danis and Jardine (2019); here, we give an informal presentation of the procedure.

First, we note that the identity of vowels in each theory is based on the string of units to which they are associated—autosegments in ARs, and qs in QRs. Thus, for example, an AR representing a vowel associated to a single high tone is equivalent to a QR with a vowel Q associated to three corresponding H-toned qs, as shown in (9a). Similarly, a vowel associated to a HL sequence in an AR corresponds to a QR with a vowel Q associated to a H-toned q and two corresponding L-toned qs. The correspondence between qs is based on our assumption of an equivalence between autosegments and correspondence chains of qs; this will be described in more detail momentarily.

(9)



In general, this method guarantees a way to convert an AR into a QR (and vice versa) based on tone strings to which each vowel is associated. As illustrated above, implementing the equivalence between autosegments and qs is a matter of assigning correspondence relations in the QR according to the number of autosegments. That is, for each tone t associated to a vowel in the AR, there is a correspondence chain c in the QR. The following gives a step-by-step method to transform ARs to QRs, and then QRs to ARs, based on this idea. Throughout, it is shown that this is possible due to SI's assumption that correspondence implies identity.

3.1 From autosegmental representations to Q-Theory representations

As illustrated in (9), we can translate individual vowels in an AR to vowel Qs in a QR based on the string of autosegments associated to each vowel. Such a translation is done out in full in (10). Correspondence between qs in (10) is indicated with indices.

(10)

Autosegments		qs	Autosegments		qs
H	↔	$H_1H_{1,2}H_2$	L	↔	$L_1L_{1,2}L_2$
HL	↔	$H_1L_2L_2$	LH	↔	$L_1H_2H_2$
HLH	↔	$H_1L_2H_3$	LHL	↔	$L_1H_2L_3$

Three assumptions are required for this transformation. The first is that correspondence implies identity in the QRs, as also assumed by SI. Without this, we could not equate q-correspondence chains to autosegments. Going from ARs to QRs, the chart in (10) assumes that for each autosegment we can create a string of corresponding, like-toned qs. Going from QRs to ARs, (10) assumes that a chain of like-toned qs can be collapsed into a single autosegment. If correspondence did not imply identity, then q-correspondence chains could include both H-toned and L-toned qs, and there would be no way to equate these with a single tonal autosegment.

The second assumption is that there is a bound on the number of tones associated to vowels in the ARs, because of the bound on the possible constrasts in QRs. SI propose the bound on qs for empirical reasons: contours (whether tonal or segmental) never appear to be more than three units long. As has been already stated, this is not an unreasonable constraint to put on ARs; for example, Jardine and Heinz (2015) show how this can be done in a natural way.

Note that not all possible correspondences among elements in the q strings are attested in the table. Some represent contrasts that appear to have no extensional consequences: for instance, a string $\mathrm{H_2H_2H_1}$ of H qs in which the first qs are in correspondence but the third is not. Such distinctions are not crucial to any of SI's analyses; indeed, they may not play any role in any grammar. Candidates consisting of identical segments but not in surface correspondence are all harmonically bounded in the systems investigated in Bennett and DelBusso (2018), supporting Bennett (2013, 33)'s claim that, "while there are many candidates that differ only in their surface correspondence structure, the majority of these are irrelevant for any given interaction". Thus, while it is true that QRs can make distinctions that ARs cannot, these distinctions do not have any extensional consequences, and therefore we ignore them for the purposes of this translation.

However, there are some contrasts that do have empirical consequences: SI entertain the possibility of distinctions between q strings such as HLL versus HHL (as reported in Dinka; SI, Remijsen 2013). This could be relaxed by allowing ARs that violate the OCP; thus, the Dinka contrast could similarly be captured as the difference between a vowels associated to a HL string and a vowel associated to a HHL string. However, as SI's analyses do not make use of the full range of these possibilities, the chart in (10) is sufficient for our purposes.

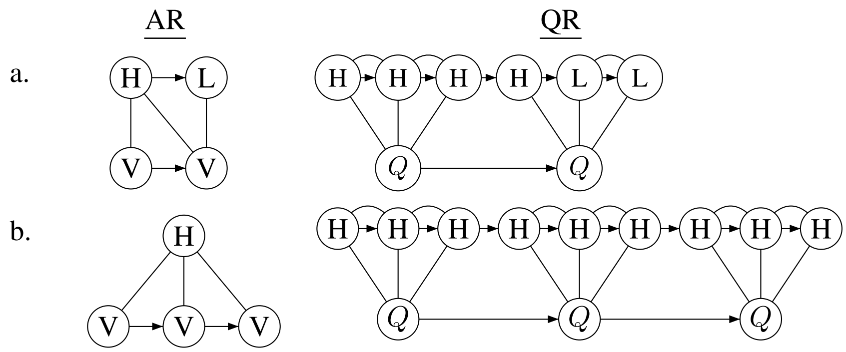
Given this chart, we can posit a step-by-step process that transforms an AR into a QR. This is given in (11).

(11) Algorithm for transforming ARs into QRs

1. For each V in the AR, place a Q in the QR associated to a string of qs according to the chart in (10).
2. For each pair of Vs V_1, V_2 in the AR associated to the same autosegment, for their equivalents Q_1 and Q_2 in the QR, draw a correspondence chain between their qs.
3. For each pair of Vs V_1, V_2 associated to identical strings of autosegments in the AR, draw a correspondence relation between their equivalents Q_1 and Q_2 in the QR.

Step (111) simply creates a string of Qs associated to qs based on the chart in (10). This step is illustrated below in (12) using two different ARs as inputs.

(12) Step (111): creating Qs and associated qs.

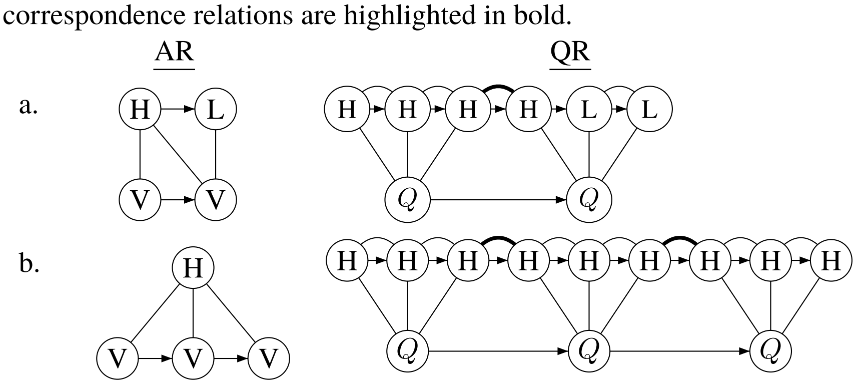


In (12a), we have the AR model for the Basaá word [hólôl] ‘ripen’. Given the instructions in Step (111), this creates two Qs. Because the first V in the AR is associated to a single H, the first Q is associated to a string $\mathrm{H}_1\mathrm{H}_{1,2}\mathrm{H}_2$ of H-toned qs belonging to a single correspondence chain, as specified in the autosegment/q-string translation chart in (10). As the second V in the AR is associated to a HL string, the second Q is associated to a string $\mathrm{H}_1\mathrm{L}_2\mathrm{L}_2$ of qs, the first being an H-toned q and the second two being L-toned qs in correspondence.

In (12b), we have an AR showing ‘plateauing’ of a H tone across three vowels. Following Step (112) and the autosegment/q-string translation chart in (10), this maps to three Qs, each associated to a string $\mathrm{H}_1\mathrm{H}_{1,2}\mathrm{H}_2$ of corresponding qs.

The next step, (112), completes the implementation of the idea that autosegments are equivalent to q-correspondence chains, which is how to equate autosegments associated to multiple vowels with q-correspondence chains that span across multiple vowel Qs. This is simple: for any two vowels V_1 and V_2 associated to the same autosegment in an AR, ensure that there is a correspondence chain between the qs of the equivalent Q_1 and Q_2 in the QR. In our diagrams, this amounts to connecting the final q of Q_1 and the initial q of Q_2 for each successive pair of Qs whose equivalent vowels share a tone in the AR.[5] This is illustrated below (13) for both our contour and plateauing example.

(13) Step (112): extending q-correspondence chains to adjacent Qs. New correspondence relations are highlighted in bold.



In (13a) and (b), the first and second vowels in the ARs are associated to the

[5]To see that this creates a transitive correspondence relation, note that shared association to a tone is transitive: if V_1 and V_2 share T_1, and V_2 and V_3 share T_1, then of course V_1 and V_3 also share T_1.

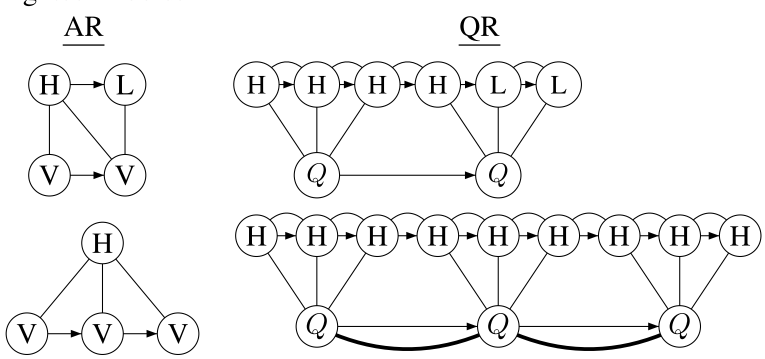
same H autosegment. Thus, in their QRs, an additional q-correspondence relation is drawn between them. In (13), this also happens between the second third vowel Qs, as the vowels in the AR also share a H autosegment. Thus, we have created a single q-correspondence chain in the QR for each autosegment in the AR. Note that the corresponding qs are all identical because they derive from the same autosegment—thus, this transformation is only possible if correspondence implies identity in the QR.[6]

Finally, Step (113) addresses correspondence between Qs. As SI explain, the tone value of a Q is based on its component qs. (More on this below in Sect. 4.1.) As correspondence of Qs is based on this value, starting from an AR, we can create correspondence between vowel Qs based on the strings of autosegments associated to their equivalent Vs in the AR. Thus, Step (113) draws correspondence relations between Qs in the QR whose equivalent Vs are associated to matching strings of autosegments. This is illustrated with our running example in (14).

(14) Step (113): Correspondence chain between Qs. New correspondence relations are highlighted in bold.

a.

b.



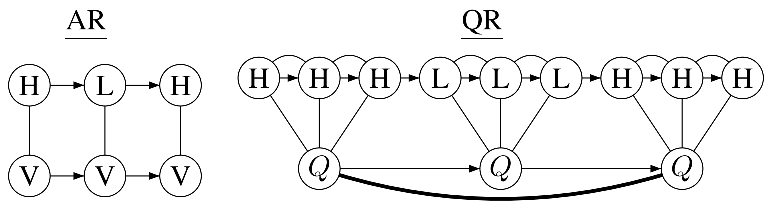
In (14a), the two vowels in the AR are associated to a H and HL string of autosegments, respectively. Thus, their corresponding Qs in the QR do *not* correspond. In contrast, in (14b), the first and second vowel in the AR are both associated to a matching string of autosegments—namely, H—and so their equivalent Qs correspond. The same goes for the second and third vowels and (though not depicted) the first and third vowels (again, we are considering transitive correspondence—see §2.3). Note that this results in QRs in which identity implies correspondence—it is impossible to generate structures in which Qs are identical but do not correspond. As we have noted already for qs in the discussion following table 10, this contrast does not play any role in the grammar and thus ommitting it has no effect on our goal of showing the two theories are equally expressive.

A third example below emphasizes that Q-correspondence is based on the autosegmental *strings* and not the sharing of autosegments. Below, two H-toned vowels are

[6]There is one more result of this transformation, which is that correspondence between qs is limited to adjacent qs. Strictly speaking, SI's definitions appear to allow long-distance correspondence of qs. However, their analyses make no explicit use of it. There is one potential case in which it is necessary, with respect to consonants, and this is discussed in 3.3. For now, as long-distance correspondence between qs does not play a crucial role in SI's analyses, we assume for the rest of the paper that q-correspondence is restricted to adjacent qs.

separated by a third L-toned vowel—Step (113) of our transformation would draw a correspondence relation between the Qs equivalent to these two H-toned vowels.

(15) Step (113) (continued): Correspondence chain between Qs based on identity of consecutive elements. The crucial correspondence relation is highlighted in bold.



Note that this, in a sense, reduces Q-correspondence to agreement. This appears to be a consequence of the assumption that correspondence implies identity, and so is entirely compatible with the analyses in SI. For more discussion see Sect. 4.1.

Thus, the algorithm in (11) produces a QR from any AR whose contours are no longer than three autosegments long. Again, this is based on the idea that correspondence chains of qs are equivalent to autosegments—an assumption made possible by the fact that correspondending elements in the QRs must be identical.

3.2 *From Q-Theory representations to autosegmental representations*

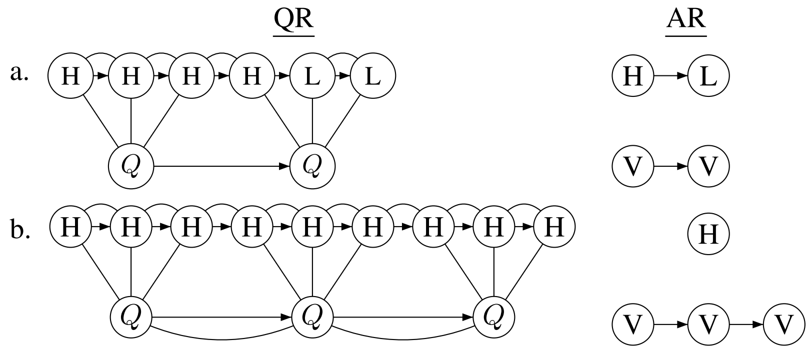
We now give the algorithm for the reverse operation: creating an AR from a QR. Following the notion that q-correspondence chains are equivalent to autosegments, we simply 'merge' members of a q-correspondence chain into a single autosegment. This is outlined explicitly in (16).

(16) Algorithm for transforming QRs into ARs

1. For each Q in the QR, create a V in the AR.
2. For each correspondence chain of qs:
   (i) Draw an autosegment of the same tone value in the AR.
   (ii) Associate that autosegment to each $V_1, V_2, ..., V_n$ for each $Q_1, Q_2, ..., Q_n$ that was associated to some q in the correspondence chain.

Step (161) is simple: vowel Qs in the QR are equivalent to Vs in the AR. Step (162) is split into two sub-steps, which are best illustrated separately. The first, which creates an autosegment for each correspondence chain, is illustrated with our running contour and plateau examples below in (17).

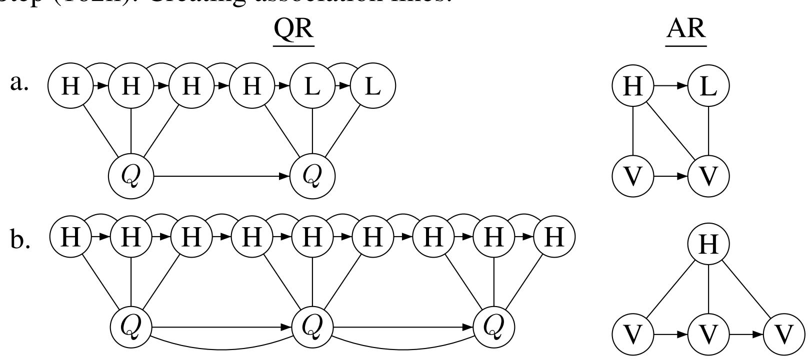
(17) Step (162i): Creating autosegments from q-correspondence chains.



In (17a), we have our QR representation of the Basaá word [hólôl] 'ripen'. There are two q-correspondence chains here: one of a series of H-toned qs, followed by another comprised of a series of L-toned qs. Thus, in the AR, we draw one H autosegment equivalent to the chain of H-toned qs, followed by one L autosegment equivalent to the chain of L-toned qs. In (17a), we have a QR representation of a plateau of H-toned vowels. As all qs correspond, Step (162i) creates a single H tone in the equivalent AR.[7]

Step (162ii) generates association lines in the AR based on associations in the QR, as illustrated below in (18).

(18) Step (162ii): Creating association lines.



Step (162ii) states that for each correspondence chain, for every Q_i that is associated to some q_i in that chain, Q_i's equivalent V_i in the AR is associated to the autosegment a_i equivalent to that correspondence chain. Thus, for example, in (18a), both vowel Qs are associated to qs in the chain of corresponding H-toned qs. Thus, both vowels in the AR are associated to that chain's corresponding H autosegment. In contrast, only the second vowel Q is associated to qs in the chain of corresponding L-toned qs, so only the second vowel in the AR is associated to the equivalent L. In (18b), all Qs are associated to some q in the chain of corresponding H-toned qs, and so all vowels in the AR are associated to the same tonal autosegment.

This illustrates how the two steps in (16) can generate an AR from any QR. Note that the algorithm does not need to make any reference to correspondence between

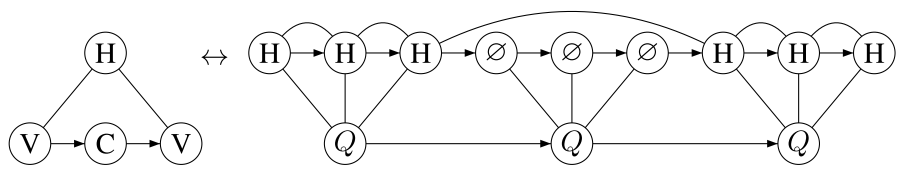
[7]This follows the assumption of the chart in (10), in which adjacent, like-toned qs correspond. This essentially boils down to the OCP: if we relax this assumption and allow, for example, a string of H-toned qs that are not in correspondence, we then get a string of H autosegments in the AR. We avoid a discussion of the universality of the OCP, as this assumption bears little on our results.

Qs. As shall be seen below in Sect. 4.1, this is due to the fact that correspondence can be replaced with tonal identity in the vowels. Thus, the function played by Q correspondence can be assumed by existing parts of the representation (namely, the association relation linking Qs/vowels to qs/tones).

3.3 Locality of correspondence and consonants

One last point with respect to the correspondence between qs bears clarification. It may be that long-distance correspondence of qs is necessary for correspondence chains of V qs to 'skip' the qs of Cs that are not specified for tone. This appears equivalent to, in ARs, Cs not projecting tones on the tonal tier. In terms of our transformation between ARs and QRs, this amounts to not assigning qs unspecified for tone in the QR to any tone in the AR. This is illustrated in (19), which explicitly adds a toneless consonant to the representation.

(19) AR QR



In the QR in (19), the middle consonant Q is unspecified for tone, indicated by the $\varnothing$ label on its qs, and the correspondence relation between the toned vowel qs 'skips' these toneles qs. This requires slight modification to our rule positing correspondence between qs of adjacent vowels, but it amounts to redefining 'adjacent' vowels as those for which only Cs intervene. If we then posit that toneless Cs in the QR project no autosegment in the AR, then we have captured this equivalence and maintained the notion that autosegments in the AR correspond to correspondence chains of qs in the QR.

It should be pointed out that SI posit that it *is* possible for Cs to bear tone: in their analysis of depressor consonants in Siswati, Cs are given L-toned qs. Given the transformation we have specified, this is equivalent in ARs to consonants that project a L tone. This was the standard analysis of depressor consonants in autosegmental phonology (Cassimjee and Kisseberth 1992). Thus, while SI argue that QRs better capture tone-consonant interactions, in actuality their argument is for a return to the segment as the TBU, as this more accurately captures tone-consonant interactions.

4 A translation between constraints

The following applies the transformation outlined in the previous section to the constraints of Q-Theory to compare how tone patterns are analyzed over QRs as opposed to ARs. We present two findings, both of which significantly weaken SI's arguments against ARs. First, the Q-correspondence at the core of SI's analysis of contour assimilation is a distinct mechanism from q-correspondence, contrary to SI's argument that only ARs must posit a separate assimilation mechanism for these patterns. Second, q-correspondence is identical to autosegmental spreading in ARs, and thus SI's constraints on 'inter-syllable similarity' and 'intra-syllable similarity' are equivalent to the AR concepts of sharing of autosegments and bans against contours, respectively.

4.1 Vowel identity and Q-correspondence

We first explore constraints concerning Qs in Q-Theory, which SI argue more insightfully capture assimilation and dissimilation patterns in tone languages in which entire contours behave as units. However, these constraints are identical to agreement constraints between vowels in ARs. Furthermore, we find that Qs cannot be 'emergent' from qs, as SI claim (p. 151). This means that Q-correspondence is necessarily distinct from q-correspondence. Thus, assimilation at the segmental level and assimilation at the sub-segmental level are captured by two distinct mechanisms in Q-Theory, for which SI criticize AP.

One type of pattern that SI claim is more directly captured in Q-Theory is reduplicative contour assimilation, such as in Changzhi (Hou 1983, cited in Duanmu 1994). In Changzhi, the diminuitive suffix /-tə$^{ʔ}_{535}$/ takes on the tone of its root; thus /təu$_{53}$-tə$^{ʔ}_{535}$/ 'bean-DIM' is realized as [təu$_{53}$-tə$^{ʔ}_{53}$]. SI analyze this as a CORR-VV constraint which forces adjacent vowels to correspond and thus be indentical. Thus, [təu$_{53}$-tə$^{ʔ}_{53}$] surfaces because *[təu$_{53}$-tə$^{ʔ}_{535}$] is ill-formed. This is shown with QRs in figure. 3, with the contours 53 and 535 represented as HL and HLH, respectively, in keeping with the notation used throughout this paper.

a.

b. *

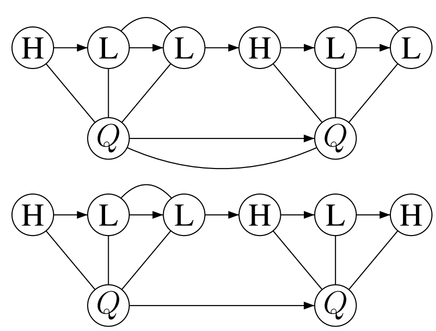


Figure 3: QRs for (a) the identical [təu$_{53}$-tə$^{ʔ}_{53}$] and non-identical (b) *[təu$_{53}$-tə$^{ʔ}_{535}$] in Changzhi

In figure. 3, (b) does not satisfy the CORR-VV requirement because the two adjacent Qs are not in correspondence. In contrast, (a) satisfies the CORR-VV constraint, and its concomitant identity requirement, because the two vowels are in correspondence, and the Qs are associated to identical strings of qs. In general, a constraint on Q identity requires that for Q_1 and Q_2 in correspondence, if the qs associated to Q_1 form a string w of tones, then the qs associated to a Q_2 must also form a string w.

This notion of identity does not change in autosegmental terms. To see, we can apply our transformation to the representations in figure. 3.

a.

b. *

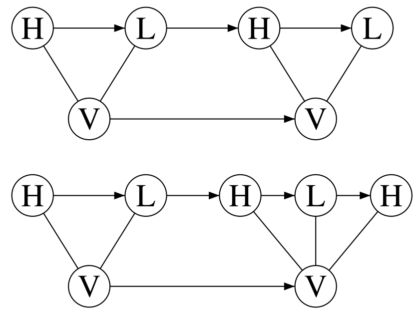


Figure 4: ARs for (a) the identical [təu$_{53}$-təˀ$_{53}$] and non-identical (b) *[təu$_{53}$-təˀ$_{535}$] in Changzhi

We can conceive of vowel identity in ARs as identical to Q-identity in QRs: two vowels are identical if they are associated to identical strings of tones. This obtains the distinction in figure. 4, as in figure. 4a both vowels are associated to HL tone strings, but in figure. 4b, the first vowel is associated to a HL tone string while the second is associated to a HLH tone string.

SI criticize a vowel-identity mechanism for assimilation in ARs, as they then do not have a unified theory of assimilation (SI, pp. 159–160). SI's reasoning is that assimilation in autosegmental phonology is operationalized as spreading, but in these tone contour cases we must resort to tone copying. However, Q-Theory also has two distinct mechanisms for accomplishing assimilation: Q-correspondence and q-correspondence. To see that Q-correspondence (and thus identity) is not derived from q-correspondence, imagine if we were to derive Q-identity from q-correspondence. To ensure the identity of the q strings, we have to check that the first q of Q_1 is the same as the first q of Q_2, and the second q of Q_1 is the same as the second q of Q_2, and the same with the third qs. This implies a one-to-one correspondence between qs as depicted in figure. 5a.

a.

b.

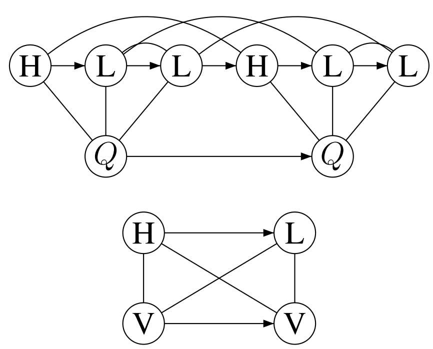


Figure 5: A QR showing identity of Qs through q-correspondence (a), and an equivalent AR with line crossing (b).

SI posit no such q-correspondence; rather, correspondence between Qs is predicated on the fact that the Qs are adjacent, and the identity between the two Qs is determined through the tonal identity of their respective q strings. Thus, Q-correspondence is a mechanism independent from q-correspondence.

Furthermore, if we apply our transformation to figure. 5a and merge all corresponding qs, we obtain a line-crossing AR in figure. 5b. SI point out that such an analysis of contour in ARs is impossible. However, our transformation shows that an an equivalent correspondence relation between qs is similarly impossible in Q-Theory. Thus, the two theories agree that contour-agreement and spreading-agreement are handled by distinct mechanisms.

4.2 Translating between q constraints and autosegmental constraints

We now further explore the equivalence of q-correspondence to autosegmental spreading, based on SI's analysis of Basaá. This is briefly illustrated in the tableau in (20) for the input /h(ó ó ó)l(ò ò ò)l/, with an all high-toned Q is followed by an all low-toned Q. Again, we focus here on surface well-formedness constraints, setting aside faithfulness (though see §5 for discussion). The key constraint for this analysis is Corr-v:\$:v, which states that vowel qs separated by a syllable boundary must be in correspondence. When Corr-v:\$:v outranks Corr-v::v, the constraint that requires adjacent qs within a Q to correspond, the winning candidate has the last q of the first vowel and the first q of the second vowel in correspondence. Since correspondence entails identity, the two qs also agree, thus creating a contour tone.

(20)

	/h(ó ó ó)l(ò ò ò)l/	Corr-v:\$:v	Corr-v::v
☞	a. h(ó$_1$ ó$_{1,2}$ ó$_{2,3}$)l(ó$_3$ ò$_4$ ò$_4$)l		*
	b. h(ó$_1$ ó$_{1,2}$ ó$_{2,3}$)l(ò$_4$ ò$_{4,5}$ ò$_5$)l	*!	

In (20), candidate (a), which has a contour, satisfies Corr-v:\$:v because of the correspondence relation of index 3 bridges two qs across syllables. This violates Corr-v::v (and input-output faithfulness), because the vowel-internal qs now disagree, but it beats the faithful candidate (b) which violates Corr-v:\$:v.

Let us now look at these constraints more carefully. For brevity, we avoid introducing syllable structure directly into the representation and simply state that vowel qs are in different syllables if they are associated to different vowel Qs. Given this, Corr-v:\$:v enforces a logical implication that states that for two adjacent qs q_1 and q_2, if they are associated to adjacent vowels then they must be in correspondence. This is illustrated in figure. 6.

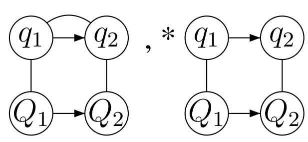
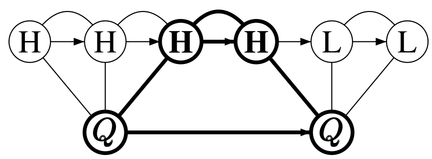


Figure 6: Visual representation of the requirement of Corr-v:\$:v.

Figure 6 depicts the requirement of CORR-v:$:v: when q_1 is immediately followed by q_2, but q_1 and q_2 are associated to distinct Qs (Q_1 and Q_2), then q_1 and q_2 must be in correspondence. A structure in which they are not in corresponence is forbidden; note the absence of the relation is crucial. To illustrate, the full models of the candidates from (20) are given below in figure. 7, with the structures targeted by CORR-v:$:v highlighted in bold.

(=20a)



(=20b) *

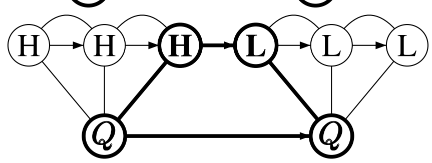
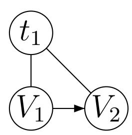


Figure 7: Examples of CORR-v:$:v. The upper figure highlights the portion of candidate (20a) that satisfies the requirement of CORR-v:$:v. This contrasts with the lower figure, which highlights the locus of the violation of CORR-v:$:v in candidate (20b).

Applying our transformation to the structures in figure. 6, we obtain the autosegmental constraint in figure. 8.



, *

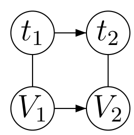
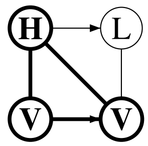


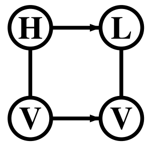
Figure 8: Autosegmental equivalent of CORR-v:$:v.

In figure. 6, the structure CORR-v:$:v required includes two qs, q_1 and q_2, in correspondence. Applying our transformation, these are merged into the single tonal autosegment t_1, yielding the required structure in figure. 8. Likewise, in the structure CORR-v:$:v identifies as marked, q_1 and q_2 are not in correspondence. Such a q_1 and q_2 are not merged by our transformation. Thus, transforming the marked structure yields an autosegmental equivalent that states that the marked structure yields t_1 and t_2 associated to adjacent vowels V_1 and V_2, respectively, with t_1 *not* associated to V_2. To illustrate, the autosegmental equivalents of candidates (20)a and (b) are given in figure. 9, with the relevant structures highlighted.

As can be seen in figure. 9, the autosegmental equivalent of CORR-v:$:v assigns a violation for adjacent vowels that do not share an autosegment. Thus, CORR-v:$:v is essentially equivalent to a constraint enforcing autosegmental spreading, such as SHARE (McCarthy 2010).



(AR equivalent of (20a))

* 

(AR equivalent of (20a))

Figure 9: AR equivalents of the candidates in (20) illustrating satisfaction and violation, respectively, of the autosegmental equivalent of Corr-v:$:v given in figure. 8.

For completeness, we briefly explore Corr-v::v the same way. As shown in (20), Corr-v::v assigns a violation for an adjacent pair of qs q_1, q_2 that belong to the same Q but are not in correspondence. This is depicted visually in figure. 10 in the same way as figure. 6 did for Corr-v:$:v.

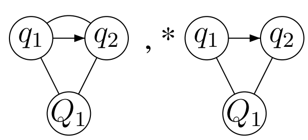


Figure 10: Visual representation of the requirement of Corr-v::v.

Applying our transformation to this constraint, we get an autosegmental constraint that states that vowels can be associated to a single tone (by merging the corresponding q_1 and q_2 in the left side of figure. 10 into a single tonal autosegment t_1), but not more than one.

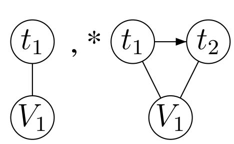


Figure 11: Autosegmental equivalent of Corr-v::v.

The translation of Corr-v::v, then, is simply a *contour constraint (like that of, e.g., Yip 2002). Our transformation, then, yields a tableau identical to that of (20), but with autosegmental constraints.

(21)

<table>
<tr><th>/ H L
| |
holol /</th><th>Share</th><th>*Contour</th></tr>
<tr><td>☞ a. H L
holol</td><td></td><td>*</td></tr>
<tr><td>b. H L
| |
holol</td><td>*!</td><td></td></tr>
</table>

Thus, we have illustrated that our transformation between ARs and QRs also implies a translation between well-formedness constraints on spreading and contour creation, such that a structure in one theory violates a constraint if and only if its equivalent structure violates the equivalent constraint in the other theory. This is significant, as constraints referring to inter-syllable identity and intra-syllable identity play a main role in SI's analysis of Mende, which SI claim is superior to an autosegmental analysis. However, the results here show that such constraints are equally expressible with autosegmental representations.

5 Discussion

In arguing for QRs, SI claim that 1) QRs dispense of the need for an association relation; 2) the q serves as the TBU; and 3) QRs capture patterns that ARs cannot. By formally investigating these representations, we show that none of these claims hold: 1) QRs require a relation identical to association; 2) Qs are the closest equivalent to TBUs in ARs, and thus QRs are similar to older autosegmental theories in which vowels are the TBU; and 3) QRs and ARs are equivalent in terms of constraints they can express. We now discuss the implications and scope of this result.

The argument here focuses mainly on markedness, or surface, constraints. This is primarily because autosegmental relations are a theory of output structure, and it is this power to which SI explicitly compare QRs. A translation between input-output correspondence constraints, however, would be similar, but more complex: as per Potts and Pullum (2002), formalizing input-output correspondence requires including input structures as well as output structures in the representation. As the purpose of this paper is to demonstrate the structural similarities between QRs and ARs, we focus on the surface correspondence constraints, but note that our results likely extend to input-output structures as well.

We have focused here on the expressive power of QRs with respect to existing ARs, as well as critically evaluating SI's arguments as to the superiority of QRs over ARs. However, one clear empirical prediction made by QRs that is not obvious with ARs is that segments, *all* segments, are made up of three parts. As SI discuss (p. 138), this idea has existed in some form with various manifestations, such as tonal complexes of Akinlabi and Liberman (2001), aperture theory of Steriade (1993, 1994) , or articulatory phonology (e.g. Gafos 2002). However, QRs are the first such formalization of segments with an across-the-board abstract tripartite structure. One consequence of this is that contrasts in affricate types (such as in Hungarian /ts/ vs. /tʃ/,

p. 150) are unified with contrasts in tone contours (such as in Dinka HHL vs. HLL, pp. 150–151). Having this three-part segment structure hard-wired into the theory allows the linguist to ask questions they might not have otherwise, questions which may lead to new insights, especially when looking beyond just tone.

Thus, we wish to express flaws not in the absolute utility of QRs, only in the argumentation of SI in comparing the power of these representations to existing ARs. In fact, our work shows that the central contribution of Q-Theory is not the abandonment of association, nor its replacement with correspondence, but rather the statement that segments have a fundamentally tripartite structure. This follows the spirit of Kornai and Pullum (1990), who provided a mathematical definition of (one set of assumptions for) X-Bar theoretic phrase structure, showing how it was no more powerful or predictive than unconstrained context-free grammars on which it was based. However, through this same method, Kornai and Pullum (1990) are able to focus on the parts of the theory that are novel or contentful, and build from there. It is our hope to do the same for Q-Theory.

Finally, what we have shown is that under a set of assumptions consistent with SI's description of Q-Theory, the computational power is no greater (or restricted) than that of classical Autosegmental Theory. However, seemingly minute changes in the definition of the theory can alter its power in perhaps unexpected ways. For example, Danis and Jardine (2019) find that QRs with unbounded correspondence is likely much more powerful than either ARs or the QRs with local correspondence, as we have defined here. In brief, given an arbitrary binary correspondence relation, one can define constraints over this relation that are much more powerful than those that appear to be attested in the phonological literature. These changes are perhaps not apparent until mathematically defined, as they are here. Future work can thus follow the model of this paper (and Danis and Jardine 2019) in formally evaluating different versions of surface correspondence.

This also applies to SI's assumption of correspondence implying identity. As discussed in the analyses above, this assumption plays a key role in the translation between ARs and QRs—essentially, it means that corresponding qs in QRs are analgous to tonal autosegments in ARs. Future work can apply the techniques used here to rigorously study the consequences of relaxing this assumption, but we briefly state our expectations here. This assumption restricts the space of representations under QRs: elements cannot be in correspondence without also agreeing. Relaxing this assumption would allow a greater range of QR structures, including ones that do not have a direct AR analogue. In terms of expressivity of the theory, this means that one could define constraints in QR theory that one could not define in AR theory. However, it is not clear that any such constraints are necessary—SI's analyses of a range of tone patterns all assume that correspondence implies identity. Further work can rigorously examine what exactly the range of constraints is allowed when correspondence does not necessarily imply identity, and whether or not it allows constraints that are more powerful than necessary (as discussed above with unbounded correspondence).

6 Conclusion

Model theory provides a method for rigorously evaluating theories of phonological representation. By doing so, we can clarify what aspects of a two theories are truly distinct. We have shown that, formally, QRs are no more or less expressive or simple than ARs in capturing tonal patterns. However, moving beyond tone, the tripartite structure of QRs provides a new avenue for investigating and unifying certain segmental phenomena, which do not have clear analogs in AP. Thus, rather than undermining QRs, investigating them in a mathematically rigorous way brings into focus their actual contribution.

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