Vilnius
University

Phonotactic well-formedness is recombinant

19th Old World Conference on Phonology (OCP 19)

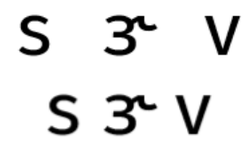
Ignas Rudaitis

28 January 2022

serve

S 3ˇ V

serve



serve

serve

verse

serve

verse

serve

verse

serve

a type of **unary** recombination

verse

a type of **ternary** recombination

spill

s p ɪ l

pillow

p ɪ l oʊ

so

s oʊ

spill

s p ɪ l

pillow

p ɪ l oʊ

so

s oʊ

spill

s p ɪ l

pillow

p ɪ l oʊ

so

s oʊ

s p ɪ l

spill

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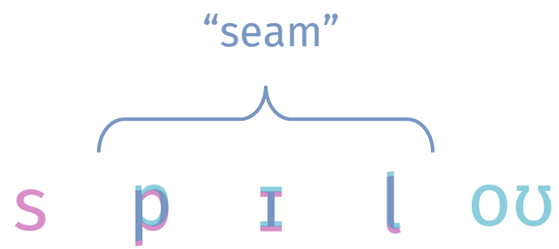


spill

s p ɪ l

pillow

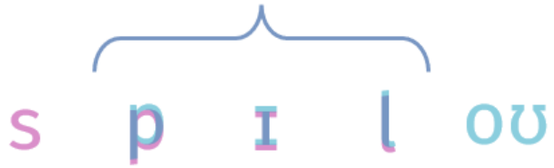
p ɪ l oʊ

so

s oʊ

s oʊ

“seam”



spill

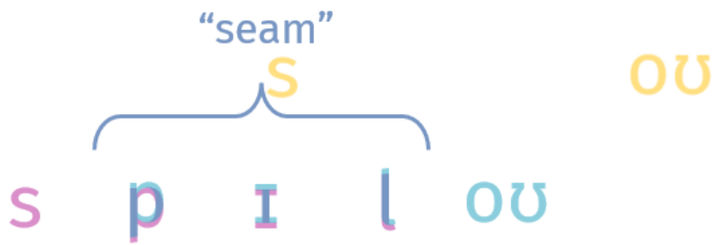
s p ɪ l

pillow

p ɪ l oʊ

so

s oʊ



°*spillow*

spill

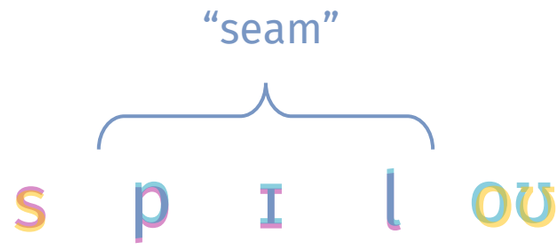
s p ɪ l

pillow

p ɪ l oʊ

so

s oʊ



°*spillow*

well-formed

spill

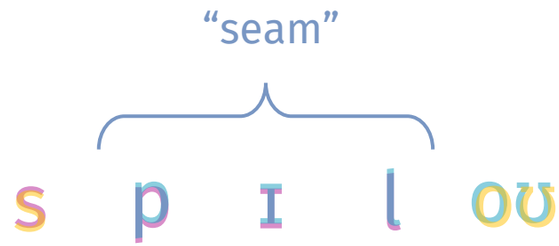
s p ɪ l

pillow

p ɪ l oʊ

so

s oʊ



°*spillow*

well-formed

spill

s p ɪ l

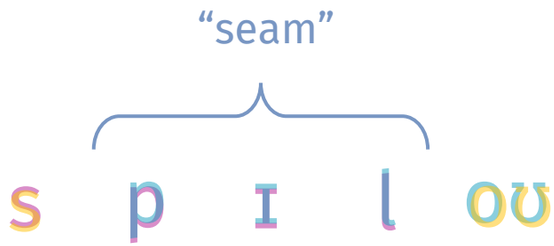
well-formed

pillow

p ɪ l oʊ

so

s oʊ



°*spillow*

well-formed

spill

s p ɪ l

well-formed

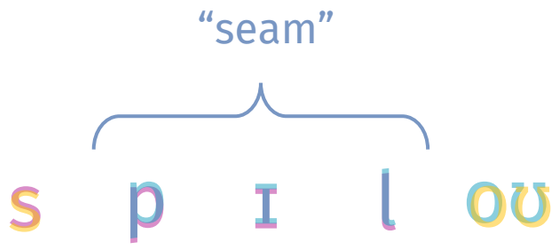
pillow

p ɪ l oʊ

well-formed

so

s oʊ



°*spillow*

well-formed

spill

s p ɪ l

well-formed

pillow

p ɪ l oʊ

well-formed

so

s oʊ

“seam”

s p ɪ l oʊ

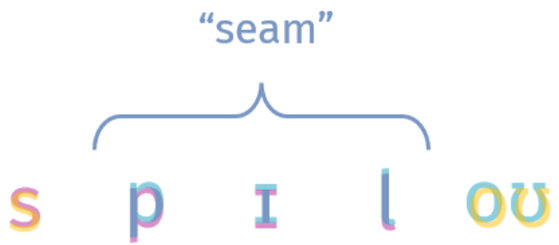
°*spillow*

well-formed

well-formed ∧ well-formed well-formed

spill *pillow* *so*

s p ɪ l p ɪ l oʊ s oʊ



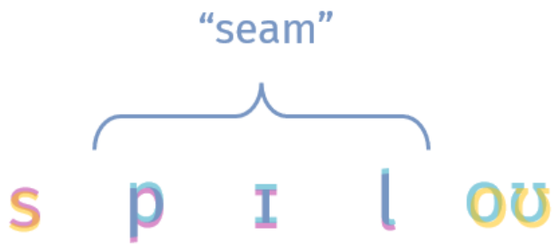
°*spillow*

well-formed

well-formed ∧ well-formed ∧ well-formed

spill *pillow* *so*

s p ɪ l p ɪ l oʊ s oʊ



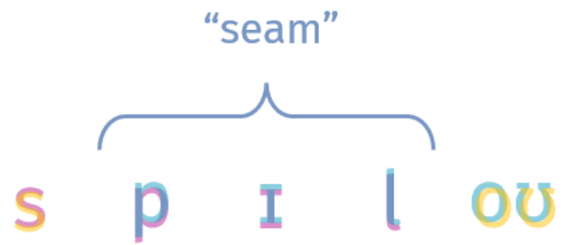
°*spillow*

well-formed

well-formed ∧ well-formed ∧ well-formed ⇒

spill *pillow* *so*

s p ɪ l p ɪ l oʊ s oʊ



°*spillow*

well-formed

well-formed ∧ well-formed ∧ well-formed ⇒

spill *pillow* *so*

s p ɪ l p ɪ l **well-formed** s oʊ

“seam”

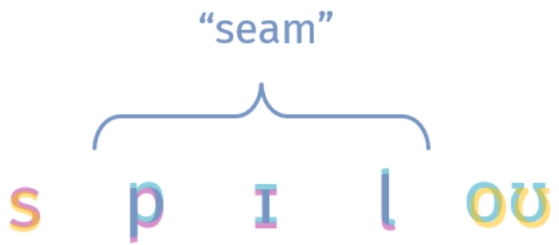
s p ɪ l oʊ

°*spillow*

well-formed ∧ well-formed ∧ well-formed ⇒ **well-formed**

spill *pillow* *so*

s p ɪ l p ɪ l oʊ s oʊ

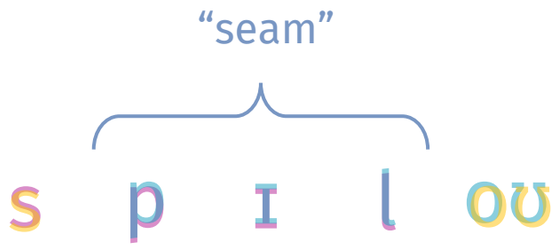


°*spillow*

well-formed ∧ well-formed ∧ well-formed ⇒ **well-formed**

spill *pillow* *so*

s p ɪ l p ɪ l oʊ s oʊ



°*spillow*

well-formed $\wedge$ well-formed $\wedge$ well-formed $\Rightarrow$ **well-formed**

XY well-formed ∧ YZ well-formed ∧ XZ well-formed ⇒ **XYZ well-formed**

XY well-formed $\wedge$ YZ well-formed $\wedge$ XZ well-formed $\Rightarrow$ **XYZ well-formed**

XY well-formed ∧ YZ well-formed ∧ XZ well-formed ⇒ **XYZ well-formed**

Hypothesis: this implication holds cross-linguistically
for nearly all segmental constraints

Coverage of the recombinant property

Bigram constraints	most local constraints
Trigram constraints	sonority sequencing
Final/initial constraints	final devoicing
Classical harmony	Finnish VH
Harmony, simple blocking	Imdlawn Tashlhiyt CH
Harmony, local bipartite blocking	Yaka CH
Harmony, distal bipartite blocking	Tutrugbu VH
Long-distance dissimilation	Latin /l ~ r/ at some stage (?)
Quantification	“at least one vowel”
Suprasegmentals	tone, stress
Any intersection of the above	multiple harmony

Outline

1. Local, initial, final constraints
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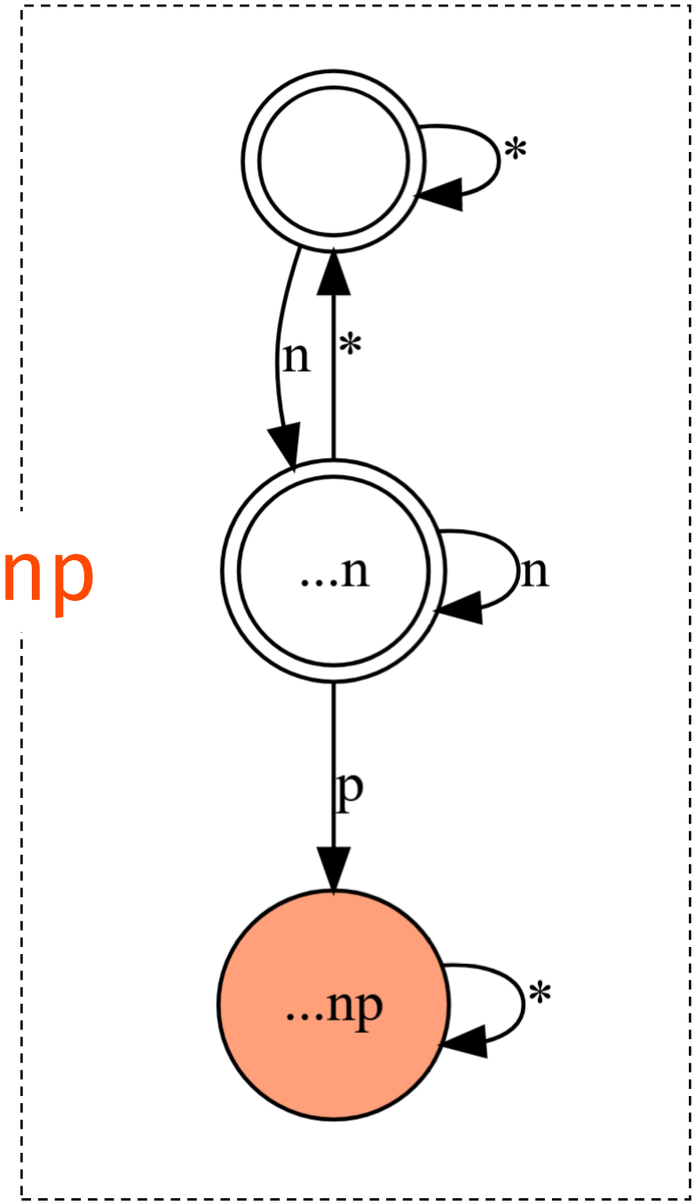
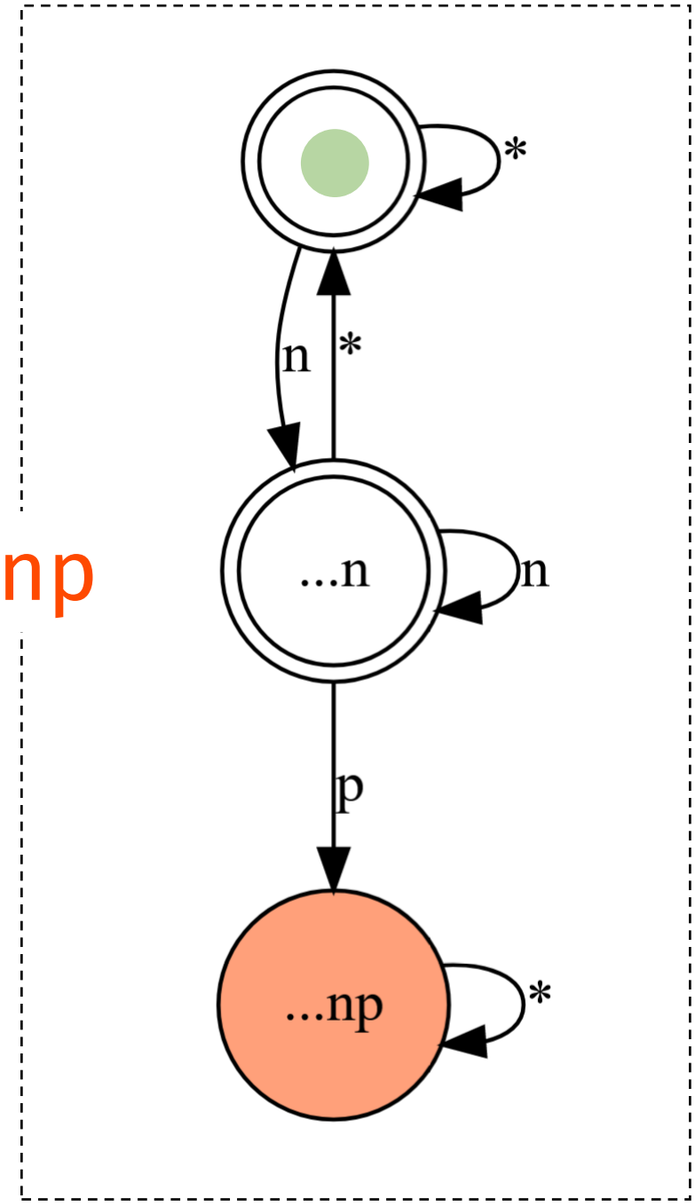
*np





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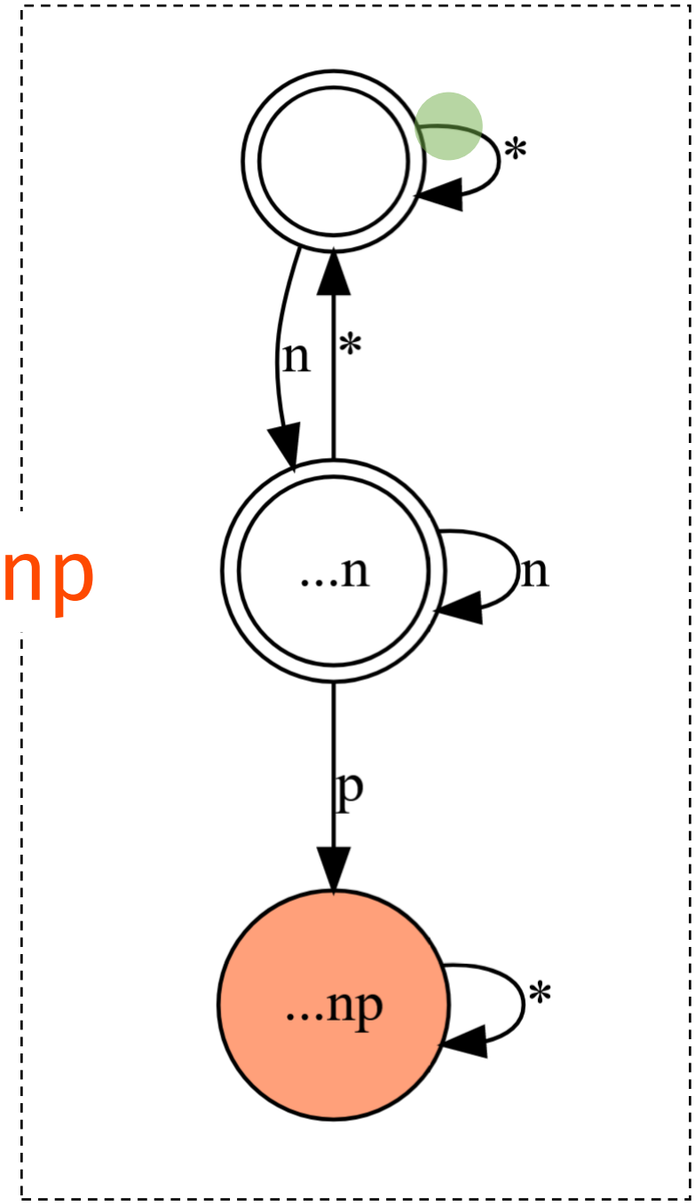
anpa



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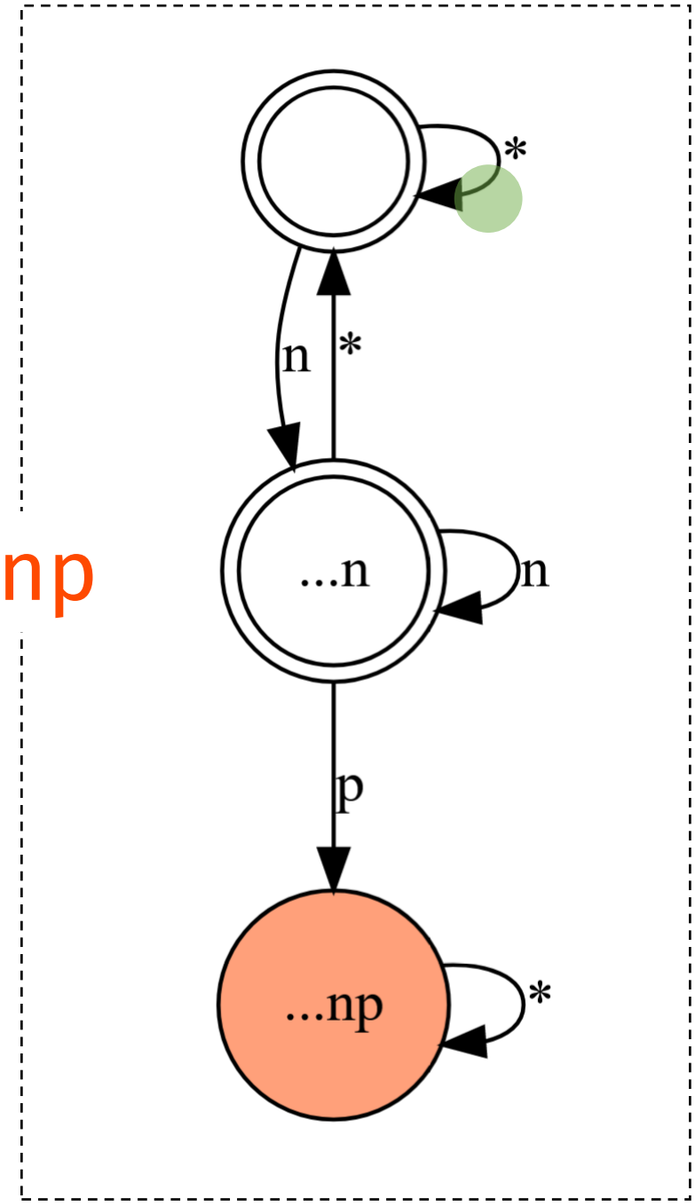
anpa

*np



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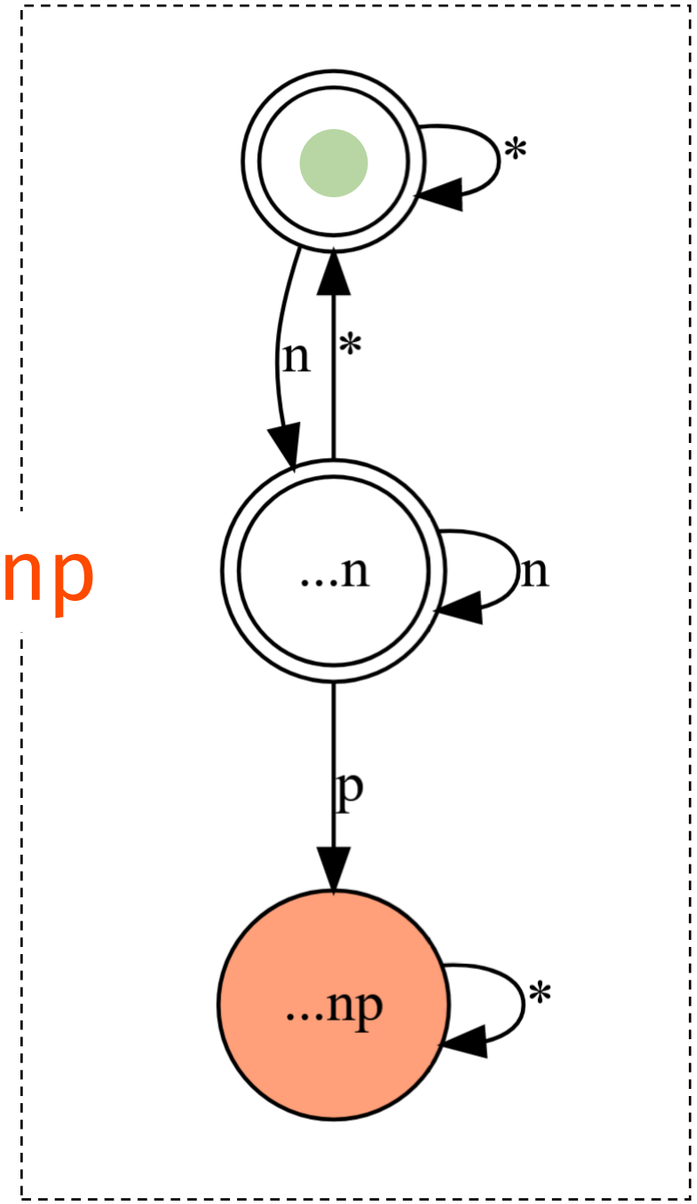

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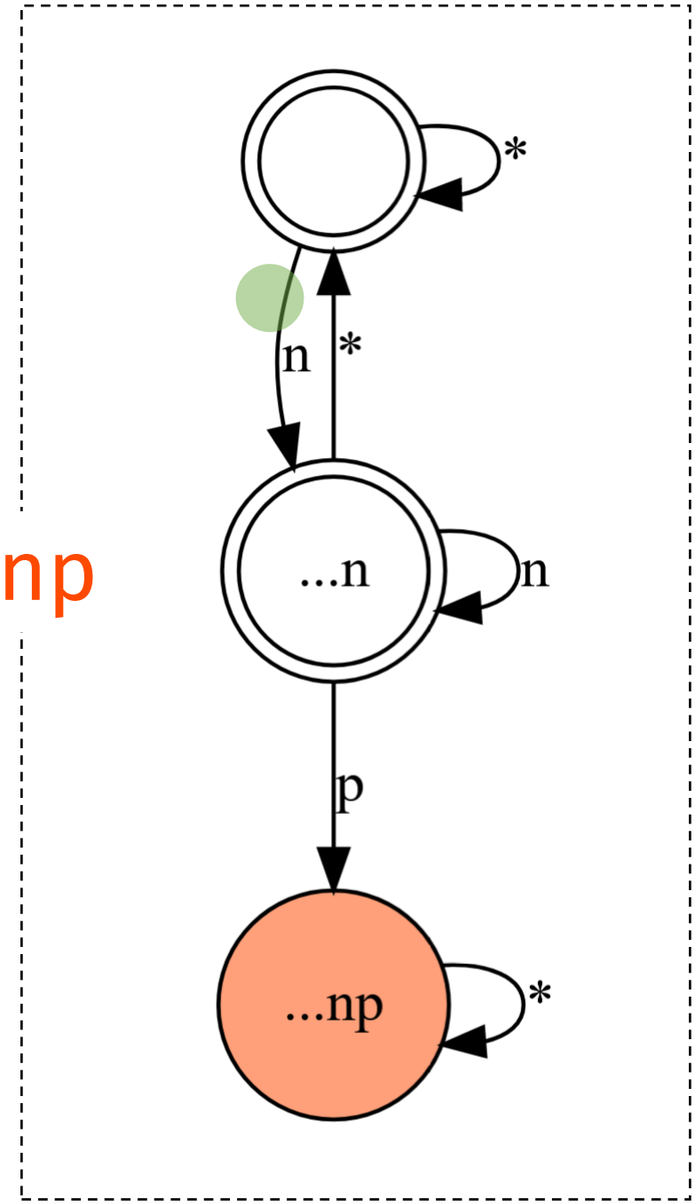
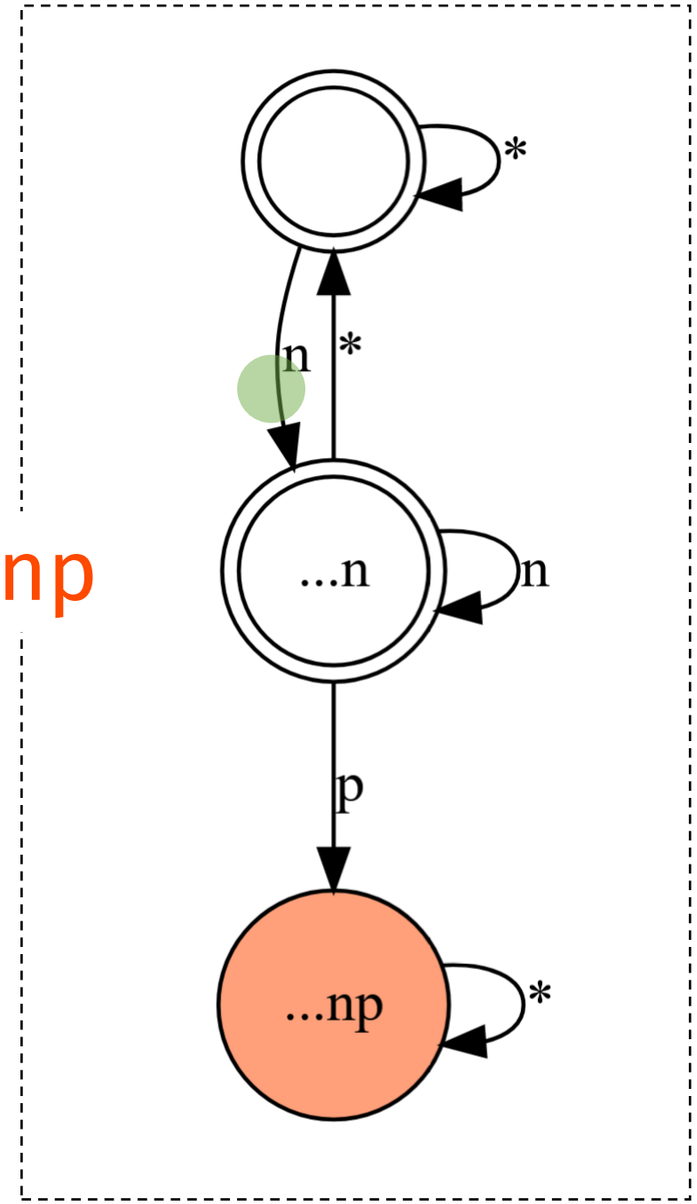


anpa

*np



anpa

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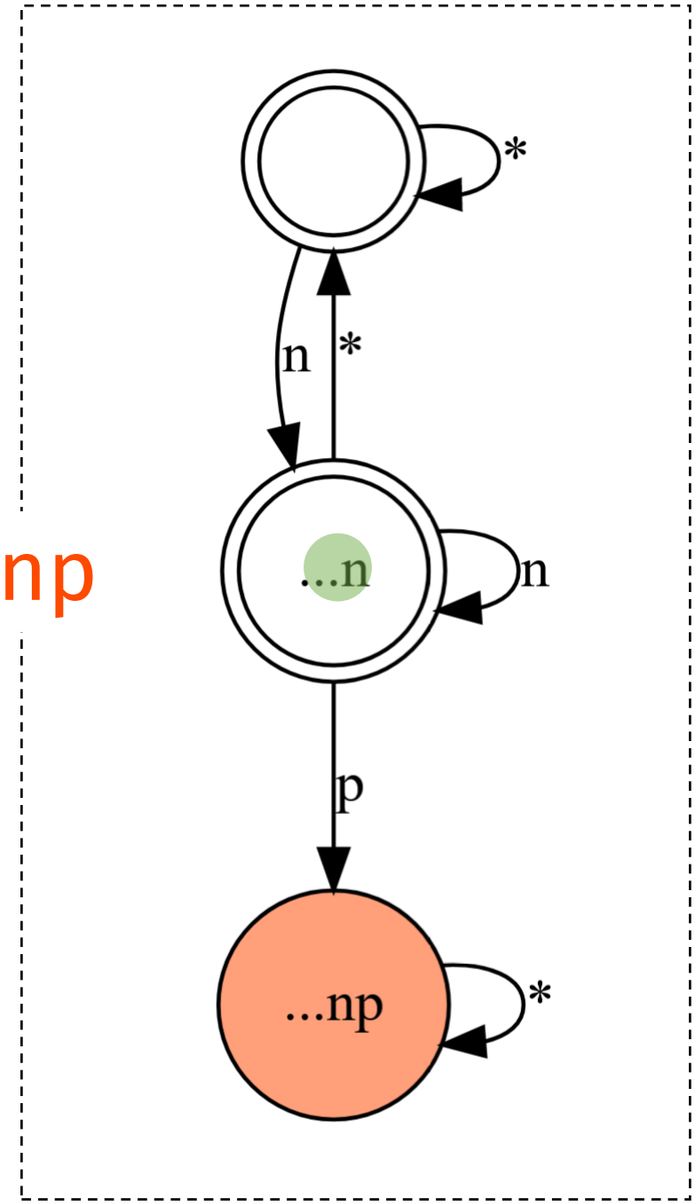

anpa

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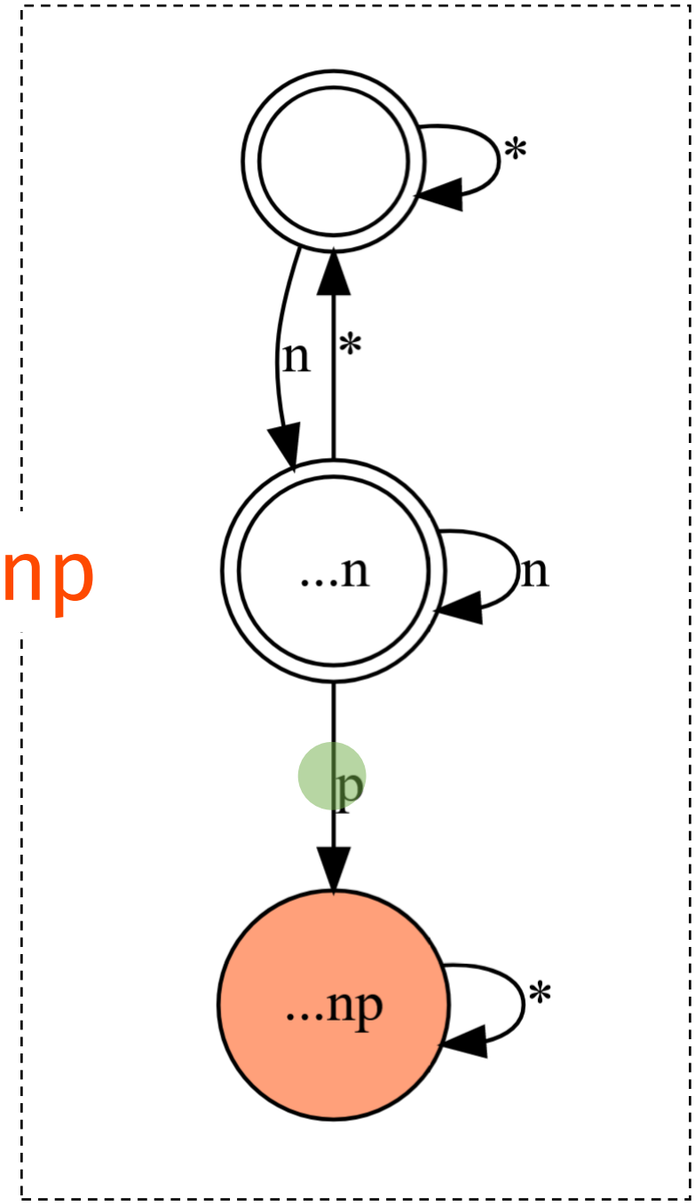

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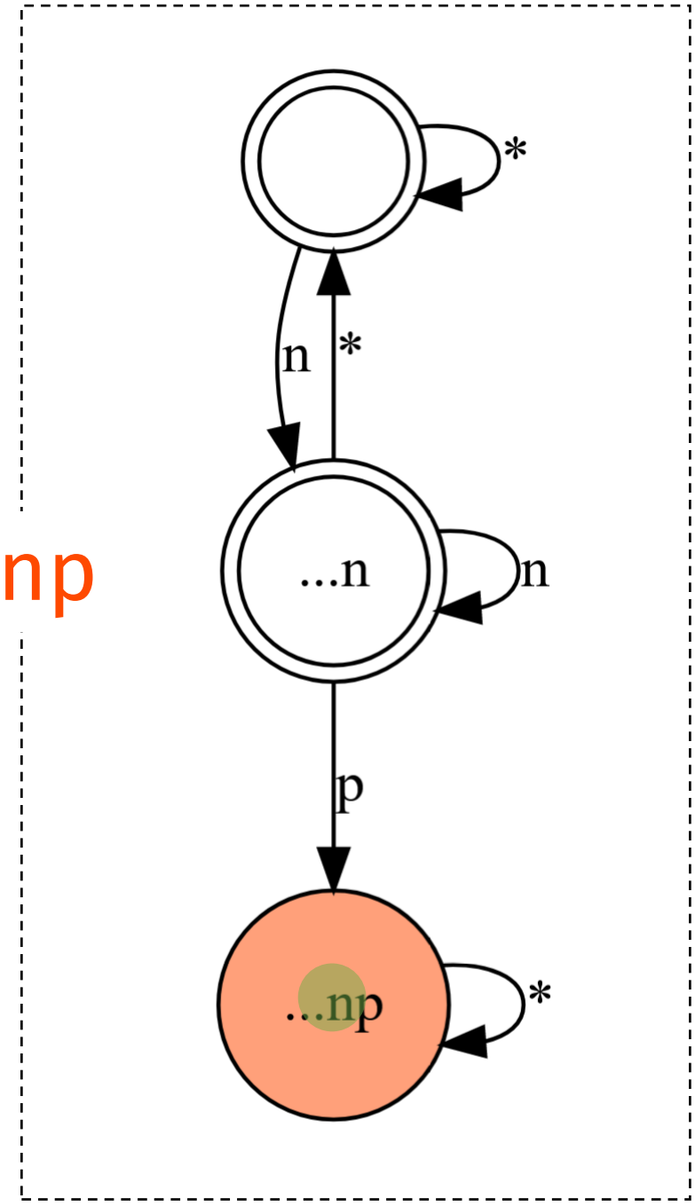

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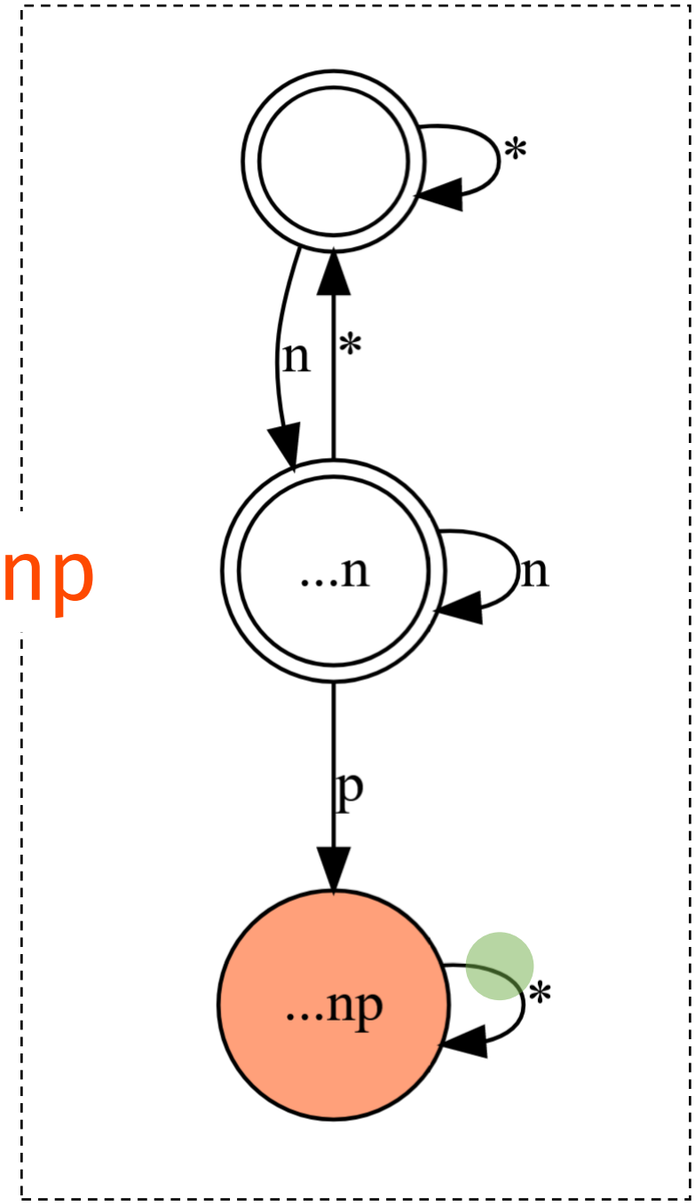

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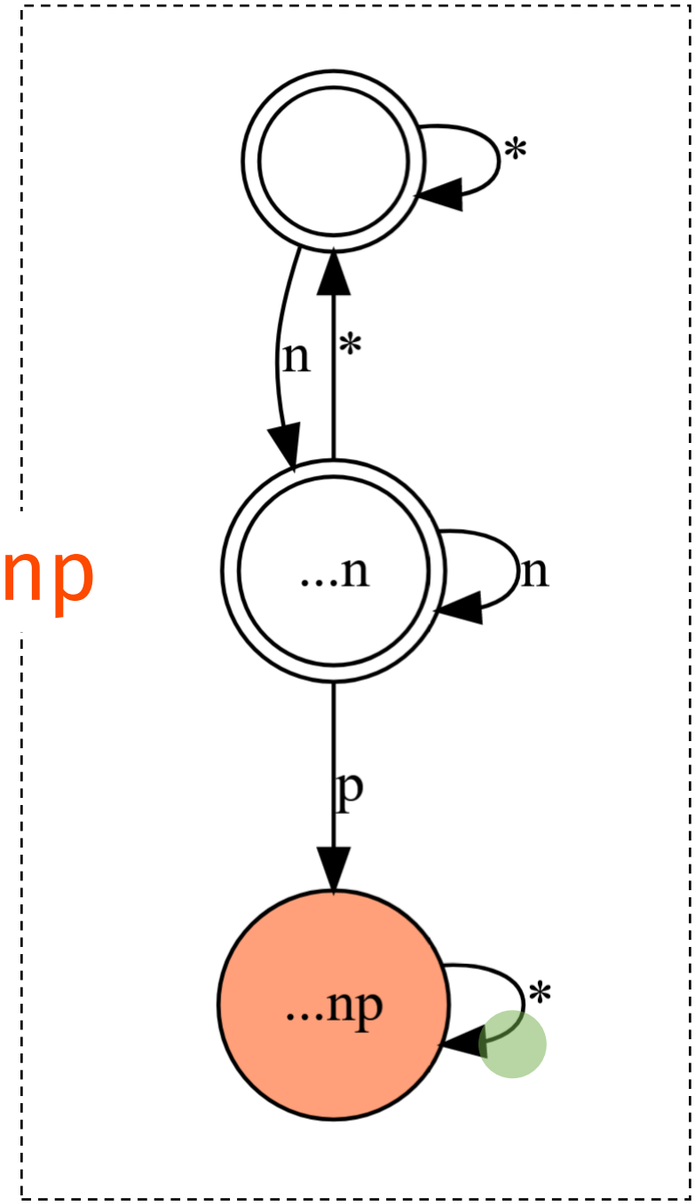

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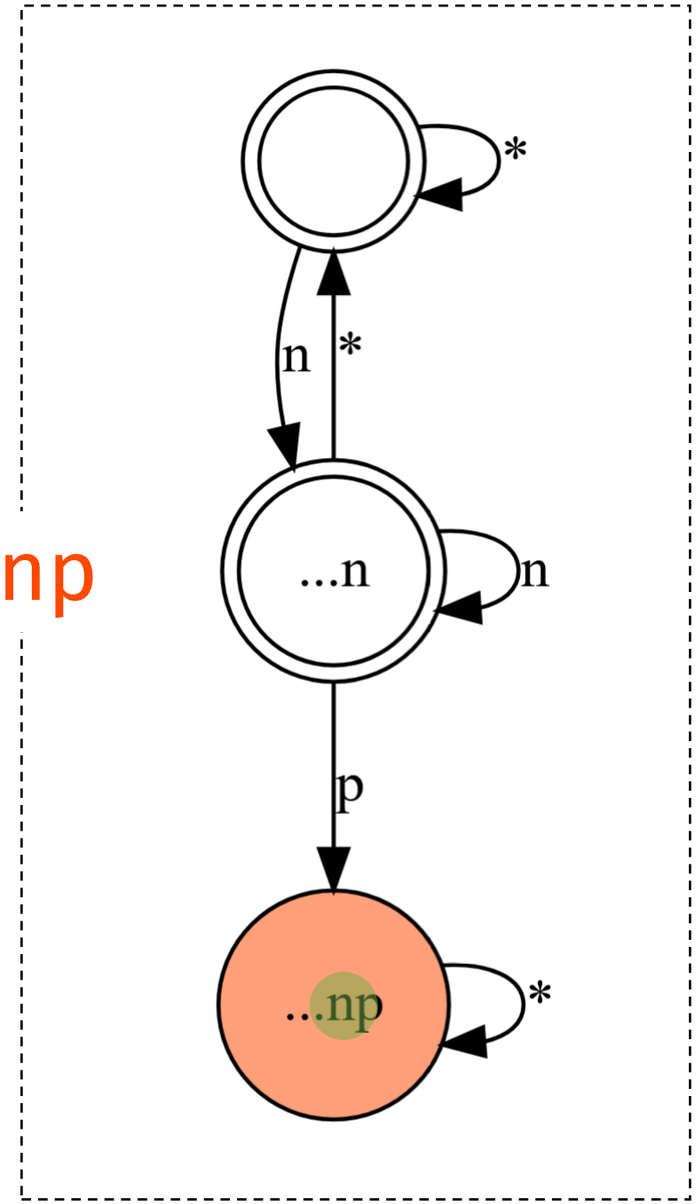

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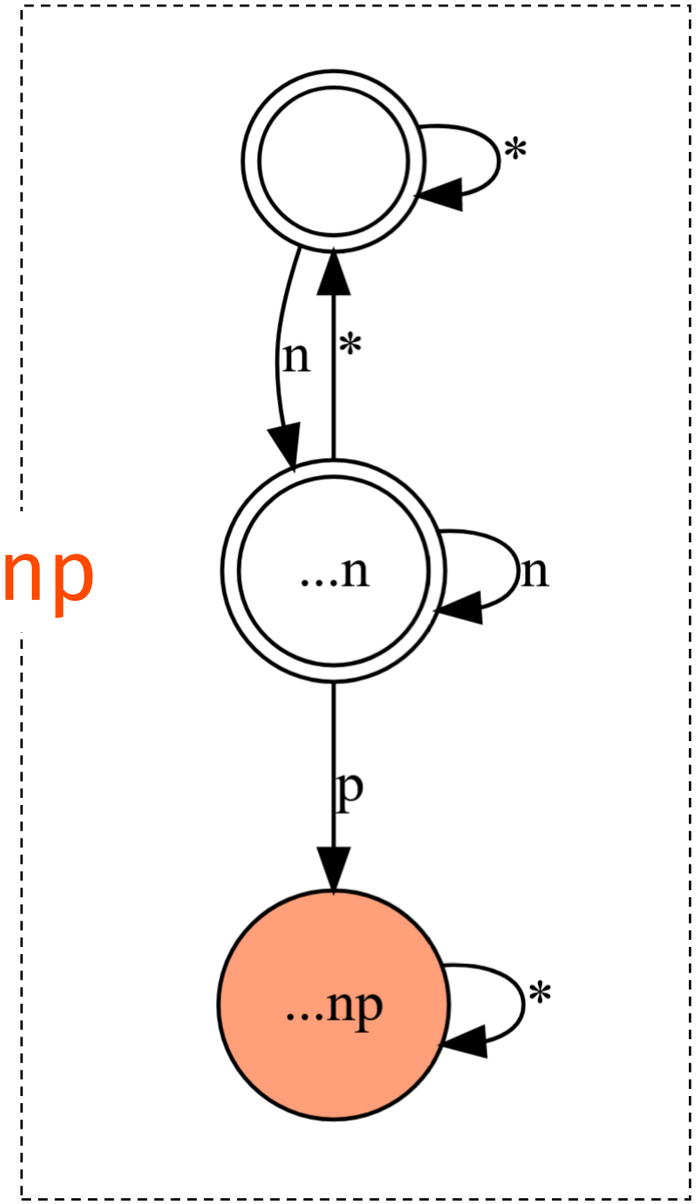

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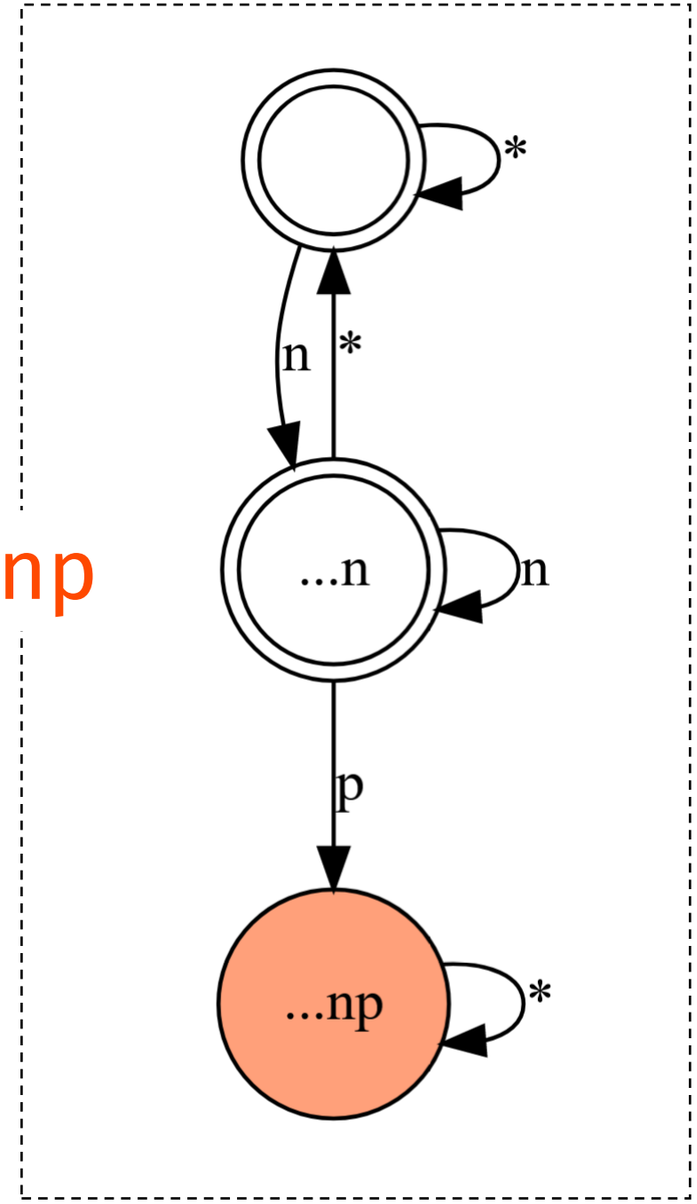
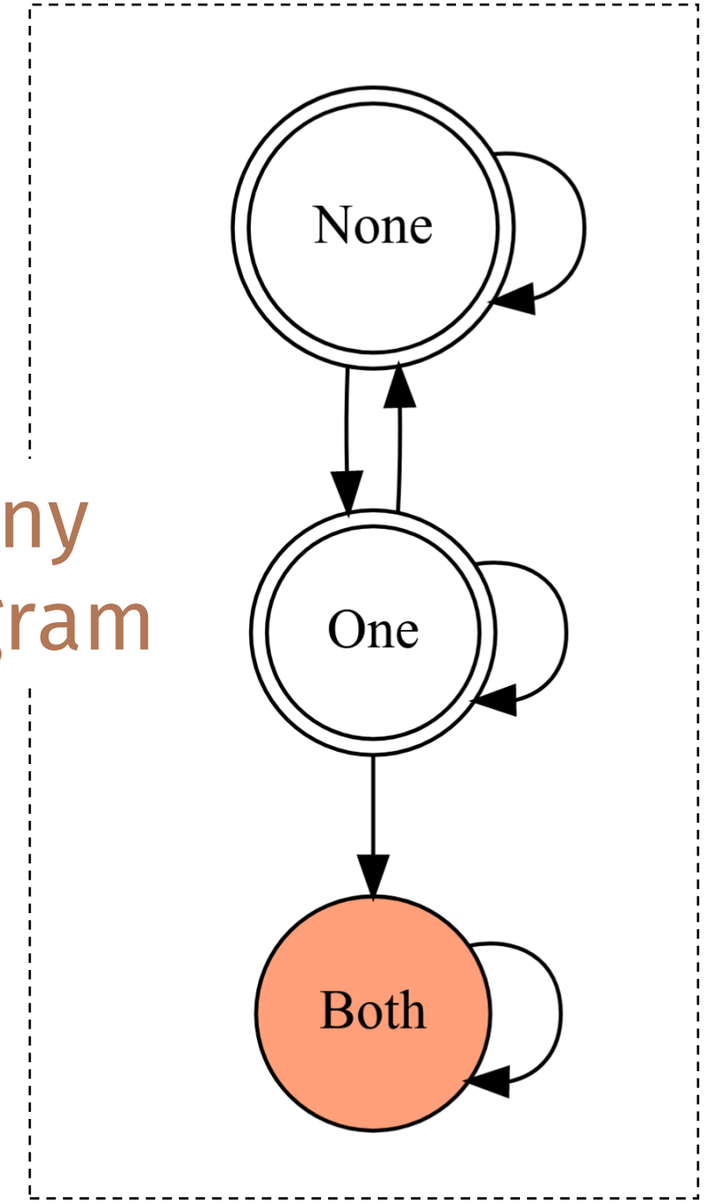
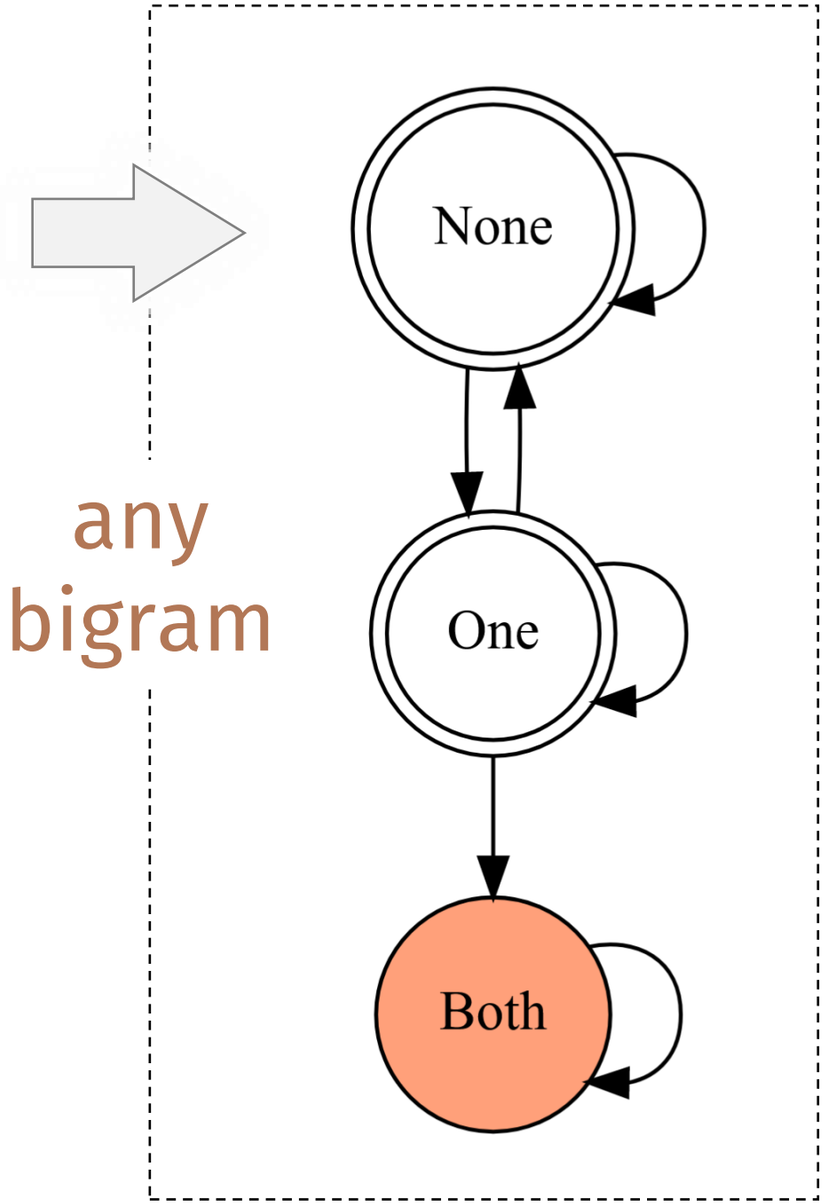


any bigram



We prove the recombinant property for this topology of automata in general.



Initial and final constraints

- Let us call XY, YZ, and XZ the (recombinant) inputs, and XYZ the output
- The output XYZ **begins** with the same bigram, as the input **XY** already did
- The output XYZ **ends** with the same bigram, as the input **YZ** already did

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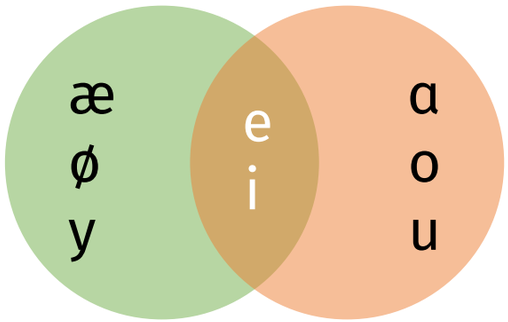
Finnish vowel harmony

Some numerals:

kɑksi	‘2’	kɑhdeksɑn	‘8’
yksi	‘1’	yhdeksæn	‘9’



Rule: do not mix green and orange vowels in one word

Finnish vowel harmony

Some numerals:

kɑksi	‘2’	kɑhdeksɑn	‘8’
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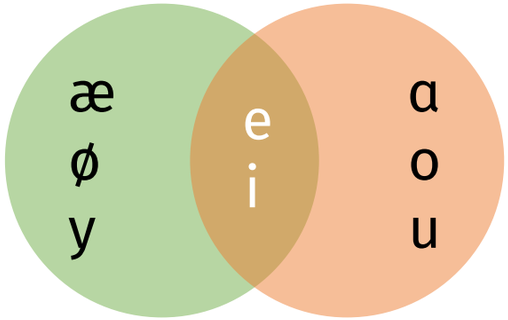
kæksi ‘?’
uksi ‘?’

pseudo-words



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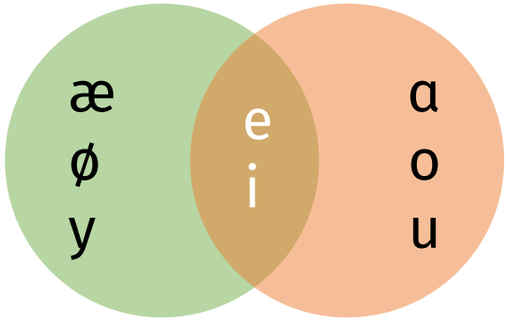
pseudo-words

kæksi	‘?’	*kæhdeksɑn	–
uksi	‘?’	*uhdeksæn	–

ill-formed



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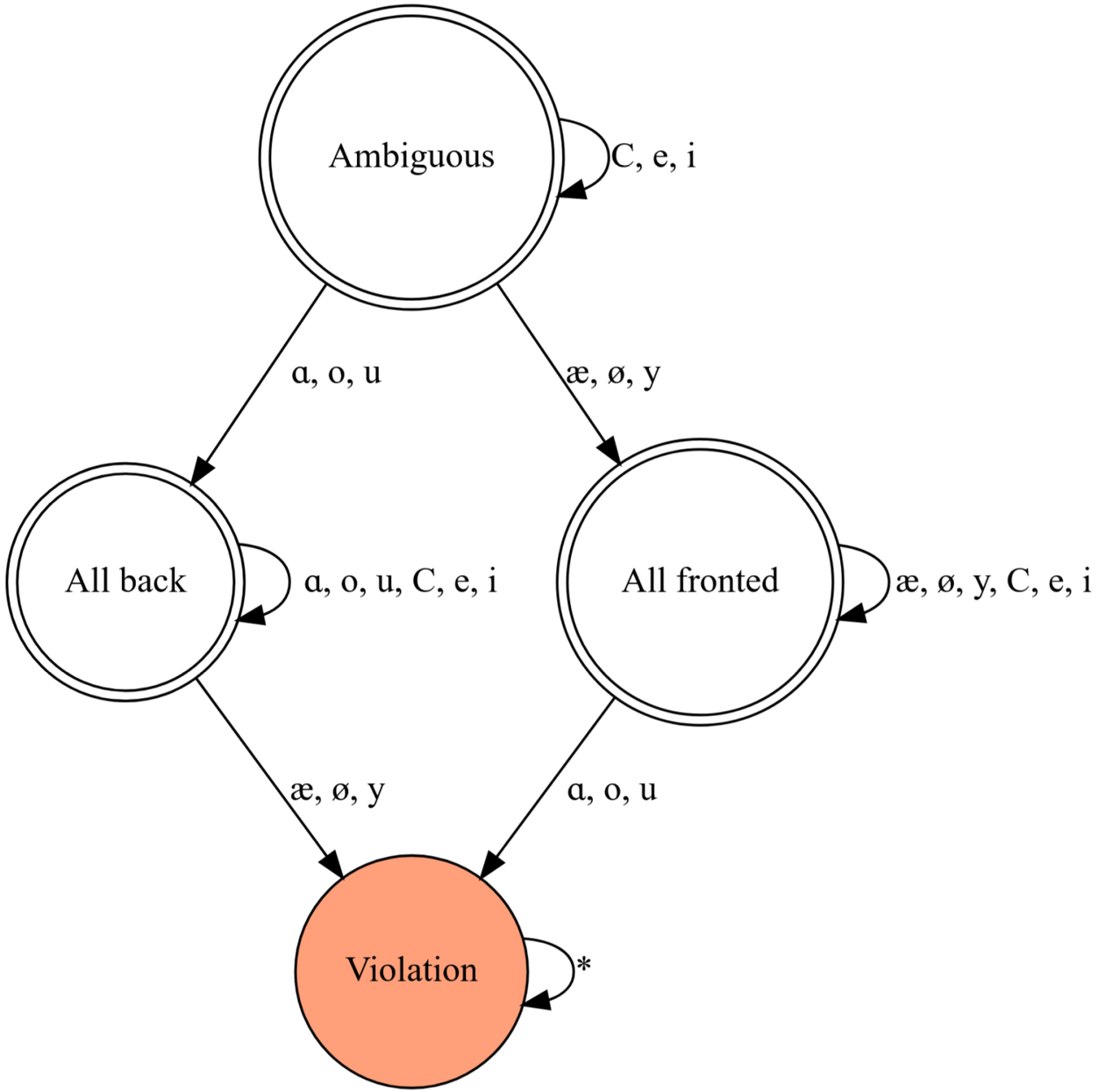
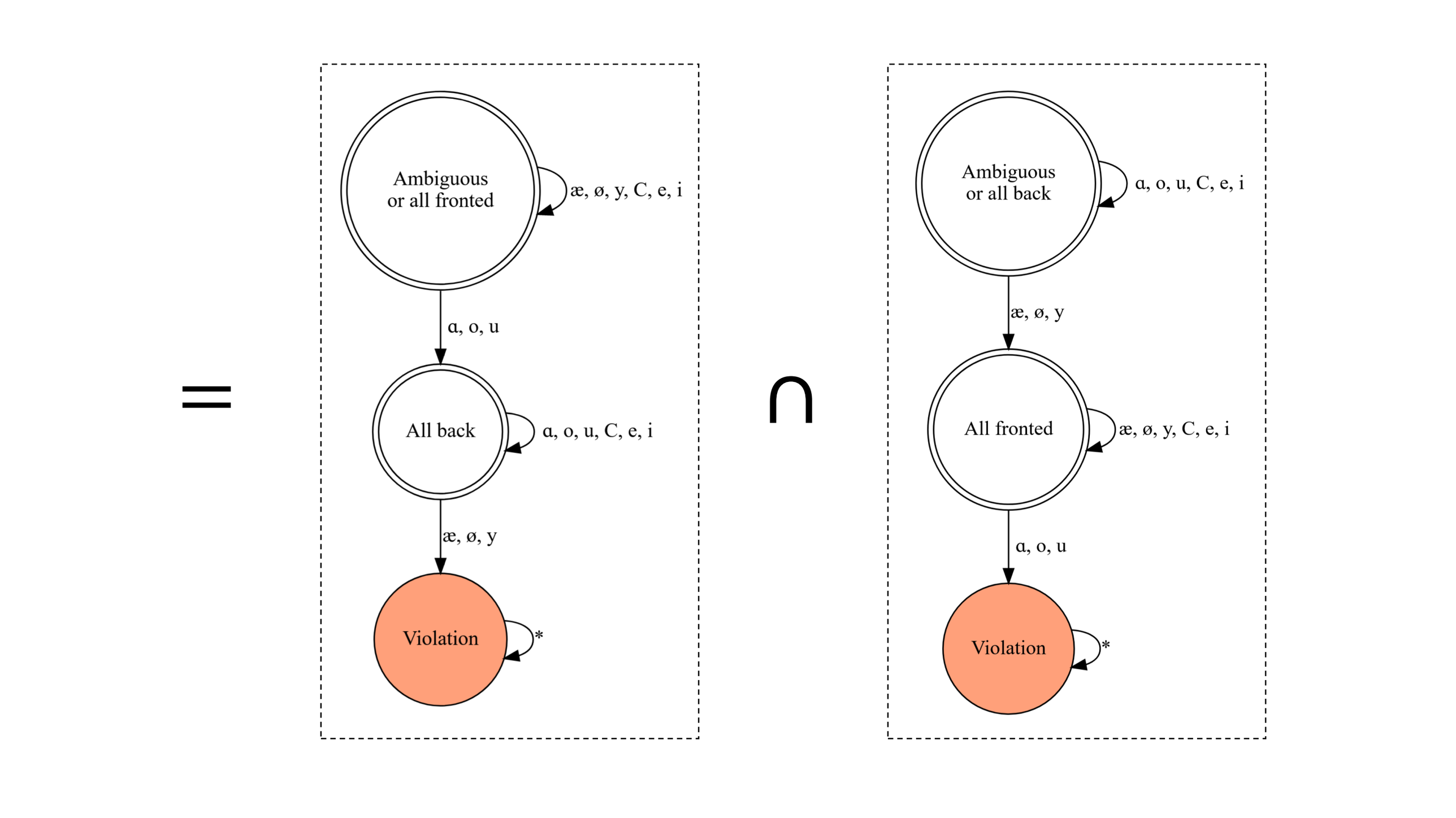

Ambiguous
C, e, i
ɑ, o, u
æ, ø, y
All back
ɑ, o, u, C, e, i
All fronted
æ, ø, y, C, e, i
æ, ø, y
ɑ, o, u
Violation
*


=
Ambiguous
or all fronted
æ, ø, y, C, e, i
ɑ, o, u
All back
ɑ, o, u, C, e, i
æ, ø, y
Violation
*
∩
Ambiguous
or all back
ɑ, o, u, C, e, i
æ, ø, y
All fronted
æ, ø, y, C, e, i
ɑ, o, u
Violation
*

The ternary recombinant property is closed under intersection.

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Three harmonies in Imdlawn Tashlhiyt

s:-uga	‘CAUS-evacuate’
s-as:twa	‘CAUS-settle’
ʃ-fiaʃr	‘CAUS-be.full.of.straw’
z-bruz:a	‘CAUS-crumble’
ʒ-m:ʒdawl	‘CAUS-stumble’
s-ħuz	‘CAUS-annex’
s:-ukz	‘CAUS-recognize’
sˤ-rˤuˤfˤzˤ	‘CAUS-appear.resistant’
s-mχazaj	‘CAUS-loathe.each.other’
ʃ-quʒ:i	‘CAUS-be.dislocated’

(Elmedlaoui 1992; Hansson 2010),
reproduced here from (Aksënova et al. 2020)

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emphasis (pharyngealization) harmony

(Elmedlaoui 1992; Hansson 2010), reproduced here from (Aksënova et al. 2020)

Two harmonies in Imdlawn Tashlhiyt

s:-uga	‘CAUS-evacuate’
s-as:twa	‘CAUS-settle’
ʃ-fiaʃr	‘CAUS-be.full.of.straw’
z-bruz:a	‘CAUS-crumble’
ʒ-m:ʒdawl	‘CAUS-stumble’
s-ħuz	‘CAUS-annex’
s:-ukz	‘CAUS-recognize’
s -r u f z	‘CAUS-appear.resistant’
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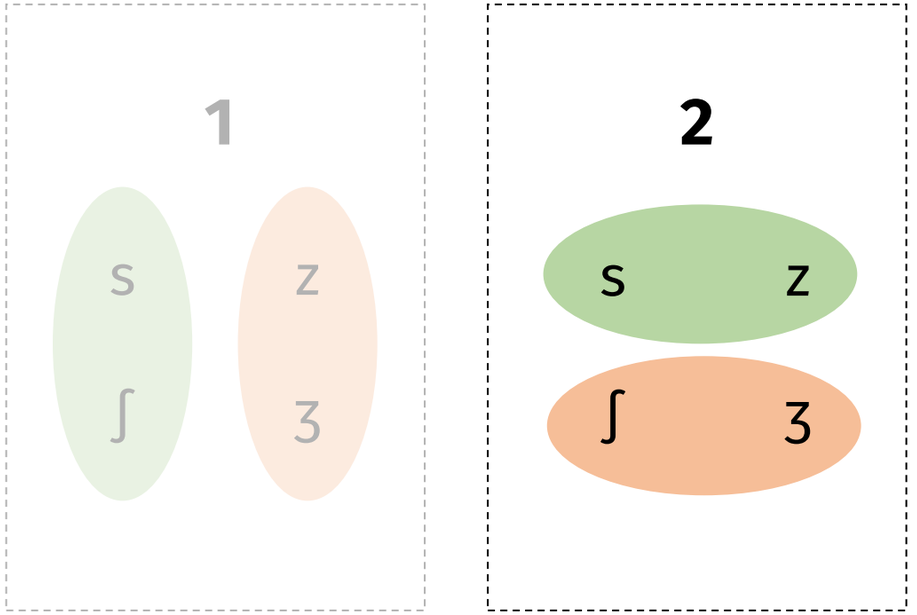

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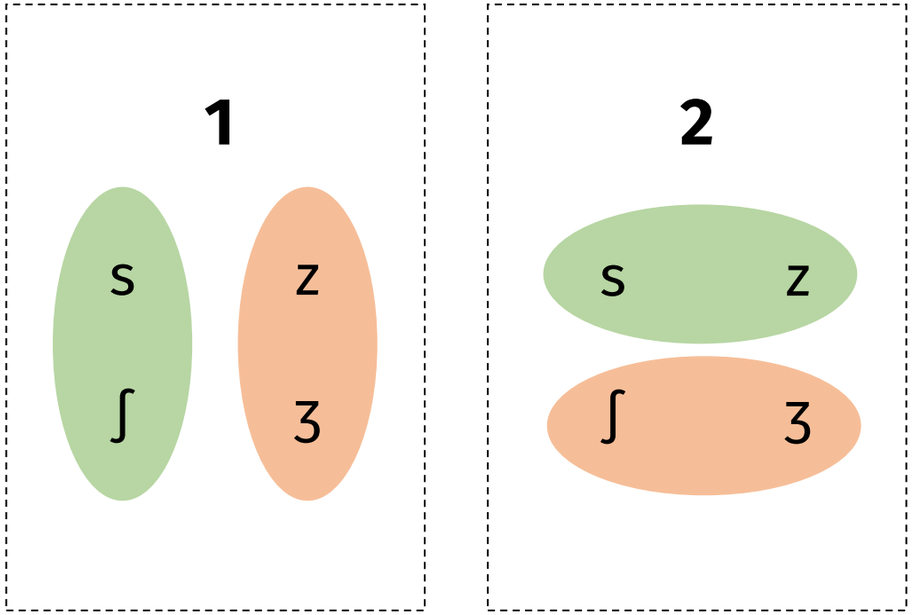

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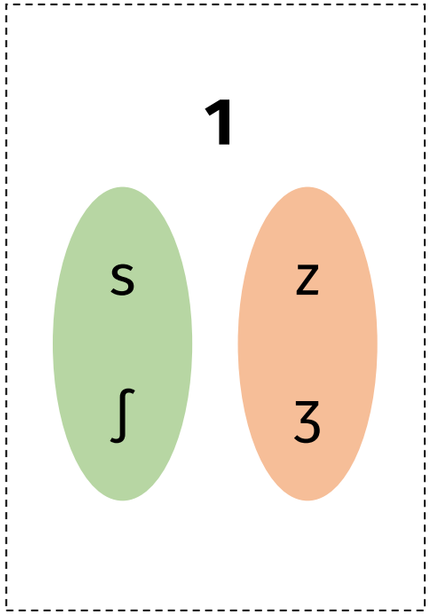

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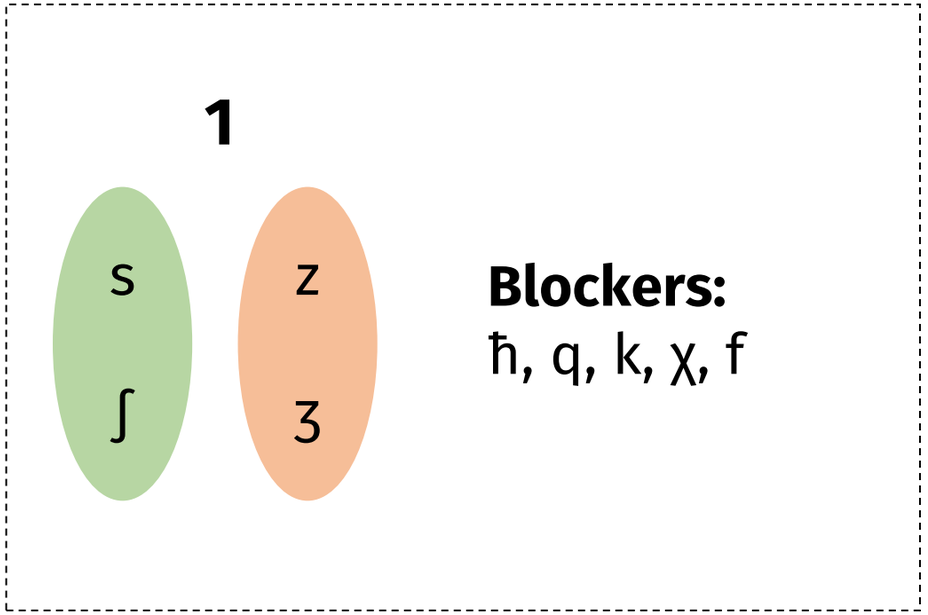

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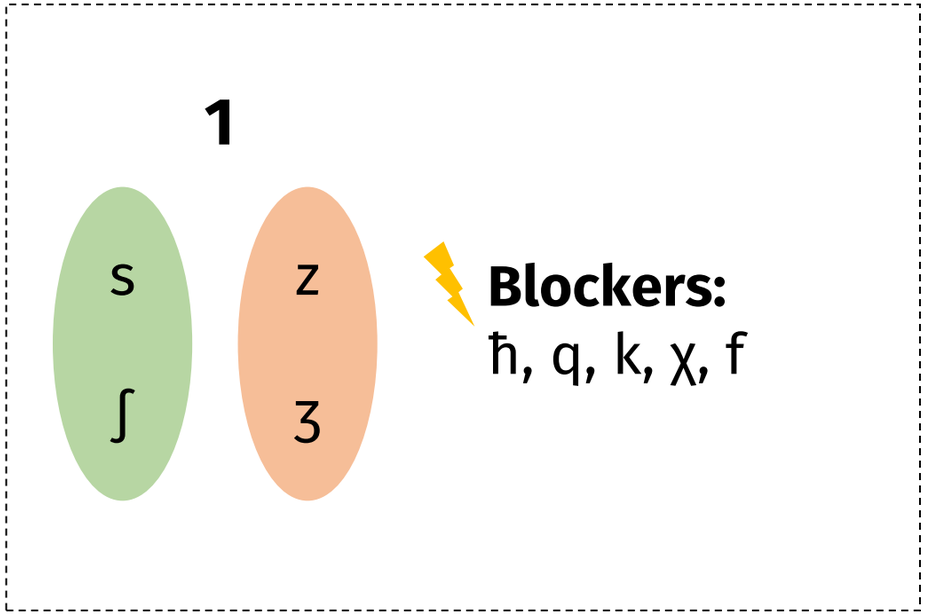

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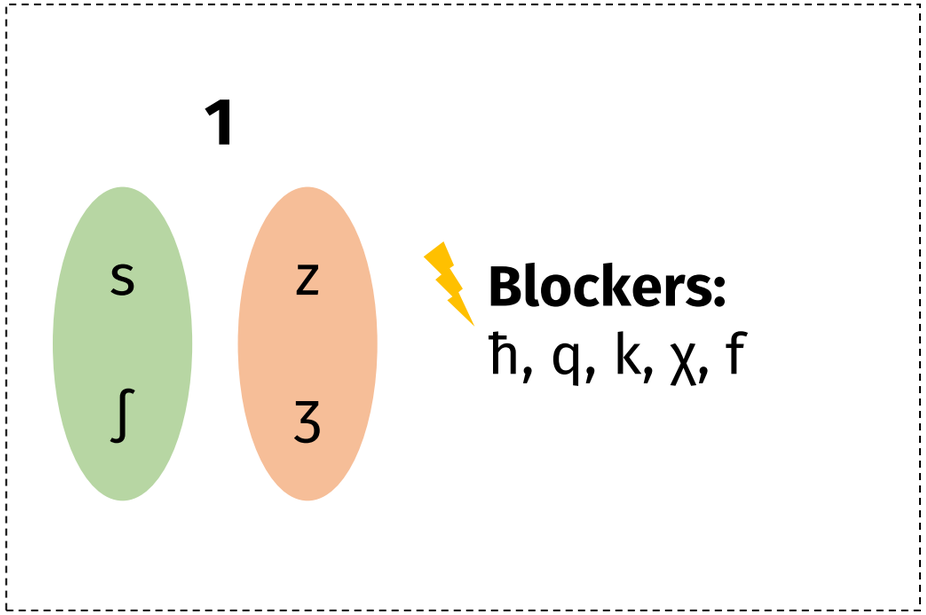
One harmony in Imdlawn Tashlhiyt

s a s: t w a

ʒ m: ʒ d a w l

s m χ a z a j

* s m a z a j



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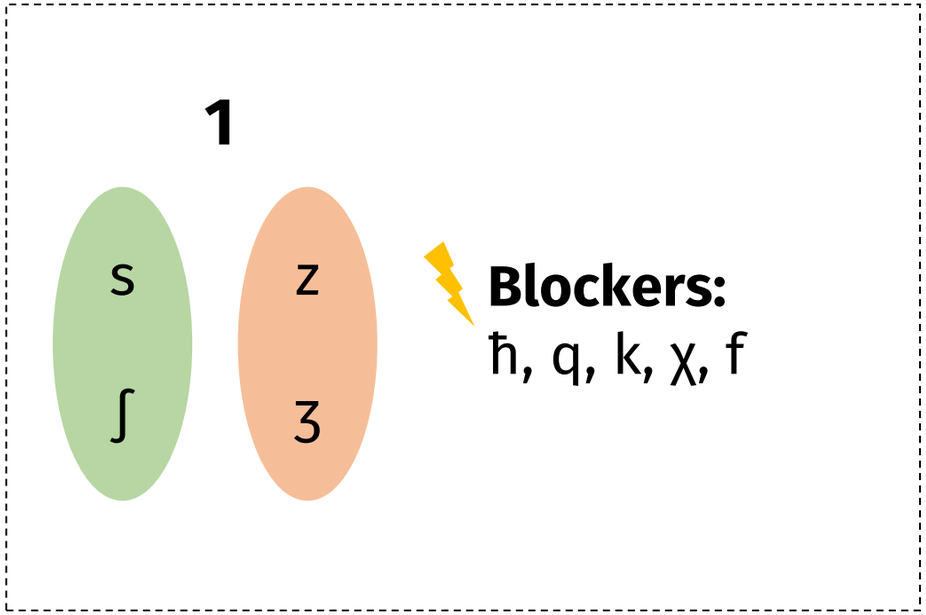

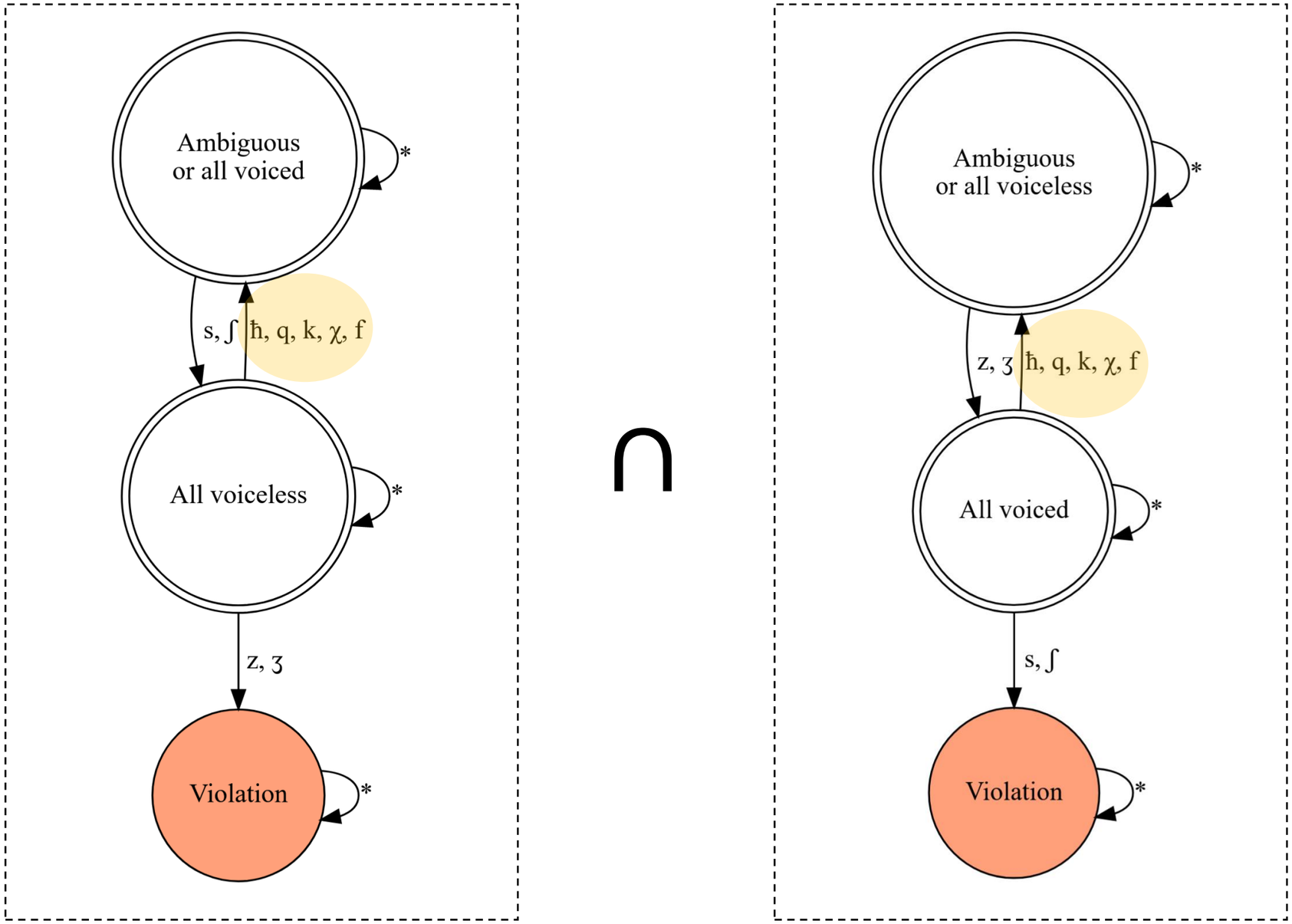
Ambiguous
or all voiced
*
s, ʃ
ħ, q, k, χ, f
All voiceless
*
z, ʒ
Violation
*
∩
Ambiguous
or all voiceless
*
z, ʒ
ħ, q, k, χ, f
All voiced
*
s, ʃ
Violation
*

?!

Phonotactics always constitutes a regular language
(Karttunen et al. 1987; Kaplan & Kay 1994).

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Moreover, Aksënova et al. (2020) have already shown that the **MTSL$_2$** (multiple tier-based strictly 2-local) subclass of regular languages suffices to account for a broad range of harmony phenomena.

The Computational Power of Harmony

Alëna Aksënova, Jonathan Rawski, Thomas Graf, Jeffrey Heinz

July 1, 2020

1 Computation and Phonology

Some of the most stunning scientific advances of the twentieth century came from the theory of computation. It is hard to think of an area in science which has not been enriched by studying a subject computationally. At the core of the theory of computation is the ability to formally define problems and processes, and ask how (or whether) they can be solved computationally, and if so, with which resources. For linguistics, the science of language, this means characterizing the cognitive processes involved in the human capacity for lan-

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On the other hand,

MTSL$_2$ =  .

Therefore, the entire MTSL$_2$ subclass has the ternary recombinant property.

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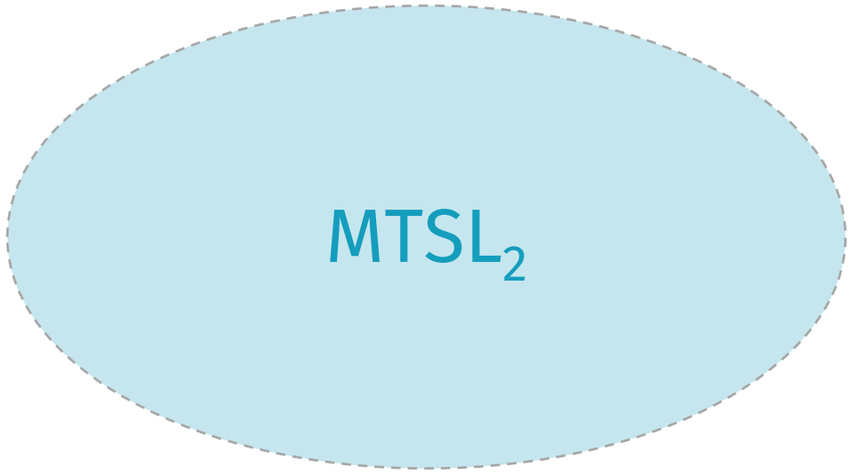
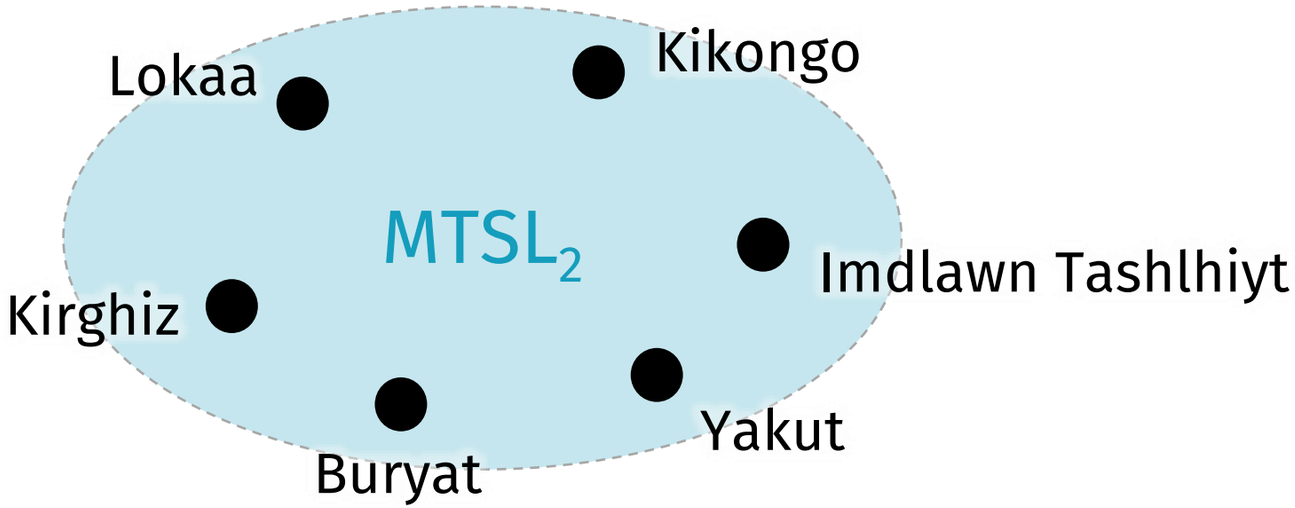
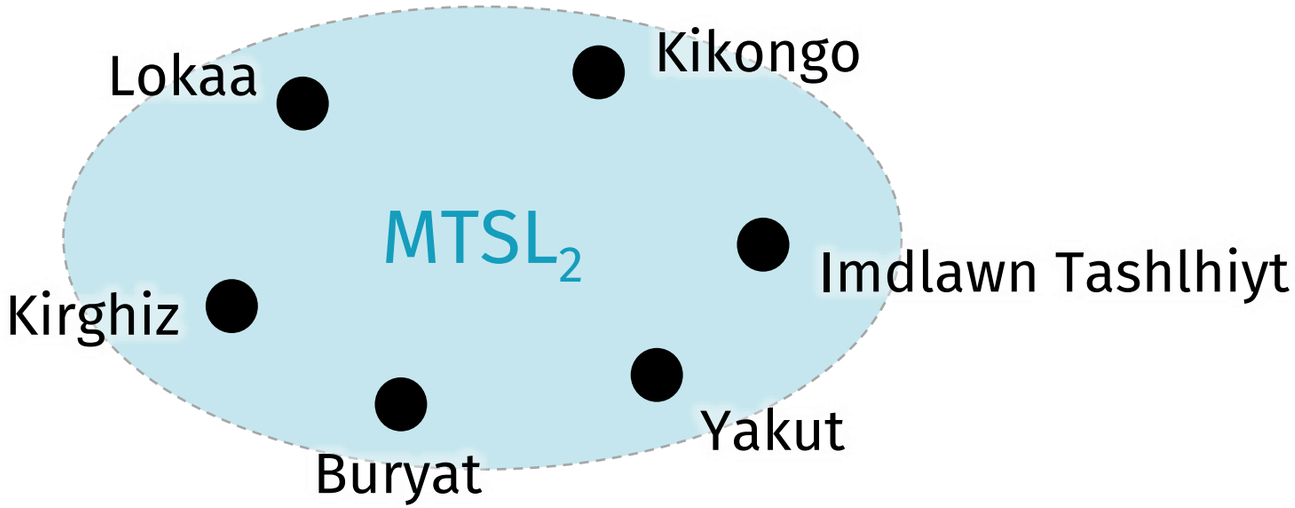

MTSL2


Lokaa
Kikongo
MTSL2
Imdlawn Tashlhiyt
Kirghiz
Yakut
Buryat

Yaka (Aksënova & De Santo 2021)



Tutrugbu (McCollum et al. 2020)

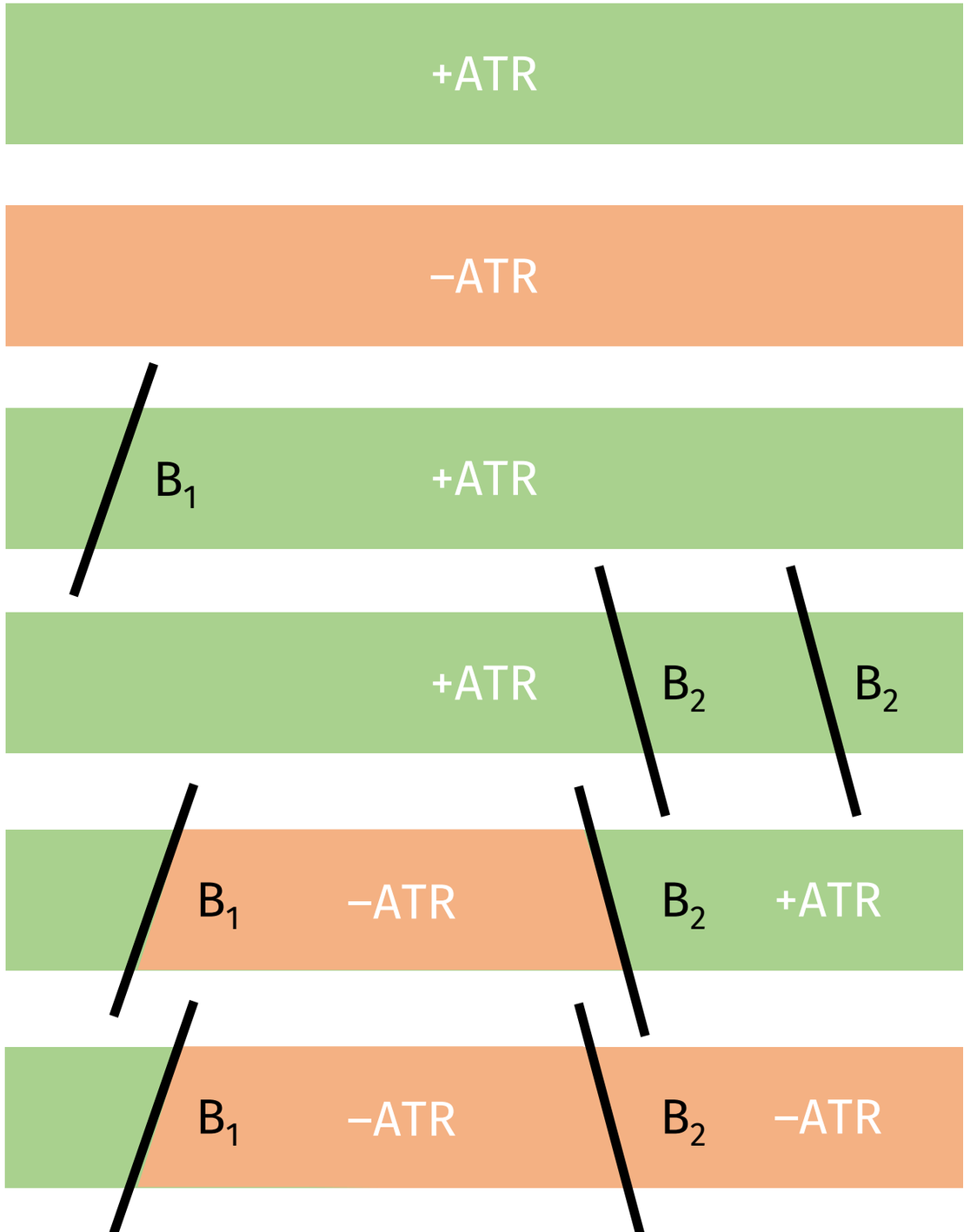
Tutrugbu (Atlantic-Congo)

ATR harmony, as per McCollum et al. (2020):

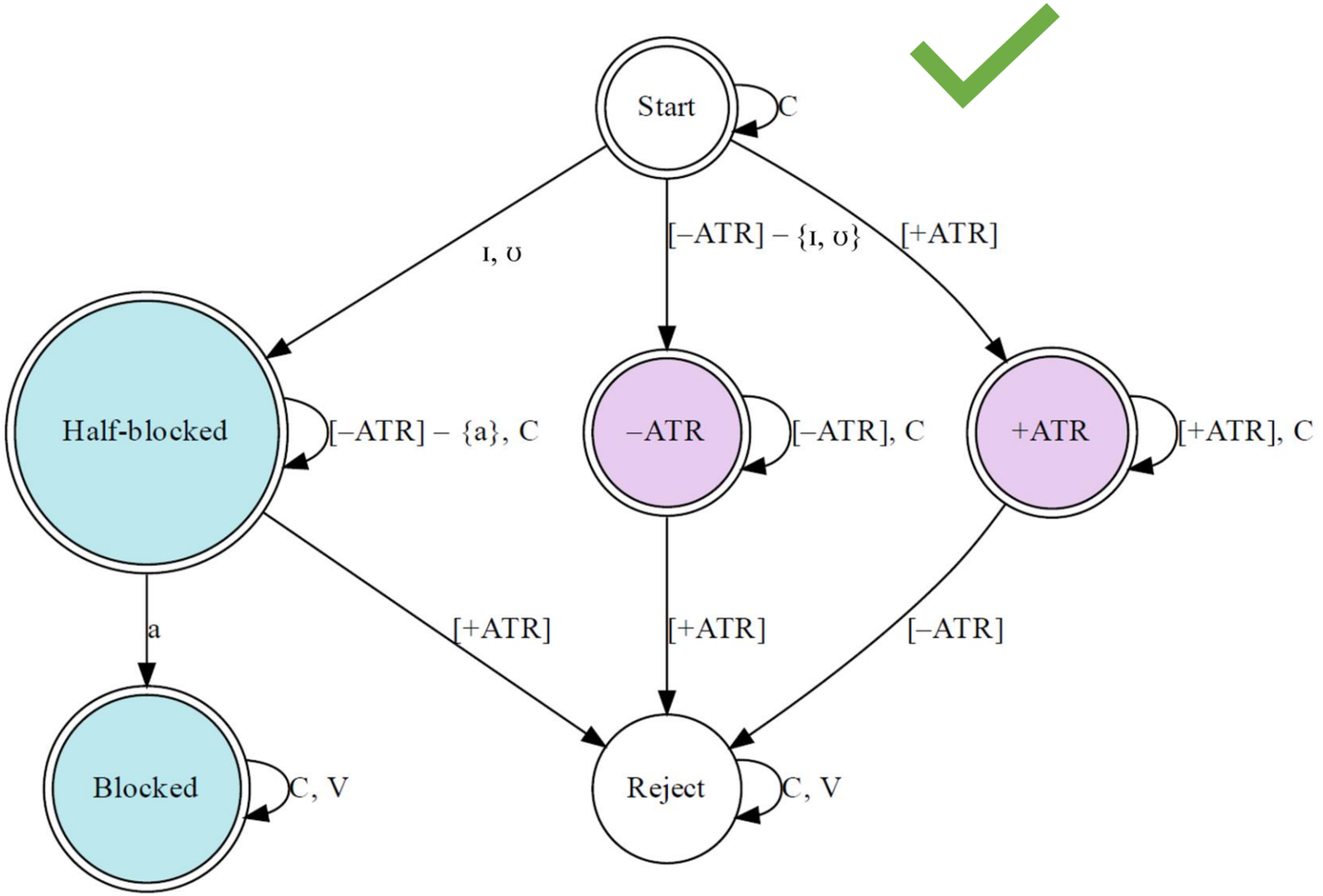
- [+ATR] = /i, e, u/
- [–ATR] = /ɪ, ɛ, a, ɔ, ʊ/

Blockers in ATR harmony:

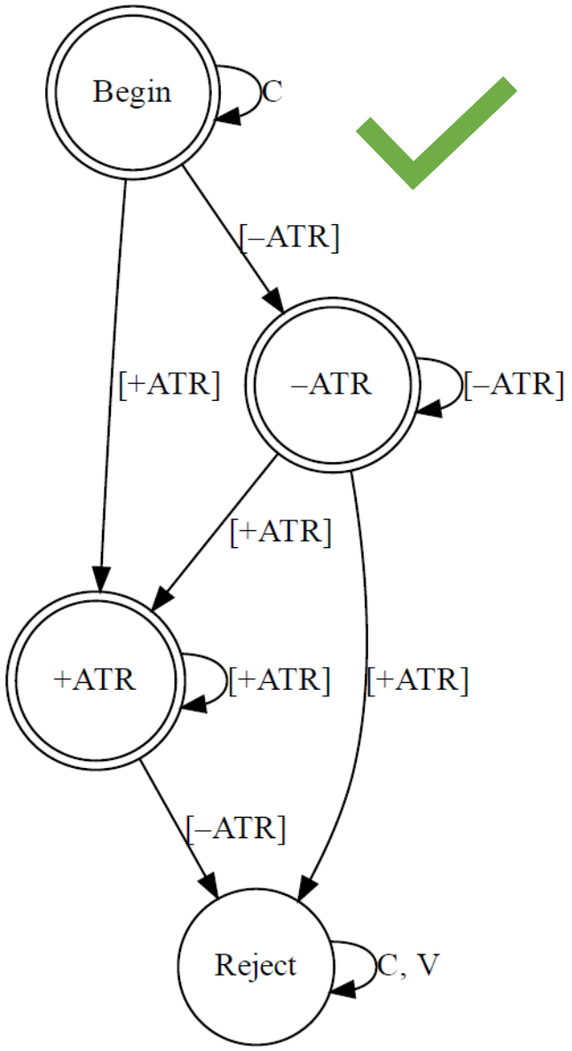
- First half: /ɪ, ʊ/ in word-initial syllable, that is, /#(C)ɪ/ or /#(C)ʊ/
- Second half: /a/


+ATR
–ATR
B1
+ATR
+ATR
B2
B2
B1
–ATR
B2
+ATR
B1
–ATR
B2
–ATR

	Transcription, harmonizing feature (±ATR), blockers (B_1, B_2)	**Gloss**
	ɪ d ɪ t ɔ – – – B_1	‘1S-ITIVE-cook’
	i d i w u + + +	‘1S-ITIVE-climb’
*	ɪ d ɪ w u – – + B_1	–
	ɪ b a w u – – + B_1 B_2	‘1S-FUT-climb’
*	ɔ b a w u – – + B_2	–
	ɪ t ɪ k a w u – – – + B_1 B_2	‘1S-NEG-PFV-climb’


Start
C
ɪ, ʊ
[–ATR] – {ɪ, ʊ}
[+ATR]
Half-blocked
[–ATR] – {a}, C
–ATR
[–ATR], C
+ATR
[+ATR], C
a
[+ATR]
[+ATR]
[–ATR]
Blocked
C, V
Reject
C, V

∩


Begin
C
[–ATR]
[+ATR]
–ATR
[–ATR]
[+ATR]
+ATR
[+ATR]
[+ATR]
[–ATR]
Reject
C, V

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*ksk in Finnish (Juho Pystynen, p. c.)

naks	onomatopoeia
skini	‘skinhead’
nakkini	‘frankfurter.1POSS’
***nakskini**	–

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naks	onomatopoeia
skini	‘skinhead’
nakkini	‘frankfurter.1POSS’
***nakskini**	–

*krt in Russian (Vladimir Panov, p. c.)

akr	‘acre’
rta	‘mouth.GEN’
akta	‘act.GEN’
***akrta**	–

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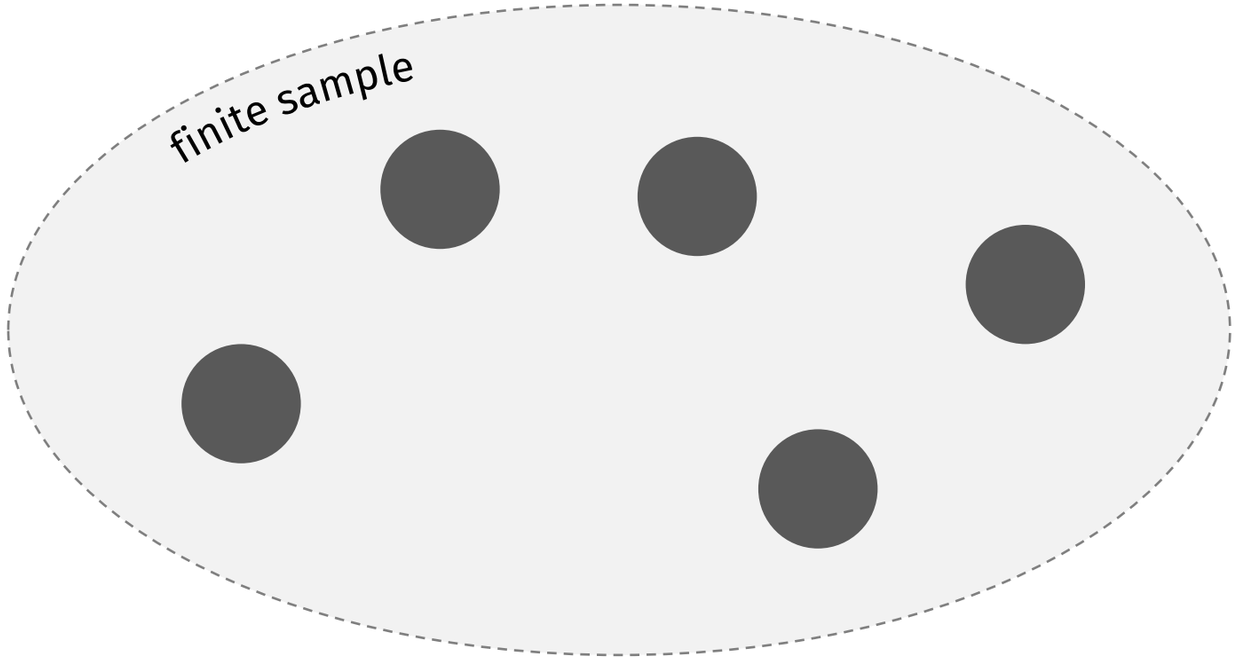
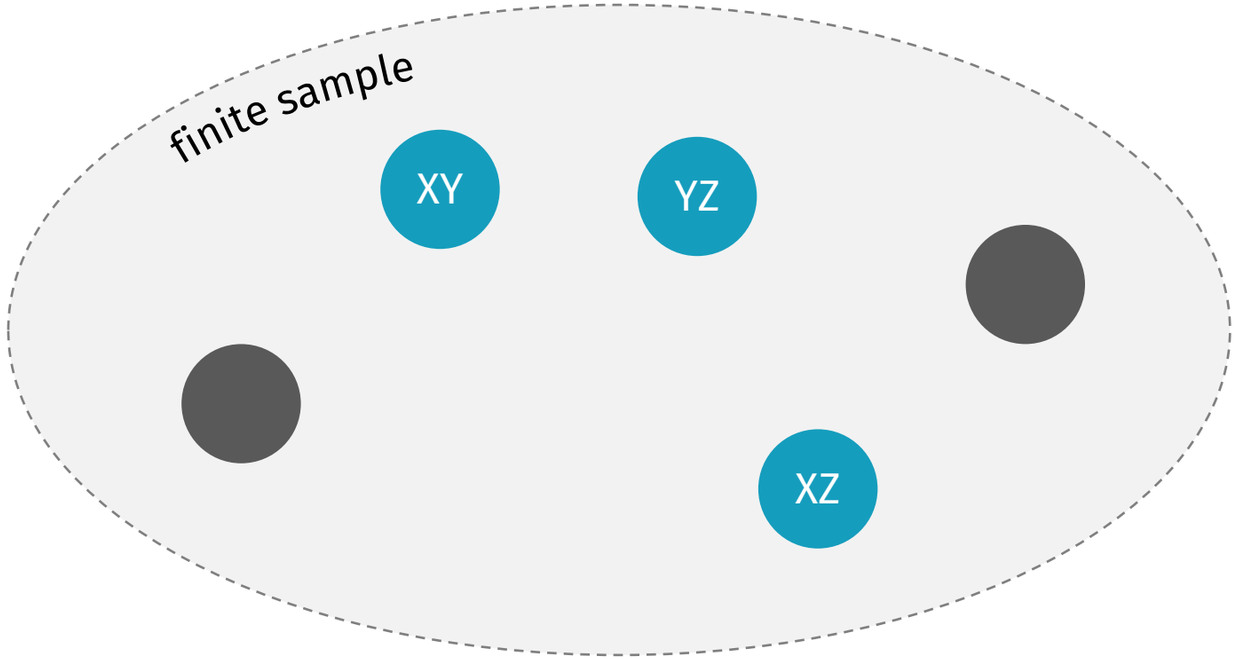
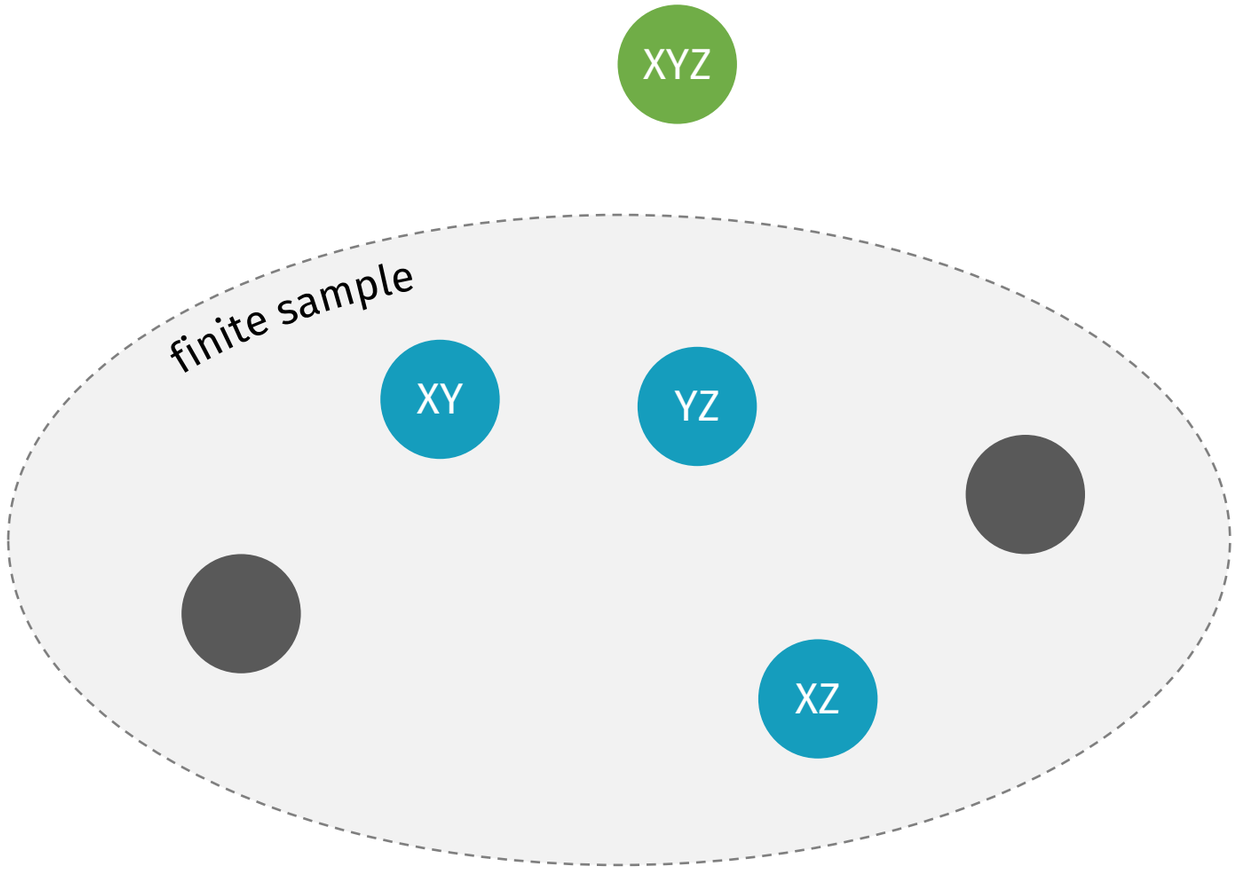

finite sample


finite sample
XY
YZ
XZ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finite sample
XY
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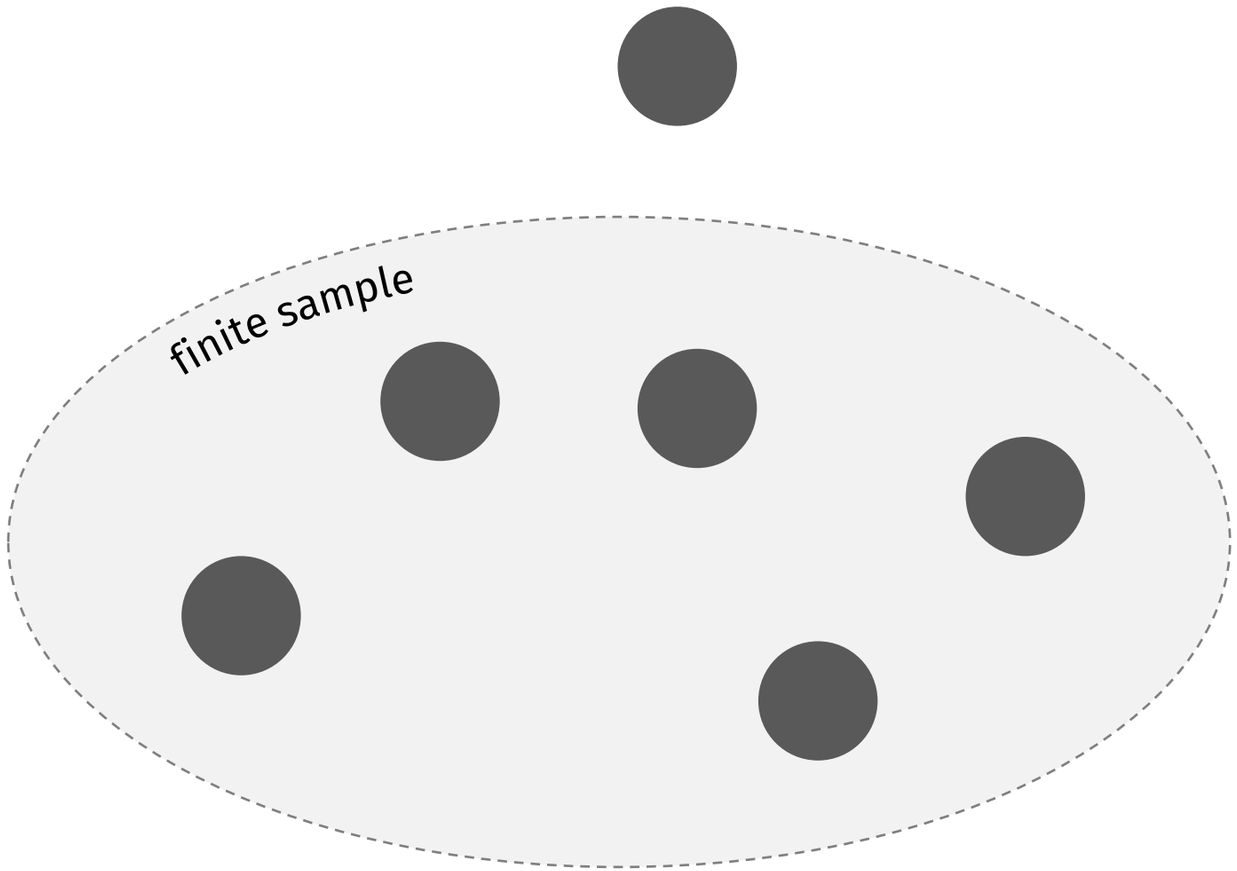
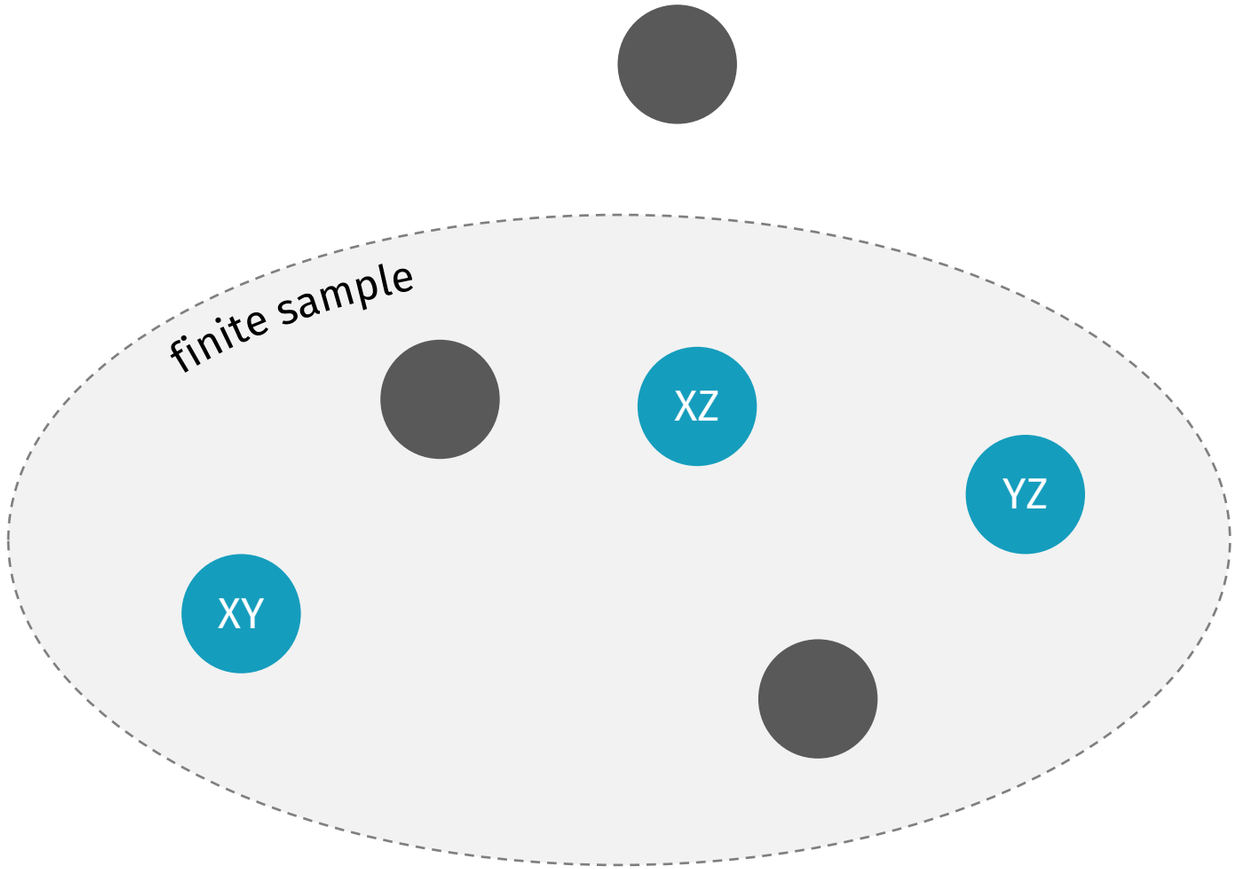
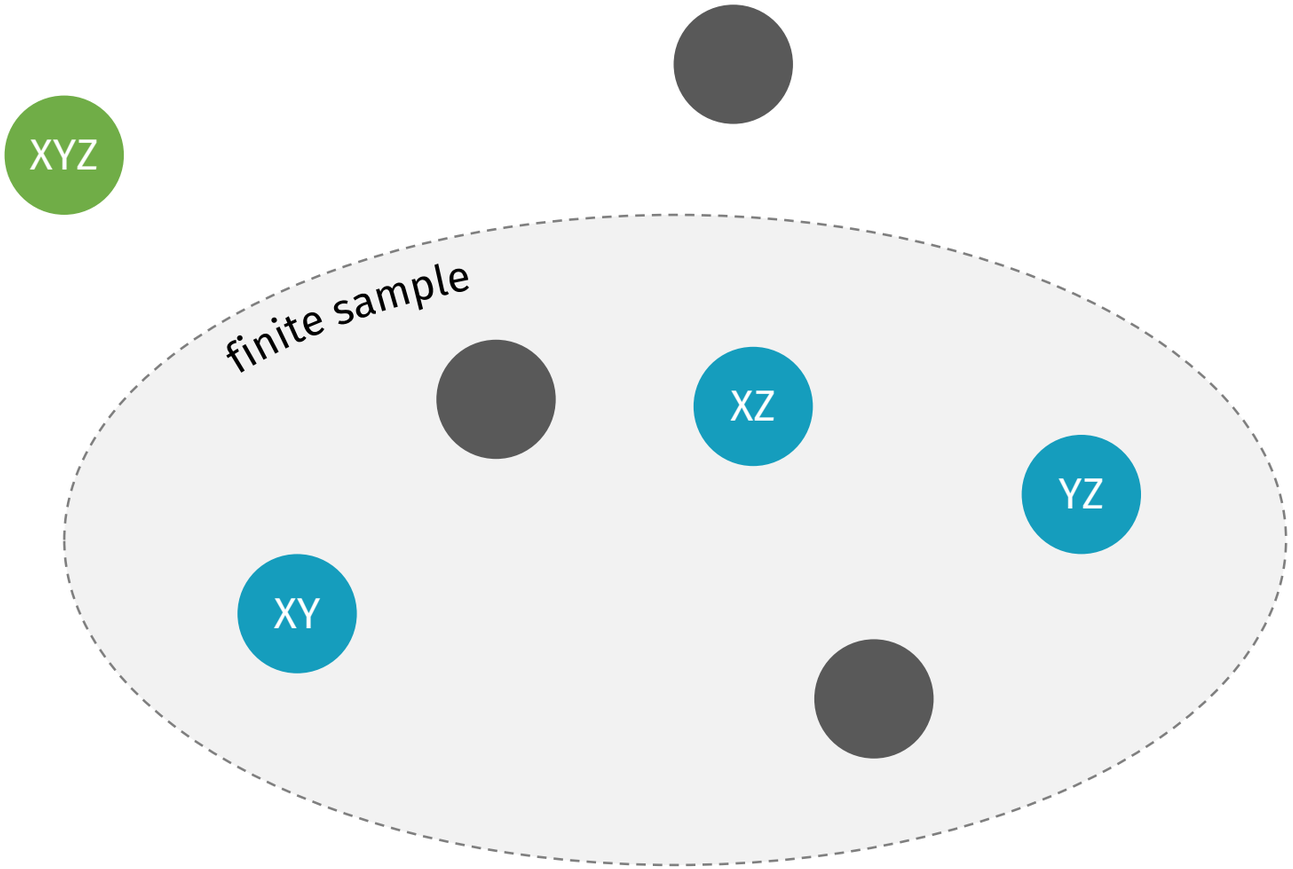
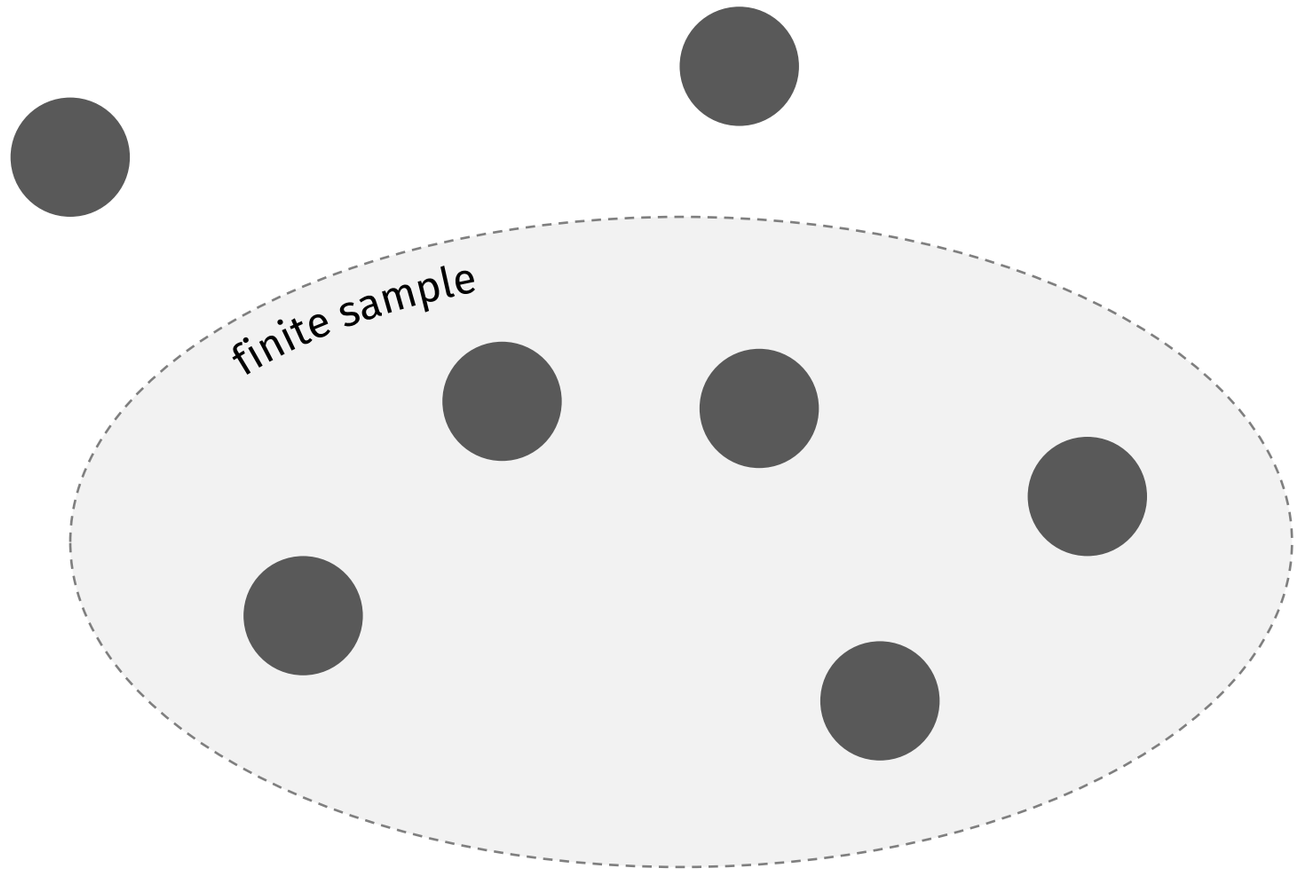
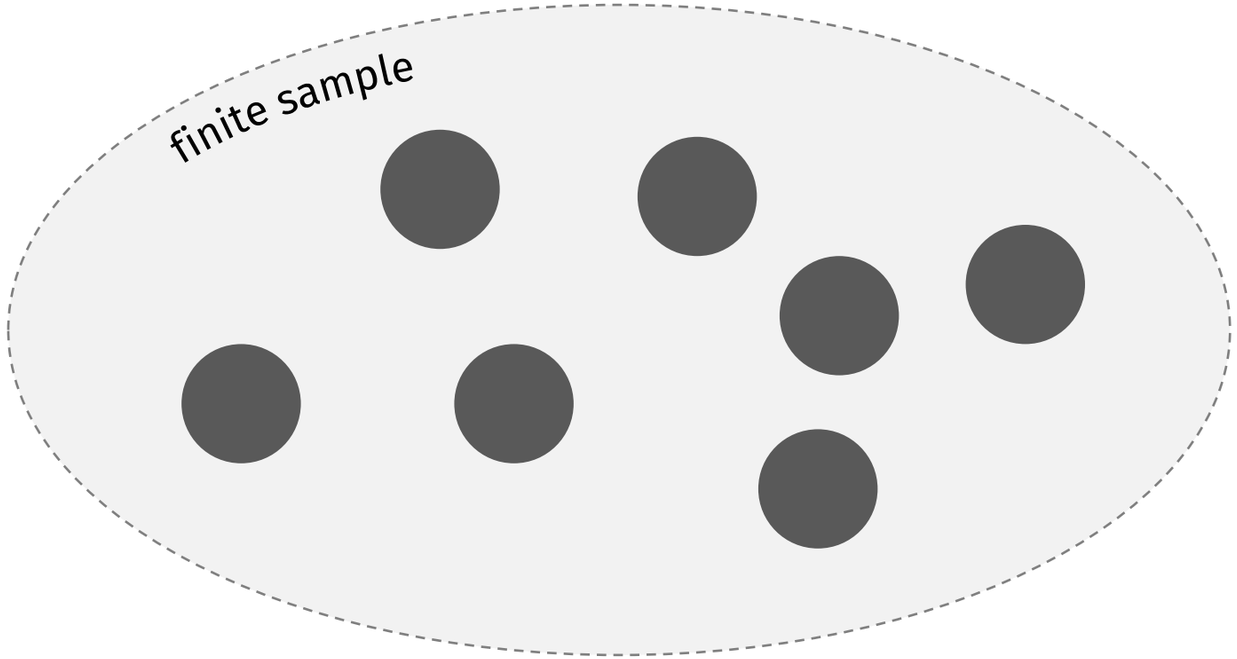

finite sample


finite sample
XZ
YZ
XY


XYZ
finite sample
XZ
YZ
XY


finite sample


finite sample

iterate

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Conclusion

- So far, everything is accounted for, except trigram constraints
  - Although requiring |Y| ≥ 2 keeps trigrams at bay
- The shape of the recombinant property is very simple
  - Subjectively, simpler to describe than MTSL_2
  - Or any particular harmony system, really
- A trivial learning algorithm follows directly
  - Not very eager to generalize
  - Exemplar-driven by design
- Future test cases: Sanskrit *n*-retroflexion (Ryan 2017), limited-distance harmony, phonologization of harmony (?)
- Wordhood is an issue, and so is representing suprasegmentals

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Thank you!