Stratified reference: the common core of distributivity, aspect, and measurement*

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Draft — comments and questions welcome!

Abstract

Why can I tell you that I ran for five minutes but not that I *ran to the store for five minutes? Why can we talk about five pounds of books but not about *five pounds of book? What keeps you from saying *sixty degrees of water when you can say sixty inches of water? And what goes wrong when I complain that *all the ants in my kitchen are numerous? The constraints on these constructions involve concepts that are generally studied separately: aspect, plural and mass reference, measurement, distributivity, and collectivity. This paper provides a unified perspective on these domains and gives a single answer to the questions above in the framework of algebraic event semantics.

1 Introduction

This paper presents a theory that builds connections between three domains which are often addressed separately: aspect, measurement, and distributivity. I aim to provide

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a synthesis of these insights in the unifying framework of algebraic event semantics. I suggest that a number of phenomena in these domains can be connected in this framework through a novel higher-order property, *stratified reference*. This property is both general enough to connect and subsume several familiar notions, and formally precise enough to transfer insights across the literature.

Stratified reference pulls together several semantic oppositions closely associated with the domains under consideration. These are the telic-atelic opposition, which is central to the study of aspect; the singular-plural opposition and count-mass opposition and the intensive-extensive opposition, which are central to the study of measurement; and the collective-distributive opposition, which is central to the study of distributivity. This paper extends and builds on algebraic semantic work on the close parallels between the nominal (count-mass) and verbal (telic-atelic) domains, such as Link (1983), Bach (1986) and Krifka (1998). I extend these parallels to include the collectivedistributive opposition.

Conceptually, singular, telic, and collective predicates are delimited or bounded in ways that plural, mass, atelic, and distributive predicates are not. This raises the question how to formally characterize the difference between the two conceptual states of boundedness and unboundedness. Answering this question amounts to specifying what property a predicate needs to have in order to qualify as atelic, distributive, plural, or to have mass reference. It is not obvious that there should be a single property that is shared by all these predicates. I will show, however, that it is indeed possible to isolate such a property. The identity of this property can be determined by analyzing a number of nominal and verbal constructions which are sensitive to the semantic oppositions in question, and which are central to the study of aspect, measurement, and distributivity. These constructions include for-adverbials, which distinguish atelic from telic predicates (1); pseudopartitives, which distinguish plurals and mass nouns from singular count nouns (2); and adverbial each, which distinguishes distributive from collective predicates (3). I refer to these three constructions collectively as distributive constructions.

- John ran for five minutes. (1) atelic
 - *John ran to the store for five minutes. *telic
- (2) thirty pounds of books a.
 - plural b. thirty liters of water mass

*collective

- *thirty pounds of **book** *singular; ok as coerced mass c.
- distributive (3)The boys each walked. a.
 - *The boys each **met**.

The main claim of this paper is these constructions all exclude bounded predicates through stratified reference, a higher-order property (a property of predicates) that is parametrized in order to account for the differences between the constructions in question. Stratified reference requires a predicate that holds of a certain entity or event to also hold of its parts along a certain dimension and down to a certain granularity. Dimension and granularity are understood as parameters which different distributive constructions can set to different values.

The dimension parameter specifies the way in which the predicate in question is distributed. Different settings of this parameter allow one and the same predicate to be atelic but not distributive, for example, or vice versa. When the dimension parameter is set to time, stratified reference applies to atelic but not to telic predicates, as in (1). When it is set to a measure function like weight or volume, stratified reference applies to mass and plural nouns but not to count nouns, as in (2). When it is set to a thematic role like agent, stratified reference applies to distributive but not to collective predicates, as in (3).

The granularity parameter specifies the size of the parts in question as measured along the relevant dimension. Depending on how it is set, the parts in question must typically be either mereological atoms (singular entities) or else very small in size. This parameter accounts, among other things, for the differences between distributive constructions over discrete (count) domains, such as adverbial-*each* constructions, and those over domains involving continuous dimensions, such as *for*-adverbials and pseudopartitives.

The names *dimension*, *granularity*, and – as I will explain shortly – *stratified reference* are derived from a visual metaphor, which is based on the idea that individuals, substances, and events occupy regions in an abstract space. The dimensions of this space include the familiar spatial and temporal dimensions as well as any measure functions and thematic roles that happen to be defined for the entity in question. To understand a thematic role as a dimension, assume that the individuals that correspond to these roles are ordered in an arbitrary but fixed way.

To give a few examples, an object whose weight is large corresponds to a region with a large extent along the weight dimension. An event whose agent is a plural entity corresponds to a region with a large extent along the agent dimension, while an event whose agent is singular corresponds to a region which is not extended along the agent dimension at all. A temporally and spatially punctual event whose thematic roles are all singular entities corresponds to a point. Finally, a temporally and spatially punctual event that has plural entities as its agent and theme corresponds to an infinitely thin rectangle that is extended along the agent and theme dimensions.

Section 2 introduces stratified reference and shows how it characterizes the opposition between atelic and telic predicates, both in time and space. Section 3 uses stratified reference to characterize several oppositions that are relevant in pseudopartitives, including those concerning singular and plural predicates, count and mass predicates, and different kinds of measure functions. Section 4 uses stratified reference to formulate meaning postulates that characterize the difference between distributive and collective predicates, and differences within the class of collective predicates. The paper closes with a discussion of open problems and possible avenues for further research.

2 Aspect

In this section I introduce and motivate stratified reference I show how it can be used to characterize the telic/atelic opposition. This opposition is linguistically relevant because it is needed in order to describe the behavior of a number of aspectually sensitive constructions. The best known examples of these constructions are measure adverbials, such as *for*-adverbials. *For*-adverbials, which are compatible with atelic predicates but not with telic predicates, setting coercion and reinterpretation aside (Vendler, 1957):

(4) a. John **walked** for five minutes.

atelic

b. *John walked to the store for five minutes.

telic

For-adverbials are usually contrasted with *in*-adverbials. In neutral contexts, they are compatible with telic predicates but not with atelic predicates:

(5) a. John **walked** for five minutes.

atelic

b. *John walked in five minutes.

telic

These are not the only aspectually sensitive adverbials. For example, *until* is also sensitive to the atelic-telic distinction (Hitzeman, 1991, 1997).

(6) a. John **ate away at his sandwich** until Mary arrived.

atelic

b. #John **finished his sandwich** until Mary arrived.

telic

Similar generalizations have been suggested to underlie the behavior of French adverbial *beaucoup* ("a lot") (Doetjes, 2007) and of German *seit* ("since"), although its interaction with the Perfect makes this more difficult to observe (von Stechow, 2002).

While most discussions of aspect focus on its interaction with tense and time, measure adverbials can be temporal as well as spatial (Moltmann, 1991; Gawron, 2005). Spatial *for*-adverbials test for a spatial counterpart of atelicity but work analogously to temporal *for*-adverbials otherwise. For example, the predicate *meander* is spatially atelic because, roughly speaking, every part of a meandering road itself meanders. For this reason, it is compatible with spatial *for*-adverbials but not with spatial emphinadverbials:

- (7) a. The road meanders for a mile.
 - b. *The road meanders in a mile.

As another example, (8a) has a stative interpretation on which it describes a crack whose width gradually increases over a stretch of five meters. By contrast, example (8b) cannot mean that the crack widens by 2cm over a stretch of five meters. It only has an irrelevant interpretation on which the crack's width over a stretch of five meters is 2cm more than elsewhere, which arguably corresponds to result-oriented interpretations of *for*-adverbials like *The Sherriff jailed Robin Hood for four years*.

(8)The crack widens for/*in 5 meters. a.

b.

spatially atelic The crack **widens 2cm** in/*for 5 meters. spatially telic

Some other adverbials also behave like spatial *for*-adverbials, as shown in the examples in (9) from (Moltmann, 1991).

- (9) Worldwide children/#a child/#1000 children suffer from hunger.
 - Throughout the country the increased air pollution caused protests /#a protest/#the protests.

From this and similar examples, I conclude that spatial for-adverbials test for a spatial counterpart of atelicity. The following minimal pair shows that spatial and temporal for-adverbials do not have the same distribution: (10a) is acceptable on an iterative interpretation, where John goes back and forth to the store many times, but (10b) is not acceptable on any interpretation.

- John pushed carts all the way to the store for fifty minutes. (10)temporally atelic
 - b. #John pushed carts all the way to the store for fifty meters. spatially telic

This contrast can be explained on the assumption that for 50 minutes checks for a property that is parametrized for time while for 50 meters checks for a property that is parametrized for space. More generally, the contrast between spatial and temporal for-adverbials supports a parametrized notion of the telic-atelic opposition, where the parameter is set either to time or to a spatial dimension.

The predicate John pushed carts all the way to the store may be used to describe to a complex sum event, in some of which John may have gone back and forth and pushed carts little by little. Such an event is depicted in the space-time diagrams in Figure 1, which builds on the spatial metaphor I described above and attempts to convey the intuition behind the explanation I will provide. The left-hand diagram corresponds to (10a) and the right-hand one to (10b).

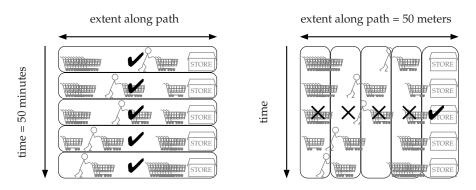


Figure 1: John pushed carts to the store is temporally atelic but spatially telic

Both sentences require the predicate to hold at very small intervals that are parts of the interval they mention. In (10a), this interval is a fifty-minute time span. As long as each subinterval of this time span is the runtime of an event that satisfies the predicate John pushed carts all the way to the store, (10a) is predicted to be acceptable. The events marked with a checkmark satisfy this predicate in a scenario where John went back and forth and pushed carts all the way to the store little by little. In (10b), the interval is a fifty-meter long path. Most of the subintervals of this path do not contain the location of the store. That (10b) is unacceptable can then be explained by the assumption that an event whose location does not contain the store does not satisfy the predicate John pushed carts all the way to the store. For example, the events marked with an X on the right-hand diagram of Figure 1 do not satisfy this predicate. Thus, we may say that push carts all the way to the store is atelic with respect to the time but not with respect to space.

One influential theory of atelicity is that for a predicate to be atelic it must have the subinterval property (e.g. Dowty, 1979). This property holds of any predicate such that, whenever it predicate holds at an interval t, it also holds at every subinterval of t, all the way down to instants. Figuratively speaking, the subinterval property requires that any event in the denotation of a predicate that has this property can be divided into infinitely thin "temporal layers" that are also in the denotation of this atelic predicate. For example, on this view, the predicate walk is atelic because we can "zoom in" to any part of the runtime of a walking event to find a part of it which is another walking event. These shorter walking events are plausibly considered parts, or subevents, of the longer walking events.

(11) $\forall e[\text{walk}(e) \rightarrow \forall t[t \leq \tau(e) \rightarrow \exists e'[\text{walk}(e') \land e' < e \land t = \tau(e')]]]$ (Whenever *walk* holds of an event *e*, then at every subinterval of the runtime of *e*, there is a subevent of which *walk* also holds.)

The subinterval property has several deficiencies. For one thing, it hard-codes the role of time in the definition of aspect, and must therefore be generalized before it can be used for spatial *for*-adverbials. Another deficiency is the well-known "minimal-parts problem": it insists that an atelic predicate must be true at all subintervals, even infinitely short ones. Not all atelic predicates satisfy this stringent criterion. It is sufficient for the predicate to be true at length intervals that count as very small relative to the length of the bigger interval. To mention a classical example, given that any waltzing event takes at least three steps to unfold, example (12) does not entail that John and Mary waltzed within every single moment of the hour, only that they waltzed within every short subinterval of the hour (Taylor, 1977; Dowty, 1979).

(12) John and Mary waltzed for an hour.

The minimal length in question is not fixed once and for all but varies relative to the length of the bigger interval, as shown by the following attested examples:

- (13) The Chinese people have created abundant folk arts ... passed on from generation to generation for thousands of years. 1
- (14) Ded'leg says: How i [sic] stop a macro for 1sec? Cog says: By creating a script that will loop for 3600 [sic] milliseconds ... ²

While it has long been accepted that the subinterval property is an idealization and that it must be softened to account for such examples, there is no consensus on the best way to soften the subinterval property. For reviews of approaches to the minimal-parts problem, see for example Krifka (1986), Mollá-Aliod (1997). Here I will develop one way to do so. I defend it against alternative approaches in Champollion (2010, ch. 5 and 6).

In the metaphor I have developed, we can diagnose the problem with the subinterval property as follows. The subinterval property requires that any event that satisfies a property P can be cut along the temporal dimension into infinitely thin "layers" each of which satisfies P. While for-adverbials do seem to constrain the event layers to be very thin, they do not have to be infinitely thin. I will call these very thin layers strata and I will amend the subinterval property in a way that will lead to stratified reference. I have chosen the names strata and stratified reference, to remind the reader of geological strata, the layers of rock which can be observed in geological formations in places such as the Grand Canyon. A geological stratum can be just a few inches thick, though not infinitely thin, and yet extend over hundreds of thousands of square miles. This aspect is mirrored in the present theory, where strata are constrained to be very thin along one dimension, but may be arbitrarily large as measured in any other dimension. This feature of the theory not only helps capture the distinction between temporal and spatial measure adverbials. It will also become important later, when I will use stratified reference to model different properties at the same time (e.g. atelicity and distributivity) that may not always hold of the same predicates. I will assume that individual constructions set the dimension parameter in different ways when they impose constraints on their constituents. For example, for-adverbials require stratified reference along the temporal dimension only. As far as *for*-adverbials are concerned, while the parts of relevant events must be short, or thin, in the temporal dimension, they may still have plural entities as agents or themes, they may be extended in space, and so on.

The subinterval property says that an atelic predicate distributes along the time dimension down to intervals of infinitely short length. Instead, as we have seen, atelic predicates distribute along a certain dimension (which may be either temporal or spatial) down to intervals of a certain length or granularity. Formally, I propose to achieve this effect by adding a dimension parameter and a granularity parameter to the subinterval property and constraining this parameter to a low but nonatomic value. I will make use of the star operator known from the literature on plural semantics (Link, 1983). The star operator is defined as follows. Here I use $\bigoplus C$ to refer to the mereo-

¹Attested example (http://www.twinbridge.com/detail.aspx?ID=315).

²Attested example (http://www.wowmacroswarcraft.com/faq/).

logical sum of the elements of the set *C*. For a general introduction to mereology in formal semantics, see Champollion & Krifka (to appear).

(15) Definition of the star operator

*
$$P \stackrel{\text{def}}{=} \{A \mid \exists C[A = \bigoplus C \land C \subseteq P]\}$$
 (A is the sum of all the elements of a subset C of P).

Intuitively, $x \in {}^*(\lambda y.P(y))$ means that x consists of one or more parts such that P holds of every one of these parts.

In the formulas in this paper, I will use the following typing conventions: t for propositions, e for ordinary objects, v for events, i for intervals, d for degrees, and n for numbers. I use the symbols x, y, z, x', y', z' and so on for variables that range over ordinary objects, e, e', e'' for events, t, t', t'' for temporal intervals, l, l', l'' for spatial intervals (locations), and n, n', n'' for numbers.

To model different levels of granularity, I will use a vague predicate ε , and I will assume that $\varepsilon(K)(x)$ holds just in case x counts as very small as compared to the comparison class K. For example, I assume that $\varepsilon(\lambda t[\text{hours}(t)=1])(t')$ is true just in case t' is very small with respect to one hour. Using this predicate, we can formally express the following statement:

(16) $\forall e[\text{waltz}(e) \rightarrow e \in {}^*\lambda e' \left(\begin{array}{c} \text{waltz}(e') \land \\ \varepsilon(\lambda t[\text{hours}(t) = 1])(\tau(e')) \end{array} \right)]$ (Every waltzing event can be divided into one or more parts, each of which is a waltzing event whose runtime is very short compared with one hour.)

Let us say that waltz has **stratified reference** with respect to the dimension runtime and the granularity $\varepsilon(\lambda t[\text{hours}(t)=1])$ (true of any interval very short as compared to an hour), formally, $SR_{\tau,\varepsilon(\lambda t[\text{hours}(t)=1])}(waltz)$, just in case (16) is true. By abstracting from this example, we arrive at the following definition:

(17) Stratified reference (Definition)
$$SR_{f,\varepsilon(K)}(P) \stackrel{\text{def}}{=} \forall x [P(x) \to x \in {}^*\lambda y \begin{pmatrix} P(y) \land \\ \varepsilon(K)(f(y)) \end{pmatrix}]$$

This definition says that stratified reference applies to a predicate P just in case the following is true: whenever P holds of an entity or event x, there is a way to divide x into strata y_1 , y_2 , etc. such that each y_i is mapped by the function f to a value which counts as very small with respect to the comparison class K. This is illustrated in the following diagram, where the vertical axis represents the dimension f. In case f is instantiated with runtime, stratified reference approximately amounts to the subinterval property, except that the minimal-parts problem is avoided.

I propose that being temporally atelic means having stratified reference with respect to time and a suitably instantiated granularity parameter. *For*-adverbials require stratified reference, not the subinterval property. For concreteness, I will assume that

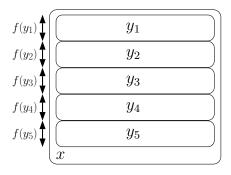


Figure 2: Stratified reference

this requirement is a presupposition, and I will represent it as a definedness requirement, but this is not crucial. I write $\lambda x:\varphi$. ψ for the partial function that is defined for all x such that φ holds, and that returns ψ wherever the function is defined (Heim & Kratzer, 1998). My entry for *for* is as follows:

(18)
$$[\![for]\!] = \lambda \tau_{\langle vi \rangle} \lambda M_{\langle it \rangle} \lambda P_{\langle vt \rangle} \lambda e : SR_{\tau, \varepsilon(M)}(P). P(e) \wedge M(\tau(e))$$

This entry combines with a temporal or spatial trace function τ (depending on whether the *for*-adverbial is temporal or spatial), which we may assume is provided by a silent head. It then combines with a measure phrase M (such as *an hour*) and a verb phrase P (such as *waltz*) and presupposes, for example, that *waltz* has stratified reference with respect to dimension τ (runtime) and granularity $\varepsilon(\lambda t.\text{hours}(t) = 1)$. This presupposition tolerates the possibility that very short waltzing events are not infinitely divisible into even shorter ones, and is therefore satisfied even though *waltz* may not have the subinterval property. This is summarized in (19). I will provide similar summaries throughout the rest of this paper.

(19) waltz for an hour

Satisfied presupposition:
$$SR_{\tau,\varepsilon(\lambda t[\text{hours}(t)=1])])}(\lambda e.\text{waltz}(e)) \Leftrightarrow \forall e[\text{waltz}(e) \to e \in {}^*\lambda e' \begin{pmatrix} \text{waltz}(e') \land \\ \varepsilon(\lambda t[\text{hours}(t)=1])(\tau(e')) \end{pmatrix}]$$

(Every waltzing event can be divided into one or more parts, each of which is a waltzing event whose runtime is very short compared with one hour.)

By assuming that different *for*-adverbials set the granularity parameter of the stratified reference conditions they impose to different values, we can model the fact that atelicity is a relative notion, as seen in examples (13) and (14). The analyses of these examples are as follows:

$$\forall e [\text{pass.on.from.generation.to.generation}(e) \rightarrow \\ e \in {}^*\lambda e' \left(\begin{array}{c} \text{pass.on.from.generation.to.generation}(e') \land \\ \varepsilon(\lambda t [\text{thousands}(\text{years}(t))])(\tau(e')) \end{array} \right)]$$

(Every event that can be described as pass on from generation to generation can be divided into one or more parts, each of which can be described as pass on from generation to generation and has a runtime that is very short compared with thousands of years.)

loop for 3600 milliseconds (21)

Satisfied presupposition:
$$SR_{\tau,\varepsilon(\lambda t[\text{milliseconds}(t)=3600])}(\lambda e.\text{loop}(e)) \Leftrightarrow \forall e[\text{loop}(e) \to e \in {}^*\lambda e' \begin{pmatrix} \text{loop}(e') \land \\ \varepsilon(\lambda t[\text{milliseconds}(t)=3600])(\tau(e')) \end{pmatrix}]$$
 (Every looping event can be divided into one or more parts, each of which is a

looping event whose runtime is very short compared with 3600 milliseconds.)

Again, these presuppositions are plausibly satisfied. Even though at least example (20) does not involve a property that has the subinterval property, it passes muster because the granularity parameter is set to a very coarse level.

Finally, stratified reference can also be used to model spatial aspect. Examples like The road meanders do not require the road in question to meander throughout its entire length, just like John walked do not require John to walk throughout his entire lifetime. To represent this fact, I assume that the relevant sentences involve underlying states and that these states may be spatiotemporally extended, just like events (Parsons, 1987). The state that verifies *The road meanders* will be coextensive to the meandering part of the road. These states play the same role in the compositional process as events, so I represent them with the same variable e. I assume that a state is mapped to its spatial extent by a function I will write as σ (Zwarts, 2006). Given this, we can translate a spatial for-adverbial as in (22). This translation imposes a stratified reference requirement that is parametrized for σ (spatial extent) instead of τ (time) and is otherwise equivalent to my translation of the temporal for-adverbial. Other spatial measure adverbials, such as worldwide and throughout the country in (9), can be represented in similar ways.

(22) [[for a mile]]
$$= \lambda P_{\langle vt \rangle} \lambda e : SR_{\sigma, \varepsilon(\lambda t[\text{miles}(t)=1])}(P).P(e) \land \text{miles}(\sigma(e)) = 1$$

With this in place, we can easily capture the requirement of spatial *for*-adverbials and related expressions, as the following examples show:

(23)meander for a mile

Satisfied presupposition:
$$SR_{\sigma,\varepsilon(\lambda l[\mathrm{miles}(l)=1])}(\lambda e.\mathrm{meander}(e)) \Leftrightarrow \forall e[\mathrm{meander}(e) \to e \in {}^*\lambda e' \left(\begin{array}{c} \mathrm{meander}(e') \land \\ \varepsilon(\lambda l[\mathrm{miles}(l)=1])(\sigma(e')) \end{array} \right)]$$
 (Every meandering state can be divided into one or more parts, each of which

is a meandering state whose spatial extent is very small compared with one mile.)

cause protests throughout the country **Satisfied presupposition:** $SR_{\sigma, \varepsilon(\lambda l[l=\sigma(\text{the.country})])}(\lambda e.\text{cause}(e) \wedge *\text{protest}(\text{theme}(e)) \Leftrightarrow$ $\forall e[\lambda e.\text{cause}(e) \wedge *\text{protest}(\text{theme}(e)) \rightarrow e \in *\lambda e' \begin{pmatrix} \text{cause}(e') \wedge *\text{protest}(\text{theme}(e')) \wedge \\ \varepsilon(\lambda l[l=\sigma(\text{the.country})])(\sigma(e')) \end{pmatrix}]$ (Every state of causing protests can be divided into one or more parts, each of which is a state of causing protests whose spatial extent is very small compared with the spatial extent of the country.)

To summarize this section, *for*-adverbials and related constructions are sensitive to the temporal/spatial opposition and to different granularities in ways that motivate generalizing the subinterval property appropriately. This generalization, stratified reference, builds on the basic intuition behind algebraic semantic accounts of aspect – namely, that atelicity can be defined in terms of a predicate applying to the parts of an event in question – but generalizes it by adding parameters that allow us to explicitly model varying dimensions and granularities.

3 Measurement

Measurement in natural language is relevant in constructions that are used to talk about an amount of some substance or event. Besides measure adverbials, Pseudopartitives are one example of such a construction (Krifka, 1998; Schwarzschild, 2006). My discussion will focus on these constructions:

(25) a. five pounds of rice

weight

b. five liters of water

volume

c. five hours of talks

duration

d. five miles of railroad tracks

spatial extent

There are two semantic parallels between pseudopartitives and *for*-adverbials. Both reject predicates that fail to apply to the parts of the entities and events in their denotation. This category includes telic predicates in the case of *for*-adverbials, as we have seen in the previous section, and count nouns in the case of pseudopartitives, as illustrated in (26). It is not possible to use a pseudopartitive like the one in (26b) to describe a single book whose weight is five pounds. (At best it can be interpreted as describing five pounds of pulp that results from shredding books, but in that case I assume the count noun has been coerced to a mass noun. I leave that interpretation aside from now on)

- (26) a. five pounds of books/rice
 - b. *five pounds of book

In addition, both pseudopartitives and *for*-adverbials reject measure functions whose value tends to stay constant across the parts of any object or event they measure. I will first illustrate this phenomenon with pseudopartitives and come back to *for*-adverbials later. Examples of such functions are speed, as illustrated by examples (27a) and (27b), and temperature, as in example (27c).

- (27) a. *run (to the store) for five miles an hour
 - b. five hours of running vs. *five miles an hour of running
 - c. five inches of snow vs. *five degrees Celsius of snow

Although I will focus on pseudopartitives and *for*-adverbials in this section, several other constructions behave analogously. For example, when comparative determiners are used with substance mass nouns, they are underspecified as to what measure function is involved (Schwarzschild, 2006). Thus, *more rope* can mean "a longer portion of rope" or "a heavier portion of rope" in different contexts, depending on what is relevant. It is not possible, however, to use comparative determiners to compare two amounts of rope by temperature. In other words is not possible to come up with a context in which *more rope* can be used felicitously to mean "a warmer portion of rope". True partitives are another construction that behaves similarly to pseudopartitives in terms of rejecting certain measure functions, as shown here:

- (28) a. *five degrees Celsius of the water in this bottle
- *temperature

b. *five miles per hour of my driving

*speed

From the examples so far, one might think that certain measure functions are never acceptable in pseudopartitives. But in fact, measure functions that are usually unacceptable can be made acceptable when the substance noun is chosen in the right way, as in the following attested example:

(29) The scientists from Princeton and Harvard universities say just *two degrees Celsius of global warming*, which is widely expected to occur in coming decades, could be enough to inundate the planet.³

So we cannot simply categorize measure functions as acceptable or unacceptable per se. What matters is whether they are acceptable on the set denoted by the substance noun of the pseudopartitive in which they appear.

The fact that pseudopartitives accept certain measure functions but reject others has previously been linked to the measure-theoretic properties of these measure functions. As discussed by Krifka (1998) and Schwarzschild (2006), the constrained corresponds to a distinction commonly made in measurement theory and in physics, namely the one between extensive and intensive measure functions. In physics, an extensive measure function is one whose magnitude is additive for subsystems; an intensive

³Attested example (Calgary Herald, December 17, 2009, article: *Two degrees is all it takes – Warming may trigger floods*).

measure function is one whose magnitude is independent of the extent of the system (Krantz et al., 1971; Mills et al., 2007). For example, when one considers the system consisting of the water in a tank, volume is an extensive measure function because the volume of the water as a whole is greater than the volume of any of its proper parts. But temperature is intensive with respect to this system because the temperature of the water as a whole is no different from the temperature of its proper parts. Krifka (1998) suggests that only extensive measure functions are admissible in pseudopartitives.

A related notion to extensive measure functions is that of a monotonic measure function. A function μ is monotonic iff for any two entities a and b in the physical world, if a is a proper part of b, then $\mu(a) < \mu(b)$. Schwarzschild (2006) suggests that only monotonic measure functions are admissible in pseudopartitives. For example, volume is monotonic, so we have an explanation of why *thirty liters of water* is acceptable. Temperature is not monotonic, which explains which **thirty degrees Celsius of water* is not acceptable.

Krifka's and Schwarzschild's accounts are attractive, and my theory will subsume their core idea, but they undergenerate in certain cases. Although *five feet of snow* is acceptable, height is not monotonic. (For example, imagine that five feet of snow fell on Berlin. The snow that fell on West Berlin is a proper part of the snow that fell on Berlin. But if height was monotonic, we would conclude that the height of the snow in West Berlin is less than five feet, contrary to fact.)

Schwarzschild (2006) is aware of the problem. From similar examples, he concludes that pseudopartitives do not test for monotonicity with respect to the mereological part-whole relation, but only with respect to a part-whole relation which he sees as contextually supplied. In our example, his assumption would be that context provides a relation according to which the snow that fell on West Berlin is not a part of the snow that fell on the entire city. Schwarzschild does not impose any formal constraints on the contextually supplied part-whole relation he assumes. However, since many measure functions like *temperature* are already correctly ruled out even without replacing the mereological part-whole relation by a contextually supplied relation, the two relations must coincide to a large extent, which suggests a certain redundancy.

To draw on the intuition behind Krifka's and Schwarzschild's account without making an appeal to context necessary, I propose to relativize monotonicity to a certain dimension, such as height in the snow example, and to a certain property, namely the one denoted by the substance noun. As we will see shortly, the theory I will adopt goes along these lines. Formally, the situation looks like a good match for stratified reference, which applies to a given property, and which can be parametrized for a given dimension. In the previous section, I motivated this parameter through the difference between temporal and spatial *for*-adverbials.

Schwarzschild (2006) suggests that that a formalization of telic-atelic opposition can be couched in terms of monotonicity. For example, *in*-adverbials can only combine with telic predicates because, as he puts it, runtime "is nonmonotonic on the relevant part-whole relation in the domain given by" that predicate. For Schwarzschild, runtime

is a dimension that is monotonic on the part-whole relation that relates events to their subevents. I propose to go in the opposite direction and use stratified reference to subsume monotonicity.

The empirical domains of measure adverbials and pseudopartitives can be linked by the following novel observation (Champollion, 2010). Those measure functions that are rejected by pseudopartitives are also rejected by *for*-adverbials:

(30) a. John waited for five hours.

duration

b. The crack widens for five meters.

spatial extent

c. *John drove for thirty miles an hour.

*speed

d. *The soup boiled for 100 degrees Celsius.

*temperature

The following modifications of examples (30c) and (30d) show that these examples cannot be ruled out on the grounds that events are not the kinds of things that have speeds and temperatures.

(31) a. John drove (at) thirty miles an hour.

speed

b. The soup boiled at 100 degrees Celsius.

temperature

So it is really something about *for*-adverbials, and not just about verbal modification in general, that rules out intensive measure functions like speed and temperature.

Just as we did in the case of pseudopartitives, we can identify properties which make it possible to use measure functions that are otherwise incompatible with *for*-adverbials. The following attested example shows this for the case of temperature:

(32) The sample continued to *cool for several degrees* to point N and then suddenly increased to a temperature between the transition points of Form I and Form I1 with no indication of the presence of Form 111. 4

These facts about *for*-adverbials suggest that it makes sense to draw on stratified reference, originally motivated by aspect, in order to solve the measurement problem. On the assumption that pseudopartitives give rise to the same kinds of parametrized presuppositions as temporal and spatial *for*-adverbials do, the relevant examples can be ruled out because the presuppositions that stratified reference generates for them can be plausibly be assumed to fail:

(33) *drive for 30 miles per hour

Failing presupposition: $SR_{speed, \epsilon(\lambda d[mph(d)=30])}(\lambda e.drive(e)) \Leftrightarrow \forall e[drive(e) \rightarrow e \in {}^*\lambda e' \begin{pmatrix} drive(e') \land \\ \epsilon(\lambda d[mph(d)=30])(speed(e')) \end{pmatrix}]$

(Every driving event can be divided into one or more parts, each of which is a driving event whose speed is very slow compared with thirty miles per hour.)

(34) *boil for 100 degrees Celsius

⁴Attested example, from Daubert & Clarke (1944).

Failing presupposition:
$$SR_{temperature, \varepsilon(\lambda d[Celsius(d)=100])}(\lambda e.boil(e)) \Leftrightarrow \forall e[boil(e) \to e \in {}^*\lambda e' \begin{pmatrix} boil(e') \land \\ \varepsilon(\lambda d[Celsius(d)=100])(temperature(e')) \end{pmatrix}]$$
 (Every boiling event can be divided into one or more parts, each of which is

(Every boiling event can be divided into one or more parts, each of which is a boiling event whose temperature is very low compared with 100 degrees Celsius.)

These presupposition fail because the subevents of a driving event typically have the same speed as that event, and similarly for the temperature of boiling events.

Since stratified reference is presupposed to hold of verb phrases, it is not surprising that specific verb phrases can rescue constructions that would otherwise be unacceptable, as in the case of temperature-based *for*-adverbials. For the following example, I assume that the relevant measure function, which I write *rel[ative] temperature*, maps any cooling event to the number of degrees of cooling that it causes.

(35) cool for five degrees

Satisfied presupposition:
$$SR_{rel-temperature, \varepsilon(\lambda d[Celsius(d)=5])}(\lambda e.cool(e)) \Leftrightarrow \forall e[cool(e) \to e \in {}^*\lambda e' \begin{pmatrix} cool(e') \land \\ \varepsilon(\lambda d[Celsius(d)=5])(rel-temperature(e')) \end{pmatrix}]$$
 (Every cooling event can be divided into one or more parts, each of which is

(Every cooling event can be divided into one or more parts, each of which is a cooling event whose relative temperature is very low compared with five degrees Celsius.)

On the assumption that every cooling process causes the temperature of the affected entity to drop continuously, this presupposition is satisfied.

It seems reasonable to assume that the same presupposition that is found in *for*-adverbials is also found in pseudopartitives. The intuition here is that a *for*-adverbial construction like *run for three hours* has essentially the same semantics as the corresponding pseudopartitive construction *three hours of running*, and that it gives rise to the same presupposition. In substance-denoting pseudopartitives, I assume that the dimension parameter is set to the appropriate measure function. For example, in *thirty liters of water*, this measure function is volume, and the resulting presupposition is plausibly satisfied:

(36) thirty liters of water

Satisfied presupposition:
$$SR_{volume, \varepsilon(\lambda d[liters(d)=30])}(\lambda x.water(x)) \Leftrightarrow \forall x[water(x) \to x \in {}^*\lambda y \begin{pmatrix} water(y) \land \\ \varepsilon(\lambda d[liters(d)=30])(volume(y)) \end{pmatrix}]$$

(Every amount of water can be divided into one or more parts, each of which is an amount of water whose volume is very small compared with thirty liters.)

Mass nouns like *water* (as we have just seen) and plural count nouns like *books* are acceptable on the plausible assumption that they have approximate divisive reference (Link, 1983; Krifka, 1998): Whenever they apply to an entity, they also apply to all of

its parts (again, leaving aside very small parts, to avoid the minimal parts problem). By contrast, singular count nouns are ruled out on the assumption that they are quantized (that is, they do not apply to any proper parts of any entity to which they apply), as proposed by Krifka (1998). This is shown in the following example:

(37) *five pounds of book

Failing presupposition:
$$SR_{weight, \varepsilon(\lambda d[pounds(d)=5])}(\lambda x.book(x)) \Leftrightarrow \forall x[book(x) \to x \in {}^*\lambda y \begin{pmatrix} book(y) \land \\ \varepsilon(\lambda d[pounds(d)=5])(weight(y)) \end{pmatrix}]$$
 (Every book can be divided into one or more parts, each of which is a book whose weight is very small compared with five pounds.)

The assumption that singular count nouns are quantized rules out the possibility, for example, that a book has any parts besides itself, and therefore *a fortiori* it is not possible for the parts of a book to be very small in weight compared with the weight of the book itself. (In many cases we could even use the weaker assumption that a book does have parts – its spine, for example – but that none of these parts qualifies as a book.)

The problematic example of the snow that fell on Berlin can now be given an account as follows:

(38) five feet of snow

Satisfied presupposition:
$$SR_{height, \epsilon(\lambda d[feet(d)=5])}(\lambda x. snow(x)) \Leftrightarrow \forall x[snow(x) \to x \in {}^*\lambda y \begin{pmatrix} snow(y) \land \\ \epsilon(\lambda d[feet(d)=5])(height(y)) \end{pmatrix}]$$

(Every amount of snow can be divided into one or more parts (horizontal layers), each of which is an amount of snow whose height is very small compared with five feet.)

This presupposition is satisfied in the Berlin scenario despite the fact that the snow on West Berlin and the snow on East Berlin have the same height. The presupposition does not require height to be monotonic, so it is no problem that certain parts of the snow that fell on Berlin have the same height.

The idea behind this account can again be understood via the visual metaphor. A plural or mass entity to which a pseudopartitive applies is divided into strata which are very small as measured in the dimension determined by the pseudopartitive, but may extend arbitrarily in other dimensions. These strata are then required to be in the denotation of the noun. Singular count nouns always fail this test because the individuals in their denotation are atomic, and cannot be further subdivided into strata. Figure 3 illustrates what (38) expresses formally: The measure function *height* is acceptable in the pseudopartitive *five feet of snow* because every amount of snow can be divided into parts (horizontal layers) of snow whose height is very small compared to five feet.

Like previous accounts, we still rule out examples involving temperature:

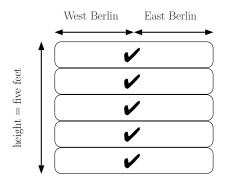


Figure 3: Accepting five feet of snow

(39) thirty degrees Celsius of water

Failing presupposition:
$$SR_{temperature, \varepsilon(\lambda d[Celsius(d)=5])}(\lambda x.water(x)) \Leftrightarrow \forall x[water(x) \to x \in {}^*\lambda y \begin{pmatrix} water(y) \land \\ \varepsilon(\lambda d[Celsius(d)=30])(temperature(y)) \end{pmatrix}]$$
 (Every amount of water can be divided into one or more parts, each of which is an amount of water whose temperature is very low compared with thirty degrees Celsius.)

This presupposition fails as desired since the parts of a given amount of water will generally have the same temperature as the entire amount. As desired, though, the constraint takes the substance noun into account, so that cases like *global warming* are predicted to be acceptable, as desired. As before, I assume that the relevant measure function, which I write *rel[ative] temperature*, maps any warming event to the number of degrees of warming that it causes.

two degrees Celsius of global warming Failing presupposition: $SR_{rel\text{-temperature}, \varepsilon(\lambda d[Celsius(d)=2])}(\lambda e. global.warming(e)) \Leftrightarrow \forall e[global warming(e) \rightarrow e \in {}^*\lambda e' \begin{pmatrix} global warming(e') \land \\ \varepsilon(\lambda d[Celsius(d)=2])(rel\text{-temperature}(e')) \end{pmatrix}]$ (Every amount of global warming can be divided into one or more parts, each of which is an amount of global warming whose relative temperature is very low compared with two degrees Celsius.)

The granularity parameter of stratified reference also makes it natural to account for an observation by Bale (2009): pseudopartitives that refer to very small quantities accept mass nouns but reject count nouns. A pseudopartitive as a whole cannot be predicated of an entity that is too small to be in the denotation of the substance noun. This is shown by the following examples:

- (41) a. Give me 500 grams of apple/apples.
 - b. Give me 100 grams of apple/?apples.

c. Give me one gram of apple/??apples.

The status of examples (41b) and (41c) can be accounted for through the vague predicate ε , on the assumption that the weight of a typical apple qualifies as very small in comparison with 500 grams but not in comparison with one gram, and that 100 grams is a borderline case.

The difference between mass and count uses of *apple* is also shown in the following example, whose acceptable version is attested (source: http://www.fowlerfarms.com/apple_a_day.htm)

(42) Lee said Americans eat the equivalent of one-fifth of a fresh apple each day, or about 19.7 pounds a year. But they should eat five times that much – at least one apple a day, he said. The Cornell researchers found that just 100 grams of **apple/?apples** have the same antioxidant activity as 1,500 milligrams of Vitamin C. (The average apple weighs 150 grams, or about 5 ounces.)

Stratified reference predicts this pattern on the assumption that the weight of the smallest apples is not very small compared to 100 grams, let alone compared to one gram. But the weight of stuff that qualifies as *apple* in the mass sense (i.e. of applesauce) can be very small down to milligrams. A similar pattern can be observed in the following squish (Eytan Zweig, p.c.):

- (43) a. *twelve pounds of twelve-pound weights
 - b. ?twelve pounds of six-pound weights
 - c. twelve pounds of four-pound weights
 - d. twelve pounds of three-pound weights
 - e. twelve pounds of two-pound weights
 - f. twelve pounds of one-pound weights

This account predicts that sentences (43a) through (43f) are only acceptable to the extent that the presupposition in (44) is satisfied. Here I write *n-pound.weights* to abbreviate the pluralized predicates denoted by *twelve-pound weights*, *six-pound weights*, etc. corresponding to the examples in (43).

(44) twelve pounds of *n*-pound weights

Presupposition:
$$SR_{weight, \varepsilon(\lambda d[pounds(d)=12])}(*\lambda x. n\text{-pound.weights}(x)) \Leftrightarrow \forall x[n\text{-pound weights}(x) \to x \in *\lambda y \begin{pmatrix} n\text{-pound weights}(y) \land \\ \varepsilon(\lambda d[pounds(d)=12])(\text{weight}(y)) \end{pmatrix}]$$
 (Every sum of one or more $n\text{-pound}$ weights can be divided into one or more parts, each of which is a sum of one or more $n\text{-pound}$ weight is very small compared with twelve pounds.)

In other words, the pseudopartitives in (43) presuppose that every sum consisting of one or more n-pound weights consists of n-pound weights that weigh much less

than twelve pounds. Whether this will turn out to be true, false, or borderline depends on the choice of n.

To summarize, we can characterize the class of admissible measure functions as follows: A pseudopartitive has to satisfy stratified reference, where the dimension parameter is specified by the measure function and the granularity parameter is specified in function of the relevant measure phrase. The constraint on measure functions is also instantiated in *for*-adverbials and other constructions. An event pseudopartitive like *three hours of running* is given the same analysis as a *for*-adverbial like *run for three hours*. This explains why the two constructions also license the same measure functions. The constraint against intensive measure functions like *temperature* observed by Krifka (1998) and Schwarzschild (2006) is subsumed by the same constraint that also prevents telic predicates from combining with *for*-adverbials.

I have exploited one of the defining features of stratified reference, namely that it pushes us towards thinking of unboundedness as relativized to a certain dimension, thematic role, or measure function. The example *five feet of snow* has played the same role as the example *push carts to the store* above. Both examples force us to consider two dimensions at once: height and width in the former case, time and space in the latter case. The insight from Schwarzschild (2006) that the pseudopartitive construction must be checked on horizontal layers of snow rather than on every subregion of snow finds a natural explanation in this framework.

4 Distributivity and Collectivity

Stratified reference not only relates the telic-atelic, intensive-extensive, singular-plural, and count-mass oppositions to each other, but also the collective-distributive opposition. This section shows how stratified reference can be used to formulate meaning postulates that capture distributivity, and that it may also be useful in order to capture a distinction within the class of collective predicates.

As is frequently done, I understand "distributivity", and its converse "collectivity", as properties of predicates. For example, predicates like *smile*, *see*, *run*, *breathe* and so on are distributive; predicates like *be numerous*, *be a good team*, *gather*, *meet*, *disperse*, or *hold hands*, are collective. Distributive predicates give rise to what will be called *distributive entailments* from pluralities to individuals; for example, (45a) entails (45b). Collective predicates lack these entailments, or give rise to them in a different way. I come back to this point below.

- (45) a. John and Mary smiled.
 - b. John smiled and Mary smiled.

It has often been suggested that the distributive-collective opposition can be captured by formulating meaning postulates for distributive predicates (e.g. Scha, 1981), at least as far as lexical (non-complex) predicates are involved (Winter, 2001). Meaning

postulates need to be formulated in somewhat different ways depending on whether or not events are involved. In semantic frameworks that lack events, a meaning postulate for a distributive predicate like *smile* could state that whenever *smile* applies to a plurality of people, it also applies to each of them. In event semantics, a predicate like *smile* does not directly apply to the smilers, but to the associated smiling events instead. This raises the question of how best to formulate distributivity-related meaning postulates in event semantics.

Because of the parameters it provides, I suggest that stratified reference is well-suited for this purpose. Its dimension parameter can be used to access the smilers via the smiling events. More importantly, this parameter can be used to relativize the notion of distributivity to a given argument position. Predicates with multiple argument positions may be distributive on all, some, or none of these positions (Lasersohn, 1988; Landman, 1996). For example, the verb *see* is distributive on both its agent and theme positions, since it follows both from *John and Mary saw Bill* and from *John saw Bill and Sue* that John saw Bill. By contrast, the verb *kill* is distributive only on its theme role but not on its agent role, as the following scenario shows. The two outlaws Bonnie and Clyde were killed by a posse of six police officers, which included Sheriff Jordan. Given this background knowledge, (46a) entails (46b) but does not entail (46c). More generally, whenever a group of people is killed then each of them is killed, but a group of people can kill a person without it being the case that each of them kills that person.

- (46) a. The police officers killed Bonnie and Clyde.
 - b. \Rightarrow The police officers killed Bonnie.
 - c. ⇒ Sheriff Jordan killed Bonnie and Clyde.

I have shown in previous sections how the dimension parameter of stratified reference can be used to model the distinction between temporal and spatial *for*-adverbials, and the different measure functions in pseudopartitives. For the present purpose, I propose to instantiate this parameter with different thematic roles. This is in keeping with the visual metaphor I introduced at the beginning of this paper, where a thematic role is just another dimension of the abstract space in which events live. We can see a (temporally) atelic predicate as a predicate that is distributive with respect to time. Similarly, we can see a distributive predicate as one that has stratified reference on the appropriate thematic position. I propose to capture lexical distributivity in event semantics by meaning postulates such as the following:

(47) Meaning postulate: *smile* is distributive on its agent position

$$SR_{agent,Atom}(\lambda e.smile(e)) \Leftrightarrow$$
 $\forall e[smile(e) \rightarrow e \in {}^*\lambda e' \begin{pmatrix} smile(e') \land \\ Atom(agent(e')) \end{pmatrix}]$

(Every smiling event can be divided into one or more parts, each of which is a smiling event whose agent is atomic.)

This meaning postulate can be read as follows. Whenever there is a smiling event e,

then that event consists of one or more smiling events e' whose agents are mereological atoms (that is, they have no proper parts). For example, if e is an event in which John and Bill smile, then that event consists of two smiling events whose agents are atoms. From this, and from the assumption that John and Bill are atoms, it follows that John smiled and that Bill smiled.

We can capture the difference between the agent and theme role of *kill* by adopting a meaning postulate analogous to (47) only for the theme position of that verb and by refraining from adopting it for the agent position. In other words, distributivity on a given argument position is stratified reference on the dimension specified by the thematic role of that argument position.

Collective predicates, such as *(be)* numerous, *(be a)* couple, gather, meet, or hold hands, do not satisfy stratified reference on the thematic role of their subjects, because none of the atomic parts of these subjects participate in events that satisfy these predicates. For example, if John and Mary are a couple then it does not follow that John is a couple. This can be modeled by refraining from adopting distributivity meaning postulates for these predicates, or by adopting their negation.

However, it may still be useful to apply a modified form of such meaning postulates to a subset of collective predicates. The view of distributivity as stratified reference suggests that not only the dimension parameter but also the granularity parameter should have a role to play. This is indeed the case: it can be used to specify "how distributive" a given predicate is on a given argument position, for example whether it distributes down to atoms or only to small but nonatomic entities.

Take for example the case of gather. Although this predicate is collective and not distributive, it still licenses entailments from larger to smaller numbers in ways that are reminiscent of a distributive predicate. For example, if A, B, and C gather, this entails that A and B gather, that A and C gather, etc. More generally, the predicate gather licenses entailments from pluralities of cardinality n to subpluralities of size n-1, n-2, and so on, down to 2. In terms of Winter (2001), gather is a 2-bounded downward closed predicate, while smile and similar distributive predicates are 1-bounded downward closed. By contrast, other collective predicates, such as be numerous or be a good team, are not even 2-bounded downward closed, since two people do not count as being numerous and since two people in a good team do not typically form a team by themselves (let alone a good one). These differences can be modeled by the granularity parameter of stratified reference. For example, gather satisfies the following meaning postulate (I use |x| for the number of atomic parts of x):

(48) Meaning postulate for gather

$$SR_{agent,\lambda x.|x|=2}(\lambda e.gather(e)) \Leftrightarrow$$
 $\forall e[gather(e) \rightarrow e \in {}^*\lambda e' \begin{pmatrix} gather(e') \land \\ |agent(e')| = 2 \end{pmatrix}]$

(Every gathering event can be divided into one or more parts, each of which is a gathering event whose agent is two people.)

The same kind of meaning postulates also applies to some other collective predicates like *meet* and *hold hands*. Yet other collective predicates, like *be numerous*, do not satisfy a corresponding meaning postulate.

Kuhn (2014), building on (Champollion, 2010, ch. 9), suggests that this distinction within the class of collective predicates might be linguistically relevant in connection with the behavior of the determiner all. As has often been observed, all rejects certain collective predicates such as be numerous, be a good team, be a group of five, but is compatible with others, such as gather, meet, be similar, and hold hands (e.g. Kroch, 1974; Dowty, 1987; Moltmann, 1997). With respect to the first class, all patterns with distributive determiners such as each, a fact that justifies treating all as a distributive determiner, as shown in (49). With respect to the second class, however, all and each come apart, as shown in (50):

- (49) a. *All the students who came to the rally are numerous.
 - b. *Each of the students who came to the rally is numerous.
- (50) a. All the students { met / gathered in the hallway / held hands }.
 - b. *Each of the students { met / gathered in the hallway / held hands }.

Stratified reference helps capture the relevant semantic distinction involved between the two kinds of collective predicates illustrated here. Distributive predicates have stratified reference down to atoms. Collective predicates like *gather*, *meet*, *hold hands*, and *be similar* are "almost distributive" in that they distribute down to entities of cardinality 2. Other collective predicates predicates like *be numerous*, *be a group of five*, or *be the team that won the tournament* are more similarl to quantized predicates, in that they do not distribute down at all.

Suppose now that *each* and *all* impose stratified-reference requirements that are similar to those imposed by *for*-adverbials and pseudopartitives, but that they set their parameters in different ways. The dimension parameter is a thematic role, and the granularity parameter is either set to *Atom* in the case of *each*, or to some low cardinality value in the case of *all*. That is, the two determiners have the entries in (51) and (52) and their presuppositions expand as in (53) and (54). (I write *agent(e) for the sum of the agents of the parts of e.)

- (53) Presupposition of each: $\forall e[\text{VP}(e) \rightarrow e \in {}^*\lambda e' \begin{pmatrix} \text{VP}(e') \land \\ \text{Atom}(\text{agent}(e')) \end{pmatrix}]$ (Every VPing event e consists of one or more VPing events whose agents are atoms.)

(54)Presupposition of all:

$$\forall e[VP(e) \rightarrow e \in {}^*\lambda e' \left(\begin{array}{c} VP(e') \land \\ \varepsilon(\lambda n.n = |\bigoplus NP|)(agent(e')) \end{array} \right)]$$

(Every VPing event e consists of one or more VPing events whose agents are very small in number compared to the number of atomic parts of the NP.)

Given these assumptions, distributive predicates like smile as well as certain collective predicates (e.g. gather, hold hands) will satisfy the presupposition of all (Kuhn, 2014). For example, when a group smiles, every one of its subgroups smiles, as does every one of its members; when a group gathers, every one of its subgroups gathers, though its individual members do not gather. For smile, this is shown in (55). For gather, this is shown in (56).

all the children smiled (55)

Satisfied presupposition:
$$SR_{agent, \epsilon(\lambda n.n=|\bigoplus child|)}(\lambda e.smile(e)) \Leftarrow$$

Satisfied presupposition:
$$SR_{agent, \varepsilon(\lambda n.n=|\bigoplus child|)}(\lambda e.smile(e)) \Leftrightarrow \forall e[smile(e) \rightarrow e \in {}^*\lambda e' \begin{pmatrix} smile(e') \land \\ \varepsilon(\lambda n.n=|\bigoplus child|)({}^*agent(e')) \end{pmatrix}]$$
 (Every smiling event can be divided into one or more parts, each of which is a

smiling event whose agents are very small in number compared with the total number of children.)

(56)all the children gathered

Satisfied presupposition:
$$SR_{agent, \epsilon(\lambda n. n = | \bigoplus child|)}(\lambda e. gather(e)) \Leftarrow$$

Satisfied presupposition:
$$SR_{agent, \varepsilon(\lambda n.n=|\bigoplus child|)}(\lambda e.gather(e)) \Leftrightarrow \forall e[gather(e) \to e \in {}^*\lambda e' \left(\begin{array}{c} gather(e') \land \\ \varepsilon(\lambda n.n=|\bigoplus child|)({}^*agent(e')) \end{array} \right)]$$
 (Every gathering event can be divided into one or more parts, each of which

is a gathering event whose agents are very low in number compared with the total number of children.)

However, those collective predicates that do not have stratified reference down to any level will not satisfy the presupposition of *all*. Here is one example:

*all the ants (in my kitchen) are numerous (57)

Failing presupposition:
$$SR_{\text{participant}, \varepsilon(\lambda n. n = | \bigoplus \text{ant}|)}(\lambda e. \text{numerous}(e)) \Leftrightarrow \forall e[\text{numerous}(e) \rightarrow e \in {}^*\lambda e' \begin{pmatrix} \text{numerous}(e') \land \\ \varepsilon(\lambda n. n = | \bigoplus \text{ant}|)({}^*\text{participant}(e')) \end{pmatrix}]$$

(Every state of being numerous can be divided into one or more parts, each of which is a state of being numerous whose participants are very low in number compared with the total number of ants in my kitchen.)

As for each, only distributive predicates will satisfy its presupposition. Even a collective predicate that has stratified reference down to granularity level 2, like meet, will not satisfy the more stringent granularity-level 1 requirement of *each*:

(58)*each of the children met

Failing presupposition:
$$SR_{agent,Atom}(\lambda e.meet(e)) \Leftrightarrow \forall e[meet(e) \rightarrow e \in {}^*\lambda e' \begin{pmatrix} meet(e') \land \\ Atom(agent(e')) \end{pmatrix}]$$

(Every meeting event can be divided into one or more parts, each of which is a meeting event whose agent is an atom.)

Stratified reference does not require that the predicate in question applies to all parts whose agent is small in the relevant way, only that there be some way of dividing the whole event into such parts. This nonexhaustive way in which stratified reference distributes the predicate was what allowed us to treat cases like *five inches of snow*. In the case of *all*, it predicts that there should be relevant cases where not every small subevent is relevant. As pointed out by Kuhn (2014), this prediction is true:

- (59) a. All the pieces of the puzzle fit together.
 - b. All the boys in the circle held hands.

In scenarios that make these sentences true, the predicates *fit together* and *hold hands* do not necessarily apply to all pairs, but only to pairs of adjacent pieces or boys. This is compatible with these predicates having stratified reference down to granularity level 2.

To summarize this section, *all* is an "almost distributive" determiner: it requires distributivity down to a small granularity level, but not all the way down to atoms. *Gather*-type predicates are "a bit distributive"; *numerous*-type predicates are not. In keeping with the broader picture in Champollion (2010), the telic-atelic opposition can be formally related to the collective-distributive opposition. To explain why *gather* distinguishes between *each* and *all*, I have followed Kuhn (2014) in suggesting that *each* distributes over events with atomic individuals while *all* distributes over events whose agents must be small in number but need not be atomic.

5 Conclusion and Outlook

I have presented a parametrized higher-order property, stratified reference, and used it to characterize a wide range of semantic oppositions: telic vs. atelic, singular vs. count/mass, intensive vs. extensive, collective vs. distributive, and following Kuhn (2014), also numerous-type vs. gather-type. More generally, I have suggested that stratified reference formalizes the bounded vs. unbounded opposition, which I have suggested as a way to subsume these semantic oppositions. Unboundedness corresponds to stratified reference, boundedness corresponds to lack of it. This has led to new answers to old questions, such as why *for*-adverbials reject telic predicates, why pseudopartitives reject singular count nouns and certain measure functions, why *each* and (to some extent) *all* reject collective predicates. I have suggested that all these constructions impose a parametrized but otherwise identical requirement on one of their arguments, and that stratified reference is well suited to capture this requirement.

In this paper, I have focused on characterizing the properties of predicates that typically consist of a single word. As for complex predicates (e.g. *see thirty zebras*), they too can be characterized as telic, atelic, distributive, collective, etc. While stratified reference is useful for this purpose as well, a full account of aspect and distributivity in these cases will need to be complemented by a theory of how a given complex predicate ends up having or not having stratified reference.

For example, distributivity/collectivity opposition also applies to complex predicates such as wear a dress (distributive), share a pizza (collective), and carry the piano upstairs (mixed distributive/collective). Phrasal predicates like wear a dress can acquire distributivity through a silent VP-level distributivity operator whose meaning is a matter of debate. Either it distributes the predicate it modifies over atoms, similar to adverbial each (Link, 1991; Winter, 2001), or it distributes them over salient nonatomic entities (Schwarzschild, 1996). Distributivity operators can be reformulated in ways that are very similar to stratified reference (Champollion, 2014). The difference between atomic and nonatomic distributivity, discussed among other places in Schwarzschild (1996), can be recast as a difference in settings of the granularity parameter of stratified reference, as argued in Champollion (2014). Certain overt modifiers, such as adverbial each and together, can also determine whether the predicate that they modify is understood distributively or collectively.

Stratified reference allows us to think about the effect of *each*, *together* and distributivity operators and about aspectual composition as two sides of the same coin. The question of how complex predicates end up being collective or distributive is analogous to the question of how complex predicates end up being atelic or telic, a process also known as aspectual composition (e.g. Krifka, 1998).

This also means that we can link problems that affect accounts of these processes. For example, certain apparently nonquantized predicates like *eat something*, *eat less than three apples*, and *drink a quantity of wine* empirically pattern with telic predicates, which present a challenge for algebraic accounts of aspect including mine (Zucchi & White, 2001). Similarly, a number of collective predicates that are incompatible with *all*, such as *be a group of less than five*, would be expected to be compatible with *all* under the account I have discussed here (Kuhn, 2014). I am not claiming to have a solution for these problems, only suggesting that if a solution to them in one domain emerges, we may well be able to adapt it to the other domain.

While I have shown that the behavior a large number of constructions can be reduced to one principle (sensitivity to stratified reference), the question arises why this principle exists and why these constructions are sensitive to it. In formal semantics, this is not the kind of question that is typically answered, or perhaps even answerable. There is no agreement on whether it even needs to be answered. On the one hand, for the purposes of comparing formal semantic theories to each other, formal semantics usually pays attention to something similar to Chomskyan explanatory adequacy: "If a number of highly complex and apparently unrelated facts are reducible to a few simple principles, then these principles explain these facts" (von Stechow, 1984) On the other

hand, we need not confine ourselves in this way: "we can seek a level of explanation deeper than explanatory adequacy, asking not only what the properties of language are but also why they are that way" (Chomsky, 2001).

The overarching theme of this paper is the strengthening of semantic relationships and parallels that hold across the nominal and verbal domain. A natural question to ask is whether the common semantics I have suggested goes hand in hand with a common syntax. For example, the basic constituent structure of for-adverbials is generally accepted as something like [V [for [three hours]]]) where V is the label of the verbal projection that the for-adverbial modifies. But there is no consensus on the constituent structure of pseudopartitives. One possibility is that the measure noun of a measure pseudopartitive forms a constituent with the determiner that precedes it, as in [[two pounds] [of tomatoes]] (Akmajian & Lehrer, 1976; Guéron, 1979; Gawron, 2002; Schwarzschild, 2002, 2006). An alternative to this analysis is the right-branching structure [two [pounds [of tomatoes]]] (Stickney, 2008; Chierchia, 2008; Bale, 2009). It is also possible that the two structures correspond to two different readings (Landman, 2004; Rothstein, 2009). It would not be difficult to formulate the semantic account presented here in a way that is consistent with either structure. However, only the first structure is analogous to the for-adverbial and therefore reflects the semantic parallel between pseudopartitives and other distributive constructions.

The theory developed in this article is deeply connected with the algebraic semantic framework developed for the modeling of measuring-out and boundedness in Krifka (1998). Just like that framework, the present theory has potential applications in morphosyntax, where in may help explain how boundedness is marked by semantic case in Finnish (Krifka, 1992; Kiparsky, 1998), by perfective prefixes in Slavic (Filip, 2000), and by accusative adverbials in Korean (Wechsler & Lee, 1996).

I do not know why there should be any constructions in language, let alone so many of them, that are sensitive to stratified reference or to the various properties it captures. To answer this question, it may be worth looking beyond formal semantics for explanations, such as first-language acquisition. Stratified reference may conceivably help first-language learners distinguish the functions of different constructions. For example, learners must distinguish constructions that specify the quantity of a substance or event, such as pseudopartitives, from superficially similar constructions that specify non-quantity-related properties, such as attributive constructions (three-pound strawberries). The latter do not impose stratified reference and are therefore compatible with intensive measure functions, as illustrated by three-degree water (Schwarzschild, 2006). Apart from sometimes misinterpreting the number word in pseudopartitives as referring to cardinality of a relevant set of objects, four-year-olds tend to correctly distinguish pseudopartitives from attributives (Syrett, 2013). Similarly, various studies have suggested that children are sensitive to the atelic-telic opposition as early as three years old, raising the question of how much of it is innately specified (Crain, 2011). If something like the boundedness-unboundedness opposition is among the building blocks of the language faculty, then we might expect that children access it early on, and possibly at the same age.

Another kind of explanation, as well as another avenue for further research, may be found in linguistic theories that study conceptual linguistic knowledge and the mental patterns and representations in which it is organized, such as cognitive semantics (Talmy, 2011) and conceptual semantics (Jackendoff, 1996). The words that introduce stratified reference constraints, such as *for*, *until*, *of*, *each* and *all*, belong to closed-class categories such as prepositions and determiners. Cognitive semantics has found that closed-class categories are highly constrained in the range of conceptual categories they can express (Talmy, 2011). The relevant conceptual category in this case would be boundedness. While cognitive semantics is sometimes seen as opposed to formal semantics, this does not have to be so (Krifka, 1998; Zwarts & Verkuyl, 1994). We can make use of formal semantic techniques such as the ones I have developed here, and assume that expressions are interpreted by elements of conceptual structures rather than entities in the real world. The present system may then be seen as a step towards a model-theoretic characterization of such frameworks.

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