Split semantics for non-monotonic quantifiers in *than*-clauses

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Abstract

Comparatives containing non-monotonic quantifiers within *than*-clauses pose non-trivial challenges for compositional semantics (e.g., *Balloon A is higher than* ***exactly two*** *of the others are*, see, e.g., Schwarzschild 2008). Following Bumford (2017a)'s split treatment for definiteness (see also Brasoveanu 2013), this paper argues for a two-stage semantic derivation for these comparatives: a bottom-up (from local to global) composition followed by a delayed top-down (from global to local) evaluation. More specifically, I propose that the semantic contribution of both these embedded non-monotonic quantifiers and the embedding morpheme *than* is twofold: first, they each introduce a discourse referent (i.e., some individual or degree-related value) during the stage of bottom-up composition; then, they simultaneously impose tests of maximality (and cardinality) during the stage of top-down evaluation. The current account further suggests that basically, the semantics of comparatives is similar to that of cumulative-reading sentences (e.g., *exactly three boys saw exactly five movies*), both involving simultaneous restrictions but no scope-taking among cardinalities or degree-related values. I show that this has profound implications for theories of comparatives.

Keywords: degree semantics, comparatives, *than*-clauses, definite descriptions, definite degree descriptions, quantification, scope, non-monotonic quantifiers, modified numerals, *than*-clause internal quantifiers, dynamic semantics, delayed evaluation.

1 Introduction

This paper aims to account for the interpretation of sentences in (1), i.e., comparatives containing non-monotonic quantifiers in their *than*-clause. Our intuitive interpretation for these sentences is sketched out in (2). Basically, each of these sentences addresses a comparison between Mary's height and the height of some boys, and it also tells us about the cardinality of all those boys who are not as tall as Mary is.

(1) a. Mary is taller than **exactly two boys** are.
b. Mary is taller than **some but not all boys** are.
c. Mary is taller than **between 2 and 4 boys** are.
d. Mary is taller than **an even number of boys** are.

(2) Mary is taller than some boys are, and the total cardinality of all these boys is

$$\begin{cases} \text{exactly 2} \\ \text{above zero and below the total number of all boys} \\ \text{between 2 and 4} \\ \text{an even number} \end{cases}$$

Intriguingly, though we have a clear intuition for the meaning of these sentences, a straightforward compositional account does not seem readily available.

According to the canonical 'A-not-A' approach to the semantics of comparatives (see Seuren 1973, 1984; Gajewski 2008; Schwarzschild 2008), basically, a comparative addresses the existence of some intermediate degree between two measurements: the **matrix subject's measurement** and the **comparative standard**. As shown in (3), here the matrix subject's measurement is expressed in terms of the set of degrees that Mary's height meets or exceeds (i.e., from 0 to 6 feet), and the comparative standard (i.e., the *than*-clause) denotes the set of degrees that John's height meets or exceeds (i.e., from 0 to 5 feet 8 inches). Then, essentially, a comparative relates these two sets of degrees with the use of an existential closure '$\exists$' and a negation operator (i.e., a set complement operator), meaning that the difference set between these two sets is a non-empty set.

(3) Suppose Mary is 6 feet tall, and John is 5 feet 8 inches tall.
Mary is taller than John is.
$\exists d[\ \{d : \text{Mary is } d\text{-tall}\} \cap \overline{\{d : \text{John is } d\text{-tall}\}}]$
There is a degree d s.t. Mary's height meets d but John's height doesn't meet d.

This semantic derivation for comparatives is typically implemented in a purely bottom-up (i.e., from local to global) compositional way. In particular, here the semantics of the *than*-clause is already fully derived before it is further used in the derivation of sentential-level meaning. As shown in (4), the *than*-clause – *than John is* (*tall*) – actually addresses a degree question: *how tall John is* (see Zhang and Ling 2017a, which analyzes a *than*-clause as a fragment answer to its corresponding degree question).[1] In other words, it is based on the measurement of John's height that this comparison under discussion is performed.

(4) Mary is taller than John is.
$[\![\text{than John is}]\!] \rightsquigarrow$ *how tall is John?*

However, when we try to follow the same 'A-not-A' recipe to derive the meaning for (1a), as shown in (5), the derived truth condition is too weak. Under the scenario in (5), our intuition is that (1a) is false, because Mary is actually taller than all the five boys are. Nevertheless, any degree d between 5′2″ and 5′4″ makes (1a) true because Mary is d-tall but exactly two boys are not d-tall. Thus, what the derivation yields is the semantics of *Mary is taller than the two shortest boys are*, not the semantics of (1a) (see Schwarzschild 2008).

[1] Fleisher (2018a), which is in this volume, adopts a similar (but not exactly the same) view.

(5) Suppose Mary is 6 feet tall, and the boys A, B, C, D, E measure 5′, 5′2″, 5′4″, 5′6″, and 5′8″, respectively.

Mary is taller than exactly two boys are. (=(1a))

There is a degree d such that Mary is d-tall but exactly two boys are not d-tall.

A potential solution is to use a maximality operator in the main clause to limit the range of Mary's height (see (6) and the discussion in Gajewski 2008, cf. Heim 2000). (6b) indeed gives the correct truth condition for (1a). Thus, given the five boys in the scenario in (5), for (1a) to be true, Mary's height should fall into the range between 5′2″ and 5′4″.

(6) $\mathsf{Max} \stackrel{\text{def}}{=} \lambda D_{dt}.\iota d[d \in D \wedge \forall d'[d' \in D \rightarrow d' \leq d]]$

a. Mary is taller than John is.

$\mathsf{Max}(\,\{d$: Mary is d-tall$\}\,) \in \{d$: John is not d-tall$\}$.

b. Mary is taller than exactly two boys are.

$\mathsf{Max}(\,\{d$: Mary is d-tall$\}\,) \in \{d$: exactly two boys are not d-tall$\}$.

However, this solution is problematic for at least two reasons. First, in terms of compositionality, it seems not fully motivated: it is unclear where this maximality operator should be located in comparatives, and why there is this asymmetry between the semantics of the two values undergoing comparison. Then, even if the first problem can be overcome (e.g., by motivating the use of two maximality operators for both parts under comparison), an even more crucial and fundamental issue is how to generate this range of values between 5′2″ and 5′4″, or in other words, how to determine the semantics of comparative standard here. As illustrated in (7), when we modify comparative morpheme *-er* by using *less* or adding a numerical differential, surprisingly, the range of values serving as comparative standard seems to co-vary. I.e., the semantic contribution of this *than*-clause – *than exactly two boys are* (*tall*) – seems unfixed.

(7) Suppose the boys A, B, C, D, E measure $5'$, $5'2''$, $5'4''$, $5'6''$, and $5'8''$, respectively.

a. Mary is **taller** than exactly two boys are. (=(1a))
Mary's height is compared with the height of the two shortest boys.
$\rightsquigarrow$ Mary's height is between $5'2''$ and $5'4''$.

b. Mary is **less tall** than exactly two boys are.
Mary's height is compared with the height of the two tallest boys.
$\rightsquigarrow$ Mary's height is between $5'4''$ and $5'6''$.

c. Mary is **between 1 and 3 inches taller** than exactly two boys are.
Mary's height is compared with the height of A and B, or that of B and C, etc.
$\rightsquigarrow$ Mary's height $\in \{5'3'', 5'5'', 5'7'', 5'9''\}$.

This observation is further supported by our intuitive judgments regarding the degree questions in (8). In contrast to felicitous degree questions (8b)–(8d), (8a) – the one corresponding to the *than*-clause of (1a) – sounds degraded and infelicitous: it is unclear which two boys are under discussion. Notice that degree question (8d) is nevertheless felicitous: intuitively, for this question, it is sufficient to choose any two random boys in a relevant set of boys and address their height. However, for (8a), it is not the case that the height of any two random boys suffices to address this question, but somehow it is elusive to determine which two specific boys are relevant. Similarly, (9) shows that those degree questions corresponding to (1b)–(1d) all sound degraded to some extent.

(8) a. ??How tall are exactly two boys?
b. How tall is John?
c. How tall is every boy?
d. How tall are two of the boys?

(9) a. ??How tall are some but not all boys?

b. ??How tall are between 2 and 4 boys?

c. ??How tall are an even number of boys?

By now, the challenge is clearer. For typical comparatives, their *than*-clause addresses its corresponding degree question, and its fixed meaning makes it possible for deriving the sentential meaning in a purely bottom-up compositional way. However, for those comparatives containing non-monotonic quantifiers in their *than*-clause, the semantics of their *than*-clause is not yet fully determined by itself, making a purely bottom-up compositional derivation for sentential meaning impossible. In some sense, it is the rest of the sentence that restricts the actual interpretation of the *than*-clause in these cases (see (7)). Therefore, no matter what approach to the semantics of comparatives is adopted, it is necessary to implement some kind of delayed, top-down (i.e., from global to local) mechanism in the semantic derivation for sentences in (1).

It turns out that these comparatives in (1) are not unique in invoking some delayed, top-down mechanism in semantic derivation. Haddock (1987) observes that under the scenario shown in Figure 1, (10) is a felicitous expression to denote R2. Even though in this context, there are multiple salient, relevant hats and multiple salient, relevant rabbits, (10) means the unique rabbit of the unique rabbit-hat pair such that the former is in the latter. The uniqueness of the rabbit and the hat in (10) can only be evaluated with a delayed, top-down mechanism, i.e., based on the introduction of multiple (interweaving) restrictions (here $\textsf{hat}\ u$, $\textsf{rabbit}\ v$, and $\textsf{in}\ u\ v$), not just based on the introduction of a single restriction (e.g., $\textsf{hat}\ u$).

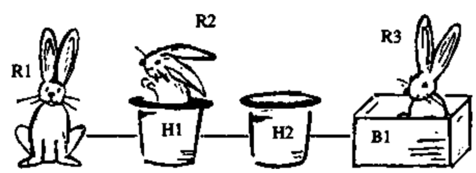


Figure 1: The rabbit in the hat

(10) the rabbit in the hat

$$[\![(10)]\!] = \nu, \text{ where } \nu = \text{ the rabbit in } u$$
$$u = \text{ the hat that } \nu \text{ is in}$$

Thus, to account for Haddock's definite descriptions and related phenomena (e.g., relative readings of superlatives), Bumford (2017a,b) proposes a split analysis for definiteness. Essentially, the semantics of definite determiner *the* comprises two parts: it first builds a set of witnesses that satisfy restrictions, and then it tests this set for uniqueness. Crucially, Bumford (2017a,b) argues that these two operations do not necessarily happen in immediate succession, and the test of uniqueness can be delayed (see also Brasoveanu 2013).

In this paper, I propose a similar split, two-stage semantics for comparatives containing non-monotonic quantifiers in their *than*-clause. More specifically, I propose that the semantic contribution of both the embedded non-monotonic quantifiers and the embedding morpheme *than* is twofold: they first introduce discourse referents (aka **dref**s) in the building of witness sets during the stage of bottom-up composition, and then they impose tests of maximality (and cardinality) during the stage of top-down evaluation. Overall, the semantics of these comparatives is as sketched out in (2) and consists of two parts: a comparison derived via a bottom-up compositional process, and some cardinality requirement imposed as top-down delayed (or post-supposed) evaluation.

In the following, I first introduce the core idea of Bumford (2017a,b)'s split approach to the semantics of definiteness (Section 2). Then I combine this split approach together with an interval-subtraction-based approach to comparatives (Zhang and Ling 2015, 2018) to analyze the core data shown in (1) (Section 3). Section 4 further compares the scopal behavior of comparatives and cumulative-reading sentences and addresses the implications of the current proposal for the theories of comparatives and degree semantics. Section 5 concludes.[2]

[2] Throughout the paper, I try to keep the formalism easy to follow and sufficiently clear. A complete compositional dynamic mechanism that I use here is developed by Charlow (2014) (see also Bumford 2017a,b).

2 Bottom-up composition and delayed evaluation

The core idea underlying Bumford (2017a,b)'s split account for definiteness has two components, both originated in the development of dynamic semantics. Within dynamic semantics, meanings are considered **updates** from an 'input' discourse context (e.g., assignment function) to 'output' discourse context(s), which potentially includes newly introduced drefs.

Thus the first component is canonical in dynamic semantics: during a bottom-up compositional derivation, indefinites introduce drefs in a **non-deterministic** way (i.e., there can be multiple salient instantiations for a variable), and predicates add restrictions on them. As illustrated in (11), this sentence makes an update such that for the set of outputs (e.g., assignment functions), there are drefs for the two variables u and ν that satisfy the restrictions giraffe ν, girl u, and see ν u. Obviously, with this kind of step-by-step bottom-up compositional derivation, the set of outputs becomes increasingly restricted.

(11) A girl saw a giraffe.

u, ν
giraffe ν girl u see ν u

Then the second component is that some kind of restrictions on drefs (most notably definiteness and quantity-related restrictions like cardinality) are not always evaluated immediately. These restrictions are imposed in a top-down, post-supposed, potentially delayed way on outputs (e.g., assignment functions) (see also Brasoveanu 2013; Charlow 2017; Zhang 2018). As a consequence, when this evaluation is delayed, it seems that linguistic expressions carrying this kind of information (e.g., cardinality) take wide scope, though the derivational mechanism involves no QR-style operation (see Charlow 2014;

Bumford 2017b for detailed discussion).

The raison d'être of this second component is primarily motivated by interpretations involving definiteness, i.e., those cases that require drefs that satsify restrictions be unique or (mereologically) maximal so that updates become **deterministic** (i.e., there can only be one unique instantiation). Moreover, in this split, two-stage approach, the effect of delayed evaluation is most evidently observed on cases involving multiple restrictions that interleave and work together to define definiteness.

For example, as illustrated in (12), *the hat* typically denotes the unique hat in a context. Here the indefinite component of *the* (which introduces a dref to be further restricted) and the definite component of *the* (which requires the dref – the one satisfying $\mathsf{hat}\ x$ – be unique) come into force in immediate succession, and thus uniqueness is evaluated on the set of outputs (here ⟨entity, assignment-function⟩ pairs) with the single restriction $\mathsf{hat}\ x$, leading to the absolute interpretation for the uniqueness of *the hat*.

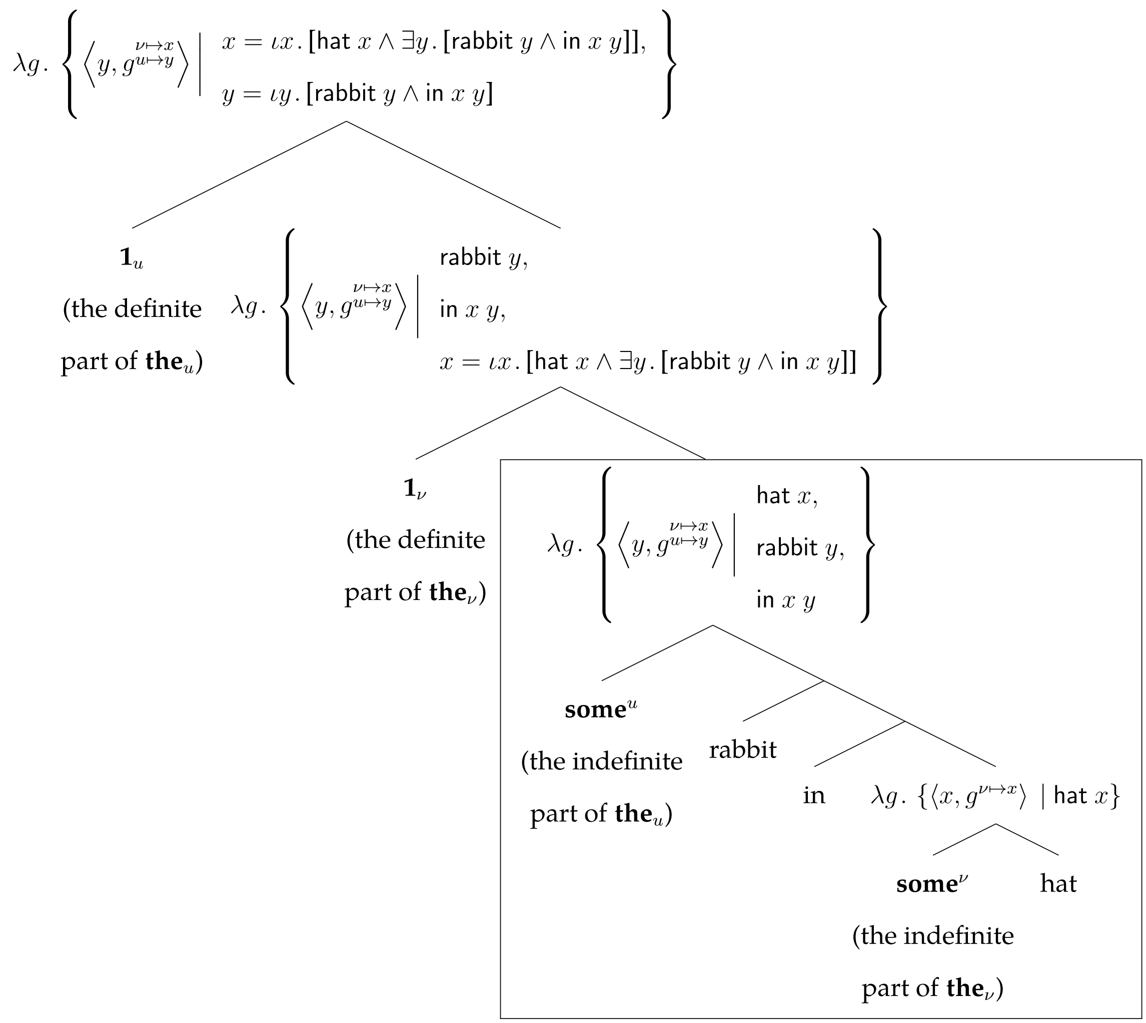
(12) $[\![\text{the hat}]\!] = \iota x \,.\, \mathsf{hat}\ x$

$\lambda g \,.\, \{\langle x, g^{\nu \mapsto x}\rangle \mid x = \iota x \,.\, \mathsf{hat}\ x\}$

- $\mathbf{1}_\nu$ (the definite part of **the**$_\nu$)
- $\lambda g \,.\, \{\langle x, g^{\nu \mapsto x}\rangle \mid \mathsf{hat}\ x\}$
  - **some**$^\nu$ (the indefinite part of **the**$_\nu$)
  - hat

However, the example of Haddock (1987) (see (10) and Figure 1) suggests that the definite component of *the* can come into play at a later stage. As shown in (13), for the two instances of *the* in *the rabbit in the hat*, during bottom-up composition (see the part in the frame in (13)), their indefinite component each introduces a dref. Then it is after the introduction of all relevant restrictions (i.e., hat x, rabbit y, and in x y) that the definite component of the two instances of *the* simultaneously requires drefs x and y (which together satisfy those restrictions) be unique. Thus uniqueness tests are imposed as delayed evaluations, on a set of outputs with more restrictions. Consequently, under our given scenario shown in Figure 1, *the rabbit in the hat* felicitously denotes the unique rabbit of the unique rabbit-hat pair such that the former is in the latter.

(13) the rabbit in the hat



It is worth emphasizing that to derive this **relative** reading of *the rabbit in the hat* in (13), the uniqueness tests of *the* are applied to a set of outputs with multiple restrictions (here $\mathsf{hat}\ x$, $\mathsf{rabbit}\ y$, and $\mathsf{in}\ x\ y$). In contrast, for the **absolute** reading of *the hat* in (12), the uniqueness test is applied to a set of outputs with a single restriction, i.e., $\mathsf{hat}\ x$. Therefore, the derivation of an absolute vs. a relative reading depends solely on the timing of applying top-down evaluations, i.e., whether it is after one or multiple restrictions have been introduced. As pointed out by Bumford (2017a,b), this immediately accounts for the relative reading of superlatives.

As shown in (14), immediately applying top-down evaluations (here $\mathbf{1}_\nu$ and $\mathbf{shortest}_\nu$) on a set of outputs with a single restriction $\mathsf{paper}\ x$ leads to the absolute reading (see (14a)). In contrast, applying top-down evaluations to a set of outputs with multiple restrictions (i.e., $\mathsf{paper}\ x$, $\mathsf{contributor}\ y$, and $\mathsf{write}\ x\ y$) leads to the relative reading (see (14b)).

(14) the contributor who wrote the shortest paper

a. The absolute reading of *the shortest paper*:

$$\lambda g\,.\,\{\langle x, g^{\nu\mapsto x}\rangle \mid x = \iota x \in G\,.\,[\neg\exists z \in G\,.\,\mathsf{shorter}\ x\ z]\},$$
where $G = \{x \mid \mathsf{paper}\ x\}$

$\mathbf{1}_\nu \circ \mathbf{shortest}_\nu$ (the definite part of $\mathbf{the}_\nu$)

$\lambda g\,.\,\{\langle x, g^{\nu\mapsto x}\rangle \mid \mathsf{paper}\ x\}$

$\mathbf{some}^\nu$ (the indefinite part of $\mathbf{the}_\nu$) paper

b. The relative reading of (14):

$$\lambda g\,.\left\{\left\langle y, g^{\substack{\nu\mapsto x\\ u\mapsto y}}\right\rangle \,\middle|\, \begin{array}{l} x = \iota x \in G\,.\,[\neg\exists z \in G\,.\,\textsf{shorter}\ x\ z], \\ y = \iota y\,.\,[\textsf{contributor}\ y \wedge \textsf{write}\ x\ y] \end{array}\right\},$$
$$\text{where } G = \left\{x \,\middle|\, \textsf{paper}\ x, \textsf{contributor}\ y, \textsf{write}\ x\ y\right\}$$

$$\mathbf{1}_u \text{ (the definite part of } \textbf{the}_u\text{)}$$

$$\lambda g\,.\left\{\left\langle y, g^{\substack{\nu\mapsto x\\ u\mapsto y}}\right\rangle \,\middle|\, \begin{array}{l} \textsf{contributor}\ y, \textsf{write}\ x\ y, \\ x = \iota x \in G\,.\,[\neg\exists z \in G\,.\,\textsf{shorter}\ x\ z] \end{array}\right\},$$
$$\text{where } G = \left\{x \,\middle|\, \textsf{paper}\ x, \textsf{contributor}\ y, \textsf{write}\ x\ y\right\}$$

$$\mathbf{1}_\nu \circ \textbf{shortest}_\nu \text{ (the definite part of } \textbf{the}_\nu\text{)}$$

$$\lambda g\,.\left\{\left\langle y, g^{\substack{\nu\mapsto x\\ u\mapsto y}}\right\rangle \,\middle|\, \begin{array}{l} \textsf{paper}\ x, \\ \textsf{contributor}\ y, \\ \textsf{write}\ x\ y \end{array}\right\}$$

For the interpretations in (14), for all the outputs already restricted in a certain way (e.g., outputs with drefs satisfying $\textsf{paper}\ x$, $\textsf{contributor}\ y$, and $\textsf{write}\ x\ y$ in (14b)), the restriction with regard to length ranking, i.e., $\textsf{shortest}\ x$, first filters out those outputs in which x's measurement is not ranked shortest, then the uniqueness requirement from the definite component of *the*$_\nu$ further imposes that this dref, which satisfies all restrictions, be unique.

Similarly, for the example in (15), modified numeral *exactly 7* brings two restrictions: the cardinality requirement is attached to the (mereological) maximality requirement of this modified numeral.[3] Intuitively, it is based on the maximal drefs satisfying the relevant

[3]See Brasoveanu (2013); Charlow (2017); Zhang (2018)'s analysis of modified numerals. The below contrast empirically shows that modified numerals introduce maximal drefs (see Szabolcsi 1997; de Swart 1999; Krifka 1999; Umbach 2005; Charlow 2014):

(i) a. Four babies cried. (✓ Perhaps there were other babies crying, but I was unsure.)
b. At least four babies cried. (# Perhaps there were other babies crying, but I was unsure.)

restrictions (i.e., the drefs denoting the total sum of papers published by some professor) that the cardinality test can be performed.

(15) the professor who published exactly 7 papers

When there are multiple instances of *exactly*, as illustrated by the cumulative-reading sentence in (16), the cardinality requirements brought by them are applied as delayed evaluations simultaneously, on two drefs that both need *a priori* to be maximal. Therefore, (16) means that maximal plural drefs X and Y are such that: their atomic members x and y satisfy restrictions $\textsf{movie}\ y$, $\textsf{boy}\ x$, and $\textsf{see}\ y\ x$, and the cardinality of X equals 3, while the cardinality of Y equals 5 (see also Brasoveanu 2013 for details).

(16) Exactly three boys saw exactly five movies. Brasoveanu (2013)

To sum up, with Bumford (2017a,b)'s split, two-stage mechanism, restricting requirements of definiteness are applied as potentially delayed top-down evaluations, at a stage when outputs have got (less or more) restricted during bottom-up composition.

3 Proposal

Based on Bumford (2017a,b)'s split approach to definiteness and cardinality, here I analyze the semantics of comparatives as a relation among three definite degree-related descriptions and propose a two-stage derivation for those comparatives containing non-monotonic quantifiers in their *than*-clause.

3.1 The semantics of comparatives: a relation among three definite degree-related descriptions

First I introduce Zhang and Ling (2015, 2018)'s interval-subtraction-based framework for the semantics of comparatives. Within this framework, the semantics of comparatives is essentially a relation among three definite degree-related descriptions that mutually restict each other. More specifically, the matrix subject's measurement and the comparative standard can be considered two definite positions on a certain scale, and the third definite description is the difference between them (see the illustration in (17)).

(17) 6 o'clock is 1 hour later than 5 o'clock is.

$$\underbrace{\text{6 o'clock}}_{\text{matrix subject's measurement: a definite position}} - \underbrace{\text{5 o'clock}}_{\text{comparative standard: another definite position}} = \underbrace{\text{1 hour}}_{\text{the differential}}$$

Then Zhang and Ling (2015, 2018) adopt interval subtraction to formally implement the relation among these three definite degree-related descriptions in a generalized way. An interval is a convex set of degrees so that it represents a position in a not-very-precise way.[4] Thus an interval like $\{x \mid a \leq x < b\}$ means a position ranging from a to b and can also be written as $[a, b)$, with a **closed lower bound** '[' and an **open upper bound** ')'.

As shown in Figure 2 and (18), a comparison between two positions can be characterized in terms of interval subtraction: subtracting the interval representing the comparative standard (here $[x_1, x_2]$) from the interval representing the matrix subject's measurement (here $[y_1, y_2]$) results in a third interval, i.e., the differential (here $[y_1 - x_2, y_2 - x_1]$). Obviously, this differential denotes the largest range of possible differences between any two random points (i.e., degrees) in the two intervals representing the two positions.

(18) Interval subtraction: (see Moore 1979)

[4] A convex totally ordered set P is a totally ordered set such that for two random elements a and b in the set, if $a \leq b$, then any element x such that $a \leq x \leq b$ is also in the set P. Evidently, sets such as $\{x \mid x \leq 2 \vee x > 4\}$ are not convex sets.

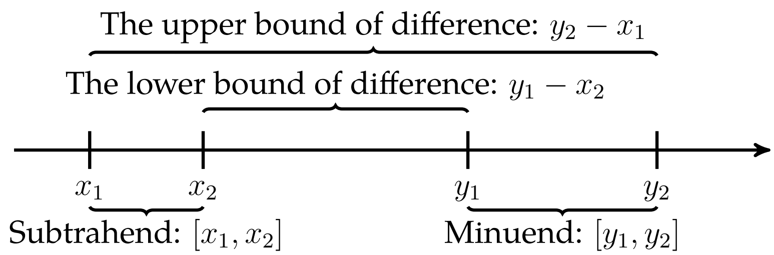


Figure 2: The subtraction between two intervals. Here $[y_1, y_2]$ is the minuend, $[x_1, x_2]$ the subtrahend, and the difference between these two intervals is the largest range of possible differences between any two random points in these two intervals, i.e., $[y_1 - x_2, y_2 - x_1]$.

$$\underbrace{[y_1, y_2]}_{\text{minuend: matrix subject's measurement}} - \underbrace{[x_1, x_2]}_{\text{subtrahend: comparative standard}} = \underbrace{[y_1 - x_2, y_2 - x_1]}_{\text{difference: differential}}$$

a. Example 1: $[5, 9] - [1, 4] = [1, 8]$ (1 and 8 are the minimum and maximum distances between the positions $[5, 9]$ and $[1, 4]$ respectively).

b. Example 2: $(5, +\infty) - [3, 4] = (1, +\infty)$ (This operation can be generalized to intervals with open and/or unbounded ends.)

Within this framework, as illustrated in (19), gradable adjectives relate atomic individuals (of type e) to positions on a relevant scale, and positions are represented as intervals (of type $\langle dt\rangle$). (19) means that the interval representing the height of individual x falls within (i.e., is a subset of) interval I.[5] For example, when measurement uncertainty is taken into consideration, the height of a certain giraffe falls at a position close to, say the value '20 feet' on a scale, with an uncertainty estimate of 0.5 feet. Thus the height of this giraffe, i.e., height(a certain giraffe), is $20' \pm 0.5'$, and this measurement can be considered an interval: $[20' - 0.5', 20' + 0.5']$. Of course, when measurement uncertainty is not considered, the height of a giraffe, say 20 feet, can still be represented as an interval, $[20', 20']$, which is actually a singleton set of degrees in which the lower and upper bounds are equivalent. Thus, any measurement result can be represented as an interval.

[5] It is worth noting that I use '$\subseteq$' to relate an interval and the measurement of an individual (see (19)), while I use '$=$' in interval subtraction (see (18)).

(19) $[\![\text{tall}]\!]_{\langle dt,et\rangle} \stackrel{\text{def}}{=} \lambda I_{\langle dt\rangle}.\lambda x_e.\mathsf{height}(x) \subseteq I$

Obviously, to account for the absolute interpretation of gradable adjectives (see (20)), we only need to assume that there is a silent context-dependent interval I^C_{pos} which ranges from the lower to the upper bound of tallness for a relevant comparison class (see e.g., Bartsch and Vennemann 1972; Cresswell 1976; Stechow 1984; Kennedy 1999). Then when the interval argument of a gradable adjective is specified with numerical values, the semantics of measurement constructions can be derived (see (21)).

(20) $[\![\text{My giraffe is tall}]\!] \Leftrightarrow \mathsf{height}(\mathsf{my\ giraffe}) \subseteq I^C_{\text{pos}}$ Absolute interpretation

(21) $[\![\text{My giraffe is between 19 and 20 feet tall}]\!]$ Measurement construction

$\Leftrightarrow \mathsf{height}(\mathsf{my\ giraffe}) \subseteq [19', 20']$

Comparative morpheme *more*/*-er* is considered the default differential in all comparative sentences. Thus, as shown in (22), it denotes the least specific positive interval, i.e., $(0, +\infty)$.

(22) $[\![\text{more}]\!]_{\langle dt\rangle} \stackrel{\text{def}}{=} (0, +\infty)$ (i.e., a least specific positive interval)

Presupposition: there is a value serving as base item (i.e., standard) for an increase.

Following Zhang and Ling (2017a), I analyze a *than*-clause as a fragment answer to its corresponding degree question. Thus, essentially, a *than*-clause denotes an interval that represents the standard for comparison. More specifically, as illustrated in (23), the semantic derivation of a *than*-clause involves two parts: generating a degree question (i.e., a set of intervals) via a silent lambda operator (Hausser and Zaefferer 1978) and picking out a definite interval via the use of *th*(*-an*) (see also Heim 1985; Beck 2010).[6]

[6]Perhaps it is worth noting that all English words starting with *th* (pronounced as ð) express definiteness: e.g., *the*, *they*, *that*, *then*, *there*, *these*, *thus*, etc. It seems reasonable that *than* contributes definiteness as well.

(23) $[\![$(that tree is taller) than my giraffe is (tall)$]\!]$

a. Generating a degree question: $\lambda I.\textsf{height}(\textsf{my giraffe}) \subseteq I$

b. Deriving its fragment answer: $\iota I[\textsf{height}(\textsf{my giraffe}) \subseteq I]$

The semantics of *th*(*-an*) will be discussed in greater detail in Section 3.3. Basically, I propose that it introduces a dref that is an interval and checks its definiteness. Obviously, the interval $(-\infty, +\infty)$ would be a trivial fragment answer to all degree questions. Thus, for a degree question, a felicitous fragment answer needs to be informative, and the definiteness of *th*(*-an*) needs to be based on informativeness. Therefore, given an individual or a group of individuals, *th*(*-an*) picks out the **narrowest** possible interval that all the relevant measurements fall into (see also Beck 2010). For example, for $[\![$than my giraffe is (tall)$]\!]$, *th*(*-an*) picks out the interval representing the measurement of my giraffe (e.g., a singleton set of degrees if the measurement is very precise); while for $[\![$than every giraffe is (tall)$]\!]$, *th*(*-an*) picks out the interval I such that the measurement of each giraffe falls into I (e.g., the interval ranging from the measurement of the shortest giraffe(s) to that of the tallest giraffe(s)).

I assume that (*th*-)*an* performs interval subtraction. As shown in (24), it takes two intervals representing the subtrahend and the difference as inputs and returns the unique interval representing the minuend. However, to facilitate reading, I will use '$\ominus$' for this operation and write *-an* along with *th*. I.e., in the following, I use *than* to mean *th*(*-an*).

(24) $[\![\text{an}/\ominus]\!]_{\langle dt,\langle dt,dt\rangle\rangle} \stackrel{\text{def}}{=} \lambda I_{\text{standard}}.\lambda I_{\text{differential}}.\iota I[I - I_{\text{standard}} = I_{\text{differential}}]$

Given the formula of interval subtraction in (18), the degree-related values serving as the comparative standard, the matrix subject's measurement, and the differential mutually constrain each other, so that knowing two of them is sufficient to deduce the third one.

Thus, in a typical step-by-step bottom-up semantic derivation for a comparative, as shown in (25), the value of the *than*-clause (i.e., the semantics of the standard) and the value of the differential are first derived (see (25a) and (25b)), and based on these two values, the value associated with the matrix subject – the minuend – is computed (see (25c)). Finally, at the level of matrix clause, the gradable adjective relates the value associated with the matrix subject and the sentence subject (see (25d)).

(25) Mary is taller than every boy is.

LF: Mary is tall $\ominus$ [-er than every boy is (tall)]

a. I_{standard} : $[\![$than every boy is (tall)$]\!]$

$= [\![\text{th-(an)}]\!][\![\lambda I.\text{every boy is } I \text{ tall}]\!]$

$= \iota I[\forall x[\mathsf{boy}(x) \rightarrow \mathsf{height}(x) \subseteq I]]$

(Roughly speaking, this is equivalent to $[\mathsf{height(shortest\ boys)}, \mathsf{height(tallest\ boys)}]$.)

b. $I_{\text{differential}}$: $[\![\text{-er}]\!] = (0, +\infty)$

c. $[\![\ominus\text{[-er than every boy is tall]}]\!]$

$= \iota I[I - I_{\text{standard}} = I_{\text{differential}}]$

$= \iota I[I - \iota I[\forall x[\mathsf{boy}(x) \rightarrow \mathsf{height}(x) \subseteq I]] = (0, +\infty)]$

d. $[\![\text{Mary is tall } \ominus \text{[-er than every boy is (tall)]}]\!]$

$\Leftrightarrow \mathsf{height(Mary)} \subseteq \iota I[I - \iota I[\forall x[\mathsf{boy}(x) \rightarrow \mathsf{height}(x) \subseteq I]] = (0, +\infty)]$

$\Leftrightarrow \mathsf{height(Mary)} \subseteq \iota I[I - [\mathsf{height(shortest\ boys)}, \mathsf{height(tallest\ boys)}] = (0, +\infty)]$

$\Leftrightarrow \mathsf{height(Mary)} \subseteq (\mathsf{height(tallest\ boys)}, +\infty)$ (see (18))

The last three lines of (25d) are equivalent. Essentially, they all mean that on the scale of height, the difference between the position representing Mary's height and the rough position that each boy's height falls into is a positive value. After simplification (see (18)), this means that Mary's height exceeds the height of the tallest boy(s), and based on the semantics of this sentence, there is no upper bound to limit her height.

3.2 The semantics of *than*-clauses containing plural individuals

To prepare for the semantic analysis presented in Section 3.3, here I address the semantics of *than*-clauses containing plural individuals and their entailment pattern.

I assume that for gradable adjectives like *tall* (see (19)), their entity argument is an atomic individual, i.e., the measurement of height is performed on atomic individuals. Thus, when a *than*-clause contains a plural individual (e.g., X in (27)), I assume a silent distributivity operator Dist (see (26)) to relate the plural individual and the predicate. As shown in (27), a *than*-clause containing a plural individual X denotes, in effect, the interval ranging from the measurement of X's least-Adj (here shortest) atomic member(s) to the measurement of X's most-Adj (here tallest) atomic member(s).

(26) $\mathsf{Dist} \stackrel{\text{def}}{=} \lambda X_e.\lambda P_{\langle et\rangle}.\forall x \sqsubseteq_{\mathsf{atom}} X[P(x)]$

i.e., for each atomic part x in the plural individual X, predicate P holds for x.

(27) $[\![\text{than } X \text{ are Dist (tall)}]\!]$

$= [\![\text{th(-an)}]\!][\![\lambda I.X \text{ are Dist } I \text{ tall}]\!]$

$= \iota I[\forall x \sqsubseteq_{\mathsf{atom}} X[\mathsf{height}(x) \subseteq I]]$

i.e., the most informative interval that the height of each atom of X falls into.

Suppose that X and Y are plural individuals and that Y is part of X. For example, X denotes the group consisting of all the boys, and Y denotes the group consisting of all the blond boys. Then for comparatives containing X or Y in their *than*-clause, (28) shows how the entailment relation of their hosting *than*-clauses follows from the part-whole relation between X and Y.

(28) If Y is part of X (i.e., $Y \sqsubseteq X$),

then $[\![\text{than } Y \text{ are Dist Adj}]\!] \subseteq [\![\text{than } X \text{ are Dist Adj}]\!]$

E.g., $[\mathsf{h(shortest\ blond\ boys)}, \mathsf{h(tallest\ blond\ boys)}] \subseteq [\mathsf{h(shortest\ boys)}, \mathsf{h(tallest\ boys)}]$

Intuitively, since Y is part of X, the most informative interval that the measurement of each atomic part of X falls into is necessarily such that the measurement of each atomic part of Y also falls into. Thus the interval $[\![$than Y are Dist ADJ$]\!]$ should be a subset of the interval $[\![$than X are Dist ADJ$]\!]$, i.e., the former entails the latter (see (28)).

For the current purpose, the entailment pattern of *than*-clauses brought by this kind of part-whole relationship (as shown in (28)) dictates how the definiteness test of a *than*-clause-internal plural individual constrains, in turn, the definiteness test for its embedding *than*-clause (i.e., the interval that the measurement of each atomic part of this plural individual falls into). More specifically, for a mereologically maximal plural individual X, $[\![$than X are Dist ADJ$]\!]$ (i.e., the narrowest interval such that the measurement of atomic members of X falls into) cannot be narrower than $[\![$than Y are Dist ADJ$]\!]$ ($Y \sqsubseteq X$) (i.e., the narrowest interval such that the measurement of atomic members of Y falls into). Therefore, with the restrictions $\mathsf{interval}\ I$ and $\forall x \sqsubseteq_{\mathsf{atom}} X[\mathsf{measurement}(x) \subseteq I]$, to guarantee that we get the largest possible cardinality of X, we need *a priori* to get the mereologically maximal X in the widest possible I. This will be crucial for the analysis presented below.

3.3 The semantics of *Mary is taller than exactly two boys*

Following Charlow (2014) and Bumford (2017a,b), I adopt a compositional dynamic semantics, in which modified numerals (e.g., *exactly*+N) and definite determiners (e.g., *the* or *than*) interact with the dynamics in two ways. Like indefinite determiner *a*, their indefinite component non-deterministically allocates a dref (i.e., discourse referent) to some variable of its input assignment function. Then their definite component tests the **definiteness** of this dref across its output assignment functions. For modified numerals, requirements of cardinality are attached to the tests of definiteness. Crucially, the tests of definiteness (and cardinality) can be imposed at a later stage, as delayed evaluations.

(29) and (30) show how the indefinite and definite components of *the* work in deriving the semantics of definite singular and plural individuals. The indefinite component **some**$^{\nu}$ combines with the restrictor *boy*/*boys* to produce a dynamic indefinite update. Given an input assignment, it returns a set of ⟨output-denotation-corresponding-to-the-constituent, output-assignment⟩ pairs (here ⟨individual, output-assignment⟩ pairs), one for each boy or each plural individual made of boys (i.e., each sum of boys).

In (29), the set of outputs is tested for uniqueness. Obviously, since there is one output for every boy in the domain, the test $\mathbf{1}_{\nu}$ fails unless the domain contains only one unique salient boy. When the test does not fail, the description denotes the determinate update that assigns the unique boy to ν.

(29) The definiteness of a singular individual means it is **unique**.

The meaning of *the boy*:

$$\lambda g\,.\begin{cases} G & \textbf{if } |G_{\nu}| = 1, \text{ where } G = \{\langle x, g^{\nu\mapsto x}\rangle \mid \mathsf{boy}\ x\} \\ & \qquad\qquad G_{\nu} = \{g'(\nu) \mid \exists\beta\,.\,\langle\beta, g'\rangle \in G\} \\ \emptyset & \textbf{otherwise} \end{cases}$$

$$\mathbf{1}_{\nu} \quad \lambda g\,.\,\{\langle x, g^{\nu\mapsto x}\rangle \mid \mathsf{boy}\ x\}$$

$$\textbf{some}^{\nu} \quad \text{boy}$$

Similarly, in (30), the set of outputs is tested for maximality. The maximality operator $\mathbf{M}_{\nu}$ filters out the outputs in which ν is not assigned the maximal sum of boys. Thus ⟦the boys⟧ means $\lambda g\,.\,\big\{\langle X, g^{\nu\mapsto X}\rangle \mid X = \Sigma\ \mathsf{boy}\big\}$, i.e., the determinate update in which ν is assigned the largest sum of boys.

(30) The definiteness of a plural individual means it is **mereologically maximal**.

The meaning of *the boys*:

$$\mathbf{M}_{\nu} \stackrel{\text{def}}{=} \lambda m \lambda g \,.\, \{\langle \alpha, h \rangle \in m(g) \mid \neg \exists \langle \beta, h' \rangle \in m(g) \,.\, h(\nu) \sqsubset h'(\nu)\}$$ [7]

$$\lambda g \,.\, \left\{ \left\langle X, g^{\nu \mapsto X} \right\rangle \mid \textsf{boys}\, X, \neg \exists Y \,.\, [\textsf{boys}\, Y \wedge X \sqsubset Y] \right\}$$

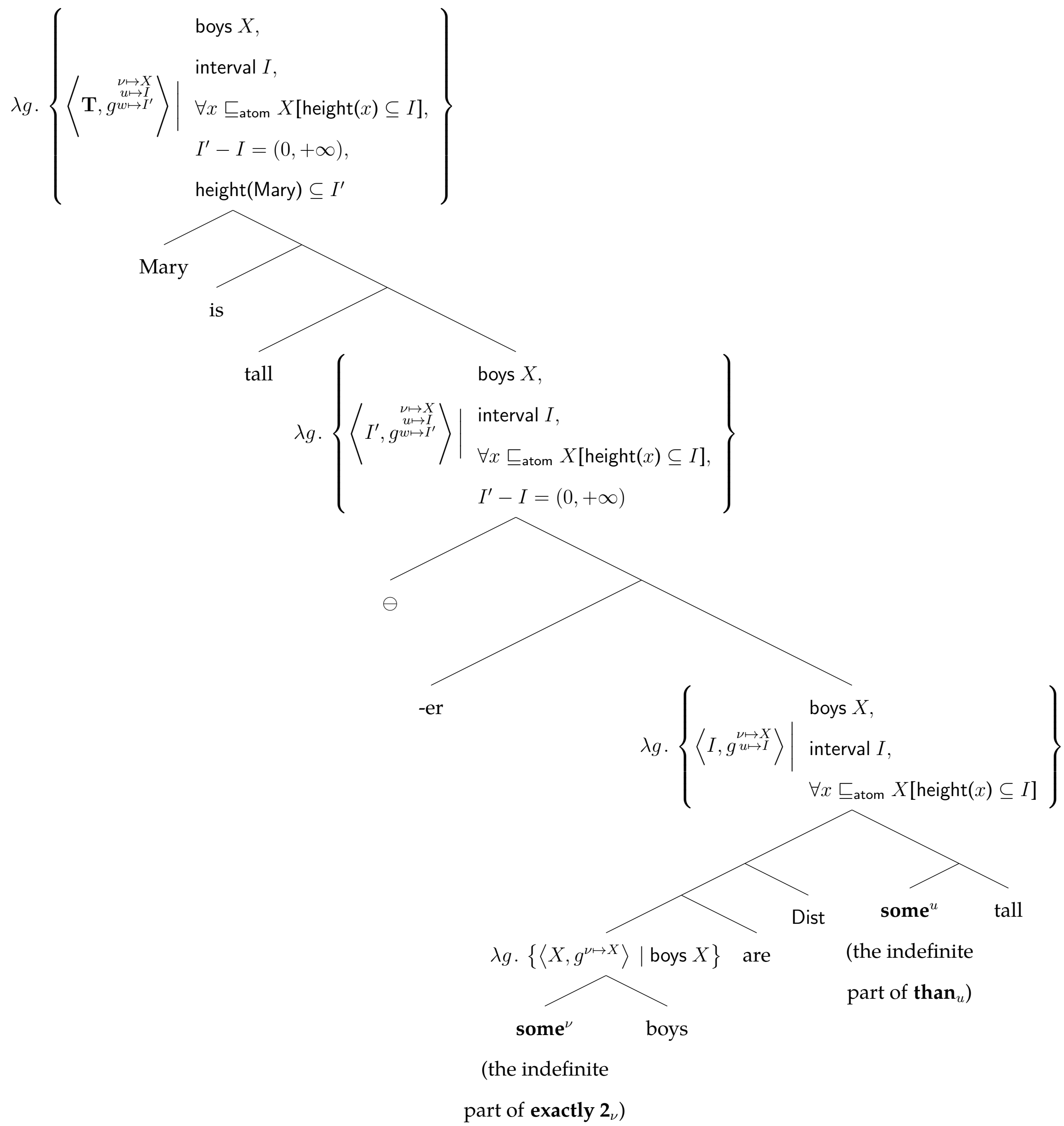
$$\mathbf{M}_{\nu} \qquad \lambda g \,.\, \left\{ \left\langle X, g^{\nu \mapsto X} \right\rangle \mid \textsf{boys}\, X \right\}$$

$$\textbf{some}^{\nu} \qquad \text{boys}$$

Now with the use of this split mechanism, (31) shows the step-by-step bottom-up composition of a comparative:

(31) ⟦Mary is taller than exactly two boys are⟧ – bottom-up composition:

[7] The type of $\mathbf{M}_{\nu}$ is $(g \to \{\langle \alpha, g \rangle\}) \to (g \to \{\langle \alpha, g \rangle\})$. Here g means the type for assignment functions, and $\{\langle \alpha, g \rangle\}$ means the type for a set of $\langle \alpha,$ assignment-function$\rangle$ pairs. The usual notation for types $\langle \alpha, \beta \rangle$ is written as $\alpha \to \beta$.



In (31), **some**$^\nu$ combines with the restrictor *boys* to produce a dynamic indefinite update. Then within the *than*-clause, I assume that the indefinite component of *than*, **some**^{u}, also produces a dynamic indefinite update, returning a set of ⟨interval, output-assignment⟩ pairs, one for each interval. With the help of Dist, gradable adjective *tall* relates each atomic member of plural individual ν with interval u so that for the outputs, there are restrictions boys X, interval I, and $\forall x \sqsubseteq_{\text{atom}} X[\text{height}(X) \subseteq I]$. Further restrictions are introduced at the matrix level: Mary's height is a subset of interval I' such that the difference between I'

and I is $(0, +\infty)$. Thus, by the end of this bottom-up compositional derivation, we obtain a set of $\langle$truth-value, output-assignment$\rangle$ pairs, and the sentence is true when eventually, there exist assignments satisfying all the restrictions.

(32) is the definition of a maximality operator for intervals. Essentially, it filters out the outputs in which their interval variable u is not assigned the widest possible interval. Thus, the application of this $\mathbf{MaxI}_u$ yields the determinate update that assigns u the widest possible interval.

(32) The maximization of an interval:

$$\mathbf{MaxI}_u \stackrel{\text{def}}{=} \lambda m \lambda g.\ \{\langle \alpha, h\rangle \in m(g) \mid \neg\exists \langle \beta, h'\rangle \in m(g).\, h(u) \subset h'(u)\}$$

(33) checks the cardinality of atomic members in a sum.

(33) Cardinality requirement:

$$\mathbf{2}_\nu \stackrel{\text{def}}{=} \lambda m g.\ \begin{cases} G & \textbf{if } |\mathsf{atoms}(\Sigma G_\nu)| = 2, \text{ where } G = m\ g \\ & \qquad\qquad\qquad\qquad G_\nu = \{g'(\nu) \mid \exists \beta.\, \langle \beta, g'\rangle \in G\} \\ \emptyset & \textbf{otherwise} \end{cases}$$

In the following, I assume that height(Mary) is a singleton set of degrees, in which both the upper and lower bounds are written as precise-height(Mary). With the use of interval maximality operator $\mathbf{MaxI}_u$, (mereological) maximality operator $\mathbf{M}_\nu$, and cardinality requirement $\mathbf{2}_\nu$, (34) shows how the tests of definiteness (and cardinality) brought by *than* and *exactly two* are applied as delayed, top-down evaluations:

(34) ⟦Mary is taller than exactly two boys are⟧ – delayed top-down evaluation:

$$\lambda g\,.\left\{\left\langle \mathbf{T}, g^{\substack{\nu\mapsto X\\ u\mapsto I\\ w\mapsto I'}}\right\rangle \middle| \begin{array}{l} I' = [\textsf{precise-height(Mary)}, \textsf{precise-height(Mary)}] \\ I = \iota I\,.\,[I = (-\infty, \textsf{precise-height(Mary)}) \wedge \\ \exists X\,.\,[\textsf{boys}\ X \wedge \forall x \sqsubseteq_{\textsf{atom}} X[\textsf{height}(x) \subseteq I]]] \\ X = \Sigma X\,.\,\textsf{boys}\ X \wedge \forall x \sqsubseteq_{\textsf{atom}} X[\textsf{height}(x) \subseteq I] \end{array}\right\}, \textbf{if } |X| = 2$$

$\mathbf{2}_\nu \circ \mathbf{M}_\nu$ (the definite part of **exactly** $\mathbf{2}_\nu$)

$$\lambda g\,.\left\{\left\langle \mathbf{T}, g^{\substack{\nu\mapsto X\\ u\mapsto I\\ w\mapsto I'}}\right\rangle \middle| \begin{array}{l} \textsf{boys}\ X, \\ \forall x \sqsubseteq_{\textsf{atom}} X[\textsf{height}(x) \subseteq I], \\ I' = [\textsf{precise-height(Mary)}, \textsf{precise-height(Mary)}] \\ I = \iota I\,.\,[I = (-\infty, \textsf{precise-height(Mary)}) \wedge \\ \exists X\,.\,[\textsf{boys}\ X \wedge \forall x \sqsubseteq_{\textsf{atom}} X[\textsf{height}(x) \subseteq I]]] \end{array}\right\}$$

$\mathbf{MaxI}_u$ (the definite part of $\mathbf{than}_u$)

$$\lambda g\,.\left\{\left\langle \mathbf{T}, g^{\substack{\nu\mapsto X\\ u\mapsto I\\ w\mapsto I'}}\right\rangle \middle| \begin{array}{l} \textsf{boys}\ X, \\ \textsf{interval}\ I, \\ \forall x \sqsubseteq_{\textsf{atom}} X[\textsf{height}(x) \subseteq I], \\ I' - I = (0, +\infty), \\ \textsf{height(Mary)} \subseteq I' \end{array}\right\}$$

As discussed earlier in Section 3.2, to guarantee that the cardinality of X (which satisfies all the relevant restrictions along with I and I') we get is the largest possible value, we need *a priori* obtain the mereologically maximal X in the widest possible I. Therefore, among the tests imposed by $\mathbf{MaxI}_u$, $\mathbf{M}_\nu$, and $\mathbf{2}_\nu$, the test of interval maximality $\mathbf{MaxI}_u$ first applies to outputs and rules out all those outputs in which the interval dref assigned to u is not maximally wide, thus yielding the determinate update such that w is assigned the unique interval I' that is equal to $[\textsf{precise-height(Mary)}, \textsf{precise-height(Mary)}]$, and u is assigned the unique interval I that is equal to $(-\infty, \textsf{precise-height(Mary)})$ (and I also needs to satisfy the restriction $\exists X\,.\,[\textsf{boys}\ X \wedge \forall x \sqsubseteq_{\textsf{atom}} X[\textsf{height}(x) \subseteq I]]$).

Then the test of $\mathbf{M}_\nu$ filters out all those outputs in which the sum of boys is not maximally large, yielding the determinate update such that ν is assigned the largest sum of boys X which satisfies the restriction $\forall x \sqsubseteq_{\text{atom}} X[\text{height}(x) \subseteq (-\infty, \text{precise-height(Mary)})]$.

Finally, the cardinality restriction $\mathbf{2}_\nu$ checks whether the cardinality of atomic members in this largest boy-sum X is equal to 2. Obviously, our intuitive interpretation for this sentences is thus derived: Mary is taller than some boys are, and the total cardinality of these boys is equal to 2 (see (2)).

In some sense, all these three tests $\mathbf{MaxI}_u$, $\mathbf{M}_\nu$, and $\mathbf{2}_\nu$ are fundamentally due to the embedded modified numeral *exactly two*. The cardinality restriction cascades down so that the dref assigned to ν is required to be the maximal plural individual and the dref assigned to u the widest possible interval.

Previously, in Section 3.1, the definiteness of ⟦th(-an)⟧ is considered based on informativeness, which essentially means that given a (plural) individual (say Y), it returns the narrowest possible interval for Y such that the measurement of each atomic part of Y falls into it. However, here, when there are no given individuals at hand, during the bottom-up composition (see (31)), a dynamic indefinite update for the interval variable u outputs many interval drefs, each of which can potentially be the narrowest possible interval for some plural individual. Then as shown in (34), it is with the use of $\mathbf{MaxI}_u$ that the widest interval is picked out among this set of potentially narrowest possible intervals. This is actually not counter-intuitive. Imagine an extreme case: Mary is taller than exactly two boys are, and the height of one of these two boys is just slightly below Mary's height, while the height of the other boy is a very low value. Obviously, the narrowest possible interval including their height is $(-\infty, \text{precise-height(Mary)})$,[8] i.e., the widest one among the set of potentially narrowest possible intervals. It is exactly for the sake of the taller boy in this extreme case that the test of **MaxI** is necessary. Without the use of $\mathbf{MaxI}_u$ in (34), I' can be any interval including height(Mary). For example, I' can be $[\text{precise-height(Mary)} - 2'', \text{precise-height(Mary)} + 2'']$, and then I is $(-\infty, \text{precise-height(Mary)} - 2'')$. Consequently,

[8] A negative value (e.g., -5 feet) for height is physically impossible, but not semantically impossible.

if a boy's height falls into $[\text{precise-height(Mary)} - 2'', \text{precise-height(Mary)})$, he would be overlooked during the tests of $\mathbf{M}_\nu$ and $\mathbf{2}_\nu$, because he is not even considered shorter than Mary.

It is evident that the semantics of the other three sentences in (1) can be accounted for in the same way, with a split, two-stage derivation and the application of three tests (i.e., $\mathbf{MaxI}_u$, $\mathbf{M}_\nu$, and a specific cardinality restriction) as delayed evaluations. (35) sketches out the cardinality restrictions specific to (1b)–(1d).

(35) Sketches of cardinality requirements for (1b)–(1d):

(Here Z is the sum of all boys, and X is the largest sum of boys such that $\forall x \sqsubseteq_{\text{atom}} X[\text{height}(x) \subseteq (-\infty, \text{precise-height(Mary)})]$.)

a. **some-but-not-all**$_\nu$: $0 < |X| < |Z|$.

b. **between-2-and-4**$_\nu$: $|X| \in [2, 4]$.

c. **an-even-number**$_\nu$: $|X|$ modulo $2 = 0$.

Overall, for all these sentences in (1), the bottom-up derivation addresses a comparison between Mary's height and the height of some boys, while the top-down evaluation addresses the cardinality of all those boys who are not as tall as Mary is. Since the tests of $\mathbf{MaxI}_u$, $\mathbf{M}_\nu$, and the relevant cardinality restrictions are all delayed evaluations within this two-stage derivation, apparently, it seems that non-monotonic quantifiers embedded within *than*-clauses take wide scope. However, delayed evaluations do not involve any QR-style operations, and thus the current account does not suffer any QR-related island issues (see Larson 1988; Gajewski 2008; van Rooij 2008; Schwarzschild 2008).[9]

[9] To argue that QR-style operations are not available for *than*-clause-internal quantifiers to take scope, Larson (1988) shows that covert or overt *wh*-movement is impossible in these cases.

(i) a. *[Which boy]$_i$ is Mary taller than t_i is?
b. *I am wondering who is taller than who else is.

3.4 Extensions

3.4.1 The effects of varying differentials

The current account can be easily extended to account for all the sentences in (7), and after all, the semantic contribution of *than exactly two boys are* (*tall*) stays constant across all these cases. Essentially, what varies across these sentences is the value of differentials. As a consequence, the application of the interval maximality operator $\mathbf{MaxI}_u$ yields different unique widest intervals.

For (7b) (repeated here as (37)), I follow Zhang and Ling (2017b, 2018) and analyze *less* as the least specific negative interval that serves as the default differential in *less-than* comparatives (see (36)). Thus, as shown in (37), in this case, the widest interval I satisfying all relevant restrictions is $(\textsf{precise-height(Mary)}, +\infty)$. Then for boy-sums whose atomic member's height falls into this interval, the cardinality of the maximal boy-sum is 2. In other words, this sentence means that Mary is less tall than some boys are, and the cardinality of all those boys taller than Mary is 2.

(36) $[\![\text{less}]\!] \stackrel{\text{def}}{=} (-\infty, 0)$ (i.e., a least specific negative interval)
Presupposition: there is a value serving as base item (i.e., standard) for a decrease.

(37) [[Mary is **less tall** than exactly two boys are]] – delayed top-down evaluation:

$$\lambda g\,.\left\{\left\langle \mathbf{T}, g^{\substack{\nu\mapsto X\\ u\mapsto I\\ w\mapsto I'}}\right\rangle \;\middle|\; \begin{array}{l} I' = [\textsf{precise-height(Mary)}, \textsf{precise-height(Mary)}] \\ I = \iota I\,.[I = (\textsf{precise-height(Mary)}, +\infty) \wedge \\ \exists X\,.[\textsf{boys } X \wedge \forall x \sqsubseteq_{\textsf{atom}} X[\textsf{height}(x) \subseteq I]]] \\ X = \Sigma X\,.\,\textsf{boys } X \wedge \forall x \sqsubseteq_{\textsf{atom}} X[\textsf{height}(x)\subseteq I]\end{array}\right\}, \textbf{if } |X| = 2$$

$\mathbf{2}_\nu \circ \mathbf{M}_\nu$ (the definite part of **exactly** $\mathbf{2}_\nu$)

$$\lambda g\,.\left\{\left\langle \mathbf{T}, g^{\substack{\nu\mapsto X\\ u\mapsto I\\ w\mapsto I'}}\right\rangle \;\middle|\; \begin{array}{l} \textsf{boys } X, \\ \forall x \sqsubseteq_{\textsf{atom}} X[\textsf{height}(x) \subseteq I], \\ I' = [\textsf{precise-height(Mary)}, \textsf{precise-height(Mary)}] \\ I = \iota I\,.[I = (\textsf{precise-height(Mary)}, +\infty) \wedge \\ \exists X\,.[\textsf{boys } X \wedge \forall x \sqsubseteq_{\textsf{atom}} X[\textsf{height}(x) \subseteq I]]] \end{array}\right\}$$

$\mathbf{MaxI}_u$ (the definite part of **than**$_u$)

$$\lambda g\,.\left\{\left\langle \mathbf{T}, g^{\substack{\nu\mapsto X\\ u\mapsto I\\ w\mapsto I'}}\right\rangle \;\middle|\; \begin{array}{l} \textsf{boys } X, \\ \textsf{interval } I, \\ \forall x \sqsubseteq_{\textsf{atom}} X[\textsf{height}(x) \subseteq I], \\ I' - I = (-\infty, 0), \\ \textsf{height(Mary)} \subseteq I' \end{array}\right\}$$

For (7c) (repeated here as (39)), the explicit numerical differential *between 1 and 3 inches* restricts the default differential $(0, +\infty)$ (see (38)). Thus, as shown in (39), in this case, the widest interval I satisfying all relevant restrictions is $[\textsf{precise-height(Mary)} - 3'', \textsf{precise-height(Mary)} - 1'']$, and then for boy-sums whose atomic member's height falls into this interval, the cardinality of the maximal boy-sum is 2.

(38) $[\![$between 1 and 3 inches …-er$]\!] = [1'', 3''] \cap (0, +\infty) = [1'', 3'']$

(39) $[\![$Mary is **between 1 and 3$''$ taller** than exactly two boys are$]\!]$ – delayed evaluation:

$$\lambda g\,.\left\{\left\langle \mathbf{T}, g^{\substack{\nu\mapsto X\\ u\mapsto I\\ w\mapsto I'}}\right\rangle \middle| \begin{array}{l} I' = [\textsf{precise-height(Mary)}, \textsf{precise-height(Mary)}] \\ I = \iota I\,.\,[I = [\textsf{precise-height(Mary)} - 3'', \textsf{precise-height(Mary)} - 1''] \wedge \\ \exists X\,.\,[\textsf{boys}\ X \wedge \forall x \sqsubseteq_{\textsf{atom}} X[\textsf{height}(x) \subseteq I]]] \\ X = \Sigma X\,.\,\textsf{boys}\ X \wedge \forall x \sqsubseteq_{\textsf{atom}} X[\textsf{height}(x) \subseteq I] \end{array}\right\}, \textbf{if } |X| = 2$$

$\mathbf{2}_\nu \circ \mathbf{M}_\nu$ (the definite part of **exactly** $\mathbf{2}_\nu$)

$$\lambda g\,.\left\{\left\langle \mathbf{T}, g^{\substack{\nu\mapsto X\\ u\mapsto I\\ w\mapsto I'}}\right\rangle \middle| \begin{array}{l} \textsf{boys}\ X, \\ \forall x \sqsubseteq_{\textsf{atom}} X[\textsf{height}(x) \subseteq I], \\ I' = [\textsf{precise-height(Mary)}, \textsf{precise-height(Mary)}] \\ I = \iota I\,.\,[I = [\textsf{precise-height(Mary)} - 3'', \textsf{precise-height(Mary)} - 1''] \wedge \\ \exists X\,.\,[\textsf{boys}\ X \wedge \forall x \sqsubseteq_{\textsf{atom}} X[\textsf{height}(x) \subseteq I]]] \end{array}\right\}$$

$\mathbf{MaxI}_u$ (the definite part of $\mathbf{than}_u$)

$$\lambda g\,.\left\{\left\langle \mathbf{T}, g^{\substack{\nu\mapsto X\\ u\mapsto I\\ w\mapsto I'}}\right\rangle \middle| \begin{array}{l} \textsf{boys}\ X, \\ \textsf{interval}\ I, \\ \forall x \sqsubseteq_{\textsf{atom}} X[\textsf{height}(x) \subseteq I], \\ I' - I = [1'', 3''], \\ \textsf{height(Mary)} \subseteq I' \end{array}\right\}$$

From (37), (39), and my above analysis for (1a), it should become evident that after the indefinite component of *exactly two* introduces a dref (i.e., a plural individual), this dref gets more and more restricted along the derivation. Thus, for these sentences in (7), the difference with regard to the value of differentials leads to different restrictions for interval drefs and eventually different maximal intervals (i.e., $I = (-\infty, \textsf{precise-height(Mary)})$, $I = (\textsf{precise-height(Mary)}, +\infty)$, and $I = [\textsf{precise-height(Mary)} - 3'', \textsf{precise-height(Mary)} - 1'']$, respectively). Then the different maximal intervals further lead to different restrictions for plural individuals and eventually different maximal plural individuals.

Therefore, apparently, under the scenario in (7), it seems that the semantics of the comparative standard is not fixed, and for each sentence, Mary is compared with a different group of boys. In fact, the semantics of *exactly two boys* thus embedded in *than*-clauses is very similar to the semantics of *the hat* in *the rabbit in the hat*: for both *exactly two boys* and *the hat*, due to delayed evaluations, the drefs receive non-local restrictions that are beyond these DPs. Without these non-local restrictions, *the rabbit* would become uninterpretable in Haddock's scenario (see Figure 1, where there are multiple hats), while the actual interpretation of *exactly two boys* cannot be fixed so that the degree question corresponding to *than exactly two boys are* (*tall*) sounds degraded (see (8)).

3.4.2 *Exactly two boys* vs. *few boys*

It is worth noting that under the current account, as shown in (40), even though degree questions containing *exactly two boys* sound degraded (see (40a)), comparatives containing this kind of non-monotonic modified numerals in their *than*-clause are nevertheless good and natural. Overall, the cardinality restriction of *exactly two* applies to outputs with more restrictions (i.e., here not just the restriction $\textsf{boys}\ X$, but also other restrictions related to measurement and comparison) so that eventually, the dref is the maximal plural individual which consists of two boys not as tall as Mary is.

(40) a. #How tall are **exactly two boys**? (=(8a))

b. Mary is taller **than exactly two boys are**.

In contrast, both degree questions and comparatives containing *no boys* or *few boys* are unacceptable (see (41) and (42)). The reason is simple: in my analysis, there is a requirement on the interval dref, i.e., $\exists X\,.\,[\textsf{boys}\ X \wedge \forall x \sqsubseteq_{\textsf{atom}} X[\textsf{height}(x) \subseteq I]]$. While the cardinality restriction of *exactly two* can eventually guarantee the satisfaction of this requirement, *no boys* or *few boys* cannot guarantee that this requirement be satisfied.[10]

(41) a. *How tall are **no boys**?

b. *Mary is taller **than no boys are**.

(42) a. *How tall are **few boys**?

[10] Notice that *few boys are above 7 feet tall* is a true and felicitous sentence under the scenario that no boys are above 7 feet tall. Thus *few boys* does not guarantee that the above existential requirement be satisfied.

b. *Mary is taller **than few boys are**.

In this sense, unacceptable comparative (42b) patterns with (43a), but not (43b). The contrast between (43a) and (43b) suggests that *few boys* is incompatible with collective predicates, but compatible with distributive predicates (here *lift the piano together*) (see Solt 2007; Zhang 2018). According to Zhang (2018), a collective predicate requires its subject be a group noun, and the very felicity of a group noun, in turn, requires that it be formed from a non-empty set of items, but expressions like *few boys* fail to guarantee this non-emptiness. Thus, the parallelism between (42b) and (43a) suggests that for plural individuals embedded within *than*-clauses (e.g., *exactly two boys*, *few boys*), restrictions like $\forall x \sqsubseteq_{\mathsf{atom}} X[\mathsf{height}(x) \subseteq I]$ (i.e., a predicate relating a plural individual with a certain interval) are similar to collective predicates. In other words, here we consider this interval I a continuous, non-dividable whole (i.e., a convex set of degrees).

(43) a. *Few boys lifted the piano together.

b. Few boys smiled.

c. A few boys lifted the piano together.

Basically, this makes the current account different from many other approaches to the semantics of comparatives, i.e., those so-called 'entanglement' theories (see Fleisher 2016's discussion). For example, according to a typical 'entanglement' theory – the degree-plurality-based approach (see Beck 2014; Dotlačil and Nouwen 2016), a *than*-clause denotes a sum of degree entities, and this sum of degree entities can be distributed over the matrix clause. For example, *Mary is taller than the boys are* means that the height of each of the boys is an atomic degree entity, and Mary's height exceeds each atomic degree entity – it is not the case that Mary's height is compared with a whole interval that each boy's height falls into. This kind of approach brings two predictions. First, the same analysis should work for both (42b) and the core sentence under discussion – *Mary is taller than exactly two boys are*. Second, (42b) should thus be judged good and pattern with acceptable sentences like (43b) (i.e., those with distributive predicates), instead of unacceptable sentences like (43a) (i.e., those with collective predicates). Neither prediction is borne out.

4 Discussion

My proposed account for the semantics of non-monotonic quantifiers embedded in *than*-clauses is crucially based on two ideas. First, comparatives mean a relation among three definite degree-related descriptions that mutually constrain one another. I.e., given the values of two of them, the value of the third is restricted. Second, definiteness is composed of two parts: one that introduces drefs and builds sets of potential witnesses, and the other that tests a set of witnesses for definiteness. Therefore, usually we use the values of the standard (i.e., the semantics of a *than*-clause) and the differential to derive the value for the matrix subject, but for *Mary is taller than exactly two boys are*, we use the value associated with the matrix subject and the differential to compute the value for the standard and furthermore, restrict the plural individual embedded in the *than*-clause.

In terms of scope, the first idea means that comparatives involve no scope interaction among their matrix subject's value, standard, and differential, and the second idea means that for those (quantificational) expressions that impose restrictions as delayed evaluations, they actually have the effects of taking wide scope without causing any island-related issues. Therefore, the only way for *than*-clause-internal quantifiers to take wide scope is exceptional scope-taking (see Charlow 2014), and universal quantifiers and distributivity operators embedded within a *than*-clause are always bounded by scope islands.

In this sense, comparatives (see (45)) are like cumulative-reading sentences (see (44)) in that there is no scope ambiguity.[11] (45) has only one reading: there exists a certain girl such that her height exceeds each boy's height (i.e., she is taller than the tallest boy is); this sentence cannot mean that for each boy, there exists a certain girl such that she is taller than him. Similarly, for the sentences in (46), only the external reading of *different* is available.[12]

(44) Exactly three boys saw exactly five movies between them. Cumulative-reading
$\rightsquigarrow$ The maximal set of boys, the cardinality of which is exactly 3, saw the maximal set of movies, the cardinality of which is exactly 5.

(45) Some girl is taller than every boy is. Unambiguous

(46) a. A different girl is taller than every boy is. Unambiguous
✓ the **external** reading of *different*: it **presupposes** that there is a certain girl x such that x's height exceeds each boy's height, and it **asserts** that there is another girl y ($y \neq x$) such that y's height also exceeds each boy's height.
\# the **internal** reading of *different*: for each boy, there is a girl such that she is taller than him, and there is a one-to-one mapping between boys and girls.
b. A different girl is not taller than every boy is. No 6-way ambiguity

[11] It seems that *few boys saw exactly five movies between them* sounds degraded as a cumulative-reading sentence. This also added to the parallelism between the semantics of *than*-clause-internal quantifiers (see (41b)) and the cumulative reading. See the discussion on *few* in Section 3.4.2 (see also Solt 2007; Zhang 2018).

[12] There is some discrepancy between the judgments I report here in (45) and (46) and those reported in Fleisher (2018b). Fleisher (2018b) claims that the internal reading of *different* (i.e., the '*every* > *different*' reading) is available for (i):

(i) A different boy is exactly six inches taller than every girl is. *every* > *different* – OK (Fleisher 2018b)

Among my informants, many of them claim that the '*every* > *different*' reading is only acceptable when the word *is* embedded in the *than*-clause is deleted, i.e., their judgments suggest that this '*every* > *different*' reading reading might be only available for phrasal comparatives, but not for clausal comparatives.

In addition, for some informants, the '*every* > *different*' reading seems more available for (i) (which contains a numerical differential, *six inches*) than for (46a). Presumably, for (46a) to be true, once there is a one-to-one mapping between boys and girls, there must be a girl taller than everyone else, thus making the '*every* > *different*' reading contradictory for (46a).

To fully settle this issue of judgments, especially with regard to whether the '*every* > *different*' reading is truly available for clausal comparatives, a rigorous large-scale judgment elicitation would be necessary, and I leave this for future research.

Only the external reading of *different* is available:

(i) $\exists > \neg$: There is another girl y, and y is not taller than every boy is.

(ii) $\neg > \exists$: There is no other girl such that she is taller than every boy is.

This parallelism between the scopal behavior of comparatives and cumulative-reading sentences has profound implications for theories of comparatives and degree semantics.

First, this challenges the parallel treatment of degrees and entities (of type e) and the quantification of degrees with the use of '∃' and '∀' (cf. the 'A-not-A' approach (see (3) and Seuren 1973, 1984; Gajewski 2008; Schwarzschild 2008) and the degree-plurality-based approach (see Beck 2014; Dotlačil and Nouwen 2016 and the discussion in Section 3.4.2)). Second, this also challenges the use of a negation operator '¬' to characterize comparison (cf. Klein 1980; Larson 1988; Alrenga and Kennedy 2014).

'∃', '∀', and '¬' all lead to scope interaction and scopal ambiguity. Moreover, with the use of these treatments, the definiteness of degrees, which is essentially similar to the definiteness of cardinalities, is overlooked. After all, the relation among the three definite degree-related descriptions in a comparative (which I analyze as three definite intervals) should just be like the relation among the three numbers in (47). I.e., the definite value '12' minus the definite value '4' is equal to the definite value '8'. After all, cardinalities are real numbers, and intervals, as shown in Section 3.1, are real numbers characterized in a generalized, not necessarily precise way.[13]

(47) 12 minus 4 is equal to 8.

What this is up to is in line with those approaches that treat comparatives along with other cardinality-related phenomena, e.g., those 'larger-than'-based or interval-based theories (e.g., Russell 1905; Cresswell 1976; Stechow 1984; Heim 1985; Rullmann 1995; Schwarzschild

[13]Whether real numbers are semantically primitive or should be constructed from other more primitive items is a totally different issue that I cannot discuss here. For relevant discussion, see Bale (2011) as well as Schwarzschild (2018); Bale (2018), which are in this volume.

and Wilkinson 2002; Heim 2006; Krasikova 2008; Beck 2010, 2011). As Beck (2010) states, 'I want to come out of the calculation of the semantics of the *than* clause holding in my hand *the* degree we will be comparing things to'. In my current account, I simply replace *the* degree with its more generalized version, i.e., *the* interval, and *than*-clauses as well as the matrix subject's measurement and the value for the differential all denote definite descriptions of cardinality-like values. Of course, as I have argued throughout this paper, we do not always hold *the* value serving as the standard beforehand, and sometimes the exact definiteness (e.g., picking out *the* value from a set of cardinality-like values) can only come into play at a later stage.

Finally, it is worth noting that the idea of decomposing an expression of definiteness into an indefinite (or existential) and a definite (or exhaustive) component appeared very early in the history of formal semantics. The distinction between these two components has already been noted by Russell (1905) for definite descriptions. The current account is actually based on a dynamicized version of this idea, which has been recently developed by Brasoveanu (2013); Charlow (2014); Bumford (2017a,b).

A solution in this same spirit for sentences in (1) is also what Gajewski (2008) was after. As shown in (48), Gajewski (2008) proposes to use a non-local exhaustive operator EXH that pragmatically strengthens the sentence meaning and rules out the weaker reading that Mary is taller than the two shortest boys are. A problem for this account is that pragmatically strengthened meanings are usually cancelable, but the exhaustiveness of modified numerals is not.

(48) Mary is taller than exactly two boys are. = (1a)

EXH [$\exists d$[Mary is d-tall and **2** students are not d-tall]] (Gajewski 2008)

$\rightsquigarrow$ Mary cannot be taller than 3 students are.

Thus, by analyzing degree-related values as definite intervals and drawing parallelism between intervals and cardinalities, the current account further extends the application of dynamic semantics into degree semantics. Further theoretical and empirical implications are for future research.[14]

5 Conclusion

With the use of existing, independently motivated mechanisms (i.e., Bumford 2017a,b's split approach to definiteness and Zhang and Ling 2015, 2018's interval-subtraction-based approach to the semantics of comparatives), I have accounted for the semantics of comparatives containing non-monotonic quantifiers (e.g., *exact two boys*) within their *than*-clause. Essentially, the semantic derivation for these sentences undergoes two stages: first, during bottom-up composition, drefs (i.e., a plural individual and an interval on a scale) are introduced and a comparison is established; then, during delayed top-down evaluations, the cardinality restriction of the embedded non-monotonic quantifier is applied to the maximal plural individual in the widest interval. This account explains why the semantics of *than*-clauses containing non-monotonic quantifiers seems unfixed or incomplete *per se*: due to delayed evaluations, plural individuals introduced by non-monotonic quantifiers like *exactly two boys* continue getting restricted from matrix clauses. This account also brings the semantics of comparatives in line with the semantics of other definiteness-related phenomena, including cardinality and cumulative-reading sentences. It seems that after all, the semantics of degrees (or intervals) can be considered a more generalized version of the semantics of cardinality.

[14] There will be many interesting topics for further exploration: e.g., the coordination of cardinalities or degree-related values (see also Bale 2018 in this volume).

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