Briefs

Merge vs. "Lerge:" Problems of Association

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Abstract

It is shown that the proposal of identifying Merge and the Leibnizian addition operator runs up against the obstacle that the latter is associative while the former is not. The confound is attributed to insufficient appreciation of the difference between a calculus for natural language syntax and a calculus of concepts.

Keywords

The proper characterization of Merge has arguably become the most important task of core minimalist theorizing, given that Merge has been singled out as "operation[] that must have evolved if language is to exist at all" (Chomsky, 2021, p. 589). Very recent contributions to this enterprise have been made, among others, by Chomsky (2019, 2020), Chomsky et al. (2019), Epstein et al. (2021), Hornstein (2017), and Svenonius (2021). All of these focus more or less directly on how to fit (variants of) Merge into a concrete grammatical "architecture" (Adger, 2021).

By contrast, Roberts and Watumull (2015) opt for abstraction, and doubly so. In an intriguing note on "Leibnizian Linguistics," they both historicize and mathematicize the debate. In fact, Roberts and Watumull go so far as to identify Merge with the famous philosopher-scientist's addition operator ⊕. Let's have a look at the crucial passage, which relies in part on the discussion by Davis in "The Universal Computer. The Road from Leibniz to Turing" (Roberts & Watumull, 2015, p. 213):

> "[Leibniz] introduced a special new symbol ⊕ to represent the com-
> bining of quite arbitrary pluralities of terms. The idea was some-

thing like the combining of two collections of things into a single collection containing all of the items in either one" (Davis 2012: 14-15). This operation is in essence formally equivalent to the *Merge* function in modern syntactic theory (Chomsky 1995); [...]. Leibniz defined some of the properties of ⊕ – call it *Lerge* – thus:

[(i)] X ⊕ Y is equivalent to Y ⊕ X.

[(ii)] X ⊕ Y = Z signifies that X and Y "compose" or "constitute" Z; this holds for any number of terms.

"Any plurality of terms, as A and B, can be added to compose a single term A ⊕ B." Restricting the plurality to two, this describes Merge exactly: it is a function that takes two arguments, α and β (e.g., lexical items), and from them constructs the set {α, β} (a phrase). (We can also see that ⊕ shares with Merge an elegant symmetry, as [(i)] states.) And according to Leibniz's principle of *the Identity of Indiscernibles*, if Merge and Lerge are formally indiscernible, they are identical: Merge *is* Lerge.

Now, undeniably, a strong case can be made for convergences between the minimalist idea of an elegant computational system underlying human language and Leibniz's ("dream" of a) "calculus ratiocinator" and "characteristica universalis." In that sense indeed "generative grammar has become increasingly Leibnizian" (Roberts & Watumull, 2015, p. 216). However, there are reasons to take issue with the above envisaged close association between operators and it should be instructive to go into some detail about why.

⊕ figures in a(n ordered) group of formal systems including a "plus-minus calculus" and a "full algebra of concepts" (Lenzen, 2004, p. 3). For the purpose at hand it suffices to focus on the so-called "plus calculus," a subsystem of the plus-minus calculus (ibid.). The following axiomatic characterization of ⊕ relies on Swoyer (1994, p. 8) (except that it presents instantiations of axioms instead of axiom schemata). Thus, in addition to commutativity, (A1)(1) (cf. Roberts & Watumull, 2015 above), ⊕ is idempotent, (A2)(2).

(1)   A ⊕ B = B ⊕ A   [(A1) Commutativity]
(2)   A ⊕ A = A   [(A2) Idempotence]

Importantly, while (2) already raises non-trivial questions regarding the set-theoretic interpretation of Merge (cf. Gärtner, 2022) to do with the status of "self-merge" (Guimarães, 2000; Sauerland & Paul, 2017, p. 30), associativity, (A3)(3), is where the Leibnizian perspective and minimalist Merge definitely part ways.

(3)   (A ⊕ B) ⊕ C = A ⊕ (B ⊕ C)   [(A3) Associativity]

Thus, clearly, since Merge is introduced into minimalist syntax to induce (hierarchical) constituency, associativity would have to be considered problematic (cf. Berwick & Chomsky, 2016, p. 127), at least, in the general case. (4) and (5) provide an elementary illustration of this point.

(4)

a.

      Leibniz

Chomsky   *admires*

b.

Chomsky
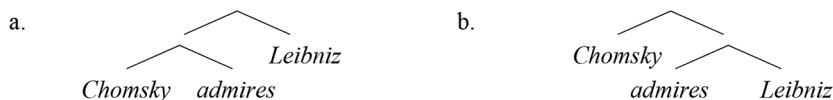
     admires   *Leibniz*

(5)   a.   Merge(Merge(*Chomsky,admires*),*Leibniz*) = { { *Chomsky, admires* }, *Leibniz* }
        b.   Merge(Merge(*admires,Leibniz*),*Chomsky*) = { { *admires, Leibniz* }, *Chomsky* }

Details of word order and case aside, (4a) and (5a) are standardly—but see, for example, Steedman (2000)—taken to differ from (4b) and (5b) in terms of thematic structure and/or grammatical functions: They underlie different construals of who admires who(m). As a consequence, given these discernible differences regarding associativity, we are forced to conclude that Merge and "Lerge" (qua counterpart of ⊕) should not be identified.

Importantly, what Roberts and Watumull may be seen to fail to stress (enough) is the distinct domains of application of the formal systems Merge and ⊕ belong to. The former is a calculus of natural language syntax, while the latter constitutes a calculus of ("semantic") concepts (cf. the idea of relating a syntactic and a semantic algebra in Montague grammar, for which, see Partee & Hendriks, 1997, Ch. 3.1). Consequently, while the distinctions in (4)/(5) matter for the enumeration of formal linguistic expressions, no comparable principle governs the conceptual realm, at least not at the level of abstraction chosen by Leibniz. If, for example, the attributes A(RROGANT,) B(OLD,) and C(UNNING) are taken to be basic characteristics of an individual, the particular order or grouping of these attributes would be immaterial.

It goes without saying that this polemical note is not in any way meant to discourage creative associations of the kind Roberts and Watumull set out to establish. On the contrary, it invites everyone to join us in the excitement of glancing through the Leibnizian window, except for the additional plea that the pane be polished somewhat more carefully first.

Three further remarks are in order to put the previous discussion into perspective.

i.   An anonymous reviewer has noted that the properties of Merge would have to be formulated more carefully if labeling were taken into account. However, addressing this issue would require going into the controversy over the status of labels within core syntax (cf. Epstein et al., 2017; Hornstein & Pietroski, 2009), a task beyond the scope of this work.

ii.   As a matter of historical accuracy it has to be made clear that Leibniz seems not to have stated (any counterpart of) (3) explicitly. Nevertheless, scholars agree that he relied on associativity in his practice of omitting brackets when writing sums of more than two terms and in several of his proofs (Lenzen, 2004, p. 24, fn. 31; Rescher, 1954, p. 11, fn. 50; Swoyer, 1994, p. 8).

iii. Certain branches of advanced mathematics, as pointed out by M. Krifka (p.c.), have introduced so-called "Leibniz algebras" (cf. Ayupov et al., 2020), whose name derives from Leibniz's work on the differentiation of functions ("Leibniz rule," "Leibniz identity"). For these structures, which are developed in the realm of fields, vectors and matrices, associativity is not a given (cf. Feldvoss, 2018). Transforming this into some kind of argument against the above reliance on (3), however, does not appear to hold much promise since commutativity is not a given for Leibniz algebras either (cf. Loday, 1993).

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Forum

Eademne Sunt?

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Abstract

This paper is a reaction to Watumull and Roberts (2023, https://doi.org/10.5964/bioling.12393).

Keywords

Leibniz, addition, merge, associativity, idempotence

In a recent squib (Gärtner, 2023), I pointed out that standard axiomatizations of the Leibnizian "real addition" operator ⊕ include associativity, (1), as one of its properties (cf. Lenzen, 2000, Section 4; Rescher, 1954, p. 11; Swoyer, 1994, p. 15):

(1)   Associativity: $(A \oplus B) \oplus C = A \oplus (B \oplus C)$

It was further maintained that associativity does not hold for Chomskyan Merge and that therefore the suggestion by Roberts and Watumull (2015, p. 213) to "identi[fy]" the two operators, the former renamed "Lerge," was problematic.

In their reply, Watumull and Roberts (2023) dismiss the associativity challenge on the basis of the idea that from a global perspective, for any (syntactic objects) *X*, *Y*, and *Z*, Merge generates all possible combinations, {{*X*,*Y*},*Z*} and {*X*,{*Y*,*Z*}} among them. And, importantly, it is assumed that "[t]he order of operations only matters when we seek to understand what parts of our knowledge we can *use*, factoring in third factors, etc." (p. 2). Effects of the structural differences are taken to be "external to the internal properties of Merge and the structures it generates" (p. 2f.).

To see what is at stake here, it has to be noted that (1) states an identity, postulating equivalence of the two sides. This usually gets interpreted as substitutability *salva veritate* in the case of ⊕, which for Merge could be changed to (at least) *salva gramma-*

ticalitate. Thus, concentrating on the latter, for any structural context κ, $\kappa[\{\{X,Y\},Z\}]$ and $\kappa[\{X,\{Y,Z\}\}]$ would both be "well-formed" by associativity. Of course, on such premises, interface effects of thematic interpretation or grammatical function assignment (mentioned in both the squib and the reply) won't constitute any obstacles. But what about the structural make up of the syntactic objects involved: featural differences, specifier vs. complement, asymmetric c-command etc.? Indeed, Watumull and Roberts (2023) are ready to flout the "grammaticality" constraint in taking it that (internal and external) "Merge generates all possible structures, only some of which converge (where convergence is dependent upon satisfying factors extrinsic to Merge)" (p. 3).

Let's turn to the axiom of idempotence, (2) (cf. Lenzen, 2000, Section 4; Rescher, 1954, p. 11; Swoyer, 1994, p. 15), which wasn't elaborated upon in the squib.

(2)   Idempotence: $A \oplus A = A$

Watumull and Roberts (2023, p. 1) consider it "unproblematic for the Merge = Lerge conjecture," assuming that it "is simply the case of Internal Merge applied to a single lexical item, yielding the successor function." In line with the exposition by Chomsky (2020, p. 24), (the initializing step of) this can be formulated as in (3a), with IM short for "Internal Merge." (3b) provides a notation with M short for generalized binary "Merge" to make the relation to (2) more transparent.

(3)   a.   IM(0) = {0,0} = {0}
        b.   M(0,0) = {0,0} = {0}

Now, clearly, the notion of a successor function demands that input and output are different, $n \neq successor(n)$, and indeed $0 \neq \{0\}$ ("zero" is unequal "one"). This, however, is in contrast to what idempotence says, where A added to itself is equivalent to (just) A. Discussing the reasons why Leibniz adopts idempotence for "real addition"—but not for its counterpart from arithmetics (Lenzen, 2004, p. 21)—would lead back to several of the points already made in the squib.[1]

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1) Watumull and Roberts (2023, p. 3) add an appeal to "scholarly scrupulous[ness]," a substantial part of which could have been avoided if they had consulted the published version of the squib rather than an earlier draft. Their conjecture that "perhaps Leibniz knew what he was doing" in omitting associativity, i.e., that he was striving for a non-associative calculus, would seem to require additional exegetic backup. It is quite compatible with the available evidence—considering both the diversity, vastness, and complexity of projects pursued by Leibniz and contemporary mathematical practice (cf. Hailperin, 2004, p. 327)—that Leibniz knew what he was doing while at the same time "overlooking" (cf. Lenzen, 2004, p. 19) some (tedious) details.

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