

The semantics of comparatives, or the measurement of differences

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Abstract

Degree semantics has been developed to study how the meanings of measurements and comparisons are encoded in natural language. Within degree semantics, this paper proposes a novel **difference-based** (or **subtraction-based**) perspective to characterize the semantics of comparatives. The main motivation is the measurability and comparability of differences involved in natural language comparatives. The main claim is that comparatives encode a subtraction relation among three items: two measurements on a certain scale and the difference between them. We analyze comparative morpheme *-er/more* as a default, least specific, positive difference, i.e., $(0, +\infty)$, and use interval arithmetic to implement subtraction. Our analysis inherits many existing insightful views on the semantics of comparatives. Moreover, the novel interval-subtraction-based implementation brings new conceptual and empirical advantages. In particular, we address the interpretation of comparatives containing numerical differentials and *than*-clause-internal quantifiers, the scope island status and the monotonicity of *than*-clauses, as well as a unified comparison-based view for various uses of gradable adjectives.

Keywords: measurement, comparison, gradable adjectives, comparatives, differentials/differences, comparative morpheme *-er/more*, measurement constructions, positive use of gradable adjectives, degrees, scales, intervals, units, orderings, interval arithmetic, interval subtraction, degree questions, definite descriptions, downward-entailing environment, additivity, anaphoricity.

1 Introduction

Humans measure objects along some dimension or scale and make comparisons among measurements. For example, we can compare how tall a giraffe is to a certain tree; we can compare some soup and coffee in terms of their temperature; and we can compare a train's arrival with the time it's supposed to arrive on a temporal scale. As shown in (1), natural language typically uses **comparatives** to encode **comparisons resulting in differences** (cf. **equatives**, which typically express **comparisons resulting in no differences**). Thus, the notion of differences should play a crucial role in the semantics of comparatives.

- | | | | |
|-----|----|---|-------------|
| (1) | a. | My giraffe is taller than that tree is. | Height |
| | b. | This soup is much hotter than that coffee seems to be. | Temperature |
| | c. | The train arrived one hour later than it should have. | Time |

Intriguingly, within the formal semantics research on comparatives, the ontology of differences (or often called differentials) has been somehow understudied. Even in those few works that focus on the semantics of numerical differentials (most notably, [Hellan 1981](#) and [Fleisher 2016](#)), differentials are not treated as an indispensable central component in comparatives. However, on the other hand, widely adopted views on this topic have already, often implicitly, made important assumptions on differences.

To begin with, a major assumption is that comparisons are performed between degrees, which result from measurement, i.e., the mapping of something, say an entity or an event, onto an abstract value – a degree – on a relevant scale (i.a., [Seuren 1973](#), [Cresswell 1976](#), [Hellan 1981](#), [Hoeksema 1983](#), [von Stechow 1984](#), [Heim 1985](#), [Lerner and Pinkal 1992](#), [1995](#), [Moltmann 1992](#), [Gawron 1995](#), [Izvorski 1995](#), [Rullmann 1995](#), [Kennedy 1999](#). See [Kennedy 1999](#) for a review and convincing defense on this). For instance, items that undergo comparison in (1b) are two values on the scale of temperature, not the soup and the coffee per se.

Furthermore, degrees involved in comparatives are often implicitly assumed to be number-like values. This assumption is obviously reflected in the use of addition for treating numerical differentials (i.a., [Hellan 1981](#), [von Stechow 1984](#)). More fundamentally, it can be considered derived from a basic empirical fact: differences yielded from comparisons can be further measured and compared, as illustrated by natural language examples like (2) (as well as (1b) and (1c)).

- | | | |
|-----|--------------------------------------|---|
| (2) | Mona is more happy than Jude is sad. | (Kennedy 1999 : Chapter 1, (89)) |
|-----|--------------------------------------|---|

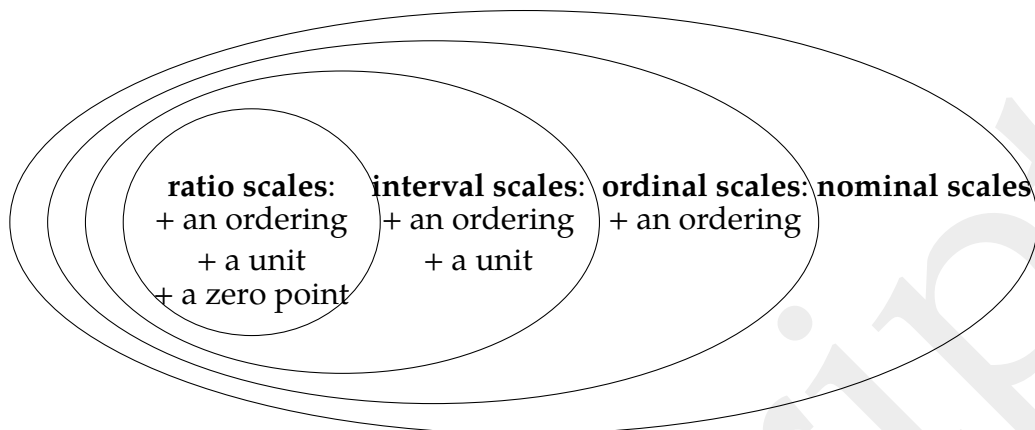


Figure 1: Four levels of scales, their entailment relationships (represented by the Venn diagram), and their defining attributes.

Presuppositions: Mona is happy and Jude is sad.

i.e., there are two comparisons here: Mona's happiness and Jude's sadness are above the (context-relevant) average level of happiness and sadness, respectively.

Assertion: The difference of 'Mona's happiness minus the average happiness' exceeds the difference of 'Jude's sadness minus the average sadness'.

i.e., the assertion addresses a third comparison between two existing differences.

According to [Stevens \(1946\)](#)'s distinction of levels of measurement and scales (see the Venn diagram in Figure 1), the very notion of measurable and comparable differences imposes a requirement on scales: they need to be equipped with not only **orderings**, but also **units**, i.e., they need to be **interval scales** or **ratio scales**.¹ The necessity of using units to measure and compare differences is most evidently demonstrated by comparisons along the temporal dimension. For the case in (1c), ordering only tells which one between the scheduled and the actual arrival times occurred first, and units (e.g., hours, minutes) are needed for the measurement of time differences (or temporal lengths). It is worth noting that units like *hours* can by no means be derived just from the ordering of equivalence classes like {the scheduled arrival time of the train, 12 o'clock, ...} or {the actual arrival time of the train, 1 o'clock, ...}.

The natural occurrence of examples like (2) and (1c) clearly indicates that comparisons encoded by comparatives need to be performed on interval scales, and degrees involved

¹See [Sassoon \(2010\)](#) for a different case of using [Stevens \(1946\)](#)'s distinction of levels of measurement and scales to explain natural language phenomena. Evidently, [Stevens \(1946\)](#)'s theory captures some crucial aspects of the conceptualization of measurements and comparisons in natural language.

in comparatives are thus number-like values on interval scales.

Then given that the notions of interval scales and the measurability and comparability of differences constitute the very foundation underlying the use of comparative constructions, making this foundational assumption explicit sheds light on the formal properties of degrees in comparatives and what operations to apply on them.

Within degree semantics, neither of the two canonical analyses of comparatives – the ‘A-not-A’ analysis (see (3), see Schwarzschild 2008 for a summary) and the ‘>’ analysis (see (4), see von Stechow 1984 and Beck 2011 for summaries) – is based on degree operations that naturally capture the measurability and comparability of differences.

(3) My giraffe is (5 inches) taller than that tree is. **the ‘A-not-A’ analysis**

a. $\exists d[\{d : \text{my giraffe is } d\text{-tall}\} \cap \{d : \text{that tree is } d\text{-tall}\}]$

The difference set between $\{d : \text{my giraffe is } d\text{-tall}\}$ and $\{d : \text{that tree is } d\text{-tall}\}$ is non-empty.

b. In the difference set between $\{d : \text{my giraffe is } d\text{-tall}\}$ and $\{d : \text{that tree is } d\text{-tall}\}$, the difference between its largest and smallest member is $\geq 5''$

(4) My giraffe is (5 inches) taller than that tree is. **the ‘>’ analysis**

a. $\max(\{d : \text{my giraffe is } d\text{-tall}\}) > \max(\{d : \text{that tree is } d\text{-tall}\})$

The largest degree that my giraffe’s height meets exceeds the largest degree that that tree’s height meets.

b. $\max(\{d : \text{my giraffe is } d\text{-tall}\}) \geq \max(\{d : \text{that tree is } d\text{-tall}\}) + 5''$

The largest degree that my giraffe’s height meets is equal to or exceeds the largest degree that that tree’s height meets plus 5 inches.

Essentially, as shown in (3a) and (4a), when there are no explicit numerical differentials (here the parenthesized part *5 inches*), these two approaches are simply based on orderings and set operations, i.e., operations that are applicable for degrees of ordinal scales and do not require degrees to be number-like values on interval scales. Then as shown in (3b) and (4b), ad hoc adjustments in the formalism are needed for analyzing comparatives that explicitly contain numerical differentials, and it is in these adjustments that the assumptions of interval scales and number-like degrees sneak into the formulas.

The use of two formulae by these canonical analyses is not only conceptually cumbersome, but also brings some detailed technical problems in composition. For the ‘A-not-A’ analysis, how does the use of a numerical expression (here *5 inches*) bring all these further

calculations on the members in the difference set? For the ‘>’ analysis, how does the use of such a numerical expression change a strict inequality like ‘>’ into a non-strict one like ‘≥’? To fix all these conceptual and technical glitches, a renovation is needed.

Therefore, in this paper, starting from the very assumption that comparisons encoded by comparatives are performed along interval scales, we propose to build the semantics of comparatives upon **subtraction**, an operation that directly captures the measurability and comparability of differences.

More specifically, we propose that a comparative always expresses a **subtraction equation among three degree-related values**: two are measurements on a certain scale that undergo comparison, and the third one is their difference (see (5a)). The notion of the measurement of differences allows us to turn inequalities into equations, which naturally fixes the above-mentioned conceptual and technical glitches with a simple step.

We also propose that comparative morpheme *-er/more* constantly contributes a least specific positive difference, which can be formally expressed as an interval $(0, +\infty)$, and adding a numerical differential further restricts this default positive difference (see (5b)).

(5) Proposals in a nutshell:

- a. The semantics of comparatives: the value associated with the matrix subject’s measurement minus the value of the standard is equal to the difference value.
- b. Comparative morpheme *-er/more* stands for a least specific positive difference, i.e., $(0, +\infty)$.

We will show that our proposal inherits important insights from the existing literature, especially on the existence of positive difference (see the ‘A-not-A’ analysis), the consideration of endpoints (see the ‘>’ analysis), the scope island status and the monotonicity of the standard in comparisons, and the definiteness of measurements (see Russell 1905). Our crucial innovation consists in the subtraction-based perspective and its interval-arithmetic-based implementation. It turns out that this innovation immediately resolves some long-standing open problems in the field, and we will address three of them.

First, in English clausal comparatives, *than*-clauses constitute a scope island, and the presence of this island creates non-trivial challenges for explaining the interpretation of comparatives containing numerical differentials and *than*-clause-internal quantifiers (see Fleisher 2016 for discussion). We will show that adopting the novel technique of interval subtraction provides a most natural solution to this issue. Then we will also show how our interval-subtraction-based treatment for the semantics of comparatives naturally ex-

138 plains the monotonicity of *than*-clauses. Finally, we will sketch a big picture on a unified
139 comparison-based view for various uses of gradable adjectives.

140 Throughout the paper, we focus on data from English clausal comparatives, but hope-
141 fully, the analysis developed here is applicable for many cross-linguistic phenomena. In
142 the following, we start with a presentation of interval subtraction (Section 2), the crucial
143 technique underlying our proposal. Based on this, Section 3 proposes to analyze (i) com-
144 parative morpheme *-er/more* as a default positive difference in comparatives and (ii) com-
145 paratives as relations among three degree-related values (two measurements and their
146 difference). Section 4 presents the detailed semantic derivation of *more-than* and *less-than*
147 comparatives containing numerical differentials. Three immediate implications of the cur-
148 rent account – the scope island issue, the monotonicity of *than*-clauses, and the bigger
149 picture for the uses of gradable adjectives – are discussed in Section 5. Section 6 further
150 compares the current analysis with existing studies on this topic. Section 7 concludes.

151 2 The technique of interval subtraction

152 In this section, we first introduce the definition of degrees, scales, and intervals, as well as
153 the notation of intervals. Then we introduce basic interval operations defined in mathe-
154 matics (Moore 1979) and present details of interval subtraction.

155 **Degrees** are considered points on an interval scale. Thus, a **scale** is actually a totally
156 ordered set of degrees (e.g., the set of real numbers \mathbb{R} is a scale). **Intervals** are defined as
157 **convex** subsets of a scale. According to the definition of convex sets (see (6)), evidently,
158 sets such as $\{x|x > 0\}$, $\{x|x \leq 4\}$, and $\{x|4 \leq x \leq 8\}$ are all convex sets, while sets like
159 $\{x|x > 10 \vee x \leq 3\}$ are not convex. Degrees are of type d , and thus intervals are of type $\langle dt \rangle$.

160 (6) The definition of a convex set:

161 A totally ordered set P is **convex** iff for any elements a and b in the set (suppose
162 $a \leq b$), any element x such that $a \leq x \leq b$ is also in the set P .

163 Since intervals are convex sets of degrees, we can rewrite an interval with its lower and
164 upper bounds. As shown in (7), we use square brackets '[' and ']' for **closed** lower and
165 upper bounds and round parentheses '(' and ')' for **open** lower and upper bounds. Obvi-
166 ously, a singleton set like $\{\delta|\delta = 3\}$ can be written as $[3, 3]$, the lower and upper bounds of
167 which are equal. We write positive and negative infinity as $+\infty$ and $-\infty$, and thus an in-
168 terval like $\{x|x \geq 4\}$ (i.e., a **left-bounded and right-unbounded** interval) can be written as

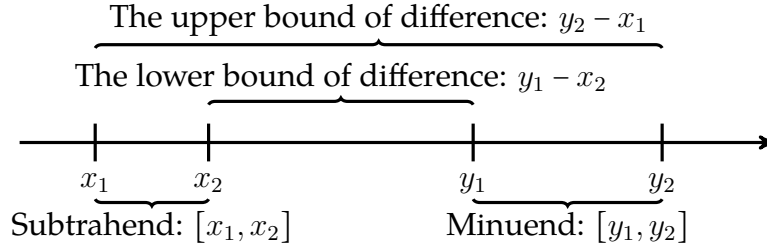


Figure 2: The subtraction between two intervals. Here $[y_1, y_2]$ means the minuend, $[x_1, x_2]$ the subtrahend, and the difference between these two intervals is the largest range of possible differences between any two random points in these two intervals, i.e., $[y_1 - x_2, y_2 - x_1]$.

$[4, +\infty)$, and an interval like $\{x|x < 3\}$ (i.e., a **left-unbounded and right-bounded** interval) can be written as $(-\infty, 3)$.

(7) Interval notation:

$$\{\delta | I_{\min} \leq \delta \leq I_{\max}\} = [I_{\min}, I_{\max}]$$

A left- and right-closed interval

$$\{\delta | I_{\min} < \delta \leq I_{\max}\} = (I_{\min}, I_{\max}]$$

A left-open and right-closed interval

$$\{\delta | I_{\min} \leq \delta < I_{\max}\} = [I_{\min}, I_{\max})$$

A left-closed and right-open interval

$$\{\delta | I_{\min} < \delta < I_{\max}\} = (I_{\min}, I_{\max})$$

A left- and right-open interval

Essentially, an interval means a range of possible values of degrees. Therefore, any operation that can be defined on the domain of degrees can also be defined on the domain of intervals, and applying an operation on two intervals results in the largest possible range of values. As shown in (8), for the four basic operations (i.e., addition, subtraction, multiplication, and division) between two intervals $[x_1, x_2]$ and $[y_1, y_2]$, the results can be defined in terms of their upper and lower bounds (i.e., x_1 , x_2 , y_1 , and y_2). All of these operations are extendable to the cases with unbounded and/or open endpoints.

(8) Basic interval operations:

$$[x_1, x_2] \langle \text{op} \rangle [y_1, y_2] = [\alpha, \beta]$$

(see Moore 1979)

$$\text{The lower bound } \alpha = \min(x_1 \langle \text{op} \rangle y_1, x_1 \langle \text{op} \rangle y_2, x_2 \langle \text{op} \rangle y_1, x_2 \langle \text{op} \rangle y_2)$$

$$\text{The upper bound } \beta = \max(x_1 \langle \text{op} \rangle y_1, x_1 \langle \text{op} \rangle y_2, x_2 \langle \text{op} \rangle y_1, x_2 \langle \text{op} \rangle y_2)$$

More specifically, as shown in (9), we can further simplify the formula of subtraction. The result of subtraction, i.e., the **difference**, is considered the largest range of possible differences between any two random points in two intervals (see Figure 2).

(9) Interval subtraction:

(see Moore 1979)

$$\underbrace{[y_1, y_2]}_{\text{minuend: matrix subject's measurement}} - \underbrace{[x_1, x_2]}_{\text{subtrahend: comparative standard}} = \underbrace{[y_1 - x_2, y_2 - x_1]}_{\text{difference: differential}}$$

- a. Example 1: $[5, 8] - [1, 3] = [2, 7]$ (2 and 7 are the minimum and maximum distances between the positions $[5, 8]$ and $[1, 3]$ respectively.)
- b. Example 2: $(4, +\infty) - [2, 3] = (1, +\infty)$ (This subtraction operation can be generalized to intervals with open and/or unbounded ends.)

From the formula (9), it is obvious that this operation of interval subtraction can be immediately used to characterize the semantics of comparatives: subtracting the interval representing the comparative standard (here $[x_1, x_2]$) from the interval representing the matrix subject's measurement (here $[y_1, y_2]$) results in a third interval, i.e., the differential (here $[y_1 - x_2, y_2 - x_1]$).

The subtraction between two intervals results in a third interval, but these three intervals are not the same kind of objects. In (9), the minuend and subtrahend intervals (i.e., $[y_1, y_2]$ and $[x_1, x_2]$) represent two not-very-precise **positions** on a scale (i.e., each position is in terms of a range), while the difference, i.e., $[y_1 - x_2, y_2 - x_1]$, represents the **distance** between the minuend and the subtrahend. This distinction between positions and distances is most evident on a temporal scale. As illustrated in (10), here *o'clock* is used to express times (i.e., positions on the temporal scale), but the unit here for temporal lengths (i.e., differences between positions on the temporal scale) is *hour*.

(10) 5 o'clock is 2 hours later than 3 o'clock is.

Some numerical examples of interval subtraction are given in (11):

- (11) a. $[5, 8] - [1, 2] = [3, 7]$
 b. $[5, 8] - [3, 7] = [-2, 5]$
 c. $[1, 2] - [5, 8] = [-7, -3]$

As shown in (11a) and (11c), when the minuend and the subtrahend are flipped, applying subtraction results in the inverse of the original difference (see (12) for details).

(12) Flipping the direction of subtraction: (see (9))

- a. $[y_1, y_2] - [x_1, x_2] = [y_1 - x_2, y_2 - x_1]$
 b. $[x_1, x_2] - [y_1, y_2] = [x_1 - y_2, x_2 - y_1] = [-(y_2 - x_1), -(y_1 - x_2)] = [0, 0] - [y_1 - x_2, y_2 - x_1]$

Then as illustrated by the contrast between (11a) and (11b), there is a crucial difference

between the subtraction defined in interval arithmetic and number arithmetic. In number arithmetic (i.e., when X , Y and Z represent numbers), if $X - Y = Z$, it follows necessarily that $X - Z = Y$ (see (13a)). However, in interval arithmetic (i.e., when X , Y and Z represent intervals), if $X - Y = Z$, generally speaking, it is not the case that $X - Z = Y$ (see (13b)).

- (13) a. Number arithmetic: $X - Y = Z \models X - Z = Y$ (e.g., $5 - 2 = 3 \models 5 - 3 = 2$)
 b. Interval arithmetic: $X - Y = Z \not\models X - Z = Y$ (see (11a) vs. (11b))

Consequently, in interval arithmetic, given $X - Y = Z$ and given the values of the subtrahend Y and the difference Z , to compute the value of the minuend X , we cannot perform interval addition on Y and Z (see (14)).

- (14) If $X - [a, b] = [c, d]$, then generally speaking, $X \neq [a + c, b + d]$.

Instead, we need to follow the formula (9) to derive the value of the minuend. As shown in (15), the minuend X is defined only when its lower bound does not exceed its upper bound. When the minuend is defined, as shown in (15b), the **upper** bound of the **subtrahend** (here b) contributes to the computation of the **lower** bound of the **minuend** X , while the **lower** bound of the **subtrahend** (here a) contributes to the computation of the **upper** bound of the **minuend** X .

- (15) If $X - [a, b] = [c, d]$,
 a. X is undefined when $b + c > a + d$;
 (i.e., when the lower bound of X exceeds the upper bound of X .)
 b. When defined, $X = [b + c, a + d]$. (see (9))
 The **lower** bound of the **minuend** X
 = the **lower** bound of the **difference** + the **upper** bound of the **subtrahend**;
 the **upper** bound of the **minuend** X
 = the **upper** bound of the **difference** + the **lower** bound of the **subtrahend**.

With the use of interval subtraction, we can now characterize a generalized comparison between two not-very-precise positions on a scale and precisely compute the distance (i.e., difference) between them. In particular, inequalities can now be turned into equations, and information with regard to the endpoints of positions and distances – including value, closedness, and boundedness – is fully taken care of with the use of this technique. Therefore, as we will show later, interval subtraction is an ideal tool for compositionally deriving the semantics of various kinds of comparatives, especially for those cases involv-

ing complex numeral differentials and/or *than*-clause internal quantifiers.

3 The semantics of *-er/more* and comparatives

3.1 Analyzing *-er/more* as a positive difference

Within the current difference-based perspective, we propose that comparative morpheme *-er/more* constantly contributes a **least specific positive difference**. As shown in (16), in terms of intervals, this morpheme denotes $(0, +\infty)$ (with a presupposition).

- (16) $[[\text{-er/more}]]_{\langle dt \rangle} \stackrel{\text{def}}{=} (0, +\infty)$ i.e., a least specific positive interval.
(Presupposing there is a discourse referent serving as base item for this increase.)

This analysis naturally captures the core semantic contribution of *-er/more*, which is **additivity**. Like other additive words in natural language (e.g., *also*, *another*), *-er/more* requires there be some existing item in a discourse. Thus, *-er/more* marks or denotes an **increase** (i.e., a positive difference) on this existing base item.

Empirically, additivity provides a most convenient, unified account for the three major uses of *-er/more*: additive constructions (Greenberg 2010, Thomas 2010), (comparative) correlatives (Brasoveanu 2008), and comparatives.

The most natural interpretation of (17a) is that the amount of alcohol Mary drank after blacking out was above zero, i.e., the amount Mary drank at a later time does not necessarily exceed the amount she drank previously (see also Greenberg 2010, Thomas 2010). Therefore, the amount Mary drank after blacking out can simply be an **increase** on the **base** of the amount she drank before blacking out, no matter how small this increase may be. This additive interpretation of *-er/more* becomes even more evident when weak NPI *any* is used along with *more* (see (17b)).

(17) Additive constructions

- a. Mary drank till she blacked out. Then she drank **more**.
- b. John refused to eat any **more**.

Then sentence (18) means that the increase of the degree of my knowledge for my dog correlates with the increase of the degree of my fondness for her. Crucially, this sentence does not tell to what extent I know about my dog or how much I like her. In other words, my total knowledge or fondness for my dog does not matter here, and what matters here

is the correlation between two **increases**.

(18) The **more** I know about my dog, the **better** I like her. Comparative correlative

Similarly, for a comparative sentence like (19), *more* can also be considered an **increase**, more specifically, an increase from her brother's height (i.e., the value of the comparative standard) to Mary's height (i.e., the value associated with the measurement of the matrix subject) (see also Greenberg 2010 and similar ideas in Kennedy and McNally 2005, Kennedy and Levin 2008).

(19) Mary is **taller** than her brother is. Comparative

This core semantic contribution of *-er* / *more* even carries to the use of *moreover*. In (20), the use of *moreover* conveys the meaning that chaos is **added** on top of depression.

(20) War brings depression. **Moreover**, it brings chaos.

There seems a difference between additive constructions (and *moreover*), on the one hand, and comparatives (and comparative correlatives), on the other hand. For additive constructions and *moreover*, we consider an increase from a part to a whole. In (17a), it is an increase from the amount of alcohol Mary drank previously to the total amount she drank; and in (20), it is an increase from depression alone to the sum of depression and chaos. For a comparative, we consider rather an increase from a standard value (i.e., the one denoted by the *than*-clause) to the value associated with the matrix subject's measurement.

However, additive constructions and comparatives demonstrate similar anaphoric behaviors, suggesting that they are fundamentally rooted in the same kind of additivity.

As illustrated in (21), additivity is distinct from mere existence in that additivity brings a presupposition: there exists something in the discourse other than the asserted item. In this sense, additive particles (e.g., *another*) are anaphoric to the Question Under Discussion (QUD) (see Thomas 2011 as well as Roberts 1996, Büring 2003, Zeevat 2004, Zeevat and Jasinskaja 2007).

(21) Additivity vs. mere existence: QUD: What does Lucy have?

a. Lucy has **a** diamond necklace. Mere existence

Assertion: There exists a diamond necklace belonging to Lucy.

b. Lucy has **another** diamond necklace. Additivity due to *another*

Assertion: There exists a diamond necklace belonging to Lucy;

Presupposition: Lucy has a diamond necklace other than this asserted one.

Evidently, tests of projection (see (23a) and (25a)) and local satisfaction (see (23b) and (25b)) show that both additive constructions and comparatives have this kind of presuppositions: i.e., there exists a certain value in the discourse other than the asserted one. In both cases, this presupposed value constitutes the base value to be increased.

(22) More alcohol was consumed. Additive construction

Presupposition: some amount of alcohol in the discourse was already consumed.

(23) Tests for presupposition in additive constructions:

a. It is possible that more alcohol was consumed. Projection

b. Either they only had one beer, or more alcohol was consumed.

Local satisfaction

(24) Sue is taller. Comparative

Presupposition: there is a certain degree of height in the discourse.

(25) Tests for presupposition in comparatives:

a. It is possible that Sue is taller. Projection

b. Either Sue is only 6 feet tall, or she is taller. Local satisfaction

Moreover, there cannot be any overlap between this base value in the discourse and the increase, as illustrated in (26) and (27). For (26), the two joint papers by Mary and Sue serve as the base value in the discourse, and the use of *more* can only be associated with the one single-authored book by Mary, i.e., Mary had one more publication, instead of three more publications. Similarly, for (27), the height of this tree (i.e., 19 feet 10 inches) serves as the base value in the discourse, and the use of *-er* can only be associated with the difference between the height of my giraffe and the height of this tree, i.e., 2 inches.

(26) Mary published a book. Then Mary and Sue published two papers together.

a. Mary and Sue published two papers. Mary had one more publication.

b. #Mary and Sue published two papers. Mary had three more publications.

(27) This tree is 19 feet 10 inches tall. My giraffe is 20 feet tall.

a. My giraffe is 2 inches taller.

b. #My giraffe is 20 feet taller.

To sum up, in its various uses, the morpheme *-er/more* always expresses additivity and marks an increase on an existing base value. Thus, it is most natural to analyze the semantics of *-er/more* as a positive difference.

3.2 The semantics of comparatives

Following Cresswell (1976), Hellan (1981), von Stechow (1984), Heim (1985), Beck (2011) and many other works in the literature on degree semantics, we consider that a gradable adjective (e.g., *tall*, *early*, *warm*) relates an individual and a certain position on the scale associated with this adjective (e.g., the scales of height, time, temperature), meaning that the measurement of this individual falls at this position on the scale. In some sense, the major semantic contribution of a gradable adjective is an interval scale that has a direction and units.

We use an interval to represent a position on a scale. Thus, as illustrated in (28), gradable adjective *tall* (of type $\langle dt, et \rangle$) relates an interval I of type $\langle dt \rangle$ and an individual x of type e , meaning that the measurement of the individual x falls at the position I on the scale of height.

$$(28) \quad \llbracket \text{tall} \rrbracket_{\langle dt, et \rangle} \stackrel{\text{def}}{=} \lambda I_{\langle dt \rangle} . \lambda x_e . [\text{HEIGHT}_{\langle e, dt \rangle}(x) \subseteq I]$$

i.e., the measurement of x falls at the position I on the scale of height.

As shown in (29), when the interval argument of a gradable adjective is specified with a range of numbers, the semantics of measurement constructions can be derived. Then as shown in (30), to account for the positive use of gradable adjectives, we follow previous works in the literature (see, e.g., Bartsch and Vennemann 1972, Cresswell 1976, von Stechow 1984, Kennedy 1999) and assume that there is a silent context-dependent interval I_{POS}^C that denotes the interval of being tall for a relevant comparison class. This interval I_{POS}^C ranges from the lower to the upper bound of being tall.

$$(29) \quad \text{My giraffe is between 19 and 20 feet tall.} \quad \text{Measurement construction}$$

$$\text{LF: } [[\text{my giraffe}] \text{ is } [[\text{between 19 and 20 feet}] \text{ tall}]]$$

$$\llbracket (29) \rrbracket \Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq [19', 20']$$

$$(30) \quad \text{My giraffe is tall.} \quad \text{Positive use}$$

$$\text{LF: } [[\text{my giraffe}] \text{ is } [I_{\text{POS}}^C \text{ tall}]]$$

$$\llbracket (30) \rrbracket \Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq I_{\text{POS}}^C$$

A sketch of the semantic analysis for comparatives is shown in (31). Evidently, the semantics of sentence (31) is parallel to that of (29) and (30).

- (31) My giraffe is taller than that tree is. Comparative
 LF: [[my giraffe] is [[-er than that tree is (tall)] tall]]
 $[[(31)]] \Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq \iota I [I - [[\text{than that tree is (tall)}]] = [[-er]]]$

Syntactically, a widely accepted view is that a *than*-clause is similar to a free relative and contains a *wh*-movement (see Bresnan 1973, 1975, Chomsky 1977, Pancheva 2007). Semantically, a *than*-clause is considered a definite description (see Russell 1905, Heim 2006a). Thus we propose to consider this definite description a fragment answer to its corresponding degree question (see also Fleisher 2018, 2019 for a similar (but not exactly the same) view).

As illustrated in (32), the LF of this *than*-clause contains an elided gradable adjective that is identical to the one used in the matrix clause (i.e., *tall*). The derivation of this *than*-clause involves two steps: forming a degree question (i.e., a set of intervals) via a silent lambda operator (Hausser and Zaefferer 1978) and picking out a definite interval via the use of *th(-an)* (see e.g., Heim 1985, Beck 2010).²

- (32) [[(my giraffe is taller) than that tree is (tall)]]
 LF: [than [λI . that tree is I tall]]
 Forming a degree question: $\lambda I. \text{HEIGHT}(\text{that tree}) \subseteq I$
 Generating its fragment answer: $\iota I [\text{HEIGHT}(\text{that tree}) \subseteq I]$

Thus we analyze (32) as a fragment answer to the degree question *how tall is that tree*. This analysis is consistent with the categorial approach to the semantics of questions and answers (Hausser and Zaefferer 1978, Hausser 1983, Von Stechow and Zimmermann 1984, Ginzburg and Sag 2000, Guerzoni and Sharvit 2007, Krifka 2011), according to which a question denotes a function, and its fragment (or short) answer denotes the argument such that applying the function to this argument results in a true proposition, i.e., the propositional (or long) answer (see (33)).

- (33) [[Who danced?]] = $\lambda x_e. x$ danced
 a. Mary. fragment answer (or short answer)

²Actually all English words starting with *th* (pronounced as ð) express definiteness: e.g., *the, they, that, then, there, these, thus*, etc. It is reasonable to assume that *than* contributes definiteness as well.

b. Mary danced.

Propositional answer (or long answer)

Obviously, the interval $(-\infty, +\infty)$ would be a trivial fragment answer to all degree questions. A felicitous fragment answer needs to be informative. Thus, the definiteness of *th(-an)* needs to be based on informativeness (see (34)). Given an individual or a group of individuals, *th(-an)* picks out the **narrowest** possible interval that the measurement of each individual falls into (see also Beck 2010). For example, for $\llbracket \text{than that tree is (tall)} \rrbracket$, *th(-an)* picks out the interval representing the measurement of a certain tree (e.g., a singleton set of degrees if the measurement is very precise); while for $\llbracket \text{than every tree is (tall)} \rrbracket$, *th(-an)* picks out the interval I such that the measurement of every tree falls into I (e.g., the interval ranging from the measurement of the shortest tree(s) to that of the tallest tree(s)).³

$$(34) \quad \llbracket \text{th(-an)} \rrbracket_{\langle \langle dt, t \rangle, dt \rangle} \stackrel{\text{def}}{=} \lambda P_{\langle dt, t \rangle} . \iota I [P(I) \wedge \neg \exists I' \neq I [P(I') \wedge P(I') >_{\text{informativeness}} P(I)]]$$

As shown in (35), we propose that *(th-)an* performs interval subtraction. It takes two intervals as inputs and returns a third interval. The inputs of $\llbracket (\text{th-})\text{an} \rrbracket$ are (i) the standard for comparison (i.e., the semantics of the *than*-clause) which serves as the subtrahend and (ii) the differential, and the output is the unique interval I representing the minuend.

$$(35) \quad \llbracket (\text{th-})\text{an} \rrbracket_{\langle \langle dt, \langle dt, dt \rangle \rangle} \stackrel{\text{def}}{=} \lambda I_{\text{standard}} . \lambda I_{\text{differential}} . \iota I [I - I_{\text{standard}} = I_{\text{differential}}]$$

We propose that comparative morpheme *-er/more* contributes the default differential: a least specific positive interval (see (36), which repeats (16)). The presuppositional requirement of *-er/more* (i.e., there is a base for increase in the discourse) is readily satisfied by the presence or accommodation of a *than*-clause.

$$(36) \quad \llbracket \text{-er/more} \rrbracket_{\langle dt \rangle} \stackrel{\text{def}}{=} (0, +\infty) \quad \text{i.e., a least specific positive interval.}$$

(Presupposition: there is a discourse referent serving as base item for an increase.)

A step-by-step derivation for the semantics of a *more-than* comparative is shown in (37). The semantics of the standard for comparison as well as that of the differential are derived respectively (see (37a) and (37b)). Then, as shown in (37c), based on the meaning of I_{standard} and $I_{\text{differential}}$, the interval $\llbracket \llbracket \text{-er than every tree is (tall)} \rrbracket \rrbracket$ actually denotes the unique interval I' such that when I' subtracts the most informative interval that every tree's height falls into (i.e., I_{standard}), the result is $(0, +\infty)$. Then, as shown in (37d), in the matrix clause,

³If we consider the precision of measurement a context-dependent parameter, then the measurement of a single tree might also be a non-singleton set of degrees, e.g., 19 feet \pm 1 inch, i.e., $[18'11'', 19'1'']$.

gradable adjective *tall* takes two arguments – the interval ‘[[[-er than every tree is (tall)]]]’ and the individual ‘[[my giraffe]]’ – and relates these two items. The last step of (37d) is to follow (15) (see Section 2) in simplifying the formula. Eventually, this sentence means that my giraffe’s height exceeds the height of the tallest tree.

(37) My giraffe is taller than every tree is.

LF: [[my giraffe] is [[-an [$\underbrace{-er}_{I_{\text{differential}}} \quad \underbrace{\text{th-} [\lambda I. \text{every tree is } I \text{ (tall)}}_{I_{\text{standard}}}]]] \text{tall}]]$

$$\underbrace{\quad \quad \quad}_{\iota I' [I' - I_{\text{standard}} = I_{\text{differential}}]}$$

a. I_{standard} : [[th-]] [[$\lambda I. \text{every tree is } I \text{ (tall)}$]]

$$= \iota I [\forall x [\text{tree}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]]$$

(Roughly speaking, this means [HEIGHT(shortest-tree), HEIGHT(tallest-tree)].)

b. $I_{\text{differential}}$: [[-er]] = $(0, +\infty)$

c. [[[-er than every tree is (tall)]]] = $\iota I' [I' - I_{\text{standard}} = I_{\text{differential}}]$

$$= \iota I' [I' - \iota I [\forall x [\text{tree}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]] = (0, +\infty)]$$

d. [[my giraffe is taller than every tree is]]

$$\Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq \iota I' [I' - \iota I [\forall x [\text{tree}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]] = (0, +\infty)]$$

$$\Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq$$

$$\iota I' [I' - [\text{HEIGHT}(\text{shortest-tree}), \text{HEIGHT}(\text{tallest-tree})] = (0, +\infty)]$$

$$\Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq (\text{HEIGHT}(\text{tallest-tree}), +\infty) \quad (\text{see (15)})$$

Overall, under the current analysis, across its various uses (i.e., in the positive use, measurement constructions, and comparatives), a gradable adjective always relates an individual with a position on a relevant scale. This position is potentially not as precise as a single number and thus best characterized with the use of an interval. Therefore, for comparatives, our intuitive interpretation for their semantics of inequalities is now addressed in terms of the description of this interval-based position.

As shown in (37d) (and the recipe of (15)), a ‘strict inequality’ interpretation is derived when this step of simplification yields a minuend that is a left-open, left-bounded, but right-unbounded interval. Moreover, given a subtrahend with both closed and bounded lower and upper bounds (here [HEIGHT(shortest-tree), HEIGHT(tallest-tree)]), the closedness and boundedness of the minuend solely depend on the closedness and boundedness of the differential, which is $(0, +\infty)$ here. It is due to the use of this default differential, $(0, +\infty)$, that comparatives with no numerical differentials express strict inequalities.

In the next section, we show that once numerical differentials restrict this default dif-

ferential and alter the closedness and boundedness of the differential, the closedness and boundedness of the minuend as well as the interpretation of the whole sentence are altered accordingly.

4 The semantics of comparatives containing numerical differentials and *than*-clause-internal quantifiers

Here we show how our proposed difference-based analysis handles numerical differentials in both *more-than* and *less-than* comparatives containing a *than*-clause-internal universal quantifier.⁴

Suppose that among all the trees in the domain, the shortest one is 18 feet tall, and the tallest one is 21 feet tall. Thus, as shown in (38), $[[\text{th}(-\text{an}) \text{ every tree is (tall)}]]$ denotes the standard of comparison, i.e., the most informative interval that every tree's height measurement falls into, i.e., $[18', 21']$. Then evidently, all the sentences in (39) and (40) are only different in terms of the value of $I_{\text{differential}}$, and a uniform analysis is sketched in (41).

$$\begin{aligned}
 (38) \quad I_{\text{standard}} &: [[\text{th- every tree is (tall)}]] \\
 &= [[\text{th}]] [[\lambda I. \text{every tree is } I \text{ tall}]] \\
 &= \iota I [\forall x [\text{tree}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]] \\
 &= [\text{HEIGHT}(\text{shortest-tree}), \text{HEIGHT}(\text{tallest-tree})] = [18', 21']
 \end{aligned}$$

- (39) a. My giraffe is **at least 5 feet taller** than every tree is.
 b. My giraffe is **at most 5 feet taller** than every tree is.
 c. My giraffe is **between 5 and 10 feet taller** than every tree is.

- (40) a. My giraffe is **at least 5 feet less tall** than every tree is.
 b. My giraffe is **at most 5 feet less tall** than every tree is.
 c. My giraffe is **between 5 and 10 feet less tall** than every tree is.

(41) LF for all the sentences in (39) and (40):

⁴The analysis of comparatives containing other types of quantifiers in their *than*-clause often requires extra mechanisms, and a full discussion is beyond the scope of this paper. The case of non-monotonic quantifiers contained in *than* clauses (e.g., *Balloon A is higher than exactly two of the others are*, see Schwarzschild 2008) is analyzed in another paper by the authors.

$$[[\text{my giraffe}] \text{ is } [-\text{an} \left\{ \begin{array}{l} \text{at least 5 feet ...-er} \\ \text{at most 5 feet ...-er} \\ \text{between 5 and 10 feet ...-er} \\ \text{at least 5 feet less} \\ \text{at most 5 feet less} \\ \text{between 5 and 10 feet less} \end{array} \right\} \text{th-} [\iota I.\text{every tree is } I \text{ (tall) }]] \text{ tall}]$$

$$[(39)/(40)] \Leftrightarrow$$

$$\text{HEIGHT}(\text{my-giraffe}) \subseteq \iota I' [I' - [\text{HEIGHT}(\text{shortest-tree}), \text{HEIGHT}(\text{tallest-tree})] = I_{\text{differential}}]$$

$$(\text{Here } [(39)/(40)] \Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq \iota I' [I' - [18', 21'] = I_{\text{differential}}].)$$

For *more-than* comparatives containing upward-entailing, downward-entailing, or non-monotonic numerical differentials (see (39)), as shown in (42), (43), and (44), their numerical differential restricts the default differential $(0, +\infty)$. With this more restricted differential, we can always use the same recipe (see (15)) to simplify the sentential semantics and derive the interval value associated with the measurement of the matrix subject.

$$(42) \quad \text{a. } [[\text{at least 5 feet ...-er}]] = [5', +\infty) \cap (0, +\infty) = [5', +\infty)$$

$$\begin{aligned} \text{b. } \iota I' [I' - [\text{HEIGHT}(\text{shortest-tree}), \text{HEIGHT}(\text{tallest-tree})] &= [[\text{at least 5 feet ...-er}]] \\ &= \iota I' [I' - [18', 21'] = [5', +\infty)] \\ &= [26', +\infty) \end{aligned}$$

$$(43) \quad \text{a. } [[\text{at most 5 feet ...-er}]] = (-\infty, 5'] \cap (0, +\infty) = (0, 5']$$

$$\begin{aligned} \text{b. } \iota I' [I' - [\text{HEIGHT}(\text{shortest-tree}), \text{HEIGHT}(\text{tallest-tree})] &= [[\text{at most 5 feet ...-er}]] \\ &= \iota I' [I' - [18', 21'] = (0, 5']] \\ &= (21', 23'] \end{aligned}$$

$$(44) \quad \text{a. } [[\text{between 5 and 10 feet ...-er}]] = [5', 10'] \cap (0, +\infty) = [5', 10']$$

$$\begin{aligned} \text{b. } \iota I' [I' - [\text{HEIGHT}(\text{shortest-tree}), \text{HEIGHT}(\text{tallest-tree})] &= [[\text{between 5 and 10 feet ...-er}]] \\ &= \iota I' [I' - [18', 21'] = [5', 10']] \\ &= [26', 28'] \end{aligned}$$

There are two issues worth noting here. First, since the subtrahend (i.e., I_{standard} , which is $[\text{HEIGHT}(\text{shortest-tree}), \text{HEIGHT}(\text{tallest-tree})]$) has both closed and bounded lower and upper bounds, the minuend directly inherits the closedness and boundedness of the differential. For example, if the differential is left-closed, left-bounded, and right-unbounded

(see (42)), then so is the minuend.

This immediately explains why on the one hand, comparatives with no numerical differential express a strict inequality – because their differential is $(0, +\infty)$ (i.e., with an open lower bound), but on the other hand, comparatives containing numerical differentials often express non-strict inequalities – because a restricted differential can have a closed lower bound. The seemingly compositional glitch brought by the use of numerical differentials is smoothly solved within interval semantics.

Moreover, this also naturally explains the observations addressed by Fleisher (2016): when a *more-than* comparative contains an upward-entailing numerical differential, the sentence has a MAX-reading (i.e., it seems that we only consider the height of the tallest tree), but when a *more-than* comparative contains a downward-entailing or non-monotonic numerical differential, the sentence has a MIN-&-MAX-reading (i.e., we need to consider both the height of the shortest and the tallest trees). Obviously, as shown in (42), for a right-unbounded differential like $[5', +\infty)$, the sum of this upper bound and the lower bound of I_{standard} is still $+\infty$, giving the impression that only the upper bound of I_{standard} (here the height of the tallest tree) can eventually be reflected in the computation of the interval associated with the measurement of the matrix subject.

Second, it is also worth noting that in order for the minuend (i.e., the interval associated with the measurement of the matrix subject) to be well defined, its upper bound should be larger than its lower bound (see (15)). Therefore, as shown in (45), the interval standing for the standard in comparison, I_{standard} , needs to be less wide than the interval standing for the differential, $I_{\text{differential}}$.

(45) For $\iota I[I - I_{\text{standard}} = I_{\text{differential}}]$ to be well defined,
 the **lower** bound of $I_{\text{differential}}$ + the **upper** bound of $I_{\text{standard}} <$
 the **upper** bound of $I_{\text{differential}}$ + the **lower** bound of I_{standard} (see (15))
 \therefore the **upper** bound of $I_{\text{standard}} -$ the **lower** bound of $I_{\text{standard}} <$
 the **upper** bound of $I_{\text{differential}} -$ the **lower** bound of $I_{\text{differential}}$
 $\therefore I_{\text{standard}}$ needs to be less wide than $I_{\text{differential}}$.

For upward-entailing differentials (e.g., (42)), i.e., those intervals with $+\infty$ as their upper bound, this requirement can always be met. However, for downward-entailing and non-monotonic differentials (e.g., (43) and (44)), this requirement brings a consequence on inference: sentences (39b) and (39c) are felicitous because these $I_{\text{differential}} - (0, 5']$ and $[5', 10']$ – are wider than the relevant I_{standard} , which is $[18', 21']$ here. This inference pat-

tern explains why for sentences like (46), in which the differential is actually an interval containing a unique degree, $[10', 10']$, we intuitively feel that it suggests that every tree should be of the same height, i.e., the interval standing for the standard includes also a unique degree. The use of interval subtraction naturally captures this intuition, and there is no need to introduce other mechanisms to deal with this inference (see also Beck 2010, Alrenga and Kennedy 2014, Fleisher 2016 for more discussion).

(46) My giraffe is exactly 10 feet taller than every tree is.
 \leadsto Every tree should be of the same height. so that I_{minuend} is defined

Then for *less-than* comparatives containing upward-entailing, downward-entailing, or non-monotonic numerical differentials (see (40)), the sketch in (41) shows that they can be analyzed in exactly the same way.

Following previous studies (e.g., Heim 2006b, Buring 2007a,b), we analyze *less* as the composition of *little* and *-er/more*. As shown in (47), we propose that *little*, which is of type $\langle dt, dt \rangle$, takes an interval as input and returns its inverse as output. In other words, *little* changes the polarity of an interval. Based on this, as shown in (48), when *little* takes *-er/more*, a least specific positive differential, as input, the output is a least specific negative differential, i.e., $(-\infty, 0)$. *Less* also brings a presupposition: it requires the existence of a discourse referent that serves as base item for a decrease.

$$(47) \quad [[\text{little}]]_{\langle dt, dt \rangle} \stackrel{\text{def}}{=} \lambda I. [[0, 0] - I]$$

$$(48) \quad [[\text{less}]]_{\langle dt \rangle} \stackrel{\text{def}}{=} [[\text{little}]] [[-\text{er/more}]] = (-\infty, 0)$$

i.e., a least specific negative interval.

(Presupposition: there is a discourse referent serving as base item for a decrease.)

A step-by-step derivation for the semantics of a *less-than* comparative is shown in (49). This derivation is totally parallel to the derivation for the semantics of *more-than* comparatives (see (37)). The only difference consists in the polarity of $I_{\text{differential}}$. For the *more-than* comparative in (37), $I_{\text{differential}}$ (i.e., $(0, +\infty)$) is right-unbounded, and thus, the measurement of the matrix subject becomes associated with a right-unbounded interval and reflects the information of the upper bound (but not the lower bound) of I_{standard} (see (37d)). Then for the *less-than* comparative in (49), $I_{\text{differential}}$ (i.e., $(-\infty, 0)$) is left-unbounded, and thus, the measurement of the matrix subject becomes associated with a left-unbounded interval and reflects the information of the lower bound (but not the upper bound) of I_{standard} .

(see (49d)). Obviously, by changing the polarity of the $I_{\text{differential}}$, *little* (or *less*) changes the direction of an inequality (i.e., *more-than* comparatives essentially express a ' $>/\geq$ ' relation, while *less-than* comparatives a ' $</\leq$ ' relation).

(49) My giraffe is less tall than every tree is.

LF: [[my giraffe] is [[-an [less th- [λI . every tree is I (tall)]]] tall]]

$$\underbrace{\underbrace{I_{\text{differential}}} \quad \underbrace{I_{\text{standard}}}}_{\iota I' [I' - I_{\text{standard}} = I_{\text{differential}}]}$$

a. I_{standard} : [[th-]] [[λI . every tree is I (tall)]]

$$= \iota I [\forall x [\text{tree}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]]$$

(Roughly speaking, this means [$\text{HEIGHT}(\text{shortest-tree}), \text{HEIGHT}(\text{tallest-tree})$].)

b. $I_{\text{differential}}$: [[less]] = $(-\infty, 0)$

c. [[[less than every tree is (tall)]]] = $\iota I' [I' - I_{\text{standard}} = I_{\text{differential}}]$

$$= \iota I' [I' - \iota I [\forall x [\text{tree}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]] = (-\infty, 0)]$$

d. [[my giraffe is less tall than every tree is]]

$$\Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq \iota I' [I' - \iota I [\forall x [\text{tree}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]] = (-\infty, 0)]$$

$$\Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq$$

$$\iota I' [I' - [\text{HEIGHT}(\text{shortest-tree}), \text{HEIGHT}(\text{tallest-tree})] = (-\infty, 0)]$$

$$\Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq (-\infty, \text{HEIGHT}(\text{shortest-tree})) \quad (\text{see (15)})$$

Similarly, as shown in (50), (51), and (52), we use the same recipe to compute the semantics of *less-than* comparatives containing upward-entailing, downward-entailing, or non-monotonic numerical differentials. Crucially, in these *less-than* comparatives, we assume that a numerical differential first combines with *more* and restricts this positive interval, and then *little* operates on this restricted positive interval and returns its inverse.

(50) a. [[at least 5 inches less]] = [[little]] [[at least 5 feet ...-er]]

$$= [[\text{little}]] [5', +\infty) = (-\infty, -5']$$

b. $\iota I' [I' - [\text{HEIGHT}(\text{shortest-tree}), \text{HEIGHT}(\text{tallest-tree})] = [[\text{at least 5 feet less}]]]$

$$= \iota I' [I' - [18', 21']] = (-\infty, -5']$$

$$= (-\infty, 13']$$

(51) a. [[at most 5 feet less]] = [[little]] [[at most 5 feet ...-er]]

$$= [[\text{little}]] (0, 5'] = [-5', 0)$$

b. $\iota I' [I' - [\text{HEIGHT}(\text{shortest-tree}), \text{HEIGHT}(\text{tallest-tree})] = [[\text{at most 5 feet less}]]]$

$$= \iota I' [I' - [18', 21']] = [-5', 0)$$

$$= [16', 18')$$

- (52) a. $[[\text{between 5 and 10 feet less}]] = [[\text{little}]] [[\text{between 5 and feet ...-er}]]$
 $= [[\text{little}]] [5', 10''] = [-10', -5']$
 b. $\iota I' [I' - [\text{HEIGHT}(\text{shortest-tree}), \text{HEIGHT}(\text{tallest-tree})]] = [[\text{between 5 and 10 feet less}]]$
 $= \iota I' [I' - [18', 21']] = [-10', -5']$
 $= [11', 13']$

Evidently, *less-than* comparatives are similar to *more-than* comparatives: the closedness and boundedness of the interval associated with the measurement of the matrix subject always follow the closedness and boundedness of $I_{\text{differential}}$, and moreover, the width of $I_{\text{differential}}$ always needs to be larger than the width of I_{standard} .

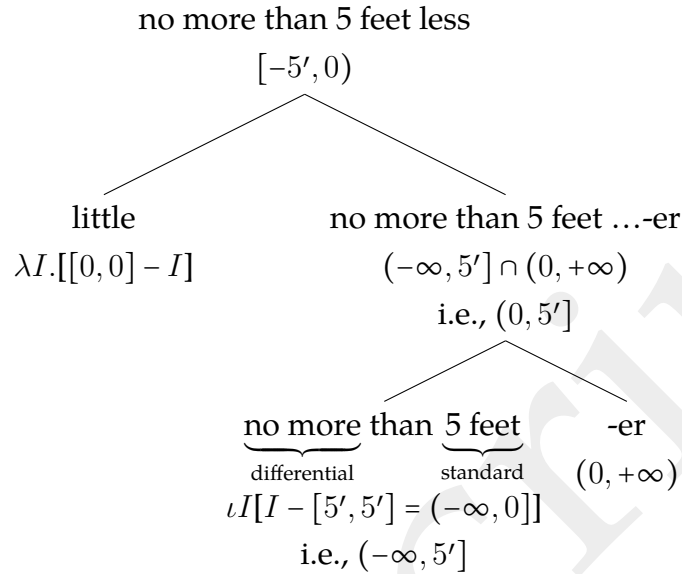
Finally, as illustrated in (53) and (54), negation operator *no* can also compose with and thus modify an interval. We analyze *little* and *no* as two distinct operators on intervals: *little* turns an interval into its inverse, while *no* negates an interval (i.e., it returns the complement of an interval). Therefore, *no more* and *no less* are slightly different from *less* and *more*: the upper bound of *no more* and the lower bound of *no less* are closed, while the upper bound of *less* and the lower bound of *more* are open.⁵

- (53) a. $[[\text{more}]] = (0, +\infty)$
 b. $[[\text{no more}]] = U \setminus (0, +\infty) = (-\infty, 0]$ $U = (-\infty, +\infty)$
 (cf. $[[\text{less}]] = [[\text{little}]] [[\text{-er/more}]] = [0, 0] - (0, +\infty) = (-\infty, 0)$.)
- (54) a. $[[\text{less}]] = (-\infty, 0)$
 b. $[[\text{no less}]] = U \setminus (-\infty, 0) = [0, +\infty)$

Based on our analysis of *no more*, (55) illustrates how to derive the meaning of a complex numerical differential: *no more than 5 feet less*.

⁵Intriguingly, *little* can compose with all positive differentials (e.g., *at most 5 inches ...-er*), but *no* can only compose with the default positive and negative intervals *more* and *less*. It is worth noting that, under the current analysis, when taking a convex interval as its input, *little* always returns another convex interval as output (i.e., the inverse of the original interval), but *no* potentially returns something that is not a convex interval (e.g., the complement of $[0, 5']$ is $\{x | x < 0 \vee x > 5'\}$). Presumably, this might explain the limited use of negation operator *no* in modifying intervals.

622 (55)



623 The upshot here is that under our current interval-subtraction-based analysis for com-
 624 paratives as well as our proposed lexical entries for interval modifiers *little* and *no*, all these
 625 cases of complex numerical differentials receive a uniform and principled treatment that
 626 naturally and precisely capture our intuitive interpretation for these expressions.

627 5 Implications of the proposal

628 Here we discuss three immediate implications of the proposed difference-based analysis
 629 that is implemented with interval subtraction. The first two focus on the semantics of the
 630 *than*-clause (i.e., the subtrahend in subtraction), while the third one addresses the role of
 631 comparative morpheme *-er* / *more* at the discourse level and suggests a unified comparison-
 632 based semantics for various uses of gradable adjectives.

633 5.1 The *than*-clause is a scope island

634 Under the current analysis, the *than*-clause, as a single whole value (i.e., an interval, or
 635 a convex set of degrees), refers to a not-necessarily-precise position on an interval scale
 636 and serves as the subtrahend in subtraction. Thus, this analysis naturally fulfills the re-
 637 quirement set by Beck (2010): ‘I want to come out of the calculation of the semantics of
 638 the *than*-clause holding in my hand *the* degree we will be comparing things to.’ The only
 639 adjustment we have made here is to characterize such a position under comparison with
 640 the notion of interval – a notion that is technically more general than that of degree.

It is worth noting that even when a *than*-clause contains a universal quantifier (see (56)), the semantics of the *than*-clause is still analyzed as a holistic position, i.e., an interval. The value, closedness, and boundedness of the lower and upper bounds of this interval is all the information that comes out of the calculation of the semantics of the *than*-clause. In other words, only the height information of the shortest and tallest trees gets projected in deriving the meaning of the matrix sentence.

$$(56) \quad \llbracket \text{th(-an) every tree is (tall)} \rrbracket = \iota I. \forall x [\text{tree}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]$$

Similarly, when a *than*-clause contains a plural individual (see (58)), we use a distributivity operator *Dist* to relate the plural individual and the predicate (see (57)). The underlying assumption is that for gradable adjectives like *tall*, their entity argument is always an atomic individual. Therefore, the meaning of the *than*-clause in (58) still refers to the interval ranging from the height of the shortest tree to the height of the tallest tree, and only the endpoint information of this interval comes out of the semantic calculation.⁶

$$(57) \quad \text{Dist} \stackrel{\text{def}}{=} \lambda X_e. \lambda P_{\langle et \rangle}. \forall x [x \sqsubseteq_{\text{atom}} X \rightarrow P(x)]$$

i.e., for each atomic part x of the plural individual X , predicate P holds for x .

$$(58) \quad \llbracket \text{th(-an) the trees are Dist (tall)} \rrbracket = \iota I. \forall x [x \sqsubseteq_{\text{atom}} \oplus \text{tree} \rightarrow \text{HEIGHT}(x) \subseteq I]$$

i.e., for each atomic part in the plural individual ‘the sum of trees’, its height measurement falls into the interval I .

As a consequence, the *than*-clause is naturally a scope island: *than*-clause-internal quantifier *every* (see (56)) and the quantifier ‘ \forall ’ used in the distributivity operator (see (57) and (58)) do not take scope outside of ‘ ι ’, i.e., they cannot take scope outside of the *than*-clause.

This is consistent with long-established empirical observations: *than*-clauses are semi-

⁶The same reasoning applies for *than*-clauses containing a free choice *any* (see (i)) or a narrow-scope disjunction (see (ii)). Briefly speaking, our intuition is that the use of free choice *any* and narrow-scope disjunction here has the flavor of universal quantification. Thus, we analyze (i) and (ii) parallel to (56) and (58). The upshot here is that the use of ‘ \forall ’ takes narrow scope within the operator ‘ ι ’, and the *than*-clause denotes a single not-necessarily-precise position on a scale, projecting only the information of endpoints of this position to the semantics of the matrix sentence. Issues such as the general distribution pattern of free choice *any* and narrow-scope disjunction as well as how they are licensed within a *than*-clause are left for upcoming research.

(i) $\llbracket \text{th(-an) any tree is (tall)} \rrbracket = \iota I. \forall x [\text{tree}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]$

(ii) $\llbracket \text{th(-an) this or that tree is (tall)} \rrbracket = \iota I. \forall x [x \in \{\text{this tree, that tree}\} \rightarrow \text{HEIGHT}(x) \subseteq I]$

lar to definite descriptions and show no scopal ambiguities (i.a., Russell 1905, Hasegawa 1972, Postal 1974, Horn 1981, von Stechow 1984, Heim 1985, Larson 1988, Rullmann 1995, Kennedy 1999). As shown in (59), quantifier raising is not possible for *than*-clause-internal quantifiers (see Larson 1988). Moreover, as shown in (60), phrasal comparative (60a) is ambiguous between two readings, due to scopal interaction between *every* and *some*. In contrast, clausal comparative (60b) has only a '*some* > *every*' reading (but no '*every* > *some*' reading), indicating that the *than*-clause is indeed a scope island and there is no scopal interaction.

(59) a. *[Which tree]_i is my giraffe taller than *t_i* is?

b. *She wants to know who was taller than who else is.

(60) a. Some giraffe is 2 inches taller **than every tree**.

Phrasal comparative: two readings.

$\checkmark \exists > \forall; \checkmark \forall > \exists$

b. Some giraffe is 2 inches taller **than every tree is**.

Clausal comparative: one reading.

$\checkmark \exists > \forall; \# \forall > \exists$

Then since we use the notion of intervals, instead of the notion of degrees, to characterize the (not-necessarily-precise) position serving as the standard in a comparison (i.e., the semantics of the *than*-clause), we naturally solve a seemingly dilemma. I.e., letting *than*-clause-internal ' \forall ' take wide scope (so that the *than*-clause can eventually lead to multiple positions for comparisons) violates the island constraint, but it is empirically problematic to let a *than*-clause like *than every tree is (tall)* simply denote a single-degree position (see the above-cited works on the island status of *than*-clauses as well as Schwarzschild and Wilkinson 2002, Schwarzschild 2008, Gajewski 2008, van Rooij 2008, Beck 2010).

This dilemma is evident for both the canonical '*A*-not-*A*' and '>' analyses. As shown in (61), since *every* takes narrow scope, both approaches analyze the semantics of the *than*-clause as the maximum degree that every tree reaches, leading to a comparison between the height of my giraffe and that of the shortest tree and generating a too-weak truth condition for the matrix clause.

(61) My giraffe is taller than every tree is.

$\exists d[\{d : \text{my giraffe is } d\text{-tall}\} \cap \overline{\{d : \text{every tree is } d\text{-tall}\}}]$

the '*A*-not-*A*' analysis

$\max(\{d : \text{my giraffe is } d\text{-tall}\}) \geq \max(\{d : \text{every tree is } d\text{-tall}\})$

the '>' analysis

\leadsto My giraffe is taller than the shortest tree is.

(Only the height information of the **shortest** tree is used in deriving the meaning

of the matrix sentence, which is problematic.)

Then if we introduce extra mechanisms and let a *than*-clause like *than every tree is (tall)* denote the maximum degree that any tree ever reaches (i.e., the height of the tallest tree, see, e.g., Beck 2010 for details), it is also empirically problematic. As illustrated in (62), when such a sentence contains a non-upward-entailing differential (here *between 3 and 5 feet*), comparing the height of my giraffe and that of the tallest tree also generates a too-weak truth condition (see also the discussion in Fleisher 2016). Suppose that the height of the trees ranges from 20 to 21 feet while my giraffe is 26 feet tall, then sentence (62) is predicted to be true under this scenario, but according to our intuition, this sentence is false, because the height of my giraffe exceeds that of the shortest tree by more than 5 feet.

- (62) My giraffe is between 3 and 5 feet taller than every tree is.
 $\max(\{d : \text{my giraffe is } d\text{-tall}\}) \geq \max(\{d : \text{any tree is } d\text{-tall}\}) + d_0$ (see Beck 2010)
 (Here d_0 represents a differential, which is between 3 and 5 feet.)
 \leadsto My giraffe is between 3 and 5 feet taller than the tallest tree is.
 (Only the height information of the **tallest** tree is used in deriving the meaning of the matrix sentence, which is also problematic.)

Taken together, the examples of (61) and (62) provide evidence showing that it is problematic for *than every tree is (tall)* to denote the height of the shortest tree alone or the height of the tallest tree alone. Thus, the semantics of the *than*-clause needs to include the height information of both the shortest and the tallest tree.

Yet it is also problematic to assume that even though the universal quantifier *every* in *than every tree is (tall)* cannot take wide scope, somehow the entire *than*-clause can take wide scope. This is the main idea behind the approach of ‘degree plurality’ (see Beck 2014, Dotlačil and Nouwen 2016 and a similar idea in Heim 2006a). According to this approach, essentially, the semantic derivation of *than every tree is (tall)* generates one single-degree height measurement for each tree and sums up these degrees, and then with the use of a distributivity operator, these height measurements are each further compared with the height of my giraffe. As shown in (63), the ‘degree plurality’ approach would still introduce scopal interaction for a clausal comparative, predicting that clausal comparative (60b) is as ambiguous as phrasal comparative (60a). Obviously, this prediction is not borne out.

- (63) Some giraffe is 2 inches taller **than every tree is**.

The ‘degree plurality’ approach: for each tree’s height measurement d , some giraffe is 2 inches taller than d . – This reading does not exist.

Therefore, an interval-based characterization for items under comparison together with the technique of interval subtraction naturally provide the best possible solution. Intervals are convex sets of degrees. By denoting an interval, a *than*-clause still denotes a single position on a scale, but this single position is potentially not as precise as a single degree, and the information of both its endpoints matters. Thus two birds can be hit with one stone: the use of the notion of intervals keeps the *than*-clause a scope island and, at the same time, addresses all the endpoint information of the semantics of the *than*-clause.

5.2 The *than*-clause contributes a downward-entailing operator

Within our current difference-based analysis implemented with interval subtraction, since a *than*-clause plays the role of a subtrahend, it also follows naturally that by playing this role of subtrahend, it (or rather the part *-an* in *than*) contributes a downward-entailing operator. There is no additional need to assume a negation operator in *than*-clauses (cf. Marques 2003, Schwarzschild 2008, Gajewski 2008, Alrenga and Kennedy 2014).

As already addressed in Section 2, within interval arithmetic, given the values of a difference and a subtrahend, we cannot directly perform interval addition on them. Instead, we need to use the formula of interval subtraction (see (9)) to compute the value of the minuend (see (15)). More specifically, as shown in (64) (which repeats (15b)), in computing the value of this minuend, it is the **upper** bound of the subtrahend that contributes to the **lower** bound of the minuend, and similarly, it is the **lower** bound of the subtrahend that contributes to the **upper** bound of the minuend.

(64) If $X - [a, b] = [c, d]$, when defined, $X = [b + c, a + d]$. I.e.,

- a. The **lower** bound of the **minuend** X
= the **lower** bound of the **difference** + the **upper** bound of the **subtrahend**;
- b. the **upper** bound of the **minuend** X
= the **upper** bound of the **difference** + the **lower** bound of the **subtrahend**.

Then since an interval means a range of values, i.e., a range of possibilities, it is evident that the larger this range is, the more possibilities it includes, and the less informative it is. Or in other words, given an interval, if we makes its lower bound lower or its upper bound higher, the interval becomes less informative.

Given (64), making the lower bound of the subtrahend lower leads to a lower upper bound for the minuend, thus decreasing the informativeness of the subtrahend (i.e., the interval standing for the subtrahend includes more possibilities) but increasing the informativeness of the minuend (i.e., the interval associated with the minuend includes fewer possibilities). Thus, more generally, lowering or raising the lower or upper bound of the subtrahend always causes the informativeness of the subtrahend and the minuend to change in opposite directions: when the subtrahend becomes more informative, the minuend becomes less informative, and vice versa.

In other words, when the interval associated with the *than*-clause (i.e., I_{standard}) becomes more informative, the sentential semantics becomes less informative, and vice versa. The informativeness of I_{standard} projects to the matrix-clause-level informativeness in a reverse way, and thus this subtrahend demonstrates exactly the defining property of a downward-entailing operator, behaving just like a negation operator that reverses the relation of entailment (see (65) and Fauconnier 1978, Ladusaw 1979, 1980).

(65) $\because \{x : x \text{ is a lizard}\} \subseteq \{x : x \text{ is a reptile}\}$ i.e., $[[\text{lizard}]] >_{\text{informativeness}} [[\text{reptile}]]$
 $\therefore \overline{\{x : x \text{ is a lizard}\}} \supseteq \overline{\{x : x \text{ is a reptile}\}}$
 i.e., $[[\text{not a reptile}]] >_{\text{informativeness}} [[\text{not a lizard}]]$
 E.g., Roo is not a reptile \models Roo is not a lizard.

As illustrated in (66), suppose that the most informative interval including every animal' height is from zero to 20 feet, while the most informative interval including every giraffe' height is from 16 to 19 feet. Then given that $[16', 19']$ is more informative than $(0, 20']$, it follows naturally that *taller than every animal is* (i.e., above the upper bound of $(0, 20']$) entails *taller than every giraffe is* (i.e., above the upper bound of $[16', 19']$).

(66) $\because \iota I. \forall x [\text{giraffe}(x) \rightarrow \text{HEIGHT}(x) \subseteq I] \subseteq \iota I. \forall x [\text{animal}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]$
 (i.e., the narrowest – most informative – interval including every giraffe's height (here $[16', 19']$) $\geq_{\text{informativeness}}$ the narrowest interval including every animal' height (here $(0, 20']$).)
 $\therefore \iota I'. [I' - \iota I. \forall x [\text{giraffe}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]] = I_{\text{differential}} \supseteq$
 $\iota I'. [I' - \iota I. \forall x [\text{animal}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]] = I_{\text{differential}}$
 (i.e., $[[\text{taller than every animal is}]] >_{\text{informativeness}} [[\text{taller than every giraffe is}]]$)
 E.g., the tree is taller than every animal is \models the tree is taller than every giraffe is.

Crucially, under the current account, a *than*-clauses is downward-entailing due to its

being a subtrahend in interval subtraction. Therefore, its downward-entailingness is with regard to the projection of the informativeness of the interval I_{standard} , not directly with regard to the projection of the informativeness of *than*-clause-internal expressions.

This naturally explains why sometimes *than*-clauses seem to be a downward-entailing environment (see (67)), but sometimes it seems that they are not a downward-entailing environment, but rather an upward-entailing environment (see (68), and see Larson 1988, Schwarzschild and Wilkinson 2002, Heim 2006b, Giannakidou and Yoon 2010 for related discussions).

- (67) a. The tree is taller than **every animal** is \models the tree is taller than **every giraffe** is.
b. The tree is taller than **any animal** is \models the tree is taller than **any giraffe** is.

- (68) a. The tree is taller than **some animal** is $\not\models$ the tree is taller than **some giraffe** is.
b. The tree is taller than **some giraffe** is \models the tree is taller than **some animal** is.

The informativeness of I_{standard} always projects to the semantics of the matrix sentence in the same reverse way. Thus, the pattern shown in (67) and (68) depends rather on how the informativeness of NP (here *animal* and *giraffe*) projects to that of their embedding DP (here ‘*every/any/some* NP’) and, eventually, to that of I_{standard} , i.e., the most informative interval containing the height measurement of each atomic individual addressed by the DP.

Obviously, as illustrated in (69), for sentences like (67), which contain *than*-clause-internal universal quantification, the informativeness projection from NP to ‘*every* NP’ is reverse, and then the informativeness projection from ‘*every* NP’ to the most informative I_{standard} is reverse again, and finally, the informativeness projection from I_{standard} to the semantics of the matrix sentence is also reverse, giving the impression that overall, the informativeness projection from NP to the semantics of matrix sentence is reverse.

- (69) $\because \{x : x \text{ is a giraffe}\} \subseteq \{x : x \text{ is an animal}\}$
(i.e., $\llbracket \text{giraffe} \rrbracket \geq_{\text{informativeness}} \llbracket \text{animal} \rrbracket$ – anything in the set of giraffes is also in the set of animals.)
 $\therefore \lambda P. \forall x [\text{giraffe}(x) \rightarrow P(x)] \supseteq \lambda P. \forall x [\text{animal}(x) \rightarrow P(x)]$
(i.e., $\llbracket \text{every animal} \rrbracket \geq_{\text{informativeness}} \llbracket \text{every giraffe} \rrbracket$ – any property P s.t. $\forall x [\text{animal}(x) \rightarrow P(x)]$ also satisfies $\forall x [\text{giraffe}(x) \rightarrow P(x)]$.)
 $\therefore \lambda I. \forall x [\text{giraffe}(x) \rightarrow \text{HEIGHT}(x) \subseteq I] \supseteq \lambda I. \forall x [\text{animal}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]$
(i.e., $\llbracket \lambda I. \text{every animal is } I \text{ tall} \rrbracket \geq_{\text{informativeness}} \llbracket \lambda I. \text{every giraffe is } I \text{ tall} \rrbracket$. – any

interval I s.t. $\forall x[\text{animal}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]$ also satisfies $\forall x[\text{giraffe}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]$.)

$\therefore \iota I. \forall x[\text{giraffe}(x) \rightarrow \text{HEIGHT}(x) \subseteq I] \subseteq \iota I. \forall x[\text{animal}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]$

(i.e., the narrowest – most informative – interval I s.t. $\forall x[\text{giraffe}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]$ is not less narrow than the narrowest one satisfying $\forall x[\text{animal}(x) \rightarrow \text{HEIGHT}(x) \subseteq I]$.)

However, as illustrated in (70), for sentences like (68), which contain *than*-clause-internal existential quantification, the informativeness projection from NP to ‘some NP’ is straightforward, but then the projection from ‘some NP’ to $[[\text{th(-an) some NP is}]]$ (i.e., I_{standard} in (68)) is reverse, and finally, the informativeness projection from I_{standard} to the semantics of the matrix sentence is again reverse, giving the impression that overall, the informativeness projection from NP to the semantics of the matrix sentence is not reverse.

- (70) $\therefore \{x : x \text{ is a giraffe}\} \subseteq \{x : x \text{ is an animal}\}$
 (i.e., $[[\text{giraffe}]] \geq_{\text{informativeness}} [[\text{animal}]]$ – anything in the set of giraffes is also in the set of animals.)
 $\therefore \lambda P. \exists x[\text{giraffe}(x) \wedge P(x)] \subseteq \lambda P. \exists x[\text{animal}(x) \wedge P(x)]$
 (i.e., $[[\text{some giraffe}]] >_{\text{informativeness}} [[\text{some animal}]]$ – any property P s.t. $\exists x[\text{giraffe}(x) \wedge P(x)]$ also satisfies $\exists x[\text{animal}(x) \wedge P(x)]$.)
 $\therefore \lambda I. \exists x[\text{giraffe}(x) \wedge \text{HEIGHT}(x) \subseteq I] \subseteq \lambda I. \exists x[\text{animal}(x) \wedge \text{HEIGHT}(x) \subseteq I]$
 (i.e., $[[\lambda I. \text{some giraffe is } I \text{ tall}]] \geq_{\text{informativeness}} [[\lambda I. \text{some animal is } I \text{ tall}]]$ – any interval I s.t. some giraffe’s height measurement falls within I also satisfies $\exists x[\text{animal}(x) \wedge \text{HEIGHT}(x) \subseteq I]$.)
 $\therefore \exists I[\exists x[\text{animal}(x) \wedge \text{HEIGHT}(x) \subseteq I] \wedge \forall I'[\exists x[\text{giraffe}(x) \wedge \text{HEIGHT}(x) \subseteq I'] \rightarrow I \subseteq I']]$
 (i.e., there exists interval I satisfying $\exists x[\text{animal}(x) \wedge \text{HEIGHT}(x) \subseteq I]$ and I is not less narrow/informative than any intervals satisfying $\exists x[\text{giraffe}(x) \wedge \text{HEIGHT}(x) \subseteq I]$.)

To sum up, subtraction behaves similarly to negation in creating a downward-entailing environment: the informativeness projection from the subtrahend to the minuend is reverse. Therefore, the current analysis naturally explains the entailment patterns for comparatives, without assuming any negation operator in *than*-clauses. The gist of the mechanism consists in the use of intervals for characterizing the standard in a comparison as well as the technique of interval subtraction shown in (64)/(15b).

5.3 Klein (1980)'s puzzle and a unified comparison-based view for various uses of gradable adjectives

Within the current proposal, comparative morpheme *-er/more* is analyzed as a least specific positive interval $(0, +\infty)$, serving as the default differential in comparatives.

Thus, as mentioned already in Section 3.1, *-er/more* actually has a discourse-level contribution: the semantics of *-er/more* can be considered the semantics of *some* plus a presuppositional requirement. As illustrated in (71), (71a) simply **asserts** existence – the amount of beer that Mary drank is above zero, and (71b) also **asserts** that the amount of beer that Mary drank is above zero. Then, in addition to this assertion, (71b) also **presupposes** that there is a salient degree of amount in the discourse, serving as the base for the asserted amount, so that the asserted amount is considered an increase on this presupposed base.

- (71) a. Mary drank **some** beer. ~ existential assertion
 b. Mary drank **more** beer. ~ presupposition + existential assertion

Similarly, the contrast shown in (72) suggests that *-er* also contributes a presuppositional requirement here. Without the use of *-er*, (72a) refers to a length measurement, a value which is 2 feet and can be written as $[2', 2']$ in our interval notation. Then with the use of *-er*, (72b) refers rather to a length measurement value serving as a difference (or an increase), and a felicitous interpretation of (72b) requires an existing salient length measurement serving as the base for this increase.

- (72) a. 2 feet long
 b. 2 feet longer

Thus these data provide clues for answering a morphological puzzle raised by Klein (1980): gradable adjectives seem to be inherently relational, but if the positive use of a gradable adjective (e.g., *my giraffe is tall*) involves already comparison (e.g., *my giraffe is tall* is often analyzed as 'my giraffe is taller than the average giraffe is'), why is the positive form *tall* morphologically simpler than the comparative form *taller*? Shouldn't the comparative meaning be more basic and thus the comparative form be morphologically simpler than the positive form?

Our proposal and discussion in Sections 3 and 4 show that the meaning of comparisons can be expressed with the use of (positive or negative) differences. Then the contrast shown in (71) and (72) suggests that the notion of differences can be contributed by

a discourse-level marker that invokes the existence of a presupposed, contextually salient measurement value serving as the base of increase or decrease (depending on the polarity of differences). In this sense, the essential contribution of *-er/more* is not to mark a comparison, but to mark the discourse salience of the value serving as the base for increase or decrease.

Now we have an answer to Klein (1980)'s puzzle: the semantics of gradable adjectives is inherently comparative, and all uses of gradable adjectives involve comparisons. Thus no additional morphological marker is needed to convey the meaning of comparisons. The use of *-er/more* is to mark the discourse salience of the value serving as the standard in a comparison. Obviously, this explains why, when uttered out of blue, only the comparative sentence (73a), but not the positive use (73b) or the measurement construction (73c), calls for accommodation of a presupposed discourse referent (dref) serving as the standard value in a comparison.

- (73) a. My giraffe is (2 feet) taller. Comparative: accommodation is needed.
 b. My giraffe is tall. The positive use: no accommodation.
 c. My giraffe is 20 feet tall. Measurement construction: no accommodation.

Then if all these uses of gradable adjectives involve comparisons, what are their standard and differential respectively?

As summarized in (74), presumably, for the positive use of gradable adjectives, their standard for comparison is the context-dependent average value, and the differential is a silent item standing for $(0, +\infty)$ (i.e., a least specific positive value), which can never be further restricted by a numerical differential, but can be modified by degree modifiers like *very*, *quite*, *relatively*, etc (yielding *very tall*, *quite tall*, *relatively tall*, etc.). For example, (73b) means that my giraffe is taller than the average height of relevant giraffes by an unspecified amount (see (75b)).

Then for measurement constructions, their differential is actually always specified by a numerical value (e.g., *20 feet* in (73c)), which stands for the distance between the measurement value (here the height measurement of my giraffe in (73c)) and the absolute zero point of the relevant scale.⁷ Thus, evidently, for measurement constructions, the standard for comparison is this absolute zero point of the relevant scale (see (75c)).

⁷Here we only consider the 'exactly' interpretation: e.g., (73c) means that my giraffe is exactly 20 feet tall. Numerical values / differentials in measurement constructions (and also comparatives) are often ambiguous between an 'exactly' reading and an 'at least' reading, but this is an orthogonal issue here.

(74) The standard and differential involved in comparison:

| Linguistic construction | Comparative | Measurement construction | Positive use |
|----------------------------------|---------------------|--------------------------|-------------------|
| Standard | <i>than</i> -clause | the zero point | the average value |
| Is the standard a dref? | yes | no | no |
| A numerical differential? | optional | yes | no |

- (75) a. $[(73a)] \Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq \iota I[I - I_{\text{standard}} = (0, 2']]$
 b. $[(73b)] \Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq \iota I[I - \text{average-of-the-height-of-giraffes} = (0, +\infty)]$
 (i.e., $I_{\text{pos}}^C = \iota I[I - \text{average-of-the-height-of-giraffes} = (0, +\infty)]$ (see also (30)).)
 c. $[(73c)] \Leftrightarrow \text{HEIGHT}(\text{my-giraffe}) \subseteq \iota I[I - [0, 0] = [20', 20']]$

This analysis for measurement constructions naturally implements Sassoon (2010)'s idea: not all gradable adjectives, but only those associated with ratio scales (i.e., scales with a meaningful, absolute zero point, see Figure 1 in Section 1) can be used to form measurement constructions (see also the discussion in Schwarzschild 2005).

Under our comparison-based analysis, measurement constructions require the existence of an absolute zero point on their relevant scale for comparison. As illustrated in (76), the scale of length naturally has a meaningful, absolute zero point, and thus (76a) is grammatical. In contrast, the scales of shortness, warmth, and earliness / lateness all lack this kind of meaningful zero point, and thus gradable adjectives like *short*, *warm*, *early*, and *late* cannot be used to form grammatical measurement constructions.

- (76) a. This tennis match was 1.5 hours **long**. Temporal length
 b. *This tennis match was 1.5 hours **short**. Temporal shortness
 c. *New York is now 70 degrees **warm**. Warmth
 d. *Our meeting time was 11 AM **early** / **late**. Earliness, lateness

On the other hand, comparatives address the measurement of differences. Though scales measuring shortness, warmth, or earliness lack a meaningful zero point, scales measuring the differences of shortness, warmth, or earliness do have their meaningful zero point: the absolute zero point stands for the absence of any differences. Therefore, gradable adjectives associated with either ratio scales or non-ratio interval scales can all be used in comparatives (see (77)).

- (77) a. This tennis match was 1.5 hours **longer** (than that one is).
 b. This tennis match was 1.5 hours **shorter** (than that one is).

- c. New York is now 70 degrees **warmer** (than Antarctic is).
 d. Our meeting time was one hour **earlier/later** (than I expected).

The parallelism between measurement constructions and comparatives is thus evident. Measurement constructions address comparisons with the absolute zero point on their relevant scale, while comparatives address measurements of differences from a contextually salient standard that is either accommodated or expressed via a *than*-construction. In both cases, numerical expressions (e.g., *1.5 hours (long/longer)* in (76a) and (77a) or modified numerals like *between 3 and 4 inches (long/longer)*) refer to differences. Thus, measurements and comparisons, as encoded in natural language, seem to suggest that they involve the same kind of conceptualization.

Overall, within the current comparison-based (or difference-based) analysis, three different uses of gradable adjectives can be characterized in a unified way. This view naturally captures our intuition that the uses of gradable adjectives are inherently related to comparisons. Moreover, our analysis also reveals the fundamental semantic contribution of *-er/more* and provides a natural answer to the seemingly morphological puzzle posed by Klein (1980).

6 Comparing the current analysis with existing works

As we mentioned at the very beginning, the main motivation of our difference-based analysis for comparatives consists in the measurability and comparability of differences. Thus, essentially, we analyze the semantics of comparatives as the measurement of differences.

This view is distinct from most existing works in two fundamental ways. On the one hand, the current analysis explicitly relies on the assumption that comparisons, as encoded in comparatives, are performed on the base of interval scales, i.e., totally ordered sets equipped with units. In this sense, we explicitly adopt a more complex ontology for degrees, allowing for generalizing measurement values from number-like points to intervals. On the other hand, once this ontology of degrees established, the current analysis characterizes comparisons with clearer and more natural operations.

In particular, we use subtraction-based equations, instead of inequalities or addition-based equations, to characterize the meaning of comparatives. (78) shows an often adopted addition-based way to extend the '*>*' account to include the semantics of numerical differentials.

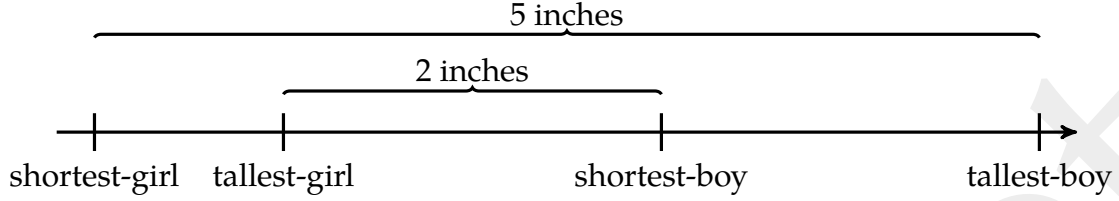


Figure 3: The boys are all between 2 and 5 inches taller than every girl is.

The compositional glitch aside (see Section 1), this addition-based analysis has a conceptual problem. As hinted in Section 2, the extension in (78) neglects that the measurements on the scale of height (i.e., the height of my giraffe and the height of that tree) are conceptually different from and prior to the notion of the difference between these measurements (i.e., the height difference). Comparison can be considered higher-order measurement – the measurement of differences between lower-order measurements. This distinction between higher-order and lower-order scales is most evidently illustrated by (79) (which repeats (10)): we use *o'clock* to express the measurements on the scale of time, but the unit *hour* to express the difference between times. Therefore, given this distinction, the notion of subtraction is more straightforward than the notion of addition in addressing the difference (or distance) between two positions (i.e., two measurements) on a scale.

(78) My giraffe is 5 inches taller than that tree is. (= (4b))

$$\max(\{d : \text{my giraffe is } d\text{-tall}\}) \geq \max(\{d : \text{that tree is } d\text{-tall}\}) + 5''$$

(79) 5 o'clock is 2 hours later than 3 o'clock is. (= (10))

When the use of interval and the use of subtraction are combined, the currently proposed account correctly derives the truth condition for (80) (see also Figure 3): essentially, the shortest boy is at least 2 inches taller than the tallest girl is, while the tallest boy is no more than 5 inches taller than the shortest girl is. However, if we do not use interval subtraction to analyze this comparison but only focus on the scale of height on which the height of girls and boys are measured, it is quite likely that we would wrongly use the distance between the height of the tallest girl and the shortest boy as the distance between two intervals (cf. previous interval-based accounts like Schwarzschild and Wilkinson 2002, Landman 2010).

(80) The boys are all between 2 and 5 inches taller than every girl is.

$$\forall x[x \sqsubseteq_{\text{atom}} \oplus \text{boy} \rightarrow \text{height}(x) \subseteq \iota I'[I' - \iota I. \forall y[\text{girl}(y) \rightarrow \text{height}(y) \subseteq I] = [2'', 5'']]]$$

It is also worth noting that the notion of intervals in our proposed analysis is fundamentally different from the notion of ‘degree plurality’. Essentially, the approach of ‘degree plurality’ is a further development of the ‘>’ analysis.

Suppose that we are comparing the height of my giraffe, which is 21 feet tall, with the height of three trees, which are 16 feet, 18 feet, and 20 feet respectively. Then as shown in (81), under the current ‘interval subtraction’ approach, the *than*-clause denotes an interval (see (81a)), while under the ‘degree plurality’ approach, the *than*-clause denotes the sum of three degrees (see (81b)).

(81) My giraffe is taller than every tree is.

- a. $[[\text{than every tree is (tall)}]] = [16', 20']$ the ‘interval subtraction’ approach
 $[[\text{(81)}]] \Leftrightarrow \text{height(my giraffe)} \subseteq \iota I' [I' - [16', 20'] = (0, +\infty)]$
- b. $[[\text{than every tree is (tall)}]] = 16' \oplus 18' \oplus 20'$ the ‘degree plurality’ approach
 $[[\text{(81)}]] \Leftrightarrow \forall d \sqsubseteq_{\text{atom}} 16' \oplus 18' \oplus 20' [\text{height(my giraffe)} > d]$

As mentioned in Section 5.1, the ‘degree plurality’ approach would still face the issue of scope island. As pointed out by Dotlačil and Nouwen (2016), the measurement of the subject of the matrix clause cannot be immediately compared with a sum of measurements, and thus the use of a distributivity operator, i.e., introducing a universal quantifier ‘ \forall ’, would be necessary (see (81b)). Therefore, as illustrated in (82), the introduction of this universal quantifier ‘ \forall ’ would lead to unattested scope ambiguity.

(82) Some giraffe is 2 feet taller than every tree is.

- a. $[[\text{(82)}]] = \exists x [\text{giraffe}(x) \wedge \forall d \sqsubseteq_{\text{atom}} 16' \oplus 18' \oplus 20' [\text{height}(x) \geq d + 2']]$ $\exists > \forall$
- b. $[[\text{(82)}]] = \forall d \sqsubseteq_{\text{atom}} 16' \oplus 18' \oplus 20' [\exists x [\text{giraffe}(x) \wedge \text{height}(x) \geq d + 2']]$ $\forall > \exists$
- \leadsto this reading is predicted by the ‘degree plurality’ approach but actually unattested.

There is one further reason that makes the ‘degree plurality’ approach doubtful in analyzing *than*-clauses like *than every tree is (tall)*. As shown in (83), the degree question *how tall are the boys* (which contains a plural DP) can be answered by a fragment answer like *5 feet, 5 feet 6 inches, and 6 feet (respectively)*, while such a fragment answer sounds much degraded for a degree question like *how tall is every boy* (which contains a universal quantifier). This contrast suggests that even if *5 feet, 5 feet 6 inches, and 6 feet* is indeed a degree plurality (i.e., a sum of degrees) and expressions like *than the boys are (tall)* indeed denote degree pluralities, it is unlikely that *than every boy is (tall)* also denotes a degree plurality.

Instead, *between 5 and 6 feet*, which indicates an interval, is a good fragment answer here. Similar observations can be made for other *wh*-questions. As illustrated in (84), while the sum *Madame Bovary, Jane Eyre, and Emma* is a felicitous fragment answer for *what did the boys read* (which contains a plural DP), it cannot be used to answer *what did every boy read* (which contains a universal quantifier). However, *a novel* is a good fragment answer for *what did every boy read* in this case. We do not delve into the details of fragment answerhood here, but the upshot is clear. For a *than*-clause containing a universal quantifier (e.g., *than every tree is (tall)*) instead of a plural DP, it is unlikely that the whole *than*-clause denotes a degree plurality.

(83) Suppose that Al, Bill, and Cal are 5 feet, 5 feet 6 inches, and 6 feet tall respectively.

- a. – How tall are **the boys**? ✓ 5 feet, 5 feet 6 inches, and 6 feet (respectively)
- b. – How tall is **every boy**? ? 5 feet, 5 feet 6 inches, and 6 feet (respectively)
- b'. – How tall is **every boy**? ✓ between 5 and 6 feet

(84) Suppose that Al read *Madame Bovary*, Bill read *Jane Eyre*, and Cal read *Emma*.

- a. – What did **the boys** read? ✓ *Madame Bovary, Jane Eyre, and Emma*
- b. – What did **every boy** read? # *Madame Bovary, Jane Eyre, and Emma*
- b'. – What did **every boy** read? ✓ a novel

Finally, we would like to cautiously point out that the very notion of 'degree plurality' might be dubious. The example in (85a) seems to give evidence that the notion of degree plurality is independently needed in natural language, since this sentence seems to have a cumulative reading (see Dotlačil and Nouwen 2016). However, it is likely that there is a silent *respectively* in this case (see (85b)). If it is so, then presumably, this kind of sentences do not have a typical cumulative reading. (86) and (87) show that in '*respectively*'-sentences, the order among the items conjoined by *and* matters, suggesting that in these cases, the use of *and* does not lead to sums of items as involved in typical cumulative reading sentences.

- (85) a. These three trees are 16 feet, 18 feet, and 20 feet tall. cumulative?
- b. These three trees are 16 feet, 18 feet, and 20 feet tall, respectively. not truly cumulative

- (86) John and Bill married Susan and Kate (respectively). not truly cumulative
- ~ John married Susan, and Bill married Kate. order matters

(87) The newborn's weight, length, and head circumference are 3.4 kg, 49.7 cm, and 33.6 cm, (respectively). not truly cumulative

To sum up, in this section, we further show how our current analysis differs from and works better than previous interval-based accounts (by adopting the interval subtraction operation) and the 'degree plurality' approach (by adopting the notion of interval and keeping canonical views on the island status of *than*-clauses). All these advantages naturally follow from the interval-subtraction-based treatment of comparatives (and the foundational assumption for the ontology of degrees in comparatives).

7 Conclusion

In this paper, we have presented a novel difference-based perspective to characterize the semantics of English clausal comparatives and implemented our analysis with the technique of interval subtraction.

Overall, we have shown that comparison can be considered the measurement of the difference between two existing measurements along a certain scale. This view is entirely motivated by the fact that the notion of measurable and comparable differences constitutes the very foundation underlying natural language comparative constructions. As we have shown throughout the paper, the use of interval subtraction is a conceptually natural way to implement this view and brings lots of empirical advantages.

Over the past few decades, the semantics of comparatives has constantly been a hot topic in formal semantics. We hope that our difference-based perspective will shed light on more empirically intriguing issues on this topic, including, for example, the distinction between phrasal vs. clausal comparatives, the semantics of cross-linguistic comparative constructions and other uses of gradable adjectives, the semantics of special comparative constructions (e.g., *Esme is more beautiful than Marie Curie was intelligent*, see Bale 2008; *Mary carried a heavier elephant a longer way than John did*, see von Stechow 1984), and the quantification and definiteness of degrees / intervals.

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