Abstract

Natural language meaning has properties of both (embodied) cognitive representations and formal/mathematical structures. But it is not clear how they actually relate to one another. This article argues that how properties of cognitive representations and formal/mathematical structures of natural language meaning can be united remains one of the puzzles in cognitive science. That is primarily because formal/mathematical structures of natural language meaning are abstract, logical and truth-conditional properties, whereas cognitive/conceptual representations are embodied and grounded in sensory-motor systems. After reviewing the current progress, this work offers, in outline, the general formulations that show how these two different kinds of representations for semantic structures can (potentially) be unified and also proposes three desiderata for testing, in brain dynamics, the mathematical equivalence between formal symbolic representations (and their transitions) and neuronal population codes (and their transitions).

1 Introduction

Linguistic meaning evinces facets of embodied cognitive representations and formal/mathematical structures. The central objective of this article is to argue that how they can be integrated and fully unified by being translated into one another both mathematically and ontologically remains one of the puzzles in cognitive science. Even though cognitive representations in classical cognitive science have been reckoned to be amodal and proposition-type (Fodor & Pylyshyn, 1988), embodied theories of cognitive representations have challenged this position (Barsalou, 2008; Shapiro, 2019; see also Cohn & Schilperoord, 2022). Within the frameworks of ecological (Turvey, 2018) and situated (Hutchins, 2010) cognition, even though mental representations are banished or downplayed, they are still cashed out in terms of action-perception cycles (Brooks, 1991). Ecological psychology does indeed admit of internal brain states (although they do not function as representations) for the encoding of information with reference to action-perception cycles (Turvey, 2018: 32), which can also enact linguistic meanings. Therefore, tensions between (embodied) mental states and formal/mathematical structures of linguistic meaning remain. This is especially because there can be gradations of embodiment (Chatterjee, 2010; Dove, 2016) and formal/mathematical structures are least embodied in not being intrinsic properties of brain structures and may have to be *indirectly* grounded in sensory-motor systems in terms of spatial encoding (Amalric & Dehaene, 2016) via a hierarchy of relations of groundedness (invariant physical constraints), embodiment (prior sensory-motor experiences) and situatedness (intentional actions through top-down and bottom-up interactions) (see Fischer & Shaki, 2018).

On the one hand, semantic structures have been analyzed in the tradition of formal semantics in terms of set-theoretic structures that have *extensions* in the world (Partee, 2004). Linguistic meaning in this tradition has been associated with denotation and truth values. So, specifying the conditions under which a sentence is true is actually a specification of the meaning of the sentence. Works in formal semantics (Chierchia & McConnell-Ginet, 1990; Larson & Segal, 1995; Heim & Kratzer, 1998) have followed these footsteps and carried forward the tradition. Further enrichments have come from Montague grammar (Dowty, 1979). Meaning is thus represented as a formal object derived compositionally from linguistic expressions in formulas of logics but devoid of any psychological anchoring. On the other hand, semantic structures in cognitive/conceptual semantics are patterns of conceptualization grounded in the mind (Jackendoff, 1990, 2002; Langacker, 1987, 1999; Talmy, 2000). On this view, semantic structures are themselves cognitive structures ultimately grounded in sensory-motor systems. Clearly, there seems to be a tension between the purely formal and abstract properties of linguistic meanings and the cognitive properties of linguistic meanings that have the grounding in the working of our cognitive machinery. Purely abstract properties appear to be non-embedded, while cognitive properties of semantic structures are embodied and also embedded. However, semantic structures in formal semantics have extensions in the real world or in some model of the real world (as in the model-theoretic view of semantics), and by virtue of this extensional orientation, abstract formal properties of semantic structures come to be related to the entities and their particular categories/grouping in the world.

This consideration notwithstanding, the tension remains, for the properties of cognitive representations are ultimately properties or categories of the cognitive organization in brains, whereas abstract formal properties of linguistic meanings are not properties or categories of brains or minds because they are categories of the outer world. Thus, their inherent properties seem to be irreconcilable. Additionally, this becomes pressing when one considers certain issues in semantic phenomena that demand references to mental states and cognitive representations (such as propositional attitudes conveyed by propositional verbs like 'believe', 'think' etc. and their propositional complements). Partee (1979) recognized this long ago.

> What I have tried to suggest is that the linguist's concern for psychological representation may be relevant to every semanticist's concern for an account of the semantics of propositional attitudes. So far I don't see how to achieve either goal; my only positive suggestion is that a good theory might be expected to achieve both at once. (Partee, 1979: 9)

In recent times, Warglien, Gärdenfors and Westera (2012) and Gärdenfors (2020) have also pointed to the necessity of integrating formal lexical-semantic properties of spatial expressions and event structures in general with conceptual spaces. The prevalent lacuna in the study of linguistic meanings is also articulated quite well by Krifka (2012).

On the one hand, the Frege/Montague research program, based on the idea that truth-conditions are the core ingredient of clause meaning and that meanings of complex expressions are computed from the meanings of the parts, has been extremely successful. On the other, it did not really address the central question: What, precisely, are the meanings of the smallest parts, the meanings of words, or rather, lexemes? (Krifka, 2012: 223)

2 Current progress and promises

Given that facets of both cognitive representations and denotative set-theoretic structures can be harmoniously found in semantic structures (Zwarts & Verkuyl, 1994; Hamm, Kamp & van Lambalgen, 2006), Zwarts and Verkuyl have notably shown that aspects of cognitive representations, especially with reference to Jackendoff's Conceptual Semantics, and denotative set-theoretic structures are compatible. However, this does not establish that cognitive representations and denotative set-theoretic structures of linguistic meanings are one and the same thing. The desired unified representation should encode aspects of both cognitive representations and set-theoretic structures in a mutually harmonious fashion. Also, it should have the explanatory import in showing how linguistic meanings that are embodied in sensory-motor systems come to simultaneously possess the formal-semantic properties that they do, and also vice versa (see Arbib, 2012). Another promising approach towards this has been *extensional superposition* (Thornton, 2021) whereby concepts that can be mapped onto their referents or extensions are combined via the operation of *superposition* that is supposed to be independent of, or neutral with respect to, semantic composition in language and conceptual combination in thought (see for detail, Appendix). This approach steers clear of the issue of how the ontological differences between cognitive representations and denotative set-theoretic structures can be reconciled and instead focuses on the *procedure* of meaning composition that can help get a handle on the expressive or combinatorial processes in both thought and language. The goal in this case is to show that the meaning-making procedures (that is, expressive or combinatorial processes) are overlapping or identical. But this leaves scope for the possibility that the outputs of such processes have divergent properties, for cognitive processes/procedures can be independent of outputs. Thus, the tensions remain between cognitive representations and denotative set-theoretic structures of linguistic meanings.

3 Future directions

A unified theoretical account of cognitive representations and formal/mathematical structures of linguistic meanings would not be simply a computational model of embodied linguistic meanings that can be described in formal-semantic terms, for (m)any embodied aspects of meaning can be described in set-theoretic terms. Rather, the theoretical unification will consist in the identification/integration of embodied cognitive/conceptual representations with formal-logical properties of linguistic meanings and vice versa (Mondal, forthcoming; see also the Appendix).

In addition, it will involve some sort of a dynamical account of the self-organization of formal-logical properties from the cognitive/conceptual representations of linguistic meanings and/or vice versa (see Steels, 2008). One way to go about this is to follow Graben, Barrett & Atmanspacher (2009) and show that the contents of embodied cognitive/conceptual representations and formal-logical properties of linguistic meanings are actually undifferentiated emergent states in brain dynamics. These emergent states can be thought of as higher-level states (cognitive/conceptual and logical representations) riding on the lower-level states of brain dynamics (dynamics of neuronal activations in neurons and neuronal assemblies). The higher-level states are thus macrostates and the lower-level ones are microstates. Such macrostates are said to be *contextually emergent*, especially when the lower-level description is necessary but not sufficient for the higher-level description (Graben, Barrett & Atmanspacher, 2009: 179). At this juncture, three significant desiderata seem important (see for formalization, Appendix).

The Mapping Condition

In the dynamical system space of the brain (modeled as a multi-dimensional continuous space), a class of microstates can be mapped onto the same macrostate (space) such that with reference to that macrostate (space) an *equivalent class* of microstates can be characterized. Now the implication of this for us is that if the same designated class of microstates is equivalently manifested as cognitive/conceptual representations and formal/logical representations for a linguistic structure, then the dynamical equivalence between them can be demonstrated.

The Uniformity/Stability Condition

Despite differences in two realizations of microstate dynamics, the macrostate dynamics must be very similar or identical in order to be stable or near-invariant over such differences in realization. What this means is simply that the macrostate dynamics for the cognitive/conceptual representations and formal/logical representations of a given linguistic structure should be very similar or identical even if their realizations in microstate dynamics are different. This will ensure a sort of uniform stability (or, simply, uniformity) in the macrostate dynamics for both cognitive/conceptual representations and formal/logical representations.

The Congruity Condition

In the continuous space of our dynamical cognitive system, one macrostate develops out of the previous stage of a macrostate as well as, of course, out of microstate dynamics. The time-dependent formal symbolic dynamics, given neural dynamics at the lower level, should display overlapping or identical attractor basin dynamics for any translation/encoding of cognitive/conceptual representations into formal/logical representations or vice versa, somewhat in line with the work in Dobosz & Duch (2010). This would ensure congruity between the brain dynamics for cognitive/conceptual representations and that for formal/logical representations of a given linguistic structure.

Since the continuous space in dynamical systems can be partitioned in order to derive symbolic dynamics realized in attractor basins (Dale & Spivey, 2005), this possibility needs to be explored further. The ultimate hope is that the unified formalism can perhaps fit and explain experimental findings in a better way than purely cognitive/conceptual approaches to linguistic meaning or purely formal/mathematical approaches.

Appendix

Extensional Superposition (Thornton, 2021):

Extensional superposition operates on extensional meanings that link to, but are not in themselves, concepts. If BOX and CONTAINER are such extensional meanings, Ξ(BOX) will be the extension of the concept attached to BOX and Ξ(CONTAINER) will be the extension of the concept attached to CONTAINER (Ξ has been used here instead of Thornton's ξ in order to avoid confusion with the *form normalizing function* in (1)). Thus, one concept via the extensional meaning can serve as a bounding or constraining concept over another in order to form a new concept. Thus, the bounding/constraining concept (placed before ⊣) is *superimposed* on the base concept (Thornton, 2021: 8-9). This is written as the following.

$$\Xi\ (\text{BOX} \dashv \text{CONTAINER})$$

This will yield the concept of containers that are boxes. Conversely, the following denotes the concept of boxes that are containers.

$$\Xi\ (\text{CONTAINER} \dashv \text{BOX})$$

Since these two may have different instantiation probabilities, the differences are shown below.

$$P(\text{BOX} \dashv \text{CONTAINER}) = \frac{|\ \Xi(\text{BOX}) \cap \Xi(\text{CONTAINER})|}{|\ \Xi(\text{CONTAINER})\ |}$$

$$P(\text{CONTAINER} \dashv \text{BOX}) = \frac{|\ \Xi(\text{BOX}) \cap \Xi(\text{CONTAINER})|}{|\ \Xi(\text{BOX})\ |}$$

The superimposition of extensional meanings can undergo self-embedding for the formation of more and more complex concepts. For instance, if we want to talk about green boxes that are containers, then we shall have that Ξ (GREEN ⊣ Ξ (CONTAINER⊣ BOX)), or more simply, GREEN ⊣ CONTAINER⊣ BOX.

The Equivalence between Conceptual/Cognitive Representations and Predicate Logic Structures (Mondal, forthcoming):

The idea here is to formulate a *general principle* of mathematical equivalence between cognitive representations of linguistic meaning and predicate logic structures (see (1)). The former is built on Jackendoff's conceptual functions and organized in terms of appropriate Trajector(TR)-Landmark(LM) alignments or Figure-Ground relations.

$$F(\langle T1, T2, \ldots \rangle) \equiv P(T1, \ldots, Ti) \vee (P'(t_1 \ldots t_j) \ldots \& \ldots P''(t_1 \ldots t_m)) \quad (1)$$

What (1) states is that a conceptual function F with its terms T1 ... Tn is formally equivalent to either a predicate P with its arguments, or to a conjoined/co-incorporated predicate. T1, T2 … can be the TR/Figure or LM/Ground. The *sequential organization* of T1, T2 … as the TR/Figure and/or LM/Ground is determined by the **control cycle** (Langacker, 2013), parallel to Talmy's (1988) **force dynamics**, which determines how an actor can exert control/force over an element that comes inside its dominion within the purview of a field. The alignment among them is specified by F on the left-hand side. It needs to be emphasized that the value of *i* in P(T1, ..., Ti) need not match the arity of F. This is exactly what the *form normalization function* in (2) enforces.

Fundamentally, what we need is generalized in (2).

$$\xi(CF^{\oplus}) \equiv PL^{R} \quad (2)$$

Here, ξ is a form normalizing function that maps a given embodied cognitive representation (CF) encoding conceptual functions (Jackendoff's) in the appropriate Trajector(TR)-Landmark(LM) alignment (Langacker's)/Figure-Ground relation (Talmy's) onto a predicate logic representation (PL^R). $CF^{\oplus}$ indicates (varying) levels of composition/concatenation of conceptual functions ('take-to-school', for instance, in PL^R has to be organized in terms of, or normalized from, a conceptual function for 'take' and a PATH function, and likewise, for 'rain', say, in the sentence 'It rained', the GO (with its lexicalized argument/term) and DOWNWARD functions specifying motion and path, respectively, have to be normalized to **rained**( )).

Therefore, if we have a sentence 'Rina opened the door', the representation in terms of conceptual functions would be the following.

$[_{Event}\, \text{CAUSE}\, (_{Object}\, \text{RINA},\, _{Object}\, \text{DOOR (+DEF)},\, _{Event}\, \text{INCH}\, (_{State}\, \text{BE}\, (_{Object}\, \text{DOOR},\, _{Property}\, \text{OPEN})\,)\,)\,]$ (3)

Here, T1 corresponds to the object 'Rina' (TR/Figure); T2 corresponds to the object DOOR (LM/Ground) and T3 to the event yielded as an output by INCH. CAUSE and INCH have to be normalized to yield **open**(T1, T2), where **open**=P and T3 does not correspond to any argument of the 2-place predicate **open**. In other cases, the form normalizing function ξ would be neutral, as in 'Rina likes the door', where the conceptual function LIKE corresponds to **like** in predicate logic and an isomorphic mapping can be established from T1, T2 of LIKE onto the terms/arguments of **like**. Furthermore, on the right-hand-side we have that P' ... & ... P'' = F, where *at least one* P from (P' ... & ... P'') *can* express a T_k such that $k \geq 1$. This is needed for cases like 'take-to-school' in PL^R, since the PATH 'to school' will be realized as a T within the argument structure of TAKE as an F on the left-hand side.

Thus, the left-hand side of (1) above defines $CF^{\oplus}$ and can also be expressed as (4).

$$F(<T1, T2,...>) \dashrightarrow F \otimes F^{+} \quad (4)$$

$F \otimes F^{+}$ designates a combination of one conceptual function with 0 or more conceptual functions. This can also be conceived of in terms of the *tensor product* of F and F^{+} if they are represented as vectors/vector spaces. The right-hand side of (1) above characterizes PL^R.

Now, in order to show how formal-semantic structures can be identified with cognitive representations in brain dynamics, we may suppose that microstates (say, L) in neuronal networks as a dynamical system can be mapped (by M_n) onto macrostates (say, H) admitting of a higher-level description, as Graben, Barrett & Atmanspacher (2009) have done. In the present case, the microstate can be microscopic activations in neurons and neural assemblies and the macrostates would be cognitive representations (CR) and formal-logical structures (PL) of linguistic meaning. CR and PL can thus be viewed as macroscopic vector spaces that are *contextually emergent* states. The three conditions/desiderata are formally framed below.

The Mapping Condition

With reference to the actual contexts of CR or PL, partitions of the neural phase spaces will give us the equivalent class E^c of microstates mapped onto the same vector h in the macrostate space H such that $M_n^{-1}(h)=E^c$, with the understanding that the mapping (φ_n) from a macrostate onto the set of all partitions of the macrostate space is available. The time-dependent iterations (as determined by the nonlinear mapping Φ_w) of E^c will help see how an equivalent class of microstates is manifested as the same macrostate space, as shown in (5).

$$\varphi_n(h)=M_n(\Phi_w(E^c)) \subset H \quad (5)$$

This means that a number of epistemically equivalent microstates should be manifested as either CR or PL. If this holds for CR as H and also for PL as H, then the dynamical equivalence between CR and PL can be established.

The Uniformity/Stability Condition

The identification of CR with PL as a macrostate warrants the *stability* of the macrostate over the microstates such that even if two random realizations of microstate dynamics are different, the macrostate dynamics will be very similar or identical.

$$\text{Probability}\{||M_n(\Phi_w(l))- M_n(\Phi_{w'}(l))|| > \varepsilon\} < \delta \quad (6)$$

Here l is a microstate vector in L, and ε, $\delta > 0$. This suggests that the macrostate dynamics for CR and PL should be similar or identical even if their realizations in random microstates are different.

The Congruity Condition

A macrostate also depends, for macrostate dynamics, on a previous macrostate derived from a microstate. Given φ: H→H, the following generalization holds.

$$h(t+1) = \varphi(h(t)) \quad (7)$$

While errors due to correlations at the level of microstates cannot be ruled out, macrostate dynamics in general need to preserve form normalization (in (2)) in both forward and inverse directions across mappings in (7) so that the macrostate dynamics for CR and PL exhibit similar/shared or identical attractor basin dynamics (see Dobosz & Duch, 2010).

$$[h(t+1) = \varphi^{\xi}(h(t))] \cong [h(t+1) = \varphi^{\xi-1}(h(t))] \quad (8)$$

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