A new defense of a classic semantics for bare numerals*

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Abstract

The semantics of numerals has been the subject of intense research, and controversy, over the last few decades, and yet there is still no consensus on what the basic, lexical meaning of a word like *three* is. The classic approach to numerical interpretation, first articulated by Horn (1972), contends that the basic meaning of *three* in a sentence like *Three people attended* is 'at least three', and that the upper bound it implies arises by way of scalar implicature. More recent approaches argue that the basic meaning of *three* is 'exactly three', implemented either with a uniqueness condition on existential quantification (Geurts 2006), or with a cardinality operator that limits the size of the individuals being existentially quantified over (Breheny 2008), or with a maximality operator that imposes an upper bound (Kennedy 2015). The goal of this article is to defend a version of the classic account and to argue against the 'exactly' accounts. The case rests on two new sets of arguments. The first set concerns numerical noun phrases that combine with collective predicates, as in *Three people lifted the piano together*: the classic account makes just the right predictions, as already argued by Koenig (1991), while the 'exactly' accounts overgenerate. The second set pertains to new data involving generically interpreted numerical noun phrases, as in *Three people can fit in the car* and *Three people can lift the piano*, which involve quasi-universal quantification over triplets of individuals (Link 1987, 1991). I show that the classic account provides a simple, uniform analysis of the interpretation of numerals across both existential and generic contexts. The 'exactly' accounts are shown to also be able to handle generic

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cases (by mirroring the classic account), but Kennedy's account overgenerates. The conclusion, taking into account the entirety of the data discussed here, is that the classic, scalar implicature approach to numerals is more empirically adequate than all of the 'exactly'-based approaches.

Keywords: numerals; plurality; distributivity; collectivity; genericity; scalar implicature

1 Introduction

This article concerns the semantics of number words, or *numerals*, such as the word *three*. Despite several decades of research on the topic, there is still no consensus on what the basic, literal meaning of a word like *three* is. One reason is that its interpretation is, in some sense, variable: in some environments, it seems to mean 'exactly three', while in other environments, it seems to mean 'at least three'.[1] For example, an utterance of (1) naturally implies that it is not the case that more than three people attended; hence, (1) is typically construed as expressing the proposition that *exactly* three people attended.

(1) Three people attended.

By contrast, an utterance of (2) does *not* imply that Ann is required not to solve more than three problems, i.e. that in order to pass, she needs to solve exactly three problems; rather, (2) is typically construed as expressing the proposition that Ann is required to solve *at least* three problems.

(2) Ann is required to solve three problems (in order to pass).

There are two broad approaches to this puzzle.[2] The 'classic' approach, first articulated by Horn (1972), contends that the basic, literal meaning of *three* in a sentence like (1) is 'at least three', and that an upper-bound inference arises by way of scalar implicature, resulting in the reading 'at least three but not at least four', i.e. 'exactly three'. No analogous implicature arises for (2). I will discuss the basic idea behind the classic view, and its merits, in §2 and present a compositional implementation of the account in §3. In presenting the classic account, I will

1 I use *italics* to refer to object-language expressions (as well as for emphasis and for introducing terminology), sans serif to refer to metalanguage expressions, 'single quotes' to informally describe particular readings or interpretations of an expression, and "double quotes" for direct quotations from the literature.

2 My discussion of the various approaches to numerals draws heavily on Spector 2013. I have simplified Spector's classification of approaches (he divides them into four types) by lumping the 'exactly'-only view and the ambiguity views into one category and by not discussing the underspecification view at all. (See Spector 2013 for arguments against the underspecification view, namely his conclusion that "there is no compelling evidence that a bare numeral could ever receive an 'at most' interpretation, and there is in fact evidence against such a view".)

point out a number of correct predictions it makes regarding the interpretation of numerals in sentences with collective predicates, such as (3). One contribution of this article is to highlight the classic account's record of success when it comes to collectivity.

(3) Three people lifted the piano together.

Another contribution of this article is to introduce new data involving generically interpreted numerical indefinites (§4). The relevant examples fall into two categories. The first category is exemplified by (4). On its generic reading, (4) means, roughly, that any group of three people can fit in the car. As in the case of (1), an utterance of (4) implies an upper bound, viz. that four people are too many.

(4) Three people can fit in the car.

The second category is exemplified by (5). On its generic reading, (5), just like (4), means, roughly, that any group of three people can lift the piano. However, in this case, a lower bound is implied, viz. that two people are too few.

(5) Three people can lift the piano.

I show that the classic account gets these facts exactly right, using well-motivated syntactic, semantic, and pragmatic mechanisms, and in a way that is completely parallel to its account of numerical interpretation in the existential domain.

Despite the broad success of the classic approach, a number of competing theories have been proposed in recent years, which contend roughly the opposite of the classic view — namely, that the basic, literal meaning of *three* in (1) is 'exactly three', and that its 'at least three' reading is somehow derived on the basis of the 'exactly three' meaning, either by semantic typeshifting operations (Geurts 2006; Kennedy 2015), or by pragmatic reasoning (Breheny 2008). One motivation for this line of analysis is that numerals can easily have their strong, 'exactly' readings even in downward-entailing environments — a fact that is inconsistent with a scalar implicature outlook. I will show that each of these alternative views makes incorrect predictions for numerical sentences involving collective predicates (they derive unattested ambiguities), and that each one is unable to provide a uniform analysis of bound inferences across both the existential and generic domains — that is, they rely on different mechanisms for different bound inferences (§5). (I also show that Kennedy's account overgenerates in the generic domain.)

Nevertheless, the question arises whether a uniform analysis is in fact desired. I will discuss in §6 the merits of a non-uniform analysis, whereby bound inferences sometimes arise as semantic entailments, and other times as scalar implicatures. The reason for entertaining this asymmetric approach is that there appears to be an asymmetry in the distribution of strong readings of numerical indefinites in downward-entailing environments: they are available in existential constructions, but not in generic ones. However, I will argue that the classic account has a potential explanation of this asymmetry. The conclusion, taking into account the entirety of the data discussed here, is that the classic, scalar implicature approach to numerals is empirically more adequate than all of the 'exactly'-based approaches.[3]

2 The classic, scalar implicature view

In this section, I describe the basic idea behind the 'at least' view of numerals and a number of correct predictions that it makes. In §3, I will present a compositional implementation of the 'at least' approach, and highlight some further correct predictions, before turning to novel data from genericity in §4.

2.1 The basic idea

The classic view is summed up best by Horn (1972) himself.

> "Let us assume that these conversational postulates [Grice's maxim of quantity] govern the interpretation of given occurrences of a cardinal number. Numbers, then, or rather sentences containing them, assert lower-boundedness — *at least n* — and given tokens of utterances containing cardinal numbers may, depending on context, implicate upper-boundedness — *at most n* — so that numbers may be interpreted as denoting an exact quantity." (Horn 1972, p. 41)

As we will see, the qualification "or rather sentences containing [numbers]" is important: many sentence types do not to imply any upper bound at all.

Consider again (1), repeated below.

(1) Three people attended.

Preliminary evidence that the upper-bound inference associated with (1) is an implicature is that it is cancellable, suspendable, etc., just like the 'not all' implicature of *some* and the 'not both' implicature of *or*.

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3 A quick note on example sentences. The literature on plurality contains many examples like *Three boys together carried a sofa up the stairs* (Koenig 1991), *Three men can lift the piano* (Link 1987, 1991), and so on. In keeping with the LSA's guidelines for inclusive language (`http://www.linguisticsociety.org/content/guidelines-inclusive-language`), I have taken the liberty of changing, without warning, occurrences of *boys* and *men* to just *people*, and of injecting female names into discussions where male names have historically been used. This choice has particularly important consequences for generic sentences with ability modals, like *Three men can lift the piano*, which unfortunately seems to imply that three women *cannot* lift it.

(6) a. Three people attended, $\left\{\begin{array}{c}\text{and possibly even}\\ \text{if not}\end{array}\right\}\left\{\begin{array}{c}\text{more}\\ \text{*fewer}\end{array}\right\}$.

b. Three people attended; in fact, four did. (cf. Horn 1972, p. 38)

(7) a. Ann read *Hamlet* or *Macbeth*, $\left\{\begin{array}{c}\text{and possibly even}\\ \text{if not}\end{array}\right\}$ both.

b. Beth solved some of the problems; in fact, she solved all of them.

It will be instructive throughout this article to compare the behavior of *three* (*some*, etc.) with that of *only three* (*only some*, etc.), in which the same inferences ('not four', 'not all', etc.) arise, but are known to be semantic entailments. In the case at hand, adding *only* renders the follow-ups infelicitous, thus lending further support to the idea that the inferences without *only* are indeed implicatures, not entailments.

(8) a. *Only three people attended; in fact, four did.

b. *Beth solved only some of the problems; in fact, she solved all of them.

Let us therefore assume that (1) is assigned the meaning 'at least three (i.e. three or more) people attended'. (In §3, I will present several ways of cashing this out compositionally.) Let us also assume that numerals form a so-called *Horn scale* (Horn 1972): ⟨. . . , *two*, *three*, *four*, . . .⟩. Then the upper-bound inference of (1) can be derived as a scalar implicature on the basis of familiar neo-Gricean reasoning, sketched out informally as follows.[4]

(9) a. The speaker uttered (1), hence believes that three or more people attended (*maxim of quality*).

b. If the speaker believed that four (or five, or . . .) people attended, it would have been preferable to utter *Four (five, . . .) people attended*, since that is more informative (*maxim of quantity*).

c. Therefore, it is not the case that the speaker believes that four (or five, or . . .) people attended.

d. Assuming that the speaker is knowledgable about whether four (or five, or . . .) people attended, it follows that the speaker believes that four (or five, or . . .) people did *not* attend, hence that *exactly* three people attended.

4 For a contemporary implementation of this reasoning process, see Sauerland 2004. Note also that the scalar implicature view of upper-bound inferences need not be cast in neo-Gricean terms. One could also assume that scalar implicatures are grammatically derived via a covert exhaustification operator (Chierchia, Fox, and Spector 2012; Fox 2007). In fact, this is precisely what Spector (2013) ultimately proposes for the analysis of numerals. Likewise, we need not rely on Horn scales, but could instead adopt the view that alternatives are structurally defined, in the sense of Katzir 2007.

2.2 Predictions

Since upper-bound inferences are treated as scalar implicatures, the prediction of this account is that numerals should pattern like other scalar items, e.g. *some* and *or*, in terms of inferences. Let us return to (2), repeated below, and recall that *three* is construed as 'at least three' here; that is, we do not infer that it is forbidden for Ann to solve more than three problems, in the way that we do for (10).[5]

(2) Ann is required to solve three problems (in order to pass).

(10) Ann is required to solve only three problems (in order to pass).

A similar pattern holds for *some* and *or*: *some* in (11a) is naturally interpreted as 'some or all' (i.e. we do not infer that it is forbidden for Ann to read all of Shakespeare's plays), and *or* in (11b) is naturally interpreted inclusively ('... or both'; i.e. we do not infer that it is forbidden for Ann to read both *Hamlet* and *Macbeth*).[6]

(11) a. Ann is required to read some of Shakespeare's plays.

b. Ann is required to read *Hamlet* or *Macbeth*.

Moreover, from (2) we infer that Ann is not required to solve more than three problems (i.e. three is sufficient). Similarly, (11a) implies that Ann is not required to read all of Shakespeare's plays, and (11b) implies that Ann is not required to read both *Hamlet* and *Macbeth*.

All of these facts follow straightforwardly from the scalar implicature view. Here is why. The relevant alternatives under consideration are those in (12), and negating these results in inferences of the form 'Ann is not required to ...', rather than 'Ann is required not to ...'.[7]

(12) a. Ann is required to solve four problems (in order to pass).

b. Ann is required to read all of Shakespeare's plays.

c. Ann is required to read *Hamlet* and *Macbeth*.

So far so good. Another prediction concerns downward-entailing (DE) envi-

5 Note that (10) also has a separate, evaluative sort of 'at least' reading (where *only* takes widest scope), which states that three problems is the most that Ann is required to solve in order to pass, and that three counts as small. On this reading, Ann is not forbidden from solving more than three problems.

6 More precisely, these are the natural interpretations under the narrow-scope readings of the scalar items. On their widescope readings, (11a) means that there are some particular Shakespeare plays that Ann is required to read, and (11b) means that either Ann is required to read *Hamlet*, or she is required to read *Macbeth*, and the speaker is unsure which.

7 Of course, the grammatical approach to scalar implicature (cf. fn. 4) would predict both inferences to be possible, depending on the scope of the covert exhaustivity operator relative to the modal.

ronments, where scalar implicatures typically disappear.[8] To illustrate, *or* in the examples in (13) is most naturally interpreted inclusively, not exclusively. For example, (13a) means that Ann read neither *Hamlet* nor *Macbeth* (hence not both, either); it does not have the meaning 'it is not true that Ann read *Hamlet* or *Macbeth* and not both', which is equivalent to 'either Ann read neither *Hamlet* nor *Macbeth*, or she read both of them'. Similar remarks hold for the other examples.[9]

(13) a. Ann did not read *Hamlet* or *Macbeth*.

b. Everyone who read *Hamlet* or *Macbeth* passed/failed.

c. Beth doubts that Ann read *Hamlet* or *Macbeth*.

d. If Ann reads *Hamlet* or *Macbeth*, she will pass/fail.

These facts have a natural explanation: in DE contexts, the relevant alternatives (e.g. *Ann did not read* Hamlet *and* Macbeth) are *weaker* than the original assertion; that is, they are entailed by the assertion and hence cannot be negated, and so no scalar strengthening occurs.

When it comes to numerals, however, the neo-Gricean account appears to make incorrect predictions: while they can certainly receive an 'at least' interpretation in DE contexts, numerals can also easily receive an 'exactly' interpretation. For example, (14a) can easily be construed as meaning that Ann did not read *exactly* three plays, as evidenced by the fact that it can be followed up with *. . . she read six!* Such a follow-up would be contradictory on an 'at least' construal of *three*.

(14) a. Ann did not read three plays.

b. Everyone who read three plays passed/failed.

c. Beth doubts that Ann read three plays.

d. If Ann reads three plays, she will pass/fail.

Horn, who later renounced his support of the classic view in light of such observations (Horn 1992, 2006), also notes the contrast in (15). Whereas the follow-up in (15a) intuitively contradicts the first sentence (because adoring entails liking),

8 The following examples involve *or*. I set aside *some* due to the complications it introduces in being a positive polarity item (i.e. in DE contexts *some* usually becomes *any*).

9 The reason for including both *passed* and *failed* is to control for contextual factors. For instance, it would perhaps be odd if everyone who read either *Hamlet* or *Macbeth* but not both passed, while not everyone who read both passed. (Typically, the more you read, the better your chances of passing.) Thus, it is arguable that *or* can indeed be construed exclusively here with *passed*, and that the contextual entailment that everyone who read both passed is responsible for the perception that *or* is interpreted inclusively. However, if we switch to *failed*, no analogous contextual entailment arises: if everyone who read either *Hamlet* or *Macbeth* but not both failed, it hardly follows that everyone who read both also failed. The fact that *or* nevertheless cannot be interpreted exclusively with *failed* suggests that it only has an inclusive interpretation in the restrictor of *everyone*. (Similar remarks hold for the antecedent of a conditional.)

the follow-up in (15b) (has a reading that) does not intuitively contradict the first sentence (because having four kids apparently does not necessarily entail having three kids).

(15) a. #Neither of us liked the movie — she adored it, and I hated it.

b. Neither of us has three kids — she has two, and I have four.

As Spector (2013) points out, there is a potential response to this shortcoming. Note first of all that even standard scalar items like *some* and *or* can receive their strengthened interpretation in DE contexts if they are focused, i.e. are pronounced with special intonation (indicated by capitalization below).

(16) a. #Ann did NOT read some of Shakespeare's plays — she read ALL of them!

b. Ann didn't read SOME of Shakespeare's plays — she read ALL of them!

(17) a. #Ann did NOT read *Hamlet* or *Macbeth* — she read BOTH!

b. Ann didn't read *Hamlet* OR *Macbeth* — she read BOTH!

Importantly, numerals do not require special intonation for their strengthened, 'exactly' construal (though they can have it, too).

(18) a. Ann did NOT read three plays — she read SIX!

b. Ann didn't read THREE plays — she read SIX!

The response, then, is that numerals are "intrinsically focused, in the sense that they automatically activate their alternatives (i.e. other numerals)" (Spector 2013). This view is also in line with the assumption of Krifka (1999) that numerals "can introduce alternatives without the help of focus".[10]

So, at this point, the scalar implicature view does quite well: it predicts an 'exactly' reading of numerals in the basic cases, it does not predict an 'exactly' reading of numerals in the scope of universal modals (and it predicts just the right inference, viz. 'not required . . . more than'), and it predicts an 'at least' reading of numerals in DE contexts (which seems accessible). In addition, its main drawback, viz. that it does not also predict the availability of an 'exactly' reading in DE contexts, can plausibly be explained by assuming that numerals are intrinsically focused, or that they can evoke alternatives without focus.

10 This view therefore predicts that numerals without overt focus should be able to receive their strong readings in basically any environment. However, as I will argue in §6, generically interpreted numerical indefinites in DE environments appear to never have strong readings, unless they are overtly focus marked (and even then, the reading is marginal); that is, they pattern just like standard scalar items in those contexts. This observation will serve as the basis for entertaining a hybrid approach to the interpretation of numerals in §6.

3 Compositional implementation of the classic view

Recall that, on the classic, scalar implicature approach to the interpretation of numerals, a sentence like (1), repeated below, is assigned a meaning that can be paraphrased as 'at least three (three or more) people attended'. The 'exactly' reading arises due to an upper-bound scalar implicature, viz. that it is not the case that four or more people attended.

(1) Three people attended.

The question we now address is how exactly such a meaning arises compositionally. We will see that there are a number of choices to make. Before proceeding to those choices, however, I will first discuss some other uses of numerals, which will narrow down the kind of analysis we want to propose, and which will additionally serve as the basis for some criticism against the alternative, 'exactly' approaches in §5.

3.1 More readings of numerals

In deciding on the basic, lexical meaning of a numeral like *three*, we need to decide whether we want to assign *three* an 'at least' meaning in *all* sentences in which it occurs, or just in certain types. It turns out that in some types of sentences (or environments), *three* never seems to mean 'at least three'.

3.1.1 Predicative uses of numerals

Consider first (19), in which *three* is in predicate position. As Landman (2004), Geurts (2006), and Rothstein (2013) all argue, *three* really seems to mean 'exactly three' here, and a theory that derives the 'exactly' meaning via scalar implicature is not a favorable theory.

(19) We are three people.

Geurts (2006) reasons as follows: while (1) and (19) appear, prima facie, to both have 'exactly' interpretations, only (1) licenses inferences to lower numerals. As he puts it (and switching now to his actual examples), "while there is no way of construing the number words so as to make [(20b)] come out valid, it is at least arguable that [(20a)] is valid in some sense".[11]

11 As we will see in §5, Geurts (2006) argues that (20a) is ambiguous between an 'at least' reading and an 'exactly' one, while (20b) has only an 'exactly' reading. His point here is that, even though the preferred reading of (20a) is the 'exactly' one, it *can* be interpreted in the 'at least' way, thus validating inferences to lower numerals; by contrast, in his system, (20b) can never be interpreted in an 'at least' way, hence never validates inferences to lower numerals.

(20) a. Five cows mooed. So: Four cows mooed.

b. These are five cows. So: These are four cows.

Landman (2004, pp. 22f), citing Partee 1987, observes furthermore that the upper bound associated with a numeral in predicate position, (21b), is not cancellable, suspendable, etc. in the way that it is for a numeral in argument position, (21a).[12]

(21) a. Three girls came in; in fact, four girls came in.

b. #The guests are three girls; in fact, they are four girls.

In sum, an empirically adequate implementation of the scalar implicature account of numerals needs to assign an 'at least' meaning when numerals occur in argument position, as in (1), (20a), and (21a), but not when they occur in predicate position, as in (19), (20b), and (21b).

3.1.2 Collective predicates

As first noted (I believe) by Koenig (1991), only sentences in which the numerical subject combines with a distributive predicate, such as *attend*, license inferences to lower numerals. When the numerical subject combines with a collective predicate, such as *carry a sofa up the stairs together* in (22), downward inferences do not hold.

(22) Three people together carried a sofa up the stairs.

(23) Three people together carried a sofa up the stairs.
$\not\Rightarrow$ Two people together carried a sofa up the stairs.

As Koenig points out further, this difference between distributive and collective predicates extends to judgments on upward compatibility, as shown by the contrasts in (24). The felicitous follow-up *in fact four* in (24a) cancels the implicature of the first part that the group in question does not consist of more than three people. By contrast, this very same follow-up is infelicitous in (24b), which suggests that the first part does *not* implicate the group in question does not consist of more than three people.

(24) a. Three people came, in fact four.

b. *Three people together carried a sofa up the stairs, in fact four.

In other words, a sentence like (22) seems to mean that (a group of) *exactly* three people carried a sofa up the stairs, not that (a group of) *at least* three people did so. To be sure, an utterance of (22) may implicate that only one group of people carried a sofa up the stairs, in which case it follows that no group of more than

12 Landman (2004) attributes this observation "to Barbara Partee, or Nirit Kadmon, or both". See also Rothstein 2013.

three people carried a sofa up the stairs. However, the sentence is still compatible with, say, a group of six people carrying a sofa up the stairs, just as long as (at least) one group of three people did so, too.[13]

In sum, an empirically adequate implementation of the scalar implicature account of numerals needs to assign an 'at least' meaning when a numerical subject combines with a distributive predicate, such as *attend* in (1), but a sort of 'exactly' meaning when it combines with a collective predicate, such as *carry a sofa up the stairs together* in (22), or *lift the piano together* in (3). (We will have more to say about the exact representation of the meaning of a collective sentence shortly.)

3.2 Syntactic-semantic implementation

Keeping in mind the observations above, let us now turn to the question of how to compositionally assign the proper interpretation to various sentences containing numerals. Like most contemporary work on quantification, we will assume that our domain of individuals contains not only ordinary, singular (or *atomic*) individuals, but also *sums* (or pluralities, or groups) of individuals (Link 1983). I will use the symbol '#' to denote the function that maps a sum x to the number of atoms that are part of x, so that '$\#x = 3$' means that x has (exactly) three atomic parts.

Given the preceding discussion, we want to assign an 'exactly' interpretation to sentences involving predicative *three*. In other words, the meaning of (19), repeated as (25a), should be represented as in (25b), not as in (25c).

(25) a. We are three people.

b. $\#(\mathsf{we}) = 3 \wedge \mathsf{people}(\mathsf{we})$

c. $\#(\mathsf{we}) \geq 3 \wedge \mathsf{people}(\mathsf{we})$

As for *three* in argument position, we want to assign an 'at least' interpretation when the verb is distributive. For (1), repeated as (26a), the representation in (26a) is perfectly suitable: it means that a group of three or more people attended, which seems to be just what we want.

(26) a. Three people attended.

b. $\exists x[\#x \geq 3 \wedge \mathsf{people}(x) \wedge \mathsf{attend}(x)]$

However, if we analyze the structurally identical case of (22), repeated as (27a), in an analogous way, with the representation in (27b), we make a wrong prediction: (27a) should have an 'at least' meaning, viz. that a group of three or more people

13 For instance, the following sentence is perfectly felicitous: *Three people together carried a sofa up the stairs; in fact, so did four people.* Here, the follow-up is not canceling the implication that first group mentioned consists of no more than three people (which, as we saw, would be infelicitous); rather, it cancels the implication that only one group of people (the group of three) carried a sofa up the stairs.

carried a sofa up the stairs. But we observed above, following Koenig (1991), that (27a) does not have this meaning.

(27) a. Three people carried a sofa up the stairs.

b. $\exists x[\#x \geq 3 \wedge \mathsf{people}(x) \wedge \mathsf{carry}(x)]$

Instead, the meaning of (27a) seems to have the representation in (28), which expresses the proposition that a group of *exactly* three people carried a sofa upstairs.

(28) $\exists x[\#x = 3 \wedge \mathsf{people}(x) \wedge \mathsf{carry}(x)]$

How do we reconcile the distributive and collective cases? It turns out that a representation of the meaning of (1) involving '=' instead of '≥', as in (29), is an 'at least' meaning, due crucially to the distributivity properties of the predicates in the sentence. Here is why. What (29) says is that a group of *exactly* three people attended. However, even if more than three people attended, then it is still true that a group of *exactly* three people attended: just take the total group of people who attended, pick any 3-membered subgroup, and that is a group of exactly three people who attended. On this analysis, then, the 'at least' meaning of (1) is thus an artifact of (i) existential quantification and (ii) distributive inferences.

(29) $\exists x[\#x = 3 \wedge \mathsf{people}(x) \wedge \mathsf{attend}(x)]$

To recap, the sentences above containing *three* should all be assigned meanings involving '='. Only when *three* occurs in argument position and the verb phrase is distributive will we derive an 'at least' reading, at which point an upper-bound scalar implicature may be triggered.

Now, to arrive compositionally at these representations, let us first assume, for simplicity, that *three* has two lexical entries, depending on whether it occurs in predicate or in argument position. Then the following two lexical entries would seem to do the trick.[14] (Note the use of '=' in both, following our discussion.)

(30) a. $[\![\text{three}_{\text{pred}}]\!] = \lambda x_e \,.\, \#x = 3$

b. $[\![\text{three}_{\text{quant}}]\!] = \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \exists x[\#x = 3 \wedge P(x) \wedge Q(x)]$

Using these entries, our running examples are assigned the right meanings on the basis of the following logical forms (LFs). Note that the type clash that occurs when *three*$_{pred}$ (type *et*) combines with *people* (type *et*) is resolved by a rule of semantic interpretation that allows two expressions of type *et* to combine intersectively (cf. the rule of *predicate modification* in Heim and Kratzer 1998).[15]

14 Rather than having the simple condition that '$\#x = 3$', we could have a more sophisticated meaning along the lines of Ionin and Matushansky 2006, involving a partition of x into three subparts of which P holds, which would allow us to compositionally analyze *three hundred* on the basis of the meanings of *three* and *hundred*.

15 Alternatively, we could assume that $[\![\text{three}]\!] = \lambda P_{et} \,.\, \lambda x_e \,.\, \#x = 3 \wedge P(x)$. A potential piece of evidence

(31) a. we [are [three$_{\text{pred}}$ people]]

b. [three$_{\text{quant}}$ people] attended

c. [three$_{\text{quant}}$ people] [carried a sofa up the stairs]

Now, it would be preferable to assign just one meaning to *three* in a way that still manages to assign to each sentence the right overall meaning. We can do this by assuming the predicative meaning to be the basic one, and, following the vast body of literature on existential indefinites, posit a separate mechanism that contributes existential quantification. There are many ways that existential quantification could be introduced, but for concreteness, let us avail ourselves of the typeshifting operator A (Partee 1987), which takes two sets and says that they have an element in common.[16]

(32) $\mathsf{A} = \lambda P_{\alpha t} \,.\, \lambda Q_{\alpha t} \,.\, \exists x_{\alpha}[P(x) \wedge Q(x)]$

The idea is that *three*$_{pred}$ combines intersectively with a noun phrase (on the basis of its predicative meaning), and then A connects everything up.

(33) a. [three$_{\text{pred}}$ people] attended

b. $\mathsf{A}(\mathsf{three}(\mathsf{people}))(\mathsf{attend})$
$\equiv \exists x[\#x = 3 \wedge \mathsf{people}(x) \wedge \mathsf{attend}(x)]$

(34) a. [three$_{\text{pred}}$ people] [carried a sofa up the stairs]

b. $\mathsf{A}(\mathsf{three}(\mathsf{people}))(\mathsf{carry})$
$\equiv \exists x[\#x = 3 \wedge \mathsf{people}(x) \wedge \mathsf{carry}(x)]$

At this point, we have what I would consider to be a suitable, contemporary version of the classic view of numerals.[17] However, let me make one small adjustment (which has been argued for in various places in the literature), mainly for presentational purposes (to maximize similarity across theories), because Kennedy

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in favor of this higher predicative type, at least for English, is the following: while a sentence like *We are three* is somewhat commonplace (e.g. uttered to the host at a restaurant), structurally identical sentences are quite bad in English, e.g. *The books on the table are three* (cf. *The number of books on the table is three*). This suggests that *three* must typically combine with a common noun in order to be used predicatively, as in *The books on the table are three dictionaries*. (Similar remarks do not hold for languages like Italian and German, in which the equivalent of *three* can be productively used in predicate position without first combining with a noun.)

16 Another possibility is to assume an *existential closure*-like operator at LF (Heim 1982). Yet another move would be to assume the presence of a null existential determiner (Link 1987; Krifka 1999).

17 "Contemporary" is maybe a stretch: this view, including its application to collective constructions, goes back to at least Link (1987) and was discussed further by Koenig (1991) (who does not cite Link 1987). But it is contemporary in the sense that (i) it is quite different from the original conception of Horn (1972), and from the view of Generalized Quantifier Theory (Barwise and Cooper 1981), and (ii) most neo-Griceans still subscribe to the same general view, modulo the different technical choices mentioned here in footnotes.

2015 contains (something like) it. The adjustment is to reduce the denotation of *three* even further by identifying it with the number 3 (type *d*), which arguably accounts for its singular term use in sentences like *The number of children Floyd has is three* and *One and two make three*.[18]

(35) $[\![\text{three}]\!] = 3$

The predicative meaning of *three* then arises by combining it with a silent ⟨*many*⟩, which shifts the number-denoting meaning of a numeral to a predicative meaning.[19, 20]

(36) $[\![\langle\text{many}\rangle]\!] = \lambda n_d \,.\, \lambda x_e \,.\, \#x = n$

4 Evidence from genericity

Now that I have presented a compositional implementation of the classic, scalar implicature view, we are finally in a position to assess how it deals with the interpretation of generic numerical indefinites. I start by presenting a very simplistic analysis of genericity, just enough to give us some theoretical scaffolding, before proceeding to the discussion of bare numerals in generic sentences.

4.1 Basic generic sentences

The example in (37) is a run-of-the-mill *generic* (or generalizing, or characterizing) sentence (Carlson 1978; Schubert and Pelletier 1987; Krifka et al. 1995): it states a generalization of some kind (in this case, about cats).

18 Frege (1884, §57) argued that a numeral denotes a "proper name" (*Eigenname*), on the basis of singular term uses like in his famous example *Die Zahl der Jupitersmonde ist vier* ('The number of Jupiter's moons is four'), in which *die Zahl der Jupitersmonde* "denotes the same object as" *vier*.

19 Rothstein (2013) proposes that the predicative (type *et*) meaning is basic and that the singular term (type *d*) meaning is the "individual property correlate" of (the set denoted by) the predicative meaning (cf. Chierchia 1985). In other words, the type *d* meaning is derived from the type *et* meaning, not the other way around, which Rothstein argues is "a direct instantiation of Frege's insight that a property has 'two modes of presentation', one unsaturated in which it applies to an argument to form a sentence, and one saturated, in which it can itself be the subject of a predication".

20 Hackl (2000) proposes that numerals combine with an existential determiner-like version of ⟨*many*⟩, given below. Clearly, $[\![[\text{three } \langle\text{many}\rangle] \text{ NP}]\!]$ for Hackl is equivalent to $\mathsf{A}([\![[\text{three } \langle\text{many}\rangle] \text{ NP}]\!])$ on the current account, so for existential sentences, the choice of ⟨*many*⟩ does not matter. However, for generic sentences (§4), it is important that existential quantification not be built into the numerical indefinite, so that a generic operator can apply instead.

(i) $[\![\langle\text{many}\rangle]\!] = \lambda n_d \,.\, \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \exists x[\#x = n \wedge P(x) \wedge Q(x)]$

(37) Cats meow.

In particular, (37) states, roughly, that in general every cat meows (possibly with exceptions), or that every typical cat meows, hence the intuitive validity of the following inference.

(38) Cats meow.
Snowball is a (typical) cat.
⇒ Snowball meows.

We can represent this reading as follows, where '$\forall_{\text{Gen}}$' is a quasi-universal (i.e. restricted universal) quantifier that quantifies over all 'typical' individuals (of some sort or other).[21]

(39) $\forall_{\text{Gen}} x[\mathsf{cats}(x) \rightarrow \mathsf{meow}(x)]$

Assuming that the extension of plural expressions like *cats* may contain both atomic and properly plural individuals (see, e.g., Krifka 1999, 2003; Sauerland, Anderssen, and Yatsushiro 2005; Spector 2007), the derived reading, viz. 'every (typical) group of cats has the property of meowing', entails that every (typical) cat meows, as desired.

A common way to derive this reading compositionally is to assume a silent generic operator, *Gen*, at LF, whose semantics involves '$\forall_{\text{Gen}}$', as shown in (40).[22]

(40) $[\![\text{Gen}]\!] = \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \forall_{\text{Gen}} x[P(x) \rightarrow Q(x)]$

(41) a. [Gen cats] meow

b. $\forall_{\text{Gen}} x[\mathsf{cats}(x) \rightarrow \mathsf{meow}(x)]$

4.2 Bare numerals in generic sentences

Generic sentences with bare numerals appear to work similarly. Consider (4) and (5), repeated from §1. (4) means, roughly, that any group of three people can fit in the car, and (5), adapted from Link 1987, 1991, means (as Link also observes) that

21 I will refrain from providing a model theory for '$\forall_{\text{Gen}}$', and in particular leave open the question of what it means for an individual to be 'typical'. See Krifka et al. 1995 for a survey of a number of proposals, any one of which could be employed here. The exact treatment of '$\forall_{\text{Gen}}$', in particular how exceptions are allowed for, is an important issue in the semantics of genericity, but is, as far as I can tell, not very important for what I will have to say about generic numerical indefinites.

22 I have assumed here that *Gen* is a dyadic operator and that it combines first with the nominal predicate and then with the verbal predicate. It may be more appropriate to assume that *Gen* is located in the verbal projection, hence combines first with the verbal predicate, or that *Gen* is a monadic (sentential) operator. As far as I can tell, none of these distinctions matter much to the discussions in this article, and so for simplicity I analyze *Gen* essentially as a silent determiner.

any group of three people can lift the piano.[23]

(4) Three people can fit in the car.

(5) Three people can lift the piano. (Link 1987, 1991)

One way to see that these readings are quasi-universal is to note the robustness of the intuitive validity of the following inferences.

(42) Three people can fit in the car.
Ann, Bill, and Carol are three (typical) people.
$\Rightarrow$ Ann, Bill, and Carol can fit in the car.

(43) Three people can lift the piano.
Ann, Bill, and Carol are three (typical) people.
$\Rightarrow$ Ann, Bill, and Carol can lift the piano.

Assuming (uncontroversially) that *three* has, or can have, a predicative meaning, then these readings fall out naturally.

(44) a. [Gen [[three ⟨many⟩] people]] [can fit in the car]

b. $\forall_{\text{Gen}} x[[\#x = 3 \wedge \mathsf{people}(x)] \rightarrow \mathsf{can.fit}(x)]$

(45) a. [Gen [[three ⟨many⟩] people]] [can lift the piano]

b. $\forall_{\text{Gen}} x[[\#x = 3 \wedge \mathsf{people}(x)] \rightarrow \mathsf{can.lift}(x)]$

These observations are not new. The earliest relevant observation that I have managed to find is by Hoeksema (1983), who writes, "I do believe that numeral-noun combinations may have a generic reading, just like ordinary bare plurals (Carlson 1978)", and gives the following examples.

(46) a. Ten people can't form a soccer team.

b. Three cars are more expensive than two.

Hoeksema furthermore observes that "the familiar equation of *n* and *at least n* leads to wrong results here [. . .] [W]ith generic readings, we need *exactly n*". He is also careful to point out that "when the bare plural is read existentially, not generically,

23 I write *can lift the piano* instead of *can lift the piano together* because *together* is now redundant: on its generic reading, (5) necessarily states a generalization about groups of people being able to do a collective action. This is why #*Three people can speak French* is completely anomalous on its generic reading. One explanation for this generalization is the following: the generic reading with a distributive predicate like *can speak French* would state that any group of three people is such that each of them can speak French, but this is tantamount to just saying that every single person can speak French. In other words, the numeral does no semantic work. Such readings would thus be ruled out by a constraint like the one that Buccola and Spector (2016) propose for independent reasons to explain the range of readings of certain modified numerals.

this amounts to the same results as in the Barwise and Cooper 1981 definition", i.e. an 'at least' interpretation, for precisely the reasons mentioned in §3.2.

Similarly, Link (1987, 1991), arguing against a Generalized Quantifier (GQ) theoretical approach to numerals, had this to say.[24]

> "Numerals play a double role within the prenominal domain. First there is a clear indication that they occur as adjectival modifiers with an intersective meaning. Consider the pair *three people lifted the piano* vs. *(any) three people can lift the piano*; if the numeral *three* is treated like a quantifier with an in-built existential force appropriate for the first sentence, then the universal (or generic) force of the second sentence cannot be accounted for." (Link 1991)

> "The first sentence [*three people lifted the piano*] expresses a particular 'historical' fact (in a certain context), the second [*three people can lift the piano*] can be used as a generic statement about the piano in question. It is really these different uses here that introduce the existential and universal force, respectively. GQ theory cannot say that, as we saw, since it considers the (existential) quantificational force as inherent in the numeral." (Link 1987)

As we will see in §5, however, this criticism does not really extend to proposals like that of Geurts (2006) or Breheny (2008), even though they, too, build existential force into the numeral — because they also have a way to shift the numeral to an adjectival type. In addition, Link (and Hoeksema) did not consider the bound inferences that generic sentences with numerals give rise to, which is the topic of the next subsection. The analysis of such inferences, and the comparison of the competing proposals discussed in this article, can therefore be seen as a natural extension of Link's investigation into the "double role" that numerals play.

4.3 Bound inferences in generic sentences

Despite their structural and even semantic similarity, the two generic sentences in (4) and (5) nevertheless pattern differently in terms of inferences, viz. whether they license inferences to lower vs. higher numerals, and whether they imply an upper vs. a lower bound.

For instance, (4), with *can fit*, licenses inferences to lower numerals. This inference is a consequence of the more general fact that *can fit in the car* licenses inferences from groups to subgroubs: if a group x can fit in the car, then it follows that every subgroup of x can, too. As a result, if every group of three people can

24 Link (1987, 1991) only considers the predicate *can lift the piano*, not *can fit in the car*. As we will see in §4.3, these two predicates have quite different properties.

fit in the car, then so can every subgroup of every group of three people, which of course includes every group of two people.

(47) Three people can fit in the car. $\rightsquigarrow$ Two people can fit in the car.

By contrast, (5), with *can lift*, licenses inferences to higher numerals. This inference is a consequence of the more general fact that *can lift the piano* licenses inferences from groups to supergroups: if a group x can lift the piano, then it follows that every supergroup of x can, too (the larger the group, the easier the piano lifting). As a result, if every group of three people can lift the piano, then so can every supergroup of every group of three people, which of course includes every group of four people.[25]

(48) Three people can lift the piano. $\rightsquigarrow$ Four people can lift the piano.

In addition, (4) and (5) each license bound inferences, but in different directions. In particular, (4) implies an upper bound, while (5) implies a lower bound.

(49) a. Three people can fit in the car. $\rightsquigarrow$ Four people cannot fit in the car.

b. Three people can lift the piano. $\dashrightarrow$ Two people cannot lift the piano.

Admittedly, the lower-bound inference for (5) is not as strong as the upper-bound inference for (4), as indicated visually by the use of different implication symbols above. (In the next subsection, I will give one plausible reason why this is the case.) Nevertheless, both bound inferences appear to be real, and furthermore they appear to be scalar implicatures (recall the hallmark traits reviewed in §1): they are cancellable, suspendable, etc., which is only felicitous if the inferences exist in the first place.

(50) a. Three people can fit in the car; in fact, $\left\{\begin{array}{c}\text{more}\\ \text{*fewer}\end{array}\right\}$ can.

b. Three people can fit in the car, if not $\left\{\begin{array}{c}\text{more}\\ \text{*fewer}\end{array}\right\}$.

25 This example is reminiscent of an example discussed in Beck and Rullmann 1999, namely *Four eggs are sufficient (to bake this cake)*, which likewise licenses inferences to higher numerals (if four eggs are sufficient, then so are five, six, and so on). For the specific purpose of investigating the meaning of numerical indefinites, I prefer to avoid such examples because it is unclear that *four eggs* should be analyzed as quantifying over individuals (i.e. groups of eggs); for example, Rett (2014) argues that determiner phrases in such sentences denote degrees. Instead, I stick to examples like (4) and (5) and work under the assumption that the relevant expression (*three people*) ranges over sums of individuals, in the spirit of Hoeksema 1983 and Link 1987, 1991.

(51) a. Three people can lift the piano; in fact, $\left\{\begin{array}{c}\text{*more}\\ \text{fewer}\end{array}\right\}$ can.

b. Three people can lift the piano, if not $\left\{\begin{array}{c}\text{*more}\\ \text{fewer}\end{array}\right\}$.

In addition, the bound inferences do not persist when the numerical indefinite occurs in the scope of a universal modal. For example, (52) means that three *or more* people must be able to fit in the car; that is, it is compatible with more than three people being able to fit. It does not require that three and *no more than three* people be able to fit in the car.

(52) It is required that three people be able to fit in the car.

Similarly, (53) means that three *or fewer* people must be able to lift the piano; that is, it is compatible with fewer than three people being able to lift it. It does not require that three and *no fewer than three* be able to lift the piano.

(53) It is required that three people be able to lift the piano.

Finally, it is easy to construct examples where the bound inference disappears when the numerical indefinite occurs in a DE environment. For instance, the most natural reading of (54) entails that Ann doubts that more than three people can fit in the car. It does not seem possible to interpret (54) as asserting that Ann doubts (the proposition) that three and no more than three can fit.

(54) Ann doubts that three people can fit in the car.

To see this more clearly, note the contrast in acceptability between the follow-up sentence in (56a), which is perfectly fine, and the follow-up sentence in (56b), which intuitively contradicts the first sentence.[26]

(55) a. Ann doubts that only three people can fit in the car — she thinks four can.

b. ??Ann doubts that three people can fit in the car — she thinks four can.

Similarly, the most natural reading of (56) entails that Ann doubts that fewer than three people can lift the piano. It does not seem possible to interpret (56) as asserting that Ann doubts (the proposition) that three and no fewer than three can lift it.

26 In §6, I will discuss the question of whether generically interpreted numerical indefinites in DE environments can ever receive their strong reading. For now, however, the point is just that there are clear cases where generically interpreted numerical indefinites in DE environments have their weak readings.

(56) Ann doubts that three people can lift the piano.

Note, again, the contrast in acceptability between the follow-up sentence in (57a), which is perfectly fine, and the follow-up sentence in (57b), which intuitively contradicts the first sentence.[27]

(57) a. Ann doubts that three, but no fewer than three, people can lift the piano — she thinks two can.

b. ??Ann doubts that three people can lift the piano — she thinks two can.

In sum, both (4) and (5) license inferences to other numerals as well as bound inferences; however, the inferences they license are the reverse of one another: (4) licenses inferences to lower numerals as well as an upper-bound inference, while (5) licenses inferences to higher numerals as well as a lower-bound inference. In addition, the bound inferences in both cases exhibit the hallmark traits of scalar implicature.

4.4 The classic view revisited

Assuming that the inferences from higher (respectively, lower) to lower (respectively, higher) numerals reported in §4.3 are valid for (4) and (5), then clearly the concomitant upper-bound (respectively, lower-bound) inferences that those sentences imply follow from standard scalar reasoning, hence can be explained in terms of scalar implicature, in exactly the same way as the upper-bound implicature for (1) arises.

More precisely, let us assume that the following statements are true.

(58) a. *n people can fit in the car* is more informative than *m people can fit in the car* iff $n > m$.

b. *n people can lift the piano* is more informative than *m people can lift the piano* iff $n < m$.

Then the upper-bound inference of (4) can be derived as a scalar implicature on the basis of familiar neo-Gricean reasoning, sketched informally as follows (cf. the derivation of the upper-bound inference for (1) in §2).

27 For some reason, *Ann doubts that only three people can lift the piano* also fails to have the reading that Ann doubts that the lower bound is three, which is why I use *three, and no fewer than three*. This fact is surely connected to the fact that *??Only three people can lift the piano* is very odd as a generic sentence: it cannot seem to be used to express the generalization that three people can lift the piano, but no fewer than three. I have no explanation for why *only* is restricted in this way, but my hunch is that for some reason *only* seems to care about the natural ordering of numbers, and hence only ever negates numerical alternatives containing a numeral that is greater than the numeral in the prejacent. I leave a more detailed investigation of this puzzle for future research.

(59) a. The speaker uttered (4), hence believes that three people can fit in the car (*maxim of quality*).

b. If the speaker believed that four (or five, or ...) people could fit in the car, it would have been preferable to utter *Four (five, ...) people can fit in the car* since that is more informative (*maxim of quantity*).

c. Therefore, it is not the case that the speaker believes four (or five, or ...) people can fit in the car.

d. Assuming that the speaker is knowledgable about whether four (or five, or ...) people can fit in the car, it follows that the speaker believes that it is *not* the case that four (or five, or ...) people can fit in the car.

The lower-bound inference for (5) can be derived in a completely analogous way by replacing each occurrence of *can fit in the car* with *can lift the piano* and by replacing each occurrence of *four (or five, or ...)* with *three (or two, or ...)*.

Recall now the difference in intuitive strength between the upper bound implied by (4) and the lower bound implied by (5). If these bound inferences are all scalar implicatures, then this variation in strength is perhaps to be expected, for the following reason: the downward inferences responsible for the upper bound of (4) are arguably logical, due in part to the distributivity (an arguably logical property) of *fit*, whereas the upward inferences responsible for the lower bound of (5) depend on encyclopedic knowledge of the world. That is, one can imagine possible worlds in which three people can lift the piano, but four cannot (e.g. because they cannot all fit around the piano), but one cannot imagine any possible world in which three people can fit in the car, but two people cannot.

Finally, let me remind the reader (§3.1.2) that the scalar implicature account of bound inferences correctly predicts no bound inference to arise in the case of an existential sentence involving a collective predicate, like (3), with *lift the piano together*. This because, in this case, there is no difference in informativity between *n people lifted the piano* and *m people lifted the piano*, i.e. neither one asymmetrically entails the other (Koenig 1991).

5 Alternative views

In this section, I present a number of recent approaches to the semantics of numerals, which differ from the classic approach in that they all take the basic meaning of a numeral like *three* to be one that gives rise to an 'exactly' interpretation in a simple existential sentence like (1), discussed in §2. In other words, the 'exactly' construal of *three* in (1) is not a scalar implicature, but rather a semantic entailment arising from the literal meaning of *three*. One advantage of such an approach is that it correctly predicts the ability of numerals, observed in §2.2, to have an 'exactly' reading even in DE environments: it is simply their normal meaning. However, I will show that this general approach also leads to some unwanted consequences when it comes to sentences with collective predicates, like (3), discussed in §3.1.2. I will also discuss how such theories deal with the interpretation of *three* in generic sentences like (4) and (5), discussed in §4.2.

5.1 Geurts 2006: A uniqueness condition on existential quantification

Geurts (2006) proposes the lexical entry in (60) for the basic meaning of *three*.

(60) $[\![\text{three}_{\text{Geurts}}]\!] = \lambda P_{et} \, . \, \lambda Q_{et} \, . \, \exists! x[\#x = 3 \wedge P(x) \wedge Q(x)]$

This entry is identical to the quantifier entry for *three* that we saw in (30b), repeated below, except for one crucial difference: the presence of '!'. The notation '$\exists x![\ldots]$' is to be read as 'there is a unique (one and only one) x such that ...'.

(30b) $[\![\text{three}_{\text{quant}}]\!] = \lambda P_{et} \, . \, \lambda Q_{et} \, . \, \exists x[\#x = 3 \wedge P(x) \wedge Q(x)]$

5.1.1 Some positive predictions for Geurts 2006

The effect of the uniqueness condition is to derive an 'exactly' reading of *three* in existential distributive contexts: (1) is assigned the meaning in (61b), which says that there is a unique group of three people who attended. Now, if more than three people attended, then there is necessarily more than one group of three people who attended (hence, no unique such group), and so the sentence is false. As a result, the sentence means that three, and no more than three, people attended.

(61) a. [three$_{\text{Geurts}}$ people] attended

b. $\exists! x[\#x = 3 \wedge \textsf{people}(x) \wedge \textsf{attend}(x)]$

To derive an 'at least' construal of numerals Geurts avails himself of two typeshifting operators from Partee 1987. The first is BE, given below.

(62) $\textsf{BE} = \lambda \mathcal{P}_{(\alpha t)t} \, . \, \lambda x_\alpha \, . \, \mathcal{P}(\lambda y_\alpha \, . \, y = x)$

Geurts calls this operation "quantifier to predicate lowering" because when BE applies to the quantifier meaning of *three*$_{\textit{Geurts}}$ *people*, the result is the same predicative, i.e. type *et*, meaning as *three*$_{\textit{pred}}$ *people*.[28] (The lexical entry for *three*$_{\textit{pred}}$ was introduced in (30a) in §3.2.)

28 The BE operator therefore allows Geurts to analyze predicative uses of *three*, as in (19) (*We are three people*), discussed in §3.1.1.

(63) $\mathsf{BE}(\llbracket \text{three}_{\text{Geurts}} \text{ people} \rrbracket)$
$\equiv \lambda x_e \,.\, \#x = 3 \wedge \mathsf{people}(x)$
$\equiv \llbracket \text{three}_{\text{pred}} \text{ people} \rrbracket$

The second typeshifting operation is A, which was introduced in §3.2 and is repeated below. Unsurprisingly, Geurts is able derive an 'at least' reading for (1) by first applying BE to *three*$_{Geurts}$ *people*, and then applying A to the resulting predicative meaning, just as we did in §3 for the classic approach.

(32) $\mathsf{A} = \lambda P_{\alpha t} \,.\, \lambda Q_{\alpha t} \,.\, \exists x_\alpha [P(x) \wedge Q(x)]$

More generally, typeshifting will allow Geurts to replicate all the same desirable results that the classic account gets right. This includes deriving a basic existential reading for sentences like (3), with the collective predicate *lift the piano together*, as well as generic readings of numerical indefinites in sentences like (4) (with *can fit in the car*) and (5) (with *can lift the piano*). Note that there is no way for Geurts to derive the bound inferences in generic contexts as semantic entailments. As I will argue in §6, this appears to be a good thing. Geurts's account therefore amounts to a hybrid view of bound inferences: upper-bound inferences in basic existential cases are semantic entailments, while both upper-and lower-bound inferences in generic cases are scalar implicatures.

5.1.2 A negative prediction for Geurts 2006

The main problem for Geurts's account has to do with existential sentences involving collective predicates, such as (3): in addition to the basic existential reading derived by typeshifting (which is attested), his account also predicts the following reading to be available, where *three*$_{Geurts}$ *people* is not typeshifted at all.

(64) $\llbracket \text{three}_{\text{Geurts}} \text{ people} \rrbracket (\llbracket \text{lifted the piano together} \rrbracket)$
$\equiv \exists! x[\#x = 3 \wedge \mathsf{people}(x) \wedge \mathsf{lift}(x)]$

This reading can be paraphrased as 'there is a unique group of three people who lifted the piano'. On this reading, the sentence is compatible with smaller and/or larger groups of people having lifted the piano, just as long as one and only one group of *three* people did so. However, no such reading appears to be accessible. For example, if Ann, Beth, and Carol lifted the piano together, and Dan, Evan, and Fred lifted it together, then the sentence is simply true — there is no reading on which it is false. To be more precise, to the extent that the sentence feels false at all, this is presumably due to the potential implicature, described in §3 (see fn. 13), that no more than one group of people lifted the piano. In other words, there is no contrast in truth value judgment for sentence (3) between a context in which two groups of three people lifted it (and no one else did) and a context in which one group of three people lifted it and another group of more or less than three people lifted it (and no one else did), but Geurts predicts there to be a contrast: the sentence should be judged true in the latter context and false in the former context.[29]

5.2 Breheny 2008: Hardwiring upper-boundedness and cumulativity

Breheny (2008) proposes that the meaning of *three* is represented as in (65).[30] What *Three P Q* means is the following: if you collect all atoms or pluralities which fall in the extension of both P and Q, and then sum them all up, the cardinality of the resulting mega-sum is 3.

(65) $[\![\text{three}_{\text{Breheny}}]\!] = \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \#(\bigsqcup\{x : P(x) \wedge Q(x)\}) = 3$

5.2.1 Some positive predictions for Breheny 2008

Let us return to (1). On Breheny's account, (1) is assigned an 'exactly' meaning, for the following reasons. If exactly three people attended (a, b, and c, say), then on the basis of (65), (1) comes out true, because the mega-sum of all sums who are people and who attended ($a \sqcup b \sqcup c$) has cardinality 3. If more than three people attended (a, b, c, and d, say), then (1) comes out false, because the mega-sum of all sums who are people and who attended ($a \sqcup b \sqcup c \sqcup d$) has cardinality greater than 3. And if fewer than three people attended (a and b, say), then (1) comes out false, because the mega-sum ($a \sqcup b$) has cardinality less than 3.

Now, it turns out that if the typeshifting operators BE and A are available, then Breheny can derive the predicative meaning for *three* and hence also an 'at least'

29 One may try to explain the lack of contrast in terms of the notion of *charity*, which hypothesizes that if a sentence is true on one reading and false on another, listeners will simply judge the sentence true (Gualmini et al. 2008). However, if we embed the sentence in a DE environment, then charity is no longer at play (or more precisely, the 'direction' of charity should be reversed). Consider (i). In a scenario where two different groups of three people lift the piano and Ann loses the bet, Geurts predicts (i) to be ambiguous between a true reading and a false one. (Moreover, if charity is at play, listeners should simply judge the sentence true.) However, the sentence intuitively false in such a scenario. In other words, Ann clearly wins the bet if two different groups of three people lift the piano.

(i) If three people lift the piano together, then Ann wins the bet.

30 Actually, for the meaning of a numerically quantified noun phrase like *three P*, Breheny proposes the representation in (ia). Translated to our current formalism, in which pluralities are modeled as sums, not sets, (ia) becomes (ib). Abstracting over P yields (65) as the meaning of *three*.

(i) a. $\lambda Q \,.\, |\bigcup\{X : P(X) \wedge Q(X)\}| = 3$

b. $\lambda Q_{et} \,.\, \#(\bigsqcup\{x : P(x) \wedge Q(x)\}) = 3$

meaning for *three* in existential distributive contexts, just like Geurts (2006) does.[31]

(66) $\mathsf{BE}(⟦\text{three}_{\text{Breheny}}\text{ people}⟧)$
$= \lambda x_e\,.\,\#x = 3 \wedge \mathsf{people}(x)$
$= ⟦\text{three}_{\text{pred}}\text{ people}⟧$

More generally, just like for Geurts (2006), typeshifting will allow Breheny to replicate all the same desirable results that the classic account gets right, including deriving a basic existential reading for sentences like (3), with the collective predicate *lift the piano together*, as well as generic readings of numerical indefinites in sentences like (4) and (5).[32] In addition, just like for Geurts, there is no way for Breheny to derive the bound inferences in generic contexts as semantic entailments,[33] which, as I will argue in §6, is a positive feature. This account therefore amounts to another hybrid view of bound inferences: upper-bound inferences in basic existential cases are semantic entailments, while both upper- and lower-bound inferences in generic cases are scalar implicatures.

5.2.2 A negative prediction for Breheny 2008

Let us return to collective predicates. As Breheny (2008, fn. 9) observes, the representation for *three* in (65) "allows for a reading of *Three people lifted the piano* which is true if John single-handedly lifted the piano and Bill and Sam also did". This is because, for Breheny, the meaning of the sentence is basically 'the number of people who single-handedly lifted the piano, or were part of a group who lifted the piano, is 3'. More precisely, in a context where John (j) single-handedly lifted the piano and Bill and Sam ($b \sqcup s$) also did so together (and no one else did any piano

31 Breheny does not actually assume any typeshifting operator. Instead, he assumes that nouns can be implicitly restricted, so that *Three people attended* means 'exactly three people who are P attended', for some arbitrary property P. If P cannot be determined by context, then the sentence winds up meaning, essentially, 'there is a P such that exactly three people who are P attended', which is equivalent to 'three or more people attended', since, if more than three people attended, we can always find a P such that exactly three of the attendees are P.

32 The facts surrounding generic readings of numerical indefinites appear to require an amendment to Breheny's original theory, which does not involve any typeshifting. Recall (fn. 31) that Breheny's way of deriving 'at least' readings in existential contexts is *not* to typeshift *three NP* into a predicative type and then existentially quantify on top; rather, he leaves *three NP* as an existential quantifier with an 'exactly' meaning but assumes that the noun phrase can be implicitly restricted: 'For some property P, exactly three NPs are P and VP'. So, to the extent that generic sentences like (4) and (5) do indeed involve a generic operator, it follows that some kind of type-lowering operation is required, which (as long as a mechanism for introducing existential quantification is also available, e.g. A) thereby renders the assumption that noun phrases are implicitly restricted superfluous.

33 This is because Breheny, like Geurts (2006), hardwires '∃' into the basic, lexical meaning of *three*. One way to see this is to note the equivalence between the formulation in (65), $\lambda P_{et}\,.\,\lambda Q_{et}\,.\,\#(\bigsqcup\{x : P(x) \wedge Q(x)\}) = 3$, and $\lambda P_{et}\,.\,\lambda Q_{et}\,.\,\exists y[\#y = 3 \wedge y = \bigsqcup\{x : P(x) \wedge Q(x)\}]$. Obviously, using either entry for a sentence of the form *Three P Q* will only ever result in an existential reading, not a generic one.

lifting), the mega-sum of all sums who are people and who lifted, or participated in lifting, the piano ($j \sqcup b \sqcup s$) has cardinality 3.

While I grant that Breheny's sentence can be judged true in the context described above, our running example, (3) (*Three people lifted the piano together*), crucially features the word *together*, and it is clear that this sentence is *false* in that same context. I therefore take this to be a wrong prediction for Breheny.

But then what about the interpretation of Breheny's example, without *together*? Let me explain what I think is going on here, and in particular how the classic account can capture what I will call 'Breheny's reading' of the sentence, which involves two components: (i) it is compatible with no group of three people actually lifting the piano (e.g. if John single-handedly lifted it, and Bill and Sam also did so together, and no one else did any piano lifting), and (ii) it implies an upper bound on the number of people who lifted, or participated in lifting, the piano.

First, let us note that *people* and *lifted the piano* both have cumulative reference (Krifka 1989): If Ann and Bill are people, and if Carol and Dan are people, then it follows that Ann, Bill, Carol, and Dan are people; and similarly, if Ann and Bill lifted the piano, and if Carol and Dan lifted it, then it follows that Ann, Bill, Carol, and Dan lifted the piano (note the absence of *together*!).[34] Now suppose that John single-handedly lifted the piano and that Bill and Sam also did so together (and that no one else did any piano lifting). Then not only are John (the atomic individual j) and Bill and Sam (the plural individual $b \sqcup s$) in the extension of both *people* and *lifted the piano*, but by the cumulativity of *people* and *lifted the piano*, so is their combined sum ($j \sqcup b \sqcup s$). As a result, it follows that there is a group of three people who 'lifted the piano' (in the cumulative sense), and so the sentence *Three people lifted the piano* comes out true in such a scenario.

What about the upper bound entailed under Breheny's analysis? Krifka (1992, 1999) has independently observed that cumulative predicates give rise to scalar inferences: *Three people ate seven apples* implies that the total number of people who ate apples is three. Since *lift the piano*, like *eat seven apples*, can be interpreted cumulatively, an upper-bound scalar implicature is expected to arise.[35] I take its

34 One way to capture the cumulative reference of *people* and *lifted the piano* is to assume that they are parsed with a pluralizing operator, usually notated as '*', which closes their extension under sum (Link 1983).

35 Nevertheless, the fact that such bound inferences arise with cumulatively interpreted predicates is actually surprising: on their cumulative readings, there is no entailment relation between *m people ate seven apples* and *n people ate seven apples* (for $m \neq n$). For instance, if a group of four people ate a group of seven apples between them, then it certainly does not follow that a group of three people ate a group of seven apples between. Similarly, on their cumulative readings, there is no entailment relation between *m people lifted the piano* and *n people lifted the piano* (for $m \neq n$). To see this clearly, suppose that a and b lifted the piano together, and so did c and d, and no one else did. Then the extension of *lifted the piano* is $\{a \sqcup b, c \sqcup d\}$; hence, the extension of **lifted the piano* (see fn. 34) is the closure of this set under sum, viz. $\{a \sqcup b, c \sqcup d, a \sqcup b \sqcup c \sqcup d\}$. As we can see, there is a group of four

status as an implicature, instead of an entailment, to be a positive feature. For example, it explains why it is hard to judge the sentence false in a scenario where Ann, Bill, and Carol lifted the piano, and at least one other person who is *not* Ann, Bill or Carol also lifted it, or was part of a group who lifted it. In addition, this upper-bound inference seems to disappear in DE environments, as is expected if it is a scalar implicature: in a scenario where Ann makes the bet described by (67), if Beth, Carol, and Dan lift the piano, then Ann clearly wins the bet, regardless of whether some other person who is not Beth, Carol, or Dan also lifts it, or is part of a group who lifts it. If the upper bound were an entailment, as it is on Breheny's account, then it should be possible to argue that Ann lost the bet, but this seems wrong.

(67) If three people lift the piano, then Ann wins the bet.

In sum, the classic, adjectival account of numerical interpretation fully replicates Breheny's reading (modulo the upper-bound entailment/implicature difference) in a way that builds on independent observations about cumulative predicates.

We can also now diagnose the problem that Breheny's account faces for (3), which features *together*: Breheny's account essentially builds cumulativity of the two arguments into the meaning of *three*, but of course not every predicate is cumulative; thus, in cases where *three* combines with a non-cumulative predicate, the account makes a wrong prediction.

5.3 Kennedy 2015: Encoding a maximality operator

Kennedy (2015) analyzes *three* as a generalized quantifier over degrees (type $(dt)t$), which encodes a maximality operator: *three P* asserts that the maximum degree in the extension of P is 3.[36]

individuals who 'lifted the piano' (in the cumulative sense), but there is no group of three individuals who did so. Krifka's explanation of the scalar implicature is the following:

> "The typical 'purpose of information exchange' in which sentences with cumulative readings and indefinite NPs occur is such that the main interest is in how many entities of each sort participate in the cumulative relation, not in the individual relations between single entities or subgroups of entities." (Krifka 1992)

However, this explanation is not very satisfying: the question 'Why is n interpreted as the (total) number of people that ate apples/lifted the piano?' cannot be adequately answered with 'Because that is the main interest'. Nevertheless, the crucial point for my purposes is that there is independent evidence that cumulative sentences *do* imply upper bounds and that, whatever the explanation for such inferences is, it carries over straightforwardly to Breheny's example.

36 Kennedy describes this semantics for numerals as "de-Fregean": for Frege (1884), *three* is a second-order property of individuals, which is true of a property of individuals just in case that property is true of exactly three people; and similarly, for Kennedy, *three* is a second-order property of degrees, which is true of a property of degrees just in case that property has a maximum of 3. Put differently, on Frege's view, *three* denotes the equivalence class of sets of individuals whose cardinality is 3, while

(68) $[\![\text{three}_{\text{Kennedy}}]\!] = \lambda P_{dt} \,.\, \max(P) = 3$

5.3.1 Some positive predictions for Kennedy 2015

The higher-order version of *three* in (68) cannot combine directly with a noun phrase like *people* (or with ⟨*many*⟩), so it must move (QR), leaving behind a trace of type *d* that can combine with ⟨*many*⟩, and thus creating a degree predicate (type *dt*) in its scope, which serves as its argument.

(69) a. $\text{three}_{\text{Kennedy}}$ [1 [[[t_1 ⟨many⟩] people] attended]]

b. $\max(\lambda n_d \,.\, \exists x[\#x = n \wedge \mathsf{people}(x) \wedge \mathsf{attend}(x)]) = n$

The meaning derived for (1) can be paraphrased as 'the maximum number *n* such that at least *n* people attended is 3', which of course is equivalent to saying that *exactly* three people attended.

To derive an 'at least' construal of numerals, Kennedy, like Geurts (2006), avails himself of two typeshifting operators: BE, which we have already encountered several times, and iota, given below.

(70) $\mathsf{iota} = \lambda P_{\alpha t} \,.\, \iota x_\alpha [P(x)]$

By applying BE and then iota to the meaning of *three*, we derive the number 3.

(71) $\mathsf{iota}(\mathsf{BE}([\![\text{three}_{\text{Kennedy}}]\!]))$
$\equiv \mathsf{iota}(\lambda m_d \,.\, m = 3)$
$\equiv 3$

At this point, an at 'at least' reading arises in the familiar way, e.g. by combining the type-lowered *three* with ⟨*many*⟩ and then shifting the numerical noun phrase to an existential quantifier, exactly as the classic account does.

More generally, just like Geurts (2006) and Breheny (2008), typeshifting will allow Kennedy to replicate all the same desirable results that the classic account gets right, including deriving a basic existential reading for sentences like (3), with the collective predicate *lift the piano together*, as well as generic readings of numerical indefinites in sentences like (4) and (5).

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on Kennedy's view, *three* denotes the equivalence class of sets of degrees whose maximum is 3. An arguably more direct instantiation of a "Fregean" semantics for numerals is provided by Rothstein (2013), who assumes that *three* is born at type *et* and denotes the class of all sums of individuals consisting of exactly three atomic parts — which of course is a version of the classic approach to numerals. (See also fn. 19.)

5.3.2 Two negative predictions for Kennedy 2015

Let us return to (3), with the collective predicate *lift the piano together*. Like Geurts (2006) and Breheny (2008), Kennedy predicts (3) to be ambiguous between the basic existential reading above (which is attested), and another reading, in which *three*$_{Kennedy}$ is not lowered, and hence has an upper-bounding effect. This reading can be paraphrased as 'the maximum number n such that a group of n people lifted the piano is 3'.

(72) a. three$_{\text{Kennedy}}$ [1 [[[t$_1$ ⟨many⟩] people] [lifted the piano together]]]

b. $\max(\lambda n_d\,.\,\exists x[\#x = 3 \wedge \mathsf{people}(x) \wedge \mathsf{lift}(x)]) = 3$

On this reading, the sentence should be judged false in a scenario where Ann, Bill, and Carol lifted the piano together, and so did Dan, Ellen, Floyd, and Gwen. However, this prediction seems not to be borne out — the sentence is simply true.[37] To be more precise, the prediction is that there is an asymmetry in judgments regarding the truth value of (3) between a context in which a group of two and another group of three people lift the piano (it should be true) and a context in which a group of three and another group of four people lift the piano (it should be false). However, neither I nor any English speaker I have consulted detects any such contrast. Most judge (3) to be true in both contexts, and the judgments of those who find (3) to be false/odd in both contexts can be explained on the basis that (3) may implicate that no more than one group lifted the piano (see fn. 13).

Let us now return to generic sentences. Kennedy, unlike Geurts (2006) and Breheny (2008), predicts the generic sentences (4) and (5) to be ambiguous between the readings described above and another reading, in which *three*$_{Kennedy}$ is not lowered, and instead scopes above *Gen*, resulting in an upper-bounding effect.[38]

(73) a. three$_{\text{Kennedy}}$ [1 [[Gen [[t$_1$ ⟨many⟩] people]] [can fit in the car]]]

b. $\max(\lambda n_d\,.\,\forall_{\text{Gen}} x[[\#x = n \wedge \mathsf{people}(x)] \rightarrow \mathsf{can.fit}(x)]) = 3$

For (4), the reading so derived can be paraphrased as 'the maximum number n such that, in general, n people can fit in the car is 3', which is precisely the upper-bound generic reading we want to capture. Notably, the LF in (73) is nearly identical to the LF in (69) for (1).

However, there is a problem: since the upper bound is a semantic entailment, it should be accessible even when the numerical indefinite occurs in a DE environment, but this appears to be a wrong prediction. I will have more to say about DE environments shortly, in §6, but in the meantime, a simple example should

37 Spector (2015) has independently pointed out this same problematic prediction.

38 This point relies, of course, on the assumption that *three* is allowed to scope above *Gen*. I see no reason (syntactic or semantic) that would prevent such movement. If such a reason could be independently established, then the criticisms raised here would no longer apply.

suffice: in a scenario where Ann makes the bet described by (74a), if it turns out that four people can fit in the car, then Ann clearly wins the bet. If the clause in the antecedent of the conditional could have its upper-bounded reading, as Kennedy's account predicts, then one should be able to argue that Ann loses the bet, but this does not seem possible. It is instructive here to contrast *three* with *only three*: if the bet is instead the one described by (74b), then Ann clearly loses (or more precisely, does not necessarily win) the bet in that same scenario. In short, the fact that (74a) cannot be interpreted as (74b) suggests that the upper bound associated with (4) is not an entailment.

(74) a. If three people can fit in the car, then Ann wins the bet.
b. If only three people can fit in the car, then Ann wins the bet.

In addition, (5) should have an analogous parse, where numeral scopes above *Gen*. The reading thus derived is paraphrasable as 'the maximum number n such that, in general, n people can lift the piano is 3'. This reading is clearly not the lower-bound generic reading we wish to derive. Moreover, if we continue to assume that (or restrict ourselves to contexts in which) (5) licenses inferences to higher numerals, then the reading derived in (75) is a (contextual) contradiction: there is no maximum number n such that n people can lift the piano. Presumably, such an LF can be ruled out on the basis of its contradictoriness.

(75) a. three$_{\text{Kennedy}}$ [1 [[Gen [[t$_1$ ⟨many⟩] people]] [can lift the piano]]]
b. $\max(\lambda n_d \,.\, \forall_{\text{Gen}} x[[\#x = n \wedge \mathsf{people}(x)] \rightarrow \mathsf{can.lift}(x)]) = 3$

To summarize, Kennedy's account is yet another instance of a hybrid view of bound inferences, but slightly different from that of Geurts (2006) and Breheny (2008): for Kennedy, upper-bound inferences in both existential and generic cases can be derived as semantic entailments, while lower-bound inferences in generic cases are scalar implicatures.

6 Discussion

6.1 Recap

Let us step back and take stock of all the observations made so far. I argued in §3 that the basic meaning of *three* is a predicative one that characterizes the set of all individuals with (exactly) three atomic parts. (Actually, I assumed that *three* denotes the number 3 and that a silent ⟨*many*⟩ maps *three* to such a predicative meaning.) I called this approach a version of the classic view because in basic distributive cases, *three* ends up meaning 'at least three', while the utterance as a whole implicates 'not more than three', just as was first proposed by Horn (1972). It also correctly predicts a basic existential reading, with no bound inference, in cases of collective predication. Finally, I showed in §4 that this view naturally explains the readings of generically interpreted sentences as well, i.e. their truth-conditional content and the bound inferences they license, which again arise as scalar implicatures.

I then presented in §5 three alternative approaches that take the basic, literal meaning of *three* to be an 'exactly' one, implemented either with a uniqueness condition on existential quantification (Geurts 2006), or with a cardinality operator that limits the size of the sums being existentially quantified over (Breheny 2008), or with a maximality operator that imposes an upper bound (Kennedy 2015).

(76) a. $[\![\text{three}]\!] = 3$ (classic view)

b. $[\![\text{three}_{\text{Geurts}}]\!] = \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \exists! x[\#x = 3 \wedge P(x) \wedge Q(x)]$

c. $[\![\text{three}_{\text{Breheny}}]\!] = \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \#(\bigsqcup\{x : P(x) \wedge Q(x)\}) = 3$

d. $[\![\text{three}_{\text{Kennedy}}]\!] = \lambda P_{dt} \,.\, \max(P) = 3$

In addition, each of these alternative approaches has a way to shift the meaning of *three* in such a way as to make *three NP* predicative, which in turn means that they can replicate all the desirable results that the classic account gets right. This includes deriving 'at least' readings for sentences with distributive predicates, basic existential readings for sentences with collective predicates, and quasi-universal readings for sentences with generic numerical indefinites.

(77) $[\![[\text{three } \langle\text{many}\rangle] \text{ people}]\!]$ (classic view)
$\equiv \mathsf{BE}([\![\text{three}_{\text{Geurts}} \text{ people}]\!])$
$\equiv \mathsf{BE}([\![\text{three}_{\text{Breheny}} \text{ people}]\!])$
$\equiv [\![\langle\text{many}\rangle]\!]\,(\mathsf{iota}(\mathsf{BE}([\![\text{three}_{\text{Kennedy}}]\!])))([\![\text{people}]\!])$

However, each of the three accounts makes wrong predictions when it comes to collective constructions. Geurts (2006) predicts sentences with collective predicates to be ambiguous between a basic existential reading (which is attested) and a kind of 'unique group of size *n*' reading (which is unattested). Breheny (2008) predicts such sentences to be true even if no group of size *n* participated in the event, and I showed that this prediction is sometimes wrong; rather, it is only the case when the predicates in the sentence can be interpreted cumulatively. Finally, Kennedy (2015) predicts collective constructions to be ambiguous between a basic existential reading and a kind of 'maximum group size is *n*' reading, which I argued is unavailable. In addition, Kennedy predicts that the upper-bound inference associated with generic sentences should be accessible even when they occur in the scope of a DE operator, which I argued is incorrect (and will revisit in more detail shortly).

6.2 Downward-entailing environments revisited

On the basis of predictions for numerical indefinites in existential sentences with collective predicates, the classic account clearly wins: the other three accounts all incorrectly derive an upper-bound inference, while the scalar implicature account correctly predicts the absence of any bound inference.

However, recall from §1 that one of the main motivations for treating the upper-bound inference associated with numerals as a semantic entailment is that it easily persists in DE environments (unlike standard scalar items like *some* an *or*) — a fact that is inconsistent with a scalar implicature explanation of the upper bound. It turns out that both upper- and lower-bound inferences in generic sentences are different: they are systematically inaccessible when the generic numerical indefinite occurs in a DE environment, hence appear to be real scalar implicatures.[39]

Take, for instance, the contrast in (78): (78a) cannot be interpreted in the same way as (78b) to mean that if three, but no more than three, people can fit in the car, then we are too many.

(78) a. #If three people can fit in the car, then we are too many.
  b. If only three people can fit in the car, then we are too many.

Similarly, (79a) cannot be interpreted in the same way as (79b).

(79) a. #If three people can lift the piano, then we are too few.
  b. If three, but no fewer than three, people can lift the piano, then we are too few.

Similar remarks hold for the following two pairs of examples containing the DE operator *doubt*.

(80) a. #Ann doubts that three people can fit in the car — she thinks four can.
  b. Ann doubts that only three people can fit in the car — she thinks four can.

(81) a. #Ann doubts that three people can lift the piano — she thinks two can.
  b. Ann doubts that three, but no fewer than three, people can lift the piano — she thinks two can.

Note also the following contrast, modeled on the example from Horn 2006 in (15) from §2.

39 Spector (2015) has independently made the same point with regard to sentences like *It is sufficient to solve three problems to pass* and *On this highway, it is forbidden to drive 60 mph*, which pattern essentially like our running generic example (5) (*Three people can lift the piano*): they license inferences to higher numbers and also imply a lower bound, viz. that it is not sufficient to solve two problems and that it is not forbidden to drive 59 mph, but when these sentences are embedded under a DE operator, these inferences disappear.

(82) a. Neither lecture was attended by three people — two people attended the first one, and four people attended the second one.

b. #Neither car fits three people — this one fits two people, and that one fits four.

c. #Neither piano can be lifted by three people — two people can lift this one, and four people can lift that one.

Prima facie, these data appear to indicate that, in fact, a non-uniform approach to numerals is warranted, in particular one in which upper-bound inferences in existential distributive contexts are semantic entailments, while upper- and lower-bound inferences in generic contexts are scalar implicatures. In short, an approach like Geurts 2006 or Breheny 2008.[40]

We therefore have a sort of tie between two approaches to numerals: a Geurts/ Breheny-style account fails in cases of collective predication by predicting an unattested ambiguity, while the classic fails to predict the distribution of strong readings of numerical indefinites in DE environments just described. The obvious question now is whether we can find a tie-breaker. As far as I can see, there is no way to break the tie in favor of Geurts/Breheny: if existential distributive sentences are ambiguous, then so necessarily are existential collective ones. Instead, I would like to argue that there are observations that may point to an amendment of the classic account that would get all the facts right.

Recall the idea from §2 that the reason that existentially interpreted numerals in DE contexts can receive their strong readings is not because they have an intrinsic 'exactly' meaning, but rather because they are intrinsically focused (Spector 2013). If this view is correct, then wherever focused scalar expressions can have their strengthened meanings, then so should (unfocused) numerals; and conversely, wherever focused scalar expressions do *not* have their strengthened meanings, then neither should (unfocused) numerals — at least, not necessarily.

So, the question now is, how do overtly focused numerals and other overtly focused scalar expressions behave when they occur in a generic construction embedded in a DE environment? Judgments are admittedly subtle, but they seem to indicate that strong readings are, surprisingly, unavailable.

For example, there is not much contrast between the two sentences in (83) or the two sentences in (84): they all sound equally bad, especially when compared to the analogous pair of existential sentences, which both sound totally fine. Similarly,

40 Such an approach need not be cashed out exactly as Geurts or Breheny does. Another option, for example, is to maintain that *three* denotes the number 3 and to posit another, 'strong' version of ⟨*many*⟩, defined below (Nouwen 2010).

(i) $[\![\langle \text{many} \rangle_{\text{strong}}]\!] = \lambda n_d \,.\, \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \exists! x[\#x = n \wedge P(x) \wedge Q(x)]$

adding overt focus to *three* in all the odd examples above does not rescue them.

(83) a. #Three people can't fit in the car — FOUR can.
b. #THREE people can't fit in the car — FOUR can.

(84) a. #Three people can't lift the piano — TWO can.
b. #THREE people can't lift the piano — TWO can.

(85) a. Three people didn't attend — FOUR did.
b. THREE people didn't attend — FOUR did.

In other words, focus — whether overt or not — does not seem to help numerals to get their strong reading in these generic contexts.

Moreover, this observation appears to extend to other scalar expressions, too. For instance, (86), on its generic reading, states that roughly any recliner can fit in the room and that any small couch can fit, and it implicates that it is not the case that a recliner *and* a small couch can fit.

(86) A recliner or a small couch can fit in the room.

Unsurprisingly, this exclusivity inference disappears when the sentence is embedded under a DE operator. What *is* surprising, however, is that, unlike in existential constructions (cf. §2), adding focus to *or* does not make the exclusive reading reappear: the following sentences all sound very strange.

(87) a. #If a recliner OR a small couch can fit in the room, then we should sell our recliner and just bring our couch.
b. #Ann doubts that a recliner OR a small couch can fit — she thinks both can.
c. #It's not true that a recliner OR a small couch can fit — both can.

Contrast the above examples with the following counterparts containing an overt *only*, which sound quite good and have the intended meaning.

(88) a. If only a recliner OR a small couch can fit in the room, then we should sell our recliner and just bring our couch.
b. Ann doubts that only a recliner OR a small couch can fit — she thinks both can.
c. It's not true that only a recliner OR a small couch can fit — both can.

When we move to *can lift the piano*, since the direction of entailment is reversed, we need to use *and* instead of *or*.[41] With this in mind, consider (89). This sentence,

41 What I mean by "the direction of entailment is reversed" is that *n people can lift the piano* entails *m people can lift the piano* (on their generic readings) just in case $n \leq m$ (rather than $m \leq n$), and *x can lift*

on its generic reading, states that roughly any pair consisting of an experienced mover and a strong helper can lift the piano, and it implicates that it is not the case that any experienced mover can lift it on her own (or that any strong helper can lift it on his own).

(89) An experienced mover and a strong helper can lift the piano.

Unsurprisingly, this latter implicature disappears when the sentence is embedded under a DE operator, but as before, adding focus to *and* does not make the strong reading reappear: the following sentences all sound very strange.

(90) a. #If an experienced mover AND a strong helper can lift the piano, then we need to find an extra person.

b. #Ann doubts that an experienced mover AND a stronger helper can lift the piano — she thinks an experienced mover can do it on her own.

c. #It's not true that an experienced mover AND a strong helper can lift the piano — an experienced mover can do it on her own.

Contrast the above examples with the following counterparts containing an overt *only*, which, as before, sound quite good and have the intended meaning.

(91) a. If only an experienced mover AND a strong helper can lift the piano, then we need to find an extra person.

b. Ann doubts that only an experienced mover AND a stronger helper can lift the piano — she thinks an experienced mover can do it on her own.

c. It's not true that only an experienced mover AND a strong helper can lift the piano — an experienced mover can do it on her own.

The generalization here is that, for some reason, scalar expressions in generic sentences that are embedded in a DE environment never seem to be able to have their strengthened interpretation, regardless of whether the scalar expression is a numeral or *and*/*or*, and regardless of whether it is overtly focused or not. I have no explanation for this fact. But the point is that the inability of numerals in such contexts to have their strong readings is a symptom of a more general phenomenon. Moreover, while an account along the lines of Geurts 2006 or Breheny 2008 does already provide an explanation of the asymmetry between numerals in existential and generic constructions, two points should be borne in mind: (i) an explanation for the generalization just uncovered is still ultimately necessary, and (ii) such an account still makes wrong predictions for existential sentences with collective predicates. Taking all of these observations together, then, the classic view appears,

the piano entails *y can lift the piano*, for any two (possibly plural) individuals x and y, whenever $x \sqsubseteq y$ (rather than $y \sqsubseteq x$).

on the whole, to come out on top.

7 Conclusion

I have presented a range of facts, some old, some new, showing that numerals have a multiplicity of construals, depending on their linguistic environment, and I have argued that a classic semantics for numerals, supplemented with a mechanism for calculating scalar implicatures, straightforwardly and uniformly captures all the construals, without overgenerating. By contrast, accounts that build an 'exactly' component into the semantics of numerals overgenerate when it comes to cases of collective predication. Considerations pertaining to the interpretation of generic numerical indefinites embedded in DE environments led us to consider whether a hybrid approach might in fact fare better than the classic account, but I argued that the 'problem' faced by the classic account is a symptom of a more general phenomenon. The argument rests on some subtle judgments, which, needless to say, deserve further scrutiny. Nevertheless, from a cost-benefit perspective, the classic account comes out on top: it gets all the basic facts right, including the collective facts, in a simple, well-motivated, and uniform way, and it still has a chance to get the embedding facts right; by contrast, a hybrid account (à la Geurts 2006 or Breheny 2008), while it gets the embedding facts right, appears to have no chance of getting the collective facts right.

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