The ups and downs of ignorance

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Abstract Plain disjunctive sentences, such as *The mystery box contains a blue ball or a yellow ball*, typically imply that the speaker does not know which of the two disjuncts is true. This is known as an IGNORANCE inference. We can distinguish between two aspects of this inference: the negated universal upper bound part (i.e., the speaker is uncertain about each disjunct), which we call UNCERTAINTY, and the existential lower bound part (i.e., the speaker considers each disjunct possible), which we call POSSIBILITY. In the traditional approach, UNCERTAINTY is derived as a primary implicature, from which POSSIBILITY follows. In this paper, we report on two experiments using a sentence-picture verification task based on the mystery box paradigm that challenge the traditional implicature approach. Our findings show that POSSIBILITY can arise without UNCERTAINTY, and thus call for a reevaluation of the traditional view of disjunction and IGNORANCE inferences. Our experimental findings are related to similar results involving disjunction in embedded contexts and pave the way for alternative theories that can account for the observed patterns of inference derivation in a unified fashion. We will discuss how recent implicature and non-implicature theories can account for the derivation of existential lower bound inferences without the presence of negated universal upper bound inferences.

Keywords: Disjunction, Ignorance inference, Implicature, Experimental Semantics & Pragmatics

1 Introduction

It is well-known that sentences with a plain disjunction like (1) typically give rise to the IGNORANCE and EXCLUSIVITY inferences, indicated in (1-a) and (1-b):

(1) Sue went to the park or the cinema. $A \vee B$

a. $\rightsquigarrow$ *The speaker doesn't know which of the two is true* IGNORANCE

b. $\rightsquigarrow$ *Sue didn't go to the park and to the cinema* EXCLUSIVITY

In this work, we will mainly focus on IGNORANCE inferences, though we will also consider the role of EXCLUSIVITY when relevant. A natural way to think about IGNORANCE is as follows: a speaker s is ignorant about A iff s does not believe that A is the case and does not believe that $\neg A$ is the case. In other words, both A and $\neg A$ are live possibilities in the speaker's doxastic state (Gazdar 1979, Hintikka 1962). We can distinguish between two aspects of an ignorance inference, where $\Box_s$ and $\Diamond_s$ quantify universally and existentially over the speaker's doxastic state: the upper bound UNCERTAINTY part in (2-a) and the lower bound POSSIBILITY part in (2-b).

(2) $\Diamond_s A \wedge \neg \Box_s A \wedge \Diamond_s B \wedge \neg \Box_s B$ IGNORANCE INFERENCE

The speaker doesn't know which of the two is true

a. $\neg \Box_s A \wedge \neg \Box_s B$ UNCERTAINTY

The speaker is not certain that Sue went to the park and the speaker is not certain that Sue went to the cinema.

b. $\Diamond_s A \wedge \Diamond_s B$ POSSIBILITY

The speaker deems possible that Sue went to the park and the speaker deems possible that Sue went to the cinema.

The traditional approach to ignorance inferences derives UNCERTAINTY as an implicature of (1) through Gricean quantity reasoning about each individual disjunct. From this, POSSIBILITY follows, given also the quality assumption that the speaker believes (1) (among others, Sauerland 2004b, Fox 2007). As such, this approach predicts that POSSIBILITY *cannot* arise in the absence of UNCERTAINTY.[1]

Recent non-implicature proposals, however, suggest that POSSIBILITY should be derived independently of UNCERTAINTY, either through a presupposition (Goldstein 2019) or as a pragmatic inference of the 'third kind' (Aloni 2022). According to these theories, a sentence like (1) can give rise to POSSIBILITY in the absence of UNCERTAINTY. Consequently, these theories, unlike the traditional implicature

1 Grammatical versions of the implicature approach make the same prediction, but they derive the corresponding inferences by means of a covert exhaustivity operator, interacting with a covert doxastic operator (among others, Meyer 2013, Buccola & Haida 2019).

approach, predict that a disjunctive sentence like (1) should be judged as felicitous when the speaker is certain about one of the disjuncts, while only considering the other possible.

In this study, we report on two experiments testing these divergent predictions. Both experiments consisted of a sentence-picture verification task based on the mystery box paradigm (Noveck 2001, Marty et al. 2023, Ramotowska et al. 2022). The results of both experiments showed that POSSIBILITY can indeed arise without UNCERTAINTY. Specifically, we found that participants judged sentences where UNCERTAINTY inferences were false as good as true controls, while more than half of them judged the sentence false when POSSIBILITY inferences were also false. These results pose a challenge for the traditional approach, which cannot predict POSSIBILITY in the absence of UNCERTAINTY.

Our experiments focused on cases of plain disjunctive sentences. Sentences involving disjunction in embedded contexts give rise to similar inference patterns and have sparked a similar debate (Ramotowska et al. 2022, Crnič et al. 2015). We will examine the relationship between our results and previous experimental research on embedded disjunction. In particular, we will interpret our experimental results from three different perspectives.

Firstly, we will explore how a recent grammatical account (Bar-Lev & Fox 2023), originally developed to capture the behavior of disjunction in embedded contexts, can be extended to deal with plain disjunctive sentences and what the consequences of this extension are.

Secondly, our experimental results are in line with the recent non-implicature accounts of POSSIBILITY mentioned above (Aloni 2022, Goldstein 2019). We will focus on the account of Aloni (2022) and discuss how it derives POSSIBILITY.

Thirdly, we will consider how our results fare with game-theoretic approaches to disjunction and modal inferences (Franke 2011a). We will conclude with potential follow-ups and a discussion on the role of different experimental tasks and protocols for the same types of inferences, emphasizing the importance of taking a holistic experimental approach when studying pragmatic phenomena.

This paper is structured as follows. Section 2 summarizes the traditional approach and its predictions. Section 3 contains an overview of the experiments, followed by a report of Experiment 1 and Experiment 2 in Section 4 and Section 5 respectively. Appendix A contains a detailed description of the instructions for the experiments. Section 6 discusses the results of our experiment, focusing on the connection with distributive inferences and the grammatical approach in Section 6.1, non-implicature accounts of disjunction in Section 6.2, and game-theoretic approaches in Section 6.3. Section 7 concludes.

2 The traditional approach and its predictions

The traditional Gricean approach (Grice 1975, 1989) derives IGNORANCE and EXCLUSIVITY inferences based on a general process of conversation that takes into account the information available to the speakers and the idea that language users behave cooperatively. The most relevant principles underlying these derivations are the maxim of quantity, which roughly says that speakers should convey all and only the most informative statement given what they know; and the maxim of quality, which roughly says that speakers should be always truthful.

These maxims can be made formal in different ways and a number of accounts have been proposed stemming from the original Gricean insights (among others, Horn 1972, Gazdar 1979, Levinson 1983, Gamut 1991). In what follows, we will rely on the following formalization of inference derivation:

(3) For all relevant alternatives ψ of an utterance ϕ:

$$\text{if } \psi \vDash \phi \text{ and } \phi \nvDash \psi, \text{ then } \neg\Box_S(\psi)$$

In words, when a speaker chooses to make a weaker statement (ϕ) instead of a stronger alternative (ψ), it implies that the speaker does not believe that the stronger statement is true ($\neg\Box_S\,\psi$). The principle above refers to the notion of *relevant* alternatives. An important question for this approach is to determine what counts as an alternative, and what ultimately counts as a relevant alternative. For what is relevant here, we will assume, as proposed in Sauerland (2004b) and maintained in subsequent literature, that a disjunction has also the individual disjuncts as alternatives. And these alternatives will end up being relevant in most cases.

(4)
$$\left\{\begin{array}{ll} \text{Sue went to the park or the cinema.} & A \vee B \\ \text{Sue went to the park.} & A \\ \text{Sue went to the cinema.} & B \\ \text{Sue went to the park and to the cinema.} & A \wedge B \end{array}\right\}$$

The Gricean account just outlined provides a systematic way to derive both IGNORANCE and EXCLUSIVITY inferences associated with a plain disjunction. As previously stated, IGNORANCE inferences are comprised of two elements: UNCERTAINTY and POSSIBILITY inferences. The standard procedure is to derive UNCERTAINTY first, and then use quality to derive POSSIBILITY.[2]

2 Another option is to close the alternatives under negation, so that the negative counterparts of the positive alternatives count as relevant (Fox 2007). Even with this approach, the negative alternatives, responsible for POSSIBILITY, are present due to the positive alternatives, responsible for UNCERTAINTY.

For instance, $\neg \Box_s A$ would be derived as in (5). A similar process with the alternative B and with the alternative $A \wedge B$ would give us $\neg \Box_s B$ and $\neg \Box_s (A \wedge B)$, respectively. Together, these are known as the primary implicatures of a plain disjunctive sentence.

(5) Derivation of UNCERTAINTY ($\neg \Box_s A$)

a. Assertion: $A \vee B$
b. A is a relevant alternative to $A \vee B$
c. A is stronger than $A \vee B$
d. From (5-b) and (5-c) above together with (3), $\neg \Box_s A$

Possibility is then derived from UNCERTAINTY together with the Quality assumption that speakers believe $A \vee B$:

(6) Derivation of POSSIBILITY

a. Assertion: $A \vee B$
b. Quality: $\Box_s(A \vee B)$
c. UNCERTAINTY: $\neg \Box_s A \wedge \neg \Box_s B$
d. From (6-b) and (6-c) above, $\Diamond_s A \wedge \Diamond_s B$

Note again that this method of deriving POSSIBILITY inferences results in their dependence on UNCERTAINTY inferences.

For the derivation of the EXCLUSIVITY inference in (1-b), a similar argument to the derivation in (5) for the alternative $A \wedge B$ yields $\neg \Box_s (A \wedge B)$. However, to obtain the stronger EXCLUSIVITY inference $\Box_s \neg(A \wedge B)$, the standard approach relies on an opinionatedness principle of the form $\Box_s \phi \vee \Box_s \neg \phi$. Applying this principle gives us $\Box_s \neg(A \wedge B)$. Sauerland (2004b) proposed that individual disjuncts are not subject to opinionatedness, as generating inferences of the form $\Box_s \neg \psi$ from $\neg \Box_s \psi$ would, in combination with the Quality assumption about the assertion, contradict previously obtained implicatures (i.e. $\neg \Box_s \psi'$).[3]

3 We note, in passing, that one of the driving factors behind the richer set of alternatives in (4) was the observation that UNCERTAINTY and POSSIBILITY inferences, unlike scalar ones, are not cancellable (Sauerland 2004b,a). However, we observe that while POSSIBILITY inferences are indeed not cancellable, UNCERTAINTY ones seem to pattern with EXCLUSIVITY inferences in terms of their cancellability:

(i) a. #I saw the Queen or the Princess of the Netherlands at the airport.
In fact, it is not possible that I saw the Queen.
b. I saw the Queen or the Princess of the Netherlands at the airport.
In fact, I saw both.
c. I saw the Queen or the Princess of the Netherlands at the airport.
In fact, I am certain that I saw the Queen and possibly the Princess.

In the following section, we will present two experimental studies that challenge the canonical view that POSSIBILITY inferences are derived from UNCERTAINTY inferences.

3 Overview of the experiments

As discussed in the previous section, UNCERTAINTY inferences are traditionally taken to be primary implicatures, and the derivation of POSSIBILITY inferences to be dependent on them.

In the following, we report on two experiments testing these hypotheses. Both experiments were built upon Marty et al. (2023)'s elaboration of Noveck (2001)'s mystery box paradigm (see also Ramotowska et al. 2022). In both experiments, participants were presented with sentence-picture items like those in Figure 1. Each item depicted a set of four boxes, three open and one covered, displayed just above an utterance produced by one of two characters. Participants were instructed that the characters could see what's inside the first three boxes but not what's inside the covered one, the so-called *mystery box*. They were also instructed that the characters had been taught the rule that the mystery box always had the same contents as one of the three open boxes (see Section 4.3 and Appendix A). The task was to decide if the character's utterance was right given the information available to them and the rule that they had learned about how the mystery box works. Participants reported their response by clicking on one of two response buttons, labelled 'Good' and 'Bad', respectively.

In the TARGET-1 conditions, test sentences were paired with pictures that make their uncertainty inferences (U-INFERENCE) false, but their possibility inferences (P-INFERENCE) true; in the TARGET-2 conditions, they were paired with pictures that make both these inferences false. We hypothesized that, if P-INFERENCES are derived from U-INFERENCES, no difference in participant's responses should be observed between both target conditions. On the other hand, if P-INFERENCES are derived independently from U-INFERENCES, then participants should reject the test sentences to a greater extent in the TARGET-2 than in the TARGET-1 conditions.

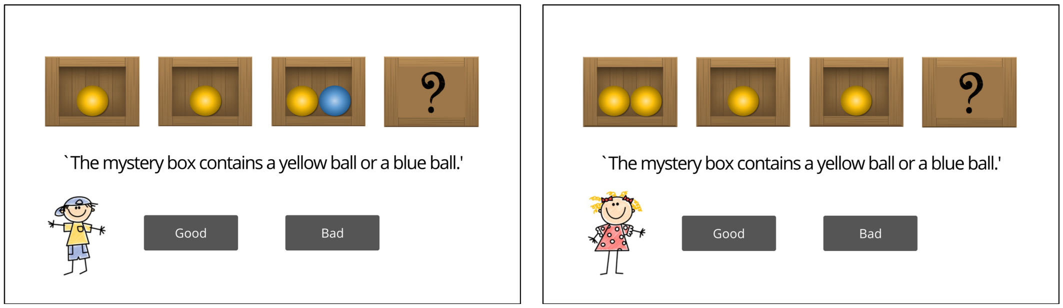


Figure 1 Example items illustrating the items' layout and task in Experiment 1 and 2. These examples are instances of the TARGET-1 (on the left) and TARGET-2 (on the right) conditions in Experiment 2.

4 Experiment 1

4.1 Participants

101 native speakers of English participated in this study (mean age 39 yrs; 51 female). Participants were recruited online through Prolific (https://www.prolific.co; see Palan & Schitter for an overview) using a suitable set of prescreening criteria (first language: English; nationality: UK/US; country of birth: UK/US; minimum approval rate: $\geq 90\%$). Participants were paid £1.20, and median completion time was about 8 minutes (hourly rate: £9/hr). All participants gave written informed consent. Data were collected and stored in accordance with the provisions of Data Protection Act 2018. The study was approved by the Research Ethics Committee at University College London and at University of Amsterdam.

4.2 Materials and Design

The experiment was based on the materials and method from Marty et al. (2023: Exp.4-6; see also Ramotowska et al.). Each item involved a sentence displayed just below a set of boxes and right above the picture of one of two characters, as exemplified in Figure 1. Sentences were constructed using the sentence frames in Table 1. The color adjectives used in the sentences, indicated by the [A] and [B] terms in Table 1, were picked at random from a list of four color terms – *yellow*, *blue*, *green* and *gray* – with replacement across items. Each sentence was placed inside simple quotation marks to show direct speech and to make it explicit that the sentence was uttered by whatever character was depicted on the item.

Every item displayed a set of four boxes horizontally arranged, each of which

TEST		'The mystery box contains a [A] ball or a [B] ball.'
CONTROL	C1	'The mystery box contains a [A] ball.'
	C2	'The mystery box does not contain a [A] ball.'
	C3	'The mystery box contains a [A] ball and a [B] ball.'
	C4	'The mystery box does not contain a [A] ball or a [B] ball.'

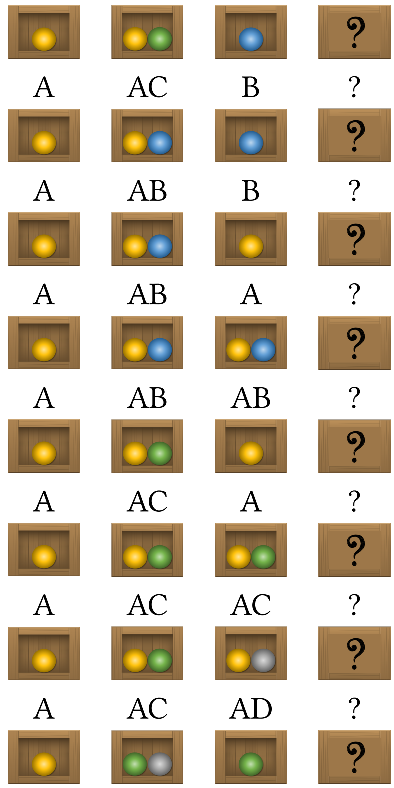
Table 1 Schematic description of the sentences tested in Experiment 1, where [A] and [B] are placeholders for different colour adjectives; for a more concrete illustration, you may read [A] as *yellow* and [B] as *blue*.

was made of three open boxes, containing one or two balls, and a covered one, marked with the symbol '?' and referred to as *the mystery box*. The position of the boxes in the row was pseudo-randomly assigned so that the mystery box always appeared at the rightmost position. The contents of the open boxes in each quadruplet were experimentally manipulated to create different picture types corresponding to the experimental conditions of the study. The test sentences under investigation were paired with five different picture types, which are described in Figure 2: the colors of A-balls and B-balls depicted in the open boxes always matched the [A] and [B] color terms used in the sentences (e.g., yellow and blue) whereas the colors of the C-balls and D-balls were randomly chosen from our list of color terms by excluding the color(s) of the matching balls (e.g., green and gray).

Target pictures were designed to make the U-INFERENCE of the test sentences always false, but their P-INFERENCE either true or false. On the TARGET-1 pictures, each of the three open boxes contained an A-ball and at least one of them also contained a B-ball, making the U-INFERENCE of the test sentences false, but their P-INFERENCE true. TARGET-2 pictures were obtained from the TARGET-1 pictures by replacing the B-ball(s) with balls of a non-matching color (i.e., a C-ball or a D-ball), thus making both the U-INFERENCE and the P-INFERENCE of the test sentences false. Different variants of the TARGET-1 and TARGET-2 pictures were constructed by varying the number of matching B-balls for the former (1 B-ball vs. 2 B-balls) and by varying both the number and color of non-matching balls for the latter (1 C-ball vs. 2 C-balls vs. 1 C-ball and 1 D-ball). For the purposes of experimental design, variants of the TARGET-1 and TARGET-2 pictures were treated as sub-conditions of the TARGET-1 and TARGET-2 conditions.[4]

FALSE, TRUE-EXCL and TRUE-ADHOC pictures were control pictures, each of which

4 As reported in Section 4.5, no contrast in participants' responses was found between the variants of the TARGET-1 pictures, nor between those of the TARGET-2 pictures, allowing us to aggregate the responses to the TARGET-1 and the TARGET-2 trials across sub-conditions without loss of information.



Condition		Example picture			
True-Adhoc		A	AC	B	?
True-Excl		A	AB	B	?
Target-1	i.	A	AB	A	?
	ii.	A	AB	AB	?
Target-2	i.	A	AC	A	?
	ii.	A	AC	AC	?
	iii.	A	AC	AD	?
False		A	CD	C	?

Test sentence: *'The mystery box contains a yellow ball or a blue ball.'* $(A \vee B)$

Table 2 Schematic description and illustration of the picture types paired with the test sentences in Experiment 1. The color of the A-balls and B-balls always matched the color adjectives used in the sentence (e.g., *yellow* and *blue*) while the color of the C-balls and D-balls never did (e.g., *green* and *gray*). Picture types are illustrated here using the following color assignment: A=yellow, B=blue, C=green and D=gray.

served a different experimental purpose. false pictures were designed to provide a clear baseline for rejection. On these pictures, one of the open boxes contained an A-ball while the other two contained balls of a non-matching color, making the test sentences unambiguously false. true-excl and true-adhoc pictures were designed to probe for the strength of two other inference types that the test sentences could give rise to: (i) ‘not-both’ exclusivity implicatures, e.g., *the mystery box does not contain both a [A] ball and a [B] ball* and (ii) ad-hoc exhaustivity inferences, e.g., *the mystery box does not contain a [C] ball or a [D] ball.* Crucially, the derivation of such inferences can affect participants’ responses in the target conditions: the first one makes the test sentences false in the TARGET-1 conditions, where one or more boxes contain two matching balls, while the second makes them false in the TARGET-2 conditions, where one or more boxes contain a non-matching ball. The goal of the TRUE-EXCL and TRUE-ADHOC conditions was to assess the extent to which participants derived these inferences so as to factor out their potential effects from the comparison of primary interest between TARGET-1 and TARGET-2 conditions. On the TRUE-EXCL pictures, one of the open boxes contained both an A-ball and a B-ball, as on the TARGET-1 pictures. These pictures made the test sentences true unless an exclusivity inference is derived (i.e., *the mystery box contains a yellow ball or a blue, but not both*). On the TRUE-ADHOC pictures, one of the open boxes contained an A-ball and a C-ball, as on the TARGET-2 pictures. These pictures made the test sentence true unless an exhaustivity inference is derived (i.e., *the mystery box contains a yellow ball or a blue, and nothing else*).

In addition to the test sentences, there were four different types of control sentences: two positive sentences (C1 and C3) and two negative ones (C2 and C4), involving either one color adjective (C1 and C2) or two (C3 and C4). Each of these sentences were paired with pictures that made them either clearly true (GOOD conditions) or clearly false (BAD conditions), as described and illustrated in Figure 3. These control items were added to the experiment to identify low-effort responses as well as to control for certain low-level response strategies in the data treatment. Specifically, we worried that some participants may perform the task superficially, simply by checking whether or not the colors mentioned in the sentence match those of the balls depicted on the pictures. We reasoned that, if a participant follows such a strategy, then they should perform relatively poorly on these items, especially in those involving negative sentences (C2 and C4).

Pairing the test and control sentences with the relevant picture types gave rise to 5 test and 8 control conditions. Each control condition was instantiated 3 times, giving rise to 24 control trials. As for the test conditions, the TARGET-1, TARGET-2 and FALSE conditions were instantiated 6 times each, and the TRUE-EXCL and TRUE-ADHOC conditions 3 times each, giving rise to 24 test trials. Instances of the TARGET-1 and TARGET-2 conditions were evenly distributed across their respective sub-conditions. Thus, each survey included 48 trials in total.

Sentence	Condition	Example picture			
C1: A	GOOD	A	A	A	?
	BAD	A	CD	C	?
C2: $\neg A$	GOOD	CD	CD	C	?
	BAD	A	CD	A	?
C3: $A \wedge B$	GOOD	AB	AB	AB	?
	BAD	AB	CD	C	?
C4: $\neg(A \vee B)$	GOOD	C	CD	C	?
	BAD	A	CD	A	?

Table 3 Schematic description and illustration of the picture types paired with the control sentences in Experiment 1. Picture types are illustrated using the same color assignment as before.

4.3 Procedure

The experiment was run as an online survey using Gorilla Experiment Builder (Anwyl-Irvine et al. 2020). In the instructions, participants were introduced to two characters, Sam and Mia, and they were presented with a short cover story. The cover story went in substance as follows (see Appendix A for details): Sam and Mia are looking at quadruplets of boxes containing balls of various colors. Each time, they can only see what's inside the first three boxes. However, they have been taught the rule that the fourth box, known as 'the mystery box', always has the same contents as one of the three open boxes and, therefore, they can make certain inferences about what's inside the mystery box. Participants were then shown two examples illustrating what the characters can and cannot infer thanks to this rule. Participants were told that the characters would be presented with many quadruplets, each of which would be followed by an utterance from either Sam or Mia about what the mystery box contains, and that their task was to decide if this utterance was right given the information available to the characters and the rule that they learned. They were instructed to click on 'Good' if they consider that the characters got it right and, otherwise, to click on 'Bad'.

Following the instructions, participants completed a short training devised to consolidate their understanding of the cover story. The training phase included one instance of each control condition, hence 8 trials. During this phase, participants received feedback on the accuracy of their responses, together with a short explanation as to why the character got it right or wrong (e.g., *Sam got it right: since every open box contains a yellow ball, one can be certain that the mystery box does too.*). After the training, the study continued with a block of 48 experimental trials. Trials were presented in random order, with a 1000 ms inter-stimulus interval. Participants reported their responses by clicking with the mouse one of two response buttons labelled 'Good' and 'Bad', respectively. The position of the response buttons (i.e., on the left or on the right) was counterbalanced across participants. Items remained on the screen until participants gave their response.

4.4 Data availability

Stimuli, data and analysis code associated with Experiment 1 are available open access on the OSF at https://osf.io/4ut2c/?view_only=9fd1ae1db3bd466bb9ade688d362b241.

4.5 Data treatment and analysis

Data treatment and analysis were carried out in the R statistical environment (R Core Team 2021) using the Hmisc (Harrell Jr 2021), lme4 (Bates et al. 2015), car (Fox & Weisberg 2019) and sjPlot (Lüdecke 2023) packages for the R statistics program.

Responses from 15 participants were removed prior to analyses because their performance in the control trials did not reach the pre-established threshold of 80% accuracy.[5] The performance of the remaining subjects (n=86) was uniformly high

5 The responses of the excluded participants were investigated to identify the possible sources of their low performance. We found little evidence for the low-level response strategy mentioned in Section 4.2. Rather, we found a discrepancy between the GOOD and BAD conditions (86% vs. 55% accuracy), showing that these participants generally (incorrectly) accepted the control sentences in the BAD conditions. Given the contents of the pictures in these conditions, it suggests that these

both in the BAD conditions (M=94.7%, 95% CI[93.2, 95.9]) and the GOOD conditions (M=95.9%, 95% CI [94.5, 96.9]), with little variation among sentence types (all Ms>92.2% across all control conditions). Responses to the TARGET-1 and TARGET-2 trials were next inspected to check for potential discrepancies among the different variants of the TARGET-1 and TARGET-2 pictures (see Table 2). For each target condition, we fitted a linear mixed-effect model with a binomial distribution, predicting responses from the fixed effect of picture sub-type. In both cases, the maximal converging model included a random intercept for participants. Both models were compared to a null model missing the fixed effect of interest. Neither model was significantly different from the null model (TARGET-1: $\chi^2(1) = 1.52$, $p = .21$; TARGET-2: $\chi^2(2) = 4.05$, $p = .13$), meaning that picture sub-type had no detectable on participants' judgements in the target conditions. Based on these results, responses to the TARGET-1 and TARGET-2 trials were aggregated across sub-conditions for the purpose of the main analysis.

Responses to the test trials were analysed by carrying out pairwise comparisons between each target condition and all other conditions. For each comparison, we fitted a linear mixed-effect model with a binomial distribution, predicting responses from the fixed effect of condition (2 levels; sum-coded). All models included random intercepts for participants and items, and random slopes for condition by participant and by item. The χ^2 and p-values we report on in the results were obtained by performing likelihood ratio tests in which the deviance of the models containing the fixed effect of condition was compared to another model without the relevant effect, but with the same random effect structure. p-values were adjusted using the Bonferroni correction method for multiple testing. Concretely, because 7 comparisons were tested, only p-values below 0.007 were treated as significant.

4.6 Results

Figure 2 shows the mean acceptance rate (i.e., the proportion of 'Good' responses) for the test sentences as a function of the experimental condition. The outputs of the statistical models and analyses are summarized in Table 4.

Acceptance rates were uniformly high in the TARGET-1 (M=94.7%, 95% CI[92.4, 96.3]) and TRUE-EXCL conditions (M=96.1%, 95% CI[93.0, 97.8]), with no significant difference between the two ($\chi^2(1) = 1.48$, adjusted $p = 1$). These results show that the participants in our study did not derive the U-INFERENCES, nor the exclusivity implicatures associated with the test sentences. By contrast, the mean acceptance

participants generally considered an utterance as right as long as it was true of at least one box, that is, if it was describing a possible outcome (rather than a certainty). Whether this bias toward acceptance stems from an application of the Charity Principle or simply from a poor understanding of the instructions, it justifies further the conservative criterion we used for inclusion in the analysis.

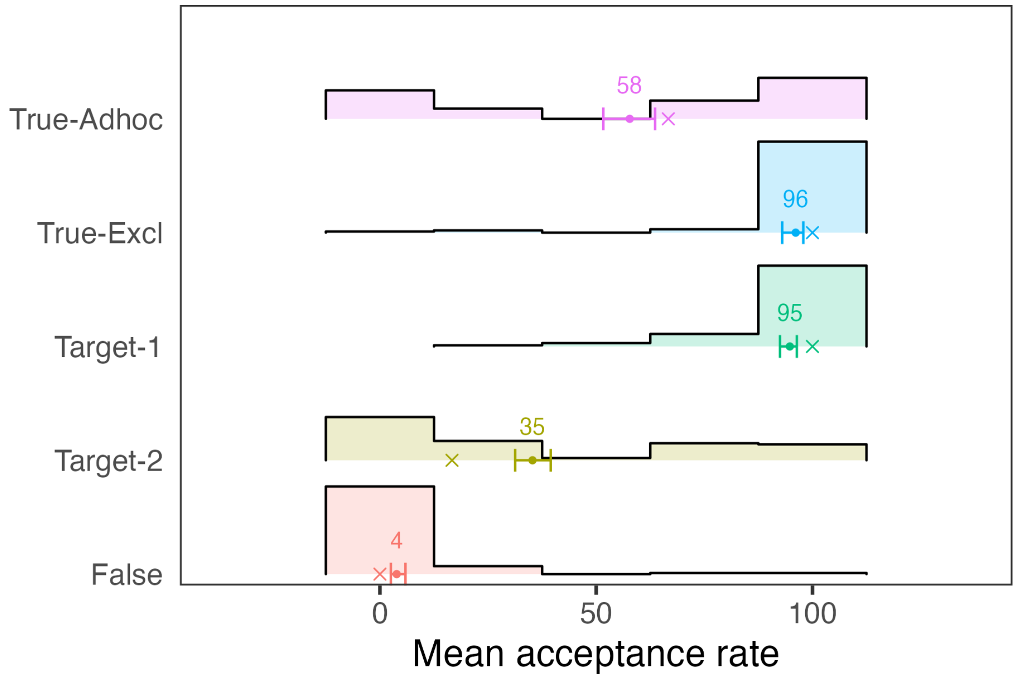


Figure 2 Mean acceptance rate (i.e., proportion of 'Good' responses) to test sentences by experimental condition in Experiment 1. For each condition, the distribution of by-participant mean rates is visualised by a histogram, the grand mean by a thick bar with its rounded value on top and the 95% CI around it, and the median by a cross.

rate for the TARGET-2 and TRUE-ADHOC conditions were somewhat intermediate (M=35.2%, 95% CI[31.2, 39.4] and M=57.7%, 95% CI[51.6, 63.6], respectively), between the high(est) rates observed in the TARGET-1 and TRUE-EXCL conditions and the low(est) rate observed in the FALSE conditions (M=3.8%, 95% CI[2.5, 5.9]), with the TARGET-2 conditions yielding significantly lower rates than the TRUE-ADHOC conditions (χ^2 = 13.93, adjusted p < .001).[6] The results from the TRUE-ADHOC conditions indicate that participants derived the ad-hoc exhaustive inferences

6 A visual inspection of the histograms in Fig. 2 suggest that, unlike in the TARGET-1 conditions, subjects' responses in the TARGET-2 and TRUE-ADHOC conditions were not uniform. To verify this impression, we carried out a post-hoc exploration of individual subjects' means by testing for unimodality of their distribution (Hartigans' dip test) in these three conditions. The results confirmed that by-participant mean rates were distributed unimodally only in the TARGET-1 conditions (TARGET-1: $D = 0.04$, $p = 0.18$; TARGET-2: $D = 0.09$, $p < .001$; TRUE-ADHOC: $D = 0.14$, $p < .001$). For completeness, a Pearson correlation coefficient was computed to assess the linear relationship between subjects' mean rates in the TARGET-2 and TRUE-ADHOC conditions. The results show a low to moderate, positive correlation ($r(84) = 0.49$, 95% CI[0.32, 0.64], $p < .001$).

associated with the test sentences to a noticeable extent. Since these inferences were false both in the TRUE-ADHOC and TARGET-2 conditions, it is possible that their derivation also affected to some extent participants' judgements in the TARGET-2 conditions. Thus, the absolute rates of acceptance for the TARGET-2 conditions in this experiment should be regarded with caution. Crucially, however, the contrast between the TARGET-2 and TRUE-ADHOC conditions show that, everything else being equal, participants rejected significantly more often the test sentences when their P-INFERENCES were also false. This contrast is explained if, in contrast to the U-INFERENCES, participants sometimes derived the P-INFERENCES associated with the test sentences, possibly in addition to deriving exhaustivity inferences.

Comparison	*Estimate*	*95% CI*	χ^2	*p*	*Adjusted p*
TARGET-1 VS. TRUE-EXCL	-1.44	[-3.99, 1.10]	1.48	0.22	1
TARGET-1 VS. TRUE-ADHOC	3.12	[1.57, 4.68]	17.30	<.001	**<.001**
TARGET-1 VS. FALSE	8.58	[6.40, 10.76]	53.26	<.001	**<.001**
TARGET-1 VS. TARGET-2	3.55	[2.41, 4.70]	37.80	<.001	**<.001**
TARGET-2 VS. TRUE-EXCL	-5.73	[-7.68, -3.77]	35.01	<.001	**<.001**
TARGET-2 VS. TRUE-ADHOC	-1.24	[-1.72, -0.76]	13.93	<.001	**<.001**
TARGET-2 VS. FALSE	4.33	[2.19, 6.47]	35.42	<.001	**<.001**

Table 4 Summary of the outputs of the statistical models and analyses. For each pairwise comparison, the model included condition as a fixed effect (level 1 vs. level 2; sum coded: 1=level 1 and -1=level 2) and a maximal random effect structure. χ^2 and p-values were obtained via model comparison. p-values were adjusted for all comparisons tested.

4.7 Discussion

The results of this experiment show that participants uniformly accepted simple disjunctive sentences of the form 'A or B' in cases where their U-INFERENCE was false, but their P-INFERENCE was true (TARGET-1 conditions), whereas they rejected these same sentences 65% of the time in cases where both their U-INFERENCE and P-INFERENCE were false (TARGET-2 conditions). Taken at face value, these findings go against the traditional approach to IGNORANCE inferences which predicts P-INFERENCES to arise from and, thus to be licensed by the derivation of U-INFERENCES. They suggest instead that P-INFERENCES remain available to speakers even in the absence of the associated U-INFERENCES and, consequently, that the former can be derived independently of the latter.

As we discussed, however, the present results also suggest that the derivation of

P-INFERENCES may not be the only driver behind the contrast between the TARGET-1 and TARGET-2 conditions and, specifically, behind the low acceptance rates observed in the TARGET-2 conditions. We also found that the test sentences were rejected 42% of the time in the TRUE-ADHOC conditions, showing that participants also derived their ad-hoc exhaustivity inferences (e.g., ‘A or B, and nothing else’) to a noticeable extent. Since deriving these inferences would also lead to rejecting the test sentences in the TARGET-2 conditions, the interpretation of the contrast between the TARGET-1 and TARGET-2 conditions is conditional on that of the further contrast between the TRUE-ADHOC and TARGET-2 conditions. For the time being, we propose that, everything else being equal, the fact that the TARGET-2 conditions gave rise to significantly lower acceptance rates than the TRUE-ADHOC conditions supports the idea that the results to the TARGET-2 conditions cannot be entirely explained by the putative effect of ad-hoc inferences, that is, without also taking into account the falsity of the P-INFERENCES in these conditions.

One potential worry with the interpretation above is that it relies on the assumption that the robustness and the frequency at which ad-hoc exhaustivity inferences were derived remained relatively constant across experimental conditions. While this assumption is certainly not implausible, the present results offer no independent evidence that would support it. In particular, we cannot exclude the possibility that (i) ad-hoc exhaustivity inferences were derived in the TARGET-2 conditions and (ii) these inferences were more robust, or simply derived more often in the TARGET-2 than in the TRUE-ADHOC conditions. As it is easy to see, making the alternative assumptions in (i) and (ii) would allow one to similarly account for the contrasts at hand without any need to refer to P-INFERENCES at all.

In the following, we report on a follow-up experiment testing this alternative explanation of the data. For these purposes, the test items from Experiment 1 were minimally modified to nullify the effect of ad-hoc inferences on participants' judgments so as to obtain more pristine TARGET-2 conditions. The results of this experiment show that the key contrast between TARGET-1 and TARGET-2 reproduces once the potential effects of ad-hoc inferences are factored out from the comparison, thus confirming our original interpretation of the data.

5 Experiment 2

5.1 Participants

100 novel participants took part in this study (mean age 42 yrs; 50 female). Participants were recruited online through Prolific using the same prescreening criteria as in Experiment 1. Participants were paid £1.20, and median completion time was about 8 minutes (hourly rate: £9/hr). The consent and data collection procedures

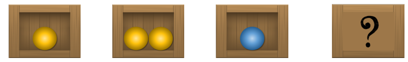
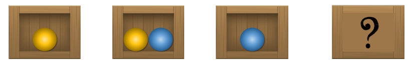
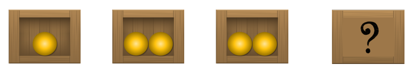
were the same as in Experiment 1.

5.2 Materials and Design

The materials were the same as in Experiment 1, except for the TARGET-2, TRUE-ADHOC and FALSE pictures, which were minimally modified to address specific issues left open by the results of Experiment 1. For simplicity, the novel conditions resulting from these picture modifications were labelled TARGET-2*, TRUE* and FALSE*, respectively. The picture types (and subtypes) used in the test trials of Experiment 2 are described and illustrated in Table 5.

First, the TARGET-2 pictures were modified to make the exhaustivity inferences associated with the test sentences true, while still making both their U-INFERENCE and P-INFERENCE false. The motivation was to create a more pristine version of these conditions enabling us to factor out the potential effect of exhaustivity inferences from the comparison of primary interest. The novel TARGET-2* pictures were obtained from the TARGET-2 pictures by replacing each ball of a non-matching color with an A-ball. As a result, all balls depicted on the TARGET-2* pictures were in effect A-balls. These modifications were applied to all variants of the TARGET-2 pictures resulting in two TARGET-2* sub-types, differing from each other only in terms of the number of A-balls they depicted. In the same vein, the TRUE-ADHOC pictures were modified to create genuinely 'true' pictures. The novel TRUE* pictures were obtained from the TRUE-ADHOC pictures through the same procedure as above – i.e., by replacing any ball of a non-matching color with an A-ball – thus making the test sentences true irrespective of the pragmatic inferences they may give to. Therefore, participants were expected to robustly and uniformly accept the test sentences in these conditions. Finally, the FALSE pictures were modified to create more challenging false controls that can also be used to probe for low-effort responses in trials involving the test sentences (as opposed to other control sentences). On the novel FALSE* conditions, one of the open boxes contained an A-ball, another one contained a B-ball while the last one contained balls of a non-matching color. Thus, if participants do not pay attention to all the open boxes in such cases (e.g., if they restrict their attention to boxes containing A-balls or B-balls), they should incorrectly accept these false controls and, therefore, we should be able to detect and quantify this behavior in our data.

The rest of the design was identical to that of Experiment 1 in all respects. Thus, in particular, Table 3 also stands as a summary of the GOOD and BAD control conditions used in Experiment 2. Each control condition was instantiated 3 times, giving rise to 24 control trials. The TARGET-1, TARGET-2* and FALSE* conditions were instantiated 6 times each, and the TRUE* and TRUE-ADHOC conditions 3 times each, giving rise to 24 test trials. As before, instances of the two target conditions

Condition		Example picture			
TRUE*					
		A	AA	B	?
TRUE-EXCL					
		A	AB	B	?
TARGET-1	i.				
		A	AB	A	?
	ii.				
		A	AB	AB	?
TARGET-2*	i.				
		A	AA	A	?
	ii.				
		A	AA	AA	?
FALSE*					
		A	CD	B	?

Test sentence: *'The mystery box contains a yellow ball or a blue ball.'* $(A \lor B)$

Table 5 Schematic description and illustration of the picture types paired with the test sentences in Experiment 2. The materials were the same as in Experiment 1, except for the pictures used in the novel TRUE*, TARGET-2* and FALSE* conditions. Picture types are illustrated using the same color assignment as before (A=yellow, B=blue, C=green and D=gray).

were evenly distributed across their respective sub-conditions. Thus, each survey included 48 trials in total, exactly as in Experiment 1.

5.3 Procedure

The procedure was identical to the one used in Experiment 1 (see Section 4.3 for details).

5.4 Data availability

Stimuli, data and analysis code associated with Experiment 2 are available open access on the OSF at https://osf.io/4ut2c/?view_only=9fd1ae1db3bd466bb9ade688d362b241.

5.5 Data treatment and analysis

Data treatment was the same as in Experiment 1. Responses from 13 participants were removed because their performance in the control trials did not reach the pre-established threshold of 80% accuracy.[7] The performance of the remaining subjects (n=87) was uniformly high both in the BAD conditions (M=95.8%, 95% CI[94.4, 96.9]) and the GOOD conditions (M=97.0%, 95% CI [95.8, 97.9]), with little variation among sentence types (all Ms>93.6% across all control conditions). As before, for each target condition (i.e., TARGET-1 and TARGET-2*), we fitted a linear mixed-effect model with a binomial distribution, predicting responses from the fixed effect of picture sub-type (see Table 5). For both models, the maximal converging included a random intercept for participants. Neither model was significantly different from the null model (TARGET-1: $\chi^2(1) = 0.54$, $p = .46$; TARGET-2*: $\chi^2(2) = 3.44$, $p = .06$). Responses to the TARGET-1 and TARGET-2* trials were thus aggregated across picture sub-conditions for the purpose of the main analysis. Responses to the test trials were analysed using the data analysis pipelines from Experiment 1. The models were the same as in Experiment 1 except in three instances where the random effect structure was simplified by removing the by-participant random slope for condition so as to avoid convergence or over-fitting issues.

5.6 Results

Figure 3 shows the mean acceptance rate (i.e., the proportion of 'Good' responses) for the test sentences as a function of the experimental condition. The outputs of the statistical models and analyses are summarized in Table 6.

The mean acceptance rate in the TARGET-1 conditions was very high (M=95.2%, 95% CI[93.0, 96.7]), similar to those found in the TRUE-EXCL conditions (M=97.7%, 95% CI[95.0, 98.9], $\chi^2(1) = 0.40$, adjusted $p = 1$) and in the novel TRUE* conditions (M=94.2%, 95% CI[90.7, 96.4], $\chi^2(1) = 0.86$, adjusted $p = 1$).[8] Thus, as in Experi-

7 For completeness, a descriptive analysis of the responses of the excluded participants was, here again, carried out (see fn. 5). As in Experiment 1, there was a discrepancy between the GOOD and BAD conditions across all control trials (81% vs. 57% accuracy). In this data set, however, the relevant discrepancy was found to be mainly driven by higher rates of incorrect responses in the BAD conditions of the negative sentences C2 and C4 (48% and 33% accuracy, respectively). These errors are those predicted by the low-level response strategy that we mentioned in Section 4.2.

8 Similar to what we observed in the TARGET-2 conditions of Experiment 1 (see fn. 6), the by-participant

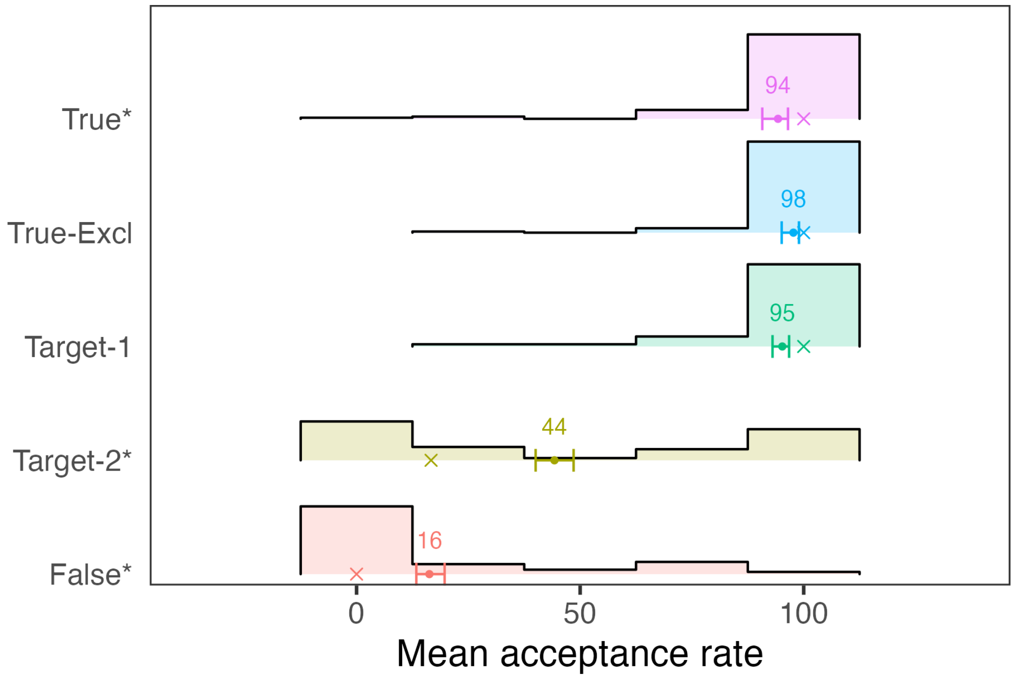


Figure 3 Mean acceptance rate (i.e., proportion of 'Good' responses) to target sentences by experimental condition in Experiment 2. For each condition, the distribution of by-participant mean rates is visualised by a histogram, the grand mean by a thick bar with its rounded value on top and the 95% CI around it, and the median by a cross.

ment 1, there is no evidence that participants in this experiment ever derived the U-INFERENCES or exclusivity implicatures associated with the test sentences. By contrast, the mean acceptance rate in the novel TARGET-2* conditions was in the mid-range (M=44.2%, 95% CI[40.0, 48.5], between the high(est) rates observed in the TARGET-1, TRUE-EXCL and TRUE* conditions (all $\chi^2(1)$s> 19.78, all adjusted ps< .001) and the low(est) rate observed in the novel FALSE* conditions (M=16.2%, 95% CI[13.3, 19.6], $\chi^2(1) = 28.75$, adjusted $p < .001$). In sum, these results show that the key contrast observed in Experiment 1 remains once the potential effects of exhaustivity inferences are factored out.

mean rates in the TARGET-2* conditions was not distributed unimodally ($D = 0.16$, $p < .001$), as shown in Fig. 3 by the presence of two peaks in their distribution. Unlike in Experiment 1, however, the distribution of the by-participant mean rates was otherwise unimodal in all other conditions.

Comparison	*Estimate*	*95% CI*	χ^2	*p*	*Adjusted p*
TARGET-1 VS. TRUE-EXCL	-0.58	[-3.10, 1.94]	0.40	0.52	1
TARGET-1 VS. TRUE*	-0.16	[-2.03, 1.69]	0.03	0.86	1
TARGET-1 VS. FALSE	7.37	[5.60, 9.14]	48.99	<.001	**<.001**
TARGET-1 VS. TARGET-2	3.99	[2.29, 5.70]	31.09	<.001	**<.001**
TARGET-2 VS. TRUE-EXCL	-4.67	[-5.65, -3.68]	22.40	<.001	**<.001**
TARGET-2 VS. TRUE*	-3.46	[-4.12, -2.80]	19.78	<.001	**<.001**
TARGET-2 VS. FALSE*	1.49	[1.23, 1.76]	28.75	<.001	**<.001**

Table 6 Summary of the outputs of the statistical models and analyses. For each pairwise comparison, the model included condition as a fixed effect (level 1 vs. level 2; sum coded: 1=level 1 and -1=level 2) and a maximal random effect structure. χ^2 and *p*-values were obtained via model comparison. *p*-values were adjusted for all comparisons tested.

5.7 Discussion

We designed Experiment 2 as a minimal variant of Experiment 1 with the aim of factoring out the potential effects of exhaustivity inferences from the comparison between the TARGET-1 and TARGET-2 conditions. The results that we obtained show that this aim was achieved and confirm the main findings from Experiment 1.

First, in line with our expectations, the novel TRUE* conditions were uniformly accepted with a mean acceptance rate of 94%, contra 57% for the TRUE-ADHOC conditions in Experiment 1. These results show that the picture modifications that we have made in Experiment 2 were successful at preventing conflicting exhaustivity inferences from affecting participants' judgments. This, in turn, secures our interpretation of the comparison between the TARGET-1 and TARGET-2* conditions as assessing, specifically, the effect of false P-INFERENCES on participants' judgments. Second, consistent with the results of Experiment 1, participants uniformly accepted the test sentences in the TARGET-1 conditions whereas they robustly rejected them in the novel TARGET-2* conditions. Taken together, these results confirm the main findings of Experiment 1 and establish, on firmer grounds, that disjunctive sentences can give rise to P-INFERENCES in the absence of U-INFERENCES.

Finally, we note that the novel FALSE* conditions gave rise to slightly higher rates of errors compared to the original FALSE conditions (16% contra 3%). These errors suggest that, on some occasions, some participants only verified whether or not balls of the matching colors were present somewhere on the pictures, i.e., disregarding whether or not these balls were present in every box. Assuming that this verification strategy was used at times, it may have magnified the contrast between the TARGET-1 and TARGET-2(*) conditions by pushing the participants further towards acceptance in the TARGET-1 conditions. The low frequency of these errors, however, clearly indicate that the effect of this low-effort strategy was, at best, marginal among the participants and cannot account in any reasonable way for the large contrast in acceptance between the TARGET-1 and TARGET-2(*) conditions.

6 General discussion

The experiments discussed in Section 4 and Section 5 pose a clear challenge to the traditional implicature approach: simple disjunctive sentences can give rise to POSSIBILITY inferences, even in cases where UNCERTAINTY inferences are not derived. For the sake of clarity, we repeat below an example with the relevant inferences.

(7) The mystery box contains a yellow or a blue ball. $A \vee B$

a. $\neg \Box_S A \wedge \neg \Box_S B$ UNCERTAINTY

b. $\Diamond_S A \wedge \Diamond_S B$ POSSIBILITY

To begin our discussion, we will first draw a connection between the findings presented in this work and similar observations that have been discussed in relation to disjunction in embedded contexts.

6.1 The connection with distributive inferences

In this work, we focused on cases on plain disjunction and its related inferences. However, embedded disjunction displays similar patterns. We illustrate this by means of a disjunction embedded under a universal modal.[9]

(8) The mystery box must contain a blue ball or a yellow ball. $\Box(A \vee B)$

a. $\neg \Box A \wedge \neg \Box B$ NEGATED UNIVERSAL

b. $\Diamond A \wedge \Diamond B$ DISTRIBUTIVE

9 Note that distributive inferences can also arise in the case of nominal quantifiers, as in (i):

(i) Every visible box contains a blue ball or a yellow ball. $\forall x(P(x) \vee Q(x))$

a. $\neg \forall x P(x) \wedge \neg \forall x Q(x)$ NEGATED UNIVERSAL

b. $\exists x P(x) \wedge \exists x Q(x)$ DISTRIBUTIVE

Crnič et al. (2015) showed that DISTRIBUTIVE inferences do indeed arise for nominal quantifier in the absence of NEGATED UNIVERSAL. They also argued that DISTRIBUTIVE inferences do not arise for the case of modals. Ramotowska et al. (2022) replicated the results of Crnič et al. (2015) for the nominal case, but they also showed that distributive inferences do indeed arise for the modal case, as discussed at the beginning of this section.

The example in (8) gives rise to the negated universal inference in (8-a), corresponding to the UNCERTAINTY inferences discussed here for plain disjunction and illustrated in (7-a). Likewise, (8) is also associated with the existential inference in (8-b), often called DISTRIBUTIVE inference, corresponding to our POSSIBILITY inferences in (7-b).

In the case of embedded disjunction, the traditional implicature approach predicts that the lower bound DISTRIBUTIVE inferences are dependent on the upper bound NEGATED UNIVERSAL part, similarly to the predictions for plain disjunction. However, experimental research (Crnič et al. 2015, Ramotowska et al. 2022) found that DISTRIBUTIVE inferences can be derived even in the absence of UNCERTAINTY ones.

On the face of the generality of the phenomena, we would want a theory that comprehensively captures the independent derivation of lower bound distributive/possibility inferences in a unified fashion.

We will start by looking at recent implicature accounts (Crnič et al. 2015, Bar-Lev & Fox 2023) which have been designed to predict DISTRIBUTIVE inferences in the absence of NEGATED UNIVERSAL. We will then consider whether these accounts can be extended to IGNORANCE inferences. Subsequently, we will shift our focus to non-implicature accounts, which do not derive POSSIBILITY as a Gricean implicature. To make the discussion concrete, we will focus on a recent proposal by Aloni (2022).

6.1.1 Recent grammatical approaches to distributive inferences

In this section, we outline how a recent account by Bar-Lev & Fox (2023) in the grammatical approach can derive DISTRIBUTIVE inferences without the need of deriving NEGATED UNIVERSAL ones.[10] The core of the proposal in Bar-Lev & Fox (2023) is the application of recursive exhaustification, building on previous work by Bar-Lev & Fox (2020), Fox (2007). In this approach implicatures are computed by applying a covert exhaustivity operator, labeled as EXH, which negates innocently excludable (IE) alternatives (i.e., those alternatives to ϕ that can be negated simultaneously without contradicting ϕ or entailing the other alternatives):

(9) a. $IE(\phi, S) := \bigcap \left\{ S' \;\middle|\; \begin{array}{l} S' \subseteq S \text{ and } S' \text{ is a maximal subset of } S \\ \text{such that } \{\neg\psi : \psi \in S\} \cup \{\phi\} \text{ is consistent} \end{array} \right\}$

b. $[\![\text{EXH}(\phi)]\!](w) = [\![\phi]\!](w) \wedge \forall \psi \in IE(\phi, ALT(\phi)) : \neg[\![\psi]\!](w)$

In the following, we illustrate the derivation of DISTRIBUTIVE inferences for the modal case in example (8). The alternatives associated with $\Box(A \vee B)$ are the full set of alternatives in (10). Note that the existential alternatives in blue ($\Diamond A$ and

10 For variants of this see Crnič et al. (2015) and for criticisms of this account see Bar-Lev & Fox (2023).

$\Diamond B$) are essential for the derivation of POSSIBILITY.

(10) $Alt(\Box(A \lor B)) = \{\Box A, \Box B, \Diamond A, \Diamond B, \Box(A \land B), \Box(A \lor B), \Diamond(A \lor B), \Diamond(A \land B)\}$

According to Bar-Lev & Fox (2023), the derivation of distributive inferences is achieved by using a double application of the exhaustivity operator EXH, along with the relevant pruning of alternatives, which we will discuss below.

In particular, the alternative in red in (10) needs to pruned to avoid the derivation of EXCLUSIVITY, which, as we discuss in more detail in Section 6.1.3, would also derive UNCERTAINITY in combination with POSSIBILITY. To also avoid weaker exclusivity inferences of the form $\neg \Box (A \land B)$, also the alternative $\Box(A \land B)$ should be pruned.[11] As a result, the pruned set of alternatives, on which EXH operates, looks as follows:

(11) $Alt_{pruned}(\Box(A \lor B)) = \{\Box A, \Box B, \Diamond A, \Diamond B, \Box(A \lor B), \Diamond(A \lor B)\}$

Distributive inferences are then obtained by recursive exhaustification, as in (12). On the first application of EXH, no alternatives are excludable. With the subsequent application of EXH, thanks to the presence of the individual existential alternatives, DISTRIBUTIVE inferences are then generated:

(12)
$$\begin{aligned}
&\textsc{exh}(\textsc{exh}\,\Box\,(A \lor B))\\
&= \Box(A \lor B) \land \neg \textsc{exh}(\Box A) \land \neg \textsc{exh}(\Box B)\\
&= \Box(A \lor B) \land \neg(\Box A \land \neg \Box B \land \neg \Diamond B) \land \neg(\Box B \land \neg \Box A \land \neg \Diamond A)\\
&= \Box(A \lor B) \land \Diamond A \land \Diamond B
\end{aligned}$$

We have seen how an implicature approach in the grammatical tradition can account for the independent derivation of DISTRIBUTIVE inferences. What remains to determine is how this account can be extended to the case of plain disjunctive sentences and POSSIBILITY inferences discussed in our experimental studies.

6.1.2 Extending the account to IGNORANCE

A natural way to extend this account to plain disjunctive sentences is the use of a silent doxastic operator (among others, Meyer 2013, Buccola & Haida 2019), which we indicate with $\Box_s$. The fundamental assumption is that every assertively used sentence is associated with a covert doxastic operator $\Box_s$, where the subscript

11 Pruning alternatives is allowed if symmetry is not broken, which is is indeed the case here. However, this pruning mechanism still raises the question of why we are allowed to prune $\Diamond(A \land B)$ but not $\Diamond A$ and $\Diamond B$. A possible explanation could link the pruning mechanism to the questions under discussion. For instance, it might be possible that conjunctive alternatives of the form $A \land B$ were not relevant to the QuD, but the individual alternatives A and B were.

refers to the doxastic source (in this case s stands for the speaker). For instance, an assertion like ‘It is raining’ will be rendered as $\Box_s p$, meaning that in all the doxastic possibilities of the speaker, it is the case that it is raining.

As a result, a plain disjunctive sentence will be associated with the alternatives in (13), where $\Diamond_s$ corresponds to the existential counterpart of $\Box_s$. As discussed for the case of distributive inferences, we have pruned the alternatives responsible for exclusivity inferences.

(13) $Alt_{pruned}(\Box_s(A \vee B)) = \{\Box_s A, \Box_s B, \Diamond_s A, \Diamond_s B, \Box_s(A \vee B), \Diamond_s(A \vee B)\}$

By virtue of this silent operator, the derivation of the POSSIBILITY inferences $\Diamond_s A$ and $\Diamond_s B$ parallels the case discussed in (12), thus capturing our results.

One main question for the extension of the account by Bar-Lev & Fox (2023) to IGNORANCE inferences concerns the nature of the silent operator involved in the alternatives. While it is not particularly controversial to assume a silent belief operator $\Box_s$ for assertively used sentences, the use of $\Diamond_s$ as an implicit or covert operator that scopes over sentences is considerably less established. One way to make sense of $\Diamond_s$ is as a weak form of assertion (Incurvati & Schlöder 2019, Dorst & Mandelkern 2022), the idea that language users can (weakly) assert something even when they have low rational credence in it. However, assuming that $\Diamond_s$ is also silent, the existence of such an operator would allow us to utter a sentence like (14) also when it is only compatible with our belief that it is raining ($\Diamond_s p$), which is arguably not the case.

(14) It is raining. $\Diamond_s / \Box_s\, p$

The use of the existential alternative $\Diamond_s$ thus poses a challenge for this approach.[12]

6.1.3 The role of EXCLUSIVITY

In the previous discussion, we pruned the alternatives to prevent the generation of EXCLUSIVITY. The reason behind this decision was that EXCLUSIVITY, when combined with POSSIBILITY or DISTRIBUTIVE inferences, leads to UNCERTAINTY or NEGATED UNIVERSAL inferences, as in (15). This would have been undesirable because we aim to generate POSSIBILITY without also generating UNCERTAINTY.

(15) The role of EXCLUSIVITY

a. $\Diamond A \wedge \Diamond B$

12 One possibility to explore for this approach in relation to this challenge is the observation that strong assertions are often unmarked, whereas weak assertions require an overt marker like ‘perhaps’, as in the sentence ‘Perhaps it is raining’ ($\Diamond_s p$), which can indeed be uttered when it is only compatible with our belief that it is raining.

b. $\Box\neg(A\wedge B)$
c. $\rightsquigarrow \neg\Box A\wedge\neg\Box B$

The derivation with the full set of alternatives in (10) is given in (16). In this case, on the first application of EXH, $\Diamond(A\wedge B)$ and $\Box(A\wedge B)$ are excludable, generating EXCLUSIVITY. As before, the subsequent application of EXH yields distributive inferences:

(16) $\text{EXH}(\text{EXH}\,\Box(A\vee B))$
$= \Box(A\vee B)\wedge\neg\Box(A\wedge B)\wedge\neg\Diamond(A\wedge B)\wedge\neg\text{EXH}(\Box A)\wedge\neg\text{EXH}(\Box B)$
$= \Box(A\vee B)\wedge\neg\Box(A\wedge B)\wedge\neg\Diamond(A\wedge B)\wedge\neg(\Box A\wedge\neg\Box B\wedge\neg\Diamond B)\wedge\neg(\Box B\wedge\neg\Box A\wedge \neg\Diamond A)$
$= \Box(A\vee B)\wedge\neg\Diamond(A\wedge B)\wedge\Diamond A\wedge\Diamond B$

As such, this approach predicts that UNCERTAINTY follows from EXCLUSIVITY (i.e., UNCERTAINTY cannot arise without the presence of EXCLUSIVITY). As regards our experiments, we found no evidence of either UNCERTAINTY or EXCLUSIVITY.[13] An important question, which we will explore in Section 6.4, is whether there can be cases in which UNCERTAINTY is derived independently of EXCLUSIVITY.

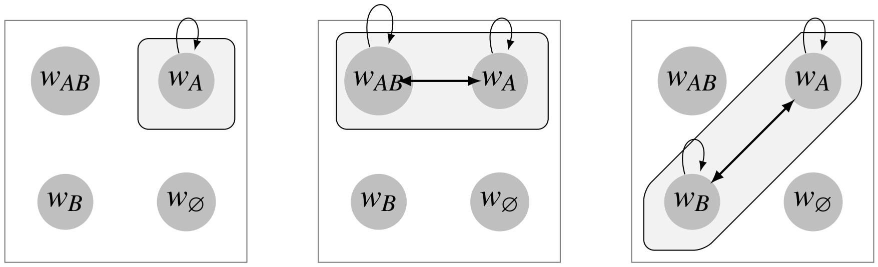
It is worth mentioning that the mechanism behind the derivation of both EXCLUSIVITY and POSSIBILITY is the same, in contrast to the non-implicature accounts presented in Section 6.2. In those cases, the derivation of exclusivity, whether weak or strong, is regarded as a type of implicature and is generally separated from distributive inferences.

We will now outline a recent non-implicature approach (Aloni 2022) to IGNORANCE inferences which accounts independently for POSSIBILITY and related inferences. We will also explore how such system can be enriched with a mechanism of implicature computation which is able to derive UNCERTAINTY via EXCLUSIVITY. It is worth noting that while our experimental data seem to suggest an independent mechanism to generate POSSIBILITY, the choice to take Aloni (2022)'s account as an example of a non-implicature approach is not crucial, and similar considerations can be explored using alternative accounts, such as the one discussed in Goldstein (2019).

6.2 A non-implicature account

On Aloni (2022)'s account, POSSIBILITY and related inferences do not arise from Gricean reasoning or the application of a covert exhaustification operator. In-

13 In our experiments, we found no evidence of strong exclusivity inferences $\Box_s\neg(A\wedge B)$. In previous experiments on distributive inferences, Crnič et al. (2015) also found no evidence of weak exclusivity inferences $\neg\Box_s(A\wedge B)$, meaning that the configuration AB-AB-AB-? was judged as good.



(a) $\vDash A \vee B$
$\nvDash (A \vee B)^+$

(b) $\vDash A \vee B$
$\vDash (A \vee B)^+$

(c) $\vDash A \vee B$
$\vDash (A \vee B)^+$

Figure 4 Illustrations. In w_{AB} both A and B hold, in w_A only A holds and so on. The accessibility relation, indicated with an arrow, is state-based/epistemic, since each world has access to the entire state.

stead, these inferences are the result of another pragmatic factor referred to as neglect-zero. In particular, Aloni (2022) puts forward the idea that language users, when interpreting a sentence, construct structures that represent reality and they systematically disregard structures that vacuously satisfy the sentence due to an empty configuration, also referred to as zero-models. Neglect-zero relates to a tendency that has been observed in number cognition and language development with regard to the number zero, as discussed in Nieder (2016).

These structures are built upon the notion of a state, a set of possible worlds, which reflects the speaker's information state and upon which formulas are interpreted. For instance, in the state s in fig. 4(c), the speaker considers possible that only A came or that only B came. Aloni (2022) formalizes this in a bilateral state-based modal logic, which we will refer as BSML.

BSML employs a split notion of disjunction, which is supported in a state s when the state can be split into two substates, each supporting one of the disjuncts. All the states in Figure 4 support a disjunction of the form $A \vee B$. For the state Figure 4(a), the singleton $\{w_A\}$, the two substates would be $\{w_A\}$ supporting A and the empty state $\varnothing$ supporting B.

6.2.1 Plain Disjunction and POSSIBILITY

As said above, Aloni (2022) proposes that language users neglect zero-models and therefore rule out the possibility of verifying a sentence by means of empty configurations, as in the last example discussed in the previous paragraph. Aloni

(2022) implements this neglect-zero effect by means of an enrichment function $(\cdot)^+$ which excludes the empty set as a possible verifier. Formally, this can be defined recursively on the complexity of the formulas, but here we will focus on the case of disjunction.

An enriched disjunction $(A \vee B)^+$ is supported in a state when the state can be split into two *non-empty* substates, each supporting one of the disjuncts. As a result, an enriched disjunction $(A \vee B)^+$ entails POSSIBILITY, but not UNCERTAINTY:

(17) a. $(A \vee B)^+ \vDash \Diamond_s A \wedge \Diamond_s B$
b. $(A \vee B)^+ \nvDash \neg \Box_s A \wedge \neg \Box_s B$

(17-a) can be formally proved in BSML. The idea is that a pragmatically enriched disjunction is supported only in states which include both A-worlds and B-worlds, and this ensure that $\Diamond_s A$ and $\Diamond_s B$ hold. In BSML $\Diamond_s$ and $\Box_s$ can be treated as standard epistemic modalities by putting the appropriate constraints in the accessibility relation (see Figure 4 for an illustration). For (17-b), a pragmatically enriched disjunction is also supported in states like Figure 4(b), which do not support $\neg \Box A$, since A is true in all words in the state.

Therefore, BSML offers a systematic explanation for the experimental findings presented in Section 4 and Section 5, namely that POSSIBILITY inferences are derived as neglect-zero effects without UNCERTAINTY inferences.

6.2.2 Embedded Disjunction and DISTRIBUTIVE Inferences

In Section 6.1, we observed that there is a strong parallelism between inferences related to plain disjunction and those in an embedded context. Therefore, a uniform theory that captures both types of inferences is desirable. BSML readily accounts for DISTRIBUTIVE inferences, since $[\Box_{(s)}(A \vee B)]^+$ entails $\Diamond_{(s)} A$ and $\Diamond_{(s)} B$ both for epistemic and other modalities (Aloni 2022):[14]

(18) a. $[\Box(A \vee B)]^+ \vDash \Diamond A \wedge \Diamond B$
b. $[\Box(A \vee B)]^+ \nvDash \neg \Box A \wedge \neg \Box B$

6.2.3 The role of EXCLUSIVITY

The non-implicature framework explored so far accounted for the derivation of POSSIBILITY and related inferences in the absence of the corresponding negated upper bound inferences and EXCLUSIVITY. While this approach makes the correct

14 Note also that first-order extensions of Aloni (2022)'s system, such as Aloni & van Ormondt (2021), can capture DISTRIBUTIVE inferences that arise in the context of nominal quantifiers.

predictions for our experimental paradigm, there are instances in which EXCLUSIVITY and UNCERTAINTY inferences are derived. We will address the issue of when and why these inferences are generated in Section 6.4.

For the moment, we observe that by adding EXCLUSIVITY to POSSIBILITY or DISTRIBUTIVE inferences, we automatically obtain the NEGATED UNIVERSAL inferences. This is similar to what was previously observed in Section 6.1.3 within the context of the grammatical approach. In particular, it would be valuable in future work to expand non-implicature accounts by including a scalarity operator $(\cdot)^{\sigma}$, which allows for the generation of scalar implicatures (e.g., EXCLUSIVITY for a plain disjunction). In this way, the interplay between the neglect-zero enrichment $(\cdot)^{+}$ and the scalar enrichment $(\cdot)^{\sigma}$ could be adequately studied.

We have seen that a non-implicature framework like the one outlined at the beginning of this section can independently account for POSSIBILITY inferences. EXCLUSIVITY and UNCERTAINTY inferences need to be derived by a separate mechanism. In the subsequent section, we will delve into an alternative theory in the context of a game-theoretic approach to pragmatics based on the Gricean original insights of cooperation between speaker and hearer. This theory will offer an additional viewpoint on the behavior of disjunction and its associated inferences.

6.3 The Speaker-Hearer perspective

In recent years, game-theoretic approaches to pragmatic inferences have become an increasingly popular tool for modeling linguistic phenomena and understanding the dynamics of communication (among others, Dekker & Van Rooy 2000, Benz et al. 2005, Franke 2011b, Frank & Goodman 2012). These models seek to capture the strategic interactions between speakers and listeners, and to provide a formal framework for analyzing how linguistic conventions and norms emerge.

In what follows, we will rely on the lifted Iterated Best Response (IBR) model discussed in Franke (2009), which allows us to model modal inferences of the kind examined in the present work.

The model includes a set of states, compatible with the BSML notion discussed in Section 6, and a set of possible messages, which we take to be A, B, $A \vee B$, $A \wedge B$. The sender is responsible for choosing an utterance, while the receiver is responsible for interpreting the utterance and inferring the intended meaning. The sender and receiver are assumed to be rational agents who are trying to behave optimally.

Figure 5 displays the optimal strategies for the sender (S^{*}) and the receiver (R^{*}). The sender sends a message based on a certain state, and in the optimal configuration we observe that the states where a disjunction $A \vee B$ is possible pattern with the results discussed in Section 4 and Section 5: only states where POSSIBILITY

$$S^* = \left\{ \begin{array}{ll} \{w_A\} & \mapsto A \\ \{w_B\} & \mapsto B \\ \{w_{AB}\} & \mapsto A \wedge B \\ \{w_A, w_{AB}\} & \mapsto A, A \vee B \\ \{w_B, w_{AB}\} & \mapsto B, A \vee B \\ \{w_A, w_B\} & \mapsto A \vee B \\ \{w_A, w_B, w_{AB}\} & \mapsto A \vee B \end{array} \right\} \quad R^* = \left\{ \begin{array}{lll} A & \mapsto & \{w_A\} \\ B & \mapsto & \{w_B\} \\ A \wedge B & \mapsto & \{w_{AB}\} \\ A \vee B & \mapsto & \{w_A, w_B\} \end{array} \right\}$$

.

Figure 5 Optimal strategies for sender and receiver in a lifted IBR model in Franke (2009)

is true are admitted. By contrast, for the receiver, disjunction is associated with a state where both UNCERTAINTY and POSSIBILITY are derived, in line with the fact that the receiver is pragmatically interpreting the disjunction $A \vee B$.[15]

This discussion leads to two observations. First, we note that the speaker is not taking disjunction to have a literal meaning, as it is not an optimal message for a state like $\{w_A\}$. As said, this aligns with treatment of disjunction in non-implicature frameworks, like in the neglect-zero enriched disjunction $(A \vee B)^+$ in BSML. Interestingly, this squares with the fact that the logical system behind BSML has been developed to model the *assertability* conditions of a sentence, rather than its truth conditions. The pragmatic neglect-zero enrichment affects this assertability dimension and is as such *speaker*-oriented, as in the game-theoretic model just outlined.[16]

Second, we believe that the distinction between speaker and hearer might shed some light on the intepretation of the results of our experiments. In particular, we may associate the speaker-oriented behavioural profile of our experiments with the observation that sentence-picture verification tasks, like the ones used in our experiments, are a production task. This is related to early observations made by Degen & Goodman (2014), who pointed out that verification tasks are a measure of production, rather than interpretation. Consequently, although further behavioral

15 There is however a notable difference with the results of our experiments. The state $\{w_{AB}\}$ is associated with the conjunction $A \wedge B$. While our experiments did not directly test this case, previous experimental studies suggest that full exclusivity is derived in a similar task. We also note that Franke (2009) relies on two fundamental assumptions. First, speakers are competent (i.e., they consider more uncertain states less likely). Second, speakers will select only the most likely messages in a state of maximal information, while they will be indifferent when the state is not of maximal information.

16 Note that in principle this effect can be blocked in some instances (e.g., in mathematical reasoning). An interesting research question is to determine which contexts and experimental tasks are tied to the different pragmatic enrichments of neglect-zero or scalarity, and their interaction.

data is needed, we might conjecture that in a sentence-picture verification paradigm, subjects reason from the point of view of the speaker: given what they know, they determine whether a certain statement can be asserted. There are of course cases where UNCERTAINTY and EXCLUSIVITY should be drawn, and we conjecture that in those cases, language users might adopt the point of view of the receiver: given an utterance, they determine the most optimal interpretation for that utterance. In the next section, we briefly overview when and why such inferences could be generated.

6.4 EXCLUSIVITY and UNCERTAINTY

Our experiments showed that disjunction could give rise to POSSIBILITY, while UNCERTAINTY and EXCLUSIVITY were not derived. Still, it is natural to assume that in certain contexts those inferences which we typically associate with disjunction are generated. In line with what was observed at the end of the last section, we could conjecture that an experiment aimed at measuring interpretation rather than production could provide valuable insights. In the current mystery box experiments, participants had to judge if a sentence was good or bad based on the information available to them. A variant of these experiments would consist in asking participants to correctly place the balls inside the boxes given a truthful sentence, thereby reflecting the hearer/interpretation viewpoint.

Another case in which EXCLUSIVITY and UNCERTAINTY seem to play a role are conversational contexts like (19) and (20). In such contexts, a disjunction results in oddity. A preliminary explanation could attribute the oddness of such continuations to the fact that exclusivity and UNCERTAINTY inferences clash with the information available in the context. In this regard, an important question is whether UNCERTAINTY inferences always correlate with EXCLUSIVITY. The seeming oddness of (20-b) suggests a negative answer.

(19) Context: We all agree that Mary is here and John is here as well.
   a. ?Mary or John are here.

(20) Context: We all agree that Mary is here, and John might be too.
   a. ?Mary or John are here.
   b. (?)I don't know whether they are both here, but either one or the other is here.

An alternative explanation for the oddness observed in cases such as (19) and (20) is that we are bringing attention to the alternatives in a linguistic form. For example, in (20), we are highlighting the alternative that 'It is certain that Mary is here and it is possible John is here'. This alternative is generally more complex, and it is typically not considered unless we emphasize it linguistically, as suggested by structure-sensitive characterizations of alternatives like Katzir (2007).

We leave the investigation of cases like the above for future work.

7 Conclusion and next steps

In this work, we have reported the results of two experiments that challenge the traditional implicature account of IGNORANCE inferences associated with plain disjunctive sentences. Our findings indicate that participants did derive POSSIBILITY inferences even in the absence of UNCERTAINTY inferences. This study thus contributes to the ongoing debate on the nature of inferences associated with disjunction. Our results call for a reevaluation of the classical view of IGNORANCE inferences and pave the way for alternative theories that can account for the observed patterns of inference derivation.

To contextualize our findings, we discussed three different perspectives: grammatical, non-implicature, and game-theoretic approaches. Furthermore, we situated our discussion within the broader context of disjunction in embedded contexts.

Further experiments could probe the robustness of our results using different experimental methodologies. For instance, experiments eliciting indirect judgments of felicity could provide converging evidence. Another potential follow-up would involve the cases we discussed in Section 6.4, where UNCERTAINTY and EXCLUSIVITY seem to play a role.

Additionally, studying the role of contextual factors in the derivation of these inferences could provide a more comprehensive understanding of the interplay between semantics and pragmatics in the interpretation of disjunctive sentences. For instance, we might conjecture that disjunction will be interpreted literally in mathematical reasoning, where even POSSIBILITY inferences are not derived. Similarly, our results showed some variation among participants, suggesting that some individuals were more prone to deriving POSSIBILITY inferences from disjunctive sentences than others. We leave exploring what drives these differences in future work.

A Instructions for Experiment 1 and 2

General Instructions

In this study, we will ask for your intuitions about certain kinds of sentences in English. These sentences will be uttered by two characters, Sam and Mia. Here they are:

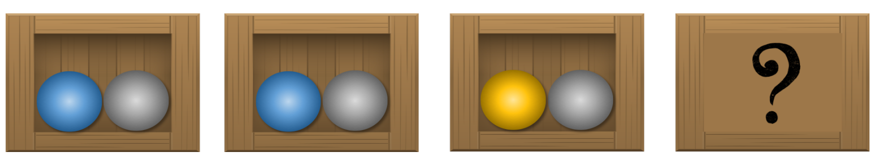
Sam

Mia

.... [continue] ....

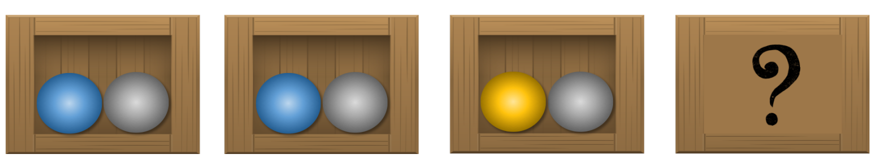
What do Sam and Mia know about the mystery box?

Sam and Mia will be presenting with quadruplets of boxes containing balls of various colors. Each time, they can see what's inside the first three boxes, but not what's inside the fourth one, the so-called 'mystery box'. However, they have been taught the rule that **the mystery box always has the same contents as one of the three open boxes.** Thanks to this rule, they can make certain inferences about what the mystery box contains and does not contain.



.... [try an example] ....

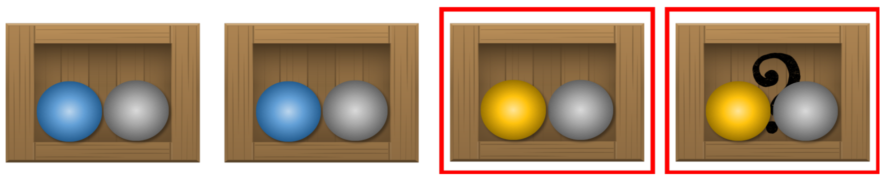
In this example, Sam knows that **every open box** contains a gray ball. Similarly, he knows that one open box also contains a yellow ball while the other two also contain a blue one.



Thanks to the rule, Sam should be certain that **the mystery box** contains a gray ball. He should also be certain that this box does not contain a green ball. However, he cannot be certain that the mystery contains a blue ball, nor can he be certain that it contains a yellow ball.

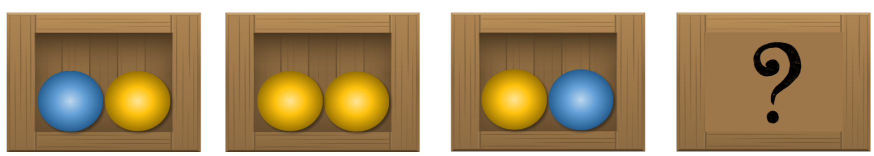
[reveal the mystery box’s contents]

In this case, the contents of the mystery box were identical to those of the third box.



[try another example]

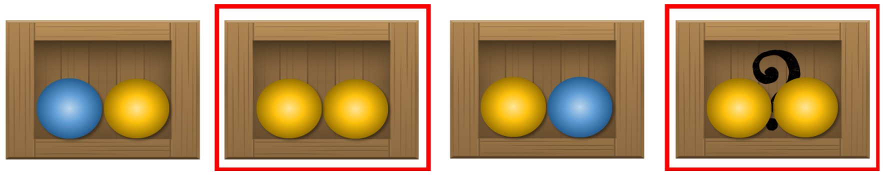
In this second example, Mia knows that **every open box** contains a yellow ball. Similarly, she knows that one open box contains another yellow ball while the other two contain a blue one.



Thanks to the rule, Mia should be certain that **the mystery box** contains a yellow ball. She should also be certain that it does not contain a gray or a green ball. However, she cannot be certain that the mystery contains two yellow balls, nor can she be certain that it contains a blue ball.

[reveal the mystery box’s contents]

In this case, the contents of the mystery box were identical to those of the second box.



[CONTINUE]

Sam and Mia will see many quadruplets like these ones, each of which will be followed by an utterance from one of them about the mystery box contains or does not contain. You task is to decide if this utterance is right given the information available to the characters and the rule that they have learned about how the mystery box works. You can click on ‘Good’ if you consider that they got it right; otherwise click on ‘Bad’.

Training Phase

You will start with a short training to get you familiar with the task. During this training, you will receive **feedback** on your responses. If you answered correctly, your will see a **green smiley face**; otherwise, you will see a **red frowning face** and be asked to try again. Use this feedback wisely to improve your answers.

We are interested in your spontaneous responses, so don’t think too long before answering.

Test Phase

As in the training, you will decide whether or not the character’s utterances are right given the information available to them and the rule that they have learned about how the mystery box works. From now on, however, **you will no longer receive feedback on your responses**.

Recall that we are interested in your spontaneous responses, so don’t think too long before answering.

References

Aloni, Maria. 2022. Logic and conversation: the case of free choice. *Semantics and Pragmatics* 15.

Aloni, Maria & Peter van Ormondt. 2021. Modified numerals and split disjunction: the first-order case. *Journal of Logic, Language and Information* .

Anwyl-Irvine, A. L., J. Massonie, A. Flitton, N. Z. Kirkham & J. K. Evershed. 2020. Gorilla in our midst: An online behavioral experiment builder. *Behavior Research Methods* 52. 388–407.

Bar-Lev, Moshe E & Danny Fox. 2020. Free choice, simplification, and innocent inclusion. *Natural Language Semantics* 28(3). 175–223.

Bar-Lev, Moshe E & Danny Fox. 2023. On fatal competition and the nature of distributive inferences .

Bates, Douglas, Martin Mächler, Ben Bolker & Steve Walker. 2015. Fitting linear mixed-effects models using lme4. *Journal of Statistical Software* 67(1). 1–48. http://dx.doi.org/10.18637/jss.v067.i01.

Benz, Anton, Gerhard Jäger, Robert Van Rooij & Robert Van Rooij. 2005. *Game theory and pragmatics.* Springer.

Buccola, Brian & Andreas Haida. 2019. Obligatory irrelevance and the computation of ignorance inferences. *Journal of Semantics* 36(4). 583–616.

Crnič, Luka, Emmanuel Chemla & Danny Fox. 2015. Scalar implicatures of embedded disjunction. *Natural Language Semantics* 23(4). 271–305.

Degen, Judith & Noah Goodman. 2014. Lost your marbles? the puzzle of dependent measures in experimental pragmatics. In *Proceedings of the annual meeting of the cognitive science society*, vol. 36 36, .

Dekker, Paul & Robert Van Rooy. 2000. Bi-directional optimality theory: An application of game theory. *Journal of Semantics* 17(3). 217–242.

Dorst, Kevin & Matthew Mandelkern. 2022. Good guesses. *Philosophy and Phenomenological Research* 105(3). 581–618.

Fox, Danny. 2007. Free choice and the theory of scalar implicatures. In Uli Sauerland & Penka Stateva (eds.), *Presupposition and Implicature in Compositional Semantics*, 71–120. Palgrave.

Fox, John & Sanford Weisberg. 2019. *An R companion to applied regression.* Thousand Oaks CA: Sage 3rd edn. https://socialsciences.mcmaster.ca/jfox/Books/Companion/.

Frank, Michael C & Noah D Goodman. 2012. Predicting pragmatic reasoning in language games. *Science* 336(6084). 998–998.

Franke, Michael. 2009. *Signal to act: Game theory in pragmatics.* University of Amsterdam.

Franke, Michael. 2011a. Quantity implicatures, exhaustive interpretation, and rational conversation. *Semantics and Pragmatics* 4(1). 1–82. http://dx.doi.org/10.3765/sp.4.1.

Franke, Michael. 2011b. Quantity implicatures, exhaustive interpretation, and rational conversation. *Semantics and Pragmatics* 4. 1–1.

Gamut. 1991. *Logic, language and meaning*. University of Chicago Press.

Gazdar, Gerald. 1979. *Pragmatics: Implicature, Presupposition, and Logical Form.* New York: Academic Press.

Goldstein, Simon. 2019. Free choice and homogeneity. *Semantics and Pragmatics* 12. 23–EA.

Grice, Paul. 1975. Logic and conversation. In The Logic of Grammar, D. Davidson and G. Harman (eds), Encino, CA: Dickenson, 64-75.

Grice, Paul. 1989. *Studies in the way of words.* Cambridge, Mass.: Harvard University Press.

Harrell Jr, Frank E. 2021. *Package 'hmisc'*. https://CRAN.R-project.org/package=Hmisc. R package version 4.5-0.

Hintikka, Jaakko. 1962. *Knowledge and belief: An introduction to the logic of the two notions.* Ithaca: Cornell University Press.

Horn, Lawrence. 1972. *On the semantic properties of logical operators in English*: UCLA dissertation.

Incurvati, Luca & Julian J Schlöder. 2019. Weak assertion. *The Philosophical Quarterly* 69(277). 741–770.

Katzir, Roni. 2007. Structurally-defined alternatives. *Linguistic and Philosophy* 30(6). 669–690. http://dx.doi.org/10.1007/s10988-008-9029-y.

Levinson, Stephen. 1983. *Pragmatics.* MIT press.

Lüdecke, Daniel. 2023. *sjplot: Data visualization for statistics in social science.* https://CRAN.R-project.org/package=sjPlot. R package version 2.8.14.

Marty, Paul, Jacopo Romoli, Yasutada Sudo & Richard Breheny. 2023. What makes an inference robust? Manuscript under review.

Meyer, Marie-Christine. 2013. *Ignorance and grammar*: MIT dissertation.

Nieder, Andreas. 2016. Representing something out of nothing: The dawning of zero. *Trends in Cognitive Sciences* 20(11). 830–842.

Noveck, Ira A. 2001. When children are more logical than adults: Experimental investigations of scalar implicature. *Cognition* 78(2). 165–188.

Palan, Stefan & Christian Schitter. 2018. Prolific.ac — a subject pool for online experiments. *Journal of Behavioral and Experimental Finance* 17. 22–27.

R Core Team. 2021. *R: A language and environment for statistical computing*. R Foundation for Statistical Computing Vienna, Austria. https://www.R-project.org/.

Ramotowska, Sonia, Paul Marty, Jacopo Romoli, Yasutada Sudo & Richard Breheny. 2022. Diversity with universality. In *Proceedings of the*

Amsterdam Colloquium 2022, https://www.dropbox.com/s/4vfhehe2q2efljs/Proceedings2022-pages-258-264.pdf?dl=0.

Sauerland, Uli. 2004a. On embedded implicatures. *Journal of cognitive science* 5(1). 107–137.

Sauerland, Uli. 2004b. Scalar implicatures in complex sentences. *Linguistics and Philosophy* 27(3). 367–391. http://dx.doi.org/10.1023/B:LING.0000023378.71748.db.