Positivity, (anti-)exahsutivity and stability*

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Abstract

Why are AND and OR the only binary connectives which are lexicalized as simplex? In this paper we point out a special property they have: They are the only 'meaningful' connectives which enable the innocent exclusion of alternatives that are logically independent from the connective's arguments. We further observe that, as a result, they are the only meaningful connectives that can block prior probabilities from leading to anti-exhaustive interpretations (Cremers et al. 2022). This property of AND and OR can explain their lexicalizability once combined with the notion of Stability from Bar-Lev & Katzir (2022), while the theory that emerges has advantages over the Stability-based account of the typology we proposed in Bar-Lev & Katzir (2022).

1 Positivity

The typology of logical connectives presents a major cross-linguistic puzzle: Among the 12 binary logical connectives in table 1 (considering here only those that are 'meaningful', i.e., non-trivial and non-redundant),[1] only AND and OR are ever lexicalized as simplex. While NOR is also apparently lexicalized sometimes, it has been argued that whenever it is, it is a morpho-syntactic composition of negation and OR (Sauerland 2000; Penka 2011; Zeijlstra 2011). For our purposes here we set aside what is involved in cases of apparent lexicalizations of NOR and why there are no apparent lexicalizations of NAND (Horn 1972), and focus on why no connective other than AND and OR is lexicalized as simplex.

	AND	OR	NOR	NAND	XOR	IFF	NOTL	NOTR	ONLYL	ONLYR	$\rightarrow$	$\leftarrow$
$p \wedge q$	1	1	0	0	0	1	0	0	0	0	1	1
$p \wedge \neg q$	0	1	0	1	1	0	0	1	1	0	0	1
$q \wedge \neg p$	0	1	0	1	1	0	1	0	0	1	1	0
$\neg p \wedge \neg q$	0	0	1	1	0	1	1	1	0	0	1	1

Table 1: All binary logical connectives which are 'meaningful' (excluding trivial and redundant connectives)

Since Horn (1972), the simplicity of AND and OR has usually been taken for granted, most commonly by assuming that 'positive' connectives, like AND and OR, are inherently less

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[1] We assume that lexicalization of the non-meaningful connectives in the following table can be ruled out on independent grounds, given arguments that there are general bans on redundancy and triviality (see Gajewski 2002; Sauerland 2004, a.o.).

	L	R	TAU	CONT
$p \wedge q$	1	1	1	0
$p \wedge \neg q$	1	0	1	0
$q \wedge \neg p$	0	1	1	0
$\neg p \wedge \neg q$	0	0	1	0

complex than 'negative' ones, like NAND (see Katzir & Singh 2013, a.o.); we will call this assumption Positivity. As Enguehard & Spector (2021) and Incurvati & Sbardolini (2020) point out, however, there is no compelling independent evidence for Positivity, and it assumes what needs to be explained. Enguehard & Spector propose an account which does not rely on Positivity, but simulations by Züfle & Katzir (2022) suggest that their account can only provide a general theory of the typological pattern if Positivity is assumed after all. Incurvati & Sbardolini also aim to account for the typological pattern without assuming Positivity, but they rely on specific assumptions about the dynamics of conversation which as far as we can see are not independently motivated.[2] More importantly, both accounts wrongly predict NOR to be lexicalizable as simplex.

Our goal is to derive Positivity by building on a property of AND and OR which falls out of an independently needed theory of alternatives and communication. The theory we assume is one where grammar incorporates an exhaustivity mechanism (with no access to real world probabilities), whereas pragmatics is governed by general rational reasoning (which takes real world probabilities into account). For recent evidence in favor of this view see especially Franke & Bergen (2020); Cremers et al. (2022). In §2 we will identify a property that distinguishes AND and OR from all other meaningful connectives based on the notion of Innocent Exclusion from Fox (2007), and in §3 we will use a simple model of communication with access to prior probabilities in order to illustrate the ramifications that this property has for communication. In §4 we will then argue that, as a result, languages with AND and OR (and no other connective) have a communicative advantage over languages with other meaningful connectives, using the notion of Stability from Bar-Lev & Katzir (2022).

2 Exhaustivity

Following Fox (2007), we assume a covert exhaustivity operator $\mathcal{E}xh$ which negates alternatives that are innocently excludable (IE), as defined in (1) (both $\mathcal{E}xh$ and Innocent Exclusion have roots in Groenendijk & Stokhof 1984; see Spector 2016).

(1) a. $[\![\mathcal{E}xh]\!](C)(p)(w) = 1$ *iff* $p(w) = 1 \wedge \forall q \in IE(C,p)[q(w) = 0]$
    b. $IE(C,p) = \bigcap\{C' : C'$ is a maximal subset of C s.t. $p \wedge \bigwedge\{\neg q : q \in C'\} \not\Leftrightarrow_L \bot\}$

Innocent Exclusion aims to avoid arbitrary choices of exclusion. We can describe what (1) does given a set of alternatives C and a prejacent p using the following procedure (see also Bar-Lev & Fox 2017):

(2) **Innocent Exclusion procedure**:
  a. Take all maximal sets of alternatives in C that can be assigned false consistently with the prejacent p.
  b. Only exclude those alternatives that are members in all such sets—the Innocently Excludable alternatives.

We observe that AND and OR have a property which sets them apart from all other connectives in table 1 and which has to do with Innocent Exclusion: When a sentence of the form P CON

[2] Specifically, they assume a dynamic system where assertion and rejection have a fundamentally different status, and as such their account relies on non-trivial assumptions about the building blocks of the dynamics of conversation (we thank Benjamin Spector for a discussion of this issue). Moreover, it is crucial for their account that rejecting a conjunction should end up equivalent to rejecting a disjunction; we however do not know of evidence that could support this assumption.

Q has a logically independent alternative (LIA) R — that is, an alternative that stands in no entailment relations to either P or Q — the alternative R is IE if CON is AND or OR, but non-IE if CON is one of the other 10 connectives in table 1. We assume that when R is salient P CON Q has the set of alternatives in (3), which is what is predicted by the theory of structural alternatives by Katzir (2007).[3]

(3) $Alt(\text{P CON Q})^{+} = \{x \text{ CON } y : x, y \in \{\text{P,Q,R}\}\} \cup \{\text{P,Q,R}\}$

For instance, R is non-IE given P NAND Q: (P NAND Q)∧¬(P NAND R) is consistent, which means there is a maximal set of alternatives whose exclusion is consistent with P NAND Q that contains P NAND R; this set cannot contain R, because (P NAND Q)∧¬(P NAND R)∧¬R is inconsistent. As a result, R is non-IE. The reader is invited to verify that similar considerations make R non-IE given any sentence of the form P CON Q for any meaningful CON other than AND or OR. Note that no addition of alternatives to $Alt(\text{P CON Q})^{+}$ can make R IE if it isn't, so exclusion of LIAs is only guaranteed in languages without any connective other than AND and OR. In other words, among all 4095 ($= 2^{12} - 1$) possible inventories of meaningful connectives, there are only 3 where LIAs are IE for every sentence of the form P CON Q: {AND}, {OR}, and {AND, OR} (see Uegaki 2022 for evidence that all of them are attested). Excludability of LIAs then distinguishes AND and OR from all other meaningful connectives:

(4) **Excludability of LIAs**: AND and OR are the only meaningful connectives which enable the innocent exclusion of LIAs.

In what follows we will argue that the excludability of LIAs has implications for communication which can explain Positivity.

3 Anti-exhaustivity

Why should the excludability of LIAs matter? We claim that it matters once we consider the implications of non-excludability of LIAs within a theory of communication based on Iterated Rationality Models (IRMs; Franke 2009; Frank & Goodman 2012; Bergen et al. 2016), due to what Cremers et al. (2022) call Anti-exhaustivity. While IRMs have often been used to derive scalar implicatures (SIs), recent work has argued that SIs are grammatically derived, and IRMs should serve as a disambiguation mechanism (see Champollion et al. 2019; Fox & Katzir 2021; Asherov et al. 2022; Cremers et al. 2022). Specifically, Cremers et al. show that, in the absence of grammatical strengthening, IRMs wrongly predict a sentence like P to sometimes end up meaning $p \wedge \neg q$ (exhaustive meaning) and sometimes end up meaning $p \wedge q$ (anti-exhaustive meaning), depending on the prior probabilities. If $\mathcal{E}xh$ applies, however, this wrong prediction disappears, because $\mathcal{E}xh$(P) excludes the LIA Q, which results in a meaning that is compatible with $p \wedge \neg q$ but incompatible with $p \wedge q$.

Let us illustrate how this problem is relevant for our discussion of connectives and LIAs by considering (a subset of) the set of alternatives in (3) when the connective CON is AND and looking at the predictions of a simple IRM (taken from Bar-Lev & Katzir 2022; this particular choice is not crucial, since as Cremers et al. 2022 pointed out, the issue of Anti-exhaustivity in the absence of grammatical strengthening arises with all IRMs they considered).

The rational hearer we assume is defined in (5-c). Given a certain message, this hearer applies Bayes' Rule to infer the likeliest state(s) given the message, on the assumption that the

[3]For simplicity we ignore replacements of the connective with other connectives; nothing hinges on this though.

message was sent by a rational speaker; if there is exactly one such state, the rational hearer chooses it, i.e., assigns it probability 1 (and if there are several likeliest states the assigned probability ends up evenly divided between them). The rational speaker we assume is defined in (5-b). If one message has the best chances of leading a naive hearer to choose the state that the rational speaker wants to convey, then they assign probability 1 to that message (if there are several best messages the assigned probability ends up evenly divided between them). The naive hearer, in turn, attributes a probability to a state upon hearing a message based on the prior probability of that state (given by the function PR) and whether the message is true in that state (that is, if the message is false in that state the assigned probability is 0).

(5) a. A naive hearer NH: $NH(s|m) \propto PR(s) \cdot [\![\text{m}]\!](s)$

b. A rational speaker RS:

$$RS(m|s) \propto \begin{cases} 1 & \text{if } m \in \underset{m'}{\text{argmax}} NH(s|m') \text{ and } \underset{m'}{\text{max}} NH(s|m') \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

c. A rational hearer RH:

$$RH(s|m) \propto \begin{cases} 1 & \text{if } s \in \underset{s'}{\text{argmax}}(RS(m|s') \cdot PR(s)) \text{ and } \underset{s'}{\text{max}} RS(m|s') \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

The set of messages given by (3) when CON is AND is as follows:[4]

(6) {P AND Q, P AND R, Q AND R, P, Q, R}

To keep the current illustration simple, we will ignore most of the messages in (6) and pretend that the possible messages are just P AND Q and its LIA R (the reader may verify that Anti-exhaustivity will arise even when the full set of messages in (6) is considered). Imagine now a rational speaker who wants to convey the state $p \wedge q \wedge r$, assuming that the prior probabilities of the states (given by the partition of logical space induced by (6)) to be as in Table 2. Naive hearer behavior in this case is predicted to be as in Table 3, and rational speaker behavior is predicted to be as in table 4.

	$p \wedge q$ $\wedge r$	$\neg p \wedge q$ $\wedge r$	$p \wedge \neg q$ $\wedge r$	$\neg p \wedge \neg q$ $\wedge r$	$p \wedge q$ $\wedge \neg r$	$\neg p \wedge q$ $\wedge \neg r$	$p \wedge \neg q$ $\wedge \neg r$	$\neg p \wedge \neg q$ $\wedge \neg r$
Priors	0.2	0.1	0.1	0.1	0.05	0.05	0.05	0.35

Table 2: Prior probabilities of states

NH	$p \wedge q$ $\wedge r$	$\neg p \wedge q$ $\wedge r$	$p \wedge \neg q$ $\wedge r$	$\neg p \wedge \neg q$ $\wedge r$	$p \wedge q$ $\wedge \neg r$	$\neg p \wedge q$ $\wedge \neg r$	$p \wedge \neg q$ $\wedge \neg r$	$\neg p \wedge \neg q$ $\wedge \neg r$
P AND Q	0.8	0	0	0	0.2	0	0	0
R	0.4	0.2	0.2	0.2	0	0	0	0

Table 3: Naive hearer behavior

[4] We only aim to provide here a demonstration of the problem of Anti-exhaustivity, so we ignore the possibility of having more complex alternatives like P AND Q AND R and the effects of message costs. As Cremers et al. (2022) show, anti-exhaustive readings are predicted even with more complex alternatives and sophisticated assumptions about the role of message costs in the model. This however depends on the assumption that message costs should play a role in the model, which we cannot argue for here and have to leave for another occasion.

RS	$p \wedge q$ $\wedge r$	$\neg p \wedge q$ $\wedge r$	$p \wedge \neg q$ $\wedge r$	$\neg p \wedge \neg q$ $\wedge r$	$p \wedge q$ $\wedge \neg r$	$\neg p \wedge q$ $\wedge \neg r$	$p \wedge \neg q$ $\wedge \neg r$	$\neg p \wedge \neg q$ $\wedge \neg r$
P AND Q	1	0	0	0	1	0	0	0
R	0	1	1	1	0	0	0	0

Table 4: Rational speaker behavior

Given a message, the rational hearer takes all states where this message would be an optimal one for the rational speaker and chooses the one which has the highest prior probability (if there is one). The rational hearer's inferences are summarized in table 5.

RH	$p \wedge q$ $\wedge r$	$\neg p \wedge q$ $\wedge r$	$p \wedge \neg q$ $\wedge r$	$\neg p \wedge \neg q$ $\wedge r$	$p \wedge q$ $\wedge \neg r$	$\neg p \wedge q$ $\wedge \neg r$	$p \wedge \neg q$ $\wedge \neg r$	$\neg p \wedge \neg q$ $\wedge \neg r$
P AND Q	1	0	0	0	0	0	0	0
R	0	1/3	1/3	1/3	0	0	0	0

Table 5: Rational hearer behavior

In this case then, the optimal choice for a rational speaker who wants to convey the state $p \wedge q \wedge r$ is P AND Q, and the rational hearer who gets this message will choose $p \wedge q \wedge r$. The model then predicts that an anti-exhaustive interpretation of P AND Q would be possible, something which never actually happens (as Cremers et al. 2022 argue for slightly different cases). Note the importance of priors here: Switching between the prior probabilities of the states $p \wedge q \wedge r$ and $p \wedge q \wedge \neg r$ in table 2 would lead a rational hearer to choose the state $p \wedge q \wedge \neg r$ when they hear P AND Q; in other words, they will have an exhaustive interpretation. Of course, if P AND Q were exhaustified, an anti-exhaustive interpretation would be impossible no matter what the priors look like, because $\mathcal{E}xh$(P AND Q) is only compatible with $p \wedge q \wedge \neg r$ given that the logically independent alternative R is IE: exhaustivity would block anti-exhaustivity.

Recall now that we have shown in §2 that $\mathcal{E}xh$ does not always allow for exclusion of LIAs for sentences with connectives: It only does so if the connective at stake is AND or OR. P NOR Q, for instance, cannot innocently exclude R when exhaustified, and as a result would sometimes end up meaning $\neg p \wedge \neg q \wedge \neg r$ (exhaustive meaning) and sometimes end up meaning $\neg p \wedge \neg q \wedge r$ (anti-exhaustive meaning), depending on the prior probabilities, even if $\mathcal{E}xh$ applies.[5] Similar results hold of any meaningful connective other than AND and OR; for reasons of space we cannot show this here in detail. Under the assumption that IRMs are a good model of communication when taken together with a grammar that incorporates $\mathcal{E}xh$ (a view defended by Franke & Bergen 2020; Cremers et al. 2022), then, we can distinguish between AND/OR and all the other 10 connectives in table 1 as follows:

(7) **Anti-exhaustivity with connectives**: For a meaningful CON, $\mathcal{E}xh$(P CON Q) can have both exhaustive and anti-exhaustive meanings (depending on prior probabilities) *iff* CON $\notin \{\text{AND}, \text{OR}\}$.

4 Stability

Using Bar-Lev & Katzir's (2022) terms (extended here to apply to hearer's choices rather than speaker's choices), a sentence whose interpretation depends on prior probabilities is *unstable*; we assume as in Bar-Lev & Katzir (2022) that instability is a bad property for communication:

[5] To keep things simple we pretend that $\mathcal{E}xh$ always applies and excludes R when it is IE. Our view is however in line with the natural assumption that exclusion of LIAs is not derived when they are irrelevant, in which case Anti-exhaustivity cannot arise (Cremers et al.).

It means that a speaker and a hearer may miscommunicate if they do not share the same assumptions about prior probabilities. We hypothesize that this, in turn, can affect language change: Unstable languages will be filtered out when languages evolve because they are not efficient for communication.

Due to Anti-exhaustivity, languages with connectives other than AND and OR are unstable: Expectations determine whether the interpretation is exhaustive or anti-exhaustive. Based on certain expectations, a speaker may utter P NAND Q, expecting the hearer to infer ¬R (exhaustive interpretation). But a hearer with different prior beliefs may infer R instead (anti-exhaustive interpretation). Languages with AND, OR, or both (but no other meaningful connective) are stable because in such languages exhaustification blocks anti-exhaustive readings: A hearer will never infer R from P AND/OR Q. The difference between AND and OR in terms of excludability of LIAs, which as we have seen has consequences for anti-exhaustivity, leads to the following generalization:

(8) **Stability**: A language (an inventory of connectives) is stable *iff* it contains no meaningful connectives other than AND and OR.

We propose then that AND and OR are the only meaningful connectives for which simplex lexicalizations are attested because they have a communicative advantage, being the only ones that can avoid anti-exhaustivity and consequently instability.

Table 6 summarizes our proposal. AND and OR enable the exclusion of LIAs through exhaustivity, which blocks anti-exhaustivity and leads to stability. Other meaningful connectives do not enable the exclusion of LIAs through exhaustivity, and as a result they give rise to anti-exhaustivity, and any inventories that include such connectives are unstable.

	Exclusion of LIAs		Anti-exhaustivity		Stability
AND and OR	succeeds	⇒	blocked	⇒	✓
Other CONs	fails	⇒	possible	⇒	✘

Table 6: Summary of our proposal

5 Comparison with Bar-Lev & Katzir (2022)

Our view, which is based on the interaction of grammatical strengthening with LIAs, avoids several shortcomings of our proposal in Bar-Lev & Katzir (2022). First, in Bar-Lev & Katzir (2022) we assumed that IRMs apply with no underlying grammatical strengthening; as a result, that system leads to anti-exhaustive readings when LIAs are considered even in a language like {AND, OR} (which then ends up as unstable from the speaker's perspective). In other words, by not assuming grammatical strengthening, the system in Bar-Lev & Katzir (2022) predicts anti-exhaustive readings to arise in English and many other languages, something which, as Cremers et al. (2022) argue, never actually happens.

Second, in Bar-Lev & Katzir (2022) non-commutative connectives are stipulated to be non-lexicalizable (as in all other attempts at explaining the typology we are aware of, see Uegaki 2022 for discussion). $\mathcal{E}xh$, which is needed anyway, makes it possible to dispense with such stipulations: All non-commutative connectives in table 1 lead to Anti-exhaustivity and instability.

Finally, in Bar-Lev & Katzir (2022) we stipulated that every language has negation, as that account is contingent on considering stability in both positive and negative sentences. Our current view singles out languages with AND and OR even without this stipulation.

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