

Monotonicity revisited: mass nouns and comparisons of purity*

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February 12, 2021

Abstract

Comparatives with *more* plus mass noun, like *John has more milk than Bill*, are naturally analyzed as referencing measure functions, functions like VOLUME or WEIGHT that map individuals to degrees. Although such measure functions vary with context as well as the choice of mass noun, there are well known grammatical limitations on this variation. In particular, Schwarzschild (2006) proposes that only *monotonic* measure functions can enter into the interpretation of comparatives with *more* plus mass noun. While this *Monotonicity Constraint* has strong empirical support, Bale and Barner (2009) have drawn attention to data that seemingly contradict it. For example, *There is more gold in the ring than in the bracelet* can be evaluated based on whether the ring is made from purer gold than the bracelet. This seems to suggest that comparatives with *more* plus mass noun can reference PURITY, yet PURITY is non-monotonic (Schwarzschild 2006, Wellwood 2015). Building on Solt (2018) and Bale and Schwarz (2019), we show here that comparisons of purity can be credited to *monotonic proportional* measure functions, thereby reconciling Bale and Barner’s observation with the Monotonicity Constraint. We provide independent support for this proposal, establishing that reference to the relevant monotonic proportional measure functions, but not to PURITY, yields meanings that accurately track speakers’ truth value judgments. Our analysis commits us to the assumption that the main clause and the comparative clause can invoke different measure functions. We propose that this is made possible by Skolemization and binding. That is, we posit function-denoting expressions which contain variables that have different binders in the two clauses.

1 Introduction

Measure functions, functions that map individuals (or events) to degrees, play a prominent role in natural language semantics, especially in the analysis of gradable predicates and the degree constructions they participate in. For example, in many semantic analyses (e.g., Bartsch and Vennemann 1972, Kennedy 1999), a gradable predicate such as *heavy* references a function that maps individuals to some measurement (such as weight) and a comparative sentence like *This thing is heavier than that thing* conveys that the function maps one thing to a greater measurement (greater weight) than another. Commenting on Schwarzschild (2006), Bale and Barner (2009), and Wellwood (2015), this paper will investigate measure functions as they relate to the meaning of *much*. It will do so by scrutinizing data where its comparative form *more* combines with a mass noun.

As Cresswell (1976) and Bale and Barner (2009) observed, *much* in combination with mass nouns can be associated with a range of different measurement scales (although variability is limited by the

*For comments and discussion, we thank the members of the Semantics Research Group at McGill University, especially Aron Hirsch, the audience at the *Amsterdam Colloquium 22* poster session, especially Maribel Romero, and two anonymous reviewers for Journal of Semantics. The authors acknowledge support provided by the Social Sciences and Humanities Research Council of Canada (SSHRC; Bale, Schwarz: Insight Grant #435-2016-1376; Schwarz, Shanks: Insight Grants #435-2013-0592, #435-2016-1448) and by McGill University’s Faculty of Arts (Shanks: Arts Research Internship Award).

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noun’s content). For example, *more butter than specified* can naturally serve to describe an excess of butter in terms of either weight or volume (although not in terms of, e.g., length); *more rope than needed* is most often understood as referencing length (although not, e.g., duration); and *more travel than expected* could be about either duration or distance (although not about, e.g., weight or volume). Despite this variability, however, certain measure functions are systematically excluded from entering the meaning of *much*. For example, *much* does not permit comparisons in terms of temperature, durability, or safeness. To illustrate, *more butter than specified*, *more rope than needed*, and *more travel than expected* cannot be understood as expressing comparisons regarding the butter’s temperature, the rope’s durability, or the travel’s safeness.

Schwarzschild (2006) proposes that there are certain constraints that limit which measure functions are permitted when *much* combines with a mass noun. In particular, he hypothesizes that such measure functions must be *monotonic* relative to the denotation of the nominal complement.¹ Broadly speaking, a measure function is monotonic with respect to a nominal complement if and only if the ordering of the outputs (the degrees) track the relevant part-whole relation inherent in the noun. More precisely, a measure function is monotonic just in case it maps any element in the nominal domain to a degree that is greater than the degrees to which it maps the element’s proper parts within that nominal domain. Consider VOLUME, the measure function that maps portions of matter to their volume. VOLUME is monotonic with respect to a noun like *water* because any given portion of water necessarily has a greater volume than any smaller portion of water it contains. In contrast, a portion of water does not in general have a greater temperature than smaller portions contained in it; TEMPERATURE, the measure function that maps its inputs to their temperature, is therefore not monotonic with respect to the noun *water*. Accordingly, the condition Schwarzschild proposes, which we will call the *Monotonicity Constraint* (following Solt 2018), correctly excludes TEMPERATURE, but not VOLUME, from the family of measure functions that can be referenced by *much* when combining with a mass noun like *water*.

Similarly, the measure function WEIGHT tracks the part-whole relation inherent in the denotation of *butter* (e.g., sub-portions of butter are less in weight than larger portions that contain it), LENGTH tracks the part-whole relation inherent in the denotation of *rope* (e.g., sub-segments of rope are shorter than larger segments that contain it), and likewise DURATION and DISTANCE track the part-whole relation inherent in the denotation of *travel* (e.g., any sub-event of travel will be shorter in duration or distance than any larger event that contains it). Hence, all of these measure functions are monotonic with respect to the nominal complement. The same is not true of DURABILITY or SAFENESS (e.g., sub-segments of rope are not necessarily less durable than larger segments that contain it, nor are sub-events of travel necessarily less safe than the larger events that contain it). The Monotonicity Constraint therefore applies correctly to the full set of observations about *much* presented above.

The Monotonicity Constraint in fact applies correctly to an impressively broad range of data. Its validity does not seem to have been explicitly questioned in the literature. And yet, an implicit challenge emerges from the way certain data have been characterized. Bale and Barner (2009) present the example in (1).

- (1) Esme has more gold in her ring than Seymour has silver in his necklace.

This sentence could certainly be understood as a comparison of the absolute weight or volume of the gold in Esme’s ring with the absolute weight or volume of the silver in Seymour’s necklace. However, Bale and Barner propose (in passing) that (1) can also have an interpretation that arises from *much* referencing a function that maps portions of matter in its domain to degrees of *purity*. We will refer to the idea that *much* can make reference to a measure of purity as the *purity hypothesis*. While not

¹Schwarzschild’s proposal builds on observations about monotonicity first discussed in Lønning 1987. As mentioned in Schwarzschild 2006, Krifka (1989) proposes a similar constraint on measure functions as they relate to nominal modifiers, however his constraint is stronger in that it requires that measure functions (operative in modifiers) be additive (as well as having the Archimedean property). Somewhat informally stated, μ is additive with respect to a partial order \leq if and only if for all x and y in the domain of \leq , if there is no non-empty element z such that $z \leq x$ and $z \leq y$ (i.e., x and y do not overlap), then $\mu(x \sqcup y) = \mu(x) + \mu(y)$. All of the monotonic measure functions we discuss in this paper also happen to be additive.

spelled out in Bale and Barner’s discussion, a rationale for this hypothesis is easy to reconstruct. (1) can be judged true in certain scenarios that do not establish its truth as a comparison of absolute weights or volumes, but that do establish that the gold in Esme’s ring is purer than the silver in Seymour’s necklace. The scenario laid out in (2) has this profile.

- (2) Esme’s ring weighs 10g and is made entirely from a gold-copper alloy, containing 9g gold and 1g copper (90% pure gold). Seymour’s necklace weighs 15g and is made entirely from a silver-copper alloy, containing 9g silver and 6g copper (60% pure silver).

While sentence (1) can clearly be judged true in (2), the scenario provides no information that renders the sentence true as a comparison of absolute amounts. However, the scenario establishes that gold constitutes a greater proportion of the alloy that makes up the ring (90%) than silver constitutes of the alloy that makes up the necklace (60%), hence it establishes that the gold (alloy) in the ring is purer than the silver (alloy) in the necklace. The purity hypothesis can therefore be motivated as an answer to the question of why (1) can be judged true in a scenario like (2).

As Wellwood (2015) notes, however, the purity hypothesis is in conflict with Schwarzschild’s Monotonicity Constraint. PURITY, the measure function that maps portions of matter in its domain to their degree of purity, fails to track the part-whole ordering of such portions. A portion of gold or silver (alloy), in particular, is intuitively not guaranteed to be of greater purity than smaller portions contained in it. So, as already noted in Schwarzschild (2006), PURITY is non-monotonic.²

What is the proper resolution to the conflict between the purity hypothesis and the Monotonicity Constraint? We will in this commentary argue that the purity hypothesis, and hence Bale and Barner’s interpretation of example (1), is incorrect. We will make this case by arguing that the purity hypothesis both undergenerates and overgenerates truth judgments for comparatives with *much*. We will moreover credit the appearance of reference to PURITY to the assumption that *much* can sometimes access certain *monotonic* proportional measure functions—measure functions that, like PURITY, output proportional degrees but, unlike PURITY, are monotonic. Such functions can be used to derive readings that we will label *relative*, readings that compare proportions rather than absolute amounts. Although in general such readings are truth-conditionally distinct from comparisons of purity, in certain common grounds a sentence may permit a relative reading that is essentially equivalent to such a comparison. This, we suggest, captures intuitions about sentences like Bale and Barner’s example (1).

Our analysis commits us to the assumption that the measure function invoked in the interpretation of the main clause can be different from the measure function invoked in the interpretation of the comparative clause. Adapting a strategy employed in different empirical domains (e.g., in von Stechow 1994 for covert quantifier domain restriction), we credit this possibility to Skolemization and to binding of Skolem arguments. That is, we propose that the two measure functions can come to differ because they are the denotations of covert object-language expressions which can contain bound variables that are bound by different binders.

Section 2 sets up assumptions about the interpretation of *much* and comparatives. Section 3 introduces the purity hypothesis in detail. Section 4 introduces an alternative analysis, which posits monotonic proportional measure functions instead of PURITY. Section 5 shows that the purity hypothesis suffers from problems of under- and overgeneration, problems that the alternative analysis evades—leading us to conclude that the purity hypothesis is not viable and that the Monotonicity Constraint can and must be maintained. Further exploring a consequence of our conclusions, Section 6 amends the analysis in Section 4 by proposing that the two measure functions that enter into the interpretation of comparatives with *much* in relative readings come to be different in virtue of variable binding into the covert expressions that denote those functions. Section 7 concludes.

²PURITY is also not additive, as noted in Krifka 1989. If *x* and *y* are two separate pieces of gold, it does not follow that the purity measure of both pieces combined is equal to the purity measure of the first piece added to the purity measure of the second.

2 *Much* and comparatives

Our case against the purity hypothesis is somewhat independent of a specific degree-based analysis of comparatives as well as the compositional nuances involved in representing the syntax and semantics of *much*. However, for the sake of exposition, it will be useful to work within a specific compositional analysis. In this section, we briefly lay out one such analysis primarily based on Heim 2000 and Hackl 2000.³ The syntactic and semantic details only matter insofar as they yield truth conditions that are dependent on the way different elements are measured within the main clause and the than-clause. Such truth conditions will allow us to more easily outline, in Section 3, a detailed proposal consistent with the purity hypothesis: one that supports an analysis of relative readings in cases like (1). The analysis we adopt here will also facilitate a comparison between the purity hypothesis and an alternative account of relative readings presented in Section 4.

A compositional semantics for comparatives with *much* can be built by adapting a proposal in Hackl (2000) for comparatives with *many*, which in turn adopts Heim’s (2000) outlook on comparatives in general. We will use sentence (3) to illustrate the account. For this example, we posit the LF in (4).

(3) There is more gold in the ring than there is in the bracelet.

(4) -er [than Op λd [[d much] gold] λx [there is x in the bracelet]]
 λd [[d much] gold] λx [there is x in the ring]]

In such an analysis, *more* is the phonological realization of *much* plus *-er*, a proposal first advanced by Bresnan (1973) and defended by Klein (1981) and Wellwood (2014, 2015), among others. As represented in the LF in (4), the comparative clause has interpreted content, viz. $[[d \text{ much}] \text{ gold}]$, that is recovered from the main clause (see e.g., Bresnan 1973, Heim 1985, Kennedy 1999). In both clauses, *much* combines with a degree-denoting expression d to form a determiner, whose restrictor is *gold*. In the comparative clause, a covert operator, *Op*, moves from *much*’s degree argument position, leaving a trace and triggering abstraction.⁴ The comparative clause accordingly denotes a set of degrees. The comparative morpheme *-er* combines with the comparative clause to form a so-called degree phrase, the constituent that occupies the first line of (4). This degree phrase has undergone covert movement, evacuating the main clause, which is shown in the second line of (4). The degree phrase’s movement is launched from *much*’s degree argument and again triggers abstraction. As a result, the main clause, like the comparative clause, also denotes a set of degrees.⁵

Covert movement of the comparative clause, although not strictly needed for the discussion that follows, allows us to assign a transparent meaning to the comparative morpheme *-er* and the (parameterized) determiner *much*. The denotation for *-er* in (5) relates the two sets of degrees furnished by the main clause and comparative clause, requiring that the former’s maximal element is greater than the latter’s.

(5) $[-er] = \lambda f_{dt}. \lambda g_{dt}. \text{MAX}(g) > \text{MAX}(f)$

The membership of the two sets of degrees, in turn, is crucially shaped by the denotation of *much*. In the lexical entry in (6), \sqcup is a meta-language operator that maps a set to its supremum—the sum of all of its elements—and μ is a meta-language variable ranging over measure functions.

³Heim’s analysis critically involves LF movement of the comparative morpheme as well as syntactic ellipsis. However, an analysis which mirrors the surface syntactic structure (as in Kennedy 1999) would do equally as well. The critical difference between the two analyses involve issues of scope which are orthogonal to our present concerns.

⁴There is overt evidence for such abstraction. For example, consider the comparatives in (i).

- (i) a. There is more gold in this ring than how much (gold) (there) is in this bracelet.
b. There is more gold in this ring than what (there) is in this bracelet.

The sentence in (ia) is acceptable in most English dialects, whereas (ib) is only acceptable in certain dialects.

⁵The abstraction of the comparative phrase is not essential to our analysis. The argument for such an abstraction depends on the interaction between scope, ellipsis and Antecedent Contained Deletion (see the discussion in Bresnan 1973, von Stechow 1984, Heim 2000, Hackl 2000, among others). Our discussion is equally compatible with a semantics and compositional structure that does not hypothesize such movement, such as the one outlined in Kennedy 1999.

$$(6) \quad \llbracket much \rrbracket = \lambda d_d. \lambda X_{et}. \lambda Y_{et}. \mu(\sqcup(X \cap Y)) \geq d$$

So the denotation of *much* takes as inputs a degree and two sets of individuals, requiring that the input degree is less than or equal to the degree to which μ maps the supremum of the two input sets' intersection. We take *than* to be semantically vacuous. With respect to the LF in (4), (5) and (6) yield the truth conditions in (7), which can be restated equivalently, and more transparently, as in (8).

$$(7) \quad \begin{aligned} & \text{MAX}(\{d : \mu(\sqcup\{x : \text{GOLD}(x) \wedge \text{IN}(x, \text{the ring})\}) \geq d\}) > \\ & \text{MAX}(\{d : \mu(\sqcup\{x : \text{GOLD}(x) \wedge \text{IN}(x, \text{the bracelet})\}) \geq d\}) \end{aligned}$$

$$(8) \quad \begin{aligned} & \mu(\sqcup\{x : \text{GOLD}(x) \wedge \text{IN}(x, \text{the ring})\}) > \\ & \mu(\sqcup\{x : \text{GOLD}(x) \wedge \text{IN}(x, \text{the bracelet})\}) \end{aligned}$$

So example (3) is predicted to be true just in case μ maps the gold in the ring to a greater degree than the gold in the bracelet. This statement yields a testable claim about the truth conditions of (3) once the measure function variable μ is assigned a value.

Critical to the discussion that follows, we take the value of μ to be underspecified. That is, its value is not conventionally determined but, subject to certain conditions, fixed by the context of utterance. In this, we follow Schwarzschild (2006), Bale and Barner (2009), Solt (2015) and Wellwood (2014, 2015), who also posit a free measure function variable in the meaning of *much*.⁶ This underspecification is needed in their analyses to capture the variability in the interpretation of *much* plus mass noun that we described at the outset. (Recall from the introduction, mass noun comparisons vary in terms of their comparative dimension, sometimes inducing comparisons by length, distance, duration, etc.) With respect to the sentence in (3), the relevant measure could be one of absolute volume or weight, much like the example in (1). This is captured by setting μ to either VOLUME or WEIGHT, deriving the reading that the weight/volume of gold in the ring is greater than the weight/volume of gold in the bracelet.

However, those two settings cannot exhaust the options. Sentence (3) permits the very same range of interpretations as Bale and Barner's (2009) example (1) above. In particular, just like (1), (3) can be judged true in scenarios that do not render the sentence true as a comparison of absolute amounts, but that would render it true as a comparison of degrees of purity. The scenario in (9), which portrays the ring as made from purer gold (90%) than the bracelet (60%), is a case in point.

- (9) The ring and the bracelet are made entirely from gold-copper alloys. The ring weighs 10g, containing 9g gold and 1g copper (90% pure gold). The bracelet weighs 15g, containing 9g gold and 6g copper (60% pure gold).

What setting of μ should this relative reading be attributed to? One possible answer consists in the purity hypothesis, a proposal that we will now introduce in detail.

3 The purity hypothesis

The purity hypothesis aims to capture the relative readings of sentences like (3) that scenarios like (9) render true. The hypothesis holds that the measure function variable in the meaning of *much* can be set to a measure function that maps portions of matter to their degree of purity. In this section, we will outline a semantics that provides more substance and detail to such a proposal. As discussed below, instead of a single measure function PURITY, a fully fleshed-out version of the purity

⁶Although Schwarzschild (2006), Bale and Barner (2009), Solt (2015) and Wellwood (2014, 2015) each address the issue of underspecification, their theoretical accounts differ significantly. Schwarzschild (2006) hypothesizes an existential quantifier over dimensions. Bale and Barner (2009) analyze *much* using a variable in the meta-language, one whose value is determined by context and a ranked set of preferred measurements. Solt (2015) and Wellwood (2014, 2015) hypothesize the existence of free measure function variables in the object language (i.e., ones that are syntactically instantiated at LF). In this section, we follow Bale and Barner (2009) in assuming a meta-language variable. However, in section 6, we argue that such variables need to be instantiated in the object-language, more in line with Solt (2015) and Wellwood (2014, 2015). See Wellwood (2014, 2020) for a discussion of the empirical and theoretical differences between metalanguage and object language variables.

hypothesis gives rise to an entire family of related measure functions. Spelling out the semantics of such functions will allow us to pinpoint the reason for their non-monotonicity, and it will help us compare the purity hypothesis with the alternative account introduced in Section 4.

One way to approach a formal analysis of purity measures is to first hypothesize that certain lexical items have both a *strict* and *loose* meaning. Empirically speaking, the word *gold* seems to fit such a characterization. Under a strict usage, *gold* is only true of portions of matter consisting of pure gold, the chemical element Au. In contrast, under a loose usage, *gold* is not only true of portions of Au but also of portions of alloys that have Au as their principal component. It is this loose meaning that plausibly allows the phrase *12 karat gold* to be non-contradictory, as only 24 karat gold is pure gold. Similarly, it is the loose meaning that permits the question *How pure is this gold?* to be non-trivial. The class of mass nouns with such a strict-loose ambiguity also includes, for example, *alcohol*. In its strict usage, this noun is only true of portions of the chemical compound ethanol. In its loose usage, it also applies to alcoholic beverages and other liquids that have ethanol as their principal component. It is in this loose sense that statements like *This whisky is alcohol* can be true or questions like *How pure is this alcohol?* are non-trivial.

A notion of *purity* can be defined via this strict-loose ambiguity. For example, the degree of purity of a portion of gold alloy is the proportion of Au contained in it. Likewise, the degree of purity of an alcoholic beverage is the proportion of ethanol contained in it. In either case, the relevant proportion could in principle be understood in terms of either weight or volume, even though conventions dictate the use of weight for the case of gold and volume for the case of alcohol. More generally, for a given mass noun α that participates in the strict-loose ambiguity, the degree of purity of a portion of matter in the extension of α in the loose sense can be characterized as the weight proportion or volume proportion of the matter contained in it that falls under α in the strict sense.

Hence, given the discussion above, a measure of purity might be best represented by a family of functions, which includes functions such as $\text{PURITY}_{\alpha, \text{VOLUME}}$ and $\text{PURITY}_{\alpha, \text{WEIGHT}}$, for each mass noun α that demonstrates a strict-loose ambiguity.⁷ For the cases of *gold* and *alcohol*, the relevant functions can be spelled out as in (10) and (11). Here \sqsubseteq is the partial ordering that relates portions of matter to portions containing them.

- (10) a. $\text{PURITY}_{\text{gold}, \text{VOLUME}} := \lambda x. \text{VOLUME}(\sqcup\{y: y \sqsubseteq x \wedge \text{Au}(y)\}) / \text{VOLUME}(x)$
b. $\text{PURITY}_{\text{gold}, \text{WEIGHT}} := \lambda x. \text{WEIGHT}(\sqcup\{y: y \sqsubseteq x \wedge \text{Au}(y)\}) / \text{WEIGHT}(x)$
- (11) a. $\text{PURITY}_{\text{alcohol}, \text{VOLUME}} := \lambda x. \text{VOLUME}(\sqcup\{y: y \sqsubseteq x \wedge \text{ETHANOL}(y)\}) / \text{VOLUME}(x)$
b. $\text{PURITY}_{\text{alcohol}, \text{WEIGHT}} := \lambda x. \text{WEIGHT}(\sqcup\{y: y \sqsubseteq x \wedge \text{ETHANOL}(y)\}) / \text{WEIGHT}(x)$

Note that the outputs of these functions are necessarily proportions between 0 and 1. This is guaranteed by the fact that the individual measured in the numerator, viz. the sum of all the Au or ethanol included in the input, is necessarily a part of the individual measured in the denominator, viz. the input in its entirety. Given that VOLUME and WEIGHT are monotonic, this ensures that the numerator is no greater than the denominator.

The purity hypothesis holds, then, that the variable μ introduced by *much* can, depending on the mass noun, be set to measure functions like those in (10) and (11). The hypothesis in particular allows for μ in (8) above, the proposed interpretation for sentence (3), to be set to the purity function $\text{PURITY}_{\text{gold}, \text{WEIGHT}}$. This setting results in the interpretation in (12).

- (12) $\frac{\text{WEIGHT}(\sqcup\{x: \text{GOLD}(x) \wedge \text{IN}(x, \text{the ring}) \wedge \text{Au}(x)\})}{\text{WEIGHT}(\sqcup\{x: \text{GOLD}(x) \wedge \text{IN}(x, \text{the ring})\})} >$
 $\frac{\text{WEIGHT}(\sqcup\{x: \text{GOLD}(x) \wedge \text{IN}(x, \text{the bracelet}) \wedge \text{Au}(x)\})}{\text{WEIGHT}(\sqcup\{x: \text{GOLD}(x) \wedge \text{IN}(x, \text{the bracelet})\})}$

If GOLD is construed under a loose meaning (where it is true of portions of gold alloy), then the truth conditions in (12) would accurately capture the purity reading of (3). In particular, (12) would be

⁷Other mass nouns include any metal term (e.g., *silver*, *copper*, etc.) and many natural kind substance terms that have a chemical essence (e.g., *water*, *cocaine*, *heroin*, etc.). In fact, any term that can coherently fit into the schema *How pure is that x?*

true in scenario (9). Recall that in that scenario, the weight proportion of Au in the gold alloy that makes up the ring is 90% (9g Au vs. 1g Cu), while the weight proportion of Au in the gold alloy that makes up the bracelet is 60% (9g Au vs. 6g Cu). In this scenario, then, the truth conditions in (12) yield the fractional comparison in (13), whose truth captures the observation that (3) can be judged true in (9).

$$(13) \quad \frac{9}{10} > \frac{9}{15} \Leftrightarrow 0.9 > 0.6$$

To provide another illustration of the purity hypothesis's utility, consider the sentence in (14) in a scenario like (15). Similar to our example with *gold*, the sentence in (14) can be true given (15) despite the fact that the absolute amount of alcohol in this drink is not greater than the absolute amount of alcohol in that drink.

(14) There is more alcohol in this drink than in that drink.

(15) This drink is 50 ml of vodka; it contains 20 ml of ethanol (i.e., its “alcohol by volume” content is 40%). That drink is 400 ml of beer; it contains 20 ml of ethanol (i.e., its “alcohol by volume” content is 5%).

The assumptions laid out in the previous section assign to (14) the interpretation in (16). The purity hypothesis allows for μ in (16) to be set to the purity function $\text{PURITY}_{\text{alcohol}, \text{VOLUME}}$. This setting will yield the truth conditions in (17).

$$(16) \quad \mu(\sqcup\{x : \text{ALCOHOL}(x) \wedge \text{IN}(x, \text{this drink})\}) > \mu(\sqcup\{x : \text{ALCOHOL}(x) \wedge \text{IN}(x, \text{that drink})\})$$

$$(17) \quad \frac{\text{VOLUME}(\sqcup\{x : \text{ALCOHOL}(x) \wedge \text{IN}(x, \text{this drink}) \wedge \text{ETHANOL}(x)\})}{\text{VOLUME}(\sqcup\{x : \text{ALCOHOL}(x) \wedge \text{IN}(x, \text{this drink})\})} > \frac{\text{VOLUME}(\sqcup\{x : \text{ALCOHOL}(x) \wedge \text{IN}(x, \text{that drink}) \wedge \text{ETHANOL}(x)\})}{\text{VOLUME}(\sqcup\{x : \text{ALCOHOL}(x) \wedge \text{IN}(x, \text{that drink})\})}$$

If ALCOHOL is construed under its loose meaning (i.e., it is true of portions of alcoholic beverages like vodka or beer), then the truth conditions in (17) in the scenario in (15) result in the fractional comparison in (18). The truth of (18) can account for the observation that (14) can be judged true in (15).

$$(18) \quad \frac{20}{50} > \frac{20}{400} \Leftrightarrow 0.4 > 0.05$$

Our present elaboration of the purity hypothesis, then, confirms its initial promise. However, it also confirms its incompatibility with the Monotonicity Constraint. Purity functions like those in (10) and (11) are non-monotonic. Such functions fail to track the part-whole relation that orders their possible inputs. That is, for any such function f and inputs x and y such that $x \sqsubset y$, nothing guarantees $f(x) < f(y)$. In particular, it is possible that $f(x) = f(y)$ even when $x \sqsubset y$. This will hold quite frequently in cases where x and y are portions of gold alloy or vodka, given that Au/ethanol is often evenly distributed through these types of substances/liquids.

What can we conclude from the incompatibility of the purity hypothesis with the Monotonicity Constraint? At least one of those two proposals will have to be abandoned. In the next two sections, we will present an argument that it is the purity hypothesis that should be abandoned. Furthermore, we will demonstrate that the readings which initially motivated the purity hypothesis are in fact compatible with the Monotonicity Constraint once the proper type of measure function is hypothesized.

4 Relative readings from monotonic measure functions

We have seen that the purity hypothesis has the benefit of capturing the observation that example (3), repeated in (19), can be judged true in the scenario in (9), repeated in (20).

(19) There is more gold in the ring than there is in the bracelet.

- (20) The ring and the bracelet are made entirely from gold-copper alloys. The ring weighs 10g, containing 9g gold and 1g copper (90% pure gold). The bracelet weighs 15g, containing 9g gold and 6g copper (60% pure gold).

We would now like to point out, however, that this judgment can in principle be accommodated without appealing to purity functions. As mentioned in Bale and Schwarz 2019, an alternative analysis becomes available if we allow the two occurrences of *much* in the LF of (19) to reference two different measure functions. To make this assumption explicit, we will use two different meta-language variables over measure functions, μ_1 for the variable in the the main clause and μ_2 for the variable in the comparative clause. In (21), the interpretation of (19) given in (8) above is updated accordingly.

- (21) $\mu_1(\sqcup\{x : \text{GOLD}(x) \wedge \text{IN}(x, \text{the ring})\}) >$
 $\mu_2(\sqcup\{x : \text{GOLD}(x) \wedge \text{IN}(x, \text{the bracelet})\})$

Suppose now that *gold* in (19) receives a strict interpretation, applying to portions of pure gold only. Hence, in (21), *GOLD* is equivalent to *Au*. With this in mind, consider the setting of μ_1 and μ_2 shown in (22). The measure function assigned to μ_1 maps any input to this input's weight divided by the ring's weight, while the one assigned to μ_2 maps any input to this input's weight divided by the bracelet's weight. Under this setting, the interpretation in (21) yields (23), which states that the ring contains a greater weight proportion of *Au* than the bracelet does.

- (22) a. $\mu_1 = \lambda x. \text{WEIGHT}(x)/\text{WEIGHT}(\text{the ring})$
b. $\mu_2 = \lambda x. \text{WEIGHT}(x)/\text{WEIGHT}(\text{the bracelet})$
- (23) $\text{WEIGHT}(\sqcup\{x : \text{GOLD}(x) \wedge \text{IN}(x, \text{the ring})\})/\text{WEIGHT}(\text{the ring}) >$
 $\text{WEIGHT}(\sqcup\{x : \text{GOLD}(x) \wedge \text{IN}(x, \text{the bracelet})\})/\text{WEIGHT}(\text{the bracelet})$

These truth conditions, with respect to the scenario in (20), yield the fractional comparisons in (24)—the very same inequality we obtained under the purity hypothesis. This comparison being true, the setting in (22) accounts for the intuition that (19) can be judged true in (20).

- (24) $\frac{9}{10} > \frac{9}{15} \Leftrightarrow 0.9 > 0.6$

In much the same way, we can reanalyze the observation that sentence (14), repeated in (25), can be judged true in scenario (15), repeated in (26).

- (25) This drink has more alcohol in it than that drink does.
- (26) This drink is 50 ml of vodka; it contains 20 ml of ethanol (i.e., its “alcohol by volume” content is 40%). That drink is 400 ml of beer; it contains 20 ml of ethanol (i.e., its “alcohol by volume” content is 5%).

The proposed reanalysis updates the analysis of (25) as in (27). Suppose now that *alcohol* in (25) receives a strict interpretation, hence that in (27), *ALCOHOL* applies to portions of the chemical compound ethanol only. Consider the variable assignments in (28). The measure functions assigned to μ_1 and μ_2 in (28) map any input to the input's weight divided by this and that drink's volume, respectively. Under this setting, the interpretation in (27) yields (29).

- (27) $\mu_1(\sqcup\{x : \text{ALCOHOL}(x) \wedge \text{IN}(x, \text{this drink})\}) >$
 $\mu_2(\sqcup\{x : \text{ALCOHOL}(x) \wedge \text{IN}(x, \text{that drink})\})$
- (28) a. $\mu_1 = \lambda x. \text{VOLUME}(x)/\text{VOLUME}(\text{this drink})$
b. $\mu_2 = \lambda x. \text{VOLUME}(x)/\text{VOLUME}(\text{that drink})$
- (29) $\text{VOLUME}(\sqcup\{x : \text{ALCOHOL}(x) \wedge \text{IN}(x, \text{this drink})\})/\text{VOLUME}(\text{this drink}) >$
 $\text{VOLUME}(\sqcup\{x : \text{ALCOHOL}(x) \wedge \text{IN}(x, \text{that drink})\})/\text{VOLUME}(\text{that drink})$

The truth conditions in (29) then convey that this drink contains a greater volume proportion of ethanol than that drink. With respect to (26), such truth conditions determine the fractional

comparison in (30)—again, the very inequality that we saw arise under the purity hypothesis. Since the fractional comparison is true, we have succeeded in capturing the observation that (25) can be judged true in this scenario.

$$(30) \quad \frac{20}{50} > \frac{20}{400} \Leftrightarrow 0.4 > 0.05$$

Let us now compare the measure functions in (22) and (28) to the purity functions in (10) and (11) above. Both types of functions are proportional measure functions in the sense that they map their inputs to fractions, and hence both types of functions yield relative readings, readings that compare proportions. Also, both types of functions are defined in terms of more basic, monotonic measure functions, WEIGHT or VOLUME. However, unlike purity functions, the measure functions in (22) and (28) are monotonic. This observation is an instance of the general fact that for any measure function m and positive real number c , if m is monotonic, then so is $\lambda x.m(x)/c$.⁸ The proposed reanalysis therefore reconciles the judgments about (19) and (25) in scenarios like (20) and (26) with the Monotonicity Constraint. Hence the truth value judgments that served as initial evidence for the purity hypothesis could in principle be captured without committing to this hypothesis, in fact by instead relying on measure functions that are consistent with the Monotonicity Constraint.

For a more detailed assessment of the relationship between the two accounts, let's take a closer look at how the truth conditions derived here compare to the truth conditions derived under the purity hypothesis. Recall that in our purity analysis, we needed to interpret GOLD in a *loose* way in order to get the right type of relative meaning. It will be useful to now make this explicit. To do so, let's update the old truth conditions for (19) in (12) with those in (31), where GOLD is replaced with the unambiguous meta-language constant GOLD_{loose}.

$$(31) \quad \frac{\text{WEIGHT}(\sqcup\{x: \text{GOLD}_{\text{loose}}(x) \wedge \text{IN}(x, \text{the ring}) \wedge \text{Au}(x)\})}{\text{WEIGHT}(\sqcup\{x: \text{GOLD}_{\text{loose}}(x) \wedge \text{IN}(x, \text{the ring})\})} > \\ \frac{\text{WEIGHT}(\sqcup\{x: \text{GOLD}_{\text{loose}}(x) \wedge \text{IN}(x, \text{the bracelet}) \wedge \text{Au}(x)\})}{\text{WEIGHT}(\sqcup\{x: \text{GOLD}_{\text{loose}}(x) \wedge \text{IN}(x, \text{the bracelet})\})}$$

In contrast, the relative truth conditions given in (23) above, the one with a monotonic proportional measure function, required that we assign a strict meaning to GOLD, a meaning which is only true of portions of Au. Maintaining this assumption, for the moment, and making it explicit as well, let's update (23) as in (32), where GOLD_{strict} is the strict meaning.

$$(32) \quad \frac{\text{WEIGHT}(\sqcup\{x: \text{GOLD}_{\text{strict}}(x) \wedge \text{IN}(x, \text{the ring})\})}{\text{WEIGHT}(\text{the ring})} > \\ \frac{\text{WEIGHT}(\sqcup\{x: \text{GOLD}_{\text{strict}}(x) \wedge \text{IN}(x, \text{the bracelet})\})}{\text{WEIGHT}(\text{the bracelet})}$$

Consider now the fractions that are being compared in (31) and (32). Given the equivalence of GOLD_{strict} and Au, the numerators in (31) have the same values as their counterparts in (32). It is only the denominators that differ, preventing the two statements from being equivalent. While in (31), WEIGHT applies to the sum of the gold (alloy) contained in the ring or bracelet, in (32) WEIGHT applies to the ring or bracelet as a whole. The fact that this is the only difference between the truth conditions in (31) and (32) clarifies why their effects coincide with respect to the scenario in (20). In that scenario, the ring and the bracelet consist entirely of gold alloy, that is, we have both GOLD_{loose}(the ring) = 1 and GOLD_{loose}(the bracelet) = 1. Under this assumption about the ring's and the bracelet's makeup, the value of the denominators in (31) also coincide with their counterparts in (32). So the fractions in (31) and their counterparts in (32) collapse into the same value, hence (31) and (32) become equivalent. So assuming the truth of both GOLD_{loose}(the ring) and GOLD_{loose}(the bracelet), the proposed analysis in terms of monotonic proportional measure functions fully replicates the effect of the purity hypothesis.

This point carries over to example (25). Making the strict and loose sense of ALCOHOL explicit, we can restate the two potential truth conditions for sentence (25) as in (33) and (34). (33) is a

⁸Suppose that m is monotonic with respect to partial order $>$. Thus, for any y and z such that $y > z$, it follows that $m(y) > m(z)$. Hence, it also follows that for any constant c greater than 0, $m(y)/c > m(z)/c$. Therefore, $\lambda x.m(x)/c$ is also monotonic.

restatement of the truth conditions with respect to the purity hypothesis, while (34) is a restatement with respect to the truth conditions derived from a monotonic proportional measure function.

$$\begin{aligned}
(33) \quad & \text{VOLUME}(\sqcup\{x: \text{ALCOHOL}_{\text{loose}}(x) \wedge \text{IN}(x, \text{this drink}) \wedge \text{ETHANOL}(x)\}) / \\
& \text{VOLUME}(\sqcup\{x: \text{ALCOHOL}_{\text{loose}}(x) \wedge \text{IN}(x, \text{this drink})\}) > \\
& \text{VOLUME}(\sqcup\{x: \text{ALCOHOL}_{\text{loose}}(x) \wedge \text{IN}(x, \text{that drink}) \wedge \text{ETHANOL}(x)\}) / \\
& \text{VOLUME}(\sqcup\{x: \text{ALCOHOL}_{\text{loose}}(x) \wedge \text{IN}(x, \text{that drink})\}) \\
(34) \quad & \text{VOLUME}(\sqcup\{x: \text{ALCOHOL}_{\text{strict}}(x) \wedge \text{IN}(x, \text{this drink})\}) / \text{VOLUME}(\text{this drink}) > \\
& \text{VOLUME}(\sqcup\{x: \text{ALCOHOL}_{\text{strict}}(x) \wedge \text{IN}(x, \text{that drink})\}) / \text{VOLUME}(\text{that drink})
\end{aligned}$$

Given the equivalence of $\text{ALCOHOL}_{\text{strict}}$ and ETHANOL , the truth conditions in (33) and (34) collapse into the same condition under the assumption that both drinks are alcoholic beverages (hence $\text{ALCOHOL}_{\text{loose}}(\text{this drink}) = 1$ and $\text{ALCOHOL}_{\text{loose}}(\text{that drink}) = 1$).

Thus, for the two examples in (19) and (25), the truth conditions derived from the two competing accounts become equivalent under certain assumptions. This means that when the context satisfies such assumptions, the truth conditions derived from monotonic proportional measure functions *are* comparisons of degrees of purity, even if the monotonic measure functions employed do not in general output such degrees. The scenarios in (9) and (15) happen to establish the conditions in question—that the rings are made of gold alloy or that the drinks are alcoholic beverages—and for this reason those scenarios cannot discriminate between the two accounts. The relevant assumptions in this case are moreover fairly plausible. Therefore, a listener confronted with (19) or (25) out of context may well accommodate a common ground that entails those assumptions. It is for this reason that Bale and Barner’s (2009) purity hypothesis can on first inspection emerge as a natural contender.

However, the competing truth conditions derived under the two analyses are of course not fully equivalent. They diverge in contexts where the relevant assumptions do *not* hold. For sentence (19), we will need to consider contexts where the ring and the bracelet are *not* made entirely of gold alloy, and for sentence (25), we will need to consider contexts where the drinks do *not* consist entirely of alcoholic beverages. In the next section, we will assess intuitions about the truth of examples (19) and (25) in scenarios of this sort.

First, though, a brief comment is in order about a conspicuous feature of the present analysis as developed above, viz. the assumption that the two occurrences of *much* in a comparative can invoke reference to two different measure functions. This assumption is not actually necessary. The same truth conditions derived above could be obtained by assuming that, instead of measure functions proper, the two occurrences of *much* can invoke reference to a so-called Skolemized measure function, and that the Skolem argument of this function can be a bound variable. We in fact believe that relative readings must ultimately be analyzed along such lines. However, since it does not bear on our main argument, for the sake of exposition we will suppress this complication for now, postponing discussion of the matter until Section 6.

5 The purity hypothesis excluded

The existence of the analysis laid out in the last section raises the question whether the purity hypothesis should be maintained. Its incompatibility with the Monotonicity Hypothesis alone might constitute a good reason for abandoning it. But we can offer more direct evidence to strengthen the case against the purity hypothesis. The evidence comes from the very type of examples that at first sight seems to invite the purity hypothesis. In Section 5.1, we will first show that those examples permit relative truth conditions other than those predicted under the purity hypothesis. From this we can conclude that the purity hypothesis is at least insufficient. Moreover, in Section 5.2, we will demonstrate that those examples in fact lack the truth conditions that the purity hypothesis predicts. Hence, not only is the purity hypothesis insufficient to capture intuitions about the meanings of comparatives with *much*, it is in fact inconsistent with those intuitions. The purity hypothesis, then, must be rejected.

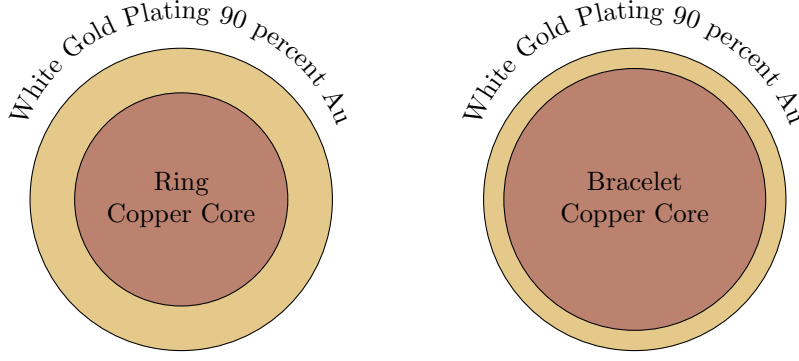


Figure 1: A visual depiction of the cross-sections of the ring (on the left) and the bracelet (on the right) in scenario (36), with the proportion of white gold plating to copper core represented by circle area.

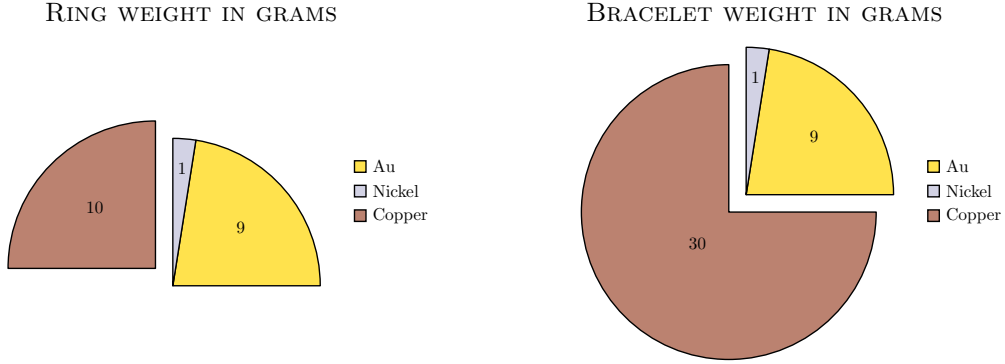


Figure 2: A representation of the makeup of the ring (left) and the bracelet (right) in scenario (36). The size of the segment represents the weight of the material in the object. The copper cores of the two objects are depicted as being separate from the white gold plating, represented by the joined segment of the Au and nickel.

5.1 The purity hypothesis undergenerates

We would like to assess truth value intuitions about sentence (19), repeated in (35), in scenarios where the ring and the bracelet are *not* entirely made up of gold alloy. A scenario of this sort is outlined in (36), and also visually represented in Figures 1 and 2.

- (35) There is more gold in the ring than there is in the bracelet.
- (36) Both the ring and the bracelet are made of copper plated with white gold. The white gold, a gold alloy, consists of 90 weight percent Au and 10 weight percent nickel. The ring consists of 10g of white gold and 10g of copper, while the bracelet consists of 10g of white gold and 30g of copper. (See Figures 1 and 2 for a visualization of this scenario.)

Since the ring and the bracelet contain the same amounts of Au (9g each) and white gold (10g each), this scenario does not render (35) true under a reading that compares absolute amounts. Moreover, since the white gold in the ring and the bracelet have identical makeups (each containing 90 weight percent of Au), the truth conditions derived from the purity hypothesis (spelled out in (31) above), are not met either: the gold in the ring is not purer than the gold in the bracelet. The purity hypothesis, then, provides no reason to expect that (19) can be judged true in (36). In contrast, the truth conditions in (32), based on monotonic proportional measure functions, *are* met in this scenario. After all, the scenario establishes that the Au in the ring (9g) constitutes 45

percent of the ring’s weight (20g), while the Au in the bracelet (9g) only constitutes 22.5 percent of the bracelet’s weight (40g). The analysis based on monotonic measure functions, then, predicts that (19) can be judged true in (36).

What are speaker intuitions about sentence (35) in the scenario in (36)? The sentence can indeed be judged true. This becomes particularly obvious when (35) is prefixed with a phrase like *relatively speaking*. We assume that this manipulation disambiguates the sentence towards a relative interpretation, eliminating a potentially competing absolute reading. However, such a reading of (36) is accessible also in the absence of such a prefix, as long as the speaker has been primed to expect a relative interpretation. We conclude, therefore, that the purity hypothesis undergenerates. The truth conditions it derives fail to capture certain truth value judgments about relative readings. Truth conditions based on monotonic measure functions are needed to capture those judgments.⁹

To confirm this conclusion with a different example, let us also examine the relevant intuitions with respect to sentence (25), repeated in (37). Recall that we are interested in scenarios where the two drinks referred to in (37) do *not* consist entirely of alcoholic beverages like vodka or beer. The scenario outlined in (38) is of the requisite type.

- (37) This drink has more alcohol in it than that drink does.
- (38) Both drinks consist of a mix of vodka and cranberry juice. The volume proportion of ethanol in the vodka (i.e., its “alcohol by volume” content) is 40%. This drink contains 50 ml of vodka and 50 ml of cranberry juice. That drink contains 50 ml of vodka and 150 ml of cranberry juice.

The features of (38) are entirely parallel to those of (36). According to (38), the two drinks contain the same amounts of ethanol and vodka, so the scenario does not render (37) true under a reading that compares absolute amounts. By the same token, the composition of the vodka in the two drinks is identical, each containing 40 volume percent of ethanol. So the truth conditions in (33), derived under the purity hypothesis, are not met: the alcohol in this drink is not purer than the alcohol in that drink. The purity hypothesis, then, provides no reason to expect that (37) can be judged true in (38). In contrast, the truth conditions in (34), based on monotonic proportional measure functions, *are* met, given that the scenario presents the ethanol in this drink (20 ml) as constituting 20 percent of the drink’s volume (100 ml), while presenting the ethanol in that drink (20 ml) as merely constituting 10 percent of that drink’s volume (200 ml). The analysis based on monotonic measure functions, then, predicts that (37) can be judged true in (36). Aligned with the judgment reported about (19), (37) can indeed be judged true in (38), most obviously so when prefixed with *relatively speaking*, again supporting the conclusion that the purity hypothesis undergenerates and that monotonic proportional measure functions are needed to capture certain truth value intuitions about relative readings.¹⁰

⁹While this conclusion strikes us as inevitable, introspection suggests that speakers typically do not actually access the truth conditions in (32) when judging (19) true in scenario (36). They are more likely to access the truth conditions in (i) instead, interpreting *gold* loosely rather than strictly. That is, speakers will judge (19) true in (36) on the grounds that the scenario establishes that the gold alloy (Au plus nickel) in the ring (10g) constitutes 50 percent of the ring’s weight (20g), while the gold alloy in the bracelet (10g) only constitutes 25 percent of the bracelet’s weight (40g).

- (i) $\text{WEIGHT}(\sqcup\{x : \text{GOLD}_{\text{loose}}(x) \wedge \text{IN}(x, \text{the ring})\}) / \text{WEIGHT}(\text{the ring}) >$
 $\text{WEIGHT}(\sqcup\{x : \text{GOLD}_{\text{loose}}(x) \wedge \text{IN}(x, \text{the bracelet})\}) / \text{WEIGHT}(\text{the bracelet})$

The intuited preference for (i) over (32) is in fact reflected in our visual representation in Figure 2, where the joined segments representing the gold alloy (Au plus nickel) are drawn as separate from the segment representing the rest (copper). We suspect that speakers have a general preference for loose interpretations. This preference is, however, easily overridden, in particular when the loose interpretation would result in pragmatic pathology. Note that (i) is contradictory in scenarios where the ring and the bracelet are made up entirely of gold alloy, as in scenario (9). The judgment that (19) is true there must be credited to the truth conditions in (32), where *gold* is interpreted strictly.

¹⁰Parallel to what we pointed out in footnote 9, it seems to us that speakers will typically access the truth conditions in (i) below, rather than those in (32), when judging sentence (25) true in scenario (38). That is, speakers will tend to interpret *alcohol* loosely, rather than strictly.

- (i) $\text{VOLUME}(\sqcup\{x : \text{ALCOHOL}_{\text{loose}}(x) \wedge \text{IN}(x, \text{this drink})\}) / \text{VOLUME}(\text{this drink}) >$
 $\text{VOLUME}(\sqcup\{x : \text{ALCOHOL}_{\text{loose}}(x) \wedge \text{IN}(x, \text{that drink})\}) / \text{VOLUME}(\text{that drink})$

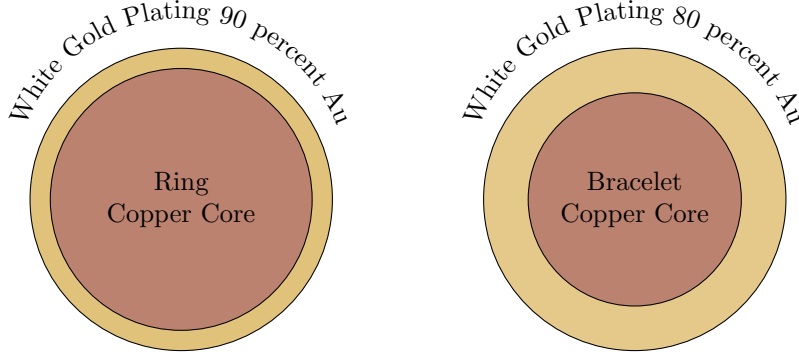


Figure 3: A visual depiction of the cross-sections of the ring (left) and the bracelet (right) in the scenario in (40), with the proportion of white gold plating to copper core represented by circle area.

In the next section, we will demonstrate that something stronger can be concluded about the purity hypothesis. Not only is it insufficient to capture the full range of intuitions about the truth conditions of relative readings, it is in fact incompatible with those intuitions.

5.2 The purity hypothesis overgenerates

We return to example (35), repeated once more in (39). We attend again to the truth conditions assigned to it under the purity hypothesis, shown in (31), as compared to the truth conditions based on monotonic measure functions, shown in (32). Consider now the scenario described in (40), which is also represented visually in Figures 3 and 4.

- (39) There is more gold in the ring than there is in the bracelet.
- (40) Both the ring and the bracelet are made of copper plated with white gold, an alloy composed of gold and nickel. The ring's white gold plating weighs 5g, and the ring's total weight is 20g; the bracelet's white gold plating weighs 20g, and the bracelet's total weight is 40g. The ring's white gold plating is composed of 90 percent Au and 10 percent nickel; the bracelet's white gold plating is 80 percent Au and 20 percent nickel. (See Figures 3 and 4 for a visualization of this scenario.)

Like in the scenario in (36), in (40) the ring and the bracelet are not made entirely of gold alloy, which makes it possible to discriminate between the truth conditions in (31) and (32). The scenario again renders (35) false as a comparison of absolute amounts. However, the truth conditions in (31), derived under the purity hypothesis, are met. This is so in virtue of the white gold in the ring containing a greater weight proportion of Au (90%) than the white gold in the bracelet (80%). In other words, the white gold in the ring is purer than the white gold in the bracelet. The purity hypothesis accordingly predicts that (39) can be judged true in (40), just like the purported paraphrase *The gold in the ring is purer than the gold in the bracelet*, whose truth in (40) is indeed uncontroversial.

In contrast, the truth conditions in (32), based on monotonic proportional measure functions, are not met. The Au in the ring constitutes a smaller weight proportion of the ring (22.5%) than the Au in the bracelet constitutes of the bracelet (40%). In addition to the truth conditions in (32), let us also consider their counterparts in (41), where $\text{GOLD}_{\text{strict}}$ is replaced by $\text{GOLD}_{\text{loose}}$. After all, in the analysis based on monotonic measure functions, there is no intrinsic requirement that the mass noun be interpreted strictly (cf. footnote 9). But (41) too is false in scenario (40), as the white gold in the ring constitutes a smaller weight proportion of the ring (25%) than the white gold in the

So speakers typically consider (25) true in (38) on the grounds that the scenario presents the vodka in this drink (50 ml) as constituting 50 percent of the drink's volume (100 ml), while presenting the vodka in that drink (50 ml) as merely constituting 25 percent of that drink's volume (200 ml).

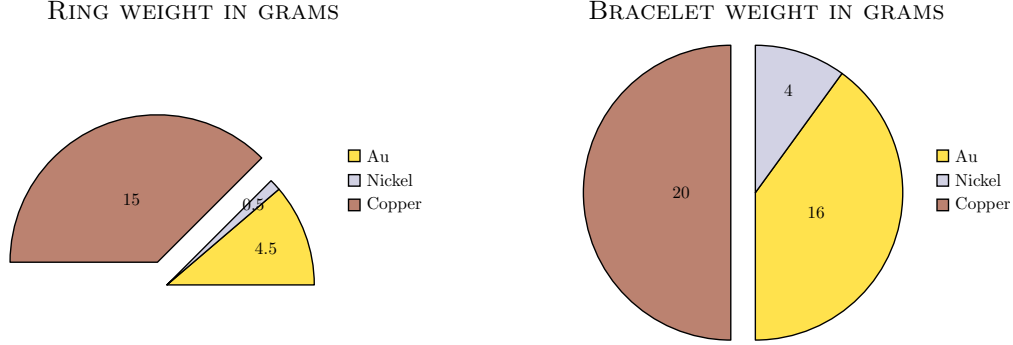


Figure 4: A representation of the makeup of the ring (left) and the bracelet (right) in the scenario in (40). The size of the segment represents the weight of the material in the object. The copper cores of the two objects are depicted as being separate from the white gold plating, represented by the joined segment of the gold and nickel.

bracelet constitutes of the bracelet (50%). So the account based on monotonic proportional measure functions does not predict that (39) can be judged true in scenario (40).

$$(41) \quad \frac{\text{WEIGHT}(\sqcup\{x : \text{GOLD}_{\text{loose}}(x) \wedge \text{IN}(x, \text{the ring})\})}{\text{WEIGHT}(\text{the ring})} > \frac{\text{WEIGHT}(\sqcup\{x : \text{GOLD}_{\text{loose}}(x) \wedge \text{IN}(x, \text{the bracelet})\})}{\text{WEIGHT}(\text{the bracelet})}$$

Turning to actual speaker judgments, it is clearly not possible to read (39) as being true in the scenario in (40), in sharp contrast to *The gold in the ring is purer than the gold in the bracelet*, even when the sentence is prefixed with *relatively speaking*. We conclude from this finding that the truth conditions in (31) are unattested, hence that the purity hypothesis overgenerates.

We can confirm this conclusion by considering the example in (37), repeated again in (42). The truth conditions based on the purity hypothesis are shown in (33), while those based on monotonic measure functions are shown in (34). Consider now the scenario in (43).

- (42) This drink has more alcohol in it than that drink does.
- (43) Both drinks consist of a mix of hard liquor and cranberry juice. This drink contains 10 ml of absinthe and 90 ml of cranberry juice. That drink contains 50 ml of vodka and 50 ml of cranberry juice. The volume proportion of ethanol in the absinthe (i.e., its “alcohol by volume” content) is 80%; the volume proportion of ethanol in the vodka (i.e., its “alcohol by volume” content) is 40%.

Like in the scenario in (38), in (43) the two drinks do not entirely consist of alcoholic beverages, which allows for the truth conditions in (33) and (34) to come apart. (43) does not make (37) true as a comparison of absolute amounts. However, the scenario meets the truth conditions in (33), based on the purity hypothesis. This is because the volume proportion of ethanol in the absinthe in this drink (80%) is greater than the volume proportion of the vodka in that drink (40%). So the purity hypothesis predicts that (42) can be judged true in (43).

In contrast, the truth conditions in (34), based on monotonic measure functions, are not met. The volume proportion of ethanol in this drink (8%) is not greater than the volume proportion of ethanol in that drink (20%). In addition, consider (44), the truth conditions that result from (34) by replacing $\text{ALCOHOL}_{\text{strict}}$ with $\text{ALCOHOL}_{\text{loose}}$, which can arise when *alcohol* in (42) is interpreted in the loose sense (cf. footnote 10).

$$(44) \quad \frac{\text{VOLUME}(\sqcup\{x : \text{ALCOHOL}_{\text{loose}}(x) \wedge \text{IN}(x, \text{this drink})\})}{\text{VOLUME}(\text{this drink})} > \frac{\text{VOLUME}(\sqcup\{x : \text{ALCOHOL}_{\text{loose}}(x) \wedge \text{IN}(x, \text{that drink})\})}{\text{VOLUME}(\text{that drink})}$$

These truth conditions are not met in (43) either, given that the volume proportion of absinthe in this drink (10%) is not greater than the volume proportion of vodka in that drink (50%). Under an analysis based on monotonic measure functions, then, it is not predicted that (42) can be judged true in (43).

Paralleling the corresponding intuitions about (39), the relevant judgments are once again clear-cut. Sentence (42) cannot be read as being true in the scenario in (43), even when the sentence is preceded by the prompt *relatively speaking*. This confirms that the purity hypothesis overgenerates, in virtue of generating readings that are not actually attested.

5.3 Interim conclusions

The relative interpretations derived from Bale and Barner’s (2009) purity hypothesis correctly describe intuitions about certain sentences in a class of scenarios. However, these interpretations do not track actual truth value judgments about these same sentences in the general case. The attested relative readings are better explained by hypothesizing that *much* references a monotonic proportional measure. The purity hypothesis, then, must be abandoned. The meaning of *much* can never reference purity functions. Given that such functions are non-monotonic, this is precisely what Schwarzschild’s (2006) Monotonicity Constraint predicts. The Monotonicity Constraint, then, can and must be maintained.

Having stated our main conclusions, we now return to an issue mentioned at the end of Section 4. The monotonic proportional analysis, as presented in Section 4, critically assumes that the (overt) main clause occurrence of *much* and the (covert) comparative clause occurrence can be associated with two different measure functions. Thus far, we have placed no constraints on how the two measure functions may differ. We will now critique these assumptions, and refine our analysis to achieve a better fit with the data.

6 On the grammar of relative readings

Thus far, we have argued that the purity hypothesis must be abandoned and that it should be replaced with an analysis involving monotonic proportional measure functions. In this section, we discuss some nuances regarding these types of proportional measurements. In particular, we first demonstrate, on empirical grounds, that we need to expand our theoretical toolbox to include variables that range over Skolemized measure functions (Section 6.1). Such an expansion allows us to account for sentences where quantified subjects “bind into” proportional measure functions. Next, we argue that such variables—and Skolemization more generally—might be a necessary ingredient to derive all relative readings. As discussed in Section 6.2, not only are LFs with such variables more consistent with well-known constraints on comparative deletion, such variables also correctly predict certain limits on relative readings. In particular, such readings do not permit cross-dimensional proportional comparisons.

6.1 Binding into measure functions

In its present form, the alternative to the purity hypothesis we presented in Section 4 suffers from systematic undergeneration. To illustrate this undergeneration, consider the example in (45). This sentence permits a relative reading conveying that the proportion of gold in each of these artefacts is greater than the proportion of gold in your pebble. Understanding proportions in terms of weight, for concreteness, the truth conditions of this reading can be stated as in (46).

(45) Each of these artefacts has more gold in it than your pebble.

(46) $\forall y[\text{OF-THESE-ARTEFS}(y) \rightarrow \text{WEIGHT}(\sqcup\{x: \text{GOLD}(x) \wedge \text{IN}(x,y)\})/\text{WEIGHT}(y) > \text{WEIGHT}(\sqcup\{x: \text{GOLD}(x) \wedge \text{IN}(x,\text{your pbl})\})/\text{WEIGHT}(\text{your pbl})]$

These truth conditions are beyond the reach of our baseline analysis in Section 4. The best we could do to try to derive them is to posit the LF in (47), which yields the interpretation in (48). But there

is no assignment of values to the measure function variables that would derive (46). Concretely, there is no way of filling in the dots in (49) that would have the intended effect. The locus of the problem centres around the fact that the interpretation in (46) effectively requires the main clause occurrence of *much* to invoke an entire family of measure functions (rather than just one), with each family member measuring the proportion of gold in one of the artefacts in the subject quantifier’s domain.

- (47) [each of these artefacts] $\lambda y[-er \lambda d[\text{than } [[d \text{ much}] \text{ gold}] \lambda x[\text{your pebble has } x \text{ in it}]$
 $\lambda d[[[d \text{ much}] \text{ gold}] \lambda x[y \text{ has } x \text{ in it}]]]$
- (48) $\forall y[\text{OF-THESE-ARTEFS}(y) \rightarrow \mu_1(\sqcup\{x: \text{GOLD}(x) \wedge \text{IN}(x,y)\}) >$
 $\mu_2(\sqcup\{x: \text{GOLD}(x) \wedge \text{IN}(x,\text{your pebble})\})]$
- (49) a. $\mu_1 = \lambda x. \text{WEIGHT}(x)/\text{WEIGHT}(\dots)$
b. $\mu_2 = \lambda x. \text{WEIGHT}(x)/\text{WEIGHT}(\text{your pebble})$

In a manner of speaking, the availability of the truth conditions in (46) reveals that it is possible for an operator, such as the subject quantifier in (45), to “bind into” a measure function parameter. Even though this is unexpected under our baseline analysis, this discovery should not actually come as a surprise. Such binding into contextually determined parameters is familiar from other empirical domains. Comparison classes referenced by the semantics of dimensional gradable adjectives like *tall* (see, e.g., Kennedy 2007) and tacit domain restrictions of quantificational determiners (see, e.g., von Stechow 1994) are known to show behaviour that is entirely parallel to what (45) illustrates for measure functions. So, while surprising from the perspective of our baseline analysis in Section 4, relative readings like the one attested in (45) are fully expected in view of what is known about contextually determined parameters in general.

The literature moreover offers a standard strategy for accommodating such binding into contextually determined parameters (e.g., von Stechow 1994, Kennedy 2007), a strategy that represents the relevant contextual parameters in the object language, allowing for the relevant binding relation to be syntactically encoded. We will now show that this standard strategy can be applied to the case at hand, thereby confirming that binding into measure functions is indeed entirely expected.

Our revision starts with the assumption that the denotation of *much* takes a measure function as one of its arguments. In (50), the lexical entry that we gave in (6) above is updated accordingly, positing that a measure function (type *ed*) must serve as *much*’s first input.

$$(50) \quad \llbracket \text{much} \rrbracket = \lambda \mu_{ed}. \lambda d_d. \lambda X_{et}. \lambda Y_{et}. \mu(\sqcup(X \cap Y)) \geq d$$

This modification allows us to syntactically represent the measure function variables in our object language (rather than the meta-language). By itself, such a modification represents only a minor change to our analysis. For example, reconsider the sentence in (19), repeated here as (51).

- (51) There is more gold in the ring than there is in the bracelet.

Under this new interpretation for *much*, (51) could be assigned an LF like (52), where each occurrence of *much* combines with a measure function variable. Such an LF would yield the interpretation in (53), where for notational convenience we use μ_1 and μ_2 as meta-language variables as well. If these variables are assigned the values in (54), then we would derive the intended relative truth conditions in (55).

- (52) $-er [\text{than Op } \lambda d[[[d [\text{much } \mu_2]] \text{ gold}] \lambda x[\text{there is } x \text{ in the bracelet}]]]$
 $\lambda d[[[d [\text{much } \mu_1]] \text{ gold}] \lambda x[\text{there is } x \text{ in the ring}]]]$
- (53) $\mu_1(\sqcup\{x : \text{GOLD}(x) \wedge \text{IN}(x, \text{the ring})\}) >$
 $\mu_2(\sqcup\{x : \text{GOLD}(x) \wedge \text{IN}(x, \text{the bracelet})\})$
- (54) a. $\mu_1 = \lambda x. \text{WEIGHT}(x)/\text{WEIGHT}(\text{the ring})$
b. $\mu_2 = \lambda x. \text{WEIGHT}(x)/\text{WEIGHT}(\text{the bracelet})$
- (55) $\text{WEIGHT}(\sqcup\{x : \text{GOLD}(x) \wedge \text{IN}(\text{the ring}, x)\})/\text{WEIGHT}(\text{the ring}) >$
 $\text{WEIGHT}(\sqcup\{x : \text{GOLD}(x) \wedge \text{IN}(\text{the bracelet}, x)\})/\text{WEIGHT}(\text{the bracelet})$

The advantage of this new representation is that we can use Skolemization to explain bound readings, such as the one exemplified in (45). For example, let's suppose that μ -variables (i.e., variables ranging over measure functions) are not the only possible syntactic complements for *much*. Perhaps it is possible to have variables that range over functions from individuals of type *e* to measure functions of type *ed*—in other words, Skolemized measure functions. (*Monotonicity* would hold of the range of such a Skolemized function, and thus of the result of the application of such a function to its Skolem argument.) We will use the symbol ν to distinguish such variables from their non-Skolemized counterparts. An example of a possible value for this new variable is given in (56). Here, a measure function is Skolemized to the measure of an entity's weight (the weight of *x*) relative to the weight of the Skolem argument (the weight of *y*).¹¹

$$(56) \quad \nu = \lambda y. \lambda x. \text{WEIGHT}(x) / \text{WEIGHT}(y)$$

This new variable allows for the (syntactic) introduction of an expression of type *e* within the complement of *much*. Of particular relevance, it is possible for the complement of *much* to be the combination of ν with a variable of type *e*, such as *y*. Furthermore, it is possible for the variable *y* to be bound by a quantifier phrase, such as *each of these artefacts*.

Given the introduction of this new variable, we can now account for the truth conditions given in (46), repeated in (57).

$$(57) \quad \forall y [\text{OF-THESE-ARTEFS}(y) \rightarrow \frac{\text{WEIGHT}(\sqcup \{x: \text{GOLD}(x) \wedge \text{IN}(x,y)\})}{\text{WEIGHT}(y)} > \frac{\text{WEIGHT}(\sqcup \{x: \text{GOLD}(x) \wedge \text{IN}(x, \text{your pbl})\})}{\text{WEIGHT}(\text{your pbl})}]$$

The truth conditions expressed in (57) can now be attributed to the LF in (58), where the subject quantifier, taking widest scope, binds the Skolem argument variable in the main clause. The LF in (58) determines the interpretation in (59), which in turn, along with the variable assignments in (60), yields the intended relative reading in (57).

$$(58) \quad [\text{each of these artefacts}] \lambda y [-\text{er } \lambda d [\text{than } [[d [\text{much } \mu] \text{ gold}] \lambda x [\text{your pebble has } x \text{ in it}]]] \lambda d [[d [\text{much } [\nu y]] \text{ gold}] \lambda x [y \text{ has } x]]]$$

$$(59) \quad \forall y [\text{OF-THESE-ARTEFS}(y) \rightarrow \nu(y) (\sqcup \{x: \text{GOLD}(x) \wedge \text{IN}(x,y)\}) > \mu(\sqcup \{x: \text{GOLD}(x) \wedge \text{IN}(x, \text{your pebble})\})]$$

$$(60) \quad \begin{array}{ll} \text{a.} & \nu = \lambda y. \lambda x. \text{WEIGHT}(x) / \text{WEIGHT}(y) \\ \text{b.} & \mu = \lambda x. \text{WEIGHT}(x) / \text{WEIGHT}(\text{your pebble}) \end{array}$$

In sum, evidence from relative readings demonstrates that our grammatical system has to provide some means to bind into measure functions. The *Skolemize-and-bind* strategy detailed here is one way to account for this type of binding. Building on this conclusion, in the next subsection we will offer a stronger hypothesis, namely that binding into measure functions is a necessary ingredient for all relative readings.

6.2 No relative readings without binding

In the previous section, we outlined a *Skolemize-and-bind* strategy to account for sentences where the measure function seems to vary with respect to a quantifier phrase. In this section, we entertain the hypothesis that *all* relative readings, not just those with quantifier phrases, involve binding into measure functions. We will argue that binding into measure functions is the only way the two occurrences of *much* can come to be associated with different measure functions.

In order to understand what is at stake, let's reconsider our example sentence in (51), repeated here as (61).

$$(61) \quad \text{There is more gold in the ring than there is in the bracelet.}$$

¹¹The basic idea of a Skolemized measure function is also captured in Solt's (2018) interpretation of the partitive morpheme.

In the previous section, we suggested that there are two instances of *much* in this sentence (one overt, the other covert) and that each instance has a measure function variable serving as its syntactic complement. We hypothesized the existence of two distinct variables, μ_1 in the main clause and μ_2 in the comparative clause. To derive the attested truth conditions, these μ -variables needed to be mapped to two different proportional measure functions.

However, the syntactic and semantic underpinnings of the *Skolemize-and-bind* strategy opens the door to an alternative analysis. In the *Skolemize-and-bind* strategy, we hypothesized that the complement of *much* could be complex, consisting of a variable ν that ranges over Skolemized measure functions in combination with a variable, such as y , that ranges over entities. One of the DP arguments in the main clause (in our example, the subject DP) was then able to raise to a position where it could bind the variable in the complement of *much*. Likewise, we could assume an equivalent structure for the sentence in (61), where both instances of *much* combine with a complement containing the variables ν and y . For each clause, one of the DP arguments could then move (covertly) to bind the y variable: the DP *[the ring]* in the main clause and the DP *[the bracelet]* in the comparative clause. The result would be an LF like the one sketched out in (62).

$$(62) \quad \text{-er [than Op } \lambda d[\text{ [the bracelet] } \lambda y[[d [\text{[much } [\nu y]]] \text{ gold}] } \lambda x[\text{there is } x \text{ in } y]]] \\ \lambda d[\text{ [the ring] } \lambda y[[[d [\text{[much } [\nu y]]] \text{ gold}] } \lambda x[\text{there is } x \text{ in } y]]]$$

Note that both occurrences of the variable ν remain free, so that the content of those occurrences will have to be fixed by context to a Skolemized measure function—the same function for both occurrences. In contrast, both instances of the input variable y are bound—by *the ring* in the main clause and by *the bracelet* in the comparative clause. This LF has the interpretation in (63), where for notational convenience we also use ν in the meta-language to refer to the semantic value of the object-language function variable. Suppose now, for example, that ν takes as its semantic value the Skolemized measure function in (64). The interpretation in (63) will then yield the corresponding truth conditions in (65).

$$(63) \quad \nu(\text{the ring})(\sqcup\{x : \text{GOLD}(x) \wedge \text{IN}(\text{the ring}, x)\}) > \\ \nu(\text{the bracelet})(\sqcup\{x : \text{GOLD}(x) \wedge \text{IN}(\text{the bracelet}, x)\})$$

$$(64) \quad \nu = \lambda y. \lambda x. \text{WEIGHT}(x) / \text{WEIGHT}(y)$$

$$(65) \quad \text{WEIGHT}(\sqcup\{x : \text{GOLD}(x) \wedge \text{IN}(\text{the ring}, x)\}) / \text{WEIGHT}(\text{the ring}) > \\ \text{WEIGHT}(\sqcup\{x : \text{GOLD}(x) \wedge \text{IN}(\text{the bracelet}, x)\}) / \text{WEIGHT}(\text{the bracelet})$$

Thus, the *Skolemize-and-bind* strategy can derive the relative readings familiar from above, although through a somewhat different compositional route.

So far, we have only demonstrated that this alternative analysis is possible and derives the correct truth conditions. However, it is important to point out that there are some empirical advantages. First, notice that in the LF in (62), the phrasal projection containing *much* in the main clause is identical to the phrasal projection containing *much* in the comparative clause: both are realized as *[much [ν y]]* and critically both contain the same variables, ν and y . This was not the case in the previous non-Skolemized analysis where the main-clause contained the phrase *[much μ₁]* and the comparative clause contained the phrase *[much μ₂]*—two distinct variables. The fact that the *Skolemize-and-bind* strategy leads to identical constituents (i.e., exact copies with identical variables) is more consistent with known syntactic constraints on comparative constructions (in particular comparative deletion, see Bresnan 1973, Heim 1985, Kennedy 1999). For example, consider the sentence in (66), where an overt variable (*him*) occurs within the degree phrase in the main clause (i.e., *prouder of him*). As with all examples of comparative deletion, the comparative clause (*than Sue is*) does not have any overt occurrences of degree-related expressions (comparative morphemes, adjectives, etc.). Rather, the interpretation of such clauses depends on the values expressed in the main clause (perhaps through some kind of ellipsis).

$$(66) \quad \text{Mary is prouder of him than Sue is.}$$

Critically, the interpretation of the comparative clause not only depends on the degree to which

Sue is proud of a particular individual, but the identity of that individual must be the same as the one referred to by *him* in the main clause. In other words, the variable represented by *him* in the main clause must have the same value as the implicit variable within the comparative clause. Given standard assumptions about variable assignments, this constraint can only be achieved if the variables in both clauses are required to be identical. It is reasonable to assume that a similar requirement holds for implicit variables as well.¹²

Another advantage of the *Skolemize-and-bind* strategy is that it places certain limits on relative readings, limits that are empirically well-attested. In particular, if we assume that such a strategy is the only means by which the main clause and comparative clause can have different measure functions (in accordance with constraints on comparative deletion mentioned above), then we would correctly exclude what we might call *cross-dimensional* proportional comparisons—comparisons of two proportions that are built from two different basic measure functions (like VOLUME and WEIGHT). To understand the importance of this exclusion, let’s reconsider the non-Skolemized analysis of sentences like *There is more gold in the ring than in the bracelet*. In the previous section, we assigned this sentence the LF in (52), repeated in (67), where μ_1 and μ_2 serve as the complements to *much*. In that section, we assigned both μ -variables to measure functions that involved relative measurements of weight. However, if (67) were a permissible LF, then, grammatically speaking, there would be nothing to prevent us from assigning a relative measure of volume to μ_1 and a relative measure of weight to μ_2 , as in (68). Such an assignment would constitute a cross-dimensional relative comparison and would yield the truth conditions in (69).¹³

(67) -er [than Op λd [[d [much μ_2] gold] λx [there is x in the bracelet]]]
 λd [[d [much μ_1] gold] λx [there is x in the ring]]

(68) a. $\mu_1 = \lambda x. \text{VOLUME}(x)/\text{VOLUME}(\text{the ring})$
b. $\mu_2 = \lambda x. \text{WEIGHT}(x)/\text{WEIGHT}(\text{the bracelet})$

(69) $\text{VOLUME}(\sqcup\{x : \text{GOLD}(x) \wedge \text{IN}(\text{the ring}, x)\})/\text{VOLUME}(\text{the ring}) >$
 $\text{WEIGHT}(\sqcup\{x : \text{GOLD}(x) \wedge \text{IN}(\text{the bracelet}, x)\})/\text{WEIGHT}(\text{the bracelet})$

We do not see any indications that such a reading is actually attested. To confirm this assessment, let us move to a slightly different type of example, a comparative about substances, such as cream, that are with equal likelihood measured in terms of either weight or volume. The example in (70) is a case in point.

(70) (Relatively speaking,) Al’s mousse has more cream in it than Bo’s.

(71) Al and Bo each made a batch of chocolate mousse consisting of whipped cream and chocolate. The two mousses have the very same composition. In terms of volume, each contains 1 cup of whipped cream and 1 cup of melted chocolate. In terms of weight, each contains 50g of whipped cream and 200g of melted chocolate.

Suppose (70) could be read as a cross-dimensional comparison analogous to (69), that is, as portraying the volume proportion of cream in Al’s mousse as greater than the weight proportion of cream in Bo’s. It should then be possible to judge (70) as true in scenario (71) on the grounds that the volume proportion of cream in Al’s mousse is 50% while the weight proportion of cream in Bo’s mousse is a mere 25%. However, regardless of how this scenario may be further fleshed out, it seems inconceivable that (70) could actually be judged true on those grounds.

This judgement remains in force even if we imagine an immediately preceding linguistic context that should be maximally conducive to supporting the truth value judgment in question. In (72), A’s

¹²Of course, identical variables do not always lead to identical semantic values when binding is involved. Indeed, when an overt variable can be interpreted as being bound, there is more flexibility with respect to the interpretation of the comparative clause. For example, [*Mary is prouder of her mother than Sue is*] can be understood as expressing that Mary’s pride for her own mother is greater than Sue’s pride for her own mother, or it could be understood as expressing that Mary’s pride for her own mother is greater than Sue’s pride for Mary’s mother.

¹³Note, unlike other types of cross-dimensional comparisons, it seems unlikely that such comparisons are ruled out due to incommensurability of the scales (see the discussion in Cresswell 1976, Kennedy 1999 and references therein). Both degrees will end up being proportions and thus in principle should be comparable. In fact, relative comparisons across different dimensions is well attested in the literature on adjectival comparisons (see Bale 2008; van Rooij 2010).

statement is an explicit unambiguous paraphrase of the intended cross-dimensional reading of (70), which could perhaps, in the right circumstances, serve as an appropriate commentary on the two mousses's composition in (71). Still, this paraphrase does nothing to draw out the relevant reading of (70)—B's assenting reply seems incoherent, and surely cannot be construed as true in (71) despite pragmatic pressure to be so interpreted.

- (72) A: Here is an observation—although one that admittedly may be rather useless—about the mousses that Al and Bo made: the volume proportion of cream in Al's mousse is 50% whereas the weight proportion of cream in Bo's mousse is 25%. Thus, the volume proportion of cream in Al's mousse is greater than the weight proportion of cream in Bo's mousse.
 B: Yes, indeed, (relatively speaking) Al's mousse has more cream in it than Bo's.

As with our ring-and-bracelet example, the non-Skolemized analysis of (70) derives the unattested reading, however the *Skolemize-and-bind* strategy does not. Let's consider the details of the non-Skolemized analysis first. By analogy to our previous examples, under this analysis the sentence in (70) should have the LF in (73), which, if interpreted under the variable assignment in (74), would yield the unattested truth conditions in (75).

- (73) -er [than Op λd [[d [much μ_2] cream] λx [there is x in Bo's mousse]]
 λd [[d [much μ_1] cream] λx [there is x in Al's mousse]]
 (74) a. $\mu_1 = \lambda x. \text{VOLUME}(x)/\text{VOLUME}(\text{Al's mousse})$
 b. $\mu_2 = \lambda x. \text{WEIGHT}(x)/\text{WEIGHT}(\text{Bo's mousse})$
 (75) $\text{VOLUME}(\sqcup\{x : \text{CREAM}(x) \wedge \text{IN}(\text{Al's mousse}, x)\})/\text{VOLUME}(\text{Al's mousse}) >$
 $\text{WEIGHT}(\sqcup\{x : \text{CREAM}(x) \wedge \text{IN}(\text{Bo's mousse}, x)\})/\text{WEIGHT}(\text{Bo's mousse})$

In contrast, the *Skolemize-and-bind* strategy predicts that (70) should have the LF in (76), which would yield the interpretation in (77).

- (76) -er [than Op λd [Bo's mousse] λy [d [[much [ν y]] cream] λx [there is x in y]]]
 λd [Al's mousse] λy [[d [much [ν y]] cream] λx [there is x in y]]]
 (77) $\nu(\text{Al's mousse})(\sqcup\{x : \text{CREAM}(x) \wedge \text{IN}(\text{Al's mousse}, x)\}) >$
 $\nu(\text{Bo's mousse})(\sqcup\{x : \text{CREAM}(x) \wedge \text{IN}(\text{Bo's mousse}, x)\})$

The ν in the main clause and the comparative clause are the same variable and thus should be assigned the same value. For example, the assignment in (78a) would yield a comparison of weight proportions—the truth conditions in (78b)—while the assignment in (79a) would yield a comparison of volume proportions—the truth conditions in (79b). No matter which assignment, the comparisons are made along the same proportional dimension.

- (78) a. $\nu = \lambda y. \lambda x. \text{WEIGHT}(x)/\text{WEIGHT}(y)$
 b. $\text{WEIGHT}(\sqcup\{x : \text{CREAM}(x) \wedge \text{IN}(\text{Al's mousse}, x)\})/\text{WEIGHT}(\text{Al's mousse}) >$
 $\text{WEIGHT}(\sqcup\{x : \text{CREAM}(x) \wedge \text{IN}(\text{Bo's mousse}, x)\})/\text{WEIGHT}(\text{Bo's mousse})$
 (79) a. $\nu = \lambda y. \lambda x. \text{VOLUME}(x)/\text{VOLUME}(y)$
 b. $\text{VOLUME}(\sqcup\{x : \text{CREAM}(x) \wedge \text{IN}(\text{Al's mousse}, x)\})/\text{VOLUME}(\text{Al's mousse}) >$
 $\text{VOLUME}(\sqcup\{x : \text{CREAM}(x) \wedge \text{IN}(\text{Bo's mousse}, x)\})/\text{VOLUME}(\text{Bo's mousse})$

Thus, if we assume that differences in measurement in the main clause and comparative clause can only be achieved through the *Skolemize-and-bind* strategy (at least in cases where there is comparative deletion), then we would have a natural explanation of why it is impossible to get a cross-dimensional relative reading.¹⁴ Constraints on comparative deletion require that the two

¹⁴Strictly speaking, this only follows if we make some reasonable assumptions about the range of possible assignments to ν . For example, the value of ν must correspond to a valid, typed λ -expression. It cannot be assigned to a function that maps some individuals to a proportional measure of weight and others to a proportional measure of volume. Although such functions can be defined extensionally, they cannot be defined intensionally with a λ -expression.

occurrences of *much* combine with identical variables. Thus, it is only in virtue of potential binders that the measure functions can come to differ (e.g., whether the binder is *[the ring]* or *[the bracelet]*, *[Al’s mousse]* or *[Bo’s mousse]*). They cannot differ in terms of what basic measurements they invoke (e.g., volume vs. weight).

We conclude that the *Skolemize-and-bind* strategy is more consistent with known syntactic constraints on comparative deletion and furthermore correctly excludes unwanted cross-dimensional comparisons.¹⁵

7 Conclusions

In our investigation of comparatives with *much* plus a mass noun, we have found that *much* is not only able to reference basic measure functions like VOLUME or WEIGHT, but also proportional measure functions. These proportional measure functions yield *relative* readings of comparative statements. Critically, such readings have truth conditions that cannot be characterized by a direct comparison of absolute amounts. Rather, such truth conditions rely on a comparison of fractions (ratios) that can be defined in terms of more basic non-proportional measurements like VOLUME or WEIGHT. We proposed that these readings arise when the two occurrences of *much* in a comparative’s LF—one in the main clause and one in the comparative clause—associate with two different measure functions. Exploring the consequences of this result, we have moreover proposed that the two measure functions can come to differ only by virtue of Skolemization and variable binding.

We were led to our proposal through a case study on comparatives with *much* that can sometimes be judged true on the basis of the degrees of purity of two portions of matter, like gold or alcohol. In our analysis, such examples show that a comparative’s content and its context of use can conspire to yield relative readings that happen to amount to comparisons of purity. But such comparisons are not, as Bale and Barner (2009) had suggested, evidence for *much* accessing measure functions that in general output degrees of purity. The hypothesis that *much* can reference purity functions predicts unattested truth conditions.

Our proposal defends Schwarzschild’s (2006) Monotonicity Constraint against the implicit challenge from comparisons of purity, and our findings in fact further strengthen the case for this constraint. The Monotonicity Constraint requires that a measure function referenced by *much* track the part-whole structure that orders the portions of matter in the extension of the mass noun that *much* combines with. This constraint explains the inability of *much* to access purity functions, since, as Schwarzschild noted, purity functions are non-monotonic. This non-monotonicity is due to purity functions taking the form $\lambda x.m(x)/n(x)$, where the output fraction’s denominator and numerator both vary with the input. In contrast, the proportional functions we have posited are monotonic, taking the form $\lambda x.m(x)/c$, where c is a constant and m is monotonic. We submit that awareness of

¹⁵In all of the Skolemized measure functions considered above, the denominator is fixed by a measurement of the Skolem argument. However, not all cases fit this pattern. For example, the sentence *[(Proportionally speaking) Al bought more Chardonnay than Bo did]* yields a relative reading that is true in the context in (i).

- (i) Al and Bo each stocked up their supplies of wine. Al bought 10 liters each of Chardonnay, Moscato and Riesling. Bo bought 10 liters of Chardonnay and 20 liters each of Moscato and Riesling.

The sentence is true by virtue of the fact that Chardonnay comprises a greater volume proportion of the wine Al purchased (33%) than it does of the wine Bo purchased (20%). Such a reading is captured by the truth conditions in (iia). Given that the most likely Skolem arguments are the DPs *Al* and *Bo* (as these are the only constituents of type e that are overtly available in the syntax), we propose that such a reading can be derived from the LF in (iib) with the interpretation in (iic) and the variable assignment in (iic).

- (ii) a. $\text{VOLUME}(\sqcup\{z : \text{CHARDONNAY}(z) \wedge \text{BOUGHT}(\text{Al}, z)\}) / \text{VOLUME}(\sqcup\{z : \text{BOUGHT}(\text{Al}, z)\}) >$
 $\text{VOLUME}(\sqcup\{z : \text{CHARDONNAY}(z) \wedge \text{BOUGHT}(\text{Bo}, z)\}) / \text{VOLUME}(\sqcup\{z : \text{BOUGHT}(\text{Bo}, z)\})$
 b. -er [than Op λd [Al λy [[d [much [ν y]] Chardonnay] λx [y bought x]]]]
 λd [Bo λy [[d [much [ν y]] Chardonnay] λx [y bought x]]]]
 c. $\nu(\text{Al})(\sqcup\{x : \text{CHARDONNAY}(x) \wedge \text{BOUGHT}(\text{Al}, x)\}) > \nu(\text{Bo})(\sqcup\{x : \text{CHARDONNAY}(x) \wedge \text{BOUGHT}(\text{Bo}, x)\})$
 d. $\nu = \lambda y.\lambda x. \text{VOLUME}(x) / \text{VOLUME}(\sqcup\{z : \text{BOUGHT}(y, z)\})$

Here the denominator in (iic) is not a measurement of the Skolem argument itself, but rather a measurement of an entity that bears a systematic relation to this Skolem argument (e.g., the aggregate of stuff bought).

proportional measure functions as possible values for *much*'s measure function variable allows one to more fully appreciate the Monotonicity Constraint's force.

References

- Bale, Alan, and David Barner. 2009. The interpretation of functional heads: Using comparatives to explore the mass/count distinction. *Journal of Semantics* 26:217–252.
- Bale, Alan, and Bernhard Schwarz. 2019. Proportional readings of *many* and *few*: the case for an underspecified measure function. *Linguistics and Philosophy* URL <https://doi.org/10.1007/s10988-019-09284-5>.
- Bale, Alan Clinton. 2008. A universal scale of comparison. *Linguistics and Philosophy* 31:1–55.
- Bartsch, Renate, and Theo Vennemann. 1972. *Semantic structures: A study in the relation between semantics and syntax*. Athenäum.
- Bresnan, Joan W. 1973. Syntax of the comparative clause construction in English. *Linguistic Inquiry* 4:275–343.
- Cresswell, Max J. 1976. The semantics of degree. In *Montague Grammar*, ed. Barbara H Partee, 261–292. New York: Academic Press.
- von Fintel, Kai. 1994. Restrictions on quantifier domains. Doctoral Dissertation, University of Massachusetts Amherst.
- Hackl, Martin. 2000. Comparative quantifiers. Doctoral Dissertation, Massachusetts Institute of Technology.
- Heim, Irene. 1985. Notes on comparatives and related matters: University of texas. manuscript, University of Texas at Austin.
- Heim, Irene. 2000. Degree operators and scope. In *Proceedings of SALT X*, ed. Brendan Jackson and Tanya Matthews, 40–64. Cornell University, Ithaca, NY: CLC Publications.
- Kennedy, Christopher. 1999. *Projecting the adjective: The syntax and semantics of gradability and comparison*. New York: Garland.
- Kennedy, Christopher. 2007. Vagueness and grammar: The semantics of relative and absolute gradable adjectives. *Linguistics and Philosophy* 30:1–45.
- Klein, Ewan. 1981. The interpretation of adjectival, nominal, and adverbial comparatives. In *Formal methods in the study of language*, ed. J. Groenendijk, T. Janssen, and M. Stokhof, 381–98. Amsterdam: Mathematical Center Tracts.
- Krifka, Manfred. 1989. Nominal reference, temporal constitution and quantification in event semantics. In *Semantics and contextual expressions*, ed. Renate Bartsch, Johan van Benthem, and Peter van Emde Boas, 75–116. Dordrecht: Foris Publications.
- Lønning, J. T. 1987. Mass terms and quantification. *Linguistics and Philosophy* 10:1–52.
- van Rooij, Robert. 2010. Measurement and Interadjective Comparisons. *Journal of Semantics* 28:335–358. URL <https://doi.org/10.1093/jos/ffq018>.
- Schwarzschild, Roger. 2006. The role of dimensions in the syntax of noun phrases. *Syntax* 9:67–110.
- Solt, Stephanie. 2015. Q-Adjectives and the Semantics of Quantity. *Journal of Semantics* 32:221–273. URL <https://doi.org/10.1093/jos/fft018>.

- Solt, Stephanie. 2018. Proportional comparatives and relative scales. In *Proceedings of Sinn und Bedeutung 21*, ed. Robert Trusswell, Chris Cummins, Caroline Heycock, Brian Rabern, and Hannah Rohde, 1123–1140. <https://semanticsarchive.net>.
- von Stechow, Arnim. 1984. Comparing semantic theories of comparison. *Journal of semantics* 3:1–77.
- Wellwood, Alexis. 2014. Measuring predicates. Doctoral Dissertation, University of Maryland.
- Wellwood, Alexis. 2015. On the semantics of comparison across categories. *Linguistics and Philosophy* 38:67–101.
- Wellwood, Alexis. 2020. *The meaning of more*. Oxford studies in semantics and pragmatics. Oxford University Press.