Linear Programming Exercise – Solution

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One of the great challenges in graduate school is management of the MA thesis. There are a number of tasks involved in this process, and rapid completion of an MA degree requires that the student pay careful attention to those activities which are "on the critical path". Figure 1 illustrates the involved activities and their interconnections.

You have been contracted by a fellow student to evaluate the factors which play a pivotal role in his degree program. He has asked your help because he knows that you are taking the linear programming course, and you understand how to interpret the writing of an MA thesis as a critical path scheduling problem. You assure your friend that you can formulate a computer model which will answer all of his questions.

a. *Formulate* a linear program which determines the minimum time cost of completing an MA thesis, initially ignoring the fact that the thesis advisors may sometimes summarily reject student work at different stages in the thesis process. Discuss how your model would determine precisely which tasks A–Q are on the "critical path".

b. Implement the model in GAMS and produce a report of the tasks on your critical path.

$$\min \quad T$$

subject to

$$\begin{aligned} T &\geq S_i + \delta_i && \forall i \\ S_j &\geq S_i + \delta_i && \forall \phi_{ij} \\ S_i &\geq 0 && \forall i \end{aligned}$$

in which T is the minimal time to complete, S_i is the starting time for task i, δ_i is the (exogenous) time required to complete task i and ϕ_{ij} is an indicator array identifying which tasks i must be completed before task j is initiated.

Having solved this problem, we can identify tasks which are "on the critical path" on the basis of binding constraints. If the Lagrange multiplier for an ϕ_{ij} constraint dictating the starting time of activity j is nonzero, then both i and j are on the critical path to completion of the project.

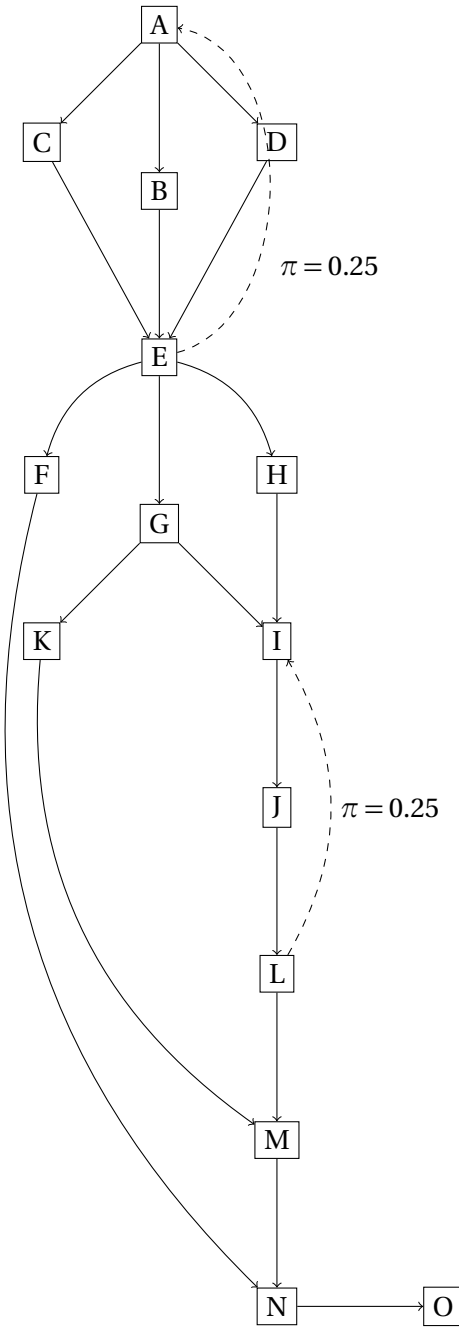
c. Formulate a second linear program in which you take into account the uncertainties regarding advisor acceptance of the idea (at node E), the simulation design (at node L) and the final draft (at node Q). At these three nodes, the *expected duration* of the task is the following equation:

$$ED_i = q_i \delta_i + \sum_j \pi_{ij} \left[S_i - S_j + ED_i \right]$$

where S_i is the expected start time of task i, δ_i is the time cost of activity i, parameter π_{ij} is the probability of regression from task i to task j ($\pi_{\mathrm{EA}} = 0.25$, $\pi_{\mathrm{LI}} = 0.25$, and $\pi_{\mathrm{QP}} = 0.15$ while all other values are zero), and q_i is the probability of successfully completing project i, $q_i = 1 - \sum_j \pi_{ij}$.

Replace δ_i in the previous problem by ED_i and add the equations which define ED_i to the model. QED.

Figure 1: Tasks to Complete an MA Thesis in Agricultural and Applied Economics



A	Formulate an idea	15 days
B	Survey policy literature	4 days
C	Code up a small illustrative example	5 days
D	Write a note on the policy issues and context	10 days
E	**Meet with advisor for idea approval**	4 days
F	Search academic literature	15 days
G	Collect empirical data	15 days
H	Formulate a more complete illustrative model	5 days
I	Meet with advisor for model approval	5 days
J	Formulate a calibrated pilot model	10 days
K	Estimate key elasticities	4 days
L	**Meet with advisor for simulation approval**	3 days
M	Perform policy simulations	4 days
N	Complete a thesis draft	20 days
O	Obtain editorial feedback and advice	5 days
P	Redraft the thesis	2 days
Q	**Meet with advisor for thesis acceptance**	10 days