AAE 706

General Equilibrium in a Nutshell: The 123 Model

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Review: Calibration of Cobb-Douglas Preferences

Suppose that we observe consumer demand prices and quantities, $(\bar{p}_i, \bar{x}_i)$. If we assume Cobb-Douglas preferences, we can *invert* the demand functions to solve for share parameters in terms of the observations:

$$\bar{M} = \sum_i \bar{p}_i \bar{x}_i$$

and

$$\theta_i = \frac{\bar{p}_i x_i}{\bar{M}}$$

A Cobb-Douglas Calibration Exercise: Question

Suzy consumes ice cream (x_1) and soda (x_2) for lunch every day, and she currently has one ice cream and two sodas per week when they both cost 1 CHF. What Cobb-Douglas utility function is consistent with Suzy's choices over ice cream and soda. Write down demand functions which could extrapolate her optimal choices to any expenditure (m) and prices (p_1 and p_2).

A Cobb-Douglas Calibration Exercise: Answer

Based current choices, we observe that Suzy's budget shares for ice cream and sodas are 1/3 and 2/3, respectively. The Cobb-Douglas utility function which describes her preferences is:

$$U(x_1, x_2) = x_1^{1/3} x_2^{2/3}$$

and demand functions are

$$x_1 = \frac{Y}{3p_1}$$

and

$$x_2 = \frac{2Y}{3p_2}$$

A Related Calibration Exercise #1: Question

Suppose that irregardless of relative prices, Suzy always has one soda before and one soda after eating an ice cream. What utility function is consistent with these choices? Write down demand functions which could extrapolate her optimal choices to any expenditure (m) and prices (p_1 and p_2).

A Related Calibration Exercise #2: Question

When Joe gets to the bar, he always asks about the price of peanuts and the price of beer. If two beers cost less than one bag of peanuts, he spends all of his money on beer. Otherwise he buys peanuts. What utility function is consistent with these choices? Write down demand functions which could extrapolate her optimal choices to any expenditure (m) and prices (p_1 and p_2).

Computable General Equilibrium Models

- Stylized (but useful): 123 model

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- Spatial price equilibrium models

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- Ramsey model and integrated assessment models (DICE)

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- Devarajan-Go-Lewis-Robinson-Sinko (1997), Chapter 6 of *Applied methods in trade policy analysis: A Handbook,* Francois and Reinert, eds., Cambridge University Press.

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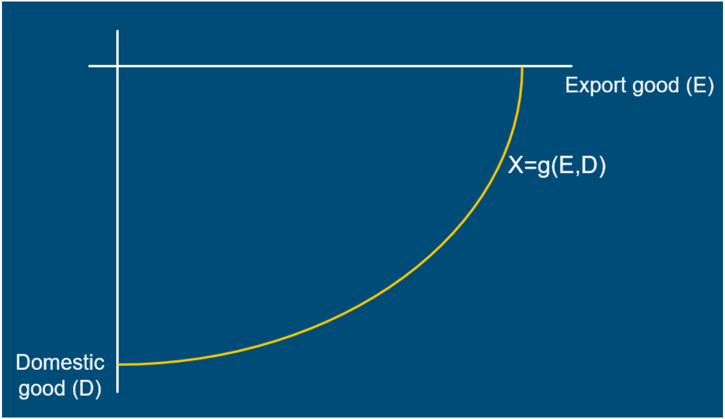
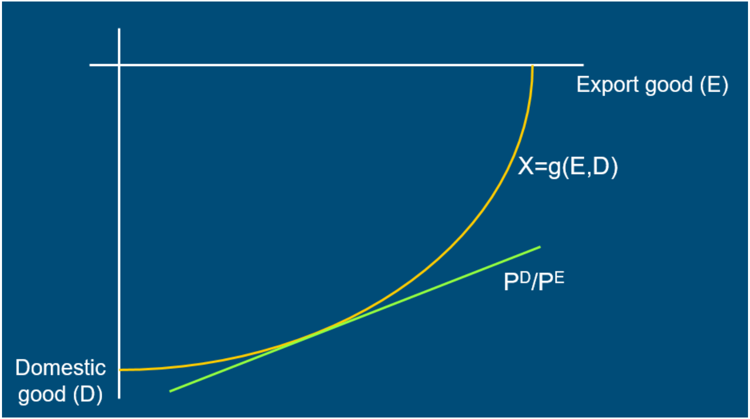
- 1 market for domestic goods (price P, market clearance)
- Balance of trade (foreign exchange price π, market clearance). N.B. Both exports and imports are denominated in units of foreign exchange.

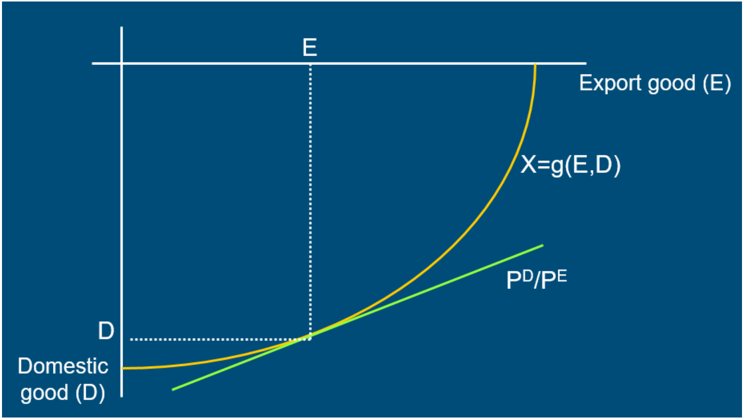
Supply Technology



Optimal Supply



Relative Prices and Optimal Supply



Differentiated Goods Supply: Details

Adopt a *constant elasticity of transformation* technology:

$$X = g(E, D) = \left(\theta_E \left(\frac{E}{\bar{e}} \right)^{\rho_E} + (1 - \theta_E) \left(\frac{D}{\bar{d}} \right)^{\rho_E} \right)^{1/\rho_E}$$

where X is an *index* of resource inputs to domestic production (when $E = \bar{e}$ and $D = \bar{d}$, $X = 1$.)

$\bar{d}$ Benchmark production for the domestic market (when we use Harberger normalization: $\bar{p}_E = 1$ and $\bar{p}_D = 1$.

$\bar{e}$ Benchmark exports

θ_E Export value share:

$$\theta_E = \frac{\bar{e}}{\bar{e} + \bar{d}}$$

Linear Homogeneity

$g(E, D)$ is linearly homogeneous, i.e.

$$g(\lambda E, \lambda D) = \lambda g(E, D) \quad \forall \lambda > 0.$$

We therefore can solve for optimal coefficients:

$$\max_{a_E, a_D} p_E a_E + p_D a_D \quad \text{s.t.} \quad g(a_E, a_D) = 1$$

Points on a unit isoquant satisfy:

$$\theta_E \left(\frac{a_E}{\bar{e}}\right)^{\rho_E} + (1 - \theta_E)\left(\frac{a_D}{\bar{d}}\right)^{\rho_E} = 1$$

Points on the Supply Frontier

Expressing a_E as a function of a_D, we have:

$$a_E = \bar{e} \left[\frac{1 - (1 - \theta_E)(a_D/\bar{d})^{\rho_E}}{\theta_E} \right]^{1/\rho_E}$$

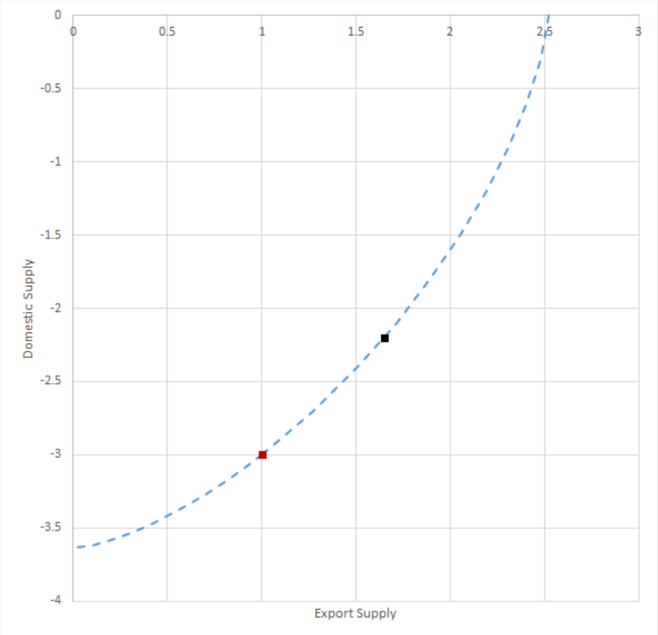
Expressing D as a function of E, we have:

$$a_D = \bar{d} \left[\frac{1 - \theta_E(a_E/\bar{e})^{\rho_E}}{1 - \theta_E} \right]^{1/\rho_E}$$

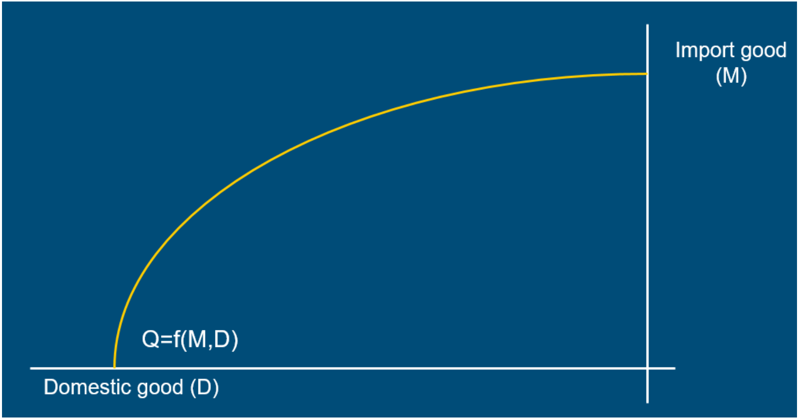
Domestic-Export Supply in Excel

123 Model Graphics				
Input Data				
Reference level of exports	e0	1		
Reference level of domestic goods	d0	3		
Elasticity of transformation	etadx	2		
	rhodx	1.5		
	thetae	0.25		
Reference level of imports	m0	1		
Elasticity of substitution	esubdm	2		
	rhodm	0.5		
	thetam	0.25		
Counterfactual Data				
Price of domestic goods	pd	1		
World price of imports	pm	1		
World price of exports	pe	1.5		
Real exchange rate	pi	1		
Counterfactual Equilibrium				
Revenue function	R	1.168082		
Export supply	E	1.649057		
Domestic Supply	D	2.198743		
Unit Isoquant (see notes)	E/e0	D/d0	E	D
	1	1	1	3
	0.9	1.032227	0.9	3.096682
	0.8	1.062254	0.8	3.186762

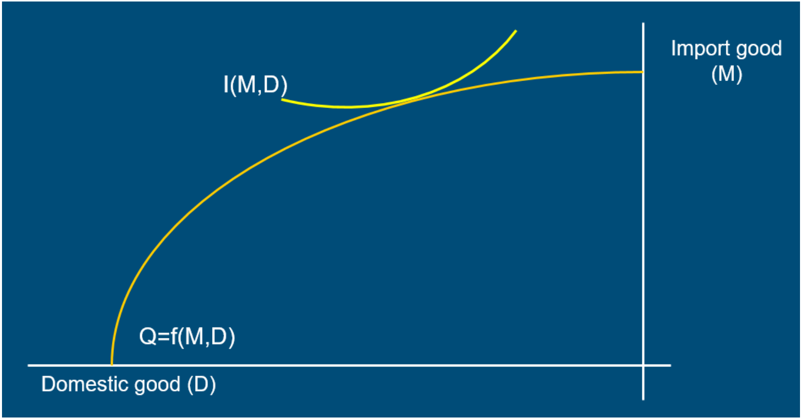
Supply Response in Excel



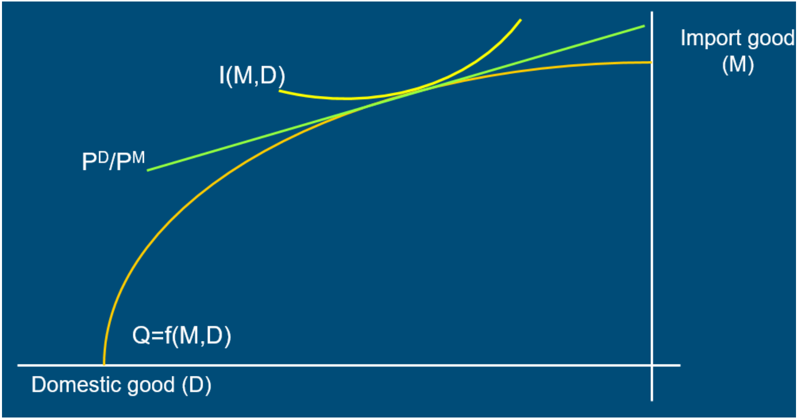
Demand Technology



Optimal Choice



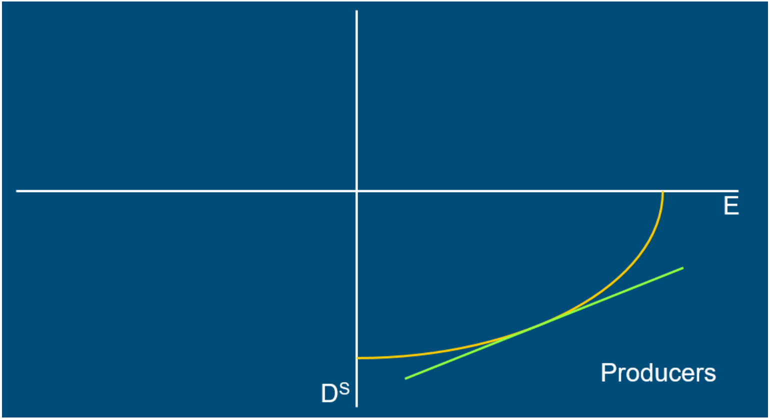
Relative Prices and Optimal Choice



Linking Elements of the Model

1. Supply to the domestic market needs to equal demand for the non-traded domestic good (D) : $D^S = D^D$
2. Current account balance (value of imports = value of exports + current account deficit (exogenous)

Linking Supply and Demand



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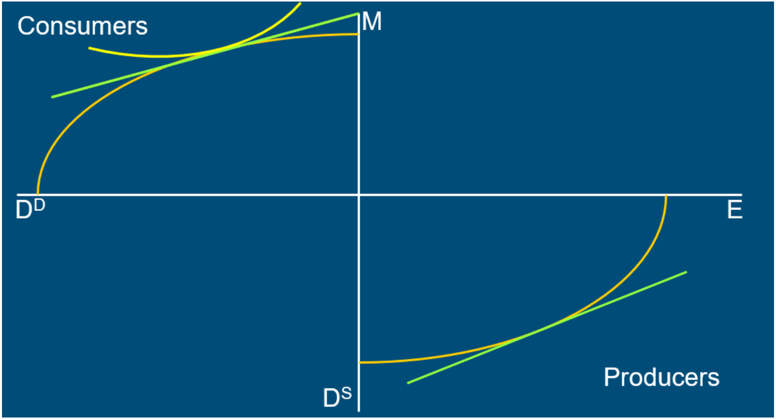

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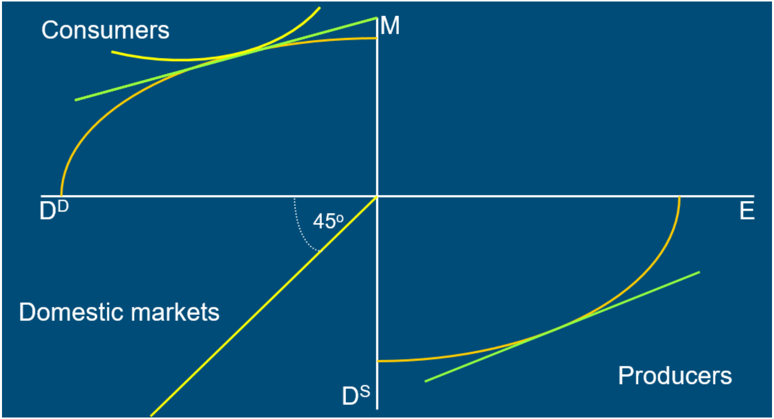
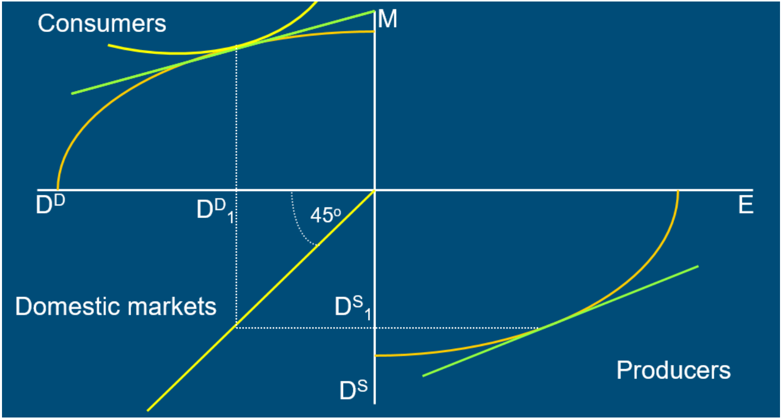


Trade Balance $\Rightarrow M = E$



Trade Balance

- Imports (M) have to be financed by exports (E) and flows of foreign money (B = current account balance):

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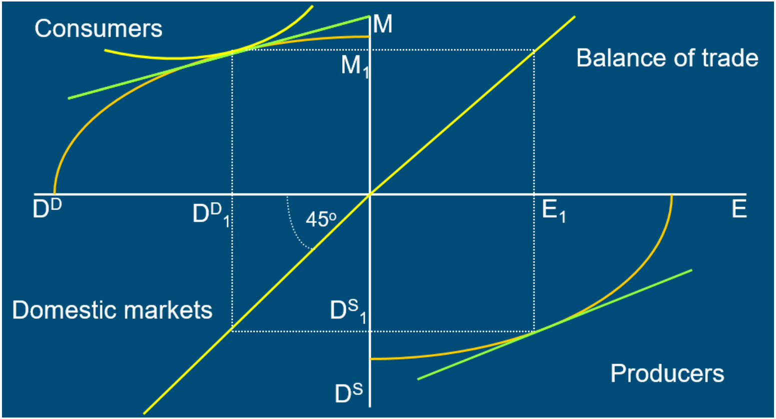
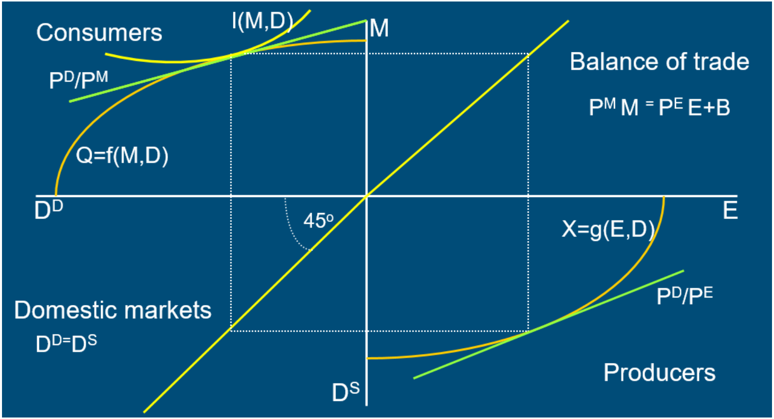
- Initially assume that B is zero (runs through origin) and that world market prices are unity (45^0 angle)


Consumers
M
M1
Balance of trade
DD
DD1
45o
E1
E
Domestic markets
DS1
Producers
DS

Basic General Equilibrium Model



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- Putting sector-effects in perspective
- Welfare analysis by including households

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Building the mathematical model

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- Add government
- Add savings and investment

Input Data

Benchmark Microconsistent Matrix	Supply	Demand	Government	Households	Investment	Chksum:
Current account	106	-145	38			0
Domestic ouputput	218	-218				0
Sales and excise taxes		-32	32			0
Import tariffs		-19	19			0
Export taxes	-1		1			0
Capital taxes	-13		13			0
Labor taxes	-4		4			0
Return to capital	-144			144		0
Wage rate	-163			163		0
Price of Armington composite		414	-36	-292	-86	0
Savings			-36	-51	86	0
Transfers			-35	35		0
Chksum:	0	0	0	0	0	

Read the Data from Excel

```
$title  Read a microconsistent matrix for a 123 model

set     mcmrow  Rows in the micro-consistent matrix /
        PFX     Current account,        PD      Domestic ouputput
        TA      Sales and excise taxes, TM      Import tariffs
        TX      Export taxes,           TK      Capital taxes
        TL      Labor taxes,            RK      Return to capital
        PL      Wage rate,              PA      Price of Armington composite /,

        mcmcol  Columns in the micro-consistent matrix /
        S       Supply,                 D       Demand,
        GOVT    Government,             HH      Households
        INVEST  Investment /;

parameter       mcm(mcmrow,mcmcol)      Microconsistent matrix;

$call gdxxrw i=123data.xlsx o=123data.gdx par=mcm rdim=1 cdim=1 rng=mcm checkdat
$gdxin 123data.gdx
$loaddc mcm
```

Elasticities

```
parameter
        etadx   Elassticity of transformation (D versus X) /4/,
        sigmadm Elasticity of substitution (D versus M) /4/,
        esubkl  Elasticity of substitution (K versus L) /1/,
        sigma   Elasticity of substitution (C versus LS) /0.4/,
```

Benchmark Data

```
        d0      Reference domestic supply
        x0      Reference exports
        kd0     Reference net capital earnings
        ld0     Reference net labor earnings
        tx      Tax on exports,
        tk      Capital tax rate
        tl      Labor tax rate,
        px0     Reference price of exports,
        rk0     Reference price of capital,
        pl0     Reference wage;

d0  = mcm("pd","s");
x0  = mcm("pfx","s");
kd0 = -mcm("rk","s");
ld0 = -mcm("pl","s");
tx  = -mcm("tx","s")/mcm("pfx","s");
tk  = mcm("tk","s")/mcm("rk","s");
tl  = mcm("tl","s")/mcm("pl","s");
px0 = 1 - tx;
rk0 = 1 + tk;
pl0 = 1 + tl;
```

Additional Parameters

```
parameters
        m0      Imports,
        tm      Import tariff rate,
        pm0     Reference price of imports,
        a0      Aggregate supply (gross of tax),
        g0      Government demand,
        ta      Excise and sales tax rate,

        bopdef  Balance of payments deficit,
        dtax    Direct tax net transfers,
        i0      Aggregate investment;

m0 = -mcm("pfx","d");
tm = mcm("tm","d")/mcm("pfx","d");
pm0 = 1 + tm;
a0 = mcm("pa","d");
g0 = -mcm("pa","govt");
ta = -mcm("ta","d")/mcm("pa","d");
bopdef = mcm("pfx","govt");
dtax = g0 - bopdef - tm*m0 - ta*a0 - tl*ld0 - tk*kd0 - tx*x0;
i0 = -mcm("pa","invest");
```

Calibration Closure: Leisure Demand

```
parameter
        l0      Leisure demand
        c0      Household consumption;

c0 = a0 - i0 - g0;
l0 = 0.75*ld0;
```

Benchmark Value Shares

```
parameter thetal  Labor share in cost function,
          thetah  Consumption share in expenditure function,
          thetam  Share parameter in Armington function
          thetaz  Share parameter in transformation function  ;

thetal = ld0*pl0 /(kd0*rk0+ld0*pl0);
thetaz = x0 *px0 /(d0+x0*px0);
thetam = m0 *pm0 /(d0+m0*pm0);
thetah = c0/(c0+l0);
```

Variable Declarations

```
NONNEGATIVE VARIABLES

*$SECTORS:
        Y        Production
        A        Armington composite
        M        Imports
        X        Exports

*$COMMODITIES:
        PD       Domestic price index
        PX       Export price index
        PM       Import price index
        PA       Armington price index
        PL       Wage rate index
        RK       Rental price index
        PFX      Foreign exchange

*$CONSUMERS:
        HH       Private households
        GOVT     Government

*$AUXILIARY:
        TAU      Replacement tax;
```

Compensated Factor Demand

```
$macro PKL   [({PL*(1+tl)/pl0}**thetal * \
               {RK*(1+tk)/rk0}**(1-thetal))$(esubkl eq 1) + \

             ({thetal * {PL*(1+tl)/pl0}**(1-esubkl) \
          + (1-thetal) * {RK*(1+tk)/rk0}**(1-esubkl) \
                                  }**{1/(1-esubkl)})$(esubkl ne 1)]

$macro LD   (ld0     * (PKL*pl0/{PL*(1+tl)})**esubkl)
$macro KD   (kd0     * (PKL*rk0/{RK*(1+tk)})**esubkl)
```

Compensated Supply: Domestic-Export

```
$macro PY   ({thetaz * {PX*(1-tx)/px0}**(1+etadx) + \
           (1-thetaz) * PD**(1+etadx)} **{1/(1+etadx)})

$macro DY   (d0      * (PD/PY)**etadx)

$macro XY   (x0      * (PX*(1-tx)/{px0*PY})**etadx)
```

Compensated Demand: Domestic-Import

```
$macro PDM  ((thetam *(PM*(1+tm)/pm0)**(1-sigmadm) +  \
            (1-thetam)*PD**(1-sigmadm))**(1/(1-sigmadm)))

$macro DA   (d0      *(PDM/PD)**sigmadm)

$macro MA   (m0      *(PDM*pm0/{PM*(1+tm)})**sigmadm)
```

Uncompensated Demand: Goods and Leisure

```
$macro PH   ({thetah *PA**(1-sigma) + \
            (1-thetah)*PL**(1-sigma)}**{1/(1-sigma)})

$macro C    (c0      *(PH/PA)**sigma *1/PH *HH/{c0+l0})

$macro L    (l0      *(PH/PL)**sigma *1/PH *HH/{c0+l0})
```

Arbitrage Conditions

```
profity..  PKL*(ld0*pl0 + kd0*rk0) =g= PY*(d0+x0*px0);

profita..  PDM*(m0*pm0 + d0) =e= PA*a0*(1-ta);

profitm..  PFX*pwm =e= PM;

profitx..  PX =e= PFX*pwx;
```

Market Clearance Conditions

```
marketd..  Y*DY =e= A*DA;

marketa..  A*a0 =g= GOVT/PA + C + i0;

marketm..  M*m0 =e= A*MA;

marketx..  Y*XY =e= X*x0;

marketfx.. X*pwx*x0 - M*pwm*m0 =e= -bopdef ;

marketk..  kd0    =e= Y*KD;

marketl..  ld0+l0 =e= Y*LD + L;
```

Budget Equations

```
incomeg..  GOVT =e= PFX*bopdef + PA*dtax +  PA*g0*TAU
                  + tx*PX*XY*Y + tk*RK*KD*Y + tl*PL*LD*Y
                  + tm*PM*MA*A + ta*PA*a0*A;

incomeh..  HH =e= PL*(ld0+l0) - PA*dtax - PA*g0*TAU + RK*kd0 - PA*i0  ;
```

Equal Yield Constraint and Model Declaration

```
taudef..   GOVT =e= PA * g0;


model mcp123 /  marketd.PD, marketa.PA, marketm.PM, marketx.PX,
                marketfx.PFX, marketk.RK, marketl.PL,
                profity.Y,  profita.A,  profitm.M,  profitx.X,
                incomeg.GOVT, incomeh.HH, taudef.TAU/;
```