



Nancy currently lives in Madison where she spends a third of her income on housing, half her income on groceries and all the remaining money on opera tickets in Chicago. She loves opera, and she has a lifetime membership at the Madison Overature Center where she is able to (freely) attend every opera which is staged, as many as she chooses to attend in Chicago. Nancy now considers a job offer in London, England. Housing in London is four times as expensive as in Madison, opera tickets are the same price as in Chicago and other goods are twice as expensive as in Madison. What is the minimum salary offer which would induce Nancy to move?

Consider the production function which combines labor (ℓ) and capital (k) with a constant elasticity of substitution equal to σ to produce output (y):

$$y = f(k, \ell) = \bar{y} \left(\theta \left(\frac{k}{\bar{k}} \right)^{\rho} + (1 - \theta) \left(\frac{\ell}{\bar{\ell}} \right)^{\rho} \right)^{1/\rho}$$

in which $\rho = 1 - 1/\sigma$ and $\theta = \bar{k}/(\bar{k} + \bar{\ell})$. Suppose that the market wage and the capital input are fixed (i.e., $p_{\ell} = 1$ and $k = \bar{k}$).

Define the price elasticity of supply as:

$$\eta = \left. \frac{\partial y}{\partial p} \frac{p}{y} \right|_{p=1}$$

Show that when the benchmark value shares of labor and capital are equal the price elasticity of supply equals the elasticity of substitution, i.e. $\eta = \sigma$.