Applied Risk in Electricity Markets

AAE 706: Applied Risk Analysis
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Background

Record cold temperatures plunged Texas into a power crisis in 2021, with millions in the state losing power. The failure demonstrates the vulnerability of power grids to shifting weather patterns that come with climate change. However, the situation in Texas was made more complex by the fact that it is the only state in the country with its own power grid, the Electric Reliability Council of Texas (ERCOT). Two other grids, the Eastern Interconnection and Western Interconnection, both administered by the Federal Energy Regulatory Commision (FERC), cover the rest of the United States.

In 2013 Texas system operators, under consultation with Harvard Professor Bill Hogan, undertook an innovation in market design to provide better pricing under increasing scarcity conditions. This scarcity pricing framework has been important in the intervening years and was a major reason for the high prices during the recent crisis. There are a number of operational details regarding the implementation of this pricing system, but the crucial idea is that in times of scarcity prices reflect the network conditions and provide incentives to increase supply or lower demand.

Last month's widespread winter storm produced low temperatures across the region. This is an unusual event, much worse than the worst case that was considered in recent prior planning. The result was loss of power plants, loss of natural gas supply, felled power transmission lines, damaged gas pipelines, damage to water systems, and so on, across the board. There was a loss of more than 50 percent of generation capacity at the same moment as electric power demand surged above the predicted peak forecast.

Power systems are designed to share across the transmission grid in order to provide nearly instantaneous support to one part when another location is in trouble. Response of the Texas network to this shock was affected by the lack of connection between ERCOT and FERC.

University of Texas Professor James K. Galbraith is skeptical about Professor Hogan's network design. His view is as follows:

> In 2002, under Governor Rick Perry, Texas deregulated its electricity system and established a free market, managed by a non-profit entity, with roughly 70 providers. But while the new system worked most of the time, people need electricity all the time.

> Harvard Kennedy School's William Hogan is credited with designing the Texas energy market. As Texans froze and their water pipes burst, he reportedly remarked that the state's energy market has functioned as designed.
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> Hogan is right, which says a lot about how some economists think.
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> ...
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> Enter the deep freeze of 2021. Water vapor in the natural gas froze at the wells, in pipes, and at generating plants. Non-weatherized windmills went offline, but they were a small part of the story. Because the Texas grid is disconnected from the rest of the country, no reserves could be imported; and, given the cold everywhere, none would have been available anyway. In the small hours of February 15, demand so outstripped supply that the entire grid reportedly came within minutes of a meltdown.
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> As this happened, the price mechanism failed completely. Wholesale prices rose a hundred-fold - but retail prices, under contract, did not rise that much, except for customers of Griddy, who got socked with bills for thousands of dollars a day. Demand rose as supply collapsed.

In this assignment we will not attempt a conclusive assessment of the factors contributing to the Texas disruption. Indeed, we will ignore several potentially important aspects of the network (e.g., the use of rolling blackouts to equilibrate supply and demand — instead assume that demand and supply are solely equilibrated through wholesale prices in each network). We will not attempt to calibrate to details of the network structure aside from assuming that the FERC and ERCOT networks are currently unconnected.

Features of the Model

We formulate a small numerical model to help us understand how risk aversion and market structure may influence the prospects of future disruptions. Specifically, we build a model in which it is possible to evaluate the impact of hedging behavior by both consumers and firms. We are primarily interested in two questions:

i. How will changes in the probabilities of of extreme events affect market conditions in the future?

ii. How might market outcomes be affected by integration of the ERCOT and FERC networks?

Key elements of our toy model are:

- Two regions: `ferc`, `ercot`
- Three states of world: `summer`, `winter`, `vortex`
- Risk averse consumers who spend their money on electricity and other goods. That is, the representative consumer in region r maximizes the utility of electric and non-electric consumption. Their utility function incorporates a parameter representing the shifts in the

state-dependent demand for electricity (λ_{rs}). The consumption aggregate of electricity use (E_{rs}) and other goods (D_{rs}) is given by:

$$C_{rs} = C(E_{rs}, D_{rs}) = \left(\theta \left(\frac{E_{rs}}{\bar{E}\lambda_{rs}} \right)^{\delta} + (1-\theta) \left(\frac{D_{rs}}{\bar{D}} \right)^{\delta} \right)^{1/\delta}$$

in which $\bar{E}$ and $\bar{D}$ are anchor points – reference demands for electricity and other goods when $\lambda = 1$ and relative prices are equal. We normalize the benchmark consumption index C to unity.[1].

Consumer *utility* is given by:

$$U_{rs} = \frac{C_{rs}^{\rho}}{\rho}$$

Substitution exponent δ in consumption aggregate C is related to ϵ (the elasticity of substitution between electricity and other goods – `esub`) as

$$\delta = 1 - 1/\epsilon,$$

and the exponent ρ in the utility function is related to σ (the degree of relative risk aversion) as[2]

$$\rho = 1 - 1/\sigma.$$

Demand shocks differ by region and weather outcome (λ_{rs}):

	summer	winter	vortex
ferc	1	1.2	1.2
ercot	1	1	1.5

The value of utility function parameter θ is such that *in both regions* at benchmark prices ($P_E = P_D = 1$) the demand for electricity (in money-metric units) is 1 and the demand for other goods equals 30, i.e. $\theta = \frac{1}{31}$. The price of other goods is invariant ($P_D = 1$) whereas the price of electricity reflects scarcity by region and state of the world.

[1]We adopt "Harberger units" here. $\bar{E}$ is the value of electricity demand at reference prices and $\bar{D}$ is expenditure on all other goods. Benchmark prices of both goods and then equal to one.

[2]Share parameter θ in the consumption aggregate is then equal to the value share of electricity at the reference point:

$$\theta = \frac{\bar{E}}{\bar{E} + \bar{D}}$$

.

Given consumption C_{rs} and electricity price p^E_{rs} the compensated consumer demands for electricity and other goods are then given by:

$$E_{rs} = C_{rs}\lambda\bar{E}\left(\frac{p^C_{rs}}{p^E_{rs}}\right)^{\epsilon}$$

and

$$D_{rs} = C_{rs}\bar{D}(p^C_{rs})^{\epsilon}$$

where p^C is the price index for a unit of consumption:

$$p^C_{rs} = \left(\theta(\lambda_{rs} p^E_{rs})^{1-\epsilon} + 1 - \theta\right)^{1/(1-\epsilon)}$$

- Expected utility depends on the subjective probability of the three states of the world (π_s). The conventional form of the expected utilioty function would be:

$$EU_r = \sum_s \pi_s \frac{C_{rs}^{\rho}}{\rho}$$

  A monotonic transformation permits us to expressed expected utility in money-metric terms:

$$EU_r = \left(\sum_s \pi_s C_{rs}^{\rho}\right)^{1/\rho}$$

  in which as noted above, ρ is related to the degree of relative risk aversion as shown above.

  We assume that prior to last month's disruption consumers and producers assumed state probabilities $(\bar{\pi}_s)$ equal to 300/365 for `summer`, 64/365 for `winter` and 1/365 for `vortex`. We calibrate our model with a "business as usual calculation" and we the investiate the consequences of a revision in subjective probabilities given by 55/365 for `winter` and 10/365 for `vortex` (i.e., a ten-fold increase in the likelihood of an extreme weather event).

- The annual household budget constraint for electricity consumers is region r is:

$$Y_r = \sum_s \pi_s \left(D_{rs} + p_{rs}^E E_{rs}\right)$$

  We assume that the representative household expenditure is $Y_r = 31$ and is unaffected by weather.

- Generation of electricity in region r, state s depends on installed capacity (K_r), weather outcome (w_{rs}) and winterization maintenance (M_r)

$$E_{rs} = K_r\,(1 - w_{rs}(1 - M_r))$$

  in which w_{rs} is the power loss associated with weather in the absence of winterization, and M_r is an index of the level of winterization undertaken $(0 \le M_r < 1)$.

  We assume weather shocks are given by:

	summer	winter	vortex
ferc	0	0.2	0.25
ercot	0	0	0.25

- The marginal cost of winterization increases monotonically from α to infinity as the level of maintenance increases from 0 to 1:

$$\mu_r = \frac{\alpha}{(1 - M_r)^{\gamma}}$$

  Maintenance cost parameters α and γ are calibrated to the marginal cost = marginal revenue arbitrage conditionin a baseline simulation with $M_{\text{ercot}} = 0.4$ and $M_{\text{ferc}} = 0.8$.

$$\gamma = \frac{\log\left(\sum_s \pi_s P^E_{\texttt{ercot},s} w_{\texttt{ercot},s}\right) - \log\left(\sum_s \pi_s P^E_{\texttt{ferc},s} w_{\texttt{ferc},s}\right)}{\log\left(1-M_{\texttt{ferc}}\right) - \log(1-M_{\texttt{ercot}})}$$

and

$$\alpha = (1-M_{\texttt{ercot}})^{\gamma} \sum_s \pi_s P^E_{\texttt{ercot},s} w_{\texttt{ercot},s}$$

Equilibrium Conditions

- Supply-demand conditions apply in each region and state of the world. Electricity supply in region r, state s depends on generation capacity (K_r), weather (w_{rs}), and maintenance (M_r). Demand depends on the state-specific consumption index (C_{rs}), the demand shock (λ_{rs}) and relative prices of aggregate consumption (P^C_{rs}) and electricity (P^E_{rs}), and net electricity exports (X_{rs})[3]:

$$K_r(1-w_{rs}(1-M_r) = C_{rs}\lambda_{rs}\left(\frac{P^C_{rs}}{P^E_{rs}\lambda_{rs}}\right)^{\epsilon} + X_{rs}$$

- The supply of generation capacity in a long-run equilibrium is determined by an decreasing returns to scale production function. Profit-maximizing supply solves:

$$\max p^K_r K_r - \xi_r$$

s.t.

$$K_r = \phi_r \sqrt{\xi_r \bar{R}_r}$$

in which ϕ_r is a calibrated scale parameter, ξ_r is variable investment expenditures and $\bar{R}_r$ is a resource input to generation capacity (i.e., regional wind, solar, natural gas resources).

The solution to this profit maximization problem can be represented by a first-order condition relating generation capacity to the rental rate on the electricity resource endowment:

$$\frac{K_r}{P^R_r} = 1$$

In a short-run equilibrium we hold generating capacity fixed and generation only varies through changes in winterization maintenance. In the long run changes in the return to generation infrastructure lead to increases or decreases in generation.

- The shadow price on sector-specific generation determines the cost of capacity supply, and the level of output by risk-netral profit maximizing firms then equates this cost with net revenue:

$$\sqrt{P^R_r} = \sum_s \pi_s P^E_{rs}(1-w_{rs}(1-M_r))$$

[3]In the benchmark equilibrium the ERCOT and FERC networks are unconnected and $X_{rs} = 0$

- Arbitrage conditions for risk-neutral firms determines the level of winterization maintenance through equation of marginal cost and marginal revenue:

$$\mu_r = \frac{\alpha}{(1-M_r)^\gamma} = \sum_s \pi_s P_{rs}^E w_{rs}$$

- Expenditure function – consumer price index – for region r in state s is

$$P_{rs}^C = \left(\theta \left(P_{rs}^E \lambda_{rs}\right)^{1-\sigma} + 1 - \theta\right)^{1/(1-\sigma)}$$

- Price index for expected utility:

$$P_r^{EU} = \left(\sum_s \pi_s (P_{rs}^C)^{1-\sigma}\right)^{1/(1-\sigma)}$$

- Given the price of a unit of expected utility, the income constraint determines the affordable quantity:

$$EU_r = \frac{1}{P_r^{EU}}$$