Homework 2: Expected Utility

AAE 706: Applied Risk Analysis
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The material covered by this problem set includes Chapter 2 of the Chavas textbook and Chapter 12 of Varian's Intermediate Microeconomics textbook and Chapter 11 of Varian's Microeconomic Analysis. See the course website for submission instructions.

1. **Farm Risk Hedging**

   A farmer's utility function for money gains and losses is approximately represented by $U(X) = 2X - 0.01X^2$, $(X \leq 100)$, where X denotes farm profit (in thousands of dollars). The farmer is currently wondering whether to spend more on fertilizer for his 1000 ha farm than last season's \$4/ha. Pertinent information is shown in the following matrix of possible dollar profits per hectare.

Type of Season	Probability	Spend \$4/ha	Spend \$8/ha	Spend \$12/ha	Spend \$16/ha
Poor	0.1	-8	-12	-16	-20
Fair	0.2	-2	-8	-12	-16
Good	0.5	2	4	6	8
Excellent	0.2	12	20	24	26

   (a) How much should the farmer spend on fertilizer?

   (b) What is the risk premium associated with his optimal decision? Interpret.

2. **Bidding Risk**

   A construction company does subcontracting on government contracts. The construction company's utility function is approximately represented by $U(X) = 2X - 0.01X^2$, $(X \leq 100)$, X being income (in thousands of dollars).

   (a) Suppose the company is considering bidding on a contract. Preparation of a bid would cost \$8000, and this would be lost if the bid failed. If the bid succeeded, the company would make \$40,000 gain. The company judges the chance of a successful bid as 0.3. What should it do?

   (b) What chance of a successful bid would make the company indifferent between bidding and not bidding for the contract? Discuss.

3. **Insider Trading**

   In the next week, the Senate is going to decide whether or not to vote on impeachment of a past president. If the president is convicted, it will be very profitable for Clinton Resorts. Indeed, if the conviction goes through, the value of stock in Clinton Resorts will rise from \$10 per share to \$15 a share, and if the impeachment vote fails, the value of the stock will fall to \$5 a share. In his capacity

as a messenger for Congressman Kickback, Buzz Condor has discovered that the impeachment conviction is much more likely than is generally thought. On the basis of what he knows, Condor has decided that the probability that the system will be approved is 3/4 and the probability that it will not be approved is 1/4. Let c_A be Condor's consumption if the system is approved and c_{NA} be his consumption if the system is not approved. Condor's von Neumann-Morgenstern utility function is $U(c_A, c_{NA}) = .75\log(c_A) + 0.25\log(c_{NA})$. Condor's total wealth is \$50,000, all of which is invested in perfectly safe assets. Condor is about to buy stock in Clinton Resorts.

Buzz Condor has no moral qualms about trading on inside information, nor does he have any concern that he will be caught and punished. To decide how much stock to buy, he simply maximizes his von Neumann-Morgenstern utility function subject to his budget. How much stock does he buy? Show your work.

4. **Clarence and Hjalmer**

Clarence Bunsen has preferences among contingent commodity bundles c_i with subjective probabilities π_i which can be represented by the utility function $u(c, pi) = \left(\sum_i \pi_i \frac{1}{\sqrt{c_i}}\right)^{-2}$.

Clarence's friend, Hjalmer Ingqvist, has offered to bet him \$1,000 on the outcome of the toss of a fair coin. If the coin comes up heads, Clarence must pay Hjalmer \$1,000 and if the coin comes up tails, Hjalmer must pay \$ x to Clarence. If he doesn't accept the bet, Clarence will have \$10,000 with certainty.

(a) Show that Clarence is an expected utility maximizer.

(b) At what value of x would Clarence be indifferent between taking the bet or not taking the bet?

(c) Suppose that if the coin comes up heads Clarence must pay Hjalmer \$ x and if tail, Hjalmer pays Clarence \$1,000. At what value of x would Clarent be indifferent between taking the bet or not taking the bet?