AAE 706:
The Allias Paradox and Producer Risk

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The Allais Paradox

The Allais Paradox, discovered by Maurice Allais, provides an example in decision theory of preferences that violate the most widely accepted normative theory of decision making, expected utility theory. What is the paradox, how does it vioate explains expected utility, and what are reasonable responses to the paradox.

The Innocuous Axioms of Choice

A1) **Completeness**: $\forall \mathbf{x}, \mathbf{y} \in \mathcal{X}$, $\mathbf{x} \succcurlyeq \mathbf{y}$ or $\mathbf{y} \succcurlyeq \mathbf{x}$. That is, among all pairs of the choices, either the first is weakly preferred to the second or the second is weakly preferred to the first, or both.

A2) **Transitivity**: $\forall \mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathcal{X}$, $\mathbf{x} \succcurlyeq \mathbf{y}$ and $\mathbf{y} \succcurlyeq \mathbf{z} \Rightarrow \mathbf{x} \succcurlyeq \mathbf{z}$. That is, if **x** is weakly preferred to **y**, and **y** is weakly preferred to **z,** then **x** must be weakly preferred to **z**.

A3c) **Archimedean**: $\forall P, Q, R \in \mathcal{P}$ if $P \succ Q \succ R$, then there exists π_1, π_2, $\in (0,1)$ and a unique $\pi^* \in (0, 1)$ such that

$$\pi_1 P(\mathbf{x}) + (1 - \pi_1) R(\mathbf{x}) \succ Q(\mathbf{x}) \succ \pi_2 P(\mathbf{x}) + (1 - \pi_2) R(\mathbf{x})$$
$$\text{and} \qquad \pi^* P(\mathbf{x}) + (1 - \pi^*) R(\mathbf{x}) \sim Q(\mathbf{x}) \,.$$

The Independence Axiom

A4) **Independence:** $\forall P, Q, R \in \mathcal{P}$ and $\forall \pi \in [0,1)$,

$$P(\mathbf{x}) \succ Q(\mathbf{x}) \quad \Leftrightarrow \quad \pi P(\mathbf{x}) + (1-\pi) R(\mathbf{x}) \succ \pi Q(\mathbf{x}) + (1-\pi) R(\mathbf{x})$$

$$P(\mathbf{x}) \sim Q(\mathbf{x}) \quad \Leftrightarrow \quad \pi P(\mathbf{x}) + (1-\pi) R(\mathbf{x}) \sim \pi Q(\mathbf{x}) + (1-\pi) R(\mathbf{x}) .$$

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The Independence Axiom

Of the four axioms underlying EUT, the Independence Axiom (A4) is the one that has sparked the most debate. A violation of this axiom seems to be the usual culprit when utility maximization fails to describe real-world applications. In fact, choices often still violate the axiom even after the discrepancy is pointed out to those making the choices and who profess that the axiom seems reasonable. Therefore, it is important to understand this axiom in detail. Mathematically, what the Independence axiom requires is that expected utility is linear in the probabilities; that is, that choices are based on an expected value of something.

The Paradox

Consider a decision maker choosing between lotteries, i.e., probability distributions over outcomes. According to expected utility theory, as long as the decision maker is rational, her preferences can be represented by a utility function of outcomes with the property that of any two lotteries she prefers the lottery with the higher expected utility value. The idea that decision makers maximize expected utility - the "expected utility hypothesis" - was put forth in part to account for the fact that many decision makers are risk averse in the sense that they would rather have, for example, a sure-thing amount of money than a lottery with the same average monetary value.

Lotteries A vs. B and C vs. D

The standard experiment demonstrating violation of the independence axiom is the Allais Paradox. This experiment has been repeated many times. The one reported here was run by Kahnemann and Tversky (1979) in the late 1970s. Seventy-two people were asked to choose between lotteries A and B[21]

$$A : \begin{cases} 33\% & \text{chance of} \quad 2500 \\ 66\% & \text{chance of} \quad 2400 \\ 1\% & \text{chance of} \quad 0 \end{cases} \quad \text{or} \quad B : \; 100\% \;\; \text{chance of } 2400 \tag{54}$$

and also between lotteries C and D.

$$C : \begin{cases} 33\% & \text{chance of} \quad 2500 \\ 67\% & \text{chance of} \quad 0 \end{cases} \quad \text{or} \quad D : \begin{cases} 34\% & \text{chance of} \quad 2400 \\ 66\% & \text{chance of} \quad 0 \end{cases} \tag{55}$$

The majority of them (61%) choose the sure thing B over the risky A and choose C over D. Either of those choices is fine on its own; however, both choices together violate the independence axiom.

The violation is usually demonstrated as follows. Mentally think of lotteries A and B as

$$A: \begin{cases} 33\% & \text{chance of} & 2500 \\ 1\% & \text{chance of} & 0 \\ 66\% & \text{chance of} & 2400 \end{cases} \quad \text{or} \quad B: \begin{cases} 34\% & \text{chance of} & 2400 \\ & & \\ 66\% & \text{chance of} & 2400 \end{cases}$$

In each lottery, there is a 66% chance of winning 2400. So the preference for B over A must be due to a preference for the 34% chance at 2400 over the 33% chance at 2500 plus a 1% chance at nothing.

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Now mentally think of lotteries C and D as

$$C: \begin{cases} 33\% & \text{chance of} & 2500 \\ 1\% & \text{chance of} & 0 \\ 66\% & \text{chance of} & 0 \end{cases} \quad \text{or} \quad D: \begin{cases} 34\% & \text{chance of} & 2400 \\ \\ 66\% & \text{chance of} & 0 \end{cases}$$

This time, each lottery has a 66% chance of 0, but otherwise C matches A and D matches B. So a preference for C over D directly contradicts the previous choice if the Independence Axiom holds.[22]

Algebraic Inconsistency

The analysis above uses the splitting to show in a simple fashion that the violation is indeed of the Independence Axiom rather than one of the other axioms. However, it is not required to demonstrate that the choices are inconsistent with EUT. If lotteries B and C are preferred to A and D, then EUT says that the utility function must satisfy

$$\begin{aligned} B \succ A \quad &\Leftrightarrow \quad u(2400) > 0.33u(2500) + 0.66u(2400) + 0.01u(0) \\ &\Leftrightarrow \quad 0 > 0.33u(2500) - 0.34u(2400) + 0.01u(0) \\ C \succ D \quad &\Leftrightarrow \quad 0.33u(2500) + 0.67u(0) > 0.34u(2400) + 0.66u(0) \\ &\Leftrightarrow \quad 0 < 0.33u(2500) - 0.34u(2400) + 0.01u(0)\,. \end{aligned} \tag{58}$$

This is an obvious contradiction. The two relations in (58) cannot be true for any utility function whether or not it is risk averse. In fact they cannot be true even for utility functions that are not increasing.

The Machina Triangle

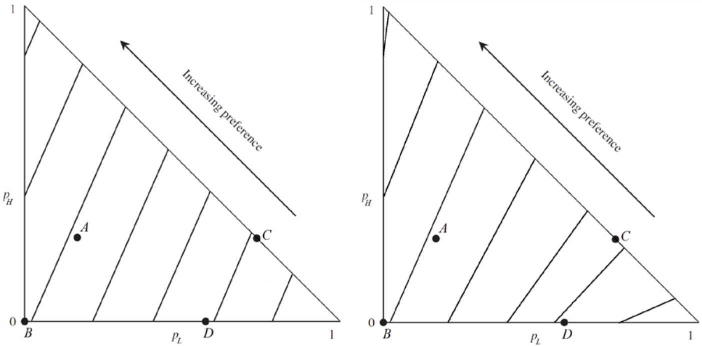
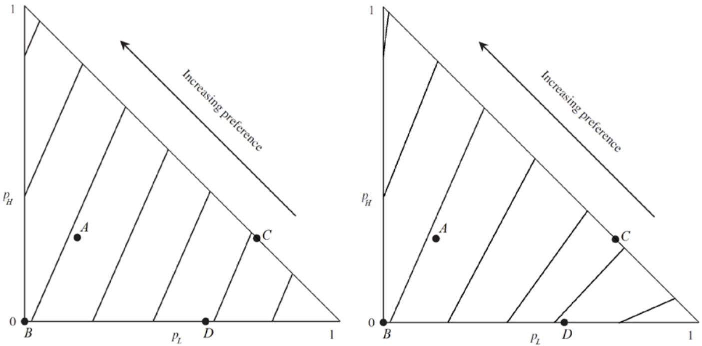


Figure 1.3 Machina Triangle Illustrating the Allais Paradox

This figure illustrates the Allais Paradox with Machina's Triangle. Under Expected Utility Theory, the indifference curves must be parallel straight lines so if *B* is preferred to *A*, then *D* must be preferred to *C*. If the indifference curves are not straight lines, then the common preferences are possible.



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Indifference curves are those points with the same expected utility, u°

$$\mathbb{E}[\tilde{u}] = u^\circ = p_L u_L + (1 - p_L - p_H) u_M + p_H u_H = u_M + (u_L - u_M) p_L + (u_H - u_M) p_H$$

$$\Rightarrow p_H = \frac{u^\circ - u_M}{u_H - u_M} + \frac{u_M - u_L}{u_H - u_M} p_L . \tag{59}$$

Axioms of Choice

While such behavior is not consistent with maximizing expected monetary value, it is consistent with maximizing expected utility, relative to a concave utility function of money. The link between a decision maker's preferences and her utility is cemented by "representation theorems": these theorems show that being (representable as) an expected utility maximizer is equivalent to having preferences that satisfy particular axioms. One of the earliest and most influential axiomatizations is that of John von Neumann and Oskar Morgenstern. The axioms of von Neumann and Morgenstern's theorem - and those employed in representation theorems in general - seem to many to be requirements of rational preferences.

The Allais Paradox is a counterexample to the expected utility hypothesis.

Responses

Broadly speaking, there are two ways to approach decision theory:

- as an analysis of the canons of instrumental rationality (*normative* decision theory), or
- as a description of actual people's preferences (*descriptive* decision theory)

For *normative* decision theorists, the standard choices in Allais's example are *paradoxical* in that seem rational to many people and yet they violate the dictates of the expected utility theory, which seems to correctly spell out the requirements of rationality.

For *descriptive* decision theorists, the Allais choices aren't so much paradoxical as they are a counterexample to the idea that expected utility theory is the correct descriptive theory.

Responses to the Paradox

Descriptive theorists have responded to the paradox by formulating alternative theories that are compatible with the Allais choices.

For normative decision theorists, there are three ways to respond to the paradox:

i. Claim that contrary to initial appearances, the Allais choices are simply irrational: although many people unreflectively have the standard Allais preferences, once an individual sees that her preferences violate the Independence Axiom, she ought to re-evaluate her preferences and bring them in line with expected utility theory.

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ii. Claim that contrary to initial appearances, the Allais choices do satisfy the expected utility hypothesis, and the apparent conflict is due to the fact that the choices have been under-described in the initial setup: the actual outcomes in the problem are not simply monetary amounts but also include the decision maker's feelings about getting those outcomes *in each situation*. Free to adjust the returns, we might claim that the irrational Allais choices maximize expected utility, they do not violate expected utility after all.

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iii. Finally, normative decision theorists might respond to the paradox by denying the expected utility hypothesis and arguing that expected utility theory is inadequate as a theory of rationality.

Price Uncertainty (Sandmo)

Under the EUH, consider the objective function of a competitive firm to be

$$EU(w + py - r^T x) = EU(w + \pi)$$

where y denotes output sold at price p, x is a vector of inputs purchased at prices r, $(r^T x)$ being the cost of production, $f(x)$ is the production function, w denotes either initial wealth, the negative of fixed cost, or exogenous income, and $\pi = (py - r^T x)$ is firm profit.

We will assume that the entrepreneur / decision maker has risk preferences represented by the utility function $U(w + \pi)$ which satisfies $U' = \frac{\partial U}{\partial w} > 0$ and $U'' = \frac{\partial^2 U}{\partial w^2} < 0$ (implying risk aversion).

Timing

Assume that, because of production lags, output price p is not known at the time of the production decisions. Thus, p is a random variable with some given subjective probability distribution. Let $\mu = E(p)$, and $p = \mu + \sigma e$, where e is a random variable with mean zero. We will characterize the probability distribution of p by the mean μ and the mean preserving spread parameter σ.

Under the EUH, the production decisions can be represented by

$$\max_{x,y} \left\{ EU(w + py - r^T x : y = f(x) \right\}$$

1. The firm minimizes cost

To see that, note that the above maximization problem can be written as

$$\max_y \left\{ \max_x \left[EU(w + py - r'x) : y = f(x) \right] \right\} = max_y \left\{ EU(w + py - C(r, y)) \right\}$$

where $C(r, y) = [\min_x r'x : y = f(x)]$ is the cost function in a standard cost minimization problem under certainty. (Production input decisions are made prior to the resolution of uncertainty.)

Choice of y

The first-order necessary condition associated with the choice of y is:[1]

$$E\left[U'(p - \frac{\partial C(r,y)}{\partial y})\right] = 0$$

$$E[U']E[p - C'] + Cov[U', p] = 0$$

$$\mu - C' = \frac{Cov[U', p]}{E[U']} = 0$$

,
where $C' = \frac{\partial C}{\partial y}$ denotes the marginal cost, and $COV(U', p) = E[U'\sigma e]$.

[1] Given correlated random variables x and y, $E(xy) = E(x)E(y) + Cov(xy)$

Relation to the Risk Premium

Consider the Arrow-Pratt risk premium $R(w, y, .)$ defined to satisfy

$$EU(w + py - C(r, y)) = U(w + \mu y - C(r, y) - R),$$

with $R > 0$ under risk aversion. Given $U' > 0$, the objective function can be equivalently written in terms of its certainty equivalent:

$$max_y \{w + \mu y - C(r, y) - R(w, y, .)\}.$$

The associated first-order condition with respect to y is

$$\mu - C' - R' = 0,$$

where $R' = \frac{\partial R}{\partial y}$ is the marginal risk premium. Comparing this result with the first-order condition derived above, it follows that $R' = \frac{-Cov(U', p)}{EU'}$.

2. The Supply Function

The supply function is the function $y^*(w, \mu, \sigma)$ that satisfies the first-order condition

$$\mu = C' + R',$$

where $R' = \frac{-Cov(U', p)}{EU'}$ is the marginal risk premium. This implies that, at the optimum supply y^*, expected price μ is equal to marginal cost C' plus marginal risk premium R'.

But $Cov(U', p) = sign(\partial U'/\partial p) = sign(U" y) < 0$ under risk aversion. It follows that the marginal risk premium is positive under risk aversion: $R' > 0$. This in turn implies that $\mu > C'$.

Thus, at the optimum, a risk averse decision maker would choose output such that expected price exceeds marginal cost. This shows that, under price risk and risk aversion, the standard marginal cost pricing rule does not apply.

3. Comparative Statics

Let $\alpha = (w, \mu, \sigma)$ be a vector of parameters of the supply function $y^*(\alpha)$. Differentiate the first order condition $F(y, \alpha) = 0$ at the optimum $y = y^*(\alpha)$ yields

$$\frac{\partial F}{\partial \alpha} + \frac{\partial F}{\partial y}\frac{\partial y^*}{\partial \alpha} = 0$$

or, with $D = \partial F / \partial y < 0$,

$$\frac{\partial y}{\partial \alpha} = -D^{-1}\frac{\partial F}{\partial \alpha} = \mathrm{sign}\left\{\frac{\partial EU'(p - C')}{\partial \alpha}\right\}$$

Initial Wealth

$$\frac{\partial y^*}{\partial w} = -D^{-1}\frac{\partial [EU'(p - C')]}{\partial w} = -D^{-1}EU''(p - C').$$

$E[U''(p - C')]$ is > 0 under decreasing absolute risk aversion (DARA), and as a consequence it can be shown that in this case we have $\frac{\partial y^*}{\partial w} > 0$.
Under DARA , changing initial wealth, fixed cost or exogenous income (w) influences supply.
Increasing initial wealth (e.g., from income transfer) would stimulate supply. At the aggregate level, a higher supply would put downward pressure on the price p (assuming a downward-sloping demand curve). The associated decline in p would make consumers better off. Such effects would not exist under certainty.

Effect of the Expected Price

$$\frac{\partial y^*}{\partial \mu} = -D^{-1}\frac{\partial[EU'(p-C')]}{\partial \mu} = -D^{-1}\{EU' + yEU"(p-C')\}.$$

Let $\frac{\partial y^c}{\partial \mu} = -D^{-1}[EU'] > 0$ denote the compensated expected price effect. We have just shown that $\frac{\partial y^*}{\partial w} = -D^{-1}E[U"(p-C')]$ This generates the following "Slutsky type" relationship

$$\frac{\partial y^*}{\partial \mu} = \frac{\partial y^c}{\partial \mu} + \left(\frac{\partial y^*}{\partial w}\right) y^*,$$

where the expected (uncompensated) price slope $\partial y^* / \partial \mu$, is equal to the compensated price slope $\partial y^c / \partial \mu > 0$, plus a wealth (or income) effect, $\frac{\partial y^*}{\partial w} y^*$.

We have also shown that $\frac{\partial y^*}{\partial w} > 0$ under DARA. A positive wealth effect is a sufficient condition for a positive uncompensated price slope, $\frac{\partial y^*}{\partial \mu} > 0$.


GIG
ECONOMY
EMPLOYMENT GENERATION
FLEXIBILITY
ABILITY
AMBIGUOUS
WORKFORCE
ALTERNATIVE
MEANS
INDEPENDENT
DIRECTLY
BEGUN
FORCED
THING
WHATEVER
COMMON
WORK-LIFE
AROUND
CONTINGENT
ENVIRONMENT
TEMPORARY
MILLENNIAL
WORK
SO-CALLED
REALITY
WORKERS
JOBS
PIECEWORK
POSITION
CONTRIBUTED
HEALTH
DECOUPLED
LOCATION
PRESSURES
STAFF
SAFETY
MOBILE APP
CONTRACT
INTUIT
LEADING
AMONG
STUDY
DECREASE
IDEALLY
DIGITAL

The “Gig Economy”

- The gig economy is based on flexible, temporary, or freelance jobs, often involving connecting with clients or customers through an online platform.
- The gig economy can benefit workers, businesses, and consumers by making work more adaptable to the needs of the moment and demand for flexible lifestyles.
- At the same time, the gig economy can have downsides due to the erosion of traditional economic relationships between workers, businesses, and clients.

Firm Decisions in the Gig Economy

Consider the market for a single commodity. Production involves inputs of entrepreneurial effort (R), capital (K), firm employees (E) and gig employee labor (L). The top level product function is Cobb-Douglas withing which we nest a constant elasticity of substitution aggregate of E and L.

$$Y_s = R^{1-\theta_K-\theta_\ell} \left(\frac{K}{\bar{K}}\right)^{\theta_K} \ell^{\theta_\ell}$$

in which

$$\ell = \left(\alpha \left(\frac{E}{\bar{E}}\right)^{\gamma} + (1-\alpha)\left(\frac{L}{\bar{L}}\right)^{\gamma}\right)^{1/\gamma}$$

Entrepreneurial Effort and Returns to Scale

Assuming $R = 1$ commodity outputs are then solely a function of capital and labor. If we were to include entrepreneurial inputs as a decision variable we would write:

$$Y_s = \left(\frac{R}{1}\right)^{1-\theta_K-\theta_\ell} \left(\frac{K}{\bar{K}}\right)^{\theta_K} \ell_s^{\theta_\ell}$$

When we fix $R = 1$, the resource input can be suppressed. Output as a function of capital and labor then exhibits decreasing returns to scale, as evidenced by $\theta_K + \theta_\ell < 1$.

Firm Employees and Gig Employees

When the elasticity of substition in the CES nest $(\mathrm{ESUB} = \frac{1}{1-\gamma})$ is less than one, then E and L_s are gross complements. When $\mathrm{ESUB} > 1$, these labor inputs are gross substitutes. The value of ESUB plays a crucial role in determining whether changes in the cost of finding gig employees increases or decreases the number of permanent employees.
Entrepreneurial, capital and employee inputs are the same in all states whereas gig employment varies by state. Calibrating to reference levels of employment $(\bar{E})$ and gig sector labor $(\bar{L})$ then index of labor inputs is given by:

$$\ell_s = \left(\alpha\left(\frac{E}{\bar{E}}\right)^{\gamma} + (1-\alpha)\left(\frac{L_s}{\bar{L}}\right)^{\gamma}\right)^{1/\gamma}$$

State-Contingent Output

When $E = \bar{E}$, $L_s = \bar{L}$ the index of labor inputs equals unity. In state s decisions K and L are fixed, and the only choice variable for the firm is L_s. Output in state s is then:

$$Y_s = \left(\frac{K}{\bar{K}}\right)^{\theta_K} \ell_s^{\theta_\ell}$$

Firm profit in state s depends on the output price, the cost of capital and employees (p_K and p_E) and the price of temporary labor (ω):

$$\pi_s = P_s Y_s - p_K K - p_E E - \omega_s L_s$$

Terminal Wealth

Terminal wealth in state s can be expressed as an index of entering wealth ($w = 1$), current profits (π_s) and long-run average profits ($\bar{\pi}$):

$$X_s = \frac{w + \pi_s}{w + \bar{\pi}}$$

Expected Utility

We assume that the firm owner is a risk averse expected utility maximizer. If the price of output in each state (P_s) is taken as exogenous, the firm is competitive and the optimal response can be calculated as a non-linear program:

$$\max_{K,E,L_s} EU = \left(\sum_s \frac{X_s^\rho}{|S|} \right)^{1/\rho}$$

Calibration of Risk Attitude

One simple way to characterize risk attitude is through the marginal utility of profit. When $X_s = 1$, the marginal value of an extra dollar of profit can be scaled to one by defining:

$$v(\pi) = \left(\frac{X(\pi)}{\bar{X}}\right)^{\rho-1} = \left(\frac{w+\pi}{w+\bar{\pi}}\right)^{-\sigma}$$

We let initial wealth equal the value of reference output ($\bar{y} = 1$) and assume a value for the marginal utility of provide with half the benchmark profit is given by: $\bar{v} = v(\bar{\pi}/2)$. Given these assumption we can compute

$$\sigma = \frac{\log(1+\bar{\pi}) - \log(1+\bar{\pi}/2)}{\log(\bar{v})}$$

GAMS Code for Inputs

```
$title    Gig Economy Model

parameter
        vref                Marginal value of profit at reference profit /2/
        piref               Reference profit calibrating sigma /0.25/

        esub                Elasticity of substitution E vs L /4/

        thetak              Value share capital /0.3/,
        thetaE              Value share of employees /0.3/,
        thetaL              Value share of hired labor /0.2/;

set     s                   States of the world /s1*s50/;

parameter
        p(s)                Output price in state s
        wage                Wage of gig workers
        sigma_p                     Variance of p /0.5/;
```

GAMS Model Declarations

```
variables
        EU                  Expected utility,
        K(s)                  Capital stock,
        E(s)                  Full time employees,
        KK                  Capital stock (stochastic model)
        EE                  Employment (stochastic model)
        L(s)                  Gig employees

        X(s)                  Ending wealth by state,
        PI(s)                  Profit (or social surplus) by state,
        Y(s)                  Supply = demand by state,
        EL(s)                  Labor-employee nest;

equations       eudef, eldef, ydef, pidef, naK, naE, xdef;

model gigeconomy /all/;
```

GAMS Model Equation Definitions

```
eudef..          EU =e= sum(s, (1/card(s)) * X(s)**rho)**(1/rho);

eldef(s)..       EL(s) =e= (thetaE/(thetaE+thetaL) * (E(s)/e0)**gamma +

                           thetaL/(thetaE+thetaL) * (L(s)/L0)**gamma)**(1/g

ydef(s)..        Y(s) =e= (K(s)/k0)**thetak * EL(s)**(thetaE+thetaL);

pidef(s)..       PI(s) =e= p(s) * Y(s)  - (K(s) + E(s) + wage*L(s));

xdef(s)..        X(s) =e= (1+PI(s))/(1+pi0);

*    Include non-anticipativity constraints in the stochastic model:

naK(s)$stochastic..    K(s) =e= KK;

naE(s)$stochastic..    E(s) =e= EE;
```