Numerical Methods

Math 3338 – Spring 2022

Homework 19

Graph Eigenvectors

In Problems 1 – 5 we're going to explore the Laplacian matrix L. This is defined as $L = D - A$ where D is the diagonal matrix with entries given by the degree of each vertex and A is the adjacency matrix.

Problem 1 (1 pt) This is not a computer question. Verify that $\vec{1}$ is an eigenvector of the Lapacian matrix with eigenvalue 0. This is a proof you'll type in LaTeX.

Problem 2 (1 pt) Here is another proof. Suppose G has k connected components. Prove the eigenvalue 0 has multiplicity k in the Lapacian.

Problem 3 (1 pt) The Lagrangian is a positive semidefinite matrix. This means all the eigenvalues should be greater than or equal to zero. Verify this (you don't need to prove it, just computationally).

Problem 4 (1 pt) Drawing graphs is extremely challenging. As long as the vertices and edges are correct, you can draw the graph however you like. Luckily, the eigenvectors of the Laplacian can help. Take the eigenvectors of the two smallest non-zero eigenvalues and use the coordinates as the (x, y) values of the vertices. This embedding is "minimal" in some sense. Write a program to draw graphs in this manner.

Problem 5 (1 pt)

The file `all_graphs.dat` contains a database of all graphs with no isolated vertices with less than or equal to 6 vertices. The file `all_graphs.pdf` is a drawing of each graph.

Use the program from the previous problem to redraw all the graphs. You should automate the creation of the LaTeX file. (I did). `matplotlib.pyplot` will be useful here. Essentially you'll be placing points and lines.