

Numerical Methods

Math 3338 – Spring 2022

Homework 13 (Due: Thursday, March 3)

Problem 1 (1 pt) Draw a picture (like with your hand) of a situation where Newton's method will fail where it cycles x values. Your picture should be moderately accurate. Figure out a way to make this a PDF to submit on Canvas. Name the image problem_1.pdf.

Problem 2 (1 pt) Draw a picture of a situation where Newton's method will converge to a root, but not the root that it's closest to.

Problem 3 (1 pt) Modify your `newton` function from last time to be more robust. Add the following two improvements,

1. Check if the derivative is 0. If it is, return `None`
2. Add a `max_iter` argument that defaults to 50. If the number of loops is larger than `max_iter`, return `None`.

Problem 4 (1 pt) The goal of this problem is to explore the basins of convergence. Let $f(x) = x^5 - 4x^3 + 2x - 1$, do the following,

1. Let x be a linspace from -2 to 2 using N points.
2. Create a `subplots` object with 5 rows (this is 2 plus the number of roots of $f(x)$.)
3. In the first row plot $f(x)$ together with the line $y = 0$.
4. For each point in x , determine which 0 (if any) the point converges to. Store this in a list or array. If it doesn't converge to a zero, store that too.
5. On each subsequent row, graph the function and $y = 0$ and the points for each zero as “.”.
6. Save the figure as a pdf.

You'll probably have to mess with the `figsize` in `subplots` to get everything to display nicely.

Submit the figure on Canvas, and describe what you see and what you find surprising. Name the PDF problem_4.pdf.

Problem 5 (1 pt) Repeat the previous problem except with,

$$f(x) = x^8 - 2x^7 - x^6 + 12x^5 - 21x^4 - 18x^3 + 41x^2 + 8x - 22$$

I recommend a domain of $[-2.05, 2.1]$. Submit the figure on Canvas, and describe what you see and what you find surprising. Name the image problem_5.pdf.