Numerical Methods

Math 3338 – Spring 2022

Worksheet 23

Animations and Chaos

1 Reading

CP	8.1, 8.2
NMEP	Chapter 7

Table 1: Sections Covered

2 Overview

Today we're going to understand the double pendulum and learn how to make an animation showing it's motion.

3 Double Pendulum

A double pendulum is a pendulum on the end of a pendulum. Figure 1 has an example of a double pendulum. This is a classic physics problem as the motion of the second pendulum is chaotic.

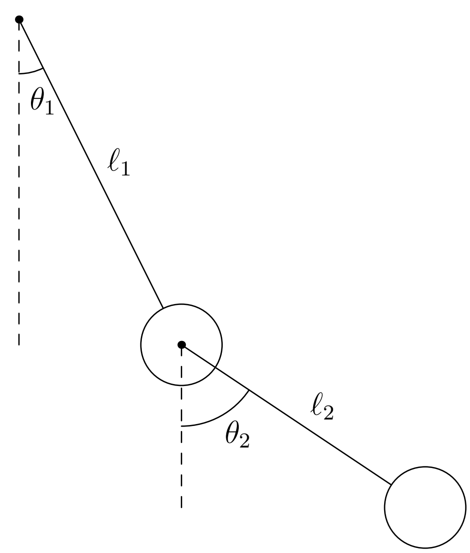

Figure 1: Double Pendulum

The equations to describe the motion are... a bit much. So we won't derive them. Before you see them, we need some notation. Dot notation is a nice simplification to Leibniz notation.

$$\frac{dx}{dt} = \dot{x} \qquad\qquad \frac{d^2x}{dt^2} = \ddot{x}$$

Each dot represents of derivative with respect to time.

We are going to assume the lengths of the rods are the same and the masses of the pendulums are the same.[1] Here are the equations.

$$2\ddot{\theta}_1 + \ddot{\theta}_2\cos(\theta_1-\theta_2) + \dot{\theta}_2^2\sin(\theta_1-\theta_2) + 2\frac{g}{\ell}\sin(\theta_1) = 0$$
$$\ddot{\theta}_2 + \ddot{\theta}_1\cos(\theta_1-\theta_2) - \dot{\theta}_1^2\sin(\theta_1-\theta_2) + \frac{g}{\ell}\sin(\theta_2) = 0$$

However these aren't actually what we solve. We actually solve the following,

$$\dot{\theta}_1 = \omega_1$$
$$\dot{\theta}_2 = \omega_2$$
$$\dot{\omega}_1 = -\frac{\omega_1^2\sin(2\theta_1-2\theta_2) + 2\omega_2^2\sin(\theta_1-\theta_2) + \frac{g}{l}(\sin(\theta_1-2\theta_2)+3\sin(\theta_1))}{3-\cos(2\theta_1-2\theta_2)}$$
$$\dot{\omega}_2 = \frac{4\omega_1^2\sin(\theta_1-\theta_2) + \omega_2^2\sin(2\theta_1-2\theta_2) + 2\frac{g}{l}(\sin(2\theta_1-\theta_2)-\sin(\theta_2))}{3-\cos(2\theta_1-2\theta_2)}$$

4 Animation

The method I'm about to describe requires FFMPEG which is an encoder algorithm. This *should* be easy to install on a Mac and is possible on a Windows computer. Alternatively, you can use a different method to create the animation[2].

At it's core, an animation is just a sequence of graphs displayed over time. We need to tell the computer what the first graph is, and the following graphs. There is an example on Canvas that should help you get started making animations.

[1]If you don't think we should, be my guest to rederive the equations
[2]This could be as easy as changing the file extension. But mp4 is generally the "best"

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Homework 23 (Due: Tuesday, April 12)

Problem 1 (1 pt) Write a function that solves the double pendulum. Your inputs should be the two initial angles, the time domain and the number of steps. You should return a tuple (θ_1, θ_2). We'll assume $g = 10$ and $\ell = 1$.

Problem 2 (1 pt) Make a graph of θ_1 vs. t and θ_2 vs t on the same axis. The initial conditions are $\theta_1 = \theta_2 = \frac{\pi}{2}$. The domain is $0 \leq t \leq 50$ with 1,000 steps. Save this graph as a PDF and include in a tex file. Explain what this graph is saying.

Problem 3 (1 pt) Make a graphs of the position of each pendulum as time progresses. This will be y vs x. The domain will be $0 \leq t \leq 50$ with 1,000 steps. Make a different plot for each initial condition.

1. $\theta_1 = \theta_2 = \frac{\pi}{2}$
2. $\theta_1 = \pi$, $\theta_2 = \frac{\pi}{2}$
3. $\theta_1 = \theta_2 = \pi$

Include these in a PDF and explain what each graph is saying.

Note: One of your curves should be a circle representing the inner pendulum.

Problem 4 (1 pt) Make an animation of the motion of this system for $1 \leq t \leq 50$, 1,000 steps, and initial conditions $\theta_1 = \pi$ and $\theta_2 = \frac{\pi}{2}$. Your plot should include the following,

1. A fixed axes, probably $-2.2 \leq x, y \leq 2.2$.
2. The pendulums. You can do this with two lines, one from the origin and the other from the end of the first.
3. The path the inner pendulum traces as it's tracing it.
4. The path the outer pendulum traces as it's tracing it.

Call this **double-1000.mp4**.

Problem 5 (1 pt) Repeat the previous with 500 steps instead of 1,000.

Problem 6 (1 pt) If you have the computation time, redo with 5,000 steps. This takes about 5 - 15 minutes on my laptop.

Problem 7 (1 pt) The three animations you created describe the exact same system, but they aren't perfectly the same. Why?