Numerical Methods

Math 3338 – Spring 2022

Homework 15 (Due: Tuesday, March 8)

Basins of Convergence

Problem 1 (1 pt) Did you know Newton's method works for complex zeros as well? Try it, set $f(x) = x^5 - 4x^3 + 2x - 1$ and run Newton's method with $x_0 = 1j$ (this is the i in Python). For this problem we are going to enhance the images we made last time, this time we'll work over the complex plane. To do this, do the following.

1. Since we're dealing with floats, create a `fuzzy_equal` function that returns True if the difference between two numbers is "small" ($< 10^{-6}$) and False otherwise.
2. Create a function `find_zeros` that will... find the zeros of a function. To do this search a 10×10 grid from $(a, b) \times (a, b)$, these are complex numbers. You should return a list containing the zeros. In general, this may not find all the zeros.
3. Create another function `find_basins`. This funtion will return an $N \times N$ grid of integers which correspond to the index of that root in the zero list. To create this, I recommend starting with an $N \times N$ grid of complex numbers, using those to find the convergence root, and then finding the index in the list.
4. Use `plt.imshow` to plot the result of `find_basins`. Save your figure as a PDF. You'll submit it on Canvas.

Problem 2 (1 pt) Use the previous problem to create a graph of the convergence of your ID polynomial. Suppose your ID number is 1234567, your ID polynomial is,

$$f(x) = x^6 + 2x^5 + 3x^4 + 4x^3 + 5x^2 + 6x + 7.$$