Numerical Methods

Math 3338 – Spring 2022

Homework 8 (Due: Thursday, February 10)

Applications of Integration

Create a file called "name_integrals.py", where name is your name (or initials or something short). In this file put the functions `latex_table`, `trapezoidal`, `trap_error`, and `romberg`. Modify the last two to only return the value of the integral (not the number of steps).

Problem 1 (1 pt) Not all integrals are over a finite domain. For example, our methods will fail if we want to evaluate $\int_0^\infty f(x)\,dx$. This is solved using a change of variables. We want a function, $F(x)$, defined on the interval (a, b) so that $\lim_{n\to a} F(x) = 0$ and $\lim_{n\to b} F(x) = \infty$. One common function is,

$$F(x) = \frac{x}{1-x}$$

on the interval $(0, 1)$. The transformation is then,

$$\int_0^1 f(F(t))F'(t)\,dt = \int_0^1 f\left(\frac{t}{1-t}\right)\cdot\frac{1}{(t-1)^2}\,dt.$$

Another function, that may be better, is,

$$F(x) = \tan(x).$$

This has the benefit of being able to transform the domain $(-\infty, \infty)$.

The *normal distribution* is given by,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}.$$

Where μ is the mean and σ is the standard deviation. This is typically standardized so that $\mu = 0$ and $\sigma = 1$. We'll assume $f(x)$ is the standardized curve.

1. Verify that $\displaystyle\int_{-\infty}^{\infty} f(x) = 1$.
2. In statistics it's important to be able to evaluate $\int_{-\infty}^{a} f(x)\,dx$. To do this they use a table, see Canvas for an example (normal.pdf). This table shows all values of that integral for $-3.49 \le a \le 3.49$ in steps of .01. Remake this table accurate to 6 decimals.

Problem 2 (1 pt) the Gamma function is a highly important function in Mathematics, Physics, Chemistry, and Statistics. The Gamma function is defined for $x > 0$,

$$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}\,dt$$

1. Write a function to evaluate $\Gamma(x)$.
2. Make a table evaluating $\Gamma(x)$ for integers $i = 1, 2, \ldots, 30$.
3. You should recognize the numbers you just computed. What are they? You may need to increase your tolerance to see this.
4. Plot both $\Gamma(x)$ and $\Gamma(x) + \sin(\pi x)$ on the same axes. Save this figure and include it in the PDF. You can see they intersect, where do you think they intersect?