Numerical Methods

Math 3338 – Spring 2022

Worksheet 22

Nonlinear Differential Equations

1 Reading

CP	8.1, 8.2
NMEP	Chapter 7

Table 1: Sections Covered

2 Overview

Believe it or not, not every thing is a first order differential equation. For example, suppose you have a function y so that $\frac{d^2y}{dt^2} = -y$. We can easily solve this $y = A\sin(t)$, but there are more general problems that are difficult to solve.

The process for these equations is to write them as a system of first order differential equations, and then solve the system. For example, suppose

$$\frac{d^2y}{dt^2} - 3y\frac{dy}{dt} - ty = 1$$

We transform this by letting a dummy variable equal the first derivative of y. Our system becomes,

$$\frac{dy}{dt} = \omega$$
$$\frac{d\omega}{dt} = 3y\omega + ty + 1$$

Where $\frac{d^2y}{dt^2} = \frac{d\omega}{dt}$ and we solved for $\frac{d\omega}{dt}$.

Then you solve this as a system of differential equations ¯_(ツ)_/¯

Numerical Methods

Math 3338 – Spring 2022

Homework 22 (Due: Thursday, April 7)

Problem 1 (1 pt) A pendulum is a weight of mass m at the end of a massless rod of length ℓ. Figure 1 shows a pendulum with a free body diagram. There is one missing force, F_T or the the force due to the

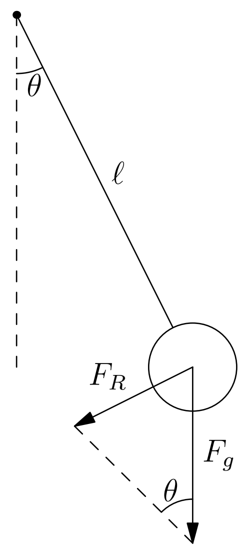

Figure 1: Pendulum!

tension in the rod. Due to physics, the sum of the forces on the mass should be 0. The only forces on the mass are F_T and F_R, the torque and restoring force. Therefore, our differential equation is given by,

$$m\ell\frac{d^2\theta}{dt} + mg\sin(\theta) = 0.$$

Or, for simplification,

$$\frac{d^2\theta}{dt} = -\frac{g}{\ell}\sin(\theta).$$

We'll assume $g = 10$ and $\ell = 1$.

Use the Runge-Kutta method to solve this system for $0 \leq t \leq 10$ and the following initial conditions,

1. $\theta_0 = \frac{\pi}{3}$, $\frac{d\theta}{dt}|_{t=0} = 0$
2. $\theta_0 = \frac{\pi}{2}$, $\frac{d\theta}{dt}|_{t=0} = 0$
3. $\theta_0 = 0$, $\frac{d\theta}{dt}|_{t=0} = \frac{\pi}{2}$
4. $\theta_0 = \pi$, $\frac{d\theta}{dt}|_{t=0} = 0$

For each initial condition, make the following graphs,

1. A plot of θ vs t
2. A plot of y vs x, the position of the pendulum. Make sure the boundaries of this graph are a square with side length 2.2, otherwise the motion won't look right.

Describe what you see in each graph and briefly why it's happening.