Aerodynamics Presentation

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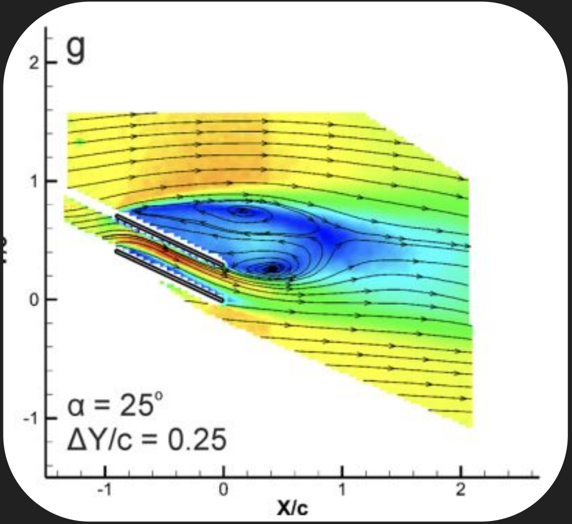
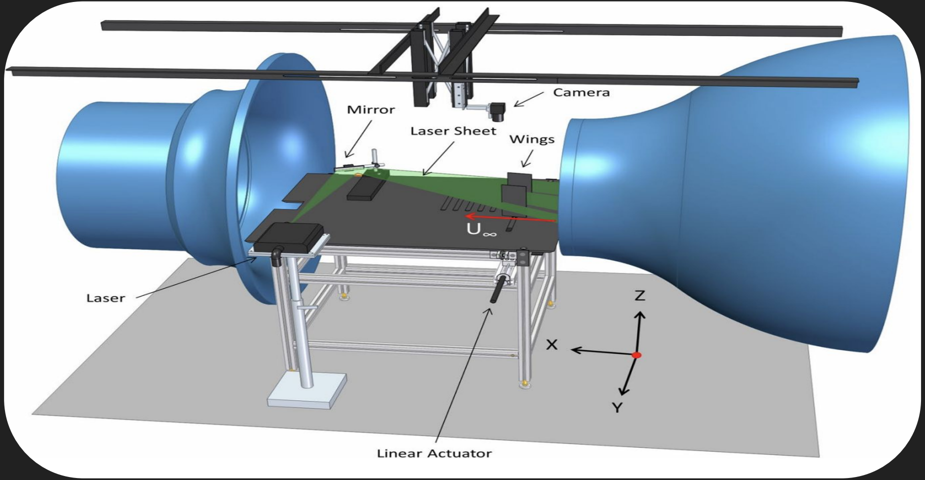
“Aerodynamics of biplanes and tandem wings at low Reynolds number”

- Motivation: improve lift & stall behavior at low Reynolds number (100 000) for MAV
- Tested biplane & tandem wings (flat plates) with varying gap / stagger
- **Two-wings worse at small angles but much better post-stall**
- Lift increases up to **~30%** and stall delayed by **10–15°**
- Flow imaging showed **five different interaction modes**

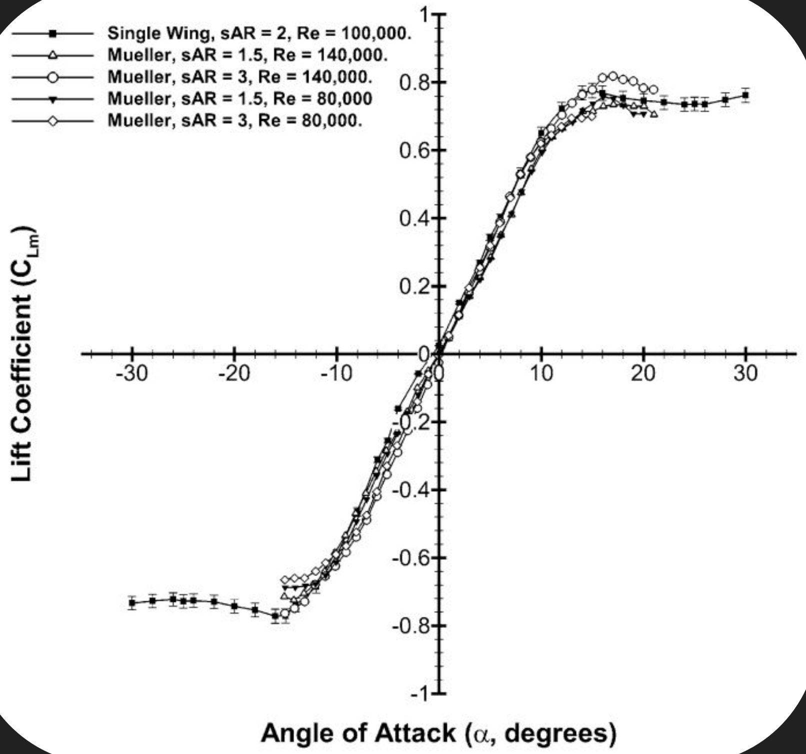




Replicated figures:

Figure 3:

- Lift coefficients for a single wing as function of the angle of attack (flat plate, sAR = 2)
- Re = 100,000.
- Measurements are compared with Mueller (2000), and Pelletier and Mueller (2000)



What it shows:

- Lift is linear at low angles of attack
- Stall occurs early (~15°)
- After stall, lift plateaus while drag rises sharply

Figure 3 - Thin Airfoil Theory, Modeling and Challenges

- Airfoil shape extrapolated from the zone without recirculation
- Challenging: different shape for α = 10°, 15°, 20°, 25°, 30°

$$y_c(x) = \frac{y_u(x) + y_l(x)}{2}$$

$$A_0 = \frac{1}{\pi}\int_0^{\pi} \frac{dy_c}{dx}\, d\theta$$

$$A_n = \frac{2}{\pi}\int_0^{\pi} \frac{dy_c}{dx} \cos(n\theta)\, d\theta$$

$$C_l = 2\pi\alpha + \pi(A_1 - 2A_0)$$

$$\alpha_{L0} = A_0 - \frac{A_1}{2}$$

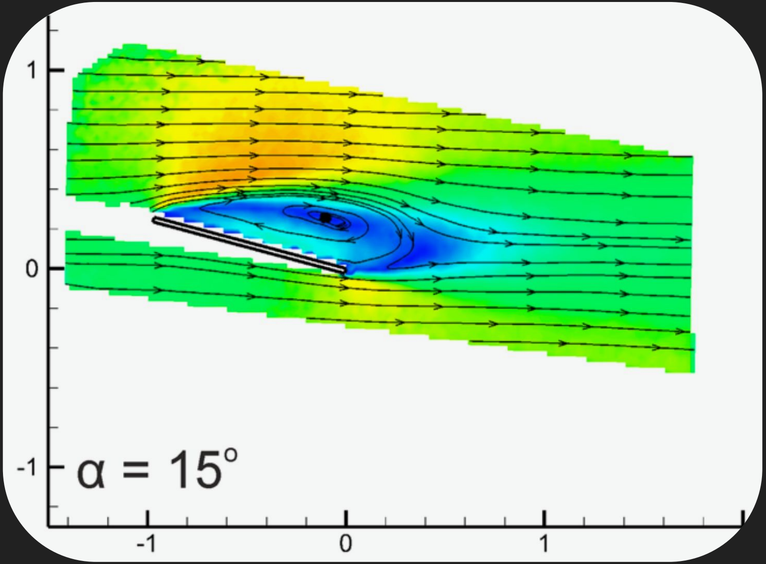
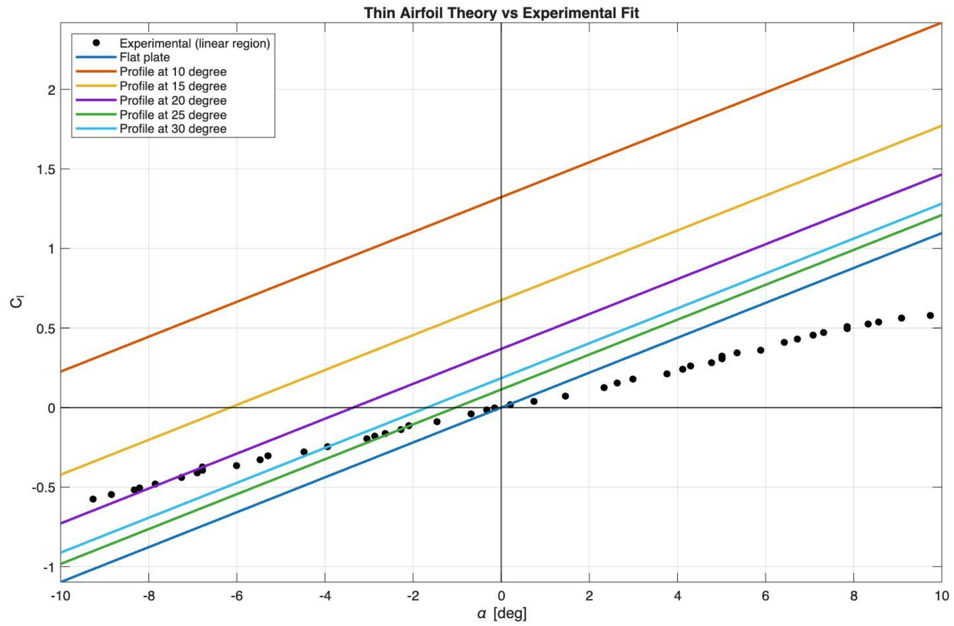


Figure 3 - Thin Airfoil Theory

- Slope always 2π
- As the chamber increases, the angle for which CL = 0 becomes increasingly negative.



Airfoil shape:	α_L0 (deg)
Flat plate	0.000
Extracted at 10°	-12.060
Extracted at 15°	-6.151
Extracted at 20°	-3.360
Extracted at 25°	-1.037
Extracted at 30°	-1.686

Figure 3 - Prandtl lifting line, Modeling and Challenges

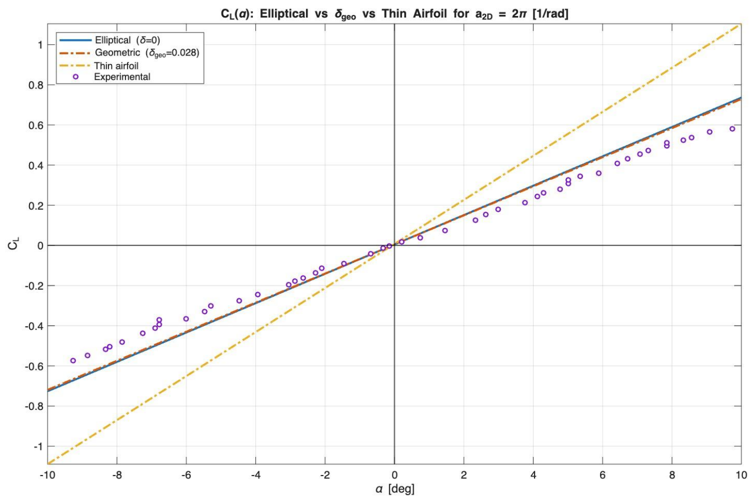
- With the lengths of chord obtained previously
- Varying the 2D slope
- Prandtl's integral formula and factor of ellipticity

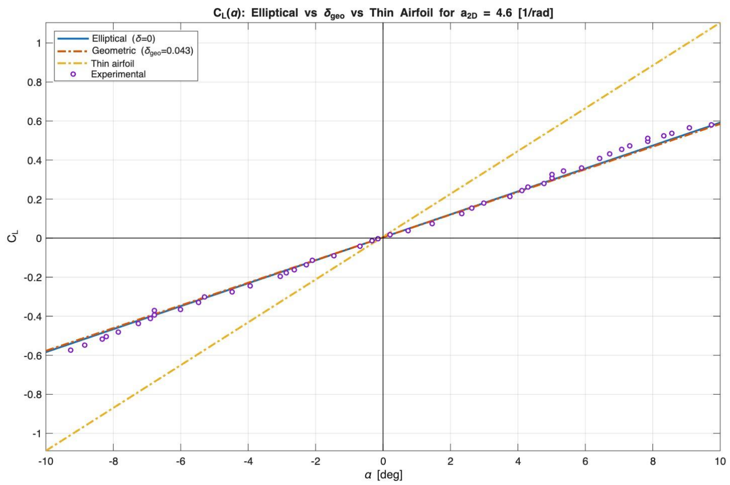
$$\frac{ca_{2D}}{4b}(\alpha - \alpha_0) = \sum_{n=1}^{\infty} A_n \sin(n\theta)\left(1 + \frac{nca_{2D}}{4b\sin\theta}\right)$$

$$\delta = \sum_{n=2}^{\infty} n\left(\frac{A_n}{A_1}\right)^2$$

$$a = \frac{a_{2D}}{1 + \frac{a_{2D}}{\pi AR}(1+\delta)}$$

Figure 3 - Prandtl lifting line





Model	a_{2D} **[1/rad]**	δ	a_{3D} **[1/rad]**
Experimental data	–	–	3.523
Using $a_{2D} = 2\pi = 6.283$			
Effective δ from experiment	6.283	0.567	3.523
Elliptical lifting-line	6.283	0	4.189
Geometric δ with $c = 0.1$ m	6.283	0.0285	4.1494
Geometric δ (10° image)	6.283	0.0292	4.1485
Geometric δ (15° image)	6.283	0.0205	4.1604
Geometric δ (20° image)	6.283	0.0133	4.1702
Geometric δ (25° image)	6.283	0.0105	4.1742
Geometric δ (30° image)	6.283	0.0086	4.1768
Glauert (1959)	6.283	0.120	4.028

Model	a_{2D} **[1/rad]**	δ	a_{3D} **[1/rad]**
Experimental data	–	–	3.523
Using $a_{2D} = 4.6$ (low-Reynolds correction)			
Effective δ from experiment	4.6	-0.165	3.523
Elliptical lifting-line	4.6	0	3.367
Geometric δ with $c = 0.1$ m	4.6	0.0430	3.3290
Geometric δ (10° image)	4.6	0.0439	3.3282
Geometric δ (15° image)	4.6	0.0318	3.3389
Geometric δ (20° image)	4.6	0.0213	3.3482
Geometric δ (25° image)	4.6	0.0170	3.3521
Geometric δ (30° image)	4.6	0.0142	3.3546
Glauert (1959)	4.6	0.120	3.262

Limitations

Thin airfoil

- Good linear slope, but too optimistic
- Real slope = 0.5-0.6 ideal slope
- By adding a thickness, an optimistic angle of attack for CL = 0 is found

Prandtl

- Lower, more realistic slope due to 3D induced effects
- LLT completely ignores Re and viscous effects
- LLT assumes smooth circulation distribution
- Both theories fail to predict stall → no viscous/separation modelling

Figure 3 Conclusion

Tried different theories and improved parameters:

- Thin airfoil modeled 2D lift trends
- Prandtl Lifting line improved our result by including 3D effects
- Considered an a_2D to account for the low reynolds number

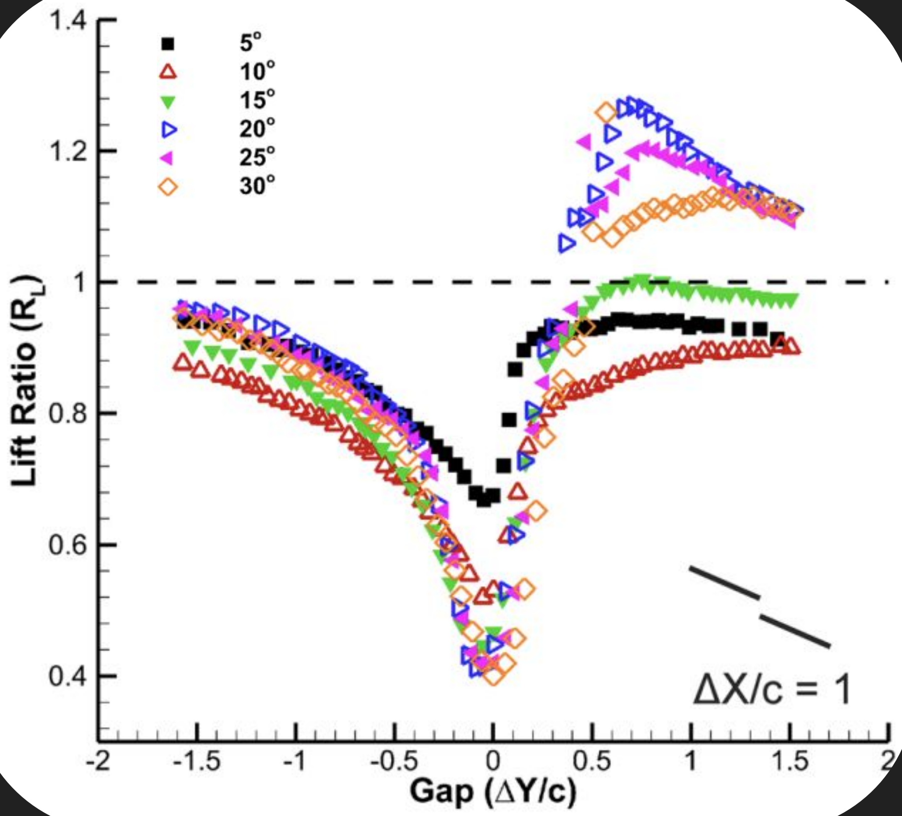
BUT

- These theories can only predict a trend in the linear region before separation
- Post stall effects can’t be observed due to the absence of viscous separation

Replicated figures:

Lift ratio: $$R_L = \frac{C_{Lt}}{C_{Lm}}$$

Figure 5. a:
Lift ratio as function of the gap between the wings at a stagger of one.



What it shows:

- two wings perform worse than one at low angle of attack
- But *much better* at high angles – up to ~30% more lift
- Stall is delayed significantly depending on spacing

Figure 5.a - Modeling and Challenges

Using Prandtl Lifting Line Theory:

1. Calculate Effective Angle of Attack due to Induced Velocities of one wing onto another
2. Calculated Total Circulation, integrating over 60 control points
3. Calculated Lift Coefficient
4. (Optional): Iterated to convergence

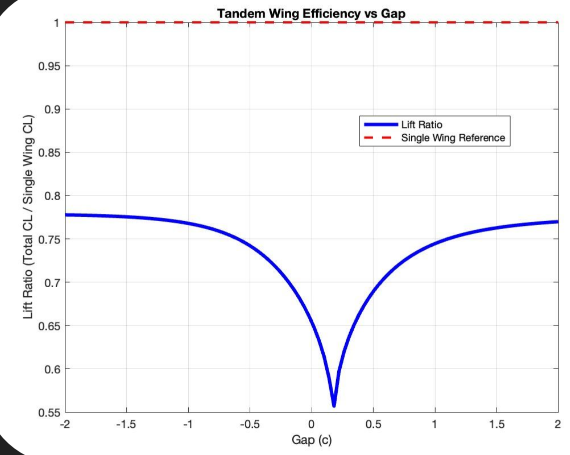
Challenges:

- Initially using Fourier Series Circulation
    - Simplified to Elliptical Distribution
        - Attempted to reiterate on Fourier Series + Circulation shape
- Convergence of the method
    - Added relaxation and modified integration methods

Figure 5.a - Prandtl lifting line

Results:

- Values ~0 Gap Similar
  - ~0.55 Lift Ratio
  - Similar Logarithmic Trend
- Values at further gaps diverge a lot
- Asymmetry:
  - Our model is “symmetrical” around ~0.25 gap instead of 0 gap



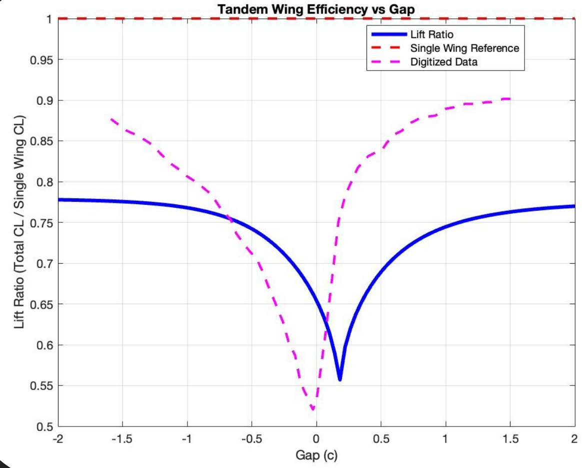
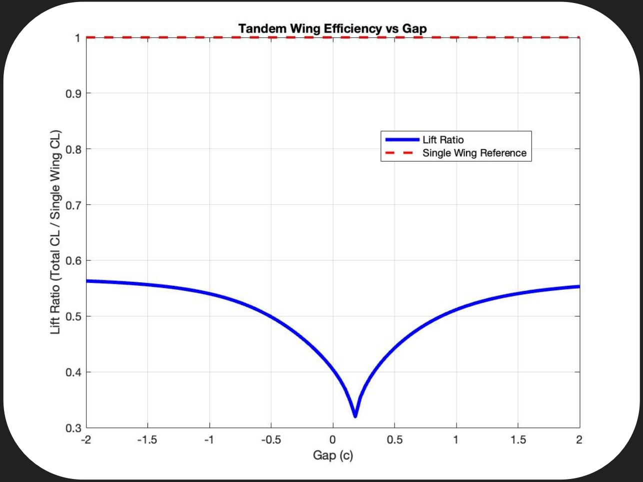


Figure 5.a - Prandtl lifting line

Simplifications/Choices:

- Elliptical Initial Distribution
  - Attempted to re-iterate on a Fourier Series, no convergence
  - Found circulation, iterated on a new initial guess: no variation (due to convergence)
- Convergence
  - Initially had “one” iteration
  - Attempted convergence of the method: yielded “damped” results
    - >1 iteration
- Simultaneous result would be better than convergence



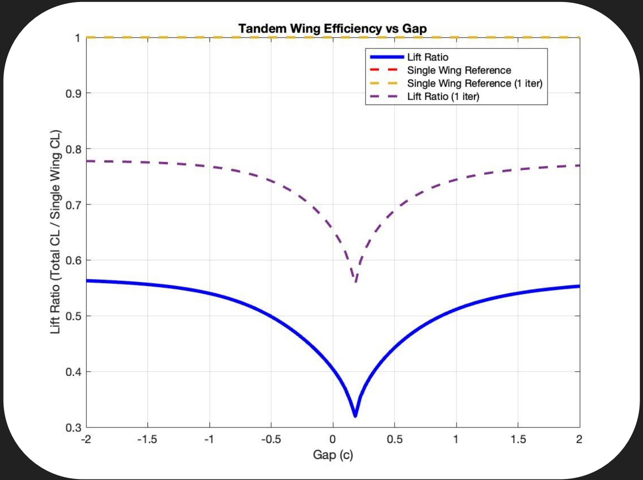


Figure 5 Conclusion

Learning:

- Better understanding of how Prandtl Lifting Line works, and its application to biplanes
- Significant improvement in MATLAB coding

Done Well:

- Attempted a range of solutions for different problems
- Changed theory

Done Better:

- Explore different "type" of model

Additional Point:

- Should we have kept the complicated theory, or was the elliptical simplification a good choice?

Sources:

Jones et al. - 2015 - Aerodynamics of biplane and tandem wings at low Reynolds numbers https://www.researchgate.net/publication/279169936_Aerodynamics_of_Biplane_and_Tandem_Wings_at_Low_Reynolds_Numbers

Cheng, Hao, Wang, Hua, Prediction of Lift Coefficient for Tandem Wing Configuration or Multiple-Lifting-Surface System Using Prandtl's Lifting-Line Theory, *International Journal of Aerospace Engineering*, 2018, 3104902, 15 pages, 2018. https://doi.org/10.1155/2018/3104902

Figliozzi, F., Mueller, T. J., Ol, M., & Kotapati, R. B. (2018). Lift curve slope variation at low Reynolds numbers. In AIAA Aviation 2018 Forum. American Institute of Aeronautics and Astronautics. https://doi.org/10.2514/6.2018-3615