ME C180, Finite Element Method. Assignment 6
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1. **Solution:**

For this problem we are again looking at a version of Poisson's equation. Since the velocity is already known, the right hand side is simply a forcing function. The formulation is as follows:

$$\nabla^2 \mathbf{P} = -\nabla \cdot (\mathbf{u} \cdot \nabla \mathbf{u}) \tag{1}$$

$$\nabla^2 \mathbf{P} - f = 0 \tag{2}$$

Again we can go through the initial derivation to obtain the weak form of this equation. From here we can use a combination of integration by parts in higher dimensions coupled with the divergence theorem. The divergence theorem is first shown below.

$$\int_\Omega \nabla \cdot \mathbf{f} \, d\Omega = \int_\Gamma \mathbf{f} \cdot \hat{n} \, d\Gamma \tag{3}$$

This is integration by parts in 2D.

$$\int_\Omega \nabla(g\mathbf{f}) d\Omega = \int_\Omega \nabla g \, \mathbf{f} \, d\Omega + \int_\Omega g \cdot \nabla \mathbf{f} \, d\Omega \tag{4}$$

With these tools at hand we can now approach the 2D problem. First we can multiply by an arbitrary weight function. We have also represented the function in a newer form for clarity later on.

$$\int_\Omega w(\nabla \cdot \nabla P) d\Omega + \int_\Omega w \, f \, d\Omega = 0 \tag{5}$$

Now we can use integration by parts on the left side term to get the following.

$$\int_\Omega \nabla^T w \nabla P \, d\Omega - \int_\Omega \nabla(w \nabla P) \, d\Omega + \int_\Omega w \, f \, d\Omega = 0 \tag{6}$$

Using the divergence theorem on the middle term we obtain the next step in the process.

$$\int_\Omega \nabla^T w \nabla P \, d\Omega - \int_\Gamma w \nabla P d\,\Gamma + \int_\Omega w \, f \, d\Omega = 0 \tag{7}$$

Since we are given that the Neumann boundary conditions are homogenous, the middle term can be removed leaving.

$$\int_\Omega \nabla^T w \nabla P \, d\Omega = -\int_\Omega w \, f \, d\Omega \tag{8}$$

From here we can use the Galerkin theorem to solve the problem in FEniCS by implementing these equations directly. We simply follow the formulation below for left and right sides (Note here that v was used as the trial function that w had represented above. This is simply convention and taken from Discussion):

$$a(P, v) = \nabla P \cdot \nabla v \tag{9}$$

$$f(u) = \nabla (u \cdot \nabla v) \tag{10}$$

$$L = f(u) \cdot v \, dx \tag{11}$$

With these equations fed directly into our program, we simply execute the solve command for a equal to L, using the given boundary conditions.

The results for the visualization obtained are shown below.

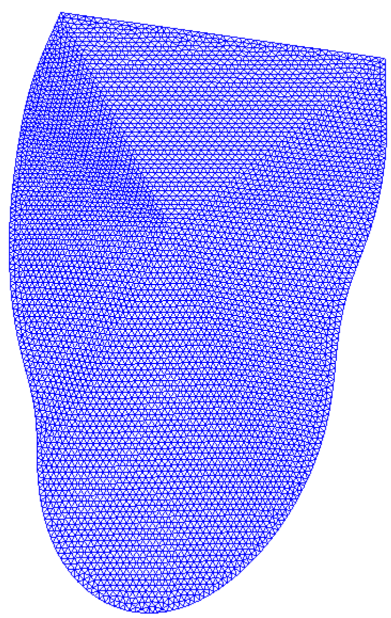

Figure 1: Above is the mesh used to solve for the finite element solution.

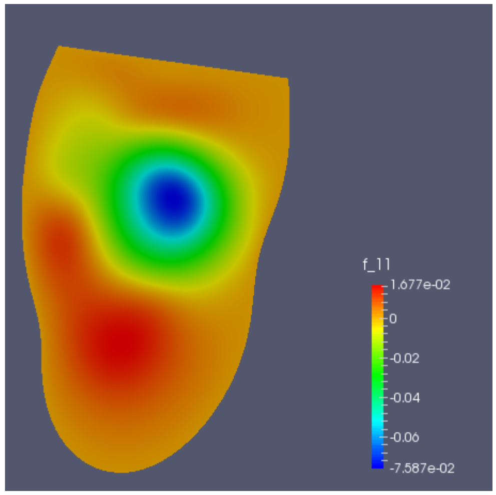

Figure 2: Above is the visualization done in Paraview for the left ventricle of a heart. As can be seen, the pressure values solved for make sense