ME C180, Finite Element Method. Assignment 7
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1. **Solution:**

The given problem was to solve for the velocity and pressure fields around a cylinder in 2D with a constant velocity. The fluid relationships are described by the Navier-Stokes equations shown below for time dependent systems.

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P + \nu \nabla^2 \mathbf{u} \tag{1}$$

$$\nabla \cdot \mathbf{u} = 0 \tag{2}$$

The cylinder was centered at (0.2,0.2) with a radius of 0.05. The dynamics viscosity was set to 0.0015. The boundary conditions for the problem were as follows: $P = 0$ on $y = 0, 0.41$ and on $x = 2.2$, u_x, $u_y = 0$ on cylinder boundary, $u_x = 3, u_y = 0$ on the left inflow boundary, and To solve the Navier-Stokes equations above, Chorin's multistep algorithm was used in conjunction with FEniCS as a Finite element solver to simulate the airflow for 1 second. Time steps of 0.0002 seconds were used.

To implement Chorin's multi-setp algorithm the projected velocity is calculated in two parts: first projecting results using solely viscous forces, and next using pressure forces next to correct the first projection. This type of method is similar to an operator splitting approach.

First we find the intermediate velocity u^* below by ignoring the pressure term, thereby making this intermediate velocity act in a divergence free field.

$$\frac{\mathbf{u}^* - \mathbf{u}^n}{\Delta t} = -(\mathbf{u}^n \cdot \nabla)\mathbf{u}^n + \nu \nabla^2 \mathbf{u}^n \tag{3}$$

Next, we add the pressure back to get the equation below.

$$\frac{\mathbf{u}^n - \mathbf{u}^*}{\Delta t} = -\nabla P \tag{4}$$

Finally, we can do a velocity correction using the previous given pressure calculation to calculate the velocity at n + 1. This correction constrains the velocity to the pressure field that we have updated. This algorithm is carried out for each time step.

Note: Simulations were intended to be done for a full 1.7 seconds; however, after using various methods to make the simulation run for this time length — including reducing the time step to 0.00005 seconds, and restructuring the algorithm — the system would always become unstable at approximately 1.005 seconds. Therefore, simulations were only done out to 1 second. The results are shown below.

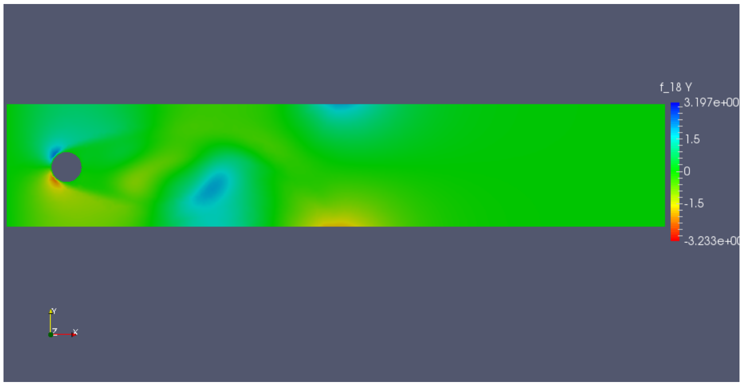

Figure 1: The figure above shows the vorticity at 0.52 seconds

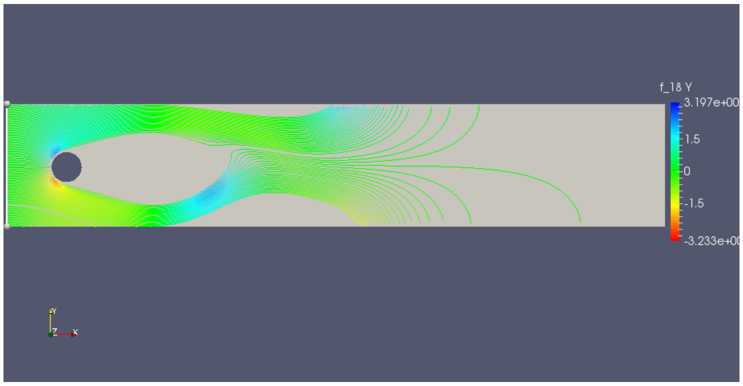

Figure 2: The figure above shows the vorticity and the streamlines at 0.52 seconds

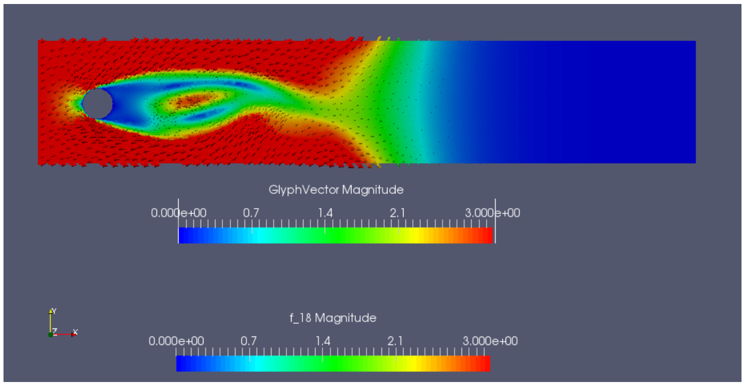

Figure 3: The figure above shows the velocity vectors at 0.52 seconds

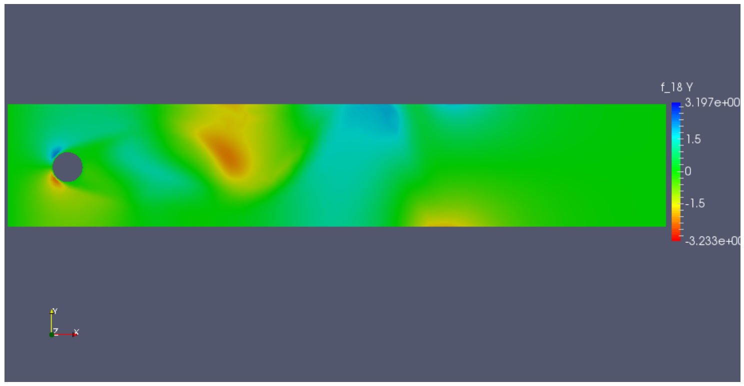

Figure 4: The figure above shows the vorticity at 0.72 seconds

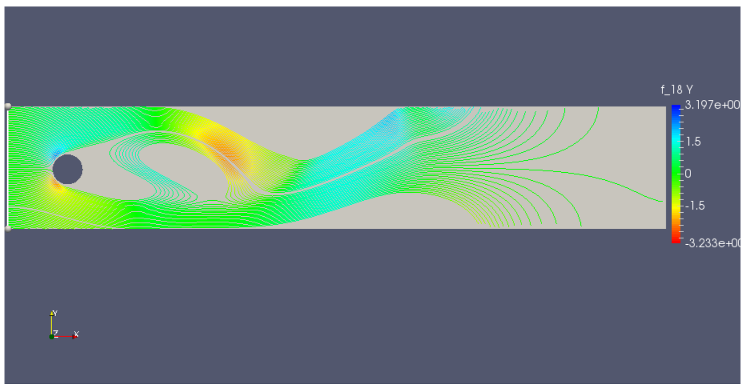

Figure 5: The figure above shows the vorticity and the streamlines at 0.72 seconds

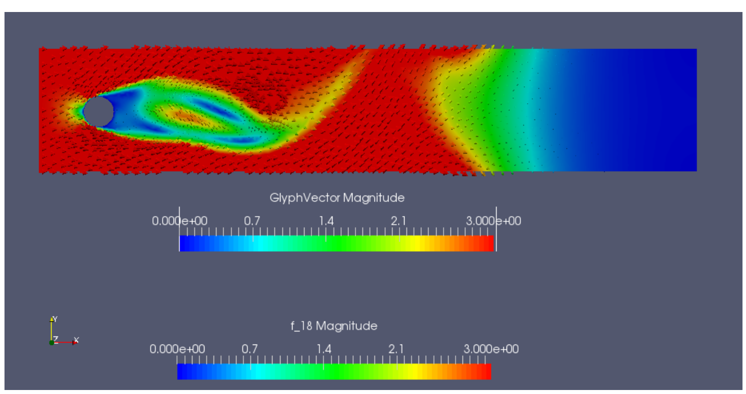

Figure 6: The figure above shows the velocity vectors at 0.72 seconds

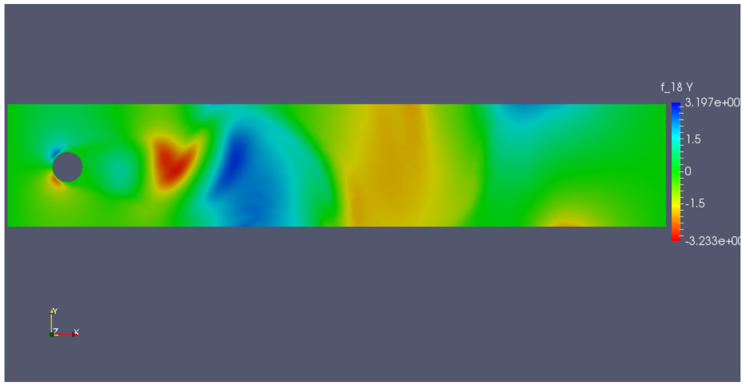

Figure 7: The figure above shows the vorticity at 0.92 seconds

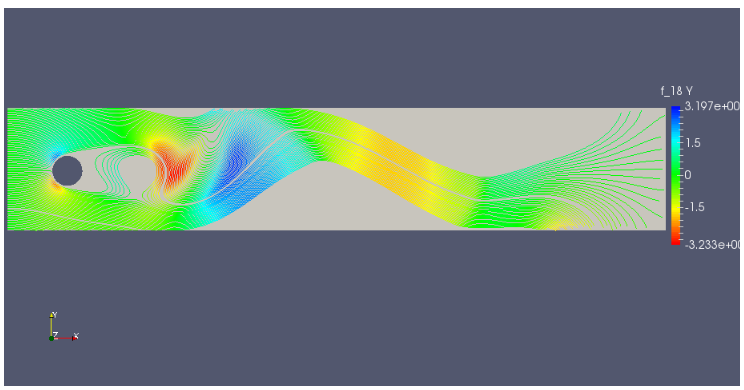

Figure 8: The figure above shows the vorticity and the streamlines at 0.92 seconds

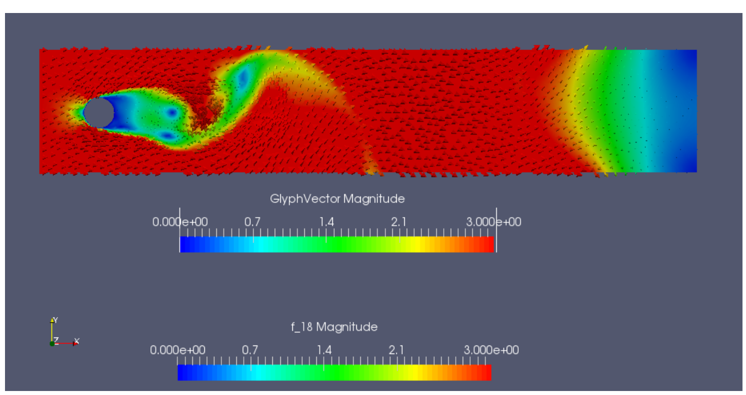

Figure 9: The figure above shows the velocity vectors at 0.92 seconds