FIN 971: Corporate Finance
Fall 2021, University of Wisconsin
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Problem Set #3 - Goal Due Date 11/23/21

Exercise 3.15 of Tirole (project riskiness and credit rationing). Consider the basic, fixed-investment model covered in Section 3.2 of Tirole (2006). In particular, investment is a fixed size I, the entrepreneur borrows $I-A$, the probability of success is either p_H (which yields no private benefit) or p_L (which yields private benefit B), success yields verifiable revenue R while failure yields 0. There are two types, "A" and "B", of the projects, which differ only with respect to "riskiness" defined by $p_H^A R^A = p_H^B R^B$, but $p_H^A > p_H^B$ so that project B is "riskier". The investment cost I is the same for both variants and furthermore, $\Delta p = p_L^A - p_L^A = p_H^B - p_L^B$. Which type of project is less prone to credit rationing?

Exercise 3.13 of Tirole (lender market power with fixed investment). The environment is similar to Section 3.2 of Tirole with one exception. An enterpreneur has internal wealth A (which could be negative because of previous debt) and wants to undertake non-negative investment $I > A$ into a fixed size project. The project yields $R > 0$ with probability p and 0 with probability $1 - p$. The probability of success is p_H if the entrepreneur works and $p_L < p_H$ if he shirks. The entrepreneur obtains private benefit B if she shirks and 0 otherwise. The borrower is protected by limited liability and everyone is risk neutral. The project is worthwhile only if the entrepreneur behaves.

The exception is that there is a single lender. This lender has access to funds that command an expected rate of return equal to 0 (so the lender would content himself with a 0 rate of return, but will use his market power to obtain a superior rate of return). Assume $V \equiv p_H R - I > 0$ and let $\overline{A}$ and $\widehat{A}$ be defined by

$$\begin{aligned}\overline{A} &\equiv I - p_H\left[R - \frac{B}{\Delta p}\right] \\ \widehat{A} &\equiv p_H \frac{B}{\Delta p}\end{aligned}$$

where $\Delta p = p_H - p_L$. Assume that $\overline{A} > 0$ and that the lender makes a take-it-or-leave-it offer to the borrower (i.e. the lender chooses R_b, the borrower's compensation in the case of success).

(i) What contract is optimal for the lender? Be sure to state the programming problem explicitly.

(ii) Is the financing decision affected by lender market power (i.e. compared to the case of competitive lenders in Section 3.2)?

(iii) Draw the borrower's net utility (i.e. net of A) as a function of A. Note that unlike the monotonic case in Section 3.2, it is nonmonotonic among the regions $(-\infty, \overline{A})$, $[\overline{A}, \widehat{A})$, $[\widehat{A}, I)$. Explain.

Exercise 3.5 of Tirole (continuous investment and decreasing returns to scale). Consider the continuous investment model of Section 3.4 of Tirole (2006) with one modification; investment I yields return $R(I)$ in the case of success and 0 in the case of failure, where $R' > 0$ and $R'' < 0$, $R'(0) > 1/p_H$, $R'(\infty) < 1/p_H$. The rest of the model is unchanged. That is, the entrepreneur starts with cash A, the probability of success is either p_H if he behaves or p_L if he misbehaves. The entrepreneur obtains private benefit BI if he misbehaves and 0 otherwise. Only the final outcome is observable. Let I^* denote the level of investment that maximizes total surplus (i.e. $p_H R'(I^*) = 1$).

(i) How does investment $I(A)$ vary with the level of cash?

(ii) How does the shadow value v of cash (the derivative of the borrower's gross utility with respect to cash) vay with the level of cash?