FIN 971: Corporate Finance
Fall 2021, University of Wisconsin
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Problem Set #4 - Goal Due Date 12/7/21

1) Exercise 5.5 of Tirole (liquidity needs and pricing of liquid assets). Consider the liquidity-needs model with a fixed investment I and only two possible liquidity shocks $\ell \in \{\ell_L, \ell_H\}$ with $\ell_L < \ell_H$. The borrower has cash A and wants to finance $I > A$ at date 0. At date 1, a cash infusion of size ℓ is needed in order for the project to continue. If ℓ is not invested at date 1, the project stops and yields nothing. If ℓ is invested, the borrower chooses between working (no private benefit with success probability p_H) and shirking (private benefit B with success probability $p_L = p_H - \Delta p$). The project then yields, at date 2, R in the case of success and 0 in the case of failure.

The liquidity shock is equal to ℓ_L with probability $(1 - \lambda)$ and to ℓ_H with probability λ, where

$$\ell_L < p_H(R - B/\Delta p) < \ell_H < p_H R.$$

Assume further that

$$p_H(R - B/\Delta p) - \ell_L > I - A. \tag{1}$$

There is a single liquid asset, Treasury bonds. A treasury bond yields 1 unit of income for certain at date 1 (and none at dates 0 and 2). It is sold at date 0 at price $q \geq 1$. The investors' rate of time preference is equal to 0 (i.e. there is no discounting between periods).

(i) Suppose that the firm has the choice between buying enough Treasury bonds to withstand the high liquidity shock and buying none. Show that it chooses to hoard liquidity if

$$\begin{aligned}(q-1)(\ell_H - p_H(R - B/\Delta p) \quad &\leq \quad (1-\lambda)(p_H(R - B/\Delta p) - \ell_L) \\ &\quad\;\; -\lambda(\ell - p_H(R - B/\Delta p) - I + A\end{aligned} \tag{2}$$

and

$$(q-1)(\ell_H - p_H(R - B/\Delta p) \leq \lambda(p_H R - \ell_H). \tag{3}$$

(ii) Suppose that the economy is composed of a continuum, with mass 1, of identical firms with characteristics as described above. The liquidity shocks of the firms are perfectly correlated. There are T Treasury bonds in the economy with $T < \ell - p_H(R - B/\Delta p)$. Show that when λ is small, the liquidity premium $(q - 1)$ commanded by Treasury bonds is proportional to the probability of a high liquidity shock. Hint: show that either (2) or (3) must be binding and use (1) to show (3) is binding.

(iii) Suppose that, in the economy considered in the previous subquestion, the government issues at date 0 not only the T Treasury bonds, but also a security that yields at date 1 a payoff equal to 1 in the good state (where the firms experience the low liquidity shock ℓ_L) and 0 in the bad state (where the firms experience the high liquidity shock ℓ_H). What is the equilibrium date 0 price q' of this new asset? Note that prices of the Treasury bonds and of this new asset must clear markets.

2) Consider a simple version of the Townsend costly state verification model in which the cash flow R obtained by the borrower can take only two values: a high value R_H with probability p_H and a low value R_L with probability $p_L = 1 - p_H$. The loan size is I. The lender and borrower are risk neutral. Unlike the case presented in class, assume lender has market power so that the optimal contract will be found by maximizing expected repayment to the lender U_L (net of auditing costs) subject to incentive compatibility and individual rationality of the borrower. The outside option for the borrower is U_B and the audit cost is K. The borrower has limited liability. The maximum penalty that can be inflicted on the borrower if he lies (reports y_L when y_H has occurred) is confiscation of y_H.

(i) Compute the optimal deterministic contract $(y(\widehat{R}), r(\widehat{R}))$ as a function of U_B. Represent the pareto frontier in the (U_B, U_L) plane. Hint: you only need consider two cases $r \leq R_L$ and $R_L < r \leq R_H$ since $r > R_H$ is inefficient because an audit would take place in state H,which is clearly dominated by the debt with $r = R_H$.

(ii) Suppose the lender can credibly commit to a stochastic auditing policy; audit with probability $q \in [0, 1]$ when the borrower reports R_L. Show that the incentive compatibility constraint is equivalent to

$$q \geq q^* = 1 - \frac{U_B}{p_H(R_H - R_L)}.$$

Represent the new Pareto Frontier. Comment.

3) Exercise 6.1 of Tirole (privately known private benefit and market breakdown). In Chapter 3 of Tirole on moral hazard, the type or "benefit B" of the entrepreneur (borrower) was known by the investor (lender). In this question, we will assume that there is a unit measure of two types $\{b, g\}$ of entrepreneurs and their type is unknown by the investor. The fraction of type g entrepreneurs is α and type b entrepreneurs is $(1 - \alpha)$. The entrepreneur want to finance a fixed-size project costing I and for simplicity has no equity ($A = 0$) so must borrow I. As in Section 3.2.1 of Tirole, the probability of success is p_H if the entrepreneur exerts high effort and p_L if the entrepreneur shirks where $\Delta p \equiv p_H - p_L > 0$. There is no private benefit $B = 0$ when exerting high effort. The private benefit when shirking is either

B_b or B_g depending on the entrepreneur's type where $B_b > B_g > 0$. Thus a "bad type" has higher private benefit when shirking. Entrepreneurs and investors are risk neutral. Except for knowing the type of entrepreneur, all other parameters are common knowledge. Assume that under asymmetric information lenders are uncertain about whether the project should be funded:

$$p_H \left(R - \frac{B_b}{\Delta p} \right) < I < p_H \left(R - \frac{B_g}{\Delta p} \right)$$

and that lenders cannot break even if the entrepreneur shirks:

$$p_L R < I.$$

Since the entrepreneurs type is unknown, the lender cannot finance only good borrowers. Due to the notational changes, denote R_e and R_l the entrepreneur and lender "returns" in the event of success and assume the entrepreneur receives no return in event of failure (this can be shown to be optimal).

i. Show that there exists $\alpha^* \in (0, 1)$ such that no financing occurs if $\alpha < \alpha^*$ and financing a pooling equilibrium exists if $\alpha \geq \alpha^*$.

ii. Describe the cross subsidies between types when borrowing arises in equilibrium.