Model

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Model

Environment

This model follows a directed search framework in the spirit of [[2010 - Guido Menzio, Shouyong Shi - Block recursive equilibria for stochastic models of search on the job||2010 - Guido Menzio, Shouyong Shi]]. The model incorporates two key sources of heterogeneity: firms differ in their remote-work efficiencies, while workers vary in their skill levels. The key mechanisms in this framework are that workers value the flexibility provided by remote work arrangements, high-skilled workers are more productive and better suited for remote work, and firms treat remote and on-site work as substitutable inputs in their production processes.

Workers are characterized by their productivity.

- [ ] Decide if we are choosing a skill vector or just a single number as skill.

They incur disutility from on-site work, which is partially compensated by their wage. We denote by $u(w, \alpha)$ the utility of a worker earning wage w with remote work arrangement $\alpha \in [0, 1]$, where α represents the fraction of time working remotely. We assume that utility continously differentiable and increasing in consumption $u_w(\cdot) > 0$ and in remote work $u_\alpha(\cdot) > 0$. We assume that workers supply their unit of labor inelastically.

Firms are characterized by their remote-work efficiency parameter ψ, which determines how effectively they can implement remote work arrangements.

☐ Consider split firm into general productivity parameter and remote work productivity parameter.

The output of a firm-worker match depends on firm productivity, worker skill and the fraction of remote work determined by the arrangement α. We asume that $A(h)$, ($A'(h) > 0$) captures the contribution to output from worker skill h. The firm split worker's labor between in-person work and remote work. However, these components aren't perfectly substitutable, remote work is adjusted by a factor $g(\psi, h)$. The function $g(\psi, h)$ captures the efficiency of remote work, with $g_\psi(\psi, h) \geq 0$ indicating that remote work efficiency increases with firm type ψ due to better technology, management practices, or the nature of the occupation. Similarly, $g_h(\psi, h) \geq 0$ suggests that remote work efficiency increases with worker skill level due to greater autonomy and technological ability.

The production function of the firm is given by:

$$Y(\alpha \mid \psi, h) = A(h)\left((1-\alpha) + \alpha g(h, \psi)\right) \tag{1}$$

Profits are determined by the difference between the output produced and the wage paid to the worker. Let x denote the total utility level that a working arrangement delivers to the worker, x is derived from both the wage received and the work arrangement (i.e., the fraction of remote work α). Since workers care only about their total utility, the firm is constrained to ensure that the worker obtains at least the utility level x. This creates a trade-off for the firm: while offering higher remote work (a larger α) may decrease productivity it might also allow the firm to pay a lower wage to meet the utility guarantee. A type ψ firm with a worker of type h chooses α to maximize the following expression:

$$\Pi(\alpha \mid \psi, h, x) = \max_{\alpha \in [0,1]} \left\{ Y(\alpha \mid \psi, h) - w(\alpha) \;\mid\; x = u(w(\alpha), \alpha) \right\} \tag{2}$$

Since $g(\psi, h)$ captures the adjustment of productivity to remote work in Equation 1 and is heterogeneous across firms and workers, the optimal remote work policy will also be heterogeneous.

Optimal Remote Work Choice

The firm's problem implies an optimal choice of remote work $\alpha^*(\psi, h, x)$ that satisfies the first-order condition:

Interior Solution: Let's start with the interior solution, where the firm chooses a positive fraction of remote work. This optimal choice $\alpha^*(\psi, h, x)$ must satisfy:(details of the derivation are in the [[Optimal Remote Policy (General Functional Forms)|Appendix]])

1. **First Order (Stationarity) Condition:**

$$A(h)\left(g(\psi, h) - 1\right) = -\frac{u_\alpha(w(\alpha^*), \alpha^*)}{u_w(w(\alpha^*), \alpha^*)}. \tag{3}$$

2. **Promise Keeping Constraint:**

$$x = u(w(\alpha^*), \alpha^*). \tag{4}$$

   The assumptions $u_w > 0$ and $u_\alpha > 0$ ensure that the utility function is strictly increasing in the wage argument and continuously differentiable, which in turn guarantees that the inversion $w(\alpha) = u^{-1}(x, \alpha)$ is well defined and unique.

Corner Solution: Notice that Equation 3 describes only the interior solutions. The firm may also choose to offer either full remote work ($\alpha = 1$) or no remote work ($\alpha = 0$). The conditions for these corner solutions are as follows:

- [ ] Check this functional forms are correct.

The optimal remote work policy is given by:

$$\alpha^*(\psi, h, x) = \begin{cases} 0 & \text{if} \quad \psi \leq \underline{\psi}(h) \\ \alpha^*(\psi, h, x) & \text{if} \quad \underline{\psi}(h) < \psi < \overline{\psi}(h) \\ 1 & \text{if} \quad \overline{\psi}(h) \leq \psi \end{cases} \tag{5}$$

where $\underline{\psi}(h)$ and $\overline{\psi}(h)$ are the thresholds for the remote work efficiency parameter ψ below which it is optimal to offer no remote work and above which it is optimal to offer full remote work, respectively. The interior solution $\alpha^*(\psi, h, x)$ satisfies the first-order condition in Equation 3.

Proposition 0.1 (Properties of the Optimal Remote Policy)**.** *The optimal remote work policy $\alpha^*(\psi, h, x)$ defined in @optimalRemoteWorkPolicy has the following properties:*

1. ***Higher-skilled workers get more remote work:*** *The interior solution $\alpha^*(h)$ is (weakly) increasing in h; that is, holding ψ and x constant, firms assign a higher remote work share to higher-skilled workers.*
2. ***Continuity and Differentiability of Thresholds:*** *The thresholds $\underline{\psi}(h)$ and $\overline{\psi}(h)$ — defined respectively as the minimal and maximal remote productivity levels for which it is optimal to offer some and full remote work — are characterized implicitly by*

$$g(\underline{\psi}(h), h) = 1 - \frac{1}{A(h)} \left[\frac{u_\alpha(w(0), 0)}{u_w(w(0), 0)} \right],$$
$$g(\overline{\psi}(h), h) = 1 - \frac{1}{A(h)} \left[\frac{u_\alpha(w(1), 1)}{u_w(w(1), 1)} \right].$$

Labor Market Search

Both firms and workers discount the future at rate β. Workers are characterized by their type h and direct their search toward submarkets distinguished by the promised utility level x. A worker of type h evaluates the different utility promises available in each submarket and chooses to search in the one that maximizes their expected value. This expected value incorporates not only the probability of being hired but also the future discounted value of the job. At the same time, firms target workers of a particular type h by posting job offers (or contracts) that promise a specific utility level x. This setup allows the market to be segmented into different submarkets. The tightness of a submarket (h, x) is defined as:

$$\theta(h, x) = \frac{v(h, x)}{u(h, x)},$$

where $v(h,x)$ denotes the number of vacancies posted by firms in the submarket and $u(h,x)$ represents the number of unemployed workers actively searching within that particular submarket. This measure of tightness directly influences the probabilities of matching: the vacancy filling rate $q(\theta(h,x))$ and the job finding rate $p(\theta(h,x))$ are both functions of θ. In our equilibrium, free entry of firms ensures that the expected profit from posting a vacancy is zero, after incurring a cost $\kappa \in \mathbb{R}_{++}$. Matches are exogenously broken at a rate δ.

Once a firm and a worker are matched, the firm delivers the promised utility x to the worker by applying the firm's optimal remote work policy. Before posting vacancies, firms face uncertainty about their remote-work efficiency parameter ψ. However, the distribution $F(\psi)$ is common knowledge among all agents in the economy. Because firms are ex-ante identical in this dimension, any worker searching in a given submarket faces the same probability of being matched with a firm having a particular productivity level ψ.

For firms, the value of posting a vacancy in a submarket characterized by (h,x) is given by

$$V(h,x) = -\kappa + q(\theta(h,x)) \int J(\psi,h,x)\, dF(\psi), \tag{6}$$

where κ is the vacancy posting cost and $J(\psi,h,x)$ is the value from an ongoing match with a firm of productivity ψ. The match value is determined by the current payoff—expressed as the output minus the wage cost plus the discounted expected continuation value:

$$J(\psi,h,x) = Y\big(\alpha^*(\psi,h) \mid \psi,h\big) - w\big(x,\alpha^*(\psi,h)\big) + \beta\Big[(1-\delta)J(\psi,h,x) + \delta\, V(h,x)\Big]. \tag{7}$$

Notice that free-entry guarantee that $V(h,x) = 0$, this means that the value function described in Equation 7 can be computed independently of the distributions of workers and vacancies across submarkets. Furthermore the value of matches pin-down the meeting rates and thus the submarket tightness. Notice that free entry condition is binding if the submarket is active in equilibrium (i.e. $\theta(h,x) > 0$), then from Equation 6:

$$\theta(h,x) = q^{-1}\left(\frac{\kappa}{\int J(\psi,h,x)\, dF(\psi)}\right) \qquad \text{if } \theta(h,x) > 0 \tag{8}$$

For workers, the value functions capture the trade-off between being unemployed and employed. The value of unemployment for a worker of type h is

$$U(h) = b + \max_x \left\{ p(\theta(h,x)) \int W(\psi, h, x) \, dF(\psi) + \Big(1 - p(\theta(h,x))\Big) U(h) \right\}, \tag{9}$$

where b denotes the unemployment benefit. Once employed, the worker's value is given by

$$W(\psi, h, x) = x + \beta \Big[(1-\delta) W(\psi, h, x) + \delta \, U(h) \Big]. \tag{10}$$

This recursive formulation encapsulates the idea that a worker receives the promised utility x while also facing the possibility of job separation.