Model Setup

Everithing is the same as in the main document.

Bellman Equations

Worker

$$U_t^j(x) = \underbrace{b(x,j,z_t)}_{\text{instant utility}} + \beta \max_{\phi_u^j(x)} \left\{ \underbrace{-c(\phi_u^j(x))}_{\text{cost of search strategy}} + \right.$$
$$\mathbb{E}_t \sum_{j' \in \mathcal{J}} \underbrace{\phi_u^j(x,j')}_{\text{weight by probability of search in } j'} \left[\overbrace{(1-p_{t+1}^{j'})U_{t+1}^{j'}(x)}^{\text{no offer, stays unemployed now in } j'} \right.$$
$$\left. \left. + p_{t+1}^{j'} \underbrace{\int \max\left\{U_{t+1}^{j'}(x), W_{0,t+1}^{j\to j'}(x,y)\right\} \frac{v_{t+1}^{j'}(y)}{V_{t+1}^{j'}} dy}_{\text{if offer, pays cost,moves to } j \text{ and then is matched randomly with some firm}} \right] \right\}$$

Value of a match

$$J_t^j(x,y) = \underbrace{f(x,y,j,z_t)}_{\text{match value added}} + \beta \mathbb{E}_t \left[\overbrace{(1-\lambda_{t+1}^j(x,y))}^{\text{match is destroyed}} \underbrace{U_{t+1}^j(x)}_{\text{worker gets unemployment value}} + \right.$$
$$\underbrace{(\lambda_t^j(x,y)}_{\text{match survives}} \max_{\phi_s^j(x)} \left\{ -c(\phi_s^j(x)) + \sum_{j' \in \mathcal{J}} \phi_s^j(x,j') \left[\overbrace{(1-sp_t^{j'})}^{\text{no new offers}} \underbrace{J_{t+1}^j(x,y)}_{\text{stays with same firm}} + \right.\right.$$
$$\left.\left. sp_t^{j'} \underbrace{\int \max\{J_{t+1}^j(x,y), W_{1,t+1}^{j\to j'}(x,y',y)\} \frac{v_{t+1}^{j'}(x)}{V_{t+1}^{j'}} dy'}_{\text{worker only accepts new offers if value is greater than current match}} \right] \right.$$

Bargaining

I use the same bargaining as in (Cahuc, Postel-Vinay, and Robin 2006) this have the following implications for the value that a worker gets from a match:

- Unemployed worker in location j when matched with firm y in location j' gets:

$$W_0^{j\to j'}(x,y) = (1-\mu)U^{j'}(x)+\mu[J^{j'}(x,y)-F^{j'\to j}] = U^j(x)+\mu\left(J^{j'}(x,y) - [U^j(x)+F^{j'\to j}]\right)$$

- Worker employed in location j by firm y when matched with firm y' in location j' gets:

$$W_1^{j\to j'}(x,y) = (1-\mu)J^j(x,y)+\mu[J^{j'}(x,y')-F^{j'\to j}] = J^j(x,y)+\mu\left(J^{j'}(x,y') - J^j(x,y) - F^{j'\to j}\right)$$

Plugging this into the Bellman equations we get:

Worker

$$\begin{aligned}
U_t^j(x) = b(x,j,z_t) + \beta \max_{\phi_u^j(x)} \Bigg\{ & -c(\phi_u^j(x)) + \mathbb{E}_t \sum_{j'\in\mathcal{J}} \phi_u^j(x,j') \Big[(1-p_{t+1}^{j'})U_{t+1}^{j'}(x) \\
& + p_{t+1}^{j'} \int \max\left\{U_{t+1}^{j'}(x), U_{t+1}^j(x) + \mu\left(J_{t+1}^{j'}(x,y) - [U_{t+1}^j(x) + F^{j'\to j}]\right)\right\} \frac{v_{t+1}^{j'}(y)}{V_{t+1}^{j'}} dy \Big] \Bigg\} \\
= b(x,j,z_t) + \beta \max_{\phi_u^j(x)} \Bigg\{ & -c(\phi_u^j(x)) + \mathbb{E}_t \sum_{j'\in\mathcal{J}} \phi_u^j(x,j') \Big[(1-p_{t+1}^{j'})U_{t+1}^{j'}(x) + p_{t+1}^{j'}U_{t+1}^{j'}(x) \\
& + p_{t+1}^{j'}\mu \int \max\left\{0, J_{t+1}^{j'}(x,y) - [U_{t+1}^j(x) + F^{j'\to j}]\right\} \frac{v_{t+1}^{j'}(y)}{V_{t+1}^{j'}} dy \Big] \Bigg\} \\
= b(x,j,z_t) + \beta \max_{\phi_u^j(x)} \Bigg\{ & \mathbb{E} - c(\phi_u^j(x)) + \mathbb{E}_t \sum_{j'\in\mathcal{J}} \phi_u^j(x,j') \Big[\\
& + p_{t+1}^{j'}\mu \int \max\left\{0, J_{t+1}^{j'}(x,y) - [U_{t+1}^j(x) + F^{j'\to j}]\right\} \frac{v_{t+1}^{j'}(y)}{V_{t+1}^{j'}} dy \Big] \Bigg\}
\end{aligned}$$

Value of a match

References

Cahuc, Pierre, Fabien Postel-Vinay, and Jean-Marc Robin. 2006. "Wage Bargaining with On-the-Job Search: Theory and Evidence." *Econometrica* 74 (2): 323–64. https://doi.org/10.1111/j.1468-0262.2006.00665.x.