A model of Sorting bewteen Workers and Firms (now in Space!)

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Model

- I adopt the framework from (Lise and Robin 2017).
- Introducing a fixed amount of locations and population distribution of workers should be an equilibrium outcome.
- Workers can “partially” direct their search to each location.
- Productivity depends on the quality of match and aggregate state as in the original paper. I want to introduce a third component which is location productivity.
  - Which in turn depends on the skill distribution of workers in each location.

Model Setup

Demographics

- There is a discrete and finite set of locations $\mathcal{J} = \{1, \ldots, |\mathcal{J}|\}$ locations indexed by $j \in \mathcal{J}$.
- Continuum of workers indexed by their ability $x \in \mathcal{X}$.
  - The total measure is normalized to 1.
  - Exogenous distribution $\ell(x)$
  - Endogenous distribution in each location $\ell^j(x)$
  - Denote μ_j the total population in location j.

Demographics

- Continuum of firms indexed by technology $y \in \mathcal{Y}$.
  - Total measure is normalized to 1.
  - Uniformly distributed.

Technology

- There is andexogenous cost of posting v job opportunities in location j is

$$c_j(v) \geq 0$$

assume it is increasing, convex, and independent of the firm

Job Search

Timing of the model

The timing is as follows:

1. At time (t), distributions of employed and unemployed workers are inherited from $t-1$.
   - $u_t^j(x)$ is the measure of type-x Unemployed workers at the location j.
   - $h_t^j(x,y)$ is the measure of type-x workers employed at the firm y at the location j.
   - Note that on each location:

$$u_t^j(x) + \int h_t^j(x,y)dy = \ell^j(x)$$

Timing of the model

2. Aggregate state changes $z_{t-1} \rightarrow z_t$.
3. At time $(t+)$ Separations occur:
   - $u_{t+}^j(x)$ is the measure of the type x Unemployed workers in the location j after the shock.
   - $h_{t+}^j(x,y)$ is the measure of type x workers that remain

Dynamic Programming

- $U_t^j(x)$ the value for an unemployed worker of type x at time t at location j.
- The value of getting an offer depends on whether the worker is employed or not:
  - $W_{0,t}^{j' \to j}(x, y)$ is the value of a type-x unemployed worker at location j' who is hired by a firm of type y at a location j.
  - $W_{1,t}^{j' \to j}(x, y, y')$ is the value offered at the time t by type y firm at location j to a type x worker employment at a type y' firm in location j'.
- $J_t^j(x, y)$ is the value of a match between a type x worker and a type y firm at time t in location j.

Unemployed Worker

Unemployed worker's value function is:

$$U_t^j(x) = b(x, j, z_t) + \beta \max_{\phi_u^j(x)} \Big\{ -c(\phi_u^j(x)) +$$

$$\mathbb{E}_t \sum \phi_u^j(x, j') \Big[(1 - p_{t+1}^{j'}) U_{t+1}^j(x)$$

Labor Market Flows

Now we characterize the flows of workers in-to and out-of unemployment at each location:

- Let
$$\eta^{j'\to j}(x,y) = \mathbb{1}_{\{S_t^{j'\to j}(x,y)>0\}}$$
and
$$\eta^{j'\to j}(x,y'\to y) = \mathbb{1}_{\{S_t^{j'\to j}(x,y)>S_t^{j'\to j'}(x,y')\}}$$

Labor Market Flows

- The law of motion of the unemployment rate is:

$$u_{t+1}^{j}(x) = \sum_{j'\in\mathcal{J}} \underbrace{\phi_u^{j'}(x,j)u_{t+}^{j'}(x)\left(1-\int \eta^{j'\to j}(x,y)p^j\frac{v^j(y)}{V^j}dy\right)}_{\text{mass of incoming unemployed workers that are not hired by any firm}} \tag{8}$$

- For ease of exposition we write the mass of employed workers as the sum of the following three terms:

Endgogenizing Location Productivity

- This section focuses on the endogenous determination of location productivity.
- The key idea is that the productivity of a location depends on the skill distribution of workers in that location.
- (Davis and Dingel 2019) in assuming that the productivity of workers in a location j is a result of idea exchange process within each location.

Worker Productivity

- Assume that a type x worker in location j has a $\Omega(x, \overline{X}^j)$ where $\overline{X}^j$ is the value of idea exchange in location j. Thus:

$$f(x, y, j, z_t) = f\left(\Omega(x, \overline{X}^j), y, z_t\right)$$

- Assume that the value of idea exchange in location j is a function of the distribution of skills in location j:

$$\overline{X}^j = \overline{X}\left(\{\ell^j(x)\}\right)$$

References

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