Industry Workforce Heterogeneity and Wage Inequality*

Mitchell Valdes-Bobes

September 6, 2022

Abstract

We examine [X]

*Valdes-Bobes: University of Wisconsin. I thank [X] for the helpful comments and discussions.

1 Introduction

Wage inequality has risen in the US since the 1980s see Acemoglu and Autor (2011). In the same period, the skill composition of the labor market in the US changed dramatically. The relative supply of skilled to unskilled labor (as defined by education) has risen from around 0.57 in 1988 to 1.06 in 2018. The fact that skilled workers made 1.1 times higher wages than unskilled workers at the beginning of the period and the ratio has steadily increased to 1.4, suggests that the relative demand for skilled labor must have also increased.

2 Literature Review

This work is inserted in several strands of literature.

- Captial-Skill complementarity:
  - Krusell et al. (2000)
  - Replications, Ohanian et al. (2021), Polgreen and Silos (2008), Maliar et al. (2020), Castex et al. (2022).
- Income Shares:
  - Karabarbounis and Neiman (2014)
  - Possible Explanations:
    - Occupations Orak (2017)
- Job Polarization stuff Song et al. (2019)
- Industry Stuff Haltiwanger et al. (2022)

3 Model

This section presents the model which is the same as (Krusell et al., 2000). There are four inputs for production in this economy: two types of capital, equipment (k_e) and structures (k_s) and two types of labor, skilled (s) and unskilled (u). Inputs are combined through a production function $G(\cdot)$ to produce three final goods: consumption (c), investment in equipment (i_e) and investment in structures (i_s). Assuming a hicks-neutral technological shock A, the aggregate production is given by

$$c_t + i_{e_t} + i_{s_t} = Y_t = A_t G(k_{s_t}, k_{e_t}, u_t, s_t) \tag{1}$$

Capital evolves following the law of motion in (9). The production function is assumed to be Cobb-Douglas in structures and a nested CES in all other inputs:

$$G(k_{s_t}, k_{e_t}, u_t, s_t) = k_{s_t}^{\alpha} \left(\mu u_t^{\sigma} + (1-\mu) \left(\lambda k_{s_t}^{\rho} + (1-\lambda) s_t^{\rho} \right)^{\frac{\sigma}{\rho}} \right)^{\frac{1-\alpha}{\sigma}} \tag{2}$$

where α is the share of capital structures in output, μ, and λ are income shares, ρ and σ govern the elasticity of substitution between capital equipment and labor:

- $\sigma_s = 1/(1-\rho)$ is the elasticity of substitution between equipment and high-skilled.
- $\sigma_u = 1/(1-\sigma)$ is the elasticity of substitution between low-skilled and equipment + high skill labor.

Labor input is defined as

$$u = \psi_t^u h_t^u$$
$$s = \psi_t^s h_t^s$$

where ψ_t^i is the (unobserved) efficiency of each type of labor and h_t^i is the number of labor hours.

3.1 Skill Premium

The model can be used to analyze the determinants of the skill premium growth, i.e. growth of the ratio of wages of skilled labor to wages of unskilled labor.

Firms solve the following profit maximization problem

$$\max_{k_{s_t}, k_{e_t}, u_t, s_t} G(k_{s_t}, k_{e_t}, u_t, s_t) - r_{s_t} k_{s_t} - r_{e_t} k_{e_t} - w_{u_t} h_{u_t} - w_{s_t} h_{s_t} \tag{3}$$

r_{s_t} and r_{e_t} are rental rates of capital and w_{u_t} and w_{s_t} are wages of unskilled and skilled workers. Assuming perfect competition, labor is paid it marginal productivity therefore the skill premium at time t, (ω_t) is given by

$$\omega_t = \frac{w_{s_t}}{w_{u_t}} = \frac{G_{h_s}(k_{s_t}, k_{e_t}, u_t, s_t)}{G_{h_u}(k_{s_t}, k_{e_t}, u_t, s_t)} \tag{4}$$

this gives the following expression for the skill premium:

$$\omega_t = \frac{(1-\mu)(1-\lambda)}{\mu}\left[\lambda\left(\frac{k_{e_t}}{s_t}\right)^{\rho} + (1-\lambda)\right]^{(\sigma-\rho)/\rho}\left(\frac{h_{u_t}}{h_{s_t}}\right)^{1-\sigma}\left(\frac{\psi_t^s}{\psi_t^u}\right)^{\sigma}. \tag{5}$$

Since the object of interest is the steady state growth of ω_t(5) can be log-linearized to obtain the following expression:

$$\ln\omega_t \simeq \lambda\frac{\sigma-\rho}{\rho}\left(\frac{k_{e_t}}{s_t}\right)^{\rho} + (1-\sigma)\ln\left(\frac{h_{u_t}}{h_{s_t}}\right) + \sigma\ln\left(\frac{\psi_t^s}{\psi_t^u}\right) \tag{6}$$

Which in turn can be written in terms of growth rates:

$$\begin{aligned} g_{\omega t} \simeq & (1-\sigma)\left(g_{h_{u_t}} - g_{h_{s_t}}\right) + \sigma\left(g_{\psi_t^s} - g_{\psi_t^u}\right) \\ & + (\sigma-\rho)\lambda\left(\frac{k_{e_t}}{s_t}\right)^{\rho}\left(g_{k_{e_t}} - g_{h_{s_t}} - g_{\psi_t^s}\right) \end{aligned} \tag{7}$$

where g_x denotes the growth rate of variable x, details on the derivations are include in Appendix A. Equation (7) has the nice property that it is a linear combination of the growth rates of the inputs in the production function, this allows us to decompose the growth rate of the skill premium into three components that are easy to analyze:

(i) $(1-\sigma)(g_{h_{u_t}} - g_{h_{s_t}})$ depends on the growth rate of one type of labor over the other. We assume that both types of labor are substitutes i.e $\sigma_u < 0 \implies (1-\sigma) < 0$. This means that if skilled labor grows at a faster rate than unskilled labor, will decrease the skill premium.

(ii) $\sigma\left(g_{\psi_t^s} - g_{\psi_t^u}\right)$ depends on the growth rate of the productivity of one type of labor over the other. I follow (Krusell et al., 2000) in making the following stochastic assumptions about labor productivity:

$$\psi_t^i = \psi_0^i + \epsilon \qquad \epsilon \sim N(0, \eta_\omega^2) \qquad i \in \{s, u\} \tag{8}$$

This assumption guarantees that on average $\sigma(g_{\psi_t^s} - g_{\psi_t^u})$ is constant over time and does not affect the skill premium growth rate.

(iii) $(\sigma-\rho)\lambda\left(\frac{k_{e_t}}{s_t}\right)^{\rho}\left(g_{k_{e_t}} - (g_{h_{s_t}} + g_{\psi_{s_t}})\right)$. This component depends on two factors:

(a) The growth rate of equipment relative to the growth rates of skilled labor input. This allows us to characterize the capital-skill complementarity hypothesis as $\sigma > \rho$, if equipment capital grows faster than skilled labor, the skill-premium will increase.

(b) The ratio of capital equipment to efficiency units of skilled labor input (given our assumptions amounts to the growth rate of skilled labor input), this effect will get larger (smaller) over time if $\rho > 0$ $(\rho < 0)$.

4 Data

Following the approach outlined in KORV Krusell et al. (2000), I constructed data series for wages, labor input, and capital input from 1963 to 2018 (the original KORV covered the period from 1963 to 1992). To create the extended series I followed Ohanian et al. (2021). I then collected similar data at the industry level covering the period from 1988 to 2018. The smaller sample size at the industry level is due to capital data availability.

4.1 Capital Data

To extend both capital series to replicate KORV I obtained investment series in equipment (I_e) and structures (I_s) from NIPA Table 5.2.5. Then the equipment (K_e) and structure (K_s) capital series were constructed using the perpetual inventory method:

$$K_{i_{t+1}} = (1 - \delta_{i_t})K_{i_t} + I_{i_t} \qquad i \in \{e, s\} \tag{9}$$

I departed from KORV by using time-varying depreciation rates δ_{i_t}, instead of constant depreciation rates for each series. As in (Ohanian et al., 2021) I deflate structures using the implicit price deflator of GDP [1], and equipment using the product of the consumption delfator[2] and the relative price of equipment [3]. Figure 1 shows the comparison between the original capital series obtained by (Krusell et al., 2000) and the updated capital series.

To obtain capital data at the industry level I used the BEA Fixed Assets dataset to obtain investment and capital consumption series by industry and type, details of which tables were used are included in Appendix **??**. Fixed Assets dataset groups industries into 76 groups. To construct a series of the labor share of output by industry, I used the BEA-BLS Integrated Industry-level Production Accounts (KLEMS)[4]. This dataset contains the data underlying the BEA/BLS Integrated Industry-level Production Account for the United States. The data covers

[1] Available https://fred.stlouisfed.org/series/GDPDEF

[2] Available at https://fred.stlouisfed.org/series/CONSDEF

[3] Available at https://fred.stlouisfed.org/series/PERIC

[4] Available at https://www.bls.gov/productivity/articles-and-research/industry-production-account-capital.xlsx

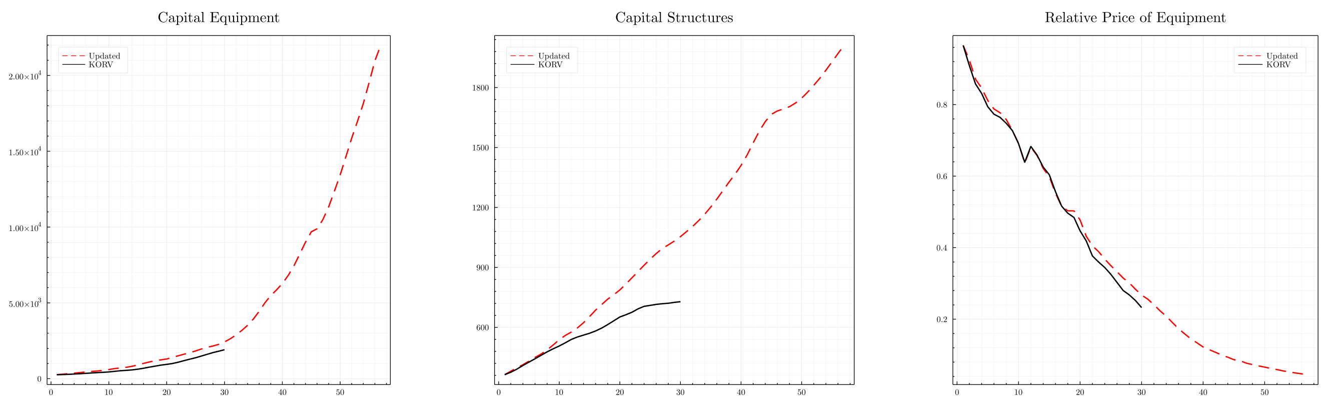

Figure 1: Capital Series

1987-2020. KLEMS data consists of 57 industry groups some of which are aggregations of industries on the BEA dataset. The table presents the crosswalk between BEA, KLEMS, and Census industry codes. I used the crosswalk provided by (Acemoglu and Restrepo, 2020). A description of the codes is in INSER TABLE AND REFERENCEIT.

4.2 Labor Data

Labor input and wages are estimated using the march supplement of the Current Population Survey (CPS), downloaded from IPUMS[5], see Flood et al. (2015). Flowing (Krusell et al., 2000) and (Ohanian et al., 2021) I include all observations excluding agents: younger than 16 or older than 70, unpaid family workers, those working in the military, those who report working less than 40 weeks a year and/or 35 hours a week, individuals with allocated income, those with hourly wages below half of the minimum federal wage rate, those did not report their education level and self-employed workers. Appendix B.1 describes in detail the cleaning process undertaken to obtain the labor input and wage series. Figure 2 displays the labor input and wage series for the 1963 - 2018 period compared with the original data.

I used the crosswalk in Table INSER TABLE AND REFERENCEIT to group Census code groups for each industry and subdivided the original CPS data. I then repeated the process described in Appendix B.1 to obtain labor input and wage series for each industry.

4.3 Labor Income Shares

To construct labor share series at the economy level I follow Krusell et al. (2000), Castex et al. (2022) and Ohanian et al. (2021) in following the Cooley and Prescott (1995). I first generate a series containing capital income (CI) consisting of the sum of

(i) net interest and miscellaneous payments, domestic industries,

[5] https://cps.ipums.org/cps/index.shtml

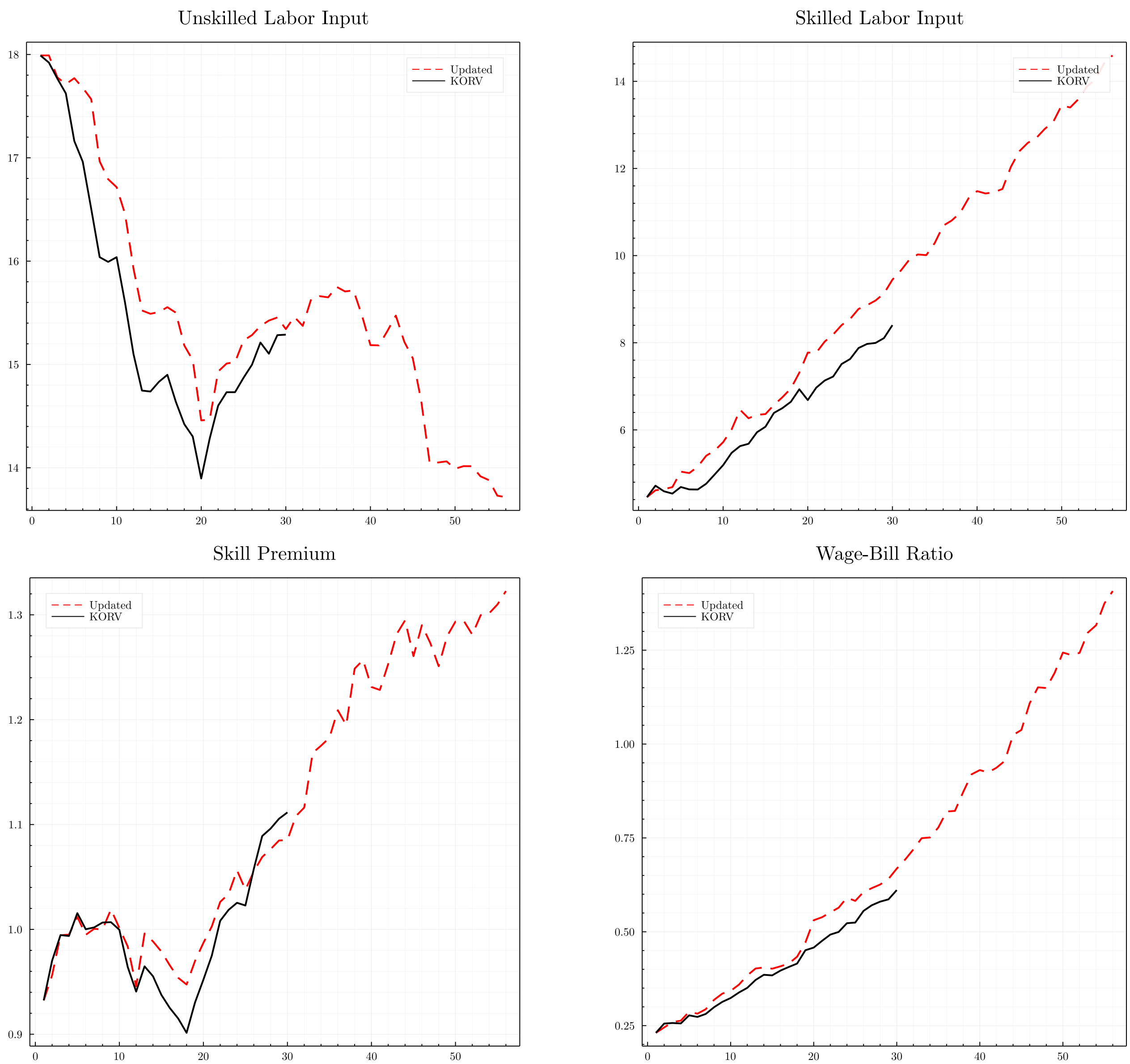

Figure 2: Labor Series

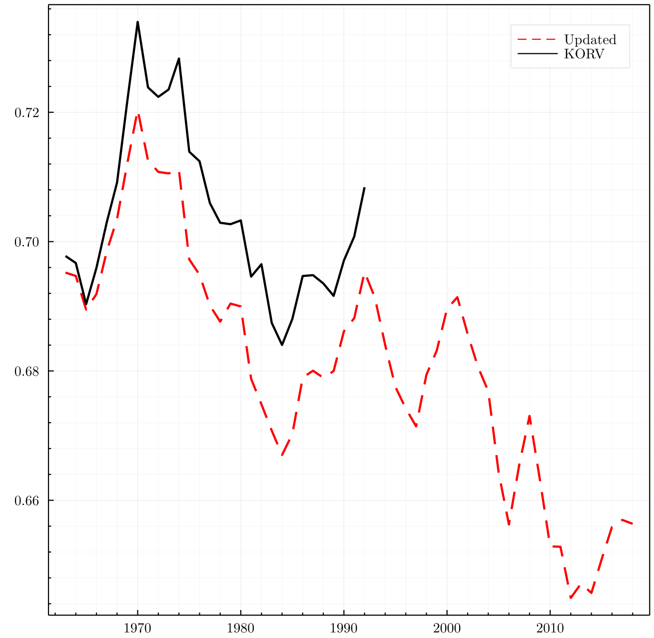

Figure 3: Labor Share of Income

(ii) corporate profits,

(iii) consumption of fixed capital.

Capital share is defined as the ratio between CI and gross domestic income (net of proprietors' income) $Y - PI$. Labor share is then calculated as

$$LI = 1 - \frac{CI}{Y - PI}$$

To construct labor share series at the industry level, I used the BEA-BLS Integrated Industry-level Production Accounts (KLEMS). KLEMS dataset contains information on the compensation of employees (with and without a college degree) and the value added by industry, I then follow (Karabarbounis and Neiman, 2014) and define the labor share as the ratio between the total compensation of employees and the total value added by industry.

4.4 Data Description

4.4.1 Industry Trends

In line with the findings of Karabarbounis and Neiman (2014) across countries, I find that the labor share of income is consistently decreasing across industries. Labor share is decreasing in 47 of 56 (87.5%). This decrease ranges between 59.7% and 0.04%. Figure 4 show the labor share of the 4 largest industries (by output) in the US. Figure 5 shows the trends of the labor input

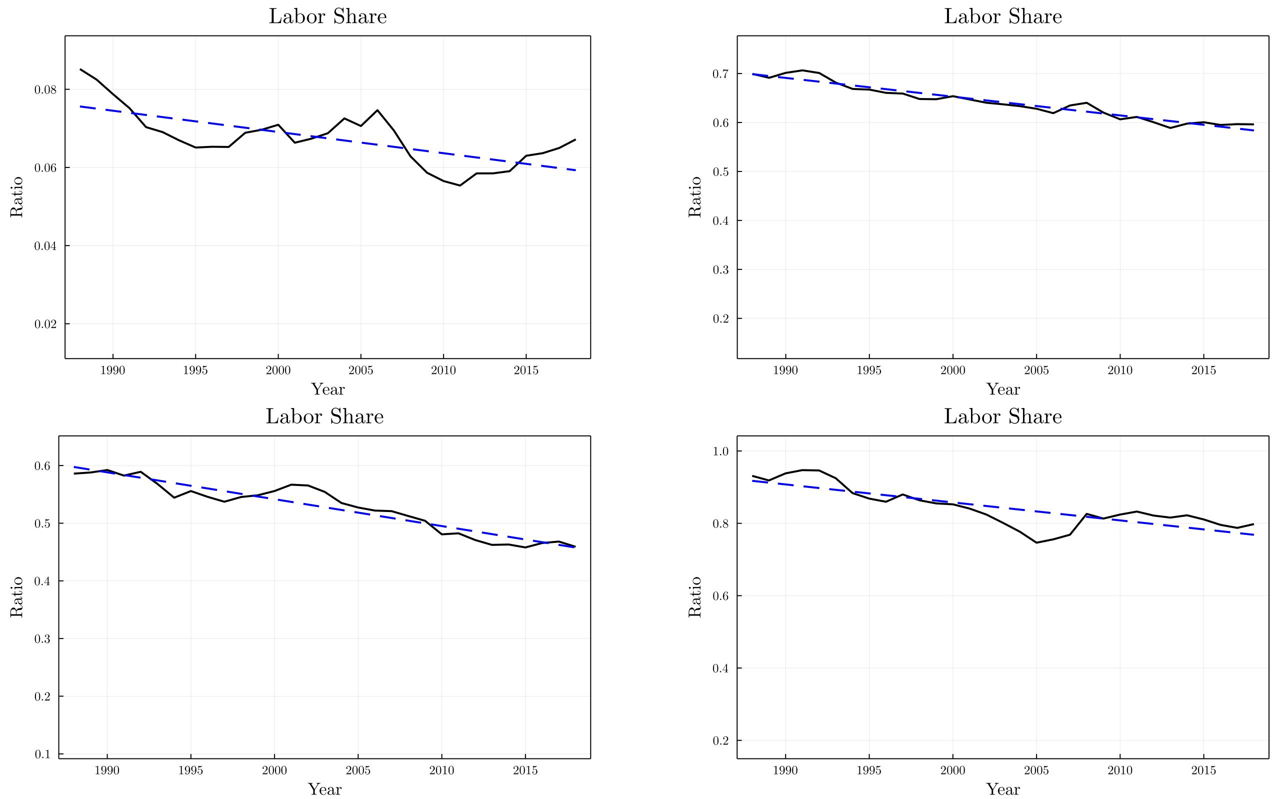

Figure 4: Declining Labor Share four larges industries

ratio in two very dissimilar industries, Construction, and Legal Services. On the one hand, in the construction industry, the ratio of skilled to unskilled workers grew from 0.15 in 1988 to 0.24 (a 60% increase) in 2018 compared to an increase from 2.1 to 5.0 (138%). On the other hand, the skill premium increased from 1.46 to 1.62 (10.2%) in the Construction industry compared to a jump from 1.87 to 2.68 (43%) in Legal Services.

Going beyond this example, I calculate the slope of the labor input for all industries and obtained that for 52 (92.8%) industries the ratio of skilled to unskilled labor grew in the period between 1988 and 2018. I then repeat the process for the skill premium: for 49 industries (87.5%) the skill premium increased in the period, for 84% of industries, both trends are increasing.

The labor input ratio and skill premium trends suggest that the relative increase in the supply of skilled labor does not correlate with the increase in its relative price at the industry level. Therefore the same puzzle that we described at the aggregate level is present when the data is segmented by industry groups: the skill premium is increasing despite the increasing relative supply of skilled labor. This suggests that the demand for skilled workers increases at a faster rate. The capita-skill complementarity hypothesis indicates that technological progress is the main driver of this increase in demand for skilled workers. As in Krusell et al. (2000) I capture technological progress as the decrease of the relative price of equipment capital relative to structure capital, It is natural to check if the trends in the ratio between the two

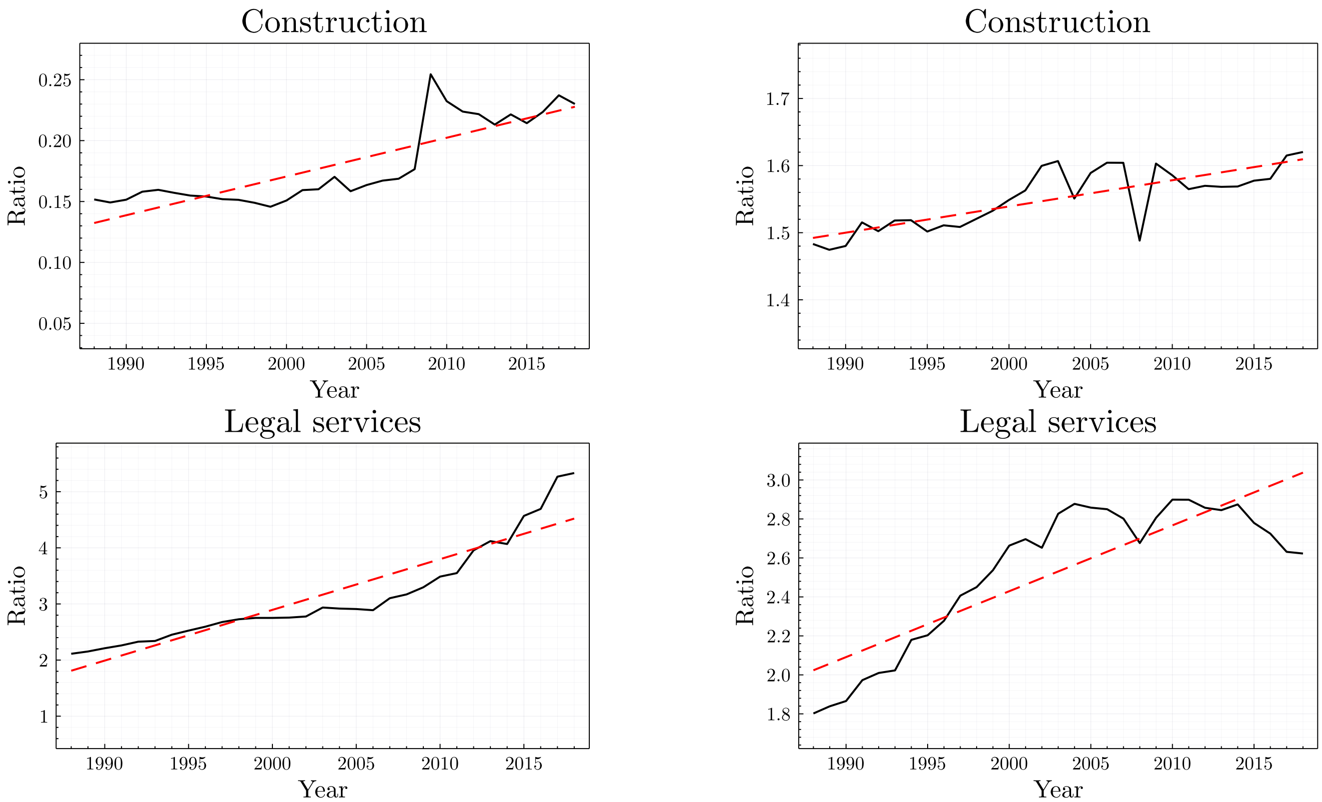

Figure 5: Trends in labor input ratio (*left*) and skill premium (*right*) for selected industries

types of capital follow similar patterns and to explore the relationship between the patterns of substitution between the two types of capital and the patterns of substitution between skilled and unskilled labor as well as the skill premium. All industries (100%) in the sample exhibit increasing patterns of substitution between equipment and structures.

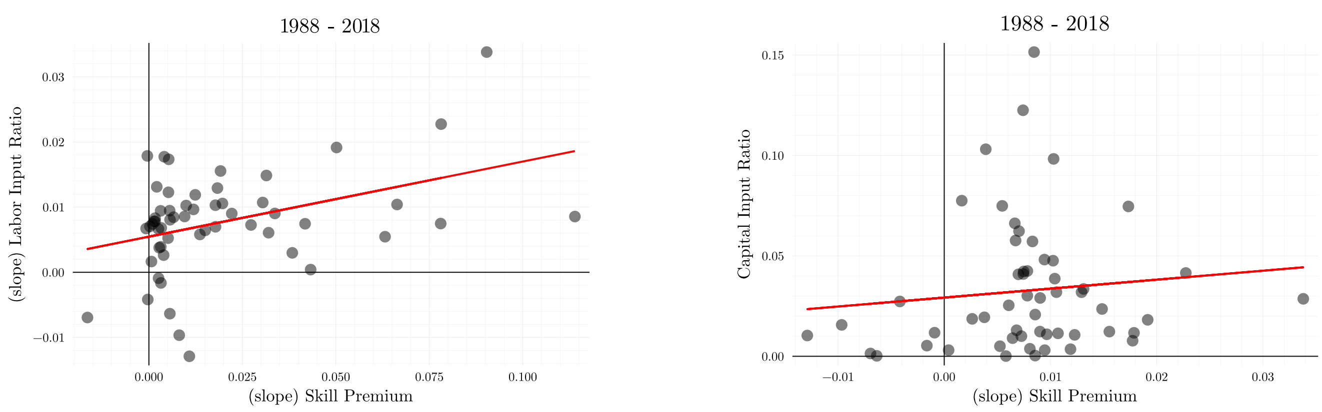

Figure 6: Relationship between the slopes of the Labor Input Ratio, Skill Premium, and Capital Input Ratio.

Figure 6 shows the relationship between the change rate of labor input and capital input with the skill premium. The important thing to notice here is that in both cases the relationship is increasing.

5 Estimation

I follow the same methodology as (Krusell et al., 2000) to estimate the model parameters. To simplify notation from now on I will refer to the unobservable labor efficiencies as, $\psi_t = \{\psi_t^u, \psi_t^s\}$, the inputs of the production function as $X_t = \{k_{s_t}, k_{e_t}, h_{s_t}, h_{u_t}\}$ and the set of parameters to be estimated as $\Phi = \{\alpha, \sigma, \rho, \mu, \lambda, \psi_0^u, \psi_0^s, \eta_\omega\}$.

Firms decide investment in structures based on expectations about future prices q_{t+1}. This is captured using a *"no arbitrage"* condition, firms equate marginal returns on investment on both types of capital. On the one hand marginal return on investment in capital structures is given by given by the sum of the marginal product of structures in $t+1$, $A_{t+1}G_{k_s}(X_{t+1}, \psi_{t+1} \mid \Phi)$ and undepreciated structures on $(1-\delta_s)$. On the other hand marginal return on investment in equipment is given by the sum of the marginal product of equipment in next period, $q_t A_{t+1} G_{k_s}(X_{t+1}, \psi_{t+1} \mid \Phi)$ and depreciated structures $\mathbb{E}(q_t/q_{t+1})(1-\delta_e)$ the term $\mathbb{E}(q_t/q_{t+1})$, as in (Krusell et al., 2000) I make the simplifying assumption of $(1-\delta_e)\mathbb{E}(q_t/q_{t+1}) = (1-\delta_e)(q_t/q_{t+1}) + \nu_t$ where ν_t is normally distributed with mean zero and variance η_ν^2 this parameter is calibrated using data on q_t.[6]. Putting everything together we have the following equation:

$$A_{t+1}G_{k_s}(X_{t+1}, \psi_{t+1} \mid \Phi) = q_t A_{t+1} G_{k_s}(X_{t+1}, \psi_{t+1} \mid \Phi) + (1-\delta_e)\left(\frac{q_t}{q_{t+1}}\right) + \nu_t \tag{10}$$

The other two structural equations used to estimate the model compare the labor share observed in the data to labor share predicted by the model $lsh(X_{t+1}, \psi_{t+1} \mid \Phi)$ and the wage-bill ratio observed in the data to wage-bill ratio predicted by the model $wbr(X_{t+1}, \psi_{t+1} \mid \Phi)$:

$$\frac{w_{s_t}h_{s_t} + w_{u_t}h_{u_t}}{Y_t} = lsh(X_t, \psi_t \mid \Phi) \tag{11}$$

$$\frac{w_{s_t}h_{s_t}}{w_{u_t}h_{u_t}} = wbr(X_t, \psi_t \mid \Phi) \tag{12}$$

Since the parameters $\mu, \lambda, \psi_0^u, \psi_0^s$ act as scaling parameters, to estimate the model one must be fixed, I follow (Krusell et al., 2000) in fixing ψ_0^s, the initial value of the productivity of skilled labor. When replicating their result on an extended sample I choose to fix $\psi_0^u = 6$ as in the original, but choose different variants when estimating each industry to improve fitness. Finally, the parameter η_ω is chosen to minimize the distance between the skill premium in the data and the skill premium predicted by the model.

The estimation process is a simulated two-stage pseudo-maximum likelihood estimation (SPMLE) developed by (White, 1996). Is reasonable that the choice of labor is influenced by the shocks in labor productivity, therefore skilled and unskilled labor is treated as endogenous.

[6]Since I use the same series of relative prices as KORV I take their calibration of $\eta_\nu = 0.02$

To allow for the possible dependence of hours worked on shocks, we use the two-stage SPML developed by, which is similar in spirit to two-stage least squares. We treat skilled and unskilled labor input as endogenous. To deal with the endogeneity, labor input is projected onto a constant, current, and lagged stock of capital equipment and structures, the lagged relative price of equipment, and a trend. The model is estimated using the instrumented labor input series, the series of capital, and prices as the inputs of the model.

The the next stage we proceed as follows: taking the variance η_ω as given, for each date t generate S realizations the stochastic components of the model φ_t use those as inputs to generate S realization of the structural equations (10), (11) and (12), to simplify notation I refer to each of those values as $\tilde{Z}_t^i(X_t, \psi_t \mid \Phi)$, (note that this is a vector of three values, one for each of the equations). Using the simulated data we obtain the first and second moments of the model:

$$m(X_t, \psi_t \mid \Phi) = \frac{1}{S} \sum_{i=1}^{S} \tilde{Z}_t^i(X_t, \psi_t \mid \Phi) \tag{13}$$

and

$$V(X_t, \psi_t \mid \Phi) = \frac{1}{S-1} \sum_{i=1}^{S} \left(\tilde{Z}_t^i(X_t, \psi_t \mid \Phi) - m(X_t, \psi_t \mid \Phi)\right) \left(\tilde{Z}_t^i(X_t, \psi_t \mid \Phi) - m(X_t, \psi_t \mid \Phi)\right)' \tag{14}$$

Finally, we minimize the same objective function as (Krusell et al., 2000):

$$\begin{aligned} \ell\left(Z^T; X_t, \psi_t \mid \Phi\right) = -\frac{1}{2T} \sum_{t=1}^{T} \Big\{ & \left[Z_t - m_S\left(\tilde{X}_t; \phi\right)\right]' \left(V_S\left(\tilde{X}_t; \phi\right)\right)^{-1} \left[Z_t - m_S\left(\tilde{X}_t; \phi\right)\right] \\ & - \log\det\left(V_S\left(\tilde{X}_t; \phi\right)\right) \Big\} \end{aligned} \tag{15}$$

where Z_t is the vector of model counterparts of $\tilde{Z}_t^i(X_t, \psi_t \mid \Phi)$.

6 Results

This section presents the results of the estimation process. I first show the result of the replication of KORV for different periods and then summarize the results of estimating the model for each industry.

6.1 KORV Replication

Table 1 compares the results obtained by Krusell et al. (2000) to this replication using their original data (1963 - 1992) [7]. I also present the estimation on the extended sample (1963 - 2018)

[7] Available at Gianluca Violante's website: http://violante.mycpanel.princeton.edu/Journals/Data_KORV.txt

and on the subset of the extended sample for which there is coverage at the industry level (1988 - 2018).

	KORV Estimation 1963 - 1992	**Replication** 1963 - 1992	**Updated Data** 1963 - 2018	**Updated Data** 1988 - 2018
α	0.117	0.113	0.118	0.08
σ	0.401	0.464	0.503	0.313
ρ	-0.495	-0.56	-0.343	-0.154
η_ω	0.043	0.043	0.083	0.043

Table 1: Parameter estimates KORV model.

First, note that the capital-skill complementarity hypothesis ($\sigma > \rho$) holds for the three samples. When the model is estimated with the full sample I found lower estimates of σ and higher estimates for ρ and α, this is consistent with the replication by Ohanian et al. (2021). Table 2 compares elasticities of substitution implied by the different parameter estimates obtained. When the initial years of the sample (1963) to (1987) are excluded, the estimates suggest that the elasticity of substitution between capital and skilled labor has increased and the elasticity of substitution of unskilled labor has decreased. A possible explanation for why skilled labor has become more substitute with equipment capital is because of the education-based classification of skill. In earlier years tasks assigned to college degree holders were more likely to be tasks that required a higher degree of specialization, while in later years a college degree has become more of a requirement for a job.

	KORV Estimation 1963 - 1992	**Replication** 1963 - 1992	**Updated Data** 1963 - 2018	**Updated Data** 1988 - 2018
σ_s	0.67	0.64	0.74	0.86
σ_u	1.67	1.86	2.01	1.45

Table 2: Implied Elastities of Substitution

Figures 7, 8 and 9 show the fit of the estimation process for three samples described. Note that the model is able to replicate the pattern and shape of the skill premium but fails to generate the volatility present in the Labor Share of Output. The model also struggles to fit the decreasing pattern of the Labor Share in longer samples.

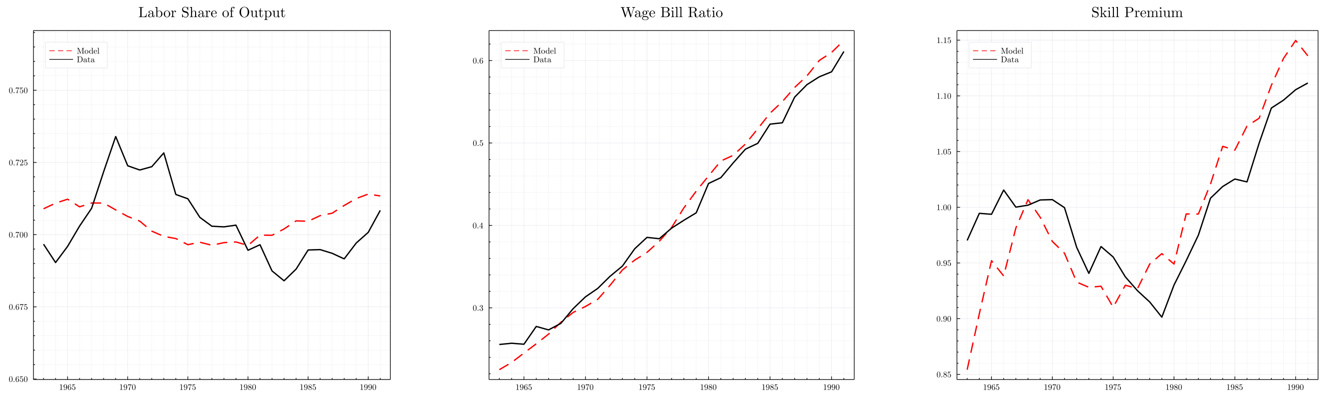

Figure 7: The model Fit for the 1963 - 1992 period with KORV Data.

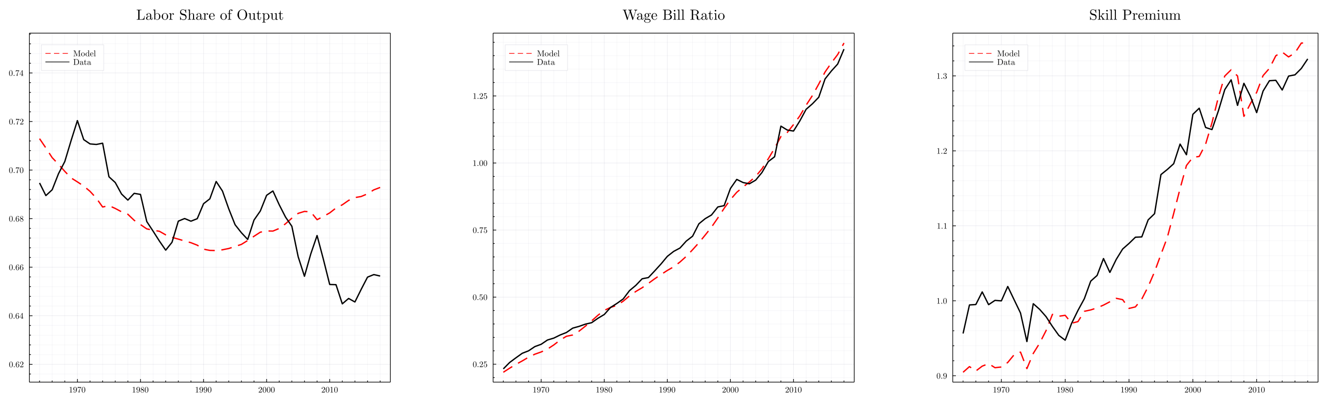

Figure 8: The model Fit for the 1963 - 2018 period with Updated Data.

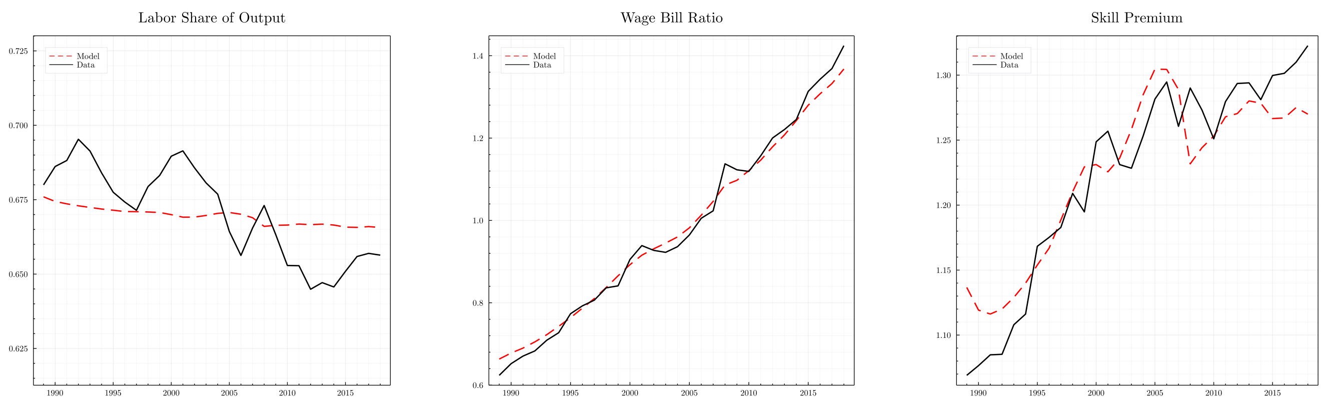

Figure 9: The model Fit for the 1988 - 2018 period with Updated Data.

6.2 Estimation by Industry

Before showing the results of the estimation at the industry level, It is important to discuss the some cavears, the first of which is that the covergence of the model is highly sensitive to the initial conditions. Figure 10 show the fit obtained by for an specific industry (Legal Services)

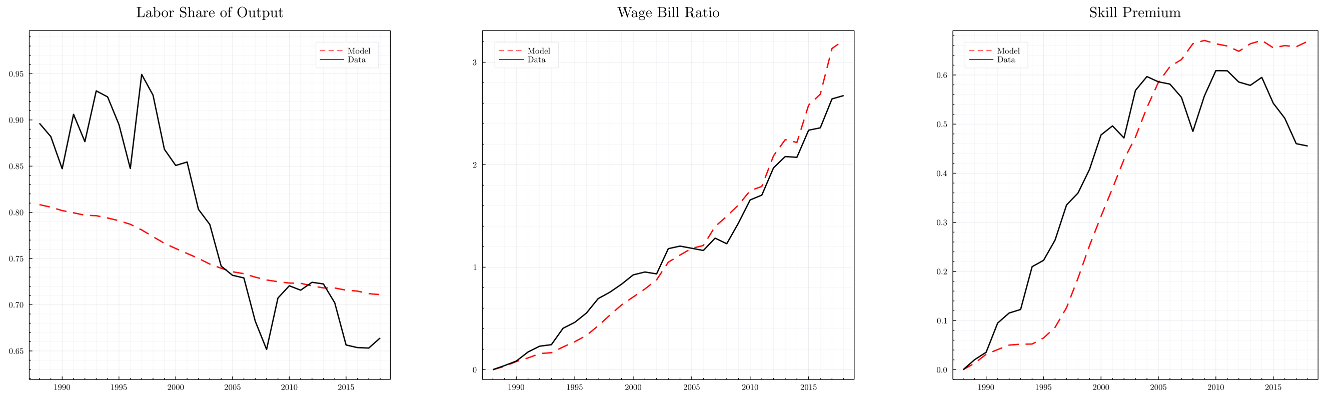

Figure 10: Fit for the 1988 - 2018 period. Legal Services Industry.

Initially I tried to sweep the parameter space to find the best initial conditions, I tried 10 values for each parameter (total of 700 points), but it took me more than a day to run the code. I settled on 10 initial conditions.

Table 3 presents a summary of the results of the estimation process at the industry level.

	Updated Data 1988 - 2018	**Industry Level** (mean)	**Industry Level** (std)
α	0.08	0.241	0.206
σ	0.313	0.483	0.710
ρ	-0.154	-0.289	0.816
η_ω	0.043	0.131	0.195

Table 3: Summary Industry Level Estimates.

On average the Capital-skill complementarity hypothesis ($\sigma > \rho$) is held at the industry level. Specifically, the hypothesis holds for 44 of 56 (78.8%) industries. In general, the point estimates of the parameters show high variance across industries. Although not statistically significant, the strength of the capital-skill complementarity hypothesis, captured as the difference $\sigma - \rho$ increases in industries with higher skill premium and a higher proportion of skilled workers, table 4 summarizes the results of the regression and Figure 11 shows the relationship between the difference $\sigma - \rho$ and the skill premium (left) and labor input ratio (right).

	Capital Skill Cpomplementarity	
	(1)	(2)
(Intercept)	0.842***	0.736***
	(0.144)	(0.124)
Skill Premium	3.779	
	(13.004)	
Labor Input Ratio		7.806
		(4.331)
Estimator	OLS	OLS
N	56	56
R^2	0.002	0.061

Table 4: Relation between the Skill Premium and the Labor Share of Output and the Capital-Skill Complementarity Hypothesis.

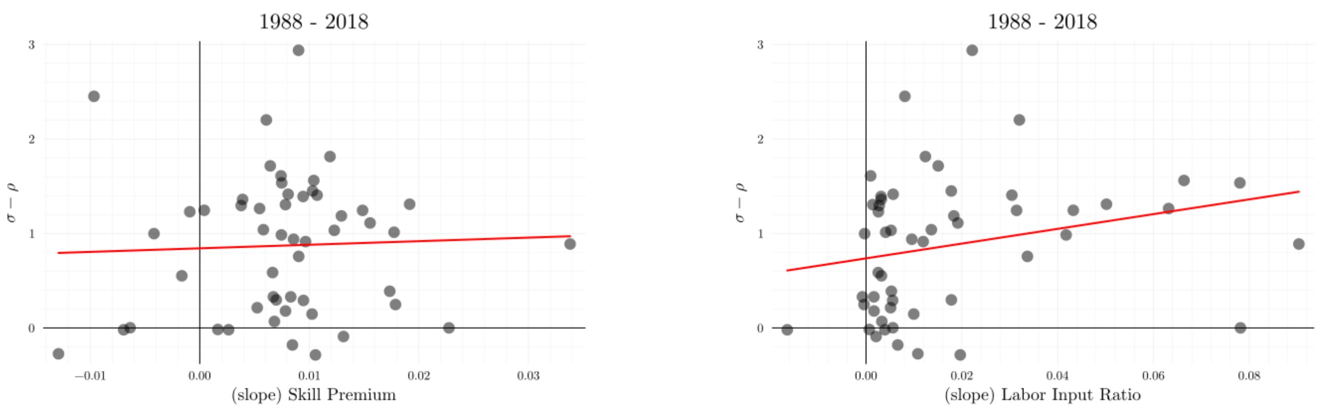

Figure 11: Relationship between the slopes of the Labor Input Ratio, Skill Premium and Capital Input Ratio.

7 Conclusion

References

Acemoglu, D. and D. Autor (2011). Skills, tasks and technologies: Implications for employment and earnings. In *Handbook of labor economics*, Volume 4, pp. 1043–1171. Elsevier.

Acemoglu, D. and P. Restrepo (2020). Unpacking skill bias: Automation and new tasks. In *aea Papers and Proceedings*, Volume 110, pp. 356–61.

Castex, G., S.-W. S. Cho, and E. Dechter (2022). The decline in capital-skill complementarity. *Journal of Economic Dynamics and Control 138*, 104363.

Cooley, T. F. and E. C. Prescott (1995). *Frontiers of business cycle research*, Volume 3. Princeton University Press Princeton, NJ.

Flood, S., M. King, S. Ruggles, and J. R. Warren (2015). Integrated public use microdata series. *Current Population Survey: Version 4*.

Haltiwanger, J. C., H. R. Hyatt, and J. Spletzer (2022). Industries, mega firms, and increasing inequality. Technical report, National Bureau of Economic Research.

Karabarbounis, L. and B. Neiman (2014). The global decline of the labor share. *The Quarterly journal of economics 129*(1), 61–103.

Krusell, P., L. E. Ohanian, J.-V. Ríos-Rull, and G. L. Violante (2000). Capital-skill complementarity and inequality: A macroeconomic analysis. *Econometrica 68*(5), 1029–1053.

Maliar, L., S. Maliar, and I. Tsener (2020). Capital-skill complementarity and inequality: Twenty years after.

Ohanian, L. E., M. Orak, and S. Shen (2021). Revisiting capital-skill complementarity, inequality, and labor share. Technical report, National Bureau of Economic Research.

Orak, M. (2017). Capital-task complementarity and the decline of the us labor share of income. *FRB International Finance Discussion Paper* (1200).

Polgreen, L. and P. Silos (2008). Capital–skill complementarity and inequality: A sensitivity analysis. *Review of Economic Dynamics 11*(2), 302–313.

Song, J., D. J. Price, F. Guvenen, N. Bloom, and T. Von Wachter (2019). Firming up inequality. *The Quarterly journal of economics 134*(1), 1–50.

White, H. (1996). *Estimation, inference and specification analysis*. Number 22. Cambridge university press.

A Ommited Derivations

Recall that the production function is given by:

$$G(k_{s_t}, k_{e_t}, u_t, s_t) = k_{s_t}^{\alpha} \left(\mu u_t^{\sigma} + (1-\mu) \left(\lambda k_{s_t}^{\rho} + (1-\lambda) s_t^{\rho} \right)^{\frac{\sigma}{\rho}} \right)^{\frac{1-\alpha}{\sigma}} \tag{16}$$

Where $u_t = \psi_t^u h_t^u$ and $s_t = \psi_t^s h_t^s$. Relevant first-order conditions are:

$$[h_t^u]: \quad w_{u_t} = (1-\alpha)\mu k_{s_t}^{\alpha} \left(\mu u_t^{\sigma} + (1-\mu) \left(\lambda k_{s_t}^{\rho} + (1-\lambda) s_t^{\rho} \right)^{\frac{\sigma}{\rho}} \right)^{\frac{1-\alpha}{\sigma}-1} u_t^{\sigma-1} \tag{17}$$

$$\begin{aligned}[h_t^s]: \quad w_{s_t} = (1-\alpha)(1-\mu)(1-\lambda) & \left(\mu u_t^{\sigma} + (1-\mu) \left(\lambda k_{s_t}^{\rho} + (1-\lambda) s_t^{\rho} \right)^{\frac{\sigma}{\rho}} \right)^{\frac{1-\alpha}{\sigma}-1} \\ & \left(\lambda k_{s_t}^{\rho} + (1-\lambda) s_t^{\rho} \right)^{\frac{\sigma}{\rho}-1} s_t^{\rho-1} \psi_t^s \end{aligned} \tag{18}$$

Dividing (18) by (17) we obtain the expression for the skill premium:

$$\begin{aligned} \omega_t &= \frac{(1-\lambda)(1-\mu)}{\mu u_t^{\sigma-1}} u_t^{\sigma-1} (\lambda k_{s_t}^{\rho} + (1-\lambda) s_t^{\rho})^{\frac{\sigma}{\rho}-1} s_t^{\rho-1} \frac{\psi_t^s}{\psi_t^u} \\ &= \frac{(1-\lambda)(1-\mu)}{\mu} \left(\lambda \left(\frac{k_{s_t}}{s_t} \right)^{\rho} + (1-\lambda) \right)^{\frac{\sigma}{\rho}-1} \left(\frac{s_t}{u_t} \right)^{\sigma-1} \frac{\psi_t^s}{\psi_t^u} \\ &= \frac{(1-\lambda)(1-\mu)}{\mu} \left(\lambda \left(\frac{k_{s_t}}{s_t} \right)^{\rho} + (1-\lambda) \right)^{\frac{\sigma}{\rho}-1} \left(\frac{h_t^u}{h_t^s} \right)^{1-\sigma} \left(\frac{\psi_t^s}{\psi_t^u} \right)^{\sigma} \end{aligned} \tag{19}$$

To obtain a version of skill premium in terms of growth rates of the log-linearized version of equation (19), start by writing a continuous time version of Equaiton (6):

$$\ln \omega(t) = \lambda \frac{\sigma - \rho}{\rho} \left(\frac{k_e(t)}{\psi^s(t) h^s(t)} \right)^{\rho} + (1-\sigma) \ln \left(\frac{h^u(t)}{h^s(t)} \right) + \sigma \ln \left(\frac{\psi^s(t)}{\psi^u(t)} \right) \tag{20}$$

Start with the first term of the sum in the RHS:

$$\begin{aligned} \frac{\partial}{\partial t} \left(\left(\frac{k_e(t)}{\psi^(t) h^s(t)} \right)^{\rho} \right) &= \rho \left(\frac{k_e'(t)}{\psi^s(t) h^s(t)} - k_e(t) \frac{\psi'^s(t) h^s(t) + \psi^s(t) h'^s(t)}{(\psi^s(t) h^s(t))^2} \right) \left(\frac{k_e(t)}{\psi^s(t) h^s(t)} \right)^{\rho-1} \\ &= \rho \left(\frac{k_e'(t) k_e(t)}{k_e(t)(\psi^s(t) h^s(t)} - k_e(t) \frac{\psi'^s(t) h^s(t) + \psi^s(t) h'^s(t)}{(\psi^s(t) h^s(t))^2} \right) \left(\frac{k_e(t)}{\psi^s(t) h^s(t)} \right)^{\rho-1} \\ &= \rho \left(\frac{k_e(t)}{\psi^s(t) h^s(t)} \right)^{\rho} \left(\frac{k_e'(t)}{k_e(t)} - \frac{\psi'^s(t) h^s(t) + \psi^s(t) h'^s(t)}{\psi^s(t) h^s(t)} \right) \\ &= \rho \left(\frac{k_e(t)}{\psi^s(t) h^s(t)} \right)^{\rho} \left(\frac{k_e'(t)}{k_e(t)} - \frac{\psi'^s(t)}{\psi^s(t)} - \frac{h'^s(t)}{h^s(t)} \right) \\ &= \rho \left(\frac{k_e(t)}{\psi^s(t) h^s(t)} \right)^{\rho} (g_{k_{e_t}} - g_{\psi_t^s} - g_{s_t}) \end{aligned}$$

The next two terms in the sum are very similar, for the first term we have:

$$\frac{\partial}{\partial t}\left(\ln\left(\frac{h^u(t)}{h^s(t)}\right)\right)=\frac{\partial}{\partial t}\left(\ln h^u(t)-\ln\left(h^s(t)\right)\right)=\frac{h'^u(t)}{h^u(t)}-\frac{h'^s(t)}{h^s(t)}=g_{u_t}-g_{s_t} \tag{21}$$

Differentiating the LHS we get:

$$\frac{\partial}{\partial t}\left(\ln\omega(t)\right)=\frac{\omega'(t)}{\omega(t)}=g_{\omega_t}$$

B Data Construction

B.1 Labor Inputs and Wage Rates

I include all observations excluding agents: younger than 16 or older than 70, unpaid family workers, those working in the military, those who report working less than 40 weeks a year and/or 35 hours a week, individuals with allocated income, those with hourly wages below half of the minimum federal wage rate, those did not report their education level and self-employed workers.

For each person, I record their characteristics age, sex, and race. Their employment statistics: employment status (`empstat`), class of worker (`classwly`), weeks worked last year (`wkswork1` and `wkswork2`), usual hours worked per week last year (`uhrsworkly` and hours work last week `ahrsworkt`). Their income: total wage and salary income `incwage` and the CPS personal supplement weights: `asecwt`.

To homogenize the data I create the following groups based on individual characteristics, age is divided into 11 five-year groups: $16-20, 21-25, 26-30, 31-35, 36-40, 41-45, 46-50$ race is divided into white black and others. Sex is divided into, male and female and education is divided into four groups without high school, high school, some college and college graduates, and beyond. Then, each person is assigned to one of 264 groups created by age, race, sex, and skill (education).

For the period between 1963 to 1975 the variables weeks worked last year (`wkswork1`) and hours worked last year (`uhrsworkly`) are not recorded so I did the following substitution: For `wkswork1` we can use the variable `wkswork2` that consist of intervals of hours worked, and then perform the substitution with the average hours worked by individuals in the same group reporting the same value of `wkswork2` for the post 1975 period. For `uhrsworkly` I used hours worked last week (`ahrsworkt`) as a proxy.

For every individual I create the following variables:

- $\ell_{i,t}$ the hours worked by individual i in year t, is the product of hours worked per week times weeks worked that year.

- $w_{i,t}$ the hourly wage of individual i in year t, obtained by dividing yearly wage income by hours worked in year t.

Let $\mathcal{G}$ be the collection of all groups we can calculate the weight of each group as $\mu_{g,t} = \sum_{i \in g} \mu_{i,t}$ where $\mu_{i,t}$ is the CPS weight of the individual. Average hours worked for each group $g \in \mathcal{G}$:

$$\ell_{g,t-1} = \frac{\sum_{i \in g} \ell_{i,t-1} \mu_{i,t}}{\mu_{g,t}}$$

and wages:

$$w_{g,t-1} = \frac{\sum_{i \in g} w_{i,t-1} \mu_{i,t}}{\mu_{g,t}}$$

Finally to obtain skilled and unskilled series labor input and wage series I partitioned the set $\mathcal{G}$ in two subsets $(\mathcal{S}, \mathcal{U})$ based on education (college graduates and non-college graduates). Let $\{H, L\}$ indicate the group type, then the total labor input is:

$$L^U_{t-1} = \sum_{g \in \mathcal{U}} \ell_{g,t-1} \mu_{g,t} w_{g,80}$$

$$L^S_{t-1} = \sum_{g \in \mathcal{S}} \ell_{g,t-1} \mu_{g,t} w_{g,80}$$

$w_{g,80}$ is the wage of the group in 1980 and is used as a scaling factor. Then wages for each skill level are obtained as:

$$W^U_{t-1} = \frac{\sum_{g \in \mathcal{U}} w_{g,t-1} \ell_{g,t-1} \mu_{g,t}}{L^U_{t-1}}$$

$$W^S_{t-1} = \frac{\sum_{g \in \mathcal{S}} w_{g,t-1} \ell_{g,t-1} \mu_{g,t}}{L^S_{t-1}}$$

C Industry Trends

D Industry Codes