The Value of Flexibility: Teleworkability, Sorting, and the Work-From-Home Wage Gap

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- ☐ Update abstract.

1 Introduction

- ☐ Update introduction.

2 Empirical Motivation: Remote Work, Wages, and Skills

- ☐ Update empirical motivation.

3 Model

This section develops a search and matching model of the labor market with two-sided heterogeneity and endogenous remote work arrangements. The model builds on the canonical ADD MAIN MODEL CITATION framework, extended to include heterogeneity in worker skills and firm capacity for remote work. This framework allows us to analyze how sorting patterns, wage differentials, and aggregate labor market outcomes are shaped by the trade-offs between productivity and the non-pecuniary benefits of remote work.

3.1 Environment

The economy is populated by a continuum of infinitely lived workers and a continuum of firms. Time is discrete and the horizon is infinite. Both workers and firms are risk-neutral and discount the future at a common rate β.

3.1.1 Agents

Workers are indexed by their skill level, $h \in \mathcal{H}$, which is exogenously distributed according to the cumulative distribution function $F_h(h)$ I NEED TO THINK ABOUT THIS MAYBE I DO UNIFORM ON THIS SIDE OF THE MARKET AND PRODUCTIVITY IS BASED ON RANK. Workers derive utility from their wage, w, and the fraction of time they work remotely, $\alpha \in$. The instantaneous utility function is given by $u(w, \alpha)$. We assume utility is quasi-linear to ensure that the choice of α is independent of the wage transfer, which simplifies the bargaining process. Specifically, we adopt the functional form:

$$u(w, \alpha) = w - c(1 - \alpha)$$

where $c(1 - \alpha)$ represents the disutility from working in the office for a fraction $1 - \alpha$ of the time. ADD PROPERTIES OF COST FUNCTION

Firms are indexed by their remote work efficiency, $\psi \in \Psi$, which is exogenously distributed according to the cumulative distribution function $F_\psi(\psi)$. Firms produce a single homogeneous good. The cost of posting v vacancies is given by an increasing and convex function $c(v)$.

Production Technology: output is generated by a match between a worker of skill h and a firm with remote efficiency ψ. The production function depends on the share of remote work, α:

$$Y(\alpha \mid h, \psi) = A(h) \cdot [(1 - \alpha) + \alpha \cdot g(h, \psi)]$$

where $A(h)$ is the baseline productivity of a worker with skill h, and $g(h, \psi)$ is the productivity of remote work relative to in-person work, which is normalized to one. We assume $A'(h) > 0$, so higher-skilled workers are more productive.

The relative productivity of remote work, $g(h, \psi)$, is a key feature of the model, capturing complementarities between worker skill and firm technology. We assume $g_\psi(h, \psi) \geq 0$ and $g_h(h, \psi) \geq 0$, meaning remote work is more effective in firms with higher remote efficiency and for higher-skilled workers. The sign of the cross-partial derivative, $g_{h\psi}$, will determine the nature of sorting in the market.

3.1.2 Searching and Matching

The labor market is characterized by search frictions. Let $u(h)$ be the measure of unemployed workers of skill h, and $v(\psi)$ be the number of vacancies posted by firms of type ψ. The aggregate measures of unemployed workers and vacancies are given respectively by:

$$L = \int u(h) dF_h(h) \quad \text{and} \quad V = \int v(\psi) dF_\psi(\psi)$$

Meetings between unemployed workers and vacancies are governed by a constant returns to scale matching function, $M(L, V)$. The rate at which an unemployed worker contacts a vacancy is $p(\theta) = M(L, V)/L$, and the rate at which a vacancy is filled is $q(\theta) = M(L, V)/V$, where $\theta = V/L$ is the labor market tightness. The probability that a searching worker meets a firm of a specific type ψ is given by the proportion of vacancies of that type, $v(\psi)/V$.

3.2 Value Functions and Bargaining

We now define the value functions for workers and firms and describe the bargaining process that determines wages and work arrangements.

3.2.1 Value of Unemployment and Vacancies

An unemployed worker of skill h receives unemployment benefits $b(h)$ and searches for a job. The value of being unemployed, $U(h)$, is described by the following Bellman equation:

$$U(h) = b(h) + \beta \mathbb{E} \left[\max\{W(h, \psi), U(h)\} \right].$$

where $W(h, \psi)$ is the value to a worker of being employed in a match with a firm of type ψ. The expectation is taken over the distribution of vacancies the worker might encounter. Assuming workers get a share ξ of the match surplus, $S(h,\psi)$, through Nash bargaining, we have:

$$W(h,\psi) = U(h) + \xi S(h,\psi).$$

The value of unemployment can then be written as:

$$U(h) = b(h) + \beta \left[U(h) + p(\theta)\xi \int S(h,\psi)^+ d\Gamma_v(\psi) \right]$$

where $\Gamma_v(\psi)$ is the endogenous distribution of vacancies and $S(h,\psi)^+ = \max\{S(h,\psi), 0\}$. Solving for $U(h)$ yields:

$$U(h) = \frac{b(h) + \beta p(\theta)\xi \int S(h,\psi)^+ d\Gamma_v(\psi)}{1-\beta}$$

The value of a vacant position is zero due to a free-entry condition, which will be detailed in the vacancy creation section.

3.2.2 Value of a Match and Surplus Sharing

A match between a worker h and a firm ψ generates a flow of output and utility. Matches are subject to an exogenous destruction shock with probability δ. We assume efficient bargaining: the firm and worker first choose the remote work share α to maximize the joint value of the match, and then use the wage w to divide the resulting surplus.

Given the quasi-linear utility, the total flow value generated by a match is the sum of output net of the worker's disutility from in-office work: $Y(\alpha \mid h,\psi) - c(1-\alpha)$. The total surplus of the match, $S(h,\psi)$, is the difference between the value of the match, $J(h,\psi)$, and the worker's outside option, $U(h)$. The surplus evolves according to the Bellman equation:

$$S(h,\psi) = s(h,\psi) + \beta(1-\delta)S(h,\psi) - \beta p(\theta)\xi \int S(h,\psi')^+ d\gamma_v(\psi')$$

where $s(h,\psi)$ is the flow surplus, defined as the maximized joint value net of the worker's flow value of unemployment:

$$s(h,\psi) = \max_{\alpha\in} \{Y(\alpha \mid h,\psi) - c(1-\alpha)\} - b(h)$$

Solving for the total match surplus gives:

$$S(h,\psi) = \frac{s(h,\psi) - \beta p(\theta)\xi \int S(h,\psi')^+ d\gamma_v(\psi')}{1-\beta(1-\delta)}$$

This equation highlights that the value of a match depends not only on its own flow surplus but also on the worker's expected future gains from searching while unemployed.

3.3 Equilibrium

An equilibrium in this economy consists of a set of value functions ($U(h)$, $S(h,\psi)$), policy functions for remote work ($\alpha^*(h,\psi)$) and wages ($w^*(h,\psi)$), vacancy posting decisions ($v(\psi)$), and a distribution of unemployed workers ($u(h)$) and employed workers ($n(h,\psi)$), such that: 1. The policy functions $\alpha^*(h,\psi)$ and $w^*(h,\psi)$ are determined by efficient bargaining. 2. Firms' vacancy creation decisions $v(\psi)$ are optimal. 3. The labor market is in a steady state, where the flows of workers into and out of unemployment are balanced.

3.3.1 Optimal Work Arrangements and Wages

The optimal remote work share, $\alpha^*(h,\psi)$, is chosen to maximize the flow surplus. The first-order condition for an interior solution is $Y'(\alpha) = c'(\alpha)$. This condition equates the marginal gain in output from an additional unit of in-person work to the marginal disutility for the worker. Analyzing this condition at the boundaries $\alpha = 0$ and $\alpha = 1$ defines two skill-dependent thresholds for firm efficiency, $\underline{\psi}(h)$ and $\overline{\psi}(h)$, which partition the market into three regimes:

$$\alpha^*(h,\psi) = \begin{cases} 0 & \text{if } \psi \leq \underline{\psi}(h) \quad \text{(Full In-Person)} \\ \alpha_{\text{interior}}(h,\psi) & \text{if } \underline{\psi}(h) < \psi < \overline{\psi}(h) \quad \text{(Hybrid)} \\ 1 & \text{if } \psi \geq \overline{\psi}(h) \quad \text{(Full Remote)} \end{cases}$$

Under the assumption of efficient bargaining, the remote work share, $\alpha^*(h,\psi)$, is chosen to maximize the total joint value of the match. The wage, w, is then determined as a purely distributional transfer to ensure that the total surplus, $S(h,\psi)$, is divided according to the exogenously given bargaining powers, ξ for the worker and $1-\xi$ for the firm.

The wage is set to deliver the worker their promised lifetime value from the match, $W(h,\psi)$. This value is composed of their outside option, the value of unemployment $U(h)$, plus their share of the total match surplus.

With the optimal physical arrangement $\alpha^*(h,\psi)$ determined, we can solve for the wage $w^*(h,\psi)$ that supports this equilibrium. The wage must be set such that the worker's value from the match equals their outside option plus their bargained share of the surplus. This implies that the wage must deliver the required utility flow and compensate for any in-office disutility.

$$w^*(h,\psi) = \underbrace{(W(h,\psi) - \beta\mathbb{E}[W'])}_{\text{Required utility flow}} + \underbrace{c(\alpha^*(h,\psi))}_{\text{Compensation for in-office work}}$$

3.3.2 Vacancy Creation

Firms post vacancies up to the point where the marginal cost of posting equals the expected marginal benefit. The expected benefit is the probability of filling a vacancy, $q(\theta)$, multiplied by the firm's share of the expected surplus from a new match. The free-entry condition is therefore:

$$c'(v(\psi)) = q(\theta)(1-\xi)\int S(h,\psi)^+ \frac{u(h)}{L} dF_h(h)$$

This condition determines the number of vacancies $v(\psi)$ for each firm type, which in turn determines the aggregate market tightness θ and the distributions of job-finding and job-filling probabilities.

3.3.3 Steady-State Flows

In a steady-state equilibrium, the mass of workers of each type and in each state (employed or unemployed) is constant. This requires that the flow of workers out of unemployment equals the flow into unemployment. The flow out of unemployment for workers of skill h is:

$$p(\theta)u(h)\int \mathbb{1}_{\{S(h,\psi)>0\}} d\Gamma_v(\psi).$$

It is the product of the mass of unemployed workers of skill h , and their effective job-finding rate, which is the probability of meeting any firm $p(\theta)$, multiplied by the probability that a meeting results in a mutually agreeable match (i.e., one with positive surplus, $S(h,\psi) > 0$). The flow into unemployment is the sum of all job destructions:

$$\delta \int n(h,\psi) dF_\psi(\psi).$$

The steady-state condition (Beveridge curve) is given by equating the aggregate flows:

$$\delta \iint n(h,\psi) dF_\psi(\psi) dF_h(h) = p(\theta) \int u(h) \left(\int \mathbb{1}_{\{S(h,\psi)>0\}} d\gamma_v(\psi) \right) dF_h(h)$$

The equilibrium is a fixed point where firms' vacancy decisions are optimal given expected surpluses, and these surpluses are consistent with the hiring probabilities that result from those same vacancy decisions.

4 Calibration

4.1 Quantitative Specification and Parameterization

To take the model to the data and analyze its quantitative implications, we must specify functional forms for the model's key components and assign numerical values to the resulting parameters. We divide the parameters into two groups: those set externally based on standard values in the literature, and those calibrated internally to match key moments of the U.S. labor market data.

4.1.1 Functional Forms

NEED TO SETTLE ON ONE FUNCTIONAL FORM FOR THE $g(h,\psi)$ FUNCTION

Production and Cost Functions: We adopt specific functional forms for the production and worker cost functions to derive an analytical solution for the optimal work arrangement, α^*.

- **In-Office Cost Function:** The worker's disutility from in-office work is assumed to be a convex function of the time spent in the office, $(1-\alpha)$.

$$c(1-\alpha) = c_0 \frac{(1-\alpha)^{1+\chi}}{1+\chi}$$

  The parameter $c_0 > 0$ scales the overall disutility, and the curvature parameter $\chi > 0$ ensures that the marginal disutility of in-office time is increasing, a standard feature that helps ensure well-behaved interior solutions for α.

Production Function: The baseline productivity, $A(h)$, is assumed to be linear in worker skill:

$$A(h) = A_1 h$$

The relative productivity of remote work, $g(h, \psi)$, is specified using a power function to capture the interaction between worker and firm characteristics in determining remote output:

$$g(h, \psi) = \psi_0 h^{\phi} \psi^{\nu}$$

Here, A_1 is a productivity scalar. In the remote productivity function, ψ_0 is a scaling factor, while ϕ and ν are the elasticities of remote productivity with respect to worker skill (h) and firm efficiency (ψ), respectively. The parameter ϕ is particularly important as it governs whether skill and firm technology are complements ($\phi > 0$) or substitutes ($\phi < 0$) in remote production.

These functional forms allow us to analytically characterize the thresholds that govern the optimal work arrangement. The upper threshold, $\overline{\psi}(h)$, which separates hybrid from full-remote work, is strictly decreasing in worker skill whenever skill and technology are complements ($\phi > 0$). This provides the intuitive result that higher-skilled workers require a lower level of firm efficiency to make full remote work optimal.

The shape of the lower threshold, $\underline{\psi}(h)$, is richer, as it is determined by a tension between the rising opportunity cost of remote work (the loss of a more productive worker from the office) and the skill-remote interaction effect. The nature of this trade-off depends critically on the skill-remote elasticity, ϕ:

- **When skill and technology are complements ($\phi > 0$):** The lower threshold has an **inverted U-shape**. For low-skill workers, the rising opportunity cost dominates, making the threshold increase with skill. For high-skill workers, the powerful complementarity effect dominates, making the threshold decrease with skill. This implies that middle-skill workers face the highest efficiency requirement to be offered a hybrid arrangement.

- **When skill and technology are weak substitutes ($-1 < \phi < 0$):** The lower threshold is **strictly increasing**. As a worker's skill increases, they become both more valuable in the office (higher opportunity cost) and relatively less effective remotely (due to substitutability). Both forces push in the same direction, requiring a progressively higher level of firm efficiency to justify a hybrid offer.

- **When skill and technology are strong substitutes ($\phi < -1$):** The lower threshold has a **U-shape**. For low-skill workers, the strong substitutability effect is dominant, causing the threshold to fall with skill. For high-skill workers, the opportunity cost effect eventually dominates again, causing the threshold to rise. This implies that middle-skill workers are the *most* likely to receive a hybrid offer.

This rich set of possibilities highlights the flexibility of the framework in capturing diverse sorting patterns. The full mathematical derivation of these cases and the formulas are presented in Section A.1

Matching and Vacancy Costs:

- We assume a standard Cobb-Douglas matching function, which exhibits constant returns to scale.

$$M(L, V) = \gamma_0 L^{\gamma_1} V^{1-\gamma_1}$$

where $\gamma_0 > 0$ is matching efficiency and $\gamma_1 \in (0, 1)$ is the elasticity of matches with respect to unemployment.

4.1.2 Firm's Vacancy Posting Problem

A firm of type ψ chooses the number of vacancies v to post to maximize its expected profit. The profit function is the expected benefit minus the total cost:

$$\Pi(v(\psi)) = qB(\psi)v - c(v)$$

where: * $B(\psi) = (1-\xi)\mathbb{E}_h[S(h, \psi)^+]$ is the firm's expected benefit *per filled vacancy*. * **Vacancy Cost Function:** The cost of posting vacancies is assumed to be a convex function, implying a rising marginal cost to posting.

$$c(v) = \frac{\kappa_0 v^{1+\kappa_1}}{1+\kappa_1}$$

where $\kappa_0 > 0$ is a cost scaling parameter and $\kappa_1 > 0$ ensures convexity.

The firm's first-order condition (FOC) for profit maximization is $\Pi'(v) = 0$, which implies that the marginal benefit must equal the marginal cost:

$$\Pi'(v) = 0 \quad \implies \quad qB(\psi) = c'(v) \quad \implies \quad qB(\psi) = \kappa_0 v(\psi)^{\kappa_1}$$

This condition allows us to derive an equilibrium condition for θ. Details of this derivation are provided in Section A.2.

4.1.3 Calibration

We choose the parameter values to align the model's steady state with key features of the pre-pandemic U.S. labor market. The model is calibrated at a monthly frequency. The parameters are summarized in Table 1.

Externally Set Parameters: Several parameters are set to standard values from the literature. The discount factor β is set to 0.996, corresponding to a 5% annual real interest rate. The elasticity of the matching function, η, is set to 0.5, a common value in the search and matching literature (Petrongolo and Pissarides, 2001).

Internally Calibrated Parameters: The remaining parameters are calibrated jointly to match a set of moments from the data.

EXAMPLE TEXT "The parameters of the production function (A_1, ν, ψ_0, ϕ) and the in-office cost function (c_0, χ) are chosen to match the average labor productivity, the observed share of workers in remote/hybrid arrangements, and the estimated compensating wage differential for remote work from survey evidence (e.g., Barrero et al., 2023a). The vacancy cost parameters (c_v, γ) and matching efficiency (M_0) are disciplined by the average labor market tightness (θ), the job-filling rate (q), and the elasticity of vacancies to productivity shocks…"

EXAMPLE TABLE

Table 1: Parameter Values

Parameter	**Description**	**Value**	**Target/Source**
Preferences & Technology			
β	Discount Factor	0.996	5% annual interest rate
χ	Curvature of in-office cost	1.5	Internally calibrated
c_0	In-office cost scale	0.25	Match compensating differential
A_1	Productivity scale	1.0	Normalization
ϕ	Skill-remote complementarity	0.1	Match skill premium
…	…	…	…
Search & Matching			
η	Matching elasticity	0.5	Petrongolo & Pissarides (2001)
δ	Exogenous separation rate	0.025	Match avg. job duration
ξ	Worker bargaining power	0.5	Shimer (2005)
…	…	…	…

5 Results

6 Conclusion

☐ Update conclusion. This is an example reference: (Aksoy et al. 2023)

References

Aksoy, Cevat Giray, Jose Maria Barrero, Nicholas Bloom, Steven Davis, Mathias Dolls, and Pablo Zarate. 2023. "Time Savings When Working from Home." Workign Paper. https://doi.org/10.3386/w30866.

A Appendix

A.1 Derivation of the Optimal Work Arrangement

This appendix derives the optimal remote work policy, $\alpha^*(h, \psi)$, and analyzes the properties of the threshold functions that partition the labor market into distinct work arrangement regimes.

A.1.1 The Maximization Problem

Under the assumption of efficient bargaining, the firm and worker choose the remote work share $\alpha \in$ to maximize the total joint flow value of the match. With quasi-linear utility, this simplifies to maximizing the sum of output $Y(\alpha)$ minus the worker's in-office disutility $c(1-\alpha)$:

$$\max_{\alpha \in [0,1]} \quad \Big\{ Y(\alpha \mid h, \psi) - c(1-\alpha) \Big\}$$

To solve this problem, we use the functional forms specified in the main text:

- **Production Function:** $Y(\alpha \mid h, \psi) = A(h) \cdot [(1-\alpha) + \alpha \cdot g(h, \psi)]$
- **Baseline Productivity:** $A(h) = A_1 h$
- **Relative Remote Productivity:** $g(h, \psi) = \psi_0 h^{\phi} \psi^{\nu}$
- **In-Office Cost Function:** $c(1-\alpha) = c_0 \frac{(1-\alpha)^{1+\chi}}{1+\chi}$

The First-Order Condition (FOC) is found by differentiating the joint surplus $S(\alpha)$ with respect to α and setting it to zero.

- **Derivative of Output:** $\frac{dY}{d\alpha} = A_1 h(g(h, \psi) - 1)$
- **Derivative of Cost:** $\frac{d}{d\alpha}\left(-c(1-\alpha)\right) = -\left(c_0(1-\alpha)^{\chi} \cdot (-1)\right) = c_0(1-\alpha)^{\chi}$

The FOC is therefore:

$$A_1 h(g(h, \psi) - 1) + c_0(1-\alpha)^{\chi} = 0$$

Rearranging this gives the fundamental equation for an interior solution:

$$A_1 h(1 - g(h, \psi)) = c_0(1-\alpha)^{\chi}$$

For an interior solution to exist, the left-hand side must be positive, which requires $1-g(h,\psi) > 0$, or $g(h,\psi) < 1$. This means remote work must be less productive than in-person work for a hybrid arrangement to be optimal.

A.1.2 Optimal Policy and Thresholds

Interior Solution

Solving the FOC for the interior solution $\alpha_{\text{interior}} \in (0,1)$:

$$\alpha_{\text{interior}}(h,\psi) = 1 - \left[\frac{A_1 h\,(1-g(h,\psi))}{c_0}\right]^{\frac{1}{\chi}} = 1 - \left[\frac{A_1 h\,(1-\psi_0 h^{\phi}\psi^{\nu})}{c_0}\right]^{\frac{1}{\chi}}$$

Threshold Derivations

The boundaries of the hybrid region are determined by analyzing the derivative of the surplus, $\frac{d(Y-c)}{d\alpha}$, at the corners $\alpha = 0$ and $\alpha = 1$.

1. **Upper Threshold, $\overline{\psi}(h)$:** The boundary for full remote work ($\alpha^* = 1$) is where

$$\begin{aligned} A_1 h(g(h,\overline{\psi}(h)) - 1) + c_0(1-1)^{\chi} &= 0 \\ A_1 h(g(h,\overline{\psi}(h)) - 1) &= 0 \\ \implies \overline{\psi}(h) &= \psi_0^{-1/\nu} h^{-\phi/\nu} \end{aligned}$$

2. **Lower Threshold, $\underline{\psi}(h)$:** The boundary for full in-person work ($\alpha^* = 0$) is where

$$
\begin{aligned}
A_1 h(g(h, \underline{\psi}(h)) - 1) + c_0(1-0)^{\chi} &= 0 \\
A_1 h(g(h, \underline{\psi}(h)) - 1) &= -c_0 \\
g(h, \underline{\psi}(h)) &= 1 - \frac{c_0}{A_1 h} \\
\psi_0 h^{\phi} [\underline{\psi}(h)]^{\nu} &= 1 - \frac{c_0}{A_1 h} \\
\implies \underline{\psi}(h) &= \left(\frac{1}{\psi_0 h^{\phi}} \left(1 - \frac{c_0}{A_1 h} \right) \right)^{1/\nu} \\
\implies \underline{\psi}(h) &= \overline{\psi}(h) \left(1 - \frac{c_0}{A_1 h} \right)^{1/\nu}
\end{aligned}
$$

A.1.3 Monotonicity and Properties

Monotonicity of the Thresholds

- **Upper Threshold $\overline{\psi}(h)$:**

  $$\frac{\partial \overline{\psi}(h)}{\partial h} = \psi_0^{-1/\nu} \left(-\frac{\phi}{\nu} \right) h^{-\frac{\phi}{\nu}-1}$$

  Assuming skill-remote complementarity ($\phi > 0$), the upper threshold is **strictly decreasing** in worker skill. Higher-skilled workers require a lower level of firm efficiency to make full remote work optimal.

- **Lower Threshold $\underline{\psi}(h)$:** The shape of this corrected threshold is now more complex, let $f(h) = \frac{1}{\psi_0} \left(h^{-\phi} - \frac{c_0}{A_1} h^{-\phi-1} \right)$, so that $\underline{\psi}(h) = [f(h)]^{1/\nu}$. The monotonicity is determined by the sign of $f'(h)$:

  $$f'(h) = \frac{1}{\psi_0} \left(-\phi h^{-\phi-1} + \frac{c_0(\phi+1)}{A_1} h^{-\phi-2} \right) = \frac{h^{-\phi-2}}{\psi_0} \left(-\phi h + \frac{c_0(\phi+1)}{A_1} \right)$$

  Setting the term in the parenthesis to zero gives the critical point $\hat{h}$:

  $$\hat{h} = \frac{c_0}{A_1} \frac{\phi+1}{\phi} = \frac{c_0}{A_1} \left(1 + \frac{1}{\phi} \right)$$

  The shape of the threshold now depends critically on the value of the skill-remote elasticity ϕ. Consider the term:

  $$T(h) = -\phi h + \frac{c_0(\phi+1)}{A_1}$$

The sign of $\frac{\partial \underline{\psi}(h)}{\partial h}$ is the same as the sign of $T(h)$. This term captures the tension between two economic forces:

1. **The Skill-Remote Interaction Effect ($-\phi h$):** This term reflects how a worker's skill directly alters their productivity in a remote setting. Its effect depends critically on the sign of ϕ.
2. **The Opportunity Cost Effect ($\frac{c_0(\phi+1)}{A_1}$):** This term reflects the cost of forgoing a worker's full baseline productivity ($A_1 h$) by having them work remotely. The firm must be compensated for this loss by the worker's increased effectiveness or willingness to accept a lower wage (captured by c_0).

Before analyzing the cases, let's define our key terms based on the cross-partial derivative of the remote productivity function, $g(h, \psi) = \psi_0 h^{\phi} \psi^{\nu}$:

$$\frac{\partial^2 g}{\partial h \partial \psi} = \psi_0 \phi \nu h^{\phi-1} \psi^{\nu-1}$$

The sign of this expression is determined entirely by the sign of ϕ.

- **Complementarity ($\phi > 0$):** Worker skill and firm remote-efficiency are complements. An increase in firm efficiency (ψ) raises the marginal productivity of worker skill (h) in remote work, and vice-versa.
- **Substitutability ($\phi < 0$):** Worker skill and firm remote-efficiency are substitutes. An increase in firm efficiency *lowers* the marginal productivity of worker skill in remote work. This can be thought of as a situation where the firm's technology is so effective that it makes the worker's innate skill less relevant for remote tasks.

- **Case 1: Strong Complementarity ($\phi > 0$):** The lower threshold $\underline{\psi}(h)$ has an **inverted U-shape**, peaking at $\hat{h} = \frac{c_0}{A_1}(1 + \frac{1}{\phi})$.
  - **Economic Intuition:** This is the most intuitive case. When skill and firm technology are complements, the threshold's shape is driven by a trade-off that changes with the skill level.
    * **For Low-Skill Workers ($h < \hat{h}$):** At low skill levels, the complementarity effect (h^{ϕ}) is weak and the worker's baseline productivity ($A_1 h$) is low. As skill h increases from a low base, the **opportunity cost effect** dominates. The loss of baseline

productivity from not being in the office grows faster than the gain from the weak complementarity. To justify offering even a small amount of remote work, the firm requires progressively higher remote efficiency (ψ). Therefore, the threshold $\underline{\psi}(h)$ is **increasing**.

    * **For High-Skill Workers ($h > \hat{h}$):** At high skill levels, the **skill-remote interaction effect** becomes dominant. The complementarity is now powerful; a small increase in h makes the worker significantly more effective with the firm's remote technology. This strong gain in remote productivity now outweighs the linear increase in opportunity cost. The firm is willing to offer a hybrid arrangement even with a lower level of its own remote efficiency (ψ) because the worker's high skill compensates for it. Therefore, the threshold $\underline{\psi}(h)$ is **decreasing**.

- **Case 2: Weak Substitutability ($-1 < \phi < 0$):** The lower threshold $\underline{\psi}(h)$ is **strictly increasing**.

  - **Economic Intuition:** In this regime, worker skill and firm technology are substitutes. An increase in worker skill has two negative consequences for the firm's incentive to offer remote work:

    1. The **opportunity cost** of not having the worker in the office ($A_1 h$) increases, making in-person work more valuable.
    2. The **relative remote productivity** (h^{ϕ}) actually *decreases* as h rises, because ϕ is negative. The worker becomes comparatively worse at remote work as their skill increases.

  - Both economic forces push in the same direction. As a worker's skill increases, they become simultaneously more valuable in the office and less effective remotely. To overcome this "double penalty" and still find it optimal to offer a hybrid arrangement, the firm must possess a substantially higher level of its own remote efficiency (ψ). Consequently, the minimum required efficiency, $\underline{\psi}(h)$, must be **strictly increasing** with worker skill.

Case 3: Strong Substitutability ($\phi < -1$) The lower threshold $\underline{\psi}(h)$ is **U-shaped**, with a minimum at $\hat{h} = \frac{c_0}{A_1}(1 + \frac{1}{\phi})$.

- **Economic Intuition:** This is the most complex case. While skill and technology are still substitutes, the relationship is so strong that it reverses the pattern seen in **Case 2**.

- **For Low-Skill Workers ($h < \hat{h}$):** At very low skill levels, the **strong substitutability** dominates. An increase in skill h makes the worker so much relatively worse at remote work (the h^{ϕ} term with $\phi < -1$ falls very rapidly) that this effect outweighs the rising opportunity cost. To compensate for this sharp decline in relative remote fitness, the firm needs less of its own efficiency (ψ) to be indifferent, as the trade-off is already heavily skewed toward in-person work. The threshold $\underline{\psi}(h)$ is **decreasing**.
- **For High-Skill Workers ($h > \hat{h}$):** As skill becomes sufficiently high, the logic from Case 2 takes over. The **opportunity cost effect** begins to dominate again. The loss of a highly productive worker from the office becomes the primary concern for the firm. Even though the worker is a substitute for technology, their high baseline productivity makes keeping them in the office very attractive. To entice the firm to offer a hybrid arrangement, the firm's own remote efficiency (ψ) must be increasingly high. Therefore, the threshold $\underline{\psi}(h)$ is **increasing**. Of course. This is an excellent next step, as the size of the hybrid region is a key outcome of the model that determines how many worker-firm pairs can even consider a non-corner solution.

A.1.4 Size and Properties of the Hybrid Region

The existence of a hybrid work arrangement is possible for a worker of skill h only if there is a non-empty range of firm efficiencies ψ such that $\underline{\psi}(h) < \psi < \overline{\psi}(h)$. We define the size of this hybrid region as the width of this interval:

$$\Delta\psi(h) = \overline{\psi}(h) - \underline{\psi}(h) = \left(\psi_0^{-1/\nu} h^{-\phi/\nu}\right) \left[1 - \left(1 - \frac{c_0}{A_1 h}\right)^{1/\nu}\right]$$

This region is well-defined for all h such that $1 - \frac{c_0}{A_1 h} > 0$, which requires $h > c_0/A_1$. For skill levels below this minimum, no remote work is ever optimal.

Analysis of the Hybrid Region's Size

To understand how the range of hybrid opportunities changes with worker skill, we analyze the derivative $\frac{d(\Delta\psi(h))}{dh}$. The behavior depends critically on the nature of the skill-remote interaction, governed by ϕ.

- **Case 1: Strong Complementarity ($\phi > 0$):** The size of the hybrid region, $\Delta\psi(h)$, is **strictly decreasing** in worker skill h.

- **Economic Intuition:** When skill and firm technology are complements, higher-skilled workers are pushed towards the full-remote corner solution more rapidly than they are pulled away from the full-in-person corner.

1. **Upper Threshold Effect:** The upper threshold $\overline{\psi}(h)$ is decreasing in h. As a worker's skill increases, their high remote productivity means they require a progressively lower level of firm efficiency to make full remote work optimal. This effect shrinks the hybrid region from above.
2. **Threshold Gap Effect:** The term in the brackets, $\left[1-\left(1-\frac{c_0}{A_1 h}\right)^{1/\nu}\right]$, represents the gap between the thresholds as a fraction of the upper threshold. This gap also shrinks as h increases. Intuitively, as a worker's baseline productivity $A_1 h$ rises, the cost of being in the office (c_0) becomes smaller in relative terms, meaning the lower threshold moves closer to the upper threshold.

- **Case 2: Substitutability ($\phi < 0$):** The monotonicity of the hybrid region's size, $\Delta\psi(h)$, is **ambiguous** and depends on the specific parameter values.
  - **Economic Intuition:** When skill and technology are substitutes, two competing economic forces are at play:
    1. **Upper Threshold Effect (Widening):** The upper threshold $\overline{\psi}(h)$ is now *increasing* in h. As a worker's skill increases, their relative remote productivity (h^{ϕ}) falls. This makes it *harder* for them to qualify for the full-remote regime, which pushes the upper boundary outwards and tends to **widen** the hybrid region.
    2. **Threshold Gap Effect (Narrowing):** The relative gap between the thresholds continues to shrink as h increases, for the same reason as in the complementarity case (the relative importance of c_0 diminishes). This effect tends to **narrow** the hybrid region.

A.2 Market Tightness Equilibrium Derivation

We assume a standard Cobb-Douglas matching function, which exhibits constant returns to scale.

$$M(L, V) = \gamma_0 L^{\gamma_1} V^{1-\gamma_1}$$

This function yields the following probabilities:

- **Job-Filling Rate:** $q(\theta) = \gamma_0 \theta^{-\gamma_1}$
- **Job-Finding Rate:** $p(\theta) = \gamma_0 \theta^{1-\gamma_1}$

Optimal Vacancy Posting

We begin with the firm's FOC and solve for the optimal number of vacancies $v(\psi)$:

$$qB(\psi) = \kappa_0 v(\psi)^{\kappa_1} \implies v(\psi) = \left(\frac{B(\psi)}{\kappa_0} q\right)^{1/\kappa_1}$$

Substituting the Cobb-Douglas job-filling rate, $q(\theta) = \gamma_0 \theta^{-\gamma_1}$, gives the expression for $v(\psi)$ as a function of market tightness θ and the firm's expected benefit $B(\psi)$:

$$v(\psi) = \left(\frac{B(\psi)}{\kappa_0} \frac{\gamma_0}{\theta^{\gamma_1}}\right)^{1/\kappa_1}$$

Equilibrium Market Tightness

The aggregate number of vacancies, V, is found by integrating $v(\psi)$ across all firm types:

$$V = \int v(\psi) dF_\psi(\psi) = \int \left(\frac{\gamma_0 B(\psi)}{\kappa_0 \theta^{\gamma_1}}\right)^{1/\kappa_1} dF_\psi(\psi)$$

We can factor out the terms that do not depend on the firm type ψ:

$$V = \left(\frac{\gamma_0}{\kappa_0 \theta^{\gamma_1}}\right)^{1/\kappa_1} \int [B(\psi)]^{1/\kappa_1} dF_\psi(\psi) = \frac{1}{\theta^{\gamma_1/\kappa_1}} \left(\frac{\gamma_0}{\kappa_0}\right)^{1/\kappa_1} \int [B(\psi)]^{1/\kappa_1} dF_\psi(\psi)$$

To find the equilibrium, we substitute the definition of market tightness, $V = \theta L$, into the left-hand side:

$$\theta L = \frac{1}{\theta^{\gamma_1/\kappa_1}} \left(\frac{\gamma_0}{\kappa_0}\right)^{1/\kappa_1} \int [B(\psi)]^{1/\kappa_1} dF_\psi(\psi)$$

Now, we solve for θ by grouping all θ terms on the left:

$$\theta^{1+\gamma_1/\kappa_1} L = \left(\frac{\gamma_0}{\kappa_0}\right)^{1/\kappa_1} \int [B(\psi)]^{1/\kappa_1} dF_\psi(\psi)$$

Finally, isolating θ yields the closed-form solution for equilibrium market tightness:

$$\theta = \left(\frac{1}{L} \left(\frac{\gamma_0}{\kappa_0}\right)^{1/\kappa_1} \int [B(\psi)]^{1/\kappa_1} dF_\psi(\psi)\right)^{\frac{\kappa_1}{\kappa_1+\gamma_1}}$$