Active Labor Market Policies in General Equilibrium: Crowd-In or Crowd-Out?

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Abstract

Recent empirical work has shown that high search costs may contribute to the low levels of wage work in many developing countries. This paper studies the aggregate effects of a job search subsidy aimed at increasing participation in the wage labor market using a a model of heterogeneous household who choose between participation in the wage sector and "subsistence self employment" and heterogeneous entrepreneurs with varied productivities. I identify four key general equilibrium channels. Congestion and capital-labor ratio reductions crowd out additional participation relative to a partial equilibrium benchmark (i.e the effect estimated experimentally) while expansions in hiring and improvements in allocative efficiency (boosting TFP and average wages) crowd in participation. I quantify the model using weekly data on individuals' search behavior and validate the model by showing that it accurately predicts the results of an experimental evaluation of labor search subsidies. The estimated model predicts the crowd-out effects dominate and the aggregate impact of such a policy is smaller than an experiment would suggest. The increase in TFP dominates the reduction in capital-labor ratios and average pre-tax wages increase by 1.9 percent, although funding the policy requires a 2.4 percent tax increase. Still, the policy increases average welfare by 0.6 percent. These results suggest that although congestion limits ability of subsidies alone to expand the wage sector in aggregate, policies aimed at reducing labor market frictions and increasing hiring could substantially increase wages and productivity.

1. Introduction

[To be written]

1.1. Related Literature

Methodologically, this paper is closely related to the macroeconomic development literature studying the interactions of workers and entrepreneurs in developing countries. The model builds on Itskhoki & Moll (2019) who study optimal Ramsey policies in a model with credit-constrained entrepreneurs and households and find that optimal policy begins by subsidizing entrepreneurship at the expense of workers to encourage growth. Buera, Kaboski & Shin (2011) show that the allocation of capital across entrepreneurs is a key determinant of productivity, a channel also present in this paper and responsible for driving the crowd-in effect through higher wages. Buera, Kaboski & Shin (2021) study the macroeconomic effects of microloans in a model of heterogeneous agents and endogenous selection into entrepreneurship.

This paper also builds on work that distinguishes between subsistence self-employment and entrepreneurship in the developing world. Feng & Ren (2021) document stark differences between the self-employed with and without employees (referred to as own-account workers and employers respectively) and show that employers' labor share is increasing in GDP while own-account work declines as GDP rises, consistent with the ALMP's goal of moving own-account workers into the wage sector. The model is closely related to the model of Herreño & Ocampo (2021) who study the macroeconomics effects of microloans and cash transfers in a heterogeneous agent model in which poor agents use less productive self-employment to cope with the risks of wage employment (as in this paper). Donovan, Lu & Schoellman (2020) construct detailed measures of worker flows between employment, unemployment, and self-employment for countries of various incomes and show that, in developing countries, self-employment and unemployment exhibit similar flows to employment and that self-employment does not help workers climb the job ladder. These results are consistent with the idea that self-employment in developing countries largely exists as a subsistence activity.

This paper contributes to a recent literature documenting and examining the macroeconomic effects of labor search frictions across the cross-country income distribution. Feng, Lagakos & Rauch (2018) document that overall unemployment rates are increasing in GDP per capita and show that skill-biased productivity dif-

ferences can explain a large fraction of the observed variation in a model with frictional labor markets and frictionless self-employment. I expand on their model by adding risk-averse households and financially-constrained entrepreneurs. Poschke (2019) shows that urban unemployment is substantially higher in developing countries and builds a model in which cross-country variation in search frictions can jointly explain cross-country variation in self-employment and urban unemployment rates, consistent with this paper's finding that individuals self-employment decisions respond strongly to changes in job-finding probabilities. In a similar vein, Banerjee, Basu & Keller (2021) find that skilled workers in developing countries exhibit higher unemployment rates, relative to unskilled workers, than in developed countries and show that this difference leads to differences in occupational choice. Finally, Porzio, Rossi & Santangelo (2021) use a model with frictional reallocation of labor from (self-employment dominated) agriculture to (wage work dominated) non-agriculture to quantify the importance of human capital in explaining the process of structural change.

This paper studies the effects of Active Labor Market Policies in general equilibrium and is thus closely related the empirical literature evaluating the effects of these policies. Abebe et al. (2021) and Franklin (2018) both study the effects of cash transfers to job searchers in extremely similar experiments and find that these subsidies increase search behavior and an individual's probability of being employed in a permanent, formal job after 16 weeks. Interesting, while they find substantial effects on job amenities and self-report job satisfaction, they find no significant effect on earnings. The results and data from these experiments play an important role in the quantitative discipline of this paper's model.

Algan et al. (2020) randomize a government program in France aimed at reducing recruitment and vacancy posting costs for firms and find that the program successfully increased vacancy posted and hirings. Similarly, De Mel, McKenzie & Woodruff (2019) find that wage subsidies effectively increase employment among microenterprises in Sri Lanka but that the impacts of the subsidy are fleeting and employment quickly returns to normal when the subsidy is removed. Alfonsi et al. (2020) evaluate the impact of free training programs, provided either directly to workers for free or provided through firms and subsidized by the experiment. Although this is less directly related to my results as there is no concept of training in the model, it is still an important experimental evaluation of ALMPs and sheds light on a main constraint preventing workers from finding wage sector employ-

ment, namely that they lack a credible mechanism through which to signal their abilities.

2. Model

The model features many properties that are characteristic of labor markets in the developing world while remaining computationally tractable. Time is discrete. There is measure one of households and an endogenous measure of entrepreneurs. Households consume, save, and choose between working in self-employment or participating in the labor market while entrepreneurs operate firms, consume profits, and accumulate capital and labor for future periods.

2.1. Search and Matching

The labor market for wage work exhibits typical search-and-matching frictions. Households must search for jobs and entrepreneurs must hire by posting vacancies. The cost of searching for a job and the cost of posting a vacancy are denoted by b and c respectively. Each period, the number of worker-firm matches is given by a homogeneous of degree 1 matching function $m(u, v)$ where u is the measure of households searching for a job and v is the measure of vacancies posted by firms. As is typical, $\theta = \frac{v}{u}$ is defined to be labor market tightness so that $p(\theta) \equiv m(\frac{1}{\theta}, 1) = \frac{m(u,v)}{v}$ is the probability that any vacancy is filled and $\theta p(\theta) = \frac{m(u,v)}{u}$ is the probability that any searcher finds a job. Finally, matches between workers and firms are separated with exogenous probability λ at the end of every period.

2.2. Households

There exists a unit measure of infinitely-lived households indexed by their wealth a, their employment status e, and their self-employment productivity z. Lifetime household utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1 - \sigma} \tag{1}$$

Households are endowed with one unit of time each period which they supply inelastically and indivisibly to either work or search each period.[1]

Any household can engage in self-employment and operate the self-employment technology $Y = A_s L$. A household's self-employment productivity y_t follows an

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[1] This assumption can be justified by the fact that a model period is one week. Additionally, I've experiment with allowing interior choices of time allocation, and it makes very little quantitative difference as most household choose to fully allocate time to either work or search anyways.

exogenous Markov process described by transition matrix M. Additionally, I normalize A_s to unity so that self-employment earnings are simple given by y. By assumption, self-employment uses only household labor and does not involve hiring workers from outside the household (which is instead done by entrepreneurs). Thus, this option most closely corresponds to the concept of "subsistence self-employment" (as in e.g. Herreño & Ocampo 2021).

Instead of engaging in self-employment, a household can choose to pay a search cost b and search for a wage job. A searching household earns nothing in the current period and finds a permanent job with probability $\theta p(\theta)$. After finding a job and becoming employed, a household can either work in their wage job or return to self-employment (of course, in equilibrium, all employed households will choose to engage in wage work). Wages are determined through bargaining (discussed later) and depend on the productivity of the entrepreneur with whom the household is matched, given by z_t.[2]

Search is undirected, and every vacancy has an equal probability of being filled. Consequently, a household's probability of matching with a job that will pay $w(z)$ conditional on matching with any job, denoted $H(z; X)$, is given by the share of vacancies posted by z-type entrepreneurs. As this depends on entrepreneur behavior, it is function of the vector of aggregate state variables X.

Households face incomplete markets a la Aiyagari (1994), Bewley (1977), and Huggett (1993) and accumulate assets for self-insurance. Each period, assets pay an exogenous rate of return r (i.e. this is a small open economy). Households cannot borrow (i.e. $a_t \geq 0$). The budget constraint for the household can be written

$$a_{t+1} + c_t = (1+r)a_t + (1-e_t)\big((1-s_t)y_t - s_t b\big) + e_t w_t(z_t) \qquad (2)$$

where $s_t \in \{0, 1\}$ is a choice variable for the household with $s_t = 1$ representing the search decision in period t and $e_t \in \{0, 1\}$ is an indicator variable with $e_t = 1$ indicating that the household is employed in period t.

As discussed above, employed households are separated from their jobs with probability λ. Additionally, the household can lose its job if the entrepreneur employing the household dies which occurs with probability $1 - \Delta$ (as discussed below) or chooses to downsize its labor force. Under generous parameter conditions

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[2]Section **??** shows that the bargained wage depends only on the productivity of the entrepreneur, rather than on other entrepreneur or household state variables, justifying the suppression of other state variables in the wage expression.

(satisfied in the estimated model), it can be shown that downsizing never occurs in equilibrium, which I assume throughout the rest of the paper. Thus the probability that an employed household retains its job at the end of the period is given by $(1 - \tilde{\lambda}) = \Delta(1 - \lambda)$.

Taking all of the above, the household's problem can be written recursively as

$$V_u(a, y, s; X) = \max_{c, a', s' \in \{0,1\}} \frac{c^{1-\sigma}}{1-\sigma} + \beta \bigg(\big(1 - s\theta p(\theta)\big) E_{y'}[V_u(a', y', s'; X')|y] +$$
$$s\theta p(\theta) \big(E_{z,y'}[V_e(a', y', z; X')|y, X]\big) \bigg)$$

$$V_e(a, y, z; X) = \max_{c, a'} \frac{c^{1-\sigma}}{1-\sigma} + \beta \bigg((1 - \tilde{\lambda}) E_{y'}[V_e(a', y', z; X')|y] +$$
$$\tilde{\lambda} E_{y'}[V_u(a', y'; X')|y] \bigg) \tag{3}$$

$$\begin{aligned}
s.t. \quad a' + c &= (1+r)a + (1-s)y - sb \qquad \text{for } V_u \\
a' + c &= (1+r)a + w(z) \qquad\qquad \text{for } V_e \\
X' &= G(X) \\
y' &\sim M(y) \\
z &\sim H(z; X)
\end{aligned}$$

where X is a vector of aggregate state variables and G is the household's perception function for the evolution of the aggregate state. V_u and V_e denote the value function of the household while unemployed and employed respectively. Here there is a small timing convention; household's must commit to search decisions one period ahead of time. The purpose of this convention is to prevent households from conditioning their search decision on contemporaneous productivity shocks. This convention simplifies some expressions in Section 3 and has little effect on quantitative results.
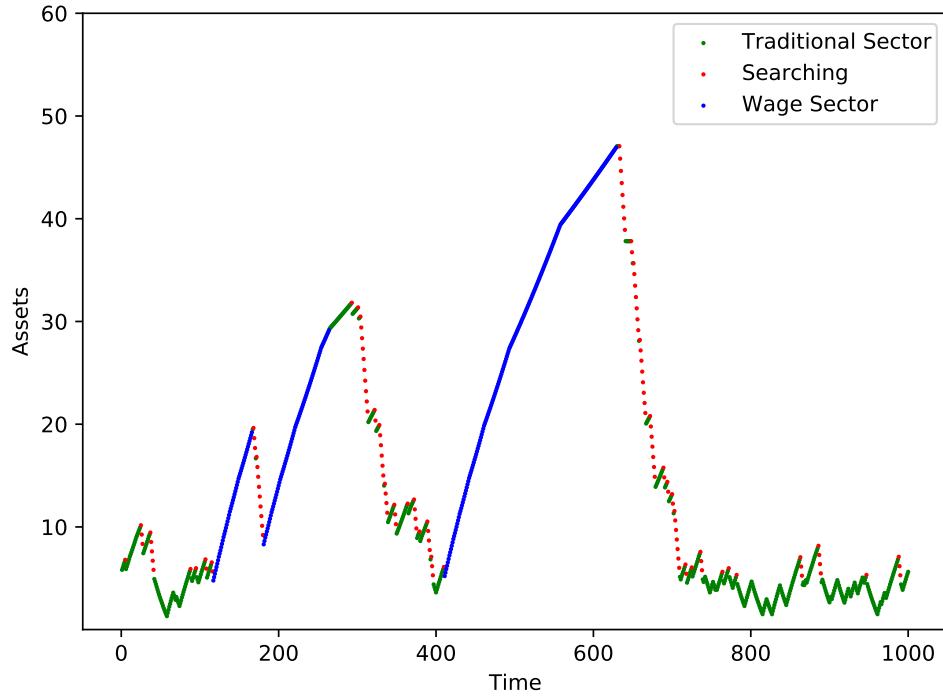
2.3. Household Behavior

Households decide whether to engage in self-employment or search for a wage job by weighing the benefits of search against the costs. In addition to the explicit search cost b and the opportunity cost of forgone self-employment earnings, the

presence of borrowing constraints means that the higher risk of job search also serves as a cost, particularly if the probability of finding a wage job is small as is the case in many developing countries.

Only households who are sufficiently self-insured will opt to pay the search cost and search for wage work, hoping for the slim probability of finding a job and achieving a large boost in earnings, while households without much self-insurance will enjoy the safety of lower but guaranteed income in self-employment. The search cost quickly diminishes the savings of searching households and reduces their self-insurance, eventually driving them to self-employment until they can restore their savings. The result is that households near the threshold of self-insurance spend a few periods working in self-employment and accumulating assets, then switch to searching for a wage job for a few periods, and return to self-employment once their savings have been depleted.

Figure 1: Household Self-Employment and Wage Sector Behavior over Time

This figure plots a simulated household's search, wage work, and self-employment behavior as well as assets over 1000 periods of the household's life. This simulation is performed using the full quantitative calibration of the model described in Section 4.

Figure 1 displays an example of this behavior for a single household simulated

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for 1000 weeks in the model (i.e. about 20 years). The x-axis displays time while the y-axis displays the household's stock of assets. The color corresponds to the household's search decision in that period; weeks in green are those where the household is engaging in self-employment, red weeks correspond to searching for wage work, and blue weeks are periods when the household is employed and working for a wage.

The figure demonstrates the household behavior described above. At the start, the household is near the threshold of self-insurance and alternates between working in self-employment and searching for wage work depending on their particular level of assets and self-employment productivity. At around week 150, the household's search is successful, and they acquire a high-earning wage job and quickly accumulate assets. They eventually separate from their employer but use their stock of assets to fund extensive search and remain in the wage sector. This behavior continues for quite some time until approximately week 700 when the household exhausts its assets without finding a job and returns to self-employment punctuated by brief periods of search.

From the perspective of a household, crowd-in and crowd-out effects must occur through changes in either the job finding probability (i.e. labor market tightness) or the expected wage. Anything that increases labor market tightness and lower the job finding probability or lowers the expected wage will cause marginal households to shift from search to self-employment, and decreased labor market tightness or higher wages will increase search. Quantitatively, at least, the crowd-in and crowd-out effects of any policy can be summarized by how it changes these two equilibrium objects. Conceptually, however, there are multiple crowd-in and crowd-out effects pushing market tightness and wages in different directions which Section 3 will explore.

2.4. Entrepreneurs

While households work in either self employment or the wage sector, entrepreneurs operate firms and employ households. The choice to include entrepreneurs as distinct agents separate from household's self employment decisions (in contrast to e.g. Buera, Kaboski & Shin 2021) reflects the qualitative difference between household self-employment-as-last-resort in and out of which households can freely flow (Donovan, Lu & Schoellman 2020) and productive entrepreneurship with the potential to grow and potential employ many households.

There are N entrepreneurs each of size $\frac{M}{N}$ born every period, and the model considers the limit $N \to \infty$.[3] At the end of a period, entrepreneurs die with probability Δ. Entrepreneurs are born with idiosyncratic ability z drawn from some distribution with bounded support $h(z)$ and an initial level of financial wealth $\underline{f}$ (taken to be exogenous). They discount the future at rate β (the same rate as households), face an exogenous death probability Δ each period, and receive lifetime utility from consumption (labeled d_t for "dividends") given by

$$\sum_{t=0}^{\infty} (\beta \Delta)^t \frac{c_t^{1-\sigma}}{1-\sigma} \tag{4}$$

Each entrepreneur operates a production technology that takes capital k and labor n and produces output y according to an ability-dependent Cobb-Douglas production function:

$$y_t = z k_t^{\alpha} n_t^{1-\alpha} \tag{5}$$

Entrepreneurs rent capital from the international capital market at an exogenous rental cost $(r + \delta)$ (i.e. this is a small open economy) and pay workers wage w_t, determined by bargaining.

Entrepreneurs are constrained in their choice of both k_t and n_t. In particular, they must use their own assets f_t as collateral and face a collateral constraint

$$k_t \leq \gamma f_t \tag{6}$$

where $\gamma \geq 1$ is a parameter summarizing the degree of financial market frictions, with $\gamma = 1$ representing the case of full self-financing and $\gamma \to \infty$ representing no financial frictions. While this constraint is exogenous, it can be thought of as arising from unenforceability of contracts or other institutional features that make uncollateralized lending risky.

To hire labor and adjust n_t, entrepreneurs post vacancies v_t. Each vacancy costs c units of output to post and is filled at the end of the period with probability $p(\theta)$.

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[3]The assumption that there are an infinite number of atomic entrepreneurs rather than a unit measure of entrepreneurs is non-standard but eliminates many technical difficulties in the discussion of wage bargaining. Other than this, there are no substantive differences between the two assumptions.

The evolution of n_t is dictated by the equation

$$n_{t+1} = (1 - \lambda)n_t + p(\theta)v_t \tag{7}$$

where λ is the exogenous separation rate.

An entrepreneur's period profits are given by

$$\pi_t(z, k_t, n_t) = zk_t^\alpha n_t^{1-\alpha} - (r + \delta)k_t - w_t n_t \tag{8}$$

Due to the constraints on the choices of k_t and n_t, an entrepreneur will earn positive profits each period. They split these profits between consumption, posting vacancies, and accumulating additional collateral f_{t+1} and face a budget constraint given by

$$d_t + f_{t+1} = \pi_t(z, k_t, n_t) + f_t - cv_t \tag{9}$$

2.5. Wage Bargaining

Each period, entrepreneurs and their hired workers bargain over wages. Because capital acts as a fixed factor of production (as the collateral constraint always binds in equilibrium), output exhibit decreasing returns to scale in labor. To accommodate this, I follow Smith (1999) and, more recently, Acemoglu & Hawkins (2014) and model production as a cooperative game between workers and entrepreneurs in which each agent is paid their Shapley value.

The entrepreneur enters the game with capital k and workforce n. Any worker that chooses not to cooperate will engage in self-employment for a period and then return to the bargaining table in the next period. That is, the outside option for the worker takes the form of a temporary strike in which the match between worker and firm is preserved (rather than being terminated). To maintain tractability, the productivity of a striking worker is assumed to be non-stochastic and identical across all workers, equal to the average productivity in the economy $\bar{y}$. This simplifies the problem dramatically as it allows the productivity of every uncooperative worker to be known a priori, rather than depending aggregate state variables such as the cross-sectional distribution of workers over productivity states and employment.

If the entrepreneur and x of the n workers choose to cooperate, they form a coalition, operate the entrepreneur's production technology, and produce $zk^\alpha x^{1-\alpha}$. The remaining $(n - x)$ workers form their own coalition and produce $(n - x)\underline{w}$.

Each agent is paid their Shapley value arising from this game, so that the wage per worker is given by

$$w = \chi z k^\alpha n^{-\alpha} + (1 - \chi)\bar{y} \tag{10}$$

where χ is a parameter governing the bargaining power of the entrepreneur relative to workers.[4] This wage determination equation is intuitive; workers are simply paid some linear combination of their marginal product of labor and their outside option $\bar{y}$, with the weight determined by bargaining power.

2.6. The Entrepreneur's Problem and Behavior

Combining equations 4 - 9 and the wage bargaining equation 10, the entrepreneur's problem can be written recursively as

$$V(z, f, n; X) = \max_{f', n', k, v, d} \frac{c^{1-\sigma}}{1 - \sigma} + \beta \Delta V(z, f', n'; X)$$

$$s.t. \quad d + f' = (1 - \chi)z k^\alpha n^{1-\alpha} - (r + \delta)k - (1 - \chi)\bar{y}n + f - cv$$

$$n' = (1 - \lambda)n + p(\theta)v$$

$$k \leq \gamma f$$

$$v \geq 0$$

$$X' = J(X)$$

where X is a vector of aggregate state variables and J is the entrepreneur's perceptions function for the evolution of the aggregate state. It is important to note that the wage bargaining equation has been substituted into the entrepreneur's budget constraint and does not depend on household state variables, eliminating the need to anything about the composition of households employed by the entrepreneur as state variables.

This problem is complex, but substantial insight into entrepreneur behavior can be gained analytically. One important result arising from the first order conditions for f' and n' is that an entrepreneur's capital-labor ratio depends only on their productivity z and aggregate state variables X, conditional on choosing positive

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[4]At a technical level, the game is between an atomistic entrepreneur and a continuum of workers; the parameter χ is the relative size of the entrepreneur.

employment (see Appendix B for the derivation).[5] Denote this value as η so that

$$\eta(z; X) = \frac{\gamma f'^*}{n'^*} \tag{11}$$

where f'^* and n'^* are the entrepreneur's optimal policy functions. The intuition for this result is straightforward; production is Cobb-Douglas and, even accounting for wage bargaining, hiring frictions, and collateral constraints, the users costs of both capital and labor are linear, resulting in a constant ratio.

This result substantially increases the computational tractability of the model. In principle the bargained wage may depend on all entrepreneur state variables as well as the state variables of the households they employ, requiring the composition of households employed by the entrepreneur to be tracked as state variables in the problems of the entrepreneur and employed households as well as well as solving for a high dimensional equilibrium wage function each period. Because the bargained wage depends only on parameters and the entrepreneur's capital-labor ratio (equation 10), this result collapses all of this heterogeneity into the single state variable z. Further, the equilibrium wage function can be found by solving a simple monotonic non-linear equation of on variable and does not require an iterative procedure.[6]

A second useful result is that entrepreneurs will pursue a constant productivity-dependent growth rate. Mathematically, f'^* will satisfy

$$f'^* = g(z; X)f \tag{12}$$
$$\frac{\partial g}{\partial z} > 0$$

for some function g. As with the capital-labor ratio, because z is fixed for an entrepreneur, this means that they will grow at a constant rate over their lifetime (in steady-state). Intuitively, g is increasing in z; more productive entrepreneurs will grow quicker. Together, the two functions η and g are sufficient to fully characterize entrepreneur behavior as a function of their productivity z and the aggregate state X.

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[5]This statement holds in universally in steady-state and holds for any transition path under the parameter restriction that $\lambda > 1 - \beta\Delta$ which is satisfied in the quantitative model.

[6]This is shown in Appendix B along with the other results discussed in this section.

2.7. Crowd-in and Crowd-out Effects on Wages

The two functions η and g can be used to gain intuition for the crowd-in and crowd-out effects of a subsidy to labor search that arise from changes in average wages. Because these operate through labor market tightness, it is useful to abuse notation and write $\hat{eta}(z;\theta)$ and $\hat{g}(z;\theta)$ to represent "the steady-state values of η and g for a z productivity entrepreneur facing steady-state labor market tightness θ. This abuse is possible because entrepreneur policy functions depend on the aggregate state X only through its implication for current and future values of θ. Intuitively, these functions can be thought of as "partial equilibrium" policy functions that take a value for the price (θ) and return the entrepreneur's optimal response, enabling comparative-static-like-statements like the following:

Proposition 1 *Let $\hat{g}$ and $\hat{\eta}$ be defined as above. Then*

$$\frac{d\hat{\eta}}{d\theta} > 0$$

$$\frac{d\hat{g}}{d\theta} < 0 \text{ and } \frac{\partial^2 \hat{g}}{\partial z \partial \theta} < 0$$

where partial derivatives denoted by ∂ are taken while holding other endogenous outcomes (i.e. $\hat{\eta}$) constant.

In words, Proposition 1 makes three claims. The first ($\frac{d\hat{\eta}}{d\theta} > 0$) is that an entrepreneur's capital-labor ratio is increasing in labor market tightness. This result is intuitive; a tighter labor market leads to higher hiring costs and thus increases the cost of labor relative to capital. Conversely, a search subsidy that induces households to shift from self employment towards search will loosen the labor market will put downwards pressure on the capital-labor ratio, leading to a reduction in wages. This will lead to crowd out (relative to a world where prices remained fixed) as lower wages will cause marginal searchers to move into self employment.

The second claim that an entrepreneur's growth rate is decreasing in market tightness ($\frac{d\hat{g}}{d\theta} < 0$) is similarly intuitive; as labor market tightness increases and hiring costs rise, the entrepreneur must spend more on hiring, reducing the profit per unit of collateral, and reducing the incentive to grow (rather than consume).

More interesting is the final statement ($\frac{\partial^2 \hat{g}}{\partial z \partial \theta} < 0$). This is best interpreted as a statement about how the magnitude of the relationship between $\hat{g}$ and θ varies

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with productivity. It says that the response of $\hat{g}$ to θ is larger (i.e. more nega-
tive) for highly productive entrepreneurs. As a result, a search subsidy increases
search and decreases labor market tightness, entrepreneur growth rates increase
(the second claim), and this increase is larger for high productivity entrepreneurs
than low productivity ones. The result is that the share of resources controlled by
high productivity entrepreneurs increases, resulting in higher TFP and — because
productive entrepreneurs pay higher wages — higher average wages, crowding in
additional workers from self employment.

The intuition for this final result is not immediately clear. It arises from the
fact that hiring costs make up a larger share of total costs for faster growing firms,
which happen to be the more productive firms. To see this, consider a compari-
son between two entrepreneurs each with a unit measure of employees, identical
capital-labor ratios η^*, but one of whom is not growing ($g = 1$) and one of whom is
growing at rate g^*.[7] Total costs for the entrepreneur without growth are given by
$r\eta^* + \chi z\eta^* + (1 - \chi)\bar{y} + \lambda\frac{c}{p(\theta)}$ where the final term represents the hiring costs. Total
costs from the growing entrepreneur are identical except they carry an addition
hiring cost term given by $(g^* - 1)\frac{c}{p(\theta)}$. From this perspective, it is clear that a reduc-
tion in $\frac{c}{p(\theta)}$ represents a larger proportional reduction in total costs for the growing
entrepreneur, freeing up relatively more resources to use for faster growth.

2.8. Crowd-in and Crowd-out Effects on Job Finding and Hiring

Before turning to quantitative analysis, it is useful to discuss the two remain-
ing crowd-in and crowd-out effects. These are formalized in the optimal policy
analysis of Section 3, but an intuitive understanding is helpful for discussion of
the quantitative results. Unlike the effects discussed previously which impacted
wages, these two effects impact job finding and hiring probabilities.

The first is very straightforward and is closely related to the congestion external-
ity in textbook labor search models. Additional workers who induced into search
by a search subsidy decrease labor market tightness and, through the matching
function, decrease the probability that any individual searcher is matched with a
job. While this increases the unemployment rate, it does not directly change the
size of relative size of the wage sector. However, as with lower wages, marginal
searchers respond to lower job finding rates by moving into self employment.[8]

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[7]This example abstracts from the fact that entrepreneurs with different productivities would
choose different capital-labor ratios, but it suffices to provide intuition.

[8]As this mechanism serves as the primary means by which the labor market achieves equilib-

The final crowd-in effect is more subtle and, unlike the previous effects, does not occur as a result of changes in labor market tightness. It arises from another model feature that is common to labor search models — namely, that bargained wages do not fully reflect a worker's marginal product. Instead, workers are paid slightly less than their marginal product with entrepreneurs collecting the difference as additional profits. A portion of these additional profits are used to grow the firm which necessitates hiring additional workers. In this sense, an additional searcher induced by a search subsidy, if they are hired, directly crowds in additional hiring in the future. One way to see this effect clearly is to consider two entrepreneur each with a unit measure of workers growing at rate g^*, one of whom is exogenously matched with an additional unit measure of workers from outside the economy (e.g. immigrants). The entrepreneur who is not matched with additional workers hires $g^* - 1$ workers on net for the next while while the matched entrepreneur hires $2(g^* - 1)$ workers on net. The unit measure of immigrants effectively crowds-in an additional $g^* - 1$ workers the next period.

3. Optimal policy and the theory of the second best

So far, I have focused on a positive description of the general equilibrium effects that are key for understanding the implementation of labor search subsidies. Lorem ipsum...

A substantial complication in analyzing the externalities present in households' labor search decisions is that is that it is not immediately clear what the appropriate social planner's problem is. As in much of the labor search literature, the problem of an all-powerful planner free from any financial constraints or labor market frictions is uninteresting (except perhaps as a benchmark); this planner would simply allocate all labor and capital to the most productive entrepreneur, assuming one exists, and divide output in a way that equalizes marginal utility across all households and entrepreneurs. This teaches us nothing about the externalities generated by households' labor search or how these externalities interact with borrowing constraints.

Instead, I follow the traditional approach and consider the problem of a constrained social planner who is subject to labor market frictions and must respect

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rium, one could argue that calling this a "crowd-out effect" is awkward.However, as Section 3 shows, this effect is not internalized by searchers and appears as a distinct term in the expression for the planner's optimal search subsidy, justifying its inclusion in the set of crowd-in and -out effects.

households' borrowing constraints (as in Davila et al. 2012). Further, because the the goal of this section is to examine the externalities exerted by *households* in their labor search decisions (as this is ultimately the outcome affected by a subsidy to labor search), I assume that the social planner can only dictate the decisions of households. The social planner cannot control the behavior of entrepreneurs (and puts zero weight on their utility), and they continue to solve their optimization problem each period. In this sense, the social planner faces an additional constraint that it cannot force entrepreneurs to act sub-optimally. Allocations satisfying these three constraints make up the set of feasible allocations for the social planner.

Definition: A path of household policy functions $\{c_t(a,y,z), a'_t(a,y,z), s_t(a,z)\}_{t=0}^{\infty}$, entrepreneur policy functions $\{g_t(z), \eta_t(z)\}_{t=0}^{\infty}$, distributions of households across savings and matched-employer productivities $\{m_t(a,z)\}_{t=1}^{\infty}$, and labor market tightness $\{\theta_t\}_{t=0}^{\infty}$ is **feasible** given an initial distribution $m_0(a,z)$ and market tightness θ_{-1} if

1. It respects the household budget constraint for all a, y, z

$$a'_t + c_t = Ra + (1-s_t)y + s_t(w_t(z_t,\theta_t) - (1-z_t)b) \quad \forall a,y,z,t \qquad (13)$$
$$a_t \geq 0$$

   Note here that the independence of s_t on y enforces the informational constraint that the planner cannot condition search policy on the realization of individual's idiosyncratic shock, mirroring the informational constraint for individuals in competitive equilibrium.

2. It respects the labor market matching technology

$$\frac{v(m_t, \eta_t, \eta_{t+1}, g_t)}{\theta_t} = \int \int s_t(a,0) m_t(a,0) j(y) dy da \qquad (14)$$
$$m_{t+1}(a',z) = (1-\tilde{\lambda}) m_t(a,z) + H(z, m_t, \eta_t, \eta_{t+1}, g_t) p(\theta_t) v(m_t, \eta_t, \eta_{t+1}, g_t)$$

   where v is the total number of posted vacancies as a function of entrepreneur policy functions, and H is the probability that an individual who finds a job is matched with a firm of productivity level z (defined for compactness and simplicity; both are further described in Appendix C)

3. The entrepreneur policy functions $\{g_t(z), \eta_t(z)\}_{t=0}^{\infty}$ solve the entrepreneurs' problem (Appendix equation 20), conditional on θ_{-1} and $\{\theta_t\}_{t=0}^{\infty}$.

The task of the social planner is maximize average household welfare subject to these feasibility conditions (and initial conditions). Formalizing the statement of this problem is straightforward but cumbersome and is relegate to Appendix C. One detail worth noting is that the constrained planner simply chooses the entire sequence $\{\theta_t\}_{t=0}^{\infty}$ simultaneously, that is, the planner's problem features full commitment.

This problem of selecting the welfare maximizing path subject to a set of dynamic constraints in a heterogeneous agent economy is similar to the Ramsey-type problems often found in the literature dealing with welfare and efficiency in heterogeneous agent models (e.g. Itskhoki & Moll 2019, Dávila & Schaab 2023). Like all Ramsey problems, the primal problem of choosing paths of consumption (or with heterogeneous agents, consumption functions) subject to feasibility constraints can be equivalently formulated as a dual problem in which the planner selects optimal tax rates from a sufficiently rich set of instruments to decentralize the optimal allocation in competitive equilibrium.

Here, the equivalent dual problem provides some intuition for the relevance of this planner's problem to the notion of an optimal labor search subsidy. The constraint that the planner cannot dictate the behavior of entrepreneurs and is restricted to paths of g_t and η_t consistent with entrepreneur optimality is equivalent to a constraint in the dual problem that the set of available tax instruments does not include any taxes levied on entrepreneurs. Similarly, the constraint that the planner must respect household budget constraints corresponds to a restriction that the set of tax instruments does not include any lump-sum transfers. Thus the only tax instruments available under the dual approach are taxes on earnings, search, and savings, all of which are potentially individual-specific.

3.1. Search Externalities without Credit Constraints

To generate some initial intuition, I start by examining externalities in the special case where household utility is given by $u(c) = c$ (i.e. $\sigma = 0$). As the exogenous small open economy interest rate is less than the household discount rate by construction, the optimal savings policies in both the competitive equilibrium and the planner problem are to save nothing and consume all disposable income every period. This essentially eliminates ex-post heterogeneity outside that of the labor market, leading to substantial simplifications.

Proposition 2 *Under the assumptions that traditional sector productivity y has no au-*

tocorrelation and $\sigma = 0$, the optimal search policies $s(a, z)$ of constrained social planner (described above) and an individual in competitive equilibrium in steady state are to search if and only if

$$\text{Individual:} \int (y + b)j(y)dy \leq \beta\bar{\theta}p(\bar{\theta}) \int_z \frac{w(z, \bar{\theta}) - \int(y + b)j(y)dy}{1 - \beta(1 - \tilde{\lambda})}dz \qquad (15)$$

$$\text{Planner:} \int (y + b)j(y)dy \leq \beta\bar{\theta}p(\bar{\theta}) \int_z \frac{w(z, \bar{\theta}) - \int(y + b)j(y)dy}{1 - \beta\Delta g(z, \bar{\theta})}dz \ + \ \mu \qquad (16)$$

$$\mu = \frac{\bar{\theta}/\bar{S}}{\frac{\partial \log v}{\partial \log g}\frac{\partial \log g}{\partial \log \theta} - 1}\left(\underbrace{\int_z \bar{\lambda}(z)\bar{\theta}p(\bar{\theta})\bar{S}\frac{\partial H}{\partial g}\frac{\partial g}{\partial \theta}dz}_{\text{Composition of Jobs}} + \right.$$

$$+ \underbrace{\int_z \bar{\lambda}(z)\bar{H}\bar{S}p(\bar{\theta})\left(1 + \frac{\partial \log p}{\partial \log \theta}\right)dz}_{\text{Congestion}} + \underbrace{\frac{1}{\beta}\int_z \frac{\partial w}{\partial \eta}\frac{\partial \eta_{t-1}}{\partial \theta}\bar{m}(z)dz + \int_z \frac{\partial w}{\partial \eta}\frac{\partial \eta_t}{\partial \theta}\bar{m}(z)dz}_{\text{Wage Changes}} + \qquad (17)$$

$$\left. +\text{Anticipation Terms} \right)$$

where bars denote steady-state values of the competitive equilibrium and planner's problem respectively, $\bar{S}$ is the steady-state number of searchers defined for compactness, and $\lambda(z)$ is the planner's shadow price denoting the marginal value of an additional worker being matched with a productivity z entrepreneur. The anticipation terms are described further in the appendix.

From the proposition, it is clear that the privately and publicly optimal decision rules follow a similar structure. Each rule weighs the costs of search, given by the goods cost of search b and the expected cost of forgone wages, against the benefits, given by the expected excess earnings while employed. In the privately optimal rule, these excess earnings are discounted by the intertemporal discount rate and probability of maintaining the job match. In the publicly optimal case, they are discounted by the intertemporal discount rate and the shutdown-inclusive growth rate of matched entrepreneur. I deferred further discussion of this difference in

discounting to the discussion of crowd-in effects.

The planner's optimal decision rule carries an additional term μ which contains all but one of the externalities present in the labor search decision. The terms contained in μ represent externalities that occur through changes in labor market tightness θ and thus are weighted by the net change in labor market tightness due to a change in the number of searchers after accounting for the response of vacancies.[9] This weight is negative, reflecting the fact that an increase in the number of searchers leads to a decreases in labor market tightness. Thus negative terms within the parentheses represent positive externalities and vice versa.

The negative externalities of search are contained in the second line of equation 17. These are the forces driving the crowd-out effects. The first term, labeled "Congestion", is typical in search models; an additional searcher pushes down labor market tightness and reduces the probability that any given searcher finds a job. As is typical, the size of this externality is proportional to the elasticity of the matching function; a high elasticity implies that an additional searcher leads to a large reduction in the job-finding probability. Intuitively, this externality is also increasing in the steady-state number of searchers $\bar{S}$ and is valued using the average shadow value of a newly hired worked (i.e. $\int_z \lambda(z) \bar{H} dz$).

The second negative externality, given by the "Wage Changes" term, captures the fact that a reduction in labor market tightness leads to reduced labor costs, and entrepreneurs respond by lowering their capital-labor ratio, resulting in lower wages (see equation 10, the wage bargaining equation). This effect is similar to the monopoly effect of Itskhoki & Moll (2019) and stems from a similar source, namely, that the planner places zero weight on the welfare of entrepreneurs. While this effect occurs contemporaneously with an increase in search behavior, it is slightly offset by a countervailing anticipation effect. When entrepreneurs foresee a reduction in labor market tightness, they respond by reducing hiring in the period before (as labor will be cheaper tomorrow), temporarily pushing up the capital-labor ratio and, consequently, wages. Because this effect occurs exactly one period before the change in labor market tightness, it is appropriately weighted by $\frac{1}{\beta}$.

The remaining term, "Composition of Jobs" is the first of two externalities corresponding to the crowd-in effects. It arises from the fact that a decrease in labor market tightness will cause entrepreneurs to grow faster (as their labor costs have

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[9]To see that this expression indeed gives the net change, note that $\theta = \frac{v(\theta)}{s} \implies \frac{d\theta}{ds} = \frac{\theta/s}{\frac{d \log v}{d \log \theta} - 1}$.

gone down, leaving more profit available to save and finance capital in the next period). This results in a positive externality because, as shown in Proposition 1, the increase in the growth rate is larger for more productive entrepreneurs. Thus the change in hiring probabilities in response to a reduction in labor market tightness $-\frac{\partial H}{\partial g}\frac{\partial g}{\theta}$ is positive for high values of z and negative for low values, leading to a net increase in allocative efficiency. While the planner internalizes this according to the shadow price $\lambda(z)$, in competitive equilibrium it manifests itself as a higher expected wage and thus induces additional labor search.

The final externality does not operate through labor market tightness. Instead, it emerges from the different discount rates exhibited in the individually and socially optimal search decision rules in 15 and 16 respectively. Under parameter assumption (made throughout this paper) that $\Delta\beta > (1-\lambda)$, we have that $\Delta g(z,\theta) > (1-\tilde\lambda)$ for all z, θ. Thus the planner's valuation of a job is higher than an individuals, even fixing labor market tightness.

Why is this the case? The key arises from the fact that the individual does not capture the entire marginal product of labor created by their job match. Instead, they earn some markdown according to 10, and the remainder is captured by the entrepreneur. Although the social planner does not value the entrepreneur's consumption, a portion of this remainder is saved and used to expand the entrepreneur's firm in the next period, including new hiring which the social planner values. Essentially, a portion of the worker's production today is used to finance the hiring of additional workers tomorrow, whose production is used to finance more workers the next period, etc. Intuitively, this effect is larger for more productive matches (appearing in 16 as a higher growth rate) and smaller when entrepreneur survival probabilities Δ are smaller.

4. Model Estimation and Quantification

In this section, I discus the estimation and quantification of the model as well as perform some model validation exercises. Broadly speaking, the parameters of the model fall into two categories. The first are parameters that can be estimated directly from data or are well-known macroeconomic parameters with standard values. These parameters I simply set equal to their estimated or standard value. The second set of parameters I estimate using the simulated method of moments to match key data moments.

4.1. Directly Estimated Parameters

Many model parameters are chosen to match values typical in the macroeconomics, are taken from external sources, or are estimated directly. These are displayed in Table 1, along with their values and sources. The discount rate β is chosen to match an annual discount rate of 0.95. Because a model period corresponds to two weeks, this is corresponds to a value of $0.95^{\frac{1}{26}}$. The rate of return on worker's savings R is taken to be exogenously equal to $0.9^{\frac{1}{26}}$. The assumption that the return to savings is less than one is typical models of developing countries (see e.g. Donovan 2021, Fujimoto, Lagakos & VanVuren 2023) and representative of the fact that households in these countries lack access to formal investment with positive returns. The value of 0.9 matches an annual inflation rate of roughly 10 percent, roughly consistent with World Bank estimates of inflation in Ethiopia over the last few years; thus the model asset a most closely reflects cash holdings. The capital share of income is set at 0.33 as is standard.

The interest rate faced by entrepreneurs is disciplined using World Bank MIX Market data containing financial information on microcredit providers in Ethiopia. Yields on loans from microfinance institutions range from 20 percent to 30 percent with negligible loan loss rates (typically less than one percent). Combining this rough average of a 25 percent annual return with 8 percent depreciation yields a depreciation-inclusive user cost of capital of 33 percent annually. This value is high relative to developed countries but is fairly typical for developing countries (see e.g. Banerjee et al. 2015, who document similar values in multiple countries including Ethiopia).

Collateral constraints are measured directly using data from the Ethiopian portion of the World Bank Enterprise Survey for the year 2015. The average collateral requirement reported by firms is slightly larger than 350 percent of loan value, meaning that a firm that owned 350,000 Birr worth of capital could pledge this as collateral and finance a loan for an additional 100,000 Birr of capital. Thus the implied value for γ is $1 + \frac{1}{3.5} = 1.29$. The Enterprise Survey is also used to estimate the entrepreneur survival probability Δ. Because productivity is constant for the life of an entrepreneur, entrepreneur death is the only reason that firms will shutdown in steady state. Consequently, the steady-state distribution of firm ages is geometric with decay parameter Δ whose value can be recovered through the simple maximum likelihood estimation. In this case, the estimate for Δ is given by $1 - \frac{1}{\hat{\mu}}$ where $\hat{\mu}$ is the sample average firm age, yielding an annual value for Δ of

0.92.

The self employment productivity process also comes directly from data. This productivity is modeled as a simple binary Markov process, drawing on the fact that earnings for those without permanent wage jobs are highly bimodal at a fortnightly frequency (seen in the high-frequency data of Abebe et al. 2021, described below). Such bimodality seems to stem from the fact that opportunities for self employment (or, often in the case of Addis Ababa, temporary "gig-style" labor that functions similarly to self employment), and many individuals report neither working nor searching in a given period, presumably earning very little.

One advantage on using a binary income process instead of a more typical AR(1) is that transitions in and out of this idle state can be observed and measured directly. Using fortnightly data on work and searcher activities (described in the next section), I estimate the transition probabilities from engaged in self employment or temporary work to idleness and back. Although there is no reason for these transitions probabilities to be identical, the estimated value for both is approximately 11 percent. While average self employment earnings (i.e. the productivity parameter A_s) are estimated using SMM, the ratio of earnings in the low productivity state to the high productivity state is chosen to match the standard deviation of self employment earnings observed in the data. In particular, I isolate the transitory, idiosyncratic variance of earnings by regressing (log) earnings on individual and week fixed effects and calculating the standard deviation of the residuals (similar to the process employed in Lagakos & Waugh 2013). Conditional on the transition probabilities, there is a one to one correspondence between the standard deviation of income and the ratio of interest.[10] The estimated ratio is 0.38 corresponding to an estimated standard deviation of .48.

Finally, the distribution from which newborn entrepreneurs draw their productivity is chosen to be an upper-truncated Pareto distribution (truncated as a bounded support for productivity is required for steady-state equilibrium to exist in the model). I set the lower bound of the distribution to a small but arbitrary number; because entrepreneurs endogeneous shut down below a threshold productivity level and the truncated Pareto distribution is scale-invariant, the lower threshold has no impact on model outcomes as long as it is below the shutdown threshold. The tail parameter is set to unite. It is worth noting that because of

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[10]For a symmetric transition matrix, as is the case here, this correspondence is given simply by $\frac{y_l}{y_h} = e^{-2\sigma}$

upper truncation, the mean and variance of productivity remain finite. The upper bound $\bar{z}$ is included in the SMM estimation, described below.

Table 1: Directly Estimated Parameters

Parameter	Value	Description	Source
β	$.95^{\frac{1}{26}}$	Discount rate	Standard value
R	$.9^{\frac{1}{26}}$	Return to savings	10% annual inflation
α	.33	Capital share	Standard value
r	$1.33^{\frac{1}{26}} - 1$	Capital cost for entrepreneurs	MIX Market
γ	1.29	Collateral constraint	World Bank ES
Δ	$.92^{\frac{1}{26}}$	Entrepreneur death prob.	World Bank ES
$M(y)$	$\begin{bmatrix} .89 & .11 \\ .11 & .89 \end{bmatrix}$	High and low y trans.	Abebe et al. (2021)
$\frac{y_l}{y_h}$	.38	Ratio low to high productivity	

This table displays the model parameters that are estimated directly as well as their values and sources. To help comparisons to typical values, parameters are displayed in annual terms. See the discussion for details on each parameter.

4.2. Parameters Estimated using the Simulate Method of Moments

The eight remaining parameters are estimated using the simulated method of moments. These parameters, along with their estimated values, are listed in Table 2. Table 3 lists the moments targeted in the estimation and their values in both the data and the model, as well as the source for each moment. While in general all eight moments are jointly determined by all eight parameters, there is a rough correspondence between parameters and moments that is worth some discussion. The parameters fall into two rough categories — those corresponding closely to household-level moments (above the dividing line in Tables 2 and 3) and those corresponding closely to firm-level moments (below the line), and I discuss each in turn.

The data for household-level moments come from two data sources. I construct the aggregate moments using the 2018-2019 wave of the Ethiopia Living Standards and Measurement Survey (LSMS), limited to individuals surveyed in Ad-

Table 2: Parameter Estimates from Simulated Method of Moments

Parameter	Estimate	Corresponding Moment
A_s	0.34	Wage sector premium
σ	5.2	% wage work
λ	0.01	Unemployment rate
b	0.05	% of expenditure on search
$\mathrm{M}\underline{f}$	.001	Control wage employment after 16 weeks
c	0.37	Cost to hire as % of wage
χ	0.62	Elas. of avg. wage to output per worker
$\bar{z}$	0.35	Avg. growth rate

This table displays the parameters estimated using simulated method of moments, their estimates, and the moment that corresponds mostly closely to each parameter. See discussion for details and intuition on these correspondences.

dis Ababa.[11] Additionally, I use fortnightly data on the behavior of job searchers collected as part of an experiment in Abebe et al. (2021). Importantly, I use only data from the control group in model estimation, setting aside the experimental treatment and data on treated individuals for post-estimation model validation.

The productivity of the self employment technology A_s is mostly pinned down by the earnings premium of wage workers (relative to the self employed and temporary laborers); the more productive the self employment technology is, the smaller this premium will be. I estimate the premium in the LSMS data using a Mincer-style regression of (log) earnings on age, the square of age, and an indicator for whether an individual reports that the earnings arise from a permanent wage job, along with a variety of controls including region, rural/urban, and education fixed effects.[12] The estimated earnings premium for wage workers is 39

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[11]While the other data sources used in estimation are from 2014-2015, the 2018 wave of the Ethiopia LSMS was the first wave capable of providing representative estimates for Addis Ababa (previous waves were not representative at a sub-national level). For this reason, I opt to use the data from 2018 rather than the 2015 wave, which would otherwise line up better with the other datasets temporally.

[12]The Ethiopian Productive Safety Net Programme (PSNP), a relatively new "workfare" program administered by the Government of Ethiopia, presents a potential complication. The program provides temporary employment and was present in some regions of Addis Ababa during the 2018 LSMS survey. It is unclear whether earnings from the PSNP should be included in estimation. Fortunately, dropping these earnings from the analysis changes the estimate by less than one per-

Table 3: Moments Targeted using the Simulated Method of Moments

Moment	Source	Data	Model
Wage sector premium	LSMS	39%	39%
% wage work	LSMS	30%	29%
Unemployment rate	LSMS	10%	12%
% of expenditure on search	Abebe et al. (2021)	15%	16%
Control wage employment after 16 weeks	Abebe et al. (2021)	12%	13%
Cost to hire as % of wage	Abebe et al. (2017)	120%	120%
Elas. of avg. wage to output per worker	World Bank ES	25%	25%
Avg. growth rate	World Bank ES	4.4%	4.4%

This table displays the moments targeted in the simulated method of moments estimation, their source, and their values in both the data and model. See the discussion for details.

percent.

The coefficient of relative risk aversion in utility σ and the job separation rate λ are jointly determined by the percentage of households engaged in wage work and the unemployment rate. The correspondence between the unemployment rate and λ is clear as λ determines the flows out of wage employment and in to unemployment. Conditional on this, the overall size of the wage sector is determined by flows into employment and thus the overall level of search. Because search is the higher-risk, higher-return option, this depends largely on household risk aversion. Using the LSMS, I estimate an unemployment rate of 10 percent and a wage work share of 30 percent. These are both higher than the values estimated by World Bank for the entire country of Ethiopia (3 percent and 15 percent respectively) which is not surprising given the sample restriction to Addis Ababa, an urban capital city.

The two remaining parameters disciplined using household moments are the goods cost of search b and the product of the size of newborn entrepreneurs M and entrepreneurs' initial assets $\underline{f}$ (which are not separately identified under constant returns to scale as there is no difference between two entrepreneurs of size x or one entrepreneur of size $2x$). Discipline for b comes directly from Abebe et al. (2021) who report that search costs amount to 5 percent of weekly expenditure

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centage point, rendering the issue quantitatively moot. I default to including all earnings from temporary employment, including those from the PSNP.

for individuals in their sample. Finally, $M\underline{f}$ closely corresponds to the level of wage employment in the control group of the experiment after 16 weeks. The amount and/or size of entrepreneurs corresponds directly to the amount of vacancies posted (i.e. doubling the number of entrepreneurs doubles the number of vacancies) which determines the job finding rate and thus (conditional on all other parameters), employment in the control group of searchers.

The remaining three parameters — the vacancy posting cost c, the wage bargaining parameter χ, and the upper bound of firm productivity $\bar{z}$ — are estimated to match firm-level moments. The moment for the vacancy post cost comes from Abebe et al. (2017) who survey firms in Addis Ababa about hiring practices and find that average reported cost of making a single hire is equal to 120 percent of the average wage paid by the firm.

The bargaining parameter and upper bound on firm productivity are chosen to match moments estimated from the World Bank Enterprise Survey. The bargaining parameter χ governs the proportion of a worker's marginal product of labor captured through the worker's wage (recall the wage bargaining equation 10); as a result, this parameter closely corresponds to the relationship between firm productivity and wages and is disciplined to match the elasticity of a firm's average wage to its output per worker. I estimate this elasticity to be 25 percent (meaning, a firm with 100 percent higher output per worker pays its workers on average 25 percent more) and use this as the target in estimation. Finally, the upper bound of firm productivity is chosen to match the average (self-reported) firm-level growth rate. Because more productive entrepreneurs choose to grow faster, the upper bound on productivity corresponds to an upper bound on growth rates which determines the average.

4.3. Model Validation

As my primary model validation exercise, I replicate the experiment performed by Abebe et al. (2021) in the model and compare the model outcomes to the experimentally estimated outcomes. As mentioned above, it is important to note that while control outcomes from the experiment are used to estimate the model, treatment outcomes and data are not. Thus comparing the model's predictions for treatment effects to those estimated in the experiment truly represents an "out-of-sample" test of the model. Below, I briefly summarize the experiment and describe how it is replicated in the model before showing the results of the validation exer-

cise.

The experiment took place in 2014-2015 and evaluate the effects of providing a cash subsidy covering some of the costs of job search to prospective searchers in Addis Ababa, Ethiopia. In the context of Addis Ababa, the majority of job search takes place in person in the city center. Particularly notable are job vacancy boards (located in the city center) which contain job postings and are consulted by the majority of searchers. Thus the cost of travel (typically by minibus) to the city center represents a large and salient cost of job search.

The experiment sampled young individuals who "(i) were between 18 and 29 years of age; (ii) had completed high school; (iii) were available to start working in the next three months; and (iv) were not currently working in a permanent job or enrolled in full time education." (Abebe et al. 2021). Individuals in the sample were randomly offered cash that could be collected in person at the city center up to three times each week. While not literally a job search subsidy as individuals could theoretically travel to the city center, collect the cash, and leave without searching, doing so would be ineffective as the cost of the subsidy is not large enough to cover the full round-trip journey.[13] Thus collecting the cash only makes sense if the individual intended to travel to the city center for other purposes (presumably job search). The cash was available for 16 weeks, and sampled individuals were survey on their search behavior through fortnightly phone surveys. After 16 weeks, treated individuals were 3.4 percentage points (p¡0.1) more likely to be employed in a permanent job.

To replicate the experiment in the model, I select a representative but small (measure 0) subset of household not employed in the wage sector from the steady-state distribution of household. In this sense, the outcomes of sampled individuals do not affect equilibrium outcomes, and the experiment happens in "partial equilibrium", reflecting the fact that providing treatment to a few hundred individuals in a city of millions is unlikely to have general equilibrium impacts. The sample is divided equally into treatment and control groups, and the cost of search parameter b is reduced by two-thirds (the median subsidy offered in the experiment) for the treatment group for 8 periods (16 weeks).

Experimental outcomes can then be observed by simulating behavior of the

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[13]In fact, the authors make sure of this by varying the subsidy offered to each individual based on the location, and thus minibus ticket cost, of the individuals home. However, I abstract from this heterogeneity and model the subsidy as uniform at the median value of subsidy offered.

treatment and control groups of household forward over time, and comparisons of means between the two groups correspond to Average Treatment Effects estimated by the experiment. For treatment households, I treat the experiment as an unanticipated MIT shock; households do not know ahead of time that they have been selected for treatment and cannot alter their behavior in response to such information. Thus difference between treatment and control groups before the treatment occurs are zero by construction.

Figure 2: Treatment Effect on Search Behavior over Time: Data and Model
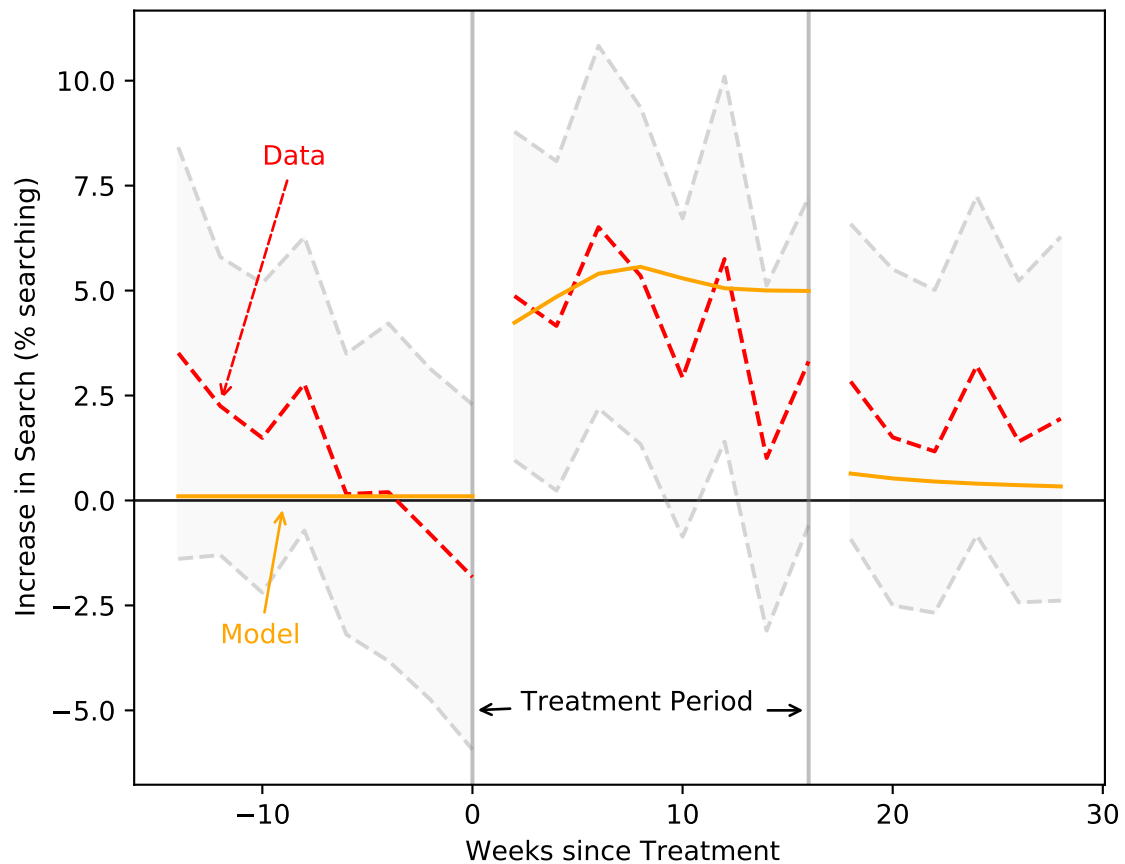
This figure displays the treatment effect on search behavior as a function of "weeks since treatment" in both the data and estimated model.

As the first validation of the model, I compare the model's predictions for the increase in search behavior when receiving the subsidy. The results are displayed in Figure 2 where the solid orange line depicts model predictions and the dotted red line depicts the experimentally estimated effects along with the associated

95 percent confidence interval. The model lines up with the experiment remarkably well. During the treatment period (between 0 and 16 weeks since treatment), treated individuals were roughly 5 percentage points more likely to search, a fact which is replicated in the model. There is a small decline in the point estimates in the last few weeks of treatment that is not quantitatively replicated by the model, but this decline is statistically insignificant, and the model continues to fall within the estimated 95 percent confidence interval.

The model also qualitatively replicates the fact that effects seem to persist for some weeks after treatment is ended, although the experimental point estimates here are noisy. The model's predictions are quantitatively smaller than these point estimates, but are well within the 95 confidence interval. One explanation for the model's under prediction of persistence is that the increase in search due to treatment resulting in some sort of learning not captured in the model, leading treated individuals to search more often even after the end of treatment.

Even if the model accurately matches the increase in search behavior due to treatment, it may not match the increase in wage employment if, for example, this increase in search was completely ineffective in generating employment. To guard against this, I also compare the model's prediction for the increase in employment after 16 weeks of treatment to that observed in the experiment. Unsurprisingly, due to the constant job finding probability, the model predicts that this roughly 5 percentage point increase in search probability results in a roughly 5 percentage point higher probability of being employed after 16 weeks. The experimental equivalent is 3.4 percentage points (95 percent confidence interval XX to YY). This is slightly lower, but the model is still reasonably accurate and well within sampling variation. The slightly small point estimate maybe be evidence of decreasing returns to scale in search, perhaps arising from the fact that job seekers go after the opportunities that they judge most likely to yield employment first.

5. Quantitative Exercise and Results

As the main quantitative experiment, I implement a cash transfer each period targeted at all individuals who are searching for wage work. I choose the size of the subsidy to be equal size used to validate the model in the previous section. In particular, this subsidy is equal to 13.7 percent of average weekly earnings (across both sectors). Recall that this subsidy size was designed to exactly offset the costs of search. As a result, the subsidy essentially sets the search cost b to zero. For

the main exercise, I assume that the subsidy is funded by a flat tax levied on wage workers, rather than a tax on all workers. This is an important distinction as it means that the tax itself serves to distort workers' choice of sector towards self-employment and, as a result, the tax contributes to the crowd-out effect. In the future, I plan to evaluate an alternative scenario where the subsidy is funded by a flat tax on all workers, eliminating this distortion, and compare how the results differ between these two cases.

Table 4 displays the results of this policy. Column (1) displays the value of moments key aggregate moments in the benchmark steady-state of the estimated model while column (2) displays the values of these moments in the post-subsidy steady-state. The policy results in a substantial increase in both GDP and welfare. Welfare increases by 0.6 percent of consumption on average while GDP increases by a little over 2 percent. This increase in GDP is the result of a 5.4 percentage point increase in the size of the wage sector, which is more productive than the self-employment sector, and an increase in wage sector earnings of 1.88 percent. This increase in earnings is the direct result of higher average wage sector TFP in the post-subsidy steady-state of the model. As the subsidy encourages wage work and the labor market slackens, entrepreneurs now dedicate fewer resources towards hiring and more resources to growth. This increase in growth is disproportionately beneficial to higher productivity entrepreneurs, allowing them to increase their market share and increasing TFP. A portion of this higher TFP is shared with workers through higher wages due to bargaining. However, it is important to note that the increase in wages due to higher TFP is not enough to overcome the increase in taxes necessary to fund the policy; post-tax earnings in the wage sector decrease by 0.5 percent

The search subsidy has only a modest impact on the size of the wage sector which increases from 34.4 percent to 39.8 percent. Labor market tightness decreases resulting in a small decrease in the job-finding probability from 3.1 percent to 2.95 percent and, consequently, an increase in the unemployment rate by 1.2 percentage points. The decrease in job-finding probability together with the decrease in post-tax earnings in the wage sector strongly suggests that the crowd-out effect dominates the crowd-in effect. To investigate this quantitatively, I perform an additional numerical experiment. Because the crowd-out and crowd-in effects operate through labor market tightness and earnings, both of which are equilibrium objects, I also compute the results of the subsidy if these equilibrium objects

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Table 4: Results of Implementing Search Subsidies

Variable	(1) Benchmark	(2) Subsidy	(3) Fixed θ
GDP		+1.5%	-
Household Welfare		-0.4%	2.9%
Size of Wage Sector	31%	34%	47%
Avg. Wage		-0.2%	-0%
Avg. Wage (incl. tax)		-3.0%	-
Job-Finding Prob.	13.6%	12.8%	13.6%
Unemployment Rate	12.2%	13.8%	15.8%

This table displays the results of the primary quantitative exercise of subsidizing search for wage jobs. Column (1) reports key aggregate parameters in the steady-state of the model before implementation while Column (2) reports these same parameters in the new steady-state of the model once the policy has been implemented. Column (3) displays the results in a hypothetical steady-state where labor market tightness θ is fixed. See the discussion for details on how to interpret these results.

were fixed to their pre-subsidy values.

The results of this numerical experiment are displayed in column (3) of Table 4. I interpret these results (when compared to the pre-subsidy model) as revealing the direct impact of the subsidy on workers' decisions and outcomes while the difference between these results with fixed labor market tightness and wages then reveals the impact of the general equilibrium effects of the subsidy. The most striking difference between this numerical experiment and the post-subsidy steady state is the size of the wage sector. When equilibrium parameters are fixed, the subsidy increases wage sector participation by a remarkable 16.1 percent points to 50.5 percent. Nearly three times as much as the 5.4 percentage point increase induced by the policy in full equilibrium. This stark difference suggests that the direct impact of the search subsidy is large; search costs serve as a substantial constraint in preventing workers from participating in the wage sector.

The large difference in wage sector participation between the full equilibrium results and the results with equilibrium values fixed also suggests that the crowd-out effects play a substantially larger quantitative role than the crowd-in effects. As can be seen from column (3), when equilibrium adjustment is shut down, the crowd-in and crowd-out channels are shut down. Labor market tightness is fixed, there is no change in the job-finding probability or in taxes that may crowd out wage workers. Similarly, because wages are fixed, there is no increase in the wage due to higher TFP that could crowd-in additional workers. Once both these channels are introduced, the size of the wage sector falls substantially, consistent with the notion that the crowd-out channels dominate.

Interestingly, the crowd-out effect seems to be large despite a fairly small decrease in the job-finding probability in the new equilibrium. The probability falls by 0.15 percentage points from 3.10 percent to 2.95 percent, a small decline. This large change in the size of the wage sector despite a small decline in job-finding probability indicates that the semi-elasticity between an individual's search choice and their probability of finding a job must be fairly large, likely a direct result of high estimated risk aversion. This behavior seems consistent with experimental interventions such as Alfonsi et al. (2020) and Abebe et al. (2017) that find large impacts on search behavior of treatments that lead individuals to substantially revise their expectations of their job-finding likelihood.

Figure 3: Welfare Effects of Search Subsidy as a Function of Household Assets
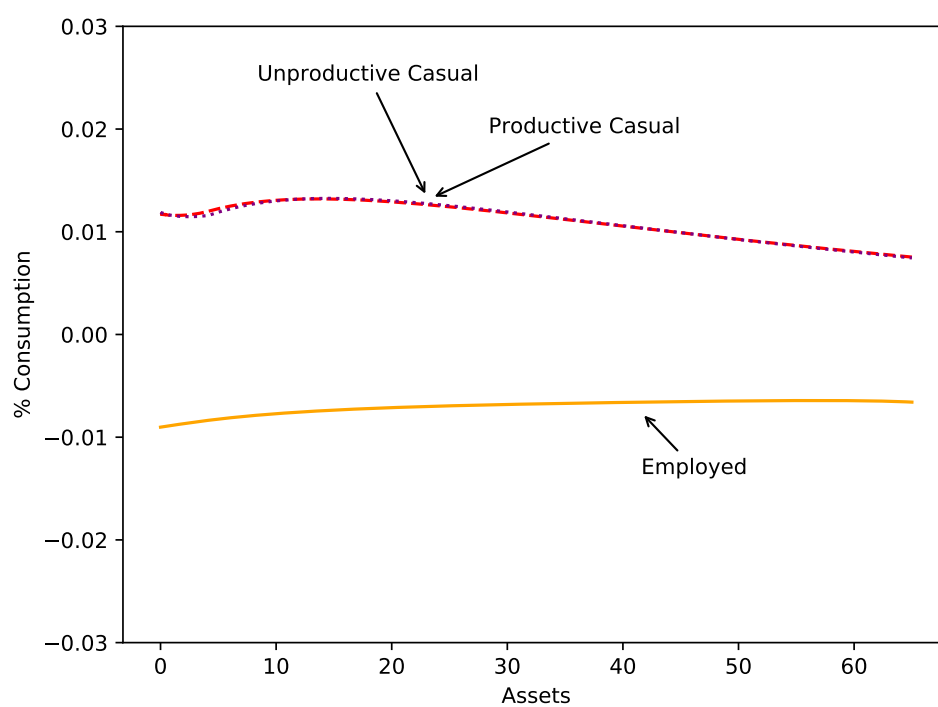
This figure displays the change in welfare, measured in consumption equivalent welfare, of the search subsidy policy as a function of a household's assets as well as their employment status and self-employment productivity.
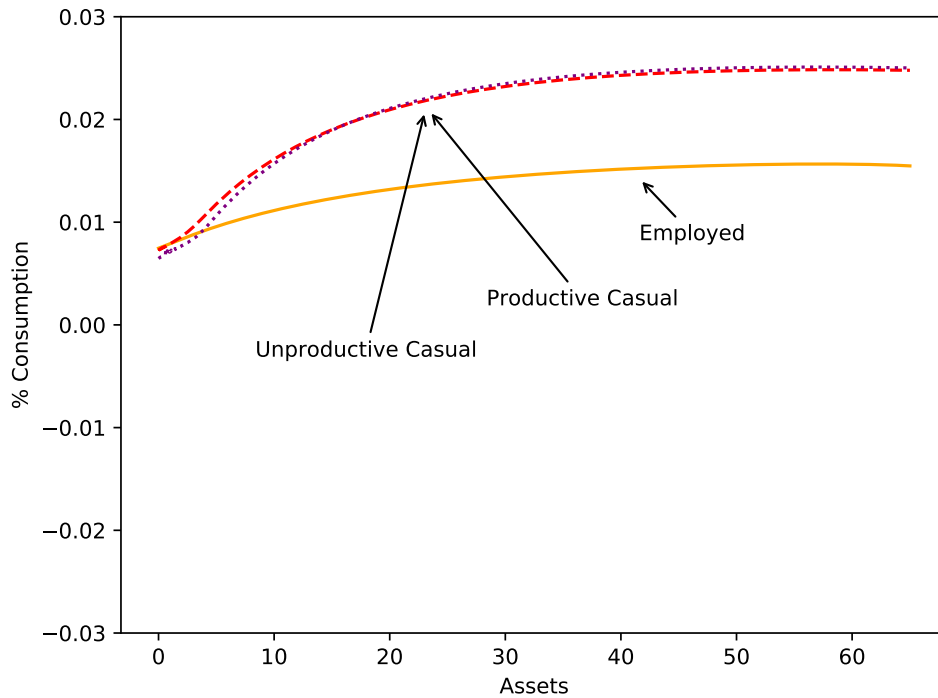
5.1. Welfare

Figure 3 displays the welfare impact of the search subsidy as a function of individual assets and employment status. For now, these numbers are calculated by comparing steady-states, although I plan to compute welfare along the transition path in the future. The red and purple lines display the welfare impact for workers without a wage sector job in the high productivity and lower productivity states respectively while the orange line displays the impact for workers matched with a wage job. Two aspects of the figure are striking. The first is that the welfare effects are highly dependent on an individual's employment state. The workers without a wage job, who switch between engaging in self-employment and searching for work, experience large welfare gains equal to around 1 percent of consumption while workers matched with an employer experience welfare loss of a little less than 1 percent. This gap is intuitive; workers without a wage job are either searching or anticipate to be searching in a few periods and thus are direct beneficiaries of the subsidy while workers already matched with a job pay a tax in order to fund the subsidy.

The second striking aspect of Figure 3 is that the welfare impacts exhibit very little heterogeneity with respect to an individual's level of wealth; individuals with zero assets experience welfare changes similar to the highest asset individuals. At first glance this result seems puzzling; however, splitting the welfare impact into the direct impact of the subsidy and the indirect impact through equilibrium objects reveals the intuition. Figure 4 displays the effect of the subsidy on welfare as a function of assets while fixing the equilibrium values of labor market tightness, wages and taxes (i.e. corresponding to column (3) of Table 4) while Figure 5 displays the difference between this counterfactual and the full results. In essence, Figure 4 displays the direct impact of the subsidy while Figure 5 displays the indirect impact.

In these figures, the impact of the policy is clearly heterogeneous with respect to individual wealth. The direct effect of the subsidy exhibits the largest welfare gains for the wealthiest individuals. Recall that households will participate in the wage sector until their self-insurance falls below a certain level, after which they will turn to self-employment until they have accumulated a buffer stock of savings. Because wealthy individuals can run down their assets for longer than poor individuals while searching for a job, they expect to collect the subsidy for more periods than poor households, who may only be able to search for a handful of

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periods before turning to self-employment. The welfare losses from the indirect effects of the policy are largest for wealthy households for a similar reason. Because wealthy households expect to participate in the wage sector the longest, they face the largest losses from a decline in the job-finding probability and an increase in taxes. Although the indirect effect and the direct effect individually exhibit substantial heterogeneity with respect to wealth, when they are combined the larger gains and larger losses for wealthy households serve to counteract each other and the overall welfare change doesn't vary much with wealth.

Figure 4: Welfare Effects of Search Subsidy as a Function of Household Assets (Fixed θ)

This figure displays the change in welfare, measured in consumption equivalent welfare, of the search subsidy policy as a function of a household's assets as well as their employment status and self-employment productivity in an alternative model where labor market tightness θ is fixed and does not change as a result of the policy. See the discussion for intuition on how to interpret these results.

6. Conclusion

[To be written]

Figure 5: Difference Between Welfare Effects of Subsidy with and without Fixed θ
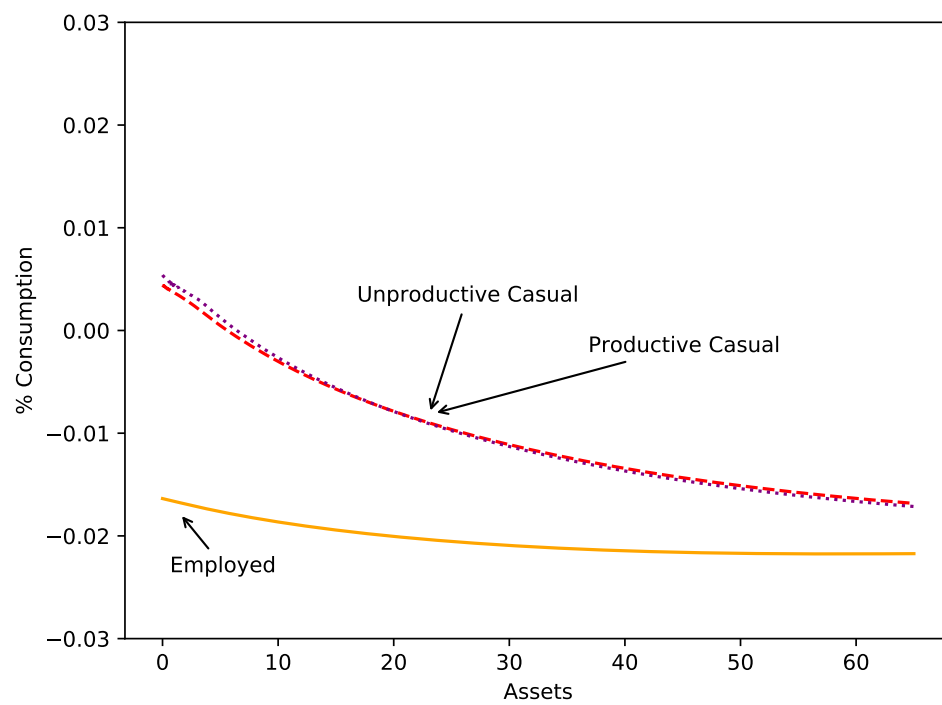
This figure the difference in the change in welfare as a function of household assets, employment status, and self-employed productivity between the full model and the alternative model with fixed θ. See the discussion for intuition on how to interpret this figure.

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Appendix

A. Additional Tables and Figure

Table A.1: Effect of Search Subsidy on Labor Market Outcomes (Abebe et al. 2021)

Outcome	Control Mean	Effect of Subsidy
Any Work	0.526	0.037
		(0.029)
Hours Worked	26.18	0.183
		(1.543)
Monthly Wages	857.9	65.88
		(63.86)
Permanent Job	0.171	0.033*
		(0.018)
Formal Job	0.224	0.054**
		(0.019)
Job Satisfaction	0.237	-0.001
		(0.027)

This table reproduces the primary results of Abebe et al. (2021) and displays the control mean for a variety of labor market outcomes as well as the experimentally estimated treatment effect of a conditional cash transfer to job seekers.

B. Derivations and Proofs from Section 2.6

The first result to show is that the entrepreneur's optimal choice of f' and n' satisfy $\eta(z; X) = \frac{\gamma f'^*}{n'^*}$ for some function η depending only on z and X. Substituting in the wage determination equation (which the entrepreneur takes as given) and the vacancy posting constraint, the first-order condition for f' and n' can be combined with the envelope condition for f and n to generate

$$\beta \Delta \mu' \left((1-\alpha)(1-\chi)z(\frac{\gamma f'}{n'})^\alpha - \left((1-\chi)\underline{w} - \frac{c}{p(\theta(X'))}(1-\lambda) \right) \right) = \frac{c}{p(\theta(X))}\mu$$

$$\beta \Delta \mu' \left(\gamma\alpha(1-\chi)z(\frac{\gamma f'}{n'})^{\alpha-1} + 1 - \gamma(r+\delta) \right) = \mu$$

where μ is the Lagrange multiplier on the budget constraint, μ' is the Lagrange multiplier on the budget constraint in the following period, and $\theta(X')$ is a price

function mapping aggregate states X to equilibrium values of θ. Combining these two equations, substituting in η, and defining A, $B(X')$, and $C(X')$ for clarity yields

$$A z \eta^{\alpha} + B(X, X') z \eta^{\alpha-1} + C(X, X') = 0$$

which, for $0 < \alpha < 1$, can be shown to have a unique and positive solution for η for any value of z, X, and X'. Call this solution $\tilde{\eta}(z; X, X')$. Finally, substituting $X' = H(X)$ and defining $\eta(z; X) = \tilde{\eta}(z; X, H(X))$ completes the derivation.

The next result to show is that entrepreneurs choose a growth rate that depends only on their z and aggregate state variables. This follows almost directly from the previous result. Substituting $n = \frac{\gamma}{\tilde{\eta}(z;X)} f$ in to the budget constraint of the entrepreneur problem reveals that the RHS of the budget constraint is now linear in f and can be written

$$d + \left(1 + \frac{c}{p(\theta(X))} \frac{\gamma}{\eta(z; X)}\right) f'$$

$$=$$

$$\left(\left(1 - \chi\right)\gamma z \tilde{\eta}(z; X)^{\alpha-1} - \left((1 - \chi)\underline{w} - \frac{c}{p(\theta(X))}(1 - \lambda)\right)\frac{\gamma}{\tilde{\eta}(z; X)} + \left(1 - \gamma(r + \delta)\right)\right) f$$

$$\Rightarrow \quad d + E(z, X)f' = D(z, X)f$$

where $D(z, X)$ and $E(z, X)$ are defined such that the second line is equivalent to the first line. E functions as the price of collateral f relative to the price of consumption d while D functions as the return to collateral. Because entrepreneurs possess log utility, the entrepreneur problem has the well-known solution of a constant growth rate in f depending on the values of D and E which are given by z and X so that $f' = g(z; X)f$.

The final result to show is the proof of Proposition 1. By assumption, θ is assumed to be constant. Let $\hat{E}(z, \theta)$ and $\hat{D}(z, \theta)$ denote E and D respectively, but with $\theta(X)$ simply replaced by θ, the argument to the function. Note that this is possible because E and D only depend on X through θ. Then we have the explicit

solution[14]

$$\hat{g}(z,\theta) = \left(\beta\Delta\frac{\hat{D}(z,\theta)}{\hat{E}(z,\theta)}\right)^{\frac{1}{\sigma}}$$

$$= \left(\beta\Delta\frac{((1-\chi)\gamma z\hat{\eta}(z;\theta)^{\alpha-1} - ((1-\chi)\underline{w} - \frac{c}{p(\theta)}(1-\lambda))\frac{\gamma}{\hat{\eta}(z;\theta)} + (1-\gamma(r+\delta)))}{\left(1 + \frac{c}{p(\theta)}\frac{\gamma}{\hat{\eta}(z;\theta)}\right)}\right)^{\frac{1}{\sigma}}$$

The chain rule yields $\frac{d\hat{g}}{d\theta} = \frac{\partial\hat{g}}{\partial c/p(\theta)}\frac{dc/p(\theta)}{d\theta} + \frac{\partial\hat{g}}{\partial\hat{\eta}}\frac{d\hat{\eta}}{dc/p(\theta)}\frac{dc/p(\theta)}{d\theta}$. Using either direct calculation of partial derivatives or implicit differentiation (in the case of $\frac{d\hat{\eta}}{dc/p(\theta)}$), we can express each individual piece as

$$\frac{\partial\hat{g}}{\partial c/p(\theta)} = -\frac{1}{\sigma}\hat{g}^{1-\sigma}\left(\frac{(\frac{\hat{g}}{\beta\Delta}-1)+\lambda}{\frac{\eta}{\gamma}+\frac{c}{p(\theta)}}\right) \le 0$$

$$\frac{\partial\hat{g}}{\partial\hat{\eta}} = \frac{1}{\sigma}\hat{g}^{1-\sigma}\left(\frac{\frac{\beta\Delta}{\hat{g}}-\frac{\hat{g}}{\beta\Delta}}{\frac{\eta}{\gamma}+\frac{c}{p(\theta)}}\right) \le 0$$

$$\frac{d\hat{\eta}}{dc/p(\theta)} = \frac{\gamma(\alpha(1-\chi)z\hat{\eta}^{\alpha-1}-(r+\delta))+\lambda}{J(\theta)} > 0$$

where $J(\theta)$ is a placeholder for a complex but unambiguously positive expression and I have made use of the first-order condition for f' in the second expression. It is worth commenting briefly on why the claimed inequalities hold. Both the first and second expressions follow directly from the fact that an optimally acting entrepreneur will ensure that $g \ge \beta\Delta$. This is clearly true as an entrepreneur can always choose to select $k = 0, n = 0$ and simply eat their cake, yielding $g = \beta\Delta$. An entrepreneur will only choose to operate if they can be weakly better off by doing so. The third and final expression follows from the first-order condition for capital which ensures that the marginal product of capital $\alpha(1-\chi)z\hat{\eta}^{\alpha-1}$ is greater than the marginal cost of capital $r + \delta$ (the MPK is greater, rather than equal to, the marginal cost due to the presence of the financing constraint). Because $\frac{dc/p(\theta)}{d\theta} > 0$ by construction, combining these inequalities with the chain rule provides the result $\frac{d\hat{g}}{d\theta} < 0$ and along the way we have shown $\frac{d\hat{\eta}}{d\theta} > 0$.

The result for $\frac{\partial\hat{g}}{\partial\theta\partial z}$ is straightforward. We have $\frac{\partial\hat{g}}{\partial z} = \frac{1}{\sigma}\hat{g}^{1-\sigma}\left(\frac{(1-\chi)\hat{\eta}^{\alpha}}{\frac{\hat{\eta}}{\gamma}+\frac{c}{p(\theta)}}\right)$ which is also

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[14]Note that this is isomorphic to a textbook cake-eating problem under CRRA utility with the addition that the cake can grow or depreciate at a constant rate. While the policy function discussed here is an intuitive generalization of the well-known textbook solution $f' = \beta^{\frac{1}{\sigma}}f$, I have not located any discussion of this exact problem to cite. Thus, the derivation is available upon request.

clearly greater than zero and decreasing in θ. Although this result holds only for partial derivatives (i.e. with $\hat{\eta}$ being held constant), it can also be shown to hold for total derivatives in the case where $\hat{\eta} \geq \alpha(1 + \frac{c}{p(\theta)}\gamma)$ by applying the chain rule as above and computing $\frac{d\hat{\eta}}{dz}$ using implicit differentiation.

C. Derivations and Proofs from Section 3

First, I formally define the functions v and H introduced in equation 14.

$$v(m_t, \eta_t, \eta_{t+1}, g_t) = \int \left[g_t(z)\Delta\frac{\eta_t(z)}{\eta_{t+1}(z)} - (1 - \tilde{\lambda}) \right] m_t(z) + \frac{\hat{D}(z, \theta_t, \eta_t(z))\gamma\underline{f}}{\eta_{t+1}(z)} h(z)\, dz \tag{18}$$

$$H(z, m_t, \eta_t, \eta_{t+1}, g_t) = \frac{\left[g_t(z)\Delta\frac{\eta_t(z)}{\eta_{t+1}(z)} - (1 - \tilde{\lambda}) \right] m_t(z) + \frac{\hat{D}(z, \theta_t, \eta_t(z))\gamma\underline{f}}{\eta_{t+1}(z)} h(z)}{v(m_t, \eta_t, \eta_{t+1}, g_t)} \tag{19}$$

The numerator is the number of matches with a productivity z entrepreneur and the denominator is the total number of matches.

The problem of the constrained social planner is given sequentially by

$$\max_{\{c_t, a'_t, s_t, \theta_t, m_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \int \int \int u(c_t) m_t(a, z) j(y)\, dy\, da\, dz$$

$$s.t.\ a'_t + c_t = Ra_t + (1 - s_t)y + s_t(w_t(z) - (1 - z)b)\ \ \forall a, y, z$$

$$a_{t+1} \geq 0 \tag{20}$$

$$\frac{v(m_t, \eta_t, \eta_{t+1}, g_t)}{\theta_t} = \int \int s_t(a, 0) m_t(a, 0) j(y)\, dy\, da$$

$$m_{t+1}(a'_t, 0) = m_t(a, 0) - \theta_t p(\theta_t) \int s_t(a, 0) m_t(a, 0) j(y)\, dy$$

$$m_{t+1}(a'_t, z) = (1 - \tilde{\lambda}) m_t(a, z) + H(z, m_t, \eta_t, \eta_{t+1}, g_t)\theta_t p(\theta_t) \int s_t(a, 0) m_t(a, 0) j(y)\, dy$$

where the functions η_t and g_t arise from the slightly modified sequential problem

of an entrepreneur:

$$\max_{\{d_t, f_{t+1}, k_t, n_t, v_t\}} \sum_{t=0}^{\infty} (\beta\Delta)^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

$$s.t. \ d_t + f_{t+1} = (1-\chi)zk_t^\alpha n_t^{1-\alpha} - (r+\delta)k_t - (1-\chi)\underline{w}n_t + f_t - cv_t$$

$$n_{t+1} = (1-\lambda)n_t + p(\theta_t)v_t \tag{21}$$

$$k_t \leq \gamma f_t$$

$$f_0 \in \mathbb{R}$$

so that $\eta_t = \frac{\gamma f_t}{n_t}$ and $g_t = \frac{f_{t+1}}{f_t}$.

Note that here I have imposed the scale-invariance of the entrepreneurs optimal capital-labor ratio and growth rate and left the initial condition f_0 arbitrary. In analysis of the problem of the social planner, it will be useful to note that while η_t and g_t are potentially functions of z and the entire sequence of labor market tightness $\{\theta\}_{t=0}^{\infty}$, solving the entrepreneur's problem reveals that they depend only on ability z and current and future tightness θ_t, θ_{t+1} and thus can be written as $\eta_t(z, \theta_t, \theta_{t+1})$ and $g(z, \theta_t, \theta_{t+1})$. The independence of entrepreneur policy functions from values of θ beyond period $t+1$ follows directly from the linearity of the hiring cost, combined with the parameter assumptions that ensure that any operating entrepreneur will choose $v_t > 0$ each period. While the continuation value of an entrepreneurs labor force depends in theory on the whole sequence of labor market tightness, the ability to re-optimize at linear cost tomorrow ensures that this continuation value is equal to the "liquidation value" of the workforce next period.

3.1. Notes and Proof for Proposition 2

The dynamic terms in equation 17 are given by

Anticipation Terms =

$$\frac{S}{\bar{\theta}}\left(\mu_{t-2}\left(\frac{\partial v_{t-2}}{\partial \eta_{t-1}}\frac{\partial \eta_{t-1}}{\partial \theta_t}\right) + \mu_{t-1}\left(\frac{\partial v_{t-1}}{\partial \eta_{t-1}}\frac{\partial \eta_{t-1}}{\partial \theta_t} + \frac{\partial v_{t-1}}{\partial \eta_t}\frac{\partial \eta_t}{\partial \theta_t}\right) + \mu_t\left(\frac{\partial v_t}{\partial \eta_t}\frac{\partial \eta_t}{\partial \theta_t}\right)\right) + \tag{22}$$

$$\bar{\theta}p(\bar{\theta})S\left(\int_z \lambda_{t-2}(z)\left(\frac{\partial H_{t-2}}{\partial \eta_{t-1}}\frac{\partial \eta_{t-1}}{\partial \theta_t}\right)dz + \int_z \lambda_{t-1}(z)\left(\frac{\partial H_{t-1}}{\partial \eta_{t-1}}\frac{\partial \eta_{t-1}}{\partial \theta_t} + \frac{\partial H_{t-1}}{\partial \eta_t}\frac{\partial \eta_t}{\partial \theta_t}\right)dz + $$

$$\int_z \lambda_t(z)\left(\frac{\partial H_t}{\partial \eta_t}\frac{\partial \eta_t}{\partial \theta_t}\right)dz\right)$$

where μ_t and $\lambda_t(z)$ are the shadow prices associated with the constraints on aggregate labor market tightness and productivity-specific matching rates respectively. These terms essentially capture the welfare gains from anticipatory hiring when labor market tightness is changed. Essentially, while the welfare changes from permanent changes in hiring are captures in the other terms of equation 17, this term captures the small gains that occur due to the fact that some of this hiring is done in anticipation of the change, shifting some hiring forward temporally.