

Job Search in Developing Countries: Crowd-In and Crowd-Out Externalities

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Abstract

Productive wage work is often difficult to find in developing countries. Many policies aim at assisting searchers and expanding the wage sector, but the rationale for intervening is unclear. This paper develops a search-and-matching model that incorporates key features of developing economies including a large self-employment sector, savings-constrained households, and capital-constrained firms. Four search externalities — two positive and two negative — emerge, leading to inefficiency. After estimating the model using an experiment that provided search subsidies to job seekers in Ethiopia, I find that the optimal policy is a *tax* that roughly doubles the cost of search, rather than a subsidy.

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1. Introduction

Movement out of self-employment and into wage work is a key feature of structural change and development, but reliable wage-sector jobs are often difficult to find (Gollin 2008, Buera, Kaboski & Shin 2015, Poschke 2019). Many individuals spend months or even years alternating between self (or marginal) employment and job search before finally finding long-term wage work (see e.g. Donovan, Lu & Schoellman 2023). As a result of these two facts, there has been substantial interest in the impacts and effectiveness of policies aimed towards expanding the wage sector from both academics and policymakers, including subsidies to labor search, transport, and (temporarily) wages (e.g. Levinsohn et al. 2014, Franklin 2018, De Mel et al. 2019, Abebe et al. 2021, and many others).

The rationale for intervening in the labor market, however, is not generally clear. Canonical models of frictional labor markets generate inefficiencies, as in Hosios (1990), and are well understood but lack many features central to developing economies, such as large shares of self-employment and substantial credit market frictions. Developing and evaluating labor market policies for developing countries requires understanding how the externalities arising from labor market frictions manifest in such an environment.

This paper generalizes these externalities to a model relevant to developing countries and studies the general equilibrium impacts and trade offs that they generate. Individuals have access to a self-employment option for subsistence and are savings constrained, limiting their ability to fund job search (as in Feng, Lagakos & Rauch 2018, Herreño & Ocampo 2021), while entrepreneurs run firms and face financial constraints that restrict their growth and distort the allocation of resources (as in Itskhoki & Moll 2019, Buera, Kaboski & Shin 2021). The two interact through a labor market exhibiting canonical Diamond-Mortensen-Pissarides search-and-matching frictions, which generates externalities from labor search and vacancy posting.

Individuals in the model desire higher-paying wage jobs but must pay a search cost (e.g. commuting) and give up a period of income in order to search. Because they face idiosyncratic job-finding risk, only sufficiently self-insured individuals will choose to search while others will opt for the guaranteed but lower income of self-employment. The model thus reproduces the empirical fact that individuals frequently and stochastically shift between self employment and job search before finding wage work.

Entrepreneurs operate a constant returns to scale production technology and consequently desire to be large but are restricted in size by a collateral constraint that prevents them from financing capital beyond some multiple of their wealth. They hire workers by posting vacancies, but any funds spent paying vacancy posting costs are funds that can no longer be used as collateral in the future. Thus labor market frictions act as a constraint to firm growth.

Despite the complexities of borrowing constrained individuals, credit constrained firms, and frictional labor markets, substantial insight into the general equilibrium effects and externalities stemming from labor market frictions can be gained analytically. Motivated by the emphasis within the development literature on worker-side interventions (often called “Active Labor Market Policies”), I consider the problem of a dynamic Ramsey planner who maximizes average worker utility subject to the constraint that it must respect individual and entrepreneur budget and credit constraints, must respect the matching technology, and cannot dictate the behavior of entrepreneurs (as in e.g. [Itskhoki & Moll 2019](#)).

Examining the planner’s problem reveals four key externalities. Because they cause in one individual’s decision to search to either discourage or encourage the search of others, I refer to these as “crowd-out” or “crowd-in” effects, and there are two of each.

The first crowd-out effect corresponds to the well-known congestion externality in typical models. An individual who searches exerts downward pressure on labor market tightness, lowers the probability that any particular individual finds a job, and causes individuals on the margin of search to move into self employment. The second crowd-out effect occurs through capital shallowing; downward pressure on labor market tightness lowers the cost of hiring for entrepreneurs and decreases the cost of labor relative to capital. As a result, entrepreneurs opt for a lower capital-labor ratio which (through a lower marginal product of labor and bargaining) leads to lower wages, again pushing marginal individuals into self-employment.

The first crowd-in effect arises from the fact that entrepreneur size is limited by collateral constraints. If an individual searches and finds a job, the resulting output is split between the worker and entrepreneur (by bargaining). The entrepreneur then uses a portion of this surplus to finance further expansion of the firm in the next period, including hiring additional workers. This effect even compounds dynamically — a portion of the output generated by these additional workers finances more expansion, and so on. In this sense, a searcher’s (potential)

employment directly crowds in future workers.

The final crowd in effect stems from a similar source. As searchers push labor market tightness down, hiring costs fall and entrepreneurs grow their firms faster. However, this increase in growth is larger for more productive entrepreneurs — they wish to grow faster (than less productive entrepreneurs) and, as a result, hiring costs make up a larger share of their total costs. A decline in these costs represents a larger (proportional) cost reduction, freeing up more resources for growth. As a result of their faster (relative) growth, the share of capital and labor allocated to the productive entrepreneurs increases over time, raising allocative efficiency, Total Factor Productivity, and wages. The result is a crowding of additional searchers from self-employment.

Despite having access to a wide variety of complex tax instruments that condition on household heterogeneity, the planner’s optimal solution balancing all four externalities can be implemented using only a single tax (or subsidy). One somewhat surprising implication of this result is that individuals’ inability to borrow does not exert additional externalities. Intuition might suggest that a planner who wants to induce individuals to search more will also want to make individuals save more in order to fund this search; however, this turns out not to be the case. Conditional on a subsidy that fully internalizes the externalities, individuals’ consumption-savings decisions align with those of the planner highlighting the fact that it is not households’ inability to borrow *per se* that justifies policy intervention.

Beyond their impact on efficiency, these four channels effects also have a direct impact on aggregate outcomes — they are relevant even for a policymaker who does not care about efficiency and values only basic outcomes, such as increasing the size of the wage sector. For any intervention aimed at increasing (or decreasing) employment, the difference between the impact that would be observed in a small-scale experimental evaluation (i.e. the partial equilibrium effect) and the impact that would occur if the policy were implemented universally (i.e. the general equilibrium effect) is exactly determined by the sum of the four channels.

In order to quantify these channels, I estimate the model using simulated method of moments to match search behavior from weekly data collected as part of an experimental evaluation of a labor search subsidy in Ethiopia ([Abebe et al. 2021](#)). I estimate the model using data from control individuals (those not receiving a subsidy) only and validate the model using its predictions for the search behavior of

treated individuals (those who receive a subsidy). In both the data and the model, the likelihood of searching in any given week increases by about 5 percent when treated.

Surprisingly, the optimal subsidy in the estimated model is equal to -101 of total search costs — that is, the optimal policy is actually a tax on search that roughly doubles the search cost (an increase equal to about 20 percent of average self employment earnings). Even though a subsidy would lead to a direct utility benefit as it reallocates income from a relatively high income state (working in self-employment) to a low income state (searching), the negative externalities from search (congestion and capital shallowing) outweigh the positive externalities (size and efficiency) to such an extent that a tax ends up maximizing welfare. The gains from the optimal policy are substantial; on average, consumers enjoy an increase in welfare equal to 1.5 percent of their consumption.

I decompose the contribution of each individual externality to the optimal tax rate by starting in a counterfactual model with all four channels shut down. In this model, the optimal policy is a subsidy equal to 95 percent of search costs. Introducing the channels one at a time (cumulatively) and computing the resulting changes in the optimal policy then measures the importance of each channel. The contribution of the size and congestion externalities are 30 percentage points and -205 percentage points respectively (i.e. if both were considered, the optimal policy would be a 80 percent tax). The capital shallowing and efficiency externalities contribute an additional -25 and 4 percentage points, resulting the in the efficient policy of a tax rate equal to 101 percent.

Turning from the efficient policy, I next examine to what extent a subsidy can be used to increase employment (i.e. for the hypothetical policymaker mentioned above). A two-thirds subsidy to search — the size evaluated in the experiment used to estimate the model — increases the size of the wage sector from 30 percent of workers to 40 percent when all four channels are shut down. A decomposition exercise similar to that above reveals that the size and congestion externalities lead to 4 and -10 percentage point changes in this increase respectively while capital shallowing and efficiency contribute -1.2pp and 0.1pp. After all channels are accounted for, the increase in employment falls to 3pp (to 33 percent).

Overall, the surprising conclusion is that, despite contradicting the intuition of policymakers and economists alike, labor markets in developing countries (at least those similar to the market in Ethiopia used to estimate the model) are character-

ized by workers who search too much rather than too little. Consequently, policies aimed at helping and encouraging workers to search, such as search subsidies, are counterproductive. While they do manage to increase the size of the wage sector and may even yield promising experimental results, they do so fairly ineffectively once equilibrium effects are taken into account and come at the cost of welfare.

Related Literature: This paper is closely related to the macroeconomic development literature studying the impact of entrepreneur-level credit constraints on growth and development such as [Buera, Kaboski & Shin \(2011\)](#), [Moll \(2014\)](#), [Itskhoki & Moll \(2019\)](#), and [Buera, Kaboski & Shin \(2021\)](#). This paper also builds on recent work draws distinction between subsistence self-employment and entrepreneurship (such as [Feng & Ren 2021](#)) or otherwise study unemployment in developing countries (such as [Feng, Lagakos & Rauch 2018](#), [Poschke 2019](#)). Closely related is [Herreño & Ocampo \(2021\)](#) who use a model in which households use self-employment to cope with the risks of wage employment (the same mechanism as this paper) to study the macroeconomic effects of microloans and cash transfers.

The model dynamics in which workers flow freely between self/marginal employment and labor search before finding a long-term wage job are very similar to those documented in [Donovan, Lu & Schoellman \(2020\)](#). In a similar vein, [Banerjee et al. \(2021\)](#) find that skilled workers in developing countries exhibit higher unemployment rates, relative to unskilled workers, and show that this differences leads to differences in occupational choice. [Porzio, Rossi & Santangelo \(2021\)](#) use a model with frictional reallocation of labor from (self-employment dominated) agriculture to (wage work dominated) non-agriculture to quantify the importance of human capital in explaining the process of structural change.

This paper is also closely related to the microeconomic literature on Active Labor Market Policies, which are intended to help grow the wage sector. [Abebe et al. \(2021\)](#) and [Franklin \(2018\)](#) both study the effects of cash transfers (the same policy studied in the quantitative portion of this paper). [De Mel et al. \(2019\)](#), [Algan et al. \(2020\)](#), and [Alfonsi et al. \(2020\)](#) all study firm-side interventions (although the latter includes an additional worker-side treatment arm) also intended to help workers find jobs. [McKenzie \(2017\)](#) provides an excellent review of this literature, which is too exhaustive to list here.

2. Model

Time is discrete. There is measure one of households and an endogenous measure of entrepreneurs. Households consume, save, and choose between working in self-employment or participating in the labor market while entrepreneurs operate firms, consume profits, and accumulate capital and labor for future periods.

2.1. Search and Matching Technology

The labor market for wage work exhibits typical search-and-matching frictions. Households must search for jobs and entrepreneurs must hire by posting vacancies. The cost of searching for a job and the cost of posting a vacancy are denoted by b and c respectively. Each period, the number of worker-firm matches is given by a homogeneous of degree 1 matching function $m(u, v)$ where u is the measure of households searching for a job and v is the number of vacancies posted by firms. As is typical, $\theta = \frac{v}{u}$ is defined to be labor market tightness so that $p(\theta) \equiv m(\frac{1}{\theta}, 1) = \frac{m(u, v)}{v}$ is the probability that any vacancy is filled and $\theta p(\theta) = \frac{m(u, v)}{u}$ is the probability that any searcher finds a job. Finally, matches between workers and firms are separated with exogenous probability λ at the end of every period.

2.2. Households

A unit measure of infinitely-lived households are indexed by their wealth a , their employment status e , and their self-employment productivity z . Lifetime household utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \quad (1)$$

Households are endowed with one unit of time each period which they supply inelastically and indivisibly to either work or search each period.¹

Labor Decisions: Any household can engage in self-employment and operate the self-employment technology

$$y_t = A_s l_t \quad (2)$$

A household's self-employment productivity l_t follows an exogenous Markov pro-

¹This assumption can be justified by the fact that a model period is one week. Additionally, quantitative experiments with allowing interior choices of time allocation suggest that the optimal policy is fairly close to "bang-bang", with households largely choosing to allocate their entire time budget to either work or search rather than a mix of the two, for reasonable parameters.

cess described by transition matrix M . For simplicity, I normalize A_s to unity so that self-employment earnings are simply given by $y_t = l_t$. By assumption, self-employment uses only household labor and does not involve hiring workers from outside the household. Thus, this option most closely corresponds to the concept of “subsistence self-employment”.

Instead of engaging in self-employment, a household can choose to pay a search cost b and search for a wage job. A searching household earns nothing in the current period and finds a permanent job with probability $\theta p(\theta)$. After finding a job and becoming employed, a household can either work in their wage job or return to self-employment (of course, in equilibrium, all employed households will choose to engage in wage work). Wages are determined through bargaining (discussed later) and depend on the productivity of the entrepreneur with whom the household is matched, given by z_t .²

Budgets: Households face incomplete markets a la [Aiyagari \(1994\)](#), [Bewley \(1977\)](#), and [Huggett \(1993\)](#) and accumulate assets for self-insurance. Each period, assets pay an exogenous rate of return r (i.e. this is a small open economy). Households cannot borrow (i.e. $a_t \geq 0$). The budget constraint for the household is

$$a_{t+1} + c_t = (1 + r)a_t + (1 - e_t)((1 - s_t)y_t - s_t b) + e_t w_t(z_t) \quad (3)$$

where $s_t \in \{0, 1\}$ is a choice variable for the household with $s_t = 1$ representing the search decision in period t and $e_t \in \{0, 1\}$ is an indicator variable with $e_t = 1$ indicating that the household is employed in period t .

Search: Search is undirected, and every vacancy has an equal probability of being filled. A household’s probability of matching with a job that will pay $w(z)$ (conditional on matching with any job), denoted $H(z; X)$ where X is a vector of aggregate state variables, is given by the share of vacancies posted by z -type entrepreneurs. Although, in principle, w and H depend on all the state variable of both the household and the entrepreneur to which they are matched, they are written here to depend only on the matched entrepreneur’s productivity z . A later section will show this to be the case, justifying this notation.

Employed households are separated from their jobs with probability λ . Additionally, the household can lose its job if the entrepreneur employing the household

²Section 2.5 shows that the bargained wage depends only on the productivity of the entrepreneur and not on other entrepreneur or household state variables.

dies which occurs with probability $1 - \Delta$ (as discussed below) or chooses to downsize its labor force. Under generous parameter conditions (satisfied in the quantified model), it can be shown that downsizing never occurs in equilibrium, which I assume throughout the rest of the paper. Thus the probability that an employed household retains its job at the end of the period is given by $(1 - \tilde{\lambda}) = \Delta(1 - \lambda)$.

Bellman Equation: Taking all of the above, the household's problem can be written recursively as

$$\begin{aligned}
V_u(a, y, s; X) &= \max_{c, a', s' \in \{0,1\}} \frac{c^{1-\sigma}}{1-\sigma} + \beta \left((1 - s\theta p(\theta)) E_{y'}[V_u(a', y', s'; X')|y] + \right. \\
&\quad \left. s\theta p(\theta) (E_z[V_e(a', z; X')]) \right) \\
V_e(a, z; X) &= \max_{c, a'} \frac{c^{1-\sigma}}{1-\sigma} + \beta \left((1 - \tilde{\lambda}) V_e(a', z; X') + \right. \\
&\quad \left. \tilde{\lambda} E_{y'}[V_u(a', y'; X')] \right) \tag{4}
\end{aligned}$$

$$\begin{aligned}
s.t. \quad a' + c &= (1 + r)a + (1 - s)y - sb && \text{for } V_u \\
a' + c &= (1 + r)a + w(z) && \text{for } V_e \\
X' &= G(X) \\
y' &\sim M(y) \\
z &\sim H(z; X)
\end{aligned}$$

where X is a vector of aggregate state variables and G is the household's perception function for the evolution of the aggregate state. V_u and V_e denote the value function of the household while unemployed and employed respectively.

Here there is a small timing convention; household's must commit to search decisions one period ahead of time. The purpose of this convention is to prevent households from conditioning their search decision on contemporaneous productivity shocks. I have also implicitly made the assumption that households moving out of employment draw their self-employment productivity from the stationary distribution of M rather than, for example, maintaining the stochastic process for y throughout employment. Both assumptions simplify some expressions in Section 3 and appear to have little quantitative impact in Section 5.

2.3. Household Behavior

Households decide whether to engage in self-employment or search for a wage job by weighing the benefits of search against the costs. In addition to the explicit search cost b and the opportunity cost of forgone self-employment earnings, the presence of borrowing constraints means that the higher risk of job search also serves as a cost, particularly if the probability of finding a wage job is small as it is in many developing countries.

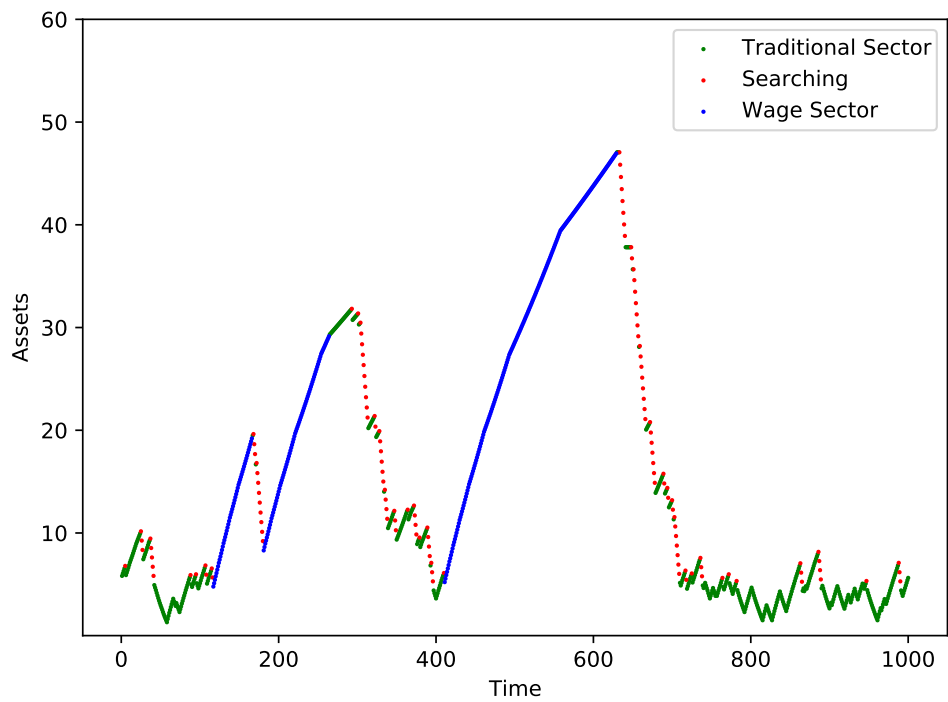
Only households who are sufficiently self-insured will opt to pay the search cost and search for wage work, hoping for the slim probability of finding a job and achieving a large boost in earnings. Households without much self-insurance will enjoy the safety of lower but guaranteed income in self-employment. For households that search, the search cost quickly diminishes their savings and reduces their self-insurance, eventually driving them to self-employment until they can re-accumulate sufficient self-insurance.

The result is that households near the threshold of self-insurance spend a few periods working in self-employment and accumulating assets, then switch to searching for a wage job for a few periods, and return to self-employment once their savings have been depleted. Of course the exact cutoff in savings above which households decide to search depends on their self-employment productivity y_t (which is stochastic), leading to some unpredictability in the exact timing of these switches.

Figure 1 displays an example of this behavior for a single household simulated for 1000 periods/weeks (about 20 years). The x-axis displays time while the y-axis displays the household's stock of assets. The color corresponds to the household's search decision in that period; weeks in green are those where the household is engaging in self-employment, red weeks correspond to searching for wage work, and blue weeks are periods when the household is employed and working for a wage.

The figure demonstrates the household behavior described above. At the start, the household is near the threshold of self-insurance and alternates between working in self-employment and searching for wage work depending on their particular level of assets and self-employment productivity. At around week 150, the household's search is successful, and they acquire a high-earning wage job and quickly accumulate assets. They eventually separate from their employer but use their stock of assets to fund extensive search and remain in the wage sector. This behavior continues for quite some time until approximately week 700 when the

Figure 1: Household Self-Employment and Wage Sector Behavior over Time



Note: This figure plots a simulated household's search, wage work, and self-employment behavior as well as assets over 1000 periods of the household's life. This simulation is performed using the quantified model described in Section 4.

household exhausts its assets without finding a job and returns to self-employment punctuated by brief periods of search.

2.4. Entrepreneurs

While households work in either self employment or the wage sector, entrepreneurs operate firms and employ households. Including entrepreneurs as distinct agents (as opposed to an occupational choice for household, as in [Buera, Kaboski & Shin 2021](#)), reflects the qualitative difference between “subsistence self-employment” (which household can flow in and out of fairly freely as in [Donovan, Lu & Schoellman 2020](#)) and productive entrepreneurship with the potential to grow and potential employ many households in addition to providing a dramatic increase in tractability.

There are N entrepreneurs each of size $\frac{M}{N}$ born every period, and the model considers the limit $N \rightarrow \infty$.³ At the end of a period, entrepreneurs die with probability Δ . Entrepreneurs are born with idiosyncratic ability z drawn from some distribution with bounded support $h(z)$ and an initial level of financial wealth \underline{f} (taken to be exogenous). They discount the future at rate β (the same rate as households), face an exogenous death probability Δ each period, and receive lifetime utility from consumption (labeled d_t for “dividends”) given by

$$\sum_{t=0}^{\infty} (\beta \Delta)^t \frac{c_t^{1-\sigma}}{1-\sigma} \quad (5)$$

Each entrepreneur operates an ability-dependent Cobb-Douglas production technology:

$$y_t = z k_t^\alpha n_t^{1-\alpha} \quad (6)$$

Entrepreneurs rent capital from the international capital market at an exogenous rental cost $(r + \delta)$ (i.e. this is a small open economy) and pay workers wage w_t , determined by bargaining.

Entrepreneurs must use their own assets f_t as collateral to finance capital and face collateral constraint

$$k_t \leq \gamma f_t \quad (7)$$

³The assumption that there are an infinite number of atomic entrepreneurs rather than a measure of non-atomic entrepreneurs is not typical but eliminates many technical difficulties in the discussion of wage bargaining. Other than this, there are no substantive differences between the two assumptions.

where $\gamma \geq 1$ is a parameter summarizing the degree of financial market frictions, with $\gamma = 1$ representing the case of full self-financing and $\gamma \rightarrow \infty$ representing no financial frictions.⁴

To hire labor and adjust n_t , entrepreneurs post vacancies v_t . Each vacancy costs c units of output to post and is filled at the end of the period with probability $p(\theta)$. The evolution of n_t is dictated by the equation

$$n_{t+1} = (1 - \lambda)n_t + p(\theta)v_t \quad (8)$$

where λ is the exogenous separation rate. Here, it is worth clarifying that while household face idiosyncratic risk in job finding and separation, entrepreneurs do not. An entrepreneur with n_t workers can ensure a labor force of precisely n_{t+1} next period by posting $\frac{n_{t+1} - (1 - \lambda)n_t}{p(\theta)}$ vacancies.

An entrepreneur's period profits are given by

$$\pi_t(z, k_t, n_t) = zk_t^\alpha n_t^{1-\alpha} - (r + \delta)k_t - w_t n_t \quad (9)$$

Due to the collateral constraint, an entrepreneur will earn positive profits each period. They split these profits between consumption, posting vacancies, and accumulating additional collateral f_{t+1} and face a budget constraint given by

$$d_t + f_{t+1} = \pi_t(z, k_t, n_t) + f_t - cv_t \quad (10)$$

2.5. Wage Bargaining

Each period, entrepreneurs and their hired workers bargain over wages. Because capital acts as a fixed factor of production (the collateral constraint always binds in equilibrium), firm output exhibits decreasing returns to scale in labor. To accommodate this, I follow [Smith \(1999\)](#) and, more recently, [Acemoglu & Hawkins \(2014\)](#) and model production as a cooperative game between workers and entrepreneurs in which each agent is paid their Shapley value.

The entrepreneur enters the game with capital k and workforce n . Any worker that chooses not to cooperate will engage in self-employment for a period and then return to the bargaining table in the next period (i.e. the outside option is a

⁴While this constraint is exogenous, it can be thought of as arising from unenforceability of contracts or other institutional features that make uncollateralized lending risky and microfounded as such (see e.g. [Buera, Kaboski & Shin 2021](#)).

shirking of duties for a period, rather than termination of the match). Defectors draw their self-employment productivity from the stationary distribution of M (as they would if they left employment). Negotiation occurs before these productivity draws are realized, however, and workers are treated symmetrically.

If the entrepreneur and x of the n workers choose to cooperate, they form a coalition, operate the entrepreneur's production technology, and produce $zk^\alpha x^{1-\alpha}$. The remaining $n - x$ workers form their own coalition and produce $(n - x)\bar{y}$ (where \bar{y} is average self-employment productivity). Each agent is paid their Shapley value arising from this game, so that the wage per worker is given by

$$w = \chi zk^\alpha n^{-\alpha} + (1 - \chi)\bar{y} \quad (11)$$

where χ is a parameter governing the bargaining power of the entrepreneur relative to workers.⁵

The resulting wage determination equation is intuitive; workers are simply paid some linear combination of their average product of labor and their outside option \bar{y} , with the weight determined by bargaining power.

2.6. The Entrepreneur's Problem and Behavior

Combining equations 5 - 10 and the wage bargaining equation 11, the entrepreneur's problem can be written recursively as

$$\begin{aligned} V(z, f, n; X) &= \max_{f', n', k, v, d} \frac{c^{1-\sigma}}{1-\sigma} + \beta \Delta V(z, f', n'; X) \\ \text{s.t. } d + f' &= (1 - \chi)zk^\alpha n^{1-\alpha} - (r + \delta)k - (1 - \chi)\bar{y}n + f - cv \\ n' &= (1 - \lambda)n + p(\theta)v \\ k &\leq \gamma f \\ v &\geq 0 \\ X' &= J(X) \end{aligned}$$

where X is a vector of aggregate state variables and J is the entrepreneur's perceptions function for the evolution of the aggregate state. It is important to note that

⁵At a technical level, the game is between an atomistic entrepreneur and a continuum of workers; the parameter χ is the relative size of the atomistic entrepreneur. It is also worth noting that because the Shapley value results in workers being paid a linear combination of their average product (rather than marginal product), the model does not nest perfectly competitive wages as a special case.

the wage bargaining equation has been substituted into the entrepreneur's budget constraint and does not depend on household state variables, eliminating the need to anything about the composition of households employed by the entrepreneur as state variables.

Entrepreneur Behavior: One important result arising from the first order conditions for f' and n' is that an entrepreneur's capital-labor ratio depends only on their productivity z and aggregate state variables X (see Appendix B for the derivation).⁶ Denote this value as η so that

$$\eta(z; X) = \frac{\gamma f'^*}{n'^*} \quad (12)$$

where f'^* and n'^* are the entrepreneur's optimal policy functions.

This result, stemming from the fact that the user costs of both capital and labor remain linear despite the model's complications, substantially increases tractability. The bargained wage w , in general, depends on all entrepreneur state variables which can change over time due to accumulation of collateral. However, combining this result with the wage bargaining equation 11 reveals that wages depend only on entrepreneurs' productivity which is fixed (at least for the lifetime of the entrepreneur), justifying the use of $w(z)$ and $H(z)$ in the household problem above.

A second useful result is that entrepreneurs will pursue a constant productivity-dependent growth rate. In essence, f'^* will satisfy

$$\begin{aligned} f'^* &= g(z; X) f \\ \frac{\partial g}{\partial z} &> 0 \end{aligned} \quad (13)$$

for some function g . As with the capital-labor ratio, because z is fixed for an entrepreneur, this means that they will grow at a constant rate over their lifetime (in steady-state). Intuitively, g is increasing in z ; more productive entrepreneurs will grow quicker. Together, the two functions η and g are sufficient to fully characterize entrepreneur behavior as a function of their productivity z and the aggregate state X .

⁶This statement holds in universally in steady-state and holds for any transition path under the parameter restriction that $\lambda > 1 - \beta\Delta$ which is satisfied in the quantitative model. It is also worth noting that entrepreneurs with sufficiently low z will choose $n = 0$ (i.e. will disengage from the economy and eat their cake rather than operate at a loss), leading to an undefined capital-labor ratio.

2.7. Crowd-in and Crowd-out Externalities

The two functions η and g can be used to gain intuition for the crowd-in and crowd-out effects, particularly the capital-shallowing and allocative efficiency channels that arise from the impact of search on wages. Because these operate through labor market tightness, it is useful to abuse notation and write $\hat{\eta}(z; \theta)$ and $\hat{g}(z; \theta)$ to represent "the steady-state values of η and g for a z productivity entrepreneur facing steady-state labor market tightness θ , which is possible because entrepreneur policy functions depend on the aggregate state only through current and future values of θ . These act as "partial equilibrium" policy functions that take a value for the price (θ) and return the entrepreneur's optimal response, yielding the following comparative-static-like-statements:

Proposition 1 *Let \hat{g} and $\hat{\eta}$ be defined as above. Then*

$$\begin{aligned} \frac{d\hat{\eta}}{d\theta} &> 0 \\ \frac{d\hat{g}}{d\theta} &< 0 \text{ and } \frac{\partial^2 \hat{g}}{\partial z \partial \theta} < 0 \end{aligned}$$

where partial derivatives denoted by ∂ are taken while holding other endogenous outcomes (i.e. $\hat{\eta}$) constant.

In words, Proposition 1 makes three claims. The first ($\frac{d\hat{\eta}}{d\theta} > 0$) is that an entrepreneur's capital-labor ratio is increasing in labor market tightness. This result is intuitive; a tighter labor market leads to higher hiring costs and thus increases the cost of labor relative to capital. Conversely, when a household chooses to search, it loosens the labor market and puts downwards pressure on the capital-labor ratio, leading to a reduction in wages — the capital shallowing channel.

The second claim that an entrepreneur's growth rate is decreasing in market tightness ($\frac{d\hat{g}}{d\theta} < 0$) is similarly intuitive; as labor market tightness increases and hiring costs rise, the entrepreneur must spend more on hiring, reducing the profit per unit of collateral, and reducing the incentive to grow (rather than consume).

More interesting is the final statement ($\frac{\partial^2 \hat{g}}{\partial z \partial \theta} < 0$) about how the magnitude of the relationship between \hat{g} and θ varies with productivity. It says that the response of \hat{g} to θ is larger (i.e. more negative) for more productive entrepreneurs. As a result, when a household decides to search and decreases labor market tightness, entrepreneur growth rates increase (the second claim), and this increase is larger

for high productivity entrepreneurs than low productivity ones. Thus the share of resources controlled by high productivity entrepreneurs increases, resulting in higher TFP and higher average wages (it can be verified that the bargained wage is strictly increasing in productivity) — the allocative efficiency channel.

This effect arises from the fact that hiring costs make up a larger share of total costs for faster growing firms, which happen to be the more productive firms. To see this, consider a comparison between two entrepreneurs each with a unit measure of employees, identical capital-labor ratios η^* , but one of whom is not growing ($g = 1$) and one of whom is growing at rate g^* .⁷ Total costs for the entrepreneur without growth are given by $r\eta^* + \chi z\eta^* + (1 - \chi)\bar{y} + \lambda \frac{c}{p(\theta)}$ (where the final term represents the hiring costs) while the growing entrepreneur carries an additional hiring cost term of $(g^* - 1)\frac{c}{p(\theta)}$. Thus a reduction in $\frac{c}{p(\theta)}$ represents a larger proportional reduction in total costs for the growing entrepreneur, freeing up relatively more resources to use for faster growth.

The Remaining Externalities: Before formalizing the externalities in Section 3, it is useful to provide some brief intuitive discussion of the two remaining externalities, both of which operate through the job finding rate (rather than the wage as was the case with the previous two).

The first is very straightforward and is essentially identical to the congestion externality in textbook labor search models. A household's decision to search decreases labor market tightness and, through the matching function, decrease the probability that other searchers are matched with jobs. While this increases the unemployment rate, it does not directly change the size of relative size of the wage sector. However, as with lower wages, marginal searchers respond to lower job finding rates by moving into self employment.

The final crowd-in externality is more subtle and is the only externality that does not occur due to changes in labor market tightness. It arises from another model feature that is common to labor search models — namely, that bargained wages do not fully reflect a worker's marginal product. Instead, workers are paid slightly less than their marginal product with entrepreneurs collecting the difference as additional profits. A portion of these additional profits are used to grow the firm which necessitates hiring additional workers. When a household decides to search, they do not internalize the fact that, if they are hired, they will directly crowd in

⁷This example abstracts from the fact that entrepreneurs with different productivities would choose different capital-labor ratios, but it suffices to provide intuition.

additional hiring in the future.

One way to clearly see this final externality is to consider two entrepreneur each with a unit measure of workers growing at rate g^* , one of whom is exogenously matched with an additional unit measure of workers from outside the economy (e.g. immigrants). The entrepreneur who is not matched with additional workers hires $g^* - 1$ workers on net for the next while while the matched entrepreneur hires $2(g^* - 1)$ workers on net. The unit measure of immigrants effectively crowds-in an additional $g^* - 1$ workers the next period.

3. Efficiency and the Social Planner

A substantial complication in analyzing the externalities present in households' labor search decisions is that it is not immediately clear what the appropriate social planner's problem is. As in much of the labor search literature, the problem of an all-powerful planner free from any financial constraints or labor market frictions is uninteresting (except perhaps as a benchmark); this planner would simply allocate all labor and capital to the most productive entrepreneur (assuming one exists) and divide output in a way that equalizes marginal utility across all households and entrepreneurs. This teaches us nothing about the externalities generated by households' labor search or how these externalities interact with borrowing constraints.

Instead, I follow the traditional approach and consider the problem of a constrained social planner who must respect both households' borrowing constraints (as in [Davila et al. 2012](#)) and the search-and-matching technology. Further, because the stated goal of most labor market policies (e.g. so-called "Active Labor Market Policies") is to improve outcomes for *household*, I focus on a social planner who values only the welfare of households. This approach has the additional benefit of being somewhat typical in macro-development models with multiple types of agents (e.g. [Itskhoki & Moll 2019](#)).

To prevent the planner from simply forcing entrepreneurs to hand over consumption to households, I also assume that the social planner can only dictate the decisions of households and cannot control the behavior of entrepreneurs, who continue to solve their optimization problem each period. In this sense, the social planner faces an additional constraint that it cannot force entrepreneurs to act sub-optimally. Allocations satisfying these three constraints make up the set of feasible allocations for the social planner.

Definition: A path of household policy functions $\{c_t(a, y, z), a'_t(a, y, z), s_t(a, z)\}_{t=0}^\infty$, entrepreneur policy functions $\{g_t(z), \eta_t(z)\}_{t=0}^\infty$, distributions of households across savings and matched-employer productivities $\{m_t(a, z)\}_{t=1}^\infty$, and labor market tightness $\{\theta_t\}_{t=0}^\infty$ is **feasible** given an initial distribution $m_0(a, z)$ and market tightness θ_{-1} if⁸

1. It respects the household budget constraint for all a, y, z

$$\begin{aligned} a'_t + c_t &= Ra + (1 - s_t)y + s_t(w_t(z_t, \theta_t) - (1 - z_t)b) \quad \forall a, y, z, t \\ a_t &\geq 0 \end{aligned} \quad (14)$$

2. It respects the labor market matching technology

$$\begin{aligned} \frac{v(m_t, \eta_t, \eta_{t+1}, g_t)}{\theta_t} &= \int \int s_t(a, 0) m_t(a, 0) j(y) dy da \\ m_{t+1}(a', z) &= (1 - \tilde{\lambda}) m_t(a, z) + H(z, m_t, \eta_t, \eta_{t+1}, g_t) p(\theta_t) v(m_t, \eta_t, \eta_{t+1}, g_t) \end{aligned} \quad (15)$$

where v is the total number of posted vacancies as a function of entrepreneur policy functions, and H is the probability that an individual who finds a job is matched with a firm of productivity level z .⁹

3. The entrepreneur policy functions $\{g_t(z), \eta_t(z)\}_{t=0}^\infty$ solve the entrepreneurs' problem (Appendix equation 21), conditional on θ_{-1} and $\{\theta_t\}_{t=0}^\infty$.

The task of the social planner is maximize average household welfare subject to these feasibility conditions. Formalizing the statement of this problem is straightforward but cumbersome and is relegated to Appendix C. One detail worth noting is that the constrained planner chooses the entire sequence $\{\theta_t\}_{t=0}^\infty$ simultaneously — that is, the planner's problem features full commitment.

This problem of selecting the welfare maximizing path subject to a set of dynamic constraints in a heterogeneous agent economy is similar to the Ramsey-type problems often found in the literature dealing with welfare and efficiency in heterogeneous agent models (e.g. [Itskhoki & Moll 2019](#), [Dávila & Schaab 2023](#)). Like

⁸Note here that the independence of s_t on y enforces the informational constraint that the planner cannot condition search policy on the realization of individual's idiosyncratic shock, mirroring the informational constraint for individuals in competitive equilibrium.

⁹Both v and H are formally defined in Appendix C.

all Ramsey problems, the primal problem of choosing paths of consumption (or consumption functions in the case of heterogeneous agents) subject to feasibility constraints can be equivalently formulated as a dual problem in which the planner selects optimal tax rates from a sufficiently rich set of instruments to decentralize the optimal allocation in competitive equilibrium.

Here, the equivalent dual problem provides some intuition for the policy relevance of this planner's problem. The constraint that the planner cannot dictate the behavior of entrepreneurs and is restricted to paths of g_t and η_t consistent with entrepreneur optimality is equivalent to a constraint in the dual problem that the set of available tax instruments does not include any taxes levied on entrepreneurs. Similarly, the constraint that the planner must respect household budget constraints corresponds to a restriction that the set of tax instruments does not include any lump-sum transfers. Thus the only tax instruments available under the dual approach are taxes on earnings, search, and savings, all of which are potentially individual-specific.

3.1. Privately- vs Socially- Optimal Search Decision Rules

With the planner's problem specified, we can finally formalize the externalities that have been discussed only intuitively up until now. This is accomplished in Proposition 2 below. I make two simplifications

To generate some initial intuition, I start by examining externalities in the special case where household utility is given by $u(c) = c$ (i.e. $\sigma = 0$). As the exogenous small open economy interest rate is less than the household discount rate by construction, the optimal savings policies in both the competitive equilibrium and the planner problem are to save nothing and consume all disposable income every period. This essentially eliminates ex-post heterogeneity outside that of the labor market, leading to substantial simplifications.

Proposition 2 *Under the assumptions that traditional sector productivity y has no autocorrelation and $\sigma = 0$, the optimal search policies $s(a, z)$ of constrained social planner (described above) and an individual in competitive equilibrium in steady state are to search*

if and only if

$$\text{Individual: } \int (y + b)j(y)dy \leq \beta \bar{\theta} p(\bar{\theta}) \int_z \frac{w(z, \bar{\theta}) - \int (y + b)j(y)dy}{1 - \beta(1 - \bar{\lambda})} dz \quad (16)$$

$$\text{Planner: } \int (y + b)j(y)dy \leq \beta \bar{\theta} p(\bar{\theta}) \int_z \frac{w(z, \bar{\theta}) - \int (y + b)j(y)dy}{1 - \beta \Delta g(z, \bar{\theta})} dz + \mu \quad (17)$$

$$\begin{aligned} \mu = & \frac{\bar{\theta}/\bar{S}}{\frac{\partial \log v}{\partial \log g} \frac{\partial \log g}{\partial \log \theta} - 1} \left(\underbrace{\int_z \bar{\lambda}(z) \bar{\theta} p(\bar{\theta}) \bar{S} \frac{\partial H}{\partial g} \frac{\partial g}{\partial \theta} dz}_{\text{Composition of Jobs}} + \right. \\ & + \underbrace{\int_z \bar{\lambda}(z) \bar{H} \bar{S} p(\bar{\theta}) \left(1 + \frac{\partial \log p}{\partial \log \theta}\right) dz}_{\text{Congestion}} + \underbrace{\frac{1}{\beta} \int_z \frac{\partial w}{\partial \eta} \frac{\partial \eta_{t-1}}{\partial \theta} \bar{m}(z) dz + \int_z \frac{\partial w}{\partial \eta} \frac{\partial \eta_t}{\partial \theta} \bar{m}(z) dz}_{\text{Wage Changes}} + \quad (18) \\ & \left. + \text{Anticipation Term} \right) \end{aligned}$$

where bars denote steady-state values of the competitive equilibrium and planner's problem respectively, \bar{S} is the steady-state number of searchers defined for compactness, and $\lambda(z)$ is the planner's shadow price denoting the marginal value of an additional worker being matched with a productivity z entrepreneur. The anticipation term is described further in the appendix.

From the proposition, it is clear that the privately and publicly optimal decision rules follow a similar structure. Each rule weighs the costs of search, given by the goods cost of search b and the expected cost of forgone wages, against the benefits, given by the expected excess earnings while employed. In the privately optimal rule, these excess earnings are discounted by the intertemporal discount rate and probability of maintaining the job match. In the publicly optimal case, they are discounted by the intertemporal discount rate and the shutdown-inclusive growth rate of matched entrepreneur. I deferred further discussion of this difference in discounting to the discussion of crowd-in effects.

The planner's optimal decision rule carries an additional term μ which con-

tains all but one of the externalities present in the labor search decision. The terms contained in μ represent externalities that occur through changes in labor market tightness θ and thus are weighted by the net change in labor market tightness due to a change in the number of searchers after accounting for the response of vacancies.¹⁰ This weight is negative, reflecting the fact that an increase in the number of searchers leads to a decrease in labor market tightness. Thus negative terms within the parentheses represent positive externalities and vice versa.

The negative externalities of search are contained in the second line of equation 18. These are the forces driving the crowd-out effects. The first term, labeled "Congestion", is typical in search models; an additional searcher pushes down labor market tightness and reduces the probability that any given searcher finds a job. As is typical, the size of this externality is proportional to the elasticity of the matching function; a high elasticity implies that an additional searcher leads to a large reduction in the job-finding probability. Intuitively, this externality is also increasing in the steady-state number of searchers \bar{S} and is valued using the average shadow value of a newly hired worker (i.e. $\int_z \lambda(z) \bar{H} dz$).

The second negative externality, given by the "Wage Changes" term, captures the fact that a reduction in labor market tightness leads to reduced labor costs, and entrepreneurs respond by lowering their capital-labor ratio, resulting in lower wages (see equation 11, the wage bargaining equation). This effect is similar to the monopoly effect of [Itskhoki & Moll \(2019\)](#) and stems from a similar source, namely, that the planner places zero weight on the welfare of entrepreneurs. While this effect occurs contemporaneously with an increase in search behavior, it is slightly offset by a countervailing anticipation effect. When entrepreneurs foresee a reduction in labor market tightness, they respond by reducing hiring in the period before (as labor will be cheaper tomorrow), temporarily pushing up the capital-labor ratio and, consequently, wages. Because this effect occurs exactly one period before the change in labor market tightness, it is appropriately weighted by $\frac{1}{\beta}$.

The remaining term, "Composition of Jobs" is the first of two externalities corresponding to the crowd-in effects. It arises from the fact that a decrease in labor market tightness will cause entrepreneurs to grow faster (as their labor costs have gone down, leaving more profit available to save and finance capital in the next period). This results in a positive externality because, as shown in Proposition 1,

¹⁰To see that this expression indeed gives the net change, note that $\theta = \frac{v(\theta)}{s} \Rightarrow \frac{d\theta}{ds} = \frac{\theta/s}{\frac{d \log v}{d \log \theta} - 1}$.

the increase in the growth rate is larger for more productive entrepreneurs. Thus the change in hiring probabilities in response to a reduction in labor market tightness $-\frac{\partial H}{\partial g} \frac{\partial g}{\partial \theta}$ is positive for high values of z and negative for low values, leading to a net increase in allocative efficiency. While the planner internalizes this according to the shadow price $\lambda(z)$, in competitive equilibrium it manifests itself as a higher expected wage and thus induces additional labor search.

The final externality does not operate through labor market tightness. Instead, it emerges from the different discount rates exhibited in the individually and socially optimal search decision rules in 16 and 17 respectively. Under parameter assumption (made throughout this paper) that $\Delta\beta > (1-\lambda)$, we have that $\Delta g(z, \theta) > (1-\tilde{\lambda})$ for all z, θ . Thus the planner's valuation of a job is higher than an individual's, even fixing labor market tightness.

Why is this the case? The key arises from the fact that the individual does not capture the entire marginal product of labor created by their job match. Instead, they earn some markdown according to 11, and the remainder is captured by the entrepreneur. Although the social planner does not value the entrepreneur's consumption, a portion of this remainder is saved and used to expand the entrepreneur's firm in the next period, including new hiring which the social planner values. Essentially, a portion of the worker's production today is used to finance the hiring of additional workers tomorrow, whose production is used to finance more workers the next period, etc. Intuitively, this effect is larger for more productive matches (appearing in 17 as a higher growth rate) and smaller when entrepreneur survival probabilities Δ are smaller.

4. Model Estimation and Quantification

In this section, I discuss the estimation and quantification of the model as well as perform some model validation exercises. Broadly speaking, the parameters of the model fall into two categories. The first are parameters that can be estimated directly from data or are well-known macroeconomic parameters with standard values. These parameters I simply set equal to their estimated or standard value. The second set of parameters I estimate using the simulated method of moments to match key data moments.

4.1. Directly Estimated Parameters

Many model parameters are chosen to match values typical in the macroeconomics, are taken from external sources, or are estimated directly. These are displayed in Table 1, along with their values and sources. The discount rate β is chosen to match an annual discount rate of 0.95. Because a model period corresponds to two weeks, this corresponds to a value of $0.95^{\frac{1}{26}}$. The rate of return on worker's savings R is taken to be exogenously equal to $0.9^{\frac{1}{26}}$. The assumption that the return to savings is less than one is typical models of developing countries (see e.g. [Donovan 2021](#), [Fujimoto](#), [Lagakos & VanVuren 2023](#)) and representative of the fact that households in these countries lack access to formal investment with positive returns. The value of 0.9 matches an annual inflation rate of roughly 10 percent, roughly consistent with World Bank estimates of inflation in Ethiopia over the last few years; thus the model asset a most closely reflects cash holdings. The capital share of income is set at 0.33 as is standard.

The interest rate faced by entrepreneurs is disciplined using World Bank MIX Market data containing financial information on microcredit providers in Ethiopia. Yields on loans from microfinance institutions range from 20 percent to 30 percent with negligible loan loss rates (typically less than one percent). Combining this rough average of a 25 percent annual return with 8 percent depreciation yields a depreciation-inclusive user cost of capital of 33 percent annually. This value is high relative to developed countries but is fairly typical for developing countries (see e.g. [Banerjee et al. 2015](#), who document similar values in multiple countries including Ethiopia).

Collateral constraints are measured directly using data from the Ethiopian portion of the World Bank Enterprise Survey for the year 2015. The average collateral requirement reported by firms is slightly larger than 350 percent of loan value, meaning that a firm that owned 350,000 Birr worth of capital could pledge this as collateral and finance a loan for an additional 100,000 Birr of capital. Thus the implied value for γ is $1 + \frac{1}{3.5} = 1.29$. The Enterprise Survey is also used to estimate the entrepreneur survival probability Δ . Because productivity is constant for the life of an entrepreneur, entrepreneur death is the only reason that firms will shutdown in steady state. Consequently, the steady-state distribution of firm ages is geometric with decay parameter Δ whose value can be recovered through the simple maximum likelihood estimation. In this case, the estimate for Δ is given by $1 - \frac{1}{\hat{\mu}}$ where $\hat{\mu}$ is the sample average firm age, yielding an annual value for Δ of

0.92.

The self employment productivity process also comes directly from data. This productivity is modeled as a simple binary Markov process, drawing on the fact that earnings for those without permanent wage jobs are highly bimodal at a fortnightly frequency (seen in the high-frequency data of [Abebe et al. 2021](#), described below). Such bimodality seems to stem from the fact that opportunities for self employment (or, often in the case of Addis Ababa, temporary “gig-style” labor that functions similarly to self employment), and many individuals report neither working nor searching in a given period, presumably earning very little.

One advantage on using a binary income process instead of a more typical AR(1) is that transitions in and out of this idle state can be observed and measured directly. Using fortnightly data on work and searcher activities (described in the next section), I estimate the transition probabilities from engaged in self employment or temporary work to idleness and back. Although there is no reason for these transitions probabilities to be identical, the estimated value for both is approximately 11 percent. While average self employment earnings (i.e. the productivity parameter A_s) are estimated using SMM, the ratio of earnings in the low productivity state to the high productivity state is chosen to match the standard deviation of self employment earnings observed in the data. In particular, I isolate the transitory, idiosyncratic variance of earnings by regressing (log) earnings on individual and week fixed effects and calculating the standard deviation of the residuals (similar to the process employed in [Lagakos & Waugh 2013](#)). Conditional on the transition probabilities, there is a one to one correspondence between the standard deviation of income and the ratio of interest.¹¹ The estimated ratio is 0.38 corresponding to an estimated standard deviation of .48.

Finally, the distribution from which newborn entrepreneurs draw their productivity is chosen to be an upper-truncated Pareto distribution (truncated as a bounded support for productivity is required for steady-state equilibrium to exist in the model). I set the lower bound of the distribution to a small but arbitrary number; because entrepreneurs endogenous shut down below a threshold productivity level and the truncated Pareto distribution is scale-invariant, the lower threshold has no impact on model outcomes as long as it is below the shutdown threshold. The tail parameter is set to unite. It is worth noting that because of

¹¹For a symmetric transition matrix, as is the case here, this correspondence is given simply by $\frac{y_l}{y_h} = e^{-2\sigma}$

upper truncation, the mean and variance of productivity remain finite. The upper bound \bar{z} is included in the SMM estimation, described below.

Table 1: Directly Estimated Parameters

Parameter	Value	Description	Source
β	$.95^{\frac{1}{26}}$	Discount rate	Standard value
R	$.9^{\frac{1}{26}}$	Return to savings	10% annual inflation
α	.33	Capital share	Standard value
r	$1.33^{\frac{1}{26}} - 1$	Capital cost for entrepreneurs	MIX Market
γ	1.29	Collateral constraint	World Bank ES
Δ	$.92^{\frac{1}{26}}$	Entrepreneur death prob.	World Bank ES
$M(y)$	$\begin{bmatrix} .89 & .11 \\ .11 & .89 \end{bmatrix}$	High and low y trans.	Abebe et al. (2021)
$\frac{y_l}{y_h}$.38	Ratio low to high productivity	

This table displays the model parameters that are estimated directly as well as their values and sources. To help comparisons to typical values, parameters are displayed in annual terms. See the discussion for details on each parameter.

4.2. Parameters Estimated using the Simulate Method of Moments

The eight remaining parameters are estimated using the simulated method of moments. These parameters, along with their estimated values, are listed in Table 2. Table 3 lists the moments targeted in the estimation and their values in both the data and the model, as well as the source for each moment. While in general all eight moments are jointly determined by all eight parameters, there is a rough correspondence between parameters and moments that is worth some discussion. The parameters fall into two rough categories — those corresponding closely to household-level moments (above the dividing line in Tables 2 and 3) and those corresponding closely to firm-level moments (below the line), and I discuss each in turn.

The data for household-level moments come from two data sources. I construct the aggregate moments using the 2018-2019 wave of the Ethiopia Living Standards and Measurement Survey (LSMS), limited to individuals surveyed in Ad-

Table 2: Parameter Estimates from Simulated Method of Moments

Parameter	Estimate	Corresponding Moment
A_s	0.34	Wage sector premium
σ	5.2	% wage work
λ	0.01	Unemployment rate
b	0.05	% of expenditure on search
Mf	.001	Control wage employment after 16 weeks
c	0.37	Cost to hire as % of wage
χ	0.62	Elas. of avg. wage to output per worker
\bar{z}	0.35	Avg. growth rate

This table displays the parameters estimated using simulated method of moments, their estimates, and the moment that corresponds mostly closely to each parameter. See discussion for details and intuition on these correspondences.

dis Ababa.¹² Additionally, I use fortnightly data on the behavior of job searchers collected as part of an experiment in [Abebe et al. \(2021\)](#). Importantly, I use only data from the control group in model estimation, setting aside the experimental treatment and data on treated individuals for post-estimation model validation.

The productivity of the self employment technology A_s is mostly pinned down by the earnings premium of wage workers (relative to the self employed and temporary laborers); the more productive the self employment technology is, the smaller this premium will be. I estimate the premium in the LSMS data using a Mincer-style regression of (log) earnings on age, the square of age, and an indicator for whether an individual reports that the earnings arise from a permanent wage job, along with a variety of controls including region, rural/urban, and education fixed effects.¹³ The estimated earnings premium for wage workers is 39

¹²While the other data sources used in estimation are from 2014-2015, the 2018 wave of the Ethiopia LSMS was the first wave capable of providing representative estimates for Addis Ababa (previous waves were not representative at a sub-national level). For this reason, I opt to use the data from 2018 rather than the 2015 wave, which would otherwise line up better with the other datasets temporally.

¹³The Ethiopian Productive Safety Net Programme (PSNP), a relatively new “workfare” program administered by the Government of Ethiopia, presents a potential complication. The program provides temporary employment and was present in some regions of Addis Ababa during the 2018 LSMS survey. It is unclear whether earnings from the PSNP should be included in estimation. Fortunately, dropping these earnings from the analysis changes the estimate by less than one per-

Table 3: Moments Targeted using the Simulated Method of Moments

Moment	Source	Data	Model
Wage sector premium	LSMS	39%	39%
% wage work	LSMS	30%	29%
Unemployment rate	LSMS	10%	12%
% of expenditure on search	Abebe et al. (2021)	15%	16%
Control wage employment after 16 weeks	Abebe et al. (2021)	12%	13%
Cost to hire as % of wage	Abebe et al. (2017)	120%	120%
Elas. of avg. wage to output per worker	World Bank ES	25%	25%
Avg. growth rate	World Bank ES	4.4%	4.4%

This table displays the moments targeted in the simulated method of moments estimation, their source, and their values in both the data and model. See the discussion for details.

percent.

The coefficient of relative risk aversion in utility σ and the job separation rate λ are jointly determined by the percentage of households engaged in wage work and the unemployment rate. The correspondence between the unemployment rate and λ is clear as λ determines the flows out of wage employment and in to unemployment. Conditional on this, the overall size of the wage sector is determined by flows into employment and thus the overall level of search. Because search is the higher-risk, higher-return option, this depends largely on household risk aversion. Using the LSMS, I estimate an unemployment rate of 10 percent and a wage work share of 30 percent. These are both higher than the values estimated by World Bank for the entire country of Ethiopia (3 percent and 15 percent respectively) which is not surprising given the sample restriction to Addis Ababa, an urban capital city.

The two remaining parameters disciplined using household moments are the goods cost of search b and the product of the size of newborn entrepreneurs M and entrepreneurs' initial assets \underline{f} (which are not separately identified under constant returns to scale as there is no difference between two entrepreneurs of size x or one entrepreneur of size $2x$). Discipline for b comes directly from [Abebe et al. \(2021\)](#) who report that search costs amount to 5 percent of weekly expenditure

centage point, rendering the issue quantitatively moot. I default to including all earnings from temporary employment, including those from the PSNP.

for individuals in their sample. Finally, Mf closely corresponds to the level of wage employment in the control group of the experiment after 16 weeks. The amount and/or size of entrepreneurs corresponds directly to the amount of vacancies posted (i.e. doubling the number of entrepreneurs doubles the number of vacancies) which determines the job finding rate and thus (conditional on all other parameters), employment in the control group of searchers.

The remaining three parameters — the vacancy posting cost c , the wage bargaining parameter χ , and the upper bound of firm productivity \bar{z} — are estimated to match firm-level moments. The moment for the vacancy post cost comes from [Abebe et al. \(2017\)](#) who survey firms in Addis Ababa about hiring practices and find that average reported cost of making a single hire is equal to 120 percent of the average wage paid by the firm.

The bargaining parameter and upper bound on firm productivity are chosen to match moments estimated from the World Bank Enterprise Survey. The bargaining parameter χ governs the proportion of a worker’s marginal product of labor captured through the worker’s wage (recall the wage bargaining equation 11); as a result, this parameter closely corresponds to the relationship between firm productivity and wages and is disciplined to match the elasticity of a firm’s average wage to its output per worker. I estimate this elasticity to be 25 percent (meaning, a firm with 100 percent higher output per worker pays its workers on average 25 percent more) and use this as the target in estimation. Finally, the upper bound of firm productivity is chosen to match the average (self-reported) firm-level growth rate. Because more productive entrepreneurs choose to grow faster, the upper bound on productivity corresponds to an upper bound on growth rates which determines the average.

4.3. Model Validation

As my primary model validation exercise, I replicate the experiment performed by [Abebe et al. \(2021\)](#) in the model and compare the model outcomes to the experimentally estimated outcomes. As mentioned above, it is important to note that while control outcomes from the experiment are used to estimate the model, treatment outcomes and data are not. Thus comparing the model’s predictions for treatment effects to those estimated in the experiment truly represents an “out-of-sample” test of the model. Below, I briefly summarize the experiment and describe how it is replicated in the model before showing the results of the validation exer-

cise.

The experiment took place in 2014-2015 and evaluate the effects of providing a cash subsidy covering some of the costs of job search to prospective searchers in Addis Ababa, Ethiopia. In the context of Addis Ababa, the majority of job search takes place in person in the city center. Particularly notable are job vacancy boards (located in the city center) which contain job postings and are consulted by the majority of searchers. Thus the cost of travel (typically by minibus) to the city center represents a large and salient cost of job search.

The experiment sampled young individuals who “(i) were between 18 and 29 years of age; (ii) had completed high school; (iii) were available to start working in the next three months; and (iv) were not currently working in a permanent job or enrolled in full time education.” (Abebe et al. 2021). Individuals in the sample were randomly offered cash that could be collected in person at the city center up to three times each week. While not literally a job search subsidy as individuals could theoretically travel to the city center, collect the cash, and leave without searching, doing so would be ineffective as the cost of the subsidy is not large enough to cover the full round-trip journey.¹⁴ Thus collecting the cash only makes sense if the individual intended to travel to the city center for other purposes (presumably job search). The cash was available for 16 weeks, and sampled individuals were surveyed on their search behavior through fortnightly phone surveys. After 16 weeks, treated individuals were 3.4 percentage points ($p < 0.1$) more likely to be employed in a permanent job.

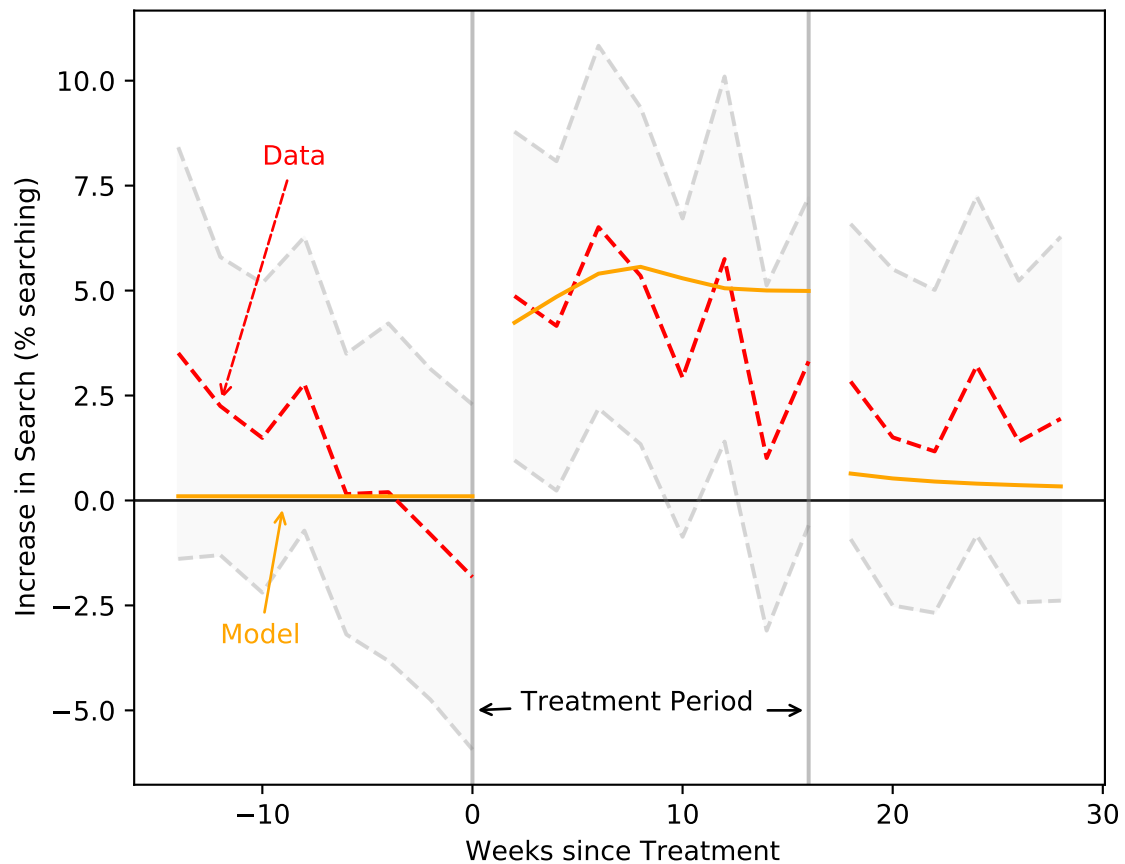
To replicate the experiment in the model, I select a representative but small (measure 0) subset of household not employed in the wage sector from the steady-state distribution of household. In this sense, the outcomes of sampled individuals do not affect equilibrium outcomes, and the experiment happens in “partial equilibrium”, reflecting the fact that providing treatment to a few hundred individuals in a city of millions is unlikely to have general equilibrium impacts. The sample is divided equally into treatment and control groups, and the cost of search parameter b is reduced by two-thirds (the median subsidy offered in the experiment) for the treatment group for 8 periods (16 weeks).

Experimental outcomes can then be observed by simulating behavior of the

¹⁴In fact, the authors make sure of this by varying the subsidy offered to each individual based on the location, and thus minibus ticket cost, of the individuals home. However, I abstract from this heterogeneity and model the subsidy as uniform at the median value of subsidy offered.

treatment and control groups of household forward over time, and comparisons of means between the two groups correspond to Average Treatment Effects estimated by the experiment. For treatment households, I treat the experiment as an unanticipated MIT shock; households do not know ahead of time that they have been selected for treatment and cannot alter their behavior in response to such information. Thus difference between treatment and control groups before the treatment occurs are zero by construction.

Figure 2: Treatment Effect on Search Behavior over Time: Data and Model



This figure displays the treatment effect on search behavior as a function of "weeks since treatment" in both the data and estimated model.

As the first validation of the model, I compare the model's predictions for the increase in search behavior when receiving the subsidy. The results are displayed in Figure 2 where the solid orange line depicts model predictions and the dotted red line depicts the experimentally estimated effects along with the associated

95 percent confidence interval. The model lines up with the experiment remarkably well. During the treatment period (between 0 and 16 weeks since treatment), treated individuals were roughly 5 percentage points more likely to search, a fact which is replicated in the model. There is a small decline in the point estimates in the last few weeks of treatment that is not quantitatively replicated by the model, but this decline is statistically insignificant, and the model continues to fall within the estimated 95 percent confidence interval.

The model also qualitatively replicates the fact that effects seem to persist for some weeks after treatment is ended, although the experimental point estimates here are noisy. The model's predictions are quantitatively smaller than these point estimates, but are well within the 95 confidence interval. One explanation for the model's under prediction of persistence is that the increase in search due to treatment resulting in some sort of learning not captured in the model, leading treated individuals to search more often even after the end of treatment.

Even if the model accurately matches the increase in search behavior due to treatment, it may not match the increase in wage employment if, for example, this increase in search was completely ineffective in generating employment. To guard against this, I also compare the model's prediction for the increase in employment after 16 weeks of treatment to that observed in the experiment. Unsurprisingly, due to the constant job finding probability, the model predicts that this roughly 5 percentage point increase in search probability results in a roughly 5 percentage point higher probability of being employed after 16 weeks. The experimental equivalent is 3.4 percentage points (95 percent confidence interval XX to YY). This is slightly lower, but the model is still reasonably accurate and well within sampling variation. The slightly small point estimate maybe be evidence of decreasing returns to scale in search, perhaps arising from the fact that job seekers go after the opportunities that they judge most likely to yield employment first.

5. Quantitative Exercise and Results

As the main quantitative experiment, I implement a cash transfer each period targeted at all individuals who are searching for wage work. I choose the size of the subsidy to be equal size used to validate the model in the previous section. In particular, this subsidy is equal to 13.7 percent of average weekly earnings (across both sectors). Recall that this subsidy size was designed to exactly offset the costs of search. As a result, the subsidy essentially sets the search cost b to zero. For

the main exercise, I assume that the subsidy is funded by a flat tax levied on wage workers, rather than a tax on all workers. This is an important distinction as it means that the tax itself serves to distort workers' choice of sector towards self-employment and, as a result, the tax contributes to the crowd-out effect. In the future, I plan to evaluate an alternative scenario where the subsidy is funded by a flat tax on all workers, eliminating this distortion, and compare how the results differ between these two cases.

Table 4 displays the results of this policy. Column (1) displays the value of moments key aggregate moments in the benchmark steady-state of the estimated model while column (2) displays the values of these moments in the post-subsidy steady-state. The policy results in a substantial increase in both GDP and welfare. Welfare increases by 0.6 percent of consumption on average while GDP increases by a little over 2 percent. This increase in GDP is the result of a 5.4 percentage point increase in the size of the wage sector, which is more productive than the self-employment sector, and an increase in wage sector earnings of 1.88 percent. This increase in earnings is the direct result of higher average wage sector TFP in the post-subsidy steady-state of the model. As the subsidy encourages wage work and the labor market slackens, entrepreneurs now dedicate fewer resources towards hiring and more resources to growth. This increase in growth is disproportionately beneficial to higher productivity entrepreneurs, allowing them to increase their market share and increasing TFP. A portion of this higher TFP is shared with workers through higher wages due to bargaining. However, it is important to note that the increase in wages due to higher TFP is not enough to overcome the increase in taxes necessary to fund the policy; post-tax earnings in the wage sector decrease by 0.5 percent

The search subsidy has only a modest impact on the size of the wage sector which increases from 34.4 percent to 39.8 percent. Labor market tightness decreases resulting in a small decrease in the job-finding probability from 3.1 percent to 2.95 percent and, consequently, an increase in the unemployment rate by 1.2 percentage points. The decrease in job-finding probability together with the decrease in post-tax earnings in the wage sector strongly suggests that the crowd-out effect dominates the crowd-in effect. To investigate this quantitatively, I perform an additional numerical experiment. Because the crowd-out and crowd-in effects operate through labor market tightness and earnings, both of which are equilibrium objects, I also compute the results of the subsidy if these equilibrium objects

Table 4: Results of Implementing Search Subsidies

Variable	(1) Benchmark	(2) Subsidy	(3) Fixed θ
GDP		+1.5%	-
Household Welfare		-0.4%	2.9%
Size of Wage Sector	31%	34%	47%
Avg. Wage		-0.2%	-0%
Avg. Wage (incl. tax)		-3.0%	-
Job-Finding Prob.	13.6%	12.8%	13.6%
Unemployment Rate	12.2%	13.8%	15.8%

This table displays the results of the primary quantitative exercise of subsidizing search for wage jobs. Column (1) reports key aggregate parameters in the steady-state of the model before implementation while Column (2) reports these same parameters in the new steady-state of the model once the policy has been implemented. Column (3) displays the results in a hypothetical steady-state where labor market tightness θ is fixed. See the discussion for details on how to interpret these results.

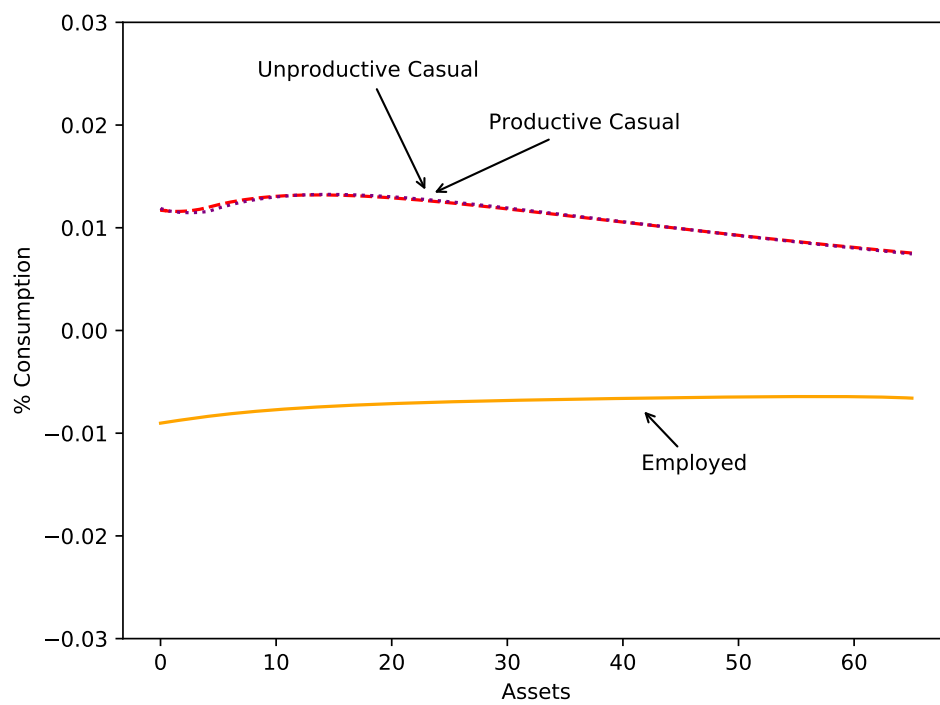
were fixed to their pre-subsidy values.

The results of this numerical experiment are displayed in column (3) of Table 4. I interpret these results (when compared to the pre-subsidy model) as revealing the direct impact of the subsidy on workers' decisions and outcomes while the difference between these results with fixed labor market tightness and wages then reveals the impact of the general equilibrium effects of the subsidy. The most striking difference between this numerical experiment and the post-subsidy steady state is the size of the wage sector. When equilibrium parameters are fixed, the subsidy increases wage sector participation by a remarkable 16.1 percent points to 50.5 percent. Nearly three times as much as the 5.4 percentage point increase induced by the policy in full equilibrium. This stark difference suggests that the direct impact of the search subsidy is large; search costs serve as a substantial constraint in preventing workers from participating in the wage sector.

The large difference in wage sector participation between the full equilibrium results and the results with equilibrium values fixed also suggests that the crowd-out effects play a substantially larger quantitative role than the crowd-in effects. As can be seen from column (3), when equilibrium adjustment is shut down, the crowd-in and crowd-out channels are shut down. Labor market tightness is fixed, there is no change in the job-finding probability or in taxes that may crowd out wage workers. Similarly, because wages are fixed, there is no increase in the wage due to higher TFP that could crowd-in additional workers. Once both these channels are introduced, the size of the wage sector falls substantially, consistent with the notion that the crowd-out channels dominate.

Interestingly, the crowd-out effect seems to be large despite a fairly small decrease in the job-finding probability in the new equilibrium. The probability falls by 0.15 percentage points from 3.10 percent to 2.95 percent, a small decline. This large change in the size of the wage sector despite a small decline in job-finding probability indicates that the semi-elasticity between an individual's search choice and their probability of finding a job must be fairly large, likely a direct result of high estimated risk aversion. This behavior seems consistent with experimental interventions such as [Alfonsi et al. \(2020\)](#) and [Abebe et al. \(2017\)](#) that find large impacts on search behavior of treatments that lead individuals to substantially revise their expectations of their job-finding likelihood.

Figure 3: Welfare Effects of Search Subsidy as a Function of Household Assets



This figure displays the change in welfare, measured in consumption equivalent welfare, of the search subsidy policy as a function of a household's assets as well as their employment status and self-employment productivity.

5.1. Welfare

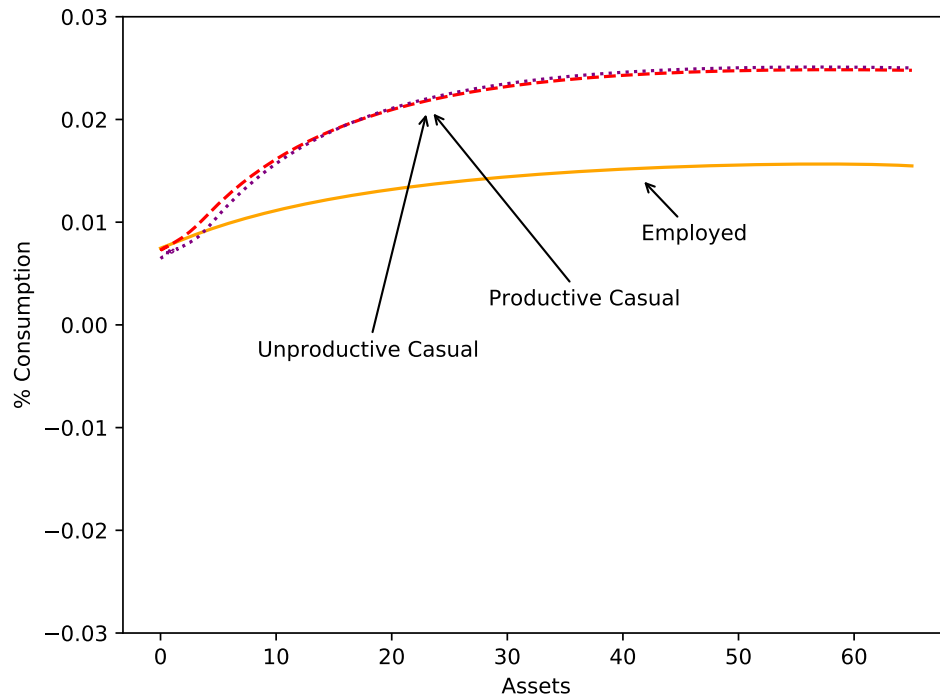
Figure 3 displays the welfare impact of the search subsidy as a function of individual assets and employment status. For now, these numbers are calculated by comparing steady-states, although I plan to compute welfare along the transition path in the future. The red and purple lines display the welfare impact for workers without a wage sector job in the high productivity and lower productivity states respectively while the orange line displays the impact for workers matched with a wage job. Two aspects of the figure are striking. The first is that the welfare effects are highly dependent on an individual's employment state. The workers without a wage job, who switch between engaging in self-employment and searching for work, experience large welfare gains equal to around 1 percent of consumption while workers matched with an employer experience welfare loss of a little less than 1 percent. This gap is intuitive; workers without a wage job are either searching or anticipate to be searching in a few periods and thus are direct beneficiaries of the subsidy while workers already matched with a job pay a tax in order to fund the subsidy.

The second striking aspect of Figure 3 is that the welfare impacts exhibit very little heterogeneity with respect to an individual's level of wealth; individuals with zero assets experience welfare changes similar to the highest asset individuals. At first glance this result seems puzzling; however, splitting the welfare impact into the direct impact of the subsidy and the indirect impact through equilibrium objects reveals the intuition. Figure 4 displays the effect of the subsidy on welfare as a function of assets while fixing the equilibrium values of labor market tightness, wages and taxes (i.e. corresponding to column (3) of Table 4) while Figure 5 displays the difference between this counterfactual and the full results. In essence, Figure 4 displays the direct impact of the subsidy while Figure 5 displays the indirect impact.

In these figures, the impact of the policy is clearly heterogeneous with respect to individual wealth. The direct effect of the subsidy exhibits the largest welfare gains for the wealthiest individuals. Recall that households will participate in the wage sector until their self-insurance falls below a certain level, after which they will turn to self-employment until they have accumulated a buffer stock of savings. Because wealthy individuals can run down their assets for longer than poor individuals while searching for a job, they expect to collect the subsidy for more periods than poor households, who may only be able to search for a handful of

periods before turning to self-employment. The welfare losses from the indirect effects of the policy are largest for wealthy households for a similar reason. Because wealthy households expect to participate in the wage sector the longest, they face the largest losses from a decline in the job-finding probability and an increase in taxes. Although the indirect effect and the direct effect individually exhibit substantial heterogeneity with respect to wealth, when they are combined the larger gains and larger losses for wealthy households serve to counteract each other and the overall welfare change doesn't vary much with wealth.

Figure 4: Welfare Effects of Search Subsidy as a Function of Household Assets (Fixed θ)

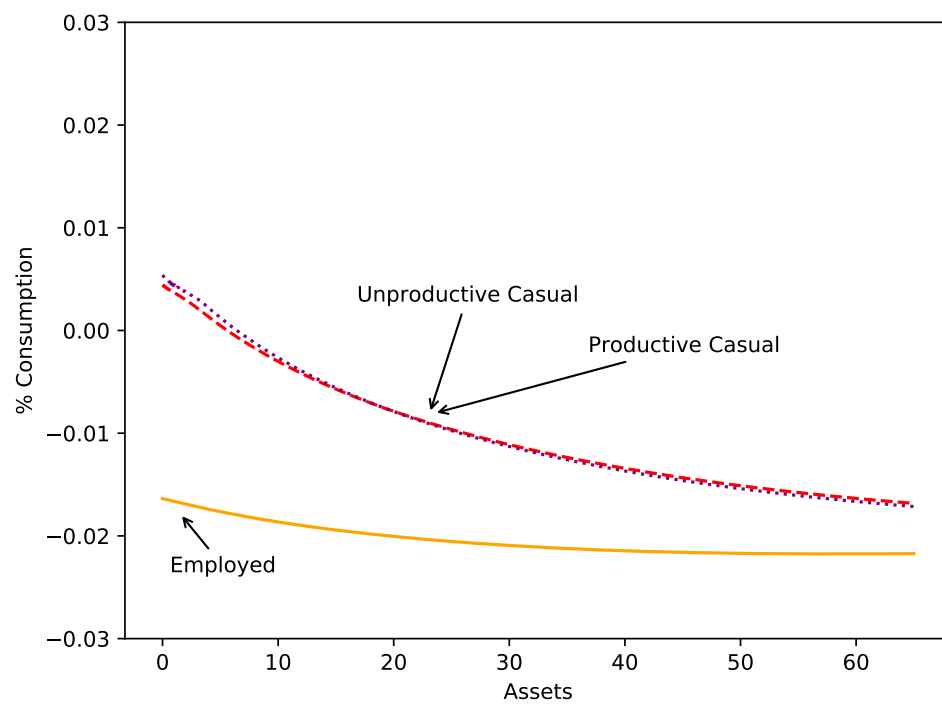


This figure displays the change in welfare, measured in consumption equivalent welfare, of the search subsidy policy as a function of a household's assets as well as their employment status and self-employment productivity in an alternative model where labor market tightness θ is fixed and does not change as a result of the policy. See the discussion for intuition on how to interpret these results.

6. Conclusion

[To be written]

Figure 5: Difference Between Welfare Effects of Subsidy with and without Fixed θ



This figure shows the difference in the change in welfare as a function of household assets, employment status, and self-employed productivity between the full model and the alternative model with fixed θ . See the discussion for intuition on how to interpret this figure.

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Appendix

A. Additional Tables and Figure

Table A.1: Effect of Search Subsidy on Labor Market Outcomes ([Abebe et al. 2021](#))

Outcome	Control Mean	Effect of Subsidy
Any Work	0.526	0.037 (0.029)
Hours Worked	26.18	0.183 (1.543)
Monthly Wages	857.9	65.88 (63.86)
Permanent Job	0.171	0.033* (0.018)
Formal Job	0.224	0.054** (0.019)
Job Satisfaction	0.237	-0.001 (0.027)

This table reproduces the primary results of [Abebe et al. \(2021\)](#) and displays the control mean for a variety of labor market outcomes as well as the experimentally estimated treatment effect of a conditional cash transfer to job seekers.

B. Derivations and Proofs from Section 2.6

The first result to show is that the entrepreneur's optimal choice of f' and n' satisfy $\eta(z; X) = \frac{\gamma f'^*}{n'^*}$ for some function η depending only on z and X . Substituting in the wage determination equation (which the entrepreneur takes as given) and the vacancy posting constraint, the first-order condition for f' and n' can be combined with the envelope condition for f and n to generate

$$\beta \Delta \mu' \left((1 - \alpha)(1 - \chi)z \left(\frac{\gamma f'}{n'} \right)^\alpha - \left((1 - \chi)\underline{w} - \frac{c}{p(\theta(X'))}(1 - \lambda) \right) \right) = \frac{c}{p(\theta(X))} \mu$$

$$\beta \Delta \mu' \left(\gamma \alpha (1 - \chi)z \left(\frac{\gamma f'}{n'} \right)^{\alpha-1} + 1 - \gamma(r + \delta) \right) = \mu$$

where μ is the Lagrange multiplier on the budget constraint, μ' is the Lagrange multiplier on the budget constraint in the following period, and $\theta(X')$ is a price

function mapping aggregate states X to equilibrium values of θ . Combining these two equations, substituting in η , and defining A , $B(X')$, and $C(X')$ for clarity yields

$$Az\eta^\alpha + B(X, X')z\eta^{\alpha-1} + C(X, X') = 0$$

which, for $0 < \alpha < 1$, can be shown to have a unique and positive solution for η for any value of z , X , and X' . Call this solution $\tilde{\eta}(z; X, X')$. Finally, substituting $X' = H(X)$ and defining $\eta(z; X) = \tilde{\eta}(z; X, H(X))$ completes the derivation.

The next result to show is that entrepreneurs choose a growth rate that depends only on their z and aggregate state variables. This follows almost directly from the previous result. Substituting $n = \frac{\gamma}{\tilde{\eta}(z; X)}f$ in to the budget constraint of the entrepreneur problem reveals that the RHS of the budget constraint is now linear in f and can be written

$$\begin{aligned} & d + \left(1 + \frac{c}{p(\theta(X))} \frac{\gamma}{\eta(z; X)}\right) f' \\ & = \\ & \left((1 - \chi)\gamma z \tilde{\eta}(z; X)^{\alpha-1} - ((1 - \chi)\underline{w} - \frac{c}{p(\theta(X))}(1 - \lambda)) \frac{\gamma}{\tilde{\eta}(z; X)} + (1 - \gamma(r + \delta)) \right) f \\ & \Rightarrow d + E(z, X)f' = D(z, X)f \end{aligned}$$

where $D(z, X)$ and $E(z, X)$ are defined such that the second line is equivalent to the first line. E functions as the price of collateral f relative to the price of consumption d while D functions as the return to collateral. Because entrepreneurs possess CRRA utility, the entrepreneur problem has the well-known solution of a constant growth rate in f depending on the values of D and E which are given by z and X so that $f' = g(z; X)f$.

The final result to show is the proof of Proposition 1. By assumption, θ is assumed to be constant. Let $\hat{E}(z, \theta)$ and $\hat{D}(z, \theta)$ denote E and D respectively, but with $\theta(X)$ simply replaced by θ , the argument to the function. Note that this is possible because E and D only depend on X through θ . Then we have the explicit

solution¹⁵

$$\begin{aligned}\hat{g}(z, \theta) &= \left(\beta \Delta \frac{\hat{D}(z, \theta)}{\hat{E}(z, \theta)} \right)^{\frac{1}{\sigma}} \\ &= \left(\beta \Delta \frac{((1 - \chi)\gamma z \hat{\eta}(z; \theta)^{\alpha-1} - ((1 - \chi)\underline{w} - \frac{c}{p(\theta)}(1 - \lambda)) \frac{\gamma}{\hat{\eta}(z; \theta)} + (1 - \gamma(r + \delta)))}{(1 + \frac{c}{p(\theta)} \frac{\gamma}{\hat{\eta}(z; \theta)})} \right)^{\frac{1}{\sigma}}\end{aligned}$$

The chain rule yields $\frac{d\hat{g}}{d\theta} = \frac{\partial \hat{g}}{\partial c/p(\theta)} \frac{dc/p(\theta)}{d\theta} + \frac{\partial \hat{g}}{\partial \hat{\eta}} \frac{d\hat{\eta}}{dc/p(\theta)} \frac{dc/p(\theta)}{d\theta}$. Using either direct calculation of partial derivatives or implicit differentiation (in the case of $\frac{d\hat{\eta}}{dc/p(\theta)}$), we can express each individual piece as

$$\begin{aligned}\frac{\partial \hat{g}}{\partial c/p(\theta)} &= -\frac{1}{\sigma} \hat{g}^{1-\sigma} \left(\frac{(\frac{\hat{g}}{\beta \Delta} - 1) + \lambda}{\frac{\eta}{\gamma} + \frac{c}{p(\theta)}} \right) \leq 0 \\ \frac{\partial \hat{g}}{\partial \hat{\eta}} &= \frac{1}{\sigma} \hat{g}^{1-\sigma} \left(\frac{\frac{\beta \Delta}{\hat{g}} - \frac{\hat{g}}{\beta \Delta}}{\frac{\eta}{\gamma} + \frac{c}{p(\theta)}} \right) \leq 0 \\ \frac{d\hat{\eta}}{dc/p(\theta)} &= \frac{\gamma(\alpha(1 - \chi)z\hat{\eta}^{\alpha-1} - (r + \delta)) + \lambda}{J(\theta)} > 0\end{aligned}$$

where $J(\theta)$ is a placeholder for a complex but unambiguously positive expression and I have made use of the first-order condition for f' in the second expression. It is worth commenting briefly on why the claimed inequalities hold. Both the first and second expressions follow directly from the fact that an optimally acting entrepreneur will ensure that $g \geq \beta \Delta$. This is clearly true as an entrepreneur can always choose to select $k = 0, n = 0$ and simply eat their cake, yielding $g = \beta \Delta$. An entrepreneur will only choose to operate if they can be weakly better off by doing so. The third and final expression follows from the first-order condition for capital which ensures that the marginal product of capital $\alpha(1 - \chi)z\hat{\eta}^{\alpha-1}$ is greater than the marginal cost of capital $r + \delta$ (the MPK is greater, rather than equal to, the marginal cost due to the presence of the financing constraint). Because $\frac{dc/p(\theta)}{d\theta} > 0$ by construction, combining these inequalities with the chain rule provides the result $\frac{d\hat{g}}{d\theta} < 0$ and along the way we have shown $\frac{d\hat{\eta}}{d\theta} > 0$.

The result for $\frac{\partial \hat{g}}{\partial \theta \partial z}$ is straightforward. We have $\frac{\partial \hat{g}}{\partial z} = \frac{1}{\sigma} \hat{g}^{1-\sigma} \left(\frac{(1-\chi)\hat{\eta}^\alpha}{\frac{\eta}{\gamma} + \frac{c}{p(\theta)}} \right)$ which is also

¹⁵Note that this is isomorphic to a textbook cake-eating problem under CRRA utility with the addition that the cake can grow or depreciate at a constant rate. While the policy function discussed here is an intuitive generalization of the well-known textbook solution $f' = \beta^{\frac{1}{\sigma}} f$, I have not located any discussion of this exact problem to cite. Thus, the derivation is available upon request.

clearly greater than zero and decreasing in θ . Although this result holds only for partial derivatives (i.e. with $\hat{\eta}$ being held constant), it can also be shown to hold for total derivatives in the case where $\hat{\eta} \geq \alpha(1 + \frac{c}{p(\theta)}\gamma)$ by applying the chain rule as above and computing $\frac{d\hat{\eta}}{dz}$ using implicit differentiation.

C. Derivations and Proofs from Section 3

First, I formally define the functions v and H introduced in equation 15.

$$v(m_t, \eta_t, \eta_{t+1}, g_t) = \frac{1}{p(\theta)} \int [g_t(z) \Delta \frac{\eta_t(z)}{\eta_{t+1}(z)} - (1 - \tilde{\lambda})] \int m_t(a, z) da + \frac{\hat{D}(z, \theta_t, \eta_t(z)) \gamma f}{\eta_{t+1}(z)} h(z) dz \quad (19)$$

$$H(z, m_t, \eta_t, \eta_{t+1}, g_t) = \frac{[g_t(z) \Delta \frac{\eta_t(z)}{\eta_{t+1}(z)} - (1 - \tilde{\lambda})] \int m_t(a, z) da + \frac{\hat{D}(z, \theta_t, \eta_t(z)) \gamma f}{\eta_{t+1}(z)} h(z)}{p(\theta) v(m_t, \eta_t, \eta_{t+1}, g_t)} \quad (20)$$

The numerator is the number of matches with a productivity z entrepreneur and the denominator is the total number of matches.

The problem of the constrained social planner is given sequentially by

$$\begin{aligned} \max_{\{c_t, a'_t, s_t, \theta_t, m_t\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \beta^t \int \int \int u(c_t) m_t(a, z) j(y) dy da dz \\ \text{s.t.} \quad & a'_t + c_t = Ra + (1 - s_t)y + s_t(w_t(z) - (1 - z)b) \quad \forall a, y, z \\ & a_{t+1} \geq 0 \\ & s_t(a, z) \in \{0, 1\} \\ & \frac{v(m_t, \eta_t, \eta_{t+1}, g_t)}{\theta_t} = \int \int s_t(a, 0) m_t(a, 0) j(y) dy da \\ & m_{t+1}(a'_t, 0) = m_t(a, 0) - \theta_t p(\theta_t) \int s_t(a, 0) m_t(a, 0) j(y) dy \\ & m_{t+1}(a'_t, z) = (1 - \tilde{\lambda}) m_t(a, z) + H(z, m_t, \eta_t, \eta_{t+1}, g_t) \theta_t p(\theta_t) \int s_t(a, 0) m_t(a, 0) j(y) dy \end{aligned} \quad (21)$$

where the functions η_t and g_t arise from the slightly modified sequential problem

of an entrepreneur:

$$\begin{aligned}
& \max_{\{d_t, f_{t+1}, k_t, n_t, v_t\}} \sum_{t=0}^{\infty} (\beta \Delta)^t \frac{c_t^{1-\sigma}}{1-\sigma} \\
& s.t. \quad d_t + f_{t+1} = (1-\chi) z k_t^\alpha n_t^{1-\alpha} - (r+\delta) k_t - (1-\chi) \underline{w} n_t + f_t - c v_t \\
& \quad n_{t+1} = (1-\lambda) n_t + p(\theta_t) v_t \\
& \quad k_t \leq \gamma f_t \\
& \quad f_0 \in \mathbb{R}
\end{aligned} \tag{22}$$

so that $\eta_t = \frac{\gamma f_t}{n_t}$ and $g_t = \frac{f_{t+1}}{f_t}$.¹⁶ Note that here I have suppressed the initial condition of the planner's problem and imposed the scale-invariance of the entrepreneurs optimal capital-labor ratio and growth rate by leaving the initial condition f_0 arbitrary.

In analysis of the problem of the social planner, it will be useful to note that while η_t and g_t are potentially functions of z and the entire sequence of labor market tightness $\{\theta\}_{t=0}^{\infty}$, solving the entrepreneur's problem reveals that they depend only on ability z and current and future tightness θ_t, θ_{t+1} and thus can be written as $\eta_t(z, \theta_t, \theta_{t+1})$ and $g_t(z, \theta_t, \theta_{t+1})$. The independence of entrepreneur policy functions from values of θ beyond period $t+1$ follows directly from the linearity of the hiring cost, combined with the parameter assumptions that ensure that any operating entrepreneur will choose $v_t > 0$ each period. While the continuation value of an entrepreneurs labor force depends in theory on the whole sequence of labor market tightness, the ability to re-optimize at linear cost tomorrow ensures that this continuation value is equal to the "liquidation value" of the workforce next period.

¹⁶Even here in the appendix I opt to write the planner's problem for the case of no autocorrelation in individuals' self-employment productivity (i.e. y is drawn from $j(y)$ each period). Including autocorrelation is conceptually simple and involves adjusting only the final two inequalities governing the evolution of the distribution m_t (and the integral in the objective function); however, doing so leads to prohibitively cumbersome notation and adds no additional insight.

3.1. Notes and Proof for Proposition 2

The dynamic terms in equation 18 are given by

Anticipation Terms =

$$\begin{aligned} & \frac{S}{\bar{\theta}} \left(\mu_{t-2} \left(\frac{\partial v_{t-2}}{\partial \eta_{t-1}} \frac{\partial \eta_{t-1}}{\partial \theta_t} \right) + \mu_{t-1} \left(\frac{\partial v_{t-1}}{\partial \eta_{t-1}} \frac{\partial \eta_{t-1}}{\partial \theta_t} + \frac{\partial v_{t-1}}{\partial \eta_t} \frac{\partial \eta_t}{\partial \theta_t} \right) + \mu_t \left(\frac{\partial v_t}{\partial \eta_t} \frac{\partial \eta_t}{\partial \theta_t} \right) \right) + \quad (23) \\ & \bar{\theta} p(\bar{\theta}) S \left(\int_z \lambda_{t-2}(z) \left(\frac{\partial H_{t-2}}{\partial \eta_{t-1}} \frac{\partial \eta_{t-1}}{\partial \theta_t} \right) dz + \int_z \lambda_{t-1}(z) \left(\frac{\partial H_{t-1}}{\partial \eta_{t-1}} \frac{\partial \eta_{t-1}}{\partial \theta_t} + \frac{\partial H_{t-1}}{\partial \eta_t} \frac{\partial \eta_t}{\partial \theta_t} \right) dz + \right. \\ & \left. \int_z \lambda_t(z) \left(\frac{\partial H_t}{\partial \eta_t} \frac{\partial \eta_t}{\partial \theta_t} \right) dz \right) \end{aligned}$$

where μ_t and $\lambda_t(z)$ are the shadow prices associated with the constraints on aggregate labor market tightness and productivity-specific matching rates respectively. These terms essentially capture the welfare gains from anticipatory hiring when labor market tightness is changed. While the welfare changes from permanent changes in hiring are captured in the other terms of equation 18, this term captures the small gains that occur due to the fact that some of this hiring is done in anticipation of the change, shifting some hiring forward temporally.

Proof: The first step is to rewrite the planner's problem to eliminate the binary choice of s_t which complicates analysis. It's fairly straightforward to show that, for utility functions exhibiting diminishing marginal utility, the optimal choice of s_t takes the form of a cutoff rule in a above which individuals search and below which they do not (this fact arises directly from the fact that c_t^* is monotonically increasing a conditional on s_t and diminishing marginal utility). Thus we can rewrite the planner's problem as selecting an optimal cutoff s_t , which is differentiable. I also rewrite the planner's problem in recursive form to simplify analysis.

$$\begin{aligned}
V(\theta_{-2}, \theta_{-1}, m) &= \max_{c, a', s, \theta, m'} \int \int \int u(c) m(a, z) j(y) dy da dz + \beta V(\theta_{-1}, \theta, m') \\
s.t. \quad a' + c &= Ra + (1 - \text{St}(a - s))y + \text{St}(a - s)(w(z) - (1 - z)b) \quad \forall a, y, z \\
a' &\geq 0 \\
\frac{v(m, \eta, \eta', g)}{\theta} &= \int_s^\infty m(a, 0) da \\
m'_e(a', 0) &= \int \tilde{\lambda} m(a, z) dz \\
m'_u(a', 0) &= m(a, 0) - \text{St}(a - s)\theta p(\theta)m(a, 0) \\
m'_e(a', z) &= (1 - \tilde{\lambda})m(a, z) \\
m'_u(a', z) &= H(z, m, \eta, \eta', g)\text{St}(a - s)\theta p(\theta)m(a, 0) \\
m'(x, 0) &= m_e(x, 0) + m_u(x, 0) \\
m'(x, z) &= m_e(x, z) + m_u(x, z)
\end{aligned} \tag{24}$$

where $\text{St}(x)$ is the step function defined via the integral of Dirac's delta δ_x .

Because the state variable describing the distribution of agents across states m is a function $\mathbb{R}^2 \rightarrow \mathbb{R}$, the value function V is technically a functional. Thus making progress requires dipping into functional analysis. I keep things relatively simple and try to align notation as closely as possible to what is typical in more standard situations. To this end, define the following shorthand to capture the notion of a “derivative of y with respect to the value of m at point (a, z) ”:

$$\frac{dy}{dm(a, z)} \equiv \left. \frac{d}{d\epsilon} y(m + \epsilon \delta_a \delta_z) \right|_{\epsilon=0}$$

With this defined, we can proceed.

The first order condition with respect to s yields

$$\frac{\lambda(s, 0)}{m(s, 0)}(y + b) = \theta p(\theta) \left(\int \mu(s, z) H(z) dz - \mu(s, 0) \right) + \tau \tag{25}$$

where λ and τ are the Lagrange multipliers on the budget and theta constraints respectively. We can then generate a pair of envelope conditions with respect to $m(s, z)$ and $m(s, 0)$ (note that I have used the first order condition for s to eliminate

τ from both).

$$\begin{aligned} \frac{1}{m(s, z)} \frac{dV}{dm(s, z)} &= \int u(c)j(y)dy + \left(g(z)\Delta\frac{\eta}{\eta'} - (1 - \tilde{\lambda})\right) \left(\int H(z)\mu(s, z)dz - \mu(s, 0)\right) \\ &\quad - \frac{\lambda(s, 0)}{\theta p(\theta)m(s, 0)}(y + b)\left(g(z)\Delta\frac{\eta}{\eta'} - (1 - \tilde{\lambda})\right) + \tilde{\lambda}\mu(s, 0) \\ &\quad + (1 - \tilde{\lambda})\mu(s, z) + \left(g(z)\Delta\frac{\eta}{\eta'} - (1 - \tilde{\lambda})\right)(\tilde{\omega}_1 - \tilde{\omega}_2) \end{aligned} \quad (26)$$

$$\frac{1}{m(s, 0)} \frac{dV}{dm(s, 0)} = \int u(c)j(y)dy + \mu(s, 0) + \lambda(s, 0)(y + b) \quad (27)$$

where $\mu(a, z)$ and $\mu(a, 0)$ are the Lagrange multiplier on the constraints governing the evolution of m' and $(\tilde{\omega}_1, \tilde{\omega}_2)$ are defined in the discussion at the end of this section.

We can then use these conditions to generate an expression for $\int H(z)\frac{1}{m(s, z)}\frac{dV}{dm(s, z)}dz - \frac{1}{m(s, 0)}\frac{dV}{dm(s, 0)}$ which should be interpreted as the planner's increase in value from moving one (normalized) unit of workers into employment while obeying the constraint that fraction $H(z)$ of workers must be matched with an entrepreneur of productivity z .

We also have from the first order conditions on $m'(s, z)$ and $m'(s, 0)$ ¹⁷

$$\int H(z)\mu(s, z)dz - \mu(s, 0) = \beta \left(\int H(z)\frac{1}{m(s, z)}\frac{dV}{dm(s, z)}dz - \frac{1}{m(s, 0)}\frac{dV}{dm(s, 0)} \right) \quad (28)$$

Combining this expression with the expression for the RHS referenced above, restricting to steady-state, and solving for the desired quantity yields

$$\int H(z)\mu(s, z)dz - \mu(s, 0) = \frac{\beta \int \int H(z)(u(c_z) - u(c_0))j(y)dydz + \text{Drift Terms}}{1 - \beta \int H(z)g(z)\Delta dz} \quad (29)$$

where c_z and c_0 are notation-saving shorthand for $c(a, z, y)$ and $c(a, 0, y)$ respectively, and the drift terms are discussed further below. This term can be plugged directly in to the first order condition with respect to s .

¹⁷This phrase should be interpreted as intuitive shorthand for the first order conditioned generated by examining a delta-perturbation of m' at (a, z) i.e. $\frac{d\mathcal{L}}{dm'(a, z)}$ in the shorthand defined above.

With the hard part done, all that remains is to use the first order condition for θ to find the following expression for τ :

$$\begin{aligned} \tau = & \frac{\theta / \int_s^\infty m(a, 0) da}{\frac{d \log v}{d \log \theta} - 1} \left(\underbrace{\left(1 + \frac{d \log p}{d \log \theta} \right) \int_s^\infty \left(\int H(z) \mu(a, z) dz - \mu(a, 0) \right) m(a, 0) p(\theta) da}_{\text{Congestion}} \right. \\ & + \underbrace{\int \int \int \lambda(a, z, y) \frac{dw}{d\theta} j(y) dy dz da}_{\text{Capital Shallowing}} + \underbrace{\theta p(\theta) \int_s^\infty \int \frac{dH(z)}{d\theta} \mu(a, z) dz m(a, 0) da}_{\text{Composition of Jobs}} \left. \right) \quad (30) \\ & + \underbrace{\beta \frac{dV}{d\theta_{-1}}}_{\text{Anticipation}} \end{aligned}$$

Finally, plugging everything in to the first order condition for s shows that the planner assigns an individual in state $(a, 0)$ to search if and only if

$$\int u'(c_0) j(y) (y + b) dy = \beta \theta p(\theta) \frac{\int \int H(z) (u(c_z) - u(c_0)) j(y) dy dz + \text{Drift Terms}}{1 - \beta \int H(z) g(z) \Delta dz} + \tau \quad (31)$$

The exact formulation of the decision rule used in Proposition 2 can be found simply by letting $\sigma \rightarrow 0$ and noting that the drift terms collapse to zero in this limit, concluding the proof.

Discussion of Drift Terms: The drift terms in the planner's decision rule serve as adjustments for the fact that the marginal job-seeker has a different level of asset holdings than the average job-seeker and, similarly, that the marginal newly employed individual has different assets than the average employed individual. Essentially, they adjust for the fact that the asset level of searchers will “drift” away from s over time.

$$\begin{aligned} \text{Drift Terms} = & \left(1 - \frac{\int_s^\infty m(a, 0) da}{m(s, 0)} \right) \lambda(s, 0) (y + b) + \left(g(z) \Delta \frac{\eta}{\eta'} - (1 - \tilde{\lambda}) \right) (\tilde{\omega}_1 - \tilde{\omega}_2) \\ \tilde{\omega}_1 = & \int \frac{\int m(a, z) \mu(a, z) \text{St}(a - s) m(a, 0) da}{m(s, z) \mu(s, z) m(s, 0)} dz \\ \tilde{\omega}_2 = & \int \frac{\int \int (g(x) \Delta - (1 - \tilde{\lambda})) H(x) m(a, x) \mu(a, x) \text{St}(a - s) m(a, 0) dadx}{(g(z) \Delta - (1 - \tilde{\lambda})) m(s, z) \mu(s, z) m(s, 0)} dz \end{aligned}$$

To see this, note that the drift terms collapse to zero when the distribution of asset

holdings among both the employed and unemployed are concentrated at s (i.e. $m(a, z) = \delta_s m$ and $m(a, 0) = \delta_s(1 - m)$). Further analysis of this term is possible but involves substantial technical complication (due to the necessity of tracking the evolution of assets over time) and provides very little additional insight.

No (Additional) Externalities in Savings Decision: Here I sketch the argument/proof of the fact that the presence of search does not induce an externality in individuals' savings decisions. That is, individuals facing a search tax/subsidy aligning their privately optimal search decision rule with that of the planner will choose the same savings policy function as the planner.

The approach follows that of [Davila et al. \(2012\)](#) and leverages a change of variables in the planner's objective function from time-space to individual-space for any finite (N period) optimization sub-problem. Consider the sub-problem of a planner facing a distribution of agents m and who has already settled on the two-period-ahead policy function a'' but must decide today's policy function a' . One could consider the maximization of the sum of today's utility (averaged over m) and tomorrow's utility (averaged over the appropriately defined m'); this is the period approach and is how the planner's problem in (21) is written. One could alternatively consider the maximization of the *two period* utility for all agents alive in the first period (i.e. averaged over m) — the individual approach. These two objects are different ways of computing the same quantity.

The approach above lets us consider the following optimization problem:

$$\begin{aligned} \max \int \int & \left(\int u(Ra - \text{Inc}(y, a, s, z) - a')j(y)dy \right. \\ & \left. + \beta \mathbb{E} \left[\int u(Ra' - \text{Inc}(y, a', s', z') - a'')j(y)dy | s, z \right] \right) m(a, z)dadz \quad (32) \\ \text{Inc} = & (1 - \text{St}(a - s))y + \text{St}(a - s)(w(z) - (1 - z)b) \end{aligned}$$

Note that all the transition dynamics across employment states z are implicit in the expectations operator (a rigorous proof would require fully specifying these details, but they can be ignored in a proof sketch).

Taking the first order condition for a' from this problem reveals that it is identical to that derived from the individual problem. Of course these first order conditions contain the policy function for s , but the assumption that the search subsidy/tax implements the planner's search policy in the decentralized economy ensures that these functions are identical. Thus the savings policies are identical, and

the proof sketch is complete.

[To add:] As written, formulas imply that choice of a' must be made *before* observing income shock. Fine for now but should probably adjust in final version.