Job Search in Developing Countries: Crowd-In and Crowd-Out Externalities

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Abstract

Productive wage work is often difficult to find in developing countries. Many policies aim at assisting searchers and expanding the wage sector, but the rationale for intervening is unclear. This paper develops a search-and-matching model that incorporates key features of developing economies including a large self-employment sector, savings-constrained households, and capital-constrained firms. Four search externalities — two positive and two negative — emerge, leading to inefficiency. After estimating the model using an experiment that provided search subsidies to job seekers in Ethiopia, I find that the optimal policy is a *tax* that roughly doubles the cost of search, rather than a subsidy.

1. Introduction

Movement out of self-employment and into wage work is a key feature of structural change and development, but reliable wage-sector jobs are often difficult to find (Gollin 2008, Buera, Kaboski & Shin 2015, Poschke 2019). Many individuals spend months or even years alternating between self (or marginal) employment and job search before finally finding long-term wage work (see e.g. Donovan, Lu & Schoellman 2023). As a result of these two facts, there has been substantial interest in the impacts and effectiveness of policies aimed towards expanding the wage sector from both academics and policymakers, including subsidies to labor search, transport, and (temporarily) wages (e.g. Levinsohn et al. 2014, Franklin 2018, De Mel et al. 2019, Abebe et al. 2021, and many others).

Canonical models of frictional labor markets generate inefficiencies, as in Hosios (1990), and are well understood but lack many features central to developing economies, such as large shares of self-employment and substantial credit market frictions. Developing and evaluating labor market policies for developing countries requires understanding how the externalities arising from labor market frictions manifest in such an environment.

This paper generalizes these externalities to a model relevant to developing countries and studies the general equilibrium impacts and trade-offs that they generate. Individuals have access to a self-employment option for subsistence and are savings-constrained, limiting their ability to fund job search. Entrepreneurs run firms and face financial constraints that restrict their growth and distort the allocation of resources, and the two interact through a labor market exhibiting canonical Diamond-Mortensen-Pissarides search-and-matching frictions, which generates externalities from labor search and vacancy posting.

Individuals in the model desire higher-paying wage jobs but must pay a search cost (e.g. commuting) and give up a period of income in order to search. Because they face idiosyncratic job-finding risk, only sufficiently self-insured individuals will choose to search while others will opt for the guaranteed but lower income of self-employment(as in Feng, Lagakos & Rauch 2018, Herreño & Ocampo 2021). The model thus reproduces the empirical fact that individuals frequently and stochastically shift between self employment and job search before finding wage work.

Entrepreneurs operate a constant returns to scale production technology and consequently desire to be large but are restricted in size by a collateral constraint

that prevents them from financing capital beyond some multiple of their wealth(as in Itskhoki & Moll 2019, Buera, Kaboski & Shin 2021). They hire workers by posting vacancies, but any funds spent paying vacancy posting costs are funds that can no longer be used as collateral in the future. Thus labor market frictions act as a constraint to firm growth.

Despite the complexities of borrowing-constrained individuals, credit-constrained firms, and frictional labor markets, substantial insight into the externalities stemming from labor market frictions can be gained analytically. Motivated by the emphasis within the development literature on worker-side interventions (often called "Active Labor Market Policies"), I consider the problem of a dynamic Ramsey planner who maximizes average worker utility subject to the constraint that it must respect individual and entrepreneur budget and credit constraints, must respect the matching technology, and cannot dictate the behavior of entrepreneurs (as in e.g. Itskhoki & Moll 2019).

Examining the planner's problem reveals four search externalities that emerge as wedges and cause the individually optimal search decision rule to differ from the planner's. I refer to negative externalities that cause one individual's search to reduce the returns to search for everyone else as "crowd-out" effects, as these lead an individual's decision to search to push marginal searchers towards self-employment. Conversely, positive search externalities that increase the returns to search are referred to as "crowd-in" effects. Overall, there are two externalities of each type.

The first crowd-out externality, Congestion, corresponds to the well-known externality of the same name from typical models. An individual who searches exerts downward pressure on labor market tightness and lowers the probability that any particular individual finds a job. The channel is offset by the Firm Size externality, which arises from the fact that entrepreneur size is limited by collateral constraints. When an individual searches and finds a job, the resulting output is split between worker and entrepreneur via bargaining. The entrepreneur then uses a portion of their share as collateral to finance further expansion of the firm, including posting additional vacancies and hiring workers. Thus, in the longer run, the individual's search decision increases the probability that future individuals find jobs.

The remaining two externalities occur through wages, rather than job-finding probabilities. The Capital Shallowing externality occurs due to the fact that downward pressure on labor market tightness (from search) lowers entrepreneurs' cost

of hiring and, consequently, decreases the cost of labor relative to capital. As a result, entrepreneurs shift to a lower capital-labor ratio which reduces the average productivity of labor and (through bargaining) wages.

The Allocative Efficiency externality is somewhat more involved. As search pushes labor market tightness down, hiring costs fall and entrepreneurs grow their firms faster, as some of these cost savings are used to finance growth. This externality arises from the fact that this increase in growth is larger for more productive entrepreneurs. Because they wish to grow faster (than less productive entrepreneurs), hiring costs make up a larger share of their total costs. Thus, a decline in hiring costs (due to falling tightness) represents a larger proportional cost reduction and frees up more resources for growth for these entrepreneurs. As a result of their faster (relative) growth, the share of capital and labor allocated to the productive entrepreneurs increases over time, raising allocative efficiency, Total Factor Productivity, and average wages, leading to a positive externality.

After identifying the four externalities, I turn to the question of optimal policy. Interestingly, despite having access to a wide variety of complex tax instruments that condition on household heterogeneity, the planner's optimal solution balancing the externalities can be implemented using only a single tax (or subsidy) on search (along with lump-sum transfers to restore budgets). In particular, the optimal policy does not involve a subsidy to savings, implying that individuals' inability to borrow does not interact with labor market frictions to exert additional externalities from savings. Intuition might suggest that a planner who wants to induce individuals to search more will also want to make individuals save more in order to fund this search; however, this turns out not to be the case. Conditional on a subsidy that fully internalizes the externalities, individuals' consumption-savings decisions align with those of the planner, highlighting the fact that it is not households' inability to borrow *per se* that justifies policy intervention.

In order to quantify the four externalities and compute the efficient policy, I estimate the model using simulated method of moments to match search behavior from weekly data collected as part of an experimental evaluation of a labor search subsidy in Ethiopia (Abebe et al. 2021). The model is estimated exclusively using data on control individuals (those not receiving a subsidy) while the outcomes of treated individuals receiving the subsidy are reserved for model validation. The model passes this validation check — while the subsidy is offered, treated individuals are about 5 percent more likely to search for work in both the data and

model.

Surprisingly, the optimal subsidy in the estimated model is equal to -101 of total search costs — that is, the optimal policy is actually a tax on search that roughly doubles the search cost (an increase equal to about 20 percent of average self-employment earnings). Even though intuition suggests that a subsidy would lead to a direct utility benefit as it reallocates income from a relatively high income state (working in self-employment) to a low income state (searching), the negative externalities from search (Congestion and Capital Shallowing) outweigh the positive externalities (Size and Allocative Efficiency) to such an extent that a tax ends up maximizing welfare.

The gains from the optimal policy are substantial — on average, consumers enjoy an increase in welfare equal to 1.5 percent of their consumption. To decompose the contribution of each externality to this overall gain, I start from optimal equilibrium and consider counterfactual equilibriums in which the size of each externality is moved marginally closer to its size in the competitive equilibrium,[1] in essence, removing the impact of the externality. The difference in welfare between the optimal equilibrium and one of these counterfactuals then quantifies the welfare impact of the corresponding externality.

This decomposition reveals two main takeaways. First, the externalities that influence employment have a quantitatively larger effect than those that influence wages. The Firm Size and Congestion externalities (impacting employment) account for -0.8 and 1.6 percent of the welfare gains respectively, while Capital Shallowing and Allocative Efficiency (impacting wages) account for 0.2 and -0.2 percent. The second, evident from the same results, is that the crowd-out externalities dominate the crowd-in. Although the Capital Shallowing (crowd-out) and Allocative Efficiency (crowd-in) channels more or less offset each other, the Congestion effect quantitatively dominates the Firm Size effect and is roughly twice as large. This, then, is the primary reason that the optimal policy takes the form of a tax, rather than a subsidy.

Although the optimal search tax increases welfare, it also shrinks the size of the wage sector (from 30 percent to 24 percent). As a consequence, such a policy could be undesirable for policymakers who may value increasing employment, even at the expense of efficiency (either for reasons outside the model, such as the need to

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[1]Using the marginal, rather than the total, change is necessary as equilibria may become degenerate when the impact of a single externality is removed completely.

attract FDI, or due to political incentives). As a second quantitative exercise, I pivot from normative to positive analysis and examine the impact of a subsidy to search aimed at expanding the wage sector. Even in this case where welfare is no longer the primary outcome of interest, the four externalities remain important as they exactly make up the difference between the impact of the policy in partial equilibrium (i.e. what would be observed or measured in a small-scale experiment) and general equilibrium (i.e. what would occur in an economy-wide implementation). This exercise, then, follows the spirit of the literature that interprets experimental results through macroeconomic models (e.g. Brooks, Donovan & Johnson 2020, Fujimoto, Lagakos & VanVuren 2023, Lagakos, Mobarak & Waugh 2023).

With the externalities shut down, the partial equilibrium results that would be observed through an experiment seem promising for the policy — the size of the wage sector increases dramatically from 30 percent to 47 percent, and this comes at almost no cost to welfare (in fact, if the experimental evaluation includes only the subsidy portion of the policy and does not include taxation, welfare increases by 1.3 percent). However, when these channels are introduced in general equilibrium, the policy is almost half as effective — the wage sector grows to only 40 percent — and carries a large welfare cost of -2.5 percent. Similar to the results of the welfare decomposition, the Congestion externality is responsible for most of this pessimism, accounting for -20 percent of difference in the size of the wage sector between partial and general equilibrium and 4.6 percent of the difference in welfare.

Overall, the surprising conclusion is that, despite contradicting the intuition of policymakers and economists alike, labor markets in developing countries (at least those similar to the market in Ethiopia used to estimate the model) are characterized by workers who search too much rather than too little. Consequently, policies aimed at helping and encouraging workers to search, such as search subsidies, are counterproductive. While they do manage to increase the size of the wage sector and may even yield promising experimental results, they do so fairly ineffectively once externalities are taken into account and carry substantial welfare costs.

Related Literature: This paper is closely related to the macroeconomic development literature studying the impact of entrepreneur-level credit constraints on growth and development such as Buera, Kaboski & Shin (2011), Moll (2014), Itskhoki & Moll (2019), and Buera, Kaboski & Shin (2021). This paper also builds on recent work drawing distinctions between subsistence self-employment and

entrepreneurship (such as Feng & Ren 2021) or otherwise studying unemployment in developing countries (such as Feng, Lagakos & Rauch 2018, Poschke 2019). Closely related is Herreño & Ocampo (2021) who use a model in which households use self-employment to cope with the risks of wage employment (the same mechanism as this paper) to study the macroeconomic effects of microloans and cash transfers.

The model dynamics in which workers flow freely between self/marginal employment and labor search before finding a long-term wage job are very similar to those documented in Donovan, Lu & Schoellman (2020). In a similar vein, Banerjee et al. (2021) find that skilled workers in developing countries exhibit higher unemployment rates relative to unskilled workers and show that this difference leads to differences in occupational choice. Porzio, Rossi & Santangelo (2021) use a model with frictional reallocation of labor from (self-employment dominated) agriculture to (wage work dominated) non-agriculture to quantify the importance of human capital in explaining the process of structural change.

This paper is also closely related to the microeconomic literature on Active Labor Market Policies, which are intended to help grow the wage sector. Abebe et al. (2021) and Franklin (2018) both study the effects of cash transfers (the same policy studied in the quantitative portion of this paper). De Mel et al. (2019), Algan et al. (2020), and Alfonsi et al. (2020) all study firm-side interventions (although the last includes an additional worker-side treatment arm) also intended to help workers find jobs. McKenzie (2017) provides an excellent review of this literature, which is too exhaustive to list here.

2. Model

Time is discrete. There is measure one of individuals (workers) and an endogenous measure of entrepreneurs. Households consume, save, and choose between working in self-employment or participating in the labor market while entrepreneurs operate firms, consume profits, and accumulate capital and labor for future periods.

2.1. Search and Matching Technology

The labor market for wage work exhibits typical search-and-matching frictions. Workers must search for jobs and entrepreneurs must hire by posting vacancies. The cost of searching for a job and the cost of posting a vacancy are denoted by b

and c respectively. Each period, the number of worker-firm matches is given by a homogeneous of degree 1 matching function $m(u, v)$ where u is the measure of individuals searching for a job and v is the number of vacancies posted by firms. As is typical, $\theta = \frac{v}{u}$ is defined to be labor market tightness so that $p(\theta) \equiv m(\frac{1}{\theta}, 1) = \frac{m(u,v)}{v}$ is the probability that any vacancy is filled and $\theta p(\theta) = \frac{m(u,v)}{u}$ is the probability that any searcher finds a job. Finally, matches between workers and firms are separated with exogenous probability λ at the end of every period.

2.2. Workers

A unit measure of infinitely-lived workers are indexed by their wealth a, their employment status e, and their self-employment productivity y. Their lifetime utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \tag{1}$$

Individuals are endowed with one unit of time each period which they supply inelastically and indivisibly to either work or search.[2]

Labor Decisions: Any individual can engage in self-employment and operate the self-employment technology

$$y_t = A_s l_t \tag{2}$$

An individual's self-employment productivity l_t follows an exogenous Markov process described by transition matrix M. For simplicity, I normalize A_s to unity so that self-employment earnings are simply given by $y_t = l_t$. By assumption, self-employment uses only an individual's own labor and does not involve hiring workers from outside the household. Thus, this option most closely corresponds to the concept of "subsistence self-employment".

Instead of engaging in self-employment, an individual can choose to pay a search cost b and search for a wage job. A searcher earns nothing in the current period and finds a permanent job with probability $\theta p(\theta)$. After finding a job and becoming employed, the individual can either work in their wage job or return to self-employment (in equilibrium, all employed workers will choose to en-

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[2]This assumption can be justified by the fact that a model period is one week. Additionally, quantitative experiments with allowing interior choices of time allocation suggest that the optimal policy is fairly close to "bang-bang", with individuals largely choosing to allocate their entire time budget to either work or search rather than a mix of the two, for reasonable parameters.

gage in wage work). Wages are determined through bargaining (discussed later) and depend on the productivity of the entrepreneur with whom the individual is matched, given by z_t.[3]

Budgets: Workers face incomplete markets a la Aiyagari (1994), Bewley (1977), and Huggett (1993) and accumulate assets for self-insurance. Each period, assets pay an exogenous rate of return r (i.e. this is a small open economy). Individuals cannot borrow (i.e. $a_t \geq 0$). Their budget constraint is then

$$a_{t+1} + c_t = (1 + r)a_t + (1 - e_t)\big((1 - s_t)y_t - s_t b\big) + e_t w_t(z_t) \tag{3}$$

where $s_t \in \{0, 1\}$ is a choice variable with $s_t = 1$ representing the search decision in period t and $e_t \in \{0, 1\}$ is an indicator variable with $e_t = 1$ indicating that the individual is employed in period t.

Search: Search is undirected, and every vacancy has an equal probability of being filled. An individual's probability of matching with a job that will pay $w(z)$ (conditional on matching with any job), denoted $H(z; X)$ where X is a vector of aggregate state variables, is given by the share of vacancies posted by z-type entrepreneurs. Although, in principle, w and H depend on all the state variables of both the individual and the entrepreneur to which they are matched, they are written here to depend only on the matched entrepreneur's productivity z. A later section will show this to be the case, justifying this notation.

Employed workers are separated from their jobs with probability λ. Additionally, an individual can lose their job if the entrepreneur employing them dies (probability $1 - \Delta$, discussed below) or chooses to downsize its labor force. Under generous parameter conditions (satisfied in the quantified model), it can be shown that downsizing never occurs in equilibrium, which I assume throughout the rest of the paper. Thus the probability that an employed worker retains their job at the end of the period is given by $(1 - \tilde{\lambda}) = \Delta(1 - \lambda)$.

Bellman Equation: Taking all of the above, the individual's optimization prob-

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[3]Section 2.5 shows that the bargained wage depends only on the productivity of the entrepreneur and not on other entrepreneur or individual state variables.

lem can be written recursively as

$$V_u(a, y; X) = \max_{c,a',s\in\{0,1\}} \frac{c^{1-\sigma}}{1-\sigma} + \beta\bigg(\big(1 - s\theta p(\theta)\big)E_{y'}[V_u(a', y'; X')|y]+$$

$$s\theta p(\theta)\big(E_z[V_e(a', z; X')]\big)\bigg)$$

$$V_e(a, z; X) = \max_{c,a'} \frac{c^{1-\sigma}}{1-\sigma} + \beta\bigg((1 - \tilde{\lambda})V_e(a', z; X')+$$

$$\tilde{\lambda}E_{y'}[V_u(a', y'; X')]\bigg) \tag{4}$$

$$
\begin{aligned}
s.t. \quad a' + c &= (1+r)a + (1-s)y - sb && \text{for } V_u \\
a' + c &= (1+r)a + w(z) && \text{for } V_e \\
X' &= G(X) \\
y' &\sim M(y) \\
z &\sim H(z; X)
\end{aligned}
$$

where X is a vector of aggregate state variables and G is the perception function for the evolution of the aggregate state. V_u and V_e denote the value function of the individual while unemployed and employed respectively. For simplicity, an individual who moves from employment to unemployment draws their self-employment productivity y from the stationary distribution of M.

2.3. Worker Behavior

Workers decide whether to engage in self-employment or search for a wage job by weighing the benefits of search against the costs. In addition to the explicit search cost b and the opportunity cost of forgone self-employment earnings, the presence of borrowing constraints means that the higher risk of job search also serves as a cost, particularly if the probability of finding a wage job is small as it is in many developing countries.

Only individuals who are sufficiently self-insured will opt to pay the search cost and search for wage work, hoping for the slim probability of finding a job and achieving a large boost in earnings. Those without much self-insurance will enjoy the safety of lower but guaranteed income in self-employment. For those

that search, the search cost quickly diminishes their savings and reduces their self-insurance, eventually driving them to self-employment until they can re-accumulate sufficient self-insurance.

The result is that individuals near the threshold of self-insurance spend a few periods working in self-employment and accumulating assets, then switch to searching for a wage job for a few periods, and return to self-employment once their savings have been depleted. Of course the exact cutoff in savings above which households decide to search depends on their self-employment productivity y_t (which is stochastic), leading to some unpredictability in the exact timing of these switches.

Figure 1: Worker Self-Employment and Wage Sector Behavior over Time
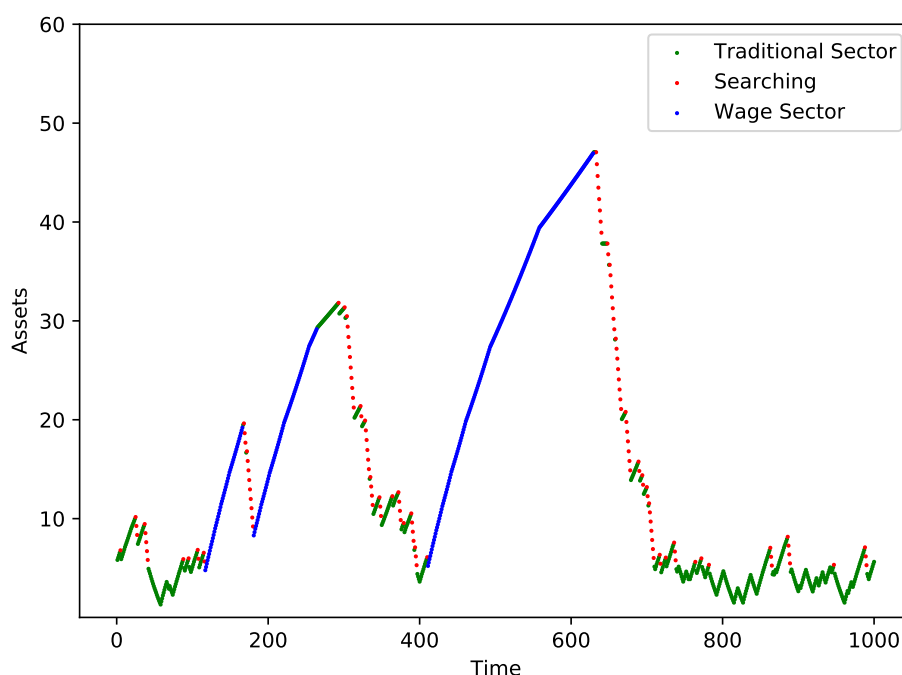
Note: This figure plots a simulated individual's search, wage work, and self-employment behavior as well as assets over 1000 periods, performed using the esimtated model described in Section 4.

Figure 1 displays an example of this behavior for a single individual simulated for 1000 periods (each period corresponds to a week, so this is about 20 years). The x-axis displays time while the y-axis displays the individual's stock of assets. The color corresponds to their search decision in that period; weeks in green are those where the individual is engaging in self-employment, red weeks correspond to searching for wage work, and blue weeks are periods when they are employed and working for a wage.

The figure demonstrates the behavior described above. At the start, the individual is near the threshold of self-insurance and alternates between working in self-employment and searching for wage work depending on their particular level of assets and self-employment productivity. At around week 150, their search is successful, and they acquire a high-earning wage job and quickly accumulate assets. They eventually separate from their employer but use their stock of assets to fund extensive search and remain in the wage sector. This behavior continues for quite some time until approximately week 700 when the individual exhausts their assets without finding a job and returns to self-employment punctuated by brief periods of search.

2.4. Entrepreneurs

While individuals work in either self-employment or the wage sector, entrepreneurs operate firms and employ workers. Including entrepreneurs as distinct agents (as opposed to an occupational choice for individuals, as in Buera, Kaboski & Shin 2021), reflects the qualitative difference between "subsistence self-employment" (which individuals can flow in and out of fairly freely as in Donovan, Lu & Schoellman 2020) and productive entrepreneurship with the potential to grow and employ many workers, in addition to providing a dramatic increase in tractability.

There are N entrepreneurs each of size $\frac{M}{N}$ born every period, and the model considers the limit $N \to \infty$.[4] At the end of a period, entrepreneurs die with probability Δ. Entrepreneurs are born with idiosyncratic ability z drawn from some distribution with bounded support $h(z)$ and an initial level of financial wealth $\underline{f}$ (taken to be exogenous). They discount the future at rate β (the same rate as workers), face an exogenous death probability Δ each period, and receive lifetime utility from consumption (labeled d_t for "dividends") given by

$$\sum_{t=0}^{\infty}(\beta\Delta)^t \frac{c_t^{1-\sigma}}{1-\sigma} \tag{5}$$

Each entrepreneur operates a Cobb-Douglas production technology that depends on their ability:

$$y_t = z k_t^\alpha n_t^{1-\alpha} \tag{6}$$

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[4]The assumption that there are an infinite number of atomic entrepreneurs rather than a measure of non-atomic entrepreneurs is not typical but eliminates many technical difficulties in the discussion of wage bargaining. Other than this, there are no substantive differences between the two assumptions.

Entrepreneurs rent capital from the international capital market at an exogenous rental cost $(r + \delta)$ (i.e. this is a small open economy) and pay workers wage w_t, determined by bargaining, but must use their own assets f_t as collateral to finance capital. Their collateral constraint is given by

$$k_t \leq \gamma f_t \tag{7}$$

where $\gamma \geq 1$ is a parameter summarizing the degree of financial market frictions, with $\gamma = 1$ representing the case of full self-financing and $\gamma \to \infty$ representing no financial frictions.[5]

To hire labor and adjust n_t, entrepreneurs post vacancies v_t. Each vacancy costs c units of output to post and is filled at the end of the period with probability $p(\theta)$. The evolution of n_t is dictated by the equation

$$n_{t+1} = (1 - \lambda)n_t + p(\theta)v_t \tag{8}$$

where λ is the exogenous separation rate. Here, it is worth clarifying that while individuals face idiosyncratic risk in job finding and separation, entrepreneurs do not — an entrepreneur with n_t workers can ensure a labor force of precisely n_{t+1} next period by posting $\frac{n_{t+1} - (1-\lambda)n_t}{p(\theta)}$ vacancies.

An entrepreneur's period profits are given by

$$\pi_t(z, k_t, n_t) = z k_t^\alpha n_t^{1-\alpha} - (r + \delta)k_t - w_t n_t \tag{9}$$

Due to the collateral constraint, an entrepreneur will earn positive profits each period. They split these profits between consumption, posting vacancies, and accumulating additional collateral f_{t+1} and face a budget constraint given by

$$d_t + f_{t+1} = \pi_t(z, k_t, n_t) + f_t - cv_t \tag{10}$$

2.5. Wage Bargaining

Each period, entrepreneurs and their hired workers bargain over wages. Because capital acts as a fixed factor of production (the collateral constraint always binds in equilibrium), firm output exhibits decreasing returns to scale in labor. To

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[5]While this constraint is exogenous, it can be thought of as arising from the unenforceability of contracts or other institutional features that make uncollateralized lending risky and microfounded as such (see e.g. Buera, Kaboski & Shin 2021).

accommodate this, I follow Smith (1999) and, more recently, Acemoglu & Hawkins (2014) and model production as a cooperative game between workers and entrepreneurs in which each agent is paid their Shapley value.

The entrepreneur enters the game with capital k and workforce n. Any worker that chooses not to cooperate will engage in self-employment for a period and then return to the bargaining table in the next period (i.e. the outside option is a shirking of duties for a period, rather than termination of the match). Defectors draw their self-employment productivity from the stationary distribution of M, but negotiation occurs before these productivity draws are realized and workers are treated symmetrically.

If the entrepreneur and x of their n workers choose to cooperate, they form a coalition, operate the entrepreneur's production technology, and produce $zk^\alpha x^{1-\alpha}$. The remaining $n-x$ workers form their own coalition and produce $(n-x)\bar{y}$ (where $\bar{y}$ is average self-employment productivity). Each agent is paid their Shapley value arising from this game, so that the wage per worker is given by

$$w = \chi z k^\alpha n^{-\alpha} + (1 - \chi)\bar{y} \tag{11}$$

where χ is a parameter governing the bargaining power of the entrepreneur relative to workers. [6]

The resulting wage determination equation is intuitive; workers are simply paid some linear combination of their average product of labor and their outside option $\bar{y}$, with the weight determined by bargaining power.

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[6] At a technical level, the game is between an atomistic entrepreneur and a continuum of workers; the parameter χ is the relative size of the atomistic entrepreneur. It is also worth noting that because the Shapley value results in workers being paid a linear combination of their average product (rather than marginal product), the model does not nest perfectly competitive wages as a special case.

2.6. The Entrepreneur's Problem and Behavior

Combining equations 5 - 10 and the wage bargaining equation 11, the entrepreneur's problem can be written recursively as

$$
V(z, f, n; X) = \max_{f', n', k, v, d} \frac{c^{1-\sigma}}{1 - \sigma} + \beta \Delta V(z, f', n'; X)
$$

$$
s.t. \ \ d + f' = (1 - \chi) z k^\alpha n^{1-\alpha} - (r + \delta)k - (1 - \chi)\bar{y}n + f - cv
$$

$$
n' = (1 - \lambda)n + p(\theta)v
$$

$$
k \leq \gamma f
$$

$$
v \geq 0
$$

$$
X' = J(X)
$$

where X is a vector of aggregate state variables and J is the entrepreneur's perceptions function for the evolution of the aggregate state. It is important to note that the wage bargaining equation has been substituted into the entrepreneur's budget constraint and does not depend on the composition of their workforce, eliminating the need to track the composition as a state variable.

Entrepreneur Behavior: One important result, from the fact that the user costs of both capital and labor are linear, is that an entrepreneur's capital-labor ratio depends only on their productivity z and aggregate state variables X (see Appendix B for the derivation).[7] Denote this value as η so that

$$
\eta(z; X) = \frac{\gamma f'^*}{n'^*} \tag{12}
$$

where f'^* and n'^* are the entrepreneur's optimal policy functions.

This result lends the model substantial tractability. In general, the bargained wage w depends on all entrepreneur state variables, which can change over time due to accumulation of collateral. However, a constant capital-labor ratio (combined with the wage bargaining equation 11) implies that wages depend only on entrepreneurs' productivity (which is fixed over their life), justifying the use of $w(z)$ and $H(z)$ in the household problem above.

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[7]This statement holds in universally in steady-state and holds for any transition path under the parameter restriction that $\lambda > 1 - \beta\Delta$ which is satisfied in the quantitative model. It is also worth noting that entrepreneurs with sufficiently low z will choose $n = 0$ (i.e. will disengage from the economy and eat their cake rather than operate at a loss), leading to an undefined capital-labor ratio.

A second useful result is that entrepreneurs will pursue a constant productivity-dependent growth rate. That is, f'^* will satisfy

$$f'^* = g(z; X)f \qquad (13)$$
$$\frac{\partial g}{\partial z} > 0$$

for some function g. Intuitively, g is increasing in z; more productive entrepreneurs will grow quicker. Together, the two functions η and g are sufficient to fully characterize entrepreneur behavior as a function of their productivity z and the aggregate state X.

2.7. Some Initial Intuition on Externalities

The two functions η and g can be used to gain some initial insight into the crowd-in and crowd-out externalities of search. Because these occur largely through changes in labor market tightness, it is useful to abuse notation and write $\hat{\eta}(z; \theta)$ and $\hat{g}(z; \theta)$ to represent "the steady-state values of η and g for a z productivity entrepreneur facing steady-state labor market tightness θ." This notation is possible because entrepreneur policy functions depend on the aggregate state only through current and future values of θ. We can then make the following comparative-static-like statements:

Proposition 1 *Let $\hat{g}$ and $\hat{\eta}$ be defined as above. Then*

$$\frac{d\hat{\eta}}{d\theta} > 0$$
$$\frac{d\hat{g}}{d\theta} < 0 \text{ and } \frac{\partial}{\partial z}\left(\frac{\partial \hat{g}}{\partial \theta}\right) < 0$$

where partial derivatives denoted by ∂ are taken while holding other endogenous outcomes (i.e. $\hat{\eta}$) constant.

The first claim of Proposition 1 ($\frac{d\hat{\eta}}{d\theta} > 0$) is that an entrepreneur's capital-labor ratio is increasing in labor market tightness (as a tighter labor market increases the cost of labor relative to capital). As a result of this, an individual choosing to search loosens the labor market and puts downwards pressure on the capital-labor ratio, leading to a reduction in wages for everyone. This is the source of the Capital Shallowing externality. Because it causes one individual's search decision

to discourage the search of others (as lower wages decrease the relative return to search), I refer to this as a "crowd-out" externality.

The proposition's second claim is that an entrepreneur's growth rate is decreasing in market tightness ($\frac{d\hat{g}}{d\theta} < 0$) and, more interestingly, that productive entrepreneurs are more sensitive to changes in θ ($\frac{\partial}{\partial z}\left(\frac{\partial \hat{g}}{\partial \theta}\right) < 0$). This effect arises from the fact that hiring costs make up a larger share of total costs for faster growing (i.e. more productive) firms. Total costs for a firm wishing to grow at rate g^* are given by

$$r\eta^* + \chi z\eta^* + (1 - \chi)\bar{y} + \left(g^* - (1 - \lambda)\right)\frac{c}{p(\theta)} \tag{14}$$

and, consequently, a reduction in hiring costs (due to a looser labor market) represents a larger proportional reduction in total costs for higher g firms. Because these cost savings are used (in part) to fund growth, this leads to larger increases in growth for firms with large baseline growth rates (i.e. the most productive firms).

The result is that reductions in labor market tightness improve allocative efficiency in the economy. As the growth rate of productive firms increases more than unproductive ones, the share of resources in the economy allocated to the productive firms increases, and misallocation is reduced. From workers' perspective, this increases expected wages (as productive firms pay more). This link between an individual's search decision and average wages is ultimately the source of the Allocative Efficiency externality. In contrast to the Capital Shallowing externality, this effect causes one individual's search to encourage the search of others and thus is a "crowd-in" externality.

The two remaining externalities (Congestion and Firm Size) are not directly apparent from the entrepreneur policy functions. The next task, accomplished in the next section, is then to formalize the problem of a social planner in order to examine an exhaustive list of externalities.

3. Efficiency and the Social Planner

A substantial complication in analyzing the externalities present in individuals' labor search decisions is that it is not immediately clear what the appropriate social planner's problem is. As in much of the labor search literature, the problem of an all-powerful planner free from any financial constraints or labor market frictions is uninteresting (except perhaps as a benchmark); this planner would simply allocate all labor and capital to the most productive entrepreneur and divide output in a

way that equalizes marginal utility across all households and entrepreneurs. This teaches us nothing about the externalities generated by households' labor search or how these externalities interact with borrowing constraints.

Instead, I follow the traditional approach and consider the problem of a constrained social planner who must respect the search-and-matching technology, as well as individuals' borrowing constraints (as in Davila et al. 2012). Further, because the stated goal of most labor market policies (e.g. so-called "Active Labor Market Policies") is to improve outcomes for workers, I focus on a social planner who values only worker welfare and places no weight on the welfare of entrepreneurs. This approach has the additional benefit of being somewhat typical in macro-development models with multiple types of agents (e.g. Itskhoki & Moll 2019).

To prevent the planner from simply forcing entrepreneurs to hand over consumption to households, I impose that the social planner can only dictate the decisions of households and cannot control the behavior of entrepreneurs, who continue to solve their optimization problem each period. In this sense, the social planner faces an additional constraint that it cannot force entrepreneurs to act suboptimally. Allocations satisfying these three constraints make up the set of feasible allocations for the planner.

Definition: A path of household policy functions $\left\{c_t(a, y, z), a'_t(a, y, z), s_t(a, y, z)\right\}_{t=0}^{\infty}$, entrepreneur policy functions $\left\{g_t(z), \eta_t(z)\right\}_{t=0}^{\infty}$, distributions of households across savings and matched-employer productivities $\left\{m_t(a, z)\right\}_{t=1}^{\infty}$, and labor market tightness $\left\{\theta_t\right\}_{t=0}^{\infty}$ is **feasible** given an initial distribution $m_0(a, z)$ and market tightness θ_{-1} if

1. It respects the household budget constraint for all a, y, z

$$
\begin{aligned}
a'_t + c_t &= Ra + w_t(z_t, \theta_t) && \forall a, y, t \text{ when } z \geq 0 \\
a'_t + c_t &= Ra + (1 - s_t)y + s_t b && \forall a, y, t \text{ when } z = 0 \qquad (15) \\
a'_t &\geq 0
\end{aligned}
$$

2. It respects the labor market matching technology

$$
\frac{v(m_t, \eta_t, \eta_{t+1}, g_t)}{\theta_t} = \int \int s_t(a, 0) m_t(a, 0) j(y) \, dy \, da \qquad (16)
$$

$$
m_{t+1}(a', z) = (1 - \tilde{\lambda}) m_t(a, z) + H(z, m_t, \eta_t, \eta_{t+1}, g_t) p(\theta_t) v(m_t, \eta_t, \eta_{t+1}, g_t)
$$

18

where v is the total number of posted vacancies as a function of entrepreneur policy functions, and H is the probability that an individual who finds a job is matched with a firm of productivity level z.[8]

3. The entrepreneur policy functions $\{g_t(z), \eta_t(z)\}_{t=0}^{\infty}$ solve the entrepreneurs' problem (Appendix equation 28), conditional on θ_{-1} and $\{\theta_t\}_{t=0}^{\infty}$.

The task of the social planner is to maximize average household welfare subject to these feasibility conditions. Formalizing the statement of this problem is straightforward but cumbersome and is relegated to Appendix C.

There are two details worth noting. The first is that this definition of the planner's problem implicitly imposes the assumption that there is no autocorrelation in individuals' self-employment productivity y (i.e. the distributions m_t are only defined over (a, z)). This assumption, maintained throughout the rest of the paper, substantially reduces notation and improves readability and does not at all change any of the core mechanisms at play. The second is that the planner's problem features full commitment (they choose the entire sequence $\{\theta_t\}_{t=0}^{\infty}$ simultaneously) abstracting from any potential complications of dynamic games between the planner and model agents.

3.1. Privately- vs Socially- Optimal Search Decision Rules

With the planner's problem specified, Proposition 2 finally formalizes the externalities that have been discussed only intuitively up until now. The proposition below makes the simplifying assumption that $\sigma \to 0$ (i.e. linear utility). This assumption is not necessary, and Appendix C provides the statement of the proposition valid for any (time-separable) utility function.[9]

Proposition 2 *Under the assumption that $\sigma \to 0$, the optimal search policies $s(y)$ of the constrained social planner and an individual in steady-state competitive equilibrium depend only on an individual's self-employment productivity and are to search if and only*

---

[8]Both v and H are formally defined in Appendix C.

[9]The fully general statement is not substantively different than the statement in Proposition 2 in the sense that the planner's optimal search decision rule differs from the individual's only by precisely the same four externalities. However, the assumption of linear utility substantially improves the readability of equations 17 and 18, providing clearer insight into the core intuition of the result.

if

$$\text{Individual: } y + b \le \beta \bar{\theta} p(\bar{\theta}) \int_z \frac{w(z, \bar{\theta}) - \left(\int_{y>s} y j(y) dy - J(s) b \right)}{1 - \beta(1 - \tilde{\lambda})} \bar{H}(z) dz \qquad (17)$$

$$\text{Planner: } y + b \le \beta \bar{\theta} p(\bar{\theta}) \int_z \frac{w(z, \bar{\theta}) - \left(\int_{y>s} y j(y) dy - J(s) b \right)}{1 - \beta \Delta g(z, \bar{\theta})} \bar{H}(z) dz \; + \; \mu \quad (18)$$

$$\mu = \frac{\bar{\theta}/\bar{S}}{\frac{\partial \log v}{\partial \log g} \frac{\partial \log g}{\partial \log \theta} - 1} \left(\underbrace{\int_z \frac{\partial w}{\partial \eta} \frac{\partial \eta}{\partial \theta} \bar{m}(z) dz}_{\text{Capital Shallowing}} + \underbrace{\int_z \bar{\lambda}(z) \bar{\theta} p(\bar{\theta}) \bar{S} \frac{\partial H}{\partial g} \frac{\partial g}{\partial \theta} dz}_{\text{Efficiency}} \right. \qquad (19)$$

$$\left. + \underbrace{\int_z \bar{\lambda}(z) \bar{H}(z) \bar{S} p(\bar{\theta}) \left(1 + \frac{\partial \log p}{\partial \log \theta} \right) dz}_{\text{Congestion}} + \text{Anticipation Term} \right)$$

where bars denote steady-state values of the competitive equilibrium and planner's problem respectively, J is the CDF of y, $\bar{S}$ is the steady-state number of searchers, and $\lambda(z)$ is the planner's shadow price denoting the marginal value of an additional worker being matched with a productivity z entrepreneur. The anticipation term is described further in the appendix.

The privately optimal search policy simply weighs the total (opportunity-cost-inclusive) cost of search $y + b$ against the benefits, which are given by the expected excess earnings while employed, discounted over the expected duration of the employment spell. Relative to this rule, the planner's decision rule differs in two ways, both of which are discussed further below. The first is that the planner discounts the excess earnings from employment using the expected growth rate of the firm (with probability Δ the entrepreneur will survive and grow their workforce by g) rather than the separation rate. The second is that the planner carries an additional term μ which internalizes changes in labor market tightness.

Search Externalities: As μ contains all but one of the externalities, it seems intuitive to start the discussion of externalities there. All three of the externalities contained in μ manifest through changes in labor market tightness. Consequently, μ is weighted by the net change in labor market tightness due to a change in the

number of searchers after accounting for the response of vacancies.[10] This weight is negative, reflecting the fact that an increase in the number of searchers leads to a decrease in labor market tightness.

The term labeled "Congestion" is typical in labor search models and reflects the fact that an additional searcher pushes down labor market tightness, reducing the job-finding probability for all searchers. As in textbook search models, the size of this externality is proportional to the elasticity of the matching function ($\frac{\partial \log p}{\partial \log \theta}$) as a high elasticity implies that an additional searcher leads to a large reduction in the job-finding probability. Intuitively, this externality is also increasing in the steady-state number of searchers $\bar{S}$.

The term labeled "Capital Shallowing" reflects the effect of labor market tightness on the capital-labor ratio and, consequently, the average wage — a looser labor market lowers the cost of labor relative to capital, inducing entrepreneurs to reduce their capital-labor ratio and thus wages. It is worth noting that the interpretation of this effect as an externality hinges on the assumption that the planner puts no weight on the consumption of entrepreneurs, as the lower capital-labor ratio would increase their profits and thus consumption. If the planner were to value this increase, this term would be more appropriately interpreted as the impact of (implicit) redistribution, rather than a pure externality.

The term labeled "Efficiency" contains the impact of the improvement in allocative efficiency that occurs when the labor market loosens as a result of search (discussed above). A looser labor market increases the growth rate of firms ($\frac{\partial g}{\partial \theta}$) but does so by disproportionately more for high-productivity firms. As a result, the probability of a worker matching with a high-productivity entrepreneur increases ($\frac{\partial H}{\partial g}$ is increasing in g) which the planner values according to the shadow price λ. At an aggregate level, this manifests as higher average wages (productive entrepreneurs pay more) and higher aggregate TFP.

The final externality, which I call the "Firm Size" externality, is not present in μ, as it does not operate through labor market tightness, and is instead present in the different discount rates used to discount the excess earning while employed in the individually- and socially- optimal search decision rules. Under the parameter assumption (made throughout this paper) that $\Delta \beta > (1 - \lambda)$, we have that $\Delta g(z, \theta) > (1 - \tilde{\lambda})$ for all z, θ. Thus the planner's "presented discounted value" of

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[10]To see that this expression indeed gives the net change, note that $\theta = \frac{v(\theta)}{s} \Rightarrow \frac{d\theta}{ds} = \frac{\theta/s}{\frac{d \log v}{d \log \theta} - 1}$.

a job is higher than an individual's, even fixing labor market tightness.

This externality is a result of the fact that individuals do not capture all the output created by their job match. Some of it is captured by the entrepreneur and used to finance future growth and, as a result, the hiring of additional workers tomorrow. This effect is easiest to see by considering an entrepreneur growing at rate g who is exogenously matched with a unit measure of workers from outside the economy (say, immigrants). Without these extra workers, the entrepreneur would hire $(g-1)n$ workers (on net) for the next period, but with these workers, the entrepreneur hires $(g-1)(n+1)$. The extra workers crowd-in $(g-1)$ additional workers in the next period (and $(g-1)^2$ the next, etc.), conditional on entrepreneur survival (probability Δ). Individuals do not value the future employment that arises from their hiring while the planner does, leading to the externality.

3.2. Implementation and Optimal Feasible Policies

The problem of selecting the welfare maximizing path subject to a set of dynamic constraints in a heterogeneous agent economy is similar to other Ramsey-type problems often found in the literature dealing with welfare and efficiency in heterogeneous agent models (e.g. Itskhoki & Moll 2019, Dávila & Schaab 2023). Like all Ramsey problems, the primal problem of choosing paths of consumption (or consumption functions in the case of heterogeneous agents) subject to feasibility constraints can be equivalently formulated as a dual problem in which the planner selects optimal tax rates from a sufficiently rich set of instruments to decentralize the optimal allocation in competitive equilibrium.

The initial impression of Proposition 2 suggests that implementing the planner's solution is simple; however, this turns out not to be the case. The competitive and planner search rules both weigh the value of self-employment (on which the two agree) against the expected value of search (on which they disagree). Although this disagreement can be resolved using only a single subsidy which aligns the planner and individual search rules, such a subsidy would also alter budget constraints. Thus the resulting competitive equilibrium does not solve the planner's problem by virtue of being infeasible, and a complex set of state-contingent lump-sum transfers are needed to restore feasibility.

As a consequence, although the planner's problem provides useful insight into the externalities of search, it is less useful in practice. Even setting aside political feasibility, policymakers do not realistically have access to the information needed

to make state-contingent lump-sum transfers perfectly enough to avoid altering individuals' incentives.

For this reason, the optimal policy analysis below instead focuses on a restricted set of feasible instruments — a subsidy to search and a tax on self-employment earnings, subject to a balanced-budget condition. For brevity, I refer to this as the "optimal feasible policy." Under the optimal feasible policy, the search decision rule s_f is given by (under the same assumptions as Proposition 2).

$$
\begin{aligned}
y + b \leq \beta\bar{\theta}p(\bar{\theta}) \int_z \frac{w(z,\bar{\theta}) - \left(\int_{y>s_f} yj(y)dy - J(s)b\right)}{1 - \beta\Delta g(z,\bar{\theta})} \bar{H}(z)dz \; + \; \mu \\
+ \frac{\int \lambda(y)\left((1 - s_f)y\tau_y + s_f b\tau_b\right)j(y)dy}{\tau_y \frac{ds_f}{d\tau_y} + \tau_b \frac{ds_f}{d\tau_b}}
\end{aligned}
\tag{20}
$$

Like the planner's solution, the optimal feasible policy balances self-employment earnings against the expected discounted value of wage employment plus the μ term which contains the externalities. The key difference is that the feasible policy carries an extra term that includes its effect on budget constraints, discounted by the responsiveness of search behavior (highly responsive search behavior means only a small subsidy is necessary, so the effect on budget constraints is small).[11]

4. Model Estimation

Getting a quantitative sense of the importance of the externalities discussed above requires bringing the model to data. This, in turn, requires focusing on a particular labor market (as many model parameters are likely to vary from country to country and even from city to city). To this end, I opt to estimate the model to match the labor market of Addis Ababa, Ethiopia, largely because some experiments useful for model estimation happened to be conducted there.

Model parameters fall into two categories. The first are parameters that are directly estimated from data (such as collateral requirement γ) or set to standard values (such as the discount rate β). The second are parameters that are more difficult to measure directly (such as the search cost b). These parameters are estimated using the simulated method of moments (SMM) to match data moments from the

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[11]Note that the assumption of linear utility (i.e. $\lambda(y) = 1$) and the balanced-budget constraint imply that this additional term is equal to zero. In other words, the competitive equilibrium under the optimal feasible policy has the same decision rule as the planner's problem. This is not true in general though, and fully expressing this term provides intuition that extends to the general model.

aforementioned experiments, as well as some aggregate moments. Before going into the details of the SMM estimation, it is worth briefly discussing a handful of the key non-SMM parameters whose values are important.[12]

4.1. Key Directly Estimated Parameters

Importantly, the rate of return on individuals' savings R is taken to be less than unity with an annual value of 0.9 (chosen to roughly match the Ethiopia inflation rate, suggesting that individuals' savings a are best thought of as cash). Because the model is estimated to a weekly frequency, this corresponds to a value of $0.9^{\frac{1}{52}}$. The assumption that the return to savings is less than one, and thus that saving is costly, is typical in models of developing countries (see e.g. Donovan 2021, Fujimoto, Lagakos & VanVuren 2023). Here, encoding this assumption is important as the difficulty of maintaining a cushion of savings is an oft-cited justification of the need for search subsidies (although the analysis in Section 3 revealed that this does not directly justify intervention).

For similar reasons, the income process of the self-employed is also important. This is measured directly using weekly data on workers and job seekers collected by Abebe et al. (2021) as part of an experiment in Addis Ababa (details below). In the context of Addis, the majority of the variation in earnings (among those without a permanent job) comes from whether an individual is currently working a temporary, gig-style job or not. Consequently, self-employment productivity is modeled as a binary Markov process (with the high state corresponding to "working" and the low state to "not working") whose transition matrix can be estimated directly (the probability of remaining in one's current state is roughly 89 percent per week for both high and low states). The ratio of earnings between the high and low state can also be measured directly and is set to 2.63. Thus the model closely matches observed volatility in self-employment earnings.

Finally, many of the entrepreneurs' parameters can be estimated using data from the World Bank. MIX Market data contains financial information on microcredit providers in Ethiopia and suggests a rough average yield of 25 percent which, combined with an 8 percent depreciation rate, suggests a user cost of capital equal to 33 percent annually.[13] The average collateral requirement in Addis Ababa (computed using the World Bank Enterprise Survey for Ethiopia in 2015) is 350 percent

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[12]Although not all parameters are discussed here, Appendix D and, in particular, Appendix Table D.1 provide an exhaustive list of these parameters, their values, and some further discussion.

[13]Loan loss rates in Ethiopia are negligible.

— a firm that owned 350,000 Birr of capital could finance a 100,000 Birr loan — suggesting a value of 1.29 $(1 + \frac{1}{3.5})$ for the collateral parameter γ. Finally, fitting a geometric distribution to the firm age distribution via maximum likelihood yields an annual entrepreneur death probability $(1 - \Delta)$ of 0.08.

4.2. Parameters Estimated using the Simulated Method of Moments

Table 1: Parameter Estimates from Simulated Method of Moments

Parameter	Estimate	Corresponding Moment
σ	5.2	% wage work
λ	0.01	Unemployment rate
A_s	0.34	Wage sector premium
b	0.05	% of expenditure on search
M$\underline{f}$	.001	Control wage employment after 16 weeks
c	0.37	Cost to hire as % of wage
χ	0.62	Elas. of avg. wage to output per worker
$\bar{z}$	0.35	Avg. growth rate

Note: This table displays the parameters estimated using simulated method of moments, their estimates, and the moment that corresponds most closely to each parameter. See discussion for details and intuition on these correspondences.

There are eight parameters estimated using the simulated method of moments to match eight data moments. Table 1 lists these eight parameters and their estimated values while Table 2 lists the eight targeted moments and their values in both the data and the model. The parameters fall into two rough categories — those corresponding closely to worker-level moments (above the dividing line in Tables 1 and 2) and those corresponding closely to firm-level moments (below the line).

Worker moments: The data for the worker-level moments come from two sources. The proportion of individuals engaged in wage work and the aggregate unemployment rate are measured using the 2018-2019 wave of the Ethiopia Living Standard and Measurement Survey (LSMS), limited to individuals in Addis Ababa. [14] The

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[14]While the other data sources used in estimation are from 2014-2015, the 2018 wave of the Ethiopia LSMS was the first wave capable of providing representative estimates for Addis Ababa

Table 2: Moments Targeted using the Simulated Method of Moments

Moment	Source	Data	Model
% wage work	LSMS	30%	29%
Unemployment rate	LSMS	10%	12%
Wage sector premium	LSMS	39%	39%
% of expenditure on search	Abebe et al. (2021)	15%	16%
Control wage emp. after 16 weeks	Abebe et al. (2021)	12%	13%
Cost to hire as % of wage	Abebe et al. (2017)	120%	120%
Elas. of avg. wage to output per worker	World Bank ES	25%	25%
Avg. growth rate	World Bank ES	4.4%	4.4%

Note: This table displays the moments targeted in the simulated method of moments estimation, their source, and their values in both the data and model. See the discussion for details.

wage sector premium is estimated on the same data by including a dummy variable indicating whether an individual is employed in a permanent wage job (vs self-employment or temporary) work in an otherwise standard Mincer regression of (log) earnings on age, as well as some controls (rural/urban, region, and section fixed effects).[15]

The two remaining household moments come from the aforementioned weekly data on job seekers from Abebe et al. (2021). The first is average expenditure on job search (for weeks in which an individual searches) as a percentage of total expenditure. This is calculated directly via survey responses (i.e. individuals are asked directly how much they spent on search and in total). The second, labeled "control wage emp. after 16 weeks", reflects the proportion of individuals in the experimental control group with wage employment 16 weeks after baseline. Although I discuss the experiment in more detail in the next sub-section, it is important to

(previous waves were not representative at a sub-national level). For this reason, I opt to use the data from 2018 rather than the 2015 wave, which would otherwise line up better with the other datasets temporally.

[15]The Ethiopian Productive Safety Net Programme (PSNP), a relatively new "workfare" program administered by the Government of Ethiopia, presents a potential complication. The program provides temporary employment and was present in some regions of Addis Ababa during the 2018 LSMS survey. It is unclear whether earnings from the PSNP should be included in estimation. Fortunately, dropping these earnings from the analysis changes the estimate by less than one percentage point, rendering the issue quantitatively moot. I default to including all earnings from temporary employment, including those from the PSNP.

note here that only data from the experimental control group is used in model estimation while data from the treatment group is reserved for model validation.

Together, these five moments pin down the five parameters above the dividing line in Table 1. The risk aversion parameter σ and the job separation rate λ are disciplined (mostly) by the size of the wage sector and the unemployment rate. While the link between the separation rate and the unemployment rate is clear, the link between risk aversion and the size of the wage sector arises from the fact that, for the worker, search is the higher-risk, higher-return option (relative to self-employment). Thus individuals' risk tolerance ends up being a primary determinant of the size of the wage sector.

The earnings premium in the wage sector naturally pins down the productivity of the self-employment technology, as a more productive technology shrinks the earnings gap between the two sectors. Similarly, the percent of total expenditure that goes towards search costs almost mechanically pins down the goods cost of search. The final moment, the employment rate of control group job seekers after 16 weeks, conceptually pins down the (weekly) job-finding rate. The parameter most directly linked to this equilibrium object is the initial size of a newborn entrepreneur (given by $M\underline{f}$, which are not separately identified) — if newborn entrepreneurs are larger, they will end up posting more vacancies, directly impacting the job-finding rate.

Firm moments: The remaining three parameters — the vacancy posting cost c, the wage bargaining parameter χ, and the upper bound of firm productivity $\bar{z}$ — are estimated to match firm-level moments. Abebe et al. (2017) survey firms in Addis about hiring practices and find that the average cost to a firm of making one additional hire is equal to 120 percent of the average wage. This moment directly pins down the vacancy posting cost.

The bargaining parameter χ, via the wage bargaining equation 11, is pinned down by the relationship between firm-level average wages and output per worker. I estimate this elasticity to be 25 percent (meaning, a firm with 100 percent higher output per worker pays its workers on average 25 percent more) in World Bank Enterprise Survey data and use this as the target in estimation. The final model object to be pinned down is the distribution of entrepreneur productivity. I choose an upper-truncate Pareto distribution with tail parameter 2.1 (as close to Zipf's law as possible while maintaining well-defined variance). The truncation point $\bar{z}$ is disciplined by the average (self-reported) annual growth rate for firms in the En-

terprise Survey as a higher $\bar{z}$ directly corresponds to a higher average growth rate due to the fact that more productive firms grow faster (at least in the model).

4.3. Model Validation and the Experiment of Abebe et al. (2021)

To validate the model, I replicate an experiment performed by Abebe et al. (2021) in the model and compare the model outcomes to the experimentally estimated outcomes. As mentioned above, it is important to note that while control outcomes from the experiment are used to estimate the model, treatment outcomes and data are not. Thus comparing the model's predicted treatment effects to those estimated in the experiment represents an "out-of-sample" test of the model.

This experiment took place in 2014-2015 and evaluated the effects of providing a cash subsidy covering some of the costs of job search to prospective searchers in Addis Ababa, Ethiopia. In the context of Addis Ababa, the majority of job search takes place in person in the city center. Particularly notable are job vacancy boards (located in the city center) which contain job postings and are consulted by the majority of searchers. Thus the cost of travel (typically by minibus) to the city center represents a large and salient cost of job search.

The experiment sampled young individuals who "(i) were between 18 and 29 years of age; (ii) had completed high school; (iii) were available to start working in the next three months; and (iv) were not currently working in a permanent job or enrolled in full time education." (Abebe et al. 2021) and randomly offered some individuals cash that could be collected in person at the city center up to three times each week. While not literally a job search subsidy as individuals could theoretically travel to the city center, collect the cash, and leave without searching, doing so would be ineffective as the cost of the subsidy is not large enough to cover the full round-trip journey.[16] Thus collecting the cash only makes sense if the individual intended to travel to the city center for other purposes (presumably job search). The cash was available for 16 weeks after which treated individuals were 3.4 percentage points (p<0.1) more likely to be employed in a permanent job.

To replicate the experiment in the model, I select a representative but small (measure 0) subset of individuals not employed in the wage sector from the steady-state distribution of individuals. In this sense, the outcomes of sampled individuals do not affect equilibrium outcomes, and the experiment happens in "partial

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[16]In fact, the authors make sure of this by varying the subsidy offered to each individual based on the location, and thus minibus ticket cost, of the individual's home. However, I abstract from this heterogeneity and model the subsidy as uniform at the median value of subsidy offered.

equilibrium" (reflecting the fact that providing treatment to a few hundred individuals in a city of millions is unlikely to have general equilibrium impacts). The sample is divided equally into treatment and control groups, and the cost of search parameter b is reduced by two-thirds (the median subsidy offered in the experiment) for the treatment group for 16 periods.

Experimental outcomes can then be observed by simulating the behavior of the treatment and control groups forward over time, and comparisons of means between the two groups correspond to Average Treatment Effects estimated by the experiment. For treatment households, I treat the experiment as an unanticipated MIT shock; households do not know ahead of time that they have been selected for treatment and cannot alter their behavior in response to such information (and are also fully aware that it will end after 16 periods). Thus differences between treatment and control groups before the treatment occurs are zero by construction.

Model vs Data: Figure 2 compares the model's predictions for the increase in search behavior as a result of the subsidy to those observed in the data. The solid orange line depicts model predictions and the dotted red line depicts the experimentally estimated effects along with the associated 95 percent confidence interval. The model lines up with the experiment remarkably well. During the treatment period (between 0 and 16 weeks since treatment), treated individuals were roughly 5 percentage points more likely to search, a fact which is replicated in the model. There is a small decline in the point estimates in the last few weeks of treatment that is not quantitatively replicated by the model, but this decline is statistically insignificant, and the model continues to fall within the estimated 95 percent confidence interval.

The model also qualitatively replicates the fact that effects seem to persist for some weeks after treatment is ended, although the experimental point estimates here are noisy. The model's predictions are quantitatively smaller than these point estimates, but are well within the 95 confidence interval. One explanation for the model's underprediction of persistence is that increasing search behavior results in some sort of learning, leading treated individuals to search more often even after the end of treatment, that is not captured in the model.

Even if the model accurately matches the increase in search behavior due to treatment, it may not match the increase in wage employment if, for example, search within a short time period exhibits substantial diminishing returns (i.e. job seekers first go after opportunities they judge most promising). Reflecting the im-

Figure 2: Treatment Effect on Search Behavior over Time: Data and Model
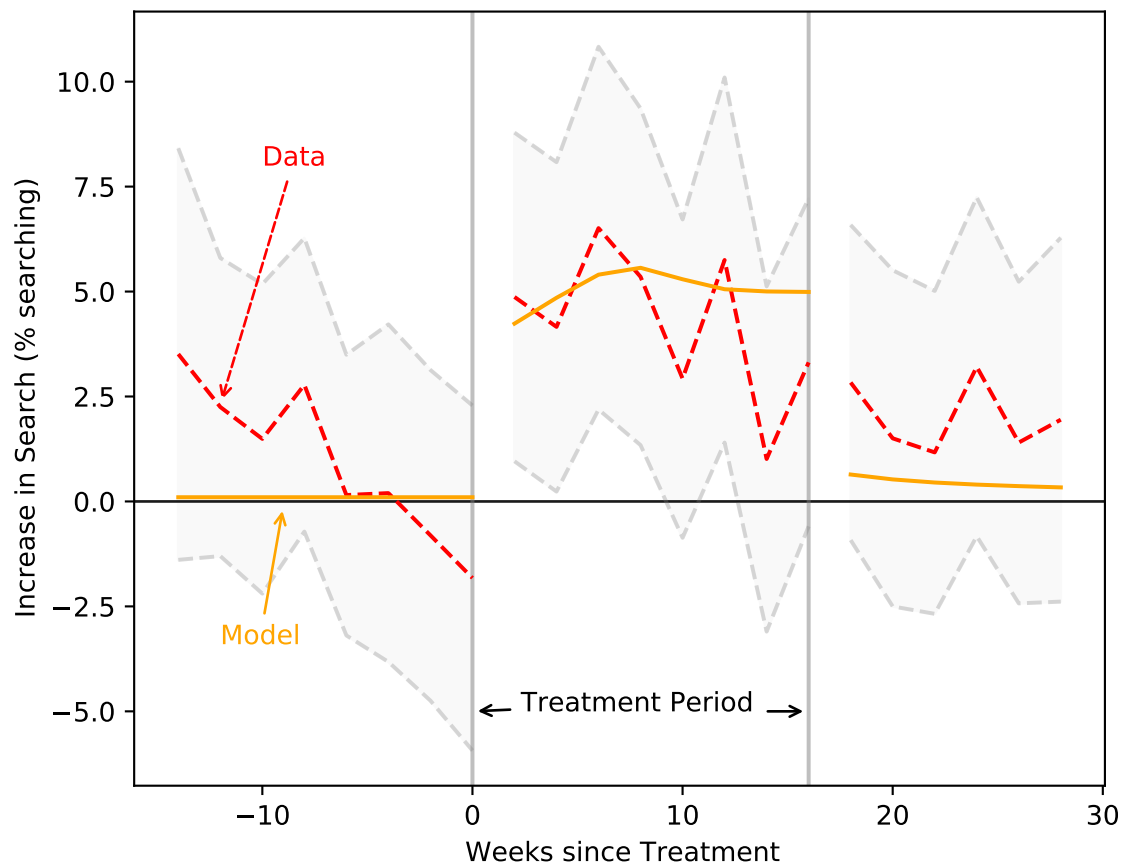
This figure displays the treatment effect on search behavior as a function of "weeks since treatment" in both the data and estimated model.

plicit assumption of constant returns, the model predicts a roughly 5 percentage point higher probability of being employed after 16 weeks, the same as the increase in search behavior. The experimental equivalent is 3.4 percentage points (90 percent confidence interval [XX] to [YY]). This is slightly lower, but the model is still reasonably accurate and well within sampling variation — the 90 percent confidence interval is not sufficient to rule out the assumption of constant returns.

5. Efficient Policy in the Estimated Model

With the estimated model in hand, we can now quantify the optimal feasible policy and investigate the relative sizes and contributions of the four externalities. The are many ways to approach this, but the simplest is to directly solve for the optimal policy tax/subsidy rates on search and self-employment and then decompose the total impact of the policy across the four externalities.[17] Here it is important to note that while equations (17) – (20) are written under the simplification of linear utility, the results in this section are computed using the estimated model which exhibits substantial curvature ($\sigma = 5.2$).

5.1. How to Decompose the Impact of the Externalities

The task of decomposing the impact of the policy across the externalities is not entirely straightforward. The Congestion, Capital Shallowing, and Allocative Efficiency externalities all operate through the adjustment of particular equilibrium values ($\theta p(\theta)$, $w(z)$, and $H(z)$; see equation 19). The impact of each channel, then, can be decomposed by examining how the policy's impact on welfare changes when the change in the appropriate equilibrium value — and thus the impact of the externality — is marginally reduced (using a marginal calculation, rather than an average, reflects the fact that the optimal policy balances the marginal impacts of each channel). Less clear, however, is the firm size externality which (at first glance) does not appear to depend on changes in any equilibrium values, instead depending only on the level of firms' growth rates $g(z)$.

The necessary insight stems from noticing that the firm size externality arises from an implicit change in the number of vacancies. To see this, consider the laws of motion for total employment at z-type firms in period $t+i$ (m_{t+i}) as a function of

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[17]This is much more straightforward than approaches centered around recomputing optimal policies with and without the presence of certain channels due to the fact that steady-state equilibria often become degenerate (i.e. no self-employment or no wage work) or fail to exist when certain channels are shut down.

search in period in t (S_t). Writing the model with $\sigma \to 0$ in order to stay consistent with Proposition 2 and simplify expressions yields

$$m_{t+1}(z; S_t) = (1 - \tilde{\lambda})m_t(z) + H(z)\theta_t p(\theta_t)S_t$$

$$m_{t+i+1}(z; S_t) = \underbrace{(1 - \tilde{\lambda})m_{t+i}(z; S_t)}_{\text{Continuing Hires}} + \underbrace{p(\theta_{t+i})v_{t+i}(S_t)}_{\text{New Hires}} \tag{21}$$

$$v_{t+i}(S_t) = \frac{1}{p(\theta_{t+i})}\left(\Delta g(z) - (1 - \tilde{\lambda})\right)m_{t+i}(z; S_t)$$

where I have suppressed the dependence of m_{t+i+1} on the entire sequence $\{\theta_{t+n}\}_{n=t}^{i}$ to focus on the impact of S_t.

From equation (21), it is clear that total employment in $t + i + 1$ depends on search today both through the probability that the searcher today will still be employed in $t + i$ and the (average) effect that the searcher will have on future hiring; this is not surprising as this is precisely the firm size externality. The key is to note how the impact of a change in S_t differs when the "New Hires" term is and is not allowed to adjust. When allowed to adjust, the difference between the "Continuing Hires" and "New Hires" terms can be ignored, yielding the simple formula depending on $\left(\Delta g(z)\right)^i$ in (22) below. When the "New Hires" term is not allow to change with changes in S_t, by replacing the impact of changes in employment in period $t + i$ on vacancies $\frac{dv_{t+i}}{m_{t+1}}$ with 0, we instead get the effect depending on $1 - \tilde{\lambda}$ written in (23) below. This substitution of $\Delta g(z)$ for $1 - \tilde{\lambda}$ is familiar — it is exactly the difference between the planner's and individuals' valuation of the benefits of search in Proposition 2.

$$\text{Both channels:} \qquad \frac{dm_{t+i+1}}{dS_t} = \left(\Delta g(z)\right)^i H(z)\frac{d}{dS_t}\left(\theta_t p(\theta_t)S_t\right) \tag{22}$$

$$\text{Continuing hires only:} \qquad \frac{dm_{t+i+1}}{dS_t} = \left(1 - \tilde{\lambda}\right)^i H(z)\frac{d}{dS_t}\left(\theta_t p(\theta_t)S_t\right) \tag{23}$$

This, at last, makes it clear how to isolate the effect of the firm size channel. Similar to the approach for other externalities, we can shut down the externality by preventing the corresponding adjustment from occurring. In this case, this amounts to shrinking the change in vacancies (and the resulting job-finding probability) in period $i > 1$ by a factor of $\frac{\sum_{t=1}^{i}\left(1 - \tilde{\lambda}\right)^t}{\sum_{t=1}\left(\int \Delta g(z)H(z)dz\right)^t}$, effectively imposing an

evolution according to (23) rather than (22).[18]

5.2. The Efficient Policy

With these computational details set aside, Table 3 reports the tax rates of the optimal feasible policy (which, recall, consists of taxes on search and self-employment earnings subject to a balanced-budget constraint) as well as the impact of the policy on welfare and the size of the wage sector. Although there is some minor variation along the transition path after the policy is implemented, the displayed rates correspond to the eventual rates in the post-policy steady-state.

Table 3: Results of the Efficient Policy

	Optimal Rates		
Search Tax:	101%	Self-emp. Tax:	-2.0%
Welfare:	+1.5%	Size of Wage Sector:	-6.0pp
(Self-employed):	+1.6%	(Pre-policy):	30%
(Employed):	+1.4%	(Post-policy):	24%

Note: This tables displays the search tax and self-employment subsidy rates that make up the optimal feasible policy in the estimated model, as well as the impact on welfare and the size of the wage sector that occurs when these rates are implemented. See text for details.

The most surprising result is that the optimal tax rate on on search is positive and large at 101 percent of baseline search costs. In other words, the competitive equilibrium exhibits too much search, and the planner finds it necessary to discourage this through a tax that shrinks the size of the wage sector from 30 percent (of workers) to 24 percent. This contradicts the intuition of many policymakers and economists that barriers to search represent a substantial problem for (potential) workers in developing countries. At least from the perspective of the model, the crowd-out externalities dominate the crowd-in externalities, and more barriers need to be erected.

The positive tax rate on search is mirrored by the negative tax rate on self-

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[18]Shrinking the change in vacancies, rather than the level, is done to remain consistent with the approach applied to the other externalities which similarly measures the impact of the changes in externalities rather than levels.

employment earnings (i.e. a subsidy) to comply with the balanced-budget constraint. This subsidy is moderate in size at 2 percent of earnings. The difference in magnitude between the tax on search and the subsidy to self-employment partially reflects the difference in size between the cost of search and average earnings (without taxes, the search cost is roughly 20 percent of average self-employment earnings) and partially reflects the difference in popularity of the two activities (there are approximately 10 times as many self-employed individuals as there are searching individuals.)

The overall impact on welfare of the policy is substantial. Average welfare increases by 1.5 percent of consumption. This impact is not particularly regressive or progressive as the average impact among the self-employed and the average impact among the employed are similar (1.6 percent and 1.4 percent respectively). Although the employed pay more of the search cost, as they are more likely to search in the near future, they also reap more of the benefits from shrinking the crowd-out externalities. On the other hand, the self-employed largely benefit through higher earnings from the subsidy.

Figure 3 displays the contribution of each of the four externalities to the overall effect, as well as the gains from the direct impact of the policy on budgets (refer to equation 20). Because the optimal policy is a tax that reduces search, the *positive* search externalities (Firm Size and Efficiency) take on negative values as a reduction in search shrinks the size of these externalities which contributes negatively to welfare. Similarly, the negative externalities of search (Congestion and Capital Shallowing) take on positive values.

Both the Firm Size and Congestion externalities contribute substantially to the welfare impact of the policy, accounting for -0.8 percent and +1.6 percent of the change respectively. The quantitative dominance of the Congestion channel sheds light into the reason why the optimal policy is a tax on search; the corresponding reduction in search behavior and increase in the job-finding probability is highly valued by individuals. Although this is somewhat offset by the firm size channel, which leads to increases in employment as a result of search (in the long run), this effect is not quantitatively large enough to overcome Congestion.

The Capital Shallowing and Efficiency externalities both contribute moderately to welfare at +0.2 percent and -0.2 percent respectively — smaller than the previous channels but still relevant. However, their impacts serve to offset each other almost exactly, and the change in welfare arising from the net change in the average wage

Figure 3: Sources of Welfare Gains from Optimal Policy
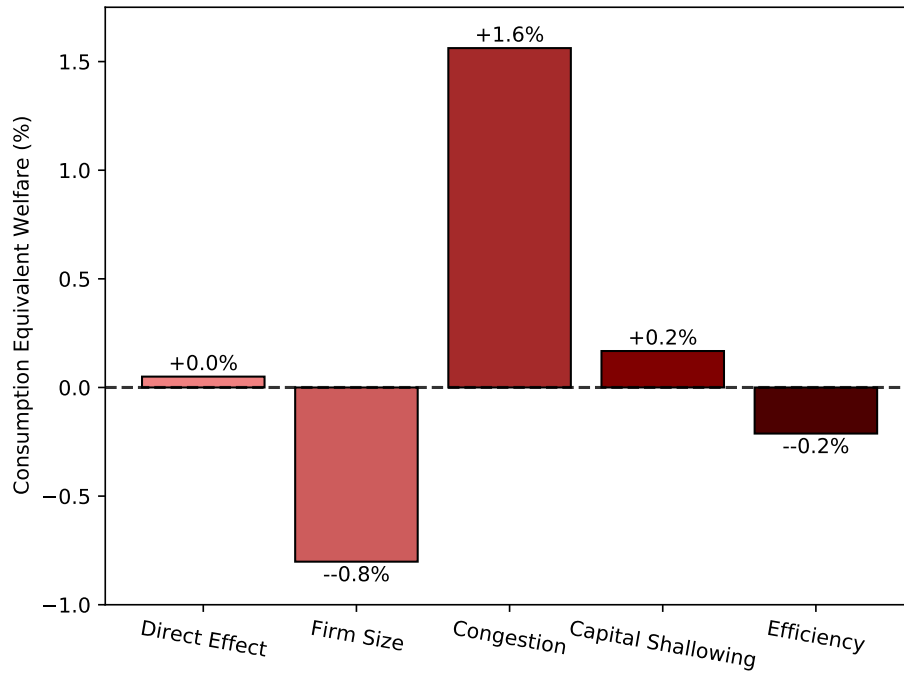
Note: This figure displays the (marginal) contribution of each of the four externalities to the overall welfare impact of the optimal feasible policy. Refer to subsection 5.1 for details on how this decomposition is performed.

is almost zero. Indeed, the change in the average wage, the combined result of the decline in wages paid by z-type entrepreneurs due to Capital Shallowing and an increased probability of matching with high-z entrepreneurs due to Efficiency, is overall an increase of less than a tenth of a percent.

The welfare impact of Direct Effect of the policy on budget is positive but negligibly small in magnitude. In theory, this effect could be either positive or negative. For an individual near the margin of search, the tax serves to move income from a low-income state (search) to a high-income state (self-employment), suggesting the effect should be negative. However, at an aggregate level, the tax serves to move income from high-income earners (the employed, who search a lot after losing their job) to low-income earners (the self-employed). Which effect dominates is a quantitative question, and it appears to be the case that they roughly cancel out.

Finally, it is worth noting that the sum of all the channels (+0.8 percent) somewhat undershoots the overall welfare effect of the policy (+1.5 percent). This is a

result of complementarities between the channels. Although all the channels interact, the largest interaction occurs between the Congestion channel and the direct effect — the cost of lost consumption in the low-income search state is substantially smaller when the probability of finding a job is higher and thus fewer periods of search are required.

6. Policy Analysis of Search Subsidies

The results of the previous section indicate that the level of search is too high and that the optimal policy is a tax on search, rather than a subsidy. Although this policy improves average welfare, it also shrinks the wage sector, which may be contradictory to the goals of many policymakers who would prefer to grow the size of the wage sector even at the expense of efficiency. Although such a goal is dubious from the perspective of the model, it may stem from practical concerns extending beyond the model's scope, such as the need to attract Foreign Direct Investment or to cement a nascent industrial base.

To accommodate this, this final section briefly pivots from normative to positive analysis and uses the estimated model to understand the impact of implementing a subsidy to job search. Even for a policymaker who values only the size of the wage sector, the crowd-in and crowd-out externalities remain important as they account for the difference between the impact of the subsidy when evaluated in partial equilibrium (i.e. on a small experimental sample) and the impact of the subsidy in general equilibrium (i.e. implemented for the entire labor market). This section, then, follows the spirit of the literature on interpreting experimental results through macroeconomic models (e.g. Brooks, Donovan & Johnson 2020, Fujimoto, Lagakos & VanVuren 2023, Lagakos, Mobarak & Waugh 2023).

The policy analyzed is a cash transfer to searchers, effectively reducing the cost of search b, as in the experiment used to estimate the model. The size of the transfers is also chosen to be the same as that used in estimation and is equal to about two-thirds of the cost of search. In the baseline evaluation, the subsidy is funded through a proportional tax on the self-employed (keeping in line with the previous section).

Table 4 shows the impact of the subsidy on the size of the wage sector, as well as welfare, in both partial and general equilibrium. The general equilibrium results correspond to the case where all equilibrium values (the job-finding probability $\theta p(\theta)$, the wage-productivity relationship $w(z)$, and the z-type match probability

$H(z)$) are allowed to adjust to their new equilibrium level, and these results represent what occurs when the policy is implemented economy wide and available to all. In contrast, the partial equilibrium results correspond to a model where these equilibrium values are fixed at their pre-policy levels. These outcomes, then, correspond to what would be observed in an experimental evaluation of the policy (as the experimental sample is small and does not influence equilbrium outcomes). Importantly, because the four externalities occur through these equilibrium adjustments, these are shut off in partial equilibrium as well.

Table 4: Result of Search Subsidies

Model	Wage Sector	Welfare
Baseline	30%	–
w/ Subsidy (Partial Eq.)	47%	-0.0%
w/ Subsidy (General Eq.)	40%	-2.5%
Only Subsidy, No Tax (PE)	43%	+1.3%
Only Congestion Channel	27%	-4.6%
Tax on Employed (GE)	32%	+0.0%

Note: This table displays the impact of implementing a subsidy for job search on the size of the wage sector and average welfare in the estimated model. Refer to the text for details on the models represented by each row.

In partial equilibrium, the results of the policy seem very promising. When the externalities are shut off, the size of the wage sector increases by 17 percentage points to 47 percent with essentially no impact on welfare (the negligible decrease mirrors the negligible increase from the optimal tax in Figure 3). These results assume that individuals are treated with both the subsidy and the tax used to fund the policy (for comparability to the previous section). If, consistent with experimental procedures, the subsidy is funded via grants from funding organizations rather than a tax of the self-employment earnings of recipients (displayed in row "Only Subsidy, No Tax"), the outcome looks even better with a welfare increase of 1.3 percent.

In general equilibrium, once the externalities are introduced, the results are sub-

stantially more pessimistic. The policy still increases the size of the wage sector by 10 percentage points; however, this comes at a substantially larger cost to welfare which decreases by 2.5 percent. Relative to the efficiency policy (which increased welfare by 1.5 percent), the subsidy results in welfare that is 4.0 percent lower. The reason is that, as in Section 5, the impact of the Congestion externality is substantial and quantitatively dominates the other three externalities. This is demonstrated in row "Only Congestion Channel" which displays the impact of the subsidy when the other three channels (Firm Size, Capital Shallowing, and Efficiency) are shut down. Relative to the partial equilibrium outcomes, introducing the Congestion channel reduces the size of the wage sector by a massive 20 percentage points and welfare by 4.6 percent.[19]

Finally, although being consistent with the previous section required a baseline policy in which the search subsidy was funded by a tax on self-employed workers, one might argue that it is more natural to fund the subsidy through a tax on wage workers. This policy effectively moves income from a high-income state (employment) to a low-income state (search). As such, there are potential welfare gains from such a policy. However, as shown in row "Tax on Employed", these gains are barely large enough to offset the losses from the decline in efficiency — the change in welfare is positive but less than a tenth of a percent. Further, because the tax on wage work discourages search (especially when compared to the tax on self-employment which encouraged it), this policy barely grows the wage sector at all, increasing its size by only 2 percentage points.

The upshot of this section is that promising experimental (partial equilibrium) results do not necessarily guarantee that a policy will be successful when scaled-up to a general equilibrium level, at least in the case of search subsidies. Such results provide no information (at least directly) about the relative sizes of the various search externalities. In the estimated model, where the Congestion externality dominates, this leads to substantially more pessimistic predictions in general equilibrium. Of course, this need not necessarily be the case — estimating the model to match a different labor market could lead the crowd-in channels to dominate, in which case the subsidy would appear *better* in general equilibrium.

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[19]Although not included for brevity's sake, decomposing the impact of the three remaining channels reveals that, as in Figure 3, the Capital Shallowing and Efficiency channels are small relative to Congestion and more or less offset each other. As a result, the impact of the Firm Size channel can be seen by comparing the results with only the Congestion channel to the full general equilibrium results.

7. Conclusion

Many policies and interventions aim to expand the wage sector by increasing the extent to which (potential) workers can search for jobs; however, frictional labor markets are known to generate search externalities. This paper develops and estimates a model that incorporates key features of developing countries in order to understand the inefficiencies that arise in this setting. Contrary to the intuition that search should be encouraged, the estimated model suggests that the optimal policy is a substantial tax increasing the cost of search.

One broad takeaway of the model and ensuing quantitative analysis, relevant to policymakers and economists alike, is that policies aimed at assisting job seekers should be very careful to distinguish between the extent to which policies encourage search (i.e. increase in individual's incentive or ability to search) versus the extent to which they improve the effectiveness of search (i.e. improve the productivity of the matching function), as improvements in search efficiency are not subject to the concern of crowding-out. Because many policies represent a combination of these two effects (e.g. government subsidies for employment agencies, discussed in Wu & Wang 2023, may encourage search by lowering the price of this service but may also improve efficiency if agencies are able to effectively streamline the matching process), experimental evaluations of these policies can productively try to distinguish between their impact on each.

The quantitative conclusions of Sections 5 and 6 should be caveated by noting that the model is estimated to the specific setting of Addis Ababa. Although quantitative exploration reveals that it is fairly difficult (though not impossible) to overturn the conclusion that the optimal policy is a tax on search, the exact level of the optimal tax can vary substantially when the targeted moments are changed. Applying the model in different settings would require new data on these moments, which may be difficult to find depending on the setting.

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Appendix

A. Additional Tables and Figure

Table A.1: Effect of Search Subsidy on Labor Market Outcomes (Abebe et al. 2021)

Outcome	Control Mean	Effect of Subsidy
Any Work	0.526	0.037
		(0.029)
Hours Worked	26.18	0.183
		(1.543)
Monthly Wages	857.9	65.88
		(63.86)
Permanent Job	0.171	0.033*
		(0.018)
Formal Job	0.224	0.054**
		(0.019)
Job Satisfaction	0.237	-0.001
		(0.027)

This table reproduces the primary results of Abebe et al. (2021) and displays the control mean for a variety of labor market outcomes as well as the experimentally estimated treatment effect of a conditional cash transfer to job seekers.

B. Derivations and Proofs from Section 2.6

The first result to show is that the entrepreneur's optimal choice of f' and n' satisfy $\eta(z; X) = \frac{\gamma f'^*}{n'^*}$ for some function η depending only on z and X. Substituting in the wage determination equation (which the entrepreneur takes as given) and the vacancy posting constraint, the first-order condition for f' and n' can be combined with the envelope condition for f and n to generate

$$\beta \Delta \mu' \left((1-\alpha)(1-\chi)z(\frac{\gamma f'}{n'})^\alpha - \left((1-\chi)\underline{w} - \frac{c}{p(\theta(X'))}(1-\lambda) \right) \right) = \frac{c}{p(\theta(X))}\mu$$

$$\beta \Delta \mu' \left(\gamma \alpha (1-\chi)z(\frac{\gamma f'}{n'})^{\alpha-1} + 1 - \gamma(r+\delta) \right) = \mu$$

where μ is the Lagrange multiplier on the budget constraint and $\theta(X')$ is a price function. Combining these two equations, substituting in η, and defining A, $B(X')$,

and $C(X')$ for clarity yields

$$Az\eta^{\alpha} + B(X, X')z\eta^{\alpha-1} + C(X, X') = 0 \qquad (24)$$

which, for $0 < \alpha < 1$, can be shown to have a unique and positive solution for η for any value of z, X, and X'. Call this solution $\tilde{\eta}(z; X, X')$. Finally, substituting $X' = H(X)$ and defining $\eta(z; X) = \tilde{\eta}(z; X, H(X))$ completes the derivation.

The next result to show is that entrepreneurs' growth rates depend only on z and aggregate state variables. This follows almost directly from the previous result. Substituting $n = \frac{\gamma}{\tilde{\eta}(z;X)} f$ in to the budget constraint of the entrepreneur problem reveals that the RHS of the budget constraint is now linear in f and can be written

$$d + E(z, X)f' = D(z, X)f \qquad (25)$$

for appropriately define functions $D(z, X)$ and $E(z, X)$ which depend only on z, X, and parameters. Because entrepreneurs possess CRRA utility, the entrepreneur problem looks similar to a cake-eating problem has the well-known solution of a constant growth rate in f depending on the values of D and E, implying that that $f' = g(z; X)f$ for some function g depending only on z, X, and parameters.

The final result is that proof of Proposition 1. By assumption, θ is now constant. Let $\hat{E}(z, \theta)$ and $\hat{D}(z, \theta)$ correspond to E and D with with $\theta(X)$ simply replaced by θ (this can be done because E and D both depend on X only through $\theta(X)$). Then we have the explicit solution[20]

$$\hat{g}(z, \theta) = \left(\beta\Delta \frac{\hat{D}(z,\theta)}{\hat{E}(z,\theta)} \right)^{\frac{1}{\sigma}}$$

$$= \left(\beta\Delta \frac{((1-\chi)\gamma z\hat{\eta}(z;\theta)^{\alpha-1} - \left((1-\chi)\underline{w} - \frac{c}{p(\theta)}(1-\lambda)\right)\frac{\gamma}{\hat{\eta}(z;\theta)} + \left(1 - \gamma(r+\delta)\right)}{\left(1 + \frac{c}{p(\theta)}\frac{\gamma}{\hat{\eta}(z;\theta)}\right)} \right)^{\frac{1}{\sigma}}$$

The chain rule yields $\frac{d\hat{g}}{d\theta} = \frac{\partial \hat{g}}{\partial c/p(\theta)} \frac{dc/p(\theta)}{d\theta} + \frac{\partial \hat{g}}{\partial \hat{\eta}} \frac{d\hat{\eta}}{dc/p(\theta)} \frac{dc/p(\theta)}{d\theta}$. Using either direct calculation of partial derivatives or implicit differentiation (in the case of $\frac{d\hat{\eta}}{dc/p(\theta)}$), we

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[20]While this solution to a "generalized cake eating problem" is straightforward, I have been unable to locate this exact formulation of the problem anywhere. As such, a derivation is available upon request.

can express each individual piece as

$$\frac{\partial \hat{g}}{\partial c/p(\theta)} = -\frac{1}{\sigma}\hat{g}^{1-\sigma}\left(\frac{\left(\frac{\hat{g}}{\beta\Delta}-1\right)+\lambda}{\frac{\eta}{\gamma}+\frac{c}{p(\theta)}}\right) \le 0$$

$$\frac{\partial \hat{g}}{\partial \hat{\eta}} = \frac{1}{\sigma}\hat{g}^{1-\sigma}\left(\frac{\frac{\beta\Delta}{\hat{g}}-\frac{\hat{g}}{\beta\Delta}}{\frac{\eta}{\gamma}+\frac{c}{p(\theta)}}\right) \le 0$$

$$\frac{d\hat{\eta}}{dc/p(\theta)} = \frac{\gamma\left(\alpha(1-\chi)z\hat{\eta}^{\alpha-1}-(r+\delta)\right)+\lambda}{J(\theta)} > 0$$

where $J(\theta)$ is a placeholder for a complex but unambiguously positive expression (note that the second expression simplifies some terms using the first order condition for f').

It is worth commenting briefly on why the claimed inequalities hold. Both the first and second inequalities follow directly from the fact that an optimizing entrepreneur will ensure that $g \ge \beta\Delta$ (an entrepreneur can always choose to select $k = 0, n = 0$ and simply eat their cake, yielding $g = \beta\Delta$, so this acts as a lower bound on all growth rates). The final expression follows from the fact that the presence of a collateral constraint ensures that the marginal product of capital $(\alpha(1-\chi)z\hat{\eta}^{\alpha-1})$ is always larger than the marginal cost of capital $(r+\delta)$.

Returning to the main results and noting that $\frac{dc/p(\theta)}{d\theta} > 0$ by assumption, combining these inequalities with the chain rule shows that $\frac{d\hat{g}}{d\theta} < 0$ and $\frac{d\hat{\eta}}{d\theta} > 0$. The result for $\frac{\partial \hat{g}}{\partial\theta\partial z}$ is straightforward. We have $\frac{\partial \hat{g}}{\partial z} = \frac{1}{\sigma}\hat{g}^{1-\sigma}\left(\frac{(1-\chi)\hat{\eta}^{\alpha}}{\frac{\hat{\eta}}{\gamma}+\frac{c}{p(\theta)}}\right)$ which is also clearly greater than zero and decreasing in θ. Although this result holds only for partial derivatives (i.e. with $\hat{\eta}$ being held constant), it can also be shown to hold for total derivatives in the case where $\hat{\eta} \ge \alpha(1+\frac{c}{p(\theta)}\gamma)$ by applying the chain rule as above and computing $\frac{d\hat{\eta}}{dz}$ using implicit differentiation.

C. Derivations and Proofs from Section 3

First, I formally define the functions v and H introduced in equation 16.

$$v(m_t, \eta_t, \eta_{t+1}, g_t) = \frac{1}{p(\theta)} \int \left[g_t(z) \Delta \frac{\eta_t(z)}{\eta_{t+1}(z)} - (1 - \tilde{\lambda}) \right] \int m_t(a, z) da + \frac{\hat{D}(z, \theta_t, \eta_t(z)) \gamma \underline{f}}{\eta_{t+1}(z)} h(z) \, dz \tag{26}$$

$$H(z, m_t, \eta_t, \eta_{t+1}, g_t) = \frac{\left[g_t(z) \Delta \frac{\eta_t(z)}{\eta_{t+1}(z)} - (1 - \tilde{\lambda}) \right] \int m_t(a, z) da + \frac{\hat{D}(z, \theta_t, \eta_t(z)) \gamma \underline{f}}{\eta_{t+1}(z)} h(z)}{p(\theta) v(m_t, \eta_t, \eta_{t+1}, g_t)} \tag{27}$$

The numerator is the number of matches with a productivity z entrepreneur and the denominator is the total number of matches.

The problem of the constrained social planner is given sequentially by

$$\max_{\{c_t, a'_t, s_t, \theta_t, m_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \int \int \int u(c_t) m_t(a, z) j(y) \, dy \, da \, dz$$

$$\text{s.t. } a'_t + c_t = Ra + (1 - s_t) y + s_t (w_t(z) - (1 - z) b) \quad \forall a, y, z$$

$$a_{t+1} \geq 0 \tag{28}$$

$$s_t(a, z) \in \{0, 1\}$$

$$\frac{v(m_t, \eta_t, \eta_{t+1}, g_t)}{\theta_t} = \int \int s_t(a, 0) m_t(a, 0) j(y) \, dy \, da$$

$$m_{t+1}(a'_t, 0) = m_t(a, 0) - \theta_t p(\theta_t) \int s_t(a, 0) m_t(a, 0) j(y) \, dy$$

$$m_{t+1}(a'_t, z) = (1 - \tilde{\lambda}) m_t(a, z) + H(z, m_t, \eta_t, \eta_{t+1}, g_t) \theta_t p(\theta_t) \int s_t(a, 0) m_t(a, 0) j(y) \, dy$$

where the functions η_t and g_t arise from the slightly modified sequential problem of an entrepreneur:

$$\max_{\{d_t, f_{t+1}, k_t, n_t, v_t\}} \sum_{t=0}^{\infty} (\beta \Delta)^t \frac{c_t^{1-\sigma}}{1 - \sigma}$$

$$\text{s.t. } d_t + f_{t+1} = (1 - \chi) z k_t^{\alpha} n_t^{1-\alpha} - (r + \delta) k_t - (1 - \chi) \underline{w} n_t + f_t - c v_t$$

$$n_{t+1} = (1 - \lambda) n_t + p(\theta_t) v_t \tag{29}$$

$$k_t \leq \gamma f_t$$

$$f_0 \in \mathbb{R}$$

so that $\eta_t = \frac{\gamma f_t}{n_t}$ and $g_t = \frac{f_{t+1}}{f_t}$.[21] Note that here I have suppressed the initial condition of the planner's problem and imposed the scale-invariance of the entrepreneurs optimal capital-labor ratio and growth rate by leaving the initial condition f_0 arbitrary.

In analysis of the problem of the social planner, it will be useful to note that while η_t and g_t are potentially functions of z and the entire sequence of labor market tightness $\{\theta\}_{t=0}^{\infty}$, solving the entrepreneur's problem reveals that they depend only on ability z and current and future tightness θ_t, θ_{t+1} and thus can be written as $\eta_t(z, \theta_t, \theta_{t+1})$ and $g(z, \theta_t, \theta_{t+1})$. The independence of entrepreneur policy functions from values of θ beyond period $t+1$ follows directly from the linearity of the hiring cost, combined with the parameter assumptions that ensure that any operating entrepreneur will choose $v_t > 0$ each period. While the continuation value of an entrepreneurs labor force depends in theory on the whole sequence of labor market tightness, the ability to re-optimize at linear cost tomorrow ensures that this continuation value is equal to the "liquidation value" of the workforce next period.

3.1. Notes and Proof for Proposition 2

The dynamic terms in equation 19 are given by

Anticipation Terms =

$$
\frac{S}{\bar{\theta}}\left(\mu_{t-2}\left(\frac{\partial v_{t-2}}{\partial \eta_{t-1}}\frac{\partial \eta_{t-1}}{\partial \theta_t}\right) + \mu_{t-1}\left(\frac{\partial v_{t-1}}{\partial \eta_{t-1}}\frac{\partial \eta_{t-1}}{\partial \theta_t} + \frac{\partial v_{t-1}}{\partial \eta_t}\frac{\partial \eta_t}{\partial \theta_t}\right) + \mu_t\left(\frac{\partial v_t}{\partial \eta_t}\frac{\partial \eta_t}{\partial \theta_t}\right)\right) + \tag{30}
$$
$$
\bar{\theta}p(\bar{\theta})S\left(\int_z \lambda_{t-2}(z)\left(\frac{\partial H_{t-2}}{\partial \eta_{t-1}}\frac{\partial \eta_{t-1}}{\partial \theta_t}\right)dz + \int_z \lambda_{t-1}(z)\left(\frac{\partial H_{t-1}}{\partial \eta_{t-1}}\frac{\partial \eta_{t-1}}{\partial \theta_t} + \frac{\partial H_{t-1}}{\partial \eta_t}\frac{\partial \eta_t}{\partial \theta_t}\right)dz + \right.
$$
$$
\left.\int_z \lambda_t(z)\left(\frac{\partial H_t}{\partial \eta_t}\frac{\partial \eta_t}{\partial \theta_t}\right)dz\right)
$$

where μ_t and $\lambda_t(z)$ are the shadow prices associated with the constraints on aggregate labor market tightness and productivity-specific matching rates respectively. These terms essentially capture the welfare gains from anticipatory hiring when labor market tightness is changed. While the welfare changes from permanent

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[21]Even here in the appendix I opt to write the planner's problem for the case of no autocorrelation in individuals' self-employment productivity (i.e. y is drawn from $j(y)$ each period). Including autocorrelation is conceptually simple and involves adjusting only the final two inequalities governing the evolution of the distribution m_t (and the integral in the objective function); however, doing so leads to prohibitively cumbersome notation and adds no additional insight.

changes in hiring are captures in the other terms of equation 19, this term captures the small gains that occur due to the fact that some of this hiring is done in anticipation of the change, shifting some hiring forward temporally.

Proof: The first step is to rewrite the planner's problem to eliminate the binary choice of s_t which complicates analysis. It's fairly straightforward to show that, for utility functions exhibiting diminishing marginal utility, the optimal choice of s_t takes the form of a cutoff rule in a above which individuals search and below which they do not (this fact arises directly from the fact that c_t^* is monotonically increasing a conditional on s_t and diminishing marginal utility). Thus we can rewrite the planner's problem as selecting an optimal cutoff s_t, which is differentiable. I also rewrite the planner's problem in recursive form to simplify analysis.

$$V(\theta_{-2}, \theta_{-1}, m) = \max_{c, a', s, \theta, m'} \int \int \int u(c) m(a, z) j(y) dy da dz + \beta V(\theta_{-1}, \theta, m')$$

$$s.t. \ a' + c = Ra + (1 - \mathsf{St}(a - s))y + \mathsf{St}(a - s)(w(z) - (1 - z)b) \ \forall a, y, z$$

$$a' \geq 0 \tag{31}$$

$$\frac{v(m, \eta, \eta', g)}{\theta} = \int_s^\infty m(a, 0) da$$

$$m_e'(a', 0) = \int \tilde{\lambda} m(a, z) dz$$

$$m_u'(a', 0) = m(a, 0) - \mathsf{St}(a - s)\theta p(\theta) m(a, 0)$$

$$m_e'(a', z) = (1 - \tilde{\lambda}) m(a, z)$$

$$m_u'(a', z) = H(z, m, \eta, \eta', g)\mathsf{St}(a - s)\theta p(\theta) m(a, 0)$$

$$m'(x, 0) = m_e(x, 0) + m_u(x, 0)$$

$$m'(x, z) = m_e(x, z) + m_u(x, z)$$

where $\mathsf{St}(x)$ is the step function defined via the integral of Dirac's delta δ_x.

Because the state variable describing the distribution of agents across states m is a function $\mathbb{R}^2 \to \mathbb{R}$, the value function V is technically a functional and making progress requires dipping into functional analysis. I keep things relatively simple and try to align notation as closely as possible to what is typical in more standard situations. To this end, define the following shorthand to capture the notion of a

"derivative of y with respect to the value of m at point (a, z)":

$$\frac{dy}{dm(a, z)} \equiv \frac{d}{d\epsilon} y(m + \epsilon \delta_a \delta_z) \Big|_{\epsilon=0}$$

With this defined, we can proceed.

The first order condition with respect to s yields

$$\frac{\lambda(s, 0)}{m(s, 0)}(y + b) = \theta p(\theta) \left(\int \mu(s, z) H(z) dz - \mu(s, 0) \right) + \tau \tag{32}$$

where λ and τ are the Lagrange multipliers on the budget and theta constraints respectively. We can then generate a pair of envelope conditions with respect to $m(s, z)$ and $m(s, 0)$ (note that I have used the first order condition for s to eliminate τ from both).

$$\frac{1}{m(s, z)} \frac{dV}{dm(s, z)} = \int u(c) j(y) dy + \left(g(z) \Delta \frac{\eta}{\eta'} - (1 - \tilde{\lambda}) \right) \left(\int H(z) \mu(s, z) dz - \mu(s, 0) \right)$$

$$- \frac{\lambda(s, 0)}{\theta p(\theta) m(s, 0)}(y + b) \left(g(z) \Delta \frac{\eta}{\eta'} - (1 - \tilde{\lambda}) \right) + \tilde{\lambda} \mu(s, 0) \tag{33}$$

$$+ (1 - \tilde{\lambda}) \mu(s, z) + \left(g(z) \Delta \frac{\eta}{\eta'} - (1 - \tilde{\lambda}) \right) (\tilde{\omega}_1 - \tilde{\omega}_2)$$

$$\frac{1}{m(s, 0)} \frac{dV}{dm(s, 0)} = \int u(c) j(y) dy + \mu(s, 0) + \lambda(s, 0)(y + b) \tag{34}$$

where $\mu(a, z)$ and $\mu(a, 0)$ are the Lagrange multiplier on the constraints governing the evolution of m' and $(\tilde{\omega}_1, \tilde{\omega}_2)$ are defined in the discussion at the endof this section.

We can then use these conditions to generate an expression for $\int H(z) \frac{1}{m(s,z)} \frac{dV}{dm(s,z)} dz - \frac{1}{m(s,0)} \frac{dV}{dm(s,0)}$ which should be interpreted as the planner's increase in value from moving one (normalized) unit of workers into employment while obeying the constraint that fracion $H(z)$ of workers must be matched with an entrepreneur of productivity z.

We also have from the first order conditions on $m'(s, z)$ and $m'(s, 0)$[22]

$$\int H(z)\mu(s, z)dz - \mu(s, 0) = \beta\left(\int H(z)\frac{1}{m(s, z)}\frac{dV}{dm'(s, z)}dz - \frac{1}{m(s, 0)}\frac{dV}{dm'(s, 0)}\right)$$
(35)

Combining this expression with the expression for the RHS referenced above, restricting to steady-state, and solving for the desired quantity yields

$$\int H(z)\mu(s, z)dz - \mu(s, 0) = \frac{\beta\int\int H(z)\big(u(c_z) - u(c_0)\big)j(y)dydz}{1 - \beta\int H(z)g(z)\Delta dz} + \text{Drift Terms}$$
(36)

where c_z and c_0 are notation-saving shorthand for $c(a, z, y)$ and $c(a, 0, y)$ respectively, and the drift terms are discussed further below. This term can be plugged directly in to the first order condition with respect to s.

With the hard part done, all that remains is to use the first order condition for θ to find the following expression for τ:

$$\tau = \frac{\theta/\int_s^\infty m(a, 0)da}{\frac{d\log v}{d\log\theta} - 1}\left(\underbrace{(1 + \frac{d\log p}{d\log\theta})\int_s^\infty\big(\int H(z)\mu(a, z)dz - \mu(a, 0)\big)m(a, 0)p(\theta)da}_{\text{Congestion}}\right.$$

$$+ \underbrace{\int\int\int\lambda(a, z, y)\frac{dw}{d\theta}j(y)dydzda}_{\text{Capital Shallowing}} + \underbrace{\theta p(\theta)\int_s^\infty\int\frac{dH(z)}{d\theta}\mu(a, z)dzm(a, 0)da}_{\text{Composition of Jobs}}\left.\right)\quad(37)$$

$$+ \underbrace{\beta\frac{dV}{d\theta_{-1}}}_{\text{Anticipation}}$$

Finally, plugging everything in to the first order condition for s shows that the planner assigns an individual in state $(a, 0)$ to search if and only if

$$u'(c_0)\big(y + b\big) \leq \beta\theta p(\theta)\frac{\int\int H(z)\big(u(c_z) - u(c_0)\big)j(y)dydz}{1 - \beta\int H(z)g(z)\Delta dz} + \text{Drift Terms} + \tau\quad(38)$$

The exact formulation of the decision rule used in Proposition 2 can be found sim-

---

[22]This phrase should be interpreted as intuitive shorthand for the first order conditioned generated by examining a delta-perturbation of m' at (a, z) i.e. $\frac{d\mathcal{L}}{dm'(a,z)}$ in the shorthand defined above.

ply by letting $\sigma \to 0$ and noting that the drift terms collapse to zero in this limit, concluding the proof.

Discussion of Drift Terms: The drift terms in the planner's decision rule serve as adjustments for the fact that the marginal job-seeker has a different level of asset holdings than the average job-seeker and, similarly, that the marginal newly employed individual has different assets than the average employed individual. Essentially, they adjust for the fact that the asset level of searchers will "drift" away from s over time.

$$\text{Drift Terms} = \frac{\left(1 - \frac{\int_s^\infty m(a,0)da}{m(s,0)}\right)\lambda(s,0)(y+b) + \left(g(z)\Delta\frac{\eta}{\eta'} - (1-\tilde{\lambda})\right)(\tilde{\omega}_1 - \tilde{\omega}_2)}{1 - \beta \int H(z)g(z)\Delta dz}$$

$$\tilde{\omega}_1 = \int \frac{\int m(a,z)\mu(a,z)\text{St}(a-s)m(a,0)da}{m(s,z)\mu(s,z)m(s,0)}dz$$

$$\tilde{\omega}_2 = \int \frac{\int\int \left(g(x)\Delta - (1-\tilde{\lambda})\right)H(x)m(a,x)\mu(a,x)\text{St}(a-s)m(a,0)dadx}{\left(g(z)\Delta - (1-\tilde{\lambda})\right)m(s,z)\mu(s,z)m(s,0)}dz$$

To see this, note that the drift terms collapse to zero when the distribution of asset holdings among both the employed and unemployed are concentrated at s (i.e. $m(a,z) = \delta_s m$ and $m(a,0) = \delta_s(1-m)$). Further analysis of this term is possible but involves substantial technical complication (due to the necessity of tracking the evolution of assets over time) and provides very little additional insight.

No (Additional) Externalities in Savings Decision: Here I sketch the argument/proof of the fact that the presence of search does not induce an externality in individuals' savings decisions. That is, individuals facing a search tax/subsidy aligning their privately optimal search decision rule with that of the planner will choose the same savings policy function as the planner.

The approach follows that of Davila et al. (2012) and leverages a change of variables in the planner's objective function from time-space to individual-space for any finite (N period) optimization sub-problem. Consider the sub-problem of a planner facing a distribution of agents m and who has already settled on the two-period-ahead policy function a'' but must decide today's policy function a'. One could consider the maximization of the sum of today's utility (averaged over m) and tomorrow's utility (averaged over the appropriately defined m'); this is the period approach and is how the planner's problem in (28) is written. One could alternatively consider the maximization of the *two period* utility for all agents alive in the first period (i.e. averaged over m) — the individual approach. These two

objects are different ways of computing the same quantity.

The approach above lets us consider the following optimization problem:

$$\max \int \int \left(\int u(Ra - \text{Inc}(y, a, s, z) - a')j(y)dy \right.$$
$$\left. + \beta \mathbb{E}\left[\int u(Ra' - \text{Inc}(y, a', s', z') - a'')j(y)dy|s, z \right] \right) m(a, z)dadz \qquad (39)$$
$$\text{Inc} = (1 - \text{St}(a - s))y + \text{St}(a - s)(w(z) - (1 - z)b)$$

Note that all the transition dynamics across employment states z are implicit in the expectations operator (a rigorous proof would require fully specifying these details, but they can be ignored in a proof sketch).

Taking the first order condition for a' from this problem reveals that it is identical to that derived from the individual problem. Of course these first order conditions contain the policy function for s, but the assumption that the search subsidy/tax implements the planner's search policy in the decentralized economy ensures that these functions are identical. Thus the savings policies are identical, and the proof sketch is complete.

Individual's Search Decision Rule: As was the case for the planner's problem, it is easy to show that individual's search decision rule is monotonic in their assets and thus the binary search choice in the individual problem can be replaced by the choice of an asset cutoff s above which the individual will search and below which they will not. Restating the relevant portion of the individual problem (4) for convenience (with auto-correlation in y removed and the aggregate state X suppressed):

$$V_u(a) = \max_{c, a', s} u(c) + \beta\left(\left(1 - \text{St}(a - s)\theta p(\theta)\right)E_y[V_u(a')] + \text{St}(a - s)\theta p(\theta)\left(E_z[V_e(a', z)]\right) \right)$$

$$V_e(a, z) = \max_{c, a'} \ u(c) + \beta\left((1 - \tilde{\lambda})V_e(a', z) + \tilde{\lambda}E_y[V_u(a')]] \right)$$

$$s.t. \ a' + c = (1 + r)a + \left(1 - \text{St}(a - s)\right)y - \text{St}(a - s)b \text{ for } V_u$$
$$a' + c = (1 + r)a + w(z) \text{ for } V_e$$

The first-order condition for s then yields

$$u'(c_u)(y+b) = \beta\theta p(\theta)\big(E_z[V_e(a'_u, z)] - E_y[V_u(a'_u)]\big) \quad (40)$$

where c_u and a'_u denote the policy functions of the "unemployed" with their dependence on (a, y) suppressed. Plugging the policy functions into the value functions above and doing some careful rearranging to the RHS yields the policy rule for search.

$$u'(c_u)(y+b) \leq \beta\theta p(\theta)\frac{E_z[u(c_e)] - E_y[u(c_u)]}{1 - \beta(1 - \tilde{\lambda})} + \text{Drift Terms}$$

$$\text{Drift Terms} = \beta(1 - \tilde{\lambda})\Delta_{a'_e, a'_u} - \Delta_{s, a'_u} + \beta\big(E_y[V_u(a'_e)] - E_y[V_u(a'_u)]\big) \quad (41)$$

$$\Delta_{x,y} = \big(E_z[V_e(x)] - E_y[V_u(x)]\big) - \big(E_z[V_e(y)] - E_y[V_u(y)]\big)$$

As in the planner's problem, the inclusion of curvature in the utility function induces some "drift terms" that account for the fact that individuals' savings drift away from the cutoff s over time.

It turns out that the drift terms in the privately optimal decision rule (41) and equivalent to the drift terms in the planner's decision rule (38) in the sense that both terms yield the same value when given the same policy function a'. This is not particularly surprising, as both terms simply exist to adjust for curvature in the utility function. The powerful implication of this fact is that the externalities contained in τ above (as well as the difference in discount rates) make up an exhaustive list of wedges between the privately and publicly optimal decision rules, even with curvature in the utility. Formally showing this equivalence is somewhat cumbersome; the quickest approach involves an awkward change-of-variables in the planner's problem (to make it look more like the individual's problem) and can be provided upon request.

D. Details on Model Estimation

Many model parameters are chosen to match values typical in the macroeconomics, are taken from external sources, or are estimated directly. These are displayed in Table D.1, along with their values and sources. The discount rate β is chosen to match an annual discount rate of 0.95. Because a model period corresponds to two weeks, this is corresponds to a value of $0.95^{\frac{1}{26}}$. The rate of return

on worker's savings R is taken to be exogenously equal to $0.9^{\frac{1}{26}}$. The assumption that the return to savings is less than one is typical models of developing countries (see e.g. Donovan 2021, Fujimoto, Lagakos & VanVuren 2023) and representative of the fact that households in these countries lack access to formal investment with positive returns. The value of 0.9 matches an annual inflation rate of roughly 10 percent, roughly consistent with World Bank estimates of inflation in Ethiopia over the last few years; thus the model asset a most closely reflects cash holdings. The capital share of income is set at 0.33 as is standard.

The interest rate faced by entrepreneurs is disciplined using World Bank MIX Market data containing financial information on microcredit providers in Ethiopia. Yields on loans from microfinance institutions range from 20 percent to 30 percent with negligible loan loss rates (typically less than one percent). Combining this rough average of a 25 percent annual return with 8 percent depreciation yields a depreciation-inclusive user cost of capital of 33 percent annually. This value is high relative to developed countries but is fairly typical for developing countries (see e.g. Banerjee et al. 2015, who document similar values in multiple countries including Ethiopia).

Collateral constraints are measured directly using data from the Ethiopian portion of the World Bank Enterprise Survey for the year 2015. The average collateral requirement reported by firms is slightly larger than 350 percent of loan value, meaning that a firm that owned 350,000 Birr worth of capital could pledge this as collateral and finance a loan for an additional 100,000 Birr of capital. Thus the implied value for γ is $1 + \frac{1}{3.5} = 1.29$. The Enterprise Survey is also used to estimate the entrepreneur survival probability Δ. Because productivity is constant for the life of an entrepreneur, entrepreneur death is the only reason that firms will shutdown in steady state. Consequently, the steady-state distribution of firm ages is geometric with decay parameter Δ whose value can be recovered through the simple maximum likelihood estimation. In this case, the estimate for Δ is given by $1 - \frac{1}{\hat{\mu}}$ where $\hat{\mu}$ is the sample average firm age, yielding an annual value for Δ of 0.92.

The self employment productivity process also comes directly from data. This productivity is modeled as a simple binary Markov process, drawing on the fact that earnings for those without permanent wage jobs are highly bimodal at a fortnightly frequency (seen in the high-frequency data of Abebe et al. 2021, described below). Such bimodality seems to stem from the fact that opportunities for self

employment (or, often in the case of Addis Ababa, temporary "gig-style" labor that functions similarly to self employment), and many individuals report neither working nor searching in a given period, presumably earning very little.

One advantage on using a binary income process instead of a more typical AR(1) is that transitions in and out of this idle state can be observed and measured directly. Using fortnightly data on work and searcher activities (described in the next section), I estimate the transition probabilities from engaged in self employment or temporary work to idleness and back. Although there is no reason for these transitions probabilities to be identical, the estimated value for both is approximately 11 percent. While average self employment earnings (i.e. the productivity parameter A_s) are estimated using SMM, the ratio of earnings in the low productivity state to the high productivity state is chosen to match the standard deviation of self employment earnings observed in the data. In particular, I isolate the transitory, idiosyncratic variance of earnings by regressing (log) earnings on individual and week fixed effects and calculating the standard deviation of the residuals (similar to the process employed in Lagakos & Waugh 2013). Conditional on the transition probabilities, there is a one to one correspondence between the standard deviation of income and the ratio of interest.[23]. The estimated ratio is 0.38 corresponding to an estimated standard deviation of .48.

Finally, the distribution from which newborn entrepreneurs draw their productivity is chosen to be an upper-truncated Pareto distribution (truncated as a bounded support for productivity is required for steady-state equilibrium to exist in the model). I set the lower bound of the distribution to a small but arbitrary number; because entrepreneurs endogeneous shut down below a threshold productivity level and the truncated Pareto distribution is scale-invariant, the lower threshold has no impact on model outcomes as long as it is below the shutdown threshold. The tail parameter is set to unite. It is worth noting that because of upper truncation, the mean and variance of productivity remain finite. The upper bound $\bar{z}$ is included in the SMM estimation, described below.

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[23]For a symmetric transition matrix, as is the case here, this correspondence is given simply by $\frac{y_l}{y_h} = e^- 2\sigma$

Table D.1: Directly Estimated Parameters

Parameter	Value	Description	Source
β	$.95^{\frac{1}{26}}$	Discount rate	Standard value
R	$.9^{\frac{1}{26}}$	Return to savings	10% annual inflation
α	$.33$	Capital share	Standard value
r	$1.33^{\frac{1}{26}} - 1$	Capital cost for entrepreneurs	MIX Market
γ	1.29	Collateral constraint	World Bank ES
Δ	$.92^{\frac{1}{26}}$	Entrepreneur death prob.	World Bank ES
$M(y)$	$\begin{bmatrix} .89 & .11 \\ .11 & .89 \end{bmatrix}$	High and low y trans.	Abebe et al. (2021)
$\frac{y_l}{y_h}$	$.38$	Ratio low to high productivity	

This table displays the model parameters that are estimated directly as well as their values and sources. To help comparisons to typical values, parameters are displayed in annual terms. See the discussion for details on each parameter.