Optimal Redistribution Through Subsidies

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Introduction

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Our approach: we pose and solve the mechanism design problem for the optimal subsidy.

Key Tradeoff

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#2. heterogeneous outside options, consumers can buy from private market.

Heterogeneous outside options are empirically relevant, *e.g.*,

- public housing (van Dijk, 2019; Waldinger, 2021),
- education (Akbarpour, Kapor, Neilson, van Dijk & Zimmerman, 2022; Kapor, Karnani & Neilson, 2024),
- healthcare (Li, 2017; Heim, Lurie, Mullen & Simon, 2021),
- SNAP (Haider, Jacknowitz & Schoeni, 2003; Ko & Moffitt, 2024; Rafkin, Solomon & Soltas, 2024).

Heterogeneous outside options lead to **lower-bound constraints** in the mechanism design problem.

Results Overview

We provide an **explicit characterization** of:

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$\rightsquigarrow$ Linear subsidies (e.g., production subsidies) are **never optimal.**

Related Literature

- **Public Finance.** Ramsey (1927), Diamond (1975), Mirrlees (1976, 1986), Atkinson & Stiglitz (1976), Nichols & Zeckhauser (1982), Blackorby & Donaldson (1988), Besley & Coate (1991), Blomquist & Christiansen (1998), Doligalski, Dworczak, Krysta & Tokarski (2023).
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type $\theta \rightsquigarrow$ demand

Producers

constant marginal cost c

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maximizing weighted total surplus:

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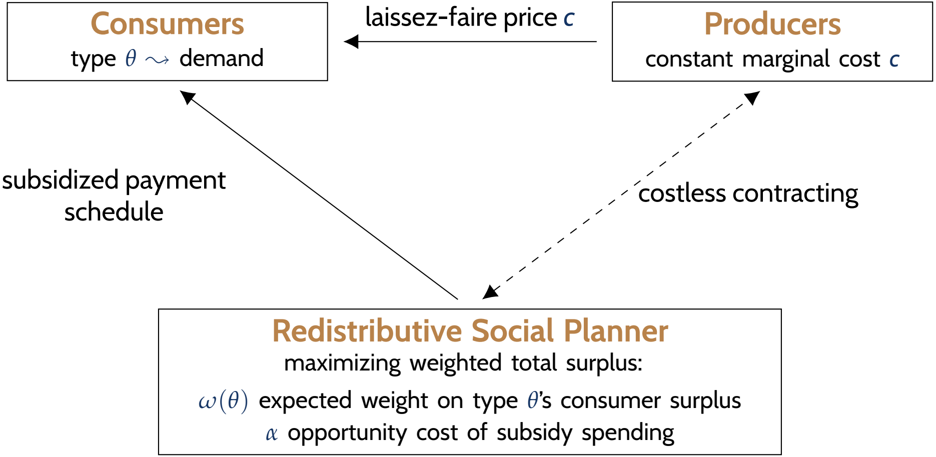

Consumers can purchase units from **both subsidized program and private market**.

Setup

Consumers:

- There is a unit mass of risk-neutral consumers in market for a divisible, homogeneous good.
- Consumers differ in type $\theta \in [\underline{\theta}, \overline{\theta}]$ with $\underline{\theta} \geq 0$, and $\theta \sim F$, continuous with density $f > 0$.
- Each consumer derives utility $\theta v(q) - t$ from quantity $q \in [0, A]$ given payment t.
  $v : [0, A] \to \mathbb{R}$ is differentiable with $v' > 0$ and $v'' < 0$.

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Extensions (not today): equilibrium effects, observable characteristics, product choice and eligibility.

Laissez-Faire Equilibrium

- Perfectly competitive private market $\rightsquigarrow$ **laissez-faire price** $p^{\mathrm{LF}} = c$ per unit.
- Each consumer solves

$$U^{\mathrm{LF}}(\theta) := \max_{q \in [0,A]} \left[\theta v(q) - cq\right].$$

v is strictly concave $\rightsquigarrow$ unique maximizer:

$$q^{\mathrm{LF}}(\theta) = (v')^{-1}\left(\frac{c}{\theta}\right) = D(c,\theta).$$

- To simplify statements of some results, assume today that $q^{\mathrm{LF}}(\underline{\theta}) > 0$.

Subsidy Design

Social planner costlessly contracts with firms and sells units at a **subsidized payment schedule** $P^\sigma(q)$.

$\rightsquigarrow \Sigma(q) = cq - P^\sigma(q)$ is the **total subsidy** as a function of q, and $\sigma(q) = \Sigma'(q)$ is the **marginal subsidy**.

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Key assumptions:

1. Each consumer can **top up** his consumption of the good, allowing him to purchase additional units in the private market at price c,

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Implementation: Consumer θ solves $U^\sigma(\theta) := \max_q [\theta v(q) - P^\sigma(q)]$, leading to **subsidized demand** $q^\sigma(\theta)$.

NLS assumption Taxation

Redistributive Objective

The social planner seeks to maximize **weighted total surplus**.

- **Consumer surplus**: social planner assigns a welfare weight $\omega(\theta) := \mathbf{E}[\omega|\theta]$ to consumer type θ.
  - $\rightsquigarrow$ $\omega(\theta)$: expected social value of giving consumer θ one unit of money.

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$\leadsto$ **Objective**:

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Remarks:

- If $\omega(\theta) > \alpha$, social planner would want to transfer a dollar to type θ.
- If $\mathbf{E}_\theta[\omega(\theta)] > \alpha$, social planner would want to make a lump-sum cash transfer to all consumers.

Endogenizing ω and α

Correlation Assumption

Two baseline cases:

"Negative Correlation": $\omega(\theta)$ is <u>decreasing</u> in θ.

- high-demand consumers tend to have lower need for redistribution.
- *e.g.*, food, education, and, if $\omega \propto 1/\text{Income}$, **normal** goods.

"Positive Correlation": $\omega(\theta)$ is <u>increasing</u> in θ.

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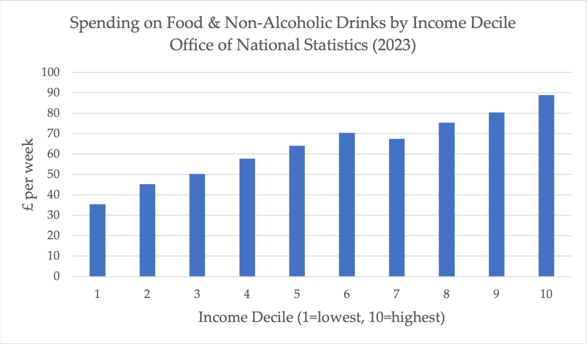

Related Literature

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  - Different objective (local linearization), incorporating a private market not in control of social planner.
  - Focus on commodity subsidization, not ruled out by Atkinson-Stiglitz because no “incentive separability.”
  - We allow for **nonlinear subsidy designs** and characterize global optimum, not FOCs à la Mirrlees.

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  - Dworczak, Akbarpour & Kominers (2021): unit demand consumers trading a fixed supply with different qualities, focusing on cross- and same-side inequality.
  - Akbarpour, Dworczak & Kominers (2022): unit demand, fixed supply, across & within-group matching.
  - Kang (2023): public option with single quality (quantity) level.
  - This paper: **partial control**, we allow consumers to consume in private market outside of planner's control.

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  - **This paper:** focus on equally efficient planner, “topping up” (cf. Kang & Watt, 2024).

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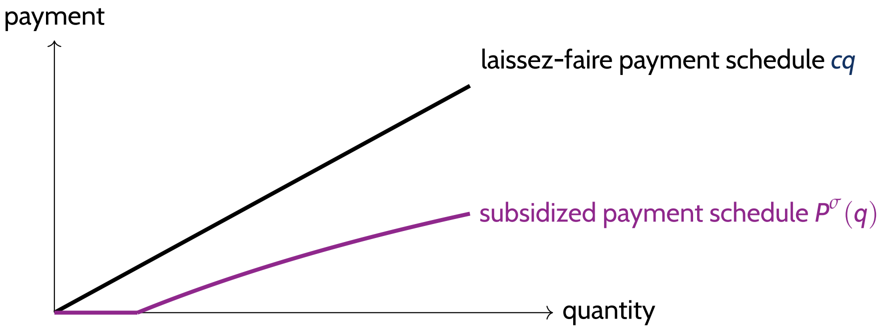
  - Yang & Zentefis (2024) study extrema of "monotone function intervals" $\rightsquigarrow$ optima of linear objectives.
  - **This paper:** explicit characterization of solution with FOSD (topping up) constraint.

When (Not) To Use Subsidies?

Private Market Access Constrains Redistribution Using Subsidies

private market access $\Longleftrightarrow$ **marginal price** $\leq c$



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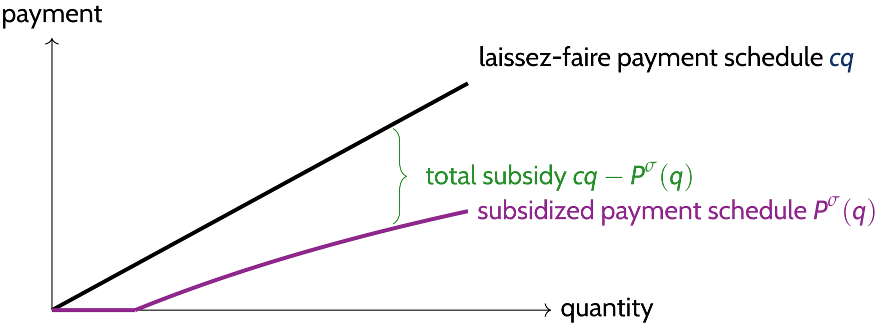

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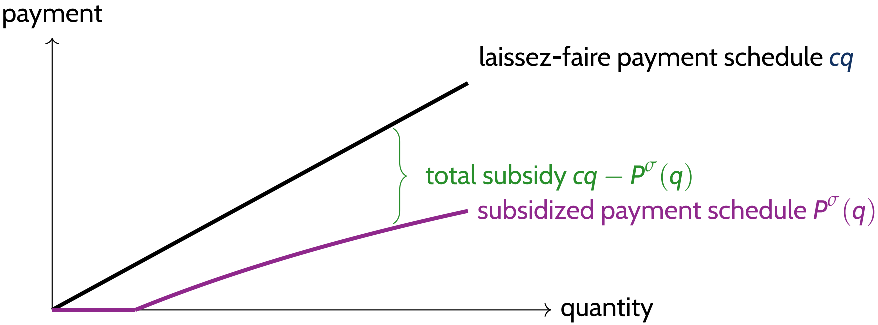

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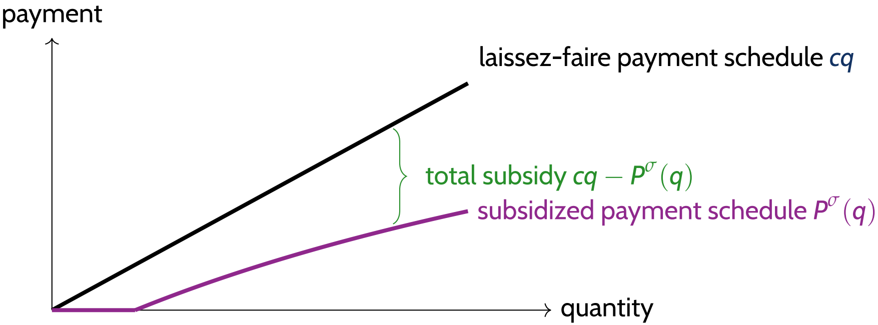

Subsidies are captured disproportionately by high θ consumers.

When Not To Subsidize?

Recall our "negative correlation" assumption: high θ consumers have lower ω.

Proposition. For any subsidy P^{σ}, the social planner would prefer to make a lump-sum transfer of $\mathbf{E}_{\theta}[\Sigma(q^{\sigma}(\theta))]$ to all consumers than the subsidy outcome.

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Proof: By definition of U^{LF} and correlation inequality,

$$\underbrace{\int_\Theta \omega(\theta)U^\sigma(\theta) - \alpha\Sigma(q^\sigma(\theta))\,\mathrm{d}F(\theta)}_{\text{objective given } P^\sigma} = \int_\Theta \omega(\theta)[\theta v(q^\sigma(\theta)) - cq^\sigma(\theta) + \Sigma(q^\sigma(\theta))] - \alpha\Sigma(q^\sigma(\theta))\,\mathrm{d}F(\theta)$$

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\end{aligned}$$

Theorem 1 (Negative Correlation, part). The social planner subsidizes consumption **only if** $\mathbf{E}_\theta[\omega(\theta)] > \alpha$ (and cash transfers are unavailable).

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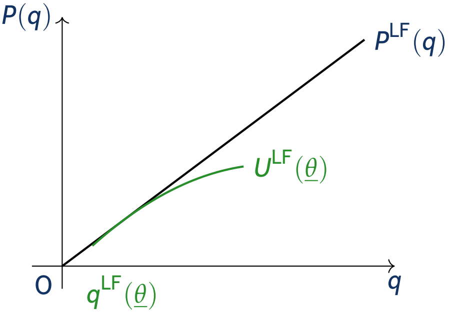
Proof of "if" direction:

Suppose $\mathbf{E}_\theta[\omega(\theta)] > \alpha$. We identify a subsidy schedule improving over laissez-faire.



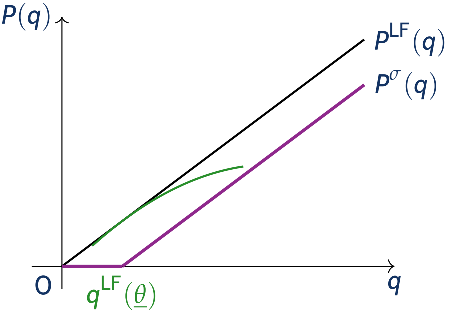
When To Subsidize?

Theorem 1 (Negative Correlation). The social planner subsidizes consumption **if and only if** $\mathbf{E}_{\theta}[\omega(\theta)] > \alpha$ (and cash transfers are unavailable).

Proof of "if" direction:

Suppose $\mathbf{E}_{\theta}[\omega(\theta)] > \alpha$. We identify a subsidy schedule improving over laissez-faire.

P^{σ} is outcome-equivalent to a cash transfer of $cq^{\text{LF}}(\underline{\theta})$ to all consumers, and improves over laissez-faire because $\mathbf{E}_{\theta}[\omega(\theta)] > \alpha$. $\square$



How to Design Subsidies?

Mechanism Design Reformulation

Revelation principle $\implies$ it suffices to consider **direct mechanisms** (q, t) consisting of:

- an **allocation function** $q : [\underline{\theta}, \overline{\theta}] \to [0, A]$ denoting *total* quantity consumed by type θ;
- a **payment rule** $t : [\underline{\theta}, \overline{\theta}] \to \mathbb{R}$ denoting *total* payment by type θ,

satisfying incentive-compatibility,

$$\theta \in \arg\max_{\hat{\theta} \in \Theta} \left\{ \theta v(q(\hat{\theta})) - t(\hat{\theta}) \right\} \text{ for all } \theta \in \Theta. \tag{IC}$$

Mechanism Design Reformulation

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Lemma (Implementation). For any (IC) mechanism (q, t), there exists a subsidy σ with $q = q^{\sigma}$ and $t = P^{\sigma} \circ q^{\sigma}$ if and only if:

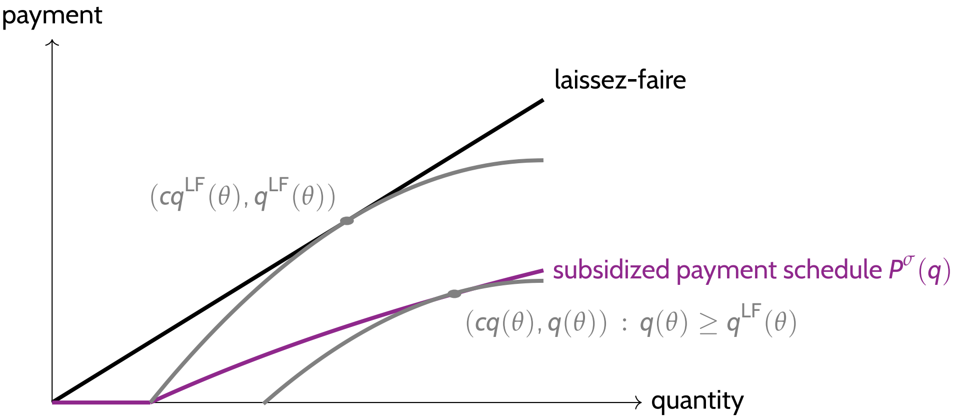
$$q(\theta) \geq q^{\text{LF}}(\theta) \text{ for all } \theta \in \Theta, \tag{LB}$$

$$t(\theta) \geq 0 \text{ for all } \theta \in \Theta, \tag{NLS}$$

$$U(\theta) \geq U^{\text{LF}}(\theta) \text{ for all } \theta \in \Theta. \tag{IR}$$

Intuition

marginal price per unit $\leq c \iff$ allocations exceed laissez-faire



Converse

Reformulating the Mechanism Design Problem

The social planner maximizes weighted total surplus

$$\max_{(q,t)} \int_{\underline{\theta}}^{\overline{\theta}} \left[\omega(\theta) \underbrace{[\theta v(q(\theta)) - t(\theta)]}_{\text{consumer surplus}} - \alpha \underbrace{[cq(\theta) - t(\theta)]}_{\text{total cost}} \right] \mathrm{d}F(\theta),$$

subject to (IC), (LB), (IR), and (NLS).

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#1. Apply Myerson (1981) Lemma and Milgrom and Segal (2002) envelope theorem to express objective in terms of $U(\underline{\theta})$ and $q(\theta)$ non-decreasing, substituting

$$t(\theta) = \theta v(q(\theta)) - \int_{\underline{\theta}}^{\theta} v(q(s))\, \mathrm{d}s - U(\underline{\theta}).$$

Reformulating the Mechanism Design Problem

The social planner maximizes weighted total surplus

$$\max_{q \text{ non-decreasing}, \, U(\underline{\theta})} \mathbf{E}_\theta[\omega(\theta) - \alpha] U(\underline{\theta}) + \alpha \int_{\underline{\theta}}^{\overline{\theta}} \left[\left(\theta + \frac{\int_\theta^{\overline{\theta}} [\omega(s) - \alpha] \mathrm{d}F(s)}{\alpha f(\theta)} \right) v(q(\theta)) - c q(\theta) \right] \mathrm{d}F(\theta),$$

subject to (LB), (IR), and (NLS).

#1. Apply Myerson (1981) Lemma and Milgrom and Segal (2002) envelope theorem to express objective in terms of $U(\underline{\theta})$ and $q(\theta)$ non-decreasing, substituting

#2. Suffices to enforce (IR) and (NLS) only for lowest type $\underline{\theta}$ because $U(\theta) - U^{\mathrm{LF}}(\theta)$ and $t(\theta)$ are nondecreasing by (IC) and (LB).

- $\rightsquigarrow$ **(NLS) binding**: if $\mathbf{E}[\omega(\theta)] > \alpha$, choose $U(\underline{\theta}) = \underline{\theta} v(q(\underline{\theta}))$.
- $\rightsquigarrow$ **(NLS) does not bind**: if $\mathbf{E}[\omega(\theta)] \leq \alpha$, choose $U(\underline{\theta}) = U^{\mathrm{LF}}(\underline{\theta})$.

Reformulating the Mechanism Design Problem

The social planner maximizes weighted total surplus

$$\max_{q \text{ non-decreasing}} \alpha \int_{\underline{\theta}}^{\overline{\theta}} \underbrace{\left[J(\theta)v(q(\theta)) - cq(\theta)\right]}_{\text{surplus of virtual type}} \mathrm{d}F(\theta) + (\text{terms independent of } q),$$

subject to (LB): $q(\theta) \geq q^{\mathrm{LF}}(\theta)$, where the **virtual type** absorbs (IC), (IR), and (NLS):

$$J(\theta) = \underbrace{\theta}_{\text{efficiency}} + \underbrace{\frac{\int_{\theta}^{\overline{\theta}} [\omega(s) - \alpha] \mathrm{d}F(s)}{\alpha f(\theta)}}_{\text{redistributive motive}} + \underbrace{\frac{\max\{\mathbf{E}_{\theta}[\omega(\theta) - \alpha], 0\} \underline{\theta} \delta_{\theta = \underline{\theta}}}{\alpha f(\theta)}}_{\text{(NLS) constraint}}$$

Call $J(\theta) - \theta$ the **distortion term**.

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Call $J(\theta) - \theta$ the **distortion term**.

Technical challenge: (LB) is a “pointwise dominance” / FOSD constraint (*cf.* Yang and Zentefis, 2024) $\rightsquigarrow$ possible interactions with the monotonicity constraint.

Characterizing the Optimal Subsidy Allocation

Theorem 2 (Negative Correlation). The optimal allocation rule is unique, continuous and satisfies

$$q^*(\theta) = \begin{cases} D\left(c, \overline{J|_{[\underline{\theta},\theta_\alpha]}}(\theta)\right) & \text{for } \theta \leq \theta_\alpha \\ q^{\text{LF}}(\theta) & \text{for } \theta \geq \theta_\alpha, \end{cases}$$

where θ_α is defined by

$$\theta_\alpha = \inf\left\{\theta \in \Theta : \overline{J|_{[\underline{\theta},\theta]}}(\theta) \leq \theta\right\}.$$

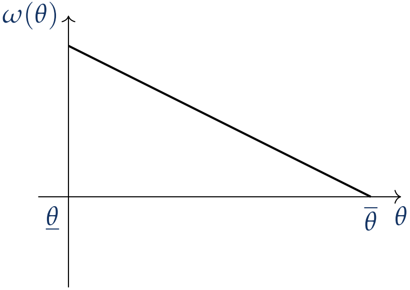
Intuition: there exists a type $\theta_\alpha \in \Theta$ (possibly $\underline{\theta}$ or $\overline{\theta}$) such that

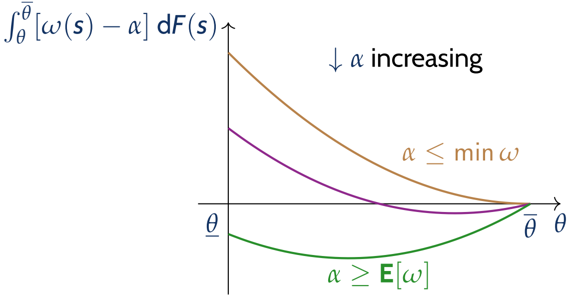
$$q^*(\theta) > q^{\text{LF}}(\theta) \text{ for all } \theta < \theta_\alpha, \text{ and}$$
$$q^*(\theta) = q^{\text{LF}}(\theta) \text{ for all } \theta \geq \theta_\alpha.$$

Ironing

Intuition: Signing the Distortion Term

Negative correlation $\rightsquigarrow \omega(\theta)$ decreasing $\rightsquigarrow$ distortion is single-crossing zero from above.





Social planner wants to distort consumption of all types down, low-demand types up and high-demand types down, or all types upwards.

Optimal Marginal Subsidy Schedule

Case 1: $\min \omega \geq \alpha$ (upward distortion for all)

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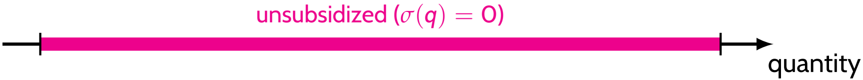
Optimal Marginal Subsidy Schedule

Case 1: $\min \omega \geq \alpha$ (upward distortion for all)

Case 2: $\min \omega \leq \alpha \leq \mathbf{E}[\omega]$ (upward distortion for low types, downward distortion for high types)

Case 3: $\mathbf{E}[\omega] \leq \alpha$ (downward distortion for all)



Economic Implications

With **negative correlation** between ω and θ:

\# 1. Lump-sum cash transfers are always **more progressive** than subsidies.

Economic Implications

With negative correlation between ω and θ:

1. Lump-sum cash transfers are always more progressive than subsidies.

2. If cash transfers are unavailable, linear subsidies are never optimal.

  # 2a. Optimal subsidies are "all or none": active subsidy programs should always incorporate a free allocation ("public option").

  # 2b. If *any* consumer has $\omega < \alpha$, optimal subsidies are capped in quantity.

Deriving the Optimal Mechanism

Solving for the Optimal Mechanism

skip

$$\max_{q} \alpha \int_{\underline{\theta}}^{\overline{\theta}} \left[J(\theta) v(q(\theta)) - c q(\theta) \right] \, \mathrm{d}F(\theta),$$

s.t. q nondecreasing and $q(\theta) \geq q^{\mathsf{LF}}(\theta)$.

Solving for the Optimal Mechanism

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Guess 1: Pointwise maximizer

$$q(\theta) = (v')^{-1}\left(\frac{c}{J(\theta)}\right) = D(c, J(\theta)).$$

Solving for the Optimal Mechanism

▸ skip

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q nondecreasing $\iff J(\theta)$ nondecreasing.
$q \geq q^{\mathrm{LF}} \iff D(c, J(\theta)) \geq D(c, \theta) \iff J(\theta) \geq \theta$.

Solving for the Optimal Mechanism

skip

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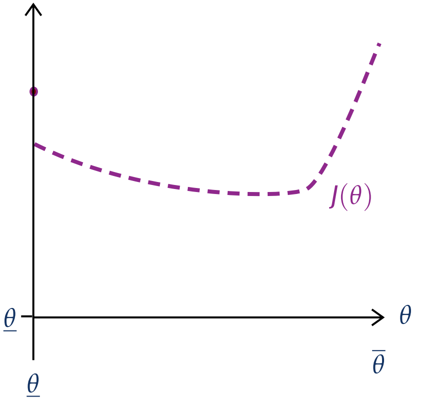

$J(\theta)$ may be non-monotone.

Solving for the Optimal Mechanism

skip

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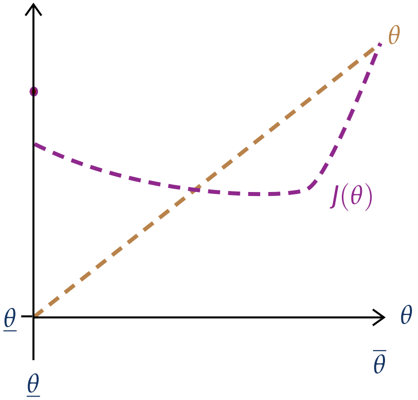


$J(\theta)$ may be smaller than θ.

Solving for the Optimal Mechanism

skip

$$\max_q \alpha \int_{\underline{\theta}}^{\overline{\theta}} [J(\theta)v(q(\theta)) - cq(\theta)] \, dF(\theta),$$

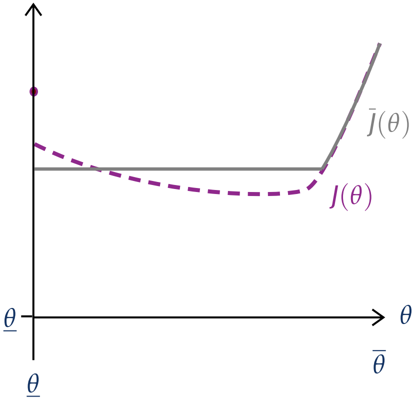
s.t. q nondecreasing and $q(\theta) \geq q^{LF}(\theta)$.

Guess 2: Relaxing the (LB) constraint

Toikka (2011); Akbarpour, Dworczak, Kominers (2021)

$$\rightsquigarrow q(\theta) = (v')^{-1}\left(\frac{c}{\bar{J}(\theta)}\right) = D(c, \bar{J}(\theta)),$$

where $\bar{J}$ is ironing of J, pooling types in any non-monotonic interval of J at its F-weighted average.



Ironing deals with non-monotonicity.

Ironing

Solving for the Optimal Mechanism

skip

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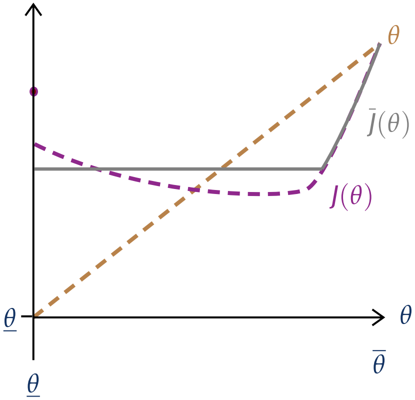


But not lower-bound constraint $\rightsquigarrow$ interaction.

Ironing

Solving for the Optimal Mechanism

skip

$$\max_q \alpha \int_{\underline{\theta}}^{\overline{\theta}} [J(\theta)v(q(\theta)) - cq(\theta)] \, dF(\theta),$$

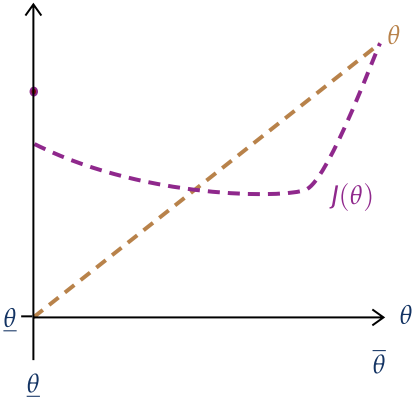
s.t. q nondecreasing and $q(\theta) \geq q^{LF}(\theta)$.

Guess 3: Our approach

Suppose solution is of the form

$$q(\theta) = D(c, H(\theta)).$$

Feasibility requires H to be nondecreasing and satisfy $H(\theta) \geq \theta$.



Need to identify nondecreasing $H \geq \theta$.

Ironing

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Theorem 2 (Negative Correlation). The optimal allocation rule is unique, continuous and satisfies

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where the subsidy type $H(\theta)$ is defined by

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▸ Ironing

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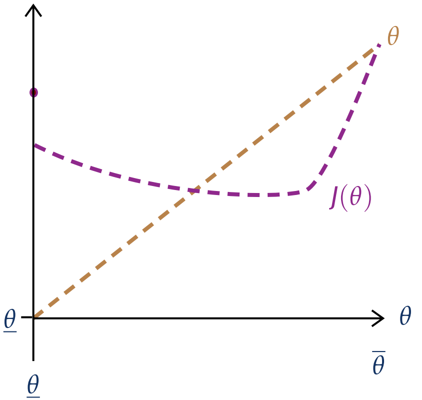

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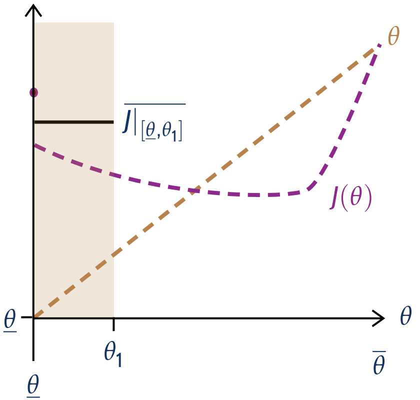

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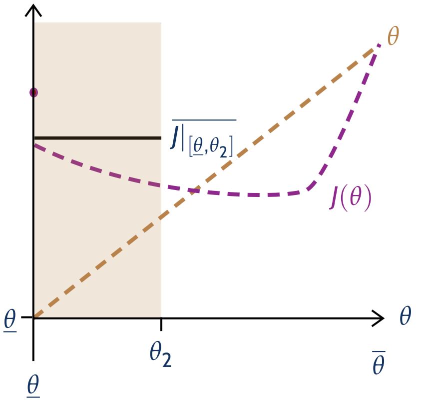

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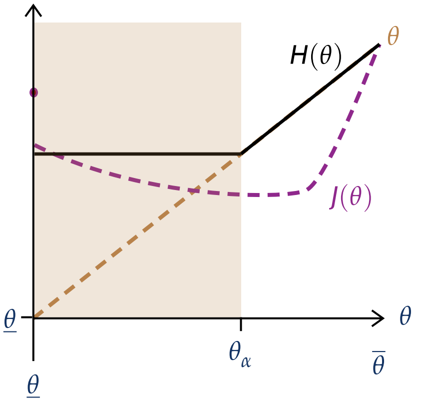

construction $\rightsquigarrow$ pooling condition and continuity

▸ Ironing

Verifying H from Theorem 2

Because $q^*(\theta) = D(c, H(\theta))$, for any feasible q

$$\int_\Theta \underbrace{[H(\theta)v(q^*(\theta)) - cq^*(\theta)]}_{\text{surplus of type } H(\theta) \text{ at } D(c,H(\theta))} \, \mathrm{d}F(\theta) \geq \int_\Theta \underbrace{[H(\theta)v(q(\theta)) - cq(\theta)]}_{\text{surplus of type } H(\theta) \text{ at } q(\theta)} \, \mathrm{d}F(\theta).$$

skip

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We want to show, for any feasible q

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Subtracting, it suffices to show, for any feasible q

$$\int_\Theta [J(\theta) - H(\theta)][v(q^*(\theta)) - v(q(\theta))] \, dF(\theta) \geq 0.$$

skip

Verifying the Variational Inequality

To show $\int_\Theta [J(\theta) - H(\theta)][v(q^*(\theta)) - v(q(\theta))] \, \mathrm{d}F(\theta) \geq 0$.

skip

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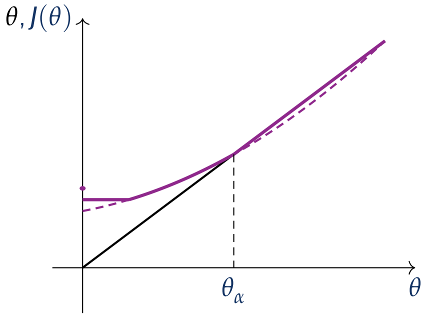
To show $\int_{\Theta}[J(\theta) - H(\theta)][v(q^*(\theta)) - v(q(\theta))]\,\mathrm{d}F(\theta) \geq 0$.

There are three possibilities for H, partitioning Θ into intervals:



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Verifying the Variational Inequality

To show $\int_{\Theta}[J(\theta) - H(\theta)][v(q^*(\theta)) - v(q(\theta))] \, \mathrm{d}F(\theta) \geq 0$.

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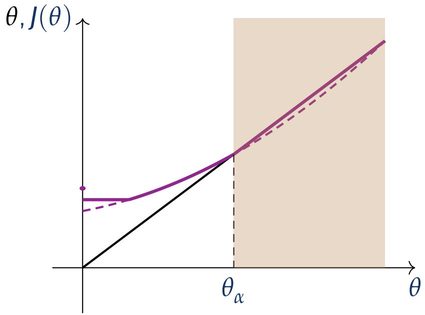
\# 1. $H(\theta) = \theta$: by construction $J(\theta) \leq \theta = H(\theta)$ and $v(q(\theta)) \geq v(q^*(\theta)) \rightsquigarrow$ integrand ≥ 0.



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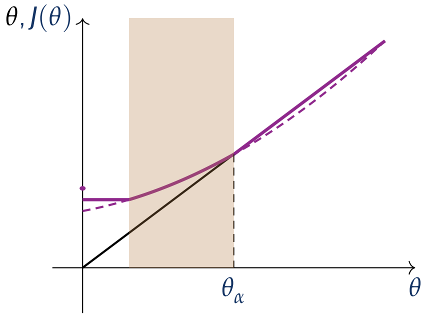
1. $H(\theta) = \theta$: by construction $J(\theta) \leq \theta = H(\theta)$ and $v(q(\theta)) \geq v(q^*(\theta)) \rightsquigarrow$ integrand ≥ 0.

2. $H(\theta) = J(\theta)$: integrand $= 0$.



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Verifying the Variational Inequality

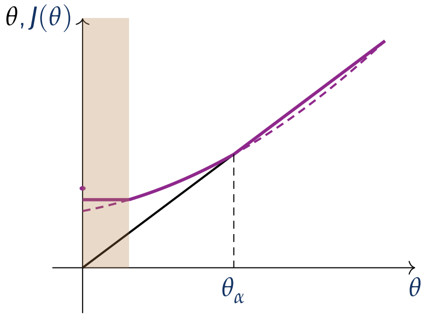
To show $\int_\Theta [J(\theta) - H(\theta)][v(q^*(\theta)) - v(q(\theta))]\, dF(\theta) \geq 0$.

There are three possibilities for H, partitioning Θ into intervals:

\# 1. $H(\theta) = \theta$: by construction $J(\theta) \leq \theta = H(\theta)$ and $v(q(\theta)) \geq v(q^*(\theta)) \rightsquigarrow$ integrand ≥ 0.

\# 2. $H(\theta) = J(\theta)$: integrand $= 0$.

\# 3. $H(\theta) = \overline{J|_{[\underline{\theta},\theta_\alpha]}}(\theta) \neq J(\theta)$:
technical lemma $\rightsquigarrow$ on any such interval Θ_i, $H = \overline{J|_{\Theta_i}}$
$\rightsquigarrow$ optimality of $D(c, H(\theta))$ in problem on Θ_i *without* (LB)
$\Longrightarrow$ same variational inequality characterizes optimality. $\square$



▸ skip

Summing Up

Proof approach:

- Guess form of solution $q^*(\theta) = D(c, H(\theta))$.
- Identify $H(\theta)$ which is continuous, $\geq \theta$, and satisfies the **pooling condition**.
- Verify optimality using **variational inequalities**.

Same method of solution works for general $\omega \rightsquigarrow$ see paper.

Generalization

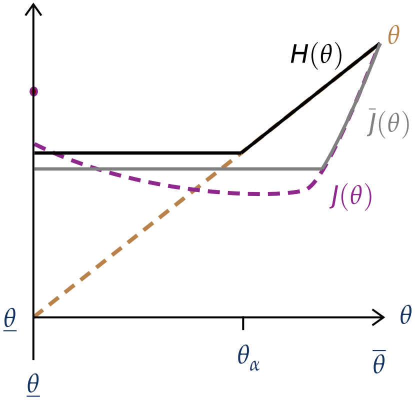
Role of Topping Up

Comparing optimum with and without (LB) constraint, $H(\theta)$ can exceed $\bar{J}$ for *all* types.

$\rightsquigarrow$ Inability to tax causes upward distortion of *all* types

It is not optimal to calculate optimal subsidy/tax and set taxes to zero.

Highlights distinction from Mirrleesian marginal approach (FOC $\not\rightsquigarrow$ optimum).



Positive Correlation

When to Subsidize?

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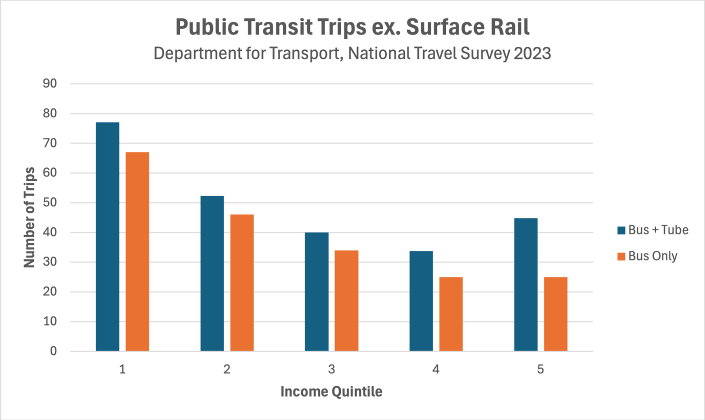

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$\rightsquigarrow$ Subsidies can be beneficial even when lump-sum cash transfers would not be.

Intuition: Social planner can always design a subsidy program with $\Sigma(q^{\sigma}(\theta)) \geq 0$ only if $\omega(\theta) \geq \alpha$.
$\rightsquigarrow$ Argument relies on nonlinearity of subsidy program.

▸ Arbitrary Correlation

How to Subsidize?

Positive Correlation

Theorem 2 (Positive Correlation). The optimal subsidy allocation rule is unique, continuous and satisfies

$$q^*(\theta) = D\left(c, H(\theta)\right), \text{ where } H(\theta) = \begin{cases} \theta & \text{if } \theta \leq \theta_\alpha, \\ \overline{J_{[\theta_\alpha, \overline{\theta}]}}(\theta) & \text{if } \theta \geq \theta_\alpha, \end{cases}$$

where $\theta_\alpha = \inf\{\theta \in \Theta : J(\theta) \geq \theta\}$.

Intuition: there exists a type $\theta_\alpha \in \Theta$ (possibly $\underline{\theta}$ or $\overline{\theta}$) such that

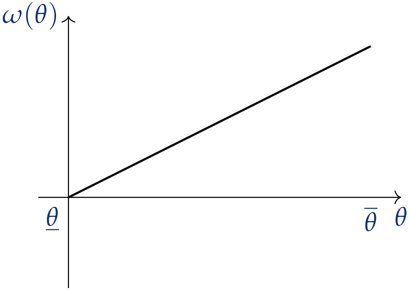
$$q^*(\theta) = q^{LF}(\theta) \text{ for all } \theta \leq \theta_\alpha, \text{ and}$$
$$q^*(\theta) \geq q^{LF}(\theta) \text{ for all } \theta > \theta_\alpha.$$

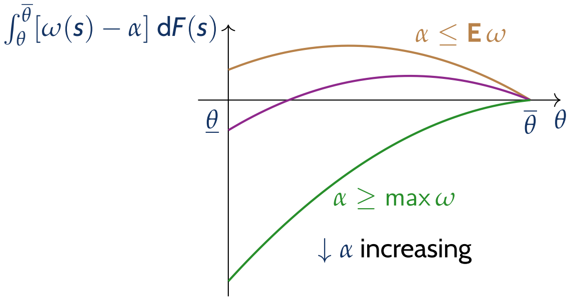
Arbitrary Correlation

How to Subsidize?

Positive Correlation

Positive correlation $\rightsquigarrow \omega(\theta)$ increasing $\rightsquigarrow$ distortion is single-crossing zero from below.





Social planner wants to distort consumption of all types down, high-demand types up and low-demand types down, or all types upwards.

Optimal Subsidy Schedule

Positive Correlation

Case 1: $\mathbf{E}[\omega] \geq \alpha$ (upward distortion for all)

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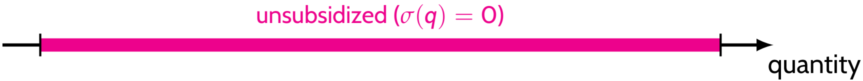

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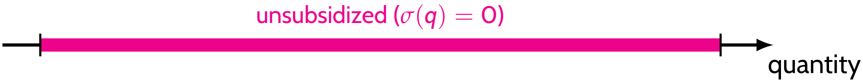

Discussion

Importance of Correlation

Negative Correlation

Subsidies dominated by cash transfers.

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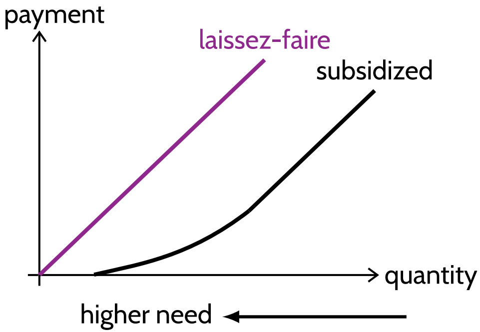

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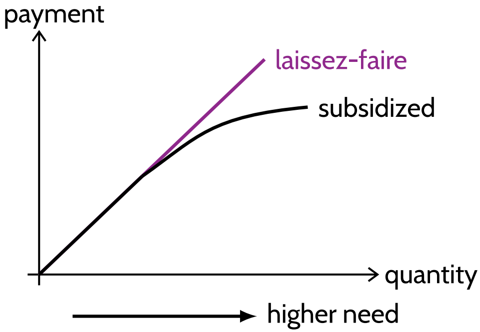

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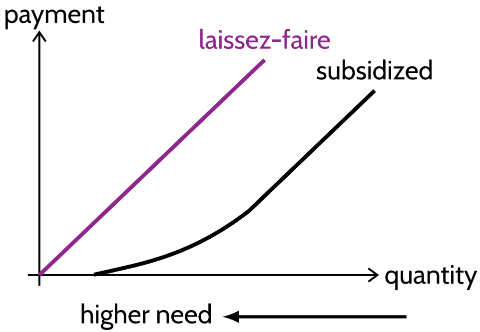

All or none subsidized.

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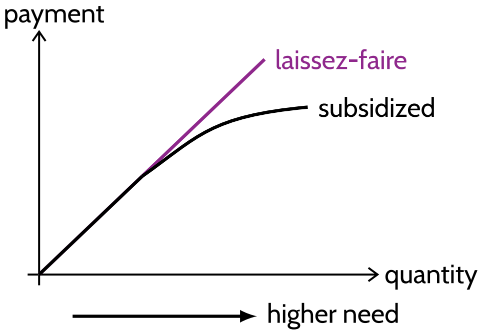

Only neediest (self-selected) consumers subsidized.

Differences In Practice

When? Theorem 1 $\rightsquigarrow$ scope of intervention larger with positive correlation ($\max \omega > \alpha$) than negative correlation ($\mathbf{E}[\omega] > \alpha$).

In practice, many government programs focused on goods consumed disproportionately by needy.

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In practice, many government programs focused on goods consumed disproportionately by needy.

How? Significant differences in marginal subsidy schedules observed in practice:

Larger subsidies for low q

- Food stamps (SNAP)
- Womens, Infants & Children (WIC) Program
- Housing Choice (Section 8) Vouchers
- Lifeline (Telecomm. Assistance) Program

Larger subsidies for high q

- Public transit fare capping
- Pharmaceutical subsidy programs
- Government-subsidized childcare places.

program details
inferior programs

How Do Optimal Subsidies Compare To Linear?

Proposition. Linear subsidies are **never optimal**.

Intuition: no distortion at the top ($J(\overline{\theta}) = \overline{\theta}$) $\rightsquigarrow$ linear subsidies never optimal.

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Implication: production subsidies are suboptimal for redistribution.

Social planner can always improve over linear subsidy by implementing:

- a **cap** on the subsidy paid to a consumer (as in negative correlation),
- a **floor** on eligibility for the subsidy (as in positive correlation), or
- a **free endowment** of the good to all consumers.

Gains from nonlinear subsidization can be **arbitrarily large** compared to gains from linear subsidization.

skip to conclusion

Comparative Statics of Subsidies

Question: How do optimal subsidies change when

(a) the social planner's desire to redistribute to each consumer increases?

(b) the correlation between demand and welfare weight increases?

(c) the marginal cost of production decreases?

▸ Details

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Details

Short Answer: Each cause the optimal subsidy program to be more generous.

But (a) and (b) cause $J(\theta)$ to increase for each $\theta \rightsquigarrow$ a larger set of consumers subsidized. (c) does not.

When Does the Planner Benefit from Private Market Restrictions?

Role of Topping Up Constraint

In some markets (e.g., public housing), the social planner may be able to restrict subsidy recipients from topping up in private market $\rightsquigarrow$ opt-in (or out) of subsidy program.

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In **Kang and Watt (2024)**, we characterize optimal subsidy mechanism under such restrictions. These lead to different (weaker) type-dependent outside option constraints:

average price $\leq$ c $\Leftrightarrow$ majorization constraint on q.

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$$\text{average price} \leq \text{c} \Leftrightarrow \text{majorization constraint on } q.$$

Proposition.

(a) "Negative Correlation": ω decreasing in $\theta \rightsquigarrow$ planner benefits from preventing topping up iff $\max \omega > \alpha$.

(b) "Positive Correlation": ω increasing in $\theta \rightsquigarrow$ planner never benefits from preventing topping up.

Intuition: Planner offers subsidies tied to consumption level favored by high ω types.

Implication: Positive correlation between demand and welfare weights reduces the need to enforce topping up restrictions.

skip to conclusion

Extensions

Equilibrium Effects

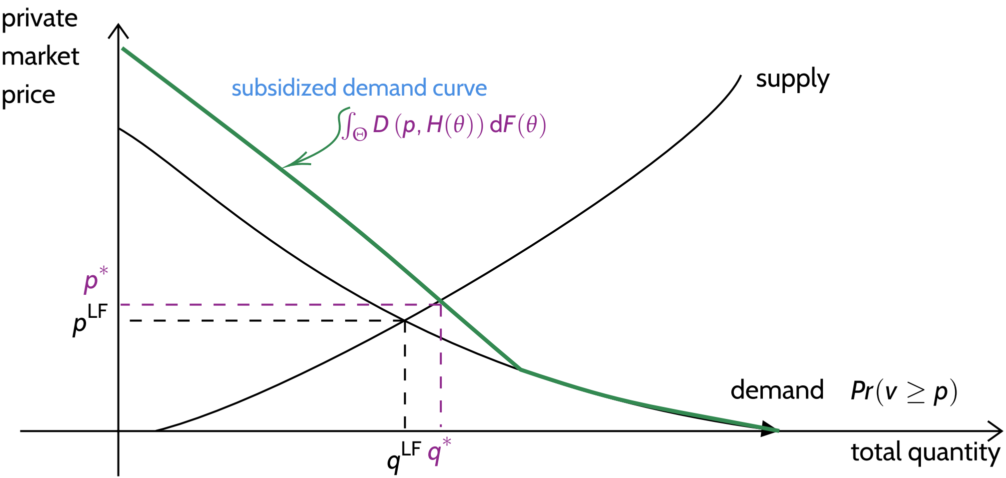
Our baseline model shuts down equilibrium effects of government subsidies on private market prices.

Empirical evidence of price effects from government subsidy programs, *e.g.*:

- public housing (Diamond and McQuade, 2019; Baum-Snow and Marion, 2009)
- pharmaceuticals (Atal et al., 2021)
- public schools (Dinerstein and Smith, 2021)
- school lunches (Handbury and Moshary, 2021)

Equilibrium Effects

Our results extend directly to imperfectly elastic supply curves:



Private Market Taxation

Our baseline model assumes the planner cannot tax the private market.

Taxation of private market reduces consumers' outside option, relaxing the (LB) constraint. If taxation is costly (*e.g.*, because of distortions on ineligible consumers):

Proposition. Suppose the planner faces a convex cost $\Gamma(\tau)$ for taxation of the private market. Then there exists an optimal tax level τ^* and subsidy program for eligible consumers satisfying

$$q^*(\theta) = D(H_{\tau^*}(\theta)),$$

where $H_{\tau^*}(\theta) \leq H(\theta)$.

Return to Model

Budget Constraints and Endogenous Welfare Weights

In our baseline model, $\omega(\cdot)$ and α are taken exogenously.

Our model can be extended to allow weights to be endogenous (cf. Pai and Strack, 2024):

- $\alpha \iff$ Lagrange multiplier on the social planner's budget constraint.
- $\omega(\theta) \iff$ the marginal value of money for a consumer with concave preferences

$$\varphi\left(\theta v(q) + I - t\right),$$

and income $I \sim G_\theta$, known but not observed by the social planner, then

$$\omega(\theta) = \mathbf{E}_{I \sim G_\theta}[\varphi'(\theta v(q(\theta)) + I - t(\theta))].$$

▸ Return to Model

Conclusion

Concluding Remarks

Takeaways for Subsidy Policy:

- Linear subsidies (e.g., production subsidies) are **never** optimal.
- When and how to subsidize depends on **correlation** between demand and need.
  - With negative correlation (many goods), why not lump-sum cash transfers? ("tortilla subsidy" vs. Progresa).
  - Goods with positive correlation are ideal candidates for subsidies (e.g., public transport) but these should have floors to improve targeting.

Technical Contribution:

- We show how to solve mechanism design problems with lower-bound constraints caused by type-dependent outside options.
- Similar mechanism design problems arise in other contexts, e.g., subsidy design with other objectives (externalities, paternalism); exclusive contracting (topping up = non-exclusive contracting, no topping up = exclusive contracting.).

Fin

Thank you for the invitation!

Assumption: No Lump-Sum Cash Transfers

Note: This constraint only binds if $\mathbf{E}_\theta[\omega(\theta)] > \alpha$.

Possible reasons:

- **Institutional**: subsidies designed by government agency without tax/transfer powers.
- **Political**: Liscow and Pershing (2022) find U.S. voters prefer in-kind redistribution to cash transfers.
- **Household Economics**: Currie (1994) finds in-kind redistribution has stronger benefits for children than cash transfer programs.
- **Pedagogical**: to contrast when the assumption is binding ($\rightsquigarrow$ cash transfers preferred to subsidies) versus non-binding (*vice versa*).
- **Model**: without NLS constraint, the social planner would want to make unbounded cash transfers when $\mathbf{E}[\omega] > \alpha$.

▸ return

When to Subsidize (General): Proof by Picture

Theorem 1. Social planner subsidizes **if and only if** there exists a type $\hat{\theta}$ for which

$$\mathbf{E}_\theta[\omega(\theta) \mid \theta \geq \hat{\theta}] \; > \; \alpha.$$

Suppose $\mathbf{E}_\theta[\omega(\theta)|\theta \geq \hat{\theta}] > \alpha$: we construct a subsidy schedule increasing weighted surplus.

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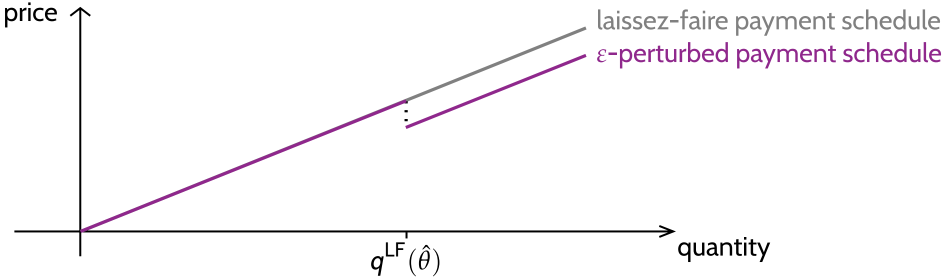

$\varepsilon-$perturbation increases utility of types $\geq \hat{\theta}$, net benefit $\varepsilon\, \mathbf{E}_\theta[\omega(\theta) - \alpha|\theta \geq \hat{\theta}]$.

return

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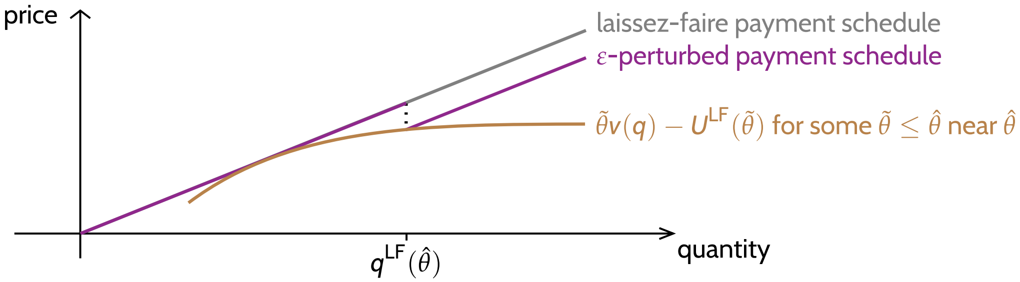


ε—perturbation increases utility of types $\geq \hat{\theta}$, net benefit $\varepsilon\, \mathbf{E}_{\theta}[\omega(\theta) - \alpha|\theta \geq \hat{\theta}]$.

But consumption is distorted for $O(\sqrt{\varepsilon})$ set of types near (but below) $\hat{\theta}$, at cost $\leq O(\sqrt{\varepsilon})\varepsilon$.

$\rightsquigarrow$ Benefits $>$ costs for small enough ε. **Note: Argument relies on nonlinearity.**

return

Topping Up $\Longleftarrow$ Lower-Bound (1/2)

Suppose $q(\theta) \geq q^{\text{LF}}(\theta)$. We want to show total subsidies $S(z)$ is increasing in z.

1. $t(\underline{\theta}) \leq cq(\underline{\theta})$ by (IR):

$$t(\underline{\theta}) \leq \underline{\theta}v(q(\underline{\theta})) - \underline{\theta}v(q^{\text{LF}}(\underline{\theta})) + cq^{\text{LF}}(\underline{\theta}),$$

and $\underline{\theta}v(q^{\text{LF}}(\underline{\theta})) - cq^{\text{LF}}(\underline{\theta}) \geq \underline{\theta}v(q(\underline{\theta})) - cq(\underline{\theta})$ by definition of q^{LF}, so $t(\underline{\theta}) \leq cq(\underline{\theta})$.

Topping Up $\Longleftarrow$ Lower-Bound (2/2)

2. The *marginal* price of any units purchased is no greater than c by (IC):

$$t(\theta') - t(\theta) = \left[\theta' v(q(\theta')) - U(\underline{\theta}) - \int_{\underline{\theta}}^{\theta'} v(q(s))\, \mathrm{d}s\right] - \left[\theta v(q(\theta)) - U(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} v(q(s))\, \mathrm{d}s\right]$$

$$= \theta' v(q(\theta')) - \theta v(q(\theta)) - \int_{\theta}^{\theta'} v(q(s))\, \mathrm{d}s$$

$$= \int_{\theta}^{\theta'} s v'(q(s))\, \mathrm{d}q(s).$$

But if $q(\theta) \geq q^{\mathrm{LF}}(\theta)$, then concavity of v implies $v'(q(\theta)) \leq v'(q^{\mathrm{LF}}(\theta)) = c/\theta$, so $t(\theta') - t(\theta) \leq c[q(\theta') - q(\theta)]$.

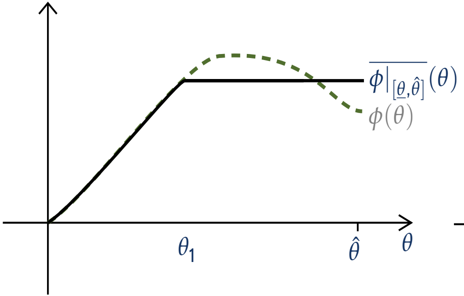
return

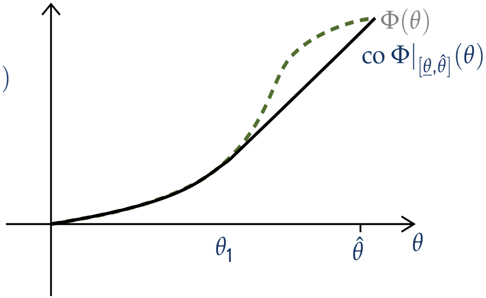
Ironing

Let ϕ be a (generalized) function and $\Phi : \theta \mapsto \int_{\underline{\theta}}^{\theta} \phi(s)\, dF(s)$. Then $\overline{\phi}$ is the monotone function satisfying

$$\text{for all } \theta \in [\underline{\theta}, \hat{\theta}], \qquad \int_{\underline{\theta}}^{\theta} \overline{\phi}(s)\, dF(s) = \text{co}\, \Phi(\theta).$$

Intuitively, $\overline{\phi}$ replaces non-monotone intervals of ϕ with F-weighted averages.





▸ Derivation ▸ Statement

Rewriting the Mechanism Design Problem

The social planner maximizes weighted total surplus

$$\max_{(q,t)} \int_{\underline{\theta}}^{\overline{\theta}} \left[\omega(\theta) \underbrace{[\theta v(q(\theta)) - t(\theta)]}_{\text{consumer surplus}} - \alpha \underbrace{[cq(\theta) - t(\theta)]}_{\text{total cost}} \right] \mathrm{d}F(\theta),$$

subject to (IC), (LB), (IR), and (NLS).

return

Rewriting the Mechanism Design Problem

$$\max_{q \text{ non-decreasing}, U(\underline{\theta}) \geq U^{\mathrm{LF}}(\underline{\theta})} \mathbf{E}_\theta [\omega(\theta) - \alpha] U(\underline{\theta}) + \alpha \int_{\underline{\theta}}^{\overline{\theta}} \left[\left(\theta + \frac{\int_\theta^{\overline{\theta}} [\omega(s) - \alpha] \mathrm{d}F(s)}{\alpha f(\theta)} \right) v(q(\theta)) - cq(\theta) \right] \mathrm{d}F(\theta),$$

subject to (LB), (IR), and (NLS).

#1. Apply Myerson (1981) Lemma and Milgrom and Segal (2002) envelope theorem to express objective in terms of $U(\underline{\theta})$ and $q(\theta)$ non-decreasing.

▸ return

Rewriting the Mechanism Design Problem

$$\max_{q \text{ non-decreasing}, U(\underline{\theta}) \geq U^{\mathrm{LF}}(\underline{\theta})} \mathbf{E}_\theta[\omega(\theta) - \alpha] U(\underline{\theta}) + \alpha \int_{\underline{\theta}}^{\overline{\theta}} \left[\left(\theta + \frac{\int_\theta^{\overline{\theta}} [\omega(s) - \alpha] \mathrm{d}F(s)}{\alpha f(\theta)} \right) v(q(\theta)) - cq(\theta) \right] \mathrm{d}F(\theta),$$

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#1. Apply Myerson (1981) Lemma and Milgrom and Segal (2002) envelope theorem to express objective in terms of $U(\underline{\theta})$ and $q(\theta)$ non-decreasing.

#2. Suffices to enforce (IR) and (NLS) only for lowest type $\underline{\theta}$ because $U(\theta) - U^{\mathrm{LF}}(\theta)$ and $t(\theta)$ are nondecreasing by (IC) and (LB).

- $\rightsquigarrow$ Low cost of public funds: if $\mathbf{E}[\omega(\theta)] > \alpha$, choose $U(\underline{\theta}) = \underline{\theta} v(q(\underline{\theta}))$.
- $\rightsquigarrow$ High cost of public funds: if $\mathbf{E}[\omega(\theta)] \leq \alpha$, choose $U(\underline{\theta}) = U^{\mathrm{LF}}(\underline{\theta})$.

return

Rewriting the Mechanism Design Problem

$$\max_{q \text{ non-decreasing}} \alpha \int_{\underline{\theta}}^{\overline{\theta}} [J(\theta)v(q(\theta)) - cq(\theta)] \, dF(\theta) + (\text{terms independent of } q),$$

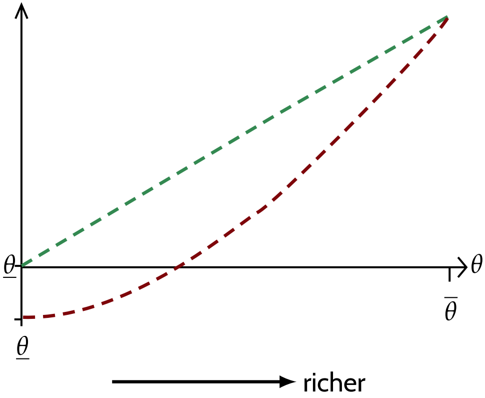
subject to (LB): $q(\theta) \geq q^{LF}(\theta)$, where the virtual type

$$J(\theta) = \underbrace{\theta}_{\text{efficiency}} + \underbrace{\frac{\int_{\theta}^{\overline{\theta}} [\omega(s) - \alpha] dF(s)}{\alpha f(\theta)}}_{\text{redistributive motive}} + \underbrace{\frac{\max\{\mathbf{E}_{\theta}[\omega(\theta) - \alpha], 0\}\underline{\theta}\delta_{\theta=\underline{\theta}}}{\alpha f(\theta)}}_{\text{(NLS) constraint}}$$

Call $J(\theta) - \theta$ the “distortion term.”

return

Decreasing Welfare Weights, High Cost of Public Funds



High cost of public funds: $\mathbf{E}[\omega(\theta)] \leq \alpha$.

$$J(\theta) = \theta + \frac{\int_{\theta}^{\overline{\theta}} [\omega(s) - \alpha] \, dF(s)}{\alpha f(\theta)}$$

is always *below* lower bound θ because distortion is single-crossing from above, negative at $\underline{\theta}$ and zero at $\overline{\theta}$.

return

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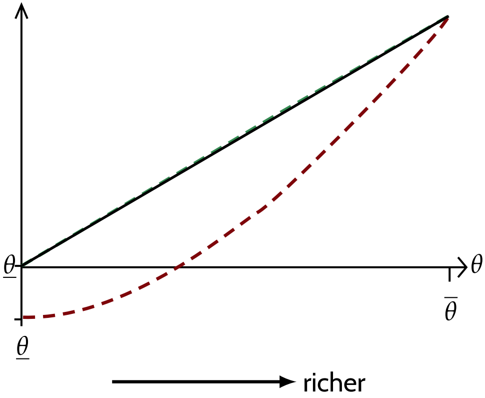

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$$J(\theta) = \theta + \frac{\int_{\theta}^{\overline{\theta}} [\omega(s) - \alpha] \; dF(s)}{\alpha f(\theta)}$$

is always *below* lower bound θ because distortion is single-crossing from above, negative at $\underline{\theta}$ and zero at $\overline{\theta}$.

$\rightsquigarrow$ Subsidy type $H(\theta) = \theta$ and optimal allocation is laissez-faire.

return

Optimal Mechanism (Arbitrary Correlation)

Theorem 2 (General). The optimal subsidy allocation rule is unique, continuous and satisfies

$$q^*(\theta) = D\left(c, H(\theta)\right), \text{ where } H(\theta) = \begin{cases} \theta & \text{if } \overline{J|_{[\underline{\theta},\theta]}}(\theta) \leq \theta, \\ \overline{J_{[\underline{\theta},\kappa_+(\theta)]}}(\theta) & \text{otherwise,} \end{cases}$$

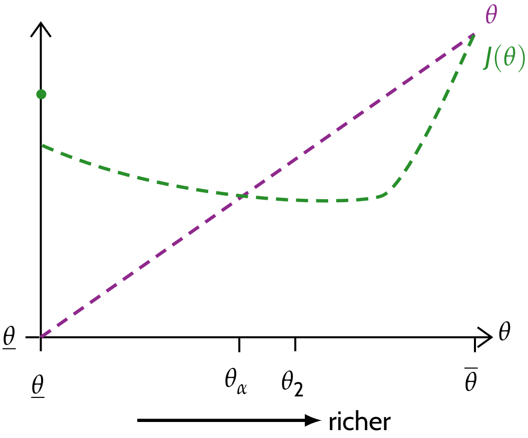
and $\kappa_+(\theta) = \inf\left\{\hat{\theta} \geq \theta : \overline{J|_{[\underline{\theta},\hat{\theta}]}}(\hat{\theta}) \leq \hat{\theta}\right\}$ or $\overline{\theta}$, if that set is empty.

"Double" ironing construction of $H(\theta)$ ensures $H(\theta) \geq \theta$, equivalent to (LB) given expression for q^*.

$\rightsquigarrow$ subsidized demand curve $\overline{D}(p) = \int_{\Theta} (v')^{-1} (p/H(\theta))\, \mathrm{d}F(\theta)$.

Ironing | return to derivation | negative correlation statement | positive correlation statement

Decreasing Welfare Weights, Low Cost of Public Funds

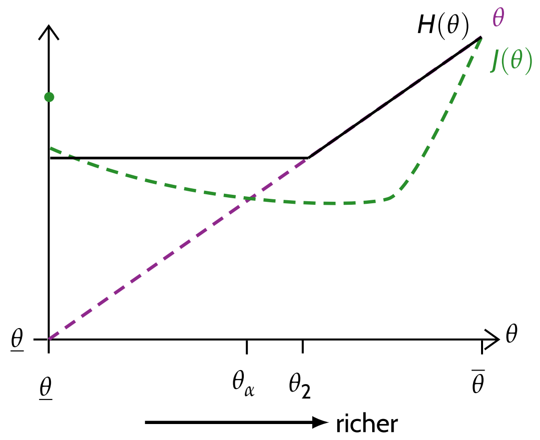


Low cost of public funds: $\mathbf{E}[\omega(\theta)] > \alpha$.

$J(\theta) = \theta + \frac{\int_{\theta}^{\overline{\theta}}[\omega(s)-\alpha]\,dF(s)+(\mathbf{E}_\theta[\omega(\theta)]-\alpha)\underline{\theta}\delta_{\theta=\underline{\theta}}}{\alpha f(\theta)}$ crosses lower bound constraint θ from *above* because distortion term is single-crossing from above, positive at $\underline{\theta}$ and zero at $\overline{\theta}$.

return

Decreasing Welfare Weights, Low Cost of Public Funds



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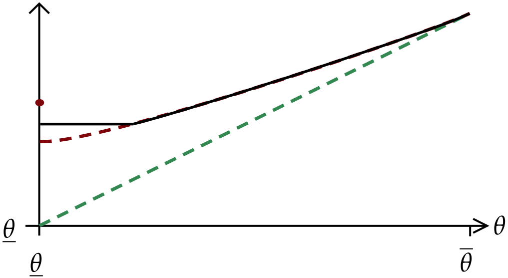
$J(\theta) = \theta + \frac{\int_\theta^{\overline{\theta}}[\omega(s)-\alpha]\, dF(s) + (\mathbf{E}_\theta[\omega(\theta)]-\alpha)\underline{\theta}\delta_{\theta=\underline{\theta}}}{\alpha f(\theta)}$ crosses lower bound constraint θ from *above* because distortion term is single-crossing from above, positive at $\underline{\theta}$ and zero at $\overline{\theta}$.

$\rightsquigarrow$ Subsidy type $H(\theta) > \theta$ for $\theta \leq \theta_2$. There is a free endowment of $q^{LF}(\theta_2)$, which strictly exceeds $q^{F}(\theta)$ for $\theta \leq \theta_2$ …*but* planner always prefers a lump-sum subsidy.

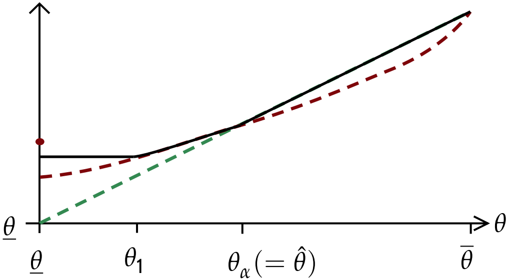
return

Decreasing Welfare Weight, Low Cost of Public Funds

Other possibilities:



Free endowment + quantity-dependent subsidies distorting all types' consumption upwards (no topping up).

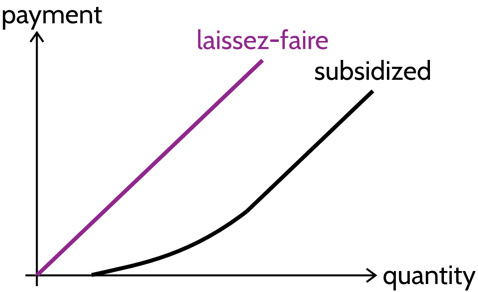


Free endowment + quantity-dependent subsidies up to a cap (high types top up in private market).

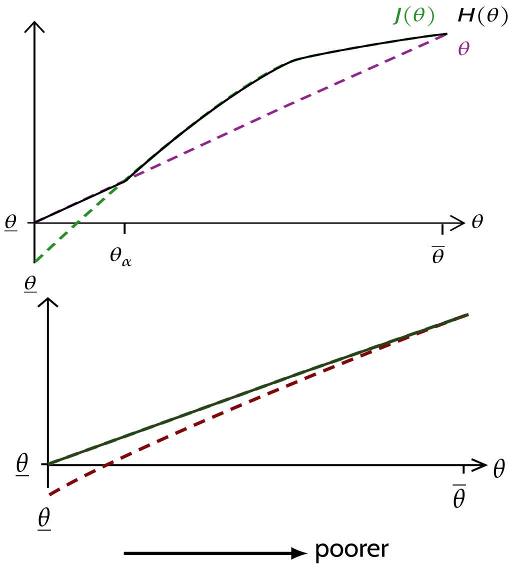
return

Decreasing Welfare Weight, Low Cost of Public Funds

Payment schedule:



Increasing Welfare Weights, High Cost of Public Funds



High cost of public funds: $\mathbf{E}[\omega(\theta)] \leq \alpha$.

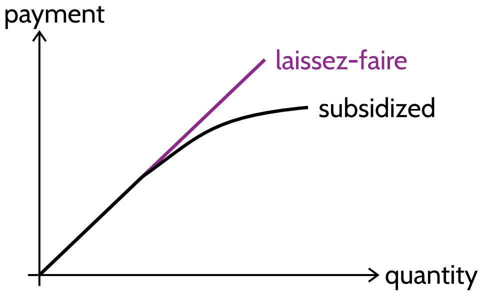
$$J(\theta) = \theta + \frac{\int_{\underline{\theta}}^{\overline{\theta}} [\omega(s) - \alpha] \, dF(s)}{\alpha f(\theta)}$$

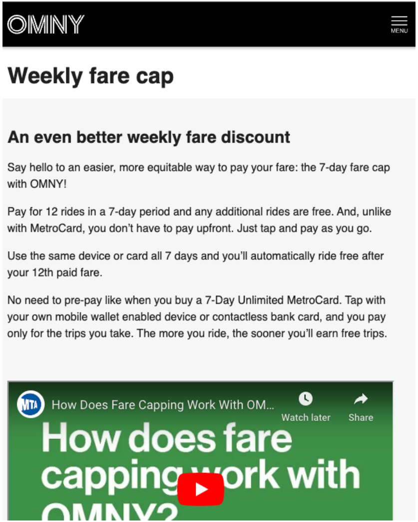
can cross lower-bound θ from *below* because the distortion term is single-crossing from below, negative at $\underline{\theta}$ and zero at $\overline{\theta}$.

$\rightsquigarrow$ Subsidy type *can* exceed θ for high types: implemented by offering discounts for consumption *above* a minimum level... *preferred* by planner to lump-sum transfer.

Increasing Welfare Weights, High Cost of Public Funds

Payment schedule:





OMNY

MENU

Weekly fare cap

An even better weekly fare discount

Say hello to an easier, more equitable way to pay your fare: the 7-day fare cap with OMNY!

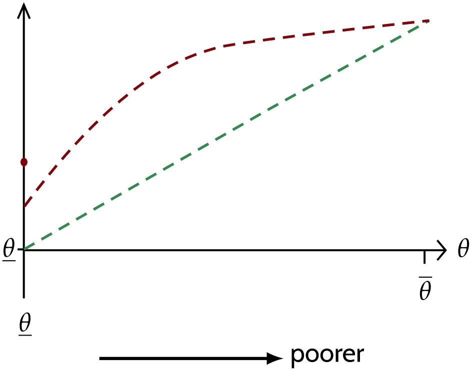
Pay for 12 rides in a 7-day period and any additional rides are free. And, unlike with MetroCard, you don't have to pay upfront. Just tap and pay as you go.

Use the same device or card all 7 days and you'll automatically ride free after your 12th paid fare.

No need to pre-pay like when you buy a 7-Day Unlimited MetroCard. Tap with your own mobile wallet enabled device or contactless bank card, and you pay only for the trips you take. The more you ride, the sooner you'll earn free trips.

return

Increasing Welfare Weights, Low Cost of Public Funds

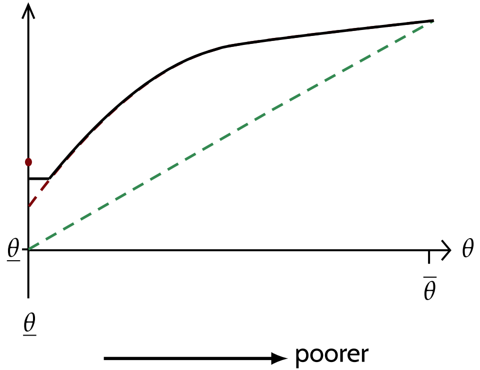


Low cost of public funds: $\mathbf{E}[\omega(\theta)] > \alpha$.

$J(\theta) = \theta + \frac{\int_{\theta}^{\overline{\theta}}[\omega(s)-\alpha]\, dF(s) + (\mathbf{E}_{\theta}[\omega(\theta)]-\alpha)\underline{\theta}\delta_{\theta=\underline{\theta}}}{\alpha f(\theta)}$ always *exceeds* lower-bound θ because the distortion term is single-crossing from below, positive at $\underline{\theta}$ and zero at $\overline{\theta}$.

$\leadsto$ Subsidy type *exceeds* θ for all types: implemented via a free allocation and discounts for additional consumption. No consumers top up.

Increasing Welfare Weights, Low Cost of Public Funds



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$\rightsquigarrow$ Subsidy type *exceeds* θ for all types: implemented via a free allocation and discounts for additional consumption. No consumers top up.

return

Discussion

Theorem 1 $\rightsquigarrow$ scope of intervention larger for "inferior goods" than "normal goods."

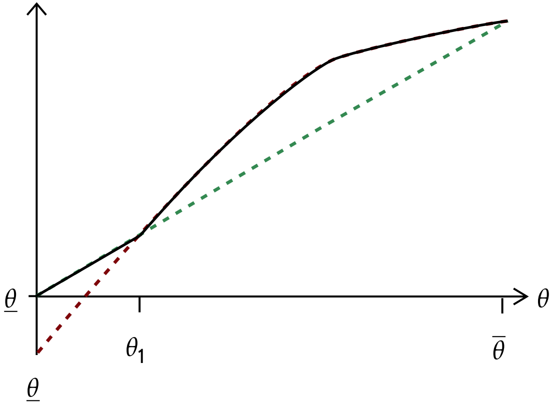
In practice, many government programs focused on goods consumed disproportionately by needy:

Examples:

- Egyptian *Tamween* food subsidy program subsidizes five loaves of *baladi* bread/day at AUD 0.01/loaf, with a cap on weights and quality of bread.
- CalFresh Restaurant Meals Program subsidizes fast food restaurants not dine-in restaurants.
- Indonesian Fuel Subsidy Program subsidizes low-octane fuel (for motorbikes) and not high-octane fuel (for cars).
- Until ~2016, Australia's Medicare subsidized amalgam fillings and not composite (tooth-coloured) fillings.

4(a) Increasing Motive for Redistribution

Suppose $\omega(\theta) \uparrow$ for each θ or, equivalently, $\alpha \downarrow$.

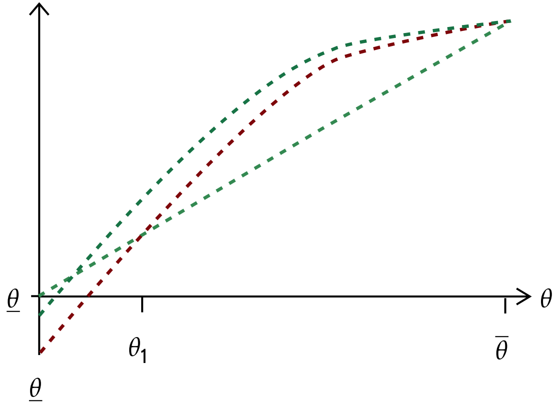


return

4(a) Increasing Motive for Redistribution

Suppose $\omega(\theta) \uparrow$ for each θ or, equivalently, $\alpha \downarrow$.

- virtual type $J(\theta) \uparrow$



return

4(a) Increasing Motive for Redistribution

Suppose $\omega(\theta)\uparrow$ for each θ or, equivalently, $\alpha\downarrow$.

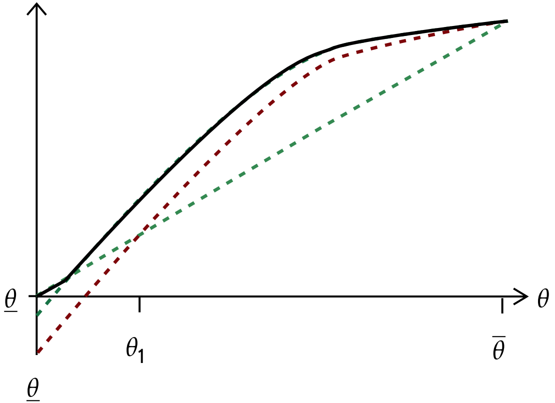
- virtual type $J(\theta)\uparrow$

$\rightsquigarrow$ each consumer's subsidy type $H(\theta)\uparrow$

$\rightsquigarrow$ each consumer's allocation $q^*(\theta)\uparrow$

$\rightsquigarrow$ set of subsidized types $\uparrow$

$\rightsquigarrow$ total subsidy per consumer $\uparrow$

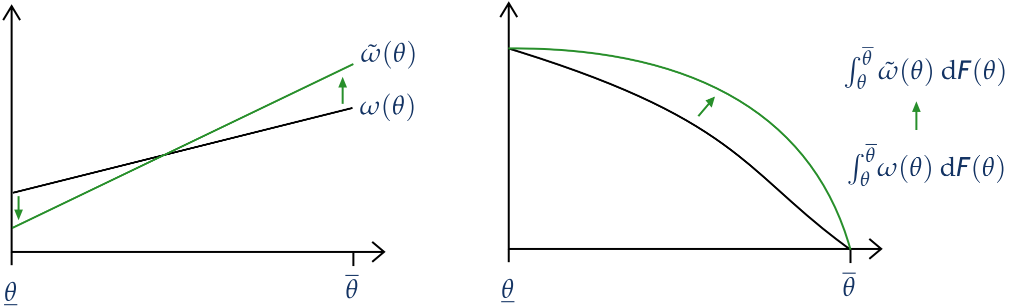


Planner & average eligible consumer prefer subsidies targeted to consumers with higher welfare weights.

return

4(b) Increasing Correlation

Suppose ω and θ become more correlated, in the sense of majorization $\rightsquigarrow$ observe higher demand expect higher ω, *i.e.*, for all $\theta \in \Theta$, $\mathbf{E}[\tilde{\omega}(\theta)|\theta \geq \hat{\theta}] \geq \mathbf{E}[\omega(\theta)|\theta \geq \hat{\theta}]$.



$\rightsquigarrow$ larger incentive to distort consumption $\rightsquigarrow$ more generous subsidies.

Planner & average eligible consumer prefer subsidies for goods with more positive correlation between demand and welfare weights.

return

4(c) Decreasing Marginal Cost

Suppose marginal cost decreases $c \downarrow$ (equiv. demand increases, so $(v') \uparrow$).

No change in virtual type $\rightsquigarrow$ no change in subsidy type.

$\rightsquigarrow$ the set of subsidized types is unchanged, while

$\rightsquigarrow$ each consumer's allocation $q^*(\theta) \uparrow$

$\rightsquigarrow$ total subsidy per subsidized consumer $\uparrow$.

▸ return

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$\rightsquigarrow$ each consumer's allocation $q^*(\theta) \uparrow$

$\rightsquigarrow$ total subsidy per subsidized consumer $\uparrow$.

Planner & average eligible consumer prefer subsidies for low cost / high demand goods.

return

Food Stamps (SNAP)

- **Overview**: U.S. program providing monthly food assistance to low-income individuals and families.
- **Initial Support**: Full subsidy up to a fixed dollar amount per month for eligible food items.
- **Free Endowment**: The subsidy starts as a full benefit and decreases after benefits are exhausted.
- **Low Consumption Focus**: Ensures a basic level of nutrition by covering initial consumption entirely.

▸ 'return

Women, Infants, and Children (WIC)

- **Overview**: Nutritional assistance for low-income pregnant women, new mothers, and young children.
- **Initial Support**: Vouchers for essential foods like milk, eggs, baby formula.
- **Free Endowment**: Recipients get fully subsidized quantities of specific foods.
- **Low Consumption Focus**: Prioritizes providing a free minimum quantity of nutritious food to families.

return

Housing Choice Voucher Program (Section 8)

- **Overview**: subsidized housing assistance for low-income renters in the U.S.
- **Initial Support**: Covers a large portion of rent (up to 70%) for qualifying households.
- **Free Endowment**: A significant rent portion is initially fully subsidized.
- **Low Consumption Focus**: Ensures low-income renters pay only a small portion of their rent.

return

Lifeline Program

- **Overview**: U.S. program offering discounted phone and internet services to low-income households.
- **Initial Support**: Monthly discounts on basic telecommunication services.
- **Free Endowment**: Full subsidy of basic services for the most disadvantaged users.
- **Low Consumption Focus**: Provides essential access to communication services with high initial subsidies.

▸ return

National School Lunch Program (NSLP)

- **Overview**: Provides free or reduced-price school meals for low-income students.
- **Initial Support**: Fully subsidized meals for eligible students based on family income.
- **Free Endowment**: Full meal subsidies provided for families below certain income thresholds.
- **Low Consumption Focus**: Ensures children receive at least one nutritious meal per day at no or low cost.

return

Australian Better Access Mental Health Initiative

- **Overview**: Australian government program subsidizing mental health services.
- **Initial Subsidy**: Up to 10 Medicare-subsidized sessions per year.
- **Additional Support**:
  - After initial sessions and doctor approval, become eligible for extra free/subsidized sessions.
  - Increased subsidy ensures access for those needing more care.

return

Australia's Child Care Subsidy (CCS)

- **Overview**: Government subsidy for childcare costs based on income and activity levels.
- **Initial Subsidy**: Covers a percentage of childcare fees up to a set number of hours.
- **Additional Support**:
  - Subsidy percentage **increases** as parents work, study, or volunteer more.
  - More hours of work/study lead to higher subsidies for additional childcare hours.
  - More children leads to higher subsidies per child.

return

Public Transit Fare Capping (Research Triangle, NC)

Fares & Passes

- Fares & Passes
- Return to Fare FAQs
- GoPass Partners
- Purchase GoPass Card
- Register GoPass Card
- Discount Fare Options
- How to use Umo
- Add Funds
- GoCary Fares
- GoDurham Fares
- GoRaleigh Fares

FIXED-ROUTE FARE OPTIONS

Fare Type	Fare	Daily Cap	Weekly Cap	Monthly Cap
Full Fare	$2.50	$5	$20	$80
Discount	$1.25	$2.50	$10	$40
Child Under 12	N/A	N/A	N/A	N/A
Youth 13-18	Qualify for free fare			
Senior 65+	Qualify for free fare			
Transit Assistance Pass	Qualify for free fare			
GoPass Partners	Learn More			

All sales are final and nonrefundable

Other Cities With Similar Programs: New York, SF Bay Area, Portland, London, Dublin, Toronto, Vancouver, Los Angeles, Singapore, Sydney, Brisbane, Melbourne, Perth, Auckland.

return

Pharmaceutical Subsidy Programs: Australia, Norway, Sweden, Denmark

- **Overview**: Government programs reducing out-of-pocket medication costs.
- **Australia (PBS)**: subsidizes prescription medicines; costs decrease after a yearly threshold (safety net) is reached.
- **Helsenorge (Norway)**: Covers up to 90% of prescription costs after reaching an annual expenditure cap.
- **Sweden**: Once a patient reaches a yearly spending threshold, additional medications are free.
- **Denmark**: Progressive subsidy structure, with higher reimbursements as individual spending increases.

return

Cost-Sharing Reductions (CSRs) and Eligibility Limits

ACA Cost-Sharing Reductions (CSRs)

- **What are CSRs?**
  - Subsidies that lower out-of-pocket costs (e.g., co-pays, deductibles).
  - Available to individuals/families with incomes between 100% and 250% of the Federal Poverty Level (FPL).
- **Eligibility Tied to Lower Insurance Plans**
  - To qualify for CSRs, you *must* purchase a **Silver-level** plan on the ACA marketplace.
  - Other plan tiers (**Bronze, Gold, or Platinum**) **do not** offer CSRs, even if you're income-eligible.
  - Silver plans have a standard **70% actuarial value**, but CSRs raise it to up to **94%** for lower-income enrollees.
- **Impact of Limiting to Silver Plans**
  - Higher-income individuals may choose other plan levels, but lose CSR eligibility.
  - Lower-income enrollees are incentivized to choose Silver plans to reduce out-of-pocket costs.

return

German Health System: Prohibition of Topping Up

- **Public Health Insurance**: Citizens covered by statutory health insurance (SHI) cannot "top up" SHI with private insurance for services already covered.
- **Supplementary Insurance**: Private insurance can only be used for services not included in SHI (e.g., private rooms, certain dental services).
- **Comprehensive Coverage**: SHI already covers essential medical services, discouraging the need for topping up with private health plans.

return

Public Education: Prohibition of Private Tutoring in China & South Korea

- **Public Education**: Both China and South Korea provide universal public education for students, with restrictions on private supplementary tutoring.
- **Prohibition**: Private tutoring and after-school programs are heavily regulated or banned to prevent parents from "topping up" public education with private instruction.
- **Equal Access**: The aim is to reduce inequality in educational opportunities and prevent wealthier families from gaining an advantage through private education.

return

Public Housing

- **Public Housing Programs**: Residents in public housing receive heavily subsidized rent, often capped at a percentage of their income.
- **Prohibition**: Participants must choose between living in public housing or renting in the private market; they cannot "top up" their public housing subsidy to rent a private apartment.
- **Example Cities and Countries**:
  - **Singapore**: The Housing & Development Board (HDB) provides subsidized flats, and participants cannot receive additional subsidies to live in private housing.
  - **Vienna, Austria**: The city's extensive public housing program offers low-cost rental units, with no option to "top up" for private market rentals.
  - **Hong Kong**: The Public Rental Housing (PRH) program offers heavily subsidized apartments, and recipients must choose between public housing and private market rentals.

▸ return

Egypt's Tamween Food Subsidy Program

- **Overview**: The Tamween program is one of the largest food subsidy systems in the world, providing essential goods to over 60 million Egyptians, mostly from low-income households.
- **Targeted Subsidy**:
  - **Bread**: Heavily subsidized at a fraction of market price (often less than 10% of the actual cost), making it affordable for the poor, who rely on it as a staple.
  - **Other Essentials**: Subsidies also cover rice, sugar, and cooking oil, basic items central to the diets of low-income families.
- **Exclusion**:
  - **Meat and Dairy**: These more expensive food items, consumed more frequently by wealthier households, are not subsidized. Consumers must pay market prices for these products.

return

Indonesian Fuel Subsidy Program: Pertamina

- **Overview**: Indonesia's fuel subsidy program supports transportation for low-income households.
- **Targeted Subsidy**: The program subsidizes low-octane fuel, which is primarily used by motorcycles, the preferred transport mode for poorer citizens.
- **Exclusion**: High-octane fuel, more commonly used by cars owned by wealthier households, is not subsidized.

return

CalFresh Restaurant Meals Program

- **Overview**: California's CalFresh program allows certain populations to use benefits for prepared meals.
- **Targeted Subsidy**: The program subsidizes meals, predominantly from fast food restaurants, providing affordable food options for homeless, elderly, and disabled individuals.
- **Exclusion**: Dine-in restaurants, typically frequented by wealthier individuals, are not included in the subsidy.

return

Public Dentistry Programs in Australia

- **Overview**: Australia's public dentistry programs provide dental care subsidies to low-income individuals.
- **Targeted Subsidy**: Prior to 2016, the program subsidized only amalgam fillings, a durable and cost-effective option used widely by lower-income patients.
- **Exclusion**: Composite (tooth-colored) fillings, which are more expensive and preferred by wealthier individuals, were not fully subsidized.
- **Post-2016**: amalgam fillings are being phased out due to mercury content.

return