Optimal Redistribution Through Subsidies

Zi Yang Kang
University of Toronto

Mitchell Watt
Stanford University
Monash University (from July 1)

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European Association of Young Economists

Introduction

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Our approach: we pose and solve the mechanism design problem for the **optimal subsidy**.

Model

Model Overview

Consumers

type $\theta \rightsquigarrow$ utility $\theta v(q) - t$,
demand $D(p, \theta)$

Producers

constant marginal cost c

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Redistributive Social Planner

maximizing weighted total surplus:

$\omega(\theta)$ expected weight on type θ's consumer surplus

α opportunity cost of subsidy spending

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costless contracting

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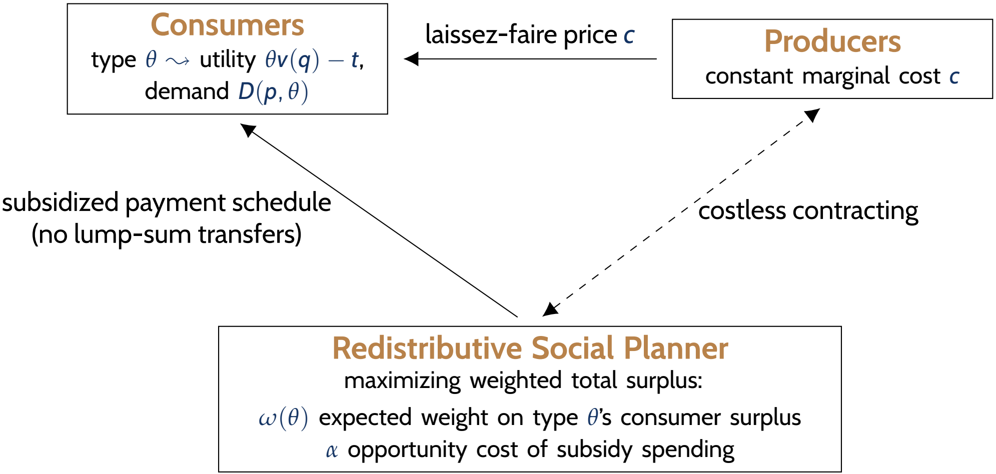

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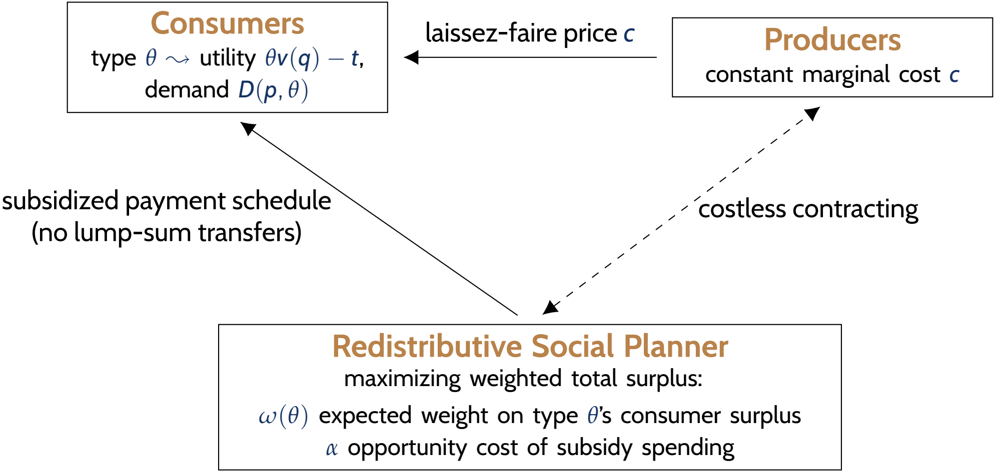

Key Assumption: Consumers can "top up," purchasing from **both subsidized program and private market**.

Mechanism Design Problem

The social planner maximizes weighted total surplus:

$$\max_{(q,t)} \int_{\underline{\theta}}^{\overline{\theta}} \left[\omega(\theta) \underbrace{[\theta v(q(\theta)) - t(\theta)]}_{\text{consumer surplus}} + \alpha \underbrace{[t(\theta) - cq(\theta)]}_{\text{total profit}} \right] \mathrm{d}F(\theta),$$

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Remark: If $\omega(\theta) > \alpha$, the planner would want to transfer cash to θ (if $\mathbf{E}[\omega(\theta)] > \alpha$, an average consumer).

Reformulation

The social planner maximizes weighted total surplus,

$$\max_{\substack{\underline{U}\leq\underline{\theta}v(q(\underline{\theta})),\\ q\text{ non-decreasing}}} \left\{ \left[\mathbf{E}[\omega]-\alpha\right]\underline{U} + \int_{\underline{\theta}}^{\overline{\theta}} \left[\left[\alpha\theta + \frac{\int_{\theta}^{\overline{\theta}} [\omega(s)-\alpha] \; dF(s)}{f(\theta)} \right] v(q(\theta)) - \alpha c q(\theta) \right] \; dF(\theta) \right\},$$

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The social planner maximizes weighted total surplus,

$$\max_{q \text{ non-decreasing}} \alpha \int_{\underline{\theta}}^{\overline{\theta}} \underbrace{[J(\theta)v(q(\theta)) - cq(\theta)]}_{\text{surplus of virtual type } J(\theta)} \, \mathrm{d}F(\theta) + (\text{terms independent of } q),$$

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In tariff space, (TU) is equivalent to marginal price $\leq$ c.

Negative Correlation

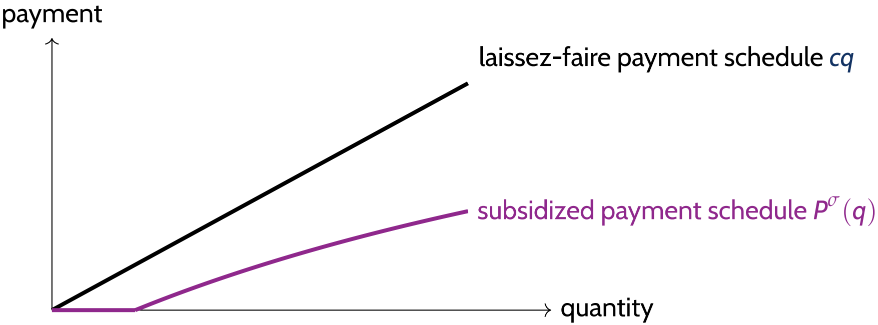
Negative Correlation Assumption

For now, assume $\omega(\theta)$ is decreasing in θ.

- high-demand consumers tend to have lower need for redistribution.
- *e.g.*, food, education, and, if $\omega \propto 1/\text{Income}$, **normal** goods.

Private Market Access Constrains Redistribution Using Subsidies

private market access $\Longleftrightarrow$ **marginal price** $\leq c$



Private Market Access Constrains Redistribution Using Subsidies

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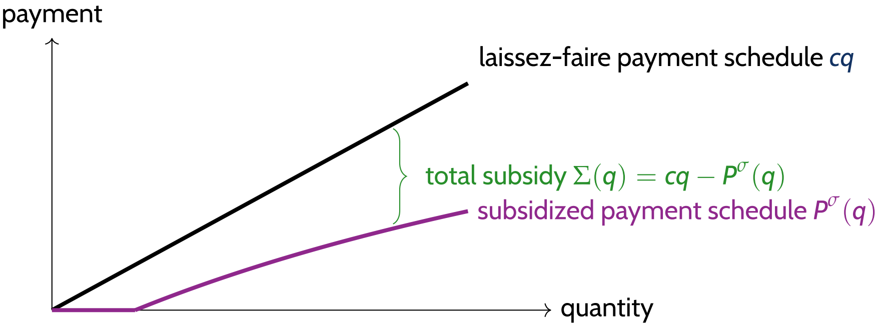

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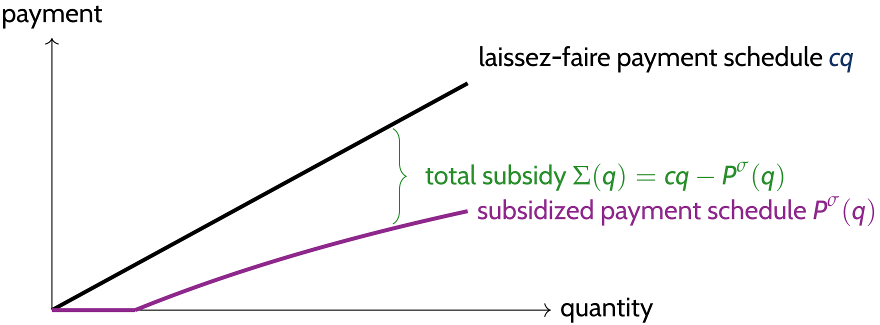

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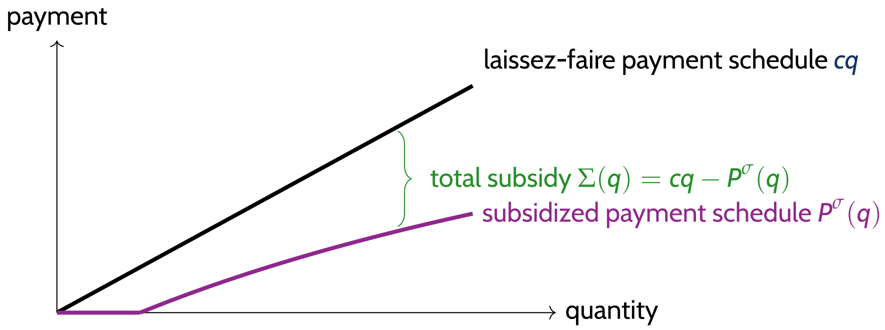

Subsidies are captured disproportionately by high θ consumers.

When to Subsidize?

Because higher θ consumers have lower welfare weights $\omega(\theta)$, we have the following:

Proposition. For any subsidy schedule P^{σ}, the social planner would prefer to make a lump-sum transfer of $\mathbf{E}_{\theta}[\Sigma(q^{\sigma}(\theta))]$ to all consumers than the subsidy outcome.

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This implies that the social planner would subsidize consumption **only if** $\mathbf{E}_\theta[\omega(\theta)] > \alpha$.

On the other hand, when $\mathbf{E}_\theta[\omega(\theta)] > \alpha$, because the social planner would always like to make a cash transfer (but cannot by assumption), we have:

Theorem 1 (Negative Correlation). The social planner offers consumption subsidies **if and only if** $\mathbf{E}_\theta[\omega(\theta)] > \alpha$ (and cash transfers are unavailable).

How to Subsidize?

Optimal marginal subsidy schedule ($\sigma(q) := \Sigma'(q)$) takes one of the following forms:

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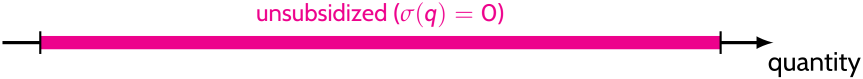
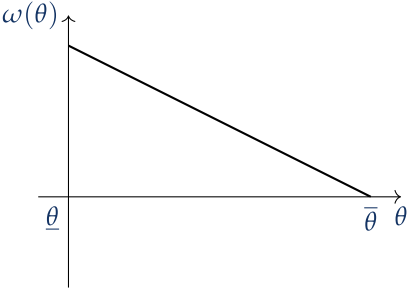
Case 2: $\min \omega \leq \alpha \leq \mathbf{E}[\omega]$ (upward distortion for low types, consumption undistorted for high types)

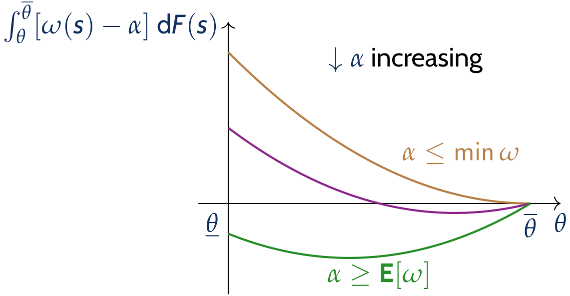
Case 3: $\mathbf{E}[\omega] \leq \alpha$ (no subsidies)



Intuition: Signing the Distortion of Virtual Type

Negative correlation ($\omega(\theta)$ decreasing) $\rightsquigarrow$ distortion $J(\theta) - \theta \overset{\text{sgn}}{=} \int_{\theta}^{\overline{\theta}} \omega(s) - \alpha \, \mathrm{d}F(s)$ is single-crossing zero from above.





Social planner wants to distort consumption of all types down, low-demand types up and high-demand types down, or all types upwards.

Solving for the Optimal Mechanism

skip

$$\max_{q} \alpha \int_{\underline{\theta}}^{\overline{\theta}} [J(\theta)v(q(\theta)) - cq(\theta)] \, \mathrm{d}F(\theta),$$

s.t. q nondecreasing and $q(\theta) \geq q^{\mathrm{LF}}(\theta)$.

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$$q(\theta) = (v')^{-1}\left(\frac{c}{J(\theta)}\right) = D(c, J(\theta)).$$

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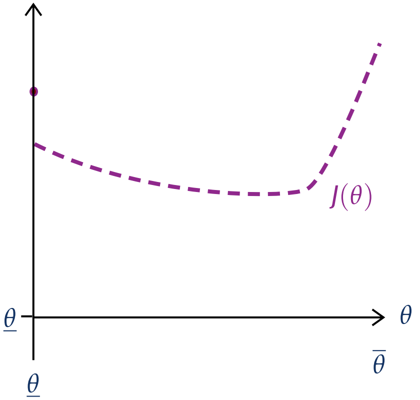

$J(\theta)$ may be non-monotone.

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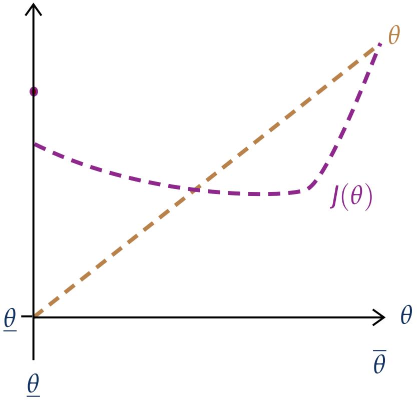


$J(\theta)$ may be smaller than θ.

Solving for the Optimal Mechanism

skip

$$\max_q \alpha \int_{\underline{\theta}}^{\overline{\theta}} [J(\theta)v(q(\theta)) - cq(\theta)] \, \mathrm{d}F(\theta),$$

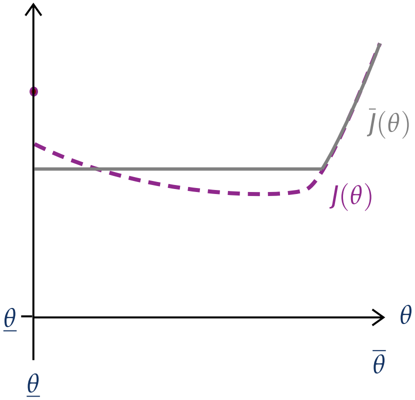
s.t. q nondecreasing and $q(\theta) \geq q^{\mathrm{LF}}(\theta)$.

Guess 2: Relaxing the (LB) constraint

Toikka (2011); Akbarpour, Dworczak, Kominers (2021)

$$\rightsquigarrow q(\theta) = (v')^{-1}\left(\frac{c}{\bar{J}(\theta)}\right) = D(c, \bar{J}(\theta)),$$

where $\bar{J}$ is ironing of J, pooling types in any non-monotonic interval of J at its F-weighted average.



Ironing deals with non-monotonicity.

Ironing

Solving for the Optimal Mechanism

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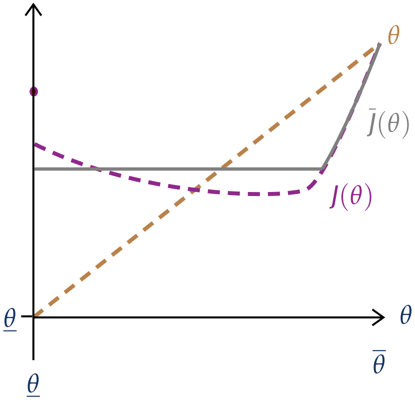

But not lower-bound constraint $\rightsquigarrow$ interaction.

Ironing

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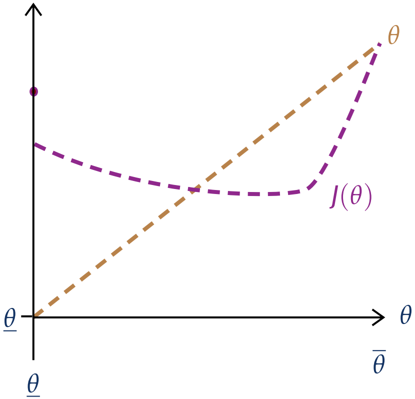
Guess 3: Our approach

Suppose solution is of the form

$$q(\theta) = D(c, H(\theta)).$$

Feasibility requires H to be nondecreasing and satisfy $H(\theta) \geq \theta$.



Need to identify nondecreasing $H \geq \theta$.

Ironing

Characterizing the Optimal Subsidy Allocation

Theorem 2 (Negative Correlation). The optimal allocation rule is unique, continuous and satisfies

$$q^*(\theta) = D(c, H(\theta)),$$

where the subsidy type $H(\theta)$ is defined by

$$H(\theta) := \begin{cases} \overline{J|_{[\underline{\theta},\theta_\alpha]}}(\theta) & \text{for } \theta \leq \theta_\alpha \\ \theta & \text{for } \theta \geq \theta_\alpha, \end{cases}$$

and θ_α is defined by

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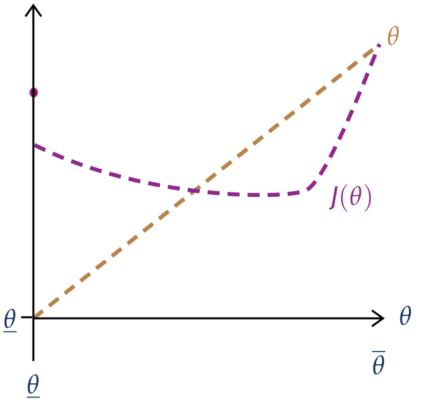

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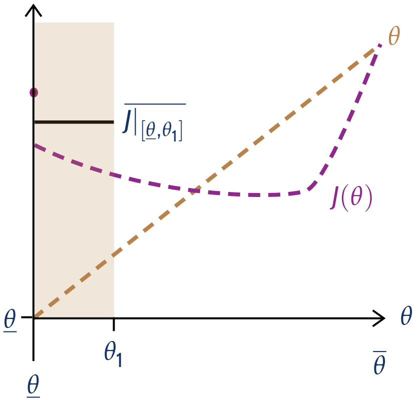

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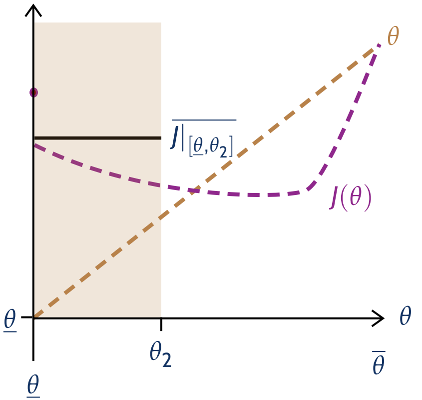

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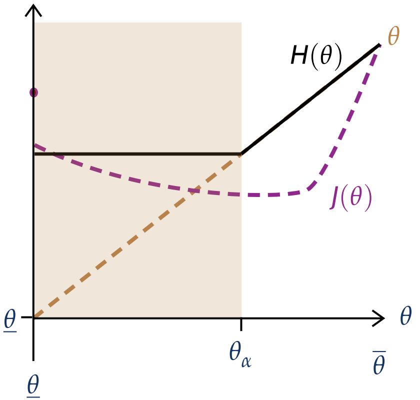

construction $\rightsquigarrow$ pooling condition and continuity

▸ Ironing

Economic Implications

With **negative correlation** between ω and θ:

1. Lump-sum cash transfers are always **more progressive** than subsidies.

Economic Implications

With negative correlation between ω and θ:

\# 1. Lump-sum cash transfers are always more progressive than subsidies.

\# 2. If cash transfers are unavailable, linear subsidies are never optimal.

- \# 2a. Optimal subsidies are "all or none": active subsidy programs should always incorporate a free allocation ("public option").

- \# 2b. If *any* consumer has $\omega < \alpha$, optimal subsidies are capped in quantity.

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When to Subsidize?

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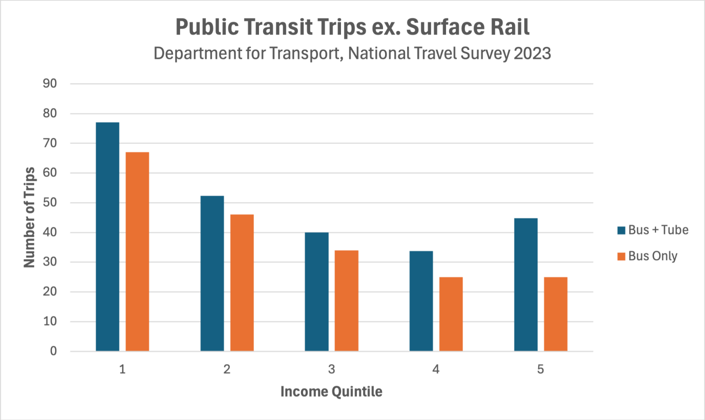

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Intuition: Social planner can always design a subsidy program with $\Sigma(q^{\sigma}(\theta)) \geq 0$ only if $\omega(\theta) \geq \alpha$.
$\rightsquigarrow$ Argument relies on nonlinearity of subsidy program.

How to Subsidize?

Positive Correlation

Optimal marginal subsidy schedule with positive correlation:

Case 1: $\mathbf{E}[\omega] \geq \alpha$ (consumption distorted upwards for all consumers)

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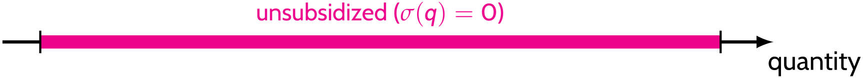

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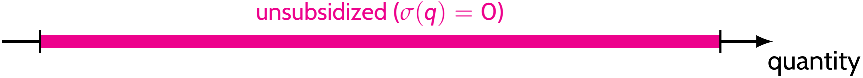

Economic Implications: Negative vs. Positive Correlation

When? Theorem 1 $\rightsquigarrow$ scope of intervention larger with positive correlation ($\max \omega > \alpha$) than negative correlation ($\mathbf{E}[\omega] > \alpha$).

In practice, many government programs focused on goods consumed disproportionately by needy.

Economic Implications: Negative vs. Positive Correlation

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How? Significant differences in marginal subsidy schedules observed in practice:

Larger subsidies for low q

- Food stamps (SNAP)
- Womens, Infants & Children (WIC) Program
- Housing Choice (Section 8) Vouchers
- Lifeline (Telecomm. Assistance) Program

Larger subsidies for high q

- Public transit fare capping
- Pharmaceutical subsidy programs
- Government-subsidized childcare places.

Conclusion

Concluding Remarks

Takeaways for Subsidy Policy:

- Linear subsidies are **never** optimal.
- When and how to subsidize depends on **correlation** between demand and need.
  - With negative correlation (many goods), why not lump-sum cash transfers? ("tortilla subsidy" vs. Progresa).
  - Goods with positive correlation are ideal candidates for subsidies (e.g., public transport) but these should have floors to improve targeting.

Technical Contribution:

- We show how to solve mechanism design problems with lower-bound constraints caused by type-dependent outside options.
- Similar mechanism design problems arise in other contexts.

Companion Paper:

- What are optimal subsidies when topping up is restricted? $\rightsquigarrow$ majorization constraint.
- Negative correlation: planner intervenes more often. Positive correlation: no change in subsidy design.

Fin

Appendices

Equilibrium Effects

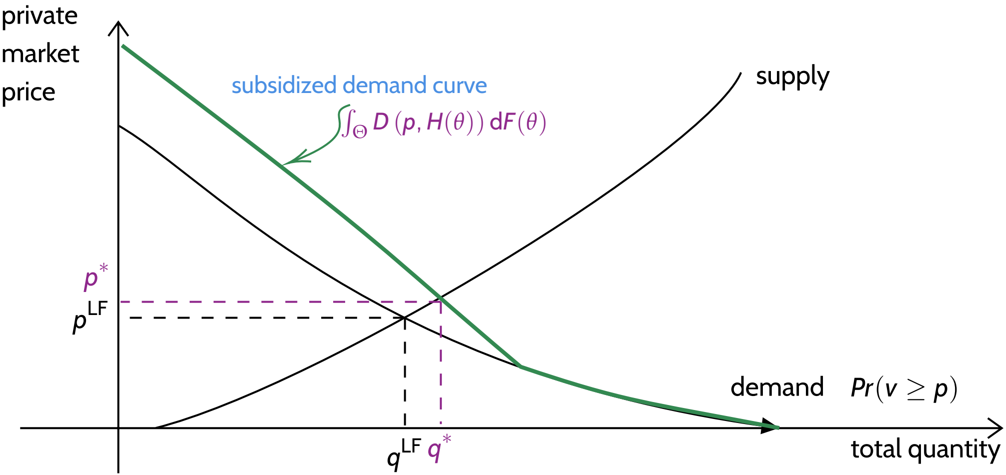
Our baseline model shuts down equilibrium effects of government subsidies on private market prices.

Empirical evidence of price effects from government subsidy programs, *e.g.*:

- public housing (Diamond and McQuade, 2019; Baum-Snow and Marion, 2009)
- pharmaceuticals (Atal et al., 2021)
- public schools (Dinerstein and Smith, 2021)
- school lunches (Handbury and Moshary, 2021)

Equilibrium Effects

Our results extend directly to imperfectly elastic supply curves:



Private Market Taxation

Our baseline model assumes the planner cannot tax the private market.

Taxation of private market reduces consumers' outside option, relaxing the (LB) constraint. If taxation is costly (*e.g.*, because of distortions on ineligible consumers):

Proposition. Suppose the planner faces a convex cost $\Gamma(\tau)$ for taxation of the private market. Then there exists an optimal tax level τ^* and subsidy program for eligible consumers satisfying

$$q^*(\theta) = D(H_{\tau^*}(\theta)),$$

where $H_{\tau^*}(\theta) \leq H(\theta)$.

Return to Model

Budget Constraints and Endogenous Welfare Weights

In our baseline model, $\omega(\cdot)$ and α are taken exogenously.

Our model can be extended to allow weights to be endogenous (cf. Pai and Strack, 2024):

- $\alpha \Longleftrightarrow$ Lagrange multiplier on the social planner's budget constraint.
- $\omega(\theta) \Longleftrightarrow$ the marginal value of money for a consumer with concave preferences

$$\varphi\left(\theta v(q) + I - t\right),$$

and income $I \sim G_\theta$, known but not observed by the social planner, then

$$\omega(\theta) = \mathbf{E}_{I\sim G_\theta}[\varphi'\left(\theta v(q(\theta)) + I - t(\theta)\right)].$$

Return to Model

Comparative Statics of Subsidies

Question: How do optimal subsidies change when

(a) the social planner's desire to redistribute to each consumer increases?

(b) the correlation between demand and welfare weight increases?

(c) the marginal cost of production decreases?

Details

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▸ Details

Short Answer: Each cause the optimal subsidy program to be more generous.

But (a) and (b) cause $J(\theta)$ to increase for each $\theta \rightsquigarrow$ a larger set of consumers subsidized. (c) does not.