

# Optimal Redistribution Through Subsidies

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# Introduction

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**Our approach:** we pose and solve the mechanism design problem for the **optimal subsidy**.

# Model

# Model Overview

## Consumers

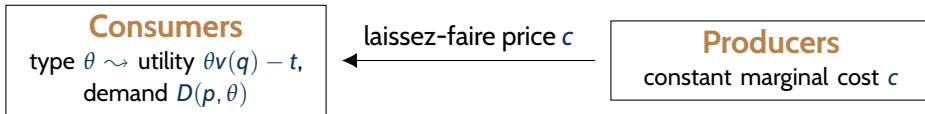
type  $\theta \rightsquigarrow$  utility  $\theta v(q) - t$ ,  
demand  $D(p, \theta)$

## Producers

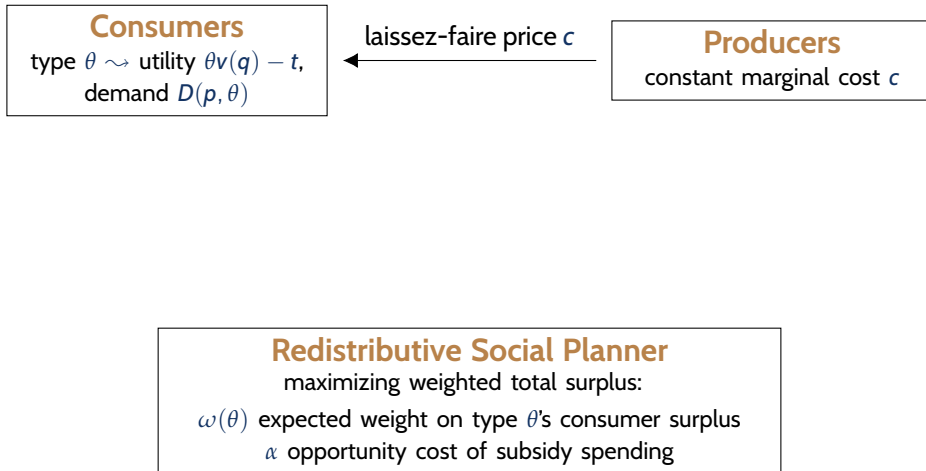
constant marginal cost  $c$



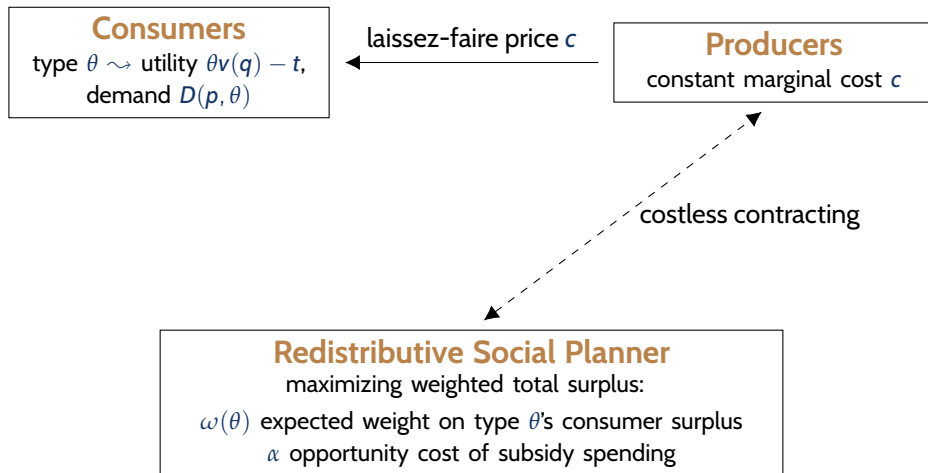
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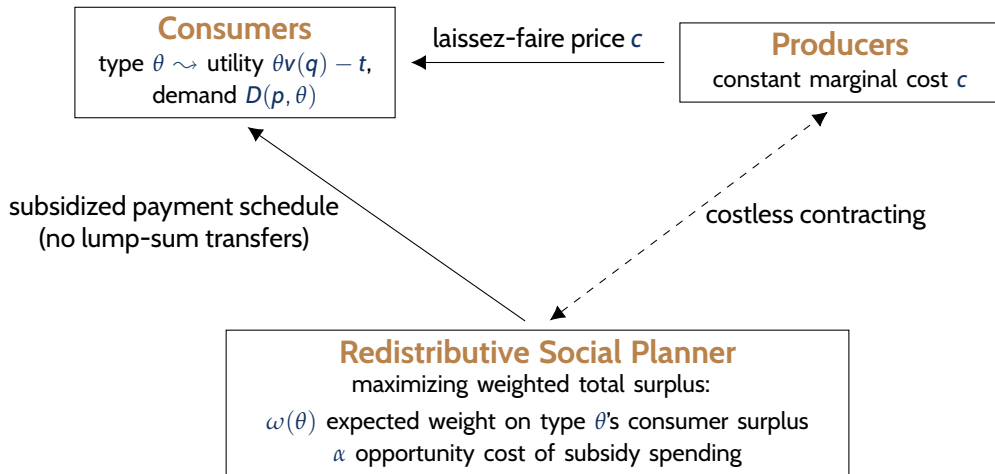
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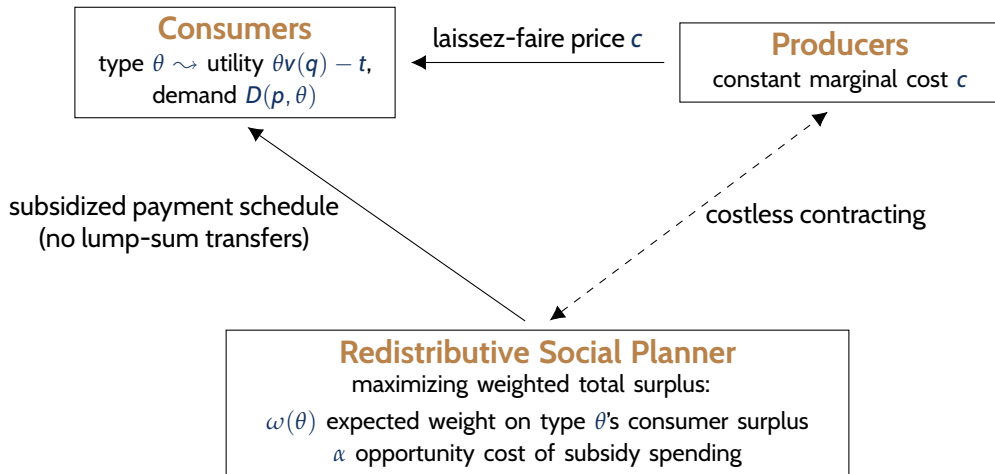
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Key Assumption: Consumers can “top up,” purchasing from **both** subsidized program and private market.

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$$\max_{(q,t)} \int_{\underline{\theta}}^{\bar{\theta}} \left[ \omega(\theta) \underbrace{[\theta v(q(\theta)) - t(\theta)]}_{\text{consumer surplus}} + \alpha \underbrace{[t(\theta) - cq(\theta)]}_{\text{total profit}} \right] dF(\theta),$$

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**Remark:** If  $\omega(\theta) > \alpha$ , the planner would want to transfer cash to  $\theta$  (if  $\mathbf{E}[\omega(\theta)] > \alpha$ , an average consumer).

# Reformulation

The social planner maximizes weighted total surplus,

$$\max_{\substack{\underline{U} \leq \underline{\theta} v(q(\underline{\theta})), \\ q \text{ non-decreasing}}} \left\{ [\mathbf{E}[\omega] - \alpha] \underline{U} + \int_{\underline{\theta}}^{\bar{\theta}} \left[ \left[ \alpha \theta + \frac{\int_{\theta}^{\bar{\theta}} [\omega(s) - \alpha] dF(s)}{f(\theta)} \right] v(q(\theta)) - \alpha c q(\theta) \right] dF(\theta) \right\},$$

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The social planner maximizes weighted total surplus,

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In tariff space, **(TU)** is equivalent to **marginal price**  $\leq c$ .

# Negative Correlation

# Negative Correlation Assumption

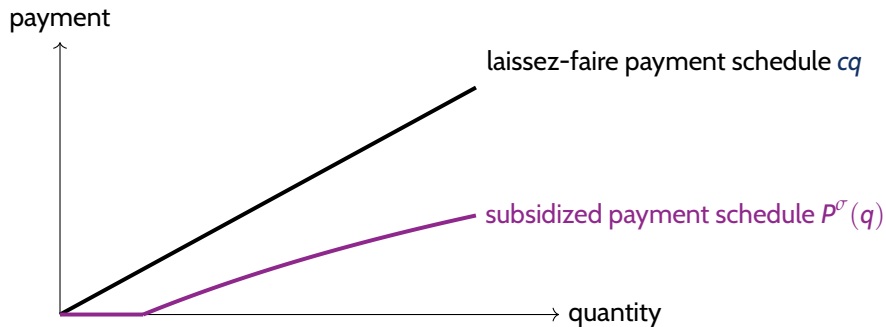
For now, assume  $\omega(\theta)$  is decreasing in  $\theta$ .

- ▶ high-demand consumers tend to have lower need for redistribution.
- ▶ e.g., food, education, and, if  $\omega \propto 1/\text{Income}$ , **normal** goods.



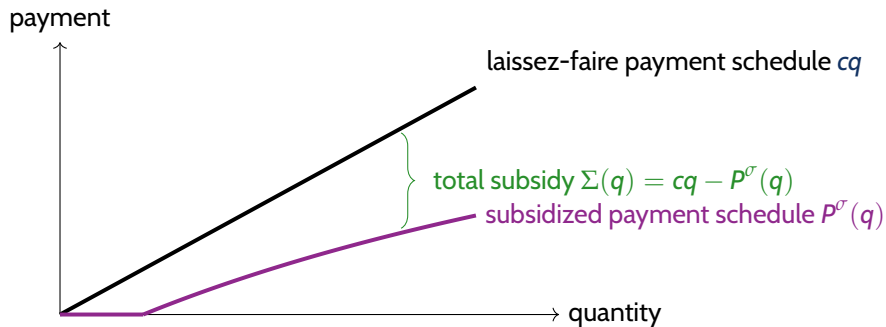
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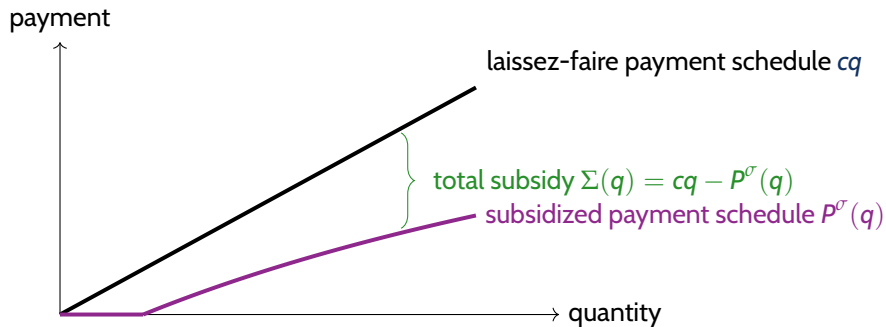


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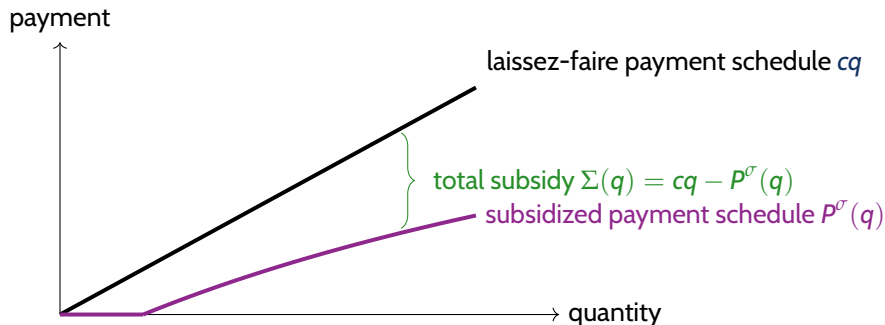


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Subsidies are captured disproportionately by **high**  $\theta$  consumers.

# When to Subsidize?

Because higher  $\theta$  consumers have lower welfare weights  $\omega(\theta)$ , we have the following:

**Proposition.** For any subsidy schedule  $P^\sigma$ , the social planner would prefer to make a lump-sum transfer of  $\mathbf{E}_\theta[\Sigma(q^\sigma(\theta))]$  to all consumers than the subsidy outcome.

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This implies that the social planner would subsidize consumption **only if**  $\mathbf{E}_\theta[\omega(\theta)] > \alpha$ .

On the other hand, when  $\mathbf{E}_\theta[\omega(\theta)] > \alpha$ , because the social planner would always like to make a cash transfer (but cannot by assumption), we have:

**Theorem 1 (Negative Correlation).** The social planner offers consumption subsidies **if and only if**  $\mathbf{E}_\theta[\omega(\theta)] > \alpha$  (and cash transfers are unavailable).

# How to Subsidize?

Optimal marginal subsidy schedule ( $\sigma(q) := \Sigma'(q)$ ) takes one of the following forms:

**Case 1:**  $\min \omega \geq \alpha$  (consumption distorted upwards for all consumers)



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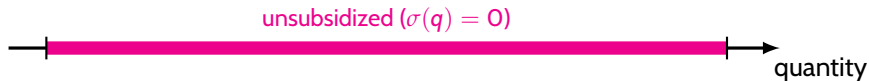
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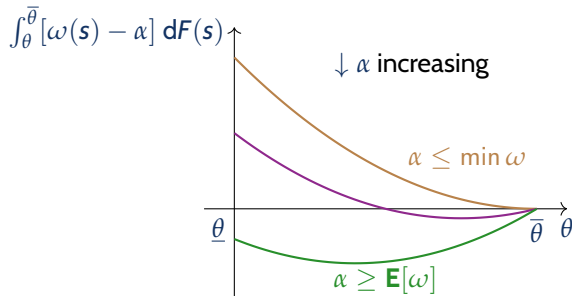
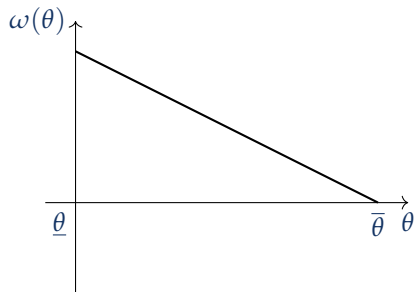


**Case 3:**  $\mathbf{E}[\omega] \leq \alpha$  (no subsidies)



# Intuition: Signing the Distortion of Virtual Type

**Negative correlation** ( $\omega(\theta)$  decreasing)  $\leadsto$  distortion  $J(\theta) - \theta \stackrel{\text{sgn}}{=} \int_{\theta}^{\bar{\theta}} \omega(s) - \alpha \, dF(s)$  is single-crossing zero from above.



Social planner wants to distort consumption of **all types down**, **low-demand types up** and **high-demand types down**, or **all types upwards**.

# Solving for the Optimal Mechanism

▶ skip

$$\begin{aligned} \max_q \alpha \int_{\underline{\theta}}^{\bar{\theta}} [J(\theta)v(q(\theta)) - cq(\theta)] dF(\theta), \\ \text{s.t. } q \text{ nondecreasing and } q(\theta) \geq q^{\text{LF}}(\theta). \end{aligned}$$

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## Guess 1: Pointwise maximizer

$$q(\theta) = (v')^{-1} \left( \frac{c}{J(\theta)} \right) = D(c, J(\theta)).$$

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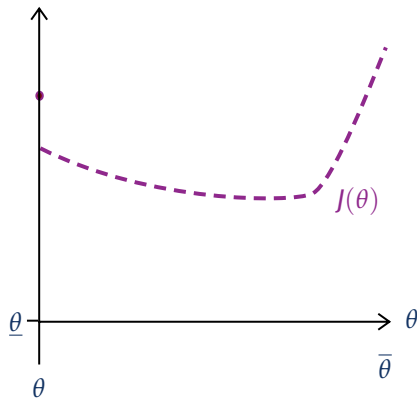
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$J(\theta)$  may be non-monotone.

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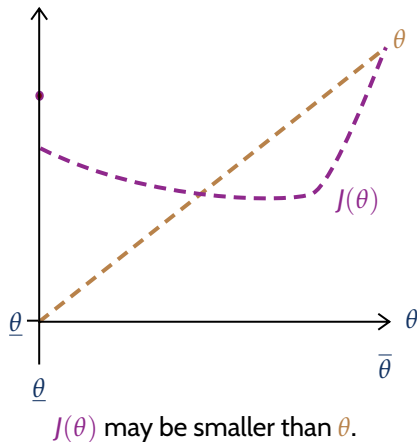
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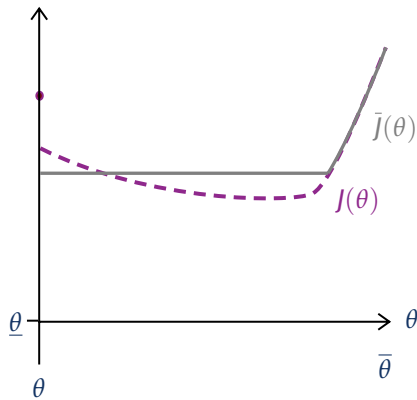
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## Guess 2: Relaxing the (LB) constraint

Toikka (2011); Akbarpour, Dworczak, Kominers (2021)

$$\leadsto q(\theta) = (v')^{-1}\left(\frac{c}{\bar{J}(\theta)}\right) = D(c, \bar{J}(\theta)),$$

where  $\bar{J}$  is ironing of  $J$ , pooling types in any non-monotonic interval of  $J$  at its  $F$ -weighted average.



Ironing deals with non-monotonicity.

► Ironing



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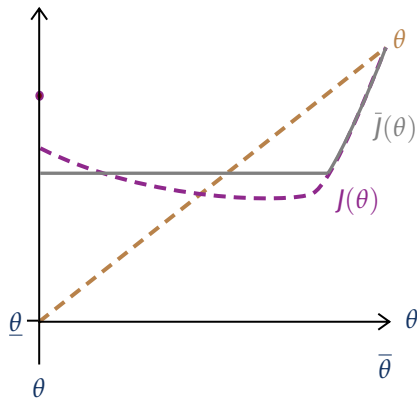
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But not lower-bound constraint  $\leadsto$  interaction.

► Ironing

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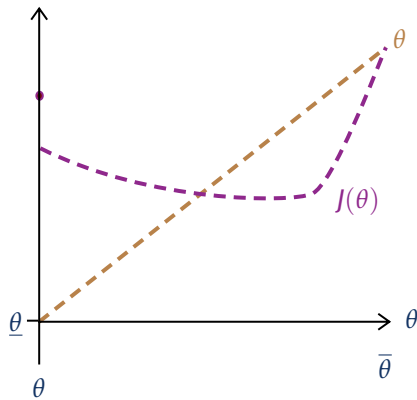
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## Guess 3: Our approach

Suppose solution is of the form

$$q(\theta) = D(c, H(\theta)).$$

Feasibility requires  $H$  to be nondecreasing and satisfy  $H(\theta) \geq \underline{\theta}$ .



Need to identify nondecreasing  $H \geq \underline{\theta}$ .

► Ironing

# Characterizing the Optimal Subsidy Allocation

**Theorem 2 (Negative Correlation).** The optimal allocation rule is unique, continuous and satisfies

$$q^*(\theta) = D(c, H(\theta)),$$

where the **subsidy type**  $H(\theta)$  is defined by

$$H(\theta) := \begin{cases} \overline{J|_{[\underline{\theta}, \theta_\alpha]}}(\theta) & \text{for } \theta \leq \theta_\alpha \\ \theta & \text{for } \theta \geq \theta_\alpha, \end{cases}$$

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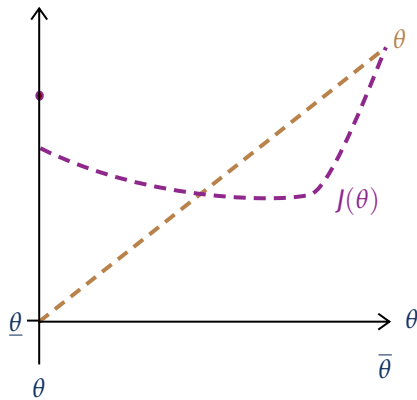
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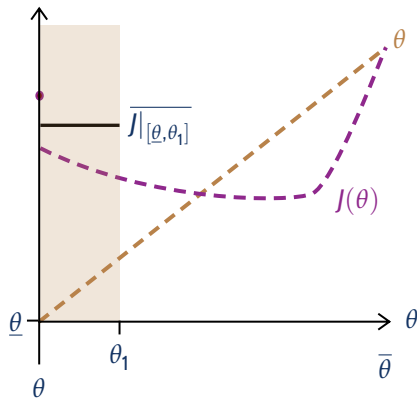
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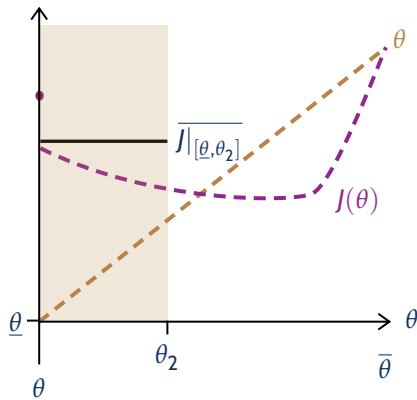
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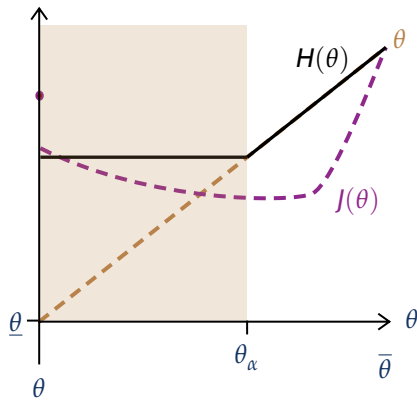
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construction  $\leadsto$  pooling condition and continuity

# Economic Implications

With **negative correlation** between  $\omega$  and  $\theta$ :

# 1. Lump-sum cash transfers are always **more progressive** than subsidies.



# Economic Implications

With **negative correlation** between  $\omega$  and  $\theta$ :

- # 1. Lump-sum cash transfers are always **more progressive** than subsidies.
- # 2. If cash transfers are unavailable, linear subsidies are **never** optimal.
  - # 2a. Optimal subsidies are “all or none”: active subsidy programs should always incorporate a **free allocation** (“public option”).
  - # 2b. If *any* consumer has  $\omega < \alpha$ , optimal subsidies are **capped** in quantity.

# Positive Correlation

# When to Subsidize?

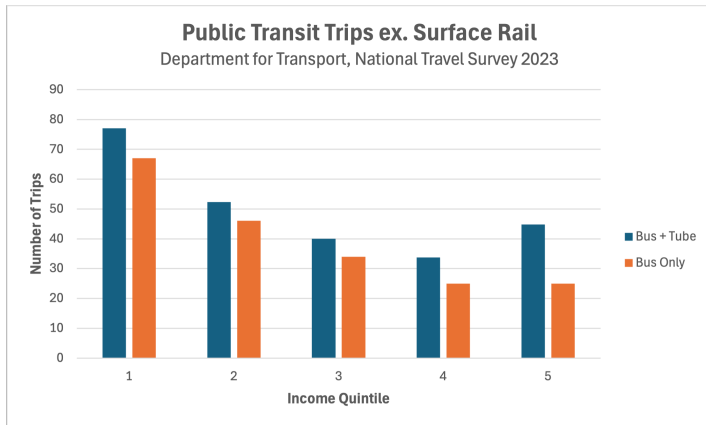
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**Intuition:** Social planner can always design a subsidy program with  $\Sigma(q^{\sigma}(\theta)) \geq 0$  only if  $\omega(\theta) \geq \alpha$ .

$\rightsquigarrow$  Argument relies on nonlinearity of subsidy program.



# How to Subsidize?

## Positive Correlation

Optimal marginal subsidy schedule with positive correlation:

**Case 1:**  $E[\omega] \geq \alpha$  (consumption distorted upwards for all consumers)



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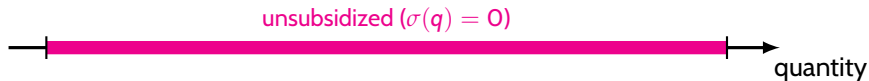
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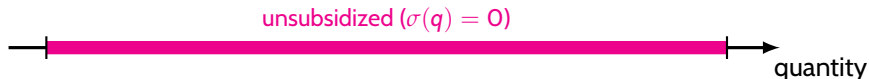
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# Economic Implications: Negative vs. Positive Correlation

**When?** Theorem 1  $\leadsto$  scope of intervention larger with positive correlation ( $\max \omega > \alpha$ ) than negative correlation ( $\mathbf{E}[\omega] > \alpha$ ).

In practice, many government programs focused on goods consumed disproportionately by needy.

# Economic Implications: Negative vs. Positive Correlation

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In practice, many government programs focused on goods consumed disproportionately by needy.

**How?** Significant differences in marginal subsidy schedules observed in practice:

## Larger subsidies for low $q$

- ▶ Food stamps (SNAP)
- ▶ Womens, Infants & Children (WIC) Program
- ▶ Housing Choice (Section 8) Vouchers
- ▶ Lifeline (Telecomm. Assistance) Program

## Larger subsidies for high $q$

- ▶ Public transit fare capping
- ▶ Pharmaceutical subsidy programs
- ▶ Government-subsidized childcare places.

# Conclusion

# Concluding Remarks

## Takeaways for Subsidy Policy:

- ▶ Linear subsidies are **never** optimal.
- ▶ When and how to subsidize depends on **correlation** between demand and need.
  - With negative correlation (many goods), why not lump-sum cash transfers? (“tortilla subsidy” vs. Progresa).
  - Goods with positive correlation are ideal candidates for subsidies (e.g., public transport) but these should have floors to improve targeting.

## Technical Contribution:

- ▶ We show how to solve mechanism design problems with lower-bound constraints caused by type-dependent outside options.
- ▶ Similar mechanism design problems arise in other contexts.

## Companion Paper:

- ▶ What are optimal subsidies when topping up is restricted?  $\rightsquigarrow$  majorization constraint.
- ▶ Negative correlation: planner intervenes more often. Positive correlation: no change in subsidy design.



# Fin

# Appendices

# Related Literature

- ▶ **Public Finance.** Ramsey (1927), Diamond (1975), Mirrlees (1976, 1986), Atkinson & Stiglitz (1976), Nichols & Zeckhauser (1982), Blackorby & Donaldson (1988), Besley & Coate (1991), Blomquist & Christiansen (1998), Doligalski, Dworczak, Krysta & Tokarski (2023).
  - Different objective (local linearization), incorporating a private market not in control of social planner.
  - Focus on commodity subsidization, not ruled out by Atkinson-Stiglitz because no “incentive separability.”
  - We allow for **nonlinear subsidy designs** and characterize global optimum, not FOCs à la Mirrlees.

# Related Literature

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- ▶ **Redistributive Mechanism Design.** Weitzman (1977), Condorelli (2013), Che, Gale & Kim (2013), Dworczak, Kominers & Akbarpour (2021, 2022), Kang (2023,2024), Akbarpour, Budish, Dworczak & Akbarpour (2024), Pai & Strack (2024), Kang & Watt (2024).
  - Dworczak, Akbarpour & Kominers (2021): unit demand consumers trading a fixed supply with different qualities, focusing on cross- and same-side inequality.
  - Akbarpour, Dworczak & Kominers (2022): unit demand, fixed supply, across & within-group matching.
  - Kang (2023): public option with single quality (quantity) level.
  - This paper: **partial control**, we allow consumers to consume in private market outside of planner's control.

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- ▶ **Partial Mechanism Design.** Jullien (2000), Philippon & Skreta (2012), Tirole (2012), Fuchs & Skrzypacz (2015), Dworczak (2020), Loertscher & Muir (2022), Loertscher & Marx (2022), Kang & Muir (2022), Kang (2023), Kang & Watt (2024).
  - **This paper:** focus on equally efficient planner, “topping up” (cf. Kang & Watt, 2024).

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- ▶ **Methodological Tools in Mechanism Design.** Jullien (2000), Toikka (2011), Corrao, Flynn & Sastry (2023), Yang & Zentefis (2024), Valenzuela-Stookey & Poggi (2024).
  - Yang & Zentefis (2024) study extrema of “monotone function intervals”  $\rightsquigarrow$  optima of linear objectives.
  - **This paper:** explicit characterization of solution with FOSD (topping up) constraint.

# Equilibrium Effects

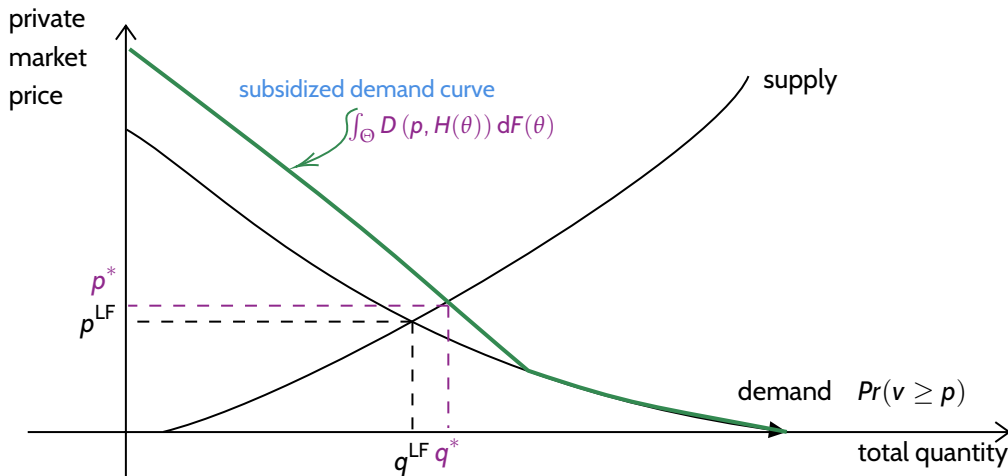
Our baseline model shuts down equilibrium effects of government subsidies on private market prices.

Empirical evidence of price effects from government subsidy programs, e.g.:

- ▶ public housing ([Diamond and McQuade, 2019](#); [Baum-Snow and Marion, 2009](#))
- ▶ pharmaceuticals ([Atal et al., 2021](#))
- ▶ public schools ([Dinerstein and Smith, 2021](#))
- ▶ school lunches ([Handbury and Moshary, 2021](#))

# Equilibrium Effects

Our results extend directly to imperfectly elastic supply curves:





# Private Market Taxation

Our baseline model assumes the planner cannot tax the private market.

Taxation of private market **reduces** consumers' outside option, relaxing the **(LB)** constraint. If taxation is costly (e.g., because of distortions on ineligible consumers):

**Proposition.** Suppose the planner faces a convex cost  $\Gamma(\tau)$  for taxation of the private market. Then there exists an optimal tax level  $\tau^*$  and subsidy program for eligible consumers satisfying

$$q^*(\theta) = D(H_{\tau^*}(\theta)),$$

where  $H_{\tau^*}(\theta) \leq H(\theta)$ .

# Budget Constraints and Endogenous Welfare Weights

In our baseline model,  $\omega(\cdot)$  and  $\alpha$  are taken exogenously.

Our model can be extended to allow weights to be endogenous (cf. [Pai and Strack, 2024](#)):

- ▶  $\alpha \iff$  Lagrange multiplier on the social planner's budget constraint.
- ▶  $\omega(\theta) \iff$  the marginal value of money for a consumer with **concave** preferences

$$\varphi(\theta v(q) + I - t),$$

and income  $I \sim G_\theta$ , known but not observed by the social planner, then

$$\omega(\theta) = \mathbf{E}_{I \sim G_\theta}[\varphi'(\theta v(q(\theta)) + I - t(\theta))].$$

# Comparative Statics of Subsidies

**Question:** How do optimal subsidies change when

- (a) the social planner's desire to redistribute to each consumer increases?
- (b) the correlation between demand and welfare weight increases?
- (c) the marginal cost of production decreases?

► Details

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► Details

**Short Answer:** Each cause the optimal subsidy program to be more generous.

But (a) and (b) cause  $J(\theta)$  to increase for each  $\theta \rightsquigarrow$  a larger set of consumers subsidized. (c) does not.