Who Gets What and When

Dynamic Incentives in Repeated Matching Markets

Mitchell Watt | mwatt@stanford.edu | 34th Stony Brook International Conference on Game Theory | July 27, 2023

A Motivating Example

Ridesharing

- Jobs appear ~ randomly over time.
- Net of payments, Castro et al. (2021) show substantial heterogeneity in job value.
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Alternative: promise better *future* positions in the queue to drivers who accept *worse* jobs *today.*

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Key results:

- Principal incentivizes undesirable allocations using promises of improved future allocations.
- Principal is “loyal”: agents with worse historical allocations are prioritized for better current allocations.

Model

A fixed population of N agents and a single principal, who owns a stream of arriving items.

In each period, $t = 0,1,2,...$:

1. N items arrive with values v_t observed by principal and agents.
   Arrival process is common knowledge, values may be negative (“bads”).

2. Principal offers an item to agent i with probability x_{it}.

3. Agent i accepts with probability y_{it}, its outside option is normalized to 0.
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Agents care about quality of matches:

$$U^A = (1 - \delta_A) \sum_t \delta_A^T x_{it} y_{it} v_{it}$$

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Principal chooses history-dependent allocation rule to maximize payoffs subject to obedience constraint for agents $\Rightarrow$ reformulate recursively using *promised utility* as state vector

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i's value today promised utility current promise

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Theorem: $\Phi(\cdot)$ exists, is unique, monotone decreasing, Schur-concave, continuous, semi-differentiable

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4. Long-run performance depends on ratio of δ^P to δ^A.

Illustrating the Optimal Contract

Two agents, $\delta^A = 0.8$ and a patient principle, $\delta^P = 0.9$.
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Agent 1 Agent 2 Unallocated

$t = 1$

Assign one to each agent: everyone is happy!

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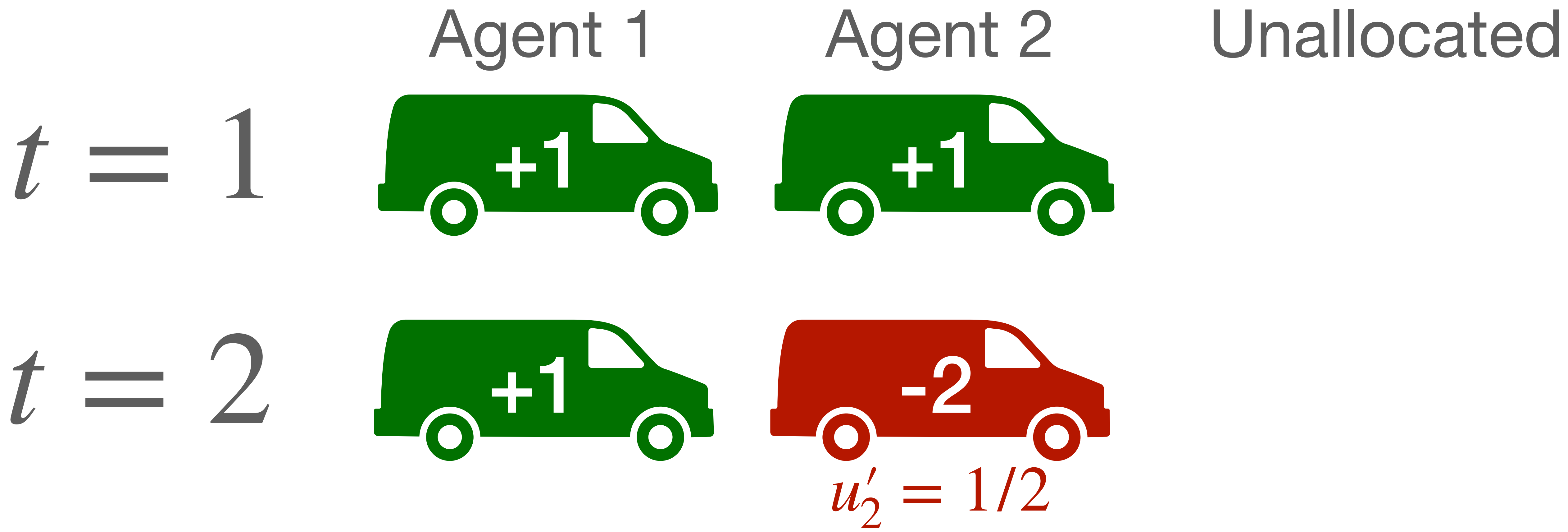

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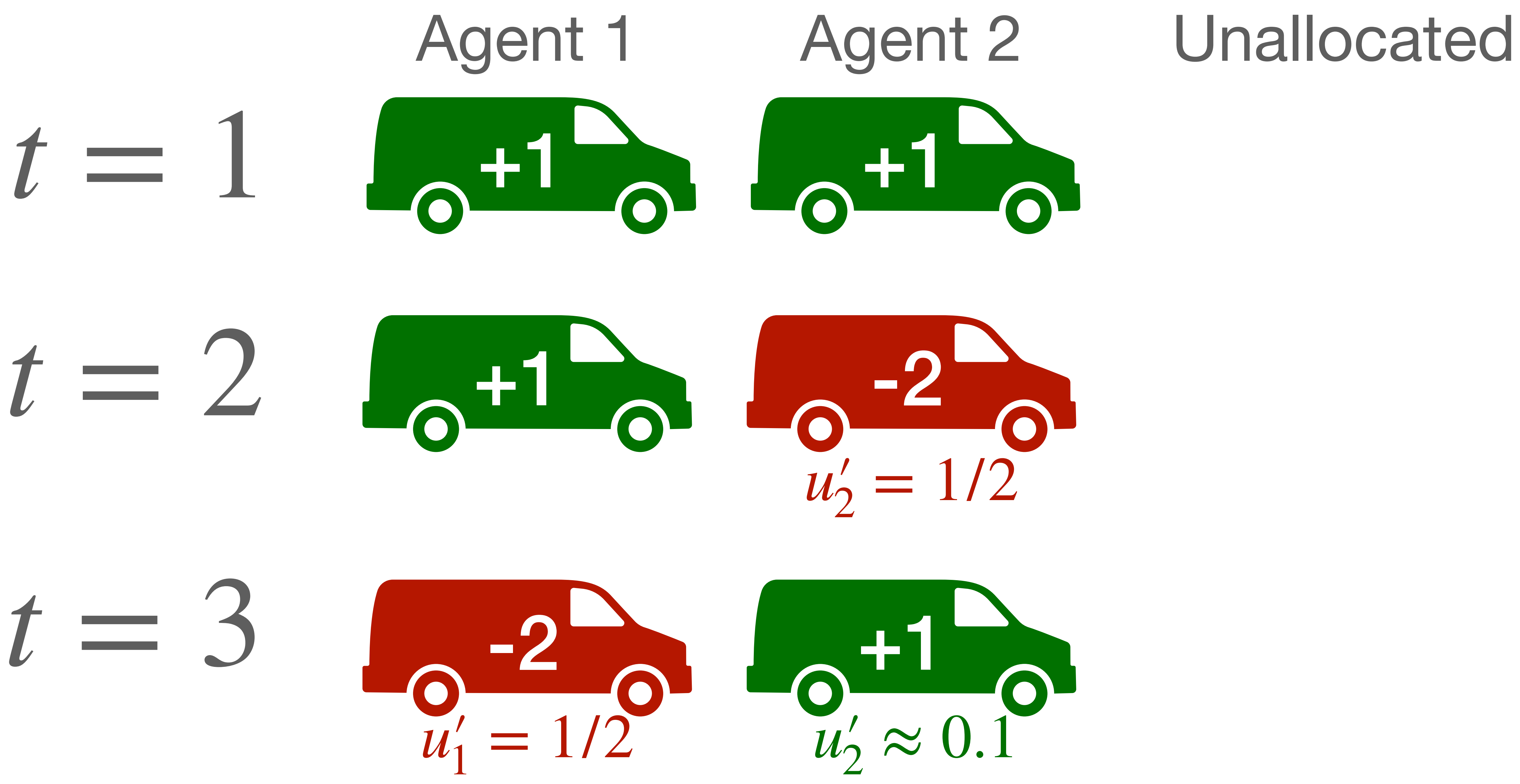

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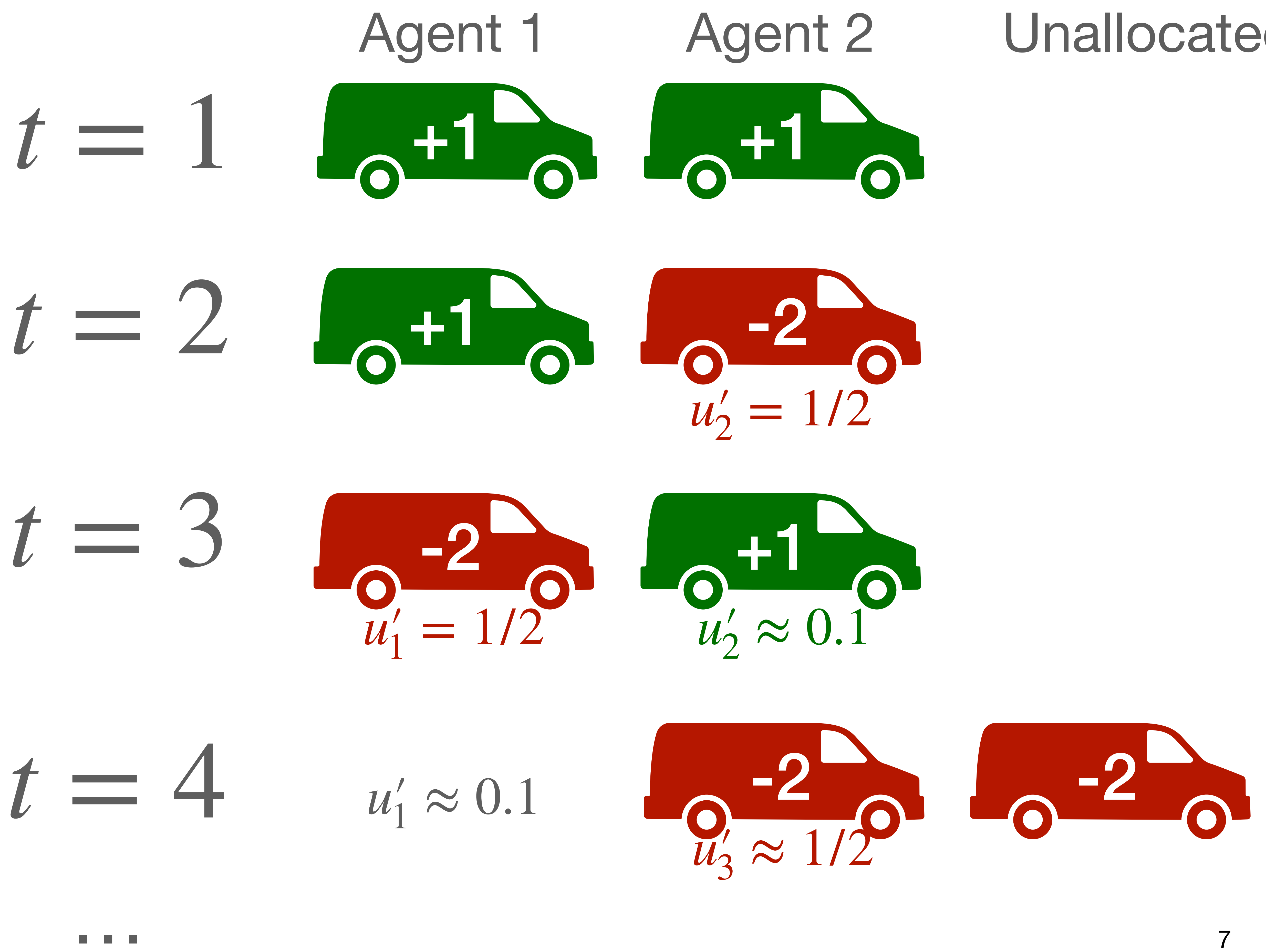

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Assign only one bad item and reduce promise to agent 1

Schur-Concavity + Intuition for Loyalty

Majorization pre-order: $u \preceq u'$ if after re-ordering components of u and u' in increasing order, we have that for all k

$$\sum_{i=1}^{k} u_i \leq \sum_{i=1}^{k} u_i', \text{ and } \sum_{i=1}^{N} u_i = \sum_{i=1}^{N} u_i'.$$

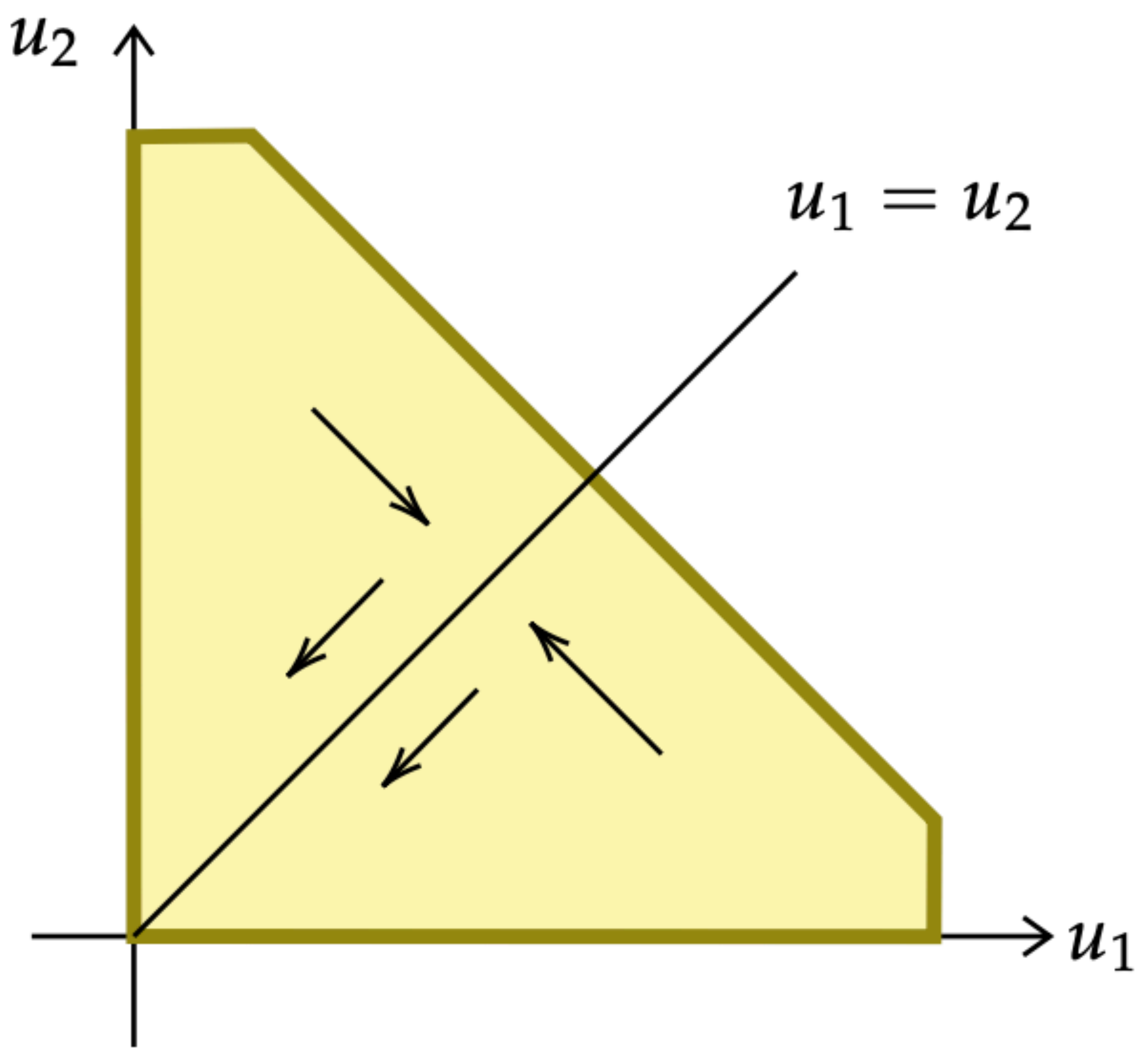
For example,

$$\left(\frac{1}{n},\ldots,\frac{1}{n}\right) \preceq \left(\frac{1}{n-1},\ldots,\frac{1}{n-1},0\right) \preceq \ldots \preceq \left(\frac{1}{2},\frac{1}{2},\ldots\right) \preceq (1,0,\ldots 0)$$

Symmetry + Concavity ⇒ Schur-concavity: Φ decreases in majorization pre-order

Schur-Ostrowski criterion:

Φ is Schur-concave iff $u_i < u_j \Rightarrow \dfrac{\partial \Phi}{\partial u_i} > \dfrac{\partial \Phi}{\partial u_j}$



Schur-Concavity of Φ implies principal prefers *equalization* of promised utilities among agents

Endogenous Loyalty

$$\Phi(u) = \max_{X(v;u)\in\mathcal{X}(v),u'(v;u)\in\mathcal{U}} \mathbb{E}_{v\sim F}\left[(1-\delta_P)\,|\,X(v;u)\,|\,+\delta_P\Phi\left(u'(v;u)\right)\right] \text{ subject to}$$

$$\mathbb{E}_{v\sim F}\left[(1-\delta_A)v_i^X(v;u)+\delta_A u_i'(v;u)\right] \geq u_i, \text{ for each } i, \quad \lambda_i(u)$$

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Optimality for X and envelope theorem implies:

$$X(v;u) \text{ solves } \max_{X\in\mathcal{X}(v)} (1-\delta_P)\,|X| + (1-\delta_A)\lambda(u)\cdot v^X + (1-\delta_A)\mu(v;u)\cdot v^X$$

$$D_u\Phi(u) = -\lambda(u)$$

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For simplicity, consider $v \gg 0$. Then $X(v;u)$ solves $\max_{X\in\mathcal{X}(v)} (1-\delta_P)\,|\,X\,|\,-(1-\delta_A)D_u\Phi(u)\cdot v^X$

But Schur-concavity implies $u_i < u_j \Rightarrow \dfrac{\partial\Phi}{\partial u_i} > \dfrac{\partial\Phi}{\partial u_j}$, but then high v_i^X should be paired with high $-\dfrac{\partial\Phi}{\partial u_i} \Rightarrow$ assortativity.

Long-Run Performance

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$$\delta_P D_u\Phi(u'(v;u)) + \delta_A\lambda(u) + \delta_A\mu(v;u) = 0 \Rightarrow D_u\Phi(u'(v;u)) = -\frac{\delta_A}{\delta_P}(\lambda(u) + \mu(v;u))$$

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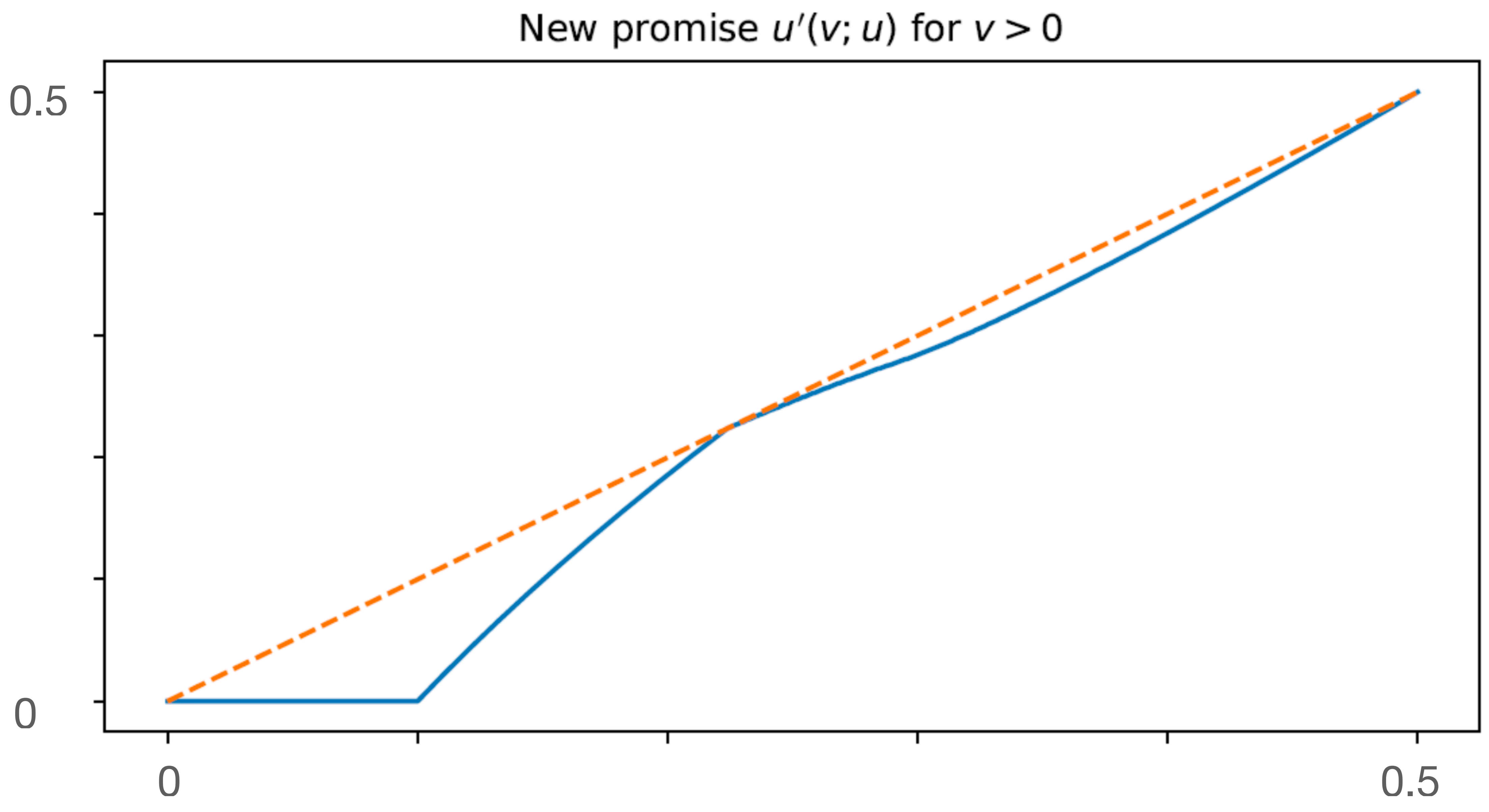
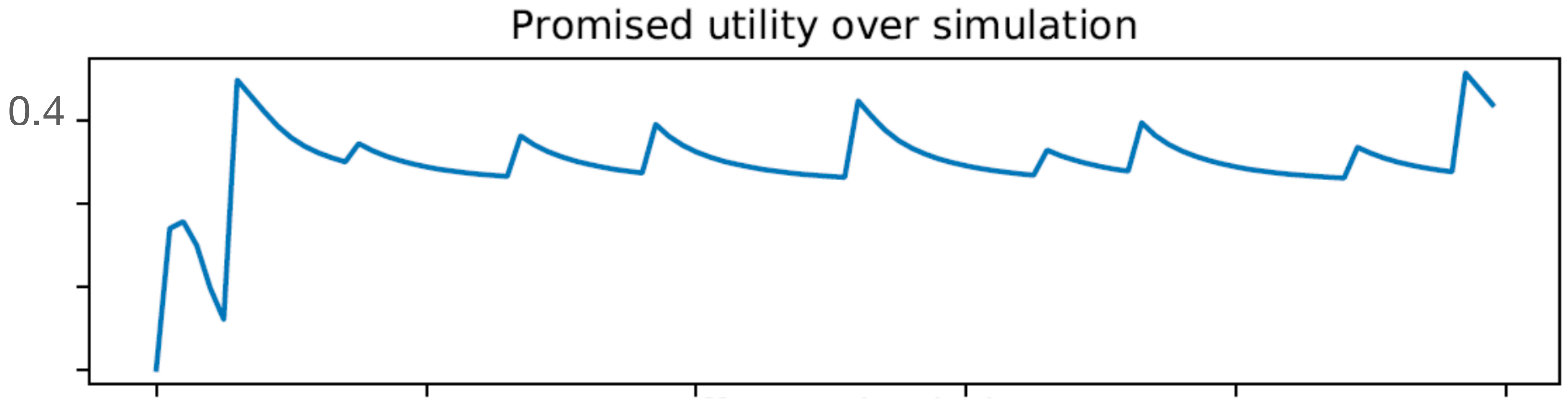
Together, implies for $v \gg 0$ that $D_u\Phi(u'(v;u)) = \frac{\delta_A}{\delta_P}D_u\Phi(u'(v;u))$

Illustrating Long-Run Performance

Consider single-agent version of previous example:

If principle is *more patient* than agent, it ‘works’ off promises over time and more bad items are allocated in long-run, e.g. $\delta_P = 0.9 > \delta_A = 0.8$

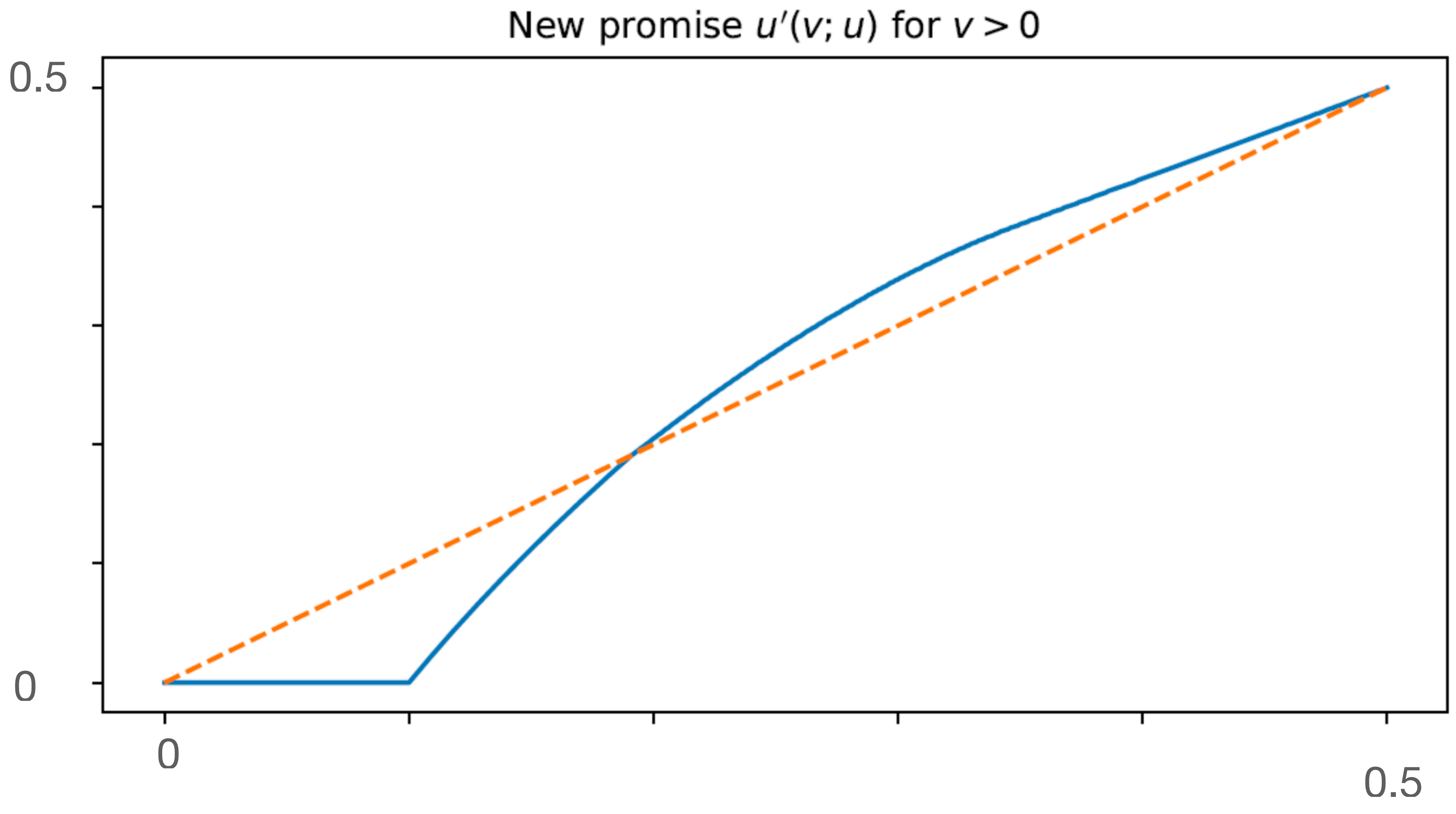


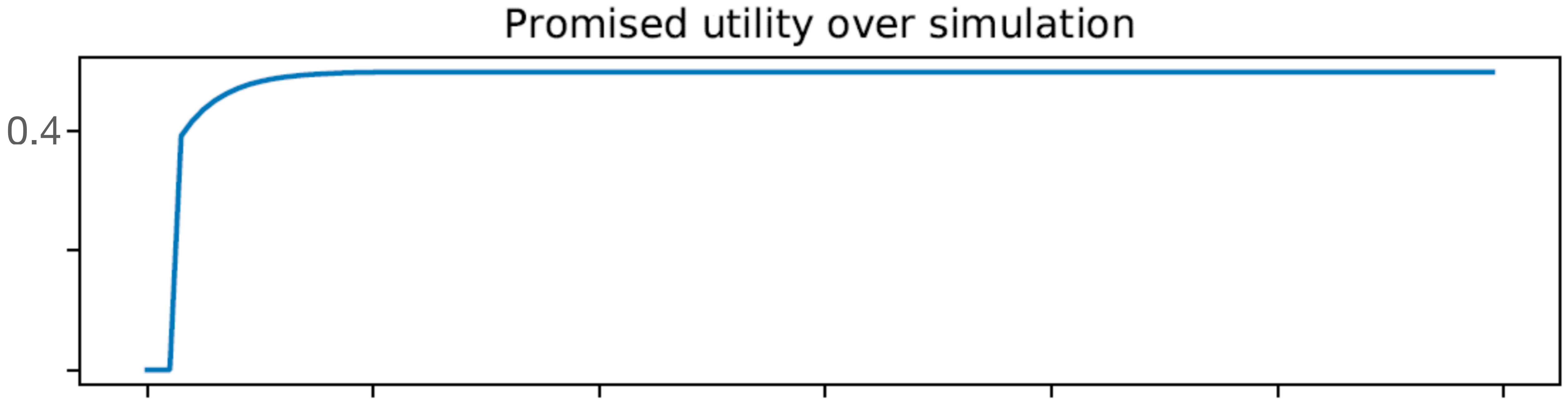


Illustrating Long-Run Performance

Consider single-agent version of previous example:

If principle is *less patient* than agents, it always promises more to incentivize bad item allocation today: eventually only good items allocated (c.f. ‘immiseration’).





Conclusion

I introduce a repeated matching model with a fixed population of agents and a period-by-period *ex post* participation constraint.

1. The principal promises better future allocations to agents who accept bad items in current period.
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Implication: My results suggest that ride-sharing apps could benefit from using dynamic incentives, transforming first-come-first-served to worst-off first-served. Practical implementation could include 'fast pass' in airport queues.