

# Optimal Redistribution Through Subsidies

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# Introduction

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**Our approach:** we pose and solve the mechanism design problem for the **optimal subsidy**.

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Heterogeneous outside options are empirically relevant, e.g.,

- ▶ public housing ([van Dijk, 2019](#); [Waldinger, 2021](#)),
- ▶ education ([Akbarpour, Kapor, Neilson, van Dijk & Zimmerman, 2022](#); [Kapor, Karnani & Neilson, 2024](#)),
- ▶ healthcare ([Li, 2017](#); [Heim, Lurie, Mullen & Simon, 2021](#)),
- ▶ SNAP ([Haider, Jacknowitz & Schoeni, 2003](#); [Ko & Moffitt, 2024](#); [Rafkin, Solomon & Soltas, 2024](#)).

Heterogeneous outside options lead to **lower-bound constraints** in the mechanism design problem.



# Results Overview

We provide an **explicit characterization** of:

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How?	Free provision and partial marginal subsidies for <b>low</b> levels of consumption.	Partial marginal subsidies for <b>higher</b> levels of consumption.

~> Linear subsidies are never optimal.

# Related Literature

- ▶ **Public Finance.** Ramsey (1927), Diamond (1975), Mirrlees (1976, 1986), Atkinson & Stiglitz (1976), Nichols & Zeckhauser (1982), Blackorby & Donaldson (1988), Besley & Coate (1991), Blomquist & Christiansen (1998), Doligalski, Dworczak, Krysta & Tokarski (2023).
  - ↪ **This paper:** allows for nonlinear subsidy designs.
- ▶ **Redistributive Mechanism Design.** Weitzman (1977), Condorelli (2013), Che, Gale & Kim (2013), Dworczak, Kominers & Akbarpour (2021, 2022), Kang (2023,2024), Akbarpour, Budish, Dworczak & Akbarpour (2024), Pai & Strack (2024).
  - ↪ **This paper:** allows consumers to consume in private market outside of planner's control.
- ▶ **Partial Mechanism Design.** Jullien (2000), Philippon & Skreta (2012), Tirole (2012), Fuchs & Skrzypacz (2015), Dworczak (2020), Loertscher & Muir (2022), Kang & Muir (2022), Kang (2023), Kang & Watt (2024).
  - ↪ **This paper:** focus on benchmark where planner is as efficient as private market, “topping up.”
- ▶ **Methodological Tools in Mechanism Design.** Jullien (2000), Toikka (2011), Corrao, Flynn & Sastry (2023), Yang & Zentefis (2024), Valenzuela-Stookey & Poggi (2024).
  - ↪ **This paper:** explicit characterization of solution with FOSD (topping up) constraint.

# Model



# Model Overview

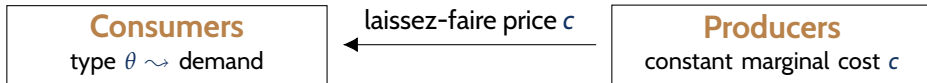
## Consumers

type  $\theta \rightsquigarrow$  demand

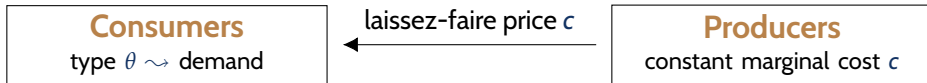
## Producers

constant marginal cost  $c$

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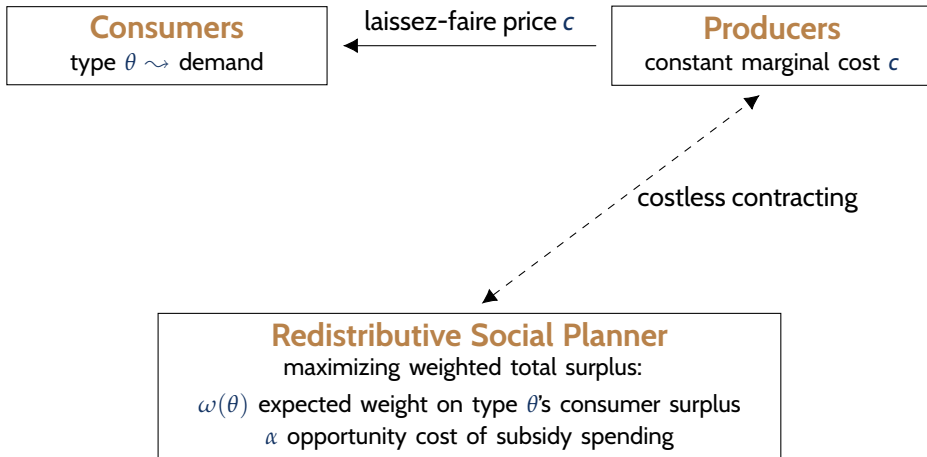
## Redistributive Social Planner

maximizing weighted total surplus:

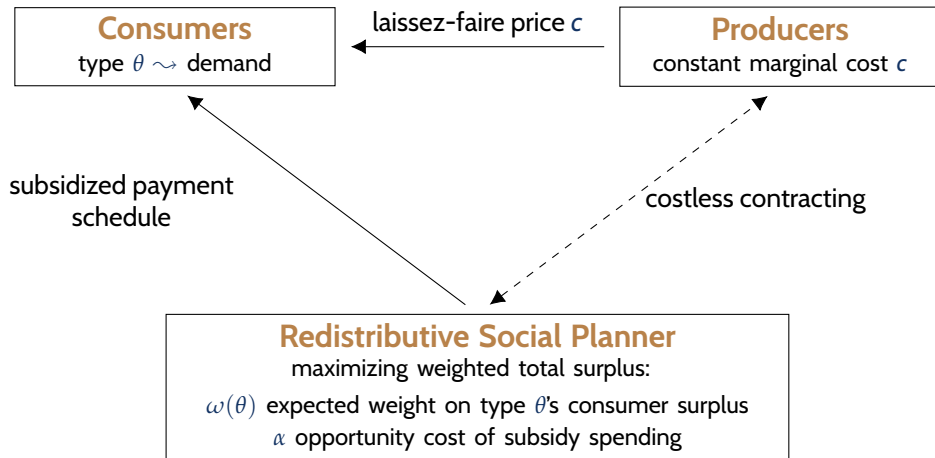
$\omega(\theta)$  expected weight on type  $\theta$ 's consumer surplus

$\alpha$  opportunity cost of subsidy spending

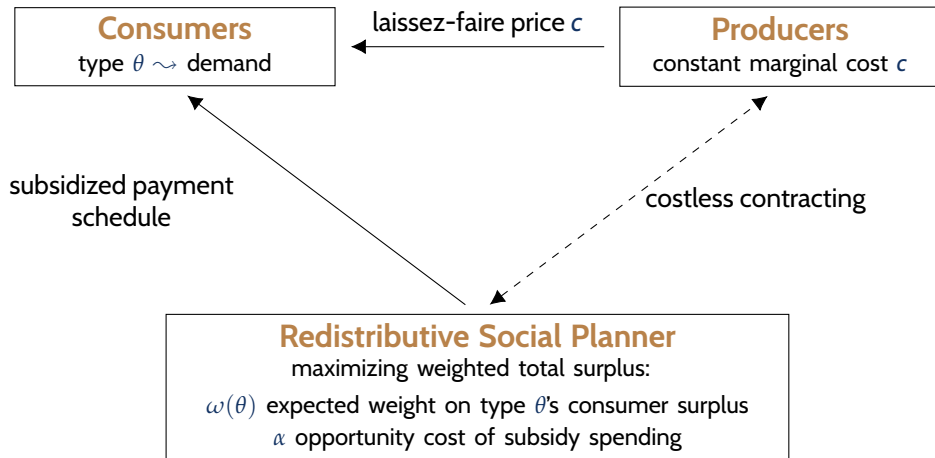
# Model Overview



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Consumers can purchase units from **both subsidized program and private market**.

# Setup

## Consumers:

- ▶ There is a unit mass of risk-neutral consumers in market for a divisible, homogeneous good.
- ▶ Consumers differ in type  $\theta \in [\underline{\theta}, \bar{\theta}]$  with  $\underline{\theta} \geq 0$ , and  $\theta \sim F$ , continuous with density  $f > 0$ .
- ▶ Each consumer derives utility  $\theta v(q) - t$  from quantity  $q \in [0, A]$  given payment  $t$ .  
 $v : [0, A] \rightarrow \mathbb{R}$  is differentiable with  $v' > 0$  and  $v'' < 0$ .

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**Extensions** (not today): equilibrium effects, observable characteristics, product choice and eligibility.

# Laissez-Faire Equilibrium

- ▶ Perfectly competitive private market  $\leadsto$  **laissez-faire price**  $p^{\text{LF}} = c$  per unit.
- ▶ Each consumer solves

$$U^{\text{LF}}(\theta) := \max_{q \in [0, A]} [\theta v(q) - cq] .$$

$v$  is strictly concave  $\leadsto$  unique maximizer:

$$q^{\text{LF}}(\theta) = (v')^{-1} \left( \frac{c}{\theta} \right) = D(c, \theta) .$$

- ▶ To simplify statements of some results, assume today that  $q^{\text{LF}}(\underline{\theta}) > 0$ .

# Subsidy Design

Social planner costlessly contracts with firms and sells units at a **subsidized payment schedule**  $P^\sigma(q)$ .

$\leadsto \Sigma(q) = cq - P^\sigma(q)$  is the **total subsidy** as a function of  $q$ , and  $\sigma(q) = \Sigma'(q)$  is the **marginal subsidy**.

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- # 1. Each consumer can **top up** his consumption of the good, allowing him to purchase additional units in the private market at price  $c$ ,

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Implementation: Consumer  $\theta$  solves  $U^\sigma(\theta) := \max_q [\theta v(q) - P^\sigma(q)]$ , leading to **subsidized demand**  $q^\sigma(\theta)$ .

# Redistributive Objective

The social planner seeks to maximize **weighted total surplus**.

- ▶ Consumer surplus: social planner assigns a welfare weight  $\omega(\theta) := \mathbf{E}[\omega|\theta]$  to consumer type  $\theta$ .  
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**Remarks:**

- ▶ If  $\omega(\theta) > \alpha$ , social planner would want to transfer a dollar to type  $\theta$ .
- ▶ If  $\mathbf{E}_{\theta}[\omega(\theta)] > \alpha$ , social planner would want to make a lump-sum cash transfer to all consumers.

# Correlation Assumption

## Two baseline cases:

“**Negative Correlation**”:  $\omega(\theta)$  is decreasing in  $\theta$ .

- ▶ high-demand consumers tend to have lower need for redistribution.
- ▶ e.g., food, education, and, if  $\omega \propto 1/\text{Income}$ , **normal** goods.

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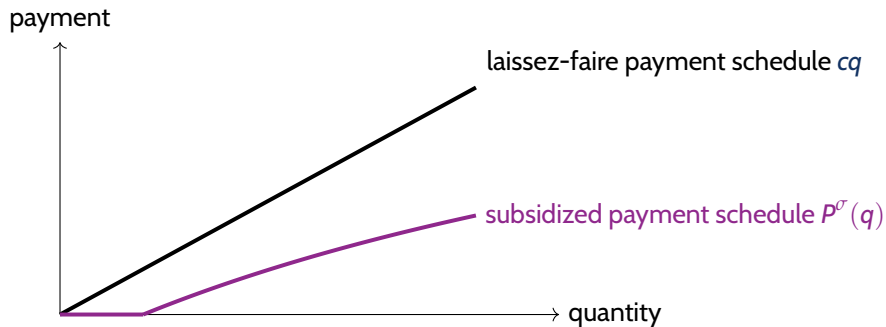
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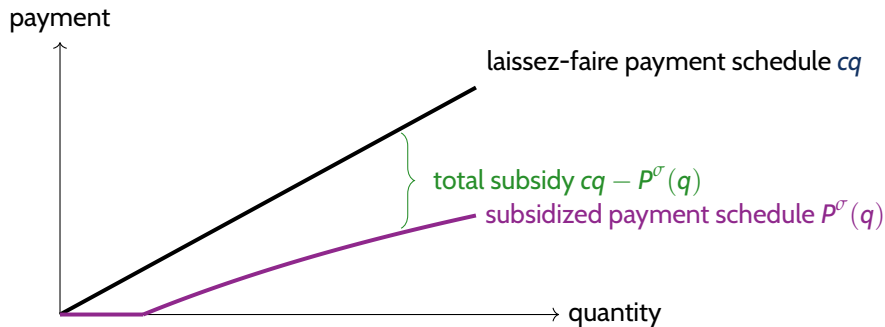
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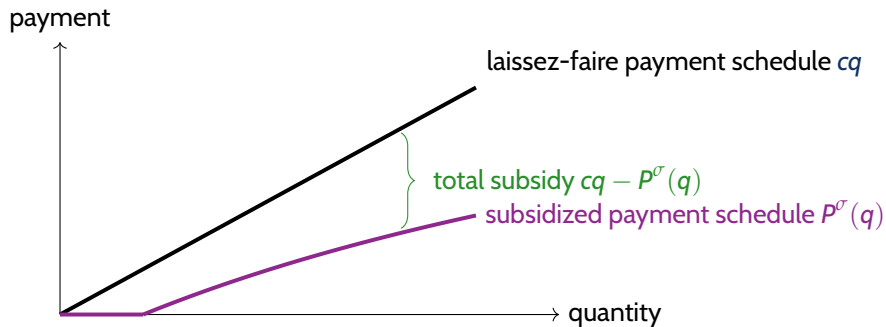


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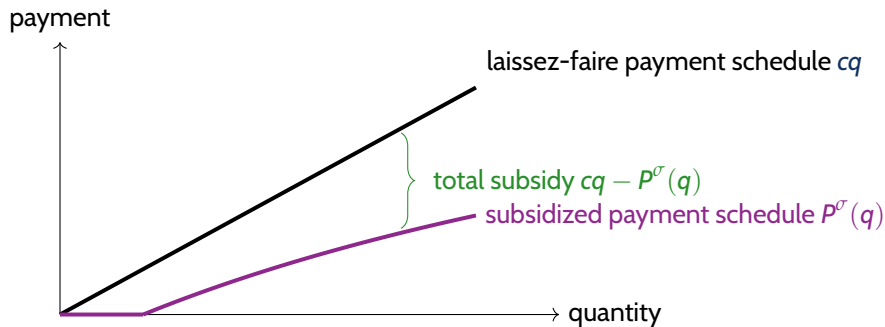


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Subsidies are captured disproportionately by **high**  $\theta$  consumers.

# When Not To Subsidize?

**Recall** our “negative correlation” assumption: high  $\theta$  consumers have lower  $\omega$ .

**Proposition.** For any subsidy  $P^\sigma$ , the social planner would prefer to make a lump-sum transfer of  $\mathbf{E}_\theta[\Sigma(q^\sigma(\theta))]$  to all consumers than the subsidy outcome.

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**Theorem 1 (Negative Correlation, part).** The social planner subsidizes consumption **only if**  $\mathbf{E}_\theta[\omega(\theta)] > \alpha$  (and cash transfers are unavailable).

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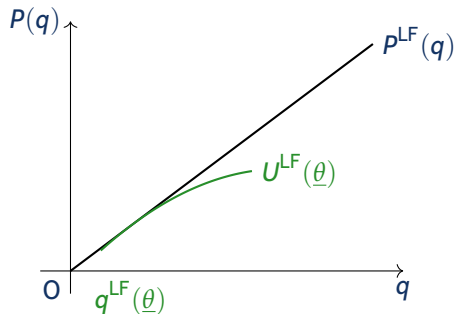
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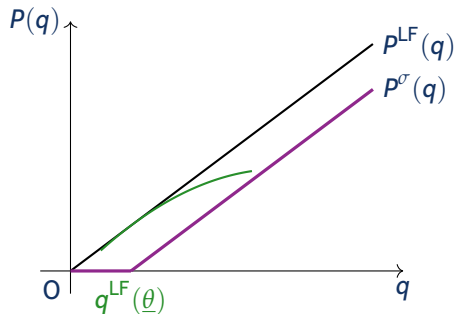
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$P^{\sigma}$  is outcome-equivalent to a cash transfer of  $cq^{\text{LF}}(\underline{\theta})$  to all consumers, and improves over laissez-faire because  $\mathbf{E}_{\theta}[\omega(\theta)] > \alpha$ . □



# How to Design Subsidies?

# Mechanism Design Reformulation

Revelation principle  $\implies$  it suffices to consider **direct mechanisms**  $(q, t)$  consisting of:

- ▶ an **allocation function**  $q : [\underline{\theta}, \bar{\theta}] \rightarrow [0, A]$  denoting *total* quantity consumed by type  $\theta$ ;
- ▶ a **payment rule**  $t : [\underline{\theta}, \bar{\theta}] \rightarrow \mathbb{R}$  denoting *total* payment by type  $\theta$ ,

satisfying incentive-compatibility,

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$$\theta \in \arg \max_{\hat{\theta} \in \Theta} \{ \theta v(q(\hat{\theta})) - t(\hat{\theta}) \} \text{ for all } \theta \in \Theta. \quad (\text{IC})$$

**Lemma (Implementation).** For any (IC) mechanism  $(q, t)$ , there exists a subsidy  $\sigma$  with  $q = q^\sigma$  and  $t = P^\sigma \circ q^\sigma$  if and only if:

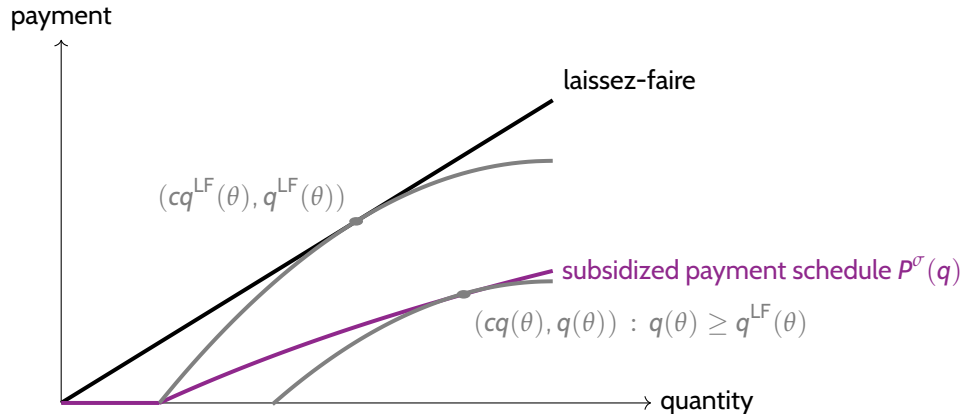
$$q(\theta) \geq q^{\text{LF}}(\theta) \text{ for all } \theta \in \Theta, \quad (\text{LB})$$

$$t(\theta) \geq 0 \text{ for all } \theta \in \Theta, \quad (\text{NLS})$$

$$U(\theta) \geq U^{\text{LF}}(\theta) \text{ for all } \theta \in \Theta. \quad (\text{IR})$$

# Intuition

marginal price per unit  $\leq c \iff$  allocations exceed laissez-faire



# Reformulating the Mechanism Design Problem

The social planner maximizes weighted total surplus

$$\max_{(q,t)} \int_{\underline{\theta}}^{\bar{\theta}} \left[ \omega(\theta) \underbrace{[\theta v(q(\theta)) - t(\theta)]}_{\text{consumer surplus}} + \alpha \underbrace{[t(\theta) - cq(\theta)]}_{\text{total cost}} \right] dF(\theta),$$

subject to (IC), (LB), (IR), and (NLS).

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**#1.** Apply **Myerson (1981)** Lemma and **Milgrom and Segal (2002)** envelope theorem to express objective in terms of  $U(\underline{\theta})$  and  $q(\theta)$  non-decreasing, substituting

$$t(\theta) = \theta v(q(\theta)) - \int_{\underline{\theta}}^{\theta} v(q(s)) ds - U(\underline{\theta}).$$

# Reformulating the Mechanism Design Problem

The social planner maximizes weighted total surplus

$$\max_{q \text{ non-decreasing, } U(\underline{\theta})} \mathbf{E}_{\theta}[\omega(\theta) - \alpha]U(\underline{\theta}) + \alpha \int_{\underline{\theta}}^{\bar{\theta}} \left[ \left( \theta + \frac{\int_{\underline{\theta}}^{\bar{\theta}} [\omega(s) - \alpha] dF(s)}{\alpha f(\theta)} \right) v(q(\theta)) - cq(\theta) \right] dF(\theta),$$

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- #1. Apply Myerson (1981) Lemma and Milgrom and Segal (2002) envelope theorem to express objective in terms of  $U(\underline{\theta})$  and  $q(\theta)$  non-decreasing, substituting
- #2. Suffices to enforce (IR) and (NLS) only for lowest type  $\underline{\theta}$  because  $U(\theta) - U^{\text{LF}}(\theta)$  and  $t(\theta)$  are nondecreasing by (IC) and (LB).
  - $\leadsto$  (NLS) binding: if  $\mathbf{E}[\omega(\theta)] > \alpha$ , choose  $U(\underline{\theta}) = \underline{\theta}v(q(\underline{\theta}))$ .
  - $\leadsto$  (NLS) does not bind: if  $\mathbf{E}[\omega(\theta)] \leq \alpha$ , choose  $U(\underline{\theta}) = U^{\text{LF}}(\underline{\theta})$ .



# Reformulating the Mechanism Design Problem

The social planner maximizes weighted total surplus

$$\max_{q \text{ non-decreasing}} \alpha \int_{\underline{\theta}}^{\bar{\theta}} \underbrace{[J(\theta)v(q(\theta)) - cq(\theta)]}_{\text{surplus of virtual type}} dF(\theta) + (\text{terms independent of } q),$$

subject to (LB):  $q(\theta) \geq q^{\text{LF}}(\theta)$ , where the **virtual type** absorbs (IC), (IR), and (NLS):

$$J(\theta) = \underbrace{\theta}_{\text{efficiency}} + \underbrace{\frac{\int_{\underline{\theta}}^{\bar{\theta}} [\omega(s) - \alpha] dF(s)}{\alpha f(\theta)}}_{\text{redistributive motive}} + \underbrace{\frac{\max\{\mathbf{E}_{\theta}[\omega(\theta) - \alpha], 0\} \theta \delta_{\theta=\underline{\theta}}}{\alpha f(\theta)}}_{\text{(NLS) constraint}}$$

Call  $J(\theta) - \theta$  the **distortion term**.

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Call  $J(\theta) - \theta$  the **distortion term**.

**Technical challenge:** (LB) is a “pointwise dominance” / FOSD constraint (cf. Yang and Zentefis, 2024)  $\leadsto$  possible interactions with the monotonicity constraint.

# Characterizing the Optimal Subsidy Allocation

**Theorem 2 (Negative Correlation).** The optimal allocation rule is unique, continuous and satisfies

$$q^*(\theta) = \begin{cases} D\left(c, \overline{J|_{[\underline{\theta}, \theta_\alpha]}}(\theta)\right) & \text{for } \theta \leq \theta_\alpha \\ q^{\text{LF}}(\theta) & \text{for } \theta \geq \theta_\alpha, \end{cases}$$

where  $\theta_\alpha$  is defined by

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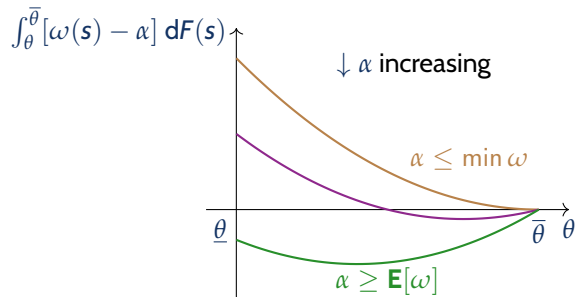
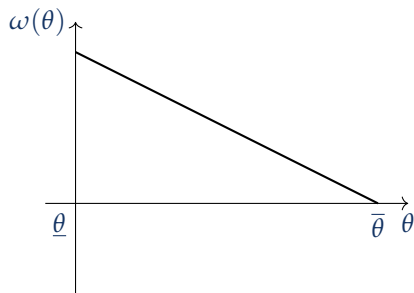
**Intuition:** there exists a type  $\theta_\alpha \in \Theta$  (possibly  $\underline{\theta}$  or  $\bar{\theta}$ ) such that

$$q^*(\theta) > q^{\text{LF}}(\theta) \text{ for all } \theta < \theta_\alpha, \text{ and}$$

$$q^*(\theta) = q^{\text{LF}}(\theta) \text{ for all } \theta \geq \theta_\alpha.$$

# Intuition: Signing the Distortion Term

**Negative correlation**  $\leadsto \omega(\theta)$  decreasing  $\leadsto$  distortion is single-crossing zero from above.



Social planner wants to distort consumption of **all types down**, **low-demand types up** and **high-demand types down**, or **all types upwards**.

# Optimal Marginal Subsidy Schedule

**Case 1:**  $\min \omega \geq \alpha$  (upward distortion for all)



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**Case 1:**  $\min \omega \geq \alpha$  (upward distortion for all)



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# Optimal Marginal Subsidy Schedule

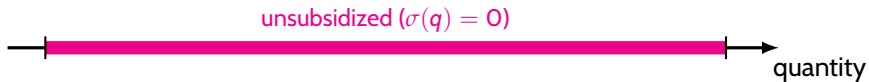
**Case 1:**  $\min \omega \geq \alpha$  (upward distortion for all)



**Case 2:**  $\min \omega \leq \alpha \leq \mathbf{E}[\omega]$  (upward distortion for low types, downward distortion for high types)



**Case 3:**  $\mathbf{E}[\omega] \leq \alpha$  (downward distortion for all)



# Economic Implications

With **negative correlation** between  $\omega$  and  $\theta$ :

# 1. Lump-sum cash transfers are always **more progressive** than subsidies.



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With **negative correlation** between  $\omega$  and  $\theta$ :

- # 1. Lump-sum cash transfers are always **more progressive** than subsidies.
- # 2. If cash transfers are unavailable, linear subsidies are **never** optimal.
  - # 2a. Optimal subsidies are “all or none”: active subsidy programs should always incorporate a **free allocation** (“public option”).
  - # 2b. If *any* consumer has  $\omega < \alpha$ , optimal subsidies are **capped** in quantity.

# Deriving the Optimal Mechanism

# Solving for the Optimal Mechanism

▶ skip

$$\begin{aligned} \max_q \alpha \int_{\underline{\theta}}^{\bar{\theta}} [J(\theta)v(q(\theta)) - cq(\theta)] dF(\theta), \\ \text{s.t. } q \text{ nondecreasing and } q(\theta) \geq q^{\text{LF}}(\theta). \end{aligned}$$

# Solving for the Optimal Mechanism

► skip

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## Guess 1: Pointwise maximizer

$$q(\theta) = (v')^{-1} \left( \frac{c}{J(\theta)} \right) = D(c, J(\theta)).$$

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Demand  $D(c, \cdot)$  is increasing, so:

$q$  nondecreasing  $\iff J(\theta)$  nondecreasing.

$q \geq q^{\text{LF}} \iff D(c, J(\theta)) \geq D(c, \theta) \iff J(\theta) \geq \theta.$

# Solving for the Optimal Mechanism

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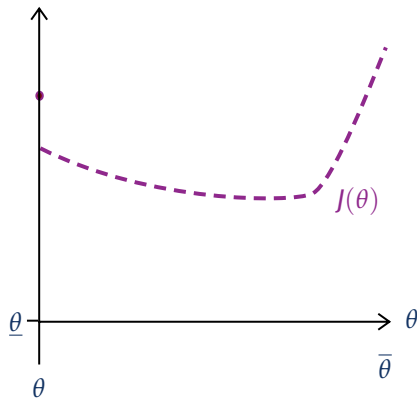
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$J(\theta)$  may be non-monotone.

# Solving for the Optimal Mechanism

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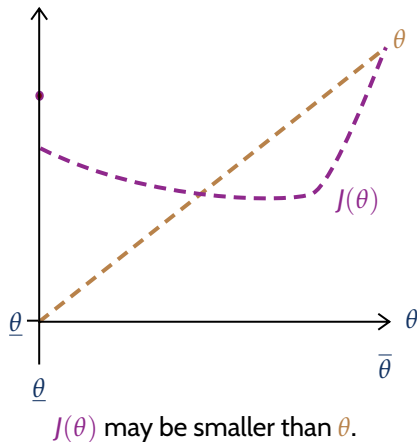
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► skip

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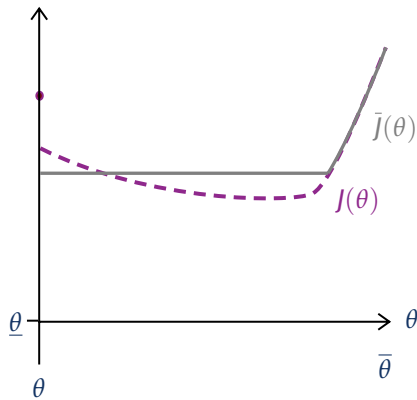
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## Guess 2: Relaxing the (LB) constraint

Toikka (2011); Akbarpour, Dworczak, Kominers (2021)

$$\leadsto q(\theta) = (v')^{-1}\left(\frac{c}{\bar{J}(\theta)}\right) = D(c, \bar{J}(\theta)),$$

where  $\bar{J}$  is ironing of  $J$ , pooling types in any non-monotonic interval of  $J$  at its  $F$ -weighted average.



Ironing deals with non-monotonicity.

► Ironing



# Solving for the Optimal Mechanism

► skip

$$\max_q \alpha \int_{\underline{\theta}}^{\bar{\theta}} [J(\theta)v(q(\theta)) - cq(\theta)] dF(\theta),$$

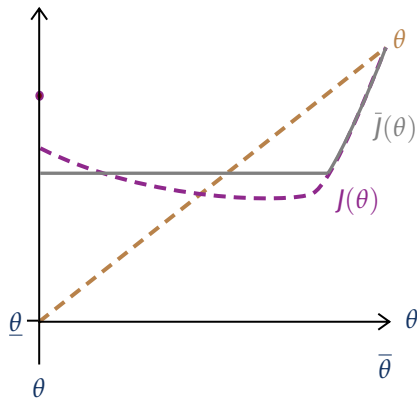
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But not lower-bound constraint  $\leadsto$  interaction.

► Ironing

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► skip

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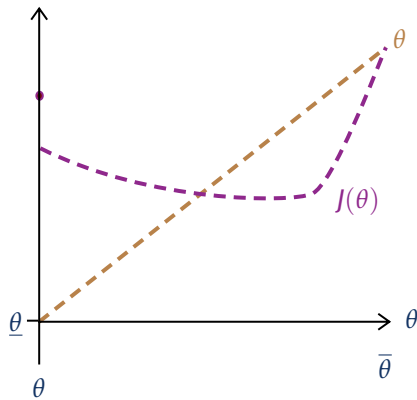
s.t.  $q$  nondecreasing and  $q(\theta) \geq q^{\text{LF}}(\theta)$ .

## Guess 3: Our approach

Suppose solution is of the form

$$q(\theta) = D(c, H(\theta)).$$

Feasibility requires  $H$  to be nondecreasing and satisfy  $H(\theta) \geq \underline{\theta}$ .



Need to identify nondecreasing  $H \geq \underline{\theta}$ .

► Ironing

# Characterizing the Optimal Subsidy Allocation

**Theorem 2 (Negative Correlation).** The optimal allocation rule is unique, continuous and satisfies

$$q^*(\theta) = D(c, H(\theta)),$$

where the **subsidy type**  $H(\theta)$  is defined by

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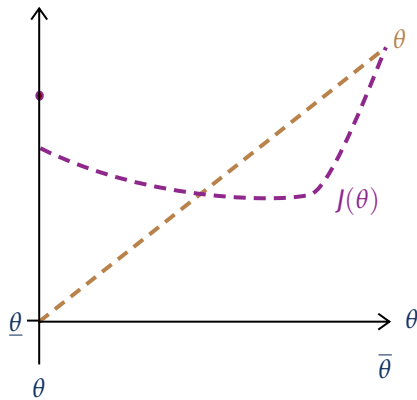
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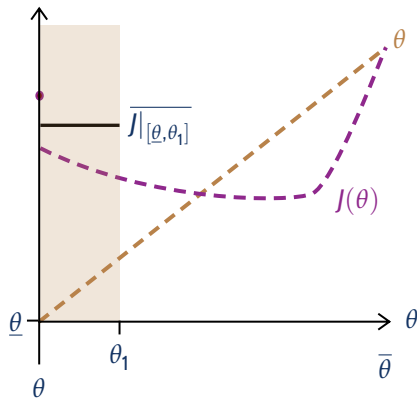
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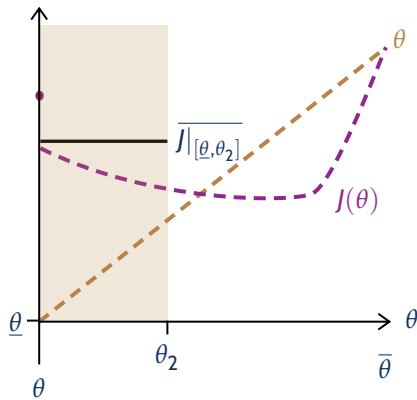
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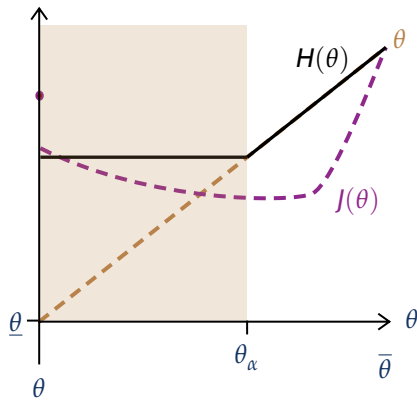
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construction  $\leadsto$  pooling condition and continuity

# Verifying $H$ from Theorem 2

Because  $q^*(\theta) = D(c, H(\theta))$ , for any feasible  $q$

$$\int_{\Theta} \underbrace{[H(\theta)v(q^*(\theta)) - cq^*(\theta)]}_{\text{surplus of type } H(\theta) \text{ at } D(c, H(\theta))} dF(\theta) \geq \int_{\Theta} \underbrace{[H(\theta)v(q(\theta)) - cq(\theta)]}_{\text{surplus of type } H(\theta) \text{ at } q(\theta)} dF(\theta).$$



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We want to show, for any feasible  $q$

$$\underbrace{\int_{\Theta} [J(\theta)v(q^*(\theta)) - cq^*(\theta)] dF(\theta)}_{\text{objective at } q^*} \geq \underbrace{\int_{\Theta} [J(\theta)v(q(\theta)) - cq(\theta)] dF(\theta)}_{\text{objective at feasible } q}.$$

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Subtracting, it suffices to show, for any feasible  $q$

$$\int_{\Theta} [J(\theta) - H(\theta)][v(q^*(\theta)) - v(q(\theta))] dF(\theta) \geq 0.$$

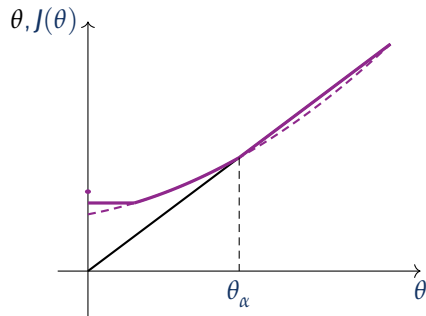
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To show  $\int_{\Theta} [J(\theta) - H(\theta)][v(q^*(\theta)) - v(q(\theta))] dF(\theta) \geq 0$ .

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There are three possibilities for  $H$ , partitioning  $\Theta$  into intervals:

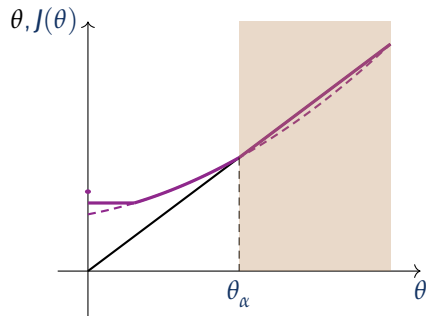


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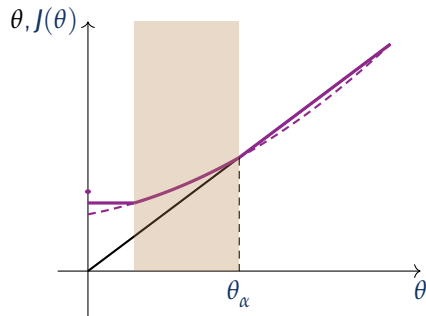


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- # 2.  $H(\theta) = J(\theta)$ : integrand = 0.



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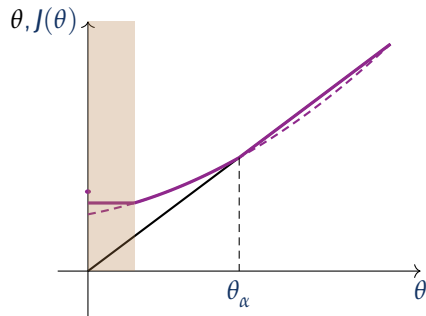
# 2.  $H(\theta) = J(\theta)$ : integrand = 0.

# 3.  $H(\theta) = \overline{J|_{[\underline{\theta}, \theta_\alpha]}}(\theta) \neq J(\theta)$ :

technical lemma  $\leadsto$  on any such interval  $\Theta_i$ ,  $H = \overline{J|_{\Theta_i}}$

$\leadsto$  optimality of  $D(c, H(\theta))$  in problem on  $\Theta_i$  *without* (LB)

$\implies$  same variational inequality characterizes optimality.  $\square$



# Summing Up

Proof approach:

- ▶ Guess form of solution  $q^*(\theta) = D(c, H(\theta))$ .
- ▶ Identify  $H(\theta)$  which is continuous,  $\geq \theta$ , and satisfies the **pooling condition**.
- ▶ Verify optimality using **variational inequalities**.

Same method of solution works for general  $\omega \rightsquigarrow$  see paper.

▶ Generalization



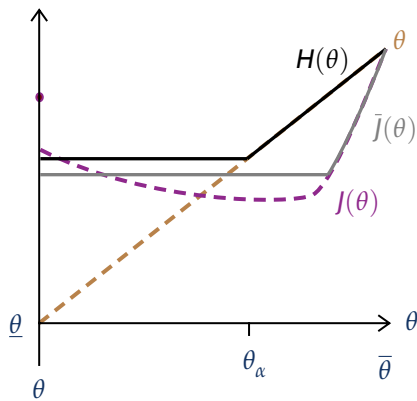
# Role of Topping Up

Comparing optimum with and without (LB) constraint,  $H(\theta)$  can exceed  $\bar{J}$  for all types.

→ Inability to tax causes upward distortion of all types

It is not optimal to calculate optimal subsidy/tax and set taxes to zero.

Highlights distinction from Mirrleesian marginal approach (FOC  $\nrightarrow$  optimum).



# Positive Correlation

# When to Subsidize?

## Positive Correlation

Suppose now that  $\omega(\theta)$  is increasing in  $\theta$  (“**positive correlation**”), e.g., public transport, staple foods.

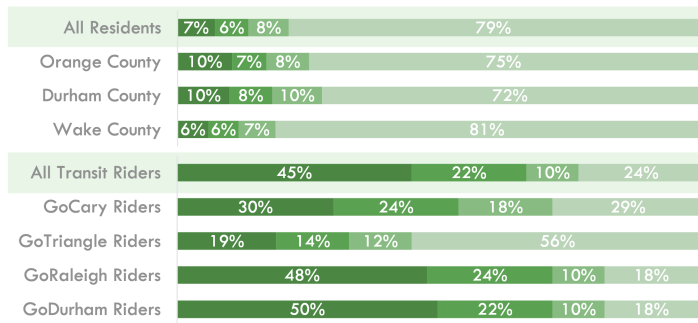
# When to Subsidize?

## Positive Correlation

Suppose now that  $\omega(\theta)$  is increasing in  $\theta$  (“**positive correlation**”), e.g., public transport, staple foods.

Household Income of Residents and Transit Riders

■ Less than \$15k ■ \$15k - \$25k ■ \$25k - \$35k ■ \$35k and above



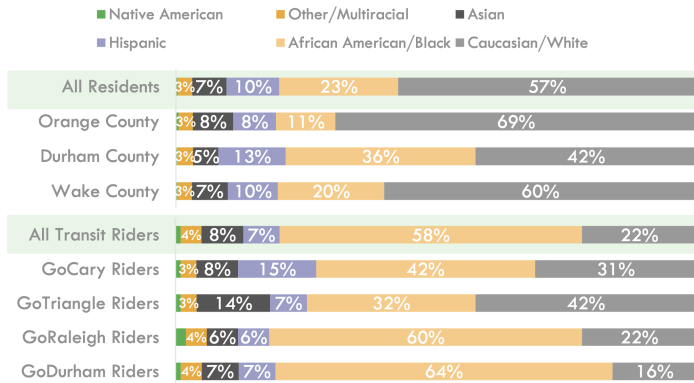
GoTriangle Onboard Customer Survey, 2019

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**Race/Ethnicities of Residents and Transit Riders**



GoTriangle Onboard Customer Survey, 2019

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**Intuition:** Social planner can always design a subsidy program with  $\Sigma(q^{\sigma}(\theta)) \geq 0$  only if  $\omega(\theta) \geq \alpha$ .

$\rightsquigarrow$  Argument relies on nonlinearity of subsidy program.

► Arbitrary Correlation

# How to Subsidize?

## Positive Correlation

**Theorem 2 (Positive Correlation).** The optimal subsidy allocation rule is unique, continuous and satisfies

$$q^*(\theta) = D(c, H(\theta)), \text{ where } H(\theta) = \begin{cases} \theta & \text{if } \theta \leq \theta_\alpha, \\ \overline{J_{[\theta_\alpha, \bar{\theta}]}}(\theta) & \text{if } \theta \geq \theta_\alpha, \end{cases}$$

where  $\theta_\alpha = \inf\{\theta \in \Theta : J(\theta) \geq \theta\}$ .

**Intuition:** there exists a type  $\theta_\alpha \in \Theta$  (possibly  $\underline{\theta}$  or  $\bar{\theta}$ ) such that

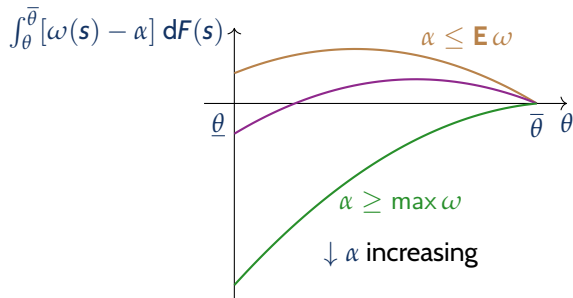
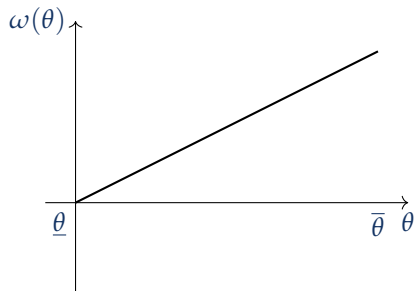
$$\begin{aligned} q^*(\theta) &= q^{\text{LF}}(\theta) \text{ for all } \theta \leq \theta_\alpha, \text{ and} \\ q^*(\theta) &\geq q^{\text{LF}}(\theta) \text{ for all } \theta > \theta_\alpha. \end{aligned}$$

► Arbitrary Correlation

# How to Subsidize?

## Positive Correlation

**Positive correlation**  $\leadsto \omega(\theta)$  increasing  $\leadsto$  distortion is single-crossing zero from below.



Social planner wants to distort consumption of **all types down**, high-demand types up and low-demand types down, or **all types upwards**.

# Optimal Subsidy Schedule

## Positive Correlation

**Case 1:**  $E[\omega] \geq \alpha$  (upward distortion for all)



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# Optimal Subsidy Schedule

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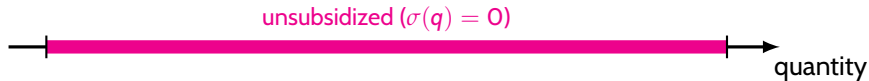
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**Case 3:**  $\max \omega \leq \alpha$  (downward distortion for all)



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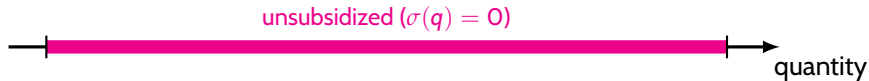
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# Discussion



# Importance of Correlation

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Subsidies dominated by cash transfers.

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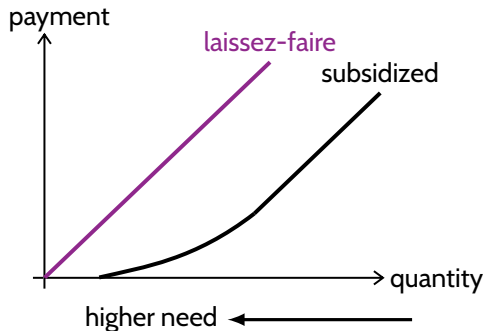
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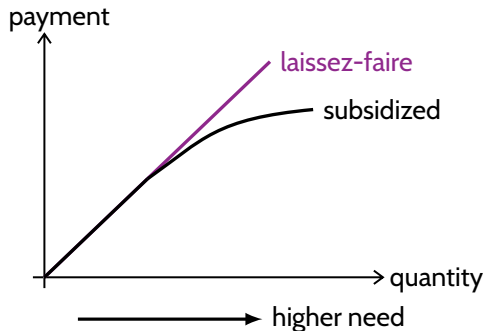
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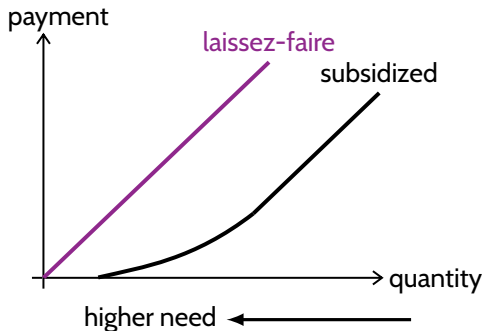


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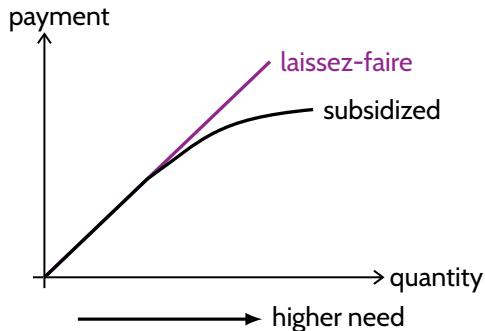


All or none subsidized.

## Positive Correlation

Subsidies dominate cash transfers.

Subsidies if and only if  $\max \omega > \alpha$ .



Only neediest (self-selected) consumers subsidized.

# Differences In Practice

**When?** Theorem 1  $\leadsto$  scope of intervention larger with positive correlation ( $\max \omega > \alpha$ ) than negative correlation ( $\mathbf{E}[\omega] > \alpha$ ).

In practice, many government programs focused on goods consumed disproportionately by needy.

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In practice, many government programs focused on goods consumed disproportionately by needy.

**How?** Significant differences in marginal subsidy schedules observed in practice:

## Larger subsidies for low $q$

- ▶ Food stamps (SNAP)
- ▶ Womens, Infants & Children (WIC) Program
- ▶ Housing Choice (Section 8) Vouchers
- ▶ Lifeline (Telecomm. Assistance) Program

## Larger subsidies for high $q$

- ▶ Public transit fare capping
- ▶ Pharmaceutical subsidy programs
- ▶ Government-subsidized childcare places.

# How Do Optimal Subsidies Compare To Linear?

**Proposition.** Linear subsidies are **never optimal**.

**Intuition:** no distortion at the top ( $J(\bar{\theta}) = \bar{\theta}$ )  $\leadsto$  linear subsidies never optimal.

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Social planner intervenes with linear subsidies only if a weighted average of all distortion terms are positive  
 $\leadsto$  more restrictive than nonlinear.

Social planner can always improve over linear subsidy by implementing:

- ▶ a **cap** on the subsidy paid to a consumer (as in negative correlation),
- ▶ a **floor** on eligibility for the subsidy (as in positive correlation), or
- ▶ a **free endowment** of the good to all consumers.

Gains from nonlinear subsidization can be **arbitrarily large** compared to gains from linear subsidization.

▶ skip to conclusion

# Comparative Statics of Subsidies

**Question:** How do optimal subsidies change when

- (a) the social planner's desire to redistribute to each consumer increases?
- (b) the correlation between demand and welfare weight increases?
- (c) the marginal cost of production decreases?

► Details

# Comparative Statics of Subsidies

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- (b) the correlation between demand and welfare weight increases?
- (c) the marginal cost of production decreases?

► Details

**Short Answer:** Each cause the optimal subsidy program to be more generous.

But (a) and (b) cause  $J(\theta)$  to increase for each  $\theta \rightsquigarrow$  a larger set of consumers subsidized. (c) does not.

# When Does the Planner Benefit from Private Market Restrictions?

## Role of Topping Up Constraint

In some markets (e.g., public housing), the social planner may be able to restrict subsidy recipients from topping up in private market  $\rightsquigarrow$  opt-in (or out) of subsidy program.

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In **Kang and Watt (2024)**, we characterize optimal subsidy mechanism under such restrictions. These lead to different (weaker) type-dependent outside option constraints:

$$\text{average price} \leq c \Leftrightarrow \text{majorization constraint on } q.$$

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**average price**  $\leq c \Leftrightarrow$  majorization constraint on  $q$ .

### Proposition.

- (a) **“Negative Correlation”**:  $\omega$  decreasing in  $\theta \leadsto$  planner benefits from preventing topping up iff  $\max \omega > \alpha$ .
- (b) **“Positive Correlation”**:  $\omega$  increasing in  $\theta \leadsto$  planner never benefits from preventing topping up.

**Intuition:** Planner offers subsidies tied to consumption level favored by high  $\omega$  types.

**Implication:** Positive correlation between demand and welfare weights reduces the need to enforce topping up restrictions.

# Extensions

# Equilibrium Effects

Our baseline model shuts down equilibrium effects of government subsidies on private market prices.

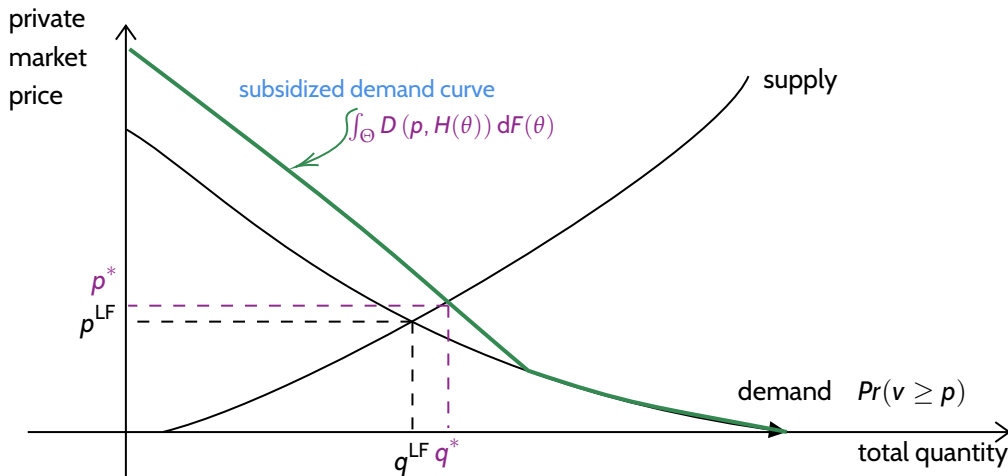
Empirical evidence of price effects from government subsidy programs, e.g.:

- ▶ public housing ([Diamond and McQuade, 2019](#); [Baum-Snow and Marion, 2009](#))
- ▶ pharmaceuticals ([Atal et al., 2021](#))
- ▶ public schools ([Dinerstein and Smith, 2021](#))
- ▶ school lunches ([Handbury and Moshary, 2021](#))



# Equilibrium Effects

Our results extend directly to imperfectly elastic supply curves:



# Private Market Taxation

Our baseline model assumes the planner cannot tax the private market.

Taxation of private market **reduces** consumers' outside option, relaxing the **(LB)** constraint. If taxation is costly (e.g., because of distortions on ineligible consumers):

**Proposition.** Suppose the planner faces a convex cost  $\Gamma(\tau)$  for taxation of the private market. Then there exists an optimal tax level  $\tau^*$  and subsidy program for eligible consumers satisfying

$$q^*(\theta) = D(H_{\tau^*}(\theta)),$$

where  $H_{\tau^*}(\theta) \leq H(\theta)$ .

# Budget Constraints and Endogenous Welfare Weights

In our baseline model,  $\omega(\cdot)$  and  $\alpha$  are taken exogenously.

Our model can be extended to allow weights to be endogenous (cf. [Pai and Strack, 2024](#)):

- ▶  $\alpha \iff$  Lagrange multiplier on the social planner's budget constraint.
- ▶  $\omega(\theta) \iff$  the marginal value of money for a consumer with **concave** preferences

$$\varphi(\theta v(q) + I - t),$$

and income  $I \sim G_\theta$ , known but not observed by the social planner, then

$$\omega(\theta) = \mathbf{E}_{I \sim G_\theta}[\varphi'(\theta v(q(\theta)) + I - t(\theta))].$$

# Conclusion

# Concluding Remarks

## Takeaways for Subsidy Policy:

- ▶ Linear subsidies are **never** optimal.
- ▶ When and how to subsidize depends on **correlation** between demand and need.
  - With negative correlation (many goods), why not lump-sum cash transfers? (“tortilla subsidy” vs. Progresa).
  - Goods with positive correlation are ideal candidates for subsidies (e.g., public transport) but these should have floors to improve targeting.

## Technical Contribution:

- ▶ We show how to solve mechanism design problems with lower-bound constraints caused by type-dependent outside options.
- ▶ Similar mechanism design problems arise in other contexts, e.g., subsidy design with other objectives (externalities, paternalism); exclusive contracting (topping up = non-exclusive contracting, no topping up = exclusive contracting.).

# Fin

Thank you for the invitation!

# Assumption: No Lump-Sum Cash Transfers

Note: This constraint only binds if  $\mathbf{E}_\theta[\omega(\theta)] > \alpha$ .

## Possible reasons:

- ▶ **Institutional:** subsidies designed by government agency without tax/transfer powers.
- ▶ **Political:** Liscow and Pershing (2022) find U.S. voters prefer in-kind redistribution to cash transfers.
- ▶ **Household Economics:** Currie (1994) finds in-kind redistribution has stronger benefits for children than cash transfer programs.
- ▶ **Pedagogical:** to contrast when the assumption is binding ( $\leadsto$  cash transfers preferred to subsidies) versus non-binding (*vice versa*).
- ▶ **Model:** without NLS constraint, the social planner would want to make unbounded cash transfers when  $\mathbf{E}[\omega] > \alpha$ .

# When to Subsidize (General): Proof by Picture

**Theorem 1.** Social planner subsidizes **if and only if** there exists a type  $\hat{\theta}$  for which

$$\mathbf{E}_{\theta}[\omega(\theta) \mid \theta \geq \hat{\theta}] > \alpha.$$

Suppose  $\mathbf{E}_{\theta}[\omega(\theta) \mid \theta \geq \hat{\theta}] > \alpha$ : we construct a subsidy schedule increasing weighted surplus.

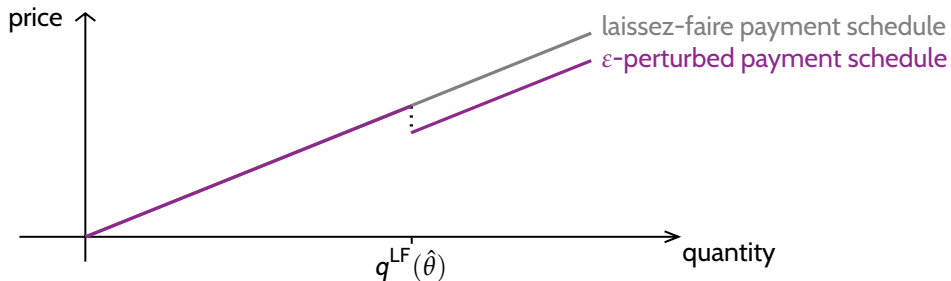


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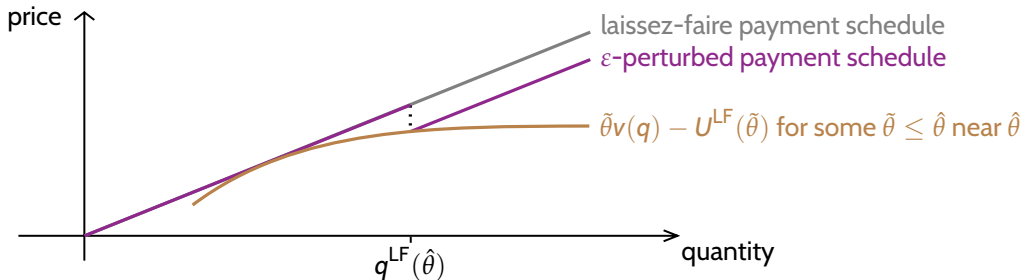
$\varepsilon$ -perturbation increases utility of types  $\geq \hat{\theta}$ , net benefit  $\varepsilon \mathbf{E}_{\theta}[\omega(\theta) - \alpha \mid \theta \geq \hat{\theta}]$ .

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$\varepsilon$ -perturbation increases utility of types  $\geq \hat{\theta}$ , net benefit  $\varepsilon \mathbf{E}_{\theta}[\omega(\theta) - \alpha \mid \theta \geq \hat{\theta}]$ .

But consumption is distorted for  $O(\sqrt{\varepsilon})$  set of types near (but below)  $\hat{\theta}$ , at cost  $\leq O(\sqrt{\varepsilon})\varepsilon$ .

$\leadsto$  Benefits  $>$  costs for small enough  $\varepsilon$ . **Note: Argument relies on nonlinearity.**

[▶ return](#)

# Topping Up $\Leftarrow$ Lower-Bound (1/2)

Suppose  $q(\theta) \geq q^{\text{LF}}(\theta)$ . We want to show total subsidies  $S(z)$  is increasing in  $z$ .

# 1.  $t(\underline{\theta}) \leq cq(\underline{\theta})$  by (IR):

$$t(\underline{\theta}) \leq \underline{\theta}v(q(\underline{\theta})) - \underline{\theta}v(q^{\text{LF}}(\underline{\theta})) + cq^{\text{LF}}(\underline{\theta}),$$

and  $\underline{\theta}v(q^{\text{LF}}(\underline{\theta})) - cq^{\text{LF}}(\underline{\theta}) \geq \underline{\theta}v(q(\underline{\theta})) - cq(\underline{\theta})$  by definition of  $q^{\text{LF}}$ , so  $t(\underline{\theta}) \leq cq(\underline{\theta})$ .

## Topping Up $\Leftarrow$ Lower-Bound (2/2)

# 2. The *marginal* price of any units purchased is no greater than  $c$  by (IC):

$$\begin{aligned} t(\theta') - t(\theta) &= \left[ \theta' v(q(\theta')) - U(\underline{\theta}) - \int_{\underline{\theta}}^{\theta'} v(q(s)) \, ds \right] - \left[ \theta v(q(\theta)) - U(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} v(q(s)) \, ds \right] \\ &= \theta' v(q(\theta')) - \theta v(q(\theta)) - \int_{\theta}^{\theta'} v(q(s)) \, ds \\ &= \int_{\theta}^{\theta'} s v'(q(s)) \, dq(s). \end{aligned}$$

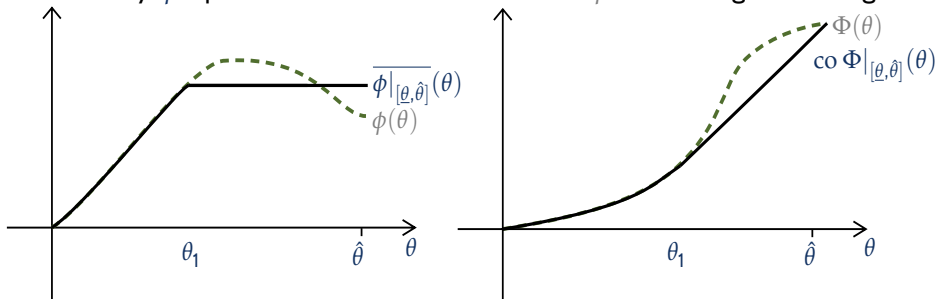
But if  $q(\theta) \geq q^{\text{LF}}(\theta)$ , then concavity of  $v$  implies  $v'(q(\theta)) \leq v'(q^{\text{LF}}(\theta)) = c/\theta$ , so  $t(\theta') - t(\theta) \leq c[q(\theta') - q(\theta)]$ .

# Ironing

Let  $\phi$  be a (generalized) function and  $\Phi : \theta \mapsto \int_{\underline{\theta}}^{\theta} \phi(s) \, dF(s)$ . Then  $\bar{\phi}$  is the monotone function satisfying

$$\text{for all } \theta \in [\underline{\theta}, \hat{\theta}], \quad \int_{\underline{\theta}}^{\theta} \bar{\phi}(s) \, dF(s) = \text{co } \Phi(\theta).$$

Intuitively,  $\bar{\phi}$  replaces non-monotone intervals of  $\phi$  with  $F$ -weighted averages.



# Rewriting the Mechanism Design Problem

The social planner maximizes weighted total surplus

$$\max_{(q,t)} \int_{\underline{\theta}}^{\bar{\theta}} \left[ \omega(\theta) \underbrace{[\theta v(q(\theta)) - t(\theta)]}_{\text{consumer surplus}} + \alpha \underbrace{[t(\theta) - cq(\theta)]}_{\text{total cost}} \right] dF(\theta),$$

subject to (IC), (LB), (IR), and (NLS).

# Rewriting the Mechanism Design Problem

$$\max_{q \text{ non-decreasing, } U(\underline{\theta}) \geq U^{\text{LF}}(\underline{\theta})} \mathbf{E}_{\theta} [\omega(\theta) - \alpha] U(\underline{\theta}) + \alpha \int_{\underline{\theta}}^{\bar{\theta}} \left[ \left( \theta + \frac{\int_{\theta}^{\bar{\theta}} [\omega(s) - \alpha] dF(s)}{\alpha f(\theta)} \right) v(q(\theta)) - cq(\theta) \right] dF(\theta),$$

subject to (LB), (IR), and (NLS).

- #1. Apply Myerson (1981) Lemma and Milgrom and Segal (2002) envelope theorem to express objective in terms of  $U(\underline{\theta})$  and  $q(\theta)$  non-decreasing.

# Rewriting the Mechanism Design Problem

$$\max_{q \text{ non-decreasing}, U(\underline{\theta}) \geq U^{\text{LF}}(\underline{\theta})} \mathbf{E}_{\theta}[\omega(\theta) - \alpha]U(\underline{\theta}) + \alpha \int_{\underline{\theta}}^{\bar{\theta}} \left[ \left( \theta + \frac{\int_{\theta}^{\bar{\theta}} [\omega(s) - \alpha] dF(s)}{\alpha f(\theta)} \right) v(q(\theta)) - cq(\theta) \right] dF(\theta),$$

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**#1.** Apply Myerson (1981) Lemma and Milgrom and Segal (2002) envelope theorem to express objective in terms of  $U(\underline{\theta})$  and  $q(\theta)$  non-decreasing.

**#2.** Suffices to enforce (IR) and (NLS) only for lowest type  $\underline{\theta}$  because  $U(\theta) - U^{\text{LF}}(\theta)$  and  $t(\theta)$  are nondecreasing by (IC) and (LB).

↪ **Low cost of public funds:** if  $\mathbf{E}[\omega(\theta)] > \alpha$ , choose  $U(\underline{\theta}) = \underline{\theta}v(q(\underline{\theta}))$ .

↪ **High cost of public funds:** if  $\mathbf{E}[\omega(\theta)] \leq \alpha$ , choose  $U(\underline{\theta}) = U^{\text{LF}}(\underline{\theta})$ .



# Rewriting the Mechanism Design Problem

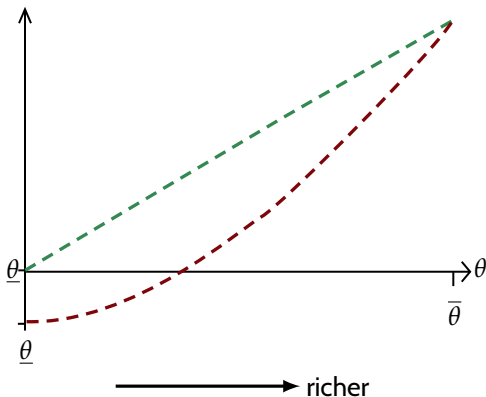
$$\max_{q \text{ non-decreasing}} \alpha \int_{\underline{\theta}}^{\bar{\theta}} [J(\theta)v(q(\theta)) - cq(\theta)] dF(\theta) + (\text{terms independent of } q),$$

subject to (LB):  $q(\theta) \geq q^{\text{LF}}(\theta)$ , where the **virtual type**

$$J(\theta) = \underbrace{\theta}_{\text{efficiency}} + \underbrace{\frac{\int_{\underline{\theta}}^{\bar{\theta}} [\omega(s) - \alpha] dF(s)}{\alpha f(\theta)}}_{\text{redistributive motive}} + \underbrace{\frac{\max\{\mathbf{E}_{\theta}[\omega(\theta) - \alpha], 0\} \theta \delta_{\theta=\underline{\theta}}}{\alpha f(\theta)}}_{\text{(NLS) constraint}}$$

Call  $J(\theta) - \theta$  the “distortion term.”

# Decreasing Welfare Weights, High Cost of Public Funds

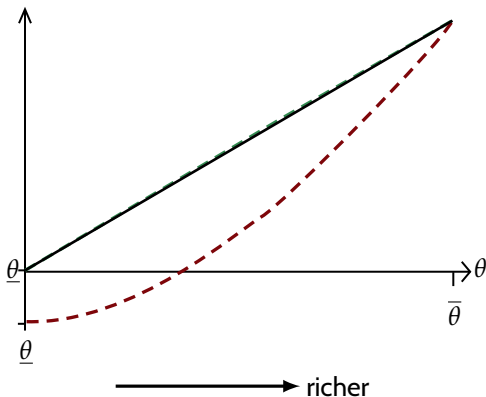


High cost of public funds:  $\mathbf{E}[\omega(\theta)] \leq \alpha$ .

$$J(\theta) = \theta + \frac{\int_{\theta}^{\bar{\theta}} [\omega(s) - \alpha] dF(s)}{\alpha f(\theta)}$$

is always *below* lower bound  $\theta$  because distortion is single-crossing from above, **negative** at  $\underline{\theta}$  and zero at  $\bar{\theta}$ .

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↪ Subsidy type  $H(\theta) = \theta$  and optimal allocation is laissez-faire.

# Optimal Mechanism (Arbitrary Correlation)

**Theorem 2 (General).** The optimal subsidy allocation rule is unique, continuous and satisfies

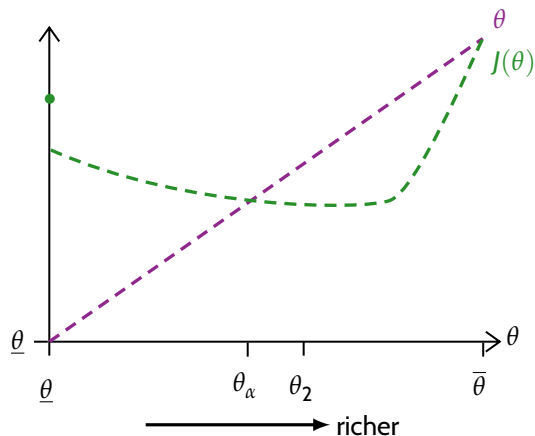
$$q^*(\theta) = D(c, H(\theta)), \text{ where } H(\theta) = \begin{cases} \theta & \text{if } \overline{J}_{[\underline{\theta}, \theta]}(\theta) \leq \theta, \\ \overline{J}_{[\underline{\theta}, \kappa_+(\theta)]}(\theta) & \text{otherwise,} \end{cases}$$

and  $\kappa_+(\theta) = \inf \left\{ \hat{\theta} \geq \theta : \overline{J}_{[\underline{\theta}, \hat{\theta}]}(\hat{\theta}) \leq \hat{\theta} \right\}$  or  $\bar{\theta}$ , if that set is empty.

“Double” ironing construction of  $H(\theta)$  ensures  $H(\theta) \geq \theta$ , equivalent to (LB) given expression for  $q^*$ .

$\leadsto$  subsidized demand curve  $\bar{D}(p) = \int_{\Theta} (v')^{-1}(p/H(\theta)) dF(\theta)$ .

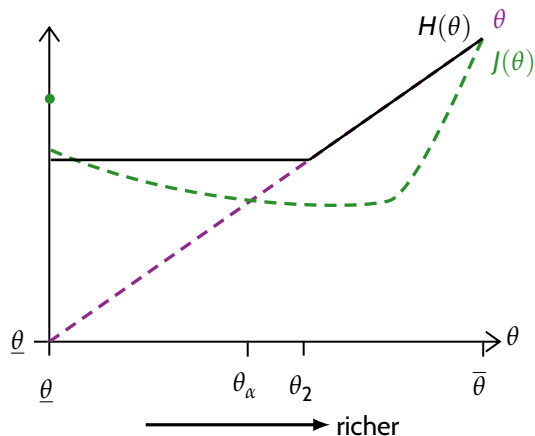
# Decreasing Welfare Weights, Low Cost of Public Funds



Low cost of public funds:  $\mathbf{E}[\omega(\theta)] > \alpha$ .

$J(\theta) = \theta + \frac{\int_{\underline{\theta}}^{\bar{\theta}} [\omega(s) - \alpha] dF(s) + (\mathbf{E}_\theta[\omega(\theta)] - \alpha) \underline{\theta} \delta_{\theta=\underline{\theta}}}{\alpha f(\theta)}$  crosses lower bound constraint  $\theta$  from *above* because distortion term is single-crossing from above, *positive* at  $\underline{\theta}$  and zero at  $\bar{\theta}$ .

# Decreasing Welfare Weights, Low Cost of Public Funds



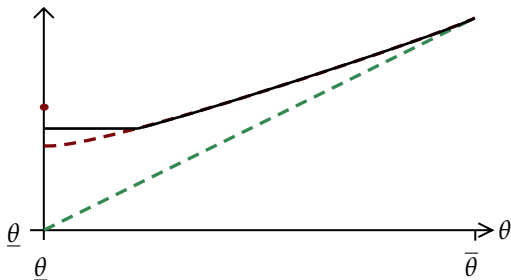
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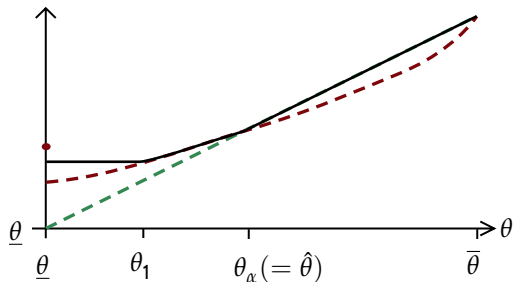
↪ Subsidy type  $H(\theta) > \theta$  for  $\theta \leq \theta_2$ . There is a free endowment of  $q^{\text{LF}}(\theta_2)$ , which strictly exceeds  $q^{\text{F}}(\theta)$  for  $\theta \leq \theta_2$  ...but planner always prefers a lump-sum subsidy.

# Decreasing Welfare Weight, Low Cost of Public Funds

Other possibilities:



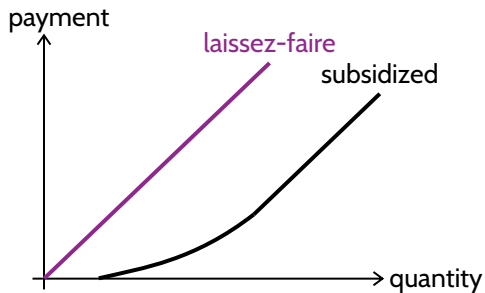
Free endowment + quantity-dependent subsidies  
distorting all types' consumption upwards (no  
topping up).



Free endowment + quantity-dependent subsidies up  
to a cap (high types top up in private market).

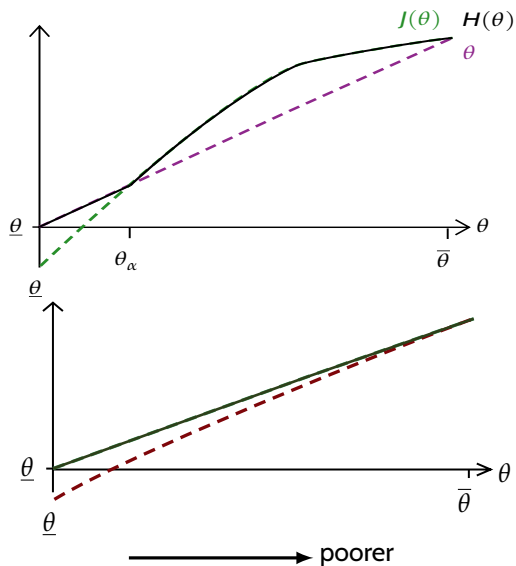
# Decreasing Welfare Weight, Low Cost of Public Funds

Payment schedule:





# Increasing Welfare Weights, High Cost of Public Funds



High cost of public funds:  $\mathbf{E}[\omega(\theta)] \leq \alpha$ .

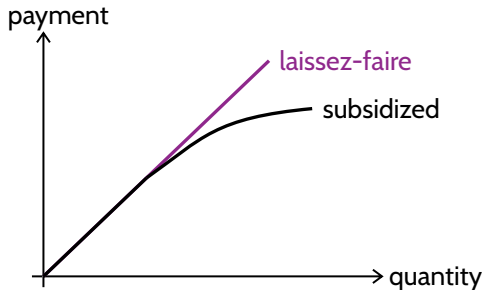
$$J(\theta) = \theta + \frac{\int_{\underline{\theta}}^{\bar{\theta}} [\omega(s) - \alpha] dF(s)}{\alpha f(\theta)}$$



can cross lower-bound  $\underline{\theta}$  from *below* because the distortion term is single-crossing from below, **negative** at  $\underline{\theta}$  and zero at  $\bar{\theta}$ .

↪ Subsidy type *can* exceed  $\theta$  for high types: implemented by offering discounts for consumption *above* a minimum level... *preferred* by planner to lump-sum transfer.

# Increasing Welfare Weights, High Cost of Public Funds

Payment schedule:





## Weekly fare cap




### An even better weekly fare discount

Say hello to an easier, more equitable way to pay your fare: the 7-day fare cap with OMNY!


Pay for 12 rides in a 7-day period and any additional rides are free. And, unlike with MetroCard, you don't have to pay upfront. Just tap and pay as you go.

Use the same device or card all 7 days and you'll automatically ride free after your 12th paid fare.

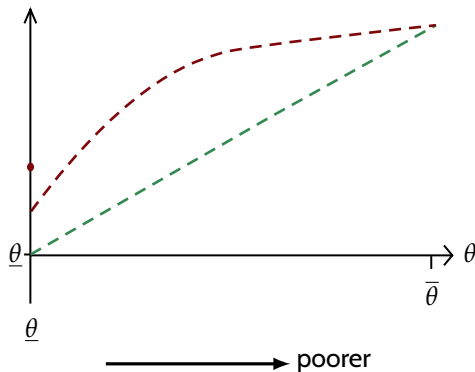
No need to pre-pay like when you buy a 7-Day Unlimited MetroCard. Tap with your own mobile wallet enabled device or contactless bank card, and you pay only for the trips you take. The more you ride, the sooner you'll earn free trips.

How Does Fare Capping Work With OM...Watch laterShare

## How does fare capping work with OMNY?



# Increasing Welfare Weights, Low Cost of Public Funds

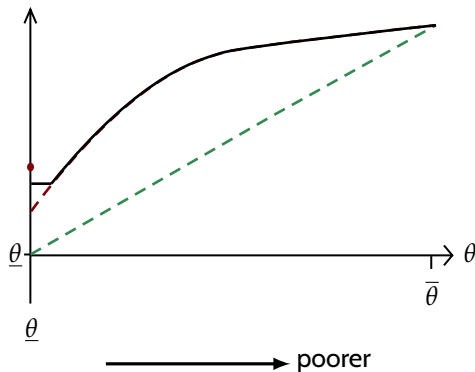


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↪ Subsidy type exceeds  $\theta$  for all types: implemented via a free allocation and discounts for additional consumption. No consumers top up.

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# Discussion

**Theorem 1**  $\rightsquigarrow$  scope of intervention larger for “inferior goods” than “normal goods.”

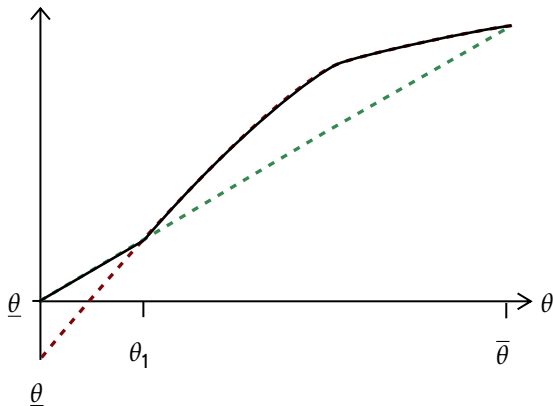
In practice, many government programs focused on goods consumed disproportionately by needy:

## Examples:

- ▶ Egyptian *Tamween* food subsidy program subsidizes five loaves of *baladi* bread/day at AUD 0.01/loaf, with a **cap** on weights and quality of bread.
- ▶ CalFresh Restaurant Meals Program subsidizes fast food restaurants not dine-in restaurants.
- ▶ Indonesian Fuel Subsidy Program subsidizes low-octane fuel (for motorbikes) and not high-octane fuel (for cars).
- ▶ Until  $\sim$ 2016, UK's NHS subsidized amalgam fillings and not composite (tooth-coloured) fillings.

## # 4(a) Increasing Motive for Redistribution

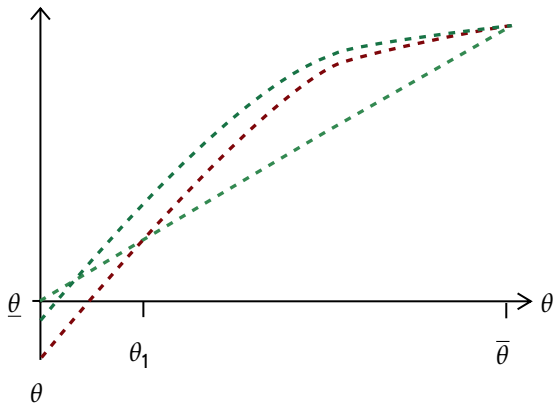
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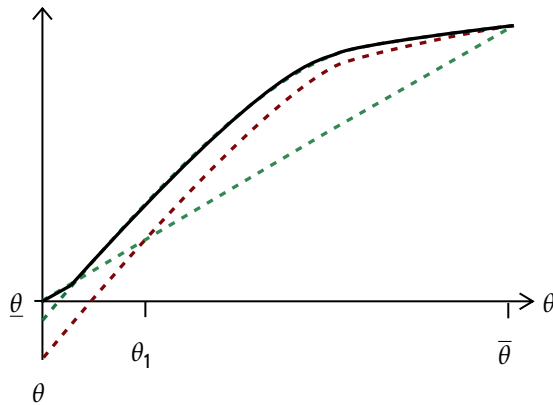
- virtual type  $J(\theta) \uparrow$



## # 4(a) Increasing Motive for Redistribution

Suppose  $\omega(\theta) \uparrow$  for each  $\theta$  or, equivalently,  $\alpha \downarrow$ .

- ▶ virtual type  $J(\theta) \uparrow$
- ↪ each consumer's subsidy type  $H(\theta) \uparrow$
- ↪ each consumer's allocation  $q^*(\theta) \uparrow$
- ↪ set of subsidized types  $\uparrow$
- ↪ total subsidy per consumer  $\uparrow$

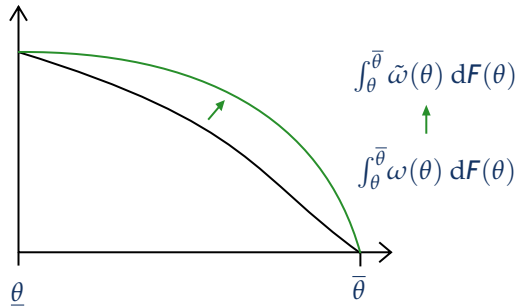
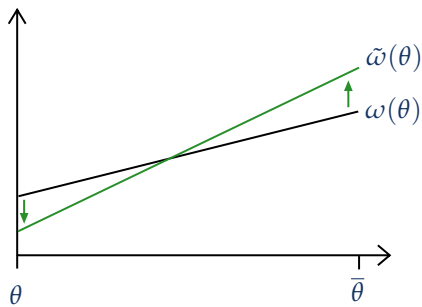


Planner & average eligible consumer prefer subsidies targeted to consumers with higher welfare weights.



## # 4(b) Increasing Correlation

Suppose  $\omega$  and  $\theta$  become more correlated, in the sense of **majorization**  $\leadsto$  observe higher demand expect higher  $\omega$ , i.e., for all  $\theta \in \Theta$ ,  $\mathbf{E}[\tilde{\omega}(\theta)|\theta \geq \hat{\theta}] \geq \mathbf{E}[\omega(\theta)|\theta \geq \hat{\theta}]$ .



$\leadsto$  larger incentive to distort consumption  $\leadsto$  more generous subsidies.

**Planner & average eligible consumer prefer subsidies for goods with more positive correlation between demand and welfare weights.**

## # 4(c) Decreasing Marginal Cost

Suppose marginal cost decreases  $c \downarrow$  (equiv. demand increases, so  $(v') \uparrow$ ).

No change in virtual type  $\leadsto$  no change in subsidy type.

- $\leadsto$  the set of subsidized types is unchanged, while
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**Planner & average eligible consumer prefer subsidies for low cost / high demand goods.**

# Food Stamps (SNAP)

- ▶ **Overview:** U.S. program providing monthly food assistance to low-income individuals and families.
- ▶ **Initial Support:** Full subsidy up to a fixed dollar amount per month for eligible food items.
- ▶ **Free Endowment:** The subsidy starts as a full benefit and decreases after benefits are exhausted.
- ▶ **Low Consumption Focus:** Ensures a basic level of nutrition by covering initial consumption entirely.

# Women, Infants, and Children (WIC)

- ▶ **Overview:** Nutritional assistance for low-income pregnant women, new mothers, and young children.
- ▶ **Initial Support:** Vouchers for essential foods like milk, eggs, baby formula.
- ▶ **Free Endowment:** Recipients get fully subsidized quantities of specific foods.
- ▶ **Low Consumption Focus:** Prioritizes providing a free minimum quantity of nutritious food to families.

# Housing Choice Voucher Program (Section 8)

- ▶ **Overview:** subsidized housing assistance for low-income renters in the U.S.
- ▶ **Initial Support:** Covers a large portion of rent (up to 70%) for qualifying households.
- ▶ **Free Endowment:** A significant rent portion is initially fully subsidized.
- ▶ **Low Consumption Focus:** Ensures low-income renters pay only a small portion of their rent.

# Lifeline Program

- ▶ **Overview:** U.S. program offering discounted phone and internet services to low-income households.
- ▶ **Initial Support:** Monthly discounts on basic telecommunication services.
- ▶ **Free Endowment:** Full subsidy of basic services for the most disadvantaged users.
- ▶ **Low Consumption Focus:** Provides essential access to communication services with high initial subsidies.



# National School Lunch Program (NSLP)

- ▶ **Overview:** Provides free or reduced-price school meals for low-income students.
- ▶ **Initial Support:** Fully subsidized meals for eligible students based on family income.
- ▶ **Free Endowment:** Full meal subsidies provided for families below certain income thresholds.
- ▶ **Low Consumption Focus:** Ensures children receive at least one nutritious meal per day at no or low cost.

# Australian Better Access Mental Health Initiative

- ▶ **Overview:** Australian government program subsidizing mental health services.
- ▶ **Initial Subsidy:** Up to 10 Medicare-subsidized sessions per year.
- ▶ **Additional Support:**
  - After initial sessions and doctor approval, become eligible for extra free/subsidized sessions.
  - Increased subsidy ensures access for those needing more care.

# Australia's Child Care Subsidy (CCS)

- ▶ **Overview:** Government subsidy for childcare costs based on income and activity levels.
- ▶ **Initial Subsidy:** Covers a percentage of childcare fees up to a set number of hours.
- ▶ **Additional Support:**
  - Subsidy percentage **increases** as parents work, study, or volunteer more.
  - More hours of work/study lead to higher subsidies for additional childcare hours.
  - More children leads to higher subsidies per child.

# Public Transit Fare Capping (Research Triangle, NC)

## Fares & Passes

Fares & Passes

Return to Fare FAQs

GoPass Partners

Purchase GoPass Card

Register GoPass Card

Discount Fare Options

How to use Umo

Add Funds

GoCary Fares

GoDurham Fares

GoRaleigh Fares

### FIXED-ROUTE FARE OPTIONS

Fare Type	Fare	Daily Cap	Weekly Cap	Monthly Cap
Full Fare	\$2.50	\$5	\$20	\$80
Discount	\$1.25	\$2.50	\$10	\$40
Child Under 12	N/A	N/A	N/A	N/A
Youth 13-18		Qualify for free fare		
Senior 65+		Qualify for free fare		
Transit Assistance Pass		Qualify for free fare		
GoPass Partners		Learn More		

*\*All sales are final and nonrefundable*

**Other Cities With Similar Programs:** New York, SF Bay Area, Portland, London, Dublin, Toronto, Vancouver, Los Angeles, Singapore, Sydney, Brisbane, Melbourne, Perth, Auckland.

# Pharmaceutical Subsidy Programs: Australia, Norway, Sweden, Denmark

- ▶ **Overview:** Government programs reducing out-of-pocket medication costs.
- ▶ **Australia (PBS):** subsidizes prescription medicines; costs decrease after a yearly threshold (safety net) is reached.
- ▶ **Helsenorge (Norway):** Covers up to 90% of prescription costs after reaching an annual expenditure cap.
- ▶ **Sweden:** Once a patient reaches a yearly spending threshold, additional medications are free.
- ▶ **Denmark:** Progressive subsidy structure, with higher reimbursements as individual spending increases.

# Cost-Sharing Reductions (CSRs) and Eligibility Limits

## ACA Cost-Sharing Reductions (CSRs)

### ► What are CSRs?

- Subsidies that lower out-of-pocket costs (e.g., co-pays, deductibles).
- Available to individuals/families with incomes between 100% and 250% of the Federal Poverty Level (FPL).

### ► Eligibility Tied to Lower Insurance Plans

- To qualify for CSRs, you *must* purchase a **Silver-level** plan on the ACA marketplace.
- Other plan tiers (**Bronze, Gold, or Platinum**) **do not** offer CSRs, even if you're income-eligible.
- Silver plans have a standard **70% actuarial value**, but CSRs raise it to up to **94%** for lower-income enrollees.

### ► Impact of Limiting to Silver Plans

- Higher-income individuals may choose other plan levels, but lose CSR eligibility.
- Lower-income enrollees are incentivized to choose Silver plans to reduce out-of-pocket costs.

# German Health System: Prohibition of Topping Up

- ▶ **Public Health Insurance:** Citizens covered by statutory health insurance (SHI) cannot "top up" SHI with private insurance for services already covered.
- ▶ **Supplementary Insurance:** Private insurance can only be used for services not included in SHI (e.g., private rooms, certain dental services).
- ▶ **Comprehensive Coverage:** SHI already covers essential medical services, discouraging the need for topping up with private health plans.

# Public Education: Prohibition of Private Tutoring in China & South Korea

- ▶ **Public Education:** Both China and South Korea provide universal public education for students, with restrictions on private supplementary tutoring.
- ▶ **Prohibition:** Private tutoring and after-school programs are heavily regulated or banned to prevent parents from "topping up" public education with private instruction.
- ▶ **Equal Access:** The aim is to reduce inequality in educational opportunities and prevent wealthier families from gaining an advantage through private education.



# Public Housing

- ▶ **Public Housing Programs:** Residents in public housing receive heavily subsidized rent, often capped at a percentage of their income.
- ▶ **Prohibition:** Participants must choose between living in public housing or renting in the private market; they cannot "top up" their public housing subsidy to rent a private apartment.
- ▶ **Example Cities and Countries:**
  - **Singapore:** The Housing & Development Board (HDB) provides subsidized flats, and participants cannot receive additional subsidies to live in private housing.
  - **Vienna, Austria:** The city's extensive public housing program offers low-cost rental units, with no option to "top up" for private market rentals.
  - **Hong Kong:** The Public Rental Housing (PRH) program offers heavily subsidized apartments, and recipients must choose between public housing and private market rentals.

# Egypt's Tamween Food Subsidy Program

- ▶ **Overview:** The Tamween program is one of the largest food subsidy systems in the world, providing essential goods to over 60 million Egyptians, mostly from low-income households.
- ▶ **Targeted Subsidy:**
  - **Bread:** Heavily subsidized at a fraction of market price (often less than 10% of the actual cost), making it affordable for the poor, who rely on it as a staple.
  - **Other Essentials:** Subsidies also cover rice, sugar, and cooking oil, basic items central to the diets of low-income families.
- ▶ **Exclusion:**
  - **Meat and Dairy:** These more expensive food items, consumed more frequently by wealthier households, are not subsidized. Consumers must pay market prices for these products.

# Indonesian Fuel Subsidy Program: Pertamina

- ▶ **Overview:** Indonesia's fuel subsidy program supports transportation for low-income households.
- ▶ **Targeted Subsidy:** The program subsidizes low-octane fuel, which is primarily used by motorcycles, the preferred transport mode for poorer citizens.
- ▶ **Exclusion:** High-octane fuel, more commonly used by cars owned by wealthier households, is not subsidized.

# CalFresh Restaurant Meals Program

- ▶ **Overview:** California's CalFresh program allows certain populations to use benefits for prepared meals.
- ▶ **Targeted Subsidy:** The program subsidizes meals, predominantly from fast food restaurants, providing affordable food options for homeless, elderly, and disabled individuals.
- ▶ **Exclusion:** Dine-in restaurants, typically frequented by wealthier individuals, are not included in the subsidy.

# Public Dentistry Programs in Australia

- ▶ **Overview:** Australia's public dentistry programs provide dental care subsidies to low-income individuals.
- ▶ **Targeted Subsidy:** Prior to 2016, the program subsidized only amalgam fillings, a durable and cost-effective option used widely by lower-income patients.
- ▶ **Exclusion:** Composite (tooth-colored) fillings, which are more expensive and preferred by wealthier individuals, were not fully subsidized.
- ▶ **Post-2016:** amalgam fillings are being phased out due to mercury content.