

Optimal Redistribution Through Subsidies

Zi Yang Kang
University of Toronto

Mitchell Watt
Stanford University

January 28, 2025

Job Talk, University of Nebraska–Lincoln

Introduction

Governments often **redistribute** by **subsidizing consumption**.

Examples: SNAP (food stamps), childcare subsidies, public transit discounts.

Introduction

Governments often **redistribute** by **subsidizing consumption**.

Examples: SNAP (food stamps), childcare subsidies, public transit discounts.

Subsidy programs in practice are often **nonlinear**: marginal subsidy depends on total quantity.

Examples: capped vouchers, co-pays, public options.

Introduction

Governments often **redistribute** by **subsidizing consumption**.

Examples: SNAP (food stamps), childcare subsidies, public transit discounts.

Subsidy programs in practice are often **nonlinear**: marginal subsidy depends on total quantity.

Examples: capped vouchers, co-pays, public options.

This paper: we characterize the **optimal nonlinear subsidy program** for redistribution.

Introduction

Governments often **redistribute** by **subsidizing consumption**.

Examples: SNAP (food stamps), childcare subsidies, public transit discounts.

Subsidy programs in practice are often **nonlinear**: marginal subsidy depends on total quantity.

Examples: capped vouchers, co-pays, public options.

This paper: we characterize the **optimal nonlinear subsidy program** for redistribution.

Key questions:

#1. When should a social planner subsidize consumption?

#2. How are subsidies optimally designed?

Introduction

Governments often **redistribute** by **subsidizing consumption**.

Examples: SNAP (food stamps), childcare subsidies, public transit discounts.

Subsidy programs in practice are often **nonlinear**: marginal subsidy depends on total quantity.

Examples: capped vouchers, co-pays, public options.

This paper: we characterize the **optimal nonlinear subsidy program** for redistribution.

Key questions:

#1. When should a social planner subsidize consumption?

#2. How are subsidies optimally designed?

Our approach: we pose and solve the mechanism design problem for the **optimal subsidy**.

Key Tradeoff

The **optimal subsidy** program trades off:

- #1. screening**, distorting consumption to redirect surplus to high-need consumers, versus
- #2. heterogeneous outside options**, consumers can buy from private market.

Key Tradeoff

The **optimal subsidy** program trades off:

- #1. **screening**, distorting consumption to redirect surplus to high-need consumers, versus
- #2. **heterogeneous outside options**, consumers can buy from private market.

Heterogeneous outside options are empirically relevant, e.g.,

- ▶ public housing (van Dijk, 2019; Waldinger, 2021),
- ▶ education (Akbarpour, Kapor, Neilson, van Dijk & Zimmerman, 2022; Kapor, Karnani & Neilson, 2024),
- ▶ healthcare (Li, 2017; Heim, Lurie, Mullen & Simon, 2021),
- ▶ SNAP (Haider, Jacknowitz & Schoeni, 2003; Ko & Moffitt, 2024; Rafkin, Solomon & Soltas, 2024).

Heterogeneous outside options lead to **lower-bound constraints** in the mechanism design problem.

Results Overview

We provide an **explicit characterization** of:

- (a) **when** the social planner strictly benefits from subsidizing consumption, and
- (b) **how** the optimal subsidy depends on quantity consumed.

Results Overview

We provide an **explicit characterization** of:

- (a) **when** the social planner strictly benefits from subsidizing consumption, and
- (b) **how** the optimal subsidy depends on quantity consumed.

Key determinant of subsidy design: correlation between demand (type θ) and need (welfare weight ω).

Results Overview

We provide an **explicit characterization** of:

- (a) **when** the social planner strictly benefits from subsidizing consumption, and
- (b) **how** the optimal subsidy depends on quantity consumed.

Key determinant of subsidy design: correlation between demand (type θ) and need (welfare weight ω).

Negative Correlation

Positive Correlation

When?

How?

Results Overview

We provide an **explicit characterization** of:

- (a) **when** the social planner strictly benefits from subsidizing consumption, and
- (b) **how** the optimal subsidy depends on quantity consumed.

Key determinant of subsidy design: correlation between demand (type θ) and need (welfare weight ω).

	Negative Correlation	Positive Correlation
When?	If and only if planner would transfer cash to the average consumer (but cannot).	
How?	Free provision and partial marginal subsidies for low levels of consumption.	

Results Overview

We provide an **explicit characterization** of:

- (a) **when** the social planner strictly benefits from subsidizing consumption, and
- (b) **how** the optimal subsidy depends on quantity consumed.

Key determinant of subsidy design: correlation between demand (type θ) and need (welfare weight ω).

	Negative Correlation	Positive Correlation
When?	If and only if planner would transfer cash to the average consumer (but cannot).	If and only if planner would transfer cash to the worst-off consumer.
How?	Free provision and partial marginal subsidies for low levels of consumption.	Partial marginal subsidies for higher levels of consumption.

Results Overview

We provide an **explicit characterization** of:

- (a) **when** the social planner strictly benefits from subsidizing consumption, and
- (b) **how** the optimal subsidy depends on quantity consumed.

Key determinant of subsidy design: correlation between demand (type θ) and need (welfare weight ω).

	Negative Correlation	Positive Correlation
When?	If and only if planner would transfer cash to the average consumer (but cannot).	If and only if planner would transfer cash to the worst-off consumer.
How?	Free provision and partial marginal subsidies for low levels of consumption.	Partial marginal subsidies for higher levels of consumption.

↪ Linear subsidies (e.g., production subsidies) are never optimal.

Related Literature

- ▶ **Public Finance.** Ramsey (1927), Diamond (1975), Mirrlees (1976, 1986), Atkinson & Stiglitz (1976), Nichols & Zeckhauser (1982), Blackorby & Donaldson (1988), Besley & Coate (1991), Blomquist & Christiansen (1998), Doligalski, Dworczak, Krysta & Tokarski (2023).
- ▶ **Redistributive Mechanism Design.** Weitzman (1977), Condorelli (2013), Che, Gale & Kim (2013), Dworczak, Kominers & Akbarpour (2021, 2022), Kang (2023,2024), Akbarpour, Budish, Dworczak & Akbarpour (2024), Pai & Strack (2024), Kang & Watt (2024).
- ▶ **Partial Mechanism Design.** Jullien (2000), Philippon & Skreta (2012), Tirole (2012), Fuchs & Skrzypacz (2015), Dworczak (2020), Loertscher & Muir (2022), Loertscher & Marx (2022), Kang & Muir (2022), Kang (2023), Kang & Watt (2024).
- ▶ **Methodological Tools in Mechanism Design.** Jullien (2000), Toikka (2011), Corrao, Flynn & Sastry (2023), Yang & Zentefis (2024), Valenzuela-Stookey & Poggi (2024).

Model

Model Overview

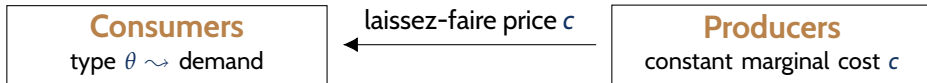
Consumers

type $\theta \rightsquigarrow$ demand

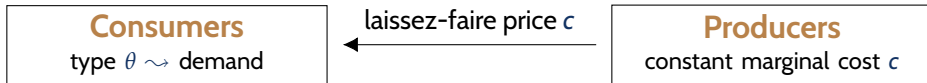
Producers

constant marginal cost c

Model Overview



Model Overview



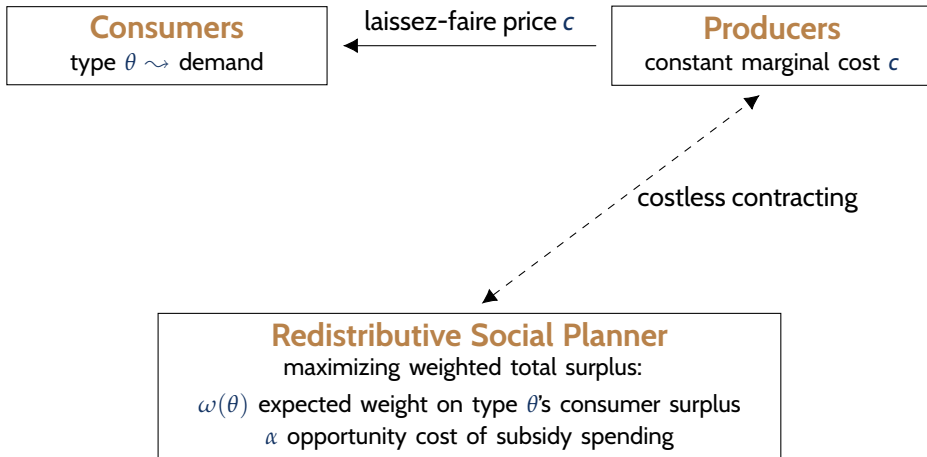
Redistributive Social Planner

maximizing weighted total surplus:

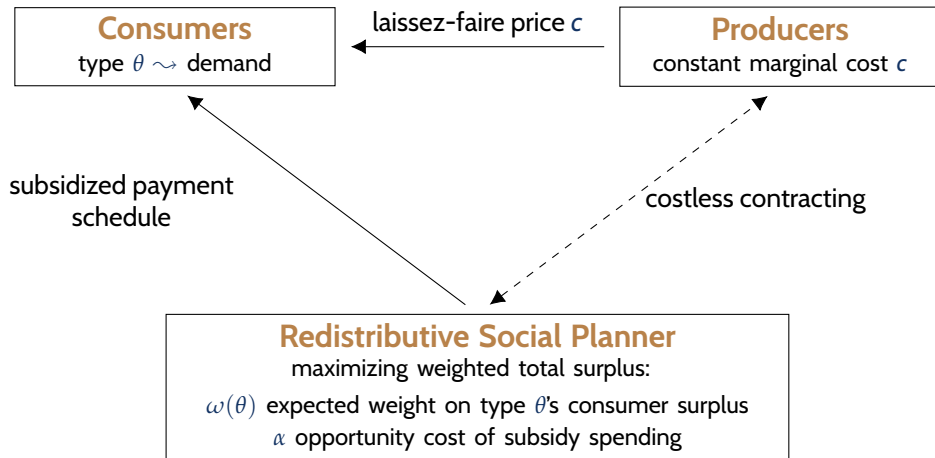
$\omega(\theta)$ expected weight on type θ 's consumer surplus

α opportunity cost of subsidy spending

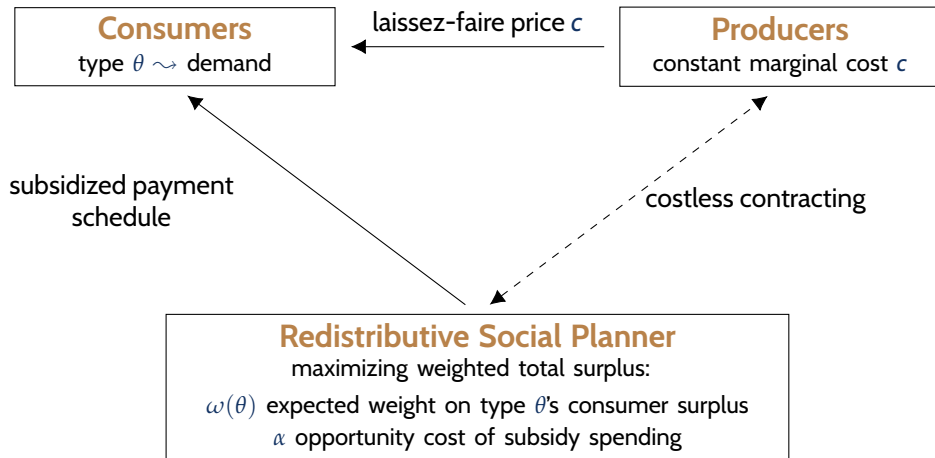
Model Overview



Model Overview



Model Overview



Consumers can purchase units from **both subsidized program and private market**.

Setup

Consumers:

- ▶ There is a unit mass of risk-neutral consumers in market for a divisible, homogeneous good.
- ▶ Consumers differ in type $\theta \in [\underline{\theta}, \bar{\theta}]$ with $\underline{\theta} \geq 0$, and $\theta \sim F$, continuous with density $f > 0$.
- ▶ Each consumer derives utility $\theta v(q) - t$ from quantity $q \in [0, A]$ given payment t .
 $v : [0, A] \rightarrow \mathbb{R}$ is differentiable with $v' > 0$, $v'' < 0$ and $\lim_{q \rightarrow 0} v'(q) = +\infty$.

Setup

Consumers:

- ▶ There is a unit mass of risk-neutral consumers in market for a divisible, homogeneous good.
- ▶ Consumers differ in type $\theta \in [\underline{\theta}, \bar{\theta}]$ with $\underline{\theta} \geq 0$, and $\theta \sim F$, continuous with density $f > 0$.
- ▶ Each consumer derives utility $\theta v(q) - t$ from quantity $q \in [0, A]$ given payment t .
 $v : [0, A] \rightarrow \mathbb{R}$ is differentiable with $v' > 0$, $v'' < 0$ and $\lim_{q \rightarrow 0} v'(q) = +\infty$.

Producers:

- ▶ The good is produced competitively at a constant marginal cost per unit, $c > 0$.

Setup

Consumers:

- ▶ There is a unit mass of risk-neutral consumers in market for a divisible, homogeneous good.
- ▶ Consumers differ in type $\theta \in [\underline{\theta}, \bar{\theta}]$ with $\underline{\theta} \geq 0$, and $\theta \sim F$, continuous with density $f > 0$.
- ▶ Each consumer derives utility $\theta v(q) - t$ from quantity $q \in [0, A]$ given payment t .
 $v : [0, A] \rightarrow \mathbb{R}$ is differentiable with $v' > 0$, $v'' < 0$ and $\lim_{q \rightarrow 0} v'(q) = +\infty$.

Producers:

- ▶ The good is produced competitively at a constant marginal cost per unit, $c > 0$.

Extensions (not today): equilibrium effects, observable characteristics, product choice and eligibility.

Laissez-Faire Equilibrium

- ▶ Perfectly competitive private market \leadsto **laissez-faire price** $p^{\text{LF}} = c$ per unit.
- ▶ Each consumer solves

$$U^{\text{LF}}(\theta) := \max_{q \in [0, A]} [\theta v(q) - cq].$$

v is strictly concave \leadsto unique maximizer:

$$q^{\text{LF}}(\theta) = (v')^{-1} \left(\frac{c}{\theta} \right) = D(c, \theta).$$

Subsidy Design

Social planner costlessly contracts with firms and sells units at a **subsidized payment schedule** $P^\sigma(q)$.

$\leadsto \Sigma(q) = cq - P^\sigma(q)$ is the **total subsidy** as a function of q , and $\sigma(q) = \Sigma'(q)$ is the **marginal subsidy**.

Subsidy Design

Social planner costlessly contracts with firms and sells units at a **subsidized payment schedule** $P^\sigma(q)$.

$\leadsto \Sigma(q) = cq - P^\sigma(q)$ is the **total subsidy** as a function of q , and $\sigma(q) = \Sigma'(q)$ is the **marginal subsidy**.

Key assumptions:

- # 1. Each consumer can **top up** his consumption of the good, allowing him to purchase additional units in the private market at price c ,

$$\leadsto \sigma(q) \geq 0 \text{ for all } q.$$

Subsidy Design

Social planner costlessly contracts with firms and sells units at a **subsidized payment schedule** $P^\sigma(q)$.

$\leadsto \Sigma(q) = cq - P^\sigma(q)$ is the **total subsidy** as a function of q , and $\sigma(q) = \Sigma'(q)$ is the **marginal subsidy**.

Key assumptions:

1. Each consumer can **top up** his consumption of the good, allowing him to purchase additional units in the private market at price c ,

$$\leadsto \sigma(q) \geq 0 \text{ for all } q.$$

2. The social planner can subsidize but **not make lump-sum cash transfers**,

$$\leadsto P^\sigma(q) \geq 0 \text{ for all } q.$$

Subsidy Design

Social planner costlessly contracts with firms and sells units at a **subsidized payment schedule** $P^\sigma(q)$.

$\leadsto \Sigma(q) = cq - P^\sigma(q)$ is the **total subsidy** as a function of q , and $\sigma(q) = \Sigma'(q)$ is the **marginal subsidy**.

Key assumptions:

1. Each consumer can **top up** his consumption of the good, allowing him to purchase additional units in the private market at price c ,

$$\leadsto \sigma(q) \geq 0 \text{ for all } q.$$

2. The social planner can subsidize but **not make lump-sum cash transfers**,

$$\leadsto P^\sigma(q) \geq 0 \text{ for all } q.$$

Implementation: Consumer θ solves $U^\sigma(\theta) := \max_q [\theta v(q) - P^\sigma(q)]$, leading to **subsidized demand** $q^\sigma(\theta)$.

Redistributive Objective

The social planner seeks to maximize **weighted total surplus**.

- ▶ Consumer surplus: social planner assigns a welfare weight $\omega(\theta) := \mathbf{E}[\omega|\theta]$ to consumer type θ .
 $\leadsto \omega(\theta)$: expected social value of giving consumer θ one unit of money.

Redistributive Objective

The social planner seeks to maximize **weighted total surplus**.

- ▶ Consumer surplus: social planner assigns a welfare weight $\omega(\theta) := \mathbf{E}[\omega|\theta]$ to consumer type θ .
 $\leadsto \omega(\theta)$: expected social value of giving consumer θ one unit of money.
- ▶ Total costs: social planner assigns a welfare weight of $\alpha \in \mathbb{R}_+$ to total costs.
 - captures **opportunity cost** of subsidy spending (cf. other redistributive programs, tax cuts).

Redistributive Objective

The social planner seeks to maximize **weighted total surplus**.

- ▶ **Consumer surplus**: social planner assigns a welfare weight $\omega(\theta) := \mathbf{E}[\omega|\theta]$ to consumer type θ .
 $\leadsto \omega(\theta)$: expected social value of giving consumer θ one unit of money.
- ▶ **Total costs**: social planner assigns a welfare weight of $\alpha \in \mathbb{R}_+$ to total costs.
 - captures **opportunity cost** of subsidy spending (cf. other redistributive programs, tax cuts).

\leadsto **Objective**:

$$\max_{P^\sigma(q) \geq 0 \text{ s.t. } \sigma(q) \geq 0} \int_{\theta} [\omega(\theta) U^\sigma(\theta) - \alpha \Sigma(q^\sigma(\theta))] dF(\theta)$$

Redistributive Objective

The social planner seeks to maximize **weighted total surplus**.

- ▶ **Consumer surplus**: social planner assigns a welfare weight $\omega(\theta) := \mathbf{E}[\omega|\theta]$ to consumer type θ .
 $\leadsto \omega(\theta)$: expected social value of giving consumer θ one unit of money.
- ▶ **Total costs**: social planner assigns a welfare weight of $\alpha \in \mathbb{R}_+$ to total costs.
 - captures **opportunity cost** of subsidy spending (cf. other redistributive programs, tax cuts).

\leadsto **Objective**:

$$\max_{P^\sigma(q) \geq 0 \text{ s.t. } \sigma(q) \geq 0} \int_{\theta} [\omega(\theta) U^\sigma(\theta) - \alpha \Sigma(q^\sigma(\theta))] dF(\theta)$$

Remarks:

- ▶ If $\omega(\theta) > \alpha$, social planner would want to transfer a dollar to type θ .
- ▶ If $\mathbf{E}_{\theta}[\omega(\theta)] > \alpha$, social planner would want to make a lump-sum cash transfer to all consumers.

Correlation Assumption

Two baseline cases:

“**Negative Correlation**”: $\omega(\theta)$ is decreasing in θ .

- ▶ high-demand consumers tend to have lower need for redistribution.
- ▶ e.g., food, education, and, if $\omega \propto 1/\text{Income}$, **normal** goods.

“**Positive Correlation**”: $\omega(\theta)$ is increasing in θ .

- ▶ high-demand consumers tend to have higher need for redistribution.
- ▶ e.g., staple foods, public transportation, and, if $\omega \propto 1/\text{Income}$, **inferior** goods.

Correlation Assumption

Two baseline cases:

“**Negative Correlation**”: $\omega(\theta)$ is decreasing in θ .

- ▶ high-demand consumers tend to have lower need for redistribution.
- ▶ e.g., food, education, and, if $\omega \propto 1/\text{Income}$, **normal** goods.

“**Positive Correlation**”: $\omega(\theta)$ is increasing in θ .

- ▶ high-demand consumers tend to have higher need for redistribution.
- ▶ e.g., staple foods, public transportation, and, if $\omega \propto 1/\text{Income}$, **inferior** goods.

Related Literature

- ▶ **Public Finance.** Ramsey (1927), Diamond (1975), Mirrlees (1976, 1986), Atkinson & Stiglitz (1976), Nichols & Zeckhauser (1982), Blackorby & Donaldson (1988), Besley & Coate (1991), Blomquist & Christiansen (1998), Doligalski, Dworczak, Krysta & Tokarski (2023).
 - We focus on commodity subsidies, not ruled out by Atkinson-Stiglitz because no “incentive separability.”
 - We allow for **nonlinear subsidy designs** and characterize global optimum, not FOCs à la Mirrlees.
 - Different objective (local linearization), incorporating a private market not in control of social planner.

Related Literature

- ▶ **Public Finance.** Ramsey (1927), Diamond (1975), Mirrlees (1976, 1986), Atkinson & Stiglitz (1976), Nichols & Zeckhauser (1982), Blackorby & Donaldson (1988), Besley & Coate (1991), Blomquist & Christiansen (1998), Doligalski, Dworczak, Krysta & Tokarski (2023).
- ▶ **Redistributive Mechanism Design.** Weitzman (1977), Condorelli (2013), Che, Gale & Kim (2013), Dworczak, Kominers & Akbarpour (2021, 2022), Kang (2023,2024), Akbarpour, Budish, Dworczak & Akbarpour (2024), Pai & Strack (2024), Kang & Watt (2024).
 - Dworczak, Akbarpour & Kominers (2021): unit demand consumers trading a fixed supply with different qualities, focusing on cross- and same-side inequality.
 - Akbarpour, Dworczak & Kominers (2022): unit demand, fixed supply, across & within-group matching.
 - This paper: **partial control**, we allow consumers to consume in private market outside of planner's control.

Related Literature

- ▶ **Public Finance.** Ramsey (1927), Diamond (1975), Mirrlees (1976, 1986), Atkinson & Stiglitz (1976), Nichols & Zeckhauser (1982), Blackorby & Donaldson (1988), Besley & Coate (1991), Blomquist & Christiansen (1998), Doligalski, Dworzak, Krysta & Tokarski (2023).
- ▶ **Redistributive Mechanism Design.** Weitzman (1977), Condorelli (2013), Che, Gale & Kim (2013), Dworzak, Kominers & Akbarpour (2021, 2022), Kang (2023,2024), Akbarpour, Budish, Dworzak & Akbarpour (2024), Pai & Strack (2024), Kang & Watt (2024).
- ▶ **Partial Mechanism Design.** Jullien (2000), Philippon & Skreta (2012), Tirole (2012), Fuchs & Skrzypacz (2015), Dworzak (2020), Loertscher & Muir (2022), Loertscher & Marx (2022), Kang & Muir (2022), Kang (2023), Kang & Watt (2024).
 - **This paper:** focus on redistribution, “topping up” (cf. Kang & Watt, 2024).
 - Compared to Kang (2023), we focus on benchmark of equally efficient social planner (cf. a public option).

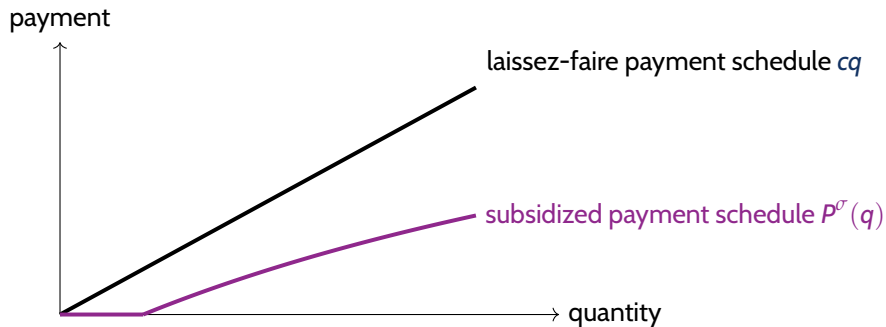
Related Literature

- ▶ **Public Finance.** Ramsey (1927), Diamond (1975), Mirrlees (1976, 1986), Atkinson & Stiglitz (1976), Nichols & Zeckhauser (1982), Blackorby & Donaldson (1988), Besley & Coate (1991), Blomquist & Christiansen (1998), Doligalski, Dworzak, Krysta & Tokarski (2023).
- ▶ **Redistributive Mechanism Design.** Weitzman (1977), Condorelli (2013), Che, Gale & Kim (2013), Dworzak, Kominers & Akbarpour (2021, 2022), Kang (2023,2024), Akbarpour, Budish, Dworzak & Akbarpour (2024), Pai & Strack (2024), Kang & Watt (2024).
- ▶ **Partial Mechanism Design.** Jullien (2000), Philippon & Skreta (2012), Tirole (2012), Fuchs & Skrzypacz (2015), Dworzak (2020), Loertscher & Muir (2022), Loertscher & Marx (2022), Kang & Muir (2022), Kang (2023), Kang & Watt (2024).
- ▶ **Methodological Tools in Mechanism Design.** Jullien (2000), Toikka (2011), Corrao, Flynn & Sastry (2023), Yang & Zentefis (2024), Valenzuela-Stookey & Poggi (2024).
 - **This paper:** explicit characterization of solution with FOSD (topping up) constraints.
 - Cf. Yang & Zentefis (2024) who study extrema of such “monotone function intervals,” optima are interior.

When (Not) To Use Subsidies?

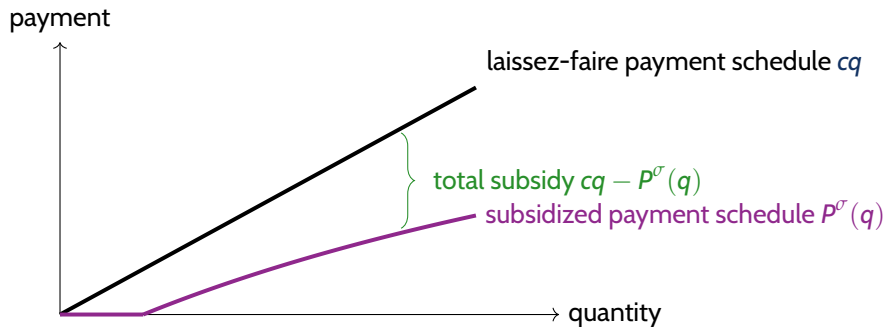
Private Market Access Constrains Redistribution Using Subsidies

private market access \iff marginal price $\leq c$



Private Market Access Constrains Redistribution Using Subsidies

private market access \iff marginal price $\leq c$
 \iff total subsidies increasing in q

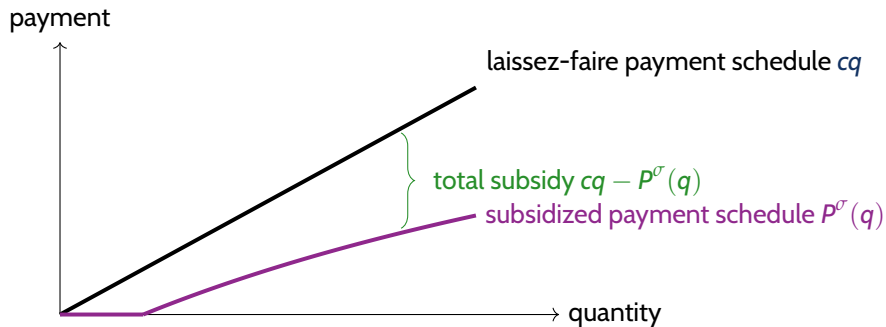


Private Market Access Constrains Redistribution Using Subsidies

private market access \iff marginal price $\leq c$

\iff total subsidies increasing in q

\iff total subsidies increasing in θ

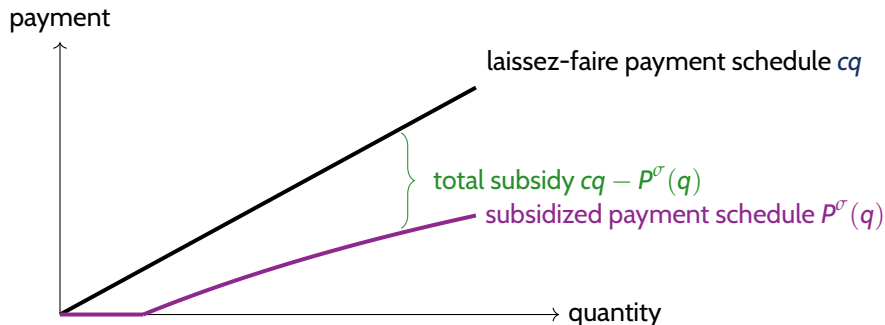


Private Market Access Constrains Redistribution Using Subsidies

private market access \iff marginal price $\leq c$

\iff total subsidies increasing in q

\iff total subsidies increasing in θ



Subsidies are captured disproportionately by **high** θ consumers.

When Not To Subsidize?

Recall our “negative correlation” assumption: high θ consumers have lower ω .

Proposition. For any subsidy P^σ , the social planner would prefer to make a lump-sum transfer of $\mathbf{E}_\theta[\Sigma(q^\sigma(\theta))]$ to all consumers than the subsidy outcome.

When Not To Subsidize?

Recall our “negative correlation” assumption: high θ consumers have lower ω .

Proposition. For any subsidy P^σ , the social planner would prefer to make a lump-sum transfer of $\mathbf{E}_\theta[\Sigma(q^\sigma(\theta))]$ to all consumers than the subsidy outcome.

Proof: By definition of U^{LF} and correlation inequality,

$$\underbrace{\int_{\Theta} \omega(\theta) U^\sigma(\theta) - \alpha \Sigma(q^\sigma(\theta)) \, dF(\theta)}_{\text{objective given } P^\sigma} = \int_{\Theta} \omega(\theta) [\theta v(q^\sigma(\theta)) - cq^\sigma(\theta) + \Sigma(q^\sigma(\theta))] - \alpha \Sigma(q^\sigma(\theta)) \, dF(\theta)$$

When Not To Subsidize?

Recall our “negative correlation” assumption: high θ consumers have lower ω .

Proposition. For any subsidy P^σ , the social planner would prefer to make a lump-sum transfer of $E_\theta[\Sigma(q^\sigma(\theta))]$ to all consumers than the subsidy outcome.

Proof: By definition of U^{LF} and correlation inequality,

$$\underbrace{\int_{\Theta} \omega(\theta) U^\sigma(\theta) - \alpha \Sigma(q^\sigma(\theta)) \, dF(\theta)}_{\text{objective given } P^\sigma} = \int_{\Theta} \omega(\theta) [\theta v(q^\sigma(\theta)) - cq^\sigma(\theta) + \Sigma(q^\sigma(\theta))] - \alpha \Sigma(q^\sigma(\theta)) \, dF(\theta) \\ \leq \int_{\Theta} \omega(\theta) [U^{LF}(\theta) + \Sigma(q^\sigma(\theta))] - \alpha \Sigma(q^\sigma(\theta)) \, dF(\theta)$$

When Not To Subsidize?

Recall our “negative correlation” assumption: high θ consumers have lower ω .

Proposition. For any subsidy P^σ , the social planner would prefer to make a lump-sum transfer of $\mathbf{E}_\theta[\Sigma(q^\sigma(\theta))]$ to all consumers than the subsidy outcome.

Proof: By definition of U^{LF} and correlation inequality,

$$\begin{aligned}
 \underbrace{\int_{\Theta} \omega(\theta) U^\sigma(\theta) - \alpha \Sigma(q^\sigma(\theta)) \, dF(\theta)}_{\text{objective given } P^\sigma} &= \int_{\Theta} \omega(\theta) [\theta v(q^\sigma(\theta)) - cq^\sigma(\theta) + \Sigma(q^\sigma(\theta))] - \alpha \Sigma(q^\sigma(\theta)) \, dF(\theta) \\
 &\leq \int_{\Theta} \omega(\theta) [U^{LF}(\theta) + \Sigma(q^\sigma(\theta))] - \alpha \Sigma(q^\sigma(\theta)) \, dF(\theta) \\
 &\leq \underbrace{\int_{\Theta} \omega(\theta) [U^{LF}(\theta) + \mathbf{E}_\theta[\Sigma(q^\sigma(\theta))]] \, dF(\theta) - \alpha \mathbf{E}_\theta[\Sigma(q^\sigma(\theta))]}_{\text{objective given cash payment } \mathbf{E}_\theta[\Sigma(q^\sigma(\theta))]} . \quad \square
 \end{aligned}$$

When Not To Subsidize?

Recall our “negative correlation” assumption: high θ consumers have lower ω .

Proposition. For any subsidy P^σ , the social planner would prefer to make a lump-sum transfer of $\mathbf{E}_\theta[\Sigma(q^\sigma(\theta))]$ to all consumers than the subsidy outcome.

Proof: By definition of U^{LF} and correlation inequality,

$$\begin{aligned} \underbrace{\int_{\Theta} \omega(\theta) U^\sigma(\theta) - \alpha \Sigma(q^\sigma(\theta)) \, dF(\theta)}_{\text{objective given } P^\sigma} &= \int_{\Theta} \omega(\theta) [\theta v(q^\sigma(\theta)) - cq^\sigma(\theta) + \Sigma(q^\sigma(\theta))] - \alpha \Sigma(q^\sigma(\theta)) \, dF(\theta) \\ &\leq \int_{\Theta} \omega(\theta) [U^{LF}(\theta) + \Sigma(q^\sigma(\theta))] - \alpha \Sigma(q^\sigma(\theta)) \, dF(\theta) \\ &\leq \underbrace{\int_{\Theta} \omega(\theta) [U^{LF}(\theta) + \mathbf{E}_\theta[\Sigma(q^\sigma(\theta))]] \, dF(\theta) - \alpha \mathbf{E}_\theta[\Sigma(q^\sigma(\theta))]}_{\text{objective given cash payment } \mathbf{E}_\theta[\Sigma(q^\sigma(\theta))]} . \quad \square \end{aligned}$$

Theorem 1 (Negative Correlation, part). The social planner subsidizes consumption **only if** $\mathbf{E}_\theta[\omega(\theta)] > \alpha$ (and cash transfers are unavailable).

When To Subsidize?

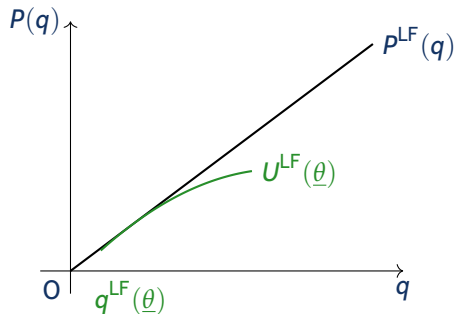
Theorem 1 (Negative Correlation). The social planner subsidizes consumption **if and only if** $\mathbf{E}_\theta[\omega(\theta)] > \alpha$ (and cash transfers are unavailable).

When To Subsidize?

Theorem 1 (Negative Correlation). The social planner subsidizes consumption **if and only if** $\mathbf{E}_\theta[\omega(\theta)] > \alpha$ (and cash transfers are unavailable).

Proof of “if” direction:

Suppose $\mathbf{E}_\theta[\omega(\theta)] > \alpha$. We identify a subsidy schedule improving over laissez-faire.



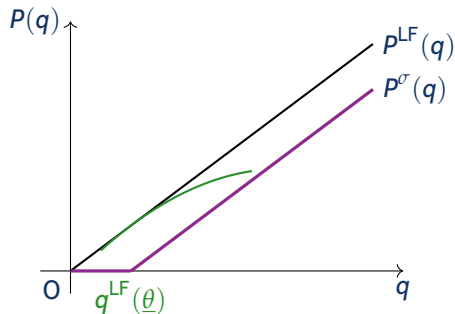
When To Subsidize?

Theorem 1 (Negative Correlation). The social planner subsidizes consumption **if and only if** $\mathbf{E}_\theta[\omega(\theta)] > \alpha$ (and cash transfers are unavailable).

Proof of “if” direction:

Suppose $\mathbf{E}_\theta[\omega(\theta)] > \alpha$. We identify a subsidy schedule improving over laissez-faire.

P^σ is outcome-equivalent to a cash transfer of $cq^{\text{LF}}(\underline{\theta})$ to all consumers, and improves over laissez-faire because $\mathbf{E}_\theta[\omega(\theta)] > \alpha$. □



How to Design Subsidies?

Mechanism Design Reformulation

Revelation principle \implies it suffices to consider **direct mechanisms** (q, t) consisting of:

- ▶ an **allocation function** $q : [\underline{\theta}, \bar{\theta}] \rightarrow [0, A]$ denoting *total* quantity consumed by type θ ;
- ▶ a **payment rule** $t : [\underline{\theta}, \bar{\theta}] \rightarrow \mathbb{R}$ denoting *total* payment by type θ ,

satisfying incentive-compatibility,

$$\theta \in \arg \max_{\hat{\theta} \in \Theta} \{ \theta v(q(\hat{\theta})) - t(\hat{\theta}) \} \text{ for all } \theta \in \Theta. \quad (\text{IC})$$

Mechanism Design Reformulation

Revelation principle \implies it suffices to consider **direct mechanisms** (q, t) consisting of:

- ▶ an **allocation function** $q : [\underline{\theta}, \bar{\theta}] \rightarrow [0, A]$ denoting *total* quantity consumed by type θ ;
- ▶ a **payment rule** $t : [\underline{\theta}, \bar{\theta}] \rightarrow \mathbb{R}$ denoting *total* payment by type θ ,

satisfying incentive-compatibility,

$$\theta \in \arg \max_{\hat{\theta} \in \Theta} \{ \theta v(q(\hat{\theta})) - t(\hat{\theta}) \} \text{ for all } \theta \in \Theta. \quad (\text{IC})$$

Lemma (Implementation). For any (IC) mechanism (q, t) , there exists a subsidy σ with $q = q^\sigma$ and $t = P^\sigma \circ q^\sigma$ if and only if:

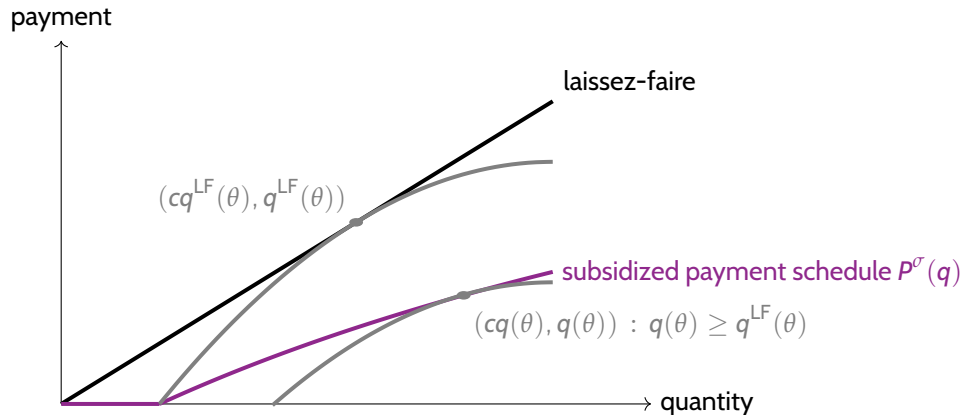
$$q(\theta) \geq q^{\text{LF}}(\theta) \text{ for all } \theta \in \Theta, \quad (\text{LB})$$

$$t(\theta) \geq 0 \text{ for all } \theta \in \Theta, \quad (\text{NLS})$$

$$U(\theta) \geq U^{\text{LF}}(\theta) \text{ for all } \theta \in \Theta. \quad (\text{IR})$$

Intuition

marginal price per unit $\leq c \iff$ allocations exceed laissez-faire



Reformulating the Mechanism Design Problem

The social planner maximizes weighted total surplus

$$\max_{(q,t)} \int_{\underline{\theta}}^{\bar{\theta}} \left[\omega(\theta) \underbrace{[\theta v(q(\theta)) - t(\theta)]}_{\text{consumer surplus}} - \alpha \underbrace{[cq(\theta) - t(\theta)]}_{\text{total cost}} \right] dF(\theta),$$

subject to (IC), (LB), (IR), and (NLS).

Reformulating the Mechanism Design Problem

The social planner maximizes weighted total surplus

$$\max_{(q,t)} \int_{\underline{\theta}}^{\bar{\theta}} \left[\omega(\theta) \underbrace{[\theta v(q(\theta)) - t(\theta)]}_{\text{consumer surplus}} - \alpha \underbrace{[cq(\theta) - t(\theta)]}_{\text{total cost}} \right] dF(\theta),$$

subject to (IC), (LB), (IR), and (NLS).

#1. Apply **Myerson (1981)** Lemma and **Milgrom and Segal (2002)** envelope theorem to express objective in terms of $U(\underline{\theta})$ and $q(\theta)$ non-decreasing, substituting

$$t(\theta) = \theta v(q(\theta)) - \int_{\underline{\theta}}^{\theta} v(q(s)) ds - U(\underline{\theta}).$$

Reformulating the Mechanism Design Problem

The social planner maximizes weighted total surplus

$$\max_{q \text{ non-decreasing, } U(\underline{\theta})} \mathbf{E}_{\theta}[\omega(\theta) - \alpha]U(\underline{\theta}) + \alpha \int_{\underline{\theta}}^{\bar{\theta}} \left[\left(\theta + \frac{\int_{\underline{\theta}}^{\theta} [\omega(s) - \alpha] dF(s)}{\alpha f(\theta)} \right) v(q(\theta)) - cq(\theta) \right] dF(\theta),$$

subject to (LB), (IR), and (NLS).

- #1. Apply Myerson (1981) Lemma and Milgrom and Segal (2002) envelope theorem to express objective in terms of $U(\underline{\theta})$ and $q(\theta)$ non-decreasing, substituting
- #2. Suffices to enforce (IR) and (NLS) only for lowest type $\underline{\theta}$ because $U(\theta) - U^{\text{LF}}(\theta)$ and $t(\theta)$ are nondecreasing by (IC) and (LB).
 - \leadsto (NLS) binding: if $\mathbf{E}[\omega(\theta)] > \alpha$, choose $U(\underline{\theta}) = \underline{\theta}v(q(\underline{\theta}))$.
 - \leadsto (NLS) does not bind: if $\mathbf{E}[\omega(\theta)] \leq \alpha$, choose $U(\underline{\theta}) = U^{\text{LF}}(\underline{\theta})$.

Reformulating the Mechanism Design Problem

The social planner maximizes weighted total surplus

$$\max_{q \text{ non-decreasing}} \alpha \int_{\underline{\theta}}^{\bar{\theta}} \underbrace{[J(\theta)v(q(\theta)) - cq(\theta)]}_{\text{surplus of virtual type}} dF(\theta) + (\text{terms independent of } q),$$

subject to (LB): $q(\theta) \geq q^{\text{LF}}(\theta)$, where the **virtual type** absorbs (IC), (IR), and (NLS):

$$J(\theta) = \underbrace{\theta}_{\text{efficiency}} + \underbrace{\frac{\int_{\underline{\theta}}^{\bar{\theta}} [\omega(s) - \alpha] dF(s)}{\alpha f(\theta)}}_{\text{redistributive motive}} + \underbrace{\frac{\max\{\mathbf{E}_{\theta}[\omega(\theta) - \alpha], 0\} \theta \delta_{\theta=\underline{\theta}}}{\alpha f(\theta)}}_{\text{(NLS) constraint}}$$

Call $J(\theta) - \theta$ the **distortion term**.

Reformulating the Mechanism Design Problem

The social planner maximizes weighted total surplus

$$\max_{q \text{ non-decreasing}} \alpha \int_{\underline{\theta}}^{\bar{\theta}} \underbrace{[J(\theta)v(q(\theta)) - cq(\theta)]}_{\text{surplus of virtual type}} dF(\theta) + (\text{terms independent of } q),$$

subject to (LB): $q(\theta) \geq q^{\text{LF}}(\theta)$, where the **virtual type** absorbs (IC), (IR), and (NLS):

$$J(\theta) = \underbrace{\theta}_{\text{efficiency}} + \underbrace{\frac{\int_{\underline{\theta}}^{\bar{\theta}} [\omega(s) - \alpha] dF(s)}{\alpha f(\theta)}}_{\text{redistributive motive}} + \underbrace{\frac{\max\{\mathbf{E}_{\theta}[\omega(\theta) - \alpha], 0\} \underline{\theta} \delta_{\theta=\underline{\theta}}}{\alpha f(\theta)}}_{\text{(NLS) constraint}}$$

Call $J(\theta) - \theta$ the **distortion term**.

Technical challenge: (LB) is a “pointwise dominance” / FOSD constraint (cf. Yang and Zentefis, 2024) \leadsto possible interactions with the monotonicity constraint.

Characterizing the Optimal Subsidy Allocation

Theorem 2 (Negative Correlation). The optimal allocation rule is unique, continuous and satisfies

$$q^*(\theta) = \begin{cases} D\left(c, \overline{J|_{[\underline{\theta}, \theta_\alpha]}}(\theta)\right) & \text{for } \theta \leq \theta_\alpha \\ q^{\text{LF}}(\theta) & \text{for } \theta \geq \theta_\alpha, \end{cases}$$

where θ_α is defined by

$$\theta_\alpha = \inf \left\{ \theta \in \Theta : \overline{J|_{[\underline{\theta}, \theta]}}(\theta) \leq \theta \right\}.$$

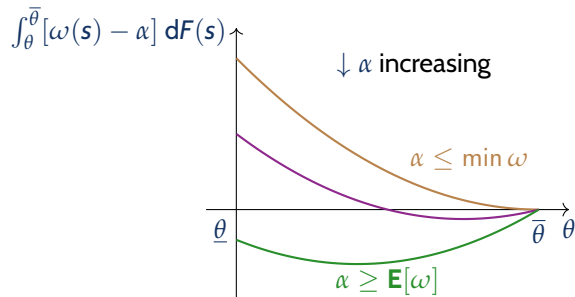
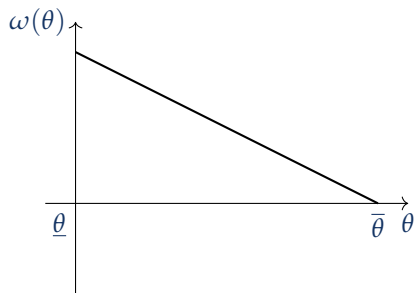
Intuition: there exists a type $\theta_\alpha \in \Theta$ (possibly $\underline{\theta}$ or $\bar{\theta}$) such that

$$q^*(\theta) > q^{\text{LF}}(\theta) \text{ for all } \theta < \theta_\alpha, \text{ and}$$

$$q^*(\theta) = q^{\text{LF}}(\theta) \text{ for all } \theta \geq \theta_\alpha.$$

Intuition: Signing the Distortion Term

Negative correlation $\leadsto \omega(\theta)$ decreasing \leadsto distortion is single-crossing zero from above.



Social planner wants to distort consumption of **all types down**, **low-demand types up** and **high-demand types down**, or **all types upwards**.

Optimal Marginal Subsidy Schedule

Case 1: $\min \omega \geq \alpha$ (upward distortion for all)



Optimal Marginal Subsidy Schedule

Case 1: $\min \omega \geq \alpha$ (upward distortion for all)



Case 2: $\min \omega \leq \alpha \leq \mathbf{E}[\omega]$ (upward distortion for low types, downward distortion for high types)



Optimal Marginal Subsidy Schedule

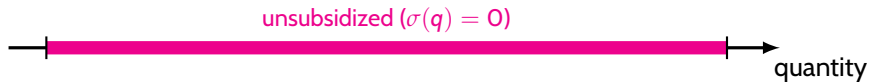
Case 1: $\min \omega \geq \alpha$ (upward distortion for all)



Case 2: $\min \omega \leq \alpha \leq \mathbf{E}[\omega]$ (upward distortion for low types, downward distortion for high types)



Case 3: $\mathbf{E}[\omega] \leq \alpha$ (downward distortion for all)



Economic Implications

With **negative correlation** between ω and θ :

1. Lump-sum cash transfers are always **more progressive** than subsidies.

Economic Implications

With **negative correlation** between ω and θ :

- # 1. Lump-sum cash transfers are always **more progressive** than subsidies.
- # 2. If cash transfers are unavailable, linear subsidies are **never** optimal.
 - # 2a. Optimal subsidies are “all or none”: active subsidy programs should always incorporate a **free allocation** (“public option”).
 - # 2b. If *any* consumer has $\omega < \alpha$, optimal subsidies are **capped** in quantity.

Deriving the Optimal Mechanism

Solving for the Optimal Mechanism

► skip

$$\max_q \alpha \int_{\underline{\theta}}^{\bar{\theta}} [J(\theta)v(q(\theta)) - cq(\theta)] dF(\theta),$$

s.t. q nondecreasing and $q(\theta) \geq q^{\text{LF}}(\theta)$.

Solving for the Optimal Mechanism

► skip

$$\max_q \alpha \int_{\underline{\theta}}^{\bar{\theta}} [J(\theta)v(q(\theta)) - cq(\theta)] dF(\theta),$$

s.t. q nondecreasing and $q(\theta) \geq q^{\text{LF}}(\theta)$.

Guess 1: Pointwise maximizer

$$q(\theta) = (v')^{-1} \left(\frac{c}{J(\theta)} \right) = D(c, J(\theta)).$$

Solving for the Optimal Mechanism

► skip

$$\max_q \alpha \int_{\underline{\theta}}^{\bar{\theta}} [J(\theta)v(q(\theta)) - cq(\theta)] dF(\theta),$$

s.t. q nondecreasing and $q(\theta) \geq q^{\text{LF}}(\theta)$.

Guess 1: Pointwise maximizer

$$q(\theta) = (v')^{-1} \left(\frac{c}{J(\theta)} \right) = D(c, J(\theta)).$$

Demand $D(c, \cdot)$ is increasing, so:

q nondecreasing $\iff J(\theta)$ nondecreasing.

$q \geq q^{\text{LF}} \iff D(c, J(\theta)) \geq D(c, \theta) \iff J(\theta) \geq \theta$.

Solving for the Optimal Mechanism

► skip

$$\max_q \alpha \int_{\underline{\theta}}^{\bar{\theta}} [J(\theta)v(q(\theta)) - cq(\theta)] dF(\theta),$$

s.t. q nondecreasing and $q(\theta) \geq q^{\text{LF}}(\theta)$.

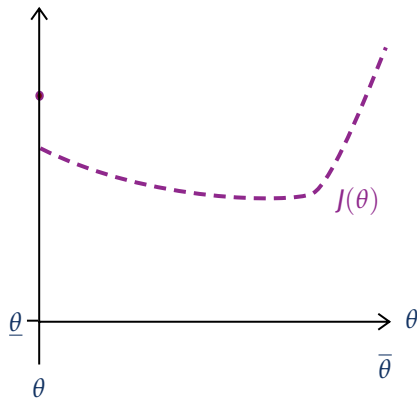
Guess 1: Pointwise maximizer

$$q(\theta) = (v')^{-1} \left(\frac{c}{J(\theta)} \right) = D(c, J(\theta)).$$

Demand $D(c, \cdot)$ is increasing, so:

q nondecreasing $\iff J(\theta)$ nondecreasing.

$q \geq q^{\text{LF}} \iff D(c, J(\theta)) \geq D(c, \theta) \iff J(\theta) \geq \theta.$



$J(\theta)$ may be non-monotone.

Solving for the Optimal Mechanism

► skip

$$\max_q \alpha \int_{\underline{\theta}}^{\bar{\theta}} [J(\theta)v(q(\theta)) - cq(\theta)] dF(\theta),$$

s.t. q nondecreasing and $q(\theta) \geq q^{\text{LF}}(\theta)$.

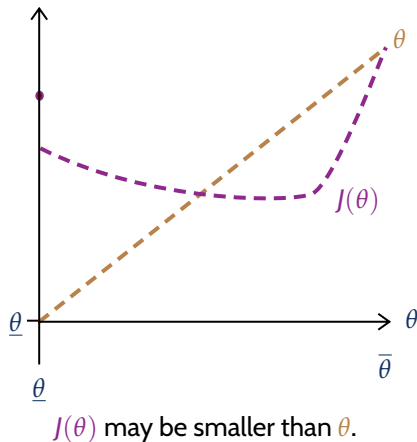
Guess 1: Pointwise maximizer

$$q(\theta) = (v')^{-1} \left(\frac{c}{J(\theta)} \right) = D(c, J(\theta)).$$

Demand $D(c, \cdot)$ is increasing, so:

q nondecreasing $\iff J(\theta)$ nondecreasing.

$q \geq q^{\text{LF}} \iff D(c, J(\theta)) \geq D(c, \theta) \iff J(\theta) \geq \theta.$



Solving for the Optimal Mechanism

► skip

$$\max_q \alpha \int_{\underline{\theta}}^{\bar{\theta}} [J(\theta)v(q(\theta)) - cq(\theta)] dF(\theta),$$

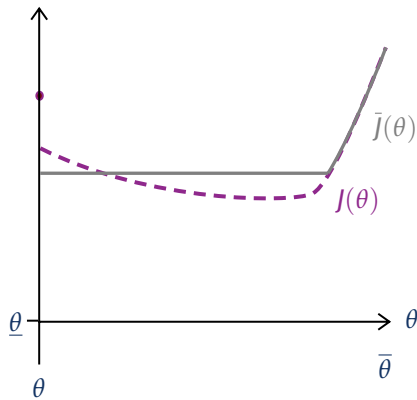
s.t. q nondecreasing and $q(\theta) \geq q^{\text{LF}}(\theta)$.

Guess 2: Relaxing the (LB) constraint

Toikka (2011); Akbarpour, Dworczak, Kominers (2021)

$$\leadsto q(\theta) = (v')^{-1}\left(\frac{c}{\bar{J}(\theta)}\right) = D(c, \bar{J}(\theta)),$$

where \bar{J} is ironing of J , pooling types in any non-monotonic interval of J at its F -weighted average.



Ironing deals with non-monotonicity.

► Ironing

Solving for the Optimal Mechanism

► skip

$$\max_q \alpha \int_{\underline{\theta}}^{\bar{\theta}} [J(\theta)v(q(\theta)) - cq(\theta)] dF(\theta),$$

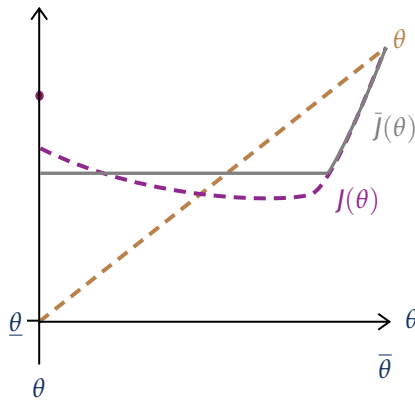
s.t. q nondecreasing and $q(\theta) \geq q^{\text{LF}}(\theta)$.

Guess 2: Relaxing the (LB) constraint

Toikka (2011); Akbarpour, Dworczak, Kominers (2021)

$$\leadsto q(\theta) = (v')^{-1}\left(\frac{c}{\bar{J}(\theta)}\right) = D(c, \bar{J}(\theta)),$$

where \bar{J} is ironing of J , pooling types in any non-monotonic interval of J at its F -weighted average.



But not lower-bound constraint \leadsto ironing.

► Ironing

Solving for the Optimal Mechanism

► skip

$$\max_q \alpha \int_{\underline{\theta}}^{\bar{\theta}} [J(\theta)v(q(\theta)) - cq(\theta)] dF(\theta),$$

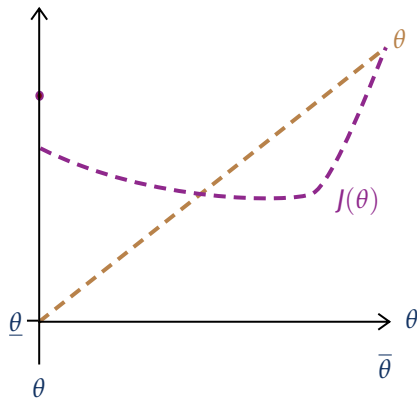
s.t. q nondecreasing and $q(\theta) \geq q^{\text{LF}}(\theta)$.

Guess 3: Our approach

Suppose solution is of the form

$$q(\theta) = D(c, H(\theta)).$$

Feasibility requires H to be nondecreasing and satisfy $H(\theta) \geq \underline{\theta}$.



Need to identify nondecreasing $H \geq \underline{\theta}$.

► Ironing

Characterizing the Optimal Subsidy Allocation

Theorem 2 (Negative Correlation). The optimal allocation rule is unique, continuous and satisfies

$$q^*(\theta) = D(c, H(\theta)),$$

where the **subsidy type** $H(\theta)$ is defined by

$$H(\theta) := \begin{cases} \overline{J|_{[\underline{\theta}, \theta_\alpha]}}(\theta) & \text{for } \theta \leq \theta_\alpha \\ \theta & \text{for } \theta \geq \theta_\alpha, \end{cases}$$

and θ_α is defined by

$$\theta_\alpha = \inf \left\{ \theta \in \Theta : \overline{J|_{[\underline{\theta}, \theta]}}(\theta) \leq \theta \right\}.$$

Characterizing the Optimal Subsidy Allocation

Theorem 2 (Negative Correlation). The optimal allocation rule is unique, continuous and satisfies

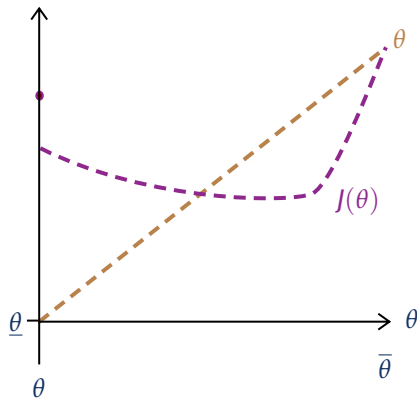
$$q^*(\theta) = D(c, H(\theta)),$$

where the **subsidy type** $H(\theta)$ is defined by

$$H(\theta) := \begin{cases} \overline{J|_{[\underline{\theta}, \theta_\alpha]}}(\theta) & \text{for } \theta \leq \theta_\alpha \\ \theta & \text{for } \theta \geq \theta_\alpha, \end{cases}$$

and θ_α is defined by

$$\theta_\alpha = \inf \left\{ \theta \in \Theta : \overline{J|_{[\underline{\theta}, \theta]}}(\theta) \leq \theta \right\}.$$



Characterizing the Optimal Subsidy Allocation

Theorem 2 (Negative Correlation). The optimal allocation rule is unique, continuous and satisfies

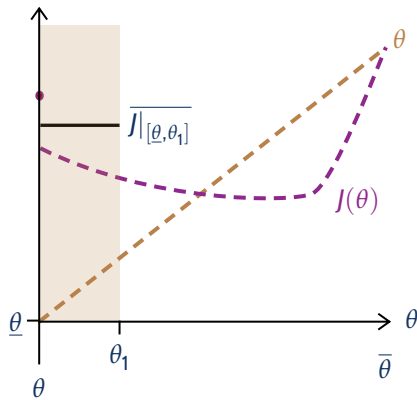
$$q^*(\theta) = D(c, H(\theta)),$$

where the **subsidy type** $H(\theta)$ is defined by

$$H(\theta) := \begin{cases} \overline{J|_{[\underline{\theta}, \theta_\alpha]}}(\theta) & \text{for } \theta \leq \theta_\alpha \\ \theta & \text{for } \theta \geq \theta_\alpha, \end{cases}$$

and θ_α is defined by

$$\theta_\alpha = \inf \left\{ \theta \in \Theta : \overline{J|_{[\underline{\theta}, \theta]}}(\theta) \leq \theta \right\}.$$



Characterizing the Optimal Subsidy Allocation

Theorem 2 (Negative Correlation). The optimal allocation rule is unique, continuous and satisfies

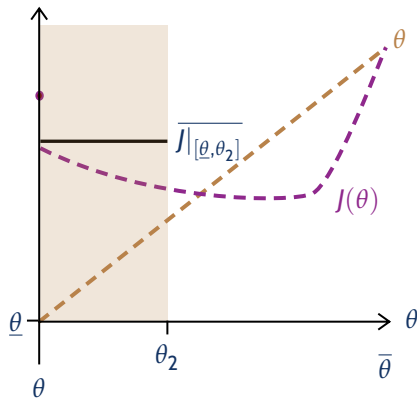
$$q^*(\theta) = D(c, H(\theta)),$$

where the **subsidy type** $H(\theta)$ is defined by

$$H(\theta) := \begin{cases} \overline{J|_{[\underline{\theta}, \theta_\alpha]}}(\theta) & \text{for } \theta \leq \theta_\alpha \\ \theta & \text{for } \theta \geq \theta_\alpha, \end{cases}$$

and θ_α is defined by

$$\theta_\alpha = \inf \left\{ \theta \in \Theta : \overline{J|_{[\underline{\theta}, \theta]}}(\theta) \leq \theta \right\}.$$



Characterizing the Optimal Subsidy Allocation

Theorem 2 (Negative Correlation). The optimal allocation rule is unique, continuous and satisfies

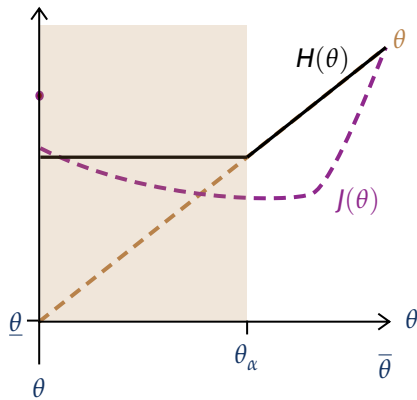
$$q^*(\theta) = D(c, H(\theta)),$$

where the **subsidy type** $H(\theta)$ is defined by

$$H(\theta) := \begin{cases} \overline{J|_{[\underline{\theta}, \theta_\alpha]}}(\theta) & \text{for } \theta \leq \theta_\alpha \\ \theta & \text{for } \theta \geq \theta_\alpha, \end{cases}$$

and θ_α is defined by

$$\theta_\alpha = \inf \left\{ \theta \in \Theta : \overline{J|_{[\underline{\theta}, \theta]}}(\theta) \leq \theta \right\}.$$



construction \leadsto pooling condition and continuity

Verifying H from Theorem 2

Because $q^*(\theta) = D(c, H(\theta))$, for any feasible q

$$\int_{\Theta} \underbrace{[H(\theta)v(q^*(\theta)) - cq^*(\theta)]}_{\text{surplus of type } H(\theta) \text{ at } D(c, H(\theta))} dF(\theta) \geq \int_{\Theta} \underbrace{[H(\theta)v(q(\theta)) - cq(\theta)]}_{\text{surplus of type } H(\theta) \text{ at } q(\theta)} dF(\theta).$$

Verifying H from Theorem 2

Because $q^*(\theta) = D(c, H(\theta))$, for any feasible q

$$\int_{\Theta} \underbrace{[H(\theta)v(q^*(\theta)) - cq^*(\theta)]}_{\text{surplus of type } H(\theta) \text{ at } D(c, H(\theta))} dF(\theta) \geq \int_{\Theta} \underbrace{[H(\theta)v(q(\theta)) - cq(\theta)]}_{\text{surplus of type } H(\theta) \text{ at } q(\theta)} dF(\theta).$$

We want to show, for any feasible q

$$\underbrace{\int_{\Theta} [J(\theta)v(q^*(\theta)) - cq^*(\theta)] dF(\theta)}_{\text{objective at } q^*} \geq \underbrace{\int_{\Theta} [J(\theta)v(q(\theta)) - cq(\theta)] dF(\theta)}_{\text{objective at feasible } q}.$$

Verifying H from Theorem 2

Because $q^*(\theta) = D(c, H(\theta))$, for any feasible q

$$\int_{\Theta} \underbrace{[H(\theta)v(q^*(\theta)) - cq^*(\theta)]}_{\text{surplus of type } H(\theta) \text{ at } D(c, H(\theta))} dF(\theta) \geq \int_{\Theta} \underbrace{[H(\theta)v(q(\theta)) - cq(\theta)]}_{\text{surplus of type } H(\theta) \text{ at } q(\theta)} dF(\theta).$$

We want to show, for any feasible q

$$\underbrace{\int_{\Theta} [J(\theta)v(q^*(\theta)) - cq^*(\theta)] dF(\theta)}_{\text{objective at } q^*} \geq \underbrace{\int_{\Theta} [J(\theta)v(q(\theta)) - cq(\theta)] dF(\theta)}_{\text{objective at feasible } q}.$$

Subtracting, it suffices to show, for any feasible q

$$\int_{\Theta} [J(\theta) - H(\theta)][v(q^*(\theta)) - v(q(\theta))] dF(\theta) \geq 0.$$

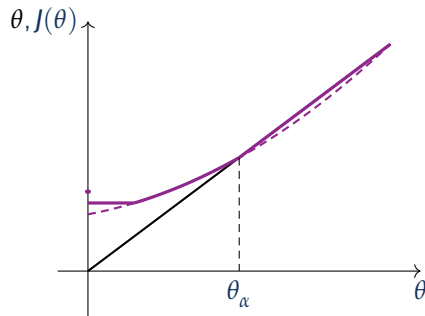
Verifying the Variational Inequality

To show $\int_{\Theta} [J(\theta) - H(\theta)][v(q^*(\theta)) - v(q(\theta))] dF(\theta) \geq 0$.

Verifying the Variational Inequality

To show $\int_{\Theta} [J(\theta) - H(\theta)][v(q^*(\theta)) - v(q(\theta))] dF(\theta) \geq 0$.

There are three possibilities for H , partitioning Θ into intervals:

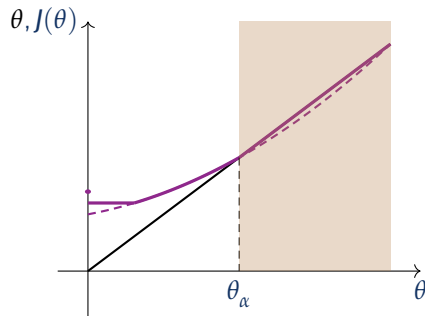


Verifying the Variational Inequality

To show $\int_{\Theta} [J(\theta) - H(\theta)][v(q^*(\theta)) - v(q(\theta))] dF(\theta) \geq 0$.

There are three possibilities for H , partitioning Θ into intervals:

1. $H(\theta) = \theta$: by construction $J(\theta) \leq \theta = H(\theta)$ and $v(q(\theta)) \geq v(q^*(\theta)) \leadsto$ integrand ≥ 0 .

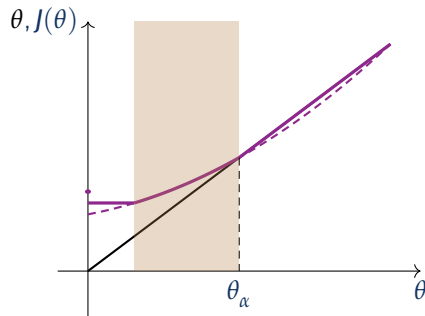


Verifying the Variational Inequality

To show $\int_{\Theta} [J(\theta) - H(\theta)][v(q^*(\theta)) - v(q(\theta))] dF(\theta) \geq 0$.

There are three possibilities for H , partitioning Θ into intervals:

- # 1. $H(\theta) = \theta$: by construction $J(\theta) \leq \theta = H(\theta)$ and $v(q(\theta)) \geq v(q^*(\theta)) \leadsto \text{integrand} \geq 0$.
- # 2. $H(\theta) = J(\theta)$: integrand = 0.



Verifying the Variational Inequality

To show $\int_{\Theta} [J(\theta) - H(\theta)][v(q^*(\theta)) - v(q(\theta))] dF(\theta) \geq 0$.

There are three possibilities for H , partitioning Θ into intervals:

1. $H(\theta) = \theta$: by construction $J(\theta) \leq \theta = H(\theta)$ and $v(q(\theta)) \geq v(q^*(\theta)) \leadsto$ integrand ≥ 0 .

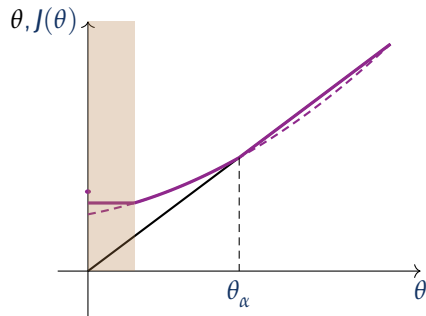
2. $H(\theta) = J(\theta)$: integrand = 0.

3. $H(\theta) = \overline{J|_{[\underline{\theta}, \theta_\alpha]}}(\theta) \neq J(\theta)$:

technical lemma \leadsto on any such interval Θ_i , $H = \overline{J|_{\Theta_i}}$

\leadsto optimality of $D(c, H(\theta))$ in problem on Θ_i *without* (LB)

\implies same variational inequality characterizes optimality. \square



Summing Up

Proof approach:

- ▶ Guess form of solution $q^*(\theta) = D(c, H(\theta))$.
- ▶ Identify $H(\theta)$ which is continuous, $\geq \theta$, and satisfies the **pooling condition**.
- ▶ Verify optimality using **variational inequalities**.

Same method of solution works for general $\omega \rightsquigarrow$ see paper.

▶ Generalization

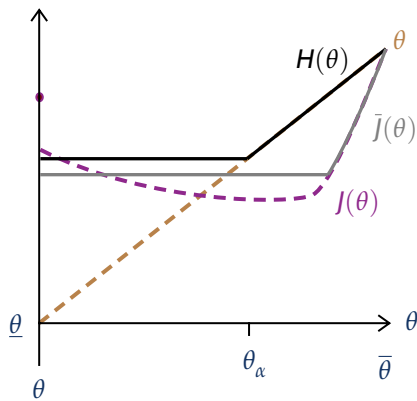
Role of Topping Up

Comparing optimum with and without (LB) constraint, $H(\theta)$ can exceed \bar{J} for *all* types.

→ Inability to tax causes upward distortion of *all* types

It is not optimal to calculate optimal subsidy/tax and set taxes to zero.

Highlights distinction from Mirrleesian marginal approach (FOC \nrightarrow optimum).



Positive Correlation

When to Subsidize?

Positive Correlation

Suppose now that $\omega(\theta)$ is increasing in θ (“positive correlation”), e.g., public transport, staple foods.

When to Subsidize?

Positive Correlation

Suppose now that $\omega(\theta)$ is increasing in θ (“**positive correlation**”), e.g., public transport, staple foods.

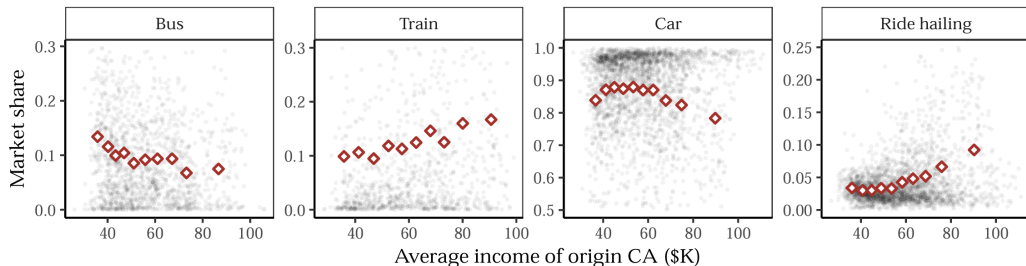


Figure 4 from Almagro, Barbieri, Castillo, Hickok and Salz (2024), constructed using cellphone data and Chicago Transit Survey, shows bus usage in Chicago is higher among lower-income Chicagoans.

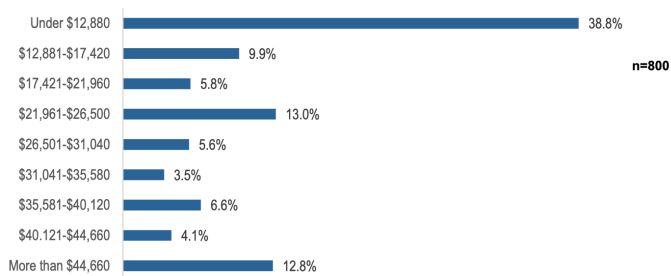
When to Subsidize?

Positive Correlation

Suppose now that $\omega(\theta)$ is increasing in θ (“**positive correlation**”), e.g., public transport, staple foods.

StarTran’s average ridership is low-income, as the plurality of respondents (39%) have a household income less than \$12,880 annually and only 13% reported they earned more than \$44,660 annually. These income levels show a heavy reliance on StarTran’s services and high levels of financial vulnerability amongst its ridership (Figure 37).

Figure 37 Household Income



City of Lincoln Transit Development Plan (2022)

When to Subsidize?

Positive Correlation

Suppose now that $\omega(\theta)$ is increasing in θ (“**positive correlation**”), e.g., public transport, staple foods.

Subsidies increasing in $\theta \rightsquigarrow$ subsidies are **more redistributive** than cash transfers.

When to Subsidize?

Positive Correlation

Suppose now that $\omega(\theta)$ is increasing in θ (“**positive correlation**”), e.g., public transport, staple foods.

Subsidies increasing in $\theta \rightsquigarrow$ subsidies are **more redistributive** than cash transfers.

Theorem 1 (Positive Correlation). The social planner subsidizes consumption **if and only if**

$$\max_{\theta} \omega(\theta) > \alpha.$$

When to Subsidize?

Positive Correlation

Suppose now that $\omega(\theta)$ is increasing in θ (“**positive correlation**”), e.g., public transport, staple foods.

Subsidies increasing in $\theta \rightsquigarrow$ subsidies are **more redistributive** than cash transfers.

Theorem 1 (Positive Correlation). The social planner subsidizes consumption **if and only if**

$$\max_{\theta} \omega(\theta) > \alpha.$$

\rightsquigarrow Subsidies can be beneficial even when lump-sum cash transfers would not be.

When to Subsidize?

Positive Correlation

Suppose now that $\omega(\theta)$ is increasing in θ (“**positive correlation**”), e.g., public transport, staple foods.

Subsidies increasing in $\theta \rightsquigarrow$ subsidies are **more redistributive** than cash transfers.

Theorem 1 (Positive Correlation). The social planner subsidizes consumption **if and only if**

$$\max_{\theta} \omega(\theta) > \alpha.$$

\rightsquigarrow Subsidies can be beneficial even when lump-sum cash transfers would not be.

Intuition: Social planner can always design a subsidy program with $\Sigma(q^{\sigma}(\theta)) \geq 0$ only if $\omega(\theta) \geq \alpha$.

\rightsquigarrow Argument relies on nonlinearity of subsidy program.

► Arbitrary Correlation

How to Subsidize?

Positive Correlation

Theorem 2 (Positive Correlation). The optimal subsidy allocation rule is unique, continuous and satisfies

$$q^*(\theta) = D(c, H(\theta)), \text{ where } H(\theta) = \begin{cases} \theta & \text{if } \theta \leq \theta_\alpha, \\ \overline{J_{[\theta_\alpha, \bar{\theta}]}}(\theta) & \text{if } \theta \geq \theta_\alpha, \end{cases}$$

where $\theta_\alpha = \inf\{\theta \in \Theta : J(\theta) \geq \theta\}$.

Intuition: there exists a type $\theta_\alpha \in \Theta$ (possibly $\underline{\theta}$ or $\bar{\theta}$) such that

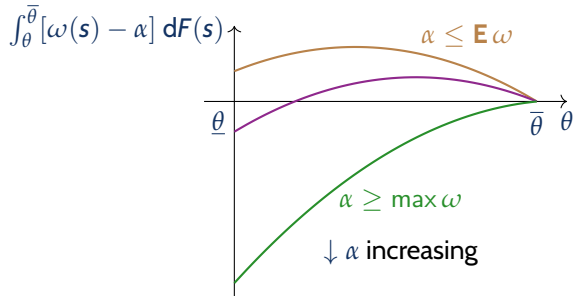
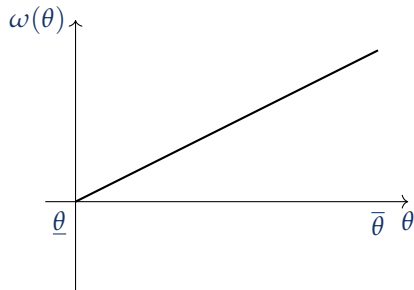
$$\begin{aligned} q^*(\theta) &= q^{\text{LF}}(\theta) \text{ for all } \theta \leq \theta_\alpha, \text{ and} \\ q^*(\theta) &\geq q^{\text{LF}}(\theta) \text{ for all } \theta > \theta_\alpha. \end{aligned}$$

► Arbitrary Correlation

How to Subsidize?

Positive Correlation

Positive correlation $\leadsto \omega(\theta)$ increasing \leadsto distortion is single-crossing zero from below.



Social planner wants to distort consumption of **all types down**, high-demand types up and low-demand types down, or **all types upwards**.

Optimal Subsidy Schedule

Positive Correlation

Case 1: $E[\omega] \geq \alpha$ (upward distortion for all)



Optimal Subsidy Schedule

Positive Correlation

Case 1: $E[\omega] \geq \alpha$ (upward distortion for all)



Case 2: $E[\omega] \leq \alpha \leq \max \omega$ (downward distortion for low types, upward distortion for high types)



Optimal Subsidy Schedule

Positive Correlation

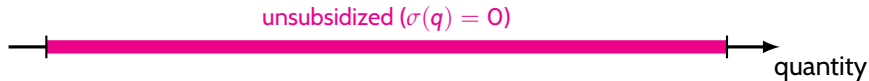
Case 1: $E[\omega] \geq \alpha$ (upward distortion for all)



Case 2: $E[\omega] \leq \alpha \leq \max \omega$ (downward distortion for low types, upward distortion for high types)



Case 3: $\max \omega \leq \alpha$ (downward distortion for all)



Optimal Subsidy Schedule

Positive Correlation

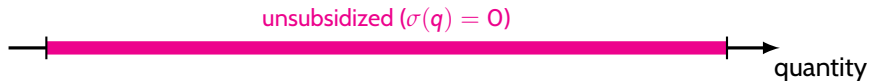
Case 1: $E[\omega] \geq \alpha$ (upward distortion for all)



Case 2: $E[\omega] \leq \alpha \leq \max \omega$ (downward distortion for low types, upward distortion for high types)



Case 3: $\max \omega \leq \alpha$ (downward distortion for all)



Importance of Correlation

Negative Correlation

Subsidies dominated by cash transfers.

Positive Correlation

Subsidies dominate cash transfers.

Importance of Correlation

Negative Correlation

Subsidies dominated by cash transfers.

Subsidies if and only if $\mathbf{E}[\omega] > \alpha$.

Positive Correlation

Subsidies dominate cash transfers.

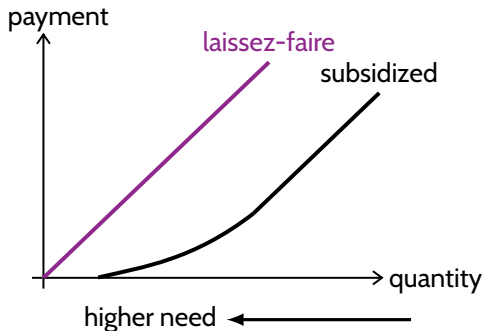
Subsidies if and only if $\max \omega > \alpha$.

Importance of Correlation

Negative Correlation

Subsidies dominated by cash transfers.

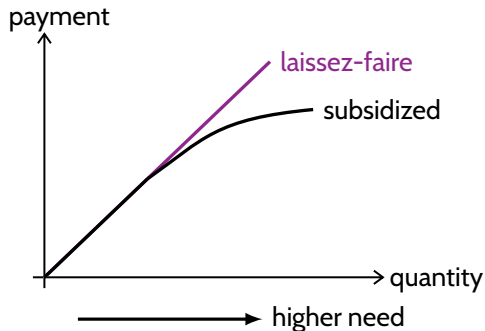
Subsidies if and only if $E[\omega] > \alpha$.



Positive Correlation

Subsidies dominate cash transfers.

Subsidies if and only if $\max \omega > \alpha$.

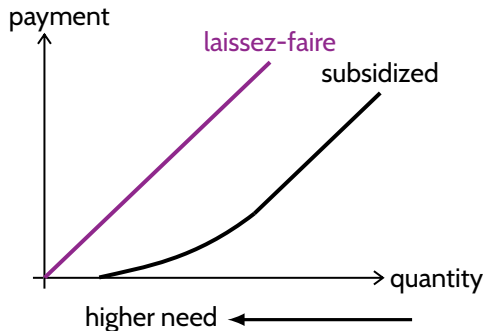


Importance of Correlation

Negative Correlation

Subsidies dominated by cash transfers.

Subsidies if and only if $E[\omega] > \alpha$.

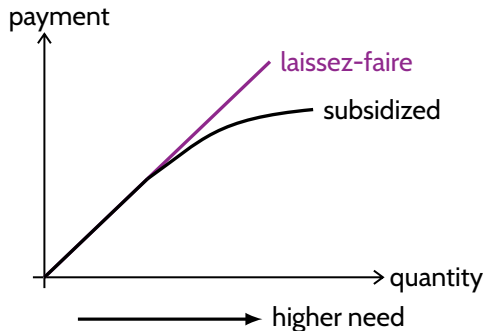


All or none subsidized.

Positive Correlation

Subsidies dominate cash transfers.

Subsidies if and only if $\max \omega > \alpha$.



Only neediest (self-selected) consumers subsidized.

Discussion

Four Questions

- # 1. What are the features of subsidy designs **in practice**?
- # 2. How do optimal subsidies **compare to linear subsidies**?
- # 3. How do **changes in market primitives** affect optimal subsidies?
- # 4. When does the planner want to **restrict** topping up?

1. What are the features of subsidy designs in practice?

1. What are the features of subsidy designs in practice?

When? Theorem 1 \leadsto scope of intervention larger with positive correlation ($\max \omega > \alpha$) than negative correlation ($\mathbf{E}[\omega] > \alpha$).

In practice, many government programs appear to select goods consumed disproportionately by needy.

1. What are the features of subsidy designs in practice?

When? Theorem 1 \leadsto scope of intervention larger with positive correlation ($\max \omega > \alpha$) than negative correlation ($\mathbf{E}[\omega] > \alpha$).

In practice, many government programs appear to select goods consumed disproportionately by needy.

How? Significant differences in marginal subsidy schedules observed in practice:

Larger subsidies for low q

- ▶ Food stamps (SNAP)
- ▶ Womens, Infants & Children (WIC) Program
- ▶ Housing Choice (Section 8) Vouchers
- ▶ Lifeline (Telecomm. Assistance) Program

Larger subsidies for high q

- ▶ Public transit fare capping
- ▶ Pharmaceutical subsidy programs
- ▶ Government-subsidized childcare places.

#2. How Do Optimal Subsidies Compare To Linear?

#2. How Do Optimal Subsidies Compare To Linear?

Proposition. Linear subsidies are **never optimal**.

Intuition: no distortion at the top ($J(\bar{\theta}) = \bar{\theta}$) \leadsto linear subsidies never optimal.

#2. How Do Optimal Subsidies Compare To Linear?

Proposition. Linear subsidies are **never optimal**.

Intuition: no distortion at the top ($J(\bar{\theta}) = \bar{\theta}$) \leadsto linear subsidies never optimal.

Implication: production subsidies are suboptimal for redistribution.

#2. How Do Optimal Subsidies Compare To Linear?

Proposition. Linear subsidies are **never optimal**.

Intuition: no distortion at the top ($J(\bar{\theta}) = \bar{\theta}$) \leadsto linear subsidies never optimal.

Implication: production subsidies are suboptimal for redistribution.

Social planner can always improve over linear subsidy by implementing:

- ▶ a **cap** on the subsidy paid to a consumer (as in negative correlation),
- ▶ a **floor** on eligibility for the subsidy (as in positive correlation), or
- ▶ a **free endowment** of the good to all consumers.

Gains from nonlinear subsidization can be **arbitrarily large** compared to gains from linear subsidization.

▶ skip to conclusion

3. How Do Changes in Market Primitives Affect Optimal Subsidies?

Question: How do optimal subsidies change when

- (a) the social planner's desire to redistribute to each consumer increases?
- (b) the correlation between demand and welfare weight increases?
- (c) the marginal cost of production decreases?

► Details

3. How Do Changes in Market Primitives Affect Optimal Subsidies?

Question: How do optimal subsidies change when

- (a) the social planner's desire to redistribute to each consumer increases?
- (b) the correlation between demand and welfare weight increases?
- (c) the marginal cost of production decreases?

► Details

Short Answer: Each cause the optimal subsidy program to be more generous.

But (a) and (b) cause $J(\theta)$ to increase for each $\theta \rightsquigarrow$ a larger set of consumers subsidized. (c) does not.

4. When Does the Planner Benefit from Restricting Topping Up?

In some markets (e.g., public housing), the social planner may be able to restrict subsidy recipients from topping up in private market \rightsquigarrow opt-in (or out) of subsidy program.

4. When Does the Planner Benefit from Restricting Topping Up?

In some markets (e.g., public housing), the social planner may be able to restrict subsidy recipients from topping up in private market \leadsto opt-in (or out) of subsidy program.

In **Kang and Watt (2024)**, we characterize optimal subsidy mechanism under such restrictions. These lead to different (weaker) type-dependent outside option constraints:

$$\text{average price} \leq c \Leftrightarrow \text{majorization constraint on } q.$$

4. When Does the Planner Benefit from Restricting Topping Up?

In some markets (e.g., public housing), the social planner may be able to restrict subsidy recipients from topping up in private market \leadsto opt-in (or out) of subsidy program.

In **Kang and Watt (2024)**, we characterize optimal subsidy mechanism under such restrictions. These lead to different (weaker) type-dependent outside option constraints:

$$\text{average price} \leq c \Leftrightarrow \text{majorization constraint on } q.$$

Proposition.

- (a) **“Negative Correlation”**: ω decreasing in $\theta \leadsto$ planner benefits from preventing topping up iff $\max \omega > \alpha$.
- (b) **“Positive Correlation”**: ω increasing in $\theta \leadsto$ planner never benefits from preventing topping up.

Intuition: Planner offers subsidies tied to consumption level favored by high ω types.

Implication: Positive correlation between demand and welfare weights reduces the need to enforce topping up restrictions.

Extensions

Equilibrium Effects

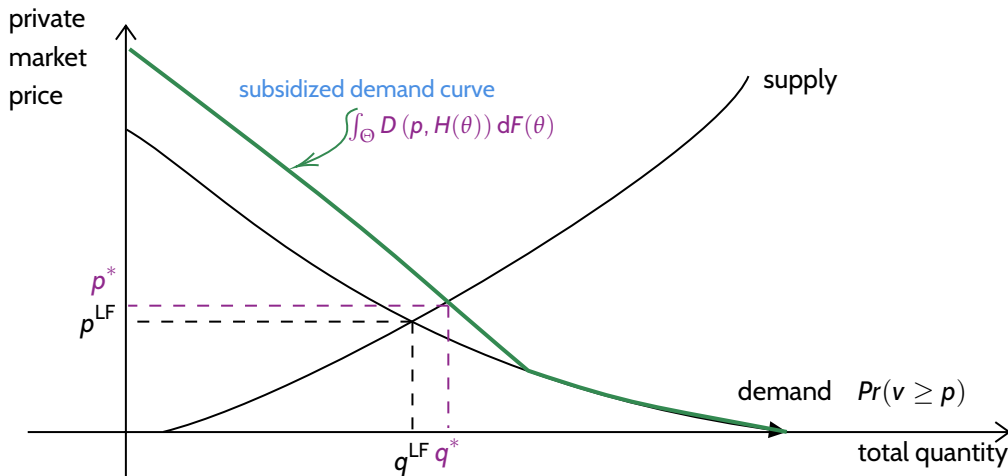
Our baseline model shuts down equilibrium effects of government subsidies on private market prices.

Empirical evidence of price effects from government subsidy programs, e.g.:

- ▶ public housing ([Diamond and McQuade, 2019](#); [Baum-Snow and Marion, 2009](#))
- ▶ pharmaceuticals ([Atal et al., 2021](#))
- ▶ public schools ([Dinerstein and Smith, 2021](#))
- ▶ school lunches ([Handbury and Moshary, 2021](#))

Equilibrium Effects

Our results extend directly to imperfectly elastic supply curves:



Private Market Taxation

Our baseline model assumes the planner cannot tax the private market.

Taxation of private market **reduces** consumers' outside option, relaxing the **(LB)** constraint. If taxation is costly (e.g., because of distortions on ineligible consumers):

Proposition. Suppose the planner faces a convex cost $\Gamma(\tau)$ for taxation of the private market. Then there exists an optimal tax level τ^* and subsidy program for eligible consumers satisfying

$$q^*(\theta) = D(H_{\tau^*}(\theta)),$$

where $H_{\tau^*}(\theta) \leq H(\theta)$.

Budget Constraints and Endogenous Welfare Weights

In our baseline model, $\omega(\cdot)$ and α are taken exogenously.

Our model can be extended to allow weights to be endogenous (cf. [Pai and Strack, 2024](#)):

- ▶ $\alpha \iff$ Lagrange multiplier on the social planner's budget constraint.
- ▶ $\omega(\theta) \iff$ the marginal value of money for a consumer with **concave** preferences

$$\varphi(\theta v(q) + I - t),$$

and income $I \sim G_\theta$, known but not observed by the social planner, then

$$\omega(\theta) = \mathbf{E}_{I \sim G_\theta}[\varphi'(\theta v(q(\theta)) + I - t(\theta))].$$

Conclusion

Concluding Remarks

Takeaways for Subsidy Policy:

- ▶ Linear subsidies (e.g., production subsidies) are **never** optimal.
- ▶ When and how to subsidize depends on **correlation** between demand and need.
 - With negative correlation (many goods), why not lump-sum cash transfers? (“tortilla subsidy” vs. Progresa).
 - Goods with positive correlation are ideal candidates for subsidies (e.g., public transport) but these should have floors to improve targeting.

Concluding Remarks

Takeaways for Subsidy Policy:

- ▶ Linear subsidies (e.g., production subsidies) are **never** optimal.
- ▶ When and how to subsidize depends on **correlation** between demand and need.
 - With negative correlation (many goods), why not lump-sum cash transfers? (“tortilla subsidy” vs. Progresas).
 - Goods with positive correlation are ideal candidates for subsidies (e.g., public transport) but these should have floors to improve targeting.


e.g. **Plan for Action on Regional Transit** (Dec. 2023, Chicago Metropolitan Agency for Planning)

Concluding Remarks

Takeaways for Subsidy Policy:

- ▶ Linear subsidies (e.g., production subsidies) are **never** optimal.
- ▶ When and how to subsidize depends on **correlation** between demand and need.
 - With negative correlation (many goods), why not lump-sum cash transfers? (“tortilla subsidy” vs. Progresa).
 - Goods with positive correlation are ideal candidates for subsidies (e.g., public transport) but these should have floors to improve targeting.

e.g. **Plan for Action on Regional Transit** (Dec. 2023, Chicago Metropolitan Agency for Planning)

 <p>Equity Transit connects people to opportunities</p>	<p>Inequity in the form of disparities — including in employment, health, educational attainment, and income — will persist for transit riders, especially from households with low income, communities of color, seniors, and residents with disabilities.</p>	<p>Leverage the transportation network to promote resiliency and inclusive growth. Long-term economic prosperity requires that the region address inequity issues and take action to increase opportunity and improve quality of life for all residents. Transit plays a key role in creating pathways to opportunity; our region and its communities are stronger when everyone has access to reliable and affordable transportation.</p>
---	---	---

Concluding Remarks

Takeaways for Subsidy Policy:

- ▶ Linear subsidies (e.g., production subsidies) are **never** optimal.
- ▶ When and how to subsidize depends on **correlation** between demand and need.
 - With negative correlation (many goods), why not lump-sum cash transfers? (“tortilla subsidy” vs. Progresa).
 - Goods with positive correlation are ideal candidates for subsidies (e.g., public transport) but these should have floors to improve targeting.

e.g. **Plan for Action on Regional Transit** (Dec. 2023, Chicago Metropolitan Agency for Planning)

Recommendation: Establish regional fare subsidy programs that include youth and riders with low income



Operating cost: Varies depending on program, with scale of costs ranging from \$15-150M annually (depending on discount, eligibility, and uptake).

Capital: N/A

Recommendation: Implement fare capping



Operating cost: Minimal but depends on implementation. There are fare revenue implications, but pass rates could be adjusted to achieve revenue neutrality if desired.

Capital: \$25-\$50M

Concluding Remarks

Takeaways for Subsidy Policy:

- ▶ Linear subsidies (e.g., production subsidies) are **never** optimal.
- ▶ When and how to subsidize depends on **correlation** between demand and need.
 - With negative correlation (many goods), why not lump-sum cash transfers? (“tortilla subsidy” vs. Progresa).
 - Goods with positive correlation are ideal candidates for subsidies (e.g., public transport) but these should have floors to improve targeting.

Technical Contribution:

- ▶ We show how to solve mechanism design problems with lower-bound constraints caused by type-dependent outside options.
- ▶ Similar mechanism design problems arise in other contexts, e.g., subsidy design with other objectives (externalities, paternalism); exclusive contracting (topping up = non-exclusive contracting, no topping up = exclusive contracting.).
- ▶ Future avenue: incorporating adverse selection, as may be needed to study healthcare subsidies.

Fin

Thank you for the invitation!

Assumption: No Lump-Sum Cash Transfers

Note: This constraint only binds if $\mathbf{E}_\theta[\omega(\theta)] > \alpha$.

Possible reasons:

- ▶ **Institutional:** subsidies designed by government agency without tax/transfer powers.
- ▶ **Political:** Liscow and Pershing (2022) find U.S. voters prefer in-kind redistribution to cash transfers.
- ▶ **Household Economics:** Currie (1994) finds in-kind redistribution has stronger benefits for children than cash transfer programs.
- ▶ **Pedagogical:** to contrast when the assumption is binding (\leadsto cash transfers preferred to subsidies) versus non-binding (*vice versa*).
- ▶ **Model:** without NLS constraint, the social planner would want to make unbounded cash transfers when $\mathbf{E}[\omega] > \alpha$.

When to Subsidize (General): Proof by Picture

Theorem 1. Social planner subsidizes **if and only if** there exists a type $\hat{\theta}$ for which

$$\mathbf{E}_{\theta}[\omega(\theta) \mid \theta \geq \hat{\theta}] > \alpha.$$

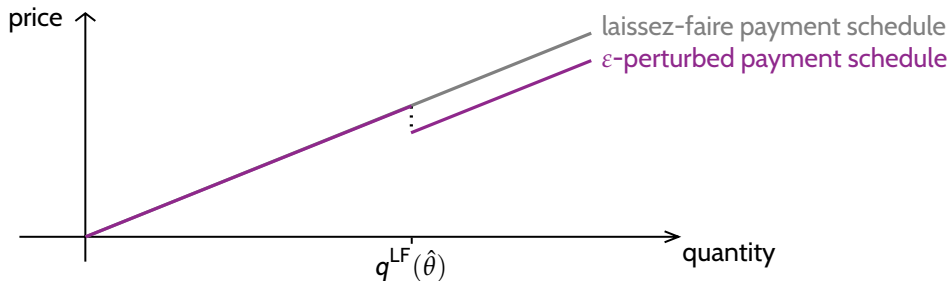
Suppose $\mathbf{E}_{\theta}[\omega(\theta) \mid \theta \geq \hat{\theta}] > \alpha$: we construct a subsidy schedule increasing weighted surplus.

When to Subsidize (General): Proof by Picture

Theorem 1. Social planner subsidizes **if and only if** there exists a type $\hat{\theta}$ for which

$$\mathbf{E}_{\theta}[\omega(\theta) \mid \theta \geq \hat{\theta}] > \alpha.$$

Suppose $\mathbf{E}_{\theta}[\omega(\theta) \mid \theta \geq \hat{\theta}] > \alpha$: we construct a subsidy schedule increasing weighted surplus.



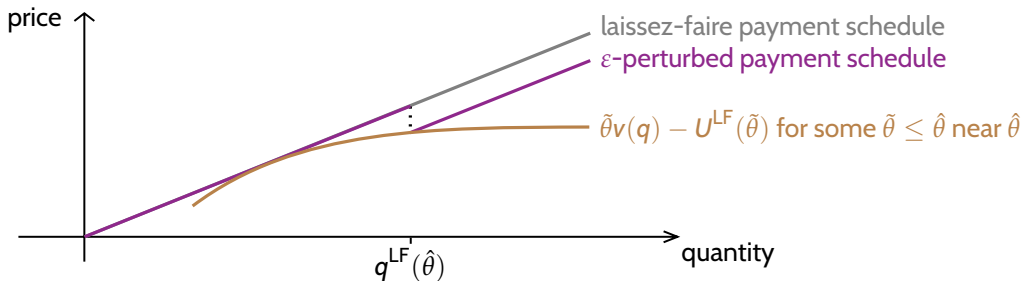
ε-perturbation increases utility of types $\geq \hat{\theta}$, net benefit $\varepsilon \mathbf{E}_{\theta}[\omega(\theta) - \alpha \mid \theta \geq \hat{\theta}]$.

When to Subsidize (General): Proof by Picture

Theorem 1. Social planner subsidizes **if and only if** there exists a type $\hat{\theta}$ for which

$$\mathbf{E}_{\theta}[\omega(\theta) \mid \theta \geq \hat{\theta}] > \alpha.$$

Suppose $\mathbf{E}_{\theta}[\omega(\theta) \mid \theta \geq \hat{\theta}] > \alpha$: we construct a subsidy schedule increasing weighted surplus.



ε -perturbation increases utility of types $\geq \hat{\theta}$, net benefit $\varepsilon \mathbf{E}_{\theta}[\omega(\theta) - \alpha \mid \theta \geq \hat{\theta}]$.

But consumption is distorted for $O(\sqrt{\varepsilon})$ set of types near (but below) $\hat{\theta}$, at cost $\leq O(\sqrt{\varepsilon})\varepsilon$.

\leadsto Benefits $>$ costs for small enough ε . **Note: Argument relies on nonlinearity.**

[▶ return](#)

Topping Up \Leftarrow Lower-Bound (1/2)

Suppose $q(\theta) \geq q^{\text{LF}}(\theta)$. We want to show total subsidies $S(z)$ is increasing in z .

1. $t(\underline{\theta}) \leq cq(\underline{\theta})$ by (IR):

$$t(\underline{\theta}) \leq \underline{\theta}v(q(\underline{\theta})) - \underline{\theta}v(q^{\text{LF}}(\underline{\theta})) + cq^{\text{LF}}(\underline{\theta}),$$

and $\underline{\theta}v(q^{\text{LF}}(\underline{\theta})) - cq^{\text{LF}}(\underline{\theta}) \geq \underline{\theta}v(q(\underline{\theta})) - cq(\underline{\theta})$ by definition of q^{LF} , so $t(\underline{\theta}) \leq cq(\underline{\theta})$.

Topping Up \Leftarrow Lower-Bound (2/2)

2. The *marginal* price of any units purchased is no greater than c by (IC):

$$\begin{aligned} t(\theta') - t(\theta) &= \left[\theta' v(q(\theta')) - U(\underline{\theta}) - \int_{\underline{\theta}}^{\theta'} v(q(s)) \, ds \right] - \left[\theta v(q(\theta)) - U(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} v(q(s)) \, ds \right] \\ &= \theta' v(q(\theta')) - \theta v(q(\theta)) - \int_{\theta}^{\theta'} v(q(s)) \, ds \\ &= \int_{\theta}^{\theta'} s v'(q(s)) \, dq(s). \end{aligned}$$

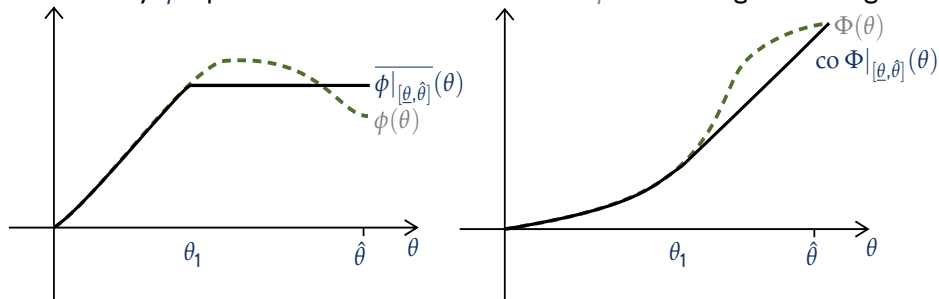
But if $q(\theta) \geq q^{\text{LF}}(\theta)$, then concavity of v implies $v'(q(\theta)) \leq v'(q^{\text{LF}}(\theta)) = c/\theta$, so $t(\theta') - t(\theta) \leq c[q(\theta') - q(\theta)]$.

Ironing

Let ϕ be a (generalized) function and $\Phi : \theta \mapsto \int_{\underline{\theta}}^{\theta} \phi(s) \, dF(s)$. Then $\bar{\phi}$ is the monotone function satisfying

$$\text{for all } \theta \in [\underline{\theta}, \hat{\theta}], \quad \int_{\underline{\theta}}^{\theta} \bar{\phi}(s) \, dF(s) = \text{co } \Phi(\theta).$$

Intuitively, $\bar{\phi}$ replaces non-monotone intervals of ϕ with F -weighted averages.



Rewriting the Mechanism Design Problem

The social planner maximizes weighted total surplus

$$\max_{(q,t)} \int_{\underline{\theta}}^{\bar{\theta}} \left[\omega(\theta) \underbrace{[\theta v(q(\theta)) - t(\theta)]}_{\text{consumer surplus}} - \alpha \underbrace{[cq(\theta) - t(\theta)]}_{\text{total cost}} \right] dF(\theta),$$

subject to (IC), (LB), (IR), and (NLS).

Rewriting the Mechanism Design Problem

$$\max_{q \text{ non-decreasing}, U(\underline{\theta}) \geq U^{\text{LF}}(\underline{\theta})} \mathbf{E}_{\theta} [\omega(\theta) - \alpha] U(\underline{\theta}) + \alpha \int_{\underline{\theta}}^{\bar{\theta}} \left[\left(\theta + \frac{\int_{\theta}^{\bar{\theta}} [\omega(s) - \alpha] dF(s)}{\alpha f(\theta)} \right) v(q(\theta)) - cq(\theta) \right] dF(\theta),$$

subject to (LB), (IR), and (NLS).

- #1. Apply Myerson's (1981) Lemma and Milgrom and Segal (2002) envelope theorem to express objective in terms of $U(\underline{\theta})$ and $q(\theta)$ non-decreasing.

Rewriting the Mechanism Design Problem

$$\max_{q \text{ non-decreasing}, U(\underline{\theta}) \geq U^{\text{LF}}(\underline{\theta})} \mathbf{E}_{\theta}[\omega(\theta) - \alpha]U(\underline{\theta}) + \alpha \int_{\underline{\theta}}^{\bar{\theta}} \left[\left(\theta + \frac{\int_{\theta}^{\bar{\theta}} [\omega(s) - \alpha] dF(s)}{\alpha f(\theta)} \right) v(q(\theta)) - cq(\theta) \right] dF(\theta),$$

subject to (LB), (IR), and (NLS).

#1. Apply **Myerson's (1981)** Lemma and **Milgrom and Segal (2002)** envelope theorem to express objective in terms of $U(\underline{\theta})$ and $q(\theta)$ non-decreasing.

#2. Suffices to enforce (IR) and (NLS) only for lowest type $\underline{\theta}$ because $U(\theta) - U^{\text{LF}}(\theta)$ and $t(\theta)$ are nondecreasing by (IC) and (LB).

↪ **Low cost of public funds:** if $\mathbf{E}[\omega(\theta)] > \alpha$, choose $U(\underline{\theta}) = \underline{\theta}v(q(\underline{\theta}))$.

↪ **High cost of public funds:** if $\mathbf{E}[\omega(\theta)] \leq \alpha$, choose $U(\underline{\theta}) = U^{\text{LF}}(\underline{\theta})$.

Rewriting the Mechanism Design Problem

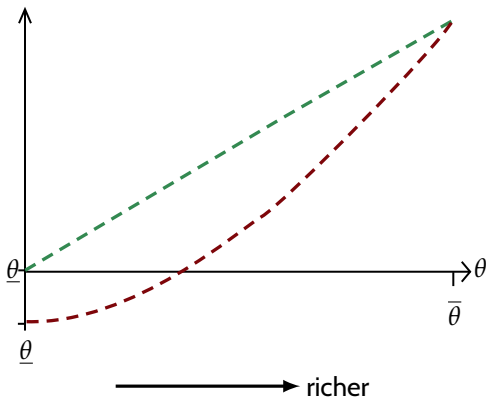
$$\max_{q \text{ non-decreasing}} \alpha \int_{\underline{\theta}}^{\bar{\theta}} [J(\theta)v(q(\theta)) - cq(\theta)] dF(\theta) + (\text{terms independent of } q),$$

subject to (LB): $q(\theta) \geq q^{\text{LF}}(\theta)$, where the **virtual type**

$$J(\theta) = \underbrace{\theta}_{\text{efficiency}} + \underbrace{\frac{\int_{\underline{\theta}}^{\bar{\theta}} [\omega(s) - \alpha] dF(s)}{\alpha f(\theta)}}_{\text{redistributive motive}} + \underbrace{\frac{\max\{\mathbf{E}_{\theta}[\omega(\theta) - \alpha], 0\} \theta \delta_{\theta=\underline{\theta}}}{\alpha f(\theta)}}_{\text{(NLS) constraint}}$$

Call $J(\theta) - \theta$ the “distortion term.”

Decreasing Welfare Weights, High Cost of Public Funds

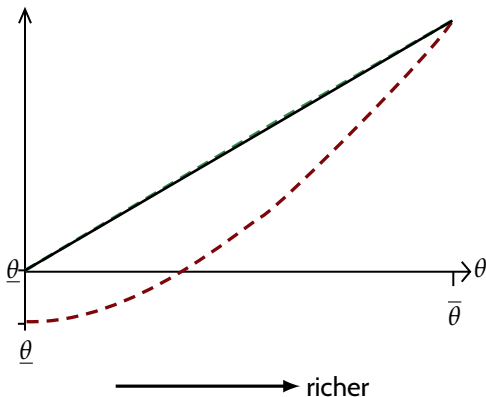


High cost of public funds: $\mathbf{E}[\omega(\theta)] \leq \alpha$.

$$J(\theta) = \theta + \frac{\int_{\theta}^{\bar{\theta}} [\omega(s) - \alpha] dF(s)}{\alpha f(\theta)}$$

is always *below* lower bound θ because distortion is single-crossing from above, **negative** at $\underline{\theta}$ and zero at $\bar{\theta}$.

Decreasing Welfare Weights, High Cost of Public Funds



High cost of public funds: $\mathbf{E}[\omega(\theta)] \leq \alpha$.

$$J(\theta) = \theta + \frac{\int_{\theta}^{\bar{\theta}} [\omega(s) - \alpha] dF(s)}{\alpha f(\theta)}$$

is always *below* lower bound θ because distortion is single-crossing from above, **negative** at $\underline{\theta}$ and zero at $\bar{\theta}$.

↪ Subsidy type $H(\theta) = \theta$ and optimal allocation is laissez-faire.

Optimal Mechanism (Arbitrary Correlation)

Theorem 2 (General). The optimal subsidy allocation rule is unique, continuous and satisfies

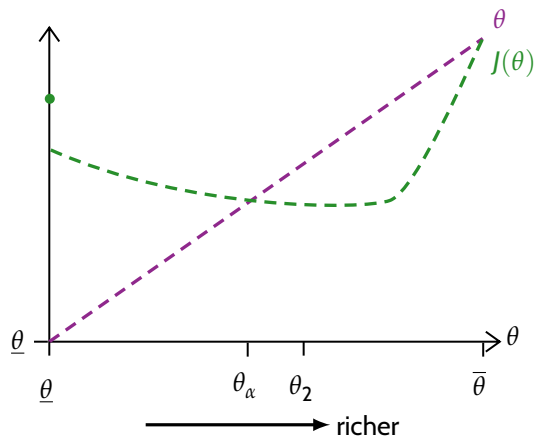
$$q^*(\theta) = D(c, H(\theta)), \text{ where } H(\theta) = \begin{cases} \theta & \text{if } \overline{J}_{[\underline{\theta}, \theta]}(\theta) \leq \theta, \\ \overline{J}_{[\underline{\theta}, \kappa_+(\theta)]}(\theta) & \text{otherwise,} \end{cases}$$

and $\kappa_+(\theta) = \inf \left\{ \hat{\theta} \geq \theta : \overline{J}_{[\underline{\theta}, \hat{\theta}]}(\hat{\theta}) \leq \hat{\theta} \right\}$ or $\bar{\theta}$, if that set is empty.

“Double” ironing construction of $H(\theta)$ ensures $H(\theta) \geq \theta$, equivalent to (LB) given expression for q^* .

\leadsto subsidized demand curve $\bar{D}(p) = \int_{\Theta} (v')^{-1}(p/H(\theta)) dF(\theta)$.

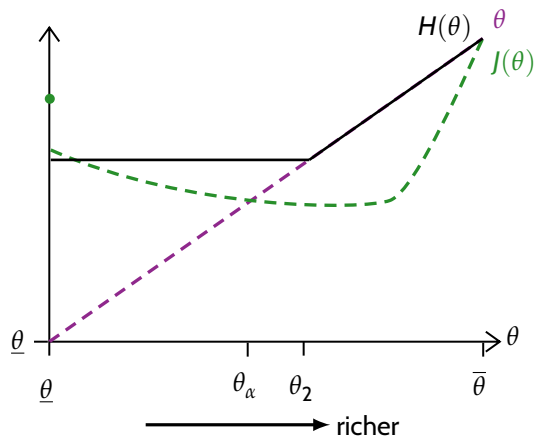
Decreasing Welfare Weights, Low Cost of Public Funds



Low cost of public funds: $\mathbf{E}[\omega(\theta)] > \alpha$.

$J(\theta) = \theta + \frac{\int_{\theta}^{\bar{\theta}} [\omega(s) - \alpha] dF(s) + (\mathbf{E}_{\theta}[\omega(\theta)] - \alpha) \underline{\theta} \delta_{\theta=\underline{\theta}}}{\alpha f(\theta)}$ crosses lower bound constraint θ from *above* because distortion term is single-crossing from above, *positive* at $\underline{\theta}$ and zero at $\bar{\theta}$.

Decreasing Welfare Weights, Low Cost of Public Funds



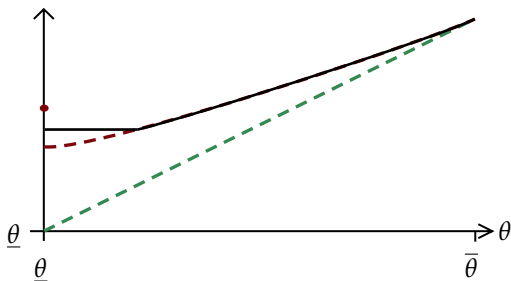
Low cost of public funds: $\mathbf{E}[\omega(\theta)] > \alpha$.

$J(\theta) = \theta + \frac{\int_{\theta}^{\bar{\theta}} [\omega(s) - \alpha] dF(s) + (\mathbf{E}_{\theta}[\omega(\theta)] - \alpha) \theta \delta_{\theta=\theta}}{\alpha f(\theta)}$ crosses lower bound constraint θ from above because distortion term is single-crossing from above, positive at θ and zero at $\bar{\theta}$.

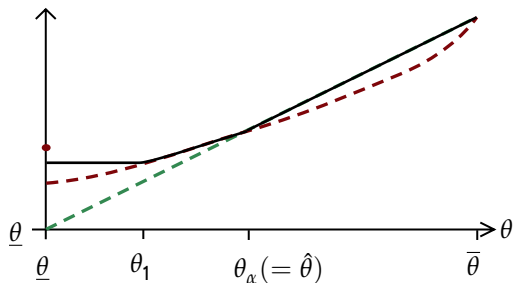
↪ Subsidy type $H(\theta) > \theta$ for $\theta \leq \theta_2$. There is a free endowment of $q^{\text{LF}}(\theta_2)$, which strictly exceeds $q^{\text{F}}(\theta)$ for $\theta \leq \theta_2$...but planner always prefers a lump-sum subsidy.

Decreasing Welfare Weight, Low Cost of Public Funds

Other possibilities:



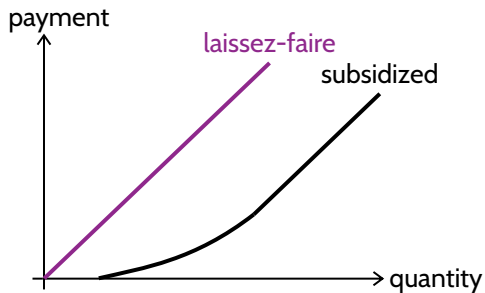
Free endowment + quantity-dependent subsidies
distorting all types' consumption upwards (no
topping up).



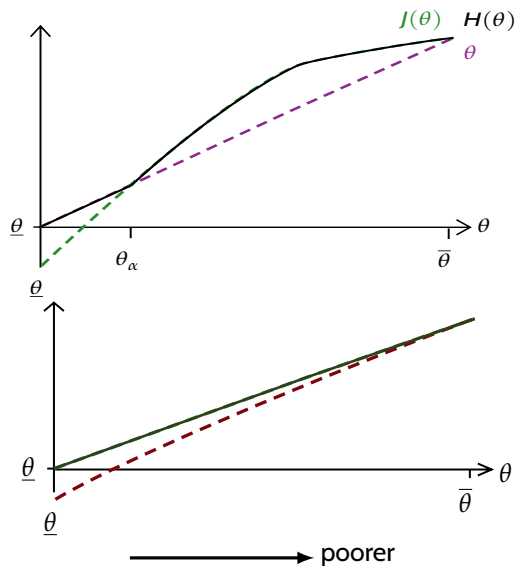
Free endowment + quantity-dependent subsidies up
to a cap (high types top up in private market).

Decreasing Welfare Weight, Low Cost of Public Funds

Payment schedule:



Increasing Welfare Weights, High Cost of Public Funds



High cost of public funds: $\mathbf{E}[\omega(\theta)] \leq \alpha$.

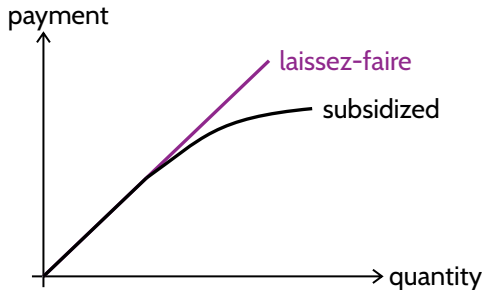
$$J(\theta) = \theta + \frac{\int_{\underline{\theta}}^{\bar{\theta}} [\omega(s) - \alpha] dF(s)}{\alpha f(\theta)}$$



can cross lower-bound $\underline{\theta}$ from *below* because the distortion term is single-crossing from below, **negative** at $\underline{\theta}$ and zero at $\bar{\theta}$.

↪ Subsidy type *can* exceed θ for high types: implemented by offering discounts for consumption *above* a minimum level... *preferred* by planner to lump-sum transfer.

Increasing Welfare Weights, High Cost of Public Funds

Payment schedule:





Weekly fare cap




An even better weekly fare discount

Say hello to an easier, more equitable way to pay your fare: the 7-day fare cap with OMNY!


Pay for 12 rides in a 7-day period and any additional rides are free. And, unlike with MetroCard, you don't have to pay upfront. Just tap and pay as you go.

Use the same device or card all 7 days and you'll automatically ride free after your 12th paid fare.

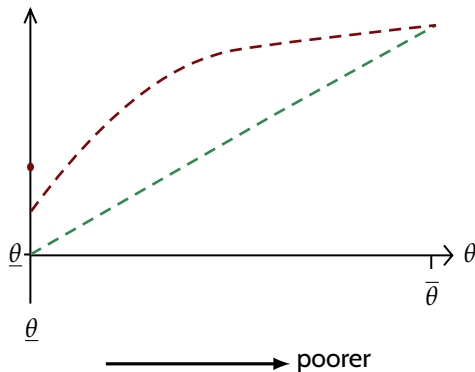
No need to pre-pay like when you buy a 7-Day Unlimited MetroCard. Tap with your own mobile wallet enabled device or contactless bank card, and you pay only for the trips you take. The more you ride, the sooner you'll earn free trips.

How Does Fare Capping Work With OM...Watch laterShare

How does fare capping work with OMNY?



Increasing Welfare Weights, Low Cost of Public Funds

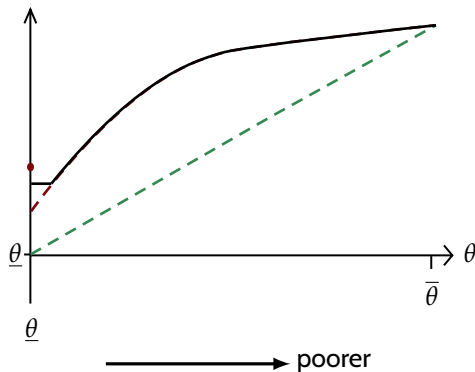


Low cost of public funds: $\mathbf{E}[\omega(\theta)] > \alpha$.

$J(\theta) = \theta + \frac{\int_{\theta}^{\bar{\theta}} [\omega(s) - \alpha] dF(s) + (\mathbf{E}_{\theta}[\omega(\theta)] - \alpha)\theta\delta_{\theta=\bar{\theta}}}{\alpha f(\theta)}$ always exceeds lower-bound θ because the distortion term is single-crossing from below, **positive** at $\underline{\theta}$ and zero at $\bar{\theta}$.

↪ Subsidy type exceeds θ for all types: implemented via a free allocation and discounts for additional consumption. No consumers top up.

Increasing Welfare Weights, Low Cost of Public Funds



Low cost of public funds: $\mathbf{E}[\omega(\theta)] > \alpha$.

$J(\theta) = \theta + \frac{\int_{\theta}^{\bar{\theta}} [\omega(s) - \alpha] dF(s) + (\mathbf{E}_{\theta}[\omega(\theta)] - \alpha)\theta\delta_{\theta=\bar{\theta}}}{\alpha f(\theta)}$ always exceeds lower-bound θ because the distortion term is single-crossing from below, **positive** at $\underline{\theta}$ and zero at $\bar{\theta}$.

↪ Subsidy type exceeds θ for all types: implemented via a free allocation and discounts for additional consumption. No consumers top up.

Discussion

Theorem 1 \rightsquigarrow scope of intervention larger for “inferior goods” than “normal goods.”

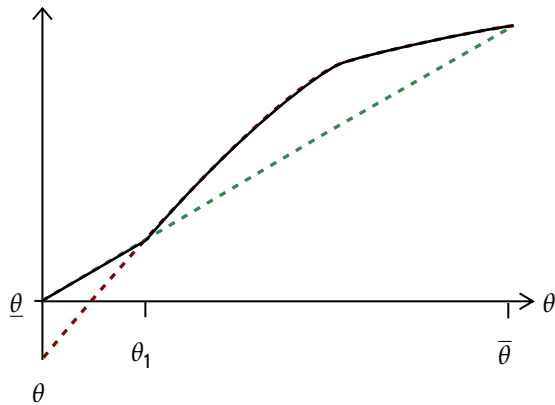
In practice, many government programs focused on goods consumed disproportionately by needy:

Examples:

- ▶ Egyptian *Tamween* food subsidy program subsidizes five loaves of *baladi* bread/day at AUD 0.01/loaf, with a **cap** on weights and quality of bread.
- ▶ CalFresh Restaurant Meals Program subsidizes fast food restaurants not dine-in restaurants.
- ▶ Indonesian Fuel Subsidy Program subsidizes low-octane fuel (for motorbikes) and not high-octane fuel (for cars).
- ▶ Until \sim 2016, Australia's Medicare subsidized amalgam fillings and not composite (tooth-coloured) fillings.

4(a) Increasing Motive for Redistribution

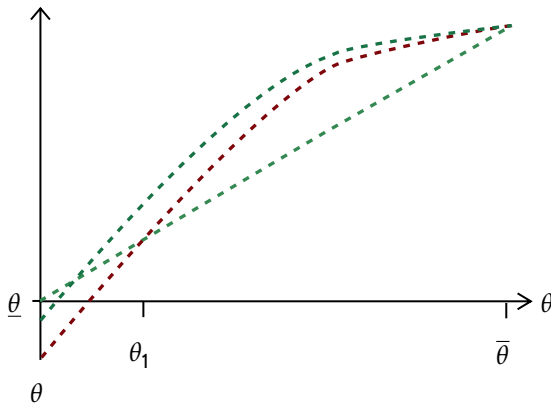
Suppose $\omega(\theta) \uparrow$ for each θ or, equivalently, $\alpha \downarrow$.



4(a) Increasing Motive for Redistribution

Suppose $\omega(\theta) \uparrow$ for each θ or, equivalently, $\alpha \downarrow$.

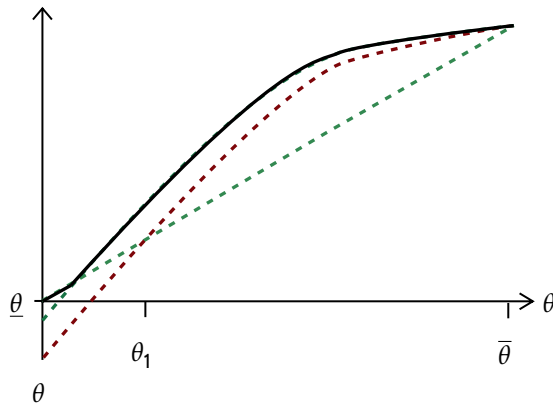
- virtual type $J(\theta) \uparrow$



4(a) Increasing Motive for Redistribution

Suppose $\omega(\theta) \uparrow$ for each θ or, equivalently, $\alpha \downarrow$.

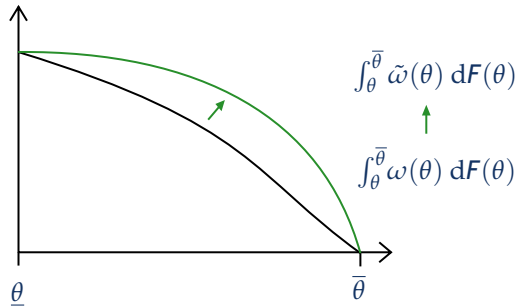
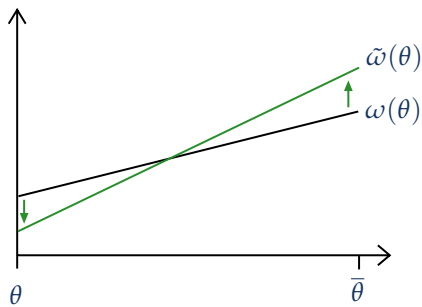
- ▶ virtual type $J(\theta) \uparrow$
- ↪ each consumer's subsidy type $H(\theta) \uparrow$
- ↪ each consumer's allocation $q^*(\theta) \uparrow$
- ↪ set of subsidized types \uparrow
- ↪ total subsidy per consumer \uparrow



Planner & average eligible consumer prefer subsidies targeted to consumers with higher welfare weights.

4(b) Increasing Correlation

Suppose ω and θ become more correlated, in the sense of **majorization** \leadsto observe higher demand expect higher ω , i.e., for all $\theta \in \Theta$, $\mathbf{E}[\tilde{\omega}(\theta)|\theta \geq \hat{\theta}] \geq \mathbf{E}[\omega(\theta)|\theta \geq \hat{\theta}]$.



\leadsto larger incentive to distort consumption \leadsto more generous subsidies.

Planner & average eligible consumer prefer subsidies for goods with more positive correlation between demand and welfare weights.

4(c) Decreasing Marginal Cost

Suppose marginal cost decreases $c \downarrow$ (equiv. demand increases, so $(v') \uparrow$).

No change in virtual type \leadsto no change in subsidy type.

- \leadsto the set of subsidized types is unchanged, while
- \leadsto each consumer's allocation $q^*(\theta) \uparrow$
- \leadsto total subsidy per subsidized consumer \uparrow .

4(c) Decreasing Marginal Cost

Suppose marginal cost decreases $c \downarrow$ (equiv. demand increases, so $(v') \uparrow$).

No change in virtual type \leadsto no change in subsidy type.

- \leadsto the set of subsidized types is unchanged, while
- \leadsto each consumer's allocation $q^*(\theta) \uparrow$
- \leadsto total subsidy per subsidized consumer \uparrow .

4(c) Decreasing Marginal Cost

Suppose marginal cost decreases $c \downarrow$ (equiv. demand increases, so $(v') \uparrow$).

No change in virtual type \rightsquigarrow no change in subsidy type.

- \rightsquigarrow the set of subsidized types is unchanged, while
- \rightsquigarrow each consumer's allocation $q^*(\theta) \uparrow$
- \rightsquigarrow total subsidy per subsidized consumer \uparrow .

Planner & average eligible consumer prefer subsidies for low cost / high demand goods.

Food Stamps (SNAP)

- ▶ **Overview:** U.S. program providing monthly food assistance to low-income individuals and families.
- ▶ **Initial Support:** Full subsidy up to a fixed dollar amount per month for eligible food items.
- ▶ **Free Endowment:** The subsidy starts as a full benefit and decreases after benefits are exhausted.
- ▶ **Low Consumption Focus:** Ensures a basic level of nutrition by covering initial consumption entirely.

Women, Infants, and Children (WIC)

- ▶ **Overview:** Nutritional assistance for low-income pregnant women, new mothers, and young children.
- ▶ **Initial Support:** Vouchers for essential foods like milk, eggs, baby formula.
- ▶ **Free Endowment:** Recipients get fully subsidized quantities of specific foods.
- ▶ **Low Consumption Focus:** Prioritizes providing a free minimum quantity of nutritious food to families.

Housing Choice Voucher Program (Section 8)

- ▶ **Overview:** subsidized housing assistance for low-income renters in the U.S.
- ▶ **Initial Support:** Covers a large portion of rent (up to 70%) for qualifying households.
- ▶ **Free Endowment:** A significant rent portion is initially fully subsidized.
- ▶ **Low Consumption Focus:** Ensures low-income renters pay only a small portion of their rent.

Lifeline Program

- ▶ **Overview:** U.S. program offering discounted phone and internet services to low-income households.
- ▶ **Initial Support:** Monthly discounts on basic telecommunication services.
- ▶ **Free Endowment:** Full subsidy of basic services for the most disadvantaged users.
- ▶ **Low Consumption Focus:** Provides essential access to communication services with high initial subsidies.

National School Lunch Program (NSLP)

- ▶ **Overview:** Provides free or reduced-price school meals for low-income students.
- ▶ **Initial Support:** Fully subsidized meals for eligible students based on family income.
- ▶ **Free Endowment:** Full meal subsidies provided for families below certain income thresholds.
- ▶ **Low Consumption Focus:** Ensures children receive at least one nutritious meal per day at no or low cost.

Australian Better Access Mental Health Initiative

- ▶ **Overview:** Australian government program subsidizing mental health services.
- ▶ **Initial Subsidy:** Up to 10 Medicare-subsidized sessions per year.
- ▶ **Additional Support:**
 - After initial sessions and doctor approval, become eligible for extra free/subsidized sessions.
 - Increased subsidy ensures access for those needing more care.

Australia's Child Care Subsidy (CCS)

- ▶ **Overview:** Government subsidy for childcare costs based on income and activity levels.
- ▶ **Initial Subsidy:** Covers a percentage of childcare fees up to a set number of hours.
- ▶ **Additional Support:**
 - Subsidy percentage **increases** as parents work, study, or volunteer more.
 - More hours of work/study lead to higher subsidies for additional childcare hours.
 - More children leads to higher subsidies per child.

Public Transit Fare Capping (Research Triangle, NC)

Fares & Passes

Fares & Passes

Return to Fare FAQs

GoPass Partners

Purchase GoPass Card

Register GoPass Card

Discount Fare Options

How to use Umo

Add Funds

GoCary Fares

GoDurham Fares

GoRaleigh Fares

FIXED-ROUTE FARE OPTIONS

Fare Type	Fare	Daily Cap	Weekly Cap	Monthly Cap
Full Fare	\$2.50	\$5	\$20	\$80
Discount	\$1.25	\$2.50	\$10	\$40
Child Under 12	N/A	N/A	N/A	N/A
Youth 13-18			Qualify for free fare	
Senior 65+			Qualify for free fare	
Transit Assistance Pass			Qualify for free fare	
GoPass Partners			Learn More	

**All sales are final and nonrefundable*

Other Cities With Similar Programs: New York, SF Bay Area, Portland, London, Dublin, Toronto, Vancouver, Los Angeles, Singapore, Sydney, Brisbane, Melbourne, Perth, Auckland.

Pharmaceutical Subsidy Programs: Australia, Norway, Sweden, Denmark

- ▶ **Overview:** Government programs reducing out-of-pocket medication costs.
- ▶ **Australia (PBS):** subsidizes prescription medicines; costs decrease after a yearly threshold (safety net) is reached.
- ▶ **Helsenorge (Norway):** Covers up to 90% of prescription costs after reaching an annual expenditure cap.
- ▶ **Sweden:** Once a patient reaches a yearly spending threshold, additional medications are free.
- ▶ **Denmark:** Progressive subsidy structure, with higher reimbursements as individual spending increases.

Cost-Sharing Reductions (CSRs) and Eligibility Limits

ACA Cost-Sharing Reductions (CSRs)

► What are CSRs?

- Subsidies that lower out-of-pocket costs (e.g., co-pays, deductibles).
- Available to individuals/families with incomes between 100% and 250% of the Federal Poverty Level (FPL).

► Eligibility Tied to Lower Insurance Plans

- To qualify for CSRs, you *must* purchase a **Silver-level** plan on the ACA marketplace.
- Other plan tiers (**Bronze, Gold, or Platinum**) **do not** offer CSRs, even if you're income-eligible.
- Silver plans have a standard **70% actuarial value**, but CSRs raise it to up to **94%** for lower-income enrollees.

► Impact of Limiting to Silver Plans

- Higher-income individuals may choose other plan levels, but lose CSR eligibility.
- Lower-income enrollees are incentivized to choose Silver plans to reduce out-of-pocket costs.

German Health System: Prohibition of Topping Up

- ▶ **Public Health Insurance:** Citizens covered by statutory health insurance (SHI) cannot "top up" SHI with private insurance for services already covered.
- ▶ **Supplementary Insurance:** Private insurance can only be used for services not included in SHI (e.g., private rooms, certain dental services).
- ▶ **Comprehensive Coverage:** SHI already covers essential medical services, discouraging the need for topping up with private health plans.

Public Education: Prohibition of Private Tutoring in China & South Korea

- ▶ **Public Education:** Both China and South Korea provide universal public education for students, with restrictions on private supplementary tutoring.
- ▶ **Prohibition:** Private tutoring and after-school programs are heavily regulated or banned to prevent parents from "topping up" public education with private instruction.
- ▶ **Equal Access:** The aim is to reduce inequality in educational opportunities and prevent wealthier families from gaining an advantage through private education.

Public Housing

- ▶ **Public Housing Programs:** Residents in public housing receive heavily subsidized rent, often capped at a percentage of their income.
- ▶ **Prohibition:** Participants must choose between living in public housing or renting in the private market; they cannot "top up" their public housing subsidy to rent a private apartment.
- ▶ **Example Cities and Countries:**
 - **Singapore:** The Housing & Development Board (HDB) provides subsidized flats, and participants cannot receive additional subsidies to live in private housing.
 - **Vienna, Austria:** The city's extensive public housing program offers low-cost rental units, with no option to "top up" for private market rentals.
 - **Hong Kong:** The Public Rental Housing (PRH) program offers heavily subsidized apartments, and recipients must choose between public housing and private market rentals.

Egypt's Tamween Food Subsidy Program

- ▶ **Overview:** The Tamween program is one of the largest food subsidy systems in the world, providing essential goods to over 60 million Egyptians, mostly from low-income households.
- ▶ **Targeted Subsidy:**
 - **Bread:** Heavily subsidized at a fraction of market price (often less than 10% of the actual cost), making it affordable for the poor, who rely on it as a staple.
 - **Other Essentials:** Subsidies also cover rice, sugar, and cooking oil, basic items central to the diets of low-income families.
- ▶ **Exclusion:**
 - **Meat and Dairy:** These more expensive food items, consumed more frequently by wealthier households, are not subsidized. Consumers must pay market prices for these products.

Indonesian Fuel Subsidy Program: Pertamina

- ▶ **Overview:** Indonesia's fuel subsidy program supports transportation for low-income households.
- ▶ **Targeted Subsidy:** The program subsidizes low-octane fuel, which is primarily used by motorcycles, the preferred transport mode for poorer citizens.
- ▶ **Exclusion:** High-octane fuel, more commonly used by cars owned by wealthier households, is not subsidized.

CalFresh Restaurant Meals Program

- ▶ **Overview:** California's CalFresh program allows certain populations to use benefits for prepared meals.
- ▶ **Targeted Subsidy:** The program subsidizes meals, predominantly from fast food restaurants, providing affordable food options for homeless, elderly, and disabled individuals.
- ▶ **Exclusion:** Dine-in restaurants, typically frequented by wealthier individuals, are not included in the subsidy.

Public Dentistry Programs in Australia

- ▶ **Overview:** Australia's public dentistry programs provide dental care subsidies to low-income individuals.
- ▶ **Targeted Subsidy:** Prior to 2016, the program subsidized only amalgam fillings, a durable and cost-effective option used widely by lower-income patients.
- ▶ **Exclusion:** Composite (tooth-colored) fillings, which are more expensive and preferred by wealthier individuals, were not fully subsidized.
- ▶ **Post-2016:** amalgam fillings are being phased out due to mercury content.