# Redistribution and Subsidy Design

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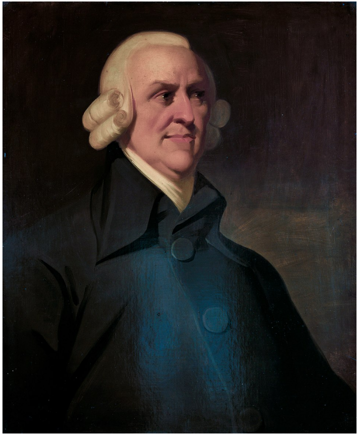
# Classical Literature on Redistribution

# Redistribution - The Classic View



**Smith's (1776)** "invisible hand" view of the market:

> *The natural effort of every individual to better his own condition, when suffered to exert itself with freedom and security, is so powerful a principle, that it is alone, and without any assistance, not only capable of carrying on the society to wealth and prosperity, but of surmounting a hundred impertinent obstructions with which the folly of human laws too often encumbers its operations.*

But **Smith** also cites concern for inequity:

> *No society can surely be flourishing and happy, of which the far greater part of the members are poor and miserable. It is but equity, besides, that they who feed, clothe, and lodge the whole body of the people, should have such a share of the produce of their own labour as to be themselves tolerably well fed, clothed, and lodged.*

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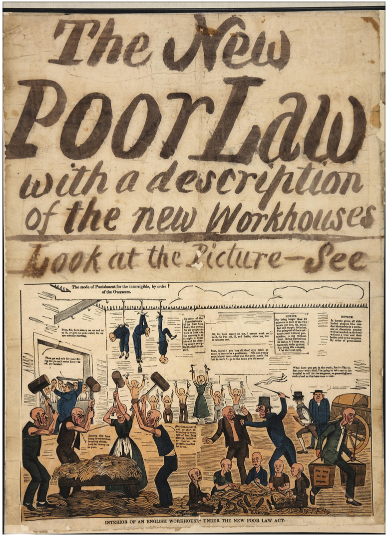


**Bentham (1834)**, **Mill (1848)** expresses a concern for incentive-compatible intervention.

Bentham's "less eligibility" principle enshrined in the design of the New Poor Law.

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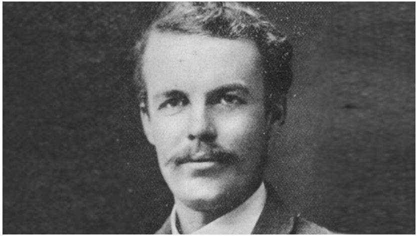

Pigou (1920) provides welfarist rationale for market intervention:

> *If income is transferred from rich persons to poor persons the proportion in which different sorts of goods and services are provided will be changed. Expensive luxuries will give place to more necessary articles, rare wines to meat and bread, new machines and factories to clothes and improved small dwellings; and there will be other changes of a like sort.*

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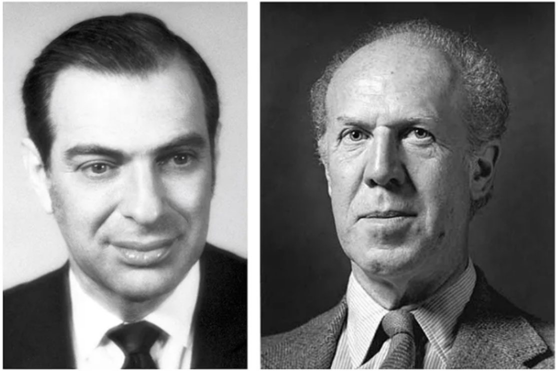

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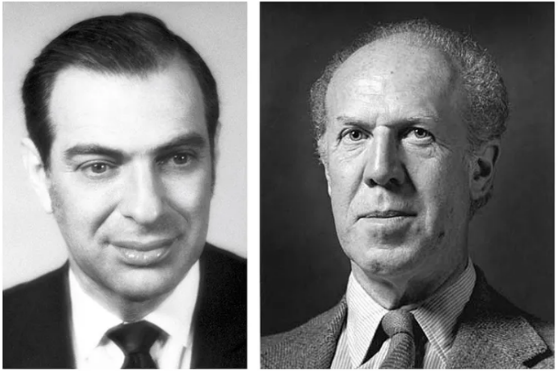

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Today: how does a designer with imperfect information and constrained redistributive power–namely, the ability to design an allocation mechanism for only one commodity–optimally intervene?

# Redistributive Mechanism Design

# Redistributive Mechanism Design: Core Idea

- Traditional mechanism design focuses on **efficiency**: allocating goods to those with highest **willingness to pay**.
- **Redistributive mechanism design** adds a normative layer:
  - Willingness to pay may reflect not only **preferences**, but also **income or wealth** (cf. Pigou).
  - Hence, observed demand can be **informative about need**.

# DKA Model

- A **continuum of buyers** $j \in J$, each with type $(v^K, v^M) \sim^{\text{i.i.d.}} G$ with support $[\underline{v}_K, \bar{v}_K] \times [\underline{v}_M, \bar{v}_M]$ :
  - $v^K$: marginal value for the good.
  - $v^M$: marginal value for money.
  - Agent utility is quasi-linear $u = v^K x^K - v^M t$
- For today (i.e., not in their paper), suppose the social planner can produce units of $K$ at **marginal cost** $c$, and has its own **value for money** ("cost of public funds") $\alpha$. It cannot give money away: $t \geq 0$.
- The planner chooses a **mechanism** to maximize expected total utility

$$\mathbb{E}_{(v^K, v^M) \sim G} \left[ (v^K - \alpha c) x^K (v_K, v_M) + (\alpha - v^M) t(v_K, v_M) \right].$$

# Mechanism Design Problem

Revelation principle $\rightsquigarrow$ suffices for the designer to choose **allocation** $X : [\underline{v}_K, \overline{v}_K] \times [\underline{v}_M, \overline{v}_M] \to [0,1]$ and **payments** $T : [\underline{v}_K, \overline{v}_K] \times [\underline{v}_M, \overline{v}_M] \to \mathbb{R}_+$ to maximize

$$\int (v^K - \alpha c) X(v_K, v_M) + (\alpha - v^M) T(v_K, v_M) \, \mathrm{d}G(v_K, v_M),$$

subject to for all $(v_K, v_M), (v'_K, v'_M) \in [\underline{v}_K, \overline{v}_K] \times [\underline{v}_M, \overline{v}_M]$:

$$v^K X(v_K, v_M) - v^M T(v_K, v_M) \geq v^K X(v'_K, v'_M) - v^M T(v'_K, v'_M) \tag{IC}$$

$$v^K X(v_K, v_M) - v^M T(v_K, v_M) \geq 0, \tag{IR}$$

$$T(v_K, v_M) \geq 0. \tag{NLS}$$

# Reducing Problem Dimensionality

Claim: the mechanism can only elicit a buyer's marginal rate of substitution between $K$ and $M$.

Intuitively, (IC) implies that a consumer with $(v_K, v_M) = (1, 5)$ must receive same outcome as consumer with $(v_K, v_M) = (2, 10)$. Formally:

**Theorem.** Suppose $X, T$ is an IC, IR mechanism, then there exists an IC, IR mechanism eliciting reports $\theta = v_K / v_M$ and $x : \left[\frac{\underline{v}_K}{\bar{v}_M}, \frac{\bar{v}_K}{\underline{v}_M}\right] \to [0, 1]$, and $t : \left[\frac{\underline{v}_K}{\bar{v}_M}, \frac{\bar{v}_K}{\underline{v}_M}\right] \to \mathbb{R}$, such that

$$X(v_K, v_M) = x(\theta), \text{ and } T(v_K, v_M) = t(\theta).$$

# Proof

Incentive-compatibility means that, for all $\left(v^K, v^M\right)$ and $\left(\hat{v}^K, \hat{v}^M\right)$ in the support of $F$, we have

$$X\left(v^K, v^M\right)\frac{v^K}{v^M} - T\left(v^K, v^M\right) \geq X\left(\hat{v}^K, \hat{v}^M\right)\frac{v^K}{v^M} - T\left(\hat{v}^K, \hat{v}^M\right)$$

as well as

$$X\left(\hat{v}^K, \hat{v}^M\right)\frac{\hat{v}^K}{\hat{v}^M} - T\left(\hat{v}^K, \hat{v}^M\right) \geq X\left(v^K, v^M\right)\frac{\hat{v}^K}{\hat{v}^M} - T\left(v^K, v^M\right).$$

Putting these together, we have

$$\left(X\left(v^K, v^M\right) - X\left(\hat{v}^K, \hat{v}^M\right)\right)\left(\frac{v^K}{v^M} - \frac{\hat{v}^K}{\hat{v}^M}\right) \geq 0.$$

It follows that

$$\frac{v^K}{v^M} > \frac{\hat{v}^K}{\hat{v}^M} \implies X\left(v^K, v^M\right) \geq X\left(\hat{v}^K, \hat{v}^M\right).$$

# Proof (continued)

**Claim** Let $X(v_K, v_M)$ be defined on $[\underline{v}_K, \bar{v}_K] \times [\underline{v}_M, \bar{v}_M]$, with $\underline{v}_K, \underline{v}_M > 0$, and suppose

$$\frac{v_K}{v_M} > \frac{v_K'}{v_M'} \quad \Rightarrow \quad X(v_K, v_M) \geq X(v_K', v_M').$$

Then there exists a non-decreasing function $x : \left[\frac{\underline{v}_K}{\bar{v}_M}, \frac{\bar{v}_K}{\underline{v}_M}\right] \to \mathbb{R}$ such that

$$X(v_K, v_M) = x\left(\frac{v_K}{v_M}\right) \quad \text{almost everywhere.}$$

**Proof Sketch:**

- Define $Y(\theta, v_M) := X(r \cdot v_M, v_M)$.
- By assumption, $Y(\theta, v_M)$ is non-decreasing in $\theta$ for each $v_M$.
- For almost all $\theta$, $Y(\theta, v_M)$ is constant in $v_M \Rightarrow Y(\theta, v_M) = x(\theta)$.
- Hence, $X(v_K, v_M) = x\left(\frac{v_K}{v_M}\right)$ almost everywhere.

Finally, if $X(v_K, v_M) = x(v_K/v_M)$, then $T(v_K, v_M) = t(v_K/v_M)$ as well (else a failure of IC).

# Rewriting the Objective

Now, rewriting the objective, we have:

$$\mathbb{E}\left[(v^K - \alpha c)X(v_K, v_M) + (\alpha - v^M)T(v_K, v_M)\right] = \mathbb{E}_{(v_K, v_M)}\left[(v^K - \alpha c)x(v_K/v_M) + (\alpha - v^M)t(v_K/v_M)\right],$$
$$= \mathbb{E}_\theta\left[\mathbb{E}[v^M | v^K/v^M = \theta]\,(\theta x(\theta) - t(\theta)) + \alpha(t(\theta) - cx(\theta))\right]$$

by law of iterated expectations.

Let $\omega(\theta) = \mathbb{E}[v^M | v^K/v^M = \theta]$ and let $F$ be the distribution of $\theta$ and $\Theta$ its support. Then the social planner's problem can be rewritten as a weighted surplus maximization problem, with Pareto weights $\omega(\theta)$:

$$\max_{x,t} \mathbb{E}_{\theta \sim G}\left[\omega(\theta)\,(\theta x(\theta) - t(\theta)) + \alpha(t(\theta) - cx(\theta))\right],$$

subject to for all $\theta, \theta' \in \Theta$: $\theta x(\theta) - t(\theta) \geq \theta x(\theta') - t(\theta')$ (IC), $\theta x(\theta) - t(\theta) \geq 0$ (IR), and $t(\theta) \geq 0$ (NLS).

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# Solving the Problem

$$\max_{x,t} \mathbb{E}_{\theta\sim G}\left[\omega(\theta)\left(\theta x(\theta)-t(\theta)\right)+\alpha(t(\theta)-cx(\theta))\right],$$

subject to for all $\theta,\theta'\in\Theta$: $\theta x(\theta)-t(\theta)\geq\theta x(\theta')-t(\theta')$ (IC), and $\theta x(\theta)-t(\theta)\geq 0$ (IR), and $t(\theta)\geq 0$ (NLS).

First, use Myerson's Lemma: (IC) equivalent to $x$ nondecreasing and an envelope theorem:

$$U(\theta)=U(\underline{\theta})+\int_{\underline{\theta}}^{\theta}x(s)\,\mathrm{d}s,\ \text{and } t(\theta)=\theta x(\theta)-U(\underline{\theta})-\int_{\underline{\theta}}^{\theta}x(s)\,\mathrm{d}s.$$

Note $U(\underline{\theta})$ and $x(\theta)\geq 0$ implies (IR), and $t(\underline{\theta})\geq 0$ implies (NLS). Substitute into the objective to obtain:

$$\max_{x\text{ nondec., }U(\underline{\theta})\geq 0}(\mathbb{E}[\omega]-\alpha)U(\underline{\theta})+\int_{\underline{\theta}}^{\overline{\theta}}\left[\omega(\theta)\int_{\underline{\theta}}^{\theta}x(s)\,\mathrm{d}s+\alpha(\theta-c)x(\theta)-\alpha\int_{\underline{\theta}}^{\theta}x(s)\,\mathrm{d}s\right]\,\mathrm{d}F(\theta)$$

Integrate by parts to obtain **linear objective** in $x$:

$$\max_{x\text{ nondec., }0\leq U(\underline{\theta})\leq\underline{\theta}x(\underline{\theta})}(\mathbb{E}[\omega]-\alpha)U(\underline{\theta})+\alpha\int_{\underline{\theta}}^{\overline{\theta}}\left[\theta+\frac{\int_{\theta}^{\overline{\theta}}\omega(s)-\alpha\,\mathrm{d}F(s)}{\alpha f(\theta)}-c\right]x(\theta)\,\mathrm{d}F(\theta)$$

# Solving the Problem (2)

So, we have:

$$\max_{x \text{ nondec.},\, 0 \le U(\underline{\theta}) \le \underline{\theta} x(\underline{\theta})} (\mathbb{E}[\omega] - \alpha) U(\underline{\theta}) + \alpha \int_{\underline{\theta}}^{\overline{\theta}} \left[ \theta + \frac{\int_{\theta}^{\overline{\theta}} \omega(s) - \alpha \, \mathrm{d}F(s)}{\alpha f(\theta)} - c \right] x(\theta) \, \mathrm{d}F(\theta).$$

Note that maximization requires $U(\underline{\theta})$ as large as possible (i.e., $t(\underline{\theta}) = 0$ so $\underline{\theta} x(\underline{\theta})$) if $\mathbb{E}[\omega] > \alpha$ and as small as possible (i.e., $0$) if $\mathbb{E}[\omega] < \alpha$. Incorporate this into the integrand as a Diract delta:

$$\max_{x \text{ nondec.}} \alpha \int_{\underline{\theta}}^{\overline{\theta}} \left[ \underbrace{\theta + \frac{\int_{\theta}^{\overline{\theta}} \omega(s) - \alpha \, \mathrm{d}F(s) + \underline{\theta} [\mathbb{E}[\omega] - \alpha]_{+} \delta_{\theta = \underline{\theta}}}{\alpha f(\theta)}}_{J(\theta)} - c \right] x(\theta) \, \mathrm{d}F(\theta)$$

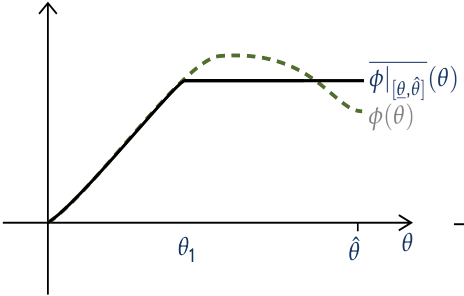
Maximize the surplus of a **virtual type** $J(\theta)$!
Solution is $q^*(\theta) = 1_{\theta \ge p}$ where $p$ satisfies $\bar{J}(p) = c$.

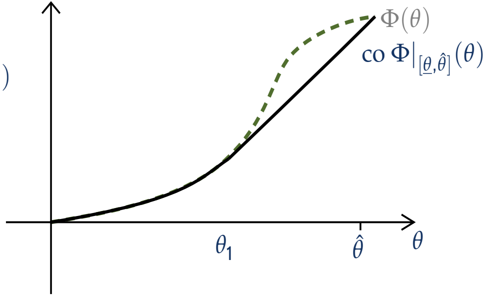
# Ironing

Let $\phi$ be a (generalized) function and $\Phi : \theta \mapsto \int_{\underline{\theta}}^{\theta} \phi(s)\, \mathrm{d}F(s)$. Then $\overline{\phi}$ is the monotone function satisfying

$$\text{for all } \theta \in [\underline{\theta}, \hat{\theta}], \qquad \int_{\underline{\theta}}^{\theta} \overline{\phi}(s)\, \mathrm{d}F(s) = \mathrm{co}\, \Phi(\theta).$$

Intuitively, $\overline{\phi}$ replaces non-monotone intervals of $\phi$ with $F$-weighted averages.




▸ Derivation ▸ Statement

# Subsidy Design with Topping Up

# Introduction

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**Our approach:** we pose and solve the mechanism design problem for the **optimal subsidy**.

# Key Tradeoff

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#2. heterogeneous outside options, consumers can buy from private market.

Heterogeneous outside options are empirically relevant, *e.g.*,

- public housing (van Dijk, 2019; Waldinger, 2021),
- education (Akbarpour, Kapor, Neilson, van Dijk & Zimmerman, 2022; Kapor, Karnani & Neilson, 2024),
- healthcare (Li, 2017; Heim, Lurie, Mullen & Simon, 2021),
- SNAP (Haider, Jacknowitz & Schoeni, 2003; Ko & Moffitt, 2024; Rafkin, Solomon & Soltas, 2024).

Heterogeneous outside options lead to lower-bound constraints in the mechanism design problem.

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$\rightsquigarrow$ Linear subsidies are **never optimal.**

# Related Literature

- **Public Finance.** Ramsey (1927), Diamond (1975), Mirrlees (1976, 1986), Atkinson & Stiglitz (1976), Nichols & Zeckhauser (1982), Blackorby & Donaldson (1988), Besley & Coate (1991), Blomquist & Christiansen (1998), Doligalski, Dworczak, Krysta & Tokarski (2023).
  - $\rightsquigarrow$ **This paper:** allows for nonlinear subsidy designs.
- **Redistributive Mechanism Design.** Weitzman (1977), Condorelli (2013), Che, Gale & Kim (2013), Dworczak, Kominers & Akbarpour (2021, 2022), Kang (2023,2024), Akbarpour, Budish, Dworczak & Akbarpour (2024), Pai & Strack (2024).
  - $\rightsquigarrow$ **This paper:** allows consumers to consume in private market outside of planner's control.
- **Partial Mechanism Design.** Jullien (2000), Philippon & Skreta (2012), Tirole (2012), Fuchs & Skrzypacz (2015), Dworczak (2020), Loertscher & Muir (2022), Loertscher & Marx (2022), Kang & Muir (2022), Kang (2023), Kang & Watt (2024).
  - $\rightsquigarrow$ **This paper:** focus on benchmark where planner is as efficient as private market, "topping up."
- **Methodological Tools in Mechanism Design.** Jullien (2000), Toikka (2011), Corrao, Flynn & Sastry (2023), Yang & Zentefis (2024), Valenzuela-Stookey & Poggi (2024).
  - $\rightsquigarrow$ **This paper:** explicit characterization of solution with FOSD (topping up) constraint.

# Model

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**Consumers**

type $\theta \rightsquigarrow$ demand

**Producers**

constant marginal cost $c$

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**Redistributive Social Planner**
maximizing weighted total surplus:
$\omega(\theta)$ expected weight on type $\theta$'s consumer surplus
$\alpha$ opportunity cost of subsidy spending

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**Consumers**

type $\theta \rightsquigarrow$ demand

laissez-faire price $c$

**Producers**

constant marginal cost $c$

costless contracting

**Redistributive Social Planner**

maximizing weighted total surplus:

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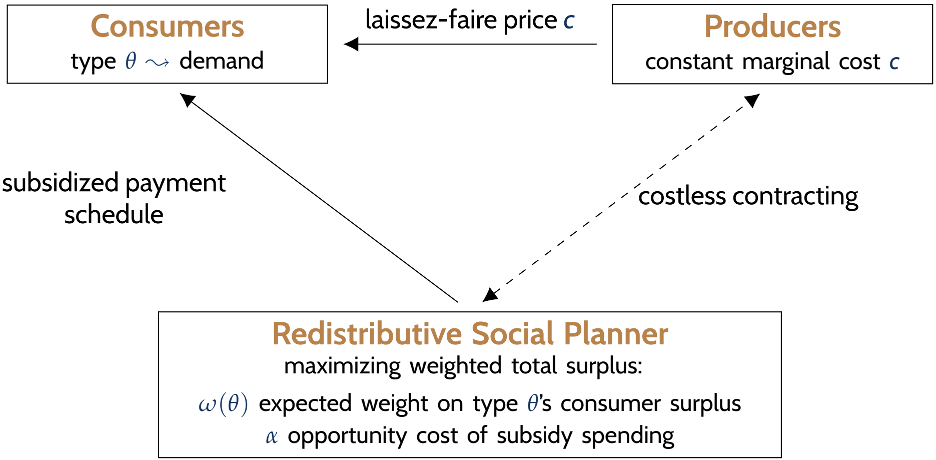

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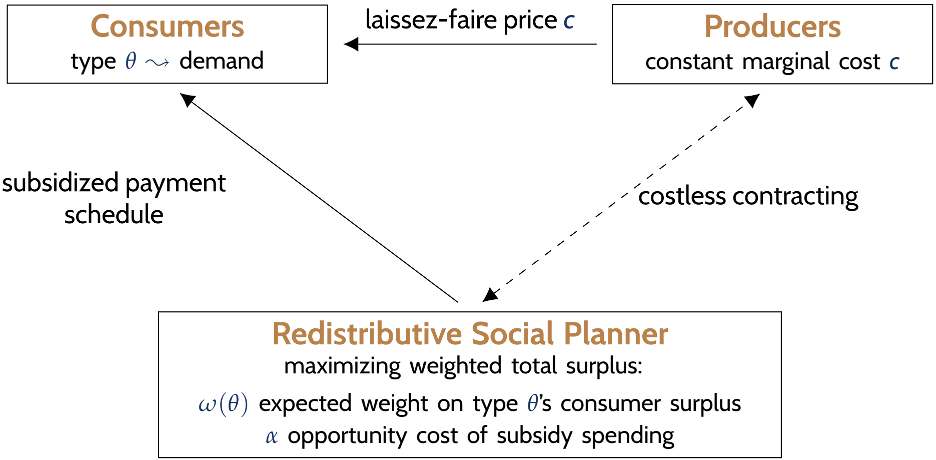


Consumers can purchase units from **both subsidized program and private market**.

# Setup

## Consumers:

- There is a unit mass of risk-neutral consumers in market for a divisible, homogeneous good.
- Consumers differ in type $\theta \in [\underline{\theta}, \overline{\theta}]$ with $\underline{\theta} \geq 0$, and $\theta \sim F$, continuous with density $f > 0$.
- Each consumer derives utility $\theta v(q) - t$ from quantity $q \in [0, A]$ given payment $t$.
  $v : [0, A] \to \mathbb{R}$ is differentiable with $v' > 0$ and $v'' < 0$.

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Extensions (not today): equilibrium effects, observable characteristics, product choice and eligibility.

# Laissez-Faire Equilibrium

- Perfectly competitive private market $\rightsquigarrow$ **laissez-faire price** $p^{\mathrm{LF}} = c$ per unit.
- Each consumer solves

$$U^{\mathrm{LF}}(\theta) := \max_{q \in [0,A]} \left[\theta v(q) - cq\right].$$

$v$ is strictly concave $\rightsquigarrow$ unique maximizer:

$$q^{\mathrm{LF}}(\theta) = (v')^{-1}\left(\frac{c}{\theta}\right) = D(c, \theta).$$

- To simplify statements of some results, assume today that $q^{\mathrm{LF}}(\underline{\theta}) > 0$.

# Subsidy Design

Social planner costlessly contracts with firms and sells units at a **subsidized payment schedule** $P^{\sigma}(q)$.

$\rightsquigarrow \Sigma(q) = cq - P^{\sigma}(q)$ is the **total subsidy** as a function of $q$, and $\sigma(q) = \Sigma'(q)$ is the **marginal subsidy**.

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**Implementation:** Consumer $\theta$ solves $U^{\sigma}(\theta) := \max_q [\theta v(q) - P^{\sigma}(q)]$, leading to **subsidized demand** $q^{\sigma}(\theta)$.

▸ NLS assumption ▸ Taxation

# Redistributive Objective

The social planner seeks to maximize **weighted total surplus**.

- **Consumer surplus**: social planner assigns a welfare weight $\omega(\theta) := \mathbf{E}[\omega|\theta]$ to consumer type $\theta$.
  - $\rightsquigarrow$ $\omega(\theta)$: expected social value of giving consumer $\theta$ one unit of money.

Endogenizing $\omega$ and $\alpha$

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Remarks:

- If $\omega(\theta) > \alpha$, social planner would want to transfer a dollar to type $\theta$.
- If $\mathbf{E}_\theta[\omega(\theta)] > \alpha$, social planner would want to make a lump-sum cash transfer to all consumers.

Endogenizing $\omega$ and $\alpha$

# Correlation Assumption

Two baseline cases:

"Negative Correlation": $\omega(\theta)$ is decreasing in $\theta$.

- high-demand consumers tend to have lower need for redistribution.
- *e.g.*, food, education, and, if $\omega \propto 1/\text{Income}$, normal goods.

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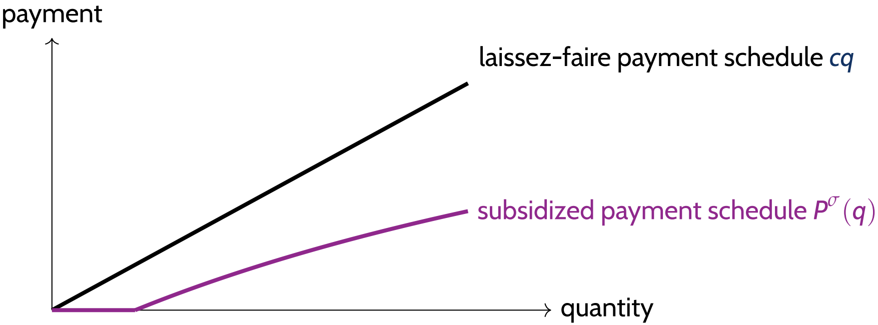
# When (Not) To Use Subsidies?

# Private Market Access Constrains Redistribution Using Subsidies

private market access $\iff$ **marginal price** $\leq c$



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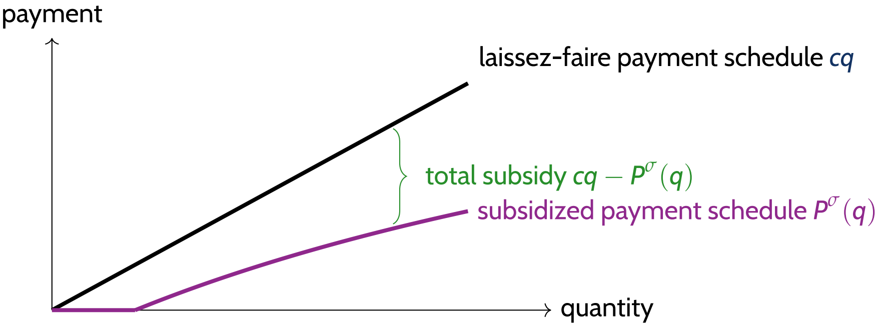

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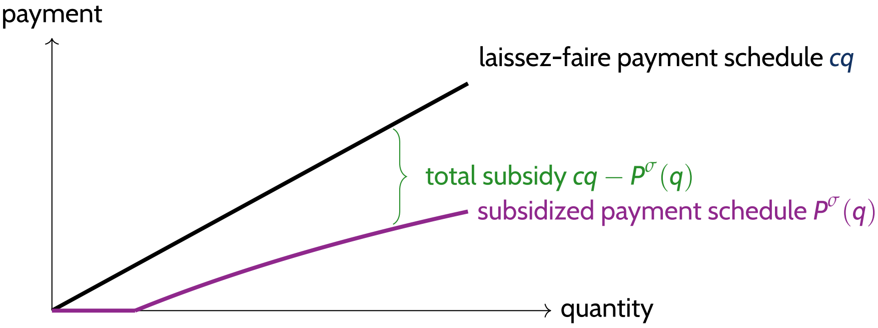

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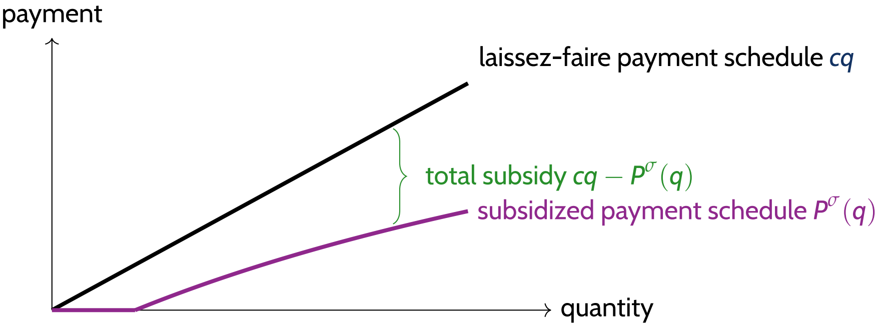


Subsidies are captured disproportionately by high $\theta$ consumers.

# When Not To Subsidize?

**Recall** our "negative correlation" assumption: high $\theta$ consumers have lower $\omega$.

**Proposition.** For any subsidy $P^{\sigma}$, the social planner would prefer to make a lump-sum transfer of $\mathbf{E}_{\theta}[\Sigma(q^{\sigma}(\theta))]$ to all consumers than the subsidy outcome.

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**Proof:** By definition of $U^{LF}$ and correlation inequality,

$$\underbrace{\int_\Theta \omega(\theta)U^\sigma(\theta) - \alpha\Sigma(q^\sigma(\theta))\, \mathrm{d}F(\theta)}_{\text{objective given } P^\sigma} = \int_\Theta \omega(\theta)[\theta v(q^\sigma(\theta)) - cq^\sigma(\theta) + \Sigma(q^\sigma(\theta))] - \alpha\Sigma(q^\sigma(\theta))\, \mathrm{d}F(\theta)$$

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**Theorem 1 (Negative Correlation, part).** The social planner subsidizes consumption only if $\mathbf{E}_\theta[\omega(\theta)] > \alpha$ (and cash transfers are unavailable).

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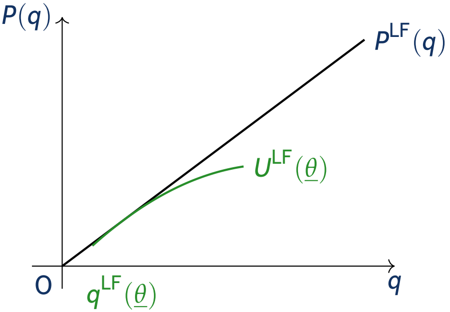
## Proof of "if" direction:

Suppose $\mathbf{E}_\theta[\omega(\theta)] > \alpha$. We identify a subsidy schedule improving over laissez-faire.



# When To Subsidize?

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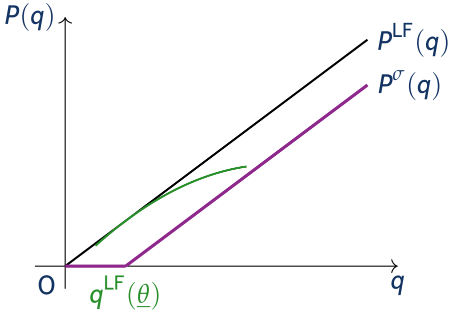
**Proof of "if" direction:**

Suppose $\mathbf{E}_\theta[\omega(\theta)] > \alpha$. We identify a subsidy schedule improving over laissez-faire.

$P^\sigma$ is outcome-equivalent to a cash transfer of $cq^{\mathrm{LF}}(\underline{\theta})$ to all consumers, and improves over laissez-faire because $\mathbf{E}_\theta[\omega(\theta)] > \alpha$. $\square$



# How to Design Subsidies?

# Mechanism Design Reformulation

Revelation principle $\Longrightarrow$ it suffices to consider **direct mechanisms** $(q, t)$ consisting of:

- an **allocation function** $q : [\underline{\theta}, \overline{\theta}] \to [0, A]$ denoting *total* quantity consumed by type $\theta$;
- a **payment rule** $t : [\underline{\theta}, \overline{\theta}] \to \mathbb{R}$ denoting *total* payment by type $\theta$,

satisfying incentive-compatibility,

$$\theta \in \arg\max_{\hat{\theta} \in \Theta} \left\{ \theta v(q(\hat{\theta})) - t(\hat{\theta}) \right\} \text{ for all } \theta \in \Theta. \tag{IC}$$

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**Lemma (Implementation).** For any (IC) mechanism $(q, t)$, there exists a subsidy $\sigma$ with $q = q^\sigma$ and $t = P^\sigma \circ q^\sigma$ if and only if:

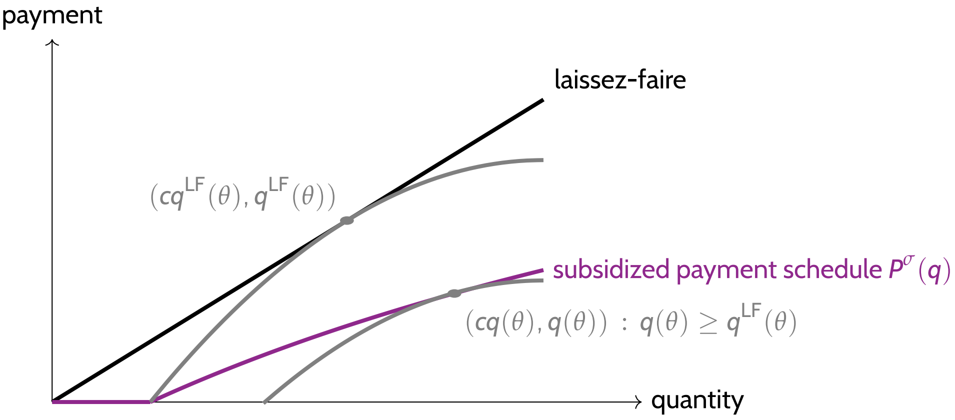
$$q(\theta) \geq q^{\mathrm{LF}}(\theta) \text{ for all } \theta \in \Theta, \tag{LB}$$

$$t(\theta) \geq 0 \text{ for all } \theta \in \Theta, \tag{NLS}$$

$$U(\theta) \geq U^{\mathrm{LF}}(\theta) \text{ for all } \theta \in \Theta. \tag{IR}$$

# Intuition

marginal price per unit $\leq c \iff$ allocations exceed laissez-faire




▸ Converse

# Reformulating the Mechanism Design Problem

The social planner maximizes weighted total surplus

$$\max_{(q,t)} \int_{\underline{\theta}}^{\overline{\theta}} \left[ \omega(\theta) \underbrace{[\theta v(q(\theta)) - t(\theta)]}_{\text{consumer surplus}} - \alpha \underbrace{[cq(\theta) - t(\theta)]}_{\text{total cost}} \right] \mathrm{d}F(\theta),$$

subject to (IC), (LB), (IR), and (NLS).

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**#1.** Apply Myerson (1981) Lemma and Milgrom and Segal (2002) envelope theorem to express objective in terms of $U(\underline{\theta})$ and $q(\theta)$ non-decreasing, substituting

$$t(\theta) = \theta v(q(\theta)) - \int_{\underline{\theta}}^{\theta} v(q(s))\,\mathrm{d}s - U(\underline{\theta}).$$

# Reformulating the Mechanism Design Problem

The social planner maximizes weighted total surplus

$$\max_{q \text{ non-decreasing}, U(\underline{\theta})} \mathbf{E}_\theta[\omega(\theta)-\alpha]U(\underline{\theta}) + \alpha \int_{\underline{\theta}}^{\overline{\theta}} \left[ \left( \theta + \frac{\int_\theta^{\overline{\theta}} [\omega(s)-\alpha] \mathrm{d}F(s)}{\alpha f(\theta)} \right) v(q(\theta)) - cq(\theta) \right] \mathrm{d}F(\theta),$$

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**#2.** Suffices to enforce (IR) and (NLS) only for lowest type $\underline{\theta}$ because $U(\theta) - U^{\mathrm{LF}}(\theta)$ and $t(\theta)$ are nondecreasing by (IC) and (LB).

- $\rightsquigarrow$ **(NLS) binding**: if $\mathbf{E}[\omega(\theta)] > \alpha$, choose $U(\underline{\theta}) = \underline{\theta} v(q(\underline{\theta}))$.
- $\rightsquigarrow$ **(NLS) does not bind**: if $\mathbf{E}[\omega(\theta)] \leq \alpha$, choose $U(\underline{\theta}) = U^{\mathrm{LF}}(\underline{\theta})$.

# Reformulating the Mechanism Design Problem

The social planner maximizes weighted total surplus

$$\max_{q \text{ non-decreasing}} \alpha \int_{\underline{\theta}}^{\overline{\theta}} \underbrace{[J(\theta)v(q(\theta)) - cq(\theta)]}_{\text{surplus of virtual type}} \, dF(\theta) + (\text{terms independent of } q),$$

subject to (LB): $q(\theta) \geq q^{\mathrm{LF}}(\theta)$, where the **virtual type** absorbs (IC), (IR), and (NLS):

$$J(\theta) = \underbrace{\theta}_{\text{efficiency}} + \underbrace{\frac{\int_{\theta}^{\overline{\theta}} [\omega(s) - \alpha] dF(s)}{\alpha f(\theta)}}_{\text{redistributive motive}} + \underbrace{\frac{\max\{\mathbf{E}_{\theta}[\omega(\theta) - \alpha], 0\} \underline{\theta} \delta_{\theta = \underline{\theta}}}{\alpha f(\theta)}}_{\text{(NLS) constraint}}$$

Call $J(\theta) - \theta$ the **distortion term**.

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Call $J(\theta) - \theta$ the **distortion term**.

**Technical challenge:** (LB) is a "pointwise dominance" / FOSD constraint (*cf.* Yang and Zentefis, 2024) $\rightsquigarrow$ possible interactions with the monotonicity constraint.

# Characterizing the Optimal Subsidy Allocation

**Theorem 2 (Negative Correlation).** The optimal allocation rule is unique, continuous and satisfies

$$q^*(\theta) = \begin{cases} D\left(c, \overline{J|_{[\underline{\theta},\theta_\alpha]}}(\theta)\right) & \text{for } \theta \leq \theta_\alpha \\ q^{\text{LF}}(\theta) & \text{for } \theta \geq \theta_\alpha, \end{cases}$$

where $\theta_\alpha$ is defined by

$$\theta_\alpha = \inf\left\{\theta \in \Theta : \overline{J|_{[\underline{\theta},\theta]}}(\theta) \leq \theta\right\}.$$

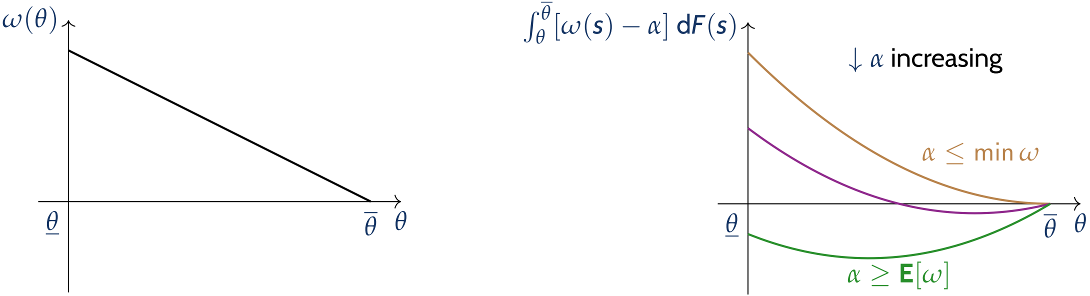
**Intuition:** there exists a type $\theta_\alpha \in \Theta$ (possibly $\underline{\theta}$ or $\overline{\theta}$) such that

$$q^*(\theta) > q^{\text{LF}}(\theta) \text{ for all } \theta < \theta_\alpha, \text{ and}$$
$$q^*(\theta) = q^{\text{LF}}(\theta) \text{ for all } \theta \geq \theta_\alpha.$$

▸ Ironing

# Intuition: Signing the Distortion Term

**Negative correlation** $\rightsquigarrow \omega(\theta)$ decreasing $\rightsquigarrow$ distortion is single-crossing zero from above.




Social planner wants to distort consumption of all types down, low-demand types up and high-demand types down, or all types upwards.

# Optimal Marginal Subsidy Schedule

**Case 1:** $\min \omega \geq \alpha$ (upward distortion for all)

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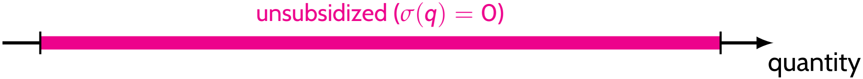
**Case 3:** $\mathbf{E}[\omega] \leq \alpha$ (downward distortion for all)



# Economic Implications

With **negative correlation** between $\omega$ and $\theta$:

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\# 2. If cash transfers are unavailable, linear subsidies are **never** optimal.

- \# 2a. Optimal subsidies are "all or none": active subsidy programs should always incorporate a **free allocation** ("public option").
- \# 2b. If *any* consumer has $\omega < \alpha$, optimal subsidies are **capped** in quantity.

# Deriving the Optimal Mechanism

# Solving for the Optimal Mechanism

skip

$$\max_{q} \alpha \int_{\underline{\theta}}^{\overline{\theta}} \left[ J(\theta) v(q(\theta)) - cq(\theta) \right] \, \mathrm{d}F(\theta),$$

s.t. $q$ nondecreasing and $q(\theta) \geq q^{\mathrm{LF}}(\theta)$.

# Solving for the Optimal Mechanism

▸ skip

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## Guess 1: Pointwise maximizer

$$q(\theta) = (v')^{-1}\left(\frac{c}{J(\theta)}\right) = D(c, J(\theta)).$$

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$q \geq q^{\mathrm{LF}} \iff D(c, J(\theta)) \geq D(c, \theta) \iff J(\theta) \geq \theta$.

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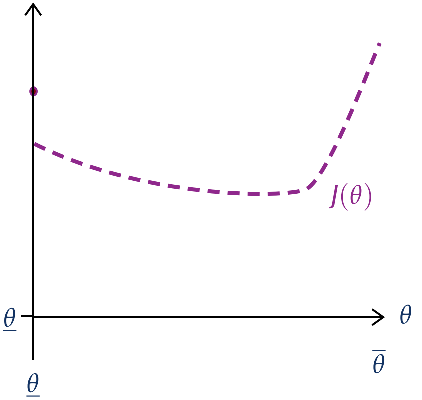


$J(\theta)$ may be non-monotone.

# Solving for the Optimal Mechanism

skip

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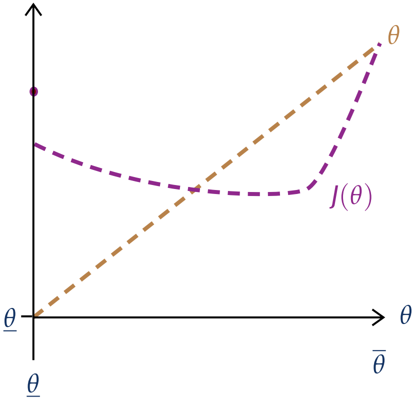
Demand $D(c, \cdot)$ is increasing, so:

$q$ nondecreasing $\iff J(\theta)$ nondecreasing.

$q \geq q^{\text{LF}} \iff D(c, J(\theta)) \geq D(c, \theta) \iff J(\theta) \geq \theta$.




$J(\theta)$ may be smaller than $\theta$.

# Solving for the Optimal Mechanism

▸ skip

$$\max_q \alpha \int_{\underline{\theta}}^{\overline{\theta}} [J(\theta)v(q(\theta)) - cq(\theta)] \, dF(\theta),$$

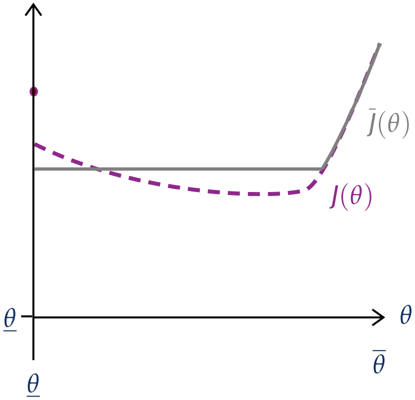
s.t. $q$ nondecreasing and $q(\theta) \geq q^{\mathrm{LF}}(\theta)$.

**Guess 2: Relaxing the (LB) constraint**

Toikka (2011); Akbarpour, Dworczak, Kominers (2021)

$$\rightsquigarrow q(\theta) = (v')^{-1}\left(\frac{c}{\bar{J}(\theta)}\right) = D(c, \bar{J}(\theta)),$$

where $\bar{J}$ is ironing of $J$, pooling types in any non-monotonic interval of $J$ at its $F$-weighted average.




Ironing deals with non-monotonicity.

▸ Ironing

# Solving for the Optimal Mechanism

skip

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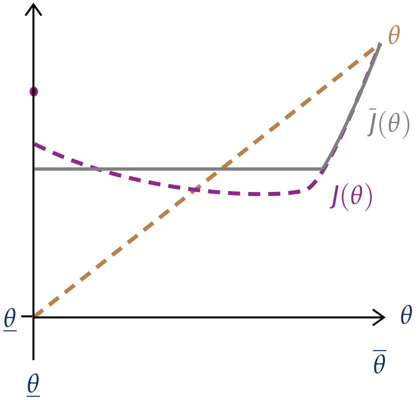


But not lower-bound constraint $\rightsquigarrow$ interaction.

Ironing

# Solving for the Optimal Mechanism

skip

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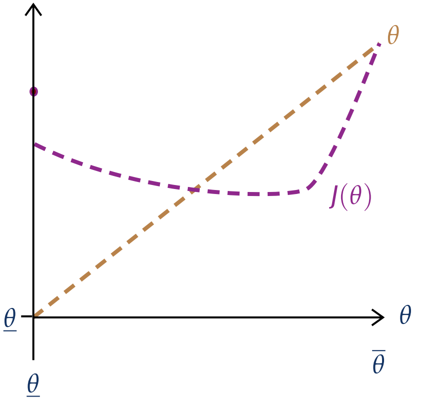
s.t. $q$ nondecreasing and $q(\theta) \geq q^{\mathrm{LF}}(\theta)$.

**Guess 3: Our approach**

Suppose solution is of the form

$$q(\theta) = D(c, H(\theta)).$$

Feasibility requires $H$ to be nondecreasing and satisfy $H(\theta) \geq \theta$.




Need to identify nondecreasing $H \geq \theta$.

Ironing

# Characterizing the Optimal Subsidy Allocation

**Theorem 2 (Negative Correlation).** The optimal allocation rule is unique, continuous and satisfies

$$q^*(\theta) = D(c, H(\theta)),$$

where the subsidy type $H(\theta)$ is defined by

$$H(\theta) := \begin{cases} \overline{J|_{[\underline{\theta},\theta_\alpha]}}(\theta) & \text{for } \theta \leq \theta_\alpha \\ \theta & \text{for } \theta \geq \theta_\alpha, \end{cases}$$

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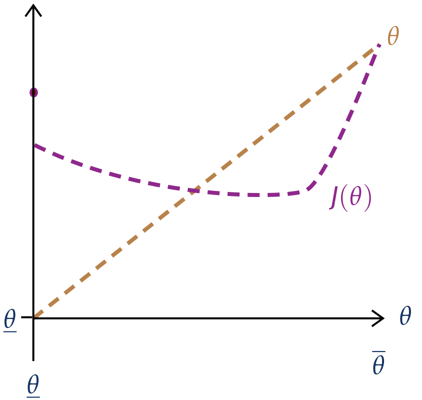


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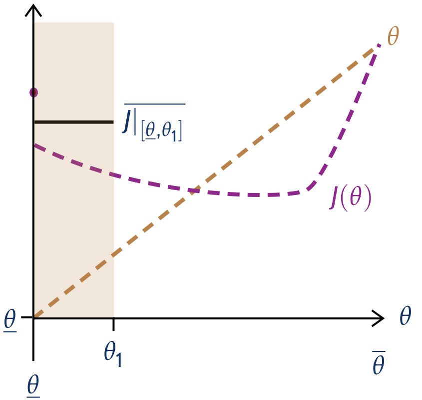


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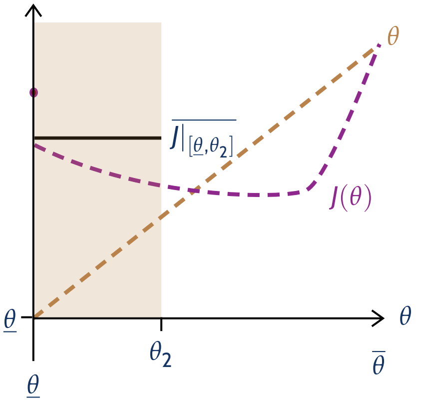


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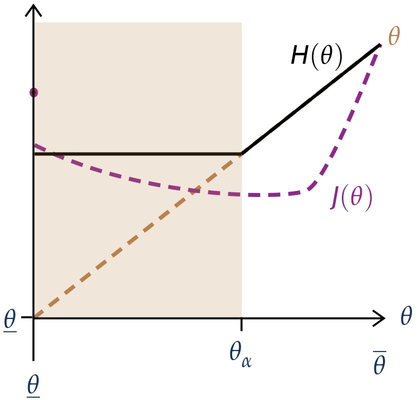


construction $\rightsquigarrow$ pooling condition and continuity

▸ Ironing

# Verifying $H$ from Theorem 2

Because $q^*(\theta) = D(c, H(\theta))$, for any feasible $q$

$$\int_\Theta \underbrace{[H(\theta)v(q^*(\theta)) - cq^*(\theta)]}_{\text{surplus of type } H(\theta) \text{ at } D(c,H(\theta))} \, \mathrm{d}F(\theta) \geq \int_\Theta \underbrace{[H(\theta)v(q(\theta)) - cq(\theta)]}_{\text{surplus of type } H(\theta) \text{ at } q(\theta)} \, \mathrm{d}F(\theta).$$

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$$\underbrace{\int_\Theta [J(\theta)v(q^*(\theta)) - cq^*(\theta)] \, \mathrm{d}F(\theta)}_{\text{objective at } q^*} \geq \underbrace{\int_\Theta [J(\theta)v(q(\theta)) - cq(\theta)] \, \mathrm{d}F(\theta)}_{\text{objective at feasible } q}.$$

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Subtracting, it suffices to show, for any feasible $q$

$$\int_\Theta [J(\theta) - H(\theta)][v(q^*(\theta)) - v(q(\theta))] \, dF(\theta) \geq 0.$$

▸ skip

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To show $\int_\Theta [J(\theta) - H(\theta)][v(q^*(\theta)) - v(q(\theta))] \, \mathrm{d}F(\theta) \geq 0$.

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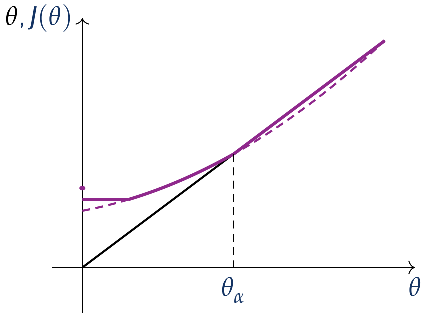


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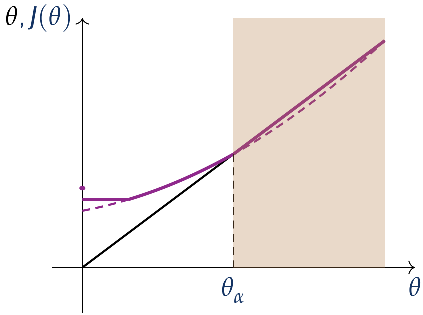


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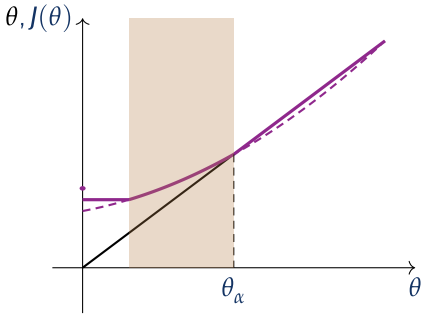


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▸ skip

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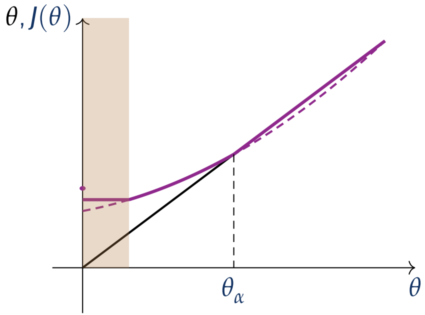
\# 3. $H(\theta) = \overline{J|_{[\underline{\theta},\theta_\alpha]}}(\theta) \neq J(\theta)$:
technical lemma $\rightsquigarrow$ on any such interval $\Theta_i$, $H = \overline{J|_{\Theta_i}}$
$\rightsquigarrow$ optimality of $D(c, H(\theta))$ in problem on $\Theta_i$ *without* (LB)
$\implies$ same variational inequality characterizes optimality. $\square$




skip

# Summing Up

Proof approach:

- Guess form of solution $q^*(\theta) = D(c, H(\theta))$.
- Identify $H(\theta)$ which is continuous, $\geq \theta$, and satisfies the **pooling condition**.
- Verify optimality using **variational inequalities**.

Same method of solution works for general $\omega \rightsquigarrow$ see paper.

Generalization

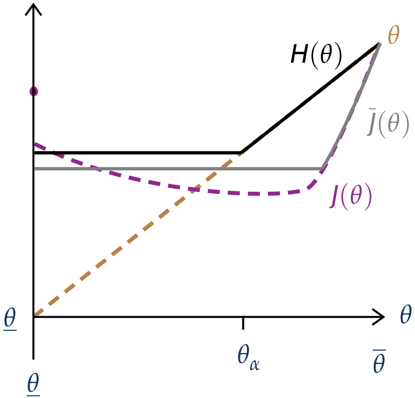
# Role of Topping Up

Comparing optimum with and without (LB) constraint, $H(\theta)$ can exceed $\bar{J}$ for *all* types.

$\rightsquigarrow$ Inability to tax causes upward distortion of *all* types

It is not optimal to calculate optimal subsidy/tax and set taxes to zero.

Highlights distinction from Mirrleesian marginal approach (FOC $\not\rightsquigarrow$ optimum).



# Positive Correlation

# When to Subsidize?

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Suppose now that $\omega(\theta)$ is increasing in $\theta$ (**"positive correlation"**), *e.g.*, public transport, staple foods.

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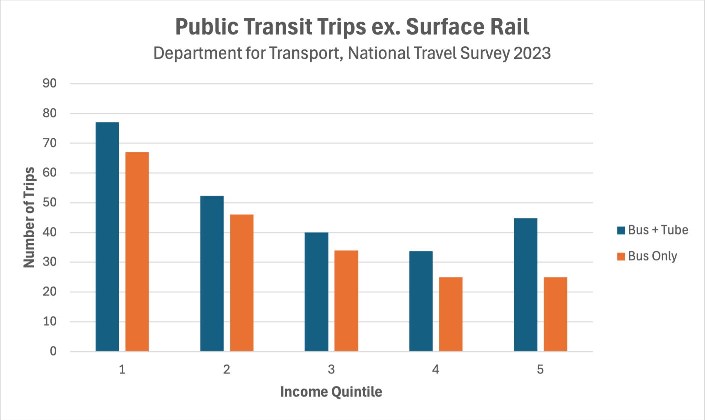

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$\rightsquigarrow$ Subsidies can be beneficial even when lump-sum cash transfers would not be.

**Intuition:** Social planner can always design a subsidy program with $\Sigma(q^{\sigma}(\theta)) \geq 0$ only if $\omega(\theta) \geq \alpha$.
$\rightsquigarrow$ Argument relies on nonlinearity of subsidy program.

▸ Arbitrary Correlation

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where $\theta_\alpha = \inf\{\theta \in \Theta : J(\theta) \geq \theta\}$.

Intuition: there exists a type $\theta_\alpha \in \Theta$ (possibly $\underline{\theta}$ or $\overline{\theta}$) such that

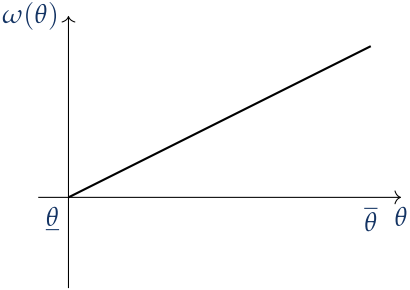
$$q^*(\theta) = q^{\mathrm{LF}}(\theta) \text{ for all } \theta \leq \theta_\alpha, \text{ and}$$
$$q^*(\theta) \geq q^{\mathrm{LF}}(\theta) \text{ for all } \theta > \theta_\alpha.$$

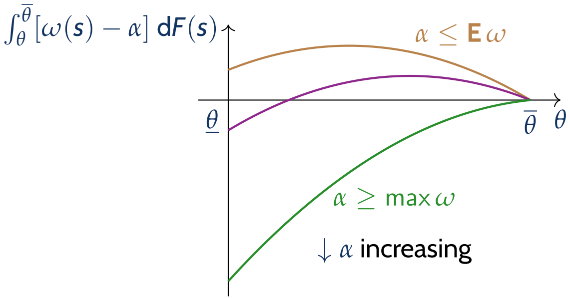
Arbitrary Correlation

# How to Subsidize?

## Positive Correlation

Positive correlation $\rightsquigarrow \omega(\theta)$ increasing $\rightsquigarrow$ distortion is single-crossing zero from below.




Social planner wants to distort consumption of all types down, high-demand types up and low-demand types down, or all types upwards.

# Optimal Subsidy Schedule

## Positive Correlation

**Case 1: $\mathbf{E}[\omega] \geq \alpha$ (upward distortion for all)**

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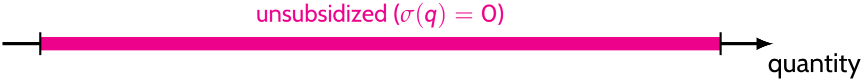

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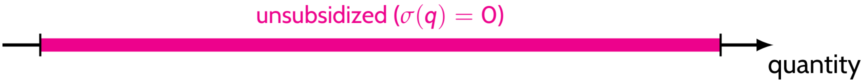

# Discussion

# Importance of Correlation

## Negative Correlation

Subsidies dominated by cash transfers.

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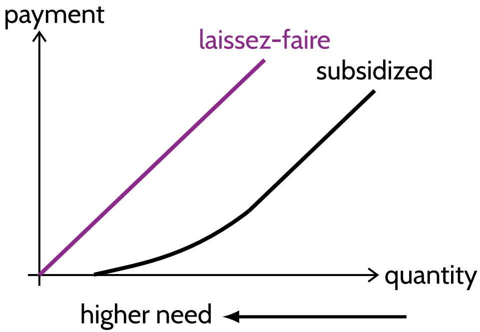


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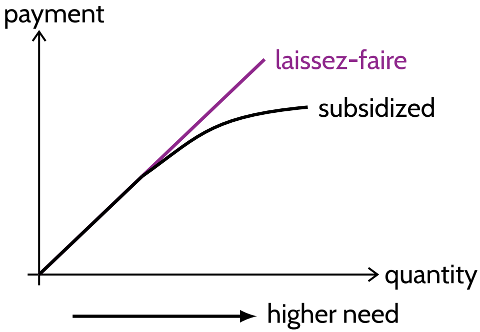

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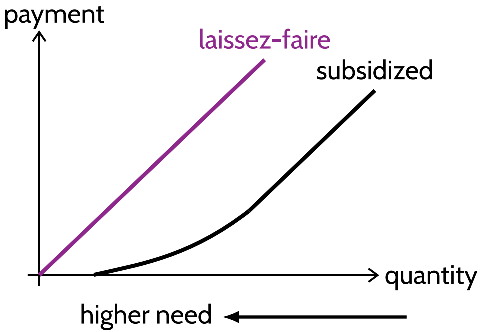


All or none subsidized.

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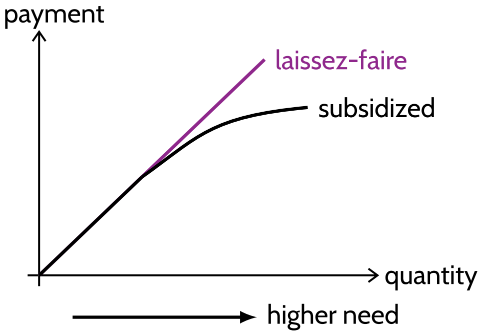


Only neediest (self-selected) consumers subsidized.

# Differences In Practice

**When?** Theorem 1 $\rightsquigarrow$ scope of intervention larger with positive correlation ($\max \omega > \alpha$) than negative correlation ($\mathbf{E}[\omega] > \alpha$).

In practice, many government programs focused on goods consumed disproportionately by needy.

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In practice, many government programs focused on goods consumed disproportionately by needy.

**How?** Significant differences in marginal subsidy schedules observed in practice:

**Larger subsidies for low $q$**

- Food stamps (SNAP)
- Womens, Infants & Children (WIC) Program
- Housing Choice (Section 8) Vouchers
- Lifeline (Telecomm. Assistance) Program

**Larger subsidies for high $q$**

- Public transit fare capping
- Pharmaceutical subsidy programs
- Government-subsidized childcare places.

program details
inferior programs

# How Do Optimal Subsidies Compare To Linear?

**Proposition.** Linear subsidies are never optimal.

Intuition: no distortion at the top ($J(\overline{\theta}) = \overline{\theta}$) $\rightsquigarrow$ linear subsidies never optimal.

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Social planner intervenes with linear subsidies only if a weighted average of all distortion terms are positive $\rightsquigarrow$ more restrictive than nonlinear.

Social planner can always improve over linear subsidy by implementing:

- a cap on the subsidy paid to a consumer (as in negative correlation),
- a floor on eligibility for the subsidy (as in positive correlation), or
- a free endowment of the good to all consumers.

Gains from nonlinear subsidization can be arbitrarily large compared to gains from linear subsidization.

skip to conclusion

# Comparative Statics of Subsidies

**Question:** How do optimal subsidies change when

(a) the social planner's desire to redistribute to each consumer increases?

(b) the correlation between demand and welfare weight increases?

(c) the marginal cost of production decreases?

Details

# Comparative Statics of Subsidies

**Question:** How do optimal subsidies change when

(a) the social planner's desire to redistribute to each consumer increases?

(b) the correlation between demand and welfare weight increases?

(c) the marginal cost of production decreases?

Details

**Short Answer:** Each cause the optimal subsidy program to be more generous.

But (a) and (b) cause $J(\theta)$ to increase for each $\theta \rightsquigarrow$ a larger set of consumers subsidized. (c) does not.

# Extensions

# Equilibrium Effects

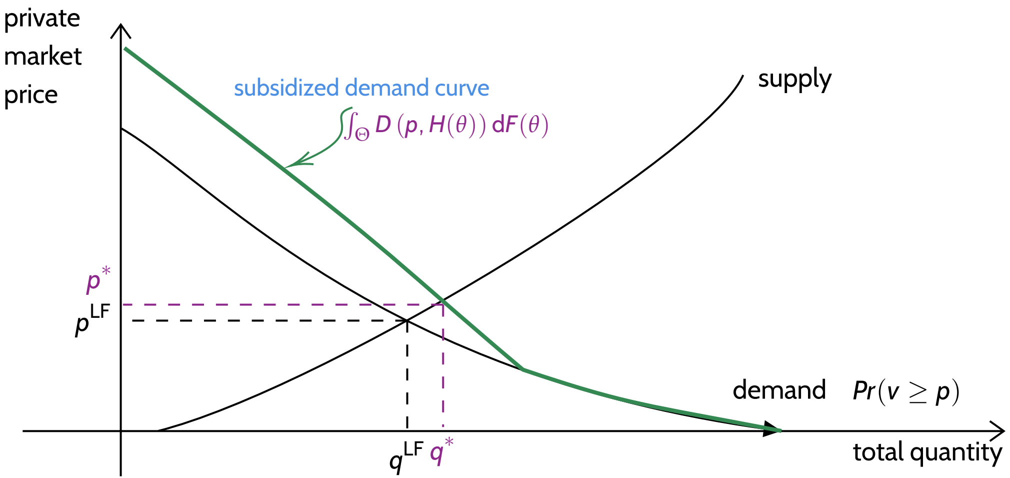
Our baseline model shuts down equilibrium effects of government subsidies on private market prices.

Empirical evidence of price effects from government subsidy programs, *e.g.*:

- public housing (Diamond and McQuade, 2019; Baum-Snow and Marion, 2009)
- pharmaceuticals (Atal et al., 2021)
- public schools (Dinerstein and Smith, 2021)
- school lunches (Handbury and Moshary, 2021)

# Equilibrium Effects

Our results extend directly to imperfectly elastic supply curves:



# Private Market Taxation

Our baseline model assumes the planner cannot tax the private market.

Taxation of private market reduces consumers' outside option, relaxing the (LB) constraint. If taxation is costly (*e.g.*, because of distortions on ineligible consumers):

**Proposition.** Suppose the planner faces a convex cost $\Gamma(\tau)$ for taxation of the private market. Then there exists an optimal tax level $\tau^*$ and subsidy program for eligible consumers satisfying

$$q^*(\theta) = D(H_{\tau^*}(\theta)),$$

where $H_{\tau^*}(\theta) \leq H(\theta)$.

Return to Model

# Budget Constraints and Endogenous Welfare Weights

In our baseline model, $\omega(\cdot)$ and $\alpha$ are taken exogenously.

Our model can be extended to allow weights to be endogenous (cf. Pai and Strack, 2024):

- $\alpha \Longleftrightarrow$ Lagrange multiplier on the social planner's budget constraint.
- $\omega(\theta) \Longleftrightarrow$ the marginal value of money for a consumer with concave preferences

$$\varphi\left(\theta v(q) + I - t\right),$$

and income $I \sim G_\theta$, known but not observed by the social planner, then

$$\omega(\theta) = \mathbf{E}_{I \sim G_\theta}\left[\varphi'\left(\theta v(q(\theta)) + I - t(\theta)\right)\right].$$

Return to Model

# Conclusion

# Concluding Remarks

## Takeaways for Subsidy Policy:

- Linear subsidies are **never** optimal.
- When and how to subsidize depends on **correlation** between demand and need.
  - With negative correlation (many goods), why not lump-sum cash transfers? ("tortilla subsidy" vs. Progresa).
  - Goods with positive correlation are ideal candidates for subsidies (e.g., public transport) but these should have floors to improve targeting.

## Technical Contribution:

- We show how to solve mechanism design problems with lower-bound constraints caused by type-dependent outside options.
- Similar mechanism design problems arise in other contexts, e.g., subsidy design with other objectives (externalities, paternalism); exclusive contracting (topping up = non-exclusive contracting, no topping up = exclusive contracting.).

# Subsidy Design Without Topping Up

# When Does the Planner Benefit from Private Market Restrictions?

## Role of Topping Up Constraint

In some markets (e.g., public housing), the social planner may be able to restrict subsidy recipients from topping up in private market $\rightsquigarrow$ opt-in (or out) of subsidy program.

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In Kang and Watt (2024), we characterize optimal subsidy mechanism under such restrictions. These lead to different (weaker) type-dependent outside option constraints:

$$\max_{q \text{ non-decreasing}} \alpha \int_{\underline{\theta}}^{\overline{\theta}} J(\theta)v(q(\theta)) - cq(\theta)\, \mathrm{d}F(\theta) \text{ subject to } U(\theta) \geq U^{\mathrm{LF}}(\theta).$$

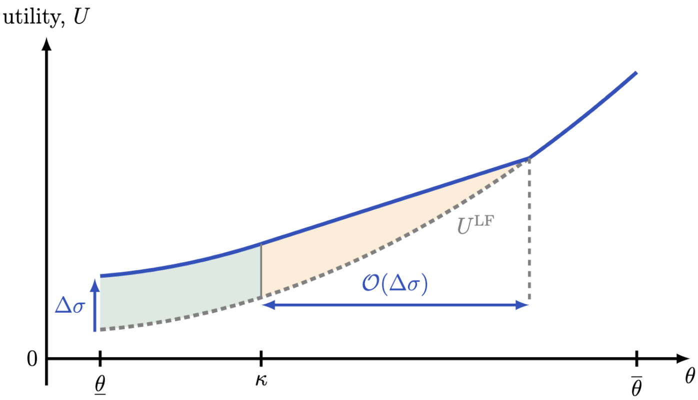
Equivalently:

$$U(\theta) \geq U^{\mathrm{LF}}(\theta) \Leftrightarrow \text{majorization constraint on } q \Leftrightarrow \text{average price} \leq c.$$

# Scope of Intervention (Without Topping Up)

**Theorem.** The optimal mechanism $(q^*, t^*)$ strictly improves on the laissez-faire outcome if and only if $\max \omega > \alpha$.

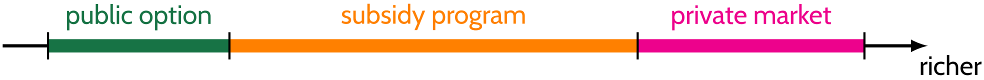
More scope for intervention than with topping up (for the negative correlation case, illustrated below).



# How to Intervene?


**(a)** $\mathbf{E}[\omega] > \alpha$


**(b)** $\mathbf{E}[\omega] \leq \alpha$ and $\mu^* = 0$




**(c)** $\mathbf{E}[\omega] \leq \alpha$ and $\mu^* > 0$

**Figure** Optimal in-kind redistribution programs under negative correlation.

# Solving the Mechanism Design Problem

Let us focus on the negative correlation case. We form the Lagrangian:

$$\mathcal{L}(q,\lambda) = \alpha \int_{\underline{\theta}}^{\overline{\theta}} J(\theta)v(q(\theta)) - cq(\theta) - \lambda(\theta)[U(\theta) - U^{\mathsf{LF}}(\theta)]dF(\theta)$$

One possibility: if $q(\theta) = D(\bar{J}(\theta), c)$ is feasible (i.e., if $\underline{\theta}v(D(\bar{J}(\underline{\theta}), c)) + \int_{\underline{\theta}}^{\theta} D(\bar{J}(s), c)\, \mathrm{d}s \geq U^{\mathsf{LF}}(\theta)$ for all $\theta \in \Theta$), then it must be optimal.

Else Lagrangian duality $\leadsto$ (IR) must bind on some interval. We show it must include $\overline{\theta}$ (else a redistributive reallocation downwards is possible).

Integrating the constraint by parts and letting $\Lambda(\theta) = \int_{\theta}^{\overline{\theta}} \lambda(\theta)\, \mathrm{d}F(\theta)$, we get

$$\mathcal{L}(q,\lambda) = \alpha \int_{\underline{\theta}}^{\overline{\theta}} J(\theta)v(q(\theta)) - cq(\theta) + \frac{\Lambda(\theta)}{f(\theta)}[v(q(\theta)) - v(q^{\mathsf{LF}}(\theta))]dF(\theta)$$

Note, wherever (IR) is non-binding, $\Lambda$ is constant! Find unique $\mu^*$ such that $D(\overline{J + \frac{\mu^*}{f}}(\theta^*), c) = D(\theta^*, c)$, where $\mu^* = (J(\theta^*) - \theta^*)f(\theta^*)$.

# Characterization of Optimal Mechanism

**Theorem 2.** For any $\mu \geq 0$, define

$$q_\mu(\theta) := (v')^{-1}\left(\frac{1}{\overline{H_\mu}(\theta)}\right), \quad \text{where } H_\mu(\theta) := \frac{\theta}{c} + \frac{\mu\underline{\theta} \cdot \delta_{\theta=\underline{\theta}} + \mu + \int_{\underline{\theta}}^{\theta} [\alpha - \omega(s)] \, \mathrm{d}F(s)}{\alpha c f(\theta)},$$

$$\theta_H(\mu) := \begin{cases} \max\left\{\theta \in [\underline{\theta}, \overline{\theta}] : \int_{\underline{\theta}}^{\theta} [\alpha - \omega(s)] \, \mathrm{d}F(s) + \mu \leq 0\right\} & \text{if } \mathbf{E}[\omega] \leq \alpha, \\ \overline{\theta} & \text{if } \mathbf{E}[\omega] > \alpha. \end{cases}$$

Under the optimal mechanism,

$$q^*(\theta) = \begin{cases} q^{\mathrm{LF}}(\theta) & \text{if } \theta \in [\theta_H(\mu^*), \overline{\theta}], \\ q_{\mu^*}(\theta) & \text{if } \theta \in [\underline{\theta}, \theta_H(\mu^*)], \end{cases}$$

where

$$\mu^* := \min\left\{\mu \in \mathbb{R}_+ : \int_{\underline{\theta}}^{\theta_H(\mu)} v(q_\mu(s)) \, \mathrm{d}s + \underline{\theta} v(q_\mu(\underline{\theta})) - U^{\mathrm{LF}}(\theta_H(\mu)) \geq 0\right\}.$$

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$\rightsquigarrow$ Participation constraints can increase scope for using non-market allocations.

# Fin

# Appendix

# Assumption: No Lump-Sum Cash Transfers

**Note:** This constraint only binds if $\mathbf{E}_\theta[\omega(\theta)] > \alpha$.

**Possible reasons:**

- **Institutional**: subsidies designed by government agency without tax/transfer powers.
- **Political**: Liscow and Pershing (2022) find U.S. voters prefer in-kind redistribution to cash transfers.
- **Household Economics**: Currie (1994) finds in-kind redistribution has stronger benefits for children than cash transfer programs.
- **Pedagogical**: to contrast when the assumption is binding ($\rightsquigarrow$ cash transfers preferred to subsidies) versus non-binding (*vice versa*).
- **Model**: without NLS constraint, the social planner would want to make unbounded cash transfers when $\mathbf{E}[\omega] > \alpha$.

return

# When to Subsidize (General): Proof by Picture

**Theorem 1.** Social planner subsidizes **if and only if** there exists a type $\hat{\theta}$ for which

$$\mathbf{E}_\theta[\omega(\theta) \mid \theta \geq \hat{\theta}] \; > \; \alpha.$$

Suppose $\mathbf{E}_\theta[\omega(\theta)|\theta \geq \hat{\theta}] > \alpha$: we construct a subsidy schedule increasing weighted surplus.

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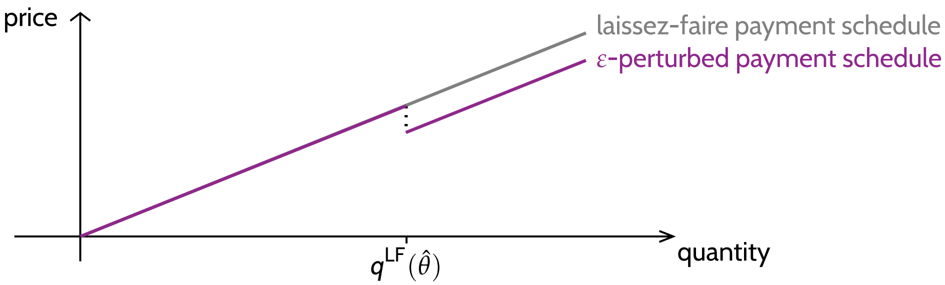


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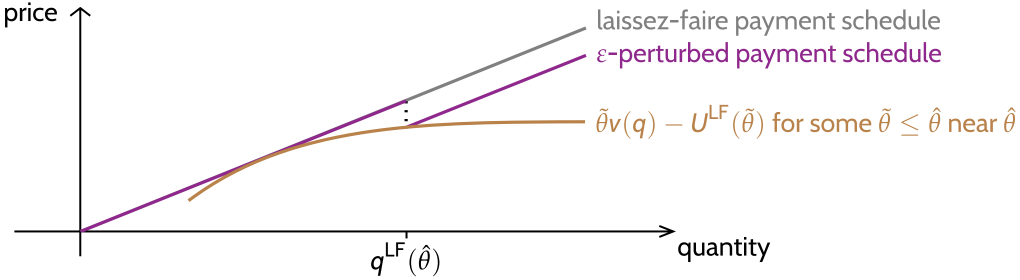


$\varepsilon-$perturbation increases utility of types $\geq \hat{\theta}$, net benefit $\varepsilon\, \mathbf{E}_{\theta}[\omega(\theta) - \alpha|\theta \geq \hat{\theta}]$.

*But* consumption is distorted for $O(\sqrt{\varepsilon})$ set of types near (but below) $\hat{\theta}$, at cost $\leq O(\sqrt{\varepsilon})\varepsilon$.

$\rightsquigarrow$ Benefits $>$ costs for small enough $\varepsilon$. **Note: Argument relies on nonlinearity.**

return

# Topping Up $\Longleftarrow$ Lower-Bound (1/2)

Suppose $q(\theta) \geq q^{\mathrm{LF}}(\theta)$. We want to show total subsidies $S(z)$ is increasing in $z$.

# 1. $t(\underline{\theta}) \leq cq(\underline{\theta})$ by (IR):

$$t(\underline{\theta}) \leq \underline{\theta}v(q(\underline{\theta})) - \underline{\theta}v(q^{\mathrm{LF}}(\underline{\theta})) + cq^{\mathrm{LF}}(\underline{\theta}),$$

and $\underline{\theta}v(q^{\mathrm{LF}}(\underline{\theta})) - cq^{\mathrm{LF}}(\underline{\theta}) \geq \underline{\theta}v(q(\underline{\theta})) - cq(\underline{\theta})$ by definition of $q^{\mathrm{LF}}$, so $t(\underline{\theta}) \leq cq(\underline{\theta})$.

# Topping Up $\Longleftarrow$ Lower-Bound (2/2)

# 2. The *marginal* price of any units purchased is no greater than $c$ by (IC):

$$t(\theta') - t(\theta) = \left[\theta' v(q(\theta')) - U(\underline{\theta}) - \int_{\underline{\theta}}^{\theta'} v(q(s))\,\mathrm{d}s\right] - \left[\theta v(q(\theta)) - U(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} v(q(s))\,\mathrm{d}s\right]$$

$$= \theta' v(q(\theta')) - \theta v(q(\theta)) - \int_{\theta}^{\theta'} v(q(s))\,\mathrm{d}s$$

$$= \int_{\theta}^{\theta'} s v'(q(s))\,\mathrm{d}q(s).$$

But if $q(\theta) \geq q^{\mathrm{LF}}(\theta)$, then concavity of $v$ implies $v'(q(\theta)) \leq v'(q^{\mathrm{LF}}(\theta)) = c/\theta$, so $t(\theta') - t(\theta) \leq c[q(\theta') - q(\theta)]$.

return

# Discussion

**Theorem 1** $\rightsquigarrow$ scope of intervention larger for “inferior goods” than “normal goods.”

In practice, many government programs focused on goods consumed disproportionately by needy:

**Examples**:

- Egyptian *Tamween* food subsidy program subsidizes five loaves of *baladi* bread/day at AUD 0.01/loaf, with a cap on weights and quality of bread.
- CalFresh Restaurant Meals Program subsidizes fast food restaurants not dine-in restaurants.
- Indonesian Fuel Subsidy Program subsidizes low-octane fuel (for motorbikes) and not high-octane fuel (for cars).
- Until ~2016, UK's NHS subsidized amalgam fillings and not composite (tooth-coloured) fillings.