A Walrasian Mechanism with Markups for Nonconvex Markets

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Abstract

We introduce markup equilibrium—an extension of Walrasian equilibrium in which consumers pay a fixed percentage markup over producer prices. In quasilinear markets, markup equilibria exist despite non-convexities. They are resource-feasible and envy-free, incur no budget deficit, and require little more communication and computation than ordinary Walrasian equilibrium. The associated markup mechanism is asymptotically incentive-compatible. We also introduce a Bound-Form First Welfare Theorem, which states that for any feasible allocation, the welfare loss compared to the first-best is bounded, using any price vector, by the sum of the resulting (i) budget surplus and (ii) rationing losses suffered by the participants. Using producer prices, this bound implies that any markup equilibrium with a small markup and few unallocated goods is nearly efficient.

Keywords: Approximate efficiency, Approximate incentive-compatibility, Market design, Non-convexity, Prices, Rationing

JEL Codes: C62, D40, D44, D47, D50, D51, D61.

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1 Introduction

Walrasian equilibrium has long served as a standard benchmark for economic outcomes due to its many desirable properties: its allocations are feasible, efficient, and envy-free; its balanced budget requires no subsidies from any third party; and these properties apply to markets with any number of products. Walrasian equilibrium was formulated as a model of an entire competitive economy, but its associated *mechanisms* are also a useful conceptual guide for organizing multi-product markets, such as electric power (where products may be distinguished by location and time of day) and fishing rights (where products vary by location and species).

From an implementation perspective, price systems offer two advantages. The first, emphasized by Hayek (1945), is that they economize on communication: when a Walrasian equilibrium exists, its prices provide the minimal information agents require to verify their proposed allocations are part of an efficient plan.[1] The second is that they provide helpful incentives: the Walrasian mechanism allows participants to misreport profitably only to the extent that they can affect prices. However, implementing even an approximate Walrasian mechanism poses multiple challenges.

The first challenge is *existence*. The aforementioned applications entail fixed costs of production and other non-convexities, which create the possibility that no Walrasian equilibrium exists. We extend Walrasian equilibrium, ensuring existence by adding a single parameter to obtain a new equilibrium concept. A *Walrasian equilibrium with markups* or just a *markup equilibrium* is a triple $(\mathbf{x}, p, \alpha)$ where $\mathbf{x}$ is a feasible allocation for buyers and sellers, p is a price vector that determines payments to sellers, and $\alpha \geq 0$ is a markup paid by buyers, so that buyer payments are determined by the price vector $(1+\alpha)p$. In a markup equilibrium, all buyers and sellers are assigned their most preferred bundles at the prices they face, and α is chosen so that total payments by buyers weakly exceed payments to sellers. A *minimal markup equilibrium* is a markup equilibrium with the smallest markup α. If there are finite, nonzero choke prices for both supply and demand, then a minimal markup equilibrium always exists. Since $(\mathbf{x}, p)$ is a Walrasian equilibrium if and only if $(\mathbf{x}, p, 0)$ is a markup equilibrium, the minimal markup equilibrium is an extension of Walrasian equilibrium: it selects a Walrasian allocation and prices whenever they exist.

Next are challenges related to *feasibility and efficiency.* A Walrasian equilibrium allocation balances supply and demand, and payments to sellers equal payments from buyers, ensuring it is always

[1] See also Nisan and Segal (2006) and Segal (2007), who prove that any decentralized communication system that implements efficient allocations must communicate a vector of supporting prices to the agents.

resource- and budget-feasible. In contrast, notions of approximate Walrasian equilibrium, including *pseudo-equilibrium* and *quasi-equilibrium* studied by Starr (1969), can specify plans that are not fully feasible. In our markup equilibrium, each firm and consumer is allocated its preferred bundle given the prices, the total production of each good weakly exceeds consumption, and the total revenue from buyers weakly exceeds payments to sellers. The markup equilibrium outcome is therefore always feasible and envy-free for both producers and consumers. Moreover, just as Walrasian allocations are efficient, markup equilibrium allocations with small markups and little excess production are nearly efficient.

A third set of challenges, emphasized by the mechanism design perspective, concerns the *incentives* for participation and truthful reporting. If producers have fixed costs, then marginal cost pricing may not cover some producers' total costs, violating their participation constraints. To address this, real-world mechanisms sometimes modify the Walrasian mechanism by adding "uplift" payments to cover producers' total costs, but these can incentivize producers to exaggerate their fixed costs. In a markup mechanism, there are no uplift payments: linear pricing with markups suffices to eliminate the participation problem without creating new incentive problems. In a markup equilibrium, production can strictly exceed consumption, but no budget deficit arises because the markups paid by consumers cover the costs of any excess production.

Fourth are challenges related to *communication and computation*. In convex markets, if the market operator announces a Walrasian equilibrium allocation and prices, then participants can verify that their allocations are part of an efficient plan. In nonconvex markets, if the planner announces the allocation, prices, and one additional parameter—the markup—then participants can verify that their allocations are part of an approximately efficient plan. In the quasilinear environments studied in this paper, a Walrasian equilibrium for convex markets can be computed by solving a convex optimization problem and its dual. While calculating exactly efficient allocations in nonconvex economies can be hard, computing an approximately efficient markup equilibrium is generally easier: it entails solving a series of convex optimizations and their duals, as detailed below.

Our analysis of the markup mechanism employs a new extension of the First Welfare Theorem for quasilinear markets, developed in this paper. We call it the *Bound-Form First Welfare Theorem* because it provides an upper bound on the welfare loss of any feasible allocation $\mathbf{x}$ using any price vector p, and it delivers a bound of *zero* when the pair $(\mathbf{x}, p)$ is a Walrasian equilibrium.

To describe the theorem, we first define several terms. The *welfare* of a feasible allocation $\mathbf{x}$ is the sum of the values enjoyed by consumers minus the sum of the costs incurred by firms; the *welfare loss* of $\mathbf{x}$ is the welfare of the first-best allocation minus the welfare of $\mathbf{x}$. Given an allocation and price vector $(\mathbf{x}, p)$, the *rationing loss* of firm f is the difference at prices p between its maximum profit and the profit from its assigned allocation x_f; we define the rationing loss for consumers similarly. The theorem states that for any price vector p and any feasible allocation $\mathbf{x}$, the welfare loss of $\mathbf{x}$ is bounded above by the sum of two terms: (i) the value at prices p of any excess of production over consumption, plus (ii) the sum at prices p of the rationing losses incurred by consumers and firms. For any Walrasian equilibrium $(\mathbf{x}, p)$, both terms are zero.

Given any markup equilibrium $(\mathbf{x}, p, \alpha)$, we can apply the Bound-Form First Welfare Theorem to allocation $\mathbf{x}$ and price vector p to bound the welfare loss of allocation $\mathbf{x}$. Because firms produce their most preferred bundles at p, *producer rationing losses are zero.* Consumer rationing losses at price vector p can be positive because, although each consumer n's bundle x_n is her most preferred bundle at the prices $(1+\alpha)p$, the consumer may prefer a different bundle at prices p. However, the envelope theorem (Milgrom and Segal, 2002) implies that *each consumer's rationing loss is of an order smaller than* α. The final term in the welfare loss is the value of excess production at prices p. By the Bound-Form First Welfare Theorem, this implies that the total welfare loss is small whenever *both* the markup α and any overproduction are small.

To compute a markup equilibrium with a small α and little excess production in a tractable way, we suggest an approach that begins with two changes to the standard Walrasian welfare maximization problem for convex economies: one affecting the constraints and the other the objective function. Our change to the constraints requires that the total production of the firms weakly exceed the total consumption *plus an operating reserve*, specified to depend on the largest nonconvexity but not on the number of producers or consumers. For a fixed markup α, the objective to be maximized is $\frac{\text{Total Utility}}{1+\alpha} - \text{Total Costs}$. These two changes distinguish our *simple markup mechanism* calculation from that for the standard Walrasian mechanism.

Next, if the specified objective is not concave, we concavify it to make the problem a convex program. Solving its dual program yields the producer price vector p; solving the primal problem yields an *approximate* markup allocation $\widehat{\mathbf{x}}$. If the approximate allocation of a producer lies outside its actual supply set at a price p, then it is rounded to a point in its supply set (and similarly for buyers at prices $(1+\alpha)p$), yielding the markup allocation $\mathbf{x}$. The Shapley-Folkman Lemma (Starr, 1969) implies that this rounding can be done so that the total supply in the markup allocation always weakly exceeds demand. By construction, the operating reserve is sufficient to replace any supply reductions resulting from the rounding process. If the resulting plan is budget-feasible, meaning that total payments by consumers (calculated by applying price vector $(1+\alpha)p$ to their rounded allocations) weakly exceed those to producers (calculated by applying price vector p to their rounded allocations), then $(\mathbf{x}, p, \alpha)$ is a markup equilibrium. We use a line search to approximate the smallest α outputting a markup equilibrium, with each search candidate requiring a convex optimization and the other steps described above.

The incentives for truthful reporting in the simple and minimal markup mechanisms are conceptually similar to those for the Walrasian mechanism in convex markets. Both mechanisms are strategy-proof in the large (Azevedo and Budish, 2019) and share the property that a participant can benefit from a false report only to the extent that the report affects the prices used to compute its payments. As a result, as the number of participants grows, the benefit to a single buyer or seller from misreporting their values or costs becomes vanishingly small.

Following an earlier draft of this paper, Ahunbay, Bichler, Dobos, and Knörr (2024) performed computational tests to assess the potential of a markup mechanism for European wholesale spot electricity markets. That paper compared markup equilibrium computations to a widely used mechanism that optimizes allocations using mixed-integer programming, computes prices using the dual of that problem's relaxation (omitting the integer constraints), and uses "uplift" payments to ensure total payments cover producers' fixed costs. The paper found that (1) markup computations "are considerably faster for relevant problem sizes" (even on a standard office laptop), (2) uplift compensation in the alternative mechanism results in substantial budget shortfalls, and (3) the markup allocation incurs only a small welfare loss relative to the full optimum. This comparison assumes that both mechanisms have access to true reports, omitting the additional losses the uplift mechanism may suffer because of its incentives for producers to exaggerate their fixed costs.

The remainder of this paper is organized as follows. Section 1.1 presents a simple single-product example illustrating how markup equilibrium can be computed and highlighting its properties, and Section 1.2 reviews the related literature. Section 2 introduces the quasilinear model and several preliminaries, including the measures of nonconvexity used. Section 3 develops the Bound-Form First Welfare Theorem. Section 4 introduces the markup equilibrium, covering its computation, feasibility, incentive properties, and efficiency guarantee. Section 5 concludes.

1.1 A single-product example

This section illustrates markup equilibrium using a single-product example of a market with nonconvexities in production but not in consumption.

Each firm f can produce zero units at zero cost or any positive quantity q_f up to its capacity K_f by incurring a fixed cost F_f. Its marginal cost of production is zero up to its capacity. If firm f produces at capacity, its average cost is $a_f = F_f/K_f$. Let $\overline{K} = \max_f K_f$ be the largest capacity among the firms. At price p, firm f's supply $S_f(p)$ is zero if $p < a_f$, K_f if $p > a_f$, and $\{0, K_f\}$ if $p = a_f$, leading to a discontinuous total supply correspondence $S(p) = \sum_f S_f(p)$.

Total consumer values $V(q)$ are strictly concave in total quantity q, leading to a continuous, downward-sloping demand function $D(p)$.

Walrasian equilibrium requires that there exists a price p such that $D(p) = S(p)$. As illustrated in Figure 1, discontinuities in the supply correspondence may imply that no Walrasian equilibrium exists. Indeed, when considering a parameterized family of demand functions $b + D(p)$, intervals of the b parameter exist for which $b + D(p)$ and $S(p)$ are disjoint. This demonstrates that there are robust examples of demand functions for which Walrasian equilibrium does not exist.

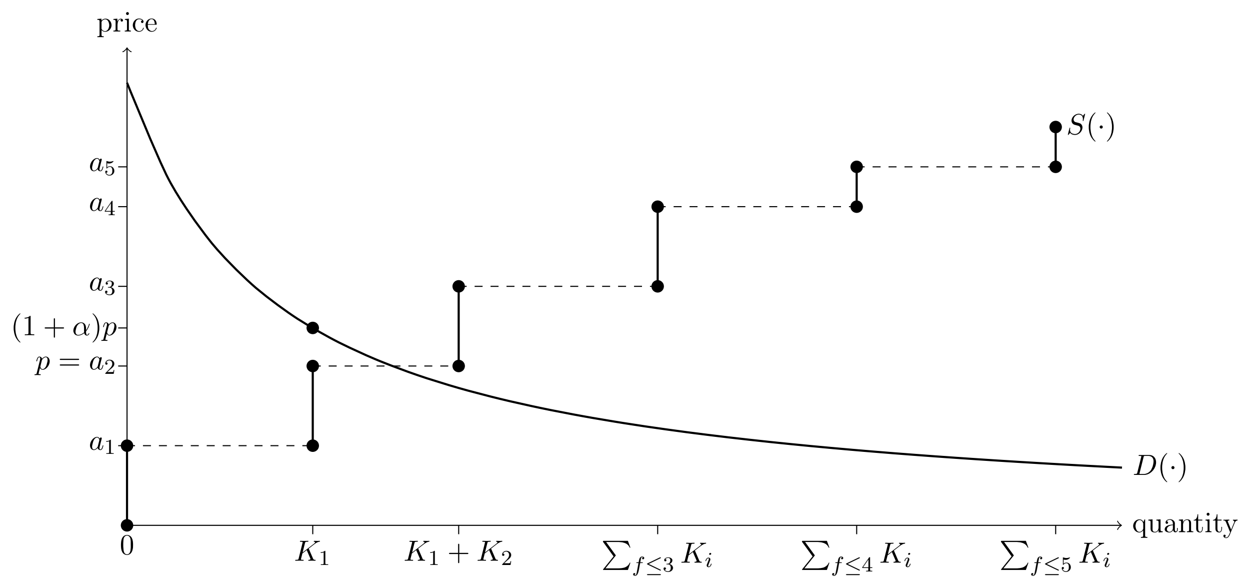

Figure 1: Five-firm example of Walrasian equilibrium nonexistence (here, firms are indexed in order of a_f). A markup equilibrium exists with total consumption and production equal to K_1, price $p = a_2$ paid to firms, and markup α on prices paid by consumers chosen so $D((1+\alpha)p) = K_1$.

The markup equilibrium, our extension of the Walrasian equilibrium, consists of a triple $(\mathbf{x}, p, \alpha)$, where $\mathbf{x}$ is the allocation, p the price per unit paid to producers, and $(1+\alpha)p$ the price per unit paid by consumers. The price and markup are chosen to avoid rationing and budget deficits: in the example in Figure 1, one markup equilibrium involves total production and consumption of K_1 units with the price $p = a_2$ paid to producers and a markup α chosen so that demand matches supply: $D((1+\alpha)p) = K_1$.[2]

This paper proposes a computational approach to identify a nearly-efficient markup equilibrium in large markets with many goods. We illustrate this approach in our single-product example. In the inner loop of our computational algorithm, we fix a candidate markup $\alpha \geq 0$ and solve a modified welfare maximization problem, incorporating adjustments to both the constraints and the objective. We modify the feasibility constraint to require total production to exceed total consumption by at least $\overline{K}$. We modify the objective by rescaling consumer values to obtain $\widehat{V}(q) = V(q)/(1+\alpha)$ and convexifying firms' cost functions, which means setting $\widehat{C}_f(q) = a_f q$ for production up to capacity, leading to convexified total costs $\widehat{C}(q)$ and supply $\widehat{S}(p)$. We then solve for q maximizing $\widehat{V}(q - \overline{K}) - \widehat{C}(q)$. The problem's first-order condition yields a price p such that $\widehat{S}(p) = D((1+\alpha)p) + \overline{K}$. At this price, firms with $a_f > p$ produce nothing, firms with $a_f < p$ produce at capacity, and one marginal firm f' with $a_{f'} = p$ may produce a fraction of its capacity. We assign firm f' to produce nothing and leave other allocations unchanged. Since $K_{f'} \leq \overline{K}$, total production is at least $D((1+\alpha)p)$, implying that the allocation is resource-feasible. In the outer loop of the computation, we perform a binary search over $\alpha \geq 0$ to find the smallest markup for which the plan budget-feasible, so that $p\widehat{S}(p) \leq (1+\alpha)D((1+\alpha)p)$.[3]

In our example, if there are many firms, no single firm has a significant incentive to exaggerate its cost or understate its capacity in the markup mechanism, as such reports have only a limited effect on the firm's price. Moreover, misreporting is risky: if the firm reports an excessively high fixed cost, its allocation will be zero. With suitable penalties for non-performance, firms are also deterred from overstating their capacities. At a markup equilibrium, no further gains from trade exist among consumers or among firms, because each group faces a single price. By the envelope theorem, adjusting the total output q yields little gain. The welfare loss in markup equilibrium

[2] In general with one good and continuous demand, the minimal markup equilibrium sets $p = \sup\{p' | D(p') \geq s \text{ for some } s \in S(p')\}$; firms' allocations are chosen as $s_f \in S_f(p)$ with $D(p) \geq \sum s_f$; α satisfies $D((1+\alpha)p) = \sum_f s_f$; and consumers' allocations are in $D((1+\alpha)p)$. While the minimal markup equilibrium is simple to compute in this case, with more goods and discontinuities in both demand and supply, the minimal markup equilibrium is generally hard to compute, necessitating the computational approach developed in this paper.

[3] To ensure a compact search space, a binary search for α can search over $\beta := 1/(1+\alpha)$ which lies in $[0, 1]$.

stems mainly from the unconsumed portion of production, an amount less than $\overline{k}$, independent of the number of participants in the market. As a percentage of the trading volume, the total welfare loss decreases to zero as market participation grows.

Although this one-dimensional example offers insights, it includes two simplifying assumptions that must be relaxed for a more general theory. Nonconvexities do not always take the form of fixed costs, requiring us to use more general measures of set nonconvexity and determine the operational reserve accordingly. In the one-dimensional problem, rounding the approximate markup allocation involves simply rounding an output up or down, but this becomes subtler in higher dimensions. For example, a firm that can produce one unit of good 1 or good 2 could be allocated $(\frac{1}{2}, \frac{1}{2})$ in the convexified optimization. Then, to avoid rationing, the output of one good may need to be rounded up while the production of the other is rounded down.

1.2 Related literature

The problem of nonconvexity for the existence of competitive equilibrium was discussed in a series of papers by Farrell (1959), Rothenberg (1960), Koopmans (1961) and Bator (1961). Much of the subsequent classical literature on nonconvexity in general equilibrium theory focused on concepts of *approximate* equilibria, which replace aggregate feasibility requirements with approximate feasibility—measured by distance in commodity space between aggregate supply and demand—while requiring that individual agents act optimally given the prices. Starr (1969) demonstrated the construction of such an approximate equilibrium in nonconvex production economies, where the maximum imbalance is proportional to the number of goods and a measure of nonconvexity. Heller (1972) proved a similar result using an alternative measure of nonconvexity. More recently, Nguyen and Vohra (2024) proved a bound for markets with indivisible goods that depends only on a measure of preference complementarity of agents. We build on some of these results (summarizing the key results we employ in Appendix B.1) but depart from this literature by requiring a feasible mechanism to always specify a feasible outcome. Influenced by computer scientists' approaches to approximations in mechanism design, we focus on approximate efficiency and truthfulness rather than approximate feasibility.[4]

A substantial literature identifies various conditions on preferences in markets with indivisibilities under which competitive equilibria exist despite nonconvexities. Contributors include Henry (1970),

[4] Scarf (1967) also includes an approximate efficiency objective.

Kelso and Crawford (1982), Bikhchandani and Mamer (1997), Gul and Stacchetti (1999), Danilov, Koshevoy, and Murota (2001), Sun and Yang (2006), Milgrom and Strulovici (2009), Hatfield, Kominers, Nichifor, Ostrovsky, and Westkamp (2013), Baldwin and Klemperer (2019), Baldwin, Jagadeesan, Klemperer, and Teytelboym (2023), and Nguyen and Vohra (2024). Milgrom (2009) and Klemperer (2010) emphasize reporting languages used when goods are substitutes. None of these papers address markets with fixed costs, such as those described above, where competitive equilibria do not generally exist. Our analysis develops practical mechanisms for those settings.

An alternative approach to equilibrium existence in nonconvex economies studies the large market limit with a continuum of agents. Aumann (1966) showed that nonconvexities at the individual firm or consumer level pose no barrier to equilibrium existence in an economy with a continuum of traders and divisible goods, while Azevedo, Weyl, and White (2013) demonstrated a similar result for quasilinear economies with indivisibilities. Our markup mechanism exists and is nearly efficient even in finite markets. Another approach permits nonlinear or personalized pricing rules, as explored by Wilson (1993), Chavas and Briec (2012), Azizan, Su, Dvijotham, and Wierman (2020), and others, but mechanisms using anonymous linear prices may be preferred for other reasons, including those related to communication, computation, familiarity, and perceived fairness.

Two recent papers propose alternative solutions to equilibrium nonexistence by specifying allocations that may not be envy-free. Feldman, Shabtai, and Wolfenfeld (2022) consider one-sided markets in which buyers allocated a good face one price, while those not allocated a good face a different price for the same good. The role of the two prices in their mechanism is to deter buyers from wanting to change their allocated bundle of goods. Goeree (2023) introduces an alternative equilibrium concept for nonconvex economies, "Yquilibrium," which involves computing an allocation and prices that minimize the difference between economic welfare and its dual. Unlike our markup equilibrium, the "Yquilibrium" can result in agents receiving allocations different from those demanded, creating an additional incentive for agents to misreport.

Our analysis of the incentive properties of markup mechanisms adapts results regarding the incentive properties of Walrasian mechanisms in large markets. Roberts and Postlewaite (1976) initiated the formal literature on this subject by studying a sequence of exchange economies with the number of agents tending to infinity, showing that continuity of the Walrasian price correspondence at the continuum limit implies that each agent's influence on the price tends to zero; that is, truthful reporting is almost optimal in large markets. Jackson (1992) showed in the same model that optimal reporting is almost truthful, that is, an agent's optimal report converges in the L^∞ norm to the agent's true demand. Watt (2025) studied the rate of convergence of price-taking incentives, showing that strong monotonicity of the demand and supply correspondences implies that the benefit to an agent of deviating from truthful reporting is approximately inversely proportional to the number of agents. While these results characterize ex post incentives, Azevedo and Budish (2019) studied interim incentives and showed that the Walrasian mechanism is *strategy-proof in the large*, implying that the benefit to any agent of misreporting against any full-support, independent and identically-distributed distribution of agent types (with finite support) tends to zero at a rate approximately inversely proportional to the square root of the number of agents in the economy.

Our study is motivated by several important applications of linear pricing mechanisms in markets with nonconvex production. We draw inspiration from the novel market design for fisheries rights in New South Wales, Australia, introduced by Bichler, Fux, and Goeree (2018, 2019), where the need to implement sustainable catches led to the exit of fishing boats and an associated loss of fixed costs.[5] Other markets with nonconvexities that have used linear prices include electricity generation (where firms have large start-up and ramping costs) and radio spectrum (where geographical complementarities can cause exposure problems).[6]

2 Model and preliminaries

2.1 Model

There is a set of *buyers* N and a set of *firms* or *sellers* F, both finite. Together, $A = N \cup F$ is the set of *agents*. There are L varieties of *goods* and a numeraire, money.

Each buyer $n \in N$ chooses a bundle $x_n = (x_n^1, \cdots, x_n^L)$ in X, a compact subset of $\mathbb{R}_+^L$ containing 0, called the *consumption possibility set*. Buyer n has quasilinear preferences[7] over bundles in X with *valuation function* $u_n : X \to \mathbb{R}$, so that the buyer's *utility* associated with receiving allocation x_n and paying t is $U_n(x_n, t) \coloneqq u_n(x_n) - t$. We assume that the valuation functions are bounded, upper semicontinuous, monotone, and satisfy $u_n(0) = 0$.

[5] We discuss the relationship between our markup mechanisms and a mechanism proposed by Bichler et al. (2018) in Appendix B.2.

[6] See Liberopoulos and Andrianesis (2016) for a summary of pricing mechanisms used in electricity markets with nonconvexities which often include "uplift" (or side-payments) alongside linear pricing and Ausubel and Milgrom (2002) for a discussion of complementarities in spectrum auctions.

[7] The quasilinearity assumption abstracts from income effects, as is standard in mechanism design. For discussion of the role of income effects see, for example, Luenberger (1992) and Morimoto and Serizawa (2015).

Each seller $f \in F$ chooses a production bundle $y_f = (y_f^1, \cdots, y_f^L)$ in the *production possibility set* Y, a compact subset of $\mathbb{R}_+^L$ containing 0. Seller f has *cost function*[8] $c_f : Y \to \mathbb{R}_+$, so that seller f's *profit* from producing $y_f \in Y$ and receiving payment t is $\pi_f(y_f, t) \coloneqq t - c_f(y_f)$. The cost functions are bounded, lower semicontinuous, monotone, and normalized so that $c_f(0) = 0$.

An *economy* $\mathscr{E}$ comprises buyers with their valuation functions and sellers with their cost functions, denoted $\mathscr{E} = \langle N, (u_n)_{n\in N}, F, (c_f)_{f\in F}\rangle$. When it is clear, we use the shorthand $\mathscr{E} = \langle N, F\rangle$.

Throughout, we assume that agent types—the valuation functions u_n of buyers and the cost functions c_f of sellers—are private information, not directly observable by the mechanism designer. The designer knows $|N|$ and $|F|$, as well as a space of possible valuation functions for the buyers, $\mathscr{U}$, and a space of cost functions for the sellers, $\mathscr{C}$, both assumed to be admissible in the sense of Aumann (1963).[9] Let the normalized counting measures μ on $\mathscr{U}$ and ν on $\mathscr{C}$ be defined by

$$\mu(u_n) \coloneqq \frac{\text{\# of buyers in } \mathscr{E} \text{ with valuation function } u_n}{|N|}$$
$$\chi(c_f) \coloneqq \frac{\text{\# of sellers in } \mathscr{E} \text{ with cost function } c_f}{|F|},$$

and let $\phi \coloneqq \frac{|F|}{|N|}$, so that $\langle N, \mu, \phi, \chi\rangle$ is an alternative specification of economy $\mathscr{E}$.

Allocations and efficiency An *allocation* $\mathbf{x} = ((x_n)_{n\in N}, (y_f)_{f\in F})$ is an assignment of consumption bundles $x_n \in X$ to each buyer $n \in N$ and production bundles $y_f \in Y$ to each seller $f \in F$. An allocation is *(resource-)feasible* if it satisfies $\sum_{n\in N} x_n \leq \sum_{f\in F} y_f$. Let $\mathbf{X}$ be the set of all resource-feasible allocations.

The *surplus* $\mathscr{S}(\mathbf{x})$ of allocation $\mathbf{x} \in \mathbf{X}$ is defined as

$$\mathscr{S}(\mathbf{x}) \coloneqq \sum_{n\in N} u_n(x_n) - \sum_{f\in F} c_f(y_f).$$

The *efficient allocation* problem is to solve $\max_{\mathbf{x}\in\mathbf{X}} \mathscr{S}(\mathbf{x})$, with a solution denoted $\mathbf{x}^* \in \arg\max_{\mathbf{x}\in\mathbf{X}} \mathscr{S}(\mathbf{x})$ and the resulting surplus $\mathscr{S}^* \coloneqq \mathscr{S}(\mathbf{x}^*)$. For any allocation $\mathbf{x} \in \mathbf{X}$, $\mathscr{S}(\mathbf{x}) - \mathscr{S}^*$ is the *deadweight loss* of $\mathbf{x}$, and the ratio $\frac{\mathscr{S}(\mathbf{x}) - \mathscr{S}^*}{\mathscr{S}^*}$ is the *percentage loss* at $\mathbf{x}$.[10]

[8] Sellers could be formulated as buyers with valuations $-c_f(y_f)$ and payments $-t$. However, to accommodate mechanisms that charge buyers and sellers different prices, we distinguish the two groups.

[9] That is, one can define a measure on $\mathscr{U}$, equipped with an appropriate σ-algebra. For example, admissible sets include the set of bounded, continuous functions on a compact subset of $\mathbb{R}^L$; the set of bounded functions with discontinuities of the first kind; or, more generally, any subset of a Baire class (Aumann, 1963).

[10] Later assumptions will rule out cases where $\mathscr{S}^* = 0$, ensuring this ratio is well-defined.

Pricing rules To prepare for our markup equilibrium, we allow two different price vectors $p^b, p^s \in \mathbb{R}_+^L$ for buyers and sellers, such that a buyer n pays $p^b \cdot x$ for bundle x, and a seller f receives $p^s \cdot y$ for supplying y.

Denote buyer n's *demand correspondence* by $D_n : \mathbb{R}_+^L \rightrightarrows X$, mapping a price vector p^b to the set of utility-maximizing bundles $D_n(p^b)$, which is well-defined because u_n is upper semicontinuous. The *indirect utility* function is $\widehat{u}_n(p^b) \coloneqq \max_{x \in X}\{u_n(x) - p^b \cdot x\}$. Similarly, denote seller f's *supply correspondence* by $S_f : \mathbb{R}_+^L \rightrightarrows Y$, mapping a price vector p^s to the set of profit-maximizing bundles $S_f(p^s)$, which is well-defined because c_f is lower semicontinuous. The *indirect profit* function is $\widehat{\pi}_f(p^s) \coloneqq \max_{y \in Y}\{p^s \cdot y - c_f(y)\}$.

2.2 Convex quasilinear economies

We study convexity as it pertains to the set of payoff-improving allocations for an agent in the economy. The $\bar{u}$-*upper contour set* of buyer $n \in N$ is

$$UC_n^{\bar{u}} \coloneqq \{(x,t) \in X \times \mathbb{R} : U_n(x,t) \geq \bar{u}\},$$

while the $\bar{\pi}$-*upper contour set* of seller $f \in F$ is

$$UC_f^{\bar{\pi}} \coloneqq \{(y,t) \in Y \times \mathbb{R} : \pi_f(y,t) \geq \bar{\pi}\}.$$

Buyer n has *convex preferences* if X is convex and the upper contour set $UC_n^{\bar{u}}$ is convex for all $\bar{u} \in \mathbb{R}$, which is equivalent to the quasiconcavity of U_n and the concavity of u_n. Seller f has *convex technology* if Y is convex and $UC_f^{\bar{\pi}}$ is convex for all $\bar{\pi} \in \mathbb{R}$, which is equivalent to the quasiconcavity of π_f and the convexity of c_f.

For quasilinear markets with convex preferences and technologies, the existence and welfare theorems of Arrow (1951), Debreu (1951), and Arrow and Debreu (1954) may be stated as follows.

Proposition 1 (Equilibrium existence and efficiency)**.** *Suppose in a quasilinear economy $\mathscr{E}$ that all buyers $n \in N$ have convex preferences and all sellers $f \in F$ have convex technologies. Then allocation $\mathbf{x} \in \mathbf{X}$ is efficient if and only if there exists $p \in \mathbb{R}_+^L, p \neq 0$ such that for all $n \in N$, $x_n \in D_n(p)$; for all $f \in F$, $y_f \in S_f(p)$; and $\sum_{n \in N} p \cdot x_n = \sum_{f \in F} p \cdot y_f$.*

Such a pair $(p, \mathbf{x})$ is a *competitive* or *Walrasian equilibrium.*

2.3 Measures of nonconvexity and approximate equilibria

The nonconvexity of a set S can be measured in several ways by comparing S to its convex hull, $\text{co}(S)$. We use the following measures of nonconvexity:

- the *inner radius* of S is $r(S) \coloneqq \sup_{x\in\text{co}(S)} \inf_{T\subseteq S: x\in\text{co}(T)} \text{rad}(T)$, and
- the *inner distance* of S is $\rho(S) \coloneqq \sup_{x\in\text{co}(S)} \inf_{y\in S} \|x-y\|$.

The two functions, illustrated in Figure 2, measure the size of the set of points in $\text{co}(S)$ that are not in S. For a convex set S, both measures are zero: $r(S) = 0 = \rho(S)$.

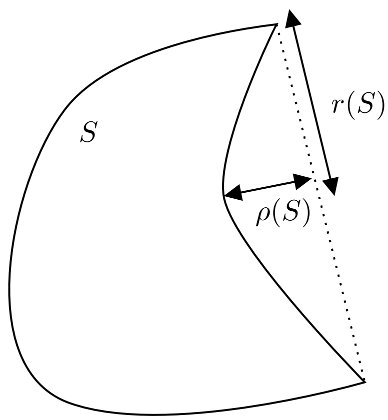

Figure 2: Measures of nonconvexity of a set

The nonconvexity of buyer n's preferences is measured by the largest inner radius or inner distance of their upper contour sets: $r_n \coloneqq \sup_{\bar{u}\in\mathbb{R}} r(UC_n^{\bar{u}})$ or $\rho_n \coloneqq \sup_{\bar{u}\in\mathbb{R}} \rho(UC_n^{\bar{u}})$. Similarly, the nonconvexity of seller f's technology is measured by $r_f \coloneqq \sup_{\bar{\pi}\in\mathbb{R}} r(UC_f^{\bar{\pi}})$ or $\rho_f \coloneqq \sup_{\bar{\pi}\in\mathbb{R}} \rho(UC_f^{\bar{\pi}})$. We let $r_{\mathscr{E}}$ and $\rho_{\mathscr{E}}$ denote the largest of these measures among all agents in economy $\mathscr{E}$.

When agents' upper contour sets are not convex, competitive equilibrium might not exist. Proposition 2, the Shapley-Folkman Lemma, helps identify allocations that are nearly competitive equilibria.[11]

Proposition 2 (Shapley-Folkman Lemma)**.** *Let $S_i \subseteq \mathbb{R}^L$ for $i = 1, ..., M$, and let $S = \bigoplus_{i=1}^M S_i$ be the Minkowski sum of those sets. Then any $x \in \text{co}(S)$ may be written as $x = \sum_{i=1}^N x_i$ where*

[11] While the "Shapley-Folkman Lemma" most accurately refers only to the result in the second sentence of Proposition 2, Starr (1969) first reported it as a result of private communication with Lloyd Shapley and Jon Folkman. Starr (1969) then proved the first part of the third sentence of Proposition 2, and Heller (1972) proved the second part. For simplicity, we refer to all of Proposition 2 as the Shapley-Folkman Lemma. Budish and Reny (2020) and Wu and Tang (2024) provide improved bounds for the Shapley-Folkman Lemma using a different measure of nonconvexity, which could also be used in our setting to improve the constant but not the asymptotic rate of convergence in some results.

$x_i \in \mathrm{co}(S_i)$ *and* $|i : x_i \in \mathrm{co}(S_i) \setminus S_i| \leq L' := \min(L, M)$*. Moreover, there exists* $y, y' \in S$ *such that* $\|x - y\| \leq (\max_i r(S_i))\sqrt{L'}$ *and* $\|x - y'\| \leq (\max_i \rho(S_i))L'$.

Proposition 2 has been used to establish results about *approximate equilibria*, which are constructed as follows. First, convexify the economy by replacing the upper contour sets of buyer preferences and seller technologies with their convex hulls. This is equivalent to replacing each buyer's valuation function u_n by its concave envelope $\mathrm{cav}(u_n)$ and each seller's cost function c_f by its convex envelope, $\mathrm{vex}(c_f)$.[12] The *convexified economy* is $\widehat{\mathscr{E}} := \langle N, (\mathrm{cav}(u_n))_{n\in N}, F, (\mathrm{vex}(c_f))_{f\in F}\rangle$.

By Proposition 1, the convexified economy has a competitive equilibrium that is efficient, according to the concavified valuation functions and convexified cost functions. Since the convexified economy's efficient allocation problem is a relaxation[13] of the same problem for the original economy, the efficient surplus of the convexified economy provides an upper bound on the efficient surplus of the original economy. The resulting price-allocation pair $(p, \mathbf{x})$ is a *pseudoequilibrium* of $\mathscr{E}$. Proposition 2 implies that one can choose $\mathbf{x}$ such that at most L' agents in $\mathscr{E}$ are not utility- or profit-maximizing at $\mathbf{x}$ given prices p and that one can find a nearby allocation $\mathbf{x}'$ in which markets may not exactly clear such that *all* agents are maximizing given prices p.[14] The price-allocation pair $(p, \mathbf{x}')$ is an *approximate equilibrium*.

Pseudoequilibria and approximate equilibria describe price-allocation pairs rather than mechanisms. These price-allocation pairs may be infeasible or impose large losses on some agents, potentially rendering them unsuitable for practical market design. We use these ideas to devise mechanisms that are computable, select feasible allocations, and possess other desirable properties akin to those of Walrasian mechanisms in convex economies.

3 Bound-Form First Welfare Theorem

When competitive equilibrium does not exist, no feasible allocation is supported by a single anonymous price vector that is the same for buyers and sellers. To restore feasibility with a single price vector,

[12] The concave envelope of a function is the pointwise smallest concave function everywhere above the function, while the convex envelope of a function is the pointwise largest convex function everywhere below the function.

[13] That is, the relaxed constraint set is weakly larger than the original, and the relaxed objective function is pointwise weakly larger than the original.

[14] To see this, note that if a buyer is assigned a bundle x_n in $\mathbf{x}$ that is not utility-maximizing at p, then x_n must be the convex combination of bundles (x_n') in X which are *exposed points* in u_n (i.e. where $\mathrm{cav}(u_n) = u_n$). Agents in the convexified economy must be indifferent between x_n and these bundles, because the concavified portions of buyers' utility functions consist of (patches of) hyperplanes, and if an agent is assigned a bundle on such a patch, then the price vector must be normal to that hyperplane.

some agent must be *rationed*, that is, assigned a bundle different from its most preferred one at the specified prices. Given an allocation and prices, we define buyers' and sellers' *rationing losses* as the excess of the payoff an agent would obtain from its most preferred bundle at the given prices over the payoff it receives in its assigned allocation.

Definition 3.1 (Rationing loss)**.** The *rationing loss* $\mathscr{R}_n(p,x)$ *of buyer* n at price p and allocation x is

$$\mathscr{R}_n(p,x) \coloneqq \hat{u}_n(p) - U_n(x, p\cdot x).$$

The *rationing loss* $\mathscr{R}_f(p,y)$ *of seller* f at price p and allocation y is

$$\mathscr{R}_f(p,y) \coloneqq \hat{\pi}_f(p) - \pi_f(y, p\cdot y).$$

The *rationing loss* of allocation $\mathbf{x} = ((x_n)_{n\in N}, (y_f)_{f\in F})$ at price p is defined by

$$\mathscr{R}(p,\mathbf{x}) \coloneqq \sum_{n\in N} \mathscr{R}_n(p,x_n) + \sum_{f\in F} \mathscr{R}_f(p,y_f).$$

If competitive equilibrium does not exist, any price-allocation pair must entail rationing, wasted supply (leading to a budget deficit), or both. Our first main result establishes that the magnitude of such rationing and budget losses fully characterizes the efficiency of the allocation.

Theorem 1 (Bound-Form First Welfare Theorem)**.** *Let* $p \in \mathbb{R}^L_+$ *be a price vector and* $\mathbf{x} \in \mathbf{X}$ *be any feasible allocation. Then, the deadweight loss of* $\mathbf{x}$ *is bounded by*

$$\underbrace{\mathscr{S}^* - \mathscr{S}(\mathbf{x})}_{\textit{deadweight loss}} \leq \underbrace{\mathscr{R}(p,\mathbf{x})}_{\textit{rationing loss}} + \underbrace{p\cdot\left(\sum_{f\in F} y_f - \sum_{n\in N} x_n\right)}_{\textit{budget deficit}}.$$

Proof. Fix any efficient allocation $\mathbf{x}^*$. By the definitions of the indirect utility and profit functions, for any prices p:

$$\hat{u}_n(p) \geq u_n(x_n^*) - p\cdot x_n^*, \text{ and}$$
$$\hat{\pi}_f(p) \geq p\cdot y_f^* - c_f(y_f^*).$$

Summing these inequalities, we obtain

$$\sum_{n\in N}\hat{u}_n(p)+\sum_{f\in F}\hat{\pi}_f(p)+p\cdot\left(\sum_{n\in N}x_n^*-\sum_{f\in F}y_f^*\right)\geq\sum_{n\in N}u_n(x_n^*)-\sum_{f\in F}c_f(y_f^*)=\mathscr{S}^*.$$

Since $\mathbf{x}^*$ is resource-feasible, the third term on the left side is nonpositive, which implies

$$\sum_{n\in N}\hat{u}_n(p)+\sum_{f\in F}\hat{\pi}_f(p)\geq\mathscr{S}^*$$

Subtracting $\mathscr{S}(\mathbf{x})$ and applying the definitions of rationing loss, we obtain

$$\mathscr{S}^*-\mathscr{S}(\mathbf{x})\leq\sum_{n\in N}\hat{u}_n(p)+\sum_{f\in F}\hat{\pi}_f(p)-\mathscr{S}(\mathbf{x})=\mathscr{R}(p,\mathbf{x})+p\cdot\left(\sum_{f\in F}y_f-\sum_{n\in N}x_n\right).$$

□

The Bound-Form First Welfare Theorem extends the First Welfare Theorem for quasilinear economies to any price-allocation pair $(p,\mathbf{x})$ rather than just Walrasian equilibria. If $(p,\mathbf{x})$ is a Walrasian equilibrium, then both the budget deficit and the rationing losses are zero, so Theorem 1 implies that the welfare loss is zero, or, equivalently, that any Walrasian equilibrium is efficient.

One interpretation of the First Welfare Theorem is that prices serve as a "certificate of optimality": if supporting prices exist for a given allocation, then it is efficient. As Scarf (1994) observed, without convexity, there is generally no such optimality test. However, Theorem 1 may be interpreted as an *approximate* optimality test: a price-allocation pair for which the sum of the rationing loss and the budget deficit is small has a correspondingly small welfare loss. Theorem 1 also suggests a link between incentives and efficiency: a pricing mechanism with little rationing and no budget deficit in which individual agents have limited influence over prices must have both good incentive properties and small deadweight losses. These observations are key to our extensions below.

4 The markup mechanism

4.1 Pricing mechanisms and approximate mechanism design

In this section, we study pricing mechanisms that map profiles of reports of sellers' cost functions $(c_f)_{f\in F}$ and buyers' value functions $(u_n)_{n\in N}$ to an allocation $\mathbf{x}\in\mathbf{X}$ and anonymous prices for buyers and sellers, $\boldsymbol{p} = (p^b, p^s)$.[15] We require that the mechanism specify outcomes that are both *resource-feasible* and *budget-feasible*, so that for all report profiles, $\sum_{f \in F} y_f \geq \sum_{n \in N} x_n$ and $p^s \cdot \sum_{f \in F} y_f \leq p^b \cdot \sum_{n \in N} x_n$.[16] How agents communicate their potentially complicated costs and values to the mechanism is beyond the scope of this paper; we instead assume that the reporting language is rich enough for buyers and sellers to report their true preferences.[17]

A pricing mechanism is *efficient* if the output allocation $\mathbf{x}$ is surplus-maximizing given the reported value and cost functions and ε-*efficient* if the deadweight loss of $\mathbf{x}$ is bounded by ε given the reports.[18] A pricing mechanism is (ex post) *individually rational* if, given reported value and cost functions, the allocation and prices determined by the mechanism deliver each agent a payoff no worse than non-participation (here, 0).

A *Walrasian mechanism* inputs reports of value and cost functions and outputs a Walrasian equilibrium price and allocation. If there are multiple Walrasian equilibria, the mechanism employs a predetermined selection rule (a similar assumption applies to the other mechanisms discussed in this paper whenever multiplicity arises). Walrasian mechanisms are efficient, individually rational, and exhibit good large-market incentive properties, but their application is limited to environments where Walrasian equilibria are guaranteed to exist. We explore extensions for settings where Walrasian equilibria may not exist.

We seek mechanisms that perform well in large markets, with approximations to efficiency that depend on the number of agents in the market. Let $\mathscr{E}_t = \langle N_t, \mu_t, \phi_t, \chi_t \rangle$ be a sequence of economies indexed by $t = 1, 2, ...$, and let $R_{\mathscr{E}_t} := \min\{r_{\mathscr{E}_t}\sqrt{L}, \rho_{\mathscr{E}_t} L\}$. The following additional assumptions apply as the economy grows large.

Assumption 1 (Existence of limit economy)**.** *As* $t \to \infty$*,* $|N_t| \to \infty$ *and* $\phi_t \to \phi \in (0, 1)$*.*

Assumption 2 (Individual nonconvexities are bounded)**.** *There exists* $\overline{R} > 0$ *such that* $R_{\mathscr{E}_t} < \overline{R}$ *for all* t*.*

[15] We do not consider randomized mechanisms, both because these are unnecessary to achieve our objectives and because they raise daunting practical issues, including stringent trust requirements in the mechanism designer and the possible failure of ex post individual rationality for ex ante individually rational lotteries.

[16] Our definition of resource-feasibility embeds a free disposal assumption, which ensures that the feasible set is nonempty. While free disposal may be a good approximation for many markets, in others (e.g., wholesale electricity production), disposal may be costly, and these costs should enter the mechanism designer's objective. However, because overproduction is bounded in the simple markup mechanism introduced in Section 4.2 below, our asymptotic efficiency and incentive compatibility results extend to a setting with (bounded) costs of disposal.

[17] Reporting languages for complex preferences have been studied by Milgrom (2009), Klemperer (2010), Bichler, Goeree, Mayer, and Shabalin (2014), Bichler, Milgrom, and Schwarz (2022) and others.

[18] We allow ε to depend on properties of $\mathscr{E}$ (and implicitly $(\mathscr{E}_t)_{t \in \mathbb{N}}$ if $\mathscr{E}$ is part of a sequence), so that, for example, $O(1/|N_t|)-$efficiency refers to a deadweight loss of $\mathbf{x}$ that is $O(1/|N_t|)$.

Assumption 3 (Growing gains from trade). *As $t \to \infty$, the efficient surplus $\mathcal{S}_t^*$ grows at least as quickly as $|N_t|$ asymptotically, or in Knuth's (1976) asymptotic notation, $\mathcal{S}_t^* = \Omega(|N_t|)$.*[19]

Assumption 4 (Prices are bounded). *There exists choke prices $\underline{p}, \overline{p} > 0$ such that for each $l = 1, \cdots, L$ and for all t, we have: (a) $D_n^l(p) = 0$ for any $n \in N_t$ and $p \in \mathbb{R}_+^L$ with $p^l > \overline{p}$, and (b) $S_f^l(p) = 0$ for any $f \in F_t$ and $p \in \mathbb{R}_+^L$ with $p^l < \underline{p}$.*

Assumption 1 asserts that for large t, the important variation among economies is their scale: the proportions of various types converge to a limit. Assumption 2 asserts that there is a uniform bound on the measure of nonconvexity across the sequence of markets, limiting the impact of any single firm's or consumer's nonconvexity in a large economy.[20] Assumption 3 is the condition that the efficient surplus per participant is bounded away from zero. Assumption 4 implies that there is a compact set of prices that can support nonzero feasible allocations.

4.2 Markup mechanisms

We now introduce *markup equilibria*, which are designed to maintain a no-rationing property similar to Walrasian equilibrium. Without rationing, finding prices such that buyers demand all supply may be impossible. To pay for firms' excess supply, a markup is applied to the prices buyers pay.

Definition 4.1 (Markup equilibrium). A *markup equilibrium* is a triple $(\alpha, p, \mathbf{x})$ consisting of a markup parameter $\alpha \geq 0$, a price $p \in \mathbb{R}_+^L$, and a resource-feasible allocation $\mathbf{x} \in \mathbf{X}$ such that:

(a) sellers' payments are determined by price vector p, and sellers are not rationed at these prices, so $y_f \in S_f(p)$;

(b) buyers' payments are determined by price vector $(1+\alpha)p$, and buyers are not rationed at these prices, so $x_n \in D_n((1+\alpha)p)$; and

(c) budgets are at least weakly balanced, so $\sum_{n \in N}(1+\alpha)p \cdot x_n - \sum_{f \in F} p \cdot y_f \geq 0$.

A *markup mechanism* is a mechanism that inputs reports of cost and value functions and outputs a markup equilibrium. With finite choke prices on supply and demand, markup equilibria with no trade and $\alpha = \infty$ always exist. However, we focus on markup mechanisms that select α close to zero and result in few unallocated goods, because their allocations are nearly efficient. This follows from Theorem 1 applied to price-allocation pair $(p, \mathbf{x})$: if few goods are unallocated, the budget deficit at

[19] Recall that $f(x) = \Omega(g(x))$ if $\liminf_{x \to \infty} |f(x)|/g(x) > 0$.

[20] It suffices to assume that the buyer's consumption possibility set X and the seller's production possibility set Y does not grow with t, since $R \leq \mathrm{rad}(X)$ and $R \leq \mathrm{rad}(Y)$.

price p is small, while if prices p and $(1+\alpha)p$ are close, the rationing losses for each buyer at price p are small. This latter claim follows by an envelope theorem argument, formalized in Proposition 3.

Proposition 3 (Rationing bound)**.** *Let $x \in D_n(p)$, and consider another price $p' \neq p$. Then the rationing loss of buyer n at allocation x given price p', $\mathscr{R}_n(x, p')$, is $O(\|p - p'\|)$.*[21]

If computational challenges were not a concern, a market designer might seek to identify a markup mechanism with the smallest loss, which we call a *minimal markup mechanism*.[22] The pair (p, x) is a Walrasian equilibrium if and only if the triple $(x, p, 0)$ is a markup equilibrium; in that case, this markup equilibrium is also the minimal markup equilibrium. In nonconvex economies, however, computing the minimal markup mechanism can be challenging. We now show that an $O(1)$-efficient markup mechanism—that is, one for which the percentage loss in welfare is at most inversely proportional to the number of agents in the economy—can be identified using only *convex* optimization problems and a one-dimensional binary search. Before providing a technical description of this *simple markup mechanism*, we outline the steps of our approach intuitively, echoing the example in the introduction.

For a fixed α, we select $(p, \mathbf{x})$ to be the equilibrium price-and-allocation pair of a related economy with three changes from the actual economy: (1) every buyer's value function is replaced by the smaller function $u_n/(1+\alpha)$, (2) all values and costs are then replaced by their concave or convex hulls, respectively,[23] and (3) we add an *operational reserve* for each good, which is a quantity demanded by the auctioneer in the amount of $R := \min\{r_{\mathscr{G}}\sqrt{L}, \rho_{\mathscr{G}} L\}$. Step (1) in this construction yields prices and allocations where buyers would demand the same allocations given their actual value functions u_n and marked-up prices $(1+\alpha)p$. We apply the Shapley-Folkman Lemma (Proposition 2) to round $\mathbf{x}$ to one of the demanded allocations for each agent while changing the net demand for each good by at most R units. This is balanced by an offsetting change to the reserve allocated to the auctioneer in step (3) to ensure feasibility.

Excluding the auctioneer's demand, the final allocation is resource-feasible, with excess supply no more than $2R$ units of each good. Excess supply can lead to efficiency loss, but the quantity allocated

[21] Recall Knuth's (1976) big O notation: $f(x) = O(g(x))$ if $\limsup_{x\to\infty} |f(x)|/g(x) < \infty$.

[22] Computing the minimal markup equilibrium is at least as hard as the Walrasian equilibrium computation problem (since the two coincide whenever the Walrasian equilibrium exist), which is known, in general, to be hard (Daskalakis, Goldberg, and Papadimitriou, 2009). We refer the reader to Lehmann, Müller, and Sandholm (2006) for a practical discussion of the difficulty of the winner determination problem in combinatorial auctions (which requires calculating an efficient allocation), a type of nonconvex market.

[23] This need not be computationally expensive. For example, if the value and cost functions are reported to the mechanism using a mixed integer program, the mechanism may simply convert integer variables to real variables to obtain the convex hulls in the form of linear or quadratic programs.

to the auctioneer is bounded by a constant independent of market size.[24] Because the price vector and the excess supply of goods are bounded, the budget surplus at price p is also bounded. As trade increases with market size, the markup, α, required to guarantee budget balance is inversely proportional to market size. The total welfare loss is thus bounded by a constant stemming from excess production, plus the sum of rationing loss terms, each inversely proportional to market size.

Definition 4.2 (Simple markup mechanism)**.** The *simple markup mechanism* is the markup mechanism with parameters $(\alpha^*, p^*, \mathbf{x}^*)$ determined as follows. If all reported values are concave and all reported costs are convex, set $\alpha^* = 0$ and choose $(p^*, \mathbf{x}^*)$ to be some Walrasian equilibrium. Otherwise, for each $\alpha > 0$, consider the following convex program:

$$\min_{p \in \mathbb{R}^L_+} \max_{x_n \in \mathrm{co}(X), y_f \in \mathrm{co}(Y)} \sum_{n \in N} \frac{\mathrm{cav}(u_n)(x_n)}{1+\alpha} - \sum_{f \in F} \mathrm{vex}(c_f)(y_f) - p \cdot \left(\sum_{n \in N} x_n + R 1_L - \sum_{f \in F} y_f \right),$$

where 1_L is the vector of ones in $\mathbb{R}^L$. Let $(p^\alpha, \tilde{\mathbf{x}}^\alpha)$ denote any solution to this program.

From $\tilde{\mathbf{x}}^\alpha \in \mathrm{co}(\mathbf{X})$, obtain, via Proposition 2, an allocation $\mathbf{x}^\alpha \in \mathbf{X}$ with $\|\mathbf{x}^\alpha - \tilde{\mathbf{x}}^\alpha\| \leq R$ such that $x_n^\alpha \in D_n((1+\alpha)p)$ for each $n \in N$ and $y_f^\alpha \in S_f(p)$ for each $f \in F$. Let

$$\alpha^* := \min \left\{ \alpha \,\middle|\, \sum_{n \in N} (1+\alpha) p^\alpha \cdot x_n^\alpha - \sum_{f \in F} p^\alpha \cdot y_f^\alpha \geq 0 \right\}, \tag{A}$$

and define $p^* = p^{\alpha^*}$ and $\mathbf{x}^* = \mathbf{x}^{\alpha^*}$.

Theorem 2 formalizes the preceding informal argument, showing that the simple markup mechanism is well-defined and $O(1/|N_t|)$-efficient.

Theorem 2 (Approximate efficiency of simple markup mechanism)**.** *Let $\mathscr{E}_t$ be a sequence of economies satisfying Assumptions 1–4. Then:*

(a) the simple markup mechanism is well-defined (that is, the minimum in (A) *is attained),*

(b) the simple markup mechanism's markup α^* is $O(1/|N_t|)$, and

[24] The choice of R units of each good as a set-aside for the auctioneer in step (3) is a theoretical guarantee. It may be possible to allocate fewer units to the auctioneer in step (3) to arrive at a more efficient feasible allocation using the same approach. Alternatively, one could check for feasible allocations with zero units set aside (corresponding to competitive equilibria) and then intelligently increase the set-aside until a budget-feasible markup mechanism is identified. We leave the details of such a mechanism for future research.

(c) *the deadweight loss of the simple markup mechanism's allocation is $O(1)$, implying a percentage loss of $O\left(1/|N_t|\right)$.*

Although the rates of convergence in Theorem 2 are stated in terms of $|N_t|$, by Assumption 1, the same asymptotic rate of convergence holds in $|F_t|$ or $|A_t|$.

Several other key properties of the simple markup mechanism are evident from its construction. First, the equilibrium is resource-feasible and budget-feasible. Second, each agent's allocation and payments are individually rational. For sellers, this is because they receive a bundle in their supply set at price p^α. For buyers, the pseudoequilibrium price p^α and consumption allocation in $\tilde{\mathbf{x}}^\alpha$ satisfy

$$\frac{1}{1+\alpha}u_n(\tilde{x}_n^\alpha) - p^\alpha \cdot \tilde{x}_n^\alpha = \frac{1}{1+\alpha}u_n(x_n^\alpha) - p^\alpha \cdot x_n^\alpha \geq 0,$$

thus buyer n's payoff, $u_n(x_n^\alpha) - (1+\alpha)p^\alpha \cdot x_n^\alpha$, is also nonnegative.

4.3 Incentives

To analyze the incentive properties of the markup mechanisms, we study an *independent private values (IPV) model* in which $(\mathscr{E}_t)_{t\in\mathbb{N}} = (\langle N_t, \mu_t, \phi_t, \chi_t\rangle)_{t\in\mathbb{N}}$ is a sequence of economies with buyer valuations and seller costs drawn i.i.d. from full-support probability distributions μ and χ defined on type spaces $\mathscr{U}$ and $\mathscr{C}$ respectively, satisfying Assumption 1 almost surely. Buyers and sellers know $\mu, \chi, |N_t|$, and $|F_t|$ and observe their realized types, but not those of other agents. Let D_μ and S_χ denote the expected demand and supply correspondences, respectively,[25] and let $\mathscr{P} \subset \mathbb{R}_+^L$ be the set of price vectors for which $D_\mu(p) > 0$ and $S_\chi(p) > 0$. We assume that $\mathscr{P}$ is compact.

We study two types of incentives in this model: interim and ex post.

Approximate interim incentive compatibility A mechanism is *interim incentive-compatible (IIC)* if truthful reporting maximizes each agent's expected payoffs under the mechanism. A mechanism is $\varepsilon-$*IIC* if, under truthful reporting, each agent's expected payoff from *any* report is no more than ε greater than the expected payoff of the truthful report.

As markup mechanisms do not ration producers or consumers at their respective prices, all markup mechanisms are envy-free. As a result, Theorem 1 of Azevedo and Budish (2019) implies that

25 Formally, given distribution μ on $\mathscr{U}$, the expected indirect utility function is defined pointwise for $p \in \mathbb{R}_+^L$ by $\mathbb{E}_\mu[\hat{u}(p)] = \int_{\mathscr{U}} \hat{u}_n(p)\,\mathrm{d}\nu(u_n)$, and the expected demand correspondence is then $D_\mu(p) = -\partial\mathbb{E}_\mu[\hat{u}(p)]$. The expected supply correspondence is defined analogously.

every markup mechanism is strategy-proof in the large, leading to the following asymptotic interim incentive compatibility result in the IPV model with finite type spaces.

Theorem 3 (Approximate IIC)**.** *In an IPV economy with finite type spaces $\mathscr{U}$ and $\mathscr{C}$, any markup mechanism is $O(1/|N_t|^{\frac{1}{2}-\varepsilon})$–IIC for any $\varepsilon > 0$.*

Approximate ex post incentive compatibility A mechanism is *ex post incentive-compatible (EPIC)* if truthful reporting is an ex post Nash equilibrium of the reporting game induced by the mechanism. A mechanism is *ε-EPIC* if truthful reporting is a ε–ex post Nash equilibrium.

To establish ex post incentive results for markup mechanisms, we exploit the fact that prices determine their allocations. Intuitively, in markup mechanisms—as in Walrasian mechanisms—buyers and sellers receive their most preferred bundles at the prices they face; consequently, a single agent profits from a false report only to the extent that it can influence its price vector. As Watt (2025) shows for Walrasian mechanisms, an agent's ability to influence prices with *any* report is approximately inversely proportional to market size when the expected demand and supply correspondences are *strongly monotone*, as defined below.

Definition 4.3 (Strong monotonicity)**.** The expected demand correspondence D_μ is *strongly monotone* if there exists some $m > 0$ such that for all $p, p' \in \mathscr{P}$, $d \in D_\mu(p)$, and $d' \in D_\mu(p')$,

$$(d - d') \cdot (p' - p) \geq m\|p - p'\|^2.$$

Similarly, the expected supply correspondence S_χ is *strongly monotone* if there exists some $m > 0$ such that for all $p, p' \in \mathscr{P}$, $s \in S_\chi(p)$, and $s' \in S_\chi(p')$,

$$(s - s') \cdot (p - p') \geq m\|p - p'\|^2.$$

Under the assumption of strong monotonicity, the expected demand and supply correspondences are responsive to small price changes, and we have the following ex post incentive properties of the simple and minimal markup mechanisms.

Theorem 4 (Approximate EPIC)**.** *In an IPV economy with strongly monotone expected demand and supply correspondences satisfying Assumption 3 almost surely, with probability $1 - O(1/|N_t|)$ over draws of $\mathscr{E}_t$, the simple and minimal markup mechanisms are $O(1/|N_t|^{1-\varepsilon})$–EPIC for any $\varepsilon > 0$.*

Our proof exploits the fact that prices in markup mechanisms are Walrasian equilibrium prices in a related convex economy, so the limited ability of agents to manipulate Walrasian prices in large markets—established by Watt (2025) using the strong monotonicity assumption—implies a similar limitation in markup mechanisms. The only additional complication is an agent's ability to affect markups, but these are always $O(1/|N_t|)$ for the simple and minimal markup mechanisms under Assumption 3. A corollary is an improved bound for interim incentives under the strong monotonicity assumption, namely that the two mechanisms are $O(1/|N_t|^{1-\varepsilon})-$IIC for any $\varepsilon > 0$.

4.4 Computational properties

While equilibrium computation is hard in general,[26] computing Walrasian equilibria in concave quasilinear economies reduces to solving a convex optimization problem and its dual. A wide class of such optimization problems can be efficiently solved, including problems with self-concordant or strongly convex objectives.[27]

In contrast, finding efficient allocations in many nonconvex economies is computationally complex, even with quasilinear preferences. For example, identifying an optimal allocation in the fisheries market of Bichler et al. (2018) involved solving a large integer programming problem, which was NP-hard, implying that no efficient optimization algorithm is known for all problem instances, although heuristics and approximations are sometimes useful.

Our approach relies on approximation. Conditional on α, the simple markup mechanism requires only solving a convex optimization problem. Identifying the optimal markup α^* in the simple markup mechanism is more challenging; however, a binary search for α could be used in practice to identify a small markup ensuring weak budget balance. Ahunbay et al. (2024) provide further details on how to adapt our markup mechanism for practical computations, focusing on an application to European wholesale spot electricity markets.

5 Conclusion

In some regulated markets with multiple closely interrelated products, market operators use Walrasian-like mechanisms despite nonconvexities in production or consumption. For example, in wholesale

[26] See, for example, Chen, Dai, Du, and Teng (2009) and Daskalakis et al. (2009).

[27] For example, Walrasian prices in economies with strongly monotone supply and demand, as introduced in Watt (2025), may be efficiently computed via tâtonnement.

electricity markets, producers often incur fixed costs to start their plants and ramping costs to adjust production, while firms buying large quantities of electricity may also start and stop production based on energy prices. Similarly, in markets for fishing catch rights, fishers incur fixed costs to staff a boat and send it to sea. Given the nonconvexities in these markets, Walrasian equilibria may not exist, so the Walrasian mechanism cannot be implemented without modification.

This paper combines two traditional perspectives to define an equilibrium and a mechanism for nonconvex markets. On the one hand, it draws on results from classic general equilibrium theory regarding approximate Walrasian equilibrium. On the other hand, it draws on concepts and traditions from mechanism and market design, aiming to ensure physical and financial feasibility as well as envy-freeness, approximate incentive feasibility, and individual rationality. Older notions like pseudo-equilibrium or quasi-equilibrium may specify outcomes that fail to be resource- or budget-feasible, impose losses on individual participants, incentivize misreporting, or lack envy-freeness. While these notions satisfy approximate market clearing in the aggregate, individual participants may find their rationing or underpayment highly significant. In contrast, our mechanism design approach to extending Walrasian equilibrium insists on prices and envy-free allocations satisfying exact resource feasibility, with no need for third-party subsidies, and satisfying participation and incentive constraints—at least approximately. In that way, our markup equilibrium and its corresponding mechanism have no close antecedent in either tradition.

Markup equilibria always exist and retain many attractive features of Walrasian equilibrium. Because they use linear prices for producers and consumers, markup equilibria economize on communication and computation and provide a robust and transparent pricing system that many market participants will find familiar and fair. When markups are small and few goods remain unsold, the resulting allocations are nearly efficient. Such equilibria can be computed in practice, with welfare losses bounded by a constant proportional to the largest relevant nonconvexity. Incentives in the markup mechanism resemble those of the Walrasian mechanism: participants can benefit from misreporting only if they can influence prices—an ability that diminishes in large markets.

When the economist's task is to reform a market, one concern may be the disruption to existing market participants. If a Walrasian-like mechanism is already employed in a nonconvex, multi-product marketplace, then implementing another Walrasian-like mechanism may limit disruption. In contrast, switching to a Vickrey-Clarke-Groves pivot mechanism would be significantly more disruptive, requiring participants to adapt their processes and affecting the values of their past capital investments. The pivot mechanism may also present other significant drawbacks: it does not guarantee that revenues weakly exceed costs, can pay more to firms that produce less output, and can require impractical levels of communication and computation.[28]

The markup mechanism exhibits inefficiency almost exclusively due to overproduction. This raises the question of whether another mechanism without excess production could perform better. Instead of setting different prices for the two sides of the market, an alternative mechanism would set a single price vector and ration, forcing some participants to accept suboptimal bundles at those prices.[29] Our Bound-Form First Welfare Theorem provides a tool to evaluate the welfare losses of such a mechanism, although assessing incentives under that alternative would require additional analysis.

We use a market design approach to address concerns related to communication, computation, and fairness. This approach adopts a different perspective on "hard" versus "soft" constraints than many older theories. Other extensions of Walrasian equilibrium treat resource constraints and budget constraints as soft, to be satisfied only approximately; in contrast, our approach requires a mechanism that satisfies these constraints exactly. While many mechanism design analyses require participants to be unable to gain from misreporting, our market design approach imposes a softer constraint, leading to mechanisms in which the potential gains from misreporting are vanishingly small, but not necessarily zero, in large markets. For some important applications, including the two discussed previously, the markup mechanism, by satisfying constraints in this way, offers an appealing approach for addressing the practical challenges of market design.

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[28] See Ausubel and Milgrom (2006) for a discussion of the fairness, participation, and revenue concerns associated with VCG pivot mechanisms, and Leyton-Brown, Milgrom, and Segal (2017) for a discussion of the computational challenges associated with VCG pivot mechanisms.

[29] For some indication of the possible benefits of this approach, consider the five-firm example in Figure 1, and suppose that firm 3 had an average cost between a_2 and $(1+\alpha)p$ and a small capacity (namely $K_3 < D(a_3) - K_1$). In that case, setting a price at which firm 3 produces while rationing firm 2 would be more efficient than the markup equilibrium we identified.

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A Proofs omitted from the main text

A.1 Proof of Proposition 3

Proof. We present two proofs of this claim: the first derives directly from the definitions, and the second demonstrates its relationship to the envelope theorem.

For the first proof, let $x' \in D_n(p')$. Then,

$$\mathscr{R}_n(p', x) = u_n(x') - p' \cdot x' - (u_n(x) - p' \cdot x).$$

Since $x \in D_n(p)$, $u(x') - p \cdot x' \leq u(x) - p \cdot x$. Thus,

$$\mathscr{R}_n(p', x) \leq p \cdot x' - p \cdot x + p' \cdot x - p' \cdot x = (p - p') \cdot (x' - x),$$

which is $O(\|p - p'\|)$ since $x', x \in X$, a compact set.

For the second proof, write

$$\begin{aligned}\mathscr{R}_n(p', x) &= \hat{u}_n(p') - (u_n(x) - p' \cdot x) \\ &= \hat{u}_n(p') - (u_n(x) - p \cdot x) - p \cdot x + p' \cdot x \\ &= \hat{u}_n(p') - \hat{u}_n(p) + (p' - p) \cdot x.\end{aligned}$$

Let $p(t) = (1-t)p + tp'$ for $t \in [0,1]$. Applying the Milgrom and Segal (2002) envelope theorem to the parametrized utility maximization problem

$$\hat{u}_n(p(t)) = \max_{x \in X}\{u_n(x) - p(t) \cdot x\},$$

gives

$$\hat{u}_n(p') = \hat{u}_n(p) - \int_0^1 (p' - p) \cdot d(t)\, \mathrm{d}t.$$

where $d(t)$ is any selection from the demand correspondence $D_n(p(t))$. Substituting this result into the expression for $\mathscr{R}_n(p', x)$ yields

$$\mathscr{R}_n(p', x) = -\int_0^1 (p' - p) \cdot d(t)\, \mathrm{d}t + (p' - p) \cdot x = \int_0^1 (p' - p) \cdot (x - d(t))\, \mathrm{d}t,$$

which is bounded by $(p' - p) \cdot (x - x')$, since $(p' - p) \cdot (x - d(t))$ increases in t by the law of demand. $\square$

A.2 Proof of Theorem 2

Proof. **Part (a)** Fix some $\mathscr{E}$ and consider any sequence $\alpha_i \to \alpha$ and selections Rev_i of revenues $\sum_{n \in N}(1 + \alpha_i)p^{\alpha_i} \cdot x_n^{\alpha_i} - \sum_{f \in F} p^{\alpha_i} \cdot y_f^{\alpha_i}$ associated with some markup mechanisms $(\alpha_i, p^{\alpha_i}, \mathbf{x}^{\alpha_i})$ constructed as in Definition 4.2. We show that $\lim_i \mathrm{Rev}_i$ is the revenue of a markup mechanism $(\alpha, p^\alpha, \mathbf{x}^\alpha)$, implying that the infimum in equation (A) is attained (and thus the minimum exists).

By the saddle point condition associated with the objective in Definition 4.2, there exist $\tilde{\mathbf{x}}^{\alpha_i}$ that maximize over $\mathrm{co}(\mathbf{X})$ the objective $\sum_{n \in N} \frac{\mathrm{cav}(u_n)(x_n)}{1+\alpha_i} - \sum_{f \in F} \mathrm{vex}(c_f)(y_f)$. As $\alpha_i \to \alpha$, this objective hypo-converges[30] (since it is continuous and bounded) to $\sum_{n \in N} \frac{\mathrm{cav}(u_n)(x_n)}{1+\alpha} - \sum_{f \in F} \mathrm{vex}(c_f)(y_f)$, so that $\tilde{\mathbf{x}}^{\alpha_i} \to \tilde{\mathbf{x}}^{\alpha}$ for some $\tilde{\mathbf{x}}^{\alpha}$ that maximizes this limiting objective. By optimality, each p^{α_i} lies in the superdifferential ∂^* of the concavified valuation functions for each buyer and in the subdifferential of the convexified cost functions of each seller at $\tilde{\mathbf{x}}^{\alpha_i}$. For these concave (and convex) functions, the superdifferential (and subdifferential) correspondences are upper hemicontinuous, so the sequence p^{α_i} converges to some p^α in the super- and subdifferentials at $\tilde{\mathbf{x}}^{\alpha}$. Finally, since the demand and supply correspondences are upper hemicontinuous, the convergence of prices implies that $\mathbf{x}^{\alpha_i}$ approaches some $\mathbf{x}^\alpha$ such that $x_n^\alpha \in D_n((1+\alpha)p^\alpha)$ and $y_f^\alpha \in S_f(p^\alpha)$. Thus, the limit of Rev_i is attained as the revenue of some markup mechanism $(\alpha, p^\alpha, \mathbf{x}^\alpha)$.

30 See Rockafellar and Wets (2009), Section 7.B.

Part (b) For notational simplicity, we drop the index t in the prices, markups and allocations.

The construction in Definition 4.2 ensures, via Proposition 2, that $\sum_{f\in F} y_f^\alpha - \sum_{n\in N} x_n^\alpha \leq (2R)1_L$. It suffices to show that for sufficiently large $|N_t|$, an α exists such that

$$\alpha \sum_{n\in N} p^\alpha \cdot x_n^\alpha \geq (2R)p^\alpha \cdot 1_L.$$

Moreover, if this α is $O\left(\frac{1}{|N_t|}\right)$, then since $\alpha^* < \alpha$, (b) will follow. This result follows from showing that for fixed $\alpha > 0$ sufficiently to zero, $\sum_{n\in N} p^\alpha \cdot x_n^\alpha$ is $\Omega(|N_t|)$, while $p^\alpha \cdot 1_L$ is $O(1)$.

Let $\mathscr{S}^\alpha$ be the value of the saddle point problem

$$\min_{p\in\mathbb{R}^L_+} \max_{\mathbf{x}\in\mathbf{X}} \sum_{n\in N} \frac{\text{cav}(u_n)(x_n)}{1+\alpha} - \sum_{f\in F} \text{vex}(c_f)(y_f) - p\cdot\left(\sum_{n\in N} x_n + R1_L - \sum_{f\in F} y_F\right).$$

First, we show that $\mathscr{S}^\alpha$ is $\Theta(|N_t|)$ for sufficiently small α.[31] To see this, write $f(|N_t|) = \sum_{n\in N_t} u_n(x_n^*)$ and $g(|N_t|) = \sum_{f\in F_t} c_f(y_f^*)$. Assumptions 1, 3 and the boundedness of utilities and costs implies that the efficient surplus is $\Theta(|N_t|)$. As a result, $\liminf_{N\to\infty} \frac{f(N)}{N} := u > 0$, and $\limsup_{N\to\infty} \frac{g(N)}{N} := c > 0$, with $u - c > 0$. Then $\mathscr{S}^\alpha \geq \liminf_{N\to\infty} \frac{f(N)}{(1+\alpha)N} - \frac{g(N)}{N} = \frac{u}{1+\alpha} - c$, which is positive for sufficiently small α.

We now show that this implies $\sum_{n\in N} p^\alpha \cdot x_n^\alpha$ is $\Omega(|N_t|)$ for small, fixed α. Since $\sum_{f\in F_t} c_f(y_f^\alpha)$ is $\Omega(|N_t|)$, individual rationality of the sellers (in the convexified economy) implies that $\sum_{f\in F_t} p^\alpha \cdot y_f^\alpha$ is $\Omega(|N_t|)$. By complementary slackness, $\sum_{n\in N_t} p^\alpha \cdot x_n^\alpha = \sum_{f\in F_t} p^\alpha \cdot y_f^\alpha - Rp^\alpha \cdot 1_L$. Since $p \in [\underline{p}, \overline{p}]^L$ by Assumption 4, we must have that $\sum_{n\in N} p^\alpha \cdot x_n^\alpha$ is $\Omega(|N_t|)$.

Since for α near zero, $\sum_{n\in N} p^\alpha \cdot x_n^\alpha$ is $\Omega(|N_t|)$ and $(2R)p^\alpha \cdot 1_L$ is $O(1)$ (where R is $O(1)$ by Assumption 2), for sufficiently large $|N_t|$, there exists an α (and thus some least α by (a)) such that

$$\alpha \sum_{n\in N} p^\alpha \cdot x_n^\alpha \geq (2R)p^\alpha \cdot 1_L,$$

and furthermore, this α is $O\left(\frac{1}{|N_t|}\right)$. Since $\alpha^* < \alpha$, we have that α^* is $O\left(\frac{1}{|N_t|}\right)$, as required.

Part (c) We apply the Bound-Form First Welfare Theorem to show that the allocation $\mathbf{x}^{\alpha^*}$ is approximately efficient. Suppose $\mathbf{x}^{\alpha^*}$ was implemented with a single price vector p^{α^*} and (therefore)

[31] Recall that $f(x) = \Theta(g(x))$ if $f(x) = O(g(x))$ and $f(x) = \Omega(g(x))$

a budget deficit. Theorem 1 implies that

$$\mathscr{S}(\mathbf{x}^*) - \mathscr{S}(\mathbf{x}^{\alpha^*}) \leq \mathscr{R}(p^{\alpha^*}, \mathbf{x}^{\alpha^*}) + p^{\alpha^*} \cdot \left(\sum_{f \in F_t} y_f^{\alpha^*} - \sum_{n \in N_t} x_n^{\alpha^*} \right). \tag{1}$$

By construction, in $\mathbf{x}^{\alpha^*}$ at prices p^{α^*}, no sellers are rationed, while at prices $(1+\alpha^*)p^{\alpha^*}$, no buyers are rationed. However, buyers are rationed at price p^{α^*}. Proposition 3 implies $\mathscr{R}_n(p^{\alpha^*}, x_n^{\alpha^*})$ is $O\left(\frac{1}{|N_t|}\right)$ because α^* is $O\left(\frac{1}{|N_t|}\right)$ and $\|p^{\alpha^*}\|$ is bounded (by Assumption 4). Thus, $\mathscr{R}(p^{\alpha^*}, \mathbf{x}^{\alpha^*}) = \sum_{n \in N_t} \mathscr{R}_n(p^{\alpha^*}, x_n^{\alpha^*})$ is $O(1)$. Finally, the budget deficit is $O(1)$ since the excess supply is bounded by construction and each component of p^{α^*} is $O(1)$. Thus, the deadweight loss is $O(1)$.

□

A.3 Proof of Theorem 4

Proof. First, we establish that the simple and minimal markup mechanisms almost surely output an $O(1/N)$ markup with $O(1)$ excess production, both under truthful reporting and after misreporting by a single agent. The remainder of this proof applies to any markup mechanism with that property. For the simple markup mechanism, these properties follow almost surely by construction and Theorem 2 (the requirements for which hold almost surely, by assumption). For the minimal markup mechanism, the markup is almost surely $O(1/N)$ by Theorem 2, and, given this, budget-feasibility ensures overproduction is almost surely $O(1)$ (otherwise, the mechanism's revenue, $\alpha p \cdot \sum x_n - p \cdot (\sum_f y_f - \sum_n x_n)$, would eventually be negative almost surely).

We now use these facts to show that the (seller) price vector output by either mechanism—both under truthful reporting and after misreporting by a single agent—minimizes an objective differing from that of the Walrasian mechanism for the convexified economy by an $O(1)$-Lipschitz convex function, constituting a perturbation of that objective, as defined by Watt (2025).

Suppose that under truthful reporting, the markup mechanism outputs a markup α, prices p^α, and has an excess supply vector $\mathbf{s}$. Suppose that buyer n_0 misreports its valuation function, and let α', $p^{\alpha'}$ and $\mathbf{s}'$ be the corresponding outputs of the mechanism given this misreport. The markup mechanism's price and allocation constitute a Walrasian equilibrium of an economy where buyers' values are scaled down by $(1+\alpha)$ and auctioneer demand is s. Thus, p^α minimizes the dual objective for that economy,

$$\frac{1}{1+\alpha} \sum_{n \in N} \hat{u}_n(p) + \sum_{f \in F} \hat{\pi}_f(p) - p \cdot s.$$

Similarly, under a buyer's misreport resulting in indirect utility $\hat{u}'_{n_0}$, prices $p^{\alpha'}$ minimize

$$\frac{1}{1+\alpha'}\sum_{n\in N\setminus\{n_0\}}\hat{u}_n(p)+\frac{1}{1+\alpha}\hat{u}'_{n_0}(p)+\sum_{f\in F}\hat{\pi}_f(p)-p\cdot s'.$$

Walrasian equilibrium prices for the convexified economy under truthful reporting minimize

$$\sum_{n\in N}\hat{u}_n(p)+\sum_{f\in F}\hat{\pi}_f(p).$$

This objective and the truthful markup mechanism's objective differ by

$$\frac{\alpha}{1+\alpha}\sum_{n\in N}\hat{u}_n(p)+p\cdot s.$$

Since $\alpha \leq O(1/N)$ almost surely, we have $\frac{\alpha}{1+\alpha} \leq O(1/N)$ as well, while $\sum_{n\in N}\hat{u}_n(p)$ is $O(N)$-Lipschitz since its subdifferential is total demand at p which is $O(N)$ (its Lipschitz constant is the largest selection from the subdifferential). Since $p\cdot s$ is Lipschitz with constant s, the perturbation above is $O(1)$-Lipschitz. Similarly, the difference between the convexified objective and that of the markup mechanism under the misreport is

$$\frac{\alpha'}{1+\alpha'}\sum_{n\in N\setminus\{n_0\}}\hat{u}_n(p)+\hat{u}_{n_0}(p)-\frac{1}{1+\alpha'}\hat{u}'_{n_0}(p)-p\cdot s',$$

which is $O(1)$ by similar reasoning.

By Theorem 4 of Watt (2025) (adapted to two-sided markets, as in Appendix C of that paper), for any $\varepsilon > 0$, with probability $1-O(1/|N_t|)$ over draws of $\mathscr{E}_t$, we have that $\|p-p^{\alpha}\|$ and $\|p-p^{\alpha'}\|$ are $O(1/|N_t|^{1-\varepsilon})$ for any $\varepsilon > 0$. Then, by the triangle inequality, $\|p^{\alpha}-p^{\alpha'}\|$ and $\|(1+\alpha)p^{\alpha}-(1+\alpha')p^{\alpha'}\|$ are $O(1/|N_t|^{1-\varepsilon})$ with the same probability, as well. But, since the ex post benefits of a misreport are bounded above by $\hat{u}_n((1+\alpha)p^{\alpha})-\hat{u}_n((1+\alpha')p^{\alpha'})$ and each buyer's indirect utility function $\hat{u}_n$ is $O(1)$-Lipschitz, those ex post benefits are $O(1/|N_t|^{1-\varepsilon})$ with high probability, as well. Similar reasoning applies to seller misreports.

□

B Additional material

B.1 Nonconvexity and approximate equilibria

This section presents a stronger statement of the Shapley-Folkman Lemma used in general equilibrium theory with nonconvexities.

Proposition 4. *Let $S_i \subseteq \mathbb{R}^L$ for $i = 1, ..., N$, $S = \bigoplus_{i=1}^N S_i$ and $L' = \min(L, N)$. Then for any $x \in \mathrm{co}(S)$:*

(a) (Shapley-Folkman Lemma) $x = \sum_{i=1}^N x_i$ where $x_i \in \mathrm{co}(S_i)$ and $|i : x_i \in \mathrm{co}(S_i) \setminus S_i| \leq L'$.

(b) (Starr, 1969) If S_i is ordered so that $r(S_i)$ is nonincreasing in i, then there is $y \in S$ such that $|x - y| \leq \sqrt{\sum_{i=1}^{L'} r(S_i)^2}$.

(c) (Heller, 1972) If S_i is ordered so that $\rho(S_i)$ is nonincreasing in i, then there is $y \in S$ such that $|x - y| \leq \sqrt{\sum_{i=1}^{L'} \rho(S_i)^2}$.

These results underpin the construction of *approximate equilibria* in general equilibrium theory. An approximate equilibrium is a price-allocation pair $(p, \mathbf{x})$ such that $x_n \in D_n(p)$ for all n, $y_f \in S_f(p)$ for all f, and $\left|\sum_{n \in N} x_n - \sum_{f \in F} y_f\right| \leq s$ for some small $s > 0$. An approximate equilibrium allocation may have excess demand, making it infeasible. An approximate equilibrium can be constructed by finding the competitive equilibrium of the convexified economy and applying to it the results of Proposition 4. Proposition 5 presents the approximate equilibrium analogs of Proposition 4.

Proposition 5. *For economy $\mathscr{E} = (N, F)$:*

(a) (Starr, 1969) There is $\mathbf{x} \in \mathrm{co}(\mathbf{X})$ and $p \in \mathbb{R}_+^L, p > 0$ such that $|n : x_n \in \mathrm{co}(X)| + |f : y_f \in \mathrm{co}(Y)| \leq L$ and for all other agents, $x_n \in D_n(p)$ and $y_f \in S_f(p)$.

(b) Let $r_n = \sup_{\bar{u} \in \mathbb{R}} r(UC_n^{\bar{u}})$ and $r_f = \sup_{\bar{\pi} \in \mathbb{R}} r(UC_f^{\bar{\pi}})$. Let $\delta \geq 0$ satisfy $r_n \leq \delta$ for all $n \in N$ and $r_f \leq \delta$ for all $f \in F$. Then there exists $p \in \mathbb{R}_+^L, p > 0$, $x_n \in X$ and $y_f \in Y$ such that $x_n \in D_n(p)$ for all $n \in N$, $y_f \in S_f(p)$ for all $f \in F$ and $\left|\sum_{n \in N} x_n - \sum_{f \in F} y_f\right| \leq \delta\sqrt{L}$.

(c) Let $\rho_n = \sup_{\bar{u} \in \mathbb{R}} \rho(UC_n^{\bar{u}})$ and $\rho_f = \sup_{\bar{\pi} \in \mathbb{R}} \rho(UC_f^{\bar{\pi}})$. Let $\delta' \geq 0$ satisfy $\rho_n \leq \delta'$ for all $n \in N$ and $\rho_f \leq \delta'$ for all $f \in F$. Then there exists $p \in \mathbb{R}_+^L, p > 0$, $x_n \in X$ and $y_f \in Y$ such that $x_n \in D_n(p)$ for all $n \in N$, $y_f \in S_f(p)$ for all $f \in F$ and $\left|\sum_{n \in N} x_n - \sum_{f \in F} y_f\right| \leq \delta' L$.

While Proposition 5(a) is standard, Propositions 5(b) and (c) offer stronger results than the classical statements by Starr (1969) and Heller (1972). The quasilinearity of agent preferences implies that agents are utility- and profit-maximizing, rather than merely expenditure-minimizing.

Nguyen and Vohra (2024) introduced the *generalized* Δ*-single improvement property*, a generalization of the well-known single improvement property in terms of perceived complementarity and substitutability of goods. Our results also extend to quasilinear preferences satisfying this definition.

Definition B.1. Buyer n's preferences satisfy the generalized Δ-single improvement property for some $\Delta > 0$ if, for any price vector $p > 0$, any two bundles $x, y \in D_n(p)$, and any price change δp with $\delta p \cdot x > \delta p \cdot y$, there exist $a \leq (x-y)^+$ and $b \leq (y-x)^+$ such that:

(a) $|a| + |b| \leq \Delta$

(b) $\delta p \cdot a > \delta p \cdot b$, and

(c) $x - a + b \in D_n(p)$.

Here $(x-y)^+$ denotes the vector whose ℓ^{th} component is $\max(x_\ell - y_\ell, 0)$.

In this definition, Δ quantifies the substitutability and complementarity between goods. Preferences with Gul and Stacchetti's 1999 single improvement property fall within the class where $\Delta = 2$.

Given the compactness assumption for X and Y, all preferences and technologies satisfy the generalized Δ-improvement property for some Δ, as noted by Nguyen and Vohra (2024). However, a stronger relationship holds between the inner radii of preferences and the Δ-single improvement property.

Proposition 6. *Let $r_n = \sup_{\bar{u} \in \mathbb{R}} r(UC_n^{\bar{u}})$. Then the preferences of buyer $n \in N$ satisfy the generalized $\Delta-$single improvement property for all $\Delta > 2\sqrt{2} r_n$.*

Proof. Let buyer n's preferences satisfy $r_n = \sup_{\bar{u} \in \mathbb{R}} r(UC_n^{\bar{u}})$. Let $x, y \in X$ and $p \in \mathbb{R}^L_+$ be given such that $x, y \in D_n(p)$. Suppose $|(x-y)^+| + |(y-x)^+| \geq 2r_n$ (otherwise, the preferences would immediately satisfy the Δ improvement property for $\Delta = 2r_n$).

For any $\epsilon > 0$, let $z \in \mathbb{R}^L_+$ be the unique convex combination of x and y such that $|x - z| = r_n + \epsilon$, and let λ satisfy $z = \lambda x + (1-\lambda) y$. By construction, $(z, p \cdot z) \in \text{co}(UC_n^{u_n(x) - p \cdot x})$. The bound on preference nonconvexity then implies the existence of a set $T \subseteq UC_n^{u(x) - p \cdot x}$ with $\text{rad}(T) \leq r_n$ such that $(z, p \cdot z) = \sum_{(x',t') \in T} \alpha_{(x',t')} (x', t')$ with $\sum_{(x',t') \in T} \alpha_{(x',t')} = 1$.

For all $(x', t') \in T$, it follows that $x' \in D_n(p)$ and $t' = p \cdot x'$. This holds because $x \in D_n(p)$ implies

$u_n(x') - p \cdot x' \leq u_n(x) - p \cdot x$. Summing yields:

$$
\begin{aligned}
u_n(x) - p \cdot x &\geq \sum_{(x',t') \in T} \alpha_{(x',t')}[u_n(x') - p \cdot x'] \\
&= \sum_{(x',t') \in T} \alpha_{(x',t')} u_n(x') - p \cdot z \\
&= \sum_{(x',t') \in T} \alpha_{(x',t')}[u_n(x') - t'].
\end{aligned}
$$

On the other hand, since $(x', t') \in UC_n^{u(x)-p\cdot x}$, $u_n(x') - t' \geq u(x) - p \cdot x$. These conditions hold simultaneously only if $u_n(x') - t' = u(x) - p \cdot x$ for all $(x', t') \in T$.

However, it then follows that $\sum_{(x',t') \in T} \alpha_{(x',t')}[u_n(x') - p \cdot x'] = u(x) - p \cdot x$. This implies that at least one of $u_n(x') - p \cdot x' \geq u_n(x) - p \cdot x$. But then $x \in D_n(p)$ implies that $u_n(x') - p \cdot x' = u_n(x) - p \cdot x$ for all x', so $x' \in D_n(p)$.

By construction, $|x - x'| \leq 2r_n + \epsilon$. Consequently, $||x - x'||_1 \leq 2\sqrt{2}r_n + \epsilon$. □

The generalized Δ-single improvement property readily extends to sellers by substituting expressions for utility with those for profits; an analogue of Proposition 6 then applies.

Nguyen and Vohra (2024) demonstrate an approximate equilibrium result in a setting with indivisibilities, i.e., where $X \subseteq \mathbb{Z}_+^L$ and $Y \subseteq \mathbb{Z}_+^L$.

Proposition 7. *Suppose all buyers' preferences and sellers' technologies satisfy the generalized Δ-improvement property and that $X \subseteq \mathbb{Z}_+^L$ and $Y \subseteq \mathbb{Z}_+^L$. Then there exists $p \in \mathbb{R}_+^L, p > 0$, $x_n \in X$ and $y_f \in Y$ such that $x_n \in D_n(p)$ for all $n \in N$, $y_f \in S_n(p)$ and for each $\ell \in L$, $\left|\sum_{n \in N} x_{nl} - \sum_{f \in F} y_{fl}\right| \leq \Delta - 1$.*

This approximate equilibrium concept is stronger than previous results because the maximum imbalance in supply and demand is bounded on a good-by-good basis, rather than by Euclidean distance in commodity space. Nonetheless, depending on the relative size of Δ, the inner radii of nonconvexity, and the inner distances of nonconvexity, any of the approximate equilibrium bounds in Proposition 5(b), 5(c) or 7 may prove strongest for our analysis.

B.2 Maximum surplus anonymous pricing

Our markup mechanisms relate to a linear pricing mechanism proposed for the allocation of commercial fisheries licenses in New South Wales, Australia: the *maximum surplus anonymous pricing* mechanism, as described by Bichler et al. (2018). That mechanism solves the standard surplus optimization problem

$$\max_{\mathbf{x}\in\mathbf{X}} \sum_{n\in N} u_n(x_n) - \sum_{f\in F} c_f(y_f),$$

but subject to the constraint that there exist prices p^b and p^s satisfying

(a) Individual rationality: for all $n \in N, f \in F$, $u_n(x_n) - p^b \cdot x_n \geq 0$ and $p^s \cdot y_f - c_f(y_f) \geq 0$.

(b) Budget balance: $\sum_{n\in N} p^b \cdot x_n \geq \sum_{f\in F} p^s \cdot y_f$.

A corollary of Theorem 2 is that the maximum surplus anonymous pricing mechanism has a deadweight loss bounded by a constant, independent of market size $|N|$. This paper introduces and analyzes markup mechanisms rather than the alternative mechanism just described for two reasons. First, the alternative mechanism is difficult to scale because it requires solving nonconvex optimization problems, whereas the simple markup mechanism can be implemented by solving convex optimization problems (plus a binary search). Second, the alternative mechanism may entail rationing at prevailing prices, which can incentivize agents to misreport. The markup mechanism avoids this problem.