Optimal Redistribution Through Subsidies

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Introduction

Governments often **redistribute** by **subsidising private market consumption**.

> **Examples**: subsidies for food, childcare, transportation, pharmaceuticals, housing, electricity.

Subsidy programs in practice are often **nonlinear**: marginal subsidy depends on total quantity.

> **Example**: food stamps (SNAP): capped $291/month in vouchers, no additional subsidies

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Three questions:

#1. When should governments use consumption subsidies for redistribution?

#2. How should consumption subsidies be designed to best target intended recipients?

#3. When does the planner benefit from restricting supplemental consumption by subsidy recipients?

Our approach: we pose and solve the **optimal nonlinear subsidy design problem** for redistribution.

Key Tradeoff

The **optimal nonlinear consumption subsidy** program trades off:

#1. screening: exploiting differences in demand to target higher-need consumers, versus

#2. type-dependent outside options: heterogeneous ability to consume in private market.

Heterogeneous outside options are empirically relevant, *e.g.*,

- public housing (van Dijk, 2019; Waldinger, 2021),
- education (Akbarpour et al., 2022),
- healthcare (Heim et al., 2021; Li, 2017),
- food stamps (Ganong and Liebman, 2018).

Heterogeneous outside options lead to new **lower-bound constraints** in the mechanism design problem.

Results Overview

We provide an **explicit characterisation** of:

(a) **when** the social planner strictly benefits from subsidising consumption, and

(b) the **optimal subsidy** as a function of quantity consumed.

Key determinants of optimal subsidy design:

- Correlation between demand for good (type θ) and need (welfare weight ω).
- Availability of other programs $\rightsquigarrow$ the opportunity cost of subsidy spending α.
- Ability to restrict private market access and associated enforcement costs.

$\rightsquigarrow$ **The optimal subsidy mechanism is never linear.**

Related Literature

- **Public Finance.** Ramsey (1927), Diamond (1975), Mirrlees (1976, 1986), Atkinson and Stiglitz (1976), Doligalski, Dworczak, Krysta and Tokarski (2023), Nichols and Zeckhauser (1982), Blackorby and Donaldson (1988), Besley and Coate (1991), Blomquist and Christiansen (1998).
  - $\rightsquigarrow$ **This paper:** does not assume design of in-kind transfer, identifies optimal **nonlinear** one.
- **Redistributive Mechanism Design.** Condorelli (2013), Dworczak, Kominers and Akbarpour (2021, 2022), Akbarpour, Budish, Dworczak and Akbarpour (2024), Pai and Strack (2024).
  - $\rightsquigarrow$ **This paper:** allows consumers to consume in private market (outside of planner's control).
- **Partial Mechanism Design.** Philippon and Skreta (2012), Tirole (2012), Fuchs and Skrzypacz (2015), Dworczak (2020), Loertscher and Muir (2022), Kang and Muir (2022), Kang (2023), Kang and Watt (2024).
  - $\rightsquigarrow$ **This paper:** focus on benchmark where planner is as efficient as private market, "topping up."
- **Methodological Tools in Mechanism Design.** Toikka (2011), Corrao et al. (2023), Yang and Zentefis (2024), Valenzuela-Stookey and Poggi (2024).
  - $\rightsquigarrow$ **This paper:** explicit characterisation of solution with FOSD (topping up) constraint.

Model

Setup

Consumers:

- There is a unit mass of risk-neutral consumers in market for a divisible, homogeneous good.
- Consumers differ in type $\theta \in [\underline{\theta}, \overline{\theta}]$, with $\theta \sim F$, which is absolutely continuous and has density $f > 0$.
- Each consumer derives utility $\theta v(q) - t$ from quantity $q \in [0, A]$ given payment t.
  $v : [0, A] \to \mathbb{R}$ is differentiable with $v' > 0$ and $v'' < 0$.

Producers:

- The good is produced competitively at a constant marginal cost per unit, $c > 0$.

Extensions (not today): equilibrium effects, product choice and eligibility.

Laissez-Faire Equilibrium

- Perfectly competitive private market $\rightsquigarrow$ laissez-faire price $p^{\mathrm{LF}} = c$ per unit.
- Each consumer solves

$$U^{\mathrm{LF}}(\theta) := \max_{q\in[0,A]} \left[\theta v(q) - cq\right].$$

v is strictly concave $\rightsquigarrow$ unique maximiser:

$$q^{\mathrm{LF}}(\theta) = (v')^{-1}\left(\frac{c}{\theta}\right) = D(\theta).$$

Redistributive Objective

The social planner seeks to maximise **weighted total surplus**:

- **Consumer surplus**: social planner assigns a welfare weight $\omega(\theta) := \mathbf{E}[\omega|\theta]$ to consumer type θ.
  - $\rightsquigarrow$ $\omega(\theta)$: expected social value of giving consumer θ one unit of money.

Endogenising ω and α

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Two baseline cases:

- **“Negative Correlation”**: $\omega(\theta)$ is decreasing in θ.
  – high-demand consumers tend to have lower need for redistribution.
  – *e.g.*, food, education, and, if $\omega \propto -\text{Income}$, **normal** goods.
- **“Positive Correlation”**: $\omega(\theta)$ is increasing in θ.
  – high-demand consumers tend to have higher need for redistribution.
  – *e.g.*, staple foods, public transportation, and, if $\omega \propto -\text{Income}$, **inferior** goods.

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- **Total costs**: social planner assigns a welfare weight of $\alpha \in \mathbb{R}_+$ to total costs.
  - captures opportunity cost of subsidy spending (*cf.* other redistributive programs, tax cuts).

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$\rightsquigarrow$ Objective: weighted total surplus $\int_\theta [\omega(\theta)U(\theta) - \alpha\mathrm{Cost}(\theta)]\ \mathrm{d}F(\theta)$

$$\omega(\theta) > \alpha \rightsquigarrow \text{social planner wants to transfer a dollar to type } \theta.$$

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$\omega(\theta) > \alpha \rightsquigarrow$ social planner wants to transfer a dollar to type θ.

$\mathbf{E}_\theta[\omega(\theta)] > \alpha \rightsquigarrow$ social planner wants to make a lump-sum cash transfer to all consumers.

Endogenising ω and α

Subsidy Design

Social planner costlessly contracts with firms and sells units at a **subsidised payment schedule** $P^{\sigma}(q)$.

- $\rightsquigarrow$ Consumer pays $P^{\sigma}(q)$ dollars for q units of the good.
- $\rightsquigarrow$ $\sigma(z) = c - (P^{\sigma})'(z)$ is the **marginal subsidy.**

Key assumption: Each consumer can **top up** consumption of the good, allowing him to purchase additional units of the item in the private market at price c.

The social planner can verify subsidised consumption but not prevent topping up.

Assumption: No Lump-Sum Cash Transfers

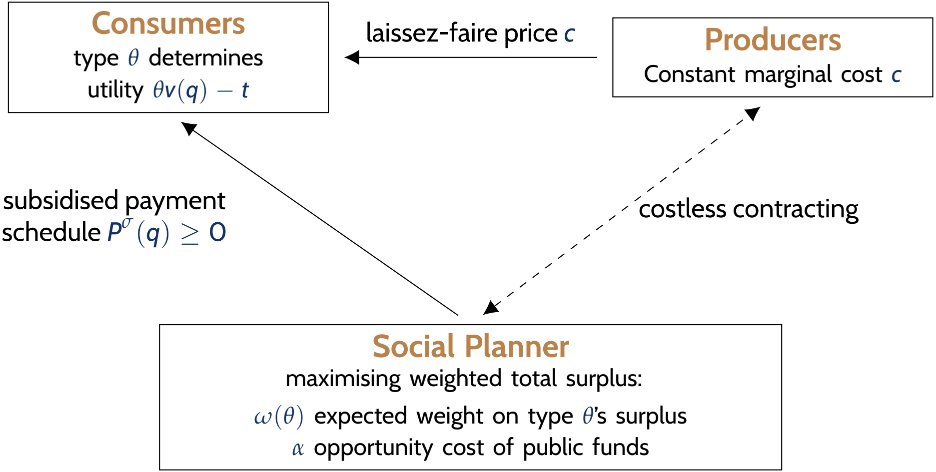
Assumption. For the rest of this talk, assume cash transfers are unavailable.

Note: This constraint only binds if $\mathbf{E}_\theta[\omega(\theta)] > \alpha$.

Possible reasons:

- Institutional: subsidies designed by government agency without tax/transfer powers.
- Political: Liscow and Pershing (2022) find U.S. voters prefer in-kind redistribution to cash transfers.
- Household Economics: Currie (1994) finds in-kind redistribution has stronger benefits for children than cash transfer programs.

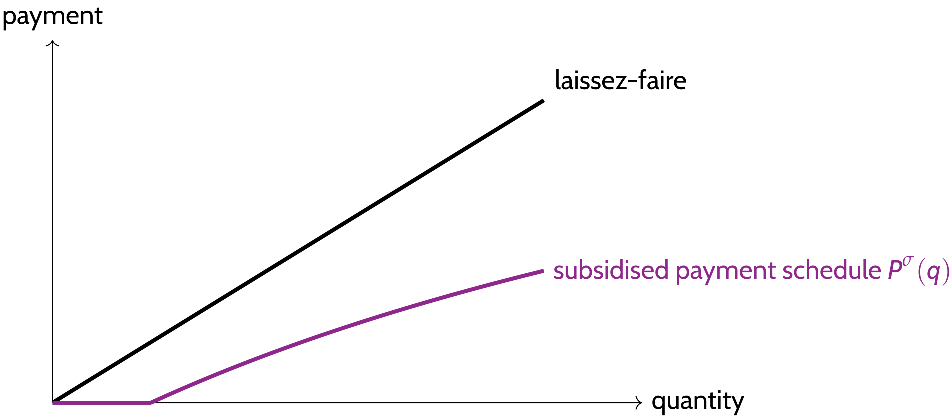
Model Overview



Consumers can purchase units from **both subsidised program and private market**.

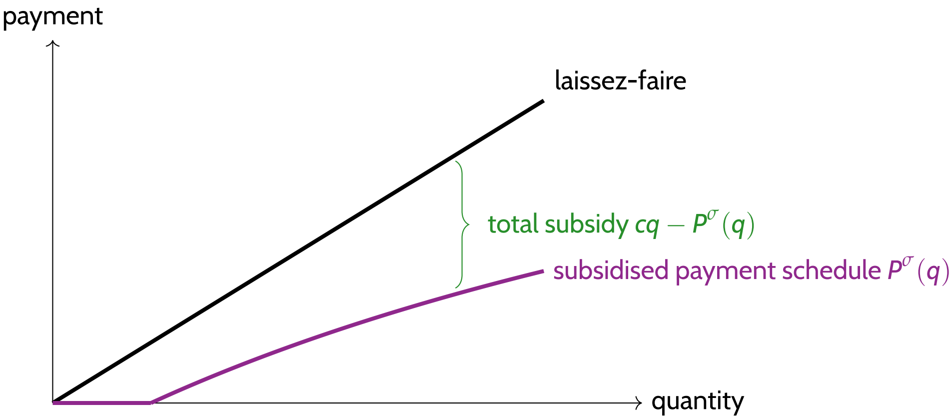
Private Market Access Constrains Implementable Price Schedules

private market access $\Longleftrightarrow$ **marginal price** $\leq c$



Private Market Access Constrains Implementable Price Schedules

private market access $\iff$ **marginal price** $\leq c$ $\iff$ **total subsidies** increase in q



When (Not) To Use Subsidies?

When Not To Subsidise

Suppose there is negative correlation between ω and θ, with $\omega(\theta)$ decreasing (e.g., "normal goods").

Theorem 1 (part). When $\omega(\theta)$ is decreasing, the social planner would always prefers to make a lump-sum cash transfer to all consumers than to offer consumption subsidies.

Intuition: Total subsidies paid is increasing in $q \Longrightarrow$ negatively correlated with ω

proof

With negative correlation, consumption subsidies are more regressive than cash transfers.

The social planner uses subsidies only if $\mathsf{E}_\theta[\omega] > \alpha$ (and cash transfers are not available).

When To Subsidise: A Sufficient Statistic

Theorem 1. The social planner can identify consumption subsidies that strictly improve on the laissez-faire outcome if and only if there exists a type $\hat{\theta}$ for which

$$\mathbf{E}_{\theta}[\omega(\theta) \mid \theta \geq \hat{\theta}] \; > \; \alpha.$$

Interpretation: The social planner is willing to offer a cash transfer to all consumers with type exceeding $\hat{\theta}$.

For negative correlation ("normal goods") with ω decreasing in θ, sufficient statistic is $\mathbf{E}_{\theta}[\omega(\theta)] > \alpha$.

For positive correlation ("inferior goods") with ω increasing in θ, sufficient statistic is $\max[\omega] > \alpha$
$\leadsto$ consumption subsidies may be <u>strictly preferred</u> to lump-sum cash transfers.

When to Subsidise: Proof by Picture

Suppose $\mathbf{E}_\theta[\omega(\theta)|\theta \geq \hat{\theta}] > \alpha$: we construct a subsidy schedule increasing weighted surplus.

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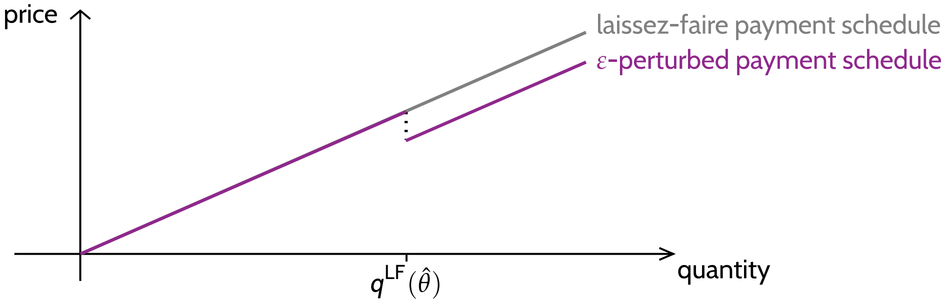

$\varepsilon-$perturbation increases utility of types $\geq \hat{\theta}$, net benefit $\varepsilon\, \mathbf{E}_\theta[\omega(\theta) - \alpha|\theta \geq \hat{\theta}]$.

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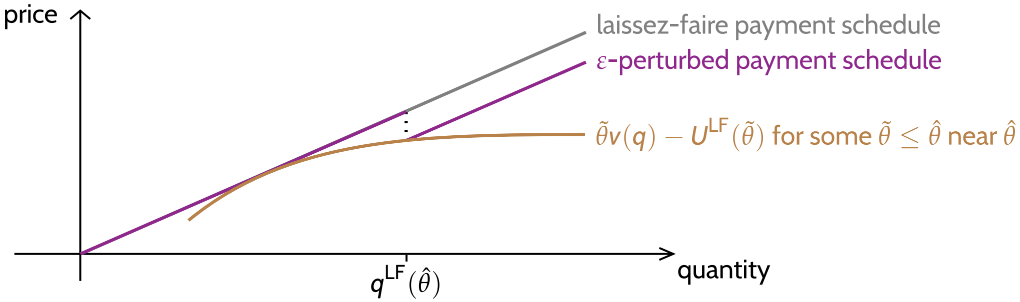


ε—perturbation increases utility of types $\geq \hat{\theta}$, net benefit $\varepsilon\, \mathbf{E}_\theta[\omega(\theta) - \alpha|\theta \geq \hat{\theta}]$.

But consumption is distorted for $O(\sqrt{\varepsilon})$ set of types near (but below) $\hat{\theta}$, at cost $\leq O(\sqrt{\varepsilon})\varepsilon$.

$\rightsquigarrow$ Benefits $>$ costs for small enough ε. **Note: Argument relies on nonlinearity.**

Discussion

Theorem 1 $\rightsquigarrow$ scope of intervention larger for “inferior goods” than “normal goods.”

In practice, many government programs focused on goods consumed disproportionately by needy:

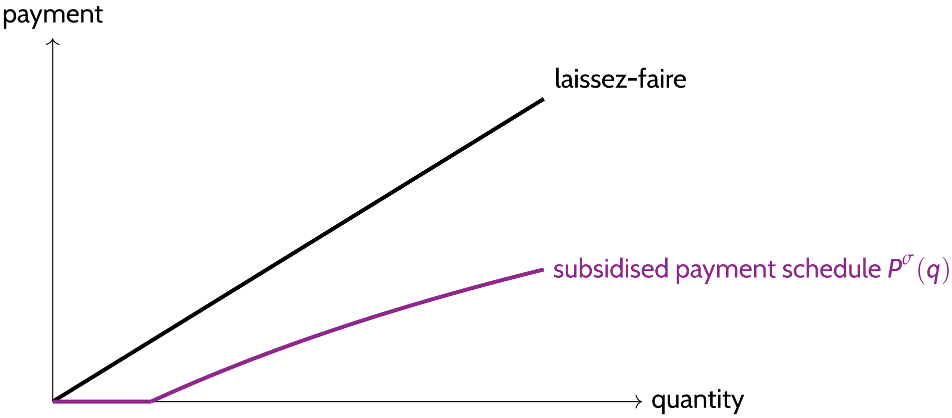
Examples:

- Egyptian *Tamween* food subsidy program subsidises five loaves of *baladi* bread/day at AUD 0.01/loaf, with a cap on weights and quality of bread.
- CalFresh Restaurant Meals Program subsidises fast food restaurants not dine-in restaurants.
- Indonesian Fuel Subsidy Program subsidises low-octane fuel (for motorbikes) and not high-octane fuel (for cars).
- Until $\sim$2016, UK’s NHS subsidised amalgam fillings and not composite (tooth-coloured) fillings.

Optimal Mechanism

Private Market Access Constrains Implementable Allocation Rules

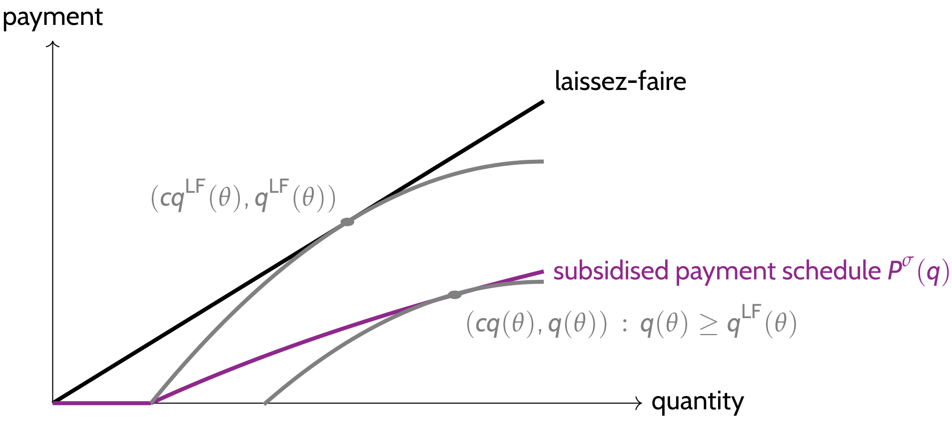
private market access $\Longleftrightarrow$ marginal price per unit $\leq c$ $\Longleftrightarrow$ subsidies increasing in q



▸ Converse

Private Market Access Constrains Implementable Allocation Rules

private market access $\Longleftrightarrow$ allocations exceed laissez-faire



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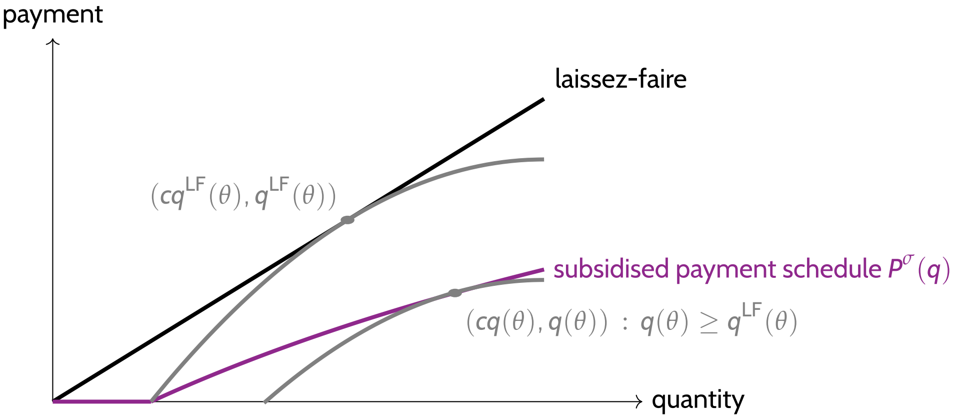
Private Market Access Constrains Implementable Allocation Rules

$\Longrightarrow$ mechanism design problem with a lower-bound constraint $q(\theta) \geq q^{LF}(\theta)$



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Mechanism Design With Topping Up

The social planner chooses a direct mechanism (q, t), consisting of:

- the allocation function $q : [\underline{\theta}, \overline{\theta}] \to [0, A]$ denoting *total* quantity consumed by type θ;
- the payment rule $t : [\underline{\theta}, \overline{\theta}] \to \mathbb{R}$ denoting *total* payment by type θ;

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to maximise weighted total surplus

$$\max_{(q,t)} \int_{\underline{\theta}}^{\overline{\theta}} \left[\omega(\theta) \underbrace{[\theta v(q(\theta)) - t(\theta)]}_{\text{consumer surplus}} - \alpha \underbrace{[cq(\theta) - t(\theta)]}_{\text{total cost}} \right] \mathrm{d}F(\theta),$$

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- individual rationality: $$\theta v(q(\theta)) - t(\theta) \geq U^{\mathrm{LF}}(\theta), \qquad \text{(IR)}$$

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- private market constraint: $q(\theta) \geq q^{LF}(\theta)$. (LB)

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Reformulating the Mechanism Design Problem

We apply standard mechanism design tools (Myerson, 1981; Milgrom and Segal, 2002) to obtain

$$\max_{q \text{ non-decreasing}} \alpha \int_{\underline{\theta}}^{\overline{\theta}} \underbrace{[J(\theta)v(q(\theta)) - cq(\theta)]}_{\text{surplus of virtual type}} \, \mathrm{d}F(\theta) + (\text{terms independent of } q),$$

subject to (LB), $q(\theta) \geq q^{\mathrm{LF}}(\theta)$, where the **virtual type** absorbs (IC), (IR), and (NLS):

$$J(\theta) = \underbrace{\theta}_{\text{efficiency}} + \underbrace{\frac{\int_{\theta}^{\overline{\theta}} [\omega(s) - \alpha] \mathrm{d}F(s)}{\alpha f(\theta)}}_{\text{redistributive motive}} + \underbrace{\frac{\max\{\mathbf{E}_{\theta}[\omega(\theta) - \alpha], 0\} \underline{\theta} \delta_{\theta = \underline{\theta}}}{\alpha f(\theta)}}_{\text{(NLS) constraint}}$$

Call $J(\theta) - \theta$ the **distortion term**.

Technical challenge: (LB) is a "pointwise dominance" / FOSD constraint (*cf.* Yang and Zentefis, 2024) $\rightsquigarrow$ possible interactions with the monotonicity constraint.

▸ Details

Solving for the Optimal Mechanism

skip

$$\max_{q} \alpha \int_{\underline{\theta}}^{\overline{\theta}} \left[J(\theta) v(q(\theta)) - c q(\theta) \right] \, \mathrm{d}F(\theta),$$

s.t. q nondecreasing and $q(\theta) \geq q^{\mathrm{LF}}(\theta)$.

Solving for the Optimal Mechanism

▸ skip

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Pointwise maximiser is just

$$q(\theta) = (v')^{-1}\left(\frac{c}{J(\theta)}\right) = D(J(\theta)).$$

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Demand $D(\cdot)$ is increasing, so:
q increasing $\iff J(\theta)$ increasing.
$q \geq q^{\mathrm{LF}} \iff D(J(\theta)) \geq D(\theta) \iff J(\theta) \geq \theta$.

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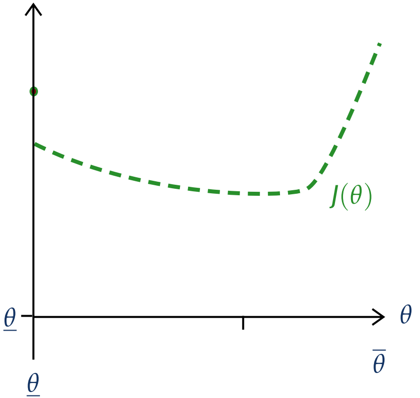

$J(\theta)$ may be non-monotone.

Solving for the Optimal Mechanism

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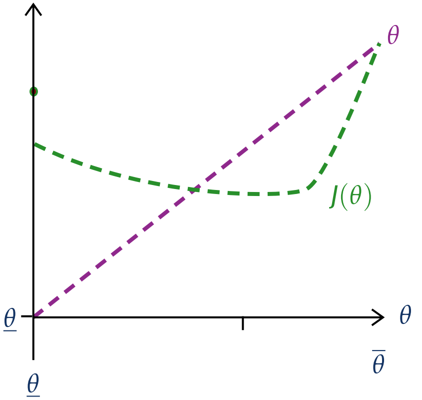

$J(\theta)$ may be smaller than θ.

Solving for the Optimal Mechanism

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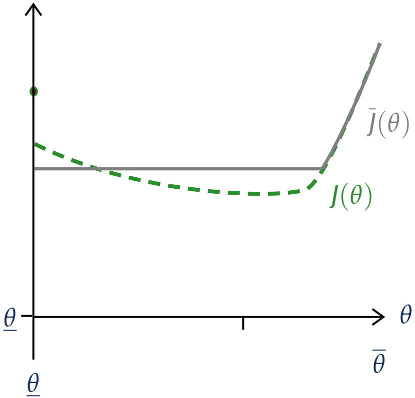

Ironing deals with non-monotonicity.

Ironing

Solving for the Optimal Mechanism

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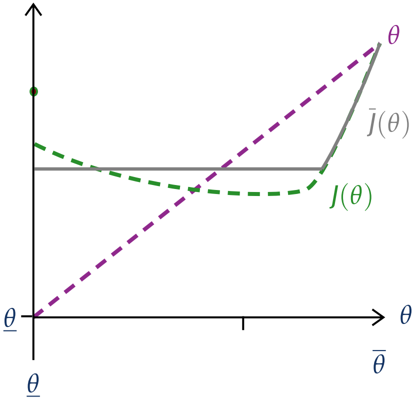


But not lower-bound constraint $\rightsquigarrow$ interaction.

▸ Ironing

Solving for the Optimal Mechanism

skip

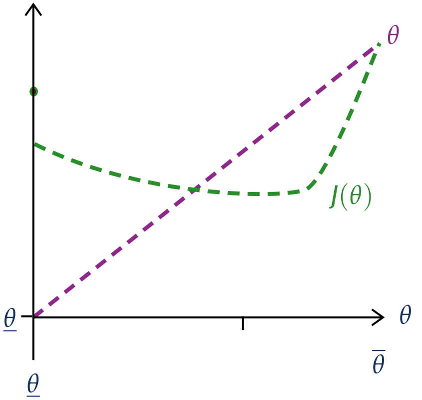
$$\max_{q} \alpha \int_{\underline{\theta}}^{\overline{\theta}} [J(\theta)v(q(\theta)) - cq(\theta)] \, \mathrm{d}F(\theta),$$

s.t. q nondecreasing and $q(\theta) \geq q^{\mathrm{LF}}(\theta)$.

Suppose solution is of the form

$$q(\theta) = D(H(\theta)),$$

chosen to be nondecreasing and satisfy $H(\theta) \geq \theta \rightsquigarrow$ feasible.



Need to identify nondecreasing $H \geq \theta$.

Ironing

Solving for the Optimal Mechanism

▸ skip

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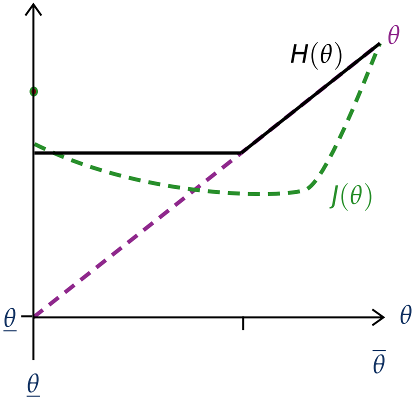

H may have pooling & binding lower bound.

▸ Ironing

Solving for the Optimal Mechanism

▸ skip

$$\max_q \alpha \int_{\underline{\theta}}^{\overline{\theta}} [J(\theta)v(q(\theta)) - cq(\theta)] \; \mathrm{d}F(\theta),$$

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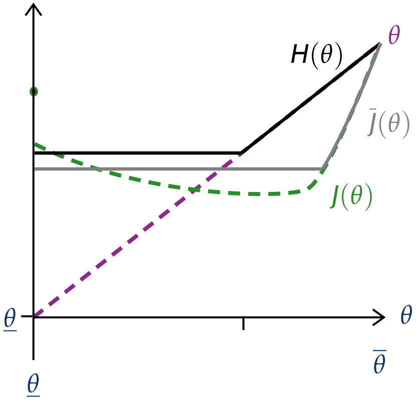

H always exceeds $\bar{J}$.

▸ Ironing

Optimal Mechanism

Theorem 2. The optimal subsidy allocation rule is unique, continuous and satisfies

$$q^*(\theta) = D\left(H(\theta)\right), \text{ where } H(\theta) = \begin{cases} \theta & \text{if } \overline{J|_{[\underline{\theta},\theta]}}(\theta) \leq \theta, \\ \overline{J_{[\underline{\theta},\kappa_+(\theta)]}}(\theta) & \text{otherwise,} \end{cases}$$

and $\kappa_+(\theta) = \inf \left\{\hat{\theta} \geq \theta : \overline{J|_{[\underline{\theta},\hat{\theta}]}}(\hat{\theta}) \leq \hat{\theta}\right\}$ or $\overline{\theta}$, if that set is empty.

Call $H(\theta)$ the **subsidy type** of type θ.

Interpretation: planner subsidises type θ to demand as much as $H(\theta)$ in the laissez-faire economy.

"Double" ironing construction of $H(\theta)$ ensures $H(\theta) \geq \theta$, equivalent to (LB) given expression for q^*.

▸ Ironing ▸ return to proof

Economic Implications

Four Questions

1. When should **non-market allocations** be used?

2. How do optimal subsidies depend on **correlation** between demand and social preferences?

3. When does the planner want to **restrict** topping up?

4. How do optimal subsidies **depend on market primitives?**

1: When Should Non-Market Allocations Be Used?

Proposition 2.

(a) If $\mathbf{E}[\omega] > \alpha$, the social planner offers all consumers a free quantity of the good.

(b) If $\mathbf{E}[\omega] < \alpha$, the initial $q^*(\underline{\theta})$ units of the good are priced at c.

(c) If $\mathbf{E}[\omega] = \alpha$, the planner is indifferent between providing $q^*(\underline{\theta})$ for free or charging price $\leq c$.

Intuition: When $\mathbf{E}[\omega] > \alpha$, the planner wants to make a cash transfer to consumers (infeasible by the (NLS) constraint), instead makes in-kind transfer.

When $\mathbf{E}[\omega] < \alpha$, the planner wants to tax all consumers (infeasible), instead charges for initial units.

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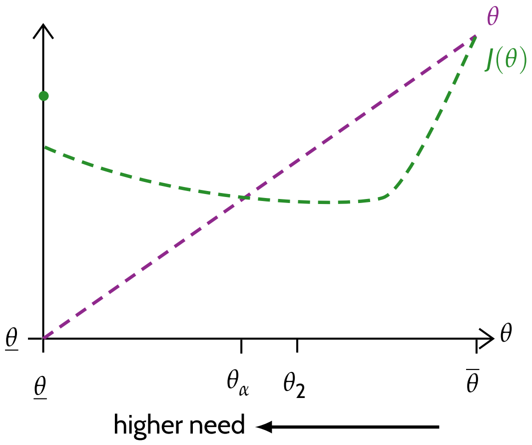
$\rightsquigarrow$ Optimal subsidy programs when demand is negatively correlated with ω **always** include public provision (cf. food stamps).

2: How Do Optimal Subsidies Depend On Correlation?

Negative Correlation ("normal goods")



▸ to point ▸ details

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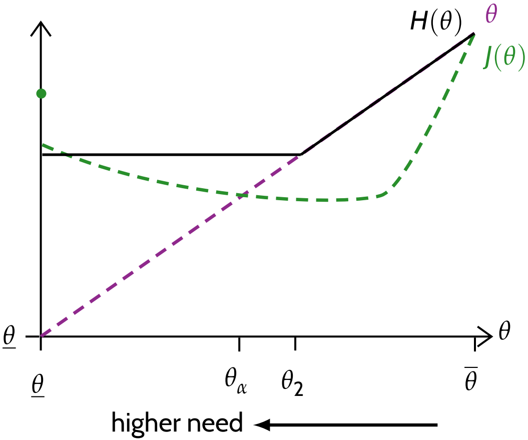

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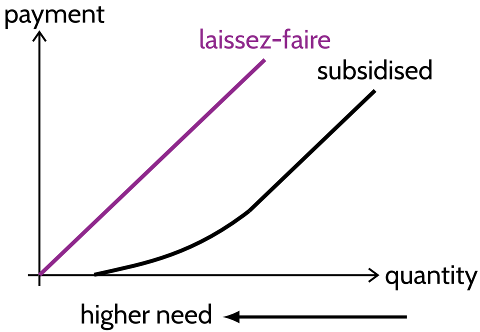

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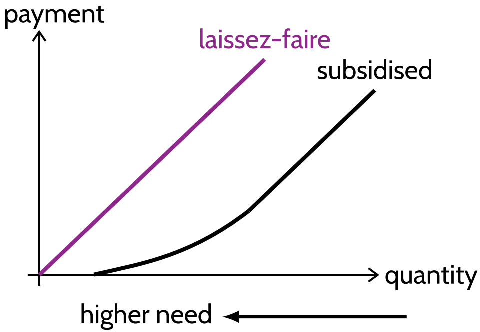
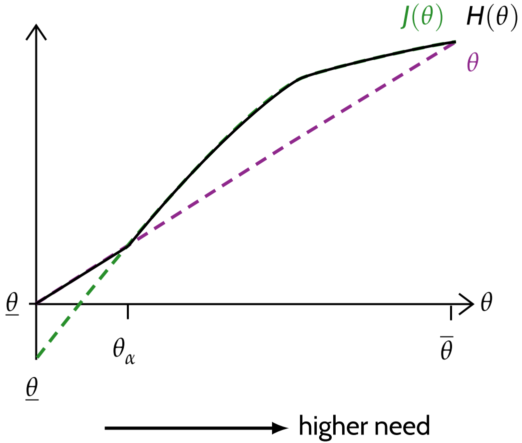


Positive Correlation ("inferior goods")



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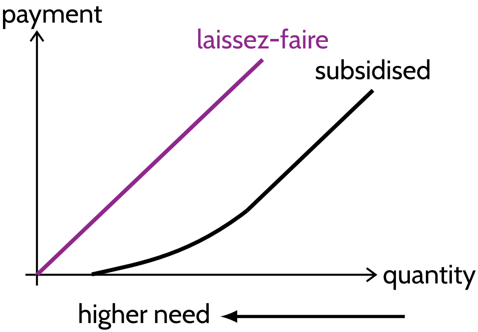

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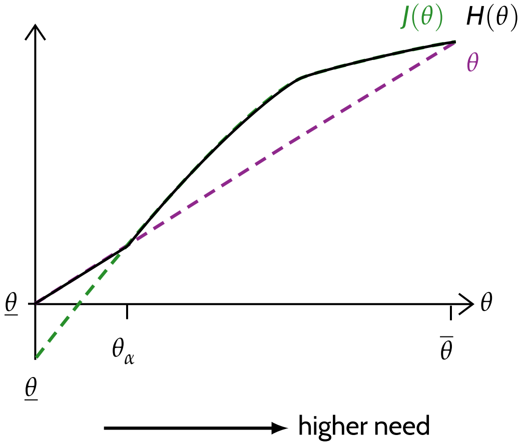

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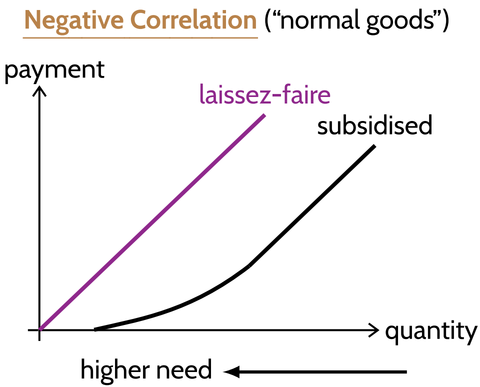

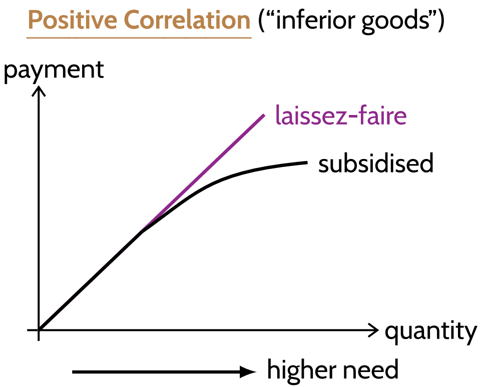

to point
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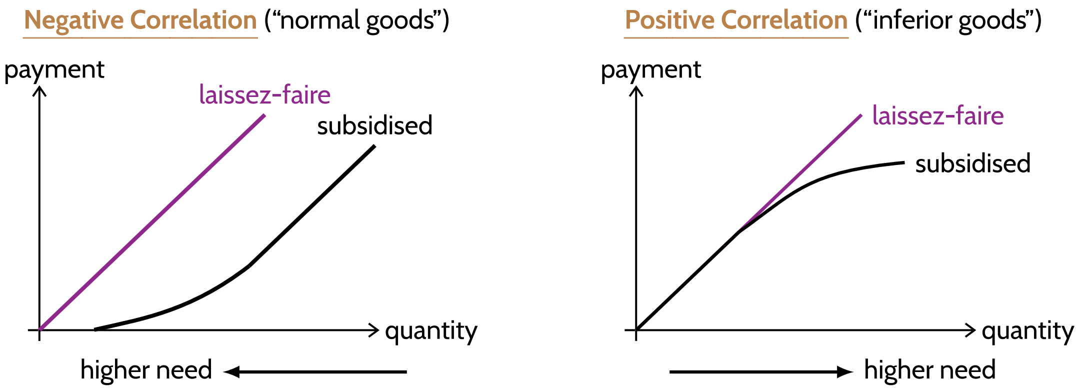

Negative Correlation: all or none of the consumers are subsidised.

Positive Correlation: subsidies only for self-selected consumers θ with $\mathbf{E}[\omega(\theta')|\theta' \geq \hat{\theta}] \geq \alpha$.

$\rightsquigarrow$ Note: Optimal subsidy scheme is **never** linear.

▸ to point ▸ details

Discussion

Significant differences in marginal subsidy schedules observed in practice:

Larger subsidies for low q

- Food stamps (SNAP)
- Womens, Infants & Children (WIC) Program
- Housing Choice (Section 8) Vouchers
- Lifeline (Telecomm. Assistance) Program

Larger subsidies for high q

- Public transit fare capping
- Pharmaceutical subsidy programs
- Government-subsidised childcare places.

program details

#3: When Does the Planner Benefit from Private Market Restrictions?

In some markets (e.g., public housing), the social planner may be able to restrict subsidy recipients from topping up in private market $\rightsquigarrow$ opt-in (or out) of subsidy program.

In Kang and Watt (2024), we characterise optimal subsidy mechanism under such restrictions. These lead to different (weaker) type-dependent outside option constraints:

$$\text{average price} \leq c \Leftrightarrow \text{majorisation constraint on } q.$$

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average price $\leq$ c $\Leftrightarrow$ majorisation constraint on q.

Proposition.

(a) "Negative Correlation": ω decreasing in $\theta \rightsquigarrow$ planner benefits from preventing topping up iff $\max \omega > \alpha$.

(b) "Positive Correlation": ω increasing in $\theta \rightsquigarrow$ planner never benefits from preventing topping up.

Intuition: Planner offers subsidies tied to consumption level favored by high ω types.

Positive correlation between demand and welfare weights reduces the need to enforce topping up restrictions.

skip to conclusion

4: How Do Optimal Subsidies Depend on Economic Primitives?

How do optimal subsidies change when:

(a) the social planner's desire to redistribute to each consumer increases?

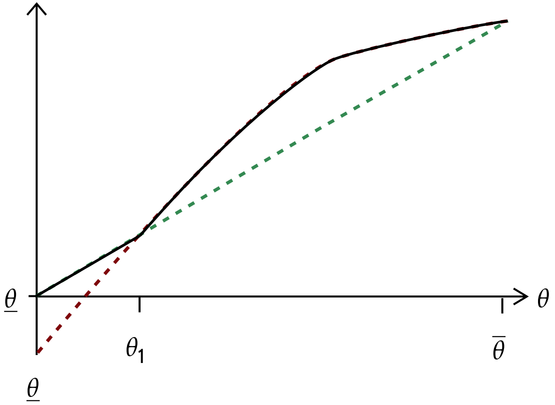
(b) the correlation between demand and welfare weight increases?

(c) the marginal cost of production decreases?

Comparative statics inform choice of product and eligibility rules for subsidy designers.

4(a) Increasing Motive for Redistribution

Suppose $\omega(\theta) \uparrow$ for each θ or, equivalently, $\alpha \downarrow$.



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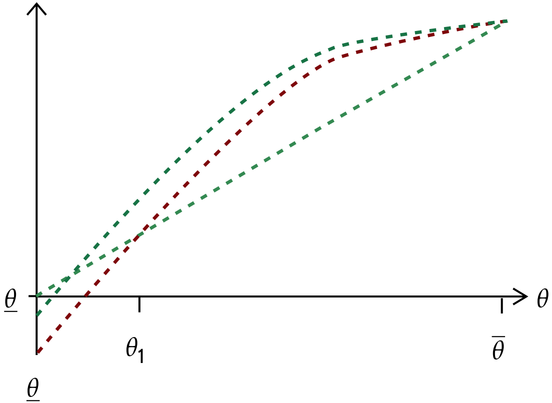
- virtual type $J(\theta)\uparrow$



4(a) Increasing Motive for Redistribution

Suppose $\omega(\theta) \uparrow$ for each θ or, equivalently, $\alpha \downarrow$.

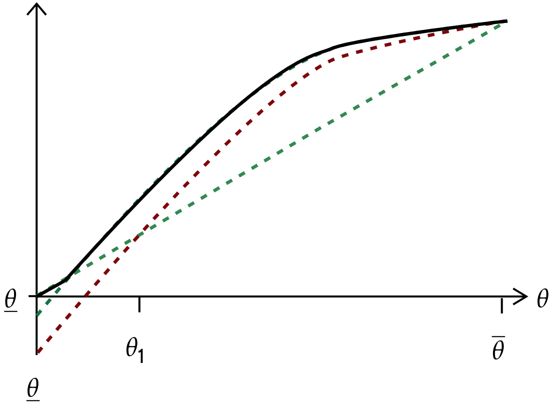
- virtual type $J(\theta) \uparrow$

$\rightsquigarrow$ each consumer's subsidy type $H(\theta) \uparrow$

$\rightsquigarrow$ each consumer's allocation $q^*(\theta) \uparrow$

$\rightsquigarrow$ set of subsidised types $\uparrow$

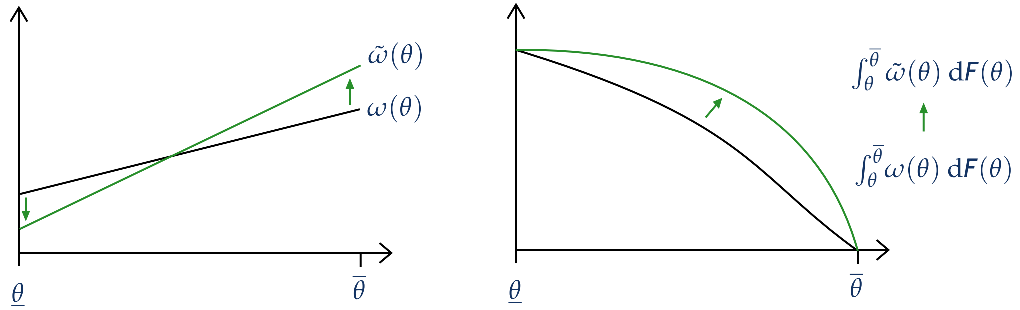
$\rightsquigarrow$ total subsidy per consumer $\uparrow$



Planner & average eligible consumer prefer subsidies targeted to consumers with higher welfare weights.

4(b) Increasing Correlation

Suppose ω and θ become more correlated, in the sense of **majorisation** $\rightsquigarrow$ observe higher demand expect higher ω, *i.e.*, for all $\theta \in \Theta$, $\mathbf{E}[\tilde{\omega}(\theta)|\theta \geq \hat{\theta}] \geq \mathbf{E}[\omega(\theta)|\theta \geq \hat{\theta}]$.



$\rightsquigarrow$ larger incentive to distort consumption $\rightsquigarrow$ more generous subsidies.

Planner & average eligible consumer prefer subsidies for goods with more positive correlation between demand and welfare weights.

4(c) Decreasing Marginal Cost

Suppose marginal cost decreases $c \downarrow$ (equiv. demand increases, so $(v') \uparrow$).

No change in virtual type $\leadsto$ no change in subsidy type.

$\leadsto$ the set of subsidised types is unchanged, while

$\leadsto$ each consumer's allocation $q^*(\theta) \uparrow$

$\leadsto$ total subsidy per subsidised consumer $\uparrow$.

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$\rightsquigarrow$ total subsidy per subsidised consumer $\uparrow$.

Planner & average eligible consumer prefer subsidies for low cost / high demand goods.

Extensions

Equilibrium Effects

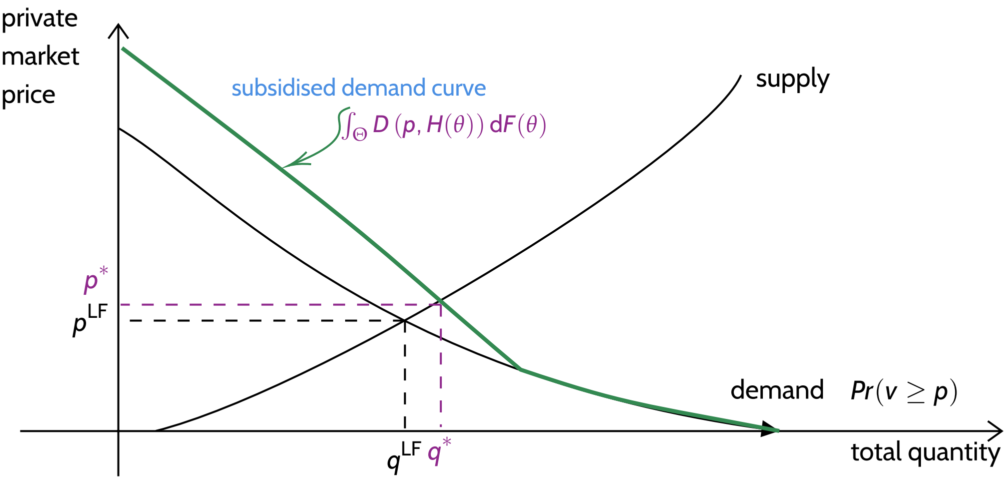
Our baseline model shuts down equilibrium effects of government subsidies on private market prices.

Empirical evidence of price effects from government subsidy programs, *e.g.*:

- public housing (Diamond and McQuade, 2019; Baum-Snow and Marion, 2009)
- pharmaceuticals (Atal et al., 2021)
- public schools (Dinerstein and Smith, 2021)
- school lunches (Handbury and Moshary, 2021)

Equilibrium Effects

Our results extend directly to imperfectly elastic supply curves:



Private Market Taxation

Our baseline model assumes the planner cannot tax the private market.

Taxation of private market reduces consumers' outside option, relaxing the (LB) constraint. If taxation is costly (*e.g.*, because of distortions on ineligible consumers):

Proposition. Suppose the planner faces a convex cost $\Gamma(\tau)$ for taxation of the private market. Then there exists an optimal tax level τ^* and subsidy program for eligible consumers satisfying

$$q^*(\theta) = D(H_{\tau^*}(\theta)),$$

where $H_{\tau^*}(\theta) \leq H(\theta)$.

Budget Constraints and Endogenous Welfare Weights

In our baseline model, $\omega(\cdot)$ and α are taken exogenously.

Our model can be extended to allow weights to be endogenous (cf. Pai and Strack, 2024):

- $\alpha \iff$ Lagrange multiplier on the social planner's budget constraint.
- $\omega(\theta) \iff$ the marginal value of money for a consumer with concave preferences

$$\varphi\left(\theta v(q) + I - t\right),$$

  and income $I \sim G_\theta$, known but not observed by the social planner, then

$$\omega(\theta) = \mathbf{E}_{I \sim G_\theta}[\varphi'\left(\theta v(q(\theta)) + I - t(\theta)\right)].$$

▸ Model

Conclusion

Concluding Remarks

Key contribution: characterisation of the optimal nonlinear subsidy mechanism when consumers have access to private markets $\rightsquigarrow$ the optimal mechanism is **never linear**, with shape depending on correlation between demand and need.

Technical contribution: how to solve mechanism design problems with lower-bound constraints caused by type-dependent outside options.

Similar mechanism design problems arise in other contexts, e.g., subsidy design with other objectives (externalities, paternalism); exclusive contracting (topping up = non-exclusive contracting, no topping up = exclusive contracting.).

Fin

Thank you for the invitation!

When Not To Subsidise: Proof

Suppose ω is decreasing in θ, then:

$$\text{total subsidies increasing in } q \implies \text{total subsidies increasing in } \theta$$

But then ω and total subsidy payments are negatively correlated so

$$\begin{aligned}
\text{weighted increase in surplus from subsidy} &= \mathbf{E}_\theta\left[\omega(\theta) \cdot \left(U(\theta) - U^{\mathsf{LF}}(\theta)\right)\right] \\
&\leq \mathbf{E}_\theta\left[\omega(\theta) \cdot \text{subsidy}(\theta)\right] \\
&< \mathbf{E}_\theta[\omega]\, \mathbf{E}_\theta\left[\text{subsidy}(\theta)\right] \\
&= \text{ weighted increase in surplus of equivalent cash transfer}
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&= \text{ weighted increase in surplus of equivalent cash transfer}
\end{aligned}$$

▸ Derivation ▸ return

Topping Up $\Longleftarrow$ Lower-Bound (1/2)

Suppose $q(\theta) \geq q^{\mathrm{LF}}(\theta)$. We want to show total subsidies $S(z)$ is increasing in z.

1. $t(\underline{\theta}) \leq cq(\underline{\theta})$ by (IR):

$$t(\underline{\theta}) \leq \underline{\theta} v(q(\underline{\theta})) - \underline{\theta} v(q^{\mathrm{LF}}(\underline{\theta})) + cq^{\mathrm{LF}}(\underline{\theta}),$$

and $\underline{\theta} v(q^{\mathrm{LF}}(\underline{\theta})) - cq^{\mathrm{LF}}(\underline{\theta}) \geq \underline{\theta} v(q(\underline{\theta})) - cq(\underline{\theta})$ by definition of q^{LF}, so $t(\underline{\theta}) \leq cq(\underline{\theta})$.

Topping Up $\Longleftarrow$ Lower-Bound (2/2)

\# 2. The *marginal* price of any units purchased is no greater than c by (IC):

$$t(\theta') - t(\theta) = \left[\theta' v(q(\theta')) - U(\underline{\theta}) - \int_{\underline{\theta}}^{\theta'} v(q(s))\, \mathrm{d}s\right] - \left[\theta v(q(\theta)) - U(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} v(q(s))\, \mathrm{d}s\right]$$

$$= \theta' v(q(\theta')) - \theta v(q(\theta)) - \int_{\theta}^{\theta'} v(q(s))\, \mathrm{d}s$$

$$= \int_{\theta}^{\theta'} s v'(q(s))\, \mathrm{d}q(s).$$

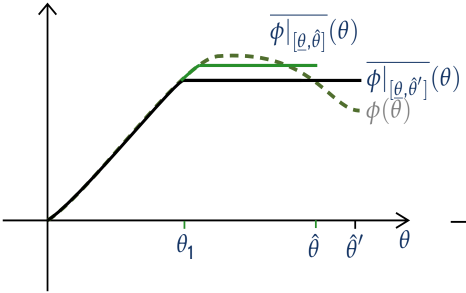
But if $q(\theta) \geq q^{\mathrm{LF}}(\theta)$, then concavity of v implies $v'(q(\theta)) \leq v'(q^{\mathrm{LF}}(\theta)) = c/\theta$, so $t(\theta') - t(\theta) \leq c[q(\theta') - q(\theta)]$.

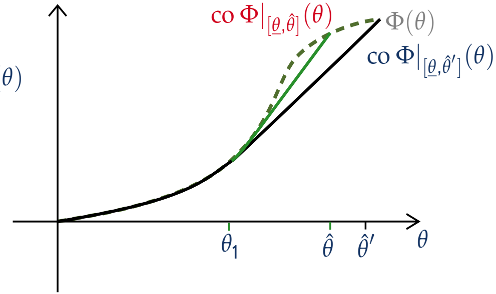
▸ return

Ironing

Let ϕ be a (generalised) function and $\Phi(\theta) = \int_{\underline{\theta}}^{\theta} \phi(s)\, \mathrm{d}F(s)$. Then $\overline{\phi}$ is the monotone function satisfying

$$\text{for all } \theta \in [\underline{\theta}, \hat{\theta}], \qquad \int_{\underline{\theta}}^{\theta} \overline{\phi}(s)\, \mathrm{d}F(s) = \mathrm{co}\, \Phi(\theta).$$





▸ Derivation ▸ Statement

Rewriting the Mechanism Design Problem

The social planner maximises weighted total surplus

$$\max_{(q,t)} \int_{\underline{\theta}}^{\overline{\theta}} \left[\omega(\theta) \underbrace{[\theta v(q(\theta)) - t(\theta)]}_{\text{consumer surplus}} + \alpha \underbrace{[t(\theta) - cq(\theta)]}_{\text{total cost}} \right] \mathrm{d}F(\theta),$$

subject to (IC), (LB), (IR), and (NLS).

▸ return

Rewriting the Mechanism Design Problem

$$\max_{q \text{ non-decreasing}} \mathbf{E}_\theta[\omega(\theta)-\alpha]U(\underline{\theta}) + \alpha \int_{\underline{\theta}}^{\overline{\theta}} \left[\left(\theta + \frac{\int_\theta^{\overline{\theta}} [\omega(s)-\alpha] \mathrm{d}F(s)}{\alpha f(\theta)} \right) v(q(\theta)) - cq(\theta) \right] \mathrm{d}F(\theta),$$

subject to (LB), (IR), and (NLS).

#1. Apply Myerson (1981) Lemma and Milgrom and Segal (2002) envelope theorem to express objective in terms of $U(\underline{\theta})$ and $q(\theta)$ non-decreasing.

▸ return

Rewriting the Mechanism Design Problem

$$\max_{q \text{ non-decreasing}} \mathbf{E}_\theta[\omega(\theta)-\alpha]U(\underline{\theta}) + \alpha \int_{\underline{\theta}}^{\overline{\theta}} \left[\left(\theta + \frac{\int_\theta^{\overline{\theta}} [\omega(s)-\alpha] \mathrm{d}F(s)}{\alpha f(\theta)} \right) v(q(\theta)) - cq(\theta) \right] \mathrm{d}F(\theta),$$

subject to (LB), (IR), and (NLS).

#1. Apply Myerson (1981) Lemma and Milgrom and Segal (2002) envelope theorem to express objective in terms of $U(\underline{\theta})$ and $q(\theta)$ non-decreasing.

#2. Suffices to enforce (IR) and (NLS) only for lowest type $\underline{\theta}$ because $U(\theta) - U^{\mathrm{LF}}(\theta)$ and $t(\theta)$ are nondecreasing by (IC) and (LB).

- $\rightsquigarrow$ **Low cost of public funds**: if $\mathbf{E}[\omega(\theta)] > \alpha$, choose $U(\underline{\theta}) = \underline{\theta} v(q(\underline{\theta}))$.
- $\rightsquigarrow$ **High cost of public funds**: if $\mathbf{E}[\omega(\theta)] \leq \alpha$, choose $U(\underline{\theta}) = U^{\mathrm{LF}}(\underline{\theta})$.

return

Rewriting the Mechanism Design Problem

$$\max_{q \text{ non-decreasing}} \alpha \int_{\underline{\theta}}^{\overline{\theta}} [J(\theta)v(q(\theta)) - cq(\theta)] \, \mathrm{d}F(\theta) + (\text{terms independent of } q),$$

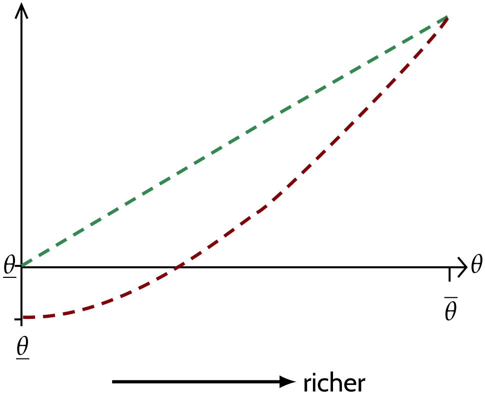
subject to (LB): $q(\theta) \geq q^{\mathrm{LF}}(\theta)$, where the virtual type

$$J(\theta) = \underbrace{\theta}_{\text{efficiency}} + \underbrace{\frac{\int_{\theta}^{\overline{\theta}} [\omega(s) - \alpha] \mathrm{d}F(s)}{\alpha f(\theta)}}_{\text{redistributive motive}} + \underbrace{\frac{\max\{\mathbf{E}_{\theta}[\omega(\theta) - \alpha], 0\} \underline{\theta} \delta_{\theta=\underline{\theta}}}{\alpha f(\theta)}}_{\text{(NLS) constraint}}$$

Call $J(\theta) - \theta$ the "distortion term."

return

Decreasing Welfare Weights, High Cost of Public Funds



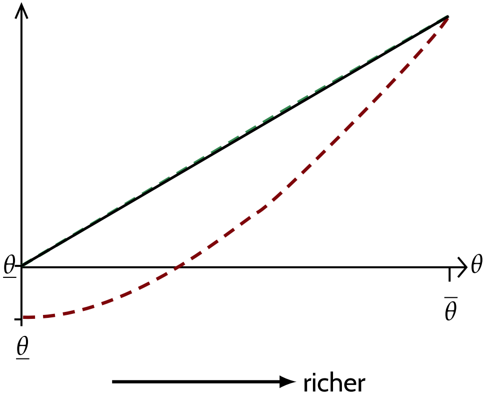
High cost of public funds: $\mathbf{E}[\omega(\theta)] \leq \alpha$.

$$J(\theta) = \theta + \frac{\int_{\theta}^{\overline{\theta}} [\omega(s) - \alpha] \, dF(s)}{\alpha f(\theta)}$$

is always *below* lower bound θ because distortion is single-crossing from above, negative at $\underline{\theta}$ and zero at $\overline{\theta}$.

▸ return

Decreasing Welfare Weights, High Cost of Public Funds



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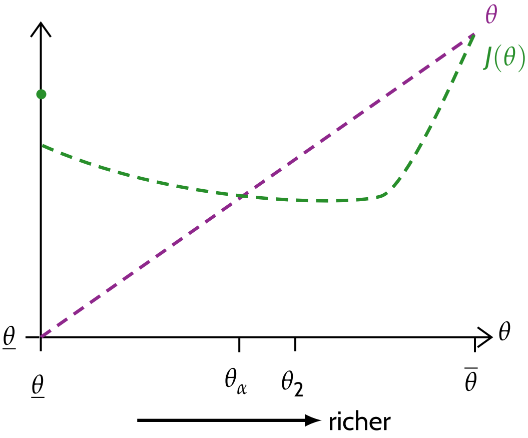
$\rightsquigarrow$ Subsidy type $H(\theta) = \theta$ and optimal allocation is laissez-faire.

return

Decreasing Welfare Weights, Low Cost of Public Funds

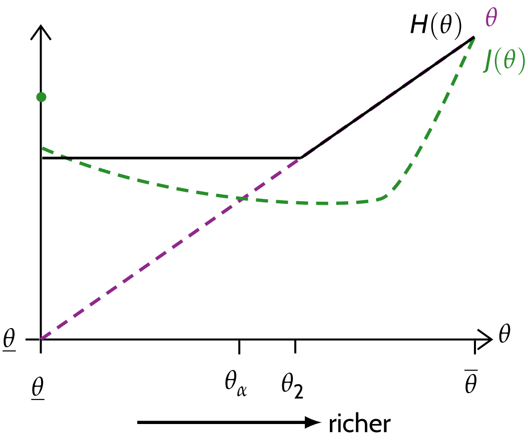


Low cost of public funds: $\mathbf{E}[\omega(\theta)] > \alpha$.

$J(\theta) = \theta + \frac{\int_{\theta}^{\overline{\theta}} [\omega(s) - \alpha] \, dF(s) + (\mathbf{E}_\theta[\omega(\theta)] - \alpha)\underline{\theta}\delta_{\theta=\underline{\theta}}}{\alpha f(\theta)}$ crosses lower bound constraint θ from *above* because distortion term is single-crossing from above, positive at $\underline{\theta}$ and zero at $\overline{\theta}$.

▸ return

Decreasing Welfare Weights, Low Cost of Public Funds



Low cost of public funds: $\mathbf{E}[\omega(\theta)] > \alpha$.

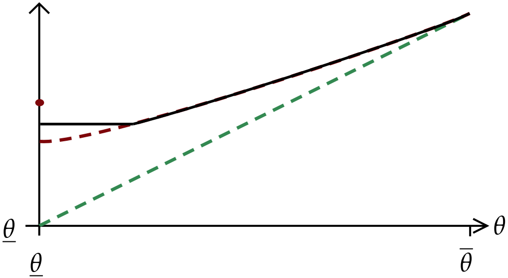
$J(\theta) = \theta + \frac{\int_{\underline{\theta}}^{\overline{\theta}}[\omega(s)-\alpha]\, dF(s) + (\mathbf{E}_\theta[\omega(\theta)]-\alpha)\underline{\theta}\delta_{\theta=\underline{\theta}}}{\alpha f(\theta)}$ crosses lower bound constraint θ from *above* because distortion term is single-crossing from above, positive at $\underline{\theta}$ and zero at $\overline{\theta}$.

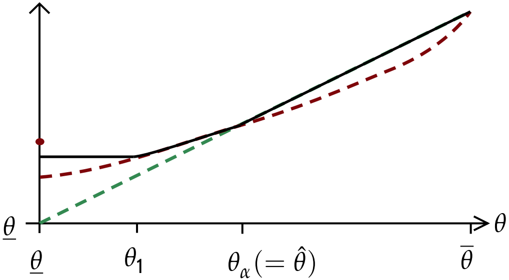
$\rightsquigarrow$ Subsidy type $H(\theta) > \theta$ for $\theta \leq \theta_2$. There is a free endowment of $q^{LF}(\theta_2)$, which strictly exceeds $q^{F}(\theta)$ for $\theta \leq \theta_2$ …*but* planner always prefers a lump-sum subsidy.

return

Decreasing Welfare Weight, Low Cost of Public Funds

Other possibilities:





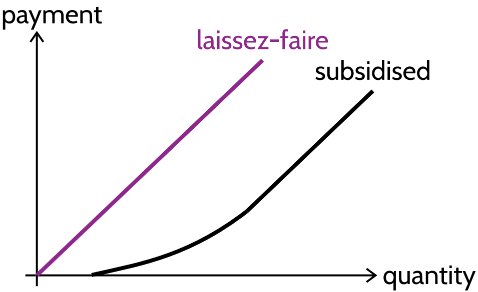
Free endowment + quantity-dependent subsidies distorting all types' consumption upwards (no topping up).

Free endowment + quantity-dependent subsidies up to a cap (high types top up in private market).

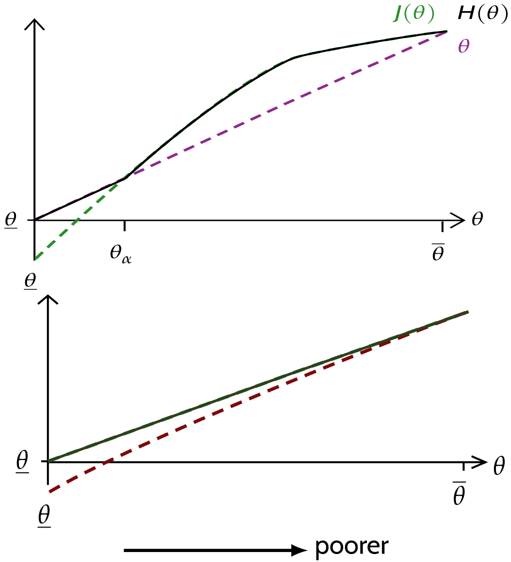
▸ return

Decreasing Welfare Weight, Low Cost of Public Funds

Payment schedule:



Increasing Welfare Weights, High Cost of Public Funds



High cost of public funds: $\mathbf{E}[\omega(\theta)] \leq \alpha$.

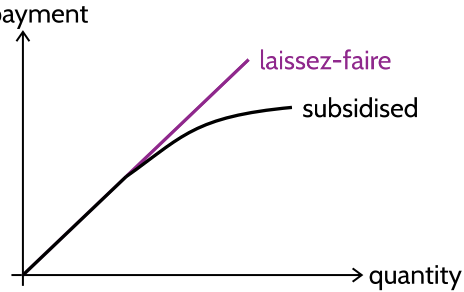
$$J(\theta) = \theta + \frac{\int_{\theta}^{\overline{\theta}} [\omega(s) - \alpha] \, \mathrm{d}F(s)}{\alpha f(\theta)}$$

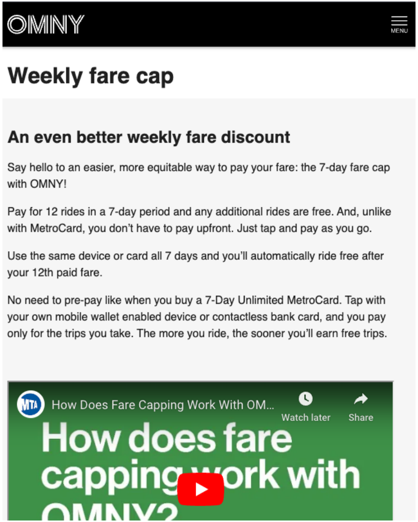
can cross lower-bound θ from *below* because the distortion term is single-crossing from below, negative at $\underline{\theta}$ and zero at $\overline{\theta}$.

$\rightsquigarrow$ Subsidy type *can* exceed θ for high types: implemented by offering discounts for consumption *above* a minimum level... *preferred* by planner to lump-sum transfer.

Increasing Welfare Weights, High Cost of Public Funds

Payment schedule:




OMNY

MENU

Weekly fare cap

An even better weekly fare discount

Say hello to an easier, more equitable way to pay your fare: the 7-day fare cap with OMNY!

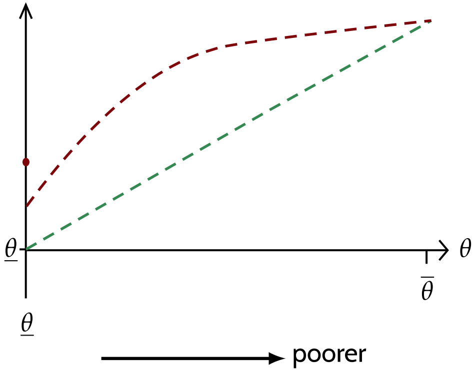
Pay for 12 rides in a 7-day period and any additional rides are free. And, unlike with MetroCard, you don't have to pay upfront. Just tap and pay as you go.

Use the same device or card all 7 days and you'll automatically ride free after your 12th paid fare.

No need to pre-pay like when you buy a 7-Day Unlimited MetroCard. Tap with your own mobile wallet enabled device or contactless bank card, and you pay only for the trips you take. The more you ride, the sooner you'll earn free trips.

return

Increasing Welfare Weights, Low Cost of Public Funds



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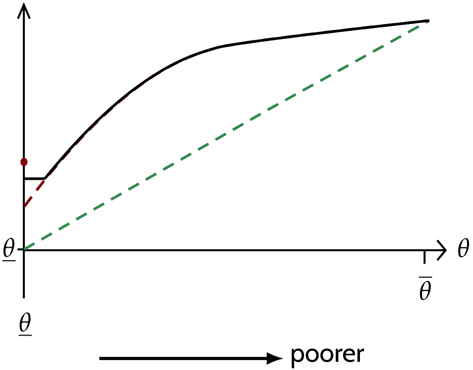
$\leadsto$ Subsidy type *exceeds* θ for all types: implemented via a free allocation and discounts for additional consumption. No consumers top up.

return

Increasing Welfare Weights, Low Cost of Public Funds



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$\rightsquigarrow$ Subsidy type *exceeds* θ for all types: implemented via a free allocation and discounts for additional consumption. No consumers top up.

return

Food Stamps (SNAP)

- **Overview**: U.S. program providing monthly food assistance to low-income individuals and families.
- **Initial Support**: Full subsidy up to a fixed dollar amount per month for eligible food items.
- **Free Endowment**: The subsidy starts as a full benefit and decreases after benefits are exhausted.
- **Low Consumption Focus**: Ensures a basic level of nutrition by covering initial consumption entirely.

'return

Women, Infants, and Children (WIC)

- **Overview**: Nutritional assistance for low-income pregnant women, new mothers, and young children.
- **Initial Support**: Vouchers for essential foods like milk, eggs, baby formula.
- **Free Endowment**: Recipients get fully subsidised quantities of specific foods.
- **Low Consumption Focus**: Prioritizes providing a free minimum quantity of nutritious food to families.

return

Housing Choice Voucher Program (Section 8)

- **Overview**: subsidised housing assistance for low-income renters in the U.S.
- **Initial Support**: Covers a large portion of rent (up to 70%) for qualifying households.
- **Free Endowment**: A significant rent portion is initially fully subsidised.
- **Low Consumption Focus**: Ensures low-income renters pay only a small portion of their rent.

return

Lifeline Program

- **Overview**: U.S. program offering discounted phone and internet services to low-income households.
- **Initial Support**: Monthly discounts on basic telecommunication services.
- **Free Endowment**: Full subsidy of basic services for the most disadvantaged users.
- **Low Consumption Focus**: Provides essential access to communication services with high initial subsidies.

return

National School Lunch Program (NSLP)

- **Overview**: Provides free or reduced-price school meals for low-income students.
- **Initial Support**: Fully subsidised meals for eligible students based on family income.
- **Free Endowment**: Full meal subsidies provided for families below certain income thresholds.
- **Low Consumption Focus**: Ensures children receive at least one nutritious meal per day at no or low cost.

return

Australian Better Access Mental Health Initiative

- **Overview**: Australian government program subsidising mental health services.
- **Initial Subsidy**: Up to 10 Medicare-subsidised sessions per year.
- **Additional Support**:
  - After initial sessions and doctor approval, become eligible for extra free/subsidised sessions.
  - Increased subsidy ensures access for those needing more care.

return

Australia's Child Care Subsidy (CCS)

- **Overview**: Government subsidy for childcare costs based on income and activity levels.
- **Initial Subsidy**: Covers a percentage of childcare fees up to a set number of hours.
- **Additional Support**:
  - Subsidy percentage **increases** as parents work, study, or volunteer more.
  - More hours of work/study lead to higher subsidies for additional childcare hours.
  - More children leads to higher subsidies per child.

return

Public Transit Fare Capping (New York)

- **Overview**: MTA system caps daily/weekly public transit fares.
- **Initial Fare**: Riders pay per trip up to a set limit.
- **Additional Support**:
  - After reaching the cap, **additional rides are free** for the rest of the day/week.
  - Frequent riders benefit from larger subsidies after hitting the cap.
- **Other Cities With Similar Programs:** SF Bay Area, Portland, London, Dublin, Toronto, Vancouver, Los Angeles, Singapore, Sydney, Brisbane, Melbourne, Perth, Auckland.

return

Pharmaceutical Subsidy Programs: Australia, Norway, Sweden, Denmark

- **Overview**: Government programs reducing out-of-pocket medication costs.
- **Australia (PBS)**: subsidises prescription medicines; costs decrease after a yearly threshold (safety net) is reached.
- **Helsenorge (Norway)**: Covers up to 90% of prescription costs after reaching an annual expenditure cap.
- **Sweden**: Once a patient reaches a yearly spending threshold, additional medications are free.
- **Denmark**: Progressive subsidy structure, with higher reimbursements as individual spending increases.

return

Cost-Sharing Reductions (CSRs) and Eligibility Limits

ACA Cost-Sharing Reductions (CSRs)

- **What are CSRs?**
  - Subsidies that lower out-of-pocket costs (e.g., co-pays, deductibles).
  - Available to individuals/families with incomes between 100% and 250% of the Federal Poverty Level (FPL).
- **Eligibility Tied to Lower Insurance Plans**
  - To qualify for CSRs, you *must* purchase a **Silver-level** plan on the ACA marketplace.
  - Other plan tiers (**Bronze, Gold, or Platinum**) **do not** offer CSRs, even if you're income-eligible.
  - Silver plans have a standard **70% actuarial value**, but CSRs raise it to up to **94%** for lower-income enrollees.
- **Impact of Limiting to Silver Plans**
  - Higher-income individuals may choose other plan levels, but lose CSR eligibility.
  - Lower-income enrollees are incentivised to choose Silver plans to reduce out-of-pocket costs.

return

German Health System: Prohibition of Topping Up

- **Public Health Insurance**: Citizens covered by statutory health insurance (SHI) cannot "top up" SHI with private insurance for services already covered.
- **Supplementary Insurance**: Private insurance can only be used for services not included in SHI (e.g., private rooms, certain dental services).
- **Comprehensive Coverage**: SHI already covers essential medical services, discouraging the need for topping up with private health plans.

return

Public Education: Prohibition of Private Tutoring in China & South Korea

- **Public Education**: Both China and South Korea provide universal public education for students, with restrictions on private supplementary tutoring.
- **Prohibition**: Private tutoring and after-school programs are heavily regulated or banned to prevent parents from "topping up" public education with private instruction.
- **Equal Access**: The aim is to reduce inequality in educational opportunities and prevent wealthier families from gaining an advantage through private education.

return

Public Housing

- **Public Housing Programs**: Residents in public housing receive heavily subsidised rent, often capped at a percentage of their income.
- **Prohibition**: Participants must choose between living in public housing or renting in the private market; they cannot "top up" their public housing subsidy to rent a private apartment.
- **Example Cities and Countries**:
  - **Singapore**: The Housing & Development Board (HDB) provides subsidised flats, and participants cannot receive additional subsidies to live in private housing.
  - **Vienna, Austria**: The city's extensive public housing program offers low-cost rental units, with no option to "top up" for private market rentals.
  - **Hong Kong**: The Public Rental Housing (PRH) program offers heavily subsidised apartments, and recipients must choose between public housing and private market rentals.

return

Egypt's Tamween Food Subsidy Program

- **Overview**: The Tamween program is one of the largest food subsidy systems in the world, providing essential goods to over 60 million Egyptians, mostly from low-income households.
- **Targeted Subsidy**:
  - **Bread**: Heavily subsidised at a fraction of market price (often less than 10% of the actual cost), making it affordable for the poor, who rely on it as a staple.
  - **Other Essentials**: Subsidies also cover rice, sugar, and cooking oil, basic items central to the diets of low-income families.
- **Exclusion**:
  - **Meat and Dairy**: These more expensive food items, consumed more frequently by wealthier households, are not subsidised. Consumers must pay market prices for these products.

return

Indonesian Fuel Subsidy Program: Pertamina

- **Overview**: Indonesia's fuel subsidy program supports transportation for low-income households.
- **Targeted Subsidy**: The program subsidises low-octane fuel, which is primarily used by motorcycles, the preferred transport mode for poorer citizens.
- **Exclusion**: High-octane fuel, more commonly used by cars owned by wealthier households, is not subsidised.

return

CalFresh Restaurant Meals Program

- **Overview**: California's CalFresh program allows certain populations to use benefits for prepared meals.
- **Targeted Subsidy**: The program subsidises meals, predominantly from fast food restaurants, providing affordable food options for homeless, elderly, and disabled individuals.
- **Exclusion**: Dine-in restaurants, typically frequented by wealthier individuals, are not included in the subsidy.

return

Public Dentistry Programs in Australia

- **Overview**: Australia's public dentistry programs provide dental care subsidies to low-income individuals.
- **Targeted Subsidy**: Prior to 2016, the program subsidised only amalgam fillings, a durable and cost-effective option used widely by lower-income patients.
- **Exclusion**: Composite (tooth-colored) fillings, which are more expensive and preferred by wealthier individuals, were not fully subsidised.
- **Post-2016**: amalgam fillings are being phased out due to mercury content.

return