Mathematical Statistics

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Contents

§1 Introduction

Teacher: Liu Liping

References:

- Chen Jiading, *Mathematical Statistics handouts*, third edition;
- D. Freedman, *Statistics*;
- Lehmann, *Theory of Point Estimation*, John Wiley and Sons;
- Lehmann, *Testing Statistical Hypothesis*, John Wiley and Sons.

Differences between Statistics and Probablistics:

- Clearer backgrounds;
- "right" or "wrong" vs. "good" or "bad";
- Different research method, more computation and simulation;
- Different ways of thinking.

Contents of this course:

- Focusing on common backgrounds, common thoughts and classic methods, don't focus on rigorous proofs.
- Chapter 2–4, first 2 sections of chapter 5, first 2 sections of chapter 7.

Statistical regularity: Law of large numbers, Central limit theorem, Law of iterated logarithm, etc.

§1.1 Basic concepts of statistics

- Totality and individuals: denoted by distribution function $F(\cdot)$ or random variable X.
- Samples: denoted by $X_1, \ldots, X_n$ or $x_1, \ldots, x_n$. We call n the sample size. If we assume independent random identically distribution (iid), call them simple random samples.
- Statistical magnitude: (measurable) sample function $g(x_1, \ldots, x_n)$.
- and more...

§1.2 Parameter estimation

Backgrounds: We already know the distribution type, but do not know the parameters. We hope to give an estimation of parameters from sample data.

Maximum Likelihood Estimate (MLE)

Definition 1.1 (Likelihood function)**.** We call

$$L(x_1, \ldots, x_n; \theta) = \prod_{i=1}^{n} f(x_i; \theta).$$

is the likelihood function of parameter θ about samples $x_1, \ldots, x_n$.

We say the maximum point of likelihood function $\hat{\theta}_n = \hat{\theta}$ is the MLE of this sample.

MLE is the most important method in classic statistics, and it should be considered first.