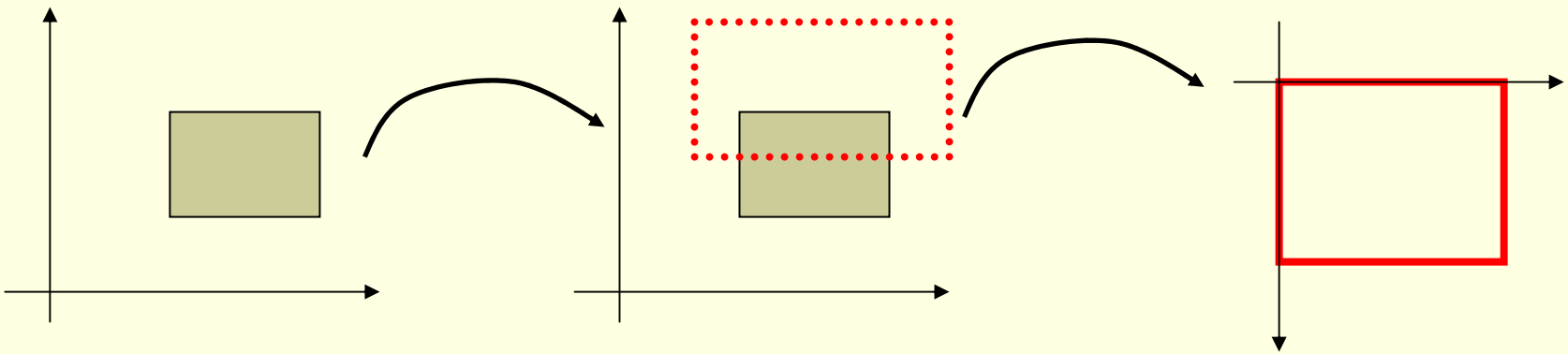
CS 335
Graphics and Multimedia

Clipping

Clipping Lines and Polygons

What is clipping?

Goal: only render what will be visible in the drawing window; eliminate from the rendering pipeline all those primitives which will not appear in the final viewport



World coordinate system Window in WCS Viewport on CRT

Clipping Approaches

Individual Pixel Tests:

- Render at the pixel level
- Check pixel for containment in the target region

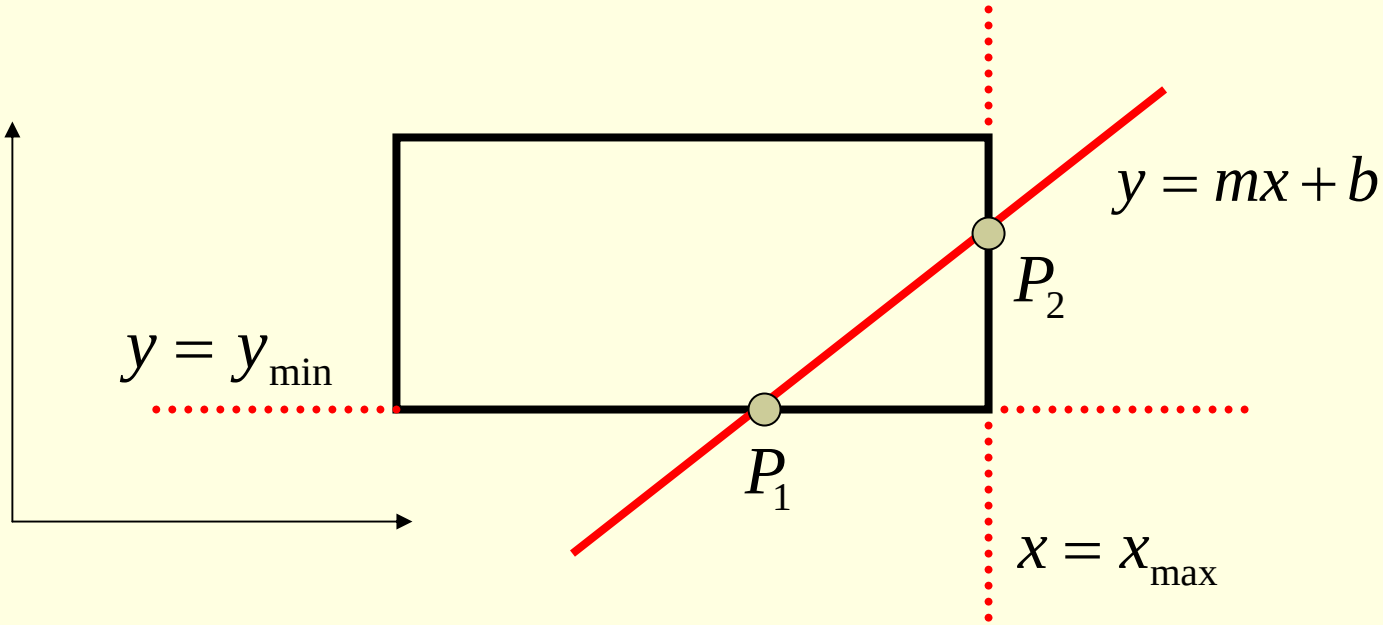
Maintain Large Drawing Area:

- Render the entire set of primitives in the WCS
- Display only the portion of the set which overlaps the viewport

Analytical Methods:

- Compute the intersections of primitives with the clip region
- Eliminate outlying portions of the primitives

Analytical Clipping



$$P_1 = \{y = y_{\min}\} \cap \{y = mx + b\}$$

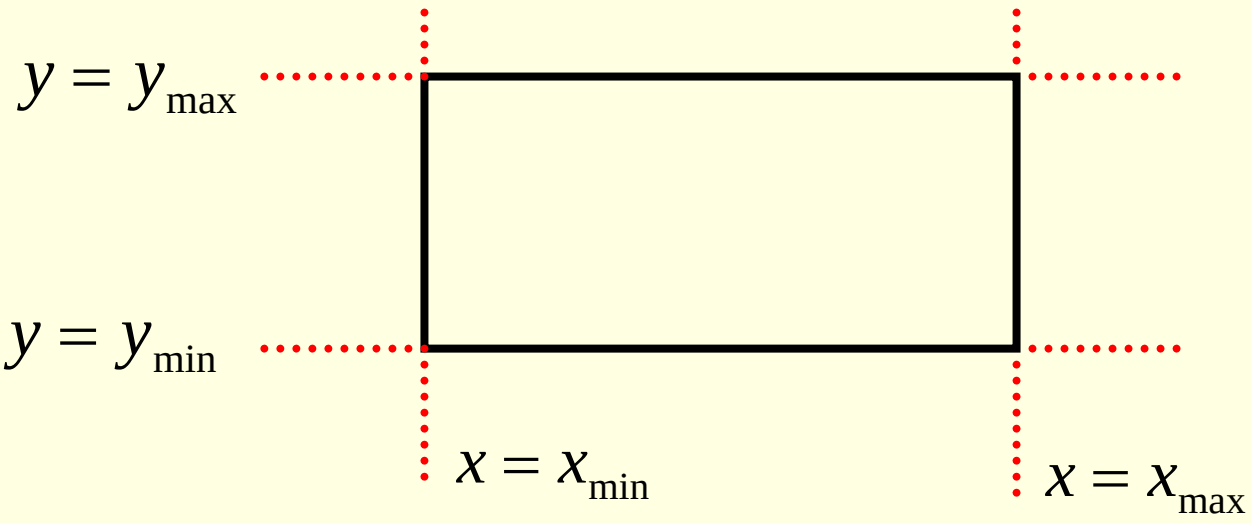
$$P_2 = \{x = x_{\max}\} \cap \{y = mx + b\}$$

Brute force method: compute all intersections of line with clip rectangle

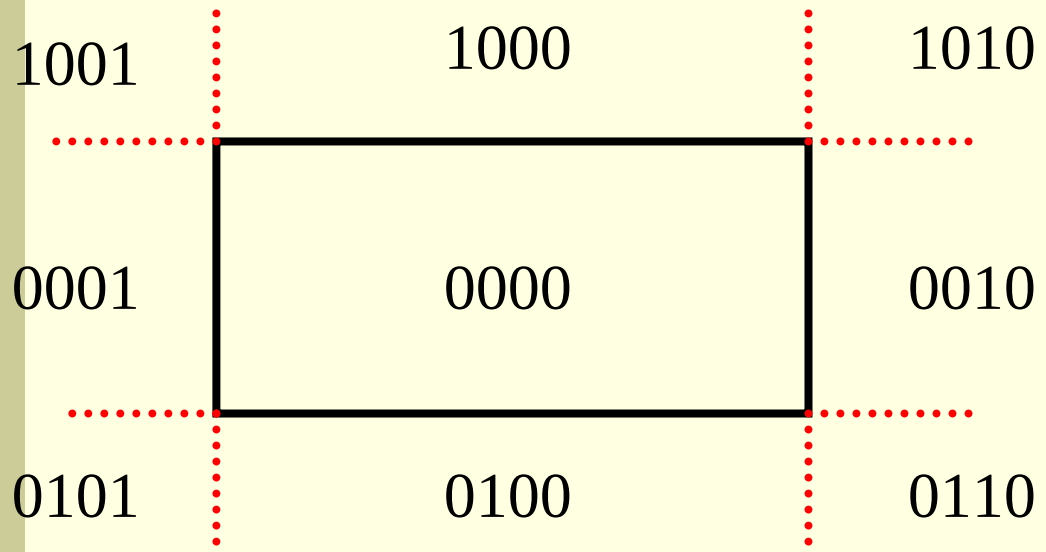
Cohen-Sutherland Clipper

- Use *regions* to accept/reject lines
- Assume the clipping region is an axis-oriented rectangle

Regions are labeled according to the half-planes defined by the clipping region:



Regions



$$\text{top}: y > y_{\max}$$
$$\text{bottom}: y < y_{\min}$$
$$\text{right}: x > x_{\max}$$
$$\text{left}: x < x_{\min}$$

Assign a bit to each condition to form a code for the nine regions

$$\text{bit } 1: \text{ sign bit of } (y_{\max} - y)$$
$$\text{bit } 2: \text{sign bit of } (y - y_{\min})$$
$$\text{bit } 3: \text{sign bit of } (x_{\max} - x)$$
$$\text{bit } 4: \text{sign bit of } (x - x_{\min})$$

The Cohen-Sutherland Algorithm

Let c1 and c2 be the region codes for the endpoints of the line to be clipped.

Trivially **Accept:**

Both codes are 0000 ⇒ segment is inside the clip region

Trivially **Reject:**

(code 1) && (code 2) are not equal to zero ⇒

Line is outside clip rectangle

(both endpoints must be on the same side of one of the four clip lines in order to yield a bitwise & that is non-zero)

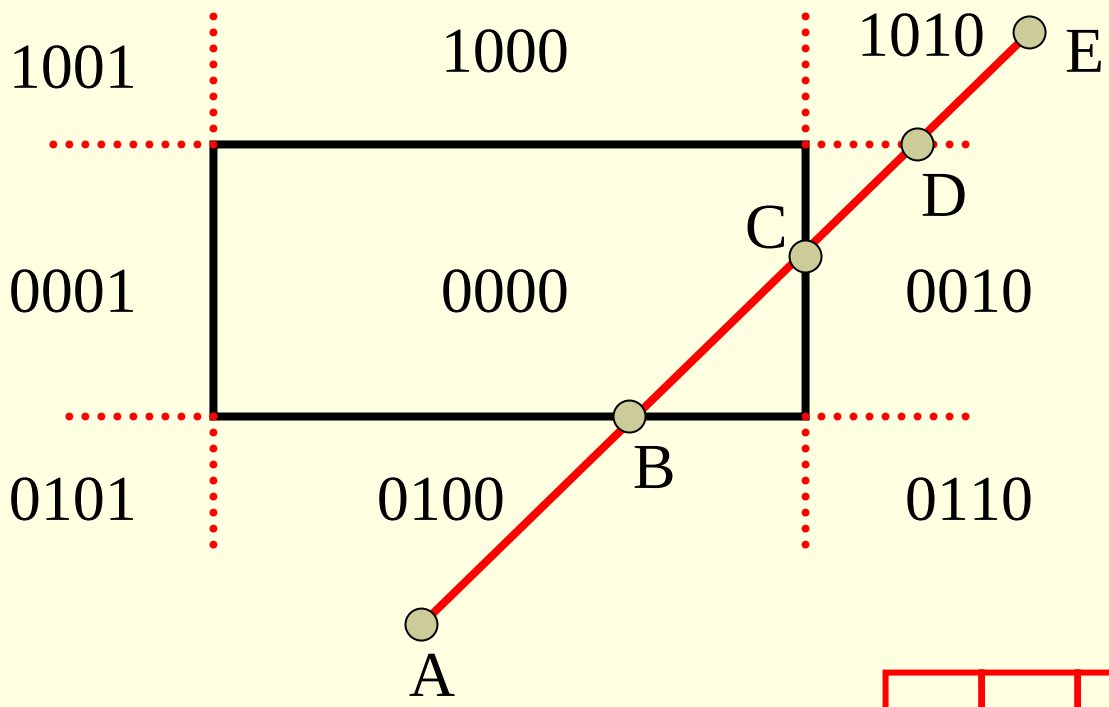
Iterative Part of the Algorithm

Repeat until the segment cannot be trivially accepted or rejected:

- Subdivide the segment:
  - Pick the endpoint which is outside the clip rectangle (one must be outside)
  - Find the first non-zero bit: this corresponds to the clip edge which intersects the line
  - Compute the intersection of the line with the edge
  - Throw away the outside vertex up to the clip rectangle

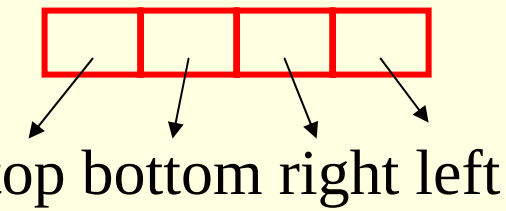
$$\{y = mx + b\} \bigcap \{y = x_{\min}\} \Rightarrow x_{\min} = mx + b \Rightarrow x = \frac{x_{\min} - b}{m}$$

An Example

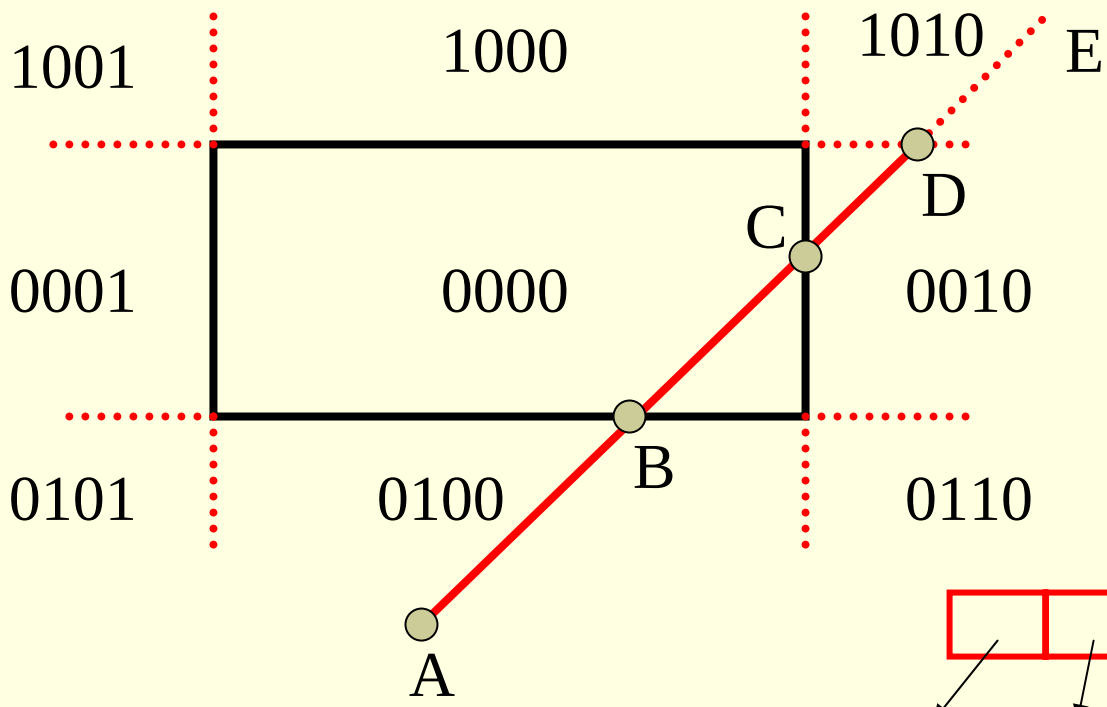


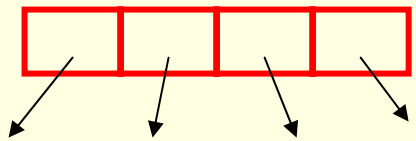
E: 1010
A: 0100
&: 0000

Subdivide using
either A or E



Example, Con't



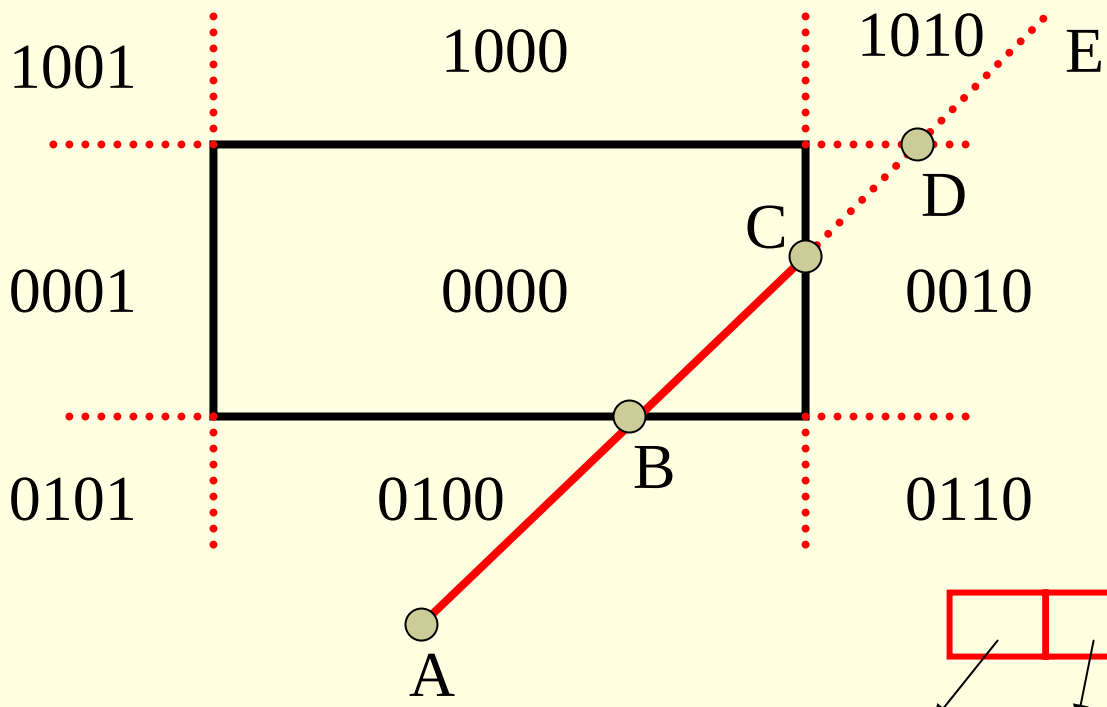


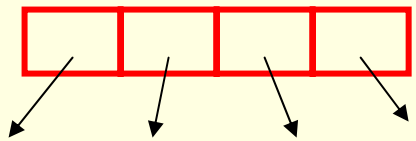
top bottom right left

D: 0010
A: 0100
&: 0000

Subdivide: Third bit of D indicates that the line intersects the right clip line

Example (Con't)



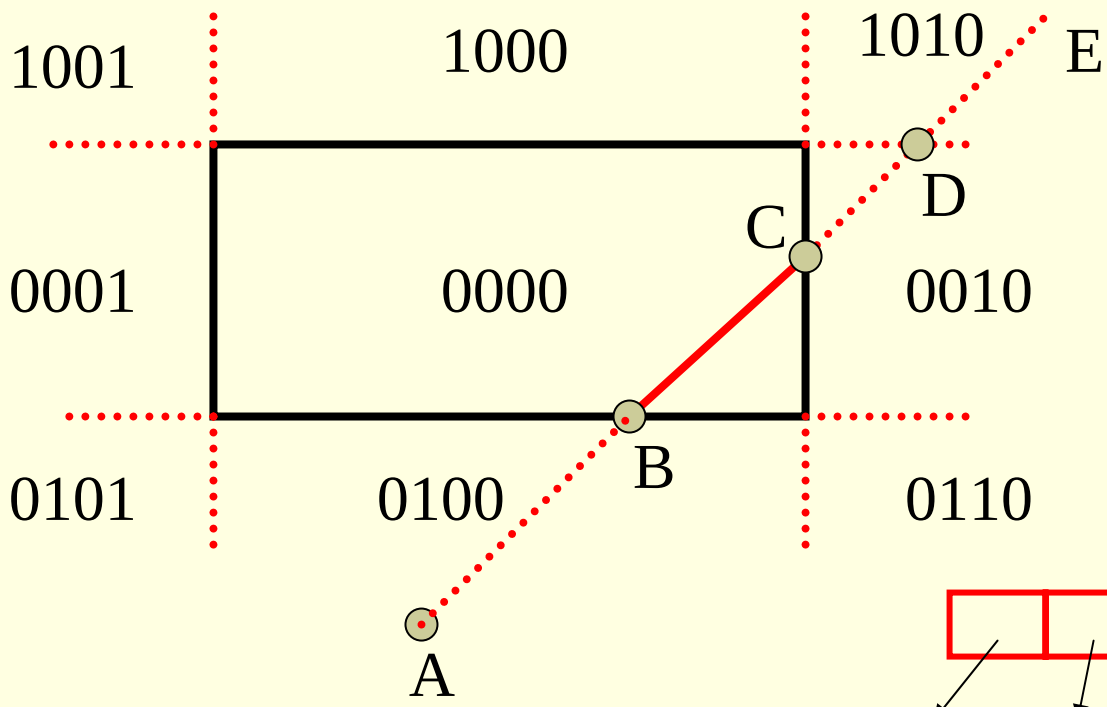


top bottom right left

C: 0000
A: 0100
&: 0000

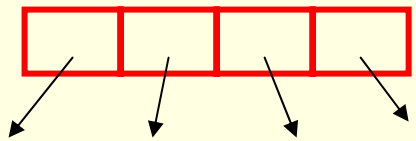
Subdivide: Second bit of A indicates that the line intersects the bottom clip line

Example: Conclusion



C: 0000
B: 0000
&: 0000

Trivially accept the clipped segment BC



Cohen-Sutherland Summary

Disadvantages:

- Fixed-order decision can do needless work
- Can improve using more regions (Nicholl-Lee-Nicholl Alg)
- Can generate more efficient rejection tests
- Clipping window must be rectangular

Advantages:

- Simple to implement
- Oriented for most simple window/viewport systems
- Extends to 3-D cubic volumes

The Liang-Barsky Line clipper (parametric line clipping)

Parametric Line

$$x = x_1 + t\Delta x$$

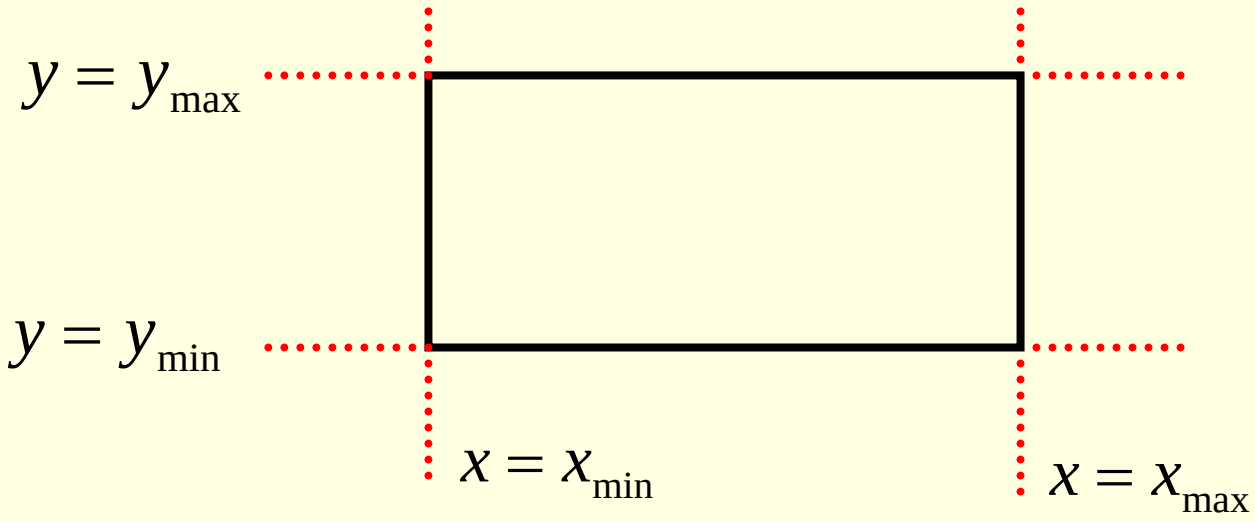
$$y = y_1 + t\Delta y$$

where

$$\Delta x = x_2 - x_1, \Delta y = y_2 - y_1$$

$$t \in [0,1]$$

Clipping Condition

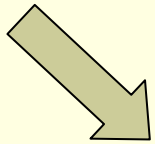


$$x_{min} \le x_1 + t\Delta x \le x_{max}$$

$$y_{min} \le y_1 + t\Delta y \le y_{max}$$

Intersection Conditions

$$x_{\min} \le x_1 + t\Delta x \le x_{\max}$$
$$y_{\min} \le y_1 + t\Delta y \le y_{\max}$$



$$tp_k \le q_k$$

where

$$p_1 = -\Delta x, \quad q_1 = x_1 - x_{\min}$$
$$p_2 = \Delta x, \quad q_2 = x_{\max} - x_1$$
$$p_3 = -\Delta y, \quad q_3 = y_1 - y_{\min}$$
$$p_4 = \Delta y, \quad q_4 = y_{\max} - y_1$$

Conditions

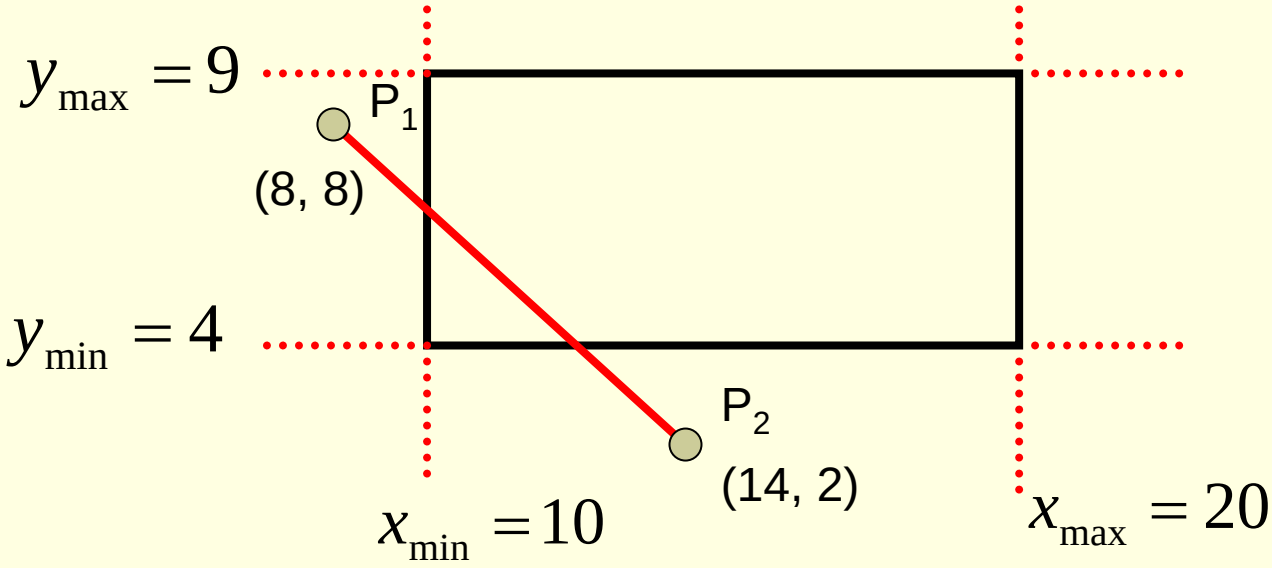
- If ($p_k = 0$) – parallel line to one of the clipping edge
  - ($q_k < 0$) – outside, reject
  - ($q_k \geqslant 0$) – inside
- If ($p_k < 0$) – from outside to inside
  - Intersection at: $t = q_k / p_k$ (OIP)
- If ($p_k > 0$) – from inside to outside
  - Intersection at: $t = q_k / p_k$ (IOP)

Key Insight:
we need to find two intersection points,
one OIP, one IOP.

Intersection Point Computation

- For ($p_k < 0$) (Outside to inside point)
  - $r_k = q_k / p_k$
  - OIP $t_1 = \max(r_k, 0)$
- For ($p_k > 0$) (Inside to outside point)
  - $r_k = q_k / p_k$
  - IOP $t_2 = \min(r_k, 1)$
- If ($t_1 > t_2$), complete outside, REJECT
- Else clipped line is between (t_1, t_2)

Liang-Barsky Line Clipping Example

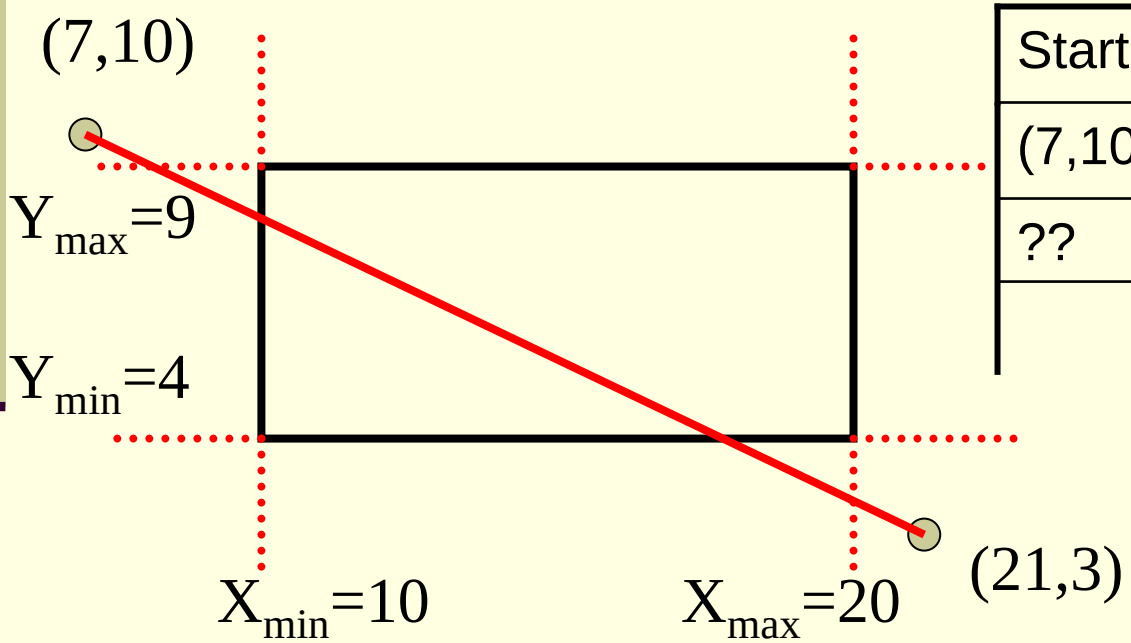


Summary: Liang-Barsky

- (x,y) coordinates are only computed for the two final intersection points
- At most 4 parameter values are computed
- This is a non-iterative algorithm
- Can be extended to 3-D

Take Home Exercise

- Work through the Cohen-Sutherland Clipping Algorithm using the following example



Start	Code	End	Code
(7,10)	1001	(21,3)	0110
??			

Take Home Exercise

- Clip the following line using the Liang-Barsky algorithm