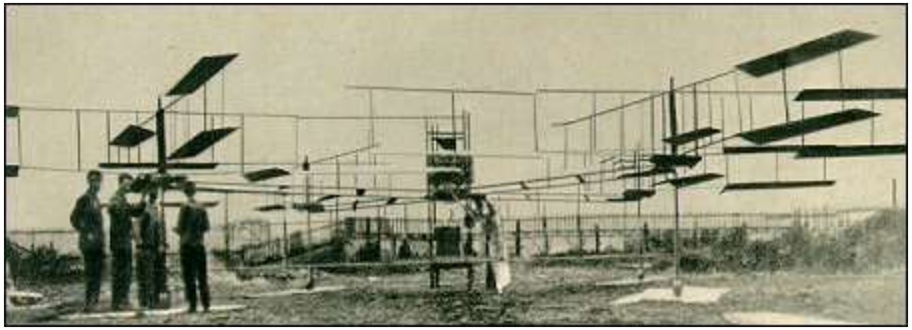
Quadcopter Dynamics

Bréguet Richet Gyroplane No. 1 1907

- Brothers Louis Bréguet and Jacques Bréguet
- Guidance of Professor Charles Richet
- The first flight demonstration of Gyroplane No. 1 with no control surfaces was achieved on 29 September 1907.



https://www.youtube.com/watch?v=dff2erD_1PU

Jerome-**de Bothezat** Flying Octopus

- Georges de Bothezat and Ivan Jerome in 1922,
- 6-bladed rotors placed at each end of an X-shaped truss structure
- Built for US Army
  - complexity, control difficulties, and high pilot workload, only capable of forward flight in a favorable wind

Étienne Œhmichen 1924

- Étienne Œhmichen in 1924
- Set distance records
  - first 1km helicopter flight. 7 mins:40 secs

UAS Categories

- Fixed wing
  - Better range
  - Better performance
- Rotary wing
  - higher degree of freedom
  - low speed flying
  - stationary flights
  - indoor usage

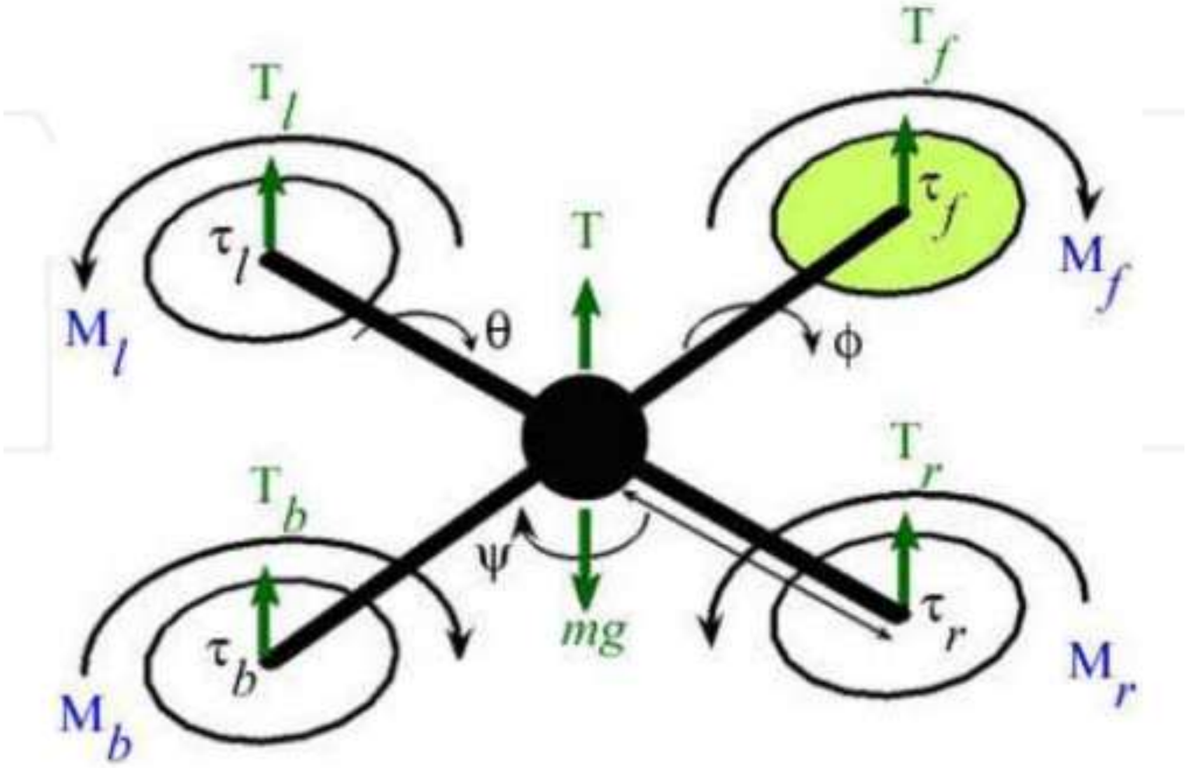
Quadcopter

- 4 rotors located at the ends of a cross structure
  - higher payload capacity
  - Maneuverability (e.g. traversing an environment with many obstacles, or landing in small areas)
- Controlled by varying the speeds of each rotor
  - Vertical Take Off and Landing (VTOL)
  - hovering capability
  - slow precise movements
  - . There are also definite advantages to having a four rotor based propulsion system, such as a

Quadcopter Attitude Control

Mx = Motor direction
Tx = Thrust force direction

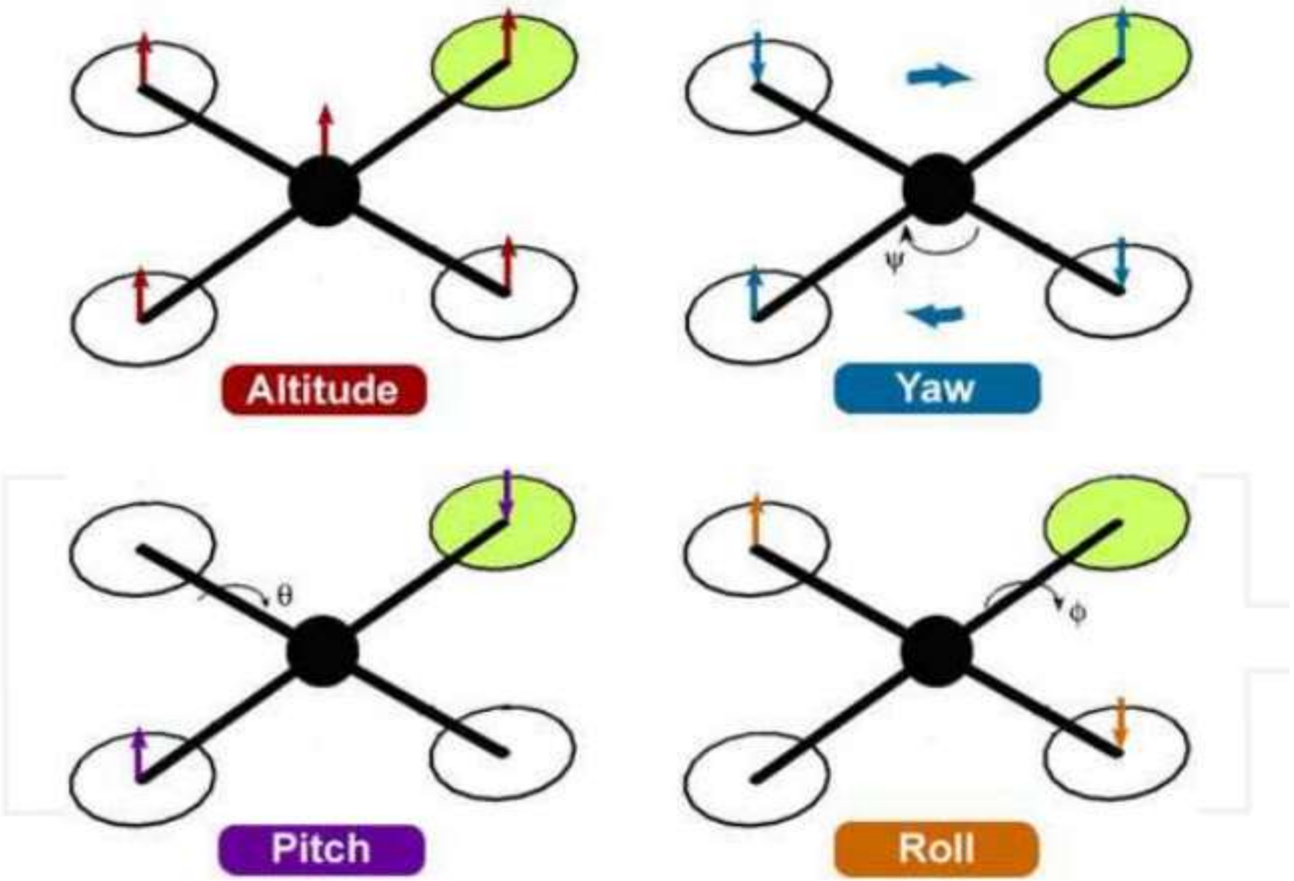
f = front
l = left
r = right
b = back



Quadcopter Attitude Control

- Vary rotation speed of each motor
- Front Rotor (Mf) and Back Rotor (Mb) pair rotates in a clockwise direction
- Right Rotor (Mr) and Left Rotor (Ml) pair rotates in a counter-clockwise direction
  - Configuration to balance the drag created by each of the spinning rotor pairs

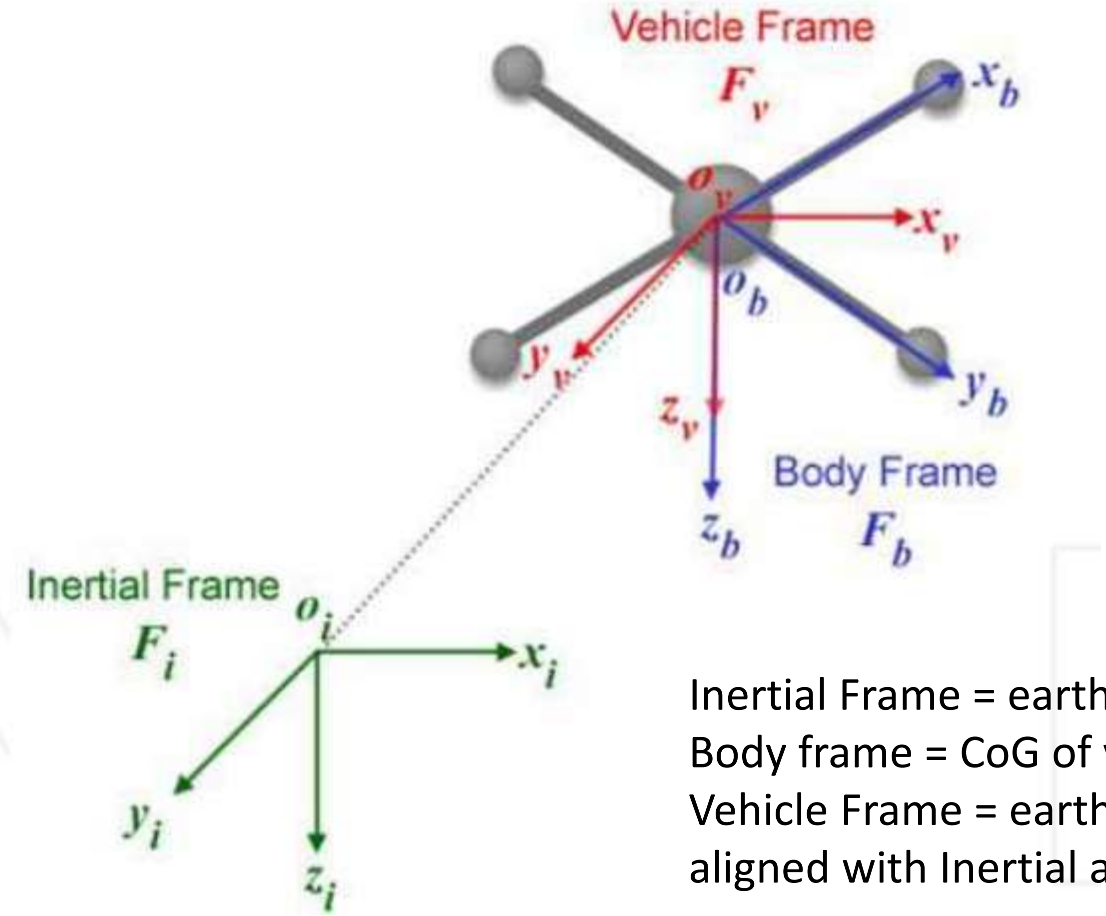
Four Maneuvers



Four Maneuvers

- Roll Angle:
  - Change relative speed of the right and left rotors
- Pitch Angle
  - Change relative speed of the front and back rotors
- Yaw Angle
  - Change speed of clockwise rotating pair and counter-clockwise rotating pair
- Vertical
  - Increasing or decreasing the speeds of all four rotors simultaneously controls the collective thrust

Reference Frames



Inertial Frame = earth-based origin at launch location
Body frame = CoG of vehicle aligned along frame
Vehicle Frame = earth-based origin at CoG of Vehicle aligned with Inertial axes

Reference Frames

- Inertial
  - earth-fixed coordinate system
  - origin located on the ground (e.g. base station)
  - Convention:
    - x-axis points towards the north
    - y-axis points towards the east
    - z-axis points towards the center of the earth.
- Body
  - origin located at the center of gravity (COG) of the quadrotor
  - axes aligned with the quadrotor structure
    - x-axis is along the arm with front motor
    - y-axis is along the arm with right motor
    - z-axis cross product of x and y
- Vehicle
  - inertial frame with the origin located at the COG of the quadrotor
  - Vehicle frame has two variations, Fϕ and Fθ
    - Fϕ is the vehicle frame
      - Fv, rotated about its z-axis by an angle ψ so that and are aligned with and , respectively.
    - Fθ is frame Fϕ rotated about its y-axis, , by a pitching angle, θ, such that xθ and yθ are aligned with and x_b and z_b

Transforming Reference Frames

- Translation and rotation matrices are used to transform one coordinate reference frame into another desired frame of reference
  - Transformation from F_i to F_v provides the displacement vector from the origin of the Inertial frame to the center of gravity (COG) of the quadrotor Vehicle
  - Transformation from F_v to F_b is rotational in nature → the roll, pitch and yaw angles.

Quadrotor Kinematics

- Quadrotor Position fro Frame F

$P^T_F = [p_x, p_y, -p_z]$

- Quadrotor Orientation for Frame F

$\Omega^T_F = [\Phi, \theta, \Psi]$

- Quadrotor Speed

$$\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ -\dot{p}_z \end{bmatrix}_{F_i} = \begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{p}_z \end{bmatrix}_{F_v} = \begin{bmatrix} R^{Fb}{}_{Fv} \end{bmatrix}^T \begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{p}_z \end{bmatrix}_{F_b}$$

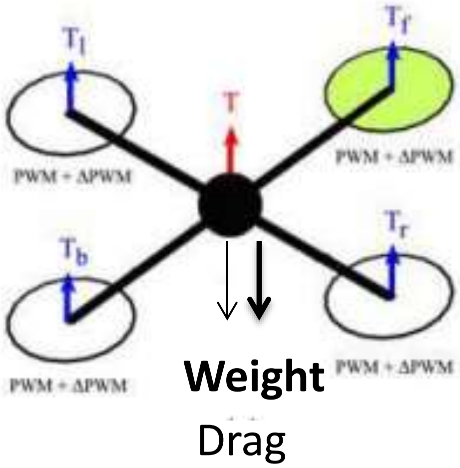
$\begin{bmatrix} R^{Fb}{}_{Fv} \end{bmatrix}^T$ = Translational matrix v → b

Quadrotor Dynamics (Vertical Axis Only)

Total Thrust =
Thrust front motor +
Thrust back motor +
Thrust left motor +
Thrust right motor

Weight (N) = mass (Kg) * gravitational constant (m/s^2) = 9.8

Drag (N) = 0.5 * ρ * V^2 * C_D * Surface Area



Forces on Quadcopter
(in body vertical axis)

Quadrotor Dynamics: Takeoff to Hover

$ma = \Sigma F$

Hover

$a_z = 0$, $a_x = a_y = 0$
Sum forces in Body Z axis

$0 = T – W$
$0 = T – mg$
$T = mg$
Thrust required to hover =
thrust to overcome weight

Vertical Takeoff (i.e. stationary to climb velocity

$a_z > 0$, $a_x = a_y = 0$
Sum forces in Body Z axis

$ma_z = T – W - D$
$ma_z = T – mg - D$
$T = mg + ma_z + D$
Thrust required to takeoff =
thrust to overcome weight +
thrust to overcome inertia +
thrust to overcome Drag

Drag = $0.5\ \rho\ V^2\ C_D$ Surface Area

Quadrotor Dynamics: Takeoff to Hover

$ma = \Sigma F$

Constant Speed Vertical Climb

$a_z = 0$, $a_x = a_y = 0$

Sum forces in Body Z axis

$0 = T – W - D$

$0 = T – mg - D$

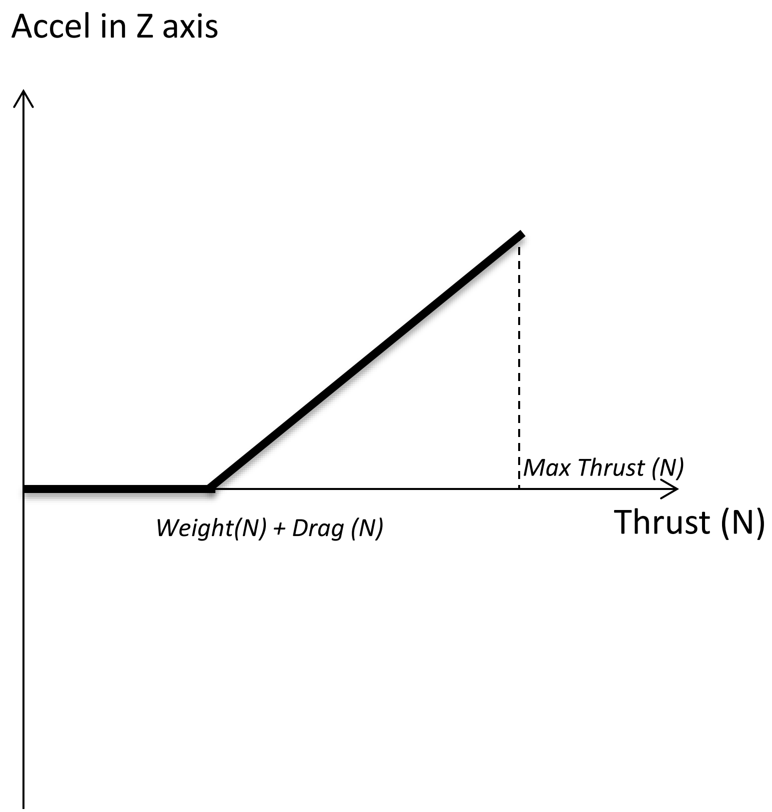
$T = mg + D$

Thrust required to climb at constant speed =

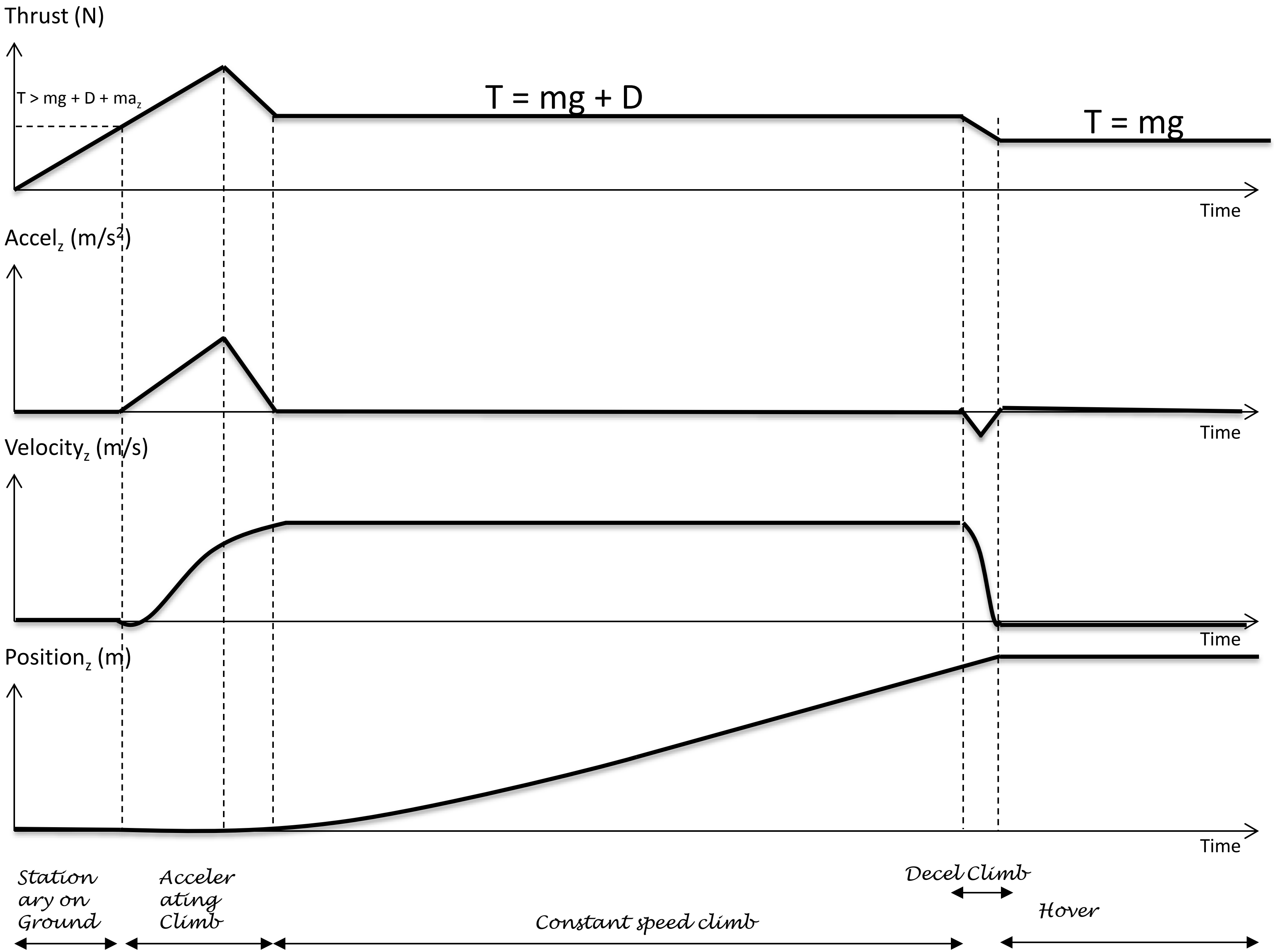
thrust overcome weight +

thrust to overcome Drag

Drag = $0.5\ \rho\ V^2\ C_D$ Surface Area



Quadrotor Dynamics: Segments of Takeoff to Hover



Build Your Own Quadcopter Simulation

Spreadsheet columns

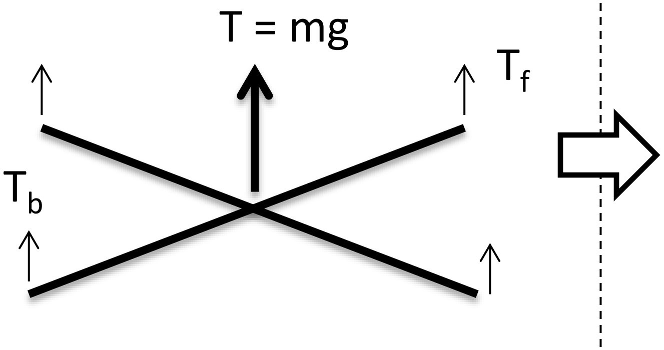
- Time (secs)
- Thrust (N)
- Weight (N)
- Drag (N)
- Sum of the Forces (N)
- Accel z (m/s^2)
- Vertical Velocity (m/s)
- Vertical Position (m)

1. Insert the correct equations in each column
   - Time increments in seconds (0 to 50 seconds)
   - Thrust is user input
   - Weight is fixed (mass = 5kg)
   - Drag is function of Vertical Velocity (use previous second Velocity), $C_D = 0.5$, $S = 0.1m^2$
   - Vertical Velocity = Accel z from previous sec * 1 sec + Vertical Velocity from previous second (i.e. integral of Accel)
   - Vertical Position = Vertical Velocity from previous sec * 1 sec + Vertical Position from previous second (i.e. integral of Vertical Velocity)

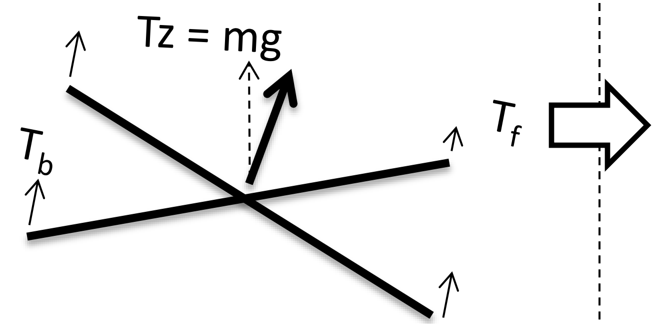
2. Enter in Thrust Values to control the Quadcopter to takeoff, accelerate to Vertical Velocity 5 m/s and then level off to hover at 15m
3. Plot Thrust, Accel z, Vert Vel, Vert Pos vs Time (see previous slide)

4. Bonus: Enter in Thrust Values to Land Quadcopter with smooth gentle landing (i.e. Kinetic Energy at landing < 250 Joules)

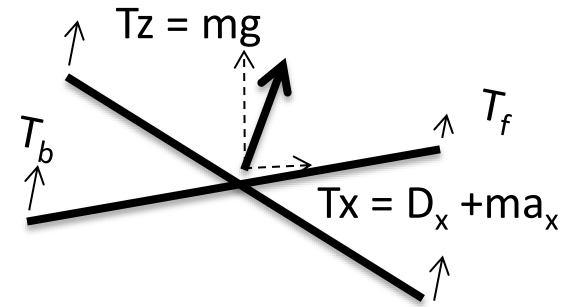
How to Move Forwards



Hover
Thrust is Vertical = Weight



Pitch Forward
Thrust is tilted forward
Vertical Thrust = Weight



Accelerate Forward
Vertical Thrust = Weight
Horizontal Thrust = thrust to overcome horizontal inertia + horizontal Drag

How to Pitch Forward

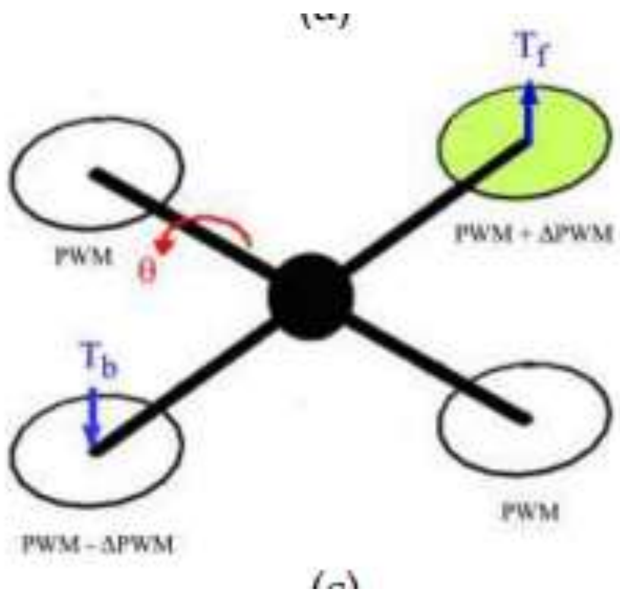
- Pitching Torque

$T_\theta = \ell\,(T_f - T_b)$

ℓ = length of boom (m)

To pitch forward

$T_b > T_f$



This diagram shows pitching backwards