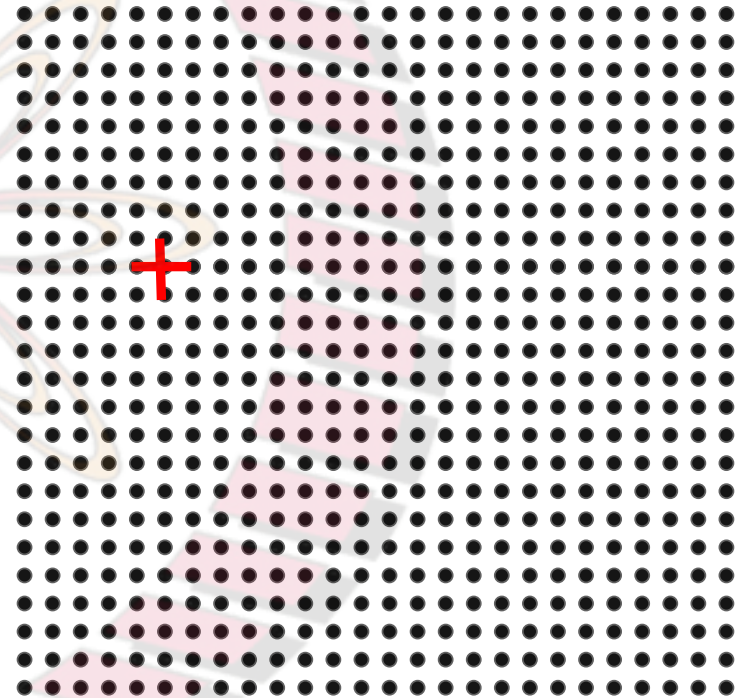


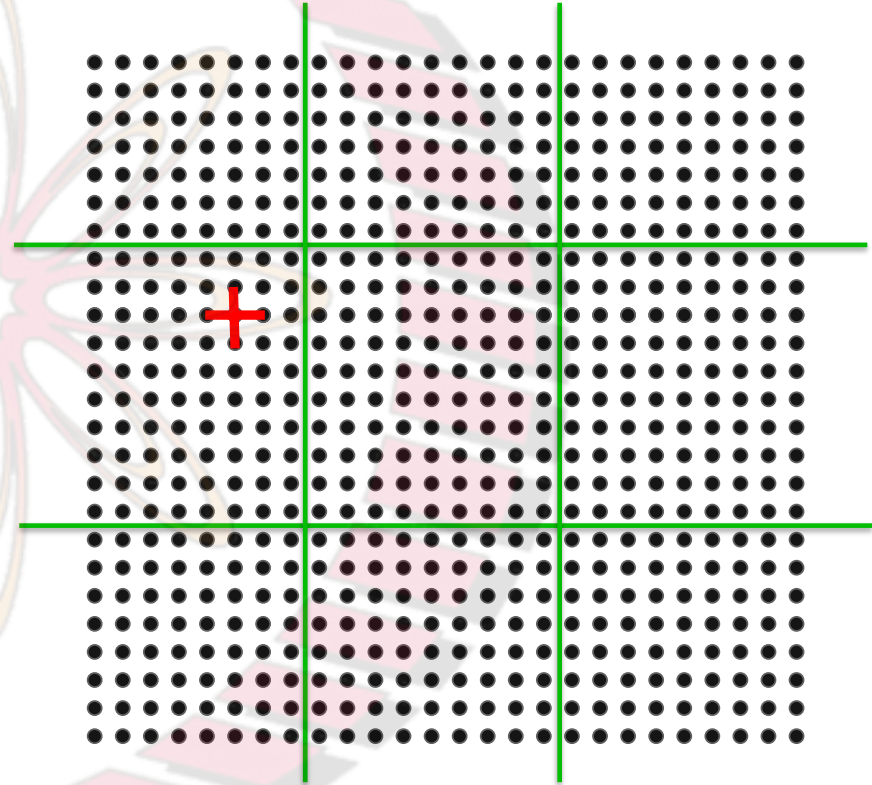
## The Global Data Structure

- Each circle is a mesh point
- Difference equation evaluated at each point involves the four neighbors
- The red “plus” is called the method’s stencil
- Good numerical algorithms form a matrix equation  $Au=f$ ; solving this requires computing  $Bv$ , where  $B$  is a matrix derived from  $A$ . These evaluations involve computations with the neighbors on the mesh.



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- Decompose mesh into equal sized (work) pieces



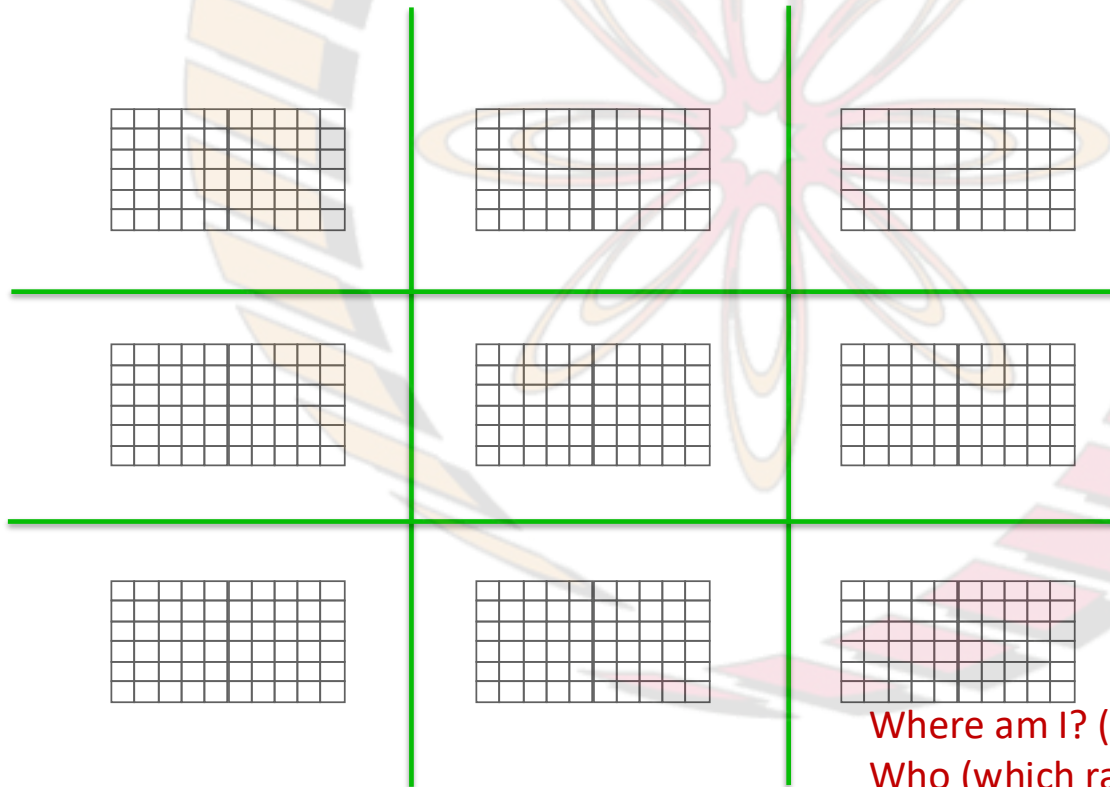
# Domain Decomposition

Parameters for domain decomposition:

$N$  = Size of the edge of the global problem domain (assuming square)

$PX, PY$  = Number of processes in X and Y dimension

$N \% PX == 0, N \% PY == 0$

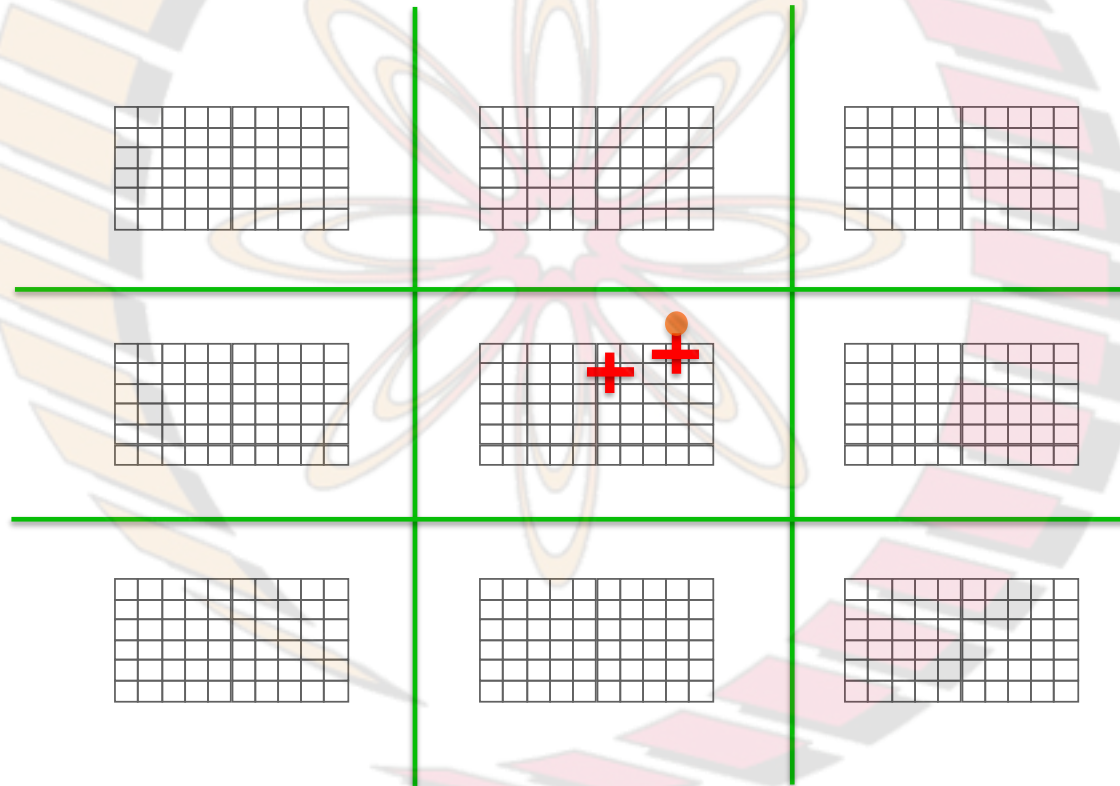


Where am I? (Global offset)

Who (which ranks) are my neighbors?

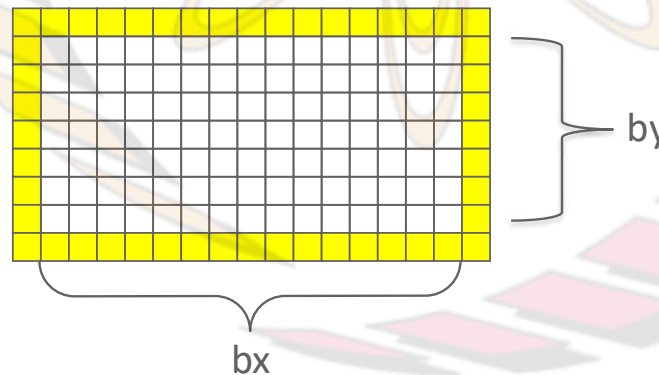
Use `MPI_PROC_NULL` for boundary

## Necessary Data Transfers



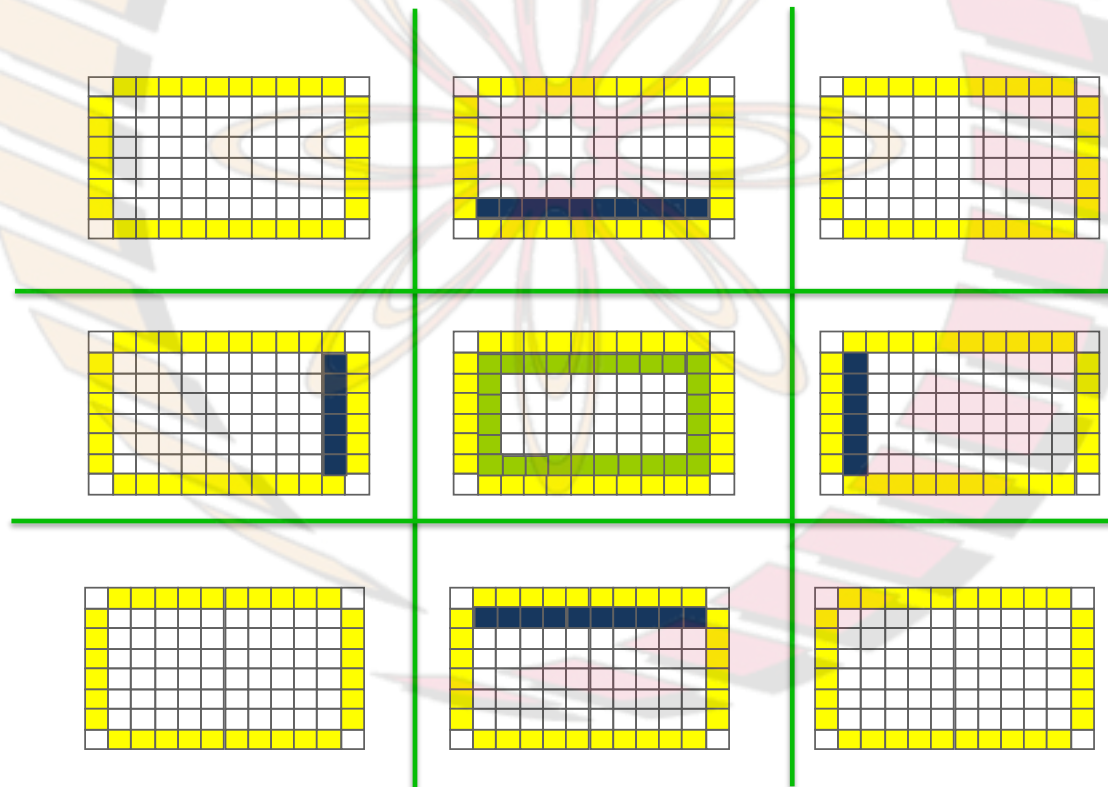
## The Local Data Structure

- Each process has its local “patch” of the global array
  - “bx” and “by” are the sizes of the local array
  - Always allocate a halo around the patch
  - Array allocated of size  $(bx+2) \times (by+2)$



## Necessary Data Transfers

- Provide access to remote data through a halo exchange (5 point stencil)

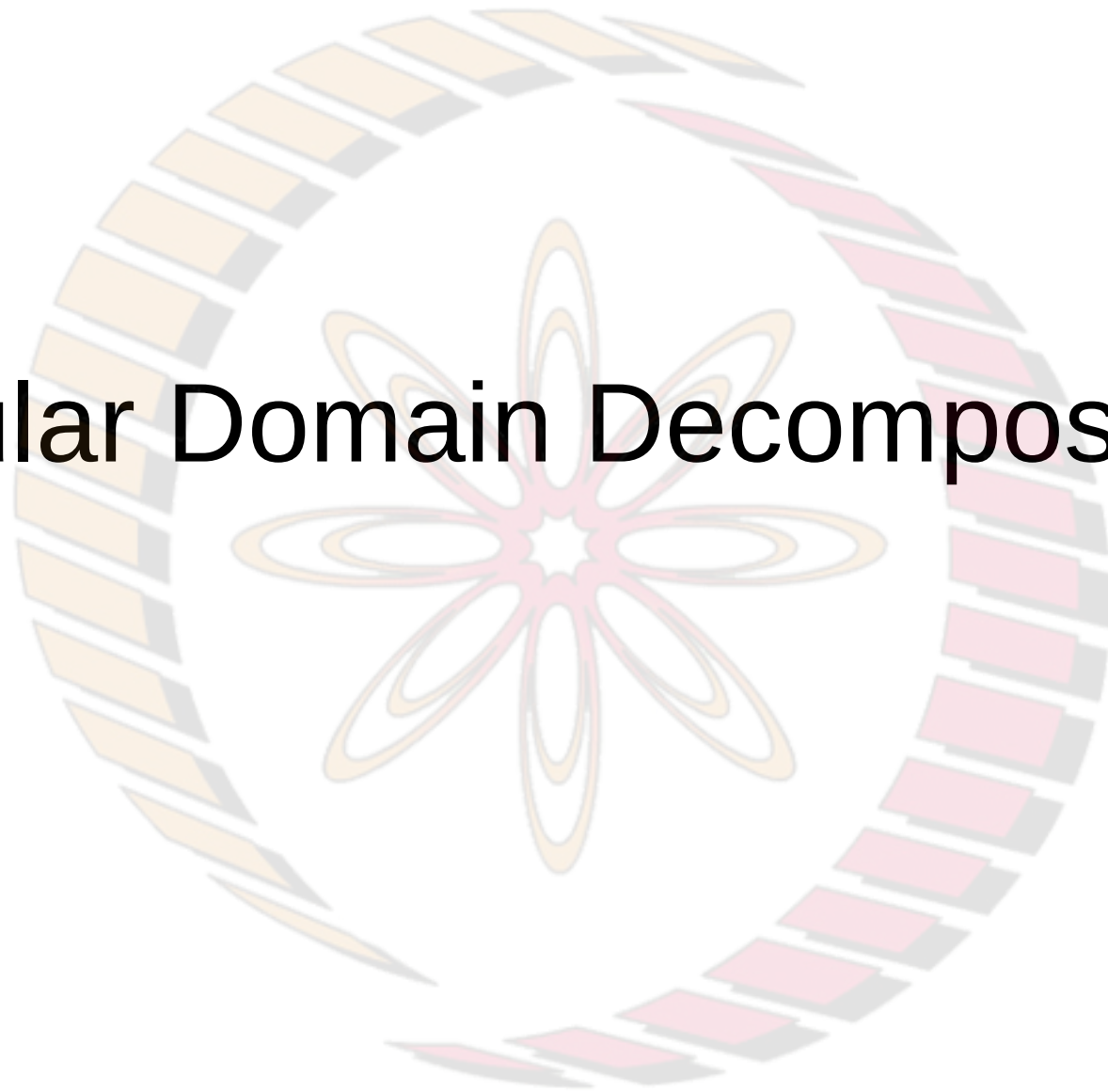




## Simple/Predefined Datatypes

- Equivalents exist for all C, C++ and Fortran native datatypes
  - C int → MPI\_INT
  - C float → MPI\_FLOAT
  - C double → MPI\_DOUBLE
  - C uint32\_t → MPI\_UINT32\_T
  - Fortran integer → MPI\_INTEGER
- For more complex or user-created datatypes, MPI provides routines to represent them as well
  - Contiguous
  - Vector/Hvector
  - Indexed/Indexed\_block/Hindexed/Hindexed\_block
  - Struct
  - Some convenience types (e.g., subarray)

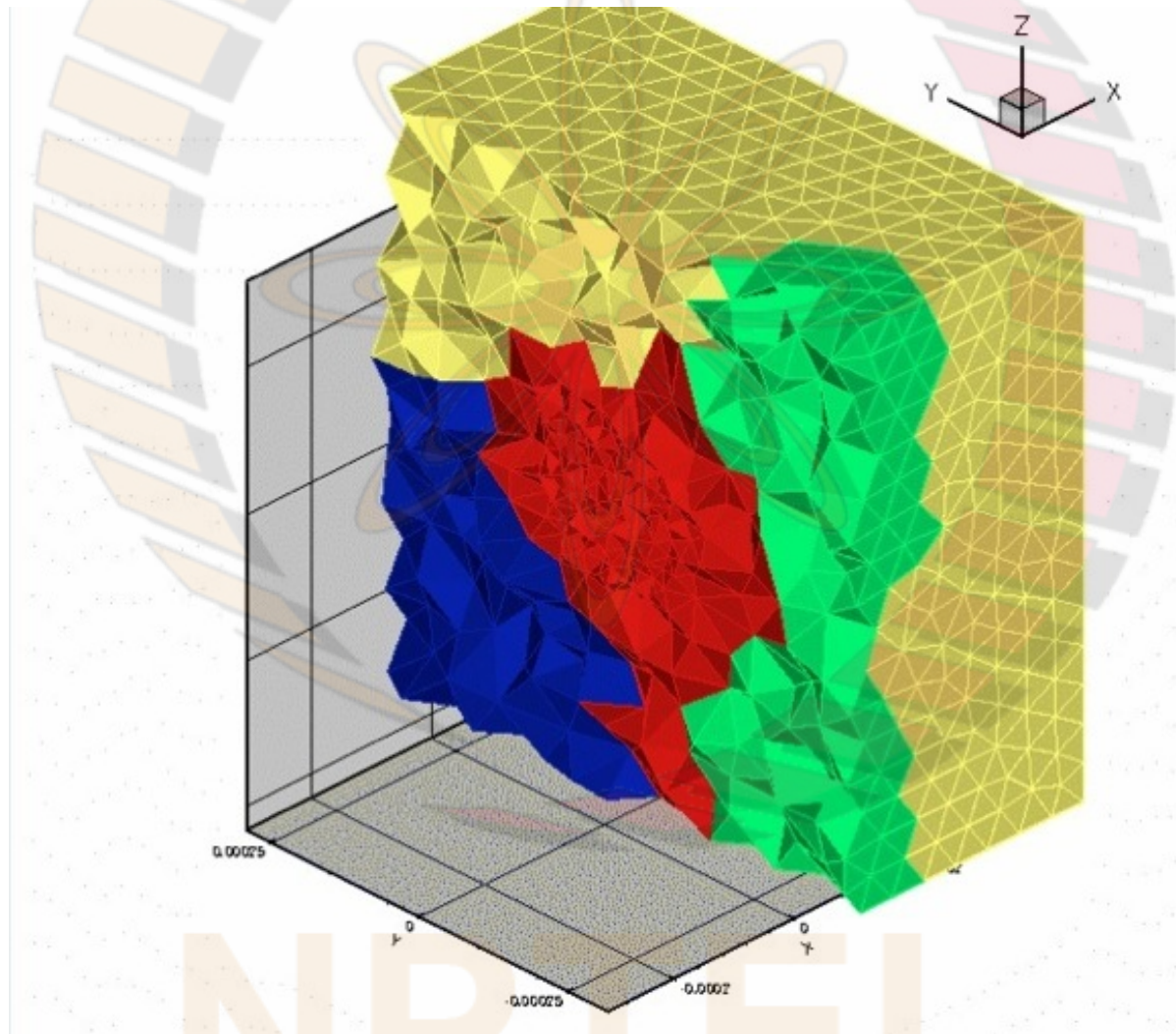
# Irregular Domain Decomposition



NPTEL

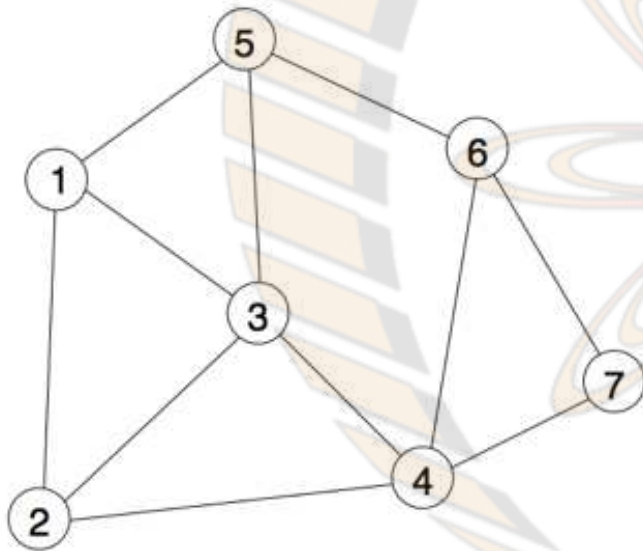


# Domain decomposed by METIS

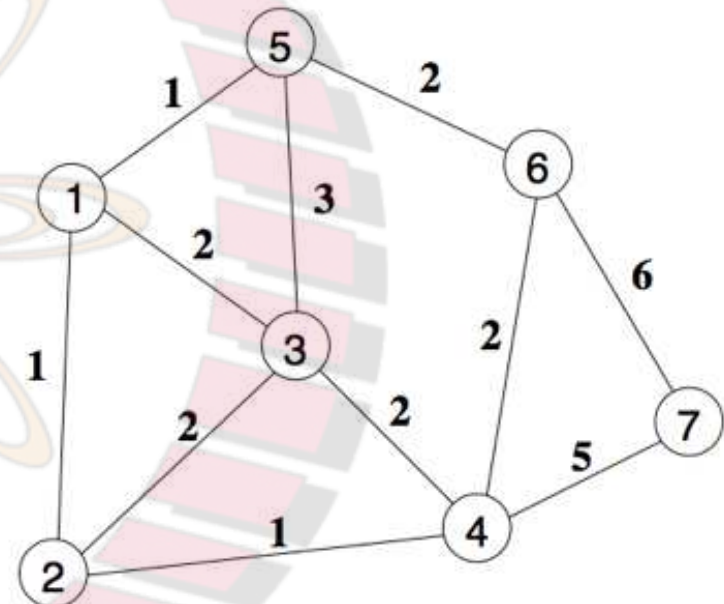


Metis is a Library for partitioning unstructured graphs / meshes

# Mesh representation as Connected graphs



Unweighted graph



Weighted graph

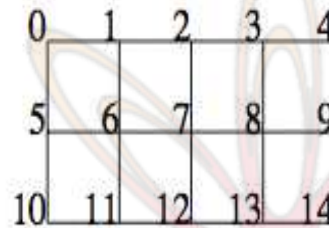
\*Images: From METIS manual

# CSR format and adjacency graph

“The adjacency structure of the graph is stored using the compressed storage format (CSR). The CSR format is a widely used scheme for storing sparse graphs. In this format the adjacency structure of a graph with  $n$  vertices and  $m$  edges is represented using two arrays **xadj** and **adjncy**. The **xadj** array is of size  $n + 1$  whereas the **adjncy** array is of size  $2m$  (this is because for each edge between vertices  $v$  and  $u$  we actually store both  $(v, u)$  and  $(u, v)$ ).” \*Q

NPTTEL

# CSR format and adjacency graph



**(a) A sample graph**

xadj	0 2 5 8 11 13 16 20 24 28 31 33 36 39 42 44
adjncy	1 5 0 2 6 1 3 7 2 4 8 3 9 0 6 10 1 5 7 11 2 6 8 12 3 7 9 13 4 8 14 5 11 6 10 12 7 11 13 8 12 14 9 13

**(b) CSR format**

\*Images: From METIS manual



# Domain decomposed by METIS



Computational domain of a automotive engine cylinder. S. Menon [2011]