IT-21633

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Problem statement: Is set of odd numbers with binary operations (+), i.e <O,+> abelian group? If not explain the reasons with necessary notations:

Ans: No, the set of odd integers under addition (O,+) is not a group (and hence not abelian) because its not closed under + and has no additive identity in O

let, O = set of all odd integers = {..., -3, -1, 1, 3, 5, ...}

operation defined: +,(addition)

we want to check whether (O,+) form an abelian group.

To be a group (O,+) must satisfy:

1. Closure
2. Associativity
3. Identity Element exists
4. Inverse exist
5. Commutativity (for abelian group)

1. Closure

if $a, b \in O$, is $a+b \in O$?

odd + odd = Even (example : $3+5=8$, even)

so closure fails immediately.

Since closure fails, it is not a group.

2. Other axioms

Even if we ignored closure, we'd check;

Associativity : Addition is associative on integers, so it would be associative on any subset

Identity: Identity would be 0, but 0 is not an odd integer, so no identity inside O.

inverse : For odd a, inverse under addition is $-a$, which is odd integers are closed under taking additive inverse but without closure and identity, irrelevant.

① let a be a group order pq, where p and q are distinct primes. Prove that a is abelian.

Ans:- Claim as stated is false.

S_3 has order $6 = 2 \cdot 3$ and is non-abelian.

Correct statement :-

let, $|a| = pq$.

with p, q Primes and $p<q$. Then by sylow theory one of the sylow subgroup is normal; hence G is a semidirect product of the sylow subgroup; in particular:

① If $p \nmid (q-1)$ then every homomorphism from a Sylow-q subgroup to Aut (sylowq) is trivial. so the semidirect.

ii) If $p \mid (q-1)$ a nontrivial semidirect product may exist and than a can be nonabelian S_3 when $p=2, q=3$.