The following is a review of the Market Risk Measurement and Management principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

The Art of Term Structure Models: Volatility and Distribution

Topic 14

Exam Focus

This topic incorporates non-constant volatility into term structure models. The generic time-dependent volatility model is very flexible and particularly useful for valuing multi-period derivatives like interest rate caps and floors. The Cox-Ingersoll-Ross (CIR) mean-reverting model suggests that the term structure of volatility increases with the level of interest rates and does not become negative. The lognormal model also has non-negative interest rates that proportionally increase with the level of the short-term rate. For the exam, you should understand how these models impact the short-term rate process, and be able to identify how a time-dependent volatility model (Model 3) differs from the models discussed in the previous topic. Also, understand the differences between the CIR and the lognormal models, as well as the differences between the lognormal models with drift and mean reversion.

Term Structure Model with Time-Dependent Volatility

LO 14.1: Describe the short-term rate process under a model with time-dependent volatility.

This topic provides a natural extension to the prior topic on modeling term structure drift by incorporating the volatility of the term structure. Following the notation convention of the previous topic, the generic continuously compounded instantaneous rate is denoted r_t and will change (over time) according to the following relationship:

$$dr = \lambda(t)dt + \sigma(t)dw$$

It is useful to note how this model augments Model 1 and the Ho-Lee model. The functional form of Model 1 (with no drift), $dr = \sigma dw$, now includes time-dependent drift and time-dependent volatility. The Ho-Lee model, $dr = \lambda(t)dt + \sigma dw$, now includes non-constant volatility. As in the earlier models, dw is normally distributed with mean 0 and standard deviation $\sqrt{dt}$.

LO 14.2: Calculate the short-term rate change and determine the behavior of the standard deviation of the rate change using a model with time dependent volatility.

The relationships between volatility in each period could take on an almost limitless number of combinations. For example, the volatility of the short-term rate in one year, $\sigma(1)$, could be 220 basis points and the volatility of the short-term rate in two years, $\sigma(2)$, could be 260 basis points. It is also entirely possible that $\sigma(1)$ could be 220 basis points and $\sigma(2)$ could be 160 basis points. To make the analysis more tractable, it is useful to assign a specific parameterization of time-dependent volatility. Consider the following model, which is known as Model 3:

$$dr = \lambda(t)dt + \sigma e^{-\alpha t}dw$$

where:
σ = volatility at t = 0, which decreases exponentially to 0 for $\alpha > 0$

To illustrate the rate change using Model 3, assume a current short-term rate, r_0, of 5%, a drift, λ, of 0.24%, and, instead of constant volatility, include time-dependent volatility of $\sigma e^{-0.3t}$ (with initial σ = 1.50%). If we also assume the *dw* realization drawn from a normal distribution is 0.2 (with mean = 0 and standard deviation = $\sqrt{1/12}$ = 0.2887), the change in the short-term rate after one month is calculated as:

$$dr = 0.24\% \times (1/12) + 1.5\% \times e^{-0.3(1/12)} \times 0.2$$

$$dr = 0.02\% + 0.29\% = 0.31\%$$

Therefore, the expected short-term rate of 5% plus the rate change (0.31%) equals 5.31%. Note that this value would be slightly less than the value assuming constant volatility (5.32%). This difference is expected given the exponential decay in the volatility.

Model 3 Effectiveness

LO 14.3: Assess the efficacy of time-dependent volatility models.

Time-dependent volatility models add flexibility to models of future short-term rates. This is particularly useful for pricing multi-period derivatives like interest rate caps and floors. Each cap and floor is made up of single period caplets and floorlets (essentially interest rate calls and puts). The payoff to each caplet or floorlet is based on the strike rate and the current short-term rate over the next period. Hence, the pricing of the cap and floor will depend critically on the forecast of $\sigma(t)$ at several future dates.

It is impossible to describe the general behavior of the standard deviation over the relevant horizon because it will depend on the deterministic model chosen. However, there are some parallels between Model 3 and the mean-reverting drift (Vasicek) model. Specifically, if the initial volatility for both models is equal and the decay rate is the same as the mean reversion rate, then the standard deviations of the terminal distributions are exactly the same. Similarly, if the time-dependent drift in Model 3 is equal to the average interest rate path in the Vasicek model, then the two terminal distributions are identical, an even stronger observation than having the same terminal standard deviation.

There are still important differences between these models. First, Model 3 will experience a parallel shift in the yield curve from a change in the short-term rate. Second, the purpose of the model drives the choice of the model. If the model is needed to price options on fixed income instruments, then volatility dependent models are preferred to interpolate between observed market prices. On the other hand, if the model is needed to value or hedge fixed income securities or options, then there is a rationale for choosing mean reversion models.

One criticism of time-dependent volatility models is that the market forecasts short-term volatility far out into the future, which is not likely. A compromise is to forecast volatility approaching a constant value (in Model 3, the volatility approaches 0). A point in favor of the mean reversion models is the downward-sloping volatility term structure.

Cox-Ingersoll-Ross (CIR) and Lognormal Models

LO 14.4: Describe the short-term rate process under the Cox-Ingersoll-Ross (CIR) and lognormal models.

LO 14.5: Calculate the short-term rate change and describe the basis point volatility using the CIR and lognormal models.

Another issue with the aforementioned models is that the basis-point volatility of the short-term rate is determined independently of the level of the short-term rate. This is questionable on two fronts. First, imagine a period of extremely high inflation (or even hyperinflation). The associated change in rates over the next period is likely to be larger than when rates are closer to their normal level. Second, if the economy is operating in an extremely low interest rate environment, then it seems natural that the volatility of rates will become smaller, as rates should be bounded below by zero or should be at most small, negative rates. In effect, interest rates of zero provide a downside barrier which dampens volatility.

A common solution to this problem is to apply a model where the basis-point volatility increases with the short-term rate. Whether the basis-point volatility will increase linearly or non-linearly is based on the particular functional form chosen. A popular model where the basis-point volatility (i.e., annualized volatility of *dr*) increases proportional to the square root of the rate (i.e., $\sigma\sqrt{r}$) is the **Cox-Ingersoll-Ross** (CIR) **model** where *dr* increases at a decreasing rate and σ is constant. The CIR model is shown as follows:

$$dr = k(\theta - r)dt + \sigma\sqrt{r}\,dw$$

As an illustration, let's continue with the example from LO 14.2, given the application of the CIR model. Assume a current short-term rate of 5%, a long-run value of the short-term rate, θ, of 24%, speed of the mean revision adjustment, k, of 0.04, and a volatility, σ, of 1.50%. As before, also assume the *dw* realization drawn from a normal distribution is 0.2. Using the CIR model, the change in the short-term rate after one month is calculated as:

$$dr = 0.04(24\% - 5\%)(1/12) + 1.5\%\sqrt{5\%} \times 0.2$$

$$dr = 0.063\% + 0.067\% = 0.13\%$$

Therefore, the expected short-term rate of 5% plus the rate change (0.13%) equals 5.13%.

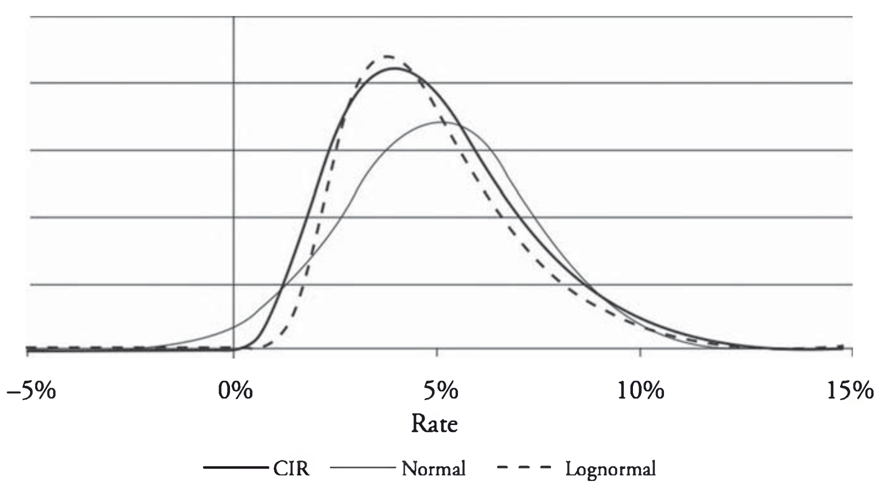
A second common specification of a model where basis-point volatility increases with the short-term rate is the **lognormal model** (Model 4). An important property of the lognormal model is that the yield volatility, σ, is constant, but basis-point volatility increases with the level of the short-term rate. Specifically, basis-point volatility is equal to σr and the functional form of the model, where σ is constant and dr increases at σr, is:

$$dr = ardt + \sigma rdw$$

For both the CIR and the lognormal models, as long as the short-term rate is not negative then a positive drift implies that the short-term rate cannot become negative. As discussed previously, this is certainly a positive feature of the models, but it actually may not be that important. For example, if a market maker feels that interest rates will be fairly flat and the possibility of negative rates would have only a marginal impact on the price, then the market maker may opt for the simpler constant volatility model rather than the more complex CIR.

The differences between the distributions of the short-term rate for the normal, CIR, and lognormal models are also important to analyze. Figure 1 compares the distributions after ten years, assuming equal means and standard deviations for all three models. As mentioned in Topic 13, the normal distribution will always imply a positive probability of negative interest rates. In addition, the longer the forecast horizon, the greater the likelihood of negative rates occurring. This can be seen directly by the left tail lying above the x-axis for rates below 0%. This is clearly a drawback to assuming a normal distribution.

Figure 1: Terminal Distributions



In contrast to the normal distribution, the lognormal and CIR terminal distributions are always non-negative and skewed right. This has important pricing implications particularly for out-of-the money options where the mass of the distributions can differ dramatically.

LO 14.6: Describe lognormal models with deterministic drift and mean reversion.

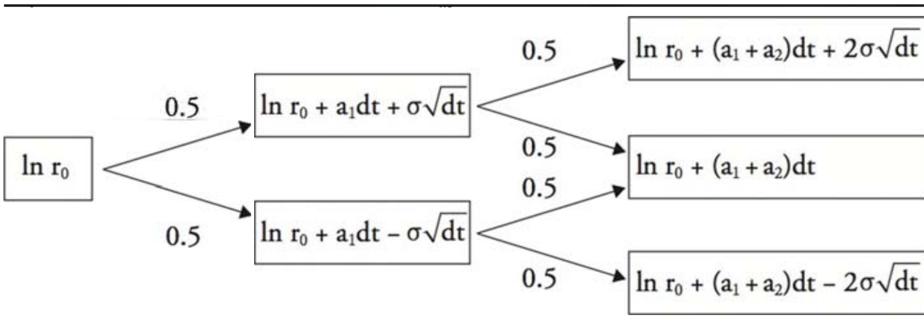
Lognormal Model with Deterministic Drift

For this LO, we detail two lognormal models, one with deterministic drift and one with mean reversion. The lognormal model with drift is shown as follows:

$$d[\ln(r)] = a(t)dt + \sigma dw$$

The natural log of the short-term rate follows a normal distribution and can be used to construct an interest rate tree based on the natural logarithm of the short-term rate. In the spirit of the Ho-Lee model, where drift can vary from period to period, the interest rate tree in Figure 2 is generated using the lognormal model with deterministic drift.

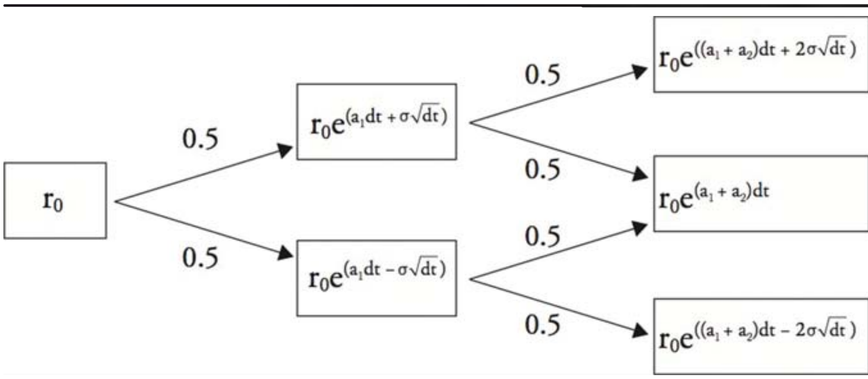
Figure 2: Interest Rate Tree with Lognormal Model (Drift)



If each node in Figure 2 is exponentiated, the tree will display the interest rates at each node. For example, the adjusted period 1 upper node would be calculated as:

$$\exp(\ln r_0 + a_1 dt + \sigma\sqrt{dt}) = r_0 e^{(a_1 dt + \sigma\sqrt{dt})}$$

Figure 3: Lognormal Model Rates at Each Node



In contrast to the Ho-Lee model, where the drift terms are additive, the drift terms in the lognormal model are multiplicative. The implication is that all rates in the tree will always be positive since $e^x > 0$ for all x. Furthermore, since $e^x \approx 1 + x$, and if we assume $a_1 = 0$ and

$dt = 1$, then: $r_0 e^{\sigma} \approx r_0(1 + \sigma)$. Hence, volatility is a percentage of the rate. For example, if $\sigma = 20\%$, then the rate in the upper node will be 20% above the current short-term rate.

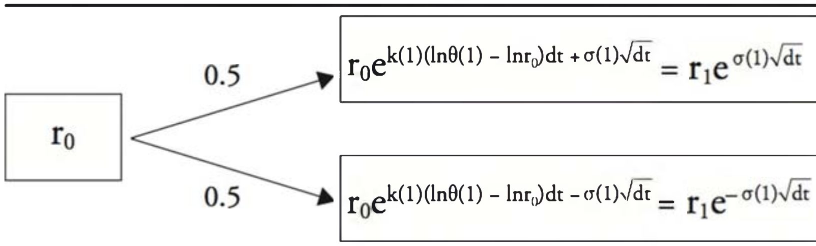
Lognormal Model with Mean Reversion

The lognormal distribution combined with a mean-reverting process is known as the Black-Karasinski model. This model is very flexible, allowing for time-varying volatility and mean reversion. In logarithmic terms, the model will appear as:

$$d[\ln(r)] = k(t)[\ln\theta(t) - \ln(r)]dt + \sigma(t)dw$$

Thus, the natural log of the short-term rate follows a normal distribution and will revert to the long-run mean of $\ln[\theta(t)]$ based on adjustment parameter $k(t)$. In addition, volatility is time-dependent, transforming the Vasicek model into a time-varying one. The interest rate tree based on this model is a bit more complicated, but it exhibits the same basic structure as previous models.

Figure 4: Interest Rate Tree with Lognormal Model (Mean Revision)



The notation r_1 is used to condense the exposition. Therefore, the ln(upper node) = $\ln r_1 + \sigma(1)\sqrt{dt}$ and ln(lower node) = $\ln r_1 - \sigma(1)\sqrt{dt}$. Following the intuition of the mean-reverting model, the tree will recombine in the second period only if:

$$k(2) = \frac{\sigma(1) - \sigma(2)}{\sigma(1)dt}$$

Recall from the previous topic that in the mean-reverting model, the nodes can be "forced" to recombine by changing the probabilities in the second period to properly value the weighted average of paths in the next period. A similar adjustment can be made in this model. However, this adjustment varies the length of time between periods (i.e., by manipulating the *dt* variable). After choosing dt_1, dt_2 is determined with the following equation:

$$k(2) = \frac{1}{dt_2}\left[1 - \frac{\sigma(2)\sqrt{dt_2}}{\sigma(1)\sqrt{dt_1}}\right]$$

Key Concepts

LO 14.1

The generic continuously compounded instantaneous rate with time-dependent drift and volatility will evolve over time according to $dr = \lambda(t)dt + \sigma(t)dw$. Special cases of this model include Model 1 ($dr = \sigma dw$) and the Ho-Lee model ($dr = \lambda(t)dt + \sigma dw$).

LO 14.2

The relationships between volatility in each period could take on an almost limitless number of combinations. To analyze this factor, it is necessary to assign a specific parameterization of time-dependent volatility such that: $dr = \lambda(t)dt + \sigma e^{-\alpha t}dw$, where σ is volatility at $t = 0$, which decreases exponentially to 0. This model is referred to as Model 3.

LO 14.3

Time-dependent volatility is very useful for pricing interest rate caps and floors that depend critically on the forecast of $\sigma(t)$ on multiple future dates. Under reasonable conditions, Model 3 and the mean-reverting drift (Vasicek) model will have the same standard deviation of the terminal distributions. One criticism of time-dependent volatility models is that the market forecasts short-term volatility far out into the future. A point in favor of the mean-reversion models is the downward-sloping volatility term structure.

LO 14.4

Two common models that avoid negative interest rates are the Cox-Ingersoll-Ross (CIR) model and lognormal model. Although avoiding negative interest rates is attractive, the non-normality of the distributions can lead to derivative mispricings.

LO 14.5

The CIR mean-reverting model has constant volatility (σ) and basis-point volatility $(\sigma\sqrt{r})$ that increases at a decreasing rate:

$$dr = k(\theta - r)dt + \sigma\sqrt{r}\,dw$$

LO 14.6

There are two lognormal models of importance: (1) lognormal with deterministic drift and (2) lognormal with mean reversion.

The lognormal model with drift is:

$$d[\ln(r)] = a(t)dt + \sigma dw$$

This model is very similar in spirit to the Ho-Lee Model with additive drift. The interest rate tree is expressed in rates, as opposed to the natural log of rates, which results in a multiplicative effect for the lognormal model with drift.

The lognormal model with mean reversion is:

$$d[\ln(r)] = k(t)[\ln\theta(t) - \ln(r)]dt + \sigma(t)dw$$

This model does not produce a naturally recombining interest rate tree. In order to force the tree to recombine, the time steps, *dt*, must be recalibrated.

Concept Checkers

1. Regarding the validity of time-dependent drift models, which of the following statements is(are) correct?

   I. Time-dependent drift models are flexible since volatility from period to period can change. However, volatility must be an increasing function of short-term rate volatilities.

   II. Time-dependent volatility functions are useful for pricing interest rate caps and floors.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

2. Which of the following choices correctly characterizes basis-point volatility and yield volatility as a function of time within the lognormal model?

	Basis-point volatility	Yield volatility
A.	increases	constant
B.	increases	decreases
C.	decreases	constant
D.	decreases	decreases

3. Which of the following statements is most likely a disadvantage of the CIR model?
   A. Interest rates are always non-negative.
   B. Option prices from the CIR distribution may differ significantly from lognormal or normal distributions.
   C. Basis-point volatility increases during periods of high inflation.
   D. Long-run interest rates hover around a mean-reverting level.

4. Which of the following statements best characterizes the differences between the Ho-Lee model with drift and the lognormal model with drift?
   A. In the Ho-Lee model and the lognormal model the drift terms are multiplicative.
   B. In the Ho-Lee model and the lognormal model the drift terms are additive.
   C. In the Ho-Lee model the drift terms are multiplicative, but in the lognormal model the drift terms are additive.
   D. In the Ho-Lee model the drift terms are additive, but in the lognormal model the drift terms are multiplicative.

5. Which of the following statements is true regarding the Black-Karasinski model?
   A. The model produces an interest rate tree that is recombining by definition.
   B. The model produces an interest rate tree that is recombining when the *dt* variable is manipulated.
   C. The model is time-varying and mean-reverting with a slow speed of adjustment.
   D. The model is time-varying and mean-reverting with a fast speed of adjustment.

Concept Checker Answers

1. **B** Time-dependent volatility models are very flexible and can incorporate increasing, decreasing, and constant short-term rate volatilities between periods. This flexibility is useful for valuing interest rate caps and floors because there is a potential payout each period, so the flexibility of changing interest rates is more appropriate than applying a constant volatility model.

2. **A** Choices B and D can be eliminated because yield volatility is constant. Basis-point volatility under the CIR model increases at a decreasing rate, whereas basis-point volatility under the lognormal model increases linearly. Therefore, basis-point volatility is an increasing function for both models.

3. **B** Choices A and C are advantages of the CIR model. Out-of-the money option prices may differ with the use of normal or lognormal distributions.

4. **D** The Ho-Lee model with drift is very flexible, allowing the drift terms each period to vary. Hence, the cumulative effect is additive. In contrast, the lognormal model with drift allows the drift terms to vary, but the cumulative effect is multiplicative.

5. **B** A feature of the time-varying, mean-reverting lognormal model is that it will not recombine naturally. Rather, the time intervals between interest rate changes are recalibrated to force the nodes to recombine. The generic model makes no prediction on the speed of the mean reversion adjustment.

The following is a review of the Market Risk Measurement and Management principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

OIS Discounting, Credit Issues, and Funding Costs

Topic 15

Exam Focus

The choice of the risk-free discount rate is important because most derivatives are discounted using the risk-free rate. In this topic, we first look at different proxies for the risk-free discount rate, including Treasury rates, LIBOR swap rates, and overnight indexed swap (OIS) rates. It is important that you understand the strengths and limitations of each choice. We then look at the OIS and OIS rate in detail and examine how to construct the zero curve from OIS rates. The final part of this topic looks at how to use OIS rates to value swaps and forward rate agreements (FRAs) and how to estimate forward LIBOR rates from OIS rates. For the exam, understand the relationship between the OIS rate and LIBOR, and be able to determine forward LIBOR rates from the OIS zero curve.

Risk-Free Rate Selection

LO 15.1: Explain the main considerations in choosing a risk-free rate for derivatives valuation.

A derivative (or a derivatives portfolio) should earn the risk-free rate if a no-arbitrage condition holds. As a result, most derivatives have historically been valued by discounting their expected cash flows using risk-free rates. This has provided the basis for valuing forwards, forward rate agreements (FRAs), and swaps, since swaps are simply portfolios of forwards or FRAs.

Given that there are numerous proxies for the risk-free rate, its choice is important. In the United States, the rates on Treasury securities including Treasury bills, notes, and bonds are often considered the closest proxy for risk-free rates. These Treasury securities are virtually free of default risk, as it is highly unlikely that the U.S. government would default on its obligations because it can always choose to increase the money supply by printing more money. Nevertheless, Treasury rates are seldom used as proxies for risk-free rates given that they are considered to be artificially low.

Professor's Note: Treasury rates are considered artificially low for three primary reasons: (1) Due to regulatory requirements, many financial institutions must purchase Treasury securities, driving up their demand and reducing their yield; (2) Banks are required to hold significantly less capital in support of Treasury securities than in support of other securities; and (3) In the U.S., Treasury securities benefit from a favorable tax treatment.

Instead of Treasury rates, most market participants used the **London Interbank Offered Rate (LIBOR)** as a proxy for the risk-free rate prior to the 2007–2009 credit crisis. LIBOR is a short-term interest rate (one year or under) that creditworthy banks (typically rated AA or stronger) charge each other. LIBOR was historically considered near risk-free because the probability of one of these well-rated banks defaulting within one year was considered to be very small. From LIBOR rates, market participants were then able to construct a zero curve that they used in valuing derivatives.

The idea that LIBOR rates represent risk-free rates changed during the recent credit crisis. LIBOR rates increased materially during this period, and the spread between LIBOR and the U.S. Treasury bill rate increased to multiples of its pre-crisis level. At the same time, the use of collateralized derivative transactions increased, which reduced credit risk in transactions. This meant that LIBOR rates were no longer considered appropriate to discount lower risk derivatives. As a result, banks continued to use LIBOR rates as the risk-free discount rates for non-collateralized transactions because these trades required a higher discount rate to account for their risk. However, for collateralized transactions, banks changed the proxy for risk-free rates from LIBOR to overnight indexed swap (OIS) rates (discussed in the next LO).

The main driver of these differing choices is that derivatives should represent a bank's average funding costs. For collateralized transactions, which are funded by collateral, OIS rates are considered a good estimate of funding costs given that the federal funds rate (which is linked to the OIS rate) is the overnight interest typically paid on collateral. For non-collateralized transactions, the average funding costs should be estimated from LIBOR (or even higher) rates.

OIS Rates and LIBOR-OIS Spread

LO 15.2: Describe the OIS rate and the LIBOR-OIS spread, and explain their uses.

The overnight rate is the rate at which large financial institutions borrow from each other in the overnight market. In the United States, this rate is called the federal funds rate and is monitored and influenced by the central bank. If a financial institution borrows (lends) funds at the overnight rate, the rate it pays (earns) during the period is the geometric average of the overnight rates.

An **overnight indexed swap (OIS)** is an interest rate swap where a fixed rate is exchanged for a floating rate, and where the floating rate is the geometric average of the overnight federal funds rates during the period. The fixed rate in the OIS is known as the **OIS rate**. If the fixed rate is greater than the geometric average of the overnight rates, the fixed side makes a payment; otherwise, the floating side makes a payment. For example, let's consider a three-month OIS with a notional principal of $25 million and an OIS rate (fixed rate) of 2.5%. If the geometric average of the overnight rates during the three-month period is 2.7%, the floating side has to make a $12,500 payment (= 3 / 12 × (0.027 – 0.025) × $25 million) to the fixed-rate payer.

Overnight indexed swaps typically have short maturities of three months or less, although maturities of up to 10 years also exist. An OIS with a maturity of one year or greater is usually divided into three-month subperiods, with settlement occurring every three months by exchanging the OIS rate with the geometric average of the overnight rates during the subperiod. The OIS rate is therefore the continually refreshed overnight rate (the rate earned on a series of overnight loans).

The difference between the LIBOR rate and the OIS rate of the same maturity (e.g., three months) is known as the **LIBOR-OIS spread**. This spread is typically used as a gauge of market stress as well as the health of the financial system. Under normal market conditions, the spread has historically averaged 10 basis points. During the 2007–2009 financial crisis, the spread rose to over 350 basis points, indicating very high stress levels in the market as liquidity dried up and financial institutions became unwilling to lend to each other for short periods.

As mentioned, in the post-financial crisis period, most financial institutions have moved away from using LIBOR rates in favor of OIS rates as the risk-free discount rates used to value derivatives in collateralized transactions. OIS rates provide good estimates of funding the cost of collateral.

LO 15.3: Evaluate the appropriateness of the OIS rate as a proxy for the risk-free rate.

The OIS rate is based on overnight federal funds rates and does not suffer from the shortcomings of LIBOR rates, which makes the OIS rate a stronger proxy for the risk-free rate, especially for collateralized derivatives. It also does not suffer from the artificially low yields of Treasury securities.

However, the OIS rate is not considered perfectly risk-free for two reasons. First, a default between two participants in an overnight loan is still possible. However, the risk of such a default is very small, and institutions with credit problems would typically be shut out of the overnight market. Second, there might be a default on the OIS transaction itself. Any credit risk could be priced into the OIS rate; however, the risk of default is considered small, especially when the OIS is collateralized.

Constructing the OIS Zero (Spot Rate) Curve

LO 15.4: Describe how to use the OIS zero curve in determining forward LIBOR rates and valuing swaps.

Constructing the OIS zero curve from OIS rates is based on the bootstrapping method similar to constructing the LIBOR zero curve from LIBOR-for-fixed swap (or simply LIBOR swap) rates. This means that the one-month OIS rate determines the one-month zero rate, the three-month OIS rate determines the three-month zero rate, etc. For longer maturity swaps using periodic settlements, the OIS rate defines a par yield bond. As an example, consider a three-year OIS rate of 3.1% with quarterly settlements. This implies that the three-year bond with a quarterly coupon payment is expected to sell for par.

A limitation of OISs is that their maturities tend to be shorter than LIBOR swap maturities, which makes it more challenging to construct the OIS zero curve for longer maturities.

However, the OIS zero curve can still be constructed for longer maturities by taking the spread between the longest maturity OIS for which there is reliable information and the corresponding LIBOR swap rate, and also assuming that this spread will remain constant for all longer maturities. For example, assume that there is no reliable data on OISs with maturities greater than four years, and that the four-year LIBOR swap rate is 3.85% and the OIS rate is 3.50%. The spread between the OIS and LIBOR swap is therefore 35 basis points, and the OIS rates are assumed to be 35 basis points lower than the LIBOR swap rates for all maturities exceeding four years.

Determining Forward LIBOR Rates

Determining the OIS zero curve is important since these OIS rates are used as the risk-free discount rates for derivatives cash flows. They are also used in deriving forward LIBOR rates to value FRAs and swaps. However, as will be shown, there is a difference in the forward LIBOR rates calculated using LIBOR and OIS discounting.

LIBOR Discounting

Before we discuss how to derive forward LIBOR rates with OIS discounting, it is helpful to understand how these forward rates are derived using LIBOR discounting. LIBOR swaps involve fixed-for-floating payments. The rate on these payments can be used to derive both LIBOR zero rates and implied forward rates. We also know that discounting the swap at LIBOR zero rates should provide par value.

For example, assume that the fixed rate in a two-year, annual pay LIBOR-for-fixed swap is 5%, and assume that a one-year LIBOR rate is 4%. The bank uses LIBOR rates for discounting. This implies that the one-year LIBOR swap zero rate is also 4%. We can use this information to compute the two-year LIBOR swap zero rate, *R*. Given that the swap should be valued at par using zero rates, we can set up the following equation per 100 value of the swap:

$$\frac{5}{1.04}+\frac{105}{(1+R)^2}=100$$

Solving for *R* gives us *R* = 5.025%, which is the two-year LIBOR swap zero rate. Given the one-year and two-year zero rates, we can then calculate the one-year period forward LIBOR rate, or *F*, beginning in one year as follows:

$$F=\frac{1.05025^2}{1.04}-1=6.06\%$$

We can check our work by setting up an equation to calculate *F* by making the value of the swap equal to zero. The values to be discounted are the payments made to the fixed by the floating side per 100 value of principal. Assuming that forward rates are realized, the swap value is:

$$\frac{5-4}{1.04}+\frac{5-100F}{1.05025^2}=0$$

This confirms our previously calculated value of F = 6.06%.

OIS Discounting

Continuing with our previous example, assume that the bank uses OIS rates, instead of LIBOR rates, for discounting. The steps to calculate the swap zero curve and the forward rates are similar to the steps described earlier. Let's assume that the bank calculated the OIS zero curve at 3.5% and 4.5% for the one- and two-year OIS zero rates assuming annual compounding. Notice that both of these rates are approximately 50 basis points lower than the LIBOR zero rates. We can now solve for F by discounting the payments made to the fixed by the floating side per 100 of principal and setting the swap value to zero. Assuming that forward rates are realized, the swap value can be expressed as:

$$\frac{5-4}{1.035}+\frac{5-100F}{1.045^2}=0$$

Solving for F yields F = 6.055%.

It is important to note that there is a difference in the forward LIBOR rate using LIBOR discounting and OIS discounting. The difference is half a basis point, which is arguably small but should not be ignored. The magnitude of the difference depends on the steepness of the zero curve and the maturity of the forward rate.

Key Concepts

LO 15.1

In a no-arbitrage world, derivatives should earn the risk-free rate by using the risk-free rate to discount their cash flows. While the true risk-free rate is not observable, there are close proxies, including the rates on Treasury securities and LIBOR rates. Both of these rates suffer from limitations: Treasury security rates are considered artificially low, while LIBOR rates are now considered artificially high for many derivative valuations.

Following the 2007–2009 financial crisis, most financial institutions continue to use LIBOR rates as risk-free discount rates in non-collateralized transactions, but have moved away from LIBOR rates to overnight indexed swap (OIS) rates for collateralized transactions (because OIS rates are more closely tied to funding costs).

LO 15.2

An OIS is an interest rate swap where a fixed rate is exchanged for a floating rate. The floating rate is the geometric average of the overnight federal funds rates during the period. The fixed rate in the OIS is known as the OIS rate.

The difference between the LIBOR rate and the OIS rate of the same maturity is known as the LIBOR-OIS spread, which is used to gauge the stress in financial markets.

LO 15.3

The OIS rate is considered a good proxy for the risk-free rate because it is based on overnight federal funds rates and does not suffer from the shortcomings of LIBOR rates. However, the OIS rate suffers from two limitations: (1) market participants might default on an overnight loan, and (2) market participants might default on the OIS transaction itself. The risk of either of these default scenarios is considered small.

LO 15.4

The OIS zero curve is constructed from OIS rates of various maturities. To construct the OIS zero curve for longer maturities for which no OISs are available, we assume a constant spread between the longest maturity OIS (for which there is reliable data) and the corresponding LIBOR swap rate for longer maturities.

Determining the OIS zero curve is important because these OIS rates are used as the risk-free discount rates for derivatives cash flows and are also used in deriving forward LIBOR rates for FRAs and swaps. Forward LIBOR rates can be estimated using both LIBOR and OIS discounting, and they require calculating the zero rates for all settlement periods. When OIS discounting is used, the forward LIBOR rates will typically be smaller than when using LIBOR rates. The magnitude of the difference depends on the steepness of the zero curve and the maturity of the forward rate.

Concept Checkers

1. The current edition of the monthly research report of an investment bank is dedicated to discussing the risk-free rate and contains the following statements:

   "In the United States, rates of Treasury securities may not be considered the best proxy for a risk-free rate because financial institutions are required to purchase Treasury securities due to various regulatory requirements, which may result in an artificially low yield for these securities. Another proxy for the risk-free rate is LIBOR, which has been increasingly used in collateralized transactions following the 2007–2009 financial crisis."

   With respect to Treasury securities and LIBOR, are these statements considered accurate?

	Treasury securities	LIBOR
A.	No	No
B.	No	Yes
C.	Yes	Yes
D.	Yes	No

2. An analyst notes in a presentation to management that the U.S. three-month LIBOR-OIS spread declined from 150 basis points a year ago to 80 basis points today. Regarding this scenario, which of the following statements is considered most accurate?
   A. The decline in spread represents a decline in credit quality in the markets.
   B. A payment of 80 basis points must be made by the floating-rate payer of the OIS.
   C. The LIBOR rate is now a better proxy for the risk-free rate than the OIS rate.
   D. Stress in the financial markets has declined.

3. Mikey Parizeau, FRM, is a fixed income analyst at a large financial institution. Parizeau states to a colleague that while the OIS rate is not entirely risk-free, it is considered the best proxy for the risk-free rate when used in valuing collateralized derivatives. Is Parizeau's observation correct?
   A. Yes.
   B. No, because the OIS rate is considered entirely risk-free.
   C. No, because the OIS rate is considered the best proxy for the risk-free rate for valuing both collateralized and non-collateralized derivatives.
   D. No, because the OIS rate is considered the best proxy for the risk-free rate for valuing only non-collateralized derivatives.

4. Assume that the one-, two- and three-year LIBOR-for-fixed swaps trade at a spread of 15, 18, and 20 basis points, respectively, above the corresponding OISs. If the 10-year LIBOR-for-fixed swap rate is 4.5%, what is the best estimate for the 10-year OIS rate?
   A. 0.2%.
   B. 4.3%.
   C. 4.5%.
   D. 4.7%.

5. A bank recently published a report on derivative valuations that contained the following statements:

   I. LIBOR-for-fixed swap rates for longer maturities where no reliable estimates exist can be estimated from OIS rates.

   II. While the calculations are different, using LIBOR rates rather than OIS rates for discounting does not change the estimates of forward LIBOR rates.

   III. When valuing a swap using OIS discounting, cash flows on the swap are calculated assuming that forward rates are realized.

   The bank's director of market risk believes some of these statements may not be accurate. How many of the statements are incorrect?

   A. 0.
   B. 1.
   C. 2.
   D. 3.

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Concept Checker Answers

1. **D** Only the first statement is correct. The second statement is incorrect because LIBOR has been increasingly used in non-collateralized transactions following the 2007–2009 financial crisis.

2. **D** The LIBOR-OIS spread is used as a measure of stress in financial markets. A decline in the spread indicates a decline in stress, or an improvement in credit quality, in markets. The spread is not a measure of payment on an overnight indexed swap. The spread also does not imply that the LIBOR rate would be superior to the OIS rate as a proxy for the risk-free rate.

3. **A** Parizeau's statement is correct. Although the OIS rate is not entirely risk-free, it is considered the best proxy for the risk-free rate to be used in valuing collateralized derivatives because the OIS rate provides a good estimate of the funding cost of collateral. LIBOR rates continue to be used for valuing non-collateralized derivatives.

4. **B** It is common for OISs not to trade for maturities that are as long as LIBOR-for-fixed swaps. Given a lack of reliable data for OIS maturities beyond three years in our example, a common approach is to assume that the spread (between the LIBOR swap rate and the OIS rate for the longest maturity with reliable data) remains constant for all longer maturities. As a result, all OIS rates beyond three years are assumed to be 20 basis points below LIBOR swap rates. The best estimate of the 10-year OIS rate is therefore 4.5% – 0.2% = 4.3%.

5. **C** Statements I and II are incorrect, and Statement III is correct. Statement I is incorrect because the logic is reversed. OIS rates for longer maturities where no reliable estimates exist can be estimated from LIBOR-for-fixed swap rates. Statement II is incorrect because using LIBOR rates rather than OIS rates for discounting will change the estimates of forward LIBOR rates.

The following is a review of the Market Risk Measurement and Management principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Volatility Smiles

Topic 16

Exam Focus

This topic discusses some of the reasons for the existence of volatility smiles, and how volatility affects option pricing as well as other option characteristics. Focus on the explanation of why volatility smiles exist in currency and equity options. Also, understand how volatility smiles impact the Greeks and how to interpret price jumps.

Put-Call Parity

LO 16.2: Explain the implications of put-call parity on the implied volatility of call and put options.

Recall that put-call parity is a no-arbitrage equilibrium relationship that relates European call and put option prices to the underlying asset's price and the present value of the option's strike price. In its simplest form, put-call parity can be represented by the following relationship:

$$c - p = S - PV(X)$$

where:
c = price of a call
p = price of a put
S = price of the underlying security
PV(X) = present value of the strike

PV(X) can be represented in continuous time by:

$$PV(X) = Xe^{-rT}$$

where:
r = risk-free rate
T = time left to expiration expressed in years

Since put-call parity is a no-arbitrage relationship, it will hold whether or not the underlying asset price distribution is lognormal, as required by the Black-Scholes-Merton (BSM) option pricing model.

If we simply rearrange put-call parity and denote subscripts for the option prices to indicate whether they are market or Black-Scholes-Merton option prices, the following two equations are generated:

$$p_{mkt} + S_0e^{-qt} = c_{mkt} + PV(X)$$

$$p_{BSM} + S_0e^{-qt} = c_{BSM} + PV(X)$$

Subtracting the second equation from the first gives us:

$$p_{mkt} - p_{BSM} = c_{mkt} - c_{BSM}$$

This relationship shows that, given the same strike price and time to expiration, option market prices that deviate from those dictated by the Black-Scholes-Merton model are going to deviate in the same amount whether they are for calls or puts. Since any deviation in prices will be the same, the implication is that the implied volatility of a call and put will be equal for the same strike price and time to expiration.

Volatility Smiles

LO 16.1: Define volatility smile and volatility skew.

LO 16.3: Compare the shape of the volatility smile (or skew) to the shape of the implied distribution of the underlying asset price and to the pricing of options on the underlying asset.

Actual option prices, in conjunction with the BSM model, can be used to generate implied volatilities which may differ from historical volatilities. When option traders allow implied volatility to depend on strike price, patterns of implied volatility are generated which resemble "**volatility smiles.**" These curves display implied volatility as a function of the option's strike (or exercise) price. In this topic, we will examine volatility smiles for both currency and equity options. In the case of equity options, the volatility smile is sometimes referred to as a **volatility skew** since, as we will see in LO 16.5, the volatility pattern for equity options is essentially an inverse relationship.

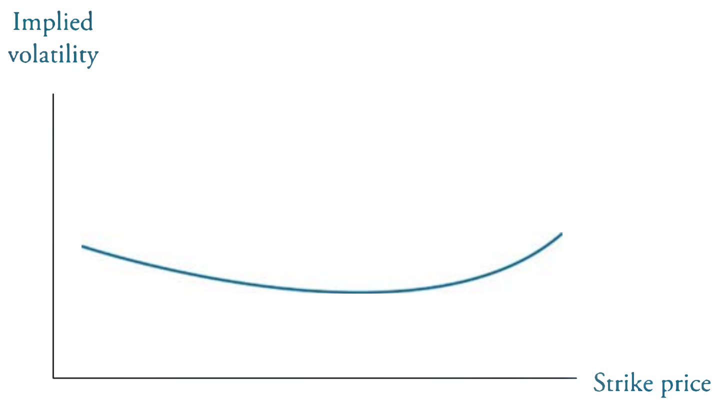
Foreign Currency Options

LO 16.4: Describe characteristics of foreign exchange rate distributions and their implications on option prices and implied volatility.

LO 16.5: Describe the volatility smile for equity options and foreign currency options and provide possible explanations for its shape.

The volatility pattern used by traders to price currency options generates implied volatilities that are higher for deep in-the-money and deep out-of-the-money options, as compared to the implied volatility for at-the-money options, as shown in Figure 1.

Figure 1: Volatility Smile for Foreign Currency Options



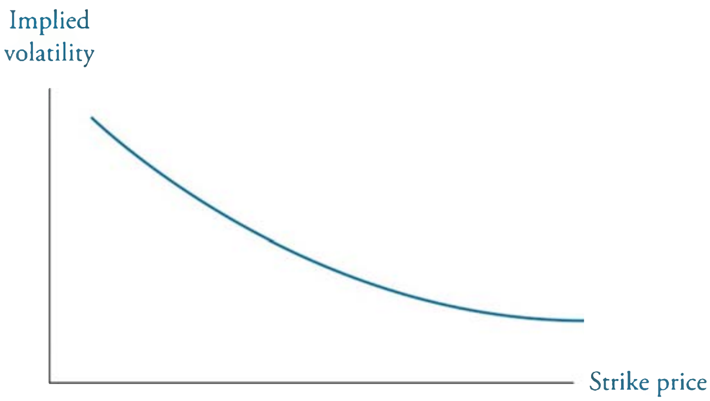
The easiest way to see why implied volatilities for away-from-the-money options are greater than at-the-money options is to consider the following call and put examples. For calls, a currency option is going to pay off only if the actual exchange rate is above the strike rate. For puts, on the other hand, a currency option is going to pay off only if the actual exchange rate is below the strike rate. If the implied volatilities for actual currency options are greater for away-from-the-money than at-the-money options, *currency traders must think there is a greater chance of extreme price movements than predicted by a lognormal distribution.* Empirical evidence indicates that this is the case.

This tendency for exchange rate changes to be more extreme is a function of the fact that exchange rate volatility is not constant and frequently jumps from one level to another, which increases the likelihood of extreme currency rate levels. However, these two effects tend to be mitigated for long-dated options, which tend to exhibit less of a volatility smile pattern than shorter-dated options.

Equity Options

The equity option volatility smile is different from the currency option pattern. The smile is more of a "smirk," or skew, that shows a higher implied volatility for low strike price options (in-the-money calls and out-of-the-money puts) than for high strike price options (in-the-money puts and out-of-the-money calls). As shown in Figure 2, there is essentially an inverse relationship between implied volatility and the strike price of equity options.

Figure 2: Volatility Smile for Equities



The volatility smirk (half-smile) exhibited by equity options translates into a left-skewed implied distribution of equity price changes. This left-skewed distribution indicates that *equity traders believe the probability of large down movements in price is greater than large up movements in price, as compared with a lognormal distribution*. Two reasons have been promoted as causing this increased implied volatility—leverage and "crashophobia."

- **Leverage.** When a firm's equity value decreases, the amount of leverage increases, which essentially increases the riskiness, or "volatility," of the underlying asset. When a firm's equity increases in value, the amount of leverage decreases, which tends to decrease the riskiness of the firm. This lowers the volatility of the underlying asset. All else held constant, there is an inverse relationship between volatility and the underlying asset's valuation.
- **Crashophobia.** The second explanation, used since the 1987 stock market crash, was coined "crashophobia" by Mark Rubinstein. Market participants are simply afraid of another market crash, so they place a premium on the probability of stock prices falling precipitously—deep out-of-the-money puts will exhibit high premiums since they provide protection against a substantial drop in equity prices. There is some support for Rubinstein's crashophobia hypothesis, because the volatility skew tends to increase when equity markets decline, but is not as noticeable when equity markets increase in value.

Alternative Methods for Studying Volatility Smiles

LO 16.6: Describe alternative ways of characterizing the volatility smile.

The volatility smiles we have characterized thus far have examined the relationship between implied volatility and strike price. Other relationships exist which allow traders to use alternative methods to study these volatility patterns. All alternatives require a replacement of the independent variable, strike price (X).

One alternative method involves replacing the strike price with strike price divided by stock price (X / S_0). This method results in a more stable volatility smile. A second alternative

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approach is to substitute the strike price with strike price divided by the forward price for the underlying asset (X / F_0). The forward price would have the same maturity date as the options being assessed. Traders sometimes view the forward price as a better gauge of at the money option prices since the forward price displays the theoretical expected stock price. A third alternative method involves replacing the strike price with the option's delta. With this approach, traders are able to study volatility smiles of options other than European and American options.

Volatility Term Structure and Volatility Surfaces

LO 16.7: Describe volatility term structures and volatility surfaces and how they may be used to price options.

The **volatility term structure** is a listing of implied volatilities as a function of time to expiration for at-the-money option contracts. When short-dated volatilities are low (from historical perspectives), volatility tends to be an increasing function of maturity. When short-dated volatilities are high, volatility tends to be an inverse function of maturity. This phenomenon is related to, but has a slightly different meaning from, the mean-reverting characteristic often exhibited by implied volatility.

A **volatility surface** is nothing other than a combination of a volatility term structure with volatility smiles (i.e., those implied volatilities away-from-the-money). The surface provides guidance in pricing options with any strike or maturity structure.

A trader's primary objective is to maintain a pricing mechanism that generates option prices consistent with market pricing. Even if the implied volatility or model pricing errors change due to shifting from one pricing model to another (which could occur if traders use an alternative model to Black-Scholes-Merton), the objective is to have consistency in model-generated pricing. The volatility term structure and volatility surfaces can be used to confirm or disprove a model's accuracy and consistency in pricing.

The Option Greeks

LO 16.8: Explain the impact of the volatility smile on the calculation of the "Greeks."

Option Greeks indicate expected changes in option prices given changes in the underlying factors that affect option prices.

The problem here is that option Greeks, including delta and vega, may be affected by the implied volatility of an option. Remember these guidelines for how implied volatility may affect the Greek calculations of an option:

- The first guideline is the **sticky strike rule**, which makes an assumption that an option's implied volatility is the same over short time periods (e.g., successive days). If this is the case, the Greek calculations of an option are assumed to be unaffected, as long as the implied volatility is unchanged. If implied volatility changes, the option sensitivity calculations may not yield the correct figures.

- The second guideline is the **sticky delta rule**, which assumes the relationship between an option's price and the ratio of underlying to strike price applies in subsequent periods. The idea here is that the implied volatility reflects the moneyness of the option, so the delta calculation includes an adjustment factor for implied volatility. If the sticky delta rule holds, the option's delta will be larger than that given by the Black-Scholes-Merton formula.

Keep in mind, however, that both rules assume the volatility smile is flat for all option maturities. If this is not the case, the rules are not internally consistent and, to correct for a non-flat volatility smile, we would have to rely on an implied volatility function or tree to correctly calculate option Greeks.

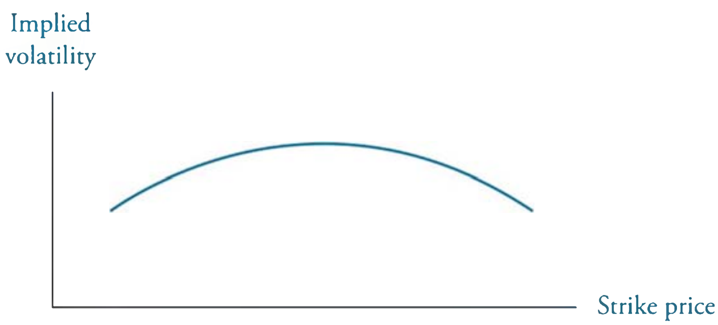
Price Jumps

LO 16.9: Explain the impact of a single asset price jump on a volatility smile.

Price jumps can occur for a number of reasons. One reason may be the expectation of a significant news event that causes the underlying asset to move either up or down by a large amount. This would cause the underlying distribution to become bimodal, but with the same expected return and standard deviation as a unimodal, or standard, price-change distribution.

Implied volatility is affected by price jumps and the probabilities assumed for either a large up or down movement. The usual result, however, is that at-the-money options tend to have a higher implied volatility than either out-of-the-money or in-the-money options. Away-from-the-money options exhibit a lower implied volatility than at-the-money options. Instead of a volatility smile, price jumps would generate a volatility frown, as in Figure 3.

Figure 3: Volatility Smile (Frown) With Price Jump



Key Concepts

LO 16.1

When option traders allow implied volatility to depend on strike price, patterns of implied volatility resemble volatility smiles.

LO 16.2

Put-call parity indicates that the deviation between market prices and Black-Scholes-Merton prices will be equivalent for calls and puts. Hence, implied volatility will be the same for calls and puts.

LO 16.3

Currency traders believe there is a greater chance of extreme price movements than predicted by a lognormal distribution. Equity traders believe the probability of large down movements in price is greater than large up movements in price, as compared with a lognormal distribution.

LO 16.4

The volatility pattern used by traders to price currency options generates implied volatilities that are higher for deep in-the-money and deep out-of-the-money options, as compared to the implied volatility for at-the-money options.

LO 16.5

The volatility smile exhibited by equity options is more of a "smirk," with implied volatility higher for low strike prices. This has been attributed to leverage and "crashophobia" effects.

LO 16.6

Alternative methods to studying volatility patterns include: replacing strike price with strike price divided by stock price, replacing strike price with strike price divided by the forward price for the underlying asset, and replacing strike price with option delta.

LO 16.7

Volatility term structures and volatility surfaces are used by traders to judge consistency in model-generated option prices.

LO 16.8

Volatility smiles that are not flat require the use of implied volatility functions or trees to correctly calculate option Greeks.

LO 16.9

Price jumps may generate volatility "frowns" instead of smiles.

Concept Checkers

1. The market price deviations for puts and calls from Black-Scholes-Merton prices indicate:
   A. equivalent put and call implied volatility.
   B. equivalent put and call moneyness.
   C. unequal put and call implied volatility.
   D. unequal put and call moneyness.

2. An empirical distribution that exhibits a fatter right tail than that of a lognormal distribution would indicate:
   A. equal implied volatilities across low and high strike prices.
   B. greater implied volatilities for low strike prices.
   C. greater implied volatilities for high strike prices.
   D. higher implied volatilities for mid-range strike prices.

3. The "sticky strike rule" assumes that implied volatility is:
   A. the same across maturities for given strike prices.
   B. the same for short time periods.
   C. the same across strike prices for given maturities.
   D. different across strike prices for given maturities.

4. Compared to at-the-money currency options, out-of-the-money currency options exhibit which of the following volatility traits?
   A. Lower implied volatility.
   B. A frown.
   C. A smirk.
   D. Higher implied volatility.

5. Which of the following regarding equity option volatility is true?
   A. There is higher implied price volatility for away-from-the-money equity options.
   B. "Crashophobia" suggests actual equity volatility increases when stock prices decline.
   C. Compared to the lognormal distribution, traders believe the probability of large down movements in price is similar to large up movements.
   D. Increasing leverage at lower equity prices suggests increasing volatility.

Concept Checker Answers

1. **A** Put-call parity indicates that the implied volatility of a call and put will be equal for the same strike price and time to expiration.

2. **C** An empirical distribution with a fat right tail generates a higher implied volatility for higher strike prices due to the increased probability of observing high underlying asset prices. The pricing indication is that in-the-money calls and out-of-the-money puts would be "expensive."

3. **B** The sticky strike rule, when applied to calculating option sensitivity measures, assumes implied volatility is the same over short time periods.

4. **D** Away-from-the-money currency options have greater implied volatility than at-the-money options. This pattern results in a volatility smile.

5. **D** There is higher implied price volatility for low strike price equity options. "Crashophobia" is based on the idea that large price declines are more likely than assumed in Black-Scholes-Merton prices, not that volatility increases when prices decline. Compared to the lognormal distribution, traders believe the probability of large down movements in price is higher than large up movements. Increasing leverage at lower equity prices suggests increasing volatility.

Self-Test: Market Risk Measurement and Management

10 Questions: 30 Minutes

1. An analyst for Z Corporation is determining the value at risk (VaR) for the corporation's profit/loss distribution that is assumed to be normally distributed. The profit/loss distribution has an annual mean of $5 million and a standard deviation of $3.5 million. Using a parametric approach, what is the VaR with a 99% confidence level?
   A. $0.775 million.
   B. $3.155 million.
   C. $5.775 million.
   D. $8.155 million.

2. The Basel Committee requires backtesting of actual losses to VaR calculations. How many exceptions would need to occur in a 250-day trading period for the capital multiplier to increase from three to four?
   A. two to five.
   B. five to seven.
   C. seven to nine.
   D. ten or more.

3. The top-down approach to risk aggregation assumes that a bank's portfolio can be cleanly subdivided according to market, credit, and operational risk measures. In contrast, a bottom-up approach attempts to account for interactions among various risk factors. In order to assess which approach is more appropriate, academic studies evaluate the ratio of integrated risks to separate risks. Regarding studies of top-down and bottom-up approaches, which of the following statements is incorrect?
   A. Top-down studies suggest that risk diversification is present.
   B. Bottom-up studies sometimes calculate the ratio of integrated risks to separate risks to be less than one.
   C. Bottom-up studies suggest that risk diversification should be questioned.
   D. Top-down studies calculate the ratio of integrated risks to separate risks to be greater than one.

4. Commercial Bank Z has a $3 million loan to company A and a $3 million loan to company B. Companies A and B each have a 5% and 4% default probability, respectively. The default correlation between companies A and B is 0.6. What is the expected loss (EL) for the commercial bank under the worst case scenario?
   a. $83,700.
   b. $133,900.
   c. $165,600.
   d. $233,800.

5. A risk manager should always pay careful attention to the limitations and advantages of applying financial models such as the value at risk (VaR) and Black-Scholes-Merton (BSM) option pricing model. Which of the following statements regarding financial models is correct?
   a. Financial models should always be calibrated using most recent market data because it is more likely to be accurate in extrapolating trends.
   b. When applying the VaR model, empirical studies imply asset returns closely follow the normal distribution.
   c. The Black-Scholes-Merton option pricing model is a good example of the advantage of using financial models because the model eliminates all mathematical inconsistences that can occur with human judgment.
   d. A good example of a limitation of a financial model is the assumption of constant volatility when applying the Black-Scholes-Merton (BSM) option pricing model.

6. Assume that a trader wishes to set up a hedge such that he sells $100,000 of a Treasury bond and buys TIPS as a hedge. Using a historical yield regression framework, assume the DV01 on the T-bond is 0.072, the DV01 on the TIPS is 0.051, and the hedge adjustment factor (regression beta coefficient) is 1.2. What is the face value of the offsetting TIPS position needed to carry out this regression hedge?
   A. $138,462.
   B. $169,412.
   C. $268,499.
   D. $280,067.

7. A constant maturity Treasury (CMT) swap pays ($1,000,000 / 2) × ($y_{CMT}$ – 9%) every six months. There is a 70% probability of an increase in the 6-month spot rate and a 60% probability of an increase in the 1-year spot rate. The rate change in all cases is 0.50% per period, and the initial y_{CMT} is 9%. What is the value of this CMT swap?
   A. $2,325.
   B. $2,229.
   C. $2,429.
   D. $905.

8. Suppose the market expects that the current 1-year rate for zero-coupon bonds with a face value of $1 will remain at 5%, but the 1-year rate in one year will be 3%. What is the 2-year spot rate for zero-coupon bonds?
   A. 3.995%.
   B. 4.088%.
   C. 4.005%.
   D. 4.115%.

9. An analyst is modeling spot rate changes using short rate term structure models. The current short-term interest rate is 5% with a volatility of 80bps. After one month passes the realization of *dw*, a normally distributed random variable with mean 0 and standard deviation $\sqrt{dt}$, is −0.5. Assume a constant interest rate drift, λ, of 0.36%. What should the analyst compute as the new spot rate?
   A. 5.37%.
   B. 4.63%.
   C. 5.76%.
   D. 4.24%.

10. Which of the following statements is incorrect regarding volatility smiles?
   A. Currency options exhibit volatility smiles because the at-the-money options have higher implied volatility than away-from-the-money options.
   B. Volatility frowns result when jumps occur in asset prices.
   C. Equity options exhibit a volatility smirk because low strike price options have greater implied volatility.
   D. Relative to currency traders, it appears that equity traders' expectations of extreme price movements are more asymmetric.

Self-Test Answers: Market Risk Measurement and Management

1. **B** The population mean and standard deviations are unknown; therefore, the standard normal z-value of 2.33 is used for a 99% confidence level.

   VaR(1%) = –5.0 million + ($3.5 million)(2.33) = –5.0 million + 8.155 million = 3.155 million (See Topic 1)

2. **D** Ten or more backtesting violations require the institution to use a capital multiplier of four. (See Topic 3)

3. **D** Top-down studies calculate this ratio to be less than one, which suggests that risk diversification is present and ignored by the separate approach. Bottom-up studies also often calculate this ratio to be less than one; however, this research has not been conclusive, and has recently found evidence of risk compounding, which produces a ratio greater than one. Thus, bottom-up studies suggests that risk diversification should be questioned. (See Topic 5)

4. **C** The default probability of company A is 5%. Thus, the standard deviation for company A is:

   $$\sqrt{0.05(1-0.05)} = 0.2179$$

   Company B has a default probability of 4% and, therefore, will have a standard deviation of 0.1960. We can now calculate the expected loss under the worst case scenario where both companies A and B are in default. Assuming that the default correlation between A and B is 0.6, the joint probability of default is:

   $$P(AB) = 0.6\sqrt{0.05(0.95) \times 0.04(0.96)} + 0.05 \times 0.04$$
   $$= 0.6\sqrt{0.001824} + 0.002 = 0.0276$$

   Thus, the expected loss for the commercial bank is $165,600 (= 0.0276 × $6,000,000). (See Topic 6)

5. **D** The Black-Scholes-Merton (BSM) option pricing model assumes strike prices have a constant volatility. However, numerous empirical studies find higher volatility for out-of-the-money options and a volatility skew in equity markets. Thus, this is a good example of a limitation of financial models. The choice of time period used to calibrate the parameter inputs for the model can have a big impact on the results. Risk managers used volatility and correlation estimates from pre-crisis periods during the recent financial crisis, and this resulted in significantly underestimating the risk for financial models. All financial models should be stress tested using scenarios of extreme economic conditions. VaR models often assume asset returns have a normal distribution. However, empirical studies find higher kurtosis in return distributions. High kurtosis implies a distribution with fatter tails than the normal distribution. Thus, the normal distribution is not the best assumption for the underlying distribution. Financial models contain mathematical inconsistencies. For example, in applying the BSM option pricing model for up-and-out calls and puts and down-and-out calls and puts, there are rare cases where the inputs make the model insensitive to changes in implied volatility and option maturity. (See Topic 8)

6. B Defining F^R and F^N as the face amounts of the real and nominal bonds, respectively, and their corresponding DV01s as DV01^R and DV01^N, a DV01 hedge is adjusted by the hedge adjustment factor, or beta, as follows:

$$F^R = F^N \times \left(\frac{\text{DV01}^N}{\text{DV01}^R}\right) \times \beta$$

$$F^R = 100{,}000 \times \left(\frac{0.072}{0.051}\right) \times 1.2 = 169{,}412$$

(See Topic 10)

7. A The payoff in each period is (\$1,000,000 / 2) × (y_{CMT} – 9%). For example, the 1-year payoff of \$5,000 in the figure below is calculated as (\$1,000,000 / 2) × (10% – 9%) = \$5,000. The other numbers in the year one cells are calculated similarly.

In six months, the payoff if interest rates increase to 9.50% is (\$1,000,000 / 2) × (9.5% – 9.0%) = \$2,500. Note that the price in this cell equals the present value of the probability weighted 1-year values plus the 6-month payoff:

$$V_{6\text{ months},U} = \frac{(\$5{,}000 \times 0.6) + (\$0 \times 0.4)}{1 + \frac{0.095}{2}} + \$2{,}500 = \$5{,}363.96$$

The other cell value in six months is calculated similarly and results in a loss of \$4,418.47.

The value of the CMT swap today is the present value of the probability weighted 6-month values:

$$V_0 = \frac{(\$5{,}363.96 \times 0.7) + (-\$4{,}418.47 \times 0.3)}{1 + \frac{0.09}{2}} = \$2{,}324.62$$

Thus the correct response is A. The other answers are incorrect because they do not correctly discount the future values or omit the 6-month payoff from the 6-month values.

(See Topic 11)

8. **A** The 2-year spot rate is computed as follows:

$$\hat{r}(2) = \sqrt[2]{(1.05)(1.03)} - 1 = 3.995\%$$

(See Topic 12)

9. **B** This short rate process has an annualized drift of 0.36%, so it requires the use of Model 2 (with constant drift). The change in the spot rate is computed as:

$dr = \lambda dt + \sigma dw$

$dr = (0.36\% / 12) + (0.8\% \times -0.5) = -0.37\% = -37$ basis points

Since the initial short-term rate was 5% and *dr* is –0.37%, the new spot rate in one month is:

5% – 0.37% = 4.63%

(See Topic 13)

10. **A** Currency options exhibit volatility smiles because the at-the-money options have lower implied volatility than away-from-the-money options.

Equity traders believe that the probability of large price decreases is greater than the probability of large price increases. Currency traders' beliefs about volatility are more symmetric as there is no large skew in the distribution of expected currency values (i.e., there is a greater chance of large price movements in either direction).

(See Topic 16)

Formulas

Topic 1

profit/loss data: $P/L_t = P_t + D_t - P_{t-1}$

arithmetic return: $r_t = \dfrac{P_t + D_t - P_{t-1}}{P_{t-1}} = \dfrac{P_t + D_t}{P_{t-1}} - 1$

geometric return: $R_t = \ln\left(\dfrac{P_t + D_t}{P_{t-1}}\right)$

delta-normal VaR: $VaR(\alpha\%) = (-\mu_r + \sigma_r \times z_\alpha) \times P_{t-1}$

lognormal VaR: $VaR(\alpha\%) = P_{t-1} \times \left(1 - e^{\mu_R - \sigma_R \times z_\alpha}\right)$

standard error of a quantile: $se(q) = \dfrac{\sqrt{p(1-p)/n}}{f(q)}$

Topic 2

age-weighted historical simulation: $w(i) = \dfrac{\lambda^{i-1}(1-\lambda)}{1-\lambda^n}$

Topic 3

model accuracy test: $z = \dfrac{x - pT}{\sqrt{p(1-p)T}}$

unconditional coverage test statistic:

$$LR_{uc} = -2\ln[(1-p)^{T-N}p^N] + 2\ln\{[1-(N/T)]^{T-N}(N/T)^N\}$$

Topic 6

portfolio mean return: $\mu_P = w_X\mu_X + w_Y\mu_Y$

portfolio standard deviation: $\sigma_P = \sqrt{w_X^2\sigma_X^2 + w_Y^2\sigma_Y^2 + 2w_X w_Y \text{cov}_{XY}}$

covariance: $\text{cov}_{XY} = \dfrac{\sum_{t=1}^{n}(X_t - \mu_X)(Y_t - \mu_Y)}{n-1}$

correlation: $\rho_{XY} = \dfrac{\text{cov}_{XY}}{\sigma_X\sigma_Y}$

realized correlation: $\rho_{\text{realized}} = \dfrac{2}{n^2 - n}\sum_{i>j}\rho_{i,j}$

correlation swap payoff: notional amount × $(\rho_{\text{realized}} - \rho_{\text{fixed}})$

joint probability of default: $P(AB) = \rho_{AB}\sqrt{PD_A(1-PD_A)\times PD_B(1-PD_B)} + PD_A \times PD_B$

Topic 7

mean reversion rate: $S_t - S_{t-1} = a(\mu - S_{t-1})$

autocorrelation: $AC(\rho_t, \rho_{t-1}) = \dfrac{\text{cov}(\rho_t, \rho_{t-1})}{\sigma(\rho_t)\times\sigma(\rho_{t-1})}$

Topic 8

correlation with expectation values: $\rho_{XY} = \dfrac{E(XY) - E(X)E(Y)}{\sqrt{E(X^2) - (E(X))^2} \times \sqrt{E(Y^2) - (E(Y))^2}}$

Spearman's rank correlation: $\rho_S = 1 - \dfrac{6\sum_{i=1}^{n} d_i^2}{n(n^2 - 1)}$

Kendall's τ: $\tau = \dfrac{n_c - n_d}{n(n-1)/2}$

Topic 12

2-year spot rate: $\hat{r}(2) = \sqrt[2]{(1+r_1)(1+r_2)} - 1$

3-year spot rate: $\hat{r}(3) = \sqrt[3]{(1+r_1)(1+r_2)(1+r_3)} - 1$

Jensen's inequality: $E\left[\dfrac{1}{(1+r)}\right] > \dfrac{1}{E[1+r]}$

Topic 13

Model 1:

$dr = \sigma dw$

where:
dr = change in interest rates over small time interval, dt
dt = small time interval (measured in years)
σ = annual basis-point volatility of rate changes
dw = normally distributed random variable with mean 0 and standard deviation $\sqrt{dt}$

Model 2: $dr = \lambda dt + \sigma dw$

Vasicek model:

$dr = k(\theta - r)dt + \sigma dw$

where:
k = a parameter that measures the speed of reversion adjustment
θ = long-run value of the short-term rate assuming risk neutrality
r = current interest rate level

long-run value of short-term rate:

$$\theta \approx r_l + \frac{\lambda}{k}$$

where:
r_l = the long-run true rate of interest

Topic 14

Model 3:

$$dr = \lambda(t)dt + \sigma e^{-\alpha t}dw$$

where:
σ = volatility at $t = 0$, which decreases exponentially to 0 for $\alpha > 0$

CIR model: $dr = k(\theta - r)dt + \sigma\sqrt{r}\,dw$

Model 4: $dr = ardt + \sigma rdw$

Topic 16

put-call parity: $c - p = S - PV(X)$

Using the Cumulative Z-Table

Probability Example

Assume that the annual earnings per share (EPS) for a large sample of firms is normally distributed with a mean of \$5.00 and a standard deviation of \$1.50. What is the approximate probability of an observed EPS value falling between \$3.00 and \$7.25?

If EPS = x = \$7.25, then z = $(x - \mu)/\sigma$ = (\$7.25 – \$5.00)/\$1.50 = +1.50

If EPS = x = \$3.00, then z = $(x - \mu)/\sigma$ = (\$3.00 – \$5.00)/\$1.50 = –1.33

For z-value of 1.50: Use the row headed 1.5 and the column headed 0 to find the value 0.9332. This represents the area under the curve to the left of the critical value 1.50.

For z-value of –1.33: Use the row headed 1.3 and the column headed 3 to find the value 0.9082. This represents the area under the curve to the left of the critical value +1.33. The area to the left of –1.33 is 1 – 0.9082 = 0.0918.

The area between these critical values is 0.9332 – 0.0918 = 0.8414, or 84.14%.

Hypothesis Testing – One-Tailed Test Example

A sample of a stock's returns on 36 non-consecutive days results in a mean return of 2.0%. Assume the population standard deviation is 20.0%. Can we say with 95% confidence that the mean return is greater than 0%?

H_0: $\mu \leq 0.0\%$, H_A: $\mu > 0.0\%$. The test statistic = *z*-statistic = $\dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$ = (2.0 – 0.0) / (20.0 / 6) = 0.60.

The significance level = 1.0 – 0.95 = 0.05, or 5%.

Since this is a one-tailed test with an alpha of 0.05, we need to find the value 0.95 in the cumulative *z*-table. The closest value is 0.9505, with a corresponding critical *z*-value of 1.65. Since the test statistic is less than the critical value, we fail to reject H_0.

Hypothesis Testing – Two-Tailed Test Example

Using the same assumptions as before, suppose that the analyst now wants to determine if he can say with 99% confidence that the stock's return is not equal to 0.0%.

H_0: $\mu = 0.0\%$, H_A: $\mu \neq 0.0\%$. The test statistic (*z*-value) = (2.0 – 0.0) / (20.0 / 6) = 0.60. The significance level = 1.0 – 0.99 = 0.01, or 1%.

Since this is a two-tailed test with an alpha of 0.01, there is a 0.005 rejection region in both tails. Thus, we need to find the value 0.995 (1.0 – 0.005) in the table. The closest value is 0.9951, which corresponds to a critical *z*-value of 2.58. Since the test statistic is less than the critical value, we fail to reject H_0 and conclude that the stock's return equals 0.0%.

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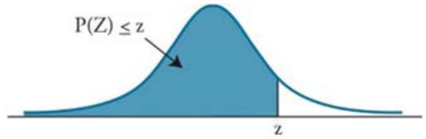
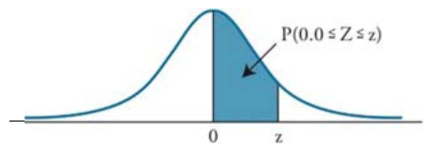


CUMULATIVE Z-TABLE

$P(Z \leq z) = N(z)$ for $z \geq 0$

$P(Z \leq -z) = 1 - N(z)$

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.937	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.983	0.9834	0.9838	0.9842	0.9846	0.985	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.989
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.994	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990



ALTERNATIVE Z-TABLE

$P(Z \leq z) = N(z)$ for $z \geq 0$

$P(Z \leq -z) = 1 - N(z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3356	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4939	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

Student's T-Distribution

Level of Significance for One-Tailed Test						
df	0.100	0.050	0.025	0.01	0.005	0.0005
Level of Significance for Two-Tailed Test						
df	0.20	0.10	0.05	0.02	0.01	0.001
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.599
3	1.638	2.353	3.182	4.541	5.841	12.294
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.869
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.408
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015
17	1.333	1.740	2.110	2.567	2.898	3.965
18	1.330	1.734	2.101	2.552	2.878	3.922
19	1.328	1.729	2.093	2.539	2.861	3.883
20	1.325	1.725	2.086	2.528	2.845	3.850
21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.768
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	2.485	2.787	3.725
26	1.315	1.706	2.056	2.479	2.779	3.707
27	1.314	1.703	2.052	2.473	2.771	3.690
28	1.313	1.701	2.048	2.467	2.763	3.674
29	1.311	1.699	2.045	2.462	2.756	3.659
30	1.310	1.697	2.042	2.457	2.750	3.646
40	1.303	1.684	2.021	2.423	2.704	3.551
60	1.296	1.671	2.000	2.390	2.660	3.460
120	1.289	1.658	1.980	2.358	2.617	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.291

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