2017 SchweserNotes™ Part II

FRM®
Exam Prep

Market Risk Measurement and Management

eBook 1

KAPLAN UNIVERSITY SCHOOL OF PROFESSIONAL AND CONTINUING EDUCATION | SCHWESER

Getting Started

Part II FRM® Exam

Welcome

As the Vice President of Product Management at Kaplan Schweser, I am pleased to have the opportunity to help you prepare for the 2017 FRM® Exam. Getting an early start on your study program is important for you to sufficiently **Prepare ▸ Practice ▸ Perform®** on exam day. Proper planning will allow you to set aside enough time to master the learning objectives in the Part II curriculum.

Now that you've received your SchweserNotes™, here's how to get started:

Step 1: Access Your Online Tools

Visit **www.schweser.com/frm** and log in to your online account using the button located in the top navigation bar. After logging in, select the appropriate part and proceed to the dashboard where you can access your online products.

Step 2: Create a Study Plan

Create a study plan with the Schweser Study Calendar (located on the Schweser dashboard). Then view the Candidate Resource Library on-demand videos for an introduction to core concepts.

Step 3: Prepare and Practice

Read your SchweserNotes™

Our clear, concise study notes will help you **prepare** for the exam. At the end of each reading, you can answer the Concept Checker questions for better understanding of the curriculum.

Attend a Weekly Class

Attend our Live Online Weekly Class or review the on-demand archives as often as you like. Our expert faculty will guide you through the FRM curriculum with a structured approach to help you **prepare** for the exam. (See our instruction packages to the right. Visit **www.schweser.com/frm** to order.)

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Maximize your retention of important concepts and **practice** answering exam-style questions in the SchweserPro™ QBank and taking several Practice Exams. Use Schweser's QuickSheet for continuous review on the go. (Visit **www.schweser.com/frm** to order.)

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A few weeks before the exam, make use of our Online Review Workshop Package. Review key curriculum concepts in every topic, **perform** by working through demonstration problems, and **practice** your exam techniques with our 8-hour live Online Review Workshop. Use Schweser's Secret Sauce® for convenient study on the go.

Step 5: Perform

As part of our Online Review Workshop Package, take a Schweser Mock Exam to ensure you are ready to **perform** on the actual FRM Exam. Put your skills and knowledge to the test and gain confidence before the exam.

Again, thank you for trusting Kaplan Schweser with your FRM Exam preparation!

Sincerely,

Derek Burkett

Derek Burkett, CFA, FRM, CAIA
VP, Product Management, Kaplan Schweser

The Kaplan Way for Learning

PREPARE
Acquire new knowledge through demonstration and examples.

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Apply new knowledge through simulation and practice.

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May Exam Instructor
Dr. John Broussard
CFA, FRM

November Exam Instructor
Dr. Greg Filbeck
CFA, FRM, CAIA

*Dates, times, and instructors subject to change

Contact us for questions about your study package, upgrading your package, purchasing additional study materials, or for additional information:

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FRM Part II Book 1: Market Risk Measurement and Management

FRM 2017 PART II BOOK 1: MARKET RISK MEASUREMENT AND MANAGEMENT

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Welcome to the 2017 SchweserNotes

Thank you for trusting Kaplan Schweser to help you reach your goals. We are very pleased to be able to help you prepare for the Part II FRM exam. In this introduction, I want to explain the resources included with the SchweserNotes, suggest how you can best use Schweser materials to prepare for the exam, and direct you toward other educational resources you will find helpful as you study for the exam.

Besides the SchweserNotes themselves, there are many educational resources available at Schweser.com. Just log in using the individual username and password you received when you purchased the SchweserNotes.

SchweserNotes™

The SchweserNotes consist of four volumes that include complete coverage of all FRM assigned topics and learning objectives (LOs), Concept Checkers (multiple-choice questions for every topic), and Self-Test questions to help you master the material and check your retention of key concepts.

Online Practice Questions

To retain what you learn, it is important that you quiz yourself often. We offer an online version of the SchweserPro™ QBank, which contains hundreds of Part II practice questions and explanations. Quizzes are available for each topic or across multiple topics. Build your own exams by specifying the topics and the number of questions.

Practice Exams

Schweser offers two full 4-hour practice exams. These exams are important tools for gaining the speed and skills you will need to pass the exam. The Practice Exams book contains answers with full explanations for self-grading and evaluation.

Schweser Study Calendar

Use your Online Access to tell us when you will start and what days of the week you can study. The online Schweser Study Calendar will create a study plan just for you, breaking the curriculum into daily and weekly tasks to keep you on track and help you monitor your study progress.

The Part II FRM exam is a formidable challenge (covering 79 assigned readings and almost 500 learning objectives), and you must devote considerable time and effort to be properly prepared. There are no shortcuts! You must learn the material, know the terminology and techniques, understand the concepts, and be able to answer 80 multiple choice questions quickly and (at least 70%) correctly. A good estimate of the study time required on average is 250 hours, but some candidates will need more or less time, depending on their individual backgrounds and experience.

To help you really master this material and be well-prepared for the FRM exam, we offer several other educational resources, including:

Online Weekly Class

Our Online Weekly Class is offered each week, beginning in February for the May exam and August for the November exam. This online class brings the personal attention of a classroom into your home or office with 30 hours of real-time instruction, led by either Dr. John Paul Broussard, CFA, FRM, or Dr. Greg Filbeck, CFA, FRM, CAIA. The class offers in-depth coverage of difficult concepts, instant feedback during lecture and Q&A sessions, and discussion of sample exam questions. Archived classes are available for viewing at any time throughout the season. Candidates enrolled in the Online Weekly Class also have full access to supplemental on-demand video instruction in the Candidate Resource Library and an e-mail address link for sending questions to the instructor at any time.

Late-Season Review

Late-season review and exam practice can make all the difference. Our Review Package helps you evaluate your exam readiness with products specifically designed for late-season studying. This Review Package includes the Online Review Workshop (8-hour live and archived online review of essential curriculum topics), the Schweser Mock Exam (one 4-hour exam), and Schweser's Secret Sauce® (concise summary of the FRM curriculum).

Part II Exam Weights

In preparing for the exam, pay attention to the weights assigned to each knowledge domain within the curriculum. The Part II exam weights are as follows:

Book	*Knowledge Domains*	*Exam Weight*	*Exam Questions*
1	Market Risk Measurement and Management	25%	20
2	Credit Risk Measurement and Management	25%	20
3	Operational and Integrated Risk Management	25%	20
4	Risk Management and Investment Management	15%	12
4	Current Issues in Financial Markets	10%	8

How to Succeed

There are no shortcuts to studying for this exam. Expect GARP to test you in a way that will reveal how well you know the Part II curriculum. You should begin studying early and stick to your study plan. You should first read the SchweserNotes and complete the Concept Checkers for each topic. At the end of each book, you should answer the provided Self-Test questions to understand how concepts may be tested on the exam. You should finish the overall curriculum at least two weeks before the FRM exam. This will allow sufficient time for Practice Exams and further review of those topics you have not yet mastered.

I would like to take this opportunity to thank the content developers, editors, and graphic designers who worked countless hours to create the 2017 FRM SchweserNotes. I would especially like to thank Adam Stueber, CAIA; Derek Burkett, CFA, FRM, CAIA; Tim Greive, CFA; Craig Prochaska, CFA; Kent Westlund, CFA; Kurt Schuldes, CFA, CAIA; Jeff Bahr, Andy Bauer, Allie Bottcher, Katherine Bourgeois, Genevieve Kretschmer, Alyssa Brunner, Lindsey Casto, Tiffany Finstuen, Laura Goetzinger, Jared Heintz, Hannah Kelley, Alissa Knop, Gretchen Panzer, Jessica Pearse, Ashley Sinclair, Ben Strong, and Debbie White for their contributions.

Best regards,

Eric Smith

Eric Smith, CFA, FRM
FRM Product Manager
Kaplan Schweser

Reading Assignments and Learning Objectives

The following material is a review of the Market Risk Measurement and Management principles designed to address the learning objectives set forth by the Global Association of Risk Professionals.

Reading Assignments

Learning Objectives

1. Estimating Market Risk Measures: An Introduction and Overview

After completing this reading, you should be able to:
1. Estimate VaR using a historical simulation approach. (page 2)
2. Estimate VaR using a parametric approach for both normal and lognormal return distributions. (page 4)
3. Estimate the expected shortfall given P/L or return data. (page 6)
4. Define coherent risk measures. (page 6)
5. Estimate risk measures by estimating quantiles. (page 6)
6. Evaluate estimators of risk measures by estimating their standard errors. (page 7)
7. Interpret QQ plots to identify the characteristics of a distribution. (page 9)

2. Non-parametric Approaches

After completing this reading, you should be able to:
1. Apply the bootstrap historical simulation approach to estimate coherent risk measures. (page 15)
2. Describe historical simulation using non-parametric density estimation. (page 16)
3. Compare and contrast the age-weighted, the volatility-weighted, the correlation-weighted, and the filtered historical simulation approaches. (page 17)
4. Identify advantages and disadvantages of non-parametric estimation methods. (page 19)

3. Backtesting VaR

After completing this reading, you should be able to:
1. Define backtesting and exceptions and explain the importance of backtesting VaR models. (page 25)
2. Explain the significant difficulties in backtesting a VaR model. (page 26)
3. Verify a model based on exceptions or failure rates. (page 26)
4. Define and identify type I and type II errors. (page 26)
5. Explain the need to consider conditional coverage in the backtesting framework. (page 29)
6. Describe the Basel rules for backtesting. (page 28)

4. VaR Mapping

After completing this reading, you should be able to:
1. Explain the principles underlying VaR mapping, and describe the mapping process. (page 34)
2. Explain how the mapping process captures general and specific risks. (page 36)
3. Differentiate among the three methods of mapping portfolios of fixed income securities. (page 37)
4. Summarize how to map a fixed income portfolio into positions of standard instruments. (page 37)
5. Describe how mapping of risk factors can support stress testing. (page 38)
6. Explain how VaR can be used as a performance benchmark. (page 39)
7. Describe the method of mapping forwards, forward rate agreements, interest rate swaps, and options. (page 39)

5. **Messages from the Academic Literature on Risk Measurement for the Trading Book**

After completing this reading, you should be able to:
1. Explain the following lessons on VaR implementation: time horizon over which VaR is estimated, the recognition of time varying volatility in VaR risk factors, and VaR backtesting. (page 44)
2. Describe exogenous and endogenous liquidity risk and explain how they might be integrated into VaR models. (page 45)
3. Compare VaR, expected shortfall, and other relevant risk measures. (page 45)
4. Compare unified and compartmentalized risk measurement. (page 46)
5. Compare the results of research on "top-down" and "bottom-up" risk aggregation methods. (page 47)
6. Describe the relationship between leverage, market value of asset, and VaR within an active balance sheet management framework. (page 48)

6. **Some Correlation Basics: Properties, Motivation, Terminology**

After completing this reading, you should be able to:
1. Describe financial correlation risk and the areas in which it appears in finance. (page 52)
2. Explain how correlation contributed to the global financial crisis of 2007 to 2009. (page 62)
3. Describe the structure, uses, and payoffs of a correlation swap. (page 58)
4. Estimate the impact of different correlations between assets in the trading book on the VaR capital charge. (page 59)
5. Explain the role of correlation risk in market risk and credit risk. (page 64)
6. Relate correlation risk to systemic and concentration risk. (page 64)

7. **Empirical Properties of Correlation: How Do Correlations Behave in the Real World?**

After completing this reading, you should be able to:
1. Describe how equity correlations and correlation volatilities behave throughout various economic states. (page 76)
2. Calculate a mean reversion rate using standard regression and calculate the corresponding autocorrelation. (page 77)
3. Identify the best-fit distribution for equity, bond, and default correlations. (page 80)

8. **Statistical Correlation Models—Can We Apply Them to Finance?**

After completing this reading, you should be able to:
1. Evaluate the limitations of financial modeling with respect to the model itself, calibration of the model, and the model's output. (page 86)
2. Assess the Pearson correlation approach, Spearman's rank correlation, and Kendall's τ, and evaluate their limitations and usefulness in finance. (page 88)

9. **Financial Correlation Modeling—Bottom-Up Approaches**

After completing this reading, you should be able to:
1. Explain the purpose of copula functions and the translation of the copula equation. (page 99)
2. Describe the Gaussian copula and explain how to use it to derive the joint probability of default of two assets. (page 100)
3. Summarize the process of finding the default time of an asset correlated to all other assets in a portfolio using the Gaussian copula. (page 103)

10. Empirical Approaches to Risk Metrics and Hedging

After completing this reading, you should be able to:

1. Explain the drawbacks to using a DV01-neutral hedge for a bond position. (page 109)
2. Describe a regression hedge and explain how it can improve a standard DV01-neutral hedge. (page 110)
3. Calculate the regression hedge adjustment factor, beta. (page 111)
4. Calculate the face value of an offsetting position needed to carry out a regression hedge. (page 111)
5. Calculate the face value of multiple offsetting swap positions needed to carry out a two-variable regression hedge. (page 112)
6. Compare and contrast level and change regressions. (page 113)
7. Describe principal component analysis and explain how it is applied to constructing a hedging portfolio. (page 113)

11. The Science of Term Structure Models

After completing this reading, you should be able to:

1. Calculate the expected discounted value of a zero-coupon security using a binomial tree. (page 120)
2. Construct and apply an arbitrage argument to price a call option on a zero-coupon security using replicating portfolios. (page 120)
3. Define risk-neutral pricing and apply it to option pricing. (page 123)
4. Distinguish between true and risk-neutral probabilities, and apply this difference to interest rate drift. (page 123)
5. Explain how the principles of arbitrage pricing of derivatives on fixed income securities can be extended over multiple periods. (page 124)
6. Define option-adjusted spread (OAS) and apply it to security pricing. (page 129)
7. Describe the rationale behind the use of recombining trees in option pricing. (page 126)
8. Calculate the value of a constant maturity Treasury swap, given an interest rate tree and the risk-neutral probabilities. (page 127)
9. Evaluate the advantages and disadvantages of reducing the size of the time steps on the pricing of derivatives on fixed income securities. (page 130)
10. Evaluate the appropriateness of the Black-Scholes-Merton model when valuing derivatives on fixed income securities. (page 130)
11. Describe the impact of embedded options on the value of fixed income securities. (page 131)

12. The Evolution of Short Rates and the Shape of the Term Structure

After completing this reading, you should be able to:

1. Explain the role of interest rate expectations in determining the shape of the term structure. (page 137)
2. Apply a risk-neutral interest rate tree to assess the effect of volatility on the shape of the term structure. (page 139)
3. Estimate the convexity effect using Jensen's inequality. (page 141)
4. Evaluate the impact of changes in maturity, yield, and volatility on the convexity of a security. (page 141)
5. Calculate the price and return of a zero coupon bond incorporating a risk premium. (page 145)

13. The Art of Term Structure Models: Drift

After completing this reading, you should be able to:

1. Construct and describe the effectiveness of a short term interest rate tree assuming normally distributed rates, both with and without drift. (page 152)
2. Calculate the short-term rate change and standard deviation of the rate change using a model with normally distributed rates and no drift. (page 153)
3. Describe methods for addressing the possibility of negative short-term rates in term structure models. (page 154)
4. Construct a short-term rate tree under the Ho-Lee Model with time-dependent drift. (page 156)
5. Describe uses and benefits of the arbitrage-free models and assess the issue of fitting models to market prices. (page 156)
6. Describe the process of constructing a simple and recombining tree for a short-term rate under the Vasicek Model with mean reversion. (page 157)
7. Calculate the Vasicek Model rate change, standard deviation of the rate change, expected rate in T years, and half life. (page 160)
8. Describe the effectiveness of the Vasicek Model. (page 161)

14. The Art of Term Structure Models: Volatility and Distribution

After completing this reading, you should be able to:

1. Describe the short-term rate process under a model with time-dependent volatility. (page 167)
2. Calculate the short-term rate change and determine the behavior of the standard deviation of the rate change using a model with time dependent volatility. (page 167)
3. Assess the efficacy of time-dependent volatility models. (page 168)
4. Describe the short-term rate process under the Cox-Ingersoll-Ross (CIR) and lognormal models. (page 169)
5. Calculate the short-term rate change and describe the basis point volatility using the CIR and lognormal models. (page 169)
6. Describe lognormal models with deterministic drift and mean reversion. (page 171)

15. OIS Discounting, Credit Issues, and Funding Costs

After completing this reading, you should be able to:

1. Explain the main considerations in choosing a risk-free rate for derivatives valuation. (page 177)
2. Describe the OIS rate and the LIBOR-OIS spread, and explain their uses. (page 178)
3. Evaluate the appropriateness of the OIS rate as a proxy for the risk-free rate. (page 179)
4. Describe how to use the OIS zero curve in determining forward LIBOR rates and valuing swaps. (page 179)

16. Volatility Smiles

After completing this reading, you should be able to:

1. Define volatility smile and volatility skew. (page 187)
2. Explain the implications of put-call parity on the implied volatility of call and put options. (page 186)

3. Compare the shape of the volatility smile (or skew) to the shape of the implied distribution of the underlying asset price and to the pricing of options on the underlying asset. (page 187)
4. Describe characteristics of foreign exchange rate distributions and their implications on option prices and implied volatility. (page 188)
5. Describe the volatility smile for equity options and foreign currency options and provide possible explanations for its shape. (page 188)
6. Describe alternative ways of characterizing the volatility smile. (page 189)
7. Describe volatility term structures and volatility surfaces and how they may be used to price options. (page 190)
8. Explain the impact of the volatility smile on the calculation of the "Greeks." (page 190)
9. Explain the impact of a single asset price jump on a volatility smile. (page 191)

The following is a review of the Market Risk Measurement and Management principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Estimating Market Risk Measures: An Introduction and Overview

Topic 1

Exam Focus

In this topic, the focus is on the estimation of market risk measures, such as value at risk (VaR). VaR identifies the probability that losses will be greater than a pre-specified threshold level. For the exam, be prepared to evaluate and calculate VaR using historical simulation and parametric models (both normal and lognormal return distributions). One drawback to VaR is that it does not estimate losses in the tail of the returns distribution. Expected shortfall (ES) does, however, estimate the loss in the tail (i.e., after the VaR threshold has been breached) by averaging loss levels at different confidence levels. Coherent risk measures incorporate personal risk aversion across the entire distribution and are more general than expected shortfall. Quantile-quantile (QQ) plots are used to visually inspect if an empirical distribution matches a theoretical distribution.

Estimating Returns

To better understand the material in this topic, it is helpful to recall the computations of arithmetic and geometric returns. Note that the convention when computing these returns (as well as VaR) is to quote return losses as positive values. For example, if a portfolio is expected to decrease in value by $1 million, we use the terminology "expected loss is $1 million" rather than "expected profit is –$1 million."

Profit/loss data: Change in value of asset/portfolio, P_t, at the end of period t plus any interim payments, D_t.

$$P/L_t = P_t + D_t - P_{t-1}$$

Arithmetic return data: Assumption is that interim payments do not earn a return (i.e., no reinvestment). Hence, this approach is not appropriate for long investment horizons.

$$r_t = \frac{P_t + D_t - P_{t-1}}{P_{t-1}} = \frac{P_t + D_t}{P_{t-1}} - 1$$

Geometric return data: Assumption is that interim payments are continuously reinvested. Note that this approach ensures that asset price can never be negative.

$$R_t = \ln\left(\frac{P_t + D_t}{P_{t-1}}\right)$$

Historical Simulation Approach

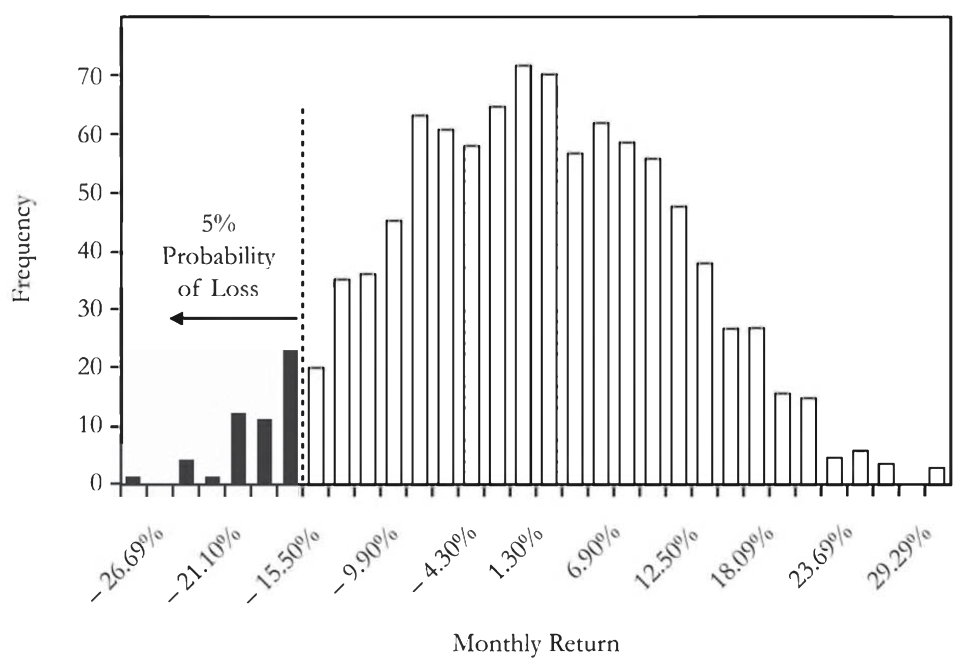
LO 1.1: Estimate VaR using a historical simulation approach.

Estimating VaR with a historical simulation approach is by far the simplest and most straightforward VaR method. To make this calculation, you simply order return observations from largest to smallest. The observation that follows the threshold loss level denotes the VaR limit. We are essentially searching for the observation that separates the tail from the body of the distribution. More generally, the observation that determines VaR for n observations at the $(1 - \alpha)$ confidence level would be: $(\alpha \times n) + 1$.

Professor's Note: Recall that the confidence level, $(1 - \alpha)$, is typically a large value (e.g., 95%) whereas the significance level, usually denoted as α, is much smaller (e.g., 5%).

To illustrate this VaR method, assume you have gathered 1,000 monthly returns for a security and produced the distribution shown in Figure 1. You decide that you want to compute the monthly VaR for this security at a confidence level of 95%. At a 95% confidence level, the lower tail displays the lowest 5% of the underlying distribution's returns. For this distribution, the value associated with a 95% confidence level is a return of –15.5%. If you have $1,000,000 invested in this security, the one-month VaR is $155,000 (–15.5% × $1,000,000).

Figure 1: Histogram of Monthly Returns



Example: Identifying the VaR limit

Identify the ordered observation in a sample of 1,000 data points that corresponds to VaR at a 95% confidence level.

Answer:

Since VaR is to be estimated at 95% confidence, this means that 5% (i.e., 50) of the ordered observations would fall in the tail of the distribution. Therefore, the 51st ordered loss observation would separate the 5% of largest losses from the remaining 95% of returns.

Professor's Note: VaR is the quantile that separates the tail from the body of the distribution. With 1,000 observations at a 95% confidence level, there is a certain level of arbitrariness in how the ordered observations relate to VaR. In other words, should VaR be the 50th observation (i.e., $\alpha \times n$), the 51st observation [i.e., $(\alpha \times n) + 1$], or some combination of these observations? In this example, using the 51st observation was the approximation for VaR, and the method used in the assigned reading. However, on past FRM exams, VaR using the historical simulation method has been calculated as just: ($\alpha \times n$), in this case, as the 50th observation.

Example: Computing VaR

A long history of profit/loss data closely approximates a standard normal distribution (mean equals zero; standard deviation equals one). **Estimate** the 5% VaR using the historical simulation approach.

Answer:

The VaR limit will be at the observation that separates the tail loss with area equal to 5% from the remainder of the distribution. Since the distribution is closely approximated by the standard normal distribution, the VaR is 1.65 (5% critical value from the *z*-table). Recall that since VaR is a one-tailed test, the entire significance level of 5% is in the left tail of the returns distribution.

From a practical perspective, the historical simulation approach is sensible only if you expect future performance to follow the same return generating process as in the past. Furthermore, this approach is unable to adjust for changing economic conditions or abrupt shifts in parameter values.

Parametric Estimation Approaches

LO 1.2: Estimate VaR using a parametric approach for both normal and lognormal return distributions.

In contrast to the historical simulation method, the parametric approach (e.g., the delta-normal approach) explicitly assumes a distribution for the underlying observations. For this LO, we will analyze two cases: (1) VaR for returns that follow a normal distribution, and (2) VaR for returns that follow a lognormal distribution.

Normal VaR

Intuitively, the VaR for a given confidence level denotes the point that separates the tail losses from the remaining distribution. The VaR cutoff will be in the left tail of the returns distribution. Hence, the calculated value at risk is negative, but is typically reported as a positive value since the negative amount is implied (i.e., it is the value that is at risk). In equation form, the VaR at significance level α is:

$$\text{VaR}(\alpha\%) = -\mu_{P/L} + \sigma_{P/L} \times z_{\alpha}$$

where μ and σ denote the mean and standard deviation of the profit/loss distribution and z denotes the critical value (i.e., quantile) of the standard normal. In practice, the population parameters μ and σ are not likely known, in which case the researcher will use the sample mean and standard deviation.

Example: Computing VaR (normal distribution)

Assume that the profit/loss distribution for XYZ is normally distributed with an annual mean of \$15 million and a standard deviation of \$10 million. **Calculate** the VaR at the 95% and 99% confidence levels using a parametric approach.

Answer:

VaR(5%) = –\$15 million + \$10 million × 1.65 = \$1.5 million. Therefore, XYZ expects to lose at most \$1.5 million over the next year with 95% confidence. Equivalently, XYZ expects to lose more than \$1.5 million with a 5% probability.

VaR(1%) = –\$15 million + \$10 million × 2.33 = \$8.3 million. Note that the VaR (at 99% confidence) is greater than the VaR (at 95% confidence) as follows from the definition of value at risk.

Now suppose that the data you are using is arithmetic return data rather than profit/loss data. The arithmetic returns follow a normal distribution as well. As you would expect, because of the relationship between prices, profits/losses, and returns, the corresponding VaR is very similar in format:

$$\text{VaR}(\alpha\%) = (-\mu_r + \sigma_r \times z_{\alpha}) \times P_{t-1}$$

Example: Computing VaR (arithmetic returns)

A portfolio has a beginning period value of $100. The arithmetic returns follow a normal distribution with a mean of 10% and a standard deviation of 20%. **Calculate** VaR at both the 95% and 99% confidence levels.

Answer:

$$\text{VaR}(5\%) = (-10\% + 1.65 \times 20\%) \times 100 = \$23.0$$

$$\text{VaR}(1\%) = (-10\% + 2.33 \times 20\%) \times 100 = \$36.6$$

Lognormal VaR

The lognormal distribution is right-skewed with positive outliers and bounded below by zero. As a result, the lognormal distribution is commonly used to counter the possibility of negative asset prices (P_t). Technically, if we assume that geometric returns follow a normal distribution (μ_R, σ_R), then the natural logarithm of asset prices follows a normal distribution and P_t follows a lognormal distribution. After some algebraic manipulation, we can derive the following expression for **lognormal VaR**:

$$\text{VaR}(\alpha\%) = P_{t-1} \times \left(1 - e^{\mu_R - \sigma_R \times z_\alpha}\right)$$

Example: Computing VaR (lognormal distribution)

A diversified portfolio exhibits a normally distributed geometric return with mean and standard deviation of 10% and 20%, respectively. **Calculate** the 5% and 1% lognormal VaR assuming the beginning period portfolio value is $100.

Answer:

$$\begin{aligned}\text{Lognormal VaR}(5\%) &= 100 \times (1 - \exp[0.1 - 0.2 \times 1.65]) \\ &= 100 \times (1 - \exp[-0.23]) \\ &= \$20.55\end{aligned}$$

$$\begin{aligned}\text{Lognormal VaR}(1\%) &= 100 \times (1 - \exp[0.1 - 0.2 \times 2.33]) \\ &= 100 \times (1 - \exp[-0.366]) \\ &= \$30.65\end{aligned}$$

Note that the calculation of lognormal VaR (geometric returns) and normal VaR (arithmetic returns) will be similar when we are dealing with short-time periods and practical return estimates.

Expected Shortfall

LO 1.3: Estimate the expected shortfall given P/L or return data.

A major limitation of the VaR measure is that it does not tell the investor the amount or magnitude of the actual loss. VaR only provides the maximum value we can lose for a given confidence level. The **expected shortfall** (ES) provides an estimate of the tail loss by averaging the VaRs for increasing confidence levels in the tail. Specifically, the tail mass is divided into *n* equal slices and the corresponding n – 1 VaRs are computed. For example, if n = 5, we can construct the following table based on the normal distribution:

Figure 2: Estimating Expected Shortfall

Confidence level	*VaR*	*Difference*
96%	1.7507	
97%	1.8808	0.1301
98%	2.0537	0.1729
99%	2.3263	0.2726
Average	2.003	
Theoretical true value	2.063	

Observe that the VaR increases (from *Difference* column) in order to maintain the same interval mass (of 1%) because the tails become thinner and thinner. The average of the four computed VaRs is 2.003 and represents the probability-weighted expected tail loss (a.k.a. expected shortfall). Note that as *n* increases, the expected shortfall will increase and approach the theoretical true loss [2.063 in this case; the average of a high number of VaRs (e.g., greater than 10,000)].

Estimating Coherent Risk Measures

LO 1.4: Define coherent risk measures.

LO 1.5: Estimate risk measures by estimating quantiles.

A more general risk measure than either VaR or ES is known as a coherent risk measure. A **coherent risk measure** is a weighted average of the quantiles of the loss distribution where the weights are user-specific based on individual risk aversion. ES (as well as VaR) is a special case of a coherent risk measure. When modeling the ES case, the weighting function is set to [1 / (1 – confidence level)] for all tail losses. All other quantiles will have a weight of zero.

Under expected shortfall estimation, the tail region is divided into equal probability slices and then multiplied by the corresponding quantiles. Under the more general coherent risk measure, the entire distribution is divided into equal probability slices weighted by the more general risk aversion (weighting) function.

This procedure is illustrated for n = 10. First, the entire return distribution is divided into nine (i.e., n – 1) equal probability mass slices at 10%, 20%, …, 90% (i.e., loss quantiles). Each breakpoint corresponds to a different quantile. For example, the 10% quantile (confidence level = 10%) relates to –1.2816, the 20% quantile (confidence level = 20%) relates to –0.8416, and the 90% quantile (confidence level = 90%) relates to 1.2816. Next, each quantile is weighted by the specific risk aversion function and then averaged to arrive at the value of the coherent risk measure.

This coherent risk measure is more sensitive to the choice of *n* than expected shortfall, but will converge to the risk measure's true value for a sufficiently large number of observations. The intuition is that as *n* increases, the quantiles will be further into the tails where more extreme values of the distribution are located.

LO 1.6: Evaluate estimators of risk measures by estimating their standard errors.

Sound risk management practice reminds us that estimators are only as useful as their precision. That is, estimators that are less precise (i.e., have large standard errors and wide confidence intervals) will have limited practical value. Therefore, it is best practice to also compute the standard error for all coherent risk measures.

Professor's Note: The process of estimating standard errors for estimators of coherent risk measures is quite complex, so your focus should be on interpretation of this concept.

First, let's start with a sample size of *n* and arbitrary bin width of *h* around quantile, *q*. Bin width is just the width of the intervals, sometimes called "bins," in a histogram. Computing standard error is done by realizing that the square root of the variance of the quantile is equal to the standard error of the quantile. After finding the standard error, a confidence interval for a risk measure such as VaR can be constructed as follows:

$$[q + se(q) \times z_\alpha] > VaR > [q - se(q) \times z_\alpha]$$

Example: Estimating standard errors

Construct a 90% confidence interval for 5% VaR (the 95% quantile) drawn from a standard normal distribution. Assume bin width = 0.1 and that the sample size is equal to 500.

Answer:

The quantile value, q, corresponds to the 5% VaR which occurs at 1.65 for the standard normal distribution. The confidence interval takes the following form:

$$[1.65 + 1.65 \times \text{se(q)}] > \text{VaR} > [1.65 - 1.65 \times \text{se(q)}]$$

Professor's Note: Recall that a confidence interval is a two-tailed test (unlike VaR), so a 90% confidence level will have 5% in each tail. Given that this is equivalent to the 5% significance level of VaR, the critical values of 1.65 will be the same in both cases.

Since bin width is 0.1, q is in the range 1.65 ± 0.1/2 = [1.7, 1.6]. Note that the left tail probability, p, is the area to the left of –1.7 for a standard normal distribution.

Next, calculate the probability mass between [1.7, 1.6], represented as $f(q)$. From the standard normal table, the probability of a loss *greater* than 1.7 is 0.045 (left tail). Similarly, the probability of a loss *less* than 1.6 (right tail) is 0.945. Collectively, f(q) = 1 – 0.045 – 0.945 = 0.01

The standard error of the quantile is derived from the variance approximation of q and is equal to:

$$\text{se}(q) = \frac{\sqrt{p(1-p)/n}}{f(q)}$$

Now we are ready to substitute in the variance approximation to calculate the confidence interval for VaR:

$$\left[1.65 + 1.65\frac{\sqrt{0.045(1-0.045)/500}}{0.01}\right] > \text{VaR} > \left[1.65 - 1.65\frac{\sqrt{0.045(1-0.045)/500}}{0.01}\right]$$
$$= 3.18 > \text{VaR} > 0.12$$

Let's return to the variance approximation and perform some basic comparative statistics. What happens if we increase the sample size holding all other factors constant? Intuitively, the larger the sample size the smaller the standard error and the narrower the confidence interval.

Now suppose we increase the bin size, h, holding all else constant. This will increase the probability mass $f(q)$ and reduce p, the probability in the left tail. The standard error will decrease and the confidence interval will again narrow.

Lastly, suppose that p increases indicating that tail probabilities are more likely. Intuitively, the estimator becomes less precise and standard errors increase, which widens the confidence interval. Note that the expression $p(1 - p)$ will be maximized at $p = 0.5$.

The above analysis was based on one quantile of the loss distribution. Just as the previous section generalized the expected shortfall to the coherent risk measure, we can do the same for the standard error computation. Thankfully, this complex process is not the focus of the LO.

Quantile-Quantile Plots

LO 1.7: Interpret QQ plots to identify the characteristics of a distribution.

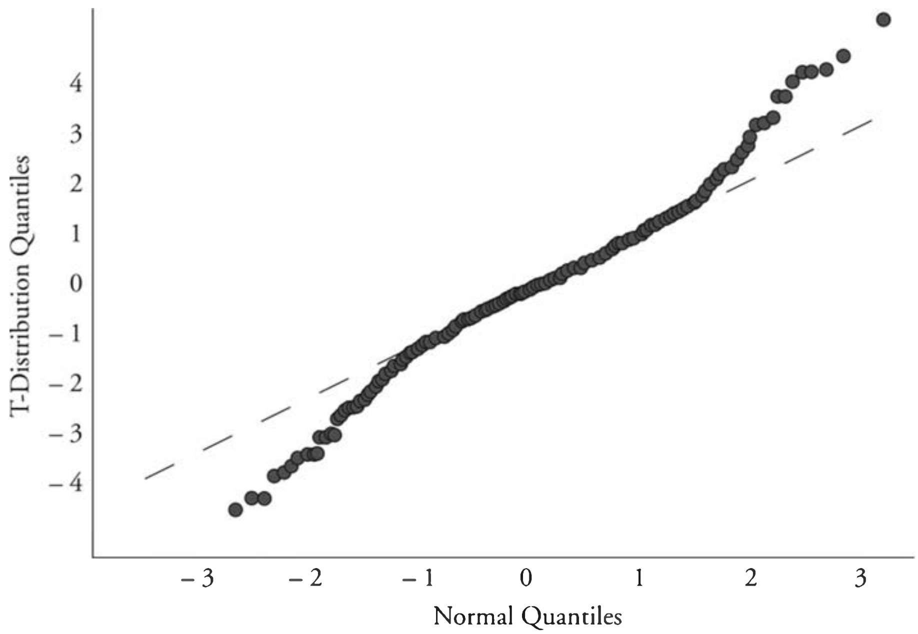
A natural question to ask in the course of our analysis is, "From what distribution is the data drawn?" The truth is that you will never really know since you only observe the realizations from random draws of an unknown distribution. However, visual inspection can be a very simple but powerful technique.

In particular, the **quantile-quantile** (QQ) **plot** is a straightforward way to visually examine if empirical data fits the reference or hypothesized theoretical distribution (assume standard normal distribution for this discussion). The process graphs the quantiles at regular confidence intervals for the empirical distribution against the theoretical distribution. As an example, if both the empirical and theoretical data are drawn from the same distribution, then the median (confidence level = 50%) of the empirical distribution would plot very close to zero, while the median of the theoretical distribution would plot exactly at zero.

Continuing in this fashion for other quantiles (40%, 60%, and so on) will map out a function. If the two distributions are very similar, the resulting QQ plot will be linear.

Let us compare a theoretical standard normal distribution relative to an empirical t-distribution (assume that the degrees of freedom for the t-distribution are sufficiently small and that there are noticeable differences from the normal distribution). We know that both distributions are symmetric, but the t-distribution will have fatter tails. Hence, the quantiles near zero (confidence level = 50%) will match up quite closely. As we move further into the tails, the quantiles between the t-distribution and the normal will diverge (see Figure 3). For example, at a confidence level of 95%, the critical z-value is –1.65, but for the t-distribution, it is closer to –1.68 (degrees of freedom of approximately 40). At 97.5% confidence, the difference is even larger, as the z-value is equal to –1.96 and the t-stat is equal to –2.02. More generally, if the middles of the QQ plot match up, but the tails do not, then the empirical distribution can be interpreted as symmetric with tails that differ from a normal distribution (either fatter or thinner).

Figure 3: QQ Plot



Key Concepts

LO 1.1

Historical simulation is the easiest method to estimate value at risk. All that is required is to reorder the profit/loss observations in increasing magnitude of losses and identify the breakpoint between the tail region and the remainder of distribution.

LO 1.2

Parametric estimation of VaR requires a specific distribution of prices or equivalently, returns. This method can be used to calculate VaR with either a normal distribution or a lognormal distribution.

Under the assumption of a normal distribution, VaR (i.e., delta-normal VaR) is calculated as follows:

$$VaR = -\mu_{P/L} + \sigma_{P/L} \times z_{\alpha}$$

Under the assumption of a lognormal distribution, lognormal VaR is calculated as follows:

$$VaR = P_{t-1} \times \left(1 - e^{\mu_R - \sigma_R \times z_{\alpha}}\right)$$

LO 1.3

VaR identifies the lower bound of the profit/loss distribution, but it does not estimate the expected tail loss. Expected shortfall overcomes this deficiency by dividing the tail region into equal probability mass slices and averaging their corresponding VaRs.

LO 1.4

A more general risk measure than either VaR or ES is known as a coherent risk measure.

LO 1.5

A coherent risk measure is a weighted average of the quantiles of the loss distribution where the weights are user-specific based on individual risk aversion. A coherent risk measure will assign each quantile (not just tail quantiles) a weight. The average of the weighted VaRs is the estimated loss.

LO 1.6

Sound risk management requires the computation of the standard error of a coherent risk measure to estimate the precision of the risk measure itself. The simplest method creates a confidence interval around the quantile in question. To compute standard error, it is necessary to find the variance of the quantile, which will require estimates from the underlying distribution.

LO 1.7

The quantile-quantile (QQ) plot is a visual inspection of an empirical quantile relative to a hypothesized theoretical distribution. If the empirical distribution closely matches the theoretical distribution, the QQ plot would be linear.

Concept Checkers

1. The VaR at a 95% confidence level is estimated to be 1.56 from a historical simulation of 1,000 observations. Which of the following statements is most likely true?
   A. The parametric assumption of normal returns is correct.
   B. The parametric assumption of lognormal returns is correct.
   C. The historical distribution has fatter tails than a normal distribution.
   D. The historical distribution has thinner tails than a normal distribution.

2. Assume the profit/loss distribution for XYZ is normally distributed with an annual mean of $20 million and a standard deviation of $10 million. The 5% VaR is calculated and interpreted as which of the following statements?
   A. 5% probability of losses of at least $3.50 million.
   B. 5% probability of earnings of at least $3.50 million.
   C. 95% probability of losses of at least $3.50 million.
   D. 95% probability of earnings of at least $3.50 million.

3. Which of the following statements about expected shortfall estimates and coherent risk measures are true?
   A. Expected shortfall and coherent risk measures estimate quantiles for the entire loss distribution.
   B. Expected shortfall and coherent risk measures estimate quantiles for the tail region.
   C. Expected shortfall estimates quantiles for the tail region and coherent risk measures estimate quantiles for the non-tail region only.
   D. Expected shortfall estimates quantiles for the entire distribution and coherent risk measures estimate quantiles for the tail region only.

4. Which of the following statements most likely increases standard errors from coherent risk measures?
   A. Increasing sample size and increasing the left tail probability.
   B. Increasing sample size and decreasing the left tail probability.
   C. Decreasing sample size and increasing the left tail probability.
   D. Decreasing sample size and decreasing the left tail probability.

5. The quantile-quantile plot is best used for what purpose?
   A. Testing an empirical distribution from a theoretical distribution.
   B. Testing a theoretical distribution from an empirical distribution.
   C. Identifying an empirical distribution from a theoretical distribution.
   D. Identifying a theoretical distribution from an empirical distribution.

Concept Checker Answers

1. **D** The historical simulation indicates that the 5% tail loss begins at 1.56, which is less than the 1.65 predicted by a standard normal distribution. Therefore, the historical simulation has thinner tails than a standard normal distribution.

2. **D** The value at risk calculation at 95% confidence is: –20 million + 1.65 × 10 million = –$3.50 million. Since the expected loss is negative and VaR is an implied negative amount, the interpretation is that XYZ will earn less than +$3.50 million with 5% probability, which is equivalent to XYZ earning at least $3.50 million with 95% probability.

3. **B** ES estimates quantiles for n – 1 equal probability masses in the tail region only. The coherent risk measure estimates quantiles for the entire distribution including the tail region.

4. **C** Decreasing sample size clearly increases the standard error of the coherent risk measure given that standard error is defined as:

$$se(q) = \frac{\sqrt{p(1-p)/n}}{f(q)}$$

As the left tail probability, *p*, increases, the probability of tail events increases, which also increases the standard error. Mathematically, p(1 – p) increases as *p* increases until *p* = 0.5. Small values of *p* imply smaller standard errors.

5. **C** Once a sample is obtained, it can be compared to a reference distribution for possible identification. The QQ plot maps the quantiles one to one. If the relationship is close to linear, then a match for the empirical distribution is found. The QQ plot is used for visual inspection only without any formal statistical test.

The following is a review of the Market Risk Measurement and Management principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Non-parametric Approaches

Topic 2

Exam Focus

This topic introduces non-parametric estimation and bootstrapping (i.e., resampling). The key difference between these approaches and parametric approaches discussed in the previous topic is that with non-parametric approaches the underlying distribution is not specified, and it is a data driven, not assumption driven, analysis. For example, historical simulation is limited by the discreteness of the data, but non-parametric analysis "smoothes" the data points to allow for any VaR confidence level between observations. For the exam, pay close attention to the description of the bootstrap historical simulation approach as well as the various weighted historical simulations approaches.

Non-parametric estimation does not make restrictive assumptions about the underlying distribution like parametric methods, which assume very specific forms such as normal or lognormal distributions. Non-parametric estimation lets the data drive the estimation. The flexibility of these methods makes them excellent candidates for VaR estimation, especially if tail events are sparse.

Bootstrap Historical Simulation Approach

LO 2.1: Apply the bootstrap historical simulation approach to estimate coherent risk measures.

The **bootstrap historical simulation** is a simple and intuitive estimation procedure. In essence, the bootstrap technique draws a sample from the original data set, records the VaR from that particular sample and "returns" the data. This procedure is repeated over and over and records multiple sample VaRs. Since the data is always "returned" to the data set, this procedure is akin to sampling with replacement. The best VaR estimate from the full data set is the average of all sample VaRs.

This same procedure can be performed to estimate the expected shortfall (ES). Each drawn sample will calculate its own ES by slicing the tail region into n slices and averaging the VaRs at each of the $n - 1$ quantiles. This is exactly the same procedure described in the previous topic. Similarly, the best estimate of the expected shortfall for the original data set is the average of all of the sample expected shortfalls.

Empirical analysis demonstrates that the bootstrapping technique consistently provides more precise estimates of coherent risk measures than historical simulation on raw data alone.

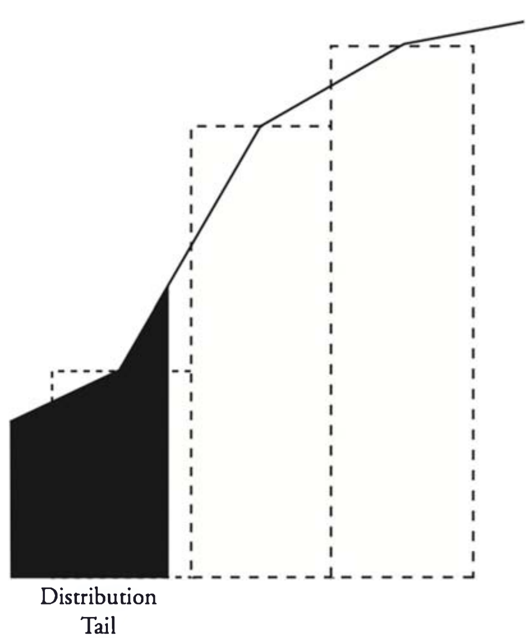
Using Non-Parametric Estimation

LO 2.2: Describe historical simulation using non-parametric density estimation.

The clear advantage of the traditional historical simulation approach is its simplicity. One obvious drawback, however, is that the discreteness of the data does not allow for estimation of VaRs between data points. If there were 100 historical observations, then it is straightforward to estimate VaR at the 95% or the 96% confidence levels, and so on. However, this method is unable to incorporate a confidence level of 95.5%, for example. More generally, with *n* observations, the historical simulation method only allows for *n* different confidence levels.

One of the advantages of non-parametric density estimation is that the underlying distribution is free from restrictive assumptions. Therefore, the existing data points can be used to “smooth” the data points to allow for VaR calculation at all confidence levels. The simplest adjustment is to connect the midpoints between successive histogram bars in the original data set’s distribution. See Figure 1 for an illustration of this **surrogate density function**. Notice that by connecting the midpoints, the lower bar “receives” area from the upper bar, which “loses” an equal amount of area. In total, no area is lost, only displaced, so we still have a probability distribution function, just with a modified shape. The shaded area in Figure 1 represents a possible confidence interval, which can be utilized regardless of the size of the data set. The major improvement of this non-parametric approach over the traditional historical simulation approach is that VaR can now be calculated for a continuum of points in the data set.

Figure 1: Surrogate Density Function



Following this logic, one can see that the linear adjustment is a simple solution to the interval problem. A more complicated adjustment would involve connecting curves, rather than lines, between successive bars to better capture the characteristics of the data.

Weighted Historical Simulation Approaches

LO 2.3: Compare and contrast the age-weighted, the volatility-weighted, the correlation-weighted, and the filtered historical simulation approaches.

The previous weighted historical simulation, discussed in Topic 1, assumed that both current and past (arbitrary) *n* observations up to a specified cutoff point are used when computing the current period VaR. Older observations beyond the cutoff date are assumed to have a zero weight and the relevant *n* observations have equal weight of (1 / n). While simple in construction, there are obvious problems with this method. Namely, why is the *n*th observation as important as all other observations, but the *(n + 1)*th observation is so unimportant that it carries no weight? Current VaR may have "ghost effects" of previous events that remain in the computation until they disappear (after *n* periods). Furthermore, this method assumes that each observation is independent and identically distributed. This is a very strong assumption, which is likely violated by data with clear seasonality (i.e., seasonal volatility). This topic identifies four improvements to the traditional historical simulation method.

Age-weighted Historical Simulation

The obvious adjustment to the equal-weighted assumption used in historical simulation is to weight recent observations more and distant observations less. One method proposed by Boudoukh, Richardson, and Whitelaw is as follows.[1] Assume w(1) is the probability weight for the observation that is one day old. Then w(2) can be defined as λw(1), w(3) can be defined as λ^2w(1), and so on. The decay parameter, λ, can take on values $0 \leq \lambda \leq 1$ where values close to 1 indicate slow decay. Since all of the weights must sum to 1, we conclude that $w(1) = (1 - \lambda) / (1 - \lambda^n)$. More generally, the weight for an observation that is *i* days old is equal to:

$$w(i) = \frac{\lambda^{i-1}(1-\lambda)}{1-\lambda^n}$$

The implication of the age-weighted simulation is to reduce the impact of ghost effects and older events that may not reoccur. Note that this more general weighting scheme suggests that historical simulation is a special case where $\lambda = 1$ (i.e., no decay) over the estimation window.

Professor's Note: This approach is also known as the hybrid approach.

1. Boudoukh, J., M. Richardson, and R. Whitelaw. 1998. "The best of both worlds: a hybrid approach to calculating value at risk." *Risk* 11: 64–67.

Volatility-weighted Historical Simulation

Another approach is to weight the individual observations by volatility rather than proximity to the current date. This was introduced by Hull and White to incorporate changing volatility in risk estimation.[2] The intuition is that if recent volatility has increased, then using historical data will underestimate the current risk level. Similarly, if current volatility is markedly reduced, the impact of older data with higher periods of volatility will overstate the current risk level.

This process is captured in the expression below for estimating VaR on day T. The expression is achieved by adjusting each daily return, $r_{t,i}$ on day t upward or downward based on the then-current volatility forecast, $\sigma_{t,i}$ (estimated from a GARCH or EWMA model) relative to the current volatility forecast on day T.

$$r^*_{t,i} = \left(\frac{\sigma_{T,i}}{\sigma_{t,i}}\right) r_{t,i}$$

where:
$r_{t,i}$ = actual return for asset i on day t
$\sigma_{t,i}$ = volatility forecast for asset i on day t (made at the end of day t − 1)
$\sigma_{T,i}$ = current forecast of volatility for asset i

Thus, the volatility-adjusted return, $r^*_{t,i}$, is replaced with a larger (smaller) expression if current volatility exceeds (is below) historical volatility on day *i*. Now, VaR, ES, and any other coherent risk measure can be calculated in the usual way after substituting historical returns with volatility-adjusted returns.

There are several advantages of the volatility-weighted method. First, it explicitly incorporates volatility into the estimation procedure in contrast to other historical methods. Second, the near-term VaR estimates are likely to be more sensible in light of current market conditions. Third, the volatility-adjusted returns allow for VaR estimates that are higher than estimates with the historical data set.

Correlation-weighted Historical Simulation

As the name suggests, this methodology incorporates updated correlations between asset pairs. This procedure is more complicated than the volatility-weighting approach, but it follows the same basic principles. Since the corresponding LO does not require calculations, the exact matrix algebra would only complicate our discussion. Intuitively, the historical correlation (or equivalently variance-covariance) matrix needs to be adjusted to the new information environment. This is accomplished, loosely speaking, by "multiplying" the historic returns by the revised correlation matrix to yield updated correlation-adjusted returns.

2. Hull, J., and A. White. 1998. "Incorporating volatility updating into the historical simulation method for value-at-risk." *Journal of Risk* 1: 5–19.

Let us look at the variance-covariance matrix more closely. In particular, we are concerned with diagonal elements and the off-diagonal elements. The off-diagonal elements represent the current covariance between asset pairs. On the other hand, the diagonal elements represent the updated variances (covariance of the asset return with itself) of the individual assets.

$$\Sigma = \begin{pmatrix} \sigma_{i,i} & \sigma_{i,j} \\ \sigma_{j,i} & \sigma_{j,j} \end{pmatrix} = \begin{pmatrix} \text{Variance}(X_i) & \text{Cov}\left(X_i, X_j\right) \\ \text{Cov}\left(X_j, X_i\right) & \text{Variance}(X_j) \end{pmatrix}$$

Notice that updated variances were utilized in the previous approach as well. Thus, correlation-weighted simulation is an even richer analytical tool than volatility-weighted simulation because it allows for updated variances (volatilities) as well as covariances (correlations).

Filtered Historical Simulation

The filtered historical simulation is the most comprehensive, and hence most complicated, of the non-parametric estimators. The process combines the historical simulation model with conditional volatility models (like GARCH or asymmetric GARCH). Thus, the method contains both the attractions of the traditional historical simulation approach with the sophistication of models that incorporate changing volatility. In simplified terms, the model is flexible enough to capture conditional volatility and volatility clustering as well as a surprise factor that could have an asymmetric effect on volatility.

The model will forecast volatility for each day in the sample period and the volatility will be standardized by dividing by realized returns. Bootstrapping is used to simulate returns which incorporate the current volatility level. Finally, the VaR is identified from the simulated distribution. The methodology can be extended over longer holding periods or for multi-asset portfolios.

In sum, the filtered historical simulation method uses bootstrapping and combines the traditional historical simulation approach with rich volatility modeling. The results are then sensitive to changing market conditions and can predict losses outside the historical range. From a computational standpoint, this method is very reasonable even for large portfolios, and empirical evidence supports its predictive ability.

Advantages and Disadvantages of Non-Parametric Methods

LO 2.4: Identify advantages and disadvantages of non-parametric estimation methods.

Any risk manager should be prepared to use non-parametric estimation techniques. There are some clear advantages to non-parametric methods, but there is some danger as well. Therefore, it is incumbent to understand the advantages, the disadvantages, and the appropriateness of the methodology for analysis.

Advantages of non-parametric methods include the following:

- Intuitive and often computationally simple (even on a spreadsheet).
- Not hindered by parametric violations of skewness, fat-tails, et cetera.
- Avoids complex variance-covariance matrices and dimension problems.
- Data is often readily available and does not require adjustments (e.g., financial statements adjustments).
- Can accommodate more complex analysis (e.g., by incorporating age-weighting with volatility-weighting).

Disadvantages of non-parametric methods include the following:

- Analysis depends critically on historical data.
- Volatile data periods lead to VaR and ES estimates that are too high.
- Quiet data periods lead to VaR and ES estimates that are too low.
- Difficult to detect structural shifts/regime changes in the data.
- Cannot accommodate plausible large impact events if they did not occur within the sample period.
- Difficult to estimate losses significantly larger than the maximum loss within the data set (historical simulation cannot; volatility-weighting can, to some degree).
- Need sufficient data, which may not be possible for new instruments or markets.

Key Concepts

LO 2.1

Bootstrapping involves resampling a subset of the original data set with replacement. Each draw (subsample) yields a coherent risk measure (VaR or ES). The average of the risk measures across all samples is then the best estimate.

LO 2.2

The discreteness of historical data reduces the number of possible VaR estimates since historical simulation cannot adjust for significance levels between ordered observations. However, non-parametric density estimation allows the original histogram to be modified to fill in these gaps. The process connects the midpoints between successive columns in the histogram. The area is then "removed" from the upper bar and "placed" in the lower bar, which creates a "smooth" function between the original data points.

LO 2.3

One important limitation to the historical simulation method is the equal-weight assumed for all data in the estimation period, and zero weight otherwise. This arbitrary methodology can be improved by using age-weighted simulation, volatility-weighted simulation, correlation-weighted simulation, and filtered historical simulation.

The age-weighted simulation method adjusts the most recent (distant) observations to be more (less) heavily weighted.

The volatility-weighting procedure incorporates the possibility that volatility may change over the estimation period, which may understate or overstate current risk by including stale data. The procedure replaces historic returns with volatility-adjusted returns; however, the actual procedure of estimating VaR is unchanged (i.e., only the data inputs change).

Correlation-weighted simulation updates the variance-covariance matrix between the assets in the portfolio. The off-diagonal elements represent the covariance pairs while the diagonal elements update the individual variance estimates. Therefore, the correlation-weighted methodology is more general than the volatility-weighting procedure by incorporating both variance and covariance adjustments.

Filtered historical simulation is the most complex estimation method. The procedure relies on bootstrapping of standardized returns based on volatility forecasts. The volatility forecasts arise from GARCH or similar models and are able to capture conditional volatility, volatility clustering, and/or asymmetry.

LO 2.4

Advantages of non-parametric models include: data can be skewed or have fat tails; they are conceptually straightforward; there is readily available data; and they can accommodate more complex analysis. Disadvantages focus mainly on the use of historical data, which limits the VaR forecast to (approximately) the maximum loss in the data set; they are slow to respond to changing market conditions; they are affected by volatile (quiet) data periods; and they cannot accommodate plausible large losses if not in the data set.

Concept Checkers

1. Johanna Roberto has collected a data set of 1,000 daily observations on equity returns. She is concerned about the appropriateness of using parametric techniques as the data appears skewed. Ultimately, she decides to use historical simulation and bootstrapping to estimate the 5% VaR. Which of the following steps is most likely to be part of the estimation procedure?
   A. Filter the data to remove the obvious outliers.
   B. Repeated sampling with replacement.
   C. Identify the tail region from reordering the original data.
   D. Apply a weighting procedure to reduce the impact of older data.

2. All of the following approaches improve the traditional historical simulation approach for estimating VaR except the:
   A. volatility-weighted historical simulation.
   B. age-weighted historical simulation.
   C. market-weighted historical simulation.
   D. correlation-weighted historical simulation.

3. Which of the following statements about age-weighting is most accurate?
   A. The age-weighting procedure incorporates estimates from GARCH models.
   B. If the decay factor in the model is close to 1, there is persistence within the data set.
   C. When using this approach, the weight assigned on day *i* is equal to: $w(i) = \lambda^{i-1} \times (1-\lambda)/(1-\lambda^{i})$.
   D. The number of observations should at least exceed 250.

4. Which of the following statements about volatility-weighting is true?
   A. Historic returns are adjusted, and the VaR calculation is more complicated.
   B. Historic returns are adjusted, and the VaR calculation procedure is the same.
   C. Current period returns are adjusted, and the VaR calculation is more complicated.
   D. Current period returns are adjusted, and the VaR calculation is the same.

5. All of the following items are generally considered advantages of non-parametric estimation methods except:
   A. ability to accommodate skewed data.
   B. availability of data.
   C. use of historical data.
   D. little or no reliance on covariance matrices.

Concept Checker Answers

1. **B** Bootstrapping from historical simulation involves repeated sampling with replacement. The 5% VaR is recorded from each sample draw. The average of the VaRs from all the draws is the VaR estimate. The bootstrapping procedure does not involve filtering the data or weighting observations. Note that the VaR from the original data set is not used in the analysis.

2. **C** Market-weighted historical simulation is not discussed in this topic. Age-weighted historical simulation weights observations higher when they appear closer to the event date. Volatility-weighted historical simulation adjusts for changing volatility levels in the data. Correlation-weighted historical simulation incorporates anticipated changes in correlation between assets in the portfolio.

3. **B** If the intensity parameter (i.e., decay factor) is close to 1, there will be persistence (i.e., slow decay) in the estimate. The expression for the weight on day i has i in the exponent when it should be n. While a large sample size is generally preferred, some of the data may no longer be representative in a large sample.

4. **B** The volatility-weighting method adjusts historic returns for current volatility. Specifically, return at time t is multiplied by (current volatility estimate / volatility estimate at time t). However, the actual procedure for calculating VaR using a historical simulation method is unchanged; it is only the inputted data that changes.

5. **C** The use of historical data in non-parametric analysis is a disadvantage, not an advantage. If the estimation period was quiet (volatile) then the estimated risk measures may understate (overstate) the current risk level. Generally, the largest VaR cannot exceed the largest loss in the historical period. On the other hand, the remaining choices are all considered advantages of non-parametric methods. For instance, the non-parametric nature of the analysis can accommodate skewed data, data points are readily available, and there is no requirement for estimates of covariance matrices.

The following is a review of the Market Risk Measurement and Management principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Backtesting VaR

Topic 3

Exam Focus

We use value at risk (VaR) methodologies to model risk. With VaR models, we seek to approximate the changes in value that our portfolio would experience in response to changes in the underlying risk factors. Model validation incorporates several methods that we use in order to determine how close our approximations are to actual changes in value. Through model validation, we are able to determine what confidence to place in our models, and we have the opportunity to improve their accuracy. Validation can be done through backtesting (also called a reality check), stress testing, and with independent review and oversight.

Backtesting

LO 3.1: Define backtesting and exceptions and explain the importance of backtesting VaR models.

Backtesting is the process of comparing losses predicted by the value at risk (VaR) model to those actually experienced over the testing period. If a model were completely accurate, we would expect VaR loss limits to be exceeded (this is called an **exception**) with the same frequency predicted by the confidence level used in the VaR model.

For example, if a VaR of $10 million is calculated at a 95% confidence level, we expect to have exceptions (losses exceeding $10 million) 5% of the time. If exceptions are occurring with greater frequency, we may be underestimating the actual risk. If exceptions are occurring less frequently, we may be overestimating risk and misallocating capital as a result.

VaR models are based on static portfolios, while actual portfolio compositions are constantly changing as relative prices change and positions are bought and sold. These risk factors affect actual profit and loss, but they are not included in the VaR model. We can minimize such effects by backtesting with a daily holding period, but the modeled returns are comparable to the hypothetical return that would be experienced had the portfolio remained constant for the holding period. Generally, we compare the VaR model returns to **cleaned returns** (actual returns adjusted for all changes that arise from changes that are not mark to market, like funding costs and fee income).

Backtesting Exceptions

LO 3.2: Explain the significant difficulties in backtesting a VaR model.

LO 3.3: Verify a model based on exceptions or failure rates.

LO 3.4: Define and identify type I and type II errors.

The very design of VaR models includes the use of confidence levels. We expect to have a frequency of exceptions that corresponds to the confidence level used for the model. If we use a 95% confidence level, we expect to find exceptions in 5% of the instances. The backtesting period constitutes a limited sample, and we would not expect to find the predicted number of exceptions in every sample. How do we determine if the actual number of exceptions is acceptable? If we expect five exceptions and find eight, is that too many? What about nine? At some level, we must reject the model, and we need to know that level.

Using Failure Rates in Model Verification

An unbiased measure of the number of exceptions as a proportion of the number of samples is called the **failure rate**. The probability of exception, *p*, equals one minus the confidence level (p = 1 – c). If we use *N* to represent the number of exceptions and *T* to represent the number of samples, then N/T is the failure rate.

A sample cannot be used to determine with absolute certainty whether the model is accurate. However, we can determine the accuracy of the model and the probability of having the number of exceptions that we experienced. When determining a range for the number of exceptions that we would accept, we must strike a balance between the chances of *rejecting an accurate model* (**Type I error**) and the chances of *accepting an inaccurate model* (**Type II error**). We can establish such ranges at different confidence levels using a binomial probability distribution and the number of samples.

Testing that the model is correctly calibrated requires the calculation of a z-score, where *x* is the number of exceptions observed. This z-score is then compared to the critical value at the chosen level of confidence (e.g., 1.96 for the 95% confidence level) to determine whether VaR is unbiased.

$$z = \frac{x - pT}{\sqrt{p(1-p)T}}$$

For example, if daily revenue fell below VaR (at the 95% confidence level) on 22 days during a 255-day period, the computed z-value of 2.66 would be larger than 1.96. In this case, we would reject the null hypothesis that VaR is unbiased and determine that the maximum number of exceptions has been exceeded.

Note that the confidence level at which we choose to accept or reject a model is not related to the confidence level at which VaR was calculated. In evaluating the accuracy of the model, we are comparing the number of exceptions observed with the maximum number of exceptions that would be expected from a correct model at a given confidence level.

Kupiec (1995)[1] determined a measure to accept or reject models using the tail points of a log-likelihood ratio:

$$LR_{uc} = -2\ln[(1-p)^{T-N}p^{N}] + 2\ln\{[1-(N/T)]^{T-N}(N/T)^{N}\}$$

where *p*, *T*, and *N* are as defined above, and LR_{uc} is the test statistic for unconditional coverage (uc).

We would *reject* the hypothesis that the model is correct if the LR > 3.84. In other words, this *LR* value is used to determine the range of acceptable exceptions without rejecting the VaR model at the 95% confidence level of the log-likelihood test.

For instance, suppose we are backtesting a daily holding period VaR model that was constructed using a 97.5% confidence level over a 255-day period. If the model is accurate, the expected number of exceptions will be 2.5% of 255, or 6.375. We know that even if our model is precise, there will be some variation in the number of exceptions between samples. The mean of the samples will approach 6.375 as the number of samples increases. However, we also know that even if the model is incorrect, we might still end up with the number of exceptions at or near 6.375.

Figure 1 shows the calculated values of *LR* with 255 samples for a number of VaR confidence levels and a sample size of 255. The bold areas in Figure 1 correspond to *LR*s greater than 3.84. We would not reject the model if the number of exceptions in our sample is greater than 2 and less than 12, since these limits correspond to an LR < 3.84. In other words, this range of exceptions would not result in rejecting the model at a 97.5% confidence level.

Figure 1: LR_{uc} Values for T = 255

Confidence Level	*N*											
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
97.5%	**7.16**	**4.19**	2.27	1.04	0.33	0.02	0.06	0.39	0.98	1.81	2.84	**4.06**
98.0%	**5.01**	2.49	1.03	0.26	0.00	0.15	0.65	1.44	2.48	3.76	**5.25**	**6.93**
99.0%	1.24	0.13	0.08	0.71	1.86	3.42	**5.32**	**7.51**	**9.97**	**12.65**	**15.55**	**18.63**

It is difficult to backtest VaR models constructed with higher levels of confidence, simply because the number of exceptions is often not high enough to provide meaningful information. As shown in Figure 1, with higher confidence levels, the range of acceptable exceptions is small. Thus, it becomes difficult to determine if the model is overstating risks (i.e., fewer than expected exceptions) or if the number of exceptions is simply at the lower range of acceptable.

1. Kupiec, Paul, 1995, Techniques for Verifying the Accuracy of Risk Measurement Models, *Journal of Derivatives*, 2 (December): 73–84.

Using VaR to Measure Potential Losses

Often the purpose of using VaR is to measure some level of potential losses. There are two theories about choosing a holding period for this application. The first theory is that the holding period should correspond to the amount of time required to either liquidate or hedge the portfolio. Thus, VaR would calculate possible losses before corrective action could take effect. The second theory is that the holding period should be chosen to match the period over which the portfolio is not expected to change due to nonrisk-related activity (e.g., trading). The two theories are not that different. For example, many banks use a daily VaR to correspond with the daily profit and loss measures. In this application, the holding period is more significant than the confidence level.

Basel Committee Rules for Backtesting

LO 3.6: Describe the Basel rules for backtesting.

In the backtesting process, we attempt to strike a balance between the probability of a Type I error and a Type II error. The Basel Committee requires that market VaR be calculated at the 99% confidence level and backtested over the past year. At the 99% confidence level, we would expect to have 2.5 exceptions (250 × 0.01) each year. In order to compensate for using inaccurate models, the committee has established a scale of the number of exceptions and corresponding increases in the capital multiplier, *k*. The multiplier is normally 3 but can be increased to as much as 4, based on the accuracy of the bank's VaR model. Figure 2 shows this scale. Increasing *k* significantly increases the amount of capital a bank must hold and lowers the bank's performance measures, like return on equity.

Figure 2: Basel Penalty Zones

Zone	*Number of Exceptions*	*Multiplier (k)*
Green	0 to 4	3.00
Yellow	5	3.40
	6	3.50
	7	3.65
	8	3.75
	9	3.85
Red	10 or more	4.00

As shown in Figure 2, the yellow zone is quite broad (five to nine exceptions). The penalty (raising the multiplier from 3 to 4) is automatically required for banks with ten or more exceptions. However, the penalty for banks with five to nine exceptions is subject to their supervisors' discretions, based on what type of model error caused the exceptions. The

Committee established four categories of causes for exceptions and guidance for supervisors for each category:

- *The basic integrity of the model is lacking.* Exceptions occurred because of incorrect data or errors in the model programming. The penalty should apply.
- *Model accuracy needs improvement.* The exceptions occurred because the model does not accurately describe risks. The penalty should apply.
- *Intraday trading activity.* The exceptions occurred due to trading activity (VaR is based on static portfolios). The penalty should be *considered*.
- *Bad luck.* The exceptions occurred because market conditions (volatility and correlations among financial instruments) significantly varied from an accepted norm. These exceptions should be expected to occur at least some of the time.

Although the yellow zone is broad, an accurate model could produce five or more exceptions 10.8% of the time. So even if a bank has an accurate model, it is subject to punishment over 10% of the time. Regulators are more concerned about the Type II errors since inaccurate models would have five errors 12.8% of the time (e.g., those with VaR calculated at the 97% confidence level rather than the required 99% confidence level). While this seems to be only a slight difference, using a 99% confidence level would result in a 1.24 times greater level of required capital, providing a powerful economic incentive for banks to use a lower confidence level.

Industry analysts have suggested lowering the required VaR confidence level to 95% and compensating by using a greater multiplier. This would result in a greater number of expected exceptions, and variances would be more statistically significant. The 1-year exception rate at the 95% level would be 13, and with more than 17 exceptions, the probability of a Type I error would be 12.5% (close to the 10.8 previously noted), but the probability of a Type II error at this level would fall to 7.4% (compared to 12.8% at a 97.5% confidence level). Thus, inaccurate models would be accepted less frequently.

Another way to make variations in the number of exceptions more significant would be to use a longer backtesting period. This approach may not be as practical, because the nature of markets, portfolios, and risk changes over time.

Conditional Coverage

LO 3.5: Explain the need to consider conditional coverage in the backtesting framework.

We have been backtesting models based on **unconditional coverage**, in which the timing of our exceptions was not considered. In addition to having a predictable number of exceptions, we also anticipate the exceptions to be fairly equally distributed across time. A bunching of exceptions may indicate that market correlations have changed or that our trading positions have been altered.

We need some guide to determine if the bunching is random or caused by one of these changes. By including a measure of the independence of exceptions, we measure **conditional**

coverage of the model. Christofferson[2] proposed extending the unconditional coverage test statistic (LR_{uc}) to allow for potential time variation of the data. He developed a statistic to determine the serial independence of deviations using a log-likelihood ratio test (LR_{ind}). The overall log-likelihood test statistic for conditional coverage (LR_{cc}) is then:

$$LR_{cc} = LR_{uc} + LR_{ind}$$

We would reject the model if $LR_{cc} > 5.99$. If exceptions are determined to be *serially dependent*, then the model needs to be revised to incorporate the correlations that are evident in the current conditions.

2. Christofferson, P.F., 1998. Evaluating Interval Forecasts, *International Economic Review,* 39, 841–862.

Key Concepts

LO 3.1

Backtesting is an important part of VaR model validation. Backtesting involves comparing the number of instances when the actual profit/loss exceeds the VaR level (called exceptions) with the number predicted by the model at the chosen level of confidence.

Cleaned returns are generally used for backtesting. Cleaned returns are actual returns adjusted for all changes that arise from changes that are not mark to market.

LO 3.2

The backtesting period constitutes a limited sample, so we do not expect to find the predicted number of exceptions in every sample. At some level, we must reject the model, so we need to find the acceptable level of exceptions.

LO 3.3

The failure rate of a model backtest is the number of exceptions divided by the number of observations: N/T.

The Basel Committee requires backtesting at the 99% confidence level over the past year (250 business days). At this level, we would expect 250×0.01, or 2.5 exceptions.

LO 3.4

In using backtesting to accept or reject a VaR model, we must balance the probabilities of two types of errors: a Type I error is rejecting an accurate model, and a Type II error is accepting an inaccurate model.

LO 3.5

Unconditional coverage testing does not evaluate the timing of exceptions, while conditional coverage tests review the number and timing of exceptions for independence. Current market or trading portfolio conditions may require changes in the model.

LO 3.6

The Basel Committee rules establish zones of number of exceptions and corresponding penalties or increases in the capital requirement multiplier from 3 to 4 (i.e., safety factor).

Concept Checkers

1. In backtesting a value at risk (VaR) model that was constructed using a 95% confidence level over a 255-day period, how many exceptions are forecasted?
   A. 5.00.
   B. 7.55.
   C. 12.75.
   D. 15.00.

2. Unconditional testing does not reflect the:
   A. size of the portfolio.
   B. number of exceptions.
   C. confidence level chosen.
   D. timing of the exceptions.

3. A Type I error occurs when:
   A. accurate models are rejected.
   B. accurate models are accepted.
   C. inaccurate models are rejected.
   D. inaccurate models are accepted.

4. Which of the following statements regarding verification of a VaR model by examining its failure rates is false?
   A. The frequency of exceptions should correspond to the confidence level used for the model.
   B. According to Kupiec (1995), we should reject the hypothesis that the model is correct if the LR > 3.84.
   C. Backtesting VaR models with lower confidence levels is difficult because the number of exceptions is not high enough to provide meaningful information.
   D. The range for the number of exceptions must strike a balance between the chances of rejecting an accurate model (a Type I error) and the chance of accepting an inaccurate model (a Type II error).

5. The Basel Committee has established four categories of causes for exceptions. Which of the following does not apply to one of those categories?
   A. Small sample.
   B. Intraday trading activity.
   C. Model accuracy needs improvement.
   D. The basic integrity of the model is lacking.

Concept Checker Answers

1. **C** $(1 - 0.95) \times 255 = 12.75$

2. **D** Unconditional testing does not capture the timing of exceptions.

3. **A** A Type I error occurs when an accurate model is rejected.

4. **C** Backtesting VaR models with *higher* confidence levels is difficult because the number of exceptions is not high enough to provide meaningful information.

5. **A** Causes include the following: bad luck, intraday trading activity, model accuracy needs improvement, and the basic integrity of the model is lacking.

The following is a review of the Market Risk Measurement and Management principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

VaR Mapping

Topic 4

Exam Focus

This topic introduces the concept of mapping a portfolio and shows how the risk of a complex, multi-asset portfolio can be separated into risk factors. We will examine the mapping of several types of portfolios, including fixed income portfolios and portfolios consisting of linear and nonlinear derivatives. Be able to describe the general mapping process and understand how it simplifies risk management for large portfolios.

Mapping a Portfolio

LO 4.1: Explain the principles underlying VaR mapping, and describe the mapping process.

Market risk is measured by first noting all of the current **positions** within a portfolio. These positions are then mapped to **risk factors** by means of factor **exposures**. Mapping involves finding common risk factors among positions in a given portfolio. If we have a portfolio consisting of a large number of positions, it may be difficult and time consuming to manage the risk of each individual position. Instead we can evaluate the value of these positions by mapping them onto common risk factors (e.g., changes in interest rates, changes in equity prices). By reducing the number of variables under consideration, we greatly simplify the risk management process.

Mapping can assist the risk manager in evaluating positions whose characteristics may change over time, such as with fixed income securities. Also, mapping can provide an effective way to manage risk when a sufficient history of data for an investment does not exist, such as with an initial public offering. In both cases, evaluating historical prices may not be relevant so the manager must evaluate those risk factors that are likely to impact the portfolio's risk profile.

By cutting down the number of risk factors needed for analysis we can greatly reduce the amount of complexity. For example, when assessing risk factors we must also assess the correlation among risk factors which requires $\{[n \times (n - 1)] / 2\}$ covariance terms, where n is the number of risk factors. The number of parameters that are needed will grow exponentially with the number of risk factors, so mapping helps to simplify the process.

After finding the common risk factors, the risk manager constructs risk factor distributions and then inputs all data into the **risk engine**. This risk engine is responsible for deriving the profit/loss distribution of the portfolio returns. This distribution of portfolio returns can then be used to compute measures such as value at risk (VaR), which is the maximum loss over a defined period of time at a stated level of confidence.

When utilizing the risk management system, it is important to recognize that this system is **position-based** (a.k.a. holdings-based analysis). This position-based method differs from measuring risk with the more traditional return-based analysis where historical returns are evaluated over time. The return-based method is easy to implement, however, it suffers from the inability to incorporate new positions. As a result, **return-based analysis** may fail to spot style drift and/or hidden risks. The position-based method is necessary for alternative investments since some investments (e.g., emerging market funds) may have a short record of existence.

As mentioned, return-based analysis may fail to account for hidden risks. Risks are hidden when the calculation of performance measures such as the **Sharpe ratio** (i.e., excess return over volatility) indicates stable performance despite the possibility of a very large negative loss. For example, a portfolio that sells out-of-the-money put options on the S&P 500 index will generate stable returns when the options remain out-of-the-money. The returns are achieved from collecting the option premiums from the buyer. However, if the index falls in value and the options are exercised, this portfolio could suffer an infrequent, but large loss. This example illustrates how the returns-based approach can be misleading when assessing volatility. With a position-based approach, hidden risk is accounted for since all new positions and markets are incorporated into the risk measure.

Measuring and managing risk with a position-based analysis is clearly superior to return-based analysis, however, there are a few items that risk managers need to be aware of when implementing this method. Relative to a return-based system, it can be more expensive and require more resources since a portfolio could potentially be made up of hundreds of positions. In addition, this method assumes that the positions are frozen over the period of evaluation. This means that dynamic trading during the time period in question could produce misleading results. Finally, the model is only as good as the data supplied. As a result, **model risk** could be present if the portfolio-based system contains errors or approximations.

Factor Exposures

Factor exposures are an important component of any portfolio-based system and are widely used in risk measurement and management. They are necessary when mapping a portfolio to risk factors since dollar factor exposures are used to replace portfolio positions. When evaluating a fixed-income portfolio, for example, the change in interest rates is the appropriate risk factor. The appropriate exposure on this risk factor is **modified duration**. When combining duration and the change in interest rates, risk managers can easily obtain the percentage change in price for a given fixed-income position.

$$\frac{\Delta P}{P} = -\text{modified duration} \times \Delta y$$

Rearranging this formula allows for the computation of dollar duration, which is simply the position value multiplied by its modified duration.

$$\Delta P = -(\text{modified duration} \times P) \times \Delta y$$

$$\Delta P = -(\text{dollar duration}) \times \Delta y$$

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This same process can also be applied to other risk factors and exposures. For example, the risk factor for equities would be the change in equity index prices. The corresponding factor exposure would be beta.

It is important to note that if all positions in a portfolio are exposed to the same risk factor, the portfolio factor exposure can be found as the weighted average of the position factor exposures. However, overall portfolio risk cannot be computed by aggregating exposures of different risk factors. For example, the modified duration of bonds cannot be combined with the beta of stocks when attempting to obtain the portfolio risk measure.

Another concern for risk manager is the potential for large movements in the risk factors. For example, with a large change in rates, duration will not be an appropriate measure of price change. In this case, second-order factor exposures such as convexity will be needed in order to more accurately reflect the actual change in bond price.

LO 4.2: Explain how the mapping process captures general and specific risks.

So how many **general (or primitive) risk factors** are appropriate for a given portfolio? In some cases, one or two risk factors may be sufficient. Of course, the more risk factors chosen, the more time consuming the modeling of a portfolio becomes. However, more risk factors could lead to a better approximation of the portfolio's risk exposure.

In our choice of general risk factors for use in VaR models, we should be aware that the types and number of risk factors we choose will have an effect on the size of residual or **specific risks**. Specific risks arise from the unsystematic risk of various positions in the portfolio. The more precisely we define risk, the smaller the specific risk will be. For example, a portfolio of bonds may include bonds of different ratings, terms, and currencies. If we use duration as our only risk factor, there will be a significant amount of variance among the bonds that we will call specific risk. If we add a risk factor for credit risk, we could expect that the amount of undefined specific risk would be smaller. If we add another risk factor for currencies, we would expect that the undefined specific risk would be even smaller.

Consider, for example, an equity portfolio with 5,000 stocks. Each stock has a market risk component and a firm-specific component. If each stock has a corresponding risk factor we would need roughly 12.5 million covariance terms (i.e., [5,000 × (5,000 – 1)] / 2) to evaluate the correlation between each risk factor. To simply the number of parameters required we need to understand that diversification will reduce the firm-specific components and leave only market risk (i.e., systematic risk or beta risk). We can then map the market risk component of each stock onto a stock index (i.e., changes in equity prices) to greatly reduce the number of parameters needed.

Mapping Approaches for Fixed-Income Portfolios

LO 4.3: Differentiate among the three methods of mapping portfolios of fixed income securities.

After we have selected our general risk factors, we must map our portfolio onto these factors. There are three systems of mapping for fixed income securities:

1. **Principal mapping.** Includes only the risk of repayment of the principal amounts. This method considers the average maturity of the portfolio.

2. **Duration mapping.** The risk of the bond is mapped to a zero-coupon bond of the same duration. Duration mapping uses the duration of the portfolio to calculate the VaR.

3. **Cash flow mapping**. The risk of the bond is decomposed into the risk of each of the bonds' cash flows. Cash flow mapping is the most precise method because we map the present value of the cash flows (face amount discounted at the spot rate for that maturity) onto the risk factors for zeros of the same maturities and include the intermaturity correlations.

LO 4.4: Summarize how to map a fixed income portfolio into positions of standard instruments.

The following two-position fixed-income portfolio will be used to illustrate the application of the fixed income systems:

- There is a short position in 1-year bonds with a $100 million face value and a 7% annual interest rate, with interest paid semiannually.
- There is a long position in 1-year bonds with a $1 billion face value and an 8% annual interest rate, with interest paid semiannually.
- The interest rate on zero-coupon bonds is 4.0% for 6-month maturity and 4.2% for 12-month maturity.

For principal mapping, VaR is calculated using the risk level from the zero-coupon bond equal to the average maturity of the portfolio. This method is the simplest of the three fixed income approaches.

For duration mapping, we calculate VaR by using the risk level of the zero-coupon bond equal to the duration of the portfolio. It may be difficult to calculate the risk level that exactly matches the duration of the portfolio.

For cash flow mapping, both the short and long positions will be decomposed into positions in two standard instruments consisting of a 6-month zero-coupon bond and a 12-month zero-coupon bond.

The short position will have the following negative cash flows:

- Cash flow 1: –$3.5 million interest in six months.
- Cash flow 2: –$3.5 million interest plus –$100 million principal, or –$103.5 million in 12 months.

The long position will have the following positive cash flows:

- Cash flow 1: $40 million interest in six months.
- Cash flow 2: $40 million interest plus $1 billion principal, or $1.04 billion in 12 months.

Both the short and long positions have now been decomposed into equivalent 6-month and 12-month zero-coupon bonds. They will now be mapped to interest rates on 6-month and 12-month zero-coupon bonds.

X_i is defined as the standard instrument of the *i*th decomposed position, and there are four standard positions that have now been defined. X_i equals the present value of the cash flow of its standard position.

For the short position:

$$X_1 = \frac{-\$3{,}500{,}000}{1+(0.04/2)} = -\$3{,}431{,}373$$

$$X_2 = \frac{-\$103{,}500{,}000}{(1+0.042/2)^2} = -\$99{,}286{,}195$$

For the long position:

$$X_1 = \frac{\$40{,}000{,}000}{1+(0.04/2)} = \$39{,}215{,}686$$

$$X_2 = \frac{\$1{,}040{,}000{,}000}{(1+0.042/2)^2} = \$997{,}658{,}381$$

The portfolio has now been mapped to the 6- and 12-month zero interest rates. In order to calculate portfolio VaR, we would need to incorporate the correlations between the zeros. Cash flow mapping is the most precise method, but is also the most complex.

Professor's Note: The previous example illustrated how to map a portfolio onto standard instruments. The calculation of VaR after the portfolio has been cash flow mapped is a complicated process that is unlikely to show up on the exam.

LO 4.5: Describe how mapping of risk factors can support stress testing.

If we assume that there is perfect correlation among maturities of the zeros, the portfolio VaR would be equal to the **undiversified VaR** (i.e., just the sum of the VaRs). Instead of calculating the undiversified VaR directly, we could reduce each zero-coupon value by its respective VaR and then revalue the portfolio. The difference between the revalued portfolio and the original portfolio value should be equal to the undiversified VaR. Stressing each zero by its VaR is a simpler approach than incorporating correlations, however, this method ceases to be viable if correlations are anything but perfect (i.e., 1).

Benchmarking a Portfolio

LO 4.6: Explain how VaR can be used as a performance benchmark.

It is often convenient to measure VaR relative to that of a benchmark portfolio. This is what is referred to as benchmarking a portfolio. Portfolios can be constructed that match the risk factors of the benchmark portfolio but have either a higher or a lower VaR. The VaR of the deviation between the two portfolios is referred to as a **tracking error VaR**. In other words, tracking error VaR is a measure of the difference between the VaR of the target portfolio and the benchmark portfolio.

LO 4.7: Describe the method of mapping forwards, forward rate agreements, interest rate swaps, and options.

Mapping Approaches for Linear Derivatives

The delta-normal method provides accurate estimates of VaR for portfolios and assets that can be expressed as linear combinations of normally distributed risk factors. Once a portfolio, or financial instrument, is expressed as a linear combination of risk factors, a covariance (correlation) matrix may be generated, and VaR can be measured using matrix multiplication.

Forwards are appropriate for application of the delta-normal method. Their values are a linear combination of a few general risk factors, which have commonly available volatility and correlation data.

To illustrate this idea, consider a forward contract to purchase pounds for dollars one year from now. This forward position is analogous to three separate risk positions:

1. A short position in a U.S. Treasury bill.
2. A long position in a 1-year U.K. bond.
3. A long position in the British pound spot market.

Given the volatilities of each of these positions and the correlation matrix for these positions along with the cash flows, the VaR for the individual risk positions, and the component VaR for each position, the VaR of the forward contract can be determined using matrix algebra.

The general procedure we've outlined applies to other types of financial instruments, such as forward rate agreements and interest rate swaps. As long as an instrument can be expressed as linear combinations of its basic components, the delta-normal VaR may be applied with reasonable accuracy.

Mapping Approaches for Nonlinear Derivatives

As you are aware by now, the delta-normal VaR method is based on linear relationships between variables. Options, however, exhibit nonlinear relationships between movements of the values of the underlying instruments and the values of the options. In many cases, the delta-normal method may still be applied because the value of an option may be expressed linearly as the product of the option delta and the underlying asset.

Unfortunately, the delta-normal VaR cannot be expected to provide an accurate estimate of true VaR over ranges where deltas are unstable. Deep out-of-the-money and deep in-the-money options have relatively stable deltas. Over these ranges, the relationship between the value of the underlying instrument and the value of the option is very much like a forward currency contract. The delta-normal VaR can be calculated by assessing the volatility of the underlying spot prices and the correlation between the price of the option and the spot price.

Professor's Note: Options are usually mapped by using Taylor series approximation and using the delta-gamma method to calculate the option VaR.

Key Concepts

LO 4.1

Portfolio exposures are broken down into general risk factors and mapped onto those factors.

LO 4.2

Undefined specific risk decreases as more risk factors are added to a VaR model.

LO 4.3

Fixed income risk mapping methods include principal mapping, duration mapping, and cash flow mapping. Each provides a different level of complexity and precision.

LO 4.4

Short and long positions in the portfolio can be decomposed and mapped to standard instruments, such as 6-month and 12-month zero-coupon bonds.

LO 4.5

Stressing each zero-coupon bond by its VaR is a simpler approach than incorporating correlations, however, this method ceases to be viable if correlations are anything other than 1.

LO 4.6

A popular use of VaR is to establish a benchmark portfolio and measure VaR of other portfolios in relation to this benchmark.

LO 4.7

Delta-normal VaR can be applied to portfolios of many types of instruments as long as the risk factors are linearly related.

Application of the delta-normal method with options and other derivatives does not provide accurate VaR measures over ranges in which deltas are unstable.

Concept Checkers

1. Delta-normal VaR will provide accurate estimates for option contracts when:
   A. deltas are stable.
   B. options are at-the-money.
   C. the correlation matrix is available.
   D. the delta-normal method can never be used for option contracts.

2. There is a short position in 1-year bonds with a $150 million face value and a 6% annual interest rate, with interest paid semiannually. The annualized interest rate on zero-coupon bonds is 3.8% for a 6-month maturity and 4.1% for a 12-month maturity. Decompose the bond into the cash flows of the two standard instruments, and then determine the present value of the cash flows of the standard instruments. What are the present values of each cash flow?

	PV of CF1	PV of CF2
A.	–$4,117,945	–$139,882,651
B.	–$4,226,094	–$143,873,919
C.	–$4,416,094	–$148,355,095
D.	–$4,879,542	–$144,224,783

3. Which of the following is not one of the three approaches for mapping portfolio fixed income securities onto risk factors?
   A. Principal mapping.
   B. Duration mapping.
   C. Cash flow mapping.
   D. Present value mapping.

4. If portfolio assets are perfectly correlated, portfolio VaR will equal:
   A. marginal VaR.
   B. component VaR.
   C. undiversified VaR.
   D. diversified VaR.

5. Which of the following can be considered a general risk factor?

   I. Exchange rates.
   II. Zero-coupon bond.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

Concept Checker Answers

1. **A** Delta-normal VaR methods will provide accurate estimates of VaR for options only over those ranges in which the deltas of the contracts are stable. Deltas are normally unstable near the money and close to expiration.

2. **C** The standard instruments are –150,000,000 × (0.06 / 2) = –\$4,500,000 for six months, and –\$4,500,000 – 150,000,000 = – \$154,500,000 for 12 months. The present values are –\$4,500,000 / 1.019 = –\$4,416,094, and –\$154,500,000 /$(1 + 0.041/2)^2$ = –\$148,355,095.

3. **D** Present value mapping is not one of the approaches. Each of the others is used.

4. **C** If we assume perfect correlation among assets, VaR would be equal to undiversified VaR.

5. **A** Exchange rates can be used as general risk factors. Zero-coupon bonds are used to map bond positions, but are not considered a risk factor. However, the interest rate on those zeroes is a risk factor.

The following is a review of the Market Risk Measurement and Management principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Messages from the Academic Literature on Risk Measurement for the Trading Book

Topic 5

Exam Focus

This topic addresses tools for risk measurement, including value at risk (VaR) and expected shortfall. Specifically, we will examine VaR implementation over different time horizons and VaR adjustments for liquidity costs. This topic also examines academic studies related to integrated risk management and discusses the importance of measuring interactions among risks due to risk diversification. Note that several concepts in this topic, such as liquidity risk, stressed VaR, and capital requirements, will be discussed in more detail in Book 3, which covers operational and integrated risk management and the Basel Accords.

Value at Risk (VaR) Implementation

LO 5.1: Explain the following lessons on VaR implementation: time horizon over which VaR is estimated, the recognition of time varying volatility in VaR risk factors, and VaR backtesting.

There is no consensus regarding the proper time horizon for risk measurement. The appropriate time horizon depends on the risk measurement purpose (e.g., setting capital limits) as well as portfolio liquidity. Thus, there is not a universally accepted approach for aggregating various VaR measures based on different time horizons.

Time-varying volatility results from volatility fluctuations over time. The effect of time-varying volatility on the accuracy of VaR measures decreases as time horizon increases. However, volatility generated by stochastic (i.e., random) jumps will reduce the accuracy of long-term VaR measures unless there is an adjustment made for stochastic jumps. It is important to recognize time-varying volatility in VaR measures since ignoring it will likely lead to an underestimation of risk. In addition to volatility fluctuations, risk managers should also account for time-varying correlations when making VaR calculations.

To simplify VaR estimation, the financial industry has a tendency to use short time horizons. This approach is computationally attractive for larger portfolios. However, a 10-day VaR time horizon, as suggested by the Basel Committee on Banking Supervision, is not always optimal. It is more preferred to instead allow the risk horizon to vary based on specific investment characteristics. When computing VaR over longer time horizons, a risk manager needs to account for the variation in a portfolio's composition over time. Thus, a longer than 10-day time horizon may be necessary for economic capital purposes.

Historically, VaR backtesting has been used to validate VaR models. However, backtesting is not effective when the number of VaR exceptions is small. In addition, backtesting is less effective over longer time horizons due to portfolio instability. VaR models tend to be more realistic if time-varying volatility is incorporated; however, this approach tends to generate a procyclical VaR measure and produces unstable risk models due to estimation issues.

Integrating Liquidity Risk into VaR Models

LO 5.2: Describe exogenous and endogenous liquidity risk and explain how they might be integrated into VaR models.

During times of a financial crisis, market liquidity conditions change, which changes the liquidity horizon of an investment (i.e., the time to unwind a position without materially affecting its price). Two types of liquidity risk are exogenous liquidity and endogenous liquidity. Both types of liquidity are important to measure; however, academic studies suggest that risk valuation models should first account for the impact of endogenous liquidity.

Professor's Note: In Book 3, we will examine the estimation of liquidity risk using the exogenous spread approach and the endogenous price approach.

Exogenous liquidity is handled through the calculation of a liquidity-adjusted VaR (LVaR) measure, and represents market-specific, average transaction costs. The LVaR measure incorporates a bid/ask spread by adding liquidity costs to the initial estimate of VaR.

Endogenous liquidity is an adjustment for the price effect of liquidating positions. It depends on trade sizes and is applicable when market orders are large enough to move prices. Endogenous liquidity is the elasticity of prices to trading volumes and is more easily observed in instances of high liquidity risk.

Poor market conditions can cause a "flight to quality," which decreases a trader's ability to unwind positions in thinly traded assets. Thus, endogenous liquidity risk is most applicable to exotic/complex trading positions and very relevant in high-stress market conditions, however, endogenous liquidity costs will be present in all market conditions.

Risk Measures

LO 5.3: Compare VaR, expected shortfall, and other relevant risk measures.

VaR estimates the maximum loss that can occur given a specified level of confidence. VaR is a useful measure of risk since it is easy to compute and readily applicable. However, it does not consider losses beyond the VaR confidence level (i.e., the threshold level). In other words, VaR does not consider the severity of losses in the tail of the returns distribution. An additional disadvantage of VaR is that it is not subadditive, meaning that the VaR of a combined portfolio can be greater than the sum of the VaRs of each asset within the portfolio.

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An alternative risk measure, frequently used by financial institutions, is **expected shortfall**. Expected shortfall is more complex and computationally intensive than VaR, however, it does correct for some of the drawbacks of VaR. Namely, it is able to account for the magnitude of losses beyond the VaR threshold and it is always subadditive. In addition, the application of expected shortfall will mitigate the impact that a specific confidence level choice will have on risk management decisions.

Spectral risk measures generalize expected shortfall and consider an investment manager's aversion to risk. These measures have select advantages over expected shortfall by including better smoothness properties when weighting observations as well as the ability to modify a risk measure to reflect an investor's specific risk aversion. Aside from the special case of expected shortfall, other spectral risk measures are rarely used in practice.

Stress Testing

It is important to incorporate stress testing into risk models by selecting various stress scenarios. Three primary applications of stress testing exercises are as follows:

1. **Historical scenarios**, which examine previous market data.

2. **Predefined scenarios**, which attempt to assess the impact on profit/loss of adverse changes in a predetermined set of risk factors.

3. **Mechanical-search stress tests**, which use automated routines to cover possible changes in risk factors.

In stress testing, it is important to "stress" the correlation matrix. However, an unreasonable assumption related to stress testing is that market shocks occur instantly and that traders cannot re-hedge or adjust their positions.

When VaR is computed and analyzed, it is generally under more normalized market conditions, so it may not be accurate in a more stressful environment. A **stressed VaR** approach, which attempts to account for a significantly financial stressed period, has not been thoroughly tested or analyzed. Thus, VaR could lead to inaccurate risk assessment under market stresses.

Professor's Note: In Book 3, we will explain the calculation of stressed VaR.

Integrated Risk Measurement

LO 5.4: Compare unified and compartmentalized risk measurement.

Unified and compartmentalized risk measurement methods aggregate risks for banks. A compartmentalized approach sums risks separately, whereas a unified, or integrated, approach considers the interaction among risks.

A **unified approach** considers all risk categories simultaneously. This approach can capture possible compounding effects that are not considered when looking at individual risk measures in isolation. For example, unified approaches may consider market, credit, and operational risks all together.

When calculating capital requirements, banks use a **compartmentalized approach**, whereby capital requirements are calculated for individual risk types, such as market risk and credit risk. These stand-alone capital requirements are then summed in order to obtain the bank's overall level of capital.

The Basel regulatory framework uses a "building block" approach, whereby a bank's regulatory capital requirement is the sum of the capital requirements for various risk categories. Pillar 1 risk categories include market, credit, and operational risks. Pillar 2 risk categories incorporate concentration risks, stress tests, and other risks, such as liquidity, residual, and business risks.

Thus, the overall Basel approach to calculating capital requirements is a non-integrated approach to risk measurement. In contrast, an integrated approach would look at capital requirements for each of the risks simultaneously and account for potential risk correlations and interactions. Note that simply calculating individual risks and adding them together will not necessarily produce an accurate measure of true risk.

Risk Aggregation

LO 5.5: Compare the results of research on "top-down" and "bottom-up" risk aggregation methods.

A bank's assets can be viewed as a series of subportfolios consisting of market, credit, and operational risk. However, these risk categories are intertwined and at times difficult to separate. For example, foreign currency loans will contain both foreign exchange risk and credit risk. Thus, interactions among various risk factors should be considered.

The **top-down approach** to risk aggregation assumes that a bank's portfolio can be cleanly subdivided according to market, credit, and operational risk measures. In contrast, a **bottom-up approach** attempts to account for interactions among various risk factors.

In order to assess which approach is more appropriate, academic studies calculate the ratio of unified capital to compartmentalized capital (i.e., the ratio of integrated risks to separate risks). Top-down studies calculate this ratio to be less than one, which suggest that **risk diversification** is present and ignored by the separate approach. Bottom-up studies also often calculate this ratio to be less than one, however, this research has not been conclusive, and has recently found evidence of risk compounding, which produces a ratio greater than one. Thus, bottom-up studies suggest that risk diversification should be questioned.

It is conservative to evaluate market risk and credit risk independently. However, most academic studies confirm that market risk and credit risk should be looked at jointly.
If a bank ignores risk interdependencies, a bank's capital requirement will be measured improperly due to the presence of risk diversification. Therefore, separate measurement of

market risk and credit risk most likely provides an upper bound on the integrated capital level.

Note that if a bank is unable to completely separate risks, the compartmentalized approach will not be conservative enough. Thus, the lack of complete separation could lead to an underestimation of risk. In this case, bank managers and regulators should conclude that the bank's overall capital level should be higher than the sum of the capital calculations derived from risks individually.

Balance Sheet Management

LO 5.6: Describe the relationship between leverage, market value of asset, and VaR within an active balance sheet management framework.

When a balance sheet is actively managed, the amount of leverage on the balance sheet becomes procyclical. This results because changes in market prices and risks force changes to risk models and capital requirements, which require adjustments to the balance sheet (i.e., greater risks require greater levels of capital). Thus, capital requirements tend to amplify boom and bust cycles (i.e., magnify financial and economic fluctuations). Academic studies have shown that balance sheet adjustments made through active risk management affect risk premiums and total financial market volatility.

Leverage (measured as total assets to equity) is inversely related to the market value of total assets. When net worth rises, leverage decreases, and when net worth declines, leverage increases. This results in a **cyclical feedback loop**: asset purchases increase when asset prices are rising, and assets are sold when asset prices are declining.

Value at risk is tied to a bank's level of economic capital. Given a target ratio of VaR to economic capital, a VaR constraint on leveraged investors can be established. An economic boom will relax this VaR constraint since a bank's level of equity is expanding. Thus, this expansion allows financial institutions to take on more risk and further increase debt. In contrast, an economic bust will tighten the VaR constraint and force investors to reduce leverage by selling assets when market prices and liquidity are declining. Therefore, despite increasingly sophisticated VaR models, current regulations intended to limit risk-taking have the potential to actually increase risk in financial markets.

Key Concepts

LO 5.1

The proper time horizon over which VaR is estimated depends on portfolio liquidity and the purpose for risk measurement. It is important to incorporate time-varying volatility into VaR models, because ignoring this factor could lead to an underestimation of risk. Backtesting VaR models is less effective over longer time horizons due to portfolio instability.

LO 5.2

Exogenous liquidity represents market-specific, average transaction costs. Endogenous liquidity is the adjustment for the price effect of liquidating specific positions. Endogenous liquidity risk is especially relevant in high-stress market conditions.

LO 5.3

VaR estimates the maximum loss that can occur given a specified level of confidence. It is a quantitative risk measure used by investment managers as a method to measure portfolio market risk. A downside of VaR is that it is not subadditive.

An alternative risk measure is expected shortfall, which is complex and computationally difficult. Spectral risk measures consider the investment manager's aversion to risk. These measures have select advantages over expected shortfall.

LO 5.4

Within a bank's risk assessment framework, a compartmentalized approach sums measured risks separately. A unified approach considers the interaction among various risk factors. Simply calculating individual risks and adding them together is not necessarily an accurate measure of true risk due to risk diversification. The Basel approach is a non-integrated approach to risk measurement.

LO 5.5

A top-down approach to risk assessment assumes that a bank's portfolio can be cleanly subdivided according to market, credit, and operational risk measures. To better account for the interaction among risk factors, a bottom-up approach should be used.

LO 5.6

When a balance sheet is actively managed, the amount of leverage on the balance sheet becomes procyclical. Leverage is inversely related to the market value of total assets. This results in a cyclical feedback loop. Financial institution capital requirements tend to amplify boom and bust cycles.

Concept Checkers

1. Which of the following statements is considered to be a drawback of the current Basel framework for risk measurement?
   A. Risk measurement focuses exclusively on VaR analysis.
   B. The current regulatory system encourages more risk-taking when times are good.
   C. There is not enough focus on a compartmentalized approach to risk assessment.
   D. There is not a feedback loop via the pricing of risk.

2. What type of liquidity risk is most troublesome for complex trading positions?
   A. Endogenous.
   B. Market-specific.
   C. Exogenous.
   D. Spectral.

3. Within the framework of risk analysis, which of the following choices would be considered most critical when looking at risks within financial institutions?
   A. Computing separate capital requirements for a bank's trading and banking books.
   B. Proper analysis of stressed VaR.
   C. Persistent use of backtesting.
   D. Consideration of interactions among risk factors.

4. What is a key weakness of the value at risk (VaR) measure? VaR:
   A. does not consider the severity of losses in the tail of the returns distribution.
   B. is quite difficult to compute.
   C. is subadditive.
   D. has an insufficient amount of backtesting data.

5. Which of the following statements is not an advantage of spectral risk measures over expected shortfall? Spectral risk measures:
   A. consider a manager's aversion to risk.
   B. are a special case of expected shortfall measures.
   C. have the ability to modify the risk measure to reflect an investor's specific risk aversion.
   D. have better smoothness properties when weighting observations.

Concept Checker Answers

1. **B** Institutions have a tendency to buy more risky assets when prices of assets are rising.

2. **A** Endogenous liquidity risk is especially relevant for complex trading positions.

3. **D** A unified approach is not used within the Basel framework, so the interaction among various risk factors is not considered when computing capital requirements for market, credit and operational risk; however, these interactions should be considered due to risk diversification.

4. **A** VaR does not consider losses beyond the VaR threshold level.

5. **B** Spectral risk measures consider aversion to risk and offer better smoothness properties. Expected shortfall is a special case of spectral risk measures.