The following is a review of the Credit Risk Measurement and Management principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Credit Risks and Credit Derivatives

Topic 21

Exam Focus

The potential failure of a party to fulfill an agreed-upon payment is an uncertainty for risk managers. The pricing of risky debt using the Merton model provides insight into predicting the probability of default and the amount of loss should default occur. Additional approaches have been developed to evaluate the credit risk of portfolios and provide estimates of a portfolio's credit value at risk. Credit derivatives provide the risk manager with financial instruments that are used to hedge credit risk exposures. This topic presents several models for evaluating and measuring credit risk, along with examples of credit derivatives used to hedge credit risk exposure. For the exam, be sure to understand the calculation of firm equity and debt under the Merton model.

Credit risk refers to the chance that a party will fail to make promised payments. Risk managers assess credit risk and determine its potential impact on income and if it should be hedged using derivatives contracts or some other means.

The two important roles that credit risk plays in risk management programs are (1) assessing the potential of default by debt claimants and (2) assessing the potential of default by counterparties of derivatives contacts.

Derivative contracts with payoffs dependent on a specified credit event are called **credit derivatives**. Risk managers use credit derivatives to hedge their exposure to credit risk.

The Merton Model

LO 21.1: Using the Merton model, calculate the value of a firm's debt and equity and the volatility of firm value.

The Merton model, based on Black-Scholes-Merton option pricing theory, evaluates various components of firm value. The simplest form of the model assumes the existence of a non-dividend paying firm with only one liability claim and that financial markets are perfect. That is, the model assumes away taxes, bankruptcy costs, and costs associated with enforcing contracts.

Suppose that the firm's only debt issue is a zero-coupon bond with a face value (or principal amount) of F, due at the maturity date of T. If the firm is unable to pay the principal at T, then the firm is bankrupt and the equity claimants receive nothing. Alternatively, if the firm value at T, V_T, is large enough to pay the principal amount, then equity holders have claim to the balance, $V_T - F$. These two payoff possibilities are the same as the payoffs for a call option, with the firm value as the underlying asset and the principal amount as the exercise price. Therefore, the value of equity at T is:

$$S_T = \text{Max}(V_T - F, 0)$$

Example: Computing the value of equity

Calculate the value of the firm's equity at T, S_T, given that principal amount due on the zero-coupon bond is \$50 million and the total value of the firm at T, V_T, is \$60 million. In addition, what is the value of equity if V_T is \$40 million?

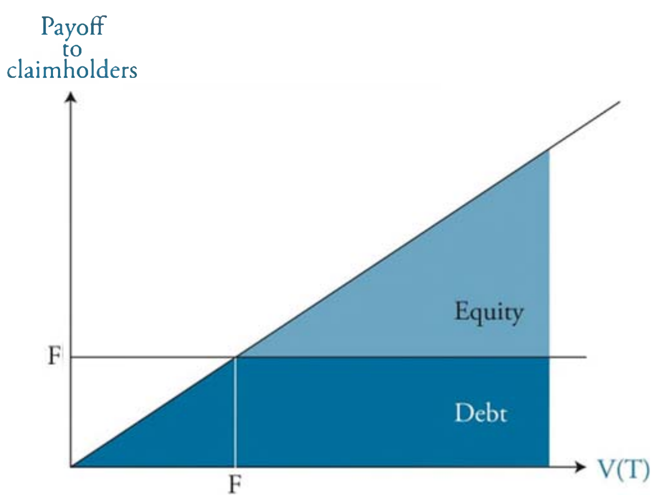
Answer:

$$S_T = \text{Max}(60 - 50, 0) = 10 = \$10 \text{ million}$$

$$S_T = \text{Max}(40 - 50, 0) = 0 = \$0 \text{ million}$$

Figure 1 depicts the payoffs at T of the claimholders relative to the total firm value. The shape of the payoff for equity is the same as the payoff of a long call option. The payoff for debt is similar to that of a risk-free bond and a short position in a put option, but the uncertainty of the payoff requires some additional analysis.

Figure 1: Debt and Equity Payoffs When Debt is Risky



If the debtholder knew with certainty that he would receive the principal amount at the maturity for the zero-coupon bond, then the payoff would be F regardless of the firm value. However, the debtholder cannot expect to receive F if the total value of the firm, V_T, is less than F. Therefore, if $F > V_T$ then the amount received by the debtholder will be reduced by

$F - V_T$. This payoff is the same as buying a Treasury bill with a face value of *F* and selling a put on the firm value with an exercise price of *F*. Therefore, the value of debt at *T* is:

$$D_T = F - \text{Max}(F - V_T, 0)$$

Example: Computing the value of debt

Calculate the value of the firm's debt at *T*, D_T, given that principal amount due on the zero-coupon bond is \$50 million and the total value of the firm at *T*, V_T, is \$40 million. In addition, what is the value of debt if V_T is \$60 million?

Answer:

$$D_T = 50 - \text{Max}(50 - 40, 0) = 50 - 10 = 40 = \$40 \text{ million}$$

$$D_T = 50 - \text{Max}(50 - 60, 0) = 50 - 0 = 50 = \$50 \text{ million}$$

Since we know that the value of the debt plus the value of equity must be equal to the total value of the firm, alternative valuation formulas can be developed. For example, since the value of the firm at *T* is:

$V_T = D_T + S_T$ then the value of debt at *T* can also be written as:

$D_T = V_T - S_T$ and by substitution:

$$D_T = V_T - \text{Max}(V_T - F, 0)$$

Therefore, the value of debt is also the difference between the value of the firm and the call option on the value of the firm with *F* as the exercise price.

The Black-Scholes-Merton option-pricing model for European options can be modified to determine the value of equity prior to *T*, $T - t$, if additional assumptions are made, which include:

- Firm value characterized by a lognormal distribution with constant volatility, σ.
- Constant interest rate, *r*.
- Perfect financial market with continuous trading.

The Value of Equity at Time *t*

Using arbitrage pricing for a portfolio of securities that replicates the value of the firm results in Merton's formula for the value of equity such that:

$$S_t = V \times N(d) - Fe^{-r(T-t)} \times N(d - \sigma\sqrt{T-t})$$

where:

$$d = \frac{\ln\left(\frac{V}{Fe^{-r(T-t)}}\right)}{\sigma\sqrt{T-t}} + \frac{1}{2}\sigma\sqrt{T-t}$$

V = value of the firm
F = face value of the firm's zero-coupon debt maturing at *T* (only liability)
σ = volatility of the value of the firm
r = annual interest rate
N(d) = cumulative normal distribution function evaluated at *d*

Example: Compute the value of equity

Using the Merton model, **calculate** the value of the firm's equity at *t* given that the current value of the firm is $60 million, the principal amount due in 3 years on the zero-coupon bond is $50 million, the annual interest rate, *r*, is 5%, and the volatility on the firm, σ, is 10%.

Answer:

$$S_t = 60 \times N(d) - 50e^{-0.05(3)} \times N(d - \sigma\sqrt{T-t})$$

$$d = \frac{\ln\left(\frac{60}{(50)(0.8607)}\right)}{(0.10)\sqrt{3-0}} + \frac{1}{2}(0.10)\sqrt{3-0} = \frac{\ln(1.3942)}{0.1732} + \frac{1}{2}(0.1732) = 2.005$$

$$S_t = 60 \times N(2.005) - 50 \times 0.8607 \times N(2.005 - 0.1732)$$

Professor's Note: N(d) can be found in a table of probability values (the z-table in the appendix).

$$S_t = 60 \times 0.9775 - 43.035 \times 0.9665 = 58.650 - 41.593 = 17.057$$

Therefore, the value of equity of the firm is $17.057 million.

The Value of Debt at Time *t*

There are two methods for valuing risky debt in this framework. Risky debt is equal to:

- Risk-free debt minus a put option on the firm.
- Firm value minus equity value.

Example: Compute the value of debt

Calculate the current value of the firm's debt as a portfolio of risk-free debt and a short position in a put on firm value with an exercise price of the face value of debt. Assume that the current value of the firm is $60 million, the principal amount due in three years on the zero-coupon bond is $50 million, the annual interest rate is 5%, and the volatility on the firm, σ, is 10%. Recall from the previous example that the value of equity is $17.057 million.

Answer:

$$D_t = Fe^{-r(T-t)} - p_t$$

$$D_t = 50 \times e^{(-0.05)(3)} - p_t$$

$$D_t = 50 \times 0.8607 - p_t$$

$$D_t = 43.035 - p_t$$

Using put-call parity, the value of the put is:

$$p_t = c_t + Fe^{-r(T-t)} - V$$

$$p_t = c_t + 43.035 - 60$$

$$p_t = 17.057 + 43.035 - 60 = 0.092$$

$$D_t = 43.035 - p_t = 43.035 - 0.092 = 42.943$$

Therefore, the value of the debt issue is $42.943 million.

Example: Compute the value of debt

Calculate the current value of the firm's debt as the difference between the total firm value and the value of equity priced as a call option. Assume that the current value of the firm is \$60 million, the principal amount due in three years on the zero-coupon bond is \$50 million, the annual interest rate is 5%, and the volatility on the firm, σ, is 10%.

Answer:

$$D_t = V - S_t$$

$$D_t = 60 - 17.057 = 42.943$$

Again, the value of debt is \$42.943 million.

Figure 2 shows the general relationships between debt and equity values according to the inputs of the Merton model.

Figure 2: Relationships Between Debt and Equity Values as Compared to the Inputs of the Merton Model

	Value of Firm, V	*Face Value of Debt, F*	*Time to Maturity, T*	*Interest Rate, r*	*Volatility of the Firm, σ*
Value of debt	+	+	–	–	–
Value of equity	+	–	+	+	+

Credit Spreads, Time to Maturity, and Interest Rates

LO 21.2: Explain the relationship between credit spreads, time to maturity, and interest rates.

A **credit spread** is the difference between the yield on a risky bond (e.g., corporate bond) and the yield on a risk-free bond (e.g., T-bond) given that the two instruments have the same maturity. For example, if a corporate bond is yielding 7% and the yield on the T-bond with the same maturity is 5%, the credit spread would be equal to 2%. This spread indicates that a higher yield is received for taking on increased risk.

The credit spread can be calculated using the following formula:

$$\text{credit spread} = -\left[\frac{1}{(T-t)}\right] \times \ln\left(\frac{D}{F}\right) - R_F$$

where:
$(T - t)$ = remaining maturity
D = current value of debt
F = face value of debt
R_F = risk-free rate

For a given risky bond, the most you can receive is the par value at maturity. However, as time increases there is greater probability that the value received will be less than par. Studies have shown that as time to maturity increases, credit spreads tend to widen (i.e., increase). This applies to both high-rated and low-rated debt. However, for very risky debt, it may be the case that credit spreads narrow since there is a greater chance of payment as maturity approaches.

In addition to time to maturity, interest rates can also impact credit spreads. As the risk-free rate increases, the expected value of the firm at maturity increases, which in turn decreases the risk of default. A reduction in the risk of default will narrow (i.e., decrease) credit spreads.

Determining Firm Value and Volatility

Since there is no single claim for the value of a levered firm, the value of the firm is unobservable. Further, it is virtually impossible to directly trade the value of the firm. These deficiencies can be solved using the Merton model if we assume that small changes in the return on equity are perfectly correlated with the value of the firm.

A portfolio consisting of a call option on the firm and a risk-free asset is equivalent to the value of the firm. Thus, a small change in firm value will change the value of equity by delta, Δ, times the change in firm value. Delta is the rate of change in the value of the call option relative to the change in the value of the underlying asset, $\Delta S/\Delta V$. The Merton model delta, Δ, is equal to $N(d)$. Therefore, if we know the parameters for calculating the value of equity as a call and the value of risk-free debt, then we can determine the firm's value and the volatility of firm value.

Figure 3: Low Firm Values and High Firm Values Using the Merton Model

	Low Firm Values			*High Firm Values*		
Value of firm per share	2.50	3.00	3.60	25.00	30.00	36.00
Value of equity per share, S_t	0.196	0.306	0.469	15.659	20.164	25.723
Change in value of equity if value of firm increases by 20%	---	56.1%	53.3%	---	28.8%	27.6%
Delta	0.197	0.245	0.299	0.886	0.914	0.937

Although delta is increasing as the value of the firm increases, the change in the value of equity decreases as firm value increases. This indicates that the distribution of equity values is not constant (which is sometimes referred to as a volatility smirk). The non-constant volatility of equity is a violation of the Black-Scholes-Merton model.

The **Geske compound option model** is appropriate for valuing the equity call option because it assumes that the value of the firm is characterized by a lognormal distribution with a constant variance.

If we know the value of equity and the value of an option on firm equity, we can use an iterative process to solve for firm value and firm volatility. A value for firm volatility and firm value is selected, and a value for equity is estimated using the Black-Scholes-Merton model. The same values for firm volatility and firm value are used in the compound option model to arrive at a value for the call option on equity. The outputs of the two models are then compared to actual values of equity and equity call option. Adjustments are made to firm value and firm volatility until the outputs of both models are the same as the actual values.

For example, suppose that the value of the firm is selected to be $25 per share, the volatility, σ, is 50%, the value of the call option using the compound option model is $6.0349 and the value of equity from the Merton model is $15.50. Also suppose the actual call option price is $6.72 and the actual equity price is $14.10.

Since the value of the option of $6.0349 is below $6.72 and the value of equity per share is above the observed price per share of $14.10, the firm value is too high and the volatility of the firm is too low. To decrease the value of equity from the Merton model, the value of the firm should be lower. To increase the value of the call option using the compound option model, the volatility measure needs to be higher. The results of the iterative process indicates that a firm value of $21 per share and a firm volatility of 68.36% produces model values that are equal to the observed values.

Subordinate Debt

LO 21.3: Explain the differences between valuing senior and subordinated debt using a contingent claim approach.

LO 21.4: Explain, from a contingent claim perspective, the impact of stochastic interest rates on the valuation of risky bonds, equity, and the risk of default.

In the event of bankruptcy, subordinate debt will receive payment only after all obligations to senior debt have been paid. Because of the uncertainty associated with financial distress, the value of subordinate debt acts more like an equity security than a debt security. Therefore, when a firm is in financial distress, the value of subordinate debt will increase as firm volatility increases, while the value of senior debt will decline.

Suppose a firm has one issue of subordinate debt (SD) and one issue of senior debt (D) where both issues have the same maturity date, *T*. *F* and *U* represent the face values of senior debt and subordinate debt, respectively. Equity, *S*, is valued as a call option on the value of the firm, *V*, with an exercise price of F + U.

Subordinate debt can be valued in a portfolio as a long position in a call option on the firm with an exercise price equal to the face value of senior debt, F, and a short position on a call option on the firm with an exercise price equal to the total principal due on all debt, U + F. Figure 4 illustrates the portfolio payoffs for the previous equations.

Figure 4: Subordinated Debt Payoff

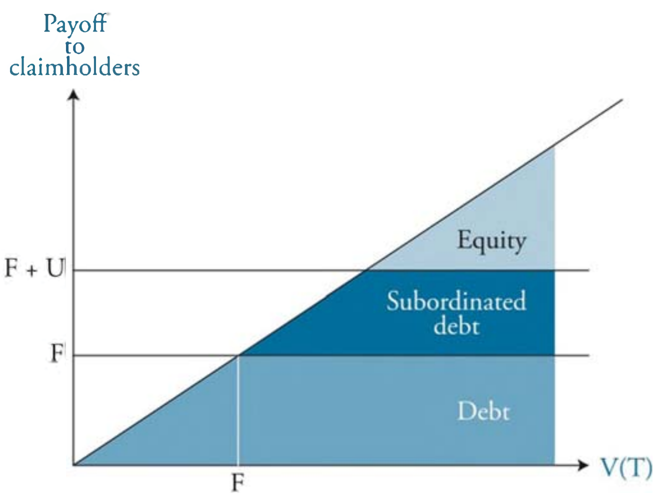


Figure 5 illustrates how subordinate debt values behave like equity when the firm has low values, as during periods of financial distress, and how they behave like senior debt when the firm is not in financial distress.

Figure 5: Relationships for Capital Components for Low vs. High Firm Values

	If Firm Value is Low Firm in Financial Distress			*If Firm Value is High Firm Not in Financial Distress*		
Variable	*Time to Maturity T*	*Firm Volatility σ*	*Annual Interest Rate r*	*Time to Maturity T*	*Firm Volatility σ*	*Annual Interest Rate r*
Senior debt	–	–	–	–	–	–
Subordinate debt	+	+	+	–	–	–
Equity	+	+	+	+	+	+

Interest Rate Dynamics

In the absence of credit risk, unanticipated changes in interest rates can affect the value of debt and the value of the firm. Increases in interest rates will decrease the value of debt because of pricing sensitivities of debt. Empirical evidence indicates that, on average, firm stock prices decline as interest rates rise. Therefore, to hedge debt, we need to account for the interactions between changing interest rates and firm value.

The **Vasicek model** allows for interest rates to revert to a long-run mean. The change in interest rates in the Vasicek model at time *t* is:

$$\Delta r_t = k(\theta - r_t)\Delta t + \sigma_r \varepsilon_t$$

where:
k = speed that interest rate reverts to the long-run mean, θ
r_t = current interest rate
σ_r = interest rate volatility
ε_t = random error term

To value debt, Shimko, Tejima, and Van Deventer (1993)[1] developed a variation of the Merton model that included the correlation between firm value and changes in interest rates, $\rho_{(V,\Delta r)}$.

Figure 6 illustrates the relationships between interest rate dynamics of a low value firm (i.e., firm in financial distress) and the value of debt.

Figure 6: Interest Rate Dynamics of Firm in Financial Distress

	$\rho_{(V,\Delta r)}$	k	σ_r	T
Value of debt	–	–	–	–

The sensitivity of the value of debt to changes in interest rates is dependent on the volatility of interest rates. When interest rate volatility is high the debt values are less sensitive to changes in interest rates. Therefore, hedging against the adverse affect of changing interest rates is dependent on the parameters of the dynamic interest rate model.

Application Difficulties

Application of the Merton model is complicated by the complexity of firms' capital structures. Most firms have a variety of debt instruments that mature at different times and have many different coupon rates (i.e., not just zero-coupons). In addition to the many different types of debt issues, the Merton model does not allow the firm value to jump. Since most defaults are surprises, the inability to have jumps in the firm value in the Merton model makes default too predictable.

Empirical research confirms the predictability of the Merton model. Jones, Mason, and Rosenfeld (1984)[2] report that a naïve model of predicting whether debt is riskless works better for investment grade bonds than the Merton model. However, the Merton model works better than the naïve model for debt below investment grade. Kim, Ramaswamy, and Sundaresan (1993)[3] report the Merton model's inability to predict credit spreads. The documented problems with the Merton model created the need for models to predict default more accurately (such as the KMV approach).

1. Shimko, David C., Naohiko Tejima, and Donald R. Van Deventer, 1993, "The Pricing of Risky Debt When Interest Rates Are Stochastic," *Journal of Fixed Income*, 3(2), 58–66.
2. Jones, Philip E., Scott P. Mason, and Eric Rosenfeld, 1984, "Contingent Claims Analysis of Corporate Capital Structures: An Empirical Investigation," *Journal of Finance*, 39(3), 611–625.
3. Kim, In Joon, Krishna Ramaswamy, and Suresh Sundaresan, 1993, "Does Default Risk in Coupons Affect the Valuation of Corporate Bonds?—A Contingent Claims Model," *Financial Management*, 22, 117–131.

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Using the Merton Model to Calculate PD and LGD

The simplest case for calculating probability of default assumes that the process for default is not correlated with the interest rate process or the recovery rate. The recovery rate is a fixed proportion of the principal that the debtholder receives in the event of default. The recovery rate is not dependent on time.

To find the value of debt, the probability of default and the recovery rate are required. If the debt is publicly traded, then the probability of default and the recovery rate can be estimated from the current price of debt; however, most debt instruments are not publicly traded.

In addition to the lack of public trading, there are four differences in measuring the risk of a debt portfolio that make estimating the probability of default and the loss due to default more challenging:

- If securities are illiquid, then the historical data is not reliable.
- The distribution of bond returns is not normal because the debtholder cannot receive more than the face amount plus the sum of the coupons.
- Debt is issued by creditors who do not have traded equity.
- Debt is not marked to market in contrast to traded securities. That is, a loss is recognized only if default occurs.

The Merton model for **probability of default** (PD) and **loss given default** (LGD) assumes that firm value is lognormally distributed with a constant volatility, and that the firm only has one liability, which is zero-coupon debt issue. The model also requires the expected return on the value of the firm, μ. The Merton model for PD is:

$$PD = N\left(\frac{\ln(F) - \ln(V) - (\mu)(T-t) + 0.5\sigma^2(T-t)}{(\sigma)\sqrt{T-t}}\right)$$

where:
N = cumulative normal distribution
F = face value of the zero-coupon bond
V = value of the firm
T = maturity date on bond
σ = volatility of firm value

Loss given default (LGD) is:

$$LGD = F \times (PD) - Ve^{\mu(T-t)} \times N\left(\frac{\ln(F) - \ln(V) - \mu(T-t) - 0.5\sigma^2(T-t)}{\sigma\sqrt{T-t}}\right)$$

Example: Compute PD and LGD

Suppose a firm with a value of $60 million has a bond outstanding with a face value of $50 million that matures in three years. The current interest rate is 6% and the volatility of the firm is 25%. What is the probability that the firm will default on its debt if the expected return on the firm, μ, is 15%? What is the expected loss given default?

Answer:

$$PD = N\left(\frac{\ln(50)-\ln(60)-(0.15)(3)+(0.5)(0.25)^2(3)}{(0.25)\sqrt{3}}\right) = N(-1.244) = 0.1069 = 10.69\%$$

$$LGD = 50(0.1069) - 60e^{0.15(3)}N\left(\frac{\ln(50)-\ln(60)-0.15(3)-0.5(0.25)^2(3)}{0.25\sqrt{3}}\right)$$

$$LGD = 5.345 - (94.099)N(-1.677) = 5.345 - (94.099)(0.0468) = 5.345 - 4.404$$
$$= 0.941 = \$941{,}000$$

Figure 7 illustrates the relationships between the inputs of the Merton model and the probability of default and then compares each relationship to loss given default.

Figure 7: Relationships for PD and LGD Relative to Variables in the Merton Model

	Value of Firm	*Firm Value Volatility, σ*	*Expected Return, μ*	*Time to Maturity, T*	*Face Value of Debt, F*
Probability of default, PD	–	+	–	–	+
Loss given default, LGD	–	+	–	–	+

Credit Risk Portfolio Models

LO 21.5: Compare and contrast different approaches to credit risk modeling, such as those related to the Merton model, CreditRisk+, CreditMetrics, and the KMV model.

Portfolio credit risk models resolve some of the difficulties of measuring a portfolio's probability of default and the amount of loss associated with default when using the Merton model. The models also allow for the inclusion of additional securities and contracts, such as swaps. Therefore, instead of having only debtholders in the model, the model includes other obligors. Obligors include all parties who have a legal obligation to the firm.

Using various methodologies, credit risk portfolio models attempt to estimate a portfolio's credit value at risk. Credit VaR (also called **credit at risk** or **default VaR**) is defined much the same as VaR (a.k.a. market VaR); the minimum credit loss at a given significance over a given time period (or alternatively, the maximum credit loss for a given confidence level over a given time period).

Credit VaR differs from market VaR in that it measures losses that are due specifically to default risk and credit deterioration risk. Like market VaR, credit VaR is measured over a specified time period at a specified probability. There are two problems, however, when calculating credit VaR. First, calculating changes in credit quality over a 1-day period is difficult. Therefore, credit VaR is usually calculated over a year, where the potential change in credit risk is more easily estimated. The second problem is that changes in credit risk are highly skewed and do not follow a normal distribution. The loss distribution of changes in credit quality for investment grade bonds closely resembles a lognormal distribution.

CreditRisk+

CreditRisk+ measures the credit risk of a portfolio using a set of common risk factors for each obligor. Each obligor has its own sensitivity to each of the common risk factors. The model allows for only two outcomes (default or nondefault) for a loss of a fixed size. The probability of default for each obligor is dependent on the credit rating and the obligor's sensitivity to each of the risk factors. Conditional on the risk factors, the model assumes that defaults are uncorrelated across obligors.

Risk factors can only have positive values and are scaled to have a mean of one. The risk factors are assumed to follow a specific distribution, such as a gamma distribution. If an obligor has a risk factor greater than one, then the probability of default for firm i increases in proportion to the obligor's exposure. After the probability of default for each obligor is calculated, the loss distribution for the portfolio can be estimated and used to assess the credit risk of the portfolio.

CreditMetrics

CreditMetrics is used for determining the credit value at risk (VaR) for large portfolios of debt claims.

Steps in calculating credit VaR using CreditMetrics:

Step 1: Determine rating class for debt claim.
Step 2: Use historical rating transition matrix to determine the probability that claim will migrate.
Step 3: Estimate the distribution of value for claim by computing the expected values for one year.
Step 4: Use the 1-year forward zero curves rates to get current price of zero-coupon bond for one year.
Step 5: Assume annual coupons to compute value of bond for each possible rating for next year.
Step 6: Compute the expected bond value $E(BV_p) = \sum_{i=1}^{N} p_i BV_i$. Then compute the credit value at risk (VaR) for a given confidence level.

where:
p = probability of migrating from a given rating
BV = the bond value plus coupon for a given rating

Example: Compute VaR using CreditMetrics[4]

Suppose your portfolio contains a senior unsecured bond issued by Triple-Bee, Inc. The bond with a credit rating of BBB matures in five years and pays a 6% coupon. If the recovery rate is 51.13%, what is the 1% credit VaR, given the following 1-year forward zero rates for the next four years and the 1-year transition probabilities of a bond with a BBB rating? Assume the bond's market price is $106.

Figure 8: One-Year Forward Zero Curves Rate for Each Rating Class (%)

Rating Class	*Year 1*	*Year 2*	*Year 3*	*Year 4*	*Bond Value Plus Coupon for Next Year*
AAA	3.60	4.17	4.73	5.12	109.37
AA	3.65	4.22	4.78	5.17	109.19
A	3.72	4.32	4.93	5.32	108.66
BBB	4.10	4.67	5.25	5.63	107.55
BB	5.55	6.02	6.78	7.27	102.02
B	6.05	7.02	8.03	8.52	98.10
C	15.05	15.02	14.03	13.52	83.64

The bond value for next year is calculated by discounting the coupons and the principal amount by the appropriate forward rate. For example, the bond value for the BB rating class is calculated as:

$$6+\frac{6}{(1+0.0555)^1}+\frac{6}{(1+0.0602)^2}+\frac{6}{(1+0.0678)^3}+\frac{6+100}{(1+0.0727)^4}=102.02$$

Figure 9: Estimation of Mean Bond Value Given the One-Year Probabilities of Migration from BBB and the Recovery Rate of 51.13% for the Triple-Bee Bond

Year-End Rating Class	*Probability of Migrating from BBB (%)*	*Cumulative Probability (%)*	*Bond Value Plus Coupon*	*(Probability) × (Bond Value Plus Coupon)*
AAA	0.02	100.00	109.37	0.022
AA	0.33	99.98	109.19	0.360
A	5.95	99.65	108.66	6.465
BBB	86.93	93.70	107.55	93.493
BB	5.30	6.77	102.02	5.407
B	1.17	1.47	98.10	1.148
C	0.12	0.30	83.64	0.100
Default	0.18	0.18	51.13	0.092
Expected bond value $E(BV_p)=\sum_{i=1}^{N} p_i BV_i$				107.087

4. The data on which this example is based can be found in the CreditMetrics® Technical Manual, available on the RiskMetrics® Web site at www.riskmetrics.com.

Answer:

The cumulative probability column in Figure 9 estimates the first percentile. The expected bond value of 98.10 is at 1.47% in the cumulative distribution and is used as a proxy for the first percentile. Since credit VaR is the difference between the current bond price and the first percentile, the credit VaR for a bond with a current price of 106 is estimated to be $7.90 (= 106 – 98.10).

Correlations are important in a portfolio. If two bonds are independent, then the probability of both bonds migrating is the product of the individual events. If the bonds are not independent, then we need to know the migration correlations. The major complexity of CreditMetrics is estimating the joint migration of bonds in a portfolio. If the historical estimates for joint probabilities are used, then additional information is required. However, if stock returns are used to estimate the correlations between bond issues, the problem is solved. CreditMetrics recommends using a factor model where stock returns depend on country and industry indices as well as unsystematic risk.

Moody's KMV Portfolio Manager

Previously, we discussed the KMV approach for estimating expected default probabilities. That approach is known as Moody's KMV Credit Monitor™. Another KMV model that is based on the Merton model, but is specifically designed for managing credit risk in a portfolio setting, is known as Moody's KMV Portfolio Manager™. KMV Credit Monitor provides data for Portfolio Manager.

The KMV model, a modified Merton model, calculates the **expected default frequencies** (EDFs) for each obligor. This modified model allows for more complicated capital structures (e.g., short-term debt, long-term debt, convertible debt, and equity). KMV solves for firm value and volatility.

The primary advantage of the KMV model is the use of current equity values in the model. This allows for the impact of a current event to immediately affect the probability of default. Ratings changes occur with a considerable lag. The use of equity values allows for probabilities of default to change continuously as equity values change. In the CreditMetrics approach, the value of the firm can change without any impact on the probability of default.

The KMV model computes the expected return from a variation of the capital asset pricing model (CAPM), which uses a factor model to simplify the correlation structure of firm returns. This provides for a direct estimation of the loss distribution without requiring the use of simulation to estimate the credit VaR of the credit portfolio.

CreditPortfolioView

CreditPortfolioView models the transition matrices using macroeconomic or economic cycle data. This is its primary distinguishing feature. Macroeconomic variables are the key drivers of default rates, and CreditPortfolioView estimates an econometric model for an index that drives the default rates of an industrial sector. The model simulates paths of the index, which produces a distribution of portfolio losses to analyze. Usually the focus is on an aggregate default rate for an entire economy.

The user can select the inputs for the econometric model. Examples of often-used inputs are GDP growth, interest rates, and unemployment. In the United States and other countries, this data is readily available from public sources. The default rates per industry and country may not always be readily available in other countries, so proxies must be used.

The procedure can be summarized in four steps:

1. Measuring the autoregressive process of the macroeconomic variables.
2. Composing sector indices for the variables.
3. Estimating a default rate based on the value of that index.
4. Comparing the simulated values to historical values to determine the transition matrix to use.

The second and third parts of the procedure consist of simulating future possible realizations of the indices and then using an appropriate transformation (e.g., logistic transformation) on the simulations so they become a distribution of default rates. If the simulations indicate that the probability of default is above (below) the historical average, the user would conclude that the sector is in recession (expansion), and they would choose a recession (expansion) transition matrix.

Limitations of the Credit Portfolio Models

Credit portfolio models have made improvements at estimating the probability of default; however, most models do not account for changes in:

- Interest rates.
- Credit spreads.
- Current economic conditions.

The state of the economy does affect probability of default for bonds. As the economy moves from an expansionary period to a recessionary period the distribution of defaults changes. In 1991 defaults were at a peak and then declined through the 1990s as the economy expanded. The correlations increase during a recession. Credit risk models that use historical correlations are not able to account for changing economic conditions.

Credit Derivatives

LO 21.6: Assess the credit risks of derivatives.

LO 21.7: Describe a credit derivative, credit default swap, and total return swap.

A credit derivative is a contract with payoffs contingent on a specified credit event.

Credit derivatives are designed as hedging instruments for credit risks. Credit derivatives are usually traded over the counter (OTC) and not on exchanges.

Credit events include:

- Bankruptcy.
- Failure to pay.
- Restructuring.
- Repudiation.
- Moratorium.
- Obligation acceleration.
- Obligation default.

One of the simplest credit derivatives is a credit default put. A **credit default put** pays on a loss of debt due to default at the maturity of the debt claim, *T*.

Example: Computing the payoffs from a credit default put

Suppose a fixed income portfolio manager buys a bond issue with a face amount of $100 million that matures in one year. The payoff of the risky bond is the same as a portfolio of owning a 1-year Treasury bill and a short position in a put written on the bond issuer's firm value with the exercise price equal to the face value of debt. To hedge the credit risk that the issuer of the debt will not pay the full amount, the debtholder can buy a credit default put on the value of the firm with an exercise price equal to the debt's face value. What is the payoff of holding a risky bond hedge with a credit default put, if the value of the risky firm's value is $60 million?

Answer:

Figure 10: Payoffs of Risky Bond Hedged With a Credit Default Put

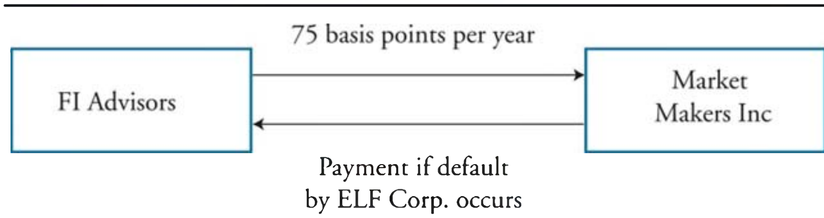
Value of the firm, *V*	60
Payoff of risk-free bond, *F*	100
Short position in put –Max(F–V,0)	–40
Risky debt is a risk-free bond + short put	60
Credit default put Max(F–V,0)	40
Hedged payoff	100

A more complex and popular credit derivative is the **credit default swap** (CDS). A CDS is similar to a typical swap in that one party makes payments to another party. The purchaser of the CDS seeks credit protection and will make fixed payments to the seller of the CDS for the life of the swap, or until a credit event occurs. This differs from a typical interest rate swap where net payments are based on some fixed and floating rates of interest. The underlying "reference" in a CDS is whether a credit event takes place. If a credit event takes place, the buyer of the CDS will make a final "accrual" payment based on the amount of time elapsed since last payment. Then the swap is settled in either physical delivery of the reference obligation, or in cash.

If the terms of the swap agreement dictate settlement by physical delivery, the buyer of the CDS delivers the reference obligation to the seller of the swap and receives the par value. If the terms of the swap agreement are for cash delivery, dealers are surveyed a specified number of days following a credit event to determine a midpoint between bid and ask prices, called Z, which will then be used to calculate the cash payment as (100 – Z)% of the notional principal.

Suppose FI Advisors owns fixed income securities issued by ELF Corp. (the reference entity issued the reference obligation) with a par value of $200 million. FI Advisors would like to protect its position against credit risk by using a credit-default swap and is able to purchase this credit protection in a credit-default swap from Market Makers, Inc., for 75 basis points of a notional principal determined to be $200 million. The life of the CDS is five years, which will require FI Advisors to pay $1.5 million to Market Makers, Inc., every year. If ELF Corp. does not default, FI Advisors receives nothing from this agreement. If ELF Corp. does default, however, Market Makers, Inc., pays FI Advisors the notional principal of $200 million. The general CDS transaction can be seen in Figure 11.

Figure 11: Example of Credit Default Swap



Total rate of return swaps (TROR) are agreements to exchange the total return of a reference asset (i.e., a risky corporate bond) for a floating rate (LIBOR) plus a specified spread. The total return of the reference asset will include both capital gains (or losses) and any flows (coupons, interest, dividends) over the life of the swap.

The total-return payer payments would be similar to those of an investment in the underlying security in exchange for LIBOR plus the spread. If the payer owns the reference asset, a total return swap would allow the owner to transfer the credit risk of the asset to the receiver. If the payer does not own the reference asset, a total return swap's cash flows would be similar to those of taking a short position in the bond. If the value of the bond declines, the payer position gains. If the value of the bond increases, the payer position loses.

Conversely, the cash flows to the receiver can be viewed as the total return on the reference asset, which is a floating rate obligation. Although the payer counterparty retains ownership of the asset, the receiver is exposed to the capital gains (or losses) and the credit risk of the asset. The spread above the floating rate the receiver is obligated to pay will depend on the credit risk of the reference asset, the creditworthiness of the receiver, and the correlation of credit quality between the reference asset issuer and the total return swap receiver.

Derivatives With Credit Risks

A **vulnerable option** is an option with default risk. An option holder receives the promised payment only if the seller of the option is able to make the payment. Without the default risk, the holder of the option at expiration receives:

$$\text{Max}(S - X, 0)$$

where:
S = underlying asset's price at expiration
X = exercise price

The vulnerable option holder receives the promised payment only if the value of the counterparty firm, *V*, is greater than the required payment on the option. The payoff of the vulnerable option is:

$$\text{Max}[\text{Min}(V, S - X), 0]$$

The correlation between the value of the firm and the underlying asset value, $\rho_{(V,S)}$, is important in the valuations of the vulnerable option. If $\rho_{(V,S)}$ is strongly negative then vulnerable option has little value because firm value is low when vulnerable option payoff is to occur. If $\rho_{(V,S)}$ is strongly positive then there is no credit risk because firm value is high when the value of equity is high.

If the option has credit risk, then a derivative contract can be written to eliminate the credit risk. If the price of the vulnerable option can be estimated then the price of the credit derivative to insure the vulnerable option can be determined. The payoff of the option used to hedge the credit risk of a vulnerable option is:

$$\text{Max}(S - X, 0) - \text{Max}[\text{Min}(V, S - X), 0]$$

An alternative approach computes the probability of default and a recovery rate estimate if default occurs. The value of the option is the weighted average of the option without default. Following this approach, the value of a vulnerable option is:

$$\text{vulnerable option} = [(1 - PD) \times c] + (PD \times RR \times c)$$

where:
c = value of the option without default
PD = probability of default
RR = recovery rate

Example: Compute the value of a vulnerable option

Suppose a firm has a debt issue with a probability of default of 10% and a recovery rate of 40%. What is the value of the vulnerable option?

Answer:

Vulnerable option value = (1 – 0.1)c + (0.1)(0.4)c = 0.90c + 0.04c = 0.94c; therefore, the vulnerable option is worth 94% of the value of the option that is free of default risk.

LO 21.8: Explain how to account for credit risk exposure in valuing a swap.

The credit risk in a swap can be reduced by requiring a margin or by netting the payments. Netting is a method where the payments are offset so that only one party needs to make a payment. The covenants of the swap agreement can affect the credit risk exposure. Suppose a counterparty that is due to receive a net payment is in default. If the swap agreement has a *full two-way payment covenant*, then the counterparty still receives the net payment. However, if the swap has a *limited two-way payment covenant*, the obligations are abolished if either party is in default. Valuing a swap can be simplified by considering a swap with only one payment.

Suppose there is only one payment to be made in a swap arrangement between Market Maker, Inc. and Risky Credit, Inc., which has no liabilities at the creation of the swap. The agreement provides for Risky Credit to receive a fixed amount, *F*, at maturity and to pay a variable amount, *S*, based on some index. The index could be based on an equity value or a floating rate. If the payments are netted, Market Maker will receive the difference between the variable payment and the fixed payment (S – F), assuming there is no default risk such that Market Maker's payoffs are:

If S < F then Market Maker pays F – S

If S > F then Market Maker receives S – F

If Risky Credit's ability to pay is uncertain (subject to default risk), then the payment to be received from Risky Credit is the smaller of (S – F) or the firm value of Risky Credit, *V*. When we consider the default risk of Risky Credit, the swap's payoff to Market Maker is:

$$(-)\text{Max}(F - S, 0) + \text{Max}[\text{Min}(S, V) - F, 0]$$

For Market Maker, the risk-free counterparty, the payoff of the swap is the same as a portfolio of a short position on a put option and a long position on a call. The put is written on an asset with a value of *S* and an exercise price of *F*. The call option is written on the lower of the variable payment, *S*, or the value of the risky counterparty, *V*, with the exercise price of the fixed payment, *F*. At the initiation of the swap agreement, *F* is selected so that the swap has no value.

The correlation between firm value, *V*, and the variable payment, *S*, is critical to the valuation of the swap. If the correlation declines, then there is no effect on the value of the put option, but the value of the option on the two risky assets declines.

Key Concepts

LO 21.1

Credit risk is the chance that a party will fail to make promised payments. The two important roles that credit risk plays in risk management programs are (1) assessing the potential of default by debt claimants and (2) assessing the potential of default by counterparties of derivatives contacts.

Given the assumptions of the Merton model, a levered firm's equity can be valued as a call option written on the value of the firm with the face value of debt as the exercise price and the time to the debt's maturity as the time to expiration. The value of equity is an increasing function of firm value, time to maturity of debt, interest rates, and volatility of the firm and a decreasing function of the face value of debt.

The value of debt, in the Merton model, is the difference between the firm value and the call option written on the value of the levered firm. The value of debt is an increasing function of firm value and the face value of debt and a decreasing function of the time to maturity of debt, interest rates, and the volatility of firm value.

LO 21.2

A credit spread is the difference between the yield on a risky bond the yield on a risk-free bond given that the two instruments have the same maturity. As time to maturity increases, credit spreads tend to widen. A reduction in the risk of default will narrow credit spreads.

LO 21.3

An extension of the Merton model provides for the pricing of subordinate debt. When a firm is experiencing financial distress (low firm values), the behavior of the values of subordinate debt are more similar to that of equity. However, if a firm is not experiencing financial distress (high firm values), the behavior of the values of subordinate debt resembles senior debt.

LO 21.4

The sensitivity of the value of debt to changes in interest rates is dependent on the volatility of interest rates. When interest rate volatility is high, the debt values are less sensitive to changes in interest rates.

LO 21.5

Credit risk models have made improvements at estimating the probability of default. CreditRisk+ measures the credit risk of a portfolio conditional on a set of common risk factors for each obligor. CreditMetrics measures the credit VAR for the portfolio. The KMV model, a modified Merton model, calculates the expected default frequencies (EDFs) for each obligor.

LO 21.6

A credit derivative is a derivatives contract with payoffs contingent on a specified credit event. Credit events include the following: failure to make required payments, restructuring that harms the creditor, invocation of cross-default clause, and bankruptcy.

LO 21.7

A credit default put is a credit derivative that pays on a loss of debt due to default at the maturity of the debt claim.

In a credit default swap (CDS), the party with the credit exposure from a debt claim will make fixed payments to a counterparty. The counterparty then agrees to pay the shortfall if the obligor is not able to meet the requirements of the debt contract (i.e., credit event). A credit event triggers the payment by the counterparty.

A total return swap involves swapping the total return from a debt obligation in exchange for a specified payment. The lending party who wants to hedge its credit risk exposure agrees to pay the interest payments and any decline in the market value of the debt instrument and receives a risk-free variable rate payment (usually based on LIBOR).

LO 21.8

In a risky swap agreement, the correlation between a risky counterparty's firm value and the variable payment, is critical to the valuation of the swap. If the correlation declines, then there is no effect on the value of the put option, but the value of the option on the two risky assets declines.

Concept Checkers

1. The role that credit risk plays in risk management programs includes assessing the potential default by:

   I. debt claimants.
   II. counterparties of a swap agreement.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

2. A non-dividend paying firm financed with 100% equity issues a zero-coupon bond with a principal amount of $50 million due in three years. What are the values of the different components of the firm's capital structure at the maturity date of the bond if the firm value at that time is $40 million?
   A. $50 million in debt and $10 million in equity.
   B. $10 million in debt and $30 million in equity.
   C. $50 million in debt and $40 million in equity.
   D. $40 million in debt and $0 in equity.

3. Suppose a firm has two debt issues outstanding. One is a senior debt issue that matures in three years with a principal amount of $100 million. The other is a subordinate debt issue that also matures in three years with a principal amount of $50 million. The annual interest rate is 5% and the volatility of the firm value is estimated to be 15%. If the volatility of the firm value declines in the Merton model, then which of the following statements is true?
   A. If the firm is experiencing financial distress (low firm value), then the value of senior debt will increase while the values of subordinate debt and equity will both decline.
   B. If the firm is not experiencing financial distress (high firm value), then the value of senior debt and subordinate debt and equity will increase.
   C. If the firm is experiencing financial distress (low firm value), then the value of senior debt and subordinate debt will increase while equity values will decline.
   D. If the firm is not experiencing financial distress (high firm value), then the value of senior debt will increase while the values of subordinate debt and equity will both decline.

4. Which of the following statements regarding the Merton model is true?
   A. A firm with numerous debt issues that mature at different times is easy to value with the Merton model.
   B. The Merton model assumes a lognormal distribution and constant variance for changes in firm value.
   C. The Merton model is able to predict default because it allows for default surprises (i.e., jumps).
   D. Empirical results indicate that the Merton model is able to predict default better than naïve models for investment grade bonds.

5. Which of the following is a characteristic of the KMV model?

    I. Each obligor has its own sensitivity to each of the common risk factors.
    II. It includes an estimate of correlation between firm values based on the correlation between observed equity values.

    A. I only.
    B. II only.
    C. Both I and II.
    D. Neither I nor II.

Concept Checker Answers

1. C Both statements are true. The two important roles that credit risk plays in risk management programs are (1) assessing the potential of default by debt claimants, and (2) assessing the potential of default by counterparties of derivatives contacts.

2. D The value of equity is the value of a call on the value of the firm with an exercise price equal to the face value of the zero-coupon bond, $S_T = \text{Max}(V_T - F, 0) = \text{Max}(40 - 50, 0) = 0$ (i.e., equity has no value). The value of debt is $D_T = F - \text{Max}(F - V_T, 0)$ or alternatively, $D_T = V_T - S_T$. Therefore, the value of debt is $40 - 0 = 40 = \$40$ million.

3. A When firms with subordinate debt are experiencing financial distress (low firm values), changes in the value of subordinate will react to changes in the model parameters in the same way as equity. Since equity is valued as a call option in the Merton model, a decline in volatility will reduce the value of equity (and subordinate debt). When firms with subordinate debt are not experiencing financial distress (high firm values), changes in the value of subordinate will react to changes in the model parameters in the same way as senior debt. Since senior debt is valued as the difference in firm value less equity valued as a call option in the Merton model, a decline in volatility will increase the value of senior debt (and subordinate debt).

4. B Most firms have a variety of debt instruments that mature at different times and have many different coupon rates (i.e., not just zero-coupons as assumed by the Merton model); therefore, Choice A is false. The Merton model assumes that the underlying asset follows a lognormal distribution with constant variance; therefore, Choice B is true. The Merton model does not allow the firm value to jump. Since most defaults are surprises, the inability to have jumps in the firm value in the Merton model makes default too predictable; therefore, Choice C is false. Jones, Mason, Rosenfeld (1984) report that a naïve model of predicting that debt is riskless works better for investment grade bonds than the Merton model. However, the Merton model works better than the naïve model for debt below investment grade; therefore, Choice D is false.

5. B Statement I is only true for CreditRisk+. Statement II is a characteristic and major advantage of the KMV model.

The following is a review of the Credit Risk Measurement and Management principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Spread Risk and Default Intensity Models

Topic 22

Exam Focus

Investors require a return for bearing credit risk, which is typically expressed relative to risk-free rates (e.g., yield spread, OAS, CDS spread). Default can be modeled with simple Bernoulli trials or more complicated intensity (hazard) models. For the exam, know the relationship between hazard rates, cumulative default probability, and conditional default probability. Also know that the credit spread is approximately equal to loss given default times probability of default.

Spread Conventions

LO 22.1: Compare the different ways of representing credit spreads.

LO 22.2: Compute one credit spread given others when possible.

Informally, a credit spread represents the difference in yields between the security of interest (e.g., corporate bond) and a reference security (typically a higher rated instrument). Ideally, these two securities would have the same maturity, so the difference in yields represents the difference in risk premiums, not compensation for the time value of money. As intuitive and attractive as this definition is, unfortunately, it can be interpreted in many different ways. Figure 1 summarizes various spread measures.

Figure 1: Various Spread Measures

Spread Measure	*Definition*
Yield spread	YTM risky bond – YTM benchmark government bond
i-spread	YTM risky bond – linearly interpolated YTM on benchmark government bond
z-spread	Basis points added to each spot rate on a benchmark curve
Asset-swap spread	Spread on floating leg of asset swap on a bond
CDS spread	Market premium of CDS of issuer bond
Option adjusted spread (OAS)	*z*-spread adjusted for optionality of embedded options. *z*-spread = OAS if no option
Discount margin	Fixed spread above current LIBOR needed to price bond correctly

The more common spread definitions (yield spread, i-spread, and *z*-spread) are demonstrated in the following examples.

Example 1: Assume the following information regarding XYZ Company and US Treasury yields.

	XYZ	*US Treasury*
Coupon rate	6% semi-annual coupon	4% semi-annual coupon
Time to maturity	20 years (7.25% YTM)	20 years (4.0% YTM)
Yield curve		4.0% flat

Based on the above information, yield spread = 7.25% – 4% = 3.25% (325 basis points)

Example 2: Assume the following information regarding XYZ Company and US Treasury yields.

	XYZ	*US Treasury*
Coupon rate	6% semi-annual coupon	4% semi-annual coupon
Time to maturity	19 years (7.25% YTM)	20 years (4.0% YTM) 18 years (3.6% YTM)
Yield curve		4.0% flat

Because the maturity of the XYZ bond does not match exactly with the maturity of the quoted Treasury bonds, the i-spread will be computed as:

i-spread = 7.25% – (4.0% + 3.6%) / 2 = 3.45%

Example 3: For this example, we consider the calculation of the *z*-spread in a continuous time framework. XYZ bond is trading at a 6% discount (94% of par) with an 8% semi-annual coupon and 10 years to maturity. Assume a flat swap curve at 10% and a spot rate of 9.6% compounded continuously for all maturities. The *z*-spread is calculated using the following expression:

$$0.94 = \left(\frac{0.08}{2}\right)\sum_{i=1}^{10\times 2} e^{-(0.096+z)0.5} + e^{-(0.096+z)10}$$

Spread '01

LO 22.3: Define and compute the Spread '01.

Recall the concept of DV01, the dollar value of a basis point, from the FRM Part I curriculum. DV01 captures the dollar price change from a one basis point change in the current yield. A similar concept for credit spreads is known as DVCS (i.e., spread '01). Here, the potential change in the bond price is estimated from a one basis point change in the *z*-spread. Specifically, the *z*-spread is shocked 0.5 basis points up and 0.5 basis points down and the difference is computed.

For example, if the current *z*-spread is 207 bps and the bond is priced at $92, we could consider incremental 0.5 basis point changes to compute the spread '01. When the *z*-spread is increased by 0.5 basis points to 207.5 bps, the new bond price is $91.93 and when the *z*-spread tightens by 0.5 basis points to 206.5 bps, the bond price increases to $92.14. Hence, given a $100 par value, the spread '01 is: 92.14 – 91.93 = 0.21 dollars per basis point.

We can further study the comparative statistics of this result. Intuitively, the smaller the *z*-spread, the larger the effect on the bond price (i.e., the greater the credit spread sensitivity). This result is straightforward because the same one basis point change represents a larger shock relative to the current *z*-spread when the *z*-spread is low. Thus, the DVCS exhibits convexity.

Binomial Distribution

LO 22.4: Explain how default risk for a single company can be modeled as a Bernoulli trial.

A Bernoulli trial is an experiment or process where the outcome can take on only two values: success or failure (i.e., a binomial distribution). Success and failure are relative terms that denote that either the event happens ("success") or does not happen ("failure"). The obvious connection to our discussion is that a firm does or does not default during a particular time period. Let us define the relevant time period as $T_2 - T_1 = \tau$ where the firm will default with probability π and remains solvent with probability $1 - \pi$. The mean and variance of a Bernoulli distribution is equal to π and $\pi(1 - \pi)$, respectively.

An important property of the Bernoulli distribution is that each trial is conditionally independent. That is, the probability of default in the next period is independent of default in any previous period. Hence, if a firm has survived until the current period, the probability of default in the next period is the same as in its first year of existence. This memoryless property is exactly the same as studying a series of coin flips. For example, if you observed that a fair coin has landed on heads 10 consecutive flips, then the best guess for another heads on the 11th flip is still 50%.

Do not confuse this concept with the cumulative probability of default. Consider a firm with probability π of default each period. The likelihood of surviving the next eight periods is $(1 - \pi)^8$. We can clearly see that as long as a firm has a positive probability of default, it will eventually default in a sufficiently long period of time.

Exponential Distribution

LO 22.5: Explain the relationship between exponential and Poisson distributions.

The exponential distribution is often used to model waiting times such as how long it takes an employee to serve a customer or the time it takes a company to default. The probability density function for this distribution is as follows:

$$f(x) = \frac{1}{\beta} \times e^{-x/\beta}, x \geq 0$$

In the above function, the scale parameter, β, is greater than zero and is the reciprocal of the "rate" parameter λ (i.e., $\lambda = 1 / \beta$). The rate parameter measures the rate at which it will take an event to occur. In the context of waiting for a company to default, the rate parameter is known as the **hazard rate** and indicates the rate at which default will arrive.

As mentioned, the exponential distribution is able to assess the time it takes a company to default. However, what if we want to evaluate the total number of defaults over a specific time period? As it turns out, the number of defaults up to a certain time period, N_t, follows a Poisson distribution with a rate parameter equal to t / β.

A **Poisson random variable** X refers to the *number of successes per unit*, the parameter lambda (λ) refers to the average or *expected number of successes per unit*. The mathematical expression for the Poisson distribution for obtaining X successes, given that λ successes are expected, is:

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

We can further examine the relationship between the exponential and Poisson distributions by considering the mean and variance of both distributions. The mean and variance of a Poisson distributed random variable is equal to λ and, as it turns out, the mean of the exponential distribution is equal to $1 / \lambda$ and the variance is equal to $1 / \lambda^2$.

Hazard Rates

LO 22.6: Define the hazard rate and use it to define probability functions for default time and conditional default probabilities.

LO 22.7: Calculate the conditional default probability given the hazard rate.

The hazard rate (i.e., default intensity) is represented by the (constant) parameter λ and the probability of default over the next, small time interval, *dt*, is λdt. Stated differently, the probability of default over $(t, t + dt) = \lambda dt$. It follows naturally that the probability of survival over the same time interval *dt* is $1 - \lambda dt$.

If the time of the default event is denoted t*, the cumulative default time distribution $F(t)$ represents the probability of default over (0, t):

$$P(t^* < t) = F(t) = 1 - e^{-\lambda t}$$

Professor's Note: This equation calculates the cumulative probability of default (cumulative PD), which is an unconditional default probability.

Similarly, the survival distribution is $P(t^* \geq t) = 1 - F(t) = e^{-\lambda t}$ and both survival and default probabilities sum to 1 at each point in time. In other words, if you have not defaulted by time t, then you have survived until this point. As t increases, the cumulative default probability approaches 1 and the survival probability approaches 0.

For completeness, we provide the marginal default probability (or default time density) function as the derivative of $F(t)$ with respect to the variable t:

$$\lambda e^{-\lambda t}$$

It is evident that this quantity is always positive indicating that the probability of default increases over time related to the intensity parameter λ.

Previously, the exponential function was used to model the default probability over (0, t). If we examine the probability of default over $(t, t + \tau)$ given survival up to time t, the function is a conditional default probability. The instantaneous conditional default probability (for small τ) is equal to $\lambda\tau$.

The conditional one-year probability of default, assuming survival during the first year, is equal to the difference between the unconditional two-year PD and the unconditional one-year PD, divided by the one-year survival probability. The following example demonstrates this calculation.

Example: Computing default probabilities

Given a hazard rate of 0.15, **compute** the one, two, and three-year cumulative default probabilities and conditional default probabilities.

Answer:

t	*Cumulative PD*	*Survival Probability*	*PD(t, t+1)*	*Conditional PD Given Survival Until Time t*
1	$1 - e^{-0.15(1)}$ = 0.1393	1 – 0.1393 = 0.8607	0.1393	
2	$1 - e^{-0.15(2)}$ = 0.2592	1 – 0.2592 = 0.7408	0.2592 – 0.1393 = 0.1199	0.1199 / 0.8607 = 0.1393
3	$1 - e^{-0.15(3)}$ = 0.3624	1 – 0.3624 = 0.6376	0.3624 – 0.2592 = 0.1032	0.1032 / 0.7408 = 0.1393

Notice that the conditional probabilities in the far right column are constant.

Risk-Neutral Hazard Rates

LO 22.8: Calculate risk-neutral default rates from spreads.

In structural models, such as the Merton model, the default probabilities are based on specific pricing functions associated with the firm's assets and liabilities (in essence, structural models implicitly assume the modeler has as much information about the firm as the firm's managers). On the other hand, reduced form models will take the market price of liquid securities such as a credit default swap (CDS) as fairly priced and back out the market's aggregated expectations of default. To calculate risk-neutral default rates from spreads we are interested in working with reduced form models, which start with market observable spreads.

Let's start by comparing zero-coupon corporate bonds to maturity-matched default-free government bonds. Since the only cash flows occur at maturity, the current prices differ based on their yields. Specifically, the price of a default-free bond maturing in τ is:

$$p_\tau = e^{-r_\tau \tau}$$

where:
r_τ = continuous discount rate

Similarly, the price of a risky (corporate) bond with spread z_τ relative to the default-free bond with maturity τ is expressed as:

$$p_\tau^{corp} = e^{-(r_\tau + z_\tau)\tau} = p_\tau e^{-z_\tau \tau}$$

If there is no default, the price between the corporate and default-free bond converge to par over τ. In case of default, creditors will recover a fraction of par, which is the recovery rate denoted as RR ($0 \le RR \le 1$).

As a simplifying assumption, suppose there will be no recovery of assets in default (i.e., RR = 0). Therefore, the corporate bond investor receives \$1 (par) if no default and \$0 if there is a default. On average, the expected value is:

$$e^{-\lambda_\tau^* \tau} \times 1 + \left(1 - e^{-\lambda_\tau^* \tau}\right) \times 0$$

On a present value basis discounting at risk-free rate generates:

$$e^{-r_\tau \tau}\left[e^{-\lambda_\tau^* \tau} \times 1 + \left(1 - e^{-\lambda_\tau^* \tau}\right) \times 0\right]$$

The final step is to equate this present value expression to the risky bond price and solve for λ^*_τ:

$$e^{-(r_\tau + z_\tau)\tau} = e^{-r_\tau \tau}\left[e^{-\lambda_\tau^* \tau} \times 1 + \left(1 - e^{-\lambda_\tau^* \tau}\right) \times 0\right]$$

Solving for the risk-neutral hazard rate when the recover rate is zero shows that $\lambda^*_T = z_T$. Thus, the interpretation of this analysis is that the credit spread (*z*-spread) is the hazard rate.

When we introduce a positive recovery rate, the analysis changes slightly:

$$e^{-(r_T+z_T)T} = e^{-r_T T}\left[e^{-\lambda^*_T T} \times 1 + \left(1 - e^{-\lambda^*_T T}\right) \times RR\right]$$

After solving for the risk-neutral hazard rate, we end up with the following approximation:

$$\lambda^*_T \approx \frac{z_T}{1 - RR}$$

Stated differently, the loss given default (1 – RR) times the default probability (hazard rate) is approximately equal to the credit spread (*z*-spread).

Example: Computing hazard rate

The three-year CDS on Bloomington Minerals and Mining has a spread of 400 basis points. The underlying nature of the business contains specialized equipment that has a limited resale potential. Thus, a credit analyst projects a 20% recovery rate in default. **Calculate** the hazard rate.

Answer:

$$\lambda^*_T \approx \frac{0.04}{1 - 0.2} = 0.05$$

LO 22.9: Describe advantages of using the CDS market to estimate hazard rates.

The primary advantage of using CDS to estimate hazard rates is that CDS spreads are observable. Although we can create a model for the hazard rate (the probability of default in the next period conditional on surviving until the current period), the estimated value would inherently be a guess. Instead, we can draw on the logic of a reduced form model to use the observable, liquid CDS to infer the estimates of the hazard rate.

Our previous analysis on estimating hazard rates did not fully capture the complexities of the bond market. First, published estimates of default probabilities are insufficient as they are typically provided for a one-year horizon which may not match the duration of the analysis. Second, few corporations issue zero-coupon bonds. One can view commercial paper as *de facto* zero-coupon bonds but the issuing universe is restricted to large, highly-rated corporations. CDS can overcome these difficulties because liquid contracts exist for several maturities (e.g., 1, 3, 5, 7 and 10 years are common). Furthermore, a large number of liquid CDS curves are available (800 in U.S. markets, 1,200 in international markets) and the contracts are more liquid than the underlying cash bonds (i.e., narrower spreads and more volume).

Hazard Rate Curves

LO 22.10: Explain how a CDS spread can be used to derive a hazard rate curve.

Constructing the hazard rate curve uses a bootstrapping methodology not that different from bootstrapping a yield curve (moving from coupon yield curve to zero-coupon yield curve). The CDS spreads provide several discrete maturities to extract hazard rates. We know from casual observation that the CDS curves can take a variety of shapes so the constant hazard rate assumption from before is not likely to hold in practice. Technically, hazard rates are measured every instant in time so the CDS data will only provide a few observable data points and will require some form of interpolation or piecewise construction to complete the curve.

Intuitively, the hazard rate can vary over time. We can represent the time-varying hazard rate as $\lambda(t)$. When the hazard rate varies, the probability of default becomes:

$$\pi_t = 1 - e^{-\int_0^t \lambda(s)ds}$$

For the special case when the hazard rate is constant (i.e., $\lambda(t) = \lambda$ for all t), then the PD expression simplifies to $\pi_t = 1 - e^{-\lambda t}$. For practical purposes, the hazard rates used in default models are not constant but will not vary each instant in time either. Therefore, using CDS speads is a reasonable compromise to accommodate changing hazard rates at discrete points in time.

Using the common CDS maturities (1, 3, 5, 7 and 10 years), the time-varying hazard rate function can be expressed in general form. In the following expression, five piecewise constant hazard rates are determined from observed CDS spreads.

$$\lambda(t) = \begin{Bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \end{Bmatrix} \quad \text{for} \quad \begin{Bmatrix} 0 < t \le 1 \\ 1 < t \le 3 \\ 3 < t \le 5 \\ 5 < t \le 7 \\ 7 < t \end{Bmatrix}$$

Subsequently, the integral in the above probability of default equation is determined as:

$$\int_0^t \lambda(s)ds = \begin{Bmatrix} \lambda_1 t \\ \lambda_1 + (t-1)\lambda_2 \\ \lambda_1 + 2\lambda_2 + (t-3)\lambda_3 \\ \lambda_1 + 2\lambda_2 + 2\lambda_3 + (t-5)\lambda_4 \\ \lambda_1 + 2\lambda_2 + 2\lambda_3 + 2\lambda_4 + (t-7)\lambda_5 \end{Bmatrix} \quad \text{for} \quad \begin{Bmatrix} 0 < t \le 1 \\ 1 < t \le 3 \\ 3 < t \le 5 \\ 5 < t \le 7 \\ 7 < t \end{Bmatrix}$$

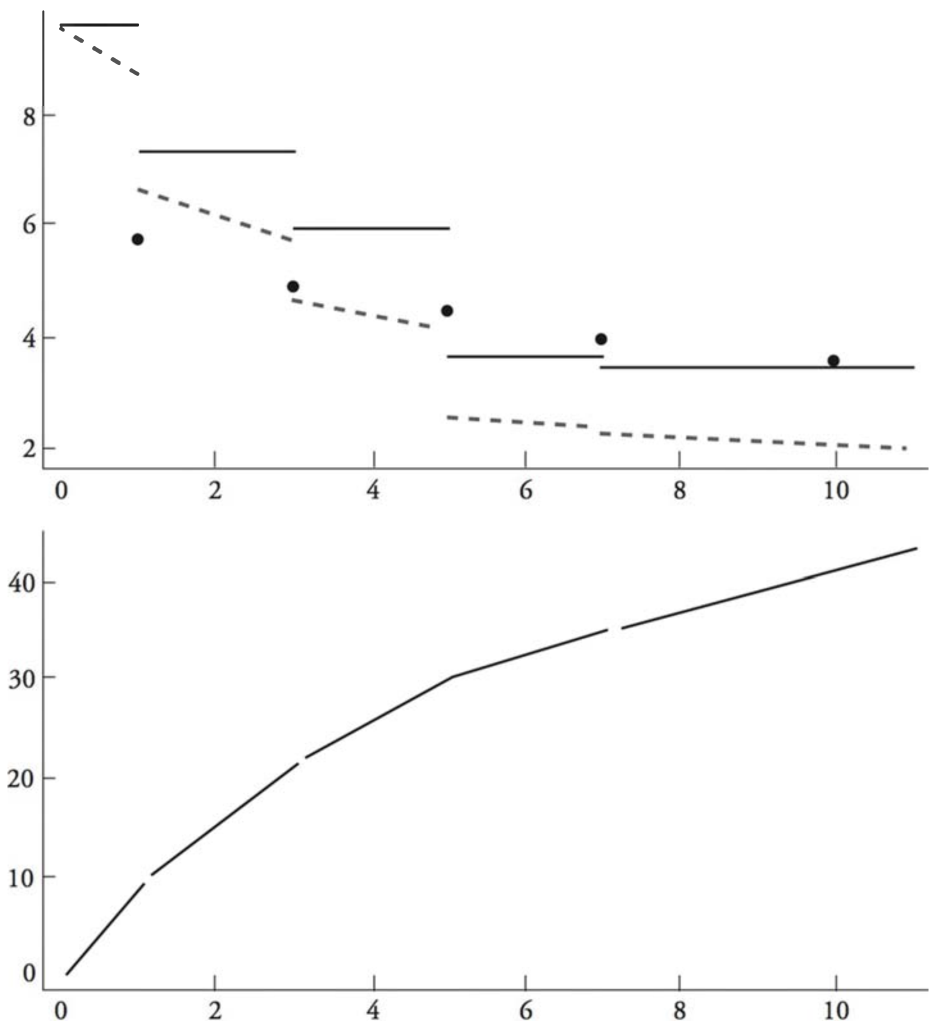
Previously, the hazard rate was extracted from the default probability. This process relied on the simple idea that (PV of expected payments in default) = (PV of expected premiums paid). At CDS swap initiation no cash transfer takes place, but afterward if there is a change in credit quality, say, an improvement, the protection seller's position gains while the protection buyer's position loses (buyer is locked into paying too much for protection based on current conditions).

The fact that the CDS swap spread is observable allows for the inference of default probability for the 1-year maturity by equating (PV of expected payments in default) and (PV of expected premiums paid). Thus, given an assumed recovery rate (usually 40%), the probability of default and, hence, the hazard rate can be inferred for the first period (using the first piecewise portion of the earlier hazard function).

The bootstrapping procedure is then employed so that the hazard rate for the first period is used to infer the hazard rate for the second period from the piecewise function (using the observable information from the second CDS contract with a 3-year maturity, a recovery rate assumption, and the swap curve). Similarly, the hazard rate from the second period is an input to find the hazard in the third period, and so on. In this fashion, a graph can be constructed showing the CDS spreads, hazard rates, and default density.

CDS spreads (single points), the hazard rate curve (solid line), and default density (dashed line) are shown in the top graph in Figure 2. The default distribution, with a discontinuous slope when moving between hazard rates, is shown in the bottom graph in Figure 2.

Figure 2: Default Curve Estimation



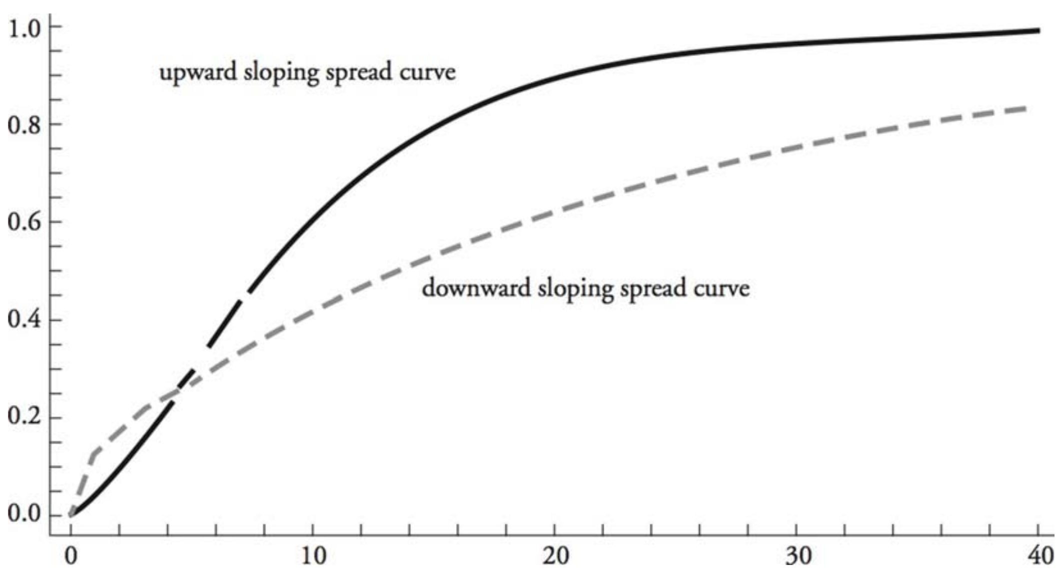
LO 22.11: Explain how the default distribution is affected by the sloping of the spread curve.

In order to explain how the slope of the spread curve affects the default distribution, it is useful to think of the term structure of the CDS spread curves. As a benchmark, consider the impact of spreads that are constant for all maturities, that is, the market's expectations for default is constant. In this case, the spread curve would be flat implying the probability of defaulting in the near term is the same as defaulting in the long run.

The most common spread curve is upward sloping. Thus, the aggregate market forecast is that default is unlikely in the near term but increases with the forecast period. In contrast, spread curves, although unusual, may be downward sloping. This phenomenon would indicate relatively high expectations of short-term default (distress) but, if the firm can right itself, it will likely survive for a sufficiently long period of time. Therefore, the longer-term spreads are lower than the short-term spreads. This situation is similar to an inverted yield curve where short-term rates are extremely high (likely from high, short-term inflation) but are expected to moderate to a more natural, lower rate in the future.

On a relative basis, a downward-sloping spread curve (dotted line) has a steeper default distribution than an upward-sloping spread curve (solid line), because the cumulative default for a short horizon is higher. Similarly, for intermediate and longer terms, the cumulative default distribution of the downward-sloping spread curve is flatter as the probability of default decreases after surviving the short term.

Figure 3: Default Distribution



Spread Risk

LO 22.12: Define spread risk and its measurement using the mark-to-market and spread volatility.

Spread risk is the risk of loss from changes in the price of securities that have a positive probability of default. By extension, the spread risk for Treasury securities is zero for any maturity. Recall that at the initiation of a CDS, the parties have implicitly agreed on a fair swap spread so that the expected payments to both are equal. Thus, no cash trades hands up-front. Rather, the protection buyer is exposed to the narrowing of spreads if the creditworthiness improves as it has essentially agreed to now-above-market premiums. Naturally, a loss that is experienced by the protection buyer (i.e., fee payer) represents a gain to the protection seller (i.e., contingent payer). Similarly, the protection seller suffers when spreads widen as it has agreed to provide contingent compensation in case of default at a rate that is now viewed as too low.

To measure spread risk, the mark-to-market of a CDS and spread volatility can be used. The mark-to-market effect is computed by shocking the entire CDS curve up and down by 0.5 basis points (similar to spread '01). Note the slight difference from spread '01 where the *z*-spread, a single value, was shocked. Thus, the entire CDS curve moves up and down by a parallel amount. An alternative measure of spread risk is to compute the volatility (standard deviation) of spreads. The spread volatility can use historical data or can be forward-looking based on a subjective probability distribution. Not surprisingly, the spread volatility spiked extremely high during the recent financial crisis for many financial services firms.

Key Concepts

LO 22.1

A credit spread represents the difference in yields between the security of interest (e.g., corporate bond) and a reference security (typically a higher rated instrument). There are several different ways to capture the concept of spread including: yield spread, i-spread, *z*-spread, asset-swap spread, CDS spread, OAS, and discount margin.

LO 22.2

The yield spread represents the difference between yield on the subject instrument and maturity-matched benchmark yield.

The i-spread (interpolated spread) uses linear interpolation when maturities do not match up precisely.

The CDS spread is the premium (percent of par) to protect against credit event.

The *z*-spread and OAS are computed rather than observed spreads. The *z*-spread is based on a hypothetical parallel shift of the benchmark curve to match the observed bond price. OAS is similar, but accommodates interest rate volatility and must be used for bonds with embedded options. *z*-spread = OAS when there are no embedded options.

LO 22.3

Analogous to DV01, the spread '01 computes the price change from a one basis point change in the *z*-spread. Computationally, the *z*-spread is increased and decreased by 0.5 basis points and the difference in resulting prices is the spread '01. This measure exhibits convexity, so as the spread increases, the marginal change in the spread '01 decreases.

LO 22.4

Bernoulli trials identify events as success (no default) and failure (insolvency) in each trial. The cumulative probability of default increases and, in the limit, all firms eventually default. The default process is memoryless – default in period (T_i, T_{i+1}) given no default until period i, is the same probability as default in the next period.

LO 22.5

Modeling defaults can be done by modeling the time to the next event using an exponential distribution. The number of defaults up to a certain time period follows a Poisson distribution.

LO 22.6

The hazard rate (or default intensity, λ) describes the likelihood of failure (default). When the hazard rate is constant, the arrival of the next default follows exponential distribution where cumulative default = $1 - e^{-\lambda t}$ and marginal probability of default is $\lambda e^{-\lambda t}$. This implies that the cumulative probability is increasing and all firms will eventually fail, and that the marginal probability of failure is decreasing.

LO 22.7

The conditional default probability computes the probability of default in the next period (distance of τ) given survival until the current period. If the hazard rate is constant over a very short time interval, then the conditional PD is calculated as $\lambda\tau$.

LO 22.8

Intuitively, risk-neutral default rates can be inferred from spreads. Given a fixed recovery rate and observable spread, the probability of default (hazard rate) can be approximated as:

$$\lambda_\tau^* \approx \frac{z_\tau}{1 - RR}$$

LO 22.9

CDS spreads are useful for estimating hazard rates because they are liquid, span multiple maturities and are standardized. These spreads provide more information about market expectations of default than typical default forecasts over the next period.

LO 22.10

More complex hazard rate models assume the hazard rate is time-varying.

The probability of default is used to back out the hazard rate for the first period (assuming the hazard rate is time-varying). The process is repeated for the next maturity (via bootstrapping) to estimate the next probability of default and hazard rate. This piecewise process is continued for several observable CDS maturities.

A default distribution curve can be constructed from a hazard rate curve. The slope of the default distribution curve will be discontinuous to reflect the movement from one hazard rate to the next.

LO 22.11

Spread curves may be upward sloping (typical shape) or downward sloping (short-term distress, but decreasing default risk if it can survive in the short term). Upward-sloping spread curves generate cumulative default distributions that are flatter in the short term but steeper afterward. Downward-sloping spread curves are steeper in the short term as the near probability of default is higher but then moderates to a flatter curve afterward.

LO 22.12

Spread risk is the change in value of risky securities from changing spreads. Similar to DV01 and spread '01 calculations, the entire CDS curve is shocked up and down by 0.5 basis points to compute the CDS mark-to-market value. Spread risk can also be measured using the historical or forward-looking standard deviation of credit spreads.

Concept Checkers

1. Which of the following statements is correct regarding spread measures?
   A. The yield spread and i-spread are equal if the benchmark yield curve is flat.
   B. The *z*-spread = OAS for callable bonds.
   C. The *z*-spread must be used for mortgage-backed securities (MBS).
   D. The CDS spread is used only with corporate bonds.

2. An analyst has noted that the default frequency in the pharmaceutical industry has been constant at 8% for an extended period of time. Based on this information, which of the following statements is most likely correct for a randomly selected firm following a Bernoulli distribution?

   I. The cumulative probability that a randomly selected firm in the pharmaceutical industry will default is constant.
   II. The probability that the firm survives for the next 6 years without default is approximately 60%.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

3. An analyst has gathered the following information about ABC Inc. and DEF Inc. The respective credit ratings are AA and BBB with 1-year CDS spreads of 200 and 300 basis points each. The associated probabilities of default based on published reports are 10% and 20%, respectively. Which of the following statements about the recovery rates is most likely correct?
   A. The market implied recovery rates are equal.
   B. The market implied recovery rate is higher for ABC.
   C. The market implied recovery rate is lower for ABC.
   D. The loss given default is higher for DEF.

4. Which of the following statements best explains the relationship between CDS spreads and hazard rates?
   A. Hazard rates are observable and can be used to infer credit spreads from backward induction.
   B. Credit spreads are observable and can be used to infer hazard rates from backward induction.
   C. Hazard rates are observable and can be used to infer credit spreads from bootstrapping.
   D. Credit spreads are observable and can be used to infer hazard rates from bootstrapping.

5. An analyst is studying the CDS spread curve for an established company. The 1-, 3- and 5-year spreads are 400 bps, 200 bps, and 150 bps, respectively. Which of the following interpretations of the data is most likely correct for the shape of the default distribution?

	Default Distribution	Near-Term Slope
A.	Upward sloping	flat slope
B.	Downward sloping	steep slope
C.	Upward sloping	steep slope
D.	Downward sloping	flat slope

Concept Checker Answers

1. A If the yield curve is flat, there is no need for interpolation. Therefore, yield spread = i-spread. *z*-spread > OAS for callable bonds. OAS must be used for MBS. CDS measures the credit risk from any security with positive probability of default including sovereign and municipal bonds.

2. B Statement I is false because the cumulative probability of default increases (i.e., even the highest rated companies will eventually fail over a long enough period). Statement II is true since the probability the firm survives over the next 6 years without default is: $(1 - 0.08)^6$ = 60.6%

3. C The approximation of credit spread = (1 – RR) × (PD). This implies:

   ABC: 200 bps = (1 – RR)(10%), so RR = 80%

   DEF: 300 bps = (1 – RR)(20%), so RR = 85%

   Thus, the market implied recovery rate is lower for ABC. Using loss given default terminology, LGD for ABC = 20% and LGD for DEF = 15%.

4. D Credit spreads are observable and, when used in conjunction with observed discount rates on swaps and the presumed recovery rate, the probability of default over the specific maturity can be inferred. The probability of default can, in turn, infer the hazard rate for the first period. Using the bootstrapped hazard rate from period 1, the second period hazard rate can be inferred using the same procedure with observable data corresponding to the longer maturity.

5. C The CDS spreads indicate a downward sloping spread curve. Note that the cumulative distribution of default is always increasing regardless of the slope of the spread curve. In addition, since the short-term probability of default is relatively high, the slope in the near term of the default distribution function is relatively steep.

The following is a review of the Credit Risk Measurement and Management principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Portfolio Credit Risk

Topic 23

Exam Focus

In this topic, we discuss the role that default correlation plays in measuring the default risk for a credit portfolio. For the exam, be prepared to list drawbacks of using default correlation and explain the single-factor model approach under the assumption that defaults are independent and returns are normally distributed. Know how to calculate the mean and standard deviation of the default distribution under the single-factor model conditional approach for correlations of 0 and 1 and the unconditional approach for correlations between 0 and 1. Lastly, be able to explain how VaR is determined using the single-factor model and copula methodology based on simulated terminal values.

Default Correlation for Credit Portfolios

LO 23.1: Define and calculate default correlation for credit portfolios.

Risks to consider when analyzing credit portfolios include default probability, loss given default (LGD), probability of deteriorating credit ratings, spread risk, and risk of loss through restructuring in bankruptcy. **Default correlation** measures the probability of multiple defaults for a credit portfolio issued by multiple obligors.

Suppose there are two firms whose probabilities of default over the next time horizon t are π_1 and π_2 for each firm, respectively. In addition, there is a joint probability that both firms will default over time horizon t equal to π_{12}.

The default correlation for this simple two firm credit portfolio can be framed around the concept of Bernoulli-distributed random variables x_i, that have four possible outcomes over a specific time horizon t. Figure 1 illustrates the four possible random outcomes where 0 denotes the event of no default and 1 denotes default. The random variables for firm 1 and 2 are x_1 and x_2. The probabilities of the four random events (firm 1 defaults, firm 2 defaults, both firms default, and neither firm defaults) are illustrated in Figure 1.

Figure 1: Default Probabilities for Two Firms

Event	x_1	x_2	$(x_1\, x_2)$	*Default Probability*
Firm 1 Defaults	1	0	0	$\pi_1 - \pi_{12}$
Firm 2 Defaults	0	1	0	$\pi_2 - \pi_{12}$
Both Default	1	1	1	π_{12}
No Default	0	0	0	$1 - \pi_1 - \pi_2 + \pi_{12}$

Thus, the probability that one of the firms defaults or both firms default equals: $\pi_1 + \pi_2 - \pi_{12}$. Since the probabilities of all four events must equal 1, the probability that no firm defaults is $1 - \pi_1 - \pi_2 + \pi_{12}$. The means of the two Bernoulli-distributed default processes are: $E[x_i] = \pi_i$, where i equals 1 or 2. The expected value of joint default is simply the product of the two denoted as: $E[x_1x_2] = \pi_{12}$. The variances are computed as: $E[x_i]^2 - (E[x_i])^2 = \pi_i(1 - \pi_i)$ and the covariance is computed as: $E[x_1x_2] - E[x_1]E[x_2] = \pi_{12} - \pi_1 \pi_2$.

Equation 1 defines the default correlation for a two firm credit portfolio as the covariance of firm 1 and 2 divided by the standard deviations of firm 1 and 2.

$$\rho_{12} = \frac{\pi_{12} - \pi_1\pi_2}{\sqrt{\pi_1(1-\pi_1)}\sqrt{\pi_2(1-\pi_2)}} \tag{1}$$

Example: Calculating Default Correlation

Assume a portfolio of two credits, one rated BBB+ and one rated BBB, whose probabilities of default over the next time horizon t are 0.002 and 0.003, respectively. In addition, assume there is a joint probability that both credits will default over time horizon t equal to 0.00015. **Calculate** the default correlation for this credit portfolio.

Answer:

Default correlation can be calculated using the following formula for a two-credit portfolio:

$$\rho_{12} = \frac{\pi_{12} - \pi_1\pi_2}{\sqrt{\pi_1(1-\pi_1)}\sqrt{\pi_2(1-\pi_2)}} = \frac{0.00015-(0.002\times 0.003)}{\sqrt{0.002(1-0.002)}\sqrt{0.003(1-0.003)}}$$

$$= \frac{0.000144}{\sqrt{0.001996}\sqrt{0.002991}} = \frac{0.000144}{(0.04468\times 0.05469)}$$

$$= 0.0589 \text{ or } 5.89\%$$

Credit Portfolio Framework

LO 23.2: Identify drawbacks in using the correlation-based credit portfolio framework.

A major drawback of using the default correlation-based credit portfolio framework is the number of required calculations. For example, to specify all possible outcome events in a three firm framework requires three individual firm default outcome probabilities, three two-default outcome probabilities, the three-default outcome probability, and the no default outcome probability. Thus, there are 2^n event outcomes with only $(n + 1) + [n(n - 1) / 2]$ conditions. If we have ten firms, there will be 1,024 event outcomes with 56 conditions. The number of pairwise correlations is equal to $n(n - 1)$. In modeling credit risk, the pairwise correlations are often set to a single, non-negative parameter.

In addition, certain characteristics of credit positions do not fit well in the default correlation credit portfolio model. Guarantees, revolving credit agreements, and other contingent liabilities have features similar to options that are not reflective of this simplistic framework. For example, credit default swap (CDS) basis trades may not be modeled simply by credit or market risk. Rather technical factors may play an important role as was evident in the subprime mortgage crisis where there was a lack of liquidity. Furthermore, convertible bonds have characteristics of credit and equity portfolios driven by market and credit risks.

Additional drawbacks in using the default correlation-based credit portfolio framework are related to the limited data for estimating defaults. Firm defaults are relatively rare events. Therefore, estimated correlations vary greatly depending on the data time horizon and industry. Most studies use an estimated correlation of 0.05. Thus, default correlations are small in magnitude, and the joint probability of two firms defaulting is even smaller.

Credit VaR

LO 23.3: Assess the impact of correlation on a credit portfolio and its Credit VaR.

The effects of default, default correlation, and loss given default are important determinants in measuring credit portfolio risk. A portfolio's credit value at risk (credit VaR) is defined as the quantile of the credit loss less the expected loss of the portfolio. Default correlation impacts the volatility and extreme quantiles of loss rather than the expected loss. Thus, default correlation affects a portfolio's credit VaR.

If default correlation is 1, then there are no credit diversification benefits, and the portfolio behaves as if there were just one credit position. A default correlation equal to 0 implies the portfolio is a binomial-distributed random variable because there is no correlation with other firms/credits.

Example: Computing credit VAR (default correlation = 1, number of credits = n)

Suppose there is a portfolio with a value of $1,000,000 that has *n* credits. Each of the credits has a default probability of π percent and a recovery rate of zero. This implies that in the event of default, the position has no value and is a total loss.

What is the extreme loss given default and credit VaR at the 95% confidence level if π is 2% and the default correlation is equal to 1?

Answer:

With the default correlation equal to 1, the portfolio will act as if there is only one credit. Viewing the portfolio as a binomial-distributed random variable, there are only two possible outcomes for a portfolio acting as one credit. Regardless of whether the number of credits in the portfolio, *n*, is 1, 20, or 1,000, it will still act as one credit when the correlation is 1.

The portfolio has a π percent probability of total loss and a (1 – π) percent probability of zero loss. Therefore, with a recovery rate of zero, the extreme loss given default is $1,000,000. The expected loss is equal to the portfolio value times π and is $20,000 in this example (0.02 × $1,000,000). There is a 98% probability that the loss will be 0, given the fact that π equals 2%. The credit VaR is defined as the quantile of the credit loss minus the expected loss of the portfolio. Therefore, at the 95% confidence level, the credit VaR is equal to –$20,000 (0 minus the expected loss of $20,000).

Note that if π was greater than (1 – confidence level), the credit VaR would have been calculated as $1,000,000 – $20,000 = $980,000.

Example: Computing credit VAR (default correlation = 0, number of credits = 50)

Again suppose there is a $1,000,000 portfolio with *n* credits that each have a default probability of *n* percent and a zero recovery rate. However, in this example the default correlation is 0, n = 50, and π = 0.02. In addition, each credit is equally weighted and has a terminal value of $20,000 if there is no default. The number of defaults is binomially distributed with parameters of n = 50 and π = 0.02. The 95th percentile of the number of defaults based on this distribution is 3. What is the credit VaR at the 95% confidence level based on these parameters?

Answer:

The expected loss in this case is also $20,000 ($1,000,000 × 0.02). If there are three defaults, the credit loss is $60,000 (3 × $20,000). The credit VaR at the 95% confidence level is $40,000 (calculated by taking the credit loss of $60,000 and subtracting the expected loss of $20,000).

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The term "granular" refers to reducing the weight of each credit as a proportion of the total portfolio by increasing the number of credits. As a credit portfolio becomes more granular, the credit VaR decreases. However, when the default probability is low, the credit VaR is not impacted as much when the portfolio becomes more granular.

Example: Computing credit VaR (default correlation = 0, number of credits = 1,000)

Suppose there is a \$1,000,000 portfolio with *n* credits that each have a default probability, π, equal to 2% and a zero recovery rate. The default correlation is 0 and n = 1,000. There is a probability of 28 defaults at the 95th percentile based on the binomial distribution with the parameters of n = 1,000 and π = 0.02. What is the credit VaR at the 95% confidence level based on these parameters?

Answer:

The 95th percentile of the credit loss distribution is \$28,000 [28 × (\$1,000,000 / 1,000)] The expected loss in this case is \$20,000 (\$1,000,000 × 0.02). The credit VaR is then \$8,000 (\$28,000 – expected loss of \$20,000).

Thus, as the credit portfolio becomes more granular, the credit VaR decreases. For very large credit portfolios with a large number of independent credit positions, the probability that the credit loss equals the expected loss eventually converges to 100%.

Conditional Default Probabilities

LO 23.4: Describe the use of a single factor model to measure portfolio credit risk, including the impact of correlation.

The **single-factor model** is used to examine the impact of varying default correlations based on a credit position's beta. Each individual firm or credit, *i*, has a beta correlation, β_i, with the market, *m*. Firm *i*'s individual asset return is defined as:

$$a_i = \beta_i m + \sqrt{1-\beta_i^2}\,\varepsilon_i \qquad (2)$$

where:

$\sqrt{1-\beta_i^2}$ = firm's standard deviation of idiosyncratic risk

ε_i = firm's idiosyncratic shock

Assuming that each ε_i is not correlated with other credits, each return on asset, a_i, is a standard normal variate. The correlation between pairs of individual asset returns between two firm's *i* and *j* is $\beta_i\beta_j$. The model assumes that firm *i* defaults if $a_i \leq k_i$, the logarithmic distance to the defaulted asset value that is measured by standard deviations.

An important property of the single-factor model is conditional independence. Conditional independence states that once asset returns for the market are realized, default risks are

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independent of each other. This is due to the assumption for the single-factor model that return and risk of assets are correlated only with the market factor. The property of conditional independence makes the single-factor model useful in estimating portfolio credit risk.

So, how can the single-factor model be used to measure default probabilities that are conditional on market movements or economic health? Suppose that the market factor, m, has a specific value of $\bar{m}$. Substituting this value $\bar{m}$ into Equation 2 and subtracting $\beta_i \bar{m}$ from both sides results in Equation 3. Default risk is measured by the distance to default, $a_i - \beta_i \bar{m}$. This distance to default either increases or decreases, and the only random parameter is the idiosyncratic shock, ε_i.

$$a_i - \beta_i \bar{m} = \sqrt{1-\beta_i^2}\,\varepsilon_i \qquad (3)$$

As a result of this conditioning, the default distribution's mean shifts based on the specific market value for any beta, β_i, that is greater than zero. The default threshold, k_i, does not change, but the standard deviation of the default distribution is reduced from 1 to $\sqrt{1-\beta_i^2}$.

The unconditional default distribution is a standard normal distribution. However, the conditional distribution is a normal distribution with a mean of $\beta_i \bar{m}$ and a standard deviation of $\sqrt{1-\beta_i^2}$. Specifying a specific value $\bar{m}$ for the market parameter, m, in the single-factor model results in the following implications:

1. The conditional probability of default will be greater or smaller than the unconditional probability of default as long as $\bar{m}$ or β_i are not equal to zero. This reduces the default triggers or number of idiosyncratic shocks, ε_i, so that it is less than or equal to $k_i - \beta_i \bar{m}$. As the market factor goes from strong to weak economies, a smaller idiosyncratic shock will trigger default.

2. The conditional standard deviation $\sqrt{1-\beta_i^2}$ is less than the unconditional standard deviation of 1.

3. Individual asset returns, a_i, and idiosyncratic shocks, ε_i, are independent from other firms' shocks and returns.

Conditional Default Distribution Variance

Suppose a firm has a beta, β_i, equal to 0.5 and a default threshold, k_i, equal to –2.33. The unconditional probability of default $\Phi(-2.33) = 0.01$. If the market return is –0.5, what is the conditional variance of the default distribution using the single-factor model?

Professor's Note: Recall that the symbol Φ represents a standard normal distribution function.

The conditional distribution is a normal distribution with a mean of $\beta_i \bar{m}$ and a conditional variance of $1-\beta_i^2$. For this example, the mean is $\beta_i \bar{m} = 0.5(-0.5) = -0.25$, and the conditional variance is $1 - 0.5^2 = 0.75$. The conditional standard deviation is then 0.866 (the square root of the variance of 0.75).

The conditional cumulative default probability function is stated as a function of *m* as follows:

$$p(m) = \Phi\left(\frac{k_i - \beta_i \bar{m}}{\sqrt{1-\beta_i^2}}\right)$$

The mean is the new distance to default based on the realized market factor, $\beta_i \bar{m}$, and the standard deviation assumes conditional independence and is equivalent to $\sqrt{1-\beta_i^2}$. Thus, given a realized market factor, $\bar{m}$, the probability of default is based on the distance of the new default trigger of idiosyncratic shocks, ε_i, measured in standard deviations below its mean of zero.

Professor's Note: For the exam, focus on how to calculate the parameters of the distribution (e.g., the mean and the standard deviation).

If we assume that distribution parameters (β, k, and π) are equal for all firms, then the probability of a joint default for two firms can be defined as:

$$\Phi\binom{k}{k} = P[-\infty \le a \le k, -\infty \le a \le k]$$

This assumption also allows us to define the default correlation for any pair of firms as follows:

$$\rho = \frac{\Phi\binom{k}{k} - \pi^2}{\pi(1-\pi)} \qquad (4)$$

Although the derivation of this default correlation equation is not required for the exam, you may wish to understand how Equation 1 (from LO 23.1) is used to derive Equation 4.

The single-factor model assumes the cumulative return distribution of any pair of credit positions *i* and *j* is distributed as a bivariate standard normal distribution with a correlation coefficient equal to $\beta_i\beta_j$. The cumulative distribution function for this pair, *i* and *j*, is $\Phi\binom{a_i}{a_j}$. We are interested in the probability of a joint default that will occur in the extreme tail of the distributions. Thus, the probability that the realized value for credit *i*, a_i, is less than the default threshold, or critical value, k_i, and is denoted for the pair of credits *i* and *j* as:

$$\Phi\binom{k_i}{k_j} = P\left[-\infty \le a_i \le k_i, -\infty \le a_j \le k_j\right]$$

In LO 23.1, Equation 1 defined the default correlation as the covariance of firm 1 and 2 divided by the standard deviations of firm 1 and 2 as follows:

$$\rho_{ij} = \frac{\pi_{ij} - \pi_i\pi_j}{\sqrt{\pi_i(1-\pi_i)}\sqrt{\pi_j(1-\pi_j)}}$$

Substituting $\Phi\begin{pmatrix} k_i \\ k_j \end{pmatrix}$ for π_{ij} results in:

$$\rho_{ij} = \frac{\Phi\begin{pmatrix} k_i \\ k_j \end{pmatrix} - \pi_i \pi_j}{\sqrt{\pi_i(1-\pi_i)}\sqrt{\pi_j(1-\pi_j)}}$$

If we assume that parameters (β, k, and π) are equal for all firms, then the pairwise asset return correlation for any two firms must equal β^2 and the previous equation simplifies to Equation 4.

Credit VaR with a Single-Factor Model

Previously, the loss distribution was estimated when the default correlation was either 0 or 1. In order to define the distribution of loss severity for values between 0 and 1, we need to determine the unconditional probability of default loss. Using the single-factor model framework, the unconditional probability of a default loss level is equal to the probability that the realized market return results in a default loss. In other words, the individual credit asset returns, a_i, are strictly a function of the market return and the asset return's correlation, or β_i, with the market. The unconditional distribution used to calculate credit VaR is determined by the following steps:

1. The default loss level is assumed to be a random variable X with realized values of x. Under this framework, x is not simulated.

2. Given a loss level of x, the value for the market factor, m, is determined at the probability of the stated loss level. The relationship between the loss level and market factor return is equal to:

$$x(m) = p(m) = \Phi\left(\frac{k - \beta\bar{m}}{\sqrt{1-\beta^2}}\right)$$

The market factor return, $\bar{m}$, for a given loss level, $\bar{x}$, is determined based on the following relationship:

$$\Phi^{-1}(\bar{x}) = \left(\frac{k - \beta\bar{m}}{\sqrt{1-\beta^2}}\right)$$

3. The market factor is assumed to be standard normal, and therefore, a loss level of 0.01 (99% confidence level) is equal to a value of –2.33 based on the standard normal distribution.

4. These steps are repeated for each individual credit to determine the loss probability distribution.

Example: Realized market value

Suppose a credit position has a correlation to the market factor of 0.25. What is the realized market value used to compute the probability of reaching a default threshold at the 99% confidence level?

Answer:

At the 99% confidence level, the default loss level has a default probability, π, of 0.01. A default loss level of 0.01 corresponds to –2.33 on the standard normal distribution. The relationship between the default loss level and the given market return, $\bar{m}$, is defined by:

$$p(\bar{m}) = 0.01 = \Phi\left(\frac{k - \beta\bar{m}}{\sqrt{1-\beta^2}}\right)$$

This is approximately equal to the probability of obtaining a realized market return of –2.33 as follows:

$$\Phi^{-1}(0.01) \approx -2.33 = \left(\frac{k - \beta\bar{m}}{\sqrt{1-\beta^2}}\right)$$

The realized market value is computed as follows:

$$\begin{aligned}
-2.33 &= \frac{-2.33 - (0.25)\bar{m}}{\sqrt{1-0.25^2}} \\
-2.33(0.9682) &= -2.33 - (0.25)\bar{m} \\
-2.256 + 2.33 &= -(0.25)\bar{m} \\
0.074 &= -(0.25)\bar{m} \\
-0.296 &= \bar{m}
\end{aligned}$$

The probability that the default threshold is reached is the same probability that the realized market return is –0.296 or lower.

The parameters play an important role in determining the unconditional loss distribution. The probability of default, π, determines the unconditional expected default value for the credit portfolio. The credit position's correlation to the market, β, determines the dispersion of the defaults based on the range of the market factor.

Credit VaR with Copulas

LO 23.5: Describe how Credit VaR can be calculated using a simulation of joint defaults.

Copulas provide a mathematical approach for determining how defaults are correlated with one another using simulated results. The following four steps are used to compute a credit VaR under the copula methodology:

1. Define the copula function.
2. Simulate default times.
3. Obtain market values and profit and loss data for each scenario using the simulated default times.
4. Compute portfolio distribution statistics by adding the simulated terminal value results.

Example: Computing credit VaR with a copula

Suppose there is a credit portfolio with two loans (rated CCC and BB) that each has a notional value of $1,000,000. Figure 2 illustrates four possible event outcomes over a default time horizon of one year for this credit portfolio. The four event outcomes are only the BB rated loan defaults, only the CCC rated loan defaults, both loans default, or no loans default.

Figure 2: Event outcomes for a two credit portfolio

Event	*Default Time*
BB Default	$(\tau_{BB,i} \leq 1, \tau_{CCC,i} > 1)$
CCC Default	$(\tau_{BB,i} > 1, \tau_{CCC,i} \leq 1)$
Both Default	$(\tau_{BB,i} \leq 1, \tau_{CCC,i} \leq 1)$
No Default	$(\tau_{BB,i} > 1, \tau_{CCC,i} > 1)$

How can credit VaR be estimated for this portfolio assuming a correlation of 0.25?

Answer:

The copula approach to estimating credit VaR is applied using the following steps:

1. The first step is to simulate 1,000 values using a copula function. The most common copula used to calculate credit VaR is the normal copula.

2. The 2,000 simulated values (1,000 pair simulations results in 2,000 values) are then mapped to their standard univariate normal quantile which results in 1,000 pairs of probability values.

3. The first and second elements of each probability pair are mapped to the BB and CCC default times, respectively.

4. A terminal value is assigned to each loan for each simulation. The values are added up for the two loans, and the sum of the no-default event value is subtracted to determine the loss. Figure 3 summarizes the sum of the terminal values and losses for 1,000 simulations.

Figure 3: Event outcomes for a two credit portfolio

Event	*Default Time*	*Terminal Value*	*Loss*
BB Default	$(\tau_{BB,i} \leq 1, \tau_{CCC,i} > 1)$	1,480,000	710,000
CCC Default	$(\tau_{BB,i} > 1, \tau_{CCC,i} \leq 1)$	1,410,000	780,000
Both Default	$(\tau_{BB,i} \leq 1, \tau_{CCC,i} \leq 1)$	700,000	1,490,000
No Default	$(\tau_{BB,i} > 1, \tau_{CCC,i} > 1)$	2,190,000	0

The loss level sums from the simulation are then used to determine the credit VaR based on the simulated distribution. In this simulation, the 99% confidence level corresponds to the $1,490,000 loss where both loans default. The 95% confidence level corresponds to the $780,000 value because the lower 5% of the simulated values resulted in defaults with a total loss of $780,000.

Key Concepts

LO 23.1

The default correlation for a two credit portfolio assuming the outcomes are Bernoulli-distributed random variables is:

$$\rho_{12} = \frac{\pi_{12} - \pi_1\pi_2}{\sqrt{\pi_1(1-\pi_1)}\sqrt{\pi_2(1-\pi_2)}}$$

LO 23.2

Drawbacks of using the default correlation-based credit portfolio framework are the number of required calculations (2^n event outcomes with $(n + 1) + [n(n - 1) / 2]$ conditions), certain characteristics of credit positions do not fit well in the default correlation credit portfolio model, and the limited data for estimating defaults due to the fact that firm defaults are relatively rare events.

LO 23.3

A portfolio's credit value at risk is defined as the quantile of the credit loss less the expected loss of the portfolio. A default correlation equal to 0 implies the portfolio is a binomially distributed random variable. As a credit portfolio becomes more granular, the credit VaR decreases.

LO 23.4

In the single-factor model, firm *i*'s individual asset return is defined as:
$a_i = \beta_i m + \sqrt{1-\beta_i^2}\varepsilon_i$ where $\sqrt{1-\beta_i^2}$ is the firm's standard deviation of idiosyncratic risk, and ε_i is the firm's idiosyncratic shock. The model assumes that firm i defaults if $a_i \leq k_i$.

The single-factor model framework states that the unconditional probability of a default loss level is equal to the probability that the realized market return results in a default loss. The market factor is assumed to be standard normal. The credit position's correlation to the market, β, determines the dispersion of the defaults based on the range of the market factor.

LO 23.5

A credit VaR under the copula methodology is computed by: defining the copula function, simulating default times, obtaining market values and profit and loss data for each scenario using the simulated default times, and computing the portfolio distribution statistics by adding the simulated terminal value results.

Concept Checkers

1. Which of the following equations best defines the default correlation for a two firm credit portfolio?
   A. $\rho_{12} = \dfrac{\pi_{12} - \pi_1\pi_2}{\sqrt{\pi_1(1-\pi_1)}\sqrt{\pi_2(1-\pi_2)}}$
   B. $\rho_{12} = \dfrac{\pi_{12}}{\sqrt{\pi_1(1-\pi_1)}\sqrt{\pi_2(1-\pi_2)}}$
   C. $\rho_{12} = \dfrac{\pi_{12}}{\sqrt{(1-\pi_1)}\sqrt{(1-\pi_2)}}$
   D. $\rho_{12} = \dfrac{\pi_{12} - \pi_1\pi_2}{\sqrt{\pi_1}\sqrt{\pi_2}}$

2. Suppose a portfolio manager is using a default correlation framework for measuring credit portfolio risk. How many unique event outcomes are there for a credit portfolio with eight different firms?
   A. 10.
   B. 56.
   C. 256.
   D. 517.

3. Suppose a portfolio has a notional value of $1,000,000 with 20 credit positions. Each of the credits has a default probability of 2% and a recovery rate of zero. Each credit position in the portfolio is an obligation from the same obligor, and therefore, the credit portfolio has a default correlation equal to 1. What is the credit value at risk at the 99% confidence level for this credit portfolio?
   A. $0.
   B. $1,000.
   C. $20,000.
   D. $980,000.

4. A portfolio manager uses the single-factor model to estimate default risk. What is the mean and standard deviation for the conditional distribution when a specific realized market value $\bar{m}$ is used?
   A. The mean and standard deviation are equivalent in the standard normal distribution.
   B. The mean is $\beta_i\bar{m}$ and the standard deviation is $\sqrt{1-\beta_i^2}$.
   C. The mean is $\bar{m}$ and the standard deviation is β_i.
   D. The mean is $\bar{m}$ and the standard deviation is 1.

5. Suppose a credit position has a correlation to the market factor of 0.5. What is the realized market value that is used to compute the probability of reaching a default threshold at the 99% confidence level?
   A. –0.2500.
   B. –0.4356.
   C. –0.5825.
   D. –0.6243.

Concept Checker Answers

1. **A** The default correlation for a two firm credit portfolio is defined as:

$$\rho_{12} = \frac{\pi_{12} - \pi_1\pi_2}{\sqrt{\pi_1(1-\pi_1)}\sqrt{\pi_2(1-\pi_2)}}$$

2. **C** There are 256 event outcomes for a credit portfolio with eight different firms calculated as: $2^8 = 256$.

3. **D** With the default correlation equal to 1, the portfolio will act as if there is only one credit. Viewing the portfolio as a binomial distributed random variable, there are only two possible outcomes for a portfolio acting as one credit. The portfolio has a 2% probability of total loss and a 98% probability of zero loss. Therefore, with a recovery rate of zero, the extreme loss given default is $1,000,000. The expected loss is equal to the portfolio value times π and is $20,000 in this example (0.02 × $1,000,000). The credit VaR is defined as the quantile of the credit loss less the expected loss of the portfolio. At the 99% confidence level, the credit VaR is equal to $980,000 ($1,000,000 minus the expected loss of $20,000).

4. **B** The conditional distribution is a normal distribution with a mean of $\beta_i \bar{m}$ and a standard deviation of $\sqrt{1-\beta_i^2}$.

5. **D** A default loss level of 0.01 corresponds to –2.33 on the standard normal distribution. The realized market value is computed as follows:

$$-2.33 = \frac{-2.33 - (0.5)\bar{m}}{\sqrt{1-0.5^2}}$$

$$-2.33(0.86603) = -2.33 - (0.5)\bar{m}$$

$$-2.01785 + 2.33 = -(0.5)\bar{m}$$

$$0.31215 = -(0.5)\bar{m}$$

$$-0.62430 = \bar{m}$$

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The following is a review of the Credit Risk Measurement and Management principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Structured Credit Risk

Topic 24

Exam Focus

In this topic, we discuss common structured products, capital structure in securitization, structured product participants, a basic waterfall structure, and the impact of correlation. For the exam, understand the qualitative impacts of changing default probability and default correlation for all tranches for mean (average) and risk (credit VaR). Default sensitivity (similar to DV01) is introduced. Understand the process to compute implied correlation extracted from observable market prices.

Types of Structured Products

LO 24.1: Describe common types of structured products.

Securitization and structured products are two of the most important financial innovations in recent memory. Securitization is basically the pooling of credit-sensitive assets and the associated creation of new securities (structured products or portfolio credit products) whose cash flows are based on underlying loans or credit claims. Each product has its own risk and return characteristics, which can vary dramatically from the original assets. For this LO, a partial list of structured products and factors that affect their valuation are discussed.

Covered bonds. Covered bonds are on-balance sheet securitizations. A pool of mortgages, which secure a bond issue, is separated from other loans into a covered pool on the originator's balance sheet. Investors have higher priority than general creditors if a bank defaults. Principal and interest is paid and guaranteed by the originator and is not based on the performance of the underlying assets themselves. Thus, covered bonds are not true securitizations since the assets are not part of a bankruptcy-remote structure and the investors have recourse against the originator.

Mortgage pass-through securities. In contrast to covered bonds, mortgage pass-through securities are true off-balance sheet securitizations. Investors receive cash flows based entirely on the performance of the pool less associated fees paid to the servicer. Most pass-throughs are agency mortgage-backed securities (MBS) that carry implicit or explicit government guarantee of performance. Thus, default risk is not a serious concern. The primary risk is due to prepayment of principal by the homeowner, most likely from refinancing after interest rate declines or home sales.

Collateralized mortgage obligations (CMOs). CMOs are MBSs that tranche (i.e., divide) cash flows into different securities based on predetermined conditions. The resulting tranches can have long or short maturities, fixed or floating cash flows, or other varieties and conditions. The most basic structure is the *waterfall* or *sequential pay structure* where Tranche 1 receives all principal and its portion of interest in each period until it is paid off. The remaining tranches will receive interest only until Tranche 1 is retired and then principal will flow down to Tranche 2, and so on. Not surprisingly, Tranche 1 will have a very low prepayment risk as it expects to receive all principal payments before other bondholders.

Structured credit products. Like other structured products, this pool of assets is backed by risky debt instruments. The difference is that structured credit products create tranches that have different amounts of credit risk. The most junior (i.e., equity) tranches bear the first losses and are most likely to be written down from defaulted assets. If the equity tranche is completely wiped out, the next most junior tranche will bear the credit risk of subsequent defaults. The most senior tranche will have the highest credit rating and the lowest probability of writedowns.

Asset-backed securities. This is the most general class of securitizations where cash-flow generating assets are pooled and subsequently tranched. Under this definition, MBS is a special case of the more general ABS. Other varieties include collateralized bond obligations (CBO), collateralized debt obligations (CDO), collateralized loan obligations (CLO), and collateralized mortgage obligations (CMO). There exist even more complex securities that pool other securitizations together such as CDO-squared (CDO of CDOs).

Structured credit products can also vary across other dimensions. First, the underlying collateral of the pool can consist of loans, bonds, credit card receivables, auto loans, and even non-debt instruments that generate cash flows, such as toll collections. Second, the size and number of tranches is specific to each transaction. Third, the pool can be passive or actively managed. In a passive pool, the existing assets, such as mortgages and auto loans, will eventually pay themselves down. On the other hand, actively managed pools will selectively add or shed assets from the pool. Managers with key insight should be able to enhance the performance of the pool by identifying overvalued and undervalued loan products. Revolving pools have a period of time where loan proceeds are reinvested in new assets. Once the revolving period ends, the asset balances are fixed (e.g., credit card balances) and will spend themselves down.

Capital Structure in Securitization

LO 24.2: Describe tranching and the distribution of credit losses in a securitization.

The capital structure of a securitization refers to the priority assigned to the different tranches. In general, the most **senior tranches** at the top of the capital structure will have the highest priority to receive principal and interest. Since these securities are perceived to be the safest, they also receive the lowest coupon.

The **equity tranche** is the slice of the cash flow distribution with the lowest priority and will absorb the first losses up to a prespecified level. These securities typically do not carry a fixed coupon but receive the residual cash flows only after the other security claims are satisfied. Therefore, the return is variable and, hence, the term "equity." Typically, the equity tranche is the smallest part of the capital structure.

Between the senior and equity tranches is the **mezzanine tranche** (i.e., the junior tranche). The mezzanine tranches will absorb losses only after the equity tranche is completely written down. Thus, the senior tranches are protected by both the equity and mezzanine debt (termed subordination or credit enhancement). Terminology-wise, the mezzanine debt attaches to the equity tranche from above and detaches from the senior tranche from below. These junior debt claims offer a relatively high coupon (if the claim is fixed) or high spread (if the claim is floating). To keep the securitization viable, the mezzanine tranches will be purposefully thin.

There are many creative ways to provide credit protection to various security classes, but this must come at the expense of shifting risk to other parts of the capital structure. In general, credit enhancement can be divided into internal and external credit enhancement mechanisms. The term **external credit enhancement** means that the credit protection takes the form of insurance or wraps purchased from a third party, typically a monoline insurer.

Two examples of **internal credit enhancements** are overcollateralization and excess spread. **Overcollateralization** is when the pool offers claims for less than the amount of the collateral. For example, consider a collateralized MBS with 101 mortgages in the collateral pool, but the face value of the bonds across all tranches only totals 100 mortgages. Overcollateralization is a *hard credit enhancement* because the protection is available at the origination of the pool.

The **excess spread** is the difference between the cash flows collected and the payments made to all bondholders. For example, if the weighted average of the collateral is 8% (net of fees) and the weighted average of the payments promised to the senior, junior, and equity tranches is 7%, then the residual 1% accumulates in a separate trust account. The excess spread will be invested and is available to make up future shortfalls. Since the excess spread is zero at origination, it is considered a *soft credit enhancement*.

Waterfall Structure

LO 24.3: Describe a waterfall structure in a securitization.

A waterfall structure outlines the rules and conditions that govern the distribution of collateral cash flows to different tranches. In the simplest example of a securitization, the senior and junior bonds will receive their promised coupons conditional on a sufficient amount of cash inflows from the underlying loans. The residual cash flow, if any, is called the excess spread. The overcollateralization triggers will decide how the excess is divided between the equity investors and the accumulating trust. Intuitively, the underlying cash flows will be largest in the earlier periods so the trust will build up a reserve against future shortfalls.

In practice, this process can be quite complex as there may be a dozen tranches or more with different coupons, maturities, and overcollateralization triggers. The waterfall is further complicated by loan defaults. A simplifying assumption would incorporate a constant default rate which can be built into the waterfall distribution. As the loans mature, the actual incidence of the loan defaults will increase or decrease the value of the respective tranches. For example, suppose that fewer loans default than previously assumed, then collateral cash flows are larger than expected and will benefit all bondholders, in particular, the equity tranche.

Let's analyze the cash flows in a waterfall structure by considering the following examples. Assume there are 1,000 identical loans with a value of $1 million each. The interest rate on the loans is floating with a rate equal to LIBOR + 300 bps, reset annually. The senior, junior, and equity tranches are 80%, 15%, and 5% of the pool, respectively. The spreads on the senior and mezzanine tranches are 1% and 5%. There is one overcollateralization trigger where the equity holders are entitled to a maximum of $15 million and any excess is diverted to the excess trust account. To begin, assume the default rate is 0%. The cash flows for the waterfall structure are detailed in Figure 1.

Figure 1: Waterfall Structure (Default Rate = 0%)

Loan Information	
# loans	1,000
$ value of identical loan	$1,000,000
Principal amount	$1,000,000,000
LIBOR	5.00%
Spread	3.00%
Coupon	8.00%
Default rate	0.00%
OC trigger	$15,000,000

Tranche Information		
Senior % of pool		80%
	LIBOR	5.00%
	Spread	1.00%
	Coupon	6.00%
Mezzanine % of pool		15%
	LIBOR	5.00%
	Spread	5.00%
	Coupon	10.00%
Equity % of pool		5%

Period	*Loan Proceeds*	
1	$80,000,000	
Senior Principal	*Senior Coupon*	*Interest*
$800,000,000	6.00%	$48,000,000
Mezzanine Principal	*Mezzanine Coupon*	*Interest*
$150,000,000	10.00%	$15,000,000
Excess CF	*CF to Equity*	*CF to Trust*
$17,000,000	$15,000,000	$2,000,000

Note that the senior tranche has a principal value of $800 million while the junior tranche has an initial principal of $150 million. Using a current LIBOR of 5%, their respective coupons are 6% (5% + 1% spread) and 10% (5% + 5% spread). The total cash flows flowing into the pool are $1 billion × 8% = $80 million, which is sufficient to pay the senior and junior claims. The residual cash flow is $80 million – ($48 million + $15 million) = $17 million.

Next, the overcollateralization test must be applied. Since the maximum the equity tranche can receive is $15 million, the equity investors will receive the full $15 million and the excess of $2 million will flow into the trust account. This is shown in the last row of Figure 1.

Now assume that the expected default rate is 4% each year. The first difference from the 0% default rate example is that the total loan proceeds is reduced by defaulted loans: $1 billion × 8% × (1 – 0.04) = $76.8 million. There is still sufficient cash flow to pay the senior and junior bondholders in full. However, when the overcollateralization test is applied, the equity holders will not reached their maximum. Therefore, the equity tranche receives only $13.8 million and there is no diversion to the trust account as shown in Figure 2.

Figure 2: Waterfall Structure (Default Rate = 4%)

Loan Information	
# loans	1,000
$ value of identical loan	$1,000,000
Principal amount	$1,000,000,000
LIBOR	5.00%
Spread	3.00%
Coupon	8.00%
Default rate	4.00%
OC trigger	$15,000,000

Tranche Information			
Senior % of pool		80%	
	LIBOR		5.00%
	Spread		1.00%
	Coupon		6.00%
Mezzanine % of pool		15%	
	LIBOR		5.00%
	Spread		5.00%
	Coupon		10.00%
Equity % of pool		5%	

Figure 2 Cont.: Waterfall Structure (Default Rate = 4%)

Period	*Loan Proceeds*	
1	$76,800,000	
Senior Principal	*Senior Coupon*	*Interest*
$800,000,000	6.00%	$48,000,000
Mezzanine Principal	*Mezzanine Coupon*	*Interest*
$150,000,000	10.00%	$15,000,000
Excess CF	*CF to Equity*	*CF to Trust*
$13,800,000	$13,800,000	$0

Securitization Participants

LO 24.4: Identify the key participants in the securitization process, and describe conflicts of interest that can arise in the process.

The nature of the securitization process from original loan to tranche issuance necessarily involves many different participants. The first step begins with the **originator** who funds the loan. The originator may be a bank, mortgage lender, or other financial intermediary. The term "sponsor" may be used if the originator supplies most of the collateral for the issue.

The **underwriter** performs a function similar to the issuance of traditional debt and equity. The underwriter structures the issue (i.e., engineers the tranche size, coupon, and triggers, and sells the bonds to investors). The underwriter warehouses the collateral and faces the risks that the issue will not be marketed or that the collateral value will drop.

The **credit rating agencies** (CRAs) are an important part of the securitization process. Without their explicit approval via credit ratings, investors would be at a severe disadvantage to assess the riskiness of the issue. The credit rating agencies can influence the size of the tranches by selecting the attachment points and thus are active participants in the process. In addition, the CRAs may influence the issue by requiring enhancements. There is a natural conflict of interest because the CRAs want to generate profit and grow their business, but it may come at the expense of allocating larger portions of the capital structure to lower interest paying senior notes. Investors can alleviate this concern by performing their own (costly) analysis or purchasing a wrap or insurance against the issue.

The role of the servicer is multifaceted and possibly understated. The servicer must collect and distribute the collateral cash flows and the associated fees. In addition, the servicer may need to provide liquidity if payments are late and resolve default situations. It is not hard to envision the conflict of interest in foreclosure: the servicer would, all else equal, like to delay foreclosure to increase their fees, while investors want as quick of a resolution as possible to minimize the damage and/or lack of maintenance from the homeowners who have no economic incentive to maintain the property.

When the pool is actively managed, another source of conflict arises. The manager naturally would like to minimize their effort to continually monitor the credit quality of the collateral unless there is a clear incentive to do so. A common feature of securitized pools is for the originator and/or manager to bear the first loss in the capital structure.

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Custodians and trustees play an administrative role verifying documents, disbursing funds, and transferring funds between accounts.

Three-Tiered Securitization Structure

LO 24.5: Compute and evaluate one or two iterations of interim cashflows in a three-tiered securitization structure.

The cash flows in a three-tiered securitization (senior, mezzanine, and equity) can be broken out into the inflows from the collateral and the outflows to the investors. The inflows prior to maturity are the interest on the collateral (L_t) plus the recovery from the sale of any defaulted assets in the current period (R_t). Assume the collateral pool has N identical loans with coupons = LIBOR + spread. The terminal cash flows in the final year are the last interest payment plus principal and recovery of defaulted assets. As an additional consideration, the recovered funds from defaults would earn interest over the remaining life of the pool at r.

The outflows are the coupon payments paid to senior and mezzanine note holders, collectively denoted B (assumed constant). The equity holder position is a bit more complicated because the excess spread trust has first priority on the cash flows to provide soft credit enhancement to the more senior tranches. Specifically, the equity holders' cash flows are dependent on the amount of inflows to the pool less any funds diverted to the excess spread account. Denote the amount diverted to the spread trust in year t as (OC_t) with maximum allowable diversion K. To determine the cash flow to equity, the following steps must be performed:

1. Is the current period interest sufficient to cover the promised coupons: $L_t - B \geq 0$? If *yes*, then the following overcollateralization test must be performed to see how much flows to trust: Is $L_t - B \geq K$? If *yes*, then K is diverted to trust, and $L_t - B - K$ flows to equity holders: $OC_t = K$. If *no*, then $L_t - B$ is diverted to trust, and nothing flows to equity holders: $OC_t = L_t - B$

2. Is the current period interest sufficient to cover the promised coupons: $L_t - B \geq 0$? If *no*, then the interest is not sufficient to pay bondholders and all L_t flows to bondholders. Therefore, the shortfall is $B - L_t$. The next step is to check if the accumulated funds in the spread trust can cover the shortfall. If the trust account has enough funds, the bondholders can be paid in full. If the trust account does not have enough funds, then the bondholders suffer a writedown.

The previous steps outlined the basic procedure for tranche cash flow distribution; however, a few more factors need to be considered. First, for each period there are possible defaults. For simplicity, assume the number of defaults (d_t) is constant for each period. Second, the amount recovered in year t (assuming a 40% recovery rate) equals:

$$R_t = 0.4d_t \times \text{loan amount}$$

Therefore, the total amount deposited into the trust account in year *t* is:

$$R_t + OC_t$$

It follows that the total amount accumulated in the trust account in year *t* is:

$$R_t + OC_t + \sum_{\tau=1}^{t-1}(1+r)^{t-\tau}OC_\tau$$

Now, if excess spread is negative ($L_t - B < 0$), the custodian must check if the trust account can cover the shortfall. Formally, the test for the custodian is:

$$R_t + \sum_{\tau=1}^{t-1}(1+r)^{t-\tau}OC_\tau > B - L_t$$

Note that there is no OC_t term to add to R_t since there is no excess spread this period. If the above test is true, then the trust account can make the bondholders whole. If it is not true, then the fund is reduced to zero and bondholders receive $R_t + \sum_{\tau=1}^{t-1}(1+r)^{t-\tau}OC_\tau$ from the trust account.

Using the previous exposition, the amount diverted to the overcollateralization account can be calculated as:

$$OC_t = \left\{\begin{matrix} \min(L_t - B, K) \\ \max\left[L_t - B, -\sum_{\tau=1}^{t-1}(1+r)^{t-\tau}OC_\tau + R_t\right] \end{matrix}\right\} \text{for} \left\{\begin{matrix} L_t \geq B \\ L_t < B \end{matrix}\right\}$$

Note that the upper condition represents inflows to the trust account while the lower condition represents outflows from the trust account.

Finally, the equity cash flows can be expressed as:

$$\max(L_t - B - OC_t, 0) \text{ for } t = 1,\ldots,T-1$$

The cash flows in the final year must be examined separately for several reasons. First, the surviving loans reach maturity and principal is returned. Second, there is no diversion to the trust account because the structure ends and all proceeds follow the waterfall. Third, since there is no diversion to the trust, there is no need to test overcollateralization triggers.

The terminal cash flows are summarized as follows:

1. Loan interest = $\left(N - \sum_{t=1}^{T} d_t\right)$ × (LIBOR + spread) × par
2. Proceeds (par) from redemption of surviving loans = $\left(N - \sum_{t=1}^{T} d_t\right) \times \text{par}$

3. Recovery in final year: $R_T = 0.4d_T \times \text{par}$

4. Residual in trust account: $\sum_{\tau=1}^{T}(1+r)^{t-\tau}OC_t$

The sum of these terminal cash flows is compared to the amount due to the senior tranche. If the sum is large enough, the senior tranche is paid off and the remainder is available for the rest of the capital structure. If the remainder is large enough to cover the junior tranche, then the residual flows to equity. If the remainder cannot meet junior claims, the junior bonds receive the excess and equity holders receive nothing.

As an example, determine the terminal cash flows to senior, junior, and equity tranches given the following information. The original loan pool included 100 loans with $1 million par value and a fixed coupon of 8%. The number of surviving loans is 90. The par for the senior and junior tranches is 75% and 20%, respectively. The equity investors contributed the remaining 5%. There were two defaults with recovery rate of 40% recovered at the end of the period. The value of the trust account at the beginning of the period is $16 million earning 4% per annum.

1. Total size of collateral pool at origination: 100 × $1,000,000 = $100,000,000

2. Senior tranche = $75,000,000
   Junior tranche = $20,000,000
   Equity tranche = $5,000,000

3. Interest from loans: 90 × 8% × $1,000,000 =	$7,200,000
4. Redemption at par: 90 × $1,000,000 =	$90,000,000
5. Recovery in final year: 2 × 40% × $1,000,000 =	$800,000
6. Value of OC at end of final year: $16,000,000 × 1.04 =	<u>$16,640,000</u>
7. Total available to satisfy all claims =	$114,640,000

8. Senior claim = $75,000,000 < $114,640,000. Senior claim is satisfied w/o impairment
9. Junior claim = $20,000,000 < $114,640,000 – $75,000,000 so junior claim is satisfied
10. Equity claim = $114,640,000 – $75,000,000 – $20,000,000 = $19,640,000

Now, continue with the same example, but change the interest rate to 5% and the beginning OC value to $3 million. The first two steps will be the same as before.

3. Interest from loans: 90 × 5% × $1,000,000 =	$4,500,000
4. Redemption at par: 90 × $1,000,000 =	$90,000,000
5. Recovery in final year: 2 × 40% × $1,000,000 =	$800,000
6. Value of OC at end of final year: $3,000,000 × 1.04 =	<u>$3,120,000</u>
7. Total available to satisfy all claims =	$98,420,000

8. Senior claim = $75,000,000 < $98,420,000. Senior claim is satisfied w/o impairment
9. Junior claim = $20,000,000 < $98,420,000 – $75,000,000 so junior claim is satisfied
10. Equity claim = $98,420,000 – $75,000,000 – $20,000,000 = $3,420,000

Finally, continue with the same example, but change the interest rate to 4% and the beginning OC value to $1 million. Assume a recovery rate of zero. Again, the first two steps are the same as before.

3.	Interest from loans: 90 × 4% × $1,000,000 =	$3,600,000
4.	Redemption at par: 90 × $1,000,000 =	$90,000,000
5.	Recovery in final year: 2 × 0% × $1,000,000 =	0
6.	Value of OC at end of final year: $1,000,000 × 1.04 =	$1,040,000
7.	Total available to satisfy all claims =	$94,640,000

8. Senior claim = $75,000,000 < $94,640,000. Senior claim is satisfied w/o impairment
9. Junior claim = $20,000,000 > $94,640,000 – $75,000,000 so junior claim is impaired
   Junior tranche receives $19,640,000
10. Equity claim = $94,640,000 – $75,000,000 – $20,000,000 < 0
    Equity tranche receives $0

Simulation Approach

LO 24.6: Describe a simulation approach to calculating credit losses for different tranches in a securitization.

The prior analysis made a few very important simplifying assumptions. In particular, the analysis assumed that the default rate was constant year over year, each loan exhibited the same default probability, and the correlation between loans was ignored. In practice, these assumptions need to be brought into the analysis and the only tractable way to do so is via simulation.

Although the technical details are well beyond the scope of the exam, we can sketch out the basic steps and intuition for the simulation approach to calculating credit losses.

Step 1: Estimate the parameters.
Step 2: Generate default time simulations.
Step 3: Compute portfolio credit losses.

The first step is to estimate the critical parameters, default intensity, and pairwise correlations. The default intensity can be estimated using market spread data to infer the hazard rate across various maturities. This piecewise-bootstrapping methodology to construct the cumulative default distribution was discussed in Topic 22. Estimating the correlation coefficients is more challenging because of a lack of usable market data. The copula correlation could be useful in theory but suffers empirical precision in practice. Instead, a sensitivity analysis is performed for various default and correlation pairs.

The second step identifies if and when the security defaults. Simulation provides information on the timing for each hypothetical outcome. The third step uses the simulation output to determine the frequency and timing of credit losses. The credit losses can be "lined up" to assess the impact on the capital structure losses. The tail of the distribution will identify the credit VaR for each tranche in the securitization.

Impact of Probability of Default and Default Correlation

LO 24.7: Explain how the default probabilities and default correlations affect the credit risk in a securitization.

There are several important comparative statistics associated with a generic securitization. The following results represent the effect of the average tranche values and writedowns. The implications of extreme tail events will be discussed shortly using VaR. The first factor to consider is the probability of default. It is straightforward to see that, for a given correlation, increasing the probability of default will negatively impact the cash flows and, thus, the values of all tranches.

The effect of changing the correlation is more subtle. Consider the stylized case where the correlation is very low, say zero, so loan performance is independent. Therefore, in a large portfolio, it is virtually impossible for none of the loans to default and it is equally unlikely that there will be a large number of defaults. Rather, the number of defaults should be very close to the probability of default times the number of loans. So, the pool would experience a level of defaults very close to its mathematical expectation and is unlikely to impair the senior tranches. The analogous situation is flipping a coin 1,000 times—the number of heads would be very close to 500. It would be virtually impossible for the number of heads to be less than 400 or greater than 600. Now, if the correlation increases, the default of one credit increases the likelihood of another default. Thus, increasing correlation decreases the value of senior tranches as the pool is now more likely to suffer extreme losses. This effect is exacerbated with a higher default probability.

Now consider the equity tranche. Recall that the equity tranche suffers the first writedowns in the pool. Therefore, a low correlation implies a predictable, but positive, number of defaults. In turn, the equity tranche will assuredly suffer writedowns. On the other hand, if the correlation increases, the behavior of the pool is more extreme, and there may be high levels of related losses or there may be very few loan losses. In sum, the equity tranche increases in value from increasing correlation as the possibility of zero (or few) credit losses increases from the high correlation.

The correlation effect on the mezzanine tranche is more complex. When default rates are low, increasing the correlation increases the likelihood of losses to the junior bonds (similar to senior bonds). However, when default rates are relatively high, increasing the correlation actually decreases the expected losses to mezzanine bonds as the possibility of few defaults is now more likely. Accordingly, the mezzanine bond mimics the return pattern of the equity tranche. In short, increasing correlation at low default rates decreases mezzanine bond values, but at high default rates it will increase mezzanine bond values.

Convexity is also an issue for default rates. For equity investors, as default rates increase from low levels, the equity tranche values decrease rapidly then moderately (a characteristic of positive convexity). Since the equity tranche is thin, small changes in default rates will disproportionately impact bond prices at first. Similarly, senior tranches exhibit negative convexity. As defaults increase, the decline in bond prices increases. As usual, the mezzanine impact is somewhere in between: negative convexity at low default rates, positive convexity at high default rates.

The previous section focused on the average (mean) value of the tranches while this section examines the distribution of possible tranche values (risk). Specifically, the goal is to analyze the impact of default probability and default correlation under extreme conditions (far into the tail). The metric used is credit VaR for various ranges of default probability and default correlation for the senior, junior, and equity tranches. The main result is that increasing default probability, while holding correlation constant, generally decreases the VaR for the equity tranches (less variation in returns) and increases the VaR for the senior tranches (more variation in returns). As usual, the mezzanine effect is mixed: VaR increases at low correlation levels (like senior bonds) then decreases at high correlation levels (like equity). These results are summarized in Figure 3.

Figure 3: Increasing Default Probability (Holding Correlation Constant)

	Mean value	*Credit VaR*
Equity tranche	↓	↓
Mezzanine tranche	↓	↑ then ↓
Senior tranche	↓	↑

The next effect to consider is the impact of a rising correlation. As a reminder, increasing correlation increases the clustering of events, either high frequency of defaults or very low frequency of defaults. Increasing correlation decreases senior bond prices as the subordination is more likely to be breached if defaults do indeed cluster. In contrast, equity returns increase as the low default scenario is more probable relative to low correlation where defaults are almost certain.

As the default correlation approaches one, the equity VaR increases steadily. The interpretation is that although the mean return is increasing so is the risk as the returns are more variable (large losses or very small losses).

All else equal, the senior VaR also increases consistently with correlation. However, we note an interesting effect: the incremental difference between high correlations (0.6 versus 0.9) is relatively small. In addition, two pairwise results are worth highlighting. If correlation is low and default frequency is relatively high, then senior bonds are well insulated. In fact, at the 10% subordination level, the senior bonds would be unaffected even at a high default rate. At the other extreme, when correlations are high (0.6 or above), then the VaRs are quite similar regardless of the default probability. Hence, generally speaking, correlation is a more important risk factor than default probability which may not be entirely intuitive.

The implications for the mezzanine tranche are, again, mixed. When default rates and correlations are lower, the mezzanine tranche behaves more like senior notes with low VaRs. However, when the default probabilities are higher and/or pairwise correlation is high, the risk profile more closely resembles the equity tranche. These results are summarized in Figure 4.

Figure 4: Increasing Correlations (Holding Default Probability Constant)

	Mean value	*Credit VaR*
Equity tranche	↑	↑
Mezzanine tranche	↓(at low default rates) ↑(at high default rates)	↑
Senior tranche	↓	↑

Measuring Default Sensitivities

LO 24.8: Explain how default sensitivities for tranches are measured.

The previous discussion highlighted the effect of increasing the probability of default, which decreases tranche values. However, this effect is not necessarily linear and also depends on the interaction with the default correlation. To analyze the marginal effects in more detail, the definition of DV01 is extended to default probabilities and is called "default '01." The default probability will be shocked up and down by the same amount (by convention 10 basis points) and each tranche will be revalued through the VaR simulations. The formulation for default '01 of each tranche is as follows:

$$1/20\ [(\text{mean value / loss based on } \pi + 0.001) - (\text{mean value / loss based on } \pi - 0.001)]$$

From this equation, there are several qualitative impacts to note. First, the default sensitivities are always positive for any default probability-correlation combination. This follows from the previous observation that all tranches are negatively affected from increasing default probabilities. Second, the default '01 will approach zero as default rates become sufficiently high as the marginal impact of increasing the default rate has minimal effect. The third result follows from the second. There will be more variation in the default sensitivities when the default rate generates losses close to the tranche's attachment point. This result is similar to the high gamma (high sensitivity in delta) for options at-the-money.

Risks for Structured Products

LO 24.9: Describe risk factors that impact structured products.

Aside from the credit portfolio modeling issues discussed before, there are at least three other risks that deserve discussion: systematic risk, tranche thinness, and loan granularity.

Similar to a well-diversified equity portfolio that cannot eliminate systematic risk, the same holds true for credit portfolios. Unfortunately, even when the collateral pool is well-diversified among lenders, terms, geography, and other factors, high systematic risk expressed in high correlations can still severely damage a portfolio. As previously discussed, with increases in pairwise correlations, the likelihood of senior tranche writedowns increases as well.

The equity and mezzanine tranches are relatively thin. This also manifests itself in the relative closeness of the 95% and 99% credit VaR. The implication is that given that the tranche has been breached, the loss is likely very large.

Loan granularity references the loan level diversification. For example, in a collateralized MBS pool, the portfolio composition is a few loans but the loans are of substantial size. This reduction in sample size increases the probability of tail events in relation to an equal sized portfolio constructed with more loans of smaller amounts.

Implied Correlation

LO 24.10: Define implied correlation and describe how it can be measured.

The implied correlation is a very similar concept to the implied volatility of an equity option. For options, the Black-Scholes-Merton model is a widely accepted valuation model and so the observable market price is associated with a unique unobserved volatility. For securitized tranches, the process is exactly the same. Starting with observed market prices and a pricing function for the tranches, it is possible to back out the unique implied correlation to calibrate the model price with the market price.

The mechanical part of the process involves several intermediate steps. First, the observable credit default swap (CDS) term structure is used to extract risk-neutral default probabilities and possibly recovery rates. Assuming constant pairwise correlation and market prices for the respective tranches, the default estimates and correlation estimates can be fed into a copula. The output is the risk-neutral implied correlation (i.e., base correlation) per tranche. The correlation estimates will vary between the tranches and are not likely to be constant giving rise to correlation skew. As an example, suppose the observed market price of the equity tranche increases from $3 million to $3.2 million, but the estimates of the risk-neutral probability of default remain the same. It can be inferred that the market's estimate of the implied correlation must have increased. The precise value must be extracted from the pricing model but qualitatively the direction is correct; increasing correlations benefit equity holders.

Motivations for Using Structured Products

LO 24.11: Identify the motivations for using structured credit products.

Identifying the motivations of loan originators and investors can provide a better understanding for why securitizations are established.

Loan originators, who help create securitizations by selling loans into a trust, are attracted to borrowing via securitization given its ability to provide a lower cost of funding. Without securitization, loans would either be retained or sold in the secondary market. These alternatives would likely be more costly than securing funding via securitization. A lower cost of funding can be obtained given the diversification of the loan pool and the reputation of the originator for underwriting high-quality loans. However, some loan pools, such as commercial mortgage pools, can be difficult to diversify. Thus, an element of systematic risk may still exist, which could lead to an underestimation of overall risk. An additional benefit of securitization for loan originators is the collection of servicing fees.

Investors, who purchase the assets in a securitization, are attracted to investing in diversified loan pools that they would not otherwise have access to without securitization, such as mortgage loans and auto loans. In addition, the ability to select a desired risk-return level via tranching offers another advantage for investors. Equity tranches will offer higher risk-return levels, while senior tranches will offer lower risk-return levels. However, it is important for investors to conduct the proper due diligence when analyzing potential tranche investments in order to understand the actual level of risk involved.

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Key Concepts

LO 24.1

Securitization is the process of pooling cash flow generating assets and reapportioning the cash flows into bonds. These structured products generate a wide range of risk-return profiles that vary in maturity, credit subordination (equity, mezzanine, and senior), type of collateral (mortgages, auto loans, and credit card balances), active or passive management, and static or revolving assets. A true securitization removes the assets from the originator's balance sheet.

LO 24.2

The capital structure of a securitization refers to the different size and priority of the tranches. In general, the senior tranches are the largest, safest, and lowest yielding bonds in the capital structure. The mezzanine tranche has lower priority than the senior tranche and is promised a higher coupon. The lowest priority tranche that bears the first loss is the equity tranche. The size of the equity and mezzanine tranches provides subordination for the senior tranche. Internal credit enhancement, such as overcollateralization and excess spread, buffers the senior tranches from losses. Likewise, external wraps and insurance also protect the senior bondholders.

LO 24.3

A waterfall structure details the distribution of collateral cash flows to the different classes of bondholders. The equity tranche typically receives the residual cash flows once the senior and mezzanine investor claims are satisfied. If the cash flows to equity holders exceed the overcollateralization trigger, the excess is diverted to a trust account. Fees and defaults will reduce the net cash flows available for distribution.

LO 24.4

Securitization is a complicated process and typically involves an originator, underwriter, credit rating agency, servicer, and manager. The originator creates the initial liability; the underwriter pools and structures the terms of the deal as well as markets the issue; the credit rating agency is an active participant suggesting/requiring sufficient subordination and enhancements to justify the ratings; the servicer collects and distributes the cash flows to investors and manages distress resolution; managers, both static and active, usually bear the first loss to mitigate conflict of interest in asset selection and credit monitoring.

LO 24.5

The three-tiered waterfall will have scheduled payments to senior and mezzanine tranches. The equity tranche receives cash flows only if excess spread > 0 (i.e., interest collected > interest owed to senior + mezzanine). The overcollateralization account increases from recovery of defaulted assets and diversion of spread (usually a maximum is predetermined) and earns the money market rate. If excess spread is negative (i.e., interest collected < interest owed to senior + mezzanine), the OC account will use all of its available funds until depleted. The terminal cash flows are more complicated: redemptions at par + interest from surviving loans + recovery in final period + terminal OC account. No funds are diverted in the final year as it all is aggregated and disbursed. Senior claims are paid first; if senior is paid in full, mezzanine claims are paid; if mezzanine is paid in full, the residual accrues to equity holders.

LO 24.6

Simulation is a useful technique to provide more insight into the performance of the collateral and, hence, cash flows to the tranches. In particular, the default intensity can be time-varying and estimated using a hazard distribution. The correlation between loans is critical to the performance of the pool, so various default probability/correlation pairs are used. Copulas could be used to simulate the timing of the defaults. Finally, simulations allow computation of VaRs for each tranche.

LO 24.7

Increasing default probability will decrease all tranches unconditionally. In contrast, increasing correlation will impact each tranche differently. In general, increasing default correlation increases the likelihood of extreme portfolio behavior (very few or many defaults).

Credit VaR can be used to measure the value of the tranches in the left tail. Increasing the correlation increases the VaR of all tranches. In contrast, increasing the probability of default decreases equity VaR and increases senior VaR.

LO 24.8

Default sensitivities are measured analogously to DV01 and spread '01 by shocking the default probability up and down by 10 basis points. Default sensitivities are always positive and are largest when the resulting loss is close to the attachment point.

LO 24.9

Similar to equity portfolios, systematic risk is present in credit portfolios. Extreme loss events are captured by high default correlations. The thinness of the equity and mezzanine tranches implies that conditional losses are likely to be large. A less granular pool (fewer but larger loans) is more likely to experience a tail event, all else equal.

LO 24.10

Implied default correlations for each tranche can be backed out of the tranche pricing model similar to how the implied volatility is calculated for the Black-Scholes-Merton model.

LO 24.11

Loan originators help create securitizations by selling loans into a trust. They are attracted to secured borrowing via securitization because it provides a lower cost of funding than alternatives such as retaining loans. Investors purchase the bonds and equity in a securitization. They are attracted to securitization because it allows them to invest in diversified loan pools that are typically reserved for banks.

Concept Checkers

1. How many of the following statements concerning the capital structure in a securitization are most likely correct?

    I. The mezzanine tranche is typically the smallest tranche size.
    II. The mezzanine and equity tranches typically offer fixed coupons.
    III. The senior tranche typically receives the lowest coupon.

    A. No statements are correct.
    B. One statement is correct.
    C. Two statements are correct.
    D. Three statements are correct.

2. Assume there are 100 identical loans with a principal balance of $500,000 each. Based on a credit analysis, a 300 basis point spread is applied to the borrowers. LIBOR is currently 4% and the coupon rate will reset annually. The senior, junior, and equity tranches are 75%, 20%, and 5% of the pool, respectively. The spreads on the senior and mezzanine tranches are 2% and 6%. Excess cash flow is diverted above $1,000,000. Assume the default rate is 2%. What are the cash flows to the mezzanine and excess trust account in the first period?

	Mezzanine	Trust account
A.	$1,000,000	$0
B.	$1,000,000	$180,000
C.	$2,250,000	$200,000
D.	$2,250,000	$250,000

3. Which of the following participants in the securitization process is least likely to face a conflict of interest?
    A. Credit rating agency and servicer.
    B. Servicer and underwriter.
    C. Custodian and trustee.
    D. Trustee and manager.

4. Which of the following statements about portfolio losses and default correlation are most likely correct?

    I. Increasing default correlation decreases senior tranche values but increases equity tranche values.
    II. At high default rates, increasing default correlation decreases mezzanine bond prices.

    A. I only.
    B. II only.
    C. Both I and II.
    D. Neither I nor II.

5. Which of the following statements best describes the calculation of implied correlation?
   A. The implied correlation for the mezzanine tranche assumes non-constant pairwise correlation.
   B. Observable market prices of credit default swaps are used to infer the tranche values.
   C. The tranche pricing function is calibrated to match the model price with the market price.
   D. The risk-adjusted default probabilities are used in model calibration.

Concept Checker Answers

1. **B** Senior tranches are perceived to be the safest, so they receive the lowest coupon. The equity tranche receives residual cash flows and no explicit coupon. Although the mezzanine tranche is often thin, the equity tranche is typically the thinnest slice.

2. **A** The interest rate on the loans = 4% (LIBOR) + 3% (spread) = 7%. Therefore, the total collateral cash flows in the first period = 100 × \$500,000 × 7% × (1 – 0.02) = \$3,430,000. The senior tranche receives \$50 million × 0.75 × (4% + 2%) = \$2,250,000. Similarly, the mezzanine tranche receives \$50 million × 0.20 × (4% + 6%) = \$1,000,000. Next, the residual cash flows are calculated: \$3,430,000 – \$2,250,000 – \$1,000,000 = \$180,000. Since \$180,000 < \$1,000,000, all cash flows are claimed by the equity investors and there is no diversion to the trust account.

3. **C** The custodian and trustee play the least important roles in the securitization process. The servicer, originator, underwriter, credit rating agency, and manager all face conflicts of interest to varying degrees.

4. **A** Statement I is true. Increasing default correlation increases the likelihood of more extreme portfolio returns (very high or very low number of defaults). The increased likelihood of high defaults negatively impacts the senior tranche. On the other hand, the increased likelihood of few defaults benefits the equity tranche as it bears first loss. Statement II is false. At high default rates, increasing the correlation increases the likelihood of more extreme portfolio returns which benefits equity investors and mezzanine investors.

5. **C** Starting with observed market prices and a pricing function for the tranches, it is possible to back out the implied correlation to calibrate the model price with the market price. The computation of implied correlation assumes constant pairwise correlation. Both credit default swap and tranche values are observed. Observed tranche values are used in conjunction with risk-neutral default probabilities to compute implied correlation.