The following is a review of the Risk Management and Investment Management principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Portfolio Construction

Topic 66

Exam Focus

This topic addresses techniques for optimal portfolio construction. We will discuss important inputs into the portfolio construction process as well as ways to modify allocations by refining the position alphas within a portfolio. This topic also goes into detail regarding transactions costs and how they influence allocation decisions with regard to portfolio monitoring and rebalancing. For the exam, pay attention to the discussions of refining alpha and the implications of transactions costs. Also, be familiar with the different techniques used to construct optimal portfolios.

The Portfolio Construction Process

LO 66.1: Distinguish among the inputs to the portfolio construction process.

The process of constructing an investment portfolio has several inputs which include:

- *Current portfolio*: The assets and weights in the current portfolio. Relative to the other inputs, the current portfolio input can be measured with the most certainty.
- *Alphas*: The excess return of each asset. This input is subject to error and bias and as a result is sometimes unreasonable.
- *Covariances*: Covariance measures how the returns of the assets in the portfolio are related. Estimates of covariance often display elements of uncertainty.
- *Transactions costs*: Like covariance, transaction costs are an important input for portfolio construction, however, these costs also contain a degree of uncertainty. Transaction costs must be amortized over the investment horizon in order to determine the optimal portfolio adjustments.
- *Active risk aversion*: This input must be consistent with the specified target active risk level. Active risk is another name for tracking error, which is the standard deviation of active return (i.e., excess return).

Refining Alphas

LO 66.2: Evaluate the methods and motivation for refining alphas in the implementation process.

The motivation for refining alpha is to address the various constraints that each investor or manager might have. For the investor, constraints might include not having any short positions and/or a restriction on the amount of cash held within the portfolio. For the manager, the constraints might include restrictions on allocations to certain stocks and/or making the portfolio neutral across sectors. The resulting portfolio will be different from a corresponding unconstrained portfolio and as a result will likely be less efficient. Constrained optimization methods for portfolio construction are often cumbersome to implement.

A method that involves refining the alphas can derive the optimal portfolio, given the consideration of portfolio constraints, in a less complicated manner. This method refines the optimal position alphas and then adjusts each position's allocation. In other words, if no short sales are allowed, then the modified alphas would be drawn closer to zero, and the optimization that would follow would call for a zero percent allocation to those short positions. If, in addition to short sales, all long position allocations were required to more closely resemble the benchmark weights, then all modified alphas would be pulled closer to zero relative to the original alphas, indicating that the constrained portfolio would more closely resemble the benchmark portfolio (i.e., since alpha is closer to zero, the returns between the benchmark and portfolio are now closer). The main idea to this approach is that refining alphas and then optimizing position allocations can replace even the most sophisticated portfolio construction process.

A manager can refine the alphas by procedures known as scaling and trimming. By considering the structure of alpha, we can understand how to use the technique of **scaling**.

$$\text{alpha} = (\text{volatility}) \times (\text{information coefficient}) \times (\text{score})$$

In this equation, score has a mean of zero and standard deviation of one. This means that alphas will have a mean zero and a range that is determined by the volatility (i.e., residual risk) and the information coefficient (i.e., correlation between actual and forecasted outcomes). The manager can rescale the alphas to make them have the proper scale for the portfolio construction process. For example, if the original alphas had a standard deviation of 2%, the rescaled alphas could have a lower standard deviation of 0.5%.

Trimming extreme values is another method of refining alpha. The manager should scrutinize alphas that are large in absolute value terms. "Large" might be defined as three times the scale of the alphas. It may be the case that such alphas are the result of questionable data, and the weights for those position allocations should be set to zero. Those extreme alphas that appear genuine may be kept but lowered to be within some limit, say, three times the scale.

LO 66.3: Describe neutralization and methods for refining alphas to be neutral.

Neutralization is the process of removing biases and undesirable bets from alpha. There are several types of neutralization: benchmark, cash, and risk-factor. In all cases, the type of neutralization and the strategy for the process should be specified before the process begins.

Benchmark neutralization involves adjusting the benchmark alpha to zero. This means the optimal position that uses the benchmark will have a beta of one. This ensures that the alphas are benchmark-neutral and avoids any issues with benchmark timing. For example, suppose that a modified alpha has a beta of 1.2. By making this alpha benchmark-neutral, a new modified alpha will be computed where the beta is reduced to one. Making the alphas cash-neutral involves adjusting the alphas so that the cash position will not be active. It is possible to simultaneously make alphas both cash and benchmark-neutral.

The risk-factor approach separates returns along several dimensions (e.g., industry). The manager can identify each dimension as a source of either risk or value added. The manager should neutralize the dimensions or factors that are a source of risk (for which the manager does not have adequate knowledge).

Transactions Costs

LO 66.4: Describe the implications of transaction costs on portfolio construction.

Transactions costs are the costs of moving from one portfolio allocation to another. They need to be considered in addition to the alpha and active risk inputs in the optimization process. When considering only alpha and active risk, any problem in setting the scale of the alphas can be offset by adjusting active risk aversion. The introduction of transactions costs increases the importance of the precision of the choice of scale. Some researchers propose that the accuracy of estimates of transactions costs is as important as the accuracy of alpha estimates. Furthermore, the existence of transactions costs increases the importance of having more accurate estimates of alpha.

When considering transactions costs, it is important to realize that these costs generally occur at a point in time while the benefits (i.e., the additional return) are realized over a time period. This means that the manager needs to have a rule concerning how to amortize the transactions costs over a given period. Beyond the implications of transactions costs, a full analysis would also consider the causes of transactions costs, how to measure them, and how to avoid them.

To illustrate the role of transactions costs and how to amortize them, we will assume forecasts can be made with certainty and the risk-free rate is zero. The cost of buying and selling stock is $0.05. The current prices of stock A and B are both $10. The forecasts are for the price of stock A to be $11 in one year and the price of stock B to be $12 in two years; therefore, the annualized alphas are the same at 10%. Also, neither stock will change in value after reaching the forecasted value. Now, assume in each successive year that the manager discovers a stock with the same properties as stock A and every two years a stock exactly like stock B. The manager would trade the stock-A type stocks each year and incur $0.10 in transactions costs at the end of each year. The alpha is 10%, and the transactions costs are 1% for type-A stocks for a net return of 9%. For the type-B stocks, the annual return is also 10%, but the transactions costs per year are only 0.5% because they are incurred every other year. Thus, on an annualized basis, the after-cost-return of type-B stocks is greater than that of type-A stocks.

Portfolio Construction Issues

LO 66.5: Assess the impact of practical issues in portfolio construction, such as determination of risk aversion, incorporation of specific risk aversion, and proper alpha coverage.

Practical issues in portfolio construction include the level of risk aversion, the optimal risk, and the alpha coverage.

Measuring the level of **risk aversion** is dependent on accurate measures of the inputs in the following expression:

$$\text{risk aversion} = \frac{\text{information ratio}}{2 \times \text{active risk}}$$

For example, assuming that the information ratio is 0.8 and the desired level of active risk is 10%, then the implied level of risk aversion is 0.04. Being able to quantify risk aversion allows the manager to understand a client's utility in a mean-variance framework. Utility can be measured as: excess return – (risk aversion × variance).

Professor's Note: Remember here that active risk is just another name for tracking error. Also note that in this risk aversion equation, the desired level of active risk is inputted as a percent instead of a decimal (i.e., 10 instead of 0.10).

Aversion to specific factor risk is important for two reasons. It can help the manager address the risks associated with having a position with the potential for huge losses, and the potential dispersion across portfolios when the manager manages more than one portfolio. This approach can help a manager decide the appropriate aversion to common and specific risk factors.

Proper **alpha coverage** refers to addressing the case where the manager has forecasts of stocks that are not in the benchmark and the manager doesn't have forecasts for assets in the benchmark. When the manager has information on stocks not in the benchmark, a benchmark weight of zero should be assigned with respect to benchmarking, but active weights can be assigned to generate active alpha.

When there is not a forecast for assets in the benchmark, alphas can be inferred from the alphas of assets for which there are forecasts. One approach is to first compute the following two measures:

$$\text{value-weighted fraction of stocks with forecasts} = \text{sum of active holdings with forecasts}$$

$$\text{average alpha for the stocks with forecasts} = \frac{\text{(weighted average of the alphas with forecasts)}}{\text{(value-weighted fraction of stocks with forecasts)}}$$

The second step is to subtract this measure from each alpha for which there is a forecast and set alpha to zero for assets that do not have forecasts. This provides a set of benchmark-neutral forecasts where assets without forecasts have an alpha of zero.

Portfolio Revisions and Rebalancing

LO 66.6: Describe portfolio revisions and rebalancing, and evaluate the tradeoffs between alpha, risk, transaction costs, and time horizon.

LO 66.7: Determine the optimal no-trade region for rebalancing with transaction costs.

If transactions costs are zero, a manager should revise a portfolio every time new information arrives. However, in a practical setting, the manager should make trading decisions based on expected active return, active risk, and transactions costs. The manager may wish to be conservative due to the uncertainties of these measures and the manager's ability to interpret them. Underestimating transactions costs, for example, will lead to trading too frequently. In addition, the frequent trading and short time-horizons would cause alpha estimates to exhibit a great deal of uncertainty. Therefore, the manager must choose an optimal time horizon where the certainty of the alpha is sufficient to justify a trade given the transactions costs.

The rebalancing decision depends on the tradeoff between transactions costs and the value added from changing the position. Portfolio managers must be aware of the existence of the no-trade region where the benefits are less than the costs. The benefit of adjusting the number of shares in a portfolio of a given asset is given by the following expression:

marginal contribution to value added = (alpha of asset) – [2 × (risk aversion) × (active risk) × (marginal contribution to active risk of asset)]

As long as this value is between the negative cost of selling and the cost of purchase, the manager would not trade that particular asset. In other words, the no-trade range is as follows:

–(cost of selling) < (marginal contribution to value added) < (cost of purchase)

Rearranging this relationship with respect to alpha gives a no-trade range for alpha:

[2 × (risk aversion) × (active risk) × (marginal contribution to active risk)] – (cost of selling) < *alpha of asset* < [2 × (risk aversion) × (active risk) × (marginal contribution to active risk)] + (cost of purchase)

The size of the no-trade region is determined by transactions costs, risk aversion, alpha and the riskiness of the assets.

Portfolio Construction Techniques

LO 66.8: Evaluate the strengths and weaknesses of the following portfolio construction techniques: screens, stratification, linear programming, and quadratic programming.

The following four generic classes of procedures cover most of the applications of institutional portfolio construction techniques: screens, stratification, linear programming, and quadratic programming. In each case the goal is the same: high alpha, low active risk, and low transactions costs. The success of a manager is determined by the value they can add minus any transaction costs:

$$(\text{portfolio alpha}) - (\text{risk aversion}) \times (\text{active risk})^2 - (\text{transactions costs})$$

Screens

Screens are accomplished by ranking the assets by alpha, choosing the top performing assets, and composing either an equally weighted or capitalization-based weighted portfolio. Screens can also rebalance portfolios; for example, the manager can sort the universe of portfolios by alpha; then, (1) divide the universe of assets into buy, hold, and sell decisions based on the rankings, (2) purchase any assets on the buy list not currently in the existing portfolio, and (3) sell any stocks in the portfolio that are on the sell list.

Screens are easy to implement and understand. There is a clear link between the cause (being in the buy/hold/sell class) and the effect (being a part of the portfolio). This technique is also robust in that extreme estimates of alpha will not bias the outcome. It enhances return by selecting high-alpha assets and controls risk by having a sufficient number of assets for diversification. Shortcomings of screening include ignoring information within the rankings, the fact there will be errors in the rankings, and excluding those categories of assets that tend to have low alphas (e.g., utility stocks). Also, other than having a large number of assets for diversification, this technique does not properly address risk management motives.

Stratification

Stratification builds on screens by ensuring that each category or stratum of assets is represented in the portfolio. The manager can choose to categorize the assets by economic sectors and/or by capitalization. If there are five categories and three capitalization levels (i.e., small, medium and large), then there will be 15 mutually exclusive categories. The manager would employ a screen on each category to choose assets. The manager could then weight the assets from each category based on their corresponding weights in the benchmark.

Stratification has the same benefits as screening and one fewer shortcoming in that it has solved the problem of the possible exclusion of some categories of assets. However, this technique still suffers from possible errors in measuring alphas.

Linear Programming

Linear programming uses a type of stratification based on characteristics such as industry, size, volatility, beta, etc. without making the categories mutually exclusive. The linear programming methodology will choose the assets that produce a portfolio which closely resembles the benchmark portfolio. This technique can also include transactions costs, reduce turnover, and set position limits.

Linear programming's strength is that the objective is to create a portfolio that closely resembles the benchmark. However, the result can be very different from the benchmark with respect to the number of assets and some risk characteristics.

Quadratic Programming

Quadratic programming explicitly considers alpha, risk, and transactions costs. Like linear programming techniques, it can also incorporate constraints. Therefore, it is considered the ultimate approach in portfolio construction. This approach is only as good as its data, however, as there are many opportunities to make mistakes. Although a small mistake could lead to a large deviation from the optimal portfolio, this is not necessarily the case since small mistakes tend to cancel out in the overall portfolio.

The following loss function provides a measure that illustrates how a certain level of mistakes may only lead to a small loss, but the losses increase dramatically when the mistakes exceed a certain level:

$$\frac{\text{loss}}{\text{value added}} = \left\{1 - \left[\frac{\text{actual market volatility}}{\text{estimated market volatility}}\right]^2\right\}^2$$

If actual market volatility is 20%, an underestimate of 1% will only produce a loss-to-value ratio of 0.0117. Underestimations of 2% and 3% will produce loss-to-value ratios equal to 0.055 and 0.1475, respectively. Thus, the increase in loss increases rapidly in response to given increases in error.

Portfolio Return Dispersion

LO 66.9: Describe dispersion, explain its causes, and describe methods for controlling forms of dispersion.

Dispersion is a measure of how much each individual client's portfolio might be different from the composite returns reported by the manager. One measure is the difference between the maximum return and minimum return for separate account portfolios. The basic causes of dispersion are the different histories and cash flows of each of the clients.

Managers can control some forms of dispersion but unfortunately not all forms. One source of dispersion beyond the manager's control is the differing constraints that each client has (e.g., not being able to invest in derivatives or other classes of assets). Managers do, however, have the ability to control the dispersion caused by different betas since this dispersion often results from the lack of proper supervision. If the assets differ between portfolios, the manager can control this source of dispersion by trying to increase the proportion of assets that are common to all the portfolios.

The existence of transactions costs implies that there is some optimal level of dispersion. To illustrate the role of transactions costs in causing dispersion, we will assume a manager has only one portfolio that is invested 60% stocks and 40% bonds. The manager knows the optimal portfolio is 62% stocks and 38% bonds, but transactions costs would reduce returns more than the gains from rebalancing the portfolio. If the manager acquires a second client, he can then choose a portfolio with weights 62% and 38% for that second client. Since one client has a 60/40 portfolio and the other has a 62/38 portfolio, there will be dispersion. Clearly, higher transactions costs can lead to a higher probability of dispersion.

A higher level of risk aversion and lower transactions costs leads to lower tracking error. Without transactions costs, there will be no tracking error or dispersion because all portfolios will be optimal. The following expression shows how dispersion is proportional to active risk:

$$E(\text{max portfolio return} - \text{min portfolio return}) = 2 \times N^{-1}(0.5^{1/J}) \times (\text{active risk})$$

where:
N^{-1} = inverse of the cumulative normal distribution function N
J = number of portfolios

Adding more portfolios will tend to increase the dispersion because there is a higher chance of an extreme value with more observations. Over time, as the portfolios are managed to pursue the same moving target, convergence will occur. However, there is no certainty as to the rate this might occur.

Key Concepts

LO 66.1

The inputs into the portfolio construction process are the current portfolio, the alphas, covariance estimates, transactions costs, and active risk aversion. With the exception of the current portfolio, all of these are subject to error and possible bias.

LO 66.2

Refining alpha is one method for including both investor constraints (e.g., no short selling) and manager constraints (e.g., proper diversification). Using refined alphas and then performing optimization can achieve the same goal as a complicated constrained optimization approach.

LO 66.3

Neutralization is the process of removing biases and undesirable bets from alphas.

Benchmark neutralization involves adjusting the benchmark alpha to zero. Cash neutralization eliminates the need for active cash management. Risk-factor neutralization neutralizes return dimensions that are only associated with risk and do not add value.

LO 66.4

Transactions costs have several implications. First, they may make it optimal not to adjust even in the face of new information. Second, transactions costs increase the importance of making alpha estimates more robust.

Including transactions costs can be complicated because they occur at one point in time, but the benefits of the portfolio adjustments are measured over the investment horizon.

LO 66.5

Practical issues in portfolio construction are the level of risk aversion, the optimal risk, and the alpha coverage. The inputs in computing the level of risk aversion need to be accurate. The aversion to a specific risk factor can help a manager address the risks of a position with a large potential loss and the dispersion across portfolios. Proper alpha coverage refers to addressing the case where the manager makes forecasts of stocks that are not in the benchmark and the manager not having forecasts for assets in the benchmark.

LO 66.6

In the process of portfolio revisions and rebalancing, there are tradeoffs between alpha, risk, transaction costs, and time horizon. The manager may wish to be conservative based on the uncertainties of the inputs. Also, the shorter the horizon, the more uncertain the alpha, which means the manager should choose an optimal time horizon where the certainty of the alpha is sufficient to justify a trade given the transactions costs.

LO 66.7

Because of transactions costs, there will be an optimal no-trade region when new information arrives concerning the alpha of an asset. That region would be determined by the level of risk aversion, active risk, the marginal contribution to active risk, and the transactions costs.

LO 66.8

Portfolio construction techniques include screens, stratification, linear programming, and quadratic programming. Stratification builds on screens, and quadratic programming builds on linear programming.

Screens simply choose assets based on raw alpha. Stratification first screens and then chooses stocks based on the screen and also attempts to include assets from all asset classes.

Linear programming attempts to construct a portfolio that closely resembles the benchmark by using such characteristics as industry, size, volatility, and beta. Quadratic programming builds on the linear programming methodology by explicitly considering alpha, risk, and transactions costs.

LO 66.9

For a manager with several portfolios, dispersion is the result of portfolio returns not being identical. The basic causes of dispersion are the different histories and cash flows of each of the clients. A manager can control this source of dispersion by trying to increase the proportion of assets that are common to all portfolios.

Concept Checkers

1. The most measurable of the inputs into the portfolio construction process is(are) the:
   A. position alphas.
   B. transactions costs.
   C. current portfolio.
   D. active risk aversion.

2. Which of the following is correct with respect to adjusting the optimal portfolio for portfolio constraints?
   A. No reliable method exists.
   B. By refining the alphas and then optimizing, it is possible to include constraints of both the investor and the manager.
   C. By refining the alphas and then optimizing, it is possible to include constraints of the investor, but not the manager.
   D. By optimizing and then refining the alphas, it is possible to include constraints of both the investor and the manager.

3. An increase in which of the following factors will increase the no-trade region for the alpha of an asset?
   I. Risk aversion.
   II. Marginal contribution to active risk.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

4. Which of the following statements most correctly describes a consideration that complicates the incorporation of transactions costs into the portfolio construction process?
   A. The transactions costs and the benefits always occur in two distinct time periods.
   B. The transactions costs are uncertain while the benefits are relatively certain.
   C. There are no complicating factors from the introduction of transactions costs.
   D. The transactions costs must be amortized over the horizon of the benefit from the trade.

5. A manager has forecasts of stocks A, B, and C, but not of stocks D and E. Stocks A, B, and D are in the benchmark portfolio. Stocks C and E are not in the benchmark portfolio. Which of the following are correct concerning specific weights the manager should assign in tracking the benchmark portfolio?
   A. $w_C = 0$.
   B. $w_D = 0$.
   C. $w_C = (w_A + w_B)/2$.
   D. $w_C = w_D = w_E$.

Concept Checker Answers

1. C The current portfolio is the only input that is directly observable.

2. B The approach of first refining alphas and then optimizing can replace even the most sophisticated portfolio construction process. With this technique both the investor and manager constraints are considered.

3. C This is evident from the definition of the no-trade region for the alpha of the asset.

   [2 × (risk aversion) × (active risk) × (marginal contribution to active risk)] – (cost of selling) < alpha of asset < [2 × (risk aversion) × (active risk) × (marginal contribution to active risk)] + (cost of purchase)

4. D A challenge is to correctly assign the transactions costs to projected future benefits. The transactions costs must be amortized over the horizon of the benefit from the trade. The benefits (e.g., the increase in alpha) occurs over time while the transactions costs generally occur at a specific time when the portfolio is adjusted.

5. A The manager should assign a tracking portfolio weight equal to zero for stocks for which there is a forecast but that are not in the benchmark. A weight should be assigned to Stock D, and it should be a function of the alphas of the other assets.

The following is a review of the Risk Management and Investment Management principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Portfolio Risk: Analytical Methods

Topic 67

Exam Focus

Due to diversification, the value at risk (VaR) of a portfolio will be less than or equal to the sum of the VaRs of the positions in the portfolio. If all positions are perfectly correlated, then the portfolio VaR equals the sum of the individual VaRs. A manager can make optimal adjustments to the risk of a portfolio with such measures as marginal VaR, incremental VaR, and component VaR. This topic is highly quantitative. Be able to find the optimal portfolio using the excess-return-to-marginal VaR ratios. For the exam, understand how correlations impact the measure of portfolio VaR. Also, it is important that you know how to compute incremental VaR and component VaR using the marginal VaR measure. We have included several examples to help with application of these concepts.

Portfolio theory depends a lot on statistical assumptions. In finance, researchers and analysts often assume returns are normally distributed. Such an assumption allows us to express relationships in concise expressions such as beta. Actually, beta and other convenient concepts can apply if returns follow an elliptical distribution, which is a broader class of distributions that includes the normal distribution. In what follows, we will assume returns follow an elliptical distribution unless otherwise stated.

LO 67.1: Define, calculate, and distinguish between the following portfolio VaR measures: individual VaR, incremental VaR, marginal VaR, component VaR, undiversified portfolio VaR, and diversified portfolio VaR.

Professor's Note: LO 67.1 is addressed throughout this topic.

Diversified Portfolio VaR

Diversified VaR is simply the VaR of the portfolio where the calculation takes into account the diversification effects. The basic formula is:

$$VaR_p = Z_c \times \sigma_p \times P$$

where:
Z_c = the z-score associated with the level of confidence c
σ_p = the standard deviation of the portfolio return
P = the nominal value invested in the portfolio

Examining the formula for the variance of the portfolio returns is important because it reveals how the correlations of the returns of the assets in the portfolio affect volatility. The variance formula is:

$$\sigma_P{}^2 = \sum_{i=1}^{N} w_i{}^2\sigma_i{}^2 + 2\sum_{i=1}^{N}\sum_{j<i}^{N} w_i w_j \rho_{i,j}\sigma_i\sigma_j$$

where:
$\sigma_P{}^2$ = the variance of the portfolio returns
w_i = the portfolio weight invested in position *i*
σ_i = the standard deviation of the return in position *i*
$\rho_{i,j}$ = the correlation between the returns of asset *i* and asset *j*

The standard deviation, denoted σ_P, is:

$$\sigma_P = \sqrt{\sigma_P{}^2} = \sqrt{\sum_{i=1}^{N} w_i{}^2\sigma_i{}^2 + 2\sum_{i=1}^{N}\sum_{j<i}^{N} w_i w_j \rho_{i,j}\sigma_i\sigma_j}$$

Clearly, the variance and standard deviation are lower when the correlations are lower.

In order to calculate delta-normal VaR with more than one risk factor, we need a covariance matrix that incorporates correlations between each risk factor in the portfolio and volatilities of each risk factor. If we know the volatilities and correlations, we can derive the standard deviation of the portfolio and the corresponding VaR measure. We will discuss how to calculate VaR using matrix multiplication later in this topic.

Individual VaR is the VaR of an individual position in isolation. If the proportion or weight in the position is w_i, then we can define the individual VaR as:

$$VaR_i = Z_c \times \sigma_i \times |P_i| = Z_c \times \sigma_i \times |w_i| \times P$$

where:
P = the portfolio value
P_i = the nominal amount invested in position i

We use the absolute value of the weight because both long and short positions pose risk.

LO 67.2: Explain the role of correlation on portfolio risk.

In a two-asset portfolio, the equation for the standard deviation is:

$$\sigma_P = \sqrt{w_1{}^2\sigma_1{}^2 + w_2{}^2\sigma_2{}^2 + 2\, w_1 w_2 \rho_{1,2}\sigma_1\sigma_2}$$

and the VaR is:

$$VaR_P = Z_c P\sqrt{w_1{}^2\sigma_1{}^2 + w_2{}^2\sigma_2{}^2 + 2\, w_1 w_2 \rho_{1,2}\sigma_1\sigma_2}$$

We can square Z_c and P and put them under the square-root sign. This allows us to express VaR_P as a function of the VaRs of the individual positions, which we express as VaR_i for each position i. For a two-asset portfolio we will have VaR_1 and VaR_2. If the correlation is zero, $\rho_{1,2} = 0$, then the third term under the radical is zero and:

$$\text{VaR for uncorrelated positions: } \text{VaR}_P = \sqrt{\text{VaR}_1^2 + \text{VaR}_2^2}$$

The other extreme is when the correlation is equal to unity, $\rho_{1,2} = \pm 1$. With perfect correlation, there is no benefit from diversification. For the two-asset portfolio, we find:

$$\text{Undiversified VaR} = \text{VaR}_P = \sqrt{\text{VaR}_1^2 + \text{VaR}_2^2 + 2\text{VaR}_1\text{VaR}_2} = \text{VaR}_1 + \text{VaR}_2$$

In general, undiversified VaR is the sum of all the VaRs of the individual positions in the portfolio when none of those positions are short positions.

Notice how evaluating VaR using both uncorrelated positions and perfectly correlated positions will place a lower and upper bound on the total (or portfolio) VaR. Total VaR will be less if the positions are uncorrelated and greater if the positions are correlated. The greatest risk is a correlation of −1 where one asset magnifies the loss of the other asset. The following examples illustrate this point.

Example: Computing portfolio VaR (part 1)

An analyst computes the VaR for the two positions in her portfolio. The VaRs: VaR_1 = \$2.4 million and VaR_2 = \$1.6 million. **Compute** VaR_P if the returns of the two assets are uncorrelated.

Answer:

For uncorrelated assets:

$$\text{VaR}_P = \sqrt{\text{VaR}_1^2 + \text{VaR}_2^2} = \sqrt{\left(2.4^2 + 1.6^2\right)(\$\text{millions})^2} = \sqrt{8.32(\$\text{millions})^2}$$

$$\text{VaR}_P = \$2.8844 \text{ million}$$

Example: Computing portfolio VaR (part 2)

An analyst computes the VaR for the two positions in her portfolio. The VaRs: VaR_1 = \$2.4 million and VaR_2 = \$1.6 million. **Compute** VaR_P if the returns of the two assets are perfectly correlated.

Answer:

For perfectly correlated assets:

$$\text{VaR}_P = \text{VaR}_1 + \text{VaR}_2 = \$2.4 \text{ million} + \$1.6 \text{ million} = \$4 \text{ million}$$

Under certain assumptions, the portfolio standard deviation of returns for a portfolio with more than two assets has a very concise formula. The assumptions are:

- The portfolio is equally weighted.
- All the individual positions have the same standard deviation of returns.
- The correlations between each pair of returns are the same.

The formula is then:

$$\sigma_P = \sigma\sqrt{\frac{1}{N} + \left(1 - \frac{1}{N}\right)\rho}$$

where:
N = the number of positions
σ = the standard deviation that is equal for all N positions
ρ = the correlation between the returns of each pair of positions

Professor's Note: This formula greatly simplifies the process of having to calculate portfolio standard deviation with a covariance matrix.

To demonstrate the benefits of diversification, we can simply set up a 2 × 2 table where there is a small and large correlation (ρ) column and a small and large sample size (N) row. Assuming that the standard deviation of returns is 20% for both assets, we see how the portfolio variance is affected by the different inputs.

Figure 1: Portfolio Standard Deviation

Sample size/correlation	ρ = *0.1*	ρ = *0.5*
N = 4	σ_P = 11.40%	σ_P = 15.81%
N = 10	σ_P = 8.72%	σ_P = 14.83%

Example: Computing portfolio VaR (part 3)

A portfolio has five positions of $2 million each. The standard deviation of the returns is 30% for each position. The correlations between each pair of returns is 0.2. **Calculate** the VaR using a Z-value of 2.33.

Answer:

The standard deviation of the portfolio returns is:

$$\sigma_P = 30\%\sqrt{\frac{1}{5} + \left(1 - \frac{1}{5}\right)0.2}$$

$$\sigma_P = 30\%\sqrt{0.36}$$

$$\sigma_P = 18\%$$

The VaR in nominal terms is:

$$\text{VaR}_p = Z_c \times \sigma_p \times V = (2.33)(18\%)(\$10 \text{ million})$$

$$\text{VaR}_p = \$4{,}194{,}000$$

Marginal VaR

Marginal VaR applies to a particular position in a portfolio, and it is the *per unit change in a portfolio VaR that occurs from an additional investment in that position*. Mathematically speaking, it is the partial derivative of the portfolio VaR with respect to the position:

$$\text{Marginal VaR} = \text{MVaR}_i = \frac{\partial \text{VaR}_P}{\partial(\text{monetary investment in i})} = Z_c \frac{\partial \sigma_P}{\partial w_i} = Z_c \frac{\text{cov}(R_i, R_P)}{\sigma_P}$$

Using CAPM methodology, we know a regression of the returns of a single asset *i* in a portfolio on the returns of the entire portfolio gives a beta, denoted β_i, which is a concise measure that includes the covariance of the position's returns with the total portfolio:

$$\beta_i = \frac{\text{cov}(R_i, R_P)}{\sigma_P^2}$$

Using the concept of beta gives another expression for marginal VaR:

$$\text{Marginal VaR} = \text{MVaR}_i = \frac{\text{VaR}_P}{\text{portfolio value}} \times \beta_i$$

Example: Computing marginal VaR

Assume Portfolio X has a VaR of €400,000. The portfolio is made up of four assets: Asset A, Asset B, Asset C, and Asset D. These assets are equally weighted within the portfolio and are each valued at €1,000,000. Asset A has a beta of 1.2. **Calculate** the marginal VaR of Asset A.

Answer

$$\text{Marginal VaR}_A = (\text{VaR}_P \,/\, \text{portfolio value}) \times \beta_A$$

$$\text{Marginal VaR}_A = (400{,}000 \,/\, 4{,}000{,}000) \times 1.2 = 0.12$$

Thus, portfolio VaR will change by 0.12 for each euro change in Asset A.

Incremental VaR

LO 67.3: Describe the challenges associated with VaR measurement as portfolio size increases.

Incremental VaR is the change in VaR from the addition of a new position in a portfolio. Since it applies to an entire position, it is generally larger than marginal VaR and may include nonlinear relationships, which marginal VaR generally assumes away. The problem with measuring incremental VaR is that, in order to be accurate, a full revaluation of the portfolio after the addition of the new position would be necessary. The incremental VaR is the difference between the new VaR from the revaluation minus the VaR before the addition. The revaluation requires not only measuring the risk of the position itself, but it also requires measuring the change in the risk of the other positions that are already in the portfolio. For a portfolio with hundreds or thousands of positions, this would be time consuming. Clearly, VaR measurement becomes more difficult as portfolio size increases given the expansion of the covariance matrix. Using a shortcut approach for computing incremental VaR would be beneficial.

For small additions to a portfolio, we can approximate the incremental VaR with the following steps:

Step 1: Estimate the risk factors of the new position and include them in a vector [η].
Step 2: For the portfolio, estimate the vector of marginal VaRs for the risk factors [MVaR_j].
Step 3: Take the cross product.

This probably requires less work and is faster to implement because it is likely the managers already have estimates of the vector of MVaR_j values in Step 2.

Before we take a look at how to calculate incremental VaR, let's review the calculation of delta-normal VaR using matrix notation (i.e., using a covariance matrix).

Example: Computing VaR using matrix notation

A portfolio consists of assets A and B. These assets are the risk factors in the portfolio. The volatilities are 6% and 14%, respectively. There are $4 million and $2 million invested in them, respectively. If we assume they are uncorrelated with each other, **compute** the VaR of the portfolio using a confidence parameter, Z, of 1.65.

Answer:

We can use matrix notation to derive the dollar variance of the portfolio:

$$\sigma_p{}^2V^2 = [\$4\ \$2]\begin{vmatrix} 0.06^2 & 0 \\ 0 & 0.14^2 \end{vmatrix}\begin{vmatrix} \$4 \\ \$2 \end{vmatrix} = 0.0576 + 0.0784 = 0.136$$

This value is in ($ millions)^2. VaR is then the square root of the portfolio variance times 1.65:

$$\text{VaR} = (1.65)(\$368{,}782) = \$608{,}490$$

Professor's Note: Matrix multiplication consists of multiplying each row by each column. For example: $(4 \times 0.06^2) + (2 \times 0) = 0.0144$; $0.0144 \times 4 = 0.0576$. Had the positions been positively correlated, some positive value would replace the zeros in the covariance matrix.

Example: Computing incremental VaR

A portfolio consists of assets A and B. The volatilities are 6% and 14%, respectively. There are $4 million and $2 million invested in them respectively. If we assume they are uncorrelated with each other, **compute** the incremental VaR for an increase of $10,000 in Asset A. Assume a Z-score of 1.65.

Answer:

To find incremental VaR, we compute the per dollar covariances of each risk factor:

$$\begin{vmatrix} \text{cov}(R_A, R_P) \\ \text{cov}(R_B, R_P) \end{vmatrix} = \begin{vmatrix} 0.06^2 & 0 \\ 0 & 0.14^2 \end{vmatrix}\begin{vmatrix} \$4 \\ \$2 \end{vmatrix} = \begin{vmatrix} 0.0144 \\ 0.0392 \end{vmatrix}$$

These per dollar covariances represent the covariance of a given risk factor with the portfolio. Thus, we can substitute these values into the marginal VaR equations for the risk factors as follows.

The marginal VaRs of the two risk factors are:

$$\text{MVaR}_A = Z_c \times \frac{\text{cov}(R_A, R_P)}{\sigma_P} = 1.65 \times \frac{0.0144}{\sqrt{0.136}} = 0.064428$$

$$\text{MVaR}_B = Z_c \times \frac{\text{cov}(R_B, R_P)}{\sigma_P} = 1.65 \times \frac{0.0392}{\sqrt{0.136}} = 0.175388$$

Since the two assets are uncorrelated, the incremental VaR of an additional \$10,000 investment in Position A would simply be \$10,000 times 0.064428, or \$644.28.

Component VaR

Component VaR for position *i*, denoted CVaR_i, is the amount of risk a particular fund contributes to a portfolio of funds. It will generally be less than the VaR of the fund by itself (i.e., stand alone VaR) because of diversification benefits at the portfolio level. In a large portfolio with many positions, the approximation is simply the marginal VaR multiplied by the dollar weight in position *i*:

$$\text{CVaR}_i = (\text{MVaR}_i) \times (w_i \times P) = \text{VaR} \times \beta_i \times w_i$$

$$= (\alpha \times \sigma_p \times P) \times \beta_i \times w_i = (\alpha \times \sigma_i \times w_i \times P) \times \rho_i = \text{VaR}_i \times \rho_i$$

The last two components consider the fact that $\text{beta}_i = (\rho_i \times \sigma_i) / \sigma_P$.

Using CVaR_i, we can express the total VaR of the portfolio as:

$$\text{VaR} = \sum_{i=1}^{N} \text{CVaR}_i = \text{VaR}\left(\sum_{i=1}^{N} w_i \times \beta_i\right)$$

Given the way the betas were computed we know: $\left(\sum_{i=1}^{N} w_i \times \beta_i\right) = 1$

Example: Computing component VaR (Example 1)

Assume Portfolio X has a VaR of €400,000. The portfolio is made up of four assets: Asset A, Asset B, Asset C, and Asset D. These assets are equally weighted within the portfolio and are each valued at €1,000,000. Asset A has a beta of 1.2. **Calculate** the component VaR of Asset A.

Answer:

Component $\text{VaR}_A = \text{VaR}_P \times \beta_A \times$ asset weight

Component $\text{VaR}_A = 400{,}000 \times 1.2 \times (1{,}000{,}000 / 4{,}000{,}000) =$ €120,000

Thus, portfolio VaR will decrease by €120,000 if Asset A is removed.

Example: Computing component VaR (Example 2, Part 1)

Recall our previous incremental VaR example of a portfolio invested $4 million in A and $2 million in B. Using their respective marginal VaRs, 0.064428 and 0.175388, **compute** the component VaRs.

Answer:

$$CVaR_A = (MVaR_A) \times (w_A \times P) = (0.064428) \times (\$4 \text{ million}) = \$257{,}713$$

$$CVaR_B = (MVaR_B) \times (w_B \times P) = (0.175388) \times (\$2 \text{ million}) = \$350{,}777$$

Professor's Note: The values have been adjusted for rounding.

Example: Computing component VaR (Example 2, Part 2)

Using the results from the previous example, **compute** the percent of contribution to VaR of each component.

Answer:

The answer is the sum of the component VaRs divided into each individual component VaR:

$$\text{\% contribution to VaR from A} = \frac{\$257{,}713}{(\$257{,}713 + \$350{,}777)} = 42.35\%$$

$$\text{\% contribution to VaR from B} = \frac{\$350{,}777}{(\$257{,}713 + \$350{,}777)} = 57.65\%$$

Normal distributions are a subset of the class of distributions called elliptical distributions. As a class, elliptical distributions have fewer assumptions than normal distributions. Risk management often assumes elliptical distributions, and the procedures to estimate component VaRs up to this point have applied to elliptical distributions.

If the returns do not follow an elliptical distribution, we can employ other procedures to compute component VaR. If the distribution is homogeneous of degree one, for example, then we can use Euler's theorem to estimate the component VaRs. The return of a portfolio of assets is homogeneous of degree one because, for some constant, *k*, we can write:

$$k \times R_P = \sum_{i=1}^{N} k \times w_i \times R_i$$

The following steps can help us find component VaRs for a non-elliptical distribution using historical returns:

Step 1: Sort the historical returns of the portfolio.
Step 2: Find the return of the portfolio, which we will designate $R_{P(VaR)}$, that corresponds to a return that would be associated with the chosen VaR.
Step 3: Find the returns of the individual positions that occurred when $R_{P(VaR)}$ occurred.
Step 4: Use each of the position returns associated with $R_{P(VaR)}$ for component VaR for that position.

To improve the estimates of the component VaRs, an analyst should probably obtain returns for each individual position for returns of the portfolio slightly above and below $R_{P(VaR)}$. For each set of returns for each position, the analyst would compute an average to better approximate the component VaR of the position.

Managing Portfolios Using VaR

LO 67.4: Apply the concept of marginal VaR to guide decisions about portfolio VaR.

LO 67.5: Explain the risk-minimizing position and the risk and return-optimizing position of a portfolio.

A manager can *lower a portfolio VaR by lowering allocations to the positions with the highest marginal VaR.* If the manager keeps the total invested capital constant, this would mean increasing allocations to positions with lower marginal VaR. Portfolio risk will be at a global minimum where all the marginal VaRs are equal for all *i* and *j*:

$$MVaR_i = MVaR_j$$

We can use our earlier example to see how we can use marginal VaRs to make decisions to lower the risk of the entire portfolio. In the earlier example, Position A has the smaller MVaR; therefore, we will compute the marginal VaRs and total VaR for a portfolio which has \$5 million invested in A and \$1 million in B. The portfolio variance is:

$$\sigma_P{}^2V^2 = [\$5\ \$1]\begin{bmatrix} 0.06^2 & 0 \\ 0 & 0.14^2 \end{bmatrix}\begin{bmatrix} \$5 \\ \$1 \end{bmatrix} = 0.0900 + 0.0196 = 0.1096$$

This value is in (\$ millions)^2. VaR is then the square root of the portfolio variance times 1.65 (95% confidence level):

$$VaR = (1.65)(\$331{,}059) = \$546{,}247$$

The VaR of \$546,247 is less than the VaR of \$608,490, which was produced when Portfolio A had a lower weight. We can see that the marginal VaRs are now much closer in value:

$$\begin{bmatrix} cov(R_A,R_P) \\ cov(R_B,R_P) \end{bmatrix} = \begin{bmatrix} 0.06^2 & 0 \\ 0 & 0.14^2 \end{bmatrix}\begin{bmatrix} \$5 \\ \$1 \end{bmatrix} = \begin{bmatrix} 0.0180 \\ 0.0196 \end{bmatrix}$$

The marginal VaRs of the two positions are:

$$\text{MVaR}_A = Z_c \times \frac{\text{cov}(R_A, R_P)}{\sigma_P} = 1.65 \times \frac{0.0180}{\sqrt{0.1096}} = 0.08971$$

$$\text{MVaR}_B = Z_c \times \frac{\text{cov}(R_B, R_P)}{\sigma_P} = 1.65 \times \frac{0.0196}{\sqrt{0.1096}} = 0.09769$$

LO 67.6: Explain the difference between risk management and portfolio management, and describe how to use marginal VaR in portfolio management.

As the name implies, risk management focuses on risk and ways to reduce risk; however, minimizing risk may not produce the optimal portfolio. Portfolio management requires assessing both risk measures and return measures to choose the optimal portfolio. Traditional efficient frontier analysis tells us that the minimum variance portfolio is not optimal. We should note that the **efficient frontier** is the plot of portfolios that have the lowest standard deviation for each expected return (or highest return for each standard deviation) when plotted on a plane with the vertical axis measuring return and the horizontal axis measuring the standard deviation. The optimal portfolio is represented by the point where a ray from the risk-free rate is just tangent to the efficient frontier. That optimal portfolio has the highest Sharpe ratio:

$$\text{Sharpe ratio} = \frac{(\text{portfolio return} - \text{risk-free rate})}{(\text{standard deviation of portfolio return})}$$

We can modify this formula by replacing the standard deviation with VaR so that the focus then becomes the excess return of the portfolio over VaR:

$$\frac{(\text{portfolio return} - \text{risk-free rate})}{(\text{VaR of portfolio})}$$

This ratio is maximized when the excess return in each position divided by its respective marginal VaR equals a constant. In other words, at the optimum:

$$\frac{(\text{Position i return} - \text{risk-free rate})}{(\text{MVaR}_i)} = \frac{(\text{Position j return} - \text{risk-free rate})}{(\text{MVaR}_j)}$$

for all positions *i* and *j*

Professor's Note: Equating the excess return/MVaR ratios will obtain the optimal portfolio. This differs from equating just the MVaRs, which obtains the portfolio with the lowest portfolio VaR.

Assuming that the returns follow elliptical distributions, we can represent the condition in a more concise fashion by employing betas, β_i, which are obtained from regressing each position's return on the portfolio return:

$$\frac{(\text{Position i return} - \text{risk-free rate})}{\beta_i} = \frac{(\text{Position j return} - \text{risk-free rate})}{\beta_j}$$

for all positions *i* and *j*

The portfolio weights that make these ratios equal will be the optimal portfolio. We now turn our attention to determining the optimal portfolio for our example portfolio of A and B. We will assume the expected excess return of A is 6% and that of B is 11%. Even without this information, we should know that the optimal portfolio will have an allocation in A less than \$5 million and in B greater than \$1 million. This is because the marginal VaRs were almost equal with those allocations. *Thus, the resulting portfolio would be close to the minimum variance,* which will not be optimal. We might want to find out how to adjust the allocation with respect to the original values of \$4 million in A and \$2 million in B. By comparing the ratios of the two assets we find:

$$\frac{\text{Excess return of A}}{\text{MVaR}_A} = \frac{0.06}{0.064428} = 0.9313$$

$$\frac{\text{Excess return of B}}{\text{MVaR}_B} = \frac{0.11}{0.175388} = 0.6272$$

We see that there is too much allocated in B. Before we adjust the portfolio, we compute the excess-return-to-VaR ratio for the entire portfolio. The return is:

$$\%\ \text{excess return on portfolio} = 7.67\% = \frac{\$4\ \text{million}}{\$6\ \text{million}}(6\%) + \frac{\$2\ \text{million}}{\$6\ \text{million}}(11\%)$$

The return to VaR (scaled by the size of the portfolio) is:

$$0.7559 = \frac{0.0767}{\$608{,}490} \times \$6\ \text{million}$$

Now, because the return to MVaR ratio was greater for A, we will increase the allocation in A to \$4.5 million and decrease that in B to \$1.5 million. With those changes, the portfolio variance is:

$$\sigma_p{}^2V^2 = [\$4.5\ \$1.5]\begin{bmatrix} 0.06^2 & 0 \\ 0 & 0.14^2 \end{bmatrix}\begin{bmatrix} \$4.5 \\ \$1.5 \end{bmatrix} = 0.0729 + 0.0441 = 0.1170$$

This value is in (\$ millions)^2. VaR is then the square root of the portfolio variance times 1.65 (95% confidence level):

$$\text{VaR} = (1.65)(\$342{,}053) = \$564{,}387$$

In this case, the marginal VaRs are found by:

$$\begin{bmatrix} \text{cov}(R_A, R_P) \\ \text{cov}(R_B, R_P) \end{bmatrix} = \begin{bmatrix} 0.06^2 & 0 \\ 0 & 0.14^2 \end{bmatrix} \begin{bmatrix} \$4.5 \\ \$1.5 \end{bmatrix} = \begin{bmatrix} 0.0162 \\ 0.0294 \end{bmatrix}$$

The marginal VaRs of the two positions are then:

$$\text{MVaR}_A = Z_c \times \frac{\text{cov}(R_A, R_P)}{\sigma_P} = 1.65 \times \frac{0.0162}{\sqrt{0.1170}} = 0.0781$$

$$\text{MVaR}_B = Z_c \times \frac{\text{cov}(R_B, R_P)}{\sigma_P} = 1.65 \times \frac{0.0294}{\sqrt{0.1170}} = 0.1418$$

We see the expected excess-return-to-marginal VaR ratios are much closer:

$$\frac{0.06}{0.0781} = 0.7678$$

$$\frac{0.11}{0.1418} = 0.7756$$

The portfolio return is now:

$$\%\text{ excess return on portfolio} = 7.25\% = \frac{\$4.5\text{ million}}{\$6\text{ million}}(6\%) + \frac{\$1.5\text{ million}}{\$6\text{ million}}(11\%)$$

The portfolio return divided by the portfolio VaR has risen. The return to VaR (scaled by the size of the portfolio) is:

$$0.7707 = \frac{0.0725}{\$564{,}387} \times \$6\text{ million}$$

This is greater than the 0.7559 value associated with the original $4 million and $2 million allocations. The result is a more optimal portfolio allocation.

Key Concepts

LO 67.1

Diversified VaR is simply the VaR of the portfolio where the calculation takes into account the diversification effects.

Individual VaR is the VaR of an individual position in isolation.

Diversified VaR is simply the VaR of the portfolio where the calculation takes into account the diversification effects. The basic formula is:

$$\text{VaR}_p = Z_c \times \sigma_p \times P$$

where:
Z_c = the *z*-score associated with the level of confidence *c*
σ_p = the standard deviation of the portfolio return
P = the nominal value invested in the portfolio

Individual VaR is the VaR of an individual position in isolation. If the proportion or weight in the position is w_i, then we can define the individual VaR as:

$$\text{VaR}_i = Z_c \times \sigma_i \times |P_i| = Z_c \times \sigma_i \times |w_i| \times P$$

where:
P = the portfolio value
P_i = the nominal amount invested in position i

Marginal VaR is the change in a portfolio VaR that occurs from an additional one unit investment in a given position. Useful representations are:

$$\text{Marginal VaR} = \text{MVaR}_i = Z_c \frac{\text{cov}(R_i, R_P)}{\sigma_P}$$

$$\text{Marginal VaR} = \text{MVaR}_i = \frac{\text{VaR}}{P} \times \beta_i$$

Incremental VaR is the change in VaR from the addition of a new position in a portfolio. It can be calculated precisely from a total revaluation of the portfolio, but this can be costly. A less costly approximation is found by (1) breaking down the new position into risk factors, (2) multiplying each new risk factor times the corresponding partial derivative of the portfolio with respect to the risk factor, and then (3) adding up all the values.

Component VaR for position *i*, denoted CVaR_i, is the amount a portfolio VaR would change from deleting that position in a portfolio. In a large portfolio with many positions, the approximation is simply the marginal VaR multiplied by the dollar weight in position *i*:

$$\text{CVaR}_i = (\text{MVaR}_i) \times (w_i \times P) = \text{VaR} \times \beta_i \times w_i$$

There is a method for computing component VaRs for distributions that are not elliptical. The procedure is to sort the historical returns of the portfolio and designate a portfolio return that corresponds to the loss associated with the VaR and then find the returns of each of the components associated with that portfolio loss. Those position returns can be used to compute component VaRs.

LO 67.2

For a two-asset portfolio, two special cases are:

1. VaR for uncorrelated positions:
   $$\text{VaR}_P = \sqrt{\text{VaR}_1^{\,2} + \text{VaR}_2^{\,2}}$$
2. VaR for perfectly correlated positions:
   $$\text{Undiversified VaR} = \text{VaR}_P = \sqrt{\text{VaR}_1^{\,2} + \text{VaR}_2^{\,2} + 2\text{VaR}_1\text{VaR}_2} = \text{VaR}_1 + \text{VaR}_2$$

LO 67.3

The incremental VaR is the difference between the new VaR from the revaluation minus the VaR before the addition. The revaluation requires not only measuring the risk of the position itself, but it also requires measuring the change in the risk of the other positions that are already in the portfolio. For a portfolio with hundreds or thousands of positions, this would be time consuming.

LO 67.4

Portfolio risk will be at a global minimum where all the marginal VaRs are equal for all *i* and *j*:

$$\text{MVaR}_i = \text{MVaR}_j$$

LO 67.5

Equating the MVaRs will obtain the portfolio with the lowest portfolio VaR. Equating the excess return/MVaR ratios will obtain the optimal portfolio.

LO 67.6

The optimal portfolio is the one for which all excess-return-to-marginal VaR ratios are equal:

$$\frac{(\text{Position i return} - \text{risk-free rate})}{(\text{MVaR}_i)} = \frac{(\text{Position j return} - \text{risk-free rate})}{(\text{MVaR}_j)}$$

Concept Checkers

1. Which of the following is the best synonym for diversified VaR?
   A. Vector VaR.
   B. Position VaR.
   C. Portfolio VaR.
   D. Incidental VaR.

2. When computing individual VaR, it is proper to:
   A. use the absolute value of the portfolio weight.
   B. use only positive weights.
   C. use only negative weights.
   D. compute VaR for each asset within the portfolio.

3. A portfolio consists of two positions. The VaR of the two positions are \$10 million and \$20 million. If the returns of the two positions are not correlated, the VaR of the portfolio would be closest to:
   A. \$5.48 million.
   B. \$15.00 million
   C. \$22.36 million.
   D. \$25.00 million.

4. Which of the following is true with respect to computing incremental VaR? Compared to using marginal VaRs, computing with full revaluation is:
   A. more costly, but less accurate.
   B. less costly, but more accurate.
   C. less costly, but also less accurate.
   D. more costly, but also more accurate.

5. A portfolio has an equal amount invested in two positions, X and Y. The expected excess return of X is 9% and that of Y is 12%. Their marginal VaRs are 0.06 and 0.075, respectively. To move toward the optimal portfolio, the manager will probably:
   A. increase the allocation in Y and/or lower that in X.
   B. increase the allocation in X and/or lower that in Y.
   C. do nothing because the information is insufficient.
   D. not change the portfolio because it is already optimal.

Concept Checker Answers

1. C Portfolio VaR should include the effects of diversification. None of the other answers are types of VaRs.

2. A The expression for individual VaR is $\text{VaR}_i = Z_c \times \sigma \times |P_i| = Z \times \sigma_i \times |w_i| \times P$. The absolute value signs indicate that we need to measure the risk of both positive and negative positions, and risk cannot be negative.

3. C For uncorrelated positions, the answer is the square root of the sum of the squared VaRs:

$$\text{VaR}_P = \sqrt{(10^2 + 20^2)} \times (\$\ \text{million}) = \$22.36 \text{ million.}$$

4. D Full revaluation means recalculating the VaR of the entire portfolio. The marginal VaRs are probably already known, so using them is probably less costly, but will not be as accurate.

5. A The expected excess-return-to-MVaR ratios for X and Y are 1.5 and 1.6, respectively. Therefore, the portfolio weight in Y should increase to move the portfolio toward the optimal portfolio.

The following is a review of the Risk Management and Investment Management principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

VaR and Risk Budgeting in Investment Management

Topic 68

Exam Focus

Banks on the "sell side" of the investment industry have long used risk budgeting and value at risk (VaR). There is a trend for the "buy side" investment firms to increasingly use VaR. One reason for increased demand for risk budgeting is the increased complexity, dynamics, and globalization of the investment industry. Use of VaR can help set better guidelines than more traditional limits. By measuring marginal and incremental VaRs, a manager can make better decisions concerning portfolio weights. For the exam, be comfortable with the concept of surplus at risk (SaR). Also, understand how to budget risk across asset classes and active managers.

Risk Budgeting

LO 68.1: Define risk budgeting.

Risk budgeting is a top-down process that involves choosing and managing exposures to risk. The main idea is that the risk manager establishes a risk budget for the entire portfolio and then allocates risk to individual positions based on a predetermined fund risk level. The risk budgeting process differs from market value allocation since it involves the allocation of risk.

Managing Risk With VaR

LO 68.2: Describe the impact of horizon, turnover and leverage on the risk management process in the investment management industry.

The "sell side" of the investment industry largely consists of banks that have developed VaR techniques and have used them for many years. Investors make up the "buy side" of the investment industry. Investors are now using VaR techniques, but they have to adapt them to the different nature of that side of the business. To understand why the needs are different, we should compare the characteristics of the two "sides." Figure 1 makes direct comparisons.

Figure 1: Sell Side and Buy Side Characteristics

Characteristic	*Sell Side*	*Buy Side*
Horizon	Short-term (days)	Long-term (month or more)
Turnover	Fast	Slow
Leverage	High	Low
Risk measures	VaR Stress tests	Asset allocation Tracking error
Risk controls	Position limits VaR limits Stop-loss rules	Diversification Benchmarking Investment guidelines

Banks trade rapidly, which is why they cannot rely on traditional measures of risk that are based on historical data. For banks, yesterday's risk may not have anything to do with today's positions. Investors usually try to hold positions for longer periods of time (e.g., years).

Having a more dynamic method for measuring risk such as VaR is also important for banks because of their high leverage. Institutional investors often have much stronger constraints with respect to leverage; therefore, they have a much lower need to control downside risk.

The Investment Process

LO 68.3: Describe the investment process of large investors such as pension funds.

The *first step* in the investment process is to determine the long-term, strategic asset allocations. Usually, the goal of the first step is to balance returns and risks using methods like mean-variance portfolio optimization. This step determines the allocations to asset classes such as domestic and foreign stocks, domestic and foreign bonds, and alternative investments such as real estate, venture capital, and hedge funds. Making this allocation relies on passive indices and other benchmarks to help measure the properties of the investment, and the availability of passive indices helps make the allocations feasible.

The *second step* in the investment process is to choose the managers who may either passively manage the fund (i.e., simply track the benchmarks) or actively manage the fund in an effort to outperform the benchmarks. The investors should review the managers' activities and performance periodically. Their activities should conform to a list of guidelines, which includes the types of investments and risk exposure restrictions such as beta and duration. Managers' performance can be evaluated by analyzing their tracking error.

VaR risk management systems are beginning to become more important because of the globalization of available investments and the increased complexity of investments. Also, investment companies are becoming more dynamic, which makes it more difficult to assess risk. With many managers, for example, each of the managers may make changes within his constraints, but the collective changes could be difficult to gauge with historical measures. In sum, because of increased globalization, complexity, and the dynamic nature of the investment industry, simply measuring risk using historical measures is no longer adequate, which has increased the need for VaR.

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Hedge Fund Issues

LO 68.4: Describe the risk management challenges associated with investments in hedge funds.

Hedge funds are a very heterogeneous class of assets that include a variety of trading strategies. Since they often use leverage and trade a great deal, their risk characteristics may be more similar to the "sell side" of the industry. Hedge funds have some other risks like liquidity and low transparency. Liquidity risk has many facets. First, there is the obvious potential loss from having to liquidate too quickly. Second, there is the difficulty of measuring the exact value of the fund to be able to ascertain its risk. Furthermore, the low liquidity tends to lower the volatility of historical prices as well as the correlations of the positions. These properties will lead to an underestimation of traditional measures of risk. In addition to these risks, there is the low level of transparency. This makes the risk measurement difficult with respect to both the size and type. Not knowing the type of risk increases the difficulty of risk management for the entire portfolio in which an investor might include hedge funds.

Absolute vs. Relative Risk and Policy Mix vs. Active Risk

LO 68.5: Distinguish among the following types of risk: absolute risk, relative risk, policy-mix risk, active management risk, funding risk, and sponsor risk.

Absolute or asset risk refers to the total possible losses over a horizon. It is simply measured by the return over the horizon. **Relative risk** is measured by excess return, which is the dollar loss relative to a benchmark. The shortfall is measured as the difference between the fund return and that of a benchmark in dollar terms. VaR techniques can apply to tracking error (i.e., standard deviation of excess return) if the excess return is normally distributed.

Professor's Note: The author's definition of tracking error differs from the definition of tracking error in other assigned readings. Jorion defines tracking error as active return minus the benchmark return. In other readings, this value is simply the excess return and tracking error is the volatility (i.e., standard deviation) of the excess return. Throughout this topic, we have expressed excess return as portfolio return minus benchmark return and tracking error as the volatility of the excess return. This methodology follows the definition of tracking error on previous FRM exams.

Distinguishing **policy mix** from **active risk** is important when an investment firm allocates funds to different managers in various asset classes. This breaks down the risk of the total portfolio into that associated with the target policy (i.e., the weights assigned to the various funds in the policy) and the risk from the fact that managers may make decisions which lead to deviations from the designated weights. VaR analysis is especially useful here because it can show the risk exposure associated with the two types of risk and how they affect the overall risk of the entire portfolio. Often, active management risk is not much of a problem for several reasons:

- For well-managed funds, it is usually fairly small for each of the individual funds.

- There will be diversification effects across the deviations.
- There can be diversification effects with the policy mix VaR to actually lower the total portfolio VaR.

Funding Risk

Funding risk refers to being able to meet the obligations of an investment company (e.g., a pension's payout to retirees). Put another way, funding risk is the risk that the value of assets will not be sufficient to cover the liabilities of the fund. The level of funding risk varies dramatically across different types of investment companies. Some have zero, while defined benefit pension plans have the highest.

The focus of this analysis is the surplus, which is the difference between the value of the assets and the liabilities, and the change in the surplus, which is the difference between the change in the assets and liabilities:

$$\text{Surplus} = \text{Assets} - \text{Liabilities}$$

$$\Delta\text{Surplus} = \Delta\text{Assets} - \Delta\text{Liabilities}$$

Typically, in managing funding risk, an analyst will transform the nominal return on the surplus into a return on the assets, and break down the return as indicated:

$$R_{\text{surplus}} = \frac{\Delta\text{Surplus}}{\text{Assets}} = \frac{\Delta\text{Assets}}{\text{Assets}} - \left(\frac{\Delta\text{Liabilities}}{\text{Liabilities}}\right)\left(\frac{\text{Liabilities}}{\text{Assets}}\right) = R_{\text{asset}} - R_{\text{liabilities}}\left(\frac{\text{Liabilities}}{\text{Assets}}\right)$$

Evaluating this expression requires assumptions about the liabilities, which are in the future and uncertain. For pension funds, liabilities represent "accumulated benefit obligations," which are the present value of pension benefits owed to the employees and other beneficiaries. Determining the present value requires a discount rate, which is usually tied to some current level of interest rates in the market. An ironic aspect of funding risk is that assets for meeting the obligations like equities and bonds usually increase in value when interest rates decline, but the present value of future obligations may increase even more. When assets and liabilities change by different amounts, this affects the surplus, and the resulting volatility of the surplus is a source of risk. If the surplus turns negative, additional contributions will be required. This is called **surplus at risk** (SaR).

One answer to this problem is to immunize the portfolio by making the duration of the assets equal that of the liabilities. This may not be possible since the necessary investments may not be available, and it may not be desirable because it may mean choosing assets with a lower return.

Example: Determining a fund's risk profile

The XYZ Retirement Fund has $200 million in assets and $180 million in liabilities. Assume that the expected return on the surplus, scaled by assets, is 4%. This means the surplus is expected to grow by $8 million over the first year. The volatility of the surplus is 10%. Using a Z-score of 1.65, **compute** VaR and the associated deficit that would occur with the loss associated with the VaR.

Answer:

First, we calculate the expected value of the surplus. The current surplus is $20 million (= $200 million – $180 million). It is expected to grow another $8 million to a value of $28 million. As for the VaR:

VaR = (1.65)(10%)($200 million) = $33 million

If this decline in value occurs, the deficit would be the difference between the VaR and the expected surplus value: $33 million – $28 million = $5 million.

Professor's Note: According to the assigned reading, the surplus at risk (SaR) is the VaR amount calculated above. Note that SaR on previous exams has been approached differently, as illustrated in the following example. Be prepared for either approach on the actual exam. In the example to follow, we will illustrate how to calculate the volatility of surplus growth. On previous FRM exams, this value has not been provided.

Example: Surplus at risk (via computing volatility of surplus)

The XYZ Retirement Fund has $200 million in assets and $180 million in liabilities. Assume that the expected annual return on the assets is 4% and the expected annual growth of the liabilities is 3%. Also assume that the volatility of the asset return is 10% and the volatility of the liability growth is 7%. **Compute** 95% surplus at risk assuming the correlation between asset return and liability growth is 0.4.

Answer:

First, compute the expected surplus growth:

$$200 \times (0.04) - 180 \times (0.03) = \$2.6 \text{ million}$$

Next, compute the volatility of the surplus growth. To compute the volatility you need to recall one of the properties of covariance discussed in the FRM Part I curriculum. The variance of assets minus liabilities [i.e., Var(A-L)] = Var(A) + Var(L) – 2 × Cov(A,L). Where covariance is equal to the standard deviation of assets times the standard deviation of liabilities times the correlation between the two. The asset and liability amounts will also need to be applied to this formula.

$$\text{Variance(A-L)} = 200^2 \times 0.10^2 + 180^2 \times 0.07^2 - 2 \times 200 \times 180 \times 0.10 \times 0.07 \times 0.4$$

$$= 400 + 158.76 - 201.6 = \$357.16 \text{ million}$$

$$\text{Standard deviation} = \sqrt{357.16} = \$18.89$$

Thus, SaR can be calculated by incorporating the expected surplus growth and standard deviation of the growth.

$$95\% \text{ SaR} = 2.6 - 1.65 \times 18.89 = \$28.57 \text{ million}$$

Professor's Note: Like VaR, SaR is a negative value since it is the surplus amount that is at risk. As a result, the negative sign is usually not presented since a negative amount is implied.

Plan Sponsor Risk

The plan sponsor risk is an extension of surplus risk and how it relates to those who ultimately bear responsibility for the pension fund. We can distinguish between the following risk measures:

- **Economic risk** is the variation in the total economic earnings of the plan sponsor. This takes into account how the risks of the various components relate to each other (e.g., the correlation between the surplus and operating profits).
- **Cash-flow risk** is the variation of contributions to the fund. Being able to absorb fluctuations in cash flow allows for a more volatile risk profile.

Ultimately, from the viewpoint of the sponsor, the focus should be on the variation of the economic value of the firm. The management should integrate the various risks associated with the movement of the assets and surplus with the overall financial goals of the sponsor. This is aligned with the current emphasis on enterprise-wide risk management.

Monitoring Risk With VaR

LO 68.6: Apply VaR to check compliance, monitor risk budgets, and reverse engineer sources of risk.

There are many types of risks that can increase dramatically in a large firm. For example, the "rogue trader" phenomenon is more likely in a large firm. This occurs when a manager of one of the accounts or funds within the larger portfolio deviates from her guidelines in terms of portfolio weights or even trades in unauthorized investments. Such deviations from compliance can be very short-term, and regular reporting measures may not catch the violations.

Risk management is necessary for all types of portfolios—even passively managed portfolios. Some analysts erroneously believe that passive investing, or benchmarking, does not require risk monitoring. This is not true because the risk profiles of the benchmarks change over time. In the late 1990s, a portfolio benchmarked to the S&P 500 would clearly have seen a change in risk exposures (e.g., an increase in the exposure to risks associated with the high-tech industry). A forward-looking risk measurement system would pick up on such trends.

Monitoring the risk of actively managed portfolios should help identify the reasons for changes in risk. Three explanations for dramatic changes in risk are (1) a manager taking on more risk, (2) different managers taking similar bets, and (3) more volatile markets. Thus, when there is an increase in the overall risk of a portfolio, top management would want to investigate the increase by asking the following questions.

Has the manager exceeded her risk budget? VaR procedures and risk management can allocate a risk budget to each manager. The procedures should give an indication if and why the manager exceeds the risk budget. Is it a temporary change from changes in the market? Has the manager unintentionally let the weights of the portfolio drift so as to increase risk? Or, more seriously, has the manager engaged in unauthorized trades?

Are managers taking too many of the same style bets? If the managers are acting independently, it is possible that they all start pursuing strategies with the same risk exposures. This could happen, for example, if all managers forecast lower interest rates. Bond managers would probably begin moving into long-term bonds, and equity managers would probably begin moving into stocks that pay a high and stable dividend like utility companies and REITs. This would drastically increase the interest rate risk of the overall portfolio.

Have markets become more volatile? If the risk characteristics of the entire market have changed, top management will have to decide if it is worth accepting the volatility or make decisions to reduce it by changing the target portfolio weights.

VaR can also be reverse engineered by utilizing the VaR tools outlined in the previous topic, such as component VaR and marginal VaR. These tools provide insight on how the overall portfolio will be affected by individual position changes. This method can be used provided that all relevant risks have been identified within the risk management system.

In the risk management process, there is a problem with measuring the risk of some unique asset classes like real estate, hedge funds, and venture capital. Also, there may be limited information on investments in a certain class (e.g., emerging markets and initial public offerings).

There is a trend in the investment industry toward management choosing a **global custodian** for the firm. Such a choice means an investor aggregates the portfolios with a single custodian, which more easily allows a consolidated picture of the total exposures of the fund. The custodian can combine reports on changes in positions with market data to produce forward-looking risk measures. Thus, the global custodian is an easy choice in pursuing centralized risk management. Along with the trend toward global custodians, there has been a trend in the "custodian industry" toward fewer custodians that can provide more services. Large custodian banks such as Citibank, Deutsche Bank, and State Street are providing risk management products.

Those that choose not to use a global custodian have done so because they feel that they have a tighter control over risk measures and can better incorporate VaR systems into operations. There are often economies of scale for larger firms in that they can spread the cost of risk management systems over a large asset base. Also, they can require tighter control when their assets are partly managed internally.

Increasingly, clients are asking money managers about their risk management systems. The clients are no longer satisfied with quarterly performance reports. Many investment managers have already incorporated VaR systems into their investment management process. Widely used risk standards for institutional investors recommend measuring the risk of the overall portfolio and measuring the risk of each instrument. It may be the case that those who do not have comprehensive risk management systems will soon be at a significant disadvantage to those who do have such systems. There also seems to be some attempt by managers to differentiate themselves with respect to risk management.

VaR Applications

LO 68.7: Explain how VaR can be used in the investment process and the development of investment guidelines.

Investment Guidelines

VaR can help move away from the ad hoc nature and overemphasis on notionals and sensitivities that characterize the guidelines many managers now use. Clearly, ad hoc procedures will generally be inferior to formal guidelines using established principles. Also, limits on notionals and sensitivities have proven insufficient when leverage and positions in derivatives exist. The limits do not account for variations in risk nor correlations. VaR limits include all of these factors.

The problem with controlling positions and not risk is that there are many rules and restrictions, which in the end may not achieve the main goal. There is no measure of the possible losses that can occur in a given time period—a good quantity to identify in order to know how much capital to have on hand to meet liquidity needs. Furthermore, simple restrictions on certain positions can be easily evaded with the many instruments that are now available. As a wider range of products develop, obviously, the traditional and cumbersome position-by-position guidelines will become even less effective.

Investment Process

VaR can help in the first step of the investment process, which is the strategic asset-allocation decision. Since this step usually uses mean-variance analysis, as does the most basic VaR measures, VaR can help in the portfolio allocation process. Furthermore, VaR can measure specific changes in risk that can result as managers subjectively adjust the weights from those recommended by pure quantitative analysis.

VaR is also useful at the trading level. A trader usually focuses on the return and stand-alone risk of a proposed position. The trader may have some idea of how the risk of the position will affect the overall portfolio, but an adequate risk management system that uses VaR can give a specific estimate of the change in risk. In fact, the risk management system should stand ready to automatically calculate the marginal VaR of each existing position and proposed position. When the trader has the choice between adding one of two positions with similar return characteristics, the trader would choose the one with the lower marginal VaR. VaR methodology can help make choices between different assets too. The optimal portfolio will be the one that has the excess-return-to-marginal VaR ratios equal for all asset types, as seen in the previous topic. Thus, when a trader is searching for the next best investment, the trader will look at securities in the asset classes that currently have the higher returns-to-marginal-VaR ratios.

Budgeting Risk

LO 68.8: Describe the risk budgeting process and calculate risk budgets across asset classes and active managers.

Risk budgeting should be a top down process. The first step is to determine the total amount of risk, as measured by VaR, that the firm is willing to accept. The next step is to choose the optimal allocation of assets for that risk exposure. As an example, a firm's management might set a return volatility target equal to 20%. If the firm has $100 million in assets under management and assuming the returns are normally distributed, at a 95% confidence level, this translates to:

$$\text{VaR} = (1.65) \times (20\%) \times (\$100 \text{ million}) = \$33 \text{ million}$$

The goal will be to choose assets for the fund that keep VaR less than this value. Unless the asset classes are perfectly correlated, the sum of the VaRs of the individual assets will be greater than the actual VaR of the portfolio. Thus, the budgeting of risk across asset classes should take into account the diversification effects. Such effects can be carried down to the next level when selecting the individual assets for the different classes.

Example: Budgeting risk across asset classes (part 1)

A manager has a portfolio with only one position: a \$500 million investment in W. The manager is considering adding a \$500 million position X or Y to the portfolio. The current volatility of W is 10%. The manager wants to limit portfolio VaR to \$200 million at the 99% confidence level. Position X has a return volatility of 9% and a correlation with W equal to 0.7. Position Y has a return volatility of 12% and a correlation with W equal to zero. **Determine** which of the two proposed additions, X or Y, will keep the manager within his risk budget.

Answer:

Currently, the VaR of the portfolio with only W is:

$$\text{VaR}_W = (2.33)(10\%)(\$500 \text{ million}) = \$116.5 \text{ million}$$

When adding X, the return volatility of the portfolio will be:

$$8.76\% = \sqrt{(0.5^2)(10\%)^2 + (0.5^2)(9\%)^2 + (2)(0.5)(0.5)(0.7)(10\%)(9\%)}$$

$$\text{VaR}_{W+X} = 2.33(8.76\%)(\$1{,}000 \text{ million}) = \$204 \text{ million}$$

When adding Y, the return volatility of the portfolio will be:

$$7.81\% = \sqrt{(0.5^2)(10\%)^2 + (0.5^2)(12\%)^2}$$

$$\text{VaR}_{W+Y} = (2.33)(7.81\%)(\$1{,}000 \text{ million}) = \$182 \text{ million}$$

Thus, Y keeps the total portfolio within the risk budget.

Example: Budgeting risk across asset classes (part 2)

In the previous example, **demonstrate** why focusing on the stand-alone VaR of X and Y would have led to the wrong choice.

Answer:

Obviously, the VaR of X is less than that of Y.

$$\text{VaR}_X = (2.33)(9\%)(\$500 \text{ million}) = \$104.9 \text{ million}$$

$$\text{VaR}_Y = (2.33)(12\%)(\$500 \text{ million}) = \$139.8 \text{ million}$$

The individual VaRs would have led the manager to select X over Y; however, the high correlation of X with W gives X a higher incremental VaR, which puts the portfolio of W and X over the limit. The zero correlation of W and Y makes the incremental VaR of Y much lower, and the portfolio of W with Y keeps the risk within the limit.

The traditional method for evaluating active managers is by measuring their excess return and tracking error and using it to derive a measure known as the information ratio. Excess return is the active return minus the benchmark return. The **information ratio** of manager *i* is:

$$IR_i = \frac{\text{(expected excess return of the manager)}}{\text{(the manager's tracking error)}}$$

For a portfolio of funds, each managed by a separate manager, the top management of the entire portfolio would be interested in the portfolio information ratio:

$$IR_P = \frac{\text{(expected excess return of the portfolio)}}{\text{(the portfolio's tracking error)}}$$

If the excess returns of the managers are independent of each other, it can be shown that the optimal allocation across managers is found by allocating weights to managers according to the following formula:

$$\text{weight of portfolio managed by manager } i = \frac{IR_i \times (\text{portfolio's tracking error})}{IR_P \times (\text{manager's tracking error})}$$

One way to use this measure is to "budget" portfolio tracking error. Given the IR_P, the IR_i, and the manager's tracking error, top management can calculate the respective weights to assign to each manager. The weights of the allocations to the managers do not necessarily have to sum to one. Any difference can be allocated to the benchmark itself because, by definition, $IR_{benchmark} = 0$.

Determining the precise weights will be an iterative process in that each selection of weights will give a different portfolio expected excess return and tracking error. Figure 2 illustrates a set of weights derived from the given inputs that satisfy the condition.

Figure 2: Budgeting Risk Across Active Managers

	Tracking Error	*Information Ratio*	*Weights*
Manager A	5.0%	0.70	51%
Manager B	5.0%	0.50	37%
Benchmark	0.0%	0.00	12%
Portfolio	3.0%	0.82	100%

Although we have skipped the derivation, we can see that the conditions for optimal allocation hold true:

$$\text{For A: } 51\% = \frac{(3\%)(0.70)}{(5\%)(0.82)}$$

$$\text{For B: } 37\% = \frac{(3\%)(0.50)}{(5\%)(0.82)}$$

The difference between 100% and the sum of the weights 51% and 37% is the 12% invested in the benchmark.

Key Concepts

LO 68.1

Risk budgeting is a top-down process that involves choosing and managing exposures to risk.

LO 68.2

Compared to banks on the "sell side," investors on the "buy side" have a longer horizon, slower turnover, and lower leverage. They have tended to use historical risk measures and focus on tracking error, benchmarking, and investment guidelines. Banks use forward-looking VaR risk measures and VaR limits. Investors seem to be using VaR more and more, but they have to adapt it to their needs.

LO 68.3

Investors are relying more on VaR because of increased globalization, complexity, and dynamics of the investment industry. They have found simply measuring risk from historical measures is no longer adequate.

LO 68.4

Hedge funds have risk characteristics that make them more similar to the "sell side" of the industry like the use of leverage and high turnover. In addition to that, they have other risks such as low liquidity and low transparency. Low liquidity leads to problems in measuring risk because it tends to put a downward bias on volatility and correlation measures.

LO 68.5

Absolute or asset risk refers to the total possible losses over a horizon. Relative risk is measured by excess return, which is the dollar loss relative to a benchmark. VaR measures can apply to both.

The risk from the policy mix is from the chosen portfolio weights, and active risk is from individual managers deviating from the chosen portfolio weights.

Funding risk is the risk that the value of assets will not be sufficient to cover the liabilities of the fund. It is important for pension funds. In applying VaR, a manager will add the expected increase in the surplus to the surplus and subtract the VaR of the assets from it. The difference between the expected surplus and the portfolio VaR is the shortfall associated with the VaR.

Two components of sponsor risk are cash-flow risk, which addresses variations of contributions to the fund, and economic risk, which is the variation of the earnings.

LO 68.6

Risk monitoring is important in large firms to catch "rogue traders" whose activities may go undetected with simple periodic statements. It is also needed for passive portfolios because the risk characteristics of the benchmarks can change. Risk monitoring can also determine why changes in risk have occurred (e.g., individual managers exceeding their budget, different managers taking on the same exposures, or the risk characteristics of the whole market changing).

There is a trend toward using a global custodian in the risk management of investment firms. It is an easy means to the goal of centralized risk management. The custodians can combine reports on changes in positions with market data to produce forward-looking risk measures. Those that choose not to use a global custodian have done so because they feel they have tighter control over risk measures and can better incorporate VaR systems into operations.

LO 68.7

There is a trend of investment managers incorporating VaR systems into their investment management process. There is evidence that money managers are differentiating themselves with respect to their risk management systems, and those that do not use such systems are at a competitive disadvantage.

VaR techniques can help move away from the ad hoc nature and overemphasis on notionals and sensitivities that characterize the guidelines many managers now use. Such guidelines are cumbersome and ineffective in that they focus on individual positions and can be easily circumvented.

VaR is useful for the investment process. When a trader has a choice between two new positions for a portfolio, the trader can compare the marginal VaRs to make the selection. When deciding whether to increase one existing position over another, the trader can compare the excess-return-to-MVaR ratios and increase the position in the one with the higher ratio.

LO 68.8

Budgeting risk across asset classes means selecting assets whose combined VaRs are less than the total allowed. The budgeting process would examine the contribution each position makes to the portfolio VaR.

For allocating across active managers, it can be shown that the optimal allocation is achieved with the following formula:

$$\text{weight of portfolio managed by manager } i = \frac{IR_i \times (\text{portfolio's tracking error})}{IR_P \times (\text{manager's tracking error})}$$

For a given group of active managers, the weights may not sum to one. The remainder of the weight can be allocated to the benchmark, which has no tracking error.

Concept Checkers

1. With respect to the buy side and sell side of the investment industry:

   I. the buy side uses more leverage.
   II. the sell side has relied more on VaR measures.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

2. Compared to policy risk, which of the following is not a reason that management risk is not much of a problem?
   A. There will be diversification effects across the deviations.
   B. Managers tend to make the same style shifts at the same time.
   C. For well-managed funds, it is usually fairly small for each of the individual funds.
   D. There can be diversification with the policy mix VaR to actually lower the total portfolio VaR.

3. Using VaR to monitor risk is important for a large firm with many types of managers because:
   A. it can help catch rogue traders and it can detect changes in risk from changes in benchmark characteristics.
   B. although it cannot help catch rogue traders, it can detect changes in risk from changes in benchmark characteristics.
   C. although it cannot detect changes in risk from changes in benchmark characteristics, it can help detect rogue traders.
   D. of no reason. VaR is not useful for monitoring risk in large firms.

4. VaR can be used to compose better guidelines for investment companies by:

   I. relying less on notionals.
   II. focusing more on overall risk.

   A. I only.
   B. II only
   C. Both I and II.
   D. Neither I nor II.

5. In making allocations across active managers, which of the following represents the formula that gives the optimal weight to allocate to a manager denoted *i*, where IR_i and IR_P are the information ratios of the manager and the total portfolio respectively?

A. $\dfrac{IR_P \times (\text{portfolio's tracking error})}{IR_i \times (\text{manager's tracking error})}$.

B. $\dfrac{IR_i \times (\text{manager's tracking error})}{IR_P \times (\text{portfolio's tracking error})}$.

C. $\dfrac{IR_i \times (\text{portfolio's tracking error})}{IR_P \times (\text{manager's tracking error})}$.

D. $\dfrac{IR_P \times (\text{manager's tracking error})}{IR_i \times (\text{portfolio's tracking error})}$.

Concept Checker Answers

1. B Compared to banks on the "sell side," investors on the "buy side" have a longer horizon, slower turnover, and lower leverage. Banks use forward-looking VaR risk measures and VaR limits.

2. B If managers make the same style shifts, then that would actually increase management risk. All the other reasons are valid.

3. A Both of these are reasons large firms find VaR and risk monitoring useful.

4. C Investment companies have been focusing on limits on notionals, which is cumbersome and has proved to be ineffective.

5. C weight of portfolio managed by manager $i = \dfrac{IR_i \times (\text{portfolio's tracking error})}{IR_P \times (\text{manager's tracking error})}$

The following is a review of the Risk Management and Investment Management principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Risk Monitoring and Performance Measurement

Topic 69

Exam Focus

Most of this topic is qualitative in nature, however, it does contain several testable concepts. Many of the concepts covered here are also covered in other assigned readings, so this topic should serve as reinforcement of those concepts. For the exam, focus on the three pillars of effective risk management: planning, budgeting, and monitoring. Understand the concept of a risk management unit (RMU) and be able to discuss its appropriate role within a company. Always keep in mind while reviewing this topic that it is the amount of risk taken that ultimately drives the level of returns—risk is the "cost" of returns.

Risk Measures

LO 69.1: Define, compare, and contrast VaR and tracking error as risk measures.

Value at risk (VaR) is defined to be the *largest* loss possible for a *certain* level of confidence over a *specific* period of time. For example, a firm could express its VaR as being 95% certain that they will lose a maximum of $5 million in the next ten days. Delta-normal VaR assumes a normal distribution, and its calculation reflects losses in the lower tail of the returns distribution.

Tracking error is defined as the standard deviation of excess returns. Excess return is defined as the portfolio return less the benchmark return (i.e., alpha). Assuming a normal distribution of excess returns, 95% of the outcomes will fall within the mean benchmark return plus or minus roughly two standard deviations.

VaR and tracking error are both measures of risk. An organization's objective is to maximize profits for a given level of risk taken. Too much risk taken (in comparison with budget) suggests a VaR level that is too high and a willingness to accept large losses to produce unnecessarily high returns. Too little risk taken suggests that there is not enough active management, and actual returns will fall short of budgeted returns.

VaR may be used to suggest the maximum dollar value of losses for a specific level of confidence over a specific time. From a portfolio management perspective, VaR could be determined for each asset class, and capital allocation decisions could be made amongst the asset classes depending on risk and return preferences. This will help to achieve targeted levels of dollar VaR. In contrast, tracking error may be used to determine the relative amount of discretion that can be taken by the portfolio manager (away from benchmark returns) in his or her attempts at active management.

Risk Planning

LO 69.2: Describe risk planning, including its objectives, effects, and the participants in its development.

There are five risk planning objectives for any entity to consider.

1. Setting expected return and expected volatility goals.

   Examples of an entity's goals could include specifying the acceptable amounts of VaR and tracking error for a given period of time. Scenario analysis could be employed to determine potential sources of failure in the plan as well as ways to respond should those sources occur.

2. Defining quantitative measures of success or failure.

   Specific guidelines should be stated. For example, one could state an acceptable level of return on equity (ROE) or return on risk capital (RORC). This would help regulatory agencies assess the entity's success or failure from a risk management perspective.

3. Generalizing how risk capital will be utilized to meet the entity's objectives.

   Objectives relating to return per unit of risk capital need to be defined. For example, the minimum acceptable RORC should be defined for each activity where risk is allocated from the budget. The correlations between the RORCs should also be considered within an entity-wide risk diversification context.

4. Defining the difference between events that cause ordinary damage versus serious damage.

   Specific steps need to be formulated to counter any event that threatens the overall long-term existence of the entity, even if the likelihood of occurrence is remote. The choice between seeking external insurance (i.e., put options) versus self-insurance for downside portfolio risk has to be considered from a cost-benefit perspective, taking into account the potential severity of the losses.

5. Identifying mission critical resources inside and outside the entity and discussing what should be done in case those resources are jeopardized.

   Examples of such resources would include key employees and financing sources. Scenario analysis should be employed to assess the impact on those resources in both good and bad times. Specifically, adverse events often occur together with other adverse (and material) events.

In general, the risk planning process frequently requires the input and approval of the entity's owners and its management team. An effective plan requires very active input from the entity's highest level of management so as to ensure risk and return issues are addressed, understood, and communicated within the entity, to key stakeholders, and to regulatory agencies.

Risk Budgeting

LO 69.3: Describe risk budgeting and the role of quantitative methods in risk budgeting.

The risk budget quantifies the risk plan. There needs to be a structured budgeting process to allocate risk capital to meet the entity's objectives and minimize deviations from the plan. Each specific allocation from the risk budget comes with a reasonable return expectation. The return expectation comes with an estimate of variability around that expectation.

With risk budgets, an amount of VaR could be calculated for each item on the income statement. This allows RORC to be calculated individually and in aggregate.

Quantitative methods (i.e., mathematical modeling) may be used in risk budgeting as follows:

1. Set the minimum acceptable levels of RORC and ROE over various time periods. This is to determine if there is sufficient compensation for the risks taken (i.e., risk-adjusted profitability).

2. Apply mean-variance optimization (or other quantitative methods) to determine the weights for each asset class.

3. Simulate the portfolio performance based on the weights above and for several time periods. Apply sensitivity analysis to the performance by considering changes in estimates of returns and covariances.

Risk Monitoring

LO 69.4: Describe risk monitoring and its role in an internal control environment.

Within an entity's internal control environment, risk monitoring attempts to seek and investigate any significant variances from budget. This is to ensure, for example, that there are no threats to meeting its ROE and RORC targets. Risk monitoring is useful in that it should detect and address any significant variances in a timely manner.

LO 69.5: Identify sources of risk consciousness within an organization.

The increasing sense of risk consciousness within and among organizations is mainly derived from the following three sources:

1. *Banks* who lend funds to investors are concerned with where those funds are invested.

2. *Boards of investment clients, senior management, and plan sponsors* have generally become more versed in risk management issues and more aware of the need for effective oversight over asset management activities.

3. *Investors* have become more knowledgeable about their investment choices. For example, beneficiaries of a defined contribution plan are responsible for selecting their individual pension investments.

LO 69.6: Describe the objectives and actions of a risk management unit in an investment management firm.

A **risk management unit** (RMU) monitors an investment management entity's portfolio risk exposure and ascertains that the exposures are authorized and consistent with the risk budgets previously set. To ensure proper segregation of duties, it is crucial that the risk management function has an independent reporting line to senior management.

The objectives of a RMU include:

- Gathering, monitoring, analyzing, and distributing risk data to managers, clients, and senior management. Accurate and relevant information must be provided to the appropriate person(s) at the appropriate time(s).
- Assisting the entity in formulating a systematic and rigorous method as to how risks are identified and dealt with. Promotion of the entity's risk culture and best risk practices is crucial here.
- Going beyond merely providing information by taking the initiative to research relevant risk topics that will affect the firm.
- Monitoring trends in risk on a continual basis and promptly reporting unusual events to management before they become significant problems.
- Promoting discussion throughout the entity and developing a process as to how risk data and issues are discussed and implemented within the entity.
- Promoting a greater sense of risk awareness (culture) within the entity.
- Ensuring that transactions that are authorized are consistent with guidance provided to management and with client expectations.
- Identifying and developing risk measurement and performance attribution analytical tools.
- Gathering risk data to be analyzed in making portfolio manager assessments and market environment assessments.
- Providing the management team with information to better comprehend risk in individual portfolios as well as the source of performance.
- Measuring risk within an entity. In other words, measuring how consistent portfolio managers are with respect to product objectives, management expectations, and client objectives. Significant deviations are brought to the attention of appropriate management to provide a basis for correction.

Professor's Note: You may see references elsewhere to an Independent Risk Oversight Unit. This is the same concept as RMU. Both measure and manage risk exposure and operate as an independent business unit.

LO 69.7: Describe how risk monitoring can confirm that investment activities are consistent with expectations.

Is the manager generating a forecasted level of tracking error that is consistent with the target?

The forecasted tracking error is an approximation of the potential risk of a portfolio using statistical methods. For each portfolio, the forecast should be compared to budget using predetermined guidelines as to how much variance is acceptable, how much variance requires further investigation, and how much variance requires immediate action. Presumably, the budget was formulated taking into account client expectations.

Tracking error forecast reports should be produced for all accounts that are managed similarly in order to gauge the consistency in risk levels taken by the portfolio manager.

Is risk capital allocated to the expected areas?

Overall tracking risk is not sufficient as a measure on its own; it is important to break down the tracking risk into "subsections." If the analysis of the risk taken per subsection does not suggest that risk is being incurred in accordance with expectations, then there may be **style drift**. Style drift may manifest itself in a value portfolio manager who attains the overall tracking error target but allocates most of the risk (and invests) in growth investments.

Therefore, by engaging in risk decomposition, the RMU may ensure that a portfolio manager's investment activities are consistent with the predetermined expectations (i.e., stated policies and manager philosophy). Also, by running the report at various levels, unreasonably large concentrations of risk (that may jeopardize the portfolio) may be detected.

Liquidity Considerations

LO 69.8: Explain the importance of liquidity considerations for a portfolio.

Liquidity considerations are important because a portfolio's liquidity profile could change significantly in the midst of a volatile market environment or an economic downturn, for instance. Therefore, measuring portfolio liquidity is a priority in stress testing.

One potential measure is **liquidity duration**. It is an approximation of the number of days necessary to dispose of a portfolio's holdings without a significant market impact. For a given security, the liquidity duration could be calculated as follows:

$$LD = \frac{Q}{(0.10 \times V)}$$

where:
LD = liquidity duration for the security on the assumption that the desired maximum daily volume of any security is 10%
Q = number of shares of the security
V = daily volume of the security

Performance Measurement

LO 69.10: Describe the objectives of performance measurement.

Performance measurement looks at a portfolio manager's actual results and compares them to relevant comparables such as benchmarks and peer groups. Therefore, performance measurement seeks to determine whether a manager can consistently outperform (through excess returns) the benchmark on a risk-adjusted basis. Similarly, it seeks to determine whether a manager consistently outperforms its peer group on a risk-adjusted basis.

Furthermore, performance measurement may help to determine whether the returns achieved are commensurate with the risk taken. Finally, performance measurement provides a basis for identifying managers who are able to generate consistent excess risk-adjusted returns. Such superior processes and performance could be replicated on an on-going basis, thereby maximizing the entity's long-run returns and profitability.

Comparison of Performance with Expectations

From a risk perspective (e.g., tracking error), portfolio managers should be assessed on the basis of being able to produce a portfolio with risk characteristics that are expected to approximate the target. In addition, they should also be assessed on their ability to actually achieve risk levels that are close to target.

From a returns perspective (e.g., performance), portfolio managers could be assessed on their ability to earn excess returns.

Goldman Sachs Asset Management utilizes a so-called "green zone" to identify instances of actual tracking error or performance that are outside of normal expectations. An acceptable amount of deviation (from a statistical perspective) is determined, and any deviations up to that amount are considered a green zone event. Unusual events that are expected to occur with some regularity are considered "yellow zone" events. Truly unusual events that require immediate investigation are considered "red zone" events. In using this simple color-coded system, the various zones are predefined and provide clear expectations for the portfolio managers. The movements of portfolios into yellow or red zones are triggering events that require further investigation and discussion.

Return Attribution

The source of returns can be attributed to specific factors or securities. For example, it is important to ensure that returns result from decisions where the manager intended to take risk and not simply from sheer luck.

Variance analysis is used to illustrate the contribution to overall portfolio performance by each security. The securities can be regrouped in various ways to conduct analysis by industry, sector, and country, for example.

In performing return attribution, factor risk analysis and factor attribution could be used. Alternatively, risk forecasting and attribution at the security level could also be used.

Sharpe and Information Ratio

The **Sharpe ratio** is calculated by taking the portfolio's actual return and subtracting the risk-free rate in the numerator. The denominator is the portfolio's standard deviation. The **information ratio** is calculated by taking the portfolio's excess returns and subtracting the benchmark's excess returns (if applicable) in the numerator. The denominator is the portfolio's tracking error. These two measures are both considered risk-adjusted return measures.

Strengths of these metrics include the following: (1) easy to use as a measure of relative performance compared to a benchmark or peer group; (2) easy to determine if the manager has generated sufficient excess returns in relation to the amount of risk taken; and (3) easy to apply to industrial sectors and countries.

Weaknesses of these metrics include the following: (1) insufficient data available to perform calculations; and (2) the use of realized risk (instead of potential risk) may result in overstated performance calculations.

Comparisons with Benchmark Portfolios and Peer Groups

LO 69.9: Describe the use of alpha, benchmark, and peer group as inputs in performance measurement tools.

One could use linear regression analysis to regress the excess returns of the investment against the excess returns of the **benchmark**. One of the outputs from this regression is **alpha**, and it could be tested for statistical significance to determine whether the excess returns are attributable to manager skill or just pure luck. The other output is **beta**, and it relates to the amount of leverage used or underweighting/overweighting in the market compared to the benchmark.

The regression also allows a comparison of the absolute amount of excess returns compared to the benchmark. Furthermore, there is the ability to separate excess returns due to leverage and excess returns due to skill. One limitation to consider is that there may not be enough data available to make a reasonable conclusion as to the manager's skill.

One could also regress the excess returns of the manager against the excess returns of the manager's **peer group**. The features of this regression are generally similar to that for the benchmark above, except that the returns of the peer group suffer from **survivorship bias**, and there is usually a wide range of funds under management amongst the peers (that reduces the comparability).

Key Concepts

LO 69.1

VaR and tracking error are both measures of risk. VaR is defined to be the largest loss possible for a certain level of confidence over a specific period of time. Tracking error is defined as the standard deviation of excess returns.

LO 69.2

There are five risk planning objectives to consider.

- Setting expected return and expected volatility goals.
- Defining quantitative measures of success or failure.
- Generalizing how risk capital will be utilized to meet the entity's objectives.
- Defining the difference between events that cause ordinary damage versus serious damage.
- Identifying mission critical resources inside and outside the entity and discussing what should be done in case those resources are jeopardized.

The risk planning process frequently requires the input and approval of the entity's owners and its management team.

LO 69.3

The risk budget quantifies the risk plan. There needs to be a structured budgeting process to allocate risk capital to meet the corporate objectives and minimize deviations from plan.

Quantitative methods may be used in risk budgeting. Activities include: setting the minimum acceptable levels of RORC and ROE, applying mean-variance optimization, simulating portfolio performance, and applying sensitivity analysis.

LO 69.4

Within an entity's internal control environment, risk monitoring attempts to seek and investigate any significant variances from budget.

LO 69.5

Sources of risk consciousness include: (1) banks, (2) boards of investment clients, senior management, and plan sponsors, and (3) investors.

LO 69.6

A risk management unit (RMU) monitors an investment management entity's portfolio risk exposure and ascertains that the exposures are authorized and consistent with the risk budgets previously set. To ensure proper segregation of duties, it is crucial that the risk management function be independent and not report to senior management.

LO 69.7

The risk monitoring process attempts to confirm that investment activities are consistent with expectations. Specifically, is the manager generating a forecasted level of tracking error that is consistent with the target? And is risk capital allocated to the expected areas?

LO 69.8

Liquidity considerations are important because a portfolio's liquidity profile could change significantly in the midst of a volatile market environment or an economic downturn, for instance.

LO 69.9

The excess returns of an investment can be regressed against the excess returns of its benchmark (e.g., S&P 500 Index). An output from this regression is alpha, which determines whether the investment's excess returns are due to skill or luck.

The excess returns of a manager can be regressed against the excess returns of the manager's peer group. This is similar to the liner regression with a benchmark portfolio, but differs since it suffers from survivorship bias.

LO 69.10

Performance measurement looks at a portfolio manager's actual results and compares them to relevant comparables such as benchmarks and peer groups.

A performance measurement framework includes: (1) comparison of performance with expectations, (2) return attribution, (3) calculation of metrics such as the Sharpe ratio and the information ratio, and (4) comparisons with benchmark portfolios and peer groups.

Concept Checkers

1. Which of the following statements about tracking error and value at risk (VaR) is least accurate?
   A. Tracking error and VaR are complementary measures of risk.
   B. Both tracking error and VaR may assume a normal distribution of returns.
   C. Tracking error is the standard deviation of the excess of portfolio returns over the return of the peer group.
   D. VaR can be defined as the maximum loss over a given time period.

2. Which of the following statements about the use of quantitative methods in risk budgeting is least accurate? They may be used:
   A. to simulate the performance of portfolios.
   B. to set levels of return on equity (ROE) and return on risk capital (RORC).
   C. in a scenario analysis context to determine the weights for each asset class.
   D. in a sensitivity analysis context to consider changes in estimates of returns and covariances.

3. A risk management unit (RMU) is most likely to be active in which of the following contexts?
   A. Risk monitoring.
   B. Risk measurement.
   C. Risk budgeting.
   D. Risk planning.

4. Which of the following statements does not help explain the purpose of risk decomposition?
   A. To ensure that there is no style drift.
   B. To detect large concentrations of risk.
   C. To detect excessive amounts of tracking risk.
   D. To ensure that investment activities are consistent with expectations.

5. Which of the following statements regarding alphas and betas is incorrect?
   A. Alpha is the excess return attributable to pure luck.
   B. Alpha is the excess return attributable to managerial skill.
   C. Beta suggests the relative amount of leverage used.
   D. Beta suggests whether some of the returns are attributable to over or under weighting the market.

Concept Checker Answers

1. **C** All of the statements are accurate with the exception of the one relating to the peer group. Tracking error is the standard deviation of the excess of portfolio returns over the return of an appropriate benchmark, not peer group.

2. **C** All of the statements are accurate with the exception of the one relating to scenario analysis. One should apply mean-variance optimization (and not scenario analysis) to determine the weights for each asset class.

3. **A** A RMU monitors an investment management firm's portfolio risk exposure and ascertains that the exposures are authorized and consistent with the risk budgets previously set.

4. **C** Risk decomposition is not designed to detect excessive amounts of tracking risk. In fact, it is the forecasted tracking error amount that should be compared to budget to ensure that there is not excessive tracking risk. All the other reasons are consistent with the purpose of risk decomposition.

5. **A** Alpha is a measure of the excess return of a manager over the peer group/benchmark that relates to skill as opposed to pure luck. Beta is a measure of the amount of leverage used compared to the peer group or a measure of the underweighting or overweighting of the market compared to the benchmark.

The following is a review of the Risk Management and Investment Management principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Portfolio Performance Evaluation

Topic 70

Exam Focus

Professional money managers are routinely evaluated using a wide array of metrics. In this topic, alternative methods of computing portfolio returns will be presented, and contrasts will be made between time-weighted and dollar-weighted returns for portfolios experiencing cash redemptions and contributions. For the exam, be sure to understand differences in the risk-adjusted performance measures, including the Sharpe ratio, Treynor ratio, Jensen's alpha, information ratio, and M^2, and how the trading practices of hedge funds complicates the evaluation process. Be able to apply Sharpe's regression-based style analysis to conduct performance attributions.

Time-Weighted and Dollar-Weighted Returns

LO 70.1: Differentiate between time-weighted and dollar-weighted returns of a portfolio and describe their appropriate uses.

The **dollar-weighted rate of return** is defined as the internal rate of return (IRR) on a portfolio, taking into account all cash inflows and outflows. The beginning value of the account is an inflow as are all deposits into the account. All withdrawals from the account are outflows, as is the ending value.

Example: Dollar-weighted rate of return

Assume an investor buys a share of stock for \$100 at t = 0, and at the end of the next year (t = 1), she buys an additional share for \$120. At the end of year 2, the investor sells both shares for \$130 each. At the end of each year in the holding period, the stock paid a \$2.00 per share dividend. What is the investor's dollar-weighted rate of return?

Answer:

Step 1: Determine the timing of each cash flow and whether the cash flow is an inflow (+) or an outflow (–).

t = 0:	purchase of first share	=	–$100
t = 1:	dividend from first share	=	+$2
	purchase of second share	=	–$120
	subtotal, t = 1		–$118
t = 2:	dividend from two shares	=	+$4
	proceeds from selling shares	=	+$260
	subtotal, t = 2		+$264

Step 2: Net the cash flows for each time period, and set the PV of cash inflows equal to the present value of cash outflows.

$$PV_{inflows} = PV_{outflows}$$

$$\$100 + \frac{\$120}{(1+r)} = \frac{\$2}{(1+r)} + \frac{\$264}{(1+r)^2}$$

Step 3: Solve for *r* to find the dollar-weighted rate of return. This can be done using trial and error or by using the IRR function on a financial calculator or spreadsheet.

The intuition here is that we deposited $100 into the account at t = 0, then added $118 to the account at t = 1 (which, with the $2 dividend, funded the purchase of one more share at $120), and ended with a total value of $264.

To compute this value with a financial calculator, use these net cash flows and follow the procedure described in Figure 1 to calculate the IRR.

Net cash flows: $CF_0 = -100$; $CF_1 = -120 + 2 = -118$; $CF_2 = 260 + 4 = 264$

Figure 1: Calculating Dollar-Weighted Return with the TI Business Analyst II Plus® Calculator

Key Strokes	*Explanation*	*Display*
[CF] [2nd] [CLR WORK]	Clear cash flow registers	CF0 = 0.00000
100 [+/–] [ENTER]	Initial cash outlay	CF0 = –100.00000
[↓] 118 [+/–] [ENTER]	Period 1 cash flow	C01 = –118.00000
[↓] [↓] 264 [ENTER]	Period 2 cash flow	C02 = 264.00000
[IRR] [CPT]	Calculate IRR	IRR = 13.86122

The dollar-weighted rate of return for this problem is 13.86%.

Time-weighted rate of return measures compound growth. It is the rate at which \$1.00 compounds over a specified time horizon. Time-weighting is the process of averaging a set of values over time. The *annual* time-weighted return for an investment may be computed by performing the following steps:

Step 1: Value the portfolio immediately preceding significant addition or withdrawals. Form subperiods over the evaluation period that correspond to the dates of deposits and withdrawals.

Step 2: Compute the holding period return (HPR) of the portfolio for each subperiod.

Step 3: Compute the product of $(1 + HPR_t)$ for each subperiod t to obtain a total return for the entire measurement period [i.e., $(1 + HPR_1) \times (1 + HPR_2) \ldots (1 + HPR_n)$]. If the total investment period is greater than one year, you must take the geometric mean of the measurement period return to find the annual time-weighted rate of return.

Example: Time-weighted rate of return

A share of stock is purchased at t = 0 for \$100. At the end of the next year, t = 1, another share is purchased for \$120. At the end of year 2, both shares are sold for \$130 each. At the end of years 1 and 2, the stock paid a \$2.00 per share dividend. What is the time-weighted rate of return for this investment? (This is the same data as presented in the dollar-weighted rate-of-return example.)

Answer:

Step 1: Break the evaluation period into two subperiods based on timing of cash flows.

Holding period 1:	beginning price	= \$100.00
	dividends paid	= \$2.00
	ending price	= \$120.00
Holding period 2:	beginning price	= \$240.00 (2 shares)
	dividends paid	= \$4.00 (\$2 per share)
	ending price	= \$260.00 (2 shares)

Step 2: Calculate the HPR for each holding period.

$$HPR_1 = [(\$120 + 2) / \$100] - 1 = 22\%$$

$$HPR_2 = [(\$260 + 4) / \$240] - 1 = 10\%$$

Step 3: Take the geometric mean of the annual returns to find the annualized time-weighted rate of return over the measurement period.

$$(1 + \text{time-weighted rate of return})^2 = (1.22)(1.10)$$

$$\text{time-weighted rate of return} = \left[\sqrt{(1.22)(1.10)}\right] - 1 = 15.84\%$$

In the investment management industry, the time-weighted rate of return is the preferred method of performance measurement for a portfolio manager because it is not affected by the timing of cash inflows and outflows, which may be beyond the manager's control.

In the preceding examples, the time-weighted rate of return for the portfolio was 15.84%, while the dollar-weighted rate of return for the same portfolio was 13.86%. The difference in the results is attributable to the fact that the procedure for determining the dollar-weighted rate of return gave a larger weight to the year 2 HPR, which was 10% versus the 22% HPR for year 1.

If funds are contributed to an investment portfolio just before a period of relatively poor portfolio performance, the dollar-weighted rate of return will tend to be depressed. Conversely, if funds are contributed to a portfolio at a favorable time, the dollar-weighted rate of return will increase. The use of the time-weighted return removes these distortions, providing a better measure of a manager's ability to select investments over the period. If a private investor has complete control over money flows into and out of an account, the dollar-weighted rate of return may be the more appropriate performance measure.

Therefore, the dollar-weighted return will exceed the time-weighted return for a manager who has superior market timing ability.

Risk-Adjusted Performance Measures

LO 70.2: Describe and distinguish between risk-adjusted performance measures, such as Sharpe's measure, Treynor's measure, Jensen's measure (Jensen's alpha), and information ratio.

LO 70.3: Describe the uses for the Modigliani-squared and Treynor's measure in comparing two portfolios, and the graphical representation of these measures.

Universe Comparisons

Portfolio rankings based merely on returns ignore differences in risk across portfolios. A popular alternative is to use a comparison universe. This approach classifies portfolios according to investment style (e.g., small cap growth, small cap value, large cap growth, large cap value) and, then, ranks portfolios based on rate of return within the appropriate style universe. The rankings are now more meaningful because they have been standardized on the investment style of the funds. This method will fail, however, if risk differences remain across the funds within a given style.

The Sharpe Ratio

The **Sharpe ratio** uses standard deviation (total risk) as the relevant measure of risk. It shows the amount of excess return (over the risk-free rate) earned per unit of total risk. Hence, the Sharpe ratio evaluates the performance of the portfolio in terms of both overall return and diversification.

The Sharpe ratio is defined as:

$$S_A = \frac{\overline{R}_A - \overline{R}_F}{\sigma_A}$$

where:

$\overline{R}_A$ = average account return
$\overline{R}_F$ = average risk-free return
σ_A = standard deviation of account returns

Professor's Note: Again, the risk measure, standard deviation, should ideally be the actual standard deviation during the measurement period.

The Treynor Measure

The **Treynor measure** is very similar to the Sharpe ratio except that it uses beta (systematic risk) as the measure of risk. It shows the excess return (over the risk-free rate) earned per unit of systematic risk.

The Treynor measure is defined as:

$$T_A = \frac{\overline{R}_A - \overline{R}_F}{\beta_A}$$

where:

$\overline{R}_A$ = average account return
$\overline{R}_F$ = average risk-free return
β_A = average beta

Professor's Note: Ideally, the Treynor measure should be calculated using the actual beta for the portfolio over the measurement period. Since beta is subject to change due to varying covariance with the market, using the premeasurement period beta may not yield reliable results. The beta for the measurement period is estimated by regressing the portfolio's returns against the market returns.

For a well-diversified portfolio, the difference in risk measurement between the Sharpe ratio and the Treynor measure becomes irrelevant as the total risk and systematic risk will be very close. For a less than well-diversified portfolio, however, the difference in rankings based on the two measures is likely due to the amount of diversification in the portfolio. Used along with the Treynor measure, the Sharpe ratio provides additional information about the degree of diversification in a portfolio.

Sharpe vs. Treynor. If a portfolio was not well-diversified over the measurement period, it may be ranked relatively higher using Treynor than using Sharpe because Treynor considers only the beta (i.e., systematic risk) of the portfolio over the period. When the Sharpe ratio is calculated for the portfolio, the excess total risk (standard deviation) due to diversifiable risk will cause rankings to be lower. Although we do not get an absolute measure of the lack of diversification, the change in the rankings shows the presence of unsystematic risk, and the greater the difference in rankings, the less diversified the portfolio.

Jensen's Alpha

Jensen's alpha, also known as Jensen's measure, is the difference between the actual return and the return required to compensate for systematic risk. To calculate the measure, we subtract the return calculated by the capital asset pricing model (CAPM) from the account return. Jensen's alpha is a direct measure of performance (i.e., it yields the performance measure without being compared to other portfolios).

$$\alpha_A = R_A - E(R_A)$$

where:
α_A = alpha
R_A = the return on the account
$E(R_A) = R_F + \beta_A[E(R_M) - R_F]$

A superior manager would have a statistically significant and positive alpha. Jensen's alpha uses the portfolio return, market return, and risk-free rate for each time period separately. The Sharpe and Treynor measures use only the average of portfolio return and risk-free rate. Furthermore, like the Treynor measure, Jensen's alpha only takes into account the systematic risk of the portfolio and, hence, gives no indication of the diversification in the portfolio.

Information Ratio

The Sharpe ratio can be changed to incorporate an appropriate benchmark instead of the risk-free rate. This form is known as the **information ratio** or **appraisal ratio**:

$$IR_A = \frac{\overline{R}_A - \overline{R}_B}{\sigma_{A-B}}$$

where:
$\overline{R}_A$ = average account return
$\overline{R}_B$ = average benchmark return
σ_{A-B} = standard deviation of excess returns measured as the difference between account and benchmark returns

The information ratio is the ratio of the surplus return (in a particular period) to its standard deviation. It indicates the amount of risk undertaken (denominator) to achieve a certain level of return above the benchmark (numerator). An active manager makes specific cognitive bets to achieve a positive surplus return. The variability in the surplus return is a measure of the risk taken to achieve the surplus. The ratio computes the surplus return relative to the risk taken. A higher information ratio indicates better performance.

Professor's Note: The version of the information ratio presented here is the most common. However, you should be aware that an alternative calculation of this ratio exists that uses alpha over the expected level of unsystematic risk over the time period, $\frac{\alpha_A}{\sigma(\varepsilon_A)}$.

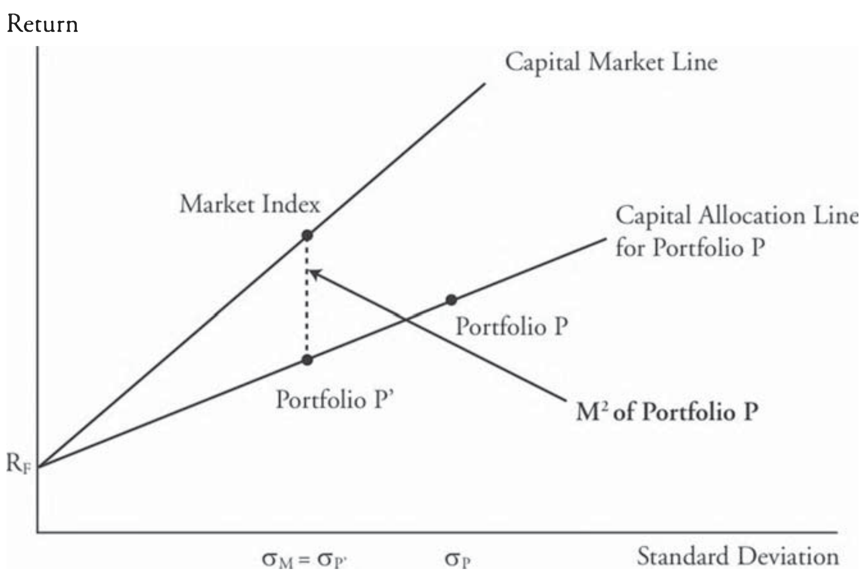
M-Squared (M^2) Measure

A relatively new measure of portfolio performance developed by Leah Modigliani and her grandfather, 1985 Nobel Prize recipient Franco Modigliani, has become quite popular. The M^2 measure compares the return earned on the managed portfolio against the market return, after adjusting for differences in standard deviations between the two portfolios.

Professor's Note: There are no squared terms in the M-squared calculation. The term "M-squared" merely refers to the last names of its originators (Leah and Franco Modigliani).

The M^2 measure can be illustrated with a graph comparing the capital market line for the market index and the capital allocation line for managed Portfolio P. In Figure 2, notice that Portfolio P has a higher standard deviation than the market index. But, we can easily create a Portfolio P' that has standard deviation equal to the market standard deviation by investing appropriate percentages in both the risk-free asset and Portfolio P. The difference in return between Portfolio P' and the market portfolio, equals the M^2 measure for Portfolio P.

Figure 2: The M^2 Measure of Portfolio Performance



Example: Calculating the M^2 performance measure

Calculate the M^2 measure for Portfolio P:

• Portfolio P mean return	10%
• Portfolio P standard deviation	40%
• Market portfolio mean return	12%
• Market portfolio standard deviation	20%
• Risk-free rate	4%

Answer:

To answer the question, first note that a portfolio, P', can be created that allocates 50/50 to the risk-free asset and to Portfolio P such that the standard deviation of Portfolio P' equals the standard deviation of the market portfolio:

$$\sigma_{P'} = w_P\sigma_P = 0.50(0.40) = 0.20$$

Therefore, a 50/50 allocation between Portfolio P and the risk-free asset provides risk identical to the market portfolio. What is the difference in return between Portfolio P' and the market portfolio? To answer this question, first we must derive the mean return on Portfolio P':

$$R_{P'} = w_F R_F + w_P R_P = 0.50(0.04) + 0.50(0.10) = 0.07$$

Alternatively, the mean return for Portfolio P' can be derived by using the equation of the capital allocation line for Portfolio P:

$$R_{P'} = R_F + \left(\frac{R_P - R_F}{\sigma_P}\right)\sigma_{P'} = R_F + \left(\frac{R_P - R_F}{\sigma_P}\right)\sigma_M$$

$$= 0.04 + \left(\frac{0.10 - 0.04}{0.40}\right)0.20 = 0.04 + (0.15)0.20 = 0.07$$

Therefore, we now have created a portfolio, P', that matches the risk of the market portfolio (standard deviation equals 20%). All that remains is to calculate the difference in returns between Portfolio P' and the market portfolio:

$$M^2 = R_{P'} - R_M = 0.07 - 0.12 = -0.05$$

Clearly, Portfolio P is a poorly performing portfolio. After controlling for risk, Portfolio P provides a return that is 5 percentage points below the market portfolio.

Professor's Note: Unfortunately, a consistent definition of M^2 does not exist. Sometimes M^2 is defined as equal to the return on the risk-adjusted Portfolio P' rather than equal to the difference in returns between P' and M. However, portfolio rankings based on the return on P' or on the difference in returns between P' and M will be identical. Therefore, both definitions provide identical portfolio performance rankings.

M^2 will produce the same conclusions as the Sharpe ratio. As stated earlier, Jensen's alpha will produce the same conclusions as the Treynor measure. However, M^2 and Sharpe may not give the same conclusion as Jensen's alpha and Treynor. A discrepancy could occur if the manager takes on a large proportion of unsystematic risk relative to systematic risk. This would lower the Sharpe ratio but leave the Treynor measure unaffected.

Example: Risk-adjusted performance appraisal measures

The data in Figure 3 has been collected to appraise the performance of four asset management firms:

Figure 3: Performance Appraisal Data

	Fund 1	*Fund 2*	*Fund 3*	*Fund 4*	*Market Index*
Return	6.45%	8.96%	9.44%	5.82%	6%
Beta	0.88	1.02	1.36	0.80	1.00
Standard deviation	2.74%	4.54%	3.72%	2.64%	2.80%
Standard deviation of excess returns	5.6%	6.1%	12.5%	5.3%	N/A

The market index return and risk-free rate of return for the relevant period were 6% and 3%, respectively. **Calculate** and **rank** the funds using Jensen's alpha, the Treynor measure, the Sharpe ratio, the information ratio, and M^2.

Answer:

Evaluation Tool	*Fund 1*	*Fund 2*	*Fund 3*	*Fund 4*
Jensen's Alpha	6.45 – 5.64 = 0.81%	8.96 – 6.06 = 2.90%	9.44 – 7.08 = 2.36%	5.82 – 5.40 = 0.42%
Rank	3	1	2	4
Treynor	$\frac{6.45-3}{0.88}=3.92$	$\frac{8.96-3}{1.02}=5.84$	$\frac{9.44-3}{1.36}=4.74$	$\frac{5.82-3}{0.80}=3.53$
Rank	3	1	2	4
Sharpe	$\frac{6.45-3}{2.74}=1.26$	$\frac{8.96-3}{4.54}=1.31$	$\frac{9.44-3}{3.72}=1.73$	$\frac{5.82-3}{2.64}=1.07$
Rank	3	2	1	4
Information Ratio	$\frac{6.45-6}{5.6}=0.08$	$\frac{8.96-6}{6.1}=0.49$	$\frac{9.44-6}{12.5}=0.28$	$\frac{5.82-6}{5.3}=-0.03$
Rank	3	1	2	4
M^2	3 + (1.26) × (2.8) = 6.53% – 6% =0.53%	3 + (1.31) × (2.8) = 6.67% – 6% =0.67%	3 + (1.73) × (2.8) = 7.84% – 6% 1.84%	3 + (1.07) × (2.8) = 6% – 6% =0
Rank	3	2	1	4

Note that Jensen's alpha and the Treynor measures give the same rankings, and the Sharpe and M^2 measures give the same rankings. However, when comparing the alpha/Treynor rankings to the Sharpe/M^2 measures, Funds 2 and 3 trade places.

Fund 2 has a much higher total risk (standard deviation) than Fund 3 but has a much lower beta. Relatively speaking, a smaller proportion of Fund 2's total risk relates to systematic risk, which is reflected in the low beta. Compared to Fund 3, it must have a bigger proportion of risk relating to non-systematic risk factors.

Hence, Fund 2 does better in the alpha/Treynor measures, as those measures only look at systematic risk (beta). It fares less well when it comes to the Sharpe/M^2 measures that look at total risk.

Statistical Significance of Alpha Returns

LO 70.4: Determine the statistical significance of a performance measure using standard error and the t-statistic.

Alpha (α) plays a critical role in determining portfolio performance. A positive alpha produces an indication of superior performance; a negative alpha produces an indication of inferior performance; and zero alpha produces an indication of normal performance matching the benchmark. The performance indicated by alpha, however, could be a result of luck and not skill. In order to assess a manager's ability to generate alpha, we conduct a *t*-test under the following hypotheses:

Null (H_0): True alpha is zero.

Alternative (H_A): True alpha is not zero.

$$t = \frac{\alpha - 0}{\sigma / \sqrt{N}}$$

where:
α = alpha estimate
σ = alpha estimate volatility
N = sample number of observations
standard error of alpha estimate = $\sigma / \sqrt{N}$

In order to compute the ***t*-statistic**, we will need to know the alpha estimate, the sample number of observations, and the alpha estimate of volatility. From the volatility and sample size estimates, we can compute the **standard error** of the alpha estimate, which is shown in the denominator of the *t*-statistic calculation.

At a 95% confidence level (5% significance level) we reject the null hypothesis if we estimate a *t*-value of 2 or larger. That is, the probability of observing such a large estimated alpha by chance is only 5%, assuming returns are normally distributed.

Professor's Note: Using a t-value of 2 is a general test of statistical significance. From the FRM Part I curriculum, we know that the actual t-value with a 95% confidence level given a large sample size is 1.96.

If we assume an excess (alpha) return of 0.09% and a standard error of the alpha of 0.093%, the *t*-statistic would be equal to 0.97 (t = 0.09% / 0.093%); therefore, we fail to reject H_0 and conclude that there is no evidence of superior (or inferior) performance.

Professor's Note: Using statistical inference when evaluating performance is extremely challenging in practice. By the time you are reasonably confident that a manager's returns are in fact due to skill, the manager may have moved elsewhere.

Measuring Hedge Fund Performance

LO 70.5: Explain the difficulties in measuring the performance of hedge funds.

Long-short hedge funds are often used to complement an investor's well-diversified portfolio. For example, the investor might allocate funds to a passively managed index fund and an actively managed long-short hedge fund. The hedge fund is designed to provide positive alpha with zero beta to the investor's overall composite portfolio. The hedge fund creates **portable alpha** in the sense that the alpha does not depend on the performance of the broad market and can be ported to any existing portfolio. Because the long-short fund is market-neutral, the alpha may be generated outside the investor's desired asset class mix.

Unfortunately, hedge fund performance evaluation is complicated because:

- Hedge fund risk is not constant over time (nonlinear risk).
- Hedge fund holdings are often illiquid (data smoothing).
- Hedge fund sensitivity with traditional markets increases in times of a market crisis and decreases in times of market strength.

The latter problem necessitates the use of estimated prices for hedge fund holdings. The values of the hedge funds, therefore, are not transactions-based. The estimation process unduly smoothes the hedge fund "values," inducing serial correlation into any statistical examination of the data.

Performance Evaluation With Dynamic Risk Levels

LO 70.6: Explain how changes in portfolio risk levels can affect the use of the Sharpe ratio to measure performance.

The Sharpe ratio is useful when evaluating the portfolio performance of a passive investment strategy, where risk and return characteristics are relatively constant over time. However, the application of the Sharpe ratio is challenged when assessing the performance of active investment strategies, where risk and return characteristics are more dynamic. Changes in volatility will likely bias the Sharpe ratio, and produce incorrect conclusions when comparing portfolio performance to a benchmark or index.

Take for example a low-risk portfolio with an alpha return of 1% and a standard deviation of 3%. The manager implements this strategy for one-year, producing quarterly returns of –2%, 4%, –2%, and 4%. The Sharpe ratio for this portfolio is calculated as: 1% / 3% = 0.3333. If the market index has a Sharpe ratio of 0.3, we would conclude that this portfolio has superior risk-adjusted performance. In the following year, the portfolio manager decides to switch to a high-risk strategy. The alpha return and risk correspondingly increase to 5% and 15%, respectively. For the second year, quarterly returns were –10%, 20%, –10%, and 20%. The Sharpe ratio in this case is still 0.3333 (= 5% / 15%), which still indicates superior performance compared to the market index. However, if the Sharpe ratio is evaluated over the two-year time frame, considering both the low-risk and high-risk strategies, the measure will drop to 0.2727 since average excess return over both years was 3% with volatility of 11%. The lower Sharpe ratio now suggests underperformance relative to the market index.

In this example, the Sharpe ratio was biased downward due to the perceived increase in risk in portfolio returns. In isolation, both the low-risk and high-risk strategies produced higher Sharpe ratios than the market index. However, when analyzed together, the Sharpe ratio suggests that the portfolio excess returns are inferior to the market. Therefore, it is important to consider changes in portfolio composition when using performance measures, as dynamic risk levels can lead to incorrect ranking conclusions.

Measuring Market Timing Ability

LO 70.7: Describe techniques to measure the market timing ability of fund managers with a regression and with a call option model, and compute return due to market timing.

Measuring Market Timing with Regression

Extending basic return regression models offers a tool to assess superior market timing skills of a portfolio manager. A market timer will include high (low) beta stocks in her portfolio if she expects an up (down) market. If her forecasts are accurate, her portfolio will outperform the benchmark portfolio. Using a market timing regression model, we can empirically test whether there is evidence of superior market timing skills exhibited by the portfolio manager. The regression equation used for this test is as follows:

$$R_P - R_F = \alpha + \beta_P(R_M - R_F) + M_P(R_M - R_F)D + \varepsilon_P$$

In this equation, *D* is a dummy variable that is assigned a value of 0 for down markets (i.e., when $R_M < R_F$) and 1 for up markets (i.e., when $R_M > R_F$). M_P is the difference between the up market and down market betas and will be positive for a successful market timer. In a bear market, beta is simply equal to β_P. In a bull market, beta is equal to $\beta_P + M_P$. Empirical evidence of mutual fund return data suggests that M_P is actual negative for most funds. Thus, researchers have concluded that fund managers exhibit little, if any, ability to correctly time the market.

Measuring Market Timing with a Call Option Model

Consider an investor who has 100% perfect market forecasting ability and holds a portfolio allocated either 100% to Treasury bills or 100% to the S&P 500 equity market index, depending on the forecast performance of the S&P 500 versus the Treasury bill return. The investor's portfolio will be:

100% invested in the S&P 500 if $E(R_M) > R_F$
100% invested in Treasury bills if $E(R_M) < R_F$

If the investor has perfect forecasting ability, then his return performance will be as follows:

R_M if $R_M > R_F$
R_F if $R_M < R_F$

Now consider an investor who invests S_0 (the current value of the S&P 500) in Treasury bills and also owns a call option on the S&P 500 with exercise price equal to the current value of the index times $(1 + R_F)$, or $S_0(1+R_F)$. Note that the exercise price equals the value of the S&P 500 if it grows at a rate equal to the risk-free rate.

What are the return possibilities for this investor? To answer this question, note that if the S&P 500 holding period return exceeds the risk-free rate, then the ending value of the call option will be:

$$S_T - X = S_0(1+R_M) - S_0(1+R_F)$$

The investor also owns Treasury bills with face value equal to $S_0(1+R_F)$. Therefore, the face value (FV) of the Treasury bills will perfectly offset the exercise price of the call option. In the up-market scenario, the ending value of the calls plus bills portfolio equals:

$$S_T - X + FV = S_0(1+R_M) - S_0(1+R_F) + S_0(1+R_F) = S_0(1+R_M)$$

Therefore, the return performance on the calls plus bills portfolio will equal:

R_M if $R_M > R_F$

If the market rises by less than the risk-free rate, the call option has no value, but the risk-free asset will still return R_F. Therefore, the down-market scenario return for the calls plus bills portfolio is:

R_F if $R_M < R_F$

In summary, the returns to the calls plus bills portfolio are identical to the 100% perfect foresight returns. Therefore, the value or appropriate fee for perfect foresight should equal the price of the call option on the market index.

Style Analysis

LO 70.8: Describe style analysis.

LO 70.9: Describe and apply performance attribution procedures, including the asset allocation decision, sector and security selection decision, and the aggregate contribution.

William Sharpe introduced the concept of style analysis. From January 1985 to December 1989 he analyzed the returns on Fidelity's Magellan Fund for style and selection bets. His study concluded that 97.3% of the fund's returns were explained by style bets (asset allocation), and 2.7% were due to selection bets (individual security selection and market timing). The importance of long-run asset allocation has been well established empirically. These results suggest that the returns to market timing and security selection are minimal at best and at worst insufficient to cover the associated operating expenses and trading costs.

The steps for Sharpe's style analysis are as follows:

1. Run a regression of portfolio returns against an exhaustive and mutually exclusive set of asset class indices:

   $$R_P = b_{P1}R_{B1} + b_{P2}R_{B2} + \ldots + b_{Pn}R_{Bn} + e_P$$

   where:
   R_P = return on the managed portfolio
   R_{Bj} = return on passive benchmark asset class n
   b_{Pj} = sensitivity or exposure of Portfolio P return to passive asset class n return
   e_P = random error term

In Sharpe's style analysis, the slopes are constrained to be non-negative and to sum to 100%. In that manner, the slopes can be interpreted to be "effective" allocations of the portfolio across the asset classes.

2. Conduct a performance attribution (return attributable to asset allocation and to selection):
   - The percent of the performance attributable to asset allocation = R^2 (the coefficient of determination).
   - The percent of the performance attributable to selection = $1 - R^2$.

The **asset allocation attribution** equals the difference in returns attributable to active asset allocation decisions of the portfolio manager:

$$[b_1R_{B1} + b_2R_{B2} + \ldots + b_nR_{Bn}] - R_B$$

Notice if the slopes (estimated allocations) for the managed portfolio equal those within the benchmark (passive asset allocation), then the asset allocation attribution will be zero.

The **selection attribution** equals the difference in returns attributable to superior individual security selection (correct selection of mispriced securities) and sector allocation (correct over and underweighting of sectors within asset classes):

$$R_P - [b_1 R_{B1} + b_2 R_{B2} + \ldots + b_n R_{Bn}]$$

Notice if the manager has no superior selection ability, then portfolio returns earned within each asset class will equal the benchmark asset class returns: $R_{Pj} = R_{Bj}$, and the selection attribution will equal zero. Also, notice that the sum of the two attribution components (asset allocation plus selection) equals the total excess return performance: $R_P - R_B$.

3. Uncover the investment style of the portfolio manager: the regression slopes are used to infer the investment style of the manager. For example, assume the following results are derived:

$$R_P = 0.75R_{LCG} + 0.15R_{LCV} + 0.05R_{SCG} + 0.05R_{SCV}$$

where:
R_{LCG} = return on the large cap growth index
R_{LCV} = return on the large cap value index
R_{SCG} = return on the small cap growth index
R_{SCV} = return on the small cap value index

The regression results indicate that the manager is pursuing primarily a large cap growth investment style.

Key Concepts

LO 70.1

The dollar-weighted rate of return is defined as the internal rate of return (IRR) on a portfolio, taking into account all cash inflows and outflows. The beginning value of the account is an inflow as are all deposits into the account. All withdrawals from the account are outflows, as is the ending value.

Time-weighted rate of return measures compound growth. It is the rate at which $1 compounds over a specified time horizon. Time-weighting is the process of averaging a set of values over time.

LO 70.2

The Sharpe ratio uses standard deviation (total risk) as the relevant measure of risk. It shows the amount of excess return (over the risk-free rate) earned per unit of total risk.

The Treynor measure is very similar to the Sharpe ratio except that it uses beta (systematic risk) as the measure of risk. It shows the excess return (over the risk-free rate) earned per unit of systematic risk.

Jensen's alpha is the difference between the actual return and the return required to compensate for systematic risk. To calculate the measure, we subtract the return calculated by the capital asset pricing model (CAPM) from the account return.

The information ratio is the ratio of the surplus return (in a particular period) to its standard deviation. It indicates the amount of risk undertaken to achieve a certain level of return above the benchmark.

LO 70.3

The M^2 measure compares the return earned on the managed portfolio against the market return, after adjusting for differences in standard deviations between the two portfolios.

LO 70.4

A positive alpha produces an indication of superior performance; a negative alpha produces an indication of inferior performance; and zero alpha produces an indication of normal performance matching the benchmark.

LO 70.5

Hedge fund performance evaluation is complicated because:

- Hedge fund risk is not constant over time (nonlinear risk).
- Hedge fund holdings are often illiquid (data smoothing).
- Hedge fund sensitivity with traditional markets increases in times of a market crisis and decreases in times of market strength.

LO 70.6

Changes in volatility will likely bias the Sharpe ratio, and produce incorrect conclusions when comparing portfolio performance to a benchmark or index.

LO 70.7

Extending basic return regression models offers a tool to assess superior market timing skills of a portfolio manager. A market timer will include high (low) beta stocks in her portfolio if she expects an up (down) market. If her forecasts are accurate, her portfolio will outperform the benchmark portfolio.

LO 70.8

William Sharpe introduced the concept of style analysis. From January 1985 to December 1989 he analyzed the returns on Fidelity's Magellan Fund for style and selection bets. His study concluded that 97.3% of the fund's returns were explained by style bets (asset allocation), and 2.7% were due to selection bets (individual security selection and market timing).

LO 70.9

The importance of long-run asset allocation has been well established empirically. Historical results suggest that the returns to market timing and security selection are minimal at best and at worst insufficient to cover the associated operating expenses and trading costs.

Concept Checkers

Use the following data to answer Questions 1 and 2.

Assume you purchase a share of stock for \$50 at time t = 0 and another share at \$65 at time t = 1, and at the end of year 1 and year 2, the stock paid a \$2.00 dividend. Also at the end of year 2, you sold both shares for \$70 each.

1. The dollar-weighted rate of return on the investment is:
   A. 10.77%.
   B. 15.45%.
   C. 15.79%.
   D. 18.02%.

2. The time-weighted rate of return on the investment is:
   A. 18.04%.
   B. 18.27%.
   C. 20.13%.
   D. 21.83%.

3. The following information is available for funds ABC, RST, JKL, and XYZ:

Fund	*Annual Rate of Return*	*Beta*	*Volatility*
ABC	15%	1.25	20%
RST	18%	1.00	25%
JKL	25%	1.20	15%
XYZ	11%	1.36	9%

The average risk-free rate was 5%. Rank the funds from best to worst according to their Treynor measure.
A. JKL, RST, ABC, XYZ.
B. JKL, RST, XYZ, ABC.
C. RST, JKL, ABC, XYZ.
D. XYZ, ABC, RST, JKL.

Use the following information to answer Question 4.

The following data has been collected to appraise funds A, B, C, and D:

	Fund A	*Fund B*	*Fund C*	*Fund D*	*Market Index*
Return	8.25%	7.21%	9.44%	10.12%	8.60%
Beta	0.91	0.84	1.02	1.34	1.00
Standard deviation	3.24%	3.88%	3.66%	3.28%	3.55%

The risk-free rate of return for the relevant period was 4%.

4. Calculate and rank the funds from best to worst using Jensen's alpha.
   A. B, D, A, C.
   B. A, C, D, B.
   C. C, A, D, B.
   D. C, D, A, B.

5. Sharpe's style analysis, used to evaluate an active portfolio manager's performance, measures performance relative to:
   A. a passive benchmark of the same style.
   B. broad-based market indices.
   C. the performance of an equity index fund.
   D. an average of similar actively managed investment funds.

Concept Checker Answers

1. **D** One way to do this problem is to set up the cash flows so that the PV of inflows = PV of outflows and then to plug in each of the multiple choices.

   $50 + 65 / (1 + IRR) = 2 / (1 + IRR) + 144 / (1 + IRR)^2 \rightarrow IRR = 18.02\%$

   Alternatively, on your financial calculator, solve for IRR: $-50 - \frac{65-2}{1+IRR} + \frac{2(70+2)}{(1+IRR)^2} = 0$

Calculating Dollar-Weighted Return With the TI Business Analyst II Plus®		
Key Strokes	Explanation	Display
[CF] [2nd] [CLR WORK]	Clear CF Memory Registers	CF0 = 0.00000
50 [+/–] [ENTER]	Initial cash inflow	CF0 = –50.00000
[↓] 63 [+/–][ENTER]	Period 1 cash inflow	C01 = –63.00000
[↓] [↓] 144 [ENTER]	Period 2 cash outflow	C02 = 144.00000
[IRR] [CPT]	Calculate IRR	IRR = 18.02210

2. **D** $HPR_1 = (65 + 2) / 50 - 1 = 34\%$, $HPR_2 = (140 + 4) / 130 - 1 = 10.77\%$

   time-weighted return = $[(1.34)(1.1077)]^{0.5} - 1 = 21.83\%$.

3. **A** Treynor measures:

$$T_{ABC} = \frac{0.15-0.05}{1.25} = 0.08 = 8$$

$$T_{RST} = \frac{0.18-0.05}{1.00} = 0.13 = 13$$

$$T_{JKL} = \frac{0.25-0.05}{1.20} = 0.1667 = 16.7$$

$$T_{XYZ} = \frac{0.11-0.05}{1.36} = 0.0441 = 4.4$$

The following table summarizes the results:

Fund	*Treynor Measure*	*Rank*
ABC	8.00%	3
RST	13.00%	2
JKL	16.67%	1
XYZ	4.41%	4

4. **C** CAPM Returns:

R_A = 4 + 0.91(8.6 – 4) = 8.19%
R_B = 4 + 0.84(8.6 – 4) = 7.86%
R_C = 4 + 1.02(8.6 – 4) = 8.69%
R_D = 4 + 1.34(8.6 – 4) = 10.16%

	Fund A	*Fund B*	*Fund C*	*Fund D*
Alpha	8.25% – 8.19% = +0.06	7.21% – 7.86% = –0.65%	9.44% – 8.69% = +0.75%	10.12% – 10.16% = –0.04%
Ranking	2	4	1	3

5. **A** Sharpe's style analysis measures performance relative to a passive benchmark of the same style.