18/11/2025

Final Exam

Lecture-01

Seperation of Variables

Standard form:

$$\frac{dy}{dx} = f(x)g(y)$$

$$\Rightarrow \frac{dy}{g(y)} = f(x)dx$$

$$\Rightarrow \int \frac{dy}{g(y)} = \int f(x)dx$$

Steps:

i) Seperate the variable

ii) Integrate both sides

iii) Write standard form:

$$y = Ax + B$$

2 types of Sol^n:

→ General Sol^n: Unknown Constants

→ Particular Sol^n: Known Constants

Ex-1:

$$\frac{dy}{dx} = 3x^2 e^{-y}$$

$$\Rightarrow \frac{dy}{e^{-y}} = 3x^2 dx$$

$$\Rightarrow \int \frac{dy}{e^{-y}} = \int 3x^2 dx$$

$$\Rightarrow \int e^{y} dy = 3 \int x^2 dx$$

$$\Rightarrow e^{y} + C_1 = 3 \frac{x^3}{3} + C_2$$

$$\Rightarrow e^{y} = x^3 + (C_2 - C_1)$$

$$\Rightarrow e^{y} = x^3 + C$$

Given, $y(0) = 1$

$\therefore x = 0, y = 1$

So, $e^1 = 0^3 + C$

$\therefore e = C$

Ex-4:

$$y^2 \frac{dy}{dx} = x$$

$$\Rightarrow \int y^2 dy = \int x dx$$

$$\Rightarrow \frac{y^3}{3} = \frac{x^2}{2} + C$$ ← General Sol^n

Given, $y(0) = 1$

$\therefore x = 0, y = 1$

So, $\frac{1^3}{3} = \frac{0^2}{2} + C$

$$\Rightarrow \frac{1}{3} = C$$

$$\therefore C = \frac{1}{3}$$

$\ln(-1)$ ✗ ① $\frac{1}{\tan y} = \cot y$

$\ln(0)$ ✗

$\ln(1) = 0$ ② $\int \cot y\, dy = \ln|\sin y|$

Ex-5:

$$\frac{dy}{dx} = e^{2x+y}$$

$$\Rightarrow \frac{dy}{dx} = e^{2x} \cdot e^{y}$$

$$\Rightarrow \int \frac{dy}{e^{y}} = \int e^{2x} dx$$

$$\Rightarrow \int e^{-y} dy = \int e^{2x} dx$$

$$\Rightarrow \frac{e^{-y}}{-1} = \frac{e^{2x}}{2} + c$$

Given,

$$x = 0,\ y = 0$$

So, $-e^{-0} = \frac{e^{0}}{2} + c$

$$\Rightarrow -1 = \frac{1}{2} + c$$

$$\Rightarrow c = -\frac{3}{2}$$

Ex-8:

$$\frac{dy}{dx} = -2x \tan y$$

$$\Rightarrow \frac{dy}{\tan y} = -2x\, dx$$

$$\Rightarrow \int \frac{dy}{\tan y} = \int -2x\, dx$$

$$\Rightarrow \int \cot y\, dy = -2 \int x\, dx$$

$$\Rightarrow \ln|\sin y| = -x^2 + C$$

Given,

$$x = 0,\ y = \frac{\pi}{2}$$

So, $\ln\left|\sin \frac{\pi}{2}\right| = -0^2 + C$

$$\therefore C = 0$$

$$\therefore \ln\left|\sin \frac{\pi}{2}\right| = -x^2 + 0 = -x^2$$

Ex-9:

$$(1+x^2)\frac{dy}{dx}+xy=0$$

$$\Rightarrow (1+x^2)\frac{dy}{dx}=-xy$$

$\bullet \int \frac{1}{y} = \ln y$

$$\Rightarrow \int \frac{dy}{-y}=\int \frac{x}{1+x^2}\,dx$$

$\bullet \int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)|+c$

$$\Rightarrow -\ln y = \frac{1}{2}\int \frac{2x}{1+x^2}\,dx$$

$$\Rightarrow -\ln y = \frac{1}{2}\ln|1+x^2|+c$$

Given, $y(0)=2$

$\therefore x=0,\ y=2$

So, $-\ln 2 = \frac{1}{2}\ln|1+0^2|+c$

$$\Rightarrow c=-\ln 2$$

$$\therefore -\ln y = \frac{1}{2}\ln|1+x^2|-\ln 2$$

Ex- 14:

$$\sec x \frac{dy}{dx} = \sec^2 y$$

$$\Rightarrow \frac{dy}{\sec^2 y} = \frac{dx}{\sec x}$$

$$\Rightarrow \frac{1}{2}\int 2\cos^2 y\, dy = \int \cos x\, dx$$

$$\Rightarrow \frac{1}{2}\int (1+\cos 2y)\, dy = \sin x + c$$

$$\Rightarrow \frac{1}{2}\left[y + \frac{\sin 2y}{2} \right] = \sin x + c$$

③ $\frac{1}{\sec^2 y} = \cos^2 y$

④ $\cos 2y = 2\cos^2 y - 1$

$\Rightarrow 2\cos^2 y = 1 + \cos 2y$

Application Type Problem:

⊛ $\frac{dT}{dt} = -k(T-25)$

$$\Rightarrow \int \frac{dT}{T-25} = \int -k \cdot dt$$

$$\Rightarrow \ln|T-25| = -kt + c$$

Given, $t=0$, $T(0) = 90°$

$\therefore \ln|90-25| = c$

$\Rightarrow c = \ln|65|$

Given, $t=4$, $T(0) = 60°$

$\therefore \ln|60-25| = -k \cdot 4 + \ln|65|$

$\Rightarrow \ln|35| = -4k + \ln|65|$

$\Rightarrow k = 0.154$

So, $\ln|T-25| = -0.154t + \ln|65|$

(2) Given, ~~t=0~~, ~~T=~~

$T = 50°$

$\therefore \ln|50-25| = -0.154t + \ln|65|$

$\Rightarrow \ln|25| = -0.154t + \ln|65|$

$\Rightarrow t = 6.20$ min

(3) Additional time needed $= (6.20 - 4)$ min

$= 2.20$ min.

Homogenous function : Power of each term is same (degree same)

How to solve it?

S.F. :

$$\frac{dy}{dx} = \frac{M(x,y)}{N(x,y)}$$

(i) Let, $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

(ii) Put the values into main D.E.

(iii) Solve it using seperation of variables

$M(x,y) = 3x^{2} + x^{1}y^{1}$ ← Homogenous funct^n

Ex-1:

$$\frac{dy}{dx} = \frac{xy+y^2}{x^2}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{x \cdot vx + (vx)^2}{x^2}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{vx^2+v^2x^2}{x^2}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{x^2(v+v^2)}{x^2}$$

$$\Rightarrow v + x\frac{dv}{dx} = v+v^2$$

$$\Rightarrow x\frac{dv}{dx} = v^2$$

$$\Rightarrow \int \frac{dv}{v^2} = \int \frac{dx}{x}$$

$$\Rightarrow -\frac{1}{v} = \ln x + c$$

$$\Rightarrow -\frac{1}{\frac{y}{x}} = \ln x + c$$

$$\Rightarrow -\frac{x}{y} = \ln x + c$$

- $\int x^n dx = \frac{x^{n+1}}{n+1}$
- $\int v^{-2} dv = \frac{v^{-2+1}}{-2+1}$
  $= -\frac{1}{v}$

Ex-9: Homework

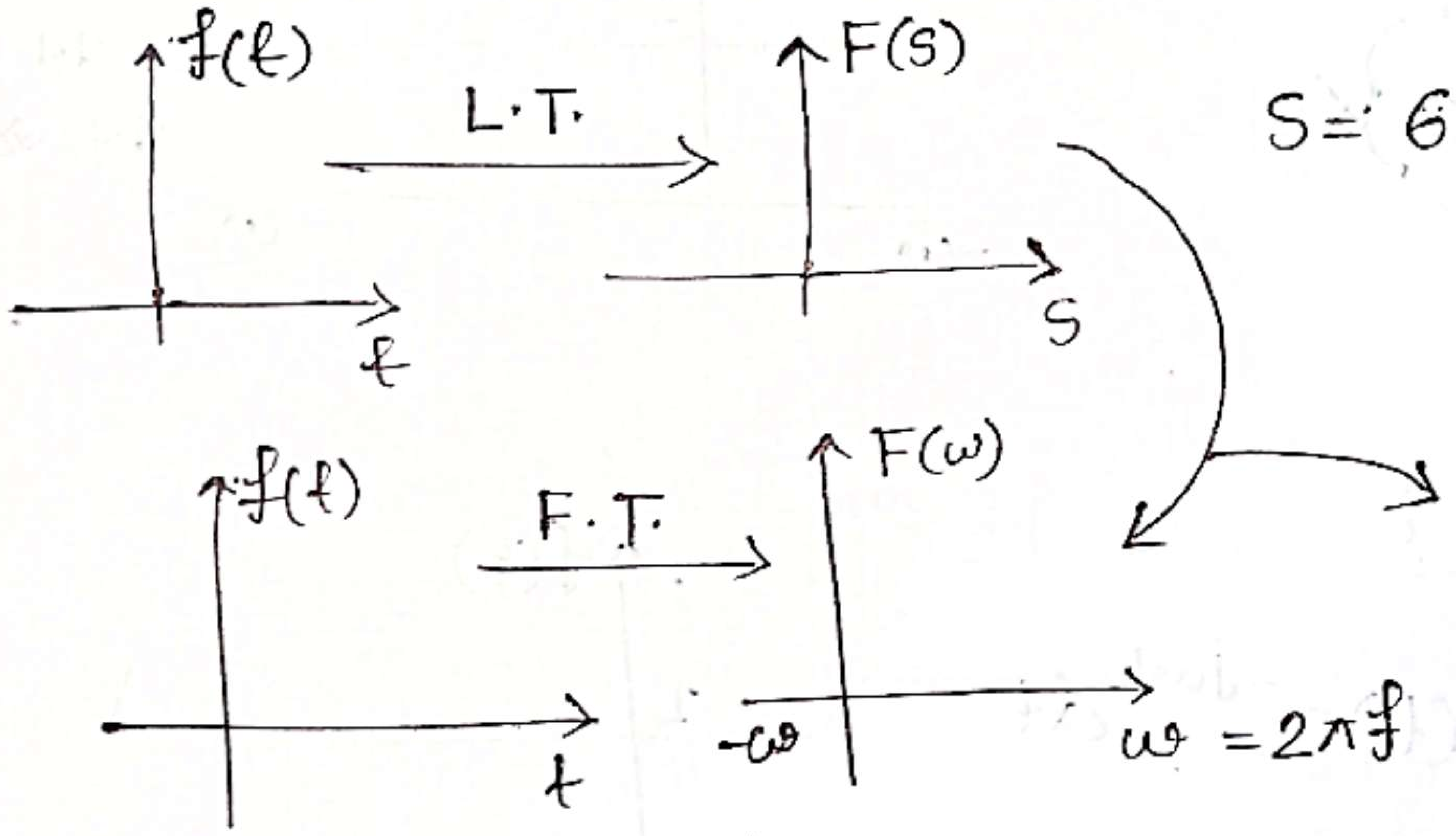
23/11/2025

Lecture-02

Fourier Transform

$$F(S) = \int_{0}^{\infty} f(t)\, e^{-St}\, dt$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt \Rightarrow \text{Fourier Transform}$$



$S = \sigma + j\omega$

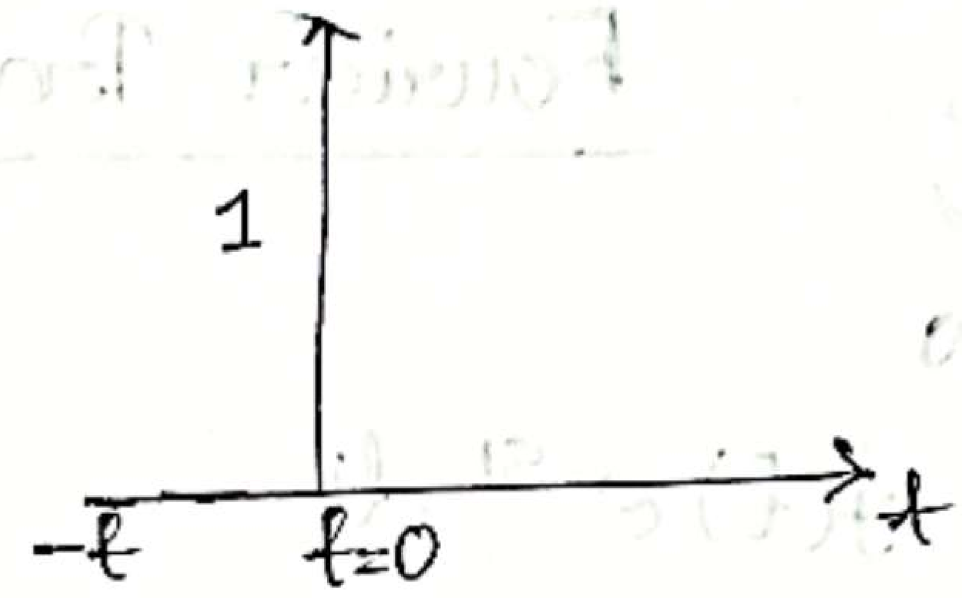
$S = j\omega$ [S replace by $j\omega$]

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega \Rightarrow \text{Inverse Fourier Transform}$$

⊞ Impulse function : $\delta(t)$

$$\delta(t) = \begin{cases} 1, & t = 0 \\ 0, & t \neq 0 \end{cases}$$

Time Domain Graph:

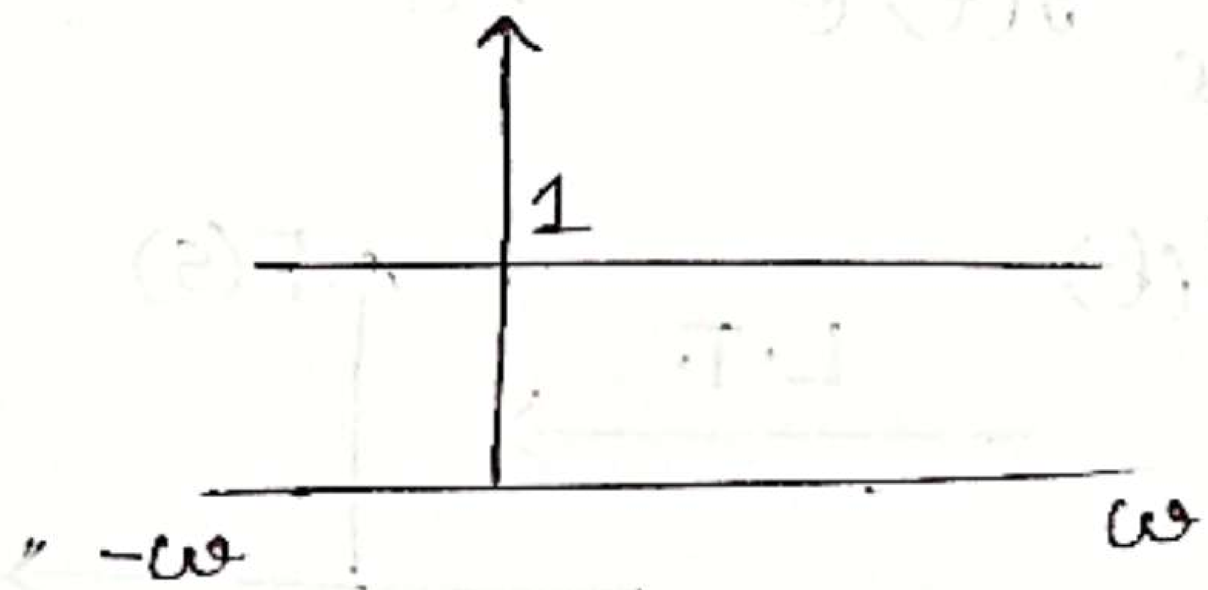


$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} \, dt$$

$$= \underline{1}$$

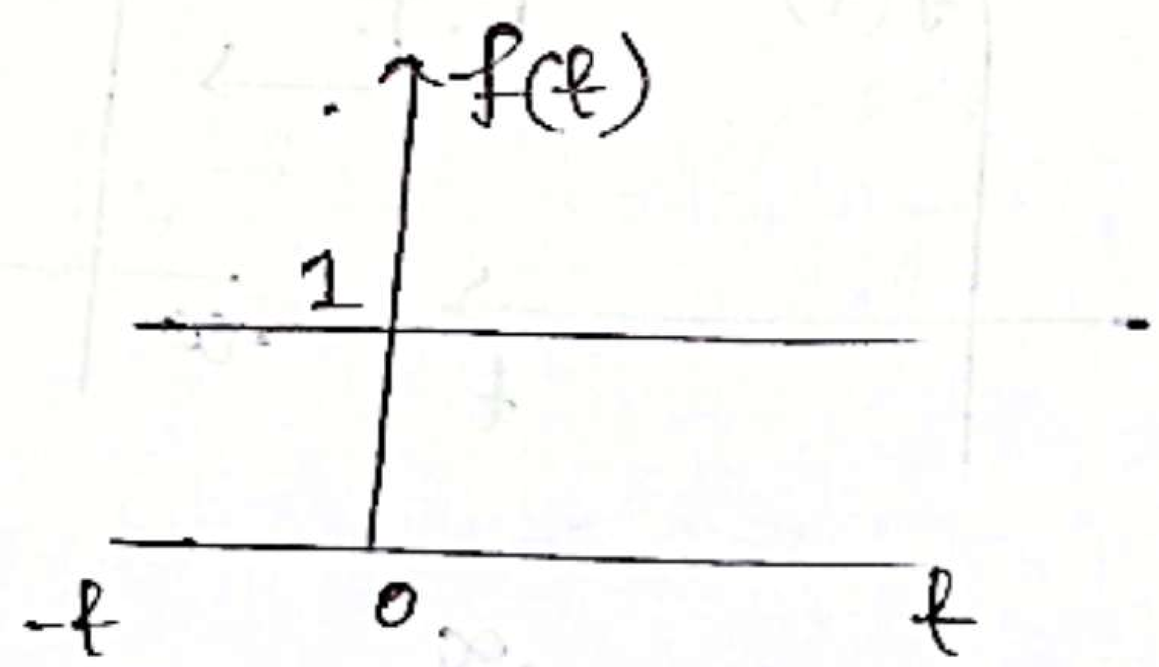
$$\delta(t) \xrightarrow{F.T.} 1$$

Fourier graph/(frequency domain graph)



ω এর সকল value এর জন্য 1

$f(t) = 1$, $F(\omega) = ?$



$$\therefore F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} \, dt$$

$$= \int_{-\infty}^{\infty} 1 \cdot e^{-j\omega t} \, dt$$

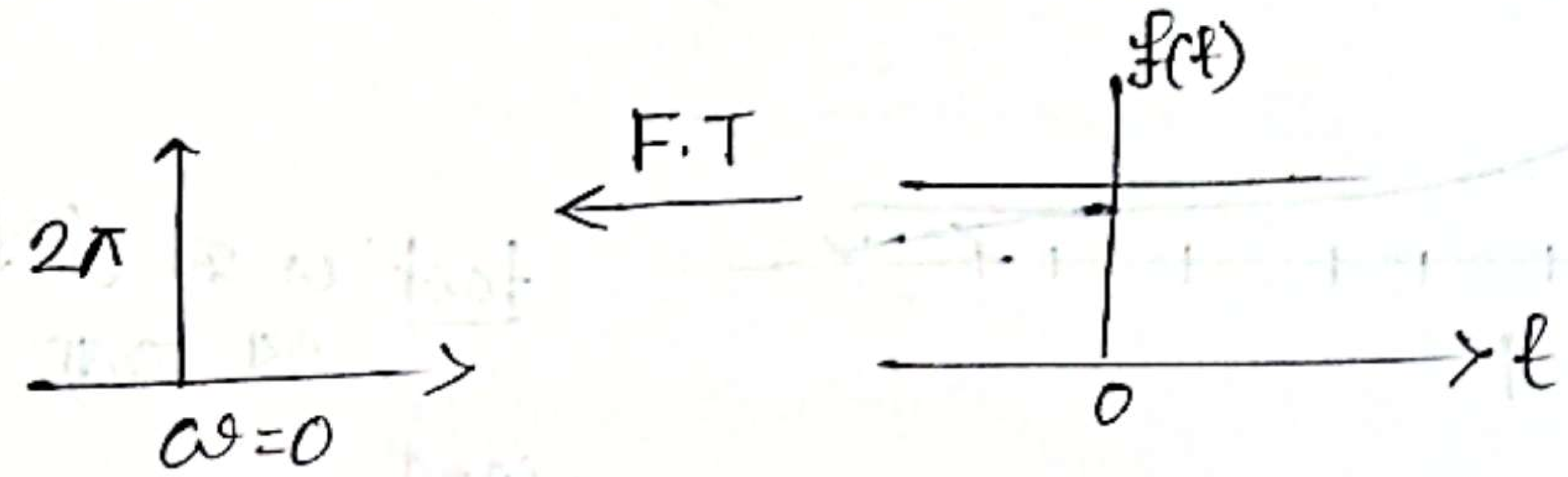
$$= \int_{-\infty}^{\infty} e^{-j\omega t} \, dt$$ ⇒ Improper Integral / Dirac Delta func

integration ∞ আসবে, তাই Value

$$= 2\pi \delta(\omega)$$

$\hookrightarrow \underline{\omega = 0}$

$1 \longrightarrow 2\pi\delta(\omega)$ $\qquad$ $\delta(t) \xrightarrow{F.T} 1$



Ex-1: $f(t) = \begin{cases} e^{-at}, & t>0 \\ 0, & t<0 \end{cases}$

$$\therefore F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-at} \cdot e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

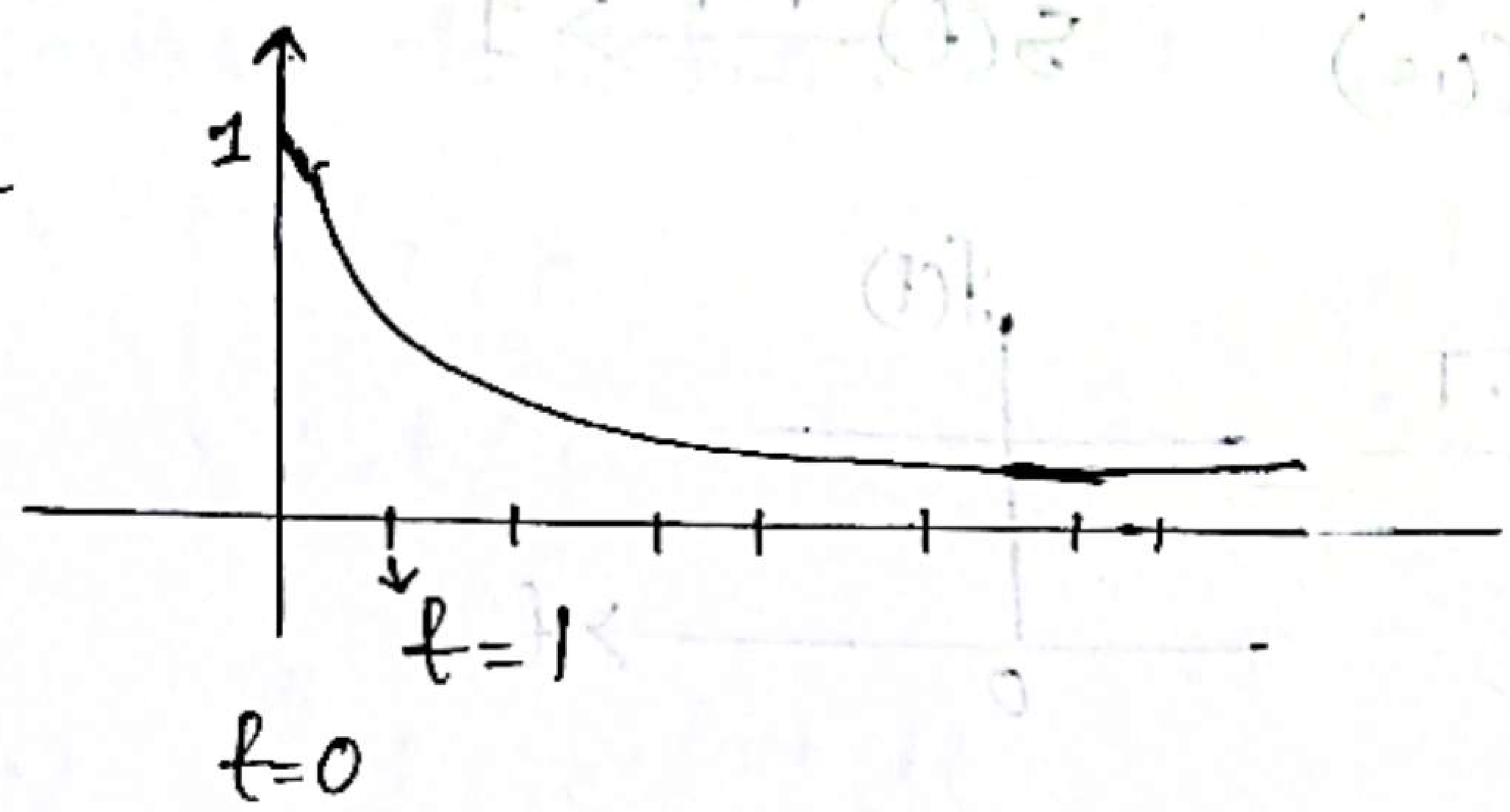
$$= \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right]_0^{\infty} = \frac{1}{-(a+j\omega)} \left[e^{-\infty} - e^{0} \right]$$

$$= \frac{1}{-(a+j\omega)} [0-1]$$

$$= \frac{1}{a+j\omega}$$

2

e^{-at}

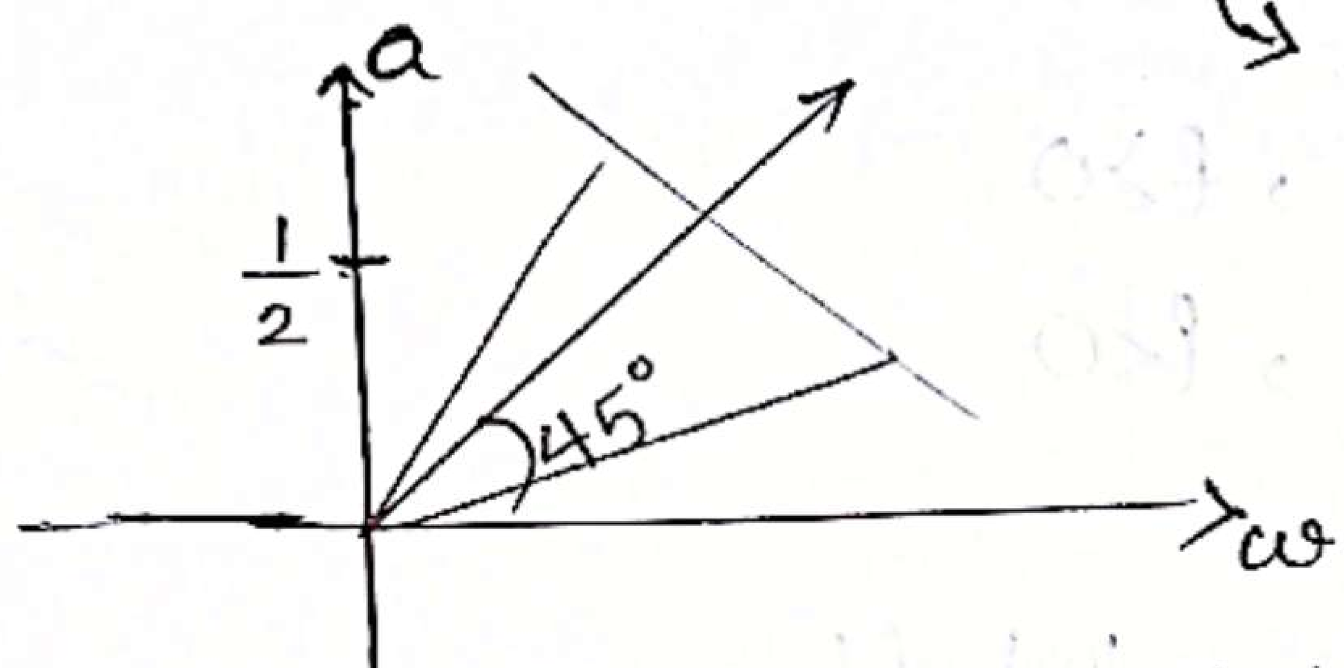


task: ω এর different value এর জন্য graph draw

$\omega=1$

$\frac{1}{2+j} = 3\angle 45^\circ$

$F(\omega) = \frac{1}{a+j\omega}$

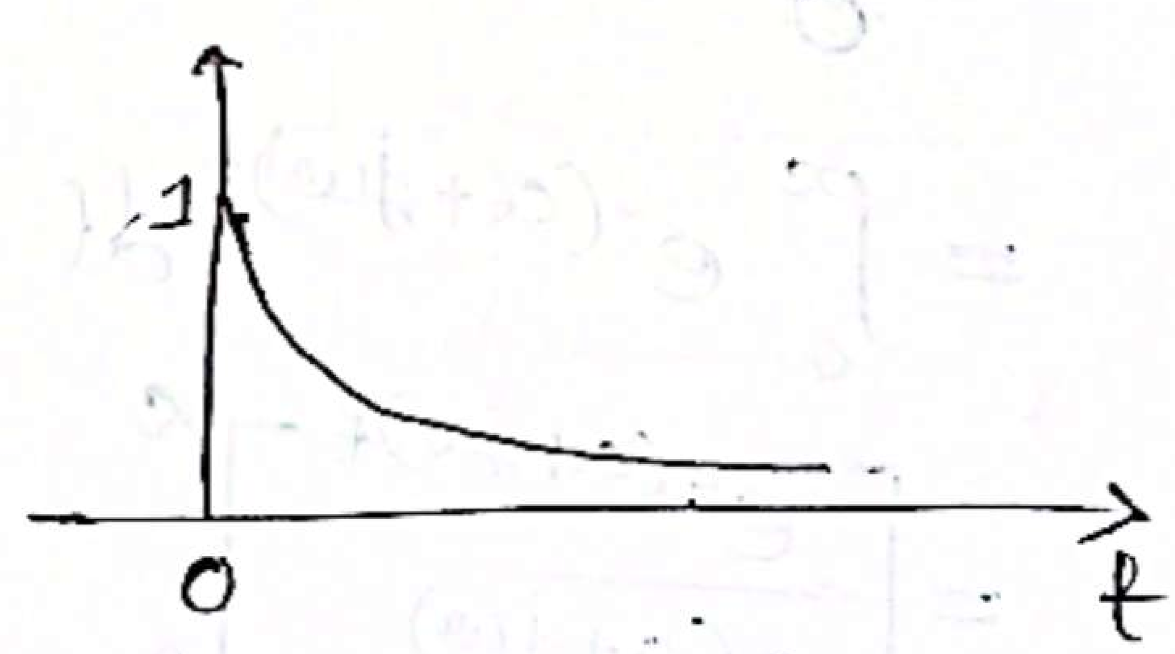


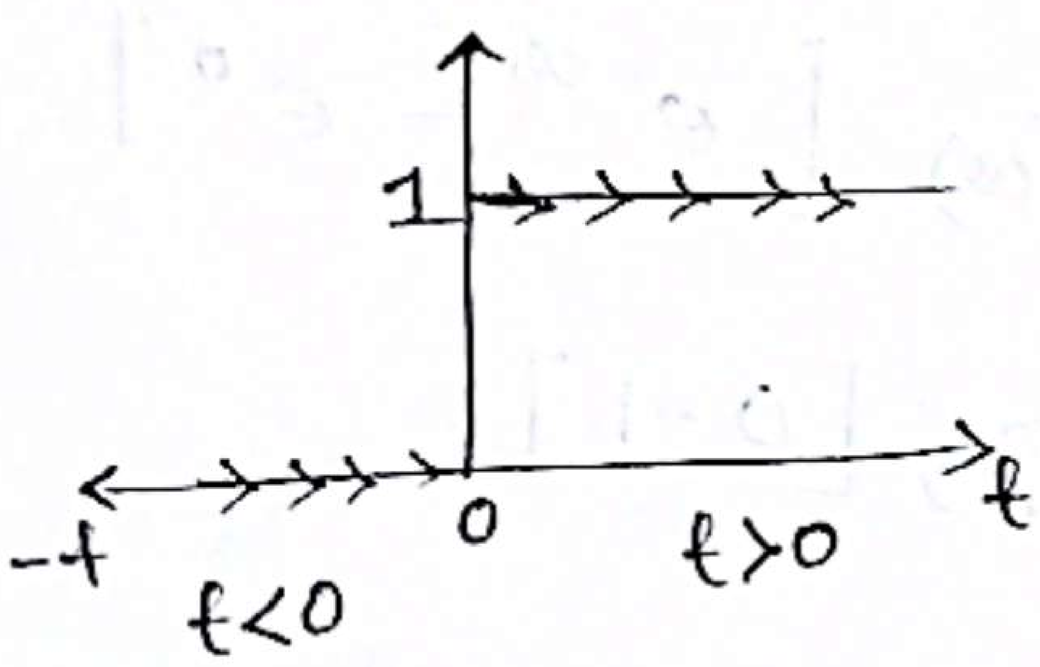
$\omega = 0 \rightarrow F(0) = \frac{1}{a+0} = \frac{1}{a} = \frac{1}{2}$

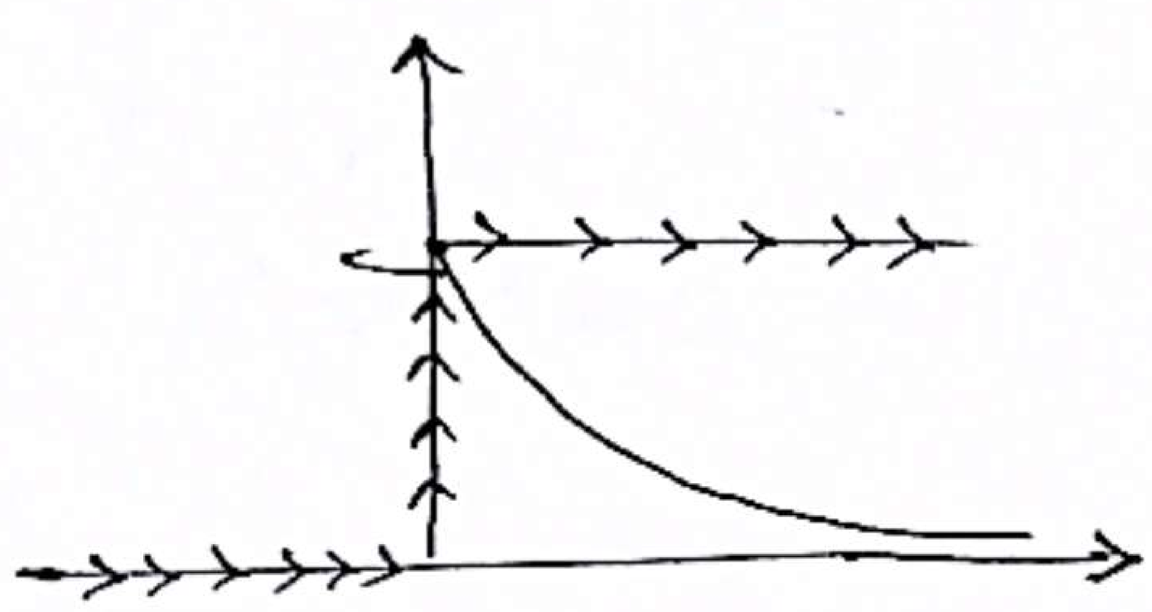
Unit-Step function:

$f(t) = e^{-at}$

$u(t) = \begin{cases} 1, & t>0 \\ 0, & t<0 \end{cases}$

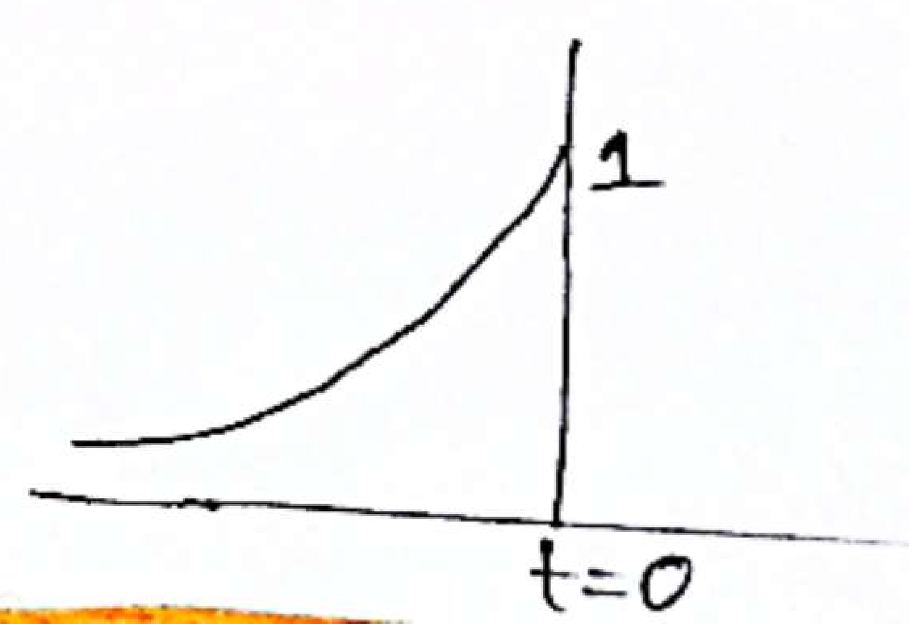




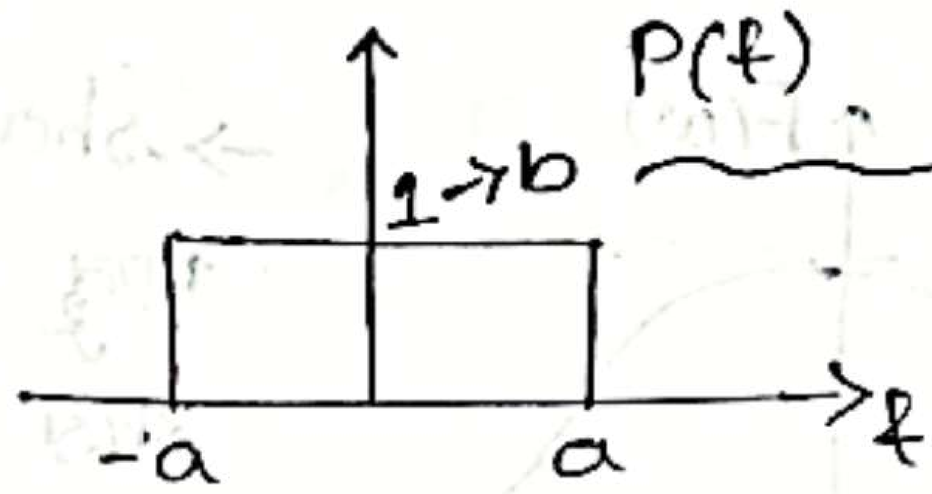


$f(t) = e^{-at}\cdot u(t)$

- $e^{-at}u(t) \longrightarrow \frac{1}{a+j\omega}$
- $e^{at}u(-t) \longrightarrow \frac{1}{a-j\omega} \longrightarrow$



Rectangular Pulse:



$\rightsquigarrow \dfrac{2b}{\omega}\sin\omega a$

$$P(\omega) = \int_{-\infty}^{\infty} P(t) \cdot e^{-j\omega t}\, dt$$

$$= \int_{-a}^{a} 1 \cdot e^{-j\omega t}\, dt$$

$$= \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-a}^{a}$$

$$= \frac{1}{j\omega}\left[e^{-j\omega a} - e^{j\omega a}\right]$$

$$= \frac{2}{\omega}\left[\frac{e^{j\omega a} - e^{-j\omega a}}{2j}\right]$$

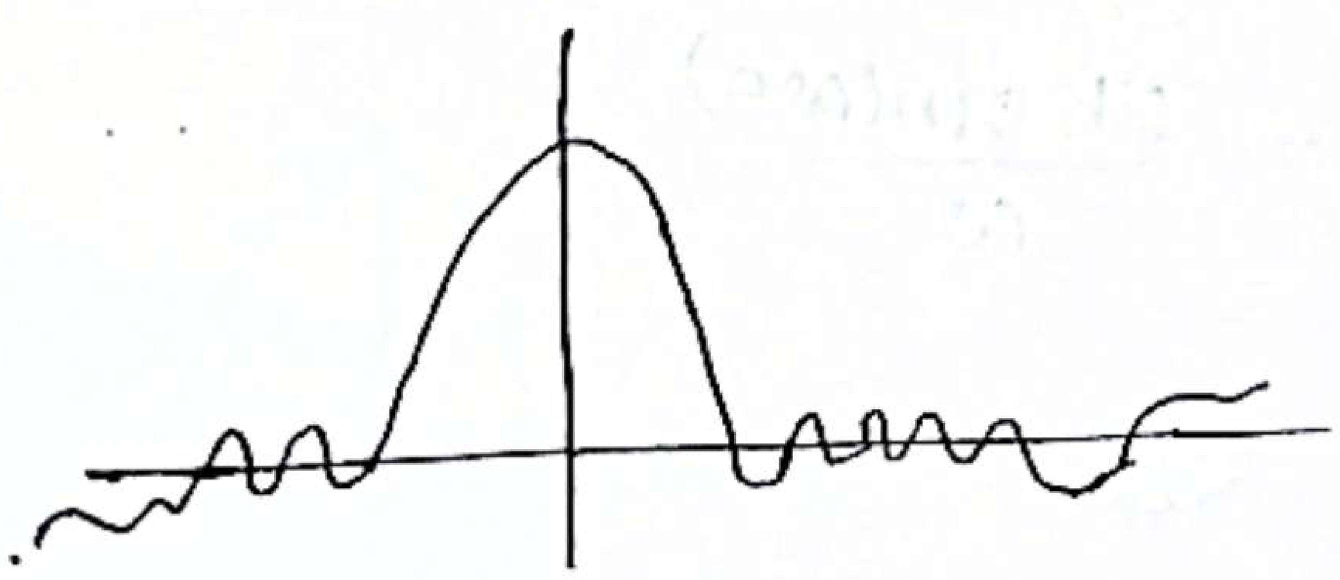
$\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$

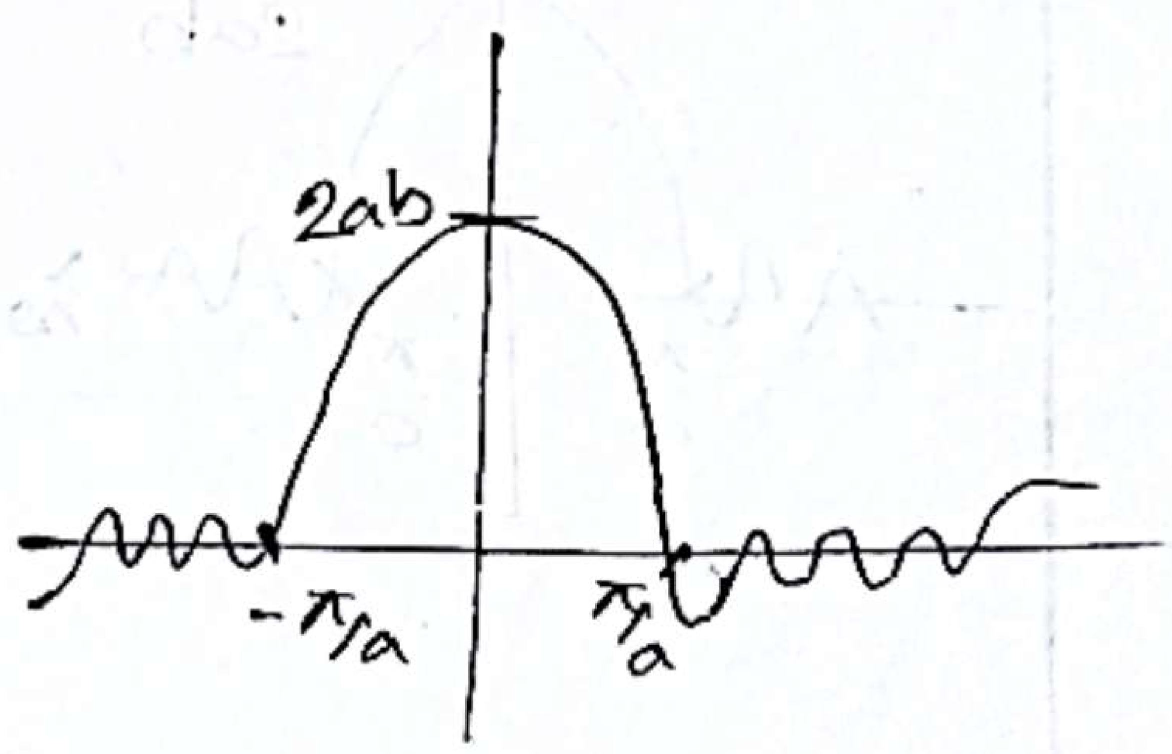
$$= \frac{2}{\omega}\sin\omega a$$

$\dfrac{2b}{\omega}\sin(\omega a)$

$= 2ab \text{ sinc}(a\omega) \rightarrow$ Plot

(*) $\dfrac{\sin x}{x} \longrightarrow \text{Sinc}(x)$





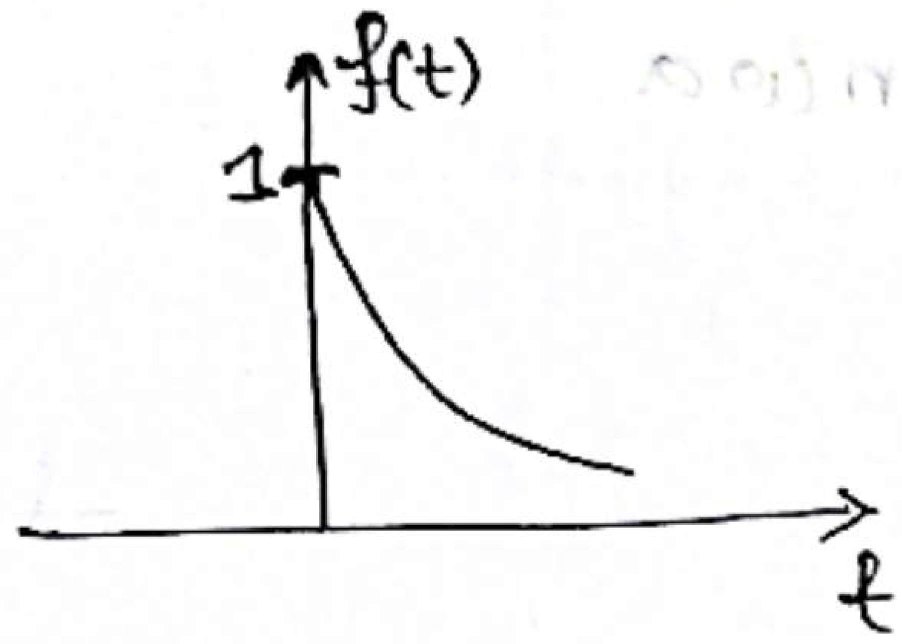
$\omega a = \pi$

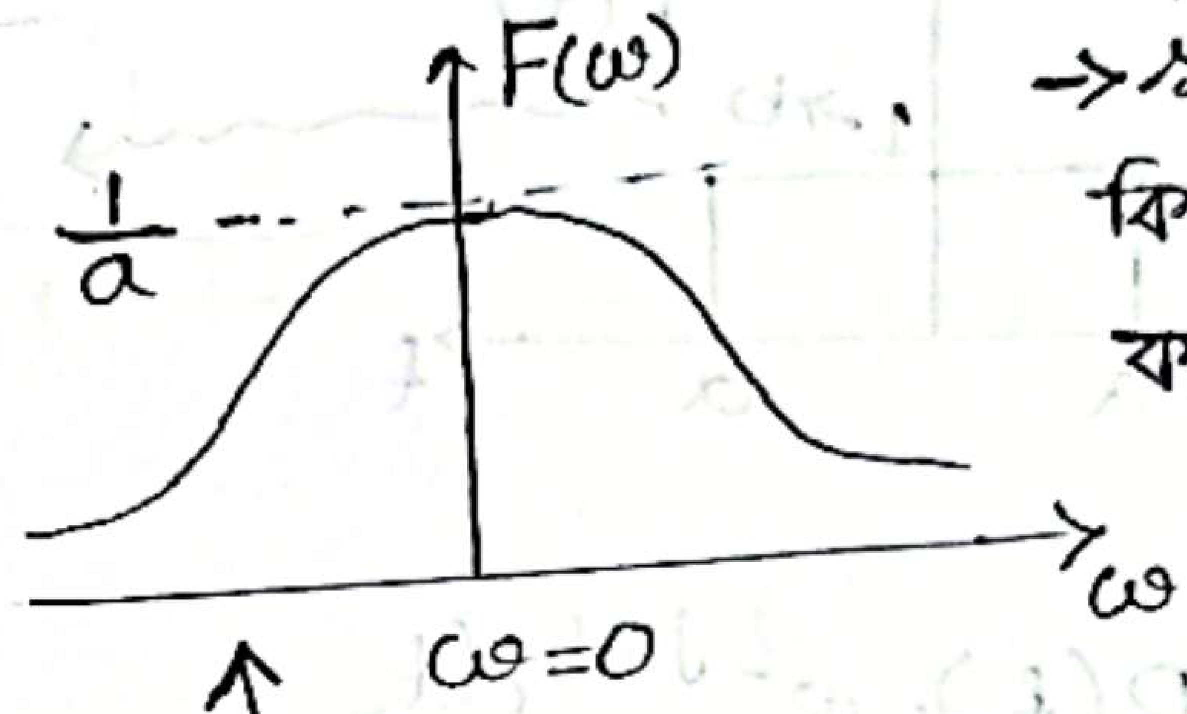
$\Rightarrow \omega = \pi/a$

25/11/2025

Lecture-03

Preview on last class:





→ shape same থাকবে, কিন্তু a এর উপর depend করে height ছোট / বড় হবে।

Formula:

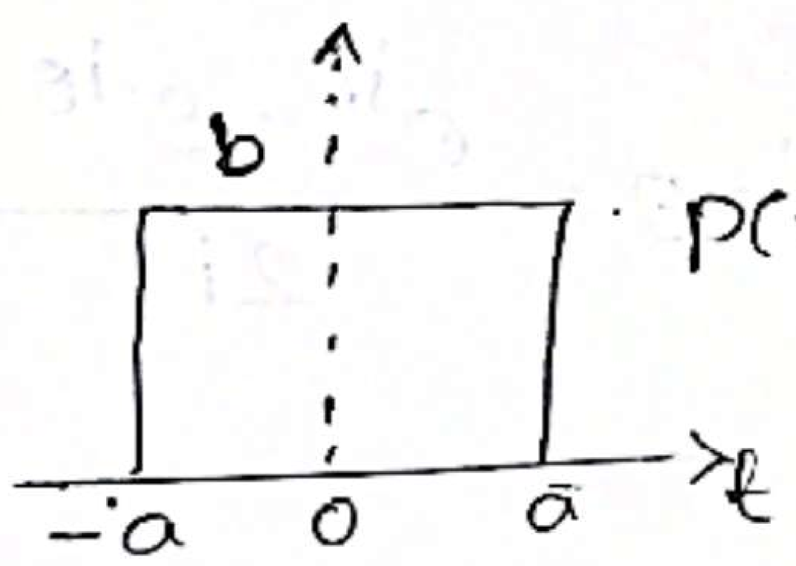
1) $e^{-at}u(t) \xrightarrow{F.T.} \frac{1}{a+j\omega}$

while $t=-t$ ← 2) $e^{at}u(-t) \xrightarrow{F.T.} \frac{1}{a-j\omega}$

3) $\delta(t) \longrightarrow 1$

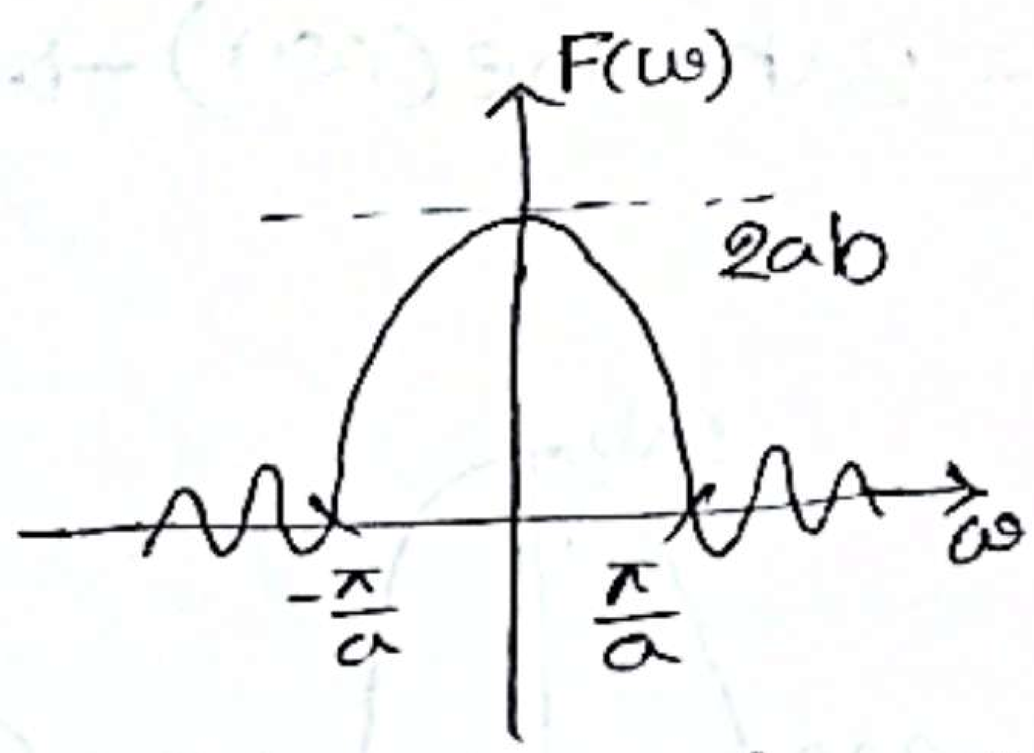
4) $1 \longrightarrow 2\pi\,\delta(\omega)$

5)



$$p(t)=\begin{cases} b, & -a\leq t\leq a \\ 0, & \text{otherwise}\end{cases}$$

↓ F.T.

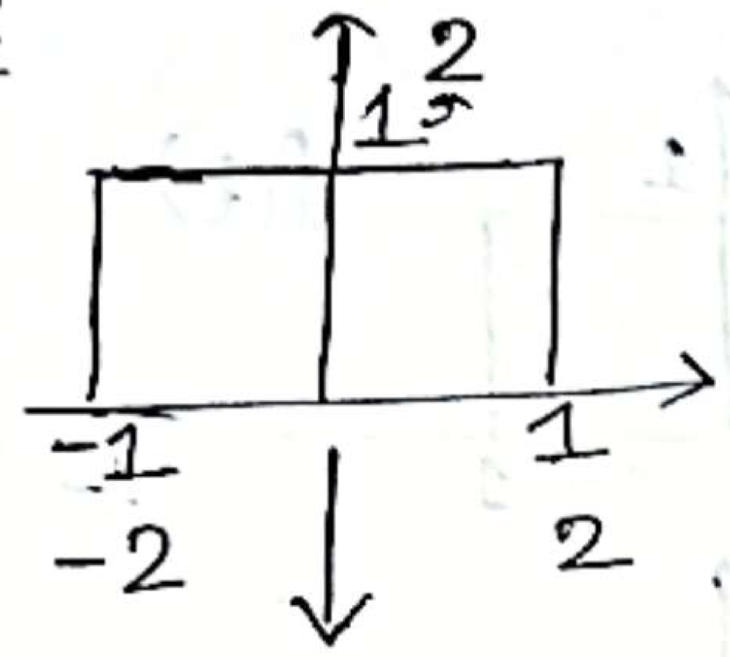


Plot করার জন্য

$$F(\omega) = 2ab\,\text{sinc}(\omega a)$$

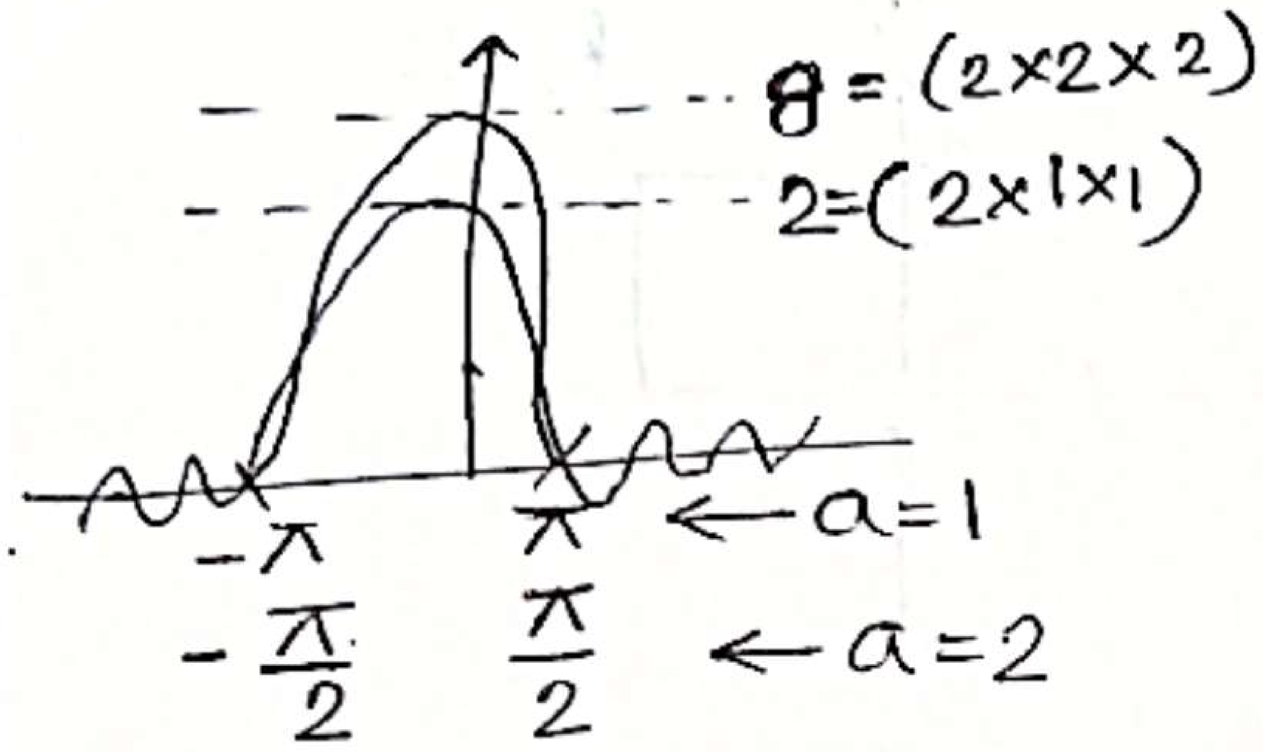
$$\text{or} = \frac{2b\sin(\omega a)}{\omega}$$

Ex:



$a=1, b=1$

$a=2, b=2$



$a=1, b=1$

$$F(\omega) = \frac{2\sin\omega}{\omega}$$

$$= 2\,\mathrm{sinc}(\omega)$$

$a=2, b=2$

$$F(\omega) = \frac{4\sin(\omega\cdot 2)}{\omega}$$

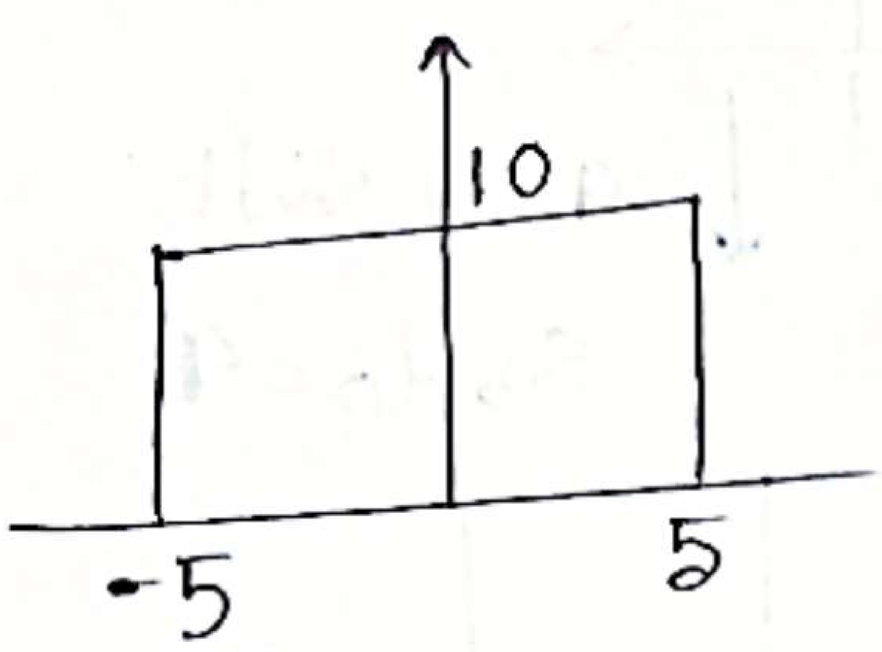
$$= 8\sin(2\omega)$$

* $F(t) = 2e^{-t}(ut) + 3e^{-2t}u(t)$

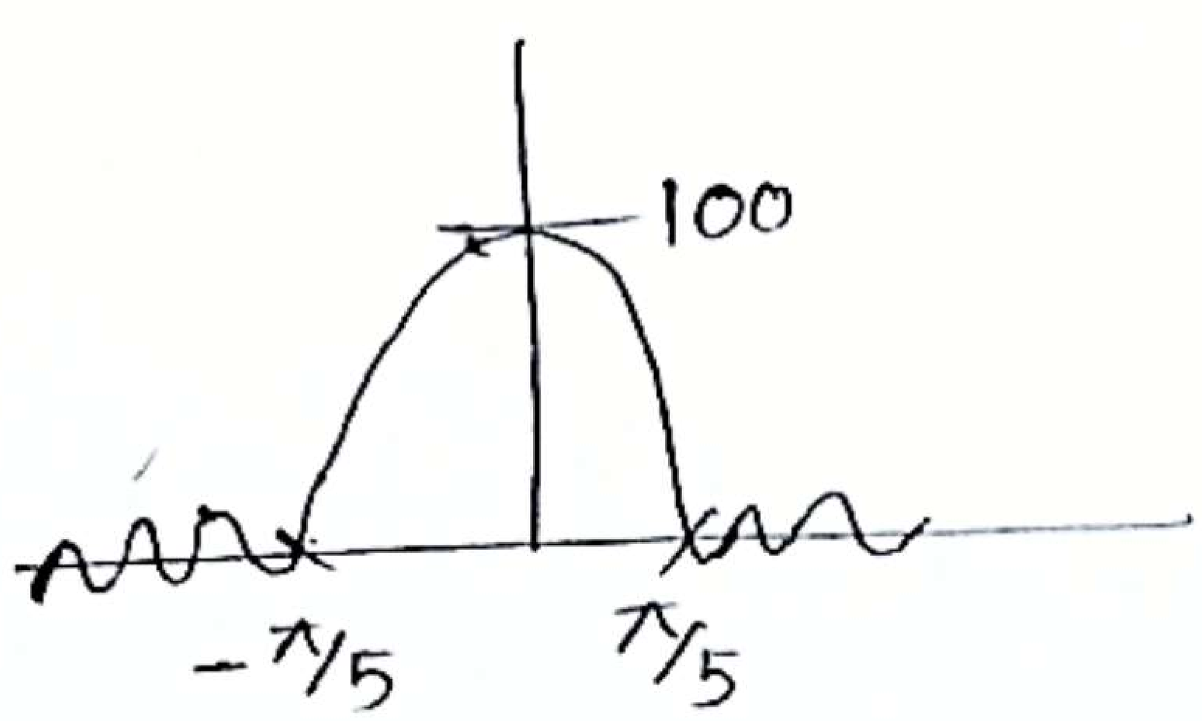
$\downarrow$

$$F(\omega) = 2\frac{1}{1+j\omega} + 3\frac{1}{2+j\omega}$$

*



$a=5, b=10$



$$F(\omega) = 100\,\mathrm{sinc}(5\omega)$$

$$= \frac{20\sin(5\omega)}{\omega}$$

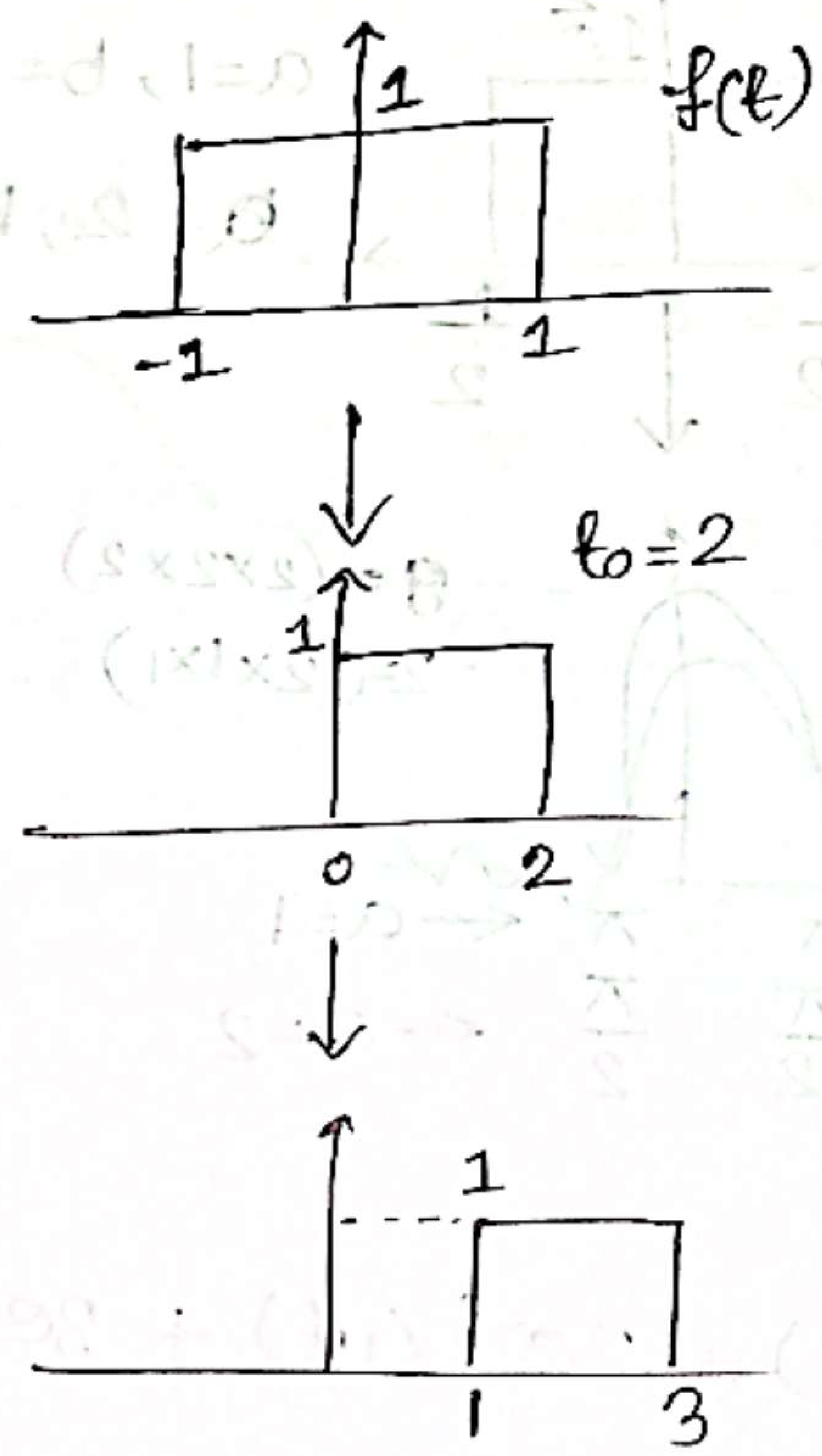
right shift → (−)
left shift → (+)

⊛ $f(t) \longrightarrow F(\omega)$

$\downarrow$

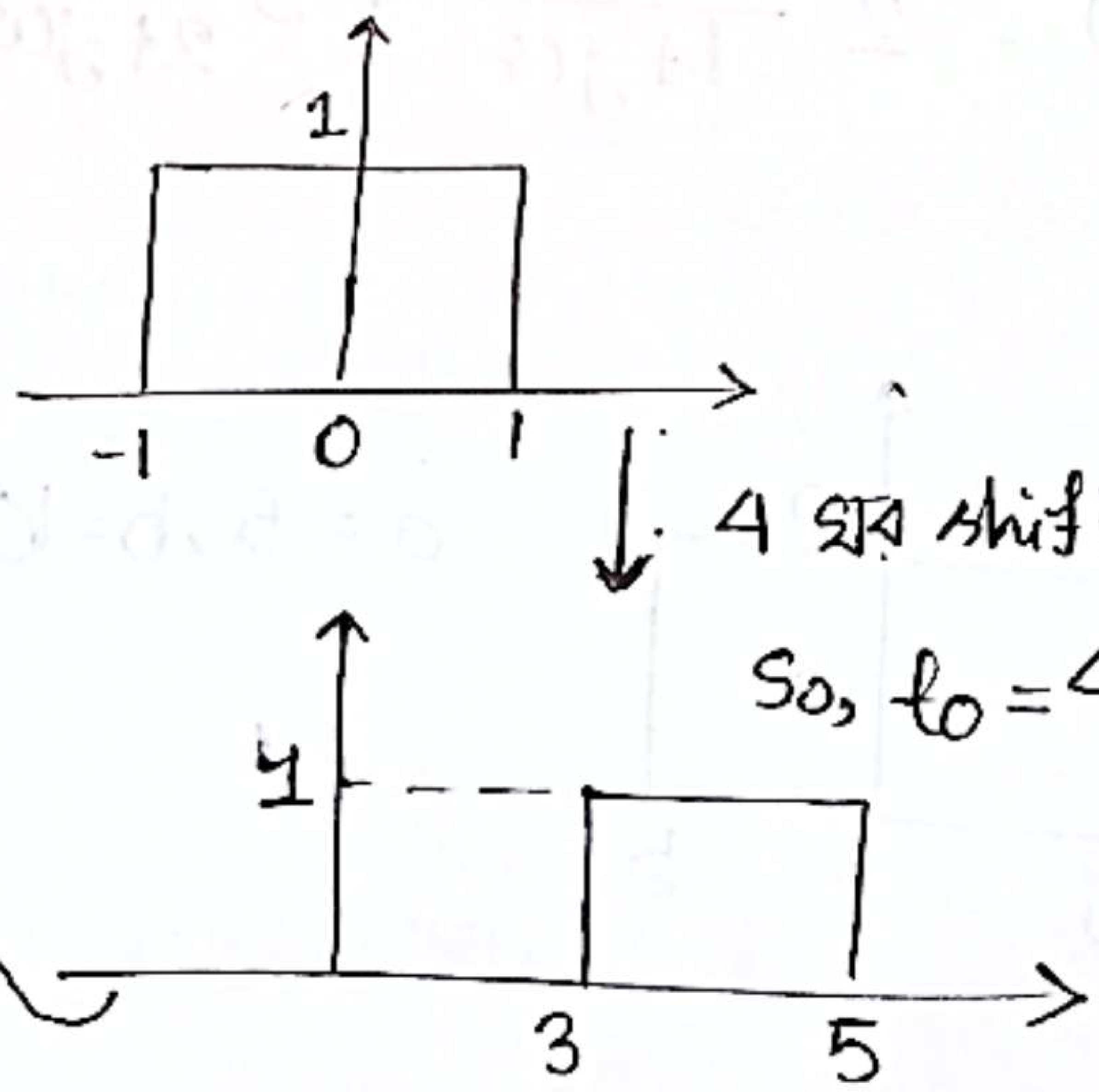
$f(t \mp t_0) \longrightarrow e^{\mp j\omega t_0} F(\omega)$

$\Rightarrow e^{-j\omega 2} \, \frac{2\sin\omega}{\omega}$



⊛ $f(t) \longrightarrow F(\omega)$

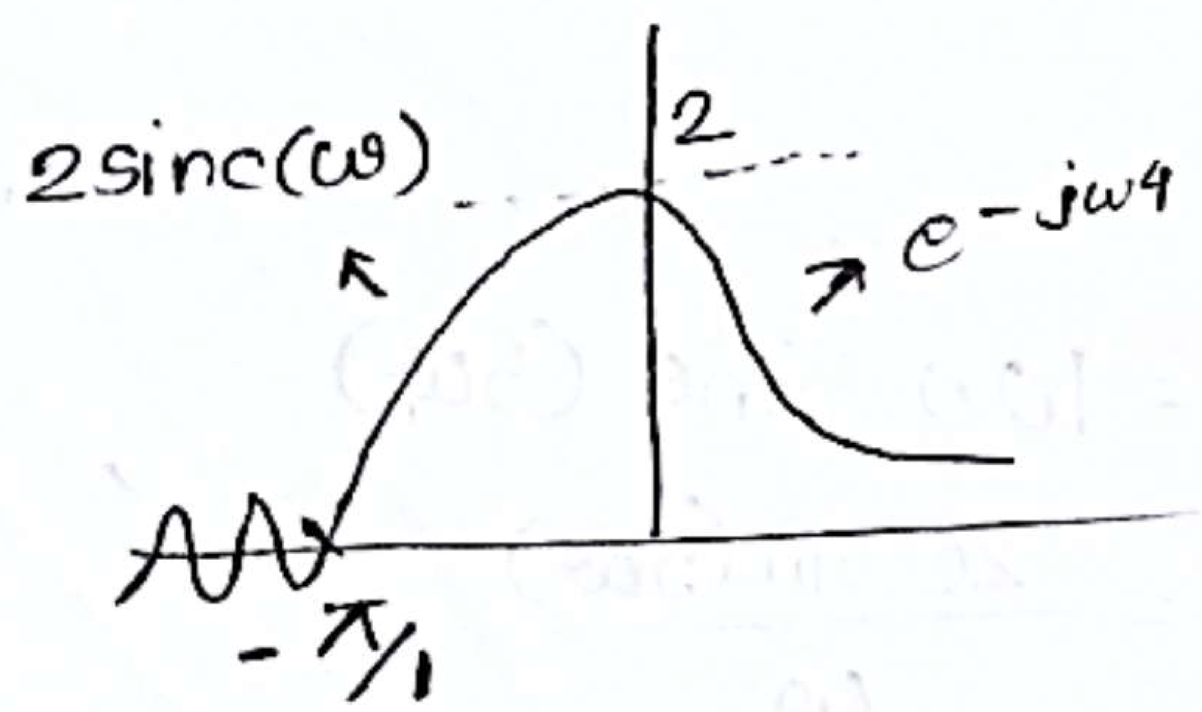
$\Rightarrow g(t) = \begin{cases} 1, & 3 \le t \le 5 \\ 0, & \text{"} \end{cases}$



4 ঘর shift,
So, $t_0 = 4$

$F(\omega) = 2\text{sinc}(\omega)$

$= e^{-j\omega 4} \cdot 2\text{sinc}(\omega)$



$F(\omega) \longrightarrow f(t)$

frequency domain to time domain:

$\int \sec x \, dx = \ln|\sec x + \tan x| + c$

$f(t) \longrightarrow F(\omega)$

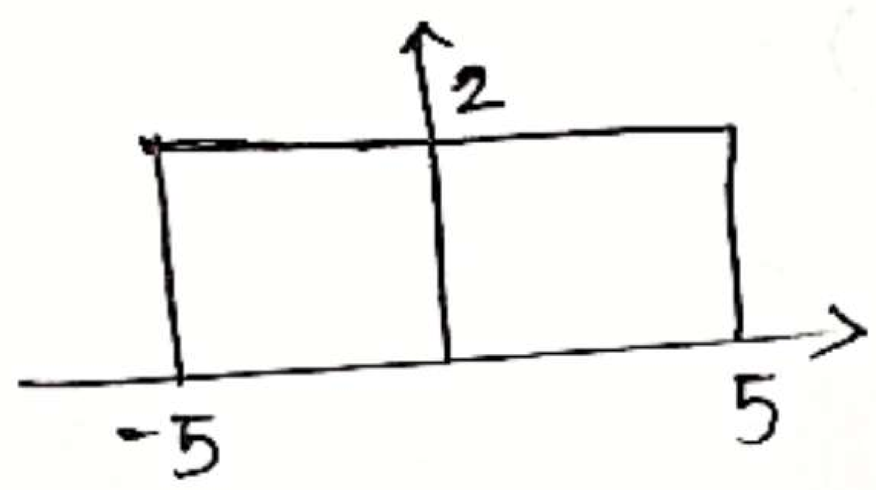
$$e^{\pm j\omega_0 t} f(t) \longrightarrow F(\omega \mp \omega_0)$$

$$F(\omega) = \underbrace{20 \frac{\sin 5\omega}{5\omega}}_{20\,\mathrm{sinc}(5\omega)} \exp(-3j\omega)$$

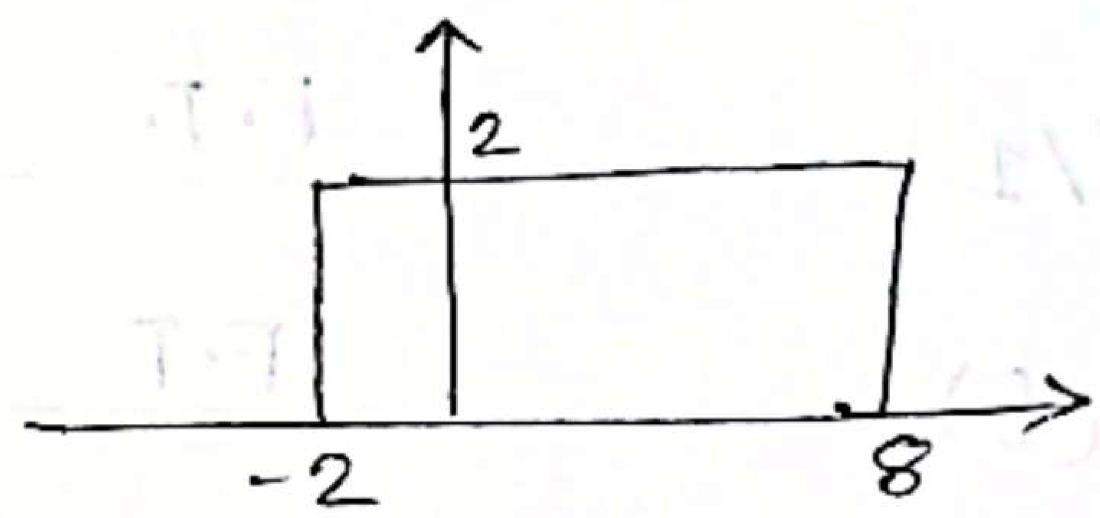
$2ab = 20$

$a = 5$

$b = 2$



$\longrightarrow$



$$f(t) = \begin{cases} 2, & -2 \leq t \leq 8 \\ 0, & \text{otherwise} \end{cases}$$

$$\frac{dQ}{dt} = -k(\sec \omega t)Q$$

$$\Rightarrow \int \frac{dQ}{Q} = \int -k(\sec \omega t)\,dt$$

$$\Rightarrow \ln Q = \frac{-k}{\omega} \ln|\sec \omega t + \tan \omega t| + c$$

$$\Rightarrow \ln Q = -\frac{k}{\omega} \ln|1+0| + c$$

$$\Rightarrow \ln Q_0 = C$$

$$\therefore C = \ln Q_0 = \ln(1000{,}000)$$

$$\ln 0.1 Q_0 = -\frac{k}{\omega} \ln|\sec \omega t + \tan \omega t| + c \quad t = ?$$

Question:

$\frac{dQ}{dt} = -k(\sec \omega t)Q$

$t = 0,\ Q(0) = Q_0$

$Q_0 = 1000{,}000$

$k = 1.2,$

$\omega = \pi/10$

$Q(5) = ?$

90% tasks

$t = ?$

4/12/2025

Lecture - 04

Formula:

time domain		Frequency domain
1. $e^{-at}u(t)$	$\xrightarrow{F.T.}$	$\frac{1}{a+j\omega}$
2. $e^{at}u(-t)$	$\xrightarrow{F.T.}$	$\frac{1}{a-j\omega}$
3. $t^0/1$	$\xrightarrow{F.T.}$	$2\pi\delta(\omega)$
4. $\delta(t)$	$\xrightarrow{F.T}$	$1/\omega^0$
5. $f(t \mp t_0)$	$\longrightarrow$	$e^{\mp j\omega t_0}F(\omega)$
6. $e^{\mp j\omega_0 t}f(t)$	$\longrightarrow$	$F(\omega \pm \omega_0)$

Ex: 1

$$X(\omega) = \frac{e^{-j3\omega}}{2+j\omega}$$

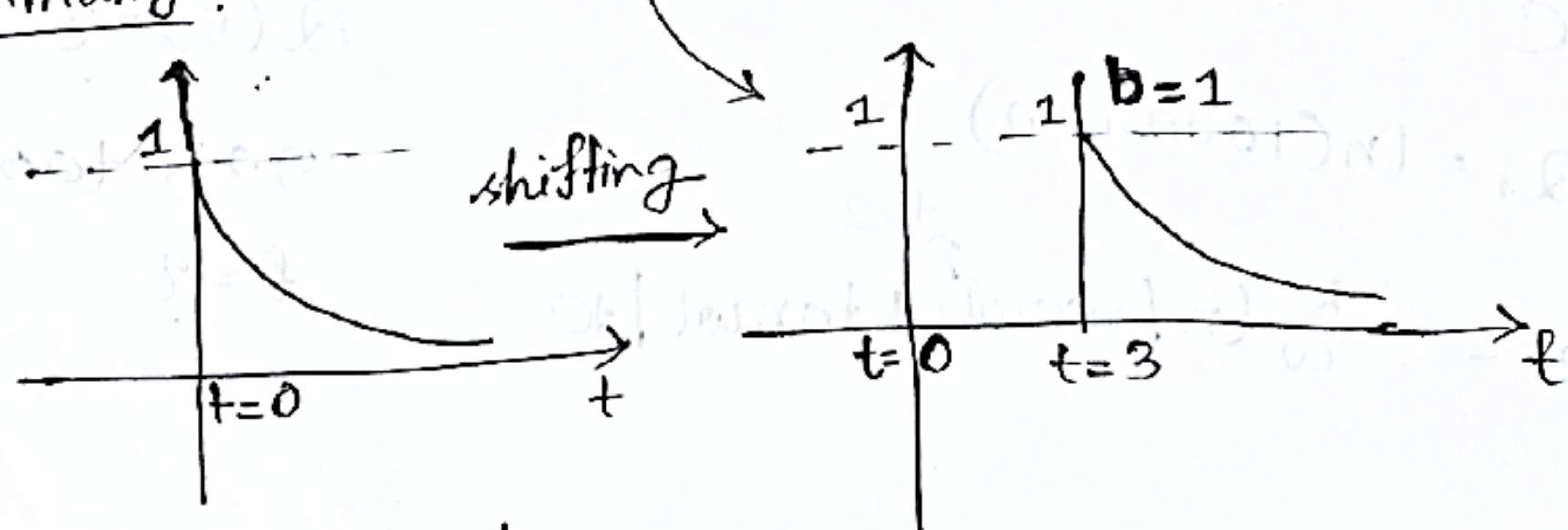
$$= e^{-j3\omega}\left[\frac{1}{2+j\omega}\right] \rightarrow F(\omega) \qquad \xrightarrow{f(t)} e^{-at}u(t) = \frac{1}{a+j\omega}$$

$$= e^{-2(t-3)}\,u(t-3)$$

→ shifting use for:

$$e^{\mp j\omega t_0}.F(\omega) = f(t \mp t_0)$$

Normally:



$e^{-at}\,u(t)$ graph

(b) $X(\omega) = \frac{j\omega}{2+j\omega}$

$= \frac{2+j\omega-2}{2+j\omega}$

$= \frac{2+j\omega}{2+j\omega} - \frac{2}{2+j\omega}$

$= 1 - 2\left[\frac{1}{2+j\omega}\right]$

FD to TD so,

$1 \xrightarrow{F.T.} \delta(t)$

$= \delta(t) - 2 \cdot e^{-2t}\, u(t)$

Property:

① $f(t) \longrightarrow F(\omega)$

$\frac{d^n}{dt^n} f(t) \longrightarrow (j\omega)^n F(\omega)$

② $t^n f(t) \longrightarrow (-1)^n \frac{d^n}{ds^n} F(s) \longleftarrow$ Laplace

$\Rightarrow (-1)^n \frac{d^n}{d(j\omega)^n} F(j\omega) \longleftarrow$ Fourier

$n=1, \Rightarrow t\, f(t) \longrightarrow (-1) \cdot \frac{d}{d(j\omega)} F(j\omega) = (-\frac{1}{j}) \frac{d}{d(\omega)} F(j\omega)$

$\Rightarrow (-jt)^n f(t) \longrightarrow \frac{d^n}{d\omega} F(j\omega)$

Formula required: 1, 3, 6, 8, 9, 12

Example:

$$f(t) = \cos\omega_0 t$$

$$\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$$

$$= \frac{1}{2}\left[e^{j\omega_0 t} + e^{-j\omega_0 t}\right]$$

$$= \frac{1}{2}\left[2\pi\delta(\omega-\omega_0) + 2\pi\delta(\omega+\omega_0)\right]$$

$$= \underbrace{\pi}_{\text{magnitude}}\left[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right] \rightarrow F(\omega)$$

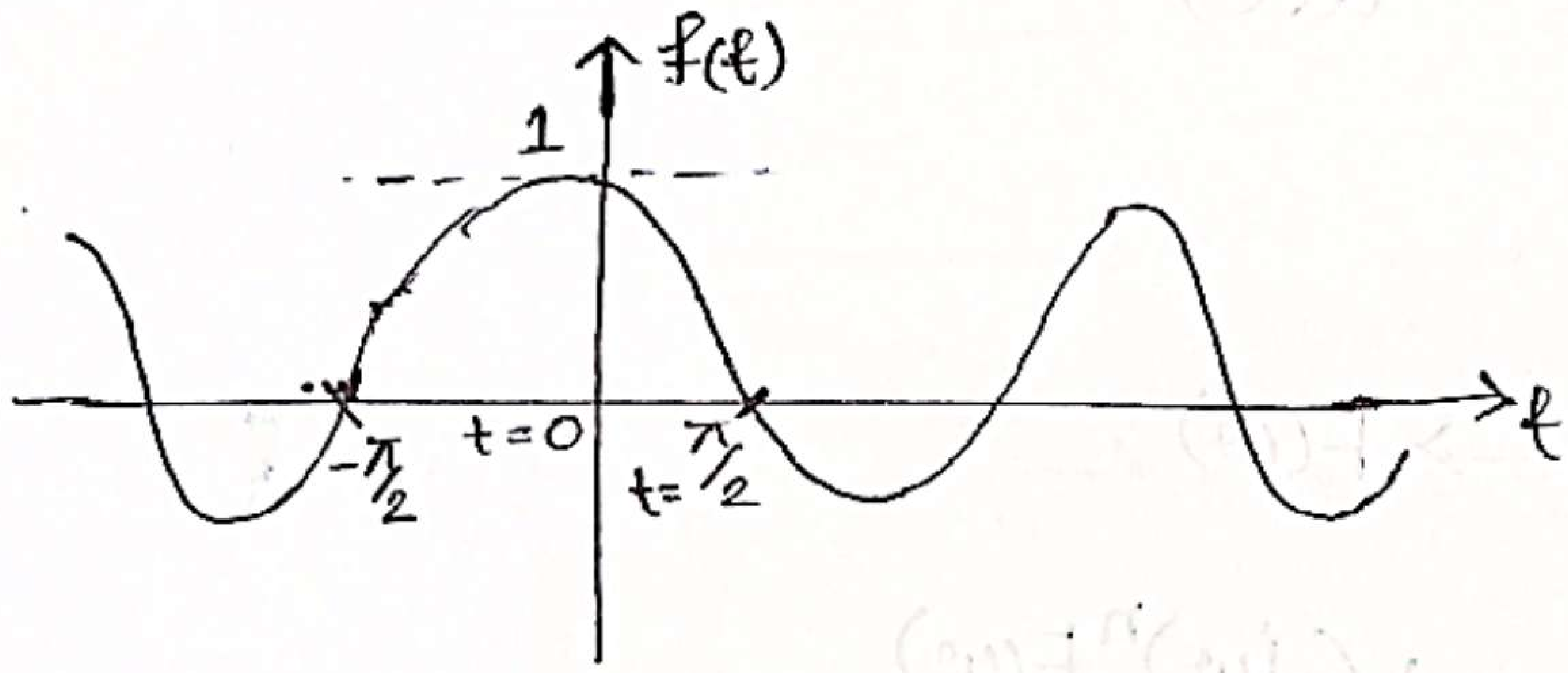
$\underbrace{f(t)}$, $\overbrace{F(\omega)}$

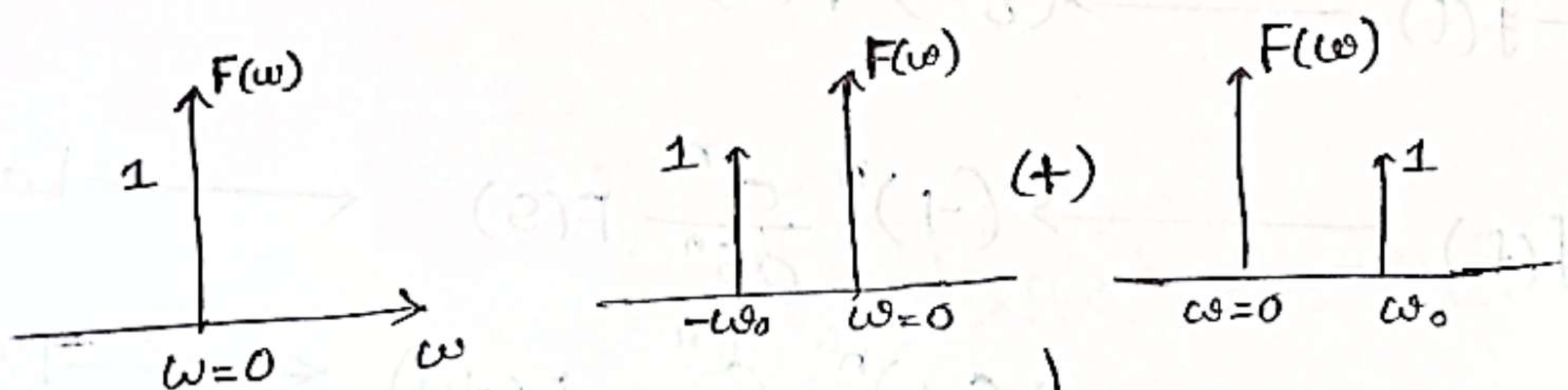
$t^0/1 \xrightarrow{F.T.} 2\pi\delta(\omega)$

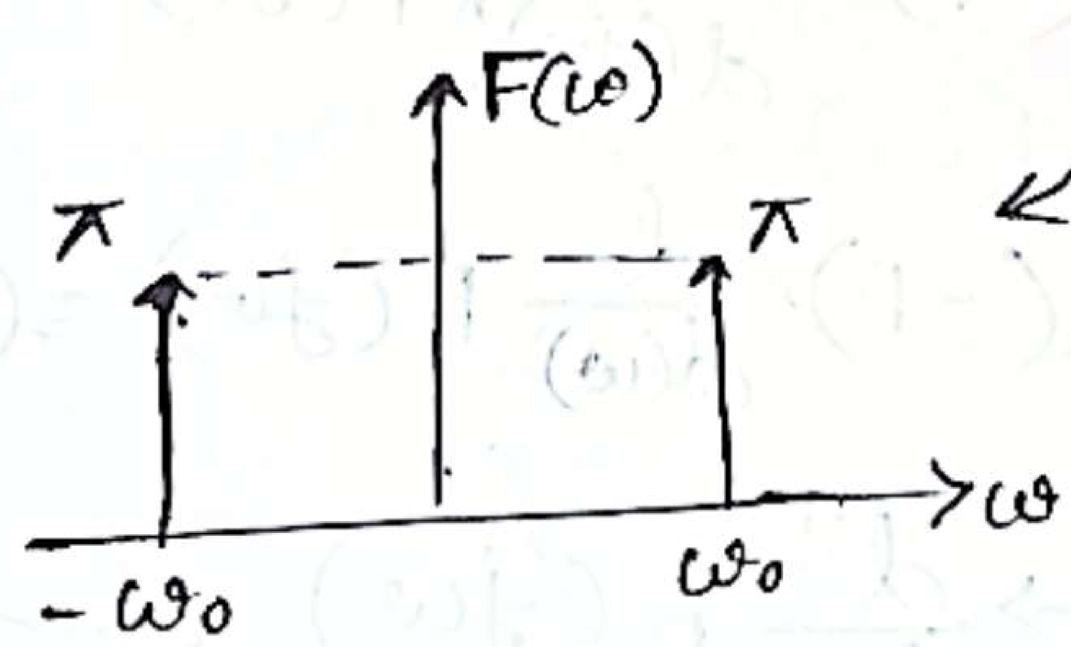
$1 \cdot e^{-j\omega_0 t} \longrightarrow 2\pi\delta(\omega+\omega_0)$

$1 \cdot e^{j\omega_0 t} \longrightarrow 2\pi\delta(\omega-\omega_0)$

Graph: $\cos\omega_0 t$







Graph: $\pi\left[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right]$

For, $f(t) = \sin\omega_0 t$

$$1+\frac{1}{j}$$

$$= 1+\frac{j}{j\cdot j}$$

$$= 1+\frac{j}{j^2}$$

$$= 1+\frac{j}{-1} = 1-j$$

$1-j$

$a+jb$

magnitude: $z = \sqrt{a^2+b^2}$ → only magnitude এর graph draw করলে হবে, phase এর লাগবে না

Phase: $\theta = \tan^{-1}\frac{b}{a}$

Modulation Theorem:

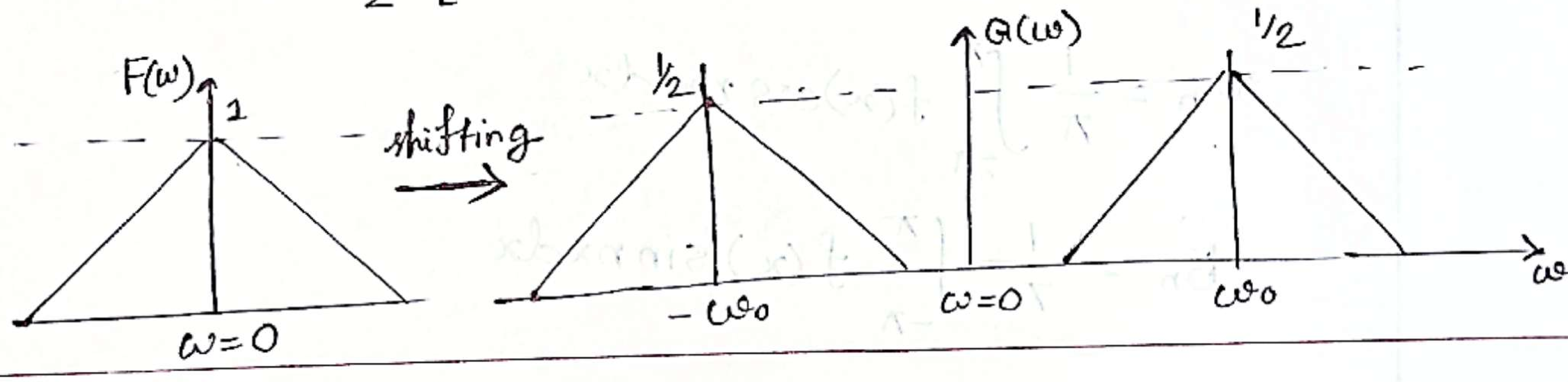
time domain এর সাথে cos/sin × থাকলে mean frequency এর domain এ shift করা

$g(t) = f(t)\cos\omega_0 t$ → Task: $G(\omega) = ?$

$$= f(t)\left[\frac{1}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)\right]$$

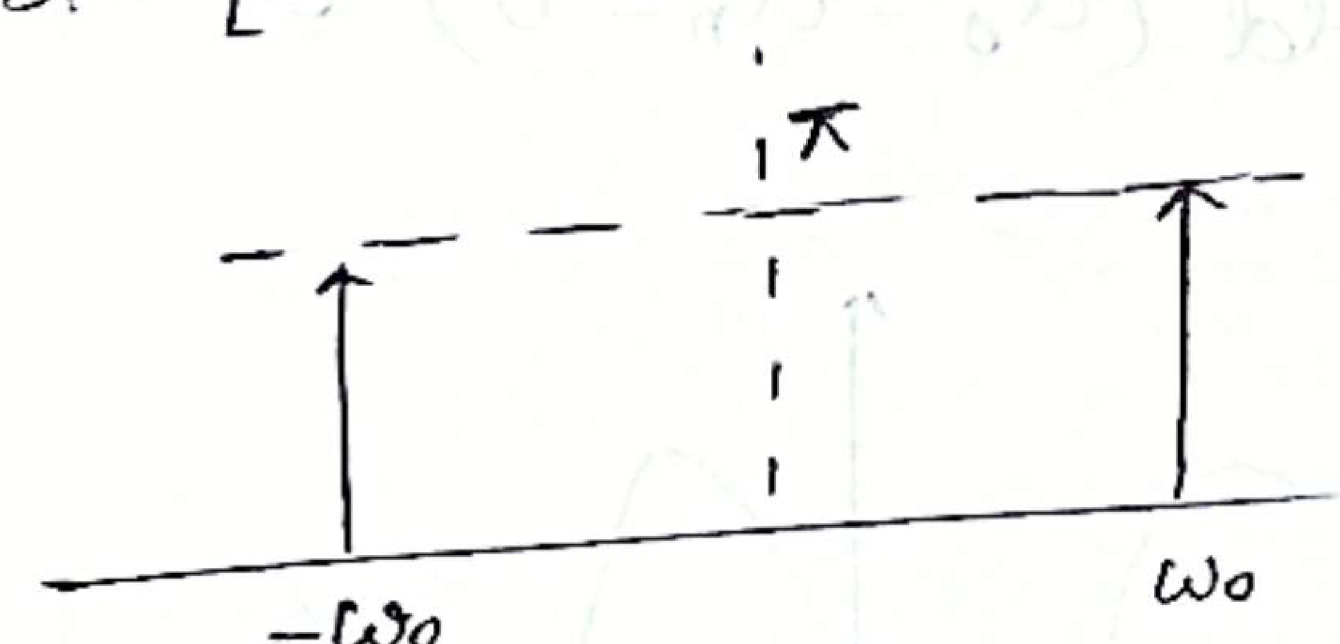
$$= \frac{1}{2}\left[f(t)e^{j\omega_0 t} + f(t)e^{-j\omega_0 t}\right]$$

$$\therefore G(\omega) = \frac{1}{2}\left[F(\omega-\omega_0) + F(\omega+\omega_0)\right]$$



Example: $f(t) = \sin\omega_0 t$

$$= -j\pi\left[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right]$$



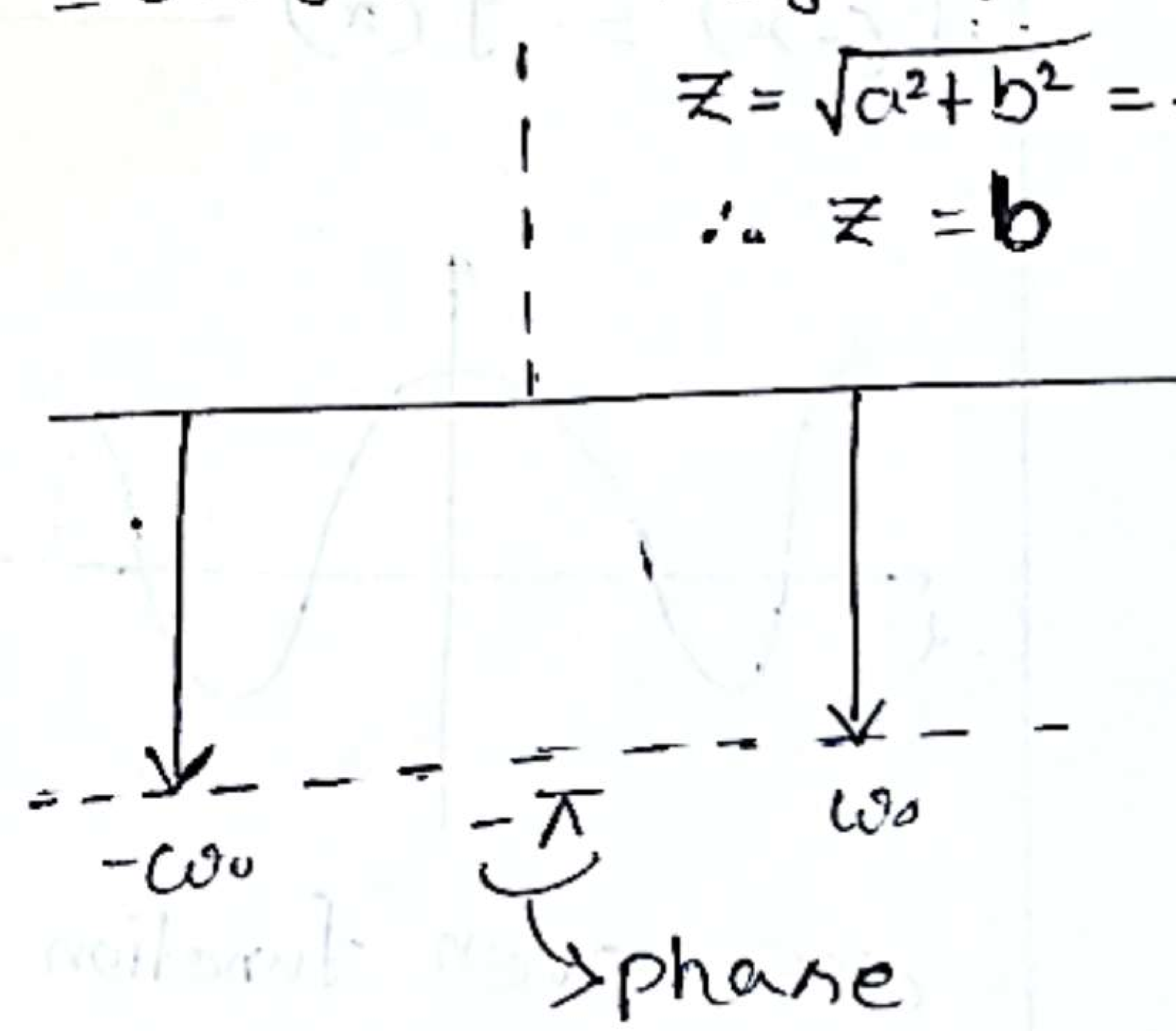
Graph: $\pi\left[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right]$

$$= 0 + (-j)\left[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right]$$

$= a + jb$ magnitude graph

$z = \sqrt{a^2+b^2} = \sqrt{b^2}$

$\therefore z = b$



Graph: $-j\pi\left[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right]$

$-j$ থাকার কারণে $y = (-)$ হয়ে যাবে, so, graph উল্টো হয়ে যাবে।

To express any periodic signal, we need fourier series:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

Step-1: find →

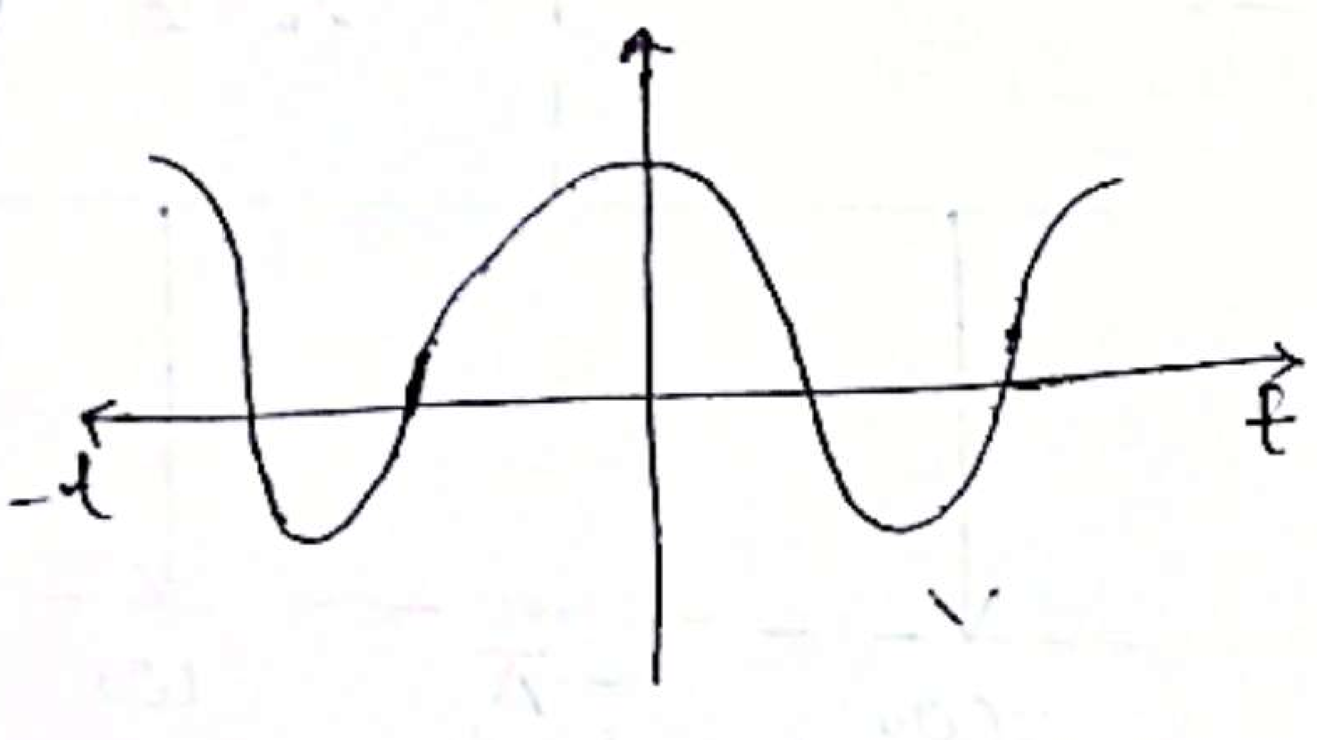
$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$$

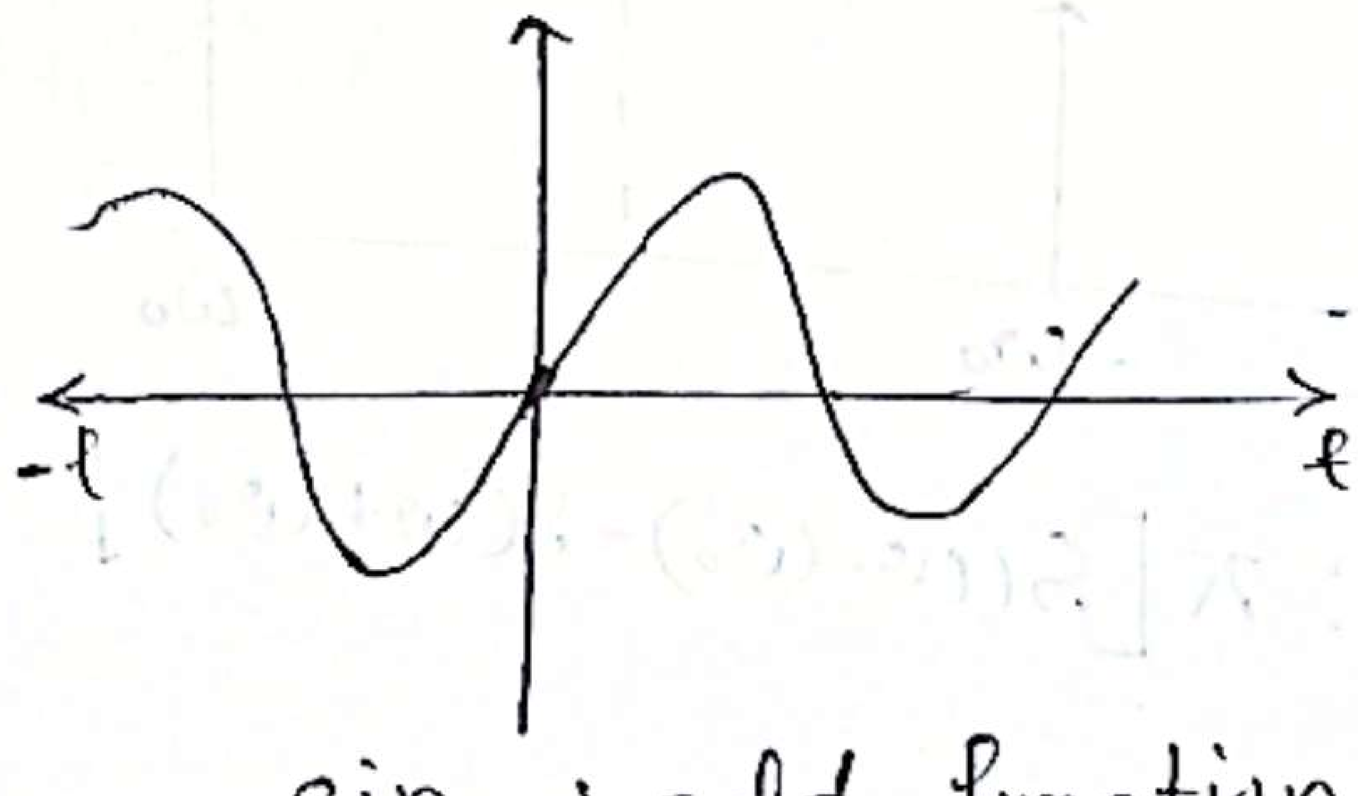
$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx \, dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx \, dx$$

$f(-x) = f(x) \longrightarrow$ even ($b_n = 0$)

$f(-x) = -f(x) \longrightarrow$ odd ($a_0 = a_n = 0$)





cos → even function

$b_n = 0$

calculate a_0, a_n

sin → odd function

$a_0 = a_n = 0$

calculate only b_n

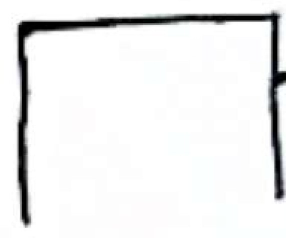
Exercise-1:

$$f(x) = \begin{cases} 1, & -\pi < x < 0 \\ 0, & 0 < x < \pi \end{cases}$$

Note: ① $<$ / $>$ দেয়া থাকলে graph draw করার সময়

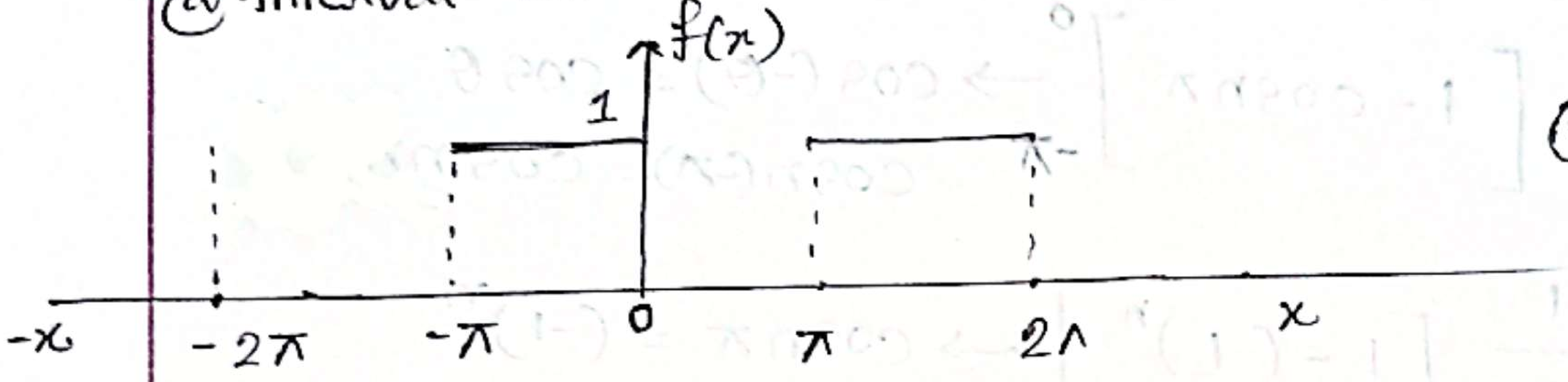
→ dotted line

② $\leqslant$ / $\geqslant$ equal দেয়া থাকলে



→ straight line হবে

ⓐ interval $-2\pi < x < 2\pi$



ⓑ ① $a_0 = \frac{1}{\pi}\int_{-\pi}^{0} 1 \cdot dx = \frac{1}{\pi}[x]_{-\pi}^{0}$

$= \frac{1}{\pi}[0 + \pi]$

$= 1$

② $a_n = \frac{1}{\pi}\int_{-\pi}^{0} \cos nx \, dx$

$= \frac{1}{\pi}\left[\frac{\sin nx}{n}\right]_{-\pi}^{0}$

$= \frac{1}{n\pi}\left[\sin 0 - \sin n(-\pi)\right]$

$= \frac{1}{n\pi}[\sin n\pi]$

$= 0$

③ $b_n = \frac{1}{\pi}\int_{-\pi}^{0} \sin nx\,dx$

$= \frac{1}{\pi}\left[\frac{-\cos nx}{}\right]_{-\pi}^{0}$

$= -\frac{1}{n\pi}\left[1-\cos n\pi\right] \rightarrow \cos(-\theta) = \cos\theta$

$\cos n(-\pi) = \cos n\pi$

$= -\frac{1}{n\pi}\left[1-(-1)^n\right] \rightarrow \cos n\pi = (-1)^n$

if, $n = odd$

$b_n = -\frac{1}{n\pi}\left[1+1\right] = -\frac{2}{n\pi}$

and, $n = even$

$b_n = 0$

$a_0 = 1$

$a_n = 0$

$$b_n = \begin{cases} -\frac{2}{n\pi}, & n = odd \\ 0, & n = even \end{cases}$$

ⓒ $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$

$\sin 3x = -1$

$x = \frac{\pi}{2}$

$x = \frac{\pi}{2}$ বসিয়ে,