CSC 439/539
Statistical Natural Language Processing
Statistical Significance: Bootstrap Resampling

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Example: Is a New Battery Really Better?

- A company has developed a new battery for use in electric cars. They believe that the new battery will allow cars to drive farther before needing to be recharged.
- The distribution of battery life in miles-to-recharge is positively skewed.
- Since the distribution is positively skewed, it might make sense to do a test of the *median*, rather than the mean.
- The median battery life for the current state-of-the-art version is 300 miles.
- What would our null and alternative hypotheses be?

Example: Is a New Battery Really Better?

- We are interested in testing

$$H_0 : \theta \leq 300 \qquad\qquad H_1 : \theta > 300$$

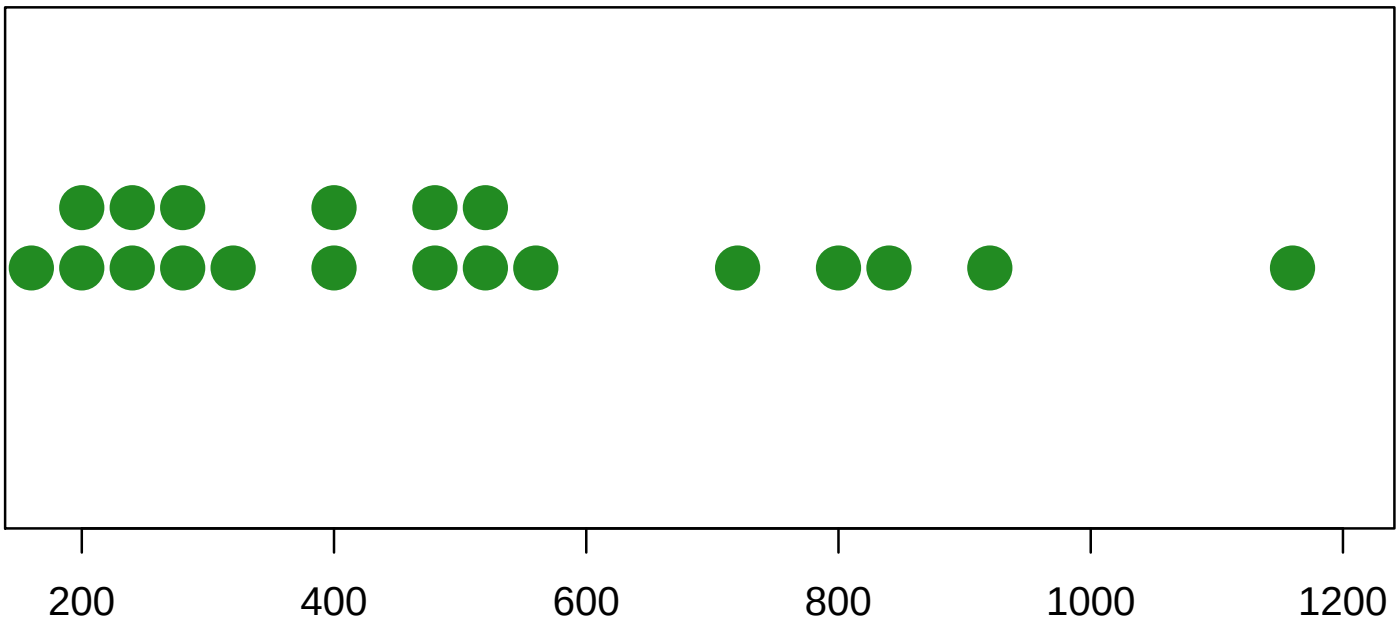
  where θ is the *population* median for the battery life distribution for the new model.
- To test this, we need a random sample of batteries from the population. We can then use the sample median, Q_2, to estimate the population median:
  - $Q_2 = \hat{\theta} \approx \theta$ (the "hat" stands for estimate)
- But because **we only have a sample**, we will be off to some extent. How can we find out the distribution of Q_2?

It Can Be Tricky

- We have no clue what the population distribution is (**same in NLP**). We know it's right skewed, so it's not normal.
- Our samples are small (evaluating batteries is expensive). – **same in NLP**

Example: Is a New Battery Really Better?

All we have is a sample of 20 batteries:



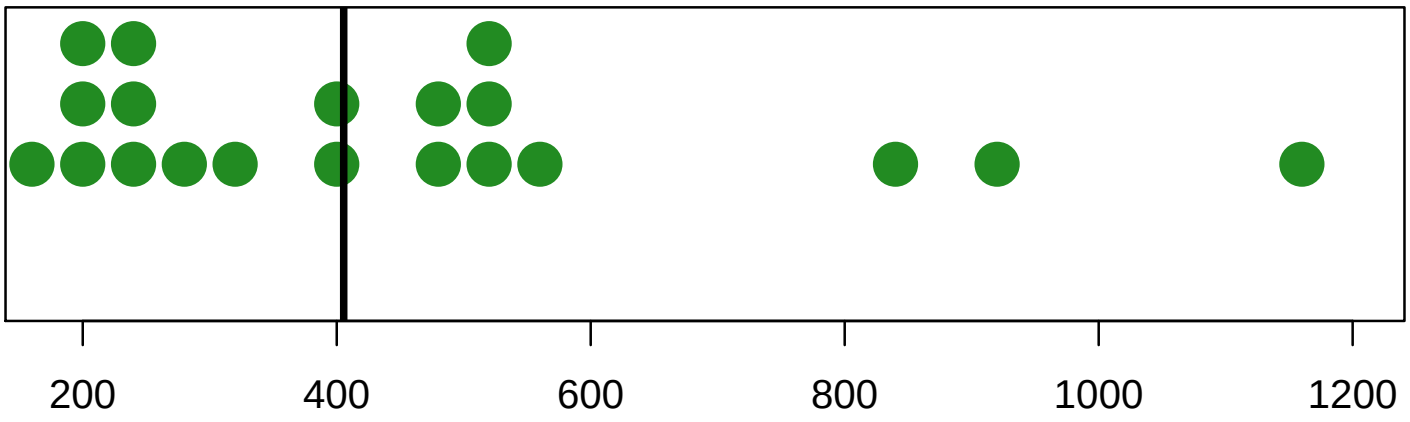
Bootstrap Resampling Intuition

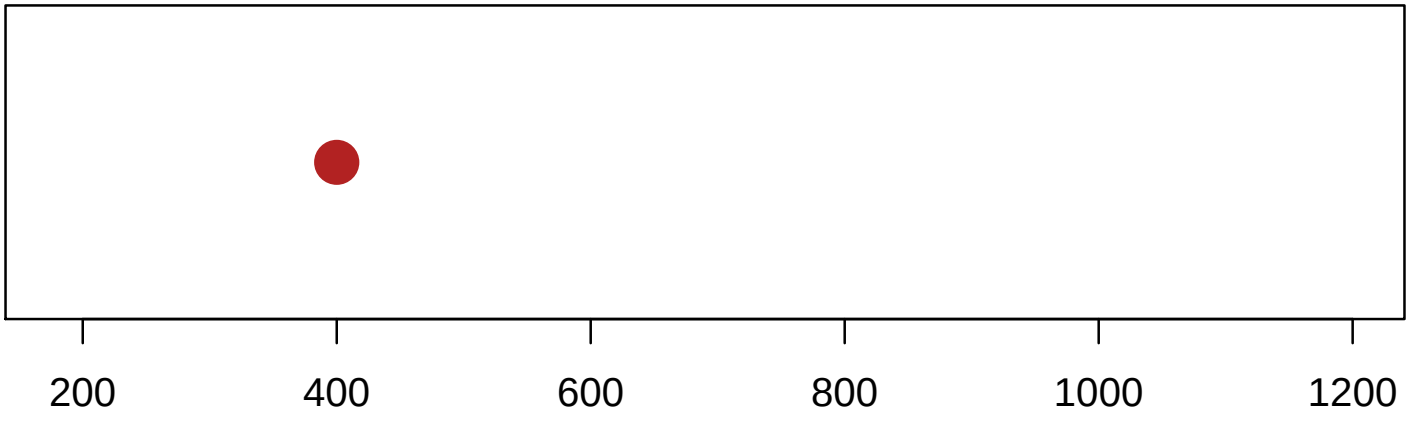
- Ideally, we would like to draw many samples from the population (of all batteries in the world) to construct the distribution of our statistic.
- But we do not have access to the entire population.
- Next best thing: **treat the sample we have as the population**. That is, draw samples from the sample.

Bootstrap Resampling Intuition

- We want our draws to be *independent* (to minimize dependencies to our sample), though, so what should we do?
- We need to sample *with replacement*, treating the sample as the population.
- If we do this repeatedly, computing our sample statistic each time, we can construct an estimated sampling distribution for that statistic.
- This procedure is known as (non-parametric) **bootstrap resampling** (or just "bootstrapping" for short), because we are using the data to "pull ourselves up by our bootstraps".

Bootstrap Resampling





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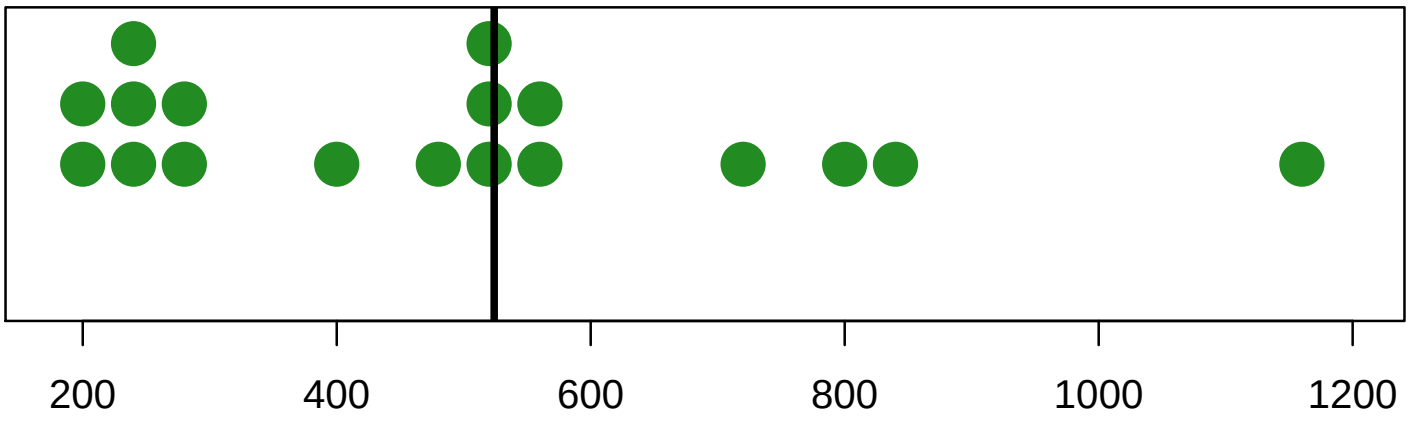

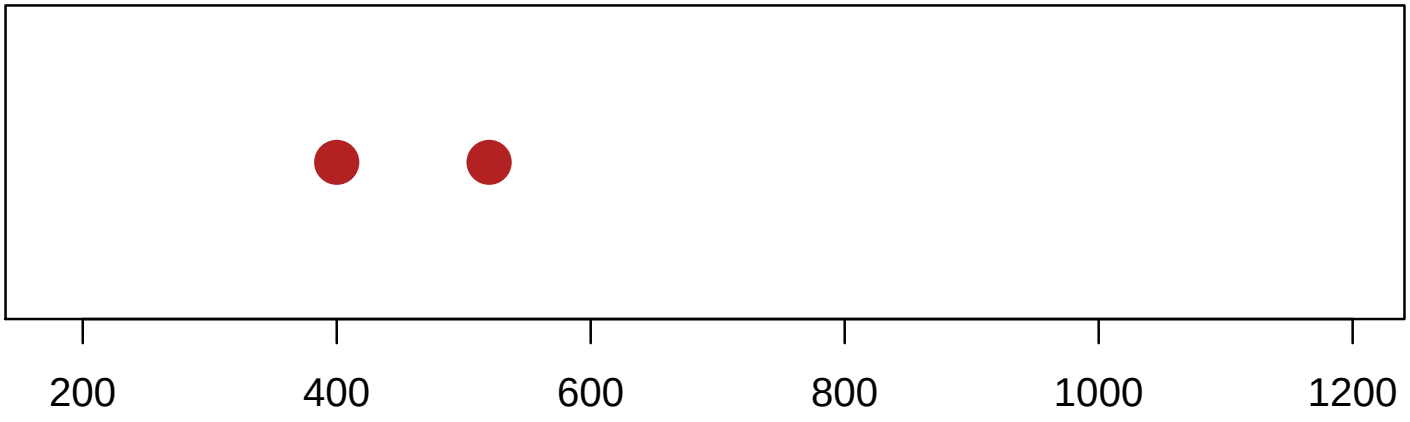

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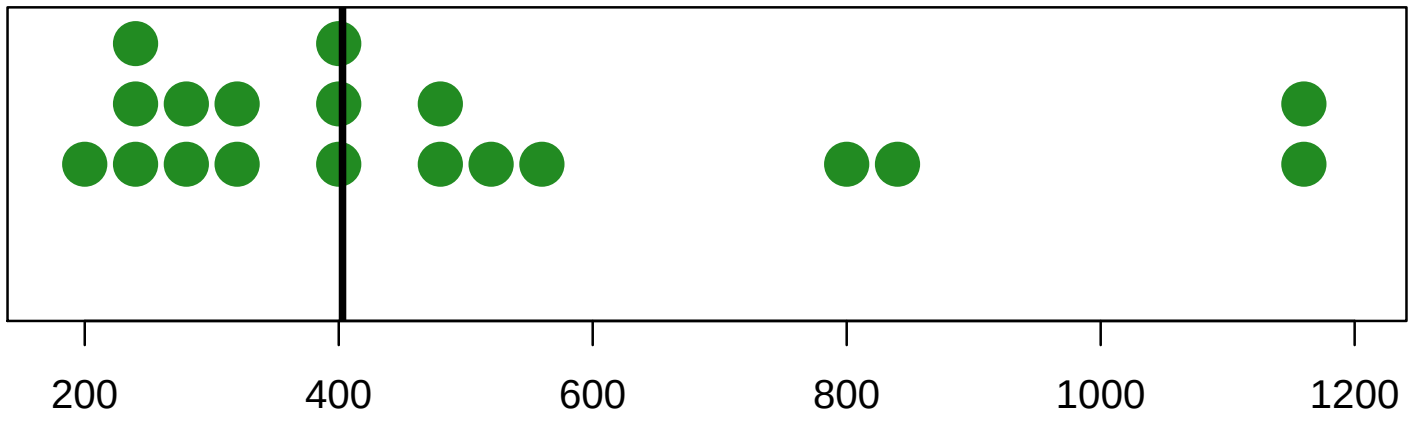

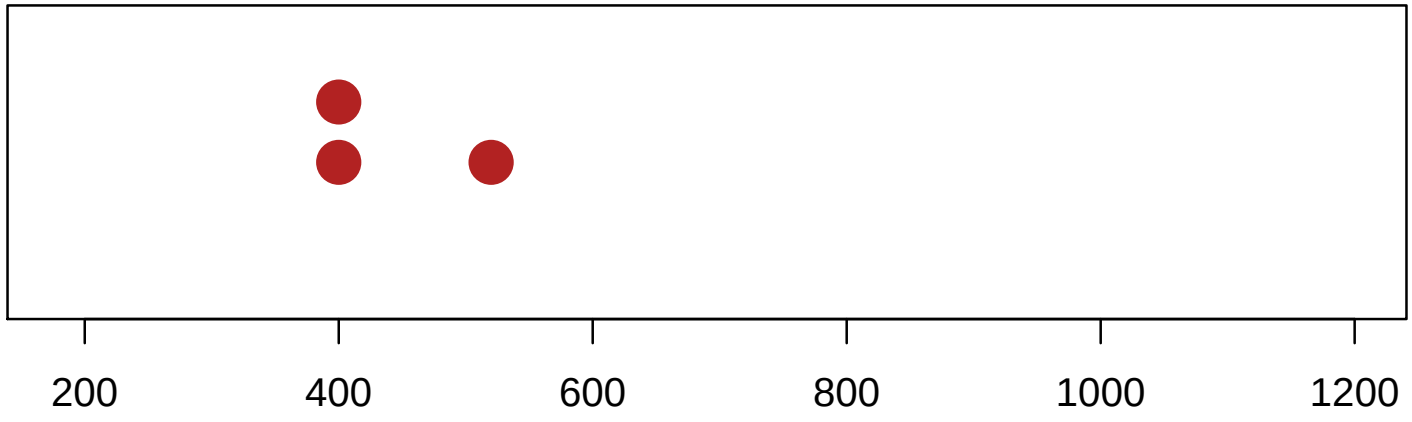

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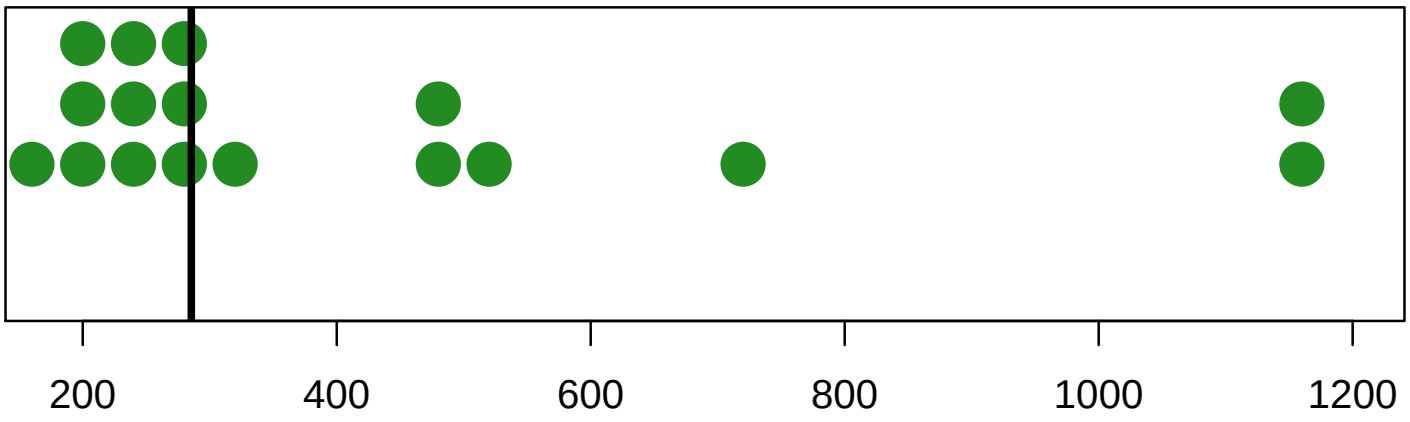

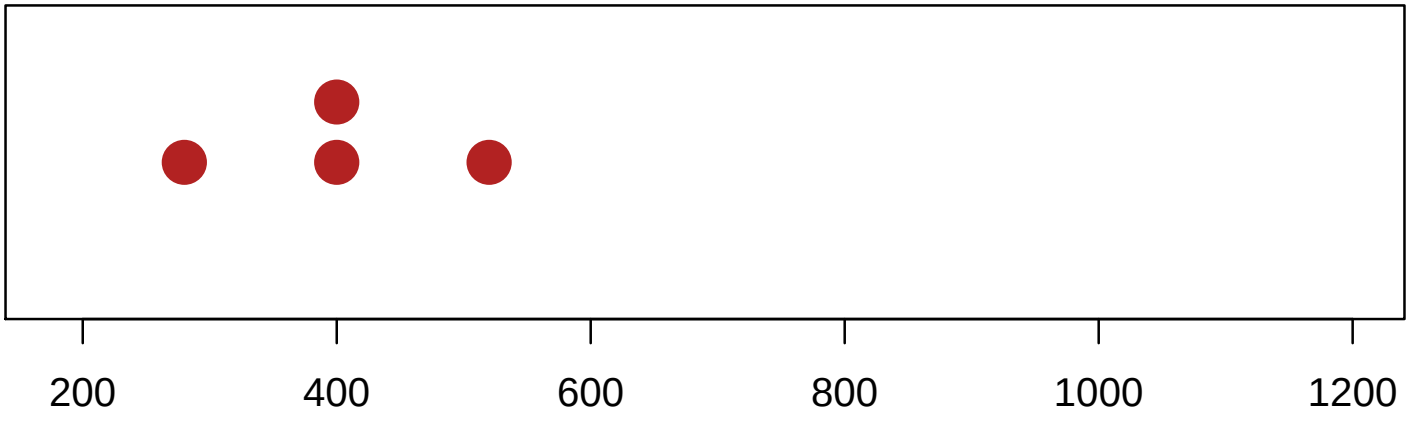

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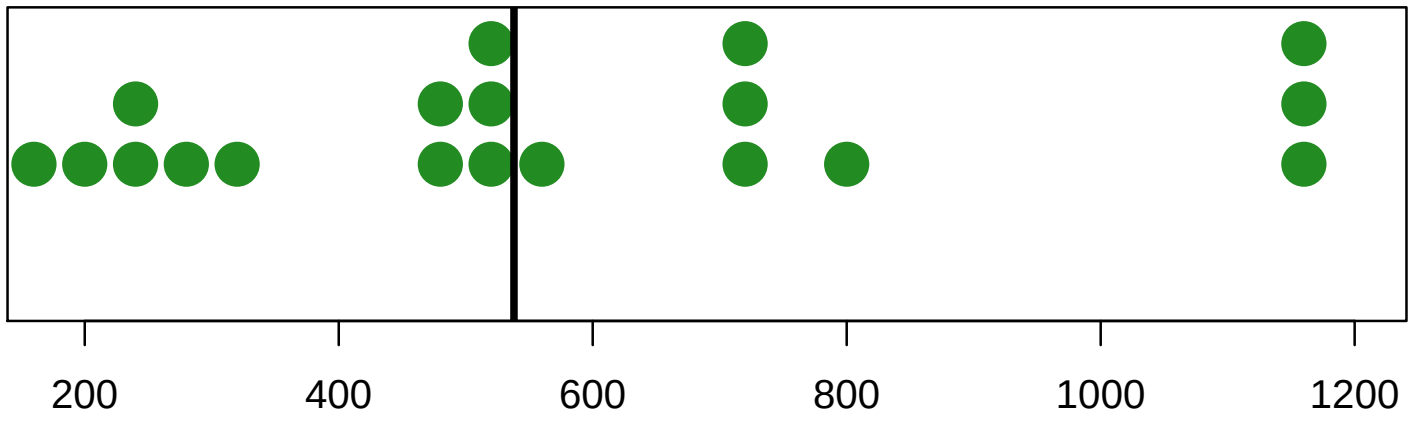

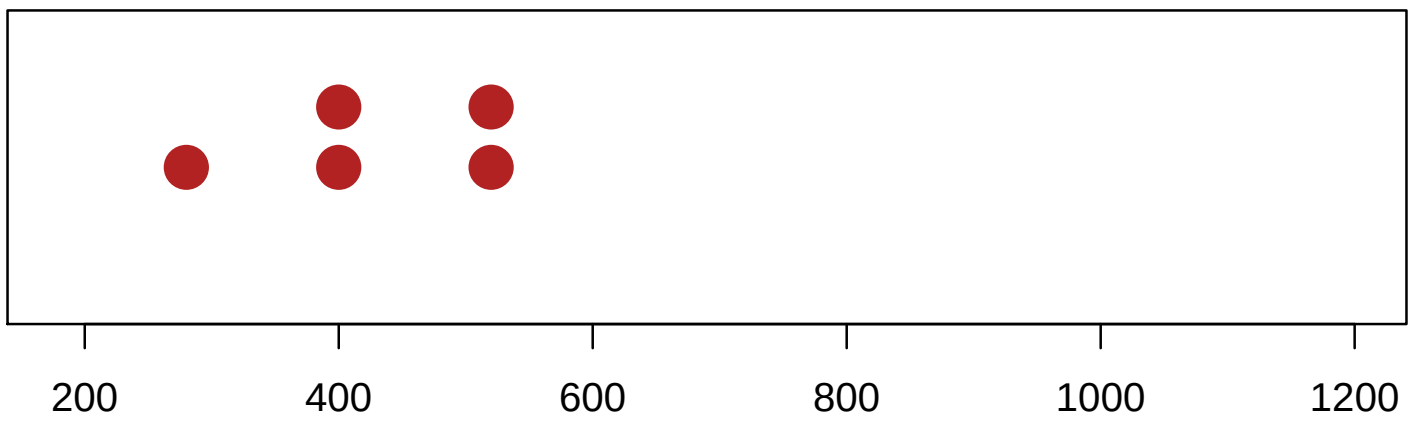

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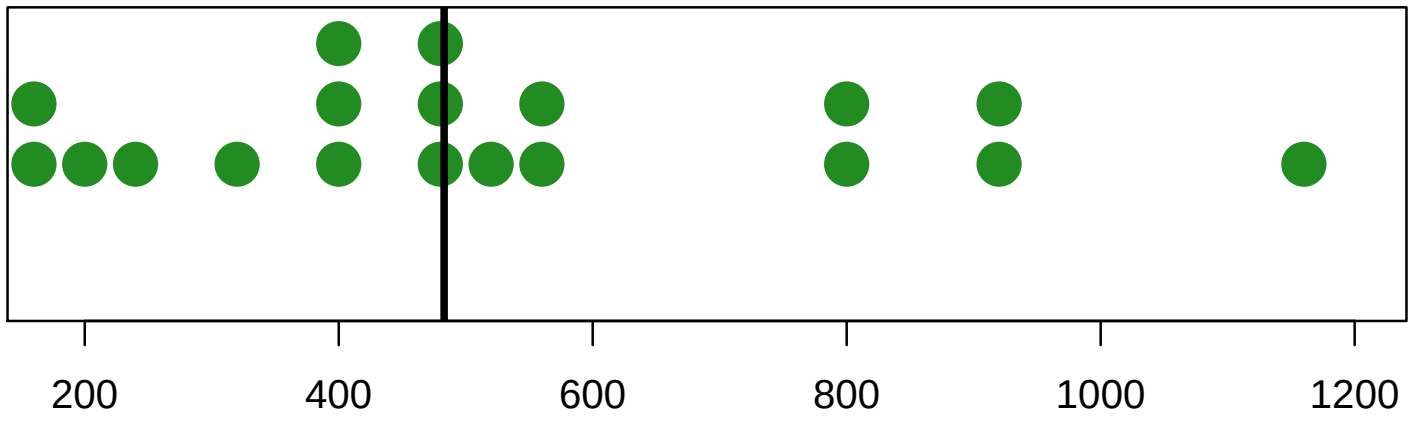

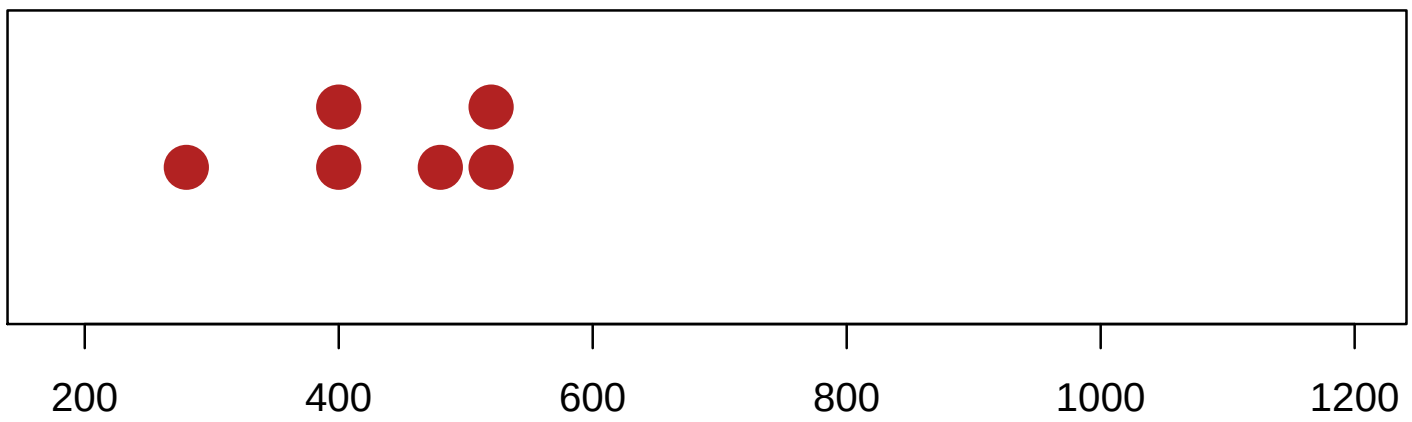

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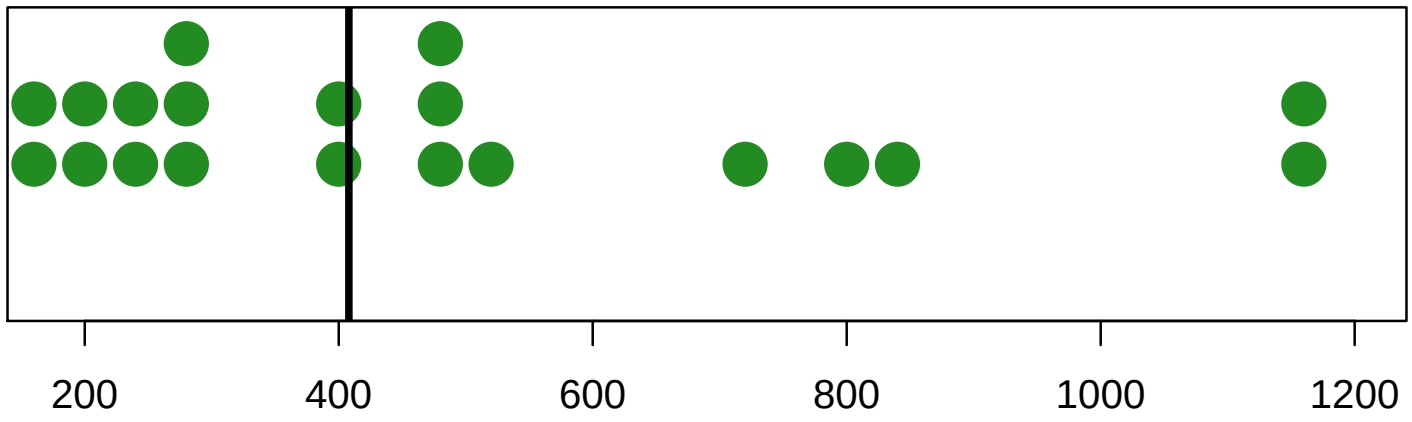

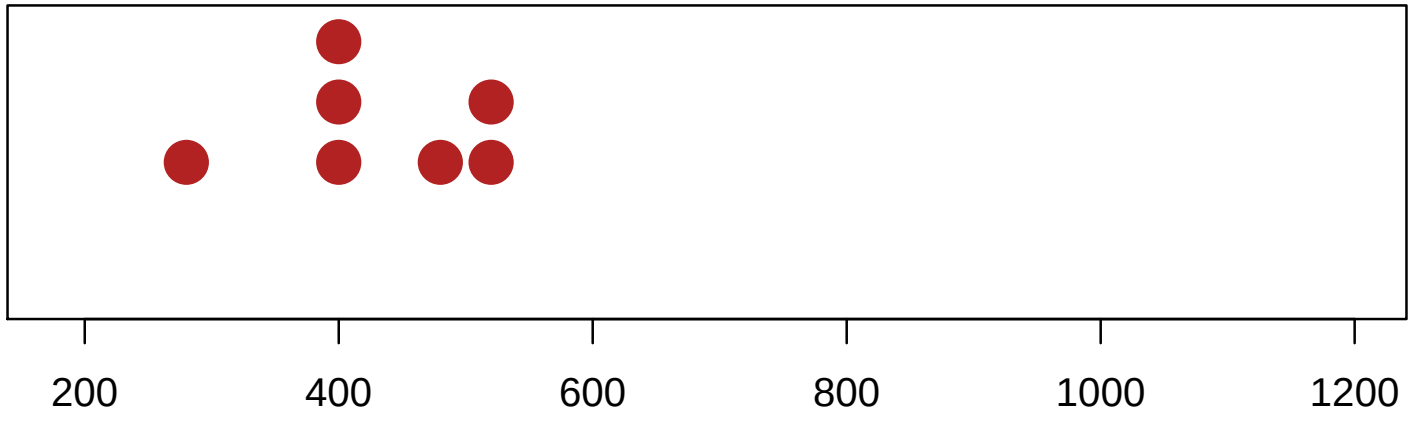

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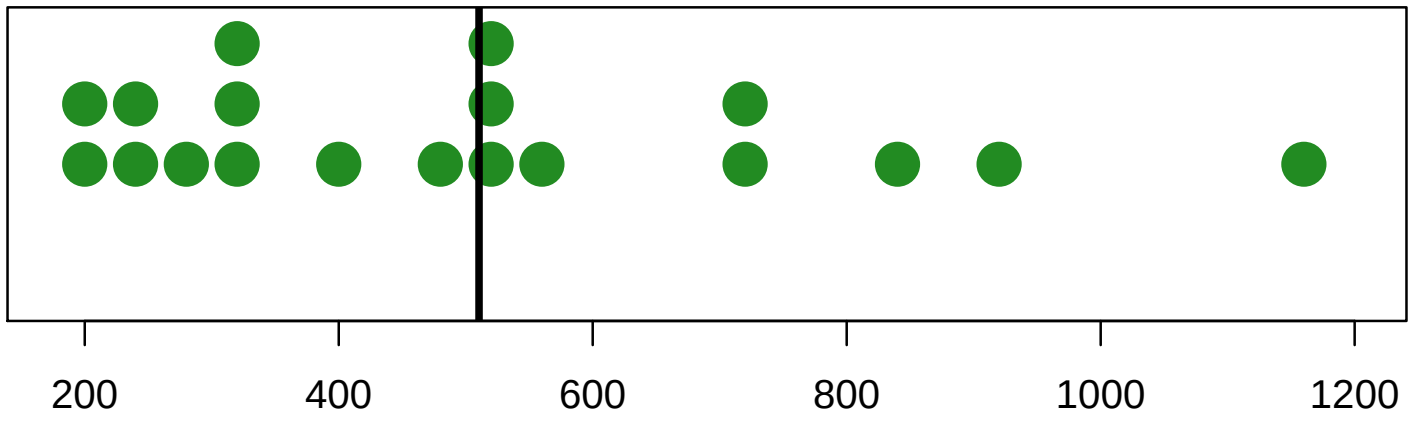

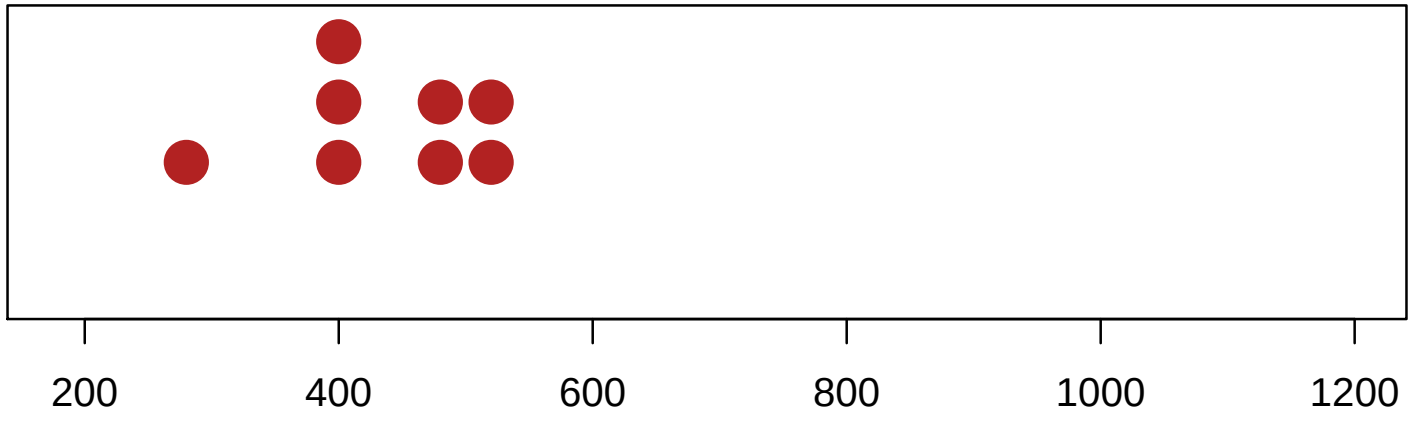

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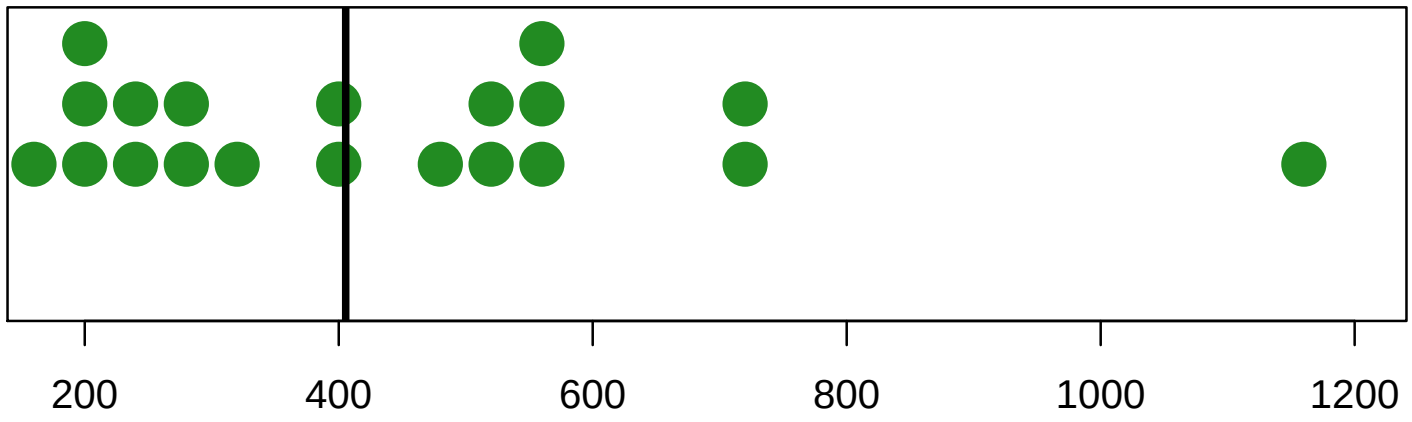

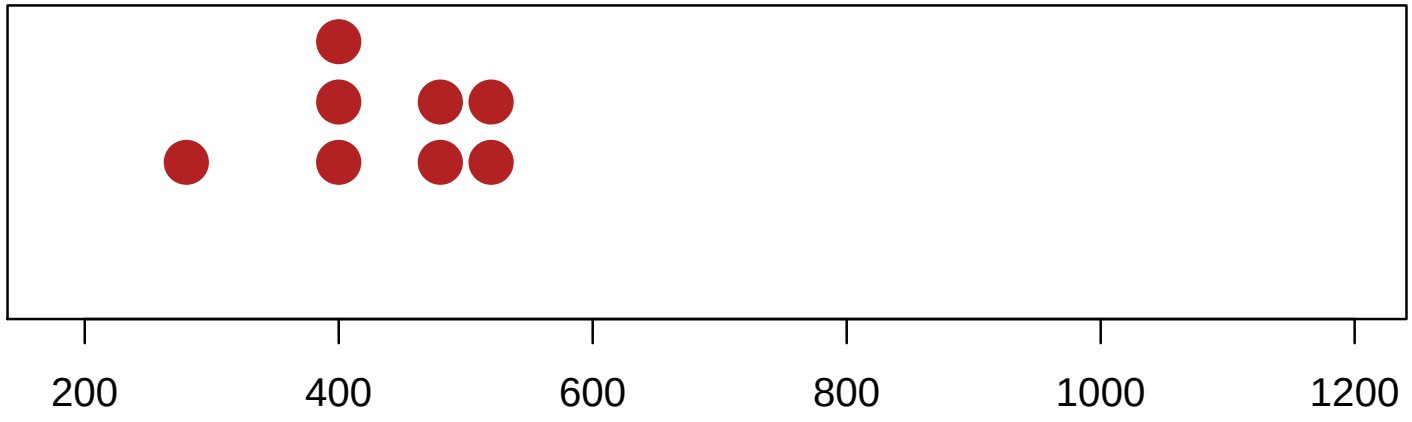

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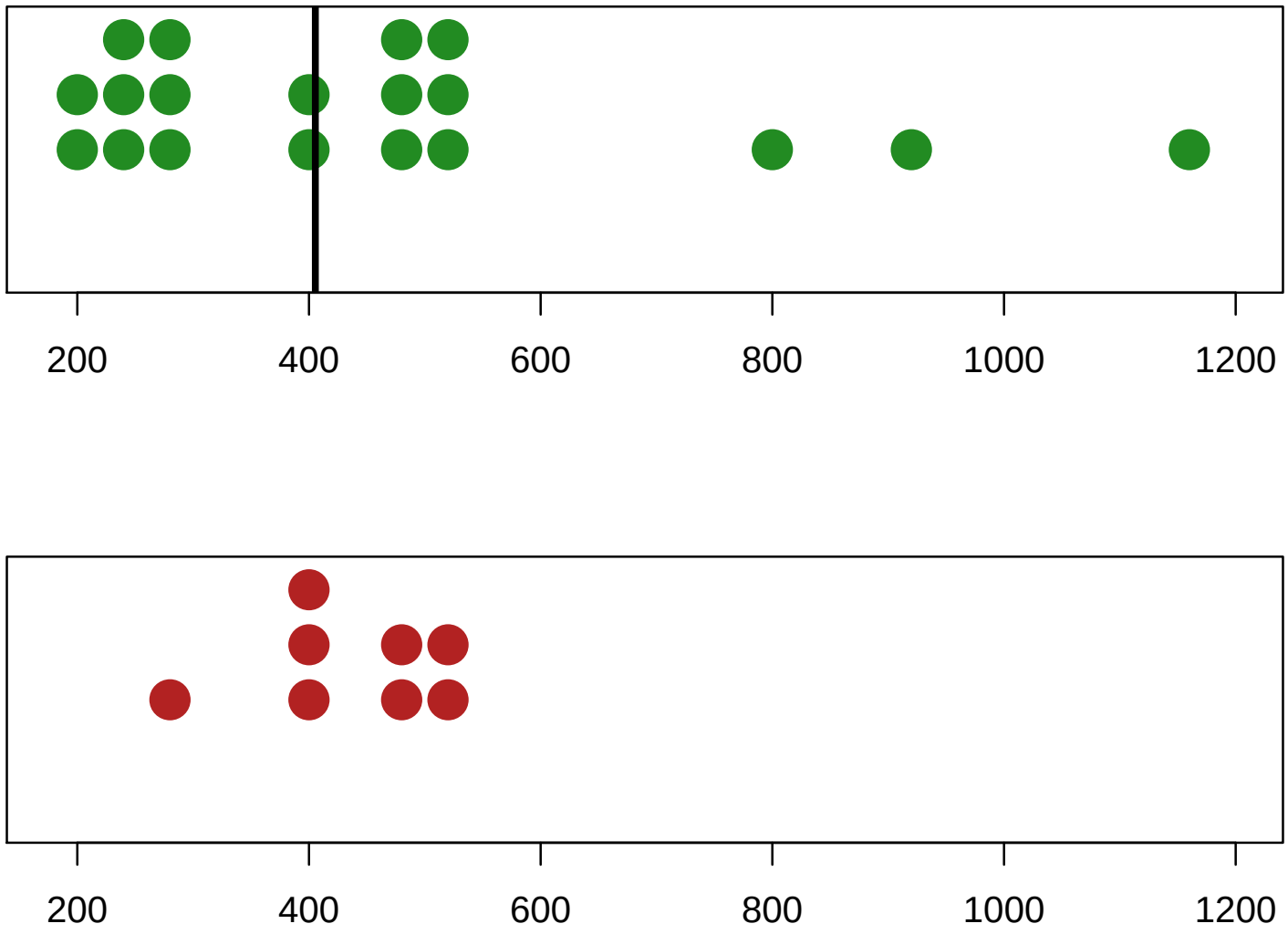

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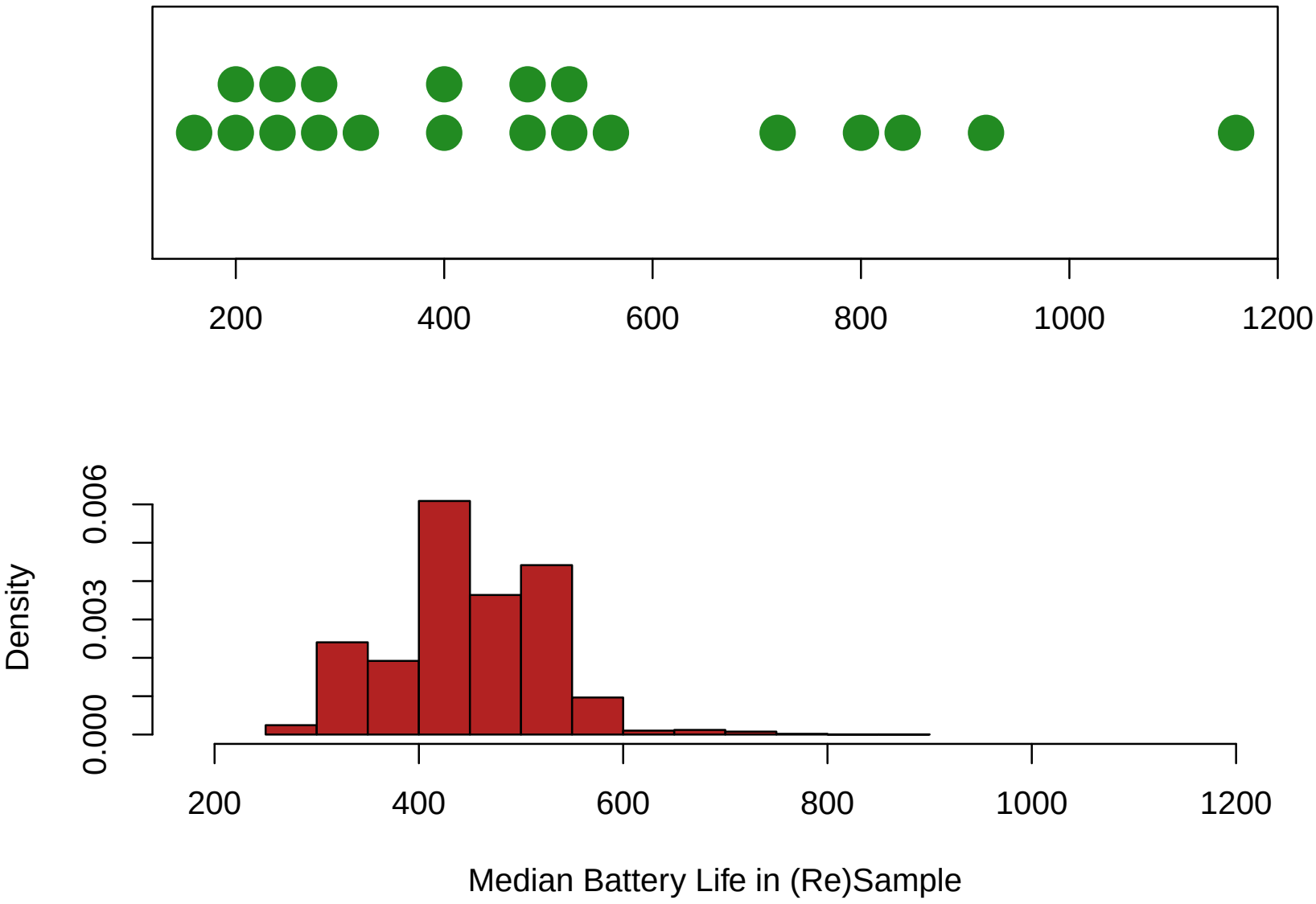

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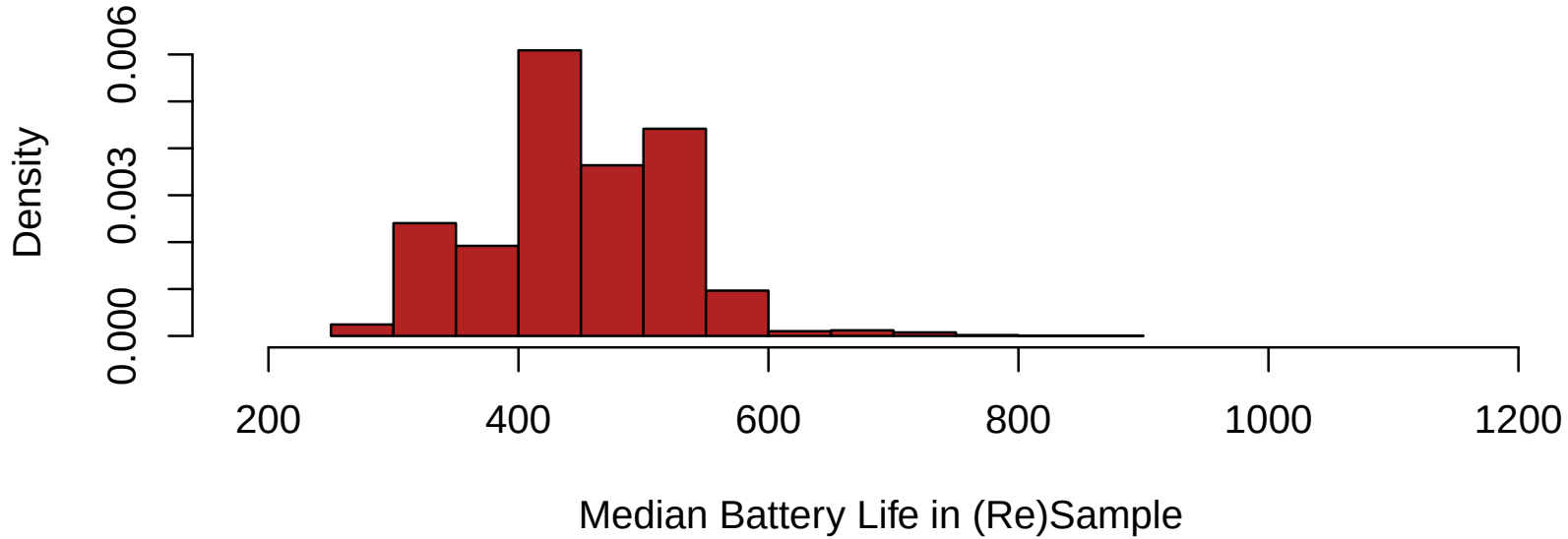

- What can we do with this distribution?
- Where did we make use of our hypotheses?

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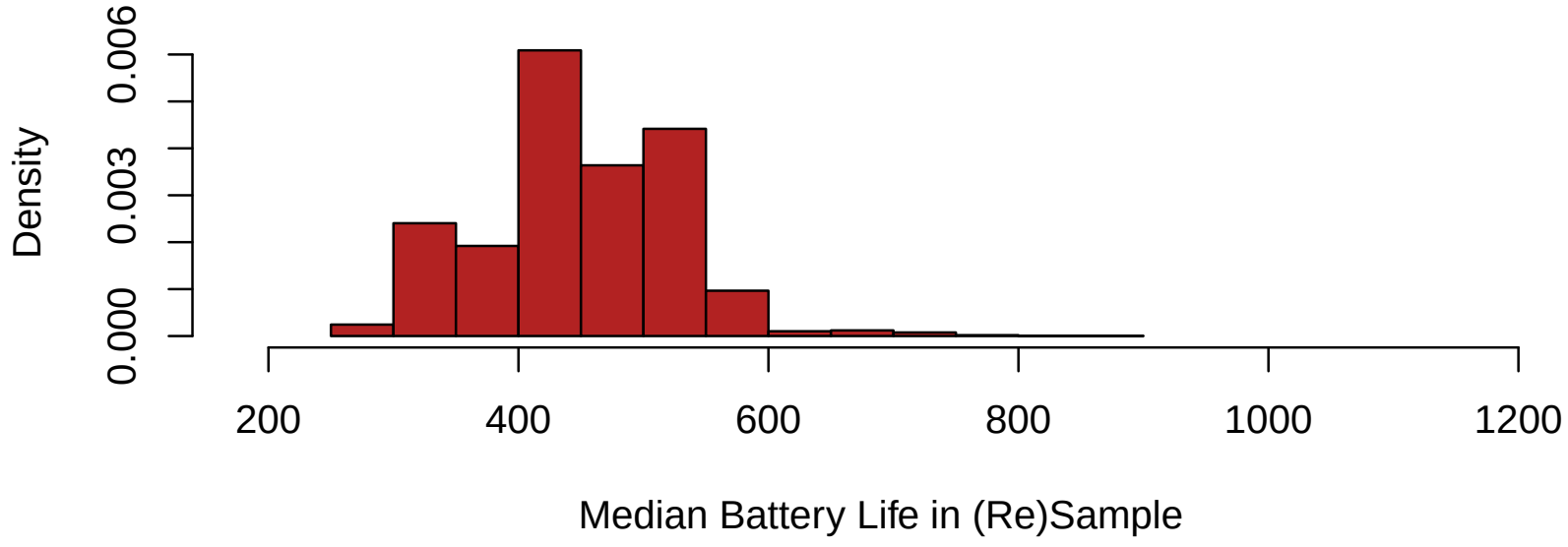
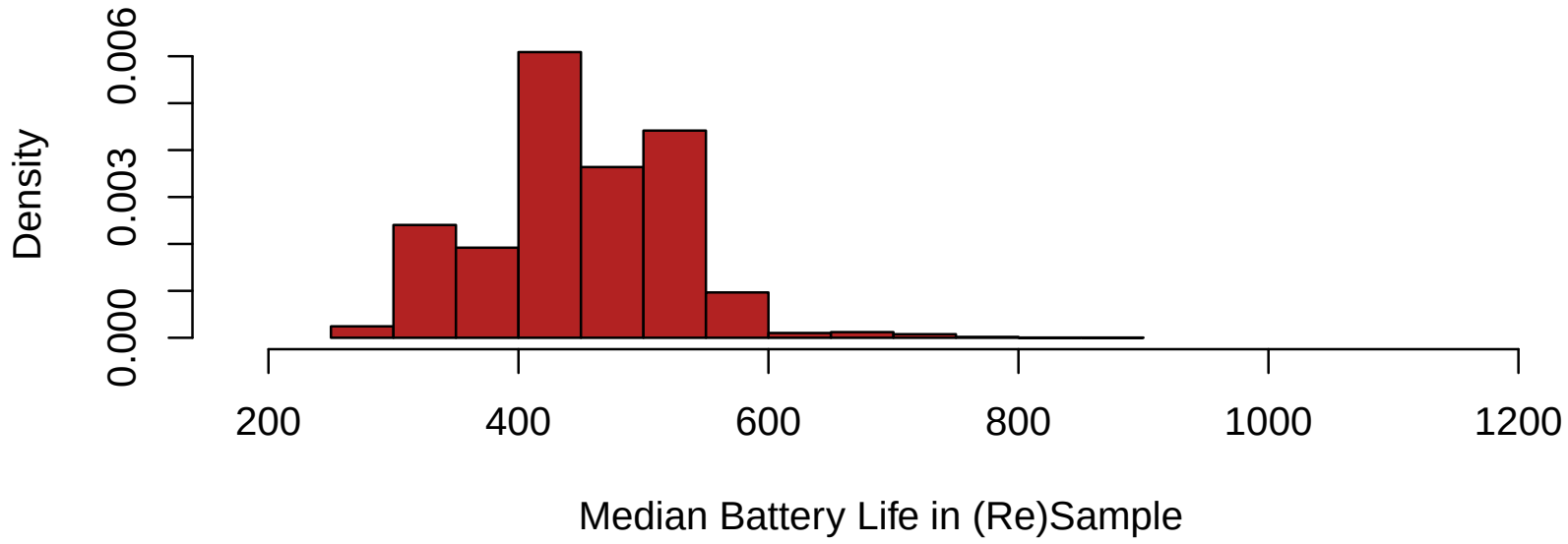


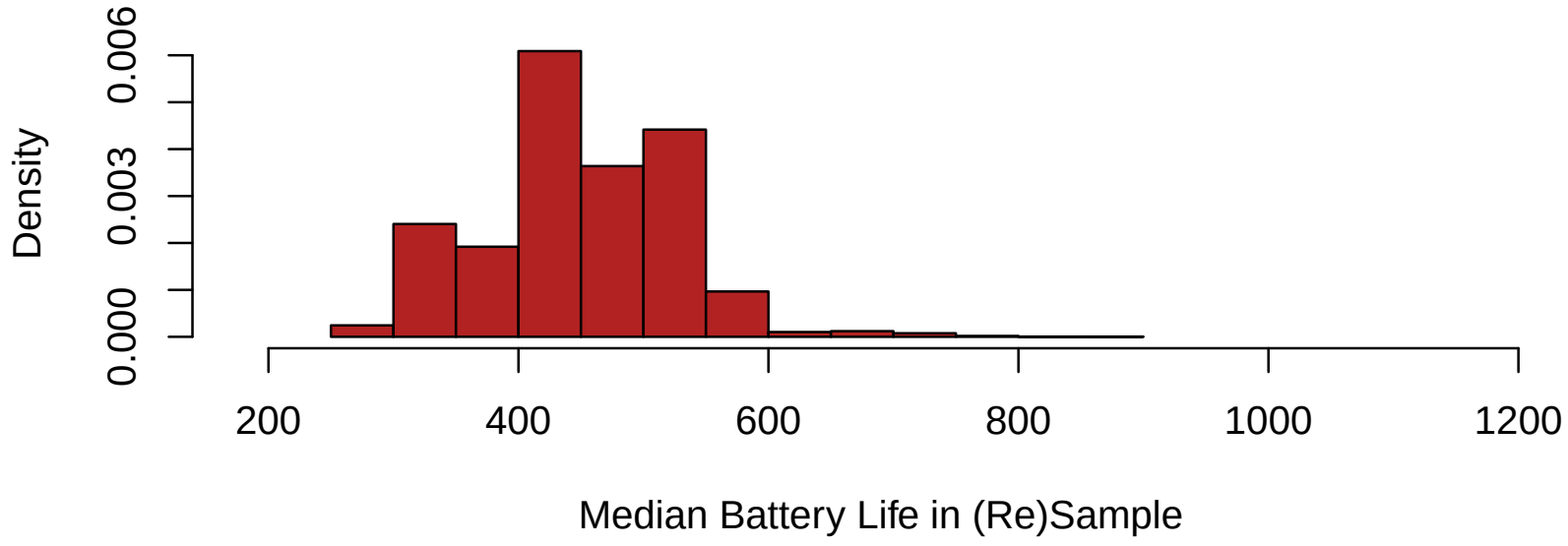
- What can we do with this distribution?
- Where did we make use of our hypotheses?
- We haven't yet! Our reference point is 300 miles. What should the (qualitative) relationship be in order to reject H_0?

Bootstrap Resampling



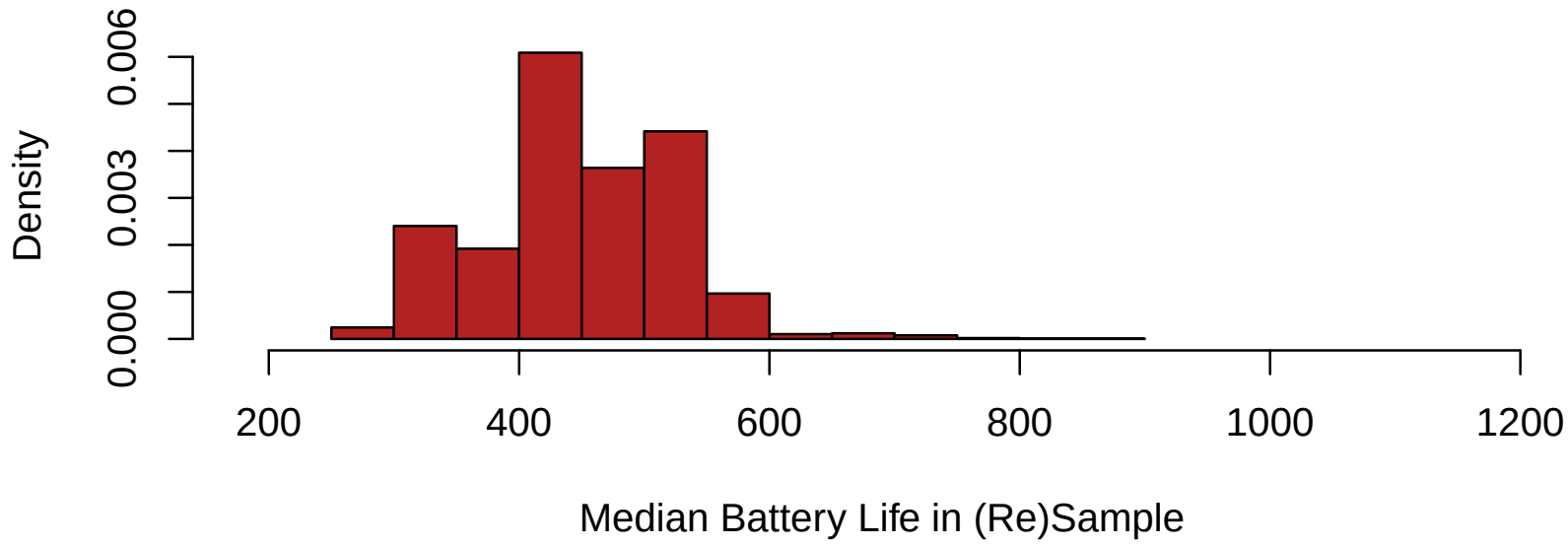
- Intuition: the farther away the H_0 value is from the mass of the bulk of the sampling distribution, the stronger the evidence that it's wrong.

Bootstrap Resampling



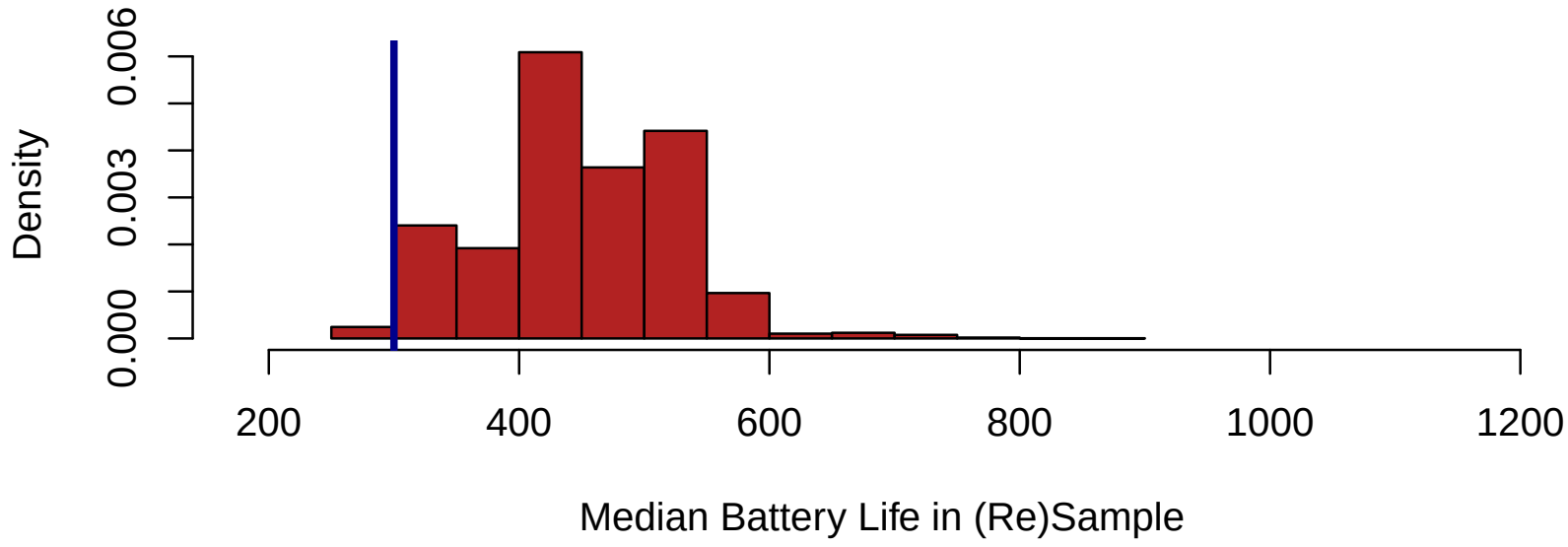
- Intuition: the farther away the H_0 value is from the mass of the bulk of the sampling distribution, the stronger the evidence that it's wrong.
- In this case, since H_1 says the median is *greater* than 300, we are looking for 300 to fall toward the *low* end of the sampling distribution.

Bootstrap Resampling



- If the H_0 value is in the extreme α (typically 0.05) of the distribution, reject H_0.

Bootstrap Resampling



- Here, 0.0122% of our simulated sample medians are below 300.
- Since this is less than α, we reject H_0.

Bootstrap Resampling

Bootstrap Resampling Procedure

1. Identify a **population value**, θ, of interest (e.g., median, 0.75 quantile, ratio of means, ...), and the boundary value, θ_0, between H_0 and H_1.
2. Identify a sample **statistic**, $\hat{\theta}$ that's a good estimator of θ (here it was the sample median, Q_2).
3. Collect a **simple random sample** of size n from the population.
4. Simulate repeated sampling from the population by repeatedly sampling n values **with replacement** from the sample. This gives an estimate of the sampling distribution of $\hat{\theta}$.
5. If θ_0 lies in the most **extreme** α of the simulated distribution (where this is defined based on directionality of H_1), reject H_0.

Bootstrap Resampling for Differences

- In NLP, we typically do bootstrap resampling to compare two groups: a baseline with my fancy new algorithm.
- In this case, the statistic of interest is the difference in means: mean(after) - mean(before).
- Assuming that your H1 says that "after" is better, then H1 is: difference > 0 and H0 is: difference ≤ 0.
- Resample using the standard algorithm and see what percentage of values ≤ 0 exist in the constructed distribution.
- If this percentage is lower than $\alpha = 0.05$ you can claim your algorithm yields statistically significant improvements.

Exercise

- You implemented a "fancy new system"^{TM} for binary classification, and want to compare it against a baseline system on a dataset of 10 points. The labels produced on this dataset are:

	1	2	3	4	5	6	7	8	9	10
Gold	A	B	A	B	A	B	A	B	A	B
Fancy new system	A	B	A	B	A	B	A	A	A	B
Baseline	A	B	A	B	B	B	A	A	A	B

- What are your H0 and H1 hypotheses?
- Can you reject H0 with $\alpha = 0.20$ using bootstrap resampling using 10 samples?