CSC 439/539
Statistical Natural Language Processing
Lecture 2: Crash Course in Machine Learning for Text Categorization

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Take-away

- Understand the task of text categorization
- A few fundamental machine learning (ML) algorithms
  - K Nearest Neighbors (kNN)
  - Perceptron
  - Logistic regression
  - Feed-forward neural networks

Overview

- Text categorization
- Introduction to categorization algorithms
- K Nearest Neighbors (kNN)
- Perceptron
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Text categorization example


```
<REUTERS NEWID="11">
<DATE>26-FEB-1987 15:18:59.34</DATE>
<TOPICS><D>earn</D></TOPICS>
<TEXT>
<TITLE>COBANCO INC &lt;CBCO> YEAR NET</TITLE>
<DATELINE>    SANTA CRUZ, Calif., Feb 26 - </DATELINE>
<BODY>Shr 34 cts vs 1.19 dlrs
    Net 807,000 vs 2,858,000
    Assets 510.2 mln vs 479.7 mln
    Deposits 472.3 mln vs 440.3 mln
    Loans 299.2 mln vs 327.2 mln
    Note: 4th qtr not available. Year includes 1985
extraordinary gain from tax carry forward of 132,000 dlrs,
or five cts per shr.
 Reuter
</BODY></TEXT>
</REUTERS>
```

What would you do with such an algorithm?

Text categorization example

- **ham** Go until jurong point, crazy.. Available only in bugis n great world la e buffet... Cine there got amore wat...
- **ham** Ok lar... Joking wif u oni...
- **spam** Free entry in 2 a wkly comp to win FA Cup final tkts 21st May 2005. Text FA to 87121 to receive entry question(std txt rate)T&C's apply 08452810075over18's
- **ham** U dun say so early hor... U c already then say...
- **ham** Nah I don't think he goes to usf, he lives around here though
- **spam** FreeMsg Hey there darling it's been 3 week's now and no word back! I'd like some fun you up for it still? Tb ok! XxX std chgs to send, £1.50 to rcv

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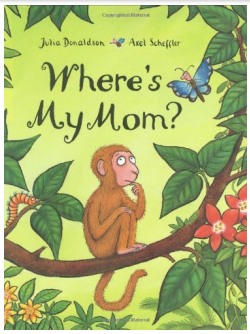
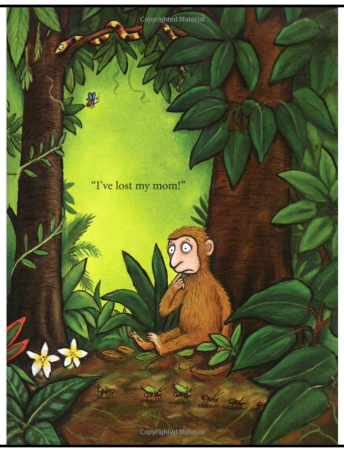
Motivation

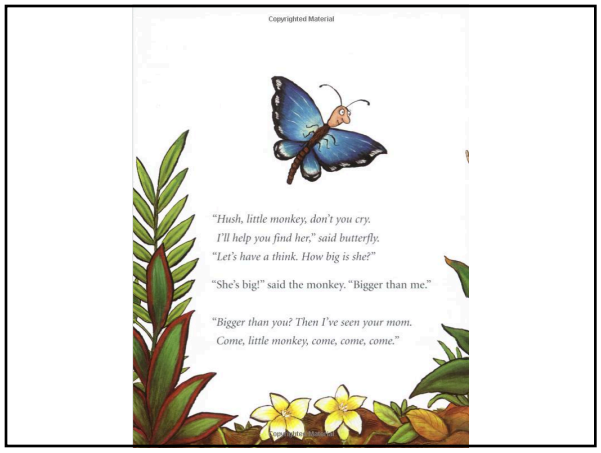
- "I don't crush the corolla of wonders of the world"
  - Lucian Blaga, 1919
- But I think that for ML, we really should destroy it
  - To help us become better users of ML software, and, hopefully, designers of new ones
  - To squash impostor syndrome. We all suffer from it. See this humorous description: http://www.isi.edu/natural-language/people/bayes-with-tears.pdf

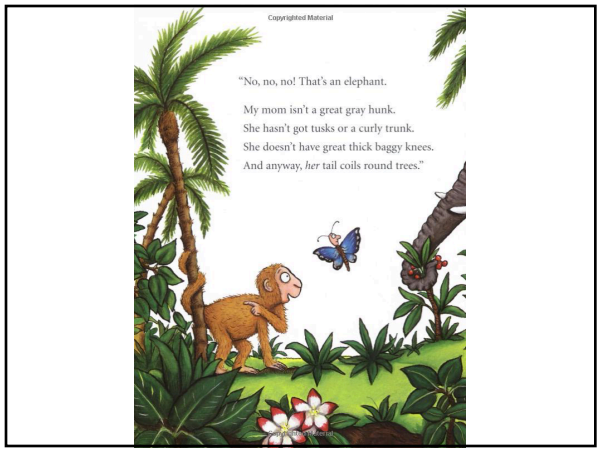
MACHINE LEARNING IS NOT HARD

Let's read a book






Copyrighted Material
"Hush, little monkey, don't you cry.
I'll help you find her," said butterfly.
"Let's have a think. How big is she?"
"She's big!" said the monkey. "Bigger than me."
"Bigger than you? Then I've seen your mom.
Come, little monkey, come, come, come."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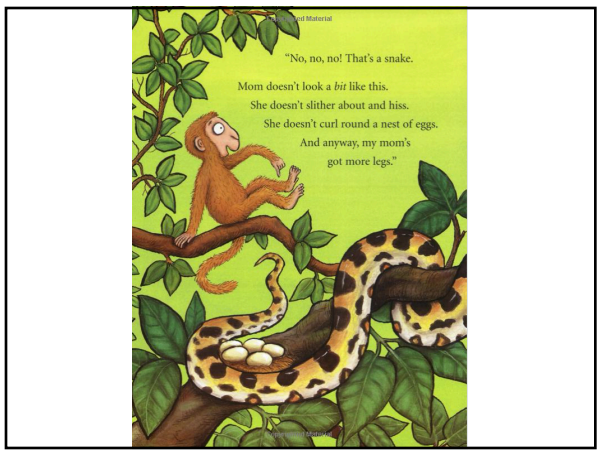
"No, no, no! That's an elephant.
My mom isn't a great gray hunk.
She hasn't got tusks or a curly trunk.
She doesn't have great thick baggy knees.
And anyway, her tail coils round trees."


"No, no, no! That's a snake.
Mom doesn't look a bit like this.
She doesn't slither about and hiss.
She doesn't curl round a nest of eggs.
And anyway, my mom's
got more legs."

Don't worry. Eventually, the baby monkey finds her mom.

What have we learned from this book?

- Objects are described by their properties, or *features*
  - Vector representation, vector length, normalization
- A classification algorithm has at its core a *similarity function*, which measures how similar properties of two objects are
  - Vector similarity
- A good algorithm aggregates its decisions over multiple properties
- Algorithms may *overfit*, if not designed well, or not trained on enough data
  - Overfitting: when an algorithm performs well in training, but poorly on unseen data (testing)

Vector length + cosine similarity

- Vector length:

$$||\vec{x}||_2 = \sqrt{\sum_i x_i^2}$$

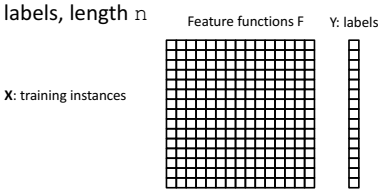
- Cosine similarity:

$$\cos(\vec{x}, \vec{y}) = \frac{\vec{x} \cdot \vec{y}}{|\vec{x}||\vec{y}|} = \frac{\sum_i x_i y_i}{\sqrt{\sum_i x_i^2}\sqrt{\sum_i y_i^2}}$$

Let's build some vectors

Representing NLP problems numerically

- **X**: instance-feature matrix, size $n \times m$ (n rows, m columns)
  - n training instances X_i
  - m feature functions f_j
  - $\mathbf{X}_{i,j} = f_j(X_i)$, i.e., the value of feature function f_j for X_i
- Y: vector of labels, length n



Slide by Erwin Chan

Definitions: instances, features, and labels

- Instances
  - $\mathbf{X} = \{ X_1, X_2, \ldots, X_n \}$: set of all instances in the data
  - X_i: an individual instance
- Features
  - $F = \{ f_1, f_2, \ldots, f_m \}$: a set of *feature functions*
  - $X_i = [f_1(X_i), f_2(X_i), \ldots, f_m(X_i)]$ is a vector of *features*, i.e., the result of applying each feature function in F to the instance X_i
- Labels
  - Set of possible labels: $Y = \{ y_1, y_2, \ldots, y_k \}$
    - Binary classification: $|Y| = 2$
    - Multiclass classification: $|Y| > 2$
  - Classification: for an instance X_i, predict a label $y_i \in Y$
  - Regression: labels are continuous numeric values

Adapted from slide by Erwin Chan

(Powerpoint) Notations

- Scalars: lower case characters
- Vectors of scalars: upper case characters
- Matrices, or vectors of vector: bold upper case characters

Have we understood?

- Features
- Feature vectors
- Vector normalization
- Cosine similarity, dot product

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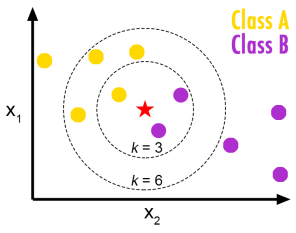
kNN

- Perhaps the simplest categorization algorithm
- Rationale: to classify a new object find the object in the training set that is most similar (1NN)
  - Not very robust: one document could be mislabeled or atypical
- Generalization: assign each object to the majority class of its *k* nearest neighbors in the training set (kNN)

Probabilistic kNN

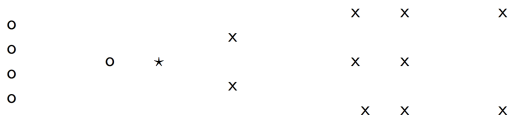
- Probabilistic version of kNN: P(c|d) = fraction of *k* neighbors of *d* that are labeled with class *c*
- kNN classification rule for probabilistic kNN: Assign *d* to class *c* with highest P(c|d)

Exercise



How is star classified for k = 3; k = 6?

Exercise



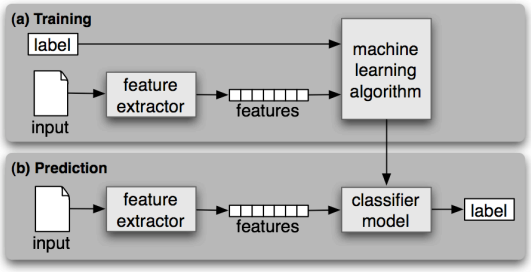
How is star classified by:

(i) 1-NN (ii) 3-NN (iii) 9-NN (iv) 15-NN

What kNN needs

- Representation of objects as vectors
- A similarity function between two vectors
  - Cosine similarity
  - Euclidian distance

Architecture of ML algorithms

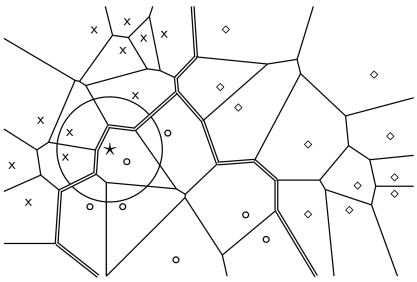


kNN algorithm

TRAIN-KNN($\mathbb{C}, \mathbb{D}$)
1 $\mathbb{D}' \leftarrow$ PREPROCESS($\mathbb{D}$)
2 $k \leftarrow$ SELECT-K($\mathbb{C}, \mathbb{D}'$)//*tuning*
3 **return** $\mathbb{D}', k$

APPLY-KNN($\mathbb{D}', k, d$)
1 $S_k \leftarrow$ COMPUTENEARESTNEIGHBORS($\mathbb{D}', k, d$)
2 **for each** $c_j \in \mathbb{C}(\mathbb{D}')$
3 **do** $p_j \leftarrow |S_k \cap c_j|/k$
4 **return** $\arg\max_j p_j$

kNN is based on Voronoi tesselation



kNN is a non-linear classifier! That is, separators are not "lines".

What is the fundamental problem with kNN?

- The overhead of prediction is high
  - We potentially have to inspect all the objects seen in training to discover the closest neighbors for a new object
- We must condense what we learn from training into something more compact, which could be applied quickly at prediction time

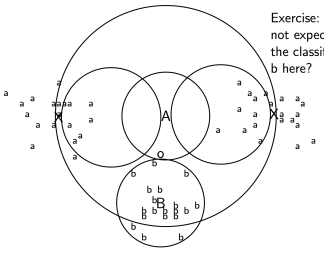
Averaging the vectors per class does not work too well

- Rocchio algorithm:
  - Compute a centroid for each class
    - The centroid is the average of all documents in the class.
  - Assign each test document to the class of its closest centroid.
- What is the runtime of this algorithm at prediction time?

Exercise

- Let's build a Rocchio classifier

Rocchio cannot handle multimodal classes



Exercise: Why is Rocchio not expected to do well for the classification task a vs. b here?

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Rationale

- Error driven learning: update what is learned only when the classifier makes a mistake in training
  - Ignore objects that are classified correctly
  - Focus on the "hard" ones

Perceptron decision

- Learns a single vector W (for binary classification)
- Decision:
  - Decide "yes" iff f(X) = W . X – Theta > 0
  - Decide "no" otherwise

Bias term

Dot product

Algorithm (1/2)

```
1  comment: Categorization Decision
2  funct decision(x⃗, w⃗, θ) ≡
3     if w⃗ · x⃗ > θ then
4                          return yes
5                    else
6                          return no
7     fi.
```

Algorithm (2/2)

```
9  comment: Initialization
10 w⃗ = 0
11 θ = 0
12 comment: Perceptron Learning Algorithm
13 while not converged yet do
14        for all elements x⃗_j in the training set  do
15           d = decision(x⃗_j, w⃗, θ)
16           if class(x⃗_j) = d then
17                                continue
18           elsif class(x⃗_j) = yes and d = no then
19                                                θ = θ − 1
20                                                w⃗ = w⃗ + x⃗_j
21           elsif class(x⃗_j) = no and d = yes then
22                                                θ = θ + 1
23                                                w⃗ = w⃗ − x⃗_j
24           fi
25        end
26 end
```

Exercise

- Let's build a Perceptron

Visualization

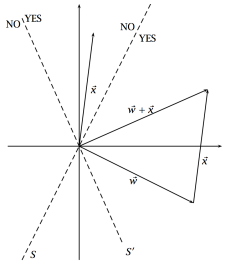


Figure 16.8 One error-correcting step of the perceptron learning algorithm. Data vector $\vec{x}$ is misclassified by the current weight vector $\vec{w}$ since it lies on the "no"-side of the decision boundary S. The correction step adds $\vec{x}$ to $\vec{w}$ and (in this case) corrects the decision since $\vec{x}$ now lies on the "yes"-side of S', the decision boundary of the new weight vector $\vec{w} + \vec{x}$.

Real-world example

Word w^i	Weight
vs	11
mln	6
cts	24
;	2
&	12
000	−4
loss	19
'	−2
"	7
3	−7
profit	31
dlrs	1
1	3
pct	−4
is	−8
s	−12
that	−1
net	8
lt	11
at	−6
θ	37

Table 16.10 Perceptron for the "earnings" category. The weight vector $\vec{w}$ and θ of a perceptron learned by the perceptron learning algorithm for the category "earnings" in Reuters.

Simplifying the bias term

- In many implementations, the bias term is folded into the vectors W and X:

$$\begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_K \\ \theta \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_K \\ -1 \end{pmatrix}$$

- The decision then becomes simply $W \cdot X > 0$
- And the updates work only with W (let's check!)

Simplifying the bias term

```
9  comment: Initialization
10 w⃗ = 0
11 θ = 0
12 comment: Perceptron Learning Algorithm
13 while not converged yet do
14     for all elements x⃗_j in the training set do
15         d = decision(x⃗_j, w⃗, θ)
16         if class(x⃗_j) = d then
17                                 continue
18         elsif class(x⃗_j) = yes and d = no then
19                                                 θ ← θ − 1
20                                                 w⃗ = w⃗ + x⃗_j
21         elsif class(x⃗_j) = no and d = yes then
22                                                 θ ← θ + 1
23                                                 w⃗ = w⃗ − x⃗_j
24         fi
25     end
26 end
```

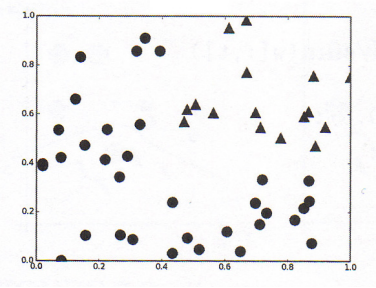
Ensemble models and average Perceptron

- In most situations, voting between multiple classifiers improves performance
  - It is also a cheap way to build a non-linear classifier from a bunch of linear ones

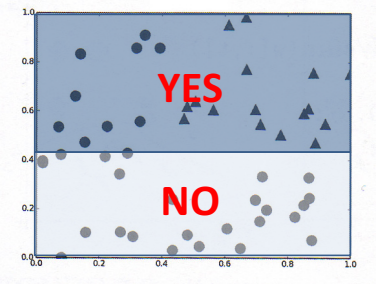
Voting

- Term typically used when the individual classifiers are not directly related
  - Different ML models; different features; etc.
  - Worth $1M: see the Netflix prize
- Types of voting
  - Unweighted voting: pick majority choice
  - Weighted voting: weigh votes by, say, accuracy over a controlled set (development) or confidence of each prediction

Example: Apply voting to this data

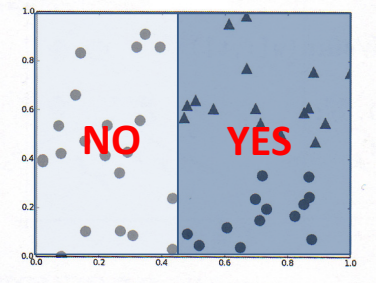


Classifier A



Slide by Erwin Chan

Classifier B



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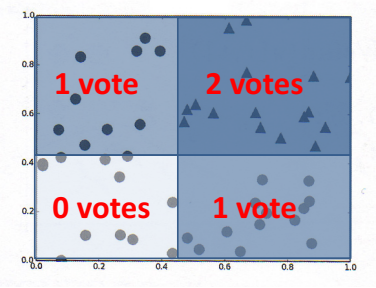
Correct decision boundary through majority vote



Slide by Erwin Chan

Prediction overhead

- But prediction overhead is high again, linear in the number of individual classifiers
- Averaging is a cheap way of reaping most benefits of voting

Averaging

```
comment: Initialization
w⃗ = 0
θ = 0
comment: Perceptron Learning Algorithm
while not converged yet do
    for all elements x⃗_j in the training set do
        d = decision(x⃗_j, w⃗, θ)
        if class(x⃗_j) = d then
                continue
        elsif class(x⃗_j) = yes and d = no then
            θ = θ
            w⃗ = w⃗ + x⃗_j
        elsif class(x⃗_j) = no and d = yes then
            θ = θ + 1
            w⃗ = w⃗ − x⃗_j
        fi
    end
end
```

avgW += W

avgW += W

avgW /= number-of-updates

Averaging

- Decision:
  - Decide “yes” iff f(X) = avgW · X > 0
  - Decide “no” otherwise

What are the fundamental problems with the Perceptron?

- The basic or averaged Perceptrons only model linearly-separable data
- No ”smooth” updates
  - The vector W is updated equally for big or small mistakes…
  - Hard to generalize, because the model “jumps around” good solutions

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Binary classification notations

Let's now talk about the classification problem. This is just like the regression problem, except that the values y we now want to predict take on only a small number of discrete values. For now, we will focus on the **binary classification** problem in which y can take on only two values, 0 and 1. (Most of what we say here will also generalize to the multiple-class case.) For instance, if we are trying to build a spam classifier for email, then $x^{(i)}$ may be some features of a piece of email, and y may be 1 if it is a piece of spam mail, and 0 otherwise. 0 is also called the **negative class**, and 1 the **positive class**, and they are sometimes also denoted by the symbols "-" and "+." Given $x^{(i)}$, the corresponding $y^{(i)}$ is also called the **label** for the training example.

All slides on logistic regression come from Andrew Ng's CS229 course at Stanford

Some notation changes

- Just as before, the bias term (Theta in the Manning textbook) is folded in the X and W vectors
- But here we will use Theta to denote the weights learned by the model (known as W in the Manning textbook)

Looking ahead: A better update rule

$$\theta := \theta + \left(y^{(i)} - h_\theta(x^{(i)})\right) x^{(i)}$$

This is what LR does and it is much better than the Perceptron update. Why?

(h is the decision function)

Let's see how we can derive this update rule

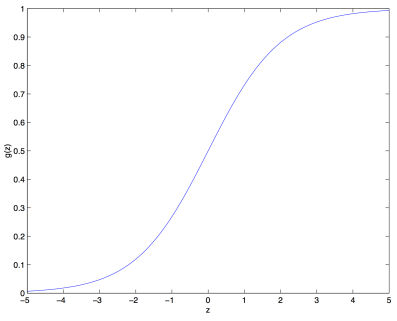
Improvement 1: Decision function

$$h_\theta(x) = g(\theta^T x) = \frac{1}{1 + e^{-\theta^T x}},$$

$$g(z) = \frac{1}{1 + e^{-z}}$$

g = sigmoid function, or logistic function

Decision function



Decision function

Let us assume that

$$P(y = 1 \mid x; \theta) = h_\theta(x)$$
$$P(y = 0 \mid x; \theta) = 1 - h_\theta(x)$$

Why does this make sense?

Note that this can be written more compactly as

$$p(y \mid x; \theta) = (h_\theta(x))^y \, (1 - h_\theta(x))^{1-y}$$

Decision function's derivative (for later)

$$\begin{aligned} g'(z) &= \frac{d}{dz} \frac{1}{1 + e^{-z}} \\ &= \frac{1}{(1 + e^{-z})^2} \left(e^{-z}\right) \\ &= \frac{1}{(1 + e^{-z})} \cdot \left(1 - \frac{1}{(1 + e^{-z})}\right) \\ &= g(z)(1 - g(z)). \end{aligned}$$

Use the differentiation rules to derive this yourself:
https://en.wikipedia.org/wiki/Differentiation_rules

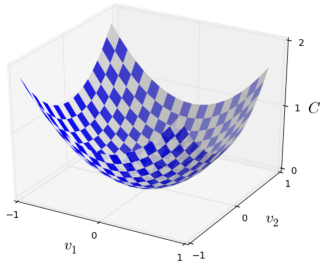
Improvement 2: Cost function and gradient descent/ascend

- Given a training set, how do we learn the parameters Theta to make h(x) close to y?
- To formalize this, we will use a *cost function*, J, which measures how close h(x) is to y
- Given such a function, we will use the gradient descent algorithm, which starts with some initial Theta, and repeatedly performs the update:

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

  - Takes a step in the direction of steepest decrease of J.

Gradient descent



Cost function for LR = Log likelihood

J = l(Theta) = log L(Theta) = log p(Y|X; Theta)

$$
\begin{aligned}
L(\theta) &= p(\vec{y} \mid X; \theta) \\
&= \prod_{i=1}^{m} p(y^{(i)} \mid x^{(i)}; \theta) \\
&= \prod_{i=1}^{m} \left(h_\theta(x^{(i)})\right)^{y^{(i)}} \left(1 - h_\theta(x^{(i)})\right)^{1-y^{(i)}}
\end{aligned}
$$

$$
\begin{aligned}
\ell(\theta) &= \log L(\theta) \\
&= \sum_{i=1}^{m} y^{(i)} \log h(x^{(i)}) + (1 - y^{(i)}) \log(1 - h(x^{(i)}))
\end{aligned}
$$

Note

- In the case of LR, we want to maximize J, so we will perform gradient ascent:

$$\theta := \theta + \alpha \nabla_\theta \ell(\theta)$$

Computing the partial derivatives

$$\begin{aligned}
\frac{\partial}{\partial \theta_j}\ell(\theta) &= \left(y\frac{1}{g(\theta^T x)} - (1-y)\frac{1}{1-g(\theta^T x)}\right)\frac{\partial}{\partial \theta_j}g(\theta^T x) \\
&= \left(y\frac{1}{g(\theta^T x)} - (1-y)\frac{1}{1-g(\theta^T x)}\right)g(\theta^T x)(1-g(\theta^T x))\frac{\partial}{\partial \theta_j}\theta^T x \\
&= \left(y(1-g(\theta^T x)) - (1-y)g(\theta^T x)\right)x_j \\
&= \left(y - h_\theta(x)\right)x_j
\end{aligned}$$

We are applying the chain rule twice here! See:
https://en.wikipedia.org/wiki/Differentiation_rules

Stochastic gradient ascent

$$\theta_j := \theta_j + \alpha\left(y^{(i)} - h_\theta(x^{(i)})\right)x_j^{(i)}$$

- "Stochastic" because we perform updates after every training example
- "Batch"
  - Perform 1 update for 1 pass over the entire training dataset
  - Almost never used in practice (why?)

The update rule

$$\theta_j := \theta_j + \alpha\left(y^{(i)} - h_\theta(x^{(i)})\right)x_j^{(i)}$$

This is much better than the Perceptron update. Why?

Exercise: write the whole LR training algorithm

Limitations

- The decision function is linear (in log space)
- Feature design is hard for most real-world NLP problems…

Overview

- Text categorization
- Introduction to categorization algorithms
- K Nearest Neighbors (kNN)
- Perceptron
- Logistic regression
- Feed-forward neural networks

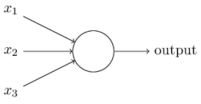
What did you find hard about ML so far?

- We need to design custom decision functions (e.g., dot product, sigmoid), and custom cost functions
- But then we have to recompute all the derivatives. Derivatives are a pain…
- Feature design is hard for real-world NLP applications

Advantages of neural networks

- Custom, non-linear decision and cost functions
- Auto-differentiation to compute derivatives automatically
- Automatic generation of feature vectors for all data points (next lecture)

Perceptron architecture



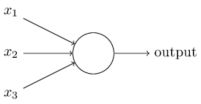
$$\text{output} = \begin{cases} 0 & \text{if } \sum_j w_j x_j \leq \text{threshold} \\ 1 & \text{if } \sum_j w_j x_j > \text{threshold} \end{cases}$$

or:

$$\text{output} = \begin{cases} 0 & \text{if } w \cdot x + b \leq 0 \\ 1 & \text{if } w \cdot x + b > 0 \end{cases}$$

All slides on logistic regression come from http://neuralnetworksanddeeplearning.com/

LR architecture

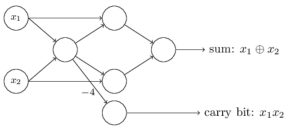


$$\sigma(z) \equiv \frac{1}{1+e^{-z}}$$

where *z* is the dot product of W and X (plus bias):

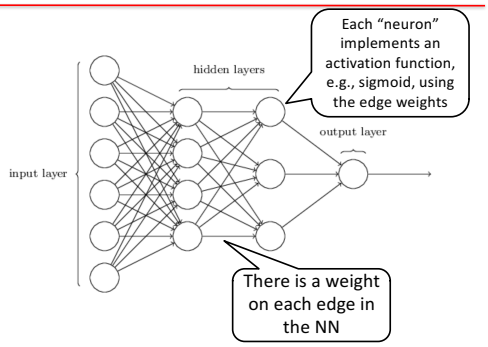
$$\frac{1}{1+\exp(-\sum_j w_j x_j - b)}$$

Many other architectures are possible…

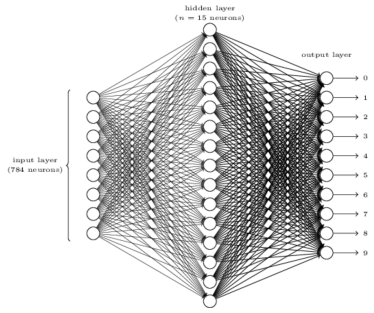


See http://neuralnetworksanddeeplearning.com/chap1.html for this discussion.

Feed forward neural networks



NN for recognizing digits



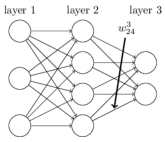
What needs to be trained

- The network (i.e., number of layers, nodes, and edges), and activation functions are static. You define them.
- During training, stochastic gradient descent (SGD) adjust the edge weights such that a cost function (J or C), e.g., negative log likelihood, is minimized.

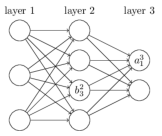
What needs to be trained

- Very similar to LR (in fact, LR is a one-layer NN…), but
- Computing the derivatives for the hidden layers (which don't exist in LR) is tricky
- We will use the backpropagation algorithm for this purpose
  - Let's do this for the feed-forward NN
  - Required for 539 only

Notations



w^l_{jk} is the weight from the k^{th} neuron in the $(l-1)^{\text{th}}$ layer to the j^{th} neuron in the l^{th} layer



$$a^l_j = \sigma\left(\sum_k w^l_{jk} a^{l-1}_k + b^l_j\right)$$

$$a^l = \sigma(w^l a^{l-1} + b^l)$$

$$z^l \equiv w^l a^{l-1} + b^l$$

Goal of backpropagation

The goal of backpropagation is to compute the partial derivatives $\partial C/\partial w$ and $\partial C/\partial b$ of the cost function C with respect to any weight w or bias b in the network.

Cost function in this example

$$C = \frac{1}{2n}\sum_x \|y(x) - a^L(x)\|^2$$

Or, for a single training example x:

$$C = \frac{1}{2}\|y - a^L\|^2 = \frac{1}{2}\sum_j (y_j - a^L_j)^2$$

- C = quadratic cost, mean squared error, MSE
- But many other cost functions are possible!
- (Incomplete) rule of thumb:
  - If binary classification: MSE
  - If multiclass classification: cross entropy

Notes: a few useful formulas

- Hadamard product: Element wise multiplication of two vectors:

$$\begin{bmatrix}1\\2\end{bmatrix} \odot \begin{bmatrix}3\\4\end{bmatrix} = \begin{bmatrix}1*3\\2*4\end{bmatrix} = \begin{bmatrix}3\\8\end{bmatrix}$$

- Error or layer l in the network:

$$\delta_j^l \equiv \frac{\partial C}{\partial z_j^l}$$

4 equations

- Which will compute the partial derivatives of C with respect to w and b, so we can perform gradient descent

Summary: the equations of backpropagation

$\delta^L = \nabla_a C \odot \sigma'(z^L)$ (BP1)

$\delta^l = ((w^{l+1})^T \delta^{l+1}) \odot \sigma'(z^l)$ (BP2)

$\frac{\partial C}{\partial b_j^l} = \delta_j^l$ (BP3)

$\frac{\partial C}{\partial w_{jk}^l} = a_k^{l-1} \delta_j^l$ (BP4)

BP1

- Compute the error in the last layer (L):

$$\delta_j^L = \frac{\partial C}{\partial a_j^L} \sigma'(z_j^L)$$

- Or, in vector form:

$$\delta^L = \nabla_a C \odot \sigma'(z^L)$$

$$\delta^L = (a^L - y) \odot \sigma'(z^L)$$

Proof of BP1

$$\delta_j^L = \frac{\partial C}{\partial z_j^L}$$

$$\delta_j^L = \sum_k \frac{\partial C}{\partial a_k^L} \frac{\partial a_k^L}{\partial z_j^L}$$

$$\delta_j^L = \frac{\partial C}{\partial a_j^L} \frac{\partial a_j^L}{\partial z_j^L}$$

$$\delta_j^L = \frac{\partial C}{\partial a_j^L} \sigma'(z_j^L).$$

BP2

- Computes the error in layer l as a function of the error in layer l + 1 (hence "backpropagation"):

$$\delta^l = ((w^{l+1})^T \delta^{l+1}) \odot \sigma'(z^l)$$

Proof of BP2 (1/2)

$$\begin{aligned}\delta_j^l &= \frac{\partial C}{\partial z_j^l} \\ &= \sum_k \frac{\partial C}{\partial z_k^{l+1}} \frac{\partial z_k^{l+1}}{\partial z_j^l} \\ &= \sum_k \frac{\partial z_k^{l+1}}{\partial z_j^l} \delta_k^{l+1},\end{aligned}$$

Proof of BP2 (2/2)

$$z_k^{l+1} = \sum_j w_{kj}^{l+1} a_j^l + b_k^{l+1} = \sum_j w_{kj}^{l+1} \sigma(z_j^l) + b_k^{l+1}$$

$$\frac{\partial z_k^{l+1}}{\partial z_j^l} = w_{kj}^{l+1} \sigma'(z_j^l)$$

$$\delta_j^l = \sum_k w_{kj}^{l+1} \delta_k^{l+1} \sigma'(z_j^l)$$

BP3 and BP4

- Rate of change of cost wrt any bias in the network:

$$\frac{\partial C}{\partial b_j^l} = \delta_j^l$$

- Rate of change of cost wrt any weight in the network:

$$\frac{\partial C}{\partial w_{jk}^l} = a_k^{l-1} \delta_j^l$$

At-home exercise: proofs of BP3 and BP4

- These could be great midterm questions (for grad students)…

Summary: the equations of backpropagation

$$\delta^L = \nabla_a C \odot \sigma'(z^L) \tag{BP1}$$

$$\delta^l = ((w^{l+1})^T \delta^{l+1}) \odot \sigma'(z^l) \tag{BP2}$$

$$\frac{\partial C}{\partial b^l_j} = \delta^l_j \tag{BP3}$$

$$\frac{\partial C}{\partial w^l_{jk}} = a^{l-1}_k \delta^l_j \tag{BP4}$$

Backpropagation + stochastic gradient descent

- SGD learns network weights (and bias) for any network, with any decision functions in the nodes, and any cost function
  - Assuming we can differentiate the decision and cost functions…
- SGD is not guaranteed to be optimal!
  - But it works reasonably well, and it is reasonably fast

Coming up

- Next lecture: Automatic feature generation through distributional similarity
- Following lecture: DyNet tutorial for feed-forward networks
- Rest of the course: structured learning, aka how to learn when the points to be categorized depend on each other?

Readings

- For kNN and Rocchio: see chapter 14 in Introduction to Information Retrieval (http://informationretrieval.org)
- For Perceptron: see chapter 16 in our textbook
- For logistic regression (LR): see Andrew Ng's notes for LR: Part II in the file cs229_notes1.pdf (in D2L)
- For neural networks: see chapters 1 and 2 in Michael Nielsen's book: http://neuralnetworksanddeeplearning.com

Take-away

- Understand the task of text categorization
- A few fundamental machine learning (ML) algorithms
  - K Nearest Neighbors (kNN)
  - Perceptron
  - Logistic regression
  - Feed-forward neural networks