

# Econometrics

## Lecture 1: Introduction

# About This Course

## Logistics

- 13 lectures, 1.5 hours each
- Office hours: TBA
- Course website: TBA

## Prerequisites

- Probability and statistics (L3 level)
- Linear algebra (matrix operations)
- Calculus (optimization, integration)

# What You Will Learn

By the end of this course, you will be able to:

- ① **Formulate** econometric models for causal questions
- ② **Estimate** parameters using OLS, IV, and panel methods
- ③ **Evaluate** assumptions and diagnose violations
- ④ **Interpret** results with statistical and economic rigor
- ⑤ **Implement** analyses in R with real data

# Course Roadmap

## Foundations

- ① **Introduction & Probability Review**
- ② Simple Regression
- ③ Multiple Regression: Estimation
- ④ Multiple Regression: Inference
- ⑤ Asymptotics for OLS

## Diagnostics

- ⑥ Heteroskedasticity
- ⑦ Specification & Functional Form

## Advanced Methods

- ⑧ Instrumental Variables
- ⑨ Simultaneous Equations
- ⑩ Panel Data
- ⑪ Binary Response Models

## Extensions

- ⑫ Time Series Basics
- ⑬ Advanced Topics & Review

# Textbooks and Resources

## Primary textbook

- Wooldridge (2019) — comprehensive, example-driven

## Complementary reading

- Angrist and Pischke (2009) — causal inference perspective
- Stock and Watson (2020) — accessible alternative
- Hansen (2022) — advanced/graduate reference

## Software

- R (with RStudio) — introduced from Lecture 2 onward

# Assessment

<b>Component</b>	<b>Weight</b>
Problem sets (4–5)	30%
Midterm exam	30%
Final exam	40%

Problem sets combine analytical derivations with R-based empirical exercises. Collaboration is encouraged; write-ups must be individual.

# Why Econometrics?

**The fundamental problem:** economic data are **observational**, not experimental.

## Correlation

- People with more education earn more
- Countries with more police have more crime
- Hospital patients are sicker than average

## Causation?

- Does education *cause* higher earnings?
- Do police *cause* crime?
- Do hospitals *make* people sick?

Econometrics provides the tools to move from correlation to causal statements — under clearly stated assumptions.

# Motivating Example: Returns to Education

The **Mincer wage equation** (Mincer, 1974):

$$\ln(\text{wage}_i) = \beta_0 + \beta_1 \text{educ}_i + \beta_2 \text{exper}_i + \beta_3 \text{exper}_i^2 + u_i$$

## Key questions:

- What does  $\beta_1$  measure? Under what conditions is it *causal*?
- What problems arise if ability is omitted?
- How do we estimate, test, and interpret  $\beta_1$ ?

We will return to this example throughout the course. By Lecture 8 (IV), you will have multiple strategies to estimate the causal return to education.

# **Part II**

# **Review of Probability**

Wooldridge, Appendix B

# Random Variables

## Random Variable

A **random variable**  $X$  is a function from a sample space  $\Omega$  to  $\mathbb{R}$ :

$$X : \Omega \rightarrow \mathbb{R}$$

### Discrete $X$

- Takes countably many values
- Characterized by its **probability mass function** (PMF):

$$f(x) = P(X = x)$$

- Example: number of children

### Continuous $X$

- Takes uncountably many values
- Characterized by its **probability density function** (PDF):

$$f(x) \geq 0, \quad \int_{-\infty}^{\infty} f(x) dx = 1$$

- Example: hourly wage

# Cumulative Distribution Function

## Cumulative Distribution Function (CDF)

The **cumulative distribution function** of  $X$  is:

$$F(x) = P(X \leq x)$$

### Properties:

- ①  $\lim_{x \rightarrow -\infty} F(x) = 0$  and  $\lim_{x \rightarrow \infty} F(x) = 1$
- ②  $F$  is non-decreasing
- ③  $F$  is right-continuous
- ④  $P(a < X \leq b) = F(b) - F(a)$

For continuous  $X$ :  $f(x) = F'(x)$  and  $F(x) = \int_{-\infty}^x f(t) dt$

# Expected Value

## Expected Value

**Discrete:**  $\mathbb{E}[X] = \sum_x x \cdot f(x)$

**Continuous:**  $\mathbb{E}[X] = \int_{-\infty}^{\infty} x f(x) dx$

## Linearity of expectation:

$$\mathbb{E}[aX + b] = a \mathbb{E}[X] + b$$

$$\mathbb{E}[X + Y] = \mathbb{E}[X] + \mathbb{E}[Y] \quad (\text{always, even if dependent})$$

**For a function  $g(X)$ :**  $\mathbb{E}[g(X)] = \int g(x) f(x) dx$

# Variance and Standard Deviation

## Variance

$$\text{Var}(X) = \mathbb{E}[(X - \mathbb{E}[X])^2] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$$

The **standard deviation** is  $\text{sd}(X) = \sqrt{\text{Var}(X)}$ .

## Properties:

- $\text{Var}(X) \geq 0$ , with equality iff  $X$  is constant a.s.
- $\text{Var}(aX + b) = a^2 \text{Var}(X)$
- $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) + 2 \text{Cov}(X, Y)$

# Covariance and Correlation

## Covariance and Correlation

$$\text{Cov}(X, Y) = \mathbb{E} [(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])] = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$$

$$\text{Corr}(X, Y) = \frac{\text{Cov}(X, Y)}{\text{sd}(X)\text{sd}(Y)}$$

### Key facts:

- $-1 \leq \text{Corr}(X, Y) \leq 1$
- $\text{Corr}(X, Y) = 0$  means **uncorrelated** (not necessarily independent)
- $\text{Cov}(X, X) = \text{Var}(X)$
- $\text{Cov}(aX + b, cY + d) = ac \text{Cov}(X, Y)$

# Key Properties of $\mathbb{E}$ and Var

## Expected value:

$$\mathbb{E}[aX + bY + c] = a \mathbb{E}[X] + b \mathbb{E}[Y] + c$$

$$\mathbb{E} \left[ \sum_{i=1}^n X_i \right] = \sum_{i=1}^n \mathbb{E}[X_i]$$

## Variance:

$$\text{Var}(aX + b) = a^2 \text{Var}(X)$$

$$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) + 2 \text{Cov}(X, Y)$$

If  $X_i$  **uncorrelated**:  $\text{Var}(\sum_i X_i) = \sum_i \text{Var}(X_i)$ ; otherwise add  $2 \sum_{i < j} \text{Cov}(X_i, X_j)$ .

# Joint, Marginal, and Conditional Distributions

**Joint PDF/PMF:**  $f_{X,Y}(x, y)$  describes the simultaneous behavior of  $X$  and  $Y$ .

**Marginal distribution:**

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x, y) dy$$

**Conditional distribution:**

$$f_{Y|X}(y | x) = \frac{f_{X,Y}(x, y)}{f_X(x)}, \quad f_X(x) > 0$$

The conditional distribution  $f_{Y|X}$  is the foundation of regression analysis: it describes how  $Y$  behaves *given* a particular value of  $X$ .

# Conditional Expectation

## Conditional Expectation

$$\mathbb{E}[Y \mid X = x] = \int y f_{Y|X}(y \mid x) dy$$

The **conditional expectation function** (CEF) is the map  $x \mapsto \mathbb{E}[Y \mid X = x]$ .

## Why it matters for econometrics:

- The regression function  $\mathbb{E}[Y \mid X]$  is the **best predictor** of  $Y$  given  $X$
- It minimizes the mean squared prediction error (assuming  $\mathbb{E}[Y^2] < \infty$ ):

$$\mathbb{E}[Y \mid X] = \arg \min_{g(X)} \mathbb{E} [(Y - g(X))^2]$$

- OLS approximates the CEF with a linear function

# Law of Iterated Expectations (LIE)

## Law of Iterated Expectations:

$$\mathbb{E}[Y] = \mathbb{E} [\mathbb{E}[Y | X]]$$

The unconditional mean of  $Y$  is the average of conditional means, weighted by  $f_X$ .

## Why this matters:

- Allows us to decompose expectations by conditioning
- Key tool in proving properties of OLS estimators
- Used repeatedly in deriving unbiasedness, omitted variable bias, IV results

**Example:**  $\mathbb{E}[\text{wage}] = \mathbb{E} [\mathbb{E}[\text{wage} | \text{educ}]]$

The overall average wage is the average of education-specific average wages.

# Conditional Variance

## Conditional Variance

$$\text{Var}(Y | X) = \mathbb{E} [(Y - \mathbb{E}[Y | X])^2 | X] = \mathbb{E}[Y^2 | X] - (\mathbb{E}[Y | X])^2$$

## Law of Total Variance:

$$\text{Var}(Y) = \underbrace{\mathbb{E} [\text{Var}(Y | X)]}_{\text{within-group}} + \underbrace{\text{Var} (\mathbb{E}[Y | X])}_{\text{between-group}}$$

Total variance decomposes into average *within-group* variance plus variance of *group means*.

# Independence

## Independence

$X$  and  $Y$  are **independent** ( $X \perp\!\!\!\perp Y$ ) if and only if:

$$f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y) \quad \text{for all } x, y$$

### Implications of independence:

- $\mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y] \implies \text{Cov}(X, Y) = 0$
- $\mathbb{E}[g(X)h(Y)] = \mathbb{E}[g(X)]\mathbb{E}[h(Y)]$
- $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$

**Warning:** Uncorrelated  $\not\Rightarrow$  independent (unless jointly Normal). Independence  $\Rightarrow$  uncorrelated always holds.

# Common Distributions in Econometrics

Distribution	Notation	Mean	Variance
Normal	$N(\mu, \sigma^2)$	$\mu$	$\sigma^2$
Standard Normal	$N(0, 1)$	0	1
Chi-squared	$\chi_k^2$	$k$	$2k$
Student's $t$	$t_k$	0 ( $k > 1$ )	$\frac{k}{k-2}$ ( $k > 2$ )
Fisher's $F$	$F_{k_1, k_2}$	$\frac{k_2}{k_2-2}$ ( $k_2 > 2$ )	(complex)

These distributions arise naturally in testing:

- $t$ -distribution → individual coefficient tests
- $F$ -distribution → joint hypothesis tests
- $\chi^2$ -distribution → specification tests, MLE

# The Normal Distribution

If  $X \sim N(\mu, \sigma^2)$ , its PDF is:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right)$$

## Key properties:

- **Standardization:**  $Z = \frac{X - \mu}{\sigma} \sim N(0, 1)$
- **Linear combinations:** if  $X \sim N(\mu_X, \sigma_X^2)$  and  $Y \sim N(\mu_Y, \sigma_Y^2)$  are independent:

$$aX + bY \sim N(a\mu_X + b\mu_Y, a^2\sigma_X^2 + b^2\sigma_Y^2)$$

- **Symmetry:**  $\Phi(-z) = 1 - \Phi(z)$ , where  $\Phi$  is the standard Normal CDF

Under classical assumptions, OLS estimators are *exactly* Normal. Under weaker assumptions, they are *asymptotically* Normal (via CLT).

# Chi-Squared, $t$ , and $F$ Distributions

## Construction from Normal random variables:

- ① If  $Z_1, \dots, Z_k \stackrel{\text{iid}}{\sim} N(0, 1)$ , then  $\sum_{i=1}^k Z_i^2 \sim \chi_k^2$
- ② If  $Z \sim N(0, 1)$  and  $V \sim \chi_k^2$  are independent, then  $\frac{Z}{\sqrt{V/k}} \sim t_k$
- ③ If  $V_1 \sim \chi_{k_1}^2$  and  $V_2 \sim \chi_{k_2}^2$  are independent, then  $\frac{V_1/k_1}{V_2/k_2} \sim F_{k_1, k_2}$

## Key relationships:

- $t_k^2 = F_{1,k}$  (squaring a  $t$  gives an  $F$ )
- As  $k \rightarrow \infty$ :  $t_k \rightarrow N(0, 1)$  and  $\chi_k^2/k \rightarrow 1$

# Moment Generating Functions

## Moment Generating Function (MGF)

The **moment generating function** of  $X$  is:

$$M_X(t) = \mathbb{E}[e^{tX}]$$

defined for  $t$  in a neighborhood of zero.

### Why useful:

- Moments:  $\mathbb{E}[X^k] = M_X^{(k)}(0)$
- **Uniqueness:** if  $M_X(t) = M_Y(t)$  in a neighborhood of 0, then  $X \stackrel{d}{=} Y$
- **Sums of independents:**  $M_{X+Y}(t) = M_X(t) \cdot M_Y(t)$

**Example:**  $X \sim N(\mu, \sigma^2) \Rightarrow M_X(t) = \exp\left(\mu t + \frac{1}{2}\sigma^2 t^2\right)$

# Probability Review: Key Results

- ①  $\mathbb{E}[aX + bY] = a\mathbb{E}[X] + b\mathbb{E}[Y]$  (always)
- ②  $\text{Var}(aX + b) = a^2 \text{Var}(X)$
- ③  $\text{Cov}(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$
- ④  $X \perp\!\!\!\perp Y \Rightarrow \text{Cov}(X, Y) = 0$  (converse is false)
- ⑤  $\mathbb{E}[Y] = \mathbb{E}[\mathbb{E}[Y | X]]$  (LIE)
- ⑥  $\text{Var}(Y) = \mathbb{E}[\text{Var}(Y | X)] + \text{Var}(\mathbb{E}[Y | X])$  (total variance)
- ⑦  $\mathbb{E}[Y | X] = \arg \min_{g(X)} \mathbb{E}[(Y - g(X))^2]$  (best predictor)

These results will be used repeatedly from Lecture 2 onward.

# **Part III**

# **Review of Statistical Inference**

Wooldridge, Appendix C

# Random Sampling

## Random Sample

A **random sample** of size  $n$  is a collection  $\{X_1, X_2, \dots, X_n\}$  of **independent and identically distributed** (i.i.d.) random variables, each with the same distribution as  $X$ .

### The i.i.d. assumption means:

- **Identical:** each  $X_i$  has the same distribution  $F$
- **Independent:** knowing  $X_i$  tells you nothing about  $X_j$  ( $i \neq j$ )

### When does i.i.d. hold?

- Cross-sectional surveys with random sampling: **typically yes**
- Time series data: **typically no** (observations are dependent)
- Panel data: **partially** (independent across  $i$ , dependent across  $t$ )

# Estimators as Random Variables

## Estimator

An **estimator**  $\hat{\theta}_n = g(X_1, \dots, X_n)$  is a function of the sample. Before the data are observed,  $\hat{\theta}_n$  is a **random variable**.

## Distinguish:

- **Parameter**  $\theta$ : fixed, unknown quantity we want to learn
- **Estimator**  $\hat{\theta}_n$ : random variable (depends on the sample)
- **Estimate**  $\hat{\theta}$ : realized value from a specific sample

Because  $\hat{\theta}_n$  is a random variable, it has a distribution — the **sampling distribution**. All of inference (bias, variance, testing) is about this distribution.

# Unbiasedness

## Unbiasedness

An estimator  $\hat{\theta}_n$  is **unbiased** for  $\theta$  if:

$$\mathbb{E}[\hat{\theta}_n] = \theta$$

The **bias** is  $\text{Bias}(\hat{\theta}_n) = \mathbb{E}[\hat{\theta}_n] - \theta$ .

**Example:** Let  $X_1, \dots, X_n \stackrel{\text{iid}}{\sim} (\mu, \sigma^2)$ .

- $\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$  is unbiased for  $\mu$ :  $\mathbb{E}[\bar{X}_n] = \mu$
- $S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X}_n)^2$  is unbiased for  $\sigma^2$
- $\tilde{S}^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X}_n)^2$  is **biased**:  $\mathbb{E}[\tilde{S}^2] = \frac{n-1}{n} \sigma^2$

# Efficiency

## Efficiency

Among all unbiased estimators,  $\hat{\theta}^*$  is **efficient** if it has the **smallest variance**:

$$\text{Var}(\hat{\theta}^*) \leq \text{Var}(\hat{\theta}) \quad \text{for all unbiased } \hat{\theta}$$

**Mean squared error** combines bias and variance:

$$\text{MSE}(\hat{\theta}_n) = \text{Var}(\hat{\theta}_n) + [\text{Bias}(\hat{\theta}_n)]^2$$

**Bias-variance tradeoff:** a slightly biased estimator with much lower variance can have smaller MSE.

Preview: the Gauss-Markov theorem (Lecture 2) establishes OLS as BLUE.

# Consistency

## Consistency

$\hat{\theta}_n$  is **consistent** for  $\theta$  if  $\hat{\theta}_n \xrightarrow{p} \theta$  as  $n \rightarrow \infty$ , i.e.,  $\lim_{n \rightarrow \infty} P(|\hat{\theta}_n - \theta| > \varepsilon) = 0$  for all  $\varepsilon > 0$ .

**Sufficient condition:**  $\text{Bias}(\hat{\theta}_n) \rightarrow 0$  and  $\text{Var}(\hat{\theta}_n) \rightarrow 0$  as  $n \rightarrow \infty$ .

### Consistency vs. unbiasedness:

- Unbiasedness is a *finite-sample* property (holds for all  $n$ )
- Consistency is an *asymptotic* property (requires  $n \rightarrow \infty$ )
- Unbiased  $\not\Rightarrow$  consistent; consistent  $\not\Rightarrow$  unbiased

# Law of Large Numbers

**Weak Law of Large Numbers (WLLN):** If  $X_1, \dots, X_n$  are i.i.d. with  $\mathbb{E}[X_i] = \mu$  and  $\text{Var}(X_i) < \infty$ , then:

$$\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i \xrightarrow{p} \mu$$

**Intuition:** as the sample grows, the sample mean converges to the population mean.

## Why it matters for econometrics:

- Justifies using sample averages to estimate population quantities
- Underpins consistency of OLS:  $\hat{\beta} \xrightarrow{p} \beta$
- Foundation for the method of moments

# Central Limit Theorem

**Central Limit Theorem (CLT):** If  $X_1, \dots, X_n$  are i.i.d. with  $\mathbb{E}[X_i] = \mu$  and  $\text{Var}(X_i) = \sigma^2 \in (0, \infty)$ , then:

$$\frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}} \xrightarrow{d} N(0, 1)$$

Equivalently:  $\sqrt{n}(\bar{X}_n - \mu) \xrightarrow{d} N(0, \sigma^2)$

## The power of the CLT:

- Works *regardless* of the distribution of  $X_i$
- Convergence is often fast (usable for  $n \geq 30$  in practice)
- Justifies Normal-based inference even for non-Normal data

# CLT: Visual Intuition

**What happens as  $n$  grows?** Consider sampling from a skewed distribution (e.g., exponential):

Sample size	Distribution of $\bar{X}_n$
$n = 1$	Same as original (skewed)
$n = 5$	Less skewed, more concentrated
$n = 30$	Approximately Normal
$n = 100$	Very close to Normal, tightly concentrated

$\bar{X}_n$  is approximately Normal for large  $n$ , no matter how non-Normal the population. This is why we can use  $t$ -tests and confidence intervals without assuming Normality.

# Properties of the Sample Mean and Variance

Let  $X_1, \dots, X_n \stackrel{\text{iid}}{\sim} (\mu, \sigma^2)$ .

**Sample mean:**  $\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$

$$\mathbb{E}[\bar{X}_n] = \mu, \quad \text{Var}(\bar{X}_n) = \frac{\sigma^2}{n}$$

**Sample variance:**  $S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X}_n)^2$

- $\mathbb{E}[S^2] = \sigma^2$  (unbiased)
- The  $n - 1$  denominator corrects for using  $\bar{X}_n$  instead of  $\mu$

**Standard error of the mean:**

$$\text{SE}(\bar{X}_n) = \frac{S}{\sqrt{n}}$$

# Confidence Intervals

## Confidence Interval

A  $100(1 - \alpha)\%$  **CI** for  $\theta$  is a random interval  $[\hat{\theta}_L, \hat{\theta}_U]$  s.t.  $P(\hat{\theta}_L \leq \theta \leq \hat{\theta}_U) = 1 - \alpha$ .

**For the mean** (large  $n$  or Normal population):

$$\bar{X}_n \pm t_{n-1, \alpha/2} \cdot \frac{S}{\sqrt{n}}$$

### Interpretation:

- **Correct:** if we repeated sampling many times,  $100(1 - \alpha)\%$  of intervals would contain  $\theta$
- **Wrong:** “there is a 95% probability that  $\theta$  is in this interval” —  $\theta$  is fixed; the interval is random

# Hypothesis Testing

## Setup:

- **Null hypothesis**  $H_0$ : a specific claim about  $\theta$  (e.g.,  $H_0 : \theta = \theta_0$ )
- **Alternative**  $H_1$ : the complement (e.g.,  $H_1 : \theta \neq \theta_0$ )
- **Test statistic:** a function of the data that measures evidence against  $H_0$

**Decision rule:** reject  $H_0$  if the test statistic falls in the **rejection region**.

### General test statistic:

$$T = \frac{\hat{\theta} - \theta_0}{\text{SE}(\hat{\theta})}$$

Under  $H_0$  and regularity conditions:  $T \sim t_{n-k}$  (or  $\approx N(0, 1)$  for large  $n$ ).

# Type I and Type II Errors

	$H_0$ true	$H_0$ false
Reject $H_0$	Type I error ( $\alpha$ )	Correct (Power)
Fail to reject $H_0$	Correct ( $1 - \alpha$ )	Type II error ( $\beta$ )

## Key concepts:

- **Size** ( $\alpha$ ): probability of rejecting  $H_0$  when it is true
- **Power** ( $1 - \beta$ ): probability of rejecting  $H_0$  when it is false
- Standard choices:  $\alpha = 0.10, 0.05, 0.01$

**Tradeoff:** decreasing  $\alpha$  (fewer false positives) increases  $\beta$  (more false negatives), holding sample size fixed.

# The $t$ -Test

**One-sample  $t$ -test:** testing  $H_0 : \mu = \mu_0$ . Under Normality of  $X_i$ :

$$t = \frac{\bar{X}_n - \mu_0}{S/\sqrt{n}} \sim t_{n-1} \quad \text{under } H_0$$

**Two-sided** ( $H_1 : \mu \neq \mu_0$ ): reject if  $|t| > t_{n-1, \alpha/2}$ .    **One-sided** ( $H_1 : \mu > \mu_0$ ): reject if  $t > t_{n-1, \alpha}$ .

Exact under Normality; approximately valid for large  $n$  by the CLT. This is the workhorse test in regression analysis (Lecture 4).

# *p*-Values

## ***p*-Value**

The ***p*-value** is the probability, under  $H_0$ , of observing a test statistic at least as extreme as the one computed from the data:

$$p = P(|T| \geq |t_{\text{obs}}| \mid H_0)$$

**Decision rule:** reject  $H_0$  at level  $\alpha$  if  $p < \alpha$ .

**Common misinterpretations (avoid these):**

- “The *p*-value is the probability that  $H_0$  is true”    **Wrong**
- “ $p = 0.03$  means a 3% chance the result is due to chance”    **Wrong**
- “ $p > 0.05$  means  $H_0$  is true”    **Wrong**

**Correct:** the *p*-value measures the *compatibility* of the data with  $H_0$ .

# From Statistics to Econometrics

**Everything we reviewed today underpins regression analysis:**

Statistical concept	Econometric application
Conditional expectation $\mathbb{E}[Y X]$	The regression function
Law of iterated expectations	Proving OLS unbiasedness
Variance decomposition	$R^2$ and model fit
Unbiasedness, efficiency	Gauss–Markov theorem
Consistency, LLN	Large-sample OLS properties
CLT	Asymptotic inference
$t$ -test and $F$ -test	Coefficient testing
Confidence intervals	Inference on $\beta$

Solid command of these foundations makes econometrics much more intuitive.

# Next Time: The Simple Regression Model

In Lecture 2, we introduce the model:

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

## What we will cover:

- Derivation of the OLS estimator via minimizing  $\sum_{i=1}^n (Y_i - \hat{Y}_i)^2$
- Interpretation of  $\beta_0$  and  $\beta_1$
- Assumptions for unbiasedness:  $\mathbb{E}[u | X] = 0$
- Properties: unbiasedness and the Gauss–Markov theorem
- First R exercise: estimating the returns to education

**Reading:** Wooldridge (2019, Chapters 1–2)

# Key Takeaways

- ① **Econometrics** = statistical methods for causal questions with observational data
- ② The **conditional expectation**  $\mathbb{E}[Y | X]$  is the regression function – the best predictor of  $Y$  given  $X$
- ③ The **LIE** and **law of total variance** are workhorses for deriving estimator properties
- ④ **Estimators are random variables** – inference is about their sampling distributions
- ⑤ The **LLN** justifies consistency; the **CLT** justifies asymptotic inference
- ⑥ **Confidence intervals** and  **$p$ -values** have precise interpretations – beware of common mistakes

## References I

- Angrist, J. D. and Pischke, J.-S. (2009). *Mostly Harmless Econometrics: An Empiricist's Companion*. Princeton University Press, Princeton, NJ.
- Hansen, B. E. (2022). *Econometrics*. Princeton University Press, Princeton, NJ.
- Mincer, J. (1974). *Schooling, Experience, and Earnings*. Columbia University Press, New York.
- Stock, J. H. and Watson, M. W. (2020). *Introduction to Econometrics*. Pearson, New York, 4th edition.
- Wooldridge, J. M. (2019). *Introductory Econometrics: A Modern Approach*. Cengage Learning, Boston, MA, 7th edition.