The course of error due to distortion of input signal to the atan2 function

Mitja Alič

Faculty of Electrical Enginering, University of Ljubljana, Tržaška cesta 25, 1000 Ljubljana
E-pošta: mitja1357@gmail.com

Distortion of sine and cosine values, used for angle determination with the atan2 function, can result in numerical error. According to the performed review of literature, error is normally presented by taking only the basic harmonic into account. This paper however presents determination of error by taking into account also higher harmonics, which are non-negligible at larger distortion of sine or cosine. Error is going to be expressed with infinite series, which expand the domain of distortion parameter.

1 Introduction

High efficient regulation of motor drives this days is presented in many applications. For quality regulated motor drives, position sensors are used[1]. Angular position is measured by incremental based sensors, resolvers, encoders [2][3][4]... Outputs from resolvers or encoders are signals of sine and cosine form. For angle needs to be calculated inverse of tangent. In applications is normally used function atan2, which returns value in range $[-\pi, \pi]$ [9].

Position sensors are not ideal. Signals from resolvers are not ideal form of sine and cosine. Nevertheless function atan2 calculate angle. Output of atan2 includes error, because of distorted sine and cosine signals. Literature [5], [6], [7] analyzed the effect of distorted sine and cosine signals evaluate to angle error. Error was described by the harmonic with highest amplitude. Nevertheless error includes higher harmonics too. In this paper angle error is presented by infinite series. Many applications have accessible angular position only. By reference encoder, error can be detected, reason for it stays unknown.

By knowing reasons, how distorted inputs signals of atan2 function impact to error, eventually from error cause can be recognized. This paper presents impact of distorted input signals of atan2 to error presented in infinite series.

2 Methodology and results

Base input signals to atan2 have form of sine and cosine. They can be distorted by offsets, different amplitude, or phase shift(1)(2)

$$Sin = B_0 + B_1 \sin(\theta + \varphi_s) \tag{1}$$
$$Cos = A_0 + A_1 \cos(\theta + \varphi_c) \tag{2}$$

Ideal case is when offsets (B_0 and A_0) and phase shifts φ_0 and φ_0 are equal to zero, amplitude are same ($B_1 = A_1$). Signals can be distorted in amplitude and phase by only one parameter (Δ_c or Δ_s) (3)(4)

$$Sin = \sin(\theta) + \Delta_c \cos(\theta) + \Delta_s \sin(\theta) \tag{3}$$
$$Cos = \cos(\theta) + \Delta_c \cos(\theta) + \Delta_s \sin(\theta) \tag{4}$$

Definition of outupt signal φ and error ε is:

$$\varphi = \mathrm{atan2d}(Sin, Cos) \tag{5}$$
$$\varepsilon = \varphi - \mathrm{atan2d}(\sin(\theta), \cos(\theta)) \tag{6}$$

where θ presents reference angle. In MATLAB is defined function atan2d(), for purpose of calculation of invert tangent in four quadrant plane. Output is in range of $[-180°, 180°]$[10]. Error is presented by infinite series (7).

$$\varepsilon(x) = C_0(x) + \sum_{n=1}^{\infty} C_n(x) \sin(n\Theta + \varphi_n(x)) \tag{7}$$

Parameter x presents independent variable, which distort input signals (1) and (2). C_0 presents offset of error, C_n amplitude of individual harmonic and φ_n presents phase of individual harmonic of error. All functions depend on x.

Here is presented approach to determinate error. One parameter (A_0, B_0, A_1, B_1, φ_s, φ_c, Δ_s, Δ_c), used in calculation separately has been limited to infinity. In limit error is expressed as example:

$$\varepsilon = \begin{cases} 90° - \theta, & \theta \in \{0°, 180°\} \\ 270° - \theta, & \theta \in \{180°, 360°\} \end{cases} \tag{8}$$

Error is transformed to Fourier series. From transform can be decided which harmonics are changing and to where they converge. Next step is finding analytic function, that describes the course of the harmonics amplitude and the phase shift of an individual harmonic when the parameter changes. Knowing the convergence of function is helpful. This study has shown that higher harmonies are potency depend on the basic harmonic of the error. The error is expressed as a potency series.

The following are the principles of error determination at different amplitudes, phase displacements, offsets and a combination of different amplitudes and phases due to one parameter.

2.1 Defining of error at different amplitudes

If both input signals are multiplied by same coefficient, output signal will not be changed. Multiply both signals with $\frac{1}{A_1}$ and ratio $\frac{B_1}{A_1}$ define as k. Setting offsets and phases to zero, the input signals are defined as:

$$Sin = k\sin(\theta) \tag{9}$$
$$Cos = \cos(\theta). \tag{10}$$

Limiting k to infinity and calculate of error get result (Figure 1),

$$\lim_{k\to\infty}(\operatorname{atan2}(k\sin\theta,\cos\theta) - \operatorname{atan2}(\sin\theta,\cos\theta)) \tag{11}$$

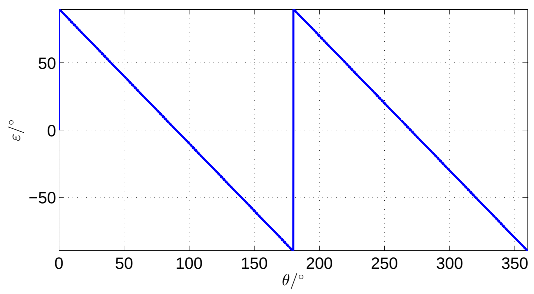

Figure 1: Error ε of limiting k to infinity

that can be transformed to Fourier series:

$$\varepsilon = \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin 2n\theta. \tag{12}$$

By calculating Fourier series of error we get only even harmonics, of which the second harmonics is the largest. Because of number 2 in argument of sine in (12), C_1 presents function of amplitude for second harmonic.

Using Curve Fitting Toolbox, best fit is rational function. Error can be expressed as:

$$\varepsilon(k) = \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{k-1}{k+1}\right)^n\sin 2n\theta \tag{13}$$

Expression is valid for k bigger than 0.

$$k \geq 0$$

V (13) namesto k vstavimo razmerje amplitud:

$$\varepsilon(k) = \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{B_1-A_1}{B_1+A_1}\right)^n\sin 2n\theta \tag{14}$$

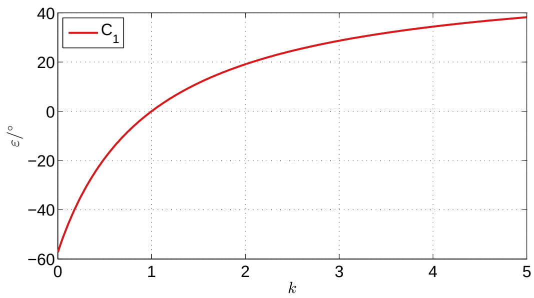

Figure 2: Potek drugega harmonika v odvistnosti od k

kar velja pri pogoju:

$$\frac{B_1}{A_1} \geq 0.$$

2.2 Defining of error at non-orthogonality

Vhodna signala imata naslednjo obliko:

$$Sin = \sin(\theta + \varphi_s) \tag{15}$$
$$Cos = \cos(\theta + \varphi_c) \tag{16}$$

Napako se določi posamično za vsakega od parametrov. Drugi je takrat enak 0. Na koncu se enačbi združi. Za določanje limite ni potrebno iti proti neskončnosti, ampak le do najslabše možnosti, ki je pri $\pm 90°$:

$$\varepsilon = \lim_{\varphi_s\to 90°}\operatorname{atan2}(Sin, Cos) - \operatorname{atan2d}(\sin(\theta),\cos(\theta)) \tag{17}$$

Potek napake ε s slike 3 predstavi vrsta (18).

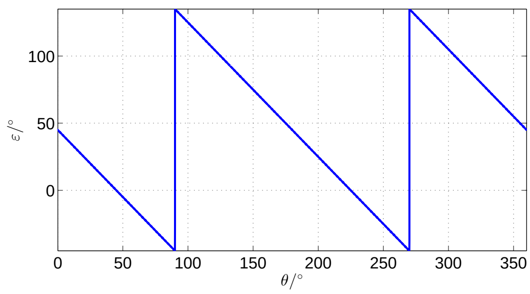

Figure 3: Napaka ε ob limiti $\varphi_s \to 90°$

$$\varepsilon = 45° - \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(2n\theta) \tag{18}$$

Iz izraza je vidno nastopanje enosmerne komponente in sodih harmonikov. Na sliki 4 so prikazani: potek enosmerne komponente C_0, potek amplitude drugega harmonika C_1, in φ_1 fazni zamik drugega harmonika glede na kosinusno obliko. Za predstavitev poteka s sinusi je potrebno prišteti še $90°$. Ordinatna skala grafa na sliki 4 je v stopinjah. Pri C_0 in C_1 stopinje predstavljajo amplitudo napake, pri φ_1 skala prikazuje fazni zamik v stopinjah. Poteku enosmerne komponente se najbolje prilega funkcija premice. Potek amplitude drugega harmonika napake ima

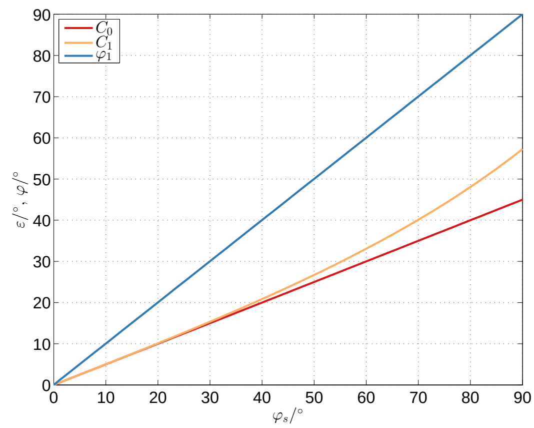

Figure 4: Potek enosmerne komponente napake C_0, amplitude drugega harmonika C_1 in faznega zamika φ_1 glede na kosinus, v odvisnosti od faznega zamika sinusa φ_s

obliko funkcije tangens. Fazni zamik drugega harmonika narašča linearno, vendar je za predstavitev s sinusi potrebno prišteti še 90°. Enako se lahko ponovi za fazni zamik kosinus signala. Enačba končnega poteka se glasi:

$$\varepsilon(\varphi_s, \varphi_c) = \frac{\varphi_s + \varphi_c}{2} + \frac{180}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} (\tan \frac{\varphi_s - \varphi_c}{2})^n \sin(2n\theta + n(90° + \varphi_s + \varphi_c)) \tag{19}$$

pri čemer, (19) velja za :

$$\varphi_s - \varphi_c \in [-90°, 90°]$$

2.3 Defining of error at offsets

Amplitudi signalov sinus in kosinus sta enaki 1. Z limito A_0 in razvojem napake v Fourierovo vrsto napako izrazimo z:

$$\varepsilon = \frac{180}{\pi} \sum_{n=1}^{\infty} \frac{2}{n} \sin(n\theta + 90°n). \tag{20}$$

Enosmerne komponente ni, največji je prvi harmonik. Potek

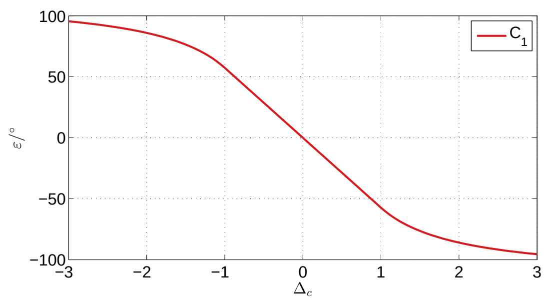

Figure 5: Potek amplitude prvega harmonika od enosmerne komponente kosinus signala, pri amplitudah enakih 1

s slike 5 se razdeli na 3 dele in aproksimira napako z naslednjim izrazom, upoštevajoč tudi enakosti amplitud:

$$\varepsilon(A_0) = \begin{cases} \frac{180}{\pi} \sum_{n=1}^{\infty} (-1)^n \frac{1}{n} (2 - |\frac{A_0}{A_1}|^{-n}) \sin(n\theta), & \frac{A_0}{A_1} \leq -1 \\ \frac{180}{\pi} \sum_{n=1}^{\infty} (-1)^n \frac{1}{n} (\frac{A_0}{A_1})^n \sin(n\theta), & |\frac{A_0}{A_1}| \leq 1 \\ \frac{180}{\pi} \sum_{n=1}^{\infty} (-1)^n \frac{1}{n} (2 - (\frac{A_0}{A_1})^{-n}) \sin(n\theta), & \frac{A_0}{A_1} \geq 1 \end{cases} \tag{21}$$

Enako se lahko stori tudi za B_0 (22) in za aproksimacijo napake, ko imata sinus in kosinus enako enosmerno komponento (23).

$$\varepsilon(B_0) = \begin{cases} \frac{180}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} (2 - |\frac{B_0}{B_1}|^{-n}) \sin(n\theta - 90°n), & \frac{B_0}{B_1} \leq -1 \\ \frac{180}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} (\frac{B_0}{B_1})^n \sin(n\theta + 90°n), & |\frac{B_0}{B_1}| \leq 1 \\ \frac{180}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} (2 - (\frac{B_0}{B_1})^{-n}) \sin(n\theta° + 90n), & \frac{B_0}{B_1} \geq 1 \end{cases} \tag{22}$$

$$\varepsilon(A_0, B_0 = A_0) = \begin{cases} \frac{180}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} (2 - |\sqrt{2}\frac{A_0}{A_1}|^{-n}) \sin(n\theta + 90°n), & \frac{A_0}{A_1} \leq -\frac{\sqrt{2}}{2} \\ \frac{180}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} (\sqrt{2}\frac{A_0}{A_1})^n \sin(n\theta - 90°n), & |\frac{A_0}{A_1}| \leq \frac{\sqrt{2}}{2} \\ \frac{180}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} (2 - (\sqrt{2}\frac{A_0}{A_1})^{-n}) \sin(n\theta - 90°n), & \frac{A_0}{A_1} \geq \frac{\sqrt{2}}{2} \end{cases} \tag{23}$$

2.4 Impact of different amplitude and phase due to one parameter

Fourierova vrsta limite (6) pri upoštevanju (3) in (4), ko gre Δ_c v neskončnost, je

$$\varepsilon = 45° - \frac{180}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin(2n\theta). \tag{24}$$

Potek drugega harmonika od Δ_c se lahko zapiše kot vsota sinusa in kosinusa.

$$C_{1s} \cdot \sin(2\theta) + C_{1c} \cdot \cos(2\theta) \tag{25}$$

Poteki enosmerne komponente ter drugega harmonika, predstavljenega kot seštevek sinusa in kosinusa, so prikazani na sliki 6.

Enosmerna komponenta ima obliko racionalne funkcije v funkciji atan. C_{1s} in C_{1c} sta racionalni funkciji. Amplituda je geometrijska vsota členov $C_n = \sqrt{C_{ns}^2 + C_{nc}^2}$, faza za sinusni potek napake je $\varphi_n = atan(\frac{C_{nc}}{C_{ns}})$. Končna enačba z upoštevanjem amplitude osnovnega signala sinus in kosinus se glasi:

$$\varepsilon(\Delta_c) = \text{atand}\frac{\Delta_c}{\Delta_c + 2A_1} + \frac{180}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} (\frac{\Delta_c}{\sqrt{\Delta_c^2 + 2A_1\Delta_c + 2A_1}})^n \sin(2n\theta + n(90° + \text{atan}(\frac{\Delta_c + A_1}{A_1}))) \tag{26}$$

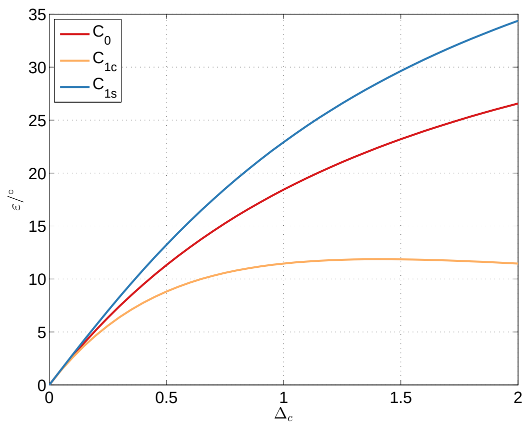

Figure 6: Potek enosmerne komponente in drugega harmonika napake, v odvisnosti od Δ_c

$$\varepsilon(\Delta_s) =$$

$$\operatorname{atand}\frac{-\Delta_s}{\Delta_s + 2A_1} + \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{\Delta_s}{\sqrt{\Delta_s^2 + 2A_1\Delta_s + 2A_1}}\right)^n$$

$$\sin(2n\theta + n(90^\circ + \operatorname{atan}(\frac{\Delta_s + A_1}{A_1}))) \quad (27)$$

$$\Delta_s, \Delta_c > -A_1$$

3 Comment on results

Pri modeliranju potekov je bilo uporabljeno prvih 15 členov potenčne vrste. Razlika med dejansko napako in predvideno napako je le numerična (slika 7). Opravil sem FFT predvidene in dejanske napake. Razlika med posameznima amplitudama harmonika je le numerična. V primeru, da parameter dosega večje vrednosti, ali se približuje robnim pogojem, dejanska napaka postane nezvezna in je s 15 členi ni mogoče povsem določiti. Omeniti je potrebno, da so kljub izpeljavam, predstavljene vrste napake posameznih deformacij med seboj odvisne.

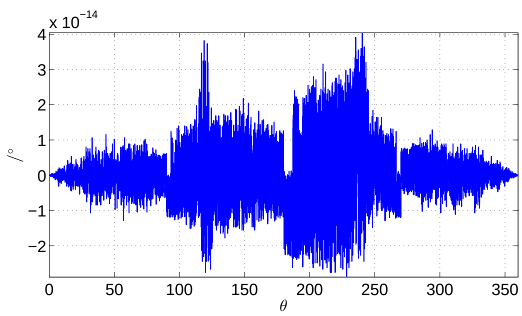

Figure 7: Razlika med predvideno (13) in dejansko napako pri $k = 1.1$

4 Conclusion

V članku so predstavljeni poteki napake v odvisnosti od različnih amplitud, enosmernih komponent, faznih zamikov in kombinacije različnih amplitud ter faznih zamikov signalov sinusa in kosinusa. Napaka vsebuje tudi višje harmonike, ki ob večjih razlikah med sinusom in kosinusom niso več zanemarljivi. Za majhna popačenja vhodnih signalov zadostuje napako izraziti z enosmerno komponento in prvim členom potenčne vrste. Napako, izraženo z enosmerno komponento in prvim členom potenčne vrste, je potrdila tudi druga literatura [5]. Izraze se lahko uporabi za ugotovitev nepravilne montaže resolverja ali enkoderja, v vgrajenih aplikacijah, kjer ni dostopa do signalov sinus in kosinus. V signalih sinus in kosinus se lahko pojavijo tudi višji harmoniki, ki prav tako vplivajo na napako. Vpliv popačenj vhodnih signalov v funkcijo atan2 na izhodno napako ponuja še veliko izzivov za nadaljne delo.

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