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Error Analysis and Compensation of the Orthogonal Magnetic Encoder

Qi Lin and Tiecai Li
department of electrical engineering
harbin institute of technology
Harbin, Heilongjiang Province, China
linqi_0813@163.com&litiecai@hit.edu.cn

Zhaoyong Zhou
department of electrical engineering
harbin institute of technology
Harbin, Heilongjiang Province, China
zhouzy@hit.edu.cn

***Abstract*—Aimed at the FPGA-based orthogonal magnetic encoder design, this paper analyzes several actual errors caused by the limitation of the processing technology and the assembly technology, which include the error caused by the imbalance of signal amplitude, the error caused by the nonorthogonality of signal phase, the error caused by the DC offset, and the error caused by harmonics. Based on ideal voltage signals, the mathematical expression of these errors cause by non-ideal characteristics and the corresponding compensation measure are derived, and through simulation and experimental, the expression and measure are verified to be right and effective.**

Keywords-magnetic encoder; error; compensation

I. INTRODUCTION

As an important component of the position servo control system, the position sensor has very broad applications. Position sensors currently used mainly include the optical encoder, the rotating transformer and the inductosyn and so on.

The transformer and inductosyn are very large. The optical encoder is mature, but expensive, it has a poor aseismatic performance and its structural principle makes it impossible to operate in the water, dust and other poor working conditions. With increasing demands of the position detection in all areas, traditional position sensors can no longer meet special requirements of some applications because of their shortcomings.

Compared to other position sensors, magnetic decoder is based on the magneto-resistance effect to transfer rotary physical quantities such as speed and angle of rotation to electrical signals, it has a faster responding speed because of better high-frequency characteristic of magneto resistance, and its contactless sensing can be perfectly applied in bad conditions such as oil, dust, greatly-variable temperature, etc. Moreover, the structure of magnetic encoder is very simple and able to endure great impacts and shocks; it is easy to be miniaturized [1] [2].

Therefore, the magnetic encoder is very suitable to be applied in systems which need high reliability and stability, nowadays it is widely applied in servo-actuator control occasions. However, in the actual production process of magnetic encoder, because of the limitation of the processing technology and the assembly technology, the air gap magnetic field and locations of Hall components are non-ideal, which will impact the calculation precision of the angular position. In this paper, a variety of error sources and their influences are analyzed so as to find the corresponding compensation measures.

II. PRINCIPLE OF ORTHOGONAL MAGNETIC ENCODER

The structure of orthogonal magnetic encoder is shown in Fig.1, when the ring magnet rotates with the motor, Hall sensors will produce two cosine voltages with a phase difference of π / 2, after the signal conditioning which can be expressed as in:

$$V_a = V_0 + V \sin\theta \tag{1}$$

$$V_b = V_0 + V \cos\theta \tag{2}$$

Where θ is the position of the encoder magnetic field, between which and the physical position of the magnetic field of the rotor of the motor there is a constant installation offset, which can be ascertained experimentally[3] [4].

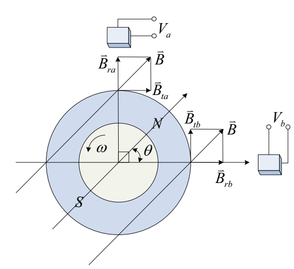

Fig.1 Structure of the orthogonal magnetic encoder

On account of the relationship: $e^{j\theta}=\cos\theta+j\sin\theta$, the two output voltages of magnetic encoder can make up a rotating vector after removing their DC components, as in (3).

$$\vec{V}_{\text{in}} = Ve^{j\theta} = V\cos\theta + jV\sin\theta \tag{3}$$

To set the tracking angle as φ, then the corresponding tracking vector is as in (4).

$$\vec{V}_{\text{f}} = Ve^{j\varphi} = V\cos\varphi + jV\sin\varphi \tag{4}$$

According to the law of vector operation, the vector $\overline{V_{in}}$ after a rotation by the angle of $-\varphi$ can be represented as in (5).

$$\begin{aligned}\vec{V} &= \vec{V}_{\text{in}} \cdot e^{-j\varphi} = Ve^{j\theta} \cdot e^{-j\varphi} = Ve^{j(\theta-\varphi)} \\ &= V\cos(\theta-\varphi) + jV\sin(\theta-\varphi)\end{aligned} \tag{5}$$

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The relationship between $\overline{V_{in}}$ and $\overline{V_f}$ can be illustrated by Fig.2, where $\overline{V}$ is the deviation vector, the phase angle of which is the phase angle difference between $\overline{V_{in}}$ and $\overline{V_f}$.

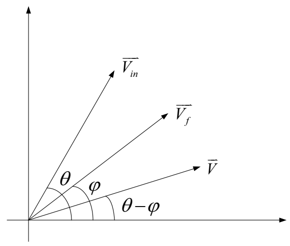

Fig.2 Vector Relation between the Input and the Tracker

From Fig.2 it is apparent that $\overline{V_f}$ can trace $\overline{V_{in}}$ completely by controlling $\varphi\rightarrow\theta$, and the direct-axis component of the deviation vector $\overline{V}$ is a sine function $V\sin(\theta-\varphi)$. In real system, if enough initialized time is allocated, the system can enter the following state before its normal operation, the later deviation becomes much smaller, and then there is the relationship as in (6).

$$\sin(\theta-\varphi)\approx\theta-\varphi \tag{6}$$

Under the approximate condition of (6), after the binary scaling, the discrete transfer function model of the tracking system can be derived, it is an elementary Ⅱ-type system, its digital decoding circuit is designed in the FPGA.

III. Error Analysis of the Magnetic Encoder

As the actual processing and assembly technology constraints, the orthogonal magnetic encoder has a variety of errors, which will affect the measurement accuracy. The first kind is the error caused by the processing of the ring magnet, including not a true round shape, uniform thickness, magnetizing equipment is not ideal and so on, these would give rise to a non-sinusoidal magnetic field, leading to greater harmonic. The second kind is the error caused by Hall sensors of inherent nonlinearity and DC voltage offset. The third kind is the assembly error, including bias magnet and two Hall sensors of not equal distance from the magnet or of nonorthogonal space locations, and so on. In addition, the temperature drift of signal conditioning circuit and power disturbances will cause errors also. These non-ideal technical characteristics will impact the sine and cosine voltage output signals in the following kinds of error: the amplitude imbalance, the nonorthogonal phase, the DC offset, and harmonics.

A. *Error caused by the amplitude imbalance*

Suppose the amplitude imbalance degree between two voltages $\overline{V_a}$ and $\overline{V_b}$ is α (-1 <α <1), then the input vector $\overline{V_{in}}$ changes to:

$$\vec{V}_{\text{in}}=(1+\alpha)V\cos\theta+jV\sin\theta=Ve^{j\theta}+\alpha V\cos\theta \tag{8}$$

The corresponding deviation vector is:

$$\vec{V}=\vec{V}_{\text{in}}\cdot e^{-j\varphi}=Ve^{j(\theta-\varphi)}+\alpha V\cos\theta\cdot e^{-j\varphi} \tag{9}$$

Its imaginary part is given to the Ⅱ-type tracking system as the deviation, that is:

$$E=V[\sin(\theta-\varphi)-\alpha\cos\theta\sin\varphi] \tag{10}$$

when $E\rightarrow0$, θ will not equal to φ, we will get a deviation $\varepsilon=\theta-\varphi$, which can be expressed as:

$$\varepsilon=\sin^{-1}\left[\alpha\sin(\theta+\phi)/\alpha+2\right] \tag{11}$$

In general α is small, so ε is small, therefore $\sin\varepsilon\approx\varepsilon$, $\theta+\varphi\approx2\theta$, then there is:

$$\varepsilon\approx\alpha\sin2\theta/2 \tag{12}$$

The equation (12) is the conversion error caused by the amplitude imbalance, which is proportional to the imbalance degree, and change with the angular position according to the second sine rule change. Fig.3 is the simulation results of the digital decoder in the FPGA when α=1/32. Both the angular position (the sawtooth wave) and the angular error (the sine wave) are binary data read from the register in FPGA, and the digital quantities of ± 32768 correspond to the actual angle of ± π, which are the same in the subsequent analyses and figures. After converting the digitized angle error into the radian, the results and the equation (12) match very well.

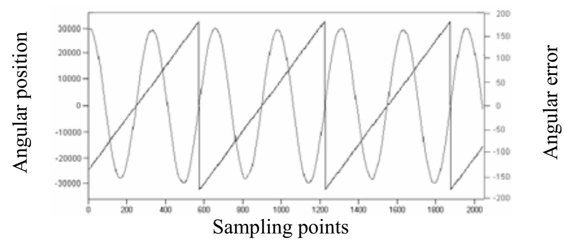

Fig.3 Error caused by the amplitude imbalance

B. *Error caused by the nonorthogonal phase*

Suppose the phase difference of two voltage signals is β, and then the input vector changes to:

$$\vec{V}_{\text{in}}=V\cos(\theta+\beta)+jV\sin\theta \tag{13}$$

The corresponding deviation vector is:

$$\begin{aligned}\vec{V}&=\vec{V}_{\text{in}}\cdot e^{-j\phi}\\&=V[\cos(\theta+\beta)\cos\phi+\sin\theta\sin\phi]\\&+jV[\sin\theta\cos\phi-\cos(\theta+\beta)\sin\phi]\end{aligned} \tag{14}$$

Its imaginary part is given to the Ⅱ-type tracking system as the deviation, that is:

$$E=V[\sin\theta\cos\varphi-\cos(\theta+\beta)\sin\varphi] \tag{15}$$

When $E\rightarrow0$, θ will not equal to φ. If E=0, then there is:

$$\sin\theta\cos\varphi-\cos(\theta+\beta)\sin\varphi=0 \tag{16}$$

Since β can be adjusted to be very small, so $\cos\beta \approx 1$, $\sin\beta \approx \beta$, therefore the equation (16) can be derived into:

$$\sin(\theta-\varphi)+\beta\sin\theta\sin\varphi=0 \tag{17}$$

And $\varepsilon = \theta - \varphi$, so there is:

$$\sin\varepsilon \approx -\beta\sin\theta\sin\varphi \tag{18}$$

For a small β, $\sin\theta \approx \sin\varphi$, so the conversion error caused by the nonorthogonal phase can be expressed as:

$$\varepsilon \approx \sin\varepsilon \approx -\beta\sin^2\theta = -\beta(1-\cos 2\theta)/2 \tag{19}$$

Fig.4 is the simulation results when β=1/32 rad, after scaling which is consistent with the equation (19).

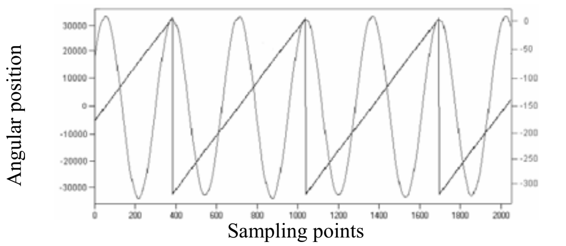

Fig.4 Error caused by the nonorthogonal phase

C. Error caused by the DC offset

Assuming the DC offset degree of the cosine voltage signal is δ, and then the input vector can be expressed as:

$$\vec{V}_{\text{in}} = V(\delta+\cos\theta)+jV\sin\theta = V\delta + Ve^{j\theta} \tag{20}$$

The corresponding deviation vector is:

$$\begin{aligned}\vec{V} &= \vec{V}_{\text{in}}\cdot e^{-j\varphi} = V\delta e^{-j\varphi}+Ve^{j(\theta-\varphi)} \\ &= V[\delta\cos\varphi+\cos(\theta-\varphi)]+jV[-\delta\sin\varphi+\sin(\theta-\varphi)]\end{aligned} \tag{21}$$

Its imaginary part is given to the Ⅱ-type tracking system as the deviation, that is:

$$E = V[-\delta\sin\varphi+\sin(\theta-\varphi)] \tag{22}$$

When $E\rightarrow 0$, θ will not equal to φ. If E=0, then there is:

$$-\delta\sin\varphi+\sin(\theta-\varphi)=0 \tag{23}$$

Considering $\varepsilon = \theta - \varphi$, so there is:

$$\sin\varepsilon = \delta\sin\varphi \tag{24}$$

Since δ can be adjusted to be very small, so ε is small, then the equation (24) can approximate to:

$$\sin\varepsilon \approx \varepsilon \tag{25}$$

$$\sin\varphi = \sin(\theta-\varepsilon) \approx \sin\theta \tag{26}$$

Therefore, the error caused by DC offset is:

$$\varepsilon \approx \delta\sin\theta \tag{27}$$

Fig.5 is the simulation results when δ=1/32, after scaling which is consistent with the equation (27).

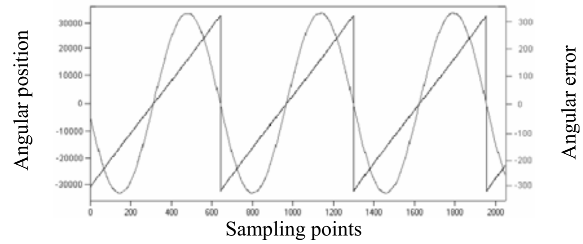

Fig.5 Error caused by the DC offset

D. Error caused by the harmonic

When voltage signals contain harmonics, the input vector can be expressed in the form of a Fourier series, that is:

$$\begin{aligned}\vec{V}_{\text{in}} &= V\left(K_0+\sum_{n=1}^{\infty}K_n\cos(n\theta)\right)+jV\left(K_0+\sum_{n=1}^{\infty}K_n\sin(n\theta)\right) \\ &= V\left(\sqrt{2}K_0e^{j\frac{\pi}{4}}+\sum_{n=1}^{\infty}K_ne^{jn\theta}\right)\end{aligned} \tag{28}$$

Where the fundamental factor K_1 equals to 1, other harmonic coefficients K_0, K_2, K_3, ..., K_n is much smaller than 1, therefore the corresponding deviation vector is:

$$\vec{V} = \vec{V}_{\text{in}}\cdot e^{-j\varphi} = V\left(\sqrt{2}K_0e^{j(\frac{\pi}{4}-\varphi)}+\sum_{n=1}^{\infty}K_ne^{j(n\theta-\varphi)}\right) \tag{29}$$

Its imaginary part is given to the Ⅱ-type tracking system as the deviation, that is:

$$E = V[\sin(\theta-\varphi)-\alpha\cos\theta\sin\varphi] \tag{30}$$

Considering $\varepsilon = \theta - \varphi$, so there is:

$$n\theta-\varphi = (n-1)\theta+(\theta-\varphi) = (n-1)\theta+\varepsilon \tag{31}$$

When $E\rightarrow 0$, that is:

$$0=\sqrt{2}K_0\cos(\varphi+\frac{\pi}{4})+\sum_{n=1}^{\infty}K_n\{\sin[(n-1)\theta]\cos\varepsilon+\cos[(n-1)\theta]\sin\varepsilon\} \tag{32}$$

When n=0 or n>1, $K_n \ll 1$, so the value of ε is very small, therefore it can be approximated that $\cos\varepsilon \approx 1$, $\sin\varepsilon \approx \varepsilon$, $\cos(\varphi+\pi/4) \approx \cos(\theta+\pi/4)$, thus there is:

$$0\approx\sqrt{2}K_0\cos(\theta+\frac{\pi}{4})+\sum_{n=1}^{\infty}K_n\sin[(n-1)\theta]+\varepsilon\sum_{n=1}^{\infty}K_n\cos[(n-1)\theta] \tag{33}$$

According to the last item of equation (33), there is:

$$\varepsilon\sum_{n=1}^{\infty}K_n\cos[(n-1)\theta]=\varepsilon+\varepsilon\sum_{n=2}^{\infty}K_n\cos[(n-1)\theta] \tag{34}$$

Since ε and K_n (n> 1) are very small, so the second item of the equation (34) can be ignored, thus there is:

$$0\approx\sqrt{2}K_0\cos(\theta+\pi/4)+\sum_{n=2}^{\infty}K_n\sin[(n-1)\theta]+\varepsilon \tag{35}$$

Therefore the error caused by the harmonic can be expressed as:

$$\varepsilon \approx -\sqrt{2}K_0 \cos(\theta + \pi/4) - \sum_{n=2}^{\infty} K_n \sin[(n-1)\theta] \qquad (36)$$

Fig.6 and Fig.7 are simulation results respectively when K_0=1/32, K_2=1/32, K_3=1/32, K_4=1/32, K_5=1/32, which are all consistent with the equation (36) after scaling.Fig.13 is the simulation result when K_5 =1/32 and β=1/32 rad, the total error after scaling basically equals to the sum of the equation (19) and the equation (36).

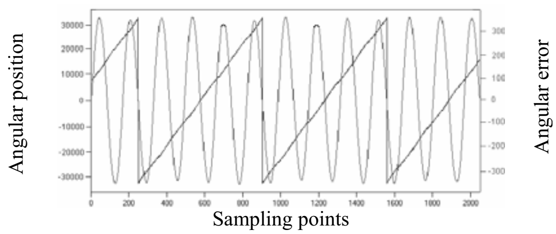

Fig.6 Error caused by the fifth harmonic

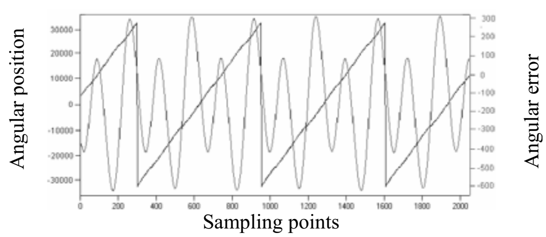

Fig.7 Error caused by the fifth harmonic and the nonorthogonal phase

IV. COMPENSATION OF THE ERROR

The five kinds of error discussed above may exist at the same time in the actual system, but the proportion of each kind of error is not the same, which is determined by the specific process [5]. To get higher conversion precision, in addition to improve the process and assembly technology, the online compensation method can be adopted. Equations (12), (19), (27) and (36) give expressions of various errors, and also provide a method of error compensation. Its basic idea is to first use the oscilloscope or a spectrum analyzer and other precision instruments to capture output voltage waveforms of Hall sensors in real time, and quantitatively detect un-ideal factors of the amplitude imbalance degree, the nonorthogonal phase radian, the DC offset and the harmonic distribution, then according to corresponding error expressions, their conversion errors can be calculated, and make the total error into a ROM table, thus the on-line compensation can be operated by looking the table.

To verify the effectiveness of the above compensation method, the PWM signal mode is adopted, in which the pulse width is proportional to the measured angle, the digital angular position is: $d=(t_{\mathrm{on}}*4097/t_{\mathrm{on}}+t_{\mathrm{off}})-1$.

The frequency of the PWM signal is 840 Hz, the period of the PWM signal is 1190 ms, its minimum pulse width is 0.29 ms, the corresponding digital angular position is 0, the corresponding angle is 0 degree, and its maximum pulse width is 1189.71 ms, the corresponding digital angular position is 4095, the corresponding angle is 359.912 degree. Fig.8 shows the minimum pulse width waveform of the PWM signal, the interior regulation precision of the PWM frequency is ±5%, by measuring the complete work cycle it can be found that the error can be eliminated.

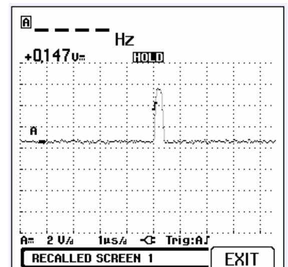

Fig.8 Mnimum pulse width waveform of the PWM signal

V. CONCLUSIONS

As an important part of the position servo control system, the position sensor is directly related to the control precision of the whole system. Therefore, in addition to use the A/D converter and FPGA of a high bit to enhance the resolution of magnetic encoder to improve the accuracy, this paper analyzes various error sources and their impacts, including the amplitude imbalance, the nonorthogonal phase, the DC offset, and harmonics. In this paper, math expressions of conversion error of angular position under various conditions are derived, therefore the amplitude and phase of these errors can be simulated in FPGA, based on the total value of these errors a ROM table is established, thus these errors can be on-line compensated by the method of looking up the table, the experimental result verifies that this method can eliminate errors.

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