The course of error due to distortion of input signal to the atan2 function

Mitja Alič

Faculty of Electrical Enginering, University of Ljubljana, Tržaška cesta 25, 1000 Ljubljana
E-pošta: mitja1357@gmail.com

Distortion of sine and cosine values, used for angle determination with the atan2 function, can result in numerical error. According to the performed review of literature, error is normally presented by taking only the basic harmonic into account. This paper however presents determination of error by taking into account also higher harmonics, which are non-negligible at larger distortion of sine or cosine. Error is going to be expressed with infinite series, which expand the domain of distortion parameter.

1 Introduction

These days the need for high quality motor regulation is present in numerous applications and has as a result become unavoidable. For a consistent and reliable measurement of rotation, position sensors are used [1], such as encoders and resolvers [2][3][4]. Because the output of such sensors is a pair of quadrature sine and cosine signals, angle must first be calculated. The easiest way of doing so is by directly calculating atan2, which returns a value between $[-\pi, \pi]$ [9].

Because position sensors aren't ideal, obtained sine and cosine signal can be deformed, phase shifted and DC offset. All of these imperfections cause the calculated angle to also include error.

Literature [5], [6], [7] and [8] analyses the impact of such imperfection for lower harmonics only and states that imperfections scale linearly. During our research it was found, that the frequency analysis also contains higher harmonics. The paper examines error waveform dependent on input signal mismatch with Fourier analysis.

2 Methodology and results

Output from a position sensor can be represented with

$$Sin = B_0 + B_1 \sin(\theta + \varphi_s) + CMM \quad (1)$$

$$Cos = A_0 + A_1 \cos(\theta + \varphi_c) + CMM \quad (2)$$

Where B_0 and A_0 represent DC offset, B_1 and A_1 signal amplitude, φ_s and φ_c phase shift and θ reference angle. Signals (1) and (2) can also have a common superimposed AC signal represented as CMM (3). CMM can be of cosine or sine form with Δ_c and Δ_s as amplitude.

$$CMM = \Delta_c \cos(\theta) + \Delta_s \sin(\theta) \quad (3)$$

By calculating atan2 for eq. (1) and (2)

$$\varphi = \operatorname{atan2}(Sin, Cos) \quad (4)$$

and then subtracting it with an unaltered signal

$$\varepsilon = \varphi - \operatorname{atan2}(\sin(\theta), \cos(\theta)) \quad (5)$$

we get error based on deformation. Because AC signal analysis is simpler in frequency domain we converted it with Fast Fourier Transform(FFT). By varying each parameter individually we examined the impact of the parameter in question on specter of error. Output of atan2 was examined by sending each parameter in eq. (1) and (2) to infinity or worst case with phase shift. In this case amplitude and phase of each harmonic approaches to a limit value.

The course of the amplitude and phase in dependence on the changing parameter was approximated by a function that best suited numerical waveform obtained from the error spectrum. By knowing the limit value to which each amplitude and phase approaches, the set of functions has decreased. We were searching for the best approximation with polynomials, rational functions, trigonometric and cyclometric functions. The best approximation was sought using the minimum squares method.

2.1 Defining of error at different amplitudes

In the first case we observed the dependence of error on the different amplitudes of the input signals. The output of the atan2 function is determined by the eq. (1) and (2) quotient. Since only the ratio of the amplitudes of the input signals needs to be preserved, we multiplied both signals by 1 / A1. Taking into account only the amplitudes in (1) and (2), inputs are:

$$Sin = k \sin(\theta), \quad (6)$$

$$Cos = \cos(\theta), \quad (7)$$

where k represents B1 / A1.

By varying the parameter k from 0 to infinity, we found that the error specter consists of even harmonics only. We also observed that phase shift didn't change. When k approaches infinity, the amplitude of harmonics approaches 180 / pi 2 / n, where n represents n-th harmonic.

S spreminjanjem parametra k od 0 do neskončno, opazovanjem napake in njegovega spektra je bilo ugotovljeno da napako ene periode sestavljajo le sodi harmoniki. Fazni zamik harmonikov je konstanten 90 degrees in je lahko predstavljen s sinusnim signalom. Amplituda posameznika harmonika se z višanjem parametra k približuje vrednosti 2/n za sode in 0 za lihe harmonike.

$$\varepsilon(k \to \infty) = \tfrac{180}{\pi} \textstyle\sum_{n=1}^{\infty} \tfrac{1}{n} \sin 2n\theta \quad (8)$$
$$\varepsilon(k \to 0) = -\tfrac{180}{\pi} \textstyle\sum_{n=1}^{\infty} \tfrac{1}{n} \sin 2n\theta \quad (9)$$

Naj izstopajočejši harmonik je drugi harmonik zato je bilo njemu namenjeno največ pozornosti. S poznanimi skrajnimi točkami in točko kjer je napaka nič, je potek harmonika z SSE 1,18e-10 opisala racionalna funkcija (10)

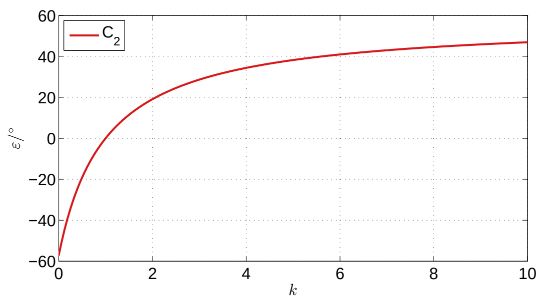

Figure 1: The course of the second harmonic depending on k

$$C_2(k) = \frac{180}{\pi} \cdot \frac{k-1}{k+1} \quad (10)$$

Predvideval sem da so višji harmoniki v korelaciji s najdeno funkcijo. Napako, ki se pojavi zaradi različnih amplitud sem zapisal v izraz Fourierove vrste (11)

$$\varepsilon(k) = \frac{180}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} (\frac{k-1}{k+1})^n \sin 2n\theta \quad (11)$$

Z vstavljanjem namesto k razmerje amplitud in množenju števca in imenovalca s A1 se končni izraz glasi:

$$\varepsilon(k) = \frac{180}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} (\frac{B_1 - A_1}{B_1 + A_1})^n \sin 2n\theta \quad (12)$$

Izraz za napako velja le ko sta A1 in B1 enakega predznaka oz je njuno razmerje večje ali enako 0.

$$\frac{B_1}{A_1} \geq 0.$$

2.2 Defining of error at non-orthogonality

Sledilo je spreminjanje parametra faz vhodnih signalov. Amplitudi se je nastavilo na 1, vhodna signala sta bila odvisna le od fs in fc(13)(14).

$$Sin = \sin(\theta + \varphi_s) \quad (13)$$
$$Cos = \cos(\theta + \varphi_c) \quad (14)$$

Error can be transform and presented in Fourier series.

$$\varepsilon(\varphi_s \to 90°) = 45° - \tfrac{180}{\pi} \textstyle\sum_{n=1}^{\infty} \tfrac{1}{n} \sin(2n\theta) \quad (15)$$
$$\varepsilon(\varphi_s \to -90°) = -45° - \tfrac{180}{\pi} \textstyle\sum_{n=1}^{\infty} \tfrac{1}{n} \sin(2n\theta) \quad (16)$$

Transform includes offset and even harmonics only. Figure 2 presents course of offset C_0, amplitude of second harmonic C_1 and phase of second harmonic φ_1 due to φ_s. y axis is in degrees. For C_0 and C_1 degrees presents amplitude of error harmonics, for φ_1 degrees presents phase. Offset is best fitted by linear function. Second harmonic is tangent function. Phase of second harmonic increases linear but for presentation with sine form must be added 90°. Same derivation can be done for phase of cosine signal. Equations can be marged and result is presented (17).

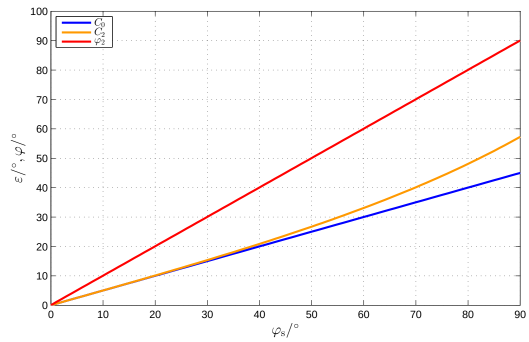

Figure 2: The course of offset component C_0, amplitude of second harmonic C_2 and phase φ_2 depend of ideal cosine signal,due to phase shift φ_s

$$\varepsilon(\varphi_s, \varphi_c) = \frac{\varphi_s + \varphi_c}{2} + \frac{180}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} (\tan \frac{\varphi_s - \varphi_c}{2})^n \sin(2n\theta + n(90° + \varphi_s + \varphi_c)) \quad (17)$$

Expression is valid only for:

$$\varphi_s - \varphi_c \in [-90°, 90°]$$

2.3 Defining of error at offsets

Define amplitudes of input signals to 1, phase shift is set to zero. Let limit parameter A_0 to infinity, error is transformed to Fourier series and expressed as:

$$\varepsilon(A_0 \to \infty) = \frac{180}{\pi} \sum_{n=1}^{\infty} \frac{2}{n} \sin(n\theta + 90° n). \quad (18)$$

Error does not include offset component, highest amplitude has first harmonic. The course from figure 3 it is

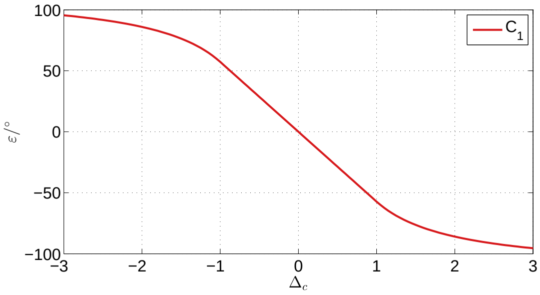

Figure 3: The course of amplitude of first harmonic due to offset A_0, where input signals have amplitude of 1

split to 3 parts and expression that best fit the curve is:

$$\varepsilon(A_0) = \begin{cases} \frac{180}{\pi}\sum_{n=1}^{\infty}(-1)^n\frac{1}{n}(2-|\frac{A_0}{A_1}|^{-n})\sin(n\theta), & \frac{A_0}{A_1}\leq -1 \\ \frac{180}{\pi}\sum_{n=1}^{\infty}(-1)^n\frac{1}{n}(\frac{A_0}{A_1})^n\sin(n\theta), & |\frac{A_0}{A_1}|\leq 1 \\ \frac{180}{\pi}\sum_{n=1}^{\infty}(-1)^n\frac{1}{n}(2-(\frac{A_0}{A_1})^{-n})\sin(n\theta), & \frac{A_0}{A_1}\geq 1 \end{cases} \quad (19)$$

Same derivation can be done for B_0 (20) and for fitting error, when sine and cosine include same offset ($A_0 = B_0$) (21).

$$\varepsilon(B_0) = \begin{cases} \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(2-|\frac{B_0}{B_1}|^{-n})\sin(n\theta-90°n), & \frac{B_0}{B_1}\leq -1 \\ \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(\frac{B_0}{B_1})^n\sin(n\theta+90°n), & |\frac{B_0}{B_1}|\leq 1 \\ \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(2-(\frac{B_0}{B_1})^{-n})\sin(n\theta°+90n), & \frac{B_0}{B_1}\geq 1 \end{cases} \quad (20)$$

$$\varepsilon(A_0, B_0 = A_0) = \begin{cases} \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(2-|\sqrt{2}\frac{A_0}{A_1}|^{-n})\sin(n\theta+90°n), & \frac{A_0}{A_1}\leq -\frac{\sqrt{2}}{2} \\ \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(\sqrt{2}\frac{A_0}{A_1})^n\sin(n\theta-90°n), & |\frac{A_0}{A_1}|\leq \frac{\sqrt{2}}{2} \\ \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(2-(\sqrt{2}\frac{A_0}{A_1})^{-n})\sin(n\theta-90°n), & \frac{A_0}{A_1}\geq \frac{\sqrt{2}}{2} \end{cases} \quad (21)$$

2.4 Impact of different amplitude and phase due to one parameter

Transform to Fourier series of limit (5) to infinity of Δ_c, where inputs are (**??**) and (**??**) is

$$\varepsilon = 45° - \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(2n\theta). \quad (22)$$

The course of second harmonic due to Δ_c, can be express as sum of sine and cosine signal. Each signal is presented in figure 4.

$$C_{1s}(\Delta_c)\cdot\sin(2\theta) + C_{1c}(\Delta_c)\cdot\cos(2\theta) \quad (23)$$

Offset of error is fitted to invert tangent function. C_{1s} and C_{1c} are best fitted to rational function. Total amplitude is geometrical summation $C_n = \sqrt{C_{ns}^2 + C_{nc}^2}$,

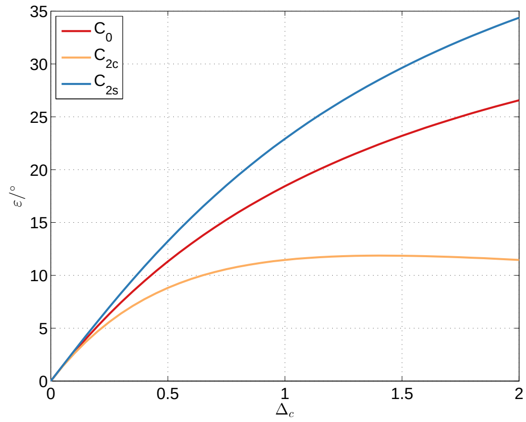

Figure 4: The course of offset and amplitude of second harmonic of error due to Δ_c

phase fitted to sine signal is calculated by $\varphi_n = atan(\frac{C_{nc}}{C_{ns}})$. Final equation due to Δ_c and $\Delta_s$is expressed in (24) and (25)

$$\varepsilon(\Delta_c) = \text{atand}\frac{\Delta_c}{\Delta_c + 2A_1} + \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{\Delta_c}{\sqrt{\Delta_c^2 + 2A_1\Delta_c + 2A_1}}\right)^n \sin(2n\theta + n(90° + \text{atan}(\frac{\Delta_c + A_1}{A_1}))) \quad (24)$$

$$\varepsilon(\Delta_s) = \text{atand}\frac{-\Delta_s}{\Delta_s + 2A_1} + \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{\Delta_s}{\sqrt{\Delta_s^2 + 2A_1\Delta_s + 2A_1}}\right)^n \sin(2n\theta + n(90° + \text{atan}(\frac{\Delta_s + A_1}{A_1}))) \quad (25)$$

$$\Delta_s, \Delta_c > -A_1$$

3 Comment on results

In test were used first 15 components of potency series. Difference between error predicted by results and actual error is only numeric (Figure 5). I made FFT of predicted error and actual error. Difference between amplitude of harmonics is numeric only. By increasing parameter error, actual error limit to discretion (nezveznosti). Error can not be fitted using first 15 components only. It is necessary to mention that despite the derivation, the presented types of errors of individual deformations still depends on each other.

4 Conclusion

This paper presents courses of error due to different amplitudes, different offsets, phase shifts and combination of parameters in input signals. Error includes higher harmonics, which become non-negligible at bigger distortion. For low distortion approximation, linear function

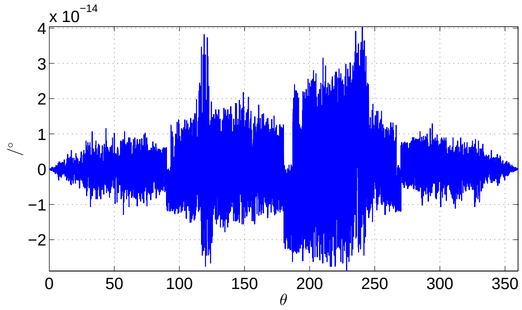

Figure 5: Difference between predicted (11) and actual error ati $k = 1.1$

can be adequatable. Literature confirmed results that was calculated at low distortion [6]. With those expression can be found reason of inappropriate installation of position sensor or actuator. Expressions can be used in applications where user do not have access to measured signals as are sine and cosine. Input signals can include higher harmonics too. Higher harmonics in input signals have impact to output signal and error. The influence of the distortion of the input signals in the atan2 function to the output error offers many challenges for further work.

Acknowledgment

This paper could not be possible without the help of some of my mentor and my professional colleagues. I am deeply grateful to their generous help in the design and experiment.

References

[1] Gachter J., Hirz M.,Seebacher R., "Impact of Rotor Position Sensor Errors on Speed Controlled Permanent Magnetized Synchronous Machines", IEEE 12th International Conference on Power Electronics and Drive Systems (PEDS), pp.822-830, 12-15 Dec. 2017

[2] Brugnano F., Concari C., Imamovic E., Savi F., Toscani A., Zanichelli R., "A simple and accurate algorithm for speed measurement in electric drives using incremental encoder", IECON 2017 - 43rd Annual Conference of the IEEE Industrial Electronics Society, pp. 8551-8556, 29 Oct.-1 Nov. 2017

[3] Reddy B.P., Murali A., Shaga G., "Low Cost Planar Coil Structure for Inductive Sensors to Measure Absolute Angular Position", 2017 2nd International Conference on Frontiers of Sensors Technologies (ICFST), pp.14-18, 14-16 April 2017

[4] Zhang Z., Ni F., Liu H., Jin M., "Theory analysis of a new absolute position sensor based on electromagnetism", International Conference on Automatic Control and Artificial Intelligence, pp.2204-208, 3-5 Mar. 2012

[5] Lara J., Chandra A., "Position Error Compensation in Quadrature Analog Magnetic Encoders through an Iterative Optimization Algorithm", IECON 2014 - 40th Annual Conference of the IEEE Industrial Electronics Society, pp.3043-3048, 29 Oct.-1 Nov. 2014

[6] Qi Lin, T. Li, Z. Zhou, "Error Analysis and Compensation of the Orthogonal Magnetic Encoder", Proceedings of IEEE ICMCC Conference, pp.11-14, 21-23 Oct. 2011

[7] Hanselman D.C., "Resolver Signal Requirements for High Accuracy Resolver-to-Digital Conversion", IEEE Transactions on Industrial Electronics, vol.37, no.6, pp.556-561, Dec. 1990

[8] Demierre M., "Improvements of CMOS Hall Microsystems and Application for Absolute Angular Position Measurements", PhD. thesis, pp. 152-161, Federal Polytechnic School of Lausanne, Switzerland, 2003

[9] https://www.mathworks.com/help/matlab/ref/atan2.html, dostop junij 2018

[10] https://www.mathworks.com/help/matlab/ref/atan2d.html, dostop junij 2018