The course of error due to distortion of input signal to the atan2 function

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Distortion of sine and cosine values, used for angle determination with the atan2 function, can result in numerical error. According to the performed review of literature, error is normally presented by taking only the basic harmonic into account. This paper however presents determination of error by taking into account also higher harmonics, which are non-negligible at larger distortion of sine or cosine. Error is going to be expressed with infinite series, which expand the domain of distortion parameter.

1 Introduction

High efficient regulation of motor drives this days is presented in many applications. For quality regulated motor drives, position sensors are used[1]. Angular position is measured by incremental based sensors, resolvers, encoders [2][3][4]... Outputs from resolvers or encoders are signals of sine and cosine form. For angle needs to be calculated inverse of tangent. In applications is normally used function atan2, which returns value in range $[-\pi, \pi]$ [9].

Position sensors are not ideal. Signals from resolvers are not ideal form of sine and cosine. Nevertheless function atan2 calculate angle. Output of atan2 includes error, because of distorted sine and cosine signals. Literature [5], [6], [7] analyzed the effect of distorted sine and cosine signals evaluate to angle error. Error was described by the harmonic with highest amplitude. Nevertheless error includes higher harmonics too. In this paper angle error is presented by infinite series. Many applications have accessible angular position only. By reference encoder, error can be detected, reason for it stays unknown.

By knowing reasons, how distorted inputs signals of atan2 function impact to error, eventually from error cause can be recognized. This paper presents impact of distorted input signals of atan2 to error presented in infinite series.

2 Methodology and results

Base input signals to atan2 have form of sine and cosine. They can be distorted by offsets, different amplitude, or phase shift(1)(2)

$$Sin = B_0 + B_1 \sin(\theta + \varphi_s) \tag{1}$$

$$Cos = A_0 + A_1 \cos(\theta + \varphi_c) \tag{2}$$

Ideal case is when offsets (B_0 and A_0) and phase shifts φ_0 and φ_0 are equal to zero, amplitude are same ($B_1 = A_1$). Signals can be distorted in amplitude and phase by only one parameter (Δ_c or Δ_s) (3)(4)

$$Sin = \sin(\theta) + \Delta_c \cos(\theta) + \Delta_s \sin(\theta) \tag{3}$$

$$Cos = \cos(\theta) + \Delta_c \cos(\theta) + \Delta_s \sin(\theta) \tag{4}$$

Definition of outupt signal φ and error ε is:

$$\varphi = \mathrm{atan2d}(Sin, Cos) \tag{5}$$

$$\varepsilon = \varphi - \mathrm{atan2d}(\sin(\theta), \cos(\theta)) \tag{6}$$

where θ presents reference angle. In MATLAB is defined function atan2d(), for purpose of calculation of invert tangent in four quadrant plane. Output is in range of $[-180^\circ, 180^\circ]$[10]. Error is presented by infinite series (7).

$$\varepsilon(x) = C_0(x) + \sum_{n=1}^{\infty} C_n(x) \sin(n\Theta + \varphi_n(x)) \tag{7}$$

Parameter x presents independent variable, which distort input signals (1) and (2). C_0 presents offset of error, C_n amplitude of individual harmonic and φ_n presents phase of individual harmonic of error. All functions depend on x.

Here is presented approach to determinate error. One parameter (A_0, B_0, A_1, B_1, φ_s, φ_c, Δ_s, Δ_c), used in calculation separately has been limited to infinity. In limit error is expressed as example:

$$\varepsilon(\theta) = \begin{cases} 90^\circ - \theta, & \theta \in \{0^\circ, 180^\circ\} \\ 270^\circ - \theta, & \theta \in \{180^\circ, 360^\circ\} \end{cases} \tag{8}$$

Error is transformed to Fourier series. From transform can be decided which harmonics are changing and to where they converge. Next step is finding analytic function, that describes the course of the harmonics amplitude and the phase shift of an individual harmonic when the parameter changes. Knowing the convergence of function is helpful. This study has shown that higher harmonies are potency depend on the basic harmonic of the error. The error is expressed as a potency series.

The following are the principles of error determination at different amplitudes, phase displacements, offsets and a combination of different amplitudes and phases due to one parameter.

2.1 Defining of error at different amplitudes

If both input signals are multiplied by same coefficient, output signal will not be changed. Multiply both signals with $\frac{1}{A_1}$ and ratio $\frac{B_1}{A_1}$ define as k. Setting offsets and phases to zero, the input signals are defined as:

$$Sin = k\sin(\theta) \tag{9}$$

$$Cos = \cos(\theta). \tag{10}$$

Limiting k to infinity and calculate of error get result (Figure 1),

$$\lim_{k\to\infty}(\operatorname{atan2}(k\sin\theta,\cos\theta) - \operatorname{atan2}(\sin\theta,\cos\theta)) \tag{11}$$

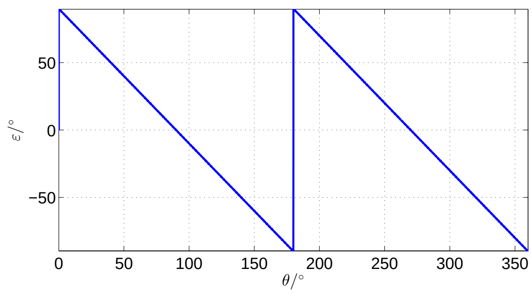

Figure 1: Error ε of limiting k to infinity

that can be transformed to Fourier series:

$$\varepsilon = \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin 2n\theta. \tag{12}$$

By calculating Fourier series of error we get only even harmonics, of which the second harmonics is the largest. Because of number 2 in argument of sine in (12), C_1 presents function of amplitude for second harmonic.

Using Curve Fitting Toolbox, best fit is rational function. Error can be expressed as:

$$\varepsilon(k) = \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(\frac{k-1}{k+1})^n\sin 2n\theta \tag{13}$$

Expression is valid for k bigger than 0.

$$k \geq 0$$

instead of k (13) inserts the ratio of amplitude:

$$\varepsilon(k) = \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(\frac{B_1 - A_1}{B_1 + A_1})^n\sin 2n\theta \tag{14}$$

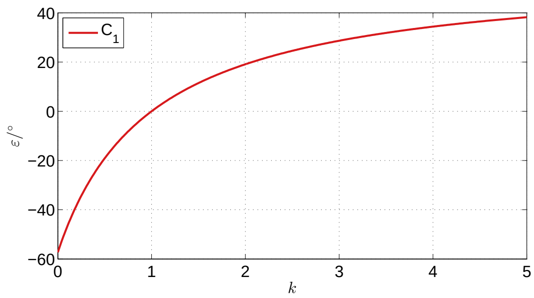

Figure 2: The course of the second accordion depending on k

(14) is valid for:

$$\frac{B_1}{A_1} \geq 0.$$

2.2 Defining of error at non-orthogonality

Inpust signals are defined as:

$$Sin = \sin(\theta + \varphi_s) \tag{15}$$

$$Cos = \cos(\theta + \varphi_c) \tag{16}$$

Error is determinate for each parameter separately. Other paramether is set to zero. At the end of equations are merged. For limitation of equation is not obligatory limit to infinity, just to the worst case. Limit is at $\pm 90^\circ$:

$$\varepsilon = \lim_{\varphi_s\to 90^\circ}\operatorname{atan2}(Sin, Cos) - \operatorname{atan2d}(\sin(\theta), \cos(\theta)) \tag{17}$$

Error can be transform and presented in Fourier series.

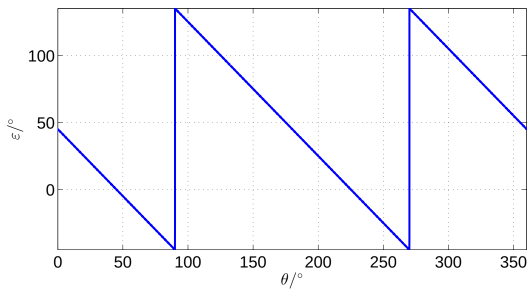

Figure 3: Error ε of limiting $\varphi_s \to 90^\circ$

$$\varepsilon = 45^\circ - \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(2n\theta) \tag{18}$$

Transform includes offset and even harmonics only. Figure 4 presents course of offset C_0, amplitude of second harmonic C_1 and phase of second harmonic φ_1 due to φ_s. y axis is in degrees. For C_0 and C_1 degrees presents amplitude of error harmonics, for φ_1 degrees presents phase. Offset is best fitted by linear function. Second harmonic is tangent function. Phase of second harmonic increases linear but for presentation with sine form must be added 90°. Same derivation can be done

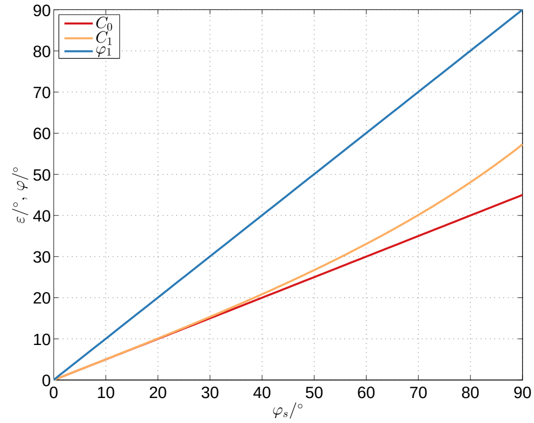

Figure 4: The course of offset component C_0, amplitude of second harmonic C_1 and phase φ_1 depend of ideal cosine signal,due to phase shift φ_s

for phase of cosine signal. Equations can be marged and result is presented (19).

$$\varepsilon(\varphi_s,\varphi_c) = \frac{\varphi_s+\varphi_c}{2} + \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(\tan\frac{\varphi_s-\varphi_c}{2})^n \sin(2n\theta+n(90°+\varphi_s+\varphi_c)) \qquad (19)$$

Expression is valid only for:

$$\varphi_s-\varphi_c \in [-90°, 90°]$$

2.3 Defining of error at offsets

Define amplitudes of input signals to 1, phase shift is set to zero. Let limit parameter A_0 to infinity, error is transformed to Fourier series and expressed as:

$$\varepsilon = \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{2}{n}\sin(n\theta+90°n). \qquad (20)$$

Error does not include offset component, highest amplitude has first harmonic. The course from figure 5 it is

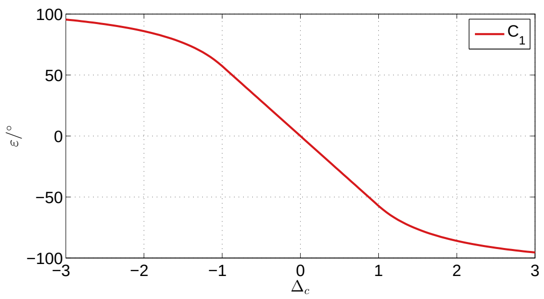

Figure 5: The course of amplitude of first harmonic due to offset A_0, where input signals have amplitude of 1

split to 3 parts and expression that best fit the curve is:

$$\varepsilon(A_0) = \begin{cases} \frac{180}{\pi}\sum_{n=1}^{\infty}(-1)^n\frac{1}{n}(2-|\frac{A_0}{A_1}|^{-n})\sin(n\theta), & \frac{A_0}{A_1}\le -1 \\ \frac{180}{\pi}\sum_{n=1}^{\infty}(-1)^n\frac{1}{n}(\frac{A_0}{A_1})^n\sin(n\theta), & |\frac{A_0}{A_1}|\le 1 \\ \frac{180}{\pi}\sum_{n=1}^{\infty}(-1)^n\frac{1}{n}(2-(\frac{A_0}{A_1})^{-n})\sin(n\theta), & \frac{A_0}{A_1}\ge 1 \end{cases} \qquad (21)$$

Same derivation can be done for B_0 (22) and for fitting error, when sine and cosine include same offset ($A_0 = B_0$) (23).

$$\varepsilon(B_0) = \begin{cases} \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(2-|\frac{B_0}{B_1}|^{-n})\sin(n\theta-90°n), & \frac{B_0}{B_1}\le -1 \\ \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(\frac{B_0}{B_1})^n\sin(n\theta+90°n), & |\frac{B_0}{B_1}|\le 1 \\ \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(2-(\frac{B_0}{B_1})^{-n})\sin(n\theta°+90n), & \frac{B_0}{B_1}\ge 1 \end{cases} \qquad (22)$$

$$\varepsilon(A_0, B_0 = A_0) = \begin{cases} \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(2-|\sqrt{2}\frac{A_0}{A_1}|^{-n})\sin(n\theta+90°n), & \frac{A_0}{A_1}\le -\frac{\sqrt{2}}{2} \\ \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(\sqrt{2}\frac{A_0}{A_1})^n\sin(n\theta-90°n), & |\frac{A_0}{A_1}|\le \frac{\sqrt{2}}{2} \\ \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(2-(\sqrt{2}\frac{A_0}{A_1})^{-n})\sin(n\theta-90°n), & \frac{A_0}{A_1}\ge \frac{\sqrt{2}}{2} \end{cases} \qquad (23)$$

2.4 Impact of different amplitude and phase due to one parameter

Transform to Fourier series of limit (6) to infinity of Δ_c, where inputs are (3) and (4) is

$$\varepsilon = 45° - \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(2n\theta). \qquad (24)$$

The course of second harmonic due to Δ_c, can be express as sum of sine and cosine signal. Each signal is presented in figure 6.

$$C_{1s}(\Delta_c)\cdot\sin(2\theta) + C_{1c}(\Delta_c)\cdot\cos(2\theta) \qquad (25)$$

Offset of error is fitted to invert tangent function. C_{1s} and C_{1c} are best fitted to rational function. Total amplitude is geometrical summation $C_n = \sqrt{C_{ns}^2 + C_{nc}^2}$, phase fitted to sine signal is calculated by $\varphi_n = atan(\frac{C_{nc}}{C_{ns}})$. Final equation due to Δ_c and $\Delta_s$is expressed in (26) and (27)

$$\varepsilon(\Delta_c) = \mathrm{atand}\frac{\Delta_c}{\Delta_c+2A_1} + \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(\frac{\Delta_c}{\sqrt{\Delta_c^2+2A_1\Delta_c+2A_1}})^n \sin(2n\theta+n(90°+\mathrm{atan}(\frac{\Delta_c+A_1}{A_1}))) \qquad (26)$$

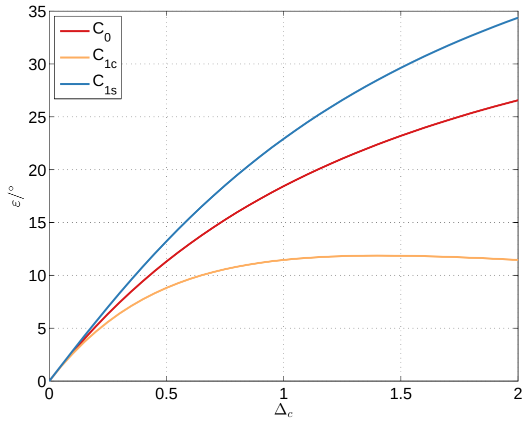

Figure 6: The course of offset and amplitude of second harmonic of error due to Δ_c

$$\varepsilon(\Delta_s) =$$

$$\text{atand}\frac{-\Delta_s}{\Delta_s + 2A_1} + \frac{180}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{\Delta_s}{\sqrt{\Delta_s^2 + 2A_1\Delta_s + 2A_1}}\right)^n$$
$$\sin(2n\theta + n(90^\circ + \text{atan}(\frac{\Delta_s + A_1}{A_1}))) \quad (27)$$

$$\Delta_s, \Delta_c > -A_1$$

3 Comment on results

In test were used first 15 components of potency series. Difference between error predicted by results and actual error is only numeric (Figure 7). I made FFT of predicted error and actual error. Difference between amplitude of harmonics is numeric only. By increasing parameter error, actual error limit to discretion (nezveznosti). Error can not be fitted using first 15 components only. It is necessary to mention that despite the derivation, the presented types of errors of individual deformations still depends on each other.

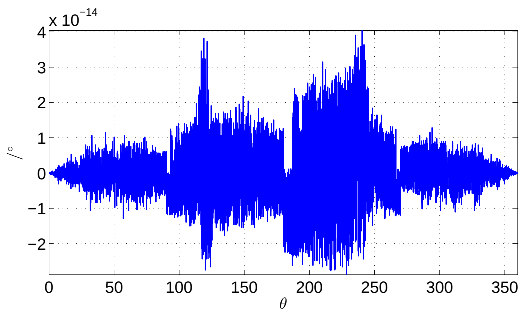

Figure 7: Difference between predicted (13) and actual error ati $k = 1.1$

4 Conclusion

This paper presents courses of error due to different amplitudes, different offsets, phase shifts and combination of parameters in input signals. Error includes higher harmonics, which become non-negligible at bigger distortion. For low distortion approximation, linear function can be adequatable. Literature confirmed results that was calculated at low distortion [5]. With those expression can be found reason of inappropriate installation of position sensor or actuator. Expressions can be used in applications where user do not have access to measured signals as are sine and cosine. Input signals can include higher harmonics too. Higher harmonics in input signals have impact to output signal and error. The influence of the distortion of the input signals in the atan2 function to the output error offers many challenges for further work.

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