Integrirani pogonski sistemi

Modeliranje električnih pogonov

as. dr. Klemen Drobnič
klemen.drobnic@fe.uni-lj.si

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Dinamični model AS

Dvoosni dinamični model asinhronskega stroja

napetostni enačbi

$$\boldsymbol{u}_s = R_s \boldsymbol{i}_s + \frac{d\boldsymbol{\psi}_s}{dt} + \mathrm{j}\omega\boldsymbol{\psi}_s$$

$$\boldsymbol{u}_r = R_r \boldsymbol{i}_r + \frac{d\boldsymbol{\psi}_r}{dt} + \mathrm{j}(\omega - \omega_r)\boldsymbol{\psi}_r$$

definicija vrtilnih hitrosti

- ω - vrtilna hitrost KS,
- ω_r - (el.) vrtilna hitrost rotorja.

sklepa izražena s tokovoma

$$\boldsymbol{\psi}_s = L_s \boldsymbol{i}_s + L_m \boldsymbol{i}_r$$

$$\boldsymbol{\psi}_r = L_r \boldsymbol{i}_r + L_m \boldsymbol{i}_s$$

tokova izražena s sklepoma

$$\boldsymbol{i}_s = \frac{\boldsymbol{\psi}_s - k_r\boldsymbol{\psi}_r}{L_{sT}}$$

$$\boldsymbol{i}_r = \frac{\boldsymbol{\psi}_r - k_s\boldsymbol{\psi}_s}{L_{rT}}$$

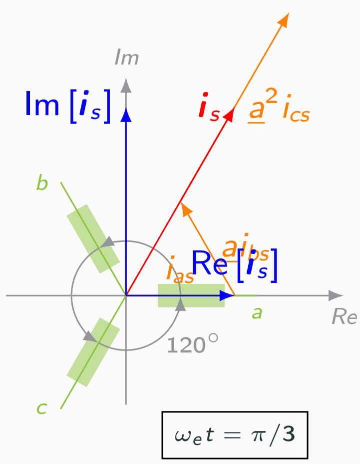
vpeljane konstante

$k_s = L_m/L_s$, $k_r = L_m/L_r$, $L_{sT} = L_s - L_m^2/L_r$, $L_{rT} = L_r - L_m^2/L_s$

Prostorski vektor

Definicija vektorja

$$\boldsymbol{i}_s = \frac{2}{3}\left(i_{as} + \underline{a} i_{bs} + \underline{a}^2 i_{cs}\right)$$



$$\begin{aligned} i_{as} &= I_{as}\cos(\omega_e t) \\ i_{bs} &= I_{bs}\cos(\omega_e t - 2\pi/3) \\ i_{cs} &= I_{cs}\cos(\omega_e t - 4\pi/3) \end{aligned}$$

$$\begin{aligned} i_{as} &= I_{as}\cos(\pi/3) = 1/2 \\ i_{bs} &= I_{bs}\cos(-\pi/3) = 1/2 \\ i_{cs} &= I_{cs}\cos(\pi) = -1 \end{aligned}$$

Lastnosti

- faze zamaknjene v prostoru $\underline{a} = e^{\mathrm{j}120^\circ}$,
- učinek faz zajamemo z enim kompl. številom,
- z izbiro $2/3$ dosežemo, da je pri simetričnem napajanju amplituda vektorja $\boldsymbol{i}_s$ enaka vršni toka faze a $i_{as} = I_s\cos(\omega_e t)$.

Vektor razdelimo na realni in imaginarni del

$$\begin{aligned} \boldsymbol{i}_s &= \frac{2}{3}(i_{as} + \underline{a} i_{bs} + \underline{a}^2 i_{cs}) \\ &= \frac{2}{3}\left[i_{as} - \frac{1}{2} i_{bs} - \frac{1}{2} i_{cs} + \mathrm{j}\left(\frac{\sqrt{3}}{2} i_{bs} - \frac{\sqrt{3}}{2} i_{cs}\right)\right], \end{aligned}$$

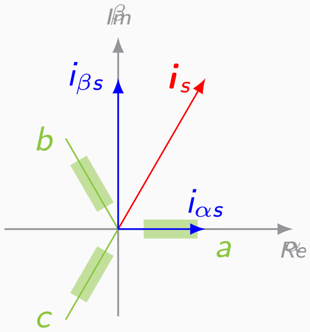
oziroma v preglednejšem zapisu

$$\mathrm{Re}\,[\boldsymbol{i}_s] = \mathrm{Re}\left[\frac{2}{3}(i_{as} + \underline{a} i_{bs} + \underline{a}^2 i_{cs})\right] = \frac{2}{3}(i_{as} - \frac{1}{2} i_{bs} - \frac{1}{2} i_{cs})$$

$$\mathrm{Im}\,[\boldsymbol{i}_s] = \mathrm{Im}\left[\frac{2}{3}(i_{as} + \underline{a} i_{bs} + \underline{a}^2 i_{cs})\right] = \frac{\sqrt{3}}{2} i_{bs} - \frac{\sqrt{3}}{2} i_{cs}.$$

Primerjava z dvoosno teorijo

Prostorski vektor lahko definiramo tudi z dvoosno teorijo (Park).



Realni del prostorskega vektorja je enak toku $i_{\alpha s}$ in imaginarni del je enak $i_{\beta s}$

$$\mathrm{Re}\left[\boldsymbol{i}_s\right] = \mathrm{Re}\left[\frac{2}{3}(i_{as} + \underline{a} i_{bs} + \underline{a}^2 i_{cs})\right] = \frac{2}{3}(i_{as} - \frac{1}{2} i_{bs} - \frac{1}{2} i_{cs}) = i_{\alpha s}$$

$$\mathrm{Im}\left[\boldsymbol{i}_s\right] = \mathrm{Im}\left[\frac{2}{3}(i_{as} + \underline{a} i_{bs} + \underline{a}^2 i_{cs})\right] = \frac{\sqrt{3}}{2} i_{bs} - \frac{\sqrt{3}}{2} i_{cs} = i_{\beta s}$$

V statorskem KS zato zapišemo $\boldsymbol{i}_s = i_{\alpha s} + \mathrm{j} i_{\beta s}$.

ničelna komponenta

Prostorski vektor ne vsebuje ničelne komponente, zato jo moramo definirati posebej.

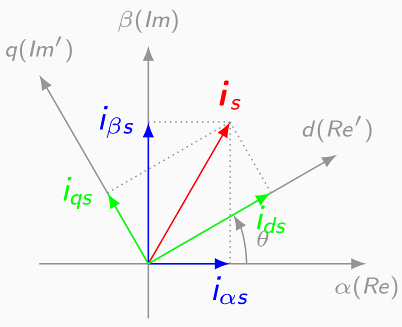
$$i_{0s} = \frac{1}{3}\left(i_{as} + i_{bs} + i_{cs}\right).$$

fazna oz. Clarkina transformacija

matrični zapis

$$\begin{pmatrix} i_{\alpha s} \\ i_{\beta s} \\ i_{0s} \end{pmatrix} = \frac{2}{3} \begin{pmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{pmatrix} \begin{pmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{pmatrix}$$

Rotacijska transformacija

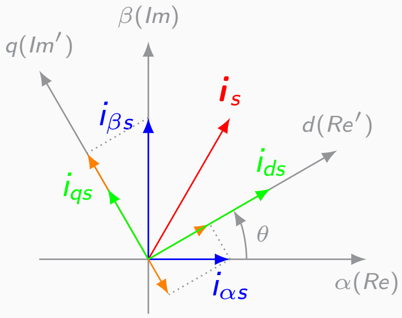


Vektor $\boldsymbol{i}_s$ zapišemo v poljubnem KS z $\boldsymbol{i}_s^K = \boldsymbol{i}_s e^{-j\theta}$

$$\boldsymbol{i}_s^K = \boldsymbol{i}_s e^{-j\theta}$$

$$\boldsymbol{i}_s^K = (i_{\alpha s} + \mathrm{j} i_{\beta s})(\cos\theta - \mathrm{j}\sin\theta)$$

$$\boldsymbol{i}_s^K = i_{\alpha s}\cos\theta + i_{\beta s}\sin\theta + \mathrm{j}(-i_{\alpha s}\sin\theta + i_{\beta s}\cos\theta)$$



interpretacija transformacije

$$\mathrm{Re}\left[\boldsymbol{i}_s^K\right] = i_{\alpha s}\cos\theta + i_{\beta s}\sin\theta = i_{ds}$$

$$\mathrm{Im}\left[\boldsymbol{i}_s^K\right] = -\,i_{\alpha s}\sin\theta + i_{\beta s}\cos\theta = i_{qs}$$

komutatorska (Parkova) transformacija

(matrični zapis)

$$\begin{pmatrix} i_{ds} \\ i_{qs} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} i_{\alpha s} \\ i_{\beta s} \end{pmatrix}$$

Izračun električne moči in navora

Trenutna moč v asinhronskem motorju je definirana z

$$P_e = u_{as} i_{as} + u_{bs} i_{bs} + u_{cs} i_{cs} + u_{ar} i_{as} + u_{br} i_{bs} + u_{cr} i_{cs}.$$

Cilj

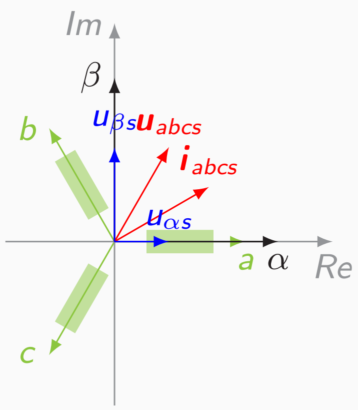
Kako trenutno moč izrazimo s prostorskimi vektorji? Kako izrazimo elektromagnetni navor?

Potrebni koraki:

- definicija prostorskih vektorjev napetosti in toka,
- kaj predstavlja produkt $\boldsymbol{u}_{abcs} \boldsymbol{i}^*_{abcs}$?
- kako lahko povežemo $\mathrm{Re}\left[\boldsymbol{u}_{abcs} \boldsymbol{i}^*_{abcs}\right]$ in P_e?
- interpretacija enačbe za električno moč (bilanca moči),
- izpeljava enačbe za elektromagnetni navor.

Vektorja statorske napetosti in toka

Vhodno moč v motor želimo izraziti z vektorji napetosti in toka.



$$\boldsymbol{u}_{abcs} = \frac{2}{3}\left(u_{as} + \underline{a} u_{bs} + \underline{a}^2 u_{cs}\right)$$

$$\boldsymbol{i}_{abcs} = \frac{2}{3}\left(i_{as} + \underline{a} i_{bs} + \underline{a}^2 i_{cs}\right), \qquad \text{kjer } \underline{a} = e^{\mathrm{j}120^\circ}$$

Vektorja sta kompleksni spremenljivki in ju lahko razdelimo na dve komponenti $\boldsymbol{u}_{abcs} = u_{\alpha s} + \mathrm{j} u_{\beta s}$.

$$\begin{aligned} \boldsymbol{u}_{abcs} &= \frac{2}{3}\left(u_{as} + \underline{a} u_{bs} + \underline{a}^2 u_{cs}\right) \\ &= \frac{2}{3}\left[u_{as} - \frac{1}{2} u_{bs} - \frac{1}{2} u_{cs} + \mathrm{j}\left(\frac{\sqrt{3}}{2} u_{bs} - \frac{\sqrt{3}}{2} u_{cs}\right)\right] \\ &= u_{\alpha s} + \mathrm{j} u_{\beta s} \end{aligned}$$

- Realna komponenta ($u_{\alpha s}$) sovpada s prostorsko orientacijo faze a,
- z izbiro faktorja $2/3$ dosežemo, da je pri simetričnem napajanju amplituda vektorja $\boldsymbol{u}_{abcs}$ enaka vršni vrednosti fazne napetosti $u_{as} = U_{as} \cos(\omega_e t)$.

Produkt prostorskega vektorja

Produkt statorskega prostorskega vektorja napetosti in toka $\boldsymbol{u}_{abcs}\boldsymbol{i}^*_{abcs}$

$$\boldsymbol{u}_{abcs}\boldsymbol{i}^*_{abcs} = \frac{2}{3}\left(u_{as} + \underline{a}u_{bs} + \underline{a}^2 u_{cs}\right)\frac{2}{3}\left(i_{as} + \underline{a}^2 i_{bs} + \underline{a}i_{cs}\right)$$

Zmnožimo prostorska vektorja

$$\begin{aligned}\boldsymbol{u}_{abcs}\boldsymbol{i}^*_{abcs} = \frac{4}{9}\Big(&u_{as}i_{as} + \underline{a}(u_{bs}i_{as} + u_{as}i_{cs})\\ &+ \underline{a}^2(u_{as}i_{bs} + u_{cs}i_{as} + u_{bs}i_{cs})\\ &+ \underline{a}^3(u_{bs}i_{bs} + u_{cs}i_{cs}) + \underline{a}^4(u_{cs}i_{bs})\Big).\end{aligned}$$

Upoštevamo lastnost kompleksnega operatorja $\underline{a}$

$$\begin{aligned}\boldsymbol{u}_{abcs}\boldsymbol{i}^*_{abcs} = \frac{4}{9}\Big[\quad &u_{as}i_{as} + u_{bs}i_{bs} + u_{cs}i_{cs}\\ &+\underline{a}(u_{as}i_{cs} + u_{bs}i_{as} + u_{cs}i_{bs})\\ &+\underline{a}^2(u_{as}i_{bs} + u_{bs}i_{cs} + u_{cs}i_{as})\Big].\end{aligned}$$

Realni del produkta $\mathrm{Re}\left[\boldsymbol{u}_{abcs}\boldsymbol{i}^*_{abcs}\right]$

Zapišimo realni del produkta

$$\begin{aligned}\mathrm{Re}\left[\boldsymbol{u}_{abcs}\boldsymbol{i}^*_{abcs}\right] = \frac{4}{9}\Bigg(&u_{as}i_{as} + u_{bs}i_{bs} + u_{cs}i_{cs}\\ &-\frac{1}{2}\Big(u_{as}\overbrace{\boxed{(i_{bs} + i_{cs})}}^{-i_{as}} + u_{bs}\overbrace{\boxed{(i_{as} + i_{cs})}}^{-i_{bs}} + u_{cs}\overbrace{\boxed{(i_{as} + i_{bs})}}^{-i_{cs}}\Big)\Bigg).\end{aligned}$$

Za trifazni sistem brez ničelnega vodnika velja $i_{as} + i_{bs} + i_{cs} = 0$,

zato se enačba poenostavi

$$\mathrm{Re}\left[\boldsymbol{u}_{abcs}\boldsymbol{i}^*_{abcs}\right] = \frac{2}{3}\overbrace{\boxed{u_{as}i_{as} + u_{bs}i_{bs} + u_{cs}i_{cs}}}^{\text{statorska vhodna moč}}$$

Produkt $\mathrm{Re}\left[\boldsymbol{u}_{abcs}\boldsymbol{i}^*_{abcs}\right]$ je enak 2/3 statorske vhodne moči.

Skupna električna moč je sestavljena iz prispevka statorja in rotorja

$$P_e = \frac{3}{2}\left(\mathrm{Re}\left[\boldsymbol{u}_{abcs}\boldsymbol{i}^*_{abcs}\right] + \mathrm{Re}\left[\boldsymbol{u}_{abcr}\boldsymbol{i}^*_{abcr}\right]\right).$$

Električna moč v dq vezju

Vektorji napetosti in toka so trenutno izraženi v naravnih koordinatah. Z rotacijsko transformacijo jih pretvorimo v dq sistem, npr. za vektorja napetosti velja

$$\boldsymbol{u}_s = e^{-\mathrm{j}\theta}\boldsymbol{u}_{abcs} \qquad \text{in} \qquad \boldsymbol{u}_r = e^{-\mathrm{j}(\theta-\theta_r)}\boldsymbol{u}_{abcr}$$

oziroma, če zapišemo obratno

$$\boldsymbol{u}_{abcs} = e^{\mathrm{j}\theta}\boldsymbol{u}_s \qquad \text{in} \qquad \boldsymbol{u}_{abcr} = e^{\mathrm{j}(\theta-\theta_r)}\boldsymbol{u}_r .$$

Če vstavimo vektorje izražene v dq koordinatah v enačbo za moč dobimo

$$\begin{aligned} P_e &= \frac{3}{2}\left(\operatorname{Re}\left[e^{\mathrm{j}\theta}\boldsymbol{u}_s e^{-\mathrm{j}\theta}\boldsymbol{i}_s^* + e^{\mathrm{j}(\theta-\theta_r)}\boldsymbol{u}_r e^{-\mathrm{j}(\theta-\theta_r)}\boldsymbol{i}_r^*\right]\right) \\ &= \frac{3}{2}\left(\operatorname{Re}\left[\boldsymbol{u}_s\boldsymbol{i}_s^* + \boldsymbol{u}_r\boldsymbol{i}_r^*\right]\right), \end{aligned}$$

oziroma v skalarni obliki

$$P_e = \frac{3}{2}\left(u_{ds}i_{ds} + u_{qs}i_{qs} + u_{dr}i_{dr} + u_{qr}i_{qr}\right).$$

Bilanca moči

V enačbi za moč vektor napetosti zamenjamo z napetostno enačbo

$$\begin{aligned} P_e = \frac{3}{2}\Bigg(&\operatorname{Re}\left[\left(R_s\boldsymbol{i}_s + (L_{ss}+L_m)\mathrm{p}\boldsymbol{i}_s + L_m\mathrm{p}\boldsymbol{i}_r + \mathrm{j}\omega\left[(L_{ss}+L_m)\boldsymbol{i}_s + L_m\boldsymbol{i}_r\right]\right)\boldsymbol{i}_s^*\right] + \\ &\operatorname{Re}\left[\left(R_r\boldsymbol{i}_r + (L_{sr}+L_m)\mathrm{p}\boldsymbol{i}_r + L_m\mathrm{p}\boldsymbol{i}_s + \mathrm{j}(\omega-\omega_r)\left[(L_{ss}+L_m)\boldsymbol{i}_r + L_m\boldsymbol{i}_s\right]\right)\boldsymbol{i}_r^*\right]\Bigg) \end{aligned}$$

Ker velja $\boldsymbol{i}_s\boldsymbol{i}_s^* = |\boldsymbol{i}_s|^2$ in $\boldsymbol{i}_r\boldsymbol{i}_r^* = |\boldsymbol{i}_r|^2$, lahko enačbo preuredimo v

$$\begin{aligned} P_e =& \frac{3}{2}R_s|\boldsymbol{i}_s|^2 + \frac{3}{2}R_r|\boldsymbol{i}_r|^2 \\ &+ \frac{3}{2}\mathrm{p}\left(\frac{L_{ss}}{2}|\boldsymbol{i}_s|^2 + \frac{L_{sr}}{2}|\boldsymbol{i}_r|^2 + L_m|\boldsymbol{i}_s + \boldsymbol{i}_r|^2\right) \\ &+ \frac{3}{2}\operatorname{Re}\left[\mathrm{j}\omega\left((L_{ss}+L_m)|\boldsymbol{i}_s|^2 + L_m\boldsymbol{i}_r\boldsymbol{i}_s^*\right) + \mathrm{j}(\omega-\omega_r)\left((L_{sr}+L_m)|\boldsymbol{i}_r|^2 + L_m\boldsymbol{i}_s\boldsymbol{i}_r^*\right)\right] \end{aligned}$$

izgube v bakru

sprememba shr. energije

mehanska energija

Mehanska moč

$$P_m = \frac{3}{2}\,\mathrm{Re}\left[\mathrm{j}\omega\Big((L_{ss}+L_m)|\boldsymbol{i}_s|^2 + \boxed{L_m\boldsymbol{i}_r\boldsymbol{i}_s^*}\Big) + \mathrm{j}(\omega-\omega_r)\Big((L_{sr}+L_m)|\boldsymbol{i}_r|^2 + \boxed{L_m\boldsymbol{i}_s\boldsymbol{i}_r^*}\Big)\right]$$

K mehanski moči prispevata samo drugi in četrti člen

$$P_m = \frac{3}{2}\,\mathrm{Re}\left[\mathrm{j}\omega L_m\boldsymbol{i}_r\boldsymbol{i}_s^* + \mathrm{j}(\omega-\omega_r)L_m\boldsymbol{i}_s\boldsymbol{i}_r^*\right]$$

Mehanska moč

Poenostavljen izraz za mehansko moč preuredimo

$$\begin{aligned} P_m &= \frac{3}{2}\,\mathrm{Re}\left[\mathrm{j}\omega L_m\boldsymbol{i}_r\boldsymbol{i}_s^* + \mathrm{j}(\omega-\omega_r)L_m\boldsymbol{i}_s\boldsymbol{i}_r^*\right] \\ &= \frac{3}{2}\,\mathrm{Re}\left[\mathrm{j}\omega L_m(\boldsymbol{i}_r\boldsymbol{i}_s^* + \boldsymbol{i}_s\boldsymbol{i}_r^*) - \mathrm{j}\omega_r L_m\boldsymbol{i}_s\boldsymbol{i}_r^*\right] \end{aligned}$$

Ker pa je izraz $\boldsymbol{i}_r\boldsymbol{i}_s^* + \boldsymbol{i}_s\boldsymbol{i}_r^*$ strogo realen,

$$\begin{aligned} \boldsymbol{i}_r\boldsymbol{i}_s^* + \boldsymbol{i}_s\boldsymbol{i}_r^* &= (i_{dr}+\mathrm{j}i_{qr})(i_{ds}-\mathrm{j}i_{qs}) + (i_{ds}+\mathrm{j}i_{qs})(i_{dr}-\mathrm{j}i_{qr}) \\ &= 2(i_{qr}i_{qs} + i_{dr}i_{ds}), \end{aligned}$$

se enačba za mehansko moč poenostavi v

$\mathrm{Re}\left[-\mathrm{j}\boldsymbol{z}\right] = \mathrm{Im}\left[\boldsymbol{z}\right]$

$$P_m = -\frac{3}{2}\omega_r L_m\,\mathrm{Re}\left[\mathrm{j}\boldsymbol{i}_s\boldsymbol{i}_r^*\right] = \frac{3}{2}\omega_r L_m\,\mathrm{Im}\left[\boldsymbol{i}_s\boldsymbol{i}_r^*\right]$$

Iz mehanske moči v elektromagnetni navor

Izraz za elektromagnetni navor dobimo iz mehanske dinamične enačbe $P_m = M_e\omega_{rm}$[1]

$$M_e = \frac{P_m}{\omega_{rm}} = \frac{3}{2} p_p L_m \operatorname{Im}\left[\boldsymbol{i}_s \boldsymbol{i}_r^*\right].$$

Enačbo lahko zapišemo tudi z vektorskim produktom

$$M_e = \frac{P_m}{\omega_{rm}} = \frac{3}{2} p_p L_m \boldsymbol{i}_r \times \boldsymbol{i}_s,$$

saj velja

$$\boldsymbol{i}_r \times \boldsymbol{i}_s = \begin{array}{c} \begin{array}{cc} d & q \end{array} \\ \begin{pmatrix} i_{dr} & i_{ds} \\ i_{qr} & i_{qs} \end{pmatrix} \end{array} = i_{dr} i_{qs} - i_{qr} i_{ds} = \operatorname{Im}\left[(i_{dr} - \mathrm{j} i_{qr})(i_{ds} + \mathrm{j} i_{qs})\right] = \operatorname{Im}\left[\boldsymbol{i}_r^* \boldsymbol{i}_s\right] = \operatorname{Im}\left[\boldsymbol{i}_s \boldsymbol{i}_r^*\right].$$

[1] Rotorska mehanska hitrost je enaka kvocientu električne rotorske hitrosti in števila polovih parov $\omega_{rm} = \omega_r / p_p$.

Produkti kompleksnih števil

definicija kompleksnih števil

$$\boldsymbol{z} = |\boldsymbol{z}| e^{\mathrm{j}\varphi_z} = a + \mathrm{j} b$$
$$\boldsymbol{w} = |\boldsymbol{w}| e^{\mathrm{j}\varphi_w} = c + \mathrm{j} d$$

kompleksni (navadni) produkt

$$\boldsymbol{z}\boldsymbol{w} = ab - bd + \mathrm{j}(ad + bc),$$

pogosteje ta produkt nastopa v obliki

$$\boldsymbol{z}\boldsymbol{w}^* = ab + bd + \mathrm{j}(bc - ad).$$

Kompleksno število lahko predstavimo kot vektor, zato lahko definiramo dva dodatna produkta.

skalarni produkt

$$\boldsymbol{z} \cdot \boldsymbol{w} = ac + bd$$

zapišemo lahko tudi

$$\boldsymbol{z} \cdot \boldsymbol{w} = \frac{1}{2}(\boldsymbol{z}\boldsymbol{w}^* + \boldsymbol{z}^*\boldsymbol{w}) = \operatorname{Re}\left[\boldsymbol{z}\boldsymbol{w}^*\right]$$
$$= |\boldsymbol{z}||\boldsymbol{w}| \cos(\varphi_z - \varphi_w)$$

vektorski produkt

$$\boldsymbol{z} \times \boldsymbol{w} = ad - bc$$

zapišemo lahko tudi

$$\boldsymbol{z} \times \boldsymbol{w} = \frac{1}{2}(\boldsymbol{z}\boldsymbol{w}^* - \boldsymbol{z}^*\boldsymbol{w}) = -\operatorname{Im}\left[\boldsymbol{z}\boldsymbol{w}^*\right]$$
$$= |\boldsymbol{z}||\boldsymbol{w}| \sin(\varphi_z - \varphi_w)$$

Izračun kompleksnega produkta

Vektorski produkt lahko izračunamo na več načinov.

$$\boldsymbol{z} \times \boldsymbol{w} = ab \sin(\varphi_b - \varphi_a)$$
$$\boldsymbol{z} \times \boldsymbol{w} = +\operatorname{Im}[\boldsymbol{z}^* \boldsymbol{w}] = -\operatorname{Im}[\boldsymbol{z}\boldsymbol{w}^*]$$
$$\boldsymbol{z} \times \boldsymbol{w} = -\operatorname{Re}[\mathrm{j}\boldsymbol{z}^* \boldsymbol{w}] = +\operatorname{Re}[\mathrm{j}\boldsymbol{z}\boldsymbol{w}^*]$$

Z upoštevanjem nekomutativnosti vektorskega produkta $\boldsymbol{z} \times \boldsymbol{w} = -(\boldsymbol{w} \times \boldsymbol{z})$ sledijo tudi preostale kombinacije.

$$\boldsymbol{w} \times \boldsymbol{z} = -\operatorname{Im}[\boldsymbol{z}^* \boldsymbol{w}] = +\operatorname{Im}[\boldsymbol{z}\boldsymbol{w}^*]$$
$$\boldsymbol{w} \times \boldsymbol{z} = +\operatorname{Re}[\mathrm{j}\boldsymbol{z}^* \boldsymbol{w}] = -\operatorname{Re}[\mathrm{j}\boldsymbol{z}\boldsymbol{w}^*]$$

Navorna enačba: Tok in fluks

statorski tok in statorski fluks

$$\begin{aligned} M_e &= \frac{3}{2} p_p L_m \boldsymbol{i}_r \times \boldsymbol{i}_s \\ &= \frac{3}{2} p_p L_m \boxed{\frac{1}{L_m}\left[-(L_{ss}+L_m)\boldsymbol{i}_s + \boldsymbol{\psi}_s\right]} \times \boldsymbol{i}_s \\ &= \frac{3}{2} p_p \boldsymbol{\psi}_s \times \boldsymbol{i}_s = \frac{3}{2} p_p \operatorname{Im}[\boldsymbol{i}_s \boldsymbol{\psi}_s^*] \end{aligned}$$

iz $\boldsymbol{\psi}_s = (L_{ss} + L_m)\boldsymbol{i}_s + L_m \boldsymbol{i}_r$ izrazimo

$$\boldsymbol{i}_r = \frac{1}{L_m}\left[-(L_{ss}+L_m)\boldsymbol{i}_s + \boldsymbol{\psi}_s\right]$$

in upoštevamo $\boldsymbol{i}_s \times \boldsymbol{i}_s = 0$.

rotorski tok in rotorski fluks

$$\begin{aligned} M_e &= \frac{3}{2} p_p L_m \boldsymbol{i}_r \times \boldsymbol{i}_s \\ &= \frac{3}{2} p_p L_m \boldsymbol{i}_r \times \boxed{\frac{1}{L_m}\left[-(L_{sr}+L_m)\boldsymbol{i}_r + \boldsymbol{\psi}_r\right]} \\ &= \frac{3}{2} p_p \boldsymbol{i}_r \times \boldsymbol{\psi}_r = \frac{3}{2} p_p \operatorname{Im}[\boldsymbol{\psi}_r \boldsymbol{i}_r^*] \end{aligned}$$

iz $\boldsymbol{\psi}_r = (L_{sr} + L_m)\boldsymbol{i}_r + L_m \boldsymbol{i}_s$ izrazimo

$$\boldsymbol{i}_s = \frac{1}{L_m}\left[-(L_{sr}+L_m)\boldsymbol{i}_r + \boldsymbol{\psi}_r\right]$$

in upoštevamo $\boldsymbol{i}_r \times \boldsymbol{i}_r = 0$.

Navorna enačba: Statorski tok in magnetilni fluks

magnetilni tok in statorski fluks

$$M_e = \frac{3}{2} p_p L_m \boldsymbol{i}_r \times \boldsymbol{i}_s$$
$$= \frac{3}{2} p_p L_m \left(\frac{1}{L_m} \boldsymbol{\psi}_r - \boldsymbol{i}_s \right) \times \boldsymbol{i}_s$$
$$= \frac{3}{2} p_p \boldsymbol{\psi}_m \times \boldsymbol{i}_s = \frac{3}{2} p_p \operatorname{Im} [\boldsymbol{i}_s \boldsymbol{\psi}_m^*]$$

iz $\boldsymbol{\psi}_m = L_m \boldsymbol{i}_r + L_m \boldsymbol{i}_s$ izrazimo

$$\boldsymbol{i}_m = \frac{1}{L_m} \boldsymbol{\psi}_r - \boldsymbol{i}_s$$

in upoštevamo $\boldsymbol{i}_s \times \boldsymbol{i}_s = 0$.

Navorna enačba: Tok in fluks

statorski tok in rotorski fluks

$$M_e = \frac{3}{2} p_p L_m \boldsymbol{i}_r \times \boldsymbol{i}_s$$
$$= \frac{3}{2} p_p L_m \left(\frac{1}{L_{sr} + L_m} \boldsymbol{\psi}_r - \frac{L_m}{L_{sr} + L_m} \boldsymbol{i}_s \right) \times \boldsymbol{i}_s$$
$$= \frac{3}{2} p_p \frac{L_m}{L_r} \boldsymbol{\psi}_r \times \boldsymbol{i}_s = \frac{3}{2} p_p \frac{L_m}{L_r} \operatorname{Im} [\boldsymbol{i}_s \boldsymbol{\psi}_r^*]$$

iz $\boldsymbol{\psi}_r = (L_{sr} + L_m) \boldsymbol{i}_r + L_m \boldsymbol{i}_s$ izrazimo

$$\boldsymbol{i}_r = \frac{1}{L_{sr} + L_m} \boldsymbol{\psi}_r - \frac{L_m}{L_{sr} + L_m} \boldsymbol{i}_s$$

in upoštevamo $\boldsymbol{i}_s \times \boldsymbol{i}_s = 0$.

rotorski tok in statorski fluks

$$M_e = \frac{3}{2} p_p L_m \boldsymbol{i}_r \times \boldsymbol{i}_s$$
$$= \frac{3}{2} p_p L_m \boldsymbol{i}_r \times \left(\frac{1}{L_{ss} + L_m} \boldsymbol{\psi}_s - \frac{L_m}{L_{ss} + L_m} \boldsymbol{i}_r \right)$$
$$= \frac{3}{2} p_p \frac{L_m}{L_s} \boldsymbol{i}_r \times \boldsymbol{\psi}_s = \frac{3}{2} p_p \frac{L_m}{L_s} \operatorname{Im} [\boldsymbol{\psi}_s \boldsymbol{i}_r^*]$$

iz $\boldsymbol{\psi}_s = (L_{ss} + L_m) \boldsymbol{i}_s + L_m \boldsymbol{i}_r$ izrazimo

$$\boldsymbol{i}_s = \frac{1}{L_{ss} + L_m} \boldsymbol{\psi}_s - \frac{L_m}{L_{ss} + L_m} \boldsymbol{i}_r$$

in upoštevamo $\boldsymbol{i}_r \times \boldsymbol{i}_r = 0$.

Navorna enačba: Rotorski fluks in statorski fluks

statorski fluks in rotorski fluks

$$
\begin{aligned}
M_e &= \frac{3}{2} p_p \frac{L_m}{L_r} \boldsymbol{\psi}_r \times \boldsymbol{i}_s \\
&= \frac{3}{2} p_p \frac{L_m}{L_r} \boldsymbol{\psi}_r \times \left[\frac{1}{\sigma L_s} \left(\boldsymbol{\psi}_s - \frac{L_m}{L_r} \boldsymbol{\psi}_r \right) \right] \\
&= \frac{3}{2} p_p \frac{L_m}{\sigma L_s L_r} \boldsymbol{\psi}_r \times \boldsymbol{\psi}_s,
\end{aligned}
$$

kjer upoštevamo $\boldsymbol{\psi}_r \times \boldsymbol{\psi}_r = 0$.

iz $\boldsymbol{\psi}_r = (L_{sr} + L_m)\boldsymbol{i}_r + L_m \boldsymbol{i}_s$ izrazimo

$$\boldsymbol{i}_r = \frac{1}{L_{sr} + L_m} \boldsymbol{\psi}_r - \frac{L_m}{L_{sr} + L_m} \boldsymbol{i}_s$$

vstavimo v $\boldsymbol{\psi}_s = (L_{ss} + L_m)\boldsymbol{i}_s + L_m \boldsymbol{i}_r$

$$\boldsymbol{\psi}_s = (L_{ss} + L_m)\boldsymbol{i}_s + L_m \left(\frac{1}{L_{sr} + L_m} \boldsymbol{\psi}_r - \frac{L_m}{L_{sr} + L_m} \boldsymbol{i}_s \right)$$

upoštevamo $L_s = L_{ss} + L_m$ in $L_r = L_{sr} + L_m$ ter delimo z L_s

$$\frac{\boldsymbol{\psi}_s}{L_s} = \boldsymbol{i}_s \left(1 - \frac{L_m^2}{L_s L_r} \right) + \frac{L_m}{L_s L_r} \boldsymbol{\psi}_r,$$

upoštevamo $\sigma = \frac{L_m^2}{L_s L_r}$ ter izrazimo $\boldsymbol{i}_s$

$$\boldsymbol{i}_s = \frac{1}{\sigma L_s} \left(\boldsymbol{\psi}_s - \frac{L_m}{L_r} \boldsymbol{\psi}_r \right).$$

Povzetek: Vse možne enačbe

$\boldsymbol{i}_r$	$\boldsymbol{i}_s$	$\frac{3}{2} p_p L_m \boldsymbol{i}_r \times \boldsymbol{i}_s$
$\boldsymbol{\psi}_s$	$\boldsymbol{i}_s$	$\frac{3}{2} p_p \boldsymbol{\psi}_s \times \boldsymbol{i}_s$
$\boldsymbol{i}_r$	$\boldsymbol{\psi}_r$	$\frac{3}{2} p_p \boldsymbol{i}_r \times \boldsymbol{\psi}_r$
$\boldsymbol{\psi}_m$	$\boldsymbol{i}_s$	$\frac{3}{2} p_p \boldsymbol{\psi}_m \times \boldsymbol{i}_s$
$\boldsymbol{i}_r$	$\boldsymbol{\psi}_m$	$\frac{3}{2} p_p \boldsymbol{i}_r \times \boldsymbol{\psi}_m$
$\boldsymbol{\psi}_r$	$\boldsymbol{i}_s$	$\frac{3}{2} p_p \frac{L_m}{L_r} \boldsymbol{\psi}_r \times \boldsymbol{i}_s$
$\boldsymbol{i}_r$	$\boldsymbol{\psi}_s$	$\frac{3}{2} p_p \frac{L_m}{L_s} \boldsymbol{i}_r \times \boldsymbol{\psi}_s$
$\boldsymbol{\psi}_r$	$\boldsymbol{\psi}_s$	$\frac{3}{2} p_p \frac{L_m}{\sigma L_s L_r} \boldsymbol{\psi}_r \times \boldsymbol{\psi}_s$

Mehanska enačba

$$M_e = J_M \frac{d\omega_{rm}}{dt} + D\omega_{rm} + M_{br}$$

pri čemer je $\omega_{rm} = \omega_r / p_p$ mehanska rotorska hitrost.