Position Error Compensation in Quadrature Analog Magnetic Encoders through an Iterative Optimization Algorithm

Jorge Lara, Student Member, IEEE, *Ambrish Chandra*, Fellow Member, IEEE
École de Technologie Supérieure, University of Québec, Montréal, QC, Canada
jorge_lara_c@yahoo.com; Ambrish.Chandra@etsmtl.ca

***Abstract* — This paper analyses the position error compensation in quadrature analog magnetic encoders through the iterative linear search optimization algorithm of "steepest descent". In current literature, the sine/cosine signals from the encoder are compensated considering only the gain mismatch, the DC offset components and the non-orthogonality. Nevertheless, the magnetic field distortion due to the non-homogeneity of the magnet and the distortion of the signals due to the mechanical misalignment between the rotation axis of the motor shaft, the magnet center and the chip sensor are generally neglected. In this work, all of these factors are taken into account. The results show that a higher order approximation for modeling the harmonic distortion contained in the encoder signals leads to a slight biasing in the convergence of the compensating parameters. However, the unconstrained and multivariate optimization algorithm of steepest descent fairly minimizes the error from the objective function $F(x_n)$ allowing achieve a compensation efficiency of up to 66%, thus increasing the overall accuracy of this kind of magnetic encoders and improving the performance of any application where they provide rotary position feedback.**

***Keywords* — quadrature analog encoders, position error compensation, harmonic distortion, iterative optimization algorithm, optimal convergence.**

I. INTRODUCTION

Rotary position sensors such as resolvers and encoders are commonly used in motor feedback control applications. Their operating principle is mainly based on reluctance, magnetoresistivity, electric field, Eddy-currents and Hall-effect. As opposed to the optical encoders, the magnetic encoders can work under harsh environment conditions of vibration, shock, moisture and dust while providing high temperature operating capabilities, good electromagnetic interference (EMI) performance and immunity to external stray fields. Because of the low-cost of Hall-effect magnetic encoders and their feasibility to provide accurate position measurements they are gaining popularity in aerospace, industrial, instrumentation and medical applications while increasing the attention of researchers to improve their performance by developing new algorithms oriented to compensate for the periodical position error commonly found in practice.

In quadrature analog magnetic encoders based on Hall-effect this error arises from the non-idealities of sine/cosine output signals due to different factors such as the non-homogeneity of the magnetic field in the linear range of the magnet and the imperfect mechanical alignment between the rotation axis of the motor shaft, the magnet center and the mid-point of the Hall-effect sensor array built-in the integrated circuit of the encoder that lead it to work in the non-linear range of the magnet.

Over the past years, different techniques have been proposed to compensate the inherent position error found in both encoders and resolvers. In [1], the information yielded by the Fast Fourier Transform (FFT) is used for this purpose. The harmonics and dc offsets obtained through this time-frequency transformation are subtracted from the non-ideal encoder signals, thus isolating the fundamental components. The non-unit amplitude and quadrature phase mismatch are then straightforwardly compensated. Since the trade-off for a better compensation is a more accurate identification of the harmonic coefficients i.e. magnitude and phase, the leakage effect must be minimized by means of a high resolution FFT in the range of milli-Hz. In practice, this means to have to control the speed of the motor with a deviation lower than 0.1 rpm from the reference speed. This requirement becomes a serious drawback for this technique. In [2], [3] the error is compensated through artificial neural networks (ANN) where the capability of modeling uncertainty and non-linearity is trade-off for the time, hardware resources and power computing required for training the network. These features limit the use of artificial intelligence in low-cost embedded applications. In [4], [5] the authors study a technique based on a pseudo-cross correlation operation that indirectly compensates the error found in the quadrature analog signals. This method identifies the Fourier series coefficients of the signal resulting from calculating the sum of squares of sine and cosine signals. The corresponding quadrature harmonic components are then passed through I controllers that ideally take them to a zero value in steady state. In practice, the output signals from the magnetic encoders present harmonic distortion, mainly the 2nd and 3rd component. Under certain conditions of magnitude, these harmonics would be reflected in the Fourier coefficients as if they come from the gain mismatch and offset, consequently leading to a mistaken correction. According to [6], this algorithm performs well in applications where the total harmonic distortion (THD) is lower than 1%. Nevertheless, in the present application, the mechanical misalignment, the airgap variations and the non-idealities of the magnet cause that the quadrature encoder signals have a THD higher than the unity. Ellipse fitting techniques have also been investigated to compensate the quadrature signals imperfections reflected as deviations from the ideal circle in the Lissajous figure [6], [7]. The minimization problem applied to this ellipse-fitting approximation is inherently incapable of modeling the harmonic distortion commonly found in magnetic analog encoders. Moreover, the presence of these harmonic components biases the convergence to non-optimal ellipse parameters that in some cases increase the position error rather than reduce it. In some other works [8], [9] the periodical position error is compensated by reducing the resultant d-axis component ideally to zero in steady state. Nevertheless, this technique disregards the effects of the harmonic distortion from the sensor signals. In [10], [11] the total position error resulting from all the non-idealities of the analog encoder signals is translated into a compensating code stored in a read-only memory (ROM). Then, a static compensation is performed by consulting the pre-built look-up table (LUT). With the aim to achieve an accurate resolver to digital conversion, the double decoupled synchronous reference frame phase locked-loop (DDSRF-PLL) has been studied as well [12]. This algorithm is based on the decomposition of the unbalanced demodulated resolver signals into their positive and negative rotating components. The systematic error is then compensated by taking the q-axis component of the positive sequence to a zero value by means of the PID controller found inside the loop of a second order state filter. The inherent delay of this structure is trade-off for better tracking capability of its input variable. The offset error exhibited when trying to follow an acceleration profile constraints its use to the low speed range. This type of algorithms that indirectly compensate the error does not allow carrying out a parametric analysis of the deviations from the ideal quadrature signals. Hence, they are incapable of compensating the error after performing an off-line calibration. The same situation occurs when using an Adaptive Notch Filter ANF-PLL or an Advanced Adaptive Digital AAD-PLL structure [13], [14]. An algorithm capable of identifying the compensating parameters is particularly attractive because the correction inherently works in real-time and does not present restrictions in neither rotor speed nor acceleration rates. In [15], [16] an interesting procedure is proposed to find the parameters that compensate the analog encoder signals by minimizing a position error function through an iterative linear search optimization algorithm. The on-line convergence of the compensating parameters and the property to adapt to real-time varying conditions are attractive characteristics for motor feedback control applications. Therefore, the present work analyses its capabilities for compensating the quadrature analog signals from Hall-effect magnetic encoders under conditions of harmonic distortion. Its convergence properties and efficiency to reduce the resulting position error are also evaluated using a higher order approximation for modeling the non-homogeneity of the magnetic field as well as the mechanical misalignment between the rotation axis of the motor shaft, the magnet center and the chip sensor.

This work is organized as follows: section II presents the effects in the resulting position error from various non-idealities commonly found in the sine/cosine encoder signals. In section III, the gradient search optimization algorithm of "steepest descent" is described. The linear compensation capabilities of this iterative algorithm are analyzed in section IV while its non-linear compensation capabilities are presented in section V. Finally, the conclusions and some important remarks are given.

II. Effects of Various Non-Idealities from Sine/Cosine Encoder Signals

The main non-idealities commonly found in the quadrature analog magnetic encoder signals, the resulting periodical position errors and their modeling equations are summarized in Table I [10], [17], [18].

TABLE I. Summary of the Main Non-Idealities of Sine/Cosine Signals from Magnetic Encoders

Non-ideality	Periodical position error	Error modeling	Units
Gain mismatch [(a)]	2nd harmonic	$\frac{\Delta G}{2}\sin(2\theta)$	rad
DC offset [(b)]	1st harmonic	$\Delta C\sin(\theta)$	rad
Non-orthogonality	2nd harmonic	$-\frac{\Phi}{2}[1-\cos(2\theta)]$	deg
Misalignment	1st harmonic	$\frac{0.1D}{100}\sin(\theta+\alpha)$	deg
Sensor tilt	Equivalent to gain mismatch	$\left[\frac{1-\cos(\beta)}{2}\right]\sin(2\theta)$	rad
Magnet tilt and airgap increasing	Reduction of magnetic field (SNR decreases)	No error	No unit
Magnetic sensor non-linearity	4th harmonic	$\frac{k_3}{4+3k_3}\sin(4\theta)$	deg
Harmonic distortion	Equivalent to (a) & (b) (n-1)th harmonic	$-\sqrt{2}K_0\cos\left(\theta+\frac{\pi}{4}\right)-\sum_{n=2}^{\infty}K_n\sin[(n-1)\theta]$	rad

These non-idealities are straightforwardly identified by looking at the geometrical effect produced on their equivalent Lissajous figure (see Fig. 1). It can be also observed the equivalent position error and the resulting speed obtained from a classical PLL.

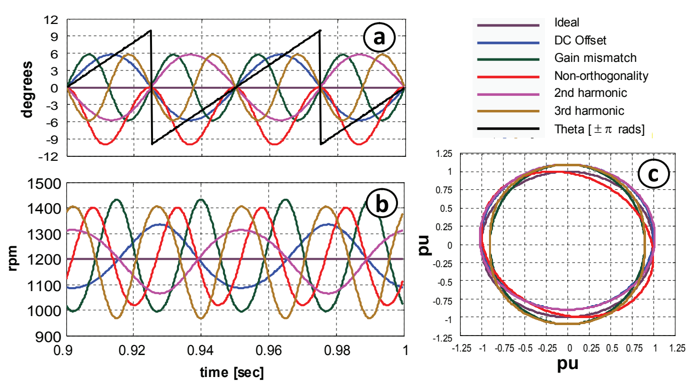

Fig. 1. Effects of the main non-idealities of quadrature encoder signals in: (a) Position error. (b) Calculated speed. (c) Lissajous figures.

In order to evaluate the maximum position error obtained from the combination of non-ideal sine gain, cosine gain, sine offset and cosine offset a sweep of these parameters has been carried out from -10% to +10% . By taking the absolute value of the partial error from the first 2 sweeps (gain and offset of sine signal), the 3D graphs presented in Fig. 2 have been obtained. It can be observed that the maxima are located at the boundary values of the evaluation where the position error can reach up to 18° degrees.

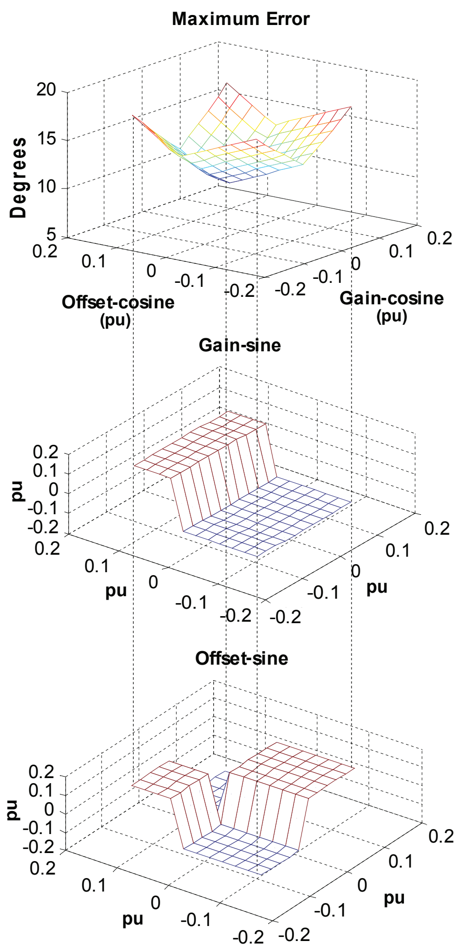

Fig. 2. Variation of sine/cosine gain mismatch and offset in the range of ±10% to locate and calculate the maximum position errors.

III. Gradient Search Optimization Algorithm of "Steepest Descent"

In this work, the iterative, unconstrained and multivariate minimization algorithm used to compensate the rotor position error is the linear search method known in literature as "*steepest descent*" wherein the search direction of the analyzed function F(x) is deduced from its negative gradient, i.e. the vector field that points in the opposite direction of biggest increase of F(x) at any given point x_n. The iterative rule is expressed as:

$$X_{n+1} = X_n - \alpha \nabla F(x_n) \quad (1)$$

where the learning rate α controls the step size to go downhill on the search direction and reach the local minimum.

The learning rate value is determined taking care of the trade-off between speed convergence and solution accuracy. By choosing a big step the algorithm converges faster, but the parameters found correspond to a coarse solution that in some cases remains oscillating around the local minimum. On the other side, if the learning rate is small, the algorithm converges to more accurate parameters equivalent to a fine solution, but at the expense of more iterations, power computing and time. On the other hand, the theoretical termination criterion is a gradient equal to zero. Nevertheless, this condition is not always reached because of the finite word length effects in digital processors. Hence, when dealing with functions that do not decrease at a very small rate along the search line, a practical criterion to stop the algorithm in a finite amount of time is given by:

$$\|\nabla F(x_n) < \varepsilon\| \quad (2)$$

where ε is a pre-determined tolerance error.

A. Compensation of the Quadrature Analog Encoder Signals by means of the Iterative Algorithm of "Steepest Descent"

The complete modeling of the quadrature analog encoder signals is expressed in (3)-(5). In this model, there are five fundamental parameters to compensate, i.e. the gains G_s and G_c , the offsets U_s and U_c, and the quadrature phase-shift Φ as well as 2(m-1) harmonic parameters, where $m \in N > 1$. The variable D represents the m order approximation for the harmonic distortion while the H stands for either the S (sine) or C (cosine) signal.

$$S = G_s \sin(\theta) + U_s + D_s \quad (3)$$

$$C = G_c \cos(\theta + \Phi) + U_c + D_c \quad (4)$$

$$D_H = c_2 H^2 + c_3 H^3 + \cdots + c_m H^m \quad (5)$$

In this study, the algorithm presented in [15], [16] has been extended by adding the quadrature phase-shift correction to the third order approximation. From here, the following matrices are deduced:

$$K_n = \begin{bmatrix} k_{c,n} & 1 & k_{c,n}^2 & k_{c,n}^3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & k_{s,n} & 1 & k_{c,n} & k_{s,n}^2 & k_{s,n}^3 \end{bmatrix} \quad (6)$$

$$X_n = \left[x_{1,n}\ x_{2,n}\ x_{3,n}\ x_{4,n}\ x_{5,n}\ x_{6,n}\ x_{7,n}\ x_{8,n}\ x_{9,n} \right]^T \quad (7)$$

where $k_{c,n}$ and $k_{s,n}$ are the sample values of the sine/cosine signals and $x_{1,n\text{-}9,n}$ are the n-th update of each of the compensating parameters found iteratively. Since the objective function is defined as:

$$F(x) = \sum_n \|e_n\|^2 \quad (8)$$

with the error e_n deduced from the trigonometric identity of the ideal circle with unity radius:

$$\sin^2(\theta) + \cos^2(\theta) = 1 \quad (9)$$

After substitution of (6) and (7) into (8) using (9) and applying the nabla operator ∇ to deduce the vector of first partial derivatives of the function F(x):

$$\nabla F(X) = \begin{bmatrix} \frac{\partial}{\partial x_1} F(X) \\ \frac{\partial}{\partial x_2} F(X) \\ \vdots \\ \frac{\partial}{\partial x_i} F(X) \end{bmatrix} \tag{10}$$

the iterative equation to update the compensating parameters results in [15]:

$$X_{n+1} = X_n - \alpha \sum_n K_n^T \cdot K_n \cdot X_n \cdot [\, X_n^T \cdot K_n^T \cdot K_n \cdot X_n - 1] \tag{11}$$

IV. Linear Compensation Capabilities of "Steepest Descent" Algorithm

In order to carry out a preliminary analysis of the linear compensation capabilities of the gradient search algorithm of "steepest descent", the harmonic distortion is considered to be zero. Hence, (3) and (4) are reduced to:

$$S = G_s \sin(\theta) + U_s \tag{12}$$

$$C = G_c \cos(\theta + \Phi) + U_c \tag{13}$$

The iterative equation (11) has been programmed in MATLAB® code and tested with a learning rate $\alpha = 0.001$ and a pair of quadrature signals modeled by (12) and (13). The obtained results are shown in Fig.3, where it can be observed how the vector X_n from (11) starts at the pre-specified initial conditions and evolves in time converging optimally to the reference values (dashed lines), thus minimizing the objective function (8) and consequently compensating the position error from the encoder.

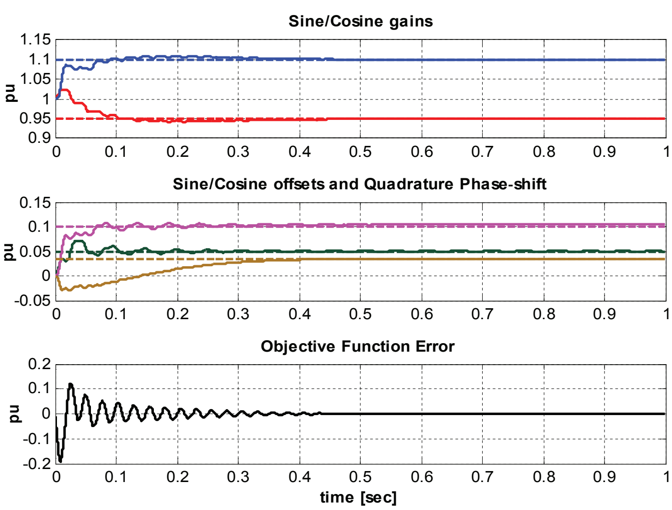

Fig. 3. References of the encoder signal parameters (dashed lines) and convergence values (solid lines) obtained from the steepest descent algorithm to compensate for the non-ideal gains, offsets and quadrature phase-shift while minimizing the error from the objective function F.

In order to evaluate the generality of convergence of this iterative gradient search compensation algorithm, 100 random simulations have been carried out allowing the quadrature analog signals parameters to vary within the limits specified in Table II.

TABLE II. Limits of the Encoder Signals Parameters to Evaluate the Convergence of the Compensation Algorithm under Linear Conditions

Parameter		Gain (%)	Offset (%)	Quadrature phase (°)
Quadrature signal	Sine	±10	±10	0
	Cosine	±10	±10	±5

Fig. 4 shows one of the five random matrices obtained when varying the main non-idealities presented in the quadrature analog encoder signals, i.e. sine gain, cosine gain, sine offset, cosine offset and quadrature phase-shift. The convergence error matrix obtained from the steepest descent algorithm is also shown. In general, the error of convergence remains below 0.005 per unit (PU) for all the parameters. In the case of the phase-shift, the equivalent error is about 0.3° degrees.

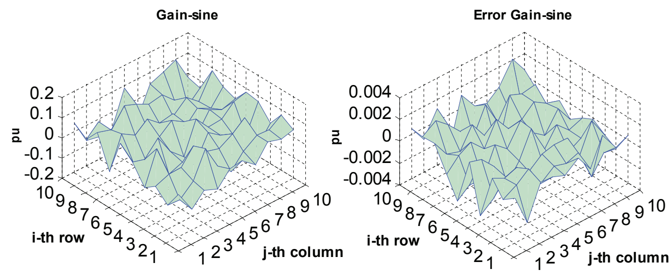

Fig. 4. (Left) 10x10 random matrix obtained when varying the five main non-idealities presented in the quadrature analog encoder signals. (Right) 10x10 convergence error matrix obtained from the steepest descent algorithm.

The linear compensation capabilities of the gradient search algorithm of steepest descent are clearly visible by looking at the resulting 10x10 position error matrices presented in Fig.5. It can be observed how the position error is reduced from 4°<e<14° to less than 0.2° degrees. The corresponding 100 Lissajous figures are also shown at the bottom. It is noteworthy that before the compensation the magnitude of their radius varies in the range of 0.8<|R|<1.2 while after the compensation this range is limited to 0.999<|R|<1.001.

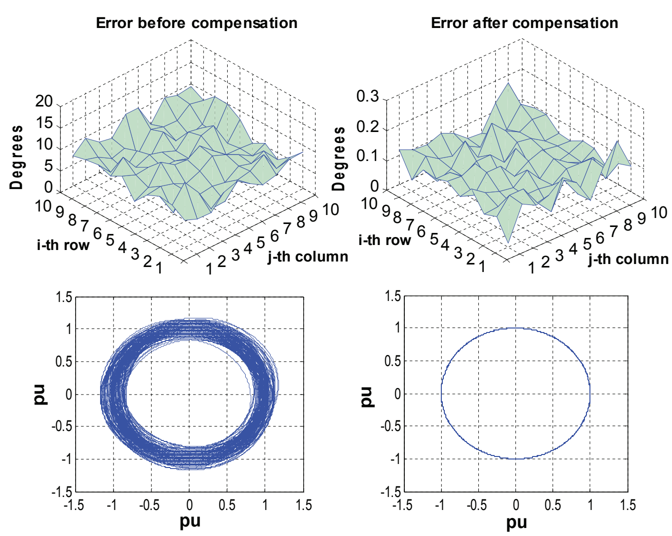

Fig. 5. (Top) 10x10 position error matrices obtained when varying the five main non-idealities in the sine/cosine encoder signals. (Bottom) Lissajous figures obtained for the 100 random simulations. (Left) Before compensation. (Right) After compensation.

V. NON-LINEAR COMPENSATION CAPABILITIES OF "STEEPEST DESCENT" ALGORITHM

The compensation capabilities of the gradient search algorithm of "steepest descent" when dealing with the harmonic distortion from the quadrature analog encoder signals are analyzed in this section.

Mainly, there exist three factors that directly influence on the performance of this gradient search algorithm:

1) The objective function (8) based on the trigonometric identity of the ideal circle with unity radius (9) is an error function dependant on both quadrature signals. Hence, the error contribution from each of the signals cannot be evaluated separately. In some cases, the quadrature signals contain a complementary harmonic distortion that reduces mutually or cancels considerably each other when evaluating (9). Under these particular conditions, the combined total error is minimized and the trigonometric identity is satisfied, nevertheless, the algorithm converges to biased compensating parameters.

2) The harmonic distortion modeled as an integer power of the fundamental component as described in (5) inherently assumes that all of the harmonics are perfectly in phase. Since it is not the case in the real quadrature encoder signals, the solution found by means of the gradient search iterative algorithm is just an approximation. As a result, the compensation of the encoder signals with these approximated parameters slightly reduces the error compensation efficiency.

3) By looking at the equivalent trigonometric identities (14), (15) and considering only the first two terms in (5), it can be observed that there exists a trade-off when compensating for the distortion coefficients because they are a function of not only the 2^{nd} and 3^{rd} harmonic, but also of an offset and a fundamental component, respectively:

$$\sin^2(\theta) = \frac{1-\cos(2\theta)}{2} \quad ; \quad \cos^2(\theta) = \frac{1+\cos(2\theta)}{2} \tag{14}$$

$$\sin^3(\theta) = \frac{3\sin(\theta)-\sin(3\theta)}{4} \quad ; \quad \cos^3(\theta) = \frac{3\cos(\theta)+\cos(3\theta)}{4} \tag{15}$$

Thus, while the optimization algorithm evolves in time trying to compensate for the distortion coefficients, the parameters of the fundamental component are disturbed and vice versa. As a result, the iterative algorithm converges to an intermediate solution that partially satisfies both conditions.

These concepts are exemplified in Fig. 6 for a third order approximation consisting of nine compensating parameters as expressed in (7).

It can be observed how some of these parameters converge to values slightly biased from the reference (dashed lines) even though the error from the objective function is considerably minimized. However, it can be also noticed that the steepest descent algorithm fairly compensates the sine/cosine encoder signals (see Fig. 6d & 6e).

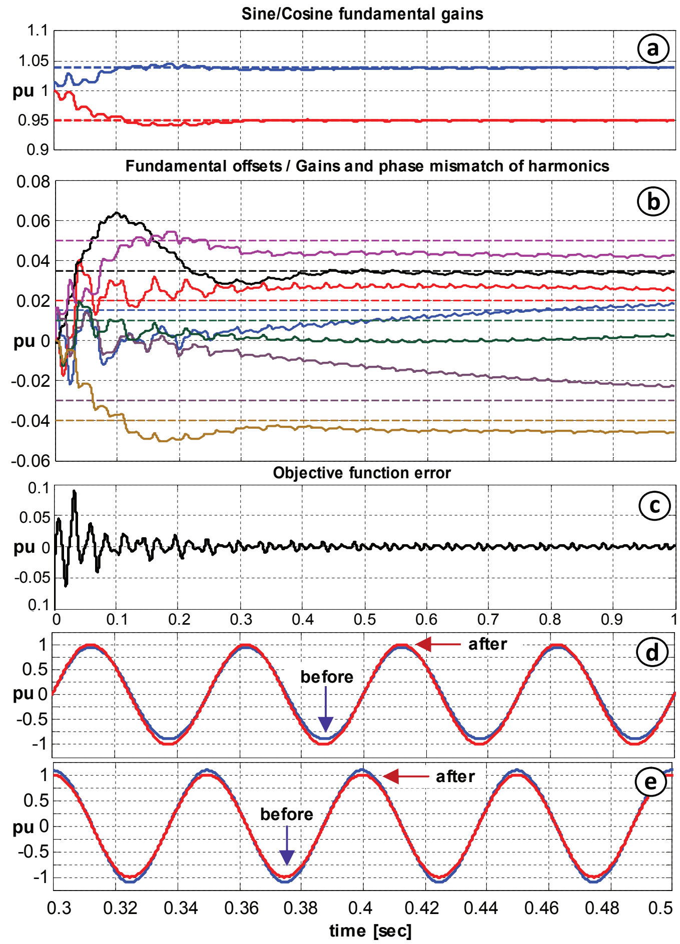

Fig. 6. (Top) Biasing of compensating parameters due to the presence of harmonic distortion in the sine/cosine encoder signals. (Bottom) Comparison of sine/cosine encoder signals before and after the compensation.

In order to perform a complete analysis of the generality of convergence of this iterative gradient search compensation algorithm under the presence of harmonic distortion, 100 random simulations have been carried out allowing the quadrature signals parameters to vary within the limits specified in Table III.

TABLE III. LIMITS OF THE ENCODER SIGNALS PARAMETERS TO EVALUATE THE CONVERGENCE OF THE COMPENSATION ALGORITHM UNDER HARMONIC DISTORTION CONDITIONS

Quadrature signal	Parameter			
	Gain (%)	Offset (%)	Quadrature phase (°)	Harmonics phase (°)
Sine (θ)	±10	±10	0	0
Sine (2θ)	± 2	0	0	±10
Sine (3θ)	± 2	0	0	±10
Cosine (θ)	±10	±10	±5	0
Cosine (2θ)	± 2	0	0	±10
Cosine (3θ)	± 2	0	0	±10

The position error matrices and the corresponding Lissajous figures obtained for these 100 random simulations are shown in Fig. 7 while the statistics of the results are summarized in Table IV. It is noteworthy that even though some of the compensating parameters present a small biasing from the optimal values, the steepest descent algorithm is capable of reducing the mean position error from 6.96° to only 2.37°. This value represents a compensation efficiency of 66%.

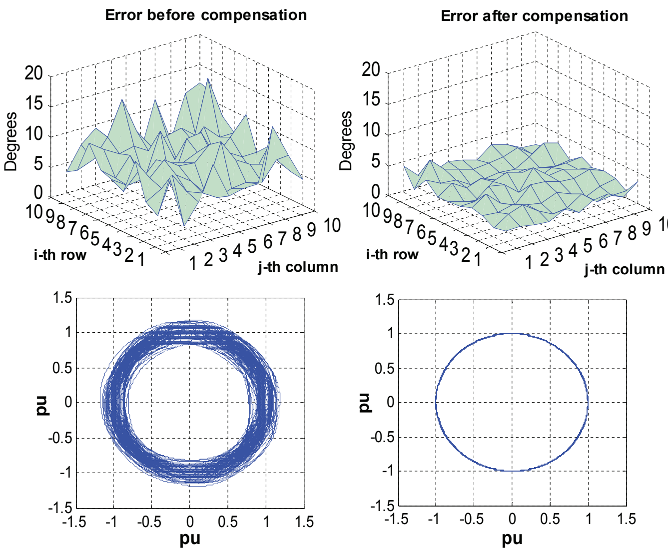

Fig. 7. (Top) 10x10 position error matrices obtained when varying the non-idealities of the encoder signals including the harmonic distortion. (Bottom) Lissajous figures obtained for the 100 random simulations. (Left) Before compensation. (Right) After compensation.

TABLE IV. SUMMARY OF THE RESULTS OBTAINED FROM THE CONVERGENCE OF THE STEEPEST DESCENT ALGORITHM UNDER HARMONIC DISTORTION CONDITIONS

	Error			Compensation efficiency
	μ [°]	σ [°]	max [°]	$[1-(\mu_{after}/\mu_{before})]x100$ [%]
Before	6.96	2.67	15.46	
After	2.37	0.96	5.34	66

VI. CONCLUSIONS

The linear and non-linear capabilities of the gradient search algorithm of "steepest descent" to compensate for the position error in analog magnetic encoders have been examined in this paper. It has been demonstrated that if the harmonic distortion contained in the encoder signals is negligible, the simplified modeling of these signals leads to an optimal compensation of the position error. On the other side, if the harmonic distortion in considered in the model, the use of a higher order approximation results in a slight biasing of the convergence parameters from the ideal values. However, in spite of these deviations, the 100 random tests carried out show that the steepest descent algorithm fairly minimizes the objective function allowing achieve a compensation efficiency of 66%, thus increasing the accuracy of this kind of quadrature analog magnetic encoders and improving its performance in rotary position feedback applications. The experimental verification of this iterative compensation algorithm running in real-time on a DSP from TI™ is part of the future work of this research.

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