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~~**Proof (ctd.)**~~

~~**Claim:** $\alpha_P(e, U_{i-1}) =$ active or $\alpha_P(e, U_i) =$ active iff (1) $\exists S \subseteq r_{3i-2}.\ e \in E(T_{P, U^S_{3i-2}})$ or (2) $\exists S \subseteq r_{3i-1}.\ e \in E(T_{P, U^S_{3i-1}})$ or (3) $\exists S \subseteq r_{3i}.\ e \in E(T_{P, U^S_{3i}})$.~~

Definition: The update sequence R_B induced by block sequence $B = (b_1, \dots, b_\ell)$ is defined as follows: Every block $b \in b_i$ is updated in rounds $i, i+1, i+2$, $i \in [\ell]$.

~~**Lemma:** A block sequence $B = \{b_1, \dots, b_\ell\}$ is feasible iff the induced update sequence $R_B = \{r_1, \dots, r_{\ell+2}\}$ is feasible.~~

~~**Proof.**~~

~~We show that (1) r_1 ~~satisfies~~ obeys the consistency rule, (2) r_{i+1} obeys the consistency rule iff inequality ~ is satisfied for i and every edge e, and (3) $r_{\ell+2}$ obeys the consistency rule iff $r_{\ell+1}$ obeys the consistency rule.~~

Ignoring capacity constraints

Lemma: For every $P \in \mathcal{P}$, every $i \in [\ell+2]$, and every $S \subseteq r_i$, P is transient for U_i^S.

Proof.

We show the following. Let $P \in \mathcal{P}$.

(1) For every $S \subseteq r_1$, P is transient for $U_1^S = S$.

(2) ~~Let~~ For every $i \in [\ell]$ and ~~For~~ every $S \subseteq r_{i+1}$, P is transient for U_i^S.

(3) For every $S \subseteq r_{\ell+2}$, P is transient for $U_{\ell+2}^S$ iff P is transient for $U_{\ell+2}^{\{\}} = U_{\ell+1}^{r_{\ell+1}}$.

(1). Let $S \subseteq r_1 \subseteq \bigcup_{P \in \mathcal{P}} V(P^u - P^o) \times \{P\}$. Hence $\alpha_P(S, 6)$ still contains the unique (s,t)-flow P^o.

(2). Suppose not. Then let $i \in [\ell]$ be minimum and $S \subseteq r_{i+1}$ be minimal such that P is not transient for U_{i+1}^S. Since $U_{i+1}^{\{\}} = U_i^{r_i}$ and i is minimum, $S \neq \{\}$. Then let (v, P') be any element in S. Since S is minimal, $P' = P$ and P is transient for $U_{i+1}^{S - \{(u,P)\}}$. ~~We consider the cases $u \in V(P$~~

Claim: For every $P \in \mathcal{P}$, $i \in [\ell+2]$, $S \subseteq r_i$, and (type-3) type-1 update $u \in S$, P is transient for U_i^S iff P is transient for U_i^{S-u}.

~~Hence v is the start of some P-block b.~~

Corollary: For every $P \in \mathcal{P}$ and every $i \in [\ell]$, P is transient for $U(B_i)$.

Proof.

Invoke above lemma for P, $i+1$, and $S := r_{i+1}|_{\leq i}$, that is r_{i+1} restricted to blocks in b_{i-1} and b_i.

Definition:

$$\alpha_P((u,v),B) = \begin{cases} \text{active if } b(u,P)\notin B \text{ and } (u,v)\in E(P^o) \\ \text{active if } b(u,P)\in B \text{ and } (u,v)\in E(P^n) \\ \text{inactive otw.} \end{cases}$$

Lemma: $e\in E(T_{P,u(B)})$ iff $\alpha_P(e,B)=$ active.

Definition: A block sequence $B=(b_1,\dots,b_\ell)$ is feasible if for every $i\in[\ell]$ and every $e\in E$,

$c(e)\geq\sum_{P:\alpha_P(e,B_{i-1})=\text{active or }\alpha_P(e,B_i)=\text{active}} d_P.$

Lemma: A block sequence $B=(b_1,\dots,b_\ell)$ is feasible iff the induced update sequence $R_B=(r_1,\dots,r_{\ell+2})$ is feasible.

Proof.