29.01.23

<u>Definition:</u> A block sequence $B = (b_1, \dots, b_c)$ is an ordered partition of the set of blocks.

<u>Notation:</u> $B_i = \bigcup_{j=1}^{i} b_j$

* $= (b_1, \dots, b_c)$

<u>Definition:</u> Let B be a block sequence. The update sequence $R_B = (r_1, \dots, r_{c+2})$ induced by B is defined as follows: For every $i \in [\cancel{1}, r+2]$, $r_i = \{(v,P) : b(v,P) \in b_i, v \in V(P^u - P^o)\} \cup \{(v,P) : b(v,P) \in b_{i-1}, v \in V(P^o \cap P^u)\} \cup \{(v,P) : b(v,P) \in b_{i-2}, v \in V(P^o - P^u)\}$, where $b_{-1} := \{\}$ ~~and~~ $b_0 := \{\}$, $b_{c+1} := \{\}$, and $b_{c+2} := \{\}$.

* We now define, given a block sequence $B = (b_1, \dots, b_c)$, the update sequence $R_B = (r_1, \dots, r_{c+2})$ induced by B. The idea is update every block in b_i in rounds r_i, r_{i+1} and r_{i+2} according to Lemma 1.

We might need a different definition, as we're ignoring capacity constraints here.

<u>Lemma:</u> Let $B = (b_1, \dots, b_c)$ be a block sequence and $R_B = (r_1, \dots, r_{c+2})$ be the induced update sequence. Then, for every $P \in \mathcal{P}$, $i \in [1, c+2]$, $S \subseteq r_i$, P is transient for U_i^S.

~~Proof.~~ <u>Definition:</u> Let $(v,P) \in V \times \mathcal{P}$. Then we call (v,P) be an update

- a type-1 update if $v \in V(P^u - P^o)$,
- 2 $V(P^o \cap P^u)$, and
- 3 $V(P^o - P^u)$.

By definition, r_i contains the type-1 updates for induced by blocks in b_i, the type-2 ...

<u>Proof.</u>

Let $P \in \mathcal{P}$.

<u>Claim:</u> Let $i \in [1, c+2]$ and $S \subseteq r_i$. Then:

(i) For every type-1 update $(v,P) \in S$, P is transient for U_i^S iff P is transient for $U_i^{S - \{(v,P)\}}$.

(ii) ↯

<u>Proof.</u>

TODO I think this proof depends significantly on my definition of blocks.

(i).

We show the following, which implies the ~~lem~~ lemma:

(1) For every $S \subseteq r_1$, P is transient for $U_1^S = S$.

(2) For every $i \in [c]$ and $S \subseteq r_{i+1}$, P is transient for U_{i+1}^S.

(3) For every $S \subseteq r_{c+2}$, P is transient for U_{c+2}^S iff P is transient for $U_{c+2}^{\{\}} = U_{c+1}^{r_{c+1}}$.

Let $S \subseteq r_1$.

(1). By definition, r_1 contains only type-1 updates. Hence, by Claim ~ (i), P is transient for U_1^S iff P is transient for $U_1^{\{\}} = \{\}$, which ~~is the case~~ holds.

Why?

(3). Let $S \subseteq r_{\ell+2}$. By definition, $r_{\ell+2}$ contains only type-3 updates. Hence, by Claim ~(ii), P is transient for $U_{\ell+2}^{S}$ iff P is transient for $U_{\ell+2}^{\{\}} = U_{\ell+1}^{r_{\ell+1}}$.

(2). ~~Let $i \in [\ell]$ and $S \subseteq r_{i+1}$.~~ Suppose not. Then let $i \in [\ell]$ be minimum and $S \subseteq r_{i+1}$ be minimal such that P is not transient for U_{i+1}^{S}. Since i is minimum, $S \neq \{\}$; otherwise $U_{i+1}^{S} = U_{i+1}^{\{\}} = U_i^{r_i}$. Then let (v, P') be any ~~element~~ update in S. Since S is minimal, $P' = P$ and P is transient for $U_{i+1}^{S - \{(v,P)\}}$. Hence, by Claim ~, (v, P) is a type-2 update. This requires that $v \in V(P)$.

By definition, all type-1 updates for $b(v, P)$ are contained in r_i .... TODO

Note that Lemma ~ implies that, for every $P \in \mathcal{P}$, $i \in [\ell+2]$, $S \subseteq r_i$, T_{P, U_i^S} exists. (s, t)-flow

<u>Definition:</u> $\alpha_P((u,v), B) = \begin{cases} \text{active if } b(u,P) \notin B \text{ and } (u,v) \in E(P^o) \\ \text{active if } b(u,P) \in B \text{ and } (u,v) \in E(P^n) \\ \text{inactive otw.} \end{cases}$

<u>Definition:</u> A block sequence $B = (b_1, \ldots, b_\ell)$ is feasible if for every $e \in E$ and every $i \in [\ell]$,

$$c(e) \geq \sum_{P \in \mathcal{P} : \alpha_P(e, B_{i-1}) = \text{active or } \alpha_P(e, B_i) = \text{active}} d_P. \quad (1)$$

<u>Lemma:</u> ~~Let $e \in E$~~ Let $\underline{B \subseteq \mathcal{B}}$, $e \in E$, $P \in \mathcal{P}$. Then $e \in E(T_{P, U(B)})$ iff $\alpha_P(e, B) =$ active. We need a new name for the set of all blocks

<u>Lemma:</u> If a block sequence $B = (b_1, \ldots, b_\ell)$ is feasible, then the induced update sequence $\mathcal{R}_B = (r_1, \ldots, r_{\ell+2})$ is feasible. Also, notation $U(B)$.

<u>Proof.</u>