15.01.23

Proof (ctd.)

We now show that there is a feasible update sequence R with l rounds for update flow network G iff there is a feasible update sequence $\tilde{R}$ with l rounds for update flow network $\tilde{G}$.

"$\Rightarrow$"

Let $R = (r_1, \dots, r_l)$ be a feasible update sequence for G.

We set $\tilde{R}(v, \tilde{F}_i) = R(v, F_i)$ for every $v \in V$, ~~$i \in [k]$~~ $F_i \in \mathcal{F} - \{F, \overline{F}\}$,

$\tilde{R}(\tilde{u}_{F^0}, \tilde{F}_i) := R(u_{F^0}, F_i), \dots$

$$\tilde{R}(v, \tilde{F}) := \begin{cases} \tilde{R}(v, \tilde{F}) & \text{if } v \in V(F^o \cup F^u) \\ \tilde{R}(v, \tilde{\overline{F}}) & \text{otw.} \end{cases}$$

$\tilde{R}(w, \tilde{F}) := R(s, F)$

Induction?

We show that $\tilde{R} = (\tilde{r}_1, \dots, \tilde{r}_l)$ is a feasible update sequence for $\tilde{G}$. Suppose not. Then let $i \in [l]$ be minimum and $S \subseteq \tilde{r}_i$ be minimal such that ~~there~~ $\mathcal{P}$ is no transient family for U_i^S.

~~Case $\mathcal{T} = \{T_{P, U_i^S}\}_{P \in \mathcal{P}}$ is not valid.~~

~~Then let $e \in E(\alpha(U_i^S, \tilde{G}))$ such that $c(e) < \sum_{P \in \mathcal{P}: e \in E(P)} d_P$.~~

If $S = \{\}$, then $i = 1$ (otw. i would not be minimum as $U_{i-1}^{\tilde{r}_{i-1}} \dots$).

BRUNNEN

15.01.23

<u>Definition:</u> A block sequence $\mathcal{B} = (b_1, \dots, b_\ell)$ is an ordered partition of the set B of blocks.

The update sequence $R_\mathcal{B}$ induced by block sequence $\mathcal{B} = (b_1, \dots, b_\ell)$ is defined as follows:

Every block $b \in b_i$ is updated in rounds $r_{3(i-1)+1}, r_{3(i-1)+2}, r_{3(i-1)+3}$, $i \in [\ell]$. Remove any rounds $r = \{\}$.

A block sequence $\mathcal{B} = (b_1, \dots, b_\ell)$ is feasible for every $i \in [\ell]$ and every $e \in E$,

$c(e) \geq \sum_{p:\, \alpha_p(e, \bigcup_{j=1}^{i-1} U(b_j)) = \text{active or } \alpha_p(e, \bigcup_i U(b_i)) = \text{active}} d_p.$

<u>Lemma:</u> A block sequence $\mathcal{B} = (b_1, \dots, b_\ell)$ is feasible iff the induced update sequence $R_\mathcal{B}$ is feasible.

<u>Proof.</u>

~~"$\Rightarrow$"~~

~~Suppose $R_\mathcal{B} = (r_1, \dots, r_{3\ell})$~~ $(r_1, \dots, r_{2\ell+1})$ ~~is feasible.~~

~~We show that above inequality is satisfied for~~

"$\Leftarrow$"

induction?

$U_i := \bigcup_{j=1}^{i} U(b_j)$

Suppose $R_\mathcal{B} = (r_1, \dots, r_{3\ell})$ is feasible, and $\mathcal{B} = (b_1, \dots, b_\ell)$ is not. Then let $i \in [\ell]$ be minimum and $e \in E$ be such that $c(e) < \sum_{p:\, \alpha_p(e, U_{i-1}) = \text{active or } \alpha_p(e, U_i) = \text{active}} d_p$.

[We have to relate $\alpha_p(e, U_i) = \text{active}$ with $e \in E(T_{p, U_j^S})$ and hence U_i with U_j^S for appropriate j, S.

<u>Claim:</u> $U_i =$ ~~for any S~~ $U_{3i}^{r_{3i}}$ for all $i \in [\ell]$

<u>Proof by induction on i.</u>

<u>Induction base, $i = 1$.</u> $U_1 = U(b_1)$.

$U_3^{r_3} = r_3 \cup \bigcup_{j=1}^{3-1} r_j = r_1 \cup r_2 \cup r_3 = U(b_1)$.

<u>Induction step $i \to i+1$.</u> $U_{3(i+1)}^{r_{3(i+1)}} = r_{3(i+1)} \cup \bigcup_{j=1}^{3(i+1)-1} r_j =$ ~~$r_{3(i+1)-2} \cup r_{3(i+1)-1} \cup r_{3(i+1)} \cup \bigcup_{j=1}^{3i} r_j$~~

$r_{3(i+1)} \cup r_{3(i+1)-1} \cup r_{3(i+1)-2} \cup r_{3i} \cup \bigcup_{j=1}^{3i-1} r_j = r_{3(i+1)} \cup r_{3(i+1)-1} \cup r_{3(i+1)-2} \cup U_{3i}^{r_{3i}} =$

$r_{3(i+1)} \cup r_{3(i+1)-1} \cup r_{3(i+1)-2} \cup U_i = U_i \cup r_{3i+1} \cup r_{3i+2} \cup r_{3i+3} = U_{i+1}$

We show that above inequality is satisfied for $i \in [\ell]$ iff r_{3i-2}, r_{3i-1}, and r_{3i} obeys the consistency rule.

"$\Leftarrow$"

Suppose r_{3i-2}, r_{3i-1}, and r_{3i} obeys the consistency rule.

[Can we get $\alpha_p(e, U_{i-1}) = \text{active}$ or $\alpha_p(e, U_i) = \text{active}$ iff $e \in E(T_{p, U_{3i}^S})$ for some $S \subseteq r_{3i}$?