18.01.23

Proof (ctd.)

"⇐"

This requires that every feasible update sequence can be transformed into a feasible block sequence.

Let $\mathcal{B}$ be a feasible block sequence for G. We set $\sigma(x_j) := 1$ iff $\mathcal{B}(b(x^j, \overline{X})) > \mathcal{B}(b(s, B))$ and show that σ is indeed a satisfying assignment for C. Let $C_i = (l_1 \vee l_2 \vee l_3 \vee l_4)$ be a clause.

We show that C_i is satisfied by σ by obtaining a literal in C_i that evaluates to 1.

Consider $\mathcal{B}(b(s,B))$. By ~~the~~ Claim ~, $\mathcal{B}(b(s,B)) > \mathcal{B}(b(u^i, L))$ or $\mathcal{B}(b(s,B)) > \mathcal{B}(b(u^i,R))$.

Case $\mathcal{B}(b(s,B)) > \mathcal{B}(b(u^i,L))$. The other case is analogous. By ~~claim~~ Claim ~, $\mathcal{B}(b(u^i,L)) > \mathcal{B}(b(\tilde{x}^i_{12}, \tilde{B}))$. By Claim ~, $\mathcal{B}(b(\tilde{x}^i_{12}, \tilde{B})) > \mathcal{B}(b(u^i_{12}, \tilde{L}))$ or $\mathcal{B}(b(\tilde{x}^i_{12}, \tilde{B})) > \mathcal{B}(b(u^i_{12}, \tilde{R}))$.

Subcase $\mathcal{B}(b(\tilde{x}^i_{12}, \tilde{B})) > \mathcal{B}(b(u^i_{12}, \tilde{R}))$. The other case is analogous. ~~By Claim ~, $\mathcal{B}(b(u^i_{12}, \tilde{R})) > \mathcal{B}(b(u^i_2, B_0))$ if l_2 is negative and $\mathcal{B}(b(u^i_2, B_1))$ else.~~

Subsubcase l_2 is negative. ~~The other case is analogous.~~ Let j be such that $l_2 = \overline{x}_j$. By Claim ~, $\mathcal{B}(b(u^i_2, B_0)) > \mathcal{B}(b(x^j_0, \overline{X}))$.

~~Hence, by definition, $\sigma(x_j)$~~ We now have

~~$\mathcal{B}(b(x^j_{01}$~~ Let x_j be such that $l_2 = x_j$ or $l_2 = \overline{x}_j$. By Claim ~, $\mathcal{B}(b(u^i_{12}, \tilde{R})) > \mathcal{B}(b(x^j_0, B_0))$ if $l_2 = \overline{x}_j$ and $\mathcal{B}(b(x^j_1, B_1))$ otw.

Subsubcase $l_2 = \overline{x}_j$. By Claim ~, $\mathcal{B}(b(x^j_0, B_0)) > \mathcal{B}(b(x^j, \overline{X}))$. Putting everything together, we now have the following inequality chain $\mathcal{B}(b(x^j, X)) < \mathcal{B}(b(x^j_{01}, B_0)) < \mathcal{B}(b(u^i_{12}, \tilde{R})) < \mathcal{B}(b(\tilde{x}^i_{12}, \tilde{B})) < \mathcal{B}(b(u^i, L)) < \mathcal{B}(b(s,B))$.

Hence, ~~By~~ by definition of assignment σ, $\sigma(x_j) = 0$. Hence $l_2 = \overline{x}_j$ evaluates to 1. Hence C_i is satisfied.

Subsubcase $l_2 = x_j$. By Claim ~, $\mathcal{B}(b(x^j_1, B_1)) > \mathcal{B}(b(x^j, X))$. Putting everything together, we now have the following inequality chain: $\mathcal{B}(b(x^j, X)) < \dots < \mathcal{B}(b(s,B))$. Hence, by Claim ~, $\mathcal{B}(b(s,B)) < \mathcal{B}(b(x^j, \overline{X}))$. Hence, by definition of assignment σ, $\sigma(x_j) = 1$. Hence $l_2 = x_j$ evaluates to 1. Hence C_i is satisfied.

Claim 1: $\mathcal{B}(b(u^i, L)) < \mathcal{B}(b(s,B))$ or $\mathcal{B}(b(u^i, R)) < \mathcal{B}(b_0(s,B))$. Suppose not. ... capacity constraint for $\mathcal{B}(b(s,B))$, (u^i, v^i) violated.

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