09.01.23

Agenda for meeting tomorrow with Prof. Dr. Räcke and Prof. Dr. Schmid

1. Reduction G from 3-SAT

$\exists \sigma . \sigma$ is a satisfying assignment for C iff $\exists R . R$ is a feasible update sequence for $G(C)$.

Def.: An update sequence is an ordered partition of $V \times P$.

Def.: An update sequence is feasible if it maintains blackhole- and congestion-freedom.

Choice: $\sigma(x) = 1$ or $\sigma(x) = 0$

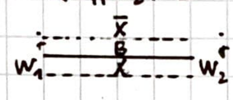
~~Choice: $R(u) < R(u')$ or $R(u)$ $(u < u')$ or R $u > u'$~~

Choice: Update u occurs before update u' in R $(R(u) < R(u'))$ $(u < u')$ or $u > u'$

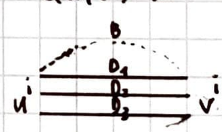
$\sigma(x) = 1$ iff $\cancel{u < u'}$ $R(u) < R(u')$

$G(C)$

$c(w_1^i, w_2^i) = 2$

For each variable x_i:  $\sigma(x_i) = 1$ iff $(w_1^i, \overline{X}) > (w_1^i, B)$

$c(u^i, v^i) = 3$

For each clause $C_i = (l_1 \vee l_2 \vee l_3)$:  $\sigma(l_1) = 1$ iff $(u^i, D_1) < (u^i, B)$

→ Draw example flow update network.

Congestion constraints:

w_1^i: $(w_1^i, B) < (w_1^i, X)$ or $(w_1^i, B) < (w_1^i, \overline{X})$.

u^i: $(u^i, D_1) < (u^i, B)$ or $(u^i, D_2) < (u^i, B)$ or $(u^i, D_3) < (u^i, B)$

u_1^i: $(u_1^i, D_1) > \begin{cases} (u_1^i, X) & \text{if the 1st literal in } C_i \text{ is positive} \\ (u_1^i, \overline{X}) & \text{otw.} \end{cases}$

Blackhole constraints:

B: $(u^i, B) < (s, B) < (w_1^i, B)$

X: $(w_1^i, X) < (u_1^i, X)$ if the ~~first~~ 1st literal in C_i is x_i

$\overline{X}$: $(w_1^i, \overline{X}) < (u_1^i, \overline{X})$ if the 1st literal in C_i is $\overline{x_i}$

D_1: $(u_1^i, D_1) < (u^i, D_1)$

2. Proof of Theorem 2

2. 1. „$\Leftarrow$"

Let $\mathcal{R}$ be a feasible update sequence. Set $\sigma(x_j) := 1$ iff $(w_1^j, \overline{X}) > (w_1^j, B)$.

Draw example flow update network immediately after performing update (s, B). ←

Alternative: Let $C_i = (x_1 \vee \overline{x}_2 \vee x_3)$ be a clause. By congestion constraint u^i, $(u^i, D_1) < (u^i, B)$ or $(u^i, D_2) < (u^i, B)$ or $(u^i, D_3) < (u^i, B)$. W.l.o.g. $(u^i, D_1) < (u^i, B)$. We show $\sigma(x_1) = 1$ which is the case iff $(w_1^1, \overline{X}) > (w_1^1, B)$.

We show $(w_1^1, X) < (w_1^1, B)$. Then, by congestion constraint w_1^j, $(w_1^1, \overline{X}) > (w_1^1, B)$.

$$(w_1^1, X) \overset{X}{<} (u_1^i, X) \overset{u_1^i}{<} (u_1^i, D_1) \overset{D_1}{<} (u^i, D_1) \overset{\text{w.l.o.g.}}{<} (u^i, B) \overset{B}{<} (s, B) \overset{B}{<} (w_1^1, B)$$

2. 2. „$\Rightarrow$"

Let σ be a satisfying assignment.

Perform all updates in the example flow update network for the satisfying assignment $\overline{x_1}$ $\sigma(x_1) = 1, \sigma(x_2) = 1, \sigma(x_3) = 0$. ←

Alternative:

(w_1^j, X) $\forall j.\ \sigma(x_j) = 1$	(u^i, D_1) $\forall i.$ 1st literal in C_i evaluates to 1	(u^i, B) $\forall i$	(s, B)	(w_1^j, B) $\forall j$	(w_1^j, X) $\forall j.\ \sigma(x_j) = 0$	(u^i, D_1) $\forall i.$ 1st literal in C_i evaluates to 0
$(w_1^j, \overline{X})$ $\forall j.\ \sigma(x_j) = 0$					$(w_1^j, \overline{X})$ $\forall j.\ \sigma(x_j) = 1$	