Advanced MCMC, Computer Classes

Mateusz K. Łącki

January 28, 2017

1 Pseudo Marginal Metropolis Hastings

Task 1 Use pseudo marginal MH to simulate a chain that target $\mathcal{N}(0, 1)$

(we will pretend that sometimes we cannot easily obtain its density $\pi(\theta)$, but sometimes we can. Please keep it a secret - it is sort of embarrassing)

- $\theta_0 = 0$
- proposal: $\theta \sim \mathcal{U}(\theta - \alpha, \theta + \alpha)$, where $\alpha > 0$ is a parameter (default $\alpha = 0.5$)
- assume that $p_\theta(x) = p(x)$ and check out the following options for the approximation $\hat{\pi}(\theta|x)$:
  - $\pi(\theta)X$, where $X \sim \text{Exp}(\lambda = 1)$
  - $\pi(\theta)X$, where $X \sim \text{Exp}(\lambda = 2)$
  - $\pi(\theta)X$, where $X \sim \mathcal{N}(\mu = 1, \sigma = 1)$
  - $\pi(\theta)X$, where $X \sim \mathcal{N}(\mu = 0, \sigma = 1)$
  - $\pi(\theta)X$, where $X \sim \mathcal{N}(\mu = 0, \sigma = 0.1 + 10 * \theta^2)$
- Use STATS.PROBPLOT to get a qq-plot for each chain.
- Plot the chain as a histogram.

Task 2 A more realistic example.

Condsider a situation where

- $\theta \sim \mathcal{U}(-20, 20)$ is the prior distribution,
- a discrete latent state X is distributed s.t. $p(X = x|\theta) \propto \exp(3\theta^2 + 20x)$, where $x \in \{-1, 1, 2, 4\}$,
- the observations $Y_{1:n}$ are generated given θ, x according to $\mathcal{N}(\theta + x, \sigma^2 = 1)^{\otimes n}$.

Implement pseudo-marginal MH for the two following cases:

1. $\hat{\pi}(\theta, x) = \pi(y_{1:n}|X, \theta)$, where $X \sim \pi(x|\theta)$.
2. $\hat{\pi}(\theta, x_{1:K}) = \frac{1}{K}\sum_{k=1}^{K} \pi(y_{1:n}|X_k, \theta)$, where $X_{1:K} \sim \pi(x|\theta)^{\otimes K}$.

Simulate the two chains.

Prepare histograms of the values estimated by both chains.

Please send the answers by the end of semester.
Best wishes, Matteo