## Monte Carlo method for option prices

1. The stock price follows, under the risk-neutral measure, the dynamic

$$S_t = S_0 \exp\left(\left(r - \frac{\sigma^2}{2}\right)t + \sigma W_t\right),$$

where  $W_t$  is a Wiener process (Brownian motion), r is an interest rate,  $\sigma$  is a volatility and  $S_0$  is a stock price at t = 0.

- 2. Write in *Octave* a function which calculates the price for an European put option by the Monte Carlo method using: a) the standard uniform generator (*rand* in *Octave*) and b) the Halton sequence, with the Box-Muller algorithm to generate normal deviates. The function, called **PutMC**(**x**,**y**,...), needs the following inputs (names of variables and values in parenthesis are default and should appear in file **CW2\_data.txt**):
  - initial stock price,  $S_0$  (S0 = 50),
  - risk-free interest rate, r (r = 0.05),
  - volatility of the stock,  $\sigma$  (sigma = 0.3),
  - time to maturity, T (T = 0.5),
  - strike, K (K = 50),
  - number of simulations,  $M (\ge 10^5 \text{ for MC and } \ge 10^3 \text{ for QMC})$ , (M = ...).
- 3. Write a program (script) which inputs data to the function **PutMC**(**x**,**y**,...) (input from the file **CW2\_data.txt**). The programme has to print out on the screen (with identification labels) the computed price for MC and QMC and 95% confidence interval (only for MC) as well as the errors of the computed prices.
- 4. What are the *numerical* convergence rates of the MC and QMC methods? Running the simulations for different M try to confirm the theoretical error bounds for both methods. Write in a comment file your findings.