Pricing American barrier options

1. In the standard Black-Scholes model calculate the prices of down-and-out American barrier call and put options.

2. Write in *Octave* a function which calculates the prices of the above instruments. The function has to use the implicit and the Crank-Nicolson schemes to solve the corresponding penalized problem in natural variables (but with the time variable reversed). The function, called **American_DO**, needs the following input (names of variables and values in parenthesis are default and should appear in the file **CW6_data.txt**):

- initial stock price, S_0 (`S0 = 100`),
- stock volatility σ (`sigma = 0.2`),
- risk-free interest rate, r (`r = 0.1`),
- time to maturity, T (`T = 0.5`),
- strike price, K (`K = 100`),
- barrier, X (`X = 90`).

In addition the following inputs are necessary for the numerical scheme to be properly settled and should also appear in the data file (a proper choice of these values is partially left to the programmer):

- number of points in the direction of t variable (`Mt = ...`),
- number of points in the direction of x variable (`Mx = ...`),
- left truncation point of the x variable, x_{min} (`xmin = 0`),
- right truncation point of the x variable, x_{max} (`xmax = 400`),
- accuracy of the penalty iterations tol, (`tol = 1e-7`).

The choice of the numerical scheme and the type of option (call/put) has to be input from the keyboard.

3. Write a program (script) which inputs data to the function **American_DO**. The data should be read from the file **CW6_data.txt**). In addition to the numerical value of the option price for a given asset price, the program should produce a graph of the option price as a function of the asset price S for $S \in [x_{min}, x_{max}]$.