Pricing European options in the Heston model

1. The stock price follows, under the risk-neutral measure, the *Heston stochastic volatility model*

$$dS_t = S_t r dt + S_t \sqrt{V_t} dW_t^S,$$
$$dV_t = a(b - V_t)dt + \sigma \sqrt{V_t} dW_t^V,$$

where r is an interest rate, σ is a volatility of volatility and a, b are positive constants. The term $(b - V_t)$ corresponds to the so called *mean reverting* property, i.e. the tendency of process V_t to return to its mean value b.

Processes W_t^S and W_t^V are the standard one-dimensional **correlated** Wiener processes. We assume a constant in time correlation

$$\mathbb{E}[dW_t^S dW_t^V] = \rho dt,$$

where ρ is a constant from interval $[-1, 1]$.

2. Write in *Octave* a function which calculates the price of a Europen call option in the Heston model by the Monte Carlo method. The function should compute S_T by calculating trajectories of S_t via the Euler-Maruyama scheme and trajectories of V_t via the Milstein scheme. The function, called **Heston_call_MC(x,y,...)**, needs the following inputs (names of variables and values in parenthesis are default and should appear in the file **CW3_data.txt**):

- initial stock price, S_0 (`S0 = 50`),
- initial *volatility* value, V_0 (`V0 = 0.06`),
- risk-free interest rate, r (`r = 0.05`),
- volatility of volatility parameter σ (`sigma = 0.4`),
- constant a (`a = 2`),
- constant b (`b = 0.04`),
- correlation ρ (`rho = -0.7`),
- time to maturity, T (`T = 2.0`),
- strike price, K (`K = 45`),
- number of points on the trajectory of Brownian motion n ($\geq$ 400), (`n = ...`),
- number of simulations, M ($\geq$ 100000), (`M = ...`).

3. Write a program (script) which inputs data to the function **Heston_call_MC**. The data should be loaded from the file **CW3_data.txt**. The program has to print out on the screen (with identification labels) the option price as well as its 95% confidence interval.

4. **Caution!** The volatility process V_t in the Heston model may reach zero if $2ab < \sigma^2$. This is often true for real market parameters, which means that due to the discretization of the processes, it can happen that the value of V_t in some time steps can be negative.

Try to propose some remedy for this problem and write the function **Heston_call_MC** in such a way that you get around it. Check the scheme using the following set of parameters (that may be encountered in long-dated equity options markets):

- initial stock price, S_0 (`S0 = 100`),
- initial *volatility* value, V_0 (`V0 = 0.09`),
- risk-free interest rate, r (`r = 0.05`),
- volatility of volatility parameter σ (`sigma = 1.0`),
- constant a (`a = 2`),
- constant b (`b = 0.09`),
- correlation ρ (`rho = -0.3`),
- time to maturity, T (`T = 5.0`),
- strike price, K (`K = 100`).

Hint: the true option price for these parameters is 34.9998.