Pricing European power options

1. The stock price follows, under the risk-neutral measure, the dynamic

$$S_t = S_0 \exp\left(\left(r - \frac{\sigma^2}{2}\right)t + \sigma W_t\right),$$

where W_t is a Wiener process (Brownian motion), r is an interest rate, σ is a volatility and S_0 is a stock price at $t = 0$.

A European capped power call option is a contract with payoff

$$h(S_T, K) = \min((S_T^i - K)^+, C),$$

where $i > 0$.

2. Write in *Octave* a function which calculates the price and delta of a European capped power call option by PDE method.

The function should compute the results in both the transformed and the natural variables of the Black-Scholes equation. Remember that after transforming the BS equation into the heat equation it is necessary to make an appropriate transformation of x_{min} and x_{max}. The function, called **CapPowerCallPDE**, needs the following inputs (names of variables and values in parenthesis are default and should appear in file **CW5_data.txt**):

- initial stock price, S_0 (`S0 = 90`),
- risk-free interest rate, r (`r = 0.05`),
- volatility of the stock, σ (`sigma = 0.3`),
- time to maturity, T (`T = 0.5`),
- strike, K (`K = 200`),
- cap level, C (`C = 250`),
- power i (`i = 1.2`).

In addition the following inputs are necessary for the numerical scheme to be properly settled and should also appear in the data file (a proper choice of these values is partially left to the programmer):

- number of points in the direction of t variable (`Mt = ...`),
- number of points in the direction of x variable (`Mx = ...`),
- left truncation point of the x variable, x_{min} (`xmin = 0`),
- right truncation point of the x variable, x_{max} (`xmax = 400`),

The values of x_{min} and x_{max} above are given in the natural variables. The function has to use both the implicit and the Crank-Nicolson schemes to solve the corresponding boundary value problem. The choice of the scheme has to be inputed from keyboard.

3. Write a program (script) which inputs data to the function **CapPowerCallPDE**. The data should be read from the file **CW5_data.txt**. In addition to the numerical values of the option price and delta for

a given asset price, the program should produce a graph of the option price and its delta as a function of the asset price S for $S \in [0, 400]$. *Warning.* The graph should use natural variables, i.e. on the horizontal line the asset price "as it is" from 0 to 400.

4. The purpose of the program is to analyse the influence of the used variable transformation on the relative accuracy of the scheme. Report in an additional file your observations on the accuracy for the different variable transformations.