Pricing Asian options using Monte Carlo method

1. The stock price follows, under the risk-neutral measure, the dynamic

$$S_t = S_0 \exp\left(\left(r - \frac{\sigma^2}{2}\right)t + \sigma W_t\right),$$

where W_t is a Wiener process (Brownian motion), r is an interest rate, σ is a volatility and S_0 is a stock price at $t = 0$.

2. Write in *Octave* a function which calculates the price of an arithmetic average call Asian option (payoff: $(\hat{S} - K)^+$) by the Monte Carlo method using *control variates* and optimize the method with respect to the θ parameter. The function, called **asian_callMC(x,y,...)**, needs the following inputs (names of variables and values in parenthesis are default and should appear in file **CW4_data.txt**):

- initial stock price, S_0 (`S0 = 50`),
- risk-free interest rate, r (`r = 0.05`),
- volatility of the stock, σ (`sigma = 0.3`),
- time to maturity, T (`T = 0.5`),
- strike, K (`K = 50`),
- number of points on the trajectory of the Brownian motion, n (`n = 100`),
- number of simulations, M ($\geq 10^5$), (`M = ...`).

3. Write a program (script) which inputs data to the function **asian_callMC** (input from the file **CW4_data.txt**). The program has to print out on the screen (with identification labels) the computed price and 95% confidence interval.

4. As control variates use geometric discrete average Asian option and geometric continuous average Asian option. Calculate prices of an arithmetic average Asian option for both choices of control variates. By outputting auxiliary values of simulated and analytic (discrete and continuous) geometric averages find out for which n there is no difference between using discrete and continuous averages as a control variate (be sure that your program works for large n). Report your finding in a separate comment file.