BIBLIOGRAPHY

Efficient Execution of DG-FEM workloads on GPUs via CUDAGraphs

Міт Котак

Senior Thesis

Faculty Mentor: Andreas Klöckner

Graduate Mentor: Kaushik Kulkarni

Submitted in partial fulfilment of the requirements

for the degree of Bachelor of Science in Engineering Physics

in the Granginer College of Engineering of the

University of Illinios Urbana-Champaign, 2023

ABSTRACT

Array programming paradigm offers routines to express the computation cleanly for for a wide variety of scientific computing applications (Finite Element Method, Stencil Codes, Image Processing, Machine Learning, etc.). While there's ongoing work to provide efficient data structures and fast library implementations for many common array operations, the performance benefits are tied to optimized method calls and vectorized array operations, both of which evaporate in larger scientific codes that do not adhere to these constraints. There have been a lot of efforts in scaling up n-d array applications through kernel and loop fusion, but little attention has been paid towards harnessing the concurrency across array operations. The dependency pattern between these array operations allow multiple array operations to be executed concurrently. This concurrency can be targeted to accelarate the application's performance. NVIDIA's CUDAGraph API offers a task programming model that can help realise this concurrency by overcoming kernel launch latencies and exploiting kernel overlap by scheduling multiple kernel executions in parallel. In this work we map the array operations onto a precise data-flow graph and expose that to a GPU via CUDAGraphs. To evaluate the soundness of this approach, we port a suite of DG-FEM operators that represent real life workloads to our framework and observe a speedup of upto 3x over a version where the array operations are executed one after the other.

ABSTRACT

ACKOWLEDGEMENT

I would like to take this opportunity to thank all of the people without whom this thesis would have never happened. First and foremost, I would like to thank my advisor, Andreas Klöckner for his guidance and always being there to support me. My graduate mentor, Kaushik Kulkarni also deserves a huge thank you for guiding me through my research journey. I'm very grateful for the time and attention they have invested in me. I consider myself lucky to have had them as my mentors.

I am grateful to the *CEESD* (Center for Exascale-Enabled Scramjet Design) team for letting me be a part of their research endevaour and the broader NCSA (National Center for Supercomputing Applications) community for the opportunity to engage with undergradaute research. I would also like to thank the Office of Undegraduate Research for travel support which helped publicize this work.

I would like to thank my parents and my extended family in the US for their unwavering support.

Thank you, all of you.

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1. Introduction

Array programming is a programming paradigm that supports a wide variety of features, including array slicing and arbitary element-wise, reduction and broadcast operators allowing the interface to correspond closely to the mathematical needs of the applications. PyCUDA[13] and several other array-based frameworks (CuPy[18], Bohrium[14], Numba[16], Legate[5]) serve as drop-in replacements for accelerating Numpy-like operations on GPUs. All of them offer array interfaces for mapping Numpy operations onto GPU memory. Incase of PyCUDA, this support is provided through the GPUArray interface. While abstractions like GPUArray's offer a very convenient abstraction for doing "stream" computing[23] on these arrays, they are not yet able to automatically schedule and manage overlapping array operations onto multiple streams. The concurrency available in the dependency pattern for these array routines can be exploited to saturate all of the available execution units [Fig. 1].

Currently the only way to tap into this concurrency is by manually scheduling array operations onto multiple CUDA streams which typically requires a lot of experimentation since information about demand resources of a kernel such as GPU threads, registers and shared memory is only accessible at runtime.

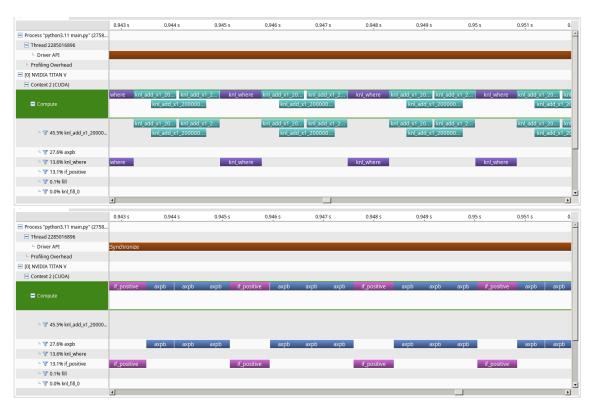


Figure 1. Profiles for CUDAGraph (top) and PyCUDA (bottom) for where (condition, if, else) + 1

Our framework realises this concurrency across array operations through NVIDIA's CUDAGraphs. CUDAGraph is a task-based programming model that allows asynchronous execution of a user-defined Directed Acyclic Graph (DAG). While the framework is generic, we look at DG-FEM workloads which have low communication cost and robust stable algorithms, thus making it easier to create compute heavy kernels that can efficiently leverage GPUs.

2 Related work 11

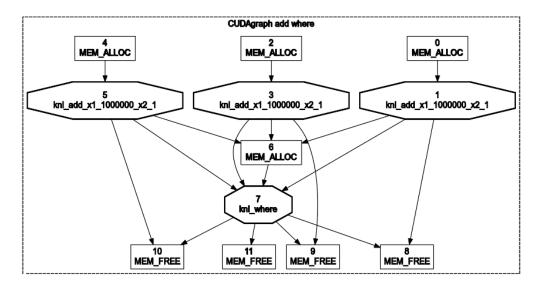


Figure 2. CUDAGraph API generated graph for where (condition, if, else) + 1

We formulate our system by building a CUDAGraph-based PyCUDA target for Pytato's IR which captures the user-defined DAG. The process is *transparent*. The key technical contributions of our system involve:

- 1. Extending PyCUDA to allow calls to the CUDAGraph API
- 2. Mapping the array operations onto a DAG through Pytato's IR to generate PyCUDA-CUDAGraph code.
- 3. Providing an evaluation for profitability of CUDAGraphs for DG-FEM workloads.

2. Related work

Castro et el [8] gives an overview of the current task-based Python computing landscape by mentioning PyCOMPs[22], Pygion[21], PyKoKKos[3] and Legion [4] that rely on decorators. A decorator is an instruction set before the definition of a function. The decorator function transforms the user function (if applicable) into a parallelization-friendly version. PyCOMPs and Pygion both rely on @task decorator to dynamically add tasks to the data dependency graph. The scheduling policy is locality-aware where the runtime system computes a score for all of the available resources and chooses the one with the highest score. The score is the number of task input parameters that are already present on that resource, thus minimizing delays between task executions. Legion uses a data-centric programming model which relies on software out-of-order processor (SOOP), for scheduling tasks which takes locality and independence properties captured by logical regions while making scheduling decisions.

In Jug [9] arguments take values or outputs of another tasks and parallelization is achieved by running more than one Jug processes for distributing the tasks. In Pydron[17], decorated functions are first translated into an intermediate representation and then analyzed by a scheduler which modifies the execution graph as each task is finished.

Since all of these frameworks rely on explicit taks declarations, they are not yet able to realise the concurrency available across array operations.

CuPy serves as a drop-in replacement to Numpy and support NVIDIA's in-house CUDA frameworks such as cuBLAS, cuDNN and cuSPARSE. Julia[6] GPU programming models use CUDA.jl to provide high level mechanics to define multidimenstional arrays (CUArray). Both CuPy and Julia offer interfaces for *implcit* graph construction which *captures* a CUDAGraph using existing stream-based APIs. Although capturing all the operations on a stream leads to terse application code, staging computations within a user-code with interleaving in-graph and out-of-graph operations cannot be expressed. The is can lead to repeated computations of the same sub-graphs.

JAX[7] optimizes GPU performance by translating high-level traces into XLA[20] HLO and then performing vectorization/parallelization and JIT compilation. Deep learning (DL) symbolic mathematical libraries such as TensorFlow[1] and PyTorch[19] allow neural networks to be specified as DAGs along which data is transformed. Current DL frameworks conduct GPU task scheduling during run time. Tensorflow represents a neural network as a computation graph of DL operators, and schedule athe GPU tasks of an operator at run time once the operator's dependencies are met. Meanwhile, for PyTorch, GPU tasks are scheduled at run time as Python code is interpreted line by line. Just like CUDAGraphs, in TensorFlow, computational DAGs are defined statically to achieve close to roofline performance for DL applications. PyTorch on the other hand offers more control at runtime by allowing the modification of executing nodes facilitating the implementation of sophosticated training routines.

Both StarPU[2] and ParSEC[10] provide excellent support for heterogenous hardware on distributed systems. Both of them share a number of common features: tasks appear to execute in program order, dependencies between tasks are determined by the arguments supplied to task calls along with the privileges requested by tasks, and tasks can be offloaded to available GPUs (with data movement managed by the system). ParSEC in particular uses a DSL compiler to read a program representation (a recursive, algebraic description of a task graph) and generate code to execute the tasks described in the program.

3. Overview

3.1. CUDA Graphs

CUDAGraphs provide a way to execute a partially ordered set of compute/memory operations on a GPU, compared to the fully ordered CUDA streams: : a stream in CUDA is a queue of copy and compute commands. Within a stream, enqueueud operations are implicitly synchronized by the GPU in order to execute them in the same order as they are placed into the stream by the programmer. Streams allow for aschnronous compute and copy, meaning that CPU cores dispatch commands without waiting for their GPU-side completition: even in asynchronous submissions, little to no control is left to the programmer with respect to when commands are inserted/fetched to/from the stream and then dispatched to the GPU engines, with these operations potentially overallaping in time.

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CUDAGraphs faciliate the mapping of independent A CUDAGraph is a set of nodes representing memory/compute operations, connected by edges representing run-after dependencies.

CUDA 10 introduced explicit APIs for creating graphs, e.g. cuGraphCreate, to create a graph; cuGraphAddMemAllocNode/cuGraphAddKernelNode/cuGraphMemFreeNode, to add a new node to the graph with the corresponding run-after dependencies with previous nodes to be exected on the GPU; cuGraphInstantiate, to create an executable graph in a stream; and a cuGraphLaunch, to launch an executable graph. We wrapped this API using PyCUDA which provided a high level Python scripting interface for GPU programming.

Operations

PyCUDA routines

Memory Allocation
Kernel Execution
Host to Device Copy
Device to Device Copy
Device to Host Copy
Memory Free
Graph Creation
Graph Instantiation
Update ExecGraph arguments
Graph Launch

add_memalloc_node add_kernel_node add_memcpy_htod_node add_memcpy_dtod_node add_memcpy_dtoh_node add_memfree_node

Graph GraphExec

batched_set_kernel_node_arguments

launch

Table 1. PyCUDA wrapper functions around CUDAGraph API

3.2. **Loopy**

Loopy[12] is a Python-based transformation toolkit to generate transformed kernels. We make use of the following components in our pipeline to generate performance tuned CUDA kernels:

- 1. Loop Domains: The upper and lower bounds of the result array's memory access pattern in the OpenCL format sourced from the shape attribute within IndexLambda and expressed using the isl library.
- 2. Statement: A set of instructions specificed in conjuction with an iteration domain which encodes an assignment to an entry of an array. The right-hand side of an assignment consists of an expression that may consist of arithmetic operations and calls to functions.
- 3. Kernel Data: A sorted list of arguments capturing all of the array node's dependencies.

3.3. Pytato

Pytato[15] is a lazy-evaluation programming based Python package that offers a Numpy-like frontend for recording array expressions.

Pytato offers an IR which encodes user defined array computations as a DAG where nodes correspond to array operations and edges represent dependencies between inputs/outputs of these operations. In this work, we are interested in the normalized form of Pytato IR, which is comprised of the following two node types:

- 1. Placeholder: A named abstract array whose shape and dtype is known with data supplied during runtime. This permits the automated gathering of a self-contained description of a piece of code without incurring the penalty faced by repeated memory transfers from the device's DRAM to lower levels of cache.
- 2. IndexLambda: Represents an array comprehension recording a scalar expression containing per index value of the array computation. This helps create a generalized

Here's a simple example demonstrating Pytato usage

```
# CreatePlaceholder node for storing array description
```

```
x = pt.make_placeholder(name="x", shape=(4,4), dtype="float64")
```

Express array computation as a scalar expression using Indexlambda

result = 2*x

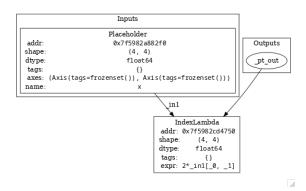


Figure 3. Pytato IR corresponding to doubling operation

4. LOWERING ARRAY OPERATIONS TO CUDAGRAPHS

Pytato provides a pt.generate_cudagraph function which triggers a two-stage code generation process that traces the array program and generates PyCUDA-CUDAGraph code. Our array programming system is based on lazy-evaluatation which offers us the flexibilty of avoiding repeated memory transfer's from the DRAM while transforming the array expressions to CUDAGraph code. Since Pytato's computation graph could be exactly mapped onto the CUDAGraph interface, we implemented a Pytato-CUDAGraph vistor for rewriting Pytato IR expressions. For the kernel nodes, instead of manually searching the parameter space, we offloaded decisions such as address spaces of buffers, mapping to device axes, loop tiling, etc. onto Loopy.

The following code is generated by passing the computation graph described in Fig 3 into pt.generate_cudagraph

```
import pycuda.driver as _pt_drv
import numpy as np
from pycuda.driver import KernelNodeParams as _pt_KernelNodeParams
from pycuda.compiler import SourceModule as _pt_SourceModule
from pycuda import gpuarray as _pt_gpuarray
from functools import cache
# {{{ Create and load kernel module
_pt_mod_0 = _pt_SourceModule("
#define bIdx(N) ((int) blockIdx.N)\n#define tIdx(N) ((int)
threadIdx.N)\n\nextern "C" __global__ void __launch_bounds__(16)
knl_mult_x1_1_x2_16(double *__restrict__ out, double const *__restrict__
_{in1}\n{ \int \int \int \int dx} dx = 0; \n out[4 * (tIdx(x) / 4) +
tIdx(x) + -4 * (tIdx(x) / 4)] = 21 * _in1[4 * (tIdx(x) / 4) + tIdx(x) +
-4 * (tIdx(x) / 4)]; \n } \n}
")
# }}}
# {{{ Stage 1: Build and cache CUDAGraph
@cache
def exec_graph_builder():
    _pt_g = _pt_drv.Graph()
    _pt_buffer_acc = {}
    _pt_node_acc = {}
    _pt_memalloc, _pt_array = _pt_g.add_memalloc_node(size=128,
dependencies=[])
    _pt_kernel_0 = _pt_g.add_kernel_node(_pt_array, 139712027164672,
func=_pt_mod_0.get_function('knl_mult_x1_1_x2_16'), block=(16, 1, 1),
grid=(1, 1, 1), dependencies=[_pt_memalloc])
    _pt_buffer_acc['_pt_array'] = _pt_array
    _pt_node_acc['_pt_kernel_0'] = _pt_kernel_0
    _pt_g.add_memfree_node(_pt_array, [_pt_kernel_0])
    return (_pt_g.get_exec_graph(), _pt_g, _pt_node_acc, _pt_buffer_acc)
# }}}
# {{{ Stage 2: Update execution graph
def _pt_kernel(allocator=cuda_allocator, dev=cuda_dev, *, a):
    _pt_result = _pt_gpuarray.GPUArray((4, 4), dtype='float64',
allocator=allocator, dev=dev)
    _pt_exec_g, _pt_g, _pt_node_acc, _pt_buffer_acc =
exec_graph_builder()
```

```
_pt_exec_g.batched_set_kernel_node_arguments({_pt_node_acc['_pt_kernel_0']:
_pt_drv.KernelNodeParams(args=[_pt_result.gpudata, a.gpudata])})
    _pt_exec_g.launch()
    _pt_tmp = {'_pt_out': _pt_result}
    return _pt_tmp['_pt_out'].get()

# }}
```

4.1. Stage 1: Build CUDAGraph

Alg 1 only gets executed only once during compilation with a $\Theta(V+E)$ complexity for Alg 2

Algorithm 1: DAG Discovery for building CUDAGraph

Step 1: Run a topological sort on Pytato IR using Kahn's algorithm [11]. This front-loads the *sink* nodes which helps avoid array recomputations during DAG discovery. Initialize a pycuda. Graph object.

Step 2:

```
for n \in \text{nodes} in Pytato IR which only have incoming edges do GraphTraverse(n) done
```

Step 3: Instantiate pycuda.Graph object and cache the resultant pycuda.GraphExec object to avoid triggering traversals of the entire graph for subsequent launches.

Algorithm 2: Pytato IR Traversal

```
function GRAPHTRAVERSE(n)

if n \in \{PlaceHolder, DataWrapper\} \{

• PLACEHOLDERMAPPER(n)
```

• Link to user provided buffers or generate new buffers via GPUArrays.

```
 \begin{array}{c} \mathbf{return} \ \{n\} \\ \\ \} \\ \mathbf{else} \ \{ \end{array}
```

- INDEXLAMBDAMAPPER(n)
- Generate kernel string and launch dimensions by plugging IndexLambda expression into lp.make_kernel.
- Add kernel node with temporary buffer arguments and corresponding result memalloc node to pycuda. Graph object with dependencies sourced from Pytato IR.
- Update Pytato IR with termporary buffer information.

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```
n\_deps \leftarrow \{\} for c \in n dependencies sourced from Pytato IR do c\_deps \leftarrow GRAPHTRAVERSE(c) n\_deps \leftarrow n\_deps \bigcup c\_deps done return n\_deps \} end function
```

4.2. Stage 2: Update CUDAGraphExec

Alg 3. gets executed for every graph launch.

Algorithm 3: Buffer update in CUDAGraphExec

for n ϵ kernel nodes in pycuda. GraphExec with temporary buffers do

• Replace temporary buffers with allocated/linked buffers from corresponding PlaceHolder nodes.

done

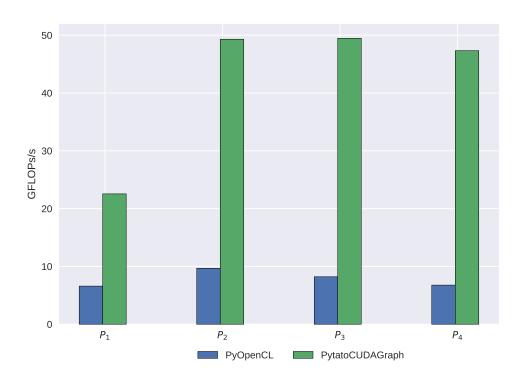
5. Results

We demonstrate the performance of our framework on two end-to-end DG-FEM operators on NVIDIA Titan V in Fig 4. We evaluated these operators on 3D meshes with tetrahedral cells and measure our speedup against PyOpenCL[13] which supports sequential stream execution. Table 2. summarizes our experimental parameters.

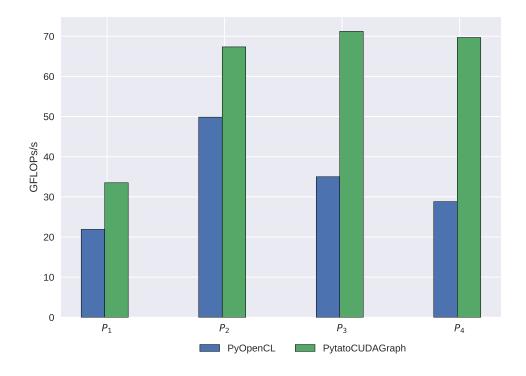
Equation	Polynomial Degree	# of tetrahedrons in the mesh
3D Wave	1	27000
	2	15625
	3	3375
	4	1000
3D Euler	1	27000
	2	15625
	3	3375
	4	1000

Table 2. Experimental parameters for DG-FEM operators

Wave Operator



Euler Operator



 $\begin{tabular}{ll} \bf Figure 4. & Performance of our framework (Pytato-PyCUDA-CUDAGraph) over sequentual stream execution (PyOpenCL) \\ \end{tabular}$

6 Conclusion 19

We report a speedup of 3-6x for Wave and 1.5-3x for Euler operators.

6. Conclusion

In this work we implement map array operations onto a precise data flow graph and expose it to a GPU via CUDAGraphs. This work enables domain experts to write array-based DG-FEM code that can be efficiently executed on GPUs through multiple streams and low kernel launch latencies. We demonstrated the performance of our system by recording a speedup for 3-6x for Wave operators and 1.5-3x for Euler Operators.

Some future extensions of this work involve (1) creating a performance model for the graph scheduling algorithm through a series of microbenchmarks, (2) providing a mechanism for hand-tuning Loopy kernels.

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