#### CS5787: Exercises 1

FULL\_CODE\_FOLDER\_URL

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### 1 Theory: Question 1 [12.5 pts]

- a. What is the input X? There are m samples in the batch with 10 features each so the input shape is (m, 10)
- b. What is the shape of the hidden layer's weight vector  $W_h$ , and the shape of its bias vector  $b_h$ ?

To transform an (m, 10) input into a hidden layer of 50 neurons we need a shape  $W_h$  to have a shape of (10, 50) so that we get a (m, 50) result. After the weight matrix is applied, we then add the bias  $b_h$  to each row of  $W_h$ , which means that the result needs to be (1, 50)

- c. What is the shape of the output layer's weight vector  $W_o$ , and its bias vector  $b_o$ ?  $W_h$ : (50,3),  $b_h$ : (1,3)
- d. What is the shape of the network's output matrix Y? (m,3)
- e. Write an equation that computes the network's output matrix Y as a function of X,  $W_h$ ,  $b_h$ ,  $W_o$ , and  $b_o$ ?

$$Y = W_o a(W_h X + b_h) + b_o$$

Where *a* is the ReLU activation function,  $a(x) = \max(0, x)$ .

## 2 Theory: Question 2 [12.5 pts]

For the first layer there are 3x3x3 parameters in each of the 100 kernels, and then we need bias parameters for each kernel as well. This totals 3x3x3x100 + 100 = 2800 parameters. In the second layer there are 3x3x100 parameters in each of the 200 kernels, and then we need bias parameters for each kernel as well. This totals 3x3x100x200 + 200 = 180200. In the final layer there are 3x3x200 parameters in each of the 200 kernels, and then we need bias parameters for each kernel as well. This totals 3x3x200x400 + 400 = 720400. Adding all the layers together we get 2800 + 180200 + 720400 = 903400 total parameters.

# 3 Theory: Question 3 [25 pts]

a. 
$$\frac{\partial f}{\partial \gamma} = \frac{\partial f}{\partial y} \frac{\partial y}{\partial \gamma} = \frac{\partial f}{\partial y} \hat{x}_i$$

b. 
$$\frac{\partial f}{\partial \beta} = \frac{\partial f}{\partial y} \frac{\partial y}{\partial \beta} = \frac{\partial f}{\partial y}$$

c. 
$$\frac{\partial f}{\partial \widehat{x_i}} = \frac{\partial f}{\partial y} \frac{\partial y}{\partial \widehat{x_i}} = \frac{\partial f}{\partial y} \gamma$$

d. 
$$\frac{\partial f}{\partial \sigma^2} = \frac{\partial f}{\partial y} \frac{\partial y}{\partial \widehat{x_i}} \frac{\partial \widehat{x_i}}{\partial \sigma^2} = \frac{\partial f}{\partial y} \gamma \frac{-1}{2} \frac{(x_i - \mu)}{\sqrt{(\sigma^2 + \epsilon)^3}}$$

e. 
$$\frac{\partial f}{\partial \mu} = \frac{\partial f}{\partial y} \frac{\partial y}{\partial \widehat{x_i}} \frac{\partial \widehat{x_i}}{\partial \mu} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial \widehat{x_i}} \frac{\partial \widehat{x_i}}{\partial \sigma^2} \frac{\partial \sigma^2}{\partial \mu} = \frac{\partial f}{\partial y} \gamma \frac{-1}{\sqrt{(\sigma^2 + \epsilon)^3}} + \frac{\partial f}{\partial y} \gamma \frac{-1}{2} \frac{(x_i - \mu)}{\sqrt{(\sigma^2 + \epsilon)^3}} \frac{-2}{m} \sum_{i=1}^m (x_i - \mu)$$

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f. 
$$\begin{split} \frac{\partial f}{\partial x_i} &= \frac{\partial f}{\partial \widehat{x_i}} \frac{\partial \widehat{x_i}}{\partial x_i} + \frac{\partial f}{\partial \sigma^2} \frac{\partial \sigma^2}{\partial x_i} + \frac{\partial f}{\partial \mu} \frac{\partial \mu}{\partial x_i} \\ \frac{\partial f}{\partial x_i} &= \frac{\partial f}{\partial y} \gamma \frac{1}{\sqrt{(\sigma^2 + \epsilon)^3}} \\ &+ \frac{\partial f}{\partial y} \gamma \frac{-1}{2} \frac{(x_i - \mu)}{\sqrt{(\sigma^2 + \epsilon)^3}} \frac{2}{m} (x_i - \mu) \\ &+ \frac{\partial f}{\partial y} \gamma \frac{-1}{\sqrt{(\sigma^2 + \epsilon)^3}} \\ &+ \frac{\partial f}{\partial y} \gamma \frac{-1}{2} \frac{(x_i - \mu)}{\sqrt{(\sigma^2 + \epsilon)^3}} \frac{-2}{m} \sum_{i=1}^m (x_i - \mu) \frac{1}{m} \end{split}$$

### 4 Practical [50 pts]

**TODO:** Provide a report detailing your experiments, results and discussion in this section.