E214 questions

May 9, 2018

1 Section 7.2.2

Question A: Decay of the Z^0 boson
Which value does the momentum of an electron have in the decay of a Z^0 boson $Z^0 \to e^+e^-$, if the Z^0 is at rest?

From four-momentum conservation:

$$p_{Z^0} = \begin{bmatrix} m_{Z^0} \\ \vec{0} \end{bmatrix} = \begin{bmatrix} E_e \\ \vec{p} \end{bmatrix} + \begin{bmatrix} E_e \\ -\vec{p} \end{bmatrix} = p_{e^+} + p_{e^-}$$

From this, we get

$$m_{Z^0} = 2\sqrt{p^2 + m_e^2} = 91.2\,\mathrm{GeV}, \quad m_e = 511\,\mathrm{GeV}$$

$$\Rightarrow p = \sqrt{\frac{m_{Z^0}^2}{4} - m_e^2} = 45.6\,\mathrm{GeV}$$

Question B: Scattering reaction $e^+e^- \to \tau^+\tau^-$
How large is the momentum of tau leptons in the reaction $e^+e^- \to \tau^+\tau^-$, if the reaction takes place in the center-of-mass system (center-of-mass energy = 5 GeV)?

In CMS frame ($\vec{p}_{e^-} = -\vec{p}_{e^+}$), the energy squared:

$$s = (p_{\tau^+} + p_{\tau^-})^2 = \begin{bmatrix} 2E_\tau \\ \vec{0} \end{bmatrix}^2 = 4E_\tau^2 = E_{CMS}^2 = 25\,\mathrm{GeV}^2$$

We can calculate ($m_\tau \approx 1.78$ GeV) the three-momentum of the tau leptons:

$$p_\tau^2 = E_\tau^2 - m_{tau}^2 = \left(\frac{25}{4} - 1.78^2\right)\mathrm{GeV}^2$$

$$p = 1.755\,\mathrm{GeV}$$

2 Section 7.4.1

How could the variable `ptw` *be constructed from other tree variables?*

We are looking at $W \to e\nu$ processes. We can use the missing transverse momentum (`ptmis_x, ptmis_y`), which we assign to the neutrino. As we know the electron transverse momentum: `el_px, el_py, el_pt`, the W-boson transverse momentum is determined by momentum conservation, if the

The correct form of the Gauss error propagation law in the presence of correlations.

The standard deviation squared of a function $f(x, y)$ is given by

$$\sigma_f^2 = \left(\frac{df}{dx}\right)^2 \sigma_x^2 + \left(\frac{df}{dy}\right)^2 \sigma_y^2 + 2\frac{df}{dx}\frac{df}{dy}\mathrm{cov}(x, y) \tag{1}$$

3 Section 7.5.1

What is the minimum invariant 4-lepton-mass, when the four leptons originate from a Z^0 pair? Why do you find 4-lepton-events with invariant mass beneath this threshold?

At threshold, the Z^0 boson decays at rest, in this case: $m_{Z^0} = 2m_l$. The minimum 4-lepton invariant mass is acquired when both Z^0's are stationary (CMS frame),and equal to $2m_{Z^0}$. When one or two of the Z^0 bosons is off-shell, the 4-lepton-mass can be lower than $2m_{Z^0}$.

Consider a Higgs boson which decays into two Z^0 bosons. How does the distribution of the 4-lepton-invariant-mass look like?

There is a Z^0 peak (at ≈ 90 GeV) and a Higgs-boson peak (at ≈ 125 GeV). These come from the $Z^0 \to 4\ell$ and the $H^0 \to ZZ \to 4\ell$ processes, respectively. One example for the background at the Higgs-peak is the $t \to bW^+$ decay.

Assume you have an ideal detector. What is the typical $\not{E}_T$ if a Z^0 pair has been produced and both Z^0 decay into electron or muon pairs? What $\not{E}_T$ will you expect when you have a real detector?

In an ideal detector, there would be no missing energy in this process. In real detectors, there are energy losses not detected due to inactive detector elements, imperfect calibration.

The Branching ratio of $t \to Wb$ is almost 100%. If you have a top anti-top pair in an event, both particles decay instantly via $t \to bW$. If both W bosons

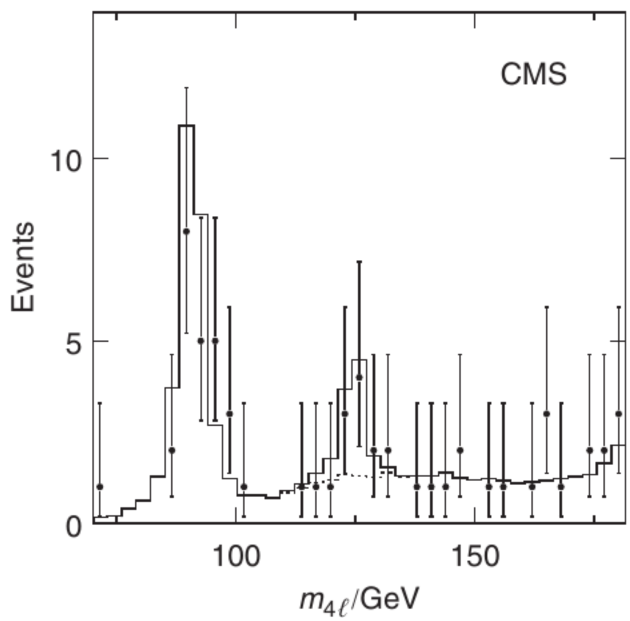

Figure 1: The invariant 4-lepton-mass distribution. *Thompson: Modern Particle Physics Fig. 17.19*

each decay leptonically ($W \to \ell\nu$), one finds two leptons in the event. What could explain the occurence of four leptons in a $t\bar{t}$ event?

The two top quarks can decay into b quarks and W bosons, both of which can further decay semi-leptonically. The process:

$$t\bar{t} \to bW^+\,\bar{b}W^- \to (l^- \ldots)\,(\nu l^+)\,(l^+ \ldots)\,(l^-\bar{\nu})$$

Gedanken-experiment: given a histogram with 2000 bins of 20000 random integers between 1 and 2000, we expect an average of 100 entries per bin. What is the statistical error for the number of entries in one bin? What is the probability of finding a bin with 130 entries? How many of such 130 entries bins (in average) do you expect to appear in 200 bins? In other words, what is the probability to find a deviation of 3 standard deviations in one of the bins of the distribution?

The error for bin entries is (counting statistics) $\sqrt{100} = 10$. Finding a bin with 130 entries (or more): $\sigma = 10 \Rightarrow 3\sigma$ is what we are looking for. The probability of higher than 130 or lower than 70 is $1 - 3\sigma = 1 - 0.9973$, so the requested probability for 1 bin is $(1 - 3\sigma)/2$. The probability that at least one of the bins has more than 130 counts:

$$P = 1 - \left(1 - \frac{1 - 0.9973}{2}\right)^{200} = 1 - 0.7532 = 0.2368. \tag{2}$$