# E214 questions

May 6, 2018

## 1 Section 7.2.2

Question A: Decay of the $Z^0$ boson
Which value does the momentum of an electron have in the decay of a $Z^0$ boson $Z^0 \rightarrow e^+e^-$, if the $Z^0$ is at rest?

From four-momentum conservation:

$$p_{Z^0} = \begin{bmatrix} m_{Z^0} \\ \vec{0} \end{bmatrix} = \begin{bmatrix} E_e \\ \vec{p} \end{bmatrix} + \begin{bmatrix} E_e \\ -\vec{p} \end{bmatrix} = p_{e^+} + p_{e^-}$$

From this, we get

$$m_{Z^0} = 2\sqrt{p^2 + m_e^2} = 91.2\,\text{GeV}, \quad m_e = 511\,\text{GeV}$$

$$\Rightarrow p = \sqrt{\frac{m_{Z^0}^2}{4} - m_e^2} = 45.6\,\text{GeV}$$

Question B: Scattering reaction $e^+e^- \rightarrow \tau^+\tau^-$

How large is the momentum of tau leptons in the reaction $e^+e^- \rightarrow \tau^+\tau^-$, if the reaction takes place in the center-of-mass system (center-of-mass energy = 5 GeV)?
In CMS frame ($\vec{p}_{e^-} = -\vec{p}_{e^+}$), the total four-momentum:

$$s = (p_{\tau^+} + p_{\tau^-})^2 = \begin{bmatrix} 2E_\tau \\ \vec{0} \end{bmatrix}^2 = 4E_\tau^2 = E_{CMS}^2 = 25\,\text{GeV}^2$$

We can calculate ($m_\tau \approx 1.78$ GeV) the three-momentum of the tau leptons:

$$p_\tau^2 = E_\tau^2 - m_{tau}^2 = \left(\frac{25}{4} - 1.78^2\right)\text{GeV}^2$$

$$p = 1.755\,\text{GeV}$$

## 2 Section 7.4.1

How could the variable `ptw` be constructed from other tree variables?

We are looking at $W \to e\nu$ processes. We can use the missing transverse momentum (`ptmis_x, ptmis_y`), which we assign to the neutrino. As we know the electron transverse momentum: `el_px, el_py, el_pt`, the W-boson transverse momentum is determined by momentum conservation, if the

The correct form of the Gauss error propagation law in the presence of correlations.

The standard deviation squared of a function $f(x, y)$ is given by

$$\sigma_f^2 = \left(\frac{df}{dx}\right)^2 \sigma_x^2 + \left(\frac{df}{dy}\right)^2 \sigma_y^2 + 2\frac{df}{dx}\frac{df}{dy}\mathrm{cov}(x, y) \tag{1}$$

**Minimize correlation: no clue. Correlation is maximal if the variables are linearly dependent ($\pm 1$), 0 if they are statistically independent.**

## 3 Section 7.5.1

Minimum invariant 4-lepton mass

At threshold: $m_{Z^0} = 2m_l$; the minimum 4-lepton invariant mass is acquired when both $Z^0$'s are stationary (CMS frame),and equal to $2m_{Z^0}$.

Lepton mass distribution

See manual p. 41, also Thomson 17.19; there is a $Z^0$ peak and a Higgs-boson peak. One example for the background at the Higgs-peak is the $t \to bW^+$ decay.

Ideal/real detector.

Ideal: there would be no missing transverse momentum. In real detectors, there are energy losses not detected due to inactive detector elements, imperfect calibration (?).

Four leptons in $t\bar{t}$ event.

Might be manual p. 46 Figure 6.2?

Bins.

The error for bin entries is (counting statistics) $\sqrt{100} = 10$. Finding a bin with 130 entries (or more): $\sigma = 10 \to 3\sigma$ is what we are looking for. The probability of higher than 130 or lower than 70 is $1 - 3\sigma = 1 - 0.9973$, so the requested probability for 1 bin is $(1 - 3\sigma)/2$. The probability that at least one of the bins has more than 130 counts:

$$P = 1 - \left(1 - \frac{1 - 0.9973}{2}\right)^{200} = 1 - 0.7532 = 0.2368. \tag{2}$$