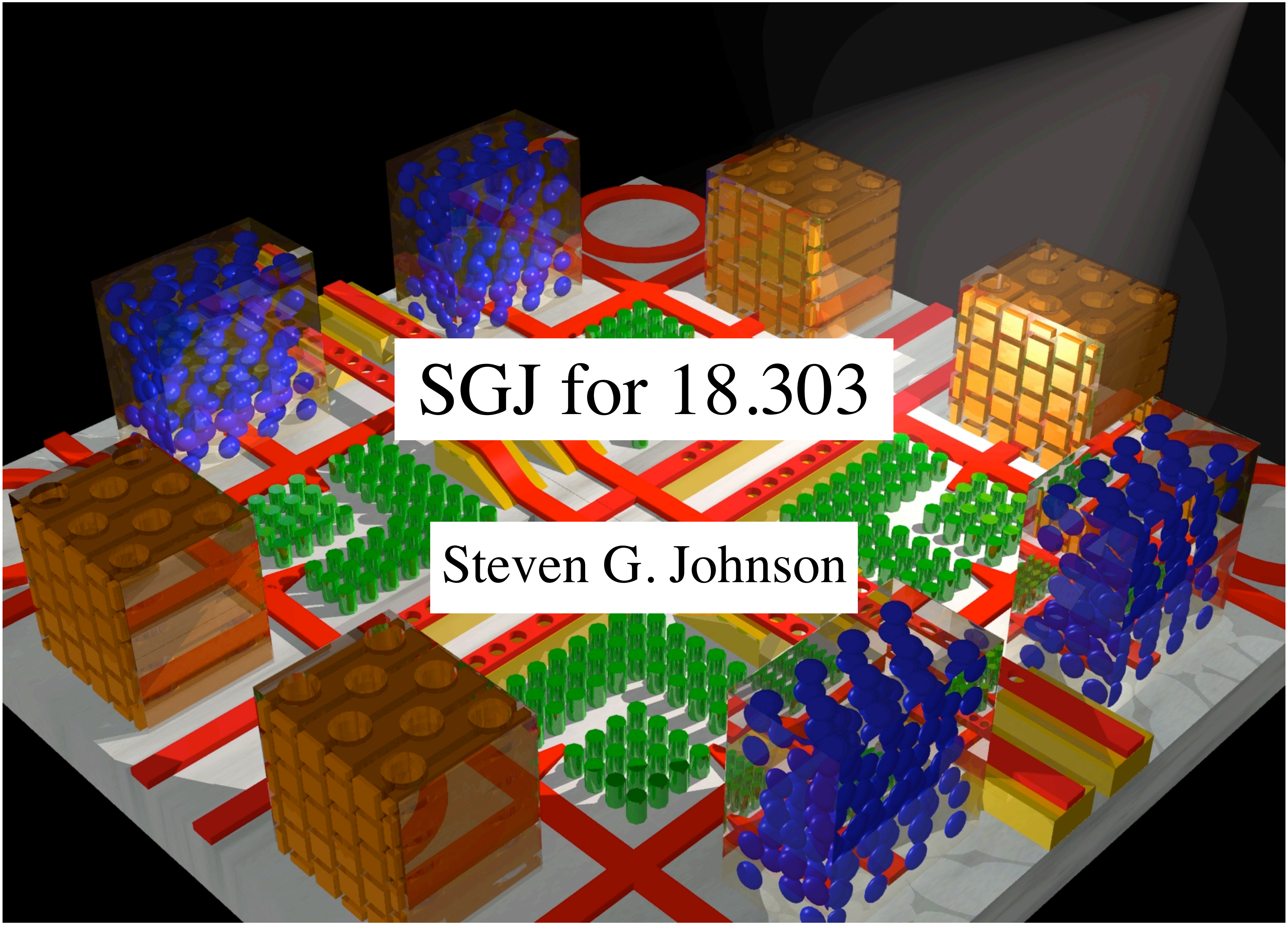
SGJ for 18.303

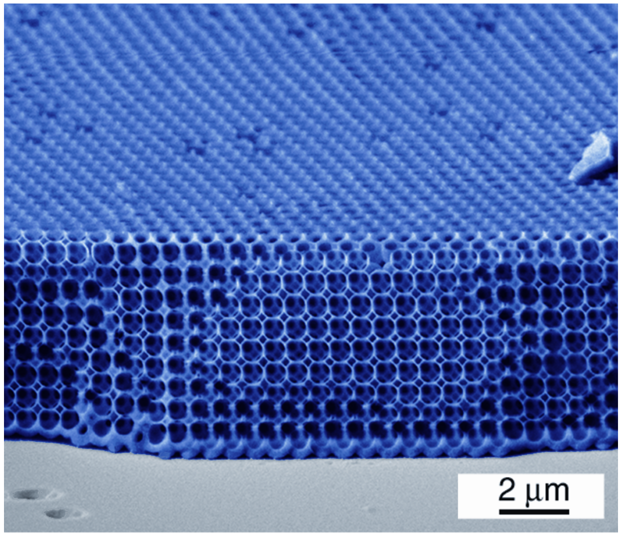
Steven G. Johnson



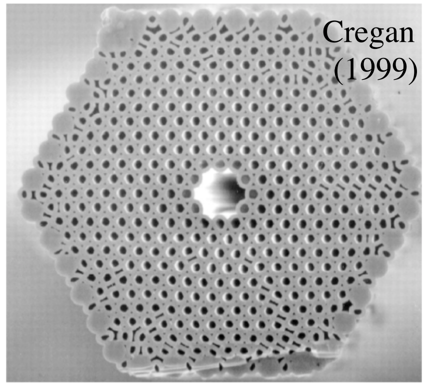
Photonics

classical electromagnetic effects can be altered by λ-scale structures

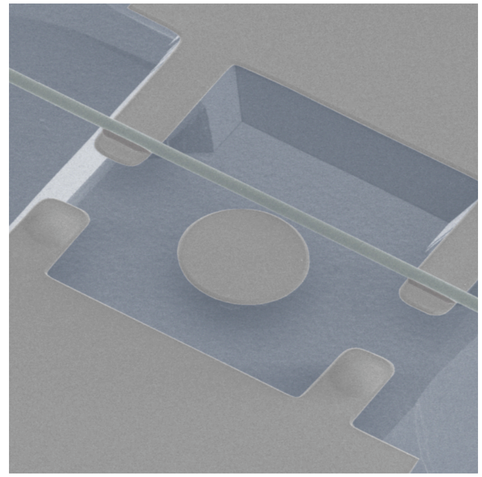
optical insulators



[D. Norris, UMN (2001)]

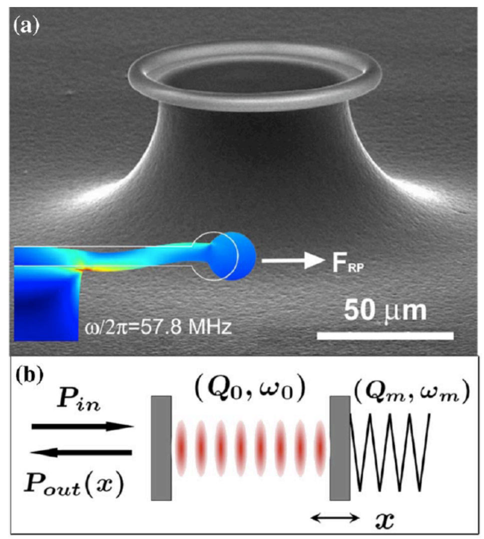


trapping/guiding light in vacuum



high-quality μ-resonances coupled to other physics (e.g. vibration)

[Eichenfield et al. *Nature Photonics* **1**, 416 (2007)]



[Schliesser et al., *PRL* **97**, 243905 (2006)]

Instead of finding new materials, *one tries to find new* geometries = new effects.

Mathematical challenges

for lengthscales a ~ wavelength λ

- Complete analytical solutions are almost never possible, so how do we understand waves in complex geometries?

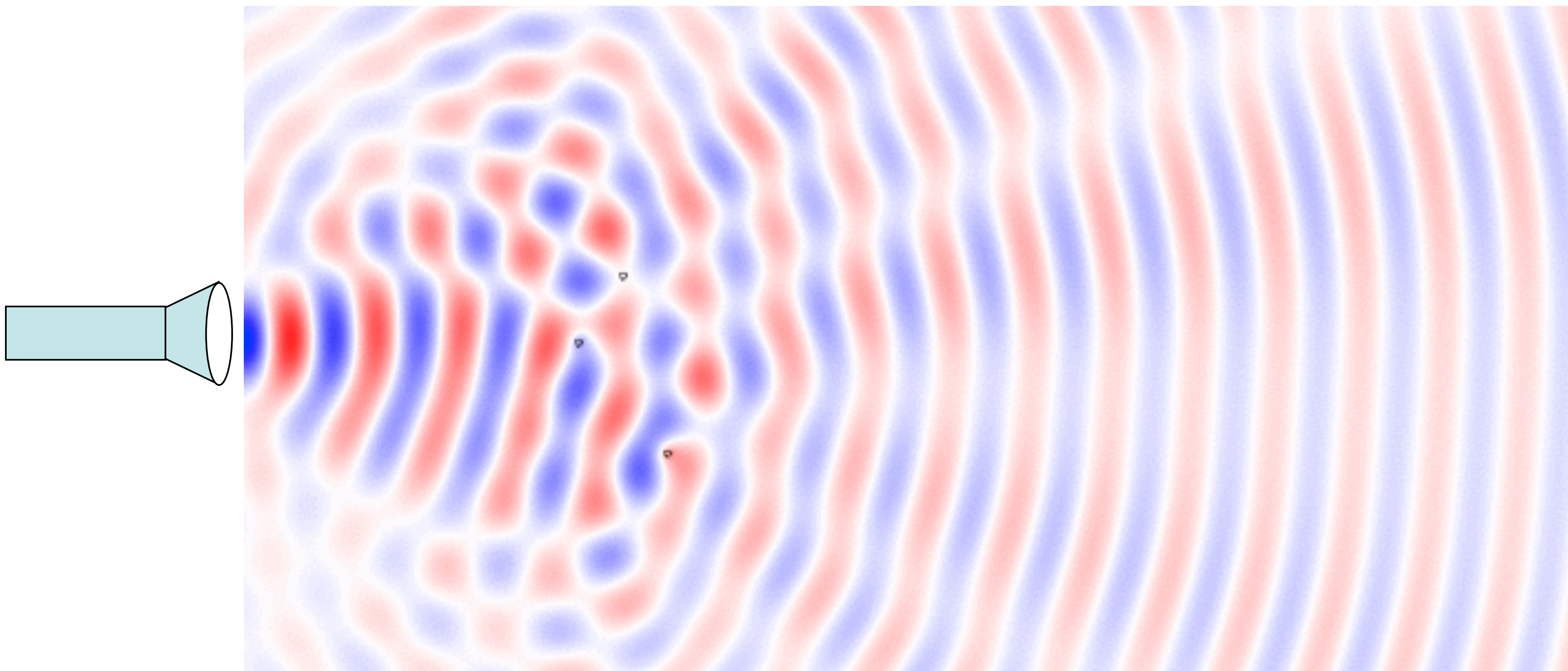
  … theorems, conservation laws, perturbatively, …

- What new phenomena are possible that wouldn't happen in homogeneous media, ray optics ($a \gg \lambda$), or circuits ($a \ll \lambda$)

- How do we design new/better optical devices (solar, lasers, fibers, …) exploiting so many degrees of freedom?

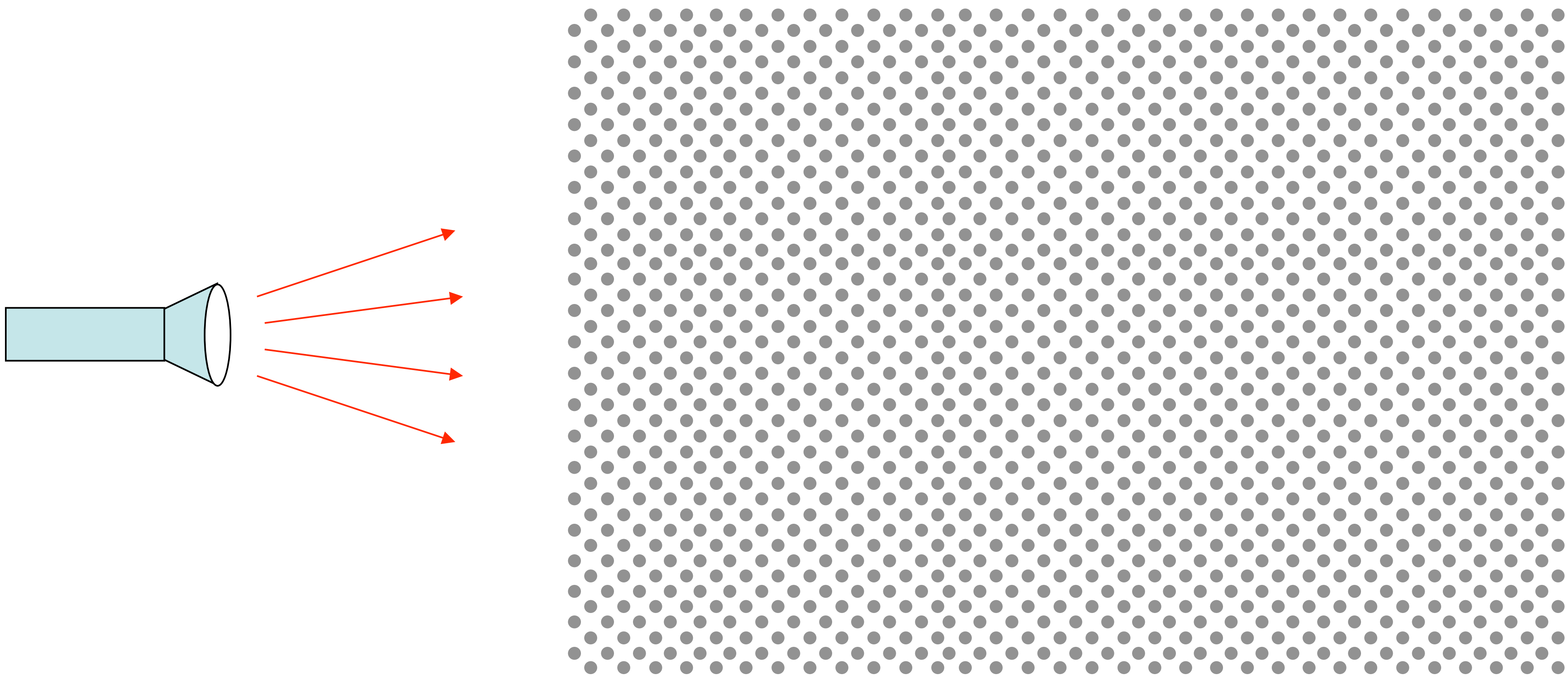
Complex geometries = messy results?

here: scattering off three specks of silicon



can be solved on a computer, but not terribly interesting…

An even bigger mess? zillons of scatterers



Blech, light will just scatter like crazy
and go all over the place … how boring!

Fun with Math

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial}{\partial t}\vec{H} = i\frac{\omega}{c}\vec{H}$$

$$\vec{\nabla} \times \vec{H} = \varepsilon\frac{1}{c}\frac{\partial}{\partial t}\vec{E} + \cancelto{0}{\vec{J}} = -i\frac{\omega}{c}\varepsilon\vec{E}$$

First task:

get rid of this mess

dielectric function $\varepsilon(\mathbf{x}) = n^2(\mathbf{x})$

$$\underbrace{\nabla \times \frac{1}{\varepsilon}\nabla \times}_{\text{eigen-operator}} \vec{H} = \underbrace{\left(\frac{\omega}{c}\right)^2}_{\text{eigen-value}} \vec{H}$$

eigen-state

+ constraint

$$\nabla \cdot \vec{H} = 0$$

Hermitian Eigenproblems

$$\underbrace{\nabla \times \frac{1}{\varepsilon} \nabla \times}_{\text{eigen-operator}} \vec{H} = \underbrace{\left(\frac{\omega}{c}\right)^2}_{\text{eigen-value}} \vec{H}$$

eigen-state

+ constraint

$$\nabla \cdot \vec{H} = 0$$

Hermitian positive semi-definite for real (lossless) $\varepsilon > 0$

→ well-known properties from linear algebra:

ω are real (lossless)

eigenfunctions are orthogonal

usually diagonalizable (eigenbasis)

…the magic of symmetry…

[Emmy Noether, 1915]

Noether's theorem:

symmetry = conservation laws

In this case, periodicity

= conserved "momentum"

= wave solutions without scattering

[Bloch waves]

Felix Bloch
(1928)

Periodic Hermitian Eigenproblems

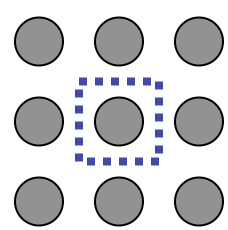
[G. Floquet, "Sur les équations différentielles linéaries à coefficients périodiques," *Ann. École Norm. Sup.* **12**, 47–88 (1883).]
[F. Bloch, "Über die quantenmechanik der electronen in kristallgittern," *Z. Physik* **52**, 555–600 (1928).]

if eigen-operator is periodic, then Bloch-Floquet theorem applies:

can choose: $$\underbrace{\vec{H}(\vec{x},t) = e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}}_{\text{planewave}}\underbrace{\vec{H}_{\vec{k}}(\vec{x})}_{\text{periodic “envelope”}}$$

Corollary 1: **k** is conserved, *i.e.* no scattering of Bloch wave

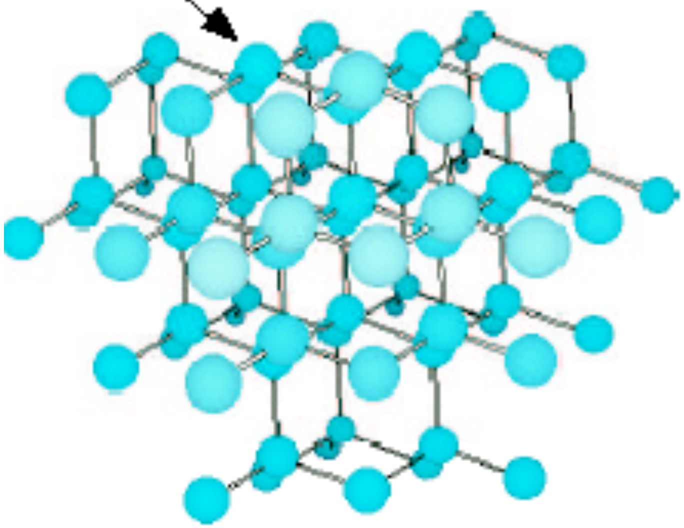
Corollary 2: $\vec{H}_{\vec{k}}$ given by finite unit cell,
so ω are discrete bands $\omega_n(\mathbf{k})$



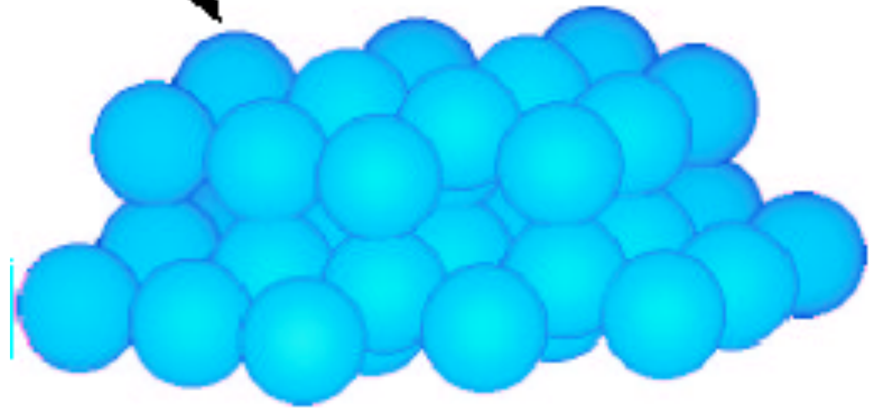
Electronic and Photonic Band Structures

Periodic Medium

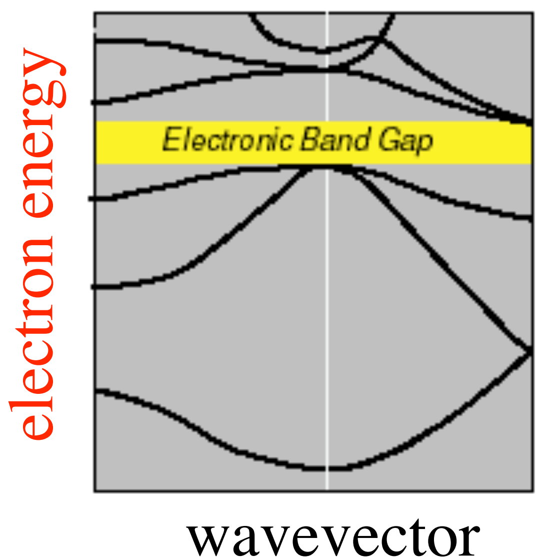
atoms in diamond structure

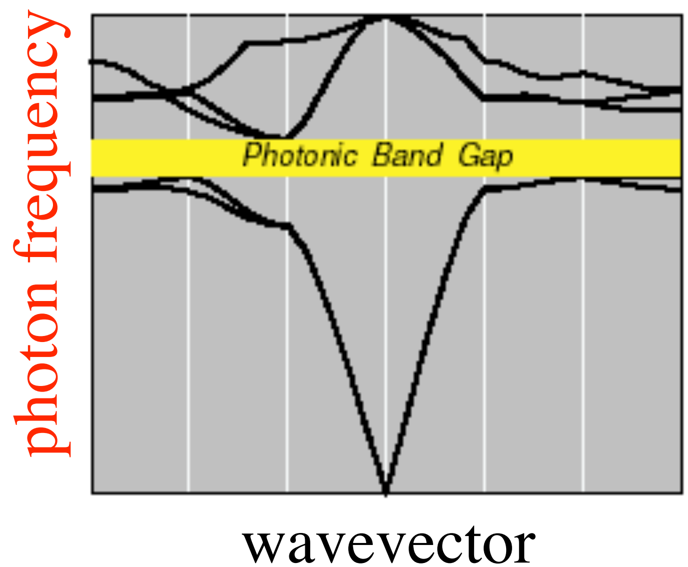


dielectric spheres, diamond lattice



Bloch waves:
Band Diagram

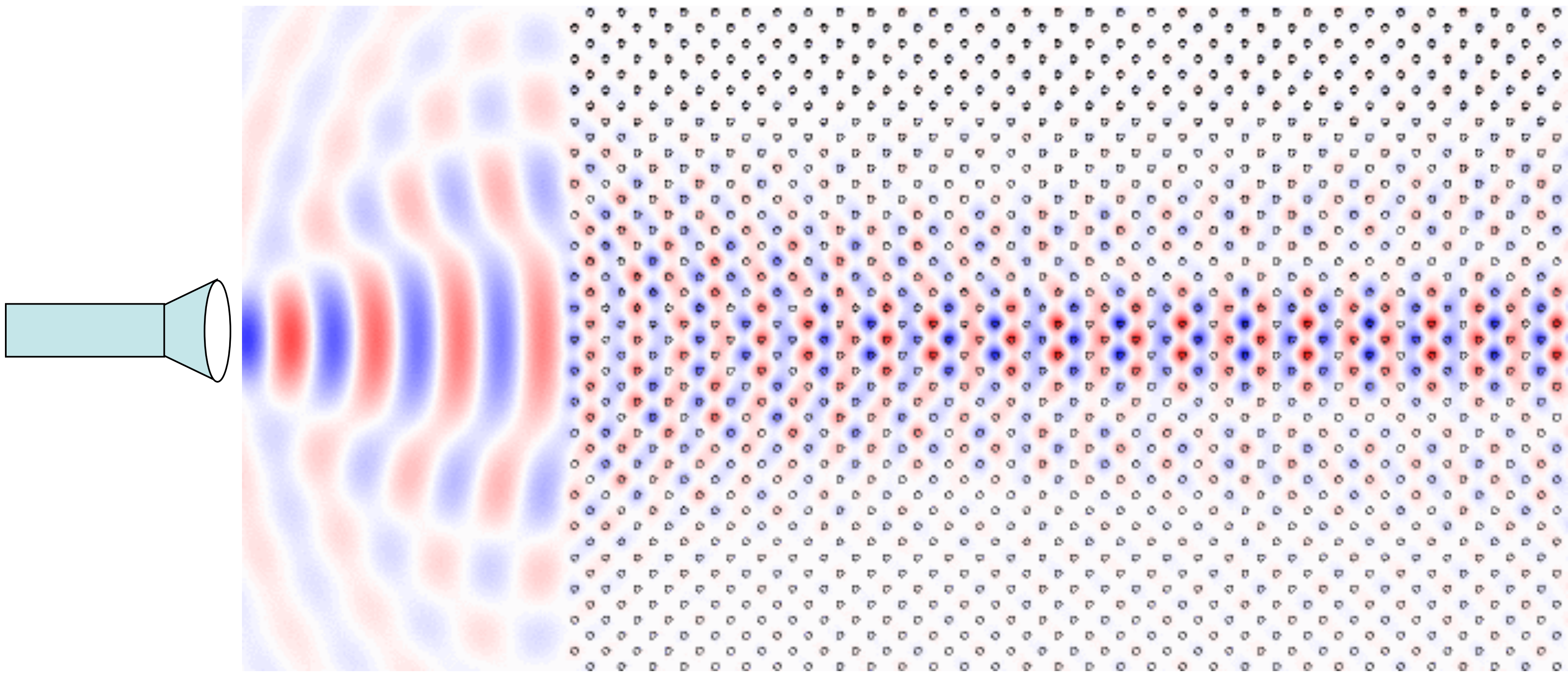




strongly interacting fermions

weakly-interacting bosons

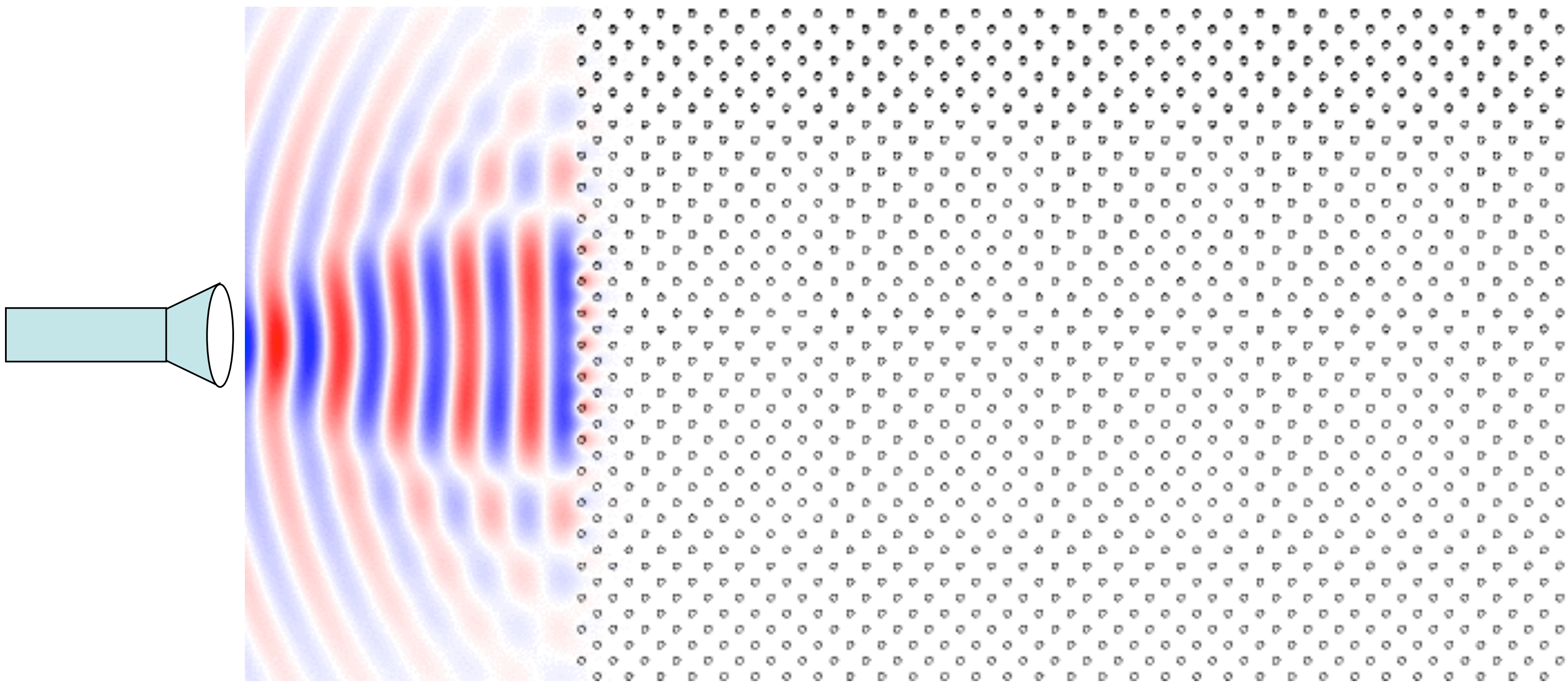
Not so messy, not so boring…



the light seems to form several *coherent beams*
that propagate *without scattering*
… and almost *without diffraction* (*supercollimation*)

A slight change? Shrink λ by 20%

an "optical insulator" (photonic bandgap)



light cannot penetrate the structure at this wavelength!

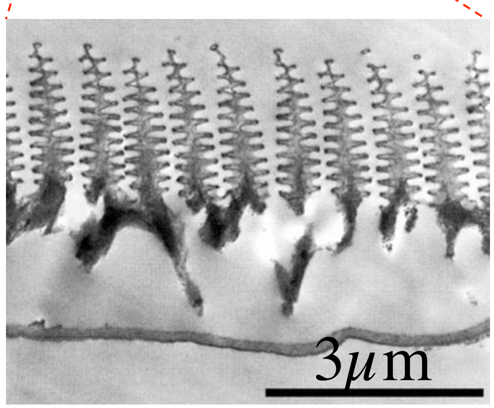
all of the scattering destructively interferes

Photonic Crystals in Nature

Morpho rhetenor butterfly

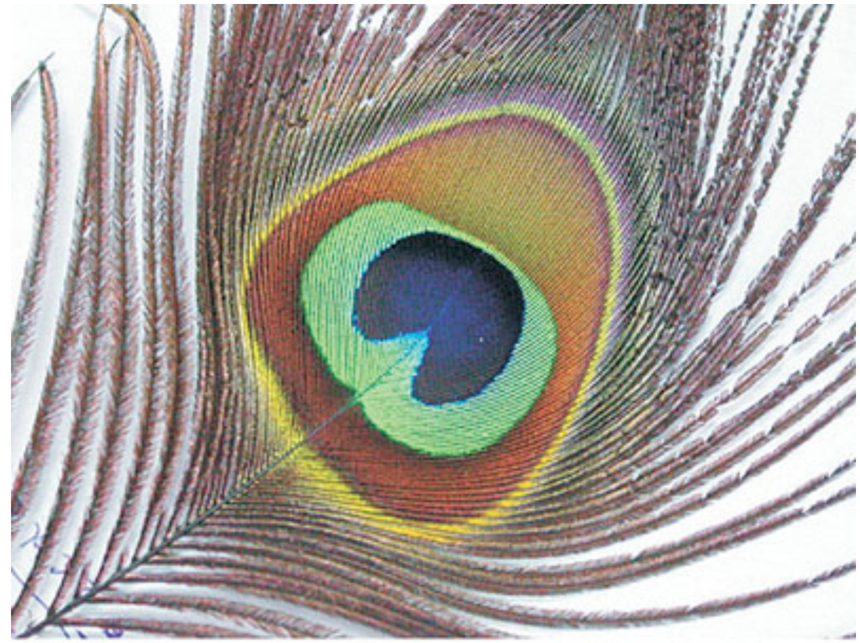
wing scale:

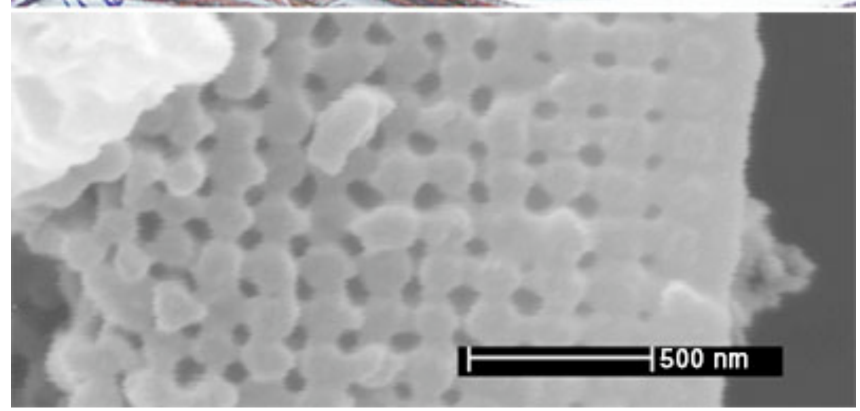
[P. Vukosic *et al.*, *Proc. Roy. Soc: Bio. Sci.* **266**, 1403 (1999)]



[also: B. Gralak *et al.*, *Opt. Express* **9**, 567 (2001)]

Peacock feather





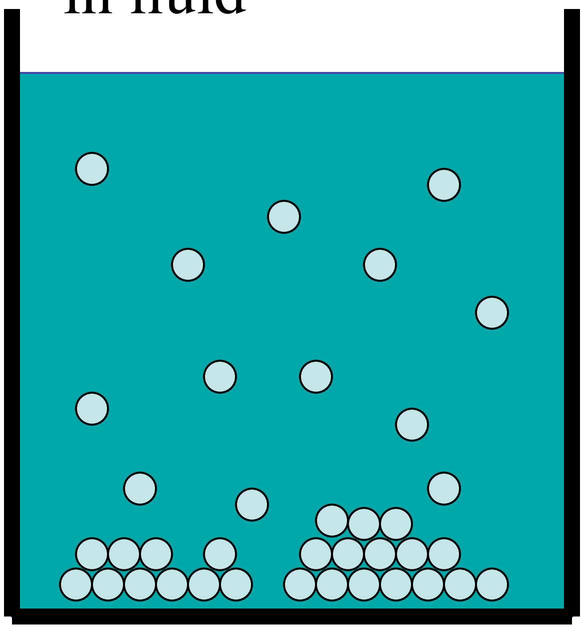
[J. Zi *et al*, *Proc. Nat. Acad. Sci. USA*, **100**, 12576 (2003)]

[figs: Blau, *Physics Today* **57**, 18 (2004)]

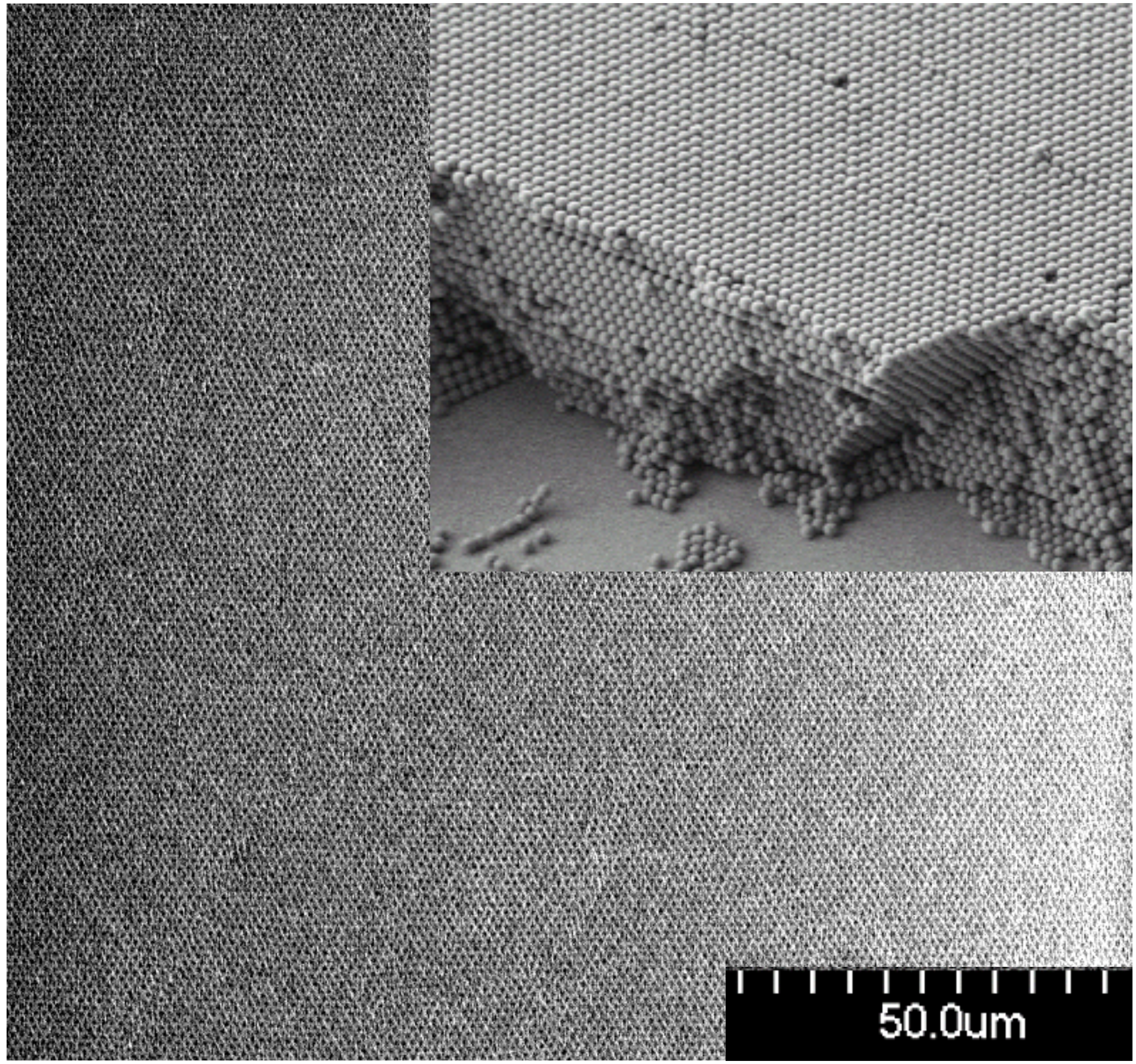
A Synthetic Opal

[fig courtesy
D. Norris, UMN]

colloid: suspension of microparticles in fluid



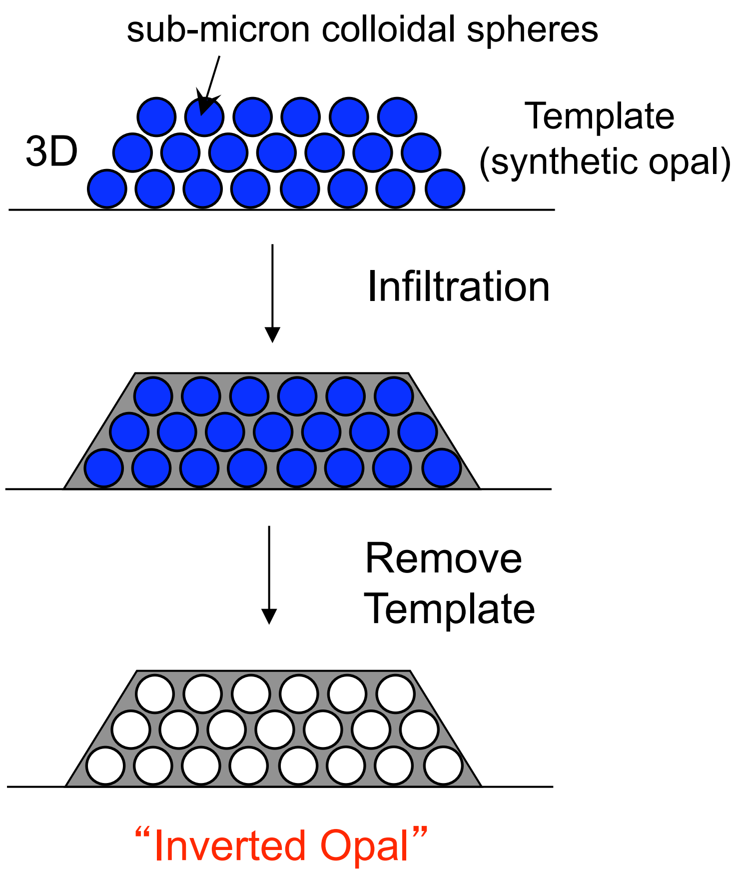
particles close-pack after fluid evaporates

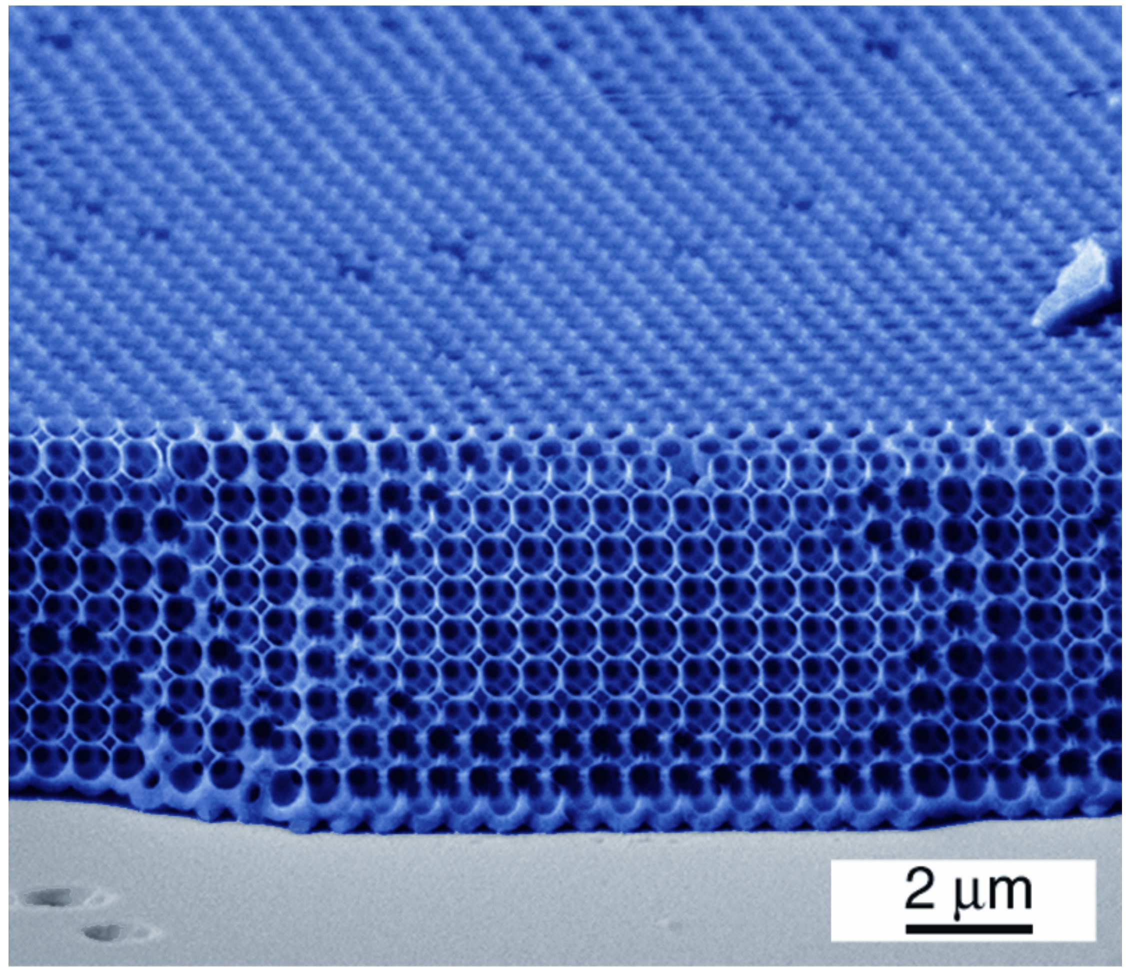


Inverse-Opal Photonic Crystal

(because opal alone has no band gap)

[fig courtesy
D. Norris, UMN]



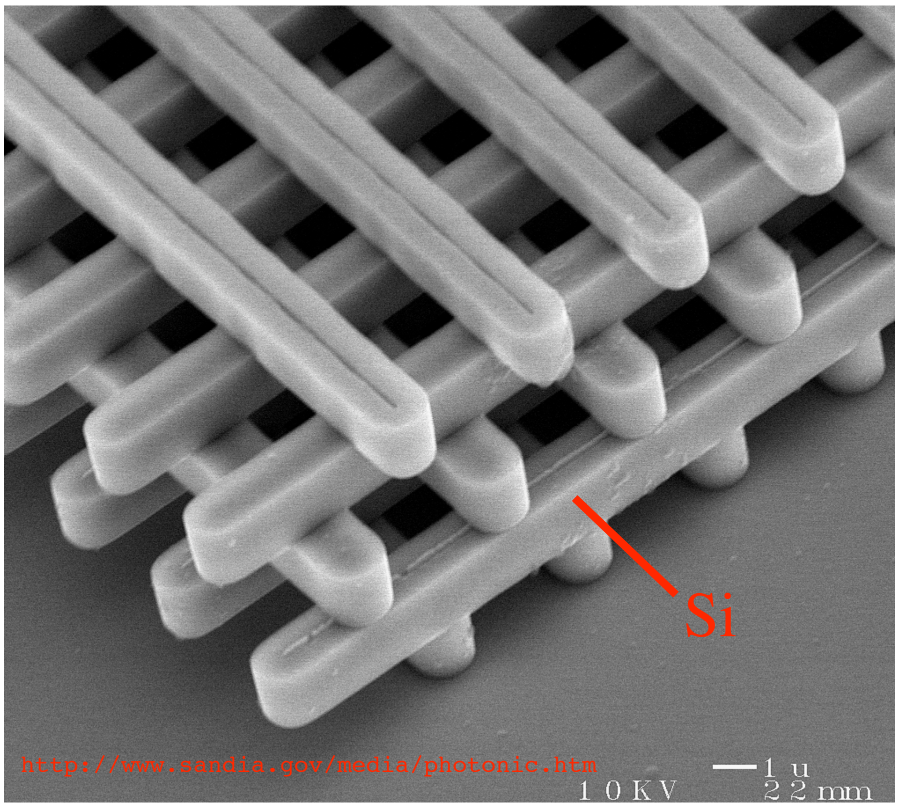


[Y. A. Vlasov *et al*., *Nature* **414**, 289 (2001).]

Fabrication by Lithography

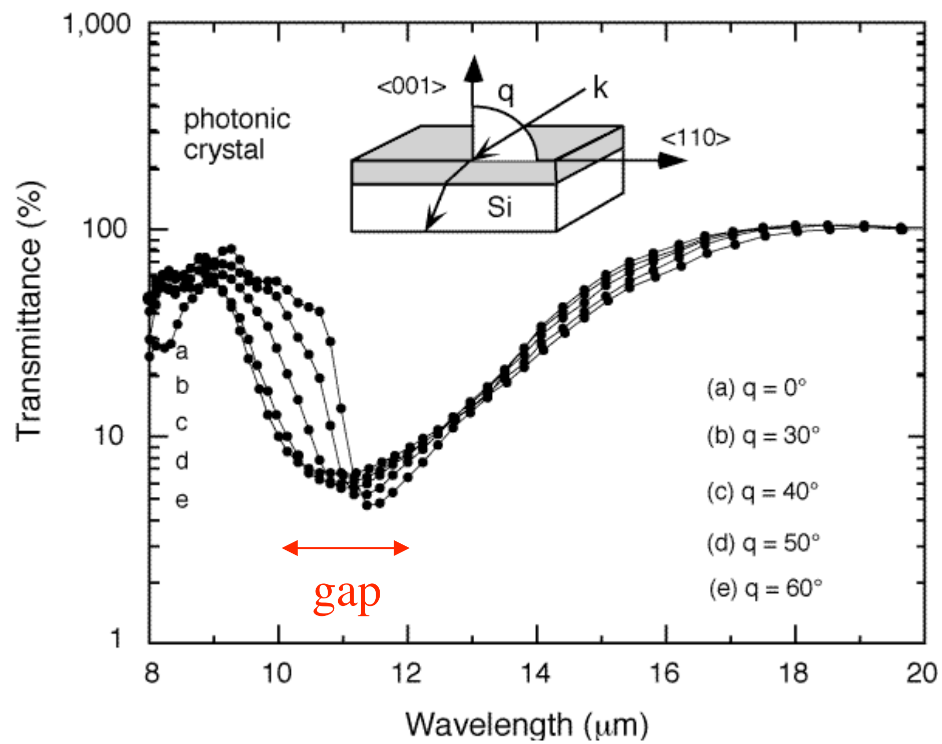
[K. Ho *et al.*, *Solid State Comm.* **89**, 413 (1994)]   [H. S. Sözüer *et al.*, *J. Mod. Opt.* **41**, 231 (1994)]

(4 "log" layers = 1 period)



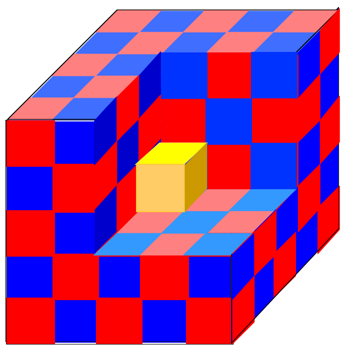
"Woodpile" crystal

[S. Y. Lin *et al.*, *Nature* **394**, 251 (1998)]

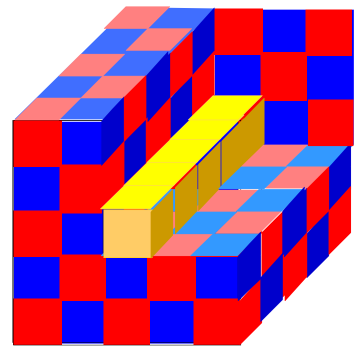


Intentional defects are good!

periodic electromagnetic media



can trap light in cavities

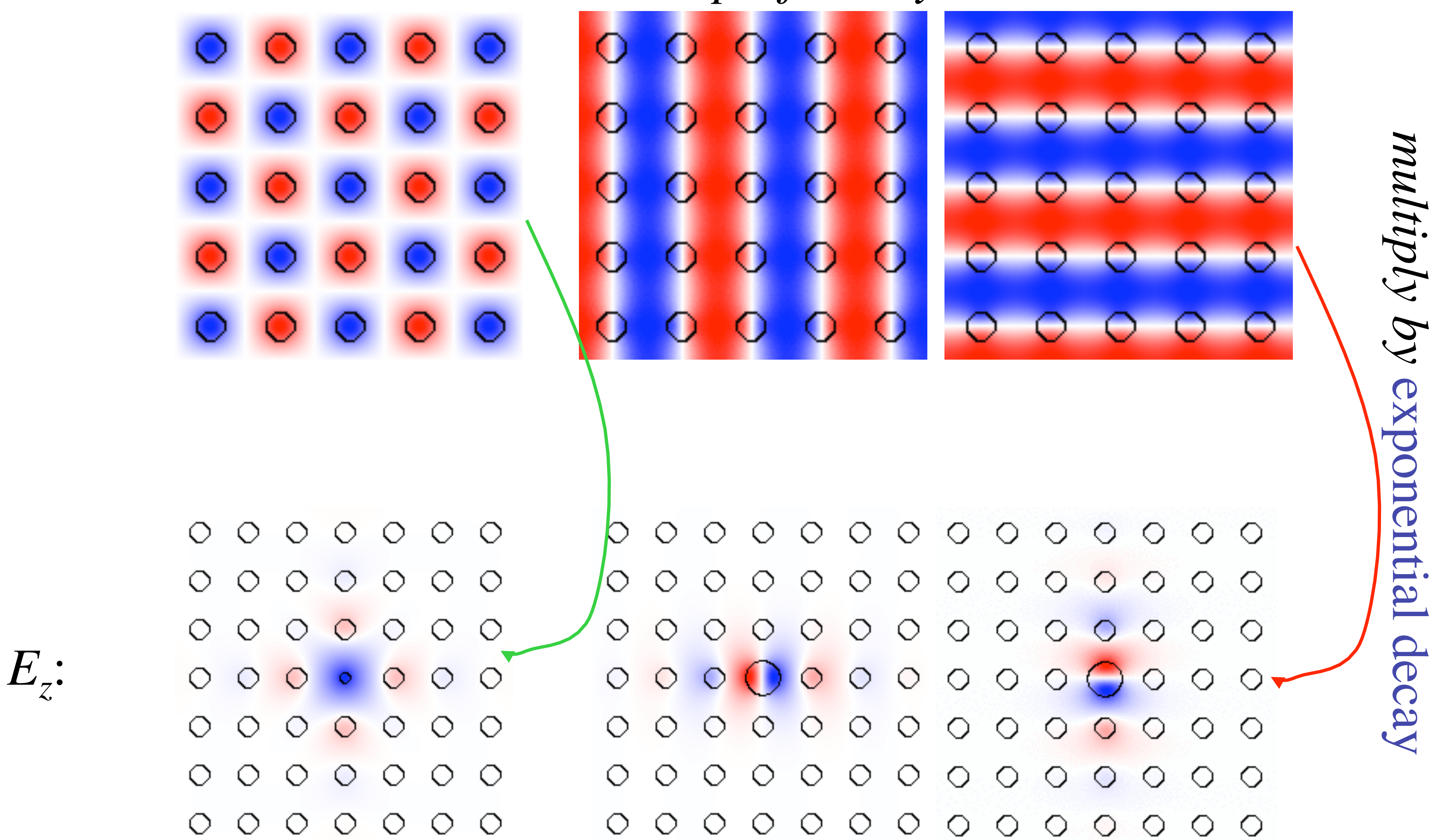


and waveguides ("wires")

with photonic band gaps:
"optical insulators"
for holding and controlling light

Dielectric microcavities

Bloch waves in perfect crystal:



shrunk rod

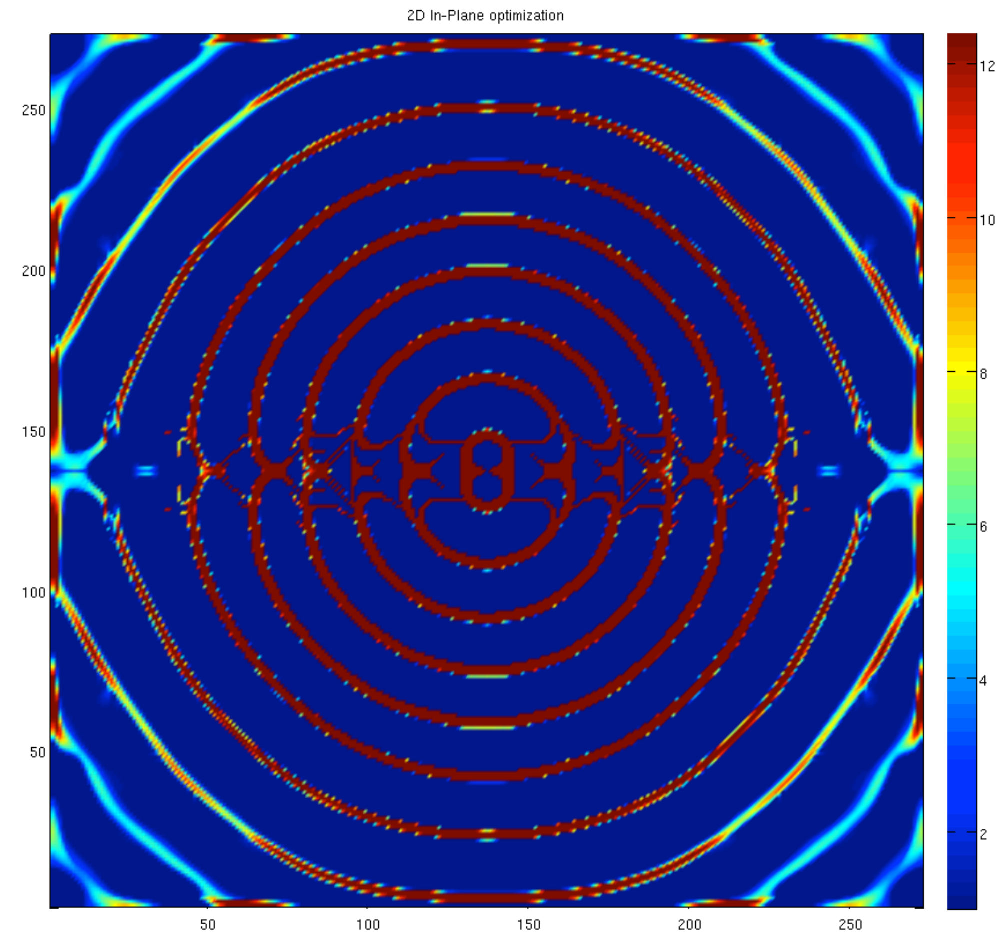
enlarged rod

Even Stronger Confinement: Topology Optimization

Let every pixel
be a degree of freedom

…

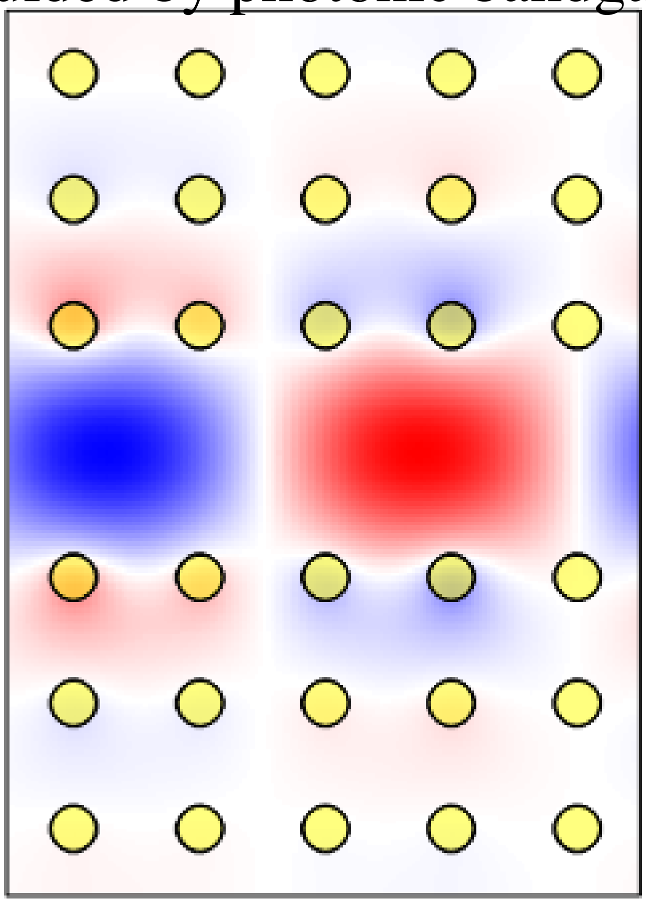
Computer searches
for best structure
(tightest confinement)
by large-scale optimization



in-plane **E** field (broken symmetry)

Waveguides

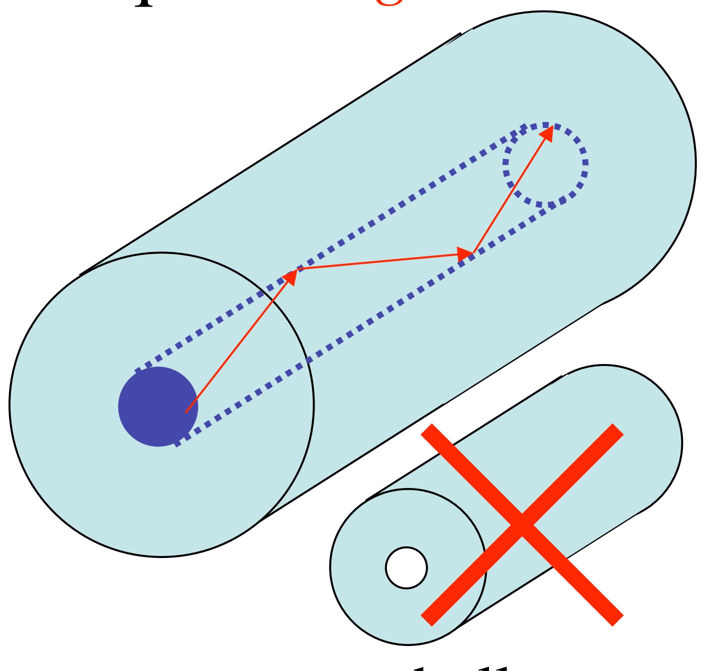
guided by photonic bandgap



vs. standard optical fiber:

"total internal reflection"

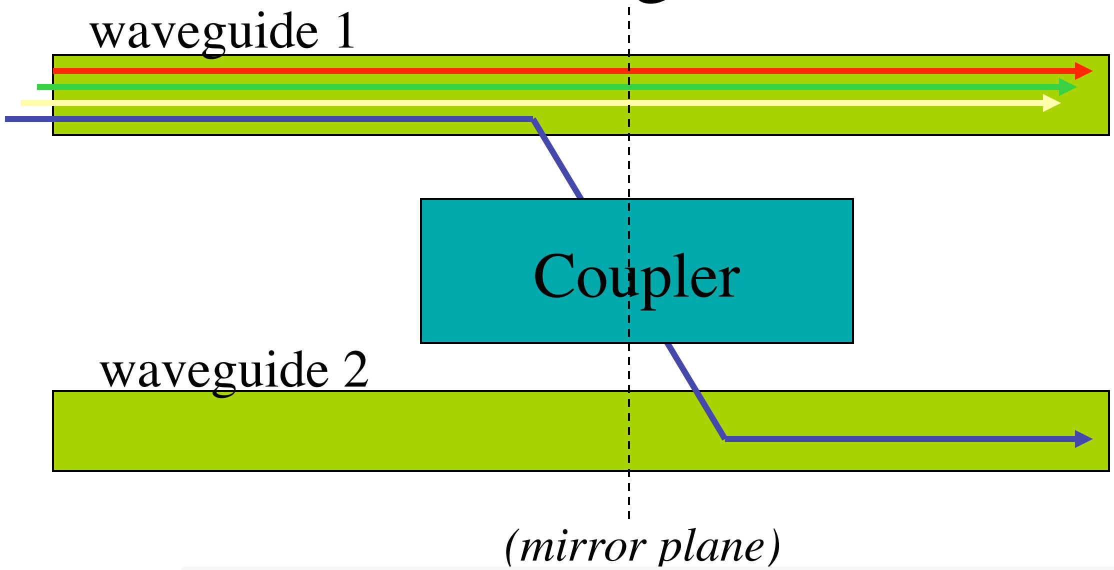
— requires *higher*-index core



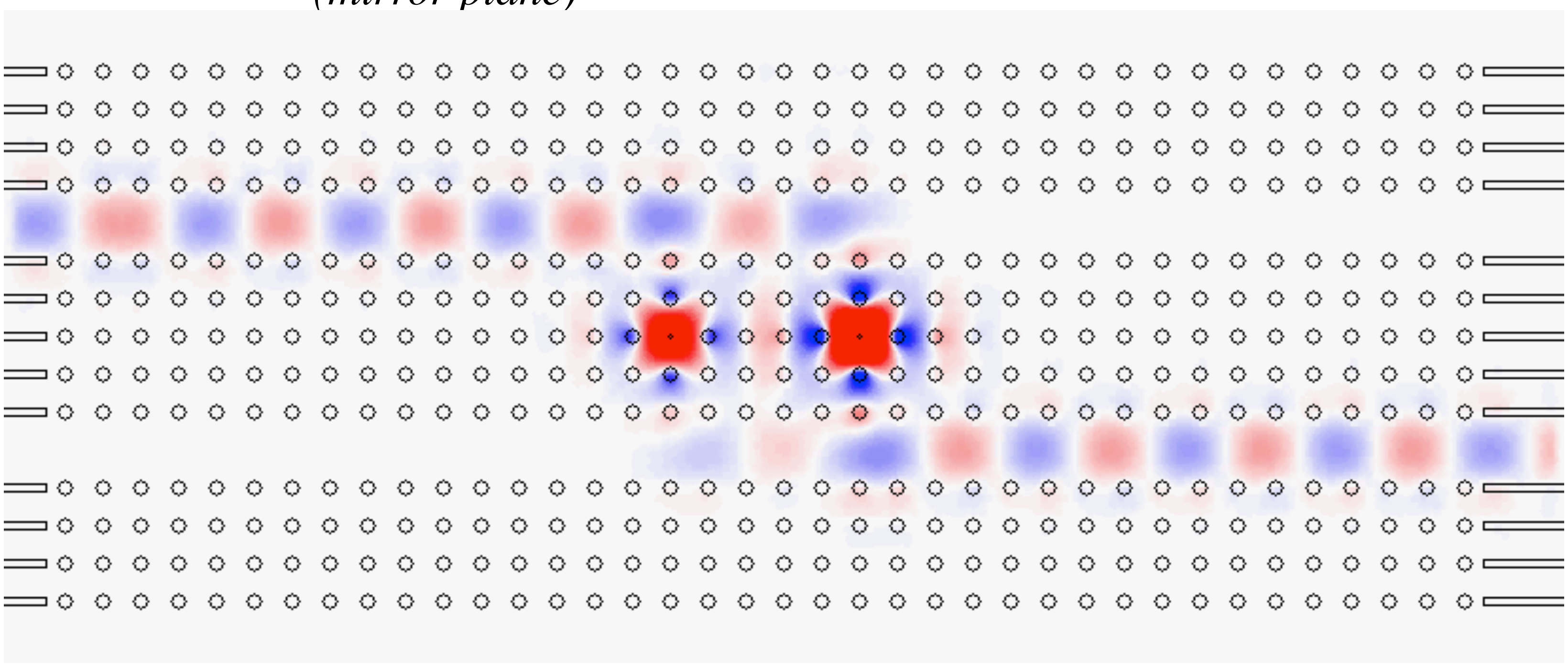
no hollow core!

hollow = lower absorption, lower nonlinearities, higher power

Waveguides + Cavities = Devices



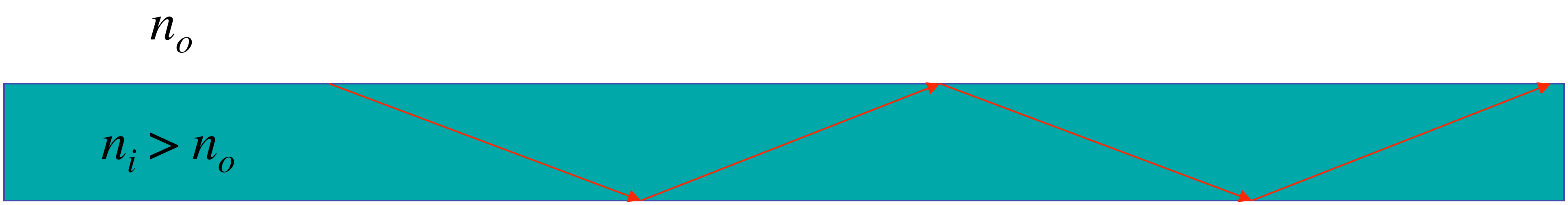
"Channel-drop filter"



[S. Fan *et al*., *Phys*. *Rev*. *Lett*. **80**, 960 (1998)]

How *else* can we confine light?

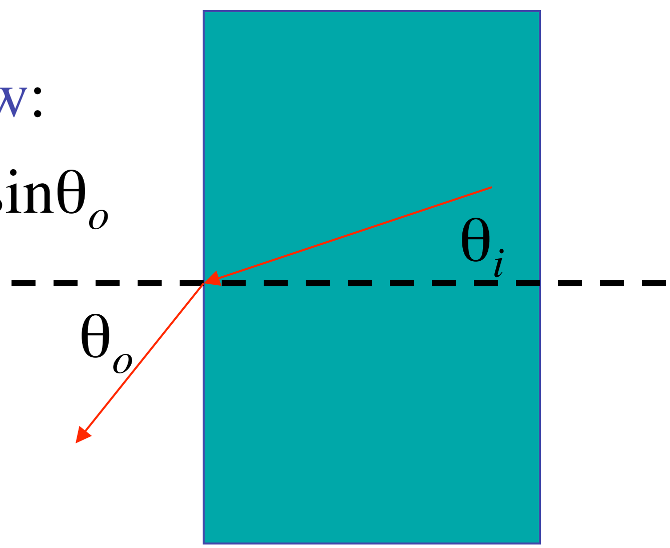
Total Internal Reflection



rays at shallow angles $> \theta_c$
are totally reflected

Snell's Law:

$$n_i \sin\theta_i = n_o \sin\theta_o$$

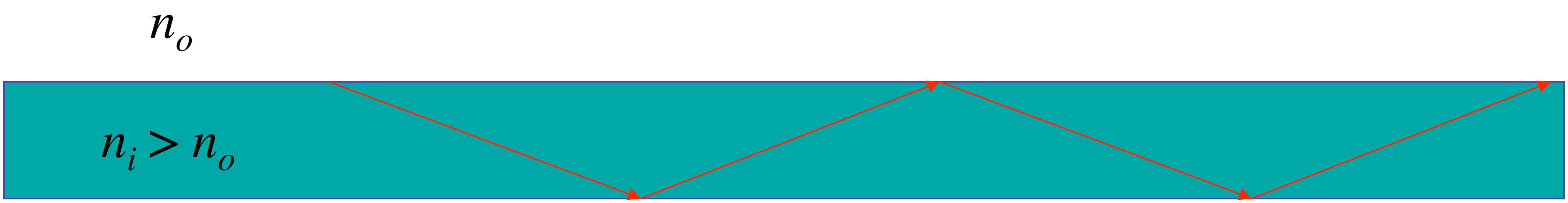


$$\sin\theta_c = n_o / n_i$$

< 1, so θ_c is real

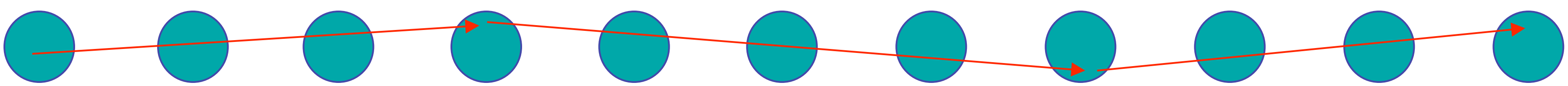
i.e. TIR can only guide
within higher index
unlike a band gap

Total Internal Reflection?



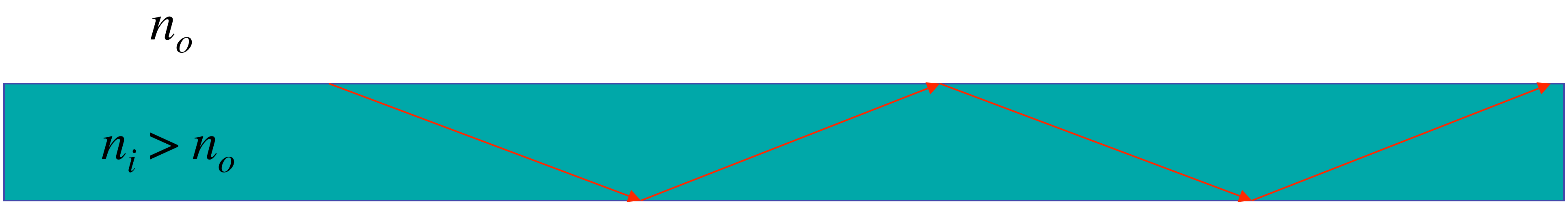
rays at shallow angles > θ_c
are totally reflected

So, for example,
a discontiguous structure can't possibly guide by TIR…

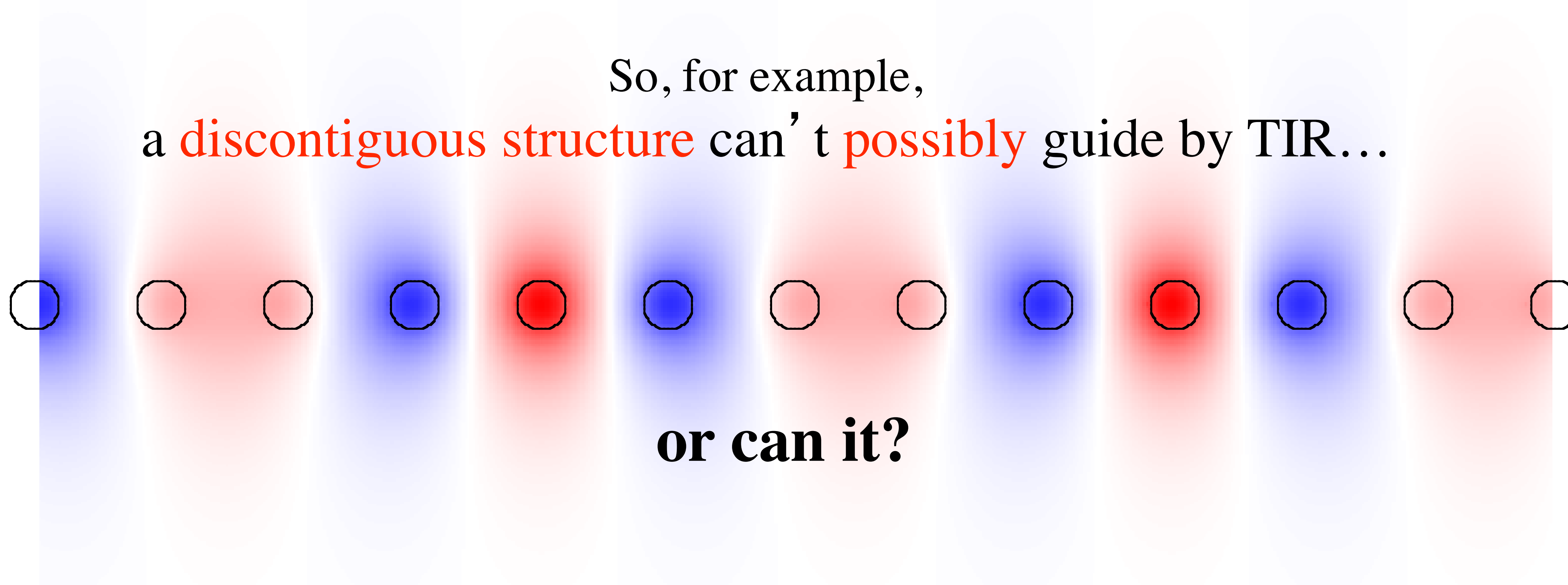


the rays can't stay inside!

Total Internal Reflection?



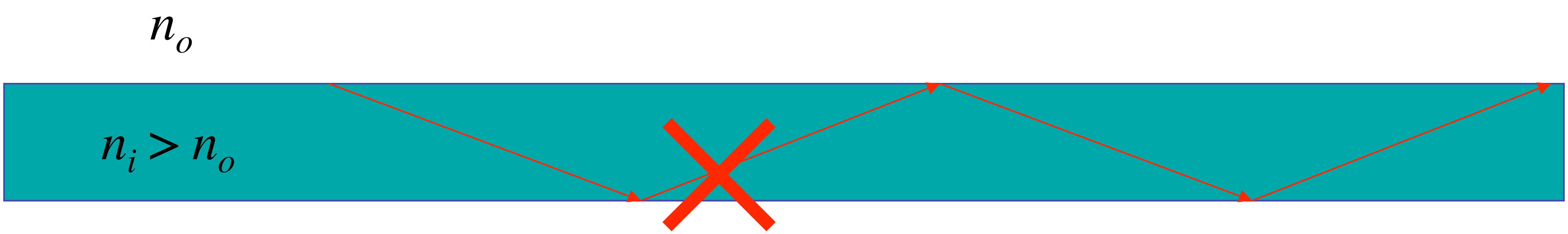
rays at shallow angles > θ_c
are totally reflected



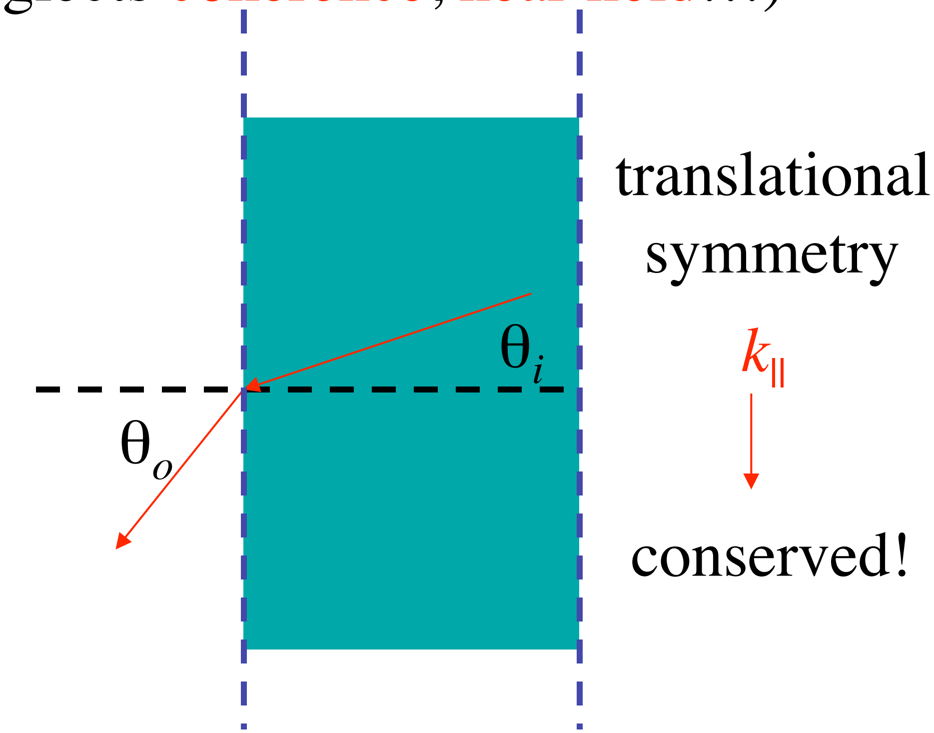
So, for example,
a discontiguous structure can't possibly guide by TIR…

or can it?

Total Internal Reflection Redux



ray-optics picture is invalid on λ scale
(neglects coherence, near field…)



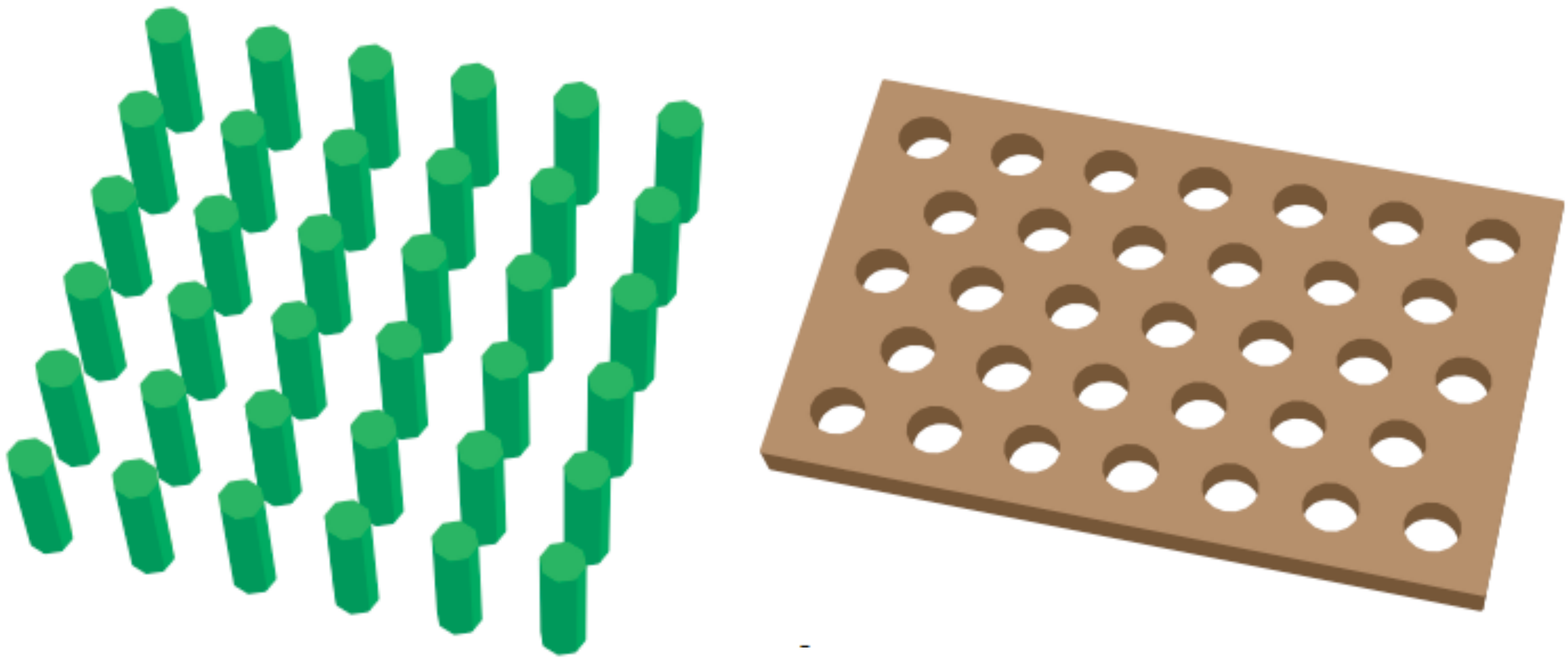
Snell's Law is really conservation of $k_{\|}$ and ω:

$$|k_i| \sin\theta_i = |k_o| \sin\theta_o$$

$$|k| = n\omega/c$$

(wavevector) (frequency)

Photonic-Crystal Slabs

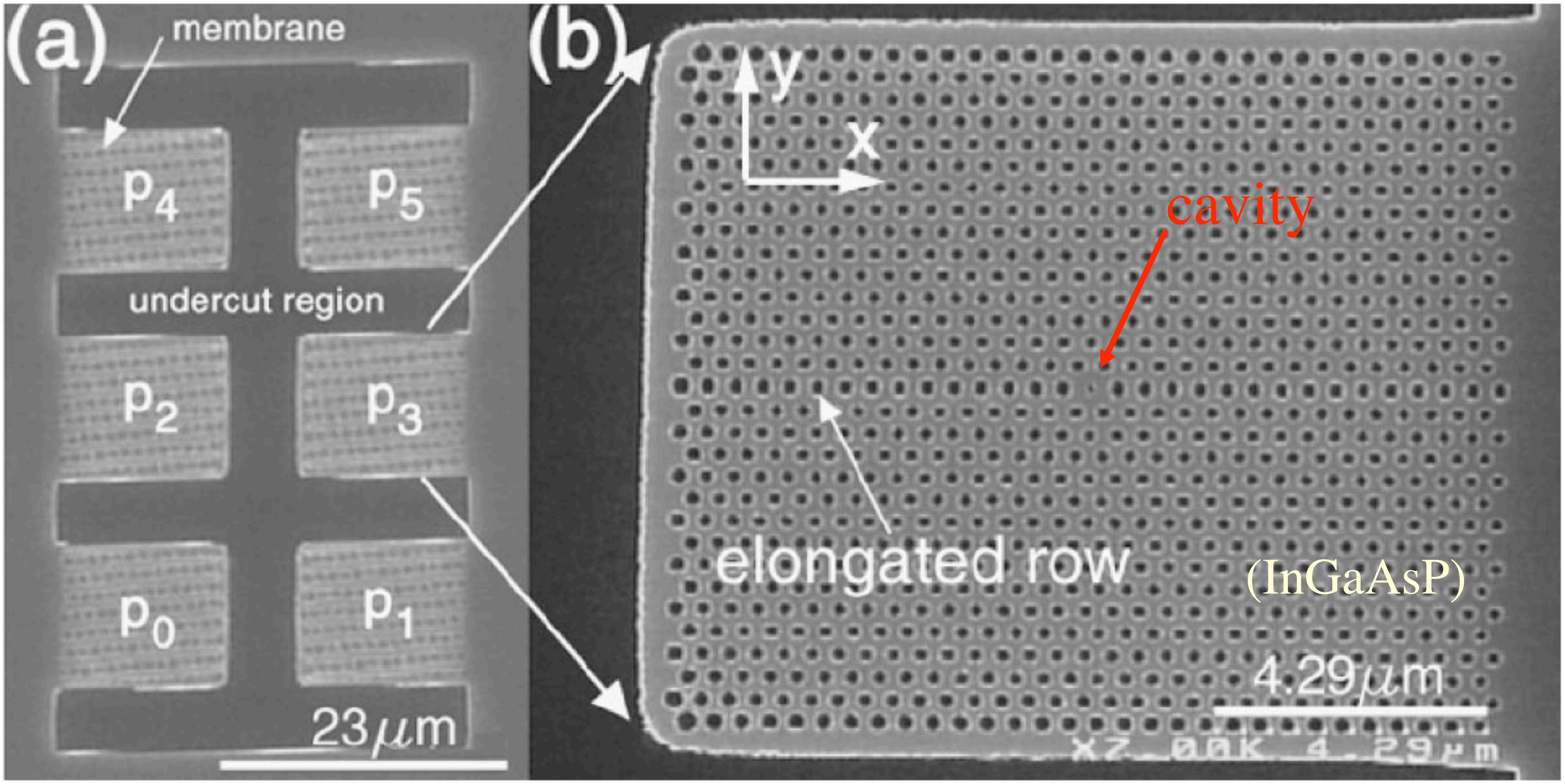


2d photonic bandgap + vertical index guiding

[S. G. Johnson and J. D. Joannopoulos, *Photonic Crystals: The Road from Theory to Practice*]

An Experimental (Laser) Cavity

[M. Loncar *et al*., *Appl*. *Phys*. *Lett*. **81**, 2680 (2002)]

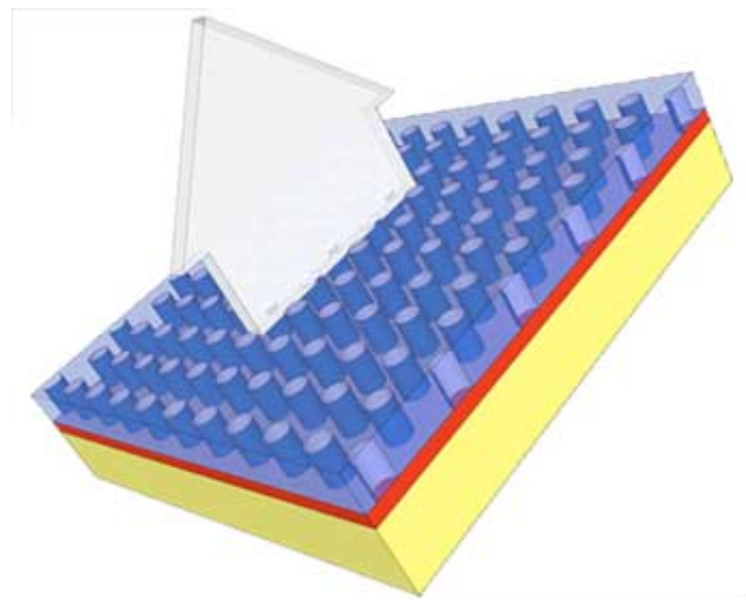


Molding Diffraction for Lighting

[another MIT startup (by a colleague): Luminus.com]

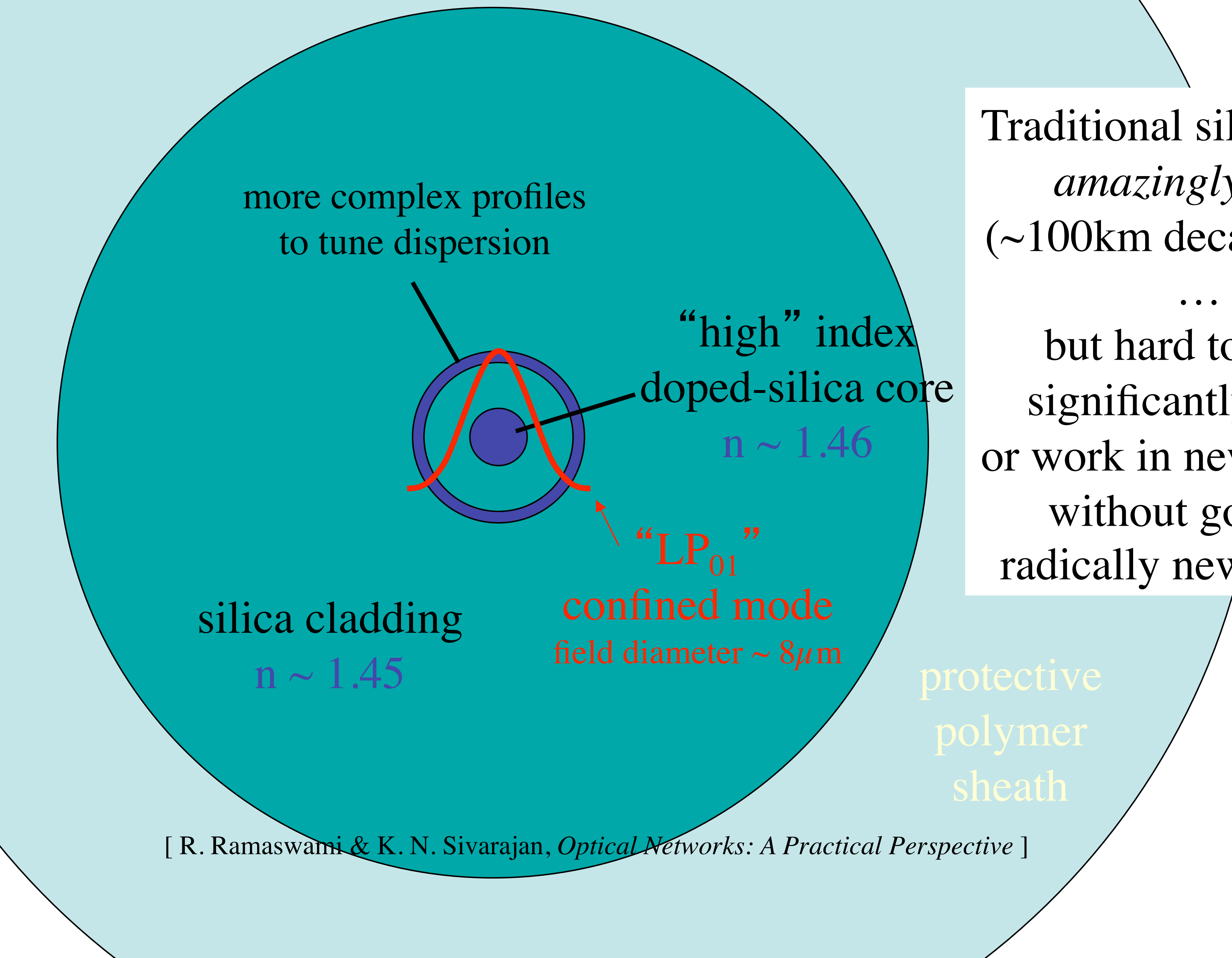
ultra-bright/efficient LEDs

periodic pattern gathers & redirects it in one direction



new projection TVs,
pocket projectors,
lighting applications,
…

Another 2d-to-3d route: Fibers

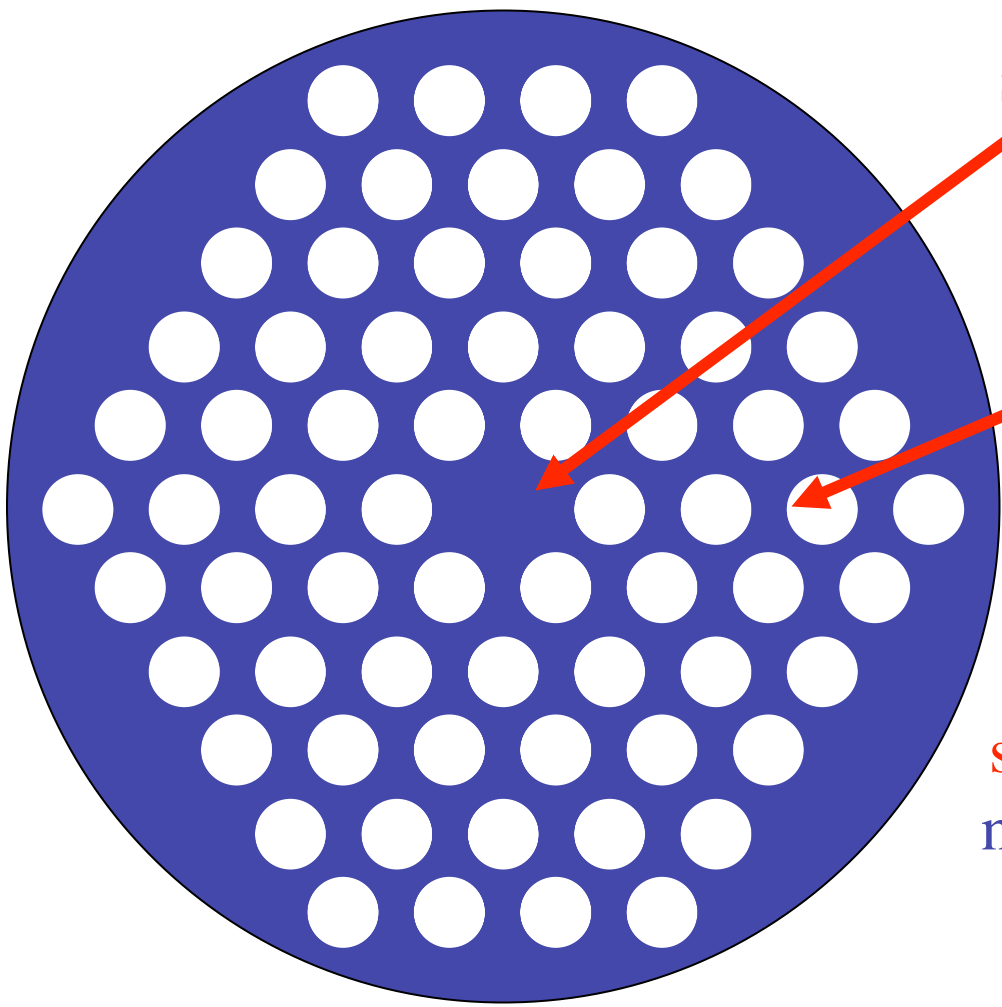


Traditional silica fibers:
amazingly good
(~100km decay length)
…
but hard to make
significantly better
or work in new regimes
without going to
radically new designs

[R. Ramaswami & K. N. Sivarajan, *Optical Networks: A Practical Perspective*]

Index-Guiding PCF & microstructured fiber:

Holey Fibers

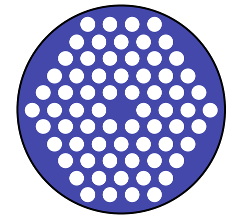


Can have much higher contrast than doped silica…

strong confinement = enhanced nonlinearities, birefringence, …

[J. C. Knight *et al*., *Opt*. *Lett*. **21**, 1547 (1996)]

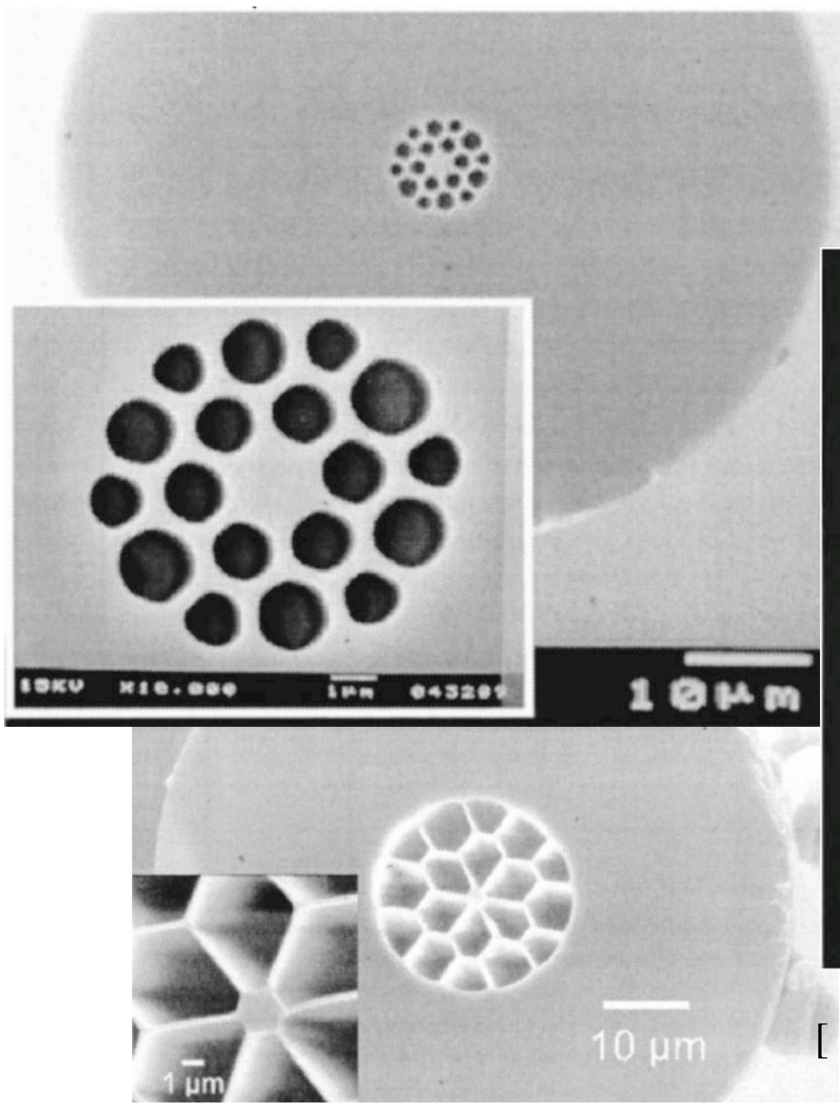
Nonlinear Holey Fibers:

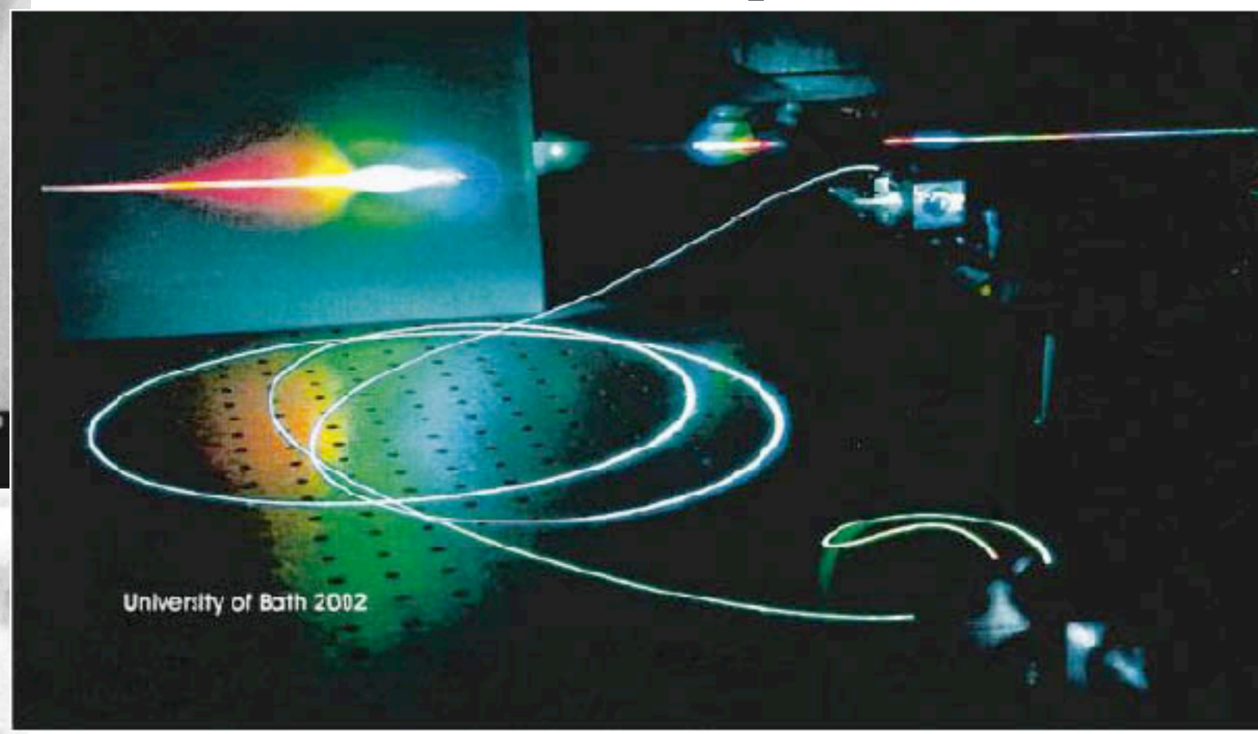


Supercontinuum Generation

(enhanced by strong confinement + unusual dispersion)

e.g. 400–1600nm “white” light:
from 850nm ~200 fs pulses (4 nJ)



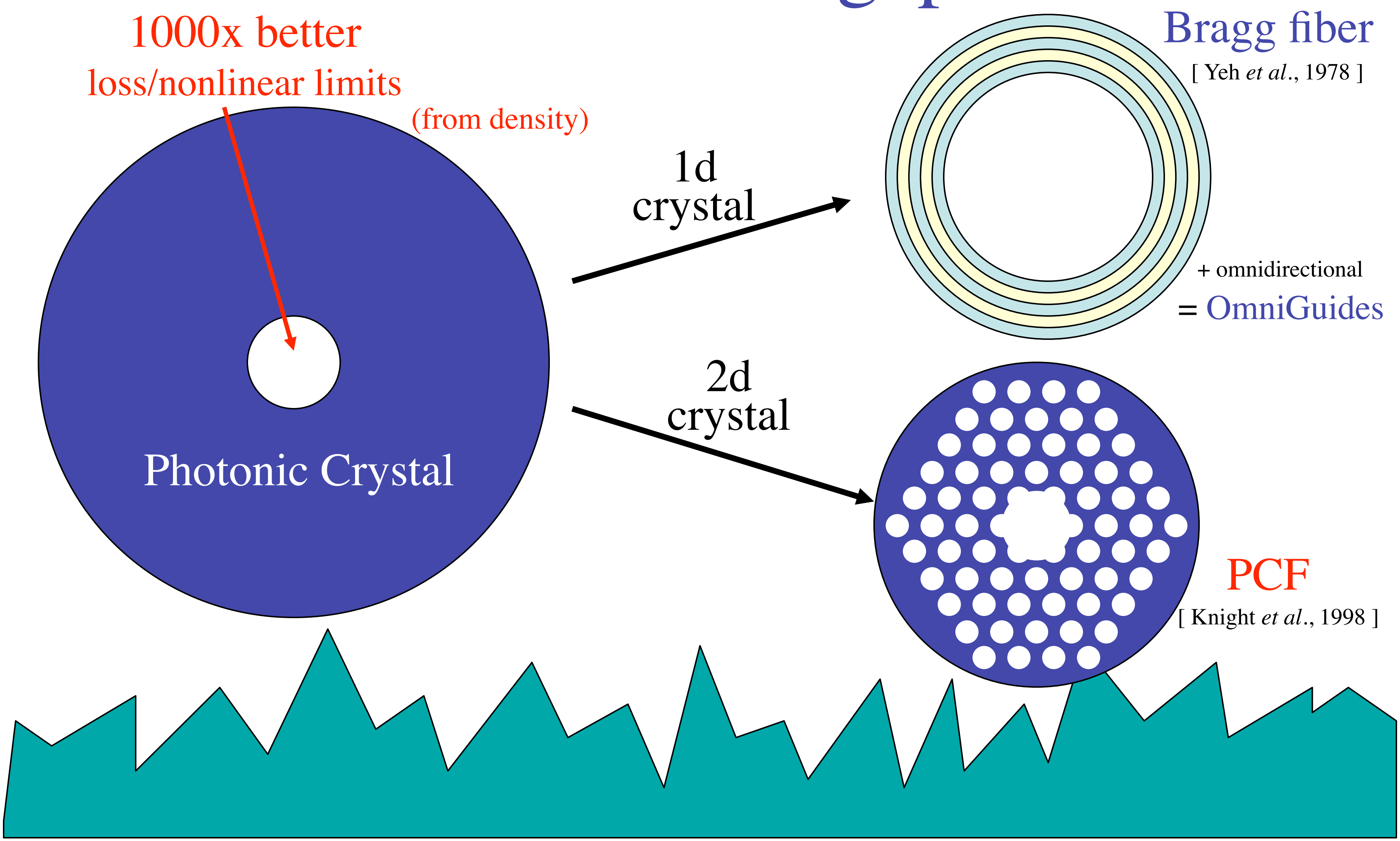


[figs: W. J. Wadsworth *et al.*, *J. Opt. Soc. Am. B* **19**, 2148 (2002)]

[earlier work: J. K. Ranka *et al.*, *Opt. Lett.* **25**, 25 (2000)]

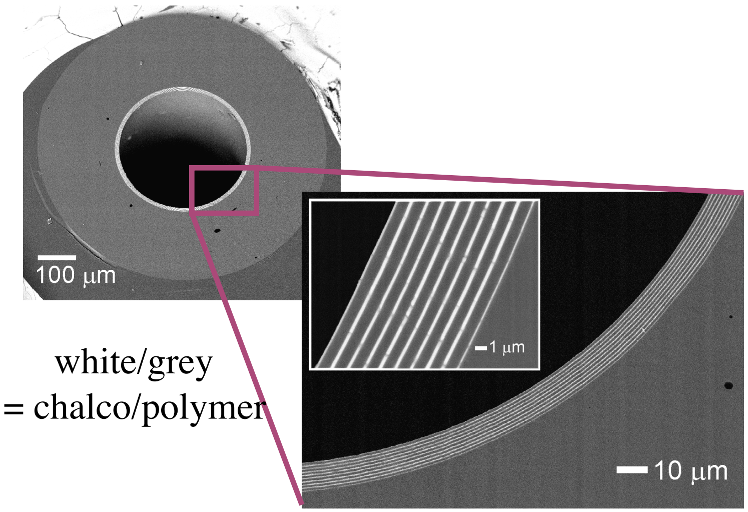
Breaking the Glass Ceiling:

Hollow-core Bandgap Fibers

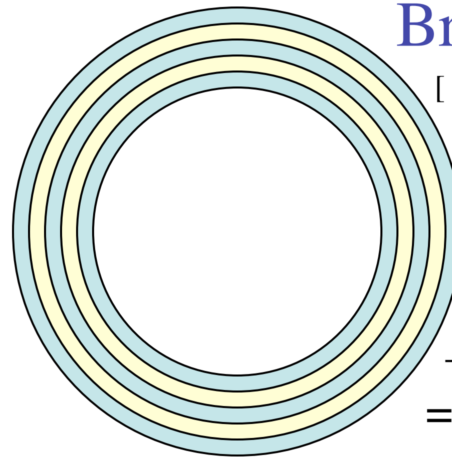


Breaking the Glass Ceiling:
Hollow-core Bandgap Fibers

[figs courtesy
Y. Fink *et al*., MIT]



white/grey
= chalco/polymer

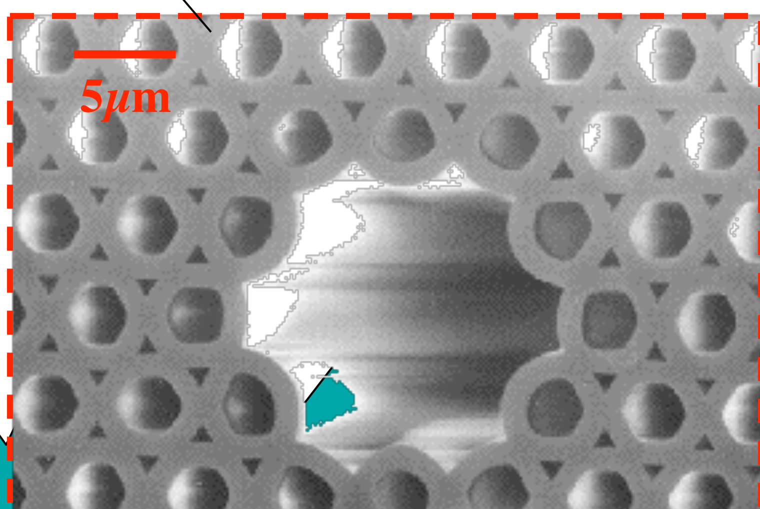


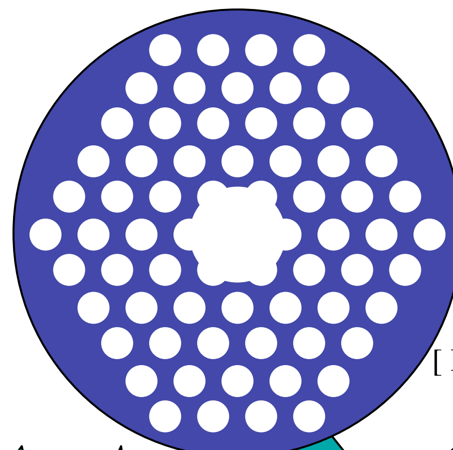
Bragg fiber
[Yeh *et al*., 1978]

+ omnidirectional
= OmniGuide
fibers

silica

[R. F. Cregan
et al.,
Science **285**,
1537 (1999)]

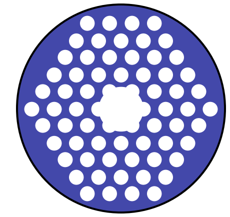


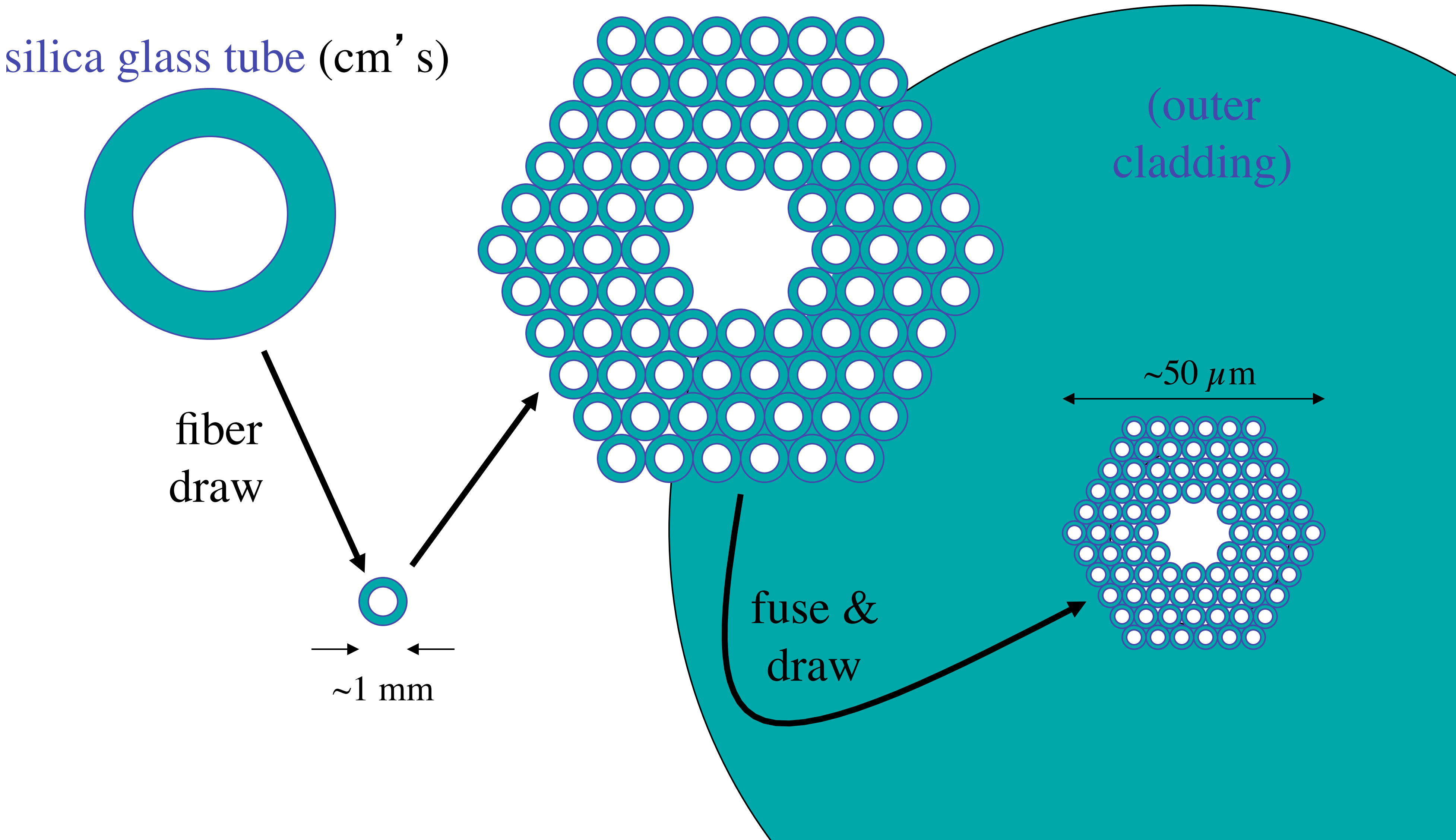


PCF
[Knight *et al*., 1998]

Experimental Air-guiding PCF

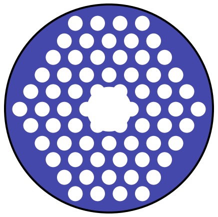
Fabrication (e.g.)

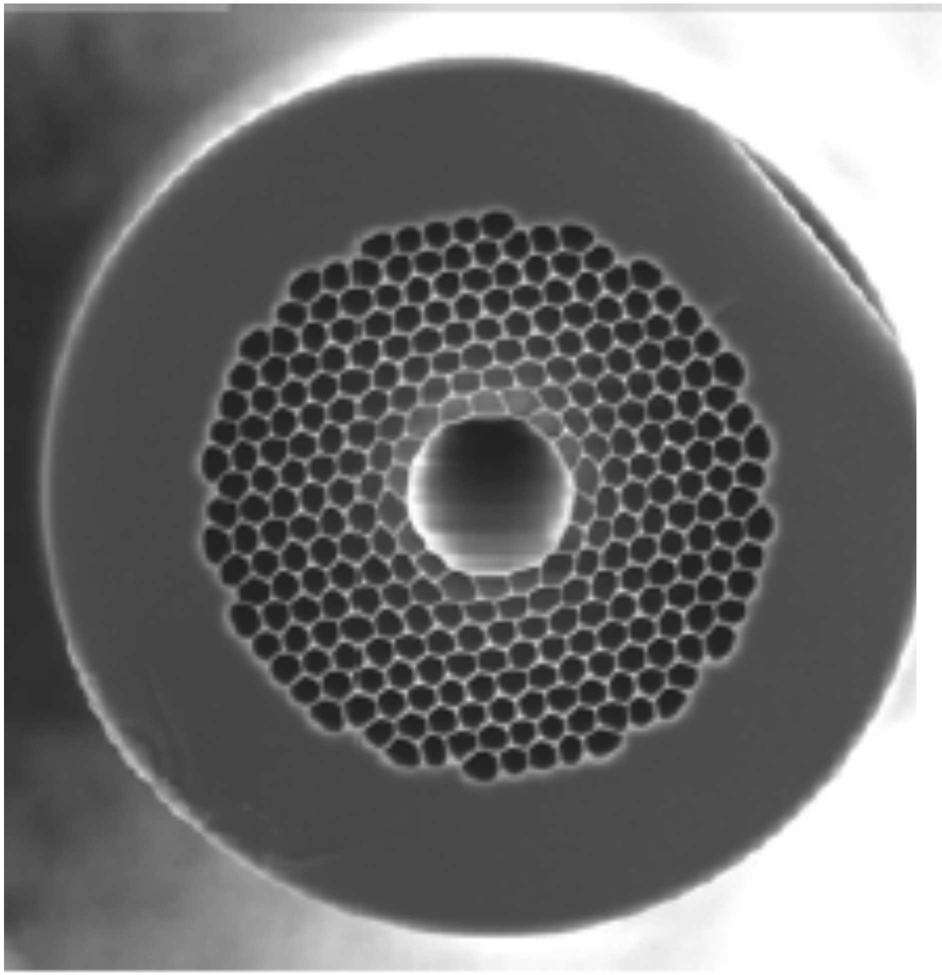




Experimental Air-guiding PCF

[Mangan, *et al*., OFC 2004 PDP24]



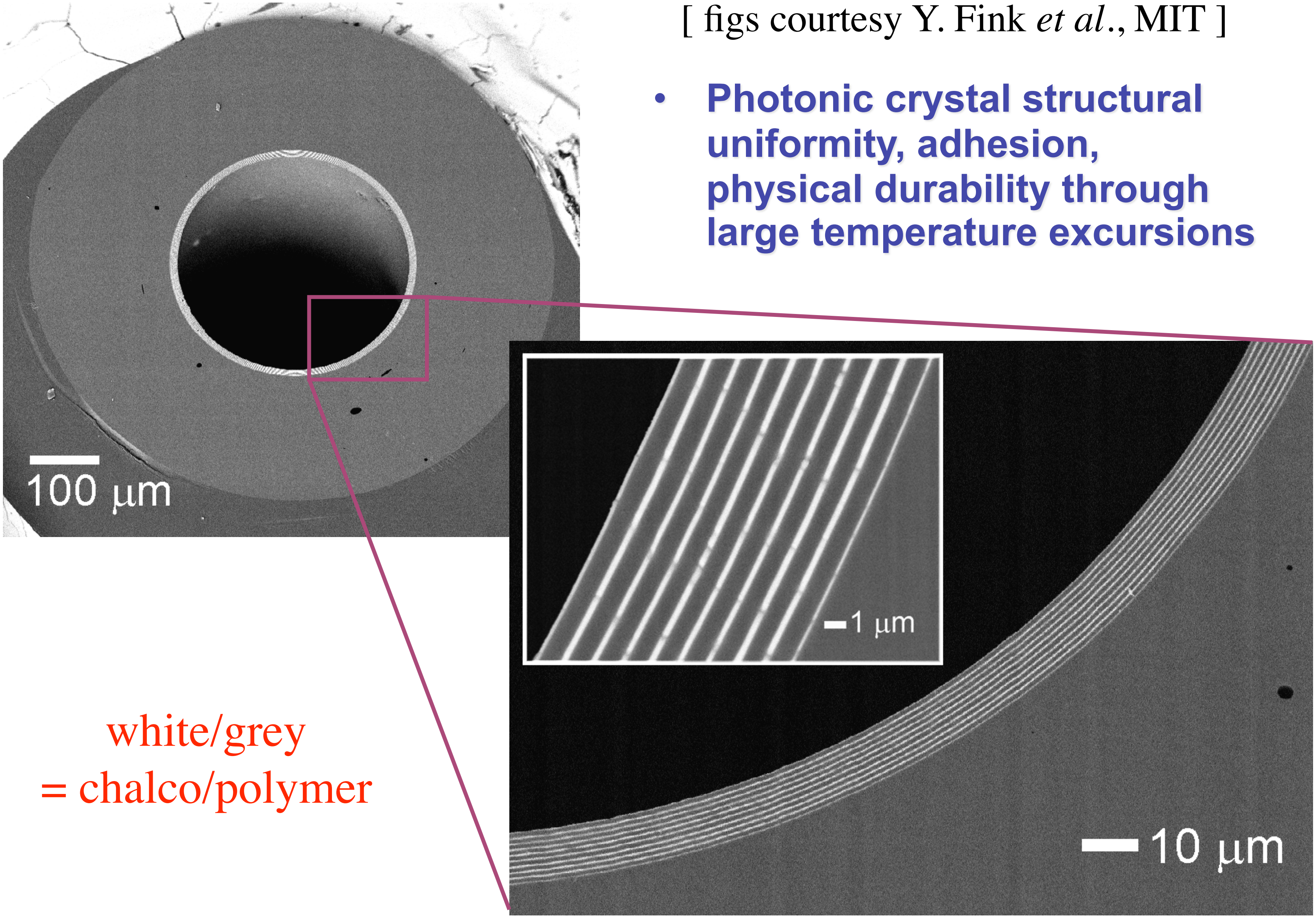


a later/better fabrication

A Cylindrical Bandgap Fiber

[figs courtesy Y. Fink *et al*., MIT]

- **Photonic crystal structural uniformity, adhesion, physical durability through large temperature excursions**



white/grey
= chalco/polymer

High-Power Fibers at New λ's

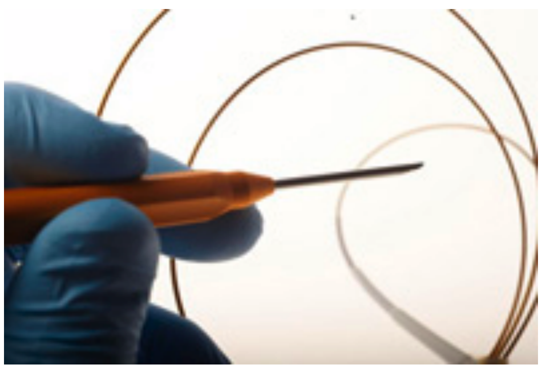
CO_2 laser: λ=10.6μm (no previous dielectric waveguide)

Polymer losses @10.6μm ~ 50,000dB/m…

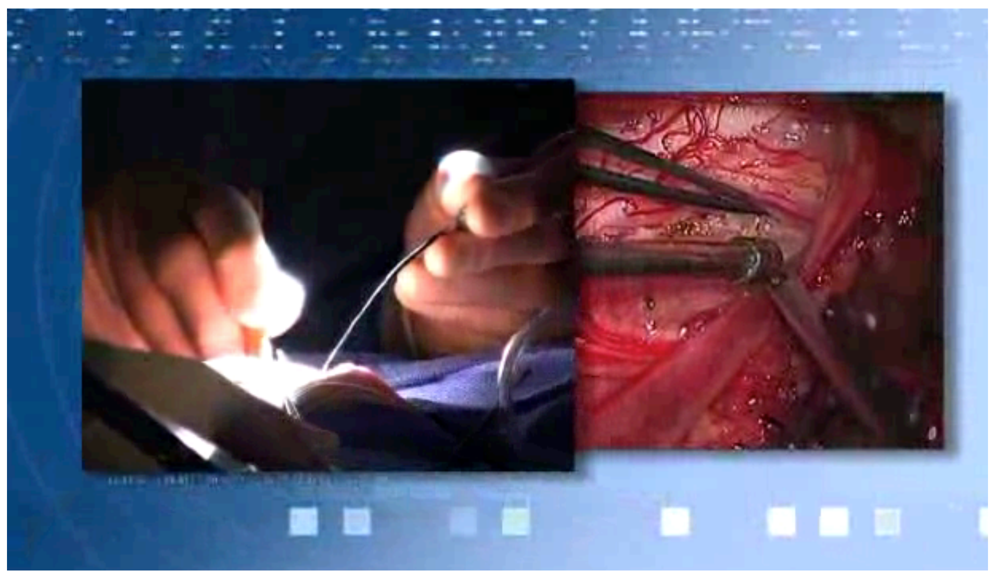
…waveguide losses < 1dB/m

[B. Temelkuran *et al*., *Nature* **420**, 650 (2002)]

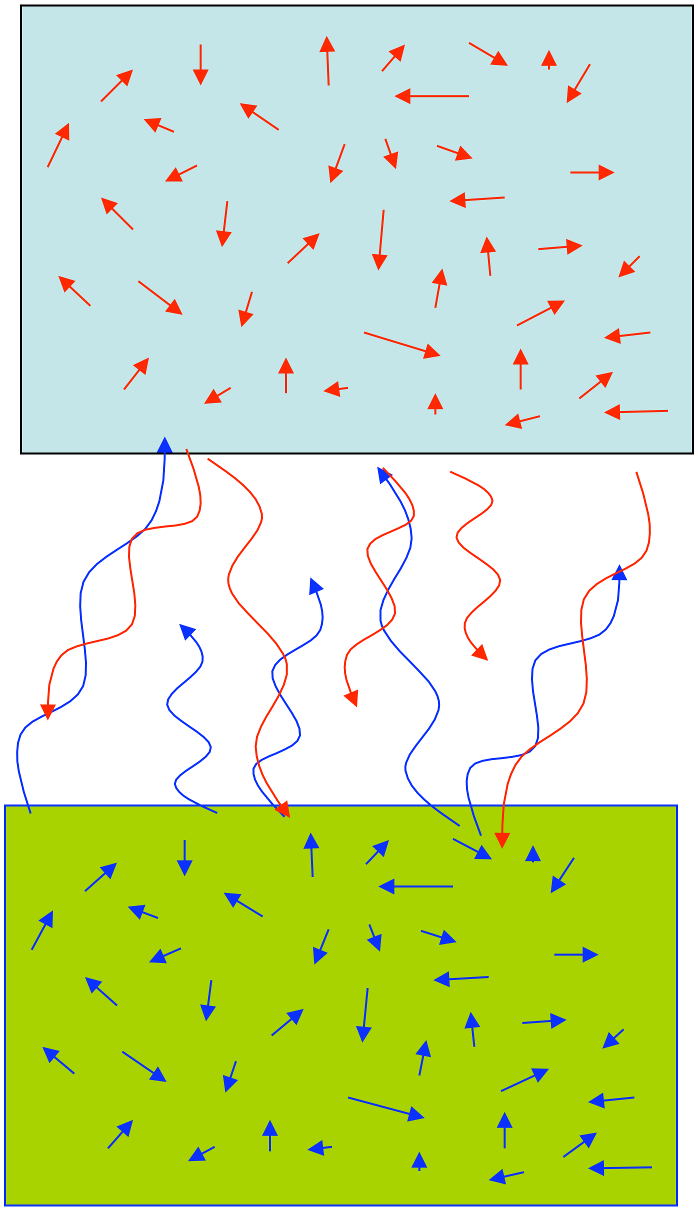
Application: Laser surgery



[www.omni-guide.com]



Electromagnetic Fluctuations



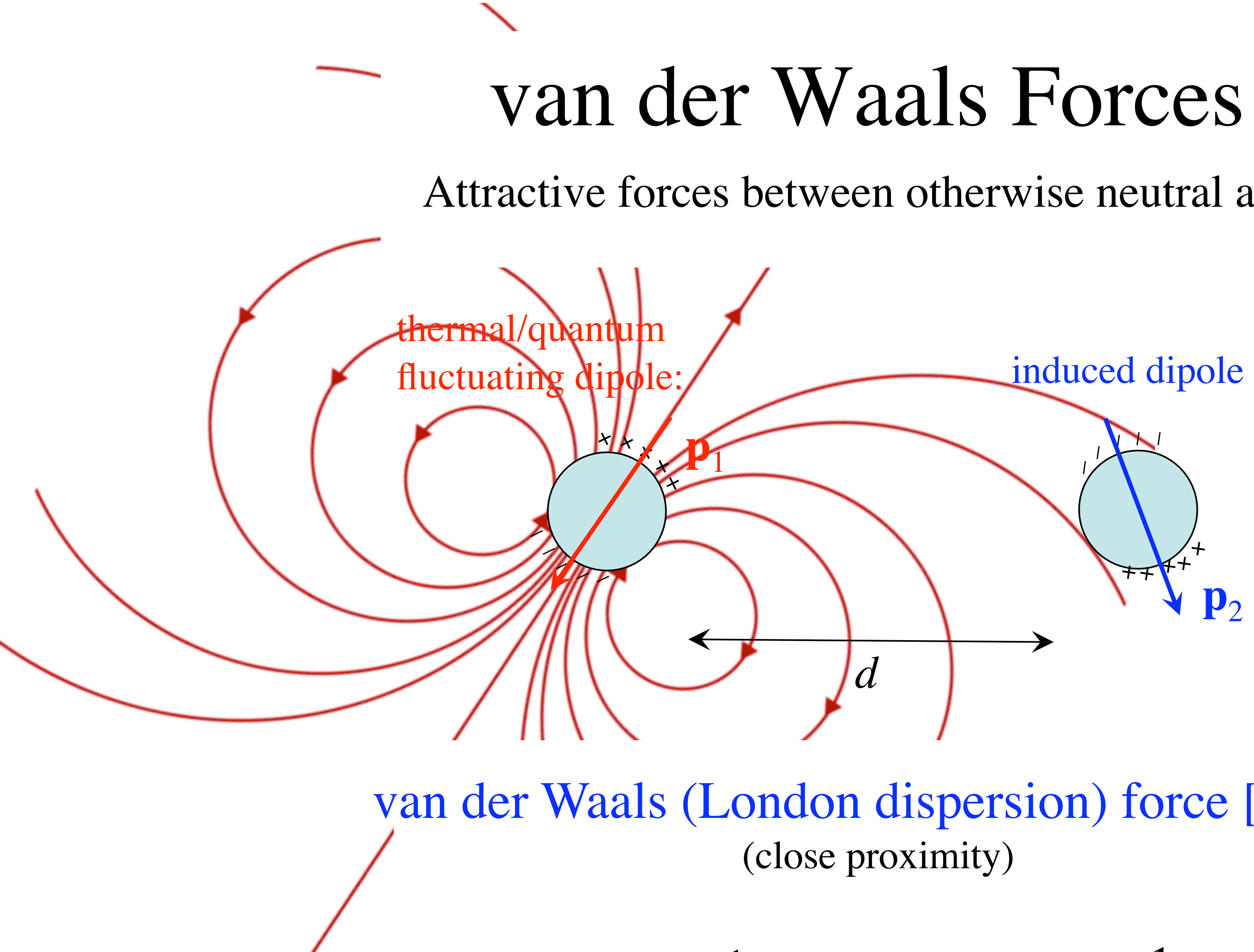
All matter is filled with constantly fluctuating current sources…

… radiating photons
= fluctuating fields everywhere
at all frequencies
[even at zero temperature
due to quantum effects]

causing nearby matter to interact

van der Waals Forces

Attractive forces between otherwise neutral atoms



van der Waals (London dispersion) force [1930]

(close proximity)

$$U \sim -\frac{1}{d^6} \quad \Rightarrow \quad F \sim -\frac{1}{d^7}$$

van der Waals Forces: Approximations

- Small separations only (e.g. < 10s of nm)

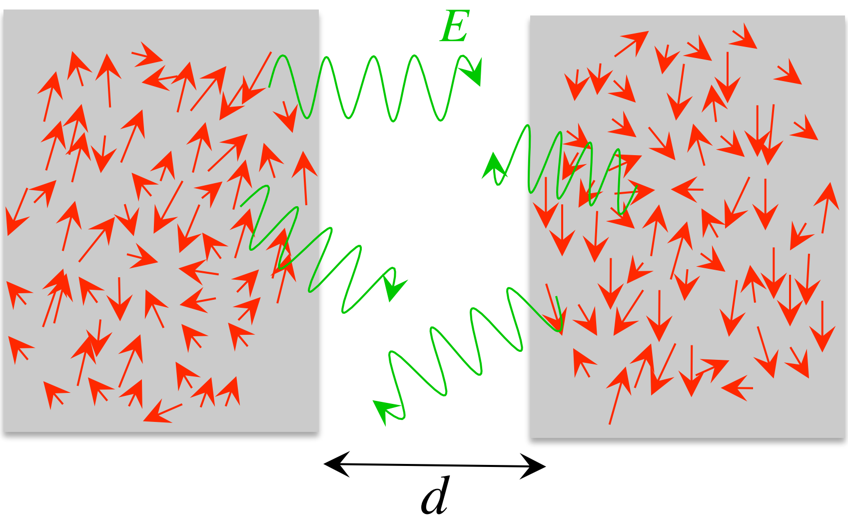
  = neglect wave effects

- Dilute / weakly-polarizable only

  = neglect multiple scattering

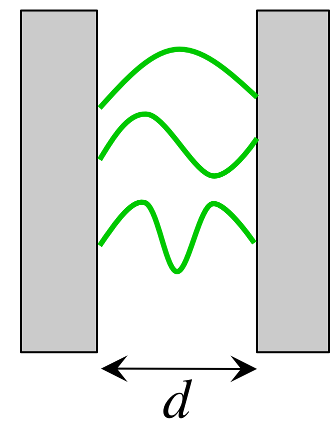
Casimir Effect

macroscopic objects
(many interacting dipoles)



Hendrik Casimir (1948)

perfect metal plates



$$F/A = -\frac{\hbar c \pi^2}{240 d^4}$$

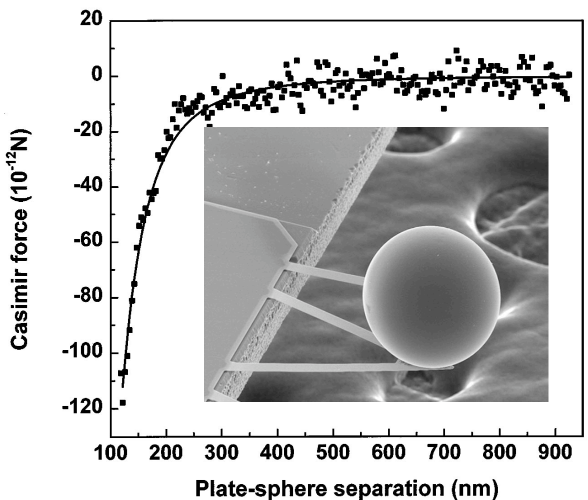
attractive, monotonically decreasing

Electromagnetic field must satisfy boundary conditions at material interfaces ⇒ designable fluctuations

pressure ~ 1 atm at d=50nm

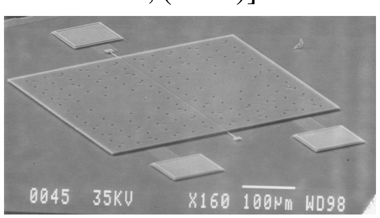
Experiments

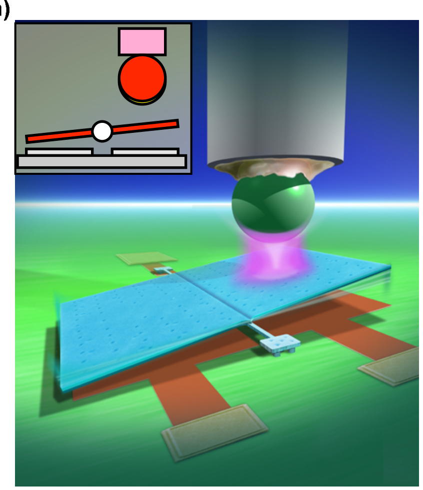
- Van Blockland, Overbeek 1978
  first clear qualitative observation
- Lamoreaux 1997 – first high-precision
- Many, many experiments since …



[U. Mohideen *et. al.* PRL, 81 (1998)]

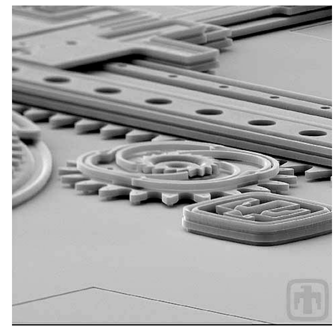
[Chan *et. al.*, *Science* **91**, (2001)]

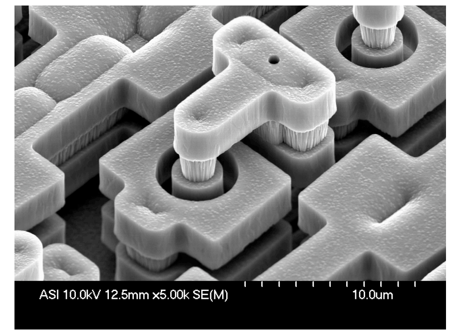


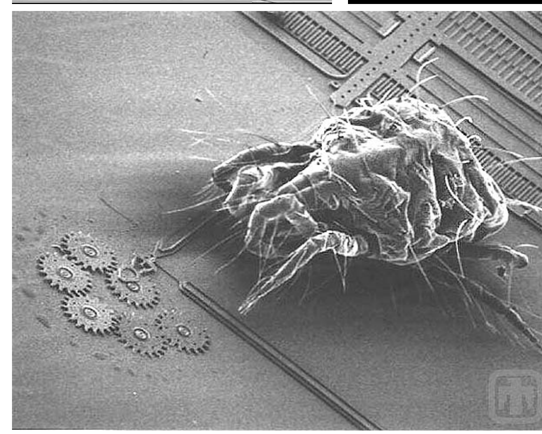


Applications

micromechanical/fluidic devices





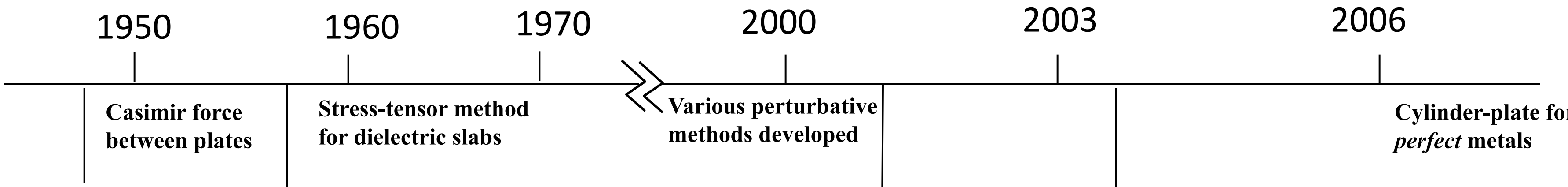


Casimir ⇒
e.g. stiction
problems

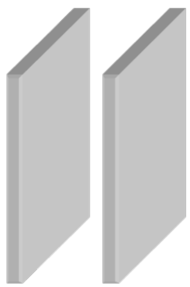
study complicated geometries:
reduce stiction? new effects?

how much can we *design* the force?

Selected pre-2007 Casimir theory

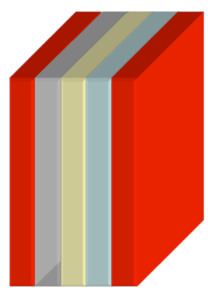


perfect-metal plates

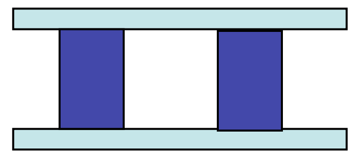


[Casimir, H. B. G, *Acta. Phys.* (1948)]

multi-layer generalizations

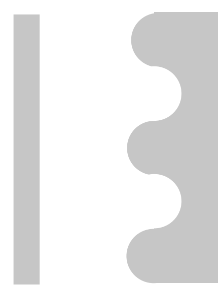


Casimir pistons



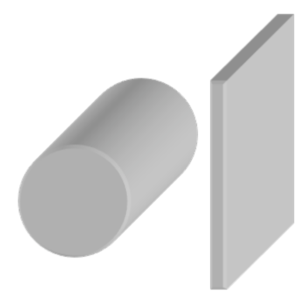
[Hertzberg, Cavalcanti (2004), Kardar, Jaffe, and others]

corrugated plates



[Emig, Kardar. *et. al.*, *PRL* **87,** 260402 (2001)]

cylinder-plate



[Emig, Jaffe, Kardar, Scardicchio *PRL* **96,** 080403 (2006)]

How can this be problem be so hard?

non-interacting bosons — linear Maxwell-like PDEs,
continuum material models
polynomial complexity

- Solution can easily involve solving PDE’s at least 1000’s of times (or much more!)

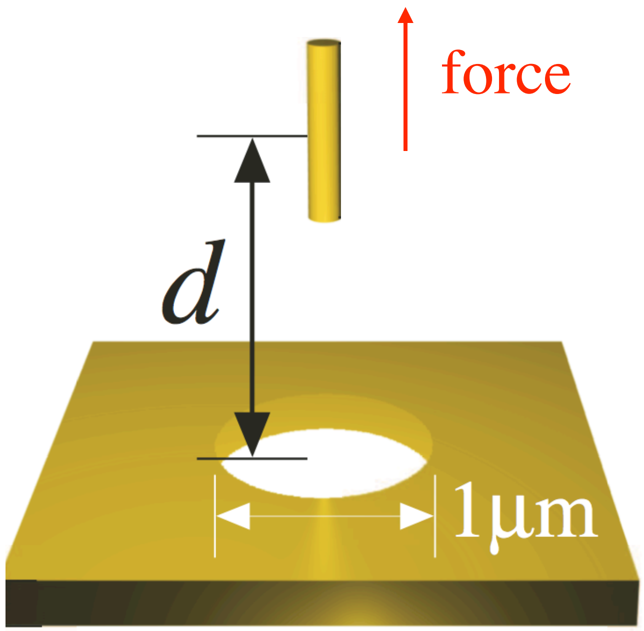
- Which PDE you solve makes a huge difference

— many equivalent formulations
… which are well suited for numerics?

Lots of tricks: good basis choices, connection to Green’s functions, rotation to imaginary frequencies, integral equations, …

1st repulsive geometry in vacuum

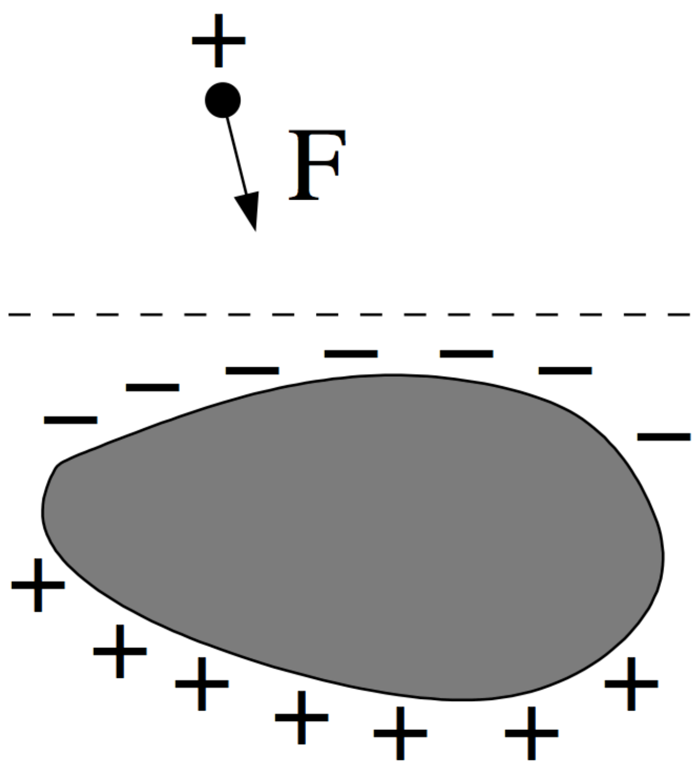
[Levin et al, *PRL* (2010)]



simple: an elongated "needle" above a metal plate with a hole

(related classical phenomena)

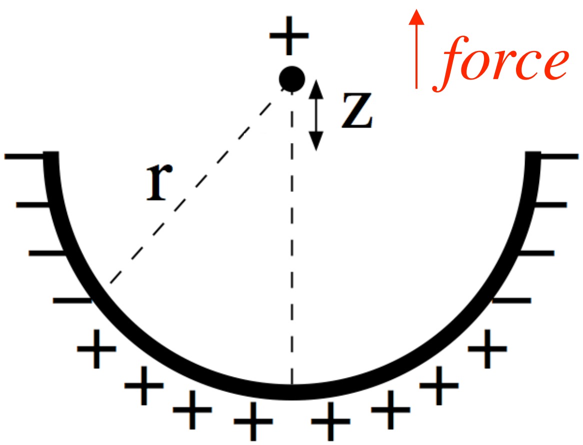
[Levin & Johnson, arXiv:1007.2175 (2010),
Am. J. Phys. **79**, 843 (2011)]

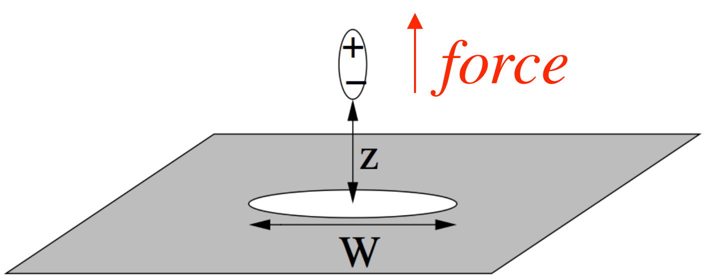


a point charge near a
neutral metallic shape
is normally attracted
…
just by changing the shape,
can we make them repel?

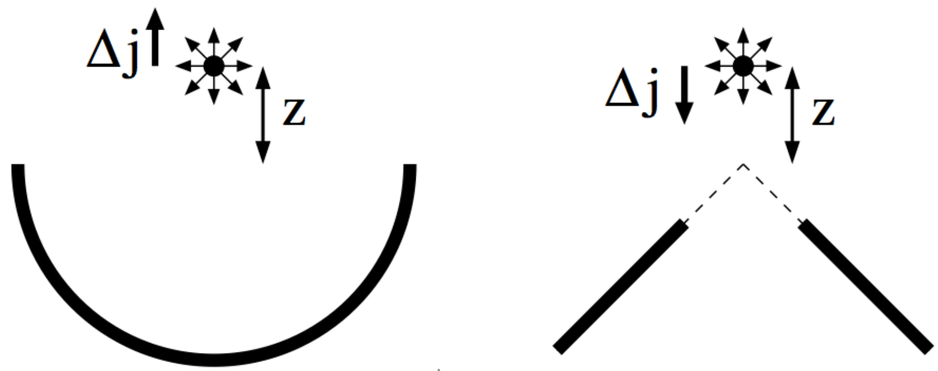
(related classical phenomena)

[Levin & Johnson, arXiv:1007.2175 (2010),
Am. J. Phys. **79**, 843 (2011)]





less obvious analogues…



pushing current away from decreased resistivity

pulling current towards increased resistivity

neutral metallic shapes that repel a point charge or a dipole

(…all just Poisson's equation)

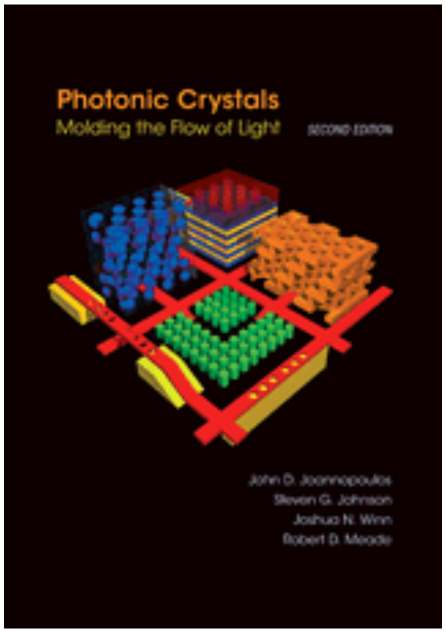
finis

A well-known PDE in complex circumstances
can produce surprising results.

There is more to learn in electromagnetism than
what is covered in 8.02, or was known to Maxwell in 1865!

Happy holidays!

Free Materials Online



Photonic Crystals book: jdj.mit.edu/book

Tutorial slides: jdj.mit.edu/photons/tutorial

Free electromagnetic simulation software
(FDTD, mode solver, etc.)
jdj.mit.edu/wiki

MIT graduate course 18.369 materials
math.mit.edu/~stevenj/18.369